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8th EDITION

OPT. MATHEMATICS SEE MANUAL

For SEE 2076/77

8th EDITION

OPTIONAL MATHEMATICS SEE MANUAL

म्यानुअल

- English र नेपाली दुवै माध्यममा
- सबै SEE/SLC का प्रश्नोत्तरहरु
- अन्य सम्भावित प्रश्नोत्तरहरु

D. R. SIMKHADA

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8th Edition

नेपाल सरकार, शिक्षा, विज्ञान तथा प्रविधि मन्त्रालय, पाठ्यक्रम विकास केन्द्र सानोठिमी, भक्तपुरद्वारा
कक्षा १० का लागि स्वीकृत पाठ्यक्रम एवम् नयाँ विशिष्टीकरण तालिका तथा नमुना प्रश्नअनुसार

अङ्ग्रेजी र नेपाली दुवै माध्यममा

OPTIONAL
MATHEMATICS
FOR
SEE MANUAL
म्यानुअल

(Class X & SEE Exam 2076)

BASED ON MODEL PATTERNS

By
D. R. Simkhada

UNIQUE FEATURES

- Each chapter starts with **FORMULAE & KEY POINTS.**
- The varieties of the problems have been illustrated by **MODEL PATTERNS.**
- **SOLUTIONS** for all the **SEE / SEE** Questions have been presented.
- **MODEL QUESTIONS & SEE QUESTIONS** have been incorporated.
- **SOLUTIONS** for the **Difficult** Questions have been presented.

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शिक्षकहरूलाई कुनै कठिन समस्याहरू भएमा !

■ आफ्नो पूर्ण परिचय र फोन नम्बरसहित सम्पर्क गर्नुहोला । ■

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आठौँ संस्करणको बारेमा

सर्वप्रथम !

प्रस्तुत पुस्तक **OPTIONAL MATHEMATICS SEE Manual** को यस अगाडिका संस्करणलाई आशातीत मन पराइदिनु भई यो संस्करण प्रकाशित गर्नमा उत्साह र उर्जा थपिदिनुहुने विद्यार्थीवर्ग र शिक्षक-शिक्षिकाज्यूहरूप्रति हार्दिक आभार तथा धन्यवाद दिँदै एसईई 2076 /77 को लागि, नेपाल सरकार पाठ्यक्रम विकास केन्द्र, सानोठिमी भक्तपुरद्वारा लागू गरिएको विशिष्टीकरण तालिका तथा नमुना प्रश्नपत्रअनुसार, तयार गरिएको यो परिमार्जित **आठौँ संस्करण** पाठकवर्ग-समक्ष प्रस्तुत गरिएको छ । विगतका वर्षहरूमा जस्तै यस पुस्तकले यहाँहरूको शैक्षिक परिलक्ष्य हासिल गर्न सहयोग पुऱ्यायो भने मेरो यो प्रयास सफल हुनेछ ।

पुस्तकमा !

एसएलसी तहको अर्थपूर्ण सिकाइको निमित्त पाठ्यक्रमद्वारा निर्देशित उपलब्धिहरू हासिल गराउन र परिणाममुखी बनाउन कसरी, कहाँबाट, के-के पढाउने भन्ने मुख्य सन्दर्भ, यो पुस्तकको अघिल्ला संस्करणहरूका प्रयोगकर्ताहरूबाट प्राप्त सुझावहरू, SEE Preparation Examinations र SEE परीक्षाका प्रश्न-संरचनाको प्रकृति तथा पाठ्यक्रम र विशिष्टीकरण तालिकालाई विचार गरी तयार गरिएको यो पुस्तकमा निम्नलिखित विशेषताहरू पाउनुहुनेछ:

☐ प्रत्येक पाठको सुरुमा आवश्यक सूत्रहरू र मुख्य बुँदाहरू समावेश गरिएको

☐ प्रत्येक पाठमा निम्नअनुसारका समाधानहरू समावेश गरिएको

SEE EXERCISES' QUESTIONS ANSWERS

यस अन्तर्गत **OPTIONAL MATHEMATICS FOR SEE COMPETITORS** पुस्तकका **SEE EXERCISE** का सबै प्रश्नहरूका समाधान समावेश गरिएको छ ।

CDC TEXTBOOK'S QUESTIONS ANSWERS

यस अर्न्तगत **CDC TEXTBOOK OF OPTIONAL MATHEMATICS** (सरकारी पुस्तक) का सबै **EXERCISE** का सबै प्रश्नहरूका समाधान समावेश गरिएको छ ।

अन्तमा !

पुस्तक प्रकाशन गरिदिने रिडमोर प्रकाशनलाई साधुवाद, पुस्तक लेखनको यात्रामा हरपल सल्लाह एवम् सुझाउ दिनुहुने शिक्षक-शिक्षिकाज्यूहरूलाई विशेष धन्यवाद ज्ञापन गर्दै पुन: पाठकवर्गबाट रचनात्मक सुझाउको अपेक्षा गर्दछु ।

काठमाडौँ

२०७६ असोज -२४

लेखक

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विषय सूची

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K = Knowledge	U = Understanding	A = Application	HA = Higher Ability	TQ = Total no. of Questions	TM = Total Marks

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एसईई / एसएलसी प्रश्नहरूको सङ्केत (THE NOTATION OF SEE / SLC QUESTIONS)

2075 R = 2075 सालको नियमित (REGULAR) परीक्षामा सोधिएको प्रश्न ।

2075 R_2 = 2075 सालको नियमित (RGULAR) पुन: परीक्षा प्रदेश नं २ मा सोधिएको प्रश्न ।

2075 R' = 2075 सालको संस्कृत मा. वि. तर्फको नियमित (REGULAR) परीक्षामा सोधिएको प्रश्न ।

2075 R'_2 = 2075 सालको संस्कृत मा. वि. तर्फको नियमित (REGULAR) पुन: परीक्षा प्रदेश नं २ मा सोधिएको प्रश्न ।

2074 S = 2074 सालको ग्रेड वृद्धि वा पुरक (SUPPLEMENTARY) परीक्षामा सोधिएको प्रश्न ।

2074 S' = 2074 सालको संस्कृत मा. वि. तर्फको ग्रेड वृद्धि वा पुरक (SUPPLEMENTARY) परीक्षामा सोधिएको प्रश्न ।

.... अन्य सालहरूको पनि सोहीअनुसार सङ्केत गरिएको छ ।

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UNIT I

बीजगणित (Algebra)

1. फलन
Function

KEY POINTS

1. यदि $f(x) = ax + b$ भए $f^{-1}(x) = \frac{x-b}{a}$ हुन्छ । (If $f(x) = ax + b$ then $f^{-1}(x) = \frac{x-b}{a}$.)
2. यदि $f(x) = \frac{ax+b}{cx+d}$ भए $f^{-1}(x) = \frac{b-dx}{cx-a}$ हुन्छ । (If $f(x) = \frac{ax+b}{cx+d}$ then $f^{-1}(x) = \frac{b-dx}{cx-a}$.)
3. $gof(x) = (gof)(x) = g(f(x))$
4. $fog(x) = (fog)(x) = f(g(x))$
5. $f^2(x) = fof = f(f(x))$
6. $f^3(x) = fofof = f(f(f(x)))$
7. $ho(gof) = ho(gof)(x) = (hog)of(x) = h(g(f(x)))$
8. $(f \pm k)(x) = f(x) 9 \pm k$; where k is a scalar.
9. $(k f)(x) = k(f(x))$; where k is a scalar.

QUESTIONS FROM SEE EXERCISES 1.1, 1.2, 1.3, 1.4

A. VERY SHORT QUESTIONS

1. त्रिकोणमितीय फलनको परिभाषा लेख्नुहोस् । (Define trigonometric function.) **[SEE MODEL 2076]**

⇨ Here, a function of an angle expressed as the ratio of two of the sides of a right angled triangle containing that angle, is called trigonometric function. For example: $f(\theta) = \sin\theta$, $g(\theta) = \cos 2\theta$, $h(\alpha) = \tan\alpha + 5$ etc. In other words, a function containing trigonometric ratio as an independent variable is called the trigonometric function.

2. बीजीय फलनको परिभाषा दिनुहोस् । (Define Algebraic Function.)

⇨ Here, the function which describes correspondence between two variables x and y which are obtained by the finite rules is called an algebraic function. For example: $y = x + 2$

3. रेखीय फलनको परिभाषा दिनुहोस् । (Define Linear Function.)

⇨ Here, a function is called linear if it can be defined by an equation of the form $f(x) = mx + c$.

4. अचर फलनको परिभाषा दिनुहोस् । (Define Constant Function.)

⇨ Here, a function of the form $f(x) = mx + c$ where $m = 0$, is called a constant function.

5. एकात्मक फलनको परिभाषा दिनुहोस् । (Define Identity Function.)

⇨ Here, a function $f: A \rightarrow B$ is said to be an identity function if it can be expressed in the form of $f(x) = x$ for all $x \in A$. It is denoted by I.

6. वर्गघातीय फलनको परिभाषा दिनुहोस् । (Define Quadratic Function.)

⇨ Here, a function of the form $f(x) = ax^2 + bx + c$; $a \neq 0$ is called a quadratic function. e.g. $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ etc. are quadratic functions.

7. घनघातीय फलनको परिभाषा दिनुहोस् । (Define Cubic Function.)

⇨ Here, a function of the form $f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$ is called a cubic function. e.g. $y = x^3$, $y = 2x^3 + 5$ etc. are cubic functions.

8. पूर्णकालको परिभाषा दिनुहोस् । (Define Period.)

⇨ Here, in the graph of trigonometric function, the distance between any of the two peaks is known as a period.

9. अबीजीय फलनको परिभाषा दिनुहोस् । (Define Transcendental function.)

⇨ Here, a transcendental function is a function that does not satisfy the properties addition, subtraction, multiplication and division like an algebraic function. For example: $f(x) = x^{\frac{1}{x}}$, $g(x) = \sin x$, $h(x) = x^x$ etc.

10. त्रिकोणमितीय फलनको परिभाषा दिनुहोस् । (Define Trigonometric Function.) ***[SEE MODEL 2076]***

⇨ Here, a function of an angle expressed as the ratio of two of the sides of a right angled triangle containing that angle, is called trigonometric function. For example: $f(\theta) = \sin\theta$, $g(\theta) = \cos 2\theta$, $h(\alpha) = \tan\alpha + 5$ etc. In other words, a function containing trigonometric ratio as an independent variable is called the trigonometric function.

11. $y = f(x) = mx + c$ मा m को मान कति हुँदा यो अचर फलन हुन्छ ?
For what value of m in $y = f(x) = mx + c$ makes it a constant function?

⇨ Here, $m = 0$ makes $y = mx + c$ a constant function.

12. $y = f(x) = mx + c$ मा m र c का मानहरू कति कति हुँदा यो एकात्मक फलन हुन्छ ?
For what values of m and c in $y = f(x) = mx + c$ make it an identity function?

⇨ Here, $m = 1$ and $c = 0$ makes $y = mx + c$ an identity function.

13. $f(x) = ax^2 + bx + c$, $a \neq 0$ लाई लेखाचित्रमा प्रस्तुत गर्दा बन्ने वक्रलाई के भनिन्छ ?
What is the name of curve obtained by sketching the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$?

⇨ Here, the curve obtained by sketching the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$ is parabola.

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14. **$f(x) = a(x-h)^2 + k, a \neq 0$ मा (*h*, *k*) ले के जनाउँदछ ? (What does (*h*, *k*) represent in the equation $f(x) = a(x-h)^2 + k, a \neq 0$?)**

⇨ Here, (*h*, *k*) represents vertex in the equation $f(x) = a(x-h)^2 + k, a \neq 0$.

15. **a को मान कति हुँदा $f(x) = a(x-h)^2 + k, a \neq 0$ ले दिने पाराबोलाको मुख माथितिर हुन्छ ?**
For what value of a, the parabola given by $f(x) = a(x-h)^2 + k, a \neq 0$ has opening upward?

⇨ Here, the positive values of *a* ($a > 0$) makes the parabola opening upward.

16. **a को मान कति हुँदा $f(x) = a(x-h)^2 + k, a \neq 0$ ले दिने पाराबोलाको मुख तलतिर हुन्छ ?**
For what value of a, the parabola given by $f(x) = a(x-h)^2 + k, a \neq 0$ has opening downward?

⇨ Here, the negative values of *a* ($a < 0$) makes the parabola opening downward.

17. **$f(x) = ax^2$ मा शीर्षबिन्दु र सममितीय रेखाको समीकरण लेख्नुहोस् ।**
In $f(x) = ax^2$, write the vertex and the equation of line of symmetry.

⇨ Here, the vertex and the equation of line of symmetry of $f(x) = ax^2$ are (0, 0) and $x = 0$ respectively.

18. **$f(x) = ax^3$ मा शीर्षबिन्दु लेख्नुहोस् । (Write the vertex of $f(x) = ax^3$.)**

⇨ Here, the vertex of $f(x) = ax^3$ is (0, 0).

19. **$y = \sin x$ को *y*-खण्ड कति हुन्छ ? (What is the *y*-intercept of $y = \sin x$?)**

⇨ Here, y-intercept of $y = \sin x$ is 0.

20. **$y = \cos x$ को *y*-खण्ड कति हुन्छ ? (What is the *y*-intercept of $y = \cos x$?)**

⇨ Here, y-intercept of $y = \cos x$ is 1.

21. **$y = \tan x$ को *y*-खण्ड कति हुन्छ ? (What is the *y*-intercept of $y = \tan x$?)**

⇨ Here, y-intercept of $y = \tan x$ is 0.

22. **$y = \sin x$ को अधिकतम मान कति हुन्छ ? (What is the maximum value of $y = \sin x$?)**

⇨ Here, the maximum value of $y = \sin x$ is 1.

23. **$y = \cos x$ को अधिकतम मान कति हुन्छ ? (What is the maximum value of $y = \cos x$?)**

⇨ Here, the maximum value of $y = \cos x$ is 1.

24. **$y = \sin x$ को न्यूनतम मान कति हुन्छ ? (What is the minimum value of $y = \sin x$?)**

⇨ Here, the minimum value of $y = \sin x$ is -1.

25. **$y = \cos x$ को न्यूनतम मान कति हुन्छ ? (What is the minimum value of $y = \cos x$?)**

⇨ Here, the minimum value of $y = \cos x$ is -1.

26. **फलन $y = \sin x$ को क्षेत्र लेख्नुहोस् । (Write the domain of function $y = \sin x$.)**

⇨ Here, the domain of $y = \sin x$ is all real numbers.

27. **फलन $y = \cos x$ को क्षेत्र लेख्नुहोस् । (Write the domain of function $y = \cos x$.)**

⇨ Here, the domain of $y = \cos x$ is all real numbers.

28. **फलन $y = \tan x$ को क्षेत्र लेख्नुहोस् । (Write the domain of function $y = \tan x$.)**

⇨ Here, the domain of $y = \tan x$ is all real numbers excluding odd multiples of $\frac{\pi^c}{2}$.

29. **फलन $y = \sin x$ को विस्तार लेख्नुहोस् । (Write the range of function $y = \sin x$.)**

⇨ Here, the range of $y = \sin x$ is all real numbers from -1 to 1.

30. **फलन $y = \cos x$ को विस्तार लेख्नुहोस् । (Write the range of function $y = \cos x$.)**

⇨ Here, the range of $y = \cos x$ is all real numbers from -1 to 1.

31. **फलन $y = \tan x$ को विस्तार लेख्नुहोस् । (Write the range of function $y = \tan x$.)**

⇨ Here, the range of $y = \tan x$ is all real numbers.

32. **संयुक्त फलनको परिभाषा दिनुहोस । (Define composite function.)**

⇨ Here, if $f: A \to B$ and $g: B \to C$ are two functions then the new function defined from A to C is called the composite function of *f* and *g*. It is denoted by $g \circ f$.

33. **संयोजन फलनको परिभाषा दिनुहोस् । (Define combination of functions)**

⇨ Here, the sum, difference, product or quotient of the functions is known as combination of the functions.

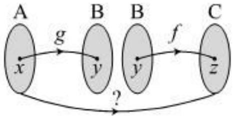
34. **यदि $f: A \to B$ र $g: B \to C$ दुई फलनहरू हुन् भने संयुक्त फलन $A \to C$ लाई के ले जनाउँदछ ?**
If $f: A \to B$ and $g: B \to C$ be the two functions then what will denote the composite function from $A \to C$?

⇨ Here, the composite function from A to C is '$g \circ f$'.

35. **यदि $h: R \to S$ र $g: Q \to R$ दुई फलनहरू हुन् भने संयुक्त फलन $Q \to S$ लाई के ले जनाउँदछ ?**
If $h: R \to S$ and $g: Q \to R$ be the two functions then what will denote the composite function from $Q \to S$?

⇨ Here, the composite function from Q to S is '$h \circ g$'.

36. **चित्रमा $g: A \to B$ र $f: B \to C$ छ । A बाट C सम्म परिभाषित फलनको नाम के हो ?**
In figure, $g: A \to B$ and $f: B \to C$. What is the name of function defined from A to C?



⇨ Here, the composite function from A to C is '$f \circ g$'.

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37. $f \circ g(x)$ को विस्तार क्षेत्र f र g मध्ये कुन फलनको विस्तार क्षेत्रसँग बराबर हुन्छ ?
Which range of the functions between f and g is equal to the range of the function $f \circ g(x)$?

⇨ Here, the range of $f \circ g(x)$ is equal to the range of f.

38. $g \circ f(x)$ को विस्तार क्षेत्र f र g मध्ये कुन फलनको विस्तार क्षेत्रसँग बराबर हुन्छ ?
Which range of the functions between f and g is equal to the range of the function $g \circ f(x)$?

⇨ Here, the range of $g \circ f(x)$ is equal to the range of g.

39. $(f \circ g)\ (x)$ फलनको क्षेत्र f र g मध्ये कुन फलनको क्षेत्रसँग बराबर हुन्छ ?
Which domain of the functions between f and g is equal to the domain of the function $f \circ g(x)$?

⇨ Here, the domain of $f \circ g(x)$ is same as domain of $g(x)$.

40. यदि $f(x) = m$ र $g(x) = n$ भए $f \circ g(x)$ को मान कति होला ? (If $f(x) = m$ and $g(x) = n$ then what is the value of $f \circ g(x)$?)

⇨ Here, $f \circ g(x) = f[g(x)] = f(n) = m$.

41. यदि $f(x) = a$ र $g(x) = b$ भए $g \circ f(x)$ को मान कति होला ? (If $f(x) = a$ and $g(x) = b$ then what is the value of $g \circ f(x)$?)

⇨ Here, $g \circ f(x) = g[f(x)] = g(a) = b$.

42. कुन अवस्थामा '$f \circ g$' र '$g \circ f$' बराबर हुन्छन् ? (In what condition, '$f \circ g$' and '$g \circ f$' are equal?)

⇨ Here, '$f \circ g$' and '$g \circ f$' are equal in the following conditions;

(i) If $f(x)$ and $g(x)$ are identity functions.

(ii) If $f(x)$ and $g(x)$ are inverse to each other.

43. दुईओटा फलनको संयुक्त फलन निकाल्नको लागि आवश्यक शर्त के हो ?
What is the necessary condition to find the composite function of two functions?

⇨ Here, the necessary condition is range of first function and the domain of second function must be equal.

44. यदि फलन $f = \{(1, 2), (2, 3), (3, 4)\}$ र $g = \{(2, a), (4, c), (3, b)\}$ भए $g \circ f(1)$ पत्ता लगाउनुहोस् ।
If the function $f = \{(1, 2), (2, 3), (3, 4)\}$ and $g = \{(2, a), (4, c), (3, b)\}$ then find function $g \circ f(1)$.

⇨ Here, $g \circ f(1) = g(f(1) = g(2)) = a$

45. यदि फलन $h = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$, भए $g \circ h(3)$ पत्ता लगाउनुहोस् ।
If $h = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find $g \circ h(3)$.

⇨ Here, $g \circ h(3) = g(h(3))= g(5) = 1$

46. यदि फलन $f = \{(1, 3), (0, 0), (-1, -3)\}$ र $g = \{(0, 2) ,(-3, -1), (3, 5)\}$ भए $g \circ f(1)$ पत्ता लगाउनुहोस् ।
If $f = \{(1, 3), (0, 0) ,(-1, -3)\}$ and $g = \{(0, 2), (-3, -1), (3, 5)\}$ then find $g \circ f(1)$.

⇨ Here, $g \circ f(1) = g(f(1)) = g(3) = 5$

47. यदि $f(x) = x + a$ र $g(x) = x$ भए $f \circ g(x)$ को मान कति हुन्छ ? (If $f(x) = x + a$ and $g(x) = x$ then what is the value of $f \circ g(x)$?)

⇨ Here, $f \circ g(x) = f(g(x)) = f(x) = x + a$.

48. यदि $f(x) = x$ र $g(x) = x + b$ भए $g\ (f(x))$ को मान कति हुन्छ ? (If $f(x) = x$ and $g(x) = x + b$ then what is the value of $g\ (f(x))$?)

⇨ Here, $g\ (f(x)) = g(f(x)) = g(x) = x + b$.

49. यदि $f = \{(2, 5)\}$ र $g = \{(5, 7)\}$ भए $g \circ f$ को क्रमजोडा के हुन्छ ? (If $f = \{(2, 5)\}$ and $g = \{(5, 7)\}$, what is the order pair of $g \circ f$?)

⇨ Here, ordered pair of $g \circ f$ is $\{2, 7\}$.

50. यदि $g = \{(5, -7)\}$ र $f = \{(-7, 2)\}$ भए $f \circ g(5)$ को मान कति हुन्छ ? (If $g = \{(5, -7)\}$ and $f = \{(-7, 2)\}$, what is the value of $f \circ g(5)$?)

⇨ Here, $f \circ g(5) = f(g(5)) = f(-7) = 2$

51. यदि $f = \{(1, 3), (2, 1)\}$ र $g = \{(1, 2), (3, 2)\}$ भए फलन $g \circ f$ को क्षेत्र कति हुन्छ ?
If $f = \{(1, 3), (2, 1)\}$ and $g = \{(1, 2), (3, 2)\}$ then what is the domain of $g \circ f$?

⇨ Here, the domain of $g \circ f$ = domain of $f = \{1, 2\}$

52. यदि $f = \{(m, n), (a, b)\}$ र $g = \{(n, a), (b, c)\}$ भए फलन $g \circ f$ को विस्तार कति हुन्छ ?
If $f = \{(m, n), (a, b)\}$ and $g = \{(n, a), (b, c)\}$ then what is the range of $g \circ f$?

⇨ Here, the range of $g \circ f$ = range of $g = \{a, c\}$

53. एक एक सम्पूर्ण फलनको परिभाषा दिनुहोस् । (Define one to one onto function.)

⇨ Here, in a function, if the codomain is equal to range and each element of range has only one pre-image in domain then the function is known as one to one onto function.

54. विपरीत फलनको परिभाषा दिनुहोस् । (Define the inverse Function.)

⇨ Here, if f is a one to one onto function from A to B then the inverse function f^{-1} is defined as a function from B to A. For example: If $f = \{(1, 3), (2, 4), (3, 5)\}$, $f^{-1} = \{(3, 1), (4, 2), (5, 3)\}$

55. कस्तो अवस्थामा कुनै फलनको विपरीत फलन सम्भव हुन्छ ?
Under what condition, the inverse function of a function is possible?

⇨ Here, if the function is one to one onto function then it's inverse is possible.

56. एकात्मक फलन र यसको विपरीतको सम्बन्ध के हुन्छ ? (What is the relation between an identity function and its inverse ?)

⇨ Here, the inverse function of an identity function is the function itself.

57. यदि $f = \{(a, b), (x, y)\}$ भए f को विपरीत फलन पत्ता लगाउनुहोस् । (If $f = \{(a, b), (x, y)\}$ then find the inverse function of f.)

⇨ Here, the inverse function of f is $f^{-1} = \{(b, a), (y, x)\}$.

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58. यदि $f^{-1} = \{(x, y), (c, d)\}$ भए फलन f पत्ता लगाउनुहोस् । (If $f^{-1} = \{(x, y), (c, d)\}$ then find the function f.)

⇨ Here, $f = \{(y, x)\ (d, c)\}$.

59. यदि $f: R \to R$ र $g: R \to R$ एक अर्काका विपरीत फलनहरू भए, प्रत्येक $x \in R$ को लागि $f \circ g(x)$ कति हुन्छ ?
If $f: R \to R$ and $g: R \to R$ are inverse to each other then for each $x \in R$, what is the value of $f \circ g(x)$?

⇨ Here, $f \circ g(x) = x$.

60. यदि $f: R \to R$ र $g: R \to R$ एक अर्काका विपरीत फलनहरू भए, प्रत्येक $a \in R$ को लागि $g \circ f(a)$ कति हुन्छ ?
If $f: R \to R$ and $g: R \to R$ are inverse to each other then for each $a \in R$, what is the value of $g \circ f(a)$?

⇨ Here, $g \circ f(a) = a$.

61. यदि $f = \{(2, -4)\}$, भए f^{-1} कति हुन्छ ? (If $f = \{(2, -4)\}$, what is f^{-1}?)

⇨ Here, $f^{-1} = \{(-4, 2)\}$.

62. यदि $g = \{(a, b)\}$, भए g^{-1} कति हुन्छ ? (If $g = \{(a, b)\}$, what is g^{-1}?)

⇨ Here, $g^{-1} = \{(b, a)\}$.

63. यदि $f(x) = x + a$, भए $f^{-1}(x)$ कति हुन्छ ? (If $f(x) = x + a$, what is $f^{-1}(x)$?)

⇨ Here, $y = x + a$ or, $x = y + a$ or, $y = x - a$ ∴ $f^{-1}(x) = x - a$.

64. यदि $p(x) = 2x$ भए $p^{-1}(x)$ कति हुन्छ ? (If $p(x) = 2x$, what is $p^{-1}(x)$?)

⇨ Here, $p(x) = 2x$ or, $y = 2x$ or, $x = 2y$ or, $y = \frac{x}{2}$ ∴ $p^{-1}(x) = \frac{x}{2}$

65. फलन $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$ को विपरीत फलन पत्ता लगाउनुहोस् ।
Find the inverse function of $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$.

⇨ Here, $f^{-1} = \{(5, 1), (6, 2), (7, 3), (8, 4)\}$.

66. फलन $g = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$ को विपरीत फलन पत्ता लगाउनुहोस् ।
Find the inverse function of $g = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$.

⇨ Here, $g^{-1} = \{(1, a), (2, b), (3, c), (4, d)\}$.

B. SHORT QUESTIONS

MODEL 1

1. यदि फलन $f(x) = 4x - 5$ को विस्तार क्षेत्र $\{-1, 7\}$ भए, त्यसको क्षेत्र निकाल्नुहोस् ।
The range of the function $f(x) = 4x - 5$ is $\{-1, 7\}$, find its domain. **[2066 S]**

⇨ Here, $f(x) = 4x - 5$ & Range $= \{-1, 7\}$
Let, $y = 4x - 5$
When $y = -1$ then, $-1 = 4x - 5$ or, $4 = 4x$ ∴ $x = 1$
When $y = 7$ then, $7 = 4x - 5$ or, $12 = 4x$ ∴ $x = 3$
Thus, the domain is $\{1, 3\}$.

2. यदि एउटा फलन $f: x \to 3x + 5$ को विस्तार क्षेत्र $\{2, 8\}$ भए त्यसको क्षेत्र निकाल्नुहोस् ।
If the range of a function $f: x \to 3x + 5$ is $\{2, 8\}$, find its domain. **[2059 R]**

⇨ Here, given function $f: x \to 3x + 5$ or, $f(x) = 3x + 5$ and Range $= \{2, 8\}$
Now, when $f(x) = 2$ then, $2 = 3x + 5$ or, $3x = 2 - 5$ ∴ $x = -1$
when $f(x) = 8$ then, $8 = 3x + 5$ or, $3x = 3$ ∴ $x = 1$
Thus, the domain is $\{-1, 1\}$.

3. फलन $f(x) = \frac{2x - 3}{5}$ को फलन क्षेत्रमा हुने कुन सदस्यको प्रतिबिम्ब 7 हुन्छ ?

What element in the domain has image 7 under the function $f(x) = \frac{2x - 3}{5}$? **[2057 R]**

⇨ Here, $f(x) = \frac{2x - 3}{5}$ and image $(y) = f(x) = 7$

i.e. $7 = \frac{2x - 3}{5}$ or, $35 = 2x - 3$ or, $2x = 38$ ∴ $x = 19$

Thus, the element 19 in the domain has the image 7.

4. फलन $f(x) = 3x + 5$ को फलन क्षेत्रमा हुने कुन सदस्यको प्रतिबिम्ब 8 हुन्छ ?
What element in the domain has the image 8 under the function $f(x) = 3x + 5$? **[2060 R]**

⇨ Here, $f(x) = 3x + 5$ and image $= 8$ or $f(x) = 8$
So, $8 = 3x + 5$ or, $3x = 8 - 5$ or, $3x = 3$ ∴ $x = 1$
Thus, the element 1 in the domain has the image 8.

5. फलन $f(x) = \sqrt{x}$ मा 4 को पूर्व प्रतिबिम्ब कति हुन्छ ? (What will be the pre-image of 4 in the function $f(x) = \sqrt{x}$?) **[2058 S, 2060S]**

⇨ Here, given function $f(x) = \sqrt{x}$ and range of function $= 4$. So, $4 = \sqrt{x}$
Squaring on both sides, we get, $16 = x$ ∴ $x = 16$
Thus, pre-image of 4 is 16.

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6. **$f(x) = \sqrt[3]{x}$ मा 8 को पूर्व प्रतिबिम्ब कति होला ? (What will be the pre-image of 8 in $f(x) = \sqrt[3]{x}$?)** **[2062 R]**

⇨ Here, given function $f(x) = \sqrt[3]{x}$ and the pre-image of 8 = ?

Since $f(x) = 8$

or, $\sqrt[3]{x} = 8$

$\therefore\ x^{\frac{1}{3}} = 8$

Cubing on both sides, we get,

$$\left(x^{\frac{1}{3}}\right)^3 = 8^3$$

$\therefore\ x = 512$

Thus, pre-image of 8 is 512.

MODEL 2

7. **संयुक्त फलनको परिभाषा दिनुहोस् । यदि f = {(3, 4), (4, 5), (5, 6)} र g = {(2, 3), (3, 4), (4, 5)} भए $f \circ g$ लाई क्रमजोडाको रुपमा व्यक्त गर्नुहोस् ।**

Define composite function. If f = {(3, 4), (4, 5), (5, 6)} and g = {(2, 3), (3, 4), (4, 5)}, express $f \circ g$ in an ordered pair form. **[2074 R]**

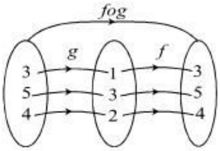
⇨ Here, if $f : A \to B$ and $g : B \to C$ are two functions then the new function defined from A to C is called the composite function of f and g. It is denoted by gof.

Here, f = {(3, 4), (4, 5), (5, 6)} and g = {(2, 3), (3, 4), (4, 5)}

So, $f \circ g$ = {(2, 4), (3, 5), (4, 6)}

8. **यदि f = {(1, 3), (2, 4), (3, 5)} र g = {(3, 1), (4, 2), (5, 3)} भए ($f \circ$g) लाई मिलान चित्रमा देखाउनुहोस् । ($f \circ$g) का क्रमजोडाहरू लेख्नुहोस् ।**

If f = {(1, 3), (2, 4), (3, 5)} and g = {(3, 1), (4, 2), (5, 3)}, then show ($f \circ$g) in arrow-diagram. Write the ordered pairs of ($f \circ$g). **[2074 S]**

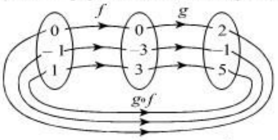


⇨ Here, f = {(1, 3), (2, 4), (3, 5)} and g = {(3, 1), (4, 2), (5, 3)}

$\therefore\ f \circ g$ = {(3, 3), (4, 4), (5, 5)}

9. **यदि f = {(1, 3), (0, 0), (– 1, – 3)} र g = {(0, 2), (– 3, – 1), (3, 5)} भए $g \circ f$ लाई मिलान चित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।**

If f = {(1, 3), (0, 0), (– 1, – 3)} and g = {(0, 2), (– 3, – 1), (3, 5)} then show the function $g \circ f$ in the arrow diagram and find it in ordered pair form. **[2065 M]**



⇨ Here, f = {(1, 3), (0, 0), (– 1, – 3) and g = {(0, 2), (– 3, – 1), (3, 5)}

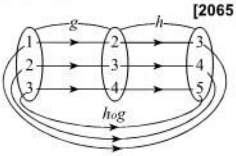
Representing the function $g \circ f$ in an arrow diagram:

From the arrow diagram,

Thus, the composite function $g \circ f$ is {(0, 2), (– 1, – 1), (1, 5)}.

10. **संयुक्त फलनलाई परिभाषित गर्नुहोस् । यदि फलनहरू g = {(1, 2), (2, 3), (3, 4)} र h = {(2, 3), (3, 4), (4, 5)} भए संयुक्त फलन $h \circ g$ लाई मिलानचित्रमा देखाउनुहोस् ।**

Define composite function. If functions g = {(1, 2), (2, 3), (3, 4)} and h = {(2, 3), (3, 4), (4, 5)}, show the composite function $h \circ g$ in an arrow diagram. **[2065 R']**



⇨ Here, if f: A → B and g : B → C are two functions then the new function defined from A to C is called the composite function of f and g. It is denoted by $g \circ f$.

Here, g = {(1, 2), (2, 3), (3, 4)} and h = {(2, 3), (3, 4), (4, 5)}

The composite function $h \circ g$ in arrow diagram is shown alongside.

Thus, the composite function: $h \circ g$ is {(1, 3), (2, 4), (3, 5)}.

11. **विपरीत फलनको परिभाषा दिनुहोस् । यदि f = {(1, 2), (3, 4), (5, 6)} र g = {(2, 5), (4, 6), (6, 8)} भए संयुक्तफलन $g \circ f$ पत्ता लगाउनुहोस् ।**

Define inverse function. If f = {(1, 2), (3, 4), (5, 6)} and g = {(2, 5), (4, 6), (6, 8)}, find the composite function $g \circ f$. **[2065 S]**

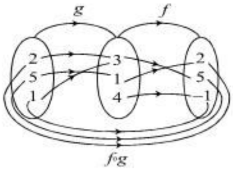
⇨ Here, let, f: A → B be one to one and onto function then a function f^{-1} : B → A such that $x \in A$ is said to be inverse function of f.

We have given, f = {(1, 2), (3, 4), (5, 6)} and g = {(2, 5), (4, 6), (6, 8)}

Thus, the composite function $g \circ f$ is {(1, 5), (3, 6), (5, 8)}.

12. **यदि f = {(1, 2), (3, 5), (4, 1)} र g = {(2, 3), (5, 1), (1, 3)} भए, $f \circ g$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।**

If f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)} then show the function $f \circ g$ in arrow diagram and find it in ordered-pair form. **[2066 R]**



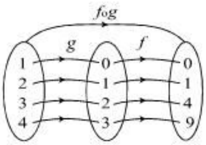
⇨ Here, the mapping diagram of $f \circ g$ are as follows:

From the above mapping diagram.

Thus, the composite function $f \circ g$ is {(2, 5), (5, 2), (1, 5)}.

13. **यदि फलनहरू f = {(0, 0), (1, 1), (2, 4), (3, 9)} र g = {(1, 0), (2, 1), (3, 2), (4, 3)} भए $f \circ g$ लाई मिलान चित्रमा देखाई क्रमजोडामा देखाउनुहोस् ।**

If f = {(0, 0), (1, 1), (2, 4), (3, 9)} and g = {(1, 0), (2, 1), (3, 2), (4, 3)} then show the function $f \circ g$ in arrow diagram and find it in order-pair form. **[2067 S]**



⇨ Here, f = {(0, 0), (1, 1), (2, 4), (3, 9)} and g = {(1, 0), (2, 1), (3, 2), (4, 3)}

We know that, the function $f \circ g$ is defined from domain of g to co-domain of f.

Thus, the composite function $f \circ g$ is {(1, 0), (2, 1), (3, 4), (4, 9)}.

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MODEL 3

14. यदि $g(x) = 2x - 1$ र $f(x) = 4x$, भए $g \circ f(x)$ को मान पत्ता लगाउनुहोस् । (If $g(x) = 2x - 1$ and $f(x) = 4x$, find the value of $g \circ f(x)$.) **[SEE MODEL 2076]**

⇨ Here, $g(x) = 2x - 1$ and $f(x) = 4x$

So, $g \circ f(x) = g(f(x))$

$= g(4x)$

$= 2(4x) - 1$

$\therefore\ g \circ f(x) = 8x - 1$

Thus, the value of $g \circ f(x)$ is $8x - 1$.

15. यदि $f(x) = 2x + 3$ भए $ff(2)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 2x + 3$ then find the value of $ff(2)$.) **[2075R$_2$]**

⇨ Here, $f(x) = 2x + 3$

So, $f(2)$

$= 2 \times 2 + 3$

$= 7$

Now, $ff(2)$

$= f[f(2)]$

$= f(7)$

$= 2 \times 7 + 3$

$= 17$

Thus, $ff(2) = 17$.

16. यदि $f(x) = 2x + 1$ भए $ff(2)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 1$ then find $ff(2)$.) **[SEE 2075 R']**

⇨ Here, $f(x) = 2x + 1$

So, $f(2)$

$= 2 \times 2 + 1$

$= 5$

Then $ff(2)$

$= f[f(2)]$

$= f(5)$

$= 2 \times 5 + 1$

$= 11$

Thus, $ff(2) = 11$.

17. यदि $fx) = x - 2$ र $g(x) = 3x + 1$ भए $g \circ f(x)$ र $g \circ f(2)$ पत्ता लगाउनुहोस् ।
If $f(x) = x - 2$ and $g(x) = 3x + 1$ then find $g \circ f(x)$ and $g \circ f(2)$. **[2074 S']**

⇨ Here, $f(x) = x - 2$ and $g(x) = 3x + 1$

So, $g \circ f(x) = g(f(x))$

$= g(x - 2)$

$= 3(x - 2) + 1$

$= 3x - 6 + 1$

$= 3x - 5$

And,

$g \circ f(2)$

$= 3 \times 2 - 5$

$= 1$

Thus, $g \circ f(x) = 3x - 5$ and $g \circ f(2) = 1$.

18. यदि $f(x) = x + 1$ र $g(x) = 2x + 1$ भए $g \circ f(2)$ को मान पत्ता लगाउनुहोस् । **[2074 R']**
If $f(x) = x + 1$ and $g(x) = 2x +1$ then find the value of $g \circ f(2)$.

⇨ Here, $f(x) = x + 1$ and $g(x) = 2x + 1$

So, $g \circ f(2) = g(f(2)) = g(2 + 1) = g(3) = 2 \times 3 + 1$

$\therefore\ g \circ f(2) = 7$

Thus, the value of $g \circ f(2)$ is 7.

19. यदि $h(x) = 2x + 3$ र $g(x) = 3x - 2$ भए $g \circ h(5)$ को मान पत्ता लगाउनुहोस् । (If $h(x) = 2x + 3$ and $g(x) = 3x - 2$, find the value of $g \circ h(5)$.) **[2073 S]**

⇨ Here, $h(x) = 2x + 3$ and $g(x) = 3x - 2$

We have, $g \circ h(5) = g(h(5))$

$= g(2 \times 5 + 3)$

$= g(13)$

$= 3 \times 13 - 2$

$= 37$

Thus, the value of $g \circ h\ (5)$ is 37.

20. यदि $f(x) = 3x + 15$ र $h(x) = 3x + 2$ भए $f \circ h(-2)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x +15$ and $h(x) = 3x + 2$, find the value of $f \circ h(-2)$.

⇨ Here, $f(x) = 3x + 15$ and $h(x) = 3x + 2$

So, $f \circ h\ (-2) = f(h(-2))$

$= f\ [3 \times (-2) + 2]$

$= f(-6 + 2)$

$= f(-4)$

$= 3 \times (-4) + 15$

$= -12 + 15$

$= 3$

Thus, the value of $f \circ h\ (-2)$ is 3.

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21. **यदि $h(x) = 2x + 3$ र $g(x) = 3x - 2$ भए $h\circ g(5)$ पत्ता लगाउनुहोस् । (If $h(x) = 2x + 3$ and $g(x) = 3x - 2$, find the $h\circ g(5)$.)** **[2073 R]**

⇨ Here, $h(x) = 2x + 3$ and $g(x) = 3x - 2$

We know that, $h\circ g(5) = h(g(5))$

$$= h(3 \times 5 - 2)$$
$$= h(13)$$
$$= 2 \times 13 + 3$$
$$\therefore \quad h\circ g(5) = 29$$

Thus, the value of $h\circ g(5)$ is 29.

22. **यदि $f(x) = x + 1$ र $g(x) = 2x + 1$ भए $g\circ f(x)$ पत्ता लगाउनुहोस् । (If $f(x) = x + 1$ and $g(x) = 2x + 1$, find $g\circ f(x)$.)** **[2072 S]**

⇨ Here, $f(x) = x + 1$ and $g(x) = 2x + 1$

So, $g\circ f(x) = g(f(x))$

$$= g(x + 1)$$
$$= 2(x + 1) + 1$$
$$= 2x + 2 + 1$$
$$\therefore \quad g\circ f(x) = 2x + 3$$

23. **यदि $f(x) = 2x + 5$ र $g(x) = 3x - 1$ भए $g\circ f(x)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 5$ and $g(x) = 3x - 1$,find $g\circ f(x)$.)** **[2071 R']**

⇨ Here, $f(x) = 2x + 5$ and $g(x) = 3x - 1$

So, $g\circ f(x) = g(f(x))$

$$= g(2x + 5)$$
$$= 3(2x + 5) - 1$$
$$= 6x + 15 - 1$$
$$\therefore \quad g\circ f(x) = 6x + 14$$

24. **यदि $g(x) = \frac{x-2}{3}$ र $h(x) = 3x + 2$ भए, $gh(x)$ एउटा एकात्मक फलन हो भनी प्रमाणित गर्नुहोस् ।**

If $g(x) = \frac{x-2}{3}$ and $h(x) = 3x + 2$, prove that $gh(x)$ is an identity function. **[2070 R]**

⇨ Here, $g(x) = \frac{x-2}{3}$ and $h(x) = 3x + 2$

We have, $gh(x) = g(h(x))$

$$= g(3x + 2)$$
$$= \frac{3x + 2 - 2}{3}$$
$$= \frac{3x}{3}$$
$$\therefore \quad gh(x) = x$$

Thus, $gh(x) = x$ shows that it is an identity function.

25. **यदि $f(x) = 2x - 3$ र $g(x) = x^2 + 1$ भए $f\circ g(3)$ को मान पत्ता लगाउनुहोस् ।**

If $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the value of $f\circ g(3)$. **[2070 R']**

⇨ Here, $f(x) = 2x - 3$ and $g(x) = x^2 + 1$

We have, $f\circ g(x)(3) = f(g(3))$

$$= f(3^2 + 1)$$
$$= f(10)$$
$$= 2 \times 10 - 3$$
$$= 17$$

Thus, the value of $f\circ g(3)$ is 17.

26. **यदि $f(x) = 3x + 2$ र $g(x) = 2x - 1$ भए, $g\circ f(x)$ पत्ता लगाउनुहोस् । (If $f(x) = 3x + 2$ and $g(x) = 2x - 1$, find $g\circ f(x)$.)** **[2065R]**

⇨ Here, $f(x) = 3x + 2$ and $g(x) = 2x - 1$

So, $g\circ f(x) = g(f(x))$

$$= g(3x + 2)$$
$$= 2(3x + 2) - 1$$
$$= 6x + 4 - 1$$

Thus, the composite function $go f(x)$ is $6x + 3$.

27. **यदि $f(x) = 2x + 3$ र $g(x) = 3x - 1$ भए $f\circ g(x)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 3$ and $g(x) = 3x - 1$, find $f\circ g(x)$.)** **[2066 R']**

⇨ Here, $f(x) = 2x + 3$ and $g(x) = 3x - 1$

So, $f\circ g(x) = f(g(x))$

$$= f(3x - 1)$$
$$= 2(3x - 1) + 3$$
$$= 6x - 2 + 3$$
$$= 6x + 1$$

Thus, the composite function $f\circ g(x)$ is $6x + 1$.

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28. **विपरित फलनको परिभाषा दिनुहोस् । यदि $f(x) = 2x - 1$ भए $ff(-1)$ पत्ता लगाउनुहोस् ।**
Define inverse function. If $f(x) = 2x - 1$, find $ff(-1)$. **[2067 R]**

⇨ Here, let, $f : A \to B$ be one to one and onto function then a function $f^{-1} : B \to A$ such that $x \in A$ is said to be inverse function of f.

We have given, $f(x) = 2x - 1$

Now, $ff(-1) = f\{2(-1) - 1\}$
$= f(-2 - 1)$
$= f(-3) = 2 \times (-3) - 1$
$= -6 - 1$
$= -7$

Thus, the required value of $ff(-1)$ is -7.

29. **यदि $g(x) = \frac{x+2}{5}$ र $h(x) = 3x - 2$ भए $gh(x)$ पत्ता लगाउनुहोस् । If $g(x) = \frac{x+2}{5}$ and $h(x) = 3x - 2$, find $gh(x)$.** **[2067R']**

⇨ Here, $g(x) = \frac{x+2}{5}$ and $h(x) = 3x - 2$

Now, $gh(x) = g(3x - 2) = \frac{3x - 2 + 2}{5} = \frac{3x}{5}$

Thus, the required value of $gh(x)$ is $\frac{3x}{5}$.

MODEL 4

30. **यदि $f : x \to 3x + b$ र $ff(2) = 12$ भए b को मान पत्ता लगाउनुहोस् । (If $f : x \to 3x + b$ and $ff(2) = 12$, find the value of b.)** **[2068 R]**

⇨ Here, $f : x \to 3x + b$
or, $f(x) = 3x + b$
or, $f(2) = 3 \times 2 + b$
$\therefore$ $f(2) = 6 + b$

Now, $ff(2) = f(f(2))$
$= f(6 + b)$
$= 3(6 + b) + b$
$= 18 + 3b + b$
or, $12 = 18 + 4b$
or, $-6 = 4b$
$\therefore$ $b = -\frac{3}{2}$

Thus, the value of b is $-\frac{3}{2}$.

31. **यदि $f(x) = 3x$, $g(x) = x + 2$ र $f{\circ}g(x) = 18$ भए x को मान पत्ता लगाउनुहोस् ।**
If $f(x) = 3x$, $g(x) = x + 2$ and $f{\circ}g(x) = 18$, find the value of x. **[2068 S]**

⇨ Here, $f(x) = 3x$, $g(x) = x + 2$ and $f{\circ}g(x) = 18$

We have, $f{\circ}g(x) = 18$
or, $f[g(x)] = 18$
or, $f[x + 2] = 18$
or, $3(x + 2) = 18$
or, $3x + 6 = 18$
or, $3x = 12$
$\therefore$ $x = \frac{12}{3} = 4$

Thus, the value of x is 4.

MODEL 5

32. **यदि $f(x) = 3x - 2$ र $f{\circ}g(x) = 6x - 2$ भए $g(x)$ पत्ता लगाउनुहोस् । (If $f(x) = 3x - 2$ and $f{\circ}g(x) = 6x - 2$, find $g(x)$.)** **[2072 R]**

⇨ Here, $f(x) = 3x - 2$ and $f{\circ}g(x) = 6x - 2$

So, $f(g(x)) = 6x - 2$
or, $3g(x) - 2 = 6x - 2$
or, $3g(x) = 6x$
$\therefore$ $g(x) = 2x$

Thus, $g(x) = 2x$.

33. **यदि $f(x) = 4x + 5$ र $f{\circ}g(x) = 8x + 13$ भए $g(x)$ पत्ता लगाउनुहोस् । (If $f(x) = 4x + 5$ and $f{\circ}g(x) = 8x + 13$ then find $g(x)$.)**

⇨ Here, $f(x) = 4x + 5$ and $f{\circ}g(x) = 8x + 13$

We have, $f{\circ}g(x) = 8x + 13$
or, $f[g(x)] = 8x + 13$
or, $4g(x) + 5 = 8x + 13$
or, $4g(x) = 8x + 13 - 5 = 8x + 8$
or, $4g(x) = 8(x + 1)$
$\therefore$ $g(x) = 2(x + 1)$

Thus, $g(x) = 2(x + 1)$.

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MODEL 6

34. यदि $f(x) = 4x + 5$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् । (Find $f^{-1}(x)$ if $f(x) = 4x + 5$.) **[SEE MODEL 2076]**

⇨ Here, $f(x) = 4x + 5$

Let $y = f(x)$ or, $y = 4x + 5$

Interchanging the position of x and y then, $x = 4y + 5$

or, $x - 5 = 4y$ $\therefore\ y = \dfrac{x-5}{4}$

Thus, $f^{-1}(x) = \dfrac{x-5}{4}$.

35. यदि $f(x) = 2x - 1$ भए $ff(-1)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 2x - 1$ then find the value of $ff(-1)$.) **[SEE 2075 R, 2067 R]**

⇨ Here, $f(x) = 2x - 1$

Now, $f(-1) = 2 \times (-1) - 1 = -3$

Then, $ff(-1) = f[f(-1)] = f(-3) = 2 \times (-3) - 1 = -6 - 1 = -7$

Thus, $ff(-1) = -7$.

36. यदि $f(x) = \dfrac{2x-3}{5}$ भए $f^{-1}\left(\dfrac{1}{5}\right)$ को मान पत्ता लगाउनुहोस् । If $f(x) = \dfrac{2x-3}{5}$, find the value of $f^{-1}\left(\dfrac{1}{5}\right)$. **[SEE 2075 R₂']**

⇨ Here, $f(x) = y = \dfrac{2x-3}{5}$

Interchanging the positions of x and y we get,

$x = \dfrac{2y-3}{5}$

or, $\dfrac{5x+3}{2} = y$

$\therefore\ f^{-1}(x) = \dfrac{5x+3}{2}$

Now, $f^{-1}\left(\dfrac{1}{5}\right) = \dfrac{5 \times \frac{1}{5} + 3}{2} = \dfrac{4}{2} = 2$

Thus, $f^{-1}\left(\dfrac{1}{5}\right) = 2$.

37. यदि $f(x) = \dfrac{2x+3}{2}$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् ।(If $f(x) = \dfrac{2x+3}{2}$, then find the value of $f^{-1}(x)$.) **[2073 R']**

⇨ Here, $f(x) = \dfrac{2x+3}{2}$

Let, $y = f(x)$

or, $y = \dfrac{2x+3}{2}$

Interchanging the position of x and y,

$x = \dfrac{2y+3}{2}$

or, $2x = 2y + 3$

or, $2x - 3 = 2y$

or, $y = \dfrac{2x-3}{2}$

$\therefore\ f^{-1}(x) = \dfrac{2x-3}{2}$

Thus, the value of $f^{-1}(x)$ is $\dfrac{2x-3}{2}$.

38. यदि $f(x) = 4x + 3$ भए $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 4x + 3$ then find the value of $f^{-1}(4)$.) **[2072 R']**

⇨ Here, $f(x) = 4x + 3$

or, $y = 4x + 3$

Interchanging the position of x and y then,

$x = 4y + 3$

or, $x - 3 = 4y$

or, $y = \dfrac{x-3}{4}$

$\therefore\ f^{-1}(x) = \dfrac{x-3}{4}$

Now, $f^{-1}(4) = \dfrac{4-3}{4} = \dfrac{1}{4}$

Thus, the value of $f^{-1}(4)$ is $\dfrac{1}{4}$.

39. यदि $f^{-1}(x) = 2x - 3$ भए $f(x)$ पत्ता लगाउनुहोस् । (If $f^{-1}(x) = 2x - 3$, find $f(x)$.)

⇨ Here, $f^{-1}(x) = 2x - 3$

Let, $y = f^{-1}(x)$

or, $y = 2x - 3$

Interchanging the values of x and y,

$x = 2y - 3$

or, $x + 3 = 2y$

$\therefore\ y = \dfrac{x+3}{2}$

Thus, $f(x) = \dfrac{x+3}{2}$.

40. यदि $f^{-1}(x) = \dfrac{x+3}{2}$ भए $f(x)$ को मान पत्ता लगाउनुहोस् । (If $f^{-1}(x) = \dfrac{x+3}{2}$, find $f(x)$.) **[2071 R]**

⇨ Here, $f^{-1}(x) = \dfrac{x+3}{2}$

Let $y = \dfrac{x+3}{2}$

Interchanging x and y then, $x = \dfrac{y+3}{2}$

or, $2x = y + 3$

or, $2x - 3 = y$

$\therefore\ y = 2x - 3$

Thus, $f(x) = 2x - 3$.

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41. फलन $f = \left\{\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right)\right\}$ को विस्तार क्षेत्र र विपरीत फलन लेख्नुहोस् ।

Write the range & inverse function of the function $f = \left\{\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right)\right\}$. **[2061 R]**

⇨ Here, given function, $f = \left\{\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right)\right\}$

Since range of a function is the set of all second elements of the order pairs, $\therefore$ Range of $f = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

A function obtained by interchanging the elements of the order pairs is called inverse the function.

Thus, Inverse of the function, $f^{-1} = \left\{\left(\frac{1}{2}, 2\right), \left(\frac{1}{3}, 3\right), \left(\frac{1}{4}, 4\right)\right\}$

42. यदि $f(x) = 3x - 11$ भए, $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् । **(If $f(x) = 3x - 11$, find the value of $f^{-1}(4)$.)** **[2068R']**

⇨ Here, $f(x) = 3x - 11$

let, $f(x) = y = 3x - 11$

Interchanging the position of x & y then,

$x = 3y - 11$

or, $x + 11 = 3y$

or, $y = \frac{x + 11}{3}$

or, $f^{-1}(x) = \frac{x + 11}{3}$

$\therefore\ f^{-1}(4) = \frac{4 + 11}{3} = 5$

Thus, the required value of $f^{-1}(4)$ is 5.

C. LONG QUESTIONS

MODEL 1

1. यदि $f(x) = 3x + 4$ र $g(x) = 2(x + 1)$ भए $(f \circ g)(x) = (g \circ f)(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = 3x + 4$ and $g(x) = 2(x + 1)$ then prove that $(f \circ g)(x) = (g \circ f)(x)$

⇨ Here, $f(x) = 3x + 4$ and $g(x) = 2(x + 1) = 2x + 2$

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(2x + 2) \\ &= 3(2x + 2) + 4 \\ &= 6x + 6 + 4 \\ \therefore\ f \circ g(x) &= 6x + 10 \end{aligned}$$

$$\begin{aligned} \text{Again, } g \circ f(x) &= g(3x + 4) \\ &= 2(3x + 4) + 2 \\ &= 6x + 8 + 2 \\ &= 6x + 10 \end{aligned}$$

Thus, $f \circ g(x) = go f(x)$ **Proved.**

2. यदि $f(x) = \frac{2x + 5}{4}$ र $g(x) = \frac{4x + 7}{5}$ भए $f \circ g(x) + \frac{9}{20} = g \circ f(x)$ हुन्छ भनी देखाउनुहोस् ।

If $f(x) = \frac{2x + 5}{4}$ and $g(x) = \frac{4x + 7}{5}$ then show that $f \circ g(x) + \frac{9}{20} = g \circ f(x)$.

⇨ Here, $f(x) = \frac{2x + 5}{4}$ and $g(x) = \frac{4x + 7}{5}$

$$\begin{aligned} \text{Now, } f \circ g(x) &= f[g(x)] \\ &= f\left(\frac{4x + 7}{5}\right) \\ &= \frac{2\left(\frac{4x + 7}{5}\right) + 5}{4} \\ &= \frac{8x + 14 + 25}{5 \times 4} \\ &= \frac{8x + 39}{20} \\ \therefore\ f \circ g(x) &= \frac{8x + 39}{20} \ \text{....(i)} \end{aligned}$$

$$\begin{aligned} \text{Again, } g \circ f(x) &= g[f(x)] \\ &= g\left(\frac{2x + 5}{4}\right) \\ &= \frac{4\left(\frac{2x + 5}{4}\right) + 7}{5} \\ &= \frac{2x + 5 + 7}{5} \\ &= \frac{2x + 12}{5} \\ \therefore\ g \circ f(x) &= \frac{2x + 12}{5} \ \text{....(ii)} \end{aligned}$$

Now, LHS $= f \circ g(x) + \frac{9}{20} = \frac{8x + 39}{20} + \frac{9}{20} = \frac{8x + 48}{20} = \frac{4(2x + 12)}{20} = \frac{2x + 12}{5} = g \circ f(x)$

Thus, LHS = RHS **Proved.**

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MODEL 2

3. यदि $g(x) = \frac{3x-1}{x-1}, x \neq 1$ र $h(x) = \frac{2x-1}{x-3}, x \neq 3$ भए $(g∘h)^{-1}x$ र $(h∘g)^{-1}x$ पत्ता लगाउनुहोस् ।

If $g(x) = \frac{3x-1}{x-1}, x \neq 1$ and $h(x) = \frac{2x-1}{x-3}, x \neq 3$, find $(g∘h)^{-1}x$ and $(h∘g)^{-1}x$.

⇨ Here, $g(x) = \frac{3x-1}{x-1}$ and $h(x) = \frac{2x-1}{x-3}$

Now, $g∘h\,(x) = g\,[h(x)] = g\left[\frac{2x-1}{x-3}\right]$

$$= \frac{3\frac{(2x-1)}{x-3} - 1}{\frac{2x-1}{x-3} - 1} = \frac{6x-3-x+3}{2x-1-x+3} = \frac{5x}{x+2}$$

$\therefore \quad g∘h(x) = y = \frac{5x}{x+2}$

Interchanging the positions of x and y then,

$x = \frac{5y}{y+2}$

or, $xy + 2x = 5y$

or, $2x = y(5 - x)$

or, $y = \frac{2x}{5-x}$

$\therefore \quad (g∘h)^{-1} = \frac{2x}{5-x}$

Again, $h∘g\,(x) = h\,[g(x)] = h\left[\frac{3x-1}{x-1}\right]$

$$= \frac{2\frac{(3x-1)}{x-1} - 1}{\frac{3x-1}{x-1} - 3} = \frac{6x-2-x+1}{3x-1-3x+3} = \frac{5x-1}{2}$$

$\therefore \; h∘g(x) = y = \frac{5x-1}{2}$

Interchanging the positions of x and y then,

$x = \frac{5y-1}{2}$

or, $2x + 1 = 5y$

or, $y = \frac{2x+1}{5}$

$\therefore \quad (h∘g)^{-1} = \frac{2x+1}{5}$

Thus, $(g∘h)^{-1} = \frac{2x}{5-x}$ and $(h∘g)^{-1} = \frac{2x+1}{5}$.

4. यदि $f(x) = \frac{x+3}{2}$ र $g(x) = 2x - 3$ भए $f^{-1}∘f\,(x) = f∘g(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{x+3}{2}$ and $g(x) = 2x - 3$, prove that $f^{-1}∘f\,(x) = f∘g\,(x)$.

⇨ Here, $f(x) = \frac{x+3}{2}$ and $g(x) = 2x - 3$

Let, $y = f(x)$

or, $y = \frac{x+3}{2}$

Interchanging the position of x and y.

Then, $x = \frac{y+3}{2}$

or, $y + 3 = 2x$

or, $y = 2x - 3$

$\therefore \quad f^{-1}(x) = 2x - 3$

Now, $f^{-1}∘f\,(x) = f^{-1}\left(\frac{x+3}{2}\right)$

$= 2\,.\,\frac{x+3}{2} - 3$

$= x + 3 - 3$

$= x$

And, $f∘g\,(x) = f(2x-3) = \frac{2x-3+3}{2} = \frac{2x}{2} = x$

Thus, $f^{-1}∘f(x) = f∘g\,(x)$

Proved.

5. यदि $f(x) = 2x + 7$ भए $f(x+2)$ र $f^{-1}(x+2)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 7$, find $f(x+2)$ and $f^{-1}(x+2)$.) [2067R']

⇨ Here, $f(x) = 2x + 7$

Now, $f(x+2) = 2(x+2) + 7$

or, $f(x+2) = 2x + 4 + 7$

$\therefore \quad f(x+2) = 2x + 11$

Now, $f(x) = (2x + 7) = y$ (let)

or, $y = 2x + 7$

Interchanging the position of x and y then, $x = 2y + 7$

or, $x - 7 = 2y$

or, $y = \frac{x-7}{2}$

or, $f^{-1}(x) = \frac{x-7}{2}$

Again, $f^{-1}(x+2) = \frac{x+2-7}{2} = \frac{x-5}{2}$

Thus, the required value of $f(x+2)$ is $2x + 11$ and $f^{-1}(x+2)$ is $\frac{x-5}{2}$.

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6. यदि $f(x) = 8 - 3x$ भए $f^{-1}(-4)$ र $f\circ f(2)$ को मान निकाल्नुहोस् । (If $f(x) = 8 - 3x$, evaluate $f^{-1}(-4)$ and $f\circ f(2)$.) **[2057S]**

⇨ Here, given function, $f(x) = 8 - 3x$

Let, $y = f(x) = 8 - 3x$

or, $y = 8 - 3x$

Interchanging x and y and solving for y,

or, $x = 8 - 3y$

or, $3y = 8 - x$

or, $y = \frac{8-x}{3}$

$\therefore\ f^{-1}(x) = \frac{8-x}{3}$

When $x = -4$ then, $f^{-1}(-4) = \frac{8-(-4)}{3} = \frac{8+4}{3} = \frac{12}{3} = 4$

$\therefore\ f^{-1}(-4) = 4$

Again, $ff(x) = f(f(x)) = f(8 - 3x)$
$= 8 - 3(8 - 3x) = 8 - 24 + 9x$
$= 9x - 16$

When putting $x = 2$, we get,
$ff(2) = 9 \times 2 - 16 = 18 - 16 = 2.$

Thus, the required value of $f^{-1}(-4)$ is 4 and $ff(2)$ is 2.

7. यदि $f(x) = 4x - 2$ र $g(x) = \frac{1}{x}$ भए $f^{-1}(6)$ र $g\circ f(2)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 4x - 2$ and $g(x) = \frac{1}{x}$, find the value of $f^{-1}(6)$ and $g\circ f(2)$. **[2057 R]**

⇨ Here, given functions $f(x) = 4x - 2$ and $g(x) = \frac{1}{x}$

Let $y = 4x - 2$

Now, interchanging x & y and solving for y, we get,

$x = 4y - 2$

or, $x + 2 = 4y$

$\therefore\ y = \frac{x+2}{4}$

i.e. $f^{-1}(x) = \frac{x+2}{4}$

Now, $f^{-1}(6) = \frac{6+2}{4} = \frac{8}{4} = 2$

Again, $g\circ f(x) = g(f(x)) = g(4x - 2) = \frac{1}{4x-2}$ $\left[\because g(x) = \frac{1}{x}\right]$

Put $x = 2$, we get, $g\circ f(2) = \frac{1}{4.2-2} = \frac{1}{8-2} = \frac{1}{6}$

Thus, the required value of $g\circ f(2)$ is $\frac{1}{6}$ and $f^{-1}(6)$ is 2.

8. $f(x) = 2x - 3$ र $g(x) = x^2 + 2$ भए पत्ता लगाउनुहोस् : (i) $f^{-1}(-2)$ (ii) $f\circ g(x)$
If $f(x) = 2x - 3$ and $g(x) = x^2 + 2$, find : (i) $f^{-1}(-2)$ (ii) $f\circ g(x)$ **[2060 S]**

⇨ Here, given functions $f(x) = 2x - 3$ and $g(x) = x^2 + 2$, $f^{-1}(-2) = ?$ and $f\circ g(x) = ?$

(i) For $f^{-1}(-2)$

Here, let $y = f(x) = 2x - 3$

Interchanging x and y and solving for y, we get,

i.e. $x = 2y - 3$ or, $x + 3 = 2y$

or, $y = \frac{x+3}{2}$ $\therefore\ f^{-1}(x) = \frac{x+3}{2}$

Now, putting $x = -2$, we get, $f^{-1}(-2) = \frac{-2+3}{2} = \frac{1}{2}$

(ii) For $f\circ g(x)$

Here, $f\circ g(x)$
$= f(g(x))$
$= f(x^2 + 2)$
$= 2(x^2 + 2) - 3$
$= 2x^2 + 4 - 3$

$\therefore\ f\circ g(x) = 2x^2 + 1$

Thus, the required value of $f^{-1}(-2)$ is $\frac{1}{2}$ and $f\circ g(x)$ is $2x^2 + 1$.

9. यदि $f(x) = 2x + 5$ र $g(x) = 3x + 1$ दुई फलनहरू भए $f^{-1}(x)$ र $gf^{-1}(5)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 5$ and $g(x) = 3x + 1$ are two functions, find $f^{-1}(x)$ and $gf^{-1}(5)$. **[2068 R']**

⇨ Here, $f(x) = 2x - 5$ and $g(x) = 3x + 1$

Now, $f(x) = 2x - 5$

or, $y = 2x - 5$

Interchanging the position of x & y then,

i.e. $x = 2y - 5$

or, $x + 5 = 2y$

or, $y = \frac{x+5}{2}$

or, $f^{-1}(x) = \frac{x+5}{2}$

$\therefore\ f^{-1}(5) = \frac{5+5}{2} = \frac{10}{2} = 5$

Then, $gf^{-1}(5) = g(5) = 3.5 + 1 = 15 + 1 = 16$

Thus, the required value of $f^{-1}(x)$ is $\frac{x+5}{2}$ and $gf^{-1}(5)$ is 16.

10. यदि $f(x) = 2x + 1$ र $g(x) = \frac{x-5}{2}$ भए $f^{-1}g^{-1}(3)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + 1$, $g(x) = \frac{x-5}{2}$ then find the value of $f^{-1}g^{-1}(3)$. **[2065S]**

⇨ Here, $f(x) = 2x + 1$ and $g(x) = \frac{x-5}{2}$

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Let, $y = f(x)$
or, $y = 2x + 1$
Interchanging the position of x and y, i.e. $x = 2y + 1$
or, $2y = x - 1$
or, $y = \frac{x-1}{2}$
$\therefore\ f^{-1}(x) = \frac{x-1}{2}$

Again, Let $y = g(x)$
or, $y = \frac{x-5}{2}$
Interchanging the position of x and y then, i.e. $x = \frac{y-5}{2}$
or, $2x = y - 5$
or, $y = 2x + 5$
$\therefore\ g^{-1}(x) = 2x + 5$

Now, $f^{-1}g^{-1}(x) = f^{-1}(g^{-1}(x)) = f^{-1}(2x + 5) = \frac{2x+5-1}{2}$
or, $f^{-1}g^{-1}(x) = x + 2$ $\quad\therefore\ f^{-1}g^{-1}(3) = 3 + 2 = 5$
Thus, the value of $f^{-1}g^{-1}(3)$ is 5.

11. यदि 3. $f(x) = 4x + 5$ र $g(x) = 5x - 4$ भए $f^{-1}\circ g^{-1}(1)$ को मान निकाल्नुहोस् ।
If 3. $f(x) = 4x + 5$ and $g(x) = 5x - 4$, find the value of $f^{-1}\circ g^{-1}(1)$. **[2068R]**

⇨ Here, $3f(x) = 4x + 5$ i.e. $f(x) = \frac{4x+5}{3}$

For f^{-1}, let $y = f(x)$
or, $y = \frac{4x+5}{3}$
Interchanging the position of x and y then, $x = \frac{4y+5}{3}$
or, $3x = 4y + 5$
or, $3x - 5 = 4y$
$\therefore\ y = \frac{3x-5}{4}$
i.e. $f^{-1}(x) = \frac{3x-5}{4}$

$g(x) = 5x - 4$
For g^{-1},
let $y = g(x)$
or, $y = 5x - 4$
Interchanging the position of x and y then,
i.e. $x = 5y - 4$
or, $x + 4 = 5y$
or, $y = \frac{x+4}{5}$
$\therefore\ g^{-1}(x) = \frac{x+4}{5}$

Now, $f^{-1}\circ g^{-1}(1) = f^{-1}(g^{-1}(1)) = f^{-1}\left(\frac{1+4}{5}\right) = f^{-1}(1) = \left(\frac{3\times 1-5}{4}\right) = \left(-\frac{2}{4}\right) = -\frac{1}{2}$

Thus, the value of $f^{-1}\circ g^{-1}(1)$ is $-\frac{1}{2}$.

12. यदि $f(x) = \frac{1}{x}, x \neq 0$ भए प्रमाणित गर्नुहोस् $f\circ f^{-1} = f^{-1}\circ f$. (If $f(x) = \frac{1}{x}, x \neq 0$ prove that: $f\circ f^{-1} = f^{-1}\circ f$.) **[2060 S]**

⇨ Here, $f(x) = \frac{1}{x}$ $\quad\therefore\ y = \frac{1}{x}$

Interchanging the position of x and y then,
i.e. $x = \frac{1}{y}$
or, $xy = 1$
or, $y = \frac{1}{x}$
$\therefore\ f^{-1}(x) = \frac{1}{x}$

Now, $f\circ f^{-1}(x) = f(f^{-1}(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$

Again, $f^{-1}\circ f = f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$

Thus, $f\circ f^{-1} = f^{-1}\circ f$ **Proved.**

13. यदि $f(x) = 1 + 2x$ र $g(x) = \frac{1}{1-x}$ भए $g^{-1}\left(\frac{1}{2}\right)$ र $f\circ g(-1)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 1 + 2x$ and $g(x) = \frac{1}{1-x}$, find the value of $g^{-1}\left(\frac{1}{2}\right)$ and $f\circ g(-1)$. **[2058 R]**

⇨ Here, given functions, $f(x) = 1 + 2x$ and $g(x) = \frac{1}{1-x}$

Let $y = g(x) = \frac{1}{1-x}$ $\quad$ So, $g^{-1}(y) = x$
Now, solving for x, we get, $y = \frac{1}{1-x}$
or, $1 - x = \frac{1}{y}$ $\quad$ or, $1 - \frac{1}{y} = x$
or, $\frac{y-1}{y} = x$
or, $g^{-1}(y) = \frac{y-1}{y}$ $(\because x = g^{-1}(y))$
Interchanging the position of x and y then,
$\therefore\ g^{-1}(x) = \frac{x-1}{x}$

Now, $g^{-1}\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-1}{\frac{1}{2}} = \frac{1-2}{2}\times\frac{2}{1} = -1$

Again, $fg(x) = f(g(x)) = f\left(\frac{1}{1-x}\right) = 1 + 2\left(\frac{1}{1-x}\right)$
$= 1 + \frac{2}{1-x} = \frac{1-x+2}{1-x} = \frac{3-x}{1-x}$

We have, $fg(-1) = \frac{3-(-1)}{1-(-1)} = \frac{3+1}{1+1} = \frac{4}{2} = 2$

Thus, the value of $g^{-1}\left(\frac{1}{2}\right)$ is -1 and $fg(-1)$ is 2.

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14. यदि फलन $f(x) = \frac{2x+3}{x+2}$ र $g(x) = x - 2$ भए $f^{-1}(x), f^{-1}(1), f\circ g(x)$ र $f\circ g(1)$ को मान निकाल्नुहोस् ।

If function $f(x) = \frac{2x+3}{x+2}$ and $g(x) = x - 2$, find the values of $f^{-1}(x)$, $f^{-1}(1)$, $f\circ g(x)$ and $f\circ g(1)$. **[2059 S]**

⇨ Here, given functions, $f(x) = \frac{2x+3}{x+2}$ and $g(x) = x - 2$

Let $y = f(x) = \frac{2x+3}{x+2}$

Interchanging the position of x and y then,

i.e. $x = \frac{2y+3}{y+2}$

or, $xy + 2x = 2y + 3$

or, $xy - 2y = 3 - 2x$

or, $y(x - 2) = 3 - 2x$

or, $y = \frac{3-2x}{x-2}$

$\therefore \quad f^{-1}(x) = \frac{3-2x}{x-2}$

Now, $f^{-1}(1) = \frac{3-2\times 1}{1-2} = \frac{3-2}{-1} = -1$

And, $fg(x) = f(g(x)) = f(x-2)$

$= \frac{2(x-2)+3}{(x-2)+2}$

$= \frac{2x-4+3}{x-2+2} = \frac{2x-1}{x}$

Again, $fg(1) = \frac{2\times 1-1}{1} = \frac{2-1}{1} = 1$

Thus, the values of $f^{-1}(x)$, $f^{-1}(1)$, $f\circ g(x)$ and $f\circ g(1)$ are $\frac{3-2x}{x-2}$, -1, $\frac{2x-1}{x}$ and 1 respectively.

15. यदि $f(x) = 3x + 4$ र $g(x) = 2(x + 1)$ भए प्रमाणित गर्नुहोस् $f\circ g = g\circ f$ र $f^{-1}(2)$ को मान निकाल्नुहोस् ।

If $f(x) = 3x + 4$ and $g(x) = 2(x + 1)$ then, prove that $f\circ g = g\circ f$ and find the value of $f^{-1}(2)$. **[2060 R]**

⇨ Here given, $f(x) = 3x + 4$ and $g(x) = 2(x + 1) = 2x + 2$

Now, $f\circ g = f\circ g(x)$

$= f(g(x))$

$= f(2x + 2)$

$= 3(2x + 2) + 4$

$= 6x + 6 + 4$

$= 6x + 10$ .... (i)

Again, $g\circ f = g\circ f(x)$

$= g(f(x))$

$= g(3x + 4)$

$= 2(3x + 4) + 2$

$= 6x + 8 + 2$

$= 6x + 10$ ...... (ii)

Hence from (i) and (ii) $f\circ g = g\circ f$ **Proved.**

Again, let $y = f(x)$

So, $y = 3x + 4$

Interchanging the position of x and y then,

i.e. $x = 3y + 4$

or, $x - 4 = 3y$

or, $y = \frac{x-4}{3}$

$\therefore \quad f^{-1}(x) = \frac{x-4}{3}$

When, x = 2 then, $f^{-1}(2) = \frac{2-4}{3} = \frac{-2}{3}$

Thus, required value of $f^{-1}(2)$ is $-\frac{2}{3}$.

16. यदि $f(x) = 2x - 3, x \in \Re$ भए $f\circ f^{-1}(x) = f^{-1}\circ f(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = 2x - 3, x \in \Re$, prove that $f\circ f^{-1}(x) = f^{-1}\circ f(x)$. **[2062 K]**

⇨ Here, $f(x) = 2x - 3$

Let $y = f(x)$

$\therefore \quad y = 2x - 3$

Interchanging the position of x and y then,

i.e. $x = 2y - 3$

or, $x + 3 = 2y$

or, $y = \frac{x+3}{2}$

or, $y = \frac{x+3}{2}$

$\therefore \quad f^{-1} = \frac{x+3}{2}$

Now, LHS $= ff^{-1}(x)$

$= f\left(\frac{x+3}{2}\right) \quad \left[\because f^{-1}(x) = \frac{x+3}{2}\right]$

$= 2\left(\frac{x+3}{2}\right) - 3$

$[\because f(x) = 2x - 3]$

$= x + 3 - 3$

$\therefore$ LHS $= x$

And, RHS $= f^{-1}f(x)$

$= f^{-1}(2x - 3) \quad [\because f(x) = 2x - 3]$

$= \frac{2x-3+3}{2} \quad \left[\because f^{-1}(x) = \frac{x+3}{2}\right]$

$= \frac{2x}{2}$

$\therefore$ RHS $= x$

Thus, LHS = RHS = x shows that $ff^{-1}(x) = f^{-1}f(x)$ **Proved.**

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MODEL 3

17. **यदि $f(x) = 2x - 5$, $g(x) = \frac{3x+5}{2}$ र $ff(x) = g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।**

If $f(x) = 2x - 5$, $g(x) = \frac{3x+5}{2}$ and $ff(x) = g^{-1}(x)$, then find the value of x. **[SEE 2075 R']**

⇨ Here, $f(x) = 2x - 5$, $g(x) = \frac{3x+5}{2}$

Let, $y = g(x) = \frac{3x+5}{2}$

Interchanging the positions of x and y, we get,

$x = \frac{3y+5}{2}$

or, $2x - 5 = 3y$

$\therefore\ y = g^{-1}(x) = \frac{2x-5}{3}$

Now, according to the question, $ff(x) = g^{-1}(x)$

or, $f[f(x)] = \frac{2x-5}{3}$

or, $f[2x-5] = \frac{2x-5}{3}$

or, $2(2x-5) - 5 = \frac{2x-5}{3}$

or, $4x - 10 - 5 = \frac{2x-5}{3}$

or, $3(4x - 15) = 2x - 5$

or, $12x - 45 = 2x - 5$

or, $10x = 40$

$\therefore\ x = 4$

Thus, the value of x is 4.

18. **यदि $f(x) = \frac{x+1}{2}$, $g(x) = \frac{x-5}{2}$ र $f{\circ}g^{-1}(x) = 6$ भए x को मान पत्ता लगाउनुहोस् ।**

If $f(x) = \frac{x+1}{2}$, $g(x) = \frac{x-5}{2}$ and $f{\circ}g^{-1}(x) = 6$, find the value of x. **[2074 S, 2072 R]**

⇨ Here, $f(x) = \frac{x+1}{2}$, $g(x) = \frac{x-5}{2}$ and $f{\circ}g^{-1}(x) = 6$

Taking $g(x) = \frac{x-5}{2}$

Let $y = g(x)$

or, $y = \frac{x-5}{2}$

Interchanging the position of x and y; $x = \frac{y-5}{2}$

or, $2x = y - 5$

$\therefore\ y = 2x + 5$

$\therefore\ g^{-1}(x) = 2x + 5$

Now, $f{\circ}g^{-1}(x) = 6$

or, $f(g^{-1}(x)) = 6$

or, $f(2x + 5) = 6$

or, $\frac{2x+5+1}{2} = 6$

or, $2x + 6 = 12$

or, $2x = 6$

$\therefore\ x = 3$

Thus, the value of x is 3.

19. **फलनहरू $f(x) = 4x - 1$ र $g(x) = 2x - 3$ दिइएका छन् । यदि $g{\circ}f^{-1}(x) = x + 1$ भए x को मान निकाल्नुहोस् ।**

It is given that the functions $f(x) = 4x - 1$ and $g(x) = 2x - 3$. If $g{\circ}f^{-1}(x) = x + 1$, find the value of x. **[2074 S']**

⇨ Here, $f(x) = 4x - 1$, $g(x) = 2x - 3$ and $g{\circ}f^{-1}(x) = x + 1$

Taking $f(x) = 4x - 1$

Let, $y = f(x) = 4x - 1$

Interchanging the position of x and y then,

$x = 4y - 1$

or, $x + 1 = 4y$

or, $y = \frac{x+1}{4}$

$\therefore\ f^{-1}(x) = \frac{x+1}{4}$

Now, $g{\circ}f^{-1}(x) = g[f^{-1}(x)]$

$= g\left(\frac{x+1}{4}\right)$

$= 2\left(\frac{x+1}{4}\right) - 3$

$= \frac{x+1}{2} - 3$

$= \frac{x+1-6}{2} = \frac{x-5}{2}$

$\therefore\ \frac{x-5}{2} = x + 1$ or, $2x + 2 = x - 5$ $\therefore\ x = -7$

Thus, the required value of x is – 7.

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20. **यदि $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ र $fg^{-1}(x) = f(x)$ भए x को मान पत्ता लगाउनुहोस् ।**

If $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ and $fg^{-1}(x) = f(x)$ then find the value of x. [2074 R']

⇨ Here, $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ and $fg^{-1}(x) = f(x)$

Let y = g(x) then,

$$y = \frac{3x-1}{2}$$

Interchanging x and y then,

$$x = \frac{3y-1}{2}$$

or, $2x = 3y - 1$

or, $3y = 2x + 1$

or, $y = \frac{2x+1}{3}$

$\therefore \quad g^{-1}(x) = \frac{2x+1}{3}$

Now, $fg^{-1}(x) = f(x)$

or, $f\left(\frac{2x+1}{3}\right) = 2x + 5$

or, $2\left(\frac{2x+1}{3}\right) + 5 = 2x + 5$

or, $\frac{4x+2}{3} + 5 = 2x + 5$

or, $\frac{4x+2}{3} = 2x$

or, $4x + 2 = 6x$

or, $2x = 2$

$\therefore \quad x = 1$

Thus, the value of x is 1.

21. **यदि $f = \{x, 5x - 13\}, g = \left\{x, \frac{2x+7}{3}\right\}$ र $g^{-1}(x) = f \circ f(x)$ भए x को मान पत्ता लगाउनुहोस् ।**

If $f = \{x, 5x - 13\}\}, g = \left\{x, \frac{2x+7}{3}\right\}$ र $g^{-1}(x) = f \circ f(x)$, find the value of x.

⇨ Here, $f = \{x, 5x - 13\}, g = \left\{x, \frac{2x+7}{3}\right\}$ and $g^{-1}(x) = f \circ f(x)$

Let, y = g(x)

or, $y = \frac{2x+7}{3}$

Interchanging the position of x and y then,

$$x = \frac{2y+7}{3}$$

or, $3x = 2y + 7$

or, $3x - 7 = 2y$

or, $y = \frac{3x-7}{2}$

$\therefore \quad g^{-1}(x) = \frac{3x-7}{2}$

Again,

$$\begin{aligned} f \circ f(x) &= f(f(x)) \\ &= f(5x - 13) \\ &= 5(5x - 13) - 13 \\ &= 25x - 65 - 13 \end{aligned}$$

$\therefore \quad f \circ f(x) = 25x - 78$

Now, $\frac{3x-7}{2} = 25x - 78$

or, $3x - 7 = 50x - 156$

or, $149 = 47x$

$\therefore \quad x = \frac{149}{47}$

Thus, the value of x is 3.17.

22. **यदि $2f(x) = kx - 3, \frac{1}{3}g(x) = \frac{1}{x+2}$ र $f \circ g^{-1}(3) = -\frac{1}{4}$ भए k को मान पत्ता लगाउनुहोस् ।**

If $2f(x) = kx - 3, \frac{1}{3}g(x) = \frac{1}{x+2}$ and $f \circ g^{-1}(3) = -\frac{1}{4}$, find the value of k. [2073 R']

⇨ Here, $2f(x) = kx - 3, \frac{1}{3}g(x) = \frac{1}{x+2}$ and $f \circ g^{-1}(3) = -\frac{1}{4}$

$\therefore \quad f(x) = \frac{kx-3}{2}, g(x) = \frac{3}{x+2}$

For $g^{-1}(x)$;

Let, y = g(x)

or, $y = \frac{3}{x+2}$

Interchanging the position of x and y, $x = \frac{3}{y+2}$

or, $xy + 2x = 3$

or, $xy = 3 - 2x$

or, $y = \frac{3-2x}{x}$

$\therefore \quad g^{-1}(x) = \frac{3-2x}{x}$

$\therefore \quad g^{-1}(3) = \frac{3-2\times3}{3} = \frac{3-6}{3} = -\frac{3}{3} = -1$

Now, $fg^{-1}(3) = -\frac{1}{4}$.

or, $f(g^{-1}(3)) = -\frac{1}{4}$

or, $f(-1) = -\frac{1}{4}$

or, $\frac{k(-1)-3}{2} = -\frac{1}{4}$

or, $-k - 3 = -\frac{1}{2}$

or, $2k + 6 = 1$

or, $2k = -5$

$\therefore \quad k = -\frac{5}{2}$

Thus, the value of k is $-\frac{5}{2}$.

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23. यदि $f(x) = 3x - 7$, र फलन $g(x) = \frac{5x+2}{3}$ दिइएको छ । यदि $g^{-1} f(x) = 8$ भए x को मान पत्ता लगाउनुहोस् ।

Given that the function $f(x) = 3x - 7$ and the function $g(x) = \frac{5x+2}{3}$. If $g^{-1} f(x) = 8$, find the value of x. **[2072 S]**

⇨ Here, $f(x) = 3x - 7$ and $g(x) = \frac{5x+2}{3}$

Let, $y = g(x) = \frac{5x+2}{3}$

Interchanging the position of x and y;

$x = \frac{5y+2}{3}$

or, $3x = 5y + 2$

or, $3x - 2 = 5y$

or, $y = \frac{3x-2}{5}$

$\therefore$ $g^{-1}(x) = \frac{3x-2}{5}$

Now,

$g^{-1}(f(x)) = g^{-1}(3x - 7)$

or, $8 = \frac{3(3x-7)-2}{5}$

or, $9x - 21 - 2 = 40$

or, $9x = 63$

$\therefore$ $x = 7$

Thus, the value of x is 7.

24. यदि $f(x) = 3x - 7$, $g(x) = \frac{4x-2}{3}$ र $f^{-1}(x) = g(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 3x - 7$, $g(x) = \frac{4x-2}{3}$ and $f^{-1}(x) = g(x)$, find the value of x. **[2071 R']**

⇨ Here, $f(x) = 3x - 7$ and $g(x) = \frac{4x-2}{3}$

Let $y = f(x)$

or, $y = 3x - 7$

Interchanging x and y then

$x = 3y - 7$

or, $x + 7 = 3y$

or, $y = \frac{x+7}{3}$

$\therefore$ $f^{-1}(x) = \frac{x+7}{3}$

We have $f^{-1}(x) = g(x)$

or, $\frac{x+7}{3} = \frac{4x-2}{3}$

or, $12x - 6 = 3x + 21$

or, $9x = 27$

$\therefore$ $x = 3$

Thus, the value of x is 3.

25. यदि $f(x) = 2x + 3$, $g(x) = \frac{x+5}{2}$ र $ff(x) = g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + 3$, $g(x) = \frac{x+5}{2}$ and $ff(x) = g^{-1}(x)$, find the value of x. **[2070 R']**

⇨ Here, $f(x) = 2x + 3$, $g(x) = \frac{x+5}{2}$ and $ff(x) = g^{-1}(x)$

So, $ff(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$

Again, let $y = g(x)$ or, $y = \frac{x+5}{2}$

Interchanging the position of x and y, $x = \frac{y+5}{2}$

or, $2x = y + 5$

or, $2x - 5 = y$

$\therefore$ $y = 2x - 5$

So, $g^{-1}(x) = 2x - 5$

Now, $ff(x) = g^{-1}(x)$

or, $4x + 9 = 2x - 5$

or, $2x = -14$

$\therefore$ $x = -7$

Thus, the value of x is -7.

26. यदि $f(x) = 12 - 3x$ र $f^{-1}(x) = 2$ भए x र $f \circ f^{-1}(P)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 12 - 3x$ and $f^{-1}(x) = 2$, find the value of x and $f \circ f^{-1}(P)$. **[2066 S]**

⇨ Here, $f(x) = 12 - 3x$

Let, $y = 12 - 3x$

Interchanging the position of x and y then,

i.e. $x = 12 - 3y$

or, $3y = 12 - x$

or, $y = \frac{12-x}{3}$

$\therefore$ $f^{-1}(x) = \frac{12-x}{3}$

We have, $f^{-1}(x) = 2$

or, $2 = \frac{12-x}{3}$

or, $12 - x = 6$

or, $12 - 6 = x$

$\therefore$ $x = 6$

Now, $f \circ f^{-1}(P) = f(f^{-1}(P)) = f\left(\frac{12-P}{3}\right) = 12 - 3\left(\frac{12-P}{3}\right) = 12 - 12 + P$

$\therefore$ $f \circ f^{-1}(P) = P$

Thus, $x = 6$ and $f \circ f^{-1}(P) = P$.

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27. **फलन $f(x) = \frac{x}{2x-3}$ र $f(x) = f^{-1}(x)$ भए x को मान निकाल्नुहोस् ।**

If $f(x) = \frac{x}{2x-3}$ and $f(x) = f^{-1}(x)$, find the value of x. **[2068 S, 2065 M]**

⇨ Here, $f(x) = \frac{x}{2x-3}$

Let $f(x) = y$

$\therefore\ y = \frac{x}{2x-3}$

Interchanging the position of x and y then,

i.e. $x = \frac{y}{2y-3}$

or, $2xy - 3x = y$

or, $2xy - y = 3x$

or, $y(2x - 1) = 3x$

or, $y = \frac{3x}{2x-1}$

$\therefore\ f^{-1}(x) = \frac{3x}{2x-1}$

Now, $f(x) = f^{-1}(x)$

or, $\frac{x}{2x-3} = \frac{3x}{2x-1}$

or, $2x^2 - x = 6x^2 - 9x$

or, $4x^2 - 8x = 0$

or, $4x(x- 2) = 0$

Either, $x = 0$

or, $x - 2 = 0 \Rightarrow x = 2$

Thus, the require value of x is 0 or 2.

28. **यदि $f(x) = 3x + 4$ र $g(x) = x + 1$ भए x को मान कति हुँदा $f^{-1}g(x) = 2$ हुन्छ ?**

If $f(x) = 3x + 4$ and $g(x) = x + 1$, for what value of x, $f^{-1}g(x) = 2$? **[2067 R]**

⇨ Here, $f(x) = 3x + 4$, $g(x) = x + 1$ and $f^{-1}g(x) = 2$

Now, $f(x) = 3x + 4$

or, $y = 3x + 4$

Interchanging the position of x and y then,

i.e. $x = 3y + 4$

or, $x - 4 = 3y$

or, $x - 4 = 3y$

or, $y = \frac{x-4}{3}$

$\therefore\ f^{-1}(x) = \frac{x-4}{3}$

Now, $f^{-1}g(x) = \frac{g(x)-4}{3}$

or, $2 = \frac{g(x)-4}{3}$

or, $6 + 4 = g(x)$

or, $10 = x + 1$

or, $x = 10 - 1$

$\therefore\ x = 9$

Thus, the required value of x is 9.

29. **यदि $f(x) = 4x + 7$, $g(x) = 3x - 5$ र $f \circ g^{-1}(x) = 15$ भए x को मान पत्ता लगाउनुहोस् ।**

If $f(x) = 4x + 7$, $g(x) = 3x - 5$ and $f \circ g^{-1}(x) = 15$, find the value of x. **[2061 S]**

⇨ Here, given function $f(x) = 4x + 7$ and $g(x) = 3x - 5$

Then to find $g^{-1}(x)$, from $g(x) = 3x - 5$

Let, $y = g(x) = 3x - 5$

or, $y = 3x - 5$

Interchanging the position of x and y then,

i.e. $x = 3y - 5$

or, $x + 5 = 3y$

or, $y = \frac{x+5}{3}$

$\therefore\ g^{-1}(x) = \frac{x+5}{3}$

$$\begin{aligned} \text{Now, } fg^{-1}(x) &= f(g^{-1}(x)) \\ &= f\left(\frac{x+5}{3}\right) \\ &= 4\left(\frac{x+5}{3}\right) + 7 \\ &= \frac{4x+20}{3} + 7 \\ &= \frac{4x+20+21}{3} \\ \therefore\ fg^{-1}(x) &= \frac{4x+41}{3} \end{aligned}$$

But by given,

$fg^{-1}(x) = 15$

So, $\frac{4x+41}{3} = 15$

or, $4x + 41 = 45$

or, $4x = 45 - 41$

or, $4x = 4$

$\therefore\ x = \frac{4}{4} = 1$

Thus, the required value of x is 1.

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30. यदि $f(x) = x^2 - 2x$, $g(x) = 2x + 3$ र $f \circ g^{-1}(x) = 3$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = x^2 - 2x$, $g(x) = 2x + 3$ and $f \circ g^{-1}(x) = 3$ then find the value of 'x'. **[2061 R, 2065 R, 2068 S']**

⇨ Here, given functions, $f(x) = x^2 - 2x$ and $g(x) = 2x + 3$

To find g^{-1},

Let $y = g(x) = 2x + 3$

Interchanging the position of x and y then,

i.e. $x = 2y + 3$

or, $x - 3 = 2y$

or, $y = \dfrac{x-3}{2}$

$\therefore \quad g^{-1}(x) = \dfrac{x-3}{2}$

But by questions, $fg^{-1}(x) = 3$

or, $f\left(\dfrac{x-3}{2}\right) = 3$

or, $\left(\dfrac{x-3}{2}\right)^2 - 2\left(\dfrac{x-3}{2}\right) = 3$

or, $\dfrac{x^2-6x+9}{4} - x + 3 = 3$

or, $\dfrac{x^2-6x+9}{4} - x = 3 - 3$

or, $\dfrac{x^2-6x+9-4x}{4} = 0$

or, $x^2 - 10x + 9 = 0$

or, $x^2 - x - 9x + 9 = 0$

or, $x(x-1) - 9(x-1) = 0$

or, $(x-1)(x-9) = 0$

Either, $x - 1 = 0$

or, $x - 9 = 0$

$\therefore \quad x = 1$ or, $x = 9$

Thus, required value of x is 1 or 9.

31. यदि $f(x) = 4x - 17$, $g(x) = \dfrac{2x+8}{5}$ र $f \circ f(x) = g^{-1}(x)$ भए, x को मान निकाल्नुहोस् ।

If $f(x) = 4x - 17$, $g(x) = \dfrac{2x+8}{5}$ and $f \circ f(x) = g^{-1}(x)$, find the value of x. **[2059 R]**

⇨ Here, given functions, $f(x) = 4x - 17$ and $g(x) = \dfrac{2x+8}{5}$

Now, $ff(x) = f(4x - 17)$

$= 4(4x - 17) - 17$

$= 16x - 68 - 17$

$= 16x - 85$

Again, let $y = g(x) = \dfrac{2x+8}{5}$ i.e. $y = \dfrac{2x+8}{5}$

Interchanging the position of x and y then,

or, $x = \dfrac{2y+8}{5}$

or, $5x = 2y + 8$

or, $5x - 8 = 2y$

or, $y = \dfrac{5x-8}{2}$

$\therefore \quad g^{-1}(x) = \dfrac{5x-8}{2}$

By given, $ff(x) = g^{-1}(x)$

or, $16x - 85 = \dfrac{5x-8}{2}$

or, $32x - 170 = 5x - 8$

or, $32x - 5x = 170 - 8$

or, $27x = 162$

$\therefore \quad x = \dfrac{162}{27} = 6$

Thus, required value of x is 6.

32. फलन $f(x) = \dfrac{x-2}{2x+1}$, $x \neq -\dfrac{1}{2}$ र $g(x) = \dfrac{1}{x}$, $x \neq 0$ दिइएका छन् । यदि $f^{-1}(x) = g \circ f(x)$ भए x को मानहरू पत्ता लगाउनुहोस् ।

Function $f(x) = \dfrac{x-2}{2x+1}$, $x \neq -\dfrac{1}{2}$ and $g(x) = \dfrac{1}{x}$, $x \neq 0$ are given. If $f^{-1}(x) = g \circ f(x)$, find the values of x. **[2062 S]**

⇨ Here, $f(x) = \dfrac{x-2}{2x+1}$, $g(x) = \dfrac{1}{x}$ and $f^{-1}(x) = gf(x)$

Let, $y = f(x)$

or, $y = \dfrac{x-2}{2x+1}$

Interchanging the position of x and y then,

$x = \dfrac{y-2}{2y+1}$

or, $2xy + x = y - 2$

or, $2xy - y = -x - 2$

or, $y(2x - 1) = -x - 2$

or, $y = \dfrac{-x-2}{2x-1}$

$\therefore \quad f^{-1}(x) = \dfrac{x+2}{1-2x}$

Now, $f^{-1}(x) = gf(x)$

or, $\dfrac{x+2}{1-2x} = g\left(\dfrac{x-2}{2x+1}\right)$

or, $\dfrac{x+2}{1-2x} = \dfrac{1}{\dfrac{x-2}{2x+1}}$

or, $\dfrac{x+2}{1-2x} = \dfrac{2x+1}{x-2}$

or, $x^2 - 4 = 1^2 - (2x)^2$

or, $x^2 - 4 = 1 - 4x^2$

or, $-4 - 1 = -4x^2 - x^2$

or, $-5 = -5x^2$

or, $x^2 = 1$

$\therefore \quad x = \pm 1$

Thus the required value of x is ± 1.

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33. यदि $f(x) = 3x - 7$, $g(x) = \frac{x+2}{5}$ र $g^{-1}\circ f(x) = f(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 3x - 7$, $g(x) = \frac{x+2}{5}$ and $g^{-1}\circ f(x) = f(x)$, find the value of x. **[2066 R]**

⇨ Here, $f(x) = 3x - 7$, $g(x) = \frac{x+2}{5}$ and $g^{-1}\circ f(x) = f(x)$.

Let $y = g(x)$

or, $y = \frac{x+2}{5}$

Interchanging the position of x and y then,

i.e. $x = \frac{y+2}{5}$

or, $5x = y + 2$

or, $5x - 2 = y$

$\therefore$ $g^{-1}(x) = 5x - 2$

Now, $g^{-1}\circ f(x) = f(x)$

or, $g^{-1}(f(x)) = f(x)$

or, $g^{-1}(3x - 7) = 3x - 7$

or, $5(3x - 7) - 2 = 3x - 7$

or, $15x - 35 - 2 = 3x - 7$

or, $12x = 37 - 7$

$\therefore$ $x = \frac{30}{12} = 2\frac{1}{2}$

Thus, the required value of x is $2\frac{1}{2}$.

34. यदि $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ र $g\circ f(4) = -8$ भए $f^{-1}(-2)$ र b का मानहरू निकाल्नुहोस् ।

If $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ and $g\circ f(4) = -8$, find the values of $f^{-1}(-2)$ & b. **[2060 C]**

⇨ Here, $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ and $gf(4) = -8$

We have, $g(f(4)) = -8$

or, $g\left(\frac{4}{2-4}\right) = -8$

or, $g\left(\frac{4}{-2}\right) = -8$

or, $g(-2) = -8$

or, $b(-2) - 2 = -8$

or, $-2b - 2 = -8$

or, $-2b = -6$

$\therefore$ $b = 3$

Again, let $y = f(x)$

or, $y = \frac{x}{2-x}$

Interchanging the position of x and y then, $x = \frac{y}{2-y}$

or, $2x - xy = y$

or, $2x = y + xy$

or, $y(1 + x) = 2x$

or, $y = \frac{2x}{1+x}$ $\therefore$ $f^{-1}(x) = \frac{2x}{1+x}$

So, $f^{-1}(-2) = \frac{2(-2)}{1-2} = \frac{-4}{-1}$

Thus, the value of $f^{-1}(-2)$ is 4 and 'b' is 3.

35. फलन $f(x) = \frac{3x+11}{x-3}$, $x \neq 3$ र $g(x) = \frac{x-3}{2}$ दिइएका छन् । $f^{-1}(x)$ पत्ता लगाउनुहोस् र $f(x) = g^{-1}(x)$ भए x का मानहरू पनि पत्ता लगाउनुहोस् ।

Functions $f(x) = \frac{3x+11}{x-3}$, $x \neq 3$ and $g(x) = \frac{x-3}{2}$ are given. Find $f^{-1}(x)$. If $f(x) = g^{-1}(x)$, find the values of x.

⇨ Here, given function, $f(x) = \frac{3x+11}{x-3}$, $x \neq 3$ and $g(x) = \frac{x-3}{2}$

Let $y = f(x) = \frac{3x+11}{x-3}$

Interchanging the position of x and y then,

i.e. $x = \frac{3y+11}{y-3}$

or, $xy - 3x = 3y + 11$

or, $xy - 3y = 3x + 11$

or, $y(x - 3) = 3x + 11$

or, $y = \frac{3x+11}{x-3}$

$\therefore$ $f^{-1}(x) = \frac{3x+11}{x-3}$, $x \neq 3$

Again, let $y = g(x) = \frac{x-3}{2}$

Interchanging the position of x and y then, i.e. $x = \frac{y-3}{2}$

or, $2x = y - 3$

or, $2x + 3 = y$

or, $y = 2x + 3$

$\therefore$ $g^{-1}(x) = 2x + 3$

But, by given $f(x) = g^{-1}(x)$

or, $\frac{3x+11}{x-3} = 2x + 3$

or, $3x + 11 = (x - 3)(2x + 3)$

or, $3x + 11 = 2x^2 + 3x - 6x - 9$

or, $2x^2 - 3x + 3x - 6x - 9 - 11 = 0$

or, $2x^2 - 6x - 20 = 0$

or, $2(x^2 - 3x - 10) = 0$

or, $x^2 - 3x - 10 = 0$

or, $x^2 + 2x - 5x - 10 = 0$

or, $x(x + 2) - 5(x + 2) = 0$

or, $(x + 2)(x - 5) = 0$

Either, $x + 2 = 0$

or, $x - 5 = 0$

$\therefore$ $x = -2, 5$

Thus, the value of x is -2 or 5.

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36. **फलन $f(x) = 3x + a$ दिइएको छ । यदि $f \circ f(6) = 10$ भए a र $f^{-1}(4)$ को मान निकाल्नुहोस् ।**
Given that function $f(x) = 3x + a$. If $f \circ f(6) = 10$, find the value of a and $f^{-1}(4)$. **[2058 S]**

⇨ Here, given function $f(x) = 3x + a$

$$\begin{aligned} \text{Now, } ff(x) &= f(f(x)) \\ &= f(3x + a) \\ &= 3\ (3x + a) + a \\ &= 9x + 3a + a \\ &= 9x + 4a \end{aligned}$$

But, $ff(6) = 10$
i.e. $10 = 9 \times 6 + 4a$
or, $10 = 54 + 4a$
or, $4a = 10 - 54$
$\therefore$ $a = -11$

Again putting the value of a in $f(x) = 3x + a$,
we get, $f(x) = 3x - 11$

Again, to find $f^{-1}(x)$
Let, $y = f(x) = 3x - 11$
Interchanging the position of x and y then,
i.e. $x = 3y - 11$
or, $x + 11 = 3y$
or, $y = \dfrac{x + 11}{3}$
$\therefore$ $f^{-1}(x) = \dfrac{x + 11}{3}$

Now, $f^{-1}(4) = \dfrac{4 + 11}{3} = \dfrac{15}{3} = 5$

Thus the value of a is -11 and $f^{-1}(4)$ is 5.

37. **यदि $f(x) = 2x + k$ र $f \circ f(4) = 10$ भए k को मान र $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् ।**
If $f(x) = 2x + k$ and $f \circ f(4) = 10$, find the k and find the value of $f^{-1}(4)$.

⇨ Here, $f(x) = 2x + k$ and $f \circ f(4) = 10$

So, $f(f(4)) = 10$
or, $f(2 \times 4 + k) = 10$
or, $f(8 + k) = 10$
or, $2(8 + k) + k = 10$
or, $16 + 2k + k = 10$
or, $3k = -6$
$\therefore$ $k = -2$

Now, $f(x) = 2x - 2$
Let, $y = 2x - 2$
Interchanging the position of x and y then,
i.e. $x = 2y - 2$
or, $\dfrac{x + 2}{2} = y$ $\qquad \therefore$ $f^{-1}(x) = \dfrac{x + 2}{2}$

Again, $f^{-1}(4) = \dfrac{4 + 2}{2} = 3$

Thus, the value of k is -2 and $f^{-1}(4)$ is 3.

38. **यदि $f(x) = 2x + k$ र $f^{-1}(4) + f(2) = 7$ भए k को मान पत्ता लगाउनुहोस् ।**
If $f(x) = 2x + k$ and $f^{-1}(4) + f(2) = 7$, find the value of 'k'.

⇨ Here, $f(x) = 2x + k$

Let, $y = f(x)$ $\qquad$ i.e. $y = 2x + k$
Interchanging the position of x and y then,
i.e. $x = 2y + k$
or, $x - k = 2y$
or, $y = \dfrac{x - k}{2}$
$\therefore$ $f^{-1}(x) = \dfrac{x - k}{2}$

Now, $f^{-1}(x) = \dfrac{x - k}{2}$ gives $f^{-1}(4) = \dfrac{4 - k}{2}$

And, $f(2) = 2 \times 2 + k = 4 + k$
From question, $f^{-1}(4) + f(2) = 7$
or, $\dfrac{4 - k}{2} + 4 + k = 7$
or, $\dfrac{4 - k}{2} + k = 3$
or, $\dfrac{4 - k + 2k}{2} = 3$
or, $4 + k = 6$
$\therefore$ $k = 2$

Thus, the required value of k is 2.

MODEL 4

39. **दुई फलनहरू $f(x) = \dfrac{2x + 5}{8}$ र $g(x) = 3x - 4$ छन् । यदि $(f \circ g)^{-1}(x)$ एउटा एकात्मक फलन हो भने x को मान पत्ता लगाउनुहोस् ।**

Two functions are $f(x) = \dfrac{2x + 5}{8}$ and $g(x) = 3x - 4$. If $(f \circ g)^{-1}(x)$ is an identity function, find the value of x.

⇨ Here, $f(x) = \dfrac{2x + 5}{8}$ and $g(x) = 3x - 4$

$$\begin{aligned} f \circ g(x) = f\big(g(x)\big) &= f(3x - 4) \\ &= \frac{2\ (3x - 4) + 5}{8} = \frac{6x - 8 + 5}{8} \\ \therefore \quad f \circ g\ (x) &= \frac{6x - 3}{8} \end{aligned}$$

For $(f \circ g)^{-1}$, let $y = f \circ g(x)$
or, $y = \dfrac{6x - 3}{8}$

Interchanging the position of x and y then, $x = \dfrac{6y - 3}{8}$

or, $8x = 6y - 3$
or, $8x + 3 = 6y$
$\therefore$ $y = \dfrac{8x + 3}{6}$

i.e. $(f \circ g)^{-1}\ (x) = \dfrac{8x + 3}{6}$

By the question; fog^{-1} is an identity function.
So, $\dfrac{8x + 3}{6} = x$
or, $8x + 3 = 6x$
or, $2x = -3$
$\therefore$ $x = -\dfrac{3}{2}$

Thus, the value of x is $-\dfrac{3}{2}$.

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40. यदि फलन $f(x) = \frac{2x+1}{4}$ भए $f^{-1}\circ f(x)$ एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् र $f^{-1}(2)$ को मान पनि पत्ता लगाउनुहोस् ।

If function $f(x) = \frac{2x+1}{4}$, prove that $f^{-1}\circ f(x)$ is an identity function and find the value of $f^{-1}(2)$. [2065 R']

⇨ Here, $f(x) = \frac{2x+1}{4}$

Let, $y = f(x)$

or, $y = \frac{2x+1}{4}$

Interchanging the position of x and y then,

i.e. $4x = 2y + 1$

or, $4x - 1 = 2y$

or, $y = \frac{4x-1}{2}$

$\therefore f^{-1}(x) = \frac{4x-1}{2}$

Now, $f^{-1}\circ f(x) = f^{-1}\left(\frac{2x+1}{4}\right)$

$= \dfrac{4.\frac{2x+1}{4} - 1}{2}$

$= \frac{2x+1-1}{2}$

$= x$

Hence, $f^{-1}\circ f(x)$ is an identity function.

Again, $f^{-1}(2) = \frac{4\times 2-1}{2} = \frac{7}{2}$

Thus, required value of $f f^{-1}(2)$ is $\frac{7}{2}$.

MODEL 5

41. यदि $g(x) = 2x - 3$ र $f\circ g(x) = 6x - 11$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् ।

If $g(x) = 2x - 3$ and $f\circ g(x) = 6x - 11$, then find $f^{-1}(x)$.

[2075 R, 2075 R$_2$]

⇨ Here, $g(x) = 2x - 3$ and $f\circ g(x) = 6x - 11$

Let $f(x) = mx + c$ then,

$f\circ g(x) = 6x - 11$

or, $f[g(x)] = 6x - 11$

or, $f(2x - 3) = 6x - 11$

or, $m(2x - 3) + c = 6x - 11$

or, $2mx - 3m + c = 6x - 11$

Comparing the coefficient of x and constants,

$2m = 6$

$\therefore m = 3$

and $-3m + c = -11$

or, $-3\times 3 + c = -11$

$\therefore c = -2$

So, $f(x) = mx + c = 3x - 2$ i.e $y = 3x - 2$

Interchanging the position of x and y, we get,

$x = 3y - 2$

or, $x + 2 = 3y$

$\therefore y = f^{-1}(x) = \frac{x+2}{3}$

Thus, $f^{-1}(x) = \frac{x+2}{3}$.

42. फलनहरू $f(x) = 3x + 5$ र $f\circ g(x) = 12x - 7$ दिइएका छन् । यदि $g^{-1}(x) = 2$ भए x को मान निकाल्नुहोस् ।

It is given that the functions $f(x) = 3x + 5$ and $f\circ g(x) = 12x - 7$. If $g^{-1}(x) = 2$, find the value of x. [2073 S']

⇨ Here, $f(x) = 3x + 5$, $g(x) = 12x - 7$ and $g^{-1}(x) = 2$

We have, again, $f\circ g(x) = 12x - 7$

or, $f(g(x)) = 12x - 7$

or, $3 . g(x) + 5 = 12x - 7$

or, $3g(x) = 12x - 12$

$\therefore g(x) = 4x - 4$

For inverse; let $y = g(x)$

or, $y = 4x - 4$

Interchanging the position of x and y then,

$x = 4y - 4$

or, $x + 4 = 4y$

$\therefore y = \frac{x+4}{4}$

i.e. $g^{-1}(x) = \frac{x+4}{4}$

or, $2 = \frac{x+4}{4}$

or, $x + 4 = 8$

$\therefore x = 4$

Thus, the value of x is 4.

43. यदि $f(x) = 3x - 1$ र $f\circ g(x) = 6x + 5$ भए $(g\circ f)^{-1}$ पत्ता लगाउनुहोस् । **(If $f(x) = 3x - 1$ and $f\circ g(x) = 6x + 5$, then find $(g\circ f)^{-1}$.))** [2071 R]

⇨ Here, $f(x) = 3x - 1$ and $f\circ g(x) = 6x + 5$

or, $f(g(x)) = 6x + 5$

or, $3g(x) - 1 = 6x + 5$

or, $3g(x) = 6x + 6$

$\therefore g(x) = 2x + 2$

Now, $g\circ f(x) = g(f(x))$

$= g(3x - 1)$

$= 2(3x - 1) + 2$

$= 6x - 2 + 2$

$\therefore g\circ f = 6x$

Let $y = 6x$

Interchanging x and y then,

$x = 6y$

$\therefore y = \frac{x}{6}$

Thus, $(g\circ f)^{-1} = \frac{x}{6}$

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44. यदि $f(x) = 3x - 4$ र $f^{-1}g(x) = \frac{3x+2}{3}$ भए $g(x)$ र $f^{-1}\left(\frac{1}{2}\right)$ निकाल्नुहोस् ।

If $f(x) = 3x - 4$ and $f^{-1}g(x) = \frac{3x+2}{3}$, find g(x) and $f^{-1}\left(\frac{1}{2}\right)$.

[2070 R]

⇨ Here, $f(x) = 3x - 4$ and $f^{-1}g(x) = \frac{3x+2}{3}$

Let $y = f(x)$

or, $y = 3x - 4$

Interchanging the position of x and y then,

$x = 3y - 4$

or, $x + 4 = 3y$

$\therefore\ y = \frac{x+4}{3}$

i.e. $f^{-1}(x) = \frac{x+4}{3}$

By the question,

$f^{-1}g(x) = \frac{3x+2}{3}$

So, $f^{-1}(g(x)) = \frac{3x+2}{3}$

or, $\frac{g(x)+4}{3} = \frac{3x+2}{3}$

or, $g(x) + 4 = 3x + 2$

$\therefore\ g(x) = 3x - 2$

Now, $f^{-1}\left(\frac{1}{2}\right) = \frac{\frac{1}{2}+4}{3} = \frac{\frac{9}{2}}{\frac{3}{1}} = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$

Thus, $g(x) = 3x - 2$ and $f^{-1}\left(\frac{1}{2}\right) = \frac{3}{2}$.

45. यदि $f(x) = 2x - 7$, र $f\circ g(x) = 4x + 3$ भए $(g\circ f)^{-1}(x)$ पत्ता लगाउनुहोस् ।

If $f(x) = 2x - 7$ and $f\circ g(x) = 4x + 3$, find $(g\circ f)^{-1}(x)$. **[2071 S]**

⇨ Here, $f(x) = 2x - 7$ and $f\circ g(x) = 4x + 3$

So, $f\circ g(x) = 4x + 3$

or, $f(g(x)) = 4x + 3$

or, $2.g(x) - 7 = 4x + 3$

or, $2g(x) = 4x + 10$

$\therefore\ g(x) = 2x + 5$

We have, $g\circ f(x) = g(f(x))$

$= g(2x - 7)$

$= 2(2x - 7) + 5$

$= 4x - 14 + 5$

$\therefore\ g\circ f(x) = 4x - 9$

Let $y = 4x - 9$

Interchanging x and y then,

$x = 4y - 9$

or, $x + 9 = 4y$

$\therefore\ y = \frac{x+9}{4}$

Thus, $(g\circ f)^{-1}(x) = \frac{x+9}{4}$.

QUESTIONS FROM CDC TEXTBOOK

1.1.1 बीजीय फलन (ALGEBRAIC FUNCTION)

EXERCISE 1.1.1

1. **तल दिइएका फलनहरूको उदाहरणसहित परिभाषा लेख्नुहोस् । (Write the definitions of following functions with examples.)**

(a) बीजीय फलन (Algebraic Function)

⇨ Here, the function which describes correspondence beetwen two variables *x* and *y* which are obtained by the finite rules is called an algebraic function. For example: $y = x + 2$

(b) रेखीय फलन (Linear Function)

⇨ Here, a function is called linear if it can be defined by an equation of the form $f(x) = mx + c$.

(c) वर्गघातीय फलन (Quadratic Function)

⇨ Here, a function of the form $f(x) = ax^2 + bx + c;\ a \neq 0$ is called a quadratic function. e.g. $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ etc. are quadratic functions.

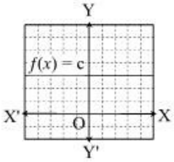
(d) घनघातीय फलन (Cubic Function)

⇨ Here, a function of the form $f(x) = ax^3 + bx^2 + cx + d;\ a \neq 0$ is called a cubic function. e.g. $y = x^3$, $y = 2x^3 + 5$ etc. are cubic functions.

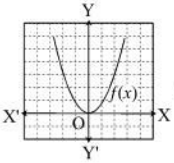
2. **तल दिइएका लेखाचित्रहरू रेखीय, वर्ग र घन घातीयमध्ये कुन प्रकारका फलनहरू हुन्, लेख्नुहोस् ।**

Which types of the functions linear, quadratic and cubic functions are given in the graph? Write it .

(a)



(b)



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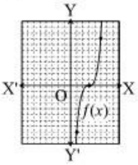
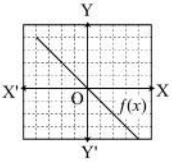
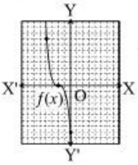


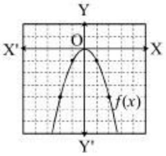
(d)



(e)



(f)



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⇨ Here,

(a) constant function (b) quadratic function (c) cubic function

(d) linear function (e) cubic function (d) quadratic function

3. **दुई ओटा नगरपालिकाहरूमा एक हप्तामा खपत हुने गुणस्तरीय खाना (quality food) को परिमाण मेट्रिक टन (*x*) र मूल्य रु. हजारमा (*y*) दिइएको छ ।**

दुवै तथ्याङ्कहरूलाई लेखाचित्रमा प्रस्तुत गर्नुहोस् र कुन रेखीय फलन हो ? पत्ता लगाउनुहोस् ।

The consumption of the quantities in metric ton (x) and price in Rs thousands (y) of the quality food for a week in two municipalities are given.

Represent both the data in a graph and which is linear function? Find it.

(a) नगरपालिका 'क' (Municipality 'a')

x	10	20	30	40	50	60	70
y	50	100	150	200	250	300	350

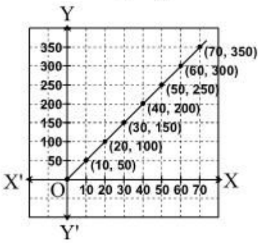
(b) नगरपालिका 'ख' (Municipality 'b')

x	10	20	30	40	50	60	70
y	10	30	70	130	210	310	430

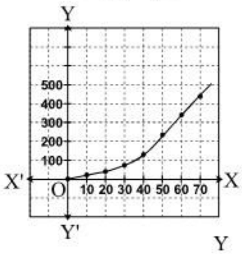
⇨ Here,

The graphical representation is shown alongside:

Municipality 'a'



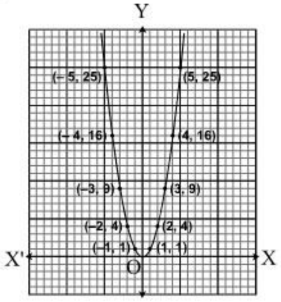
Municipality 'b'



From the graph, the graph (a) represents the linear function.

4. **(a) – 5 देखि 5 सम्मका पूर्णाङ्कहरूलाई *x* र तिनीहरूको वर्गलाई *y* अथवा *f*(*x*) मानी फलन *f*(*x*) = x^2 लाई लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

Taking the integers from – 5 to 5 as x and their squares as y or *f*(*x*), represent the function *f*(*x*) = x^2 in a graph.



⇨ Here, $f(x) = x^2$

x	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4	5
y = f(x)	25	16	9	4	1	0	1	4	9	16	25

The graphical representation is shown alongside:

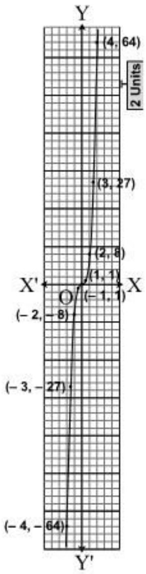
(b) – 4 देखि 4 सम्मका पूर्णाङ्कहरूलाई *x* र तिनीहरूका घन (cube) लाई *y* अथवा *g* (*x*) मानी फलन *g* (*x*) = x^3 लाई लेखाचित्रमा देखाउनुहोस् ।

Taking the integers from – 4 to 4 as *x* and their cubes as *y* or *g*(*x*), represent the function *g*(*x*) = x^3 in a graph.

⇨ Here, $g(x) = x^3$

x	– 4	– 3	– 2	– 1	0	1	2	3	4
y = g(x)	– 64	– 27	– 8	– 1	0	1	8	27	64

The graphical representation is shown alongside:



5. **आफ्नो शरीरको तापक्रम लगातार एक हप्तासम्म एउटै समयमा नाप्नुहोस् । दिनलाई *x*-अक्षमा र तापक्रमलाई *y*-अक्षमा देखाई प्राप्त विवरणलाई लेखाचित्रमा प्रस्तुत गरी कस्तो लेखाचित्र बन्छ ? छलफल गर्नुहोस् ।**

Measure the temperature of your body continuously for a week at the same interval of time. Taking days on x-axis and temperature on y-axis, present the data on graph and discuss the nature of graph.

⇨ Show to your teacher.

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1.1.2 त्रिकोणमितीय फलन (TRIGONOMETRIC FUNCTION)

EXERCISE 1.1.2

1. **तल दिइएका फलनहरूको विस्तार क्षेत्र लेख्नुहोस् (Write the range of following functions) :**

(a) $f(x) = \sin x$

⇨ Here, the range of $f(x) = \sin x$ is all real numbers between – 1 and 1 inclusive.

(b) $f(x) = \cos x$

⇨ Here, the range of $f(x) = \cos x$ is all real numbers between – 1 and 1 inclusive.

(c) $f(x) = \tan x$

⇨ Here, the range of $f(x) = \tan x$ is all the real numbers.

2. **तल दिइएका फलनहरूको पिरियड (period) लेख्नुहोस् । (Write the period of following functions.)**

(a) $y = \sin x$ ⇨ Here, the period of $y = \sin x$ is $2\pi^c$.

(b) $y = \cos x$ ⇨ Here, the period of $y = \cos x$ is $2\pi^c$.

(c) $y = \tan x$ ⇨ Here, the period of $y = \tan x$ is π^c.

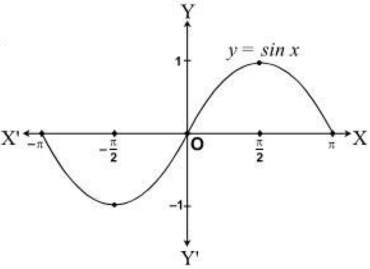
3. **लेखाचित्रमा देखाउनुहोस् । (Show in the graph.)**

(a) $f(x) = \sin x\ (-\pi \le x \le \pi)$

⇨ Here, $y = f(x) = \sin x\ (-\pi^c \le x \le \pi^c)$

x	$-\pi^c = -180°$	$-\dfrac{\pi^c}{2} = -90°$	0	$\dfrac{\pi^c}{2} = 90°$	$\pi^c = 180°$
$y = f(x)$	0	– 1	0	1	0

The graph is shown alongside:

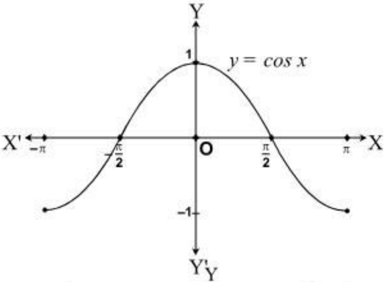


(b) $f(x) = \cos x\ (-\pi \le x \le \pi)$

⇨ Here, $y = f(x) = \cos x\ (-\pi^c \le x \le \pi^c)$

x	$-\pi^c = -180°$	$-\dfrac{\pi^c}{2} = -90°$	0	$\dfrac{\pi^c}{2} = 90°$	$\pi^c = 180°$
$y = f(x)$	– 1	0	1	0	– 1

The graph is shown alongside:

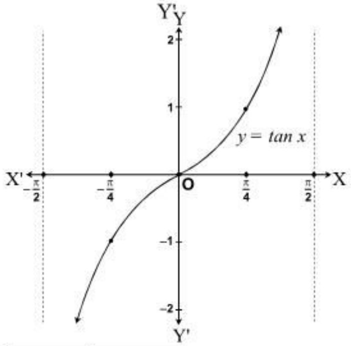


(c) $f(x) = \tan x\ \left(-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}\right)$

⇨ Here, $y = f(x) = \tan x\ \left(-\dfrac{\pi^c}{2} \le x \le \dfrac{\pi^c}{2}\right)$

x	$-\dfrac{\pi^c}{2} = -90°$	$-\dfrac{\pi^c}{4} = -45°$	0	$\dfrac{\pi^c}{4} = 45°$	$\dfrac{\pi^c}{2} = 90°$
$y = f(x)$	∞	– 1	0	1	∞

The graph is shown alongside:

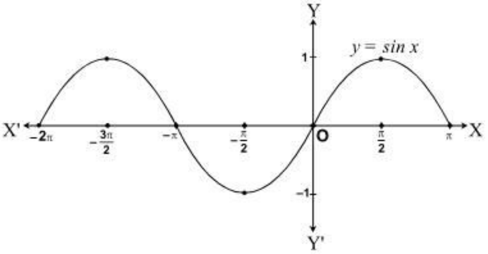


(d) $g(x) = \sin x\ (-2\pi \le x \le \pi)$

⇨ Here, $y = g(x) = \sin x\ (-2\pi^c \le x \le \pi^c)$

x	$-2\pi^c = -360°$	$-\dfrac{3\pi^c}{2} = -270°$	$-\pi^c = -180°$	$-\dfrac{\pi^c}{2} = -90°$	0°	$\dfrac{\pi^c}{2} = 90°$	$\pi^c = 180°$
$y = g(x)$	0	1	0	– 1	0	1	0

The graph is shown alongside:



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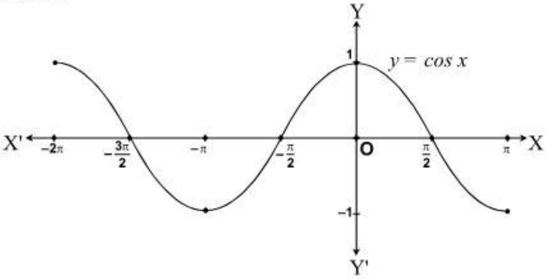
(e) **$g(x) = \cos x$ $(-2\pi \le x \le \pi)$**

⇨ Here, $y = g(x) = \cos x$ $(-2\pi^c \le x \le \pi^c)$

x	$-2\pi^c = -360°$	$-\dfrac{3\pi^c}{2} = -270°$	$-\pi^c = -180°$	$-\dfrac{\pi^c}{2} = -90°$	0°	$\dfrac{\pi^c}{2} = 90°$	$\pi^c = 180°$
$y = g(x)$	1	0	−1	0	1	0	−1

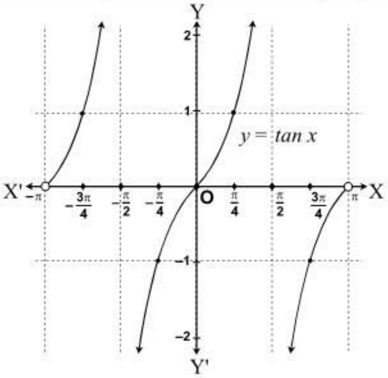
The graph is shown alongside:



(f) **$g(x) = \tan x$ $(-\pi < x < \pi)$**

⇨ Here, $y = g(x) = \tan x$ $(-\pi^c < x < \pi^c)$

x	$-\pi^c = -180°$	$-\dfrac{3\pi^c}{4} = -135°$	$-\dfrac{\pi^c}{2} = -90°$	$-\dfrac{\pi^c}{4} = -45°$	0	$\dfrac{\pi^c}{4} = 45°$	$\dfrac{\pi^c}{2} = 90°$	$\dfrac{3\pi^c}{4} = 135°$	$\pi^c = 180°$
$g(x)$	0	1	∞	−1	0	1	∞	−1	0



4. **दैनिक जीवनमा $y = \sin x$ र $y = \cos x$ को लेखाचित्र कहाँ कहाँ प्रयोग भएको हुन्छ ? खोजी गरी प्रतिवेदन तयार गरी कक्षाकोठामा प्रस्तुत गर्नुहोस् ।**

Where is the use of graph of $y = \sin x$ and $y = \cos x$ in the daily life? Investigate and make a report in the classroom.

⇨ Here, the similar nature of graph of $y = \sin x$ and $\cos x$ can be seen in the following:

(i) When a roll of wire is expanded.
(ii) When heart beat is shown in the screen.
(iii) When bouncing a ball on the ground etc.

1.1.3 संयुक्त फलन (COMPOSITE FUNCTIONS)

EXERCISE 1.1.3

1. **(a)** **संयुक्त फलनको परिभाषा लेख्नुहोस् । (Define composite function.)**

⇨ Here, if $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions then the new function defined from A to C is called the composite function of f and g. It is denoted by $g\circ f$.

(b) **$(f\circ g)(x)$ फलनको क्षेत्र f र g मध्ये कुन फलनको क्षेत्रसँग बराबर हुन्छ ?**

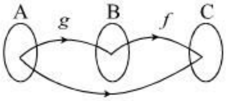
Which domain of the functions between f and g is equal to the domain of the function $f\circ g(x)$?

⇨ Here, the domain of g is equal to the domain of $f\circ g(x)$.

(c) **चित्रमा $g: A \longrightarrow B$ र $f: B \longrightarrow C$ छ । (In figure, $g: A \longrightarrow B$ and $f: B \longrightarrow C$).**

A बाट C सम्म परिभाषित फलनको नाम के हो ? लेख्नुहोस् ।

What is the name of function defined from A to C? Write.



⇨ Here, the function defined from A to C is $f\circ g$.

(d) **यदि $f = \{(1, 3), (2, 1), (3, 2)\}$ र $g = \{(1, 2), (2, 3), (3, 1)\}$ भए फलन $(g\circ f)$ को क्षेत्र कति हुन्छ ?**

If $f = \{(1, 3), (2, 1), (3, 2)\}$ and $g = \{(1, 2), (2, 3), (3, 1)\}$ then what is the domain of function $(g\circ f)$?

⇨ Here, $g\circ f = \{(1, 1), (2, 2), (3, 3)\}$ $\quad\therefore$ Domain of $g\circ f = \{1, 2, 3\}$.

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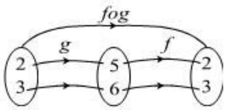
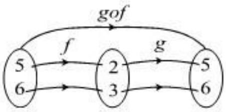
2. (a) **यदि $f = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$ भए $(f∘g)$ र $(g∘f)$ लाई क्रमजोडाका रूपमा लेख्नुहोस् ।**
If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ then write $(f∘g)$ and $(g∘f)$ in ordered pair form.

⇨ Here, $f∘g = \{(2, 5), (5, 2), (1, 5)\}$ and $(g∘f) = \{(1, 3), (3, 1), (4, 3)\}$.

(b) **यदि $f = \{(5, 2), (6, 3)\}$ र $g = \{(2, 5), (3, 6)\}$ भए $(f∘g)$ र $(g∘f)$ लाई मिलान चित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।**
If $f = \{(5, 2), (6, 3)\}$ and $g = \{(2, 5), (3, 6)\}$ then show $(f∘g)$ and $(g∘f)$ in an arrow diagram and write in ordered pair form.

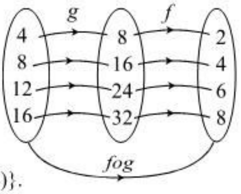
⇨ Here, the mapping diagrams are as follows,





From the mapping diagrams: $f∘g = \{(2, 2), (3, 3)\}$, $g∘f = \{(5, 5), (6, 6)\}$.

(c) **यदि $(f∘g) = \{(4, 2), (8, 4), (12, 6), (16, 8)\}$ र $g = \{(4, 8), (8, 16), (12, 24), (16, 32)\}$ भए $(f∘g)$ लाई मिलान चित्रमा देखाउनुहोस् । f लाई क्रमजोडाहरूका रूपमा लेख्नुहोस् ।**
If $(f∘g) = \{(4, 2), (8, 4), (12, 6), (16, 8)\}$ and $g = \{(4, 8), (8, 16), (12, 24), (16, 32)\}$ then show $(f∘g)$ in an arrow diagram. Write f in ordered pair form.



⇨ Here, the mapping diagram of $f∘g$ is shown alongside
From the mapping diagram, the ordered pairs of $f = \{(8, 2), (16, 4), (24, 6), (32, 8)\}$.

3. (a) **यदि $f : R \to R : f(x) = x + 2$ र $g : R \to R : g(x) = 4 - x^2$ भए $(f∘g)(x)$ र $(g∘f)(x)$ पत्ता लगाउनुहोस् ।**
If $f : R \to R : f(x) = x + 2$ and $g : R \to R : g(x) = 4 - x^2$ then find $(f∘g)(x)$ and $(g∘f)(x)$.

⇨ Here, $f(x) = x + 2$ and $g(x) = 4 - x^2$
Then, $f∘g(x) = f(4 - x^2) = 4 - x^2 + 2 = 6 - x^2$
$g∘f(x) = g(x + 2) = 4 - (x + 2)^2$
$= 4 - (x^2 + 4x + 4)$
$= 4 - x^2 - 4x - 4$
$\therefore\ g∘f(x) = - x^2 - 4x$
Thus, $f∘g(x) = 6 - x^2$ and $g∘f(x) = - x^2 - 4x$.

(b) **यदि $g : R \to R : g(x) = 2 + 3x$ र $h : R \to R : h(x) = x^2$ भए $(g∘h)(x)$ र $(h∘g)(x)$ पत्ता लगाउनुहोस् ।**
If $g : R \to R : g(x) = 2 + 3x$ and $h : R \to R : h(x) = x^2$ then find $(g∘h)(x)$ and $(h∘g)(x)$.

⇨ Here, $g(x) = 2 + 3x$ and $h(x) = x^2$
So, $(g∘h)(x) = g(h(x) = g(x^2) = 2 + 3 \times x^2$
$\therefore\ (g∘h)(x) = 2 + 3x^2$
And, $h∘g(x) = h(g(x))$
$= h(2 + 3x) = (2 + 3x)^2$
$\therefore\ h∘g(x) = 4 + 12x + 9x^2$
Thus, $(g∘h)(x) = 2 + 3x^2$ and $h∘g(x) = 4 + 12x + 9x^2$.

(c) **R ले वास्तविक सङ्ख्याहरूको समूहलाई जनाउँछ र फलनहरू $f: R \to R : f(x) = 5x - 3$ र $g : R \to R : g(x) = 2x + 5$ परिभाषित छन् । $(f∘g)(x)$ र $(g∘f)(x)$ के एक अर्कासँग बराबर हुन्छन् ? पत्ता लगाउनुहोस् ।**
R represents the set of real numbers and function $f: R \to R : f(x) = 5x - 3$ and $g : R \to R : g(x) = 2x + 5$ are defined. Are the functions $(f∘g)(x)$ and $(g∘f)(x)$ equal to each other ? Find.

⇨ Here, $f(x) = 5x - 3$ and $g(x) = 2x + 5$
So, $(f∘g)(x) = f(g(x))$
$= f(2x + 5)$
$= 5(2x + 5) - 3$
$= 10x + 25 - 3$
$\therefore\ (f∘g)(x) = 10x + 22$

And, $(g∘f)(x) = g(f(x))$
$= g(5x - 3)$
$= 2(5x - 3) + 5$
$= 10x - 6 + 5$
$\therefore\ (g∘f)(x) = 10x - 1$

Thus, $(f∘g)(x) = 10x + 22$ and $(g∘f)(x) = 10x - 1$ shows that the functions are not equal to each other.

4. (a) **यदि $f : R \to R : f(x) = 2x + 3$ र $g : R \to R : g(x) = 2x - 1$ भए $(f∘g)(5)$ र $(g∘f)(- 2)$ को मान पत्ता लगाउनुहोस् ।**
If $f : R \to R : f(x) = 2x + 3$ and $g : R \to R : g(x) = 2x - 1$ then find the value of $(f∘g)(5)$ and $(g∘f)(- 2)$.

⇨ Here, $f(x) = 2x + 3$ and $g(x) = 2x - 1$
So, $f∘g(5) = f(g(5))$
$= f(2 \times 5 - 1) = f(9) = 2 \times 9 + 3$
$\therefore\ f∘g(5) = 21$
And, $g∘f(- 2) = g(f(- 2))$
$= g[2 \times (- 2) + 3]$
$= g(- 4 + 3)$
$= g(- 1) = 2 \times (- 1) - 1 = - 2 - 1$
$\therefore\ g∘f(- 2) = - 3$
Thus, the values are $f∘g(5) = 21$ and $g∘f(- 2) = - 3$.

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(b) **यदि $f: N \to R : f(x) = 4x - 2$ र $g : R \to R : g(x) = x^2$ भए $(f \circ g)$ (1) र $(g \circ f)$ (4) को मान पत्ता लगाउनुहोस् ।**
If $f: N \to R : f(x) = 4x - 2$ and $g : R \to R : g(x) = x^2$ then find the value of $(f \circ g)$ (1) and $(g \circ f)$ (4).

⇨ Here, $f(x) = 4x - 2$ and $g(x) = x^2$

So, $(f \circ g)(1) = f(g(1))$
$= f(1^2)$
$= f(1)$
$= 4 \times 1 - 2$
$\therefore \quad (f \circ g)(1) = 2$

And, $(g \circ f)(4) = g(f(4))$
$= g(4 \times 4 - 2)$
$= g(14)$
$= 14^2$
$\therefore \quad (g \circ f)(4) = 196$

Thus, the values are $(f \circ g)(1) = 2$ and $(g \circ f)(4) = 196$.

(c) **यदि $f(x) = 3 - 4x, x \in R$ र $g(x) = 2x + 3, x \in R$ भए $(g \circ f)$ (1) र $(f \circ g)$ (4) को मान पत्ता लगाउनुहोस् ।**
If $f(x) = 3 - 4x, x \in R$ and $g(x) = 2x + 3, x \in R$ then find the value of $(g \circ f)$ (1)and $(f \circ g)$ (4).

⇨ Here, $f(x) = 3 - 4x$ and $g(x) = 2x + 3$

So, $(g \circ f)(1) = g(f(1))$
$= g(3 - 4 \times 1)$
$= g(-1)$
$= 2 \times (-1) + 3$
$= -2 + 3$
$\therefore \quad (g \circ f)(1) = 1$

And, $(f \circ g)(4) = f(g(4))$
$= f(2 \times 4 + 3)$
$= f(11)$
$= 3 - 4 \times 11$
$= 3 - 44$
$\therefore \quad (f \circ g)(4) = -41$

Thus, the values are $(g \circ f)(1) = 1$ and $(f \circ g)(4) = -41$.

5. (a) **यदि $f: R \to R : f(x) = \frac{1}{2}(x - 3)$ र $(f \circ g) : R \to R : (f \circ g(x)) = x$ भए $g(x)$ पत्ता लगाउनुहोस् ।**

If $f: R \to R : f(x) = \frac{1}{2}(x - 3)$ and $(f \circ g) : R \to R : (f \circ g(x)) = x$ then find $g(x)$.

⇨ Here, $f(x) = \frac{1}{2}(x - 3)$ and $(f \circ g)(x) = x$

So, $(f \circ g)(x) = x$
or, $f(g(x)) = x$
or, $\frac{1}{2}[g(x) - 3] = x$
or, $g(x) - 3 = 2x$
$\therefore \quad g(x) = 2x + 3$

(b) **यदि $g : R \to R : g(x) = 4 - x$ र $(f \circ g)(x) = 11 - 2x$ भए $f(x)$ पत्ता लगाउनुहोस् ।**
If $g : R \to R : g(x) = 4 - x$ and $(f \circ g)(x) = 11 - 2x$ then find $f(x)$.

⇨ Here, $g(x) = 4 - x$ and $(f \circ g)(x) = 11 - 2x$
So, $(f \circ g)(x) = 11 - 2x$
or, $f(g(x)) = 11 - 2x$
or, $f(4 - x) = 11 - 2x$
Let, $4 - x = a$ then $x = 4 - a$
So, $f(a) = 11 - 2(4 - a)$
or, $f(a) = 11 - 8 + 2a$
or, $f(a) = 3 + 2a$
$\therefore \quad f(x) = 3 + 2x = 2x + 3$.

(c) **यदि $f : N \to R : f(x) = x^2$ र $f \circ g : R \to R\ (f \circ g)(x) = x^2 - 2x + 1$ भए $g(x)$ पत्ता लगाउनुहोस् ।**
If $f : N \to R : f(x) = x^2$ and $f \circ g : R \to R\ (f \circ g)(x) = x^2 - 2x + 1$ then find $g(x)$.

⇨ Here, $f(x) = x^2$ and $(f \circ g)(x) = x^2 - 2x + 1$
So, $(f \circ g)(x) = x^2 - 2x + 1$
or, $f(g(x)) = x^2 - 2x + 1$
or, $[g(x)]^2 = (x - 1)^2$
$\therefore \quad g(x) = x - 1$

6. (a) **यदि $h(x) = \frac{1}{(x + 3)^3}$ $(x \neq -3)$ र $h(x) = (f \circ g)(x)$ भए फलन $f(x)$ र $g(x)$ का सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।**

If $h(x) = \frac{1}{(x + 3)^3}$ $(x \neq -3)$ and $h(x) = (f \circ g)(x)$ then find the possible formulae of $f(x)$ and $g(x)$.

⇨ Here, $h(x) = \frac{1}{(x + 3)^3}$ and $h(x) = f \circ g(x)$

So, $f \circ g(x) = \frac{1}{(x + 3)^3}$

$\therefore$ Outside function $= f(x) = \frac{1}{x^3}$

$\therefore$ Inside function $= g(x) = (x + 3)$

Thus, the possible formulae are; $f(x) = \frac{1}{x^3}$ and $g(x) = x + 3$.

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(b) यदि $h(x) = (2x - 3)^5$, $h(x) = (f \circ g)(x)$ भए $f(x)$ र $g(x)$ का कुनै दुई ओटा सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।
If $h(x) = (2x - 3)^5$, $h(x) = (f \circ g)(x)$ then find the possible formulae of $f(x)$ and $g(x)$.

⇨ Here, $h(x) = (2x - 3)^5$ and $h(x) = (f \circ g)(x)$
So, $(f \circ g)(x) = (2x - 3)^5$
$\therefore$ Outside function $= f(x) = x^5$
$\therefore$ Inside function $= g(x) = (2x - 3)$
Thus, the possible formulae are; $f(x) = x^5$ and $g(x) = (2x - 3)$.

7. (a) यदि $f(x) = \dfrac{6}{x-2}$ $(x \neq 2, x \in R)$, $g(x) = ax^2 - 1$ र $(g \circ f)(5) = 7$ भए a को मान पत्ता लगाउनुहोस् ।
If $f(x) = \dfrac{6}{x-2}$ $(x \neq 2, x \in R)$, $g(x) = ax^2 - 1$ and $(g \circ f)(5) = 7$ then find the value of a.

⇨ Here, $f(x) = \dfrac{6}{x-2}$ and $g(x) = ax^2 - 1$
So, $(g \circ f)(5) = 7$
or, $g(f(5)) = 7$
or, $g\left(\dfrac{6}{5-2}\right) = 7$
or, $g(2) = 7$
or, $a \times (2)^2 - 1 = 7$
or, $a \times 4 = 8$ $\qquad \therefore a = 2$
Thus, the value of a is 2.

(b) यदि $f: R \to R: f(x) = (ax + 5)$, $g(x) = 8x + 13$ र $(g \circ f)(5) = 93$ भए a को मान पत्ता लगाउनुहोस् ।
If $f: R \to R: f(x) = (ax + 5)$, $g(x) = 8x + 13$ and $(g \circ f)(5) = 93$ then find the value of a.

⇨ Here, $f(x) = (ax + 5)$, $g(x) = 8x + 13$ and $g \circ f(5) = 93$.
So, $(g \circ f)(5) = 93$
or, $g(f(5)) = 93$
or, $g(a \times 5 + 5) = 93$
or, $g(5a + 5) = 93$
or, $8(5a + 5) + 13 = 93$
or, $8(5a + 5) = 80$
or, $5a + 5 = 10$
or, $5a = 5$ $\qquad \therefore a = 1$
Thus, the value of a is 1.

8. एउटा रेफ्रिजेरेटरमा राखिएको खानामा ब्याक्टेरियाहरूको सङ्ख्या, $N(T) = 20T^2 - 80T + 500$ $(2 \leq T \leq 14)$ को रूपमा व्यक्त गर्न सकिन्छ । जहाँ, T ले खानाको तापक्रमलाई जनाउँछ र $T(t) = 4t + 2$ $(0 \leq t \leq 3)$; जहाँ t ले घण्टामा हुने समयलाई जनाउँछ भने
The number of bacteria in a food kept in a refrigerator can be expressed as $N(T) = 20T^2 - 80T + 500$ $(2 \leq T \leq 14)$. Where, T represents the temperature of food and $T(t) = 4t + 2$ $(0 \leq t \leq 3)$; where t represents time in hours.

(a) $(N \circ T)(t)$ पत्ता लगाउनुहोस् । (Find $(N \circ T)(t)$.)

⇨ Here,
$$\begin{aligned}(N \circ T)(t) &= N[T(t)]\\ &= N(4t + 2)\\ &= 20(4t + 2)^2 - 80(4t + 2) + 500\\ &= 20(16t^2 + 16t + 4) - 320t - 160 + 500\\ &= 320t^2 + 320t + 80 - 320t - 160 + 500\\ &= 320t^2 + 420\end{aligned}$$
$\therefore (N \circ T)(t) = 320t^2 + 420$

(b) फ्रिजमा राखेको 2 घण्टामा उक्त खानामा कति ब्याक्टेरिया हुन्छन् ? पत्ता लगाउनुहोस् ।
How many bacteria will be there in the food when it is kept in refrigerator for two hours? Find it.

⇨ Here, we have, $(N \circ T)(t) = 320t^2 + 420$
So, when time (t) = 2 hours then
No. of bacteria $(N \circ T)(t) = 320 \times 2^2 + 420 = 1280 + 420 = 1700$
Thus, no. of bacteria in 2 hours = 1700

(c) खानामा कति घण्टामा 3300 ओटा ब्याक्टेरिया हुन्छन् ? पत्ता लगाउनुहोस् ।
In how many hours will the no. of bacteria be 3300 in food? find it.

⇨ Here, no. of bacteria (N) = 3300, time (t) = ?
We have, $N = 320t^2 + 420$
or, $3300 = 320\,t^2 + 420$
or, $320t^2 = 3300 - 420 = 2880$
or, $t^2 = \dfrac{2880}{320} = 9$
$\therefore t = 3$ $[\because 0 \leq t \leq 3]$
Thus, required time is 3 hours.

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1.1.4 विपरीत फलन (INVERSE OF A FUNCTION)

EXERCISE 1.1.4

1. **परिभाषा लेख्नुहोस् (Define) :**

(a) एक एक फलन (One to one Function)

⇨ Here, Let $f : A \to B$ be a function. If each elements of B has only one pre-image in A then f is called one to one function.

(b) सम्पूर्ण फलन (Onto Function)

⇨ Here, in a function f, if the co-domain of f is equal to range of f then f is called an onto function.

(c) विपरीत फलन (Inverse of Function)

⇨ Here, if f is a one to one onto function from A to B then the inverse function f^{-1} is defined as a function from B to A. For example: If $f = \{(1, 3), (2, 4), (3, 5)\}$, $f^{-1} = \{(3, 1), (4, 2), (5, 3)\}$.

2. **(a) फलन $f: A \to B$ को कुन अवस्थामा विपरीत फलन f^{-1} परिभाषित हुन्छ ? लेख्नुहोस् ।**
In what condition, the inverse f^{-1} of $f: A \to B$ is defined? Write it.

⇨ Here, if f is one to one onto function then the inverse of f is defined.

(b) $f : R \to R$ र $g : R \to R$ एक अर्काका विपरीत फलनहरू हुन् । प्रत्येक $x \in R$ का लागि $(f \circ g)(x)$ र $(g \circ f)(x)$ कति हुन्छ ? लेख्नुहोस् ।
$f : R \to R$ and $g : R \to R$ are inverse to each other. For each $x \in R$, what are the values of $(f \circ g)(x)$ and $(g \circ f)(x)$? Write it.

⇨ Here, if f and g are the inverse to each other then,
$f \circ g(x) = x$ and $g \circ f(x) = x$.

3. **तल दिइएका फलनहरू एक एक सम्पूर्ण फलनहरू हुन् । प्रत्येकको विपरीत फलन लेख्नुहोस् ।**
Given functions are one to one onto functions. Write the inverse of each function.

(a) $f = \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)\}$ ⇨ Here, $f^{-1} = \{(1, 1), (4, 2), (9, 3), (16, 4), (25, 5)\}$

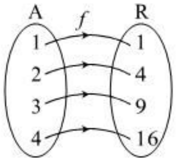
(b) $g = \{(1, 4), (3, 6), (4, 7), (5, 8)\}$ ⇨ Here, $g^{-1} = \{(4, 1), (6, 3), (7, 4), (8, 5)\}$

(c) $h = \{(8, 2), (27, 3), (64, 4), (125, 5), (216, 6)\}$ ⇨ Here, $h^{-1} = \{(2, 8), (3, 27), (4, 64), (5, 125), (6, 216)\}$

4. **तल दिइएका फलनहरू एक एक सम्पूर्ण फलनहरू हुन्/होइनन् पत्ता लगाउनुहोस् । एक एक सम्पूर्ण फलन भएमा तिनीहरूको विपरीत फलन पनि पत्ता लगाउनुहोस् ।**
Find whether the following functions are one to one onto functions or not. If they are one to one onto then find their inverse function.

(a) $f : \{1, 2, 3, 4\} \to R : f(x) = x^2$

⇨ Here, $f : \{1, 2, 3, 4\} \to R : f(x) = x^2$
When $x = 1$ then $y = f(1) = 1^2 = 1$; (1, 1)
When $x = 2$ then $y = f(2) = 2^2 = 4$; (2, 4)
When $x = 3$ then $y = f(3) = 3^2 = 9$; (3, 9)
When $x = 4$ then $y = f(4) = 4^2 = 16$; (4, 16)
Now, the mapping diagram;



It shows that f is one to one onto function because the real number other than {1, 4, 9, 16} do not have the pre-image.

(b) $f : N \to N : f(x) = 3x + 1$

⇨ Here, $f : N \to N : f(x) = 3x + 1$
Let, $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$
So, $f(x_1) = 3x_1 + 1$ and $f(x_2) = 3x_2 + 1$
i.e. $3x_1 + 1 = 3x_2 + 1$
or, $3x_1 = 3x_2$
$\therefore \quad x_1 = x_2$
It shows that if two images are equal then their pre-images are also equal. So, the function is one to one onto function.
Now, let $y = f(x)$
or, $y = 3x + 1$
Interchanging the position of x and y then,
$x = 3y + 1$
or, $x - 1 = 3y$
$\therefore \quad y = \frac{x-1}{3}$
i.e. $f^{-1}(x) = \frac{x-1}{3}$

Thus, the function f is one to one onto and $f^{-1}(x) = \frac{x-1}{3}$.

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(c) $g : R \to R : g(x) = \frac{x-1}{4}$

⇨ Here, $g : R \to R : g(x) = \frac{x-1}{4}$

Let, $x_1, x_2 \in R$ such that $g(x_1) = g(x_2)$

So, $g(x_1) = \frac{x_1 - 1}{4}$ and $g(x_2) = \frac{x_2 - 1}{4}$

$\therefore \quad \frac{x_1 - 1}{4} = \frac{x_2 - 1}{4}$

or, $x_1 - 1 = x_2 - 1$

$\therefore \quad x_1 = x_2$

It shows that if two images are equal then their pre images are also equal. So the function is one to one onto function.

Now, let $y = g(x)$

or, $y = \frac{x-1}{4}$

Interchanging the role of x and y then,

$x = \frac{y-1}{4}$

or, $4x = y - 1$

or, $4x + 1 = y$

$\therefore \quad g^{-1}(x) = 4x + 1$

Thus, the function is one to one onto and $g^{-1}(x) = 4x + 1$.

(d) **h : Q → Q : h(x) = 5 + 2x**

⇨ Here, $h : Q \to Q : h(x) = 5 + 2x$

Let, $x_1, x_2 \in Q$ such that $h(x_1) = h(x_2)$

So, $h(x_1) = 5 + 2x_1$ and $h(x_2) = 5 + 2x_2$

$\therefore \quad 5 + 2x_1 = 5 + 2x_2$

or, $2x_1 = 2x_2$

$\therefore \quad x_1 = x_2$

It shows that if two images are equal then their pre-images are also equal.

So, the function is one to one onto function.

Now, let $y = h(x)$

or, $y = 5 + 2x$

Interchanging the position of x and y then,

$x = 5 + 2y$

or, $x - 5 = 2y$

$\therefore \quad y = \frac{x-5}{2}$

i.e $h^{-1}(x) = \frac{x-5}{2}$

Thus, the function is one to one onto and $h^{-1}(x) = \frac{x-5}{2}$.

(e) **k : R → R : k(x) = x^2**

⇨ Here, $k : R \to R : k(x) = x^2$

Let, $x_1, x_2 \in R$ such that $k(x_1) = k(x_2)$

So, $k(x_1) = x_1^2$ and $k(x_2) = x_2^2$ i.e. $x_1^2 = x_2^2$

The above equation has $x_1 = 1$ and $x_2 = -1$ also a solution. In this case $x_1 \neq x_2$ with $k(x_1) = k(x_2)$.

It shows that if two images are equal their pre-images may not be equal.

So, the function is not one to one onto function and thus, it's inverse donot exist.

5. (a) **यदि एउटा एक एक सम्पूर्ण फलन, $f(x) = 3x - 5$ भए $f^{-1}(4)$ पत्ता लगाउनुहोस् ।**

If $f(x) = 3x - 5$ is a one to one onto function then find $f^{-1}(4)$.

⇨ Here, $f(x) = 3x - 5$

or, $y = 3x - 5$

Interchanging the role of x and y then, $x = 3y - 5$

or, $x + 5 = 3y$

$\therefore \quad y = \frac{x+5}{3}$

i.e. $f^{-1}(x) = \frac{x+5}{3}$

So, $f^{-1}(4) = \frac{4+5}{3} = \frac{9}{3} = 3$

Thus, the value of $f^{-1}(4)$ is 3.

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(b) **यदि एउटा एक एक सम्पूर्ण फलन, $g(x) = 4x - 3$ भए $g^{-1}(7)$ पत्ता लगाउनुहोस् ।**

If $g(x) = 4x - 3$ is a one to one onto function then find $g^{-1}(7)$.

⇨ Here, $g(x) = 4x - 3$

or, $y = 4x - 3$

Interchanging the role of x and y then,

$x = 4y - 3$

or, $x + 3 = 4y$

$\therefore \quad y = \frac{x+3}{4}$

i.e. $g^{-1}(x) = \frac{x+3}{4}$

So, $g^{-1}(7) = \frac{7+3}{4} = \frac{10}{4} = \frac{5}{2}$

Thus, the value of $g^{-1}(7)$ is $\frac{5}{2}$.

(c) **यदि एउटा एक एक सम्पूर्ण फलन, $h(x) = \frac{2x-3}{4}$ भए $h^{-1}(2)$ पत्ता लगाउनुहोस् ।**

If $h(x) = \frac{2x-3}{4}$ is a one to one onto function then find $h^{-1}(2)$.

⇨ Here, $h(x) = \frac{2x-3}{4}$

or, $y = \frac{2x-3}{4}$

Interchanging the role of x and y then,

$x = \frac{2y-3}{4}$

or, $4x = 2y - 3$

or, $4x + 3 = 2y$

$\therefore \quad y = \frac{4x+3}{2}$

i.e. $h^{-1}(x) = \frac{4x+3}{2}$

Now, $h^{-1}(2) = \frac{4 \times 2 + 3}{2} = \frac{11}{2}$

Thus, the value of $h^{-1}(2)$ is $\frac{11}{2}$.

6. **तल दिइएका फलनहरू एक एक सम्पूर्ण फलनहरू हुन् भने दिइएको समीकरण हल गरी x को मान पत्ता लगाउनुहोस् ।**

If the following functions are one to one onto function then find the value of x by solving the given equation.

(a) **$f(x) = 2x - 7$; $g(x) = \frac{x+2}{3}$ and $(f \circ g)(x) = g^{-1}(x)$**

⇨ Here, $f(x) = 2x - 7$; $g(x) = \frac{x+2}{3}$ and $(f \circ g)(x) = g^{-1}(x)$

$$\begin{aligned}
\text{So, } f(g(x)) &= f\left(\frac{x+2}{3}\right) \\
&= 2\left(\frac{x+2}{3}\right) - 7 \\
&= \frac{2x+4}{3} - \frac{7}{1} \\
&= \frac{2x+4-21}{3} \\
\therefore \quad f(g(x)) &= \frac{2x-17}{3} \quad \text{........(i)}
\end{aligned}$$

Again, let $y = g(x)$

or, $y = \frac{x+2}{3}$

Interchanging the role of x and y then,

$x = \frac{y+2}{3}$

or, $3x = y + 2$

or, $3x - 2 = y$

$\therefore \quad g^{-1}(x) = 3x - 2$ ......(ii)

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By the question and from (i) and (ii);

$f(g(x)) = g^{-1}(x)$

or, $\frac{2x-17}{3} = 3x-2$

or, $2x - 17 = 9x - 6$

or, $-11 = 7x$

$\therefore$ $x = -\frac{11}{7}$

Thus, the value of x is $-\frac{11}{7}$

(b) $f(x) = 2x + 7$; $g(x) = x^2 - 2x$ and $(g\circ f^{-1})(x) = 3$

⇨ Here, $f(x) = 2x + 7$; $g(x) = x^2 - 2x$ and $(g\circ f^{-1})(x) = 3$

For $f^{-1}(x)$,

let $y = f(x)$

or, $y = 2x + 7$

Interchanging the role of x and y then,

$x = 2y + 7$

or, $\frac{x-7}{2} = y$

$\therefore$ $f^{-1}(x) = \frac{x-7}{2}$

Now, $(g\circ f^{-1})(x) = 3$

or, $g(f^{-1}(x)) = 3$

or, $g\left(\frac{x-7}{2}\right) = 3$

or, $\left(\frac{x-7}{2}\right)^2 - 2\left(\frac{x-7}{2}\right) = 3$

or, $\frac{x^2-14x+49}{4} - (x-7) = 3$

or, $\frac{x^2-14x+49}{4} = 3 + x - 7$

or, $\frac{x^2-14x+49}{4} = x - 4$

or, $x^2 - 14x + 49 = 4x - 16$

or, $x^2 - 18x + 65 = 0$

or, $x^2 - 13x - 5x + 65 = 0$

or, $(x - 13)(x - 5) = 0$

Either, $x - 13 = 0$ $\quad\therefore$ $x = 13$

or, $x - 5 = 0$ $\quad\therefore$ $x = 5$

Thus, the values of x are 5 or 13.

(c) $f(x) = x + 5$; $g(x) = \frac{x-2}{3}$ and $(f\circ f)(x) = g^{-1}(x)$

⇨ Here, $f(x) = x + 5$; $g(x) = \frac{x-2}{3}$ and $(f\circ f)(x) = g^{-1}(x)$

For $g^{-1}(x)$,

let $y = g(x)$

or, $y = \frac{x-2}{3}$

Interchanging the role of x and y then,

$x = \frac{y-2}{3}$

or, $3x = y - 2$

or, $3x + 2 = y$

$\therefore$ $g^{-1}(x) = 3x + 2$

Now, $(f\circ f)(x) = g^{-1}(x)$

or, $f(f(x)) = 3x + 2$

or, $f(x + 5) = 3x + 2$

or, $(x + 5) + 5 = 3x + 2$

or, $x + 10 = 3x + 2$

or, $10 - 2 = 3x - x$

or, $2x = 8$

$\therefore$ $x = 4$

Thus, the value of x is 4.

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7. (a) यदि फलन $f: R \rightarrow R: f(x) = 4x - 3$ र $g: R \rightarrow R: g(x) = \frac{x+2}{5}$ भए $(f^{-1} \circ g^{-1})$ (2) को मान पत्ता लगाउनुहोस् ।

If the function $f: R \rightarrow R: f(x) = 4x - 3$ and $g: R \rightarrow R: g(x) = \frac{x+2}{5}$ then find the value of $(f^{-1} \circ g^{-1})$ (2).

⇨ Here, $f(x) = 4x - 3$ and $g(x) = \frac{x+2}{5}$

For $f^{-1}(x)$;
Let $y = f(x)$
or, $y = 4x - 3$
Interchanging the role of x and y then,
$x = 4y - 3$
or, $x + 3 = 4y$
$\therefore\ y = \frac{x+3}{4}$
i.e. $f^{-1}(x) = \frac{x+3}{4}$
Again, for $g^{-1}(x)$;
Let $y = g(x)$
or, $y = \frac{x+2}{5}$
Interchanging the role of x and y then,
$x = \frac{y+2}{5}$
or, $5x = y + 2$
or, $5x - 2 = y$
$\therefore\ g^{-1}(x) = 5x - 2$

Now, $(f^{-1} \circ g^{-1})(2) = f^{-1}(g^{-1}(2))$
$= f^{-1}(5 \times 2 - 2)$
$= f^{-1}(10 - 2)$
$= f^{-1}(8)$
$= \frac{8+3}{4}$
$\therefore\ (f^{-1} \circ g^{-1})(2) = \frac{11}{4}$

Thus, the value of $(f^{-1} \circ g^{-1})(2)$ is $\frac{11}{4}$

(b) यदि $f: R \rightarrow R: f(x) = \frac{x+2}{3}$ र $g: R \rightarrow R: g(x) = x - 5$ भए $(f^{-1} \circ g^{-1}(1))$ को मान पत्ता लगाउनुहोस् ।

If $f: R \rightarrow R: f(x) = \frac{x+2}{3}$ and $g: R \rightarrow R: g(x) = x - 5$ then find the value of $(f^{-1} \circ g^{-1}(1))$.

⇨ Here, $f(x) = \frac{x+2}{3}$ and $g(x) = x - 5$

For $f^{-1}(x)$,
Let $y = f(x)$
or, $y = \frac{x+2}{3}$
Interchanging the role of x and y then,
$x = \frac{y+2}{3}$
or, $3x = y + 2$
or, $3x - 2 = y$
$\therefore\ f^{-1}(x) = 3x - 2$

Again, for $g^{-1}(x)$,
Let, $y = g(x)$
or, $y = x - 5$
Interchanging the role of x and y then,
$x = y - 5$
$\therefore\ y = x + 5$
i.e. $g^{-1}(x) = x + 5$
Now, $(f^{-1} \circ g^{-1}(1)) = f^{-1}(g^{-1}(1))$
$= f^{-1}(1 + 5)$
$= f^{-1}(6)$
$= 3 \times 6 - 2$
$[\because f^{-1}(x) = 3x - 2]$
$= 18 - 2 = 16$
Thus, the value of $(f^{-1} \circ g^{-1}(1)) = 16$.

8. सावालाई 'x', र साधरण ब्याजको मिश्रधनलाई $f(x)$ मानी आफ्नो घर नजिकको बैङ्क अथवा वित्तीय संस्थाले दिने ब्याजदरमा 5 वर्षका लागि मिश्रधन पत्ता लगाउने फलन $f(x)$ खोजी गर्नुहोस् । $(f \circ f^{-1}(x))$ र $f^{-1}(400)$ को मान पनि पत्ता लगाउनुहोस् ।
Investigate the function $f(x)$ to find the amount based on simple interest given by a bank or financial institution close to your house of principal 'x' for 5 years. Also, find the value of $(f \circ f^{-1}(x))$ and $f^{-1}(400)$.

⇨ Show to your teacher.

9. आफ्नो कक्षा अथवा अगिल्लो कक्षाको विज्ञान विषयको अध्ययन गरी तापक्रम मापनमा डिग्री सेल्सियस र डिग्री फरेनहाइटको सम्बन्ध दर्साउने फलनको खोजी गर्नुहोस् । उक्त फलनको विपरीत फलन पनि पत्ता लगाउनुहोस् ।
Investigate the function to denote the relation between degree celcius and degree fahrenheit studing the science text book of your class or previous class. Find the inverse function of the function.

⇨ Show to your teacher.

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2. बहुपदीय
Polynomial

KEY POINTS

➢ **शेषसाध्य (Remainder Theorem):**

यदि p(x) लाई (x – a) ले भाग गरियो भने शेष = p(a) हुन्छ । (If p(x) is divided by (x – a), the remainder = p(a).)

भाजक (Divisor)	शेष (Remainder)
x + a *i.e.* x – (– a)	p(– a)
ax – b *i.e.* $a\left(x-\frac{b}{a}\right)$	$p\left(\frac{b}{a}\right)$
ax + b *i.e.* $a\left(x+\frac{b}{a}\right)$	$p\left(-\frac{b}{a}\right)$
b – ax *i.e.* $-a\left(x-\frac{b}{a}\right)$	$p\left(\frac{b}{a}\right)$

➢ **गुणनखण्ड साध्य (Factor Theorem)**

(i) यदि p(x) को एउटा गुणनखण्ड (x – a) भए p(a) = 0 हुन्छ । (If (x – a) is a factor of p(x) then p(a) = 0.)

(ii) यदि p(a) = 0 भए p(x) को एउटा गुणनखण्ड (x – a) हुन्छ । (If p(a) = 0 then (x – a) is a factor of p(x).)

QUESTIONS FROM SEE EXERCISE 2

A. VERY SHORT QUESTIONS

1. बहुपदीयको परिभाषा दिनुहोस् । (Define the Polynomial.)

⇨ Here, a polynomial is a rational expression each of whose terms consists a constant multiplied by a positive power of a variable. Its standard form is : $p(x) = a_0 + a_1x^1 + a_2x^2 + \ldots\ldots + a_nx^n$, where a_0, a_1, a_2 ……. , a_n are constants and x is variable, 1, 2, 3, ….., n are the powers of variable.

2. सङ्क्षिप्त भाग विधिको परिभाषा दिनुहोस् । (Define the synthetic Division method.)

⇨ Here, a simplified method for dividing a polynomial of degree > 1 by another polynomial of the first degree by writing down only the coefficients of the several powers of the variable and changing the sign of the constant term, is known as synthetic division method.

3. भागको सिद्धान्तको परिभाषा दिनुहोस् । (Define the division Algorithm.)

⇨ Here, if a polynomial P(x) is divided by D(x) so that the quotient is Q(x) and remainder R then, P(x) = D(x) × Q(x) + R.

4. बहुपदीय समीकरणको परिभाषा दिनुहोस् । (Define the polynomial equation.)

⇨ Here, a polynomial equation is an equation that has multiple terms made up of numbers and variables.

5. बहुपदीयको मूल वा शुन्यको परिभाषा दिनुहोस् । (Define the root or zero of a polynomial.)

⇨ Here, the roots or zeros of a polynomial are those values of the variable that cause the polynomial to evaluate to zero.

6. यदि $f(x)$ को डिग्री '4m' र $g(x)$ को डिग्री 'm' भए $f(x) \div g(x)$ को डिग्री कति होला ?
If the degree of $f(x)$ is '4m' and degree of $g(x)$ is 'm,' what is the degree of $f(x) \div g(x)$?

⇨ Here, degree of $f(x) \div g(x)$ = degree of $f(x)$ – degree of $g(x)$ = 4 m – m = 3 m

7. यदि $f(x)$ को डिग्री 'm' र $g(x)$ को डिग्री 'n', $m > n$ भए $f(x) \div g(x)$ को डिग्री कति होला ?
If the degree of $f(x)$ is 'm' and degree of $g(x)$ is 'n', $m > n$ then what is the degree of $f(x) \div g(x)$?

⇨ Here, degree of $f(x) \div g(x)$ = degree of $f(x)$ – degree of $g(x)$ = m – n.

8. यदि $f(x)$ र $g(x)$ दुईओटा बहुपदीयहरू भए कस्तो अवस्थामा $f(x) \div g(x)$ अपरिभाषीत हुन्छ ?
If $f(x)$ and $g(x)$ be the two polynomials, under what condition $f(x) \div g(x)$ will be undefined?

⇨ Here, if $g(x) = 0$ then $f(x) \div g(x)$ will be undefined.

9. यदि भागफल $q(x)$, शेष $r(x)$, र भाजक $d(x)$ भए बहुपदीय $f(x)$ लाई $q(x)$, $r(x)$ र $d(x)$ को रूपमा लेख्नुहोस् ।
If the quotient $q(x)$, remainder $r(x)$, and divisor $d(x)$ then write the polynomial $f(x)$ in terms of $q(x)$, $r(x)$ and $d(x)$.

⇨ Here, $f(x) = d(x) \times q(x) + r(x)$.

10. यदि भागफल Q, शेष R, र भाजक D भए बहुपदीय $f(x)$ लाई Q, R र D को रूपमा लेख्नुहोस् ।
If the quotient Q, remainder R, and divisor D then write the polynomial $f(x)$ in terms of Q, R and D.

⇨ Here, $f(x)$ = D × Q + R.

11. $f(x) = q(x) \times d(x) + r(x)$ मा हरेक पदको अर्थ लेख्नुहोस् । (In $f(x) = q(x) \times d(x) + r(x)$, write the meaning of each term.)

⇨ Here, $f(x)$ = dividend, $q(x)$ = quotient, $d(x)$ = divisor and $r(x)$ = remainder.

12. सङ्क्षिप्त भाग विधिमा भाज्य र भागफलको डिग्रीहरूको अन्तर कति हुनुपर्छ ?
What should be the difference of degree of dividend and quotient in the Synthetic division method ?

⇨ Here, the difference of the degree of dividend and quotient in the synthetic division method is 1.

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13. सङ्क्षिप्त भाग विधिमा भाजक र भागफलको डिग्रीको योगफल कति हुन्छ ?
What should be the sum of degree of divisor and quotient in the Synthetic division method ?

⇨ Here, the sum of degree of divisor and quotient in synthetic division method is equal to the degree of dividend.

14. सङ्क्षिप्त भाग विधिमा भाजक (divisor) को डिग्री कति हुन्छ ?
What is the degree of divisor in synthetic division method?

⇨ Here, degree of divisor in synthetic division method is 1.

15. शेष साध्य उल्लेख गर्नुहोस् । (State Remainder Theorem.)

⇨ Here, if $p(x)$ is a polynomial of degree n and $(x - a)$ is a divisor of $p(x)$ then $p(a)$ is remainder, where the degree of quotient will be $(n - 1)$.

16. यदि एउटा बहुपदीय $f(x)$ लाई $(x - a)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(x - a)$, what will be its remainder?

⇨ Here, if $f(x)$ is divided by $(x - a)$ then the remainder $= f(a)$.

17. यदि एउटा बहुपदीय $f(x)$ लाई $(x + b)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(x + b)$, what will be its remainder?

⇨ Here, if $f(x)$ is divided by $(x + b)$ then the remainder $= f(-b)$.

18. यदि एउटा बहुपदीय $q(x)$ लाई $(ax - b)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $q(x)$ is divided by $(ax - b)$, what will be its remainder?

⇨ Here, if $q(x)$ is divided by $(ax - b)$ then the remainder $= q\left(\frac{b}{a}\right)$.

19. यदि एउटा बहुपदीय $f(x)$ लाई $(px + q)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(px + q)$, what is its remainder?

⇨ Here, if $f(x)$ is divided by $(px + q)$ then the remainder $= f\left(\frac{-q}{p}\right)$.

20. गुणनखण्ड साध्य उल्लेख गर्नुहोस् । (State the Factor Theorem.)

⇨ Here, if $p(x)$ be a polynomial (degree > 0) and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$. Conversely, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$.

21. जब एउटा बहुपदीय $q(x)$ लाई शेष $q(- a) = 0$ हुने गरी अर्को बहुपदीयले भाग गरिन्छ भने एउटा गुणनखण्ड के होला ?
When a polynomial $q(x)$ is divided by another polynomial such that remainder $q(- a) = 0$, what will be its one of the factor?

⇨ Here, when a polynomial $q(x)$ is divided by another polynomial such that $q(- a) = 0$ then $(x + a)$ is a factor.

22. यदि बहुपदीय $p(x)$ को एउटा गुणनखण्ड $(x - m)$ भए $p(m)$ को मान कति होला ?
If $(x - m)$ is a factor of polynomial $p(x)$, what is the value of $p(m)$?

⇨ Here, if $(x - m)$ is a factor of polynomial $p(x)$, then the value of $p(m)$ is 0.

23. एउटा बहुपदीय $f(x)$ मा $f\left(\frac{b}{a}\right) = 0$ भए यसका गुणनखण्डहरू मध्य एउटा गुणनखण्ड पत्ता लगाउनुहोस् ।

In a polynomial $f(x)$, $f\left(\frac{b}{a}\right) = 0$, what will be its one of the factors?

⇨ Here, in a polynomial $f(x)$ if $f\left(\frac{b}{a}\right) = 0$ then $(ax - b)$ will be a factor.

24. $f\left(-\frac{m}{n}\right) = 0$ हुनेगरी एउटा बहुपदीय $f(x)$ छ । यसका गुणखनण्डहरू मध्य एउटा गुणनखण्ड पत्ता लगाउनुहोस् ।

$f(x)$ is a polynomial such that $f\left(-\frac{m}{n}\right) = 0$. Find its one of the factors.

⇨ Here, in a polynomial $f(x)$ if $f\left(-\frac{m}{n}\right) = 0$ then $(nx + m)$ will be a factor.

25. $f(x) = (x - c) \times q(x) + r(x)$ मा $f(x)$ को एउटा गुणनखण्ड $x - c$ भए शेष $r(x)$ कति हुन्छ ?

In $f(x) = (x - c) \times q(x) + r(x)$, $x - c$ is a factor of $f(x)$ then what will be the remainder $r(x)$?

⇨ Here, if $(x - c)$ is a factor in $f(x) = (x - c) \times q(x) + r(x)$ then the value of $r(x) = 0$.

26. $f(x)$, $d(x)$ र $q(x)$ मा $f(x)$ ले n डिग्रीको बहुपदीय, $d(x)$ ले भाजक र $q(x)$ ले भागफललाई जनाउँछ भने यिनीहरू बिचको सम्बन्ध लेख्नुहोस्, जहाँ $d(x)$ ले $f(x)$ को गुणनखण्डलाई जनाउँछ ।

In $f(x)$, $d(x)$ and $q(x)$, $f(x)$ represents a polynomial of degree n, $d(x)$ is divisor and $q(x)$ is quotient then write the relation between them where $d(x)$ represents a factor of $f(x)$.

⇨ Here, $f(x) = d(x) \times q(x)$ if $d(x)$ is a factor of $f(x)$.

27. बहुपदीय $f(x) = x^2 - a^2$ का मूलहरू के के हुन् ? (What are the roots of the polynomial $f(x) = x^2 - a^2$?)

⇨ Here, the roots of $f(x) = x^2 - a^2 = 0$ is $x = \pm a$.

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28. **बहुपदीय $p(x) = x^2 - m^2$ का शुन्यहरू के के हुन् ? (What are the zeros of the polynomial $p(x) = x^2 - m^2$?)**

⇨ Here, $x = \pm m$ are the zeros of $p(x) = x^2 - m^2$.

29. **बहुपदीय $x^3 + x^2 + 2x + 5$ को डिग्री कति हुन्छ ? (What is the degree of polynomial $x^3 + x^2 + 2x + 5$?)**

⇨ Here, the degree of $x^3 + x^2 + 2x + 5$ is 3.

30. **बहुपदीय $f(x) = 2x^3 + 4x^2 + 6x + 7$ को डिग्री कति हुन्छ ? (What is the degree of polynomial $f(x) = 2x^3 + 4x^2 + 6x + 7$?)**

⇨ Here, the degree of $f(x) = 3$.

31. **यदि $f(x) = x^5 - 2x^3 + 7$ र $g(x) = x^2 - 3$ भए $f(x) \div g(x)$ को भागफलको डिग्री पत्ता लगाउनुहोस् ।**
If $f(x) = x^5 - 2x^3 + 7$ and $g(x) = x^2 - 3$ then what is the degree of quotient in $f(x) \div g(x)$?

⇨ Here, the degree of $f(x) \div g(x) = 5 - 2 = 3$.

32. **यदि $p(x) = x^3 - 8$ र $q(x) = x - 2$ भए $p(x) \div q(x)$, को भागफलको डिग्री पत्ता लगाउनुहोस् ।**
If $p(x) = x^3 - 8$ and $q(x) = x - 2$ then what is the degree of quotient when $p(x) \div q(x)$?

⇨ Here, the degree of $p(x) \div q(x) = 3 - 1 = 2$.

33. **शेष कति होला जब एउटा बहुपदीय $f(x)$ लाई $(2x + 3)$ ले भाग गरिएको छ ?**
What is the remainder when a polynomial $f(x)$ is divided by $(2x + 3)$?

⇨ Here, $2x + 3 = 0$ $\quad\therefore\ x = -\frac{3}{2}$ $\quad$ So, $f\left(-\frac{3}{2}\right)$ is the remainder.

34. **शेष कति होला जब एउटा बहुपदीय $x^3 - 1$ लाई $(x - 2)$ ले भाग गरिएको छ ?**
What is the remainder when a polynomial $x^3 - 1$ is divided by $(x - 2)$?

⇨ Here, $x - 2 = 0$ $\quad\therefore\ x = 2$ $\quad$ So, $f(2) = 2^3 - 1 = 7$ is the remainder.

35. **शेष कति होला जब एउटा बहुपदीय $x^3 - 4$ लाई $(x + 1)$ ले भाग गरिएको छ ?**
What is the remainder when a polynomial $x^3 - 4$ is divided by $(x + 1)$?

⇨ Here, $x + 1 = 0$ $\quad\therefore\ x = -1$
So, $f(-1) = (-1)^3 - 4 = -1 - 4 = -5$ is the remainder.

36. **$f(x) = x^2 - 16$ का शुन्यहरू अनुमान गर्नुहोस् । (Guess the zeros of the polynomial $f(x) = x^2 - 16$.)**

⇨ Here, $f(x) = x^2 - 16 = 0$ $\quad$ or, $x^2 = 16$ $\quad\therefore\ x = \pm 4$
The roots of $f(x)$ are ± 4.

37. **बहुपदीय $p(x) = (x + 3)(x - 4)$ का मूलहरू पत्ता लगाउनुहोस् । (Find the roots of the polynomial $p(x) = (x + 3)(x - 4)$.)**

⇨ Here, $p(x) = (x + 3)(x - 4) = 0$ $\quad$ The roots of $p(x)$ are -3, and 4.

B. SHORT QUESTIONS

MODEL 1

1. **यदि $x^3 - 19x - 30 = (x + 2)$. Q(x) भए सङ्क्षिप्त भाग विधिको प्रयोग गरेर Q(x) पत्ता लगाउनुहोस् ।**
If $x^3 - 19x - 30 = (x + 2).Q(X)$, find Q(x) by using synthetic division method.

⇨ Here, $x^3 - 19x - 30 = (x + 2).\ Q(x)$
Comparing $(x + 2)$ with $(x - a)$ then $a = -2$.
Using synthetic division,

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & \downarrow & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \multicolumn{1}{|r|}{0} \\ \cline{5-5} \end{array}$$

$\therefore\ Q(x) = x^2 - 2x - 15$

2. **यदि $x^3 - 21x - 20 = (x + 1)$. Q(x) भए सङ्क्षिप्त भाग विधि प्रयोग गरेर Q(x) पत्ता लगाउनुहोस् ।**
If $x^3 - 21x - 20 = (x + 1)$. Q(x), find Q(x) by using synthetic division method. **[2071 S]**

⇨ Here, $x^3 - 21x - 20 = (x + 1).\ Q(x)$
Comparing $(x - a)$ with $(x + 1)$ then $a = -1$
Using the synthetic division,

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -21 & -20 \\ & \downarrow & -1 & 1 & 20 \\ \hline & 1 & -1 & -20 & \multicolumn{1}{|r|}{0} \\ \cline{5-5} \end{array}$$

Thus, $Q(x) = x^2 - x - 20$.

3. **यदि $2x^3 - 7x^2 + x + 10 = (x - 1)\ Q(x) + R$ भए शेष R र बहुपदीय Q(x) पत्ता लगाउनुहोस् ।**
If $2x^3 - 7x^2 + x + 10 = (x - 1)\ Q(x) + R$, find the remainder R and polynomial Q(x).

⇨ Here, given, $2x^3 - 7x^2 + x + 10 = (x - 1)\ Q(x) + R$
When given expression is divided by $(x - 1)$, then quotient = Q (x) and remainder = R.
So using synthetic division method for $x = +1$, we get,

$$\begin{array}{r|rrrr} +1 & 2 & -7 & 1 & 10 \\ & \downarrow & +2 & -5 & -4 \\ \hline & 2 & -5 & -4 & \multicolumn{1}{|r}{6} \\ \hline & \multicolumn{3}{c}{Q} & \multicolumn{1}{|c}{R} \end{array}$$

Thus, the quotient $Q(x) = 2x^2 - 5x - 4$ and remainder $(R) = 6$.

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MODEL 2

4. शेषसाध्य प्रयोग गरेर $8x^3 - 4x^2 + 2x - 5$ लाई $2x - 1$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
Using Remainder theorem, find the remainder when $8x^3 - 4x^2 + 2x - 5$ is divided by $2x - 1$. **[2072 R]**

⇨ Here, $p(x) = 8x^3 - 4x^2 + 2x - 5$ and $d(x) = 2x - 1$

Since, $2x - 1 = 0 \quad \Rightarrow \quad 2x = 1 \quad \therefore \quad x = \frac{1}{2}$

Now, $P\left(\frac{1}{2}\right) = 8 \times \left(\frac{1}{2}\right)^3 - 4 \times \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} - 5$

or, $R = 8 \times \frac{1}{8} - 4 \times \frac{1}{4} + 1 - 5$

or, $R = 1 - 1 + 1 - 5 \quad \therefore \quad R = -4$

Thus, the remainder is -4.

5. शेष साध्यको कथन लेख्नुहोस् । सो साध्य प्रयोग गरी $x^6 - 1$ लाई $x + 1$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
State remainder theorem. Use it to find the remainder when the polynomial $x^6 - 1$ is divided by $x + 1$. **[2068 R]**

⇨ **Remainder Theorem states that:** When the polynomial f (x) of degree n is divided by the binomial x – a, then the remainder R is given by the value f (a) of the polynomial and degree of the quotient Q(x) is n – 1.

$\therefore \quad p(x) = x^6 - 1$ and $d(x) = x + 1$

Comparing $(x + 1)$ with $(x - a)$ then $a = -1$

We know that,

Reminder $= p(a) = p(-1) = (-1)^6 - 1 = 1 - 1 = 0$

Thus, the remainder is 0.

6. शेष साध्यको परिभाषा दिनुहोस् । शेष साध्यको प्रयोग गरी $3x^3 - 5x^2 + 2x - 3$ लाई $(x - 2)$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
Define the remainder theorem. Find the remainder by using the remainder theorem when $3x^3 - 5x^2 + 2x - 3$ is divided by $(x - 2)$. **[2074 S, 2057 R]**

⇨ For first part: refer Q.No. 5 above.

Here, given $f(x) = 3x^3 - 5x^2 + 2x - 3$ and $g(x) = x - 2$

Comparing $(x - 2)$ with $(x - a)$ then, $a = 2$

Now, $f(2) = 3.2^3 - 5.2^2 + 2.2 - 3 \qquad = 24 - 20 + 4 - 3$

$= 28 - 23 = 5$

Thus, the remainder is 5.

7. शेष साध्यको परिभाषा दिनुहोस् । शेष साध्यको प्रयोग गरी $2x^3 - 7x^2 + 5x + 4$ लाई $(x - 3)$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
Define the remainder theorem. Find the remainder by using the remainder theorem when $2x^3 - 7x^2 + 5x + 4$ is divided by $(x - 3)$. **[SEE 2074 R, 2060 S]**

⇨ For first part: refer Q.No. 5 above.

Here, given polynomial $f(x) = 2x^3 - 7x^2 + 5x + 4$ and divisor $= (x - 3)$

Comparing $(x - 3)$ with $(x - a)$ then $a = 3$

Since, $f(3)$ is the remainder of $f(x)$ when divided by $(x - 3)$.

So, $f(3) = 2.3^3 - 7.3^2 + 5.3 + 4 \qquad = 54 - 63 + 15 + 4$

$= 73 - 63 = 10$

Thus, the remainder is 10.

MODEL 3

8. यदि $f(x) = 4x^3 - 3x^2 + 3x - k$ लाई $(x - 2)$ ले भाग गर्दा शेष 12 हुन्छ भने k को मान पत्ता लगाउनुहोस् ।
If $f(x) = 4x^3 - 3x^2 + 3x - k$ is divided by $(x - 2)$, the remainder is 12. Find the value of k. **[2075 R']**

⇨ Here, $f(x) = 4x^3 - 3x^2 + 3x - k$

Since $f(x)$ leaves remainder 12 when divided by $x - 2$,

then $f(2) = 12$

or, $4 . 2^3 - 3 . 2^2 + 3 . 2 - k = 12$

or, $32 - 12 + 6 - k = 12$

$\therefore \quad k = 14$

Thus, the value of k is 14.

9. यदि $2x^3 - 4x^2 + kx + 10$ लाई $(x + 2)$ ले भाग गर्दा शेष 4 आउँछ भने शेषसाध्य प्रयोग गरेर k को मान पत्ता लगाउनुहोस् ।
If $2x^3 - 4x^2 + kx + 10$ is divided by $(x + 2)$, the remainder is 4. Find the value of k using remainder theorem. **[2073 R]**

⇨ Here, $p(x) = 2x^3 - 4x^2 + kx + 10$, $d(x) = x + 2$ and $R = 4$

Since $x + 2 = 0 \quad \Rightarrow x = -2$

Now, $p(-2) = 2 \times (-2)^3 - 4 \times (-2)^2 + k(-2) + 10$

or, $4 = -16 - 16 - 2k + 10$

or, $26 = -2k \quad \therefore \quad k = -13$

Thus, the value of k is -13.

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10. यदि $x^3 - kx^2 - 13x + 10$ लाई $(x + 2)$ ले भाग गर्दा शेष 4 हुन्छ भने k को मान शेषसाध्य प्रयोग गरेर पत्ता लगाउनुहोस् ।
If $x^3 - kx^2 - 13x + 10$ is divided by $(x + 2)$, the remainder is 4. Find the value of k using the remainder theorem.

⇨ Here, let $P(x) = x^3 - kx^2 - 13x + 10$ $\quad d(x) = (x + 2) = 0$
$\therefore\ x = -2$ and $R = 4$
Now, $P(-2) = (-2)^3 - k(-2)^2 - 13(-2) + 10$
or, $4 = -8 - 4k + 26 + 10$
or, $4k = 24$ $\quad \therefore\ k = 6$
Thus, the value of k is 6.

11. यदि $2x^3 + 3x^2 - px + 4$ लाई $(x + 3)$ ले भाग गर्दा शेष 10 हुन्छ भने शेषसाध्य प्रयोग गरेर p को मान पत्ता लगाउनुहोस् ।
If $2x^3 + 3x^2 - px + 4$ is divided by $(x + 3)$, the remainder is 10. Find the value of p using the remainder theorem.

⇨ Here, let, $f(x) = 2x^3 + 3x^2 - px + 4$, $d(x) = x + 3$ and $R = 10$
Since, $x + 3 = 0 \Rightarrow x = -3$
So, by remainder theorem,
$f(-3) = 2(-3)^3 + 3(-3)^2 - p(-3) + 4$
or, $10 = 2 \times (-27) + 3 \times 9 + 3p + 4$
or, $10 = -54 + 27 + 3p + 4$
or, $10 = -23 + 3p$
or, $33 = 3p$ $\quad \therefore\ p = 11$
Thus, the value of p is 11.

12. यदि बहुपदीय $x^3 + 6x^2 + kx + 10$ लाई $(x + 2)$ ले भाग गर्दा शेष 4 हुन्छ भने k को मान शेष साध्य प्रयोग गरेर पत्ता लगाउनुहोस् ।
If the polynomial $x^3 + 6x^2 + kx + 10$ is divided by $(x + 2)$, the remainder is 4, find the value of k using the remainder theorem. **[2070 R]**

⇨ Here, $p(x) = x^3 + 6x^2 + kx + 10$, $d(x) = x + 2$, $R = 4$
Since $x + 2 = 0$ $\quad \Rightarrow x = -2$
Now, $p(-2) = (-2)^3 + 6 \times (-2)^2 + k(-2) + 10$
or, $4 = -8 + 24 - 2k + 10$
or, $2k = 26 - 4$
or, $2k = 22$ $\quad \therefore\ k = 11$
Thus, the value of k is 11.

13. फलन $f(x) = 2x^4 - 3x^2 + 6x + k$ दिइएको छ । यदि $f(1) = 0$ भए k को मान निकाल्नुहोस् ।
Given that the function $f(x) = 2x^4 - 3x^2 + 6x + k$. If $f(1) = 0$, find the value of k. **[2060 R]**

⇨ Here given, $f(x) = 2x^4 - 3x^2 + 6x + k$, and $f(1) = 0$
Put $x = 1$ in $f(x)$, we get,
$f(1) = 2 \times 1^4 - 3 \times 1^2 + 6 \times 1 + k$
or, $0 = 2 - 3 + 6 + k$
or, $0 = 5 + k$ $\quad$ or, $k = -5$
Thus, the required value of k is – 5.

14. यदि $x^3 + ax^2 - x + 7$ लाई $x - 3$ ले भाग गर्दा शेष 4 रहन्छ भने a को मान पत्ता लगाउनुहोस् ।
If $x^3 + ax^2 - x + 7$ leaves the remainder 4 when it is divided by $x - 3$, find the value of a. **[2062K]**

⇨ Here, $p(x) = x^3 + ax^2 - x + 7$, Divisor $= x - 3$ and Remainder $= 4$
Comparing $(x - 3)$ with $(x - c)$ then $c = 3$
We have, Remainder $= p(c) = 4$
or, $p(3) = 4$
or, $3^3 + a \times 3^2 - 3 + 7 = 4$
or, $27 + 9a + 4 = 4$
or, $9a = -27$ $\quad \therefore\ a = -3$
Thus the required value of a is – 3.

15. बहुपदीय $f(x) = x^3 - (P - 2)x^2 - Px + 28$ लाई $x + 3$ ले भाग गर्दा शेष 10 आउँछ भने शेष साध्य प्रयोग गरी P को मान पत्ता लगाउनुहोस् ।
The polynomial $f(x) = x^3 - (P - 2)x^2 - Px + 28$ leaves a remainder 10 when divided by $x + 3$. Find the value of P, using the remainder theorem. **[2065 R]**

⇨ Here, $f(x) = x^3 - (P - 2)x^2 - Px + 28$, $\quad$ Divisor $= x + 3$ and Remainder $= 10$
Comparing $(x + 3)$ with $(x - a)$ then $a = -3$
Now, Remainder $= f(-3)$
or, $10 = (-3)^3 - (P - 2)(-3)^2 - P(-3) + 28$
or, $10 = -27 - (P - 2)9 + 3P + 28$
or, $10 = 1 - 9P + 18 + 3P$
or, $6P = 18 + 1 - 10$
or, $6P = 9$
$\therefore\ P = \frac{9}{6} = \frac{3}{2}$

Thus, the required value of P is $\frac{3}{2}$.

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16. **$2x^3 + 3x^2 - kx + 4$ लाई $(x - 2)$ ले भाग गर्दा शेष $2k$ हुन्छ भने k को मान पत्ता लगाउनुहोस् ।**
$2x^3 + 3x^2 - kx + 4$ is divided by $(x - 2)$ the remainder is $2k$, find the value of k. **[2065 R']**

⇨ Here, $p(x) = 2x^3 + 3x^2 - kx + 4,$ $d(x) = x - 2$ and $R = 2k$
Comparing $(x - 2)$ with $(x - a)$ then, $a = 2$
Now, $P(2) = 2 \times 2^3 + 3 \times 2^2 - k \times 2 + 4$
or, $P(2) = 16 + 12 - 2k + 4$
or, $2k = 32 - 2k$
or, $4k = 32$ $\therefore$ $k = 8$
Thus, the value of k is 8.

MODEL 4

17. **बहुपदीय $x^3 - 9x^2 + 24x - 20$ को एउटा गुणनखण्ड $(x - 2)$ हो भनी प्रमाणित गर्नुहोस् ।**
Prove that $(x - 2)$ is a factor of a polynomial $x^3 - 9x^2 + 24x - 20$. **[2074 S']**

⇨ Here, let, $P(x) = x^3 - 9x^2 + 24x - 20$ and factor $= (x - 2)$
comparing $(x - 2)$ with $(x - a)$ then $a = 2$
Now, $P(a) = P(2) = 2^3 - 9 \times 2^2 + 24 \times 2 - 20$
$= 8 - 36 + 48 - 20$
$= 56 - 56 = 0$
Thus, $P(2) = 0$ shows that $(x - 2)$ is a factor of $P(x)$.

18. **गुणनखण्ड साध्यको कथन लेख्नुहोस् । गुणनखण्ड साध्य प्रयोग गरी $x + 3$ बहुपदीय $x^3 - 8x + 3$ को गुणनखण्ड हो होइन यकिन गर्नुहोस् ।**
State factor theorem. Use factor theorem to determine whether $x + 3$ is a factor of the polynomial $x^3 - 8x + 3$. **[2071 R']**

⇨ **Statement of factor theorem:** If a polynomial $p(x)$ is divided by $(x - a)$ and $f(a) = R = 0$ then $(x - a)$ is a factor of $p(x)$.
Here, $p(x) = x^3 - 8x + 3$ and $d(x) = x + 3$
Comparing $(x + 3)$ with $(x - a)$ then $a = -3$
Now, $P(a) = P(3) = (-3)^3 - 8 \times (-3) + 3 = -27 + 24 + 3 = 0$
Since $P(3) = 0$ shows that $(x + 3)$ is a factor of $P(x)$.

19. **शेष साध्यलाई परिभाषित गर्नुहोस् । शेष साध्यको प्रयोग गरी $(x - 2)$ बहुपदीय $3x^3 - 2x^2 + 5x - 14$ को एउटा गुणनखण्ड हो या होइन पत्ता लगाउनुहोस् ।**
State Remainder Theorem. Use Remainder Theorem to determine whether $(x - 2)$ is a factor of the polynomial $3x^3 - 2x^2 + 5x - 14$. **[2066 S]**

⇨ Here, Remainder Theorem states that, if $p(x)$ is a polynomial of degree n and $(x - a)$ is a divisor of $p(x)$ then $p(a)$ is remainder, where the degree of quotient will be $(n - 1)$.
Here, $p(x) = 3x^3 - 2x^2 + 5x - 14$ $d(x) = x - 2$
Comparing $(x - 2)$ with $(x - a)$ then, $a = 2$
Now, $p(a) = p(2) = 3 \times 2^3 - 2 \times 2^2 + 5 \times 2 - 14 = 24 - 8 + 10 - 14 = 12$
Thus, $p(a) \neq 0$ shows that $(x - 2)$ is not a factor of given polynomial.

20. **गुणनखण्ड साध्य प्रयोग गरी $(x - 1)$ बहुपदीय $3x^3 + 2x - 5$ को गुणनखण्ड हो होइन यकिन गर्नुहोस् ।**
Use factor theorem to determine whether $(x - 1)$ is a factor of the polynomial $3x^3 + 2x - 5$. **[2065 M]**

⇨ Here, If a polynomial $p(x)$ is divided by $(x - a)$ and $p(a) = R = 0$ then $(x - a)$ is a factor of $p(x)$.
Comparing $(x - 1)$ with $(x - a)$ then $a = 1$.
Testing factor, $p(a) = p(1) = 3(1)^3 + 2(1) - 5 = 3 + 2 - 5 = 0$
Thus, $(x - 1)$ is a factor.

21. **$f(x) = x^3 - 3x^2 - 4x + 12$ को गुणनखण्ड $(x + 2)$ हो भनी देखाउनुहोस् ।(Show that $(x + 2)$ is a factor of $f(x) = x^3 - 3x^2 - 4x + 12$.)** **[2066 R]**

⇨ Here, if $p(x)$ is a polynomial with degree > 0 and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$.
Conversely, if $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$.
Here, $f(x) = x^3 - 3x^2 - 4x + 12$Factor $= (x + 2)$
Comparing $(x + 2)$ with $(x - a)$ then $a = -2$
Now, $f(-2) = (-2)^3 - 3(-2)^2 - 4(-2) + 12 = -8 - 3 \times 4 + 8 + 12 = -20 + 20$
$\therefore$ $f(-2) = 0$
Thus, $(x + 2)$ is a factor of $f(x)$.

22. **$3x^3 - 15x + 6$ को एउटा गुणनखण्ड $(x - 2)$ हुन्छ भनी देखाउनुहोस् । (Show that $(x - 2)$ is a factor of $3x^3 - 15x + 6$.)** **[2067 R']**

⇨ Here, $p(x) = 3x^3 - 15x + 6$ and $p(x)$ is divided by $(x - 2)$
Comparing $(x - 2)$ with $(x - a)$ then $a = 2$
Now, by using remainder theorem, $p(2) = 3.(2)^3 - 15.2 + 6 = 3 \times 8 - 30 + 6 = 24 - 30 + 6 = 0$
Where, remainder is 0.
Thus, $(x - 2)$ is a factor of given polynomial.

23. **गुणनखण्ड साध्यको कथन लेख्नुहोस् । $f(x) = x^3 + 6x^2 + 7x + 2$ को एउटा गुणनखण्ड $(x + 1)$ हो भनी देखाउनुहोस् ।**
State the factor theorem. Show that $(x + 1)$ is a factor of $f(x) = x^3 + 6x^2 + 7x + 2$. **[2067S]**

⇨ **The statement of factor theorem:**
If a polynomial $p(x)$ is divided by $(x - a)$ and $f(a) = R = 0$ then $(x - a)$ is a factor of $p(x)$.
Here, $f(x) = x^3 + 6x^2 + 7x + 2$ and a factor is $(x + 1)$
Comparing $(x + 1)$ with $(x - a)$ then $a = -1$
Now, $f(-1) = (-1)^3 + 6(-1)^2 + 7(-1) + 2 = -1 + 6 - 7 + 2$
$\therefore$ $f(-1) = 0$
Thus, $f(-1) = 0$ shows that $(x + 1)$ is a factor of $f(x)$.

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MODEL 5

24. यदि एउटा बहुपदीय $p(x) = 2x^3 - 6x^2 - 5m - 2$ को एउटा गुणनखण्ड $x - 2$ हो भने m को मान पत्ता लगाउनुहोस् ।

If a factor of a polynomial $p(x) = 2x^3 - 6x^2 - 5m - 2$ is $x - 2$, find the value of m. **[2075 R]**

⇨ Here, $p(x) = 2x^3 - 6x^2 - 5m - 2$ and factor $= x - 2$

since $x - 2 = 0$ $\Rightarrow$ $x = 2$

Now, $p(2) = 0$

or, $2 \times 2^3 - 6 \times 2^2 - 5m - 2 = 0$

or, $16 - 24 - 5m - 2 = 0$

or, $5m = -10$

$\therefore$ $m = -2$

Thus, the value of m is -2.

25. यदि बहुपदीय $2x^3 + 3x^2 - mx + 4$ को एउटा गुणनखण्ड $(x + 2)$ भए m को मान निकाल्नुहोस् ।

If $(x + 2)$ is a factor of the polynomial $2x^3 + 3x^2 - mx + 4$, find the value of m. **[2075 R$_2$]**

⇨ Here, let $f(x) = 2x^3 + 3x^2 - mx + 4$ and factor $= x + 2$

Since, $x + 2 = 0$ $\therefore$ $x = -2$

Now, $f(-2) = 0$

or, $2 \times (-2)^3 + 3 \times (-2)^2 - m(-2) + 4 = 0$

or, $-16 + 12 + 2m + 4 = 0$

or, $2m = 0$

$\therefore$ $m = 0$

Thus, the value of m is 0.

26. यदि $f(x) = x^3 - 3x^2 - 4x + k$ को एउटा गुणनखण्ड $(x - 2)$ भए k को मान पत्ता लगाउनुहोस् ।

If $(x - 2)$ is a factor of $f(x) = x^3 - 3x^2 - 4x + k$, find the value of k. **[2075 R'$_2$]**

⇨ Here, $f(x) = x^3 - 3x^2 - 4x + k$ and factor $= x - 2$

Since $x - 2 = 0$ $\therefore$ $x = 2$

So, $f(2) = 0$

or, $2^3 - 3 . 2^2 - 4 \times 2 + k = 0$

or, $8 - 12 - 8 + k = 0$

$\therefore$ $k = 12$

Thus, the value of k is 12.

27. यदि $2x^3 - x^2 + 10x - k$ को एउटा गुणनखण्ड $x - 3$ भए k को मान पत्ता लगाउनुहोस् ।

If $x - 3$ is a factor of $2x^3 - x^2 + 10x - k$ then find the value of k. **[2073 R']**

⇨ Here, $p(x) = 2x^3 - x^2 + 10x - k$ and a factor $= (x - 3)$

So, $x - 3 = 0$ $\therefore$ $x = 3$

Now, $p(3) = 2 \times 3^3 - 3^2 + 10 \times 3 - k$

or, $0 = 54 - 9 + 30 - k$

$\therefore$ $k = 75$ Thus, the value of k is 75.

28. यदि $x^3 + px^2 + 4x + 5$ को एउटा गुणनखण्ड $x - 5$ भए p को मान पत्ता लगाउनुहोस् ।

If $x - 5$ is a factor of $x^3 + px^2 + 4x + 5$, find the value of p. **[2072 S]**

⇨ Here, $p(x) = x^3 + px^2 + 4x + 5$ and a factor $= (x - 5)$

So, $x - 5 = 0$ $\therefore$ $x = 5$

Now, $p(5) = 5^3 + p \times 5^2 + 4 \times 5 + 5$

or, $0 = 125 + 25p + 25$

or, $25p = -150$ $\therefore$ $p = -6$

Thus, the value of p is -6.

29. यदि $2x^3 - 7px + (p - 12)$ को गुणनखण्ड $x - 5$ भए p को मान पत्ता लगाउनुहोस् ।

If $x - 5$ is a factor of $2x^3 - 7px + (p - 12)$, find the value of p. **[2072 R']**

⇨ Here, $p(x) = 2x^3 - 7px + (p - 12)$ & factor $= x - 5$

Since, $x - 5 = 0$ $\therefore$ $x = 5$

Now, $p(5) = 2 \times 5^3 - 7p \times 5 + p - 12$

or, $0 = 250 - 35p + p - 12$

or, $34p = 238$

$\therefore$ $p = 7$

Thus, the value of p is 7.

30. यदि $x^3 - 19x - p$ को गुणनखण्ड $(x + 2)$ भए p को मान पत्ता लगाउनुहोस् ।

If $(x + 2)$ is a factor of $x^3 - 19x - p$, find the value of p. **[2070 R']**

⇨ Here, $p(x) = x^3 - 19x - p$ and $x + 2 \Rightarrow x = -2$

So, $p(-2) = (-2)^3 - 19(-2) - p$

or, $0 = -8 + 38 - p$

$\therefore$ $p = 30$

Thus, the value of p is 30.

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31. यदि $2x^3 - kx^2 - 8x + 5$ को एउटा गुणनखण्ड $x + 1$ भए k को मान निकाल्नुहोस् ।
If $x + 1$ is a factor of $2x^3 - kx^2 - 8x + 5$, find the value of k. **[2061 R, 2068 R']**

⇨ Here given polynomial, $f(x) = 2x^3 - kx^2 - 8x + 5$ and $(x + 1)$ is a factor of $f(x)$
Comparing $(x + 1)$ with $(x - a)$ then, $a = -1$
We know that, remainder $(R) = f(-1)$
i.e. $R = f(-1) = 2(-1)^3 - k(-1)^2 - 8(-1) + 5$
or, $0 = -2 - k + 8 + 5$ [Since $(x + 1)$ is a factor of $f(x)$.]
or, $0 = -k + 11$
$\therefore$ $k = 11$
Thus, the value of k is 11.

32. यदि $f(x) = x^3 + kx^2 - 4x + 12$ को एउटा गुणनखण्ड $x + 2$ भए, k को मान निकाल्नुहोस् ।
If $(x + 2)$ is a factor of $f(x) = x^3 + kx^2 - 4x + 12$, find the value of k. **[SEE 2074 R', 2057 S]**

⇨ Here, given expression, $f(x) = x^3 + kx^2 - 4x + 12$
Since, $(x + 2)$ is a factor.
Comparing $(x + 2)$ with $(x - a)$ then, $a = -2$
So when putting $x = -2$, the given expression becomes zero.
or, $f(-2) = 0$
or, $f(-2) = (-2)^3 + k(-2)^2 - 4 \times (-2) + 12$
or, $0 = -8 + 4k + 8 + 12$ or, $0 = 12 + 4k$
or, $4k = -12$ $\therefore$ $k = -3$
Thus, the value of k is – 3.

33. यदि $P(x) = 2x^3 + 3x^2 - kx + 4$ को एउटा गुणनखण्ड $(x + 3)$ भए k को मान पत्ता लगाउनुहोस् ।
If $x + 3$ is a factor of $P(x) = 2x^3 + 3x^2 - kx + 4$, find the value of k. **[2065 S]**

⇨ Here, polynomial: $P(x) = 2x^3 + 3x^2 - kx + 4$ and a factor $= (x + 3)$
Comparing $(x + 3)$ with $(x - a)$ then, $a = -3$
Now, $P(-3) = 2(-3)^3 + 3(-3)^2 - k(-3) + 4$
or, $0 = -54 + 27 + 3k + 4$
or, $23 = 3k$
$\therefore$ $k = \frac{23}{3}$ Thus, the value of k is $\frac{23}{3}$.

34. यदि $2x^3 + ax^2 + x + 2$ को एउटा गुणनखण्ड $2x + 1$ भए, a को मान पत्ता लगाउनुहोस् ।
If $2x + 1$ is a factor of $2x^3 + ax^2 + x + 2$, find the value of a.

⇨ Here, given expression, $2x^3 + ax^2 + x + 2$

Since, $2x + 1 = 2\left(x + \frac{1}{2}\right)$, is a factor of given expression.

Comparing $\left(x + \frac{1}{2}\right)$ with $(x - a)$ then $a = -\frac{1}{2}$.

So, the given expression is satisfied by $x = -\frac{1}{2}$.

$$\text{Now, } 2 . \left(\frac{-1}{2}\right)^3 + a\left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 2 = 0$$

$$\text{or, } 2 \times \left(-\frac{1}{8}\right) + a . \frac{1}{4} - \frac{1}{2} + 2 = 0$$

$$\text{or, } -\frac{1}{4} + \frac{a}{4} - \frac{1}{2} + 2 = 0$$

$$\text{or, } \frac{a}{4} = \frac{1}{4} + \frac{1}{2} - 2$$

$$\text{or, } \frac{a}{4} = -\frac{5}{4} \qquad \therefore \quad a = -5$$

Thus, the required value of a is – 5.

35. यदि $x^3 - (k - 1)x^2 + kx + 54$ को एउटा गुणनखण्ड $x + 3$ भए k को मान पत्ता लगाउनुहोस् ।
If $x + 3$ is a factor of $x^3 - (k - 1)x^2 + kx + 54$ find, the value of k.

⇨ Here, given expression $x^3 - (k - 1)x^2 + kx + 54$ .... (i)
Since, $x + 3$ is a factor of the given expression.
Comparing $(x + 3)$ with $(x - a)$ then $a = -3$
Then, $x = -3$ must satisfy the given expression.
So, $(-3)^3 - (k - 1)(-3)^2 + k(-3) + 54 = 0$ or, $-27 - (k - 1)\,9 - 3k + 54 = 0$
or, $-27 - 9k + 9 - 3k + 54 = 0$ or, $-12k + 36 = 0$
or, $k = \frac{-36}{-12} = 3$ $\therefore$ $k = 3$

Thus, the required value of k is 3.

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36. यदि $x^3 + 4x^2 + kx - 30$ को एउटा गुणनखण्ड $x - 3$ भए k को मान निकाल्नुहोस् ।
If $x - 3$ is a factor of $x^3 + 4x^2 + kx - 30$, find the value of k.

⇨ Here, given expression, $f(x) = x^3 + 4x^2 + kx - 30$
Since $(x - 3)$ is a factor of given expression.
Comparing $(x - 3)$ with $(x - a)$ then $a = 3$
So putting $x = 3$, the given expression becomes zero.
Now, $f(3) = 0$ or, $3^3 + 4.3^2 + k.3 - 30 = 0$
or, $27 + 36 + 3k - 30 = 0$ or, $3k + 33 = 0$
or, $3k = -33$ $\therefore$ $k = \frac{-33}{3} = -11$
Thus, the value of k is – 11.

37. यदि बहुपदीय $P(x) = 2x^3 + 3x^2 + kx + 4$ को एउटा गुणनखण्ड $x - 3$ भए k को मान निकाल्नुहोस् ।
If $x - 3$ is a factor of polynomial $P(x) = 2x^3 + 3x^2 + kx + 4$, find the value of k.

⇨ Here, $P(x) = 2x^3 + 3x^2 + kx + 4$ and factor $(x - 3) = 0$
Comparing $(x - 3)$ with $(x - a)$ then $a = 3$
So, $P(3) = 2 \times 3^3 + 3 \times 3^2 + k \times 3 + 4$
or, $0 = 54 + 27 + 3k + 4$
or, $0 = 85 + 3k$
or, $3k = -85$ $\therefore$ $k = -\frac{85}{3} = -28\frac{1}{3}$
Thus, the value of k is $-28\frac{1}{3}$.

38. यदि $2x^3 + x^2 - kx - 1 = 0$ को एउटा गुणनखण्ड $x - 1$ भए k को मान पत्ता लगाउनुहोस् ।
If $x - 1$ is a factor of $2x^3 + x^2 - kx - 1 = 0$, find the value of k.

⇨ Here, $P(x) = 2x^3 + x^2 - kx - 1$ and factor $= (x - 1)$
Since $(x - 1)$ is a factor of $p(x)$.
Comparing $(x - 1)$ with $(x - a)$ then $a = 1$
So, $p(1) = 0$ or, $2(1)^3 + 1^2 - k(1) - 1 = 0$
or, $2 + 1 - k - 1 = 0$ or, $2 - k = 0$
$\therefore$ $k = 2$
Thus, the value of k is 2.

39. यदि $3x^3 + Kx^2 - 2x - 8$ को एउटा गुणनखण्ड $x + 2$ भए, K को मान निकाल्नुहोस् ।
Calculate the value of K if $x + 2$ is a factor of $3x^3 + Kx^2 - 2x - 8$.

⇨ Here, given, expression $f(x) = 3x^3 + Kx^2 - 2x - 8$ and one of the factor of $f(x) = (x + 2)$
Comparing $(x + 2)$ with $(x - a)$ then $a = -2$
So, $f(-2) = 0$
But, $f(-2) = 3(-2)^3 + K(-2)^2 - 2(-2) - 8$
or, $0 = -3 \times 8 + 4K + 4 - 8$
or, $4K - 24 - 4 = 0$
or, $4K - 28 = 0$
or, $4K = 28$ $\therefore$ $K = 7$
Thus, the required value of K is 7.

40. यदि $6x^3 - (k + 6)x^2 + 2kx - 25$ को एउटा गुणनखण्ड $2x - 5$ भए k को मान निकाल्नुहोस् ।
If $2x - 5$ is a factor of $6x^3 - (k + 6)x^2 + 2kx - 25$, find the value of k.

⇨ Here, given expression $f(x) = 6x^3 - (k + 6)x^2 + 2kx - 25$ and factor of $f(x) = (2x - 5) = 2\left(x - \frac{5}{2}\right)$

Comparing $\left(x - \frac{5}{2}\right)$ with $(x - a)$ then $a = \frac{5}{2}$.

So, $f\left(\frac{5}{2}\right) = 0$.

or, $f\left(\frac{5}{2}\right) = 6\left(\frac{5}{2}\right)^3 - (k+6)\left(\frac{5}{2}\right)^2 + 2k\left(\frac{5}{2}\right) - 25$

or, $0 = 6.\frac{125}{8} - (k + 6).\frac{25}{4} + \frac{10k}{2} - 25$

or, $\frac{3 \times 125}{4} - (k+6).\frac{25}{4} + \frac{10k}{2} - 25 = 0$

or, $\frac{375 - 25k - 150 + 20k - 100}{4} = 0$

or, $375 - 250 - 5k = 0$
or, $125 = 5k$ $\therefore$ $k = 25$
Thus, the required value of k is 25.

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41. $a^4 - 3a^3 - 2a^2 + a + P$ को एउटा गुणनखण्ड $a + 1$ भए P को मान पत्ता लगाउनुहोस् ।
If $a + 1$ is a factor of $a^4 - 3a^3 - 2a^2 + a + P$, find the value of P.

⇨ Here, given function $f(a) = a^4 - 3a^3 - 2a^2 + a + P$ and $(a + 1)$ is a factor of $f(a)$.
Comparing $(a + 1)$ with $(a - A)$ then $A = -1$.
Now, by using factor theorem,
$f(A) = f(-1) = (-1)^4 - 3(-1)^3 - 2(-1)^2 + (-1) + P$
or, $0 = 1 + 3 - 2 - 1 + P$
or, $0 = P + 1$
∴ $P = -1$
Thus, the required value P is – 1.

C. LONG QUESTIONS

MODEL 1

1. हल गर्नुहोस् (Solve) : $x^3 - 3x^2 - 4x + 12 = 0$ **[SEE MODEL 2076]**

⇨ Here, $x^3 - 3x^2 - 4x + 12 = 0$
The factors of 12 are; ± 1, ± 2, ± 3, ± 4, ± 6 and ± 12
At $x = 2$, using synthetic division,

2	1	– 3	– 4	12
	↓	2	– 2	– 12
	1	– 1	– 6	0

∴ R = 0 shows that $(x - 2)$ is a factor and $x^2 - x - 6$ is the quotient.

So, Factor × Quotient = 0
or, $(x - 2)(x^2 - x - 6) = 0$
or, $(x - 2)(x^2 - 3x + 2x - 6) = 0$
or, $(x - 2)\{x(x - 3) + 2(x - 3)\} = 0$
or, $(x - 2)(x - 3)(x + 2) = 0$
Either, $x - 2 = 0$ or, $x - 3 = 0$
or, $x + 2 = 0$ ∴ $x = 2$
∴ $x = 3$ ∴ $x = -2$

Thus, $x = 2$ or 3 or – 2 is the solution.

2. हल गर्नुहोस् (Solve): $2x^3 - 9x^2 + 7x + 6 = 0$ **[2075 R]**

⇨ Here, let $f(x) = 2x^3 - 9x^2 + 7x + 6 = 0$
Probable factors of 6 are ± 1, ± 2, and ± 3.
Now, testing the factors by using synthetic division method:
At $x = 2$ using synthetic division,

2	2	– 9	7	6
	↓	4	– 10	– 6
	2	– 5	– 3	0
				R

So, remainder = 0
Hence by the factor theorem, $x - 2$ is a factor of $f(x)$ and Quotient $= 2x^2 - 5x - 3$

We know that,
Given polynomial = factor × quotient
Hence, $2x^3 - 9x^2 + 7x + 6$
$= (x - 2)(2x^2 - 5x - 3)$
$= (x - 2)(2x^2 - 6x + x - 3)$
$= (x - 2)[2x(x - 3) + 1(x - 3)]$
$= (x - 2)(2x + 1)(x - 3)$
Either, $x - 2 = 0$ ....(i) ∴ $x = 2$
or, $2x + 1 = 0$ ....(ii) ∴ $x = -\frac{1}{2}$
or, $x - 3 = 0$.......(iii) ∴ $x = 3$

Thus, from (i), (ii) and (iii), $x = 2, -\frac{1}{2}$ or 3 are the required solutions.

3. हल गर्नुहोस् (Solve): $x(x^2 + 3) - 2(3x^2 - 5) = 0$ **[2075 R'$_2$]**

⇨ Here, $x(x^2 + 3) - 2(3x^2 - 5) = 0$
or, $x^3 + 3x - 6x^2 + 10 = 0$
or, $x^3 - 6x^2 + 3x + 10 = 0$
The possible factors of 10 are ± 1, ± 2, ± 5 and ± 10
Now testing the factors by using synthetic division method:
At $x = 2$ using synthetic division,

2	1	– 6	3	10
	↓	2	– 8	– 10
	1	– 4	– 5	0
				R

∴ R = 0 shows that $(x - 2)$ is a factor.
So, Quotient $= x^2 - 4x - 5$

We know,
given polynomial = Factor × Quotient
So, $x^3 - 6x^2 + 3x + 10 = 0$
or, $(x - 2)(x^2 - 4x - 5) = 0$
or, $(x - 2)(x^2 - 5x + x - 5) = 0$
or, $(x - 2)[x(x - 5) + 1(x - 5)] = 0$
or, $(x - 2)(x + 1)(x - 5) = 0$
Either, $x - 2 = 0$ ∴ $x = 2$
or, $x + 1 = 0$ ∴ $x = -1$
or, $x - 5 = 0$ ∴ $x = 5$

Thus, $x = -1$ or 2 or 5 are the solutions of given polynomial equation.

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4. हल गर्नुहोस् (Solve): $2x^3 + x^2 - 5x + 2 = 0$ **[2074 S]**

⇨ Here, $2x^3 + x^2 - 5x + 2 = 0$

The factors of 2 are ± 1, ± 2.

At x = 1, using synthetic division;

$$\begin{array}{c|ccc|c} 1 & 2 & 1 & -5 & 2 \\ & \downarrow & 2 & 3 & -2 \\ \hline & 2 & 3 & -2 & 0 \\ \hline & & Q(x) & & R \end{array}$$

Since, R = 0 so, $(x - 1)$ is a factor and $2x^2 + 3x - 2$ is the quotient.

Now, $2x^3 + x^2 - 5x + 2$

$= \text{Factor} \times \text{quotient}$

$= (x - 1)(2x^2 + 3x - 2)$

or, $(x - 1)(2x^2 + 4x - x - 2) = 0$

or, $(x - 1)\{2x(x + 2) - 1(x + 2)\} = 0$

or, $(x - 1)(x + 2)(2x - 1) = 0$

Either, $x - 1 = 0 \quad \Rightarrow \quad x = 1$

or, $x + 2 = 0 \quad \Rightarrow \quad x = -2$

or, $2x - 1 = 0 \quad \Rightarrow \quad x = \frac{1}{2}$

Thus, $x = 1$ or -2 or $\frac{1}{2}$ is the solution.

5. हल गर्नुहोस् (Solve): $2x^3 + 9x^2 + 7x - 6 = 0$ **[2074 R]**

⇨ Here, $2x^3 + 9x^2 + 7x - 6 = 0$

Factors of 6 are ± 1, ± 2, ± 3, ± 6.

At x = – 2, using synthetic division;

$$\begin{array}{c|ccc|c} -2 & 2 & 9 & 7 & -6 \\ & \downarrow & -4 & -10 & 6 \\ \hline & 2 & 5 & -3 & 0 \\ \hline & & Q(x) & & R \end{array}$$

Since, R = 0 shows that $(x + 2)$ is a factor of given polynomial and quotient $(Q(x)) = 2x^2 + 5x - 3$.

Now, Factor × Quotient = 0

or, $(x + 2)(2x^2 + 5x - 3) = 0$

or, $(x + 2)(2x^2 + 6x - 1x - 3) = 0$

or, $(x + 2)\{2x(x + 3) - 1(x + 3)\} = 0$

or, $(x + 2)(x + 3)(2x - 1) = 0$

Either, $x + 2 = 0 \quad \therefore \quad x = -2$

or, $x + 3 = 0 \quad \therefore \quad x = -3$

or, $2x - 1 = 0 \quad \therefore \quad x = \frac{1}{2}$

Thus, $x = -2$ or -3 or $\frac{1}{2}$ is the solution.

6. हल गर्नुहोस् (Solve): $x^2(2x + 3) - (11x + 6) = 0$ **[2073 S]**

⇨ Here, $x^2(2x + 3) - (11x + 6) = 0$

We know that, given polynomial = factor × quotient

So, $f(x) = 2x^3 + 3x^2 - 11x - 6$

$= 2x^3 - 4x^2 + 7x^2 - 14x + 3x - 6$

$= 2x^3 + 3x^2 - 11x - 6$

Factors of 6 are ± 1, ± 2, ± 3, ± 6.

At x = 2, using synthetic division;

$$\begin{array}{c|ccc|c} 2 & 2 & 3 & -11 & -6 \\ & \downarrow & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & 0 \\ \hline & & Q(x) & & R \end{array}$$

Since, R = 0 shows that $(x - 2)$ is a factor of given polynomial and quotient $(Q(x)) = 2x^2 + 7x + 3$.

Now, $f(x) = \text{Factor} \times \text{Quotient}$

$= (x - 2)(2x^2 + 7x + 3)$

$= (x - 2)(2x^2 + x + 6x + 3)$

$= (x - 2)\{x(2x + 1) + 3(2x + 1)\}$

$= (x - 2)(x + 3)(2x + 1)$

But $f(x) = 0$

So, $(x - 2)(x + 3)(2x + 1) = 0$

Either, $x - 2 = 0$ ....(i)

or, $x + 3 = 0$ ....(ii)

or, $2x + 1 = 0$ ....(iii)

Now, from (i), (ii) and (iii);

$x = 2, x = -3$ and $x = -\frac{1}{2}$

Thus, $x = 2, -3, -\frac{1}{2}$ are the solutions of the given polynomial equation.

7. हल गर्नुहोस् (Solve): $x^2(x - 1) = 2(7x - 12)$ **[2073 S']**

⇨ Here, $x^2(x - 1) = 2(7x - 12)$

or, $x^3 - x^2 = 14x - 24$

or, $x^3 - x^2 - 14x + 24 = 0$

Factors of 24 are ± 1, ± 2, ± 3, ± 4, ± 6, ± 8, ± 12, ± 24

At x = 2, using synthetic division,

$$\begin{array}{c|ccc|c} 2 & 1 & -1 & -14 & 24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

∴ R = 0 shows that $(x - 2)$ is a factor and Quotient $= x^2 + x - 12$

Now, Factor × Quotient = 0

or, $(x - 2)(x^2 + x - 12) = 0$

or, $(x - 2)(x^2 + 4x - 3x - 12) = 0$

or, $(x - 2)\{x(x + 4) - 3(x + 4)\} = 0$

or, $(x - 2)(x + 4)(x - 3) = 0$

Either $x - 2 = 0 \quad \therefore \quad x = 2$

or, $x + 4 = 0 \quad \therefore \quad x = -4$

or, $x - 3 = 0 \quad \therefore \quad x = 3$

Thus, $x = 2$ or 3 or -4 is the solution.

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8. हल गर्नुहोस् (Solve): $6x^3 + x^2 - 19x + 6 = 0$ **[2072 S]**

⇨ Here, $p(x) = 6x^3 + x^2 - 19x + 6 = 0$

Factor of 6 are ± 1, ± 2, ± 3, ± 6

At $x = -2$,

$$\begin{array}{r|rrr|r} -2 & 6 & 1 & -19 & 6 \\ & \downarrow & -12 & 22 & -6 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

∴ R = 0 shows that $(x + 2)$ is a factor and

Quotient = $6x^2 - 11x + 3$

Now, Factor × Quotient = 0

or, $(x + 2)(6x^2 - 11x + 3) = 0$

or, $(x - 2)(6x^2 - 9x - 2x + 3) = 0$

or, $(x - 2)\{3x(2x - 3) - 1(2x - 3)\} = 0$

or, $(x - 2)(2x - 3)(3x - 1) = 0$

Either $x + 2 = 0 \Rightarrow x = -2$

or, $2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

or, $3x - 1 = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$

Thus, $x = -2, \frac{3}{2}, \frac{1}{3}$ is the solution.

9. हल गर्नुहोस् (Solve) : $x^3 - 4x^2 + x + 6 = 0$ **[2070 R']**

⇨ Here, $x^3 - 4x^2 + x + 6 = 0$

Factors of 6 are ± 1, ± 2, ± 3, ± 6

At $x = -1$

$$\begin{array}{r|rrr|r} -1 & 1 & -4 & 1 & 6 \\ & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

Since R = 0 so $(x + 1)$ is a factor.

Factor = $(x + 1)$ and Quotient = $(x^2 - 5x + 6)$

Thus, $x = -1, 3, 2$ is the solution.

We have, p(x) = Factor × Quotient

or, $0 = (x + 1)(x^2 - 5x + 6)$

$= (x + 1)(x^2 - 3x - 2x + 6)$

$= (x + 1)\{x(x - 3) - 2(x - 3)\}$

∴ $0 = (x + 1)(x - 3)(x - 2)$

Either, $x + 1 = 0 \Rightarrow x = -1$

or $x - 3 = 0 \Rightarrow x = 3$

or $x - 3 = 0 \Rightarrow x = 2$

10. हल गर्नुहोस् (Solve): $x^3 - 7x^2 + 7x + 15 = 0$ **[2057 R, 2066R']**

⇨ Here, given: $x^3 - 7x^2 + 7x + 15 = 0$

Where, possible factors of 15 are ± 1, ± 3, ± 5, ± 15

Now, testing the factors by using the synthetic division method:

At x = 3 using synthetic division,

$$\begin{array}{r|rrr|r} 3 & 1 & -7 & 7 & 15 \\ & \downarrow & +3 & -12 & -15 \\ \hline & 1 & -4 & -5 & 0 \\ \hline & & Q & & R \end{array}$$

So, remainder = 0

∴ $x - 3$ is a factor.

Quotient = $x^2 - 4x - 5$

We know that,

given polynomial = factor × quotient

Hence, $x^3 - 7x^2 + 7x + 15$

$= (x - 3)(x^2 - 4x - 5)$

$= (x - 3)(x^2 + x - 5x - 5)$

$= (x - 3)(x + 1)(x - 5)$

But, $x^3 - 7x^2 + 7x + 15 = 0$

∴ $(x - 3)(x + 1)(x - 5) = 0$

Either, $x - 3 = 0$ .... (i),

or, $x + 1 = 0$ .... (ii),

or, $x - 5 = 0$ .... (iii)

Now, From (i), (ii) & (iii)

Thus, x = 3, – 1, 5 are the solutions of the given polynomial equation.

11. हल गर्नुहोस् (Solve): $x^3 - 4x^2 - 7x + 10 = 0$ **[2057 S]**

⇨ Here, Given equation, $f(x) = x^3 - 4x^2 - 7x + 10 = 0$

Where, possible factors of 10 are ± 1, ± 2, ± 5, ± 10

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

$$\begin{array}{r|rrr|r} 1 & 1 & -4 & -7 & 10 \\ & \downarrow & +1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \\ \hline & & Q & & R \end{array}$$

So, remainder = 0

∴ $x - 1$ is a factor.

Quotient = $x^2 - 3x - 10$

We know that,

given polynomial = factor × quotient

Hence, $f(x) = x^3 - x^2 - 3x^2 + 3x - 10x + 10$

$= (x - 1)(x^2 - 3x - 10)$

$= (x - 1)(x^2 + 2x - 5x - 10)$

$= (x - 1)\{x(x + 2) - 5(x + 2)\}$

$= (x - 1)(x + 2)(x - 5)$

But $f(x) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

Either, $x - 1 = 0$ ..... (i),

or, $x + 2 = 0$ .... (ii),

or, $x - 5 = 0$ .... (iii)

Now, From (i), (ii) & (iii); x = 1, x = – 2 and x = 5

Thus, x = 1, – 2, 5 are the solutions of the given polynomial equation.

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12. हल गर्नुहोस् (Solve): $x^3 - 8x^2 + 19x - 12 = 0$ **[2066 R, 2065 S]**

⇨ Here, $x^3 - 8x^2 + 19x - 12 = 0$

The factors of 12 are: ± 1, ± 2, ± 3, ± 4, ± 6, ± 12

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

1	1	– 8	19	– 12
	↓	1	– 7	12
	1	– 7	12	0
		Q		R

Since remainder is zero.

So, $(x - 1)$ is a factor and the quotient is $x^2 - 7x + 12$.

We know that,

given polynomial = factor × quotient

Now, $x^3 - 8x^2 + 19x - 12 = 0$

or, $(x - 1)(x^2 - 7x + 12) = 0$

or, $(x - 1)(x^2 - 4x - 3x + 12) = 0$

or, $(x - 1)\{x(x - 4) - 3(x - 4)\} = 0$

or, $(x - 1)(x - 4)(x - 3) = 0$

Either, $x - 1 = 0$ ..... (i),

or, $x - 4 = 0$ .... (ii),

or, $x - 3 = 0$ .... (iii)

Now, From (i), (ii) & (iii); x = 1, x = 3 and x = 4

Thus, x = 1 or 3 or 4 are the solutions of the given polynomial equation.

13. हल गर्नुहोस् (Solve): $x^3 - 3x^2 - 10x + 24 = 0$ **[2072 R', 2065 R, 2067 R']**

⇨ Here, $P(x) = x^3 - 3x^2 - 10x + 24 = 0$

The factors of 24 are: ± 1, ± 2, ± 3, ± 4, ± 6, ± 12, ± 24

Now, testing the factors by using the synthetic division method:

At x = 2 using synthetic division,

2	1	– 3	– 10	24
	↓	2	– 2	– 24
	1	– 1	– 12	0
		Q		

∴ R = 0 shows that $(x - 2)$ is a factor of P(x)

Quotient = $x^2 - x - 12$

We know that,

given polynomial = factor × quotient

So, $x^3 - 3x^2 - 10x + 24 = 0$

or, $(x - 2)(x^2 - x - 12) = 0$

or, $(x - 2)(x^2 - 4x + 3x - 12) = 0$

or, $(x - 2)\{x(x - 4) + 3(x - 4)\} = 0$

or, $(x - 2)(x - 4)(x + 3) = 0$

Either, $x - 2 = 0$ .... (i)

or, $x - 4 = 0$ .... (ii)

or, $x + 3 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = 2, x = 4 and x = – 3

Thus, x = 2, 4 and – 3 are the solutions of the given polynomial equation.

14. हल गर्नुहोस् (Solve): $x^3 - 6x^2 + 11x - 6 = 0$ **[2075 R', 2073 R', 2068 S', 2067 S, 2066 S]**

⇨ Here, $x^3 - 6x^2 + 11x - 6 = 0$

The factors of 6 are; ± 1, ± 2, ± 3 and ± 6

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

1	1	– 6	11	– 6
	↓	1	– 5	6
	1	– 5	6	0
		Q		R

∴ R = 0 shows that $(x - 1)$ is a factor.

$Q(x) = x^2 - 5x + 6$

We know that,

given polynomial = factor × quotient

So, $x^3 - 6x^2 + 11x - 6 = 0$

or, $(x - 1)(x^2 - 5x + 6) = 0$

or, $(x - 1)(x^2 - 3x - 2x + 6) = 0$

or, $(x - 1)\{x(x - 3) - 2(x - 3)\} = 0$

or, $(x - 1)(x - 3)(x - 2) = 0$

Either, $x - 1 = 0$ .... (i)

or, $x - 3 = 0$ .... (ii)

or, $x - 2 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = 1, x = 3 and x = 2

Thus, x = 1 or 3 or 2 are the solutions of the given polynomial equation.

15. हल गर्नुहोस् (Solve): $2x^3 + 3x^2 - 11x - 6 = 0$ **[2069 S, 2071 R', 2074 R', 2059 R]**

⇨ Here, $2x^3 + 3x^2 - 11x - 6 = 0$

The factors of 6 are; ± 1, ± 2, ± 3 and ± 6

Now, testing the factors by using the synthetic division method:

At x = 2 using synthetic division,

2	2	3	– 11	– 6
	↓	4	14	6
	2	7	3	0
		Q		R

∴ R = 0 shows that $(x - 2)$ is a factor.

$Q(x) = 2x^2 + 7x + 3$

We know that,

given polynomial = factor × quotient

So, $f(x) = 2x^3 - 4x^2 + 7x^2 - 14x + 3x - 6$

$= (x - 2)(2x^2 + 7x + 3)$

$= (x - 2)(2x^2 + x + 6x + 3)$

$= (x - 2)\{x(2x + 1) + 3(2x + 1)\}$

$= (x - 2)(x + 3)(2x + 1)$

But $f(x) = 0$

So, $(x - 2)(x + 3)(2x + 1) = 0$

Either, $x - 2 = 0$ ...... (i)

or, $x + 3 = 0$ .... (ii)

or, $2x + 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = 2, x = – 3, and $x = -\frac{1}{2}$

Thus, $x = 2, -3, -\frac{1}{2}$ are the solutions of the given polynomial equation.

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16. हल गर्नुहोस् (Solve): $2x^3 + x^2 - 2x - 1 = 0$ **[2060 R]**

⇨ Here, given equation, $2x^3 + x^2 - 2x - 1 = 0$

The factors of 6 are; ± 1, ± 2, ± 3 and ± 6

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

$$\begin{array}{r|rrr|r} 1 & 2 & 1 & -2 & -1 \\ & \downarrow & 2 & 3 & 1 \\ \hline & 2 & 3 & 1 & 0 \\ \hline & & Q & & R \end{array}$$

Here, remainder = 0.

So, x – 1 is a factor of the given expression.

$Q(x) = 2x^2 + 3x + 1$

We have, given polynomial = factor × quotient

So, $2x^3 + x^2 - 2x - 1$

$= (x - 1)(2x^2 + 3x + 1)$

$= (x - 1)(2x^2 + 2x + x + 1)$

$= (x - 1)[2x(x + 1) + 1(x + 1)]$

$= (x - 1)(x + 1)(2x + 1)$

But, $2x^3 + x^2 - 2x - 1 = 0$

So, $(x - 1)(x + 1)(2x + 1) = 0$

Either, $x - 1 = 0$ .... (i),

or, $x + 1 = 0$ ..... (ii)

or, $2x + 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 1, -1$ and $-\frac{1}{2}$

Thus, $x = 1, -1$ and $-\frac{1}{2}$ are the solutions of the given polynomial equation.

17. हल गर्नुहोस् (Solve): $2x^3 - 3x^2 - 3x + 2 = 0$ **[2074 S', 2060 S, 2065 M]**

⇨ Here, Given equation, $f(x) = 2x^3 - 3x^2 - 3x + 2 = 0$

The factors of 2 are; ± 1 and ± 2

Now,

Testing the factors by using the synthetic division method:

At x = – 1 using synthetic division,

$$\begin{array}{r|rrr|r} -1 & 2 & -3 & -3 & 2 \\ & \downarrow & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 0 \\ \hline & & Q & & R \end{array}$$

Here, remainder = 0.

So, x + 1 is a factor of the given expression.

$Q(x) = 2x^2 - 5x + 2$

We know that,

given polynomial = factor × quotient

So, $f(x) = 2x^3 - 3x^2 - 3x + 2$

$= (x + 1)(2x^2 - 5x + 2)$

$= (x + 1)(2x^2 - x - 4x + 2)$

$= (x + 1)\{x(2x - 1) - 2(2x - 1)\}$

$= (x + 1)(2x - 1)(x - 2)$

But, $f(x) = 0$

$\therefore\ (x + 1)(2x - 1)(x - 2) = 0$

Either, $x + 1 = 0$ ..... (i)

or, $2x - 1 = 0$ ..... (ii)

or, $x - 2 = 0$ ..... (iii)

Now, From (i), (ii) and (iii); $x = -1,\ x = \frac{1}{2}$ and $x = 2$

Thus, $x = -1, \frac{1}{2}, 2$ are the solutions of the given polynomial equation.

18. हल गर्नुहोस् (Solve): $2x^3 + 5x^2 - 4x - 3 = 0$ **[2061 S]**

⇨ Here, given equation $2x^3 + 5x^2 - 4x - 3 = 0$

The factors of 3 are; ± 1 and ± 3

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

$$\begin{array}{r|rrr|r} 1 & 2 & 5 & -4 & -3 \\ & \downarrow & 2 & 7 & 3 \\ \hline & 2 & 7 & 3 & 0 \\ \hline & & Q & & R \end{array}$$

Here, remainder = 0.

So, x – 1 is a factor of the given expression.

$Q(x) = 2x^2 + 7x + 3$

We know that,

given polynomial = factor × quotient

So, $2x^3 + 5x^2 - 4x - 3 = 0$

or, $(x - 1)(2x^2 + 7x + 3) = 0$

or, $(x - 1)(2x^2 + x + 6x + 3) = 0$

or, $(x - 1)\{x(2x + 1) + 3(2x + 1)\} = 0$

or, $(x - 1)(2x + 1)(x + 3) = 0$

Either, $x - 1 = 0$ .... (i)

or, $2x + 1 = 0$ .... (ii)

or, $x + 3 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 1$, or, $x = \frac{-1}{2}$ or, $x = -3$

Thus, $x = -3, \frac{-1}{2}$, 1 are the solutions of the given polynomial equation.

19. हल गर्नुहोस् (Solve): $2x^3 - 5x^2 - 6x + 9 = 0$ **[2060 C]**

⇨ Here, $p(x) = 2x^3 - 5x^2 - 6x + 9 = 0$

The factors of 9 are ± 1, ± 3, ± 9,

Now, testing the factors by using synthetic division method:

At x = 1 using synthetic division,

$$\begin{array}{r|rrr|r} 1 & 2 & -5 & -6 & 9 \\ & \downarrow & 2 & -3 & -9 \\ \hline & 2 & -3 & -9 & 0 \\ & Q & & & R \end{array}$$

Since Remainder = 0, So, (x – 1) is a factor of p(x).

$Q(x) = 2x^2 - 3x - 9$

We have, given polynomial = factor × quotient

So, $p(x) = (x - 1)(2x^2 - 3x - 9)$

$= (x - 1)(2x^2 - 6x + 3x - 9)$

$= (x - 1)\{2x(x - 3) + 3(x - 3)\}$

$= (x - 1)(x - 3)(2x + 3)$

Since, $p(x) = 0$;

Either, $x - 1 = 0$ .... (i)

or, $x - 3 = 0$ .... (ii)

or, $2x + 3 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 1$, or, $x = 3$, or, $x = -\frac{3}{2}$

Thus, $x = 1$ or 3 or $-\frac{3}{2}$ are the solutions of the given polynomial equation.

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20. हल गर्नुहोस् (Solve): $2x^3 - 3x^2 - 11x + 6 = 0$ **[2070 S', 2062 R]**

⇨ Here, given equation, $f(x) = 2x^3 - 3x^2 - 11x + 6 = 0$

The factors of 6 are ± 1, ± 2, ± 3 and ± 6

Now, testing the factors by using the synthetic division method:

At x = – 2 using synthetic division,

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & \downarrow & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

Since Remainder = 0

So, (x + 2) is a factor of f (x).

$Q(x) = 2x^2 - 7x + 3$

We know that,

given polynomial = factor × quotient

So, $f(x) = 2x^3 - 3x^2 - 11x + 6$

$= (x + 2)(2x^2 - 7x + 3)$

$= (x + 2)(2x^2 - x - 6x + 3)$

$= (x + 2)\{x(2x - 1) - 3(2x - 1)\}$

$= (x + 2)(2x - 1)(x - 3)$

But $f(x) = 0$ So, $(x + 2)(2x - 1)(x - 3) = 0$

Either, $x + 2 = 0$ .... (i)

or, $2x - 1 = 0$ .... (ii)

or, $x - 3 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = -2$, $x = \frac{1}{2}$ and $x = 3$

Thus, $x = -2, \frac{1}{2}, 3$ are the solutions of the given polynomial equation.

21. हल गर्नुहोस् (Solve): $3y^3 - 14y^2 - 7y + 10 = 0$ **[2068 R']**

⇨ Here, let, $P(y) = 3y^3 - 14y^2 - 7y + 10 = 0$

Possible factors of 10 are ± 1, ± 2, ± 5, ± 10

Now, testing the factors by using the synthetic division method:

At y = – 1 using synthetic division,

$$\begin{array}{r|rrrr} -1 & 3 & -14 & -7 & 10 \\ & \downarrow & -3 & 17 & -10 \\ \hline & 3 & -17 & 10 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

Since Remainder = 0

So, (y + 1) is a factor of P(y).

$Q(y) = 3y^2 - 17y + 10$

We know that,

given polynomial = factor × quotient

Now, $3y^3 - 14y^2 - 7y + 10 = 0$

or, $(y + 1)(3y^2 - 17y + 10) = 0$

or, $(y + 1)(3y^2 - 15y - 2y + 10) = 0$

or, $(y + 1)\{3y(y - 5) - 2(y - 5)\} = 0$

or, $(y + 1)(y - 5)(3y - 2) = 0$

Either, $y + 1 = 0$ .... (i)

or, $y - 5 = 0$ .... (ii)

or, $3y - 2 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $y = -1$, $y = 5$ and $y = \frac{2}{3}$

Thus, $y = -1, 5$ and $\frac{2}{3}$ are the solutions of the given polynomial equation.

22. हल गर्नुहोस् (Solve): $6x^3 + 7x^2 - x - 2 = 0$ **[2058 S]**

⇨ Here, given equation $f(x) = 6x^3 + 7x^2 - x - 2 = 0$

The factors of 2 are ± 1 and ± 2

Now, testing the factors by using the synthetic division method:

At x = – 1 using synthetic division,

$$\begin{array}{r|rrrr} -1 & 6 & 7 & -1 & -2 \\ & \downarrow & -6 & -1 & 2 \\ \hline & 6 & 1 & -2 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

Since Remainder = 0

So, (x + 1) is a factor of f (x).

$Q(x) = 6x^2 + x - 2$

We know that,

given polynomial = factor × quotient

So, $f(x) = 6x^3 + 7x^2 - x - 2$

$= (x + 1)\ (6x^2 + x - 2)$

$= (x + 1)\ (6x^2 + 4x - 3x - 2)$

$= (x + 1)\{\ 2x(3x + 2) - 1(3x + 2)\}$

$= (x + 1)(3x + 2)(2x - 1)$

But, $f(x) = 0$ So, $(x + 1)(3x + 2)(2x - 1) = 0$

Either, $x + 1 = 0$ ..... (i)

or, $3x + 2 = 0$ .... (ii)

or, $2x - 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = -1$, $x = -\frac{2}{3}$ and $x = \frac{1}{2}$

Thus, $x = -1, -\frac{2}{3}$ and $\frac{1}{2}$ are the solutions of the given polynomial equation.

23. हल गर्नुहोस् (Solve): $6x^3 - 7x^2 - 7x + 6 = 0$ **[2059 S]**

⇨ Here, given equation $f(x) = 6x^3 - 7x^2 - 7x + 6 = 0$ ...... (i)

The factors of 6 are ± 1, ± 2, ± 3 and ± 6

Now, testing the factors by using the synthetic division method:

At x = – 1 using synthetic division,

$$\begin{array}{r|rrrr} -1 & 6 & -7 & -7 & 6 \\ & \downarrow & -6 & 13 & -6 \\ \hline & 6 & -13 & 6 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

Since Remainder = 0

So, (x + 1) is a factor of f (x).

$Q(x) = 6x^2 - 13x + 6$

We have, given polynomial = factor × quotient

So, $f(x) = 6x^3 - 7x^2 - 7x + 6$

$= (x + 1)(6x^2 - 13x + 6)$

$= (x + 1)(6x^2 - 9x - 4x + 6)$

$= (x + 1)\{3x(2x - 3) - 2(2x - 3)\}$

$= (x + 1)(2x - 3)(3x - 2)$

But, $f(x) = 0$,

So, $(x + 1)(2x - 3)(3x - 2) = 0$

Either, $x + 1 = 0$ .... (i)

or, $2x - 3 = 0$ .... (ii)

or, $3x - 2 = 0$, ... (iii)

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Now, From (i), (ii) and (iii); $x = -1$, $x = \frac{3}{2}$ or, $x = \frac{2}{3}$

Thus, $x = -1, \frac{2}{3}, \frac{3}{2}$ are the solutions of the given polynomial equation.

24. हल गर्नुहोस् (Solve): $6x^3 - 13x^2 + x + 2 = 0$ **[2062 S]**

⇨ Here, Let, $p(x) = 6x^3 - 13x^2 + x + 2 = 0$

The factors of 2 are; ± 1, ± 2

Now, testing the factors by using the synthetic division method:

At x = 2 using synthetic division,

2	6	– 13	1	2
	↓	12	– 2	– 2
	6	– 1	– 1	0
		Q		R

Since remainder = 0

So, (x – 2) is a factor p(x).

$Q(x) = 6x^2 - x - 1$

We know that,

given polynomial = factor × quotient

So, $p(x) = (x - 2)(6x^2 - x - 1)$

$= (x - 2)(6x^2 - 3x + 2x - 1)$

$= (x - 2)\{3x(2x - 1) + 1(2x - 1)\}$

$= (x - 2)(2x - 1)(3x + 1)$

Since, p(x) = 0,

$\therefore \quad (x - 2)(2x - 1)(3x + 1) = 0$

Either, $x - 2 = 0$ .... (i)

or, $2x - 1 = 0$ .... (ii)

or, $3x + 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 2$, $x = \frac{1}{2}$ or, $x = -\frac{1}{3}$

Thus, $x = 2, \frac{1}{2}, -\frac{1}{3}$ are the solutions of the given polynomial equation.

25. हल गर्नुहोस् (Solve): $8x^3 - 2x^2 - 5x - 1 = 0$ **[2058 R]**

⇨ Here, given equation, $8x^3 - 2x^2 - 5x - 1 = 0$

Probable factors of 1 are 1 and – 1.

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

1	8	– 2	– 5	– 1
	↓	+ 8	+ 6	+ 1
	8	+ 6	+ 1	0
		Q		R

Since, remainder (R) = 0.

So, x – 1 is a factor of the given equation.

$Q(x) = 8x^2 + 6x + 1$

We know that,

given polynomial = factor × quotient

So, we have $8x^3 - 2x^2 - 5x - 1$

$= (x - 1)(8x^2 + 6x + 1)$

So, $8x^3 - 2x^2 - 5x - 1 = 0$ gives

$(x - 1)(8x^2 + 6x + 1) = 0$

or, $(x - 1)[8x^2 + 4x + 2x + 1] = 0$

or, $(x - 1)\{4x(2x + 1) + 1(2x + 1)\} = 0$

or, $(x - 1)(2x + 1)(4x + 1) = 0$

Either, $x - 1 = 0$ .... (i)

or, $2x + 1 = 0$ .... (ii)

or, $4x + 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 1$, $x = -\frac{1}{2}$, $x = -\frac{1}{4}$

Thus, $x = 1, -\frac{1}{2}, -\frac{1}{4}$ are the solutions.

26. हल गर्नुहोस् (Solve): $6x^3 - 5x^2 - 3x + 2 = 0$ **[2067 R]**

⇨ Here, $6x^3 - 5x^2 - 3x + 2 = 0$

Probable factors of 2 are ± 1 and ± 2.

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

1	6	– 5	– 3	2
	↓	6	1	– 2
	6	1	– 2	0
		Q		R

Since, remainder (R) = 0.

So, x – 1 is a factor of the given equation.

$Q(x) = 6x^2 + x - 2$

We know that,

given polynomial = factor × quotient

So, we have $6x^3 - 5x^2 - 3x + 2 = 0$

or, $(x - 1)(6x^2 + x - 2) = 0$

or, $(x - 1)(6x^2 + 4x - 3x - 2) = 0$

or, $(x - 1)\{2x(3x + 2) - 1(3x + 2)\} = 0$

or, $(x - 1)(3x + 2)(2x - 1) = 0$

Either, $x - 1 = 0$ .... (i)

or, $3x + 2 = 0$ .... (ii)

or, $2x - 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); $x = 1$, $x = -\frac{2}{3}$, $x = \frac{1}{2}$

Thus, $x = 1, -\frac{2}{3}, \frac{1}{2}$ are the solutions of the given polynomial equation.

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27. हल गर्नुहोस् (Solve): $2x^3 + 6 = 3x^2 + 11x$ **[2072 R, 2068 R]**

⇨ Here, $2x^3 + 6 = 3x^2 + 11x$

or, $2x^3 - 3x^2 - 11x + 6 = 0$

The factors of 6 are; ± 1, ± 2, ± 3, ± 6

Now, testing the factors by using the synthetic division method:

At x = – 2 using the synthetic division,

$$\begin{array}{r|rrr|r} -2 & 2 & -3 & -11 & 6\\ & \downarrow & -4 & +14 & -6\\ \hline & 2 & -7 & 3 & 0\\ \hline & & Q & & R \end{array}$$

Since, remainder (R) = 0.

So, x + 2 is a factor of the given equation.

$Q(x) = 2x^2 - 7x + 3$

We know that,

given polynomial = factor × quotient

Now, $2x^3 - 3x^2 - 11x + 6 = 0$

or, $(x + 2)(2x^2 - 7x + 3) = 0$

or, $(x + 2)(2x^2 - 6x - x + 3) = 0$

or, $(x + 2)\{2x(x - 3) - 1(x - 3)\} = 0$

or, $(x + 2)(x - 3)(2x - 1) = 0$

Either, x + 2 = 0 .... (i)

or, x – 3 = 0 .... (ii)

or, 2x – 1 = 0 .... (iii)

Now, From (i), (ii) and (iii); $x = -2$, $x = 3$, $x = \frac{1}{2}$

Thus, $x = -2$, 3 and $\frac{1}{2}$ are the solutions of the given polynomial equation.

MODEL 2

28. गुणन साध्य प्रयोग गरेर हल गर्नुहोस् (Solve by using factor theorem): $2x^3 + 13x^2 = 36$ **[SEE 2075 R_2]**

⇨ Here, $2x^3 + 13x^2 - 36 = 0$

The possible factors of 36 are;

± 1, ± 2, ± 3, ± 4, ± 6, ± 9

Now, testing the factors by using synthetic division method:

At $x = -2$, using synthetic division,

$$\begin{array}{r|rrr|r} -2 & 2 & 13 & 0 & -36\\ & \downarrow & -4 & -18 & 36\\ \hline & 2 & 9 & -18 & 0\\ \hline & & & & R \end{array}$$

Since remainder (R) = 0

So, $x + 2$ is a factor of given polynomial.

Thus, $x = -2$ or $\frac{3}{2}$ or – 6.

We know that,

given polynomial = factor × quotient

So, $2x^3 + 13x^2 - 36 = 0$

or, $(x - 2)(2x^2 + 9x - 18) = 0$

or, $(x + 2)(2x^2 + 12x - 3x - 18) = 0$

or, $(x + 2)[2x(x + 6) - 3(x + 6)] = 0$

or, $(x + 2)(2x - 3)(x + 6) = 0$

Either, $x + 2 = 0$ or, $2x - 3 = 0$ or, $x + 6 = 0$

$\therefore$ $x = -2$ $\therefore$ $x = \frac{3}{2}$ $\therefore$ $x = -6$

29. हल गर्नुहोस् (Solve) : $3x^3 = 7x^2 - 4$ **[2073 R]**

⇨ Here, $3x^3 = 7x^2 - 4$

or, $3x^3 - 7x^2 + 4 = 0$

Factors of 4 are ± 1, ± 2, ± 4

At x = 1 then,

$$\begin{array}{r|rrr|r} 1 & 3 & -7 & 0 & 4\\ & & 3 & -4 & -4\\ \hline & 3 & -4 & -4 & 0 \end{array}$$

R = 0 shows that (x – 1) is a factor.

$\therefore$ $Q(x) = 3x^2 - 4x - 4$.

Thus, x = 1 or 2 or $-\frac{2}{3}$ is the solution

So, Factor × Q(x) = 0

or, $(x - 1)(3x^2 - 4x - 4) = 0$

or, $(x - 1)(3x^2 - 6x + 2x - 4) = 0$

or, $(x - 1)\{3x(x - 2) + 2(x - 2)\} = 0$

or, $(x - 1)(x - 2)(3x + 2) = 0$

Either, x – 1 = 0 or, x – 2 = 0 or, 3x + 2 = 0

$\therefore$ x = 1 or, x = 2 $\therefore$ $x = -\frac{2}{3}$

30. हल गर्नुहोस् (Solve) : $6x^3 = 4 - 13x^2$ **[2071 R]**

⇨ Here, $6x^3 = 4 - 13x^2$

or, $6x^3 + 13x^2 - 4 = 0$

Factors of 4 are ± 1, ± 2, ± 4

Using synthetic division taking x = – 2

$$\begin{array}{r|rrr|r} -2 & 6 & 13 & 0 & -4\\ & \downarrow & -12 & -2 & 4\\ \hline & 6 & 1 & -2 & 0 \end{array}$$

R = 0 shows that (x + 2) is a factor and $6x^2 - x - 2$ is a quotient.

Thus, $x = -2, \frac{2}{3}, -\frac{1}{2}$ is the solution.

Now, $6x^3 + 13x^2 - 4 = 0$

or, $(x + 2)(6x^2 - x - 2) = 0$

or, $(x + 2)(6x^2 - 4x + 3x - 2) = 0$

or, $(x + 2)\{2x(3x - 2) + 1(3x - 2)\} = 0$

$\therefore$ $(x + 2)(3x - 2)(2x + 1) = 0$

Either x + 2 = 0 or, 3x – 2 = 0 or, 2x + 1 = 0

$\therefore$ $x = -2$ $\therefore$ $x = \frac{2}{3}$ $\therefore$ $x = -\frac{1}{2}$

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31. हल गर्नुहोस् (Solve): $x^3 = 7x^2 - 36$ **[2071 S]**

⇨ Here, $x^3 = 7x^2 - 36$ or, $x^3 - 7x^2 + 36 = 0$

Factors of 36 are ± 1, ± 2, ± 3, ± 4, ± 9, ± 12, ± 18, ± 36

Using synthetic division at x = – 2

– 2	1	– 7	0	36
	↓	– 2	18	– 36
	1	– 9	18	0

R = 0 shows that (x + 2) is a factor.

Quotient = $x^2 - 9x + 18$

So, $x^3 - 7x^2 + 36 = 0$

or, $(x + 2)(x^2 - 9x + 18) = 0$

or, $(x + 2)(x^2 - 6x - 3x + 18) = 0$

or, $(x + 2)\{x(x - 6) - 3(x - 6)\} = 0$

or, $(x + 2)(x - 6)(x - 3) = 0$

Either $x + 2 = 0$ or, $x - 6 = 0$ or, $x - 3 = 0$

∴ $x = -2$ ∴ $x = 6$ ∴ $x = 3$

Thus, x = – 2 or 3 or 6 is the solution.

32. हल गर्नुहोस् (Solve): $x^3 - 21x - 20 = 0$ **[2070 R]**

⇨ Here, $x^3 - 21x - 20 = 0$

Factors of 20 are ± 1, ± 2, ± 4, ± 5, ± 10, ± 20

At x = – 1

– 1	1	0	– 21	– 20
	0 ↓	– 1	1	20
	1	– 1	– 20	0 = R

Since R = 0 so (x + 1) is a factor.

Quotient $Q(x) = x^2 - x - 20$

Now, factor × Q(x) = P(x)

or, $(x + 1)(x^2 - x - 20) = 0$

or, $(x + 1)(x^2 - 5x + 4x - 20) = 0$

or, $(x + 1)\{x(x - 5) + 4(x - 5)\} = 0$

or, $(x + 1)(x - 5)(x + 4) = 0$

Either $x + 1 = 0$ ∴ $x = -1$

or, $x - 5 = 0$ ∴ $x = 5$

or, $x + 4 = 0$ ∴ $x = -4$

Thus, x = – 1 or 5 or – 4 is the solution.

33. हल गर्नुहोस् (Solve): $x^3 - 3x - 2 = 0$ **[2060 S']**

⇨ Here, $x^3 - 3x - 2 = 0$

Possible factors of 2 are ± 1, ± 2

Now, testing the factors by using the synthetic division method:

At x = – 1 using the synthetic division,

– 1	1	0	– 3	– 2
	↓	– 1	1	+ 2
	1	– 1	– 2	0
		Q		R

Since, remainder (R) = 0.

So, x + 1 is a factor of the given equation.

$Q(x) = x^2 - x - 2$

We know that,

given polynomial = factor × quotient

So, $p(x) = x^3 - 3x - 2$

$= (x + 1)(x^2 - x - 2)$

$= (x + 1)(x^2 - 2x + x - 2)$

$= (x + 1)\{x(x - 2) + 1(x - 2)\}$

$= (x + 1)(x - 2)(x + 1)$

Now, $(x + 1)(x - 2)(x + 1) = 0$

Either, $x + 1 = 0$ ..... (i)

or, $x - 2 = 0$ .... (ii)

or, $x + 1 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = – 1, x = 2, x = – 1

Thus, x = – 1 and 2 are the solutions of the given polynomial equation.

34. हल गर्नुहोस् (Solve): $z^3 - 19z - 30 = 0$ **[2065 R']**

⇨ Here, $z^3 - 19z - 30 = 0$

Factors of 30 are; ± 1, ± 2, ± 3, ± 5, ± 6, ± 10, ± 15, ± 30

Now, Using the synthetic division method:

At z = – 2 using synthetic division,

– 2	1	0	– 19	– 30
	↓	– 2	4	30
	1	– 2	– 15	0
		Q		R

Since, remainder (R) = 0.

So, z + 2 is a factor of the given equation.

$Q(x) = z^2 - 2z - 15$

We know that,

given polynomial = factor × quotient

Now, $z^3 - 19z - 30 = 0$

or, $(z + 2)(z^2 - 2z - 15) = 0$

or, $(z + 2)(z^2 - 5z + 3z - 15)\ 0$

or, $(z + 2)\{z(z - 5) + 3(z - 5)\} = 0$

or, $(z + 2)(z - 5)(z + 3) = 0$

Either, $z + 2 = 0$ .... (i)

or, $z - 5 = 0$ .... (ii)

or, $z + 3 = 0$ .... (iii)

Now, From (i), (ii) and (iii); z = – 2, z = 5 and z = – 3.

Thus, z = – 2, – 3, 5 are the solutions of the given polynomial equation.

35. हल गर्नुहोस् (Solve): $3x^3 - 13x^2 + 16 = 0$ **[2069 R', 2061 R]**

⇨ Here, given equation $3x^3 - 13x^2 + 16 = 0$

Factors of 16 are; ± 1, ± 2, ± 4, ± 8, ± 16

Now, testing the factors by using the synthetic division method:

At x = – 1 using synthetic division,

– 1	3	– 13	0	16
	↓	– 3	16	– 16
	3	– 16	16	0
		Q		R

Since, remainder (R) = 0.

So, x + 1 is a factor of the given equation.

$Q(x) = 3x^2 - 16x + 16$

We have, given polynomial = factor × quotient

Now, $f(x) = 3x^3 - 13x^2 + 16$

$= (x + 1)(3x^2 - 16x + 16)$

$= (x + 1)(3x^2 - 4x - 12x + 16)$

$= (x + 1)\{x(3x - 4) - 4(3x - 4)\}$

$= (x + 1)(3x - 4)(x - 4)$

But $f(x) = 0$,

So, $(x + 1)(3x - 4)(x - 4) = 0$

Either, $x + 1 = 0$ ...... (i)

or, $3x - 4 = 0$ .... (ii)

or, $x - 4 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = – 1, $x = \frac{4}{3}$ and x = 4.

Thus, x = – 1, $\frac{4}{3}$, 4 are the solutions of the given polynomial equation.

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MODEL 3

36. हल गर्नुहोस् (Solve): $y = x^3 - 4x^2 + x + 8$ and $y = 2$ **[2062K]**

⇨ Here, $y = x^3 - 4x^2 + x + 8$ and $y = 2$

$\therefore\ 2 = x^3 - 4x^2 + x + 8$

or, $x^3 - 4x^2 + x + 6 = 0$

Let $p(x) = x^3 - 4x^2 + x + 6 = 0$

Factors of 6 are ± 1, ± 2, ± 3, ± 6

Now, testing the factors by using the synthetic division method:

At x = – 1 using synthetic division,

$$\begin{array}{r|rrr|r} -1 & 1 & -4 & 1 & 6 \\ & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \\ \hline & & Q & & R \end{array}$$

Since remainder = 0 so (x + 1) is a factor of p(x).

$Q(x) = x^2 - 5x + 6$

We know that,

given polynomial = factor × quotient

$\therefore\ p(x) = (x + 1)(x^2 - 5x + 6)$

$= (x + 1)(x^2 - 3x - 2x + 6)$

$= (x + 1)\{x(x - 3) - 2(x - 3)\}$

$= (x + 1)(x - 3)(x - 2)$

Since p(x) = 0

$\therefore\ (x + 1)(x - 3)(x - 2) = 0$

Either $x + 1 = 0$ ...... (i)

or, $x - 3 = 0$ ..... (ii)

or, $x - 2 = 0$ .... (iii)

Now, From (i), (ii) and (iii); x = – 1, x = 3 and x = 2.

Thus, x = – 1, 3, 2 are the solutions of the given polynomial equation.

37. हल गर्नुहोस् (Solve): $y = 7$ and $y = x^3 + 7x^2 - 21x - 20$

⇨ Here, $y = x^3 + 7x^2 - 21x - 20$

$\therefore\ 7 = x^3 + 7x^2 - 21x - 20$

or, $x^3 + 7x^2 - 21x - 27 = 0$

Let $P(x) = x^3 + 7x^2 - 21x - 27 = 0$

Factors of 27 are ± 1, ± 3, ± 9, ± 27

Now, testing the factors by using the synthetic division method:

At $x = 3$ using synthetic division,

$$\begin{array}{r|rrr|r} 3 & 1 & 7 & -21 & -27 \\ & \downarrow & 3 & 30 & 27 \\ \hline & 1 & 10 & 9 & 0 \\ \hline & & & & R \end{array}$$

Since remainder = 0 so $(x - 3)$ is a factor of $P(x)$.

$\therefore\ Q(x) = x^2 + 10x + 9$

We know that,

Given polynomial = factor × quotient

$\therefore\ P(x) = (x - 3)(x^2 + 10x + 9)$

$= (x - 3)(x^2 + 9x + x + 9)$

$= (x - 3)\{x(x + 9) + 1(x + 9)\}$

$= (x - 3)(x + 1)(x + 9)$

Since $P(x) = 0$

$\therefore\ (x - 3)(x + 1)(x + 9) = 0$

Either, $x - 3 = 0$ $\quad\therefore\ x = 3$

or, $x + 1 = 0$ $\quad\therefore\ x = -1$

or, $x + 9 = 0$ $\quad\therefore\ x = -9$

Thus, $x = -1$ or 3 or – 9 are the required solutions.

QUESTIONS FROM CDC TEXTBOOK

1.2.1 बहुपदीयहरू (POLYNOMIALS)

EXERCISE 1.2.1

1. (a) बहुपदीय $x^3 + x^2 + 2x + 5$ को डिग्री कति हुन्छ ? (What is the degree of polynomial $x^3 + x^2 + 2x + 5$?)

⇨ Here, the degree of given polynomial is 3.

(b) बहुपदीय $f(x)$ लाई d(x) ले भाग गर्दा भागफल q(x) र शेष r(x) भए $f(x)$ लाई d(x), q(x) र r(x) को पदमा व्यक्त गर्नुहोस् । When a polynomial $f(x)$ is divided by d(x) then quotient is q(x) and remainder is r(x). Express $f(x)$ in terms of d(x), q(x) and r(x).

⇨ Here, the required relation is $f(x) = d(x) \times q(x) + r(x)$

(c) दुई ओटा बहुपदीयको भागफलबाट प्राप्त हुने भागफल र शेषमा कसको डिग्री बढी हुन्छ ? In the division of two polynomials, which of quotient or remainder has more degree?

⇨ Here, this question is wrong.

2. भाग गर्नुहोस् (Divide):

(a) $(x^4 + 2x^2 + 3x + 5) \div x$

⇨ Here, $(x^4 + 2x^2 + 3x + 5) \div x$

$$\begin{array}{r|l} & \quad x^3 + 2x + 3 \\ \hline x & x^4 + 2x^2 + 3x + 5 \\ & -x^4 \\ \hline & \quad 2x^2 \\ & \quad -2x^2 \\ \hline & \qquad 3x \\ & \qquad -3x \\ \hline & \qquad\quad 5 \end{array}$$

$\therefore$ Quotient $= x^3 + 2x + 3$ and Remainder = 5.

(b) $(2x^3 + 4x^2 + 6x + 7) \div x^2$

⇨ Here, $(2x^3 + 4x^2 + 6x + 7) \div x^2$

$$\begin{array}{r|l} & \quad 2x + 4 \\ \hline x^2 & 2x^3 + 4x^2 + 6x + 7 \\ & -2x^3 \\ \hline & \quad 4x^2 \\ & \quad -4x^2 \\ \hline & \qquad 6x + 7 \end{array}$$

$\therefore$ Quotient $= 2x + 4$ and Remainder $= 6x + 7$.

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(c) $(x^5 + 4x^4 + 3x^3 + 7x^2 + 6x + 9) \div x^2$

⇨ Here, $(x^5 + 4x^4 + 3x^3 + 7x^2 + 6x + 9) \div x^2$

$$
\begin{array}{rl}
 & x^3 + 4x^2 + 3x + 7 \\ \cline{2-2}
x^2 \big) & x^5 + 4x^4 + 3x^3 + 7x^2 + 6x + 9 \\
 & x^5 \\
 & - \\ \cline{2-2}
 & 4x^4 \\
 & 4x^4 \\
 & - \\ \cline{2-2}
 & 3x^3 \\
 & 3x^3 \\
 & - \\ \cline{2-2}
 & 7x^2 \\
 & 7x^2 \\
 & - \\ \cline{2-2}
 & 6x + 9
\end{array}
$$

∴ Quotient $= x^3 + 4x^2 + 3x + 7$ and Remainder $= 6x + 9$.

3. भाग गर्नुहोस् **(Divide):**

(a) $(x^3 - 27) \div (x - 3)$

⇨ Here, $(x^3 - 27) \div (x - 3)$

$$
\begin{array}{rl}
 & x^2 + 3x + 9 \\ \cline{2-2}
x - 3 \big) & x^3 - 27 \\
 & x^3 - 3x^2 \\
 & - \quad + \\ \cline{2-2}
 & 3x^2 - 27 \\
 & 3x^2 - 9x \\
 & - \quad + \\ \cline{2-2}
 & 9x - 27 \\
 & 9x - 27 \\
 & - \quad + \\ \cline{2-2}
 & 0
\end{array}
$$

∴ Quotient $= x^2 + 3x + 9$ and Remainder $= 0$.

(b) $(x^3 + 6x^2 + 12x + 8) \div (x + 2)$

⇨ Here, $(x^3 + 6x^2 + 12x + 8) \div (x + 2)$

$$
\begin{array}{rl}
 & x^2 + 4x + 4 \\ \cline{2-2}
x + 2 \big) & x^3 + 6x^2 + 12x + 8 \\
 & x^3 + 2x^2 \\
 & - \quad - \\ \cline{2-2}
 & 4x^2 + 12x \\
 & 4x^2 + 8x \\
 & - \quad - \\ \cline{2-2}
 & 4x + 8 \\
 & 4x + 8 \\
 & - \quad - \\ \cline{2-2}
 & 0
\end{array}
$$

∴ Quotient $= x^2 + 4x + 4$ and Remainder $= 0$.

(c) $(x^4 - 16) \div (x - 2)$

⇨ Here, $(x^4 - 16) \div (x - 2)$

$$
\begin{array}{rl}
 & x^3 + 2x^2 + 4x + 8 \\ \cline{2-2}
x - 2 \big) & x^4 - 16 \\
 & x^4 - 2x^3 \\
 & - \quad + \\ \cline{2-2}
 & 2x^3 - 16 \\
 & 2x^3 - 4x^2 \\
 & - \quad + \\ \cline{2-2}
 & 4x^2 - 16 \\
 & 4x^2 - 8x \\
 & - \quad + \\ \cline{2-2}
 & 8x - 16 \\
 & 8x - 16 \\
 & - \quad + \\ \cline{2-2}
 & 0
\end{array}
$$

∴ Quotient $= x^3 + 2x^2 + 4x + 8$ and Remainder $= 0$.

(d) $(x^4 - 7x^2 + 1) \div (x^2 + 3x + 1)$

⇨ Here, $(x^4 - 7x^2 + 1) \div (x^2 + 3x + 1)$

$$
\begin{array}{rl}
 & x^2 - 3x + 1 \\ \cline{2-2}
x^2 + 3x + 1 \big) & x^4 - 7x^2 + 1 \\
 & x^4 + 3x^3 + x^2 \\
 & - \quad - \quad - \\ \cline{2-2}
 & -3x^3 - 8x^2 + 1 \\
 & -3x^3 - 9x^2 - 3x \\
 & + \quad + \quad + \\ \cline{2-2}
 & x^2 + 3x + 1 \\
 & x^2 + 3x + 1 \\
 & - \quad - \quad - \\ \cline{2-2}
 & 0
\end{array}
$$

∴ Quotient $= x^2 - 3x + 1$ and remainder $= 0$.

(e) $(x^4 + x^2 + 1) \div (x^2 - x + 1)$

⇨ Here, $(x^4 + x^2 + 1) \div (x^2 - x + 1)$

$$
\begin{array}{rl}
 & x^2 + x + 1 \\ \cline{2-2}
x^2 - x + 1 \big) & x^4 + x^2 + 1 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\ \cline{2-2}
 & x^3 + 1 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\ \cline{2-2}
 & x^2 - x + 1 \\
 & x^2 - x + 1 \\
 & - \quad + \quad - \\ \cline{2-2}
 & 0
\end{array}
$$

∴ Quotient $= x^2 + x + 1$ and remainder $= 0$.

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4. भागफल q(x), शेष r(x), र भाजक d(x) दिइएको अवस्थामा बहुपदीय $f(x)$ पत्ता लगाउनुहोस् :
Find the polynomial $f(x)$ when quotient q(x), remainder r(x) and divisor d(x) are given.

(a) d(x) = (x – 1); q(x) = 4x + 5; r(x) = 7

⇨ Here, $d(x) = (x - 1)$; $q(x) = 4x + 5$ and $r(x) = 7$

We know that,

$$\begin{aligned} f(x) &= d(x) \times q(x) + r(x) \\ &= (x-1)(4x+5)+7 \\ &= 4x^2+5x-4x-5+7 \end{aligned}$$

$\therefore \quad f(x) = 4x^2 + x + 2$

(b) d(x) = (2x – 3); q(x) = 2x + 3; r(x) = 5

⇨ Here, $d(x) = (2x - 3)$; $q(x) = 2x + 3$; $r(x) = 5$

We know that, $f(x) = d(x) \times q(x) + r(x)$

$$\begin{aligned} &= (2x-3)(2x+3)+5 \\ &= (2x)^2-3^2+5 \\ &= 4x^2-9+5 \end{aligned}$$

$\therefore \quad f(x) = 4x^2 - 4$

(c) d(x) = (7 – x); q(x) = x^2 + x + 1; r(x) = 7

⇨ Here, $d(x) = (7 - x)$; $q(x) = x^2 + x + 1$; $r(x) = 7$

We know that,

$$\begin{aligned} f(x) &= d(x) \times q(x) + r(x) \\ &= (7-x)(x^2+x+1)+7 \\ &= 7x^2+7x+7-x^3-x^2-x+7 \end{aligned}$$

$\therefore \quad f(x) = -x^3 + 6x^2 + 6x + 14$

(d) d(x) = (x^2 + 3); q(x) = 3x – 5; r(x) = 1 – x

⇨ Here, $d(x) = (x^2 + 3)$; $q(x) = 3x - 5$; $r(x) = 1 - x$

We know that,

$$\begin{aligned} f(x) &= d(x) \times q(x) + r(x) \\ &= (x^2+3)(3x-5)+1-x \\ &= 3x^3-5x^2+9x-15+1-x \end{aligned}$$

$\therefore \quad f(x) = 3x^3 - 5x^2 + 8x - 14$

5. x^3 + 2x^2 – 5x – 6 लाई क्रमशः (x + 1) र (x – 3) ले भाग गर्नुहोस् । दुवैले भाग गरेपछि प्राप्त हुने शेषका आधारमा दिइएको बहुपदीय (x + 1) र (x – 3) बिच के सम्बन्ध छ ? पत्ता लगाउनुहोस् ।
Divide x^3 + 2x^2 – 5x – 6 respectively by (x + 1) and (x – 3). On the basis of remainder thus obtained, find the relation between (x + 1) and (x – 3).

⇨ Here, $(x^3 + 2x^2 - 5x - 6) \div (x + 1)$ and $(x^3 + 2x^2 - 5x - 6) \div (x - 3)$

$$\begin{array}{r|l} & x^2+x-6 \\ \hline x+1 & x^3+2x^2-5x-6 \\ & x^3+x^2 \\ & - \quad - \\ \hline & \quad x^2-5x \\ & \quad x^2+x \\ & \quad - \quad - \\ \hline & \qquad -6x-6 \\ & \qquad -6x-6 \\ & \qquad + \quad + \\ \hline & \qquad\quad 0 \end{array} \qquad\qquad \begin{array}{r|l} & x^2+5x+10 \\ \hline x-3 & x^3+2x^2-5x-6 \\ & x^3-3x^2 \\ & - \quad + \\ \hline & \quad 5x^2-5x \\ & \quad 5x^2-15x \\ & \quad - \quad + \\ \hline & \qquad 10x-6 \\ & \qquad 10x-30 \\ & \qquad - \quad + \\ \hline & \qquad\quad 24 \end{array}$$

$(x + 1)$ divides $(x^3 + 2x^2 - 5x - 6)$ exactly. i.e. R = 0 and $(x - 3)$ doesnot divide $(x^3 + 2x^2 - 5x - 6)$ exactly i.e. R = 24. Thus, $(x + 1)$ is a factor of $(x^3 + 2x^2 - 5x - 6)$ and $(x - 3)$ is not a factor.

1.2.2 सङ्क्षिप्त भाग विधि (SYNTHETIC DIVISION)

EXERCISE 1.2.2

1. (a) सङ्क्षिप्त भाग विधिमा भाजक (divisor) को डिग्री कति हुन्छ ?
What is the degree of divisor in synthetic division method?

⇨ Here, the degree of the divisor is 1.

(b) सङ्क्षिप्त भाग विधिमा भाज्य (dividend) र भागफल (quotient) को डिग्रीको फरक कति हुन्छ ?
What is the difference of degree of dividend and quotient in synthetic division method?

⇨ Here, the difference of degree of dividend and quotient is 1.

2. सङ्क्षिप्त भाग विधिबाट भागफल र शेष पत्ता लगाउनुहोस् :
Find the quotient and remainder using synthetic division method.

(a) (x^3 – 7x^2 + 13x + 3) ÷ (x – 2)

⇨ Here, $(x^3 - 7x^2 + 13x + 3) \div (x - 2)$

Comparing $(x - 2)$ with $(x - a)$ then, a = 2

Using synthetic division method:

$$\begin{array}{r|rrr|r} 2 & 1 & -7 & 13 & 3 \\ & \downarrow & 2 & -10 & 6 \\ \hline & 1 & -5 & 3 & 9 \\ \hline & & Q & & R \end{array}$$

$\therefore$ Quotient $= x^2 - 5x + 3$ and remainder = 9.

(b) (x^3 – 3x + 10) ÷ (x + 1)

⇨ Here, $(x^3 - 3x + 10) \div (x + 1)$

Comparing $(x + 1)$ with $(x - a)$ then, a = – 1

Using synthetic division method:

$$\begin{array}{r|rrr|r} -1 & 1 & 0 & -3 & 10 \\ & \downarrow & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & 12 \\ \hline & & Q & & R \end{array}$$

$\therefore$ Quotient $= x^2 - x - 2$ and remainder = 12.

(c) (5x^4 – 2x + 5) ÷ (x + 3)

⇨ Here, $(5x^4 - 2x + 5) \div (x + 3)$

Comparing $(x + 3)$ with $(x - a)$ then, a = – 3

Using synthetic division method:

$$\begin{array}{r|rrrr|r} -3 & 5 & 0 & 0 & -2 & 5 \\ & \downarrow & -15 & 45 & -135 & 411 \\ \hline & 5 & -15 & 45 & -137 & 416 \\ \hline & & & Q & & R \end{array}$$

$\therefore$ Quotient $= 5x^3 - 15x^2 + 45x - 137$ and Remainder = 416.

(d) (x^7 + x^6 – x^5 – 2x^4 + 2) ÷ (x + 1)

⇨ Here, **(x^7 + x^6 – x^5 – 2x^4 + 2) ÷ (x + 1)**

Comparing $(x + 1)$ with $(x - a)$ then, a = – 1

Using synthetic division method:

$$\begin{array}{r|rrrrrrr|r} -1 & 1 & 1 & -1 & -2 & 0 & 0 & 0 & 2 \\ & \downarrow & -1 & 0 & 1 & 1 & -1 & 1 & -1 \\ \hline & 1 & 0 & -1 & -1 & 1 & -1 & 1 & 1 \\ \hline & & & & Q & & & & R \end{array}$$

$\therefore$ Quotient $= x^6 - x^4 - x^3 + x^2 - x +$ 1and remainder = 1.

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3. **सङ्क्षिप्त भाग विधिबाट भागफल र शेष पत्ता लगाउनुहोस् ।**
Find the quotient and remainder using synthetic division method.

(a) **$(2x^4 - 3x^2 - 1) \div (2x - 1)$**

⇨ Here, $(2x^4 - 3x^2 - 1) \div (2x - 1)$

So, $2x - 1 = 2\left(x - \frac{1}{2}\right)$

Comparing $\left(x - \frac{1}{2}\right)$ with $(x - a)$ then, $a = \frac{1}{2}$

Using synthetic division method:

$$\begin{array}{c|cccc|c} \frac{1}{2} & 2 & 0 & -3 & 0 & -1 \\ & \downarrow & 1 & \frac{1}{2} & -\frac{5}{4} & -\frac{5}{8} \\ \hline & 2 & 1 & -\frac{5}{2} & -\frac{5}{4} & -\frac{13}{8} \\ \hline & & & Q & & R \end{array}$$

$\therefore$ Quotient $= (2x^3 + x^2 - \frac{5}{2}x - \frac{5}{4}) \div 2$

$= x^3 + \frac{1}{2}x^2 - \frac{5}{4}x - \frac{5}{8}$ and

Remainder $= -\frac{13}{8}$

(b) **$(4x^4 - 3x^2 + 2) \div (4x - 1)$**

⇨ Here, $(4x^4 - 3x^2 + 2) \div (4x - 1)$

So, $4x - 1 = 4\left(x - \frac{1}{4}\right)$

Comparing $\left(x - \frac{1}{4}\right)$ with $(x - a)$ then, $a = \frac{1}{4}$

Using synthetic division method:

$$\begin{array}{c|cccc|c} \frac{1}{4} & 4 & 0 & -3 & 0 & 2 \\ & \downarrow & 1 & \frac{1}{4} & -\frac{11}{16} & -\frac{11}{64} \\ \hline & 4 & 1 & -\frac{11}{4} & -\frac{11}{16} & \frac{117}{64} \\ \hline & & & Q & & R \end{array}$$

$\therefore$ Quotient $= (4x^3 + x^2 - \frac{11}{4}x - \frac{11}{16}) \div 4$

$= x^3 + \frac{1}{4}x^2 - \frac{11}{16}x - \frac{11}{64}$ and

Remainder $= \frac{117}{64}$

(c) **$(3x^4 - 7x^2 + 6x - 2) \div (3x + 2)$**

⇨ Here, $(3x^4 - 7x^2 + 6x - 2) \div (3x + 2)$

So, $(3x + 2) = 3\left(x + \frac{2}{3}\right)$

Comparing $\left(x + \frac{2}{3}\right)$ with $(x - a)$ then, $a = -\frac{2}{3}$

Using synthetic division method:

$$\begin{array}{c|cccc|c} -\frac{2}{3} & 3 & 0 & -7 & 6 & -2 \\ & \downarrow & -2 & \frac{4}{3} & \frac{34}{9} & -\frac{176}{27} \\ \hline & 3 & -2 & -\frac{17}{3} & \frac{88}{9} & -\frac{230}{27} \\ \hline & & & Q & & R \end{array}$$

$\therefore$ Quotient $= (3x^3 - 2x^2 - \frac{17}{3}x + \frac{88}{9}) \div 3$

$= x^3 - \frac{2}{3}x^2 - \frac{17}{9}x + \frac{88}{27}$ and

Remainder $= -\frac{230}{27}$

(d) **$(4x^4 - 3x^2 + 7x + 8) \div (2x + 3)$**

⇨ Here, $(4x^4 - 3x^2 + 7x + 8) \div (2x + 3)$

So, $(2x + 3) = 2\left(x + \frac{3}{2}\right)$

Comparing $\left(x + \frac{3}{2}\right)$ with $(x - a)$ then, $a = -\frac{3}{2}$

Using synthetic division method:

$$\begin{array}{c|cccc|c} -\frac{3}{2} & 4 & 0 & -3 & 7 & 8 \\ & \downarrow & -6 & 9 & -9 & 3 \\ \hline & 4 & -6 & 6 & -2 & 11 \\ \hline & & & Q & & R \end{array}$$

$\therefore$ Quotient $= (4x^3 - 6x^2 + 6x - 2) \div 2$

$= 2x^3 - 3x^2 + 3x - 1$ and

Remainder $= 11$.

1.2.3 शेष साध्य (REMAINDER THEOREM)

EXERCISE 1.2.3

1. (a) **शेष साध्यको कथन लेख्नुहोस् । (Write the statement of remainder theorem.)**

⇨ Here, if $p(x)$ is a polynomial of degree n and $(x - a)$ is a divisor of $p(x)$ then $p(a)$ is remainder, where the degree of quotient will be $(n - 1)$.

(b) **बहुपदीय, $f(x)$ लाई cx + d ले भाग गर्दा शेष कति हुन्छ ?**
What will be the remainder if polynomial $f(x)$ is divided by cx + d?

⇨ Here, if the polynomial $f(x)$ is divided by $(cx + d)$ then $f\left(-\frac{d}{c}\right)$ is the remainder.

2. (a) **यदि $f(x) = x^3 - x^2 + 1$ र $g(x) = x + 1$ भए $f(x)$ लाई $g(x)$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।**
If $f(x) = x^3 - x^2 + 1$ and $g(x) = x + 1$ then find the remainder when $f(x)$ is divided by $g(x)$.

⇨ Here, divisor $= g(x) = x + 1$ and dividend $= f(x) = x^3 - x^2 + 1$
Taking $g(x) = 0$ then, $x + 1 = 0$ $\quad \therefore \quad x = -1$
Now, remainder $f(-1) = (-1)^3 - (-1)^2 + 1 = -1 - 1 + 1$
$\therefore \quad f(-1) = -1$
Thus, the remainder is -1.

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(b) **शेष साध्य प्रयोग गरी $x^3 - x^2 + 1$ लाई $x - 2$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।**
Find the remainder when $x^3 - x^2 + 1$ is divided by $x - 2$, using remainder theorem.

⇨ Here, dividend $= f(x) = x^3 - x^2 + 1$ and divisor $= x - 2$
So, taking $x - 2 = 0$ then $x = 2$
Now, remainder $= f(2) = 2^3 - 2^2 + 1 = 5$
Thus, the remainder is 5.

3. **तल दिइएको अवस्थामा शेष साध्य प्रयोग गरी शेष पत्ता लगाउनुहोस् :**
Find the remainder using remainder theorem in the following conditions.

(a) **$(4x^2 + 6x + 8) \div (2x - 1)$**

⇨ Here, dividend $= f(x) = 4x^2 + 6x + 8$ and divisor $= 2x - 1$

So, taking $2x - 1 = 0$ then $x = \frac{1}{2}$

$$\text{Now, remainder} = f\left(\frac{1}{2}\right)$$
$$= 4 \times \left(\frac{1}{2}\right)^2 + 6 \times \left(\frac{1}{2}\right) + 8$$
$$= 4 \times \frac{1}{4} + 6 \times \frac{1}{2} + 8 = 1 + 3 + 8$$
$$\therefore \quad f\left(\frac{1}{2}\right) = 12$$

Thus, the remainder is 12.

(b) **$(6x^3 + 4x^2 + 3x + 4) \div (3x - 4)$**

⇨ Here, dividend $= f(x) = 6x^3 + 4x^2 + 3x + 4$ and divisor $= 3x - 4$

So, taking $3x - 4 = 0$ then $x = \frac{4}{3}$

$$\text{Now, remainder} = f\left(\frac{4}{3}\right)$$
$$= 6 \times \left(\frac{4}{3}\right)^3 + 4 \times \left(\frac{4}{3}\right)^2 + 3 \times \frac{4}{3} + 4$$
$$= 6 \times \frac{64}{27} + 4 \times \frac{16}{9} + 4 + 4$$
$$= \frac{128}{9} + \frac{64}{9} + 8 = \frac{88}{3}$$

Thus, the remainder is $\frac{88}{3}$.

(c) **$(8x^4 + 5x^3 - 2x^2 + 7x - 1) \div (2x + 6)$**

⇨ Here, dividend $= f(x) = 8x^4 + 5x^3 - 2x^2 + 7x - 1$
And, divisor $= 2x + 6$
So, taking $2x + 6 = 0$ then $2x = -6$
$\therefore \quad x = -3$
Now, remainder $= f(-3)$
$= 8(-3)^4 + 5(-3)^3 - 2(-3)^2 + 7(-3) - 1$
$= 8 \times 81 - 5 \times 27 - 2 \times 9 - 21 - 1$
$= 648 - 135 - 18 - 22$
$= 473$
Thus, the remainder is 473.

(d) **$(5x^4 - 6x^3 + 8x^2 - 10x + 12) \div (3x + 9)$**

⇨ Here, dividend $= f(x) = 5x^4 - 6x^3 + 8x^2 - 10x + 12$
And, divisor $= 3x + 9$
So, taking $3x + 9 = 0$ then $3x = -9$
$\therefore \quad x = -3$
Now, remainder $= f(-3)$
$= 5(-3)^4 - 6(-3)^3 + 8(-3)^2 - 10(-3) + 12$
$= 5 \times 81 + 6 \times 27 + 8 \times 9 + 30 + 12$
$= 681$
Thus, the remainder is 681.

4. **(a)** **यदि बहुपदीय $2x^3 + 3x^2 - kx + 4$ लाई $(x + 2)$ ले भाग गर्दा शेष 16 रहन्छ भने k को मान पत्ता लगाउनुहोस् ।**
If polynomial $2x^3 + 3x^2 - kx + 4$ is divided by $(x + 2)$, the remainder is 16. Find the value of k.

⇨ Here, polynomial $f(x) = 2x^3 + 3x^2 - kx + 4$, divisor $= x + 2$ and remainder $= 16$
So, taking $x + 2 = 0$ then $x = -2$
Now, remainder $= f(-2)$
or, $16 = 2(-2)^3 + 3(-2)^2 - k(-2) + 4$
or, $16 = -16 + 12 + 2k + 4$
or, $16 = 2k$ $\qquad \therefore \quad k = 8$
Thus, the value of k is 8.

(b) **यदि बहुपदीय $x^4 + 5x^3 - kx^2 + 7x + 10$ लाई $(x + 1)$ ले भाग गर्दा शेष 12 रहन्छ भने k को मान पत्ता लगाउनुहोस् ।**
If polynomial $x^4 + 5x^3 - kx^2 + 7x + 10$ is divided by $(x + 1)$, the remainder is 12. Find the value of k.

⇨ Here, polynomial $p(x) = x^4 + 5x^3 - kx^2 + 7x + 10$, divisor $= (x + 1)$ and remainder $= 12$
So, taking $(x + 1) = 0$ then $x = -1$
Now, remainder $= p(-1)$
or, $12 = (-1)^4 + 5(-1)^3 - k(-1)^2 + 7(-1) + 10$
or, $12 = 1 - 5 - k - 7 + 10$
or, $12 = -1 - k$ $\qquad \therefore \quad k = -13$
Thus, the value of k is -13.

(c) **यदि $4x^3 - 3mx + 5$ लाई $(x - 1)$ ले भाग गर्दा शेष 10 रहन्छ भने m को मान पत्ता लगाउनुहोस् ।**
If $4x^3 - 3mx + 5$ is divided by $(x - 1)$, the remainder is 10. Find the value of m.

⇨ Here, polynomial $p(x) = 4x^3 - 3mx + 5$, divisor $= (x - 1)$ and remainder $= 10$
So, taking $(x - 1) = 0$ then $x = 1$
Now, remainder $= p(1)$
or, $10 = 4(1)^3 - 3m(1) + 5$
or, $10 = 4 - 3m + 5$
or, $10 = 9 - 3m$ $\qquad$ or, $3m = 9 - 10 = -1$ $\qquad \therefore \quad m = -\frac{1}{3}$

Thus, the value of m is $-\frac{1}{3}$.

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(d) यदि $x^3 - 9x^2 + (k + 1)x - 7$ को एउटा गुणनखण्ड $(x - 7)$ भए k को मान पत्ता लगाउनुहोस् ।
If $(x - 7)$ is a factor of polynomial $f(x) = x^3 - 9x^2 + (k + 1)x - 7$, find the value of k.

⇨ Here, polynomial = $p(x) = x^3 - 9x^2 + (k + 1)x - 7$ and factor = $x - 7$, So, R = 0
Taking $x - 7 = 0$ then $x = 7$
Now, remainder $= p(7)$
or, $0 = 7^3 - 9 \times 7^2 + (k + 1)7 - 7$
or, $0 = 343 - 9 \times 49 + 7k + 7 - 7$
or, $0 = -98 + 7k$
or, $7k = 98$
$\therefore$ $k = 14$
Thus, the value of k is 14.

5. (a) यदि $2x^2 - 5x + a$ र $x^3 - x^2 + ax + 5$ दुवैलाई $(x + 2)$ ले भाग गर्दा बराबर शेष आउँछ भने a को मान पत्ता लगाउनुहोस् ।
If $2x^2 - 5x + a$ and $x^3 - x^2 + ax + 5$ leave equal remainder when divided by $(x + 2)$, find the value of a.

⇨ Here, let $p(x) = 2x^2 - 5x + a$ and $f(x) = x^3 - x^2 + ax + 5$ divisor = $x + 2$
So, taking $x + 2 = 0$ $\therefore$ $x = -2$
Now, remainders are equal
i.e. $p(-2) = f(-2)$
So, $2(-2)^2 - 5(-2) + a = (-2)^3 - (-2)^2 + a(-2) + 5$
or, $8 + 10 + a = -8 - 4 - 2a + 5$
or, $18 + a = -7 - 2a$
or, $3a = -25$
$\therefore$ $a = -\frac{25}{3}$

Thus, the value of a is $-\frac{25}{3}$.

(b) यदि $x^3 - ax^2 + 8x + 11$ र $2x^3 - ax^2 + 7ax + 13$ दुवैलाई $(x - 1)$ ले भाग गर्दा बराबर शेष आउँछ भने a को मान पत्ता लगाउनुहोस् ।
If $x^3 - ax^2 + 8x + 11$ and $2x^3 - ax^2 + 7ax + 13$ leave equal remainder when divided by $(x - 1)$, find the value of a.

⇨ Here, let $p(x) = x^3 - ax^2 + 8x + 11$ and $f(x) = 2x^3 - ax^2 + 7ax + 13$ divisor = $x - 1$
So, taking $x - 1 = 0$ $\therefore$ $x = 1$
Now, remainders are equal. i.e. $p(1) = f(1)$
So, $1^3 - a \times 1^2 + 8 \times 1 + 11 = 2 \times 1^3 - a \times 1^2 + 7a \times 1 + 13$
or, $1 - a + 8 + 11 = 2 - a + 7a + 13$
or, $20 - a = 15 + 6a$
or, $-7a = 15 - 20$
or, $7a = 5$
$\therefore$ $a = \frac{5}{7}$

Thus, the value of a is $\frac{5}{7}$.

1.2.4 गुणनखण्ड साध्य (FACTOR THEOREM)

EXERCISE 1.2.4

1. (a) गुणनखण्ड साध्यको कथन लेख्नुहोस् । (Write the statement of factor theorem.)

⇨ Here, if $p(x)$ be a polynomial (degree > 0) and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$. Conversely, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$.

(b) $f(x) = (x - c) \times q(x) + r(x)$ मा $f(x)$ को एउटा गुणनखण्ड $x - c$ भए शेष $r(x)$ कति हुन्छ ?
In $f(x) = (x - c) \times q(x) + r(x)$, $x - c$ is a factor of $f(x)$ then what will be the remainder $r(x)$?

⇨ Here, if $(x - c)$ is a factor of $f(x)$ then the remainder $r(x) = 0$.

(c) $f(x)$, $d(x)$ र $q(x)$ मा $f(x)$ ले n डिग्रीको बहुपदीय, $d(x)$ ले भाजक र $q(x)$ ले भागफललाई जनाउँछ भने यिनीहरूबिचको सम्बन्ध लेख्नुहोस्, जहाँ $d(x)$ ले $f(x)$ को गुणनखण्डलाई जनाउँछ ।
In $f(x)$, $d(x)$ and $q(x)$, $f(x)$ represents a polynomial of degree n, $d(x)$ is divisor and $q(x)$ is quotient then write the relation between them where $d(x)$ represents a factor of $f(x)$.

⇨ Here, the required relation is; $f(x) = d(x) \times q(x)$

2. (a) $f(x) = x^3 - 27$ को एउटा गुणनखण्ड $(x - 3)$ हुन्छ भनी गुणनखण्ड साध्यको प्रयोग गरी प्रमाणित गर्नुहोस् ।
Prove that $x - 3$ is a factor of $f(x) = x^3 - 27$ by using factor theorem.

⇨ Here, $f(x) = x^3 - 27$ and divisor = $x - 3$
Taking $x - 3 = 0$ then $x = 3$
So, $f(3) = 3^3 - 27 = 27 - 27 = 0$
Thus, $f(3) = 0$ shows that $(x - 3)$ is a factor.

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(b) **गुणनखण्ड साध्यको प्रयोग गरी $f(x) = 2x^3 + 3x^2 - 11x - 6$ को गुणनखण्ड $x - 2$ हो/होइन भनी यकिन गर्नुहोस् ।**
Determine whether $x - 2$ is a factor of $f(x) = 2x^3 + 3x^2 - 11x - 6$ by using factor theorem.

⇨ Here, $f(x) = 2x^3 + 3x^2 - 11x - 6$ and divisor $= x - 2$
Taking $x - 2 = 0$ then $x = 2$
So, $f(2) = 2 \times 2^3 + 3 \times 2^2 - 11 \times 2 - 6 = 16 + 12 - 22 - 6$
$\therefore \quad f(2) = 0$
Thus, $f(2) = 0$ shows that $(x - 2)$ is a factor.

(c) **गुणनखण्ड साध्यको प्रयोग गरी $f(x) = 6x^3 + 11x^2 - 26x - 15$ का गुणनखण्डहरू $(x + 3)$, $(2x + 1)$, $(3x - 5)$ मध्ये कुन कुन हुन् ? पत्ता लगाउनुहोस् ।**
Out of $(x + 3)$, $(2x + 1)$ and $(3x - 5)$, which are the factors of $f(x) = 6x^3 + 11x^2 - 26x - 15$? Find by using factor theorem.

⇨ Here, $f(x) = 6x^3 + 11x^2 - 26x - 15$ and divisors are ; $(x + 3)$, $(2x + 1)$, $(3x - 5)$
Taking $x + 3 = 0$ then $x = -3$
So, $f(-3) = 6(-3)^3 + 11(-3)^2 - 26(-3) - 15$
$= 6 \times (-27) + 11 \times 9 + 78 - 15$
$= -162 + 99 + 78 - 15$
$\therefore \quad f(-3) = 0$
Thus, $f(-3) = 0$ shows that $(x + 3)$ is a factor of $f(x)$.

Again, Taking $2x + 1 = 0$ then $2x = -1 \quad \therefore \quad x = -\frac{1}{2}$

$$\text{So, } f\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^3 + 11\left(-\frac{1}{2}\right)^2 - 26\left(-\frac{1}{2}\right) - 15$$
$$= -6 \times \frac{1}{8} + 11 \times \frac{1}{4} + 13 - 15$$
$$= -\frac{3}{4} + \frac{11}{4} - 2$$
$$\therefore \quad f\left(-\frac{1}{2}\right) = 0$$

Thus, $f\left(-\frac{1}{2}\right) = 0$ shows that $(2x + 1)$ is a factor of $f(x)$.

Now, Taking $3x - 5 = 0$ then $3x = 5 \therefore \quad x = \frac{5}{3}$

$$\text{So, } f\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right)^3 + 11\left(\frac{5}{3}\right)^2 - 26\left(\frac{5}{3}\right) - 15$$
$$= \frac{250}{9} + \frac{275}{9} - \frac{130}{3} - 15 \qquad \therefore \quad f\left(\frac{5}{3}\right) = 0$$

Thus, $f\left(\frac{5}{3}\right) = 0$ shows that $(3x - 5)$ is a factor of $f(x)$.

3. **(a)** **यदि बहुपदीय $4x^2 + mx + 8$ को एउटा गुणनखण्ड $(x + 2)$ भए m को मान पत्ता लगाउनुहोस् ।**
If $(x + 2)$ is a factor of polynomial $4x^2 + mx + 8$, find the value of m.

⇨ Here, $p(x) = 4x^2 + mx + 8$ and factor $= x + 2$
Taking $x + 2 = 0 \qquad \therefore \quad x = -2$
So, $p(-2) = 4(-2)^2 + m(-2) + 8$
or, $0 = 4 \times 4 - 2m + 8$
or, $2m = 16 + 8 \qquad$ or, $2m = 24 \qquad \therefore \quad m = 12$
Thus, the value of m is 12.

(b) **यदि बहुपदीय $x^3 - kx^2 + 3x + 6$ को एउटा गुणनखण्ड $(x + 1)$ भए k को मान पत्ता लगाउनुहोस् ।**
If $(x + 1)$ is a factor of polynomial $x^3 - kx^2 + 3x + 6$, find the value of k.

⇨ Here, $p(x) = x^3 - kx^2 + 3x + 6$ and factor $= x + 1$
Taking $x + 1 = 0 \qquad \therefore \quad x = -1$
So, $p(-1) = (-1)^3 - k(-1)^2 + 3(-1) + 6$
or, $0 = -1 - k - 3 + 6$
or, $0 = -k + 2 \qquad \therefore \quad k = 2$
Thus, the value of k is 2.

(c) **यदि बहुपदीय, $f(x) = 2x^3 - ax^2 - 8x + 5$ को एउटा गुणनखण्ड $(x + 1)$ भए a को मान पत्ता लगाउनुहोस् ।**
If $(x + 1)$ is a factor of polynomial $f(x) = 2x^3 - ax^2 - 8x + 5$, find the value of a.

⇨ Here, $f(x) = 2x^3 - ax^2 - 8x + 5$ and factor $= x + 1$
Taking $x + 1 = 0 \qquad \therefore \quad x = -1$
So, $f(-1) = 2(-1)^3 - a(-1)^2 - 8(-1) + 5$
or, $0 = 2 \times (-1) - a + 8 + 5$
or, $0 = -2 - a + 13 \qquad \therefore \quad a = 11$
Thus, the value of a is 11.

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4. (a) बहुपदीय $f(x) = 2x^3 + 6x^2 + 7x + 5$ मा कति जोड्दा $f(x)$ को एउटा गुणनखण्ड $(x + 3)$ हुन्छ ? पत्ता लगाउनुहोस् ।
What should be added to the polynomial $f(x) = 2x^3 + 6x^2 + 7x + 5$ so that $(x + 3)$ is a factor of $f(x)$? Find it.

⇨ Here, let m should be added then, $f(x) = 2x^3 + 6x^2 + 7x + 5 + m$

Factor = $(x + 3) = 0$ ∴ $x = -3$

So, $f(-3) = 2(-3)^3 + 6(-3)^2 + 7(-3) + 5 + m$

or, $0 = 2 \times (-27) + 54 - 21 + 5 + m$

or, $0 = -54 + 54 - 16 + m$

∴ $m = 16$

Thus, 16 should be added.

(b) बहुपदीय $g(x) = x^3 + 28$ बाट कति घटाउँदा $g(x)$ को एउटा गुणनखण्ड $(x + 4)$ हुन्छ ? पत्ता लगाउनुहोस् ।
What should be subtracted from the polynomial $g(x) = x^3 + 28$ so that $(x + 4)$ is a factor of $g(x)$? Find it.

⇨ Here, let m should be subtracted then, $g(x) = x^3 + 28 - m$ and factor = $(x + 4)$

Taking $x + 4 = 0$ ∴ $x = -4$

So, $g(-4) = (-4)^3 + 28 - m$

or, $0 = -64 + 28 - m$

or, $0 = -36 - m$

∴ $m = -36$

Thus, – 36 should be subtracted.

(c) बहुपदीय $3x^3 + 5x^2 - 5x + 7$ बाट कति घटाउँदा उक्त बहुपदीयको एउटा गुणन खण्ड $(x + 2)$ हुन्छ ? पत्ता लगाउनुहोस् ।
What should be subtracted from the polynomial $3x^3 + 5x^2 - 5x + 7$ so that $(x + 2)$ is a factor of the polynomial?

⇨ Here, let m should be subtracted then, $p(x) = 3x^3 + 5x^2 - 5x + 7 - m$ then factor = $(x + 2)$

Taking $x + 2 = 0$ ∴ $x = -2$

So, $p(-2) = 3(-2)^3 + 5(-2)^2 - 5(-2) + 7 - m$

or, $0 = 3 \times (-8) + 5 \times 4 + 10 + 7 - m$

or, $0 = -24 + 20 + 17 - m$

∴ $m = 13$

Thus, the required number is 13.

5. गुणनखण्ड साध्य र खण्डीकरण सम्बन्धी एउटा छोटो रिपोर्ट तयार गरी कक्षाकोठामा प्रस्तुत गर्नुहोस् ।
Prepare a short report on factor theorem and factorization and present it in your classroom.

⇨ Show to your teacher.

1.2.5 शेष साध्य र गुणनखण्ड साध्यको प्रयोग

(USE OF THE REMAINDER THEOREM AND FACTOR THEOREM)

EXERCISE 1.2.5

1. (a) यदि $f(x)$ को एउटा गुणनखण्ड a भए $f(a)$ को मान कति हुन्छ ? (If a factor of $f(x)$ is a, what is the value of $f(a)$?)

⇨ Here, if $(x - a)$ is a factor of $f(x)$ then $f(a) = 0$.

(b) बहुपदीय $f(x)$ को समाधान पत्ता लगाउनु भन्नाले के बुझिन्छ ?
What do you mean by finding the solution of polynomial $f(x)$?

⇨ Here, finding the solution of polynomial is finding the value of variable which satisfies the polynomial.

2. खण्डीकरण गर्नुहोस् (Factorise) :

(a) $3x^3 - 19x^2 + 32x - 16$

⇨ Here, $p(x) = 3x^3 - 19x^2 + 32x - 16$

The possible factors of 16 are;

± 1, ± 2, ± 4, ± 8, ± 16.

Using the synthetic division at $x = 1$ then,

1	3	– 19	32	– 16
	↓	3	– 16	16
	3	– 16	16	0
		Q		R

∴ $R = 0$ shows that $(x - 1)$ is a factor and $3x^2 - 16x + 16$ is a quotient.

Now, $p(x)$ = Factor × Quotient

$= (x - 1)(3x^2 - 16x + 16)$

$= (x - 1)(3x^2 - 12x - 4x + 16)$

$= (x - 1)\{3x(x - 4) - 4(x - 4)\}$

$= (x - 1)(x - 4)(3x - 4)$

Thus, $p(x) = (x - 1)(x - 4)(3x - 4)$

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(b) **$x^3 - 19x - 30$**

⇨ Here, $p(x) = x^3 - 19x - 30$
The possible factors of 30 are
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$.
Using the synthetic division at $x = -2$ then,

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & \downarrow & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

∴ R = 0 shows that $(x + 2)$ is a factor of $p(x)$ and the quotient $= x^2 - 2x - 15$

Now, $p(x)$ = Factor × Quotient
$= (x + 2)(x^2 - 2x - 15)$
$= (x + 2)(x^2 - 5x + 3x - 15)$
$= (x + 2)\{x(x - 5) + 3(x - 5)\}$
Thus, $p(x) = (x + 2)(x - 5)(x + 3)$.

(c) **$x^3 - 9x^2 + 26x - 24$**

⇨ Here, $p(x) = x^3 - 9x^2 + 26x - 24$
The possible factors of 24 are
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$.
Using the synthetic division at $x = 2$ then,

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & \downarrow & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

∴ R = 0 shows that $(x - 2)$ is a factor of $p(x)$ and the quotient is $x^2 - 7x + 12$

Now, $p(x)$ = Factor × Quotient
$= (x - 2)(x^2 - 7x + 12)$
$= (x - 2)(x^2 - 4x - 3x + 12)$
$= (x - 2)\{x(x - 4) - 3(x - 4)\}$
Thus, $p(x) = (x - 2)(x - 4)(x - 3)$

3. **खण्डीकरण गर्नुहोस् (Factorise) :**

(a) **$(x + 1)(x^2 - 5x + 10) - 12$**

⇨ Here, $p(x) = (x + 1)(x^2 - 5x + 10) - 12$
$= x^3 - 5x^2 + 10x + x^2 - 5x + 10 - 12$
$= x^3 + x^2 - 5x^2 + 10x - 5x - 2$
$= x^3 - 4x^2 + 5x - 2$

The possible factors of 2 are; $\pm 1, \pm 2$
Using the synthetic division at $x = 1$.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 5 & -2 \\ & \downarrow & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

∴ R = 0 shows that $(x - 1)$ is a factor of $p(x)$ and the quotient is $(x^2 - 3x + 2)$

Now, $p(x)$ = Factor × Quotient
$= (x - 1)(x^2 - 3x + 2)$
$= (x - 1)(x^2 - 2x - x + 2)$
$= (x - 1)\{x(x - 2) - 1(x - 2)\}$
Thus, $p(x) = (x - 1)(x - 2)(x - 1)$

(b) **$(x - 1)(2x^2 + 15x + 15) - 21$**

⇨ Here, $p(x) = (x - 1)(2x^2 + 15x + 15) - 21$
$= 2x^3 + 15x^2 + 15x - 2x^2 - 15x - 15 - 21$
∴ $p(x) = 2x^3 + 13x^2 - 36$
The possible factors of 36 are;
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$.
Using the synthetic division at $x = -2$.

$$\begin{array}{r|rrrr} -2 & 2 & 13 & 0 & -36 \\ & \downarrow & -4 & -18 & 36 \\ \hline & 2 & 9 & -18 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

∴ R = 0 shows that $(x + 2)$ is a factor of $p(x)$ and the quotient is $(2x^2 + 9x - 18)$

Now, $p(x)$ = Factor × Qutient
$= (x + 2)(2x^2 + 9x - 18)$
$= (x + 2)(2x^2 + 12x - 3x - 18)$
$= (x + 2)\{2x(x + 6) - 3(x + 6)\}$
Thus, $p(x) = (x + 2)(x + 6)(2x - 3)$.

(c) **$(x - 3)(x^2 - 5x + 8) - 4x + 12$**

⇨ Here, $p(x) = (x - 3)(x^2 - 5x + 8) - 4x + 12$
$= x^3 - 5x^2 + 8x - 3x^2 + 15x - 24 - 4x + 12$
∴ $p(x) = x^3 - 8x^2 + 19x - 12$
The possible factors of 12 are;
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
Using the synthetic division at $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 19 & -12 \\ & \downarrow & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & \multicolumn{1}{|r}{0} \\ \hline & & Q & & \multicolumn{1}{|r}{R} \end{array}$$

∴ R = 0 shows that $(x - 1)$ is a factor of $p(x)$ and the quotient is $(x^2 - 7x + 12)$

Now, $p(x)$ = Factor × Qutient
$= (x - 1)(x^2 - 7x + 12)$
$= (x - 1)(x^2 - 4x - 3x + 12)$
$= (x - 1)\{x(x - 4) - 3(x - 4)\}$
Thus, $p(x) = (x - 1)(x - 4)(x - 3)$.

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4. **हल गर्नुहोस् (Solve) :**

(a) $x^3 - 3x^2 - 4x + 12 = 0$

⇨ Here, $x^3 - 3x^2 - 4x + 12 = 0$

The factors of 12 are;

± 1, ± 2, ± 3, ± 4, ± 6 and ± 12

At $x = 2$, using synthetic division,

$$\begin{array}{c|cccc} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & \multicolumn{1}{|c}{0} \end{array}$$

∴ R = 0 shows that $(x - 2)$ is a factor and $x^2 - x - 6$ is the quotient.

So, Factor × Quotient = 0

or, $(x - 2)(x^2 - x - 6) = 0$

or, $(x - 2)(x^2 - 3x + 2x - 6) = 0$

or, $(x - 2)\{x(x - 3) + 2(x - 3)\} = 0$

or, $(x - 2)(x - 3)(x + 2) = 0$

Either, $x - 2 = 0$ or, $x - 3 = 0$ or, $x + 2 = 0$

∴ $x = 2$ ∴ $x = 3$ ∴ $x = -2$

Thus, $x = 2$ or 3 or -2 is the solution.

(b) $x^3 - 8x^2 + 19x - 12 = 0$

⇨ Here, $x^3 - 8x^2 + 19x - 12 = 0$

The factors of 12 are: ± 1, ± 2, ± 3, ± 4, ± 6, ± 12

Now, testing the factors by using the synthetic division method:

At x = 1 using synthetic division,

$$\begin{array}{c|ccc|c} 1 & 1 & -8 & 19 & -12 \\ & \downarrow & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & 0 \\ \hline & & Q & & R \end{array}$$

Since remainder is zero.

So, $(x - 1)$ is a factor and the quotient is $x^2 - 7x + 12$.

We know that,

given polynomial = factor × quotient

Now, $x^3 - 8x^2 + 19x - 12 = 0$

or, $(x - 1)(x^2 - 7x + 12) = 0$

or, $(x - 1)(x^2 - 4x - 3x + 12) = 0$

or, $(x - 1)\{x(x - 4) - 3(x - 4)\} = 0$

or, $(x - 1)(x - 4)(x - 3) = 0$

Either, $x - 1 = 0$ ..... (i),

or, $x - 4 = 0$ .... (ii),

or, $x - 3 = 0$ .... (iii)

Now, From (i), (ii) & (iii); x = 1, x = 3 and x = 4

Thus, x = 1 or 3 or 4 are the solutions of the given polynomial equation.

(c) $2y^3 + 6 = 3y^2 + 11y$

⇨ Here, $2y^3 + 6 = 3y^2 + 11y$

or, $2y^3 - 3y^2 - 11y + 6 = 0$

Let, $p(y) = 2y^3 - 3y^2 - 11y + 6 = 0$

The possible factors of 6 are ± 1, ± 2, ± 3, ± 6

Using the synthetic division at $y = -2$

$$\begin{array}{c|ccc|c} -2 & 2 & -3 & -11 & 6 \\ & \downarrow & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \\ \hline & & Q & & R \end{array}$$

∴ R = 0 shows that $(y + 2)$ is a factor of $p(y)$ and the quotient is $(2y^2 - 7y + 3)$.

Now, $p(y) = \text{Factor} \times \text{Quotient}$

$= (y + 2)(2y^2 - 7y + 3)$

$= (y + 2)(2y^2 - 6y - y + 3)$

$= (y + 2)\{2y(y - 3) - 1(y - 3)\}$

∴ $p(y) = (y + 2)(y - 3)(2y - 1) = 0$

Either, $y + 2 = 0$ ∴ $y = -2$

or, $y - 3 = 0$ ∴ $y = 3$

or, $2y - 1 = 0$ or, $2y = 1$ ∴ $y = \frac{1}{2}$

Thus, $y = -2, 3, \frac{1}{2}$ are the solutions

(d) $y^3 + 11y = 6y^2 + 6$

⇨ Here, $y^3 + 11y = 6y^2 + 6$

or, $y^3 + 11y - 6y^2 - 6 = 0$

∴ $y^3 - 6y^2 + 11y - 6 = 0$

Let, $p(y) = y^3 - 6y^2 + 11y - 6 = 0$

The possible factors of 6 are ± 1, ± 2, ± 3, ± 6.

Using the synthetic division at $y = 1$

$$\begin{array}{c|ccc|c} 1 & 1 & -6 & 11 & -6 \\ & \downarrow & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \\ \hline & & Q & & R \end{array}$$

∴ R = 0 shows that $(y - 1)$ is a factor of $p(y)$ and the quotient is $(y^2 - 5y + 6)$.

Now, $y^3 - 6y^2 + 11y - 6 = 0$

or, $(y - 1)(y^2 - 5y + 6) = 0$

or, $(y - 1)(y^2 - 3y - 2y + 6) = 0$

or, $(y - 1)\{y(y - 3) - 2(y - 3)\} = 0$

or, $(y - 1)(y - 3)(y - 2) = 0$

Either, $y - 1 = 0$ or, $y - 3 = 0$ or, $y - 2 = 0$

∴ $y = 1$ ∴ $y = 3$ ∴ $y = 2$

Thus, $y = 1$ or 3 or 2 is the solution.

5. **दुईओटा साझा गुणनखण्डहरू भएका र डिग्री 4 भएका बहुपदीयहरू $f(x)$ र $g(x)$ लाई फलनको स्वरूपमा लेख्नुहोस् । $f(x)$ र $g(x)$ का अन्य गुणनखण्डहरू पनि पत्ता लगाउनुहोस् ।**

Write the polynomials $f(x)$ and $g(x)$ in the form of function which have two common factors and degree of four. Find other factors of $f(x)$ and $g(x)$ also.

⇨ Here, let $f(x) = x^4 - 1$ and $g(x) = (x - 1)(x + 1)x^2 = (x^2 - 1)x^2 = x^4 - x^2$

So, $f(x) = x^4 - 1$

$= (x^2 - 1)(x^2 + 1)$

$= (x + 1)(x - 1)(x^2 + 1)$

$g(x) = x^4 - x^2$

$= x^2(x^2 - 1)$

$= x \cdot x \cdot (x + 1)(x - 1)$

∴ Common factor = $(x + 1)(x - 1)$

Thus, other factors of $f(x) = (x^2 + 1)$ and other factors of $g(x) = x \cdot x$.

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3. समानान्तरीय अनुक्रम
Arithmetic Sequence

Formulae and Key Points

	समानान्तरीय अनुक्रम वा श्रेणी (AS) Arithmetic Sequence or Series (AS)	सङ्केत Index
➢ साधारण पद General term	$t_n = a + (n - 1)d$	a = पहिलो पद (First term) n = पदहरूको सङ्ख्या No. of terms d = समान अन्तर Common difference b = अन्तिम पद Last term m_n = n औं मध्यमा n^{th} mean
➢ मध्यमा Mean	$AM = \frac{a+b}{2}$	
➢ मध्यमाहरू Means	$d = \frac{b-a}{n+1}$ $m_1 = a + d, m_2 = a + 2d, \ldots, m_n = a + nd$	
➢ योगफल (Sum)	$S_n = \frac{n}{2}\{2a + (n - 1)d\}$ **or** $S_n = \frac{n}{2}(a + b)$	

- पहिला n ओटा प्राकृतिक सङ्ख्याहरूको योगफल (The sum of first n-natural numbers) $= \frac{n}{2}(n + 1)$
- पहिला n ओटा बिजोर सङ्ख्याहरूको योगफल (The sum of first n-odd numbers) $= n^2$
- पहिला n ओटा जोर सङ्ख्याहरूको योगफल (The sum of first n-even numbers) $= n(n + 1)$

AS का पदहरूलाई सङ्केत गर्ने सजिलो तरिका (The easy method to denote the terms of AS):

पदहरूको सङ्ख्या (Number of terms)	3	4	5
पदहरू (Terms)	a – d, a, a + d	a – 3d, a – d, a + d, a + 3d	a – 2d, a – d, a, a + d, a + 2d
समान अन्तर (Common Difference)	d	2d	d

QUESTIONS FROM SEE EXERCISE 3

A. VERY SHORT QUESTIONS

1. दुई सङ्ख्याहरू *a* र *b* बिचको अङ्कगणितीय मध्यक के हुन्छ ?
What is the arithmetic mean between two numbers *a* and *b*? **[SEE MODEL 2076]**

⇨ Here, the arithmetic mean between two numbers *a* and *b* is given by $\frac{a+b}{2}$.

2. अनुक्रमको परिभाषा दिनुहोस् । **(Define Sequence.)**

⇨ Here, a list of numbers in definite order is called sequence. e.g. 3,5, 7…………

3. श्रेणीको परिभाषा दिनुहोस् । **(Define Series.)**

⇨ Here, when the terms of sequence are connected by addition or subtraction signs: e.g. 2 + 4 + 6 + 8 …. which is a series.

4. समानान्तरीय अनुक्रम र श्रेणीको परिभाषा दिनुहोस् । **(Define Arithmetic sequence and series.)**

⇨ Here, a sequence is said to be an arithmetic sequence when its terms increase or decrease continually by a common difference. The corresponding series of this sequence is called the arithmetic series. e.g. if 3, 7, 11, 15, …………is an arithmetic sequence then 3 + 7 + 11 + 15 + ... is an arithmetic series.

5. समानान्तरीय मध्यमा को परिभाषा दिनुहोस् । **(Define Arithmetic mean.)**

⇨ Here, in arithmetic sequence, the term/terms between the first term and the last term is/are called arithmetic mean/means, e.g. if 3, 7, 11, 15, 19, 23 are in arithmetic sequence then 7, 11, 15, 19 are arithmetic means.

6. एउटा AS को *n* औं पद के हुन्छ, जसको पहिलो पद '*a*' र समान अन्तर '*d*' छ ?
What is the n^{th} term of an AS whose first term is '*a*' and common difference is '*d*'?

⇨ Here, n^{th} term of an AS = a + (n – 1) d.

7. सूत्र $t_n = a + (n - 1)d$ मा प्रत्येक पदको अर्थ उल्लेख गर्नुहोस् ।
State the meaning of each term in the formula: $t_n = a + (n - 1)d$.

⇨ Here, $t_n = n^{th}$ term, a = first term, n = number of terms, d = common difference.

8. पहिलो पद '*a*' र समान अन्तर '*d*' भएको AS को *n* औं समानान्तरीय मध्यमा के हुन्छ ?
What is the n^{th} arithmetic mean of an AS whose first term is '*a*' and common difference is '*d*'?

⇨ Here, the n^{th} arithmetic mean = $a + nd$.

9. पहिलो पद '*a*' र समान अन्तर '*d*' भएको AS को छैटौं समानान्तरीय मध्यमा के हुन्छ ?
What is the 6^{th} arithmetic mean of an AS whose first term is '*a*' and common difference is '*d*'?

⇨ Here, the 6^{th} arithmetic mean = $a + 6d$.

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10. **n ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 'a' र अन्तिम पद 'b' छ भने समान अन्तर कति होला ?**
What is the common difference of an AS whose first term is 'a' and last term is 'b' and having 'n' arithmetic means?

⇨ Here, the common difference $(d) = \frac{b-a}{n+1}$.

11. **पहिलो n ओटा प्राकृतिक सङ्ख्याहरूको योगफल कति हुन्छ ? (What is the sum of first n natural numbers?)**

⇨ Here, the sum of first n-natural numbers $= \frac{n}{2}(n+1)$.

12. **पहिलो n ओटा जोर प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।**
Find the sum of first n even natural numbers.

⇨ Here, the sum of first n-even natural numbers $= n(n+1)$.

13. **पहिलो n ओटा बिजोर प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् । (Write the sum of first n odd natural numbers).**

⇨ Here, the sum of first n-odd natural numbers $= n^2$.

14. **पहिलो पद 'a' र अन्तिम पद 'b' भएको समानान्तरीय अनुक्रमको n ओटा पदहरूको योगफल कति हुन्छ ?**
What is the sum of n terms of arithmetic sequence whose first term is 'a' and last term is 'b'?

⇨ Here, sum of n terms $= \frac{n}{2}(a+b)$.

15. **पहिलो पद 'a' र समान अन्तर 'd' भएको समानान्तरीय अनुक्रम को n ओटा पदहरूको योगफल लेख्नुहोस् ।**
Write the sum of n terms of an AS having first term 'a' and common difference 'd'.

⇨ Here, sum of n terms $= \frac{n}{2}[2a + (n-1)d]$.

16. **दिइएको सूत्रमा हरेक पदको अर्थ लेख्नुहोस् । (Write the meaning of each term in the formula): $S_n = \frac{n}{2}\{2a + (n-1)d\}$.**

⇨ Here, S_n = sum of n terms, n = number of terms, a = first term and d = common difference.

17. **दिइएको सूत्रमा हरेक पदको अर्थ लेख्नुहोस् । (Write the meaning of each term in the formula): $S_n = \frac{n}{2}(a+\ell)$.**

⇨ Here, S_n = sum of n terms, a = first term and ℓ = last term.

18. **अनुक्रम 3, 4, 5, 6, 7 मा समान अन्तर पत्ता लगाउनुहोस् । (Find the common difference in the sequence 3, 4, 5, 6, 7.)**

⇨ Here, common difference $= t_2 - t_1 = 4 - 3 = 1$.

19. **अनुक्रम 10, 12, 14, 16 मा समान अन्तर पत्ता लगाउनुहोस् । (Find the common difference in the sequence 10, 12, 14, 16.)**

⇨ Here, common difference $= t_2 - t_1 = 12 - 10 = 2$.

20. **एउटा AP को छैटौं पद 19 र समान अन्तर 3 छ । यसको सातौं पद पत्ता लगाउनुहोस् ।**
6th term of an AP is 19 and the common difference is 3. Find its 7th term.

⇨ Here, seventh term = 6th term + common difference = 19 + 3 = 22

21. **एउटा AP को आठौं पद 44 र समान अन्तर 4 छ । यसको सातौं पद पत्ता लगाउनुहोस् ।**
8th term of an AP is 44 and the common difference is 4. Find its 7th term.

⇨ Here, seventh term = 8th term – common difference = 44 – 4 = 40

22. **19 र 25 को समानान्तरीय मध्यमा पत्ता लगाउनुहोस् । (Find the arithmetic mean between 19 and 25.)**

⇨ Here, Arithmetic mean $= \frac{a+b}{2} = \frac{19+25}{2} = \frac{44}{2} = 22$

23. **22 र 8 को समानान्तरीय मध्यमा पत्ता लगाउनुहोस् । (Find the arithmetic mean between 22 and 8.)**

⇨ Here, Arithmetic mean $= \frac{a+b}{2} = \frac{22+8}{2} = \frac{30}{2} = 15$

24. **एउटा AS को पहिलो पद र तेस्रो पद क्रमशः 84 र 72 भए दोस्रो पद पत्ता लगाउनुहोस् ।**
Find the second term of an AS whose first and third terms are 84 and 72 respectively.

⇨ Here, second term $= \frac{\text{first term} + \text{third term}}{2} = \frac{84+72}{2} = \frac{156}{2} = 78$

25. **एउटा AS को पहिलो पद र तेस्रो पद क्रमशः 90 र 80 भए दोस्रो पद पत्ता लगाउनुहोस् ।**
Find the second term of an AS whose first and third terms are 90 and 80 respectively.

⇨ Here, second term $= \frac{\text{first term} + \text{third term}}{2} = \frac{90+80}{2} = \frac{170}{2} = 85$

26. **एउटा AS को पहिलो पद 8 र समान अन्तर 2 भए $8^{\text{औं}}$ मध्यमा कति हुन्छ ?**
What is the 8th mean of an AS whose 1st term is 8 and common difference is 2?

⇨ Here, eighth mean $= a + 8d = 8 + 8 \times 2 = 24$

27. **एउटा AS मा पहिलो पद = समान अन्तर = 5 भए $5^{\text{औं}}$ मध्यमा कति होला ?**
In an AS, first term = common difference = 5 then what is the 5th mean?

⇨ Here, fifth mean $= a + 5d = 5 + 5 \times 5 = 30$

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28. 6 ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 5 र अन्तिम पद 40 भए समान अन्तर कति होला ?
What is the common difference of an AS whose first term is 5 and last term is 40 and having 6 arithmetic means?

⇨ Here, common difference $= \frac{b-a}{m+1} = \frac{40-5}{6+1} = \frac{35}{7} = 5$

29. 7 ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 4 र अन्तिम पद 44 भए समान अन्तर कति होला ?
What is the common difference of an AS whose first term is 4 and last term is 44 and having 7 arithmetic means?

⇨ Here, common difference $= \frac{b-a}{m+1} = \frac{44-4}{7+1} = \frac{40}{8} = 5$

30. पहिलो पद 2 र अन्तिम पद 20 भएको समानान्तरीय अनुक्रमको पहिलो 10 ओटा पदहरूको योगफल कति हुन्छ ?
What is the sum of first 10 terms of an arithmetic sequence whose first term is 2 and last term is 20?

⇨ Here, sum of 10 terms (S_{10}) $= \frac{n}{2}(a + \ell) = \frac{10}{2}(2 + 20) = 5 \times 22 = 110$

31. पहिलो पद '3' र समान अन्तर '2' भएको AS को पहिलो 10 ओटा पदहरूको योगफल कति हुन्छ ?
Write the sum of first 10 terms of an AS having first term '3' and common difference '2'.

⇨ Here, sum of 10 terms (S_{10}) $= \frac{n}{2}[2a + (n-1)d] = \frac{10}{2}[2 \times 3 + 9 \times 2] = 5 \times 24 = 120$

B. SHORT QUESTIONS

MODEL 1

1. समानान्तर श्रेणीको एउटा उदाहरणसहित परिभाषा दिनुहोस् । (Define arithmetic series with an example.) **[2073 S]**

⇨ Here, a sequence is said to be an arithmetic sequence when its terms increase or decrease continually by a common difference. The corresponding series of this sequence is called the arithmetic series.
e.g. 3 + 7 + 11 + 15 + ………… is an arithmetic series.

2. समानान्तरीय अनुक्रम 10, $12\frac{1}{2}$, 15, $17\frac{1}{2}$ .... को कति औं पदको मान 30 हुन्छ ?

Which term of the arithmetic sequence 10, $12\frac{1}{2}$, 15, $17\frac{1}{2}$ ....is 30? **[2075 R'_2]**

⇨ Here, the given AS is 10, $12\frac{1}{2}$, 15, $17\frac{1}{2}$, ...

So, first term a = 10 and common difference (d) $= t_2 - t_1 = 12.5 - 10 = 2.5$
We know, $t_n = a + (n-1)d$
or, $30 = 10 + (n-1)\,2.5$
or, $20 = 2.5\,n - 2.5$
or, $\frac{22.5}{2.5} = n$ $\therefore$ $n = 9$
Thus, 9th term of the given AS is 30.

3. यदि श्रेणी 80 + 70 + 60 + ... को n औं पद शून्य भए n को मान निकाल्नुहोस् ।
If the n^{th} term of the series 80 + 70 + 60 + ... is zero, find the value of n. **[2074 R]**

⇨ Here, 80 + 70 + 60 + ...
First term (a) = 80, second term (t_2) = 70
Common difference (d) $= t_2 - t_1 = 70 - 80 = -10$
We have, n^{th} term (t_n) $= a + (n-1)d$
or, $0 = 80 + (n-1)(-10)$
or, $0 = 80 - 10n + 10$
or, $0 = 90 - 10n$
or, $10n = 90$ $\therefore$ $n = 9$
Thus, the required value of n is 9.

4. यदि 25, m + 1, 35 एउटा समानान्तरीय अनुक्रम भए m को मान पत्ता लगाउनुहोस् ।
If 25, m + 1, 35 is an arithmetic sequence then find the value of m. **[2074 R']**

⇨ Here, 25, m + 1, 35 are in AS.
So, we have, $t_2 - t_1 = t_3 - t_2$
or, $m + 1 - 25 = 35 - m - 1$
or, $2m - 24 = 34$
or, $2m = 58$
$\therefore$ $m = 29$
Thus, the value of m is 29.

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5. श्रेणी 84 + 78 + 72 + ... को कुन पद शुन्य हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series 84 + 78 + 72 + ... is zero? Find it. [2074 S]

⇨ Here, 84 + 78 + 72 + ..., $t_n = 0$
So, first term (a) = 84 and common difference (d) = $t_2 - t_1 = 78 - 84 = -6$
We know that, $t_n = a + (n - 1)d$
or, $0 = 84 + (n - 1)(-6)$
or, $6(n - 1) = 84$
or, $n - 1 = 14$
$\therefore$ $n = 15$
Thus, 15th term of the given series is zero.

6. श्रेणी 5 + 9 + 13 + ... को कुनचाहिँ पद 85 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series 5 + 9 + 13 + ... is 85? Find it. [2071 S]

⇨ Here, 5 + 9 + 13 + ... and $t_n = 85$
We know that, a = 5 and $d = t_2 - t_1 = 9 - 5 = 4$
By the formula, $t_n = a + (n - 1)d$
or, $85 = 5 + (n - 1)4$
or, $80 = (n - 1)4$
or, $20 = n - 1$
$\therefore$ $n = 21$
Thus, 21st terms is 85.

7. अनुक्रम 3, 6, 9, 12,....... को 15औं पद पत्ता लगाउनुहोस् । (Find the fifteenth term of the sequence 3, 6, 9, 12,........) [2068 R']

⇨ Here, first term (t_1) or (a) = 3 Second term (t_2) = 6
Common difference (d) = $t_2 - t_1 = 6 - 3 = 3$
Fifteenth term (t_{15}) = ?
Now, by using formula, $t_n = a(n - 1)d$
or, $t_{15} = 3(15 - 1)3$
$= 3 + 14 \times 3$
$= 3 + 42$
$= 45$
Thus, the fifteenth term of the sequence is 45.

8. श्रेणी 5 + 9 + 13 + ... + 77 मा कतिओटा पदहरू छन्, पत्ता लगाउनुहोस् ।
How many terms are there in the series 5 + 9 + 13 + ... + 77 ? Find. [2065 R]

⇨ Here, 5 + 9 + 13 + ... + 77 First term (a) = 5
Second term (t_2) = 9
Common difference (d) = $t_2 - t_1 = 9 - 5 = 4$
We have, $t_n = a + (n - 1)d$
or, $77 = 5 + (n - 1)4$
or, $72 = (n - 1)4$
or, $18 = n - 1$
$\therefore$ $n = 19$
Thus, there are 19 terms.

9. श्रेणी 3 + 6 + 9 + 12 + ... को कति औँ पद 96 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series 3 + 6 + 9 + 12 + ... is 96 ? Find it. [2066 R']

⇨ Here, 3 + 6 + 9 + 12 + ... and $t_n = 96$
So, $a = 3$, $d = t_2 - t_1 = 6 - 3 = 3$
We know that, $t_n = a + (n - 1)d$
or, $96 = 3 + (n - 1)3$
or, $93 = (n - 1)3$
or, $(n - 1) = 31$
$\therefore$ $n = 32$
Thus, the 32nd term is 96.

10. श्रेणी 24 + 28 + 32 + ... को कुन पद 52 हुन्छ ? (Which term is 52 in a series 24 + 28 + 32 + ... ?) [2067 S]

⇨ Here, 24 + 28 + 32 + ...and $t_n = 52$ It is an AS, $a = 24$, $d = t_2 - t_1 = 28 - 24 = 4$
We know that, $t_n = a + (n - 1)d$
or, $52 = 24 + (n - 1)4$
or, $28 = (n - 1)4$
or, $n - 1 = 7$
$\therefore$ $n = 8$
Thus, the eighth term is 8.

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11. यदि 6, p, q र 18 अङ्कगणितीय अनुक्रममा छन् भने p र q का मानहरू पत्ता लगाउनुहोस् ।
If 6, p, q and 18 are in an arithmetic sequence, find the value of p and q. **[2064 S]**

⇨ Here, 6, p, q and 18 are in AS

$\therefore$ First term (a) = 6 and 4th term $(t_4) = 18$

Now, $t_n = a + (n - 1)d$

or, $t_4 = a + (4 - 1)\,d$

So, $a + 3d = 18$

or, $6 + 3d = 18$

or, $3d = 12$

$\therefore$ $d = 4$

Now, $p = a + d = 6 + 4 = 10$

and, $q = a + 2d = 6 + 2 \times 4 = 14$

Thus, the value of p and q are 10 and 14.

12. 5, x, y, 11 एउटा समानान्तरीय श्रेणी हो भने x र y को मान पत्ता लगाउनुहोस् ।
5, x, y, 11 is an arithmetic series. Find the values of x and y. **[2066 S]**

⇨ Here, 5, x, y, 11 are in AP. And a = 5 $\therefore$ $t_4 = 11$

or, $a + 3d = 11$

or, $5 + 3d = 11$

or, $3d = 6$

$\therefore$ $d = 2$

Now, $x = t_2 = a + d = 5 + 2 = 7$

$y = t_3 = a + 2d = 5 + 2 \times 2 = 9$

Thus, the values of x and y are 7 and 9.

MODEL 2

13. श्रेणी 2 + 6 + 10 + ... को पहिलो 10 ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 10 terms of the series 2 + 6 + 10 + ... **[2074 S']**

⇨ Here, 2 + 6 + 10 + ...

This is an AP. So, $a = 2$, $d = t_2 - t_1 = 6 - 2 = 4$

$$\begin{aligned} \text{We know that, } S_n &= \frac{n}{2}[2a + (n - 1)d] \\ &= \frac{10}{2}[2 \times 2 + (10 - 1)4] \\ &= 5[4 + 36] \\ &= 200 \end{aligned}$$

Thus, the sum of first ten terms is 200.

14. 2 + 4 + 6 + ... 20 ओटा पदहरू भएको श्रेणीको योगफल निकाल्नुहोस् । (Find the sum of the series 2 + 4 + 6 + ... 20 terms.) **[2065 M]**

⇨ Here, the given series is arithmetic series of even natural numbers.

$$\begin{aligned} \text{We know that, Sum } (S_n) &= n\,(n + 1) \\ &= 20\,(20 + 1) \\ &= 20 \times 21 \end{aligned}$$

Thus, the sum $(S_n) = 420$.

15. 5 + 11 + 17 + ...सातौं पदसम्मको योगफल निकाल्नुहोस् । (Find the sum of 5 + 11 + 17 + ... seventh term.) **[2064 R]**

⇨ Here, 5 + 11 + 17 + ... seventh term

First term (a) = 5,

Common difference (d) = $t_2 - t_1 = 11 - 5 = 6$

Number of terms (n) = 7

$$\begin{aligned} \text{We know that, } S_n &= \frac{n}{2}[2a + (n - 1)d] \\ &= \frac{7}{2}[2 \times 5 + (7 - 1)6] \\ &= \frac{7}{2}[10 + 36] \\ &= \frac{7}{2} \times 46 \end{aligned}$$

Thus, the seventh term (S_7) is 161.

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16. श्रेणी $\sum_{n=4}^{7}(3n-2)$ को योगफल निकाल्नुहोस् । (Find the sum of the series $\sum_{n=4}^{7}(3n-2)$.) **[2057R]**

⇨ Here, given $\sum_{n=4}^{7}(3n-2)$ Putting n = 4, 5, 6, 7 and writing in the form of sum,

$$\sum_{n=4}^{7}(3n-2) = (3\times4-2)+(3\times5-2)+(3\times6-2)+(3\times7-2)$$
$$= (12-2)+(15-2)+(18-2)+(21-2)$$
$$= 10+13+16+19 = 58$$

Thus, the sum of the series is 58.

C. LONG QUESTIONS

MODEL 1

1. **एउटा अङ्कगणितीय अनुक्रमको तेस्रो पद – 40 र तेह्रौं पद 0 भए अठ्ठाइसौं पद कति हुन्छ ?**
The 3rd term and the 13th terms of an arithmetic sequence are – 40 and 0 respectively, what is the 28th term of the sequence ? **[2065 M]**

⇨ Here, third term (t_3) = – 40 and

Thirteenth term (t_{13}) = 0

$\therefore\ -40 = a + 2d$ ....(i) $[\because t_3 = -40]$

$\therefore\ 0 = a + 12d$ ....(ii) $[\because t_{13} = 0]$

Solving equations (i) and (ii);

$$\begin{array}{l} -40 = a + 2d \\ \quad 0 = a + 12d \\ \underline{-\quad\ -\quad -} \\ -40 = \quad -10d \end{array}$$

$\therefore\ d = 4$

Putting the value of d in (i) then,

$-40 = a + 2 \times 4$

or, $-40 = a + 8$

or, $-48 = a$

$\therefore\ a = -48$

Now, $t_{28} = a + 27d = -48 + 27 \times 4$

$\therefore\ t_{28} = 60$

Thus, the 28th term is 60.

2. **यदि एउटा समानान्तरीय श्रेणीको पाँचौं र दशौं पदहरू क्रमशः 17 र 42 भए यसको बीसौं पद पत्ता लगाउनुहोस् ।**
If 5th and 10th term of an arithmetic series are 17 and 42 respectively, find its 20th term. **[2060 S']**

⇨ Here, given $t_5 = 17$ & $t_{10} = 42$

$\therefore\ 17 = a + 4d$ …….......... (i)

$42 = a + 9d$ …………. (ii)

From (i) & (ii)

$$\begin{array}{l} 17 = a + 4d \\ 42 = a + 9d \\ \underline{-\quad -\quad -} \\ -25 = -5d \end{array}$$

$\therefore\ d = 5$

Putting value of d in equation (i) 17 = a + 4d

or, $17 = a + 4 \times 5$

or, $17 - 20 = a$

or, $-3 = a$

$\therefore\ a = -3$

Then, $t_{20} = a + 19d = -3 + 19 \times 5 = -3 + 95 = 92$

Thus, the required value of 20th term is 92.

3. **कुनै समानान्तर श्रेणीको छैटौं पद 23 र यसको नवौं पद 35 भए कुन पद 67 होला ?**
If the 6th term of an arithmetic series is 23 and its 9th term 35, which term will be 67? **[2063 R']**

⇨ Here, sixth term (t_6) = 23 Ninth term (t_9) = 35

If $t_n = 67$ then n = ?

We know that, $t_6 = 23$

$\therefore\ a + 5d = 23$ .... (i)

Again, $t_9 = 35$

$\therefore\ a + 8d = 35$ .... (ii)

Solving (i) and (ii) then,

$$\begin{array}{l} a + 5d = 23 \\ a + 8d = 35 \\ \underline{-\ \ -\quad\ -} \\ \ -3d = -12 \end{array}$$

$\therefore\ d = 4$

Putting d = 4 in (i) then, $a + 5 \times 4 = 23$

or, $a = 23 - 20$

$\therefore\ a = 3$

Now, $t_n = a + (n-1)d$

or, $67 = 3 + (n-1)4$

or, $67 - 3 = (n-1)4$

or, $n - 1 = 16$

$\therefore\ n = 17$

Thus, 17th term is 67.

MODEL 2

4. **समानान्तरीय अनुक्रममा रहेका तीन पदहरूको योगफल 21 र गुणनफल 315 भए, ती पदहरू निकाल्नुहोस् ।**
Three terms in an arithmetic progression have sum 21 and product 315. Find the terms. **[2070 R]**

⇨ Here, let the three numbers are a – d, a, a + d respectively.

Then, $a - d + a + a + d = 21$

or, $3a = 21$ $\therefore\ a = 7$

Again, product = 315

or, $(a-d)(a+d).\ a = 315$

or, $(7-d)(7+d) \times 7 = 315$

or, $(49 - d^2) = 45$

or, $4 = d^2$

$\therefore\ d = \pm 2$

When a = 7 and d = – 2 then,

Three terms; $(a-d), a, (a+d)$

$= (7+2), 7, (7-2) = 9, 7, 5$

When a = 7 and d = 2 then,

Three terms $= (a-d), a, (a+d)$

$= (7-2), 7, (7+2)$

$= 5, 7, 9$

Thus, the three terms are 5, 7, 9 or 9, 7, 5.

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5. **यदि कुनै समानान्तर श्रेणीको चौथो पद 1 र पहिलो आठौं पदसम्मको योगफल 18 भए सो श्रेणीको दसौं पद पत्ता लगाउनुहोस् ।**
If the fourth term of an AP is 1 and the sum of its first eight terms is 18, find the tenth term of the series. **[2060 S]**

⇨ Here given, 4th term of an A.P. $(t_4) = 1$

Sum of first 8 terms $(S_8) = 18$

If a be the first term and d be common ratio, then

Using formula, $t_n = a + (n - 1)d$,

we have, $t_4 = a + (4 - 1)d$

or, $1 = a + 3d$

$\therefore\ a = 1 - 3d$ ..... (i)

Again, using formula $S_n = \frac{n}{2}\ [2a + (n - 1)d]$,

we get, $S_8 = \frac{8}{2}\ [2a + (8 - 1)d]$

or, $18 = 4(2a + 7d)$

or, $18 = 4[2(1 - 3d) + 7d]$

or, $18 = 4[2 - 6d + 7d]$

or, $18 = 4(2 + d)$

or, $18 = 8 + 4d$

or, $4d = 18 - 8 = 10$

$\therefore\ d = \frac{10}{4} = \frac{5}{2}$

Putting the value of d in (i),

we get, $a = 1 - 3d = 1 - 3 \times \frac{5}{2} = \frac{2 - 15}{2} = -\frac{13}{2}$

Now, 10^{th} term $(t_{10}) = a + (10 - 1)d$

$= -\frac{13}{2} + 9 \times \frac{5}{2}$

$= \frac{-13 + 45}{2}$

$= \frac{32}{2}$

Thus, the tenth term is 16.

6. **एउटा समानान्तर श्रेणीको पहिला तीन पदहरूको योग 21 छ । यदि तेस्रो पदबाट पहिलो दुई पदहरूको योग घटाइयो भने 9 हुन्छ भने सो श्रेणीका तीन पदहरू निकाल्नुहोस् ।**
The sum of the first three terms of an AS is 21. If the sum of the first two terms is subtracted from the third term then it would be 9, find the three terms of the series. **[2059S]**

⇨ Here, let, the three terms of an AS be $a - d,\ a,\ a + d$

According to question,

$a - d + a + a + d = 21$

or, $3a = 21$

$\therefore\ a = 7$

Again, by question,

$(a + d) - (a - d + a) = 9$

or, $a + d - a + d - a = 9$

or, $2d - a = 9$

or, $2d - 7 = 9\quad (\because a = 7)$

or, $2d = 9 + 7$

$\therefore\ d = \frac{16}{2} = 8$

Hence, the putting the values of a and d in;

$a - d$, a and $a + d$,

We get the three terms of an AS as;

$7 - 8,\ 7,\ 7 + 8$

$= -1,\ 7,\ 15$

Thus, the three terms of the series are -1, 7, 15.

7. **एउटा समानान्तरीय श्रेणीको पहिला तीन पदहरूको योगफल 42 र पहिलो पाँच पदहरूको योगफल 80 भए त्यस श्रेणीको बिसौं पद निकाल्नुहोस् ।**
If the sum of the first three terms of an arithmetic series is 42 and that of the first five terms is 80, find the twentieth term of the series. **[2062 S]**

⇨ Here, sum of first 3 terms $(S_3) = 42$

Sum of first 5 terms $(S_5) = 80$

We know that; $S_3 = 42$

or, $\frac{3}{2}[2a + (3 - 1)d] = 42$

or, $\frac{3}{2}[2a + 2d] = 42$

or, $a + d = 14$ .... (i)

Again, $S_5 = 80$

or, $\frac{5}{2}[2a + (5 - 1)d] = 80$

or, $\frac{5}{2}[2a + 4d] = 80$

or, $a + 2d = 16$ .... (ii)

Solving (i) and (ii) then,

$$\begin{array}{r} a + d = 14 \\ a + 2d = 16 \\ - \quad - \qquad - \\ \hline -d = -2 \end{array}$$

$\therefore\ d = 2$

Putting $d = 2$ in (i) $a + 2 = 14$

$\therefore\ a = 12$

Now, $t_{20} = a + 19d$

$= 12 + 19 \times 2$

$\therefore\ t_{20} = 50$

Thus, the twentieth term is 50.

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8. **यदि कुनै समानान्तरीय श्रेणीको पहिलो सातौं पदहरूको योगफल 14 र पहिलो दसौं पदहरूको योगफल 125 भए चौथो पद पत्ता लगाउनुहोस् ।**
If the sum of the first seven terms of an arithmetic series is 14 and the sum of the first ten terms is 125 then find the fourth term of the series. **[2063 R]**

⇨ Here, sum of first seven terms of A.S. $(S_7) = 14$

Sum of first ten terms of A.S. $(S_{10}) = 125$

We know, $S_n = \frac{n}{2}\,[2a + (n-1)d]$

or, $S_7 = \frac{7}{2}\,[2a + (7-1)d]$

or, $14 = \frac{7}{2}\,[2a + 6d]$

or, $28 = 14a + 42d$

or, $14a + 42d = 28$

or, $14a + 42d - 28 = 0$

or, $2(7a + 21d - 14) = 0$

or, $7a + 21d - 14 = 0$

$\therefore$ $a + 3d = 2$ .... (i)

Again, $S_{10} = \frac{10}{2}\,[2a + (10-1)d]$

or, $125 = 5\,[2a + 9d]$

or, $125 = 10a + 45d$

or, $10a + 45d - 125 = 0$

or, $2a + 9d - 25 = 0$

$\therefore$ $2a + 9d = 25$ .... (ii)

Now, multiplying equation (i) by 2 and subtracting equation (ii) from (i), we get,

$$\begin{array}{r} 2a + 6d = 4 \\ 2a + 9d = 25 \\ -\quad -\quad\quad - \\ \hline -3d = -21 \end{array}$$

or, $d = \frac{21}{3} = 7$

Putting the value of d in equation (i),

we get, $a + 3 \times 7 = 2$

or, $a + 21 = 2$

or, $a = 2 - 21$

or, $a = -19$

Then, fourth term (t_4) $= a + 3d$

$= -19 + 3 \times 7$

$= -19 + 21$

$= 2$

Thus, the fourth term of the series is 2.

9. **समानान्तरीय श्रेणीमा तीनओटा पदहरूको योग 36 र तिनीहरूको गुणनफल 1140 छ भने ती पदहरू पत्ता लगाउनुहोस् ।**
The sum of three terms in arithmetic series is 36 and their product is 1140, find these terms. **[2065 R']**

⇨ Here, let, the three terms of an AP be $a - d$, a and $a + d$.

By the question, $a - d + a + a + d = 36$

or, $3a = 36$

$\therefore$ $a = 12$

Putting the value of a in above then, the terms are,

$12 - d$, 12 and $12 + d$

Again, by the question, product = 1140

i.e. $(12 - d)12\,(12 + d) = 1140$

or, $12(144 - d^2) = 1140$

or, $144 - d^2 = 95$

or, $49 = d^2$

$\therefore$ $d = \pm 7$

When $d = -7$ then the terms are;

$12 + 7$, 12 and $12 - 7$

i.e. 19, 12 and 5

When $d = 7$ then the terms are;

$12 - 7$, 12 and $12 + 7$

i.e. 5, 12 and 19

Thus, the three terms are;

5, 12, 19 or 19, 12, 5.

MODEL 3

10. **यदि एउटा समानान्तरीय श्रेणीको 4 औं र 15 औं पदहरू क्रमशः 11 र 44 भए यसका प्रथम 20 पदहरूको योग पत्ता लगाउनुहोस् ।**
If the 4th and 15th terms of an arithmetic series are 11 and 44 respectively then find the sum of its first 20 terms. **[SEE 2075 R']**

⇨ Here, $t_4 = 11$ and $t_{15} = 44$

or, $a + 3d = 11$ .... (i) and

$[\because t_n = a + (n-1)d]$

$a + 14\,d = 44$ ....(ii)

Solving (i) and (ii), then

$$\begin{array}{r} a + 3d = 11 \\ a + 14\,d = 44 \\ -\quad -\quad\quad - \\ \hline -11d = -33 \end{array}$$

$\therefore$ $d = 3$

Putting the value of d in (i),

$a + 3 \times 3 = 11$

$\therefore$ $a = 2$

We know that,

Sum of n terms of AS is given by;

or, $S_n = \frac{n}{2}\,[2a + (n-1)d]$

or, $S_{20} = \frac{20}{2}\,[2 \times 2 + 19 \times 3]$

$= 10 \times 61$

$= 610$

Thus, the sum of first 20 terms of AS is 610.

11. **एउटा समानान्तरीय श्रेणीको चौथो पद पहिलो पदको तेब्बर छ र सातौं पद तेस्रो पदको दोब्बर भन्दा पनि 1 ले बढी छ । सो श्रेणीका पहिला दश पदहरूको योगफल निकाल्नुहोस् ।**
The fourth term of an arithmetic series is three times the first term and seventh term exceeds twice the third term by 1. Find the sum of the first ten terms of the series. **[SEE 2075 R'_2]**

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⇨ Here, let a be the first term and d be the common difference of the arithmetic series.

By question, $t_4 = 3a$ ......(i) and

$2t_3 + 1 = t_7$ ....(ii)

From (i), $a + 3d = 3a$ $[\because t_n = a + (n - 1)d]$

$\therefore$ $3d = 2a$ ......(iii)

From (ii), $2(a + 2d) + 1 = a + 6d$

or, $2a + 4d + 1 = a + 6d$

or, $a = 2d - 1$

or, $2a = 4d - 2$

or, $3d = 4d - 2$

$[\because$ from (iii), $2a = 3d]$

$\therefore$ $d = 2$

Putting d = 2 in (iii), $a = \dfrac{3 \times 2}{2}$ $\therefore$ $a = 3$

We know that, sum of n terms of AS is,

$S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$S_{10} = \dfrac{10}{2}[2 \times 3 + 9 \times 2]$

$= 5[6 + 18] = 120$

Thus, the sum of the first ten terms of the series is 120.

12. कुनै एउटा समानान्तरीय श्रेणीको तेस्रो र सातौं पदहरू क्रमशः 18 र 30 भए प्रथम 20 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The 3^{rd} and 7^{th} terms of an arithmetic series are 18 and 30 respectively, find the sum of the first 20 terms. **[2071 R']**

⇨ Here, third term $(t_3) = 18$

Seventh term $(t_7) = 30$

Sum of first 20 terms $(S_{20}) = ?$

We have, $t_n = a + (n - 1)\,d$

So, $t_3 = a + 2d$

$\therefore$ $18 = a + 2d$ .... (i)

Again, $t_7 = a + 6d$

$\therefore$ $30 = a + 6d$ .... (ii)

Solving equation (i) and (ii) then,

$30 = a + 6d$

$18 = a + 2d$

$\underline{-\quad\ -\ -}$

$12 = 4d$ $\qquad\therefore$ $d = 3$

From equation (i) $18 = a + 2d$

or, $18 = a + 2 \times 3$

$\therefore$ $a = 12$

Now, $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$= \dfrac{20}{2}[2 \times 12 + (20 - 1)3]$

$= 10[24 + 57] = 10 \times 81 = 810$

Thus, the sum of first 20 terms is 810.

13. एउटा समानान्तर श्रेणीको नवौं पद र उन्नाइसौं पदहरू क्रमशः 40 र 60 छन् भने सो श्रेणीको पहिलो 25 पदहरूको योगफल निकाल्नुहोस् ।
The ninth and nineteenth terms of an arithmetic series are 40 and 60 respectively. Find the sum of the first 25 terms of the series. **[2070 R']**

⇨ Here, ninth term $(t_9) = 40$ and nineteenth term $(t_{19}) = 60$

We know that, $t_9 = a + 8d$ or, $40 = a + 8d$ .... (i)

Again, $t_{19} = a + 18d$ or, $60 = a + 18d$ .... (ii)

Solving equation (i) and (ii),

$40 = a + 8d$

$60 = a + 18d$

$\underline{-\quad -\quad -}$

$-20 = -10d$

$\therefore$ $d = 2$

Putting the value of d = 2 in (i) then,

$40 = a + 8d$

or, $40 = a + 8 \times 2$

$\therefore$ $a = 24$

Now, $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$S_{25} = \dfrac{25}{2}[2a + 24d] = \dfrac{25}{2}[2 \times 24 + 24 \times 2] = \dfrac{25}{2} \times 96$

$\therefore$ $S_{25} = 1200$

Thus, the sum of first 25 terms is 1200.

14. यदि कुनै समानान्तर श्रेणीको तेस्रो पद 1 र पाचौं पद 7 भए सो श्रेणीका पहिला दस पदहरूको योगफल निकाल्नुहोस् ।
If the third term of an arithmetic series is 1 and fifth term is 7, find the sum of first ten terms of the series.

⇨ Here, $t_3 = 1$ or, $a + 2d = 1$

or, $a = 1 - 2d$ .... (i)

Again, $t_5 = 7$ or, $a + 4d = 7$

or, $(1 - 2d) + 4d = 7$ or, $1 - 2d + 4d = 7$

or, $2d = 6$ or, $d = 3$

Now, $a = 1 - 2d = 1 - 6 = -5$ Then, $S_n = \dfrac{n}{2}(2a + (n - 1)d)$

or, $S_{10} = \dfrac{10}{2}\{-10 + 9 \times 3\} = 5(-10 + 27) = 5 \times 17 = 85$

Thus, the sum of first ten terms is 85.

15. यदि एउटा समानान्तरीय श्रेणीको दसौं पद पद 39 र सत्रौ पद 67 छन् भने पहिलो 26 पदहरूको योगफल निकाल्नुहोस् ।
If the 10^{th} term of an arithmetic series is 39 and the 17^{th} term 67, find the sum of the first 26 terms. **[2073 S', 2065 S]**

⇨ Here, in the given AP, 10^{th} term $(t_{10}) = 39$ and 17^{th} term $(t_{17}) = 67$

We know that; $t_n = a + (n - 1)d$

So, $t_{10} = a + 9d$

$\therefore$ $39 = a + 9d$ .... (i)

Substituting the value of d in equation (i) then,

$39 = a + 9d$

or, $39 = a + 9 \times 4$

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And, $t_{17} = a + 16d$

$\therefore$ $67 = a + 16d$ .... (ii)

Solving equations (i) and (ii) then;

$$\begin{array}{l} 39 = a + 9d \\ 67 = a + 16d \\ - \quad\; - \quad - \\ \hline -28 = -7d \end{array}$$

$\therefore$ $d = 4$

or, $39 = a + 36$

$\therefore$ $a = 3$

Now, the sum of first n terms $(S_n) = \frac{n}{2}[2a + (n - 1)d]$

So, the sum of first 26 terms $(S_{26}) = \frac{26}{2}[2 \times 3 + 25 \times 4]$

$= 13 \times 106 = 1378$

Thus, the sum of first 26 terms is 1378.

16. यदि कुनै समानान्तर श्रेणीको तेस्रो र एघारौँ पदहरू क्रमशः 8 र – 8 भए सो श्रेणीको पहिला सात ओटा पदहरूको योगफल निकाल्नुहोस् ।

If the third and eleventh terms of an arithmetic series are 8 and – 8 respectively, find the sum of first seven terms of the series. **[2067 R']**

⇨ Here, in any arithmetic series, Third term $(t_3) = 8$

or, $a + (3 - 1)d = 8$

or, $a + 2d = 8$ ………... (i)

Eleventh term $(t_{11}) = -8$

or, $a + (11 - 1)\,d = -8$

or, $a + 10d = -8$ ……… (ii)

or, $a + 2d + 8d = -8$

or, $8 + 8d = -8$ $[\because a + 2d = 8]$

or, $8d = -8 - 8$

or, $d = -\frac{16}{8} = -2$

Now, putting the value of d in equation (i);

$a + 2(-2) = 8$

or, $a - 4 = 8$

$\therefore$ $a = 8 + 4 = 12$

Then, sum of first seven terms $(S_7) = ?$

We know ,

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

or, $S_7 = \frac{7}{2}\{2 \times 12 + (7 - 1)(-2)\}$

$= \frac{7}{2}\{24 + 6 \times (-2)\}$

$= \frac{7}{2}(24 - 12)$

$= \frac{7}{2} \times 12 = 42$

Thus, the sum of first seven terms of the series is 42.

17. एउटा समानान्तरीय श्रेणीको चौधौं पद 2 र पहिला दस पदहरूको योगफल – 150 भए सो श्रेणीको पहिला पच्चिस पदहरूको योगफल निकाल्नुहोस् ।

Fourteenth term of an arithmetic series is 2 and the sum of its first ten terms is – 150. Find the sum of the first twenty five terms of the series. **[2060 C, 2065 E]**

⇨ Here, fourteenth term $(t_{14}) = 2$

Sum of 1st 10 terms $(S_{10}) = -150$

We know that, $t_{14} = a + 13d$

or, $2 = a + 13d$ .... (i)

Again, $S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

or, $-150 = 5[2a + 9d]$

$\therefore$ $-30 = 2a + 9d$ .... (ii)

Solving (i) and (ii) then,

$$\begin{array}{l} \;\;4 = 2a + 26d \\ -30 = 2a + 9d \\ + \quad\;\; - \quad - \\ \hline \;\;34 \;\;= 17d \end{array}$$

$\therefore$ $d = 2$

Putting $d = 2$ in (i) then,

$a + 13d = 2$

or, $a + 13 \times 2 = 2$

$\therefore$ $a = -24$

Now, $S_{25} = \frac{25}{2}[2a + (25 - 1)d]$

$= \frac{25}{2} \times [2 \times (-24) + 24 \times 2]$

$\therefore$ $S_{25} = 0$

Thus, the sum of the first twenty five terms is 0.

18. ओटा पदहरू भएको एउटा समानान्तर श्रेणीको अन्तिम पद 195 र समान अन्तर 5 भए त्यो श्रेणीको योगफल निकाल्नुहोस् ।

The last term of an arithmetic series of 20 terms is 195 and the common difference 5. Calculate the sum of the series.

⇨ Here, given number of terms in A.S. $(n) = 20$

Last term $(\ell) = 195$

Common difference $(d) = 5$

We have, $\ell = a + (n - 1)\,d$

or, $195 = a + (20 - 1)\,5$

or, $195 = a + 19 \times 5$

or, $195 = a + 95$

or, $a = 195 - 95$

$\therefore$ $a = 100$

Thus, the sum of the series (S_{20}) is 2950.

Again, we have, $S_n = \frac{n}{2}[2a + (n - 1)\,d]$

$\therefore$ $S_{20} = \frac{20}{2}[2 \,.\, 100 + (20 - 1) \,.\, 5]$

$= 10\,[200 + 19 \times 5]$

$= 10\,[200 + 95]$

$= 10 \times 295$

$= 2950$

19. एउटा समानान्तर श्रेणीको पहिलो दसौं पदहरूको योगफल 50 र पाँचौं पद दोस्रो पदको तेब्बर भए पहिलो पद र पहिलो बिसौं पदहरूको योगफल निकाल्नुहोस् ।

The sum of the first ten terms of an arithmetic progression is 50 and its fifth term is treble of the second term. Calculate the first term and the sum of the first twenty terms. **[2064 S, 2067 R]**

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⇨ Here, $S_{10} = 50$, $t_5 = 3t_2$, a = ? and S_{20} = ?

We know that, $S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $50 = \frac{10}{2}[2a + (10 - 1)d]$

or, $50 = 5[2a + 9d]$

∴ $2a + 9d = 10$ .... (i)

Again, $t_5 = 3t_2$

or, $a + 4d = 3(a + d)$

or, $a + 4d = 3a + 3d$

or, $2a - d = 0$ .... (ii)

Solving equations (i) and (ii)

$$\begin{array}{l} 2a + 9d = 10 \\ 2a - d = 0 \\ \underline{-\quad +\quad -} \\ 10d = 10 \end{array}$$

∴ $d = 1$

Putting the value of d in (i),

$2a + 9d = 10$

or, $2a + 9 \times 1 = 10$

or, $2a = 1$

∴ $a = \frac{1}{2}$

Now, sum of 20 terms $(S_{20}) = \frac{20}{2}[2a + (20 - 1)d]$

$= 10\left[2 \times \frac{1}{2} + 19 \times 1\right]$

$= 10[1 + 19]$

$= 200$

Thus, $a = \frac{1}{2}$ and $S_{20} = 200$.

MODEL 4

20. यदि एउटा समानान्तरीय श्रेणीका प्रथम 9 पदहरूको योग 72 र प्रथम 17 पदहरूको योग 289 भए प्रथम 25 पदहरूको योग पत्ता लगाउनुहोस् ।

If the sum of the first 9 terms of an arithmetic series is 72 and the sum of the first 17 terms is 289 then the sum of the first 25 terms. **[2073 R']**

⇨ Here, $S_9 = 72$, $S_{17} = 289$ S_{25} = ?

We know that, $S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $S_9 = \frac{9}{2}[2a + 8d]$

or, $72 = \frac{9}{2} \times 2(a + 4d)$

∴ $a + 4d = 8$ ….. (i)

Again, $S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $S_{17} = \frac{17}{2}[2a + 16d]$

or, $289 = 17[a + 8d]$

∴ $a + 8d = 17$ …….. (ii)

Solving equation (i) and (ii) then,

$$\begin{array}{l} a + 4d = 8 \\ a + 8d = 17 \\ \underline{-\ -\quad -} \\ -4d = -9 \end{array}$$

∴ $d = \frac{9}{4}$

Putting the value of d in (i) then,

$a + 4 \times \left(\frac{9}{4}\right) = 8$

or, $a + 9 = 8$

∴ $a = -1$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{25}{2}[2a + 24d]$

$= \frac{25}{2}\left[2 \times (-1) + 24 \times \frac{9}{4}\right]$

$= \frac{25}{2}[-2 + 54]$

$= 650$

Thus, the sum of first 25 terms is 650.

21. यदि एउटा समानान्तरीय श्रेणीमा प्रथम 7 पदहरूको योगफल 14 छ र प्रथम 11 पदहरूको योगफल 66 भए प्रथम 25 पदहरूको योगफल पत्ता लगाउनुहोस् ।

The sum of first 7 terms of an arithmetic series is 14 and the sum of first 11 terms is 66. Find the sum of first 25 terms.

⇨ Here, $S_7 = 14$, $S_{11} = 66$, S_{25} = ?

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $14 = \frac{7}{2}[2a + 6d]$

or, $4 = 2a + 6d$

∴ $a + 3d = 2$ ......(i)

Again, $S_{11} = \frac{11}{2}[2a + 10d]$

or, $66 = \frac{11}{2}[2a + 10d]$

or, $12 = 2a + 10d$

∴ $a + 5d = 6$ ......(ii)

Solving equation (i) and (ii) then,

$$\begin{array}{l} a + 3d = 2 \\ a + 5d = 6 \\ \underline{-\quad -\quad -} \\ -2d = -4 \end{array}$$

∴ $d = 2$

Putting value of d in (i) then $a + 3 \times 2 = 2$

or, $a = 2 - 6 = -4$ ∴ $a = -4$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $S_{25} = \frac{25}{2}[2 \times (-4) + 24 \times 2]$

or, $\frac{25}{2}[-8 + 48] = 500$

Thus, the sum of first 25 terms is 500.

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MODEL 5

22. **एउटा समानान्तर श्रेणीको पहिलो पद र अन्तिम पद क्रमशः – 24 र 72 छन् । यदि सबै पदहरूको योगफल 600 भए सो श्रेणीको पदहरूको सङ्ख्या र समान अन्तर निकाल्नुहोस् ।**

The first and last terms of an arithmetic series are – 24 and 72 respectively. If sum of all terms of the series is 600, find the number of terms and the common difference of the series. **[2059 R]**

⇨ Here given, First term of A.S. (a) = – 24

Last term of A.S. (ℓ) = 72

Sum of n terms (S_n) = 600

We have,

$\ell = a + (n - 1)\,d$

or, $72 = -24 + (n - 1)\,d$

or, $(n - 1)\,d = 72 + 24$

$\therefore$ $(n - 1)\,d = 96$ .... (i)

Again, $S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$

or, $600 = \frac{n}{2}\,[2 \times (-24) + (n - 1)\,d]$

or, $600 \times 2 = n[-48 + 96]$ [$\because$ from (i)]

or, $1200 = n \times 48$

or, $n = \frac{1200}{48} = 25$

$\therefore$ $n = 25$

Again, put the value of n = 25 in (i) we get, $(25 - 1)\,d = 96$

or, $24\,d = 96$

or, $d = \frac{96}{24} = 4$

Thus, the number of terms (n) = 25 and common difference (d) = 4.

23. **एउटा समानान्तर श्रेणीका पहिला दस पदहरूको योगफल 520 छ । यदि यसको सातौं पद तेस्रो पदको दोब्बर भए सो श्रेणीको पहिलो पद र समान अन्तर पत्ता लगाउनुहोस् ।**

In an AS, the sum of the first ten terms is 520. If its seventh term is double of its third term, calculate the first term and the common difference of the series. **[2061 R]**

⇨ Here, given sum of 10 terms of an A.P. (S_{10}) = 520

If first term = a and common difference = d

Then nth term $t_n = a + (n - 1)d$

So, 7th term $t_7 = a + (7 - 1)d$

$\therefore$ $t_7 = a + 6d$

3rd term $t_3 = a + (3 - 1)d$

$\therefore$ $t_3 = a + 2d$

But by given, $t_7 = 2t_3$

or, $a + 6d = 2(a + 2d)$

or, $a + 6d = 2a + 4d$

or, $6d - 4d = 2a - a$

$\therefore$ $a = 2d$

Again, using formula,

$S_n = \frac{n}{2}\,[2a + (n - 1)d]$

or, $S_{10} = \frac{10}{2}\,[2a + (10 - 1)d]$

or, $520 = 5(2a + 9d)$

or, $\frac{520}{5} = 2 \times 2d + 9d$ [$\because$ a = 2d]

or, $104 = 4d + 9d$

or, $104 = 13d$

$\therefore$ $d = \frac{104}{13} = 8$

Again, $a = 2d = 2 \times 8 = 16$

Thus, the first term (a) = 16 and common difference (d) = 8.

MODEL 6

24. **5 र 25 को बीचमा चारओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert four arithmetic means between 5 and 25.)** **[2072 R']**

⇨ Here,

$5 \ldots \boxed{m_1} \ldots \boxed{m_2} \ldots\ldots \boxed{m_3} \ldots\ldots \boxed{m_4} \ldots\ldots 25$

First term (a) = 5

Sixth term (t_6) = 25

or, $a + 5d = 25$

or, $5 + 5d = 25$

or, $5d = 20$

$\therefore$ $d = 4$

Now,

$m_1 = a + d = 5 + 4 = 9$

$m_2 = a + 2d = 5 + 2 \times 4 = 13$

$m_3 = a + 3d = 5 + 3 \times 4 = 17$

$m_4 = a + 4d = 5 + 4 \times 4 = 21$

Thus, the four means are 9, 13, 17 and 21.

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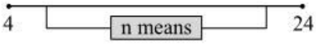
25. 4 र 24 का बीचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा र अन्तिम मध्यमाको अनुपात 4 : 5 भए उक्त श्रेणीमा जम्मा कतिओटा पदहरू छन् ? पत्ता लगाउनुहोस् ।

There are n arithmetic means between 4 and 24. If the ratio of third mean to the last mean is 4 : 5 then find the number of terms in the series. **[2071 R]**

⇨ Here, first term (a) = 4, last term (b) = 24



We know that, $d = \frac{b - a}{n + 1} = \frac{24 - 4}{n + 1} = \frac{20}{n + 1}$

We have given, $\frac{m_3}{m_n} = \frac{4}{5}$

or, $\frac{a + 3d}{a + nd} = \frac{4}{5}$

or, $\frac{4 + 3 \times \frac{20}{n + 1}}{4 + n \times \frac{20}{n + 1}} = \frac{4}{5}$

or, $\frac{\frac{4n + 4 + 60}{n + 1}}{\frac{4n + 4 + 20n}{n + 1}} = \frac{4}{5}$

or, $\frac{4n + 64}{24n + 4} = \frac{4}{5}$

or, $\frac{4(n + 16)}{4(6n + 1)} = \frac{4}{5}$

or, $24n + 4 = 5n + 80$

or, $19n = 76$

$\therefore$ $n = 4$

Thus, the total numbers of terms = 4 + 2 = 6.

MODEL 7

26. एकजना ठेकेदारले कुनै काम निश्चित समय भन्दा ढिलो गरे वापत पहिलो दिनको रु. 200, दोस्रो दिनको रु. 250 र तेस्रो दिनको रु. 300 गर्दै प्रत्येक पछिल्लो दिनमा पहिलो दिनको भन्दा रु. 50 बढि जरिवाना तिर्नुपर्छ । यदि उक्त ठेकेदारले कुनै काम 30 दिन ढिला गरेमा जम्मा कति जरिवाना तिर्नुपर्ला ?

A contractor on construction job specifies a penalty for delay of completion beyond a certain date as: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day and so on. The penalty for each succeding day being Rs 50 more than that of the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days? **[SEE MODEL 2076]**

⇨ Here, the penalties in order;

First	Second	Third	....
Rs 200	Rs 250	Rs 300	....

It forms an AS.

First term (a) = 200 and Common difference (d) = 50

No. of late days (n) = 30

We know that, $S_n = \frac{n}{2}[2a + (n - 1) d]$

or, $S_{30} = \frac{30}{2}[2 \times 200 + (30 - 1) 50]$

$= 15 [400 + 29 \times 50]$

$= 15 \times 1850$

$\therefore$ $S_{30} = 27750$

Thus, the required amount of penalties is Rs 27750.

27. एकजना मानिसको सुरूको तलब रु. 15000 छ र हरेक वर्ष रु. 1500 का दरले बढ्छ । 10$^{\text{औं}}$ वर्षको तलब र 10 वर्ष सम्मको जम्मा तलब पत्ता लगाउनुहोस् ।

The starting salary of a man is Rs 15000 and it increases by Rs 1500 every year. Find the salary of 10th year and total salary for 10 years.

⇨ Here, starting salary (a) = Rs 15000

Common difference (d) = Rs 1500

No. of years (n) = 10,

We know that,

$t_n = a + (n - 1)d$

or, $t_{10} = 15000 + 9 \times 1500$

$= 15000 + 13500$

$\therefore$ $t_{10} =$ Rs 28500

Again, we know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

or, $S_{10} = \frac{10}{2}[2 \times 15000 + 9 \times 1500]$

$= 5[30000 + 13500]$

$\therefore$ $S_{10} = 217500$

Thus, the salary of 10th year is Rs 28500 and total salary for 10 years is Rs 217500.

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QUESTIONS FROM CDC TEXTBOOK

1.3 समानान्तर श्रेणीको साधारण पद (GENERAL TERM OF ARITHMETIC SEQUENCE)

EXERCISE 1.3.1

1. **(a) अनुक्रम भनेको के हो ? उदाहरणसहित व्याख्या गर्नुहोस् । (What do you mean by sequence? Explain with examples.)**

⇨ Here, a list of numbers in definite order is called sequence. e.g. 3,5, 7…………

(b) अनुक्रम र श्रेणीमा के भिन्नता छ ? (What is the difference between sequence and series?)

⇨ Here, the terms of sequence are connected with comma (,) and the terms of series are connected with (+) or (–) sign.

(c) समानान्तरीय अनुक्रमलाई परिभाषित गर्नुहोस् । (Define arithmetic sequence.)

⇨ Here, a sequence is said to be an arithmetic sequence when its terms increase or decrease continually by a common difference. The corresponding series of this sequence is called the arithmetic series. e.g. if 3, 7, 11, 15, …………is an arithmetic sequence then 3 + 7 + 11 + 15 + ... is an arithmetic series.

(d) समानान्तरीय अनुक्रमका विशेषताहरू उल्लेख गर्नुहोस् । (List down the characteristics of arithmetic sequence.)

⇨ Here, the characteristics of arithmetic sequence are as follows:

(i) Terms are either in ascending or descending order.
(ii) The difference between the consecutive terms are equal.

(e) समानान्तरीय मध्यमा भनेको के हो ? समानान्तरीय मध्यमा निकाल्ने सूत्रहरू उल्लेख गर्नुहोस् ।
What do you mean by arithmetic mean? Note down the formulas to calculate arithmetic mean.

⇨ Here, in arithmetic sequence, the term/terms between the first term and the last term is/are called arithmetic mean/means, e.g. if 3, 7, 11, 15, 19, 23 are in arithmetic sequence then 7, 11, 15, 19 are arithmetic means. The formula to calculate an arithmetic mean is; $AM = \frac{a+b}{2}$

The formulae to calculate arithmetic means are: $m_1 = a + d$, $m_2 = a + 2d$, $m_3 = a + 3d$, ..., $m_n = a + nd$.

2. **तल दिइएका मध्ये कुन कुन समानान्तरीय अनुक्रम हुन् र कुन कुन होइनन् ? कारण पनि उल्लेख गर्नुहोस् ।**
Which of the following sequences are arithmetic and which are not? Also, state the reason.

(a) 8, 13, 18, 23, ...

⇨ Here, 8, 13, 18, 23, ...

$t_2 - t_1 = 13 - 8 = 5$
$t_3 - t_2 = 18 - 13 = 5$
$t_4 - t_3 = 23 - 18 = 5$

This is an arithmetic sequence because the differences of the consecutive terms are equal.

(b) 6, 3, 0, – 3, – 6, ...

⇨ Here, 6, 3, 0, – 3, – 6, ...

$t_2 - t_1 = 3 - 6 = -3$
$t_3 - t_2 = 0 - 3 = -3$
$t_4 - t_3 = -3 - 0 = -3$
$t_5 - t_4 = -6 + 3 = -3$

This is an arithmetic sequence because the differences of the consecutive terms are equal.

(c) $7, 6\frac{1}{3}, 5\frac{1}{3}, 4\frac{2}{3}, \ldots$

⇨ Here, $7, 6\frac{1}{3}, 5\frac{1}{3}, 4\frac{2}{3}$, ...

$$t_2 - t_1 = 6\frac{1}{3} - 7 = -\frac{2}{3}$$
$$t_3 - t_2 = 5\frac{1}{3} - 6\frac{1}{3} = -1$$
$$t_4 - t_3 = 4\frac{2}{3} - 5\frac{1}{3} = -\frac{2}{3}$$

This is not an arithmetic sequence because the differences of the consecutive terms are not equal.

(d) $2^2, 3^2, 4^2, 5^2, 6^2, 7^2$

⇨ Here, $2^2, 3^2, 4^2, 5^2, 6^2, 7^2$

$t_2 - t_1 = 3^2 - 2^2 = 9 - 4 = 5$
$t_3 - t_2 = 4^2 - 3^2 = 16 - 9 = 7$
$t_4 - t_3 = 5^2 - 4^2 = 25 - 16 = 9$
$t_5 - t_4 = 6^2 - 5^2 = 36 - 25 = 11$
$t_6 - t_5 = 7^2 - 6^2 = 49 - 36 = 13$

This is not an arithmetic sequence because the differences of the consecutive terms are not equal.

(e) 18, 15, 12, 9, 6, 3

⇨ Here, 18, 15, 12, 9, 6, 3

$t_2 - t_1 = 15 - 18 = -3$
$t_3 - t_2 = 12 - 15 = -3$
$t_4 - t_3 = 9 - 12 = -3$
$t_5 - t_4 = 6 - 9 = -3$
$t_6 - t_5 = 3 - 6 = -3$

This is an arithmetic sequence because the differences of the consecutive terms are equal.

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3. दिइएका समानान्तरीय अनुक्रमको समान अन्तर, साधारण पद (t_n), र दसौं पद निकाल्नुहोस् :
Find the common difference, general term (t_n) and 10th term of the following arithmetic sequences.

(a) 20, 26, 32, 38, ...

⇨ Here, 20, 26, 32, 38, ...

First term (a) = 20

Common difference (d) = $t_2 - t_1 = 26 - 20 = 6$

We know that,

General term (t_n) $= a + (n - 1) d$

$= 20 + (n - 1) 6$

$= 20 + 6n - 6$

$\therefore \quad t_n = 6n + 14$

Again, 10th term (t_{10}) $= 6 \times 10 + 14$

$= 60 + 14$

$= 74$

Thus, the common difference, general term and tenth term are 6, 6n + 14 and 74 respectively.

(b) 1, – 2, – 5, – 8, ...

⇨ Here, 1, – 2, – 5, – 8, ...

We know that,

Common difference $= t_2 - t_1$

$= -2 - 1$

$= -3$

We have, general term (t_n)$= a + (n - 1) d$

$= 1 + (n - 1) (-3)$

$= 1 - 3n + 3$

$\therefore \quad t_n = 4 - 3n$

Now, 10th term (t_{10}) $= 4 - 3 \times 10$

$= 4 - 30$

$\therefore \quad t_{10} = -26$

Thus, common difference, general term and 10th term are – 3, 4 – 3n & – 26 respectively.

(c) 1, 5, 9, 13, ...

⇨ Here, 1, 5, 9, 13, ...

First term (a) = $t_1 = 1$

Common difference (d) = $t_2 - t_1 = 5 - 1 = 4$

We know that,

General term (t_n) $= a + (n - 1) d$

$= 1 + (n - 1) 4$

$= 1 + 4n - 4$

$\therefore \quad t_n = 4n - 3$

Now, 10th term (t_{10}) $= 4 \times 10 - 3 = 37$

Thus, common difference, general term and 10th term are 4, 4n – 3 and 37 respectively.

(d) $\frac{1}{3}, -\frac{8}{3}, -\frac{17}{3}, \ldots$

⇨ Here, $\frac{1}{3}, -\frac{8}{3}, -\frac{17}{3}$, ...

First term (a) = $\frac{1}{3}$

Common difference (d) = $t_2 - t_1 = -\frac{8}{3} - \frac{1}{3} = -3$

We know that,

General term (t_n) $= a + (n - 1) d$

$= \frac{1}{3} + (n - 1) (-3)$

$= \frac{1}{3} + (-3n) + 3$

$= \frac{1}{3} + 3 - 3n$

$= \frac{10}{3} - 3n$

Now, 10th term (t_{10}) $= \frac{10}{3} - 3 \times 10 = \frac{10}{3} - 30 = -\frac{80}{3}$

Thus, common difference, general term and 10th term are; – 3, $\frac{10}{3}$ – 3n and – $\frac{80}{3}$ respectively.

4. दिइएको अवस्थामा समानान्तरीय अनुक्रमको समान अन्तर (d) अथवा पहिलो पद (a) पत्ता लगाउनुहोस् ।
Find the common difference (d) or first term (a) of the arithmetic sequence in the following conditions.

(a) समान अन्तर (d) = 2 र सातौँ पद (t_7) = 14
Common difference (d) = 2 and 7th term (t_7) = 14

⇨ Here, $d = 2, t_7 = 14, a = ?$

We know that, $t_n = a + (n - 1) d$

So, $t_7 = a + 6 d$

or, $14 = a + 6 \times 2$

or, $14 = a + 12$

or, $14 - 12 = a$

$\therefore \quad a = 2$

Thus, the value of a is 2.

(b) समान अन्तर (d) = 3 र दसौँ पद (t_{10}) = 29
Common difference (d) = 3 and 10th term (t_{10}) = 29

⇨ Here, $d = 3$ and $t_{10} = 29; a = ?$

We know that,

$t_n = a + (n - 1) d$

So, $t_{10} = a + 9d$

or, $29 = a + 9 \times 3$

or, $29 = a + 27$

$\therefore \quad a = 2$

Thus, the first term is 2.

(c) पहिलो पद (a) = 6 र छैटौं पद (t_6) = 21
First term (a) = 6 and 6th term (t_6) = 21

⇨ Here, first term (a) = 6 and sixth term (t_6) = 21, d = ?

We know that,

$t_n = a + (n - 1) d$

So, $t_6 = a + 5d$

or, $21 = 6 + 5d$

or, $15 = 5d$

$\therefore \quad d = 3$

Thus, the common difference is 3.

(d) पहिलो पद (a) = – 2 र बिसौँ पद (t_{20}) = 74
First term (a) = – 2 and 20th term (t_{20}) = 74

⇨ Here, $a = -2$ and $t_{20} = 74; d = ?$

We know that,

$t_n = a + (n - 1) d$

or, $t_{20} = a + 19d$

or, $74 = -2 + 19d$

or, $76 = 19d$

$\therefore \quad d = 4$

Thus, the common difference is 4.

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(e) **पहिलो पद (a) = $\frac{4}{5}$ र पन्ध्रौं पद (t_{15}) = $8\frac{4}{5}$**

First term (a) = $\frac{4}{5}$ and 15th term (t_{15}) = $8\frac{4}{5}$

⇨ Here, $a = \frac{4}{5}$, $t_{15} = 8\frac{4}{5}$, d = ?

We know that, $t_n = a + (n - 1)\,d$

or, $t_{15} = a + 14d$

or, $8\frac{4}{5} = \frac{4}{5} + 14d$

or, $8\frac{4}{5} - \frac{4}{5} = 14d$

or, $8 = 14d$

or, $d = \frac{8}{14} = \frac{4}{7}$

Thus, the common difference is $\frac{4}{7}$.

(b) **$7 + 5\frac{1}{2} + 4 + 2\frac{1}{2} + \ldots - 23$**

⇨ Here, $7 + 5\frac{1}{2} + 4 + 2\frac{1}{2} + \ldots\ldots\ldots\ldots - 23$

First term (a) = 7

Common difference (d) = $t_2 - t_1 = 5\frac{1}{2} - 7 = -\frac{3}{2}$

Last term (t_n) = – 23

We know that, $t_n = a + (n - 1)\,d$

or, $-23 = 7 + (n - 1)\,(-)\frac{3}{2}$

or, $-30 = (1 - n)\frac{3}{2}$

or, $-60 = 3 - 3n$

or, $3n = 63$

∴ $n = 21$

Thus, the number of terms is 21.

(d) **$2 + 8 + 14 + 20 + \ldots + 80$**

⇨ Here, $2 + 8 + 14 + 20 + \ldots + 80$

First term (a) = 2

Common difference (d) = $t_2 - t_1 = 8 - 2 = 6$

We have,

$t_n = a + (n - 1)d$

or, $80 = 2 + (n - 1)6$

or, $80 - 2 = 6n - 6$

or, $6n = 78 + 6$

or, $n = \frac{84}{6} = 14$

∴ $n = 14$

Thus, the number of terms is 14.

5. **तल दिइएका श्रेणीहरूको पद सङ्ख्या पत्ता लगाउनुहोस् ।**
(Find the No. of terms of the following series.)

(a) **$5 + 8 + 11 + \ldots + 320$**

⇨ Here, $5 + 8 + 11 + \ldots\ldots\ldots\ldots + 320$

First term (a) = 5

Common difference (d) = $t_2 - t_1 = 8 - 5 = 3$

Last term (t_n) = 320

We know that,

$t_n = a + (n - 1)\,d$

or, $320 = 5 + (n - 1)\,3$

or, $315 = (n - 1)\,3$

or, $105 = n - 1$

∴ $n = 106$

Thus, the required number of terms is 106.

(c) **$4 + 11 + 18 + \ldots + 74$**

⇨ Here, $4 + 11 + 18 + \ldots\ldots\ldots\ldots + 74$

First term (a) = 4

Common difference (d) = $t_2 - t_1$
$= 11 - 4$
$= 7$

We have,

$t_n = a + (n - 1)d$

or, $74 = 4 + (n - 1)\,7$

or, $70 = (n - 1)\,7$

or, $10 = n - 1$

∴ $n = 11$

Thus, the number of terms is 11.

(e) **$\frac{4}{5} + \frac{38}{35} + \frac{48}{35} + \ldots + 8\frac{4}{5}$**

⇨ Here, $\frac{4}{5} + \frac{38}{35} + \frac{48}{35} + \ldots\ldots\ldots + 8\frac{4}{5}$

First term (a) = $\frac{4}{5}$

Common difference (d) = $t_2 - t_1$
$= \frac{38}{35} - \frac{4}{5} = \frac{2}{7}$

We have, $t_n = a + (n - 1)d$

or, $8\frac{4}{5} = \frac{4}{5} + (n - 1)\frac{2}{7}$

or, $8\frac{4}{5} - \frac{4}{5} = (n - 1)\frac{2}{7}$

or, $8 = \frac{2n - 2}{7}$

or, $2n - 2 = 56$

or, $2n = 58$

∴ $n = 29$

Thus, the number of terms is 29.

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6. निम्नलिखित अवस्थामा समानान्तरीय अनुक्रमको पहिलो पद (a) र समान अन्तर (d) पत्ता लगाउनुहोस् ।
Find the first term (a) and common difference (d) of arithmetic sequence in following conditions.

(a) पाँचौँ पद (t_5) = 22 र आठौँ पद (t_8) = 34
5th term (t_5) = 22 and 8th term (t_8) = 34

⇨ Here, fifth term (t_5) = 22 and eighth term (t_8) = 34
We know that,
$t_n = a + (n - 1) d$
So, $t_5 = a + 4d$ and $t_8 = a + 7d$
∴ $22 = a + 4d$ ....(i) and
$34 = a + 7d$ ....(ii)
Solving equations (i) and (ii) then,
$22 = a + 4d$
$34 = a + 7d$
$- \quad - \quad -$
$-12 = -3d$
∴ $d = 4$
From (i),
$22 = a + 4d$
or, $22 = a + 4 \times 4$
or, $22 = a + 16$
∴ $a = 6$
Thus, the first term and the common difference are 6 and 4 respectively.

(b) चौथो पद (t_4) = 13 र छैटौँ पद (t_6) = 7
4th term (t_4) = 13 and 6th term (t_6) = 7

⇨ Here, fourth term (t_4) = 13 and sixth term (t_6) = 7
We know that,
$t_n = a + (n - 1) d$
So, $t_4 = a + 3d$ and $t_6 = a + 5d$
∴ $13 = a + 3d$ ....(i) and
$7 = a + 5d$ ....(ii)
Solving equations (i) and (ii) then,
$13 = a + 3d$
$7 = a + 5d$
$- \quad - \quad -$
$6 = -2d$
∴ $d = -3$
Substituting $d = -3$ in (i);
$13 = a + 3d$
or, $13 = a + 3(-3)$
or, $13 = a - 9$
∴ $a = 22$
Thus, the first term and the common difference are 22 and – 3 respectively.

(c) पाँचौँ पद (t_5) = 13 र दसौँ पद (t_{10}) = 28
5th term (t_5) = 13 and 10th term (t_{10}) = 28

⇨ Here, fifth term (t_5) = 13 and
tenth term (t_{10}) = 28
We know that,
$t_n = a + (n - 1)d$
So, $t_5 = a + 4d$ and $t_{10} = a + 9d$
∴ $13 = a + 4d$ ....(i) and
$28 = a + 9d$ ....(ii)
Solving equations (i) and (ii) then,
$13 = a + 4d$
$28 = a + 9d$
$- \quad - \quad -$
$-15 = -5d$
∴ $d = 3$
From (i);
$13 = a + 4d$
or, $13 = a + 4 \times 3$
or, $13 = a + 12$
∴ $a = 1$
Thus, the first term and the common difference are 1 and 3 respectively.

(d) दसौँ पद (t_{10}) = 23 र बत्तिसौँ पद (t_{32}) = 67
10th term (t_{10}) = 23 and 32nd term (t_{32}) = 67

⇨ Here, tenth term (t_{10}) = 23 and thirty second term (t_{32}) = 67
We know that,
$t_n = a + (n - 1)d$
So, $t_{10} = a + 9d$ and $t_{32} = a + 31d$
∴ $23 = a + 9d$ ....(i) and
$67 = a + 31d$ ....(ii)
Solving equations (i) and (ii) then,
$23 = a + 9d$
$67 = a + 31d$
$- \quad - \quad -$
$-44 = -22d$
∴ $d = 2$
From (i); $23 = a + 9d$
or, $23 = a + 9 \times 2$
or, $23 = a + 18$
or, $5 = a$
∴ $a = 5$
Thus, the first term and the common difference are 5 and 2 respectively.

7. (a) समानान्तरीय अनुक्रमको तेस्रो र तेह्रौँ पद क्रमशः 40 र 0 भए कुन पदको मान 28 हुन्छ ? पत्ता लगाउनुहोस् ।
If the 3rd and 13th terms of an arithmetic sequence are 40 and 0 respectively then the value of which term is 28? Find out.

⇨ Here, $t_3 = 40$, $t_{13} = 0$, $t_m = 28$; $m = ?$
We know that, $t_n = a + (n - 1)d$
So, $t_3 = a + 2d$ and $t_{13} = a + 12d$
∴ $40 = a + 2d$ ....(i) and $0 = a + 12d$ ....(ii)
Solving equations (i) and (ii) then,
$40 = a + 2d$
$0 = a + 12d$
$- \quad - \quad -$
$40 = -10d$
∴ $d = -4$
From (i);
$40 = a + 2d$
or, $40 = a + 2 \times (-4)$
or, $40 = a - 8$
∴ $a = 48$

Now, $t_m = a + (m - 1)d$
or, $28 = 48 + (m - 1)(-4)$
or, $-20 = -4(m - 1)$
or, $5 = m - 1$
∴ $m = 6$
Thus, the required term is 6th term.

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(b) समानान्तरीय अनुक्रमको तेस्रो र बयालिसौं पद क्रमशः 10 र 88 भए कुन पदको मान 24 हुन्छ ? पत्ता लगाउनुहोस् ।
If the 3^{rd} and 42^{nd} terms of an arithmetic sequence are 10 and 88 respectively then the value of which term is 24? Find out.

⇨ Here, $t_3 = 10$, $t_{42} = 88$, $t_m = 24$, $m = ?$

We have, $t_n = a + (n - 1)d$

So, $t_3 = a + 2d$ and $t_{42} = a + 41d$

$\therefore$ $10 = a + 2d$ ....(i) and

$88 = a + 41d$ ....(ii)

Solving equations (i) and (ii) then,

$$\begin{array}{l} 10 = a + 2d \\ 88 = a + 41d \\ \underline{-\quad\ -\quad -} \\ -78 = -39d \end{array}$$

$\therefore$ $d = 2$

From equation (i) then,

$10 = a + 2d$

or, $10 = a + 2 \times 2$

or, $10 - 4 = a$

$\therefore$ $a = 6$

Now, $t_m = a + (m - 1) d$

or, $24 = 6 + (m - 1) 2$

or, $18 = (m - 1) 2$

or, $9 = m - 1$

$\therefore$ $m = 10$

Thus, the required term is 10^{th} term.

(d) समानान्तरीय अनुक्रमको सातौं र एकाउन्नौं पद क्रमशः – 3 र – 355 भए बिसौं पदको मान कति हुन्छ ? पत्ता लगाउनुहोस् ।
If the 7^{th} and 51^{st} terms of an arithmetic sequence are – 3 and – 355 respectively then find the value of 20^{th} term.

⇨ Here, $t_7 = -3$ and $t_{51} = -355$, $t_{20} = ?$

We know that, $t_n = a + (n - 1) d$

So, $t_7 = a + 6d$ and $t_{51} = a + 50 d$

$\therefore$ $-3 = a + 6d$ ....(i) and

$-355 = a + 50 d$ ....(ii)

Solving equations (i) and (ii) then,

$$\begin{array}{l} -3 \quad = a + 6d \\ -355 = a + 50d \\ \underline{+ \qquad\quad - \quad -} \\ 352 = -44d \end{array}$$

$\therefore$ $d = -8$

Substituting the value of d in equation (i) then,

$-3 = a + 6d$

or, $-3 = a + 6 (-8)$

or, $-3 + 48 = a$

$\therefore$ $a = 45$

Now,

$t_{20} = a + 19d$

$= 45 + 19 \times (-8)$

$= 45 - 152$

$\therefore$ $t_{20} = -107$

Thus, 20^{th} term is – 107.

(c) समानान्तरीय अनुक्रमको छैटौं र सत्रौं पद क्रमशः 19 र 41 भए सयौं पदको मान कति हुन्छ ? पत्ता लगाउनुहोस् ।
If the 6^{th} and 17^{th} terms of an arithmetic sequence are 19 and 41 respectively then find the value of 100^{th} term.

⇨ Here, $t_6 = 19$, $t_{17} = 41$, $t_{100} = ?$

We have, $t_n = a + (n - 1)d$

So, $t_6 = a + 5d$ and $t_{17} = a + 16d$

$\therefore$ $19 = a + 5d$ ....(i) and

$41 = a + 16 d$ ....(ii)

Solving equations (i) and (ii) then,

$$\begin{array}{l} 19 = a + 5d \\ 41 = a + 16d \\ \underline{-\quad\ -\quad -} \\ -22 = -11d \end{array}$$

$\therefore$ $d = 2$

Substituting the value of d in equation (i) then,

$19 = a + 5d$

or, $19 = a + 5 \times 2$

or, $19 = a + 10$

$\therefore$ $a = 9$

Now,

$t_{100} = a + 99 d$

$= 9 + 99 \times 2$

$= 9 + 198$

$\therefore$ $t_{100} = 207$

Thus, the 100^{th} term is 207.

8. (a) कुनै समानान्तरीय अनुक्रममा $5t_5 = 9t_9$ भए प्रमाणित गर्नुहोस् : $t_{14} = 0$
In an arithmetic sequence, $5t_5 = 9t_9$ then prove that: $t_{14} = 0$.

⇨ Here, $5t_5 = 9t_9$

We have, $t_n = a + (n - 1) d$ then, $5t_5 = 9t_9$ becomes;

$5 (a + 4d) = 9 (a + 8d)$

or, $5a + 20 d = 9a + 72d$

or, $-4a = 52d$

or, $a = -13d$

$\therefore$ $a + 13d = 0$

i.e. $t_{14} = 0$ **Proved.**

(b) कुनै समानान्तरीय अनुक्रममा $\frac{t_7}{t_{11}} = \frac{11}{7}$ भए प्रमाणित गर्नुहोस् : $t_{18} = 0$ (In an arithmetic sequence, $\frac{t_7}{t_{11}} = \frac{11}{7}$ then prove that: $t_{18} = 0$.)

⇨ Here, $\frac{t_7}{t_{11}} = \frac{11}{7}$

or, $7t_7 = 11t_{11}$

We have, $t_n = a + (n - 1) d$

So, $7 (a + 6d) = 11 (a + 10d)$

or, $7a + 42d = 11a + 110d$

or, $-4a = 68d$

or, $a = -17d$

or, $a + 17d = 0$

i.e. $t_{18} = 0$ **Proved.**

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9. यदि समानान्तरीय श्रेणीको दसौँ पदको दस गुणासँग पन्ध्रौँ पदको पन्ध्र गुणा बराबर छ र पहिलो पद 48 छ भने उक्त अनुक्रमको समान अन्तर पत्ता लगाउनुहोस् ।

In an arithmetic series, 10 times of 10th term is equals to 15 times of 15th term and first term is 48. Find the common difference of the sequence.

⇨ Here, $10t_{10} = 15t_{15}$; $a = 48$, $d = ?$

We have, $t_n = a + (n - 1)d$ then $10t_{10} = 15t_{15}$ becomes;

So, $10 (a + 9d) = 15 (a + 14d)$

or, $10a + 90d = 15a + 210d$

or, $-5a = 120d$

or, $a = -24d$

or, $48 = -24d$

$\therefore$ $d = -2$

Thus, the common difference is – 2.

10. (a) एउटा समानान्तरीय श्रेणीमा रहेका तीन ओटा पदहरूको योगफल 36 छ र तिनीहरूको गुणनफल 1140 छ भने ती पदहरू पत्ता लगाउनुहोस् ।

If the sum of three terms in an arithmetic series is 36 and their product is 1140, find the terms.

⇨ Here, let $(a - d)$, a and $(a + d)$ are the three terms of an A.P. Then, sum = 36

i.e. $a - d + a + a + d = 36$

or, $3a = 36$

$\therefore$ $a = 12$

Again, product = 1140

or, $(a - d)a (a + d) = 1140$

or, $(a^2 - d^2)a = 1140$

or, $(12^2 - d^2) 12 = 1140$

or, $(144 - d^2) = 95$

or, $144 - d^2 = 95$

or, $49 = d^2$

$\therefore$ $d = \pm 7$

When a = 12 and d = 7 then the three terms are;

$(a - d), a, (a + d)$

$= (12 - 7), 12, (12 + 7)$

$= 5, 12, 19$

When a = 12 and d = – 7 then the three terms are

$(a - d), a, (a + d)$

$= (12 + 7), 12, (12 - 7)$

$= 19, 12, 5$

Thus, the three numbers are 5, 12, 19 or 19, 12, 5.

(b) एउटा समानान्तरीय श्रेणीमा रहेका तीन ओटा सङ्ख्याहरूको योगफल 45 छ र तिनीहरूको गुणनफल 1875 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

If the sum of three number in an arithmetic series is 45 and their product is 1875, find the numbers.

⇨ Here, sum of 3 terms = 45 and product of 3 terms = 1875.

Let $(a - d)$, a & $(a + d)$ are the three terms of A.P.

By the question,

$a - d + a + a + d = 45$

or, $3a = 45$

$\therefore$ $a = 15$

Again, $(a - d) a(a + d) = 1875$

or, $(15 - d) 15 (15 + d) = 1875$

or, $15^2 - d^2 = 125$

or, $-d^2 = 125 - 225$

or, $-d^2 = -100$

$\therefore$ $d^2 = (\pm 10)^2$

$\therefore$ $d = \pm 10$

When a = 15 and d = 10 then the three terms are;

$(a - d), a, (a + d)$

$= (15 - 10), 15, (15 + 10)$

$= 5, 15, 25$

When a = 15 and d = – 10 then the three terms are; $(a - d), a, (a + d)$

$= (15 + 10), 15, (15 - 10)$

$= 25, 15, 5$

Thus, the three terms are 5, 15, 25 or 25, 15, 5.

11. (a) श्रेणी 14 + 12 + 10 + ... को n औँ पदसँग श्रेणी 20 + 17 + 14 + ... को n औँ पद बराबर छ भने n को मान पत्ता लगाउनुहोस् ।

If the nth term of series 14 + 12 + 10 + ... is equals to the nth term of series 20 + 17 + 14 + ... then find the value of n.

⇨ Here, 14 + 12 + 10 + ... and 20 + 17 + 14 + ... are given series.

14 + 12 + 10 + ...	**20 + 17 + 14 + ...**
$a = 14$ $d = t_2 - t_1 = 12 - 14 = -2$ We have, $t_n = a + (n - 1)d$ $= 14 + (n - 1) (-2)$ $= 14 - 2 (n - 1)$ $= 14 - 2n + 2$ $\therefore$ $t_n = 16 - 2n$	$a = 20$ $d = t_2 - t_1 = 17 - 20 = -3$ We know that, $t_n = a + (n - 1) d$ $= 20 + (n - 1) (-3)$ $= 20 - 3 (n - 1)$ $= 20 - 3n + 3$ $\therefore$ $t_n = 23 - 3n$

By the question,

$16 - 2n = 23 - 3n$

or, $3n - 2n = 23 - 16$

$\therefore$ $n = 7$

Thus, the value of n is 7.

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(b) **श्रेणी – 9 – 6 – 3 – ... को n औं पदसँग श्रेणी 16 + 14 + 12 + ... को n औं पद बराबर छ भने n को मान पत्ता लगाउनुहोस् ।**
If the n^{th} term of series – 9 – 6 – 3 – ... is equals to the n^{th} term of series 16 + 14 + 12 + ... then find the value of n.

⇨ Here, – 9 – 6 – 3 – .... and 16 + 14 + 12 + ... are given series.

– 9 – 6 – 3 –	16 + 14 + 12 + ...
$a = -9$ $d = t_2 - t_1 = -6 + 9 = 3$ We have, $t_n = a + (n-1)d$ $= -9 + (n-1)\,3$ $= -9 + 3n - 3$ $\therefore\ t_n = -12 + 3n$	$a = 16$ $d = t_2 - t_1 = 14 - 16 = -2$ We know that, $t_n = a + (n-1)\,d$ $= 16 + (n-1)(-2)$ $= 16 - 2(n-1)$ $= 16 - 2n + 2$ $\therefore\ t_n = 18 - 2n$

By the question, $-12 + 3n = 18 - 2n$
or, $3n + 2n = 18 + 12$
or, $5n = 30$
$\therefore\ n = 6$
Thus, the value of n is 6.

12. दिइएको अवस्थामा दोस्रो पदको मान निकाल्नुहोस् (Find the value of 2^{nd} terms in following conditions) :

(a) पहिलो पद = 7 र तेस्रो पद = 17
First term = 7 and third term = 17

⇨ Here, first term $(t_1) = 7$ and third term $(t_3) = 17$
We know that,

$$t_2 = \frac{t_1 + t_3}{2} = \frac{7 + 17}{2} = \frac{24}{2} = 12$$

$\therefore\ t_2 = 12$
Thus, the second term is 12.

(c) पहिलो पद = $\frac{4}{5}$ र तेस्रो पद = $\frac{11}{5}$

First term = $\frac{4}{5}$ and third term = $\frac{11}{5}$

⇨ Here, $t_1 = \frac{4}{5}$ and $t_3 = \frac{11}{5}$
We know that,

$$\text{Second term } (t_2) = \frac{t_1 + t_3}{2} = \frac{\frac{4}{5} + \frac{11}{5}}{2} = \frac{3}{2}$$

Thus, the second term is $\frac{3}{2}$.

(b) चौधौं पद = 30 र सोह्रौं पद = 40, पन्ध्रौं पद = ?
14^{th} term = 30 and 16^{th} term = 40, 15^{th} term = ?

⇨ Here, $t_{14} = 30$ and $t_{16} = 40$
We know that,

$$t_{15} = \frac{1}{2}(t_{14} + t_{16})$$
$$= \frac{1}{2}(30 + 40)$$
$$= \frac{1}{2} \times 70$$
$$= 35$$

Thus, the 15th term is 35.

(b) पहिलो पद = – 20 र तेस्रो पद = 60
First term = – 20 and third term = 60

⇨ Here, $t_1 = -20$, $t_3 = 60$
We know that,

$$\text{Second term } (t_2) = \frac{t_1 + t_3}{2} = \frac{-20 + 60}{2} = 20$$

Thus, the second term is 20.

13. तलको अवस्थामा तोकिएको समानान्तरीय मध्यमा निकाल्नुहोस् ।
Find the indicated arithmetic mean in the following conditions.

(a) छैटौं पद = – 3 र आठौं पद = 9, सातौं पद = ?
6^{th} term = – 3 and 8^{th} term = 9, 7^{th} term = ?

⇨ Here, $t_6 = -3$ and $t_8 = 9$
We know that,

$$t_7 = \frac{t_6 + t_8}{2} = \frac{-3 + 9}{2} = \frac{6}{2} = 3$$

Thus, the 7th term is 3.

(c) नवौं पद = 28 र एघारौं पद = 36, दसौं पद = ?
9^{th} term = 28 and 11^{th} term = 36, 10^{th} term = ?

⇨ Here, $t_9 = 28$ and $t_{11} = 36$
We know that,

$$t_{10} = \frac{t_9 + t_{11}}{2}$$
$$= \frac{28 + 36}{2}$$
$$= \frac{64}{2}$$
$$= 32$$

Thus, the tenth term is 32.

14. निम्नअनुसार समानान्तरीय मध्यमा निकाल्नुहोस् (Find the arithmetic means as below):

(a) 2 र 20 का बिचमा 5 ओटा (5 between 2 and 20)

⇨ Here, first term (a) = 2 and last term (b) = 20.
No. of AM = 5
We know that,

$$\text{Common difference (d)} = \frac{b - a}{n + 1}$$
$$= \frac{20 - 2}{5 + 1}$$
$$= \frac{18}{6}$$
$$= 3$$

Now,

First mean $(m_1) = a + d = 2 + 3 = 5$

Second mean $(m_2) = a + 2d = 2 + 2 \times 3 = 8$

Third mean $(m_3) = a + 3d = 2 + 3 \times 3 = 11$

Fourth mean $(m_4) = a + 4d = 2 + 4 \times 3 = 14$

Fifth mean $(m_5) = a + 5d = 2 + 5 \times 3 = 17$

Thus, the means are; 5, 8, 11, 14 and 17.

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(b) – 18 र 2 का बिचमा 4 ओटा (4 between – 18 and 2)

⇨ Here, first term (a) = – 18, and last term (b) = 2

No. of AM (n) = 4

We know that,

$$\text{Common difference (d)} = \frac{b-a}{n+1} = \frac{2+18}{4+1} = \frac{20}{5} = 4$$

Now, $m_1 = a + d = -18 + 4 = -14$

$m_2 = a + 2d = -18 + 2 \times 4 = -10$

$m_3 = a + 3d = -18 + 3 \times 4 = -6$

$m_4 = a + 4d = -18 + 4 \times 4 = -2$

Thus, the means are; – 14, – 10, – 6, – 2.

(d) $\frac{1}{2}$ र $\frac{7}{2}$ का बिचमा 5 ओटा (5 between $\frac{1}{2}$ and $\frac{7}{2}$)

⇨ Here, first term (a) = $\frac{1}{2}$ and last term (b) = $\frac{7}{2}$

No. of means (n) = 5

We have, $d = \frac{b-a}{n+1} = \frac{\frac{7}{2}-\frac{1}{2}}{5+1} = \frac{\frac{6}{2}}{\frac{6}{1}} = \frac{6}{2} \times \frac{1}{6} = \frac{1}{2}$

Now,

$m_1 = a + d = \frac{1}{2} + \frac{1}{2} = 1$

$m_2 = a + 2d = \frac{1}{2} + 2 \times \frac{1}{2} = \frac{3}{2}$

$m_3 = a + 3d = \frac{1}{2} + 3 \times \frac{1}{2} = \frac{1}{2} + \frac{3}{2} = 2$

$m_4 = a + 4d = \frac{1}{2} + 4 \times \frac{1}{2} = \frac{1}{2} + 2 = \frac{5}{2}$

$m_5 = a + 5d = \frac{1}{2} + 5 \times \frac{1}{2} = \frac{6}{2} = 3$

Thus, the means are; 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3.

(c) 1 र 16 का बिचमा 2 ओटा (2 between 1 and 16.)

⇨ Here, first term (a) = 1 and last term (b) = 16.

No. of means (n) = 2

We have,

$$d = \frac{b-a}{n+1} = \frac{16-1}{2+1} = \frac{15}{3} = 5$$

Now,

$m_1 = a + d = 1 + 5 = 6$

$m_2 = a + 2d = 1 + 2 \times 5 = 11$

Thus, the means are 6 and 11.

15. दिइएको समानान्तर अनुक्रमबाट, *x* को मान निकाल्नुहोस् :
Find the value of x from the following arithmetic sequence.

(a) 7, *x*, 11

⇨ Here, 7, x, 11

First term (a) = 7 & Last term (b) = 11

AM = x

We have,

$$AM = \frac{a+b}{2} = \frac{7+11}{2} = \frac{18}{2} = 9$$

∴ x = 9

Thus, the value of x is 9.

(b) 2*x* + 1, 2*x* – 1, 3*x* + 4

⇨ Here, 2x + 1, 2x – 1, 3x + 4

Given terms are in AP.

So, $t_2 - t_1 = t_3 - t_2$

or, $(2x - 1) - (2x + 1) = (3x + 4) - (2x - 1)$

or, $2x - 1 - 2x - 1 = 3x + 4 - 2x + 1$

or, $2x - 2 = 3x + 5$

or, $-7 = x$

∴ x = – 7

Thus, the value of x is – 7.

(c) *x* + 1, *x* + 5, 3*x* + 1

⇨ Here, x + 1, x + 5, 3x + 1

Given terms are in AP.

So, $t_2 - t_1 = t_3 - t_2$

or, $(x + 5) - (x + 1) = (3x + 1) - (x + 5)$

or, $x + 5 - x - 1 = 3x + 1 - x - 5$

or, $4 = 2x - 4$

or, $2x = 8$

∴ x = 4

Thus, the value of x is 4.

16. (a) पदहरू 2 र 11 का बिचमा पर्ने मध्यमाहरूको सङ्ख्या निकाल्नुहोस्, जहाँ पहिलो र अन्तिम मध्यमाको अनुपात 7 : 19 छ।
Find the number of means between 2 and 11 where the ratio of first mean to the last mean is 7 : 19.

⇨ Here, first term (a) = 2, last term (b) = 11

First mean : last mean = 7 : 19

No. of means (n) = ?

We know that,

$$d = \frac{b-a}{n+1} = \frac{11-2}{n+1} = \frac{9}{n+1}$$

By the question,

$$\frac{m_1}{m_n} = \frac{7}{19}$$

or, $\frac{a+d}{a+nd} = \frac{7}{19}$

or, $\frac{2 + \frac{9}{n+1}}{2 + n \cdot \frac{9}{n+1}} = \frac{7}{19}$

(b) पदहरू 5 र 35 का बिचमा n ओटा मध्यमाहरू छन्। दोस्रो र अन्तिम मध्यमाको अनुपात 1 : 4 छ भने n को मान पत्ता लगाउनुहोस्।
There are n arithmetic means between 5 and 35. If the ratio of second mean to the last mean is 1 : 4 then find the value of n.

⇨ Here, first term (a) = 5, last term (b) = 35, second mean : last mean = $m_2 : m_n$ = 1 : 4

No. of means (n) = ?

We have,

$$\text{Common difference (d)} = \frac{b-a}{n+1} = \frac{35-5}{n+1} = \frac{30}{n+1}$$

Again,

$$\frac{m_2}{m_n} = \frac{1}{4}$$

or, $\frac{a+2d}{a+nd} = \frac{1}{4}$

or, $\frac{5 + 2 \times \frac{30}{n+1}}{5 + n \times \frac{30}{n+1}} = \frac{1}{4}$

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or, $\dfrac{\dfrac{2n+2+9}{n+1}}{\dfrac{2n+2+9n}{n+1}} = \dfrac{7}{19}$

or, $\dfrac{2n+11}{11n+2} = \dfrac{7}{19}$

or, $7(11n+2) = 19(2n+11)$

or, $77n + 14 = 38n + 209$

or, $39n = 195$

$\therefore$ $n = 5$

Thus, the number of means is 5.

or, $\dfrac{\dfrac{5n+5+60}{n+1}}{\dfrac{5n+5+30n}{n+1}} = \dfrac{1}{4}$

or, $\dfrac{5n+65}{35n+5} = \dfrac{1}{4}$

or, $35n + 5 = 20n + 260$

or, $15n = 255$

$\therefore$ $n = 17$

Thus, the number of means is 17.

17. (a) एउटा मिटर ट्याक्सीमा सुरुमा रु. 5 र त्यसपछि प्रत्येक 1 km मा रु. 9 का दरले भाडा उठ्छ भने 10 km यात्रा गर्दा जम्मा कति रुपियाँ तिर्नुपर्ला ?

In a meter taxi, Rs 5 is added at first and the fare goes on at the rate of Rs 9 per km. How much taxi fare is required to travel 10 km distance?

⇨ Here, taxi fare in the beginning (a) = Rs 5

The difference in each km (d) = Rs 9

If 10 km distance is covered then t_{11} is required.

We know that,

$t_n = a + (n - 1)\,d$

So, $t_{11} = a + 10d$

$= \text{Rs } 5 + 10 \times \text{Rs } 9$

$= \text{Rs } 5 + \text{Rs } 90$

$\therefore$ $t_{11} = \text{Rs } 95$

Thus, Rs 95 is required to travel 10 km distance.

(b) एक जना कर्मचारीको मासिक तलब रू. 40,000 छ । वार्षिक रू. 2,000 का दरले उसको तलबमा वृद्धि हुँदै जान्छ भने एघारौँ वर्षमा उसको मासिक तलब कति पुग्छ ? पत्ता लगाउनुहोस् ।

The monthly income of a civil servant is Rs 40,000. If his income increases by Rs 2000 per year, what will be his monthly income in 11th year? find out.

⇨ Here, salary in the beginning (a) = Rs 40000

Increased salary (d) = Rs 2000

We know that,

$t_n = a + (n - 1)d$

or, $t_{11} = 40000 + (11 - 1)\,2000$

$= 40000 + 10 \times 2000$

$= 40000 + 20000$

$= 60000$

Thus, the monthly income in eleventh year is Rs 60000.

18. यदि कुनै समानान्तरीय अनुक्रमको p औँ, q औँ र r औँ पदहरू क्रमशः a, b र c भए प्रमाणित गर्नुहोस् :p(b – c) + q(c – a) + r(a – b) = 0

If pth, qth and rth terms of an arithmetic sequence are a, b and c respectively then prove that: p(b – c) + q(c – a) + r(a – b) = 0

⇨ Here, $t_p = a$, $t_q = b$, $t_r = c$

We have, $t_n = a + (n - 1)d$

So, $t_p = t_1 + (p - 1)d$

$\therefore$ $a = t_1 + (p - 1)d$

And, $t_q = t_1 + (q - 1)d$

$\therefore$ $b = t_1 + (q - 1)d$

Similarly, $t_r = t_1 + (r - 1)d$

$\therefore$ $c = t_1 + (r - 1)d$

Now, $p\,(b - c) = p\,[t_1 + (q - 1)\,d - (t_1 + (r - 1)\,d)]$

$= p\,[t_1 + qd - d - t_1 - rd + d]$

$\therefore$ $p\,(b - c) = p\,(qd - rd) = pqd - prd$ ....(i)

Again, $q(c - a) = q\,[t_1 + (r - 1)d - t_1 - (p - 1)\,d]$

$= q\,[t_1 + rd - d - t_1 - pd + d]$

$= q\,(rd - pd)$

$\therefore$ $q\,(c - a) = qrd - pqd$ ....(ii)

Similarly, $r(a - b) = r(t_1 + (p - 1)d - t_1 - (q - 1)\,d]$

$= r[t_1 + pd - d - t_1 - qd + d]$

$\therefore$ $r(a - b) = r[pd - qd]$ ....(iii)

$= prd - qrd$

Adding (i), (ii) and (iii);

$p(b - c) + q(c - a) + r(a - b) = pqd - prd + qrd - pqd + prd - qrd$

$= 0$

$\therefore$ $p(b - c) + q(c - a) + r(a - b) = 0$ **Proved.**

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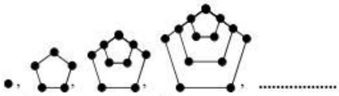
19. **दिइएको सङ्ख्याहरूको ढाँचामा (In the following number pattern)**

(a) n औँ पद पत्ता लगाउनुहोस् । (Find n^{th} term)

(a)



(a) ⇨ Here, terms of AP are;

1, 5, 9, 13, ...

So, first term (a) = 1

Common difference (d) = $t_2 - t_1 = 5 - 1 = 4$

We know that,

$t_n = a + (n - 1)d$

$= 1 + (n - 1) 4$

$= 1 + 4n - 4$

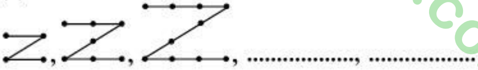
$\therefore \quad t_n = 4n - 3$

So, $t_{10} = 4 \times 10 - 3 = 40 - 3 = 37$

Thus, n^{th} term and 10^{th} term are (4n – 3) and 37 respectively.

(b) दसौँ पद निकाल्नुहोस् । (Find 10^{th} term)

(b)



(b) ⇨ Here, the terms of AP are;

4, 7, 10, 13, ...

So, first term (a) = 4

Common difference (d) = $t_2 - t_1 = 7 - 4 = 3$

We know that,

$t_n = a + (n - 1)d$

$= 4 + (n - 1) 3$

$= 4 + 3n - 3$

$\therefore \quad t_n = 1 + 3n$

Now, $t_{10} = 1 + 3 \times 10 = 31$

Thus, n^{th} term and 10^{th} term are (1 + 3n) and 31 respectively.

1.3.2 समानान्तर श्रेणीको योगफल (SUM OF ARITHMETIC SERIES)

EXERCISE 1.3.2

1. **निम्न श्रेणीहरूको योगफल निकाल्नुहोस् (Find the sum of following series):**

(a) 4 + 7 + 10 + ... 10 ओटा पदहरू

4 + 7 + 10 +..... 10 terms

⇨ Here,

First term (a) = 4

Common difference (d) = $t_2 - t_1 = 7 - 4 = 3$

Number of terms (n) = 10

Sum $(S_{10}) = ?$

We have,

$$S_n = \frac{n}{2}\{2a + (n - 1) d\}$$

$$S_{10} = \frac{10}{2}\{2 \times 4 + (10 - 1) 3\}$$

$= 5 (8 + 9 \times 3)$

$= 5 (8 + 27)$

$= 5 \times 35$

$\therefore \quad S_{10} = 175$

Thus, the sum of 10 terms is 175.

(c) 12 + 9 + 6 + ... 32 ओटा पदहरू

12 + 9 + 6 +..... 32 terms

⇨ Here, 12 + 9 + 6 + 32 + ... 32 terms

First term (a) = 12

Common difference (d) = $t_2 - t_1 = 9 - 12 = -3$

No. of terms (n) = 32

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1) d]$$

$$= \frac{32}{2}[2 \times 12 + (32 - 1) (-3)]$$

$= 16 [24 - 3 \times 31]$

$= 16 \times (-) 69$

$\therefore \quad S_{32} = -1104$

Thus, the sum of 32 terms is – 1104.

(b) 8 + 5 + 2 + ... 17 ओटा पदहरू

8 + 5 + 2 +..... 17 terms

⇨ Here, First term (a) = 8

Common difference (d) = $t_2 - t_1 = 5 - 8 = -3$

Number of terms (n) = 17

We have, $S_n = \frac{n}{2}\{2a + (n - 1) d\}$

$$\text{or,} \quad S_{17} = \frac{17}{2}\{2 . 8 + (17 - 1) (-3)\}$$

$$= \frac{17}{2}\{16 - 16 \times 3\}$$

$$= \frac{17}{2}\{16 - 48\}$$

$$= \frac{17}{2} \times (-32)$$

$\therefore \quad S_{17} = -272$

Thus, the sum of 17 terms is – 272.

(d) 1 + 4 + 7 + ... + 34

⇨ Here, First term (a) = 1

Common difference (d) = $t_2 - t_1 = 4 - 1 = 3$

Last term $(\ell) = 34$

We have, $t_n = a + (n - 1)d$

or, $34 = 1 + (n - 1) 3$

or, $34 = 1 + 3n - 3$

or, $34 = -2 + 3n$

or, $36 = 3n$

$\therefore \quad n = 12$

Now, $S_n = \frac{n}{2}(a + \ell)$

$$\text{or,} \quad S_{12} = \frac{12}{2}(1 + 34)$$

$= 6 \times 35$

$\therefore \quad S_{12} = 210$

Thus, the sum of 12 terms is 210.

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(e) 2.01 + 2.02 + 2.03 + ... + 3.00

⇨ Here, First term $(a) = 2.01$
Common difference $(d) = t_2 - t_1 = 2.02 - 2.01 = 0 .01$
Last term $(\ell) = 3.00$
Sum $(S_n) = ?$
We have,
$t_n = a + (n - 1)d$
or, $3 = 2.01 + (n - 1)(0.01)$
or, $3 = 2.01 + 0.01n - 0.01$
or, $3 = 2 + 0.01\,n$
or, $1 = 0.01n$
$\therefore\ n = \frac{1}{0.01} = 100$

Now, $S_n = \frac{n}{2}\ (a + \ell)$

$S_{100} = \frac{100}{2}(2.01 + 3.0)$
$= 50 \times 5.\,01$
$\therefore\ S_{100} = 250.5$
Thus, the sum of 100 terms is 250.5.

(f) $7 + 8\frac{1}{4} + 9\frac{1}{2} + ... + 17$

⇨ Here, first term $(a) = 7$ and last term $(b) = 17$.
Common difference $(d) = t_2 - t_1$
$= 8\frac{1}{4} - 7 = 1\frac{1}{4}$

We know that, $t_n = a + (n - 1)d$
or, $17 = 7 + (n - 1)\ 1\frac{1}{4}$
or, $10 = (n - 1)\frac{5}{4}$
or, $40 = 5n - 5$
or, $5n = 45$
$\therefore\ n = 9$
Now, $S_n = \frac{n}{2}(a + b)$
$= \frac{9}{2}(7 + 17)$
$= \frac{9}{2} \times 24$
$\therefore\ S_9 = 108$
Thus, the sum of 9 terms is 108.

2. (a) पहिलो पद 4 र समान अन्तर 5 भएको समानान्तरीय श्रेणीका पहिला 20 पदहरूको योगफल निकाल्नुहोस् ।
Find the sum of first 20 terms of an arithmetic series having first term 4 and common difference 5.

⇨ Here, First term $(a) = 4$
Common difference$(d) = 5$
Number of terms $(n) = 20$
Sum of series $(S_{20}) = ?$

We have, $S_n = \frac{n}{2}\{\ 2a + (n - 1)d\}$

or, $S_{20} = \frac{20}{2}\{\ 2 \times 4 + (20 - 1)\ 5\}$
$= 10\{\ 8 + 19 \times 5\}$
$= 10\{\ 8 + 95\}$
$\therefore\ S_{20} = 1030$
Thus, the required sum is 1030.

(b) यदि कुनै समानान्तरीय श्रेणीको पहिलो पद 3 र अन्तिम पद 98 छ भने पहिलो 20 ओटा पदहरूको योगफल निकाल्नुहोस् ।
If the first term of an arithmetic series is 3 and the last term is 98, find the sum of first 20 terms.

⇨ Here, first term $(a) = 3$ and last term $(b) = 98$
Sum of 20 terms $(S_{20}) = ?$
We know that,
$S_n = \frac{n}{2}(a + b)$
or, $S_{20} = \frac{20}{2}(3 + 98)$
$= 10 \times 101$
$\therefore\ S_{20} = 1010$
Thus, the required sum is 1010.

(c) पहिलो पद 3 र 10 ओटा पदहरूको योगफल 210 भएको समानान्तरीय श्रेणीको समान अन्तर पत्ता लगाउनुहोस् ।
Find the common difference of an arithmetic series having first term 3 and the sum of 10 terms is 210.

⇨ Here, first term $(a) = 3$, sum of 10 terms $(S_{10}) = 210$

We have, $S_n = \frac{n}{2}[2a + (n - 1)\ d]$

or, $S_{10} = \frac{10}{2}[2 \times 3 + (10 - 1)\ d]$

or, $210 = 5\ [6 + 9d]$
or, $42 = 6 + 9d$
or, $36 = 9d$ $\therefore$ $d = 4$
Thus, the common difference is 4.

3. (a) चौथो पद 7 र बाह्रौं पद 39 भएको एउटा समानान्तरीय श्रेणीको (In an arithmetic series, 4th term is 7 and 12th terms is 39.)
(i) पहिलो पद र समान अन्तर निकाल्नुहोस् । (Find the first term and common difference.)

⇨ Here, Fourth term $(t_4) = 7$
Twelth term $(t_{12}) = 39$
Common difference $(d) = ?$
First term $(a) = ?$
We have, $t_n = a + (n - 1)d$
$t_4 = a + (4 - 1)d$
or, $7 = a + 3d$ ....(1)
and $t_{12} = a + (12 - 1)d$
or, $39 = a + 11d$ ....(2)
Thus, $d = 4$ and $a = -5$.

Subtracting equation (2) from equation (1)

$$\begin{array}{rcl} 7 &=& a + 3d \\ 39 &=& a + 11d \\ - & & -\ \ - \\ \hline -32 &=& -8d \end{array}$$

$\therefore\ d = \frac{-32}{-8} = 4$
Substituting the value of d in equation (1) then,
$7 = a + 3 \times 4$
$\therefore\ a = 7 - 12 = -5$

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(ii) पहिला 15 पदहरूको योगफल निकाल्नुहोस् । (Find the sum of first 15 terms.)

⇨ Here, We have, $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$$S_{15} = \frac{15}{2}\{2 \times (-5) + (15 - 1)(4)\}$$
$$= \frac{15}{2}(-10 + 14 \times 4)$$
$$= \frac{15}{2}(-10 + 56) = \frac{15}{2} \times 46$$
$$= 15 \times 23$$
$$\therefore \quad S_{15} = 345$$

Thus, the sum of first 15 terms is 345.

(b) पाँचौं पद र दसौं पद क्रमशः 17 र 42 भएको समानान्तरीय श्रेणीको,
In an arithmetic series having 5th term and 10th term respectively 17 and 42.

(i) समान अन्तर र पहिलो पद निकाल्नुहोस् । (Find the common difference and first term.)

(ii) पहिला 20 ओटा पदहरूको योगफल निकाल्नुहोस् । (Find the sum of first twenty terms.)

⇨ Here, $t_5 = 17$ and $t_{10} = 42$

We know that,

$$t_n = a + (n - 1)\,d$$

So, $t_5 = a + 4d$ and $t_{10} = a + 9d$

$\therefore \quad 17 = a + 4d$ ....(i) and

$42 = a + 9d$ ....(ii)

Solving equation (i) and (ii) then,

$$\begin{array}{r} 17 = a + 4d \\ 42 = a + 9d \\ - \quad - \quad - \\ \hline -25 = -5d \end{array}$$

$\therefore \quad d = 5$

Substituting d = 5 in (i) then,

$17 = a + 4 \times 5$

or, $17 = a + 20$

$\therefore \quad a = -3$

Now, sum of n terms $= \frac{n}{2}[2a + (n - 1)\,d]$

So, sum of 20 terms $= \frac{20}{2}[2 \times (-3) + 19 \times 5]$

$$= 10(-6 + 95)$$
$$= 10 \times 89$$
$$\therefore \quad S_{20} = 890$$

Thus, $d = 5$, $a = -3$ and $S_{20} = 890$.

4. (a) यदि कुनै समानान्तरीय अनुक्रमको छैटौं पद 64 छ भने पहिलो 11 ओटा पदहरूको योगफल कति हुन्छ ?
If the 6th term of an arithmetic sequence is 64, what will be the sum of first 11 terms?

⇨ Here, $t_6 = 64$, $S_{11} = ?$

We have, $t_n = a + (n - 1)\,d$

So, $t_6 = a + (6 - 1)\,d$

$\therefore \quad 64 = a + 5d$ ....(i)

Again,

$$S_n = \frac{n}{2}[2a + (n - 1)\,d]$$
$$\therefore \quad S_{11} = \frac{11}{2}[2a + 10\,d]$$
$$= \frac{11 \times 2}{2}[a + 5d]$$
$$= 11 \times (a + 5d)$$
$$= 11 \times 64 \qquad \text{[From (i)]}$$
$$\therefore \quad S_{11} = 704$$

Thus, the sum of eleven terms is 704.

(b) यदि कुनै समानान्तरीय अनुक्रमको सोह्रौं पद 59 छ भने पहिलो 31 ओटा पदहरूको योगफल कति हुन्छ ?
If the 16th term of an arithmetic sequence is 59, what will be the sum of first 31 terms?

⇨ Here, $t_{16} = 59$ and $S_{31} = ?$

We have,

$$t_n = a + (n - 1)d$$

So, $t_{16} = a + 15\,d$

or, $59 = a + 15\,d$ ....(i)

Again,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
$$\therefore \quad S_{31} = \frac{31}{2}[2a + 30d]$$
$$= \frac{31 \times 2}{2}[a + 15\,d]$$
$$= 31 \times 59 \qquad \text{[From (i)]}$$
$$\therefore \quad S_{31} = 1829$$

Thus, the sum of 31 terms is 1829.

5. एउटा समानान्तरीय श्रेणीको पहिलो र अन्तिम पदहरू क्रमशः 2 र 29 छन् । यदि त्यो श्रेणीको योगफल 155 भए,
The first term and last term of an arithmetic series are 2 and 29 respectively. If the sum of the series is 155,

(a) पद सङ्ख्या र समान अन्तर पत्ता लगाउनुहोस् । (Find the no. of terms and common difference.)

(b) यदि सो श्रेणीमा थप 3 ओटा पदहरू भएका भए अन्तिम पद र योगफल निकाल्नुहोस् ।
If three more terms were added to the series, find the last term and the sum of terms.

⇨ Here, First term $(a) = 2$ & Last term $(\ell) = 29$

We have, $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

or, $155 = \frac{n}{2}[2.2 + (n - 1)d]$

or, $310 = n\{4 + (n - 1)\,d\}$ ....(1)

Again, We have,

$t_n = a + (n - 1)d$

or, $29 = 2 + (n - 1)\,d$

or, $29 + 2 = 2 + 2 + (n - 1)d$

or, $31 = 4 + (n - 1)d$ ....(2)

From equation (1) & (2),

$310 = n \times 31$

$\therefore \quad n = \frac{310}{31} = 10$

Substituting the value of n in equation (2)

$31 = 4 + (10 - 1)d$

or, $27 = 9d$

$\therefore \quad d = 3$

Thus, number of terms is 10 & common difference is 3.

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If 3 more terms are added then, number of terms (n) = 10 + 3 = 13

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{13} = \frac{13}{2}\{2a + (13-1)d\}$

$= \frac{13}{2}\{2.2 + 12.3\}$

$= \frac{13}{2} \times 40$

$\therefore\ S_{13} = 260$

$t_{13} = a + (13-1)d$

$= 2 + 12 \times 3$

$= 2 + 36$

$\therefore\ t_{13} = 38$

Thus, 13th term is 38 & sum of 13 terms is 260.

6. (a) एउटा समानान्तरीय अनुक्रमको पहिला 6 ओटा पदहरूको योगफल 42 छ । दसौं पद र तिसौं पदको अनुपात 1 : 3 छ भने पहिलो पद र तेह्रौं पद निकाल्नुहोस् ।

The sum of first six terms of an arithmetic sequence is 42. If the ratio of 10th term to 30th term is 1 : 3, find the first term and 13th term.

⇨ Here, sum of first 6 terms (S_6) = 42

$t_{10} : t_{30} = 1 : 3$, a = ? and t_{13} = ?

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $S_6 = \frac{6}{2}[2a + 5d]$

or, $42 = 3(2a + 5d)$

or, $14 = 2a + 5d$ ....(i)

Again, $t_{10} : t_{30} = 1 : 3$

or, $\frac{a + 9d}{a + 29d} = \frac{1}{3}$

or, $3a + 27d = a + 29d$

or, $2a = 2d$

$\therefore\ a = d$ ....(ii)

From (i) & (ii),

$14 = 2a + 5a$

or, $14 = 7a$

$\therefore\ a = 2$

Now, $t_n = a + (n-1)d$

or, $t_{13} = a + 12d$

$= 2 + 12 \times 2$

$= 2 + 24$

$\therefore\ t_{13} = 26$

Thus, the 13th term is 26 and first term is 2.

(b) एउटा समानान्तरीय श्रेणीको पहिलो दस पदहरूको योगफल 50 छ र पाँचौं पद दोस्रो पदको तेब्बर भए पहिलो पद र पहिलो 20 पदहरूको योगफल निकाल्नुहोस् ।

The sum of first ten terms of an arithmetic series is 50. If the 5th term is three times of 2nd term, find the first term and sum of first twenty terms.

⇨ Here, $S_{10} = 50$, $t_5 = 3t_2$, S_{20} = ?

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $S_{10} = \frac{10}{2}[2a + 9d]$

or, $50 = 5(2a + 9d)$

$\therefore\ 2a + 9d = 10$ ....(i)

Again, $t_5 = 3t_2$

or, $a + 4d = 3(a + d)$ $[\because t_n = a + (n-1)d]$

or, $a + 4d = 3a + 3d$

or, $-2a = -d$

$\therefore\ 2a = d$ ....(ii)

From (i) and (ii), $d + 9d = 10$

or, $10d = 10$

$\therefore\ d = 1$

From (ii); $2a = d$ or, $2a = 1$ $\therefore\ a = \frac{1}{2}$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $S_{20} = \frac{20}{2}\left[2 \times \frac{1}{2} + 19 \times 1\right]$

$= 10(1 + 19)$

$= 10 \times 20$

$\therefore\ S_{20} = 200$

Thus, the first term = $\frac{1}{2}$ & sum of 20 terms = 200.

7. योगफल निकाल्नुहोस् (Find the sum of):

(a) सुरुका 50 ओटा प्राकृतिक सङ्ख्याहरूको

First fifty natural numbers

⇨ Here,

First term (a) = 1

Common difference (d) = $t_2 - t_1 = 2 - 1 = 1$

Number of terms (n) = 50

Sum (S_{50}) = ?

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{50} = \frac{50}{2}\{2a + (50-1)d\}$

$= 25\{2 + 49 \times 1\}$

$= 25\{2 + 49\}$

$= 25 \times 51$

$\therefore\ S_{50} = 1275$

Thus, the required sum is 1275.

(b) 1 देखि 100 सम्मका 5 ले निःशेष भाग जाने सङ्ख्याहरूको

The numbers from 1 to 100 which are exactly divisible by 5.

⇨ Here, the numbers from 1 to 100 divisible by 5 are; 5, 10, 15, 20, ..., 100

First term (a) = 5

Common difference (d) = $t_2 - t_1 = 10 - 5 = 5$

Last term (t_n) = 100

We know that, $t_n = a + (n-1)d$

or, $100 = 5 + (n-1)5$

or, $95 = (n-1)5$

or, $n - 1 = 19$

$\therefore\ n = 20$

Again, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ S_{20} = \frac{20}{2}[2 \times 5 + 19 \times 5]$

$= 10(10 + 95)$

$= 10 \times 105$

$\therefore\ S_{20} = 1050$

Thus, the required sum is 1050.

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(c) सुरूका 40 ओटा जोर सङ्ख्याहरूको
First fourty even numbers.

⇨ Here, first 40 even numbers
i.e. 2, 4, 6, 8, ...
First term (a) = 2
Common difference (d) = $t_2 - t_1 = 4 - 2 = 2$
We have,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{or, } S_{40} = \frac{40}{2}[2 \times 2 + 39 \times 2]$$

$$= 20[4 + 78]$$

$$= 20 \times 82$$

$$\therefore S_{40} = 1640$$

Thus, the required sum is 1640.

(b) $\sum\limits_{k=3}^{8}(4k - 1)$

⇨ Here, $\sum\limits_{k=3}^{8}(4k - 1)$

So, first term (a) = $4 \times 3 - 1 = 11$
Last term (b) = $4 \times 8 - 1 = 31$
No. of terms (n) = $8 - 2 = 6$
We know that,

$$S_n = \frac{n}{2}(a + b)$$

$$\text{or, } S_6 = \frac{6}{2}(11 + 31)$$

$$\text{or, } S_6 = 3 \times 42$$

$$\therefore S_6 = 126$$

Thus, the sum of terms is 126.

8. मान निकाल्नुहोस् (Evaluate) :

(a) $\sum\limits_{n=2}^{11}(n + 7)$

⇨ Here, $\sum\limits_{n=2}^{11}(n + 7)$

So, first term (a) = $2 + 7 = 9$
Last term (b) = $11 + 7 = 18$
No. of terms (n) = $11 - 1 = 10$
We have,

$$S_n = \frac{n}{2}(a + b)$$

$$= \frac{10}{2}(9 + 18)$$

$$= 5 \times 27$$

$$\therefore S_{10} = 135$$

Thus, the sum of series is 135.

(c) $\sum\limits_{n=1}^{6}(5n^2 + 2)$

⇨ Here, $\sum\limits_{n=1}^{6}(5n^2 + 2)$

So, first term (a) = $5 \times 1^2 + 2 = 5 + 2 = 7$
Last term (b) = $5 \times 6^2 + 2 = 180 + 2 = 182$
No. of terms (n) = 6
We know that,

$$S_n = \frac{n}{2}(a + b)$$

$$\text{or, } S_6 = \frac{6}{2}(7 + 182)$$

$$= 3 \times 189$$

$$\therefore S_6 = 567$$

Thus, the sum of terms is 567.

9. (a) एउटा समानान्तरीय श्रेणीका सुरुका तीन पदहरू p + 2, 2p – 1 र p + 6 भए p को मान र सुरुका 5 ओटा पदहरूको योगफल निकाल्नुहोस् ।
If the first three terms of an arithmetic series are p + 2, 2p – 1 and p + 6, find the value of p and the sum of first five terms.

⇨ Here, $p + 2$, $2p - 1$ and $p + 6$ are given terms.
We have, $t_2 - t_1 = t_3 - t_2$
or, $(2p - 1) - (p + 2) = (p + 6) - (2p - 1)$
or, $2p - 1 - p - 2 = p + 6 - 2p + 1$
or, $p - 3 = 7 - p$
or, $2p = 10$
$\therefore p = 5$
So, $t_1 = p + 2 = 5 + 2 = 7$
and, $t_2 = 2p - 1 = 2 \times 5 - 1 = 9$
We know that, $d = t_2 - t_1 = 9 - 7 = 2$
We have,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}(2 \times 7 + 4 \times 2)$$

$$= \frac{5}{2}(14 + 8)$$

$$= \frac{5}{2} \times 22$$

$$\therefore S_5 = 55$$

Thus, the value of p is 5 and sum of first five terms is 55.

(b) 2(k – 1), k + 2 र 3k एउटा समानान्तरीय श्रेणीका तीन ओटा क्रमागत पदहरू भए k को मान निकाल्नुहोस् । उक्त श्रेणीका पहिला 10 ओटा पदहरूको योगफल पनि निकाल्नुहोस् ।
If 2(k – 1), k + 2 and 3k are three consecutive terms of an arithmetic series, find the value of k. Also, find the sum of first ten terms of the series.

⇨ Here, $2(k - 1)$, $k + 2$ and $3k$ are three consecutive terms.
First term (a) = $2(k - 1)$
Second term (t_2) = $k + 2$
Third term (t_3) = $3k$
We have, $t_2 - t_1 = t_3 - t_2$
or, $(k + 2) - 2(k - 1) = 3k - (k + 2)$
or, $k + 2 - 2k + 2 = 3k - k - 2$
or, $-k + 4 = 2k - 2$
or, $-3k = -6$ $\quad\therefore k = 2$
Substituting k = 2 in above,
Then, $a = 2(k - 1) = 2(2 - 1) = 2$
$t_2 = (k + 2) = 2 + 2 = 4$
$t_3 = 3k = 3 \times 2 = 6$
So, common difference (d) = $t_2 - t_1 = 4 - 2 = 2$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2a + 9d] = 5[2 \times 2 + 9 \times 2] = 5 \times 22$$

$$\therefore S_{10} = 110$$

Thus, the value of k is 2 and the sum of first 10 terms is 110.

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10. (a) **एकजना महिलाले एक महिनामा रु. 32 बचत गर्छिन् । अर्को महिनामा रु. 36 र तेस्रो महिनामा रु. 40 बचत गर्छिन् । यदि उनले यही क्रममा बचत गर्दै जाँदा कति महिनामा जम्मा रू. 2000 बचत गर्छिन् ?**

A woman saves Rs 32 during one month, Rs 36 in the next month and Rs 40 in the third month. If she continues her savings in this sequence, in how many months will she save Rs 2000?

⇨ Here, first term (a) = 32, second term $(t_2) = 36$

So, common difference $(d) = t_2 - a = 36 - 32 = 4$

Sum of terms $(S_n) = 2000$

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $2000 = \frac{n}{2}[2 \times 32 + (n-1)4]$

or, $2000 = \frac{n}{2}[64 + 4n - 4]$

or, $4000 = n(60 + 4n)$

or, $4000 = 60n + 4n^2$

or, $4n^2 + 60n - 4000 = 0$

or, $n^2 + 15n - 1000 = 0$

or, $n^2 + 40n - 25n - 1000 = 0$

or, $n(n + 40) - 25(n + 40) = 0$

or, $(n + 40)(n - 25) = 0$

Either, $n + 40 = 0$ or $n - 25 = 0$

$\therefore \quad n = -40$ (rejected)

$\therefore \quad n = 25$

Thus, the required number of months is 25.

(b) **कृषि कार्यका लागि एउटा फाइनान्स कम्पनीबाट लिएको ऋणको साँवा र ब्याज गरेर जम्मा रु. 29000 तिर्नुपर्नेमा मासिक किस्ताबन्दीका दरले तिर्दै जाँदा यो रकम 20 महिनामा चुक्ता हुने रहेछ । यदि प्रत्येक किस्तामा रु. 100 बढी रकम तिर्दै जानुपर्ने सर्त भए पहिलो किस्ताको रकम कति होला ?**

A farmer has to pay Rs 29,000 to clear the debt he had taken for agricultural works from a finance company . He is supposed to pay the debt in 20 monthly installments increasing every subsequent installment by Rs 100. Find the amount he has to pay as the first installment.

⇨ Here, total payment (S_{20}) = Rs 29,000

Common difference (d) = Rs 100

First term (a) = ?

We have,

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

or, $S_{20} = \frac{20}{2}\{2a + (20-1)100\}$

or, $29{,}000 = 10\{2a + 1900\}$

or, $2900 = 2a + 1900$

or, $1000 = 2a$

$\therefore \quad a = \frac{1000}{2} = 500$

Thus, the amount required for the first installment = Rs 500.

4. गुणोत्तर अनुक्रम

Geometric Sequence

Formulae and Key Points

	गुणोत्तर अनुक्रम वा श्रेणी (GS) Geometric Sequence or Series (GS)	सङ्केत Index
साधारण पद General term	$t_n = ar^{n-1}$	a = **पहिलो पद** First term n = **पदहरूको सङ्ख्या** No. of terms r = **समान अनुपात** Common ratio b = **अन्तिम पद** Last term m_n = n **औँ मध्यमा** n^{th} mean
मध्यमा Mean	$GM = \sqrt{ab}$	
मध्यमाहरू Means	$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ $m_1 = ar, m_2 = ar^2, \ldots\ldots.., m_n = ar^n$	
योगफल (Sum)	$S_n = \frac{a(r^n - 1)}{r - 1}$ **Or** $S_n = \frac{br - a}{r - 1}$	

➢ **यदि दुई सङ्ख्याहरू** a **र** b **बीचको समानान्तरीय मध्यक** = AM **र गुणोत्तर मध्यक** = GM **भए,**
पहिलो सङ्ख्या $(a) = AM - \sqrt{AM^2 - GM^2}$ **र दोस्रो सङ्ख्या** $(b) = AM + \sqrt{AM^2 - GM^2}$ **हुन्छ ।**

If a and b are two numbers and their arithmetic mean = AM and geometric mean = GM, the 1st number $(a) = AM - \sqrt{AM^2 - GM^2}$ and 2nd number $(b) = AM + \sqrt{AM^2 - GM^2}$

➢ **GS का पदहरूलाई सङ्केत गर्ने सजिलो तरिका (The easy method to denote the terms of GS):**

पदहरूको सङ्ख्या (Number of terms)	3	4	5	6
पदहरू (Terms)	$\frac{a}{r}, a, ar$	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}\ ar, ar^3, ar^5$
समान अनुपात (Common ratio)	r	r^2	r	r^2

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QUESTIONS FROM SEE EXERCISE 5

A. VERY SHORT QUESTIONS

1. गुणोत्तर अनुक्रम/श्रेणी को परिभाषा दिनुहोस् । (Define Geometric sequence/series.)

⇨ Here, quantities are said to be in geometric sequence when they increase or decrease by a constant factor. The constant factor is called the common ratio.
e.g. 1, 2, 4, 8……….and 16, 8, 4 ……… etc. are in geometric sequence.

2. गुणोत्तर मध्यमा को परिभाषा दिनुहोस् । (Define Geometric mean.)

⇨ Here, in geometric sequence the term/terms between the first term and the last term is/are called the geometric mean/means.

3. पहिलो पद '*a*' र समान अनुपात '*r*' भएको गुणोत्तर अनुक्रमको साधारण पद लेख्नुहोस् ।
Write the general term of a geometric sequence having first term '*a*' and common ratio '*r*'.

⇨ Here, the general term of the geometric sequence $(t_n) = ar^{n-1}$

4. पहिलो पद '*a*', अन्तिम पद '*b*' र पदहरूको सङ्ख्या '*n*' भएको *GS* को समान अनुपात कति हुन्छ ?
What is the common ratio of a *GS* having first term '*a*', last term '*b*' and number of terms '*n*'?

⇨ Here, the common ratio of GS = $\left(\frac{b}{a}\right)^{\frac{1}{n-1}}$

5. दुईओटा धनात्मक सङ्ख्याहरू *a* र *b* बिचको गुणोत्तर मध्यमा कति हुन्छ ?
What is the geometric mean between two positive numbers *a* and *b*?

⇨ Here, the geometric mean between a and b is $\sqrt{ab}$.

6. यदि पहिलो पद '*a*' र समान अनुपात '*r*' भए गुणोत्तर अनुक्रमको $n^{\text{औं}}$ मध्यमा कति हुन्छ ?
If '*a*' is first term and '*r*' is common ratio then what is n^{th} mean of geometric sequence?

⇨ Here, the n^{th} mean $(m_n) = ar^n$

7. यदि '*a*' र '*b*' दुई धनात्मक सङ्ख्याहरू भए तिनीहरूका *AM* र *GM* बीचको सम्बन्ध लेख्नुहोस् ।
If '*a*' and '*b*' are the two positive numbers, write the relation between their *AM* and *GM*.

⇨ Here, $AM \geq GM$ is the required relation.

8. यदि '*a*' र '*b*' बीचको गुणोत्तर मध्यमा '*G*' भए *a*, *b* र *G* को सम्बन्ध पत्ता लगाउनुहोस् ।
If '*G*' is the geometric mean between '*a*' and '*b*', find the relation of *a*, *b* and *G*.

⇨ Here, $G = \sqrt{ab}$ is the required relation.

9. पहिलो पद '*a*' र समान अनुपात '*r*' भएको गुणोत्तर श्रेणीमा '*n*' ओटा पदहरू भए अन्तिम पद (*b*) कति होला ?
What is the last term '*b*' of a *GP* of *n* terms having first term '*a*' and common ratio '*r*'?

⇨ Here, the last term $(b) = ar^{n-1}$

10. पहिलो पद '*a*' र समान अनुपात '*r*' भएको गुणोत्तर श्रेणीको '*n*' ओटा पदहरूको योगफल 'S_n' भए S_n, *a*, *r* र *n* को सम्बन्ध पत्ता लगाउनुहोस् ।
If 'S_n' is the sum of *n* terms of a *GS* with first term '*a*' and common ratio '*r*' then find the relation between S_n, *a*, *r* and *n*.

⇨ Here, the sum of n terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

11. पहिलो पद '*a*', अन्तिम पद '*ℓ*' र समान अनुपात '*r*' भएको *GS* को योगफल कति हुन्छ ?
What is the sum of *GS* having first term '*a*', last term '*ℓ*' and common ratio '*r*'?

⇨ Here, the sum of n terms $(S_n) = \frac{\ell r - a}{r - 1}$

12. सूत्र $S_n = \frac{a(r^n - 1)}{r - 1}$ मा प्रत्येक पदहरूको अर्थ लेख्नुहोस् ।

Write the meaning of each term in the formula: $S_n = \frac{a(r^n - 1)}{r - 1}$

⇨ Here, S_n = sum of n terms, a = first term, r = common ratio and n = number of terms

13. श्रेणी 4, 8, 16, 32, ...को समान अनुपात कति हुन्छ ? (What is the common ratio of the series 4, 8, 16, 32, ...?)

⇨ Here, the common ratio $= \frac{t_2}{t_1} = \frac{8}{4} = 2$

14. श्रेणी 81, 27, 9, 3, ...को समान अनुपात कति हुन्छ ? (What is the common ratio of the series 81, 27, 9, 3, ...?)

⇨ Here, common ratio $= \frac{t_2}{t_1} = \frac{27}{81} = \frac{1}{3}$

15. चौथो पद 16 र समान अनुपात 2 भएको GS को पाचौँ पद पत्ता लगाउनुहोस् ।
Find the fifth term of GS whose fourth term is 16 and common ratio is 2.

⇨ Here, fifth term = 4th term × common ratio = 16 × 2 = 32

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16. नवौं पद 27 र समान अनुपात 3 भएको GS को दसौं पद पत्ता लगाउनुहोस् ।
Find the tenth term of GS whose ninth term is 27 and common ratio is 3.

⇨ Here, tenth term = ninth term × common ratio = 27 × 3 = 81

17. तेह्रौं पद 625 र समान अनुपात 5 भएको GS को 12 औं पद पत्ता लगाउनुहोस् ।
Find the 12th term of GS whose 13th term is 625 and common ratio is 5.

⇨ Here, 12^{th} term $= \frac{13^{th}\text{ term}}{r} = \frac{625}{5} = 125$

18. पहिलो पद '5' र समान अनुपात '2' भएको गुणोत्तर अनुक्रमको पाचौं पद लेख्नुहोस् ।
Write the 5th term of geometric sequence having first term '5' and common ratio '2'.

⇨ Here, 5^{th} term $= ar^4 = 5 \times (2)^4 = 5 \times 16 = 80$

19. पहिलो पद '2' र समान अनुपात '3' भएको गुणोत्तर अनुक्रमको चौथो पद लेख्नुहोस् ।
Write the 4th term of geometric sequence having first term '2' and common ratio '3'.

⇨ Here, 4^{th} term $= ar^3 = 2 \times 3^3 = 54$

20. दुई सङ्ख्याहरू 1 र 4 को गुणोत्तर मध्यमा पत्ता लगाउनुहोस् । (Find the geometric mean of two numbers 1 and 4.)

⇨ Here, geometric mean $= \sqrt{ab} = \sqrt{1 \times 4} = 2$

21. दुई सङ्ख्याहरू 1 र 9 को गुणोत्तर मध्यमा पत्ता लगाउनुहोस् । (Find the geometric mean of two numbers 1 and 9.)

⇨ Here, geometric mean $= \sqrt{ab} = \sqrt{1 \times 9} = 3$

22. यदि एउटा GS को पहिलो पद 4 र समान अनुपात 2 भए दोस्रो मध्यमा पत्ता लगाउनुहोस् ।
If a GS has first term 4 and common ratio 2 then find the 2nd mean.

⇨ Here, second mean $= ar^2 = 4 \times 2^2 = 4 \times 4 = 16$

23. यदि एउटा GS को पहिलो पद 3 र समान अनुपात 5 भए दोस्रो मध्यमा पत्ता लगाउनुहोस् ।
If a GS has first term 3 and common ratio 5 then find the 2nd mean.

⇨ Here, second mean $= ar^2 = 3 \times 5^2 = 3 \times 25 = 75$

24. एउटा गुणोत्तर श्रेणीको पहिला 5 पदहरूको योगफल 124 र पहिला 4 पदहरूको योगफल 60 छ । पाचौं पद पत्ता लगाउनुहोस् ।
The sum of first 5 terms of a geometric series is 124 and the sum of first 4 terms of the geometric series is 60. Find the 5th term.

⇨ Here, fifth term $= S_5 - S_4 = 124 - 60 = 64$

25. एउटा गुणोत्तर श्रेणीको *n* पदहरूको योगफल 3577 र (*n* – 1) पदहरूको योगफल 1785 छ । *n* औं पद पत्ता लगाउनुहोस् ।
The sum of *n* terms of a geometric series is 3577 and the sum of (*n* – 1) terms of the geometric series is 1785. Find the *n*th term.

⇨ Here, n^{th} term $= S_n - S_{n-1} = 3577 - 1785 = 1792$

B. SHORT QUESTIONS

MODEL 1

1. गुणोत्तर श्रेणीको परिभाषा एउटा उदाहरणसहित दिनुहोस् । (Define Geometric series with an example.) **[2073 S']**

⇨ Here, the sum of terms of geometric sequence is called the geometric series. For example:
If 3, 6, 12, 24, 48, ... be a geometric sequence then 3 + 6 + 12 + 24 + 48 + ... is the geometric series.

2. यदि 4, a र 16 गुणोत्तरीय अनुक्रममा भए a को मान पत्ता लगाउनुहोस् ।
If 4, a and 16 are in the geometric sequence, find the value of a. **[2073 R']**

⇨ Here, 4, a and 16 are in GS
So, $\text{GM} = \sqrt{\text{AB}}$
or, $a = \sqrt{4 \times 16}$
$\therefore\ a = \pm 8$
Since the sequence is increasing so a = 8
Thus, the value of a is 8.

3. यदि m + 2, m + 8 र 17 + m गुणोत्तर अनुक्रममा भए m को मान पत्ता लगाउनुहोस् ।
If m + 2, m + 8 and 17 + m are in geometric sequence, find the value of m. **[2072 R']**

⇨ Here, the terms of a GS are; m + 2, m + 8 and 17 + m
So, $(t_2)^2 = t_1 \times t_3$
or, $(m + 8)^2 = (m + 2)(17 + m)$
or, $m^2 + 16m + 64 = 17m + m^2 + 34 + 2m$
or, $16m + 64 = 19m + 34$
or, $30 = 3m$
$\therefore\ m = 10$
Thus, the value of m is 10.

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4. यदि गुणोत्तर श्रेणीको तेस्रो पद 3 छ भने यसका प्रथम पाँच पदहरूको गुणनफल निकाल्नुहोस् ।

If the third term of a geometric series is 3, find the product of its first five terms. **[2070 R]**

⇨ Here, third term $(t_3) = 3$

We know that,

$$\begin{aligned}\text{product of first five terms} &= a \times ar \times ar^2 \times ar^3 \times ar^4\\ &= a^5r^{10}\\ &= (ar^2)^5\\ &= (3)^5\\ &= 243\end{aligned}$$

Thus, the product of first five term is 243.

5. यदि a, a – 2, a + 1 गुणोत्तरीय अनुक्रममा भए a को मान पत्ता लगाउनुहोस् ।

If a, a – 2, a + 1 are in geometric sequence, find the value of a. **[2068 R]**

⇨ Here, the terms of GS are a, $a - 2$, $a + 1$

We know that, $\frac{t_2}{t_1} = \frac{t_3}{t_2}$

or, $\frac{a-2}{a} = \frac{a+1}{a-2}$

or, $(a-2)^2 = a(a+1)$

or, $a^2 - 4a + 4 = a^2 + a$

or, $-4a - a = -4$

or, $-5a = -4$ $\therefore$ $a = \frac{4}{5}$

Thus, the value of a is $\frac{4}{5}$.

6. यदि x + 4, x – 2 र x – 6 ज्यामितीय अनुक्रममा भए x को मान निकाल्नुहोस् ।

If x + 4, x – 2 and x – 6 are in geometric sequence, find the value of x. **[2067 R]**

⇨ Here, $x + 4$, $x - 2$ & $x - 6$ are in G.P.

So, $t_1 = x + 4$, $t_2 = x - 2$ & $t_3 = x - 6$

Now, $\frac{t_2}{t_1} = \frac{t_3}{t_2}$

or, $\frac{x-2}{x+4} = \frac{x-6}{x-2}$

or, $(x-2)^2 = (x-6)(x+4)$

or, $x^2 - 4x + 4 = x^2 - 2x - 24$

or, $-4x + 2x = -24 - 4$

or, $-2x = -28$

$\therefore$ $x = \frac{28}{2} = 14$

Thus, the required value of x is 14.

7. गुणोत्तर अनुक्रमको पहिलो र दोस्रो पद क्रमशः 32 र 8 भए छैटौं पद कति होला ?

First and second terms of a geometrical sequence are 32 and 8 respectively. What will be the sixth term? **[2064 R]**

⇨ Here, first term $(a) = 32$ Second term $(t_2) = 8$

We have common ratio $= \frac{t_2}{a} = \frac{8}{32} = \frac{1}{4}$

$$\begin{aligned}\text{Now, sixth term } (t_6) &= ar^5\\ &= 32 \times \left(\frac{1}{4}\right)^5\\ &= 32 \times \frac{1}{1024}\\ &= \frac{1}{32}\end{aligned}$$

Thus, the sixth term (t_6) is $\frac{1}{32}$.

MODEL 2

8. 4 र 16 बिचको समानान्तरीय मध्यमा र गुणोत्तर मध्यमा निकाल्नुहोस् ।

Calculate the arithmetic mean and geometric mean between 4 and 16. **[SEE 2075 R]**

⇨ Here, let $a = 4$ and $b = 16$

We know that, $AM = \frac{a+b}{2} = \frac{4+16}{2} = 10$

Also, $GM = \sqrt{ab} = \sqrt{4 \times 16} = \sqrt{64} = 8$

Thus, AM is 10 and GM is 8.

9. यदि 2, 4, *m* एउटा गुणोत्तर अनुक्रम भए *m* को मान पत्ता लगाउनुहोस् ।

If 2, 4, *m* is a geometric sequence then find the value of *m*. **[SEE 2075 R']**

⇨ Here, 2, 4, m is a geometric sequence,

then $\sqrt{4} = 2m$

or, $4 = 4m^2$

or, $m^2 = 1$ $\therefore$ $m = 1$

Thus, the value of m is 1.

10. 3 र 27 बिचको समानान्तरीय र गुणोत्तर मध्यमा पत्ता लगाउनुहोस् ।

Find the arithmetic and geometric mean between 3 and 27. **[SEE 2075 R'$_2$, 2070 R']**

⇨ Here, let $a = 3$ and $b = 27$

We know that,

$AM = \frac{a+b}{2} = \frac{3+27}{2} = 15$

Also,

$GM = \sqrt{ab} = \sqrt{3 \times 27} = \sqrt{81} = 9$

Thus, AM is 15 and GM is 9.

11. यदि 2 र x को गुणोत्तर मध्यमा 4 भए तिनीहरूको समानान्तरीय मध्यमा पत्ता लगाउनुहोस् ।

If the geometric mean of 2 and x is 4, find their arithmetic mean. **[2072 R]**

⇨ Here, $a = 2$, $b = 2$ and $GM = 4$

We know that, $GM = \sqrt{ab}$

or, $4 = \sqrt{2 \times x}$ or, $16 = 2x$

$\therefore$ $x = 8$

Now, $AM = \frac{a+b}{2} = \frac{2+x}{2} = \frac{2+8}{2} = 5$

Thus, the AM is 5.

12. यदि 4 र x को समानान्तरीय मध्यमा 34 भए तिनीहरूको गुणोत्तर मध्यमा निकाल्नुहोस् ।

If the arithmetic mean between 4 and x is 34, find their geometric mean. **[2072 S]**

⇨ Here, $AM = 34$

We know that, $AM = \frac{a+b}{2}$ or, $34 = \frac{x+4}{2}$ or, $x = 64$

Now, $GM = \sqrt{ab} = \sqrt{4 \times x} = \sqrt{64 \times 4} = 16$

Thus, the GM is 16.

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13. श्रेणी 3 + 6 + 12 + ... को कुनचाहिँ पद 192 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series 3 + 6 + 12 + ... is 192 ? Find it. **[2071 R]**

⇨ Here, $3 + 6 + 12 + ...$ and $t_n = 192$ & $n = ?$

$\therefore \quad a = 3$ and $r = \frac{t_2}{t_1} = \frac{6}{3} = 2$

We know that,

$t_n = ar^{n-1}$

or, $192 = 3 \times 2^{n-1}$

or, $64 = 2^{n-1}$

or, $2^6 = 2^{n-1}$

or, $n - 1 = 6$

$\therefore \quad n = 7$

Thus, the required term is 7th term.

14. दिइएको श्रेणीको पदसङ्ख्या पत्ता लगाउनुहोस् :
Find the number of terms of the given series: **[2071 R']**
$2 - 2\sqrt{2} + 4 - 4\sqrt{2} + ... + 16$.

⇨ Here, $2 - 2\sqrt{2} + 4 - 4\sqrt{2} + ... + 16$ and

The first term (a) = 2

Common ratio $(r) = \frac{t_2}{t_1} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$ and

The last term $(t_n) = 16$

or, $ar^{n-1} = 16$

or, $2 \times (-\sqrt{2})^{n-1} = 16$

or, $(-\sqrt{2})^{n-1} = 8$

or, $(-\sqrt{2})^{n-1} = (-\sqrt{2})^6$

or, $n - 1 = 6$

$\therefore \quad n = 7$

Thus, there are 7 terms.

15. यदि 4 र x को बीचमा गुणोत्तर मध्यमा 8 भए x को मान पत्ता लगाउनुहोस् ।
If geometric mean is 8 between of 4 and x, find the value of x. **[2065 R']**

⇨ Here, two numbers are a = 4 and b = x and their GM = 8

We know that, $GM = \sqrt{ab}$

or, $8 = \sqrt{4 \times x}$

or, $64 = 4x$ (Squaring both sides)

$\therefore \quad x = 16$

Thus, the value of x is 16.

16. कुनै गुणोत्तर अनुक्रमको पहिलो पद 8 र अन्तिम पद 1/2 भए सो अनुक्रमको तीनओटा गुणोत्तर मध्यमाहरू पत्ता लगाउनुहोस् ।
If the first term is 8 and the last term is 1/2 of a geometrical sequence, find three geometrical means. **[2065 E]**

⇨ Here, first term (a) = 8 Last term $(\ell) = \frac{1}{2}$

No. of means (n) = 3

$$\text{Now, } r = \left(\frac{\ell}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{1}{2}}{8}\right)^{\frac{1}{3+1}}$$

$$= \left(\frac{1}{16}\right)^{\frac{1}{4}} = \left(\frac{1}{2}\right)^{4 \times \frac{1}{4}} = \frac{1}{2}$$

Then, $m_1 = ar = 8.\frac{1}{2} = 4$

$m_2 = ar^2 = 8.\frac{1}{4} = 2$

$m_3 = ar^3 = 8.\frac{1}{8} = 1$

Thus, the required means are 4, 2 and 1 respectively.

17. दिइएको गुणोत्तर अनुक्रममा x, y र z का मानहरू पत्ता लगाउनुहोस्: $\frac{1}{8}$, x, y, z, 2
Find the values of x, y and z from the given geometrical sequence: $\frac{1}{8}$, x, y, z, 2. **[2060 C]**

⇨ Here, $\frac{1}{8}, x, y, z, 2$

First term $(a) = \frac{1}{8}$

Fifth term $(t_5) = 2$

We have, $t_5 = ar^4$

or, $2 = \frac{1}{8} \times r^4$

or, $16 = r^4$

$\therefore \quad r = 2$

Now, $x = ar = \frac{1}{8} \times 2 = \frac{1}{4}$

$y = ar^2 = \frac{1}{8} \times 2^2 = \frac{1}{2}$

$z = ar^3 = \frac{1}{8} \times 2^3 = 1$

Thus, the values of x, y and z are; $\frac{1}{4}, \frac{1}{2}$, 1 resp.

MODEL 3

18. 81+ 27 + 9 + ... को पहिलो पाँचौं पदसम्मको योगफल निकाल्नुहोस् । (Find the sum of first five terms of 81 + 27 + 9 + ...) **[2063 R]**

⇨ Here, given series, 81 + 27 + 9 + ...

Since the given series is the geometric series

So, first term (a) = 81 Common ratio $(r) = \frac{27}{81} = \frac{1}{3}$

No. of terms (n) = 5 Sum of first five terms $(S_5) = ?$

$$\text{By the formula, } S_n = \frac{a(r^n - 1)}{r - 1} = \frac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1} = \frac{81\left(\frac{1}{243} - 1\right)}{\frac{1-3}{3}} = \frac{81\left(\frac{1-243}{243}\right)}{-\frac{2}{3}} = \frac{81\left(-\frac{242}{243}\right)}{-\frac{2}{3}}$$

$$= 81 \times \left(-\frac{242}{243}\right) \times \left(-\frac{3}{2}\right) = \frac{58806}{486} = 121$$

Thus, the required sum is 121.

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19. निम्नलिखित श्रेणीको योगफल निकाल्नुहोस् : 2 + 4 + 8 + ... 8 ओटा पदहरू सम्म
Find the sum of the following series : 2 + 4 + 8 + ... upto 8 terms. **[2066 R]**

⇨ Here, $r = \frac{t_2}{t_1} = \frac{4}{2} = 2$ and N = 8

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

or, $S_8 = \frac{2(2^8 - 1)}{2 - 1}$

or, $S_8 = \frac{2 \times 255}{1}$

∴ $S_8 = 510$

Thus, the sum of 8 terms is 510.

20. गुणोत्तर अनुक्रम 48, 24, 12, ... को पहिलो पाँच पदहरूको योगफल निकाल्नुहोस् ।
Find the sum of the first five terms of the geometric sequence 48, 24, 12, ... **[2065 S]**

⇨ Here, the given GS is: 48, 24, 12, ...
First term (a) = 48,

Common ratio(r) = $\frac{t_2}{t_1} = \frac{24}{48} = \frac{1}{2}$

No. of terms of (n) = 5
We have,

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{48\left\{\left(\frac{1}{2}\right)^5 - 1\right\}}{\frac{1}{2} - 1} = \frac{48 \times (-)\frac{31}{32}}{-\frac{1}{2}} = 93$$

Thus, the sum of the first five terms is 93.

21. दिइएको गुणोत्तर श्रेणीको योगफल निकाल्नुहोस्: 3 + 6 + 12 + ... + 768
Find the sum of the geometric series given: 3 + 6 + 12 + ... + 768 **[2067 R']**

⇨ Here, first term (t_1) or (a) = 3 Second term (t_2) = 6
Last term (ℓ) = = 768 Sum of n^{th} term $(S_{n\,)-?}$

Common ratio (r) = $\frac{t_2}{t_1} = \frac{6}{3} = 2$

Now, $S_n = \frac{\ell r - a}{r - 1} = \frac{768 \times 2 - 3}{2 - 1} = \frac{1536 - 3}{1} = 1533$

Thus, the sum of the given series is 1533.

C. LONG QUESTIONS

MODEL 1

1. गुणोत्तर श्रेणीमा भएका तीनओटा धनात्मक सङ्ख्याहरूको योगफल 104 छ र त्यसका पहिलो र तेस्रो सङ्ख्याहरूको गुणनफल 576 भए उक्त सङ्ख्याहरू निकाल्नुहोस् ।
The sum of three positive numbers in geometric series is 104 and the product of the first and third numbers is 576, find the numbers. **[2074 S]**

⇨ Here, let the three terms of a GS be $\frac{a}{r}$, a and ar.

By the question, Sum = 104

or, $\frac{a}{r} + a + ar = 104$

∴ $a + ar + ar^2 = 104r$ ......(i)

Again, the second condition is ; $\frac{a}{r} \times ar = 576$

or, $a^2 = 576$

∴ $a = \pm 24 = 24$ [∴ Positive nos.]

From equation (i),

$24 + 24r + 24r^2 = 104r$

or, $24r^2 - 80r + 24 = 0$

or, $3r^2 - 10r + 3 = 0$

or, $3r^2 - 9r - r + 3 = 0$

or, $3r(r - 3) - 1(r - 3) = 0$

or, $(r - 3)(3r - 1) = 0$

Either, $r - 3 = 0 \Rightarrow r = 3$

or, $3r - 1 = 0 \Rightarrow r = \frac{1}{3}$

When r = 3 the numbers $\frac{a}{r}$, a, ar becomes;

$\frac{24}{3}$, 24, 24 × 3 = 8, 24, 72

When $r = \frac{1}{3}$ the numbers $\frac{a}{r}$, a, ar becomes;

$\frac{24}{\frac{1}{3}}$, 24, $24 \times \frac{1}{3}$ = 72, 24, 8

Thus, the numbers are 8, 24, 72 or 72, 24, 8.

2. एउटा ज्यामितीय श्रेणीको पहिला तीन पदहरू x + 6, x र x – 3 भए x र पाँचौं पदको मान निकाल्नुहोस् ।
x + 6, x and x – 3 are the first three terms of a geometric series. Find the value of x and its fifth term. **[2058 S]**

⇨ Here given,

Three terms of a G.S. are : x + 6, x and x – 3.

So, $\frac{x}{x + 6} = \frac{x - 3}{x}$

or, $x^2 = (x - 3)(x + 6)$

or, $x^2 = x^2 + 6x - 3x - 18$

or, $x^2 = x^2 + 3x - 18$

or, $3x = 18$

∴ $x = \frac{18}{3} = 6$

Here, first term (a) = x + 6 = 6 + 6 = 12

Second term (t_2) = x = 6

Hence, common ratio (r) = $\frac{t_2}{a} = \frac{6}{12} = \frac{1}{2}$

Now, fifth term (t_5) = ?

Using formula, $t_n = ar^{n-1}$

$t_5 = ar^{5-1}$

$= 12.\left(\frac{1}{2}\right)^4$

$= 12 \times \frac{1}{16}$

$= \frac{3}{4}$

Thus, the value of x is 6 and fifth term t_5 is $\frac{3}{4}$.

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3. गुणोत्तर श्रेणीको लगातार आउने तीनओटा पदहरूको योगफल 62 र तिनीहरूको गुणनफल 1000 भए, ती पदहरू पत्ता लगाउनुहोस् ।
The sum of three consecutive terms in G.P. is 62 and their product is 1000, find the terms. **[2063 R]**

⇨ Here, let the three consecutive terms in G.P. are $\frac{a}{r}$, a, ar

According to question, $\frac{a}{r} . a. ar = 1000$ or, $a^3 = 1000$

or, $a = \sqrt[3]{1000}$ or, $a = 10$

Again, $\frac{a}{r} + a + ar = 62$ or, $\frac{10}{r} + 10 + 10r = 62$ or, $10 + 10r + 10r^2 = 62r$

or, $10r^2 + 10r - 62r + 10 = 0$ or, $10r^2 - 52r + 10 = 0$ or, $5r^2 - 26r + 5 = 0$

or, $5r^2 - (25 + 1)r + 5 = 0$ or, $5r^2 - 25r - r + 5 = 0$ or, $5r(r - 5) - 1\,(r - 5) = 0$

or, $(r - 5)\,(5r - 1) = 0$

Either, $r - 5 = 0$ or, $5r - 1 = 0$

$\therefore$ $r = 5$ $\therefore$ $r = \frac{1}{5}$

Putting the value of a and r, we get,

When $a = 10$ and $r = 5$, $\frac{a}{r} = \frac{10}{5} = 2$, $a = 10$, $ar = 10 \times 5 = 50$

When $a = 10$ and $r = \frac{1}{5}$, $\frac{a}{r} = \frac{10}{\frac{1}{5}} = 10 \times 5 = 50$, $a = 10$, $ar = 10 \times \frac{1}{5} = 2$

Thus, the required numbers are 2, 10, 50 or 50, 10 and 2.

MODEL 2

4. यदि a र b को बिचमा पाँचओटा गुणोत्तर मध्यमाहरू छन् । यदि दोस्रो र अन्तिम मध्यमा क्रमशः 2 र 16 छन् भने a र b का मानहरू पत्ता लगाउनुहोस् ।
There are five geometric means between a and b. If second and last mean are 2 and 16 respectively, find the values of a and b. **[2073 R]**

⇨ Here, a, m_1, $m_2 = 2$, m_3, m_4, $m_5 = 16$, b

We have, $m_2 = 2$

Again, $m_5 = 16$

or, $ar^2 = 2$ ……… (i)

or, $ar^5 = 16$ …….. (ii)

Dividing (ii) by (i) then,

$$\frac{ar^5}{ar^2} = \frac{16}{2}$$

or, $r^3 = 8$

or, $r^3 = 2^3$

$\therefore$ $r = 2$

Taking equation (i) then,

$ar^2 = 2$

or, $a \times 2^2 = 2$

$\therefore$ $a = \frac{1}{2}$

Now, $b = t_7 = ar^6 = \frac{1}{2} \times 2^6$

$\therefore$ $b = 32$

Thus, the values of a and b are $\frac{1}{2}$ and 32 respectively.

5. $\frac{1}{9}$ र 9 को बीचमा हुने 3 ओटा गुणोत्तर मध्यमाहरू निकाल्नुहोस् ।

Insert 3 geometric means between $\frac{1}{9}$ and 9. **[2057 S]**

⇨ Here, given first term of a G.S. $(a) = \frac{1}{9}$

Last term of the G.S. $(b) = 9$

Number of geometric means $(n) = 3$

If r be the common ratio, then using formula,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{9}{\frac{1}{9}}\right)^{\frac{1}{3+1}}$$

$$= (81)^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3$$

If G_1, G_2 and G_3 be geometric means, then

$G_1 = ar = \frac{1}{9} \times 3 = \frac{1}{3}$

$G_2 = ar^2 = \frac{1}{9} \times 3^2 = 1$

$G_3 = ar^3 = \frac{1}{9} \times 3^3 = 3$

Thus, the required G.M.'s are $\frac{1}{3}$, 1, 3.

6. $\frac{2}{3}$ र 162 को बीचमा 4 ओटा गुणोत्तर मध्यमाहरू भर्नुहोस् । (Insert 4 GMs between $\frac{2}{3}$ and 162.) **[2066 R]**

⇨ Here, $a = \frac{2}{3}$, $b = 162$ and $n = 4$

We know that, $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{162}{2} \times 3\right)^{\frac{1}{4+1}} = (243)^{1/5} = 3^{5 \times 1/5} = 3$

Now, 1st mean $= m_1 = ar = \frac{2}{3} \times 3 = 2$ 2nd mean $= m_2 = ar^2 = \frac{2}{3} \times 3^2 = 6$

3rd mean $= m_3 = ar^3 = \frac{2}{3} \times 3^3 = 18$ 4th mean $= m_4 = ar^4 = \frac{2}{3} \times 3^4 = 54$

Thus, the 4 means are 2, 6, 18 and 54.

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7. 5 र 80 को बीचमा पर्ने केही ज्यामितीय मध्यमाहरू छन् । यदि दोश्रो मध्यमा 20 भए ती दुई सङ्ख्याभित्र पर्ने मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् र अन्तिम मध्यमा पनि पत्ता लगाउनुहोस् ।
There are some geometric means between 5 and 80. If the second mean be 20, find the number of means between the two numbers. Also find the last mean. **[2062 R]**

⇨ Here given, First term of G.S. (a) = 5
Last term of G.S. $(\ell) = 80$
Second means $(G_2) = 20$
Number of mean (n) = ?
Last G.M. $(G_n) = ?$
Since middle term of first term (a) and second mean (G_2) is first mean (G_1).
So $G_1 = \sqrt{aG_2} = \sqrt{5 \times 20} = \sqrt{100} = 10$
Now, common ratio $(r) = \frac{G_1}{a} = \frac{10}{5} = 2$
Now, $a = 5, r = 2, \ell = 80$
Using formula, $\ell = ar^{n-1}$
or, $80 = 5 . (2)^{n+2-1}$
or, $\frac{80}{5} = 2^{n+1}$
or, $16 = 2^{n+1}$
or, $2^4 = 2^{n+1}$
So, $n + 1 = 4$
$\therefore \quad n = 3$
Thus, number of means (n) is 3.
Now, First G.M. $(G_1) = ar = 5 \times 2 = 10$
Second G.M. $(G_2) = ar^2 = 5 \times 2^2 = 20$
Third (last) G.M. $(G_3) = ar^3 = 5 \times 2^3 = 40.$
Thus, the last mean is 40.

8. 1/2 र 16 को बीचमा पर्ने केही ज्यामितीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा 4 भए ती दुई सङ्ख्याहरूभित्र पर्ने मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् र अन्तिम मध्यमा पनि पत्ता लगाउनुहोस् ।
There are some geometric means between 1/2 and 16. If the third mean be 4, find the number of means between two numbers. Also find the last mean. **[2063 R']**

⇨ Here, let there are (n – 2) geometric means
$\therefore$ First term $(a) = \frac{1}{2}$ and
last term $(t_n) = 16$
Taking,
Third mean $(m_3) = 4$
or, $ar^3 = 4$
or, $\frac{1}{2} r^3 = 4$
or, $r^3 = 8$
$\therefore \quad r = 2$
Now,
last term $(t_n) - 16$
or, $ar^{n-1} = 16$
or, $\frac{1}{2} \times 2^{n-1} = 16$
or, $2^{n-2} = 2^4$
or, $n - 2 = 4$
$\therefore \quad n = 6$
So, no. of means = 6 – 2 = 4
Last mean $= m_4 = ar^4 = \frac{1}{2} \times 2^4$
$\therefore$ mean = 8
Thus, no. of means is 4 & last mean is 8.

9. 1 र 64 का बीचमा गुणोत्तर मध्यमाहरूको सङ्ख्या निकाल्नुहोस् जसको पहिलो र अन्तिम मध्यमाको अनुपात 1:16 छ ।
Find the number of geometric means inserted between 1 and 64 in which the ratio of first mean to the last mean is 1:16.[2068 R]

⇨ Here, $1, m_1, \ldots\ldots, m_n, 64$
So, a = 1 and b = 64

We know that, $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$
$= \left(\frac{64}{1}\right)^{\frac{1}{n+1}}$
$\therefore \quad r = 64^{\frac{1}{n+1}}$

Now, $\frac{m_1}{m_n} = \frac{1}{16}$
or, $\frac{ar}{ar^n} = \frac{1}{16}$
or, $r^{(1-n)} = \frac{1}{16}$
or, $64^{\frac{1-n}{n+1}} = 16^{-1}$ $[\because r = 64^{\frac{1}{n+1}}]$
or, $4^{3\left(\frac{1-n}{n+1}\right)} = 4^{-2}$
or, $3\left(\frac{1-n}{n+1}\right) = -2$
or, $3 - 3n = -2n - 2$
or, $5 = n$
$\therefore \quad n = 5$

Thus, the required no. of G.M.s is 5.

MODEL 3

10. यदि एउटा गुणोत्तर श्रेणीको दोस्रो पद 48 र पाँचौ पद 6 छ भने प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the second term of a geometric series is 48 and its fifth term is 6 then find the sum of the first 6 terms. **[2074 R']**

⇨ Here, second term $(t_2) = 48$ and fifth term $(t_5) = 6$

We know that,
$t_n = ar^{n-1}$
or, $t_2 = ar^{2-1}$
$\therefore \quad ar = 48$ ....(i)

Again,
$t_n = ar^{n-1}$
or, $t_5 = ar^4$
$\therefore \quad ar^4 = 6$...............(ii)

Dividing equation (ii) by (i) then,
$\frac{6}{48} = \frac{ar^4}{ar}$
or, $\frac{1}{8} = r^3$
$\therefore \quad r = \frac{1}{2}$

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From (i); ar = 48

or, $a \times \frac{1}{2} = 48$

$\therefore$ a = 96

Now, sum of first n terms $(s_n) = \frac{a(r^n - 1)}{r - 1}$

So, sum of first 6 terms $(s_6) = \frac{96\left[\left(\frac{1}{2}\right)^6 - 1\right]}{\frac{1}{2} - 1} = \frac{96\left[\frac{1}{64} - 1\right]}{-\frac{1}{2}} = \frac{96 \times (-)\frac{63}{64}}{-\frac{1}{2}} = 189$

Thus, the sum of first 6 terms is 189.

11. एउटा गुणोत्तर श्रेणीको चौथो पद 24 र समान अनुपात 2 भए उक्त श्रेणीको पहिलो 9 ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 9 terms of a geometric series whose fourth term is 24 and common ratio is 2. **[2074 S']**

⇨ Here, fourth term $(t_4) = 24$ and common ratio $(r) = 2$

We know that ,

$t_4 = ar^3$

or, $24 = a \times (2)^3$

or, $24 = a \times 8$

$\therefore$ a = 3

Now, $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_9 = \frac{3(2^9 - 1)}{2 - 1} = \frac{3 \times 511}{1}$

$\therefore$ $S_9 = 1533$

Thus, the required sum is 1533.

12. एउटा गुणोत्तर श्रेणीको तेस्रो पद र सातौँ पद क्रमशः 20 र 320 भए उक्त श्रेणीको प्रथम 9 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 9 terms of a geometric series whose third term and seventh term are 20 and 320 respectively **[2072 R]**

⇨ Here, third term $(t_3) = 20$ and
seventh term $(t_7) = 320$

We know that, $t_3 = ar^2$

or, $20 = ar^2$ .... (i)

Again, $t_7 = ar^6$

or, $320 = ar^6$ .... (ii)

Dividing (ii) by (i) then,

$\frac{320}{20} = \frac{ar^6}{ar^2}$

or, $16 = r^4$

or, $(\pm 2)^4 = r^4$

$\therefore$ $r = \pm 2$

Since the term of the series is increasing

So, r = 2, putting r = 2 in (i) then,

$ar^2 = 20$

or, $a \times 2^2 = 20$

$\therefore$ a = 5

Now, we have $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_9 = \frac{a(r^9 - 1)}{r - 1}$

$= \frac{5(2^9 - 1)}{2 - 1}$

$= 5 \times (512 - 1)$

$= 5 \times 511$

$= 2555$

Thus, the value of S_9 is 2555.

13. एउटा गुणोत्तर श्रेणीको दोस्रो पद र पाचौँ पद क्रमशः 15 र 405 भए उक्त श्रेणीको प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the second term and fifth term of a geometric series are 15 and 405 respectively, find the sum of the series from first to 6 terms. **[2072 S]**

⇨ Here, second term $(t_2) = 15$ and
fifth term $(t_5) = 405$

We have, $t_2 = a + d$

$\therefore$ $15 = ar$ .... (i)

Again, $t_5 = ar^4$

or, $405 = ar^4$ .... (ii)

Dividing equation (ii) by equation (i);

$\frac{405}{15} = \frac{ar^4}{ar}$

or, $27 = r^3$

or, $3^3 = r^3$

$\therefore$ r = 3

Putting r = 3 in (i) then,

$15 = ar$

or, $15 = a \times 3$

$\therefore$ a = 5

Now,

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{5(3^6 - 1)}{3 - 1}$

$= \frac{5 \times 728}{2}$

$\therefore$ $S_6 = 1820$

Thus, the sum of first 6 terms is 1820.

14. एउटा गुणोत्तर श्रेणीको चौथो पद 54 र समान अनुपात 3 भए उक्त श्रेणीको प्रथम 8 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 8 terms of a geometric series whose fourth term is 54 and common ratio is 3. **[2071 S]**

⇨ Here, fourth term $(t_4) = 54$Common ratio $(r) = 3$ Sum of first 8 terms $(S_8) = ?$

We know that, $t_n = ar^{n-1}$ or, $t_4 = ar^3$ or, $54 = a \times 3^3$

or, $54 = a \times 27$ $\therefore$ a = 2

Again, sum of terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

Sum of 8 terms (S_8) $= \frac{a(r^8 - 1)}{3 - 1} = \frac{2(3^8 - 1)}{2} = 3^8 - 1 = 6561 - 1$ $\therefore$ $S_8 = 6560$

Thus, the sum of 8 terms is 6560.

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15. तेस्रो पद 12 र सातौँ पद 192 भएको कुनै गुणोत्तर श्रेणीको पहिलो दसौँ पदसम्मको योगफल निकाल्नुहोस् ।
The third term is 12 and seventh term is 192 of a geometric series, find the sum of its first ten terms. **[2064 S]**

⇨ Here, 3rd term $(t_3) = 12$ 7th term $(t_7) = 192$

Sum of first 10 terms $(S_{10}) = ?$

We have, $t_3 = ar^2$ $\therefore$ $12 = ar^2$ .... (i)

And, $t_7 = ar^6$ $\therefore$ $192 = ar^6$ .... (ii)

Dividing (ii) by (i); $\frac{192}{12} = \frac{ar^6}{ar^2}$

or, $16 = r^4$

or, $r^4 = (\pm 2)^4$

$\therefore$ $r = \pm 2$

When $r = \pm 2$ then from (i);

$12 = a \times 4$

$\therefore$ $a = 3$

Now, sum of first 10 terms

$$= \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{3\{(\pm 2)^{10} - 1\}}{\pm 2 - 1}$$

$$= \frac{3(1024 - 1)}{\pm 2 - 1}$$

Taking (+) ve sign, $S_{10} = \frac{3 \times 1023}{1}$

$\therefore$ $S_{10} = 3069$

Taking (–) ve sign, $S_{10} = \frac{3 \times 1023}{-3}$

$\therefore$ $S_{10} = -1023$

Thus, the sum of 10 terms (S_{10}) 3069 or – 1023.

16. कुनै GP को तेस्रो पद 12 र छैटौँ पद 96 भए पहिलो आठौँ पदसम्मको योगफल निकाल्नुहोस् ।
If the third term of a GP is 12 and sixth term is 96, find the sum of the first eight terms. **[2065 E]**

⇨ Here, $t_3 = 12$, $t_6 = 96$, $S_8 = ?$

So, $t_3 = ar^2 = 12$ .... (i)

$t_6 = ar^5 = 96$ .... (ii)

Taking the ratio of (ii) & (i) equation,

$$\frac{ar^5}{ar^2} = \frac{96}{12}$$

or, $r^3 = 8$

$\therefore$ $r = 2$

When r in equation (i)

or, $ar^2 = 12$

or, $a \times 4 = 12$

$\therefore$ $a = 3$

Now, $S_8 = \frac{a(r^n - 1)}{r - 1}$

$$= \frac{3(2^8 - 1)}{2 - 1}$$

$= 3 \times 255$

$= 765$

Thus, the required sum of first eight terms is 765.

17. 3 + 6 + 12 + ...+ 768 दिइएको श्रेणी छ । (3 + 6 + 12 + ...+ 768 is the given series.) **[2063 M]**

(a) यो श्रेणीको पद सङ्ख्या पत्ता लगाउनुहोस् । (Find the number of terms of this series.)

⇨ Here, given series is; 3 + 6 + 12 + ……………..+ 768

$$r = \frac{t_2}{t_1} = \frac{6}{3} = \frac{12}{6} = 2$$

$\therefore$ The series in a G.P. where,

First term (a) = 3,

Common ratio (r) = 2 and

Last term $(l) = 768$

Now, Last term $(l) = ar^{n-1}$

or, $768 = 3(2)^{n-1}$

or, $256 = 2^{n-1}$

or, $2^8 = 2^{n-1}$

or, $n - 1 = 8$

$\therefore$ $n = 9$

Thus, the number of terms in the series is 9.

(b) यो श्रेणीको योगफल निकाल्नुहोस् । (Find the sum of the series.)

⇨ Here, Sum $(S_n) = \frac{lr - a}{r - 1} = \frac{768 \times 2 - 3}{2 - 1} = 1533$

Thus, the sum is 1533.

18. धनात्मक समान अनुपात भएको एउटा गुणोत्तर श्रेणीको पहिला चार पदहरूको योगफल 40 र पहिला दुई पदहरूको योगफल 4 छ भने सो श्रेणीको पहिला आठ पदहरूको योगफल निकाल्नुहोस् ।
The sum of first four terms is 40 and the sum of first two terms is 4 of a geometric series whose common ratio is positive, find the sum of first eight terms.

[2060 R, 2065 R]

⇨ Here, given sum of first four terms of a G.S. $(s_4) = 40$

Sum of first two terms of a G.S. $(S_2) = 4$

Sum of first eight terms of a G.S. $(S_8) = ?$

If a be the first term and r be the common ratio, then using formula

We have, $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_4 = \frac{a(r^4 - 1)}{r - 1}$

or, $40 = \frac{a(r^4 - 1)}{r - 1}$ .... (i)

We have, $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_2 = \frac{a(r^2 - 1)}{r - 1}$

or, $4 = \frac{a(r^2 - 1)}{r - 1}$ .... (ii)

Dividing eq^n (i) by (ii) we get, $\frac{40}{4} = \frac{\frac{a(r^4 - 1)}{r - 1}}{\frac{a(r^2 - 1)}{r - 1}} = \frac{a(r^4 - 1)}{r - 1} \times \frac{r - 1}{a(r^2 - 1)} = \frac{(r^2 - 1)(r^2 + 1)}{r^2 - 1} = r^2 + 1$

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or, $10 = r^2 + 1$

or, $r^2 = 10 - 1$

or, $r^2 = 9$

$\therefore$ $r = \pm 3$

But, by given, common ratio is positive, So, r = 3

Putting the value of r in eq^n (ii) , we get,

$$4 = \frac{a(3^2 - 1)}{3 - 1}$$

or, $4 = \frac{a\,(9 - 1)}{2}$

or, $4 = \frac{a.\,8}{2}$

or, $4 = 4a$

$\therefore$ $a = 1$

Now, sum of first eight terms (S_8) $= \frac{a(r^8 - 1)}{r - 1}$

$= \frac{1\,(3^8 - 1)}{3 - 1}$

$= \frac{6561 - 1}{2}$

$= \frac{6560}{2}$

$= 3280.$

Thus, the sum first eight terms (S_8) is 3280.

MODEL 4

19. यदि एउटा गुणोत्तर श्रेणीको पहिलो तीन पदहरूको योगफल 1 र यसको पहिलो छ पदहरूको योगफल 28 भए सो श्रेणीको समानुपात र पहिलो पद पत्ता लगाउनुहोस् ।

In a geometric series the sum of the first three terms is 1 and sum of its first 6 terms is 28, find the common ratio and the first term. **[2067 S, 2064 R']**

⇨ Here, sum of the first three terms $(S_3) = 1$

We have , $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_3 = \frac{a(r^3 - 1)}{r - 1}$

$\therefore$ $1 = \frac{a(r^3 - 1)}{r - 1}$ .... (i)

Sum of the first six terms $(S_6) = 28$

We have , $S_n = \frac{a(r^n - 1)}{r - 1}$

or, $S_6 = \frac{a(r^6 - 1)}{r - 1}$

or, $28 = \frac{a(r^6 - 1)}{r - 1}$ .... (ii)

Dividing (ii) by (i) $\frac{28}{1} = \frac{\frac{a(r^6 - 1)}{r - 1}}{a\frac{(r^3 - 1)}{r - 1}}$

or, $28 = \frac{(r^6 - 1)}{(r^3 - 1)}$

or, $28 = \frac{(r^3 + 1)\,(r^3 - 1)}{(r^3 - 1)}$

or, $r^3 + 1 = 28$

or, $r^3 = 27$

$\therefore$ $r = 3$

Putting r = 3 in (i) $1 = \frac{a(3^3 - 1)}{3 - 1}$

or, $2 = a \times 26$

$\therefore$ $a = \frac{1}{13}$

Thus, the first term is $\frac{1}{13}$ and common ratio is 3.

20. एउटा समानान्तर श्रेणीका दोश्रो, चौथो र नवौं पदहरु गुणोत्तर श्रेणीमा भए सो गुणोत्तर श्रेणीको समानुपात निकाल्नुहोस् ।

The second, fourth and ninth terms of an arithmetic progression are in geometric progression. Calculate the common ratio of the geometric progression. **[2064 R]**

⇨ Here, let, the 2nd , 4th and 9th term of an AS are; a + d, a + 3d and a + 8d respectively.

By the question they are in GS.

So, $\frac{a + 3d}{a + d} = \frac{a + 8d}{a + 3d}$ $\left[\because \frac{t_2}{t_1} = \frac{t_3}{t_2}\right]$

or, $a^2 + 8ad + ad + 8d^2 = a^2 + 6ad + 9d^2$

or, $3ad = d^2$

or, $d^2 - 3ad = 0$

or, $d(d - 3a) = 0$

$\therefore$ $d = 3a$ $[d \neq 0]$

So, the terms are;

$a + d,\ a + 3d,\ a + 8d$

or, $a + 3a,\ a + 3 \times 3a,\ a + 8 \times 3a$

or, $4a,\ 10a,\ 25a$

Now, common ratio $= \frac{t_2}{t_1} = \frac{10a}{4a} = \frac{5}{2}$

Thus, the common ratio is $\frac{5}{2}$.

21. एउटा गुणोत्तर श्रेणीको छैटौं पद यसको दोस्रो पदको 16 गुणा र पहिला सात पदहरूको योगफल $\frac{127}{4}$ छ भने यो श्रेणीको धनात्मक समानुपात र पहिलो पद पत्ता लगाउनुहोस् ।

In a geometric series if the sixth term is 16 times the second term and the sum of first seven terms is $\frac{127}{4}$; then find positive common ratio and the first term of the series. **[2063 S]**

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⇨ Here, $t_6 = 16t_2$ and $S_7 = \frac{127}{4}$

We have, $ar^5 = 16(ar)$

or, $\frac{ar^5}{ar} = 16$

or, $r^4 = (\pm 2)^4$

or, $r = \pm 2$

$\therefore$ $r = 2$ [+ve value only]

But r is positive, so r = 2

Again, $S_7 = \frac{a(r^7 - 1)}{r - 1}$

or, $\frac{127}{4} = \frac{a(2^7 - 1)}{2 - 1}$

or, $\frac{127}{4} = a \times 127$

$\therefore$ $a = \frac{1}{4}$

Thus, common ratio is 2 and first term is $\frac{1}{4}$.

MODEL 5

22. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 25 र तिनीहरूको गुणोत्तर मध्यमा 20 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।

The arithmetic mean of two numbers is 25 and their geometric mean is 20, find the numbers. **[2073 S]**

⇨ Here, AM = 25 and GM = 20

Let, a and b be two numbers then, $AM = \frac{a+b}{2}$

or, $25 = \frac{a+b}{2}$

$\therefore$ $a + b = 50$ ....(i)

And, $GM = \sqrt{ab}$

or, $20 = \sqrt{ab}$

$\therefore$ $ab = 400$ ....(ii)

We know that,

$$a - b = \sqrt{(a+b)^2 - 4ab}$$
$$= \sqrt{50^2 - 4 \times 400}$$
$$= \sqrt{900}$$
$$= \pm 30$$

Taking (+) ve sign then, $a - b = 30$ ....(iii)

Solving equation (i) and (ii) then,

$$\begin{array}{r} a + b = 50 \\ a - b = 30 \\ \hline 2a = 80 \end{array}$$

$\therefore$ $a = 40$

From (i);

$a + b = 50$

or, $40 + b = 50$

$\therefore$ $b = 10$

When (–) ve sign sign is taken then,

a = 10 and b = 40.

Thus, the two numbes are 10 and 40 or 40 and 10.

23. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 5 र गुणोत्तर मध्यमा 4 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।

Find two numbers whose arithmetic mean is 5 and geometric mean is 4. **[2057 R]**

⇨ Here, Let the two numbers are a and b.

Now, Arithmetic mean $= \frac{a+b}{2}$

or, $\frac{a+b}{2} = 5$

or, $a + b = 10$

$\therefore$ $a = 10 - b$ .... (i)

Again,

Geometric mean $= \sqrt{ab}$

or, $\sqrt{ab} = 4$

or, $ab = 16$

or, $(10 - b)\, b = 16$ ($\because$ from (i))

or, $10b - b^2 = 16$

or, $b^2 - 10b + 16 = 0$

or, $b^2 - 2b - 8b + 16 = 0$

or, $b(b - 2) - 8(b - 2) = 0$

or $(b - 2)(b - 8) = 0$

or, $b - 2 = 0$ and $b - 8 = 0$

$\therefore$ $b = 2$ and $b = 8$

When $b = 2$, $a = 10 - 2 = 8$

When $b = 8$, $a = 10 - 8 = 2$

Thus, the required two numbers are;

8 and 2 or 2 and 8.

24. दुई सङ्ख्याको समानान्तरीय मध्यमा 10 र गुणोत्तर मध्यमा 8 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

Find two numbers whose arithmetic mean is 10 and geometric mean is 8. **[2062 K, 2068R']**

⇨ Here, Arithmetic mean (AM) = 10,

Geometric mean (GM) = 8

Let, the numbers are a and b.

We know that;

1st number (a) $= AM + \sqrt{AM^2 - GM^2}$
$= 10 + \sqrt{10^2 - 8^2}$
$= 10 + 6$
$= 16$

And, 2nd number (b) $= AM - \sqrt{AM^2 - GM^2}$
$= 10 - \sqrt{10^2 - 8^2}$
$= 10 - 6$
$= 4$

Thus, the numbers are 16 and 4 or 4 and 16.

25. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 50 र गुणोत्तर मध्यमा 40 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

Find two numbers whose arithmetic mean is 50 and geometric mean is 40. **[2066 R', 2066S]**

⇨ Here, AM = 50 and GM = 40

Let, a and b be the two numbers

Then, 1st number (a) $= AM + \sqrt{AM^2 - GM^2}$
$= 50 + \sqrt{50^2 - 40^2}$
$= 50 + 30$

$\therefore$ a $= 80$

2nd number (b) $= AM - \sqrt{AM^2 - GM^2}$
$= 50 - \sqrt{50^2 - 40^2}$
$= 50 - 30$

$\therefore$ b $= 20$

Thus, the two numbers are 80 and 20 or 20 and 80.

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MODEL 6

26. एउटा गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 243 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद एउटा समानान्तर श्रेणीको दशौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको पहिला 19 पदहरूको योगफल पत्ता लगाउनुहोस् ।

The product of the first five terms of a geometric series is 243. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of the first 19 terms of the arithmetic series.

[SEE 2075 R, 2075 R$_2$]

⇨ Here, let the first five terms of a G.S be,

a, ar, ar^2, ar^3 and ar^4 respectively.

According to the question,

$a \,.\, ar \,.\, ar^2 \,.\, ar^3 \,.\, ar^4 = 243$

or, $a^5r^{10} = 243$

or, $(ar^2)^5 = 3^5$

$\therefore$ $ar^2\ (t_3) = 3$

By the question,

t_3 of GS = t_{10} of A.S

i.e. In AS, $t_{10} = 3$

We have,

$t_n = a + (n-1)d$

i.e. $t_{10} = a + (10-1)d$

$\therefore$ $a + 9d = 3$ .....(i)

Again, we know,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

So, Sum of first 19 terms (S_{19}) is ;

$$S_{19} = \frac{19}{2}[2a + (19-1)d]$$
$$= \frac{19}{2}[2a + 18d]$$
$$= \frac{19}{2}\,.\,2[a + 9d][\because \text{ from (i)}]$$
$$= 19 \times 3$$
$$= 57$$

Thus, the sum of first 19 terms of AS is 57.

27. एउटा समानान्तरीय श्रेणीमा तीनओटा पदहरूको योगफल 24 छ । यदि ती पदहरूमा क्रमशः 1, 6 र 18 जोड्दा परिणाम गुणोत्तर श्रेणीमा हुन्छ भने ती पदहरू निकाल्नुहोस् ।

The sum of three terms in an arithmetic series is 24. If 1, 6 and 18 are added to them respectively, the results are in geometrical series, find the terms. [SEE 2074 R]

⇨ Here; sum of 3 terms of an AP = 24.

Let $a - d$, a and $a + d$ be the three terms of an AP.

By the question,

sum = 24

or, $a - d + a + a + d = 24$

or, $3a = 24$

$\therefore$ $a = 8$

So, the terms are; $8 - d$, 8 and $8 + d$

Again, from the question,

$(8 - d + 1)$, $(8 + 6)$ and $(8 + d + 18)$ are in GP.

$\therefore$ $9 - d$, 14 and $26 + d$ are in GP.

So, $\frac{t_2}{t_1} = \frac{t_3}{t_2}$

or, $\frac{14}{9-d} = \frac{26+d}{14}$

or, $196 = 234 + 9d - 26d - d^2$

or, $d^2 + 17d - 38 = 0$

or, $d^2 + 19d - 2d - 38 = 0$

or, $d(d + 19) - 2(d + 19) = 0$

or, $(d + 19)(d - 2) = 0$

$\therefore$ $d = -19$ or 2

When $d = -19$ and $a = 8$;

First term = $a - d = 8 + 19 = 27$

Second term = $a = 8$

Third term = $a + d = 8 - 19 = -11$

When $d = 2$ and $a = 8$;

First term = $a - d = 8 - 2 = 6$

Second term = $a = 8$

Third term = $a + d = 8 + 2 = 10$

Thus, the three terms are 27, 8, – 11 or 6, 8, 10.

QUESTIONS FROM CDC TEXTBOOK

1.3.3 गुणोत्तर अनुक्रम र श्रेणी (GEOMETRIC SEQUENCE AND SERIES)

EXERCISE 1.3.3

1. **(a) गुणोत्तर अनुक्रम भनेको के हो ? उदाहरणसहित व्याख्या गर्नुहोस् ।**

What do you mean by geometric sequence? Explain with examples.

⇨ Here, quantities are said to be in geometric sequence when they increase or decrease by a constant factor. The constant factor is called the common ratio.

e.g. 1, 2, 4, 8..........and 16, 8, 4 .... etc. are in geometric sequence.

(b) गुणोत्तर अनुक्रमका विशेषताहरू उल्लेख गर्नुहोस् । (List the characteristics of geometric sequence.)

⇨ Here, the characteristics of geometric sequence are as follows:

(i) The ratio of any two consecutive terms are equal.

(ii) The geometric sequence is in the form of a, ar, ar^2, ar^3, ...

(iii) The geometric sequence is an ordered list of numbers in which each term after the first is found by multiplying the previous one by a constant.

(c) गुणोत्तर अनुक्रम र गुणोत्तर श्रेणीबिचको सम्बन्ध लेख्नुहोस् ।

Write the relation between geometric sequence and geometric series.

⇨ Here, the general form of a geometric sequence is a, ar, ar^2, ar^3, ar^4, ... then its corresponding geometric series is $a + ar + ar^2 + ar^3 + ar^4 +$ ....

In other words, if the terms of geometric sequence are connected by the sign (+) or (–) then it is geometric series.

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(d) गुणोत्तर मध्यमा भनेको के हो ? गुणोत्तर मध्यमा पत्ता लगाउने तरिका लेख्नुहोस् ।
What is geometric mean? Write the way to calculate geometric mean.

⇨ Here, in geometric sequence the term/terms between the first term and the last term is/are called the geometric mean/means.

(i) If a, m, b are in GS then GM is; $m = \sqrt{ab}$

(ii) If a, m_1, m_2, m_3, ..., b are in GS then the geometric means: $m_1 = ar$, $m_2 = ar^2$, $m_3 = ar^3$ .... are known as geometric means between a and b.

2. दिइएका अनुक्रमहरू मध्ये कुन कुन गुणोत्तर अनुक्रम हुन् र कुन कुन होइनन् ? छुट्याई कारण पनि लेख्नुहोस् ।
Which of the following sequences are geometric and which are not? Also, state the reason.

(a) 7, 14, 28

⇨ Here, 7, 14, 28

$$r_1 = \frac{t_2}{t_1} = \frac{14}{7} = 2$$

$$r = \frac{t_3}{t_2} = \frac{28}{14} = 2$$

The ratios of the consecutive terms are equal.

Thus, the sequence is a geometric.

(c) 25, 5, 1, ...

⇨ Here, 25, 5, 1, ...

So, $t_1 = 25$, $t_2 = 5$ and $t_3 = 1$

$$r_1 = \frac{t_2}{t_1} = \frac{5}{25} = \frac{1}{5}$$

$$r_2 = \frac{t_3}{t_2} = \frac{1}{5}$$

The ratios of the consecutive terms are equal.

Thus, the sequence is geometric.

(b) a, a^2, a^3

⇨ Here, a, a^2, a^3

$$r_1 = \frac{t_2}{t_1} = \frac{a^2}{a} = a$$

$$r_2 = \frac{t_3}{t_2} = \frac{a^3}{a^2} = a$$

The ratios of the consecutive terms are equal.

Thus, the sequence is a geometric.

(d) $7, -1, \frac{1}{4}, -\frac{1}{16}, ...$

⇨ Here, $7, -1, \frac{1}{4}, -\frac{1}{16}$ ....

So, $t_1 = 7$, $t_2 = -1$, $t_3 = \frac{1}{4}$ and $t_4 = -\frac{1}{16}$

$$r_1 = \frac{t_2}{t_1} = \frac{-1}{7} = -\frac{1}{7}, \qquad r_2 = \frac{t_3}{t_2} = \frac{\frac{1}{4}}{-1} = -\frac{1}{4},$$

$$r_3 = \frac{t_4}{t_3} = \frac{-\frac{1}{16}}{\frac{1}{4}} = -\frac{1}{16} \times \frac{4}{1} = -\frac{1}{4}$$

All the ratios of the consecutive terms are not equal.

Thus, the sequence is not geometric.

3. दिइएका गुणोत्तर अनुक्रमको पहिलो पद (a) र समान अनुपात (r) का आधारमा छैटौँ पद र बाह्रौँ पद निकाल्नुहोस् ।
Find the 6th term and 12th term on the basis of first term (a) and common ratio (r) of the geometric sequence.

(a) पहिलो पद (a) = 120 र समान अनुपात (r) = $\frac{1}{2}$ (First term (a) = 120 and common ratio (r) = $\frac{1}{2}$)

⇨ Here, a = 120 and $r = \frac{1}{2}$

We have, $t_n = ar^{n-1}$

So, $t_6 = ar^{6-1}$

$$= 120 \times \left(\frac{1}{2}\right)^5$$

$$= 120 \times \frac{1}{32}$$

$$\therefore \quad t_6 = \frac{15}{4}$$

and $t_{12} = ar^{12-1}$

$$= 120 \times \left(\frac{1}{2}\right)^{11}$$

$$= 120 \times \frac{1}{2048}$$

$$\therefore \quad t_{12} = \frac{15}{256}$$

Thus, the required terms are $\frac{15}{4}$ and $\frac{15}{256}$.

(b) पहिलो पद (a) = – 3 र समान अनुपात (r) = 2 (First term (a) = – 3 and common ratio (r) = 2)

⇨ Here, a = – 3 and r = 2

We have, $t_n = ar^{n-1}$

So, $t_6 = ar^{6-1}$

$$= -3\,(2)^5$$

$$= -3 \times 32$$

$$\therefore \quad t_6 = -96$$

and $t_{12} = ar^{12-1}$

$$= (-3)\,2^{11}$$

$$= -3 \times 2048$$

$$\therefore \quad t_{12} = -6144$$

Thus, the required terms are – 96 and – 6144.

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(c) पहिलो पद (a) = $\frac{1}{3}$ र समान अनुपात (r) = 3 (First term (a) = $\frac{1}{3}$ and common ratio (r) = 3)

⇨ Here, $a = \frac{1}{3}$ and $r = 3$

We have, $t_n = ar^{n-1}$

So, $t_6 = ar^{6-1}$

$= \frac{1}{3}(3)^5$

$= \frac{243}{3}$

$\therefore \quad t_6 = 81$

and $t_{12} = ar^{12-1}$

$= \frac{1}{3}(3)^{11}$

$= \frac{177147}{3}$

$\therefore \quad t_{12} = 59049$

Thus, the required terms are 81 and 59049.

4. निम्न लिखित गुणोत्तर अनुक्रमका पद सङ्ख्या निकाल्नुहोस् । (Find the No. of terms of the following geometric sequence.)

(a) 5, – 15, 45, ..., – 10935

⇨ Here, First term (a) = 5

Common ratio $(r) = \frac{-15}{5} = -3$

Last term $(t_n) = -10935$

Number of terms (n) = ?

We have, $t_n = ar^{n-1}$

or, $-10935 = 5(-3)^{n-1}$

or, $-2187 = (-3)^{n-1}$

or, $(-3)^7 = (-3)^{n-1}$

or, $7 = n - 1$

$\therefore \quad n = 8$

Thus, number of terms in the sequence is 8.

(b) 1, 3, 9, ..., 243

⇨ Here, first term (a) = 1

Common ratio $(r) = \frac{t_2}{t_1} = \frac{3}{1} = 3$

Last term $(t_n) = 243$

We know that,

$t_n = ar^{n-1}$

or, $243 = 1 \times 3^{n-1}$

or, $243 = 3^{n-1}$

or, $3^5 = 3^{n-1}$

or, $n - 1 = 5$

$\therefore \quad n = 6$

Thus, the number of terms is 6.

(c) 4, 6, 9, ..., $\frac{243}{8}$

⇨ Here, first term (a) = 4

Common ratio $(r) = \frac{t_2}{a} = \frac{6}{4} = \frac{3}{2}$

Last term $(t_n) = \frac{243}{8}$

We know that,

$t_n = ar^{n-1}$

or, $\frac{243}{8} = 4 \times \left(\frac{3}{2}\right)^{n-1}$

or, $\frac{243}{32} = \left(\frac{3}{2}\right)^{n-1}$

or, $\left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{n-1}$

or, $n - 1 = 5$

$\therefore \quad n = 6$

Thus, the number of terms is 6.

(d) $\frac{1}{4}, \frac{1}{2}$, 1, 2, ..., 128

⇨ Here, First term $(a) = \frac{1}{4}$

Common ratio $(r) = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \times \frac{4}{1} = 2$

Last term $(t_n) = 128$

Number of terms (n) = ?

We have, $t_n = ar^{n-1}$

or, $128 = \frac{1}{4}(2)^{n-1}$

or, $128 \times 4 = 2^{n-1}$

or, $2^9 = 2^{n-1}$

or, $9 = n - 1$

$\therefore \quad n = 9 + 1 = 10$

Thus, number of terms in the sequence is 10.

5. दिइएका अवस्थामा गुणोत्तर अनुक्रमको पहिलो पद (a) अथवा समान अनुपात (r) निकाल्नुहोस् ।
Find the first term (a) or common ratio (r) of geometric sequence in following conditions.

(a) समान अनुपात (r) = 2, दसौँ पद (t_{10}) = 1536

Common ratio (r) = 2, 10th term (t_{10}) = 1536

⇨ Here, $r = 2$ and $t_{10} = 1536$

We know that,

$t_n = ar^{n-1}$

or, $t_{10} = a(2)^{10-1}$

or, $1536 = a \times 2^9$

or, $a = \frac{1536}{512}$

$\therefore \quad a = 3$

Thus, the first term (a) is 3.

(b) समान अनुपात (r) = $\frac{1}{3}$, आठौँ पद (t_8) = $\frac{1}{729}$

Common ratio (r) = $\frac{1}{3}$, 8th term (t_8) = $\frac{1}{729}$

⇨ Here, $r = \frac{1}{3}$, $t_8 = \frac{1}{729}$, a = ?

We know that, $t_n = ar^{n-1}$

or, $t_8 = ar^7$

or, $\frac{1}{729} = a\left(\frac{1}{3}\right)^7$

or, $\frac{1}{729} = a \times \frac{1}{2187}$

$\therefore \quad a = \frac{2187}{729} = 3$

Thus, the first term (a) = 3.

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(c) **पहिलो पद (a) = 7 र एघारौं पद (t_{11}) = 11**

First term (a) = 7 and 11th term (t_{11}) = 11

⇨ Here, first term (a) = 7 and $t_{11} = 11$

We know that, $t_n = ar^{n-1}$

or, $t_{11} = 7 \times r^{11-1}$

or, $11 = 7 \times r^{10}$

or, $\frac{11}{7} = r^{10}$

$\therefore \quad r = \sqrt[10]{\frac{11}{7}} = \left(\frac{11}{7}\right)^{\frac{1}{10}}$

Thus, the value of r is $\left(\frac{11}{7}\right)^{\frac{1}{10}}$.

(d) **पहिलो पद (a) = 2 र आठौं पद (t_8) = 4374**

First term (a) = 2 and 8th term (t_8) = 4374

⇨ Here, a = 2 and eighth term (t_8) = 4374

We know that,

$t_n = ar^{n-1}$

or, $t_8 = 2 \times r^{8-1}$

or, $4374 = 2 \times r^7$

or, $2187 = r^7$

or, $3^7 = r^7$

$\therefore \quad r = 3$

Thus, the value of r is 3.

6. (a) **यदि 2k + 2, 2k + 6 र 7k + 10 गुणोत्तर अनुक्रममा भए k को मान पत्ता लगाउनुहोस् ।**

If 2k + 2, 2k + 6 and 7k + 10 are in geometric sequence, find the value of k.

⇨ Here, a = 2k + 2, GM = 2k + 6 & b = 7k + 10

We know that, $GM^2 = ab$

or, $(2k + 6)^2 = (2k + 2)(7k + 10)$

or, $4k^2 + 24k + 36 = 14k^2 + 20k + 14k + 20$

or, $4k^2 + 24k + 36 = 14k^2 + 34k + 20$

or, $-10k^2 - 10k + 16 = 0$

or, $-2(5k^2 + 5k - 8) = 0$

or, $5k^2 + 5k - 8 = 0$

Comparing it with $ak^2 + bk + c = 0$ then

$a = 5, b = 5 \ \& \ c = -8$

We have,

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 + 4 \times 5 \times 8}}{2 \times 5}$$

$$\therefore \quad k = \frac{-5 \pm \sqrt{185}}{10}$$

Thus, the value of k is $\frac{-5 \pm \sqrt{185}}{10}$.

(b) **यदि 5x – 2, x + 2 र x गुणोत्तर अनुक्रममा भए x को मान निकाल्नुहोस् ।**

If 5x – 2, x + 2 and x are in geometric sequence, find the value of x.

⇨ Here, a = 5x – 2, GM = x + 2 & b = x

We know that,

$GM^2 = ab$

or, $(x + 2)^2 = (5x - 2)x$

or, $x^2 + 4x + 4 = 5x^2 - 2x$

or, $4x^2 - 6x - 4 = 0$

or, $2(2x^2 - 3x - 2) = 0$

or, $2x^2 - 3x - 2 = 0$

or, $2x^2 - 4x + x - 2 = 0$

or, $2x(x - 2) + 1(x - 2) = 0$

or, $(x - 2)(2x + 1) = 0$

Either, $x - 2 = 0$ or, $2x + 1 = 0$

$\therefore \quad x = 2$ $\quad \therefore \ x = -\frac{1}{2}$

Thus, x = 2 or $-\frac{1}{2}$ is the required value.

(c) **यदि 6, x, y, 162 गुणोत्तर अनुक्रममा भए x र y को मान निकाल्नुहोस् ।**

If 6, x, y, 162 are in geometric sequence, find the value of x and y.

⇨ Here, 6, x, y, 162 are in G.S.

We have, a = 6, b = 162,

no. of means (n) = 2

We know that, $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$= \left(\frac{162}{6}\right)^{\frac{1}{2+1}}$

$= (27)^{\frac{1}{3}}$

$= 3^{3 \times \frac{1}{3}}$

$= 3$

Now, $x = ar = 6 \times 3 = 18$

$y = ar^2 = 6 \times 3^2 = 54$

Thus, the values of x and y are 18 and 54 resp.

7. (a) **एउटा गुणोत्तर अनुक्रमको पाँचौं पद र आठौं पद क्रमश: 162 र 4374 भए दसौं पद पत्ता लगाउनुहोस् ।**

If the 5th term and the 8th term of a geometric sequence are 162 and 4374 respectively, find the 10th term.

⇨ Here, $t_5 = 162$ and $t_8 = 4374$

We know that, $t_n = ar^{n-1}$

So, $t_5 = ar^4$ and $t_8 = ar^7$

$\therefore \quad 162 = ar^4$ ....(i) and

$4374 = ar^7$ ....(ii)

Dividing equation (ii) by (i) then, $\frac{4374}{162} = \frac{ar^7}{ar^4}$

or, $27 = r^3$

or, $r^3 = 3^3$

$\therefore \quad r = 3$

From (i), $162 = a \times 3^4$

or, $162 = a \times 81$

$\therefore \quad a = 2$

Now, 10th term (t_{10}) $= ar^9$

$= 2 \times 3^9$

$= 39366$

Thus, the 10th term is 39366.

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(b) **एउटा गुणोत्तर अनुक्रमको चौथो र सातौँ पद क्रमशः 18 र 144 भए पन्ध्रौँ पद पत्ता लगाउनुहोस् ।**
If the 4th term and the 7th term of a geometric sequence are 18 and 144 respectively, find the 15th term.

⇨ Here, $t_4 = 18$ and $t_7 = 144$

We know that, $t_n = ar^{n-1}$

So, $t_4 = ar^3$ and $t_7 = ar^6$

$\therefore\ 18 = ar^3$ ....(i) and

$144 = ar^6$ ....(ii)

Dividing equation (ii) by (i) then,

$$\frac{144}{18} = \frac{ar^6}{ar^3}$$

or, $8 = r^3$

or, $r^3 = 2^3$

$\therefore\ r = 2$

From (i), $18 = a \times 2^3$

or, $a = \frac{18}{8}$

$\therefore\ a = \frac{9}{4}$

Now, 15th term $(t_{15})\ = ar^{14}$

$= \frac{9}{4} \times 2^{14}$

$= 36864$

Thus, the 15th term is 36864.

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8. (a) **गुणोत्तर अनुक्रमको चौथो पद पहिलो पदको 8 गुणासँग बराबर छ र छैटौँ पद 64 छ भने दसौँ पद पत्ता लगाउनुहोस् ।**
If the 4th term of a geometric sequence is equals to 8 times of 1st term and the 6th term is 64, find the 10th term.

⇨ Here, $t_4 = 8$ (First term) and $t_6 = 64$

So, $ar^3 = 8a$ and $ar^5 = 64$

or, $r^3 = 8$

$\therefore\ r = 2$

Again, $ar^5 = 64$

or, $a \times 2^5 = 64$

or, $a \times 32 = 64$

$\therefore\ a = 2$

Now, $t_{10} = ar^9$

$= 2 \times 2^9 = 2 \times 512$

$= 1024$

Thus, the 10th term is 1024.

(b) **यदि गुणोत्तर अनुक्रमको पहिलो पदको 128 गुणा सातौँ पदको 2 गुणासँग बराबर छ र तेस्रो पद 8 छ भने आठौँ पद निकाल्नुहोस् ।**
If 128 times of first term of a geometric sequence is equals to two times of 7th term and 3rd term is 8, find the 8th term.

⇨ Here, $128a = 2t_7$ and $t_3 = 8$; $t_8 = ?$

We know that, $t_n = ar^{n-1}$

So, $128a = 2 \times ar^6$

or, $64 = r^6$

or, $(\pm 2)^6 = r^6$

$\therefore\ r = \pm 2$

Again, $t_3 = ar^2$

or, $8 = a(\pm 2)^2$

or, $8 = a \times 4$

$\therefore\ a = 2$

Now, $t_8 = ar^7 = 2 \times (\pm 2)^7$

Taking (+) ve sign then $t_8 = 2 \times 2^7 = 256$

Taking (–) ve sign then $t_8 = 2 \times (-2)^7 = -256$

Thus, the eighth term is – 256 or 256.

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9. (a) **गुणोत्तर श्रेणीका तीन ओटा पदहरूको योगफल 114 र गुणनफल 46656 छ भने ती पदहरू पत्ता लगाउनुहोस् ।**
If the sum of three terms of a geometric series is 114 and their product is 46656, find the terms.

⇨ Here, sum = 114 and product = 46656

Let the three terms be $\frac{a}{r}$, a, ar

By the question, product = 46656

or, $\frac{a}{r} \times a \times ar = 46656$

or, $a^3 = 46656$

$\therefore\ a = 36$

Again, sum = 114

or, $\frac{a}{r} + a + ar = 114$

or, $a\left(\frac{1}{r} + 1 + r\right) = 114$

or, $36\left(\frac{1 + r + r^2}{r}\right) = 114$

or, $\frac{1 + r + r^2}{r} = \frac{114}{36}$

or, $36 + 36r + 36r^2 = 114r$

or, $36r^2 - 78r + 36 = 0$

or, $6(6r^2 - 13r + 6) = 0$

or, $6r^2 - 13r + 6 = 0$

or, $6r^2 - 9r - 4r + 6 = 0$

or, $3r\,(2r - 3) - 2(2r - 3) = 0$

or, $(2r - 3)\,(3r - 2) = 0$

Either, $2r - 3 = 0$ or $3r - 2 = 0$

or, $2r = 3$ or, $3r = 2$

$\therefore\ r = \frac{3}{2}$ $\therefore\ r = \frac{2}{3}$

When $r = \frac{3}{2}$ and $a = 36$

Then the three terms are $= \frac{a}{r}$, a, ar

$= \frac{36}{\frac{3}{2}}, 36, 36 \times \frac{3}{2}$

$= \frac{36 \times 2}{3}, 36, 36 \times \frac{3}{2}$

$= 24, 36, 54$

When $r = \frac{2}{3}$ and $a = 36$

Then the three terms are $= \frac{a}{r}$, a, ar

$= \frac{36}{\frac{2}{3}}, 36, 36 \times \frac{2}{3}$

$= \frac{36 \times 3}{2}, 36, 36 \times \frac{2}{3}$

$= 54, 36, 24$

Thus, the three numbers are: 24, 36, 54 or 54, 36, 24.

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(b) गुणोत्तर श्रेणीका तीन ओटा पदहरूको योगफल 38 र गुणनफल 1728 छ भने ती पदहरू पत्ता लगाउनुहोस् ।

If the sum of three terms of a geometric series is 38 and their product is 1728, find the terms.

⇨ Here, sum = 38 and product = 1728

Let the terms be $\frac{a}{r}$, a, ar

By the question, product = 1728

or, $\frac{a}{r} \times a \times ar = 1728$

or, $a^3 = 1728$

or, $a^3 = 12^3$

$\therefore$ a = 12

Again, sum = 38

or, $\frac{a}{r} + a + ar = 38$

or, $a\left(\frac{1}{r} + 1 + r\right) = 38$

or, $12\left(\frac{1 + r + r^2}{r}\right) = 38$

or, $\frac{1 + r + r^2}{r} = \frac{38}{12}$

or, $\frac{1 + r + r^2}{r} = \frac{19}{6}$

or, $6 + 6r + 6r^2 = 19r$

or, $6 - 13r + 6r^2 = 0$

or, $6r^2 - 13r + 6 = 0$

or, $6r^2 - 9r - 4r + 6 = 0$

or, $3r(2r - 3) - 2(2r - 3) = 0$

or, $(2r - 3)(3r - 2) = 0$

Either, $2r - 3 = 0$ or $3r - 2 = 0$

or, $2r = 3$ or, $3r = 2$

$\therefore r = \frac{3}{2}$ $\therefore r = \frac{2}{3}$

When, $r = \frac{3}{2}$ and a = 12 then,

The three terms are = $\frac{a}{r}$, a, ar

$= \frac{12}{\frac{3}{2}}, 12, 12 \times \frac{3}{2}$

$= 12 \times \frac{2}{3}, 12, 12 \times \frac{3}{2}$

= 8, 12, 18

When, $r = \frac{2}{3}$ and a = 12 then,

The three terms are = $\frac{a}{r}$, a, ar

$= \frac{12}{\frac{2}{3}}, 12, 12 \times \frac{2}{3}$

$= 12 \times \frac{3}{2}, 12, 12 \times \frac{2}{3}$

= 18, 12, 8

Thus, the three terms are 8, 12, 18 or 18, 12, 8.

10. दिइएका दुई पदहरूबिच पर्ने गुणोत्तर मध्यमा निकाल्नुहोस् ।

Find the geometric mean between the following two terms.

(a) 9 and 16

⇨ Here, first term (a) = 9 and

last term (b) = 16

GM = ?

We have,

$$GM = \sqrt{ab}$$
$$= \sqrt{9 \times 16}$$
$$= 3 \times 4 = 12$$
$$\therefore GM = 12$$

Thus, the GM is 12.

(b) 54 and 6

⇨ Here, first term (a) = 54,

last term (b) = 6 and GM = ?

We have,

$$GM = \sqrt{ab}$$
$$= \sqrt{54 \times 6}$$
$$= \sqrt{9 \times 6 \times 6}$$
$$= 3 \times 6 = 18$$
$$\therefore GM = 18$$

Thus, GM is 18.

(c) $\frac{1}{64}$ and $\frac{1}{16}$

⇨ Here, $a = \frac{1}{64}$ and $b = \frac{1}{16}$

So, $GM = \sqrt{ab}$

$$= \sqrt{\frac{1}{64} \times \frac{1}{16}}$$
$$= \sqrt{\left(\frac{1}{8}\right)^2 \times \left(\frac{1}{4}\right)^2}$$
$$= \frac{1}{8} \times \frac{1}{4}$$
$$= \frac{1}{32}$$

Thus, the GM is $\frac{1}{32}$.

(d) 8 and $\frac{32}{3}$

⇨ Here, a = 8 and $b = \frac{32}{3}$

We know that,

$$GM = \sqrt{ab}$$
$$= \sqrt{8 \times \frac{32}{3}}$$
$$= \sqrt{\frac{8 \times 8 \times 4}{3}}$$
$$= \frac{8 \times 2}{\sqrt{3}}$$
$$= \frac{16}{\sqrt{3}}$$

Thus, the GM is $\frac{16}{\sqrt{3}}$.

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11. निम्नअनुसारका गुणोत्तर मध्यमाहरू भर्नुहोस् (Insert the geometric means) :

(a) 3 र 243 का बिचमा 3 ओटा मध्यमाहरू
3 means between 3 and 243

⇨ Here, $a = 3$, $b = 243$ and no. of means $(n) = 3$

We know that,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{\frac{1}{4}} = 3^{4 \times \frac{1}{4}}$$

$\therefore \quad r = 3$

Now, first mean $(m_1) = ar = 3 \times 3 = 9$
Second mean $(m_2) = ar^2 = 3 \times 3^2 = 27$
Third mean $(m_3) = ar^3 = 3 \times 3^3 = 81$

Thus, the three means are 9, 27, 81.

(b) 2 र 64 का बिचमा 4 ओटा मध्यमाहरू
4 means between 2 and 64

⇨ Here, $a = 2$, $b = 64$ & no. of means $(n) = 4$

We have,

$$\text{Common ratio } (r) = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{64}{2}\right)^{\frac{1}{4+1}} = (32)^{\frac{1}{5}} = 2^{5 \times \frac{1}{5}}$$

$\therefore \quad r = 2$

Now, first mean $(m_1) = ar = 2 \times 2 = 4$
Second mean $(m_2) = ar^2 = 2 \times 2^2 = 8$
Third mean $(m_3) = ar^3 = 2 \times 2^3 = 16$
Fourth mean $(m_4) = ar^4 = 2 \times 2^4 = 32$

Thus, the means are 4, 8, 16, 32.

(c) 8 र $\frac{1}{8}$ का बिचमा 5 ओटा मध्यमाहरू (5 means between 8 and $\frac{1}{8}$)

⇨ Here, $a = 8$, $b = \frac{1}{8}$ & No. of means $(n) = 5$

We have,

$$\text{Common ratio } (r) = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{\frac{1}{8}}{8}\right)^{\frac{1}{5+1}} = \left(\frac{1}{64}\right)^{\frac{1}{6}} = \left(\frac{1}{2}\right)^{6 \times \frac{1}{6}}$$

$\therefore \quad r = \frac{1}{2}$

Now, $m_1 = ar = 8 \times \frac{1}{2} = 4$

$m_2 = ar^2 = 8 \times \left(\frac{1}{2}\right)^2 = 8 \times \frac{1}{4} = 2$

$m_3 = ar^3 = 8 \times \left(\frac{1}{2}\right)^3 = 8 \times \frac{1}{8} = 1$

$m_4 = ar^4 = 8 \times \left(\frac{1}{2}\right)^4 = 8 \times \frac{1}{16} = \frac{1}{2}$

$m_5 = ar^5 = 8 \times \left(\frac{1}{2}\right)^5 = 8 \times \frac{1}{32} = \frac{1}{4}$

Thus, the means are 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$.

12. (a) पदहरू 10 र 1280 का बिचमा पर्ने गुणोत्तर मध्यमा सङ्ख्या निकाल्नुहोस्, जहाँ पहिलो मध्यमा र अन्तिम मध्यमाको अनुपात 1 : 32 छ।
Find the no. of geometric means between 10 and 1280 where the ratio of first mean to the last mean is 1 : 32.

⇨ Here, $a = 10$, $b = 1280$, No. of means $= n$

We know that,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{1280}{10}\right)^{\frac{1}{n+1}} = 128^{\frac{1}{n+1}}$$

By the question, first mean : last mean = 1 : 32

or, $ar : ar^n = 1 : 32$

or, $\frac{ar}{ar^n} = \frac{1}{32}$

or, $\frac{r}{r^n} = \frac{1}{2^5}$

or, $r^{1-n} = 2^{-5}$

or, $\left(128^{\frac{1}{n+1}}\right)^{1-n} = 2^{-5}$

or, $2^{7 \times \frac{1}{n+1} \times 1-n} = 2^{-5}$

or, $2^{\frac{7(1-n)}{n+1}} = 2^{-5}$

or, $\frac{7(1-n)}{n+1} = -5$

or, $7 - 7n = -5n - 5$

or, $-2n = -12$

$\therefore \quad n = 6$

Thus, the number of means = 6.

(b) पदहरू 3 र 192 को बिचमा केही गुणोत्तर मध्यमाहरू छन्। यदि पाँचौँ गुणोत्तर मध्यमा 96 छ भने मध्यमा सङ्ख्या निकाल्नुहोस्।
There are some geometric means between 3 and 192. If the 5th geometric mean is 96, find the number of means.

⇨ Here, $a = 3$, $b = 192$, No. of means $= n$

We know that,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{192}{3}\right)^{\frac{1}{n+1}} = (64)^{\frac{1}{n+1}}$$

Again, Fifth mean = 96

or, $ar^5 = 96$

or, $3 \times \left[(64)^{\frac{1}{n+1}}\right]^5 = 96$

or, $(64)^{\frac{5}{n+1}} = 32$

or, $2^{\frac{6 \times 5}{n+1}} = 2^5$

or, $\frac{6 \times 5}{n+1} = 5$

or, $\frac{6}{n+1} = 1$

or, $n + 1 = 6$

$\therefore n = 5$

Thus, the no. of means is 5.

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13. (a) **समानान्तरीय मध्यमा 34 र गुणोत्तर मध्यमा 16 हुने दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।**
Find the two numbers whose arithmetic mean is 34 and geometric mean is 16.

⇨ Here, let a & b are the numbers.
AM = 34 and GM = 16
We know that;

$$\begin{aligned} 1^{st} \text{ number (a)} &= AM + \sqrt{AM^2 - GM^2} \\ &= 34 + \sqrt{34^2 - 16^2} \\ &= 34 + \sqrt{1156 - 256} \\ &= 34 + \sqrt{900} \\ &= 34 + 30 \end{aligned}$$

$\therefore$ 1^{st} number (a) = 64

$$\begin{aligned} \text{And, Second number (b)} &= AM - \sqrt{AM^2 - GM^2} \\ &= 34 - \sqrt{34^2 - 16^2} \\ &= 34 - \sqrt{1156 - 256} \\ &= 34 - \sqrt{900} \\ &= 34 - 30 \end{aligned}$$

$\therefore$ Second number (b) = 4
Thus, required numbers are 64 and 4.

14. (a) **कुनै दुई सङ्ख्याहरूको अनुपात 2 : 1 छ । तिनीहरूको गुणोत्तर मध्यमा 4 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।**
The ratio of any two numbers is 2 : 1. If their geometric mean is 4 then find the numbers.

⇨ Here, ratio of two numbers
i.e. a : b = 2 : 1; GM = 4
So, a = 2x and b = x
We have given,

$GM = \sqrt{ab}$
or, $4 = \sqrt{ab}$
or, $4^2 = ab$
or, $16 = 2x . x$
or, $16 = 2x^2$
or, $x^2 = 8$
$\therefore$ $x = 2\sqrt{2}$

$$\begin{aligned} \text{Now, first number} &= 2x \\ &= 2 \times 2\sqrt{2} \\ &= 4\sqrt{2} \\ \text{Second number} &= x \\ &= 2\sqrt{2} \end{aligned}$$

Thus, the two numbers are $4\sqrt{2}$ and $2\sqrt{2}$.

(b) **समानान्तरीय मध्यमा 25 र गुणोत्तर मध्यमा 20 हुने दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।**
Find the two numbers whose arithmetic mean is 25 and geometric mean is 20.

⇨ Here, let a & b are the numbers.
AM = 25 and GM = 20
We know that,

$$\begin{aligned} \text{First Number (a)} &= AM + \sqrt{AM^2 - GM^2} \\ &= 25 + \sqrt{25^2 - 20^2} \\ &= 25 + \sqrt{625 - 400} \\ &= 25 + \sqrt{225} \\ &= 25 + 15 \\ &= 40 \end{aligned}$$

$$\begin{aligned} \text{Second number (b)} &= AM - \sqrt{AM^2 - GM^2} \\ &= 25 - \sqrt{25^2 - 20^2} \\ &= 25 - \sqrt{625 - 400} \\ &= 25 - \sqrt{225} \\ &= 25 - 15 \\ &= 10 \end{aligned}$$

Thus, the two numbers are 40 and 10.

(b) **कुनै दुई सङ्ख्याहरूको अनुपात 1 : 16 छ । तिनीहरूको गुणोत्तर मध्यमा $\frac{1}{4}$ छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।**
The ratio of any two numbers is 1 : 16. If their geometric mean is $\frac{1}{4}$ then find the numbers.

⇨ Here, ratio of two numbers
i.e. a : b = 1 : 16; GM = $\frac{1}{4}$
$\therefore$ a = x and b = 16x
We know that,

$GM = \sqrt{ab}$
or, $GM^2 = ab$
or, $\left(\frac{1}{4}\right)^2 = x . 16x$
or, $\frac{1}{16} = x . 16x$
or, $x . 16x . 16 = 1$
or, $(16x)^2 = 1$
or, $(16x)^2 = (\pm 1)^2$
$\therefore$ $x = \pm\frac{1}{16}$

Thus, the numbers are $\frac{1}{16}$ and 1 or $-\frac{1}{16}$ and -1.

1.3.4 गुणोत्तर श्रेणीको योगफल (SUM OF GEOMETRIC SERIES)

EXERCISE 1.3.4

1. **निम्न लिखित श्रेणीहरूको योगफल निकाल्नुहोस् (Find the sum of folowing series) :**

(a) **1 + 3 + 9 + ... 7 ओटा पदहरू (1 + 3 + 5 + ...7 terms)**

⇨ Here, a = 1 and No. of terms = 7
We have, common ratio (r) $= \frac{t_2}{t_1} = \frac{3}{1} = 3$

So, sum of n terms $= \frac{a(r^n - 1)}{r - 1}$

or, Sum of 7 terms $= \frac{a(r^7 - 1)}{r - 1} = \frac{1\,((3)^7 - 1)}{3 - 1} = \frac{3^7 - 1}{3 - 1} = \frac{2186}{2}$

$\therefore$ $S_7 = 1093$
Thus, sum of first seven terms = 1093

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(b) **128 + 64 + 32 + ... 10 ओटा पदहरू**
128 + 64 + 32 + ... 10 terms

⇨ Here, $a = 128$,
No. of terms $(n) = 10$

We have, common ratio $(r) = \frac{t_2}{t_1} = \frac{64}{128} = \frac{1}{2}$

Now, sum of n terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

$$\text{or,}\quad S_{10} = \frac{128\left[\left(\frac{1}{2}\right)^{10} - 1\right]}{\frac{1}{2} - 1}$$

$$= \frac{128 \times \left(\frac{1}{1024} - 1\right)}{-\frac{1}{2}}$$

$$= 128 \times \left(-\frac{1023}{1024}\right) \times \left(-\frac{2}{1}\right)$$

$$\therefore\quad S_{10} = \frac{1023}{4}$$

Thus, the sum of 10 terms is $\frac{1023}{4}$.

(d) **3 + 6 + 12 + ... + 1536**

⇨ Here, first term $(a) = 3$ and

common ratio $(r) = \frac{t_2}{a}$

$$= \frac{6}{3}$$

$$= 2$$

We know that,

$$S_n = \frac{\ell r - a}{r - 1}$$

$$= \frac{1536 \times 2 - 3}{2 - 1}$$

$$= \frac{3069}{1}$$

$$= 3069$$

Thus, the sum of the series is 3069.

(c) $\mathbf{1 + \frac{1}{2} + \frac{1}{4} + \ldots}$ **8 ओटा पदहरू** $\mathbf{(1 + \frac{1}{2} + \frac{1}{4} + \ldots}$ **8 terms)**

⇨ Here, first term $(a) = 1$,

Common ratio $(r) = \frac{t_2}{t_1} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

We have, Sum of n terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

$$= \frac{1\left[\left(\frac{1}{2}\right)^{8} - 1\right]}{\frac{1}{2} - 1}$$

$$= \frac{1\left[\frac{1}{2^8} - 1\right]}{-\frac{1}{2}}$$

$$= 1 \times (-)\frac{255}{256}(-)\frac{2}{1}$$

$$= \frac{255}{128}$$

Thus, the sum of 8 terms is $\frac{255}{128}$.

(e) $\mathbf{\sqrt{2} - 2 + 2\sqrt{2} - \ldots + 64\sqrt{2}}$

⇨ Here, first term $(a) = \sqrt{2}$,
Second term $(t_2) = -2$
Last term $(\ell) = 64\sqrt{2}$

We know that,

Common ratio $(r) = \frac{t_2}{a} = -\frac{2}{\sqrt{2}} = \frac{-\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = -\sqrt{2}$

We have,

Sum of the series $(S_n) = \frac{\ell r - a}{r - 1}$

$$= \frac{64\sqrt{2} \times (-)\sqrt{2} - \sqrt{2}}{-\sqrt{2} - 1}$$

$$= \frac{-128 - \sqrt{2}}{-\sqrt{2} - 1}$$

$$= \frac{\sqrt{2} + 128}{\sqrt{2} + 1}$$

$$= 53.61$$

Thus, the sum of the series is 53.61.

(f) $\mathbf{\frac{1}{9} + \frac{2}{3} + 4 + \ldots + 24}$

⇨ Here, $\frac{1}{9} + \frac{2}{3} + 4$ .... $+ 24$

First term $(a) = \frac{1}{9}$

Common ratio $(r) = \frac{\frac{2}{3}}{\frac{1}{9}} = \frac{2}{3} \times \frac{9}{1} = 6$

Last term $(\ell) = 24$

Sum $(S_n) =$?

We have,

$$S_n = \frac{\ell r - a}{r - 1}$$

$$= \frac{24 \times 6 - 1/9}{6 - 1} = \frac{144 - 1/9}{5} = \frac{1296 - 1}{9 \times 5} = \frac{1295}{9 \times 5} = 28\,\frac{7}{9}$$

$$\therefore\quad S_n = 28\,\frac{7}{9}$$

Thus, the sum of n terms is $\frac{259}{9} = 28\frac{7}{9}$.

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2. मान निकाल्नुहोस् (Evaluate):

(a) $\sum_{n=2}^{6} 2^{3n}$

⇨ Here, $\sum_{n=2}^{6} 2^{3n}$

First term (a) = $2^{3\times2} = 2^6 = 64$

Second term $(t_2) = 2^{3\times3} = 2^9 = 512$

Last term $(\ell) = 2^{3\times6} = 2^{18} = 262144$

We have, Common ratio $(r) = \frac{t_2}{t_1} = \frac{512}{64} = 8$

No. of terms (n) = 6 – 1 = 5

Now, $S_n = \frac{\ell r - a}{r-1} = \frac{262144\times8-64}{8-1} = 299584$

Thus, the sum of the terms is 299584.

Alternative Method

$$S_n = 2^{3\times2} + 2^{3\times3} + 2^{3\times4} + 2^{3\times5} + 2^{3\times6}$$
$$= 2^6 + 2^9 + 2^{12} + 2^{15} + 2^{18}$$
$$= 2^6 (1 + 2^3 + 2^6 + 2^9 + 2^{12})$$
$$= 64 \times 4681$$
$$\therefore \quad S_n = 299584$$

(b) $\sum_{k=2}^{6} 2(-2)^k$

⇨ Here, first term (a) = $2(-2)^2 = 2\times4 = 8$

Second term $(t_2) = 2(-2)^3 = 2\times(-8) = -16$

Last term $(\ell) = 2(-2)^6 = 2\times64 = 128$

We have, common ratio $= \frac{t_2}{t_1} = \frac{-16}{8} = -2$

Now,

$$S_n = \frac{\ell r - a}{r-1} = \frac{128\times(-2)-8}{-2-1} = \frac{-256-8}{-3} = \frac{-264}{-3} = 88$$

Thus, sum of the terms is 88.

(c) $\sum_{m=1}^{5} (3^m + 2)$

⇨ Here, $\sum_{m=1}^{5} (3^m + 2)$

$$= (3^1+2) + (3^2+2) + (3^3+2) + (3^4+2) + (3^5+2)$$
$$= 5 + 11 + (27+2) + (81+2) + (243+2)$$
$$= 5 + 11 + 29 + 83 + 245$$
$$= 373$$

Thus, the sum of the series is 373.

3. एउटा गुणोत्तर श्रेणी 3 + 6 + 12 + ... + 768 भए

3 + 6 + 12 + ... + 768 is a geometric series.

(a) उक्त श्रेणीमा भएका पद सङ्ख्या निकाल्नुहोस् ।

Find the No. of terms of given series.

(b) उक्त श्रेणीको योगफल पत्ता लगाउनुहोस् ।

Find the sum of given series.

⇨ Here, first term (a) = 3

Second term $(t_2) = 6$

So, common ratio $(r) = \frac{t_2}{a} = \frac{6}{3} = 2$

We know that,

$$t_n = ar^{n-1}$$
$$\text{or,} \quad 768 = 3 \times 2^{n-1}$$
$$\text{or,} \quad 256 = 2^{n-1}$$
$$\text{or,} \quad 2^8 = 2^{n-1}$$
$$\text{or,} \quad n - 1 = 8$$
$$\therefore \quad n = 9$$

Again, sum of n terms $(S_n) = \frac{a(r^n-1)}{r-1}$

$$= \frac{3(2^9-1)}{2-1}$$
$$= \frac{3\times(512-1)}{1}$$
$$= 3 \times 511$$
$$= 1533$$

Thus, the sum of n terms is 1533.

4. (a) श्रेणी 64 + 96 + 144 + ... मा कति ओटा पदहरूको योगफल 1330 हुन्छ ?

In the series 64 + 96 + 144 + ..., the sum of how many terms is 1330?

⇨ Here, first term (a) = 64

Common ratio $(r) = \frac{t_2}{a} = \frac{96}{64} = \frac{3}{2}$

Sum $(S_n) = 1330$

We know that, $S_n = \frac{a(r^n-1)}{r-1}$

$$\text{or,} \quad 1330 = \frac{64\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2}-1}$$
$$\text{or,} \quad \frac{1330}{64} = \frac{\left(\frac{3}{2}\right)^n - 1}{\frac{1}{2}}$$
$$\text{or,} \quad \frac{665}{32} \times \frac{1}{2} = \left(\frac{3}{2}\right)^n - 1$$
$$\text{or,} \quad \frac{665}{64} + 1 = \left(\frac{3}{2}\right)^n$$
$$\text{or,} \quad \frac{729}{64} = \left(\frac{3}{2}\right)^n$$
$$\text{or,} \quad \left(\frac{3}{2}\right)^6 = \left(\frac{3}{2}\right)^n$$
$$\therefore \quad n = 6$$

Thus, the number of terms is 6.

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(b) श्रेणी 9 + 3 + 1 + ... मा कति ओटा पदहरूको योगफल $\frac{121}{9}$ हुन्छ ?

In the series 9 + 3 + 1 + ... , the sum of how many terms is $\frac{121}{9}$?

⇨ Here, first term (a) = 9

Second term $(t_2) = 3$

Sum of the series $(S_n) = \frac{121}{9}$

We know that,

Common ratio $(r) = \frac{t_2}{a} = \frac{3}{9} = \frac{1}{3}$

Now,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\text{or, } \frac{121}{9} = \frac{9\left\{\left(\frac{1}{3}\right)^n - 1\right\}}{\left(\frac{1}{3} - 1\right)}$$

$$\text{or, } \frac{121}{81} = \frac{\left(\frac{1}{3}\right)^n - 1}{-\frac{2}{3}}$$

$$\text{or, } \frac{121}{81} \times (-)\frac{2}{3} = \left(\frac{1}{3}\right)^n - 1$$

$$\text{or, } -\frac{242}{243} + 1 = \left(\frac{1}{3}\right)^n$$

$$\text{or, } \frac{1}{243} = \left(\frac{1}{3}\right)^n$$

$$\text{or, } \left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^n$$

$\therefore$ n = 5

Thus, the number of terms is 5.

5. (a) पहिलो पद 1 र समान अनुपात 2 भएको गुणोत्तर श्रेणीका पहिला पाँच पदहरूको योगफल निकाल्नुहोस् ।

Find the sum of first five terms of a geometric series whose first term is 1 and common ratio is 2.

⇨ Here, first term (a) = 1

Common ratio (r) = 2

Sum of first five terms $(S_5) = ?$

We have, $S_n = \frac{a(r^n - 1)}{r - 1}$

$$\text{or, } S_5 = \frac{a(r^5 - 1)}{r - 1} = \frac{1((2)^5 - 1)}{2 - 1} = \frac{1(32 - 1)}{1} = 31$$

Thus, sum of first five terms = 31

(b) पहिलो पद 3 र समान अनुपात $\frac{3}{5}$ भएको गुणोत्तर श्रेणीको आठौँ पदसम्मको योगफल निकाल्नुहोस् ।

Find the sum upto 8th term of a geometric series whose first term is 3 and common ratio is $\frac{3}{5}$.

⇨ Here, first term (a) = 3, common ratio $(r) = \frac{3}{5}$

Sum of 8 terms $(S_8) = ?$

We know that,

$$S_n = \frac{a(1 - r^n)}{1 - r} \qquad [\because r < 1]$$

$$= \frac{3\left\{1 - \left(\frac{3}{5}\right)^8\right\}}{1 - \frac{3}{5}}$$

$$= \frac{3\left(1 - \frac{6561}{390625}\right)}{\frac{2}{5}}$$

$$= 3 \times \frac{384064}{390625} \times \frac{5}{2}$$

$$= \frac{576096}{78125} = 7.374$$

Thus, the sum of the series is 7.374

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6. एउटा गुणोत्तर श्रेणीको पहिलो पद 3, अन्तिम पद 384 र तिनीहरूको योगफल 765 छ भने पद सङ्ख्या र समान अनुपात पत्ता लगाउनुहोस् ।

In a geometric series, first term is 3, last term is 384 and the sum is 765. Find the no. of terms and common ratio.

⇨ Here, first term (a) = 3, Last term $(\ell) = 384$

Sum of series $(S_n) = 765$

We know that, $S_n = \frac{\ell r - a}{r - 1}$

$$\text{or, } 765 = \frac{384r - 3}{r - 1}$$

or, 765r – 765 = 384r – 3

or, 381r = 762

$$\text{or, } r = \frac{762}{381} = 2$$

Again, last term = 384

or, $ar^{n-1} = 384$

or, $a\,(2)^{n-1} = 384$

or, $3\,(2)^{n-1} = 384$

or, $2^{n-1} = 128$

or, $2^{n-1} = 2^7$

or, n – 1 = 7

$\therefore$ n = 8

Thus, the no. of terms is 8 and the common ratio is 2.

7. (a) पहिलो पद 5 र अन्तिम पद 160 भएको गुणोत्तर श्रेणीको योगफल 315 भए समान अनुपात निकाल्नुहोस् ।

The sum of a geometric series is 315 whose first term is 5 and last term is 160. Find the common ratio.

⇨ Here, first term (a) = 5

Last term $(\ell) = 160$

Sum $(S_n) = 315$

Common ratio (r) = ?

We have,

$$S_n = \frac{a - r\ell}{1 - r}$$

$$\text{or, } 315 = \frac{5 - r \times 160}{1 - r}$$

or, 315 – 315r = 5 – 160r

or, – 315r + 160r = 5 – 315

or, – 155 r = – 310

$$\text{or, } r = \frac{310}{155}$$

$\therefore$ r = 2

$\therefore$ Common ratio = 2

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(b) **समान अनुपात 3 र अन्तिम पद 189 भएको गुणोत्तर श्रेणीको योगफल 280 भए पहिलो पद निकाल्नुहोस् ।**
The sum of a geometric series is 280 whose common ratio is 3 and last term is 189. Find the first term.

⇨ Here, Common ratio (r) = 3 Last term (ℓ) = 189

Sum (S_n) = 280 First term (a) = ?

We have, $S_n = \dfrac{a - r\ell}{1 - r}$

or, $280 = \dfrac{a - 3 \times 189}{1 - 3}$

or, $280 = \dfrac{a - 567}{-2}$

or, $-560 = a - 567$

or, $a = 567 - 560$

$\therefore$ $a = 7$

Thus, first term = 7

8. **(a)** **दोस्रो पद 3 र पाँचौं पद 81 भएको एउटा गुणोत्तर श्रेणीका पहिलो 7 ओटा पदहरूको योगफल निकाल्नुहोस् ।**
Find the sum of first seven terms of a geometric series whose 2nd term is 3 and 5th term is 81.

⇨ Here, second term (t_2) = 3

or, $ar = 3$ ....(i)

Fifth term (t_5) = 81

or, $ar^4 = 81$ ....(ii)

Dividing (ii) by (i) then, $\dfrac{ar^4}{ar} = \dfrac{81}{3}$

or, $r^3 = 27$

or, $r^3 = 3^3$

$\therefore$ $r = 3$

Again, from (i);

$ar = 3$

or, $a \times 3 = 3$

$\therefore$ $a = 1$

Now, Sum (S_n) = $\dfrac{a(r^n - 1)}{r - 1}$

So, $S_7 = \dfrac{a(r^7 - 1)}{r - 1}$

$= \dfrac{1(3^7 - 1)}{3 - 1}$

$= \dfrac{2187 - 1}{2}$

$= 1093$

Thus, the sum of 7 terms is 1093.

(b) **तेस्रो पद $\frac{1}{3}$ र छैटौं पद $\frac{1}{81}$ भएको गुणोत्तर श्रेणीका पहिला 6 ओटा पदहरूको योगफल निकाल्नुहोस् ।**

Find the sum of first six terms of a geometric series whose 3rd term is $\frac{1}{3}$ and 6th term is $\frac{1}{81}$.

⇨ Here, third term (t_3) = $\dfrac{1}{3}$

Sixth term (t_6) = $\dfrac{1}{81}$

Sum of 6 terms (S_6) = ?

We have,

$t_3 = ar^{3-1}$

or, $\dfrac{1}{3} = ar^2$ ....(1)

Again, we have,

$t_6 = ar^{6-1}$

or, $\dfrac{1}{81} = ar^5$ ....(2)

Dividing equation no (2) by equation (1)

or, $\dfrac{\frac{1}{81}}{\frac{1}{3}} = \dfrac{ar^5}{ar^2}$

or, $\dfrac{1}{81} \times \dfrac{3}{1} = r^3$

or, $\dfrac{1}{27} = r^3$

or, $\left(\dfrac{1}{3}\right)^3 = r^3$

$\therefore$ $r = \dfrac{1}{3}$

Substituting the value of r in equation no (1)

$\dfrac{1}{3} = a\left(\dfrac{1}{3}\right)^2$

or, $\dfrac{1}{3} = a \cdot \dfrac{1}{9}$

or, $a = \dfrac{1}{3} \times \dfrac{9}{1}$

$\therefore$ $a = 3$

Now ,

$S_6 = \dfrac{a - ar^6}{1 - r} = \dfrac{3 - 3 \cdot \left(\frac{1}{3}\right)^6}{1 - \frac{1}{3}}$

$= \dfrac{3 - 3 \times \frac{1}{729}}{\frac{3-1}{3}} = \dfrac{3 - \frac{1}{243}}{\frac{3-1}{3}}$

$= \dfrac{729 - 1}{243} \times \dfrac{3}{2}$

$= \dfrac{728}{81 \times 2}$

$= \dfrac{364}{81}$

$= 4.49$

$\therefore$ $S_n = 4.49$

Thus, the sum of first 6 terms is $\dfrac{364}{81}$.

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9. (a) **एउटा गुणोत्तर श्रेणीका पहिलो 2 ओटा पदहरूको योगफल 3 र पहिलो 4 ओटा पदहरूको योगफल 15 छ । यदि समान अनुपात धनात्मक भए पहिलो 6 ओटा पदहरूको योगफल निकाल्नुहोस् ।**

In a geometric series, the sum of first two terms is 3 and the sum of first four terms is 15. If the common ratio is +ve, find the sum of first six terms.

⇨ Here, sum of first two terms $(S_2) = 3$

Sum of first four terms $(S_4) = 15$

Sum of first six terms $(S_6) = ?$

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

So, $S_2 = \frac{a(r^2 - 1)}{r - 1}$ ....(i) and

$S_4 = \frac{a(r^4 - 1)}{r - 1}$ ....(ii)

Dividing equation (ii) by (i) then,

$$\frac{S_4}{S_2} = \frac{\frac{a(r^4 - 1)}{r - 1}}{\frac{a(r^2 - 1)}{r - 1}}$$

or, $\frac{15}{3} = \frac{r^4 - 1}{r^2 - 1}$

or, $5 = \frac{(r^2 - 1)(r^2 + 1)}{(r^2 - 1)}$

or, $5 = r^2 + 1$

or, $4 = r^2$

or, $r^2 = (\pm 2)^2$

$\therefore$ $r = 2$ [$\because$ By the questions r is positive.]

Putting r = 2 in (i), then

$$S_2 = \frac{a(r^2 - 1)}{(r - 1)}$$

$$= \frac{a(r - 1)(r + 1)}{(r - 1)}$$

or, $3 = a(r + 1)$

or, $3 = a(2 + 1)$

or, $3 = 3a$

$\therefore$ $a = 1$

Now, sum $(S_6) = \frac{a(r^n - 1)}{r - 1}$

$$= \frac{1(2^6 - 1)}{2 - 1}$$

$$= \frac{64 - 1}{1}$$

$$= 63$$

Thus, the sum of 6 terms is 63.

(b) **एउटा धनात्मक समान अनुपात भएको गुणोत्तर श्रेणीका पहिला चार पदहरूको योगफल 40 र पहिला दुई पदहरूको योगफल 4 छ भने सो श्रेणीका पहिला आठ पदहरूको योगफल निकाल्नुहोस् ।**

In a geometric series having +ve value of common ratio, the sum of first four terms is 40 and the sum of first two terms is 4. Find the sum of first eight terms of the series.

⇨ Here, sum of first 4 terms $(S_4) = 40$

Sum of first 2 terms $(S_2) = 4$

Sum of first 8 terms $(S_8) = ?$

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

So, $S_4 = \frac{a(r^4 - 1)}{r - 1}$ ....(i)

and $S_2 = \frac{a(r^2 - 1)}{r - 1}$ ....(ii)

Dividing (ii) by (i) then,

$$\frac{S_4}{S_2} = \frac{\frac{a(r^4 - 1)}{r - 1}}{\frac{a(r^2 - 1)}{r - 1}}$$

or, $\frac{40}{4} = \frac{(r^2 + 1)(r^2 - 1)}{r^2 - 1}$

or, $10 = r^2 + 1$

or, $r^2 = 9$

or, $r^2 = (\pm 3)^2$

$\therefore$ $r = 3$ [r is only positive by the question.]

From (ii);

$$S_2 = \frac{a(r^2 - 1)}{r - 1}$$

or, $4 = \frac{a(3^2 - 1)}{3 - 1}$

or, $4 = \frac{a \times 8}{2}$

$\therefore$ $a = 1$

Now,

$$S_8 = \frac{a(r^8 - 1)}{r - 1}$$

$$= \frac{1(3^8 - 1)}{3 - 1}$$

$$= \frac{6560}{2}$$

$$= 3280$$

Thus, the sum is 3280.

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5. रेखीय योजना
Linear Programming

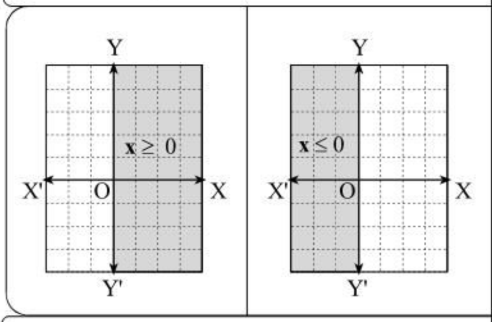
Formulae and Key Points

1. $ax + by \leq c$ र $ax + by \geq c$ को सीमारेखाको समीकरण $ax + by = c$ हुन्छ ।
   The equation of boundary line of $ax + by \leq c$ and $ax + by \geq c$ is $ax + by = c$.

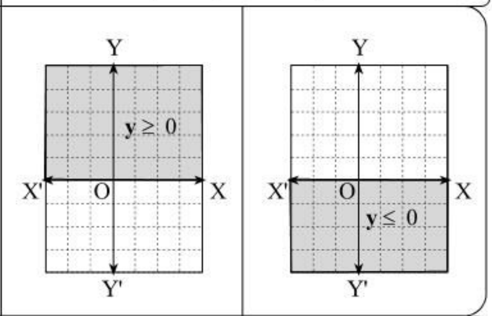
2. $ax + by \leq c$ र $ax + by \geq c$ को हल क्षेत्रहरू परीक्षण बिन्दु लिएर पहिचान गर्नुपर्दछ ।
   The solution sets of $ax + by \leq c$ and $ax + by \geq c$ should be identified by taking a test point.

3. हल क्षेत्र (Solution Set)

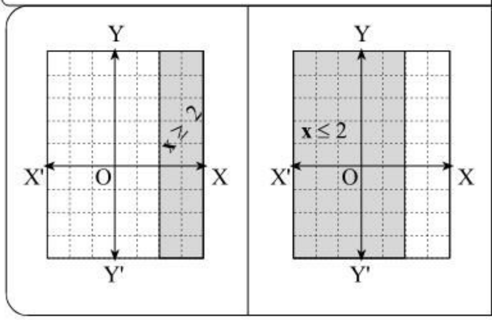
$x \geq 0$ र $x \leq 0$ को लेखाचित्र
Graph of $x \geq 0$ and $x \leq 0$



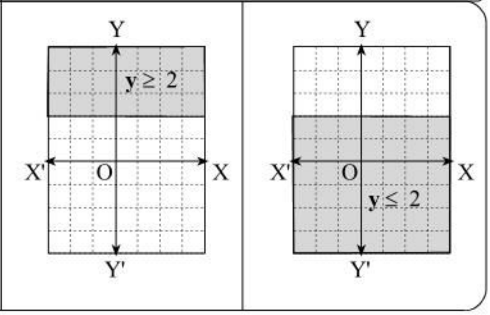
$y \geq 0$ र $y \leq 0$ को लेखाचित्र
Graph of $y \geq 0$ and $y \leq 0$



$x \geq 2$ र $x \leq 2$ को लेखाचित्र
Graph of $x \geq 2$ and $x \leq 2$



$y \geq 2$ र $y \leq 2$ को लेखाचित्र
Graph of $y \geq 2$ and $y \leq 2$



4. **मुख्य बुदाँहरू (Key Points)**

असमानताहरू (Inequalities)	हल क्षेत्र (Solution Set)
$x \geq 0$ and $y \geq 0$	पहिलो चतुर्थांश (First quadrant)
$x \leq 0$ and $y \geq 0$	दोस्रो चतुर्थांश (Second quadrant)
$x \leq 0$ and $y \leq 0$	तेस्रो चतुर्थांश (Third quadrant)
$x \geq 0$ and $y \leq 0$	चौथो चतुर्थांश (Fourth quadrant)
$x \geq k$	रेखा $x = k$ देखि धनात्मक दिशा (Positive direction from the line $x = k$)
$y \geq k$	रेखा $y = k$ देखि धनात्मक दिशा (Positive direction from the line $y = k$)
$x \leq k$	रेखा $x = k$ देखि ऋणात्मक दिशा (Negative direction from the line $x = k$)
$y \leq k$	रेखा $y = k$ देखि ऋणात्मक दिशा (Negative direction from the line $y = k$)

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QUESTIONS FROM SEE EXERCISE 5

A. VERY SHORT QUESTIONS

1. समीकरण को परिभाषा दिनुहोस् । (Define Equation.)

⇨ Here, a mathematical statement containing the equal sign (=) is called an equation.

2. असमानता को परिभाषा दिनुहोस् । (Define Inequality.)

⇨ Here, a mathematical statement containing one of the signs ($<$, $>$, $\leq$ or $\geq$) is called an inequality or inequation.

3. सीमा रेखा को परिभाषा दिनुहोस् । (Define Boundary line.)

⇨ Here, the line which is the corresponding equation of given inequality and it divides the co-ordinate graph (plane) into two half planes is called the boundary line.

4. सर्तहरूको परिभाषा दिनुहोस् । (Define Constraints.)

⇨ Here, the conditions to be satisfied by the variables of the objective function are said to be constraints. They are the conditions related to inequalities.

5. रेखीय योजना को परिभाषा दिनुहोस् । (Define Linear programming.)

⇨ Here, linear programming is the mathematical technique for finding the maximum or minimum value of a linear function subject to the set of linear constraints.

6. उद्देश्य फलन को परिभाषा दिनुहोस् । (Define Objective function.)

⇨ Here, the linear function which is to be maximized or minimized with respect to given constraints is called objective function.

7. रेखीय असमानता प्रणालीको परिभाषा दिनुहोस् । (Define System of linear inequalities.)

⇨ Here, linear inequality is a relation represented by $ax + by + c \geq 0$, $ax + by + c \leq 0$, $ax + by + c > 0$, $ax + by + c < 0$, (i.e. $ax + by + c \neq 0$).

8. लेखाचित्रमा असमानता $x \geq 0$ ले कुन कुन चतुर्थांशहरू जनाउँदछ ?
Which quadrants do the inequality $x \geq 0$ represent in a graph?

⇨ Here, the inequality $x \geq 0$ represents first and fourth quadrants.

9. लेखाचित्रमा असमानता $y \geq 0$ ले कुन कुन चतुर्थांशहरू जनाउँदछ ?
Which quadrants do the inequality $y \geq 0$ represent in a graph?

⇨ Here, the inequality $y \geq 0$ represents first and second quadrants.

10. असमानता $ax + by \leq c$ को सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $ax + by \leq c$.

⇨ Here, the equation of boundary line of the inequality $ax + by \leq c$ is $ax + by = c$.

11. असमानता $ax + by \geq c$ को सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $ax + by \geq c$.

⇨ Here, the equation of boundary line of the inequality $ax + by \geq c$ is $ax + by = c$.

12. असमानता $2x + 3y \leq 6$ को सङ्गति सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $2x + 3y \leq 6$.

⇨ Here, the equation of the boundary line of $2x + 3y \leq 6$ is $2x + 3y = 6$.

13. असमानता $2x + y > 10$ को सङ्गति सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $2x + y > 10$.

⇨ Here, the equation of the boundary line of $2x + y > 10$ is $2x + y = 10$.

14. $x \geq 2$ को हल क्षेत्र रेखा $x = 2$ बाट कता पट्टी पर्छ ? (In which side from the line $x = 2$ is the solution region of $x \geq 2$ lie?)

⇨ Here, the right side inclusive the line $x = 2$ is the solution set of $x \geq 2$.

15. $y \leq 5$ को हल क्षेत्र रेखा $y = 5$ बाट कता पट्टी पर्छ ? (In which side from the line $y = 5$ is the solution region of $y \leq 5$ lie?)

⇨ Here, downward side inclusive the line $y = 5$ is the solution set of $y \leq 5$.

16. के असमानता $2x + 3y \geq 6$ को हल समूहमा (2, 3) पर्दछ ? (Does (2, 3) lie on the solution set of inequality $2x + 3y \geq 6$?)

⇨ Here, $2x + 3y = 2 \times 2 + 3 \times 3 = 13 > 6$. So, (2, 3) lies in $2x + 3y \geq 6$.

17. के असमानता $3x + 2y \geq 8$ को हल समूहमा (0, 0) पर्दछ ? (Does (0, 0) lie on the solution set of inequality $3x + 2y \geq 8$?)

⇨ Here, $3x + 2y = 3 \times 0 + 2 \times 0 = 0 \geq 8$(false) . So, (0, 0) doesnot lie in $3x + 2y \geq 8$.

18. सीमा रेखाको समीकरण $2x + 3y = 6$ र हल समूहमा बिन्दु (0, 0) भएको असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $2x + 3y = 6$ and the solution set contains (0, 0).

⇨ Here, $2x + 3y = 2 \times 0 + 3 \times 0 = 0 < 6$ $\therefore$ $2x + 3y \leq 6$ is the required inequality.

19. सीमा रेखाको समीकरण $7x + 5y = 20$ र हल समूहमा बिन्दु (0, 0) नहुने असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $7x + 5y = 20$ and the solution set does not contain (0, 0).

⇨ Here, $7x + 5y = 7 \times 0 + 5 \times 0 = 0 \geq 20$ (for not containing origin)

$\therefore$ $7x + 5y \geq 20$ is the required inequality.

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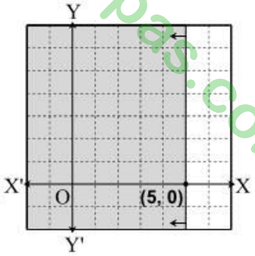
B. SHORT QUESTIONS

1. असमानता $2x + 5 \leq 15$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :

Solve the inequality $2x + 5 \leq 15$ and show the values in graph by shading:



⇨ Here, $2x + 5 \leq 15$

or, $2x \leq 15 - 5$

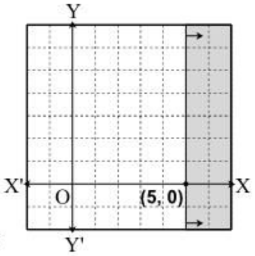
or, $2x \leq 10$

$\therefore$ $x \leq 5$

The boundary line of this inequality is x = 5. The region represented by the given inequality is shown in the graph. The solution is shown by the shaded portion.

2. असमानता $(2x - 1) \geq \frac{1}{2}(x + 13)$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :

Solve the inequality $(2x - 1) \geq \frac{1}{2}(x + 13)$ and show the values in graph by shading:



⇨ Here, $2x - 1 \geq \frac{1}{2}(x + 13)$

or, $4x - 2 \geq x + 13$

or, $4x - x \geq 13 + 2$

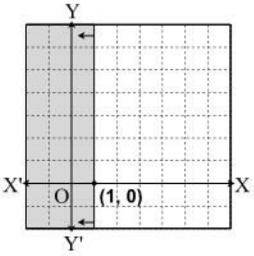
or, $3x \geq 15$

$\therefore$ $x \geq 5$

This inequality contains the straight line x = 5 and all the plane region on the right side of this line which is shown by the shaded region on the graph.

3. असमानता $\frac{1}{5}(2x + 3) \leq \frac{1}{3}(x + 2)$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :

Solve the inequality $\frac{1}{5}(2x + 3) \leq \frac{1}{3}(x + 2)$ and show the values in graph by shading:



⇨ Here, Given inequality is $\frac{1}{5}(2x + 3) \leq \frac{1}{3}(x + 2)$

or, $3(2x + 3) \leq 5(x + 2)$

or, $6x + 9 \leq 5x + 10$

or, $6x - 5x \leq 10 - 9$

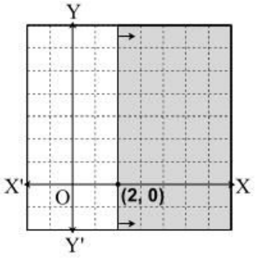
or, $x \leq 1$

Presentation of solution of the inequality on graph.

Here, the boundary line is x = 1

4. असमानता $(2x + 4) \geq \frac{1}{2}(x + 14)$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :

Solve the inequality $(2x + 4) \geq \frac{1}{2}(x + 14)$ and show the values in graph by shading:



⇨ Here, $(2x + 4) \geq \frac{1}{2}(x + 14)$

or, $2(2x + 4) \geq (x + 14)$

or, $4x + 8 \geq x + 14$

or, $4x - x \geq 14 - 8$

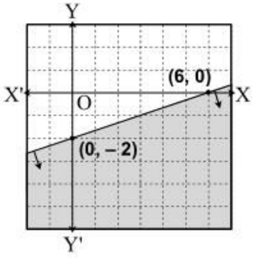
or, $3x \geq 6$

$\therefore$ $x \geq 2$

The boundary line of this inequality is x = 2, which is the line parallel to y - axis and passing through the point (2, 0) on x - axis. Since $x \geq 2$, the half plane of the inequality lies to the right side of the boundary line. The graph is shown alongside.

5. असमानता $\frac{1}{2}(x - y) - \frac{1}{3}x \geq 1$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :

Solve the inequality $\frac{1}{2}(x - y) - \frac{1}{3}x \geq 1$ and show the values in graph by shading:



⇨ Here, $\frac{1}{2}(x - y) - \frac{1}{3}x \geq 1$

or, $x - y - \frac{2}{3}x \geq 2$ [∵ Multiplying both sides by 2.]

or, $3x - 3y - 2x \geq 6$ [∵ Multiplying both sides by 3.]

$\therefore$ $x - 3y \geq 6$

The boundary line of this inequality is $x - 3y = 6$

x	0	6	– 3
y	– 2	0	– 3

If we put (0, 0) on $x - 3y \geq 6$, we get $0 - 0 \geq 6$ which is false.

Therefore, the half plane determined by the given inequality does not contain the origin. The graph of the inequality is shown alongside.

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6. **असमानता $\frac{3x+1}{2} \le \frac{2x-1}{3}$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :**

Solve the inequality $\frac{3x+1}{2} \le \frac{2x-1}{3}$ and show the values in graph by shading:

⇨ Here, $\frac{3x+1}{2} \le \frac{2x-1}{3}$

or, $3(3x+1) \le 2(2x-1)$

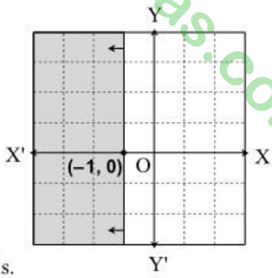
or, $9x + 3 \le 4x - 2$

or, $9x - 4x \le -2 - 3$

or, $5x \le -5$

$\therefore$ $x \le -1$

The boundary line of this inequality is x = – 1, which is a line parallel to y-axis. This inequality represents the plane to the left side of the boundary line which is shown by shading in the graph.



7. **असमानता $\frac{4x+1}{3} \ge \frac{3x-1}{2}$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :**

Solve the inequality $\frac{4x+1}{3} \ge \frac{3x-1}{2}$ and show the values in graph by shading:

⇨ Here, $\frac{4x+1}{3} \ge \frac{3x-1}{2}$

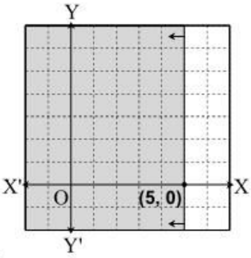
or, $8x + 2 \ge 9x - 3$

or, $8x - 9x \ge -3 - 2$

or, $-x \ge -5$

$\therefore$ $x \le 5$

The boundary line of this inequality is x = 5. This line is parallel to y-axis and meets x-axis at (5, 0). The half plane determined by the inequality lies on the left side of the boundary line which is shown by shading the region in graph.



8. **असमानता $y \le x + 5$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :**

Solve the inequality $y \le x + 5$ and show the values in graph by shading:

⇨ Here, the boundary line of given inequality is the equation, $y = x + 5$

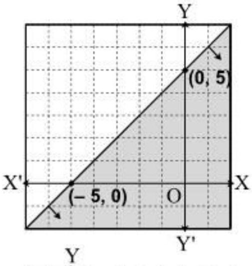
x	0	–5	–1	–3
y	5	0	4	2

Let us choose origin 'i.e. (0, 0) as the test point.

If we put x = 0 & y = 0 in $y \le x + 5$,

We get $0 \le 0 + 5$, or $0 \le 5$ which is true.

So the half plane contains the origin which is shown in the graph.



9. **असमानता $2x + y \ge 7$ हल गरी मानहरू लेखाचित्रमा छाया पारी देखाउनुहोस् :**

Solve the inequality $2x + y \ge 7$ and show the values in graph by shading:

⇨ Here, the boundary line of the given inequality is $2x + y = 7$.

or, $y = 7 - 2x$

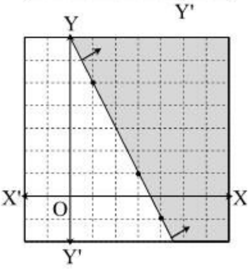
The table for this line is given as below :

x	1	4	3	– 1
y	5	–1	1	9

Let us take the origin i.e. (0, 0) as test point. If we put x = 0 & y = 0 in the given inequality, we get

$2 \times 0 + 0 \ge 7$, or, $0 \ge 7$, which is false.

So, the inequality represents the region in which the origin does not lie.



10. **दिइएको चित्रमा छाया पारेको भाग जनाउने असमानता पत्ता लगाउनुहोस् ।**

Find the inequality that represents the shaded region in the given figure.

⇨ Here, From the given figure,

X-intercept (a) = 3 and Y-intercept (b) = 5

We know that,

The equation of line in double intercepts form is;

$$\frac{x}{a} + \frac{y}{b} = 1$$

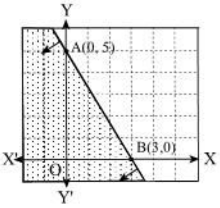
or, $\frac{x}{3} + \frac{y}{5} = 1$

or, $\frac{5x + 3y}{15} = 1$

$\therefore$ $5x + 3y = 15$

Let (0, 0) be the testing point. Since $5(0) + 3(0) = 0 \le 15$,

Thus, the required inequality is $5x + 3y \le 15$.



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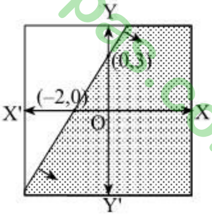
11. दिइएको चित्रमा छाया पारेको भाग जनाउने असमानता पत्ता लगाउनुहोस् ।
Find the inequality that represents the shaded region in the given figure.



⇨ Here, From the given figure,
X-intercept (a) = – 2 and Y-intercept (b) = 3
We know that,
The equation of line in double intercepts form is;

$$\frac{x}{a}+\frac{y}{b}=1$$

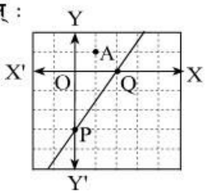
or, $\frac{x}{-2}+\frac{y}{3}=1$

or, $\frac{-3x+2y}{6}=1$

$\therefore\ -3x+2y=6$

Let (0, 0) be the testing point. Since $-3(0)+2(0)=0<6$,
Thus, the required inequality is $-3x+2y\le 6$.

12. रेखा PQ लाई प्रसङ्गको रेखामानी बिन्दु A सन्तुलन गर्ने अर्धसमतलको असमानता पत्ता लगाउनुहोस् :
Find the inequality of half-plane that satisfies the point A with respect to the line PQ:



⇨ Here, From the given figure, the line PQ has;
X-intercept (a) = 2 and Y-intercept (b) = – 3
We know that,
The equation of line in double intercepts form is;

$$\frac{x}{a}+\frac{y}{b}=1$$

or, $\frac{x}{2}+\frac{y}{-3}=1$

or, $\frac{3x-2y}{6}=1$

$\therefore\ 3x-2y=6$

Let (0, 0) be the testing point. Since $3(0)-2(0)=0<6$,
Thus, the required inequality is $3x-2y\le 6$.

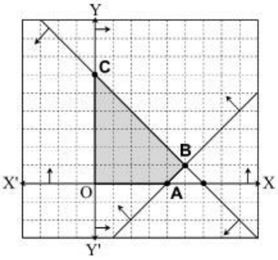
C. LONG QUESTIONS

MODEL 1

1. Z = 10x + 12y को निम्न लिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।
Find the maximum value of Z = 10x + 12y under the following constraints: $x+y\le 6$, $x-y\le 4$, $x\ge 0$ and $y\ge 0$ **[2075 R'$_2$]**

⇨ Here, given constraints are, $x+y\le 6$, $x-y\le 4$, $x\ge 0$ and $y\ge 0$
The boundary line of $x+y\le 6$ is,
$x+y=6$

x	0	6	2	4	3	5
y	6	0	4	2	3	1



Let, (0, 0) be the test point then $x+y<6$
or, $0+0<6$ or, $0<6$ (true)
So, the boundary line of $x+y\le 6$ lies towards the origin.
The boundary line of $x-y\le 4$ is,
$x-y=4$

x	4	0	5	3
y	0	– 4	1	– 1

Let (0, 0) be the test point then $x-y<4$
or, $0-0<4$
or, $0<4$ (true)
So, the boundary line of $x-y\le 4$ lies towards the origin.
$(x\ge 0)\cap(y\ge 0)$ represents the first quadrant.
From the graph, OABC is the required polygonal region where the coordinates of vertices are O(0, 0), C(0, 6), B(5, 1) and A(4, 0)

Vertices	x	y	$Z_{max}=10x+12y$
O(0, 0)	0	0	$10\times 0+12\times 0=0$
C(0, 6)	0	6	$10\times 0+12\times 6=72$
B(5, 1)	5	1	$10\times 5+12\times 1=62$
A(4, 0)	4	0	$10\times 4+12\times 0=40$

Thus, the maximum value of Z is 72 at (0, 6).

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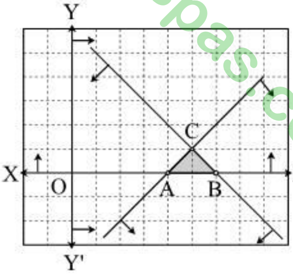
2. उद्देश्यफलन P = 3x + 2y को निम्न लिखित अवस्थाहरूमा अधिकतम मान निकाल्नुहोस् ।
Maximize the objective function P = 3x + 2y under the following constraints.
$x + y \le 6$, $x - y \ge 4$, $x \ge 0$ and $y \ge 0$ **[2074 S]**



⇨ Here, $x + y \le 6$, $x - y \ge 4$, $x \ge 0$, $y \ge 0$

The corresponding equation of $x + y \le 6$ is $x + y = 6$

x	0	6
y	6	0

(0, 6), (6, 0)

Let (0, 0) be the test point then,

$x + y \le 6$

or, $0 + 0 \le 6$

∴ $0 < 6$ [True]

The corresponding equation of $x - y \ge 4$ is $x - y = 4$

x	0	4
y	– 4	0

(0, – 4), (4, 0)

Let, (0, 0) be the test point then,

$x - y \ge 4$

or, $0 - 0 \ge 4$

$0 > 4$ [False]

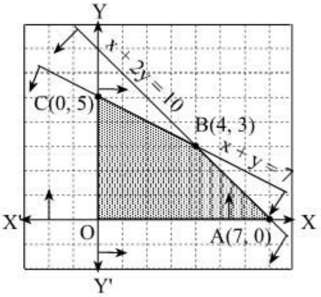
$(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

From the graph, the vertices of the feasible region are: A(4, 0), B(6, 0) and C(5, 1).

Vertices	Objective function: P = 3x + 2y	Remarks
A(4, 0)	$3 \times 4 + 2 \times 0 = 12$	Minimum
B(6, 0)	$3 \times 6 + 2 \times 0 = 18$	Maximum
C(5, 1)	$3 \times 5 + 2 \times 1 = 17$	

Thus, the maximum value is 18 at (6, 0).

3. तलका सर्तहरू मान्य हुने गरी उद्देश्य फलन P = 7x + 3y को अधिकतम मान निर्धारण गर्नुहोस् ।
Maximize the objective function P = 7x + 3y under the following constraints. **[2072 R]**
$x + y \le 7$, $x + 2y \le 10$, $x \ge 0$ and $y \ge 0$



⇨ Here, The corresponding equation of line $x + y \le 7$ is $x + y = 7$.

x	0	7
y	7	0

Let (0, 0) be the test point then,

$x + y \le 7$

or, $0 + 0 \le 7$

∴ $0 < 7$ (True)

Again, the corresponding equation of $x + 2y \le 10$ is $x + 2y = 10$

x	0	10
y	5	0

Let (0, 0) be the test point then, $x + 2y \le 10$

or, $0 + 0 \le 10$

or, $0 < 10$ (True)

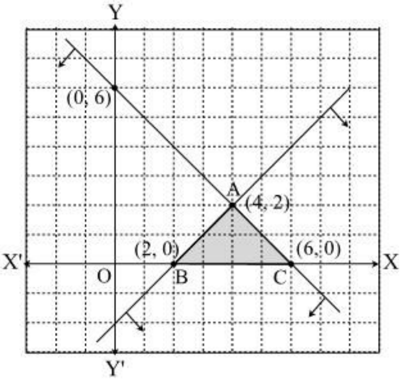
We have, $(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

From the graph the coordinates of vertices of feasible region are; O(0, 0), C(0, 5), B(4, 3) and A(7, 0).

Vertices	P = 7x + 3y	Value	Remarks
O(0, 0)	$P = 7 \times 0 + 3 \times 0$	0	
C(0, 5)	$P = 7 \times 0 + 3 \times 5$	15	
B(4, 3)	$P = 7 \times 4 + 3 \times 3$	37	
A(7, 0)	$P = 7 \times 7 + 3 \times 0$	49	Maximum

Thus, the maximum value of P = 7x + 3y is 49 at the point A(7, 0).

4. तलका सर्तहरू मान्य हुने गरी उद्देश्य फलन P = 3x + 5y को अधिकतम मान निर्धारण गर्नुहोस् : $x + y \le 6$, $x - y \ge 2$, $x \ge 0$ and $y \ge 0$
Maximize the objective function P = 3x + 5y under the following constraints: $x + y \le 6$, $x - y \ge 2$, $x \ge 0$ and $y \ge 0$ **[2072 S]**



⇨ Here, The corresponding equation of $x + y \le 6$ is $x + y = 6$

x	0	6
y	6	0

(0, 6) (6, 0)

Let (0, 0) be the test point then; $x + y \le 6$

or, $0 + 0 \le 6$

∴ $0 < 6$ (True)

The corresponding equation of $x - y \ge 2$ is $x - 2 = 2$

x	2	0
y	0	– 2

(2, 0), (0, – 2)

Let (0, 0) be a test point then, $x - y \ge 2$

or, $0 - 0 \ge 2$

or, $0 \ge 2$ (False)

∴ $(x \ge 0)$ and $(y \ge 0)$ represents the first quadrant.

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From the graph, the feasible region is a triangle ABC with co-ordinates A(4, 2), B(2, 0) and C(6, 0).

Now,

Vertices	P = 3x + 5y	Value	Remarks
(4, 2)	$3 \times 4 + 5 \times 2$	22	Maximum
(2, 0)	$3 \times 2 + 5 \times 0$	6	
(6, 0)	$3 \times 6 + 5 \times 0$	18	

Thus, from the table the maximum value is 22 at (4, 2).

5. दिइएका शर्तहरू $2x + y \le 8$, $2x + 4y \le 14$, $x \ge 0$ र $y \ge 0$ पुरा गरी उद्देश्य फलन $Z = 6x + 5y$ को अधिकतम् मान निकाल्नुहोस् ।

Maximize the objective function $Z = 6x + 5y$ under the given conditions $2x + y \le 8$, $2x + 4y \le 14$, $x \ge 0$ and $y \ge 0$.

[2070 R']

⇨ Here, given constraints; $2x + y \le 8$, $2x + 4y \le 14$, $x \ge 0$ and $y \ge 0$

The corresponding equation of $2x + y \le 8$ is $2x + y = 8$.

x	0	4
y	8	0

Let (0, 0) be the test point then, $2x + y \le 8$

or, $2 \times 0 + 0 \le 8$

∴ $0 < 8$ (True)

The corresponding equation of $2x + 4y \le 14$ is $2x + 4y = 14$.

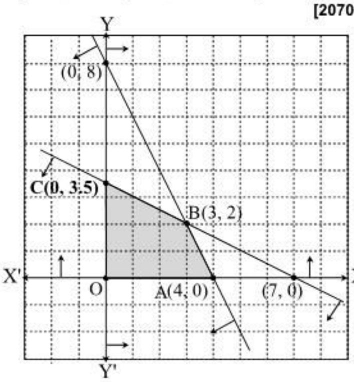
x	1	7
y	3	0

Let (0, 0) be the test point then, $2x + 4y \le 14$

or, $2 \times 0 + 4 \times 0 \le 14$

or, $0 < 14$ (True)

$(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.



From the graph the vertices of feasible region are; O(0, 0), A(4, 0), B(3, 2), C(0, 3.5).

Vertices	Z = 6x + 5y	Remarks
O(0, 0)	$Z = 6 \times 0 + 5 \times 0 = 0$	
A(4, 0)	$Z = 6 \times 4 + 5 \times 0 = 24$	
B(3, 2)	$Z = 6 \times 3 + 5 \times 2 = 28$	Maximum
C(0, 3.5)	$Z = 6 \times 0 + 5 \times 3.5 = 17.5$	

Thus, the maximum value is 28 at (3, 2).

6. उद्देश्य फलन $P = 6x + 5y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + y \le 6$, $x - y \ge -2$, $x \ge 0$, $y \ge 0$

Maximize $P = 6x + 5y$ under the following constraints: $x + y \le 6$, $x - y \ge -2$, $x \ge 0$, $y \ge 0$ **[2057 R, 2065 E]**

⇨ Here, given inequalities are: $x + y \le 6$, $x - y \ge -2$, $x \ge 0$, $y \ge 0$

The boundary lines of the above first two inequalities are:

$x + y = 6$ .... (i) $x - y = -2$ .... (ii)

From (i); $x + y = 6$

x	0	6
y	6	0

∴ The line (i) passes through the points (0, 6) and (6, 0).

Let (0, 0) be the test point,

$x + y < 6$

or, $0 + 0 < 6$

∴ $0 < 6$ (True)

From (ii) $x - y = -2$

x	0	– 2
y	2	0

∴ The line (ii) passes through the points (0, 2) and (– 2, 0).

Let (0, 0) be the test point,

$x - y > -2$

or, $0 - 0 > -2$

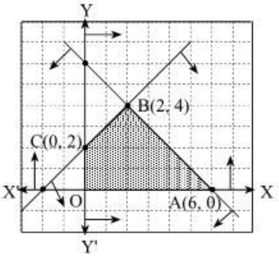
∴ $0 > -2$ (True)

∴ $(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

The graph of the above inequalities is as shown alongside

Here the shaded region OABC is the solution set of the given inequalities.

So the vertices of the shaded region are. O(0,0), A(6, 0) B(2, 4) and C(0, 2).



Points (x, y)	P(x, y) = 6x + 5y	Value	Remarks
O(0, 0)	$6 \times 0 + 5 \times 0$	0	
A(6, 0)	$6 \times 6 + 5 \times 0$	36	Maximum
B(2, 4)	$6 \times 2 + 5 \times 4$	32	
C(0, 2)	$6 \times 0 + 5 \times 2$	10	

Thus, the maximum value is 36 at the point A(6, 0).

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7. उद्देश्य फलन P = 9x + 7y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \le 7$, $x - y \ge 4$, $x \ge 0$, $y \ge 0$
Maximize P = 9x + 7y under the following constraints: $x + 2y \le 7$, $x - y \ge 4$, $x \ge 0$, $y \ge 0$ **[2057 S]**

⇨ Here given inequalities are $x + 2y \le 7$, $x - y \ge 4$, $x \ge 0$ and $y \ge 0$

The boundary line of $x + 2y \le 7$ is

$x + 2y = 7$

x	1	3
y	3	2

So, the boundary line passes through (1, 3) and (3, 2).

Let (0, 0) be the test point in

$x + 2y < 7$

or, $0 + 0 < 7$

∴ $0 < 7$ (True) so, boundary line is toward origin.

The boundary line of $x - y \ge 4$ is;

$x - y = 4$

x	0	4
y	– 4	0

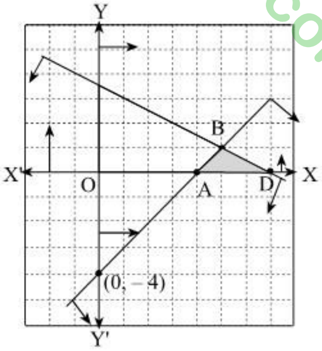
So the boundary line passes through (0, – 4) and (4, 0).

Let (0, 0) be the test point in

$x - y > 4$,

or, $0 - 0 > 4$

∴ $0 > 4$ (False) so, boundary line is opposite to the origin.



∴ $(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

Now, the graph of the boundary lines of the given inequalities is as shown.

Here, the shaded region ADB is a solution region.

The vertices of the regions are: A(4, 0), D(7, 0) and B(5, 1)

Points (x, y)	P(x, y) = 9x + 7y	Value	Remarks
A(4, 0)	$9 \times 4 + 7 \times 0$	36	
B(5, 1)	$9 \times 5 + 7 \times 1$	52	
D(7, 0)	$9 \times 7 + 7 \times 0$	63	Maximum

Thus, the maximum value of P = 9x + 7y is 63 at D(7, 0).

8. उद्देश्य फलन P = 3x + y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2y \ge x - 1$, $x + y \le 4$; $x \ge 0$, $y \ge 0$
Maximize P = 3x + y under the following constraints: $2y \ge x - 1$, $x + y \le 4$; $x \ge 0$, $y \ge 0$ **[2058 R]**

⇨ Here, given inequalities, $2y \ge x - 1$, $x + y \le 4$, $x \ge 0$, $y \ge 0$

The boundary lines of the above inequalities are

$2y = x - 1$ ...... (i), $x + y = 4$ ...... (ii), $x = 0$ .... (iii), $y = 0$ .... (iv)

From (i) $2y = x - 1$

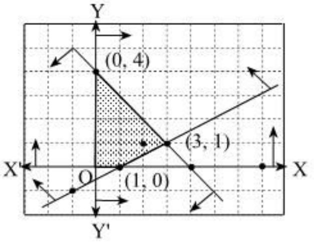
x	– 1	1
y	– 1	0

(– 1,– 1) and (1, 0).

From (ii) $x + y = 4$

x	0	4
y	4	0

(0, 4) and (4, 0).



Taking (0, 0) as the testing point for inequalities 1st and 2nd, we get $0 > -1$ and $0 < 4$ respectively.

So, the inequalities 1st and 2nd contain the origin.

∴ $(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

Now, drawing the graph of the given inequalities as shown.

Here, the shadded region OABC is the solution region.

The coordinates of the shadded region OABC are O(0, 0), A(1, 0), B(3, 1) and C(0, 4)

Points (x, y)	P(x, y) = 3x + y	Value	Remarks
O(0, 0)	$3 \times 0 + 0$	0	
A(1, 0)	$3 \times 1 + 0$	3	
B(3, 1)	$3 \times 3 + 1$	10	Maximum
C(0, 4)	$3 \times 0 + 4$	4	

Thus, the maximum value of P = 3x + y is 10 at B(3, 1).

9. उद्देश्य फलन F = 3x + 5y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x - 2y \le 1$, $x + y \le 4$, $x \ge 0$, $y \ge 0$
Maximize F = 3x + 5y under the following constraints: $x - 2y \le 1$, $x + y \le 4$, $x \ge 0$, $y \ge 0$ **[2067 R]**

⇨ Here, given inequalities are; $x - 2y \le 1$ .... (i) $x + y \le 4$ .... (ii)

$x \ge 0$, $y \ge 0$

Now, equation of boundary line from (i) inequality

$x - 2y = 1$

x	1	3	– 1
y	0	1	– 1

Let (0, 0) be a testing point

$0 - 2.0 < 1$

or, $0 < 1$ (T)

So, (0, 0) lies in shaded region

Equation of boundary line

From (ii) inequality

$x + y = 4$

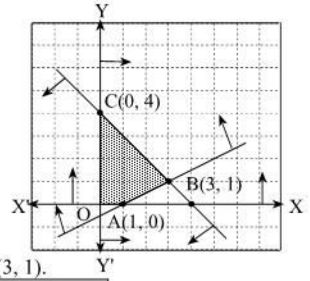
x	1	4	2
y	3	0	2

Let (0, 0) be a testing point

$0 + 0 < 4$

or, $0 < 4$ (T)

So, (0, 0) lies in shaded region.



Then, from graph paper vertices of the shaded region (0, 4), (1, 0), (0, 0) and (3, 1).

Vertices	X-com	Y-com	F = 3x + 5y	Result
(0, 4)	0	4	F = 0 + 20 = 20	Maximum
(1, 0)	1	0	F = 3 + 0 = 3	
(0, 0)	0	0	F = 0 + 0 = 0	
(3, 1)	3	1	F = 9 + 5 = 14	

Thus, 20 is a maximum value at point (0, 4).

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10. उद्देश्य फलन $P = 5x + 6y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \leq 6$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$
Maximize $P = 5x + 6y$ under the following constraints: $x + 2y \leq 6$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$ **[2061 S]**

⇨ Here, given inequalities are : $x + 2y \leq 6$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$

The boundary lines of the above inequalities are :

$x + 2y = 6$ .... (i) $\quad$ $3x + 2y = 12$ .... (ii)
$x = 0$ .... (iii) and $\quad$ $y = 0$ .... (iv)

From (i); $x + 2y = 6$

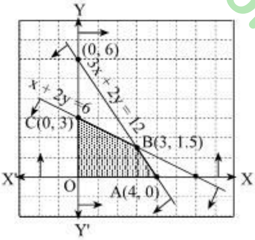
x	0	6
y	3	0

Let (0, 0) be the test point,
$x + 2y < 6$
or, $0 + 0 < 6$
$\therefore$ $0 < 6$ (True)

From (ii) $3x + 2y = 12$

x	0	4
y	6	0

Let (0, 0) be the test point,
$3x + 2y < 12$
or, $3 \times 0 + 2 \times 0 < 12$
$\therefore$ $0 < 12$ (True)



Here, the shaded region OABC is a solution region.

The coordinates of the vertices of the region OABC are O(0, 0), A(4, 0), $B\left(3, \frac{3}{2}\right)$ and C(0, 3)

Points (x, y)	P(x, y) = 5x + 6y	Value	Remarks
O(0, 0)	$5 \times 0 + 6 \times 0$	0	
A(4, 0)	$5 \times 4 + 6 \times 0$	20	
$B\left(3, \frac{3}{2}\right)$	$5 \times 3 + 6 \times \frac{3}{2}$	24	Maximum
C(0, 3)	$5 \times 0 + 6 \times 3$	18	

Thus, the maximum value is 24 at $B\left(3, \frac{3}{2}\right)$.

11. उद्देश्य फलन $P = 3x + 4y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + y \leq 7$, $x - y \geq - 1$, $x \geq 0$, $y \geq 0$
Maximize $P = 3x + 4y$ under the following constraints: $x + y \leq 7$, $x - y \geq - 1$, $x \geq 0$, $y \geq 0$ **[2059 S]**

⇨ Here, given inequalities are;

$x + y \leq 7$, $x - y \geq - 1$, $x \geq 0$ and $y \geq 0$

The boundary line of $x + y \leq 7$ is
$x + y = 7$

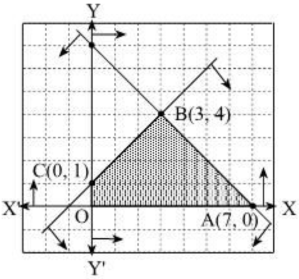
x	0	7
y	7	0

So, the boundary line passes through (0, 7) and (7, 0).
Let (0, 0) be the test point,
$x + y < 7$
or, $0 + 0 < 7$
$\therefore$ $0 < 7$ (True)

The boundary line of $x - y \geq - 1$ is
$x - y = - 1$

x	0	– 1
y	1	0

So the boundary line passes through (0, 1) and (– 1, 0).
Let (0, 0) be the test point,
$x - y > - 1$
or, $0 - 0 > - 1$
$\therefore$ $0 > - 1$ (True)



$\therefore$ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

Here, the shaded region OABC is the solution region where the vertices of OABC are O(0, 0), A (7,0), B(3,4) and C(0,1). The maximum or minimum values are found at these points as :

Points (x, y)	P(x, y) = 3x + 4y	Value	Remarks
O(0, 0)	$3 \times 0 + 4 \times 0$	0	
A(7, 0)	$3 \times 7 + 4 \times 0$	21	
B(3, 4)	$3 \times 3 + 4 \times 4$	25	Maximum
C(0, 1)	$3 \times 0 + 6 \times 1$	6	

Thus, the maximum value of P is 25 at (3,4).

12. उद्देश्य फलन $P = 10x + 12y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \leq 12$, $3x + 2y \leq 24$, $x \geq 0$, $y \geq 0$
Maximize $P = 10x + 12y$ under the following constraints: $x + 2y \leq 12$, $3x + 2y \leq 24$, $x \geq 0$, $y \geq 0$ **[2060 R]**

⇨ Here, given inequalities are $x + 2y \leq 12$, $3x + 2y \leq 24$, $x \geq 0$ and $y \geq 0$

The boundary line of $x + 2y \leq 12$ is
$x + 2y = 12$

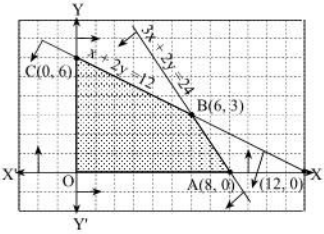
x	0	12
y	6	0

So, the boundary line passes through (0, 6) and (12, 0)
Let, (0, 0) be the test point,
$x + 2y \leq 12$
or, $0 + 2 \times 0 < 12$
$\therefore$ $0 < 12$ (True)

The boundary line of $3x + 2y \leq 24$ is
$3x + 2y = 24$

x	0	8
y	12	0

So, the boundary line passes through (0, 12) and (8, 0).
Let (0, 0) be the test point,
$3x + 2y < 24$
or, $3 \times 0 + 2 \times 0 < 24$
$\therefore$ $0 < 24$ (True)



$\therefore$ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

Then, from the graph paper vertices of the shaded region are O(0, 0), C(0, 6), B(6, 3) and A(8, 0).

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Vertices	x-com	y-com	P = 10x + 12y	Result
O(0, 0)	0	0	0 + 0 = 0	
C(0, 6)	0	6	0 + 72 = 72	
B(6, 3)	6	3	60 + 36 = 96	Maximum
A(8, 0)	8	0	80 + 0 = 80	

Thus, 96 is maximum value at B(6, 3).

13. उद्देश्य फलन $P(x, y) = 3x + 5y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + y \leq 6, x - y \leq 4, x \geq 0, y \geq 0$
Maximize $P(x,y) = 3x + 5y$ under the following constraints: $x + y \leq 6, x - y \leq 4, x \geq 0, y \geq 0$ **[2062 K]**

⇨ Here, given inequalities are; $x + y \leq 6$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$

The boundary line of $x + y \leq 6$ is
$x + y = 6$

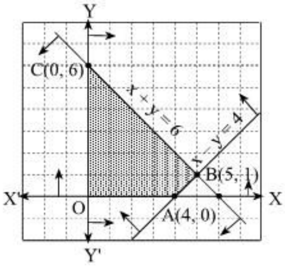
x	0	6
y	6	0

So, the boundary line passes through (0, 6) and (6, 0).
Let, (0, 0) be the test point,
$x + y < 6$
or, $0 + 0 < 6$
∴ $0 < 6$ (True)

The boundary line of $x - y \leq 4$ is
$x - y = 4$

x	0	4
y	– 4	0

So, the boundary line passes through (0, – 4) and (4, 0).
Let (0, 0) be the test point,
$x - y < 4$
or, $0 - 0 < 4$
∴ $0 < 4$ (True)



∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.
From the graph the vertices of polygonal region OABC are; O(0, 0), A(4, 0), B(5, 1) and C(0, 6).

Points (x, y)	P(x, y) = 3x + 5y	Value	Remarks
O(0, 0)	$3 \times 0 + 5 \times 0$	0	
A(4, 0)	$3 \times 4 + 5 \times 0$	12	
B(5, 1)	$3 \times 5 + 5 \times 1$	20	
C(0, 6)	$3 \times 0 + 5 \times 6$	30	Maximum

Thus, the maximum value is 30 at (0, 6).

14. उद्देश्य फलन $Z = 3x + 4y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + y \leq 6, x - y \leq 4, x \geq 0 , y \geq 0$
Maximize $Z = 3x + 4y$ under the following constraints: $x + y \leq 6, x - y \leq 4, x \geq 0 , y \geq 0$ **[2066 R]**

⇨ Here, given inequalities are; $x + y \leq 6$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$

The boundary line of $x + y \leq 6$ is
$x + y = 6$

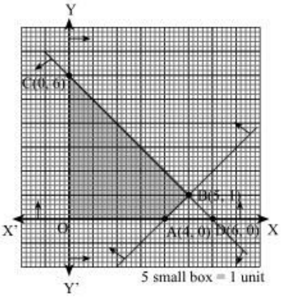
x	0	6
y	6	0

So, the boundary line passes through (0, 6) and (6, 0).
Let, (0, 0) be the test point,
$x + y < 6$
or, $0 + 0 < 6$
∴ $0 < 6$ (True)

The boundary line of $x - y \leq 4$ is
$x - y = 4$

x	0	4
y	– 4	0

So, the boundary line passes through (0, – 4) and (4, 0).
Let (0, 0) be the test point,
$x - y < 4$
or, $0 - 0 < 4$
∴ $0 < 4$ (True)



5 small box = 1 unit

∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.
From the graph the vertices of polygonal region OABC are;
O(0, 0), A(4, 0), B(5, 1) and C(0, 6).
OABC is the shaded and feasible region.

Points (x, y)	P(x, y) = 3x + 4y	Value	Remarks
O(0, 0)	$3 \times 0 + 4 \times 0$	0	
A(4, 0)	$3 \times 4 + 4 \times 0$	12	
B(5, 1)	$3 \times 5 + 4 \times 1$	19	
C(0, 6)	$3 \times 0 + 4 \times 6$	24	Maximum

Thus, the maximum value of Z is 24 at (0, 6).

15. उद्देश्य फलन $P = 2x + 5y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + y \leq 5, y - x \geq 1, x \geq 0, y \geq 0$
Maximize $P = 2x + 5y$ under the following constraints: $2x + y \leq 5, y - x \geq 1, x \geq 0, y \geq 0$ **[2060 C]**

⇨ Here, given inequalities are; $2x + y \leq 5$, $y - x \geq 1$, $x \geq 0$, $y \geq 0$

The boundary line of $2x + y \leq 5$ is
$2x + y = 5$

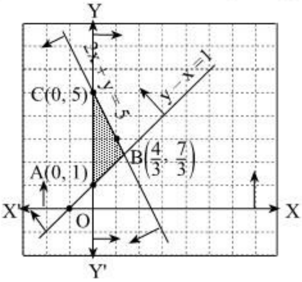
x	0	1
y	5	3

So, the boundary line passes through (0, 5) and (1, 3).
Let (0, 0) be the test point,
$2x + y < 5$
or, $2 \times 0 + 0 < 5$
∴ $0 < 5$ (True)

The boundary line of $y - x \geq 1$ is
$y - x = 1$

x	0	– 1
y	1	0

So the boundary line passes through (0, 1) and (– 1, 0).
Let (0, 0) be the test point,
$y - x > 1$
or, $0 - 0 > 1$
∴ $0 > 1$ (False)



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∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph the vertices of polygonal region of ΔABC are:

A(0, 1), $B\left(\frac{4}{3}, \frac{7}{3}\right)$ and C(0, 5)

Points (x, y)	P = 2x + 5y	Value	Remarks
A(0, 1)	$2 \times 0 + 5 \times 1$	5	
$B\left(\frac{4}{3}, \frac{7}{3}\right)$	$2 \times \frac{4}{3} + 5 \times \frac{7}{3}$	$\frac{43}{3}$	
C(0, 5)	$2 \times 0 + 5 \times 5$	25	Maximum

Thus, the maximum value is 25 at C(0, 5).

16. **उद्देश्य फलन P = 2x + 3y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \leq 10$, $2x + y \leq 14$, $x \geq 0$, $y \geq 0$**

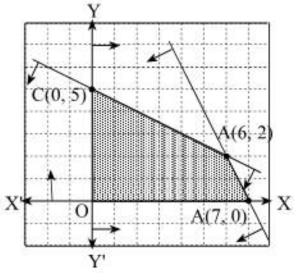
Maximize P = 2x + 3y under the following constraints: $x + 2y \leq 10$, $2x + y \leq 14$, $x \geq 0$, $y \geq 0$ **[2063 R]**

⇨ Here, the given inequalities are: $x + 2y \leq 10$, $2x + y \leq 14$, $x \geq 0$, $y \geq 0$

Corresponding boundary lines of given inequalities are:

$x + 2y = 10$ .... (i) $2x + y = 14$ .... (ii)

$x = 0$ .... (iii) $y = 0$ .... (iv)



Values for equation (i)

x	0	6
y	5	2

∴ The boundary line (i) passes through (0, 5) and (6, 2).

Values for equation (ii)

x	7	6
y	0	2

∴ The boundary line (ii) passes through (7, 0) and (6, 2).

Taking (0, 0) as a test point for inequality first we get $0 < 10$ (True)

Taking (0, 0) as a test point for inequality second we get, $0 < 14$ (True)

∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

The graph is as shown alongside:

The shaded region OABC as shown in the graph is the solution region. Whose co-ordinates are O(0, 0), A(7, 0), B(6, 2) and C(0, 5) at which maximum or minimum value of P = 2x + 3y is found as follow:

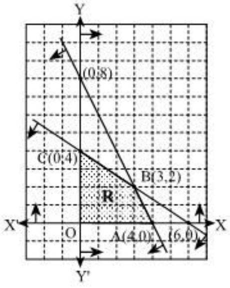
Vertices	x	y	Values of P = 2x + 3y	Remarks
O(0, 0)	0	0	$2 \times 0 + 3 \times 0 = 0$	
A(7, 0)	7	0	$2 \times 7 + 3 \times 0 = 14 + 0 = 14$	
B(6, 2)	6	2	$2 \times 6 + 3 \times 2 = 12 + 6 = 18$	Maximum
C(0, 5)	0	5	$2 \times 0 + 3 \times 5 = 0 + 15 = 15$	

Thus, the maximum value is 18 at (6, 2).

17. **उद्देश्य फलन P = x + 2y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + y \leq 8$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$**

Maximize P = x + 2y under the following constraints: $2x + y \leq 8$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$ **[2063 M]**

⇨ Here, given inequlities are $2x + y \leq 8$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$.



The corresponding equation of

$2x + y \leq 8$ is; $2x + y = 8$

x	0	4
y	8	0

So the line passing through (0, 8) and (4, 0) is drawn.

Let, (0, 0) be the test point then,

$2x + y \leq 8$

or, $2 \times 0 + 0 \leq 8$

or, $0 < 8$ (true)

Again, the corresponding equation of

$2x + 3y \leq 12$ is $2x + 3y = 12$

x	0	6
y	4	0

So the line passing through (0, 4) and (6, 0) is drawn.

Let, (0, 0) be the test point then,

$2x + 3y \leq 12$

or, $2 \times 0 + 3 \times 0 \leq 12$

or, $0 < 12$ (true)

$(x \geq 0) \cap (y \geq 0)$ represents the 1^{st} quadrant.

Now, graphing the constraints O(0, 0), A(4, 0), B(3, 2) & C(0, 4) we get the following feasible region R.

The value of the objective function P in the vertices of the fesible region is tabulated below.

Vertex	x	y	P = x + 2y	Remarks
O(0, 0)	0	0	0	
A(4, 0)	4	0	4	
B(3, 2)	3	2	7	
C(0, 4)	0	4	8	Maximum

Thus, the objective function is maximum at C(0,4) and the maximum value is 8.

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18. उद्देश्य फलन $P = 3x + 2y + 5$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \geq 1, x + y \leq 5, x \geq 0 , y \geq 0$

Maximize $P = 3x + 2y + 5$ under the following constraints: $x + 2y \geq 1, x + y \leq 5, x \geq 0 , y \geq 0$ **[2064 R]**

⇨ Here, given inequalities are $x + 2y \geq 1, x + y \leq 5, x \geq 0$ and $y \geq 0$.

The boundary line of $x + 2y \geq 1$ is

$x + 2y = 1$.

x	1	– 1
y	0	1

So, the boundary line passes through (1, 0) and (– 1, 1).

Let, (0, 0) be the test point,

$x + 2y > 1$

or, $0 + 2 \times 0 > 1$

∴ $0 > 1$ (False)

The boundary line of $x + y \leq 5$ is

$x + y = 5$.

x	0	5
y	5	0

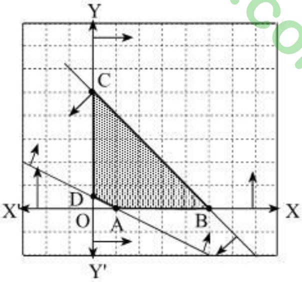
So, the boundary line passes through (0, 5) and (5, 0).

Let, (0, 0) be the test point,

$x + y < 5$

or, $0 + 0 < 5$

∴ $0 < 5$ (True)



∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph the vertices of the polygonal region ABCD are:

A(1, 0), B(5, 0), C(0, 5) and D(0, 0.5).

Now,

Vertices (x, y)	P = 3x + 2y + 5	Value	Remarks
A(1, 0)	$3 \times 1 + 2 \times 0 + 5$	8	
B(5, 0)	$3 \times 5 + 2 \times 0 + 5$	20	Maximum
C(0, 5)	$3 \times 0 + 2 \times 5 + 5$	15	
D(0, 0.5)	$3 \times 0 + 2 \times 0.5 + 5$	6	

Thus, the maximum value is 20 at B(5, 0).

19. उद्देश्य फलन $Z = 8x + 3y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + y \leq 7, x + 2y \leq 10, x \geq 0 , y \geq 0$

Maximize $Z = 8x + 3y$ under the following constraints: $x + y \leq 7, x + 2y \leq 10, x \geq 0 , y \geq 0$ **[2063 S]**

⇨ Here, given inequalities are: $x + y \leq 7, x + 2y \leq 10, x \geq 0$ and $y \geq 0$

The boundary line of $x + y \leq 7$ is

$x + y = 7$.

x	0	7
y	7	0

So, the boundary line passes through (0, 7) and (7, 0).

Let, (0, 0) be the test point.

$x + y < 7$

or, $0 + 0 < 7$

∴ $0 < 7$ (True)

The boundary line of $x + 2y \leq 10$ is

$x + 2y = 10$

x	0	6
y	5	2

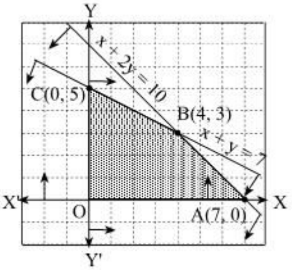
So, the boundary line passes through (0, 5) and (6, 2).

Let the (0, 0) be the test point

$x + 2y < 10$

or, $0 + 2 \times 0 < 10$

∴ $0 < 10$ (True)



∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph the vertices of the polygonal region i.e. of quad. OABC are:

O(0, 0), A(7, 0), B(4, 3) and C(0, 5).

Vertices (x, y)	Z = 8x + 3y	Value	Remarks
O(0, 0)	$8 \times 0 + 3 \times 0$	0	
A(7, 0)	$8 \times 7 + 3 \times 0$	56	Maximum
B(4, 3)	$8 \times 4 + 3 \times 3$	41	
C(0, 5)	$8 \times 0 + 3 \times 5$	15	

Thus, the maximum value is 56 at A(7, 0).

20. उद्देश्य फलन $P = 5x + 3y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + y \leq 20, 2x + 3y \leq 24, x \geq 0, y \geq 0$

Maximize $P = 5x + 3y$ under the following constraints: $2x + y \leq 20, 2x + 3y \leq 24, x \geq 0, y \geq 0$ **[2065 M]**

⇨ Here, given inequalities $2x + y \leq 20, 2x + 3y \leq 24, x \geq 0, y \geq 0$

Corresponding equation of

$2x + y \leq 20$ is $2x + y = 20$.

∴ $y = 20 – 2x$

x	0	10
y	20	0

(0, 20), (10, 0)

Let (0, 0) be the test point,

$2x + y < 20$

or, $2 \times 0 + 0 < 20$

∴ $0 < 20$ (True)

Corresponding equation of $2x + 3y \leq 24$

is; $2x + 3y = 24$

or, $3y = 24 – 2x$

∴ $y = \frac{24 – 2x}{3}$

x	0	12
y	8	0

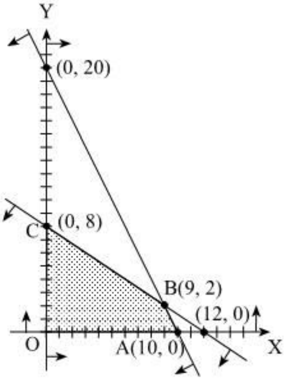
(0, 8), (12, 0)

Let (0, 0) be the test point in $2x + 3y < 24$

or, $2 \times 0 + 3 \times 0 < 24$

or, $0 + 0 < 24$

∴ $0 < 24$ (True)



∴ $x \geq 0$ and $y \geq 0$ represents first quadrant.

Shaded and feasible region is polygon OABC.

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From the graph the vertices of feasible region O(0, 0), A(10, 0), B(9, 2) and C(0, 8).

Vertices (x, y)	Z = 5x + 3y	Value	Remarks
O(0, 0)	$5 \times 0 + 3 \times 0$	0	
A(10, 0)	$5 \times 10 + 3 \times 0$	50	
B(9, 2)	$5 \times 9 + 3 \times 2$	51	Maximum
C(0, 8)	$5 \times 0 + 3 \times 8$	24	

Thus, the maximum value of P is 51 at B (9, 2)

21. उद्देश्य फलन P = 5x + 4y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + 5y \le 16, 2x + y \le 8, x \ge 0, y \ge 0$

Maximize P = 5x + 4y under the following constraints: $2x + 5y \le 16, 2x + y \le 8, x \ge 0, y \ge 0$ **[2066 S]**

⇨ Here, the given constraints are; $2x + 5y \le 16$, $2x + y \le 8$, $x \ge 0$, $y \ge 0$ and P = 5x + 4y

The corresponding equation of

$2x + 5y \le 16$ is;

or, $2x + 5y = 16$

x	3	8
y	2	0

(3, 0) (8, 0)

Let (0, 0) be the test point then,

$2x + 5y \le 16$

or, $2 \times 0 + 5 \times 0 \le 16$

∴ 0 < 16 (True)

The corresponding equation of

$2x + y \le 8$ is;

or, $2x + y = 8$

x	0	2
y	8	4

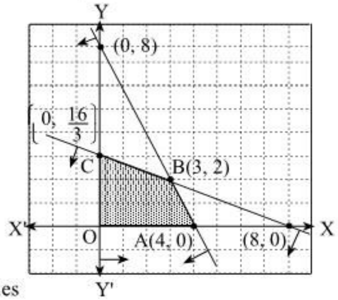
(0, 8), (2, 4)

Let, (0, 0) be the test point then,

$2x + y \le 8$

or, $2 \times 0 + 0 \le 8$

∴ 0 < 8 (True)



$(x \ge 0) \cap (y \ge 0)$ represents the first quadrant. Now, graphing the inequalities From the graph OABC is the polygonal region.

So, the vertices of polygonal region are; O(0, 0), A(4, 0), B(3, 2) and C$\left(0, 3\frac{1}{5}\right)$

Vertices	P = 5x + 4y	Value	Remarks
O(0, 0)	$5 \times 0 + 4 \times 0$	0	Minimum
A(4, 0)	$5 \times 4 + 4 \times 0$	20	
B(3, 2)	$5 \times 3 + 4 \times 2$	23	Maximum
C$\left(0, 3\frac{1}{5}\right)$	$5 \times 0 + 4 \times \frac{16}{5}$	12.8	

Thus, the required maximum value is 23.

22. उद्देश्य फलन Q = 5x + 4y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + y \ge 5, 3x + y \le 7, x \ge 0, y \ge 0$

Maximize Q = 5x + 4y under the following constraints: $2x + y \ge 5, 3x + y \le 7, x \ge 0, y \ge 0$ **[2067 R']**

⇨ Here given inequalities are; $2x + y \ge 5$ ……. (i),

$3x + y \le 7$ …………. (ii) and $x \ge 0$, $y \ge 0$

Now, equation of the boundary line of inequality (i) is $2x + y = 5$

x	0	2	3
y	5	1	– 1

Let (0, 0) be a testing point

2. 0 + 0 > 5

or, 0 > 5 (F)

So, (0, 0) does not lie in shaded region.

Equation of the boundary line of inequality (ii) is $3x + y = 7$

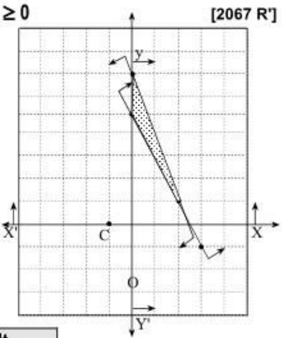
x	0	2	3
y	7	1	– 2

Let (0, 0) be a testing point,

3.0 + 0 < 7

or, 0 < 7 (T)

So, (0, 0) lies in shaded region.



From the graph paper, vertices of the shaded region are (2, 1), (0, 5) & (0, 7)

Again, for maximize function Q = 5x + 4y

Vertices	x-com	y-com	Q = 5x + 4y	Result
(2, 1)	2	1	10 + 4 = 14	
(0, 5)	0	5	0 + 20 = 20	
(0, 7)	0	7	0 + 28 = 28	Maximum

Thus, the 28 is a maximum value at point (0, 7).

23. उद्देश्य फलन P = 3x + 2y को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।

Maximize P = 3x + 2y under the following constraints.

$x + y \ge 0, x - y \le 0, y \le 2, x \ge -1$ **[SEE 2074 R, 2060 S']**

⇨ Here, given inequalities are: $x + y \ge 0$, $x - y \le 0$, $y \le 2$ and $x \ge -1$

Now, equation of boundary line of

$x + y \ge 0$ is $x + y = 0$

x	0	1	– 1
y	0	– 1	1

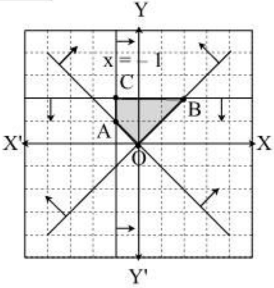
Line passes through (0, 0) so testing point be (1, 0) → alternative point

equation of boundary line of

$x - y \le 0$ is $x - y = 0$

x	0	1	2
y	0	1	2

Lines passes through (0, 0) so testing point be (1, 0) → alternative point



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$1 + 0 > 0$
or, $1 > 0$ (True)

So, (1, 0) lies in shaded region

$1 - 0 < 0$
or, $1 < 0$ (false)
So, (1, 0) does not lies in shaded region.

Equation of boundary lines of $y \leq 2$ & $x \geq -1$ are $y = 2$ & $x = -1$.

From the graph paper vertices of shaded region are O (0, 0), A (– 1, 1), C (– 1, 2) and B (2, 2).

Again, for maximize function $P = 3x + 2y$

Vertices	x-com	y-com	P = 3x + 2y	Result
O (0, 0)	0	0	$3 \times 0 + 2 \times 0 = 0$	
A (– 1, 1)	– 1	1	$3 \times (-1) + 2 \times 1 = -1$	
C (– 1, 2)	**– 1**	**2**	**$3 \times (-1) + 2 \times 2 = 1$**	
B (2, 2)	2	2	$3 \times 2 + 2 \times 2 = 10$	Maximum

Thus, 10 is maximum value at (2, 2).

24. उद्देश्य फलन $P = 4x + y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $2x + 3y \geq 6$, $2x - 3y \leq 6$, $x \geq 0$, $y \leq 2$

Maximize $P = 4x + y$ under the following constraints: $2x + 3y \geq 6$, $2x - 3y \leq 6$, $x \geq 0$, $y \leq 2$ **[2063 R']**

⇨ Here, the given inequalities are: $2x + 3y \geq 6$, $2x - 3y \leq 6$, $x \geq 0$, $y \leq 2$

The boundary line of
$2x + 3y \geq 6$ is
$2x + 3y = 6$.

x	0	3
y	2	0

So, the boundary line passes through (0, 2) and (3, 0).

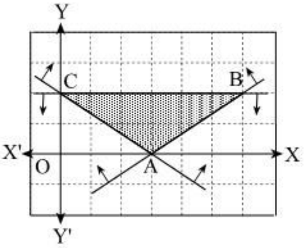
Let (0, 0) be the test point,
$2x + 3y > 6$
or, $2 \times 0 + 3 \times 0 > 6$
∴ $0 > 6$ (False)

The boundary line of
$2x - 3y \leq 6$ is
$2x - 3y = 6$.

x	0	3
y	– 2	0

So, the boundary line passes through (0, – 2) and (3, 0).

Let (0, 0) be the test point,
$2x - 3y < 6$
or, $2 \times 0 - 3 \times 0 < 6$
∴ $0 < 6$ (True)



$y \leq 2$ represents the region downward from $y = 2$.

$x \geq 0$ represents the region right from $x = 0$.

From the graph the vertices of polygonal region ΔABC are A(3, 0), B(6, 2) and C(0, 2).

Points (x, y)	P = 4x + y	Value	Remarks
A(3, 0)	$4 \times 3 + 0$	12	
B(6, 2)	$4 \times 6 + 2$	26	Maximum
C(0, 2)	$4 \times 0 + 2$	2	

Thus, the maximum value is 26 at (6, 2).

25. उद्देश्य फलन $Z = 5x + 3y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस्: $x + 2y \leq 6$, $x - 2y \leq 2$, $x \geq -2$

Maximize $Z = 5x + 3y$ under the following constraints: $x + 2y \leq 6$, $x - 2y \leq 2$, $x \geq -2$ **[2065 R']**

⇨ Here, given inequalities are $x + 2y \leq 6$, $x - 2y \leq 2$ and $x \geq -2$

The equation of boundary line of
$x + 2y \leq 6$ is $x + 2y = 6$.

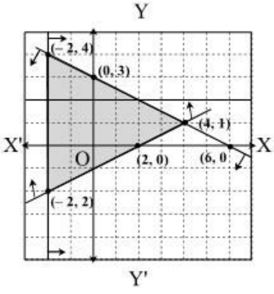
x	0	6
y	3	0

Let (0, 0) be the test point then,
$x + 2y \leq 6$
or, $0 + 2 \times 0 \leq 6$
or, $0 < 6$ [True]

Again, the equation of boundary line
of $x - 2y \leq 2$ is $x - 2y = 2$

x	0	2
y	– 1	0

Let, (0, 0) be the test point then,
$x - 2y \leq 2$
or, $0 - 2 \times 0 \leq 2$
or, $0 < 2$ (True)



$x \geq -2$ represents positive direction from the line $x = -2$.

From the graph,

The vertices of polygonal region are; (– 2, – 2), (4, 1) and (– 2, 4)

Vertices	Z = 5x + 3y	Value	Remarks
(– 2, – 2)	$5 \times (-2) + 3(-2)$	– 16	
(4, 1)	$5 \times 4 + 3 \times 1$	23	Maximum
(– 2, 4)	$5 \times (-2) + 3 \times 4$	2	

Thus, $z = 5x + 3y$ has maximum value 23 at (4, 1).

26. उद्देश्य फलन $Z = -2x + 5y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।

Maximize $Z = -2x + 5y$ under the following constraints: $5x + 2y \leq 45$, $4x + 5y \leq 53$, $x \geq 2$, $y \geq 0$ **[2064 R']**

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⇨ Here, given inequalities are: $5x + 2y \le 45$, $4x + 5y \le 53$, $x \ge 2$ and $y \ge 0$

The equation of boundary line of $5x + 2y \le 45$ is $5x + 2y = 45$.

x	9	5
y	0	10

The boundary line passes through (9, 0) and (5, 10).

Let (0, 0) be the test point:

$5x + 2y < 45$

or, $5 \times 0 + 2 \times 0 < 45$

∴ $0 < 45$ (True)

The equation of the boundary line of $4x + 5y \le 53$ is $4x + 5y = 53$.

x	2	7
y	9	5

So, the boundary line passes through (2, 9) and (7, 5).

Let (0, 0) be the test point;

$4x + 5y < 53$

or, $4 \times 0 + 5 \times 0 < 53$

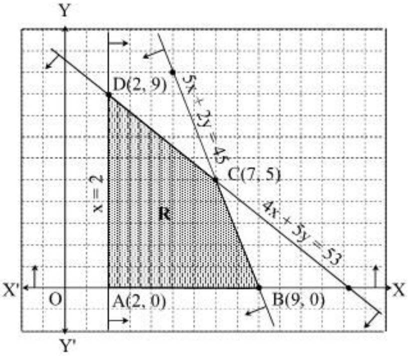
∴ $0 < 53$ (True)

∴ $x \ge 2$ represents the right side from $x = 2$.

∴ $y \ge 0$ represents the upward from $y = 0$.

Graphing the inequalities.

From the graph the vertices of the polygonal region are: A(2, 0), B(9, 0), C(7, 5) and D(2, 9).



Vertices (x, y)	Z = – 2x + 5y	Values	Remarks
A(2, 0)	$-2 \times 2 + 5 \times 0$	– 4	
B(9, 0)	$-2 \times 9 + 5 \times 0$	– 18	
C(7, 5)	$-2 \times 7 + 5 \times 5$	11	
D(2, 9)	$-2 \times 2 + 5 \times 9$	41	Maximum

Thus, the maximum value is 41 at D(2, 9).

MODEL 2

27. सर्तहरू $x - 2y \le 1, x + y \le 4, x \ge 0, y \ge 0$ को आधारमा $P = 5x + 4y$ को अधिकतम र न्यूनतम मान पत्ता लगाउनुहोस् ।

Optimize $P = 5x + 4y$ under the given constraints: $x - 2y \le 1, x + y \le 4, x \ge 0, y \ge 0$ **[SEE MODEL 2076]**

⇨ Here, given constraints are; $x - 2y \le 1$, $x + y \le 4$, $x \ge 0$, $y \ge 0$.

The corresponding equation of $x - 2y \le 1$ is $x - 2y = 1$.

x	1	3
y	0	1

(1, 0) and (3, 1)

Let (0, 0) be the test point then,

LHS = $x - 2y = 0 - 2 \times 0 = 0 < 1$ (True)

The test point is true for $x - 2y \le 1$.

The corresponding equation of $x + y \le 4$ is $x + y = 4$.

x	0	4
y	4	0

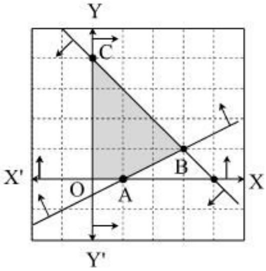
(0, 4) and (4, 0)

Let (0, 0) be the test point then, LHS = $x + y = 0 + 0 = 0 < 4$ (True)

The test point is true for $x + y \le 4$.

$(x \ge 0) \cap (y \ge 0)$ represents the first quadrant.

The graph is shown alongside:



From the graph, the vertices of the polygonal region are: A(1, 0), B(3, 1), C(0, 4) and O(0, 0)

Calculation of maximum and minimum value.

Vertices	P = 5x + 4y	Value	Remarks
A (1, 0)	$5 \times 1 + 4 \times 0$	5	
B (3, 1)	$5 \times 3 + 4 \times 1$	19	Maximum
C (0, 4)	$5 \times 0 + 4 \times 4$	16	
O (0, 0)	$5 \times 0 + 4 \times 0$	0	Minimum

Thus, the maximum value is 19 at B (3, 1) and the minimum value is 0 at O (0, 0).

28. उद्देश्य फलन $Z = 2x - 3y$ को निम्नलिखित अवस्थामा न्युनतम मान निकाल्नुहोस्: $x + y \ge 0, x - y \le 0, x \ge -1$ and $y \le 2$

Minimize $Z = 2x - 3y$, under the following constraints: $x + y \ge 0, x - y \le 0, x \ge -1$ and $y \le 2$ **[2064 S]**

⇨ Here, given inequalities are: $x + y \ge 0$, $x - y \le 0$, $x \ge -1$ and $y \le 2$

The boundary line of $x + y \ge 0$ is $x + y = 0$.

x	0	2	– 1	– 2
y	0	– 2	1	2

So, the boundary line passes through (0, 0) and (2, – 2).

Let (1, 0) be the test point,

$x + y > 0$

or, $1 + 0 > 0$

∴ $1 > 0$ (True)

The boundary line of $x - y \le 0$ is $x - y = 0$.

x	0	1
y	0	1

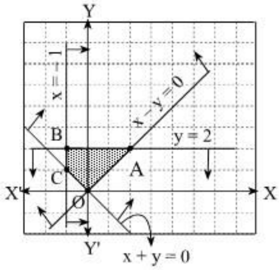
So, the boundary line passes though (0, 0) and (1, 1).

Let (1, 0) be the test point:

$x - y < 0$

or, $1 - 0 < 0$

or, $1 < 0$ (False)



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$x \geq -1$ represents right hand side region from $x = -1$.
$y \leq 2$ represents the downward region from the $y = 2$.

From the graph the vertices of the polygonal region i.e. of quadrilateral OABC are: O(0, 0), A(2, 2), B(– 1, 2) and C(– 1, 1).

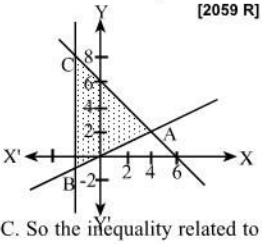
Vertices (x, y)	Z = 2x – 3y	Values	Remarks
O(0, 0)	$2 \times 0 - 3 \times 0$	0	
A(2, 2)	$2 \times 2 - 3 \times 2$	– 2	
B(– 1, 2)	$2 \times (-1) - 3 \times 2$	– 8	Minimum
C(– 1, 1)	$2 \times (-1) - 3 \times 1$	– 5	

Thus, the minimum value is – 8 at (– 1, 2).

MODEL 3

29. दिइएको चित्रमा B र C को निर्देशाङ्क क्रमशः (– 2,– 1) र (– 2, 8) छन् । त्रिभुज ABC भित्र छाया परेको भाग तीनओटा असमान समीकरणले जनाएको छ । तिनीहरूमध्ये एउटा x + y ≤ 6 छ भने A को निर्देशाङ्क र अरू दुई असमान समीकरण पत्ता लगाउनुहोस् । तीनओटै असमान समीकरणलाई मान्य हुने मानबाट x + 2y को अधिकतम मान निकाल्नुहोस् ।

In the given diagram, the coordinates of B and C are (– 2, – 1) and (– 2, 8) respectively. The shaded region inside the triangle ABC is represented by three inequalities, one of these is x + y ≤ 6. Write down the co-ordinates of A and other two inequalities. Also calculate the maximum value of x + 2y from the values which satisfy all three inequalities. **[2059 R]**



⇨ Here, given coordinates of B & C are (– 2, – 1) and (– 2, 8) respectively.

Then equation of line BC is $y + 1 = \frac{8+1}{-2+2}(x + 2)$

or, $y + 1 = \frac{9}{0}(x + 2)$ or, $9(x + 2) = 0$ $\therefore$ $x + 2 = 0$ .... (i)

Alternatively, since the x-coordinates of points B and C is same,
So, the line BC is parallel to y-axis
$\therefore$ Equation of BC is $x = -2$ $\therefore$ $x + 2 = 0$

Since, from figure origin (0, 0) lies with in the inequality represented by the line BC. So the inequality related to line BC is $x + 2 \leq 0$ ..... (ii)

Again, the line BA passes through the points B(– 2, – 1) and O(0, 0).

So, equation of AB is $y + 1 = \frac{0+1}{0+2}(x + 2)$

or, $y + 1 = \frac{1}{2}(x + 2)$ or, $2y + 2 = x + 2$ $\therefore$ $x - 2y = 0$ .... (iii)

Also, from figure, since origin lies within the inequality represented by the line AB. So (2, 0) be a testing point on shaded region $2 - 2.0 = 2 > 0$

So, $x - 2y \geq 0$ ……….. (iii)

Then the given inequality $x + y \leq 6$ is related to the line AC, since it contains the origin.
So, the boundary line of AC is $x + y = 6$ ...... (iv)
Since A is the point of intersection of the line AC and AB. So, solving equations (iii) and (iv) we get.
From (iii) $x = 2y$ in (iv) gives;

$2y + y = 6$ or, $3y = 6$ $\therefore$ $y = 2$ and $x = 2 \times 2 = 4$

$\therefore$ The coordinates of A is (4, 2)

Now, the values of function $x + 2y$ at the points A(4, 2), B(– 2, – 1) and C(– 2, 8) are:

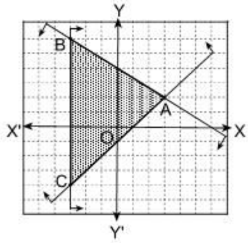
At A (4, 2), $x + 2y = 4 + 2 \times 2 = 4 + 4 = 8$ At B (– 2, – 1), $x + 2y = -2 + 2 \times (-1) = -2 - 2 = -4$

At C (– 2, 8), $x + 2y = -2 + 2 \times 8 = -2 + 16 = 14$ $\therefore$ The maximum value = 14 at C(– 2, 8).

Thus, the required results are: Coordinates of A = (4, 2), two inequalities $x + 2 \leq 0$ and $x - 2y \geq 0$ and maximum value = 14 at C(– 2, 8).

30. दिइएको चित्रमा A, B र C का निर्देशाङ्कहरू क्रमशः (3, 2), (– 3, 6) र (– 3, – 4) छन् । ABC भित्र छाया परेको भाग तीनओटा असमान समीकरणहरूले जनाएको छ । ती तीन असमान समीकरणहरू पत्ता लगाउनुहोस् र ती तीन असमान समीकरणहरूलाई मान्य हुने मानबाट 4x – 5y को न्यूनतम मान पनि निकाल्नुहोस् ।

In the given diagram, the co-ordinates of A, B and C are (3, 2), (– 3, 6) and (– 3, – 4) respectively. The shaded region inside the ΔABC is represented by three inequalities. Write down the equations of these three inequalities and also calculate the minimum value of 4x – 5y from the values which satisfy all the three inequalities. **[2062 R]**



⇨ Here, the coordinates of the vertices of the shaded region ΔABC are A (3, 2), B(– 3, 6) and C(– 3, – 4)

Here, equation of the boundary line AB representing one inequality is

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ or, $y - 2 = \frac{6-2}{-3-3}(x - 3)$ or, $y - 2 = \frac{4}{-6}(x - 3)$

or, $y - 2 = \frac{-2}{3}(x - 3)$ or, $3y - 6 = -2x + 6$ $\therefore$ $2x + 3y = 12$ .... (i)

Here, the inequality of the boundary line (i) contains the origin as in the given figure. So, the inequality related to boundary line (i) is $2x + 3y \leq 12$ and AB is a solid line.

Similarly, the boundary line AC representing another inequality is

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$y - 2 = \frac{-4-2}{-3-3}(x - 3)$ or, $y - 2 = \frac{-6}{-6}(x - 3)$

or, $y - 2 = x - 3$ ∴ $x - y = 1$ .... (ii)

Here the inequality of the boundary line (ii) contains the origin as in given figure, and AC is a solid line. So the inequality related to boundary line (ii) is $x - y \le 1$.

Similarly, as in figure, the boundary line BC of another inequality passes through (– 3, 0) and parallel to y– axis is $x = -3$ .... (iii). Also the inequality of boundary line (iii) contains the origin and in a solid line. So the inequality related to boundary line (iii) is $x \ge -3$.

∴ The required inequalities are $2x + 3y \le 12$, $x - y \le 1$, $x \ge -3$

Since the solution region of the given inequalities is the ΔABC whose vertices are A(3, 2), B(– 3, 6) and C(– 3, – 4), on which maximum and minimum values of $4x - 5y$ is found as follow :

at A (3, 2), $4x - 5y = 4 . 3 - 5 . 2 = 12 - 10 = 2$

at B (– 3, 6), $4x - 5y = 4 \times (-3) - 5 . 6 = -12 - 30 = -42$

at C (– 3, – 4), $4x - 5y = 4 \times (-3) - 5 \times (-4) = -12 + 20 = 8$

Thus, the minimum value = – 42 at B(– 3, 6)

QUESTIONS FROM CDC TEXTBOOK

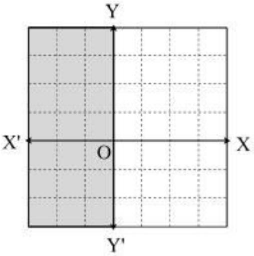
1.4 रेखीय योजना (LINEAR PROGRAMMING)

EXERCISE 1.4

1. दिइएका असमानताको लेखाचित्र खिच्नुहोस् (Draw the graph of following inequalities):

(a) $x \le 0$

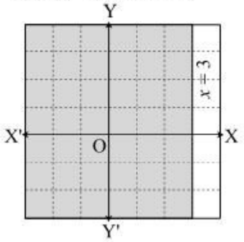
⇨ Here, the solution of $x \le 0$ is left from $x = 0$ inclusive.



(b) $x \le 3$

⇨ Here, $x \le 3$

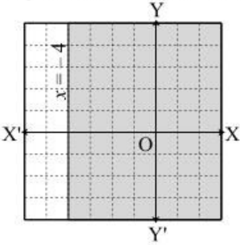
The solution of $x \le 3$ is left from $x = 3$ inclusive.



(c) $x \ge -4$

⇨ Here, $x \ge -4$

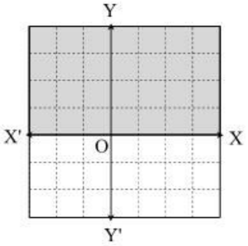
The solution of $x \ge -4$ is right from $x = -4$ inclusive.



(d) $y \ge 0$

⇨ Here, $y \ge 0$

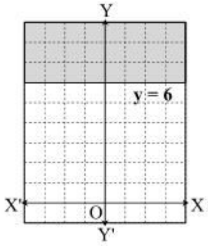
The solution of $y \ge 0$ is upward from $y = 0$ inclusive.



(e) $y \ge 6$

⇨ Here, $y \ge 6$

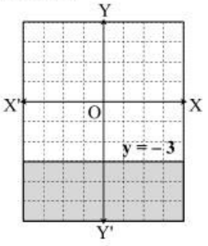
The solution of $y \ge 6$ is upward from $y = 6$ inclusive.



(f) $y \le -3$

⇨ Here, $y \le -3$

The solution of $y \le -3$ is downward from $y = -3$ inclusive.



2. दिइएका असमानताको लेखाचित्र खिच्नुहोस् (Draw the graph of following inequalities):

(a) $2x + 3y \ge 6$

⇨ Here, $2x + 3y \ge 6$

The corresponding equation of $2x + 3y \ge 6$ is $2x + 3y = 6$

x	0	3
y	2	0

(0, 2), (3, 0)

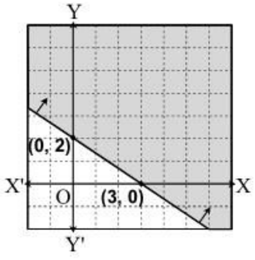
Taking (0, 0) as a test point then,

$2x + 3y \ge 6$

or, $2 \times 0 + 3 \times 0 \ge 6$

∴ $0 \ge 6$ (false)

Thus, the opposite side to the test point is shaded.



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(b) $x - y \geq 4$

⇨ Here, $x - y \geq 4$

The corresponding equation of $x - y \geq 4$ is $x - y = 4$

x	0	4
y	– 4	0

(0, – 4), (4, 0)

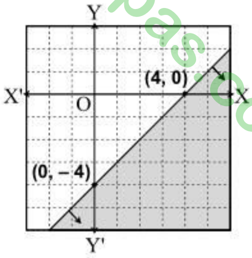
Let, (0, 0) be the test point.

Then, $x - y \geq 4$

or, $0 - 0 \geq 4$

$\therefore$ $0 \geq 4$ (false)

Thus, the opposite side to the test point is shaded.



(c) $x + 2y \leq 8$

⇨ Here, $x + 2y \leq 8$

Corresponding equation is $x + 2y = 8$

When $x = 0$, then $y = 4$ $\therefore$ point is (0, 4)

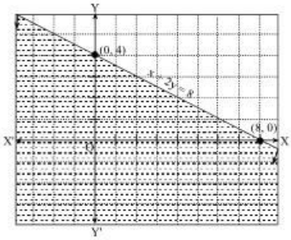
and when $y = 0$, then $x = 8$, $\therefore$ point is (8, 0)

The graph is shown in the figure,

Let, (0, 0) be the test point.

$0 + 2 \times 0 \leq 8$ $0 < 8$ (true)

Thus, the corresponding side to the test point is shaded.



(d) $4x + 3y \leq - 12$

⇨ Here, $4x + 3y \leq - 12$

The corresponding equation of $4x + 3y \leq - 12$ is $4x + 3y = - 12$

x	0	– 3
y	– 4	0

(0, – 4), (– 3, 0)

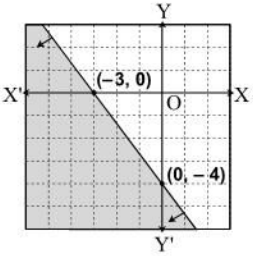
Let, (0, 0) be the test point.

Then, $4x + 3y \leq - 12$

or, $4 \times 0 + 3 \times 0 \leq - 12$

$\therefore$ $0 \leq - 12$ (false)

Thus, opposite side to the test point is shaded.



3. **दिइएका असमानता पद्धतिको लेखाचित्र खिची साझा हल क्षेत्र देखाउनुहोस् :**

Show the common solution region of given inequalities system by drawing the graph.

(a) $2x + 2y \geq 6$ and $y \geq 0$

⇨ Here, $2x + 2y \geq 6$ and $y \geq 0$

When $2x + 2y = 6$

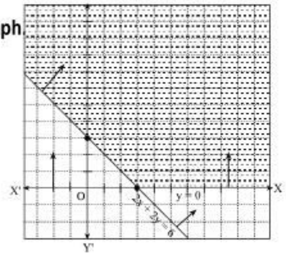
x	0	3
y	3	0

Let, (0, 0) be the test point.

Then, $2 \times 0 + 2 \times 0 \geq 6$ or, $0 \geq 6$ (false)

So, opposite side to the test point is shaded.

Double shaded portion in the figure is solution region.



(b) $x + y \leq 1$ and $x - y \geq 1$

⇨ Here, $x + y \leq 1$ and $x - y \geq 1$

When $x + y = 1$

x	0	1
y	1	0

Let, (1, 1) be the test point then $x + y \leq 1$

i.e. $1 + 1 \leq 1$ or, $2 \leq 1$ (false)

So, opposite side to the test point is shaded.

When $x - y = 1$

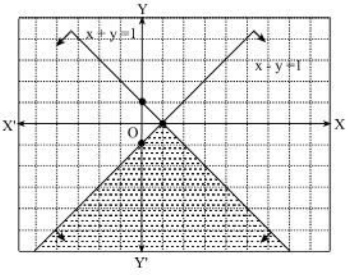
x	0	1
y	– 1	0

Let, (1, 1) be the test point then $x - y \geq 1$

i.e. $1 - 1 \geq 1$ or, $0 \geq 1$ (fasle)

So, opposite side to the test point is shaded.

Double shaded portion in the figure is solution region.



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(c) $2x + 3y \leq 6$ and $3x - y \leq 0$

⇨ Here, $2x + 3y \leq 6$ and $3x - y \leq 0$

The corresponding equation of $2x + 3y \leq 6$ is $2x + 3y = 6$

x	0	3
y	2	0

(0, 2), (3, 0)

Let, (0, 0) be the test point then,

$2x + 3y \leq 6$

or, $2 \times 0 + 2 \times 0 \leq 6$

$\therefore$ $0 < 6$ (true)

Again, the corresponding equation of $3x - y \leq 0$ is $3x - y = 0$

x	0	1
y	0	3

(0, 0), (1, 3)

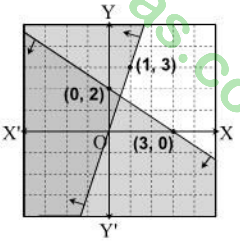
Let, (0, 2) be the test point then,

$3x - y \leq 0$

or, $3 \times 0 - 2 \leq 0$

$\therefore$ $-2 < 0$ (true)

Thus, the solution is shown by shading.



(d) $x + y \leq 2$, $2x - 3y \leq 6$ and $y \leq 2$

⇨ Here, $x + y \leq 2$, $2x - 3y \leq 6$ and $y \leq 2$

Taking $x + y \leq 2$, its corresponding equation is,

$x + y = 2$

x	0	2
y	2	0

(0, 2), (2, 0)

Let, (0, 0) be the test point then, $x + y \leq 2$

or, $0 + 0 \leq 2$

$\therefore$ $0 < 2$ (true) (So, the solution towards (0, 0) is shaded.)

Taking $2x - 3y \leq 6$, its corresponding equation is $2x - 3y = 6$.

x	0	3
y	– 2	0

(0, – 2), (3, 0)

Let, (0, 0) be the test point then,

$2x - 3y \leq 6$

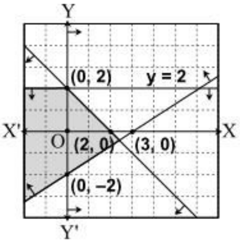
or, $2 \times 0 - 3 \times 0 \leq 6$

or, $0 < 6$ (true)

So, the solution towards (0, 0) is shaded.

And, $y \leq 2$ is the downward from $y = 2$.

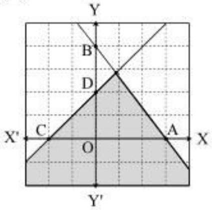
Thus, the solution set is shown by shading.



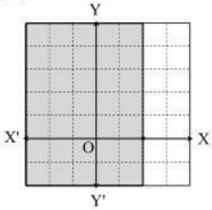
4. दिइएको लेखाचित्रमा छाया पारिएको क्षेत्रले जनाउने असमानता लेख्नुहोस् :

Write the inequality denoted by the shaded portion in the given graph.

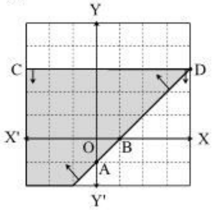
(a)



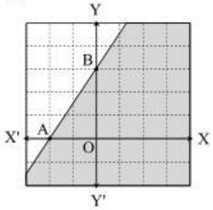
(b)



(c)



(d)



(a)

⇨ Here, from the graph, the co-ordinates of A, B, C and D are; A(3, 0), B(0, 4), C(– 2, 0) and D(0, 2).

The shaded portion is bounded by AB and CD.

So, equation of AB; $\frac{x}{a} + \frac{y}{b} = 1$

or, $\frac{x}{3} + \frac{y}{4} = 1$

or, $4x + 3y = 12$

Let (0, 0) be the test point; then,

LHS $= 4x + 3y$

$= 4 \times 0 + 3 \times 0$

$= 0 < 12$

So, $4x + 3y \leq 12$ is the corresponding inequality.

Again, for equation of DC, $\frac{x}{a} + \frac{y}{b} = 1$

or, $\frac{x}{-2} + \frac{y}{2} = 1$

or, $\frac{-x}{2} + \frac{y}{2} = 1$

$\therefore$ $-x + y = 2$

Let (0, 0) be the test point then,

LHS $= -x + y = 0 + 0 < 2$

So, $-x + y \leq 2$ is the corresponding inequality.

Thus, $4x + 3y \leq 12$ and $-x + y \leq 2$ are the required inequalities.

(b)

⇨ Here, shaded region is left side from $x = 2$ inclusive.

Thus, the required inequality is $x \leq 2$.

(c)

⇨ Here, the shaded region is bounded by CD and AB.

For equation of AB,

$\frac{x}{a} + \frac{y}{b} = 1$ or, $\frac{x}{1} + \frac{y}{-1} = 1$

or, $x - y = 1$

Let (0, 0) be the test point then,

LHS $= x - y = 0 - 0 = 0 < 1$

So, $x - y \leq 1$ is the corresponding inequality.

Again, equation of CD is $y = 3$.

The inequality represented by shaded part, is $y \leq 3$.

Thus, $x - y \leq 1$ and $y \leq 3$ are the required inequalities.

(d)

⇨ Here, for the equation of AB, $\frac{x}{a} + \frac{y}{b} = 1$

or, $\frac{x}{-2} + \frac{y}{3} = 1$ or, $\frac{-x}{2} + \frac{y}{3} = 1$

or, $\frac{-3x + 2y}{6} = 1$ $\therefore$ $-3x + 2y = 6$

Let (0, 0) be the test point then,

LHS $= -3x + 2y$

$= -3 \times 0 + 2 \times 0$

$= 0 < 6$

Thus, $-3x + 2y \leq 6$ is the required inequality.

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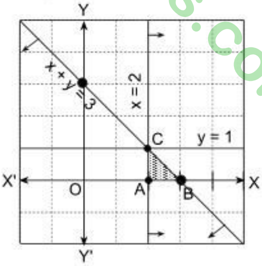
5. दिइएका असमानता पद्धतिको लेखाचित्र खिची तयार भएको बहुभुजका कुनाहरूको निर्देशाङ्क निकाल्नुहोस् । Write the co-ordiantes of corners of quadrilateral obtained by drawing the graph of following inequalities.

(a) $x + y \leq 3, x \geq 2, y \leq 1$



⇨ Here, When $x + y = 3$

x	0	3	2
y	3	0	1

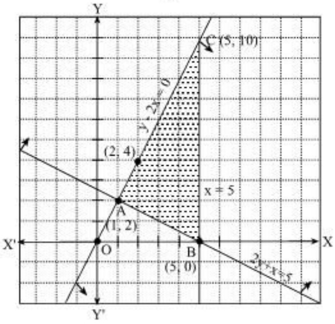
Let, (0, 0) be the test point then,

$x + y \leq 3$ or, $0 + 0 \leq 3$ or, $0 < 3$ (true)

$x \geq 2$ represents that the solution is right from $x = 2$ and, $y \leq 1$ represents that the solution is downward from $y = 1$.

Thus, co-ordinates of vertices of solution region A, B & C are (2, 0), (3, 0) & (2, 1) respectively.

(b) $y - 2x \leq 0, 2y + x \geq 5, x \leq 5$



⇨ Here, When, $y - 2x = 0$

x	0	2
y	0	4

Let, (1, 0) be test point then,

$y - 2x \leq 0$ or, $0 - 2 \times 1 \leq 0$

or, $-2 \leq 0$ (false)

When $2y + x = 5$

x	1	3
y	2	1

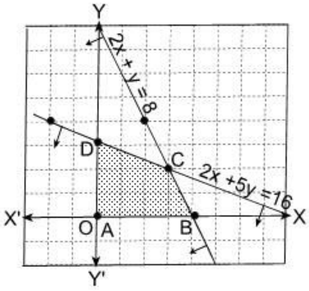
Let, (0, 0) be test point then,

$2y + x \geq 5$ or, $2 \times 0 + 0 \geq 5$ or, $0 \geq 5$(false)

$x \leq 5$ represents that the solution is left from $x = 5$.

Thus, co-ordinates of vetices of solution region are A(1, 2), B(5, 0) & C(5, 10).

(c) $2x + 5y \leq 16, 2x + y \leq 8, x \geq 0, y \geq 0$



⇨ Here, When $2x + 5y = 16$

x	3	– 2
y	2	4

Let, (0, 0) be the test point then, $2x + 5y \leq 16$,

or, $2 \times 0 + 5 \times 0 \leq 16$ or, $0 < 16$ (true)

When $2x + y = 8$

x	4	2
y	0	4

Again, let (0, 0) be the test point then,

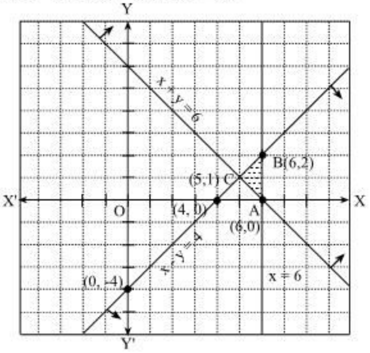
$2x + y \leq 8$ or, $2 \times 0 + 0 \leq 8$ or, $0 < 8$ (true)

$(x \geq 0) \cap (y \geq 0)$ represents first quadrant.

Thus, co-ordinates of vertices of solution region are A(0, 0), B(4, 0), C(3, 2) & D$\left(0, 3\frac{1}{5}\right)$.

6. निम्न सर्तहरू पूरा गरी उद्देश्य फलनको अधिकतम र न्यूनतम मान निकाल्नुहोस् : Find the maximum and minimum value of objective function under the following constraints.

(a) P = 2x + y लाई सर्तहरू $x + y \geq 6, x - y \geq 4, x \leq 6$ (P = 2x + y; constraints $x + y \geq 6, x - y \geq 4, x \leq 6$)



⇨ Here, $x + y \geq 6, x - y \geq 4, x \leq 6$

The corresponding equation of $x + y \geq 6$ is $x + y = 6$

x	0	6	5
y	6	0	1

Let, (0, 0) be the test point then,

$0 + 0 \geq 6$ or, $0 \geq 6$ (false)

The corresponding equation of $x - y \geq 4$ is $x - y = 4$

x	0	4
y	– 4	0

Let, (0, 0) be the test point then,

$0 - 0 \geq 4$ or, $0 \geq 4$ (false)

$x \leq 6$ represents the solution is left from $x = 6$.

From the graph ΔABC, is the polygonal region.

Vertices	x	y	P = 2x + y	Result
A (6, 0)	6	0	P = 2.6 + 0 = 12	
B (6, 2)	6	2	P = 2.6 + 2 = 14	Maximum
C (5, 1)	5	1	P = 2.5 + 1 = 11	Minimum

Thus, the objective function P is maximum at (6, 2) & value is 14 and the minimum value is 11 at (5, 1).

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(b) Q = 5*x* + 4*y* लाई सर्तहरू 2*x* + 4*y* ≥ 8, 3*x* + *y* ≤ 4, *x* ≥ 0, *y* ≥ 0
Q = 5*x* + 4*y* ; constraints 2*x* + 4*y* ≥ 8, 3*x* + *y* ≤ 4, *x* ≥ 0, *y* ≥ 0

⇨ Here, $2x + 4y \geq 8$, $3x + y \leq 4$, $x \geq 0$, $y \geq 0$.

The corresponding equation of $2x + 4y \geq 8$ is $2x + 4y = 8$.

x	0	4
y	2	0

(0, 2), (4, 0)

Let, (0, 0) be the test point then,

$2x + 4y \geq 8$

or, $2 \times 0 + 4 \times 0 \geq 8$

or, $0 \geq 8$ (False)

Again, taking $3x + y \leq 4$ then its corresponding equation is; $3x + y = 4$

x	0	1
y	4	1

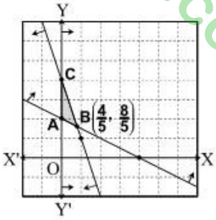
(0, 4), (1, 1)

Let (0, 0) be the test point then,

$3x + y \leq 4$

or, $3 \times 0 + 0 \leq 4$

∴ $0 < 4$ (True)



∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph, ΔABC is the solution set. So, A(0, 2), B(0.8, 1.6) and C(0, 4)

Now,

Vertices	Q = 5x + 4y	Remarks
A(0, 2)	$5 \times 0 + 4 \times 2 = 8$	Minimum
B(0.8, 1.6)	$5 \times 0.8 + 4 \times 1.6 = 10.4$	
C(0, 4)	$5 \times 0 + 4 \times 4 = 16$	Maximum

Thus, the minimum value is 8 at A(0, 2) and the maximum value is 16 at C(0, 4).

(c) L = 2*x* + 4*y* लाई सर्तहरू 4*x* + 3*y* ≤ 12, *x* + 2*y* ≤ 4, *x* ≥ 0, *y* ≥ 0
L = 2*x* + 4*y*; constraints 4*x* + 3*y* ≤ 12, *x* + 2*y* ≤ 4, *x* ≥ 0, *y* ≥ 0

⇨ Here, $4x + 3y \leq 12$, $x + 2y \leq 4$, $x \geq 0$, $y \geq 0$

The corresponding equation of $4x + 3y \leq 12$ is; $4x + 3y = 12$

x	0	3
y	4	0

Let, (0, 0) be the test point then,

$4x + 3y \leq 12$

or, $4 \times 0 + 3 \times 0 \leq 12$

or, $0 < 12$ (true)

Again, taking $x + 2y \leq 4$ then its corresponding equation is; $x + 2y = 4$

x	0	4
y	2	0

Let, (0, 0) be the test point then,

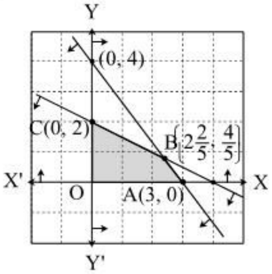
$x + 2y \leq 4$

or, $0 + 2 \times 0 \leq 4$

or, $0 < 4$ (true)

∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph, OABC is the polygonal region.



Vertices	x	y	L = 2x + 4y	Conclusion
O (0, 0)	0	0	L = 2.0 + 4.0 = 0	Minimum
A (3, 0)	3	0	L = 2.3 + 4.0 = 6	
B $\left(2\frac{2}{5}, \frac{4}{5}\right)$	$2\frac{2}{5}$	$\frac{4}{5}$	$L = 2.\frac{12}{5} + 4.\frac{4}{5} = 8$	Maximum
C (0, 2)	0	2	L = 2.0 + 4.2 = 8	Maximum

Thus, the objective function is minimum at origin & value is 0 and the maximum value is 8 at C(0, 2) or at B$\left(\frac{12}{5}, \frac{4}{5}\right)$.

(d) P = 5*x* + 4*y* लाई सर्तहरू *x* + 2*y* ≥ 3, 3*x* + *y* ≤ 3, *x* ≥ 0, *y* ≥ 0 (P = 5*x* + 4*y*; constraints *x* + 2*y* ≥ 3, 3*x* + *y* ≤ 3, *x* ≥ 0, *y* ≥ 0)

⇨ Here, $x + 2y \geq 3$, $3x + y \leq 3$, $x \geq 0$, $y \geq 0$

The corresponding equation of $x + 2y \geq 3$ is $x + 2y = 3$

x	0	3
y	1.5	0

(0, 1.5), (3, 0)

Let (0, 0) be the test point then,

$x + 2y \geq 3$

or, $0 + 2 \times 0 \geq 3$

∴ $0 \geq 3$ (False)

Again, the corresponding equation of $3x + y \leq 3$ is $3x + y = 3$

x	0	1
y	3	0

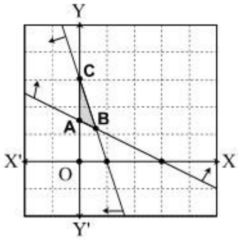
(0, 3), (1, 0)

Let (0, 0) be the test point then,

$3x + y \leq 3$

or, $3 \times 0 + 0 \leq 3$

∴ $0 < 3$ (True)



∴ $(x \geq 0) \cap (y \geq 0)$ represents the first quadrant.

From the graph, ΔABC is the solution set having A(0, 1.5), B(0.6, 1.2), C(0, 3).

Vertices	P = 5x + 4y	Remarks
A(0, 1.5)	$5 \times 0 + 4 \times 1.5 = 6$	Minimum
B(0.6, 1.2)	$5 \times 0.6 + 4 \times 1.2 = 7.8$	
C(0, 3)	$5 \times 0 + 4 \times 3 = 12$	Maximum

Thus, the maximum value is 12 at C(0, 3) and minimum value is 6 at A(0, 1.5).

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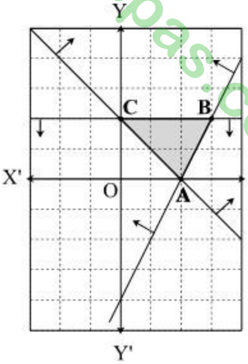
7. (a) **दिइएको लेखाचित्रबाट छाँया पारिएको भागले जनाउने 5 ओटा असमानताहरू लेखी फलन $P = 4x + 9y$ को अधिकतम मान पत्ता लगाउनुहोस् ।**
Write down five inequalities that describes the shaded region in the graph and find the maximum value of function of $P = 4x + 9y$



⇨ Here, the shaded part is only in first quadrant so the two inequalities are; $x \geq 0$ and $y \geq 0$. The equation of BC is $y = 2$

So, the corresponding inequality with respect to the solution set is $y \leq 2$.

Equation of AC; $\frac{x}{a} + \frac{y}{b} = 1$

or, $\frac{x}{2} + \frac{y}{2} = 1$

$\therefore$ $x + y = 2$

Let (2, 1) be the test point then;

LHS $= x + y = 2 + 1 = 3 > 2$

$\therefore$ $x + y \geq 2$ is the corresponding inequality with respect to the shaded part.

Again, equation of AB; $\frac{x}{a} + \frac{y}{b} = 1$

or, $\frac{x}{2} + \frac{y}{-4} = 1$

or, $\frac{x}{2} - \frac{y}{4} = 1$

or, $2x - y = 4$

Let (2, 1) be the test point;

Then, LHS $= 2x - y = 2 \times 2 - 1 = 3 < 4$

$\therefore$ $2x - y \leq 4$ is the corresponding inequality with respect to the shaded part.

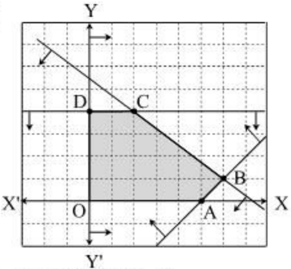
Thus, $x \geq 0$, $y \geq 0$, $y \leq 2$, $x + y \geq 2$ and $2x - y \leq 4$ are the required inequalities.

From the graph the vertices of feasible region are A(2, 0), B(3, 2) and C(0, 2).

Vertices	$P = 4x + 9y$	Remarks
A(2, 0)	$4 \times 2 + 9 \times 0 = 8$	Minimum
B(3, 2)	$4 \times 3 + 9 \times 2 = 30$	Maximum
C(0, 2)	$4 \times 0 + 9 \times 2 = 18$	

Thus, the maximum value is 30 at (3, 2).

(b) **दिइएको लेखाचित्रबाट छाँया पारिएको भागले जनाउने पाँच ओटा असमानताहरू लेखी फलन $P = 2x + y - 4$ को अधिकतम मान पत्ता लगाउनुहोस् ।**
Write down five inequalities that describes the shaded region in the graph and find the maximum value of function of $P = 2x + y - 4$



⇨ Here, the shaded part is only in first quadrant so the two inequalities are; $x \geq 0$ and $y \geq 0$.

The shaded part is downward from the line $y = 4$ so the inequality corresponding to the line CD is $y \leq 4$.

The equation of BC; B(6, 1) and C(2, 4) is;

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y - 1 = \frac{4 - 1}{2 - 6}(x - 6)$

or, $y - 1 = \frac{3}{-4}(x - 6)$

or, $-4y + 4 = 3x - 18$

or, $3x + 4y = 22$

Let (1, 1) be the test point then,

LHS $= 3x + 4y$
$= 3 \times 1 + 4 \times 1$
$= 7 < 22$

So, the corresponding inequality is $3x + 4y \leq 22$.

For the equation of AB; A(5, 0) and B(6, 1)

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y - 0 = \frac{1 - 0}{6 - 5}(x - 5)$

or, $y = \frac{1}{1}(x - 5)$

or, $y = x - 5$

$\therefore$ $x - y = 5$

Let (1, 1) be the test point then,

LHS $= x - y$
$= 1 - 1$
$= 0 < 5$

So, the corresponding inequality is $x - y \leq 5$.

Now, from the graph the vertices of polygonal region are;

Vertices	$P = 2x + y - 4$	Remarks
O(0, 0)	$2 \times 0 + 0 - 4 = -4$	
A(5, 0)	$2 \times 5 + 0 - 4 = 6$	
B(6, 1)	$2 \times 6 + 1 - 4 = 9$	Maximum
C(2, 4)	$2 \times 2 + 4 - 4 = 4$	
D(0, 4)	$2 \times 0 + 4 - 4 = 0$	

Thus, the five inequalities are as follows:

$x \geq 0$, $y \geq 0$, $y \leq 4$, $3x + 4y \leq 22$, $x - y \leq 5$ and, the maximum value is 9 at (6, 1).

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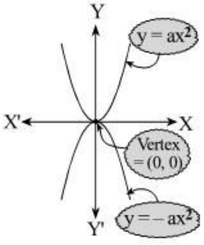
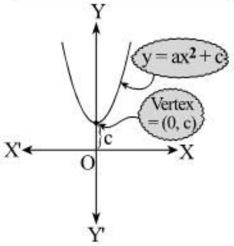
6. समीकरण र लेखाचित्र
Equation and Graph

Formulae and Key Points

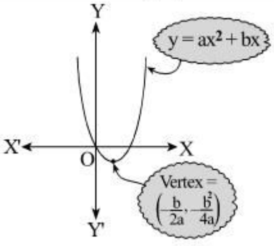
S.N.	पाराबोलाको समीकरण (Equation of Parabola)	शीर्षबिन्दु (Vertex)
1.	$y = ax^2 + bx + c, a \neq 0$	$\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$
2.	$y = ax^2 + c, a \neq 0$	(0, c)
3.	$y = ax^2 + bx; a \neq 0$	$\left(\frac{-b}{2a}, \frac{-b^2}{4a}\right)$
4.	$y = ax^2; a \neq 0$	(0, 0)
5.	$y = a(x - h)^2 + k$	(h, k)



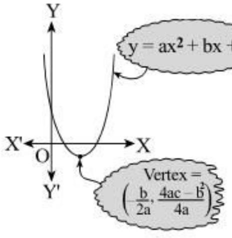
$y = ax^2; a \neq 0$



$y = ax^2 + c, a \neq 0$



$y = ax^2 + bx; a \neq 0$



$y = ax^2 + bx + c, a \neq 0$

QUESTIONS FROM SEE EXERCISE 6

A. VERY SHORT QUESTIONS

1. वर्गघातीय समीकरणको परिभाषा दिनुहोस् । (Define Quadratic equation.)

⇨ Here, a function of the form $f(x) = ax^2 + bx + c$; $a \neq 0$ is called a quadratic function. e.g. $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ etc. are quadratic functions.

2. पाराबोला वा अनुवृत्तको परिभाषा दिनुहोस् । (Define Parabola.)

⇨ Here, the smooth curve obtained by the graph of a quadratic equation is called the parabola.

3. पाराबोलाको शीर्षको परिभाषा दिनुहोस् । (Define Vertex of parabola.)

⇨ Here, the turning point of the parabola is called its vertex.

4. घनघातीय समीकरणको परिभाषा दिनुहोस् । (Define Cubic equation.)

⇨ Here, a function of the form $f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$ is called a cubic function. e.g. $y = x^3$, $y = 2x^3 + 5$ etc. are cubic functions.

5. अचलहरू p, q र r प्रयोग भएको एउटा पूर्ण वर्ग समीकरण लेख्नुहोस् ।
Write a complete quadratic equation having constants p, q, and r.

⇨ Here, the required quadratic equation is $px^2 + qx + r = 0$.

6. अचलहरू a, b, c र d को प्रयोग गरी एउटा पूर्ण घन फलनको उदाहरण दिनुहोस् ।
Using the constants a, b, c and d, give an example of a complete cubic equation.

⇨ Here, the required cubic equation is $ax^3 + bx^2 + cx + d = 0$.

7. समीकरण $y = ax^2 + bx + c, a \neq 0$ भएको पाराबोलाको शीर्षको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2 + bx + c, a \neq 0$?

⇨ Here, the vertex of the parabola is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

8. समीकरण $y = ax^2 + c = 0$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2 + c = 0$?

⇨ Here, the vertex of the parabola is (0, c).

9. समीकरण $y = ax^2$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2$?

⇨ Here, the vertex of the parabola is (0, 0).

10. समीकरण $y = a(x - h)^2 + k$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = a(x - h)^2 + k$?

⇨ Here, the vertex of the parabola is (h, k).

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11. वर्ग फलन $y = ax^2$ मा a को मान धनात्मक वा ऋणात्मक के हुँदा शीर्षबिन्दु न्यूनतम हुन्छ ?
Which value of a, positive or negative, in the quadratic equation $y = ax^2$ is the vertex point minimum?

⇨ Here, if a is positive then vertex point is minimum.

12. वर्ग फलन $y = ax^2$ मा a को मान धनात्मक वा ऋणात्मक के हुँदा शीर्षबिन्दु अधिकतम हुन्छ ?
Which value of a, positive or negative, in the quadratic equation $y = ax^2$ is the vertex point maximum?

⇨ Here, if a is negative then vertex point is maximum.

13. वर्ग फलन $y = ax^2$ मा $a > 0$ हुँदा पाराबोलाको खुलेको भाग माथि फर्कन्छ वा तल फर्कन्छ ?
In which side, above or below, the opening of the parabola $y = ax^2$ faces when $a > 0$?

⇨ Here, when $a > 0$ then the opening of $y = ax^2$ is upward.

14. वर्ग फलन $y = ax^2$ मा $a < 0$ हुँदा पाराबोलाको खुलेको भाग माथि फर्कन्छ वा तल फर्कन्छ ?
In which side, above or below, the opening of the parabola $y = ax^2$ faces when $a < 0$?

⇨ Here, when $a < 0$ then the opening of $y = ax^2$ is downward.

15. यदि $h = 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if $h = 0$? Write it.

⇨ Here, when $h = 0$ in $y = a(x - h)^2 + k$ then the vertex of parabola lies in y - axis.

16. यदि $h > 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if $h > 0$? Write it.

⇨ Here, when $h > 0$ in $y = a(x - h)^2 + k$ then the vertex of the parabola lies right of y - axis by h units.

17. यदि $h < 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if h < 0? Write it.

⇨ Here, when $h < 0$ in $y = a\ (x - h)^2 + k$ then the vertex of the parabola lies left of y-axis by h-units.

18. यदि $k = 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if $k = 0$? Write it.

⇨ Here, when k = 0 in $y = a(x - h)^2 + k$ then the vertex of the parabola lies at the x-axis.

19. यदि $k > 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if $k > 0$? Write it.

⇨ Here, when $k > 0$ in y = $a(x - h)^2 + k$ then the vertex of the parabola lies above x-axis by k units.

20. यदि $k < 0$ भए वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्षबिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie if $k < 0$? Write it.

⇨ Here, when $k < 0$ in $y = a(x - h)^2 + k$ then the vertex of the parabola lies below x-axis by k units.

21. समीकरण $ax^2 + bx + c = 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $ax^2 + bx + c = 0$?

⇨ Here, the equation of line of symmetry of $ax^2 + bx + c = 0$ is $x = -\frac{b}{2a}$. i.e. $x + \frac{b}{2a} = 0$.

22. समीकरण $y = ax^2 + c, a \neq 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $y = ax^2 + c, a \neq 0$?

⇨ Here, the equation of line of symmetry of the parabola $y = ax^2 + c, a \neq 0$ is $x = 0$.

23. समीकरण $y = ax^2 + bx, a \neq 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $y = ax^2 + bx, a \neq 0$?

⇨ Here, the equation of line of symmetry of the parabola $y = ax^2 + bx$ is $x = -\frac{b}{2a}$.

24. वर्ग समीकरण $y = ax^2 + c$ मा c ले के जनाउँदछ ? **(What does c represent in the quadratic equation $y = ax^2 + c$?)**

⇨ Here, in the quadratic equation $y = ax^2 + c$, c represents y - intercept.

25. घन समीकरण $ax^3 + bx^2 + cx + d = 0$ मा 'd' ले के जनाउँदछ ?
What does 'd' represent in the cubic equation $ax^3 + bx^2 + cx + d = 0$?

⇨ Here, in the cubic equation $ax^3 + bx^2 + cx + d = 0$, d represents y - intercept.

26. समीकरणहरू $y = 2x^2$ र $y = 3x^2$ मध्य कुनको वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = 2x^2$ and $y = 3x^2$ has the wider curve?

⇨ Here, $y = 2x^2$ has wider curve because 2 is less than 3.

27. समीकरणहरू $y = -3x^2$ र $y = -4x^2$ मध्य कुनको वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = -3x^2$ and $y = -4x^2$ has the wider curve ?

⇨ Here, $y = -3x^2$ has wider curve because 3 is less than 4.

28. समीकरण $y = 2x^2$ को न्यूनतम मान कति होला ? **(What is the minimum value of $y = 2x^2$?)**

⇨ Here, the minimum value of $y = 2x^2$ is (0, 0).

29. समीकरण $y = -3x^2$ को अधिकतम मान कति होला ? **(What is the maximum value of $y = -3x^2$?)**

⇨ Here, the maximum value of $y = -3x^2$ is (0, 0).

30. समीकरण $y = x^2 + 5$ हुने पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the coordinates of vertex of parabola whose equation is $y = x^2 + 5$.

⇨ Here, the co-ordinates of vertex of parabola is (0, 5).

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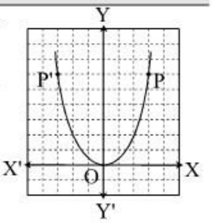
31. समीकरण $y = 2x^2 – 3$ हुने पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the coordinates of vertex of parabola whose equation is $y = 2x^2 – 3$.

⇨ Here, the co-ordinates of vertex of parabola is (0, – 3).

B. SHORT QUESTIONS

MODEL 1

1. सँगै दिइएको लेखाचित्रबाट आवश्यक समीकरण लेख्नुहोस् र तोकिएका दुई बिन्दुहरू P र P' का निर्देशाङ्कहरू समेत उल्लेख गर्नुहोस् ।
Write the required equation from the given graph and determine the co-ordinates of two points P and P'. [SLC 2063 R]

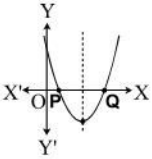


⇨ Here, the given parabola passes through origin and symmetric to the y-axis.
So, the equation of parabola is; $y = ax^2$ ....(i)
The co-ordinates of P and P' are (3, 6) and (– 3, 6). i.e. equation (i) passes through (3, 6)
$\therefore \quad 6 = a \times 3^2$
or, $\quad 6 = a \times 9$
$\therefore \quad a = \frac{2}{3}$

Now, putting the value of a in (i) then, $y = \frac{2}{3}x^2$

Thus, the required equation is $y = \frac{2}{3}x^2$ and the points are P(3, 6) and P'(– 3, 6).

2. सँगैको चित्रमा $y = ax^2 + bx + c$ को लेखाचित्र देखाइएको छ । लेखाचित्रले X-अक्षलाई P र Q मा काटेको छ । P र Q का निर्देशाङ्कहरू के–के होलान् र यी निर्देशाङ्कका X-निर्देशाङ्कहरूले के–के जनाउँदछन् ?
Figure given alongside represents the sketch of the graph $y = ax^2 + bx + c$. The graph intersects X-axis at P and Q. What are the co-ordinates of P and Q and what do their X-co-ordinates represent ? [SLC 2063 M]

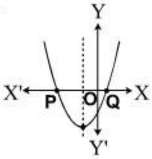


⇨ Here, the co-ordinates of P is; $\left(\frac{-b-\sqrt{b^2-4ac}}{2a}, 0\right)$

The co-ordinates of Q is; $\left(\frac{-b+\sqrt{b^2-4ac}}{2a}, 0\right)$

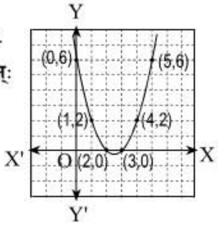
The X-co-ordinates of P and Q represent the solution of the quadratic equation $ax^2 + bx + c = 0$.

3. सँगैको चित्रमा $y = x^2 + 4x – 5$ को लेखाचित्र देखाइएको छ । यसले X-अक्षको बिन्दुहरू P र Q मा काटेको छ । P र Q का निर्देशाङ्कहरू के–के हुन् ? यी निर्देशाङ्कको X-निर्देशाङ्कहरूले के जनाउँदछन् ?
Figure along the side represents the sketch of the graph $y = x^2 + 4x – 5$. It cuts the X-axis at P and Q. What are the co-ordinates of P and Q? What do these X-co-ordinates denote? [SLC 2063 R']



⇨ Here, $y = x^2 + 4x – 5$
The points P and Q are the points at x-axis.
So, y = 0
i.e. $x^2 + 4x – 5 = 0$
or, $x^2 + 5x – x – 5 = 0$
or, $x(x + 5) – 1 (x + 5) = 0$
or, $(x + 5) (x – 1) = 0$
Either, $x + 5 = 0 \qquad \therefore \quad x = – 5$
or, $x – 1 = 0 \qquad \therefore \quad x = 1$
So, the points of x-axis are (– 5, 0) and (1, 0).
∴ P(– 5, 0) and Q(1, 0)
The x-components of the points represent the roots of the equation.
Thus, P(– 5, 0) and Q(1, 0) are the required points and x-components represent the roots.

4. दिइएको लेखाचित्रबाट पाराबोलाको Y-खण्ड, X-खण्ड र सममिति अक्षको समीकरण पत्ता लगाउनुहोस्:
Find the Y-intercept, X-intercept and equation of line of symmetry in the given graph:



⇨ Here, from the graph, point at y-axis is (0, 6)
∴ y-intercept = 6
Points at x axis are (2, 0) and (3, 0).
∴ x-intercepts = 2 and 3.
The x-component of the vertex of the parabola is 2.5.
∴ Equation of line of symmetry is x = 2.5
Thus, y-intercept is 6, x-intercepts are 2 and 3 and the line of symmetry is x = 2.5

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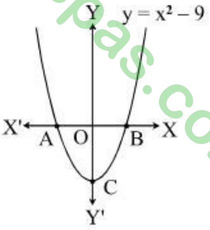
5. चित्रमा देखाइएको जानकारीबाट बिन्दुहरू A, B र C का निर्देशाङ्कहरू पत्ता लगाउनुहोस्:
From the given information in the figure, find the co-ordinates of points A, B and C:



⇨ Here, equation of parabola is $y = x^2 - 9$.
A and B are the points at x-axis.
So, $y = 0$
$\therefore\ y = x^2 - 9$ becomes $x^2 - 9 = 0$
or, $x^2 = 9$
$\therefore\ x = \pm 3$
So, A(– 3, 0) and B(3, 0)
Vertex is at y axis so, x-component of vertex is 0.
i.e. $y = x^2 - 9$ becomes $y = 0^2 - 9$
$\therefore\ y = -9$
i.e. the co-ordinates of C is (0, – 9).
Thus, A(– 3, 0), B(3, 0) and C(0, – 9) are the required points.

6. पाराबोलाको मानक समीकरण लेखी $y = x^2 - 2x + 1$ को शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Write the standard equation of a parabola and find the vertex of $y = x^2 - 2x + 1$.
Find the vertex of parabola, which is formed from $f(x) = 5x^2 + 6$.

⇨ Here, the equation $y = a(x - h)^2 - k$ is the standard equation of parabola.
Comparing given equation $y = x^2 - 2x + 1$ with $y = ax^2 + bx + c$ then, a = 1, b = – 2 and c = 1

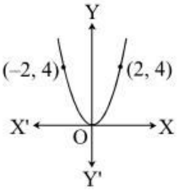
By the formula of vertex; $(h, k) = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{-2}{2 \times 1}, \frac{4 \times 1 \times 1 - (-2)^2}{4 \times 1}\right) = \left(1, \frac{4-4}{4}\right)$

$\therefore\ (h, k) = (1, 0)$
Thus, the vertex is (1, 0).

Alternative method

Given equation of parabola is; $y = x^2 - 2x + 1$
or, $y = x^2 - 2 \,.\, x \,.\, 1 + 1^2$
or, $y = (x - 1)^2$
or, $y = 1 \,.\, (x - 1)^2 + 0$
Comparing it with $y = a(x - h)^2 + k$ then, Vertex = (h, k) = (1, 0)
Thus, the vertex is (1, 0)

7. चित्रमा देखाइएको पाराबोलाको समीकरण पत्ता लगाउनुहोस्:
Find the equation of the parabola shown in the graph:



⇨ Here, from the graph,
The given parabola passes through the origin and symmetric to the y-axis.
So, the equation of the parabola is; $y = ax^2$ ....(i)
Equation (i) passes through (2, 4);
So, $4 = a \times 2^2$
or, $4 = a \times 4$
$\therefore\ a = 1$
Now, from (i) $y = ax^2$ becomes $y = x^2$
Thus, the required equation of the parabola is $y = x^2$.

8. उद्गमबिन्दुमा शीर्ष भएको एउटा पाराबोला बिन्दु (4, 32) भएर जान्छ भने सो पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the parabola which pass through the point (4, 32) and vertex lies at the origin.

⇨ Here, the given parabola has vertex at the origin.
So, the equation of the parabola is; $y = ax^2$ ....(i)
Equation (i) passes through the point (4, 32) so it becomes; $y = ax^2$
or, $32 = a \times 4^2$
or, $32 = a \times 16$
$\therefore\ a = 2$
Putting the value of a in (i) then $y = 2x^2$
Thus, the required equation of the parabola is $y = 2x^2$.

9. उद्गमबिन्दु भएर जाने एउटा घन समीकरणको वक्रमा एउटा बिन्दु (2, 32) पर्दछ भने सो घन समीकरण पत्ता लगाउनुहोस् ।
The curve of a cubic equation passes through the origin contains the point (2, 32). Find the cubic equation.

⇨ Here, the cubic equation passes through the origin.
So, the cubic equation is $y = ax^3$ ....(i)
Equation (i) passes through (2, 32)
So, $32 = a \times 2^3$
or, $32 = a \times 8$
$\therefore\ a = 4$
Now, from (i); $y = 4x^3$
Thus, the required cubic equation is $y = 4x^3$.

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10. **$f(x) = x^2 - 1$ र $f(x) = 3$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।**
What will be the points of intersection of the curve $f(x) = x^2 - 1$ and $f(x) = 3$? **[SEE MODEL 2076]**

⇨ Here, $f(x) = x^2 - 1$ and $f(x) = 3$

So, $3 = x^2 - 1$

or, $x^2 = 4$

or, $x^2 = (\pm 2)^2$

$\therefore \quad x = \pm 2$

Thus, the points of intersection are (– 2, 3) and (2, 3).

11. **$f(x) = x^2 - 5x$ र $f(x) = -6$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।**
What will be the points of intersection of the curve $f(x) = x^2 - 5x$ and $f(x) = -6$?

⇨ Here, $f(x) = x^2 - 5x$ .... (i) and $f(x) = -6$ ....(ii)

So, from (i) and (ii), $x^2 - 5x = -6$

or, $x^2 - 5x + 6 = 0$

or, $x^2 - 2x - 3x + 6 = 0$

or, $x(x - 2) - 3(x - 2) = 0$

or, $(x - 2)(x - 3) = 0$

Either, $x - 2 = 0$ $\quad\therefore \quad x = 2$

or, $x - 3 = 0$ $\quad\therefore \quad x = 3$

Thus, the required points are (2, – 6) and (3, – 6).

C. LONG QUESTIONS

MODEL 1

1. **वर्ग समीकरण $x^2 - 3x = 10$ लाई लेखाचित्रद्वारा हल गर्नुहोस् । (Solve graphically the quadratic equation $x^2 - 3x = 10$.)** **[SEE 2075 R]**

⇨ Here, $x^2 - 3x - 10 = 0$

or, $x^2 = 3x + 10 = y$ (let)

$\therefore \quad y = x^2$ .... (i)

$\therefore \quad y = 3x + 10$ .... (ii)

From (i), $y = x^2$

x	2	1	0	– 1	– 2
y	4	1	0	1	4

(2, 4), (1, 1), (0, 0), (– 1, 1), (– 2, 4)

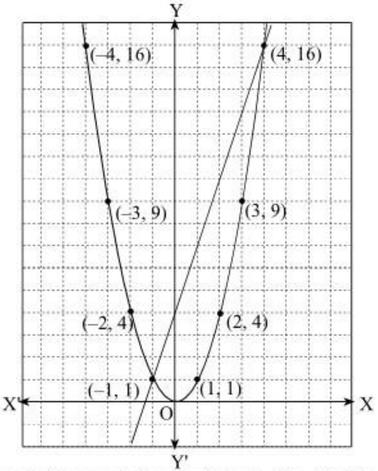
From (ii), $y = 3x + 10$

x	– 3	– 1	– 2
y	1	7	4

(3, 1), (– 1, 7), (– 2, 4)

From the graph the points of intersection are;
(– 2, 4) & (5, 25) whose x-co-ordinates are 5, – 2.

Thus, the solution of given equation is 5, – 2.



2. **वर्ग समीकरण $x^2 - 2x - 3 = 0$ लाई लेखाचित्रद्वारा हल गर्नुहोस् । (Solve graphically the quadratic equation $x^2 - 2x - 3 = 0$.)** **[SEE 2075 R', 2073 R, 2069 R']**

⇨ Here, $x^2 - 2x - 3 = 0$

or, $x^2 = 2x + 3$

Let, $y = x^2 = 2x + 3$

So, $y = x^2$ …… (i)

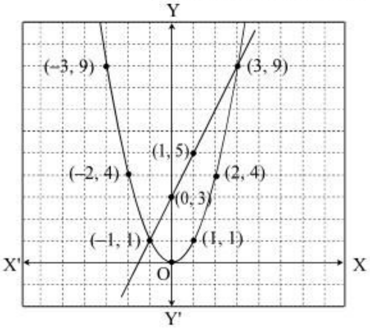
x	0	1	2	– 1	– 2	3	– 3
y	0	1	4	1	4	9	9

$y = 2x + 3$ ….. (ii)

x	1	0	3	– 1
y	5	3	9	1

From graph paper curve line and straight line are intersect at the point (– 1, 1) & (3, 9).

Thus, value of x are – 1 or 3.



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3. **लेखाचित्र विधिबाट हल गर्नुहोस् (Solve by graphical method): $x^2 + x - 2 = 0$** **[2073 S', 2071 S]**

⇨ Here, $x^2 + x - 2 = 0$

or, $x^2 = 2 - x$

Let $y = x^2 = 2 - x$

Taking $y = x^2$ then,

x	– 2	– 1	0	1	2
y	4	1	0	1	4

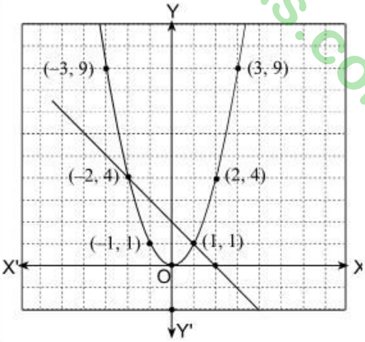
Thus, (– 2, 4), (– 1, 1), (0, 0), (1, 1), (2, 4) are the obtained points.

Taking $y = 2 - x$ then,

x	2	1	0	– 2
y	0	1	2	4

Thus, (2, 0), (1, 1), (0, 2) are the obtained points.

Graphical representation of above equations is shown alongside.



From the graph, the point of intersection are (1, 1) and (– 2, 4).

Thus, x = 1 or – 2 is the solution.

4. **वर्ग समीकरण $x^2 - 3x - 4 = 0$ लाई लेखा चित्रद्वारा हल गर्नुहोस् ।**

Solve graphically the quadratic equation $x^2 - 3x - 4 = 0$. [2073 S]

⇨ Here, $x^2 - 3x - 4 = 0$

or, $x^2 = 3x + 4$

Let, $y = x^2 = 3x + 4$ then, $y = x^2$ ....(i)

and $y = 3x + 4$ ....(ii)

From (i), $y = x^2$

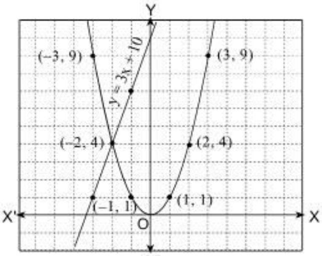
x	– 3	– 2	– 1	0	1	2	3	4
y	9	4	1	0	1	4	9	16

Thus, (– 3, 9) (– 2, 4), (– 1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16) are the obtained points.

From (ii), $y = 3x + 4$

x	0	1	– 1	4	– 2	– 3	– 4
y	4	7	1	16	– 2	– 5	– 8

Thus, (0, 4), (1, 7), (– 1, 1), (4, 16), (– 2, – 2), (– 3, – 5), (– 4, – 8) are the obtained points.



From the graph, the points of intersection are (– 1, 1) and (4, 16).

So, the solution is x = 4, – 1

Thus, x = 4, – 1 is the solution.

5. **वर्ग समीकरण $x^2 + 7x + 12 = 0$ लाई लेखाचित्रद्वारा हल गर्नुहोस् ।**

(Solve graphically the quadratic equation $x^2 + 7x + 12 = 0$.) **[2070 R]**

⇨ Here, given equation is $x^2 + 7x + 12 = 0$

or, $x^2 = -7x - 12$

Let $y = x^2 = -7x - 12$ then, $y = x^2$ ……. (i)

$y = -7x - 12$ ……….. (ii)

From (i); $y = x^2$

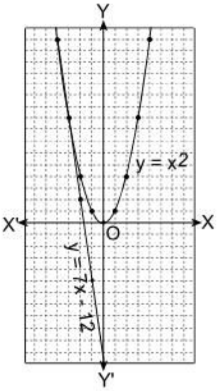
x	3	2	1	0	– 1	– 2	– 3
y	9	4	1	0	1	4	9

(3, 9), (2, 4), (1, 1), (0, 0), (– 1, 1), (– 2, 4), (– 3, 9)

From (ii); $y = -7x - 12$

x	0	– 1	– 3	– 4
y	–12	– 5	9	16

(0, – 12), (– 1, – 5), (– 3, 9) and (– 4, 16)



From the graph paper, (– 3, 9) and (– 4, 16) are the point of intersection. Thus, x = – 3 or – 4.

6. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 + 2x - 3 = 0$** **[2075 R_2, 2074 S', 2058 S, 2063 S, 2065E, 2066 R']**

⇨ Here, Quadratic equations $x^2 + 2x - 3 = 0$.

This can be written as $x^2 = -2x + 3$

Let, $x^2 = -2x + 3 = y$

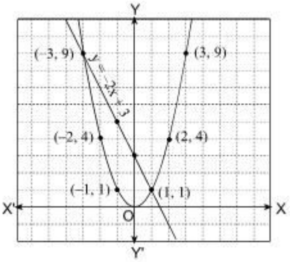
Hence, $y = x^2$ ...... (i) and $y = -2x + 3$ ..... (ii)

The different values of x and y from equation (i) :

x	– 3	– 2	– 1	0	1	2	3
y	9	4	1	0	1	4	9

The diffemt values of x and y from equation (ii) :

x	– 1	0	1
y	5	3	1



Now, plotting the points of equations (i) and (ii) and joining them.

From the graph,

The point of intersection of equation (i) and (ii) are (– 3, 9) and (1, 1) whose x-coordinates are – 3 and 1.

Thus, the solution of quadratic equation $x^2 + 2x - 3 = 0$ are – 3, 1.

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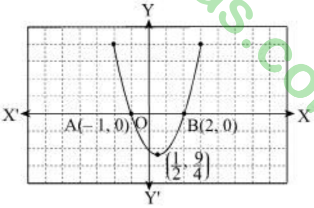
7. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - x - 2 = 0$** **[SEE 2074 R', 2061 R, 2063 M, 2068 R']**

⇨ Here, let $y = x^2 - x - 2$

Comparing it with $y = ax^2 + bx + c$ then we get, $-\frac{b}{2a} = \frac{-(-1)}{2.1} = \frac{1}{2}$

When $x = \frac{1}{2}$ then $y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 2 = \frac{1}{4} - \frac{1}{2} - 2 = \frac{-9}{4}$

$\therefore$ Vertex $= \left(\frac{1}{2}, \frac{-9}{4}\right)$



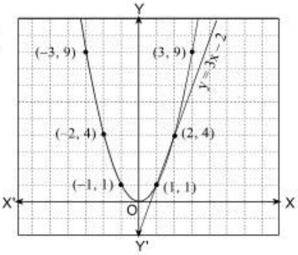
Making the value table,

x	3	2	1	0	– 1	– 2
y	4	0	– 2	– 2	0	4

Graphing these ordered pairs, we get the following graph.
As the graph intersects X-axis at (– 1, 0) and B(2, 0)
Thus, x = – 1 and x = 2 are the solution of the given quadratic equation.

8. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - 3x + 2 = 0$** **[2071 R', 2073 R', 2063 R', 2065 S]**

⇨ Here, $x^2 - 3x + 2 = 0$
Let $x^2 = 3x - 2 = y$
So that, $y = x^2$ .... (i) & $y = 3x - 2$ .... (ii)
From (i), $y = x^2$



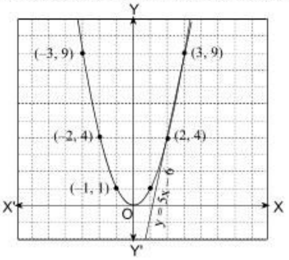
x	2	1	0	– 1	– 2
y	4	1	0	1	4

From (ii) $y = 3x - 2$

x	0	1	2
y	– 2	1	4

From the graph $y = x^2$ and $y = 3x - 2$ are intersected at the points whose x-co-ordinates are 1 and 2.
Thus, the solution of given equation is x = 1, 2.

9. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - 5x + 6 = 0$** **[2072 R', 2064 S]**



⇨ Here, $x^2 - 5x + 6 = 0$
Let $x^2 = 5x - 6 = y$
So that, $y = x^2$ .... (i) and $y = 5x - 6$ .... (ii)
From (i), $y = x^2$

x	2	1	0	– 1	– 2
y	4	1	0	1	4

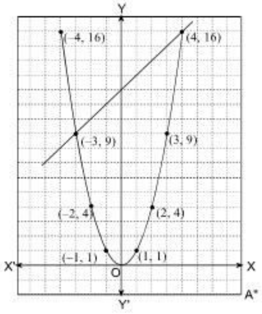
From (ii) $y = 5x - 6$

x	0	1	2
y	– 6	– 1	4

From the graph $y = x^2$ and $y = 5x - 6$ are intersected at the points whose x-co-ordinates are 2 and 3.
Thus, the solution of given equation is 2, 3.

10. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - x - 12 = 0$** **[2065 R]**

⇨ Here, $x^2 - x - 12 = 0$ or, $x^2 = x + 12 = y$
$\therefore$ $y = x^2$ .... (i) and $y = x + 12$ .... (ii)
From (i)



x	± 2	± 1	0	± 3	± 4
y	4	1	0	9	16

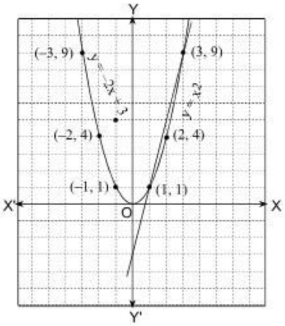
From (ii)

x	0	– 2
y	12	10

From (i) and (ii) drawing the graph.
From the graph the points of intersection are (– 3, 9) and (4, 16).
Thus, the solution of given equation is x = – 3 or 4.

11. **लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - 4x + 3 = 0$** **[2067 S]**

⇨ Here, $x^2 - 4x + 3 = 0$
Let, $x^2 = 4x - 3 = y$ then.
$y = x^2$ .... (i) and $y = 4x - 3$ .... (ii)
From equation (i), $y = x^2$



x	0	±1	±2	±3
y	0	1	4	3

(0, 0), (– 1, 1), (1, 1) (2, 4), (– 2, 4), (3, 9) (– 3, 9)
From equation (ii) $y = 4x - 3$

x	0	1
y	– 3	1

(0, – 3), (1, 1)
Now, graphing the equation
From the graph the point of intersections are (1, 1) and (3, 9).
Thus, the solution of given equation is x = 1 or 3.

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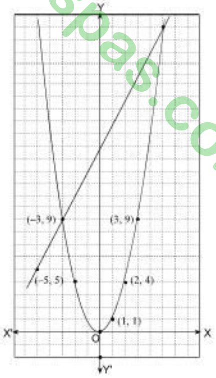
12. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $x^2 - 2x - 15 = 0$ **[2068 R]**



⇨ Here, $x^2 - 2x - 15 = 0$

or, $x^2 = 2x + 15 = y$ (say)

So, $y = x^2$ …….. (i) and $y = 2x + 15$ ……… (ii)

From (i), $y = x^2$

x	± 1	±2	±3	±4	±5
y	1	4	9	16	25

From (ii) $y = 2x + 15$

x	0	– 5
y	15	5

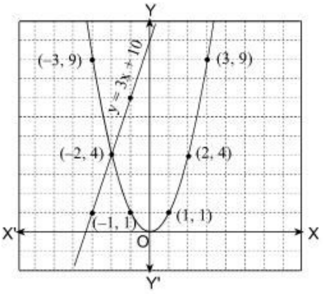
Now, graphing them.

From the graph the point of intersection are (– 3, 9) and (5, 25).

Thus, the solution of given equation is x = – 3 and 5.

13. वर्ग समीकरण $x^2 - 3x - 10 = 0$ लाई लेखाचित्र विधिद्वारा हल गर्नुहोस् । साथै पारावोला र सीधा रेखाबीचमा प्रतिच्छेदन बिन्दुहरू समेत पत्ता लगाउनुहोस् ।

Solve graphically the quadratic equation $x^2 - 3x - 10 = 0$. Also find the intersecting points of parabola and straight line. **[2064 R]**



⇨ Here, $x^2 - 3x - 10 = 0$

or, $x^2 = 3x + 10 = y$ (let)

∴ $y = x^2$ ……….. (i)

∴ $y = 3x + 10$ ……… (ii)

From (i), $y = x^2$

x	2	1	0	– 1	– 2
y	4	1	0	1	4

(2, 4), (1, 1), (0, 0), (– 1, 1), (– 2, 4)

From (ii), $y = 3x + 10$

x	– 3	– 1	– 2
y	1	7	4

(– 3, 1), (– 1, 7), (– 2, 4)

From the graph the points of intersection are (– 2, 4) & (5, 25) whose x-co-ordinates are 5, – 2.

Thus, the solution of given equation is 5, – 2.

MODEL 2

14. लेखाचित्र विधिबाट हल गर्नुहोस् (Solve graphically) : $y = x^2$; $y = 2 - x$ **[2071 R]**

⇨ Here, $y = x^2$ and $y = 2 - x$

Value table of $y = x^2$

x	– 2	– 1	0	1	2
y	4	1	0	1	4

(– 2, 4), (– 1, 1), (0, 0), (1, 1), (2, 4)

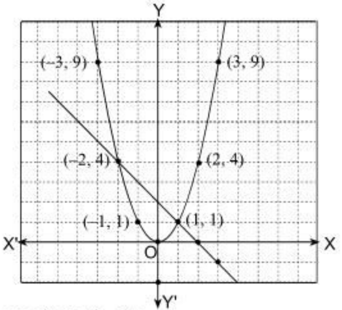
Value table of $y = 2 - x$

x	1	2	3
y	1	0	– 1

(1, 1), (2, 0), (3, – 1)

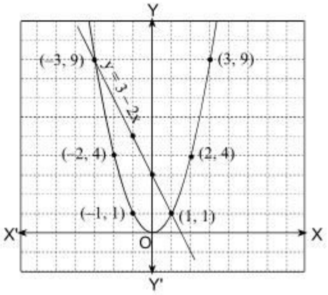
Graph of above equations is shown alongside.

From the graph (– 2, 4) and (1, 1) are the solution.



15. तलका समीकरणहरू हल गर्नुहोस् र साथै प्रतिच्छेदन बिन्दु पनि देखाउनुहोस्: $y = x^2$, $y = 3 - 2x$

Solve the following equations graphically and also indicate the point of intersection: $y = x^2$, $y = 3 - 2x$ **[2060 S]**



⇨ Here given equations :

$y = x^2$ ….. (i) and $y = 3 - 2x$ …… (ii)

From equation (i) $y = x^2$

x	– 3	– 2	– 1	0	1	2	3
y	9	4	1	0	1	4	9

Again, from equation (ii) $y = 3 - 2x$

x	– 1	0	1
y	5	3	1

Now, putting the points of x and y from equation (i) and (ii),

Here, from the graph, the points of intersection of the equation (i) and (ii) are (– 3, 9) and (1, 1).

Thus, the solutions of given equations are (– 3, 9) and (1, 1).

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16. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically): $y = x^2$ and $y = 2x - 1$ [2064 R']

⇨ Here, $y = x^2$ .... (i) and

$y = 2x - 1$ .... (ii)

From (i), $y = x^2$

x	2	1	0	– 1	– 2
y	4	1	0	1	4

(2, 4), (1, 1), (0, 0), (– 1, 1), (– 2, 4)

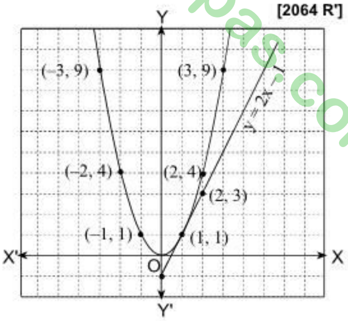
From (ii), $y = 2x - 1$

x	1	0	2
y	1	– 1	3

(1, 1), (0, – 1), (2, 3)

From the graph $y = x^2$ and $y = 2x - 1$ are intersected at the points (1, 1)

Thus, the solution of given equation is (1, 1).



QUESTIONS FROM CDC TEXTBOOK

1.5 वर्ग समीकरण र लेखाचित्र (QUADRATIC EQUATION AND GRAPHS)

EXERCISE 1.5

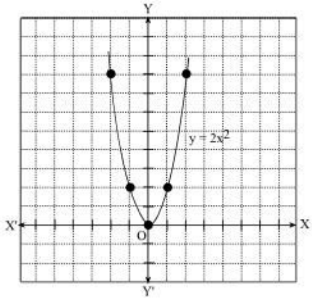
1. निम्न लिखित समीकरणको लेखाचित्र खिच्नुहोस् । (Draw the graph of following equation.)

(a) $y = 2x^2$

⇨ Here, $y = 2x^2$, the vertex of this equation is (0, 0)

x	– 2	– 1	0	1	2
y	8	2	0	2	8

Now, the graph of given equation,



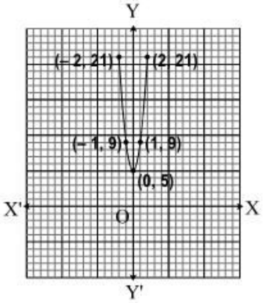
(b) $y = 4x^2 + 5$

⇨ Here,

$y = 4x^2 + 5$, the vertex of this equation is (0, 5)

x	0	1	– 1	2	– 2
y	5	9	9	21	21

Now, the graph of above equation;



(c) $y = x^2 - 1$

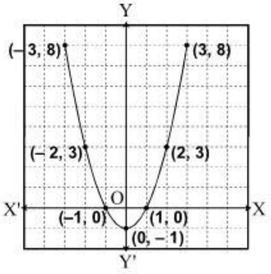
⇨ Here, $y = x^2 - 1$

We have, the equation of parabola $y = ax^2 + c$ has the vertex (0, c).

So, the vertex of $y = x^2 - 1$ is (0, – 1)

x	– 3	– 2	– 1	0	1	2	3
y	8	3	0	– 1	0	3	8

The graph of $y = x^2 - 1$ is shown alongside.



(d) $y = \frac{1}{4}(x^2 + 2)$

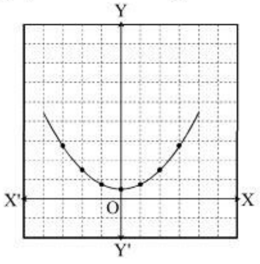
⇨ Here, $y = \frac{1}{4}(x^2 + 2) = \frac{1}{4}x^2 + \frac{1}{2} = \frac{1}{4}(x - 0)^2 + \frac{1}{2}$

Comparing it with $y = a(x - h)^2 + k$ then,

Vertex = (h, k) = $\left(0, \frac{1}{2}\right)$ = (0, 0.5)

x	– 3	– 2	– 1	0	1	2	3
y	2.75	1.5	0.75	0.5	0.75	1.5	2.75

The graph is shown alongside.



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(e) $y = x^2 + x + 6$

⇨ Here, $y = x^2 + x + 6$

Comparing $y = x^2 + x + 6$ with $y = ax^2 + bx + c$ then

$a = 1, b = 1$ and $c = 6$

We have,

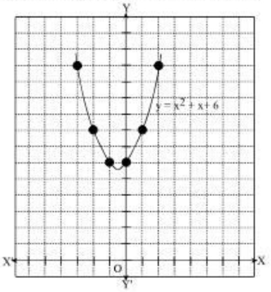
$$\text{Co-ordinates of vertex} = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$= \left(\frac{-1}{2}, \frac{4 \times 1 \times 6 - 1}{4}\right)$$

$$\therefore \text{ Vertex} = \left(\frac{-1}{2}, \frac{23}{4}\right)$$

x	– 3	– 2	– 1	0	1	2
y	12	8	6	6	8	12

Now, the graph of above equation;



(f) $y = x^2 + x - 2$

⇨ Here, $y = x^2 + x - 2$

$$\text{or, } y = x^2 + 2 \cdot x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 2$$

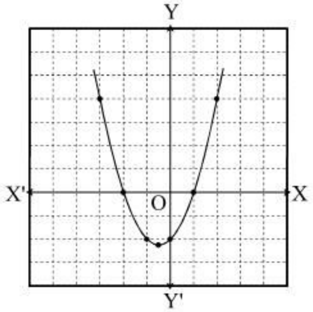
$$\therefore y = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$$

Comparing it with $y = a(x - h)^2 + k$ then,

$$\text{Vertex} = (h, k) = \left(-\frac{1}{2}, -\frac{9}{4}\right) = (-0.5, -2.25)$$

x	– 3	– 2	– 1	– 0.5	0	1	2
y	4	0	– 2	– 2.25	– 2	0	4

The graph is shown below:



2. **निम्न लिखित समीकरणको लेखाचित्र खिच्नुहोस् (Draw the graph of following equation):**

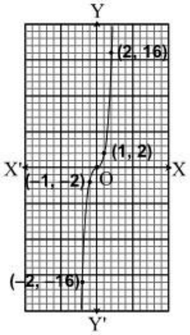
(a) $y = 2x^3$

⇨ Here, $y = 2x^3$

The vertex of $y = ax^3$ is (0, 0).

x	– 3	– 2	– 1	0	1	2	3
y	– 54	– 16	– 2	0	2	16	54

The graph is shown below:



(b) $y = 3x^3 - 10$

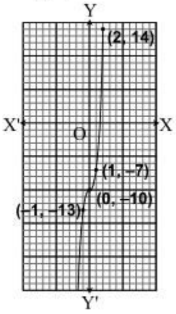
⇨ Here, $y = 3x^3 - 10$

We have, $y = ax^3 + c$ has vertex at (0, c).

So, the vertex of $y = 3x^3 - 10$ is (0, – 10).

x	– 3	– 2	– 1	0	1	2	3
y	– 91	– 34	– 13	–10	– 7	14	71

The graph is show alongside:



(c) $y = 4x^3 - 15$

⇨ Here, $y = 4x^3 - 15$

We have, $y = ax^3 + c$ has vertex at (0, c).

So, the vertex of $y = 4x^3 - 15$ is (0, – 15).

x	– 3	– 2	– 1	0	1	2	3
y	– 123	– 47	– 19	– 15	– 11	17	93

The graph is shown alongside.

(d) $y = \frac{1}{2}x^3 + \frac{1}{2}$

⇨ Here, $y = \frac{1}{2}x^3 + \frac{1}{2}$

We have, $y = ax^3 + c$ has vertex at (0, c).

So, the vertex of $y = \frac{1}{2}x^3 + \frac{1}{2}$ is $\left(0, \frac{1}{2}\right) = (0, 0.5)$

x	– 3	– 2	– 1	0	1	2	3
y	– 13	– 3.5	0	0.5	1	4.5	14

Now, the graph is shown alongside:

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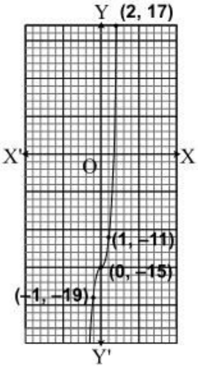
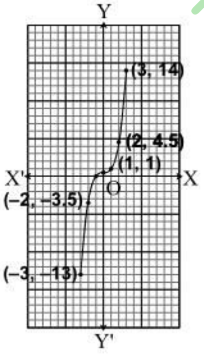




3. निम्न लिखित समीकरणको लेखाचित्र खिच्नुहोस् (Draw the graph of following equation):

(a) $y = 4x^2 + 8x + 5$ and $x + y = 3$

⇨ Here, $y = 4x^2 + 8x + 5$ & $x + y = 3$

$y = 4x^2 + 8x + 5$............ (1)

Comparing $y = 4x^2 + 8x + 5$ with $y = ax^2 + bx + c$ then, a = 4, b = 8 and c = 5

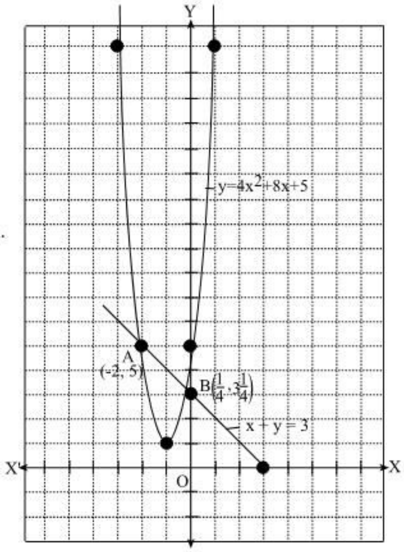
Now, vertex $= \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(\frac{-8}{2 \times 4}, \frac{4 \times 4 \times 5 - 8^2}{4 \times 4}\right)$

$\therefore$ Vertex = (–1, 1)

x	– 3	– 2	– 1	0	1
y	17	5	1	5	17

$x + y = 3$ ....(2)

x	0	3
y	3	0



A (– 2, 5) & B $\left(-\frac{1}{4}, 3\frac{1}{4}\right)$ are the solution points from graph.

From equation (2); $y = 3 – x$

Substituting the value of y in equation (1)

$3 – x = 4x^2 + 8x + 5$

or, $4x^2 + 9x + 2 = 0$

or, $4x^2 + 8x + x + 2 = 0$

or, $4x (x + 2) + 1 (x + 2) = 0$

or, $(x + 2) (4x + 1) = 0$

$\therefore$ Either, $x = – 2$

or, $x = -\frac{1}{4}$

Substituting these values in equation (2); $y = 3 + 2 = 5$

Again, $y = 3 + \frac{1}{4} = \frac{13}{4} = 3\frac{1}{4}$

Thus, solution points are (– 2, 5) & $\left(-\frac{1}{4}, 3\frac{1}{4}\right)$

(b) $y = x^2 – x – 3$ and $x = y$

⇨ Here, $y = x^2 – x – 3$ and $x = y$

or, $y = x^2 - 2 . x . \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 3 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 3$

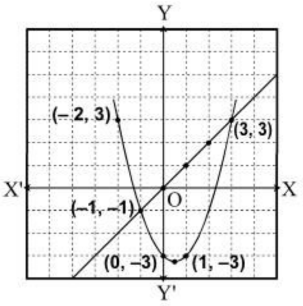
$\therefore$ $y = \left(x - \frac{1}{2}\right)^2 - \frac{13}{4}$

Comparing it with $y = a(x – h)^2 + k$ then,

Vertex = (h, k) $= \left(\frac{1}{2}, -\frac{13}{4}\right) = (0.5, – 3.25)$

For $y = x^2 – x – 3$

x	– 2	– 1	0	0.5	1	2	3
y	3	– 1	– 3	– 3.25	– 3	– 1	3



For the line x = y the points are (1, 1), (2, 2) etc.

Now, the graph is shown alongside:

From the graph, the point of intersection of parabola and straight line are: (– 1, – 1) and (3, 3)

Thus, (– 1, – 1) & (3, 3) is the solution.

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(c) $y = 6x^2 - 2x - 15$ **and** $y = 4x - 3$

⇨ Here, $y = 6x^2 - 2x - 15$ and $y = 4x - 3$

Taking

$$y = 6x^2 - 2x - 15$$

$$= 6\left(x^2 - \frac{1}{3}x\right) - 15$$

$$= 6\left\{x^2 - 2.x.\frac{1}{6} + \left(\frac{1}{6}\right)^2\right\} - 6 \times \left(\frac{1}{6}\right)^2 - 15$$

$$= 6\left(x - \frac{1}{6}\right)^2 - 6 \times \frac{1}{36} - 15$$

$$\therefore \quad y = 6\left(x - \frac{1}{6}\right)^2 - \frac{91}{6}$$

Comparing it with $y = a(x - h)^2 + k$ then,

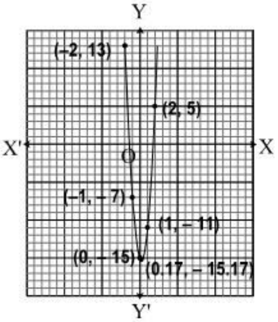
$$\text{Vertex} = (h, k) = \left(\frac{1}{6}, -\frac{91}{6}\right)$$

$$= (0.17, -15.17)$$

x	– 2	– 1	0	0.17	1	2	3
y	13	– 7	– 15	– 15.17	– 11	5	33

Again, taking $y = 4x - 3$

x	0	1
y	– 3	1



From the graph, the point of intersections are (– 1, – 7) and (2, 5)

Thus, (– 1, – 7) and (2, 5) is the solution.

(d) $y = x^2 + 2x - 8$ **and** $y = -5$

⇨ Here, $y = x^2 + 2x - 8$ & $y = -5$

$y = x^2 + 2x - 8$ ....(1)

Comparing $y = x^2 + 2x - 8$ with $y = ax^2 + bx + c$

then, $a = 1$, $b = 2$ & $c = -8$

We know that;

$$\text{Vertex} = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$$

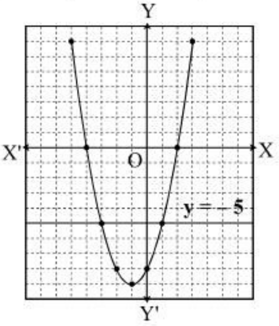
$$= \left\{\frac{-2}{2 \times 1}, \frac{4 \times 1 \times (-8) - 2^2}{4 \times 1}\right\}$$

$$\therefore \text{ Vertex} = (-1, -9)$$

x	– 5	– 4	– 3	– 2	– 1	0	1	2	3
y	7	0	– 5	– 8	– 9	– 8	– 5	0	7

$y = -5$ ....(2)

x	– 3	1
y	– 5	– 5



From graph intersected points are;

(–3, –5) & (1, –5).

Substituting the value of y in equation (1)

$-5 = x^2 + 2x - 8$

or, $x^2 + 2x - 3 = 0$

or, $x^2 + 3x - x - x = 0$

or, $x(x + 3) - 1(x + 3) = 0$

or, $(x + 3)(x - 1) = 0$

Either, $x + 3 = 0$ or, $x - 1 = 0$

$\therefore x = -3$ $\therefore x = 1$

Thus, solution points are (– 3, – 5) & (1, – 5).

4. तलका वर्ग समीकरणलाई लेखाचित्र विधिद्वारा हल गर्नुहोस् । (Solve the following quadratic equations by graphical method.)

(a) $x^2 + 2x - 3 = 0$

⇨ Here, given quadratic equation is; $x^2 + 2x - 3 = 0$.

This can be written as $x^2 = -2x + 3$

Let, $x^2 = -2x + 3 = y$

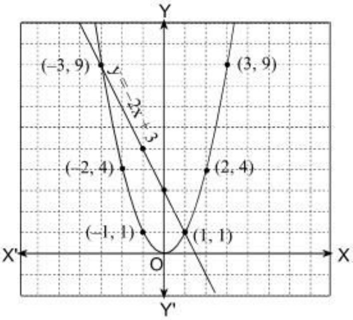
Hence, $y = x^2$ ...... (i) and $y = -2x + 3$ ..... (ii)

The different values of x and y from equation (i) :

x	– 3	– 2	– 1	0	1	2	3
y	9	4	1	0	1	4	9

The different values of x and y from equation (ii) :

x	– 1	0	1
y	5	3	1



Now, plotting the points of equations (i) and (ii) and joining them.

From the graph,

The point of intersection of equation (i) and (ii) are (– 3, 9) and (1, 1) whose x-coordinates are – 3 and 1.

Thus, the solution of quadratic equation $x^2 + 2x - 3 = 0$ are – 3, 1.

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(b) $3x^2 + 5x + 2 = 0$

⇨ Here, $3x^2 + 5x + 2 = 0$

Let $y = 3x^2$ ....(1)

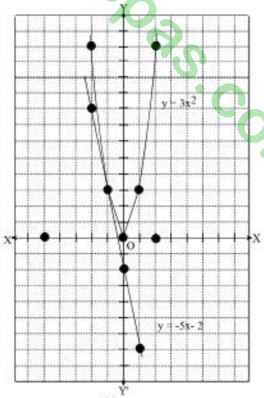
Given equation reduces to $y = -5x - 2$ ....(2)

From equation (1)

x	– 2	– 1	0	1	2
y	12	3	0	3	12

From equation (2)

x	– 2	0	1
y	8	– 2	– 7



From graph, points of intersections are (– 1, 3) and $\left(-\frac{2}{3}, 1\frac{1}{3}\right)$.

Thus, the soluion is $x = -1$ or $-\frac{2}{3}$.

(c) $x^2 - 5x + 6 = 0$

⇨ Here, $x^2 - 5x + 6 = 0$

Let $x^2 = 5x - 6 = y$

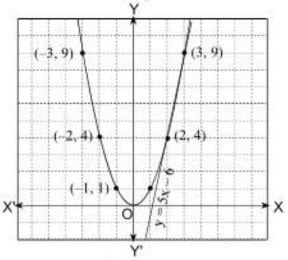
So that, $y = x^2$ .... (i) and $y = 5x - 6$ .... (ii)

From (i), $y = x^2$

x	2	1	0	– 1	– 2
y	4	1	0	1	4

From (ii) $y = 5x - 6$

x	0	1	2
y	– 6	– 1	4



From the graph $y = x^2$ and $y = 5x - 6$ are intersected at the points (2, 4) and (3, 9) whose x-co-ordinates are 2 and 3.

Thus, the solution of given equation is 2, 3.

(d) $x^2 - 4x + 4 = 0$

⇨ Here, $x^2 - 4x + 4 = 0$

Let $y = x^2$ ....(1)

∴ $y - 4x + 4 = 0$

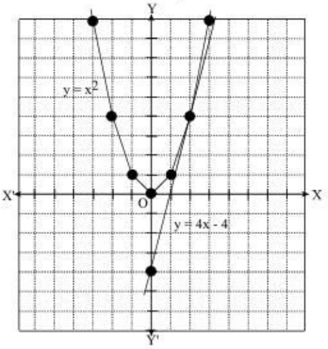
or, $y = 4x - 4$ ....(2)

From equation (1); its vertex is (0, 0)

x	– 3	– 2	– 1	0	1	2	3
y	9	4	1	0	1	4	9

From equation (2)

x	0	1
y	– 4	0

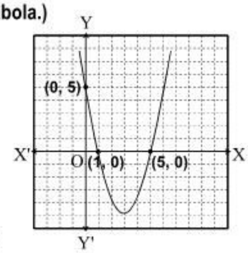


From graph, point of contact is (2, 4)

Thus, the solution is $x = 2$.

5. तल दिइएका पाराबोलाको समीकरण पत्ता लगाउनुहोस् । (Find the equation of following parabola.)

(a)



⇨ Here, the points at x - axis are; (1, 0) and (5, 0).

So, $x = 1$ and $x = 5$ is the solution.

Equation having solution $x = 1$ and $x = 5$ is

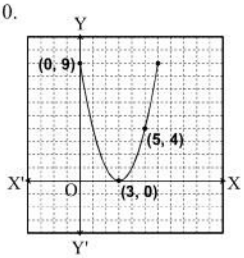
$(x - 1)(x - 5) = 0$

or, $x^2 - 5x - x + 5 = 0$

∴ $x^2 - 6x + 5 = 0$

Thus, the required quadratic equation is $x^2 - 6x + 5 = 0$.

(b)



⇨ Here, the point at x - axis is (3, 0)

So, $x = 3$ and $x = 3$ is the solution of the parabola.

Now, Equation having solution $x = 3$ and $x = 3$ is,

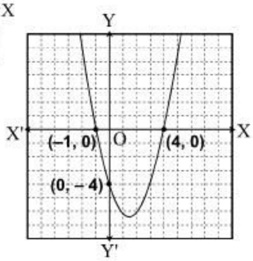
$(x - 3)(x - 3) = 0$

or, $(x - 3)^2 = 0$

or, $x^2 - 6x + 9 = 0$

Thus, the required quadratic equation is $x^2 - 6x + 9 = 0$.

(c)



⇨ Here, the points at x - axis are;

(– 1, 0) and (4, 0)

So, $x = -1$ and $x = 4$ is the solution.

Now, equation having solution $x = -1$ and $x = 4$ is;

$(x + 1)(x - 4) = 0$

or, $x^2 - 4x + x - 4 = 0$

∴ $x^2 - 3x - 4 = 0$

Thus, $x^2 - 3x - 4 = 0$ is the required quadratic equation.

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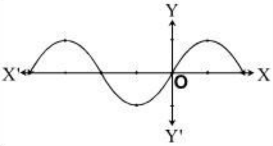
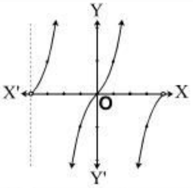
निरन्तरता (Continuity)

UNIT II

1. सीमान्त मान र निरन्तरता
Limit and Continuity

KEY POINTS

2.1 निरन्तरताको धारणा (Concept of continuity)

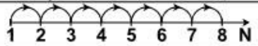
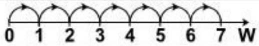
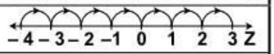
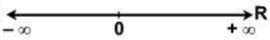
निरन्तरता Continuity				
विच्छिन्नता Discontinuity				

यदि कुनै फलनको लेखाचित्र खिच्दा सिसाकलमलाई नउचालीकन लगातार खिच्न सकिन्छ भने यसलाई निरन्तरता भनिन्छ । If the graph of a function can be drawn without lifting the pencil from the paper then it is called continuity.

निरन्तरताका उदाहरणहरू (Examples of continuity)

- विरुवाको वृद्धि (Growth of plant)
- पानीको बहाव (Water flow)
- चलिरहेको गाडी (Bus movement)
- वास्तविक सङ्ख्याहरूको सङ्ख्यारेखा (Number line of real numbers) etc.

2.2 सङ्ख्याहरूको समूहको क्रममा निरन्तरता (Continuity in the order of set of numbers)

सङ्ख्याहरू (Numbers)	सङ्ख्यारेखा (Number line)
प्राकृतिक सङ्ख्याहरू (Natural numbers): $\{1, 2, 3, 4, 5, ...\}$	
पूर्ण सङ्ख्याहरू (Whole numbers): $\{0, 1, 2, 3, 4, 5, ...\}$	
पूर्णाङ्कहरू (Integers): $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$	
वास्तविक सङ्ख्याहरू (Real Numbers): प्राकृतिक सङ्ख्याहरू (Natural numbers), पूर्ण सङ्ख्याहरू (Whole numbers), पूर्णाङ्कहरू (Integers), आनुपातिक सङ्ख्याहरू (Rational Numbers), अनानुपातिक सङ्ख्याहरू (Irrational numbers)	

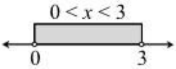
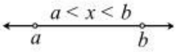
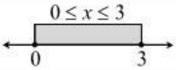
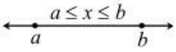
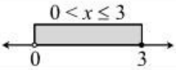
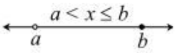
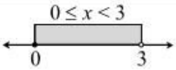
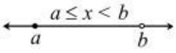
(a) प्राकृतिक सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन । The number line of natural numbers does not show continuity.

(b) पूर्ण सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन । (The number line of whole numbers does not show continuity.)

(c) पूर्णाङ्कहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन । (The number line of integers does not show continuity.)

(d) वास्तविक सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरताको प्रतिनिधित्त्व गर्दछ । किनकी दुईओटा वास्तविक सङ्ख्याहरूको बीचमा अनगिन्ती वास्तविक सङ्ख्याहरू हुन्छन् । The number line of real numbers shows the continuity because there are many real numbers between any two real numbers.

2.3 अन्तराल (Interval)

अन्तराल सङ्केत Interval Notation		समूह सङ्केत Set Notation	लेखाचित्र वा सङ्ख्या रेखा Graph or Number Line		अन्तरालको नाम Interval name
(a)	(a, b)	$\{x : a < x < b\}$			खुल्ला अन्तराल Open Interval
(b)	$[a, b]$	$\{x : a \le x \le b\}$			बन्द अन्तराल Closed Interval
(c)	$(a, b]$	$\{x : a < x \le b\}$			बायाँ खुल्ला अन्तराल Left open Interval
(d)	$[a, b)$	$\{x : a \le x < b\}$			दायाँ खुल्ला अन्तराल Right open Interval

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(a) खुल्ला अन्तराल : अन्तिम बिन्दुहरू a र b दुवै समावेश नहुने अन्तराललाई खुल्ला अन्तराल भनिन्छ । यसलाई (a, b) ले जनाइन्छ ।
Open Interval: An interval not containing both the end points a and b is known as an open interval. It is denoted by (a, b).

(b) बन्द अन्तराल: अन्तिम बिन्दुहरू a र b दुवै समावेश हुने अन्तराललाई बन्द अन्तराल भनिन्छ । यसलाई $[a, b]$ ले जनाइन्छ ।
Closed Interval: An interval containing both the end points a and b is known as a closed interval. It is denoted by $[a, b]$.

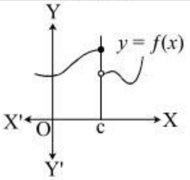
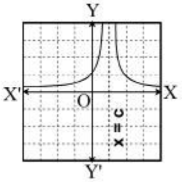
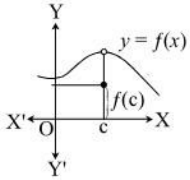
(c) बायाँ खुल्ला अन्तराल: अन्तिम बिन्दु a नपर्ने तर अन्तिम बिन्दु b पर्ने अन्तराललाई बायाँ खुल्ला अन्तराल भनिन्छ । यसलाई $(a, b]$ ले जनाइन्छ ।
Left open Interval: An interval not containing the end point a but containing the end point b is known as the left open interval. It is denoted by $(a, $b].

(d) दायाँ खुल्ला अन्तराल : अन्तिम बिन्दु a पर्ने तर अन्तिम बिन्दु b नपर्ने अन्तराललाई दायाँ खुल्ला अन्तराल भनिन्छ । यसलाई $[a, b)$ ले जनाइन्छ ।
Right open Interval: An interval containing the end point a but not containing the end point b is known as the right open interval. It is denoted by $[a, b)$.

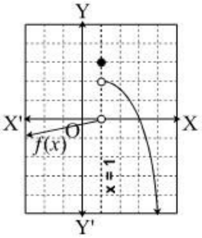
2.4 लेखाचित्रमा विच्छिन्नताको खोजी (Investigation of discontinuity in graph)

यदि कुनै पनि फलनको लेखाचित्रको कुनै बिन्दुमा छलाङ (Jump), छिद्र (hole), रिक्त (gap), उछाल (cusp) र टुटेको (break) छ भने सो बिन्दुमा विच्छिन्नता छ भनिन्छ ।

If the graph of any function has 'jump', hole, gap, cusp and break in a point then the function is said to be discontinuous at that point.

Graph I	Graph II	Graph III
		
यो फलन $x = c$ मा विच्छिन्न छ । यसलाई छलाङ (Jump) विच्छिन्नता भनिन्छ । This function is discontinuous at $x =$ c. This is jump discontinuity	यो फलन $x = c$ मा विच्छिन्न छ । यस्तो विच्छिन्नतालाई असीमित विच्छिन्नता भनिन्छ । This function is discontinuous at $x = c$. This type of discontinuity is called infinite discontinuity.	यो फलन $x = c$ मा विच्छिन्न छ । यस्तो विच्छिन्नतालाई निष्कासन विच्छिन्नता भनिन्छ । This function is discontinuous at $x = c$. This type of discontinuity is called removable discontinuity.

2.5 बायाँबाट सीमान्त मान (Left hand limit)



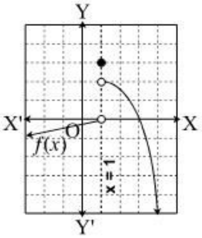
जब x बायाँबाट बिन्दु a को नजिक पुग्दछ, यसलाइ $x \longrightarrow a^-$ अथवा $x \longrightarrow a - 0$ ले जनाइन्छ र यस्तो अवस्थामा फलन $f(x)$ का लागि बायाँबाट बिन्दु a मा सीमान्त मानलाई $\lim\limits_{x \to a^-} f(x)$ अथवा $\lim\limits_{x \to a-0} f(x)$ द्वारा परिभाषित गरिन्छ ।

When x approaches to a point a from the left, then it is denoted by $x \longrightarrow a^-$ or $x \longrightarrow a - 0$ and the left hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^-} f(x)$ or $\lim\limits_{x \to a-0} f(x)$.

चित्रमा फलन $f(x)$ का लागि बायाँबाट बिन्दु $x = 1$ मा सीमान्त मान $= \lim\limits_{x \to 1^-} f(x) = 0$ हुन्छ ।

In the figure, the left hand limit of a function $f(x)$ at a point $x = 1$ is $= \lim\limits_{x \to 1^-} f(x) = 0$.

2.6 दायाँबाट सीमान्त मान (Right hand limit)



जब x दायाँबाट बिन्दु a को नजिक पुग्दछ, यसलाइ $x \to a^+$ अथवा $x \to a + 0$ ले जनाइन्छ र यस्तो अवस्थामा फलन $f(x)$ का लागि दायाँबाट बिन्दु a मा सीमान्त मानलाई $\lim\limits_{x \to a^+} f(x)$ अथवा $\lim\limits_{x \to a+0} f(x)$ द्वारा परिभाषित गरिन्छ ।

When x approaches to a point 'a' from the right, then it is denoted by $x \to a^+$ or $x \to a + 0$ and the right hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^+} f(x)$ or $\lim\limits_{x \to a+0} f(x)$.

चित्रमा फलन $f(x)$ का लागि दायाँबाट बिन्दु $x = 1$ मा सीमान्त मान $= \lim\limits_{x \to 1^+} f(x) = 2$ हुन्छ ।

In the figure, the right hand limit of a function $f(x)$ at a point $x = 1$ is $= \lim\limits_{x \to 1^+} f(x) = 2$.

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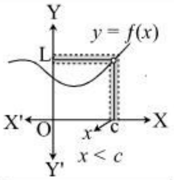
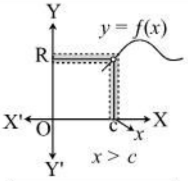
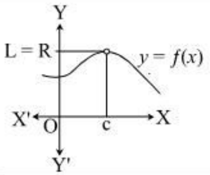
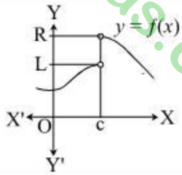
बायाँबाट सीमान्त मान र दायाँबाट सीमान्त मानका उदाहरणहरू (Examples of left hand limit and right hand limit)

 $\lim_{x \to c^-} f(x) = \text{L}$	 $\lim_{x \to c^+} f(x) = \text{R}$	 $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$	 $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

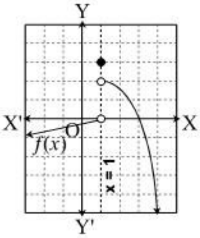
2.7 फलनको मान (Value of function)

यदि f एउटा फलन भए यसको क्षेत्रका हरेक सङ्ख्या x को सङ्गति प्रतिबिम्ब विस्तारमा $f(x)$ हुन्छ र 'f of x' अथवा 'f at x' भनेर पढिन्छ । $f(x)$ लाई फलन f को x मा हुने मान भनिन्छ ।

If f is a function, then for each number x in its domain, the corresponding image in the range is designated by the symbol $f(x)$, read as 'f of x' or as 'f at x'. We refer to $f(x)$ as the value of function f at the number x.

चित्रमा फलन $f(x)$ का लागि बिन्दु $x = 1$ मा फलनको मान $= f(1) = 3$ हुन्छ ।

In the figure, the value of the function $f(x)$ at a point $x = 1$ is $f(1) = 3$



2.8 निरन्तरता (Continuity)

कुनै फलन $x = a$ मा निरन्तर हुनको लागि निम्न अवस्थाहरू मान्य हुनु पर्दछ ।

For a function to be continuous at $x = a$, the following conditions must be satisfied.

(i) $f(a)$ को अस्तित्व हुनुपर्छ वा $f(a)$ परिभाषीत हुनुपर्छ । ($f(a)$ must exists.)

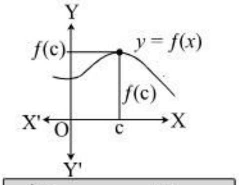
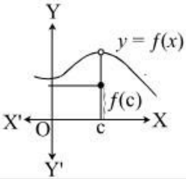
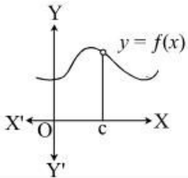
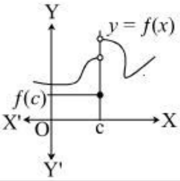
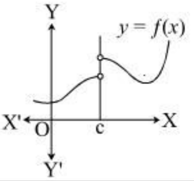
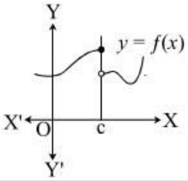
(ii) बायाँ पक्षबाट सीमान्त मान = दायाँ पक्षबाट सीमान्त मान (Left hand limit = Right hand limit)

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

(iii) $\lim_{x \to a} f(x) = f(a)$ अर्को शब्दमा, कुनै बिन्दु 'a' मा फलनको मान र फलनको सीमान्त मान बराबर भए सो फलनलाई बिन्दु 'a' मा निरन्तर छ भनिन्छ ।

In other words , if value of function and the limit value of the function at a point a are equal then the function is continuous at 'a'.

निरन्तरता र विच्छिन्नताका उदाहरणहरू (Examples of continuity and discontinuity):

निरन्तरता (Continuity)  $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ $\lim_{x \to c} f(x) = f(c)$	**विच्छिन्नता (Discontinuity)**  $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ $\lim_{x \to c} f(x) \neq f(c)$	**विच्छिन्नता (Discontinuity)**  $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ $f(c)$ is not defined.
विच्छिन्नता (Discontinuity)  $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$ $f(c)$ is defined.	**विच्छिन्नता (Discontinuity)**  $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$ $f(c)$ is not defined.	**विच्छिन्नता (Discontinuity)**  $\lim_{x \to c^-} f(x) = f(c) \neq \lim_{x \to c^+} f(x)$ $f(c)$ is defined.

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QUESTIONS FROM SEE EXERCISE 1

A. VERY SHORT QUESTIONS

1. निरन्तरताको परिभाषा दिनुहोस् । (Define the continuity)

⇨ Here, if the graph of a function can be drawn without lifting the pencil from the paper then it is called continuity.

2. विच्छिन्नता को परिभाषा दिनुहोस् । (Define the discontinuity)

⇨ Here, if the graph of any function has 'jump', hole, gap, cusp and break in a point then the function is said to be discontinuous at that point.

3. खुल्ला अन्तरालको परिभाषा दिनुहोस् । (Define the open interval)

⇨ Here, open Interval: An interval not containing both the end points a and b is known as an open interval. It is denoted by (a, b).

4. बन्द अन्तरालको परिभाषा दिनुहोस् । (Define the closed interval)

⇨ Here, closed Interval: An interval containing both the end points a and b is known as a closed interval. It is denoted by $[a, b]$.

5. बायाँ पक्षबाट सीमान्त मानको परिभाषा दिनुहोस् । (Define the left hand limit)

⇨ Here, when x approaches to a point a from the left, then it is denoted by $x \longrightarrow a^-$ or $x \longrightarrow a - 0$ and the left hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^-} f(x)$ or $\lim\limits_{x \to a - 0} f(x)$.

6. दायाँ पक्षबाट सीमान्त मानको परिभाषा दिनुहोस् । (Define the right hand limit)

⇨ Here, when x approaches to a point 'a' from the right, then it is denoted by $x \to a^+$ or $x \to a + 0$ and the right hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^+} f(x)$ or $\lim\limits_{x \to a + 0} f(x)$.

7. फलनको मानको परिभाषा दिनुहोस् । (Define the functional value)

⇨ Here, if f is a function, then for each number x in its domain, the corresponding image in the range is designated by the symbol $f(x)$, read as 'f of x' or as 'f at x'. We refer to $f(x)$ as the value of function f at the number x.

8. अन्तराल (interval) [*a*, *b*] को अर्थ के हुन्छ ? (What is the meaning of the interval [*a*, *b*]?)

⇨ Here, an interval containing both the end points a and b is known as a closed interval. It is denoted by $[a, b]$.

9. अन्तराल (interval) (*a*, *b*) को अर्थ के हुन्छ ? (What do you meant by the interval (*a*, *b*)?)

⇨ Here, an interval not containing both the end points a and b is known as an open interval. It is denoted by (a, b).

10. "$-3 \le x \le 3$ को लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् । (Write the interval notation for "$-4 < x < 4$.)

⇨ Here, the interval notation of $-3 \le x \le 3$ is $[-3, 3]$.

11. "$-4 < x < 4$ को लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् । (Write the interval notation for "$-4 < x < 4$.)

⇨ Here, the interval notation of $-4 < x < 4$ is $(-4, 4)$.

**12. $\{x : -14 \le x < -11\}$ को लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् ।
Write the interval notation for $\{x : -14 \le x < -11\}$.)**

⇨ Here, the interval notation of $\{x : -14 \le x < -11\}$ is $[-14, 11)$.

**13. $\{x : 6 < x \le 20\}$ को लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् ।
Write the interval notation for $\{x : 6 < x \le 20\}$.**

⇨ Here, the interval notation of $\{x : 6 < x \le 20\}$ is $(6, 20]$.

14. बायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् ।(Write down the notational representation of left hand limit.)

⇨ Here, the notational representation of left hand limit at $x = a$ is $\lim\limits_{x \to a^-} f(x)$.

15. दायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् । (Write down the notational representation of right hand limit.)

⇨ Here, the notational representation of right hand limit at $x = a$ is $\lim\limits_{x \to a^+} f(x)$.

⇨ Here, condition in notation of continuity is; $\lim\limits_{x \to a} f(x) = f(a)$.

**16. सङ्ख्या रेखामा अविच्छिन्नता हुने सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers which is continuous in number line. [SEE MODEL 2076]**

⇨ Here, the set of numbers which is continuous in number line is set of real numbers.

**17. सङ्ख्या रेखामा प्राकृतिक सङ्ख्याहरूको समूह र वास्तविक सङ्ख्याहरूको समूह मध्ये कुन समूहमा निरन्तरता हुन्छ ?
In which set of numbers, do you find continuity between set of natural numbers and set of real numbers in number line?**

⇨ Here, the set of real numbers in number line has continuity.

**18. सङ्ख्या रेखामा विच्छिन्नता जनाउने कुनै एक सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers whose number line is discontinuous.**

⇨ Here, the set of numbers whose number line is discontinuous are as follows:

- Set of natural numbers,
- Set of rational numbers
- Set of whole numbers
- Set of irrational numbers etc.
- Set of integers

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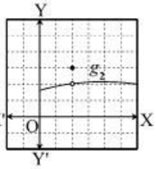
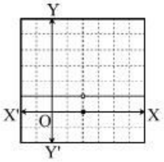
19. **दिइएको फलन कुन बिन्दुमा विच्छिन्नता भएको छ ?**
In which point, the given function is discontinuous?



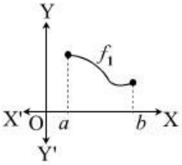
⇨ Here, the function is discontinuous at $x = 2$.

21. **दिइएको फलन कुन बिन्दुमा विच्छिन्नता भएको छ ?**
In which point, the given function is discontinuous?



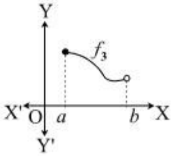
⇨ Here, the function is discontinuous at $x = 2$.

23. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



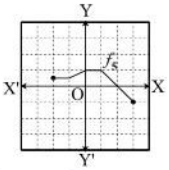
⇨ Here, the interval of the function is $[a, b]$.

25. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



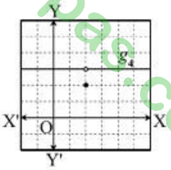
⇨ Here, the interval of the function is $[a, b)$.

27. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



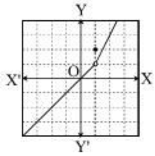
⇨ Here, the interval of the function is $[-2, 3]$.

20. **दिइएको फलन कुन बिन्दुमा विच्छिन्नता भएको छ ?**
In which point, the given function is discontinuous?



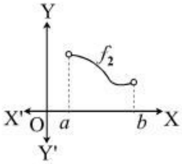
⇨ Here, the function is discontinuous at $x = 2$.

22. **दिइएको फलन कुन बिन्दुमा विच्छिन्नता भएको छ ?**
In which point, the given function is discontinuous?



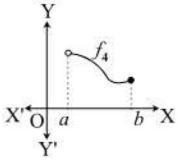
⇨ Here, the function is discontinuous at $x = 1$.

24. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



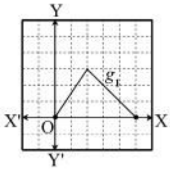
⇨ Here, the interval of the function is (a, b).

26. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



⇨ Here, the interval of the function is $(a, b]$.

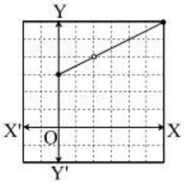
28. **दिइएको फलनको अन्तराल लेख्नुहोस् ।**
Find the interval of the given function.



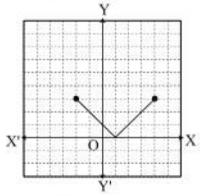
⇨ Here, the interval of the function is $[0, 5]$.

29. **दिइएको लेखाचित्रबाट (From the given graph):**

(i)



(ii)



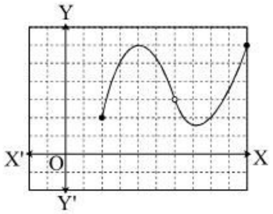
A. **निरन्तरताका अन्तरालहरू पत्ता लगाउनुहोस् ।**
Find the intervals of continuity.

B. **विच्छिन्नताको बिन्दु पत्ता लगाउनुहोस् ।**
Find the point of discontinuity.

⇨ Here,(i) The graph is defined for $x = 0$ to $x = 6$ except $x = 2$.
A. So intervals of continuity are; $[0, 2)$ and $(2, 6]$
B. Points of discontinuity : $x = 2$
(ii) The graph is defined for $x = -2$ to $x = 4$.
A. So the interval of continuity : $[-2, 4]$
B. There is no point of discontinuity.

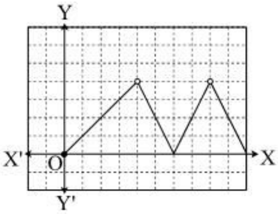
30. **दिइएका लेखाचित्रहरूबाट (i) निरन्तरताका अन्तरालहरू र (ii) विच्छिन्नताका बिन्दुहरू पत्ता लगाउनुहोस् ।**
From the following graph, find the (i) continuity interval and (ii) points of discontinuity.

(a)



⇨ Here, from the graph,
Continuity intervals : $[2, 6), (6, 10]$
Point of discontinuity : $x = 6$

(b)



⇨ Here, from the graph,
Continuity intervals : $[0, 4), (4, 8), (8, 10]$
Point of discontinuity : $x = 4$ and $x = 8$

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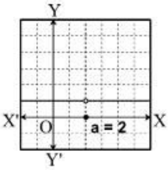
31. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा बायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

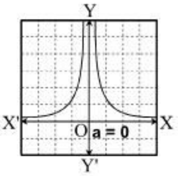
From the graph, find the left hand limit of the function at $x = a$.



⇨ Here, the left hand limit at a = 2 is ; 1

33. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा बायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

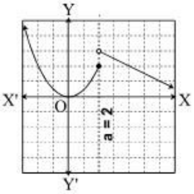
From the graph, find the left hand limit of the function at $x = a$.



⇨ Here, the left hand limit at a = 0 is ; ∞

35. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा दायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

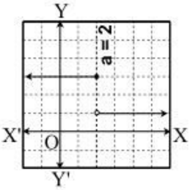
From the graph, find the right hand limit of the function at $x = a$.



⇨ Here, the right hand limit at $a = 2$ is ; 3

37. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा दायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

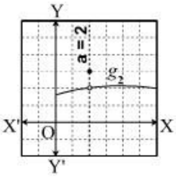
From the graph, find the right hand limit of the function at $x = a$.



⇨ Here, the right hand limit at $a = 2$ is ; 1

39. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा मान पत्ता लगाउनुहोस् ।**

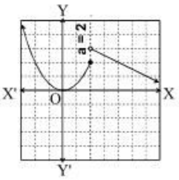
Find the functional value of the function at $x = a$, from the graph.



⇨ Here, the value of function at $x = a = 2$ is ; 3

41. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा मान पत्ता लगाउनुहोस् ।**

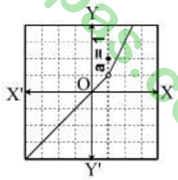
Find the functional value of the function at $x = a$, from the graph.



⇨ Here, the value of function at $x = a = 2$ is ; 2

32. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा बायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

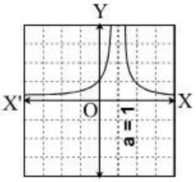
From the graph, find the left hand limit of the function at $x = a$.



⇨ Here, the left hand limit at a = 1 is ; 1

34. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा बायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

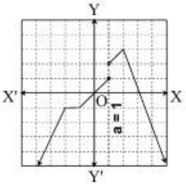
From the graph, find the left hand limit of the function at $x = a$.



⇨ Here, the left hand limit at a = 1 is ; ∞

36. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा दायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

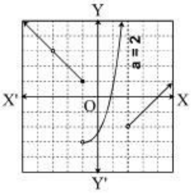
From the graph, find the right hand limit of the function at $x = a$.



⇨ Here, the right hand limit at $a = 1$ is ; 2

38. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा दायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।**

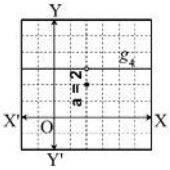
From the graph, find the right hand limit of the function at $x = a$.



⇨ Here, the right hand limit at $a = 2$ is ; – 2

40. **दिइएको लेखाचित्रबाट फलनको $x = a$ मा मान पत्ता लगाउनुहोस् ।**

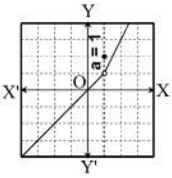
Find the functional value of the function at $x = a$, from the graph.



⇨ Here, the value of function at $x = a = 2$ is ; 2

42. दिइएको लेखाचित्रबाट फलनको $x = a$ मा मान पत्ता लगाउनुहोस् ।

Find the functional value of the function at $x = a$, from the graph.



⇨ Here, the value of function at $x = a = 1$ is ; 2

43. **फलन $f(x) = \frac{1}{x-3}$ को $x = 3$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।**

Examine the continuity or discontinuity of $f(x) = \frac{1}{x-3}$ at $x = 3$.

⇨ Here, $f(x) = \frac{1}{x-3}$

Functional value at $x = 3$,

$$f(3) = \frac{1}{3-3} = \frac{1}{0} = \infty$$

So, given function is discontinuous at $x = 3$.

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44. एउटा बिरुवा 44 cm अग्लो छ । यदि बिरुवाको उचाइ हरेक दिन 8 cm बढ्दै जान्छ भने 3 दिन पछि बिरुवाको उचाइ कति होला ? के यो वृद्धिले निरन्तरताको प्रतिनिधित्व गर्दछ ? उत्तरको पुष्टि गर्नुहोस् ।

A plant is 44 cm tall. If the height of the plant is increasing 8 cm each day, what will be the height of the plant after 3 days? Does this increment represent continuity? Justify your answer.

⇨ Here, the height of the plant = 44 cm,

Rate of increment in each day = 8 cm

Now, the height of the plant after 3 days

= 44 cm + 8 cm + 8 cm + 8 cm = 68 cm

Since the plant is living thing, so it increases continuously with the same rate in the certain time.

Thus, the increasing height of plant is continuity.

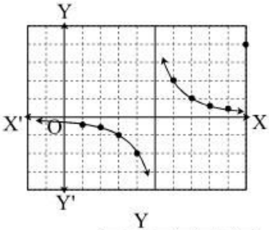
B. LONG QUESTIONS

1. $f(x) = \frac{2}{x-5}$ को लेखाचित्र खिच्नुहोस् । कुन बिन्दुमा सो फलन विच्छिन्न हुन्छ ? पत्ता लगाउनुहोस् ।

Draw the graph of $f(x) = \frac{2}{x-5}$ and find, in which point the function is discontinuous?

⇨ Here, $f(x) = \frac{2}{x-5}$ i.e. $y = \frac{2}{x-5}$

x	-1	0	1	2	3	4	5	6	7	9
y	$-0.3\overline{3}$	-0.4	-0.5	$-0.6\overline{6}$	-1	-2	∞	2	1	0.5



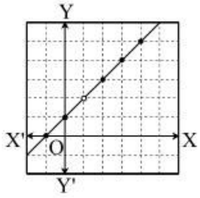
From the graph, we can say that the function is discontinuous at $x = 5$.

2. $g(x) = \frac{x^2-1}{x-1}$ को लेखाचित्र खिच्नुहोस् र विच्छिन्नताको बिन्दु खोजी गर्नुहोस् ।

Draw the graph of $g(x) = \frac{x^2-1}{x-1}$ and investigate the point of discontinuity.

⇨ Here, $g(x) = \frac{x^2-1}{x-1}$, i.e. $y = \frac{x^2-1}{x-1}$

x	-3	-2	-1	0	1	2	3	4
y	-2	-1	0	1	∞	3	4	5

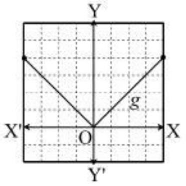


From the figure, we can say that the function $g(x)$ is discontinuous at $x = 1$.

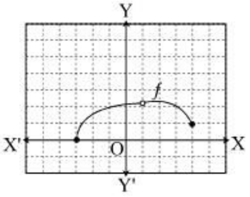
3. लेखाचित्रमा वक्रहरूको निरन्तरता वा विच्छिन्नताको खोजी गर्नुहोस् ।

Investigate continuity or discontinuity of the curve in the graph.

Graph I



Graph II



⇨ (a) Here, the graph is drawn from $x = -4$ to $x = 4$.

There are no jumps or holes in the graph.

We say that the function g is continuous over $[-4, 4]$.

That is, this graph can be traced without lifting the pencil from the paper.

(b) Here, the graph is drawn from $x = -3$ to $x = 4$. There is a hole in the graph at $x = 1$.

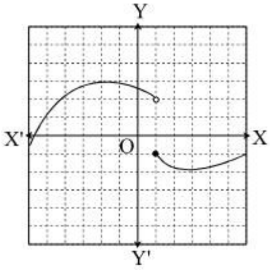
We say that the function f is discontinuous at $x = 1$.

Similarly, the function f is continuous at $[-3, 1)$ and $(1, 4]$.

In other words, f is continuous at $x = -3, x = -2, x = -1, x = 0, x = 2, x = 3$ etc.

4. सँगैको लेखाचित्र प्रयोग गरी तलका प्रश्नहरूको उत्तर दिनुहोस् ।

Use the adjoining graph and answer the following questions:



(a) पत्ता लगाउनुहोस् (Find) : $\lim_{x \to 1^+} f(x)$, $\lim_{x \to 1^-} f(x)$ & $\lim_{x \to 1} f(x)$

(b) $f(1)$ पत्ता लगाउनुहोस् । (Find $f(1)$.)

(c) के $x = 1$ मा f निरन्तर छ ? (Is f continuous at $x = 1$?)

(d) $\lim_{x \to -2} f(x)$ पत्ता लगाउनुहोस् । (Find $\lim_{x \to -2} f(x)$.)

(e) $f(-2)$ पत्ता लगाउनुहोस् । (Find $f(-2)$.)

(f) के $x = -2$ मा f निरन्तर छ ? (Is f continuous at $x = -2$?)

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⇨ (a) For $\lim_{x \to 1^+} f(x)$ This is the right hand limit.

So observe the graph from the right side then, $\lim_{x \to 1^+} f(x) = -1$

Again, for $\lim_{x \to 1^-} f(x)$

This is the left hand limit. So observe the graph from the left side then, $\lim_{x \to 1^-} f(x) = 2$

Now, $\lim_{x \to 1} f(x)$ does not exist.

That is, $f(x)$ is discontinuous at $x = 1$.

(b) From the graph, $f(1) = -1$

(c) $f(x)$ is not continuous at $x = 1$

(d) Here, $\lim_{x \to -2} f(x)$ From the graph, $\lim_{x \to -2} f(x) = 3$

(e) From the graph, $f(-2) = 3$

(f) From the graph, f is continuous at $x = -2$

5. फलन $f(x) = x^2$ को $x = 4$ मा निरन्तरता वा विच्छिन्नता पत्ता लगाउनुहोस् ।
Find the continuity or discontinuity of the function $f(x) = x^2$ at $x = 4$.

⇨ Here, $f(x) = x^2$ Functional value at $x = 4$ is ; $f(4) = 4^2 = 16$

For left hand limit of $f(x)$ at $x = 4$	**For right hand limit of $f(x)$ at $x = 4$**
Choose $x = 3.9, 3.99, 3.999,$	Choose $x = 4.1, 4.01, 4.001,$................
So, $f(3.9) = 3.9^2 = 15.21$	So, $f(4.1) = 4.1^2 = 16.81$
$f(3.99) = 3.99^2 = 15.92$	$f(4.01) = 4.01^2 = 16.08$
$f(3.999) = 3.999^2 = 15.99$	$f(4.001) = 4.001^2 = 16.008$
....	....
$\therefore \lim_{x \to 4^-} f(x) = 16$	$\therefore \lim_{x \to 4^+} f(x) = 16$

This shows that, functional value = left hand limit = right hand limit.
Thus, the given function is continuous at $x = 4$.

6. $f(x) = 2 - 3x^2$ को $x = 0$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Test the continuity or discontinuity of $f(x) = 2 - 3x^2$ at $x = 0$

⇨ Here, $f(x) = 2 - 3x^2$ Functional value at $x = 0$ is ; $f(0) = 2 - 3 \times 0^2 = 2 - 0 = 2$

For left hand limit of $f(x)$ at $x = 0$	**For right hand limit of $f(x)$ at $x = 0$**
Choose $x = -0.1, -0.01, -0.001,$................	Choose $x = 0.1, 0.01, 0.001,$................
So, $f(-0.1) = 2 - 3(-0.1)^2 = 1.97$	So, $f(0.1) = 2 - 3(0.1)^2 = 1.97$
$f(-0.01) = 2 - 3(-0.01)^2 = 1.999$	$f(0.01) = 2 - 3(0.01)^2 = 1.999$
$f(-0.001) = 2 - 3(-0.001)^2 = 1.999$	$f(0.001) = 2 - 3(0.001)^2 = 1.999$
....	....
$\therefore \lim_{x \to 0^-} f(x) = 2$	$\therefore \lim_{x \to 0^+} f(x) = 2$

This shows that, functional value = left hand limit = right hand limit.
Thus, the given function is continuous at $x = 0$.

7. $f(x) = \frac{1}{x-2}$ को $x \neq 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Test the continuity or discontinuity of $f(x) = \frac{1}{x-2}$ at $x \neq 2$.

⇨ Here, $f(x) = \frac{1}{x-2}$ and $x \neq 0$ Choose any one number other than 2, say 3.

Functional value at $x = 3$, $f(3) = \frac{1}{3-2} = \frac{1}{1} = 1$

For left hand limit of $f(x)$ at $x = 3$	**For right hand limit of $f(x)$ at $x = 3$**
Choose $x = 2.9, 2.99, 2.999,$................	Choose $x = 3.01, 3.001, 3.0001,$................
So, $f(2.9) = \frac{1}{2.9-2} = \frac{1}{0.9} = 1.11$	So, $f(3.01) = \frac{1}{3.01-2} = \frac{1}{1.01} = 0.990$
$f(2.99) = \frac{1}{2.99-2} = \frac{1}{0.99} = 1.01$	$f(3.001) = \frac{1}{3.001-2} = \frac{1}{1.001} = 0.999$
$f(2.999) = \frac{1}{2.999-2} = \frac{1}{0.999} = 1.001$	$f(3.0001) = \frac{1}{3.0001-2} = \frac{1}{1.0001} = 0.999$
....	....
$\therefore \lim_{x \to 3^-} f(x) = 1$	$\therefore \lim_{x \to 3^+} f(x) = 1$

This shows that, functional value = left hand limit = right hand limit.
Thus, the given function is continuous at $x = 3$.

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8. $f(x) = \frac{1}{1-x}$ **को $x = 1$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।**

Test the continuity or discontinuity of $f(x) = \frac{1}{1-x}$ at $x = 1$.

⇨ Here, $f(x) = \frac{1}{1-x}$ Functional value at $x = 1$,

Now, $f(1) = \frac{1}{1-1} = \frac{1}{0} = \infty$ Functional value is not defined.

Thus, $f(x) = \frac{1}{1-x}$ is discontinuous at $x = 1$.

9. $f(x) = \frac{x^2-9}{x-3}$ **को $x = 3$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।**

Test the continuity or discontinuity of $f(x) = \frac{x^2-9}{x-3}$ at $x = 3$.

⇨ Here, $f(x) = \frac{x^2-9}{x-3}$ Functional value at $x = 3$,

Now, $f(3) = \frac{3^2-9}{3-3} = \frac{0}{0}$ which is indeterminate form.

So, the given function is discontinuous at $x = 3$.

10. $f(x) \left.\begin{matrix} = 2 - x^2, x \le 2 \\ = x - 4, x > 2 \end{matrix}\right\}$ **को $x = 2$ मा निरन्तरता परीक्षण गर्नुहोस् ।(Examine the continuity of function $f(x) \left.\begin{matrix} = 2 - x^2, x \le 2 \\ = x - 4, x > 2 \end{matrix}\right\}$ at $x = 2$)**

⇨ Here, For $x = 2$, the function is $f(x) = 2 - x^2$

Functional value at $x = 2$ is; $f(2) = 2 - 2^2 = 2 - 4 = -2$

For left hand limit of $f(x)$ at $x = 2$	**For right hand limit of $f(x)$ at $x = 2$**
Choose the numbers $x = 1.9, 1.99, 1.999$, ...	Choose the numbers $x = 2.01, 2.001, 2.0001$, ...
For the numbers less than 2,	For the numbers greater than 2,
the function is $f(x) = 2 - x^2$	the function is $f(x) = x - 4$
So, $f(1.9) = 2 - (1.9)^2 = -1.61$	So, $f(2.01) = 2.01 - 4 = -1.99$
$f(1.99) = 2 - (1.99)^2 = -1.9601$	$f(2.001) = 2.001 - 4 = -1.999$
$f(1.999) = 2 - (1.999)^2 = -1.996$	$f(2.0001) = 2.0001 - 4 = -1.9999$
....	....
$\therefore \lim\limits_{x \to 2^-} f(x) = -2$	$\therefore \lim\limits_{x \to 2^+} f(x) = -2$

This shows that, functional value = left hand limit = right hand limit

Thus, the given function is continuous at $x = 2$.

11. **फलन $f(x) \left.\begin{matrix} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{matrix}\right\}$ को $x = 5$ मा निरन्तरता परीक्षण गर्नुहोस् ।**

Examine the continuity of function $f(x) \left.\begin{matrix} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{matrix}\right\}$ at $x = 5$

⇨ Here, for $x = 5$, the function is $f(x) = x^2 + 2$

So, functional value at $x = 5$ is; $f(5) = 5^2 + 2 = 25 + 2 = 27$

For left hand limit of $f(x)$ at $x = 5$	**For right hand limit of $f(x)$ at $x = 5$**
Choose the numbers $x = 4.9, 4.99, 4.999$, ...	Choose the numbers $x = 5.01, 5.001, 5.0001$, ...
For the numbers less than 5,	For the numbers greater than 5,
the function is $f(x) = x^2 + 2$	the function is $f(x) = 3x + 12$
So, $f(4.9) = 4.9^2 + 2 = 26.01$	So, $f(5.01) = 3 \times 5.01 + 12 = 27.03$
$f(4.99) = 4.99^2 + 2 = 26.90$	$f(5.001) = 3 \times 5.001 + 12 = 27.003$
$f(4.999) = 4.999^2 + 2 = 26.99$	$f(5.0001) = 3 \times 5.0001 + 12 = 27.0003$
....	....
$\therefore \lim\limits_{x \to 5^-} f(x) = 27$	$\therefore \lim\limits_{x \to 5^+} f(x) = 27$

This shows that, functional value = left hand limit = right hand limit.

Thus, the given function is continuous at $x = 5$.

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12. फलन $f(x)$, $x = 2$ मा निरन्तर हुन्छ भनी देखाउनुहोस् । (Show that the function $f(x)$ is continuous at $x = 2$.) $f(x) = \begin{cases} 2x - 1, & x < 2 \\ 3, & x = 2 \\ x + 1, & x > 2 \end{cases}$

⇨ Here, For functional value at $x = 2$, $f(2) = 3$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
Choose $x = 1.9, 1.99, 1.999,$	Choose $x = 2.01, 2.001, 2.0001,$ ....
For the numbers less than 2,	For the numbers greater than 2,
the function is $f(x) = 2x - 1$	the function is; $f(x) = x + 1$
So, $f(1.9) = 2 \times 1.9 - 1 = 2.8$	So, $f(2.01) = 2.01 + 1 = 3.01$
$f(1.99) = 2 \times 1.99 - 1 = 2.98$	$f(2.001) = 2.001 + 1 = 3.001$
$f(1.999) = 2 \times 1.999 - 1 = 2.998$	$f(2.0001) = 2.0001 + 1 = 3.0001$
....	....
$\therefore \lim\limits_{x \to 2^-} f(x) = 3$	$\therefore \lim\limits_{x \to 2^+} f(x) = 3$

This shows that, functional value = LHL = RHL Proved.

Thus, the function is continuous at $x = 2$.

13. एउटा फलन $f(x)$ लाई निम्नानुसार परिभाषीत गरिएको छ (A function $f(x)$ is defined as follows): $f(x) = \begin{cases} 2x^2 + 1, & x < 3 \\ 5, & x = 3 \\ 6x + 1, & x > 3 \end{cases}$

उल्लेख गरिएको बिन्दुमा माथिको फलनको निरन्तरता परीक्षण गर्नुहोस् । यदि $f(x)$ निरन्तर छैन भने कसरी निरन्तर बनाउन सकिन्छ ?

Investigate the continuity of the above function at the point mentioned. If $f(x)$ is not continuous, how can it be made continuous?

⇨ Here, For functional value at $x = 3$, $f(3) = 5$

For left hand limit of $f(x)$ at $x = 3$	For right hand limit of $f(x)$ at $x = 3$
Choose $x = 2.9, 2.99, 2.999,$	Choose $x = 3.01, 3.001, 3.0001,$ .....
For 'x' less than 3,	For 'x' greater than 3,
the function is $f(x) = 2x^2 + 1$	the function is $f(x) = 6x + 1$
So, $f(2.9) = 2 \times (2.9)^2 + 1 = 17.82$	So, $f(3.01) = 6 \times 3.01 + 1 = 19.06$
$f(2.99) = 2 \times (2.99)^2 + 1 = 18.88$	$f(3.001) = 6 \times 3.001 + 1 = 19.006$
$f(2.999) = 2 \times (2.999)^2 + 1 = 18.98$	$f(3.0001) = 6 \times 3.0001 + 1 = 19.0006$
....	....
$\therefore \lim\limits_{x \to 3^-} f(x) = 19$	$\therefore \lim\limits_{x \to 3^+} f(x) = 19$

$\therefore$ LHL = RHL So the limit exists. So, $\lim\limits_{x \to 3} f(x) = 19$ but $f(3) = 5$

Hence the given function is discontinuous at $x = 3$.

Thus, to make above function continuous, $f(3)$ must be 19. i.e. $f(3) = 19$.

14. यदि दिइएको फलन $x = 2$ मा निरन्तर भए k को मान पत्ता लगाउनुहोस् : $f(x) = \begin{cases} kx - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

If the given function is continuous at $x = 2$ then find the value of k : $f(x) = \begin{cases} kx - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

⇨ Here, $f(x) = \begin{cases} kx - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

The functional value at $x = 2$; $\therefore f(2) = 2 \times 2 - 3 = 1$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
Choose $x = 1.9, 1.99, 1.999,$	Choose $x = 2.01, 2.001, 2.0001,$....
For x less than 2,	For 'x' greater than 2,
the function is $f(x) = kx - 1$	the function is $f(x) = 2x - 3$
So, $f(1.9) = 1.9k - 1$	So, $f(2.01) = 2 \times 2.01 - 3 = 1.02$
$f(1.99) = 1.99k - 1$	$f(2.001) = 2 \times 2.001 - 3 = 1.002$
$f(1.999) = 1.999k - 1$	$f(2.0001) = 2 \times 2.0001 - 3 = 1.0002$
....	....
$\therefore \lim\limits_{x \to 2^-} f(x) = 2k - 1$	$\therefore \lim\limits_{x \to 2^+} f(x) = 1$

For the function $f(x)$ to be continuous at $x = 2$,

We have, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

or, $2k - 1 = 1 = 1$

or, $2k = 2$

$\therefore k = 1$

Thus, the value of k is 1.

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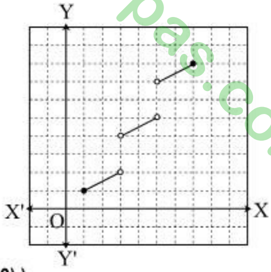
15. सँगै दिइएको लेखाचित्रबाट निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of the graph given aside.

Here, the graph alongside is drawn from $x = 1$ to $x = 7$.
There are gaps at $x = 3$ and $x = 5$ and the function is continuous for all other points from $x = 1$ to $x = 7$.



Continuous in	Discontinuous at
[1, 3)	$x = 3$
(3, 5)	$x = 5$
(5, 7]	

16. यदि $f(x) = 2x - 3$ भए (If $f(x) = 2x - 3$ then)

(i) $f(1.99)$, $f(1.999)$ र $f(1.9999)$ पत्ता लगाउनुहोस् । (Find $f(1.99)$, $f(1.999)$ and $f(1.9999)$.)

(ii) $f(2.001)$, $f(2.0001)$ र $f(2.00001)$ पत्ता लगाउनुहोस् । (Find $f(2.001)$, $f(2.0001)$ and $f(2.00001)$.)

(iii) $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$ र $f(2)$ पत्ता लगाउनुहोस् । (Find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$ and $f(2)$.)

(iv) के $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$ हुन्छ ? (Is $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$?)

(v) के निश्कर्ष निकाल्न सक्नुहुन्छ ? (What conclusion can be drawn?)

⇨ Here, $f(x) = 2x - 3$

So, (i) $f(1.99) = 2 \times 1.99 - 3 = 0.98$
$f(1.999) = 2 \times 1.999 - 3 = 0.998$
$f(1.9999) = 2 \times 1.9999 - 3 = 0.9998$

(ii) $f(x) = 2x - 3$
So, $f(2.001) = 2 \times 2.001 - 3 = 1.002$
$f(2.0001) = 2 \times 2.0001 - 3 = 1.0002$
$f(2.00001) = 2 \times 2.00001 - 3 = 1.00002$

(iii) $\therefore \lim_{x\to 2^-} f(x) = 1$
$\lim_{x\to 2^+} f(x) = 1$ and $f(2) = 2 \times 2 - 3 = 4 - 3 = 1$

(iv) Yes, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

(v) Given function is continuous at $x = 2$

17. फलन $f(x) = x + 2$ को $x = 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine continuity or discontinuity of $f(x) = x + 2$ at $x = 2$.

⇨ Here, $f(x) = x + 2$ Functional value at $x = 2$; $f(2) = 2 + 2 = 4$

For left hand limit at $x = 2$;	For right hand limit at $x = 2$;
$f(1.9) = 1.9 + 2 = 3.9$	$f(2.01) = 2.01 + 2 = 4.01$
$f(1.99) = 1.99 + 2 = 3.99$	$f(2.001) = 2.001 + 2 = 4.001$
$f(1.999) = 1.999 + 2 = 3.999$	$f(2.0001) = 2.0001 + 2 = 4.0001$
……………………………………	……………………………………
$\therefore \lim_{x\to 2^-} f(x) = 4$	$\therefore \lim_{x\to 2^+} f(x) = 4$

It shows that left hand limit = right hand limit = functional value.
Thus, the given function is continuous at $x = 2$.

18. फलन $f(x) = \frac{x+1}{4x+4}$ को $x = 4$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine continuity or discontinuity of $f(x) = \frac{x+1}{4x+4}$ at $x = 4$.

⇨ Here, $f(x) = \frac{x+1}{4x+4}$ The functional value at $x = 4$; $f(4) = \frac{4+1}{4\times4+4} = \frac{5}{20} = \frac{1}{4} = 0.25$

For left hand limit at $x = 4$;	For right hand limit at $x = 4$;
$f(3.9) = \frac{3.9+1}{4\times3.9+4} = 0.25$	$f(4.01) = \frac{4.01+1}{4\times4.01+4} = 0.25$
$f(3.99) = \frac{3.99+1}{4\times3.99+4} = 0.25$	$f(4.001) = \frac{4.001+1}{4\times4.001+4} = 0.25$
$f(3.999) = \frac{3.999+1}{4\times3.999+4} = 0.25$	$f(4.0001) = \frac{4.0001+1}{4\times4.0001+4} = 0.25$
……………………………………	……………………………………
$\therefore \lim_{x\to 4^-} f(x) = 0.25$	$\therefore \lim_{x\to 4^+} f(x) = 0.25$

It shows that left hand limit = right hand limit = functional value.
Thus, the given function is continuous at $x = 4$.

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19. फलन $f(x)=\begin{cases}\dfrac{x^2-4x}{x-4}, & x\neq 4\\ 3, & x=4\end{cases}$ को बिन्दु $x=1$ मा निरन्तरता परीक्षण गर्नुहोस् ।

Examine the continuity of function $f(x)=\begin{cases}\dfrac{x^2-4x}{x-4}, & x\neq 4\\ 3, & x=4\end{cases}$ at the point $x=1$.

⇨ Here, $f(x)=\begin{cases}\dfrac{x^2-4x}{x-4}, & x\neq 4\\ 3, & x=4\end{cases}$

So, functional value at $x = 1$; $f(x)=\dfrac{x^2-4x}{x-4}$ or, $f(1)=\dfrac{1^2-4\times 1}{1-4}=\dfrac{1-4}{1-4}=\dfrac{-3}{-3}=1$

For left hand limit of $f(x)$ at $x = 1$;	For right hand limit of $f(x)$ at $x = 1$;
Choose $x = 0.9, 0.99, 0.999, \ldots\ldots\ldots$	Choose $x = 1.01, 1.001, 1.0001, \ldots\ldots\ldots\ldots\ldots$
$f(0.9) = \dfrac{0.9^2-4\times 0.9}{0.9-4} = 0.9$	$f(1.01) = \dfrac{1.01^2-4\times 1.01}{1.01-4} = 1.01$
$f(0.99) = \dfrac{0.99^2-4\times 0.99}{0.99-4} = 0.99$	$f(1.001) = \dfrac{1.001^2-4\times 1.001}{1.001-4} = 1.001$
$f(0.999) = \dfrac{0.999^2-4\times 0.999}{0.999-4} = 0.999$	$f(1.0001) = \dfrac{1.0001^2-4\times 1.0001}{1.0001-4} = 1.0001$
....	....
$\therefore \lim\limits_{x\to 1^-} f(x) = 1$	$\therefore \lim\limits_{x\to 1^+} f(x) = 1$

Now, $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = f(1) = 1$

Thus, the function is continuous at $x = 1$

20. फलन $f(x)=\begin{cases}\dfrac{x^2-7x}{x-7}, & x\neq 7\\ 5, & x=7\end{cases}$ को बिन्दु $x=7$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of function $f(x)=\begin{cases}\dfrac{x^2-7x}{x-7}, & x\neq 7\\ 5, & x=7\end{cases}$ at the point $x=7$.

⇨ Here, $f(x)=\begin{cases}\dfrac{x^2-7x}{x-7}, & x\neq 7\\ 5, & x=7\end{cases}$

So, functional value at $x = 7$; $f(7) = 5$

For left hand limit at $x = 7$;	For right hand limit at $x = 7$;
$f(6.9) = \dfrac{6.9^2-7\times 6.9}{6.9-7} = 6.9$	$f(7.01) = \dfrac{7.01^2-7\times 7.01}{7.01-7} = 7.01$
$f(6.99) = \dfrac{6.99^2-7\times 6.99}{6.99-7} = 6.99$	$f(7.001) = \dfrac{7.001^2-7\times 7.001}{7.001-7} = 7.001$
$f(6.999) = \dfrac{6.999^2-7\times 6.999}{6.999-7} = 6.999$	$f(7.0001) = \dfrac{7.0001^2-7\times 7.0001}{7.0001-7} = 7.0001$
....	....
$\therefore \lim\limits_{x\to 7^-} f(x) = 7$	$\therefore \lim\limits_{x\to 7^+} f(x) = 7$

Now, $\lim\limits_{x\to 7^-} f(x) = \lim\limits_{x\to 7^+} f(x) \neq f(7)$

Thus, the function is discontinuous at $x = 7$.

21. फलन $f(x)=\begin{cases}2x-1, & x<2\\ 3, & x=2\\ x+1, & x>2\end{cases}$ को बिन्दु $x=2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of function $f(x)=\begin{cases}2x-1, & x<2\\ 3, & x=2\\ x+1, & x>2\end{cases}$ at the point $x=2$.

⇨ Here, $f(x)=\begin{cases}2x-1, & x<2\\ 3, & x=2\\ x+1, & x>2\end{cases}$

Functional value at $x = 2$ $f(x) = 3$ $\therefore f(2) = 3$

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For left hand limit at $x = 2$, $f(x) = 2x - 1$	For right hand limit at $x = 2$, $f(x) = x + 1$
$f(1.9) = 2 \times 1.9 - 1 = 2.8$ $f(1.99) = 2 \times 1.99 - 1 = 2.98$ $f(1.999) = 2 \times 1.999 - 1 = 2.998$ .... $\therefore \lim\limits_{x \to 2^-} f(x) = 3$	$f(2.01) = 2.01 + 1 = 3.01$ $f(2.001) = 2.001 + 1 = 3.001$ $f(2.0001) = 2.0001 + 1 = 3.0001$ .... $\therefore \lim\limits_{x \to 2^+} f(x) = 3$

Now, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2) = 3$

Thus, the function is continuous at $x = 2$.

22. **एउटा फलन $f(x) = \begin{cases} 2x^2 + 1, & x < 2 \\ 5, & x = 2 \\ 4x + 1, & x > 2 \end{cases}$ द्वारा परिभाषीत गरिएको छ । $x = 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् । यदि $f(x)$ निरन्तर नभए कसरी निरन्तर बनाउन सकिन्छ ?**

A function is defined by $f(x) = \begin{cases} 2x^2 + 1, & x < 2 \\ 5, & x = 2 \\ 4x + 1, & x > 2 \end{cases}$. Examine the continuity or discontinuity at the point $x = 2$. If $f(x)$ is not continuous, how can it be made continuous?

⇨ Here, $f(x) = \begin{cases} 2x^2 + 1, & x < 2 \\ 5, & x = 2 \\ 4x + 1, & x > 2 \end{cases}$

The functional value at $x = 2$ $\quad f(x) = 5 \quad \therefore f(2) = 5$

For left hand limit at $x = 2$, $f(x) = 2x^2 + 1$	For right hand limit at $x = 2$, $f(x) = 4x + 1$
$f(1.9) = 2 \times 1.9^2 + 1 = 8.22$ $f(1.99) = 2 \times 1.99^2 + 1 = 8.9202$ $f(1.999) = 2 \times 1.999^2 + 1 = 8.992002$ .... $\therefore \lim\limits_{x \to 2^-} f(x) = 9$	$f(2.01) = 4 \times 2.01 + 1 = 9.04$ $f(2.001) = 4 \times 2.001 + 1 = 9.004$ $f(2.0001) = 4 \times 2.0001 + 1 = 9.0004$ .... $\therefore \lim\limits_{x \to 2^+} f(x) = 9$

Now, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) \neq f(2)$

Thus, the function is not continuous at $x = 2$. i.e. the function is discontinuous at $x = 2$.

Here, the left hand limit and right hand limit are equal but the functional value is not equal to LHL and RHL.

So, to make $f(x)$ continuous; $f(2)$ must be 9. i.e. $f(2) = 9$.

QUESTIONS FROM CDC TEXTBOOK

2.1 सङ्ख्याहरूको क्रमको समूहमा निरन्तरता (CONTINUITY IN THE ORDER OF SET OF NUMBERS)

EXERCISE 2.1

1. **तल दिइएका सङ्ख्याहरूलाई चित्रद्वारा सङ्ख्या रेखामा देखाउनुहोस् :**
Show the following numbers in the number line:

(a) **1 देखी 5 सम्मका प्राकृतिक सङ्ख्याहरू (Natural numbers from 1 to 5)**

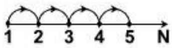
(b) **8 देखि 6 सम्मका पूर्णाङ्कहरू (Integers from 8 to 6)**

(c) **– 4 देखी + 6 सम्मका पूर्णाङ्कहरू (Integers from – 4 to + 6)**

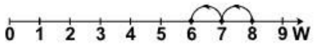
(d) **– 4 देखि + 4 सम्मका वास्तविक सङ्ख्याहरू (Real numbers from – 4 to + 4)**

⇨ Here,

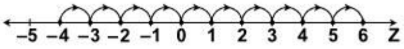
(a)



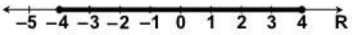
(b)



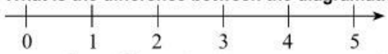
(c)

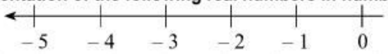


(d)



2. **(a)** **तल दिइएका वास्तविक सङ्ख्याहरूको चित्रात्मक प्रस्तुतिमा के फरक छ ?**
What is the difference between the diagramatic representation of the following real numbers in number line?





⇨ Here, 1st and 2nd number lines represent positive real numbers and negative real numbers respectively.
In other words, the first number line is in increasing order and the second number line is in decreasing order.

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(b) **प्राकृतिक सङ्ख्याको सुरुको सङ्ख्या कति हुन्छ ? (What is the first natural number?)**

⇨ Here, 1 is the first natural number.

(c) **प्राकृतिक सङ्ख्याको अन्तिम सङ्ख्या कति हुन्छ ? (What is the last natural number?)**

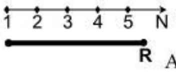
⇨ Infinitive (∞)

(d) **के समतलीय सतहमा प्राकृतिक सङ्ख्याहरूलाई सिधा रेखाले जोड्न सकिन्छ ?**
Can the natural numbers in plane surface be connected by the straight line?

⇨ No

(e) **वास्तविक सङ्ख्याहरू र प्राकृतिक सङ्ख्याहरूको चित्रात्मक प्रस्तुतिमा के फरक छ ?**
What is the difference between the diagramatic representation of real numbers and natural numbers in the number line?

⇨ Here,



Just the numbers are marked.

All the parts are marked.

3. **निरन्तर र निरन्तरता शब्दको अर्थ स्पष्ट पार्दै हाम्रो दैनिक जीवनमा प्रयोग हुने गरेका एउटा-एउटा उदाहरण प्रस्तुत गर्नुहोस् ।**
Clearify the meaning of continuous and continuity and give one/one example which are used in our daily life.

⇨ Here,

Continuous: The meaning of continuous is going on without being interrupted.

In other words, continuous is going on or extending without interruption or break.

Examples of continuous in daily life:

(i) My computer makes a continuous low buzzing noise.

(ii) There is a continuous white traffic line on the road.

Continuity: The fact of something continuing for a period of time without being changed or stopped.

In other words, uninterrupted duration or continuation especially without essential change, is called continuity.

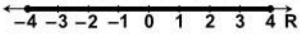
Examples of continuity:

(i) There is a problem with the movie's continuity.

(ii) There is continuity in the growth of a plant.

4. **वास्तविक सङ्ख्याहरूलाई जनाउने गरी एउटा सङ्ख्या रेखा खिच्नुहोस् । उक्त सङ्ख्या रेखामा पूर्णाङ्कहरू र वास्तविक सङ्ख्याहरूमध्ये कुन कुनमा निरन्तरता देख्न सकिन्छ ? व्याख्या गर्नुहोस् ।**
Draw a number line to represent real number. Which of the numbers, integers or real numbers has continuity in number line? Explain.

⇨ Here, the number line of real numbers is shown below:



The integers has discontinuity and real numbers has continuity in number line.

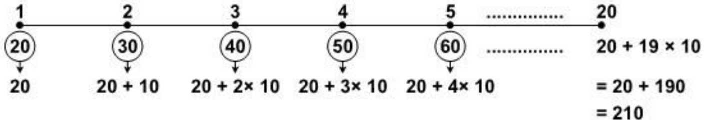
5. **(a)** **एउटा बिरूवाको आइतबारको उचाइ 3 मि.मि. छ । उक्त बिरूवा प्रत्येक दिन निरन्तर रूपमा 2 मि.मि. बढ्दै जान्छ । त्यही हप्ताको शनिबार उक्त बिरूवाको उचाइ कति होला ? पत्ता लगाउनुहोस् ।**
On sunday, the height of a plant is 3 mm. The height of the plant increases continuously by 2 mm everyday. What will be the height of the plant on saturday of the same week? Find it.

⇨ Here, height on sunday = 3 mm
Rate of increment = 2 mm everyday
Height on saturday = 3 mm + 2 mm × 6 (monday→saturday)
= 3 mm + 12 mm
= 15 mm

Thus, the height of the plant on saturday is 15 mm.

(b) **एउटा विद्यार्थीको खुत्रुकेमा महिनाको पहिलो दिन रु. 20 छ । प्रत्येक दिन निरन्तर रूपमा उसले रु. 10 रकम सो खुत्रुकेमा जम्मा गर्दै जान्छ । 20 दिनसम्म जम्मा भएको रकमलाई सङ्ख्या रेखामा देखाउनुहोस् ।**
In a piggy pocket, a student has Rs 20 in the first day of a month. He puts Rs 10 everyday in the piggy pocket continuously. Show the collected money up to 20 days in numbe line.

⇨ Here,



Thus, the amount collected upto 20 days is Rs 210.

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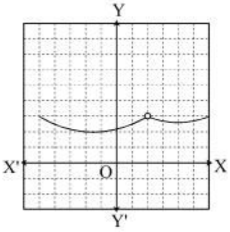
2.2 लेखाचित्रबाट फलनको विच्छिन्नताको खोजी (INVESTIGATION OF DISCONTINUITY IN GRAPH)

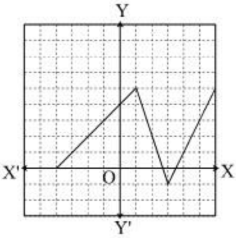
EXERCISE 2.2

1. **तल दिइएका वक्रहरू (i) कुन बिन्दुदेखि कुन बिन्दुसम्म (ii) कुन बिन्दुमा विछिन्न (Discontinuous) छन् ? लेख्नुहोस् ।**
   In the given curves, (i) at which points it is defined and (ii) at which point it is discontinuous? Write it.

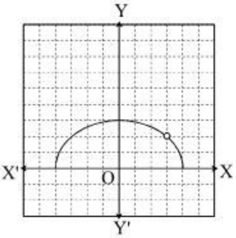
(a)



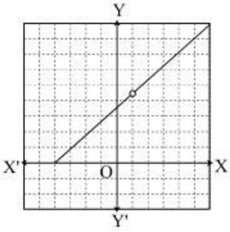
(b)



(c)



(d)



⇨ Here,

(a) The function is defined in the intervals [– 5, 2) and (2, 6].
The function is discontinuous at x = 2.

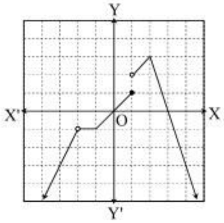
(b) The function is defined in the interval [– 4, 6].
The function is not discontinuous.

(c) The function is defined in the intervals [– 4, 3) and (3, 4].
The function is discontinuous at x = 3.

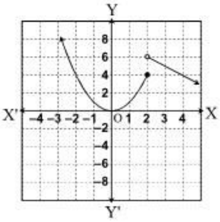
(d) The function is defined in the intervals [– 4, 1) and (1, 6].
The function is discontinuous at x = 1.

2. **तल दिइएका वक्रहरू – 4 देखि + 4 सम्म कुन बिन्दुमा निरन्तर (Continuous) र कुन बिन्दुमा विछिन्न (discontinuous) छन्, लेख्नुहोस् ।**
   In the given curves, at which points from – 4 to + 4 is continuous and in which point it is discontinuous? Write it.

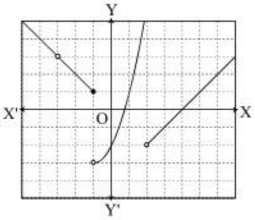
(a)



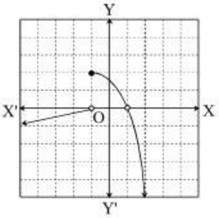
(b)



(c)



(d)



⇨ Here,

(a) Intervals of continuity are: (– 4, – 2), (– 2, 1] and (1, 4].
Point of discontinuity is x = 1.

(b) Intervals of continuity are: [– 3, 2] and (2, 4].
Point of discontinuity is x = 2.

(c) Intervals of continuity are: [– 4, – 3), (– 3, – 1], (– 1, 2) and (2, 4].
Points of discontinuity are: x = – 3, x = – 1 and x = 2.

(d) Intervals of continuity are: (– 4, – 1), [– 1, 1) and (1, 2).
Points of discontinuity are: x = – 1, x = 1 and x = 2.

3. **आफ्नो टोल अथवा छिमेकमा बस्ने मानिसहरूको उमेर सोधी तल दिइएको तालिका भर्नुहोस् ।**
   Fill up the given table by asking the age of people of your locality.

उमेर वर्षमा (Age in yrs)	0-20	20-40	40-60	60-80	80 **भन्दा माथि**
मानिसहरूको सङ्ख्या (No. of people)					

उक्त तथ्याङ्कका आधारमा भन्दा कम (is less than) र भन्दा बढी (is more than) सञ्चित बारम्बारता वक्र खिच्नुहोस् । कुनै निश्चित बिन्दुमा उक्त वक्रहरूको निरन्तरता (Continuity) र विछिन्नता (discontinuity) को प्रतिवेदन तयार पारी उक्त प्रतिवेदनलाई कक्षाकोठामा प्रस्तुत गर्नुहोस् ।

Draw less than ogive and more than ogive cumulative frequency curve on the basis of above data. Prepare a report on continuity and discontinuity of above curves at a certain point and present the report on your classroom.

⇨ Show to your teacher.

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2.3 निरन्तरताको साङ्केतिक प्रस्तुति (NOTATIONAL REPRESENTATION OF CONTINUITY)

EXERCISE 2.3

1. **(a)** $\lim\limits_{x \to a^-} f(x)$ **लाई वाक्यमा लेख्नुहोस् । (Write** $\lim\limits_{x \to a^-} f(x)$ **into sentence.)**

⇨ Here, the input 'x' approaches 'a' from the left in the function $f(x)$.
In other words, the left hand limit of $f(x)$ at $x = a$.

(b) $\lim\limits_{x \to a^+} f(x)$ **लाई वाक्यमा लेख्नुहोस् । (Write** $\lim\limits_{x \to a^+} f(x)$ **into sentence.)**

⇨ Here, the input 'x' approaches 'a' from the right in the function $f(x)$.
In other words, the right hand limit of $f(x)$ at $x = a$.

(c) $\lim\limits_{x \to a} f(x)$ **लाई वाक्यमा लेख्नुहोस् । (Write** $\lim\limits_{x \to a} f(x)$ **into sentence.)**

⇨ Here, the input 'x' approaches 'a' in the function $f(x)$.
In other words, the limit of $f(x)$ at $x = a$.

(d) बिन्दु $x = a$ मा फलन $f(x)$ को निरन्तरता हुने अवस्थालाई सङ्केतमा लेख्नुहोस् ।
Write the condition of $f(x)$ is continuous at $x = a$ in notation.

⇨ Here, the conditions of $f(x)$ is continuous at x = a are as follows:

(i) $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$ (ii) $\lim\limits_{x \to a} f(x) = f(a)$.

2. **$f(x) = x + 1$ एउटा वास्तविक मान भएको फलन छ । ($f(x) = x + 1$ is a real valued function.)**

(a) x = 1.9, 1.99, 1.999 र 1.9999 मा $f(x)$ को मान कति हुन्छ ? पत्ता लगाउनुहोस् ।
What are the values of $f(x)$ at x = 1.9, 1.99, 1.999 and 1.9999? Find it.

⇨ Here, $f(x) = x + 1$

$f(1.9) = 1.9 + 1 = 2.9$
$f(1.99) = 1.99 + 1 = 2.99$
$f(1.999) = 1.999 + 1 = 2.999$
$f(1.9999) = 1.9999 + 1 = 2.9999$

(b) x = 2.1, 2.01, 2.001, र 2.0001 मा $f(x)$ को मान कति हुन्छ ? पत्ता लगाउनुहोस् ।
What are the values of $f(x)$ at x = 2.1, 2.01, 2.001, and 2.0001? Find it.

⇨ Here, $f(2.1) = 2.1 + 1 = 3.1$
$f(2.01) = 2.01 + 1 = 3.01$
$f(2.001) = 2.001 + 1 = 3.001$
$f(2.0001) = 2.0001 + 1 = 3.0001$

(c) $f(2)$ कति हुन्छ ? पत्ता लगाउनुहोस् । (Find $f(2)$.)

⇨ Here, $f(2) = 2 + 1 = 3$

(e) के बिन्दु $x = 2$ मा f(x) निरन्तर हुन्छ ? पत्ता लगाउनुहोस् ।

Is $f(x)$ continuous at point x = 2? Find it.

⇨ Here, from (c) and (d);

We have, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

Thus, $f(x)$ is continuous at x = 2.

(d) $\lim\limits_{x \to 2^-} f(x)$ **र** $\lim\limits_{x \to 2^+} f(x)$ **को मान कति कति हुन्छन् ? पत्ता लगाउनुहोस् ।**

What are the values of $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$**?**

⇨ Here, from (a); $\lim\limits_{x \to 2^-} f(x) = 3$

From (b); $\lim\limits_{x \to 2^+} f(x) = 3$

3. **$f(x) = \begin{cases} x + 2, & 1 \le x \le 2 \\ 4x - 2, & x \ge 2 \end{cases}$ परिभाषित छ । ($f(x)$ is defined as $f(x) = \begin{cases} x + 2, & 1 \le x \le 2 \\ 4x - 2, & x \ge 2 \end{cases}$)**

(a) x = 1.99 हुँदा $f(x)$ को मान पत्ता लगाउनुहोस् । (Find the value of $f(x)$ at x = 1.99.)

⇨ Here, at x = 1.99 the function is $f(x) = x + 2$ So, $f(1.99) = 1.99 + 2 = 3.99$

(b) x = 2.01 हुँदा $f(x)$ को मान पत्ता लगाउनुहोस् । (Find the value of $f(x)$ at x = 2.01.)

⇨ Here, at x = 2.01 the function is $f(x) = 4x - 2$ So, $f(2.01) = 4 \times 2.01 - 2 = 8.04 - 2 = 6.04$

(c) $\lim\limits_{x \to 2^-} f(x)$ **र** $\lim\limits_{x \to 2^+} f(x)$ **को मान कति कति हुन्छन् ? पत्ता लगाउनुहोस् । (Find the value of** $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$**.)**

⇨ Here, from (a); $\lim\limits_{x \to 2^-} f(x) = 4$ From (b); $\lim\limits_{x \to 2^+} f(x) = 6$

(d) के $x = 2$ मा फलन $f(x)$ निरन्तर हुन्छ ? पत्ता लगाउनुहोस् । (Is the function $f(x)$ continuous at x = 2? Find it.)

⇨ Here, from (c) $\lim\limits_{x \to 2^-} f(x) \ne \lim\limits_{x \to 2^+} f(x)$ Thus, the function is not continuous at x = 2.

4. **हाम्रो दैनिक जीवनमा निरन्तरता (Continuity) भन्ने शब्द कहाँ कहाँ प्रयोग भएको छ ? पाठ्यपुस्तकको अध्ययन गरी अथवा इन्टरनेटबाट खोजी गरी अथवा आफूभन्दा माथिल्लो कक्षामा गणित विषय लिएर पढ्ने साथीहरूसँग सोधी पत्ता लगाउनुहोस् । प्राप्त नतिजालाई प्रतिवेदनका रूपमा कक्षाकोठामा प्रस्तुत गर्नुहोस् ।**
Where is the use of continuity in our daily life? Find by studying books, from internet or asking the friend of senior classes. Represent the report so obtained in the class. ⇨ Show to your teacher.

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UNIT III

मेट्रिक्स (Matrix)

1. डिटरमिन्यान्ट, विपरीत मेट्रिक्स र समीकरण
Determinant, Inverse matrix and Equation

KEY POINTS

1. **मेट्रिक्सको डिटरमिनान्ट (Determinant of a matrix)**

मानौँ $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ एउटा 2×2 मेट्रिक्स भए वास्तविक सङ्ख्या $(ad - bc)$ लाई मेट्रिक्स A को डिटरमिनान्ट भनिन्छ र

यसलाई $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ले जनाइन्छ ।

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 matrix then its determinant is the real number $(ad - bc)$.

We write, Det. $A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Note: यदि $A = [a]$ भए $|A| = a$ हुन्छ । (If $A = [a]$ then $|A| = a$.)

2. **विपरीत मेट्रिक्सहरू (Inverse matrices)** **[SLC 2065 M, 2071 R']**

यदि A र B समान क्रमको वर्गाकार मेट्रिक्सहरू, I सोही क्रमको एकाइ मेट्रिक्स र $AB = BA = I$ भए मेट्रिक्सहरू A र B लाई आपसमा विपरीत मेट्रिक्सहरू भनिन्छ । A को विपरीत मेट्रिक्सलाई A^{-1} ले जनाइन्छ । त्यसैले $A^{-1} = B$ हुन्छ ।

If A and B are two square matrices of same order, I is an identity matrix of the same order and $AB = BA = I$ then A and B are said to be inverse to each other. The inverse of A is denoted by A^{-1}. So $A^{-1} = B$.

3. **एकल मेट्रिक्स र स्वामित्वहिन एकल मेट्रिक्स (Singular Matrix and Non singular matrix)**

एउटा वर्गाकार मेट्रिक्स A मा $|A| = 0$ भए सो मेट्रिक्सलाई एकल मेट्रिक्स भनिन्छ । यदि $|A| \neq 0$ भए A लाई स्वामित्वहिन एकल मेट्रिक्स भनिन्छ ।

A square matrix A is called a singular matrix if $|A| = 0$. If $|A| \neq 0$ then A is non-singular matrix.

4. **आसन्न मेट्रिक्स (Adjoint matrix)**

2×2 क्रम भएको वर्ग मेट्रिक्समा मुख्य विकर्णका सदस्यहरूको स्थान परिवर्तन र सहायक विकर्णका सदस्यहरूको चिन्ह परिवर्तन गर्दा प्राप्त हुने नयाँ मेट्रिक्सलाई सो मेट्रिक्सको आसन्न मेट्रिक्स भनिन्छ ।

A new matrix, formed by interchanging the position of the elements of the principal diagonal and changing the sign of the elements of the secondary diagonal of a square matrix of order 2×2, is called an adjoint matrix. For example: If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then adj.(A) $= \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

5. यदि $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ भए $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$ हुन्छ । (If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$.)

6. यदि $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ भए Adj (A) $= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ हुन्छ । (If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then Adj (A) $= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$)

7. $A^{-1} = \frac{1}{|A|}$ Adj (A)
8. $AX = B \Rightarrow X = A^{-1}B$
9. $XA = B \Rightarrow X = BA^{-1}$

QUESTIONS FROM SEE EXCERCISE 1

A. VERT SHORT QUESTIONS

1. **तलका पदहरूको उदाहरण सहित परिभाषा दिनुहोस् । (Define the following terms with example.)**

A. एकल मेट्रिक्स (Singular matrix)

⇨ Here, a square matrix A is called a singular matrix if $|A| = 0$.

B. स्वामित्वहिन एकल मेट्रिक्स (Non singular matrix)

⇨ Here, a square matrix A is called non-singular matrix if $|A| \neq 0$.

C. विपरीत मेट्रिक्स (Inverse matrix)

⇨ Here, if A and B are two square matrices of same order, I is an identity matrix of the same order and $AB = BA = I$ then A and B are said to be inverse to each other. The inverse of A is denoted by A^{-1}. So $A^{-1} = B$.

2. **कस्तो अवस्थामा युगपथ रेखीय समीकरणहरूको एकल समाधान हुँदैन ?**
Under what condition, the system of simultaneous equation has no unique solution?

⇨ Here, if $|A| = 0$ then there is no unique solution.

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3. **कस्तो अवस्थामा एउटा मेट्रिक्सको विपरीत सम्भव हुन्छ ?**
Under what condition, the inverse of a matrix is possible?

⇨ Here, if the determinant of a matrix is non-zero then the inverse is possible.

4. **A, B र X तीनओटा मेट्रिक्सहरू छन् । यदि AX = B र |A| ≠ 0 भए X को मान पत्ता लगाउनुहोस् ।**
A, B and X are three matrices. If AX = B and |A| ≠ 0, what is the value of X?

⇨ Here, if AX = B then $X = A^{-1}B$.

5. **यदि A = [*m*] भए |A| कति हुन्छ ? (If A = [*m*] then what is the value of |A| ?)**

⇨ Here, if A = [*m*] then |A| = m.

6. **यदि AB = BA = I भए A र B कस्तो मेट्रिक्सहरू हुन् ? (If AB = BA = I then what types of matrices are A and B?)**

⇨ Here, if AB = BA = I then A and B are inverse matrix to each other.

7. **मेट्रिक्स $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ का मुख्य विकर्णका सदस्यहरू के के हुन् ?**

What are the elements of principal / leading / major diagonal of a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

⇨ Here, the elements of principal diagonal are a and d.

8. **समीकरणहरू $ax + by = c$ र $mx + ny = p$ लाई मेट्रिक्सको रूपमा व्यक्त गर्नुहोस् ।**
Express the equations $ax + by = c$ and $mx + ny = p$ into matrix form.

⇨ Here, the matrix form of given equations is $\begin{bmatrix} a & b \\ m & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ p \end{bmatrix}$

9. **2 × 2 एकाइ मेट्रिक्सको डिटरमिन्यान्ट के हुन्छ ? (What is the determinant of a 2 × 2 identity matrix?)**

⇨ Here, the determinant of 2 × 2 identity matrix is 1.

10. **यदि A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ भए $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ले के जनाउँदछ ? (If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then what does $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ denote?)**

⇨ Here, $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ denotes the adjoint of A.

11. **यदि $\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 3$ भए *k* को मान पत्ता लगाउनुहोस् । (If $\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 3$, find the value of *k*.)**

⇨ Here, $\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 3$

or, 12 – 3k = 3
or, 9 = 3k
∴ k = 3
Thus, the value of k is 3.

12. **यदि A = $\begin{bmatrix} 2 & 0 \\ b & x \end{bmatrix}$ र A एकल मेट्रिक्स भए *x* को मान पत्ता लगाउनुहोस् ।**

If A = $\begin{bmatrix} 2 & 0 \\ b & x \end{bmatrix}$ and A is a singular matrix then find the value of *x*.

⇨ Here, $\begin{vmatrix} 2 & 0 \\ b & x \end{vmatrix} = 0$

or, 2x – 0 = 0
or, 2x = 0
∴ x = 0
Thus the value of x is 0.

13. **यदि $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ भए X को मान पत्ता लगाउनुहोस् । (If $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find the value of X.)**

⇨ Here, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ∴ $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ [∵ IX = X]

14. **यदि A = [4] भए |A| पत्ता लगाउनुहोस् । (If A = [4] then find |A| .)**

⇨ Here, A = [4] ∴ A = 4

15. **यदि A = $\begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ भए Adj. (A) पत्ता लगाउनुहोस् । (If A = $\begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ then find Adj. (A).)**

⇨ Here, A = $\begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ ∴ Adj (A) = $\begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix}$

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16. समीकरणहरू $2x + 3y = 5$ र $3x + y = 4$ लाई मेट्रिक्स रूपमा लेख्नुहोस् ।
Write the equations $2x + 3y = 5$ and $3x + y = 4$ in matrix form.

⇨ Here, the matrix form of given equations is $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

17. यदि मेट्रिक्स $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ भए |A| को मान कति हुन्छ ? (If matrix $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, what is the value of |A| ?)

[SEE MODEL 2076]

⇨ Here, $|A| = \begin{vmatrix} p & q \\ r & s \end{vmatrix} = ps - qr$.

18. यदि $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ भए |A| कति हुन्छ ? (If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ then what is the value of |A| ?

⇨ Here, $|A| = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2$.

19. $A = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$ को आसन्न (Adjoint) मेट्रिक्स पत्ता लगाउनुहोस् । (Find the adjoint matrix of $A = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$.)

⇨ Here, adjoint matrix of $A = \begin{bmatrix} b & -a \\ -b & a \end{bmatrix}$.

B. SHORT QUESTIONS

MODEL 1

1. यदि $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ भए |A| पत्ता लगाई A^{-1} परिभाषित हुन्छ, हुँदैन लेख्नुहोस् ।
If $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ find |A| and write A^{-1} is defined or not. **[SEE MODEL 2076]**

⇨ Here, $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

So, $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2 + 3 = 5$

Since $|A| \neq 0$ so, A^{-1} exists or defined.

2. यदि $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ र $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ भए |AB| पत्ता लगाउनुहोस् ।
If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, find |AB|.) **[2074 S']**

⇨ Here, $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

So, $AB = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2+6 & 0+(-3) \\ -1+8 & 0+4(-1) \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 8 & -3 \\ 7 & -4 \end{bmatrix}$

Now, $|AB| = \begin{vmatrix} 8 & -3 \\ 7 & -4 \end{vmatrix} = -32 + 21 = -11$

Thus, the value of |AB| is – 11.

3. यदि $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ र $B = \begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix}$ भए A – 3B को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।
If $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix}$ find the determinant of A – 3B. **[2066 R']**

⇨ Here, $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix}$

So, $A - 3B = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} - 3\begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -15 \\ 18 & 6 \end{pmatrix} = \begin{pmatrix} 2-3 & 4+15 \\ 3-18 & 2-6 \end{pmatrix}$

$\therefore A - 3B = \begin{pmatrix} -1 & 19 \\ -15 & -4 \end{pmatrix}$

Now, $|A - 3B| = \begin{vmatrix} -1 & 19 \\ -15 & -4 \end{vmatrix} = (-1) \times (-4) - (-15) \times 19 = 289$

Thus, the determinant of A – 3B is 289.

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4. यदि $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ र $B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ भए 3A – 4B को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ find the determinant of 3A – 4B. [2057 R, 2068 R']

⇨ Here, given matrices are;

$$A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix} \text{and } B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

Now, $3A - 4B = 3\begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix} - 4\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 9 & 12 \\ -6 & 12 \end{pmatrix} + \begin{pmatrix} -8 & -4 \\ 4 & -12 \end{pmatrix}$$

$$= \begin{pmatrix} 9-8 & 12-4 \\ -6+4 & 12-12 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 8 \\ -2 & 0 \end{pmatrix}$$

Now, determinant of $3A - 4B = |3A - 4B|$

$$= \begin{vmatrix} 1 & 8 \\ -2 & 0 \end{vmatrix} = (1 \times 0 - (-2) \times 8) = 0 + 16 = 16$$

Thus, the determinant of 3A – 4B is 16.

5. यदि $P = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ र $Q = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ भए 2P – 3Q को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ find the determinant of 2P – 3Q. [2057 S]

⇨ Here, given matrices are;

$$P = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} \text{ and } Q = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$$

Now, $2P - 3Q = 2\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} - 3\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 2 & 4 \\ 6 & -8 \end{pmatrix} + \begin{pmatrix} -3 & -9 \\ -6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2-3 & 4-9 \\ 6-6 & -8+3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -5 \\ 0 & -5 \end{pmatrix}$$

Now, determinant of 2P – 3Q is |2P – 3Q|

$$= \begin{vmatrix} -1 & -5 \\ 0 & -5 \end{vmatrix} = -1 \times (-5) - 0 \times (-5) = 5 + 0 = 5$$

Thus, the determinant of 2P – 3Q is 5.

6. यदि $P = \begin{pmatrix} 3 & -4 \\ 2 & 1 \end{pmatrix}$ र $Q = \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}$ भए 3P – 2Q को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 3 & -4 \\ 2 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}$ find the determinant of 3P – 2Q. [2061 S]

⇨ Here, given matrices are;

$$P = \begin{pmatrix} 3 & -4 \\ 2 & 1 \end{pmatrix} \text{and } Q = \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}$$

Now, $3P - 2Q = 3\begin{pmatrix} 3 & -4 \\ 2 & 1 \end{pmatrix} - 2\begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 9 & -12 \\ 6 & 3 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-2 & -12+4 \\ 6-6 & 3+2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -8 \\ 0 & 5 \end{pmatrix}$$

Now, determinant of 3P – 2Q is | 3P – 2Q| .

$$= \begin{vmatrix} 7 & -8 \\ 0 & 5 \end{vmatrix} = 7 \times 5 - 0 \times (-8) = 35 + 0 = 35$$

Thus, required determinant is 35.

7. यदि $P = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ र $Q = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ भए PQ को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ find the determinant of PQ. [2058 S]

⇨ Here, given matrices are;

$$P = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \text{and } Q = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

Now, $PQ = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 1 \times 2 - 1 \times 1 & 1 \times 3 - 1 \times 4 \\ -1 \times 2 + 1 \times 1 & -1 \times 3 + 1 \times 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & 3-4 \\ -2+1 & -3+4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Again, determinant of PQ is |PQ|.

$$= \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 1 \times 1 - (-1) \times (-1) = 1 - 1 = 0$$

Thus, the determinant of PQ is 0.

8. यदि $P = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$ र $Q = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ भए PQ को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If matrix $P = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$ and matrix $Q = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$, find the determinant of the matrix PQ. [2066 R]

⇨ Here, given matrices $P = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

So, $PQ = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 6+6 & -3+4 \\ 8-3 & -4-2 \end{pmatrix}$

$\therefore$ $PQ = \begin{pmatrix} 12 & 1 \\ 5 & -6 \end{pmatrix}$

Now, $|PQ| = \begin{vmatrix} 12 & 1 \\ 5 & -6 \end{vmatrix} = -72 - 5 = -77$

Thus, determinant of matrix PQ is – 77.

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9. यदि $M = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ र $N = \begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$ भए MN को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If matrix $M = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and matrix $N = \begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$ find the determinant of MN. **[2067 R]**

⇨ Here, $M = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$

$$\text{Now, } MN = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -8+15 & -12+10 \\ -4+9 & -6+6 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -2 \\ 5 & 0 \end{pmatrix}$$

$$\text{Then, } |MN| = \begin{vmatrix} 7 & -2 \\ 5 & 0 \end{vmatrix} = 0 + 10 = 10$$

Thus, determinant of MN is 10.

10. यदि $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ र $B = \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$ भए MN को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$, find the determinant of MN. **[2067 S]**

⇨ Here, $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$

$$\text{So, } AB = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+6 & -2-9 \\ 8+2 & -4-3 \end{bmatrix}$$

$$\therefore \ AB = \begin{bmatrix} 10 & -11 \\ 10 & -7 \end{bmatrix}$$

$$\text{Now, } |AB| = \begin{vmatrix} 10 & -11 \\ 10 & -7 \end{vmatrix} = -70 + 110 = 40$$

Thus, the determinant of AB is 40.

11. यदि $A = \begin{pmatrix} 3 & 5 \\ -6 & 0 \end{pmatrix}$ र $B = \begin{pmatrix} 18 & -6 \\ 12 & 3 \end{pmatrix}$ भए $3A - \frac{1}{3}B$ को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 3 & 5 \\ -6 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 18 & -6 \\ 12 & 3 \end{pmatrix}$ find the determinant of $3A - \frac{1}{3}B$. **[2059 S]**

⇨ Here, given matrices are;

$$A = \begin{pmatrix} 3 & 5 \\ -6 & 0 \end{pmatrix}, B = \begin{pmatrix} 18 & -6 \\ 12 & 3 \end{pmatrix}$$

$$\text{Now, } 3A - \frac{1}{3}B = 3\begin{pmatrix} 3 & 5 \\ -6 & 0 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 18 & -6 \\ 12 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 15 \\ -18 & 0 \end{pmatrix} - \begin{pmatrix} 6 & -2 \\ 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9-6 & 15+2 \\ -18-4 & 0-1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 17 \\ -22 & -1 \end{pmatrix}$$

Now, determinant of $3A - \frac{1}{3}B$ is $\left|3A - \frac{1}{3}B\right|$.

$$= \begin{vmatrix} 3 & 17 \\ -22 & -1 \end{vmatrix}$$

$$= 3 \times (-1) - 17 \times (-22)$$

$$= -3 + 374 = 371$$

Thus, determinant of $3A - \frac{1}{3}B$ is 371.

12. यदि $P = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$ र $Q = \begin{pmatrix} 2 & 4 \\ -6 & 2 \end{pmatrix}$ भए $5P - \frac{1}{2}Q + 2I$ को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 4 \\ -6 & 2 \end{pmatrix}$ find the determinant of $5P - \frac{1}{2}Q + 2I$. **[2059 R]**

⇨ Here, given matrices are;

$$P = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}, Q = \begin{pmatrix} 2 & 4 \\ -6 & 2 \end{pmatrix} \text{ and } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{Now, } 5P - \frac{1}{2}Q + 2I$$

$$= 5\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2 & 4 \\ -6 & 2 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 0 \\ -5 & 15 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 10-1+2 & 0-2+0 \\ -5+3+0 & 15-1+2 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -2 \\ -2 & 16 \end{pmatrix}$$

The determinant of $5P - \frac{1}{2}Q + 2I$ is $\left|5P - \frac{1}{2}Q + 2I\right|$

$$= \begin{vmatrix} 11 & -2 \\ -2 & 16 \end{vmatrix}$$

$$= 11 \times 16 - (-2) \times -2 = 176 - 4 = 172$$

Thus, required determinant is 172.

13. यदि $P = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ र $Q = \begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix}$ भए $5P - 2Q - 3I$ को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix}$ find the determinant of $5P - 2Q - 3I$. **[2061 R]**

⇨ Here, given matrices $P = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$, $Q = \begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\text{Now, } 5P - 2Q - 3I = 5\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} - 2\begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix} - \begin{pmatrix} -6 & 0 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

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$$= \begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ -2 & 4 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+6-3 & -10+0+0 \\ 15-2+0 & 20+4-3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -10 \\ 13 & 21 \end{pmatrix}$$

Then determinant $|5P - 2Q - 3I| = \begin{vmatrix} 8 & -10 \\ 13 & 21 \end{vmatrix} = 8 \times 21 - 13 \times (-10) = 168 + 130 = 298$

MODEL 2

14. यदि मेट्रिक्स A = $\begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ र A को डिटरमिन्यान्ट 4 भए *x* को मान पत्ता लगाउनुहोस् ।

If the matrix A = $\begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ and determinant of A is 4 then, find the value of *x*. **[SEE 2075 R']**

⇨ Here, $A = \begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$

By the question, $|A| = 4$

or, $\begin{vmatrix} 2x & 6 \\ 4 & 1 \end{vmatrix} = 4$

or, $2x - 24 = 4$

or, $2x = 28$

$\therefore \quad x = 14$

Thus, the value of x is 14.

16. यदि मेट्रिक्स P = $\begin{bmatrix} 2x & -6 \\ 3 & 4 \end{bmatrix}$ र P को डिटरमिन्यान्ट 10 भए x को मान पत्ता लगाउनुहोस् ।

If matrix P = $\begin{bmatrix} 2x & -6 \\ 3 & 4 \end{bmatrix}$ and determinant of P is 10 then find the value of x. **[2074 R']**

⇨ Here, $P = \begin{pmatrix} 2x & -6 \\ 3 & 4 \end{pmatrix}$ and $|P| = 10$

So, $|P| = \begin{vmatrix} 2x & -6 \\ 3 & 4 \end{vmatrix}$

or, $10 = 8x + 18$

or, $-8 = 8x$

$\therefore \quad x = -1$

Thus, the value of x is – 1.

18. यदि K = $\begin{bmatrix} 2 & -3 \\ 5 & m \end{bmatrix}$ र |K| = 23 भए, m को मान पत्ता लगाउनुहोस् ।

If K = $\begin{bmatrix} 2 & -3 \\ 5 & m \end{bmatrix}$ and |K| = 23 then find the value of m. **[2073 R']**

⇨ Here, $K = \begin{bmatrix} 2 & -3 \\ 5 & m \end{bmatrix}$

So, $|K| = \begin{vmatrix} 2 & -3 \\ 5 & m \end{vmatrix} = 2m + 15$

or, $23 = 2m + 15$

or, $8 = 2m \quad \therefore \quad m = 4$

Thus, the value of m is 4.

15. यदि M = $\begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, N = $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ र |MN| = 10 भए P को मान पत्ता लगाउनुहोस् ।

If M = $\begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, N = $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ and |MN| = 10, find the value of P. **[SEE 2075 R'2]**

⇨ Here, $M = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, $N = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ and $|MN| = 10$

Now, $MN = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix} \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 14+3 & 8+2 \\ 21+3P & 12+2P \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 10 \\ 21+3P & 12+2P \end{bmatrix}$$

since $|MN| = 10$

or, $\begin{vmatrix} 17 & 10 \\ 21+3P & 12+2P \end{vmatrix} = 10$

or, $204 + 34P - 210 - 30P = 10$

or, $4P - 6 = 10$

or, $4P = 16$

$\therefore \quad P = 4$

Thus, the value of P is 4.

17. यदि $\begin{vmatrix} 3x & -6 \\ 4 & 2 \end{vmatrix}$ = 0 भए x को मान पत्ता लगाउनुहोस् ।

If $\begin{vmatrix} 3x & -6 \\ 4 & 2 \end{vmatrix}$ = 0, find the value of x. **[2073 R]**

⇨ Here, $\begin{vmatrix} 3x & -6 \\ 4 & 2 \end{vmatrix} = 0$

or, $6x + 24 = 0$

or, $6x = -24$

$\therefore \quad x = -4$

Thus, the value of x is – 4.

19. यदि $\begin{vmatrix} 2p & 7 \\ -5 & 9 \end{vmatrix}$ = – 1 भए p को मान निकाल्नुहोस् ।

If $\begin{vmatrix} 2p & 7 \\ -5 & 9 \end{vmatrix}$ = – 1, find the value of p. **[2073 S]**

⇨ Here, $\begin{vmatrix} 2p & 7 \\ -5 & 9 \end{vmatrix} = -1$

or, $2p \times 9 - 7 \times (-5) = -1$

or, $18p + 35 = -1$

or, $18p = -36$

$\therefore \quad p = -2$

Thus, the value of p is – 2.

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20. यदि $\begin{vmatrix} 3p & 2 \\ -4 & 3 \end{vmatrix} = 8$ भए p को मान निकाल्नुहोस् ।

If $\begin{vmatrix} 3p & 2 \\ -4 & 3 \end{vmatrix} = 8$, find the value of p. **[2073 S']**

⇨ Here, $\begin{vmatrix} 3p & 2 \\ -4 & 3 \end{vmatrix} = 8$

or, $3p \times 3 - 2(-4) = 8$

or, $9p + 8 = 8$

or, $9p = 0$

$\therefore$ $p = 0$

Thus, the value of p is 0.

21. यदि मेट्रिक्स $M = \begin{bmatrix} -5 & -8 \\ p & 12 \end{bmatrix}$ को determinant 20 भए, p को मान पत्ता लगाउनुहोस् ।

If the determinant of matrix $M = \begin{bmatrix} -5 & -8 \\ p & 12 \end{bmatrix}$ is 20, find the value of p. **[2070 R]**

⇨ Here, $M = \begin{bmatrix} -5 & -8 \\ p & 12 \end{bmatrix}$ and $|M| = 20$

We have, $|M| = \begin{vmatrix} -5 & -8 \\ p & 12 \end{vmatrix}$

or, $20 = -60 + 8p$

or, $80 = 8p$

$\therefore$ $p = 10$

Thus, the value of p is 10.

22. यदि $M = \begin{bmatrix} 5 & 3 \\ 6 & K \end{bmatrix}$ र $|M| = 27$ भए K को मान पत्ता लगाउनुहोस् । **[2072 R']**

If $M = \begin{bmatrix} 5 & 3 \\ 6 & K \end{bmatrix}$ and $|M| = 27$ then find the value of K.

⇨ Here, $M = \begin{bmatrix} 5 & 3 \\ 6 & K \end{bmatrix}$

So, $|M| = \begin{vmatrix} 5 & 3 \\ 6 & K \end{vmatrix}$

or, $27 = 5K - 18$

or, $45 = 5K$

$\therefore$ $K = 9$

Thus, the value of K is 9.

23. यदि $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}$ र $|BA| = -5$ भए m को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}$ and $|BA| = -5$, find the value of m. **[2070 R']**

⇨ Here, $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}$

and $|BA| = -5$

$$\text{So, } BA = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4+3 & 2+9 \\ 2+m & 1+3m \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 11 \\ 2+m & 1+3m \end{pmatrix}$$

Now, $|BA| = \begin{vmatrix} 7 & 11 \\ 2+m & 1+3m \end{vmatrix} = 7 + 21m - 22 - 11m$

or, $-5 = 10m - 15$

or, $10 = 10m$

$\therefore$ $m = 1$

Thus, the value of m is 1.

24. यदि $A = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix}$ र $|A - B - 5I| = 14$ भए x को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix}$ and the determinant of $A - B - 5I$ is 14, calculate the value of x. **[2062R]**

⇨ Here, given matrices $A = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix}$ and $|A - B - 5I| = 14$, $x = ?$

$$\text{Now, } A - B - 5I = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix} - \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -4-4 & 5-6 \\ 7-x & 8-3 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & -1 \\ 7-x & 5 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -8-5 & -1 \\ 7-x & 5-5 \end{pmatrix} = \begin{pmatrix} -13 & -1 \\ 7-x & 0 \end{pmatrix}$$

Now, determinant of $A - B - 5I = |A - B - 5I| = 14$

or $\begin{vmatrix} -13 & -1 \\ 7-x & 0 \end{vmatrix} = 14$

or, $-13 \times 0 - (7 - x) \times (-1) = 14$

$\therefore$ $7 - x = 14$

or, $-x = 14 - 7$ $\therefore$ $x = -7$

Thus, the required value of x is – 7.

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25. यदि $A = \begin{pmatrix} 3p & -2 \\ 4 & 3 \end{pmatrix}$ र डिटरमिन्यान्ट 71 भए p को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 3p & -2 \\ 4 & 3 \end{pmatrix}$ and the value of its determinant is 71, find the value of 'p'. **[2060CP]**

⇨ Here, $A = \begin{pmatrix} 3p & -2 \\ 4 & 3 \end{pmatrix}$

or, $|A| = \begin{vmatrix} 3p & -2 \\ 4 & 3 \end{vmatrix}$

or, $71 = 9p + 8$

or, $71 - 8 = 9p$

or, $63 = 9p$

$\therefore \quad p = 7$

Thus, the required value of p is 7.

26. यदि $A = \begin{pmatrix} 3 & 1 \\ -2 & x \end{pmatrix}$ र डिटरमिन्यान्ट $|A| = 14$ भए x को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 3 & 1 \\ -2 & x \end{pmatrix}$ and $|A| = 14$, find the value of x. **[2063S]**

⇨ Here, $A = \begin{pmatrix} 3 & 1 \\ -2 & x \end{pmatrix}$

or, $|A| = \begin{vmatrix} 3 & 1 \\ -2 & x \end{vmatrix}$

or, $14 = 3x + 2$

or, $14 - 2 = 3x$

or, $12 = 3x$

$\therefore \quad x = 4$

Thus, the required value of x is 4.

27. यदि $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$ डिटरमिन्यान्ट $|AB| = 5$ भए x को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$ and $|AB| = 5$, find the value of x. **[2065 R']**

⇨ Here, $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$ and $|AB| = 5$

So, $AB = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$

or, $AB = \begin{pmatrix} 2+2 & 3+2x \\ 6+1 & 9+x \end{pmatrix}$

$\therefore \quad AB = \begin{pmatrix} 4 & 3+2x \\ 7 & 9+x \end{pmatrix}$

Now, $|AB| = \begin{vmatrix} 4 & 3+2x \\ 7 & 9+x \end{vmatrix}$

$= 4(9 + x) - 7(3 + 2x)$

or, $|AB| = 36 + 4x - 21 - 14x = 15 - 10x$

or, $5 = 15 - 10x$

or, $10x = 10$

$\therefore \quad x = 1$

Thus, the required value of x is 1.

28. यदि $\begin{vmatrix} x & -2x \\ 3x & 4x \end{vmatrix} = 100$ भए x को मान पत्ता लगाउनुहोस् ।

If $\begin{vmatrix} x & -2x \\ 3x & 4x \end{vmatrix} = 100$ then find the value of x. **[2065 E]**

⇨ Here, $\begin{vmatrix} x & -2x \\ 3x & 4x \end{vmatrix} = 100$

or, $4x^2 + 6x^2 = 100$

or, $10x^2 = 100$

or, $x^2 = \frac{100}{10}$

or, $x^2 = 10$

$\therefore \quad x = \sqrt{10}$

Thus, the required value of x is $\sqrt{10}$.

MODEL 3

29. विपरीत मेट्रिक्सको परिभाषा लेख्नुहोस् । कुन अवस्थामा विपरीत मेट्रिक्स परिभाषित हुँदैन ? लेख्नुहोस् ।

Write the definition of inverse matrix. In which condition the inverse of the matrix cannot be defined? Write it. **[SEE 2075 R, R2]**

⇨ If A and B are two square matrices of same order, I is an identity matrix of the same order and AB = BA = I then A and B are said to be inverse to each other. The inverse of A is denoted by A^{-1}. So $A^{-1} = B$. If the determinant of a matrix is 0, then inverse of the matrix cannot be defined.

30. x को मान कति हुँदा मेट्रिक्स $\begin{pmatrix} 4 & -3 \\ x & 3 \end{pmatrix}$ को विपरीत मेट्रिक्स परिभाषित गर्न सकिँदैन ? पत्ता लगाउनुहोस् ।

For what value of x, the inverse of matrix $\begin{pmatrix} 4 & -3 \\ x & 3 \end{pmatrix}$ cannot be defined? Find it. **[2074 R]**

⇨ Here, $\begin{pmatrix} 4 & -3 \\ x & 3 \end{pmatrix}$ So, $\begin{vmatrix} 4 & -3 \\ x & 3 \end{vmatrix} = 12 + 3x$

If $|A| = 0$ then A^{-1} is not defined.

So, $12 + 3x = 0$

or, $3x = -12$

$\therefore \quad x = -4$

Thus, the value of x is – 4.

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31. यदि मेट्रिक्स A = $\begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix}$ भए A^{-1} पत्ता लगाउनुहोस् ।

If matrix A = $\begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix}$ then find A^{-1}. [2074 R']

⇨ Here, $A = \begin{pmatrix} 2 & 3 \\ -3 & -5 \end{pmatrix}$

So, $|A| = \begin{vmatrix} 2 & 3 \\ -3 & -5 \end{vmatrix} = -10 + 9 = -1$

We have, $A^{-1} = \frac{1}{|A|}$ (Adj. (A))

$= \frac{1}{-1}\begin{pmatrix} -5 & -3 \\ 3 & 2 \end{pmatrix}$

$\therefore\ A^{-1} = \begin{pmatrix} 5 & 3 \\ -3 & -2 \end{pmatrix}$

Thus, the inverse of matrix A is $A^{-1}\begin{pmatrix} 5 & 3 \\ -3 & -2 \end{pmatrix}$

32. यदि मेट्रिक्स A = $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ भए A^{-1} पत्ता लगाउनुहोस् ।

If matrix A = $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ then find A^{-1}. [2073 R']

⇨ Here, $A = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

We have, $|A| = \begin{vmatrix} 5 & 3 \\ 7 & 4 \end{vmatrix} = 20 - 21 = -1$

We know that, $A^{-1} = \frac{1}{|A|}$ [Adj. (A)]

$= \frac{1}{-1}\begin{bmatrix} 4 & -3 \\ -7 & 5 \end{bmatrix}$

$= -1\begin{bmatrix} 4 & -3 \\ -7 & 5 \end{bmatrix}$

$\therefore\ A^{-1} = \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$

Thus, the inverse of A is $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$.

33. यदि A को विपरित मेट्रिक्स $A^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।

If the inverse of matrix A is $A^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$, find the matrix A. [2071 R]

⇨ Here, $A^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$

We have, $|A^{-1}| = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 12 - 10 = 2$

We know that, $A = (A^{-1})^{-1} = \frac{1}{|A^{-1}|}$ Adj. (A^{-1})

$= \frac{1}{2}\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$

$\therefore\ A = \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}$

Thus, the required matrix $A = \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}$

34. a को मान कति हुँदा A = $\begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$ को विपरित मेट्रिक्स परिभाषित हुँदैन ? पत्ता लगाउनुहोस् ।

For what value of a, the inverse of A = $\begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$ is not defined? Find it. [2074 S]

⇨ Here, $A = \begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$

So, $|A| = \begin{vmatrix} 4 & 3 \\ a & 3 \end{vmatrix} = 12 - 3a$

If the inverse of A is not defined then, $|A| = 0$

i.e. $12 - 3a = 0$

or, $3a = 12$ $\quad\therefore\ a = 4$

Thus, the value of a is 4.

35. यदि मेट्रिक्स $\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$ को विपरित मेट्रिक्स परिभाषित गर्न सकिँदैन भने x को मान पत्ता लगाउनुहोस् ।

If the inverse of the matrix $\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$ can not be defined, find the value of x. [2072 R]

⇨ Here, given matrix = $\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$

So, $\begin{vmatrix} x & 3 \\ 4 & 2 \end{vmatrix} = 0$

or, $2x - 12 = 0$

or, $2x = 12$

$\therefore\ x = 6$

Thus, the value of x is 6.

36. विपरीत मेट्रिक्सको परिभाषा दिनुहोस् । यदि A = $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$ भए A को विपरीत मेट्रिक्स A^{-1} पत्ता लगाउनुहोस् ।

Define inverse of a matrix. Find the inverse A^{-1} to the matrix A if A = $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$ [2071 R']

⇨ **Inverse matrix:** If A and B are two square matrices of same order , I is an identity matrix of the same order and AB = BA = I then A and B are said to be inverse to each other. The inverse of A is denoted by A^{-1}. So $A^{-1} = B$.

Here, $A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$

So, $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 5 - 8 = -3$

And, Adj.(A) = $\begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix}$

We know that, $A^{-1} = \frac{1}{|A|}$ Adj. (A)

$= -\frac{1}{3}\begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix}$

$\therefore\ A^{-1} = \begin{pmatrix} -\frac{5}{3} & \frac{2}{3} \\ \frac{4}{3} & -\frac{1}{3} \end{pmatrix}$

Thus, the inverse A^{-1} is $\begin{pmatrix} -\frac{5}{3} & \frac{2}{3} \\ \frac{4}{3} & -\frac{1}{3} \end{pmatrix}$

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37. यदि मेट्रिक्स A को विपरित मेट्रिक्स $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।

If an inverse matrix of a matrix A is $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, find the matrix A. **[2071 S]**

⇨ Here, $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$\text{Adj. }(A^{-1}) = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

We know that,

$$A = (A^{-1})^{-1} = \frac{1}{|A^{-1}|}(\text{Adj. }(A^{-1}))$$

$$= \frac{1}{-2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{bmatrix}$$

Thus, the matrix A is $\begin{bmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{bmatrix}$.

38. यदि $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ भए A^{-1} को विपरीत मेट्रिक्स A पत्ता लगाउनुहोस् ।

Find the inverse A^{-1} to matrix A if $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ **[2065 M]**

⇨ Here, we have, $|A| = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$

So, $|A| = 2 \times 4 - 3 \times 1 = 5$

$$\text{Adj. }(A) = \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$$

We know that,

$$A^{-1} = \frac{1}{|A|}\cdot \text{Adj. of } A$$

Now, $A^{-1} = \frac{1}{5}\begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$

39. यदि $P = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ भए विपरीत मेट्रिक्स P^{-1} पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$, find P^{-1}. **[2066 S]**

⇨ Here, $P = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

$$\therefore \quad |P| = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$$

Now, $P^{-1} = \frac{1}{|P|}\text{Adj.}(A) = \frac{1}{6}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

Thus, $P^{-1} = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

40. यदि $A = \begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}$ भए विपरीत मेट्रिक्स A^{-1} पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}$ then find inverse matrix A^{-1}. **[2060 R]**

⇨ Here, given $A = \begin{pmatrix} 7 & 4 \\ 3 & 2 \end{pmatrix}$

or, $|A| = \begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix}$

$$= 7 \times 2 - 3 \times 4 = 14 - 12 = 2 \neq 0$$

Hence A^{-1} exists.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then, $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Here, a = 7, b = 4, c = 3, d = 2

Thus, $A^{-1} = \frac{1}{2}\begin{pmatrix} 2 & -4 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ \frac{-3}{2} & \frac{7}{2} \end{pmatrix}$

41. यदि $P = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$ भए P को विपरीत मेट्रिक्स पत्ता लगाउनुहोस् ।

If $P = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$ find the inverse matrix of P. **[2065 S]**

⇨ Here, $P = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$

So, $|P| = \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} = 2 \times 1 - 0 \times 3 = 2 \neq 0$

Hence, P^{-1} exists

Now, $P^{-1} = \frac{1}{|P|}\cdot \text{Adj (P)}$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix}$$

Thus, the inverse matrix of P is $\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & 1 \end{bmatrix}$.

42. मेट्रिक्स $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ को विपरीत मेट्रिक्स पत्ता लगाउनुहोस् ।

Find the inverse matrix of matrix $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ **[2068 R']**

⇨ Here, Let $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ be the given matrix.

Now, Adj of $A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$$

Then, $A^{-1} = \frac{1}{|A|}\text{Adj. of } A = \frac{1}{1}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

Thus, inverse of given matrix is $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

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43. यदि विपरीत मेट्रिक्स $A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ भए मेट्रिक्स A को मान निकाल्नुहोस् ।

If inverse matrix $A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find the value of matrix A. **[2065 E]**

⇨ Here, Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a square matrix, $A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is identity matrix

Now, $A.A^{-1} = I$

or, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Taking the relation of corresponding elements from equal matrices,

$a + 3b = 1$ or, $a = 1 - 3b$	$2a + 4b = 0$ or, $2(1 - 3b) + 4b = 0$ or, $2 - 6b + 4b = 0$ or, $2 - 2b = 0$ or, $2b = 2$ $\therefore$ $b = 1$	$c + 3d = 0$ or, $c = -3d$	$2c + 4d = 1$ or, $-6d + 4d = 1$ or, $-2d = 1$ $\therefore$ $d = -\frac{1}{2}$
or, $a = 1 - 3.1$ $= 1 - 3$ $\therefore$ $a = -2$		$\therefore$ $c = -3\left(-\frac{1}{2}\right) = \frac{3}{2}$	

Thus, the required matrix is $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

MODEL 4

44. यदि मेट्रिक्सहरू $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ र $\begin{pmatrix} x & -1 \\ -\frac{5}{2} & 2 \end{pmatrix}$ आपसमा एक अर्काका विपरीत मेट्रिक्स हुन् भने x को मान निकाल्नुहोस् ।

If the matrices $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} x & -1 \\ -\frac{5}{2} & 2 \end{pmatrix}$ are inverse matrix to each other, calculate the value of x. [2074 S']

⇨ Here, $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} x & -1 \\ -\frac{5}{2} & 2 \end{pmatrix}$ are inverse to each other,

So, $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} x & -1 \\ -\frac{5}{2} & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 4x-5 & -4+4 \\ 5x-\frac{15}{2} & -5+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 4x-5 & 0 \\ 5x-\frac{15}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Equating the corresponding elements then,

$4x - 5 = 1$ and $5x - \frac{15}{2} = 0$

or, $4x = 6$ or, $5x = \frac{15}{2}$

$\therefore$ $x = \frac{6}{4} = \frac{3}{2}$ $\therefore$ $x = \frac{3}{2}$

Thus, the value of x is $\frac{3}{2}$.

45. मेट्रिक्स $\begin{bmatrix} m & 2 \\ 7 & 3 \end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix} 3 & -2 \\ -7 & m \end{bmatrix}$ भए m को मान निकाल्नुहोस् ।

If the inverse of matrix $\begin{bmatrix} m & 2 \\ 7 & 3 \end{bmatrix}$ is the matrix $\begin{bmatrix} 3 & -2 \\ -7 & m \end{bmatrix}$, find the value of m. [2072 S]

⇨ Here, $\begin{bmatrix} m & 2 \\ 7 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ -7 & m \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or, $\begin{bmatrix} 3m-14 & -2m+2m \\ 21-21 & -14+3m \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or, $\begin{bmatrix} 3m-14 & 0 \\ 0 & -14+3m \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equating the corresponding components;

$3m - 14 = 1$

or, $3m = 15$

$\therefore$ $m = 5$

$-14 + 3m = 1$

or, $3m = 15$

$\therefore$ $m = 5$

Thus, the value of m is 5.

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46. यदि $\begin{pmatrix}2m & 7\\ 5 & 9\end{pmatrix}$ को विपरीत मेट्रिक्स $\begin{pmatrix}9 & n\\ -5 & 4\end{pmatrix}$ भए m र n का मानहरू पत्ता लगाउनुहोस् ।

If the inverse of the matrix $\begin{pmatrix}2m & 7\\ 5 & 9\end{pmatrix}$ is the matrix $\begin{pmatrix}9 & n\\ -5 & 4\end{pmatrix}$ calculate the value of m and n.[2061 R]

⇨ Here, the inverse marix of $\begin{pmatrix}2m & 7\\ 5 & 9\end{pmatrix}$ is $\begin{pmatrix}9 & n\\ -5 & 4\end{pmatrix}$

So, $\begin{pmatrix}2m & 7\\ 5 & 9\end{pmatrix}\begin{pmatrix}9 & n\\ -5 & 4\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}2m \times 9 + 7 \times (-5) & 2m \times n + 7 \times 4\\ 5 \times 9 + 9 \times (-5) & 5n + 9 \times 4\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}18m - 35 & 2mn + 28\\ 45 - 45 & 5n + 36\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}18m - 35 & 2mn + 28\\ 0 & 5n + 36\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $18m - 35 = 1$ .... (i)

$2mn + 28 = 0$ .... (ii)

$5n + 36 = 1$ .... (iii)

From equation (i)

$18m = 36$

or, $m = \frac{36}{18} = 2$

$\therefore$ $m = 2$

From equation (iii)

$5n = 1 - 36$

or, $5n = -35$

or, $n = \frac{-35}{5} = -7$

$\therefore$ $m = 2$ and $n = -7$

Thus required value of m and n are 2 and − 7 respectively.

47. यदि $A = \begin{pmatrix}1 & -2\\ 0 & x\end{pmatrix}$ को विपरीत मेट्रिक्स $B = \begin{pmatrix}1 & 4\\ y & 2\end{pmatrix}$ भए x र y का मानहरू पत्ता लगाउनुहोस् ।

If the inverse of matrix $A = \begin{pmatrix}1 & -2\\ 0 & x\end{pmatrix}$ is the matrix $B = \begin{pmatrix}1 & 4\\ y & 2\end{pmatrix}$ determine the values of x and y.

[2059 R, 2068R]

⇨ Here, $A = \begin{pmatrix}1 & -2\\ 0 & x\end{pmatrix}$ and $B = \begin{pmatrix}1 & 4\\ y & 2\end{pmatrix}$

Since, B is the inverse of the matrix A, then

AB = I where $I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is the identity matrix.

So, $\begin{pmatrix}1 & -2\\ 0 & x\end{pmatrix}\begin{pmatrix}1 & 4\\ y & 2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}1 \times 1 + (-2) \times y & 1 \times 4 + (-2) \times 2\\ 0 \times 1 + x \times y & 0 \times 4 + x \times 2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}-2y + 1 & 4 - 4\\ 0 + xy & 0 + 2x\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}-2y + 1 & 0\\ xy & 2x\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Equating the corresponding terms, we get,

$-2y + 1 = 1$ .... (i)

$xy = 0$ .... (ii)

$2x = 1$ .... (iii)

From (iii) $x = \frac{1}{2}$ and

From (i) $-2y = 0$

$\therefore$ $y = 0$

Thus, the required value of x & y are $\frac{1}{2}$ and 0 respectively.

48. यदि $\begin{pmatrix}x & 2x - 9\\ -y & 3\end{pmatrix}$ को विपरीत मेट्रिक्स $\begin{pmatrix}3 & 5\\ y & x\end{pmatrix}$ भए x र y का मानहरू पत्ता लगाउनुहोस् ।

If the inverse of the matrix $\begin{pmatrix}x & 2x - 9\\ -y & 3\end{pmatrix}$ is the matrix $\begin{pmatrix}3 & 5\\ y & x\end{pmatrix}$ find the values of x and y. [2062R]

⇨ Here, inverse of the matrix $\begin{pmatrix}x & 2x - 9\\ -y & 3\end{pmatrix}$ is matrix $\begin{pmatrix}3 & 5\\ y & x\end{pmatrix}$

So, $\begin{pmatrix}x & 2x - 9\\ -y & 3\end{pmatrix}\begin{pmatrix}3 & 5\\ y & x\end{pmatrix} = I$ [∵ Where I is a unit matrix of order 2 × 2]

or, $\begin{pmatrix}3x + y(2x - 9) & 5x + x(2x - 9)\\ -3y + 3y & -5y + 3x\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}3x + 2xy - 9y & 5x + 2x^2 - 9x\\ 0 & 3x - 5y\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

or, $\begin{pmatrix}3x + 2xy - 9y & 2x^2 - 4x\\ 0 & 3x - 5y\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$\therefore$ $3x + 2xy - 9y = 1$, ... (i)

$2x^2 - 4x = 0$ .... (ii)

$3x - 5y = 1$ .... (iii)

Now, from equation (ii) $2x(x - 2) = 0$

Either, $2x = 0$ or $x - 2 = 0$

$\therefore$ $x = 0$ or $x = 2$

Putting x = 0 in equation (iii) we get, $3.0 - 5y = 1$

or, $-5y = 1$

$\therefore$ $y = \frac{-1}{5}$

Again, putting x = 2 in equation (iii) we get,

$3.2 - 5y = 1$

or, $6 - 5y = 1$

or, $-5y = 1 - 6$

or, $-5y = -5$

$\therefore$ $y = \frac{5}{5} = 1$

Here, Since equation (i) is not satisfied by x = 0 and $y = \frac{-1}{5}$ but satisfied by x = 2 and y = 1.

Thus, the required value of x and y are 2 and 1 respectively.

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49. यदि $\begin{pmatrix} x & 3 \\ -1 & -1 \end{pmatrix}$ को विपरीत मेट्रिक्स $\begin{pmatrix} -1 & -3 \\ 1 & 2 \end{pmatrix}$ भए x को मान पत्ता लगाउनुहोस् ।

If the matrices $\begin{pmatrix} x & 3 \\ -1 & -1 \end{pmatrix}$ and $\begin{pmatrix} -1 & -3 \\ 1 & 2 \end{pmatrix}$ are inverse to each other, find the value of x. **[2067R']**

⇨ Here, given matrices $\begin{pmatrix} x & 3 \\ -1 & -1 \end{pmatrix}$ and $\begin{pmatrix} -1 & -3 \\ 1 & 2 \end{pmatrix}$ are inverse matrix to each other.

So, $\begin{pmatrix} x & 3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} -x+3 & -3x+6 \\ 1-1 & 3-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Taking corresponding elements; $-x + 3 = 1$

or, $-x = 1 - 3$

or, $-x = -2$

$\therefore\ x = 2$

Thus, the required value of x is 2.

C. LONG QUESTIONS

MODEL1

1. **मेट्रिक्स विधिबाट हल गर्नुहोस् :**

Solve by matrix method:

$3x + 5y = 11$, $2x - 3y = 1$. **[SEE MODEL 2076]**

⇨ Here, $3x + 5y = 11$ and $2x - 3y = 1$

The matrix form of above equations;

$$\underset{A}{\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 11 \\ 1 \end{pmatrix}}$$

We have,

$AX = B$

So, $X = A^{-1}B$.

So, $|A| = \begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix}$

$= -9 - 10$

$= -19$

$\therefore$ Adj. (A) $= \begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}$

We know that,

$A^{-1} = \frac{1}{|A|}$ Adj (A)

$= -\frac{1}{19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}$

Now, $X = A^{-1}B$

$= -\frac{1}{19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ 1 \end{pmatrix}$

$= -\frac{1}{19}\begin{pmatrix} -3 \times 11 + (-5) \times 1 \\ -2 \times 11 + 3 \times 1 \end{pmatrix}$

$= -\frac{1}{19}\begin{pmatrix} -33 - 5 \\ -22 + 3 \end{pmatrix}$

$= -\frac{1}{19}\begin{pmatrix} -38 \\ -19 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

i.e. $x = 2$, $y = 1$

Thus, $x = 2$ and $y = 1$ is the solution.

2. **मेट्रिक्स विधिबाट हल गर्नुहोस् :**

Solve by matrix method:

$\frac{3}{y} = \frac{2}{x} + 2$, $\frac{6}{x} - \frac{2}{y} = 1$ **[SEE 2075 R]**

⇨ Here, the given equations are $\frac{3}{y} = \frac{2}{x} + 2$, $\frac{6}{x} - \frac{2}{y} = 1$

The matrix form of above equations is;

$$\underset{A}{\begin{pmatrix} -2 & 3 \\ 6 & -2 \end{pmatrix}}\underset{X}{\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}} = \underset{B}{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

$\therefore\ X = A^{-1}B$ ....(i)

So, $|A| = \begin{vmatrix} -2 & 3 \\ 6 & -2 \end{vmatrix} = |4 - 18| = -14$

And, Adj. (A) $= \begin{pmatrix} -2 & -3 \\ -6 & -2 \end{pmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ Adj. (A)

$= -\frac{1}{14}\begin{pmatrix} -2 & -3 \\ -6 & -2 \end{pmatrix}$

We have, $X = A^{-1}B$ [from (i)]

$= -\frac{1}{14}\begin{pmatrix} -2 & -3 \\ -6 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \frac{1}{14}\begin{pmatrix} 2 & 3 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \frac{1}{14}\begin{pmatrix} 4+3 \\ 12+2 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 7 \\ 14 \end{pmatrix}$

or, $\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$

So, $\frac{1}{x} = \frac{1}{2}$ $\therefore\ x = 2$

and $\frac{1}{y} = 1$ $\therefore\ y = 1$

Thus, $x = 2$ and $y = 1$ are the solutions.

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3. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$\frac{4}{x}+\frac{3}{y}=1, \frac{3}{x}-\frac{2}{y}=\frac{1}{24}$ **[SEE 2075 R$_2$, 2064 S]**

⇨ Here, the given equations are

$\frac{4}{x}+\frac{3}{y}=1, \frac{3}{x}-\frac{2}{y}=\frac{1}{24}$

The matrix form of above equations are:

$$\underset{A}{\begin{pmatrix} 4 & 3 \\ 3 & -2 \end{pmatrix}}\underset{X}{\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}}=\underset{B}{\begin{pmatrix} 1 \\ \frac{1}{24} \end{pmatrix}}$$

$AX = B$

or, $X = A^{-1}B$

Where,

$$A=\begin{pmatrix} 4 & 3 \\ 3 & -2 \end{pmatrix}, X=\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} \ \& \ B=\begin{pmatrix} 1 \\ \frac{1}{24} \end{pmatrix}$$

$$\therefore \ |A| = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix}$$
$$= -8-9$$
$$= -17$$

$$\therefore \ A^{-1} = \frac{1}{|A|}\text{Adj (A)}$$
$$= \frac{1}{-17}\begin{pmatrix} -2 & -3 \\ -3 & 4 \end{pmatrix}$$
$$= \frac{1}{17}\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{17}\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ \frac{1}{24} \end{pmatrix}$$
$$= \frac{1}{17}\begin{pmatrix} 2+\frac{1}{8} \\ 3-\frac{1}{6} \end{pmatrix}$$
$$= \frac{1}{17}\begin{pmatrix} \frac{17}{8} \\ \frac{17}{6} \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}=\begin{pmatrix} \frac{1}{8} \\ \frac{1}{6} \end{pmatrix}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{8}$$

$\therefore$ x = 8 and $\frac{1}{y}=\frac{1}{6} \therefore$ y = 6

Thus, the required value are x = 8 and y = 6.

4. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

2x + 3y = 3 , x + 2y = 1 **[SEE 2075 R', 2073 R']**

⇨ Here, given equations are

2x + 3y = 3 and x + 2y = 1

$$\underset{A}{\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 3 \\ 1 \end{pmatrix}}$$

We have, $|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4-3 = 1$

Adj. (A) $=\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

We know that,

$$A^{-1}=\frac{1}{|A|}\text{Adj (A)} = \frac{1}{1}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$
$$=\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$
$$=\begin{pmatrix} 6-3 \\ -3+2 \end{pmatrix}$$
$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Thus, x = 3 and y = – 1 is the solution.

5. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$\frac{5}{x}+3y=7, 7y-\frac{10}{x}=12$ **[2074 S, 2073 R]**

⇨ Here, $\frac{5}{x}+3y=7$ ………… (i)

and $-\frac{10}{x}+7y=12$ ………… (ii)

Writing equations (i) and (ii) in matrix form,

$$\underset{A}{\begin{pmatrix} 5 & 3 \\ -10 & 7 \end{pmatrix}}\underset{X}{\begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 7 \\ 12 \end{pmatrix}}$$

$$\text{We have, } |A| = \begin{vmatrix} 5 & 3 \\ -10 & 7 \end{vmatrix}$$
$$= 35+30$$
$$= 65$$

$$\text{Adj. (A)} = \begin{pmatrix} 7 & -3 \\ 10 & 5 \end{pmatrix}$$

$$\text{We have, } X = \frac{1}{|A|}\text{Adj. (A). B}$$
$$=\frac{1}{65}\begin{pmatrix} 7 & -3 \\ 10 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 12 \end{pmatrix}$$
$$=\frac{1}{65}\begin{pmatrix} 49-36 \\ 70+60 \end{pmatrix}$$
$$=\frac{1}{65}\begin{pmatrix} 13 \\ 130 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix}=\begin{pmatrix} \frac{1}{5} \\ 2 \end{pmatrix}$$

Thus, x = 5 and y = 2 is the solution.

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6. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $5x - 2y = 10$ and $4x - 3y = -6$ [SEE 2075 R'$_2$]

⇨ Here, given equations are:

$5x - 2y = 10$ and $4x - 3y = -6$

Writing these equations in matrix form then,

$$\underset{A}{\begin{pmatrix} 5 & -2 \\ 4 & -3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 10 \\ -6 \end{pmatrix}}$$

$\therefore \quad X = A^{-1}B$ ....(i)

$$\text{Now, } |A| = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix} = |-15 + 8| = -7$$

$$\text{Adj (A)} = \begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|}\,.\text{ Adj (A)}$$

$$= -\frac{1}{7}\begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}$$

We have, $X = A^{-1}B$ [From (i)]

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ -6 \end{pmatrix}$$

$$= -\frac{1}{7}\begin{pmatrix} -30 - 12 \\ -40 - 30 \end{pmatrix}$$

$$= -\frac{1}{7}\begin{pmatrix} -42 \\ -70 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$\therefore \quad x = 6$ and $y = 10$

Thus, $x = 6$ and $y = 10$ are the solutions.

7. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $2x + 3y = 3$, $x - 4y = 7$ [2074 S']

⇨ Here, given equations are;

$2x + 3y = 3$, $x - 4y = 7$

The matrix form of given equations are;

$$\underset{A}{\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 3 \\ 7 \end{pmatrix}}$$

$$\text{So, } |A| = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -8 - 3 = -11$$

$$\text{We have, } A^{-1} = \frac{1}{|A|}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{-11}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{11}\begin{pmatrix} 4 & 3 \\ 1 & -2 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{11}\begin{pmatrix} 4 & 3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

$$= \frac{1}{11}\begin{pmatrix} 12 + 21 \\ 3 - 14 \end{pmatrix}$$

$$= \frac{1}{11}\begin{pmatrix} 33 \\ -11 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

i.e. $x = 3$ and $y = -1$

Thus, $x = 3$ and $y = -1$ is the solution.

8. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $2x - 3y = 7$, $4y - 3x + 10 = 0$ [2074 R, 2067 R]

⇨ Here, given equations are

$2x - 3y - 7 = 0$ and $4y - 3x = -10$

Above equation's can be written as matrix form

$$\text{or, } \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -10 \end{pmatrix}$$

or, $AX = B$ or, $X = A^{-1}B$....(i)

Where,

$$A = \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix} \text{and } B = \begin{pmatrix} 7 \\ -10 \end{pmatrix}$$

$$\text{Now, Inverse of A} \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}$$

$$= \frac{1}{8 - 9}\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -3 \\ -3 & -2 \end{pmatrix}$$

$$\left[\because A^{-1} = \frac{1}{|A|}.\text{ adj A}\right]$$

From equation (i),

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 7 \\ -10 \end{pmatrix}$$

$$= \begin{pmatrix} -28 + 30 \\ -21 + 20 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Thus, the required value of x & y are 2 & – 1 respectively.

9. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $9x - 8y = 12$, $2x + 3y = 17$ [2074 R', 2071 R']

⇨ Here, given equation of lines are;

$9x - 8y = 12$ and $2x + 3y = 17$

The matrix form of given equations:

$$\underset{A}{\begin{pmatrix} 9 & -8 \\ 2 & 3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 12 \\ 17 \end{pmatrix}}$$

$$|A| = \begin{vmatrix} 9 & -8 \\ 2 & 3 \end{vmatrix} = 27 + 16 = 43$$

$$\text{Adj (A)} = \begin{pmatrix} 3 & 8 \\ -2 & 9 \end{pmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|}\text{Adj. (A)} = \frac{1}{43}\begin{pmatrix} 3 & 8 \\ -2 & 9 \end{pmatrix}$$

$$\text{We have, } X = A^{-1}B = \frac{1}{43}\begin{pmatrix} 3 & 8 \\ -2 & 9 \end{pmatrix}\begin{pmatrix} 12 \\ 17 \end{pmatrix}$$

$$= \frac{1}{43}\begin{pmatrix} 3 \times 12 + 8 \times 17 \\ -2 \times 12 + 9 \times 17 \end{pmatrix}$$

$$= \frac{1}{43}\begin{pmatrix} 36 + 136 \\ -24 + 153 \end{pmatrix}$$

$$= \frac{1}{43}\begin{pmatrix} 172 \\ 129 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Thus, $x = 4$ and $y = 3$ is the solution.

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10. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\frac{4}{x}+\frac{1}{y}=3, \frac{2}{x}-\frac{5}{y}=-4$ **[2073 S, 2073 S']**

⇨ Here, $\frac{4}{x}+\frac{1}{y}=3$ ....(i)

and $\frac{2}{x}-\frac{5}{y}=-4$ ....(ii)

Given equations in matrix form can be written as:

$$\underset{A}{\begin{pmatrix} 4 & 1 \\ 2 & -5 \end{pmatrix}}\underset{X}{\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}}=\underset{B}{\begin{pmatrix} 3 \\ -4 \end{pmatrix}}$$

So, $|A|=\begin{vmatrix} 4 & 1 \\ 2 & -5 \end{vmatrix}=-20-2=-22.$

Now, adjoint (A) $=\begin{pmatrix} -5 & -1 \\ -2 & 4 \end{pmatrix}$

We have, $A^{-1}=\frac{1}{|A|}$ Adj. (A)

$$=\frac{1}{-22}\begin{pmatrix} -5 & -1 \\ -2 & 4 \end{pmatrix}$$

$$=\frac{1}{22}\begin{pmatrix} 5 & 1 \\ 2 & -4 \end{pmatrix}$$

Now,

$$X=A^{-1}B=\frac{1}{22}\begin{pmatrix} 5 & 1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$=\frac{1}{22}\begin{pmatrix} 15-4 \\ 6+16 \end{pmatrix}$$

$$=\frac{1}{22}\begin{pmatrix} 11 \\ 22 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}=\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

Thus, x = 2 and y = 1 is the solution.

12. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
5x + 3y = 9; 4y + 7x = 13 **[2072 R]**

⇨ Here, given equation of lines are;
5x + 3y = 9 and 7x + 4y = 13
The matrix form of given equations are ;

$$\underset{A}{\begin{pmatrix} 5 & 3 \\ 7 & 4 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 9 \\ 13 \end{pmatrix}}$$

We have, $|A|=\begin{vmatrix} 5 & 3 \\ 7 & 4 \end{vmatrix}=20-21=-1$

We know that, $A^{-1}=\frac{1}{|A|}\begin{pmatrix} 4 & -3 \\ -7 & 5 \end{pmatrix}$

$$=\frac{1}{-1}\begin{pmatrix} 4 & -3 \\ -7 & 5 \end{pmatrix}$$

Now, $X=A^{-1}B$

or, $\begin{pmatrix} x \\ y \end{pmatrix}=-1\begin{pmatrix} 4 & -3 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} 9 \\ 13 \end{pmatrix}$

$$=-1\begin{pmatrix} 36-39 \\ -63+65 \end{pmatrix}=-1\begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Thus, x = 3 and y = – 2 are the solutions.

11. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
5x – 9y = 47; 2y + 3x = 6 **[2072 S]**

⇨ Here, given equations are:

5x – 9y = 47;

2y + 3x = 6

Writing these equations in matrix form then;

$$\underset{A}{\begin{pmatrix} 5 & -9 \\ 3 & 2 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 47 \\ 6 \end{pmatrix}}$$

$$|A|=\begin{vmatrix} 5 & -9 \\ 3 & 2 \end{vmatrix}$$

$$=10+27$$

$$=37$$

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix} 2 & 9 \\ -3 & 5 \end{pmatrix}$$

$$=\frac{1}{37}\begin{pmatrix} 2 & 9 \\ -3 & 5 \end{pmatrix}$$

We have, $X=A^{-1}B$

or, $\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{37}\begin{pmatrix} 2 & 9 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 47 \\ 6 \end{pmatrix}$

$$=\frac{1}{37}\begin{pmatrix} 94+54 \\ -141+30 \end{pmatrix}$$

$$=\frac{1}{34}\begin{pmatrix} 148 \\ 111 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Thus, x = 4 and y = 3 are the solutions.

13. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
x – 5y = 8, 6x + 11y = 7 **[2072 R']**

⇨ Here, given equations are; x – 5y = 8 & 6x + 11y = 7

$$\underset{A}{\begin{pmatrix} 1 & -5 \\ 6 & 11 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 8 \\ 7 \end{pmatrix}}$$

We have, $|A|=\begin{vmatrix} 1 & -5 \\ 6 & 11 \end{vmatrix}=11+30=41$

We know that, $A^{-1}=\frac{1}{41}\begin{pmatrix} 11 & 5 \\ -6 & 1 \end{pmatrix}$

Now, $X=A^{-1}B=\frac{1}{41}\begin{pmatrix} 11 & 5 \\ -6 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 7 \end{pmatrix}$

$$=\frac{1}{41}\begin{pmatrix} 88+35 \\ -48+7 \end{pmatrix}$$

$$=\frac{1}{41}\begin{pmatrix} 123 \\ -41 \end{pmatrix}$$

$$=\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Thus, x = 3 and y = – 1 are the solution.

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14. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
3x + 5y = 24, 5x = 2y + 9 **[2071 R, 2068 S]**

⇨ Here, 3x + 5y = 24 and 5x = 2y + 9

Given equation can be written as;

3x + 5y = 24 and 5x – 2y = 9

The matrix form of above equations:

$$\underset{A}{\begin{pmatrix} 3 & 5 \\ 5 & -2 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}}=\underset{B}{\begin{pmatrix} 24 \\ 9 \end{pmatrix}}$$

$$|A| = \begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix} = -6-25 = -31$$

$$\text{Adj }(A) = \begin{pmatrix} -2 & -5 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}(A) = \frac{1}{-31}\begin{pmatrix} -2 & -5 \\ -5 & 3 \end{pmatrix}$$

$$= \frac{1}{31}\begin{pmatrix} 2 & 5 \\ 5 & -3 \end{pmatrix}$$

We know that,

$$X = A^{-1}B \quad = \frac{1}{31}\begin{pmatrix} 2 & 5 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} 24 \\ 9 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} \quad = \frac{1}{31}\begin{pmatrix} 48+45 \\ 120-27 \end{pmatrix}$$

$$= \frac{1}{31}\begin{pmatrix} 93 \\ 93 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

∴ x = 3 and y = 3

Thus x = 3 and y = 3 is the solution.

16. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\mathbf{\frac{10}{x}+\frac{12}{y}=6}$ **and** $\mathbf{\frac{25}{x}-\frac{2}{y}=2}$ **[2070 R']**

⇨ Here given equations are:

$\frac{10}{x}+\frac{12}{y}=6$ ……….. (i) and $\frac{25}{x}-\frac{2}{y}=2$ ………. (ii)

The matrix form of above equations:

$$\underset{A}{\begin{pmatrix} 10 & 12 \\ 25 & -2 \end{pmatrix}}\underset{X}{\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}}=\underset{B}{\begin{pmatrix} 6 \\ 2 \end{pmatrix}}$$

$$\text{We have, } |A| = \begin{vmatrix} 10 & 12 \\ 25 & -2 \end{vmatrix} = -20-300 = -320$$

$$\text{We know that, } A^{-1} = \frac{1}{|A|}\begin{pmatrix} -2 & -12 \\ -25 & 10 \end{pmatrix}$$

$$= \frac{1}{-320}\begin{pmatrix} -2 & -12 \\ -25 & 10 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B \quad = -\frac{1}{320}\begin{pmatrix} -2 & -12 \\ -25 & 10 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$= -\frac{1}{320}\begin{pmatrix} -12-24 \\ -150+20 \end{pmatrix}$$

$$= -\frac{1}{320}\begin{pmatrix} -36 \\ -130 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} \frac{9}{80} \\ \frac{13}{32} \end{pmatrix}$$

Thus, $x = \frac{80}{9}$ and $y = \frac{32}{13}$ are the solution.

15. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\mathbf{x-\frac{2}{y}=4;\ 2x+\frac{3}{y}=1}$ **[2070 R]**

⇨ Here, $x-\frac{2}{y}=4$ and $2x+\frac{3}{y}=1$

The matrix form of given equations are

$$\underset{A}{\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix}}=\underset{B}{\begin{pmatrix} 4 \\ 1 \end{pmatrix}}$$

$$\text{We have, } |A| = \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 3+4 = 7$$

$$\text{We know that, } A^{-1} = \frac{1}{|A|}\begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}$$

$$= \frac{1}{7}\begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \frac{1}{7}\begin{pmatrix} 12+8 \\ -8+1 \end{pmatrix}$$

$$= \frac{1}{7}\begin{pmatrix} 14 \\ -7 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Thus, x = 2 and y = – 1 are the solution.

17. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
3x – 5y = 3, 4x + 3y = 4 **[2059 R]**

⇨ Here, given equations:

3x – 5y = 3   4x + 3y = 4

The matrix form of the above equations is

$$\begin{pmatrix} 3 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

or, AX = B

$$\text{where, } A = \begin{pmatrix} 3 & -5 \\ 4 & 3 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix} \text{and } B = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\text{Here, } |A| = \begin{vmatrix} 3 & -5 \\ 4 & 3 \end{vmatrix}$$

$$= 3 \times 3 - 4 \times (-5)$$

$$= 9 + 20$$

$$= 29$$

Now, form AX = B,

$$X = A^{-1}B$$

$$= \frac{1}{|A|}\begin{pmatrix} 3 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$= \frac{1}{29}\begin{pmatrix} 3 \times 3 + 5 \times 4 \\ -4 \times 3 + 3 \times 4 \end{pmatrix}$$

$$= \frac{1}{29}\begin{pmatrix} 9+20 \\ -12+12 \end{pmatrix}$$

$$= \frac{1}{29}\begin{pmatrix} 29 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Thus, the required value of x & y are 1 & 0 respectively.

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18. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $4x - 3y = 11,\ 3x + 7y = -1$ [2058R, 2060R]

⇨ Here, given equations:

$4x - 3y = 11 \qquad 3x + 7y = -1$

Writting the given equations in the form of matrix,

$$\begin{pmatrix}4 & -3\\ 3 & 7\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}11\\ -1\end{pmatrix}$$

or, AX = B ....(i)

where, $A=\begin{pmatrix}4 & -3\\ 3 & 7\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ & $B=\begin{pmatrix}11\\ -1\end{pmatrix}$

Now, determinant of A i.e. $|A| = \begin{vmatrix}4 & -3\\ 3 & 7\end{vmatrix}$

$= 28 - (-9)$

$= 28 + 9$

$= 37 \neq 0$

Hence, the solution exists.

$X = A^{-1}B$, where $A^{-1} = \frac{1}{|A|}$ Adj. $A = \frac{1}{37}\begin{pmatrix}7 & 3\\ -3 & 4\end{pmatrix}$

$$\therefore \quad X = A^{-1}B = \frac{1}{37}\begin{pmatrix}7 & 3\\ -3 & 4\end{pmatrix}\begin{pmatrix}11\\ -1\end{pmatrix}$$

$$= \frac{1}{37}\begin{pmatrix}77-3\\ -33-4\end{pmatrix}$$

$$= \frac{1}{37}\begin{pmatrix}74\\ -37\end{pmatrix}$$

$$= \begin{pmatrix}2\\ -1\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}2\\ -1\end{pmatrix}$$

Thus, the required value of x & y are 2 & – 1 respectively.

19. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $3x + 5y = 21,\ 2x + 3y = 13$ [2057 R]

⇨ Here, given equations:

$3x + 5y = 21 \qquad 2x + 3y = 13$

Writting the given equations in the form of matrix,

$$\begin{pmatrix}3 & 5\\ 2 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}21\\ 13\end{pmatrix}$$

or, AX = B where,

$A=\begin{pmatrix}3 & 5\\ 2 & 3\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}21\\ 13\end{pmatrix}$

Here, $|A| = \begin{vmatrix}3 & 5\\ 2 & 3\end{vmatrix}$

$= 3 \times 3 - 2 \times 5$

$= 9 - 10 = -1 \neq 0$

∴ The given equations have unique solution.

We have, $X = A^{-1}B$,

where, $A^{-1} = \frac{1}{-1}\begin{pmatrix}3 & -5\\ -2 & 3\end{pmatrix}$

$$= \begin{pmatrix}-3 & 5\\ 2 & -3\end{pmatrix}$$

$$\therefore \quad X = \begin{pmatrix}x\\ y\end{pmatrix} = A^{-1}B$$

$$= \begin{pmatrix}-3 & 5\\ 2 & -3\end{pmatrix}\begin{pmatrix}21\\ 13\end{pmatrix}$$

$$= \begin{pmatrix}-63+65\\ 42-39\end{pmatrix}$$

$$= \begin{pmatrix}2\\ 3\end{pmatrix}$$

Thus, the required value of x & y are 2 & 3 respectively.

20. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $3x - 2y = 5,\ x + y = 5$ [2057 S]

⇨ Here, given equations, $3x - 2y = 5$ and $x + y = 5$

Writing the given equations in matrix form :

$$\begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}5\\ 5\end{pmatrix}$$

or, AX = B

Where, $A=\begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}5\\ 5\end{pmatrix}$

Here $|A| = \begin{vmatrix}3 & -2\\ 1 & 1\end{vmatrix}$

$= 3 \times 1 - 1 \times (-2)$

$= 3 + 2 = 5 \neq 0$ [∴ unique solution exists]

We have, $X = A^{-1}B$,

where $A^{-1} = \frac{1}{|A|}$ Adj. $A = \frac{1}{5}\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$

$$\text{Hence } X = \begin{pmatrix}x\\ y\end{pmatrix} = A^{-1}B$$

$$= \frac{1}{5}\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}\begin{pmatrix}5\\ 5\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}5+10\\ -5+15\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}15\\ 10\end{pmatrix}$$

$$= \begin{pmatrix}3\\ 2\end{pmatrix}$$

Thus, the required value of x & y are 3 & 2 respectively.

21. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $2x + y = 3,\ 3x + 2y = 2$ [2058 S]

⇨ Here, given equations: $2x + y = 3$ and $3x + 2y = 2$

Matrix form of given equations is

$$\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}3\\ 2\end{pmatrix}$$

or, AX = B

Where, $A=\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}3\\ 2\end{pmatrix}$

Here, $|A| = \begin{vmatrix}2 & 1\\ 3 & 2\end{vmatrix}$

$= 4 - 3 = 1 \neq 0$

∴ unique solution exists.

Now, from AX = B

We have, $X = A^{-1}B$

where, $A^{-1} = \frac{1}{|A|}$ Adj. $A = \frac{1}{1}\begin{pmatrix}2 & -1\\ -3 & 2\end{pmatrix}$

$$\text{Hence, } X = \begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{1}\begin{pmatrix}2 & -1\\ -3 & 2\end{pmatrix}\begin{pmatrix}3\\ 2\end{pmatrix}$$

$$= \begin{pmatrix}2 \times 3 - 1 \times 2\\ -3 \times 3 + 2 \times 2\end{pmatrix}$$

$$= \begin{pmatrix}6-2\\ -9+4\end{pmatrix}$$

$$= \begin{pmatrix}4\\ -5\end{pmatrix}$$

Thus, the required value of x & y are 4 & – 5 respectively.

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22. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$x + 2y = 8, 2x + 3y = 11$ **[2061 S]**

⇨ Here, given equations are

$x + 2y = 8$ and $2x + 3y = 11$

The matrix form of given equations is :

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$$

or, $AX = B$ .... (i)

Where, $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$

Here, $|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

$= 3 - 4 = -1 \neq 0 \therefore$ Unique solution exists.

So, $A^{-1} = \frac{1}{|A|}$ Adj $A = \frac{1}{-1}\begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}$

Now, from equation (i) $X = A^{-1} B$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 8 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \times 8 + 2 \times 11 \\ 2 \times 8 - 1 \times 11 \end{pmatrix}$$

$$= \begin{pmatrix} -24 + 22 \\ 16 - 11 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

Thus, the required value of x & y are – 2 & 5 respectively.

23. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$2x + 3y = 5, 5x - 2y = 3$ **[2060 S]**

⇨ Here, given equations are;

$2x + 3y = 5$ & $5x - 2y = 3$

The matrix form of given equations is :

$$\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

or, $AX = B$ ...... (i)

Where, $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

Here, $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix}$

$= 2 \times (-2) - 5 \times 3$

$= -4 - 15$

$= -19 \neq 0 \therefore$ Unique solution exists.

Here, from (i)

$X = A^{-1}B$

where, $A^{-1} = \frac{1}{|A|}$ Adj. $A = \frac{1}{-19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}$

$$\text{Hence, } X = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$= \frac{1}{-19}\begin{pmatrix} -10 - 9 \\ -25 + 6 \end{pmatrix}$$

$$= \frac{1}{-19}\begin{pmatrix} -19 \\ -19 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Thus, the required value of x & y are 1 & 1 respectively.

24. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$3x + y = 51, 4x - 3y = 3$ **[2061 R]**

⇨ Here, given equations are:

$3x + y = 51$ and $4x - 3y = 3$

The matrix form of given equations is

$$\begin{pmatrix} 3 & 1 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 51 \\ 3 \end{pmatrix}$$

or, $AX = B$ ...... (i)

Where $A = \begin{pmatrix} 3 & 1 \\ 4 & -3 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 51 \\ 3 \end{pmatrix}$

Here, $|A| = \begin{vmatrix} 3 & 1 \\ 4 & -3 \end{vmatrix} = 3 \times (-3) - 4 \times 1$

$= -9 - 4$

$= -13$

Again, $A^{-1} = \frac{1}{|A|}$ Adj. A

where Adj. $A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

So from (i)

$$X = A^{-1}B = -\frac{1}{13}\begin{pmatrix} -3 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 51 \\ 3 \end{pmatrix}$$

$$= -\frac{1}{13}\begin{pmatrix} -3 \times 51 + (-1) \times 3 \\ -4 \times 51 + 3 \times 3 \end{pmatrix}$$

$$= -\frac{1}{13}\begin{pmatrix} -153 - 3 \\ -204 + 9 \end{pmatrix}$$

$$= -\frac{1}{13}\begin{pmatrix} -156 \\ -195 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 15 \end{pmatrix}$$

Thus, the required value of x & y are 12 & 15 respectively.

25. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$x + 3y = 12, 4x - 3y = 3$ **[2062 K]**

⇨ Here, given equations are

$x + 3y = 12$ and $4x - 3y = 3$

The matrix form of above equations is

$$\underset{A}{\begin{pmatrix} 1 & 3 \\ 4 & -3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 12 \\ 3 \end{pmatrix}} \text{ i.e. } AX = B$$

$|A| = \begin{vmatrix} 1 & 3 \\ 4 & -3 \end{vmatrix}$

$= -3 - 12$

$= -15 \neq 0$

$\therefore$ unique solution exists.

$A^{-1} = \frac{1}{|A|}$ (Adj. A)

$\therefore A^{-1} = \frac{1}{-15}\begin{pmatrix} -3 & -3 \\ -4 & 1 \end{pmatrix}$

$$\text{Now, } X = A^{-1}B = -\frac{1}{15}\begin{pmatrix} -3 & -3 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 12 \\ 3 \end{pmatrix}$$

$$= -\frac{1}{15}\begin{pmatrix} -36 - 9 \\ -48 + 3 \end{pmatrix}$$

$$= -\frac{1}{15}\begin{pmatrix} -45 \\ -45 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Thus, the required value of x & y are 3 and 3 respectively.

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26. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$3x + 2y = 1,\ 7x + 5y = 4$ [2062 S]

⇨ Here, given equations are:

$3x + 2y = 1$ and $7x + 5y = 4$

The matrix form of above equations are;

$$\underset{A}{\begin{pmatrix}3 & 2\\ 7 & 5\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}1\\ 4\end{pmatrix}}\text{ i.e. } AX = B \text{ ....(i)}$$

$$\therefore\quad |A| = \begin{vmatrix}3 & 2\\ 7 & 5\end{vmatrix}$$

$$= 15 - 14$$

$$= 1 \neq 0$$

$\therefore$ unique solution exists.

We have, $A^{-1} = \dfrac{1}{|A|}(\text{Adj. }(A))$

$$= \frac{1}{1}\begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}$$

$$= \begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}\begin{pmatrix}1\\ 4\end{pmatrix}$$

$$= \begin{pmatrix}5-8\\ -7+12\end{pmatrix}$$

$$\therefore\quad \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-3\\ 5\end{pmatrix}$$

Thus, the required value of x & y are – 3 & 5 respectively.

27. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$4x - 5y = 2,\ 3x + 4y = 48$ [2064 R']

⇨ Here, given equations are:

$4x - 5y = 2$ and $3x + 4y = 48$

The matrix form of above equations;

$$\underset{A}{\begin{pmatrix}4 & -5\\ 3 & 4\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}2\\ 48\end{pmatrix}}\text{ i.e. } AX = B$$

$$\therefore\quad |A| = \begin{vmatrix}4 & -5\\ 3 & 4\end{vmatrix}$$

$$= (16 + 15)$$

$$= 31 \neq 0$$

$\therefore$ unique solution exists.

We have, $A^{-1} = \dfrac{1}{|A|}\text{Adj }(A)$

$$= \frac{1}{31}\begin{pmatrix}4 & 5\\ -3 & 4\end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{31}\begin{pmatrix}4 & 5\\ -3 & 4\end{pmatrix}\begin{pmatrix}2\\ 48\end{pmatrix}$$

$$= \frac{1}{31}\begin{pmatrix}8+240\\ -6+192\end{pmatrix}$$

$$= \frac{1}{31}\begin{pmatrix}248\\ 186\end{pmatrix}$$

$$\therefore\quad \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}8\\ 6\end{pmatrix}$$

Thus, the required value of x & y are 8 & 6 respectively.

28. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$2x - y = 5,\ x - 2y = 1$ [2065 R]

⇨ Here, given equations are:

$2x - y = 5$ .... (i) and $x - 2y = 1$ .... (ii)

Writing these equation in matrix form

$$\underset{A}{\begin{pmatrix}2 & -1\\ 1 & -2\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}5\\ 1\end{pmatrix}}$$

or, $AX = B$

or, $X = A^{-1}B$

$$\text{Where } A = \begin{pmatrix}2 & -1\\ 1 & -2\end{pmatrix} B = \begin{pmatrix}5\\ 1\end{pmatrix} \& \ X = \begin{pmatrix}x\\ y\end{pmatrix}$$

$$|A| = \begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix} = -4 + 1 = -3$$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix}-2 & 1\\ -1 & 2\end{pmatrix}$$

$$\therefore\quad A^{-1} = \frac{1}{-3}\begin{pmatrix}-2 & 1\\ -1 & 2\end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-3}\begin{pmatrix}-2 & 1\\ -1 & 2\end{pmatrix}\begin{pmatrix}5\\ 1\end{pmatrix}$$

$$= -\frac{1}{3}\begin{pmatrix}-10+1\\ -5+2\end{pmatrix}$$

$$= -\frac{1}{3}\begin{pmatrix}-9\\ -3\end{pmatrix}$$

$$\therefore\quad \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}3\\ 1\end{pmatrix}$$

Thus, the required value of x & y are 3 & 1 respectively.

29. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$x - 2y = -7,\ 3x + 7y = 5$ [2065 E]

⇨ Here given equations are:

$x - 2y = -7$ .... (i)

$3x + 7y = 5$ .... (ii)

Equation (i) & (ii) can be written as matrix form

$$\begin{pmatrix}1 & -2\\ 3 & 7\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-7\\ 5\end{pmatrix}$$

or, $AX = B$

or, $X = A^{-1}B$

$$\text{Where } A = \begin{pmatrix}1 & -2\\ 3 & 7\end{pmatrix} X = \begin{pmatrix}x\\ y\end{pmatrix} \& \ B = \begin{pmatrix}-7\\ 5\end{pmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}\text{Adj. A}$

$$= \frac{1}{7+6}\begin{pmatrix}7 & 2\\ -3 & 1\end{pmatrix}$$

$$\text{or, } A^{-1} = \frac{1}{13}\begin{pmatrix}7 & 2\\ -3 & 1\end{pmatrix}$$

$$\text{Then, } X = A^{-1}B = \frac{1}{13}\begin{pmatrix}7 & 2\\ -3 & 1\end{pmatrix}\begin{pmatrix}-7\\ 5\end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix}-49+10\\ +21+5\end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix}-39\\ 26\end{pmatrix}$$

$$\text{or, } \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-3\\ 2\end{pmatrix}$$

Thus, the required value of x & y are – 3 & 2 respectively.

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30. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$5x - 3y = 13,\ x - 5y = 7$ **[2065 S]**

⇨ Here, the given equations are:

$5x - 3y = 13$ and $x - 5y = 7$

Writing these equations in matrix form

$$\begin{pmatrix}5 & -3\\ 1 & -5\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}13\\ 7\end{pmatrix}$$

Let, $A=\begin{pmatrix}5 & -3\\ 1 & -5\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}13\\ 7\end{pmatrix}$

$$|A| = \begin{vmatrix}5 & -3\\ 1 & -5\end{vmatrix}$$

$$= -25 + 3$$

$$= -22 \neq 0$$

So, A^{-1} exists.

We know that; $A^{-1}=\dfrac{1}{|A|}(\text{Adj. }(A))$

$$= -\frac{1}{22}\begin{bmatrix}-5 & 3\\ -1 & 5\end{bmatrix}$$

Now, we have, $X = A^{-1}B$

$$\text{So, }\begin{pmatrix}x\\ y\end{pmatrix} = -\frac{1}{22}\begin{bmatrix}-5 & 3\\ -1 & 5\end{bmatrix}\begin{bmatrix}13\\ 7\end{bmatrix}$$

$$= -\frac{1}{22}\begin{bmatrix}-65+21\\ -13+35\end{bmatrix}$$

$$= -\frac{1}{22}\begin{pmatrix}-44\\ 22\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}2\\ -1\end{pmatrix}$$

Thus, the $x = 2$ and $y = -1$ are the required solution.

31. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$7x - 2y = 3,\ \ x + 4y = 9$ **[2066 R']**

⇨ Here, given equations are,

$7x - 2y = 3$ and $x + 4y = 9$

The matrix form of above equations is;

$$\underset{A}{\begin{pmatrix}7 & -2\\ 1 & 4\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}3\\ 9\end{pmatrix}}$$

or, $AX = B$

Where, $A=\begin{pmatrix}7 & -2\\ 1 & 4\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ & $B=\begin{pmatrix}3\\ 9\end{pmatrix}$

$$|A| = \begin{vmatrix}7 & -2\\ 1 & 4\end{vmatrix} = 28 + 2 = 30$$

We have, $A^{-1} = \dfrac{1}{|A|}(\text{Adj. }(A))$

$$= \frac{1}{30}\begin{pmatrix}4 & 2\\ -1 & 7\end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{30}\begin{pmatrix}4 & 2\\ -1 & 7\end{pmatrix}\begin{pmatrix}3\\ 9\end{pmatrix}$$

$$= \frac{1}{30}\begin{pmatrix}12+18\\ -3+63\end{pmatrix}$$

$$= \frac{1}{30}\begin{pmatrix}30\\ 60\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}1\\ 2\end{pmatrix}$$

Thus, $x = 1$ and $y = 2$ is the required solution.

32. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$2x - 3y - 7 = 0,\ 4y - 3x = -10$ **[2067 R]**

⇨ Here, given equations are,

$2x - 3y - 7 = 0$ or, $2x - 3y = 7$

and $4y - 3x = -10$

Above equations can be written as matrix form

$$\text{or, }\begin{pmatrix}2 & -3\\ -3 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}7\\ -10\end{pmatrix}$$

or, $AX = B$ or, $X = A^{-1}B$

Where, $A=\begin{pmatrix}2 & -3\\ -3 & 4\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}7\\ -10\end{pmatrix}$

or, Inverse of $\begin{pmatrix}2 & -3\\ -3 & 4\end{pmatrix}$

i.e. $A^{-1} = \dfrac{1}{8-9}\begin{pmatrix}4 & 3\\ 3 & 2\end{pmatrix}$

$$= \begin{pmatrix}-4 & -3\\ -3 & -2\end{pmatrix}\left[\because A^{-1} = \frac{1}{|A|}.\text{ adj }A\right]$$

or, Now, $X = A^{-1}B$

$$\Rightarrow \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-4 & -3\\ -3 & -2\end{pmatrix}\begin{pmatrix}7\\ -10\end{pmatrix}$$

$$= \begin{pmatrix}-28+30\\ -21+20\end{pmatrix}$$

$$= \begin{pmatrix}2\\ -1\end{pmatrix}$$

Thus, the required value of x & y are 2 & – 1 respectively.

33. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$3x - y = 2,\ \ x + 2y = 3$ **[2067 R']**

⇨ Here, given equations are,

$3x - y = 2$ and $x + 2y = 3$

Above equation can be written as matrix form,

$$\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2\\ 3\end{pmatrix}$$

or, $AX = B$ or, $X = A^{-1}B$

Where, $A=\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}, X=\begin{pmatrix}x\\ y\end{pmatrix}$ & $B=\begin{pmatrix}2\\ 3\end{pmatrix}$

$$\text{Now, } A^{-1} = \frac{1}{|A|}\text{Adj. }A\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix}$$

$$\text{or, } A^{-1} = \frac{1}{\begin{vmatrix}3 & -1\\ 1 & 2\end{vmatrix}}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}$$

$$= \frac{1}{6+1}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix} = \frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}$$

Then, $X = A^{-1}B$

$$\text{or, } X = \frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}2\\ 3\end{pmatrix}$$

$$\text{or, } X = \frac{1}{7}\begin{pmatrix}4+3\\ -2+9\end{pmatrix} \qquad \text{or, } X = \frac{1}{7}\begin{pmatrix}7\\ 7\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}$$

Thus, the required value of x is 1 & y is 1.

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34. **मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): 2x + 3y = 10, 5x + 3y = 16 [2068 R']**

⇨ Here, given equations are

2x + 3y = 10 ……… (i)

and 5x + 3y = 16 …….. (ii)

Writing the given equations in matrix form we get,

$$\begin{pmatrix}2 & 3\\ 5 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}10\\ 16\end{pmatrix}$$

or, AX = B   or, $X = A^{-1}B$

Where, $A=\begin{pmatrix}2 & 3\\ 5 & 3\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}10\\ 16\end{pmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ Adj. of A

$$=\frac{1}{\begin{vmatrix}2 & 3\\ 5 & 3\end{vmatrix}}\begin{bmatrix}3 & -3\\ -5 & 2\end{bmatrix}$$

$$=\frac{1}{6-15}\begin{bmatrix}3 & -3\\ -5 & 2\end{bmatrix}=\frac{1}{-9}\begin{pmatrix}3 & -3\\ -5 & 2\end{pmatrix}$$

Then, $X = A^{-1}B$

or, $X=-\frac{1}{9}\begin{pmatrix}3 & -3\\ -5 & 2\end{pmatrix}\begin{pmatrix}10\\ 16\end{pmatrix}$

or, $X=-\frac{1}{9}\begin{pmatrix}30-48\\ -50+32\end{pmatrix}$

or, $X=-\frac{1}{9}\begin{pmatrix}-18\\ -18\end{pmatrix}$ or, $\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2\\ 2\end{pmatrix}$

Thus, the required value of x & y are 2 & 2 respectively.

35. **मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): 3x = 5y + 3, 4x – 4 = – 3y [2067 S]**

⇨ Here, 3x = 5y + 3 and 4x – 4 = – 3y

⇒ 3x – 5y = 3 and 4x + 3y = 4

Writing these equations in matrix form then,

$$\underset{A}{\begin{pmatrix}3 & -5\\ 4 & 3\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}3\\ 4\end{pmatrix}}$$

∴ AX = B

or, $X = A^{-1}B$

Where, $A=\begin{pmatrix}3 & -5\\ 4 & 3\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\end{pmatrix}$ & $B=\begin{pmatrix}3\\ 4\end{pmatrix}$

$$|A| = \begin{vmatrix}3 & -5\\ 4 & 3\end{vmatrix}$$

$= 9 + 20 = 29$

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}3 & 5\\ -4 & 3\end{pmatrix}$$

We have, $X = A^{-1}B=\frac{1}{29}\begin{pmatrix}3 & 5\\ -4 & 3\end{pmatrix}\begin{pmatrix}3\\ 4\end{pmatrix}$

$$=\frac{1}{29}\begin{pmatrix}9+20\\ -12+12\end{pmatrix}$$

$$=\frac{1}{29}\begin{pmatrix}29\\ 0\end{pmatrix}$$

or, $\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix}$

∴ x = 1 and y = 0

Thus, x = 1 and y = 0 are the solutions of given equations.

36. **मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): 2x + 3y – 18 = 0, 3x – 2y – 1 = 0 [2059 S]**

⇨ Here, given equations are

2x + 3y = 18 and 3x – 2y = 1

The matrix form of the equations are

$$\begin{pmatrix}2 & 3\\ 3 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}18\\ 1\end{pmatrix}$$

or, AX = B .... (i)

where, $A=\begin{pmatrix}2 & 3\\ 3 & -2\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\end{pmatrix}$ and $B=\begin{pmatrix}18\\ 1\end{pmatrix}$

Here $|A| = \begin{vmatrix}2 & 3\\ 3 & -2\end{vmatrix}$

$= 2 \times (-2) - 3 \times 3$

$= -4 - 9$

$= -13 \neq 0$

[∴ Unique solution exists.]

Now, from (i) $X = A^{-1}B$

where $A^{-1}=\frac{1}{|A|}$ Adj. $A=\frac{1}{-13}\begin{pmatrix}-2 & -3\\ -3 & 2\end{pmatrix}$

Hence, $X=\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{-13}\begin{pmatrix}-2 & -3\\ -3 & 2\end{pmatrix}\begin{pmatrix}18\\ 1\end{pmatrix}$

$$=\frac{1}{-13}\begin{pmatrix}-36-3\\ -54+2\end{pmatrix}$$

$$=\frac{1}{-13}\begin{pmatrix}-39\\ -52\end{pmatrix}$$

$$=\begin{pmatrix}3\\ 4\end{pmatrix}$$

Thus, the required values are x = 3 and y = 4.

37. **मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): 5x + 2y – 3 = 0, x + 2y = 5 [2060 CP]**

⇨ Here, given equations can be written as;

5x + 2y = 3 and x + 2y = 5

The matrix form of above equations;

$$\underset{A}{\begin{pmatrix}5 & 2\\ 1 & 2\end{pmatrix}}\underset{X}{\begin{pmatrix}x\\ y\end{pmatrix}}=\underset{B}{\begin{pmatrix}3\\ 5\end{pmatrix}}$$

or, AX = B

or, $X = A^{-1}B$

Where, $A=\begin{pmatrix}5 & 2\\ 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}x\\ y\end{pmatrix}$ & $X=\begin{pmatrix}3\\ 5\end{pmatrix}$

∴ $|A| = \begin{vmatrix}5 & 2\\ 1 & 2\end{vmatrix}$

$= 10 - 2$

$= 8$

$$A^{-1}=\frac{1}{|A|}\text{Adj }(A)=\frac{1}{8}\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix}$$

We know that,

$$X = A^{-1}B=\frac{1}{8}\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix}\begin{pmatrix}3\\ 5\end{pmatrix}$$

$$=\frac{1}{8}\begin{pmatrix}6-10\\ -3+25\end{pmatrix}$$

$$=\frac{1}{8}\begin{pmatrix}-4\\ 22\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-0.5\\ 2.75\end{pmatrix}$$

Thus, the required values are x = – 0.5 and y = 2.75

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38. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$2x + 3y + 4 = 0, -5x + 4y + 13 = 0$ [2062 R]

⇨ Here, given equations;

$2x + 3y + 4 = 0 \quad \therefore \quad 2x + 3y = -4$

and $-5x + 4y + 13 = 0 \quad \therefore \quad -5x + 4y = -13$

The matrix form of above equations are;

$$\begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -13 \end{pmatrix}$$

or, $AX = B$ .... (i)

Where, $A = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} -4 \\ -13 \end{pmatrix}$

Here, $|A| = \begin{vmatrix} 2 & 3 \\ -5 & 4 \end{vmatrix} = 8 + 15 = 23 \neq 0$

[$\therefore$ unique solution exists.]

Now, from equation (i) $X = A^{-1}B$

where $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{23}\begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} -4 \\ -13 \end{pmatrix}$$

$$= \frac{1}{23}\begin{pmatrix} 4 \times (-4) + (-3) \times (-13) \\ 5 \times (-4) + 2 \times (-13) \end{pmatrix}$$

$$= \frac{1}{23}\begin{pmatrix} -16 + 39 \\ -20 - 26 \end{pmatrix}$$

$$= \frac{1}{23}\begin{pmatrix} 23 \\ -46 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Thus, the required values are $x = 1$ and $y = -2$.

40. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$2x - y = 1, 2y + x = 3$ [2063 R]

⇨ Here, given equations are

$2x - y = 1$ and, $2y + x = 3 \qquad$ or, $x + 2y = 3$

The matrix form of above equations are;

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

or, $AX = B$ ..... (i),

where $A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Now, $|A| = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$

$= 4 + 1 = 5 \neq 0$ (Unique solution exists)

Now, $A^{-1} = \frac{1}{|A|}$ Adj. A

or, $A^{-1} = \frac{1}{|A|}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

Now, from (i) $\qquad X = A^{-1}B$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 2 \times 1 + 1 \times 3 \\ -1 \times 1 + 2 \times 3 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 2 + 3 \\ -1 + 6 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Thus, the required value of x is 1 and y is 1.

39. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$5x + 7y = 1, x + 4y + 5 = 0$ [2066 R]

⇨ Here, given equations are,

$5x + 7y = 1$ .... (i) and $x + 4y = -5$ .... (ii)

The matrix form of above equations is;

$$\begin{array}{ccc} \begin{pmatrix} 5 & 7 \\ 1 & 4 \end{pmatrix} & \begin{pmatrix} x \\ y \end{pmatrix} = & \begin{pmatrix} 1 \\ -5 \end{pmatrix} \\ A & X & B \end{array}$$

We know that; $AX = B$

or, $X = A^{-1}B$

So, $|A| = \begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix}$

$= 20 - 7$

$= 13 \neq 0$

[$\therefore$ unique solution exists.]

And, $A^{-1} = \frac{1}{|A|}$ Adj (A)

$$= \frac{1}{13}\begin{pmatrix} 4 & -7 \\ -1 & 5 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{13}\begin{pmatrix} 4 & -7 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix} 39 \\ -26 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Thus, $x = 3$ and $y = -2$ are the required solution.

41. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$4x + 3y = 5, y - 3x = -7$ [2064 R]

⇨ Here, given equations can be written as;

$4x + 3y = 5$ and $3x - y = 7$

The matrix form of above equations are;

$$\begin{array}{ccc} \begin{pmatrix} 4 & 3 \\ 3 & -1 \end{pmatrix} & \begin{pmatrix} x \\ y \end{pmatrix} = & \begin{pmatrix} 5 \\ 7 \end{pmatrix} \\ A & X & B \end{array}$$

or, $AX = B$

or, $X = A^{-1}B$

Where, $A = \begin{pmatrix} 4 & 3 \\ 3 & -1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ & $B = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix}$

$= -4 - 3 \times 3$

$= -13$

$\therefore \quad A^{-1} = \frac{1}{|A|}$Adj of A

$$= \frac{1}{-13}\begin{pmatrix} -1 & -3 \\ -3 & 4 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = -\frac{1}{13}\begin{pmatrix} -1 & -3 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{13}\begin{pmatrix} -5 - 21 \\ -15 + 28 \end{pmatrix}$$

$$= -\frac{1}{13}\begin{pmatrix} -26 \\ 13 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Thus, the required values are $x = 2$ and $y = -1$.

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42. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$x - 3y = 2,\ \ 2y + x = 7$ **[2063 S]**

⇨ Here, given equations can be written as;
$x - 3y = 2$ and $x + 2y = 7$
The matrix form of above equations are:

$$\underset{A}{\begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 2 \\ 7 \end{pmatrix}}$$

$AX = B$
or, $X = A^{-1}B$

Where, $A = \begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$$\therefore\ |A| = \begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} = 2 + 3 = 5$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{Adj (A)} = \frac{1}{5}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{5}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 4+21 \\ -2+7 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 25 \\ 5 \end{pmatrix}$$

$$\therefore\ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Thus, the required values are $x = 5$ and $y = 1$.

44. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\frac{6}{x} + \frac{3}{y} = 4,\ \ \frac{4}{x} - \frac{3}{y} = 1$ **[2066 S]**

⇨ Here, the given equations are:
$\frac{6}{x} + \frac{3}{y} = 4$ .... (i) and $\frac{4}{x} - \frac{3}{y} = 1$ .... (ii)
The matrix form of above equations are;

$$\begin{pmatrix} 6 & 3 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \text{i.e. } AX = B$$

Where, $A = \begin{pmatrix} 6 & 3 \\ 4 & -3 \end{pmatrix}, X = \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix}, B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$$\therefore\ |A| = \begin{vmatrix} 6 & 3 \\ 4 & -3 \end{vmatrix} = -18 - 12 = -30$$

$$A^{-1} = \frac{1}{|A|}(\text{Adj (A)}) = \frac{1}{-30}\begin{pmatrix} -3 & -3 \\ -4 & 6 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = -\frac{1}{30}\begin{pmatrix} -3 & -3 \\ -4 & 6 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = -\frac{1}{30}\begin{pmatrix} -3\times4+(-3)\times1 \\ -4\times4+6\times1 \end{pmatrix} = -\frac{1}{30}\begin{pmatrix} -12-3 \\ -16+6 \end{pmatrix} = -\frac{1}{30}\begin{pmatrix} -15 \\ -10 \end{pmatrix}$$

$$\therefore\ \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \end{pmatrix} \Rightarrow x = 2 \text{ and } y = 3.$$

Thus, the required value of x is 2 & y is 3.

43. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$3x + \frac{4}{y} = 7, x + \frac{1}{y} = 3$ **[2063 R']**

⇨ Here, given equations are: $3x + \frac{4}{y} = 7, x + \frac{1}{y} = 3$
The matrix form of above equations is;

$$\underset{A}{\begin{pmatrix} 3 & 4 \\ 1 & 1 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix}} = \underset{B}{\begin{pmatrix} 7 \\ 3 \end{pmatrix}}$$

$\therefore\ AX = B$ or, $X = A^{-1}B$

Where, $A = \begin{pmatrix} 3 & 4 \\ 1 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix}$ & $B = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$$\therefore\ |A| = \begin{pmatrix} 3 & 4 \\ 1 & 1 \end{pmatrix} = 3 - 4 = -1$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{Adj (A)} = \frac{1}{-1}\begin{pmatrix} 1 & -4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 1 & -3 \end{pmatrix}$$

$$\text{Now, } X = A^{-1}B = \begin{pmatrix} -1 & 4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -7+12 \\ 7-9 \end{pmatrix}$$

$$\therefore\ \begin{pmatrix} x \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \Rightarrow \frac{1}{y} = -2 \quad \therefore\ y = -\frac{1}{2}$$

Thus, the required values are $x = 5$ and $y = -\frac{1}{2}$.

45. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$x = \frac{2}{3}y,\ \ 4x - 3y = 1$ **[2071 S, 2063 M]**

⇨ Here, we have, $x = \frac{2}{3}y$ or, $3x = 2y$
$\therefore\ 3x - 2y = 0$ ....(i)
and $4x - 3y = 1$ ....(ii)
Expressing in matrix form we have.

$$\begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{or, } AX = B$$

or, $A^{-1}.AX = A^{-1}B$
$X = A^{-1}B$ .... (a)

Where $X = \begin{pmatrix} x \\ y \end{pmatrix}, A = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\therefore\ |A| = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix} = -9 + 8 = -1 \neq 0$$

So, the system of given equation have unique solution.

$$\text{Now, } A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{-1}\begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$$

Therefore from (a) we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 0 & -3 \end{pmatrix}$$

$$\therefore\ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

Thus, $x = -2$ and $y = -3$ are the required solution.

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46. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$\frac{3}{2}x + 2y = 1,\ \frac{x}{3} - \frac{y}{3} = 1$ **[2065 M]**

⇨ Here, given equations are:

$$\frac{3}{2}x + 2y = 1 \Rightarrow \frac{3x+4y}{2} = 1$$

∴ $3x + 4y = 2$ .... (i)

$$\frac{x}{3} - \frac{y}{3} = 1 \quad \Rightarrow \frac{x-y}{3} = 1$$

∴ $x - y = 3$ .... (ii)

Expressing the equations in matrix form,

$$\begin{pmatrix} 3 & 4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$AX = B$

or, $X = A^{-1}B$ ....(a)

Where, $A = \begin{pmatrix} 3 & 4 \\ 1 & -1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

We have; $|A| = \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix}$

$= 3 \times (-1) - 4 \times 1$

$= -3 - 4$

∴ $|A| = -7$

∴ $A^{-1} = \frac{1}{-7}\begin{pmatrix} -1 & -4 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{7}\begin{pmatrix} 1 & 4 \\ 1 & -3 \end{pmatrix}$

Now, from equation (a),

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 1 & 4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \frac{1}{7}\begin{pmatrix} 1 \times 2 + 4 \times 3 \\ 1 \times 2 - 3 \times 3 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 14 \\ -7 \end{pmatrix}$

∴ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Thus the required value of x is 2 & y is – 1.

47. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$\frac{5}{2}x - y = 5,\ \frac{4}{3}x - y = -2$ **[2065 R']**

⇨ Here, $\frac{5}{2}x - y = 5$ and $\frac{4}{3}x - y = -2$

or, $5x - 2y = 10$ and $4x - 3y = -6$

The matrix form of above equations is;

$$\underset{A}{\begin{pmatrix} 5 & -2 \\ 4 & -3 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 10 \\ -6 \end{pmatrix}}$$

$AX = B$ or, $X = A^{-1}B$

Where, $A = \begin{pmatrix} 5 & -2 \\ 4 & -3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ & $B = \begin{pmatrix} 10 \\ -6 \end{pmatrix}$

$|A| = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix} = -15 + 8 = -7$

$\text{Adj}(A) = \begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|}\text{Adj}(A) = \frac{1}{-7}\begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}$

Now, $X = A^{-1}B = -\frac{1}{7}\begin{pmatrix} -3 & 2 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ -6 \end{pmatrix}$

$= -\frac{1}{7}\begin{pmatrix} -3 \times 10 + 2 \times (-6) \\ -4 \times 10 + 5 \times (-6) \end{pmatrix}$

$= -\frac{1}{7}\begin{pmatrix} -30 - 12 \\ -40 - 30 \end{pmatrix}$

$= -\frac{1}{7}\begin{pmatrix} -42 \\ -70 \end{pmatrix}$

∴ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

Thus, x = 6 and y = 10 is the solution of given equations.

48. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

$\frac{3x+5y}{8} = \frac{5x-2y}{3} = 3$ **[2068 R]**

⇨ Here, $\frac{3x+5y}{8} = \frac{5x-2y}{3} = 3$

∴ $3x + 5y = 24$ .... (i) and

$5x - 2y = 9$ .... (ii)

The matrix form of above equations:

$$\underset{A}{\begin{pmatrix} 3 & 5 \\ 5 & -2 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 24 \\ 9 \end{pmatrix}}$$

or, $X = A^{-1}B$

Where, $A = \begin{pmatrix} 3 & 5 \\ 5 & -2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 24 \\ 9 \end{pmatrix}$

∴ $|A| = \begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix}$

$= -6 - 25$

$= -31$

So, $A^{-1} = \frac{1}{|A|}\begin{pmatrix} -2 & -5 \\ -5 & 3 \end{pmatrix}$

$= \frac{1}{-31}\begin{pmatrix} -2 & -5 \\ -5 & 3 \end{pmatrix}$

We know that, $X = A^{-1}B$

$= -\frac{1}{31}\begin{pmatrix} -2 & -5 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 24 \\ 9 \end{pmatrix}$

$= \frac{1}{31}\begin{pmatrix} 2 & 5 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} 24 \\ 9 \end{pmatrix}$

$= \frac{1}{31}\begin{pmatrix} 48 + 45 \\ 120 - 27 \end{pmatrix}$

$= \frac{1}{31}\begin{pmatrix} 93 \\ 93 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

∴ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

Thus, x = 3 and y = 3 are the solutions.

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MODEL 2

49. यदि $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ भए $A^2 + 5A^{-1} - 14I$ को डिटरमिन्यान्ट निकाल्नुहोस्, जसमा I एउटा 2 × 2 एकाइ मेट्रिक्स हो ।

If $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$, find the determinant of $A^2 + 5A^{-1} - 14I$, where I is a 2 × 2 unit matrix. **[2063 R']**

⇨ Here, $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{Adj. } (A) = \frac{1}{1}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$$

Now, $A^2 + 5A^{-1} - 14I$

$$= \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} + 5\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} - 14\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9+5 & 15+10 \\ 3+2 & 5+4 \end{pmatrix} + \begin{pmatrix} 10 & -25 \\ -5 & 15 \end{pmatrix} - \begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} 14+10-14 & 25-25+0 \\ 5-5+0 & 9+15-14 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

Determinant of $A^2 + 5A^{-1} - 14I$

$$= |A^2 + 5A^{-1} - 14I|$$

$$= \begin{vmatrix} 10 & 0 \\ 0 & 10 \end{vmatrix}$$

$$= 100 - 0$$

$$= 100$$

Thus, $|A^2 + 5A^{-1} - 14I| = 100$

50. यदि $M = \begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix}$ र $N = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ भए $(2M + 3N)^T$ को मेट्रिक्स पत्ता लगाई त्यसको डिटरमिन्यान्ट निकाल्नुहोस् ।

If $M = \begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$, find the matrix of $(2M + 3N)^T$ and then find its determinant. **[2064 R'**

⇨ Here, $M = \begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

$$\therefore \quad 2M + 3N = 2\begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix} + 3\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 8 \\ -4 & 10 \end{pmatrix} + \begin{pmatrix} 6 & 3 \\ -3 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 6+6 & 8+3 \\ -4-3 & 10+9 \end{pmatrix}$$

$$\therefore \quad 2M + 3N = \begin{pmatrix} 12 & 11 \\ -7 & 19 \end{pmatrix}$$

$$\text{or,} \quad (2M + 3N)^T = \begin{pmatrix} 12 & -7 \\ 11 & 19 \end{pmatrix}$$

Now Determinant of $(2M + 3N)^T$

$$= |(3M + 3N)^T|$$

$$= \begin{vmatrix} 12 & -7 \\ 11 & 19 \end{vmatrix}$$

$$= 228 + 77$$

Thus, $|(3M + 3N)^T| = 305$

QUESTIONS FROM CDC TEXTBOOK

3.1 मेट्रिक्सको डिटरमिनेन्ट (DETERMINANT OF A MATRIX)

EXERCISE 3.1

1. दिइएका डिटरमिनेन्टको मान निकाल्नुहोस् । (Find the value of following determinants.)

(a) $|-8|$

⇨ Here, $|-8| = -8$

(b) $\begin{vmatrix} 6 & -3 \\ 4 & 7 \end{vmatrix}$

⇨ Here, $\begin{vmatrix} 6 & -3 \\ 4 & 7 \end{vmatrix} = 42 + 12 = 54$

(c) $\begin{vmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{vmatrix}$

⇨ Here, $\begin{vmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{vmatrix} = \cos^2 A + \sin^2 A = 1$

(d) $\begin{vmatrix} -3 & -2 \\ 4 & 5 \end{vmatrix}$

⇨ Here, $\begin{vmatrix} -3 & -2 \\ 4 & 5 \end{vmatrix} = -15 + 8 = -7$

(e) $\begin{vmatrix} 4 & 8 \\ 0 & -5 \end{vmatrix}$

⇨ Here, $\begin{vmatrix} 4 & 8 \\ 0 & -5 \end{vmatrix} = -20 - 0 = -20$

(f) $\begin{vmatrix} x+y & x-y \\ x-y & x+y \end{vmatrix}$

⇨ Here, $\begin{vmatrix} x+y & x-y \\ x-y & x+y \end{vmatrix} = (x+y)^2 - (x-y)^2$

$$= x^2 + 2xy + y^2 - (x^2 - 2xy + y^2)$$

$$= x^2 + 2xy + y^2 - x^2 + 2xy - y^2$$

$$= 4xy$$

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2. दिइएका मेट्रिक्सको डिटरमिनेन्टको मान निकाल्नुहोस् । (Find the determinant of following matrices.)

(a) [6] ⇨ Here, Determinant of [6] = |6| = 6

(b) [– 12] ⇨ Here, Determinant of [– 12] = |– 12| = – 12

(c) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ ⇨ Here, determinant of $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} = \begin{vmatrix} a & b \\ -b & a \end{vmatrix} = a^2 + b^2$

(d) $\begin{bmatrix} 3 & -2 \\ 2 & 8 \end{bmatrix}$ ⇨ Here, determinant of $\begin{bmatrix} 3 & -2 \\ 2 & 8 \end{bmatrix} = \begin{vmatrix} 3 & -2 \\ 2 & 8 \end{vmatrix} = 24 + 4 = 28$

(e) $\begin{bmatrix} -1 & -8 \\ 8 & 22 \end{bmatrix}$ ⇨ Here, determinant of $\begin{bmatrix} -1 & -8 \\ 8 & 22 \end{bmatrix} = \begin{vmatrix} -1 & -8 \\ 8 & 22 \end{vmatrix} = -22 + 64 = 42$

(f) $\begin{bmatrix} 0 & 2 \\ -4 & 5 \end{bmatrix}$ ⇨ Here, determinant of $\begin{bmatrix} 0 & 2 \\ -4 & 5 \end{bmatrix} = \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 0 + 8 = 8$

3. निम्न अवस्थामा *x* को मान पत्ता लगाउनुहोस् । (Find the value of x in given conditions.)

(a) $\begin{vmatrix} -3 & x \\ 5 & 2 \end{vmatrix} = 9$

⇨ Here, $\begin{vmatrix} -3 & x \\ 5 & 2 \end{vmatrix} = 9$

or, $-6 - 5x = 9$

or, $-5x = 15$

$\therefore\ x = -3$

Thus, the value of x is – 3.

(b) $\begin{vmatrix} 5 & -2 \\ 2x & -3 \end{vmatrix} = 1$

⇨ Here, $\begin{vmatrix} 5 & -2 \\ 2x & -3 \end{vmatrix} = 1$

or, $5(-3) - 2x(-2) = 1$

or, $-15 + 4x = 1$

or, $4x = 16$

$\therefore\ x = 4$

Thus, the value of x is 4.

(c) $\begin{vmatrix} 3x & 4 \\ 9x & 2x \end{vmatrix} = 0$

⇨ Here, $\begin{vmatrix} 3x & 4 \\ 9x & 2x \end{vmatrix} = 0$

or, $6x^2 - 36x = 0$

or, $6x(x - 6) = 0$

Either, $6x = 0 \quad \therefore\ x = 0$

or, $x - 6 = 0 \quad \therefore\ x = 6$

Thus, x = 0 or 6 is the required value of x.

(d) $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$

⇨ Here, $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$

or, $2x^2 - 15 = 15 + 20$

or, $2x^2 = 50$

or, $x^2 = 25$

$\therefore\ x = \pm 5$

Thus, the value of x is ± 5.

4. यदि P = $\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ र Q = $\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$ भए, निम्न मेट्रिक्सहरूको डिटरमिनेन्ट निकाल्नुहोस् ।

If P = $\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ and Q = $\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$ then find the determinant of following matrices.

(a) 2P + 3Q **(b)** 4P – 2Q **(c)** 3PQ

⇨ Here, P = $\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ and Q = $\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$

(a) 2P + 3Q

$$= 2\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix} + 3\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 4 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 21 & 24 \\ 0 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 21 & 4 + 24 \\ 8 + 0 & 6 - 15 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 28 \\ 8 & -9 \end{bmatrix}$$

Now, $|2P + 3Q| = \begin{vmatrix} 31 & 28 \\ 8 & -9 \end{vmatrix} = -279 - 224 = -503$

Thus, the determinant of 2P + 3Q is – 503.

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(b) 4P – 2Q

$$= 4\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix} - 2\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 8 \\ 16 & 12 \end{bmatrix} - \begin{bmatrix} 14 & 16 \\ 0 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 20-14 & 8-16 \\ 16-0 & 12+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -8 \\ 16 & 22 \end{bmatrix}$$

Now, $|4P-2Q| = \begin{vmatrix} 6 & -8 \\ 16 & 22 \end{vmatrix}$

$= 132 + 128$

$= 260$

Thus, the determinant of (4P – 2Q) is 260.

(c) 3PQ

$$PQ = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 35+0 & 40-10 \\ 28+0 & 32-15 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 30 \\ 28 & 17 \end{bmatrix}$$

$$\therefore\ 3PQ = 3\begin{bmatrix} 35 & 30 \\ 28 & 17 \end{bmatrix} = \begin{bmatrix} 105 & 90 \\ 84 & 51 \end{bmatrix}$$

Now, $|3PQ| = \begin{vmatrix} 105 & 90 \\ 84 & 51 \end{vmatrix}$

$= 105 \times 51 - 90 \times 84$

$= 5355 - 7560$

$= -2205$

Thus, the determinant of 3PQ is – 2205.

5. (a) यदि $A = \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix}$ र $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$ भए, परीक्षण गर्नुहोस् : |AB| = |A| |B|

If $A = \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$ then verify: |AB| = |A| |B|

⇨ Here, $A = \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$

$$\text{So, } AB = \begin{bmatrix} 9 & -3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 27-12 & -18-18 \\ 6+16 & -4+24 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & -36 \\ 22 & 20 \end{bmatrix}$$

$$\therefore\ AB = \begin{bmatrix} 15 & -36 \\ 22 & 20 \end{bmatrix}$$

So, $|AB| = \begin{vmatrix} 15 & -36 \\ 22 & 20 \end{vmatrix} = 300 + 792 = 1092$

Again,

$|A| = \begin{vmatrix} 9 & -3 \\ 2 & 4 \end{vmatrix} = 36 + 6 = 42$ and

$|B| = \begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix} = 18 + 8 = 26$

Now, |A|.|B| = 42 × 26 = 1092 = |AB|

Thus, |AB| = |A||B| **Proved.**

(b) यदि $P = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ र $Q = \begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix}$ भए, परीक्षण गर्नुहोस् : |PQ| = |P| |Q|

If $P = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix}$ then verify: |PQ| = |P| |Q|

⇨ Here, $P = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix}$

$$\text{So, } PQ = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5+12 & -15+4 \\ 4+18 & -12+6 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -11 \\ 22 & -6 \end{bmatrix}$$

$$\therefore\ PQ = \begin{bmatrix} 17 & -11 \\ 22 & -6 \end{bmatrix}$$

So, $|PQ| = \begin{vmatrix} 17 & -11 \\ 22 & -6 \end{vmatrix}$

$= 17(-6) + 11 \times 22$

$= -102 + 242$

$= 140$

Again, $|P| = \begin{vmatrix} 5 & 2 \\ 4 & 3 \end{vmatrix} = 15 - 8 = 7$

$|Q| = \begin{vmatrix} 1 & -3 \\ 6 & 2 \end{vmatrix} = 2 + 18 = 20$

Now, |P||Q| = 7 × 20 = 140 = |PQ|

Thus, |P||Q| = |PQ| **Proved.**

6. यदि $M = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ र $N = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}$ भए, MN र NM को डिटरमिनेन्ट पत्ता लगाउनुहोस् ।

If $M = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}$ then find the determinant of MN and NM.

⇨ Here, $M = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}$

$$\text{So, } MN = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -4+12 & 6-3 \\ -8-4 & 12+1 \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -12 & 13 \end{bmatrix}$$

$\therefore\ |MN| = \begin{vmatrix} 8 & 3 \\ -12 & 13 \end{vmatrix} = 104 + 36 = 140$

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$$\text{Again, NM} = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -2\times2+3\times4 & +2\times3+3\times1 \\ -4\times2+1\times4 & +4\times3+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -4+12 & 6+3 \\ -8+4 & 12+1 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ -4 & 13 \end{bmatrix}$$

$$\therefore \quad |NM| = \begin{vmatrix} 8 & 9 \\ -4 & 13 \end{vmatrix} = 104 + 36 = 140$$

Thus, |MN| = 140 and |NM| = 140 are the required values.

7. (a) यदि $A = \begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}$ भए, $2A^2 - 5A + 2I$ को डिटरमिनेन्ट पत्ता लगाउनुहोस् । जहाँ I एउटा 2 × 2 को एकाइ मेट्रिक्स हो ।

If $A = \begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}$ then find the determinant of $2A^2 - 5A + 2I$, where I is an identity matrix of order 2 × 2.

⇨ Here, $A = \begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}$

So, $2A^2 - 5A + 2I$

$$= 2\begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 0-2\times3 & 0-2\times4 \\ 0+4\times3 & -3\times2+4\times4 \end{bmatrix} - \begin{bmatrix} 0 & -10 \\ 15 & 20 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= 2\begin{bmatrix} -6 & -8 \\ 12 & 10 \end{bmatrix} - \begin{bmatrix} 0 & -10 \\ 15 & 20 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -16 \\ 24 & 20 \end{bmatrix} - \begin{bmatrix} 0 & -10 \\ 15 & 20 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12-0+2 & -16+10+0 \\ 24-15+0 & 20-20+2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -6 \\ 9 & 2 \end{bmatrix}$$

$$\text{Now, } |2A^2 - 5A + 2I| = \begin{vmatrix} -10 & -6 \\ 9 & 2 \end{vmatrix} = -20 + 54 = 34$$

Thus, the determinant of $2A^2 - 5A + 2I$ is 34.

(b) यदि $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ र $B = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}$ भए, $5A - 2B + 3I$ को डिटरमिनेन्ट पत्ता लगाउनुहोस्, जहाँ I एउटा 2 × 2 को एकाइ मेट्रिक्स हो ।

If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}$ then find the determinant of $5A - 2B + 3I$, where I is an identity matrix of order 2 × 2.

⇨ Here, given matrices $A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\text{Now, } 5A - 2B + 3I = 5\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} - 2\begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix} - \begin{pmatrix} -6 & 0 \\ 2 & -4 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ -2 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+6+3 & -10+0+0 \\ 15-2+0 & 20+4+3 \end{pmatrix} = \begin{pmatrix} 14 & -10 \\ 13 & 27 \end{pmatrix}$$

$$\text{Then determinant } |5A - 2B + 3I| = \begin{vmatrix} 14 & -10 \\ 13 & 27 \end{vmatrix} = 14\times27 - 13\times(-10) = 378 + 130 = 508$$

Thus, the determinant of 5A – 2B + 31 is 508.

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3.2 विपरीत मेट्रिक्स (INVERSE MATRIX)

EXERCISE 3.2

1. **दिइएका मेट्रिक्सहरू गुणन गर्नुहोस् र तिनीहरू एक आपसमा विपरीत मेट्रिक्स छन् भनी देखाउनुहोस् ।**
Multiply the following matrices and show that they are inverse of each other.

(a) $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

⇨ Here, $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

So, $AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2\times3-1\times5 & -2\times1+1\times2 \\ 5\times3-3\times5 & -5\times1+3\times2 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Again, $BA = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3\times2-1\times5 & 3\times1-1\times3 \\ -5\times2+2\times5 & -5\times1+2\times3 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & 3-3 \\ -10+10 & -5+6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, AB = BA = I shows that A and B are inverse to each other.

(b) $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

⇨ Here, $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

So, $AB = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3\times1-1\times2 & -3\times1+1\times3 \\ 2\times1-1\times2 & -2\times1+1\times3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Again, $BA = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times3-1\times2 & 1\times1-1\times1 \\ -2\times3+3\times2 & -2\times1+3\times1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, AB = BA = I shows that A and B are inverse to each other.

(c) $A = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

⇨ Here, $A = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

So, $AB = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 5\times2-3\times3 & 5\times3-3\times5 \\ -3\times2+2\times3 & -3\times3+2\times5 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Again, $BA = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2\times5-3\times3 & -2\times3+3\times2 \\ 3\times5-5\times3 & -3\times3+5\times2 \end{bmatrix}$

$\therefore \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, AB = BA = I shows that A and B are inverse to each other.

2. **तल दिइएका मेट्रिक्सहरूको विपरीत मेट्रिक्स निकाल्नुहोस् । (Find the inverse matrix of following matrices.)**

(a) $\begin{bmatrix} 2 & 7 \\ 5 & 9 \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} 2 & 7 \\ 5 & 9 \end{bmatrix}$ So, $|A| = \begin{vmatrix} 2 & 7 \\ 5 & 9 \end{vmatrix} = 18 - 35 = -17$ Adj. $(A) = \begin{bmatrix} 9 & -7 \\ -5 & 2 \end{bmatrix}$

We know that $A^{-1} = \frac{1}{|A|}$ Adj. $(A) = \frac{1}{-17}\begin{bmatrix} 9 & -7 \\ -5 & 2 \end{bmatrix}$

Thus, the inverse of the given matrix is $\begin{bmatrix} -\frac{9}{17} & \frac{7}{17} \\ \frac{5}{17} & -\frac{2}{17} \end{bmatrix}$.

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(b) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} = 6 - 5 = 1$

$\text{Adj. }(A) = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

We know that $A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$

$$= \frac{1}{1}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

(d) $\begin{bmatrix} 3 & 0 \\ 6 & 8 \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} 3 & 0 \\ 6 & 8 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 3 & 0 \\ 6 & 8 \end{vmatrix} = 24 - 0 = 24$

$\text{Adj. }(A) = \begin{bmatrix} 8 & 0 \\ -6 & 3 \end{bmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$$

$$= \frac{1}{24}\begin{bmatrix} 8 & 0 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{pmatrix} \frac{8}{24} & 0 \\ -\frac{6}{24} & \frac{3}{24} \end{pmatrix}$$

$$\therefore\ A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{4} & \frac{1}{8} \end{pmatrix}$$

Thus, the inverse of the given matrix is $\begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{4} & \frac{1}{8} \end{pmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix} = 8 - 15 = -7$

$\text{Adj. }(A) = \begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$$

$$= -\frac{1}{7}\begin{pmatrix} 4 & -3 \\ -5 & 2 \end{pmatrix}$$

$$\therefore\ A^{-1} = \begin{pmatrix} -\frac{4}{7} & \frac{3}{7} \\ \frac{5}{7} & -\frac{2}{7} \end{pmatrix}$$

Thus, the inverse of the given matrix is $\begin{pmatrix} -\frac{4}{7} & \frac{3}{7} \\ \frac{5}{7} & -\frac{2}{7} \end{pmatrix}$

(e) $\begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{bmatrix}$

$$\therefore\ |A| = \begin{vmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{vmatrix}$$

$$= \cos^2 A + \sin^2 A = 1$$

$$\text{Adj. }(A) = \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$$

$$= \frac{1}{1}\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

Thus, the inverse of the given matrix is;

$\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$.

(f) $\begin{bmatrix} \frac{1}{3} & 2 \\ 2 & 6 \end{bmatrix}$

⇨ Here, let $A = \begin{bmatrix} \frac{1}{3} & 2 \\ 2 & 6 \end{bmatrix}$ $\therefore\ |A| = \begin{vmatrix} \frac{1}{3} & 2 \\ 2 & 6 \end{vmatrix} = \frac{1}{3} \times 6 - 2 \times 2 = 2 - 4 = -2$

$\text{Adj. }(A) = \begin{bmatrix} 6 & -2 \\ -2 & \frac{1}{3} \end{bmatrix}$ We have, $A^{-1} = \frac{1}{|A|}\text{ Adj. }(A) = \frac{1}{-2}\begin{bmatrix} 6 & -2 \\ -2 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & -\frac{1}{6} \end{bmatrix}$

Thus, the inverse of the given matrix is $\begin{bmatrix} -3 & 1 \\ 1 & -\frac{1}{6} \end{bmatrix}$.

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3. (a) यदि मेट्रिक्स $\begin{bmatrix} p & 1 \\ 5 & 2 \end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix} 2 & -1 \\ -5 & q \end{bmatrix}$ भए, p र q को मानहरू निकाल्नुहोस् ।

If matrix $\begin{bmatrix} 2 & -1 \\ -5 & q \end{bmatrix}$ is the inverse of matrix $\begin{bmatrix} p & 1 \\ 5 & 2 \end{bmatrix}$ then find the values of p and q.

⇨ Here, inverse of $\begin{bmatrix} p & 1 \\ 5 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2 & -1 \\ -5 & q \end{bmatrix}$

So, $\begin{bmatrix} p & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -5 & q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or, $\begin{bmatrix} 2p-5 & -p+q \\ 10-10 & -5+2q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or, $\begin{bmatrix} 2p-5 & -p+q \\ 0 & -5+2q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equating the corresponding elements,

$2p - 5 = 1$ and $-p + q = 0$

or, $2p = 6$ or, $-3 + q = 0$

$\therefore$ $p = 3$ $\therefore$ $q = 3$

Thus, the values of p and q are 3 and 3.

(b) यदि मेट्रिक्स $\begin{bmatrix} x & 2x-9 \\ -y & 3 \end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix} 3 & 5 \\ y & x \end{bmatrix}$ भए, x र y को मानहरू निकाल्नुहोस् ।

If matrix $\begin{bmatrix} 3 & 5 \\ y & x \end{bmatrix}$ is the inverse of matrix $\begin{bmatrix} x & 2x-9 \\ -y & 3 \end{bmatrix}$ then find the values of x and y.

⇨ Here, the inverse of $\begin{pmatrix} x & 2x-9 \\ -y & 3 \end{pmatrix}$ is matrix $\begin{pmatrix} 3 & 5 \\ y & x \end{pmatrix}$

So, $\begin{pmatrix} x & 2x-9 \\ -y & 3 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ y & x \end{pmatrix} = I$ |∵ Where I is a unit matrix of order 2 × 2|

or, $\begin{pmatrix} 3x + y(2x-9) & 5x + x(2x-9) \\ -3y + 3y & -5y + 3x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 3x + 2xy - 9y & 5x + 2x^2 - 9x \\ 0 & 3x - 5y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 3x + 2xy - 9y & 2x^2 - 4x \\ 0 & 3x - 5y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $3x + 2xy - 9y = 1$, ... (i)

$2x^2 - 4x = 0$ .... (ii)

$3x - 5y = 1$ .... (iii)

Now, from equation (ii); $2x(x-2) = 0$

Either, $2x = 0$ or $x - 2 = 0$

$\therefore$ $x = 0$ or $x = 2$

Putting x = 0 in equation (iii) we get ; $3.0 - 5y = 1$

or, $-5y = 1$

$\therefore$ $y = -\frac{1}{5}$

Again, putting x = 2 in (iii) we get, $3.2 - 5y = 1$

or, $6 - 5y = 1$

or, $-5y = 1 - 6$

or, $-5y = -5$

$\therefore$ $y = 1$

Since equation (i) is not satisfied by $x = 0$ and $y = -\frac{1}{5}$ but satisfied by $x = 2$ and $y = 1$.

Thus, the required value of x and y are 2 and 1 respectively.

(c) यदि मेट्रिक्स $\begin{bmatrix} 2m & 7 \\ 5 & 9 \end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix} 9 & n \\ -5 & 4 \end{bmatrix}$ भए, m र n को मानहरू पत्ता लगाउनुहोस् ।

If matrix $\begin{bmatrix} 9 & n \\ -5 & 4 \end{bmatrix}$ is the inverse of matrix $\begin{bmatrix} 2m & 7 \\ 5 & 9 \end{bmatrix}$ then find the values of m and n.

⇨ Here, given inverse marix of $\begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}$ is $\begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix}$

So, $\begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}\begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 2m \times 9 + 7 \times (-5) & 2m \times n + 7 \times 4 \\ 5 \times 9 + 9 \times (-5) & 5n + 9 \times 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 18m - 35 & 2mn + 28 \\ 45 - 45 & 5n + 36 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 18m - 35 & 2mn + 28 \\ 0 & 5n + 36 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $18m - 35 = 1$ .... (i)

$2mn + 28 = 0$ .... (ii)

$5n + 36 = 1$ .... (iii)

From equation (i)

$18m = 36$

or, $m = \frac{36}{18} = 2$

$\therefore$ $m = 2$

From equation (iii)

$5n = 1 - 36$

or, $5n = -35$

or, $n = \frac{-35}{5} = -7$

$\therefore$ $m = 2$ and $n = -7$

Thus required value of m and n are 2 and – 7 respectively.

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4. यदि $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$ र $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ भए, (If $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then):

(a) A^{-1} र B^{-1} निकाल्नुहोस् । (Find A^{-1} and B^{-1}.)

⇨ Here, $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

For A^{-1}, $|A| = \begin{vmatrix} 2 & 0 \\ 5 & 3 \end{vmatrix} = 6 - 0 = 6$

$\text{Adj. }(A) = \begin{bmatrix} 3 & 0 \\ -5 & 2 \end{bmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$$

$$= \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -5 & 2 \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{6} & \frac{1}{3} \end{bmatrix}$$

Again, $|B| = \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} = 8 - 7 = 1$

$\text{Adj. }(B) = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

We know that,

$$B^{-1} = \frac{1}{|B|}\text{ Adj. }(B)$$

$$= \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$$

$$\therefore\ B^{-1} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$$

(b) $(AB)^{-1}$ निकाल्नुहोस् । (Find $(AB)^{-1}$.)

⇨ Here, $AB = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2\times4+0\times7 & 2\times1+0\times2 \\ 5\times4+3\times7 & 5\times1+3\times2 \end{bmatrix}$$

$$\therefore\ AB = \begin{bmatrix} 8 & 2 \\ 41 & 11 \end{bmatrix}$$

We have, $|AB| = \begin{vmatrix} 8 & 2 \\ 41 & 11 \end{vmatrix} = 88 - 82 = 6$

$\text{Adj. }(AB) = \begin{bmatrix} 11 & -2 \\ -41 & 8 \end{bmatrix}$

We know that,

$$(AB)^{-1} = \frac{1}{|AB|}\text{ Adj. }(AB)$$

$$\therefore\ (AB)^{-1} = \frac{1}{6}\begin{bmatrix} 11 & -2 \\ -41 & 8 \end{bmatrix}$$

(c) $(AB)^{-1} = B^{-1} . A^{-1}$ परीक्षण गर्नुहोस् ।

Verify: $(AB)^{-1} = B^{-1}.A^{-1}$

⇨ From (a) we have,

$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -5 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

From (b),

$$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 11 & -2 \\ -41 & 8 \end{bmatrix}$$

Now,

$$B^{-1}.A^{-1} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\frac{1}{6}\begin{bmatrix} 3 & 0 \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 2\times3+1\times5 & 2\times0-1\times2 \\ -7\times3-4\times5 & -7\times0+4\times2 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 11 & -2 \\ -41 & 8 \end{bmatrix}$$

$$\therefore\ B^{-1}.A^{-1} = \frac{1}{6}\begin{bmatrix} 11 & -2 \\ -41 & 8 \end{bmatrix}$$

$\therefore\ (AB)^{-1} = B^{-1}.A^{-1}$ **Proved.**

5. यदि $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ भए, परीक्षण गर्नुहोस्

If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ then verify that:

(a) $A . A^{-1} = A^{-1} . A = I$

⇨ Here, $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

For A^{-1}, $= |A| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 6 - 4 = 2$

$\text{Adj. }(A) = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

We know that $A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$

$$\therefore\ A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$$\text{Now, } A.A^{-1} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}.\frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6-4 & -8+8 \\ 3-3 & -4+6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore\ A.A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{ .... (i)}$$

Again,

$$A^{-1}.A = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6-4 & 12-12 \\ -2+2 & -4+6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore\ A^{-1}.A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \text{ .... (ii)}$$

From (i) and (ii); $A.A^{-1} = A^{-1}.A = I$ **Proved.**

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(b) $(A^T)^{-1} = (A^{-1})^T$

⇨ Here, $A^T = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$|A^T| = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 6 - 4 = 2$$

$$\text{Adj. }(A^T) = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

We know that $(A^T)^{-1} = \frac{1}{|A^T|}$ Adj of (A^T)

$\therefore \quad (A^T)^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ .... (i)

From (a) $A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$\therefore \quad (A^{-1})^T = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ .... (ii)

From (i) and (ii); $(A^T)^{-1} = (A^{-1})^T$ **Proved.**

3.3 दुई चलयुक्त युगपत रेखीय समीकरणको मेट्रिक्स विधिबाट हल

SOLVING SIMULTANEOUS EQUATION OF TWO VARIABLES BY MATRIX METOHD

EXERCISE 3.3

1. निम्न लिखित अवस्थामा विपरीत मेट्रिक्सको प्रयोग गरी मेट्रिक्स $X = \begin{bmatrix} x \\ y \end{bmatrix}$ को मान पत्ता लगाउनुहोस् :

Find the value of matrix $X = \begin{bmatrix} x \\ y \end{bmatrix}$ by using inverse matrix method in following conditions.

(a) $\begin{bmatrix} 1 & -8 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 39 \\ 13 \end{bmatrix}$

⇨ Here, we have, $\begin{bmatrix} 1 & -8 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 39 \\ 13 \end{bmatrix}$

Let, $A = \begin{bmatrix} 1 & -8 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ & $C = \begin{bmatrix} 39 \\ 13 \end{bmatrix}$

$\therefore \; |A| = \begin{vmatrix} 1 & -8 \\ 2 & 3 \end{vmatrix} = 3 + 16 = 19$

Now,

$$A^{-1} = \frac{1}{19}\begin{bmatrix} 3 & 8 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{8}{19} \\ -\frac{2}{19} & \frac{1}{19} \end{bmatrix}$$

We have,

$$X = A^{-1}C = \begin{bmatrix} \frac{3}{19} & \frac{8}{19} \\ -\frac{2}{19} & \frac{1}{19} \end{bmatrix}\begin{bmatrix} 39 \\ 13 \end{bmatrix}$$

$$\text{or, } X = \begin{bmatrix} \frac{3}{19} \times 39 + \frac{8}{19} \times 13 \\ -\frac{2}{19} \times 39 + \frac{1}{19} \times 13 \end{bmatrix} = \begin{bmatrix} \frac{221}{19} \\ -\frac{65}{19} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{221}{19} \\ -\frac{65}{19} \end{bmatrix}$$

(b) $\begin{bmatrix} 5 & 7 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

⇨ Here, we have given, $\begin{bmatrix} 5 & 7 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

Let $A = \begin{bmatrix} 5 & 7 \\ 1 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ & $C = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

$\therefore \; |A| = \begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix} = 20 - 7 = 13$

$$\text{Now, } A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} 4 & -7 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{13} & -\frac{7}{13} \\ -\frac{1}{13} & \frac{5}{13} \end{bmatrix}$$

We have,

$$X = A^{-1}C$$

$$= \begin{bmatrix} \frac{4}{13} & -\frac{7}{13} \\ -\frac{1}{13} & \frac{5}{13} \end{bmatrix}\begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{13} \times 1 + \left(\frac{-7}{13}\right) \times (-5) \\ -\frac{1}{13} \times 1 + \frac{5}{13} \times (-5) \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{4}{13} + \frac{35}{13} \\ -\frac{1}{13} + \left(\frac{-25}{13}\right) \end{bmatrix} = \begin{bmatrix} \frac{39}{13} \\ \frac{-26}{13} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

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(c) $\begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 14 \\ 28 \end{bmatrix}$

⇨ Here, $\underset{A}{\begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 14 \\ 28 \end{bmatrix}}$

So, $|A| = \begin{vmatrix} 2 & 5 \\ 4 & 10 \end{vmatrix} = 20 - 20 = 0$

Since, |A| = 0 so there is no unique solution.

(d) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$

⇨ Here, $\underset{A}{\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}}$

So, $|A| = \begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta + \cos^2\theta = 1$

Adj. (A) = $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

We know that, $A^{-1} = \frac{1}{|A|}$ Adj. (A)

$$= \frac{1}{1}\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$$

Now, $X = A^{-1}B$

$$= \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta + \cos^2\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Thus, x = 1 and y = 0 are the solutions.

2. **दिइएका जोडा समीकरणहरूको मेट्रिक्स विधिबाट हल गर्नुहोस् :**

Solve the given pair of equations by matrix method:

(a) $x - 3y = 5,\ 2x - 5y = 9$

⇨ Here, $x - 3y = 5,\ 2x - 5y = 9$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 5 \\ 9 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix} = -5 + 6 = 1$

Adj. (A) = $\begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj (A)}$$

$$= \frac{1}{1}\begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} -5\times5+3\times9 \\ -2\times5+1\times9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Thus, x = 2 and y = – 1 is the solution.

(b) $x - 2y = 4,\ 3x - 5y - 7 = 0$

⇨ Here, $x - 2y = 4,\ 3x - 5y = 7$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 4 \\ 7 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 1 & -2 \\ 3 & -5 \end{vmatrix} = -5 + 6 = 1$

Adj. (A) = $\begin{pmatrix} -5 & 2 \\ -3 & 1 \end{pmatrix}$

We know that

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{1}\begin{pmatrix} -5 & 2 \\ -3 & 1 \end{pmatrix}$$

Now, $X = A^{-1}B = \begin{pmatrix} -5 & 2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$$= \begin{pmatrix} -5\times4+2\times7 \\ -3\times4+1\times7 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix}$$

Thus, x = – 6 and y = – 5 is the solution.

(c) $x + y = 6,\ 2x - y = 3$

⇨ Here, $x + y = 6,\ 2x - y = 3$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 6 \\ 3 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3$

Adj. (A) = $\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{-3}\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

Now, $X = A^{-1}B = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$$= \frac{1}{3}\begin{pmatrix} 6+3 \\ 12-3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 9 \\ 9 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Thus, x = 3 and y = 3 is the solution.

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(d) **$2x - 3y = 1, 4y + 3x = 10$**

⇨ Here, $2x - 3y = 1, 4y + 3x = 10$

The matrix form of above equations is;

$$\underset{A}{\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}}\underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{B}{\begin{pmatrix} 1 \\ 10 \end{pmatrix}}$$

We have, $|A| = \begin{vmatrix} 2 & -3 \\ 3 & 4 \end{vmatrix} = 8 + 9 = 17$

$\text{Adj. (A)} = \begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{17}\begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{17}\begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 10 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 4 + 30 \\ -3 + 20 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 34 \\ 17 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Thus, $x = 2$ and $y = 1$ is the solution.

(e) **$4x - 3y = 11, 3x = 5 - y$**

⇨ Here, $4x - 3y = 11, 3x + y = 5$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 11 \\ 5 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 4 & -3 \\ 3 & 1 \end{vmatrix} = 4 + 9 = 13$

$\text{Adj.(A)} = \begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{13}\begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{13}\begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 11 \\ 5 \end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix} 11 + 15 \\ -33 + 20 \end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix} 26 \\ -13 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Thus, $x = 2$ and $y = -1$ is the solution.

(f) **$2x + 5 = 4(y + 1) - 1, 3x + 4 = 5(y + 1) - 3$**

⇨ Here, $2x + 5 = 4(y + 1) - 1$

or, $2x + 5 = 4y + 4 - 1$

or, $2x - 4y = -2$

And, $3x + 4 = 5(y + 1) - 3$

or, $3x + 4 = 5y + 5 - 3$

or, $3x - 5y = -2$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 2 & -4 \\ 3 & -5 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} -2 \\ -2 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 2 & -4 \\ 3 & -5 \end{vmatrix} = -10 + 12 = 2$

$\text{Adj.(A)} = \begin{pmatrix} -5 & 4 \\ -3 & 2 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{2}\begin{pmatrix} -5 & 4 \\ -3 & 2 \end{pmatrix}$$

Now,

$X = A^{-1}B$

$$= \frac{1}{2}\begin{pmatrix} -5 & 4 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 5 \times 2 - 4 \times 2 \\ 3 \times 2 - 2 \times 2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Thus, $x = 1$ and $y = 1$ is the solution.

3. **दिइएका जोडा समीकरणहरूको मेट्रिक्स विधिबाट हल गर्नुहोस्**

Solve the given pair of equations by matrix method:

(a) $\frac{x}{3} - \frac{4y}{3} = -2, \frac{3x}{4} - 4y = 2$

⇨ Here, $\frac{x}{3} - \frac{4y}{3} = -2$ and $\frac{3x}{4} - 4y = 2$

or, $\frac{x - 4y}{3} = -2$ and $\frac{3x - 16y}{4} = 2$

$\therefore$ $x - 4y = -6$ and $3x - 16y = 8$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 1 & -4 \\ 3 & -16 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} -6 \\ 8 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 1 & -4 \\ 3 & -16 \end{vmatrix} = -16 + 12 = -4$

$\text{Adj.(A)} = \begin{pmatrix} -16 & 4 \\ -3 & 1 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. (A)} = \frac{1}{-4}\begin{pmatrix} -16 & 4 \\ -3 & 1 \end{pmatrix}$$

Now, $X = A^{-1}B = \frac{1}{-4}\begin{pmatrix} -16 & 4 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -6 \\ 8 \end{pmatrix}$

$$= -\frac{1}{4}\begin{pmatrix} 96 + 32 \\ 18 + 8 \end{pmatrix}$$

$$= -\frac{1}{4}\begin{pmatrix} 128 \\ 26 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -32 \\ -\frac{13}{2} \end{pmatrix}$$

Thus, $x = -32$ and $y = -\frac{13}{2}$ is the solution.

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(b) $\boldsymbol{\frac{3}{x}+\frac{2}{y}=13, \frac{5}{x}-\frac{3}{y}=9}$

⇨ Here, $\frac{3}{x}+\frac{2}{y}=13$ and $\frac{5}{x}-\frac{3}{y}=9$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}}\underset{X}{\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix}}=\underset{B}{\begin{bmatrix} 13 \\ 9 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} = -9-10=-19$

$\text{Adj.}(A) = \begin{pmatrix} -3 & -2 \\ -5 & 3 \end{pmatrix}$

We know that, $A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$

$$= \frac{1}{-19}\begin{pmatrix} -3 & -2 \\ -5 & 3 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= -\frac{1}{19}\begin{pmatrix} -3 & -2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 13 \\ 9 \end{pmatrix}$$

$$= -\frac{1}{19}\begin{pmatrix} -39-18 \\ -65+27 \end{pmatrix}$$

$$= -\frac{1}{19}\begin{pmatrix} -57 \\ -38 \end{pmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Thus, $\frac{1}{x}=3$ i.e $x=\frac{1}{3}$ and $\frac{1}{y}=2$ i.e $y=\frac{1}{2}$ is the solution.

(c) $\boldsymbol{\frac{2x+4}{5}=y=\frac{40-3x}{4}}$

⇨ Here, $\frac{2x+4}{5}=y=\frac{40-3x}{4}$

or, $\frac{2x+4}{5}=y$ and $y=\frac{40-3x}{4}$

or, $2x+4=5y$ and $4y=40-3x$

$\therefore$ $2x-5y=-4$ and $3x+4y=40$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} -4 \\ 40 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 2 & -5 \\ 3 & 4 \end{vmatrix} = 8+15=23$

$\text{Adj.}(A) = \begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}$

We know that,

$$A^{-1} = \frac{1}{|A|}\text{ Adj. }(A)$$

$$= \frac{1}{23}\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{23}\begin{pmatrix} 4 & 5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -4 \\ 40 \end{pmatrix}$$

$$= \frac{1}{23}\begin{pmatrix} -16+200 \\ 12+80 \end{pmatrix}$$

$$= \frac{1}{23}\begin{pmatrix} 184 \\ 92 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

Thus, x = 8 and y = 4 is the solution.

(d) $\boldsymbol{\frac{5}{y}=\frac{1}{x}-1, \frac{5}{y}=\frac{2}{x}-4}$

⇨ Here, $\frac{5}{y}=\frac{1}{x}-1$ and $\frac{5}{y}=\frac{2}{x}-4$

$\therefore$ $\frac{1}{x}-\frac{5}{y}=1$ and $\frac{2}{x}-\frac{5}{y}=4$

The matrix form of above equations is;

$$\underset{A}{\begin{bmatrix} 1 & -5 \\ 2 & -5 \end{bmatrix}}\underset{X}{\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix}}=\underset{B}{\begin{bmatrix} 1 \\ 4 \end{bmatrix}}$$

We have, $|A| = \begin{vmatrix} 1 & -5 \\ 2 & -5 \end{vmatrix} = -5+10=5$

$\text{Adj.}(A) = \begin{pmatrix} -5 & 5 \\ -2 & 1 \end{pmatrix}$

We know that

$$A^{-1} = \frac{1}{|A|}$$

$$\text{Adj. }(A) = \frac{1}{5}\begin{pmatrix} -5 & 5 \\ -2 & 1 \end{pmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{5}\begin{pmatrix} -5 & 5 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} -5+20 \\ -2+4 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 15 \\ 2 \end{pmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix} = \begin{pmatrix} 3 \\ \frac{2}{5} \end{pmatrix}$$

Thus, $\frac{1}{x}=3$ i.e $x=\frac{1}{3}$ and $\frac{1}{y}=\frac{2}{5}$ i.e $y=\frac{5}{2}$ is the solution.

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4. (a) जोडा समीकरणहरू $x + 3y = 5$ र $2x - 3y = 1$ लाई (Given pair of equations $x + 3y = 5$ and $2x - 3y = 1$.)

(i) **मेट्रिक्सका रूपमा लेख्नुहोस् । (Write in matrix form.)**

(ii) **के यी समीकरणहरूको एकल समाधान हुन्छ ? (Is there a single solution for these equations?)**

(iii) **उक्त समीकरणहरू हल गर्नुहोस् । (Solve the above equations.)**

⇨ Here, $x + 3y = 5$ and $2x - 3y = 1$

The matrix form of above equations is; $\underset{A}{\begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 5 \\ 1 \end{bmatrix}}$

We have, $|A| = \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = -3 - 6 = -9$

Since $|A| \neq 0$ so the given equations have the unique solution.

For A^{-1}, $\text{Adj.}(A) = \begin{pmatrix} -3 & -3 \\ -2 & 1 \end{pmatrix}$

We know that $A^{-1} = \frac{1}{|A|}$ Adj. $(A) = \frac{1}{-9}\begin{pmatrix} -3 & -3 \\ -2 & 1 \end{pmatrix}$

$$\begin{aligned} \text{Now,}\quad X &= A^{-1}B \\ &= -\frac{1}{9}\begin{pmatrix} -3 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} \\ &= -\frac{1}{9}\begin{pmatrix} -15-3 \\ -10+1 \end{pmatrix} \\ &= -\frac{1}{9}\begin{pmatrix} -18 \\ -9 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{aligned}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Thus, $x = 2$ and $y = 1$ is the solution.

(b) जोडा समीकरणहरू $2x + 5y = 2$ र $3x - 5y = 3$ लाई (Given pair of equations $2x + 5y = 5$ and $3x - 5y = 3$.)

(i) **मेट्रिक्सको रूपमा लेख्नुहोस् । (Write in matrix form.)**

(ii) **के यी समीकरणहरूको एकल समाधान हुन्छ ? (Is there a single solution for these equations?)**

(iii) **उक्त समीकरणहरू हल गर्नुहोस् । (Solve the above equations.)**

⇨ Here, $2x + 5y = 2$ and $3x - 5y = 3$

The matrix form of above equations is; $\underset{A}{\begin{bmatrix} 2 & 5 \\ 3 & -5 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

We have, $|A| = \begin{vmatrix} 2 & 5 \\ 3 & -5 \end{vmatrix} = -10 - 15 = -25$

Since, $|A| \neq 0$ so the given equations have the unique solution.

For A^{-1}, $\text{Adj.}(A) = \begin{pmatrix} -5 & -5 \\ -3 & 2 \end{pmatrix}$

We know that, $A^{-1} = \frac{1}{|A|}$ Adj. $(A) = \frac{1}{-25}\begin{pmatrix} -5 & -5 \\ -3 & 2 \end{pmatrix}$

$$\begin{aligned} \text{Now,}\quad X &= A^{-1}B \\ &= -\frac{1}{25}\begin{pmatrix} -5 & -5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ &= -\frac{1}{25}\begin{pmatrix} -10-15 \\ -6+6 \end{pmatrix} \\ &= -\frac{1}{25}\begin{pmatrix} -25 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Thus, $x = 1$ and $y = 0$ is the solution.

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2. क्रामरको नियम
Cramer's Rule

KEY POINTS

2.1 **क्रामरको नियम** (Cramer's rule)

रेखीय समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ र $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ भए

सुत्रहरू $x = \frac{D_x}{D}$ र $y = \frac{D_y}{D}$ लाई क्रामरको नियम भनिन्छ ।

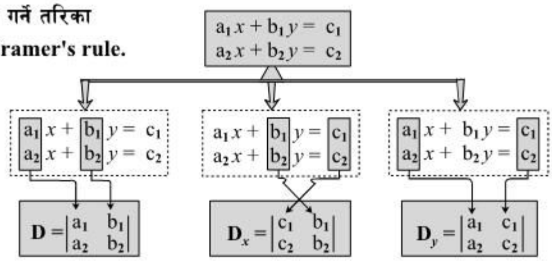
In the linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ if $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ then the formulae $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$ is known as Cramer's rule.

2.2 **क्रामरको नियम प्रयोग गरी रेखीय समीकरणहरू हल गर्ने तरिका**
Method of solving linear equations using Cramer's rule.

यदि $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ भए,

If $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ then,

Then $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.



Note: D_x and D_y can also be represented by D_1 and D_2 respectively.

QUESTIONS FROM SEE EXCERCISE 2

A. VERY SHORT QUESTIONS

1. **क्रामरको नियम प्रयोग गरी कस्ता समीकरणहरू हल गर्न सकिन्छ ?**
What type of equations can be solved by using Cramer's rule?

⇨ Here, the simultaneous linear equations can be solved by using Cramer's rule.

2. **समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा क्रामरको नियम अनुसार D को मान लेख्नुहोस् ।**
According to the Cramer's rule, write the value of D in the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

⇨ Here, the value of $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$.

3. **क्रामरको नियम अनुसार समीकरणहरू $2x + 3y = 5$ र $x + y = 2$ बाट D को मान पत्ता लगाउनुहोस् ।**
Find the value of D using Cramer's rule from the equations: $2x + 3y = 5$ and $x + y = 2$

⇨ Here, the value of $D = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2 - 3 = -1$

4. **यदि क्रामरको नियम अनुसार x र y का गुणाङ्कहरूको डिटरमिन्यान्ट (D) = 18, अचर पद र y को गुणाङ्कहरूको डिटरमिन्यान्ट (D_x) = – 54 भए x को मान पत्ता लगाउनुहोस् ।**
If the determinant of coefficient of x and y i.e. (D) = 18, determinant of constant term and coefficient of y (D_x) = – 54 then using Cramer's rule, find the value of x.

⇨ Here, $D = 18$ and $D_x = -54$

So, $x = \frac{D_x}{D} = \frac{-54}{18} = -3$

5. **क्रामरको नियम अनुसार, $D = 2$, $D_x = 8$ र $D_y = 6$ भए y को मान पत्ता लगाउनुहोस् ।**
According to Cramer's rule, $D = 2$, $D_x = 8$ and $D_y = 6$ then find the value of y.

⇨ Here, $D = 2$ and $D_y = 6$

We have, $y = \frac{D_y}{D} = \frac{6}{2} = 3$

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B. SHORT QUESTIONS

1. **यदि $D = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$, $D_x = \begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix}$ र $Dy = \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}$ भए x को मान पत्ता लगाउनुहोस् ।**

If $D = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$, $D_x = \begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix}$ and $Dy = \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}$ then, find the value of x.

⇨ Here, $D = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$ and $D_x = \begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix}$

So, $D = 1 - 2 = -1$

$D_x = 4 - 6 = -2$

Now, $x = \frac{D_x}{D} = \frac{-2}{-1} = 2$

Thus, the value of x is 2.

2. **क्रामरको नियम अनुसार $ax + by = c$ र $px + qy = r$ मा D_1 र D_2 को मान पत्ता लगाउनुहोस् ।**

According to Cramer's rule, find the values of D_1 and D_2 for $ax + by = c$ and $px + qy = r$. [SEE MODEL 2076]

⇨ Here, $ax + by = c$ and $px + qy = r$

So, $D_1 = \begin{vmatrix} c & b \\ r & q \end{vmatrix} = cq - br$

And, $D_2 = \begin{vmatrix} a & c \\ p & r \end{vmatrix} = ar - cp$

Thus, D_1 and D_2 are (cq – br) and (ar – cp) respectively.

3. **यदि $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix}$ र $D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix}$ भए क्रामरको नियम अनुसार x र y का मानहरू पत्ता लगाउनुहोस् ।**

If $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix}$ and $D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix}$ then find the values of x and y using Cramer's rule.

⇨ Here, $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix}$,

$D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix}$ and

$D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix}$

$\therefore \quad D = 7 + 6 = 13$

$\therefore \quad D_x = -49 + 10 = -39$

$\therefore \quad D_y = 5 + 21 = 26$

Now, $x = \frac{D_x}{D} = \frac{-39}{13} = -3$

$y = \frac{D_y}{D} = \frac{26}{13} = 2$

Thus, the values of x and y are – 3 and 2 respectively.

4. **समीकरणहरू $x + 3y = 12$ र $4x - 3y = 3$ मा क्रामरको नियम अनुसार D र D_x का मानहरू पत्ता लगाउनुहोस् ।**

Find the values of D and D_x using Cramer's rule from the equations $x + 3y = 12$ and $4x - 3y = 3$.

⇨ Here, $x + 3y = 12$ and $4x - 3y = 3$

So, $D = \begin{vmatrix} 1 & 3 \\ 4 & -3 \end{vmatrix} = -3 - 12 = -15$

And, $D_x = \begin{vmatrix} 12 & 3 \\ 3 & -3 \end{vmatrix} = -36 - 9 = -45$

Thus, D and D_x are – 15 and – 45 respectively.

5. **समीकरणहरू $3x + 2y = 1$ र $7x + 5y = 4$ मा क्रामरको नियम अनुसार D र D_y का मानहरू पत्ता लगाउनुहोस् ।**

Find the values of D and D_y using Cramer's rule from the equations $3x + 2y = 1$ and $7x + 5y = 4$.

⇨ Here, $3x + 2y = 1$ and $7x + 5y = 4$

So, $D = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 15 - 14 = 1$

And, $D_y = \begin{vmatrix} 3 & 1 \\ 7 & 4 \end{vmatrix} = 12 - 7 = 5$

Thus, D and D_y are 1 and 5 respectively.

6. **समीकरणहरू $4x - 5y = 2$ र $3x + 4y = 48$ मा क्रामरको नियम प्रयोग गरी D_x र D_y का मानहरू पत्ता लगाउनुहोस् ।**

Find the values of D_x and D_y using Cramer's rule from the equations $4x - 5y = 2$ and $3x + 4y = 48$.

⇨ Here, $4x - 5y = 2$ and $3x + 4y = 48$

So, $D_x = \begin{vmatrix} 2 & -5 \\ 48 & 4 \end{vmatrix} = 8 + 240 = 248$

And, $D_y = \begin{vmatrix} 4 & 2 \\ 3 & 48 \end{vmatrix} = 192 - 6 = 186$

Thus, the values of D_x and D_y are 248 and 186 respectively.

7. **समीकरणहरू $3x + 5y - 21 = 0$ र $2x + 3y = 13$ मा क्रामरको नियम प्रयोग गर्दा $x = \frac{D_1}{D} = \frac{D_1}{(-1)}$ र $y = \frac{D_2}{D} = \frac{D_2}{(-1)}$ भए D_1 र D_2 का मानहरू पत्ता लगाउनुहोस् ।**

In Cramer's rule for solving equations $3x + 5y - 21 = 0$ and $2x + 3y = 13$, $x = \frac{D_1}{D} = \frac{D_1}{(-1)}$ and $y = \frac{D_2}{D} = \frac{D_2}{(-1)}$. Find the values of D_1 and D_2.

⇨ Here, $3x + 5y - 21 = 0$ and $2x + 3y = 13$

So, $D_1 = \begin{vmatrix} 21 & 5 \\ 13 & 3 \end{vmatrix} = 63 - 65 = -2$

And, $D_2 = \begin{vmatrix} 3 & 21 \\ 2 & 13 \end{vmatrix} = 39 - 42 = -3$

Thus, $D_1 = -2$ and $D_2 = -3$.

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8. **यदि $D = \begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} 9 & 3 \\ 13 & 7 \end{vmatrix}$ र $D_y = \begin{vmatrix} 5 & 9 \\ 4 & 13 \end{vmatrix}$ भए यी मानहरूसँग सम्बन्धित समीकरणहरू पत्ता लगाउनुहोस् ।**

If $D = \begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} 9 & 3 \\ 13 & 7 \end{vmatrix}$ and $D_y = \begin{vmatrix} 5 & 9 \\ 4 & 13 \end{vmatrix}$ then write the corresponding equations to these values.

⇨ Here, $D = \begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and,

$$D_x = \begin{vmatrix} 9 & 3 \\ 13 & 7 \end{vmatrix} = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$\therefore$ $a_1 = 5, b_1 = 3, c_1 = 9, a_2 = 4, b_2 = 7$ and $c_2 = 13$

So, $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

$\therefore$ $5x + 3y = 9$ and $4x + 7y = 13$

Thus, the required equations are;

$5x + 3y = 9$ and $4x + 7y = 13$.

9. **यदि समीकरणहरू $ax - 5y = 3$ र $4x + 3y = 4$ मा $D = 29$ भए क्रामरको नियम प्रयोग गरी a को मान पत्ता लगाउनुहोस् ।**

If the equations $ax - 5y = 3$ and $4x + 3y = 4$ have $D = 29$ then find the value of a by using Cramer's rule.

⇨ Here, $ax - 5y = 3$ and $4x + 3y = 4$; $D = 29$

So, $D = \begin{vmatrix} a & -5 \\ 4 & 3 \end{vmatrix}$

or, $29 = 3a + 20$

or, $9 = 3a$

$\therefore$ $a = 3$

Thus, the value of a is 3.

10. **यदि समीकरणहरू $mx + 5y = 21$ र $2x + 3y = 13$ मा $D_y = -3$ भए क्रामरको नियम प्रयोग गरी m को मान पत्ता लगाउनुहोस् ।**

If the equations $mx + 5y = 21$ and $2x + 3y = 13$ have $D_y = -3$ then find the value of m by using Cramer's rule.

⇨ Here, $mx + 5y = 21$ and $2x + 3y = 13$; $D_y = -3$

So, $D_y = \begin{vmatrix} m & 21 \\ 2 & 13 \end{vmatrix}$

or, $-3 = 13m - 42$

or, $39 = 13m$

$\therefore$ $m = 3$

Thus the value of m is 3.

11. **k को मान कति हुँदा समीकरणहरू $2x + y = k$ र $kx + 2y = 2$ मा $D_y + 5 = 0$ हुन्छ ?**

For what value of k, the equations $2x + y = k$ and $kx + 2y = 2$ have $D_y + 5 = 0$?

⇨ Here, $2x + y = k$ and $kx + 2y = 2$; $D_y + 5 = 0$

So, $D_y = \begin{vmatrix} 2 & k \\ k & 2 \end{vmatrix} = 4 - k^2$

Now, $D_y + 5 = 0$

or, $4 - k^2 + 5 = 0$

or, $k^2 = 9$

$\therefore$ $k = \pm 3$

Thus the value of k is ± 3.

C. LONG QUESTIONS

1. **क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।**

Solve the equations using Cramer's rule.

$x + y = \frac{7}{6}$, $2x - 3y = -1$

⇨ Here, given equation, are:

$x + y = \frac{7}{6}$ and $2x - 3y = -1$

Coeff. of x	Coeff. of y	Constant term
1	1	7/6
2	– 3	– 1

$$D = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 2 = -5$$

$$D_1 = \begin{vmatrix} 7/6 & 1 \\ -1 & -3 \end{vmatrix}$$

$$= \frac{7}{6} \times (-3) - 1 \times (-1) = -\frac{7}{2} + 1 = \frac{-5}{2}$$

$$D_2 = \begin{vmatrix} 1 & 7/6 \\ 2 & -1 \end{vmatrix}$$

$$= 1 \times (-1) - \frac{7}{6} \times 2 = -1 - \frac{7}{3} = \frac{-10}{3}$$

Using Cramer's rule, we have,

$$x = \frac{D_1}{D} = \frac{-5/2}{-5} = \frac{1}{2},$$

$$y = \frac{D_2}{D} = \frac{-10/3}{-5} = \frac{2}{3}.$$

Thus, $x = \frac{1}{2}$; $y = \frac{2}{3}$.

2. **क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।**

Solve the equations using Cramer's rule.

$3x - 4 = 4y$; $2x - 12y = -72$

⇨ Here, given equations are:

$3x - 4y = 4$ and $x - 6y = -36$

Now,

Coefficient of x	Coefficient of y	Constant
3	– 4	4
1	– 6	– 36

$$D = \begin{vmatrix} 3 & -4 \\ 1 & -6 \end{vmatrix}$$

$$= -18 + 4 = -14$$

$$D_1 = \begin{vmatrix} 4 & -4 \\ -36 & -6 \end{vmatrix}$$

$$= -24 - 144 = -168$$

$$D_2 = \begin{vmatrix} 3 & 4 \\ 1 & -36 \end{vmatrix}$$

$$= -108 - 4 = -112$$

Now using Cramer's rule, we get,

$$x = \frac{D_1}{D} = \frac{-168}{-14} = 12$$

$$y = \frac{D_2}{D} = \frac{-112}{-14} = 8$$

Thus, x = 12 and y = 8.

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3. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।

Solve the equations using Cramer's rule.

5x – 3y = 8 and 2x + 5y = 59

⇨ Here,

Coefficient of x	Coefficient of y	Constant
5	– 3	8
2	5	59

D = determinant of coefficients

$$= \begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix} = 25 + 6 = 31$$

Replacing C_1 by constants, we have,

$$D_1 = \begin{vmatrix} 8 & -3 \\ 59 & 5 \end{vmatrix} = 40 + 177 = 217$$

Replacing C_2 by constants, we have,

$$D_2 = \begin{vmatrix} 5 & 8 \\ 2 & 59 \end{vmatrix} = 295 - 16 = 279$$

Now, By using Cramer's rule,

$$x = \frac{D_1}{D} = \frac{217}{31} = 7 \text{ and } y = \frac{D_2}{D} = \frac{279}{31} = 9$$

Thus, x = 7 & y = 9

4. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।

Solve the equations using Cramer's rule.

x + 2y = 7 and 2x – y = 4

⇨ Here, $x + 2y = 7$ ....(i) and

$2x - y = 4$ ....(ii)

Coefficient of x	Coefficient of y	Constant term
1	2	7
2	– 1	4

$$\text{Now, } D = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$$= 1 \times (-1) - 2 \times 2$$

$$= -5$$

$$D_x = \begin{vmatrix} 7 & 2 \\ 4 & -1 \end{vmatrix}$$

$$= 7 \times (-1) - 4 \times 2$$

$$= -15 \text{ and}$$

$$D_y = \begin{vmatrix} 1 & 7 \\ 2 & 4 \end{vmatrix}$$

$$= 1 \times 4 - 7 \times 2$$

$$= -10$$

Using Cramer's rule,

$$x = \frac{D_x}{D} = \frac{-15}{-5} = 3 \text{ and}$$

$$y = \frac{D_y}{D} = \frac{-10}{-5} = 2$$

Thus, $x = 3$ and $y = 2$.

5. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।

Solve the equations using Cramer's rule.

3x – 2y = 1, – x + 4y = 3

⇨ Here,

Coefficient of x	Coefficient of y	Constant
3	– 2	1
– 1	4	3

The determinant of coefficients of x and y is

$$D = \begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix} = 12 - 2 = 10$$

Replacing C_1 by constants, we have,

$$D_1 = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 4 + 6 = 10$$

Replacing C_2 by constants, we have,

$$D_2 = \begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} = 9 + 1 = 10$$

By Cramer's rule,

$$x = \frac{D_1}{D} = \frac{10}{10} = 1 \text{ and } y = \frac{D_2}{D} = \frac{10}{10} = 1$$

Thus, x = 1 and y = 1.

6. यदि $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ र $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ भए $x = \frac{D_1}{D}$ र $y = \frac{D_2}{D}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, if $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ then prove that $x = \frac{D_1}{D}$ and $y = \frac{D_2}{D}$.

⇨ Here, $a_1x + b_1y = c_1$ .....(1) and $a_2x + b_2y = c_2$ ....(2).

Multiplying equation (1) by b_2 and equation (2) by b_1 then,

$b_2 \times [a_1x + b_1y = c_1] \Rightarrow a_1b_2x + b_1b_2y = b_2c_1$ ....(i)

$b_1 \times [a_2x + b_2y = c_2] \Rightarrow a_2b_1x + b_1b_2y = b_1c_2$ ....(ii)

Subtracting equation (ii) from (i),

$$a_1b_2x - a_2b_1x = b_2c_1 - b_1c_2$$

or, $(a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2$

or, $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

or, $Dx = D_1 \qquad \therefore \quad x = \frac{D_1}{D}$

Again, multiplying equation (1) by a_2 and equation (2) by a_1 then,

$$a_2 \times [a_1x + b_1y = c_1]$$

or, $a_1a_2x + b_1a_2y = a_2c_1$ ....(iii)

$$a_1 \times [a_2x + b_2y = c_2]$$

or, $a_1a_2x + a_1b_2y = a_1c_2$ ....(iv)

Subtracting equation (iv) from (iii) then,

$$b_1a_2y - a_1b_2y = a_2c_1 - a_1c_2$$

or, $(b_1a_2 - a_1b_2)y = a_2c_1 - a_1c_2$

or, $(a_1b_2 - b_1a_2)y = a_1c_2 - a_2c_1$

or, $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

or, $D\,y = D_2 \qquad \therefore \quad y = \frac{D_2}{D}$

Thus, $x = \frac{D_1}{D}$ and $y = \frac{D_2}{D}$ **Proved.**

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7. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$2x - y = 5,\ x - 2y = 1$

⇨ Here, writing the coefficients and constants in order,
We have,

Coefficient of *x*	Coefficient of *y*	Constant term
2	– 1	5
1	– 2	1

$$D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = -4 + 1 = -3$$

$$D_1 = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = -10 + 1 = -9$$

$$D_2 = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2 - 5 = -3$$

Since $D \neq 0$, by Cramer's rule, the above system of equations has the unique solution given by

$$x = \frac{D_1}{D} = \frac{-9}{-3} = 3 \quad \text{and}$$

$$y = \frac{D_2}{D} = \frac{-3}{-3} = 1$$

Thus, the required solution is (3, 1).

---

8. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$2x + 5y = 17$ and $5x - 2y = -1$

⇨ Here, $2x + 5y = 17$ and $5x - 2y = -1$
We have,

Coefficient of *x*	Coefficient of *y*	Constant term
2	5	17
5	– 2	– 1

Using Cramers rule:

$$D = \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix} = -4 - 25 = -29$$

$$D_1 = \begin{vmatrix} 17 & 5 \\ -1 & -2 \end{vmatrix} = -34 + 5 = -29$$

$$D_2 = \begin{vmatrix} 2 & 17 \\ 5 & -1 \end{vmatrix} = -2 - 85 = -87$$

We know that,

$$x = \frac{D_1}{D} = \frac{-29}{-29} = 1$$

$$y = \frac{D_2}{D} = \frac{-87}{-29} = 3$$

Thus, x = 1 and y = 3.

---

9. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$3x + 4y = -2$ and $5x - 7y = 24$

⇨ Here, $3x + 4y = -2$ and $5x - 7y = 24$

Coefficient of *x*	Coefficient of *y*	Constant term
3	4	– 2
5	– 7	24

$$D = \begin{vmatrix} 3 & 4 \\ 5 & -7 \end{vmatrix} = -21 - 20 = -41$$

$$D_1 = \begin{vmatrix} -2 & 4 \\ 24 & -7 \end{vmatrix} = +14 - 96 = -82$$

$$D_2 = \begin{vmatrix} 3 & -2 \\ 5 & 24 \end{vmatrix} = 72 + 10 = 82$$

We know that, $x = \frac{D_1}{D} = \frac{-82}{-41} = 2$

$$y = \frac{D_2}{D} = \frac{82}{-41} = -2$$

Thus, x = 2 and y = – 2.

---

10. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$-2x + 4y = 3,\ 3x - 7y = 1$

⇨ Here, writing the coefficients and constants in order;

Coefficient of *x*	Coefficient of *y*	Constant term
– 2	4	3
3	– 7	1

Now, $D = \begin{vmatrix} -2 & 4 \\ 3 & -7 \end{vmatrix} = 14 - 12 = 2$

$$D_1 = \begin{vmatrix} 3 & 4 \\ 1 & -7 \end{vmatrix} = -21 - 4 = -25$$

$$D_2 = \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = -2 - 9 = -11$$

Since $D \neq 0$, by Cramer's rule, above system of equations has unique solution given by;

$$x = \frac{D_1}{D} = \frac{-25}{2} \text{ and } y = \frac{D_2}{D} = \frac{-11}{2}$$

Thus, the required solution is $\left(-\frac{25}{2}, \frac{-11}{2}\right)$.

---

11. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$5x - 4y = -3$ and $7x + 2y = 49$

⇨ Here, $5x - 4y = -3$ and $7x + 2y = 49$
Using Cramers rule,

Coefficient of *x*	Coefficient of *y*	Constant term
5	– 4	– 3
7	2	49

$$D = \begin{vmatrix} 5 & -4 \\ 7 & 2 \end{vmatrix} = 10 + 28 = 38$$

$$D_1 = \begin{vmatrix} -3 & -4 \\ 49 & 2 \end{vmatrix} = -6 + 196 = 190$$

$$D_2 = \begin{vmatrix} 5 & -3 \\ 7 & 49 \end{vmatrix} = 245 + 21 = 266$$

We know that, $x = \frac{D_1}{D} = \frac{190}{38} = 5$

$$y = \frac{D_2}{D} = \frac{266}{38} = 7$$

Thus, x = 5 and y = 7.

---

12. क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।
Solve the equations using Cramer's rule.
$2x + 5y = 24$ and $2x + 3y = 12$

⇨ Here, writing the coefficients and constant in order, we have,

Coefficient of *x*	Coefficient of *y*	Constant term
2	5	24
2	3	12

Now, $D = \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = 6 - 10 = -4$

$$D_1 = \begin{vmatrix} 24 & 5 \\ 12 & 3 \end{vmatrix} = 72 - 60 = 12$$

$$D_2 = \begin{vmatrix} 2 & 24 \\ 2 & 12 \end{vmatrix} = 24 - 48 = -24$$

Since $D \neq 0$, by Cramer's rule, above system of equations has unique solution given by

$$x = \frac{D_1}{D} = \frac{12}{-4} = -3 \quad \text{and } y = \frac{D_2}{D} = \frac{-24}{-4} = 6$$

Thus, the required solution is (– 3, 6).

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13. **क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।**
Solve the equations using Cramer's rule.
$\frac{2x}{3} + y = 16$ **and** $x + \frac{y}{4} = 14$

⇨ Here, $\frac{2x}{3} + y = 16 \Rightarrow 2x + 3y = 48$

And $x + \frac{y}{4} = 14 \Rightarrow 4x + y = 56$

Coefficient of *x*	Coefficient of *y*	Constant term
2	3	48
4	1	56

By the Cramers rule

$$D = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = 2 - 12 = -10$$

$$D_1 = \begin{vmatrix} 48 & 3 \\ 56 & 1 \end{vmatrix} = 48 - 168 = -120$$

$$D_2 = \begin{vmatrix} 2 & 48 \\ 4 & 56 \end{vmatrix} = 112 - 192 = -80$$

Now, $x = \frac{D_1}{D} = \frac{-120}{-10} = 12$ and

$y = \frac{D_2}{D} = \frac{-80}{-10} = 8$

Thus, x = 12 and y = 8

14. **क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् ।**
Solve the equations using Cramer's rule.
$3x + \frac{4}{y} = 10$ **and** $-2x + \frac{3}{y} = -1$

⇨ Here, $3x + \frac{4}{y} = 10$ and $-2x + \frac{3}{y} = -1$

Coefficient of *x*	Coefficient of $\frac{1}{y}$	Constant term
3	4	10
−2	3	−1

By the Cramer's rule method,

$$D = \begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix} = 9 + 8 = 17$$

$$D_1 = \begin{vmatrix} 10 & 4 \\ -1 & 3 \end{vmatrix} = 30 + 4 = 34$$

$$D_2 = \begin{vmatrix} 3 & 10 \\ -2 & -1 \end{vmatrix} = -3 + 20 = 17$$

We know that, $x = \frac{D_1}{D} = \frac{34}{17} = 2$

$\frac{1}{y} = \frac{D_2}{D} = \frac{17}{17} = 1$

$\therefore\ y = 1$

Thus, x = 2 and y = 1

3.4 क्रामरको नियम (CRAMER'S RULE)

EXERCISE 3.4

1. **दिइएका समीकरणहरूको क्रामर नियम (Cramer's Rule) प्रयोग गरी हल गर्नुहोस् ।**
Solve the given equations by using cramer's Rule.

(a) $4x - 3y = -1,\ 3x + 4y = 18$

⇨ Here, given equations are;
$4x - 3y = -1$ and $3x + 4y = 18$

Coefficient of x	Coefficient of y	Constant Term
4	−3	−1
3	4	18

So, $D = \begin{vmatrix} 4 & -3 \\ 3 & 4 \end{vmatrix} = 16 + 9 = 25$

$$D_x = \begin{vmatrix} -1 & -3 \\ 18 & 4 \end{vmatrix} = -4 + 54 = 50$$

$$D_y = \begin{vmatrix} 4 & -1 \\ 3 & 18 \end{vmatrix} = 72 + 3 = 75$$

By the Cramer's rule,

$x = \frac{D_x}{D} = \frac{50}{25} = 2$

$y = \frac{D_y}{D} = \frac{75}{25} = 3$

Thus, x = 2 and y = 3 is the solution.

(b) $2x - 3y = 3,\ 4x - y = 1$

⇨ Here, the given equations are;
$2x - 3y = 3,\ 4x - y = 1$

Coefficient of x	Coefficient of y	Constant Term
2	−3	3
4	−1	1

So, $D = \begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = -2 + 12 = 10$

$$D_x = \begin{vmatrix} 3 & -3 \\ 1 & -1 \end{vmatrix} = -3 + 3 = 0$$

$$D_y = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = 2 - 12 = -10$$

By the Cramer's rule,

$x = \frac{D_x}{D} = \frac{0}{10} = 0$

$y = \frac{D_y}{D} = -\frac{10}{10} = -1$

Thus, x = 0 and y = −1 is the solution.

(c) $2x - 5y = 4,\ 4x + y = 30$

⇨ Here, the given equations are;
$2x - 5y = 4,\ 4x + y = 30$

Coefficient of x	Coefficient of y	Constant Term
2	−5	4
4	1	30

So, $D = \begin{vmatrix} 2 & -5 \\ 4 & 1 \end{vmatrix} = 2 + 20 = 22$

$$D_x = \begin{vmatrix} 4 & -5 \\ 30 & 1 \end{vmatrix} = 4 + 150 = 154$$

$$D_y = \begin{vmatrix} 2 & 4 \\ 4 & 30 \end{vmatrix} = 60 - 16 = 44$$

By the Cramer's rule,

$x = \frac{D_x}{D} = \frac{154}{22} = 7$

$y = \frac{D_y}{D} = \frac{44}{22} = 2$

Thus, x = 7 and y = 2 is the solution.

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(d) $3x + 2y + 9 = 0,\ 2x - 3y = -6$

⇨ Here, the given equations are;

$3x + 2y + 9 = 0,\ 2x - 3y = -6$

Coefficient of x	Coefficient of y	Constant Term
3	2	– 9
2	– 3	– 6

So, $D = \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -9 - 4 = -13$

$D_x = \begin{vmatrix} -9 & 2 \\ -6 & -3 \end{vmatrix} = 27 + 12 = 39$

$D_y = \begin{vmatrix} 3 & -9 \\ 2 & -6 \end{vmatrix} = -18 + 18 = 0$

By the Cramer's rule,

$x = \frac{D_x}{D} = \frac{39}{-13} = -3$ and $y = \frac{D_y}{D} = \frac{0}{-13} = 0$

Thus, x = – 3 and y = 0 is the solution.

(e) $2(x - 1) = y$ and $3(x - 1) = -4y$

⇨ Here, $2(x - 1) = y$ and $3(x - 1) = -4y$

or, $2x - 2 = y$ and $3x - 3 = -4y$

$\therefore$ $2x - y = 2$ and $3x + 4y = 3$

Coefficient of x	Coefficient of y	Constant Term
2	– 1	2
3	4	3

So, $D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11$

$D_x = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11$

$D_y = \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix} = 6 - 6 = 0$

By the Cramer's rule,

$x = \frac{D_x}{D} = \frac{11}{11} = 1$ and $y = \frac{D_y}{D} = \frac{0}{11} = 0$

Thus, x = 1 and y = 0 is the solution.

(f) $8x + 11 = 3y - 20,\ 6y - 15 = -2x + 11$

⇨ Here, $8x + 11 = 3y - 20$

or, $8x - 3y = -31$

And, $6y - 15 = -2x + 11$

or, $2x + 6y = 26$

Coefficient of x	Coefficient of y	Constant Term
8	– 3	– 31
2	6	26

So, $D = \begin{vmatrix} 8 & -3 \\ 2 & 6 \end{vmatrix} = 48 + 6 = 54$

$D_x = \begin{vmatrix} -31 & -3 \\ 26 & 6 \end{vmatrix} = -186 + 78 = -108$

$D_y = \begin{vmatrix} 8 & -31 \\ 2 & 26 \end{vmatrix} = 208 + 62 = 270$

By the Cramer's rule,

$x = \frac{D_x}{D} = -\frac{108}{54} = -2$

$y = \frac{D_y}{D} = \frac{270}{54} = 5$

Thus, x = – 2 and y = 5 is the solution.

2 दिइएका समीकरणहरूको क्रामर नियम (Cramer's Rule) प्रयोग गरी हल गर्नुहोस् :

Solve the given equations by using cramer's Rule.

(a) $\frac{x}{7} - \frac{2y}{7} = -1,\ \frac{3x}{5} + \frac{7y}{5} = 1$

⇨ Here, $\frac{x}{7} - \frac{2y}{7} = -1$ and $\frac{3x}{5} + \frac{7y}{5} = 1$

or, $\frac{x - 2y}{7} = -1$ and $\frac{3x + 7y}{5} = 1$

$\therefore$ $x - 2y = -7$ and $3x + 7y = 5$

Coefficient of x	Coefficient of y	Constant Term
1	– 2	– 7
3	7	5

So, $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix} = 7 + 6 = 13$

$D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix} = -49 + 10 = -39$

$D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix} = 5 + 21 = 26$

By the Cramer's rule,

$x = \frac{D_x}{D} = -\frac{39}{13} = -3$ and $y = \frac{D_y}{D} = \frac{26}{13} = 2$

Thus, x = – 3 and y = 2 is the solution.

(b) $\frac{4}{x} + \frac{5}{y} = 58,\ \frac{7}{x} + \frac{3}{y} = 67$

⇨ Here, equations are; $\frac{4}{x} + \frac{5}{y} = 58$ and $\frac{7}{x} + \frac{3}{y} = 67$

Let $\frac{1}{x} = u$; $\frac{1}{y} = v$, then the above equations become;

$4u + 5v = 58$ and $7u + 3v = 67$

Coefficient of u	Coefficient of v	Constant
4	5	58
7	3	67

$D = \begin{vmatrix} 4 & 5 \\ 7 & 3 \end{vmatrix} = 12 - 35 = -23$

$D_1 = \begin{vmatrix} 58 & 5 \\ 67 & 3 \end{vmatrix} = 174 - 335 = -161$

$D_2 = \begin{vmatrix} 4 & 58 \\ 7 & 67 \end{vmatrix} = 268 - 406 = -138$

Using Cramer's rule,

$u = \frac{D_1}{D} = \frac{-161}{-23} = 7$ and $v = \frac{D_2}{D} = \frac{-138}{-23} = 6$

But $\frac{1}{x} = u$ so, $\frac{1}{x} = 7$

or, $7x = 1$

$\therefore$ $x = \frac{1}{7}$

and $\frac{1}{y} = v$

So, $\frac{1}{y} = 6$

or, $1 = 6y$

$\therefore$ $y = \frac{1}{6}$

Thus, $x = \frac{1}{7}$; $y = \frac{1}{6}$

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(c) $\frac{x+1}{8}=\frac{y+3}{5}=\frac{x-y}{4}$

⇨ Here, $\frac{x+1}{8}=\frac{y+3}{5}=\frac{x-y}{4}$

Taking first two ratios then,

$$\frac{x+1}{8}=\frac{y+3}{5}$$

or, $5x + 5 = 8y + 24$

or, $5x - 8y = 19$ .... (i)

Taking last two ratios then,

$$\frac{y+3}{5}=\frac{x-y}{4}$$

or, $5x - 5y = 4y + 12$

or, $5x - 9y = 12$ .... (ii)

Coefficient of x	Coefficient of y	Constant Term
5	– 8	19
5	– 9	12

So, $D = \begin{vmatrix} 5 & -8 \\ 5 & -9 \end{vmatrix} = -45 + 40 = -5$

$D_x = \begin{vmatrix} 19 & -8 \\ 12 & -9 \end{vmatrix} = -171 + 96 = -75$

$D_y = \begin{vmatrix} 5 & 19 \\ 5 & 12 \end{vmatrix} = 60 - 95 = -35$

By the Cramer's rule,

$$x = \frac{D_x}{D} = -\frac{-75}{-5} = 15$$

$$y = \frac{D_y}{D} = \frac{-35}{-5} = 7$$

Thus, x = 15 and y = 7 is the solution.

(d) $\frac{2}{3}x + y = 1, \frac{1}{2}x + y = \frac{1}{2}$

⇨ Here, $\frac{2}{3}x + y = 1$ and $\frac{1}{2}x + y = \frac{1}{2}$

or, $\frac{2x+3y}{3} = 1$ and $\frac{x+2y}{2} = \frac{1}{2}$

or, $2x + 3y = 3$ and $x + 2y = 1$

Coefficient of x	Coefficient of y	Constant Term
2	3	3
1	2	1

So, $D = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$

$D_x = \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} = 6 - 3 = 3$

$D_y = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2 - 3 = -1$

By the Cramer's rule,

$$x = \frac{D_x}{D} = \frac{3}{1} = 3$$

$$y = \frac{D_y}{D} = -\frac{1}{1} = -1$$

Thus, x = 3 and y = – 1 is the solution.

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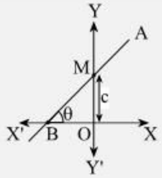
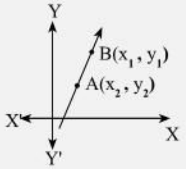
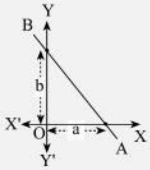
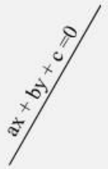
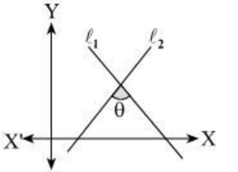
निर्देशाङ्क ज्यामिति (Co-ordinate Geometry)

1. दुई सीधा रेखाहरूबीचको कोण
Angle between Two Straight Lines

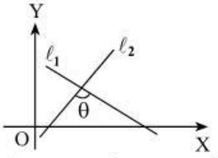
KEY POINTS

रेखा AB को Line AB				
झुकाव (Slope)	$m = \tan\theta$	$m = \frac{y_2 - y_1}{x_2 - x_1}$	$m = -\frac{b}{a}$	$m = -\frac{a}{b} = -\frac{\text{x-को गुणांक}}{\text{y-को गुणांक}}$
समीकरण Equation	$y = mx + c$	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$	$\frac{x}{a} + \frac{y}{b} = 1$	$ax + by + c = 0$

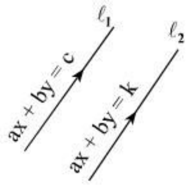
	रेखाहरूको समीकरण Equation of Lines	**रेखाहरू बीचको कोण** Angle between Lines	**रेखाहरूको अवस्था** (Condition of Lines) **समानान्तर हुने** being parallel	**लम्ब हुने** being perpendicular
1.	$y = m_1x + c_1$ $y = m_2x + c_2$	$\theta = \tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$	$m_1 = m_2$	$m_1.\ m_2 = -1$
2.	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\theta = \tan^{-1}\left(\pm\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right)$	$a_1b_2 = a_2b_1$	$a_1a_2 + b_1b_2 = 0$

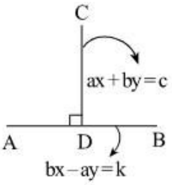


$\ell_1 \longrightarrow y = m_1x + c_1$
$\ell_2 \longrightarrow y = m_2x + c_2$



$\ell_1 \longrightarrow a_1x + b_1y + c_1 = 0$
$\ell_2 \longrightarrow a_2x + b_2y + c_2 = 0$





रेखाको समीकरण Equation of Line		**झुकाव** slope	**समानान्तर हुने रेखाको समीकरण** Equation of parallel line	**लम्ब हुने रेखाको समीकरण** Equation of perpendicular line
1.	$ax + by = c$	$m = -\frac{a}{b}$	$ax + by = k$	$bx - ay = k$
eg	$6x + 3y = 8$	$m = -\frac{6}{3} = -2$	$6x + 3y = k$	$3x - 6y = k$
2.	$y = mx + c$	m	$y = mx + k$	$y = -\frac{1}{m}x + k$
eg	$y = 5x + 4$	5	$y = 5x + k$	$y = -\frac{1}{5}x + k$

Note : जहाँ k कुनै अचल सङ्ख्या हो । (Where k, is any constant.)

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QUESTIONS FROM SEE EXERCISE 1

A. VERY SHORT QUESTIONS

1. झुकावको आधारमा तलका पदहरूको परिभाषा दिनुहोस् । (Define the following terms on the basis of slope.)

   A. **समानान्तर रेखाहरू (Parallel lines)**

   ⇨ Here, the lines having equal slopes are called parallel lines.

   B. **लम्ब रेखाहरू (Perpendicular lines)**

   ⇨ Here, the lines are said to be perpendicular to each other if the product of their slopes is – 1.

2. यदि दुई रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ आपसमा समानान्तर भए यिनीहरूको झुकावको सम्बन्ध लेख्नुहोस् ।
   If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel to each other, write the relation between their slopes.

⇨ Here, if $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel then $m_1 = m_2$.

3. $ax + by + c = 0$ सँग समानान्तर हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
   Find the equation of straight line parallel to $ax + by + c = 0$.

⇨ Here, the equation of parallel line to $ax + by + c = 0$ is $ax + by = k$ where k is any number.

4. झुकाव $-\frac{1}{m}$ भएको रेखासँग लम्ब हुने रेखाको झुकाव कति हुन्छ ?

   What is the slope of a line which is perpendicular to the line having slope $-\frac{1}{m}$?

⇨ Here, the slope of line is m which is perpendicular to the line having slope $-\frac{1}{m}$.

5. रेखाहरू $x = 0$ र $y = 0$ बीचको कोण कति हुन्छ ? **(What is the angle between the lines $x = 0$ and $y = 0$?)**

⇨ Here, the angle between $x = 0$ and $y = 0$ is 90°.

6. एउटा रेखा (ℓ_1) को झुकाव $\frac{3}{4}$ छ । रेखा (ℓ_1) सँग समानान्तर हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।

   The slope of a line (ℓ_1) is $\frac{3}{4}$. Find the slope of another line parallel to the line (ℓ_1).

⇨ Here, the slope of line parallel to ℓ_1 is also $\frac{3}{4}$.

7. रेखा $2x – 3y = 5$ को झुकाव $\frac{2}{3}$ छ । रेखा $2x – 3y = 5$ सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।

   The slope of a line $2x – 3y = 5$ is $\frac{2}{3}$. Find the slope of line perpendicular to $2x – 3y = 5$.

⇨ Here, the slope of line perpendicular to the given line $= -\frac{1}{m_1} = -\frac{1}{\frac{2}{3}} = -\frac{3}{2}$

8. यदि दुई सिधा रेखाहरूका झुकावहरू क्रमशः m_1 र m_2 छन् र तिनीहरू बिचको कोण θ भए tan θ को मान पत्ता लगाउने सुत्र लेख्नुहोस् ।
   If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for tan θ. **[SEE MODEL 2076]**

⇨ Here, the formula to calculate tan θ is; $\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$.

B. SHORT QUESTIONS

MODEL 1

1. यदि दुई सीधा रेखाहरूको झुकावहरू क्रमशः $\sqrt{3}$ र $\frac{1}{\sqrt{3}}$ भए तिनीहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।

   If the slopes of two straight lines are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ respectively then find the acute angle between them. **[2075 R']**

⇨ Here, let $m_1 = \sqrt{3}$ and $m_2 = \frac{1}{\sqrt{3}}$

We know that, angle between two lines is given by, $\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan\theta = \pm\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \pm\left(\frac{\frac{3-1}{\sqrt{3}}}{2}\right) = \pm\frac{2}{2\sqrt{3}} = \pm\frac{1}{\sqrt{3}}$

or, $\tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$ [for acute angle]

∴ θ = 30°

Thus, the acute angle between the lines is 30°.

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2. रेखाहरू MN र PQ का झुकावहरू क्रमशः 2 र – 3 छन् भने ती रेखाहरूबीचको अधिककोण पत्ता लगाउनुहोस् ।
The slopes of the lines MN and PQ are 2 and – 3 respectively. Find the obtuse angle between the lines. **[2074 S']**

⇨ Here, $m_1 = 2$ and $m_2 = -3$

Let θ be angle between the lines then,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$
$$= \pm\frac{2+3}{1-2\times 3}$$
$$= \pm\frac{5}{-5} = \pm 1$$

or, $\tan\theta = \tan 45° \text{ or } \tan(180° - 45°)$

∴ $\theta = 45° \text{ or } 135°$.

Thus, the obtuse angle is 135°.

3. यदि रेखाहरू AB र CD का झुकावहरू क्रमशः 3 र – 2 भए तिनीहरू बीचको न्यूनकोण पत्ता लगाउनुहोस् ।
If the slopes of the lines AB and CD are 3 and – 2 respectively, find the acute angle between them.[2071 R]

⇨ Here, slope of AB $(m_1) = 3$ and
slope of CD $(m_2) = -2$

Let θ be angle between AB and CD then,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

or, $\tan\theta = \pm\dfrac{3+2}{1+3\times(-2)}$

or, $\tan\theta = \pm\dfrac{5}{-5} = \pm 1$

or, $\tan\theta = \tan 45°$ [Being acute angle.]

∴ $\theta = 45°$

Thus, the acute angle between the lines is 45°.

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4. झुकावहरू क्रमशः $-\sqrt{3}$ र $\sqrt{3}$ भएका रेखाहरूबीचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines having slopes $-\sqrt{3}$ and $\sqrt{3}$ respectively. **[2071 S]**

⇨ Here, Slope $(m_1) = -\sqrt{3}$ and Slope $(m_2) = \sqrt{3}$

Let θ be an acute angle then,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

or, $\tan\theta = \pm\dfrac{-\sqrt{3}-\sqrt{3}}{1+(-\sqrt{3})\sqrt{3}}$

or, $\tan\theta = \pm\dfrac{-2\sqrt{3}}{1-3}$

or, $\tan\theta = \pm\sqrt{3}$

or, $\tan\theta = \tan 60°$ [since θ is acute angle.]

∴ $\theta = 60°$

Thus, the required acute angle is 60°.

5. रेखाहरू 2x – y + 4 = 0 र 3x + y + 3 = 0 बीचको अधिककोण निकाल्नुहोस् ।
Find the obtuse angle between two lines 2x – y + 4 = 0 and 3x + y + 3 = 0. **[2065 R]**

⇨ Here, the given equations are;
$2x - y + 4 = 0$ .... (i) and
$3x + y + 3 = 0$ .... (ii)

Slope of line (i); $m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2}{-1} = 2$

Slope of line (ii); $m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{3}{1} = -3$

Let θ be angle between given lines.

Then, $\theta = \tan^{-1}\left(\pm\dfrac{m_1 - m_2}{1 + m_1m_2}\right)$

$= \tan^{-1}\left(\pm\dfrac{2+3}{1-6}\right)$

$= \tan^{-1}\left(\pm\dfrac{5}{-5}\right)$

∴ $\theta = \tan^{-1}(\pm 1) = 45° \text{ or } 135°$

Thus, the obtuse angle is 135°.

6. समीकरणहरू $\sqrt{3}$x – y + 6 = 0 र y + 3 = 0 भएका सीधारेखाहरू बीचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between two straight lines having equations $\sqrt{3}$x – y + 6 = 0 and y + 3 = 0. **[2066 S]**

⇨ Here, given equations are;
$\sqrt{3}x - y + 6 = 0$ .... (i) and $y + 3 = 0$ .... (ii)

Let m_1 and m_2 be the slopes of line (i) & line (ii) then, $m_1 = \sqrt{3}$ and $m_2 = \dfrac{0}{1} = 0$

Now, we have, $\tan\theta = \pm\dfrac{m_1 - m_2}{1 + m_1m_2} = \pm\dfrac{\sqrt{3}-0}{1+\sqrt{3}\times 0}$

or, $\tan\theta = \pm\sqrt{3} = \tan 60° \text{ or } \tan 120°$

∴ $\theta = 60° \text{ or } 120°$

Thus, the acute angle is 60°.

MODEL 2

7. समीकरणहरू 3x + 4y + 5 = 0 र 6x + 8y + 7 = 0 का झुकाव पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् ।
Find the slopes of two straight lines 3x + 4y + 5 = 0 and 6x + 8y + 7 = 0 and write the relationship between them.
[SEE MODEL 2076]

⇨ Here, given equations are; $3x + 4y + 5 = 0$..........(i) and $6x + 8y + 7 = 0$ ....(ii)

Now, slope of line (i) is, $m_1 = \dfrac{-\text{Coeff. of } x}{\text{Coeff. of } y}$

∴ $m_1 = \dfrac{-3}{4}$

Similarly, slope of line (ii) is, $m_2 = \dfrac{-6}{8} = \dfrac{-3}{4}$

Since $m_1 = m_2 = \dfrac{-3}{4}$ So, the lines are parallel to each other.

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8. यदि सीधा रेखाहरू $lx + my + c = 0$ र $px + qy + r = 0$ एक आपसमा समानान्तर भए प्रमाणित गर्नुहोस् : $lq = mp$.
If the straight lines $lx + my + c = 0$ and $px + qy + r = 0$ are parallel to each other then prove that: $lq = mp$. [2074 R']

⇨ Here, given lines are $\ell x + my + c = 0$ … …(i) and $px + qy + r = 0$ … …(ii)

Slope of line (i): $m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = \frac{-\ell}{m}$

Slope of line (ii): $m_2 = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = \frac{-p}{q}$

Since the lines are parallel so, $m_1 = m_2$

i.e. $\frac{-\ell}{m} = \frac{-p}{q}$

$\therefore \quad \ell q = mp$

Thus, $\ell q = mp$ completes the proof.

9. बिन्दुहरू (3, – 4) र (– 2, 6) भएर जाने सीधारेखा र समीकरण $2x + y + 3 = 0$ भएको सीधारेखा समानान्तर हुन्छन् भनी प्रमाणित गर्नुहोस् ।
Prove that straight line passing through the points (3, – 4) and (– 2, 6) and the straight line having equation $2x + y + 3 = 0$ are parallel. [2073 R]

⇨ Here, slope of line passing through (3, – 4) & (– 2, 6).

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 + 4}{-2 - 3} = \frac{10}{-5} = -2$$

Slope of line having equation $2x + y + 3 = 0$

$$m_2 = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{2}{1} = -2$$

Since the slopes of the lines are equal so they are parallel.

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10. यदि रेखाहरू $3x + my = 5$ र $\frac{x}{2} + \frac{y}{3} = 1$ एक आपसमा समानान्तर छन् भने m को मान पत्ता लगाउनुहोस् ।
If the lines $3x + my = 5$ and $\frac{x}{2} + \frac{y}{3} = 1$ are parallel to each other, find the value of m. [2073 R']

⇨ Here, given equation of lines are;

$3x + my = 5$ …. (i) and $\frac{x}{2} + \frac{y}{3} = 1$ ………. (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$

$\therefore \quad m_1 = -\frac{3}{m}$

Slope of line (ii);

$$m_2 = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{\frac{1}{2}}{\frac{1}{3}} = -\frac{1}{2} \times \frac{3}{1} = -\frac{3}{2}$$

Since, the lines are parallel,
So, $m_1 = m_2$
or, $-\frac{3}{m} = -\frac{3}{2}$
or, $3m = 6$
$\therefore \quad m = 2$
Thus, the value of m is 2.

11. दुई सरल रेखाहरू $P x + 3y - 12 = 0$ र $4y - 3x + 7 = 0$ समानान्तर भए P को मान पत्ता लगाउनुहोस् ।
If the straight lines $Px + 3y - 12 = 0$ and $4y - 3x + 7 = 0$ are parallel to each other, find the value of P. [2071 R']

⇨ Here, given equation of lines are;

$Px + 3y - 12 = 0$ ……… (i) and
$4y - 3x + 7 = 0$ ………. (ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y} = -\frac{P}{3}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y} = -\frac{-3}{4} = \frac{3}{4}$$

Since, the lines are parallel.
So, $m_1 = m_2$

i.e. $-\frac{P}{3} = \frac{3}{4}$

$\therefore \quad P = -\frac{9}{4}$

Thus, the value of P is $-\frac{9}{4}$.

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12. $2x - 8y + 6 = 0$ र $3x - 12y - 4 = 0$ रेखाहरू समानान्तर छन् भनी देखाउनुहोस् ।
Show that the lines $2x - 8y + 6 = 0$ and $3x - 12y - 4 = 0$ are parallel. [2066 R']

⇨ Here, the given lines are; $2x - 8y + 6 = 0$ ……….. (i) and $3x - 12y - 4 = 0$ ………. (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff of } x}{\text{coeff. of } y} = -\frac{2}{-8} = \frac{1}{4}$

Slope of line (ii); $m_2 = -\frac{\text{coeff of } x}{\text{coeff. of } y} = -\frac{3}{-12} = \frac{1}{4}$

Where, $m_1 = m_2$ shows that the lines are parallel.

13. समीकरणहरू $3x + 5y = 4$ र $6x + 10y = 3$ भएका रेखाहरू समानान्तर हुन्छन् भनी देखाउनुहोस् ।
Prove that the lines with equations $3x + 5y = 4$ and $6x + 10y = 3$ are parallel to each other. [2067 R']

⇨ Here, the given lines are: $3x + 5y = 4$ ……… (i) and $6x + 10y = 3$ ………. (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff of } x}{\text{coeff. of } y} = \frac{-3}{5}$

Slope of line (ii); $m_2 = -\frac{\text{coeff of } x}{\text{coeff. of } y} = \frac{-3}{5}$

Where, $m_1 = m_2$ shows that lines are parallel.

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14. यदि बिन्दुहरू (3, –4) र (–2, a) भएर जाने रेखा समीकरण $y + 2x + 3 = 0$ भएको रेखासँग समानान्तर हुन्छ भने a को मान पत्ता लगाउनुहोस् ।
If the line passing through (3, – 4) and (– 2, a) is parallel to the line given by the equation $y + 2x + 3 = 0$, find the value of a. [2065 M]

⇨ Here, where slope of the line through (3,– 4), (– 2, a) is; $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a + 4}{-5}$

Slope of the line $y + 2x + 3 = 0$ is; $m_2 = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -2$

Comparing slopes, $m_1 = m_2$ or, $\frac{a + 4}{-5} = -2$

or, $a + 4 = 10$ $\quad\therefore\quad a = 6$

Thus, the value of a is 6.

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15. बिन्दुहरू (a, 4) र (2, – 5) जोड्ने सीधा रेखा 2x – 3y = 12 समीकरण हुने सीधा रेखासँग समानान्तर भए a को मान पत्ता लगाउनुहोस् । **[2068 R']**

If the line joining the points (a, 4) and (2, –5) is parallel to the line with given equation 2x – 3y = 12, find the value of a.

⇨ Here, slope of line joining the points (a, 4) and (2, – 5)

So, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-4}{2-a} = \frac{-9}{2-a}$

Slope of the line 2x – 3y = 12, $(m_2) = -\frac{2}{-3} = \frac{2}{3}$

Where, two lines are parallel. So, $m_1 = m_2$

or, $\frac{-9}{2-a} = \frac{2}{3}$

or, $-27 = 4 - 2a$

or, $2a = 4 + 27$

$\therefore \quad a = \frac{31}{2}$

Thus, the required value of a is $\frac{31}{2}$.

MODEL 3

16. सीधारेखा 3x – 4y = 10 सँग लम्ब हुने रेखाको झुकाव निकाल्नुहोस् ।

Find the slope of the line perpendicular to the straight line 3x – 4y = 10. **[2072 R]**

⇨ Here, given equation of line is 3x – 4y = 10 ……. (i)

Slope of equation (i) is;

$$m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = -\frac{3}{-4} = \frac{3}{4}$$

Slope of perpendicular line to the line (i)

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$$

Thus, the slope of required line is $-\frac{4}{3}$.

17. रेखा 3x + 2y + 4 = 0 सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।

Find the slope of the line which is perpendicular to the line 3x + 2y + 4 = 0. **[2067 S]**

⇨ Here, given equation of line is 3x + 2y + 4 = 0

Slope of this line $(m_1) = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{3}{2}$

We know that, Condition for perpendicularity is;

$m_1 \times m_2 = -1$

or, $-\frac{3}{2} \times m_2 = -1$

$\therefore \quad m_2 = \frac{2}{3}$

Thus, required value of slope of second line is $\frac{2}{3}$.

18. सीधा रेखा 4x + 3y = 12 सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।

Find the slope of the straight line perpendicular to 4x + 3y = 12. **[2064 R']**

⇨ Here, given equation of straight line is 4x + 3y = 12 .... (i)

$\therefore$ Slope $(m_1) = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{4}{3}$

Slope of the straight line ⊥r to line (i) is;

$\therefore$ Slope $(m_2) = -\frac{1}{m_1} = -\frac{1}{-\frac{4}{3}} = \frac{3}{4}$

Thus, the required slope $(m_2) = \frac{3}{4}$.

MODEL 4

19. रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ बिचको कोण पत्ता लगाउने सूत्र लेख्नुहोस् । साथै ती रेखाहरू आपसमा लम्ब हुने अवस्था पनि लेख्नुहोस् ।

Write down the formula to find the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Also write the condition of perpendicularity of these lines. **[2075 R, 2075 R_2]**

⇨ Here, given lines are; $y = m_1x + c_1$ and $y = m_2x + c_2$.

Angle between the given lines is given by;

$$\theta = \tan^{-1}\left(\frac{\pm\, m_1 - m_2}{1 + m_1 m_2}\right);$$

Where m_1 and m_2 are the slopes of 1st and 2nd line respectively.

The condition of perpendicularity is : $m_1 \times m_2 = -1$.

20. रेखाहरू $x - 3y - 5 = 0$ र $3x + y - 15 = 0$ एक आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।

Show that the lines $x - 3y - 5 = 0$ and $3x + y - 15 = 0$ are perpendicular to each other. **[2075 R_2']**

⇨ Here, given lines are,

$x - 3y - 5 = 0$ ....(i) and

$3x + y - 15 = 0$ .... (ii)

Slope of line (i) is $(m_1) = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = \frac{-1}{-3} = \frac{1}{3}$

Slope of line (ii) is $(m_2) = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = \frac{-3}{1}$

Now, $m_1 \times m_2 = \frac{1}{3} \times \frac{-3}{1} = -1$

Since the product of their slopes is – 1

So, the lines are perpendicular.

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21. यदि रेखाहरू $l_1x + m_1y + n_1 = 0$ र $l_2x + m_2y + n_2 = 0$ एक आपसमा लम्ब भए $l_1l_2 + m_1m_2 = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ are perpendicular to each other, prove that $l_1l_2 + m_1m_2 = 0$. [2074 R]

⇨ Here, given equation of lines are;

$\ell_1x + m_1y + n_1 = 0$ ....(i) and

$\ell_2x + m_2y + n_2 = 0$ ....(ii)

Slope of line (i); $m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{\ell_1}{m_1}$

Slope of line (ii); $m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{\ell_2}{m_2}$

Since, the lines are perpendicular,

So, $m_1 \times m_2 = -1$

or, $-\dfrac{\ell_1}{m_1} \times (-)\dfrac{\ell_2}{m_2} = -1$

or, $\dfrac{\ell_1\ell_2}{m_1m_2} = -1$

or, $\ell_1\ell_2 = -m_1m_2$

$\therefore$ $\ell_1\ell_2 + m_1m_2 = 0$

Thus, the required condition is $\ell_1\ell_2 + m_1m_2 = 0$.

Proved.

22. समीकरणहरू $2x - y + 1 = 0$ र $x + my - 7 = 0$ भएका दुई रेखाहरू आपसमा लम्ब छन् भने m को मान पत्ता लगाउनुहोस् ।

The two straight lines having equations $2x - y + 1 = 0$ and $x + my - 7 = 0$ are perpendicular to each other. Find the value of m. [2074 S]

⇨ Here, the given lines are;

$2x - y + 1 = 0$ ....(i) and

$x + my - 7 = 0$ .....(ii)

Slope of line (i); $m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2}{-1} = 2$

Slope of line (ii); $m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{1}{m}$

Since the lines are perpendicular so,

$m_1 \times m_2 = -1$

i.e. $2 \times \left(-\dfrac{1}{m}\right) = -1$

$\therefore$ $m = 2$

Thus, the value of m is 2.

23. समीकरण $px + qy + r = 0$ र $qx - py + s = 0$ भएका सीधा रेखाहरू आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the straight lines having equations $px + qy + r = 0$ and $qx - py + s = 0$ are perpendicular to each other. [2073 S]

⇨ Here, $px + qy + r = 0$ ....(i) and

$qx - py + s = 0$ ....(ii)

Slope of line (i); $m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{p}{q}$

Slope of line (ii); $m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{q}{-p} = \dfrac{q}{p}$

Now, $m_1 \times m_2 = -\dfrac{p}{q} \times \dfrac{q}{p} = -1$

Thus, $m_1 \times m_2 = -1$ proves the lines are perpendicular.

24. समीकरणहरू $(2 + \sqrt{3})x - y + 1 = 0$ र $(2 - \sqrt{3})x + y = 0$ भएका दुई सीधा रेखाहरू आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the two straight lines having equations $(2 + \sqrt{3})x - y + 1 = 0$ and $(2 - \sqrt{3})x + y = 0$ are perpendicular to each other.

⇨ Here, given equation of lines are;

$(2 + \sqrt{3})x - y + 1 = 0$ ....(i) and

$(2 - \sqrt{3})x + y = 0$ ....(ii)

Slope of line (i);

$m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2 + \sqrt{3}}{-1} = 2 + \sqrt{3}$

Slope of line (ii);

$m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2 - \sqrt{3}}{1} = \sqrt{3} - 2$

Now,

$m_1 \times m_2 = (2 + \sqrt{3})\,(\sqrt{3} - 2) = (\sqrt{3})^2 - 2^2 = 3 - 4 = -1$

Thus, $m_1 \times m_2 = -1$ shows that given lines are perpendicular to each other.

25. यदि बिन्दुहरू (3, – 4) र (– 2, k) भएर जाने रेखा समीकरण $5x + y = -3$ भएको रेखासँग लम्ब हुन्छ भने k को मान पत्ता लगाउनुहोस् ।

If the line passing through the points (3, – 4) and (– 2, k) is perpendicular to the line having equation $5x + y = -3$, find the value of k.

⇨ Here, slope of line passing through (3, – 4) and (– 2, k).

$m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{k + 4}{-2 - 3} = \dfrac{k + 4}{-5}$

Slope of line having equation $5x + y = -3$

$m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{5}{1} = -5$

Since, the lines are perpendicular

So, $m_1 \times m_2 = -1$

or, $-\dfrac{k + 4}{5} \times (-5) = -1$

or, $k + 4 = -1$

$\therefore$ $k = -5$

Thus, the value of k is – 5.

26. समीकरणहरू $3x + 4y = 12$ र $2x - py = 5$ भएका सीधा रेखाहरू लम्ब छन् भने p को मान पत्ता लगाउनुहोस् ।

The straight lines having equations $3x + 4y = 12$ and $2x - py = 5$ are perpendicular to each other, find the value of p. [

⇨ Here, the perpendicular lines are;

$3x + 4y = 12$ ……… (i) and $2x - py = 5$ …….. (ii)

Slope of line (i) is; $m_1 = \dfrac{-\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{3}{4}$

Slope of line (ii) is; $m_2 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2}{-p} = \dfrac{2}{p}$

Now, the condition of perpendicular is; $m_1 \times m_2 = -1$

or, $-\dfrac{3}{4} \times \dfrac{2}{p} = -1$

or, $\dfrac{3}{2p} = 1$

or, $2p = 3$

$\therefore$ $p = \dfrac{3}{2}$

Thus, the value of p is $\dfrac{3}{2}$.

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27. **यदि दुईओटा सरल रेखाहरू $px + qy + r = 0$ र $\ell x - my + n = 0$ एक आपसमा लम्ब भए, $p\ell = qm$ हुन्छ भनी देखाउनुहोस् ।**
If two straight lines $px + qy + r = 0$ and $\ell x - my + n = 0$ are perpendicular to each other, show that: $p\ell = qm$. [2070 R]

⇨ Here, given equation of lines are;

$px + qy + r = 0$ ….. (i) and

$\ell x - my + n = 0$ ……… (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{p}{q}$

Slope of line (ii); $m_2 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{\ell}{-m} = \frac{\ell}{m}$

Since the lines are perpendicular.

So, $m_1 \times m_2 = -1$

or, $-\frac{p}{q} \times \frac{\ell}{m} = -1$

$\therefore$ $p\ell = qm$ **Proved.**

28. **दुई सरल रेखाहरू $px + 3y - 12 = 0$ र $4y - 3x + 7 = 0$ एक आपसमा लम्ब भए p को मान निकाल्नुहोस् ।**
Two straight lines $px + 3y - 12 = 0$ and $4y - 3x + 7 = 0$ are perpendicular to each other, find the value of p. [2070 R']

⇨ Here, given equation of lines;

$px + 3y - 12 = 0$ …….. (i) and

$-3x + 4y + 7 = 0$ ……. (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{p}{3}$

Slope of line (ii); $m_2 - \frac{\text{coeff of x}}{\text{coeff of y}} = -\frac{-3}{4} = \frac{3}{4}$

Since the lines are perpendicular to each other.

$m_1 \times m_2 = -1$

or, $-\frac{p}{3} \times \frac{3}{4} = -1$

or, $p = 4$

Thus, the value of p is 4.

29. **यदि सीधा रेखाहरू $2x + 3y + 6 = 0$ र $ax - 5y + 20 = 0$ एक आपसमा लम्ब छन् भने a को मान पत्ता लगाउनुहोस् ।**
If the straight lines $2x + 3y + 6 = 0$ and $ax - 5y + 20 = 0$ are perpendicular to each other, find the value of a. [2068 R]

⇨ Here, given equation $2x + 3y = 6 = 0$ …….. (i)

$ax - 5y + 20 = 0$ ……… (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}}$ $\therefore$ $m_1 = -\frac{2}{3}$

Slope of line (ii); $m_2 = -\frac{\text{coeff. of x}}{\text{coeff. of y}}$ $\therefore$ $m_2 = \frac{a}{5}$

Since lines are perpendicular to each other,

So, $m_1 \times m_2 = -1$

or, $-\frac{2}{3} \times \frac{a}{5} = -1$

or, $\frac{2a}{15} = 1$

$\therefore$ $a = \frac{15}{2} = 7\frac{1}{2}$

Thus, the value of a is $7\frac{1}{2}$.

30. **यदि समीकरणहरू $3x - 4y + 7 = 0$ र $ax + 3y - 5 = 0$ ले दिने रेखाहरू आपसमा लम्ब हुन्छन् भने a को मान पत्ता लगाउनुहोस् ।**
Find the value of a if the lines given by the equations $3x - 4y + 7 = 0$ and $ax + 3y - 5 = 0$ are perpendicular to each other. [2063 S]

⇨ Here, given lines are;

$3x - 4y + 7 = 0$ .... (i) and

$ax + 3y - 5 = 0$ .... (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = \frac{-3}{-4} = \frac{3}{4}$

Slope of line (ii); $m_2 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{a}{3}$

Lines (i) and (ii) are perpendicular to each other.

So, $m_1 \times m_2 = -1$

or, $\frac{3}{4} \times (-)\frac{a}{3} = -1$

or, $\frac{a}{4} = 1$

$\therefore$ $a = 4$

Thus, required value of a is 4.

31. **यदि $kx - 3y - 31 = 0$ र $3x + 4y + 5 = 0$ आपसमा लम्ब भए k को मान निकाल्नुहोस् ।**
If $kx - 3y - 31 = 0$ & $3x + 4y + 5 = 0$ are perpendicular to each other, find the value of k. [2066 R]

⇨ Here, the given equations are;

$kx - 3y - 31 = 0$ .... (i) &

$3x + 4y + 5 = 0$ .... (ii)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{k}{-3} = \frac{k}{3}$

Slope of line (ii); $m_2 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{3}{4}$

Since lines are perpendicular so, $m_1 \times m_2 = -1$

So, $\frac{k}{3}\left(-\frac{3}{4}\right) = -1$

or, $-\frac{k}{4} = -1$

$\therefore$ $k = 4$

Thus, the value of k is 4.

32. **रेखा $kx - 3y + 6 = 0$ र बिन्दुहरू (4, 3) र (5, – 3) जोड्ने रेखा आपसमा लम्ब छन् भने k को मान कति हुन्छ ?**
For what value of k, the line $kx - 3y + 6 = 0$ is perpendicular to the line joining (4, 3) and (5, – 3) ? [2064 R]

⇨ Here, slope of line $kx - 3y + 6 = 0$ is;

$m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{k}{-3} = \frac{k}{3}$

Slope of line joining the points (4, 3) and (5, – 3) is;

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-3}{5-4} = \frac{-6}{1} = -6$

Since lines are perpendicular to each other,

$m_1 \times m_2 = -1$

or, $\frac{k}{3} \times (-)\, 6 = -1$

or, $k \times 2 = 1$

$\therefore$ $k = \frac{1}{2}$

Thus, required value of k is $\frac{1}{2}$.

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33. रेखा ax + 3y + 5 = 0 र बिन्दुहरू (4, 3) र (6, – 3) जोड्ने रेखा आपसमा लम्ब छन् भने a को मान पत्ता लगाउनुहोस् ।

The line ax + 3y + 5 = 0 is perpendicular to the line joining the points (4, 3) and (6, – 3), find the value of a. [2065 S]

⇨ Here, the given equation of line is: $ax + 3y + 5 = 0$.

Its slope $(m_1) = -\dfrac{\text{coefficient of x}}{\text{coefficient of y}} = -\dfrac{a}{3}$

The slope of line joining the points (4, 3) and (6, – 3): $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-3}{6-4} = -\dfrac{6}{2} = -3$

Since the lines are perpendicular to each other,

So, $m_1 \times m_2 = -1$

or, $-\dfrac{a}{3} \times (-3) = -1$

$\therefore\ a = -1$

Thus, the value of a is – 1.

MODEL 5

34. यदि सरल रेखा 3*x* + 5*y* = 17 ले सरल रेखा 3*x* – *ky* = 8 सँग 45° को कोण बनाउँदछ भने *k* को मान पत्ता लगाउनुहोस् ।

Find the value of *k* if the straight line 3*x* + 5*y* = 17 makes an angle of 45° with the straight line 3*x* – *ky* = 8. [2075 R₂']

⇨ Here, given lines are, $3x + 5y = 17$ .....(i) and

$3x - ky = 8$ ....(ii)

slope of line (i) is $(m_1) = -\dfrac{\text{Coeff. of x}}{\text{Coeff. of y}}$

$\therefore\ m_1 = -\dfrac{3}{5}$

slope of line (ii) is $(m_2) = -\dfrac{\text{Coeff. of x}}{\text{Coeff. of y}} = \dfrac{-3}{-k}$

$\therefore\ m_2 = \dfrac{3}{k}$

We know that, angle between two lines is given by,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\text{or, } \tan 45^\circ = \pm\frac{\left(-\frac{3}{5} - \frac{3}{k}\right)}{1 + \left(-\frac{3}{5}\right) \times \frac{3}{k}}$$

$$= \pm\left(\frac{-3k-15}{5k-9}\right)$$

$$\text{or, } 1 = \pm\left(\frac{-3k-15}{5k-9}\right)$$

Taking +ve sign,

$5k - 9 = -3k - 15$

or, $8k = -6$

$\therefore\ k = -\dfrac{6}{8} = -\dfrac{3}{4}$

Taking –ve sign,

$5k - 9 = 3k + 15$

or, $2k = 24$

$\therefore\ k = 12$

Thus, the required value of k is 12 or $-\dfrac{3}{4}$.

35. सरल रेखा 3x – my = 19 ले सरल रेखा 3x + 5y = 7 सँग 45° को कोण बनाउँछ भने m को मान निकाल्नुहोस् ।

Calculate the value of m if the line 3x – my = 19 makes an angle of 45° with the line 3x + 5y = 7. [2067 R]

⇨ Here, $3x - my = 19$ .... (i)

Slope $(m_1) = -\dfrac{3}{-m} = \dfrac{3}{m}$

or, $3x + 5y = 7$ .... (ii)

Slope $(m_2) = -\dfrac{3}{5}$

Where equation (i) & (ii) makes an angle 45°.

So, $\tan 45^\circ = \pm\dfrac{m_1 - m_2}{1 + m_1.m_2}$

$$\text{or, } 1 = \pm\frac{\frac{3}{m} + \frac{3}{5}}{1 + \frac{3}{m}\left(-\frac{3}{5}\right)}$$

$$\text{or, } 1 = \pm\frac{\frac{15+3m}{5m}}{\frac{5m-9}{5m}}$$

$\therefore\ (5m - 9) = \pm(15 + 3m)$

Taking (+ve) sign,

$5m - 3m = 15 + 9$

or, $2m = 24$

or, $m = 12$

Taking (–ve) sign,

$5m + 3m = -15 + 9$

or, $8m = -6$

or, $m = -\dfrac{6}{8} = -\dfrac{3}{4}$

Thus, the possible value of m is 12 or $-\dfrac{3}{4}$.

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C. LONG QUESTIONS

MODEL 1

1. समीकरण 2x + y – 4 = 0 भएको रेखासँग समानान्तर हुने र y-अक्षमा बनाएको खण्डको लम्बाइ 2 एकाइ भएको सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line which is parallel to the line having equation 2x + y – 4 = 0 and making an intercept of length 2 units along y-axis. [2072 R]

⇨ Here, given equation of line is $2x + y - 4 = 0$

Slope of the line $(m_1) = -\dfrac{\text{coeff. of } x}{\text{coeff of } y} = -\dfrac{2}{1} = -2$

Slope of the line parallel to the line having slope $(m_1) = -2$ is $m_2 = -2$

Passing point is (0, 2).

We have, equation of the line having slope m_2 and passing point (x_1, y_1) is;

$y - y_1 = m_2(x - x_1)$

or, $y - 2 = -2(x - 0)$

or, $y - 2 = -2x$

or, $2x + y = 2$

Thus, the required equation of line is $2x + y = 2$.

2. समीकरण x – 2y – 4 = 0 भएको रेखासँग समानान्तर हुने र बिन्दु (3, 2) बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through the point (3, 2) and parallel to the line having equation x – 2y – 4 = 0. [2072 R']

⇨ Here, given equation of line is $x - 2y = 4$ …. (i)

Equation of line parallel to line (i) is;

$x - 2y = k$ ………. (ii)

Equation (ii) passes through a point (3, 2).

So, $3 - 2 \times 2 = k$

$\therefore$ $k = -1$

Putting the value of k in (ii) then,

$x - 2y = -1$

Thus, $x - 2y + 1 = 0$ is the required equation of the line.

3. शीर्षबिन्दुहरू P(3, 3), Q(–2, –6) र R(5, –3) भएको ΔPQR को भारकेन्द्र भएर जाने अनि QR रेखासँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the line passing through the centroid of ΔPQR with vertices P(3, 3), Q(–2, –6) and R(5, –3) and parallel to the line QR. [2070 R]

⇨ Here, P(3, 3), Q(– 2, – 6) and R(5, – 3)

The centroid of ΔPQR

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{3 - 2 + 5}{3}, \frac{3 - 6 - 3}{3}\right)$$

$$= (2, -2)$$

Slope of line QR; $(m_1) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 + 6}{5 + 2} = \dfrac{3}{7}$

Slope of line parallel to QR is $\dfrac{3}{7}$.

We know that,

Equation of line is;

$y - y_1 = m(x - x_1)$

or, $y + 2 = \dfrac{3}{7}(x - 2)$

or, $7y + 14 = 3x - 6$

or, $3x - 7y = 20$

Thus, the required equation of the line is $3x - 7y = 20$.

4. A(3, 2), B(1, – 1) र C(5, – 5) त्रिभुज ABC का शीर्षबिन्दुहरू हुन् । ΔABC को गुरुत्वकेन्द्रबाट जाने र BC भुजासँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।

A(3, 2), B(1, – 1) and C(5, – 5) are the vertices of a triangle ABC. Find the equation of a straight line passing through the centroid of ΔABC and parallel to the side BC. [2070 R']

⇨ Here, vertices of ΔABC are; A(3, 2), B(1, – 1) and C(5, – 5)

Centroid of ΔABC; (x, y)

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{3 + 1 + 5}{3}, \frac{2 - 1 - 5}{3}\right)$$

$$= \left(3, -\frac{4}{3}\right)$$

Slope of line BC is; $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5 + 1}{5 - 1} = -1$

Now, equation of line passing through $\left(3, -\dfrac{4}{3}\right)$ and having slope – 1 is; $y - y_1 = m(x - x_1)$

or, $y + \dfrac{4}{3} = -1\,(x - 3)$

or, $\dfrac{3y + 4}{3} = -x + 3$

or, $3y + 4 = -3x + 9$

$\therefore$ $3x + 3y = 5$

Thus, the equation of line is $3x + 3y = 5$.

5. बिन्दु (– 2, – 3) बाट जाने र रेखा 5x + 7y = 14 सँग समानान्तर हुने सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line which is parallel to the line with equation 5x + 7y = 14 and passes through the point (– 2, –3). [2068 R']

⇨ Here, the line parallel to line $5x + 7y = 14$ is $5x + 7y = k$

Line $5x + 7y = k$ passes through the point $(-2, -3)$,

So, $5 \times (-2) + 7 \times (-3) = k$

$\therefore$ $k = -31$

Thus, required equation of the straight line is $5x + 7y + 31 = 0$.

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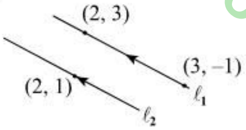
6. **बिन्दुहरू (2, 3) र (3, –1) जोड्ने रेखासँग समानान्तर भई बिन्दु (2, 1) भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the line which passes through the point (2, 1) and is parallel to the line joining the points (2, 3) and (3, – 1). **[2065 S]**

⇨ Here, the slope of line (ℓ_1) passing through (2, 3) and (3, – 1) is; $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{3-2} = -4$



So, the slope of required line (ℓ_2); $m = m_1 = -4$
And, the line passing through the point (2, 1).
We know that; Line passing through (x_1, y_1) and slope m is; $y - y_1 = m(x - x_1)$
or, $y - 1 = -4(x - 2)$
or, $y - 1 = -4x + 8$
∴ $4x + y = 9$
Thus, the equation of required line is $4x + y = 9$.

MODEL 2

7. **बिन्दु (3, 2) भएर जाने र रेखा 4x – 3y – 10 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of a straight line passing through the point (3, 2) and perpendicular to the line 4x – 3y – 10 = 0. **[2075 R']**

⇨ Here, the line perpendicular to $4x - 3y - 10 = 0$ .... (i) is;
$3x + 4y + k = 0$ .... (ii)
Since line (ii) passes through the point (3, 2)
So, we have,
$3.3 + 4.2 + k = 0$
or, $9 + 8 + k = 0$
∴ $k = -17$
Substituting the value of k in (ii) then,
$3x + 4y - 17 = 0.$
Thus, $3x + 4y - 17 = 0$ is the requird equation.

8. **समीकरण 7x – 5y – 6 = 0 भएको रेखासँग लम्ब हुने र बिन्दु (– 1, – 2) बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equations of the line which is perpendicular to the line 7x – 5y – 6 = 0 and passing through the point (– 1, – 2). **[2072 S]**

⇨ Here, given equation of line is $7x - 5y - 6 = 0$ …. (i)
Equation of line perpendicular to (i) is;
$5x + 7y = k$ ……...(ii)
Equation (ii) passes through the point (– 1, – 2).
So, $5 \times (-1) + 7(-2) = k$
or, $-5 - 14 = k$
∴ $k = -19$
Putting $k = -19$ in equation (ii) then; $5x + 7y = -19$
∴ $5x + 7y + 19 = 0$
Thus, the required equation of the line is $5x + 7y + 19 = 0$.

9. **बिन्दु (2, 3) बाट जाने र रेखा 4x – 3y = 10 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the line passing through (2, 3) and perpendicular to the line 4x – 3y = 10. **[2057 R]**

⇨ Here, any line perpendicular to
$4x - 3y = 10$ .... (i) is
$3x + 4y + k = 0$ .... (ii)
Since, line (ii) passes through the point (2, 3),
So, we have, $3.2 + 4.3 + k = 0$
or, $6 + 12 + k = 0$
∴ $k = -18$
Thus, the required equation of the straight line is;
$3x + 4y - 18 = 0$

10. **बिन्दु (2, – 3) बाट जाने र रेखा 3x + 4y + 18 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the line passing through the point (2, – 3) and perpendicular to the line 3x + 4y + 18 = 0. **[2057S]**

⇨ Here, given straight line $3x + 4y + 18 = 0$ .... (i)
Any line perpendicular to line (i) is;
$4x - 3y + k = 0$ .... (ii)
But equation (ii) passes through the point (2, –3).
So, $4 \times 2 - 3 \times (-3) + k = 0$
or, $8 + 9 + k = 0$
or, $17 + k = 0$
∴ $k = -17$
Putting the value of k in equation (ii), we get $4x - 3y - 17 = 0$ is the required equation.

11. **बिन्दु (4, 6) बाट जाने र रेखा x – 2y – 2 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equations of the line which passes through the point (4, 6) and is perpendicular to the line x – 2y – 2 = 0. **[2058 R]**

⇨ Here, given line $x - 2y - 2 = 0$ .... (i)
Any line perpendicular to (i) is $2x + y + k = 0$ — (ii)
But the line (ii) passes through the point (4, 6) so, substituting (4, 6) in equation (ii) we get
$2 \times 4 + 6 + k = 0$
or, $8 + 6 + k = 0$
∴ $k = -14$
Putting $k = -14$ in equation (ii), we get the required equation $2x + y - 14 = 0$

12. **बिन्दु (– 2, – 3) बाट जाने र रेखा 5x + 7y = 14 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the straight line which passes through the point (– 2, – 3) and is perpendicular to the line 5x + 7y = 14. **[2058 S, 2066 R']**

⇨ Here, given line, $5x + 7y = 14$ ..... (i)
Any line perpendicular to line (i) is;
$7x - 5y + k = 0$ ...... (ii)
But line (ii) passes through the point (– 2, – 3),
So, $7 \times (-2) - 5 \times (-3) + k = 0$
or, $-14 + 15 + k = 0$
or, $1 + k = 0$
∴ $k = -1$
Now, putting the value of k in equation (ii),
We get, $7x - 5y - 1 = 0$ is the required equation.

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13. बिन्दु (– 6, 4) बाट जाने र रेखा 3x – 4y + 9 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the straight line, which passes through the point (–6, 4) and perpendicular to 3x – 4y + 9 = 0. [2059 S]

⇨ Here, given equation of st. line is;

$3x - 4y + 9 = 0$ ...... (i)

Any line perpendicular to line (i) is;

$4x + 3y + k = 0$ ...... (ii)

But line (ii) passes through the point (– 6, 4);

So, $4 \times (-6) + 3 \times 4 + k = 0$

or, $-24 + 12 + k = 0$

or, $-12 + k = 0$

$\therefore$ $k = 12$

Now, putting the value of k in equation (ii) we get the required equation as $4x + 3y + 12 = 0$.

14. बिन्दु (2, 3) बाट जाने र रेखा x – 3y – 2 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through the point (2, 3) and perpendicular to the straight line x – 3y – 2 = 0. [2060 R]

⇨ Here, given line $x - 3y - 2 = 0$ .... (i) and

Point (2, 3).

Any line perpendicular to line (i) is;

$3x + y + k = 0$ .... (ii)

But this line (ii) passes through the point (2, 3).

So, $3 \times 2 + 3 + k = 0$

or, $6 + 3 + k = 0$

$\therefore$ $k = -9$

Putting the value of k in equation (ii) we get,

$3x + y - 9 = 0$

Thus, $3x + y - 9 = 0$ is the required equation.

15. बिन्दु (2, –3) बाट जाने र रेखा 5x – 4y + 19 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the line passing through the point (2, – 3) and perpendicular to the line 5x – 4y + 19 = 0. [2061S]

⇨ Here, given line $5x - 4y + 19 = 0$ .... (i)

Any line perpendicular to (i) is;

$4x + 5y + K = 0$ .... (ii)

But line (ii) passes through the point (2, –3),

So $4 . 2 + 5. (-3) + K = 0$

or, $8 - 15 + K = 0$

or, $-7 + K = 0$

$\therefore$ $K = 7$

Now, putting the value of k in equation (ii), we get,

$4x + 5y + 7 = 0$

Thus, $4x + 5y + 7 = 0$ is the required equation.

16. बिन्दु (7, 1) बाट जाने र रेखा 5x + 7y + 12 = 0 सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a line passing through (7, 1) and perpendicular to 5x + 7y + 12 = 0. [2063 R–II]

⇨ Here, given equation of the line is;

$5x + 7y + 12 = 0$ .... (i)

Equation of the line ⊥ to equation (i) is;

$7x - 5y = k$ .... (ii)

Since line (ii) passes through (7, 1).

So, $7 \times 7 - 5 \times 1 = k$

or, $49 - 5 = k$

$\therefore$ $k = 44$

Putting the value of k in (ii);

$7x - 5y = 44$

Thus, the $7x - 5y = 44$ is the required equation of the line.

17. बिन्दु (2, 3) बाट जाने र रेखा x – 2y – 5 = 0 सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस्

Find the equation of a line passing through the point (2, 3) and perpendicular to the line x – 2y – 5 = 0. [2063 S]

⇨ Here, given equation of the line is;

$x - 2y - 5 = 0$ .... (i)

Equation of the line perpendicular to line (i) is;

$2x + y = k$ .... (ii)

Since line (ii) passes through (2, 3).

So, $2 \times 2 + 3 = k$

$\therefore$ $k = 7$

Putting the value of k in (ii) then, $2x + y = 7$ is the required equation of the line.

18. बिन्दु P (– 2, 4) बाट रेखा 7x – 24y + 10 = 0 मा लम्ब PQ खिचिएको छ भने PQ रेखाको समीकरण पत्ता लगाउनुहोस् र PQ को लम्बाइ पनि पत्ता लगाउनुहोस् ।

From the point P(– 2, 4), if PQ is drawn perpendicular to the line 7x – 24 y + 10 = 0, find the equation of the line PQ. Also determine the length of PQ. [2061 R]

⇨ Here, given equation $7x - 24y + 10 = 0$ ..... (i)

Any line PQ which is perpendicular to (i) is;

$24x + 7y + k = 0$ ...... (ii)

But this line (ii) passes through the point P(– 2, 4)

So, we have $24 \times (-2) + 7 \times 4 + k = 0$

or, $-48 + 28 + k = 0$

or, $-20 + k = 0$

$\therefore$ $k = 20$

Putting the value of $k = 20$ in equation (ii) we get,

$24x + 7y + 20 = 0$ ..... (ii) is the required equation of the line PQ.

Also the perpendicular distance PQ from P(– 2, 4) on line (i) is $\left[\because\ d = \frac{ax_1 + by_1 + c}{\sqrt{a_2 + b_2}}\right]$

$$PQ = \left|\frac{7 \times (-2) - 24 \times 4 + 10}{\sqrt{7^2 + (-24)^2}}\right|$$

$$= \left|\frac{-14 - 96 + 10}{\sqrt{49 + 576}}\right| = \left|\frac{-100}{\sqrt{625}}\right| = \left|\frac{-100}{25}\right|$$

$$= |-4| = 4$$

Thus, the length of PQ = 4 units.

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19. M(4, 7) र N(5, – 2) दुईओटा बिन्दुहरू हुन् । MN मा लम्ब हुने र बिन्दु (2, 3) भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।

M(4, 7) and N(5, – 2) are two points. Find the equation of the straight line perpendicular to MN and passing through the point (2, 3). [2067 S]

⇨ Here, the given points are M(4, 7) and N(5, – 2).

Slope of line MN is $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{5-4} = \frac{-9}{1}$

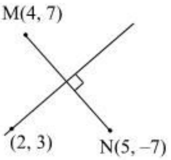
Since lines are perpendicular to each other,

So, $m_1 \times m_2 = -1$

i.e. $-9 \times m_2 = -1$

$\therefore \quad m_2 = \frac{1}{9}$

Now, equation of line passing through (x_1, y_1) and having slope $\frac{1}{9}$ is;



$$y - y_1 = \frac{1}{9}(x - x_1)$$

It passes through (2, 3);

$$y - 3 = \frac{1}{9}(x - 2)$$

or, $9(y - 3) = x - 2$

or, $9y - 27 = x - 2$

$\therefore \quad x - 9y + 25 = 0$

Thus, the required equation of the line is $x - 9y + 25 = 0$.

20. बिन्दुहरू (3, 4) र (– 1, – 6) जोड्ने रेखाको मध्यबिन्दुबाट जाने र रेखा 3x – 8y + 7 = 0 सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Determine the equation of the line passing through the mid-points of the line joining the points (3, 4) and (– 1, – 6) and perpendicular to the line 3x – 8y + 7 = 0. [2062 R]

⇨ Here, given line is $3x - 8y + 7 = 0$ ......(i)

Any line perpendicular to line (i) is;

$8x + 3y + k = 0$ ..... (ii)

Mid-point of the line joining two points (3, 4) and (–1, –6) is(x, y) where

$$x = \frac{x_1 + x_2}{2} = \frac{3-1}{2} = \frac{2}{2} = 1 \text{ and}$$

$$y = \frac{y_1 + y_2}{2} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

$\therefore$ The mid-point is (1, – 1)

But the line (ii) passes through the point (1, –1)

So, $8 \times 1 + 3 \times (-1) + k = 0$

or, $8 - 3 + k = 0$

or, $5 + k = 0$

$\therefore \quad k = -5$

Putting the value of k in eq^n (ii) we get,

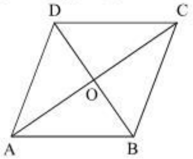
Thus, $8x + 3y - 5 = 0$ is the required equation.

MODEL 3

21. बिन्दुहरू (2, 4) र (8, 10) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the perpendicular bisector of the line segment joining the points (2, 4) and (8, 10). [2074 R']

⇨ Here, A(2, 4) and C(8, 10)



(a) Mid-point of AC $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+8}{2}, \frac{4+10}{2}\right)$

$= (5, 7)$

(b) Slope of AC; $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-4}{8-2} = \frac{6}{6} = 1$

(c) For slope of BD, BD is perpendicular bisector of AC.

So, $m_1 \times m_2 = -1$

or, $1 \times m_2 = -1$

or, $1 \times m_2 = -1$

$\therefore \quad m_2 = -1$ is the slope of BD

Passing point of BD = (5, 7)

Now, using one point formula, $y - y_1 = m(x - x_1)$

or, $y - 7 = -1(x - 5)$

or, $y - 7 = -x + 5$

$\therefore \quad x + y = 12$

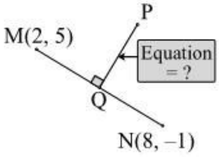
Thus, the equation of perpendicular bisector is $x + y = 12$.

22. बिन्दुहरू M(2, 5) र N(8, – 1) लाई जोडेर बनेको रेखाको लम्बार्धकको समीकरण निकाल्नुहोस् ।

Find the equation of the perpendicular bisector of a line which is formed by joining the points M(2, 5) and N(8, – 1). [2074 S']

⇨ Here, given points are M(2, 5) and N(8, – 1).

Let, PQ is the perpendicular bisector of MN.



Slope of MN $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{8-2} = -\frac{6}{6} = -1$

Since PQ ⊥ MN,

So slope of PQ $= -\frac{1}{m_1} = -\frac{1}{-1} = 1 = m$

Mid point of MN is ; $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+8}{2}, \frac{5-1}{2}\right)$

$= (5, 2) = (x_1, y_1)$

We have, equation of PQ;

$y - y_1 = m(x - x_1)$

or, $y - 2 = 1\ (x - 5)$

or, $y - 2 = x - 5$

$\therefore \quad x - y = 3$

Thus, the required equation is $x - y = 3$.

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23. **बिन्दुहरू (3, 5) र (– 7, 3) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the perpendicular bisector of the line segment joining the points (3, 5) and (– 7, 3).[2073 R']

⇨ Here, given points are (3, 5) and (– 7, 3).

$$\text{So, } m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-7 - 3} = \frac{-2}{-10} = \frac{1}{5}$$

Mid-point of line segment joining given points;

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{or, } (x, y) = \left(\frac{3 - 7}{2}, \frac{5 + 3}{2}\right)$$

or, $(x, y) = (-2, 4)$

Since the lines are perpendicular.

So, $m_1 \times m_2 = -1$

or, $\frac{1}{5} \times m_2 = -1$

∴ $m_2 = -5$

We know that,

Equations of line having slope $(m_2) = -5$ and passing through the point (– 2, 4) is given by,

$y - y_1 = m_2(x - x_1)$

or, $y - 4 = -5(x + 2)$

or, $y - 4 = -5x - 10$

∴ $5x + y + 6 = 0$

Thus, the required equation of line is $5x + y + 6 = 0$.

25. **दिइएका बिन्दुहरू A(3, – 7) र B(– 5, 3) जोड्ने रेखाखण्डसँग लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the perpendicular bisector of the line segment joining the given points A(3, – 7) and B(– 5, 3). [2071 R']

⇨ Here, given points are A(3, – 7) and B(– 5, 3).

Slope of line joining A(3, – 7) & B(– 5, 3):

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 + 7}{-5 - 3} = \frac{10}{-8} = -\frac{5}{4}$$

Mid-point of A(3, – 7) and B(– 5, 3):

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 - 5}{2}, \frac{-7 + 3}{2}\right) = (-1, -2)$$

Required line is perpendicular to given line,

So, $m_1 \times m_2 = -1$

or, $-\frac{5}{4} \times m_2 = -1$

∴ $m_2 = \frac{4}{5}$

We know that, Equation of line passing through (– 1, – 2) and slope $\frac{4}{5}$ is; $y - y_1 = m(x - x_1)$

i.e. $y + 2 = \frac{4}{5}(x + 1)$

or, $5y + 10 = 4x + 4$

or, $6 = 4x - 5y$

∴ $4x - 5y = 6$

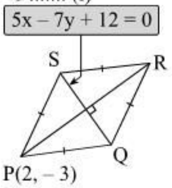
Thus, the required equation of line is $4x - 5y = 6$.

24. **दिइएको चित्रमा PQRS एउटा समबाहु चतुर्भुज हो । यदि विकर्ण QS को समीकरण 5x – 7y + 12 = 0 छ र बिन्दु P को निर्देशाङ्कहरू (2, – 3) भए विकर्ण PR को समीकरण पत्ता लगाउनुहोस् ।**

In the given figure, PQRS is a rhombus. If the equation of a diagonal QS is 5x – 7y + 12 = 0 and the coordinates of the point P are (2, – 3), find the equation of the diagonal PR. [2071 S]

⇨ Here, given equation of line QS is;

$5x - 7y + 12 = 0$ .... (i)



Equation of line PR perpendicular to equation (i) is;

$7x + 5y = k$ .... (ii)

Equation (ii) passes through P(2, – 3)

So, $7x + 5y = k$

or, $7 \times 2 + 5(-3) = k$

or, $14 - 15 = k$

∴ $k = -1$

Thus, equation of line PR is $7x + 5y = -1$.

i.e. $7x + 5y + 1 = 0$.

26. **बिन्दुहरू (3, –7) र (– 5, 3) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the perpendicular bisector of the line joining points (3, – 7) and (– 5, 3). [2060 S]

⇨ Here, $(x_1, y_1) = (3, -7)$ & $(x_2, y_2) = (-5, 3)$

If (x, y) be the mid point of the line joining the given two points, then using formula,

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\text{We have, } x = \frac{3 - 5}{2}, y = \frac{-7 + 3}{2}$$

∴ $x = -1, y = -2$

∴ The required mid-point is $(x, y) = (-1, -2)$

Again, equation of st. line joining two given points is obtained by using formula,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } y - (-7) = \frac{3 - (-7)}{-5 - 3}(x - 3)$$

$$\text{or, } y + 7 = \frac{3 + 7}{-8}(x - 3)$$

$$\text{or, } y + 7 = \frac{10}{-8}(x - 3)$$

$$\text{or, } y + 7 = -\frac{5}{4}(x - 3)$$

or, $4y + 28 = -5x + 15$

∴ $5x + 4y + 13 = 0$ .... (i)

Again any line perpendicular to equation (i) is

$4x - 5y + k = 0$ .... (ii)

But line (ii) passes through the point (– 1, – 2)

Hence, $4 \times (-1) - 5 \times (-2) + k = 0$

or, $-4 + 10 + k = 0$

or, $6 + k = 0$

∴ $k = -6$

Now putting the value of k in equation (ii), we get the required equation as: $4x - 5y - 6 = 0$.

Thus, the required equation is $4x - 5y - 6 = 0$.

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27. **बिन्दु A र B को निर्देशाङ्क क्रमश: (3, – 1) र (7, 1) भए AB को लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**
The points A and B have co-ordinates (3, –1) and (7, 1) respectively. Find the equation of the perpendicular bisector of AB. [2059 R, 2065 R]

⇨ Here, given points A(3, – 1) and B(7, 1)
Now, equation of line AB is;

$$y + 1 = \frac{1+1}{7-3}(x-3) \quad \because \left(y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right)$$

or, $y + 1 = \frac{2}{4}(x - 3)$

or, $y + 1 = \frac{1}{2}(x - 3)$

or, $2y + 2 = x - 3$

$\therefore$ $x - 2y - 5 = 0$ .... (i)

Now the mid-points of AB is;

$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}$$

or, $x = \frac{3+7}{2}$ and $y = \frac{-1+1}{2}$

$\therefore$ $x = \frac{10}{2} = 5$ and $y = \frac{0}{2} = 0$

The mid-points of AB (5, 0).
Any line perpendicular to (i) is;
$2x + y + k = 0$ .... (ii)
But for the line (ii) to be perpendicular bisector of line AB, equation (ii) must pass through (5, 0),
So, $2 \times 5 + 0 + k = 0$
or, $10 + k = 0$
$\therefore$ $k = – 10$
Put k = – 10 in (ii) we get,
Thus, the required equation is $2x + y – 10 = 0$.

28. **बिन्दुहरू (3, 5) र (9, 3) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the perpendicular bisector of the line joining the points (3, 5) and (9, 3).[2061 S, 2063 R]

⇨ Here, the slope of line joining (3, 5) and (9, 3) is;

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{9-3} = -\frac{2}{6} = -\frac{1}{3}$$

The mid-point of the line joining (3, 5) and (9, 3) is;

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{3+9}{2}, \frac{5+3}{2}\right)$$
$$= (6, 4)$$

The equation of line passing through mid-point (6, 4) is;
$y – 4 = m(x – 6)$ .... (i)
The slope of the line (i) is m.
Since the line (i) is the perpendicular bisector of line joining given two points,
So, $m_1 \times m_2 = – 1$

or, $-\frac{1}{3} \times m = – 1$

$\therefore$ $m = \frac{-3}{-1} = 3$

Putting the value of m in equation (i)
$y – 4 = 3(x – 6)$
or, $y – 4 = 3x – 18$
or, $y – 3x – 4 + 18 = 0$
or, $y – 3x + 14 = 0$
$\therefore$ $3x – y – 14 = 0$
Thus, the required equation is $3x – y – 14 = 0$.

29. **बिन्दुहरू (4, – 2) र (6, 2) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the perpendicular bisector of the line joining the points (4, – 2) and (6, 2). [2060 CP]

⇨ Here, slope of the line joining (4, – 2) and (6, 2) is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2+2}{6-4} = \frac{4}{2} = 2$$

The mid-point of the line joining (4, – 2) and (6, 2).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4+6}{2}, \frac{-2+2}{2}\right)$$
$$= (5, 0)$$

The equation of the line passing through (5, 0) is;
$y – 0 = m_2(x – 5)$ .... (i)
Since line (i) and given line are perpendicular to each other
So, $m_1 \times m_2 = – 1$
or, $2 \times m_2 = – 1$

$\therefore$ $m_2 = -\frac{1}{2}$

Putting the value of m_2 in equation (i)

$$y - 0 = -\frac{1}{2}(x - 5)$$

or, $y - 0 = -\frac{1(x-5)}{2}$

$\therefore$ $2y = – x + 5$
Thus, the $x + 2y = 5$ is the required equation of the line.

30. **बिन्दुहरू (3, 2) र (7, 6) जोड्ने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of a perpendicular bisector of a line segment joining the points (3, 2) & (7, 6). [2067 R']

⇨ Here, A(3, 2) = (x_1, y_1) & B(7, 6) = (x_2, y_2) are any two points
Now, mid-point of AB is;

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{3+7}{2}, \frac{2+6}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{8}{2}\right)$$
$$= (5, 4)$$

Slope of AB $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{7-3} = \frac{4}{4} = 1$

Two lines are perpendicular so, $m_1.m_2 = – 1$
or, $1.m_2 = – 1$
or, $m_2 = – 1$
Then the line passes through (5, 4) & slope is – 1.
So equation is $y – y_1 = m(x – x_1)$
or, $y – 4 = – 1(x – 5)$
or, $x + y – 4 – 5 = 0$
$\therefore$ $x + y – 9 = 0$
Thus, the $x + y – 9 = 0$ is the equation of the perpendicular bisector.

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31. बिन्दुहरू M र N का निर्देशाङ्कहरू क्रमशः (4, – 3) र (8, 5) भए रेखा MN को लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।
The points M and N have the co-ordinates (4, – 3) and (8, 5) respectively. Find the equation of the perpendicular bisector of the line MN. [2064 S]

⇨ Here, given the co-ordinates of M and N are, (4, – 3) and (8, 5).

The slope of MN $(m_1) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5+3}{8-4} = 2$

Mid-point of MN $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_2 + y_1}{2}\right)$

$= \left(\dfrac{4+8}{2}, \dfrac{-3+5}{2}\right)$

$= (6, 1)$

Now, equation of line passing through (6, 1) is;

$(x - 6) = m_2(y - 1)$ .... (i)

The line (i) and the line MN are perpendicular to each other. So, $m_1 \times m_2 = -1$

or, $2 \times m_2 = -1$

$\therefore$ $m_2 = -\dfrac{1}{2}$

Now, putting the value of m_2 in equation (i)

$y - y_1 = m(x - x_1)$

or, $y - 1 = -\dfrac{1}{2}(x - 6)$

or, $2y - 2 = -(x - 6)$

or, $x + 2y - 2 - 6 = 0$

$\therefore$ $x + 2y - 8 = 0$

Thus, $x + 2y - 8 = 0$ is a required equation of line.

32. एउटा वर्गको विकर्णका अन्तिम बिन्दुहरू (2, 3) र (– 6, 5) हुन् भने अर्को विकर्णको समीकरण पत्ता लगाउनुहोस् ।
If (2, 3) and (–6, 5) are the end points of one of the diagonal of a square, find the equation of the other diagonal. [2065 M]

⇨ Here, the diagonal of the square bisect each other at right angle.

Therefore, Slope of one diagonal, $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-3}{-6-2}$

$= \dfrac{2}{-8}$

$= -\dfrac{1}{4}$

Slope of another diagonal, $m_2 = 4$. $\left[\because m_2 = -\dfrac{1}{m_1}\right]$

Mid point of the given diagonal

$= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{2-6}{2}, \dfrac{3+5}{2}\right)$

$= (-2, 4)$

Equation of the required diagonal,

$y - y_1 = m_2(x - x_1)$

i.e. $y - 4 = 4(x + 2)$

or, $y - 4 = 4x + 8$

Thus, $4x - y + 12 = 0$ is the required equation of another diagonal.

33. समबाहु चतुर्भुज ABCD का विपरीत शीर्षबिन्दुहरू A(2, 4) र C(8, 10) भए पत्ता लगाउनुहोस्:
(a) AC विकर्णको मध्यबिन्दु
(b) AC भुजाको झुकाव
(c) BD विकर्णको समीकरण
A(2, 4) and C(8, 10) are the opposite vertices of a rhombus ABCD, find:
(a) the mid point of diagonal AC.
(b) Slope of AC.
(c) the equation of the diagonal BD. [2065 R']

⇨ Here, A(2, 4) and C(8, 10)

(a) Mid-point of AC $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

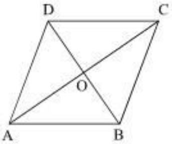
$= \left(\dfrac{2+8}{2}, \dfrac{4+10}{2}\right)$

$= (5, 7)$

(b) Slope of AC; $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10-4}{8-2}$

$= 1$



(c) For slope of BD,

BD is perpendicular bisector of AC.

So, $m_1 \times m_2 = -1$

or, $1 \times m_2 = -1$

or, $1 \times m_2 = -1$

$\therefore$ $m_2 = -1$ is the slope of BD

Passing point of BD = (5, 7)

Now, using one point formula,

$y - y_1 = m(x - x_1)$

or, $y - 7 = -1(x - 5)$

or, $y - 7 = -x + 5$

$\therefore$ $x + y = 12$

Thus, the equation of BD is $x + y = 12$.

34. समबाहु चतुर्भुज ABCD को दुई विपरीत शीर्षबिन्दुहरू A(3, 5) र C(7, 9) भए विकर्ण BD को समीकरण पत्ता लगाउनुहोस् ।
A(3, 5), and C(7, 9) are the opposite vertices of a rhombus ABCD. Find the equation of the diagonal BD. [2067 R]

⇨ Here, A(3, 5) & C(7, 9) are the opposite vertices of the diagonal of a rhombus ABCD.

Now, mid point of AC is $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{3+7}{2}, \dfrac{5+9}{2}\right)$

$= (5, 7)$

Slope of AC is $(m_1) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9-5}{7-3}$

$= \dfrac{4}{4}$

$= 1$

Where, diagonals of rhombus are perpendicularly bisect each other.

So, $m_1.m_2 = 1$ or $m_2 = -1$.

Then, diagonal BD passes through (5, 7) &

Slope is – 1.

So, equation of the diagonal BD is,

$y - y_1 = m(x - x_1)$

or, $y - 7 = -1(x - 5)$

or, $x + y - 7 - 5 = 0$

$\therefore$ $x + y - 12 = 0$

Thus, $x + y - 12 = 0$ is a required equation of another diagonal of given rhombus.

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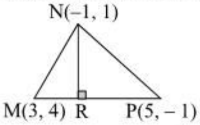
MODEL 4

35. बिन्दुहरू M(3, 4), N(–1, 1) र P(5, –1) एउटा त्रिभुज MNP का शीर्षबिन्दुहरू हुन् । बिन्दु N(–1, 1) बाट खिचिएको त्रिभुज MNP को उचाइको समीकरण पत्ता लगाउनुहोस् ।
The points M(3, 4), N(–1, 1) and P(5, –1) are the vertices of a triangle MNP. Find the equation of the altitude of the triangle MNP drawn from the point N(–1, 1). [2074 R]

⇨ Here, given vertices of ΔMNP are;
M(3, 4), N(–1, 1) and P(5, – 1) and
Let, R be the point of altitude drawn from point N.



Slope of MP $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-4}{5-3}$

$= \frac{-5}{2} = -\frac{5}{2}$

Slope of NR $(m_2) = -\frac{1}{m_1}$

$= -\frac{1}{-\frac{5}{2}}$

$= \frac{2}{5}$ $[\because m_1 \times m_2 = -1]$

We have, Equation of line NR;

$y - y_1 = m_2(x - x_1)$

or, $y - 1 = \frac{2}{5}(x + 1)$

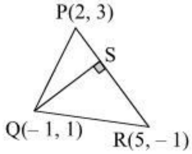
or, $5y - 5 = 2x + 2$

$\therefore$ $2x - 5y + 7 = 0$

Thus, the required equation of altitude is $2x - 5y + 7 = 0$.

36. बिन्दुहरू P(2, 3), Q(– 1, 1) र R(5, – 1) त्रिभुज PQR का शीर्षबिन्दुहरू हुन् । बिन्दु Q(– 1, 1) बाट खिचिएको त्रिभुज PQR को उचाइको समीकरण पत्ता लगाउनुहोस् ।
The points P(2, 3), Q(– 1, 1) and R(5, – 1) are the vertices of a triangle PQR. Find the equation of the altitude of the triangle PQR drawn from the point Q(– 1, 1). [2074 S]

⇨ Here, the vertices of ΔPQR are; P(2, 3), Q(– 1, 1) and R(5, – 1).
Let, S be the point of altitude drawn from Q and m_2 be the slope of QS.



Slope of PR $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-3}{5-2} = -\frac{4}{3}$

Slope of QS $(m_2) = -\frac{1}{m_1}$

$= -\frac{1}{-\frac{4}{3}}$

$= \frac{3}{4}$ $[\because m_1 \times m_2 = -1]$

We have, equation of line QS;

$y - y_1 = m_2(x - x_1)$

or, $y - 1 = \frac{3}{4}(x + 1)$

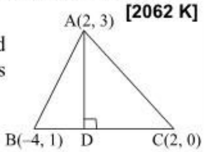
or, $4y - 4 = 3x + 3$

$\therefore$ $3x - 4y + 7 = 0$

Thus, $3x - 4y + 7 = 0$ is the required equation.

37. A(2, 3), B(– 4, 1) र C(2, 0) शीर्षबिन्दुहरू भएका त्रिभुज ABC को शीर्षबिन्दु A(2, 3) बाट खिचिएको उचाइको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the altitude of triangle ABC with vertices A(2, 3), B(– 4, 1) and C(2, 0) drawn from the vertex A(2, 3). [2062 K]

⇨ Here, A(2, 3), B(– 4, 1) and C(2, 0) be the given vertices and AD ⊥ BC is drawn.



Slope of BC is; $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-1}{2+4}$

$= -\frac{1}{6}$

Since AD ⊥ BC so;

slope of AD × Slope of BC = – 1

or, Slope of AD $\times \frac{-1}{6} = -1$

$\therefore$ Slope of AD$(m_2) = 6$

Now, equation of the line AD is; $y - y_1 = m_2(x - x_1)$

Since it passes though (2, 3).

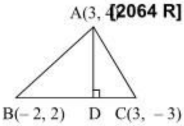
So, $y - 3 = 6(x - 2)$

or, $y - 3 = 6x - 12$

Thus, $6x - y = 9$ is the required equation of the line.

38. ΔABC का शीर्षबिन्दुहरू A(3, 4), B(– 2, 2) र C(3, – 3) छन् । शीर्षबिन्दु A बाट सम्मुख भुजा BC मा AD लम्ब खिचिएको छ । AD को समीकरण पत्ता लगाउनुहोस् ।
The vertices of a ΔABC are A(3, 4), B(– 2, 2) and C(3, – 3). AD is also perpendicular drawn from the vertex A on the opposite side BC. Find the equation of the AD. [2064 R]

⇨ Here, given vertices of ΔABC be A(3, 4), B(– 2, 2) and C(3, – 3) and AD ⊥ BC is drawn.



Slope of BC is $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-2}{3+2} = \frac{-5}{5} = -1$

Since AD ⊥ BC,

So, slope of BC × slope of AD = – 1

or, – 1 × slope of AD = – 1

$\therefore$ Slope of AD = 1

$\therefore$ Equation of line having slope 1 and passing through A(3, 4) is;

$y - y_1 = m_2(x - x_1)$

or, $y - 4 = 1(x - 3)$

or, $y - 4 = x - 3$

$\therefore$ $x - y + 1 = 0$

Thus, $x - y + 1 = 0$ is the required equation of AD.

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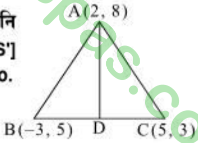
39. दिइएको चित्रमा BC को मध्यबिन्दु D भए AD ⊥ BC हुन्छ भनी प्रमाणित गर्नुहोस् । AD को समीकरण पनि पत्ता लगाउनुहोस् । **[2060 S']**

In the given figure, D is the mid-point of BC. Prove that AD ⊥ BC. Find the equation of AD also.



⇨ Here, A(2, 8), B(– 3, 5) & C(5, 3) are the given vertices of ΔABC & D is mid-point of line BC.

Now, co-ordinates of D $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3 + 5}{2}, \frac{5 + 3}{2}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

Slope of AD (m_2) $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-4}{-1}$

$= 4$

Where $m_1.m_2 = -\frac{1}{4}.4 = -1$

Then, slope of BC (m_1) $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3 - 5}{5 + 3}$

$= -\frac{2}{8} = -\frac{1}{4}$

So, AD & BC are $\perp_r$ to each others.

Again, equation of line AD is;

$y - y_1 = m(x - x_1)$

or, $y - 8 = 4(x - 2)$

or, $y - 8 = 4x - 8$

∴ $4x - y = 0$

Thus, $4x - y = 0$ is a required equation of line AD.

MODEL 5

40. बिन्दु (4, – 1) भएर जाने र रेखा 2*x* – 3*y* = 5 सँग 45° को कोण बनाउने कुनै एउटा सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of any one straight line passing through the point (4, – 1) and making an angle of 45° with the line 2*x* – 3*y* = 5. **[2075 R, 2075 R₂]**

⇨ Here, the given equation of line is ; $2x - 3y = 5$

It's slope $(m_1) = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{-3} = \frac{2}{3}$

Let m_2 be the slope of required line then,

$\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan 45° = \pm \frac{\frac{2}{3} - m_2}{1 + \frac{2}{3}m_2}$

or, $1 = \pm \left(\frac{2 - 3m_2}{3 + 2m_2}\right)$

Taking +ve sign, $3 + 2m_2 = 2 - 3m_2$

or, $5m_2 = -1$

∴ $m_2 = -\frac{1}{5}$

Taking –ve sign, $3 + 2m_2 = -2 + 3m_2$

or, $5 = m_2$

∴ $m_2 = 5$

We have, the one point formula, $y - y_1 = m(x - x_1)$

So, when $m_2 = 5$ and passing point is (4, – 1) then,

$y + 1 = 5(x - 4)$

or, $y + 1 = 5x - 20$

∴ $5x - y - 21 = 0$

Similarly, when $m_2 = -\frac{1}{5}$ and passing point is (4, – 1),

$y + 1 = -\frac{1}{5}(x - 4)$

or, $5y + 5 = -x + 4$

∴ $x + 5y + 1 = 0$

Thus, $5x - y - 21 = 0$ and $x + 5y + 1 = 0$ are the required equations.

41. बिन्दु (2, – 1) भएर जाने र समीकरण *6x + 5y – 2 = 0* भएको रेखासँग *45°* को कोण बनाउने एउटा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through a point (2, – 1) and making an angle of *45°* with the straight line having equation *6x + 5y – 2 = 0.* **[2073 S']**

⇨ Here, equation of line is; $6x + 5y - 2 = 0$ ....(i)

Slope of line (i); $m_1 = -\frac{\text{coeff. of x}}{\text{coeff. of y}} = -\frac{6}{5}$

We have, given angle between the lines is 45°.

So, $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan 45° = \pm \frac{\frac{-6}{5} - m_2}{1 - \frac{6}{5} \times m_2}$

or, $1 = \pm \frac{-6 - 5m_2}{5 - 6m_2}$

Taking (+) ve sign; $1 = \frac{-6 - 5m_2}{5 - 6m_2}$

or, $5 - 6m_2 = -6 - 5m_2$

or, $-m_2 = -11$

∴ $m_2 = 11$

Taking (–) ve sign; $1 = \frac{6 + 5m_2}{5 - 6m_2}$

or, $6 + 5m_2 = 5 - 6m_2$

or, $11m_2 = -1$

∴ $m_2 = -\frac{1}{11}$

We have, passing point $(x_1, y_1) = (2, -1)$.

Equation of line passing through (x_1, y_1) and slope (m) is; $(y - y_1) = m(x - x_1)$

or, $y + 1 = 11(x - 2)$ [when m = 11]

or, $y + 1 = 11x - 22$

or, $11x - y = 23$ ....(ii)

Again, taking slope $(m) = -\frac{1}{11}$ then,

$y + 1 = -\frac{1}{11}(x - 2)$

or, $11y + 11 = -x + 2$

or, $x + 11y + 9 = 0$ ....(iii)

Thus, $11x - y = 23$ and $x + 11y + 9 = 0$ are the required equations of the lines.

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42. **दिइएको वर्ग ABCD मा विकर्ण AC को समीकरण 3x – 4y + 10 = 0 र शीर्षबिन्दु B को निर्देशाङ्कहरू (4, – 5) भए विकर्ण BD को समीकरण पत्ता लगाउनुहोस् ।**

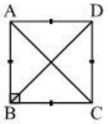
The equation of a diagonal AC of given square ABCD is 3x – 4y + 10 = 0 and the coordinates of vertex B are (4, – 5). Find the equation of diagonal BD. [2071 R]

⇨ Here, equation of AC is $3x - 4y + 10 = 0$ .... (i)

Co-ordinates of B is (4, – 5).

We know that,

AC and BD are the perpendicular bisector to each other.



So equation of line perpendicular to equation (i) is

$4x + 3y = k$ ...... (ii)

The line (ii) passes through (4, – 5).

So, $4 \times 4 + 3 \times (-5) = k$

or, $16 - 15 = k$

$\therefore$ $k = 1$

Putting k = 1 in equation (i) then,

$4x + 3y = 1$

Which is the required equation of the line BD.

43. **रेखा x – 3y = 2 सँग 45° को कोण बनाउने र बिन्दु (2, 3) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the straight lines, which passes through the point (2, 3) and making an angle of 45° with the line x – 3y = 2. [2063 M, 2069 R']

⇨ Here, the equation of any line through (2, 3) is;

$y - 3 = m(x - 2)$………………….(i)

The slope of the line $x - 3y = 2$ ……(ii) is;

$$m_2 = -\frac{1}{-3} = \frac{1}{3}$$

Line (i) and (ii) are intersecting at an angle of 45°.

So, $\tan\theta = \pm \dfrac{m_1 - m_2}{1 + m_1.m_2}$

or, $\tan 45° = \pm \dfrac{m - \frac{1}{3}}{1 + m\left(\frac{1}{3}\right)}$

$\therefore$ $1 = \pm \dfrac{3m - 1}{3 + m}$

Taking +ve sign, we get, $1 = \dfrac{3m - 1}{3 + m}$

or, $3m - 1 = 3 + m$

or, $2m = 4$

$\therefore$ $m = 2$

Taking –ve sign, we get, $1 = -\dfrac{3m - 1}{3 + m}$

or, $3m - 1 = -3 - m$

or, $4m = -2$

$\therefore$ $m = \dfrac{-1}{2}$

Substituting the values of m in (i) we get

$y - 3 = 2(x - 2)$

or, $y - 3 = 2x - 4$

or, $4 - 3 = 2x - y$

$\therefore$ $2x - y = 1$ ………(a)

and $y - 3 = -\dfrac{1}{2}(x - 2)$

or, $x + 2y - 8 = 0$ …………………(b)

Thus, (a) and (b) are required equations.

44. **समीकरण 4x – 5y + 9 = 0 भएको रेखासँग 45° को कोण बनाउने र बिन्दु (5, 0) बाट जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the straight lines passing through the point (5, 0) and making an angle of 45° with the line 4x – 5y + 9 = 0. [2066 R]

⇨ Here, given equation of line is; $4x - 5y + 9 = 0$

Angle with the line $(\theta) = 45°$

Slope of the given line is; $(m_1) = -\dfrac{\text{coefficient of x}}{\text{coefficient of y}} = -\dfrac{4}{-5} = \dfrac{4}{5}$

We have, $\tan\theta = \pm \dfrac{m_1 - m_2}{1 + m_1\, m_2}$

or, $\tan 45° = \pm \dfrac{\frac{4}{5} - m_2}{1 + \frac{4}{5}.\, m_2} = \pm \dfrac{\frac{4 - 5m_2}{5}}{\frac{5 + 4m_2}{5}}$

or, $1 = \pm \dfrac{4 - 5m_2}{5 + 4m_2}$

or, $5 + 4m_2 = \pm (4 - 5m_2)$

Taking (+)ve sign, $5 + 4m_2 = 4 - 5m_2$

or, $9m_2 = -1$

$\therefore$ $m_2 = -\dfrac{1}{9}$

Taking (–)ve sign, $5 + 4m_2 = -4 + 5m_2$

or, $-m_2 = -9$

$\therefore$ $m_2 = 9$

Equation of the line with slope $(m_2) = -\dfrac{1}{9}$ and passing point (5, 0).

$y - y_1 = m_2(x - x_1)$

or, $y - 0 = -\dfrac{1}{9}(x - 5)$

or, $9y = -x + 5$

$\therefore$ $x + 9y = 5$

Equation of the line with slope $(m_2) = 9$ and passing through (5, 0).

$y - y_1 = m_2(x - x_1)$

or, $y - 0 = 9(x - 5)$

or, $y = 9x - 45$

$\therefore$ $9x - y = 45$

Thus, the equation of required lines are; $x + 9y = 5$ and $9x - y = 45$.

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45. **समीकरण $x - \sqrt{3}y = 4$ भएको रेखासँग 30° को कोण बनाउने र बिन्दु (1, 0) बाट जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the straight lines passing through the point (1, 0) and making an angle of 30° with the line $x - \sqrt{3}y = 4$. **[2065 E]**

⇨ Here, given equation of the line is;

$x - \sqrt{3}\,y = 4$ ……… (i)

Slope $(m_1) = -\dfrac{1}{-\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

Equation (i) & (ii) lines makes an angle 30°,

So, $\tan 30° = \pm\dfrac{m_1 - m_2}{1 + m_1.m_2}$

or, $\dfrac{1}{\sqrt{3}} = \pm\dfrac{\frac{1}{\sqrt{3}} - m_2}{1 + \frac{1}{\sqrt{3}}m_2}$

or, $\dfrac{1}{\sqrt{3}} = \pm\dfrac{1 - \sqrt{3}\,m_2}{\sqrt{3} + m_2}$

or, $\sqrt{3} + m_2 = \pm(\sqrt{3} - 3m_2)$

Taking (+ve), $\sqrt{3} + m_2 = \sqrt{3} - 3m_2$

or, $m_2 + 3m_2 = \sqrt{3} - \sqrt{3}$

or, $4m_2 = 0$

$\therefore\ m_2 = 0$

Taking (–ve), $\sqrt{3} + m_2 = -\sqrt{3} + 3m_2$

or, $\sqrt{3} + \sqrt{3} = 3m_2 - m_2$

or, $2m_2 = 2\sqrt{3}$

or, $m_2 = \sqrt{3}$

Then, the equation of second line passes through the point (1, 0)

So, $y - y_1 = m(x - x_1)$

or, $y - 0 = m(x - 1)$

or, $y = m(x - 1)$ ……. (ii)

Putting the value of $m_2 = 0$ in equation (ii),

$y = 0(x - 1)$

$\therefore\ y = 0$

Putting the value of $m_2 = \sqrt{3}$ in equation (ii),

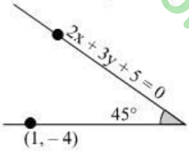
$y = \sqrt{3}(x - 1)$

or, $y = \sqrt{3}\,x - \sqrt{3}$

$\therefore\ \sqrt{3}x - y - \sqrt{3} = 0$

Thus, $y = 0$ & $\sqrt{3}x - y - \sqrt{3} = 0$ are required equations of the line.

46. **बिन्दु (1, – 4) बाट जाने र रेखा 2x + 3y + 5 = 0 सँग 45° को कोण बनाउने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the straight lines passing through (1, – 4) and making an angle of 45° with the line 2x + 3y + 5 = 0. **[2066 S]**



⇨ Here, given equation of line is;

$2x + 3y + 5 = 0$ .... (i)

Its slope $(m_1) = -\dfrac{\text{coeff. of x}}{\text{coeff of y}}$

$\therefore\ m_1 = -\dfrac{2}{3}$

Let m_2 be the slope of required line then,

$\tan\theta = \pm\dfrac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan 45° = \pm\dfrac{-\frac{2}{3} - m_2}{1 - \frac{2}{3}m_2}$

or, $1 = \pm\left(\dfrac{-2 - 3m_2}{3 - 2m_2}\right)$

Taking +ve sign,

$1 = \dfrac{-2 - 3m_2}{3 - 2m_2}$

or, $3 - 2m_2 = -2 - 3m_2$

$\therefore\ m_2 = -5$

Taking –ve sign,

$1 = -\left(\dfrac{-2 - 3m_2}{3 - 2m_2}\right)$

or, $3 - 2m_2 = 2 + 3m_2$

or, $1 = 5m_2$

$\therefore\ m_2 = \dfrac{1}{5}$

When $m_2 = -5$ and passing point is (1, – 4) then,

$y + 4 = -5(x - 1)$

or, $y + 4 = -5x + 5$

$\therefore\ 5x + y = 1$

When $m_2 = \dfrac{1}{5}$ and passing point is (1, – 4) then,

$y + 4 = \dfrac{1}{5}(x - 1)$

or, $5y + 20 = x - 1$

$\therefore\ x - 5y = 21$

Thus, $5x + y = 1$ and $x - 5y = 21$ are the required equations.

MODEL 6

47. **सीधारेखाहरू 3x + 4y = 7 र 5x – 2y = 3 को प्रतिच्छेदन बिन्दु भएर जाने र सीधारेखा 2x + 3y = 5 सँग लम्ब हुने एउटा सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of a straight line passing through the point of intersection of the straight lines 3x + 4y = 7 and 5x – 2y = 3 and perpendicular to the straight line 2x + 3y = 5. **[2073 R]**

⇨ Here, given equation of lines are;

$3x + 4y = 7$ ……. (i) and $5x - 2y = 3$ ………… (ii)

Solving equation (i) and (ii) × 2 then

$$\begin{array}{r} 3x + 4y = 7 \\ \underline{10x - 4y = 6} \\ 13x = 13 \end{array}$$

$\therefore\ x = 1$

Putting x = 1 in (i) then,

$3 \times 1 + 4y = 7$

or, $4y = 4$ $\quad\therefore\ y = 1$

So, the point of intersection is (1, 1).

Given equation of line is $2x + 3y = 5$ ……. (iii)

Slope of line (iii) is $m_1 = -\dfrac{\text{coeff. of x}}{\text{coeff. of y}} = -\dfrac{2}{3}$

Since the lines are perpendicular,

So, $m_1 \times m_2 = -1$

or, $-\dfrac{2}{3} \times m_2 = -1$

$\therefore\ m_2 = \dfrac{3}{2}$

Now, equation of line having slope $(m) = \dfrac{3}{2}$ and passing point $(x_1, y_1) = (1, 1)$ is given by, $y - y_1 = m(x - x_1)$

or, $y - 1 = \dfrac{3}{2}(x - 1)$

or, $2y - 2 = 3x - 3$

or, $3x - 2y = 1$

Thus, the required equation of the line is $3x - 2y = 1$.

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48. **रेखा $2x - y = 3$ सँग समानान्तर भई रेखाहरू $3x + y = 7$ र $3y = 4x - 5$ को प्रतिच्छेदन बिन्दुबाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the line passing through the point of intersection of the lines $3x + y = 7$ and $3y = 4x - 5$ and parallel to the line $2x - y = 3$.

⇨ Here, given lines are;

$2x - y = 3$ ....(i)

$3x + y = 7$ ....(ii) and

$3y = 4x - 5$ ....(iii)

Solving equation (ii) × 3 and (iii) then

$$\begin{array}{r} 9x + 3y = 21 \\ 4x - 3y = 5 \\ \hline 13x = 26 \end{array}$$

$\therefore \quad x = 2$

From (iii),

$3y = 4x - 5$

or, $3y = 4 \times 2 - 5$

or, $3y = 3$

$\therefore \quad y = 1$

So, the point of intersection is (2, 1).

We have, equation of the line parallel to line (i) is $2x - y = k$

Since it passes through (2, 1)

So, $2 \times 2 - 1 = k$

$\therefore \quad k = 3$

Thus, the reuqired equation is $2x - y = 3$.

49. **यदि सरल रेखा $\frac{x}{a} + \frac{y}{b} = 1$ रेखाहरू $x + y = 3$ र $2x - 3y = 1$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $y = x - 6$ सँग समानान्तर हुन्छ भने a र b को मान पत्ता लगाउनुहोस् ।**

If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point of intersection of the lines $x + y = 3$ and $2x - 3y = 1$ and is parallel to the line $y = x - 6$, then find the values of a and b.

⇨ Here, given equations of lines are;

$x + y = 3$ …….. (i)

$2x - 3y = 1$ ……. (ii)

$x - y = 6$ ………….. (iii)

Solving equation (i) × 2 & (ii) then

$$\begin{array}{l} 2x + 2y = 6 \\ 2x - 3y = 1 \\ \underline{- \quad + \quad -} \\ \quad 5y = 5 \end{array}$$

$\therefore \quad y = 1$

From (i) $x + y = 3$

or, $x + 1 = 3$

$\therefore \quad x = 2$

So, the point of intersection is (2, 1).

Equation of line parallel to $y = x - 6$

i.e. $x - y = 6$ is; $x - y = k$ it passes through (2, 1).

So, the equation of line parallel to $x - y = 6$ is;

$x - y = 1$

Now, $x - y = 1$

or, $\frac{x}{1} + \frac{y}{(-1)} = 1$

Comparing it with $\frac{x}{a} + \frac{y}{b} = 1$ then $a = 1$ and $b = -1$

Thus, the values of a and b are 1 and – 1 respectively.

QUESTIONS FROM CDC TEXTBOOK

4.1 दुई सरल रेखाहरूबिचको कोण (ANGLE BETWEEN TWO STRAIGHT LINES)

EXERCISE 4.1

1. **(a)** **दुई सरल रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ बिचको कोण कति हुन्छ ?**

What is the angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$?

⇨ Here, the required angle between the lines is

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\therefore \quad \theta = \tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$$

(c) **सरल रेखा $4x + 3y + 5 = 0$ को झुकाव पत्ता लगाउनुहोस् ।**

Find the slope of straight line $4x + 3y + 5 = 0$.

⇨ Here, $4x + 3y + 5 = 0$

We have,

$$\text{Slope (m)} = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}} = -\frac{4}{3}$$

Thus, the slope of the line is $-\frac{4}{3}$.

(b) **दुई सरल रेखाहरू आपसमा लम्ब हुने र समानान्तर हुने अवस्थाहरू लेख्नुहोस् ।**

Write the conditions for two straight lines to be perpendicular and parallel to each other.

⇨ Here, the conditions of lines

(i) to be parallel is $m_1 = m_2$

(ii) to be perpendicular is $m_1 \times m_2 = -1$

(d) **बिन्दुहरू (4, – 5) र (– 8, 9) जोड्ने रेखाको मध्यबिन्दु र झुकाव पत्ता लगाउनुहोस् ।**

Find the mid point and slope of line joining the points (4, – 5) and (– 8, 9).

⇨ Here, given points are (4, – 5) and (– 8, 9).

We know that, the midpoint formula;

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{4 - 8}{2}, \frac{-5 + 9}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{4}{2}\right)$$

$\therefore \quad (x, y) = (-2, 2)$

$$\text{Slope (m)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 + 5}{-8 - 4} = \frac{14}{-12} = -\frac{7}{6}$$

Thus, the midpoint is (– 2, 2) and slope is $-\frac{7}{6}$.

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(e) **रेखा $y = 3x + 7$ सँग लम्ब हुने र समानान्तर हुने रेखाहरूको झुकाव पत्ता लगाउनुहोस् ।**
Find the slope of lines which are perpendicular and parallel to the line $y = 3x + 7$.

⇨ Here, given line is $y = 3x + 7$

Comparing it with $y = m_1x + c$ then $m_1 = 3$.

If the lines are perpendicular then,

$m_1 \times m_2 = -1$

or, $3 \times m_2 = -1$

$\therefore\ m_2 = -\frac{1}{3}$

So, the slope of perpendicular line is $-\frac{1}{3}$.

If the lines are parallel then,

$m_1 = m_2$

or, $3 = m_2$

$\therefore\ m_2 = 3$

Thus, the slope of the parallel line is 3.

2. **तलका रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् । (Find the acute angle between the following pair of lines.)**

(a) **$y = \sqrt{3}x + 8$ and $y + 10 = 0$**

⇨ Here, $y = \sqrt{3}\,x + 8$ and $y + 10 = 0$

Now, taking first equation ,

$y = \sqrt{3}\,x + 8$

Comparing with $y = m_1x + c_1$

$m_1 = \sqrt{3}$

Taking second equation ,

$y + 10 = 0$

or, $y = 0.\,x - 10$

Comparing with $y = m_2x + c_2$

$\therefore\ m_2 = 0$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \pm\frac{\sqrt{3} - 0}{1 + \sqrt{3}.0}$$

$$= \pm\sqrt{3}$$

For acute angle, taking (+) ve sign,

or, $\tan\theta = \tan 60°$

$\therefore\ \theta = 60°$

Thus, the required acute angle is 60°.

(b) **$x - y - 5 = 0$ and $x - 7y + 7 = 0$**

⇨ Here, $x - y - 5 = 0$ and $x - 7y + 7 = 0$

Now, taking first equation, $x - y - 5 = 0$

or, $-y = -x + 5$

or, $y = x - 5$

Comparing with $y = m_1x + c_1$ then $m_1 = 1$

Taking second equation,

$x - 7y + 7 = 0$

or, $-7y = -x - 7$

or, $y = \frac{1}{7}x + 1$

Comparing with $y = m_2\,x + c_2$ then $m_2 = \frac{1}{7}$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2} = \pm\frac{1 - \frac{1}{7}}{1 + 1\,.\frac{1}{7}} = \pm\frac{\frac{7-1}{7}}{\frac{7+1}{7}} = \pm\frac{6}{8} = \pm\frac{3}{4}$$

For acute angle, taking (+) ve sign,

or, $\tan\theta = \pm\frac{3}{4}$

$\therefore\ \theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$

Thus, the required acute angle is 36.87°.

(c) **$3x + 4y + 4 = 0$ and $5x + 12y + 4 = 0$**

⇨ Here, $3x + 4y + 4 = 0$ and $5x + 12y + 4 = 0$

Taking first equation,

$3x + 4y + 4 = 0$

or, $4y = -3x - 4$

or $y = -\frac{3}{4}x - 1$

Comparing with $y = m_1x + c_1$, $m_1 = -\frac{3}{4}$

Taking second equation, $5x + 12y + 4 = 0$

or, $12y = -5x - 4$

or, $y = -\frac{5}{12}x - \frac{1}{3}$

Comparing with $y = m_2x + c_2$, $m_2 = -\frac{5}{12}$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2} = \pm\frac{\frac{-3}{4} - \left(\frac{-5}{12}\right)}{1 + \left(\frac{-3}{4}\right)\left(\frac{-5}{12}\right)}$$

$$= \pm\frac{\frac{-9+5}{12}}{\frac{48+15}{48}} = \pm\frac{-4}{1} \times \frac{4}{63} = \pm\frac{16}{63}$$

Taking positive sign, $\tan\theta = \frac{16}{63}$

$\therefore\ \theta = \tan^{-1}\left(\frac{16}{63}\right) = 14.25°$

Thus, the required acute angle is 14.25°.

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(d) $y-\sqrt{3}x-4=0$ and $x-\sqrt{3}y-5=0$

⇨ Here, $y-\sqrt{3}x-4=0$ and $x-\sqrt{3}y=5$

Taking first equation,

$y=\sqrt{3}x+4$

Comparing with $y=m_1x+c_1$, $m_1=\sqrt{3}$

Taking second equation ,

$x-\sqrt{3}y=5$

or, $-\sqrt{3}y=-x+5$

or, $y=\frac{1}{\sqrt{3}}x+\frac{5}{-\sqrt{3}}$

Comparing with $y=m_2x+c_2$, $m_2=\frac{1}{\sqrt{3}}$

We have,

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}.\frac{1}{\sqrt{3}}}$$

$$=\pm\frac{3-1}{\sqrt{3}\times 2}=\pm\frac{2}{2\sqrt{3}}=\pm\frac{1}{\sqrt{3}}$$

For acute angle, taking (+) ve sign; $\tan\theta=\frac{1}{\sqrt{3}}$

or, $\tan\theta=\tan 30°$

$\therefore$ $\theta=30°$

Thus, the required acute angle is 30°.

(e) $\sqrt{3}x-y+6=0$ and $y+3=0$

⇨ Here, $\sqrt{3}x-y+6=0$ and $y+3=0$

Taking first equation,

$\sqrt{3}x-y+6=0$

or, $-y=-\sqrt{3}x-6$

or, $y=\sqrt{3}x+6$

Comparing with $y=m_1x+c_1$,

$m_1=\sqrt{3}$

Taking second equation ,

$y+3=0$

or, $y=0\,.\,x-3$

Comparing with $y=m_2x+c_2$,

$m_2=0$

We have,

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{\sqrt{3}-0}{1+\sqrt{3}\,.0}$$

$$=\pm\frac{\sqrt{3}}{1}$$

$$=\pm\sqrt{3}$$

For acute angle; taking (+)ve sign,

$\tan\theta=\sqrt{3}$

or, $\tan\theta=\tan 60°$

$\therefore$ $\theta=60°$

Thus, the required acute angle is 60°.

3. तलका रेखाहरूबिचको अधिककोण पत्ता लगाउनुहोस् । (Find the obtuse angle between the following pair of lines.)

(a) $3x+2y-1=0$ and $2x+3y+4=0$

⇨ Here, $3x+2y-1=0$ and $2x+3y+4=0$

Taking first equation,

$3x+2y-1=0$

or, $2y=-3x+1$

or, $y=-\frac{3}{2}x+\frac{1}{2}$

Comparing with $y=m_1x+c_1$, $m_1=-\frac{3}{2}$

Taking second equation ,

$2x+3y+4=0$

or, $3y=-2x-4$

or, $y=-\frac{2}{3}x-\frac{4}{3}$

Comparing with $y=m_2x+c_2$, $m_2=-\frac{2}{3}$

We have,

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}$$

$$=\pm\frac{-\frac{3}{2}-\left(\frac{-2}{3}\right)}{1+\left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right)}$$

$$=\pm\frac{\frac{-9+4}{6}}{(1+1)}=\pm\frac{-5}{6}\times\frac{1}{2}=\pm\frac{5}{12}$$

For obtuse angle, taking negative sign,

$\tan\theta=-\frac{5}{12}$

$=-0.416$

$=\tan 157°$ (from table)

$\therefore$ $\theta=157°$

Thus, the required obtuse angle is 157°.

(b) $2x-7y+11=0$ and $x-3y-8=0$

⇨ Here, $2x-7y+11=0$ and $x-3y-8=0$

Taking first equation ,

$2x-7y+11=0$

or, $-7y=-2x-11$

or, $y=\frac{2}{7}x+\frac{11}{7}$

Comparing with $y=m_1x+c_1$, $m_1=\frac{2}{7}$

Taking second equation,

$x-3y-8=0$

or, $-3y=-x+8$

or, $y=\frac{1}{3}x-\frac{8}{3}$

Comparing with $y=m_2x+c_2$, $m_2=\frac{1}{3}$

We have,

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{\frac{2}{7}-\frac{1}{3}}{1+\frac{2}{7}\times\frac{1}{3}}$$

$$=\pm\frac{\frac{6-7}{21}}{\frac{21+2}{21}}=\pm\frac{-1}{23}=\pm\frac{1}{23}$$

For obtuse angle, taking (–)ve sign,

$\tan\theta=-\frac{1}{23}$

$=-0.043$

$=\tan(180°-2°)$

$=\tan 178°$

$\therefore$ $\tan\theta=\tan 178°$

$\therefore$ $\theta=178°$

Thus, the required obtuse angle is 178°.

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(c) $2x + 3y = 4$ and $x + 2y = 3$

⇨ Here, $2x + 3y = 4$ and $x + 2y = 3$

From first equation,

$2x + 3y = 4$

or, $3y = -2x + 4$

or, $y = -\frac{2}{3}x + \frac{4}{3}$

Comparing with $y = m_1x + c_1$, $m_1 = -\frac{2}{3}$

From second equation,

$x + 2y = 3$

or, $2y = -x + 3$

or, $y = -\frac{1}{2}x + \frac{3}{2}$

Comparing with $y = m_2x + c_2$, $m_2 = -\frac{1}{2}$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \pm\frac{\frac{-2}{3} - \left(\frac{-1}{2}\right)}{1 + \left(\frac{-2}{3}\right)\left(\frac{-1}{2}\right)}$$

$$= \pm\frac{\frac{-2}{3} + \frac{1}{2}}{1 + \frac{1}{3}}$$

$$= \pm\frac{\frac{-4+3}{6}}{\frac{3+1}{3}} = \pm\frac{-1}{6} \times \frac{3}{4} = \pm\frac{1}{8}$$

For obtuse angle, taking (–)ve sign,

$\tan\theta = -\frac{1}{8}$

$= -0.125$

$= \tan(180° - 7°)$

or, $\theta = 180° - 7° = 173°$

Thus, the required obtuse angle is 173°.

(e) $y = \sqrt{3}x + 5$ and $y + 10 = 0$

⇨ Here, $y = \sqrt{3}x + 5$ and $y + 10 = 0$

Taking first equation,

$y = \sqrt{3}x + 5$

Comparing with $y = m_1x + c_1$ then $m_1 = \sqrt{3}$

Taking second equation,

$y + 10 = 0$

$y = 0.x - 10$

Comparing with $y = m_2x + c_2$ then $m_2 = 0$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \pm\frac{\sqrt{3} - 0}{1 + (\sqrt{3})(0)}$$

$$= \pm\frac{\sqrt{3}}{1} = \pm(\sqrt{3})$$

For obtuse angle, taking –ve sign,

or, $\tan\theta = -\sqrt{3}$

$= \tan(180° - 60°)$

$= \tan 120°$

$\therefore\ \theta = 120°$

Thus, the required obtuse angle is 120°.

(d) $2x + y = 3$ and $3x + 2y = 1$

⇨ Here, $2x + y = 3$ and $3x + 2y = 1$

Taking first equation,

$2x + y = 3$

or, $y = -2x + 3$

Comparing with $y = m_1x + c_1$, $m_1 = -2$

Taking second equation,

$3x + 2y = 1$

or, $2y = -3x + 1$

or, $y = -\frac{3}{2}x + \frac{1}{2}$

Comparing with $y = m_2x + c_2$, $m_2 = -\frac{3}{2}$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1\,m_2}$$

$$= \pm\frac{-2 - \left(-\frac{3}{2}\right)}{1 + (-2)\left(-\frac{3}{2}\right)}$$

$$= \pm\frac{-2 + \frac{3}{2}}{1 + 3}$$

$$= \pm\frac{-4+3}{2} \times \frac{1}{4}$$

$$= \pm(-1) \times \frac{1}{8}$$

$$= \pm\left(-\frac{1}{8}\right)$$

$$= \pm\frac{1}{8}$$

For obtuse angle, taking –ve sign,

or, $\tan\theta = -\frac{1}{8}$

$= \tan(180° - 7.12°)$

$= \tan 173°$

$\therefore\ \theta = 173°$

Thus, the required obtuse angle is 173°.

4. तलका रेखाहरू आपसमा समानान्तर छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the following lines are parallel to each other.

(a) $x - 2y + 3 = 0$ and $2x - 4y + 9 = 0$

⇨ Here, $x - 2y + 3 = 0$ and $2x - 4y + 9 = 0$

Taking first equation,

$x - 2y + 3 = 0$

or, $-2y = -x - 3$

or, $y = \frac{1}{2}x + \frac{3}{2}$

Comparing with $y = m_1x + c_1$, $m_1 = \frac{1}{2}$

Taking second equation,

$2x - 4y + 9 = 0$

or, $-4y = -2x - 9$

or, $y = \frac{1}{2}x + \frac{9}{4}$

Comparing with $y = m_2x + c_2$, $m_2 = \frac{1}{2}$

Here, $m_1 = m_2 = \frac{1}{2}$

Thus, given pair of lines are parallel.

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(b) $3x - 4y = 7$ and $4y = 3x + 11$

⇨ Here, $3x - 4y = 7$ and $4y = 3x + 11$

Taking first equation,

$3x - 4y = 7$

or, $-4y = -3x + 7$

or, $y = \frac{3}{4}x + \left(\frac{7}{-4}\right)$

Comparing with $y = m_1x + c_1$, $m_1 = \frac{3}{4}$

Taking second equation,

$4y = 3x + 11$

or, $y = \frac{3}{4}x + \frac{11}{4}$

Comparing with $y = m_2x + c_2$, $m_2 = \frac{3}{4}$

Here, $m_1 = m_2 = \frac{3}{4}$

Thus, given pair of lines are parallel.

(d) $2x - 3y = 5$ and $2x - 3y - 7 = 0$

⇨ Here, $2x - 3y = 5$ and $2x - 3y - 7 = 0$

Taking first equation,

$2x - 3y = 5$

or, $3y = 2x - 5$

or, $y = \frac{2}{3}x - \frac{5}{3}$

Comparing with $y = m_1x + c_1$, $m_1 = \frac{2}{3}$

Again, taking second equation,

$2x - 3y - 7 = 0$

or, $2x - 7 = 3y$

or, $y = \frac{2}{3}x - \frac{7}{3}$

Comparing with $y = m_2x + c_2$, $m_2 = \frac{2}{3}$

Here, $m_1 = m_2 = \frac{2}{3}$

Thus, given pair of lines are parallel.

(b) $3y - 2x = 1$ and $3x + 2y = 15$

⇨ Here, $3y - 2x = 1$ and $3x + 2y = 15$

Taking first equation, $3y - 2x = 1$

or, $3y = 2x + 1$

or, $y = \frac{2}{3}x + \frac{1}{3}$

$\therefore$ $m_1 = \frac{2}{3}$

Taking second equation,

$3x + 2y = 15$

or, $2y = -3x + 15$

or, $y = -\frac{3}{2}x + \frac{15}{2}$

$\therefore$ $m_2 = \frac{-3}{2}$

Now, $m_1m_2 = \frac{2}{3} \times \left(-\frac{3}{2}\right) = -1$

Thus, $m_1 \times m_2 = -1$ shows that the given lines are perpendicular to each other.

(c) $x - 5y - 3 = 0$ and $10y = 2x + 13$

⇨ Here, $x - 5y - 3 = 0$ and $10y = 2x + 13$

Taking first equation,

$x - 5y - 3 = 0$

or, $-5y = -x + 3$

or, $y = \frac{1}{5}x - \frac{3}{5}$

Comparing with $y = m_1x + c_1$, $m_1 = \frac{1}{5}$

Again, taking second equation,

$10y = 2x + 13$

or, $y = \frac{1}{5}x + \frac{13}{10}$

Comparing with $y = m_2x + c_2$, $m_2 = \frac{1}{5}$

Here, $m_1 = m_2 = \frac{1}{5}$

Thus, given pair of lines are parallel.

5. तलका रेखाहरू आपसमा लम्ब छन् भनी प्रमाणित गर्नुहोस् ।
Prove that the following lines are perpendicular to each other.

(a) $5x + 12y = 0$ and $12x - 5y = 17$

⇨ Here, $5x + 12y = 0$ and $12x - 5y = 17$

Taking first equation , $5x + 12y = 0$

or, $12y = -5x$

or, $y = -\frac{5}{12}x + \frac{0}{12}$

$\therefore$ $m_1 = -\frac{5}{12}$

Taking second equation , $12x - 5y = 17$

or, $-5y = -12x + 17$

or, $y = \frac{12}{5}x - \frac{17}{5}$

$\therefore$ $m_2 = \frac{12}{5}$

Now, $m_1.m_2 = -\frac{5}{12} \times \frac{12}{5} = -1$

Thus, $m_1 \times m_2 = -1$ shows that the given lines are perpendicular to each other.

(c) $4x - 3y - 3 = 10$ and $3x + 4y = 18$

⇨ Here, $4x - 3y - 3 = 10$ and $3x + 4y = 18$

Taking first equation,

$-3y = -4x + 10 + 3$

or, $y = \frac{4}{3}x - \frac{13}{3}$

$\therefore$ $m_1 = \frac{4}{3}$

Taking second equation,

$3x + 4y = 18$

or, $4y = -3x + 18$

or, $y = -\frac{3}{4}x + \frac{9}{2}$

$\therefore$ $m_2 = -\frac{3}{4}$

Now, $m_1m_2 = \frac{4}{3} \times -\frac{3}{4} = -1$

Thus, $m_1 \times m_2 = -1$ shows that the given lines are perpendicular to each other.

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(d) $7x + 8y = 63$ **and** $8x - 7y = 1$

⇨ Here, $7x + 8y = 63$ ....(i) and $8x - 7y = 1$ ....(ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{7}{8}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{8}{-7}$$

$$= \frac{8}{7}$$

Now, $m_1 \times m_2 = -\frac{7}{8} \times \frac{8}{7} = -1$

Thus, $m_1 \times m_2 = -1$ shows that the given lines are perpendicular to each other.

6. तल दिइएको अवस्थामा a को मान पत्ता लगाउनुहोस् । (Find the value of a in the following given conditions.)

(a) $4x + 3y = 0$ **र** $3x + ay = 5$ **आपसमा लम्ब छन् ।**
$4x + 3y = 0$ and $3x + ay = 5$ are perpendicular to each other.

⇨ Here, $4x + 3y = 0$ ....(i) and
$3x + ay = 5$ ....(ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{4}{3}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{3}{a}$$

Since the lines are perpendicular,

So, $m_1 \times m_2 = -1$

or, $-\frac{4}{3} \times (-)\frac{3}{a} = -1$

or, $\frac{4}{a} = -1$

$\therefore$ $a = -4$

Thus, the value of a is – 4.

(b) $ax + 5y = 16$ **र** $6x + 10y - 9 = 0$ **आपसमा लम्ब छन् ।**
$ax + 5y = 16$ and $6x + 10y - 9 = 0$ are perpendicular to each other.

⇨ Here, $ax + 5y = 16$ ....(i) and
$6x + 10y = 9$ ....(ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}} = -\frac{a}{5}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}} = -\frac{6}{10}$$

Since the lines are perpendicular,

So, $m_1 \times m_2 = -1$

or, $-\frac{a}{5} \times (-)\frac{6}{10} = -1$

or, $\frac{6a}{50} = -1$

or, $\frac{3a}{25} = -1$

$\therefore$ $a = -\frac{25}{3}$

Thus, the value of a is $-\frac{25}{3}$.

(c) $ax + 3y = 4$ **र** $3x + 9y = 5$ **आपसमा समानान्तर छन् ।**
$ax + 3y = 4$ and $3x + 9y = 5$ are parallel to each other.

⇨ Here, $ax + 3y = 4$ ....(i) and
$3x + 9y = 5$ ....(ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{a}{3}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{3}{9}$$

Since the lines are parallel,

So, $m_1 = m_2$

or, $-\frac{a}{3} = -\frac{3}{9}$

$\therefore$ $a = 1$

Thus, the value of a is 1.

(d) $5x + ay - 6 = 0$ **र** $5x - 3y - 8 = 0$ **आपसमा समानान्तर छन् ।**
$5x + ay - 6 = 0$ and $5x - 3y - 8 = 0$ are parallel to each other.

⇨ Here, $5x + ay = 6$ ....(i) and
$5x - 3y = 8$ ....(ii)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{5}{a}$$

Slope of line (ii);

$$m_2 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{5}{-3} = \frac{5}{3}$$

Since the lines are parallel,

So, $m_1 = m_2$

or, $-\frac{5}{a} = \frac{5}{3}$

$\therefore$ $a = -3$

Thus, the value of a is – 3.

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7\. (a) **बिन्दु (3, 4) भएर जाने र रेखा 3*x* + 4*y* = 12 सँग समानान्तर हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line passing through the point (3, 4) and parallel to the line 3*x* + 4*y* = 12.

⇨ Here, Taking, $3x + 4y = 12$

or, $4y = -3x + 12$

or, $y = -\frac{3}{4}x + 3$

$\therefore \quad m = -\frac{3}{4}$

We have, equation of st. line passing through the point (3, 4) and slope 'm' is,

$y - y_1 = m\ (x - x_1)$

or, $y - 4 = -\frac{3}{4}(x - 3)$

or, $4y - 16 = -3x + 9$

or, $3x + 4y - 25 = 0$

Thus, $3x + 4y = 25$ is the required equation.

(c) **बिन्दुहरू (2, 3) र (3, – 1) जोड्ने रेखासँग समानान्तर हुने र बिन्दु (2, 1) भएर जाने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line which is parallel to the line joining the points (2, 3) and (3, – 1) and passing through the point (2, 1).

⇨ Here, Slope of line which passes through (2, 3) and (3, –1) is;

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-3}{3-2} = \frac{-4}{1} = -4$

Line passing through a point (2, 1) is,

$y - y_1 = m\ (x - x_1)$

or, $y - 1 = -4\ (x - 2)$

or, $y - 1 = -4x + 8$

Thus, $4x + y = 9$ is the required equation.

8\. (a) **रेखा 2*x* + 5*y* + 31 = 0 सँग लम्ब हुने र बिन्दु (2, 5) भएर जाने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line which is perpendicular to the line 2*x* + 5*y* + 31 = 0 and passing through the point (2, 5).

⇨ Here, taking $2x + 5y + 31 = 0$

or, $5y = -2x - 31$

or, $y = -\frac{2}{5}x - \frac{31}{5}$

$\therefore \quad m_1 = -\frac{2}{5}$

Since the given line and the required line are perpendicular.

So, $m_1 m_2 = -1$

$\therefore \quad m_2 = -\frac{1}{m_1} = -\frac{1}{-\frac{2}{5}} = \frac{5}{2}$

Now, we have, equation of line passing through the point (2, 5) and having slope $\frac{5}{2}$ is,

$y - y_1 = m_2\ (x - x_1)$

or, $y - 5 = \frac{5}{2}\ (x - 2)$

or, $2y - 10 = 5x - 10$

Thus, $5x - 2y = 0$ is the required equation.

(b) **बिन्दु (2, 5) भएर जाने र रेखा 2*x* + 5*y* + 31 = 0 सँग समानान्तर हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line passing through the point (2, 5) and parallel to the line 2*x* + 5*y* + 31 = 0.

⇨ Here,

Taking, $2x + 5y + 31 = 0$

or, $5y = -2x - 31$

or, $y = \frac{-2}{5}x - \frac{31}{5}$

$\therefore \quad m = -\frac{2}{5}$

We have,

Equation of straight line passing through the point (2, 5) and having slope $-\frac{2}{5}$ is,

$y - y_1 = m\ (x - x_1)$

or, $y - 5 = -\frac{2}{5}(x - 2)$

or, $5y - 25 = -2x + 4$

Thus, $2x + 5y = 29$ is the required equation.

(d) **बिन्दुहरू (– 7, 5) र (2, 2) जोड्ने रेखासँग समानान्तर हुने र बिन्दु (– 4, 1) भएर जाने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line which is parallel to the line joining the points (– 7 , 5) and (2, 2) and passing through the point (– 4, 1).

⇨ Here, Slope of line which passes through (–7, 5) and (2, 2) is,

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{2+7} = -\frac{3}{9} = -\frac{1}{3}$

Line passing through a point (– 4, 1) is,

$y - y_1 = m\ (x - x_1)$

or, $y - 1 = -\frac{1}{3}(x + 4)$

or, $3y - 3 = -x - 4$

or, $x + 3y = -1$

Thus, $x + 3y + 1 = 0$ is the required equation.

(b) **बिन्दु (2, – 4) भई जाने र रेखा 5*x* + 7*y* + 12 = 0 सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a straight line which is perpendicular to the line 5*x* + 7*y* + 12 = 0 and passing through the point (2, – 4).

⇨ Here,

$5x + 7y + 12 = 0$

or, $7y = -5x - 12$

or, $y = -\frac{5}{7}x - \frac{12}{7}$

$\therefore \quad m_1 = \frac{-5}{7}$

Since the given line and the required line are perpendicular. So, $m_1 m_2 = -1$

$m_2 = -\frac{1}{m_1} = -\frac{1}{-\frac{5}{7}} = \frac{7}{5}$

Now, we have, equation of line passing through the point (2, – 4) and having slope $\frac{7}{5}$ is; $y - y_1 = m_2\ (x - x_1)$

or, $y + 4 = \frac{7}{5}\ (x - 2)$

or, $5y + 20 = 7x - 14$

Thus, $7x - 5y = 34$ is the required equation.

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(c) **बिन्दुहरू (– 4, – 7) र (5, – 2) जोड्ने रेखासँग लम्ब भई बिन्दु (2, 3) भएर जाने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of a straight line which is perpendicular to the line joining the points (– 4, – 7) and (5, – 2) and passing through the point (2, 3).

⇨ Here, Slope of the line which joins (– 4, – 7) and (5, – 2) is;

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 + 7}{5 + 4} = \frac{5}{9}$$

Now, $m_1m_2 = -1$

$$\text{or,}\quad m_2 = -\frac{1}{m_1} = \frac{-1}{\frac{5}{9}} = \frac{-9}{5}$$

Line passing through a point (2, 3) is,

$$y - y_1 = m_2(x - x_1)$$

$$\text{or,}\quad y - 3 = -\frac{9}{5}(x - 2)\ [\because (x_1, y_1) = (2, 3)]$$

or, $5y - 15 = -9x + 18$

Thus, $9x + 5y = 33$ is the required equation.

9. (a) **बिन्दु (1, – 4) बाट जाने र रेखा $2x + 3y = 5$ सँग 45° कोण बनाउने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**
Find the equations of lines passing through the point (1, – 4) and making and angle of 45° with the line $2x + 3y = 5$.

⇨ Here, given equation of line is;

$2x + 3y = 5$ ....(i)

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}} = -\frac{2}{3}$$

Let, m_2 be the slope of required line then,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$\text{or,}\quad \tan 45° = \pm\frac{-\frac{2}{3} - m_2}{1 - \frac{2}{3}m_2}$$

$$\text{or,}\quad 1 = \pm\left(\frac{-2 - 3m_2}{3 - 2m_2}\right)$$

$$\text{Taking +ve sign,}\ 1 = \frac{-2 - 3m_2}{3 - 2m_2}$$

or, $3 - 2m_2 = -2 - 3m_2$

∴ $m_2 = -5$

$$\therefore\quad m_2 = \frac{1}{5}$$

$$\text{Taking –ve sign,}\ 1 = -\left(\frac{-2 - 3m_2}{3 - 2m_2}\right)$$

or, $3 - 2m_2 = 2 + 3m_2$

or, $1 = 5m_2$

When $m_2 = -5$ and passing point is (1, – 4) then,

$y + 4 = -5(x - 1)$

or, $y + 4 = -5x + 5$

∴ $5x + y = 1$

When $m_2 = \frac{1}{5}$ and passing point is (1, – 4) then,

$$y + 4 = \frac{1}{5}(x - 1)$$

or, $5y + 20 = x - 1$

∴ $x - 5y = 21$

Thus, $5x + y = 1$ and $x - 5y = 21$ are the required equations.

(d) **बिन्दु (2, – 3) भई जाने र बिन्दुहरू (5, 7) र (– 6, 3) जोड्दा हुने रेखासँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of a straight line passing through the point (2, – 3) and perpendicular to the line joining the points (5, 7) and (– 6, 3).

⇨ Here, slope of the line which passes through the points, (5, 7) and (– 6, 3) is;

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{-6 - 5} = \frac{-4}{-11} = \frac{4}{11}$$

Now, $m_1m_2 = -1$

$$\text{or,}\quad m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{4}{11}} = -\frac{11}{4}$$

Line passing through a point (2, – 3) is,

$$y - y_1 = m_2(x - x_1)$$

$$\text{or,}\quad y + 3 = \frac{-11}{4}(x - 2)\ [\because (x_1, y_1) = (2, -3)]$$

or, $4y + 12 = -11x + 22$

Thus, $11x + 4y = 10$ is the required equation.

(b) **रेखा $6x + 5y - 1 = 0$ सँग 45° कोण बनाउने र बिन्दु (2, – 1) भएर जाने दुई रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of two lines which make an angle of 45° with the line $6x + 5y - 1 = 0$ and passing through the point (2, – 1).

⇨ Here, for the slope of line, $6x + 5y - 1 = 0$

or, $5y = -6x + 1$

$$\text{or,}\quad y = -\frac{6}{5}x + \frac{1}{5}$$

$$\therefore\quad m_1 = -\frac{6}{5}$$

Let, m_2 be the slope of the other line.

$$\text{We have,}\ \tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$\text{or,}\quad \tan 45° = \pm\frac{\frac{-6}{5} - m_2}{1 - \frac{6}{5}.m_2}$$

$$\text{or,}\quad 1 = \pm\frac{\frac{-6 - 5m_2}{5}}{\frac{5 - 6\,m_2}{5}}$$

or, $5 - 6m_2 = \pm(-6 - 5m_2)$

Taking + ve sign, $5 - 6m_2 = -6 - 5m_2$

or, $-m_2 = -11$

or, $m_2 = 11$

Taking –ve sign,

$5 - 6m_2 = -(-6 - 5m_2)$

or, $5 - 6m_2 = 6 + 5m_2$

or, $-11m_2 = 1$

$$\text{or,}\quad m_2 = -\frac{1}{11}$$

A line passing through the point (2, – 1) and $m_2 = -\frac{1}{11}$ is

$$y + 1 = -\frac{1}{11}(x - 2)$$

or, $11y + 11 = -x + 2$

or, $x + 11y + 9 = 0$

A line passing through the point (2, – 1) and $m_2 = 11$ is

$y + 1 = 11(x - 2)$

or, $y + 1 = 11x - 22$

Thus, $11x - y = 23$ and $x + 11y + 9 = 0$ are the required equations.

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(c) रेखा $2x - 3y + 10 = 0$ सँग 45° कोण बनाई बिन्दु (2, – 1) भएर जाने दुई रेखाहरूको समीकरण निकाल्नुहोस् ।

Find the equation of two lines which make an angle of 45° with the line $2x - 3y + 10 = 0$ and passing through the point (2, – 1).

⇨ Here, given equation of line is;

$$2x - 3y + 10 = 0 \text{ ....(i)}$$

Slope of line (i);

$$m_1 = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{2}{-3}$$

$$= \frac{2}{3}$$

We have, angle between the lines (θ) = 45°

Let, m_2 be the slope of required line then by using formula,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\text{or,}\quad \tan 45° = \pm\frac{\frac{2}{3} - m_2}{1 + \frac{2}{3}m_2}$$

$$\text{or,}\quad 1 = \pm\frac{\frac{2 - 3m_2}{3}}{\frac{3 + 2m_2}{3}}$$

$$\text{or,}\quad 1 = \pm\frac{2 - 3m_2}{3 + 2m_2}$$

$$\text{or,}\quad 3 + 2m_2 = \pm(2 - 3m_2)$$

Taking + ve sign,

$$3 + 2m_2 = 2 - 3m_2$$

$$\text{or,}\quad 5m_2 = -1$$

$$\therefore\quad m_2 = -\frac{1}{5}$$

Taking –ve sign,

$$3 + 2m_2 = -2 + 3m_2$$

$$\text{or,}\quad 5 = m_2$$

$$\therefore\quad m_2 = 5$$

Now, equation of line passing through (2, – 1) and slope $= -\frac{1}{5}$ is,

$$y - y_1 = m(x - x_1)$$

$$\text{or,}\quad y + 1 = -\frac{1}{5}(x - 2)$$

$$\text{or,}\quad 5y + 5 = -x + 2$$

$$\therefore\quad x + 5y + 3 = 0$$

Again, equation of line passing through (2, – 1) and slope = 5 is,

$$y - y_1 = m(x - x_1)$$

$$\text{or,}\quad y + 1 = 5(x - 2)$$

$$\text{or,}\quad y + 1 = 5x - 10$$

$$\therefore\quad 5x - y = 11$$

Thus, the required equation of lines are;

$x + 5y + 3 = 0$ and $5x - y = 11$.

(d) उद्गम बिन्दुबाट जाने र रेखा $x + y + 3 = 0$ सँग 60° को कोण बनाउने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of lines passing through the origin and making an angle of 60° with the line $x + y + 3 = 0$.

⇨ Here, given equation of line is;

$$x + y + 3 = 0 \text{ ....(i)}$$

$$\text{Slope of line } (m_1) = -\frac{\text{Coefficient of x}}{\text{Coefficient of y}}$$

$$= -\frac{1}{1}$$

$$= -1$$

We have, angle between the lines (θ) = 60°

Using formula,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\text{or,}\quad \tan 60° = \pm\frac{-1 - m_2}{1 - 1\,m_2}$$

$$\text{or,}\quad \sqrt{3} = \pm\frac{-1 - m_2}{1 - m_2}$$

Taking (+) ve sign,

$$\sqrt{3}\,(1 - m_2) = -1 - m_2$$

$$\text{or,}\quad \sqrt{3} - \sqrt{3}m_2 = -1 - m_2$$

$$\text{or,}\quad \sqrt{3} + 1 = \sqrt{3}m_2 - m_2$$

$$\text{or,}\quad \sqrt{3} + 1 = (\sqrt{3} - 1)\,m_2$$

$$\text{or,}\quad \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = m_2$$

$$\text{or,}\quad m_2 = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1} = \frac{4 + 2\sqrt{3}}{2}$$

$$\therefore\quad m_2 = 2 + \sqrt{3}$$

Taking (–) ve sign,

$$\sqrt{3}\,(1 - m_2) = -(-1 - m_2)$$

$$\text{or,}\quad \sqrt{3} - \sqrt{3}m_2 = 1 + m_2$$

$$\text{or,}\quad \sqrt{3} - 1 = m_2 + \sqrt{3}\,m_2$$

$$\text{or,}\quad \sqrt{3} - 1 = m_2\,(1 + \sqrt{3})$$

$$\text{or,}\quad \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = m_2$$

$$\text{So,}\quad m_2 = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$\text{or,}\quad m_2 = \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2}$$

$$\therefore\quad m_2 = 2 - \sqrt{3}$$

Now, equation of line passing through (0, 0) and slope $= 2 + \sqrt{3}$ is;

$$y - y_1 = m(x - x_1)$$

$$\text{or,}\quad y - 0 = 2 + \sqrt{3}\,(x - 0)$$

$$\therefore\quad y = (2 + \sqrt{3})\,x$$

Again, equation of line passing through (0, 0) and slope $= 2 - \sqrt{3}$ is;

$$y - y_1 = m(x - x_1)$$

$$\text{or,}\quad y - 0 = (2 - \sqrt{3})\,x$$

$$\therefore\quad y = (2 - \sqrt{3})x$$

Thus, the required equation of lines are;

$y = (2 + \sqrt{3})x$ and $y = (2 - \sqrt{3})\,x$.

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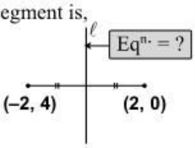
10. (a) बिन्दुहरू (– 2, 4) र (2, 0) जोड्ने रेखाको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of perpendicular bisector of the line joining the points (– 2, 4) and (2, 0).

⇨ Here, slope of given line segment is,



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{2+2}$$

∴ $m_1 = -1$

We have,

$m_1 \times m_2 = -1$

or, $-1 \times m_2 = -1$ [Where m_2 is the slope of required line]

∴ $m_2 = 1$

Midpoint of given line segment is,

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2+2}{2}, \frac{4+0}{2}\right)$$

∴ $(x, y) = (0, 2)$

Now, equation of line passing through (0, 2) and slope 1 is;

$y - y_1 = m(x - x_1)$

or, $y - 2 = 1(x - 0)$

or, $y - 2 = x$

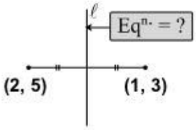
∴ $x - y + 2 = 0$

Thus, the required equation of line is $x - y + 2 = 0$.

(c) बिन्दुहरू (2, 5) र (1, 3) जोड्ने रेखाको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of perpendicular bisector of the line joining the points (2, 5) and (1, 3).

⇨ Here, slope of given line segment is,



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{1-2} = \frac{-2}{-1} = 2$$

Slope of perpendicular bisector $(m_2) = -\frac{1}{m_1} = -\frac{1}{2}$

We have, midpoint of given line segment is,

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+1}{2}, \frac{5+3}{2}\right)$$

∴ $(x, y) = \left(\frac{3}{2}, 4\right)$

Now, equation of line passing through $\left(\frac{3}{2}, 4\right)$ and having slope $(m) = -\frac{1}{2}$ is,

$y - y_1 = m(x - x_1)$

or, $y - 4 = -\frac{1}{2}\left(x - \frac{3}{2}\right)$

or, $2y - 8 = -\left(\frac{2x-3}{2}\right)$

or, $4y - 16 = -2x + 3$

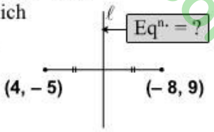
or, $2x + 4y = 19$

Thus, the required equation of perpendicular bisector is $2x + 4y = 19$.

(b) बिन्दुहरू (4, – 5) र (– 8, 9) जोड्ने रेखाको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of perpendicular bisector of the line joining the points (4, – 5) and (– 8, 9).

⇨ Here, slope of the line which joins (4, – 5) and (– 8, 9) is,



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9+5}{-8-4} = \frac{14}{-12} = \frac{7}{-6}$$

Let, m_2 be the slope of second line,

Since they are perpendicular,

So, $m_1 m_2 = -1$

or, $\frac{7}{-6} m_2 = -1$

or, $m_2 = \frac{6}{7}$

Mid point of line segment joining (4, – 5) and (– 8, 9) is,

$$\text{Mid-point} = \left(\frac{4-8}{2}, \frac{-5+9}{2}\right) = \left(\frac{-4}{2}, \frac{4}{2}\right)$$

$= (-2, 2)$

Equation of a line passing through the point (– 2, 2) is,

$y - y_1 = m_2(x - x_1)$

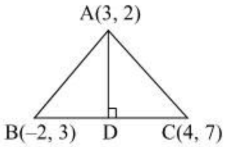
or, $y - 2 = \frac{6}{7}(x + 2)$ [∵ $(x_1, y_1) = (-2, 2)$]

or, $7y - 14 = 6x + 12$

Thus, $6x - 7y + 26 = 0$ is the required equation.

(d) तलको चित्रबाट रेखा AD को समीकरण पत्ता लगाउनुहोस् ।

Find the equation of line AD from the following figure.



⇨ Here, slope of BC

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4+2} = \frac{4}{6} = \frac{2}{3}$$

We have,

slope of AD × slope of BC = – 1

or, slope of AD × $\frac{2}{3} = -1$

∴ slope of AD = $-\frac{3}{2}$

Now, equation of AD passing through A(3, 2) and slope $-\frac{3}{2}$ is,

$y - y_1 = m(x - x_1)$

or, $y - 2 = -\frac{3}{2}(x - 3)$

or, $2y - 4 = -3x + 9$

or, $3x + 2y = 13$

Thus, the required equation of AD is $3x + 2y = 13$.

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OTHER IMPORTANT QUESTIONS

1. रेखा 2x + 3y + 5 = 0 सँग 45° को कोण बनाउने र बिन्दु (1, – 4) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् । Find the equations of the line that passes through the point (1, – 4) & are inclined at 45° to the line 2x + 3y + 5 = 0.

⇨ Here, for the slope of $2x + 3y + 5 = 0$

or, $3y = -2x - 5$

or, $y = -\frac{2}{3}x - \frac{5}{3}$

$\because \quad m_1 = \frac{-2}{3}$

Let m_2 be the slope of the other line.

We have, $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan 45° = \pm \frac{\frac{-2}{3} - m_2}{1 - \frac{2}{3}m_2}$

or, $1 = \pm \frac{\frac{-2-3m_2}{3}}{\frac{3-2m_2}{3}}$

$\therefore \quad 3 - 2m_2 = \pm(-2 - 3m_2)$

Taking +ve sign,

$3 - 2m_2 = -2 - 3m_2$

or, $m_2 = -5$

A line passing through a point,

$y - y_1 = m_2(x - x_1)$

or, $y + 4 = -5(x - 1)$ $[\because (x_1, y_1) = (1, -4)]$

or, $y + 4 = -5x + 5$

or, $5x + y = 1$

$\therefore \quad 5x + y - 1 = 0$

Taking –ve sign, $3 - 2m_2 = -(-2 - 3m_2)$

or, $3 - 2m_2 = 2 + 3m_2$

or, $-5m_2 = -1$

or, $m_2 = \frac{1}{5}$

A line passing through the point (1, – 4) and having slope $\frac{1}{5}$ is;

$y - y_1 = m_2(x - x_2)$

or, $y + 4 = \frac{1}{5}(x - 1)$ $[\because (x_1, y_1) = (1, -4)]$

$\therefore \quad x - 5y = 21$

Thus, $5x + y - 1 = 0$ and $x - 5y = 21$ are the required equations.

2. रेखा 6x + 5y –1 = 0 सँग 45° को कोण बनाउने र बिन्दु (2, – 1) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् । Find the equation of two lines which passes through (2 , – 1) & make an angle of 45° with the line 6x + 5y –1 = 0.

⇨ Here, $6x + 5y - 1 = 0$

For the slope of line, $6x + 5y - 1 = 0$

or, $5y = -6x + 1$

or, $y = -\frac{6}{5}x + \frac{1}{5}$

$\therefore \quad m_1 = -\frac{6}{5}$

Let m_2 be the slope of the other line.

We have, $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1m_2}$

or, $\tan 45° = \pm \frac{\frac{-6}{5} - m_2}{1 - \frac{6}{5}.m_2}$

or, $1 = \pm \frac{\frac{-6-5m_2}{5}}{\frac{5-6m_2}{5}}$

$\therefore \quad 5 - 6m_2 = \pm(-6 - 5m_2)$

Taking + ve sign,

$5 - 6m_2 = -6 - 5m_2$

or, $-m_2 = -11$

or, $m_2 = 11$

Taking –ve sign,

$5 - 6m_2 = -(-6 - 5m_2)$

or, $5 - 6m_2 = 6 + 5m_2$

or, $-11m_2 = 1$

or, $m_2 = -\frac{1}{11}$

A line passing through the point (2, –1) and

$m_1 = -\frac{1}{11}$ is;

$y + 1 = -\frac{1}{11}(x - 2)$

or, $11y + 11 = -x + 2$

$\therefore \quad x + 11y + 9 = 0$

A line passing through a point (2, –1) & $m_2 = 11$ is

$y + 1 = 11(x - 2)$

or, $y + 1 = 11x - 22$

$\therefore \quad 11x - y = 23$

Thus, $x + 11y + 9 = 0$ and $11x - y = 23$ are the required equations.

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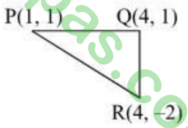
3. यदि P(1, 1), Q(4, 1) र R(4, –2) हरू ∆PQR का शीर्षबिन्दुहरू भए ∡PQR को नाप पत्ता लगाउनुहोस् ।

If P(1, 1), Q(4, 1) and R(4, –2) are the vertices of ∆PQR. Find the measure of ∡PQR.



⇨ Here, P(1, 1), Q(4, 1) and R(4, – 2) are the vertices of ∆PQR.

y-components of both co-ordinates is 1 in P and Q.

$\therefore$ $y = 1$ is the equation of PQ.

Again; Q(4,1) and R(4, – 2)

x-components of both co-ordinates is 4.

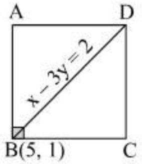
$\therefore$ $x = 4$ is the equation of QR.

We know that,

Angle between $x = a$ and $y = b$ is 90°.

So, the angle between $x = 4$ and $y = 1$ is 90°.

Thus, the value of ∡PQR is 90°.



4. ABCD एउटा वर्ग हो । यसको विकर्ण BD को समीकरण x – 3y = 2 भए AB र BC का समीकरणहरू पत्ता लगाउनुहोस् ।

ABCD is a square. If the equation of its diagonal BD is x – 3y = 2, find the equations of AB and BC.

⇨ Here, given equation of diagonal is $x - 3y = 2$

$$\text{slope of given diagonal } (m_1) = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$$

$$= -\frac{1}{-3}$$

$$= \frac{1}{3}$$

Let θ be an angle made by the diagonal with sides then, $\tan\theta = \pm\dfrac{m_1 - m_2}{1 + m_1m_2}$

$$\text{or, } \tan 45^0 = \pm\frac{\frac{1}{3} - m_2}{1 + \frac{1}{3}m_2}$$

$$= \pm\frac{\frac{1 - 3m_2}{3}}{\frac{3 + m_2}{3}}$$

$$= \pm\frac{1 - 3m_2}{3 + m_2}$$

or, $3 + m_2 = \pm(1 - 3m_2)$

Taking (+) ve sign	Taking (–) ve sign
$1 - 3m_2 = 3 + m_2$ or, $-2 = 4m_2$ $\therefore$ $m_2 = -\frac{1}{2}$	$3 + m_2 = -1 + 3m_2$ or, $-2m_2 = -4$ $\therefore$ $m_2 = 2$

We know that, equation of straight line with slope 'm' and passing through the point (x_1, y_1) is ; $y - y_1 = m(x - x_1)$

When slope is $-\frac{1}{2}$ and the passing point is (5, 1) then, $y - 1 = -\frac{1}{2}(x - 5)$

or, $2y - 2 = -x + 5$

$\therefore$ $x + 2y = 7$

When slope is 2 and the passing point is (5, 1) then, $y - 1 = 2(x - 5)$

or, $y - 1 = 2x - 10$

$\therefore$ $2x - y = 9$

Thus, the required equations of sides of square are; $x + 2y = 7$ and $2x - y = 9$.

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2. जोडा रेखाहरूको समीकरण
Pair of Straight Lines

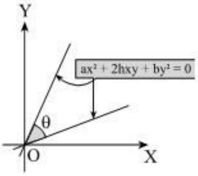
Formulae and Key points

रेखाहरूको समीकरण Equation of Lines		रेखाहरू बीचको कोण Angle between Lines	रेखाहरूको अवस्था (Condition of Lines)	
			सम्पाती हुने being coincident	लम्ब हुने being perpendicular
1.	$ax^2 + 2hxy + by^2 = 0$	$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a+b}\right)$	$h^2 = ab$	$a + b = 0$
2.	$ax^2 + 2hxy + by^2$ $+ 2gx + 2fy + c = 0$	$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a+b}\right)$	$h^2 = ab$	$a + b = 0$

3. $ax^2 + 2hxy + by^2 = 0$ ले जनाउने सीधारेखाहरू :
The straight lines represented by $ax^2 + 2hxy + by^2 = 0$ are:

(a) $ax + hy + y\sqrt{h^2 - ab} = 0$

(b) $ax + hy - y\sqrt{h^2 - ab} = 0$



QUESTIONS FROM SEE EXERCISE 2

A. VERY SHORT QUESTIONS

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):

A. समघातीय वर्ग समीकरण (Homogeneous equation)

⇨ Here, if the degree of each term of an equation is same then the equation is called homogeneous equation. The equation of the form $ax^2 + 2hxy + by^2 = 0$ is called the homogeneous equation of second degree in x and y which represent a pair of straight lines passing through the origin.

B. दुई डिग्रीको साधारण समीकरण (General equation of second degree)

⇨ Here, the equation of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is the general equation of second degree in x and y. It represents a pair of straight lines.

2. कस्तो अवस्थामा जोडा सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ लम्ब हुन्छन् ?
Under what condition, the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will be perpendicular?

⇨ Here, if $a + b = 0$ then the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular.

3. x र y मा रहेको उद्गम बिन्दु भएर जाने समघातीय वर्ग समीकरण के हुन्छ ?
What is the homogeneous equation of second degree in x and y representing a pair of straight lines through origin?

⇨ Here, the homogeneous equation of second degree in x and y is; $ax^2 + 2hxy + by^2 = 0$

4. $ax^2 + 2hxy + by^2 = 0$ सँग लम्ब भई उद्गम बिन्दु भएर जाने जोडा रेखाहरूको एकल समीकरण के हुन्छ ?
What is the single equation of pair of lines perpendicular to $ax^2 + 2hxy + by^2 = 0$ and passing through origin?

⇨ Here, the single equation of pair of lines perpendicular to $ax^2 + 2hxy + by^2 = 0$ and passing through origin is $bx^2 - 2hxy + ay^2 = 0$.

5. यदि जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ को झुकावहरू m_1 र m_2 भए $(m_1 + m_2)$ को मान लेख्नुहोस् ।
If m_1 and m_2 are the slopes of two lines $ax^2 + 2hxy + by^2 = 0$ then write the value of $(m_1 + m_2)$

⇨ Here, the sum of slopes $(m_1 + m_2)$ in $ax^2 + 2hxy + by^2 = 0$ is $-\frac{2h}{b}$.

6. जोडा रेखाहरू $x - 2 = 0$ र $x + 2 = 0$ ले जनाउने एकल समीकरण के हुन्छ ?
What is the single equation to represent the pair of lines $x - 2 = 0$ and $x + 2 = 0$?

⇨ Here, the single equation is; $(x + 2)(x - 2) = 0$
i.e. $x^2 - 4 = 0$

7. जोडा रेखाको एकल समीकरण $(x - y)(x + y) = 0$ भए बेग्ला बेग्लै समीकरण पत्ता लगाउनुहोस् ।
The single equation of pair of lines is $(x - y)(x + y) = 0$. Find the separate equations of the lines.

⇨ Here, the separate equations of lines represented by $(x + y)(x - y) = 0$ are; $x + y = 0$ and $x - y = 0$

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B. SHORT QUESTIONS

MODEL 1

1. $2x^2 + 7xy + 3y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equation of the lines represented by a pair of lines $2x^2 + 7xy + 3y^2 = 0$. [2057 R]

⇨ Here, given equations of line pair is;

$2x^2 + 7xy + 3y^2 = 0$

or, $2x^2 + xy + 6xy + 3y^2 = 0$

or, $x(2x + y) + 3y (2x + y) = 0$

$\therefore$ $(2x + y) (x + 3y) = 0$

Thus, the required separate equations are $2x + y = 0$ and $x + 3y = 0$.

2. $6x^2 + 5xy - 6y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of the lines represented by a pair of lines $6x^2 + 5xy - 6y^2 = 0$. [2074 R, 2060 S]

⇨ Here, given equation of pair of st. lines;

$6x^2 + 5xy - 6y^2 = 0$ ...... (i)

Now, from equation (i)

$6x^2 - 4xy + 9xy - 6y^2 = 0$

or, $2x(3x - 2y) + 3y(3x - 2y) = 0$

$\therefore$ $(3x - 2y) (2x + 3y) = 0$

Thus, $3x - 2y = 0$ and $2x + 3y = 0$ are the required separate equations of st. lines.

3. $2x^2 - 5xy + 3y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equation of the lines represented by the equation $2x^2 - 5xy + 3y^2 = 0$. [2064 R']

⇨ Here, given equation of pair of straight lines;

$2x^2 - 5xy + 3y^2 = 0$ .... (i)

Now, from equation (i),

$2x^2 - 3xy - 2xy + 3y^2 = 0$

or, $x(2x - 3y) - y(2x - 3y) = 0$

$\therefore$ $(x - y) (2x - 3y) = 0$

Thus, $x - y = 0$ and $2x - 3y = 0$ are the required separate equations of the pair of straight lines.

4. $2x^2 + xy - 6y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of the lines represented by the equation $2x^2 + xy - 6y^2 = 0$.[2063

⇨ Here, given equation of pair of st. lines;

$2x^2 + xy - 6y^2 = 0$ .... (i)

Now, from equation (i),

$2x^2 + 4xy - 3xy - 6y^2 = 0$

or, $2x(x + 2y) - 3y(x + 2y) = 0$

$\therefore$ $(x + 2y) (2x - 3y) = 0$

Thus, $x + 2y = 0$ and $2x - 3y = 0$ are the required separate equations of the pair of straight lines.

5. $x^2 - 3xy + 2y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of the lines by the equation $x^2 - 3xy + 2y^2 = 0$. [2065 M]

⇨ Here, given equation of pair of straight lines;

$x^2 - 3xy + 2y^2 = 0$

or, $x^2 - 2xy - xy + 2y^2 = 0$

or, $x(x - 2y) - y(x - 2y) = 0$

$\therefore$ $(x - y) (x - 2y) = 0$

Thus, $x - y = 0$ and $x - 2y = 0$ are the required equations.

6. $3x^2 - 4xy + y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of a pair of straight lines represented by the equation $3x^2 - 4xy + y^2 = 0$.[2067 R'

⇨ Here, given equation of pair of straight lines;

$3x^2 - 4xy + y^2 = 0$

or, $3x^2 - 3xy - xy + y^2 = 0$

or, $3x(x - y) - y(x - y) = 0$

$\therefore$ $(x - y) (3x - y) = 0$

Either, $x - y = 0$ .... (i)

or, $3x - y = 0$ .... (ii)

Thus, $x - y = 0$ & $3x - y = 0$ are the separate equations of the pair of lines.

7. $x^2 + x - y - y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of line given by the equation $x^2 + x - y - y^2 = 0$. [2066 S]

⇨ Here, given equation of pair of straight lines;

$x^2 + x - y - y^2 = 0$

or, $x^2 - y^2 + x - y = 0$

or, $(x + y) (x - y) + 1(x - y) = 0$

$\therefore$ $(x - y) (x + y + 1) = 0$

Either, $x - y = 0$ .... (i)

or, $x + y + 1 = 0$ .... (ii)

Thus, equation (i) and (ii) are the required separate equations.

8. $x^2 - y^2 - 2x + 2y = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of the pair of line represented by the equation $x^2 - y^2 - 2x + 2y = 0$. [2072 R', 2071 R', 2070 S, 2061 R]

⇨ Here, given single equation $x^2 - y^2 - 2x + 2y = 0$ .... (i)

Resolving equation (i) into two linear factors, we get, $x^2 - y^2 - 2x + 2y = 0$

or, $(x + y) (x - y) - 2(x - y) = 0$

$\therefore$ $(x - y) (x + y - 2) = 0$

Thus, $x - y = 0$ and $x + y - 2 = 0$ are the required separated equations of the pair of straight lines.

9. $x^2 - y^2 + 3x - 3y = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।

Find the separate equations of the pair of lines represented by the equation $x^2 - y^2 + 3x - 3y = 0$.[2061

⇨ Here, given single equation of a pair of lines;

$x^2 - y^2 + 3x - 3y = 0$ ...... (i)

From equation (i) $x^2 - y^2 + 3x - 3y = 0$

or, $(x - y) (x + y) + 3 (x - y) = 0$

or, $(x - y) (x + y + 3) = 0$

or, $x - y = 0$ and $x + y + 3 = 0$

Thus, the separate equations are :

$x - y = 0$ and $x + y + 3 = 0$.

10. $3x^2 - 5xy - 2y^2 - x + 2y = 0$ ले दिने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equations of the lines represented by $3x^2 - 5xy - 2y^2 - x + 2y = 0$. [2066 R']

⇨ Here, $3x^2 - 5xy - 2y^2 - x + 2y = 0$

or, $3x^2 - 6xy + xy - 2y^2 - x + 2y = 0$

or, $3x(x - 2y) + y(x - 2y) - (x - 2y) = 0$

or, $(x - 2y) (3x + y - 1) = 0$

Either, $x - 2y = 0$ .... (i)

or, $3x + y - 1 = 0$ .... (ii)

Thus, equation (i) and (ii) are the required equations of lines.

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MODEL 2

11. **जोडा रेखाहरू $3x + 2y = 0$ र $2x - 3y = 0$ लाई जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।**

Find the single equation for the pair of lines represented by $3x + 2y = 0$ and $2x - 3y = 0$.

⇨ Here, $3x + 2y = 0$ ....(i) and

$2x - 3y = 0$ ....(ii)

Combining equation (i) and (ii) then,

$(3x + 2y) (2x - 3y) = 0$

or, $3x(2x - 3y) + 2y(2x - 3y) = 0$

or, $6x^2 - 9xy + 4xy - 6y^2 = 0$

or, $6x^2 - 5xy - 6y^2 = 0$

Thus, the required single equation is $6x^2 - 5xy - 6y^2 = 0$.

12. **एक जोडा सरल रेखाहरू $x = 3y$ र $3x = y$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।**

Find the single equation which represents a pair of straight lines $x = 3y$ and $3x = y$.

⇨ Here, given lines are: $x = 3y$ and $3x = y$

$\Rightarrow x - 3y = 0$ and $3x - y = 0$

Combining these lines then,

$(x - 3y) (3x - y) = 0$

or, $3x^2 - xy - 9xy + 3y^2 = 0$

or, $3x^2 - 10xy + 3y^2 = 0$

Thus, the required single equation of the line is;

$3x^2 - 10xy + 3y^2 = 0$.

13. **कुनै दुई जोडा रेखाहरू $x = 2y$ र $2x = y$ द्वारा बन्ने वर्ग समघातीय समीकरण पत्ता लगाउनुहोस् ।**

Find the homogeneous equation of 2nd degree from the pair of lines $x = 2y$ & $2x = y$. **[2063 R]**

⇨ Here, the given equations are $x = 2y$ and $2x = y$

or, $x - 2y = 0$ .... (i) and $2x - y = 0$ .... (ii)

Multiplying equation (i) and (ii) we get,

$(x - 2y) (2x - y) = 0$

or, $2x^2 - xy - 4xy + 2y^2 = 0$

or, $2x^2 - 5xy + 2y^2 = 0$

Thus, the homogeneous equation of the second degree is $2x^2 - 5xy + 2y^2 = 0$.

MODEL 3

14. **समीकरण $2x^2 - 7xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।**

Find the acute angle between the lines represented by the equation $2x^2 - 7xy + 3y^2 = 0$. **[2075 R₂']**

⇨ Here, given equations of lines is $2x^2 - 7xy + 3y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 2, \quad 2h = -7 \Rightarrow h = \frac{-7}{2}$ and $b = 3$

We know that, angle between two lines is,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm 2\frac{\sqrt{\left(\frac{-7}{2}\right)^2 - 2 \times 3}}{2 + 3}$$

$$= \pm\frac{2\sqrt{49 - 24}}{2 \times 5}$$

$$= \pm\frac{\sqrt{25}}{5} = \pm 1$$

or, $\tan\theta = \tan 45°$

$\therefore \quad \theta = 45°$

Thus, the required acute angle is 45°.

15. **समीकरण $x^2 + 4xy + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा सरल रेखाबिचको कोण पत्ता लगाउनुहोस् ।**

Find the angle between the pair of lines represented by the equation $x^2 + 4xy + y^2 = 0$. **[2075 R', 2073 R']**

⇨ Here, given equation of lines is $x^2 + 4xy + y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get,

$a = 1, h = 2$ and $b = 1$

Let θ be the angle between the lines then,

$$\tan\theta = \left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

$$= \left(\pm\frac{2\sqrt{2^2 - 1 \times 1}}{1 + 1}\right)$$

$$= \pm\frac{2 \times \sqrt{3}}{2}$$

$$= \pm\sqrt{3}$$

or, $\tan\theta = \tan 60°, \tan \ 120°$

$\therefore \quad \theta = 60°, 120°$

Thus, required angles are 60° and 120°.

16. **जोडा रेखाहरू $6x^2 - xy - y^2 = 0$ बीचको कोणहरू पत्ता लगाउनुहोस् ।**

Find the angles between the pair of lines represented by $6x^2 - xy - y^2 = 0$. **[2058 S]**

⇨ Here given, equation of pair of straight lines is $6x^2 - xy - y^2 = 0$ .... (i)

Comparing this equation with $ax^2 + 2hxy + by^2 = 0$, we get, $a = 6, 2h = -1$ and $b = -1$

$\therefore \quad a = 6, \ h = \frac{-1}{2}$ and $b = -1$

If θ is the angle between two lines, then using formula $\tan\theta = \frac{\pm 2\sqrt{h^2 - ab}}{a + b}$

$$\tan\theta = \frac{\pm 2\sqrt{\left(\frac{-1}{2}\right)^2 - 6 \times (-1)}}{6 - 1} = \frac{\pm 2\sqrt{\frac{1}{4} + 6}}{5} = \frac{\pm 2\sqrt{\frac{1 + 24}{4}}}{5} = \frac{\pm 2\sqrt{\frac{25}{4}}}{5} = \pm\frac{5}{5} = \pm 1$$

Taking the (+ve) sign,

$\tan\theta = 1 = \tan 45°$

$\therefore \quad \theta = 45°$

Again, taking –ve sign,

$\tan\theta = -1 = \tan 135°$

$\therefore \quad \theta = 135°$

$\therefore \quad \theta = 45°, 135°$

Thus, the angle between the lines is 45° or 135°.

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17. जोडा रेखाहरू $3x^2 + 2y^2 - 5xy = 0$ बीचको कोणहरू पत्ता लगाउनुहोस् ।

Find the angles between the pair of lines represented by $3x^2 + 2y^2 - 5xy = 0$.

⇨ Here, equation of given line $3x^2 - 5xy + 2y^2 = 0$ is the form of $ax^2 + 2hxy + by^2 = 0$

Where, $a = 3, b = 2, 2h = -5$

or $h = -\frac{5}{2}$

Now, $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

or, $\tan\theta = \pm\frac{2\sqrt{\frac{25}{4} - 6}}{3 + 2}$

or, $\tan\theta = \pm\frac{2\frac{\sqrt{25 - 24}}{2}}{5}$

or, $\tan\theta = \pm\frac{1}{5}$

$\therefore\ \theta = \tan^{-1}\left(\pm\frac{1}{5}\right)$

Thus, required angle of the line is $\tan^{-1}\left(\pm\frac{1}{5}\right)$.

19. समीकरण $x^2 - 4xy + y^2 = 0$ ले जनाउने जोडा रेखाबीचको अधिककोण पत्ता लगाउनुहोस् ।

$x^2 - 4xy + y^2 = 0$ is the equation which represents a pair straight lines. Find the obtuse angle between the pair of lines.

⇨ Here given, equation of pair of st. lines are;

$x^2 - 4xy + y^2 = 0$ .... (i)

Comparing this equation with $ax^2 + 2hxy + by^2 = 0$

where, $a = 1, 2h = -4, b = 1$

$\therefore\ a = 1, h = -2, b = 1$

If θ be the angle between pair of lines, then using formula, $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

or, $\tan\theta = \pm\frac{2\sqrt{(-2)^2 - 1.1}}{1 + 1}$

$= \pm\frac{2\sqrt{4 - 1}}{2}$

$= \pm\sqrt{3}$

For obtuse angle, taking –ve sign, we get,

$\tan\theta = -\sqrt{3} = \tan 120°$

Thus, $\theta = 120°$ is the required obtuse angle.

18. समीकरण $x^2 - 2xy\operatorname{cosec}\alpha + y^2 = 0$ ले जनाउने एक जोडा सरल रेखाहरूबीचको कोण पत्ता लगाउनुहोस् ।

Find the angle between a pair of straight lines represented by an equation $x^2 - 2xy\operatorname{cosec}\alpha + y^2 = 0$.

⇨ Here, $x^2 - 2xy\operatorname{cosec}\alpha + y^2 = 0$ is the form of $ax^2 + 2hxy + by^2 = 0$, where, $a = 1, b = 1$

$2h = -2\operatorname{cosec}\alpha$

or, $h = -\operatorname{cosec}\alpha$

Now, θ be an angle between pair of lines

So, $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

or, $\tan\theta = \pm\frac{2\sqrt{\operatorname{cosec}^2\alpha - 1.1}}{1 + 1}$

or, $\tan\theta = \pm\frac{2\sqrt{\operatorname{cosec}^2\alpha - 1}}{2}$

or, $\tan\theta = \pm\sqrt{\cot^2\alpha}$

or, $\tan\theta = \pm\cot\alpha$

$\therefore\ \theta = \tan^{-1}(\pm\cot\alpha)$

Thus, the required angle between pair of lines is $\tan^{-1}(\pm\cot\alpha)$.

20. समीकरण $12x^2 - 23xy + 5y^2 = 0$ ले जनाउने जोडा रेखाबीचको अधिककोण पत्ता लगाउनुहोस् ।

Find the obtuse angle between the lines represented by the equation $12x^2 - 23xy + 5y^2 = 0$.

⇨ Here, given equation of pair of lines is;

$12x^2 - 23xy + 5y^2 = 0$..........(i)

Comparing eq^n (i) with $ax^2 + 2hxy + by^2 = 0$, we get,

$a = 12, 2h = -23$

$\therefore\ h = \frac{-23}{2}$ and $b = 5$

If θ be the angle between pair of lines, then,

$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

$$\text{or, } \tan\theta = \pm\frac{2\sqrt{\left(-\frac{23}{2}\right)^2 - 12.5}}{12 + 5}$$

$$= \pm\frac{2\sqrt{\frac{529}{4} - 60}}{17} = \pm\frac{2\sqrt{\frac{529 - 240}{4}}}{17}$$

$$= \pm\frac{\frac{2\sqrt{289}}{2}}{17} = \pm\frac{17}{17} = \pm 1$$

Here, taking +ve sign,

$\tan\theta = 1 = \tan 45°$

$\therefore\ \theta = 45°$

Again taking –ve sign,

$\tan\theta = -1 = \tan 135°$

$\therefore\ \theta = 135°$

Thus, the obtuse angle between the pair of lines $(\theta) = 135°$.

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21. समीकरण $3x^2 - 7xy + 2y^2 = 0$ ले जनाउने जोडा रेखाबीचको अधिककोण पत्ता लगाउनुहोस् ।
Find the obtuse angle between the lines represented by the equation $3x^2 - 7xy + 2y^2 = 0$. [2063 R']

⇨ Here, given equation of the lines;

$3x^2 - 7xy + 2y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$

Where, $a = 3, h = \frac{-7}{2}$ and $b = 2$

Let, θ be the angle between the lines

$$\therefore \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{\left(-\frac{7}{2}\right)^2 - 3 \times 2}}{3 + 2}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{\frac{49}{4} - 6}}{5}\right) = \tan^{-1}\left(\pm\frac{2 \times \frac{5}{2}}{5}\right)$$

$$\therefore \theta = \tan^{-1}(\pm 1)$$

Thus, θ = 135° is the required obtuse angle.

23. समीकरण $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2 = 0$ ले जनाउने जोडा रेखाबीचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines represented by the equation $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2 = 0$. [2062 CP, 2065 E]

⇨ Here, given equation of lines; $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$

Where, $a = \frac{1}{2}, h = 1$ and $b = \frac{1}{2}$

Let θ be the angle between given lines

$$\text{or, } \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{1 - \frac{1}{2} \times \frac{1}{2}}}{\frac{1}{2} + \frac{1}{2}}\right)$$

$$= \tan^{-1}\left(\pm\frac{2 \times \frac{\sqrt{3}}{2}}{1}\right)$$

$$\therefore \theta = \tan^{-1}(\pm\sqrt{3})$$

Thus, θ = 60° is the required acute angle between given lines.

22. समीकरण $x^2 - 4xy + y^2 = 0$ ले जनाउने जोडा रेखाबीचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines represented by the equation $x^2 - 4xy + y^2 = 0$. [2062 K, 2068 R]

⇨ Here, given equation of lines; $x^2 - 4xy + y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$

$a = 1, 2h = -4$ and $b = 1$

$\therefore h = -2$

Let θ be the angle between lines;

$$\text{or, } \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{4 - 1}}{1 + 1}\right)$$

$$\therefore \theta = \tan^{-1}(\pm\sqrt{3})$$

Thus, θ = 60° is the required acute angle between the given lines.

24. समीकरण $6x^2 + xy - y^2 = 0$ ले जनाउने जोडा रेखाबीचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines represented by the equation $6x^2 + xy - y^2 = 0$. [2062 S]

⇨ Here, given equation of lines; $6x^2 + xy - y^2 = 0$

Comparing it with $ax^2 + 2xhy + by^2 = 0$

Where, $a = 6,\ h = \frac{1}{2}$ and $b = -1$

Let θ be the angle between lines

$$\text{or, } \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{\frac{1}{4} - 6(-1)}}{6 - 1}\right)$$

$$= \tan^{-1}\left(\pm\frac{2 \times \sqrt{\frac{1}{4} + 6}}{5}\right)$$

$$= \tan^{-1}\left(\pm\frac{2 \times \frac{5}{2}}{5}\right)$$

$$\therefore \theta = \tan^{-1}(\pm 1)$$

Thus, θ = 45° is the required acute angle between the lines.

MODEL 4

25. यदि समीकरण $16x^2 - kxy + 9y^2 = 0$ ले जनाउने रेखाहरू सम्पाती छन् भने k को मान पत्ता लगाउनुहोस् ।
If the lines represented by $16x^2 - kxy + 9y^2 = 0$ are coincident, find the value of k. [2075 R]

⇨ Here, $16x^2 - kxy + 9y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 16, 2h = -k$ and $b = 9$ $\quad\therefore\ h = -\frac{k}{2}$

For the lines to be coincident, $h^2 = ab$

$$\text{or, } \left(-\frac{k}{2}\right)^2 = 16 \times 9$$

$$\text{or, } \frac{k^2}{4} = 144$$

$$\text{or, } k^2 = 144 \times 4 = 576$$

$$\therefore k = \pm 24$$

Thus, the value of k is ± 24.

26. यदि समीकरण $kx^2 - 24xy + 16y^2 = 0$ ले जनाउने रेखाहरू सम्पाती भए k को मान पत्ता लगाउनुहोस् ।
If the lines represented by $kx^2 - 24xy + 16y^2 = 0$ are coincident, then find the value of k. [2075 R_2]

⇨ Here, given equation of line is;

$kx^2 - 24xy + 16y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = k, 2h = -24 \Rightarrow h = -12$ and $b = 16$

For the lines to be coincident,

$h^2 = ab$

or, $(-12)^2 = k \times 16$

$\therefore k = \frac{144}{16} = 9$

Thus, the value of k is 9.

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27. m को मान कति भए $3x^2 + 8xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने दुई रेखाहरू आपसमा लम्ब हुन्छन् ? पत्ता लगाउनुहोस् ।
For what value of m, the two lines represented by $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other? Find it. [2075 R']

⇨ Here, the given equation is; $3x^2 + 8xy + my^2 = 0$.
Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get,
$a = 3,\ 2h = 8 \Rightarrow h = 4$ and $b = m$
For the lines to be perpendicular, $a + b = 0$
or, $3 + m = 0$
$\therefore\ m = -3$
Thus, for m = – 3, the lines are perpendicular to each other.

28. यदि $3x^2 - 5xy + py^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबीचको कोण एक समकोण भए p को मान निकाल्नुहोस् ।
If the angle between the lines represented by $3x^2 - 5xy + py^2 = 0$ is a right angle, find the value of p. [2074 S]

⇨ Here, the given equation is ;
$3x^2 - 5xy + py^2 = 0$ ....(i)
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,
$a = 3,\ 2h = -5$ and $b = p$
Now, $a + b = 0$ ($\because$ since the lines are perpendicular.)
or, $3 + p = 0$
$\therefore\ p = -3$
Thus, the value of p is – 3.

29. समीकरण $5x^2 + 24xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा रेखाहरू आपसमा लम्ब छन् भने m को मान पत्ता लगाउनुहोस् ।
An equation $5x^2 + 24xy + my^2 = 0$ represents a pair of straight lines which are perpendicular to each other. Find the value of m. [2074 S']

⇨ Here, $5x^2 + 24xy + my^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 5$, $b = m$
Since the lines are perpendicular,
So, $a + b = 0$
or, $5 + m = 0$
$\therefore\ m = -5$
Thus, the value of m is – 5.

30. यदि समीकरण $2x^2 + 8xy - (m + 1)\,y^2 = 0$ ले जनाउने रेखाहरू सम्पाती छन् भने m को मान निकाल्नुहोस् ।
If the lines represented by the equation $2x^2 + 8xy - (m + 1)y^2 = 0$ are coincident, find the value of m. [2073 R]

⇨ Here, given equation of lines; $2x^2 + 8xy - (m + 1)y^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 2$, $2h = 8 \Rightarrow h = 4$ and $b = -(m + 1)$
Since the lines are coincident;
So, $h^2 = ab$
or, $(4)^2 = 2\,(-)\,(m + 1)$
or, $16 = -2m - 2$
or, $18 = -2m$
$\therefore\ m = -9$
Thus, the value of m is – 9.

31. यदि $x^2 + 2kxy + 4y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू सम्पाती छन् भने k को मान पत्ता लगाउनुहोस् ।
If the pair of lines represented by the equation $x^2 + 2kxy + 4y^2 = 0$ are coincident, find the value of k. [2073 R']

⇨ Here, given equation of lines $x^2 + 2kxy + 4y^2 = 0$.
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then $a = 1$, $2h = 2k$ and $b = 4$
$\therefore\ h = k$
Since the lines are coincident,
So, $h^2 = ab$
or, $k^2 = 1 \times 4$
or, $k^2 = 4$
$\therefore\ k = \pm 2$
Thus, the value of k is ± 2.

32. यदि समीकरण $9x^2 + 12xy + (k + 1)y^2 = 0$ ले जनाउने रेखाहरू सम्पाती छन् भने k को मान निकाल्नुहोस् ।
If the lines represented by the equation $9x^2 + 12xy + (k + 1)\,y^2 = 0$ are coincident, find the value of k. [2073 S]

⇨ Here, $9x^2 + 12xy + (k + 1)y^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 9$, $2h = 12 \Rightarrow h = 6$ and $b = k + 1$
Since the lines are coincident,
So, $h^2 = ab$
or, $6^2 = 9(k + 1)$
or, $36 = 9(k + 1)$
or, $k + 1 = 4$
$\therefore\ k = 3$
Thus, the value of k is 3.

33. यदि समीकरण $9x^2 + (k + 1)xy + 4y^2 = 0$ ले जनाउने रेखाहरू सम्पाती छन् भने k को मान निकाल्नुहोस् ।
If the lines represented by the equation $9x^2 + (k + 1)xy + 4y^2 = 0$ are coincident, find the value of k. [2073 S']

⇨ Here, $9x^2 + (k + 1)xy + 4y^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 9$,
$2h = k + 1$ and $b = 4$ $\therefore\ h = \frac{k+1}{2}$
Since the lines are coincident, so, $h^2 = ab$
i.e. $\left(\frac{k+1}{2}\right)^2 = 9 \times 4$
or, $(k + 1)^2 = 9 \times 4 \times 4$
or, $k^2 + 2k + 1 = 144$
or, $k^2 + 2k - 143 = 0$
or, $k^2 + 13k - 11k - 143 = 0$
or, $k(k + 13) - 11(k + 13) = 0$
or, $(k - 11)(k + 13) = 0$
Either $k - 11 = 0$ $\therefore\ k = 11$
or, $k + 13 = 0$ $\therefore\ k = -13$
Thus, the value of k is 11 or – 13.

34. $3x^2 - 6xy + (k + 4)y^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा रेखाहरू आपसमा सम्पाती भए k को मान पत्ता लगाउनुहोस् ।
Find the value of k if the pair of lines represented by $3x^2 - 6xy + (k + 4)y^2 = 0$ are coincident to each other. [2072 S]

⇨ Here, $3x^2 - 6xy + (k + 4)y^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 3$,
$2h = -6$ and $b = k + 4$
$\therefore\ h = -3$
The condition of perpendicularity: $h^2 = ab$
or, $(-3)^2 = 3 \times (k + 4)$
or, $9 = 3 \times (k + 4)$
or, $k + 4 = 3$
$\therefore\ k = -1$
Thus, the value of k is – 1.

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35. समीकरण $16x^2 - 24xy + 9y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा रेखाहरू आपसमा सम्पाती हुन्छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the pair of lines represented by the equation $16x^2 - 24xy + 9y^2 = 0$ are coincident. [2071 R]

⇨ Here, $16x^2 - 24xy + 9y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 16, 2h = -24, \ h = -12, b = 9$

We have, $h^2 = (-12)^2 = 144$

$ab = 16 \times 9 = 144$

Thus, $h^2 = ab$ shows that the lines are coincident.

37. $(m - 6)\, x^2 - 8xy + my^2 = 0$ समीकरणले जनाउने रेखाहरू सम्पाति हुन्छन् भने m को मान पत्ता लगाउनुहोस् । [2070 R]

Find the value of m, if the lines represented by equation $(m - 6)\, x^2 - 8xy + my^2 = 0$ are coincident.

⇨ Here, given equation of lines,

$(m - 6)x^2 - 8xy + my^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = m - 6, 2h - 8 \Rightarrow h = -4, b = m$

Since the lines are coincident,

So, $h^2 = ab$

or, $(-4)^2 = (m - 6)m$

or, $16 = m^2 - 6m$

or, $m^2 - 6m - 16 = 0$

or, $m^2 - 8m + 2m - 16 = 0$

or, $m(m - 8) + 2(m - 8) = 0$

or, $(m - 8)\,(m + 2) = 0$

Either, $m - 8 = 0 \quad \therefore \quad m = 8$

or $m + 2 = 0 \quad \therefore \quad m = -2$

Thus, the value of m is 8 or – 2.

39. $9x^2 - 24xy + 16y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू आपसमा सम्पाती हुन्छन् भनी देखाउनुहोस् ।

Show that the pair of straight lines represented by $9x^2 - 24xy + 16y^2 = 0$ are coincident. [2074 R', 2065 R]

⇨ Here, $9x^2 - 24xy + 16y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 9, 2h = -24$ and $b = 16$

Now, $h^2 = (-12)^2 = 144$ and

$ab = 9 \times 16 = 144$

Thus, $h^2 = ab$ shows that the lines represented by $9x^2 - 24xy + 16y^2 = 0$ are coincident. **Proved.**

40. समीकरण $6x^2 + 5xy - 6y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू बीचको कोण एक समकोण हुन्छ भनी प्रमाणित गर्नुहोस् ।

Prove that the angle between the lines represented by an equation $6x^2 + 5xy - 6y^2 = 0$ is a right angle. [2067 S]

⇨ Here, $6x^2 + 5xy - 6y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then

$a = 6, 2h = 5$ and $b = -6$

Now, $a + b = 6 - 6 = 0$

$\therefore$ $a + b = 0$ shows that the angle between the lines is a right angle.

36. $5x^2 - 6xy - 5y^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा रेखाहरूबीचको कोण एक समकोण हुन्छ भनी प्रमाणित गर्नुहोस् । [2071 S]

Prove that the angle between a pair of lines represented by $5x^2 - 6xy - 5y^2 = 0$ is a right angle.

⇨ Here, given equation of lines is $5x^2 - 6xy - 5y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$

Then, $a = 5$ and $b = -5$

Now, $a + b = 5 - 5 = 0$

Thus, it shows that the line represented by given equation are perpendicular to each other.

38. यदि समीकरण $kx^2 - 12xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा सरल रेखाहरू आपसमा सम्पाती छन् भने k को मान पत्ता लगाउनुहोस् ।

If a pair of straight lines represented by the equation $kx^2 - 12xy + 3y^2 = 0$ are coincident, find the value of k. [2070 R']

⇨ Here, equation of pair of lines $kx^2 - 12xy + 3y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = k$, $2h = -12 \Rightarrow h = -6$ and $b = 3$

Since the lines are coincident.

So, $h^2 = ab$

or, $(-6)^2 = k \times 3$

or, $36 = 3k$

$\therefore \quad k = 12$

Thus, the value of k is 12.

41. $6x^2 - 5xy - 6y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू बीचको कोण एक समकोण हुन्छ भनी प्रमाणित गर्नुहोस् ।

Prove that the angle between the lines represented by $6x^2 - 5xy - 6y^2 = 0$ is at right angle. [2065 R']

⇨ Here, given equation of lines is;

$6x^2 - 5xy - 6y^2 = 0$ .... (i)

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 6, 2h = -5$ and $b = -6$

Now, $a + b = 6 - 6 = 0$

So, the lines are perpendicular to each other.

Thus, the angle between the lines is 90°.

42. यदि समीकरण $2x^2 - 3xy + my^2 = 0$ ले जनाउने रेखाहरू सम्पाती हुन्छन् भने m को मान पत्ता लगाउनुहोस् ।

If a pair of straight lines represented by $2x^2 - 3xy + my^2 = 0$ are coincident, find the value of m. [2068 R']

⇨ Here, $2x^2 - 3xy + my^2 = 0$ is the form of

$ax^2 + 2hxy + by^2 = 0$

Where, $a = 2, b = m, 2h = -3$ or $h = \frac{-3}{2}$

Now, If pair of lines are coincident (parallel) then,

$h^2 = ab$

or, $\left(-\frac{3}{2}\right)^2 = 2m$

or, $\frac{9}{4} = 2m$

$\therefore \quad m = \frac{9}{8}$

Thus, required value of m is $\frac{9}{8}$.

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43. $5x^2 - 8xy + ky^2 = 0$ ले दिने दुई रेखाहरू आपसमा लम्ब भए, k पत्ता लगाउनुहोस् ।

Find k so that $5x^2 - 8xy + ky^2 = 0$ may represent two mutually perpendicular lines.

⇨ Here, the given equation of lines; $5x^2 - 8xy + ky^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then

$a = 5, 2h = -8$ and $b = k$

Since, the lines are perpendicular to each other

So, $a + b = 0$

or, $5 + k = 0$

∴ $k = -5$

Thus, the value of k is – 5.

44. यदि समीकरण $3x^2 - 12xy + my^2 = 0$ ले जनाउने रेखाहरू सम्पाती हुन्छन् भने m को मान पत्ता लगाउनुहोस् ।

If a pair of straight lines represented by $3x^2 - 12xy + my^2 = 0$ are coincident, find the value of m.

[2066 R]

⇨ Here, given equation of lines $3x^2 - 12xy + my^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$

then, $a = 3, 2h = -12$ and $b = m \Rightarrow h = -6$

Since, the lines are coincident so $h^2 = ab$

or, $(-6)^2 = 3 \times m$

or, $36 = 3m$

∴ $m = 12$

Thus, the value of m is 12.

C. LONG QUESTIONS

MODEL 1

1. $3x^2 - 8xy - 3y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् र यसको कोण पनि पत्ता लगाउनुहोस् ।

Find the separate equation of the pair of line represented by the equation $3x^2 - 8xy - 3y^2 = 0$. Also find the angle between them. [2075 R_2', 2062 R]

⇨ Here, given equation of a pair of straight lines :

$3x^2 - 8xy - 3y^2 = 0$ .... (i)

or, $3x^2 - 9xy + xy - 3y^2 = 0$

or, $3x(x - 3y) + y(x - 3y) = 0$

or, $(x - 3y)(3x + y) = 0$

∴ The required separate equations of pair of st. lines are $x - 3y = 0$ and $3x + y = 0$.

Again comparing eq^n (i) with $ax^2 + 2hxy + by^2 = 0$ we get, $a = 3, 2h = -8$ and $b = -3$

∴ $a = 3, h = -4$ and $b = -3$

$a + b = 0$ shows that the lines are perpendicular.

∴ $\theta = 90°$

Thus, the angle between them is 90°.

2. समीकरण $6x^2 + xy - y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा सरल रेखाहरूको समीकरण पत्ता लगाउनुहोस् । साथै ती रेखाहरूबीचको कोणहरू पनि पत्ता लगाउनुहोस् ।

Find the equation of a pair of straight lines represented by an equation $6x^2 + xy - y^2 = 0$. Also find the angles between them. [2074 S']

⇨ Here, given equation of lines is $6x^2 + xy - y^2 = 0$

So, $6x^2 + xy - y^2 = 0$

or, $6x^2 + 3xy - 2xy - y^2 = 0$

or, $3x(2x + y) - y(2x + y) = 0$

or, $(2x + y)(3x - y) = 0$

Either, $2x + y = 0$ ....(i)

or, $3x - y = 0$ ......(ii)

∴ $2x + y = 0$ and $3x - y = 0$ are the required equation of lines.

Again, comparing given equation with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 6, \quad 2h = 1 \quad \therefore h = \frac{1}{2}$ and $b = -1$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b} = \pm\frac{2\sqrt{\frac{1}{4} + 6}}{6 - 1} = \pm\frac{2\sqrt{\frac{25}{4}}}{5}$$

$$\text{or, } \tan\theta = \pm\frac{2 \times \frac{5}{2}}{5} = \pm 1$$

or, $\tan\theta = \tan 45°$ or $135°$

∴ $\theta = 45°$ or $135°$

Thus, the required lines are $2x + y = 0$ and $3x - y = 0$ and angles are 45° and 135°.

3. समीकरण $2x^2 + 3xy + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरूबीचको कोण पनि पत्ता लगाउनुहोस् ।

Find the equations of the pair of lines represented by the equation $2x^2 + 3xy + y^2 = 0$ and find the angle between them. [2072 R']

⇨ Here, given equation of the line is; $2x^2 + 3xy + y^2 = 0$

$2x^2 + 3xy + y^2 = 0$

or, $2x^2 + 2xy + xy + y^2 = 0$

or, $2x(x + y) + y(x + y) = 0$

or, $(x + y)(2x + y) = 0$

Either, $x + y = 0$ …. (i)

or, $2x + y = 0$ ………….. (ii)

Equation (i) and (ii) are the required equation of the lines.

Comparing given equation with $ax^2 + 2hxy + by^2 = 0$ then, $a = 2, 2h = 3$ and $b = 1$

Let θ be angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b} = \pm\frac{2\sqrt{\left(\frac{3}{2}\right)^2 - 2 \times 1}}{2 + 1}$$

$$= \pm\frac{2\sqrt{\frac{9}{4} - 2}}{3} = \pm 2 \times \frac{\frac{1}{2}}{\frac{3}{1}} = \pm\frac{1}{3}$$

or, $\tan\theta = \tan 18.43°$ or $161.56°$

∴ $\theta = 18.43°$ or $161.56°$

Thus, the value of θ is 18.43° or 161.56°.

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4. **समीकरण $6x^2 - xy - y^2 = 0$ ले प्रतिनिधित्व गर्ने बेग्लाबेग्लै रेखाहरूको समीकरण पत्ता लगाउनुहोस् । ती रेखाहरू बीचको कोण पनि निकाल्नुहोस् ।**

   Find the separate equation of the pair of lines represented by the equation $6x^2 - xy - y^2 = 0$. Also find the angle between them. [2072 S]

⇨ Here, given equation of lines; $6x^2 - xy - y^2 = 0$

or, $6x^2 - 3xy + 2xy - y^2 = 0$

or, $3x(2x - y) + y(2x - y) = 0$

or, $(2x - y)(3x + y) = 0$

Either, $2x - y = 0$ ....(i)

or, $3x + y = 0$ ....(ii)

Again, comparing given equation with $ax^2 + 2hxy + by^2 = 0$ then, $a = 6$, $2h = -1$ and $b = -1$

Let θ be the angle between lines then,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \pm\frac{2\sqrt{\left(-\frac{1}{2}\right)^2 - 6(-1)}}{6 - 1}$$

$$= \pm\frac{2\sqrt{\frac{1}{4} + 6}}{5} = \pm 2 \times \frac{5}{2} \times \frac{1}{5}$$

or, $\tan\theta = \pm 1$

or, $\tan\theta = \tan 45°$ or $135°$

Thus, equation (i) and (ii) are required equations and $\theta = 45°$ or $135°$ is the angle between the lines.

6. **यदि समीकरण $3x^2 + 8xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने दुई सरल रेखाहरू एक आपसमा लम्ब छन् भने ती दुई रेखाहरूको बेग्लाबेग्लै समीकरण पत्ता लगाउनुहोस् ।**

   If two straight lines represented by an equation $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other, find the separate equation of two lines. [2070 R']

⇨ Here, given equation of lines; $3x^2 + 8xy + my^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, then, $a = 3$, $2h = 8$ and $b = m$

Since the lines are perpendicular so, $a + b = 0$

or, $3 + m = 0$

∴ $m = -3$

Now, $3x^2 + 8xy - 3y^2 = 0$

or, $3x^2 + 9xy - xy - 3y^2 = 0$

or, $3x(x + 3y) - y(x + 3y) = 0$

or, $(x + 3y)(3x - y) = 0$

Either, $x + 3y = 0$

or, $3x - y = 0$

Thus, $x + 3y = 0$ and $3x - y = 0$ are the required equation of lines.

5. **$5x^2 - 8xy + py^2 = 0$ ले दुईओटा परस्पर लम्ब रेखाहरूलाई जनाउँछ भने p को मान पत्ता लगाउनुहोस् । साथै $5x^2 - 8xy + (p + 8)y^2 = 0$ ले जनाउँने सिधा रेखाहरूको समीकरण पनि पत्ता लगाउनुहोस् ।**

   If the straight lines represented by $5x^2 - 8xy + py^2 = 0$ are perpendicular to each other, then find the value of p. Also find the equations of straight lines represented by $5x^2 - 8xy + (p + 8)y^2 = 0$. [2071 R]

⇨ Here, given equation of lines $5x^2 - 8xy + py^2 = 0$.

Comparing it with $ax^2 + 2xhy + by^2 = 0$ then,

$a = 5$, $2h = -8$ i.e. $h = -4$ and $b = p$

Since the lines are perpendicular to each other,

$a + b = 0$

or, $5 + p = 0$

∴ $p = -5$

Now, $5x^2 - 8xy + (p + 8)y^2 = 0$

or, $5x^2 - 8xy + 3y^2 = 0$

or, $5x^2 - 5xy - 3xy + 3y^2 = 0$

or, $5x(x - y) - 3y(x - y) = 0$

or, $(x - y)(5x - 3y) = 0$

Either, $x - y = 0$

or, $5x - 3y = 0$

Thus, $x - y = 0$ and $5x - 3y = 0$ are the required equations of the lines.

7. **समीकरण $2x^2 + 7xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने दुई रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरूबीचको कोण पनि पत्ता लगाउनुहोस् ।**

   Find the equations of two lines represented by the equation $2x^2 + 7xy + 3y^2 = 0$. Also find the angle between them. [2058 R, 2068 R']

⇨ Here, given equation, $2x^2 + 7xy + 3y^2 = 0$ .... (i)

Resolving this equation (i) into two linear factors, we have $2x^2 + xy + 6xy + 3y^2 = 0$

or, $x(2x + y) + 3y(2x + y) = 0$

or, $(2x + y)(x + 3y) = 0$

Either, $2x + y = 0$ or, $x + 3y = 0$

∴ The required two lines represented by equation (i) are $2x + y = 0$ and $x + 3y = 0$

Again, comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$,

We get, $a = 2$, $2h = 7$

or, $h = \frac{7}{2}$, $b = 3$

If θ be the angle between two lines represented by equation (i), then we have, $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

$$\text{or, } \tan\theta = \pm\frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2.3}}{2 + 3} = \pm\frac{2\sqrt{\frac{49}{4} - 6}}{5}$$

$$= \pm\frac{2\sqrt{\frac{49 - 24}{4}}}{5} = \pm\frac{2\sqrt{\frac{25}{4}}}{5} = \pm\frac{2}{5} \times \frac{5}{2} = \pm 1$$

Now taking positive sign, we get,

$\tan\theta = 1 = \tan 45°$

∴ $\theta = 45°$

Taking negative sign, we get,

$\tan\theta = -1 = \tan 135°$

∴ $\theta = 135°$

Thus, the required angles are 45° & 135°.

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8. समीकरण $2x^2 + 5xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरू बीचको कोण पनि पत्ता लगाउनुहोस् ।

Find the equation of the pair of lines represented by the equation $2x^2 + 5xy + 3y^2 = 0$. Also find the angle between them. [2064 R]

⇨ Here, given equation of the lines;

$2x^2 + 5xy + 3y^2 = 0$ .... (i)

or, $2x^2 + 2xy + 3xy + 3y^2 = 0$

or, $2x(x + y) + 3y(x + y) = 0$

$\therefore$ $(x + y)(2x + 3y) = 0$

$\therefore$ $x + y = 0$ and $2x + 3y = 0$ are the required lines represented by given equation.

Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$

$\therefore$ $a = 2, h = \frac{5}{2}$ and $b = 3$

Let θ be the angle between lines;

$$\therefore \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\sqrt{\frac{25}{4}-2\times 3}}{2+3}\right)$$

$$= \tan^{-1}\left(\pm\frac{2\times\frac{1}{2}}{5}\right)$$

$$= \tan^{-1}\left(\pm\frac{1}{5}\right)$$

Thus, θ = 11.31° or 168.69° is the required angle between lines.

9. समीकरण $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरू बीचको कोण पनि पत्ता लगाउनुहोस् ।

Find the equation of a pair of lines represented by the equation $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$. Also, find the angle between them. [2067 S]

⇨ Here, $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$

or, $abx^2 - aby^2 + a^2xy - b^2xy = 0$

or, $abx^2 + a^2xy - aby^2 - b^2xy = 0$

or, $ax(bx + ay) - by(ay + bx) = 0$

$\therefore$ $(bx + ay)(ax - by) = 0$

Either, $bx + ay = 0$ or $ax - by = 0$ are the required equations of lines.

Again, $abx^2 - aby^2 + (a^2 - b^2)xy = 0$

or, $abx^2 + (a^2 - b^2)xy - aby^2 = 0$

Comparing it with $Ax^2 + 2hxy + By^2 = 0$ then $A = ab$, $2h = a^2 - b^2$ and $B = -ab$

Now, $A + B = ab - ab = 0$

Thus, $A + B = 0$ shows the angle between the lines is 90°.

10. $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू पत्ता लगाउनुहोस् ।

Determine the lines represented by the equation $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$. [2068 R]

⇨ Here, given equation of lines is $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$

or, $(x + y)^2 - 2(x + y) - 15 = 0$

or, $(x + y)^2 - 5(x + y) + 3(x + y) - 15 = 0$

or, $(x + y)\{(x + y) - 5\} + 3\{(x + y) - 5\} = 0$

$\therefore$ $(x + y - 5)(x + y + 3) = 0$

Either, $x + y - 5 = 0$ .... (i)

or, $x + y + 3 = 0$ .... (ii)

Thus, the lines represented by given equation are $x + y - 5 = 0$ and $x + y + 3 = 0$.

11. दिइएको समीकरण $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$ ले जनाउने एक जोडी सरल रेखाहरूको समीकरण निकाल्नुहोस् ।

Find the equation of a pair of straight lines represented by the given equation $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$. [2066 S]

⇨ Here, given equation of lines is; $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$

or, $2x^2 - 5xy + 3x - 3y^2 + 19y - 20 = 0$

or, $2x^2 - (5y - 3)x - (3y^2 - 19y + 20) = 0$

Which is the form of $ax^2 + bx + c = 0$ where $a = 2$, $b = -(5y - 3)$ & $c = -(3y^2 - 19y + 20)$

Using the quadratic formula,

Now, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Then,

$$x = \frac{5y-3\pm\sqrt{(5y-3)^2+4\times 2(3y^2-19y+20)}}{2\times 2}$$

$$= \frac{5y-3\pm\sqrt{25y^2-30y+9+24y^2-152y+160}}{4}$$

$$= \frac{5y-3\pm\sqrt{49y^2-182y+169}}{4}$$

$$= \frac{5y-3\pm\sqrt{(7y-13)^2}}{4}$$

$$\therefore x = \frac{5y-3\pm(7y-13)}{4}$$

Taking (+)ve sign then,

$$x = \frac{5y-3+7y-13}{4}$$

or, $4x = 12y - 16$

or, $4x - 12y + 16 = 0$

$\therefore$ $x - 3y + 4 = 0$ .... (i)

Taking (–)ve sign then,

$$x = \frac{5y-3-7y+13}{4}$$

or, $4x = -2y + 10$

or, $4x + 2y = 10$

$\therefore$ $2x + y = 5$ .... (ii)

Thus, equation (i) and (ii) are the required equations of lines.

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MODEL 2

12. यदि $x^2 + 2xy \operatorname{cosec}\theta + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको न्यूनकोण α भए, प्रमाणित गर्नुहोस् :
If the acute angle between the pair of lines represented by $x^2 + 2xy \operatorname{cosec}\theta + y^2 = 0$ is α then prove that: $\alpha = 90° - \theta$. **[2074 R', 2073 S, 2073 S']**

⇨ Here, given equation of lines is;

$$x^2 + 2xy \operatorname{cosec}\theta + y^2 = 0 \text{ ....(i)}$$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$$a = 1,\ 2h = 2\operatorname{cosec}\theta \text{ and } b = 1$$

$\therefore\ h = \operatorname{cosec}\theta$

Let α be an acute angle between the lines then,

$$\tan\alpha = \frac{2\sqrt{h^2 - ab}}{a + b}$$

or, $\tan\alpha = \dfrac{2\sqrt{\operatorname{cosec}^2\theta - 1 \times 1}}{1 + 1} = \dfrac{2 \times \sqrt{\cot^2\theta}}{2}$

or, $\tan\alpha = \cot\theta$

or, $\tan\alpha = \tan(90° - \theta)$

$\therefore\ \alpha = 90° - \theta$

Thus, $\alpha = (90° - \theta)$ **Proved.**

13. यदि $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा सरल रेखाहरूबीचको न्यूनकोण α भए प्रमाणित गर्नुहोस्: $\alpha = 90° - \theta$
If the acute angle between the pair of straight lines represented by $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ is α then prove that: $\alpha = 90° - \theta$ **[2073 R]**

⇨ Here, given equation of lines is;

$$x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0 \text{ ....(i)}$$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$$a = 1,\ 2h = -2\operatorname{cosec}\theta \text{ and } b = 1$$

$\therefore\ h = -\operatorname{cosec}\theta$

Let α be an acute angle between the lines then,

$$\tan\alpha = \frac{2\sqrt{h^2 - ab}}{a + b}$$

or, $\tan\alpha = \dfrac{2\sqrt{\operatorname{cosec}^2\theta - 1 \times 1}}{1 + 1} = \dfrac{2 \times \sqrt{\cot^2\theta}}{2}$

or, $\tan\alpha = 2\sqrt{(-\operatorname{cosec}\theta)^2 - 1 \times 1}$

or, $\tan\alpha = \tan(90° - \theta)$

or, $\tan\alpha = \cot\theta$

$\therefore\ \alpha = 90° - \theta$

Thus, $\alpha = (90° - \theta)$ **Proved.**

14. समीकरण $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरूबीचको कोण पनि पत्ता लगाउनुहोस् ।
Find the equation of the pair of lines represented by the equation $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$. Also find the angle between them. **[2059 R]**

⇨ Here, given equation of lines;

$$x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$$

or, $x^2 - 2.x.y\operatorname{cosec}\theta + (y\operatorname{cosec}\theta)^2 - (y\operatorname{cosec}\theta)^2 + y^2 = 0$

or, $(x - y\operatorname{cosec}\theta)^2 - y^2\operatorname{cosec}^2\theta + y^2 = 0$

or, $(x - y\operatorname{cosec}\theta)^2 - y^2(\operatorname{cosec}^2\theta - 1) = 0$

or, $(x - y\operatorname{cosec}\theta)^2 - (y\cot\theta)^2 = 0$

$\therefore\ (x - y\operatorname{cosec}\theta + y\cot\theta)(x - y\operatorname{cosec}\theta - y\cot\theta) = 0$

Comparing given equation with $ax^2 + 2hxy + by^2 = 0$,
we get, $a = 1$, $h = -\operatorname{cosec}\theta$ and $b = 1$

Let α be the angle between the lines then,

$$\tan\alpha = \pm 2\frac{\sqrt{h^2 - ab}}{a + b} = \pm 2\frac{\sqrt{(-\operatorname{cosec}\theta)^2 - 1}}{2}$$

$$= \sqrt{\operatorname{cosec}^2\theta - 1} = \pm\cot\theta$$

$\therefore\ \alpha = \tan^{-1}(\pm\cot\theta)$

Thus, $x - y\operatorname{cosec}\theta + y\cot\theta = 0$ and $x - y\operatorname{cosec}\theta - y\cot\theta = 0$ are the required equations of lines and angle between them is $\tan^{-1}(\pm\cot\theta)$

15. समीकरण $x^2 - 2xy\cot 2\alpha - y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाको समीकरण पत्ता लगाउनुहोस् साथै ती रेखाहरू बीचको कोण पत्ता लगाउनुहोस् ।
Find the equation of straight lines of the equation represented by $x^2 - 2xy\cot 2\alpha - y^2 = 0$ and find the angle between lines. **[2065 R']**

⇨ Here, given equation is $x^2 - 2xy\cot 2\alpha - y^2 = 0$ …(i)

or, $x^2 + (-2y\cot 2\alpha)x - y^2 = 0$

Comparing it with $ax^2 + bx + c = 0$,
we get $a = 1$, $b = -2y\cot 2\alpha$ and $c = -y^2$

$$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2y\cot 2\alpha \pm \sqrt{(-2y\cot 2\alpha)^2 - 4.1.(-y^2)}}{2.1}$$

$$= \frac{2y\cot 2\alpha \pm \sqrt{4y^2\cot^2 2\alpha + 4y^2}}{2}$$

$$= \frac{2y(\cot 2\alpha \pm \sqrt{\cot^2 2\alpha + 1})}{2}$$

$$= y(\cot 2\alpha \pm \sqrt{\operatorname{cosec}^2 2\alpha})$$

$$= y(\cot 2\alpha \pm \operatorname{cosec} 2\alpha)$$

Taking +ve sign,

$$x = y(\cot 2\alpha + \operatorname{cosec} 2\alpha)$$

$\therefore\ x - y(\cot 2\alpha + \operatorname{cosec} 2\alpha) = 0$ .... (ii)

Taking –ve sign,

$$x = y(\cot 2\alpha - \operatorname{cosec} 2\alpha)$$

$\therefore\ x - y(\cot 2\alpha - \operatorname{cosec} 2\alpha) = 0$ …….. (iii)

Hence equation (ii) and (iii) are the required equations of the straight lines represented by equation (i).

Again, $x^2 - 2xy\cot 2\alpha - y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1$, $2h = -2\cot 2\alpha$ and $b = -1 \Rightarrow h = -\cot 2\alpha$

Let, θ be angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm\frac{2\sqrt{\cot^2 2\alpha - 1(-1)}}{1 - 1}$$

$$= \pm\frac{2\sqrt{\cot^2 2\alpha + 1}}{0} = \infty$$

$\therefore\ \theta = \tan^{-1}\infty = 90°$

Thus, the value of θ is 90°.

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16. समीकरण $x^2 + 2xy \sec\theta + y^2 = 0$ ले जनाउने रेखाहरू बीचको न्यूनकोण α भए $\alpha = \theta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If α be the acute angle made by the straight lines represented by the equation $x^2 + 2xy \sec\theta + y^2 = 0$, prove that $\alpha = \theta$. **[2070 S', 2067 R']**

⇨ Here, $x^2 + 2xy \sec\theta + y^2 = 0$ is the form of $ax^2 + 2hxy + by^2 = 0$,

Where $a = 1, b = 1, 2h = 2\sec\theta$ or $h = \sec\theta$ & angles between the pair of lines be α .

Now, by using formula, $\tan\alpha = \pm\dfrac{2\sqrt{h^2 - ab}}{a + b}$

or, $\tan\alpha = \pm\dfrac{2\sqrt{\sec^2\theta - 1.1}}{1 + 1}$

or, $\tan\alpha = \pm\dfrac{2\sqrt{\sec^2\theta - 1}}{2}$

or, $\tan\alpha = \pm\sqrt{\tan^2\theta}$

or, $\tan\alpha = \tan\theta$ or, $-\tan\theta$

or, $\alpha = \theta$ ($\because$ α is acute angle) Thus, $\alpha = \theta$ **Proved**

MODEL 3

17. उद्गम बिन्दुबाट जाने र $2x^2 - 5xy + 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$.
[SEE MODEL 2076]

⇨ Here, $2x^2 - 5xy + 2y^2 = 0$

or, $2x^2 - 4xy - xy + 2y^2 = 0$

or, $2x(x - 2y) - y(x - 2y) = 0$

or, $(x - 2y)(2x - y) = 0$

Either,

$x - 2y = 0$ ....(i)

or, $2x - y = 0$ ....(ii)

Equation of the line perpendicular to (i) and passing through origin is

$2x + y = 0$ ....(iii)

Equation of the line perpendicuar to (ii) and passing through origin is;

$x + 2y = 0$ ....(iv)

Combining (iii) and (iv) then,

$(2x + y)(x + 2y) = 0$

or, $2x^2 + 4xy + xy + 2y^2 = 0$

$\therefore$ $2x^2 + 5xy + 2y^2 = 0$

Thus, $2x^2 + 5xy + 2y^2 = 0$ is the required equation of pair of lines.

18. $x^2 - 5xy + 4y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भइ बिन्दु (1, 2) बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of lines which pass through the point (1, 2) and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$. **[2071 R']**

⇨ Here, given equation of lines is $x^2 - 5xy + 4y^2 = 0$

or, $x^2 - 4xy - xy + 4y^2 = 0$

or, $x(x - 4y) - y(x - 4y) = 0$

or, $(x - 4y)(x - y) = 0$

Either, $x - 4y = 0$ .... (i) and

$x - y = 0$ .... (ii)

Equation of line perpendicular to line (i) is;

$4x + y = k$

It passes through the point (1, 2).

So, $4 \times 1 + 2 = k$

$\therefore$ $k = 6$

Hence, $4x + y = 6$ is a line perpendicular to (i).

Again, equation of line perpendicular to line (ii) is;

$x + y = k$

It passes through the point (1, 2).

So, $1 + 2 = k$

$\therefore$ $k = 3$

Hence, $x + y = 3$ is a line perpendicular to (ii).

Combining $4x + y = 6$ and $x + y = 3$

or, $4x + y - 6 = 0$ and $x + y - 3 = 0$

or, $(4x + y - 6)(x + y - 3) = 0$

or, $4x(x + y - 3) + y(x + y - 3) - 6(x + y - 3) = 0$

or, $4x^2 + 4xy - 12x + xy + y^2 - 3y - 6x - 6y + 18 = 0$

or, $4x^2 + 5xy + y^2 - 18x - 9y + 18 = 0$

Thus, $4x^2 + 5xy + y^2 - 18x - 9y + 18 = 0$ is the required equation of the lines.

19. समीकरण $2x^2 - 3xy - 5y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दुबाट जाने एक जोडा रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the origin and perpendicular to the lines represented by the equation $2x^2 - 3xy - 5y^2 = 0$. **[2070 R]**

⇨ Here, given equation of lines $2x^2 - 3xy - 5y^2 = 0$

or, $2x^2 - 5xy + 2xy - 5y^2 = 0$

or, $x(2x - 5y) + y(2x - 5y) = 0$

or, $(2x - 5y)(x + y) = 0$

Either, $2x - 5y = 0$ . ... (i)

or, $x + y = 0$ .... (ii)

Equation of line perpendicular to line (i) and passing through the origin is; $5x + 2y = 0$ .... (iii)

Equation of line perpendicular to line (ii) and passing through the origin is; $x - y = 0$ .... (iv)

Combining equation (iii) and (iv) then, $(5x + 2y)(x - y) = 0$

or, $5x^2 - 5xy + 2xy - 2y^2 = 0$

or, $5x^2 - 3xy - 2y^2 = 0$

Thus, the required equation of the lines is $5x^2 - 3xy - 2y^2 = 0$.

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20. **$x^2 - 5xy + 4y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भई उद्गम बिन्दुबाट जाने रेखाहरूको एउटै समीकरण के हुन्छ ?**

What is the single equation of straight lines through the origin and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$? **[2064 R', 2065 R]**

⇨ Here, given equation of lines;

$x^2 - 5xy + 4y^2 = 0$ .... (i)

or, $x^2 - 4xy - xy + 4y^2 = 0$

or, $x(x - 4y) - y(x - 4y) = 0$

$\therefore$ $(x - y)(x - 4y) = 0$

So, $x - y = 0$ and $x - 4y = 0$ are the lines represented by equation (i).

The lines $\perp_r$ to lines (i) and passing through origin are: $x + y = 0$ and $4x + y = 0$

Combining them; $(x + y)(4x + y) = 0$

or, $4x^2 + xy + 4xy + y^2 = 0$

Thus, $4x^2 + 5xy + y^2 = 0$ is the required equation of lines perpendicular to (i) and passing through origin.

21. **समीकरण $x^2 - xy - 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दु भएर जाने एक जोडा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।**

Find the single equation of a pair of straight lines through the origin and perpendicular to the lines pairs represented by $x^2 - xy - 2y^2 = 0$.

⇨ Here, given equation of lines, $x^2 - xy - 2y^2 = 0$

or, $x^2 - 2xy + xy - 2y^2 = 0$

or, $x(x - 2y) + y(x - 2y) = 0$

$\therefore$ $(x - 2y)(x + y) = 0$

Either, $x - 2y = 0$ or $x + y = 0$

Equations of the lines perpendicular to the lines $x - 2y = 0$ and $x + y = 0$ are;

$2x + y + c_1 = 0$ and $x - y + c_2 = 0$ respectively.

Since the lines pass through the point (0, 0).

So, $2 \times 0 + 0 + c_1 = 0$ and $0 - 0 + c_2 = 0$

$\therefore$ $c_1 = 0$ and $c_2 = 0$

Hence, the required equation of the pairs of lines is

$(2x + y)(x - y) = 0$

or, $2x^2 - 2xy + xy - y^2 = 0$

$\therefore$ $2x^2 - xy - y^2 = 0$

Thus, $2x^2 - xy - y^2 = 0$ is the required equation.

22. **समीकरण $2x^2 - xy - 3y^2 = 0$ ले जनाउने एक जोडी रेखासँग लम्ब हुने र उद्गम बिन्दुबाट जाने एक जोडी रेखाको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a pair of straight line passing through the origin and perpendicular to the pair of lines represented by $2x^2 - xy - 3y^2 = 0$. **[2066 R]**

⇨ Here, given equation of lines is; $2x^2 - xy - 3y^2 = 0$

or, $2x^2 - 3xy + 2xy - 3y^2 = 0$

or, $x(2x - 3y) + y(2x - 3y) = 0$

$\therefore$ $(2x - 3y)(x + y) = 0$

Either, $2x - 3y = 0$ .... (i)

or, $x + y = 0$ .... (ii)

Equation of line perpendicular to $2x - 3y = 0$ and passing through origin is $3x + 2y = 0$ ...... (iii)

Equation of line perpendicular to $x + y = 0$ and passing through origin is $x - y = 0$ .... (iv)

Combining equation (iii) and (iv);

$(3x + 2y)(x - y) = 0$

$\therefore$ $3x^2 - 3xy + 2xy - 2y^2 = 0$

Thus, $3x^2 - xy - 2y^2 = 0$ is the required equation of the lines.

23. **समीकरण $2x^2 - 7xy + 5y^2 = 0$ ले जनाउने रेखाहरूसँग लम्ब भई उद्गम बिन्दु भएर जाने जोडा रेखाको एकल समीकरण पत्ता लगाउनुहोस् ।**

Find the single equation of a pair of straight lines passing through the origin and perpendicular to the lines represented by the equation $2x^2 - 7xy + 5y^2 = 0$.

⇨ Here, given equation is $2x^2 - 7xy + 5y^2 = 0$

or, $2x^2 - 5yx - 2xy + 5y^2 = 0$

or, $x(2x - 5y) - y(2x - 5y) = 0$

or, $(2x - 5y)(x - y) = 0$

Either, $2x - 5y = 0$ .... (i) &

$x - y = 0$ .... (ii)

Now, equation of line perpendicular with $2x - 5y = 0$ and passes through (0, 0) is;

$-5x - 2y = 0$

or, $5x + 2y = 0$

Equation of line perpendicular with $x - y = 0$ & passes through (0, 0) is;

$-x - y = 0$

or, $x + y = 0$

Then, combined equation is;

$(5x + 2y)(x + y) = 0$

or, $5x^2 + 5xy + 2xy + 2y^2 = 0$

$\therefore$ $5x^2 + 7xy + 2y^2 = 0$

Thus, $5x^2 + 7xy + 2y^2 = 0$ is a required equation under the given condition.

MODEL 4

24. **यदि समीकरण $2x^2 + kxy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको कोण 45° भए k को धनात्मक मान पत्ता लगाई रेखाहरूका बेग्लाबेग्लै समीकरणहरू पनि पत्ता लगाउनुहोस् ।**

If an angle between the pair of lines represented by the equation $2x^2 + kxy + 3y^2 = 0$ is 45°, then find the positive value of k and also find the separate equations of the lines. **[2075 R, 2075 R_2]**

⇨ Here, given equation is, $2x^2 + kxy + 3y^2 = 0$ ....(i)

Comparing given equation with $ax^2 + 2hxy + by^2 = 0$, we get, $a = 2$, $2h = k \Rightarrow h = \frac{k}{2}$ and $b = 3$

We know that, Angle between two lines is given by,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

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or, $\tan 45° = \pm \dfrac{2\sqrt{\left(\frac{k}{2}\right)^2 - 2 \times 3}}{2+3}$

or, $1 = \pm \dfrac{2\sqrt{\frac{k^2 - 6 \times 4}{4}}}{5}$

or, $1 = \pm \dfrac{2\sqrt{k^2 - 24}}{2 \times 5}$

or, $5 = \pm\sqrt{k^2 - 24}$

or, $k^2 - 24 = 25$

or, $k^2 = 49$

∴ $k = 7$(+ve value only)

25. समीकरण $x^2 + xy - Ky^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा सरल रेखाहरूबिचको कोण 45° छ भने K को मान पत्ता लगाउनुहोस् ।

The angle between a pair of straight lines represented by the equation $x^2 + xy - Ky^2 = 0$ is 45°. Find the value of K. [2072 R]

⇨ Here, given equations of pair of lines;

$x^2 + xy - ky^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then, $a = 1$, $2h = 1 \Rightarrow h = \frac{1}{2}$ and $b = -k$

Let, θ be angle between the lines then,

$\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a+b}$

or, $\tan 45° = \dfrac{2\sqrt{\left(\frac{1}{2}\right)^2 + 1 \times K}}{1-K}$

or, $1 = \dfrac{2\sqrt{\frac{1}{4} + K}}{1-K}$

or, $1 - K = 2\dfrac{\sqrt{1+4K}}{2}$

or, $1 - K = \sqrt{1+4K}$

or, $1 - 2K + K^2 = 1 + 4K$

or, $K^2 - 6K = 0$

or, $K(K-6) = 0$

Either, $K = 0$ or, $K - 6 = 0$

∴ $K = 6$

Thus, the value of K is 0 or 6.

Substituting the value of k in equation (i), then

$2x^2 + 7xy + 3y^2 = 0$

or, $2x^2 + xy + 6xy + 3y^2 = 0$

or, $x(2x + y) + 3y(2x + y) = 0$

or, $(x + 3y)(2x + y) = 0$

Either, $x + 3y = 0$

or, $2x + y = 0$

Thus, the +ve value of k is 7 and separate equations are $(x + 3y) = 0$ and $(2x + y) = 0$.

26. यदि समीकरण $2x^2 + kxy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबीचको कोण 45° भए k को मान पत्ता लगाउनुहोस् ।

If the angle between the lines represented by $2x^2 + kxy + 3y^2 = 0$ is 45°, find the value of k. [2071 S]

⇨ Here, given equation of the lines is $2x^2 + kxy + 3y^2 = 0$

Angle between the lines is 45°, k = ?

Comparing given equation with $ax^2 + 2hxy + by^2 = 0$

then, $a = 2$, $2h = k$, i.e. $h = \frac{k}{2}$ and $b = 3$

Let θ be angle between the lines then,

$\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a+b}$

or, $\tan 45° = \pm \dfrac{2\sqrt{\left(\frac{k}{2}\right)^2 - 2 \times 3}}{2+3}$

or, $1 = \pm \dfrac{2\sqrt{\frac{k^2}{4} - 6}}{5}$

or, $1 = \pm 2\sqrt{\dfrac{k^2 - 24}{4}} \times \dfrac{1}{5}$

or, $1 = \pm \dfrac{\sqrt{k^2 - 24}}{5}$

or, $5 = \pm\sqrt{k^2 - 24}$

or, $(5)^2 = (\pm\sqrt{k^2 - 24})^2$

or, $25 = k^2 - 24$

or, $k^2 = 49$

∴ $k = \pm 7$

Thus, the value of k is ± 7.

MODEL 5

27. समीकरण $3x^2 + xy - 10y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग समानान्तर हुने र बिन्दु (1, 0) भएर जाने एक जोडी रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a pair of lines passing through the point (1, 0) and parallel to the lines represented by the equation $3x^2 + xy - 10y^2 = 0$. [2074 R]

⇨ Here, given equation of pair of lines

$3x^2 + xy - 10y^2 = 0$

or, $3x^2 + 6xy - 5xy - 10y^2 = 0$

or, $3x(x + 2y) - 5y(x + 2y) = 0$

or, $(x + 2y)(3x - 5y) = 0$

Either, $x + 2y = 0$ ....(i)

or, $3x - 5y = 0$ ....(ii)

Equation of line parallel to line (i) is $x + 2y = k$

It passes through (1, 0).

So, $1 + 2 \times 0 = k$

∴ $k = 1$

So, the equation of line parallel to (i) is

$x + 2y = 1$

∴ $x + 2y - 1 = 0$...............(iii)

Again, equation of line parallel to (ii) is $3x - 5y = k$.

It passes through (1, 0).

So, $3 \times 1 - 5 \times 0 = k$

∴ $k = 3$

So, the equation of line parallel to the line (ii) is

$3x - 5y = 3$

∴ $3x - 5y - 3 = 0$....................(iv)

Combining (iii) and (iv) then,

$(x + 2y - 1)(3x - 5y - 3) = 0$

or, $x(3x - 5y - 3) + 2y(3x - 5y - 3) - 1(3x - 5y - 3) = 0$

or, $3x^2 - 5xy - 3x + 6xy - 10y^2 - 6y - 3x + 5y + 3 = 0$

∴ $3x^2 + xy - 10y^2 - 6x - y + 3 = 0$

Thus, the required equation of pair of lines is :

$3x^2 + xy - 10y^2 - 6x - y + 3 = 0.$

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28. **समीकरण $x^2 + 3xy - 4y^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा सरलरेखाहरूसँग समानान्तर हुने र बिन्दु (0, 1) भएर जाने एकजोडा सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।**
Find the single equation of a pair of straight lines passing through the point (0, 1) and parallel to the pair of straight lines represented by the equation $x^2 + 3xy - 4y^2 = 0$. **[2074 S]**

⇨ Here, Given equation of pair of lines is;
$x^2 + 3xy - 4y^2 = 0$
or, $x^2 + 4xy - xy - 4y^2 = 0$
or, $x(x + 4y) - y(x + 4y) = 0$
or, $(x + 4y)(x - y) = 0$
Either, $x + 4y = 0$ ....(i)
or, $x - y = 0$ ....(ii)
Equation of line parallel to line (i) is $x + 4y = k$.
It passes through (0, 1)
so, $0 + 4 . 1 = k$
$\therefore$ $k = 4$
Hence equation of parallel line to line (i) is $x + 4y = 4$
$\therefore$ $x + 4y - 4 = 0$ ....(iii)

Equation of line parallel to line (ii) is ; $x - y = k$
It passes through (0, 1). So, $0 - 1 = k$ $\therefore$ $k = -1$
Hence equation of parallel line to line (ii) is ;
$x - y = -1$
$\therefore$ $x - y + 1 = 0$ ....(iv)
Now, combining equation (iii) and (iv) then,
$(x + 4y - 4)(x - y + 1) = 0$
or, $x(x - y + 1) + 4y(x - y + 1) - 4(x - y + 1) = 0$
or, $x^2 - xy + x + 4xy - 4y^2 + 4y - 4x + 4y - 4 = 0$
or, $x^2 + 3xy - 4y^2 - 3x + 8y - 4 = 0$
Thus, the required equation of lines is;
$x^2 + 3xy - 4y^2 - 3x + 8y - 4 = 0$.

QUESTIONS FROM CDC TEXTBOOK

4.2 जोडा रेखाहरूका समीकरण (EQUATION OF PAIR OF STRAIGHT LINES)

EXERCISE 4.2

1. (a) **समघातीय वर्ग समीकरणको दुई ओटा उदाहरण दिनुहोस् । (Give two examples of homogeneous equation of second degree.)**

⇨ Here, two examples of homogeneous equations are as follows:
(i) $4x^2 + 5xy + y^2 = 0$ and (ii) $8x^2 + 9xy + y^2 = 0$

(b) **समीकरण $ax^2 + 2hxy + by^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको कोण कति हुन्छ ?**
What is the angle between a pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$?

⇨ Here, the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$.

(c) **समीकरण $ax^2 + 2hxy + by^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू आपसमा लम्ब हुने र सम्पाती हुने अवस्थाहरू लेख्नुहोस् ।**
Write the conditions for a pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ to be perpendicular and parallel to each other.

⇨ Here, the conditions of perpendicularity and parallelism of lines represented by $ax^2 + 2hxy + by^2 = 0$ is;
(i) $a + b = 0$ (condition of perpendicular) and (ii) $h^2 - ab = 0$ (condition of parallel)

2. **तलका जोडा समीकरणहरूलाई प्रतिनिधित्व गर्ने एउटै समीकरण लेख्नुहोस् ।**
Write a single equation representing the following pair of equations.

(a) **$ax = by$ and $bx + ay = 0$**

⇨ Here, $ax = by$ and $bx + ay = 0$
First equation can be written as, $ax - by = 0$
Second equation,$bx + ay = 0$
Combining both equations,
$(ax - by)(bx + ay) = 0$
or, $abx^2 + a^2xy - b^2xy - aby^2 = 0$
or, $abx^2 + (a^2 - b^2)xy - aby^2 = 0$
Thus, the required equation of lines is $abx^2 + (a^2 - b^2)xy - aby^2 = 0$.

(b) **$2x + y = 0$ and $x + y = 0$**

⇨ Here, $2x + y = 0$ and $x + y = 0$
Combining given equations then,
$(2x + y)(x + y) = 0$
or, $2x^2 + 2xy + xy + y^2 = 0$
or, $2x^2 + 3xy + y^2 = 0$
Thus, the required equation of lines is;
$2x^2 + 3xy + y^2 = 0$.

(c) **$\sqrt{3}x = y$ and $y = 0$**

⇨ Here, $\sqrt{3}x = y$ and $y = 0$
Combining these equations then,
$(\sqrt{3}x - y)y = 0$
or, $\sqrt{3}xy - y^2 = 0$
Thus, the required single equation is $\sqrt{3}xy - y^2 = 0$.

(d) **$x + y + 2 = 0$ and $x + 2y - 1 = 0$**

⇨ Here, $x + y + 2 = 0$ and $x + 2y - 1 = 0$
Combining both equations,
$(x + y + 2)(x + 2y - 1) = 0$
or, $x^2 + 2xy - x + xy + 2y^2 - y + 2x + 4y - 2 = 0$
or, $x^2 + 3xy + 2y^2 + x + 3y - 2 = 0$ ....(i)
Thus, equation (i) represents pair of straight lines.

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3. **तलका समीकरणहरूले जनाउने दुई सरल रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of two straight lines represented by the following equations.

(a) $4x^2 + 5xy + y^2 = 0$

⇨ Here, $4x^2 + 5xy + y^2 = 0$
or, $4x^2 + 4xy + xy + y^2 = 0$
or, $4x(x + y) + y(x + y) = 0$
or, $(x + y)(4x + y) = 0$
Either, $x + y = 0$
or, $4x + y = 0$
Thus, separate equations are $x + y = 0$ and $4x + y = 0$.

(b) $4x^2 - 17xy + 4y^2 = 0$

⇨ Here, $4x^2 - 17xy + 4y^2 = 0$
or, $4x^2 - 16xy - xy + 4y^2 = 0$
or, $4x(x - 4y) - y(x - 4y) = 0$
or, $(x - 4y)(4x - y) = 0$
Either, $x - 4y = 0$
or, $4x - y = 0$
Thus, separate equations are $x - 4y = 0$ and $4x - y = 0$.

(c) $x^2 - 5xy + 4y^2 = 0$

⇨ Here, , $x^2 - 5xy + 4y^2 = 0$
or, $x^2 - 4xy - xy + 4y^2 = 0$
or, $x(x - 4y) - y(x - 4y) = 0$
or, $(x - 4y)(x - y) = 0$
Either, $x - 4y = 0$
or, $x - y = 0$
Thus, separate equations are $x - 4y = 0$ and $x - y = 0$.

(e) $33x^2 - 44xy + 11y^2 = 0$

⇨ Here, $33x^2 - 44xy + 11y^2 = 0$
or, $11(3x^2 - 4xy + y^2) = 0$
or, $3x^2 - 4xy + y^2 = 0$
or, $3x^2 - xy - 3xy + y^2 = 0$
or, $x(3x - y) - y(3x - y) = 0$
or, $(3x - y)(x - y) = 0$
Either, $3x - y = 0$ ....(i)
or, $x - y = 0$ ....(ii)
Thus, $3x - y = 0$ and $x - y = 0$ are the required separate equations of lines.

(d) $y^2 - 3xy - 2x^2 = 0$

⇨ Here, $y^2 - 3xy - 2x^2 = 0$
or, $-2x^2 - 3xy + y^2 = 0$
or, $-(2x^2 + 3xy - y^2) = 0$
or, $2x^2 + 3xy - y^2 = 0$
Comparing it with $ax^2 + bx + c = 0$ then,
$a = 2$, $b = 3y$ and $c = -y^2$

We have, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

or, $x = \dfrac{-3y \pm \sqrt{9y^2 + 4 \times 2 \times y^2}}{2 \times 2}$

or, $x = \dfrac{-3y \pm \sqrt{17y^2}}{4}$

or, $x = \dfrac{-3y \pm \sqrt{17}\,y}{4}$

Taking +ve sign,
$4x = -3y + \sqrt{17}\,y$
or, $4x + 3y - \sqrt{17}\,y = 0$
$\therefore$ $4x + (3 - \sqrt{17})y = 0$ ....(i)
Taking (–) ve sign,

$x = \dfrac{-3y - \sqrt{17}\,y}{4}$

or, $4x = -3y - \sqrt{17}\,y$
or, $4x + 3y + \sqrt{17}\,y = 0$
$\therefore$ $4x + (3 + \sqrt{17})y = 0$ ....(ii)
Thus, the equations (i) and (ii) are the required separate equations.

(f) $x^2 - y^2 = 0$

⇨ Here, $x^2 - y^2 = 0$
or, $(x + y)(x - y) = 0$
Either, $x + y = 0$
or, $x - y = 0$
Thus, $x + y = 0$ and $x - y = 0$ are the required equations of the lines.

4. **तलका समीकरणहरूले दिने सरल रेखाबिचका कोणहरू निकाल्नुहोस् ।**
Find the angles between the two straight lines given by the following equations.

(a) $6x^2 + xy - y^2 = 0$

⇨ Here, $6x^2 + xy - y^2 = 0$
Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 6$, $b = -1$ and $2h = 1$, $\therefore$ $h = \dfrac{1}{2}$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm\frac{2\sqrt{\left(\frac{1}{2}\right)^2 + 6 \times 1}}{6 - 1}$$

$$= \pm\frac{2\sqrt{\frac{1}{4} + 6}}{5}$$

$$= \pm\frac{2 \times \frac{5}{2}}{5}$$

or, $\tan\theta = \pm 1$
or, $\tan\theta = \tan 45°$ or $135°$
$\therefore$ $\theta = 45°$ or $135°$
Thus, the angle between the lines is 45° or 135°.

(b) $2x^2 + 7xy + 3y^2 = 0$

⇨ Here, $2x^2 + 7xy + 3y^2 = 0$ ....(i)
Comparing equation (i) with $ax^2 + 2hxy + by^2 = 0$,

We get, $a = 2$, $2h = 7$ $\Rightarrow$ $h = \dfrac{7}{2}$, $b = 3$

If θ be the angle between two lines represented by equation (i), then we have

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b} = \pm\frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2.3}}{2 + 3}$$

$$\text{or, } \tan\theta = \pm\frac{2\sqrt{\frac{49}{4} - 6}}{5} = \pm\frac{2\sqrt{\frac{49 - 24}{4}}}{5}$$

$$= \pm\frac{2\sqrt{\frac{25}{4}}}{5} = \pm\frac{2}{5} \times \frac{5}{2} = \pm 1$$

Now taking positive sign, we get,
$\tan\theta = 1 = \tan 45°$
$\therefore$ $\theta = 45°$
Taking negative sign, we get,
$\tan\theta = -1 = \tan 135°$
$\therefore$ $\theta = 135°$
Thus, the required angles are 45° and 135°.

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(c) $x^2 - 2\cot\alpha\, xy - y^2 = 0$

⇨ Here, $x^2 - 2\cot\alpha\, xy - y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1, b = -1$ and $2h = -2\cot\alpha$

$\therefore \quad h = -\cot\alpha$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \pm\frac{2\sqrt{(-\cot\alpha)^2 - 1\times(-1)}}{1-1}$$

$\tan\theta = \pm\infty$

or, $\tan\theta = \tan 90°$

$\therefore \quad \theta = 90°$

Alternative method:

Here $a + b = 1 - 1 = 0$

Thus, $a + b = 0$ shows that the angle between the lines is 90°.

(d) $x^2 + 2\sec\alpha\, xy + y^2 = 0$

⇨ Here, $x^2 + 2\sec\alpha\, xy + y^2 = 0$

or, $x^2 + 2\sec\alpha\,.\,xy + y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1, 2h = 2\sec\alpha$ and $b = 1$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$$

or, $\tan\theta = \pm\dfrac{2\sqrt{(\sec\alpha)^2 - 1\times 1}}{1+1}$

$= \pm\sqrt{\sec^2\alpha - 1}$

or, $\tan\theta = \pm\tan\alpha$

$\therefore \quad \theta = \alpha$ or $(180° - \alpha)$

Thus, the angle between the lines is;

α or (180° – α).

(e) $x^2 + 5xy + 6y^2 = 0$

⇨ Here, $x^2 + 5xy + 6y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1, b = 6$ and $2h = 5, \quad \therefore \quad h = \dfrac{5}{2}$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \pm\frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 1\times 6}}{1+6}$$

or, $\tan\theta = \pm\dfrac{2\sqrt{\frac{25}{4} - 6}}{7} = \pm\dfrac{2\sqrt{\frac{1}{4}}}{7} = \pm\dfrac{1}{7}$

or, $\theta = \tan^{-1}\left(\pm\dfrac{1}{7}\right) = 8°$ or $172°$

Thus, the angle between the lines is 8° or 172°.

(f) $3x^2 - 4xy + y^2 = 0$

⇨ Here, $3x^2 - 4xy + y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 3, b = 1$ and $2h = -4, \quad \therefore h = -2$

Let θ be the angle between the lines then,

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \pm\frac{2\sqrt{4 - 3\times 1}}{3+1}$$

or, $\tan\theta = \pm\dfrac{2\times 1}{4}$

or, $\tan\theta = \pm\dfrac{1}{2}$

or, $\theta = \tan^{-1}\left(\pm\dfrac{1}{2}\right) = 26.6°$ or $153.4°$

Thus, the angle between the lines is 26.6° or 153.4°.

5. तलका समीकरणहरूले दिने सरल रेखाहरू आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the straight lines given by the following equations are perpendicular to each other.

(a) $3x^2 + 8xy - 3y^2 = 0$

⇨ Here, $3x^2 + 8xy - 3y^2 = 0$

Comparing it with the equation $ax^2 + 2hxy + by^2 = 0$ then, $a = 3, \quad b = -3$ and $2h = 8$

$\therefore \quad h = \dfrac{8}{2} = 4$

Angle between lines is 90°, when

$a + b = 0$

or, $3 + (-3) = 0$

or, $0 = 0$

LHS = RHS

Thus, the lines represented by above equation are perpendicular to each other.

(b) $2x^2 - 3xy - 2y^2 = 0$

⇨ Here, $2x^2 - 3xy - 2y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 2, b = -2, 2h = -3,$

$\therefore \quad h = -\dfrac{3}{2}$

$a + b = 2 - 2 = 0$

Thus, $a + b = 0$ shows that the angle between the lines is 90°.

(c) $6x^2 + 9xy - 6y^2 = 0$

⇨ Here, $6x^2 + 9xy - 6y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 6, 2h = 9$ and $b = -6$

Now, $a + b = 6 - 6 = 0$

Thus, $a + b = 0$ shows that the angle between the lines is 90°.

(d) $5x^2 + 24xy - 5y^2 = 0$

⇨ Here, $5x^2 + 24xy - 5y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 5, 2h = 24$ and $b = -5$

Now, $a + b = 5 - 5 = 0$

Thus, $a + b = 0$ shows that the angle between the lines is 90°.

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6. **तलका समीकरणहरूले दिने सरल रेखाहरू आपसमा सम्पाती हुन्छन् भनी प्रमाणित गर्नुहोस् ।**
Prove that the straight lines given by the following equations are parallel to each other.

(a) $\boldsymbol{9x^2 - 24xy + 16y^2 = 0}$

⇨ Here, $9x^2 - 24xy + 16y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 9,\ 2h = -24$

$\therefore\ h = -12$ and $b = 16$

Now, $h^2 - ab = (-12)^2 - 9 \times 16$

$= 144 - 144 = 0$

Thus, $h^2 - ab = 0$ shows that the lines are coincident.

(b) $\boldsymbol{x^2 - 4xy + 4y^2 = 0}$

⇨ Here, $x^2 - 4xy + 4y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1,\ b = 4$ and $2h = -4$

$\therefore\ h = -2$

Now, $h^2 - ab = (-2)^2 - 1 \times 4$

$= 4 - 4 = 0$

Thus, $h^2 - ab = 0$ shows that the lines are coincident.

(c) $\boldsymbol{4x^2 - 12xy + 9y^2 = 0}$

⇨ Here, $4x^2 - 12xy + 9y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 4,\ b = 9$ and $2h = -12$

$\therefore\ h = -6$

Now, $h^2 - ab = (-6)^2 - 4 \times 9$

$= 36 - 36 = 0$

Thus, $h^2 - ab = 0$ shows that the lines are coincident.

(d) $\boldsymbol{x^2 + 6xy + 9y^2 = 0}$

⇨ Here, $x^2 + 6xy + 9y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 1,\ b = 9$ and $2h = 6$

$\therefore\ h = 3$

Now, $h^2 - ab = (3)^2 - 1 \times 9$

$= 9 - 9 = 0$

Thus, $h^2 - ab = 0$ shows that the lines are coincident.

7. **तलका समीकरणहरूले दिने सरल रेखाहरू आपसमा लम्ब छन् भने p र q को मान पत्ता लगाउनुहोस् ।**
If the straight lines given by the following equations are perpendicular to each other, find the values of p and q.

(a) $\boldsymbol{11x^2 - \frac{5}{3}xy + py^2 = 0}$

⇨ Here, $11x^2 - \frac{5}{3}xy + py^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 11$ and $b = p$

Since lines are perpendicular,

So, $a + b = 0$

or, $11 + p = 0$

$\therefore\ p = -11$

Thus, the value of p is – 11.

(b) $\boldsymbol{p^2x^2 - 5xy - 9y^2 = 0}$

⇨ Here, $p^2x^2 - 5xy - 9y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = p^2$ and $b = -9$

Since the lines are perpendicular.

So, $a + b = 0$

or, $p^2 - 9 = 0$

or, $p^2 = 9$

$\therefore\ p = \pm 3$

Thus, the value of p is ± 3.

(c) $\boldsymbol{\frac{7}{2}x^2 + 5xy + qy^2 = 0}$

⇨ Here, $\frac{7}{2}x^2 + 5xy + qy^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = \frac{7}{2}$ and $b = q$

Since the lines are perpendicular,

So, $a + b = 0$

or, $\frac{7}{2} + q = 0$

$\therefore\ q = -\frac{7}{2}$

Thus, the value of q is $-\frac{7}{2}$.

(d) $\boldsymbol{(q^2 - 1)x^2 + 2xy - (3q - 3)y^2 = 0}$

⇨ Here, $(q^2 - 1)x^2 + 2xy - (3q - 3)y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = q^2 - 1$ and $b = -(3q - 3)$

Since the lines are perpendicular,

So, $a + b = 0$

or, $q^2 - 1 - (3q - 3) = 0$

or, $q^2 - 1 - 3q + 3 = 0$

or, $q^2 - 3q + 2 = 0$

or, $q^2 - 2q - q + 2 = 0$

or, $q(q - 2) - 1(q - 2) = 0$

or, $(q - 2)(q - 1) = 0$

Either, $q - 2 = 0 \quad \therefore\ q = 2$

or, $q - 1 = 0 \quad \therefore\ q = 1$

Thus, the values of q are 2 or 1.

8. **तलका समीकरणहरूले दिने सरल रेखाहरू आपसमा सम्पाती छन् भने k को मान पत्ता लगाउनुहोस् ।**
If the straight lines given by the following equations are parallel to each other, find the value of k.

(a) $\boldsymbol{kx^2 - 8xy + 8y^2 = 0}$

⇨ Here, $kx^2 - 8xy + 8y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = k,\ b = 8$ and $2h = -8$

$\therefore\ h = -4$

If the lines are coincident then,

$h^2 - ab = 0$

or, $(-4)^2 - k \times 8 = 0$

or, $16 = 8k$

$\therefore\ k = 2$

Thus, the value of k is 2.

(b) $\boldsymbol{9x^2 - 24xy + ky^2 = 0}$

⇨ Here, $9x^2 - 24xy + ky^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 9,\ b = k$ and $2h = -24$

$\therefore\ h = -12$

If the lines are coincident then, $h^2 - ab = 0$

or, $(-12)^2 - 9 \times k = 0$

or, $144 - 9k = 0$

or, $9k = 144$

$\therefore\ k = 16$

Thus, the value of k is 16.

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(c) $2x^2 + kxy + 2y^2 = 0$

⇨ Here, $2x^2 + kxy + 2y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = 2$, $b = 2$ and $2h = k$

$\therefore\ h = \frac{k}{2}$

If the lines are coincident then, $h^2 - ab = 0$

or, $\left(\frac{k}{2}\right)^2 - 2 \times 2 = 0$

or, $\left(\frac{k}{2}\right)^2 = 4$

or, $\frac{k^2}{4} = 4$

or, $k^2 = 16$

or, $k^2 = (\pm 4)^2$

$\therefore\ k = \pm 4$

Thus, the value of k is ± 4.

9. तलका समीकरणहरूले दिने सरल रेखासँग लम्ब हुने र उद्गम बिन्दुबाट जाने जोडी रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equations of a pair of lines perpendicular to the straight lines given by the following equations and passing through the origin.

(a) $2x^2 - 3xy + y^2 = 0$

⇨ Here, $2x^2 - 3xy + y^2 = 0$

or, $2x^2 - 2xy - xy + y^2 = 0$

or, $2x(x - y) - y(x - y) = 0$

or, $(2x - y)(x - y) = 0$

Either, $2x - y = 0$ ....(i)

or, $x - y = 0$ ....(ii)

Equation of line perpendicular to (i) and passing through origin is;

$x + 2y = 0$ ....(iii)

Equation of line perpendicular to (ii) and passing through origin is;

$x + y = 0$ ....(iv)

Combining equations (iii) and (iv) then,

$(x + 2y)(x + y) = 0$

or, $x^2 + xy + 2xy + 2y^2 = 0$

$\therefore\ x^2 + 3xy + 2y^2 = 0$

Thus, the required equation is $x^2 + 3xy + 2y^2 = 0$.

(c) $x^2 - 5xy + 6y^2 = 0$

⇨ Here, $x^2 - 5xy + 6y^2 = 0$

or, $x^2 - 3xy - 2xy + 6y^2 = 0$

or, $x(x - 3y) - 2y(x - 3y) = 0$

or, $(x - 3y)(x - 2y) = 0$

Either, $x - 3y = 0$ ....(i)

or, $x - 2y = 0$ ....(ii)

Equation of line perpendicular to equation (i) and passing through origin is;

$3x + y = 0$ ....(iii)

Equation of line perpendicular to equation (ii) and passing through origin is;

$2x + y = 0$ ....(iv)

Combining equation (iii) and (iv) then,

$(3x + y)(2x + y) = 0$

or, $6x^2 + 3xy + 2xy + y^2 = 0$

or, $6x^2 + 5xy + y^2 = 0$

Thus, the required equation of lines is;
$6x^2 + 5xy + y^2 = 0$.

(d) $(k - 9)x^2 - (k - 10)xy + ky^2 = 0$

⇨ Here, $(k - 9)x^2 - (k - 10)\,xy + ky^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$ then,

$a = k - 9$, $b = k$ and $2h = -(k - 10)$

$\therefore\ h = \frac{10 - k}{2}$

If the lines are coincident then, $h^2 - ab = 0$

or, $\left(\frac{10 - k}{2}\right)^2 - (k - 9)\,k = 0$

or, $\frac{100 - 20\,k + k^2}{4} - \frac{k^2}{1} + \frac{9k}{1} = 0$

or, $\frac{100 - 20k + k^2 - 4k^2 + 36\,k}{4} = 0$

or, $-3k^2 + 16k + 100 = 0$

or, $3k^2 - 16k - 100 = 0$

Comparing it with $ax^2 + bx + c = 0$ then,

$x = k$, $a = 3$, $b = -16$ and $c = -100$

We have,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{16 \pm \sqrt{16^2 + 4 \times 3 \times 100}}{2 \times 3}$$

or, $$k = \frac{16 \pm \sqrt{256 + 1200}}{6}$$

$$= \frac{16 \pm \sqrt{1456}}{6}$$

$$= \frac{16 \pm 4\sqrt{91}}{6}$$

$$= \frac{8 \pm 2\sqrt{91}}{3}$$

Thus, the value of k is $\frac{8 \pm 2\sqrt{91}}{3}$.

(b) $2x^2 - 7xy + 5y^2 = 0$

⇨ Here, given equation is $2x^2 - 7xy + 5y^2 = 0$

or, $2x^2 - 5xy - 2xy + 5y^2 = 0$

or, $x(2x - 5y) - y(2x - 5y) = 0$

or, $(2x - 5y)(x - y) = 0$

Either, $2x - 5y = 0$ …….. (i)

& $x - y = 0$ ………… (ii)

Now, equation of line perpendicular to $2x - 5y = 0$ and passing through (0, 0) is;

$-5x - 2y = 0$

or, $5x + 2y = 0$

Equation of line perpendicular to $x - y = 0$ and passing through (0, 0) is;

$-x - y = 0$

or, $x + y = 0$

Then, combined equation is;

$(5x + 2y)(x + y) = 0$

or, $5x^2 + 5xy + 2xy + 2y^2 = 0$

$\therefore\ 5x^2 + 7xy + 2y^2 = 0$

Thus, $5x^2 + 7xy + 2y^2 = 0$ is the required equation under the given condition.

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(d) $3x^2 + 8xy + 5y^2 = 0$

⇨ Here, $3x^2 + 8xy + 5y^2 = 0$

or, $3x^2 + 3xy + 5xy + 5y^2 = 0$

or, $3x(x + y) + 5y(x + y) = 0$

or, $(x + y)(3x + 5y) = 0$

Either, $x + y = 0$ ....(i) or, $3x + 5y = 0$ ....(ii)

Equation of line perpendicular to (i) and passing through origin is; $x - y = 0$ ....(iii)

Equation of line perpendicular to (ii) and passing through origin is; $5x - 3y = 0$ ....(iv)

Combining (iii) and (iv) then,

$(x - y)(5x - 3y) = 0$

or, $5x^2 - 3xy - 5xy + 3y^2 = 0$

or, $5x^2 - 8xy + 3y^2 = 0$

Thus, the required equation of lines is $5x^2 - 8xy + 3y^2 = 0$.

10. माथि प्रश्न नं. 9 बाट प्राप्त जोडा सरल रेखाको लेखाचित्र बनाई कक्षाकोठामा प्रस्तुत गर्नुहोस् ।

Find the graph of a pair of straight lines obtained from question no.9 (above) and present it to your classroom.

⇨ Show to your teacher.

OTHER IMPORTANT QUESTIONS

1. समीकरण $6x^2 + xy - 25x - y^2 + 25 = 0$ ले जनाउने रेखाहरूको समीकरणहरू पत्ता लगाउनुहोस् र ती रेखाहरूको प्रतिच्छेदन बिन्दु पत्ता लगाउनुहोस् ।

Find the separate equations of the lines represented by $6x^2 + xy - 25x - y^2 + 25 = 0$ and find their point of intersection.

⇨ Here, $6x^2 + xy - 25x - y^2 + 25 = 0$

or, $6x^2 + (xy - 25x) + (25 - y^2) = 0$

or, $6x^2 + (y - 25)x + (25 - y^2) = 0$

Comparing it with $ax^2 + bx + c = 0$ then,

$a = 6$, $b = y - 25$ and $c = 25 - y^2$

By formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

or, $x = \frac{-(y - 25) \pm \sqrt{(y - 25)^2 - 4 \times 6(25 - y^2)}}{2 \times 6}$

or, $12x = -y + 25 \pm \sqrt{y^2 - 50y + 625 - 600 + 24y^2}$

or, $12x = -y + 25 \pm \sqrt{25y^2 - 50y + 25}$

or, $12x = -y + 25 \pm \sqrt{(5y - 5)^2}$

$\therefore$ $12x = -y + 25 \pm (5y - 5)$

Taking (+) ve sign,	Taking (–) ve sign,
$12x = -y + 25 + (5y - 5)$ or, $12x - 4y = 20$ $\therefore$ $3x - y = 5$ ......(i)	$12x = -y + 25 - (5y - 5)$ or, $12x = -y + 25 - 5y + 5$ or, $12x + 6y = 30$ $\therefore$ $2x + y = 5$ ......(ii)

Thus, the required equations are ; $3x - y = 5$ and $2x + y = 5$.

Solving equations (i) and (ii) then, $x = 2$, $y = 1$

Thus, their point of intersection is (2, 1).

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3. शाङ्किक क्षेत्र
Conic Section

KEY POINTS

शाङ्किक क्षेत्र (Conic section)

सोलीको सतहलाई समतल सतहले प्रतिच्छेदन गर्दा प्राप्त हुने वक्रलाई शाङ्किक क्षेत्र भनिन्छ ।

A conic section is a curve obtained by the intersection of the surface of a cone with a plane.

QUESTIONS FROM SEE EXERCISE 3

A. VERY SHORT QUESTIONS

1. तलका पदहरू परिभाषीत गर्नुहोस् (Define the following terms):

A. सोली (Cone)

⇨ Here, a 3-dimensional solid object that has a circular base joined to a point by a curved surface, is called cone.

In other words, rotating a right angled triangle around one of its shortest sides as fast as possible will produce a cone.

B. शाङ्किक क्षेत्र (Conic section)

⇨ Here, a conic section is a curve obtained by the intersection of the surface of a cone with a plane.

C. पाराबोला (Parabola)

⇨ Here, the plane curve formed by intersecting a cone with a plane surface in such a way that the plane surface is parallel to the generator and the angle made by the plane surface with the axis of cone (θ) is equal to the semi-vertical angle α ($\theta = \alpha$) is called parabola.

In other words, if a plane cuts the cone such that it is parallel to the generator of the cone, then the open curve or the section formed will be a parabola.

D. दीर्घ वृत्त (Ellipse)

⇨ Here, the plane curve formed by intersecting a cone with a plane surface making an angle of θ with the axis of cone in such a way that the value of θ is greater than the semi vertical angle (α) and less than 90° ($\alpha < \theta < 90°$) is called an ellipse.

In other words, if a plane cuts the cone such that the angle made by the plane with the axis is greater than the semi-vertical angle, then the closed curve or the section formed is an ellipse.

E. अतिपरावलय (Hyperbola)

⇨ Here, a plane curve formed by intersecting a double napped cone with a plane in such a way that the angle made by the plane surface with the axis of cone (θ) is less than the semi-vertical angle α ($\theta < \alpha$) is called hyperbola.

In other words, if a plane cuts the double right cone such that the angle between the axis and the plane is less than the semi-vertical angle, then their intersection forms a curve or a section which is a hyperbola.

F. सोलीको अक्ष (Axis of cone)

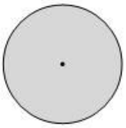
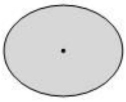
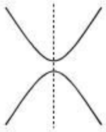
⇨ Here, the line joining vertex and the centre of the circular base is called the axis of cone.

G. वृत्त (Circle)

⇨ Here, the plane curve formed by intersecting a cone with a plane surface making an angle of 90° with the axis of cone or parallel to the base of the cone is called circle.

In other words, if a plane cuts the cone perpendicularly to its axis, then the section formed by the intersection of the plane and cone is a circle.

2. तलका वक्रहरूको नामाकरण गर्नुहोस् (Name the following curves):

(a) Circle	**(b) Ellipse**	**(c) Parabola**	**(d) Hyperbola**
			

3. एउटा सोलीलाई समतलीय सतहले आधारको समानान्तर हुनेगरि प्रतिच्छेदन गर्दा के बन्दछ ?

What will be formed if a plane intersects a cone parallel to its base? [SEE MODEL 2076]

⇨ Here, if a plane intersects a cone making parallel to the base of the cone then the conic so formed is the circle.

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4. **एउटा सतहले सोलीको अक्षसँग लम्ब हुने गरी र सोलीको शीर्षबिन्दुबाट नजाने गरी प्रतिच्छेदित गर्दछ । यसरी बन्ने शाङ्किक भागको नाम लेख्नुहोस् ।**
A plane perpendicular to the axis of cone intersects a cone not passing through the vertex. Name the section so formed.

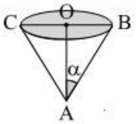
⇨ Here, if a plane perpendicular to the axis of cone not passing through the vertex intersects the cone then the conic so formed is circle.

5. **सोलीको अर्धशीर्षकोण α र सोलीको अक्षसँग समतल सतहले बनाएको कोण θ छ । यदि θ = α भए समतलीय सतह र सोलीको प्रतिच्छेदनबाट बन्ने शाङ्किकलाई के भनिन्छ ?**
The semi vertical angle of a cone is α and the angle made by the plane with the axis of cone is θ . If θ = α , what is the name of the conic section formed by the intersection of a plane surface and cone?

⇨ Here, if θ = α then the conic so formed is parabola.

6. **सँगैको चित्रमा तलका प्रत्येकले के जनाउँदछ ? (What does each of the following represent in the figure?)**

A. AO B. AB C. α D. A



⇨ Here,

A. AO = axis of cone

B. AB = Generator of cone

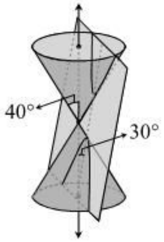
C. α = semi-vertical angle

D. A = vertex of cone

7. **कस्तो अवस्थामा दीर्घ वृत्त बन्दछ ? (In which condition, ellipse is formed?)**

⇨ Here, when the angle made by the plane (θ) with axis is greater than the semi-vertical angle (α) of cone then the ellipse is formed.

8. **एउटा सोलीको अर्ध शीर्षकोण 40° छ । समतल सतहले सोलीको अक्षसँग 30° को कोण बनाएर सो सोलीलाई प्रतिच्छेदन गर्दा बन्ने शाङ्किकको नाम लेख्नुहोस् ।**
The semi vertical angle of a cone is 40°. If the plane surface intersects the cone making an angle of 30° with the axis of the cone, write the name of the conic section so formed.



⇨ Here, the semivertical angle (α) is greater than the angle made by plane with axis of cone (θ) so the conic is hyperbola.

B. SHORT QUESTIONS

1. **एउटा सोलीको शीर्षकोण 80° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा पाराबोला बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोण पत्ता लगाउनुहोस् ।**
The vertical angle of a cone is 80°. If the cone is intersected by a plane so that a parabola is formed, find the angle between the axis of cone and the plane surface.

⇨ Here, vertical angle (2α) = 80°

So, the semi-vertical angle $(\alpha) = \frac{80°}{2} = 40°$

The conic is parabola.
So, semi-vertical angle = angle between axis of cone and plane.
or, 40° = angle between axis of cone and plane.
Thus the required angle is 40°.

2. **एउटा सोलीको शीर्षकोण 90° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा दीर्घ वृत्त बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?**
The vertical angle of a cone is 90°. If the cone is intersected by a plane so that an ellipse is formed, what will be the possible value of angle between axis of cone and the plane?

⇨ Here, vertical angle (2α) = 90°

So, semi-vertical angle $(\alpha) = \frac{90°}{2} = 45°$

When ellipse is formed then,
$\theta > \alpha$ where θ is angle between axis of cone & plane.
or, $\theta > 45°$
Thus the possible value of angle is greater than 45° and less than 90°.

3. **एउटा सोलीको शीर्षकोण 70° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा अति परावलय बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?**
The vertical angle of a cone is 70°. If the cone is intersected by a plane so that an hyperbola is formed, what will be the possible value of angle between axis of cone and the plane?

⇨ Here, vertical angle (2α) = 70°

So, semi-vertical angle $(\alpha) = \frac{70°}{2} = 35°$

If the hyberbola is formed then, $\theta < \alpha$ where θ is angle between axis of cone and plane or, $\theta < 35°$
Thus the possible value of angle is less than 35° and greater than 0°.

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4. वृत्त र दीर्घ वृत्तमा फरक लेख्नुहोस् । (Write the differences between circle and ellipse.)

⇨ The difference between circle and ellipse are as follows:

Circle	Ellipse
The plane curve formed by intersecting a cone with a plane surface making an angle of 90° with the axis of cone or parallel to the base of the cone is called circle. In other words, if a plane cuts the cone perpendicularly to its axis, then the section formed by the intersection of the plane and cone is a circle.	The plane curve formed by intersecting a cone with a plane surface making an angle of θ with the axis of cone in such a way that the value of θ is greater than the semi vertical angle (α) and less than 90° (α < θ < 90°) is called an ellipse. In other words, if a plane cuts the cone such that the angle made by the plane with the axis is greater than the semi-vertical angle, then the closed curve or the section formed is an ellipse.

5. सोलीको सचित्र परिभाषा दिनुहोस् । (Define cone with figure.)

A 3 - dimensional solid object that has a circular base joined to a point by a curved surface, is called cone. In other words, rotating a right angled triangle around one of its shortest sides as fast as possible will produce a cone.

QUESTIONS FROM CDC TEXTBOOK

4.3 शाङ्किक (CONIC SECTIONS)

EXERCISE 4.3

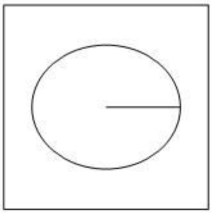
1. (a) कुनै समतलीय सतहले सोलीलाई काट्दा यदि सो सतह सोलीको अक्षसँग समानान्तर भए कुन शाङ्किक भाग बन्छ ?
If an intersecting plane cuts the cone by making the plane parallel to the axis of cone then what conic section does it form?

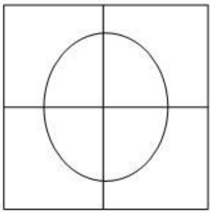
⇨ Here, the conic section is hyperbola.

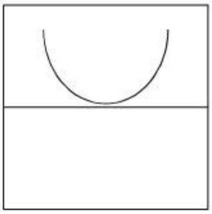
(b) कुनै समतलीय सतहले सोलीलाई काट्दा यदि सो सतह सोलीको जेनेरेटर (Generator) सँग समानान्तर भए कुन शाङ्किक भाग बन्छ ?
If an intersecting plane is parallel to the generator of the cone then what conic section does it form?

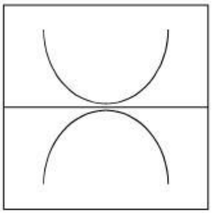
⇨ Here, the conic section is the parabola.

2. तल दिइएका ज्यामितीय आकृतिहरूको नाम लेख्नुहोस् । (Name the following geometric shapes.)









⇨ Here,
(a) Circle (b) Ellipse (c) Parabola (d) Hyperbola

3. चित्रमा दिइएका स्ट्रिपहरू प्रयोग गरी दीर्घवृत्त (Ellipse) कसरी बनाउन सकिन्छ ? समूहमा छलफल गरी नतिजा कक्षाकोठामा प्रस्तुत गर्नुहोस् ।

⇨ Show to your teacher.

4. आलु काटेर, कागज फोल्ड गरेर र माटाका डल्लो प्रयोग गरी पाराबोला (Parabola) इलिप्स (Ellipse) र हाइपरबोला (Hyperbola) कसरी बनाउन सकिन्छ ? छलफल गर्नुहोस् ।

⇨ Show to your teacher.

5. कुनै ठोस वस्तु माटो, मुला, आलु वा गाजर लिई एउटा सोलीको आधारमा हुने गरी बनाउनुहोस् । त्यसको ठाडो अक्षसँग लम्ब, समानान्तर विभिन्न कोणमा छड्के पारी काट्नुहोस् । यसरी काट्दा बन्ने सतह पहिचान गरी चित्र बनाउनुहोस् वा फोटो लिनुहोस् । यसलाई सामग्रीसहित कक्षामा प्रस्तुत गर्नुहोस् ।

⇨ Show to your teacher.

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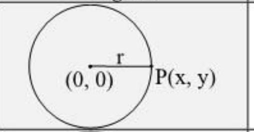
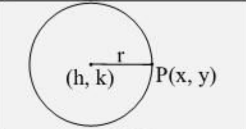
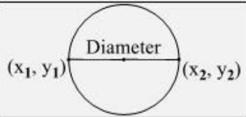
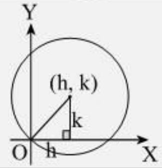
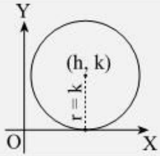
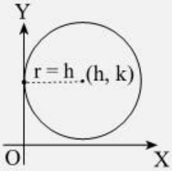
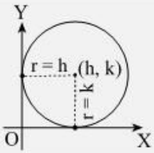
4. वृत्त
Circle

Formulae and Key Points

क्र.स. S.N.	चित्र Figure	वृत्तको अवस्था Condition of Circle	वृत्तको समीकरण Equation of Circle
1.		वृत्तको केन्द्र उद्‌गम् बिन्दुमा हुँदा । The centre of circle is at origin.	$x^2 + y^2 = r^2$
2.		वृत्तको केन्द्र बिन्दु (h, k) हुँदा । The centre of circle is (h, k).	$(x – h)^2 + (y – k)^2 = r^2$
3.		व्यासका दुई छेउका बिन्दुहरू दिँदा । Two ends of diameter are given.	$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
4.		उद्‌गम् बिन्दु भएर जाने । Passing through origin ($r^2 = h^2 + k^2$).	$(x – h)^2 + (y – k)^2 = h^2 + k^2$ $(\because r^2 = h^2 + k^2)$
5.		X-अक्षलाई छुँदा । Circle touches X-axis (r = k).	$(x – h)^2 + (y – k)^2 = k^2$ $(\because r = k)$
6.		Y-अक्षलाई छुँदा । Circle touches Y-axis (r = h).	$(x – h)^2 + (y – k)^2 = h^2$ $(\because r = h)$
7.		दुवै अक्षलाई छुँदा । Circle touches both the axes (r = h = k).	$(x – h)^2 + (y – h)^2 = h^2$ or, $(x – k)^2 + (y – k)^2 = k^2$ or, $(x – r)^2 + (y – r)^2 = r^2$ $(\because r = h = k)$

8. वृत्तको समीकरण $x^2 + y^2 + 2gx + 2fy + c = 0$ को रूपमा भए
If the equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$:

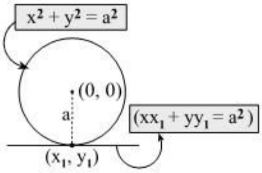
(i) केन्द्र $= (– g, – f)$ or, $(-\frac{1}{2}\,x\text{ को गुणाङ्क}, -\frac{1}{2}\,y\text{ को गुणाङ्क})$

Centre $= (– g, – f)$ or, $(-\frac{1}{2}\text{coefficient of }x, -\frac{1}{2}\text{coefficient of }y)$

(ii) अर्धव्यास $= \sqrt{g^2 + f^2 – c}$ or, $\frac{1}{2}\sqrt{(x\text{ को गुणाङ्क})^2 + (y\text{ को गुणाङ्क})^2 – 4c}$

Radius $= \sqrt{g^2 + f^2 – c}$ or, $\frac{1}{2}\sqrt{(\text{coefficient of }x)^2 + (\text{coefficient of }y)^2 – 4c}$

9. वृत्त $x^2 + y^2 = a^2$ को बिन्दु (x_1, y_1) मा खिचिएको स्पर्शरेखाको समीकरण $xx_1 + yy_1 = a^2$ हुन्छ ।
The equation of tangent in a circle $x^2 + y^2 = a^2$ at a point (x_1, y_1) is $xx_1 + yy_1 = a^2$.



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QUESTIONS FROM SEE EXERCISE 4

A. VERY SHORT QUESTIONS

1. **शाङ्किक क्षेत्रको आधारमा वृत्तको परिभाषा दिनुहोस् । (Define circle with respect to the conic section.)**

⇨ Here, the plane curve formed by intersecting a cone with a plane surface making an angle of 90° with the axis of cone or parallel to the base of the cone is called circle.

In other words, if a plane cuts the cone perpendicularly to its axis, then the section formed by the intersection of the plane and cone is a circle.

2. **समीकरण $x^2 + y^2 = r^2$ ले जनाउने बिन्दुपथको नाम लेख्नुहोस् ।**
Write the name of the locus represented by the equation $x^2 + y^2 = r^2$.

⇨ Here, the name of the locus is a circle having centre is at origin.

3. **वृत्तको व्यास रूपको समीकरण लेख्नुहोस् । (State the diameter form of equation of circle.)**

⇨ Here, the equation in diameter form is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

4. **केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' एकाइ भएको वृत्तमा कुन अवस्थामा r = k हुन्छ ?**
Under what condition will $r = k$ of a circle with centre (h, k) and radius 'r' units?

⇨ Here, when the circle touches x - axis then r = k.

5. **यदि P र Q केन्द्रबिन्दु भएका दुई वृत्तहरू बाहिरपट्टी बिन्दु A मा स्पर्श भएका छन् भने PA, QA र PQ को सम्बन्ध लेख्नुहोस् ।**
If two circles with centres P and Q touch externally at a point A then write the relation among PA, QA and PQ

⇨ Here, when the circles having centres P and Q touch externally at A then the relation among PA, QA and PQ is PQ = PA + QA

6. **एउटा वृत्तको केन्द्र (0, 0) र अर्धव्यास 3 एकाइ छ । यसको समीकरण लेख्नुहोस् ।**
A circle has its centre at (0, 0) and radius 3 units. Write its equation.

⇨ Here, equation of circle is $x^2 + y^2 = r^2$

or, $x^2 + y^2 = 3^2$

∴ $x^2 + y^2 = 9$

7. **वृत्त $(x - 1)^2 + (y - 2)^2 = 2^2$ ले कुन अक्षलाई छुन्छ, किन ? (Which axis does the circle $(x - 1)^2 + (y - 2)^2 = 2^2$ touch, why?)**

⇨ Here, equation of circle is $(x - 1)^2 + (y - 2)^2 = 2^2$. It shows that k = r

So, the circle touches x - axis.

8. **वृत्त $(x - 2)^2 + (y - 3)^2 = 5^2$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।**
Find the centre and radius of circle $(x - 2)^2 + (y - 3)^2 = 5^2$.

⇨ Here, comparing $(x - 2)^2 + (y - 3)^2 = 5^2$ with $(x - h)^2 + (y - k)^2 = r^2$ then,

Centre (h, k) = (2, 3) and radius (r) = 5 units

B. SHORT QUESTIONS

MODEL 1

1. **यदि एउटा वृत्तको समीकरण $(x + 5)^2 + y^2 = 64$ भए त्यसको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।**
If the equation of a circle is $(x + 5)^2 + y^2 = 64$, find its centre and radius. **[2075 R_2']**

⇨ Here, given equation of circle is,

$(x + 5)^2 + y^2 = 64$

or, $(x + 5)^2 + (y - 0)^2 = 8^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

We get, h = – 5, k = 0 and r = 8

∴ centre = (h, k) = (– 5, 0) and

radius = 8 units.

Thus, centre of given circle is (– 5, 0) and radius is 8 units.

2. **x + y = 5 र 2x – y = 1 वृत्तका दुई व्यासहरूका समीकरणहरू भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।**
Find the co-ordinates of the centre of a circle having equations of two diameters x + y = 5 and 2x – y = 1. **[2073 R']**

⇨ Here, given equations of diameters are

x + y = 5 ……. (i) and 2x – y = 1 …….. (ii)

Solving equation (i) and (ii) then,

$$\begin{aligned} x + y &= 5 \\ \underline{2x - y} &\underline{= 1} \\ 3x &= 6 \qquad \therefore\ x = 2 \end{aligned}$$

Putting the value of x in (i) then, x + y = 5

or, 2x + y = 5

∴ y = 3

Thus, the co-ordinates of the centre is (2, 3).

3. **यदि वृत्त $x^2 + y^2 - 4x - 6y - k = 0$ को अर्धव्यास 4 एकाइ भए k को मान पत्ता लगाउनुहोस् ।**
If the radius of the circle $x^2 + y^2 - 4x - 6y - k = 0$ is 4 units, find the value of k. **[2072 R, 2071 S]**

⇨ Here, given equation of circle is,

$x^2 + y^2 - 4x - 6y - k = 0$

or, $x^2 - 4x + y^2 - 6y - k = 0$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = k + 4 + 9$

or, $(x - 2)^2 + (y - 3)^2 = k + 13$

or, $(x - 2)^2 + (y - 3)^2 = (\sqrt{k + 13})^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,

radius (r) = $\sqrt{k + 13}$

or, $4 = \sqrt{k + 13}$

or, 16 = k + 13

∴ k = 3

Thus, the value of k is 3.

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4. **समीकरण $x^2 + y^2 - 10x - 4y = 7$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।**

Find the centre and radius of a circle having the equation $x^2 + y^2 - 10x - 4y = 7$. [2072 S]

⇨ Here, $x^2 + y^2 - 10x - 4y = 7$

or, $x^2 - 10x + y^2 - 4y = 7$

or, $x^2 - 2.x.5 + 5^2 + y^2 - 2.y.2 + 2^2 = 36$

or, $(x - 5)^2 + (y - 2)^2 = 6^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

Then, centre $(h, k) = (5, 2)$ and radius $(r) = 6$

Thus, the centre and radius are $(5, 2)$ and 6 units.

5. **समीकरण $x^2 + y^2 - 20y + 75 = 0$ भएको वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।**

Find the co-ordinates of the centre of the circle $x^2 + y^2 - 20y + 75 = 0$. [2063 R']

⇨ Here, given equation of circle is;

$x^2 + y^2 - 20y + 75 = 0$

or, $x^2 + y^2 - 2.y.10 + 10^2 - 10^2 + 75 = 0$

or, $(x - 0)^2 + (y - 10)^2 = 25$

or, $(x - 0)^2 + (y - 10)^2 = 5^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,

Centre $= (h, k) = (0, 10)$ and radius $= r = 5$ units.

Thus, the centre of given circle is $(0, 10)$.

6. **समीकरण $x^2 + y^2 - 4x + 3 = 0$ भएको वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।**

Find the centre of the circle $x^2 + y^2 - 4x + 3 = 0$. [2064 R']

⇨ Here, given equation of the circle is;

$x^2 + y^2 - 4x + 3 = 0$

or, $x^2 - 4x + 4 + y^2 - 4 + 3 = 0$

or, $(x - 2)^2 + (y - 0)^2 = 1^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

∴ Centre $= (h, k) = (2, 0)$

Thus, the centre of the circle is $(2, 0)$

7. **समीकरण $x^2 + y^2 + 4x - 6y + 4 = 0$ भएको वृत्तको अर्धव्यासको लम्बाइ पत्ता लगाउनुहोस् ।**

Calculate the length of the radius of the circle $x^2 + y^2 + 4x - 6y + 4 = 0$. [2063 R]

⇨ Here, given equation of circle is,

$x^2 + y^2 + 4x - 6y + 4 = 0$

or, $x^2 + 4x + y^2 - 6y + 4 = 0$

or, $(x)^2 + 2.x.2 + 2^2 - 2^2 + (y)^2 - 2.y.3 + 3^2 - 3^2 + 4 = 0$

or, $(x + 2)^2 + (y - 3)^2 = -4 + 2^2 + 3^2$

or, $(x + 2)^2 + (y - 3)^2 = 9$

Comparing this equation with the equation of circle

$(x - h)^2 + (y - k)^2 = r^2$

We get, $r^2 = 9$

or, $r = \sqrt{9} = 3$

Thus, the radius is 3 units.

8. **यदि वृत्त $x^2 + y^2 - ax - by - 12 = 0$ को केन्द्र (2, 3) भए a र b का मानहरू निकाल्नुहोस् ।**

If the centre of the circle $x^2 + y^2 - ax - by - 12 = 0$ is (2, 3), find the values of a and b. [2071 R]

⇨ Here, $x^2 + y^2 - ax - by - 12 = 0$

or, $x^2 - ax + y^2 - by - 12 = 0$

or, $x^2 - 2.x.\frac{a}{2} + \left(\frac{a}{2}\right)^2 + y^2 - 2.y.\frac{b}{2} + \left(\frac{b}{2}\right)^2$

$= 12 + \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$

or, $\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + 12$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,

centre $(h, k) = \left(\frac{a}{2}, \frac{b}{2}\right) = (2, 3)$

$\Rightarrow \frac{a}{2} = 2$ and $\frac{b}{2} = 3$ ∴ $a = 4$ and $b = 6$

Thus, the values of a and b are 4 and 6 respectively.

9. **समीकरण $x^2 + y^2 + 2x - 1 = 0$ भएको वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।**

Find the centre of the circle $x^2 + y^2 + 2x - 1 = 0$. [2063 M]

⇨ Here, given equation of circle is;

$x^2 + y^2 + 2x - 1 = 0$

or, $x^2 + 2x + 1 + y^2 - 2 = 0$

or, $(x^2 + 2x + 1) + y^2 = 2$

∴ $(x + 1)^2 + y^2 = (\sqrt{2})^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then, centre $(h, k) = (-1, 0)$

Thus, the centre of given circle is $(-1, 0)$.

10. **समीकरण $x^2 + y^2 - 6x + 2y + 1 = 0$ भएको वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।**

Find the centre of the circle whose equation is $x^2 + y^2 - 6x + 2y + 1 = 0$. [2068 R]

⇨ Here, given equation of circle is;

$x^2 + y^2 - 6x + 2y + 1 = 0$

or, $x^2 - 6x + y^2 + 2y + 1 = 0$

or, $x^2 - 2.3.x + 3^2 + y^2 + 2.y.1 + 1^2 = 3^2$

or, $(x - 3)^2 + (y + 1)^2 = 3^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then

centre $= (h, k) = (3 - 1)$

Thus, the centre is $(3, -1)$.

11. **समीकरण $x^2 + y^2 - 4x + 10y - 7 = 0$ भएको वृत्तको अर्धव्यासको लम्बाइ पत्ता लगाउनुहोस् ।**

What will be the length of the radius of the circle whose equation is $x^2 + y^2 - 4x + 10y - 7 = 0$? [2064 R]

⇨ Given equation of circle is;

$x^2 + y^2 - 4x + 10y - 7 = 0$

or, $x^2 - 4x + y^2 + 10y - 7 = 0$

or, $x^2 - 4x + 4 + y^2 + 10y + 25 = 7 + 4 + 25$

or, $(x - 2)^2 + (y + 5)^2 = 36$

or, $(x - 2)^2 + \{y - (-5)\}^2 = 6^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

Thus, the radius $= r = 6$ units.

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12. एउटा वृत्तको समीकरण $x^2 + y^2 - 2y = 24$ भए त्यसको अर्धव्यास र केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
If the equation of a circle is $x^2 + y^2 - 2y = 24$, find its radius and centre. **[2058 R, 2066 R']**

⇨ Here, given equation of the circle;

$x^2 + y^2 - 2y = 24$ .... (i)

Changing this equation in the form of

$(x - h)^2 + (y - k)^2 = r^2$

We get, $x^2 + y^2 - 2.y.1 + 1^2 = 24 + 1$

or, $(x - 0)^2 + (y - 1)^2 = 25$

$\therefore$ $(x - 0)^2 + (y - 1)^2 = 5^2$

Comparing this equation with the equation of the circle $(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre & r is the radius.

We get, h = 0, k = 1 and r = 5

Thus, the coordinates of the centre of the circle (h, k) = (0, 1) and radius (r) = 5 units.

13. समीकरण $x^2 + (y + 2)^2 = 49$ ले प्रतिनिधित्व गर्ने वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle represented by $x^2 + (y + 2)^2 = 49$. **[2067 R']**

⇨ Here, $x^2 + (y + 2)^2 = 49$

or, $(x - 0)^2 + (y + 2)^2 = 7^2$

It is in the form of $(x - h)^2 + (y - k)^2 = r^2$

where, centre (h, k) = (0, – 2)

Radius (r) = 7

Thus, (0, – 2) is a centre & 7 units is the radius of given circle.

14. एउटा वृत्तको समीकरण $(x + 5)^2 + y^2 = 121$ भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र व्यास निकाल्नुहोस् ।
If the equation of a circle is $(x + 5)^2 + y^2 = 121$, find the co-ordinates of the centre of the circle and its diameter. **[2059 R]**

⇨ Here, given equation of circle is;

$(x + 5)^2 + y^2 = 121$ .... (i)

Changing equation (i) in the form of equation of circle $(x - h)^2 + (y - k)^2 = r^2$

and comparing, we get,

$[x - (-5)]^2 + (y - 0)^2 = 11^2$

Where, h = – 5, k = 0 and r = 11

Thus, centre of the circle (h, k) = (–5, 0) and diameter = 2r = 2 × 11 = 22 units.

15. समीकरण $x^2 + y^2 - 2y - 48 = 0$ भएको एउटा वृत्तको परिधिको लम्बाइ पत्ता लगाउनुहोस् ।
Find the length of the circumference of a circle having the equation $x^2 + y^2 - 2y - 48 = 0$. **[2067 R]**

⇨ Here, equation of given circle is;

$x^2 + y^2 - 2y - 48 = 0$

or, $x^2 + y^2 - 2.y.1 + 1^2 - 1 - 48 = 0$

or, $(x - 0)^2 + (y - 1)^2 = 49$

$\therefore$ $(x - 0)^2 + (y - 1)^2 = (7)^2$

Which is the form of $(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) = (0, 1) & r = 7

Now, circumference of circle

$(c) = 2\pi r = 2.\frac{22}{7} \times 7 = 44$ units

Thus, circumference of given circle is 44 units.

MODEL 2

16. यदि वृत्तको केन्द्र (2, 3) र अर्धव्यास 5 एकाइ भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।
If the centre of the circle is (2, 3) and its radius is 5 units then find the equation of the circle.
[2074 S', 2070 R', 2074 R']

⇨ Here, centre = (h, k) = (2, 3) and radius (r) = 5 units

We have, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 2)^2 + (y - 3)^2 = 5^2$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 25$

$\therefore$ $x^2 + y^2 - 4x - 6y = 12$

Thus, the equation of circle is $x^2 + y^2 - 4x - 6y - 12 = 0$.

17. केन्द्र (1, –2) र ब्यास $2\sqrt{5}$ एकाइ भएको एउटा वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of a circle having centre (1, –2) and the diameter $2\sqrt{5}$ units. **[2070 R]**

⇨ Here, centre (h, k) = (1, – 2) and radius (r) = $\frac{2\sqrt{5}}{2}$ = $\sqrt{5}$ units.

We know that, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 1)^2 + (y + 2)^2 = (\sqrt{5})^2$

or, $x^2 - 2x + 1 + y^2 + 4y + 4 = 5$

or, $x^2 + y^2 - 2x + 4y = 0$

Thus, the equation of the circle is $x^2 + y^2 - 2x + 4y = 0$.

18. केन्द्रबिन्दु (0, 0) र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre (0, 0) and radius 3 units. **[2063 M]**

⇨ Here, centre of a circle (h, k) = (0, 0)

Radius of a circle (r) = 3 units

Now, equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 0)^2 + (y - 0)^2 = 3^2$

or, $x^2 + y^2 - 9 = 0$

Thus, required equation of a circle is $x^2 + y^2 - 9 = 0$.

19. केन्द्रबिन्दु (3, 0) र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having the centre (3, 0) and radius 5 units. **[2065 M]**

⇨ Here, centre = (h, k) = (3, 0), Radius = r = 5

We know that,

equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 3)^2 + (y - 0)^2 = 5^2$

or, $x^2 - 6x + 9 + y^2 = 25$

Thus, $x^2 + y^2 - 6x - 16 = 0$ is the required equation of a circle.

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20. केन्द्रबिन्दु (0, 2) र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre (0, 2) and radius is 3 units. [2064 S]

⇨ Here, centre = (0, 2) = (h, k)

Radius = 3 = r

We have, when centre is (h, k) and radius r then equation of circle,

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 0)^2 + (y - 2)^2 = 3^2$

or, $x^2 + y^2 - 4y + 4 = 9$

Thus, $x^2 + y^2 - 4y - 5 = 0$ is the required equation of circle.

21. केन्द्रबिन्दु (2, – 1) र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle with centre (2, – 1) and radius 3 units. [2065 S]

⇨ Here, in the given circle;

centre = (2, – 1) = (h, k) and Radius (r) = 3 units

We know that; the equation of circle with centre (h, k) & radius r is;

$(x - h)^2 + (y - k)^2 = r^2$

So, $(x - 2)^2 + (y + 1)^2 = 3^2$

or, $x^2 - 4x + 4 + y^2 + 2y + 1 = 9$

$\therefore$ $x^2 + y^2 - 4x + 2y = 4$

Thus, the required equation of the circle is;

$x^2 + y^2 - 4x + 2y = 4.$

22. केन्द्रबिन्दु (– 4, 1) र अर्धव्यास 4 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre at (– 4, 1) and radius 4 units. [2066 R]

⇨ Here, centre (h, k) = (– 4, 1) and radius (r) = 4 units

We know that, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x + 4)^2 + (y - 1)^2 = 4^2$

or, $x^2 + 8x + 16 + y^2 - 2y + 1 = 16$

or, $x^2 + y^2 + 8x - 2y + 1 = 0$

Thus, the equation of circle is $x^2 + y^2 + 8x - 2y + 1 = 0$.

MODEL 3

23. केन्द्रबिन्दु (2, 3) र बिन्दु (5, 6) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre at (2, 3) and passing through the point (5, 6). [2075 R']

⇨ Here, centre of circle (h, k) = (2, 3) and passing point (x, y) = (5, 6)

Now, radius of circle $= \sqrt{(x - h)^2 + (y - k)^2}$

$= \sqrt{(5 - 2)^2 + (6 - 3)^2}$

$= \sqrt{9 + 9} = \sqrt{18}$ units

We know that, equation of circle having centre (2, 3) and radius $\sqrt{18}$ units is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 2)^2 + (y - 3)^2 = (\sqrt{18})^2$

or, $x^2 + y^2 - 4x - 6y + 4 + 9 = 18$

$\therefore$ $x^2 + y^2 - 4x - 6y - 5 = 0$

Thus, the required equation of circle is;

$x^2 + y^2 - 4x - 6y - 5 = 0.$

24. केन्द्रबिन्दु (0, 0) भएको र बिन्दु (2, –3) भएर जाने एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre (0, 0) and passing through the point (2, –3). [2074 R]

⇨ Here, centre (h, k) = (0, 0),

passing point = (2, – 3)

We have,

radius (r) $= \sqrt{x^2 + y^2}$

$= \sqrt{2^2 + (-3)^2}$

$= \sqrt{13}$ units

Now, equation of circle is;

$x^2 + y^2 = r^2$

$\therefore$ $x^2 + y^2 = 13$

Thus, the required equation of circle is $x^2 + y^2 = 13$.

25. केन्द्रबिन्दु (3, 4) भएको र बिन्दु (7, 7) भएर जाने एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre (3, 4) and passing through the point (7, 7). [2074 S]

⇨ Here, centre (h, k) = (3, 4) and passing point = (7, 7)

So, Radius = distance between (3, 4) and (7, 7)

$= \sqrt{(7 - 3)^2 + (7 - 4)^2}$

$= \sqrt{4^2 + 3^2}$

$\therefore$ r = 5 unit.

Now, equation of the circle is ;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 3)^2 + (y - 4)^2 = 5^2$

or, $x^2 - 6x + 9 + y^2 - 8y + 16 = 25$

or, $x^2 + y^2 - 6x - 8y = 0$

Thus, the equation of the circle is;

$x^2 + y^2 - 6x - 8y = 0.$

26. केन्द्रबिन्दु (2, 3) भएको र बिन्दु (– 2, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre (2, 3) and passing through the point (– 2, 0). [2066 S]

⇨ Here, centre = (h, k) = (2, 3) and

Passing point = (– 2, 0)

Radius = distance between (2, 3) and (– 2, 0)

$= \sqrt{(2 + 2)^2 + (3 - 0)^2}$

$= \sqrt{4^2 + 3^2}$

= 5 units

Now, equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 2)^2 + (y - 3)^2 = 5^2$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 25$

Thus, $x^2 + y^2 - 4x - 6y = 12$ is the required equation of circle.

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27. केन्द्रबिन्दु (2, 3) भएको र बिन्दु (6, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre (2, 3) and passing through the point (6, 0). **[2068 R']**

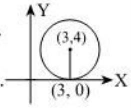
⇨ Here, $(h, k) = (2, 3)$ and passing points is $(x, y) = (6, 0)$
Now, equation of circle whose centre (h, k) is
$(x - h)^2 + (y - k)^2 = r^2$
or, $(6 - 2)^2 + (0 - 3)^2 = r^2$
or, $4^2 + (-3)^2 = r^2$
or, $16 + 9 = r^2$
or, $25 = r^2$
$\therefore$ $r = 5$

Then, putting the value of r in standard equation
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 2)^2 + (y - 3)^2 = 5^2$
or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 25$
or, $x^2 + y^2 - 4x - 6y + 13 - 25 = 0$
or, $x^2 + y^2 - 4x - 6y - 12 = 0$
Thus, the required equation of a circle is:
$x^2 + y^2 - 4x - 6y - 12 = 0.$

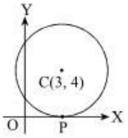
MODEL 4

28. P(3, 4) केन्द्रबिन्दु भएको वृत्तले X-अक्षमा बिन्दु (3, 0) मा छोएर जान्छ भने यो वृत्तको अर्धव्यासको नाप कति हुन्छ ?
What is the length of the radius of the circle having centre at P(3, 4) and that touches the X-axis at (3, 0) ? **[2063 S]**

⇨ Here, center $(h, k) = (3, 4)$
When circle touches X-axis then $r = k$.
$\therefore$ $r = 4$
Thus, the radius of the circle is 4 units.



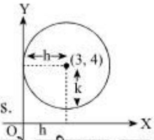
30. सँगैको चित्रमा C(3, 4) वृत्तको केन्द्रबिन्दु हो । सो वृत्तले X-अक्षको P बिन्दुमा छोएको छ भने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
In the adjoining figure, C(3, 4) is the centre of the circle. The circle touches X-axis at P. Find the equation of the circle.



⇨ Here, centre $(h, k) = (3, 4)$. The circle touches X-axis
So, radius $(r) = k = 4$ units
We know that, equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 3)^2 + (y - 4)^2 = 4^2$
or, $x^2 - 6x + 9 + y^2 - 8y + 16 = 16$
Thus, $x^2 + y^2 - 6x - 8y + 9 = 0$ is the required equation of the circle.

29. P(3, 4) केन्द्रबिन्दु भएको वृत्तले Y-अक्षमा बिन्दु (0, 4) मा छोएर जान्छ भने यो वृत्तको अर्धव्यासको नाप कति होला ?
What is the length of radius of the circle having centre at P(3, 4) and that touches the Y-axis at (0, 4) ?

⇨ Here, centre of a circle;
$(h, k) = (3, 4)$
When circle touches y-axis, $h = r$,
So, radius $(r) = 3$ units
Thus, required length of radius is 3 units.



31. केन्द्र (3, 6) भएको र x-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle having centre (3, 6) and touching the x-axis.

⇨ Here, centre $= (h, k) = (3, 6)$

The circle touches x-axis so $k = r = 6$

We know that, equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$

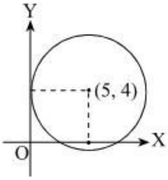
i.e. $(x - 3)^2 + (y - 6)^2 = 6^2$

or, $x^2 - 6x + 9 + y^2 - 12y + 36 = 36$

$\therefore$ $x^2 + y^2 - 6x - 12y + 9 = 0$

Thus, the equation of circle is;
$x^2 + y^2 - 6x - 12y + 9 = 0.$

32. Y-अक्षलाई स्पर्श गर्ने र केन्द्रबिन्दु (5, 4) भएको वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle having centre at (5, 4) and that touches the Y-axis. **[2063 M]**



⇨ Here, Center of circle $(h, k) = (5, 4)$ i.e. $h = 5$ and $k = 4$
Given circle touches Y-axis;
So, Radius $(r) = h = 5$
We know that; the equation of circle; $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 5)^2 + (y - 4)^2 = 5^2$
or, $x^2 - 2.x.5 + 5^2 + y^2 - 2.y.4 + 4^2 = 25$
or, $x^2 - 10x + 25 + y^2 - 8y + 16 = 25$
$\therefore$ $x^2 + y^2 - 10x - 8y + 16 = 0$
Thus, $x^2 + y^2 - 10x - 8y + 16 = 0$ is the required equation of circle.

MODEL 5

33. कुनै वृत्तको एउटा व्यासका दुई छेउका बिन्दुहरू (0, 6) र (8, 0) छन् भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The end points of a diameter of a circle are (0, 6) and (8, 0). Find the equation of the circle. **[2075 R]**

⇨ Here, let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (8, 0)$
We know that,
Equation of a circle in diameter form is;
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
or, $(x - 0)(x - 8) + (y - 6)(y - 0) = 0$
or, $x^2 - 8x + y^2 - 6y = 0$
or, $x^2 + y^2 - 8x - 6y = 0$
Thus, the equation of the circle is $x^2 + y^2 - 8x - 6y = 0$.

34. कुनै वृत्तको एउटा व्यासका दुई छेउका बिन्दुहरू (3, 0) र (0, 4) छन् भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The end points of a diameter of a circle are (3, 0) and (0, 4). Find the equation of the circle. **[2075 R$_2$]**

⇨ Here, let $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (0, 4)$
We know that,
Equation of circle in diameter form is;
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
or, $(x - 3)(x - 0) + (y - 0)(y - 4) = 0$
or, $x^2 - 3x + y^2 - 4y = 0$
or, $x^2 + y^2 - 3x - 4y = 0$
Thus, the equation of circle is $x^2 + y^2 - 3x - 4y = 0$.

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35. यदि वृत्तको एउटा व्यासका छेउ बिन्दुहरू क्रमशः (3, 4) र (– 3, – 4) भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।

If the end points of a diameter of a circle are (3, 4) and (– 3, – 4) respectively, find the equation of the circle. [2075 R']

⇨ Here, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-3, -4)$

We know that equation of circle in diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 3)(x + 3) + (y - 4)(y + 4) = 0$

or, $x^2 - 9 + y^2 - 16 = 0$

∴ $x^2 + y^2 = 25$

Thus, equation of circle is, $x^2 + y^2 = 25$.

36. एउटा व्यासको छेउछेउका बिन्दुहरू (1, 3) र (2, – 5) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle having (1, 3) and (2, – 5) as the end points of a diameter. [2073 R]

⇨ Here, end points of diameter are;

$(1, 3) = (x_1, y_1)$ and $(2, -5) = (x_2, y_2)$

We know that, equation of circle in diameter form;

$x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 1)(x - 2) + (y - 3)(y + 5) = 0$

or, $x^2 - 2x - x + 2 + y^2 + 5y - 3y - 15 = 0$

or, $x^2 + y^2 - 3x + 2y - 13 = 0$

Thus, the equation of circle is $x^2 + y^2 - 3x + 2y - 13 = 0$.

37. एउटा व्यासको छेउ छेउका बिन्दुहरू (2, 3) र (4, 0) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle having (2, 3) and (4, 0) as the end points of a diameter. [2073 S]

⇨ Here, $(2, 3) = (x_1, y_1)$ and $(4, 0) = (x_2, y_2)$

We know that,

Equation of circle in diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 2)(x - 4) + (y - 3)(y - 0) = 0$

or, $x^2 - 4x - 2x + 8 + y^2 - 0 - 3y + 0 = 0$

∴ $x^2 + y^2 - 6x - 3y + 8 = 0$

Thus, the required equation of circle is;

$x^2 + y^2 - 6x - 3y + 8 = 0$.

38. एउटा व्यासको छेउ छेउका बिन्दुहरू (– 1, 2) र (0, – 4) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle having (– 1, 2) and (0, – 4) as the end points of a diameter. [2073 S']

⇨ Here, $(-1, 2) = (x_1, y_1)$ and $(0, -4) = (x_2, y_2)$

We have, equation of circle in diameter form:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x + 1)(x - 0) + (y - 2)(y + 4) = 0$

or, $x(x + 1) + y^2 + 4y - 2y - 8 = 0$

or, $x^2 + x + y^2 + 2y - 8 = 0$

Thus, the equation of circle is $x^2 + y^2 + x + 2y - 8 = 0$.

39. बिन्दुहरू (4, 1) र (6, 5) वृत्तको एउटा व्यासका छेउछेउका बिन्दुहरू हुन् भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a circle having end points of a diameter (4, 1) and (6, 5). [2072 R']

⇨ Here, $(4, 1) = (x_1, y_1)$ and $(6, 5) = (x_2, y_2)$

We know that, diameter from of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 4)(x - 6) + (y - 1)(y - 5) = 0$

or, $x^2 - 4x - 6x + 24 + y^2 - 5y - y + 5 = 0$

or, $x^2 + y^2 - 10x - 6y + 29 = 0$

Thus, the required equation of the circle is;

$x^2 + y^2 - 10x - 6y + 29 = 0$.

40. व्यासको छेउ-छेउमा बिन्दुहरू A(5, 6) र B(3, 4) भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle having the end points of a diameter A(5, 6) and B(3, 4). [2071 R']

⇨ Here, end points of diameters are;

$A(5, 6) = (x_1, y_1)$ and $B(3, 4) = (x_2, y_2)$

We know that,

The equation of circle is diameter form;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 5)(x - 3) + (y - 6)(y - 4) = 0$

or, $x^2 - 3x - 5x + 15 + y^2 - 4y - 6y + 24 = 0$

∴ $x^2 + y^2 - 8x - 10y + 39 = 0$

Thus, $x^2 + y^2 - 8x - 10y + 39 = 0$ is the required equation of circle.

41. बिन्दुहरू (3, 2) र (– 1, 6) वृत्तको एउटा व्यासका छेउछेउको बिन्दुहरू हुन् भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a circle having end points of a diameter (3, 2) and (– 1, 6). [2065 R]

⇨ Here, end points of diameter are; (3, 2) and (– 1, 6)

We have, the diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 3)(x + 1) + (y - 2)(y - 6) = 0$

or, $x^2 + x - 3x - 3 + y^2 - 6y - 2y + 12 = 0$

Thus, $x^2 + y^2 - 2x - 8y + 9 = 0$ is the required equation of circle.

42. व्यासको छेउछेउको बिन्दुहरूको निर्देशाङ्क (– 1, 5) र (3, – 3) छन् भने वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of circle when the co-ordinates of the end of diameter are (– 1, 5) and (3, – 3). [2065 R']

⇨ Here, end points of diameter are;

$(-1, 5) = (x_1, y_1)$ and $(3, -3) = (x_2, y_2)$

We have, $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x + 1)(x - 3) + (y - 5)(y + 3) = 0$

or, $x^2 - 3x + x - 3 + y^2 - 5y + 3y - 15 = 0$

∴ $x^2 + y^2 - 2x - 2y - 18 = 0$

Thus, $x^2 + y^2 - 2x - 2y - 18 = 0$ is the required equation of circle.

43. बिन्दु (1, 2) र (3, 6) जोड्ने रेखा एउटा वृत्तको व्यास भए सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।

If the line joining the points (1, 2) and (3, 6) is the diameter of a circle, find the equation of the circle. [2058 S]

⇨ Here, given ends of the diameter of a circle are $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 6)$

We know that the equations of the circle in diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

So, $(x - 1)(x - 3) + (y - 2)(y - 6) = 0$

or, $x^2 - 3x - x + 3 + y^2 - 6y - 2y + 12 = 0$

Thus, $x^2 + y^2 - 4x - 8y + 15 = 0$ is the required equation of the circle.

44. व्यासको छेउछेउका बिन्दुहरू (– 1, 0) र (7, 4) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a circle whose diameter ends at the points (– 1, 0) and (7, 4). [2063 S]

⇨ Here, given ends of diameter are;

$(-1, 0) = (x_1, y_1)$ and $(7, 4) = (x_2, y_2)$

We know that; the diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x + 1)(x - 7) + (y - 0)(y - 4) = 0$

or, $(x^2 - 7x + x - 7) + y^2 - 4y = 0$

or, $x^2 - 6x - 7 + y^2 - 4y = 0$

∴ $x^2 + y^2 - 6x - 4y - 7 = 0$

Thus, $x^2 + y^2 - 6x - 4y - 7 = 0$ is the required equation of the circle.

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C. LONG QUESTIONS

MODEL 1

1. वृत्त $2x^2 + 2y^2 - 20x - 28y + 98 = 0$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle $2x^2 + 2y^2 - 20x - 28y + 98 = 0$. [2075 R₂']

⇨ Here, given equation of circle is,
$2x^2 + 2y^2 - 20x - 28y + 98 = 0$
or, $x^2 + y^2 - 10x - 14y + 49 = 0$
or, $x^2 - 10x + y^2 - 14y + 49 = 0$
or, $x^2 - 2 . 5 . x + 25 + y^2 - 2 . 7 . y + 49 = 25$
or, $(x - 5)^2 + (y - 7)^2 = 5^2$
Comparing this equation with the equation of Circle $(x - h)^2 + (y - k)^2 = r^2$, we get
$h = 5, k = 7$ and $r = 5$
∴ Centre $(h, k) = (5, 7)$ and radius = 5 units.
Thus, coordinates of centre is (5, 7) and radius is 5 units.

3. समीकरण $x^2 + y^2 + 6x - 4y - 3 = 0$ भएको वृत्तको केन्द्रबिन्दु र व्यासको लम्बाइ पत्ता लगाउनुहोस् ।
Find the centre and length of the diameter of a circle having an equation $x^2 + y^2 + 6x - 4y - 3 = 0$. [2074 S']

⇨ Here, the given equation of circle is ;
$x^2 + y^2 + 6x - 4y - 3 = 0$
or, $x^2 + 6x + y^2 - 4y - 3 = 0$
or, $x^2 + 2 . x . 3 + 3^2 + y^2 - 2 . y . 2 + 2^2 = 3 + 3^2 + 2^2$
or, $(x + 3)^2 + (y - 2)^2 = 16$
or, $\{x - (-3)\}^2 + (y - 2)^2 = 4^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,
Centre $= (h, k) = (-3, 2)$ and Radius $= r = 4$ units
We have, diameter $= 2r = 2 \times 4$ units $= 8$ units
Thus, the centre is $(-3, 2)$ and diameter is 8 units.

4. समीकरण $2x - 6y - x^2 - y^2 = 1$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the coordinates of the centre and the radius of a circle whose equation is $2x - 6y - x^2 - y^2 = 1$. [2057 S]

⇨ Here given equation of circle, $2x - 6y - x^2 - y^2 = 1$
or, $x^2 + y^2 - 2x + 6y + 1 = 0$
or, $x^2 - 2x + 1 + y^2 + 6y + 9 - 9 = 0$
or, $(x - 1)^2 + (y + 3)^2 = 9$
or, $(x - 1)^2 + [y - (-3)]^2 = 3^2$
Comparing this equation with the equation of the circle $(x - h)^2 + (y - k)^2 = r^2$ we get,
$h = 1, k = -3$ and $r = 3$ where (h, k) is the centre and r is the radius.
Thus, the co-ordinates of centre of the circle:
$(h, k) = (1, -3)$ and radius $(r) = 3$ unit.

6. समीकरण $x^2 + y^2 - 2x - 6y + 1 = 0$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
The equation of a circle is $x^2 + y^2 - 2x - 6y + 1 = 0$, find the co–ordinates of its centre and radius. [2059 S]

⇨ Here, given equation of circle is;
$x^2 + y^2 - 2x - 6y + 1 = 0$
Changing the given equation in the form $(x - h)^2 + (y - k)^2 = r^2$ of equation of circle, we get,
$x^2 - 2x + 1 + y^2 - 6y + 9 - 9 = 0$
or, $(x - 1)^2 + (y - 3)^2 = 9$
∴ $(x - 1)^2 + (y - 3)^2 = 3^2$
Comparing this equation with the equation of circle $(x - h)^2 + (y - k)^2 = r^2$ we get,
Centre $(h, k) = (1, 3)$ and radius $(r) = 3$ units.
Thus, the co-ordinates of centre of the circle $(h, k) = (1, 3)$ and radius $(r) = 3$ units.

2. वृत्त $x^2 + y^2 - 10x + 4y + 13 = 0$ को केन्द्र र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$. [2074 R', 2069 R']

⇨ Here, $x^2 + y^2 - 10x + 4y + 13 = 0$
or, $(x^2 - 2.5x + 5^2) + (y^2 + 2 .2y + 2^2) = 25 + 4 - 13$
or, $(x - 5)^2 + (y + 2)^2 = 16$
or, $(x - 5)^2 + \{ y - (-2)\}^2 = 4^2$
Comparing with $(x - h)^2 + (y - k)^2 = r^2$,
We get, $h = 5, k = -2$ & $r = 4$
∴ Centre $= (5, -2)$ & $r = 4$ unit.

5. समीकरण $2x^2 + 2y^2 - 2x + 6y = 45$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
The equation of a circle is $2x^2 + 2y^2 - 2x + 6y = 45$. Find the coordinates of its center and its radius. [2057 R]

⇨ Here given equation of a circle is $2x^2 + 2y^2 - 2x + 6y = 45$
or, $x^2 + y^2 - x + 3y = \frac{45}{2}$ (Dividing by 2 on both sides.)
or, $x^2 - 2.x.\frac{1}{2} + \frac{1}{4} + y^2 + 2y.\frac{3}{2} + \frac{9}{4} - \frac{1}{4} - \frac{9}{4} = \frac{45}{2}$
or, $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{45}{2} + \frac{1}{4} + \frac{9}{4}$
or, $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{90 + 1 + 9}{4}$
or, $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{100}{4}$
or, $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 25$
∴ $\left(x - \frac{1}{2}\right)^2 + \left\{y - \left(-\frac{3}{2}\right)^2\right\} = 5^2$
Comparing this equation with equation of the circle $(x - h)^2 + (y - k)^2 = r^2$
We get centre $(h, k) = \left(\frac{1}{2}, -\frac{3}{2}\right)$ and radius $(r) = 5$
Thus, centre $(h, k) = \left(\frac{1}{2}, -\frac{3}{2}\right)$ and radius $(r) = 5$ units.

7. समीकरण $x^2 + y^2 - 10x - 14y + 49 = 0$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the coordinates of the centre and the radius of a circle whose equation is $x^2 + y^2 - 10x - 14y + 49 = 0$. [2061 S]

⇨ Here, given equation of circle;
$x^2 + y^2 - 10x - 14y + 49 = 0$ .... (i)
or, $x^2 - 10x + y^2 - 14y + 49 = 0$
or, $x^2 - 10x + 25 + y^2 - 14y + 49 - 25 = 0$
or, $(x - 5)^2 + (y - 7)^2 = 25$
∴ $(x - 5)^2 + (y - 7)^2 = 5^2$
Comparing this equation with the equation of circle $(x - h)^2 + (y - k)^2 = r^2$, we get,
$h = 5, k = 7$ and $r = 5$
Thus, centre of circle $(h, k) = (5, 7)$ and radius $(r) =$ 5 units.

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8. समीकरण $9x^2 + 9y^2 - 36x + 6y = 107$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
If the equation of a circle be $9x^2 + 9y^2 - 36x + 6y = 107$, find the coordinates of the centre and the radius of the circle. **[2062 R, 2065 R]**

⇨ Here given, equation of circle;

$$9x^2 + 9y^2 - 36x + 6y = 107 \text{ .... (i)}$$

Dividing both sides by 9, we get,

$$x^2 + y^2 - 4x + \frac{2}{3}y = \frac{107}{9}$$

or, $x^2 - 4x + y^2 + \frac{2}{3}y = \frac{107}{9}$

or, $x^2 - 2 . 2x + 4 + y^2 + 2 . \frac{1}{3}y + \frac{1}{9} = \frac{107}{9} + 4 + \frac{1}{9}$

or, $(x-2)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{107+36+1}{9}$

or, $(x-2)^2 + \left[y - \left(\frac{-1}{3}\right)\right]^2 = \frac{144}{9} = 16$

∴ $(x-2)^2 + \left[y - \left(\frac{-1}{3}\right)\right]^2 = (4)^2$

This equation is of the form $(x-h)^2 + (y-k)^2 = r^2$

where, $h = 2$, $k = \frac{-1}{3}$ and $r = 4$

Thus, centre of the circle $(h, k) = (2, \frac{-1}{3})$ and radius $(r) = 4$ units.

9. एउटा वृत्तको समीकरण $4x^2 + 4y^2 - 24x - 20y - 3 = 0$ भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र व्यास पत्ता लगाउनुहोस् ।
If the equation of a circle is $4x^2 + 4y^2 - 24x - 20y - 3 = 0$, find the co-ordinates of its centre and the diameter of the circle. **[2070 R', 2062 S]**

⇨ Here, given equation of the circle is;

$$4x^2 + 4y^2 - 24x - 20y - 3 = 0$$

Dividing both the sides by 4

or, $x^2 + y^2 - 6x - 5y - \frac{3}{4} = 0$

or, $x^2 - 6x + y^2 - 5y = \frac{3}{4}$

or, $x^2 - 2.x.3 + 3^2 + y^2 - 2.y.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = \frac{3}{4} + 3^2 + \left(\frac{5}{2}\right)^2$

or, $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{3}{4} + 9 + \frac{25}{4}$

or, $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = (4)^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

Then, centre $= (h, k) = \left(3, \frac{5}{2}\right)$ and

radius $= r = 4$ units.

Then, diameter $= 2r = 2 \times 4 = 8$ units.

Thus, centre $(h, k) = \left(3, \frac{5}{2}\right)$ and diameter $= 8$ units.

10. $2x^2 + 2y^2 - 8x - 12y + 1 = 0$ समीकरण भएको वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the coordinates of the centre and the radius of the circle given by the equation $2x^2 + 2y^2 - 8x - 12y + 1 = 0$. **[2065 M]**

⇨ Here given equation of a circle is $2x^2 + 2y^2 - 8x - 12y + 1 = 0$

Dividing both the sides by 2 then, $x^2 + y^2 - 4x - 6y + \frac{1}{2} = 0$

or, $x^2 - 4x + y^2 - 6y + \frac{1}{2} = 0$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 4 + 9 - \frac{1}{2}$

or, $(x-2)^2 + (y-3)^2 = \frac{25}{2}$

∴ $(x-2)^2 + (y-3)^2 = \left(\frac{5}{\sqrt{2}}\right)^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

Thus, centre is (2, 3) and radius is $\frac{5}{\sqrt{2}}$ units.

MODEL 2

11. यदि समीकरण $x^2 + y^2 + 4x - 6y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्कहरू (0, 2) भए उक्त व्यासको अर्को छेउको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 + 4x - 6y + 8 = 0$ is (0, 2) then find the co-ordinates of the other end of the diameter. **[2074 S, 2073 S]**

⇨ Here, given equation of circle is;

$$x^2 + y^2 + 4x - 6y + 8 = 0$$

or, $x^2 + 4x + y^2 - 6y + 8 = 0$

or, $x^2 + 4x + 4 + y^2 - 6y + 9 = 9 + 4 - 8$

or, $(x+2)^2 + (y-3)^2 = 5$

∴ $(x+2)^2 + (y-3)^2 = (\sqrt{5})^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$ then, centre $(h, k) = (-2, 3)$

By the question, one end of diameter is (0, 2)

Let other end be (x_2, y_2).

Centre is the midpoint of diameter, so using formula,

$x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$

or, $-2 = \frac{0 + x_2}{2}$ or, $3 = \frac{2 + y_2}{2}$

∴ $x_2 = -4$ or, $2 + y_2 = 6$

∴ $y_2 = 4$

Thus, the co-ordinates of other end is (– 4, 4).

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12. यदि समीकरण $x^2 + y^2 - 6x + 4y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्कहरू (1, – 1) भए उक्त व्यासको अर्को छेउको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 - 6x + 4y + 8 = 0$ is (1, – 1) then find the co-ordinates of the other end of the diameter.

⇨ Here, given equation of circle is;

$x^2 + y^2 - 6x + 4y + 8 = 0$

or, $x^2 - 6x + y^2 + 4y + 8 = 0$

or, $x^2 - 6x + 9 + y^2 + 4y + 4 = -8 + 4 + 9$

or, $(x - 3)^2 + (y + 2)^2 = 5$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then centre $(h, k) = (3, -2)$

Let the co-ordinates of other point be (x_2, y_2) then, we have, $(3, -2)$ is the midpoint of $(1, -1)$ and (x_2, y_2).

Using formula,

$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}$$

or, $3 = \frac{1 + x_2}{2}$ and $-2 = \frac{-1 + y_2}{2}$

or, $6 = 1 + x_2$ and $-4 = -1 + y_2$

$\therefore\ x_2 = 5 \qquad \therefore\ y_2 = -3$

Thus, the co-ordinates of other end is (5, – 3)

13. यदि समीकरण $x^2 + y^2 + 4x - 6y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्क (– 4, 4) भए उक्त व्यासको अर्को छेउका निर्देशाङ्क पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 + 4x - 6y + 8 = 0$ is (– 4, 4), then find the co-ordinates of the other end of the diameter.

⇨ Here, given equation of circle is $x^2 + y^2 + 4x - 6y + 8 = 0$

or, $x^2 + 4x + y^2 - 6y + 8 = 0$

or, $x^2 + 4x + 4 + y^2 - 6y + 9 = 9 + 4 - 8$

or, $(x + 2)^2 + (y - 3)^2 = 5$

$\therefore\ (x + 2)^2 + (y - 3)^2 = (\sqrt{5})^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then, centre $(h, k) = (-2, 3)$

One end of diameter = (– 4, 4)

Let other end be (x_2, y_2)

Centre is the mid-point of diameter, so using formula,

$$x = \frac{x_1 + x_2}{2} \qquad \text{and } y = \frac{y_1 + y_2}{2}$$

or, $-2 = \frac{-4 + x_2}{2}$ $\qquad$ or, $3 = \frac{4 + y_2}{2}$

or, $-4 = -4 + x_2$ $\qquad$ or, $6 = 4 + y_2$

$\therefore\ x_2 = 0 \qquad \therefore\ y_2 = 2$

Thus, the co-ordinates of other end is (0, 2).

14. वृत्त $x^2 + y^2 - 2x - 2y = 8$ को व्यासको एउटा छेउको निर्देशाङ्क (2, 4) भए अर्को छेउको निर्देशाङ्क पत्ता लगाउनुहोस् ।
The co-ordinates of an end point of the diameter of circle $x^2 + y^2 - 2x - 2y = 8$ is (2, 4), find the coordinate of other end.

⇨ Here, equation of a given circle is $x^2 + y^2 - 2x - 2y = 8$

or, $x^2 - 2x + y^2 - 2y = 8$

or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 8 + 2$

or, $(x - 1)^2 + (y - 1)^2 = (\sqrt{10})^2$

Which is the form $(x - h)^2 + (y - k)^2 = r^2$

So, centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{10}$

Now, one end of the diameter of a circle

$(x_1, y_1) = (2, 4)$ other end is (x_2, y_2).

Where, centre (h, k) is a mid-point of diameter of a circle.

Then, $h = \frac{x_1 + x_2}{2}$ & $k = \frac{y_1 + y_2}{2}$

or, $1 = \frac{2 + x_2}{2}$, $1 = \frac{4 + y_2}{2}$

or, $2 = 2 + x_2$, $2 = 4 + y_2$

or, $x_2 = 0$ & $y_2 = -2$

Thus, other end point of diameter of a circle is (0, – 2).

15. समीकरण $x^2 + y^2 - 4x - 6y + 11 = 0$ भएको वृत्तको एउटा व्यासको एक छेउ (1, 2) भए अर्को छेउ पनि पत्ता लगाउनुहोस् ।
If (1, 2) is one end of a diameter of the circle with equation $x^2 + y^2 - 4x - 6y + 11 = 0$, find the other end.

⇨ Here, $x^2 + y^2 - 4x - 6y + 11 = 0$ is a equation of a circle.

$(1, 2) = (x_1, y_1)$ is a one end of a diameter of a circle.

Now, $x^2 - 4x + y^2 - 6y + 11 = 0$

or, $x^2 - 2.x.2 + 2^2 + y^2 - 2.y.3 + 3^2 - 4 - 9 + 11 = 0$

or, $(x - 2)^2 + (y - 3)^2 = 2$

or, $(x - 2)^2 + (y - 3)^2 = (\sqrt{2})^2$

It is in the form of equation of a circle

$(x - h)^2 + (y - k)^2 = r^2$

Where centre $(h, k) = (2, 3)$ & radius $(r) = \sqrt{2}$

Then, let other end of diameter of a circle be (x_2, y_2) and centre $(h, k) = (2, 3)$ be a mid-point of diameter.

So, $h = \frac{x_1 + x_2}{2}$ $\qquad$ and $k = \frac{y_1 + y_2}{2}$

or, $2 = \frac{1 + x_2}{2}$ $\qquad$ or, $3 = \frac{2 + y_2}{2}$

or, $4 - 1 = x_2$ $\qquad$ or, $6 - 2 = y_2$

or, $x_2 = 3$ $\qquad$ or, $y_2 = 4$

Thus, other ending point of a diameter is (3, 4).

MODEL 3

16. केन्द्र बिन्दु (1, 2) भएको र रेखाहरू $x + 2y = 3$ र $3x + y = 4$ को प्रतिच्छेदन बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre (1, 2) and passing through the point of intersection of the lines $x + 2y = 3$ and $3x + y = 4$. **[2072 R]**

⇨ Here, given equations are;

$x + 2y = 3$ … (i) and $3x + y = 4$ …….. (ii)

Solving equation (i) and (ii) then,

$$\begin{array}{ll} & (x + 2y = 3) \times 3 \\ \Rightarrow & 3x + 6y = 9 \\ \text{and} & 3x + y = 4 \\ & \underline{-\quad -\quad -} \\ & 5y \quad = 5 \end{array} \qquad \therefore\ y = 1$$

Putting the value of y = 1 in (i) then; $x + 2y = 3$

or, $x + 2 \times 1 = 3 \qquad \therefore\ x = 1$

So, the passing point is (1, 1).

Centre $(h, k) = (1, 2)$

We have, radius $= \sqrt{(2 - 1)^2 + (1 - 1)^2}$

$= \sqrt{1^2 + 0^2} = 1$ unit

Now, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 1)^2 + (y - 2)^2 = 1^2$

or, $x^2 - 2x + 1 + y^2 - 4y + 4 = 1$

or, $x^2 + y^2 - 2x - 4y + 4 = 0$

Thus, the equation of the circle is

$x^2 + y^2 - 2x - 4y + 4 = 0.$

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17. बिन्दुहरू (– 1, 3) र (3, 1) जोड्ने रेखाको मध्य-बिन्दुबाट जाने केन्द्रबिन्दु (4, 6) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre (4, 6) and passing through the midpoint of the line joining the points (– 1, 3) and (3, 1). [2072 S]

⇨ Here, centre of circle (h, k) = (4, 6) and passing point is the midpoint of (–1 , 3) and (3, 1).

So, passing point = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1+3}{2}, \frac{3+1}{2}\right) = (1, 2)$

Now, radius = distance between (1, 2) and (4, 6) $= \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = 5$ units

We know that, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 4)^2 + (y - 6)^2 = 5^2$

or, $x^2 - 8x + 16 + y^2 - 12y + 36 = 25$

or, $x^2 + y^2 - 8x - 12y + 27 = 0$

Thus, the required equation of circle is $x^2 + y^2 - 8x - 12y + 27 = 0$.

MODEL 4

18. रेखाहरू x – y = 4 र 2x + 3 y + 7 = 0 को प्रतिच्छेदन बिन्दु केन्द्रबिन्दु भएको र बिन्दु (2, 4) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre as the point of intersection of the lines x – y = 4 and 2x + 3y + 7 = 0 and passing through the point (2, 4). [2068 R]

⇨ Here, given equation of lines are,

$x - y = 4$ .... (i) $2x + 3y = -7$ .... (ii)

Solving equation (i) × 2 and subtracting equation (ii) then,

$$\begin{array}{r} 2x - 2y = 8 \\ 2x + 3y = -7 \\ \underline{-\quad -\qquad +} \\ -5y = 15 \end{array}$$

∴ $y = -3$

Putting $y = -3$ in equation (i) then, $x - y = 4$

or, $x + 3 = 4$

∴ $x = 1$

So, the centre of circle (h, k) = (1, – 3)

Radius = distance between (1, – 3) & (2, 4)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2-1)^2 + (4+3)^2}$

$= \sqrt{(1)^2 + 7^2}$

∴ $r = \sqrt{50}$

We know that, equation of circle with centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 1)^2 + (y + 3)^2 = (\sqrt{50})^2$

or, $x^2 - 2x + 1 + y^2 + 6y + 9 = 50$

or, $x^2 + y^2 - 2x + 6y = 40$

Thus, the equation of circle is $x^2 + y^2 - 2x + 6y - 40 = 0$.

19. बिन्दु (3, 4) बाट जाने एउटा वृत्तको दुई व्यासहरूका समीकरणहरू x + y = 14 र 2x – y = 4 भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle passing through the point (3, 4) are x + y = 14 and 2x – y = 4. Find the equation of the circle. [2061 R]

⇨ Here, given equation of the two diameter of a circle are ; $x + y = 14$ ...... (i), $2x - y = 4$ .... (ii)

Adding equations (i) and (ii) we get, $3x = 18$

∴ $x = \frac{18}{3} = 6$

Put x = 6 in equation (i), we get, $6 + y = 14$

∴ $y = 14 - 6 = 8$

∴ The point of intersection of two diameters is; (6, 8).

But two diameters intersect at the centre only.

∴ Centre of the circle (h, k) = (6, 8).

Also circle passes through the point (3, 4).

So, radius (r) = distance between (6, 8) and (3, 4)

$= \sqrt{(6-3)^2 + (8-4)^2}$

$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$

$= \sqrt{25}$

= 5 units.

We have, equation of the circle is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 6)^2 + (y - 8)^2 = 5^2$

or, $x^2 - 12x + 36 + y^2 - 16y + 64 = 25$

or, $x^2 + y^2 - 12x - 16y + 100 - 25 = 0$

Thus, $x^2 + y^2 - 12x - 16y + 75 = 0$ is the required equation of the circle.

20. सरल रेखाहरू 2x + y = 4 र 2y – x = 3 का प्रतिच्छेदन बिन्दुमा केन्द्रबिन्दु पर्ने तथा बिन्दु (4, 6) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle whose centre is at the point of intersection of 2x + y = 4 and 2y – x = 3 and passing the point (4, 6). [2065 S, 2066 R]

⇨ Here, give equation of lines are; $2x + y = 4$ .... (i) and $2y - x = 3$ .... (ii)

Solving the equations then,

$$\begin{array}{r} 2x + y = 4 \\ \underline{-\ 2x + 4y = 6} \\ 5y = 10 \end{array}$$

∴ $y = 2$

Subsisting the value of y in (i) then, $2x + 2 = 4$

or, $2x = 2$ ∴ $x = 1$

Hence, centre (h, k) = (1, 2)

Now, radius $= \sqrt{(x_2 - h)^2 + (y_2 - k)^2}$ distance between points (1, 2) & (4, 6)

∴ $r = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = 5$ units

We know that, equation of circle with centre (h, k) and radius r is; $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 1)^2 + (y - 2)^2 = 5^2$

or, $x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

∴ $x^2 + y^2 - 2x - 4y = 20$

Thus, the equation of circle is $x^2 + y^2 - 2x - 4y = 20$.

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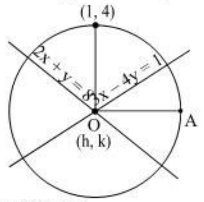
21. वृत्तको व्यासहरू $3x - 4y - 1 = 0$ र $2x + y - 8 = 0$ हुने र बिन्दु (1, 4) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through (1, 4) and the diameters of the circle are $3x - 4y - 1 = 0$ and $2x + y - 8 = 0$. [2064 S]

⇨ Here, given equation of diameters are;
$3x - 4y = 1$ .... (i) and $2x + y = 8$ .... (ii)



Solving (i) and (ii) × 4
$3x - 4y = 1$
$\underline{8x + 4y = 32}$
$11x \quad = 33$
$\therefore \quad x = 3$
Putting $x = 3$ in (i) then; $3x - 4y = 1$
or, $3 \times 3 - 4y = 1$ or, $8 = 4y$
$\therefore \quad y = 2$
$\therefore$ Centre $(h, k) = (x, y) = (3, 2)$.
Points at the circumference (1, 4).
Now, radius = distance between points (3, 2) & (1, 4)
$= \sqrt{(1-3)^2 + (4-2)^2} = \sqrt{(-2)^2 + (2)^2}$
$\therefore$ radius (r) $= 2\sqrt{2}$ units
We know that; equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$.
or, $(x - 3)^2 + (y - 2)^2 = (2\sqrt{2})^2$
or, $x^2 - 6x + 9 + y^2 - 4y + 4 = 8$
Thus, $x^2 + y^2 - 6x - 4y + 5 = 0$ is the required equation of the circle.

22. बिन्दु (5, 1) बाट जाने एउटा वृत्तका दुई व्यासहरूका समीकरणहरू क्रमशः $x - y = 3$ र $2x + y = 21$ भए सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle passing through the point (5, 1) are $x - y = 3$ and $2x + y = 21$ respectively. Find the equation of the circle. [2067 R]

⇨ Here, equation of a diameters of a circle are
$x - y = 3$ .... (i) & $2x + y = 21$ .... (ii)
Adding equation (i) & (ii) we get
$3x = 24$ or, $x = 8$
Putting the value of x in $x - y = 3$
Where $x - y = 3$ or $8 - y = 3$ or $y = 5$
Now, intersection point of a diameter is centre of a circle so, $(h, k) = (8, 5)$ & passes through equation of a circle from point (5, 1).
So, $(x - h)^2 + (y - k)^2 = r^2$
or, $(5 - 8)^2 + (1 - 5)^2 = r^2$
or, $(-3)^2 + (-4)^2 = r^2$
or, $9 + 16 = r^2$
or, $25 = r^2$
$\therefore \quad r = 5$
Then, putting the value of $(h, k) = (8, 5)$ & $r = 5$ is equation of a circle
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 8)^2 + (y - 5)^2 = 5^2$
or, $x^2 - 16x + 64 + y^2 - 10y + 25 = 25$
or, $x^2 + y^2 - 16x - 10y + 64 = 0$
Thus, the required equation of the circle is $x^2 + y^2 - 16x - 10y + 64 = 0$.

23. वृत्तको दुई व्यासको समीकरण क्रमशः $2x + y = 5$ र $x - y = 1$ र बिन्दु (1, 4) भएर जाने वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle which passes through the points (1, 4) and two diameters whose equations are $2x + y = 5$ and $x - y = 1$. [2065 E]

⇨ Here, given equation of diameters are $2x + y = 5$
or, $y = 5 - 2x$ ……. (i) $x - y = 1$ ……. (ii)
Putting the value of y from equation (i) to equation (ii) we get,
$x - (5 - 2x) = 1$
or, $x - 5 + 2x = 1$
or, $3x = 6$
or, $x = \frac{6}{3} = 2$
Then, $y = 5 - 2x = 5 - 4 = 1$
Where, intersection point of diameter of a circle is centre so, $(h, k) = (2, 1)$
Now, point at circumference is (1, 4)
So, r $= \sqrt{(1-2)^2 + (4-1)^2}$
$= \sqrt{(-1)^2 + (3)^2} = \sqrt{1 + 9}$
$= \sqrt{10} \quad [\because r = \sqrt{(x-h)^2 + (y-k)^2}]$
So, equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 2)^2 + (y - 1)^2 = (\sqrt{10})^2$
or, $x^2 - 4x + 4 + y^2 - 2y + 1 = 10$
or, $x^2 + y^2 - 4x - 2y - 5 = 0$
Thus, required equation of a circle is;
$x^2 + y^2 - 4x - 2y - 5 = 0$.

24. एउटा वृत्तको अर्धव्यास 5 एकाइ छ र दुई व्यासहरूका समीकरणहरू $2x - y = 5$ र $x - 3y + 5 = 0$ छन् । सो वृत्तको समीकरण निकाल्नुहोस् र उक्त वृत्त उद्‌गम बिन्दुबाट जान्छ भनी प्रमाणित गर्नुहोस् ।
A circle has radius 5 units & the equations of its 2 diameters are $2x - y = 5$ & $x - 3y + 5 = 0$. Find the equation of the circle and prove that it passes through the origin. [2060 R]

⇨ Here, given radius of the circle (r) = 5 units
Equations of two diameters are
$2x - y = 5$ .... (i) & $x - 3y + 5 = 0$ .... (ii)
Since the point of intersection of the two diameters is the center. So, solving equations (i) & (ii), we get,
From eq^n (ii) $x = 3y - 5$ in (i) we get,
$2(3y - 5) - y = 5$
or, $6y - 10 - y = 5$
or, $5y = 5 + 10$
or, $y = \frac{15}{5} = 3$
$\therefore \quad y = 3$
Put $y = 3$ in (i), we get, $2x - 3 = 5$
or, $2x = 5 + 3$
or, $x = \frac{8}{2} = 4$
$\therefore \quad x = 4$
$\therefore$ The centre of the circle $(h, k) = (4, 3)$
We have, eq^n of the circle with centre at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 4)^2 + (y - 3)^2 = 5^2$
or, $x^2 - 2.x.4 + 4^2 + y^2 - 2.y.3 + 3^2 = 25$
or, $x^2 - 8x + 16 + y^2 - 6y + 9 - 25 = 0$
Thus, $x^2 + y^2 - 8x - 6y = 0$ is the required eq^n of the circle.
Again, substituting (0, 0) in eq^n of the circle,
we get, $0^2 + 0^2 - 8.0 - 6.0 = 0$
$\therefore \quad 0 = 0$ which is true.
Thus, the circle passes through the origin. **Proved.**

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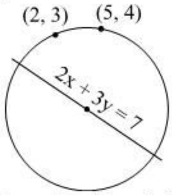
25. **बिन्दुहरू (2, 3) र (5, 4) भएर जाने तथा केन्द्रबिन्दु रेखा 2x + 3y – 7 = 0 मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of the circle passing through the points (2, 3) and (5, 4) and centre on the line 2x + 3y – 7 = 0. **[2073 R']**

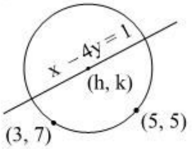
⇨ Here, given equations of centre line is $2x + 3y = 7$.



Let (h, k) be the centre of the circle then,
$2h + 3k = 7$ …. (i)
Distance between (2, 3) and (h, k)
= Distance between (5, 4) and (h, k)
So, $(h - 2)^2 + (k - 3)^2 = (h - 5)^2 + (k - 4)^2$
or, $h^2 - 4h + 4 + k^2 - 6k + 9 = h^2 - 10h + 25 + k^2 - 8k + 16$
or, $6h + 2k = 28$
∴ $3h + k = 14$ …… (ii)
Solving equations (i) and (ii) × 3 then,
$2h + 3k = 7$
$9h + 3k = 42$
$-\ \ -\ \ \ -$
$-7h = -35$
∴ $h = 5$
Putting the value of h in (ii) then, $3 \times 5 + k = 14$
∴ $k = -1$
So, centre (h, k) = (5, – 1)
Radius = Distance between (2, 3) and (5, – 1)
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5 - 2)^2 + (-1 - 3)^2}$
$= \sqrt{3^2 + 4^2}$
∴ Radius = 5 units
Now, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 5)^2 + (y + 1)^2 = 5^2$
or, $x^2 - 10x + 25 + y^2 + 2y + 1 = 25$
or, $x^2 + y^2 - 10x + 2y + 1 = 0$
Thus, the equation of circle is;
$x^2 + y^2 - 10x + 2y + 1 = 0.$

26. **एउटा वृत्तको केन्द्रबिन्दु, रेखा x – 4y = 1 मा पर्दछ । सो वृत्त बिन्दुहरू (3, 7) र (5, 5) भएर जान्छ भने उक्त वृत्तको समीकरण निकाल्नुहोस् ।**
Find the equation of the circle whose centre lies on the line x – 4y = 1 and which passes through the points (3, 7) and (5, 5). **[2071 R]**

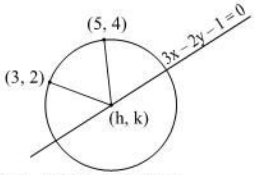
⇨ Here, let (h, k) be the centre of the circle.



Given line x – 4y = 1 passes through the centre.
So, $h - 4k = 1$
∴ $h = 4k + 1$ ...... (i)
We have,
Distance between (h, k) and (3, 7)
= Distance (h, k) and (5, 5)
So, $\sqrt{(h - 3)^2 + (k - 7)^2} = \sqrt{(h - 5)^2 + (k - 5)^2}$
or, $h^2 - 6h + 9 + k^2 - 14k + 49$
$= h^2 - 10h + 25 + k^2 - 10k + 25$
or, $4h - 4k = -8$
or, $h - k = -2$
or, $4k + 1 - k = -2$ [From (i)]
or, $3k = -3$
∴ $k = -1$
From (i) $h = 4k + 1 = 4 \times (-1) + 1 = -3$
So, the centre (h, k) = (– 1, – 3)
Now, radius $= \sqrt{(h - 3)^2 + (k - 7)^2}$
$= \sqrt{(-3 - 3)^2 + (-1 - 7)^2}$
$= \sqrt{36 + 64} = \sqrt{100}$
= 10 unit.
We know that,
Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x + 3)^2 + (y + 1)^2 = 100$
or, $x^2 + 6x + 9 + y^2 + 2y + 1 = 100$
or, $x^2 + y^2 + 6x + 2y = 90$
∴ $x^2 + y^2 + 6x + 2y - 90 = 0$
Thus, the required equation of the circle is;
$x^2 + y^2 + 6x + 2y - 90 = 0.$

27. **बिन्दुहरू (3, 2) र (5, 4) भएर जाने र केन्द्र 3x – 2y – 1 = 0 मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।** **[2071 S]**
Find the equation of the circle which passes through the points (3, 2) and (5, 4) and its centre lies on the line 3x – 2y – 1= 0.

⇨ Here, given equation of line is $3x - 2y = 1$ ...... (i)
It passes through (h, k).



So, $3 \times h - 2 \times k = 1$
∴ $3h - 2k = 1$ .... (ii)
Since (3, 2) and (5, 4) are equidistant from (6, 2).
So, $(3 - h)^2 + (2 - k)^2 = (5 - h)^2 + (4 - k)^2$
or, $9 - 6h + h^2 + 4 - 4k + k^2$
$= 25 - 10h + h^2 + 16 - 8k + k^2$
or, $-6h + 10h - 4k + 8k = 41 - 13$
or, $4h + 4k = 28$
∴ $h + k = 7$ ...... (iii)

Solving equation (ii) and (iii) × 2 then,
$3h - 2k = 1$
$\underline{2h + 2k = 14}$
$5h = 15$
∴ $h = 3$
From (iii) $h + k = 7$
or, $3 + k = 7$
∴ $k = 4$
So, centre (h, k) = (3, 4)
We have, radius (r) $= \sqrt{(5 - h)^2 + (4 - k)^2}$
$= \sqrt{(5 - 3)^2 + (4 - 4)^2} = 2$
Now equation of circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 3)^2 + (y - 4)^2 = 2^2$
or, $x^2 - 6x + 9 + y^2 - 8y + 16 = 4$
or, $x^2 + y^2 - 6x - 8y + 21 = 0$
Thus, the required equation of circle is;
$x^2 + y^2 - 6x - 8y + 21 = 0.$

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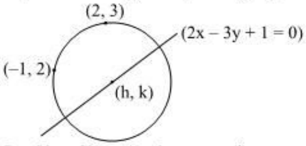
28. बिन्दुहरू (2, 3) र (– 1, 2) भएर जाने र वृत्तको केन्द्रबिन्दु रेखा 2x – 3y + 1 = 0 मा पर्दछ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle passing through the points (2, 3) and (– 1, 2) and centre lies on the line 2x – 3y + 1 = 0. [2067 S]

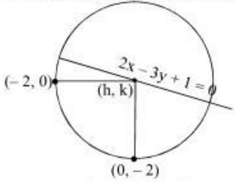
⇨ Here, give equation of central line is $2x - 3y + 1 = 0$
It passes through the point (h, k)



So, $2h - 3k + 1 = 0$ .... (i)
We know that,
Distance between (– 1, 2) and (h, k) = distance between (2, 3) and (h, k)
i.e. $\sqrt{(h+1)^2 + (k-2)^2} = \sqrt{(h-2)^2 + (k-3)^2}$
or, $h^2 + 2h + 1 + k^2 - 4k + 4 = h^2 - 4h + 4 + k^2 - 6k + 9$
or, $6h + 2k = 8$
$\therefore$ $3h + k = 4$ .... (ii)
Solving equation (i) and equation (ii) × 3
$2h - 3k + 1 = 0$
$\underline{9h + 3k - 12 = 0}$
$11h - 11 = 0$
$\therefore$ $h = 1$
From (ii); $3h + k = 4$
or, $3 \times 1 + k = 4$
$\therefore$ $k = 1$
So the centre of circle = (1, 1)
For radius $= \sqrt{(h+1)^2 + (k-2)^2}$
$= \sqrt{(1+1)^2 + (1-2)^2} = \sqrt{5}$ units
Now, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$
i.e. $(x - 1)^2 + (y - 1)^2 = 5$
or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 5$
or, $x^2 + y^2 - 2x - 2y - 3 = 0$
Thus, the required equation of circle is;
$x^2 + y^2 - 2x - 2y - 3 = 0.$

29. बिन्दुहरू (– 2, 0) र (0, – 2) भएर जाने र केन्द्रबिन्दु सीधारेखा 2x – 3y + 1 = 0 मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the points (– 2, 0) and (0, – 2) and its centre lies on the straight line 2x – 3y + 1 = 0. [2063 R']

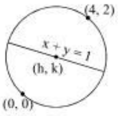
⇨ Here, given equation of diameter is $2x - 3y + 1 = 0$.



It passes through (h, k)
So, $2h - 3k + 1 = 0$ .... (i)
Again, d{(– 2, 0) and (h, k)} = d{(0, – 2) and (h, k)}
or, $\sqrt{(h+2)^2 + (k-0)^2} = \sqrt{(h-0)^2 + (k+2)^2}$
or, $h^2 + 4h + 4 + k^2 = h^2 + k^2 + 4k + 4$
or, $4h = 4k$
$\therefore$ $h = k$
Putting h = k in (i) then
$2h - 3h + 1 = 0$
or, $-h = -1$
$\therefore$ $h = 1, k = 1$
Now, centre = (h, k) = (1, 1)
radius $= \sqrt{(h+2)^2 + (k-0)^2}$
$= \sqrt{(1+2)^2 + (1)^2}$
$= \sqrt{3^2 + 1^2}$
$= \sqrt{10}$ units
We know that, equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 1)^2 + (y - 1)^2 = 10$
or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 10$
Thus, $x^2 + y^2 - 2x - 2y = 8$ is the required equation of the circle.

30. उद्गम बिन्दु र बिन्दु (4, 2) बाट जाने वृत्तको केन्द्रबिन्दु रेखा x + y = 1 मा पर्दछ भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle which passes through the origin and the point (4, 2) lies on the line x + y = 1. Find the equation of the circle. [2064 R]

⇨ Here, given equation of diameter is $x + y = 1$ .... (i)
It passes through (h, k) so $h + k = 1$ .... (ii)
Again d{(0, 0) and (h, k)} = d{(h, k) and (4, 2)}



or, $\sqrt{(h-0)^2 + (k-0)^2} = \sqrt{(4-h)^2 + (2-k)^2}$
or, $\sqrt{h^2 + k^2} = \sqrt{16 - 8h + h^2 + 4 - 4k + k^2}$
or, $h^2 + k^2 = h^2 + k^2 - 8h - 4k + 20$
or, $8h + 4k = 20$
or, $2h + k = 5$ .... (iii)
Solving (ii) and (iii)
$h + k = 1$
$\underline{-2h + k = 5}$
or, $-h = -4$ $\therefore$ $h = 4$
Putting the value of h in (ii) then
$h + k = 1$
or, $4 + k = 1$ $\therefore$ $k = -3$
Now, radius (r) = d{(h, k) and (0, 0)} = d{(4, – 3) and (0, 0)}
$= \sqrt{(4-0)^2 + (-3-0)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5$ units
We know that, equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 4)^2 + (y + 3)^2 = 5^2$
or, $x^2 - 8x + 16 + y^2 + 6y + 9 = 25$
Thus, $x^2 + y^2 - 8x + 6y = 0$ is the required equation of circle.

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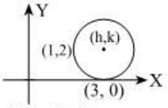
MODEL 6

31. X-अक्षको बिन्दु (3, 0) मा छुने र (1, 2) बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle which touches the X-axis at a point (3, 0) and passing through point (1, 2). **[2064 R']**

⇨ Here, the circle touches X-axis at (3, 0).

So the radius r = k and h = 3



$\therefore$ Centre = (h, k) = (3, k)

We know that; equation of circle with centre (3, k) and radius (r) = k is

$(x-3)^2+(y-k)^2=k^2$ .... (i)

Since, the circle passes through (1, 2).

So, $(1-3)^2+(2-k)^2=k^2$

or, $(-2)^2+4-4k+k^2=k^2$

or, $4+4-4k=0$

or, $4k=8$

$\therefore$ $k=2$

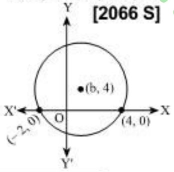
Putting the value of k in (i) then $(x-3)^2+(y-2)^2=2^2$

or, $x^2-6x+9+y^2-4y+4=4$

Thus, $x^2+y^2-6x-4y+9=0$ is the required equation of circle.

32. केन्द्रबिन्दु (b, 4) तथा X-अक्षको (– 2, 0) र (4, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a circle having the centre at (b, 4) and passing through the points (– 2, 0) and (4, 0) of X-axis. **[2066 S]**



⇨ Here, given points at the circumference are; (– 2, 0) and (4, 0).

We have,

distance of (– 2, 0) and (b, 4)

= distance of (4, 0) and (b, 4)

i.e. $\sqrt{(b+2)^2+(4-0)^2}=\sqrt{(b-4)^2+(4-0)^2}$

or, $b^2+4b+4+4^2=b^2-8b+16+4^2$

or, $12b=12$

$\therefore$ $b=1$

So, the centre = (1, 4)

And radius $=\sqrt{(b+2)^2+(4-0)^2}$

$=\sqrt{(1+2)^2+4^2}=5$ units

Now, equation of circle is; $(x-h)^2+(y-k)^2=r^2$

or, $(x-1)^2+(y-4)^2=5^2$

or, $x^2-2x+1+y^2-8y+16=25$

or, $x^2+y^2-2x-8y=8$

Thus, $x^2+y^2-2x-8y=8$ is the required equation of circle.

MODEL 7

33. एउटा पाङ्ग्राको परिधिमा रहेका तीनओटा बिन्दुहरू (5, 7) , (– 1, 7) र (5, – 1) एउटा निश्चित बिन्दुबाट बराबर दुरीमा पर्दछन् । उक्त निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै उक्त तीन बिन्दुहरू पर्ने गरी बिन्दुपथ पत्ता लगाउनुहोस् ।

On a wheel, there are three points (5, 7) , (– 1, 7) and (5, – 1) located such that the distance from a fixed point to these points is always equal. Find the coordinate of the fixed point and then derive the equation representing the locus that contains all three points. **[SEE 2076 M]**

⇨ Here, let the given points be;

A(– 1, 7), B(5, – 1) and C(5, 7).

Let P(h, k) be the point which is equidistant from the given points.

So, AP = BP = CP

or, $AP^2=BP^2=CP^2$

Taking $AP^2=CP^2$ then,

$(h+1)^2+(k-7)^2=(h-5)^2+(k-7)^2$

or, $(h+1)^2=(h-5)^2$

or, $h^2+2h+1=h^2-10h+25$

or, $12h=24$ $\therefore$ $h=2$

Again, taking $BP^2=CP^2$ then,

$(h-5)^2+(k+1)^2=(h-5)^2+(k-7)^2$

or, $(k+1)^2=(k-7)^2$

or, $k^2+2k+1=k^2-14k+49$

or, $16k=48$ $\therefore$ $k=3$

So the point of equidistant = (h, k) = (2, 3)

i.e. the centre of circle = (2, 3)

For radius; the distance between (2, 3) and (5, 7)

i.e. $r=\sqrt{(5-2)^2+(7-3)^2}=\sqrt{3^2+4^2}$

$\therefore$ $r=5$ units

Now, equation of the locus is;

$(x-h)^2+(y-k)^2=r^2$

or, $(x-2)^2+(y-3)^2=5^2$

or, $x^2-4x+4+y^2-6y+9=25$

$\therefore$ $x^2+y^2-4x-6y+13=25$

$\therefore$ $x^2+y^2-4x-6y=12$

Thus, the equation of the locus is $x^2+y^2-4x-6y=12$.

34. बिन्दुहरू A(– 4, – 2), B(2, 6) र C(2, – 2) भएर जाने वृत्तको अर्धव्यास निकाल्नुहोस् ।

Find the radius of the circle which passes through the points A(– 4, – 2), B(2, 6) and C(2, – 2). **[2060 S]**

⇨ Here, given points are A(– 4, – 2), B(2, 6), C(2, – 2)

We have equation of the circle is;

$(x-h)^2+(y-k)^2=r^2$ ..... (i)

Since equation (i) passes through the points;

A(– 4, – 2), B(2, 6) and C(2, – 2).

So, $(-4-h)^2+(-2-k)^2=r^2$ .... (ii)

$(2-h)^2+(6-k)^2=r^2$ .... (iii)

and $(2-h)^2+(-2-k)^2=r^2$ ...... (iv)

From equation (ii) and (iii)

$(-4-h)^2+(-2-k)^2=(2-h)^2+(6-k)^2$

or, $16+8h+h^2+4+4k+k^2$

$=4-4h+h^2+36-12k+k^2$

or, $12h+16k-20=0$

$\therefore$ $3h+4k-5=0$ .... (v)

Again from equation (iii) and (i)

$(2-h)^2+(6-k)^2=(2-h)^2+(-2-k)^2$

or, $4-4h+h^2+36-12k+k^2$

$=4-4h+h^2+4+4k+k^2$

or, $16k-32=0$

or, $16k=32$

$\therefore$ $k=2$

Putting the value of k in equation (v), we get,

$3h+4\times2-5=0$

or, $3h+8-5=0$

or, $3h+3=0$

or, $3h=-3$ $\therefore$ $h=-1$

Putting the values of h and k in equation (ii), we get,

$[-4-(-1)]^2+(-2-2)^2=r^2$

or, $(-4+1)^2+(-4)^2=r^2$

or, $(-3)^2+16=r^2$ or, $9+16=r^2$

or, $25=r^2$

$\therefore$ $r=5$

Thus, the required radius (r) = 5 units.

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35. बिन्दुहरू (1, 2), (3, – 4) र (5, – 6) भएर जाने वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।

Find the radius to the circle passing through the points (1, 2), (3, –4) & (5, – 6). [2063 R]

⇨ Here, given points are;

A(1, 2), B(3, – 4) and C(5, – 6).

Let, (h, k) be the centre and r be the radius of the circle then equation of circle is

$(x – h)^2 + (y – k)^2 = r^2$ .... (i)

But the circle (i) passes through the given points A, B and C, so we have,

$(1 – h)^2 + (2 – k)^2 = r^2$ .... (ii)

$(3 – h)^2 + (– 4 – k)^2 = r^2$ .... (iii)

$(5 – h)^2 + (– 6 – k)^2 = r^2$ .... (iv)

Now, from equation (ii) and (iii) we get,

$(1 – h)^2 + (2 – k)^2 = (3 – h)^2 + (– 4 – k)^2$

or, $1^2 – 2.1.h + h^2 + 2^2 – 2.2.k. + k^2$

$= 3^2 – 2.3.h + h^2 + (– 4)^2 – 2.(– 4).k + k^2$

or, $1 – 2h + h^2 + 4 – 4k + k^2$

$= 9 – 6h + h^2 + 16 + 8k + k^2$

or, $1 – 2h + 4 – 4k = 9 – 6h + 16 + 8k$

or, $– 4k – 8k – 2h + 6h + 1 + 4 – 9 – 16 = 0$

or, $– 12k + 4h – 20 = 0$

or, $– 3k + h – 5 = 0$

or, $h – 3k = 5$ .... (v)

From equation (ii) and (iv) we get,

$(1 – h)^2 + (2 – k)^2 = (5 – h)^2 + (– 6 – k)^2$

or, $1^2 – 2.1.h + h^2 + 2^2 – 2.2.k + k^2$

$= 5^2 – 2.5.h + h^2 + (– 6)^2 – 2.(– 6).k + k^2$

or, $1 – 2h + h^2 + 4 – 4k + k^2$

$= 25 – 10h + h^2 + 36 + 12k + k^2$

or, $1 – 2h + 4 – 4k = 25 – 10h + 36 + 12k$ or, $– 4k – 12k – 2h + 10h + 1 + 4 – 25 – 36 = 0$

or, $– 16k + 8h – 56 = 0$

or, $– 8(2k – h + 7) = 0$

or, $2k – h + 7 = 0$

or, $2k – h = – 7$ .... (vi)

Solving equation (v) and (vi)

$h – 3k = 5$

$\underline{– h + 2k = – 7}$

$– k = – 2$

∴ $k = 2$

Putting the value of k is (v), we get,

$h – 3 \times 2 = 5$

or, $h – 6 = 5$

or, $h = 5 + 6$

or, $h = 11$

Again, putting the values of h and k in equation (ii) we get,

$(1 – 11)^2 + (2 – 2)^2 = r^2$

or, $(– 10)^2 = r^2$

or, $100 = r^2$

or, $r = \sqrt{100}$

or, $r = 10$

Thus, the required radius of the circle is 10 units.

36. बिन्दुहरू (– 6, 5) (– 3, – 4) र (2, 1) बाट जाने वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।

Find the centre point of a circle which passes through the points (– 6, 5), (– 3, – 4) and (2, 1). [2065 R']

⇨ Here, the points of circle are;

A(– 6, 5), B(– 3, – 4) and C(2, 1)

Let, P(h, k) be the centre of circle.

Then, AP = BP = CP

⇒ $AP^2 = BP^2 = CP^2$

Taking $AP^2 = BP^2$ then,

or, $(h + 6)^2 + (k – 5)^2 = (h + 3)^2 + (k + 4)^2$

or, $h^2 + 12h + 36 + k^2 – 10k + 25$

$= h^2 + 6h + 9 + k^2 + 8k + 16$

or, $6h – 18k = – 36$

∴ $h – 3k = – 6$ .... (i)

Taking $BP^2 = CP^2$ then,

or, $(h + 3)^2 + (k + 4)^2 = (h – 2)^2 + (k – 1)^2$

or, $h^2 + 6h + 9 + k^2 + 8k + 16$

$= h^2 – 4h + 4 + k^2 – 2k + 1$

or, $10h + 10k = – 20$

∴ $h + k = – 2$ .... (ii)

Solving equation (i) and (ii),

$h – 3k = – 6$

$h + k = – 2$

$\underline{– \quad – \quad +}$

$– 4k = – 4$

∴ $k = 1$

Putting the value of k in (ii) then, $h + k = – 2$

or, $h + 1 = – 2$

∴ $h = – 3$

Thus, the centre is (– 3, 1).

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MODEL 8

37. **यदि रेखा $x + y = 1$ ले वृत्त $x^2 + y^2 = 1$ लाई दुई बिन्दुहरूमा काट्दछ भने उक्त बिन्दुहरूबीचको दूरी पत्ता लगाउनुहोस् ।**

If the line $x + y = 1$ cuts a circle $x^2 + y^2 =1$ at two points, find the distance between the two points.

[2062 K]

⇨ Here, given equation of line; $x + y = 1$ .... (i)

Given equation of circle, $x^2 + y^2 = 1$

or, $x^2 + (1 – x)^2 = 1$ [From (i)]

or, $x^2 + 1 – 2x + x^2 = 1$

or, $2x^2 – 2x = 0$

or, $2x(x – 1) = 0$

∴ $x = 0$ or 1

When $x = 0$ then from (i) $0 + y = 1$

∴ $y = 1$

So, one point is (0, 1).

When $x = 1$ then from (i) $1 + y = 1$

or, $y = 1 – 1$

∴ $y = 0$

So, other point is (1, 0).

Now, the distance between two points;

(0, 1) and (1, 0) is

$$d = \sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$$

$$= \sqrt{(1 – 0)^2 + (0 – 1)^2}$$

$$= \sqrt{1 + 1}$$

$$= \sqrt{2} \text{ units.}$$

Thus, distance between the two points is $\sqrt{2}$ units.

38. **केन्द्रबिन्दु (0, 0) र अर्धव्यास 2 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् । उक्त वृत्त र रेखा $x + y = 2$ को प्रतिच्छेदन बिन्दुहरूको निर्देशाङ्कहरू पनि पत्ता लगाउनुहोस् ।**

Find the equation of a circle with centre (0, 0) whose radius is 2 units. Also, find the co-ordinates of the points of intersection of the circle and the line $x + y = 2$.

⇨ Here, centre of circle = (0, 0) and radius = 2 units.

We know that, equations of circle is;

$(x – h)^2 + (y – k)^2 = r^2$

or, $(x – 0)^2 + (y – 0)^2 = 2^2$

∴ $x^2 + y^2 = 4$ .... (i)

Again, given line is $x + y = 2$ .... (ii)

Now solving equation (i) and (ii) we get,

$x^2 + y^2 = 4$

or, $(x + y)^2 – 2xy = 4$

or, $(2)^2 – 2xy = 4$

or, $4 – 4 = 2xy$

or, $2xy = 0$

or, $xy = 0$

Either $x = 0$ Or $y = 0$ ....(iii)

From (ii) and (iii),

When $x = 0$ then $0 + y = 2$

∴ $y = 2$ i.e point is (0, 2)

When $y = 0$ then $x + 0 = 2$

∴ $x = 2$ i.e point is (2, 0)

Thus, the equation of circle is $x^2 + y^2 = 4$ and points of intersection are (0, 2) and (2, 0).

MODEL 9

39. **समीकरण $x^2 + y^2 – 6x + y = 1$ भएको वृत्तसँग केन्द्रित हुने र बिन्दु (4, – 2) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a circle concentric with the circle $x^2 + y^2 – 6x + y = 1$ and passing through the point (4, – 2). [2072 R']

⇨ Here, equation of circle is; $x^2 + y^2 – 6x + y = 1$

or, $x^2 – 6x + y^2 + y = 1$

or, $x^2 – 6x + 3^2 + y^2 + 2.y.\frac{1}{2} + \left(\frac{1}{2}\right)^2 = 3^2 + \left(\frac{1}{2}\right)^2 + 1$

or, $(x – 3)^2 + \left(y + \frac{1}{2}\right)^2 = 9 + \frac{1}{4} + 1$

or, $(x – 3)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{37}{4} + 1$

or, $(x – 3)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{41}{4}$

Comparing it with $(x – h)^2 + (y – k)^2 = r^2$ then,

centre $(h, k) = \left(3, –\frac{1}{2}\right)$ passing point = (4, – 2)

$$\text{Radius } = \sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2} = \sqrt{(4 – 3)^2 + \left(– 2 + \frac{1}{2}\right)^2} = \sqrt{1^2 + \left(\frac{–3}{2}\right)^2} = \sqrt{1 + \frac{9}{4}} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$$

Now, equation of circle is; $(x – h)^2 + (y – k)^2 = r^2$

or, $(x – 3)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{13}{4}$

or, $x^2 – 6x + 9 + y^2 + 2.y.\frac{1}{2} + \frac{1}{4} = \frac{13}{4}$

or, $x^2 + y^2 – 6x + y + 9 + \frac{1}{4} = \frac{13}{4}$

∴ $x^2 + y^2 – 6x + y + 6 = 0$

Thus, the required equation of circle is $x^2 + y^2 – 6x + y + 6 = 0$.

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40. **समीकरण $x^2 + y^2 + 6x - 8y - 11 = 0$ भएको एउटा वृत्तको केन्द्रमा केन्द्रीत हुने र बिन्दु (4, 3) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a circle passing through the point (4, 3) and concentric with the circle having equation $x^2 + y^2 + 6x - 8y - 11 = 0$. **[2070 R]**

⇨ Here, given equation of circle is;

$x^2 + y^2 + 6x - 8y - 11 = 0$

or, $x^2 + 6x + y^2 - 8y - 11 = 0$

or, $x^2 + 6x + 3^2 + y^2 - 8y + 4^2 = 11 + 3^2 + 4^2$

or, $(x + 3)^2 + (y - 4)^2 = 36$

or, $(x + 3)^2 + (y - 4)^2 = 6^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

Centre (h, k) = (– 3, 4) passing point = (4, 3)

We know that,

Radius (r) $= \sqrt{(3-4)^2 + (4+3)^2}$

$= \sqrt{1 + 49}$

$= \sqrt{50}$

$= 5\sqrt{2}$

We have, $(x - h)^2 + (y - k)^2 = r^2$

or, $(x + 3)^2 + (y - 4)^2 = (5\sqrt{2})^2$

or, $x^2 + 6x + 9 + y^2 - 8y + 16 = 50$

or, $x^2 + y^2 + 6x - 8y = 25$

Thus, the required equation of the circle is;

$x^2 + y^2 + 6x - 8y = 25$.

41. **बिन्दु (5, 4) बाट जाने र वृत्त $x^2 + y^2 - 8x + 12y + 15 = 0$ सँग एक केन्द्रित हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of the circle which is concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through the point (5, 4). **[2068 R']**

⇨ Here, equation of a given circle is;

$x^2 + y^2 - 8x + 12y + 15 = 0$

or, $x^2 - 8x + y^2 + 12y + 15 = 0$

or, $x^2 - 2.x.4 + 4^2 + y^2 + 2.y.6 + 6^2 - 4^2 - 6^2 + 15 = 0$

or, $(x - 4)^2 + (y + 6)^2 = 16 + 36 - 15$

or, $(x - 4)^2 + (y + 6)^2 = 37$

or, $(x - 4)^2 + (y + 6)^2 = (\sqrt{37})^2$ is the form of

$(x - h)^2 + (y - k)^2 = r^2$ so, (h, k) = (4, – 6)

Now, equation of another circle is passes through (5, 4) & whose centre is (4, – 6)

So, (x, y) = (5, 4) & (h, k) = (4, – 6)

Then, $(x - h)^2 + (y - k)^2 = r^2$

or, $(5 - 4)^2 + (4 + 6)^2 = r^2$

or, $1 + (10)^2 = r^2$

or, $1 + 100 = r^2$

or, $r = \sqrt{101}$

Again, equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 4)^2 + (y + 6)^2 = (\sqrt{101})^2$

or, $x^2 - 8x + 16 + y^2 + 12y + 36 = 101$

or, $x^2 + y^2 - 8x + 12y = 101 - 52$

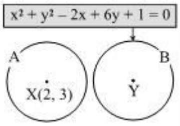
or, $x^2 + y^2 - 8x + 12y = 49$

Thus, $x^2 + y^2 - 8x + 12y = 49$ is the required equation of the circle.

MODEL 10

42. **दिइएको चित्रमा बराबर वृत्तहरू A र B का केन्द्रबिन्दुहरू क्रमशः X र Y छन् । यदि X को निर्देशाङ्कहरू (2, 3) र वृत्त B को समीकरण $x^2 + y^2 - 2x + 6y + 1 = 0$ छ भने वृत्त A को समीकरण निकाल्नुहोस् ।**

In the given figure, the centres of two equal circles A and B are X and Y respectively. If the coordinates of X are (2, 3) and the equation of circle B is $x^2 + y^2 - 2x + 6y + 1 = 0$ then find the equation of the circle A. **[2074 R]**



⇨ Here, given equation of circle B is;

$x^2 + y^2 - 2x + 6y + 1 = 0$

or, $x^2 - 2x + y^2 + 6y + 1 = 0$

or, $x^2 - 2 . x . 1 + 1^2 + y^2 + 2 . 3y + 3^2 = 1^2 + 3^2 - 1$

or, $(x - 1)^2 + (y + 3)^2 = 3^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we get, radius (r) = 3 units

which is also the radius of circle A.

So, in a circle A,

radius (r) = 3 units and centre (h, k) = (2, 3)

We have the equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 2)^2 + (y - 3)^2 = 3^2$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 9$

$\therefore$ $x^2 + y^2 - 4x - 6y + 4 = 0$

Thus, the required equation of circle A is;

$x^2 + y^2 - 4x - 6y + 4 = 0$.

43. **अर्धव्यास वृत्त $x^2 + y^2 - 2x - 6y + 1 = 0$ को अर्धव्याससँग बराबर हुने र (2, 3) केन्द्र भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of circle whose centre is (2, 3) and which has the same radius as the circle $x^2 + y^2 - 2x - 6y + 1 = 0$.

⇨ Here, given equation of circle is;

$x^2 + y^2 - 2x - 6y + 1 = 0$

or, $x^2 - 2x + 1 + y^2 - 6y = 0$

or, $x^2 - 2.x.1 + 1^2 + y^2 - 2.y.3 + 3^2 = 3^2$

or, $(x - 1)^2 + (y - 3)^2 = 3^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$

We get, h = 1, k = 3 and r = 3

So, the radius of required circle is 3 unit and given centre is (2, 3).

Now, equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 2)^2 + (y - 3)^2 = 3^2$

or, $x^2 - 4x + 4 + y^2 - 6y + 9 = 9$

$\therefore$ $x^2 + y^2 - 4x - 6y + 4 = 0$

Thus, the required equation of circle is;

$x^2 + y^2 - 4x - 6y + 4 = 0$.

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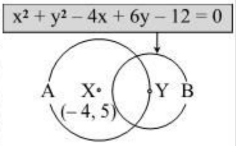
MODEL 11

44. **दिइएको चित्रमा केन्द्रबिन्दु X भएको वृत्त A वृत्त B को केन्द्रबिन्दु Y भएर गएको छ । यदि वृत्त B को समीकरण $x^2 + y^2 - 4x + 6y - 12 = 0$ र X को निर्देशाङ्कहरू (– 4, 5) छन् भने वृत्त A को समीकरण पत्ता लगाउनुहोस् ।**

In the given figure, the circle A with centre X passes through the centre Y of the circle B. If the equation of circle B is $x^2 + y^2 - 4x + 6y - 12 = 0$ and the co-ordinates of X are (– 4, 5), then find the equation of the circle A. **[2075 R]**



⇨ Here, equation of circle B is;

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Comparing it with general equation of circle;

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

We get, g = – 2 and f = 3

So, centre of circle B i.e. Y = (– g, – f) = (2, – 3)

Circle A passes through point (2, – 3) and its centre is (– 4, 5). Then, radius of circle A is,

$$r = \sqrt{(-4-2)^2 + (5+3)^2} = \sqrt{6^2 + 8^2} = 10 \text{ units}$$

Now, equation of circle having centre at (– 4, 5) and radius 10 units is given by;

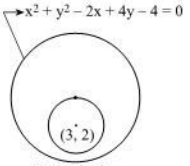
$$(x-h)^2 + (y-k)^2 = r^2$$
or, $(x+4)^2 + (y-5)^2 = 10^2$
or, $x^2 + y^2 + 8x - 10y + 16 + 25 = 100$
or, $x^2 + y^2 + 8x - 10y - 59 = 0$

Thus, $x^2 + y^2 + 8x - 10y - 59 = 0$ is the equation of the circle A.

45. **केन्द्रबिन्दु (3, 2) भएको र समीकरण $x^2 + y^2 - 2x + 4y - 4 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।** **[2071 R']**

Find the equation of a circle with centre (3, 2) and passing through the centre of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$.

⇨ Here,



Given equation of circle is;

$$x^2 + y^2 - 2x + 4y - 4 = 0 \text{ ...... (i)}$$
or, $x^2 - 2x + y^2 + 4y - 4 = 0$
or, $x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$
or, $(x-1)^2 + (y+2)^2 = 9$
∴ $(x-1)^2 + (y+2)^2 = 9$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$ then centre of given circle = (h, k) = (1, – 2)

Which is the passing point of required circle.

Radius of required circle (r)

= Distance (1, – 2) and (3, 2)
$= \sqrt{(3-1)^2 + (2+2)^2}$
$= \sqrt{2^2 + 4^2} = \sqrt{20}$

Now, equation or circle is $(x-h)^2 + (y-k)^2 = r^2$

or, $(x-3)^2 + (y-2)^2 = (\sqrt{20})^2$
or, $x^2 - 6x + 9 + y^2 - 4y + 4 = 20$
∴ $x^2 + y^2 - 6x - 4y - 7 = 0$

Thus, the equation of circle is $x^2 + y^2 - 6x - 4y - 7 = 0$.

46. **केन्द्र (– 1, 2) भएको र समीकरण $x^2 + y^2 - 6x - 10y - 2 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।**

Find the equation of a circle with centre (– 1, 2) and passing through the centre of the circle having equation $x^2 + y^2 - 6x - 10y - 2 = 0$. **[2075 R$_2$]**

⇨ Here, given equation of circle is $x^2 + y^2 - 6x - 10y - 2 = 0$

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

g = – 3 and f = – 5

So, centre = (– g, – f) = (3, 5)

Now, radius of circle having centre at (– 1, 2) and passing point (3, 5) is,

$r = \sqrt{(3+1)^2 + (5-2)^2} = \sqrt{16+9} = 5$ units

Equation of circle having centre at (– 1, 2) and radius 5 units is:

$$(x-h)^2 + (y-k)^2 = r^2$$
or, $(x+1)^2 + (y-2)^2 = 5^2$
or, $x^2 + y^2 + 2x - 4y + 5 = 25$
∴ $x^2 + y^2 + 2x - 4y - 20 = 0$

Thus, the equation of circle is, $x^2 + y^2 + 2x - 4y - 20 = 0$.

QUESTIONS FROM CDC TEXTBOOK

4.4 वृत्त (CIRCLE)

EXERCISE 4.4

1. (a) **उद्गम बिन्दु केन्द्र र अर्धव्यास r एकाइ भएको वृत्तको समीकरण लेख्नुहोस् ।**

Find the equation of a circle with centre at origin and radius r units.

⇨ Here, the equation of circle is $x^2 + y^2 = r^2$.

(b) **केन्द्र (p, q) र अर्धव्यास r एकाइ भएको वृत्तको समीकरण लेख्नुहोस् ।**

Find the equation of a circle with centre at (p, q) and radius r units.

⇨ Here, the equation of circle is $(x-p)^2 + (y-q)^2 = r^2$.

(c) **$x^2 + y^2 + 2gx + 2fy + c = 0$ समीकरण भएको वृत्तको केन्द्र र अर्धव्यास लेख्नुहोस् ।**

Write the centre and radius of circle having equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

⇨ Here, $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre = (– g, – f) and radius = $\sqrt{g^2 + f^2 - c}$

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2. **निम्न लिखित केन्द्र र अर्धव्यास भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।**
Find the equation of circle having centre and radius as below.

क्र.सं. (S.N.)	केन्द्र (centre)	अर्धव्यास (radius)
(a)	(0, 0)	5
(b)	(2, 3)	4
(c)	(– 3, – 4)	6
(d)	(0, 1)	4
(e)	(2, – 5)	7
(f)	(5, 0)	3
(g)	(– 2, 3)	4
(h)	(3, 4)	5
(i)	(a, a)	$a\sqrt{2}$
(j)	(a, b)	a

(c) Centre (– 3, – 4), radius 6

⇨ Here, radius (r) = 6, center (h, k) = (– 3, – 4)
We have, equation of circle having centre (h, k) and radius r is,
$(x – h)^2 + (y – k)^2 = r^2$
or, $(x + 3)^2 + (y + 4)^2 = 6^2$
or, $x^2 + 6x + 9 + y^2 + 8y + 16 = 36$
or, $x^2 + y^2 + 6x + 8y + 25 = 36$
Thus, $x^2 + y^2 + 6x + 8y = 11$ is the required equation.

(e) Centre (2, – 5), radius (7)

⇨ Here, centre (h, k) = (2, – 5) and radius (r) = 7
We know that,
equation of circle having centre (h, k) and radius r is,
$(x – h)^2 + (y – k)^2 = r^2$
or, $(x – 2)^2 + (y + 5)^2 = 7^2$
or, $x^2 – 4x + 4 + y^2 + 10y + 25 = 49$
$\therefore$ $x^2 + y^2 – 4x + 10y = 20$
Thus, the required equation of circle is;
$x^2 + y^2 – 4x + 10y = 20.$

(g) Centre (– 2, 3) and radius (4)

⇨ Here, centre (h, k) = (– 2, 3) and radius (r) = 4
We have, equation of circle is;
$(x – h)^2 + (y – k)^2 = r^2$
So, $(x + 2)^2 + (y – 3)^2 = 4^2$
or, $x^2 + 4x + 4 + y^2 – 6y + 9 = 16$
$\therefore$ $x^2 + y^2 + 4x – 6y = 3$
Thus, the equation of circle is $x^2 + y^2 + 4x – 6y = 3.$

(i) Center (a, a) , radius $a\sqrt{2}$

⇨ Here, radius (r) = $a\sqrt{2}$ and center (h, k) = (a, a)
We have, equation of circle is;
$(x – h)^2 + (y – k)^2 = r^2$
or, $(x – a)^2 + (y – a)^2 = (a\sqrt{2})^2$
or, $x^2 – 2ax + a^2 + y^2 – 2ay + a^2 = 2a^2$
or, $x^2 + y^2 – 2ax – 2ay + 2a^2 = 2a^2$
Thus, $x^2 + y^2 – 2ax – 2ay = 0$ is the required equation.

(a) Center (0, 0), radius 5

⇨ Here, center = (0, 0) and radius = 5
We have, equation of circle is, $x^2 + y^2 = r^2$
or, $x^2 + y^2 = 5^2$
Thus, $x^2 + y^2 = 25$ is the required equation.

(b) Centre (2, 3), radius 4

⇨ Here, radius (r) = 4, center (h , k) = (2, 3)
We have, equation of circle having centre (h, k) and radius r is, $(x – h)^2 + (y – k)^2 = r^2$
or, $(x – 2)^2 + (y – 3)^2 = 4^2$
or, $x^2 – 4x + 4 + y^2 – 6y + 9 = 16$
or, $x^2 – 4x + y^2 – 6y + 13 = 16$
Thus, $x^2 + y^2 – 4x – 6y = 3$ is the required equation.

(d) Centre (0, 1), radius (4)

⇨ Here, radius (r) = 4 and center (h, k) = (0, 1)
We have,
equation of circle having centre (h, k) and radius r is,
$(x – h)^2 + (y – k)^2 = r^2$
or, $(x – 0)^2 + (y – 1)^2 = 4^2$
or, $x^2 + y^2 – 2y + 1 = 16$
Thus, $x^2 + y^2 – 2y = 15$ is the required equation.

(f) Centre (5, 0), radius (3)

⇨ Here, centre (h, k) = (5, 0) and radius (r) = 3
We have, equation of circle is;
$(x – h)^2 + (y – k)^2 = r^2$
So, $(x – 5)^2 + (y – 0)^2 = 3^2$
or, $x^2 – 10x + 25 + y^2 = 9$
or, $x^2 + y^2 – 10x + 25 – 9 = 0$
$\therefore$ $x^2 + y^2 – 10x + 16 = 0$
Thus, the equation of circle is $x^2 + y^2 – 10x + 16 = 0.$

(h) Centre (3, 4) and radius (5)

⇨ Here, centre (h, k) = (3, 4) and radius (r) = 5
We have, equation of circle is;
$(x – h)^2 + (y – k)^2 = r^2$
So, $(x – 3)^2 + (y – 4)^2 = 5^2$
or, $x^2 – 6x + 9 + y^2 – 8y + 16 = 25$
$\therefore$ $x^2 + y^2 – 6x – 8y = 0$
Thus, the equation of circle is $x^2 + y^2 – 6x – 8y = 0.$

(j) Centre (a, b) and radius (a)

⇨ Here, centre (h, k) = (a, b) and radius (r) = a
We have, equation of circle is;
$(x – h)^2 + (y – k)^2 = r^2$
So, $(x – a)^2 + (y – b)^2 = a^2$
or, $x^2 – 2ax + a^2 + y^2 – 2by + b^2 = a^2$
$\therefore$ $x^2 + y^2 – 2ax – 2by + b^2 = 0$
Thus, the equation of circle is;
$x^2 + y^2 – 2ax – 2by + b^2 = 0.$

3. **तल दिइएका वृत्तको केन्द्र र अर्धव्यास पत्ता लगाउनुहोस् । (Find the centre and radius of following circles.)**

(a) $x^2 + y^2 = 16$

⇨ Here, $x^2 + y^2 = 16$
or, $(x – 0)^2 + (y – 0)^2 = 4^2$
Comparing with $(x – h)^2 + (y – k)^2 = r^2$ then,
$\therefore$ h = 0 , k = 0 and r = 4
$\therefore$ Center = (0, 0) and r = 4
Thus, the centre is (0, 0) and radius is 4 units.

(b) $x^2 + (y + 2)^2 = 49$

⇨ Here, $x^2 + (y + 2)^2 = 49$
or, $(x – 0)^2 + \{ y – (– 2)\}^2 = 7^2$
Comparing with $(x – h)^2 + (y – k)^2 = r^2$ then,
h = 0 , k = – 2 & r = 7
$\therefore$ Center (h, k) = (0, –2) & r = 7
Thus, centre is (0, – 2) and radius is 7 units.

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(c) $(x-4)^2 + (y-2)^2 = 36$

⇨ Here, $(x-4)^2 + (y-2)^2 = 6^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = 4, k = 2 \;\&\; r = 6$

$\therefore$ Centre $= (4, 2) \;\&\; r = 6$

Thus, the centre is (4, 2) and radius is 6 units.

(e) $(x-2)^2 + (y-3)^2 = 25$

⇨ Here, $(x-2)^2 + (y-3)^2 = 25$

or, $(x-2)^2 + (y-3)^2 = 5^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = 2, k = 3 \;\&\; r = 5$

$\therefore$ Centre $= (2, 3)$ radius $= 5$

Thus, the centre is (2, 3) and radius is 5 units.

(g) $x^2 + y^2 - 10x + 4y + 13 = 0$

⇨ Here, $x^2 + y^2 - 10x + 4y + 13 = 0$

or, $(x^2 - 2.5x + 5^2) + (y^2 + 2.2y + 2^2) = 25 + 4 - 13$

or, $(x-5)^2 + (y+2)^2 = 16$

or, $(x-5)^2 + \{y-(-2)\}^2 = 4^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = 5, k = -2 \;\&\; r = 4$

$\therefore$ Centre $= (5, -2) \;\&\; r = 4$

Thus, the centre is (5, – 2) and radius is 4 units.

(i) $x^2 + y^2 - 3x - y - \frac{13}{2} = 0$

⇨ Here, $x^2 + y^2 - 3x - y - \frac{13}{2} = 0$

or, $x^2 - 2.\frac{3}{2}x + \left(\frac{3}{2}\right)^2 + y^2 - 2.\frac{1}{2}y + \left(\frac{1}{2}\right)^2 = \frac{9}{4} + \frac{1}{4} + \frac{13}{2}$

or, $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9+1+26}{4}$

or, $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{36}{4}$

or, $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{6}{2}\right)^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = \frac{3}{2}, k = \frac{1}{2} \;\&\; r = \frac{6}{2} = 3$

$\therefore$ Center $= \left(\frac{3}{2}, \frac{1}{2}\right)$ and $r = 3$

Thus, the centre is $\left(\frac{3}{2}, \frac{1}{2}\right)$ and radius is 3 units.

(d) $(x+5)^2 + (y+3)^2 = 25$

⇨ Here, $(x+5)^2 + (y+3)^2 = 25$

or, $\{x-(-5)\}^2 + \{y-(-3)\}^2 = (5)^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = -5, k = -3 \;\&\; r = 5$

$\therefore$ Centre $= (-5, -3) \;\&\; r = 5$

Thus, the centre is (– 5, – 3) and radius is 5 units.

(f) $(x-a)^2 + (y+b)^2 = k^2$

⇨ Here, $(x-a)^2 + (y+b)^2 = k^2$

or, $(x-a)^2 + \{y-(-b)\}^2 = k^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$\therefore$ $h = a, k = -b \;\&\; r = k$

Centre $= (a, -b) \;\&\; r = k$

Thus, the centre is (a, – b) and radius is k units.

(h) $x^2 + y^2 + 6x - 8y - 11 = 0$

⇨ Here, $x^2 + y^2 + 6x - 8y - 11 = 0$

or, $(x^2 + 2.3x + 3^2) + (y^2 - 2.4y + 4^2) = 9 + 16 + 11$

or, $(x+3)^2 + (y-4)^2 = 36$

or, $\{x-(-3)\}^2 + (y-4)^2 = 6^2$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$ then,

$h = -3, k = 4 \;\&\; r = 6$

$\therefore$ Center $= (-3, 4) \;\&\; r = 6$

Thus, the centre is (– 3, 4) and radius is 6 units.

(j) $2x^2 + 2y^2 + 10x + 6y + 7 = 0$

⇨ Here, $2x^2 + 2y^2 + 10x + 6y + 7 = 0$

or, $\frac{2x^2}{2} + \frac{2y^2}{2} + \frac{10x}{2} + \frac{6y}{2} + \frac{7}{2} = 0$

or, $x^2 + y^2 + 5x + 3y + \frac{7}{2} = 0$

or, $x^2 + 5x + y^2 + 3y + \frac{7}{2} = 0$

or, $x^2 + 2.x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 + y^2 + 2.y.\frac{3}{2} + \left(\frac{3}{2}\right)^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 - \frac{7}{2}$

or, $\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4} + \frac{9}{4} - \frac{7}{2}$

or, $\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25+9-14}{4}$

or, $\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 5$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$ then,

Centre $(h, k) = \left(-\frac{5}{2}, -\frac{3}{2}\right)$

radius $(r) = \sqrt{5}$ units

Thus, the centre is $\left(-\frac{5}{2}, -\frac{3}{2}\right)$ and radius $\sqrt{5}$ units.

4. दिइएका बिन्दुहरू वृत्तका व्यासका छेउ छेउ बिन्दुहरू हुन् भने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Given points are the end points of a diameter. Find the equation of the circle.

(a) (3, 4) and (2, – 7)

⇨ Here, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (2, -7)$ are the end points of diameter.

We have, equation of circle in diameter form is;

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

or, $(x-3)(x-2) + (y-4)(y+7) = 0$

or, $x^2 - 2x - 3x + 6 + y^2 + 7y - 4y - 28 = 0$

$\therefore$ $x^2 + y^2 - 5x + 3y - 22 = 0$

Thus, $x^2 + y^2 - 5x + 3y - 22 = 0$ is the required equation.

(b) (4, 1) and (6, 5)

⇨ Here, let $(x_1, y_1) = (4, 1)$ and $(x_2, y_2) = (6, 5)$ are the end points of diameter.

We have, equation of circle in diameter form is;

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

or, $(x-4)(x-6) + (y-1)(y-5) = 0$

or, $x^2 - 6x - 4x + 24 + y^2 - 5y - y + 5 = 0$

$\therefore$ $x^2 + y^2 - 10x - 6y + 29 = 0$

Thus, $x^2 + y^2 - 10x - 6y + 29 = 0$ is the required equation.

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(c) (a, 0) and (– a, 0)

⇨ Here, let $(x_1, y_1) = (a, 0)$ and $(x_2, y_2) = (-a, 0)$ are the end points of diameter.

We have, equation of circle in diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - a)(x + a) + (y - 0)(y - 0) = 0$

or, $x^2 - a^2 + y^2 = 0$

$\therefore$ $x^2 + y^2 = a^2$

Thus, $x^2 + y^2 = a^2$ is the required equation.

(e) (5, 0) and (0, 5)

⇨ Here, $(5, 0) = (x_1, y_1)$ and $(0, 5) = (x_2, y_2)$

We have, equation of circle in diameter form;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 5)(x - 0) + (y - 0)(y - 5) = 0$

or, $x(x - 5) + y(y - 5) = 0$

or, $x^2 - 5x + y^2 - 5y = 0$

$\therefore$ $x^2 + y^2 - 5x - 5y = 0$

Thus, the equation of the circle is $x^2 + y^2 - 5x - 5y = 0$.

(d) (3, 0) and (– 1, 0)

⇨ Here, (3, 0), (– 1, 0)

So, $(3, 0) = (x_1, y_1)$ and $(-1, 0) = (x_2, y_2)$

We have, equation of circle in diameter form;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 3)(x + 1) + (y - 0)(y - 0) = 0$

or, $x^2 + x - 3x - 3 + y^2 = 0$

or, $x^2 + y^2 - 2x - 3 = 0$

Thus, the equation of circle is $x^2 + y^2 - 2x - 3 = 0$.

(f) (– 3, – 2) and (3, 2)

⇨ Here, $(-3, -2) = (x_1, y_1)$ and $(3, 2) = (x_2, y_2)$

We know that,

Equation of circle in diameter form is;

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x + 3)(x - 3) + (y + 2)(y - 2) = 0$

or, $x^2 - 9 + y^2 - 4 = 0$

or, $x^2 + y^2 = 13$

Thus, the equation of the circle is $x^2 + y^2 = 13$.

5. निम्न लिखित बिन्दुहरू भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle passing through the following points.

(a) (– 2, 2), (2, 4) and (4, 0)

⇨ Here, (– 2, 2), (2, 4), (4, 0)

Let, $(x - h)^2 + (y - k)^2 = r^2$ be the equation of circle which passes through the points (– 2, 2), (2, 4) and (4, 0).

So, $(-2 - h)^2 + (2 - k)^2 = r^2$ ....(1)

$(2 - h)^2 + (4 - k)^2 = r^2$ .... (2)

$(4 - h)^2 + (0 - k)^2 = r^2$ ....(3)

Equating equation no. (1) and equation no. (2) then,

$(-2 - h)^2 + (2 - k)^2 = (2 - h)^2 + (4 - k)^2$

or, $4 + 4h + h^2 + 4 - 4k + k^2$
$= 4 - 4h + h^2 + 16 - 8k + k^2$

or, $8h + 4k = 20 - 8$

or, $8h + 4k = 12$

or, $2h + k = 3$ ....(4)

Equating equation no. (2) and (3)

$(2 - h)^2 + (4 - k)^2 = (4 - h)^2 + (0 - k)^2$

or, $4 - 4h + h^2 + 16 - 8k + k^2$
$= 16 - 8h + h^2 + k^2$

or, $4h - 8k = -4$

or, $h - 2k = -1$ ....(5)

Multiplying equation (5) by 2 then subtracting from (4) we get,

$$\begin{array}{l} 2h + k = 3 \\ 2h - 4k = -2 \\ \underline{-\quad + \quad +} \\ 5k = 5 \qquad \text{or, } k = 1 \end{array}$$

Putting the value of k in (5) then $h - 2 = -1$

$\therefore$ $h = 1$ and Centre = (1, 1)

Substituting value of h and k in equation (1),

$(-2 - 1)^2 + (2 - 1)^2 = r^2$

or, $(-3)^2 + (1)^2 = r^2$

or, $9 + 1 = r^2$

or, $10 = r^2$

$\therefore$ $r = \sqrt{10}$

So, required equation is,

or, $(x - 1)^2 + (y - 1)^2 = (\sqrt{10})^2$

or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 10$

Thus, $x^2 + y^2 - 2x - 2y = 8$ is the required equation.

(b) (1, 1), (4, 4) and (5, 1)

⇨ Here, (1, 1), (4, 4), (5, 1)

Let, (h, k) be the centre and r be the radius of a circle.

Then equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$ ....(i)

Equation (i) passes through (1, 1), (4, 4) & (5, 1).

So, $(1 - h)^2 + (1 - k)^2 = r^2$ ....(ii)

or, $(4 - h)^2 + (4 - k)^2 = r^2$ ....(iii)

or, $(5 - h)^2 + (1 - k)^2 = r^2$ ....(iv)

From (ii) and (iv);

$(1 - h)^2 + (1 - k)^2 = (5 - h)^2 + (1 - k)^2$

or, $(1 - h)^2 = (5 - h)^2$

or, $1 - 2h + h^2 = 25 - 10h + h^2$

or, $1 - 2h = 25 - 10h$

or, $8h = 24$

$\therefore$ $h = 3$

Again, from (iii) and (iv)

$(4 - h)^2 + (4 - k)^2 = (5 - h)^2 + (1 - k)^2$

or, $(4 - 3)^2 + (4 - k)^2 = (5 - 3)^2 + (1 - k)^2$

or, $1 + (4 - k)^2 = 2^2 + (1 - k)^2$

or, $1 + 16 - 8k + k^2 = 4 + 1 - 2k + k^2$

or, $-6k = -12$

$\therefore$ $k = 2$

So, centre (h, k) = (3, 2)

Radius (r) = distance between (3, 2) and (1, 1)

$= \sqrt{(1 - 3)^2 + (1 - 2)^2}$

$= \sqrt{(-2)^2 + (-1)^2}$

$= \sqrt{4 + 1}$

$= \sqrt{5}$ units

Now, equation of circle is;

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 3)^2 + (y - 2)^2 = 5$

or, $x^2 - 6x + 9 + y^2 - 4y + 4 = 5$

or, $x^2 + y^2 - 6x - 4y + 8 = 0$

Thus, the equation of the circle is;

$x^2 + y^2 - 6x - 4y + 8 = 0$

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(c) (0, 0), (0, b) and (a, 0)

⇨ Here, (0, 0) , (0, b) and (a, 0)

Let, $(x-h)^2+(y-k)^2=r^2$ passes through the points (0, 0), (0, b) and (a, 0).

So, $(0-h)^2+(0-k)^2=r^2$ ....(1)

$(0-h)^2+(b-k)^2=r^2$ ....(2)

$(a-h)^2+(0-k)^2=r^2$..........(3)

Equating equation no. (1) & equation no. (2) then,

$(0-h)^2+(0-k)^2=(0-h)^2+(b-k)^2$

or, $k^2=b^2-2bk+k^2$

or, $k=\frac{b^2}{2b}=\frac{b}{2}$

Equating equation no. (2) and equation no. (3) then,

$(0-h)^2+(b-k)^2=(a-h)^2+(0-k)^2$

or, $h^2+b^2-2bk+k^2=a^2-2ah+h^2+k^2$

or, $b^2-2bk=-2ah+a^2$

or, $2ah=2bk+a^2-b^2$

or, $2ah=2b.\frac{b}{2}+a^2-b^2$

or, $2ah=b^2+a^2-b^2$

∴ $h=\frac{a^2}{2a}=\frac{a}{2}$

∴ Centre of the circle is $\left(\frac{a}{2},\frac{b}{2}\right)$

Substituting the value of h and k in equation no. (1)

then, $\left(0-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2=r^2$

or, $\frac{a^2}{4}+\frac{b^2}{4}=r^2$

∴ $\frac{a^2+b^2}{4}=r^2$

Now, required equation is, $(x-h)^2+(y-k)^2=r^2$

or, $\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{a^2+b^2}{4}$

or, $x^2-xa+\frac{a^2}{4}+y^2-yb+\frac{b^2}{4}=\frac{a^2+b^2}{4}$

or, $x^2-xa+y^2-by=\frac{a^2}{4}+\frac{b^2}{4}-\frac{a^2}{4}-\frac{b^2}{4}$

∴ $x^2+y^2-ax-by=0$

Thus, the equation of circle is $x^2+y^2-ax-by=0$.

(d) (1, – 1), (3, 1) and (3, – 3)

⇨ Here, (1, – 1), (3, 1) and (3, – 3)

Let, the equation of circle be;

$(x-h)^2+(y-k)^2=r^2$ ....(i)

So, centre = (h, k) and radius = r.

Equation (i) passes through (1, – 1), (3, 1) and (3, – 3).

So, $(1-h)^2+(-1-k)^2=r^2$ ....(ii)

$(3-h)^2+(1-k)^2=r^2$ ....(iii)

$(3-h)^2+(-3-k)^2=r^2$ ....(iv)

Equating equation (ii) and (iii) then,

$(1-h)^2+\{-(1+k)\}^2=(3-h)^2+(1-k)^2$

or, $1-2h+h^2+(1+k)^2=9-6h+h^2+1-2k+k^2$

or, $1-2h+1+2k+k^2=9-6h+1-2k+k^2$

or, $4h+4k=8$

∴ $h+k=2$ ....(v)

Again, taking (iii) and (iv) then,

$(3-h)^2+(1-k)^2=(3-h)^2+(-3-k)^2$

or, $(1-k)^2=\{-(3+k)\}^2$

or, $1-2k+k^2=9+6k+k^2$

or, $-8k=8$

∴ $k=-1$

From (v);

$h+k=2$

or, $h-1=2$

∴ $h=3$

So, centre = (h, k) = (3, – 1)

Radius = distance between (3, – 1) and (1 – 1)

$=\sqrt{(1-3)^2+(-1+1)^2}$

$=\sqrt{(-2)^2+0^2}$

∴ r = 2 units

Now, equation of circle is, $(x-h)^2+(y-k)^2=r^2$

or, $(x-3)^2+(y+1)^2=2^2$

or, $x^2-6x+9+y^2+2y+1=4$

or, $x^2+y^2-6x+2y+10=4$

∴ $x^2+y^2-6x+2y+6=0$

Thus, the equation of circle is $x^2+y^2-6x+2y+6=0$.

(e) (2, 6), (6, 4) and (– 3, 1)

⇨ Here, (2, 6), (6, 4), (– 3, 1)

Let, $(x-h)^2+(y-k)^2=r^2$ ....(i) be the equation of a circle whose centre is (h, k) and radius r.

Then, equation (i) passes through (2, 6), (6, 4) and (– 3, 1)

So, $(2-h)^2+(6-k)^2=r^2$ ....(ii)

$(6-h)^2+(4-k)^2=r^2$ ....(iii)

$(-3-h)^2+(1-k)^2=r^2$ ....(iv)

From (ii) and (iii);

$(2-h)^2+(6-k)^2=(6-h)^2+(4-k)^2$

or, $4-4h+h^2+36-12k+k^2$

$=36-12h+h^2+16-8k+k^2$

or, $40+8h-4k=52$

or, $8h-4k=12$

∴ $2h-k=3$ ....(v)

Again, taking (ii) and (iv) then,

$(2-h)^2+(6-k)^2=(-3-h)^2+(1-k)^2$

or, $4-4h+h^2+36-12k+k^2$

$=9+6h+h^2+1-2k+k^2$

or, $-10h-10k+40=10$

or, $-10h-10k=-30$

∴ $h+k=3$ ....(vi)

Solving (v) and (vi);

$2h-k=3$

$\underline{h+k=3}$

$3h=6$

∴ $h=2$

From (vi);

$h+k=3$

or, $2+k=3$

∴ $k=1$

So, centre (h, k) = (2, 1)

Radius (r) = Distance between (2, 1) and (2, 6)

$=\sqrt{(2-2)^2+(6-1)^2}$

$=\sqrt{0^2+5^2}$

= 5 units

Now, equation of the circle is;

$(x-h)^2+(y-k)^2=r^2$

or, $(x-2)^2+(y-1)^2=5^2$

or, $x^2-4x+4+y^2-2y+1=25$

or, $x^2+y^2-4x-2y+5=25$

∴ $x^2+y^2-4x-2y=20$

Thus, the equation of the circle is $x^2+y^2-4x-2y=20$.

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(f) (1, 0), (– 1, 0) and (0, 1)

⇨ Here, (1, 0), (– 1, 0), (0, 1)
Let $(x-h)^2+(y-k)^2=r^2$ ....(i) be the equation of cirlce. Where (h, k) is centre and r is radius.
Then equation (i) passes through (1, 0), (– 1, 0) & (0, 1)
So, $(1-h)^2+(0-k)^2=r^2$
∴ $(1-h)^2+k^2=r^2$ ....(ii)
$(-1-h)^2+(0-k)^2=r^2$
∴ $(1+h)^2+k^2=r^2$ ....(iii)
$(0-h)^2+(1-k)^2=r^2$
∴ $h^2+(1-k)^2=r^2$ ....(iv)
From (ii) and (iii);
$(1-h)^2+k^2=(1+h)^2+k^2$
or, $1-2h+h^2=1+2h+h^2$
or, $0-4h=0$
∴ $h=0$
From (iii) and (iv);
$(1+h)^2+k^2=h^2+(1-k)^2$
or, $1+2h+h^2+k^2=h^2+1-2k+k^2$
or, $2h+2k=0$
or, $2\times 0+2k=0$
or, $2k=0$
∴ $k=0$
So, centre (h, k) = (0, 0)
Radius (r) = distance between (0, 0) and (0, 1)
$=\sqrt{(0-0)^2+(1-0)^2}=\sqrt{1^2}=1$
Now, equation of circle is;
$x^2+y^2=r^2$
or, $x^2+y^2=1^2$
∴ $x^2+y^2=1$
Thus, the equation of circle is $x^2+y^2=1$.

6. तलका बिन्दुहरू एउटै वृत्तमा पर्छन् भनी प्रमाणित गर्नुहोस् । (Prove that the following points lie on a same circle.)

(a) (3, 3), (6, 4), (7, 1) and (4, 0)

⇨ Here, (3, 3), (6, 4), (7, 1) and (4, 0) are given points.
Let, $(x-h)^2+(y-k)^2=r^2$ ....(i) be the equation of circle.
Equation (i) passes through (3, 3), (6, 4) and (7, 1).
So, $(3-h)^2+(3-k)^2=r^2$ ....(ii)
$(6-h)^2+(4-k)^2=r^2$..............(iii)
$(7-h)^2+(1-k)^2=r^2$ ....(iv)
From (ii) and (iii);
$(3-h)^2+(3-k)^2=(6-h)^2+(4-k)^2$
or, $9-6h+h^2+9-6k+k^2$
$=36-12h+h^2+16-8k+k^2$
or, $18+6h+2k=52$
or, $6h+2k=34$
∴ $3h+k=17$ ....(v)
From (iii) and (iv);
$(6-h)^2+(4-k)^2=(7-h)^2+(1-k)^2$
or, $36-12h+h^2+16-8k+k^2$
$=49-14h+h^2+1-2k+k^2$
or, $52+2h-6k=50$
or, $2h-6k=-2$
or, $h-3k=-1$
∴ $h=3k-1$
From (v);
$3(3k-1)+k=17$
or, $9k-3+k=17$
or, $10k=20$
∴ $k=2$
∴ $h=3k-1=6-1=5$
∴ Centre = (h, k) = (5, 2)
Radius = Distance between (5, 2) and (6, 4)
$=\sqrt{(6-5)^2+(4-2)^2}$
$=\sqrt{1^2+(2)^2}$
$=\sqrt{1+4}=\sqrt{5}$ units
Now, equation of circle; $(x-h)^2+(y-k)^2=r^2$
or, $(x-5)^2+(y-2)^2=5$
or, $x^2-10x+25+y^2-4y+4=5$
∴ $x^2+y^2-10x-4y+24=0$ ....(vi)
Remaining point is (4, 0).
So, putting x = 4 and y = 0 in equation (vi);
Then, LHS $=x^2+y^2-10x-4y+24$
$=4^2+0^2-10\times 4-4\times 0+24$
$=16+0-40-0+24$
$=40-40$
$=0$ = RHS
Thus, the four points (3, 3), (6, 4), (7, 1) and (4, 0) lie on the same circle.

(b) (1, 0), (2, – 7), (8, 1) and (9, – 6)

⇨ Here, (1, 0), (2, – 7), (8, 1) and (9, – 6) are given points.
Let, $(x-h)^2+(y-k)^2=r^2$ ....(i) be the equation of circle.
Let us find the equation of circle using only three points (1, 0), (2, – 7) and (8, 1).
So, $(1-h)^2+(0-k)^2=r^2$ ....(ii)
$(2-h)^2+(-7-k)^2=r^2$ ....(iii)
$(8-h)^2+(1-k)^2=r^2$ ....(iv)
From (ii) and (iii);
$(1-h)^2+(0-k)^2=(2-h)^2+(-7-k)^2$
or, $1-2h+h^2+k^2=4-4h+h^2+49+14k+k^2$
or, $1-2h=53-4h+14k$
or, $2h-14k=52$
or, $h-7k=26$
∴ $h=7k+26$ ....(v)
From (ii) and (iv);
$(1-h)^2+(0-k)^2=(8-h)^2+(1-k)^2$
or, $1-2h+h^2+k^2=64-16h+h^2+1-2k+k^2$
or, $14h+2k=64$
∴ $7h+k=32$ ....(vi)
From (v) and (vi); $7h+k=32$
or, $7(7k+26)+k=32$
or, $49k+182+k=32$
or, $50k=-150$
∴ $k=-3$
From (v); $h = 7k+26$
$= 7\times(-3)+26$
$= -21+26$
∴ $h = 5$
So, centre = (h, k) = (5, – 3)
Radius = Distance between (5, – 3) and (1, 0)
$=\sqrt{(1-5)^2+(0+3)^2}$
$=\sqrt{4^2+3^2}=\sqrt{25}=5$ units
We have, equation of circle is;
$(x-h)^2+(y-k)^2=r^2$
or, $(x-5)^2+(y+3)^2=5^2$
or, $x^2-10x+25+y^2+6y+9=25$
or, $x^2+y^2-10x+6y+9=0$ ....(vii)
Remaining point is (9, – 6);
So, LHS $=x^2+y^2-10x+6y+9$
$=9^2+(-6)^2-10\times 9-6\times 6+9$
$=81+36-90-36+9$
$=90-90$
$=0$ = RHS
Thus, the four points (1, 0), (2, – 7), (8, 1) and (9, – 6) lie on the same circle.

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7. **तलका वृत्तको समीकरण पत्ता लगाउनुहोस् । (Find the equation of following circles.)**

(a) **केन्द्र (3, 4) र x-अक्षलाई छुने**
Centre at (3, 4) and touches the x-axis.

⇨ Here, centre $(h, k) = (3, 4)$
Circle touches x - axis.
So, $r = k = 4$
We have, equation of circle is; $(x - h)^2 + (y - k)^2 = r^2$
So, $(x - 3)^2 + (y - 4)^2 = 4^2$
or, $x^2 - 6x + 9 + y^2 - 8y + 16 = 16$
or, $x^2 + y^2 - 6x - 8y + 9 = 0$
Thus, the equation of circle is;
$x^2 + y^2 - 6x - 8y + 9 = 0.$

(b) **केन्द्र (4, 5) र x-अक्षलाई छुने**
Center at (4, 5) and touches the x-axis.

⇨ Here, centre $(h, k) = (4, 5)$
The circle touches x - axis.
So, $r = k = 5$
We have, equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 4)^2 + (y - 5)^2 = 5^2$
or, $x^2 - 8x + 16 + y^2 - 10y + 25 = 25$
or, $x^2 + y^2 - 8x - 10y + 16 = 0$
Thus, the equation of the circle is;
$x^2 + y^2 - 8x - 10y + 16 = 0.$

(c) **केन्द्र (4, – 3) र y-अक्षलाई छुने**
Center at (4, – 3) and touches the y-axis.

⇨ Here, centre $(h, k) = (4, -3)$
The circle touches y - axis.
So, $r = h = 4$
We have, equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 4)^2 + (y + 3)^2 = 4^2$
or, $x^2 - 8x + 16 + y^2 + 6y + 9 = 16$
$\therefore$ $x^2 + y^2 - 8x + 6y + 9 = 0$
Thus, the equation of the circle is;
$x^2 + y^2 - 8x + 6y + 9 = 0.$

(d) **केन्द्र (– 5, – 4) र y-अक्षलाई छुने**
Center at (– 5, – 4) and touches the y-axis.

⇨ Here, centre $(h, k) = (-5, -4)$
The circle touches y - axis.
So, $r = h = 5$
We have, the equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x + 5)^2 + (y + 4)^2 = 5^2$
or, $x^2 + 10x + 25 + y^2 + 8y + 16 = 25$
or, $x^2 + y^2 + 10x + 8y + 16 = 0$
Thus, the equation of the circle is;
$x^2 + y^2 + 10x + 8y + 16 = 0.$

(e) **केन्द्र (2, 2) र दुवै अक्षलाई छुने**
Center at (2, 2) and touches both the axes.

⇨ Here, centre $(h, k) = (2, 2)$
The circle touches both the axes.
So, $r = h = k = 2$
We have, the equation of circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 2)^2 + (y - 2)^2 = 2^2$
or, $x^2 - 4x + 4 + y^2 - 4y + 4 = 4$
$\therefore$ $x^2 + y^2 - 4x - 4y + 4 = 0$
Thus, the equation of the circle is
$x^2 + y^2 - 4x - 4y + 4 = 0.$

(f) **अर्धव्यास 5 एकाइ दुवै धनात्मक अक्षलाई छुने ।**
Radius 5 unit and touches both positive axes.

⇨ Here, radius $(r) = 5$ unit
The circle touches both the axes.
So, $h = k = r = 5$
$\therefore$ Centre $= (h, k) = (5, 5)$ and $r = 5$ unit
We have, the equation of the circle is;
$(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 5)^2 + (y - 5)^2 = 5^2$
or, $x^2 - 10x + 25 + y^2 - 10y + 25 = 25$
$\therefore$ $x^2 + y^2 - 10x - 10y + 25 = 0$
Thus, the equation of circle is;
$x^2 + y^2 - 10x - 10y + 25 = 0.$

8. **वृत्तको समीकरणका विभिन्न स्वरूपहरूबिच सम्बन्ध खोजी कक्षाकोठामा प्रस्तुत गर्नुहोस् ।**
Find the relation between various forms of circle and present in your classroom.

⇨ Show to your teacher.

OTHER IMPORTANT QUESTIONS

1. **बिन्दुहरू (1 ,3) , (2, – 1) र (– 1, 1) भएर जाने वृत्तको केन्दबिन्दु पत्ता लगाउनुहोस् ।**
Find the center of the circle passing through the points ; (1 ,3) , (2, – 1) & (– 1, 1).

⇨ Here, (1, 3), (2, – 1) & (– 1, 1)
Let circle $(x - h)^2 + (y - k)^2 = r^2$ passes through the points (1 ,3), (2, – 1) & (– 1, 1)

$(1 - h)^2 + (3 - k)^2 = r^2$ ....(1)
$(2 - h)^2 + (-1 - k)^2 = r^2$ ....(2)
$(-1 - h)^2 + (1 - k)^2 = r^2$ ....(3)

Equating equation (1) & equation (2)
$(1 - h)^2 + (3 - k)^2 = (2 - h)^2 + (-1 - k)^2$
or, $1 - 2h + h^2 + 9 - 6k + k^2 = 4 - 4h + h^2 + 1 + 2k + k^2$
or, $2h - 8k = 5 - 10 = -5$
or, $2h - 8k = -5$ ....(4)

Equating equation (2) & equation (3)
$(2 - h)^2 + (-1 - k)^2 = (-1 - h)^2 + (1 - k)^2$
or, $4 - 4h + h^2 + 1 + 2k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$
or, $-6h + 4k = 2 - 5$
or, $6h - 4k = 3$ ....(5)

Multiplying equation (4) by 3 then subtracting equation (5) from it.

$$\begin{array}{l} 6h - 24k = -15 \\ 6h - 4k = 3 \\ \underline{- \quad + \quad -} \\ -20k = -18 \end{array}$$

$\therefore$ $k = \frac{18}{20} = \frac{9}{10}$

Multiplying equation no (5) by 2 then subtracting equation no (4) from it.

$$\begin{array}{l} 12h - 8k = 6 \\ 2h - 8k = -5 \\ \underline{- \quad + \quad +} \\ 10h = 11 \end{array}$$

$\therefore$ $h = \frac{11}{10}$

Thus, center of the circle is $\left(\frac{11}{10}, \frac{9}{10}\right)$

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2. **बिन्दुहरू (2 ,– 1), (2, 3) र (4, –1) भएर जाने वृत्तको केन्दबिन्दु पत्ता लगाउनुहोस् ।**

Find the center of the circle passing through the points; (2 ,– 1), (2, 3) & (4, –1).

⇨ Here, $(2, -1), (2, 3)$ & $(4, -1)$

Let $(x-h)^2+(y-k)^2=r^2$ passes through the points $(2,-1), (2,3)$ & $(4,-1)$

$(2-h)^2+(-1-k)^2=r^2$ ....(1)

$(2-h)^2+(3-k)^2 = r^2$ ....(2)

$(4-h)^2+(-1-k)^2=r^2$ ....(3)

Equating equation (1) & equation (2)

$(2-h)^2+(-1-k)^2=(2-h)^2+(3-k)^2$

or, $4-4h+h^2+1+2k+k^2$

$= 4-4h+h^2+9-6k+k^2$

or, $8k = 13-5$

or, $8k = 8$

$\therefore$ $k = 1$

Equating equation no (2) & (3)

$(2-h)^2+(3-k)^2=(4-h)^2+(-1-k)^2$

or, $4-4h+h^2+9-6k+k^2$

$= 16-8h+h^2+1+2k+k^2$

or, $4h - 8k = 17-13$

or, $4h-8k=4$

or, $h-2k = 1$

or, $h-2.1=1$

or, $h = 1+2 = 3$

Thus, centre of the circle is (3, 1) .

3. **बिन्दुहरू (– 2, 2) , (2, 4) र (4 , 0) भएर जाने वृत्तको केन्द्रबिन्दु र समीकरण पत्ता लगाउनुहोस् ।**

Find the centre and equation of the circle which passes through the points; (– 2, 2) , (2, 4) & (4 , 0).

⇨ Here, $(-2, 2), (2, 4)$ & $(4, 0)$

Let $(x-h)^2+(y-k)^2=r^2$ passes through the points $(-2, 2), (2, 4)$ & $(4, 0)$.

$(-2-h)^2+(2-k)^2=r^2$ ....(1)

$(2-h)^2+(4-k)^2=r^2$ .... (2)

$(4-h)^2+(0-k)^2=r^2$ ....(3)

Equating equation (1) & equation (2)

$(-2-h)^2+(2-k)^2=(2-h)^2+(4-k)^2$

or, $4+4h+h^2+4-4k+k^2=4-4h+h^2+16-8k+k^2$

or, $8h+4k=20-8$

or, $8h+4k=12$

or, $2h+k=3$ ....(4)

Equating equation (2) & (3)

$(2-h)^2+(4-k)^2=(4-h)^2+(0-k)^2$

or, $4-4h+h^2+16-8k+k^2=16-8h+h^2+k^2$

or, $4h-8k=-4$

or, $h-2k=-1$ ....(5)

Multiplying equation (5) by 2 then subtracting from (4)

$2h+k=3$

$2h-4k=-2$

$-\quad+\quad+$

$5k=5$

or, $k=1$ $\quad\therefore\ h=1$ $\quad\therefore$ Centre = (1, 1)

Substituting value of h & k in equation (1)

$(-2-1)^2+(2-1)^2=r^2$

or, $(-3)^2+(1)^2=r^2$

or, $9+1=r^2$

or, $10=r^2$ $\qquad\therefore\ r=\sqrt{10}$

$\therefore$ Required equation is, $(x-1)^2+(y-1)^2=(\sqrt{10})^2$

or, $x^2-2x+1+y^2-2y+1=10$

Thus, $x^2+y^2-2x-2y=8$ is the required equation of the circle having centre (1, 1).

4. **बिन्दुहरू (1, 1) , (2, – 1) र (3, – 2) भएर जाने वृत्तको केन्द्रबिन्दु र समीकरण पत्ता लगाउनुहोस् ।**

Find the centre and equation of the circle which passes through the points; (1, 1) , (2, –1) & (3, – 2).

⇨ Here, $(1, 1), (2, -1)$ & $(3, -2)$

Let $(x-h)^2+(y-k)^2=r^2$ passes through the points $(1, 1), (2, -1)$ & $(3, -2)$

or, $(1-h)^2+(1-k)^2=r^2$ ....(1)

or, $(2-h)^2+(-1-k)^2=r^2$ ..... (2)

or, $(3-h)^2+(-2-k)^2=r^2$ ....(3)

Equating equation (1) & (2)

$(1-h)^2+(1-k)^2=(2-h)^2+(-1-k)^2$

or, $1-2h+h^2+1-2k+k^2=4-4h+h^2+1+2k+k^2$

or, $2h-4k=5-2$

or, $2h-4k=3$ ....(4)

Equating equation (2) & (3)

$(2-h)^2+(-1-k)^2=(3-h)^2+(-2-k)^2$

or, $4-4h+h^2+1+2k+k^2=9-6h+h^2+4+4k+k^2$

or, $2h-2k=13-5$

or, $2h-2k=8$

or, $h-k=4$ ....(5)

Multiplying equation (5) by 2 then subtracting from (4)

$2h-4k=3$

$2h-2k=8$

$-\quad+\quad-$

$-2k=-5$ $\qquad\therefore\ k=\dfrac{5}{2}$

Multiplying equation (5) by 4 then subtracting from (4)

$2h-4k=3$

$4h-4k=16$

$-\quad+\quad-$

$-2h=-13$ $\qquad\therefore\ h=\dfrac{13}{2}$ $\qquad\therefore$ Center $\left(\dfrac{13}{2},\dfrac{5}{2}\right)$

Substituting value of h & k in equation no (1)

$$\left(1-\frac{13}{2}\right)^2+\left(1-\frac{5}{2}\right)^2=r^2$$

or, $\left(\dfrac{2-13}{2}\right)^2+\left(\dfrac{2-5}{2}\right)^2=r^2$

or, $\left(\dfrac{-11}{2}\right)^2+\left(\dfrac{-3}{2}\right)^2=r^2$

or, $\dfrac{121}{4}+\dfrac{9}{4}=r^2$

or, $\dfrac{121+9}{4}=r^2$

or, $\dfrac{130}{4}=r^2$ $\qquad\therefore\ r=\sqrt{\dfrac{130}{4}}=\sqrt{\dfrac{65}{2}}$

Required equation is $\left(x-\dfrac{13}{2}\right)^2+\left(y-\dfrac{5}{2}\right)^2=\sqrt{\left(\dfrac{65}{2}\right)^2}$

or, $\left(\dfrac{2x-13}{2}\right)^2+\left(\dfrac{2y-5}{2}\right)^2=\dfrac{65}{2}$

or, $\dfrac{4x^2-52x+169}{4}+\dfrac{4y^2-20y+25}{4}=\dfrac{65}{2}$

or, $4x^2-52x+169+4y^2-20y+25=130$

or, $4x^2+4y^2-52x-20y=130-194$

or, $4x^2+4y^2-52x-20y=-64$

Thus, $x^2+y^2-13x-5y+16=0$ is the required equation of the circle with centre $\left(\dfrac{13}{2},\dfrac{5}{2}\right)$.

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5. बिन्दुहरू (0, 0) , (0, b) र (a, 0) भएर जाने वृत्तको केन्द्रबिन्दु र समीकरण पत्ता लगाउनुहोस् ।
Find the centre and equation of the circle which passes through the points; (0, 0) , (0, b) & (a, 0).

⇨ Here, Let $(x - h)^2 + (y - k)^2 = r^2$ passes through the points (0, 0), (0, b) & (a, 0).
or, $(0 - h)^2 + (0 - k)^2 = r^2$ .....(1)
or, $(0 - h)^2 + (b - k)^2 = r^2$ ....(2)
or, $(a - h)^2 + (0 - k)^2 = r^2$.....(3)
Equating equation (1) & equation (2)
$(0 - h)^2 + (0 - k)^2 = (0 - h)^2 + (b - k)^2$
or, $k^2 = b^2 - 2bk + k^2$
$\therefore\ k = \frac{b^2}{2b} = \frac{b}{2}$

Equating equation (2) & equation (3)
$(0 - h)^2 + (b - k)^2 = (a - h)^2 + (0 - k)^2$
or, $h^2 + b^2 - 2bk + k^2 = a^2 - 2ah + h^2 + k^2$
or, $b^2 - 2bk = - 2ah + a^2$
or, $2ah = 2b\,k + a^2 - b^2$
or, $2ah = 2b \cdot \frac{b}{2} + a^2 - b^2$
or, $2ah = b^2 + a^2 - b^2$
$\therefore\ h = \frac{a^2}{2a} = \frac{a}{2}$ $\qquad\therefore$ Centre of the circle is $\left(\frac{a}{2}, \frac{b}{2}\right)$

Substituting the value of h & k in equation (1); $\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2 = r^2$
or, $\frac{a^2}{4} + \frac{b^2}{4} = r^2$
or, $\frac{a^2 + b^2}{4} = r^2$
$\therefore\ r = \sqrt{\frac{a^2 + b^2}{4}}$

$\therefore$ Required equation is, $\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{\sqrt{a^2 + b^2}}{2}\right)^2$
or, $x^2 - xa + \frac{a^2}{4} + y^2 - yb + \frac{b^2}{4} = \frac{a^2 + b^2}{4}$
or, $x^2 - xa + y^2 - by + = \frac{a^2}{4} + \frac{b^2}{4} - \frac{a^2}{4} - \frac{b^2}{4}$

Thus, $x^2 + y^2 - ax - bx = 0$ is the required equation and $\left(\frac{a}{2}, \frac{b}{2}\right)$ is required centre.

6.. अर्धव्यास 3 र केन्द्रबिन्दु (2, 1) मा भएको एउटा वृत्त खिच । (Draw a circle whose radius is 3 & centre is at (2 , 1))
(a) सो वृत्तको समीकरण लेख्नुहोस् । (Write the equation of the circle.)
(b) सो वृत्तमा A(2, – 2) र B(5, 1) पर्दछन् भनी प्रमाणित गर्नुहोस् । (Prove that the points A(2, – 2) & B(5, 1) lie on the circle.)

⇨ (a) Here, radius (r) = 3
centre (h, k) = (2, 1)
We have, $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 2)^2 + (y - 1)^2 = 3^2$
or, $x^2 - 4x + 4 + y^2 - 2y + 1 = 9$
or, $x^2 + y^2 - 4x - 2y = 4$
Thus, required equation is $x^2 + y^2 - 4x - 2y = 4$

(b) If the circle passes through the point A (2, – 2),
⇨ Here, $(2)^2 + (-2)^2 - 4(2) - 2(-2) = 4$
or, $4 + 4 - 8 + 4 = 4$
or, $12 - 8 = 4$
or, $4 = 4$ $\qquad\therefore$ LHS = RHS
If the circle passes through the point B (5 ,1),
$(5)^2 + (1)^2 - 4(5) - 2(1) = 4$
or, $25 + 1 - 20 - 2 = 4$
or, $26 - 22 = 4$
or, $4 = 4$ $\qquad\therefore$ LHS = RHS
Thus, A(2, –2) & B(5, 1) lie on the circle.

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त्रिकोणमिति (Trigonometry)

1. अपवर्तीय र अर्ध अपवर्तीय कोणहरू Multiple and Sub-multiple Angles

FORMULAE

अपवर्त्य कोणहरू (Multiple Angles)		अपवर्तक कोणहरू (Submultiple Angles)	
1.	$\sin 2A = 2 \sin A \cos A$	1.	$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$
2.	$\cos 2A = \cos^2 A - \sin^2 A$	2.	$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$
3.	$\cos 2A = 2 \cos^2 A - 1$	3.	$\cos A = 2 \cos^2 \frac{A}{2} - 1$
4.	$\cos 2A = 1 - 2 \sin^2 A$	4.	$\cos A = 1 - 2 \sin^2 \frac{A}{2}$
5.	$1 + \cos 2A = 2 \cos^2 A$	5.	$1 + \cos A = 2 \cos^2 \frac{A}{2}$
6.	$1 - \cos 2A = 2 \sin^2 A$	6.	$1 - \cos A = 2 \sin^2 \frac{A}{2}$
7.	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$	7.	$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$
8.	$\frac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A$	8.	$\frac{1 - \cos A}{1 + \cos A} = \tan^2 \frac{A}{2}$
9.	$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$	9.	$\sin A = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$
10.	$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$	10.	$\cos A = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$
11.	$\sin 3A = 3 \sin A - 4 \sin^3 A$	11.	$\sin A = 3 \sin \frac{A}{3} - 4 \sin^3 \frac{A}{3}$
12.	$4 \sin^3 A = 3 \sin A - \sin 3A$	12.	$4 \sin^3 \frac{A}{3} = 3 \sin \frac{A}{3} - 3 \sin A$
13.	$\cos 3A = 4 \cos^3 A - 3 \cos A$	13.	$\cos A = 4 \cos^3 \frac{A}{3} - 3 \cos \frac{A}{3}$
14.	$4 \cos^3 A = 3 \cos A + \cos 3A$	14.	$4 \cos^3 \frac{A}{3} = 3 \cos \frac{A}{3} + \cos A$

QUESTIONS FROM SEE EXERCISE 1

A. VERY SHORT QUESTIONS

1. sin 2A लाई tan A को रूपमा व्यक्त गर्नुहोस् । (Express sin 2A in terms of tan A.) **[SEE 2076 M]**

⇨ Here, $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$ is the required relation.

2. cos 2A लाई cos A को रूपमा लेख्नुहोस् । (Write cos 2A interms of cos A.)

⇨ Here, $\cos 2A = 2 \cos^2 A - 1$

3. tan 2A लाई tan A को रूपमा लेख्नुहोस् । (Write tan 2A interms of tan A.)

⇨ Here, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

4. sin (A + B) = sin A cos B + cos A sin B, मा A = B भए कुन सूत्र बन्छ ?
In sin (A + B) = sin A cos B + cos A sin B, if A = B then which formula will be formed?

⇨ Here, if A = B in the given formula then, $\sin 2A = 2 \sin A \cos A$

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5. यदि sin A = x भए cos 2A कति हुन्छ ? (If sin A = x then what is cos 2A?)

⇨ Here, $\sin A = x$ So, $\cos 2A = 1 - 2\sin^2 A = 1 - 2 \times x^2$

$\therefore \cos 2A = 1 - 2x^2$

6. यदि sin A = m र cos A = n भए sin 2A कति हुन्छ ? (If sin A = m and cos A = n then what is sin 2A?)

⇨ Here, sin A = m and cos A = n

So, sin 2A = 2 sin A cos A = 2mn

7. sin 4A लाई 2A को रूपमा व्यक्त गर्नुहोस् । (Express sin 4A interms of 2A.)

⇨ Here, sin 4A = sin 2. (2A) = 2 sin 2A cos 2A

8. यदि $\sin\frac{A}{2} = m$ भए cos A कति हुन्छ ? (If $\sin\frac{A}{2} = m$ then what is cos A?)

⇨ Here, $\sin\frac{A}{2} = m$ So, $\cos A = 1 - 2\sin^2\frac{A}{2} = 1 - 2 \times m^2 = 1 - 2m^2$

9. $\sin\frac{A}{2}$ लाई $\frac{A}{4}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\sin\frac{A}{2}$ interms of $\frac{A}{4}$.)

⇨ Here, $\sin\frac{A}{2} = \sin 2 . \frac{A}{4} = 2\sin\frac{A}{4}\cos\frac{A}{4}$

10. $\cos^2\frac{A}{2} - \sin^2\frac{A}{2}$ सँग बराबर हुने त्रिकोणमितीय अनुपात लेख्नुहोस् ।

Write the trigonometric ratio equal to the trigonometric ratio $\cos^2\frac{A}{2} - \sin^2\frac{A}{2}$.

⇨ Here, $\cos^2\frac{A}{2} - \sin^2\frac{A}{2} = \cos A$

11. यदि sin A = $\frac{1}{2}$ भए cos 2A को मान पत्ता लगाउनुहोस् । (If sin A = $\frac{1}{2}$, find the value of cos 2A.)

⇨ Here, $\sin A = \frac{1}{2}$ So, $\cos 2A = 1 - 2\sin^2 A = 1 - 2 \times \left(\frac{1}{2}\right)^2 = 1 - 2 \times \frac{1}{4} = \frac{1}{2}$

12. यदि $\cos\frac{A}{3} = \frac{\sqrt{3}}{2}$ भए cos A को मान पत्ता लगाउनुहोस् । (If $\cos\frac{A}{3} = \frac{\sqrt{3}}{2}$, find the value of cos A.)

⇨ Here, $\cos\frac{A}{3} = \frac{\sqrt{3}}{2}$ or, $\cos\frac{A}{3} = \cos 30°$ or, $\frac{A}{3} = 30°$ $\therefore$ A = 90°

So, cos A = cos 90°

B. SHORT QUESTIONS

MODEL 1

1. यदि sin A = $\frac{3}{4}$ भए cos 2A को मान निकाल्नुहोस् ।

If sin A = $\frac{3}{4}$, find the value of cos 2A. [2068R]

⇨ Here, $\sin A = \frac{3}{4}$

We know that,
$$\begin{aligned}\cos 2A &= 1 - 2\sin^2 A\\ &= 1 - 2 \times \left(\frac{3}{4}\right)^2\\ &= 1 - 2 \times \frac{9}{16} = 1 - \frac{9}{8} = -\frac{1}{8}\end{aligned}$$

Thus, the value of cos 2A is $-\frac{1}{8}$.

3. यदि 5sin θ = 4 भए cos 2θ को मान निकाल्नुहोस् ।

If 5sin θ = 4, find the value of cos 2θ. [2063M]

⇨ Here, $5\sin\theta = 4$

or, $\sin\theta = \frac{4}{5}$

Now,
$$\begin{aligned}\cos 2\theta &= 1 - 2\sin^2\theta\\ &= 1 - 2 \times \left(\frac{4}{5}\right)^2\\ &= 1 - 2 \times \frac{16}{25}\end{aligned}$$

$$\therefore \cos 2\theta = -\frac{7}{25}$$

2. यदि sin A = $\frac{3}{5}$ भए cos 2A को मान निकाल्नुहोस् ।

If sin A = $\frac{3}{5}$, find the value of cos 2A. [2071 R', 2064 R']

⇨ Here, $\sin A = \frac{3}{5}$

$$\begin{aligned}\therefore \cos 2A &= 1 - 2\sin^2 A\\ &= 1 - 2\left(\frac{3}{5}\right)^2\\ &= 1 - 2 \times \frac{9}{25} = \frac{7}{25}\end{aligned}$$

Thus, the value of cos 2A is $\frac{7}{25}$.

4. यदि sin θ = $\frac{3}{5}$ भए sin 2θ को मान निकाल्नुहोस् ।

If sin θ = $\frac{3}{5}$, find the value of sin 2θ. [2063R']

⇨ Here, $\sin\theta = \frac{3}{5}$

$$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

Now, $\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{3}{5} \times \frac{4}{5}$

$$\therefore \sin 2\theta = \frac{24}{25}$$

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5. यदि $\cos\theta = \frac{3}{4}$ भए $\sin 2\theta$ को मान निकाल्नुहोस् ।

If $\cos\theta = \frac{3}{4}$, find the value of $\sin 2\theta$.

⇨ Here, $\cos\theta = \frac{3}{4}$

$$\therefore\ \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{3}{4}\right)^2}$$

$$= \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Now, $\sin 2\theta = 2\sin\theta\cos\theta = 2\times\frac{\sqrt{7}}{4}\times\frac{3}{4}$

$$\therefore\ \sin 2\theta = \frac{3\sqrt{7}}{8}$$

6. यदि $\tan A = \frac{3}{4}$ भए $\sin 2A$ को मान निकाल्नुहोस् ।

If $\tan A = \frac{3}{4}$, find the value of $\sin 2A$.

⇨ Here, given $\tan A = \frac{3}{4}$

We know,

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1+\frac{9}{16}}$$

$$= \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$$

$$\therefore\ \sin 2A = \frac{24}{25}.$$

7. यदि $\cos\theta = \frac{12}{13}$ र $\sin\alpha = \frac{4}{5}$ भए $\sin 2\theta$ and $\cos 2\alpha$ को मान निकाल्नुहोस् ।

If $\cos\theta = \frac{12}{13}$ and $\sin\alpha = \frac{4}{5}$, find the value of $\sin 2\theta$ and $\cos 2\alpha$. **[2059S]**

⇨ Here, given $\cos\theta = \frac{12}{13}$ and $\sin\alpha = \frac{4}{5}$

$\sin 2\theta = ?$ and $\cos 2\alpha = ?$

Here, $\cos\theta = \frac{12}{13}$ so, $\sin\theta = \sqrt{1-\cos^2\theta}$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$= \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{169-144}{169}}$$

$$= \sqrt{\frac{25}{169}}$$

$$= \frac{5}{13}$$

Now, $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2.\frac{5}{13}.\frac{12}{13} = \frac{120}{169}$$

and $\cos 2\alpha = 1 - 2\sin^2\alpha$

$$= 1 - 2.\left(\frac{4}{5}\right)^2$$

$$= 1 - 2\times\frac{16}{25}$$

$$= 1 - \frac{32}{25}$$

$$= \frac{25-32}{25}$$

$$= \frac{-7}{25}$$

Thus, $\sin 2\theta = \frac{5}{13}$ & $\cos 2\alpha = \frac{-7}{25}$.

MODEL 2

8. यदि $\sin A = \frac{1}{2}$ भए $\sin 3A$ को मान पत्ता लगाउनुहोस् ।

If $\sin A = \frac{1}{2}$, find the value of $\sin 3A$. **[2072 R']**

⇨ Here, $\sin A = \frac{1}{2}$

We know that,

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$= 3\times\frac{1}{2} - 4\times\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2}$$

$\therefore\ \sin 3A = 1$

Thus, the value of sin 3A is 1.

9. यदि $\cos\theta = \frac{3}{5}$ भए $\cos 3\theta$ को मान पत्ता लगाउनुहोस् ।

If $\cos\theta = \frac{3}{5}$, find the value of $\cos 3\theta$. **[2064 S]**

⇨ Here, $\cos\theta = \frac{3}{5}$

We have,

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$= 4\times\left(\frac{3}{5}\right)^3 - 3\times\frac{3}{5}$$

$$= 4\times\frac{27}{125} - \frac{9}{5} = \frac{4\times27 - 9\times25}{125}$$

$$\therefore\ \cos 3\theta = -\frac{117}{125}$$

10. यदि $\sin\alpha = \frac{3}{4}$ भए $\cos 2\alpha$ र $\sin 3\alpha$ को मान पत्ता लगाउनुहोस् । If $\sin\alpha = \frac{3}{4}$, find the value of $\cos 2\alpha$ and $\sin 3\alpha$ **[2059 R]**

⇨ Here, given, $\sin\alpha = \frac{3}{4}$ and $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{3}{4}\right)^2} = \sqrt{1-\frac{9}{16}} = \sqrt{\frac{16-9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$

Now, $\cos 2\alpha = 2\cos^2\alpha - 1 = 2.\left(\frac{\sqrt{7}}{4}\right)^2 - 1 = \frac{2\times7}{16} - 1 = \frac{14-16}{16} = \frac{-2}{16} = \frac{-1}{8}$

And, $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha = 3.\frac{3}{4} - 4.\left(\frac{3}{4}\right)^3 = \frac{9}{4} - \frac{4\times27}{64} = \frac{9\times4-27}{16} = \frac{9}{16}$

Thus, $\cos 2\alpha = \frac{-1}{8}$ & $\sin 3\alpha = \frac{9}{16}$.

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MODEL 3

11. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ हुन्छ भनी देखाउनुहोस् ।

If $\cos 30° = \frac{\sqrt{3}}{2}$, show that $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ **[2075 R'₂, 2074 S']**

⇨ Here, $\cos 30° = \frac{\sqrt{3}}{2}$

We know that, $\cos 2A = 2\cos^2 A - 1$

$$\therefore \quad \cos A = \sqrt{\frac{1+\cos 2A}{2}}$$

For A = 15°;

$$\cos 15° = \sqrt{\frac{1+\cos 2\,.\,15°}{2}}$$

$$= \sqrt{\frac{1+\cos 30°}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} \quad = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{4}} \quad = \sqrt{\frac{(2+\sqrt{3})\times 2}{4\times 2}}$$

$$= \sqrt{\frac{4+2\sqrt{3}}{8}} \quad = \sqrt{\frac{3+1+2\sqrt{3}}{8}}$$

$$= \sqrt{\frac{(\sqrt{3})^2 + 2\,.\sqrt{3}\,.\,1 + 1^2}{8}}$$

$$= \sqrt{\frac{(\sqrt{3}+1)^2}{8}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \text{RHS.}$$ **Proved.**

12. यदि $\sin\frac{\theta}{2} = \frac{1}{2}$ भए $\cos\theta$ को मान निकाल्नुहोस् ।

If $\sin\frac{\theta}{2} = \frac{1}{2}$, find the value of $\cos\theta$. **[2074 S]**

⇨ Here, $\sin\frac{\theta}{2} = \frac{1}{2}$

We know that,

$$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$$

$$= 1 - 2\times\left(\frac{1}{2}\right)^2$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the value of $\cos\theta$ is $\frac{1}{2}$.

13. यदि $\tan\frac{A}{2} = \frac{3}{4}$ भए $\sin A$ को मान पत्ता लगाउनुहोस् ।

If $\tan\frac{A}{2} = \frac{3}{4}$ then find the value of sin A. **[2073 R]**

⇨ Here, $\tan\frac{A}{2} = \frac{3}{4}$,

$$\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}} = \frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{3}{2}}{1+\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{25}{16}} = \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$$

Thus, the value of sin A is $\frac{24}{25}$.

14. यदि $\cos 45° = \frac{1}{\sqrt{2}}$ भए, प्रमाणित गर्नुहोस् : $\tan 22\frac{1°}{2} = \sqrt{2} - 1$.

If $\cos 45° = \frac{1}{\sqrt{2}}$, prove that: $\tan 22\frac{1°}{2} = \sqrt{2} - 1$ **[2073 S']**

⇨ Here, we have, $\cos 45° = \frac{1}{\sqrt{2}}$

$$\text{or,}\quad \frac{1-\tan^2\frac{45°}{2}}{1+\tan^2\frac{45°}{2}} = \frac{1}{\sqrt{2}}$$

$$\text{or,}\quad \sqrt{2} - \sqrt{2}\tan^2\frac{45°}{2} = 1 + \tan^2\frac{45°}{2}$$

$$\text{or,}\quad \sqrt{2} - 1 = \sqrt{2}\tan^2\frac{45°}{2} + \tan^2\frac{45°}{2}$$

$$\text{or,}\quad \tan^2\frac{45°}{2}\times(\sqrt{2}+1) = \sqrt{2}-1$$

$$\text{or,}\quad \tan^2\frac{45°}{2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$\text{or,}\quad \tan^2\frac{45°}{2} = (\sqrt{2}-1)^2$$

$$\therefore \quad \tan 22\frac{1°}{2} = \sqrt{2} - 1$$ **Proved.**

15. यदि $\tan\frac{A}{2} = \frac{3}{4}$ भए tan A को सङ्ख्यात्मक मान पत्ता लगाउनुहोस् ।

If $\tan\frac{A}{2} = \frac{3}{4}$, find the numerical value of tan A. **[2064 R]**

⇨ Here, $\tan\frac{A}{2} = \frac{3}{4}$

$$\therefore \quad \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$

$$= \frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{6}{4}\times\frac{16}{7}$$

$$= \frac{24}{7}$$

$$\therefore \quad \tan A = \frac{24}{7}$$

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16. यदि $\sin\frac{\alpha}{2} = \frac{3}{5}$ भए $\sin\alpha$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\alpha}{2} = \frac{3}{5}$, find the value of $\sin\alpha$. [2062 S]

⇨ Here, $\sin\frac{\alpha}{2} = \frac{3}{5}$

$$\therefore \quad \cos\frac{\alpha}{2} = \sqrt{1-\left(\sin\frac{\alpha}{2}\right)^2} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\text{Now, } \sin\alpha = 2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2} = 2\times\frac{3}{5}\times\frac{4}{5}$$

$$\therefore \quad \sin\alpha = \frac{24}{25}$$

18. यदि $\cos 330° = \frac{\sqrt{3}}{2}$ भए $\sin 165° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos 330° = \frac{\sqrt{3}}{2}$, prove that: $\sin 165° = \frac{\sqrt{3}-1}{2\sqrt{2}}$. [2060 C]

⇨ Here, $\cos 330° = \frac{\sqrt{3}}{2}$

We know that, $\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$

Putting $\theta = 165°$,

$$\sin 165° = \sqrt{\frac{1-\cos 330°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2-\sqrt{3}}{2}\times\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{(2-\sqrt{3})\times 2}{4\times 2}} = \sqrt{\frac{4-2\sqrt{3}}{8}} = \frac{\sqrt{\sqrt{3}^2-2.\sqrt{3}.1+1^2}}{2\sqrt{2}} = \frac{(\sqrt{3}-1)}{2\sqrt{2}}$$

Thus, $\sin 165° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ **Proved.**

17. यदि $\cos\frac{A}{2} = \frac{3}{5}$ भए $\cos A$ को मान पत्ता लगाउनुहोस् ।

If $\cos\frac{A}{2} = \frac{3}{5}$, find the value of $\cos A$. [2068 R']

⇨ Here, $\cos\frac{A}{2} = \frac{3}{5}$

$$\text{Now, } \cos A = 2\cos^2\frac{A}{2} - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = \frac{2\times 9}{25} - 1 = \frac{18-25}{25} = -\frac{7}{25}$$

Thus, required value of $\cos A$ is $-\frac{7}{25}$ under the given condition.

19. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए, $\sin 15°$ को मान निकाल्नुहोस् ।

Find the value of $\sin 15°$ if $\cos 30° = \frac{\sqrt{3}}{2}$ [2066 S]

⇨ Here, $\sin 15° = \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$$\therefore \sin 15° = \sqrt{\frac{2-\sqrt{3}}{2\times 2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

20. $\cos A$ लाई $\cot\frac{A}{2}$ को रुपमा व्यक्त गर्नुहोस् ।

Express $\cos A$ in terms of $\cot\frac{A}{2}$. [2067 R]

⇨ Here,

$$\cos A = \cos 2\left(\frac{A}{2}\right)$$

$$= \frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{\cos^2\frac{A}{2} + \sin^2\frac{A}{2}} \quad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{\frac{\cos^2\frac{A}{2}}{\sin^2\frac{A}{2}} - \frac{\sin^2\frac{A}{2}}{\sin^2\frac{A}{2}}}{\frac{\cos^2\frac{A}{2}}{\sin^2\frac{A}{2}} - \frac{\sin^2\frac{A}{2}}{\sin^2\frac{A}{2}}} \quad \left[\because \text{Divided by } \sin^2\frac{A}{2}\right]$$

$$= \frac{\cot^2\frac{A}{2} - 1}{\cot^2\frac{A}{2} + 1}$$

Thus, $\cos A = \frac{\cot^2\frac{A}{2} - 1}{\cot^2\frac{A}{2} + 1}$ as a form of $\cot\frac{A}{2}$

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MODEL 4

21. यदि $\cos\frac{\alpha}{3}=\frac{1}{2}$ भए $\sin\alpha$ को मान निकाल्नुहोस् । (If $\cos\frac{\alpha}{3}=\frac{1}{2}$, find the value of $\sin\alpha$.) **[2070 R]**

⇨ Here, $\cos\frac{\alpha}{3}=\frac{1}{2}$

So, $\sin\frac{\alpha}{3} = \sqrt{1-\cos^2\frac{\alpha}{3}}$

$= \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \sqrt{1-\frac{1}{4}}$

$= \frac{\sqrt{3}}{2}$

We know that,

$\sin\alpha = 3\sin\frac{\alpha}{3} - 4\sin^3\frac{\alpha}{3}$

$= 3\times\frac{\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3}{2}\sqrt{3} - \frac{3\sqrt{3}}{2}$

$\therefore\ \sin\alpha = 0$

Thus, the value of $\sin\alpha$ is 0.

22. यदि $\cos\frac{\theta}{3} = \frac{3}{5}$ भए $\cos\theta$ को मान पत्ता लगाउनुहोस् ।

If $\cos\frac{\theta}{3} = \frac{3}{5}$, find the value of $\cos\theta$. **[2058 S]**

⇨ Here, given, $\cos\frac{\theta}{3} = \frac{3}{5}$, $\cos\theta = ?$

We know that, $\cos\theta = 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}$

or, $\cos\theta = 4 . \left(\frac{3}{5}\right)^3 - 3\left(\frac{3}{5}\right)$

$= 4\times\frac{27}{125} - \frac{9}{5}$

$= \frac{108-225}{125}$

$\therefore\ \cos\theta = \frac{-117}{125}$

23. यदि $\sin\frac{\theta}{3} = \frac{4}{5}$ भए $\sin\theta$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\theta}{3} = \frac{4}{5}$, find the value of $\sin\theta$.

[2073 R', 2057 S, 2065 E]

⇨ Here, given, $\sin\frac{\theta}{3} = \frac{4}{5}$, $\sin\theta = ?$

We know that, $\sin A = 3\sin\frac{A}{3} - 4\sin^{3}\frac{A}{3}$

So, $\sin\theta = 3\sin\frac{\theta}{3} - 4\sin^3\frac{\theta}{3}$ $= 3.\frac{4}{5} - 4\left(\frac{4}{5}\right)^3$

$= \frac{12}{5} - \frac{4\times64}{125}$ $= \frac{12\times25-256}{125}$

$= \frac{300-256}{125}$ $= \frac{44}{125}$

Thus, value of $\sin\theta = \frac{44}{125}$.

24. यदि $\sin\frac{\alpha}{3} = \frac{3}{5}$ भए $\sin\alpha$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\alpha}{3} = \frac{3}{5}$, find the value of $\sin\alpha$.

[2074 R, 2073 S, 2060 R, 2066 R']

⇨ Here, $\sin\frac{\alpha}{3} = \frac{3}{5}$

Now, $\sin\alpha = 3\sin\frac{\alpha}{3} - 4\sin^3\frac{\alpha}{3}$

$= 3\times\frac{3}{5} - 4\times\left(\frac{3}{5}\right)^3$

$= \frac{9}{5} - \frac{108}{125}$

$= \frac{117}{125}$

Thus, the value of $\sin\alpha$ is $\frac{117}{125}$.

25. यदि $\sin\frac{\theta}{3} = \frac{1}{2}$ भए $\sin\theta$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\theta}{3} = \frac{1}{2}$, find the value of $\sin\theta$.

[2061 S, 2065 S, 2066 R]

⇨ Here, given $\sin\frac{\theta}{3} = \frac{1}{2}$ $\sin\theta = ?$

Using formula, $\sin\theta = 3\sin\frac{\theta}{3} - 4\sin^3\frac{\theta}{3}$

So, $\sin\theta = 3.\frac{1}{2} - 4.\left(\frac{1}{2}\right)^3$ $= \frac{3}{2} - \frac{4}{8}$

$= \frac{3}{2} - \frac{1}{2}$ $= \frac{3-1}{2}$

$= \frac{2}{2}$ $= 1$

Thus, $\sin\theta = 1$.

MODEL 5

26. प्रमाणित गर्नुहोस् (Prove that):

$\sin^4 B + \cos^4 B = 1 - \frac{1}{2}\sin^2 2B$ **[2075 R, 2075 R']**

⇨ Here, LHS $= \sin^4 B + \cos^4 B$

$= (\sin^2 B)^2 + (\cos^2 B)^2$

$= (\sin^2 B + \cos^2 B)^2 - 2\sin^2 B\cos^2 B$

$= 1 - 2\sin^2 B\cos^2 B$

$= 1 - \frac{1}{2}(2\sin B\cos B)^2$

$= 1 - \frac{1}{2}\sin^2 2B = \text{RHS}$ **Proved.**

27. प्रमाणित गर्नुहोस् (Prove that):

$\frac{1+\cos A+\cos 2A}{\sin A+\sin 2A} = \cot A$ **[2075 R_2']**

⇨ Here,

LHS $= \frac{1+\cos A+\cos 2A}{\sin A+\sin 2A}$

$= \frac{1+\cos A+2\cos^2 A-1}{\sin A+2\sin A . \cos A}$

$= \frac{\cos A(1+2\cos A)}{\sin A(1+2\cos A)} = \frac{\cos A}{\sin A} = \cot A = \text{RHS}.$

Proved.

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28. प्रमाणित गर्नुहोस् (Prove that):

$\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$ [2070 R']

⇨ Here,

$$\text{LHS} = \operatorname{cosec} 2\theta - \cot 2\theta$$
$$= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$$
$$= \frac{1-\cos 2\theta}{\sin 2\theta}$$
$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta = \text{RHS}$$

Proved.

29. प्रमाणित गर्नुहोस् (Prove that):

$\sin^2(45° - \theta) + \sin^2(45° + \theta) = 1$ [2060C, 2063R]

⇨ Here, $\text{LHS} = \sin^2(45° + \theta) + \sin^2(45° - \theta)$

$$= (\sin 45°.\cos\theta + \cos 45°.\sin\theta)^2 + (\sin 45°.\cos\theta - \cos 45°\sin\theta)^2$$
$$= (\sin 45°\cos\theta)^2 + 2\sin 45°\cos\theta\cos 45°\sin\theta + (\cos 45°\sin\theta)^2 + (\sin 45°\cos\theta)^2 - 2.\sin 45°\cos\theta\cos 45°\sin\theta + (\cos 45°\sin\theta)^2$$
$$= \sin^2 45°\cos^2\theta + \cos^2 45°\sin^2\theta + \sin^2 45°.\cos^2\theta + \cos^2 45°\sin^2\theta$$
$$= 2\sin^2 45°.\cos^2\theta + 2\cos^2 45°.\sin^2\theta$$
$$= 2.\left(\frac{1}{\sqrt{2}}\right)^2\cos^2\theta + 2\left(\frac{1}{\sqrt{2}}\right)^2\sin^2\theta$$
$$= 2.\frac{1}{2}\cos^2\theta + 2.\frac{1}{2}\sin^2\theta$$
$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

Thus, LHS = RHS **Proved.**

30. प्रमाणित गर्नुहोस् (Prove that):

$1 - \sin 2A = 2\sin^2(45° - A)$ [2063M, 2065R]

⇨ Here, $\text{RHS} = 2\sin^2(45° - A)$

$$= 1 - \cos 2(45° - A)$$
$$= 1 - \cos(90° - 2A)$$
$$= 1 - \sin 2A$$
$$= \text{L.H.S}$$

Thus, RHS = LHS completes the proof.

31. प्रमाणित गर्नुहोस् (Prove that):

$1 + \cos 2A = 2\sin^2(90° - A)$ [2064 R']

⇨ Here,

$$\text{RHS} = 2\sin^2(90° - A)$$
$$= 1 - \cos 2(90° - A)\ [\because 1 - \cos 2A = 2\sin^2 A]$$
$$= 1 - \cos(180° - 2A)$$
$$= 1 + \cos 2A = \text{LHS}$$

Proved.

32. प्रमाणित गर्नुहोस् (Prove that):

$2\cos^2(45° - \theta) = 1 + \sin 2\theta$ [2066 R']

⇨ Here,

$$\text{LHS} = 2\cos^2(45° - \theta)$$
$$= 1 + \cos 2(45° - \theta)\ [\because 2\cos^2 A = 1 + \cos 2A]$$
$$= 1 + \cos(90° - 2\theta)$$
$$= 1 + \sin 2\theta = \text{RHS}$$

Proved.

33. प्रमाणित गर्नुहोस् (Prove that):

$\dfrac{1 + \tan^2(45° - \beta)}{1 - \tan^2(45° - \beta)} = \operatorname{cosec} 2\beta$ [2067 R']

⇨ Here,

$$\text{LHS} = \frac{1 + \tan^2(45° - \beta)}{1 - \tan^2(45° - \beta)}\ \left[\because \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}\right]$$
$$= \sec 2(45° - \beta)$$
$$= \sec(90° - 2\beta)$$
$$= \operatorname{cosec} 2\beta$$

$\therefore$ LHS = RHS **Proved.**

34. प्रमाणित गर्नुहोस् (Prove that):

$\dfrac{\sin 2A - \sin A}{1 - \cos A + \cos 2A} = \tan A$ [2064 S]

⇨
$$\text{LHS} = \frac{\sin 2A - \sin A}{1 - \cos A + \cos 2A}$$
$$= \frac{2\sin A\cos A - \sin A}{1 - \cos A + 2\cos^2 A - 1}$$
$$= \frac{\sin A(2\cos A - 1)}{2\cos^2 A - \cos A}$$
$$= \frac{\sin A(2\cos A - 1)}{\cos A(2\cos A - 1)} = \frac{\sin A}{\cos A}$$
$$= \tan A\ \text{RHS}$$

Proved.

35. प्रमाणित गर्नुहोस् (Prove that):

$\dfrac{\sin\alpha + \sin 2\alpha}{1 + \cos\alpha + \cos 2\alpha} = \tan\alpha$ [2072 S, 2066 S, 2060 S']

⇨ Here,
$$\text{LHS} = \frac{\sin\alpha + \sin 2\alpha}{1 + \cos\alpha + \cos 2\alpha}$$
$$= \frac{\sin\alpha + 2\sin\alpha\cos\alpha}{1 + \cos\alpha + 2\cos^2\alpha - 1}$$
$$= \frac{\sin\alpha(1 + 2\cos\alpha)}{\cos\alpha(1 + 2\cos\alpha)}$$
$$= \tan\alpha$$
$$= \text{RHS}$$

Proved.

36. प्रमाणित गर्नुहोस् (Prove that):

$\sin A.\cos 2A = \dfrac{1}{4}\sin 4A.\sec A$ [2065 S]

⇨ Here, $\text{LHS} = \sin A\cos 2A$

$$= \frac{2\sin A\cos A\cos 2A}{2\cos A}$$
$$= \frac{\sin 2A\cos 2A}{2\cos A}$$
$$= \frac{2\sin 2A\cos 2A}{4\cos A}$$
$$= \frac{1}{4}\sin 4A\sec A = \text{RHS}$$

$\therefore$ LHS = RHS **Proved.**

37. प्रमाणित गर्नुहोस् (Prove that):

$\dfrac{1 - \cos 2A}{\sin 2A} = \tan A$ [2068 R']

⇨ Here,

$$\text{LHS} = \frac{1 - \cos 2A}{\sin 2A}$$
$$= \frac{2\sin^2 A}{2\sin A.\cos A}\ \left[\begin{array}{l}\because 1 - \cos 2A = 2\sin^2 A\\ \sin 2A = 2\sin A.\cos A\end{array}\right]$$
$$= \frac{\sin A}{\cos A} = \tan A$$

$\therefore$ LHS = RHS **Proved.**

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MODEL 6

38. यदि $\sin\frac{A}{3} = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए $\sin A = -\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् । **[2075 R]**

If $\sin\frac{A}{3} = \frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that: $\sin A = -\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

⇨ Here, $\sin\frac{A}{3} = \frac{1}{2}\left(a+\frac{1}{a}\right)$

Now,

$$\begin{aligned}
\text{LHS} &= \sin A \\
&= 3\sin\frac{A}{3} - 4\sin^3\frac{A}{3} \\
&= 3\times\frac{1}{2}\left(a+\frac{1}{a}\right) - 4\times\left[\frac{1}{2}\left(a+\frac{1}{a}\right)\right]^3 \\
&= \frac{3}{2}\left(a+\frac{1}{a}\right) - 4\cdot\frac{1}{8}\left(a+\frac{1}{a}\right)^3 \\
&= \frac{1}{2}\left(a+\frac{1}{a}\right)\left[3-\left(a+\frac{1}{a}\right)^2\right] \\
&= \frac{1}{2}\left(a+\frac{1}{a}\right)\left(3-a^2-2-\frac{1}{a^2}\right) \\
&= \frac{1}{2}\left(a+\frac{1}{a}\right)\left(-a^2+1-\frac{1}{a^2}\right) \\
&= -\frac{1}{2}\left(a+\frac{1}{a}\right)\left(a^2-1+\frac{1}{a^2}\right) \\
&= -\frac{1}{2}\left(a^3+\frac{1}{a^3}\right) = \text{RHS}
\end{aligned}$$

Proved.

40. प्रमाणित गुर्नहोस् (Prove that):

$\frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} = \cot\frac{\theta}{2}$[2075 R', 2074 S', 2071 R', 2069

⇨ Here,

$$\begin{aligned}
\text{LHS} &= \frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} \\
&= \frac{1+2\cos^2\frac{\theta}{2}-1+2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{1-1+2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \\
&= \frac{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]} \\
&= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2} = \text{RHS}
\end{aligned}$$

Proved.

39. यदि $\cos\frac{B}{3} = \frac{1}{2}\left(m+\frac{1}{m}\right)$ भए प्रमाणित गर्नुहोस् :

If $\cos\frac{B}{3} = \frac{1}{2}\left(m+\frac{1}{m}\right)$ then, prove that:

$\cos B = \frac{1}{2}\left(m^3+\frac{1}{m^3}\right)$ **[SEE 2075 R_2]**

⇨ Here, $\cos\frac{B}{3} = \frac{1}{2}\left(m+\frac{1}{m}\right)$

Now,

$$\begin{aligned}
\text{LHS} &= \cos B \\
&= 4\cos^3\frac{B}{3} - 3\cos\frac{B}{3} \\
&= 4\left\{\frac{1}{2}\left(m+\frac{1}{m}\right)\right\}^3 - 3\cdot\frac{1}{2}\left(m+\frac{1}{m}\right) \\
&= \frac{4}{8}\left(m+\frac{1}{m}\right)^3 - \frac{3}{2}\left(m+\frac{1}{m}\right) \\
&= \frac{1}{2}\left(m+\frac{1}{m}\right)\left[\left(m+\frac{1}{m}\right)^2-3\right] \\
&= \frac{1}{2}\left(m+\frac{1}{m}\right)\left[m^2+2+\frac{1}{m^2}-3\right] \\
&= \frac{1}{2}\left(m+\frac{1}{m}\right)\left(m^2-1+\frac{1}{m^2}\right) \\
&= \frac{1}{2}\left(m^3+\frac{1}{m^3}\right) = \text{RHS}
\end{aligned}$$

Proved.

41. प्रमाणित गर्नुहोस् (Prove that):

$\frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$ **[2074 S']**

⇨ Here,

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta}{1-\cos\theta} \\
&= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} \\
&= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \\
&= \cot\frac{\theta}{2} = \text{RHS}
\end{aligned}$$

Proved.

42. यदि $\sin\frac{\theta}{2} = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए प्रमाणित गर्नुहोस् : $\cos\theta = \frac{-1}{2}\left(a^2+\frac{1}{a^2}\right)$ [If $\sin\frac{\theta}{2} = \frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that: $\cos\theta = \frac{-1}{2}\left(a^2+\frac{1}{a^2}\right)$] **[2074 R']**

⇨ Here, $\sin\frac{\theta}{2} = \frac{1}{2}\left(a+\frac{1}{a}\right)$

$$\begin{aligned}
\text{So, LHS} = \cos\theta &= 1-2\sin^2\frac{\theta}{2} &&= 1-2\left[\frac{1}{2}\left(a+\frac{1}{a}\right)\right]^2 \\
&= 1-2\times\frac{1}{4}\left(a+\frac{1}{a}\right)^2 &&= 1-\frac{1}{2}\left(a^2+2.\ a.\frac{1}{a}+\frac{1}{a^2}\right) \\
&= 1-\frac{1}{2}\left(a^2+\frac{1}{a^2}+2\right) &&= 1-\frac{1}{2}\left(a^2+\frac{1}{a^2}\right)-2\times\frac{1}{2} \\
&= -\frac{1}{2}\left(a^2+\frac{1}{a^2}\right) &&= \text{RHS}
\end{aligned}$$

Thus, LHS = RHS completes the proof.

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43. प्रमाणित गर्नुहोस् (Prove that):

$\cot\frac{A}{2} - \tan\frac{A}{2} = 2\cot A$

[2071 R]

⇨ Here, LHS

$= \cot\frac{A}{2} - \tan\frac{A}{2} = \cot\frac{A}{2} - \frac{1}{\cot\frac{A}{2}}$

$= \frac{\cot^2\frac{A}{2} - 1}{\cot\frac{A}{2}} = \frac{\frac{\cos^2\frac{A}{2}}{\sin^2\frac{A}{2}} - 1}{\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}}$

$= \frac{\left(\cos^2\frac{A}{2} - \sin^2\frac{A}{2}\right)}{\sin^2\frac{A}{2}} \times \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} =$

$\frac{\cos A \times 2}{2 \times \sin\frac{A}{2} . \cos\frac{A}{2}} = \frac{2\cos A}{\sin A}$

$= 2\cot A = \text{RHS}$ **Proved.**

44. प्रमाणित गर्नुहोस् (Prove that):

$\text{cosec}\,\theta - \cot\theta = \tan\frac{\theta}{2}$

[2071 S]

⇨ Here, LHS $= \text{cosec}\,\theta - \cot\theta$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1 - \cos\theta}{\sin\theta}$

$= \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \tan\frac{\theta}{2}$

$= \text{RHS}$ **Proved.**

45. $\frac{\sin A}{1 + \cos A}$ **लाई tangent अनुपातको अपवर्तकको रूपमा व्यक्त गर्नुहोस् ।**

Express $\frac{\sin A}{1 + \cos A}$ in terms of sub-multiple angle of tangent.

[2076 Model]

⇨ Here, $\frac{\sin A}{1 + \cos A}$

$= \frac{2\sin\frac{A}{2} . \cos\frac{A}{2}}{1 + 2\cos^2\frac{A}{2} - 1}$

$= \frac{2\sin\frac{A}{2} . \cos\frac{A}{2}}{2\cos^2\frac{A}{2}}$

$= \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$

$= \tan\frac{A}{2}$

Thus, the required term is $\tan\frac{A}{2}$

46. प्रमाणित गर्नुहोस् (Prove that):

$\frac{2\sin\beta + \sin 2\beta}{2\sin\beta - \sin 2\beta} = \cot^2\frac{\beta}{2}$

[2059 R]

⇨ Here, LHS $= \frac{2\sin\beta + \sin 2\beta}{2\sin\beta - \sin 2\beta}$

$= \frac{2\sin\beta + 2\sin\beta\cos\beta}{2\sin\beta - 2\sin\beta\cos\beta}$

$= \frac{2\sin\beta\,(1 + \cos\beta)}{2\sin\beta\,(1 - \cos\beta)}$

$= \frac{2\cos^2\frac{\beta}{2}}{2\sin^2\frac{\beta}{2}}$

$= \cot^2\frac{\beta}{2}$

$= \text{RHS}$ **Proved.**

47. प्रमाणित गर्नुहोस् (Prove that):

$\cot A = \frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$

[2060 R]

⇨ Here, RHS $= \frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$

$= \frac{1}{2}\left(\frac{1}{\tan\frac{A}{2}} - \tan\frac{A}{2}\right)$

$= \frac{1}{2}\left(\frac{1 - \tan^2\frac{A}{2}}{\tan\frac{A}{2}}\right)$

$= \frac{1 - \tan^2\frac{A}{2}}{2\tan\frac{A}{2}} = \frac{1}{\frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}} = \frac{1}{\tan A}$

$= \cot A = \text{LHS}$ **Proved.**

48. प्रमाणित गर्नुहोस् (Prove that):

$\frac{\cos\alpha}{1 - \sin\alpha} = \frac{1 + \tan\frac{\alpha}{2}}{1 - \tan\frac{\alpha}{2}}$

[2059 S]

⇨ Here,

LHS $= \frac{\cos\alpha}{1 - \sin\alpha} = \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$= \frac{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)}{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)^2}$

$= \frac{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}} = \frac{\frac{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{\frac{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}$

$= \frac{\frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{\frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} - \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}$

$= \frac{1 + \tan\frac{\alpha}{2}}{1 - \tan\frac{\alpha}{2}} = \text{RHS}$ **Proved.**

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49. प्रमाणित गर्नुहोस् (Prove that):

$$\cos^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)-\sin^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)=\sin\frac{\theta}{2}$$ **[2061R]**

⇨ Here,

$$\text{LHS} = \cos^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)-\sin^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)$$

$$= 1-\sin^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)-\sin^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right) \quad [\because \cos^2 = 1-\sin^2\theta]$$

$$= 1-2\sin^2\left(\frac{\pi-\theta}{4}\right)$$

$$= \cos 2\left(\frac{\pi-\theta}{4}\right) \quad [\because 1-2\sin^2\theta = \cos 2\theta]$$

$$= \cos\left(\frac{\pi-\theta}{2}\right)$$

$$= \cos\left(\frac{\pi}{2}-\frac{\theta}{2}\right)$$

$$= \sin\frac{\theta}{2} = \text{RHS}$$

Proved.

50. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{1-\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)}{1+\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)}=\sin\frac{\theta}{2}$$ **[2064 R]**

⇨ Here, LHS

$$= \frac{1-\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)}{1+\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)}$$

$$= \cos 2\left(\frac{\pi^c}{4}-\frac{\theta}{4}\right)$$

$$= \cos\left(\frac{\pi^c}{2}-\frac{\theta}{2}\right)$$

$$= \cos\left(90^\circ-\frac{\theta}{2}\right)$$

$$= \sin\frac{\theta}{2}$$

$$= \text{RHS}$$

Proved.

51. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\sin\frac{\theta}{2}-\sqrt{1+\sin\theta}}{\cos\frac{\theta}{2}-\sqrt{1+\sin\theta}}=\cot\frac{\theta}{2}$$ **[2063 R']**

⇨ Here,

$$\text{LHS} = \frac{\sin\frac{\theta}{2}-\sqrt{1+\sin\theta}}{\cos\frac{\theta}{2}-\sqrt{1+\sin\theta}}$$

$$= \frac{\sin\frac{\theta}{2}-\sqrt{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}{\cos\frac{\theta}{2}-\sqrt{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}$$

$$= \frac{\sin\frac{\theta}{2}-\sqrt{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}{\cos\frac{\theta}{2}-\sqrt{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}$$

$$= \frac{\sin\frac{\theta}{2}-\sqrt{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}}{\cos\frac{\theta}{2}-\sqrt{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}}$$

$$= \frac{\sin\frac{\theta}{2}-\sin\frac{\theta}{2}-\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}-\cos\frac{\theta}{2}}$$

$$= \frac{-\cos\frac{\theta}{2}}{-\sin\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2} = \text{RHS}$$

Proved.

52. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\sin\frac{A}{2}+\sin A}{1+\cos\frac{A}{2}+\cos A}=\tan\frac{A}{2}$$ **[2072 R', 2065 M]**

⇨ Here, LHS

$$= \frac{\sin\frac{A}{2}+\sin A}{1+\cos\frac{A}{2}+\cos A}$$

$$= \frac{\sin\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}{1+\cos\frac{A}{2}+2\cos^2\frac{A}{2}-1}$$

$$= \frac{\sin\frac{A}{2}\left(1+2\cos\frac{A}{2}\right)}{\cos\frac{A}{2}\left(1+2\cos\frac{A}{2}\right)} = \tan\frac{A}{2}$$

$$= \text{RHS}$$

Proved.

53. प्रमाणित गर्नुहोस् (Prove that):

$$\tan\frac{\theta}{2}=\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$$ **[2073 R', 2072 R, 2072 S, 2067 S]**

⇨ Here, RHS

$$= \frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$$

$$= \frac{(1-\cos\theta)+\sin\theta}{(1+\cos\theta)+\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2} = \text{LHS}$$

Proved.

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54. प्रमाणित गर्नुहोस् (Prove that): $\frac{1 + \cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$ [2065 R]

⇨ Here, LHS $= \frac{1+\cos\theta}{\sin\theta} = \frac{1 + \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}{\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}} = \frac{\frac{1+\tan^2\frac{\theta}{2}+1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}{\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}} = \frac{2}{2\tan\frac{\theta}{2}} = \frac{1}{\tan\frac{\theta}{2}} = \cot\frac{\theta}{2} = \text{RHS}$ **Proved.**

C. LONG QUESTIONS

MODEL 1

1. प्रमाणित गर्नुहोस् (Prove that):
cosec 2A + cosec 4A = cot A – cot 4A[2074 R, 2074 S]

⇨ Here,

$$
\begin{aligned}
\text{LHS} &= \text{cosec } 2A + \text{cosec } 4A\\
&= \cot A - \cot A + \text{cosec } 2A + \text{cosec } 4A\\
&= \cot A - \frac{\cos A}{\sin A} + \frac{1}{\sin 2A} + \text{cosec } 4A\\
&= \cot A - \left(\frac{\cos A}{\sin A} - \frac{1}{2\sin A\cos A}\right) + \text{cosec } 4\,A\\
&= \cot A - \frac{2\cos^2 A - 1}{2\sin A\cos A} + \text{cosec } 4A\\
&= \cot A - \frac{\cos 2A}{\sin 2A} + \text{cosec } 4A\\
&= \cot A - \frac{\cos 2A}{\sin 2A} + \frac{1}{\sin 4A}\\
&= \cot A - \left(\frac{\cos 2A}{\sin 2A} - \frac{1}{2\sin 2A\cos 2A}\right)\\
&= \cot A - \frac{2\cos^2 2A - 1}{2\sin 2A\cos 2A}\\
&= \cot A - \frac{\cos 4A}{\sin 4A}\\
&= \cot A - \cot 4A \;= \text{RHS}
\end{aligned}
$$

Thus, LHS = RHS completes the proof.

2. प्रमाणित गर्नुहोस् (Prove that) :
$8(\sin^6 p + \cos^6 p) = 5 + 3\cos 4p$ [2073 R']

⇨ Here,

$$
\begin{aligned}
\text{LHS} &= 8(\sin^6 p + \cos^6 p)\\
&= 8[(\cos^2 p)^3 + (\sin^2 p)^3]\\
&= 8[(\cos^2 p + \sin^2 p)^3 - 3\cos^2 p\sin^2 p(\cos^2 p + \sin^2 p)]\\
&= 8[1 - 3\cos^2 p\sin^2 p \times 1]\\
&= 8 - 6(2\sin p\cos p)^2\\
&= 8 - 6(\sin 2p)^2]\\
&= 8 - 6\sin^2 2p\\
&= 8 - 3(2\sin^2 2p)\\
&= 8 - 3\,[1 - \cos 4p]\\
&= 8 - 3 + 3\cos 4p\\
&= 5 + 3\cos 4p \;= \text{RHS}
\end{aligned}
$$

Proved.

3. प्रमाणित गर्नुहोस् (Prove that):
$\sin^2 A - \cos^2 A.\cos 2B = \sin^2 B - \cos^2 B.\cos 2A$ [2074 R']

⇨ Here,

$$
\begin{aligned}
\text{LHS} &= \sin^2 A - \cos^2 A\cos 2B\\
&= \sin^2 A - \cos^2 A(1 - 2\sin^2 B)\\
&= \sin^2 A - \cos^2 A + 2\cos^2 A\sin^2 B\\
&= -(\cos^2 A - \sin^2 A) + (1 + \cos 2A)\sin^2 B\\
&= -\cos 2A + \sin^2 B + \cos 2A\sin^2 B\\
&= \sin^2 B - \cos 2A + \cos 2A\sin^2 B\\
&= \sin^2 B - \cos 2A(1 - \sin^2 B)\\
&= \sin^2 B - \cos 2A\,.\cos^2 B\\
&= \sin^2 B - \cos^2 B\cos 2A = \text{RHS}
\end{aligned}
$$

Thus, LHS = RHS completes the proof.

4. प्रमाणित गर्नुहोस् (Prove that):
$\frac{\sec 4\theta - 1}{\sec 2\theta - 1} = \tan 4\theta.\cot\theta$ [2073 R, 2073 S, 2073 S']

⇨ Here, LHS $= \frac{\sec 4\theta - 1}{\sec 2\theta - 1}$

$$
\begin{aligned}
&= \frac{\frac{1}{\cos 4\theta} - 1}{\frac{1}{\cos 2\theta} - 1}\\
&= \frac{1 - \cos 4\theta}{\cos 4\theta} \times \frac{\cos 2\theta}{1 - \cos 2\theta}\\
&= \frac{2\sin^2 2\theta \times \cos 2\theta}{\cos 4\theta \times 2\sin^2\theta}\\
&= \frac{2\sin 2\theta.\cos 2\theta}{\cos 4\theta}.\frac{\sin 2\theta}{2\sin^2\theta}\\
&= \frac{\sin 4\theta}{\cos 4\theta}.\frac{2\sin\theta\cos\theta}{2\sin^2\theta}\\
&= \tan 4\theta.\frac{\cos\theta}{\sin\theta}\\
&= \tan 4\theta.\cot\theta \;= \text{RHS}
\end{aligned}
$$

Proved.

5. प्रमाणित गर्नुहोस् (Prove that): $\cos^3\theta.\cos 3\theta + \sin^3\theta.\sin 3\theta = \cos^3 2\theta$ [2071 R, 2071 S, 2070 S']

⇨ Here,

LHS $= \cos^3\theta.\cos 3\theta + \sin^3\theta\sin 3\theta$

We know that,

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

or, $4\cos^3\theta = \cos 3\theta + 3\cos\theta$

∴ $\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$ and

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

or, $4\sin^3\theta = 3\sin\theta - \sin 3\theta$

∴ $\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

Now, LHS

$$
\begin{aligned}
&= \frac{1}{4}(\cos 3\theta + 3\cos\theta)\cos 3\theta + \frac{1}{4}(3\sin\theta - \sin 3\theta)\sin 3\theta\\
&= \frac{1}{4}[\cos^2 3\theta + 3\cos\theta\cos 3\theta + 3\sin\theta\sin 3\theta - \sin^2 3\theta)\\
&= \frac{1}{4}[\cos^2 3\theta - \sin^2 3\theta + 3(\cos\theta\cos 3\theta + \sin\theta\sin 3\theta)]\\
&= \frac{1}{4}[\cos 2\times 3\theta + 3\cos(\theta - 3\theta)] = \frac{1}{4}[\cos 6\theta + 3\cos 2\theta]\\
&= \frac{1}{4}[\cos 3\times 2\theta + 3\cos 2\theta]\\
&= \frac{1}{4}[4\cos^3 2\theta - 3\cos 2\theta + 3\cos 2\theta]\\
&= \frac{1}{4}\times 4\cos^3 2\theta = \cos^3 2\theta = \text{RHS}
\end{aligned}
$$

Proved.

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6. **प्रमाणित गर्नुहोस् (Prove that):**

$\mathbf{\tan\theta + 2\tan 2\theta + 4\cot 4\theta = \cot\theta}$ **[2072 R]**

⇨ Here, LHS $= \tan\theta + 2\tan 2\theta + 4\cot 4\theta$

$$= \tan\theta + 2\tan 2\theta + \frac{4}{\tan 4\theta}$$

$$= \tan\theta + 2\tan 2\,\theta + \frac{4(1-\tan^2 2\theta)}{2\tan 2\theta}$$

$$= \tan\theta + \frac{2\tan^2 2\theta + 2 - 2\tan^2 2\theta}{\tan 2\theta}$$

$$= \tan\theta + \frac{2}{\tan 2\theta}$$

$$= \tan\theta + \frac{2}{\frac{2\tan\theta}{1-\tan^2\theta}}$$

$$= \tan\theta + \frac{1-\tan^2\theta}{\tan\theta}$$

$$= \frac{\tan^2\theta + 1 - \tan^2\theta}{\tan\theta}$$

$$= \frac{1}{\tan\theta}$$

$= \cot\theta =$ RHS **Proved.**

8. **प्रमाणित गर्नुहोस् (Prove that):**

$\mathbf{\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = 4}$ **OR** $\mathbf{\sqrt{3}\operatorname{cosec} 20° - \sec 20° = 4}$

[2061 R, 2065 E, 2065 S, 2067 R]

⇨ Here, LHS

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$$

$$= \frac{\tan 60°.\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$$

$$= \frac{\frac{\sin 60°}{\cos 60°}.\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$$

$$= \frac{\sin 60°.\cos 20° - \cos 60°.\sin 20°}{\sin 20°.\cos 20°.\cos 60°}$$

$$= \frac{\sin(60° - 20°)}{\sin 20°.\cos 20° \times \frac{1}{2}}$$

$$= \frac{2\sin 40°}{\frac{1}{2}.2\sin 20°.\cos 20°} = \frac{4\sin 40°}{\sin 2.20°} = \frac{4\sin 40°}{\sin 40°}$$

$= 4 =$ RHS **Proved.**

9. $\mathbf{\cos^8\theta + \sin^8\theta = 1 - \sin^2 2\theta + \frac{1}{8}\sin^4 2\theta}$ **[2064 R]**

⇨ Here, LHS

$$= \sin^8\theta + \cos^8\theta$$

$$= (\sin^4\theta)^2 + (\cos^4\theta)^2$$

$$= (\sin^4\theta - \cos^4\theta)^2 + 2\sin^4\theta.\cos^4\theta$$

$$=[(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)]^2 + \frac{2}{16}(16\sin^4\theta\cos^4\theta)$$

$$= 1(\sin^2\theta - \cos^2\theta)^2 + \frac{2}{16}(2\sin\theta\cos\theta)^4$$

$$= (\cos^2\theta - \sin^2\theta)^2 + \frac{2}{16}(\sin 2\theta)^4$$

$$= (\cos 2\theta)^2 + \frac{1}{8}\sin^4 2\theta = \cos^2 2\theta + \frac{1}{8}\sin^4 2\theta$$

$= 1 - \sin^2 2\theta + \frac{1}{8}\sin^4 2\theta =$ RHS **Proved.**

7. **प्रमाणित गर्नुहोस् (Prove that):**

$\mathbf{\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = 4}$ **OR** $\mathbf{\operatorname{cosec} 10° - \sqrt{3}\sec 10° = 4}$**[2**

⇨ Here, LHS $= \frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$

$$= \frac{1}{\sin 10°} - \frac{\tan 60°}{\cos 10°}\ (\because \tan 60° = \sqrt{3})$$

$$= \frac{1}{\sin 10°} - \frac{\sin 60°}{\cos 60°.\cos 10°}$$

$$= \frac{\cos 60°.\cos 10° - \sin 60°.\sin 10°}{\sin 10°.\cos 10°.\cos 60°}$$

$$= \frac{\cos(60° + 10°)}{\sin 10°.\cos 10° \times \frac{1}{2}}$$

$$= \frac{2\cos 70°}{2\sin 10°.\cos 10° \times \frac{1}{2}}$$

$$= \frac{2 \times 2\cos(90° - 20°)}{\sin 2.10°}$$

$$= \frac{4\sin 20°}{\sin 20°}$$

$= 4 =$ RHS **Proved.**

10. **प्रमाणित गर्नुहोस् (Prove that):**

$\mathbf{\cos^6\theta - \sin^6\theta = \cos 2\theta\left(1 - \frac{1}{4}\sin^2 2\theta\right)}$**[2058 S, 2062R]**

⇨ Here,

LHS

$$= \cos^6\theta - \sin^6\theta$$

$$= (\cos^2\theta)^3 - (\sin^2\theta)^3$$

$$= (\cos^2\theta - \sin^2\theta)(\cos^4\theta + \cos^2\theta.\sin^2\theta + \sin^4\theta)$$

$$= \cos 2\theta(\cos^4\theta + 2\cos^2\theta.\sin^2\theta + \sin^4\theta - \cos^2\theta.\sin^2\theta)$$

$$= \cos 2\theta[(\cos^2\theta + \sin^2\theta)^2 - \frac{4}{4}\sin^2\theta.\cos^2\theta]$$

$$= \cos 2\theta[(1^2 - \frac{1}{4}(2\sin\theta.\cos\theta)^2]$$

$= \cos 2\theta(1 - \frac{1}{4}\sin^2 2\theta) =$ RHS **Proved.**

11. **प्रमाणित गर्नुहोस् (Prove that):**

$\mathbf{\cos^6\theta + \sin^6\theta = \frac{1}{8}(5 + 3\cos 4\theta)}$ **[2063 R']**

⇨ Here,

LHS

$$= \cos^6\theta + \sin^6\theta$$

$$= (\cos^2\theta)^3 + (\sin^2\theta)^3$$

$$= (\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)$$

$$= 1 - 3\cos^2\theta\sin^2\theta \times 1$$

$$= 1 - \frac{3}{4}(2\sin\theta\cos\theta)^2$$

$$= 1 - \frac{3}{4}(\sin 2\theta)^2$$

$$= 1 - \frac{3}{4}\sin^2 2\theta$$

$$= 1 - \frac{3}{8}(2\sin^2 2\theta)$$

$$= 1 - \frac{3}{8}[1 - \cos 4\theta]$$

$$= 1 - \frac{3}{8} + \frac{3}{8}\cos 4\theta$$

$= \frac{5}{8} + \frac{3}{8}\cos 4\theta = \frac{1}{8}(5 + 3\cos 4\theta) =$ RHS **Proved.**

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12. प्रमाणित गर्नुहोस् (Prove that):

$\frac{1}{\sin 2A} + \frac{\cos 4A}{\sin 4A} = \cot A - \text{cosec } 4A$ [2060CP]

⇨ Here,

$$\begin{aligned}\text{LHS} &= \frac{1}{\sin 2A} + \frac{\cos 4A}{\sin 4A}\\ &= \frac{1}{\sin 2A} + \frac{\cos 4A}{2\sin 2A \cos 2A}\\ &= \frac{2\cos 2A + \cos 4A}{2\sin 2A \cos 2A}\\ &= \frac{2\cos 2A + 2\cos^2 2A - 1}{2\sin 2A \cos 2A}\\ &= \frac{2\cos 2A\,(1 + \cos 2A) - 1}{2\sin 2A \cos 2A}\\ &= \frac{2\cos 2A(1 + \cos 2A)}{2\sin 2A \cos 2A} - \frac{1}{2\sin 2A.\cos 2A}\\ &= \frac{2\cos^2 A}{2\sin A.\cos A} - \frac{1}{\sin 4A}\\ &= \cot A - \text{cosec } 4A = \text{RHS}\end{aligned}$$

Thus, LHS = RHS **Proved.**

14. प्रमाणित गर्नुहोस् (Prove that):

$(1 + \sin 2A + \cos 2A)^2 = 4\cos^2 A\,(1 + \sin 2A)$ [2064 S]

⇨ Here, LHS

$= (1 + \sin 2A + \cos 2A)^2$

$= (1 + \sin 2A)^2 + 2(1 + \sin 2A)\cos 2A + (\cos 2A)^2$

$= (1 + \sin 2A)\,(1 + \sin 2A + 2\cos 2A) + (\cos^2 2A)$

$= (1 + \sin 2A)\,(1 + \sin 2A + 2\cos 2A) + 1 - \sin^2 2A$

$= (1+\sin 2A)\,(1+\sin 2A+2\cos 2A)+(1+\sin 2A)\,(1-\sin 2A)$

$= (1 + \sin 2A)\,[(1 + \sin 2A + 2\cos 2A + 1 - \sin 2A)]$

$= (1 + \sin 2A)\,(2 + 2\cos 2A)$

$= (1 + \sin 2A)\,2(1 + \cos 2A)$

$= (1 + \sin 2A)\,2 \times 2\cos^2 4A$

$= 4\cos^2 4A\,(1 + \sin 2A) = \text{RHS}$ **Proved.**

16. यदि 2tan α = 3 tan β भए $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If 2tan α = 3 tan β, prove that: $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$

[2065 R]

⇨ Here, $2\tan\alpha = 3\tan\beta$ or, $\frac{\tan\alpha}{\tan\beta} = \frac{3}{2}$

∴ $\tan\alpha = 3k$ and $\tan\beta = 2k$

$$\text{LHS} = \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= \frac{3k - 2k}{1 + 3k \times 2k} = \frac{k}{1 + 6k^2}$$

$$\text{RHS} = \frac{\sin 2\beta}{5 - \cos 2\beta} = \frac{\frac{2\tan\beta}{1 + \tan^2\beta}}{5 - \frac{1 - \tan^2\beta}{1 + \tan^2\beta}}$$

$$= \frac{\frac{2\tan\beta}{1 + \tan^2\beta}}{\frac{5 + 5\tan^2\beta - 1 + \tan^2\beta}{1 + \tan^2\beta}}$$

$$= \frac{2\tan\beta}{4 + 6\tan^2\beta} = \frac{\tan\beta}{2 + 3\tan^2\beta}$$

$$= \frac{2k}{2 + 3 \times (2k)^2} = \frac{2k}{2 + 12k^2}$$

$= \frac{k}{1 + 6k^2}$ Thus, LHS = RHS **Proved.**

13. प्रमाणित गर्नुहोस् (Prove that):

$4\,\text{cosec } 2A.\cot 2A = \text{cosec}^2 A - \sec^2 A$ [2061 S]

⇨ Here,

$$\begin{aligned}\text{RHS} &= \text{cosec}^2 A - \sec^2 A\\ &= (\text{cosec } A - \sec A)\,(\text{cosec } A + \sec A)\\ &= \left(\frac{1}{\sin A} - \frac{1}{\cos A}\right)\left(\frac{1}{\sin A} + \frac{1}{\cos A}\right)\\ &= \left(\frac{\cos A - \sin A}{\sin A.\cos A}\right)\left(\frac{\cos A + \sin A}{\sin A.\cos A}\right)\\ &= \frac{4(\cos^2 A - \sin^2 A)}{(2\sin A.\cos A)^2}\\ &= \frac{4(\cos^2 A - \sin^2 A)}{\sin^2 2A}\\ &= \frac{4\cos 2A}{\sin 2A.\sin 2A}\\ &= \frac{4}{\sin 2A}.\frac{\cos 2A}{\sin 2A}\\ &= 4\text{cosec } 2A.\cot 2A\\ &= \text{RHS}\end{aligned}$$

Thus, LHS = RHS **Proved.**

15. प्रमाणित गर्नुहोस् (Prove that):

$\cos^2\alpha + \sin^2\alpha.\cos 2\beta = \cos^2\beta + \sin^2\beta.\cos 2\alpha$[2071 R',

⇨ Here,

$$\begin{aligned}\text{LHS} &= \cos^2\alpha + \sin^2\alpha.\cos 2\beta\\ &= 1 - \sin^2\alpha + \sin^2\alpha.\cos 2\beta\\ &= 1 - \sin^2\alpha\,(1 - \cos 2\beta)\\ &= 1 - \sin^2\alpha.2\sin^2\beta\\ &= 1 - (2\sin^2\alpha)\sin^2\beta\\ &= 1 - (1 - \cos 2\alpha)\sin^2\beta\\ &= \cos^2\beta + \sin^2\beta - (1 - \cos 2\alpha)\sin^2\beta\\ &= \cos^2\beta + \sin^2\beta\,(1 - 1 + \cos 2\alpha)\\ &= \cos^2\beta + \sin^2\beta.\cos 2\alpha\\ &= \text{RHS}\end{aligned}$$

Proved.

17. प्रमाणित गर्नुहोस् (Prove that):

$\sin^4 x = \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$ [2066 S]

⇨ Here,

$$\begin{aligned}\text{RHS} &= \frac{1}{8}(3 - 4\cos 2x + \cos 4x)\\ &= \frac{1}{8}[3 - 4(1 - 2\sin^2 x) + 1 - 2\sin^2 2x]\\ &= \frac{1}{8}(3 - 4 + 8\sin^2 x + 1 - 2\sin^2 2x)\\ &= \frac{1}{8}[8\sin^2 x - 2(2\sin x\cos x)^2]\\ &= \frac{1}{8}[8\sin^2 x - 8\sin^2 x\cos^2 x]\\ &= \frac{1}{8}[8\sin^2 x - 8\sin^2 x\,(1 - \sin^2 x)]\\ &= \frac{1}{8}[8\sin^2 x - 8\sin^2 x + 8\sin^4 x]\\ &= \frac{1}{8} \times 8\sin^4 x\\ &= \sin^4 x\\ &= \text{LHS}\end{aligned}$$

Proved.

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MODEL 2

18. प्रमाणित गर्नुहोस् (Prove that):

$$(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\left(\frac{A-B}{2}\right)$$

[2065 E]

⇨ Here, LHS

$= (\cos A + \cos B)^2 + (\sin A + \sin B)^2$

$= \cos^2 A + 2.\cos A.\ \cos B + \cos^2 B$

$\quad + \sin^2 A + 2.\sin A.\ \sin B + \sin^2 B$

$= 2 + 2(\cos A.\ \cos B + \sin A.\ \sin B)$

$= 2 + 2.\cos (A - B)$

$= 2[1 + \cos(A - B)] \qquad [\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}]$

$= 2\left[2\cos^2\left(\frac{A-B}{2}\right)\right]$

$= 4\cos^2\left(\frac{A-B}{2}\right)$

Thus, LHS = RHS **Proved.**

19. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{1+\cos 2\theta}{\sin 2\theta}\cdot\frac{1+\cos\theta}{\cos\theta} = \cot\frac{\theta}{2}$$

[2065 R']

⇨ Here, LHS $= \frac{1+\cos 2\theta}{\sin 2\theta}\cdot\frac{1+\cos\theta}{\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}\cdot\frac{1+\cos\theta}{\cos\theta}$

$= \frac{1+\cos\theta}{\sin\theta}$

$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2} = \text{RHS}$ **Proved.**

20. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sin 2A}{1+\cos 2A}\cdot\frac{\cos A}{1+\cos A} = \tan\frac{A}{2}$ **[2060 S]**

⇨ Here, LHS $= \frac{\sin 2A}{1+\cos 2A}\cdot\frac{\cos A}{1+\cos A} \qquad = \frac{2\sin A.\cos A}{1+2\cos^2 A-1}\cdot\frac{\cos A}{1+\cos A} \qquad = \frac{2\sin A.\cos A}{2\cos^2 A}\cdot\frac{\cos A}{1+\cos A}$

$= \frac{\sin A}{1+\cos A} \qquad = \frac{2\sin\frac{A}{2}.\cos\frac{A}{2}}{1+2\cos^2\frac{A}{2}-1} \qquad = \frac{2\sin\frac{A}{2}.\cos\frac{A}{2}}{2\cos^2\frac{A}{2}}$

$= \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} \qquad = \tan\frac{A}{2} = \text{RHS}$ **Proved.**

MODEL 3

21. प्रमाणित गर्नुहोस् (Prove that) : $8\left(1+\sin\frac{\pi^c}{8}\right)\left(1+\sin\frac{3\pi^c}{8}\right)\left(1-\sin\frac{5\pi^c}{8}\right)\left(1-\sin\frac{7\pi^c}{8}\right) = 1$ **[2070 R]**

⇨ Here, LHS $= 8\left(1+\sin\frac{\pi^c}{8}\right)\left(1+\sin\frac{3\pi^c}{8}\right)\left(1-\sin\frac{5\pi^c}{8}\right)\left(1-\sin\frac{7\pi^c}{8}\right)$

$= 8\left(1+\sin\frac{\pi^c}{8}\right)\left(1+\sin\frac{3\pi^c}{8}\right)\left[1-\sin\left(\pi^c-\frac{3\pi^c}{8}\right)\right]\left[1-\sin\left(\pi^c-\frac{\pi^c}{8}\right)\right]$

$= 8\left(1+\sin\frac{\pi^c}{8}\right)\left(1+\sin\frac{3\pi^c}{8}\right)\left(1-\sin\frac{3\pi^c}{8}\right)\left(1-\sin\frac{\pi^c}{8}\right)$

$= 8\left(1-\sin^2\frac{\pi^c}{8}\right)\left(1-\sin^2\frac{3\pi^c}{8}\right)$

$= 8\cos^2\frac{\pi^c}{8}\times\cos^2\frac{3\pi^c}{8} \qquad = 8\cos^2\frac{\pi^c}{8}\times\left[\cos\left(\frac{\pi^c}{2}-\frac{\pi^c}{8}\right)\right]^2$

$= 8\cos^2\frac{\pi^c}{8}\times\sin^2\frac{\pi^c}{8} \qquad = 2\times 4\cos^2\frac{\pi^c}{8}\times\sin^2\frac{\pi^c}{8}$

$= 2\left(2\cos\frac{\pi^c}{8}\sin\frac{\pi^c}{8}\right)^2 \qquad = 2\left(\sin 2\times\frac{\pi^c}{8}\right)^2$

$= 2\sin^2\frac{\pi^c}{4} \qquad = 2\times\sin^2 45°$

$= 2\times\left(\frac{1}{\sqrt{2}}\right)^2 \qquad = 2\times\frac{1}{2} = 1 = \text{RHS}$ **Proved.**

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22. प्रमाणित गर्नुहोस् (Prove that) : $\left(1+\cos\frac{\pi^c}{8}\right)\left(1+\cos\frac{3\pi^c}{8}\right)\left(1+\cos\frac{5\pi^c}{8}\right)\left(1+\cos\frac{7\pi^c}{8}\right)=\frac{1}{8}$

⇨ Here, LHS $=\left(1+\cos\frac{\pi^c}{8}\right)\left(1+\cos\frac{3\pi^c}{8}\right)\left(1+\cos\frac{5\pi^c}{8}\right)\left(1+\cos\frac{7\pi^c}{8}\right)$

$=\left(1+\cos\frac{\pi^c}{8}\right)\left(1+\cos\frac{3\pi^c}{8}\right)\left[1+\cos\left(\pi-\frac{3\pi^c}{8}\right)\right]\left[1+\cos\left(\pi-\frac{\pi^c}{8}\right)\right]$

$=\left(1+\cos\frac{\pi^c}{8}\right)\left(1+\cos\frac{3\pi^c}{8}\right)\left(1-\cos\frac{3\pi^c}{8}\right)\left(1-\cos\frac{\pi^c}{8}\right)$

$=\left(1-\cos^2\frac{\pi^c}{8}\right)\left(1-\cos^2\frac{3\pi^c}{8}\right)$

$=\sin^2\frac{\pi^c}{8}\times\sin^2\frac{3\pi^c}{8}$ $\qquad=\frac{1}{4}\left[2\sin\frac{3\pi^c}{8}.\sin\frac{\pi^c}{8}\right]^2$

$=\frac{1}{4}\left[\cos\left(\frac{3\pi^c-\pi^c}{8}\right)-\cos\left(\frac{3\pi^c+\pi^c}{8}\right)\right]^2$ $\qquad=\frac{1}{4}\left[\cos\frac{\pi^c}{4}-\cos\frac{\pi^c}{2}\right]^2$

$=\frac{1}{4}[\cos 45°-\cos 90°]^2$ $\qquad=\frac{1}{4}\left[\frac{1}{\sqrt{2}}-0\right]^2$

$=\frac{1}{4}\times\frac{1}{2}$ $\qquad=\frac{1}{8}=\text{RHS}$ **Proved.**

QUESTIONS FROM CDC TEXTBOOK

5.1 अपवर्त्यकोणका त्रिकोणमितीय अनुपातहरू (TRIGONOMETRIC RATIONS OF MULTIPLE ANGLES)

EXERCISE 5.1

1. (a) अपवर्त्यकोण भनेको के हो ? उदाहरणसहित प्रष्ट पार्नुहोस् ।
What do you mean by multiple angles? Clarify with examples.

⇨ Here, if A be an angle, then 2A, 3A, 4A, ... etc are called the multiple angles of A.

(b) **sin 2A लाई sin A र cos A का रूपमा लेख्नुहोस् । (Write sin 2A in terms of sin A and cos A.)**

⇨ Here, $\sin 2A = 2\sin A\cos A$

(c) **tan 2A लाई sin A र cos A का रूपमा लेख्नुहोस् । (Write tan 2A in terms of sin A and cos A.)**

⇨ Here, $\tan 2A=\frac{\sin 2A}{\cos 2A}=\frac{2\sin A\cos A}{\cos^2 A-\sin^2 A}$

(d) **cos 2A लाई, (Write cos 2A)**

(i) **sin A र cos A का रूपमा लेख्नुहोस् । (In terms of sin A and cos A.)** ⇨ Here, $\cos 2A=\cos^2 A-\sin^2 A$

(ii) **sin A का रूपमा लेख्नुहोस् । (In terms of sin A.)** ⇨ Here, $\cos 2A = 1-2\sin^2 A$

(iii) **cos A का रूपमा लेख्नुहोस् । (In terms of cos A.)** ⇨ Here, $\cos 2A = 2\cos^2 A-1$

(iv) **tan A का रूपमा लेख्नुहोस् । (In terms of tan A.)** ⇨ Here, $\cos 2A=\frac{1-\tan^2 A}{1+\tan^2 A}$

(e) **sin 3A लाई sin A का रूपमा लेख्नुहोस् । (Write sin 3A in terms of sin A.)** ⇨ Here, $\sin 3A = 3\sin A-4\sin^3 A$

(f) **cos 3*x* लाई cos *x* का रूपमा लेख्नुहोस् । (Write cos 3*x* in terms of cos *x*.)** ⇨ Here, $\cos 3x = 4\cos^3 x-3\cos x$

(g) **tan 3*y* लाई tan *y* का रूपमा लेख्नुहोस् । (Write tan 3*y* in terms of tan *y*.)** ⇨ Here, $\tan 3y=\frac{3\tan y-\tan^3 y}{1-3\tan^2 y}$

2. (a) **यदि $\sin A=\frac{4}{5}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।**

If $\sin A=\frac{4}{5}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\sin A=\frac{4}{5}$ So, $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{25-16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

(i) $\sin 2A = 2\sin A\cos A = 2\times\frac{4}{5}\times\frac{3}{5}=\frac{24}{25}$ (ii) $\cos 2A = 1-2\sin^2 A = 1-2\times\left(\frac{4}{5}\right)^2=1-\frac{32}{25}=-\frac{7}{25}$

(iii) $\tan 2A=\frac{\sin 2A}{\cos 2A}=\frac{\frac{24}{25}}{-\frac{7}{25}}=-\frac{24}{7}$ Thus, $\sin 2A=\frac{24}{25}$, $\cos 2A=-\frac{7}{25}$ and $\tan 2A=-\frac{24}{7}$.

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(b) यदि sin A = $\frac{5}{13}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।

If sin A = $\frac{5}{13}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\sin A = \frac{5}{13}$

So, $\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

(i) $\sin 2A = 2\sin A\cos A = 2\times\frac{5}{13}\times\frac{12}{13} = \frac{120}{169}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \frac{144-25}{169} = \frac{119}{169}$

(iii) $\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\frac{120}{169}}{\frac{119}{169}} = \frac{120}{119}$

Thus, $\sin 2A = \frac{120}{169}$, $\cos 2A = \frac{119}{169}$ and $\tan 2A = \frac{120}{119}$.

(c) यदि cos A = $\frac{\sqrt{3}}{2}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।

If cos A = $\frac{\sqrt{3}}{2}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\cos A = \frac{\sqrt{3}}{2}$ or, $\cos A = \cos 30°$ $\therefore A = 30°$

Now,(i) $\sin 2A = \sin 2\times 30° = \sin 60° = \frac{\sqrt{3}}{2}$

(ii) $\cos 2A = \cos 2\times 30° = \cos 60° = \frac{1}{2}$

(iii) $\tan 2A = \tan 2\times 30° = \tan 60° = \sqrt{3}$

Thus, $\sin 2A = \frac{\sqrt{3}}{2}$, $\cos 2A = \frac{1}{2}$ and $\tan 2A = \sqrt{3}$.

(d) यदि cos A = $\frac{7}{25}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।

If cos A = $\frac{7}{25}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\cos A = \frac{7}{25}$

So, $\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{7}{25}\right)^2} = \sqrt{1-\frac{49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$

Now, (i) $\sin 2A = 2\sin A\cos A = 2\times\frac{24}{25}\times\frac{7}{25} = \frac{336}{625}$

(ii) $\cos 2A = 1-2\sin^2 A = 1-2\left(\frac{24}{25}\right)^2 = 1-2\times\frac{576}{625} = 1-\frac{1152}{625} = -\frac{527}{625}$

(iii) $\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\frac{336}{625}}{-\frac{527}{625}} = -\frac{336}{527}$

Thus, $\sin 2A = \frac{336}{625}$, $\cos 2A = -\frac{527}{625}$ and $\tan 2A = -\frac{336}{527}$.

(e) यदि tan A = $\frac{3}{4}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।

If tan A = $\frac{3}{4}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1+\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16+9}{16}} = \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$

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$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16-9}{16}} = \frac{3}{2}\times\frac{16}{7} = \frac{24}{7}$$

Thus, $\sin 2A = \frac{24}{25}$, $\cos 2A = \frac{7}{25}$ and $\tan 2A = \frac{24}{7}$.

(f) यदि tan A = $\frac{1}{\sqrt{3}}$ भए sin 2A, cos 2A र tan 2A को मान पत्ता लगाउनुहोस् ।

If tan A = $\frac{1}{\sqrt{3}}$ then find the value of sin 2A, cos 2A and tan 2A.

⇨ Here, $\tan A = \frac{1}{\sqrt{3}}$ or, $\tan A = \tan 30°$ $\therefore A = 30°$

Now, (i) $\sin 2A = \sin 2\times 30° = \sin 60° = \frac{\sqrt{3}}{2}$

(ii) $\cos 2A = \cos 2\times 30° = \cos 60° = \frac{1}{2}$

(iii) $\tan 2A = \tan 2\times 30° = \tan 60° = \sqrt{3}$

Thus, $\sin 2A = \frac{\sqrt{3}}{2}$, $\cos 2A = \frac{1}{2}$ and $\tan 2A = \sqrt{3}$.

(g) यदि sin θ = $\frac{1}{2}$ भए sin 3θ र cos 3θ को मान पत्ता लगाउनुहोस् ।

If sin θ = $\frac{1}{2}$ then find the value of sin 3θ and cos 3θ.

⇨ Here, $\sin\theta = \frac{1}{2}$

So, $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Now, (i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - 4\times\frac{1}{8} = \frac{3-1}{2} = \frac{2}{2} = 1$

(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right) = 4\times\frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

Thus, sin 3θ = 1 and cos 3θ = 0.

(h) यदि cos θ = $\frac{\sqrt{3}}{2}$ भए cos 3θ र sin 3θ को मान पत्ता लगाउनुहोस् ।

If cos θ = $\frac{\sqrt{3}}{2}$ then find the value of cos 3θ and sin 3θ.

⇨ Here, $\cos\theta = \frac{\sqrt{3}}{2}$

So, $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

(i) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right) = 4\times\frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

Now, (ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - 4\times\frac{1}{8} = \frac{3-1}{2} = \frac{2}{2} = 1$

Thus, sin 3θ = 1 and cos 3θ = 0.

(i) यदि cos β = $\frac{4}{5}$ भए sin 3β र cos 3β को मान पत्ता लगाउनुहोस् ।

If cos β = $\frac{4}{5}$ then find the value of sin 3β and cos 3β.

⇨ Here, $\cos\beta = \frac{4}{5}$

So, $\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

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We have, $\sin 3\beta = 3\sin\beta - 4\sin^3\beta = 3 \times \frac{3}{5} - 4 \times \left(\frac{3}{5}\right)^3 = \frac{9}{5} - 4 \times \frac{27}{125} = \frac{9}{5} - \frac{108}{125} = \frac{117}{125}$

Again, $\cos 3\beta = 4\cos^3\beta - 3\cos\beta = 4\left(\frac{4}{5}\right)^3 - 3 \times \frac{4}{5} = 4 \times \frac{64}{125} - \frac{12}{5} = \frac{256}{125} - \frac{12}{5} = -\frac{44}{125}$

Thus, $\sin 3\beta = \frac{117}{125}$ and $\cos 3\beta = -\frac{44}{125}$.

(j) यदि $\sin x = \frac{\sqrt{3}}{2}$ भए $\cos 3x$ र $\sin 3x$ को मान पत्ता लगाउनुहोस् ।

If $\sin x = \frac{\sqrt{3}}{2}$ then find the value of $\cos 3x$ and $\sin 3x$.

⇨ Here, $\sin x = \frac{\sqrt{3}}{2}$ or, $\sin x = \sin 60°$ $\therefore$ $x = 60°$

Now, $\cos 3x = \cos 3 \times 60° = \cos 180° = -1$

$\sin 3x = \sin 3 \times 60° = \sin 180° = 0$

Thus, $\cos 3x = -1$ and $\sin 3x = 0$.

(k) यदि $\tan\alpha = \frac{1}{2}$ भए $\tan 3\alpha$ को मान पत्ता लगाउनुहोस् । (If $\tan\alpha = \frac{1}{2}$ then find the value of $\tan 3\alpha$.)

⇨ Here, $\tan\alpha = \frac{1}{2}$

We have, $\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha} = \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3\left(\frac{1}{2}\right)^2} = \frac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{2}$

Thus, the value of $\tan 3\alpha$ is $\frac{11}{2}$.

(ℓ) यदि $\tan\theta = \frac{1}{\sqrt{2}}$ भए $\tan 3\theta$ को मान पत्ता लगाउनुहोस् । (If $\tan\theta = \frac{1}{\sqrt{2}}$ then find the value of $\tan 3\theta$.)

⇨ Here, $\tan\theta = \frac{1}{\sqrt{2}}$

So, $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{3 \times \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}}\right)^3}{1 - 3\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}}}{1 - 3 \times \frac{1}{2}} = \frac{\frac{6-1}{2\sqrt{2}}}{\frac{2-3}{2}} = \frac{\frac{5}{2\sqrt{2}}}{-\frac{1}{2}} = -\frac{5}{2\sqrt{2}} \times \frac{2}{1} = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$

Thus, the value of $\tan 3\theta$ is $-\frac{5\sqrt{2}}{2}$.

3. प्रमाणित गर्नुहोस् (Prove that):

(a) $\sin A = \pm\sqrt{\frac{1 - \cos 2A}{2}}$

⇨ Here, $\cos 2A = 1 - 2\sin^2 A$

or, $2\sin^2 A = 1 - \cos 2A$

or, $\sin^2 A = \frac{1 - \cos 2A}{2}$

$\therefore$ $\sin A = \pm\sqrt{\frac{1 - \cos 2A}{2}}$ **Proved.**

(c) $\tan A = \pm\sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}$

⇨ Here, $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

or, $\cos 2A + \cos 2A \tan^2 A = 1 - \tan^2 A$

or, $\cos 2A \tan^2 A + \tan^2 A = 1 - \cos 2A$

or, $\tan^2 A (1 + \cos 2A) = 1 - \cos 2A$

or, $\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$

$\therefore$ $\tan A = \pm\sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}$ **Proved.**

(b) $\cos A = \pm\sqrt{\frac{1 + \cos 2A}{2}}$

⇨ Here, $\cos 2A = 2\cos^2 A - 1$

or, $\cos 2A + 1 = 2\cos^2 A$

or, $\cos^2 A = \frac{1 + \cos 2A}{2}$

$\therefore$ $\cos A = \pm\sqrt{\frac{1 + \cos 2A}{2}}$ **Proved.**

(d) $\cot 2A = \frac{\cot^2 A - 1}{2\cot A}$

⇨ Here,

LHS $= \cot 2A = \frac{1}{\tan 2A} = \frac{1}{\frac{2\tan A}{1 - \tan^2 A}}$

$= \frac{1 - \tan^2 A}{2\tan A} = \frac{1 - \frac{1}{\cot^2 A}}{2\frac{1}{\cot A}} = \frac{\frac{\cot^2 A - 1}{\cot^2 A}}{\frac{2}{\cot A}}$

$= \frac{\cot^2 A - 1}{\cot^2 A} \times \frac{\cot A}{2} = \frac{\cot^2 A - 1}{2\cot A}$

$=$ RHS **Proved.**

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(e) $\cos 2A = \dfrac{\cot^2 A - 1}{\cot^2 A + 1}$

⇨ Here, LHS $= \cos 2A$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \dfrac{1}{\cot^2 A}}{1 + \dfrac{1}{\cot^2 A}}$$

$$= \frac{\dfrac{\cot^2 A - 1}{\cot^2 A}}{\dfrac{\cot^2 A + 1}{\cot^2 A}}$$

$$= \frac{\cot^2 A - 1}{\cot^2 A + 1}$$

= RHS **Proved.**

(f) $\operatorname{cosec} 2A = \dfrac{\cot^2 A + 1}{2 \cot A}$

⇨ Here,

LHS $= \operatorname{cosec} 2A = \dfrac{1}{\sin 2A}$

$$= \frac{1}{\dfrac{2 \tan A}{1 + \tan^2 A}} = \frac{1 + \tan^2 A}{2 \tan A}$$

$$= \frac{1 + \dfrac{1}{\cot^2 A}}{2 \dfrac{1}{\cot A}} = \frac{\cot^2 A + 1}{\cot^2 A} \times \frac{\cot A}{2}$$

$$= \frac{\cot^2 A + 1}{2 \cot A} = \text{RHS}$$

Proved.

4. प्रमाणित गर्नुहोस् (Prove that):

(a) $\dfrac{\sin 2A}{1 - \cos 2A} = \cot A$

⇨ Here, LHS $= \dfrac{\sin 2A}{1 - \cos 2A}$

$$= \frac{\sin 2A}{1 - (\cos^2 A - \sin^2 A)}$$

$$= \frac{\sin 2A}{1 - \cos^2 A + \sin^2 A}$$

$$= \frac{2 \sin A \cos A}{\sin^2 A + \sin^2 A}$$

$$= \frac{2 \sin A \cos A}{2 \sin^2 A}$$

$$= \frac{\sin A \cos A}{\sin A \sin A} = \cot A$$

= RHS **Proved.**

(c) $\dfrac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A$

⇨ Here, LHS $= \dfrac{1 - \cos 2A}{1 + \cos 2A}$

$$= \frac{1 - (\cos^2 A - \sin^2 A)}{1 + (\cos^2 A - \sin^2 A)}$$

$$= \frac{1 - \cos^2 A + \sin^2 A}{1 + \cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A + \sin^2 A}{\cos^2 A + \cos^2 A}$$

$$= \frac{2 \sin^2 A}{2 \cos^2 A} = \tan^2 A$$

= RHS **Proved.**

(e) $\dfrac{\cos 2A}{1 + \sin 2A} = \dfrac{1 - \tan A}{1 + \tan A}$

⇨ Here,

LHS $= \dfrac{\cos 2A}{1 + \sin 2A}$

$$= \frac{\cos^2 A - \sin^2 A}{\sin^2 A + \cos^2 A + 2 \sin A \cos A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{(\sin A + \cos A)^2}$$

$$= \frac{\cos A - \sin A}{\cos A + \sin A}$$

$$= \frac{\dfrac{\cos A}{\cos A} - \dfrac{\sin A}{\cos A}}{\dfrac{\cos A}{\cos A} + \dfrac{\sin A}{\cos A}}$$

$$= \frac{1 - \tan A}{1 + \tan A} = \text{RHS}$$

Proved.

(b) $\dfrac{\sin 2A}{1 + \cos 2A} = \tan A$

⇨ Here, LHS $= \dfrac{\sin 2A}{1 + \cos 2A}$

$$= \frac{\sin 2A}{1 + \cos^2 A - \sin^2 A}$$

$$= \frac{2 \sin A \cos A}{\cos^2 A + \cos^2 A}$$

$$= \frac{2 \sin A \cos A}{2 \cos^2 A}$$

$= \tan A$

= RHS **Proved.**

(d) $\dfrac{1 + \sin 2A}{\cos 2A} = \dfrac{\cos A + \sin A}{\cos A - \sin A}$

⇨ Here,

LHS $= \dfrac{1 + \sin 2A}{\cos 2A}$

$$= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{(\sin A + \cos A)^2}{(\cos A + \sin A)(\cos A - \sin A)}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$

= RHS **Proved.**

(f) $\dfrac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A} = \tan A$

⇨ Here, LHS $= \dfrac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A}$

$$= \frac{1 - 1 + 2 \sin^2 A + 2 \sin A \cos A}{1 + 2 \cos^2 A - 1 + 2 \sin A \cos A}$$

$$= \frac{2 \sin^2 A + 2 \sin A \cos A}{2 \cos^2 A + 2 \sin A \cos A}$$

$$= \frac{2 \sin A (\sin A + \cos A)}{2 \cos A (\cos A + \sin A)}$$

$$= \frac{\sin A}{\cos A}$$

$= \tan A =$ RHS **Proved.**

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(g) $\dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$

⇨ Here,

$$\text{LHS} = \frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$$

$$= \frac{\sin A + 2\sin A \,.\, \cos A}{1 + \cos A + 2\cos^2 A - 1}$$

$$= \frac{\sin A(1 + 2\cos A)}{\cos A\,(1 + 2\cos A)}$$

$$= \frac{\sin A}{\cos A}$$

$= \tan A = \text{RHS}$ **Proved.**

(i) $\tan\alpha + \cot\alpha = 2\,\text{cosec}\, 2\alpha$

⇨ Here, LHS $= \tan\alpha + \cot\alpha$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$$

$$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{1}{\sin\alpha\cos\alpha}$$

$$= \frac{2}{2\sin\alpha\cos\alpha}$$

$$= \frac{2}{\sin 2\alpha}$$

$= 2\,\text{cosec}\, 2\alpha$

$= \text{RHS}$ **Proved.**

(h) $\cos 2A = \dfrac{\cot A - \tan A}{\cot A + \tan A}$

⇨ Here, RHS $= \dfrac{\cot A - \tan A}{\cot A + \tan A}$

$$= \frac{\dfrac{1}{\tan A} - \tan A}{\dfrac{1}{\tan A} + \tan A}$$

$$= \frac{\dfrac{1 - \tan^2 A}{\tan A}}{\dfrac{1 + \tan^2 A}{\tan A}} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$= \cos 2A = \text{LHS}$ **Proved.**

(j) $\tan\theta - \cot\theta = -\,2\cot 2\theta$

⇨ Here, LHS $= \tan\theta - \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{-(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{-\,2\cos 2\theta}{2\sin\theta\cos\theta}$$

$$= -\,2\frac{\cos 2\theta}{\sin 2\theta}$$

$= -\,2\cot 2\theta = \text{RHS}$ **Proved.**

(Note: the question is corrected.)

5. **प्रमाणित गर्नुहोस् (Prove that):**

(a) $\cot\left(A + \dfrac{\pi}{4}\right) - \tan\left(A - \dfrac{\pi}{4}\right) = \dfrac{2\cos 2A}{1 + \sin 2A}$

⇨ Here, LHS $= \cot\left(A + \dfrac{\pi}{4}\right) - \tan\left(A - \dfrac{\pi}{4}\right)$

$= \cot(A + 45°) - \tan(A - 45°)$

$$= \frac{\cos(A + 45°)}{\sin(A + 45°)} - \frac{\sin(A - 45°)}{\cos(A - 45°)}$$

$$= \frac{\cos(A + 45°)\cos(A - 45°) - \sin(A + 45°)\sin(A - 45°)}{\sin(A + 45°)\cos(A - 45°)}$$

$$= \frac{\cos\{(A + 45°) + (A - 45°)\}}{\sin(A + 45°)\cos(A - 45°)}$$

$$= \frac{\cos 2A}{(\sin A\cos 45° + \cos A\sin 45°)(\cos A\cos 45° + \sin A\sin 45°)}$$

$$= \frac{\cos 2A}{\left(\sin A \times \dfrac{1}{\sqrt{2}} + \cos A \times \dfrac{1}{\sqrt{2}}\right)\left(\cos A \times \dfrac{1}{\sqrt{2}} + \sin A \times \dfrac{1}{\sqrt{2}}\right)}$$

$$= \frac{\cos 2A}{\dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}}(\sin A + \cos A)(\cos A + \sin A)}$$

$$= \frac{\cos 2A}{\dfrac{1}{2}(\cos A + \sin A)^2}$$

$$= \frac{2\cos 2A}{\cos^2 A + \sin^2 A + 2\sin A\cos A}$$

$= \dfrac{2\cos 2A}{1 + \sin 2A} = \text{RHS}$ **Proved.**

(b) $\cos^2\left(\dfrac{\pi}{4} - B\right) - \sin^2\left(\dfrac{\pi}{4} - B\right) = \sin 2B$

⇨ Here, LHS $= \cos^2\left(\dfrac{\pi}{4} - B\right) - \sin^2\left(\dfrac{\pi}{4} - B\right)$

$= \cos^2(45° - B) - \sin^2(45° - B)$

$= \cos 2(45° - B)$

$= \cos(90° - 2B)$

$= \sin 2B = \text{RHS}$ **Proved.**

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(c) $\tan\left(C+\frac{\pi}{4}\right)+\tan\left(C-\frac{\pi}{4}\right)=2\tan 2C$

⇨ Here, LHS

$$=\tan\left(C+\frac{\pi}{4}\right)+\tan\left(C-\frac{\pi}{4}\right)$$

$$=\tan(C+45°)+\tan(C-45°)$$

$$=\frac{\tan C+\tan 45°}{1-\tan C\tan 45°}+\frac{\tan C-\tan 45°}{1+\tan C\tan 45°}$$

$$=\frac{\tan C+1}{1-\tan C}+\frac{\tan C-1}{1+\tan C}$$

$$=\frac{\tan C+1}{1-\tan C}-\frac{1-\tan C}{1+\tan C}$$

$$=\frac{(1+\tan C)^2-(1-\tan C)^2}{(1+\tan C)(1-\tan C)}$$

$$=\frac{1+2\tan C+\tan^2 C-(1-2\tan C+\tan^2 C)}{1-\tan^2 C}$$

$$=\frac{1+2\tan C+\tan^2 C-1+2\tan C-\tan^2 C}{1-\tan^2 C}$$

$$=\frac{4\tan C}{1-\tan^2 C}$$

$$=2\frac{2\tan C}{1-\tan^2 C}$$

$= 2\tan 2C =$ RHS **Proved.**

(e) $\tan\left(\frac{\pi}{4}+B\right)=\frac{\cos 2B}{1-\sin 2B}$

⇨ Here,

$$\text{LHS}=\tan\left(\frac{\pi}{4}+B\right)$$

$$=\tan(45°+B)$$

$$=\frac{\sin(45°+B)}{\cos(45°+B)}$$

$$=\frac{\sin 45°\cos B+\cos 45°\sin B}{\cos 45°\cos B-\sin 45°\sin B}$$

$$=\frac{\frac{1}{\sqrt{2}}\cos B+\frac{1}{\sqrt{2}}\sin B}{\frac{1}{\sqrt{2}}\cos B-\frac{1}{\sqrt{2}}\sin B}$$

$$=\frac{\frac{1}{\sqrt{2}}(\cos B+\sin B)}{\frac{1}{\sqrt{2}}(\cos B-\sin B)}$$

$$=\frac{\cos B+\sin B}{\cos B-\sin B}\times\frac{\cos B-\sin B}{\cos B-\sin B}$$

$$=\frac{\cos^2 B-\sin^2 B}{(\cos B-\sin B)^2}$$

$$=\frac{\cos 2B}{\cos^2 B-2\cos B\sin B+\sin^2 B}$$

$=\frac{\cos 2B}{1-\sin 2B}=$ RHS **Proved.**

(d) $\tan\left(\alpha+\frac{\pi}{4}\right)-\tan\left(\alpha-\frac{\pi}{4}\right)=2\sec 2\alpha$

⇨ Here,

$$\text{LHS}=\tan\left(\alpha+\frac{\pi}{4}\right)-\tan\left(\alpha-\frac{\pi}{4}\right)$$

$$=\tan(\alpha+45°)-\tan(\alpha-45°)$$

$$=\frac{\tan\alpha+\tan 45°}{1-\tan\alpha\tan 45°}-\frac{\tan\alpha-\tan 45°}{1+\tan\alpha\tan 45°}$$

$$=\frac{\tan\alpha+1}{1-\tan\alpha}-\frac{\tan\alpha-1}{1+\tan\alpha}$$

$$=\frac{1+\tan\alpha}{1-\tan\alpha}+\frac{1-\tan\alpha}{1+\tan\alpha}$$

$$=\frac{(1+\tan\alpha)^2+(1-\tan\alpha)^2}{(1-\tan\alpha)(1+\tan\alpha)}$$

$$=\frac{1+2\tan\alpha+\tan^2\alpha+1-2\tan\alpha+\tan^2\alpha}{1-\tan^2\alpha}$$

$$=\frac{2+2\tan^2\alpha}{1-\tan^2\alpha}=\frac{2(1+\tan^2\alpha)}{1-\tan^2\alpha}$$

$$=2\times\frac{\sec^2\alpha}{1-\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\times\frac{1}{\cos^2\alpha}}{\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}$$

$$=2\times\frac{1}{\cos 2\alpha}=2\sec 2\alpha$$

= RHS **Proved.**

(f) $\frac{1-\tan^2\left(\frac{\pi}{4}-\theta\right)}{1+\tan^2\left(\frac{\pi}{4}-\theta\right)}=\sin 2\theta$

⇨ Here,

$$\text{LHS}=\frac{1-\tan^2\left(\frac{\pi}{4}-\theta\right)}{1+\tan^2\left(\frac{\pi}{4}-\theta\right)}$$

$$=\frac{1-\tan^2(45°-\theta)}{1+\tan^2(45°-\theta)}$$

$$=\frac{1-\frac{\sin^2(45°-\theta)}{\cos^2(45°-\theta)}}{1+\frac{\sin^2(45°-\theta)}{\cos^2(45°-\theta)}}$$

$$=\frac{\frac{\cos^2(45°-\theta)-\sin^2(45°-\theta)}{\cos^2(45°-\theta)}}{\frac{\cos^2(45°-\theta)+\sin^2(45°-\theta)}{\cos^2(45°-\theta)}}$$

$$=\frac{\cos 2(45°-\theta)}{\cos^2(45°-\theta)}\times\frac{\cos^2(45°-\theta)}{1}$$

$$=\frac{\cos(90°-2\theta)}{1}$$

$$=\sin 2\theta$$

= RHS **Proved.**

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6. प्रमाणित गर्नुहोस् (Prove that):

(a) $\frac{1+\sin 2A}{1-\sin 2A} = \left(\frac{\cot A+1}{\cot A-1}\right)^2$

⇨ Here,

$$\begin{aligned}
\text{LHS} &= \frac{1+\sin 2A}{1-\sin 2A} = \frac{1+\frac{2\tan A}{1+\tan^2 A}}{1-\frac{2\tan A}{1+\tan^2 A}}\\
&= \frac{\frac{1+\tan^2 A+2\tan A}{1+\tan^2 A}}{\frac{1+\tan^2 A-2\tan A}{1+\tan^2 A}}\\
&= \frac{(1+\tan A)^2}{(1-\tan A)^2} = \left(\frac{1+\tan A}{1-\tan A}\right)^2\\
&= \left(\frac{1+\frac{1}{\cot A}}{1-\frac{1}{\cot A}}\right)^2 = \left(\frac{\frac{\cot A+1}{\cot A}}{\frac{\cot A-1}{\cot A}}\right)^2\\
&= \left(\frac{\cot A+1}{\cot A-1}\right)^2 = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(d) $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1) = 2\cos 4\theta+1$

⇨ Here,

$$\begin{aligned}
\text{LHS} &= (2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1)\\
&= (4\cos^2\theta-1)(2\cos 2\theta-1)\\
&= (2\times 2\cos^2\theta-1)(2\cos 2\theta-1)\\
&= \{2(\cos 2\theta+1)-1\}(2\cos 2\theta-1)\\
&= (2\cos 2\theta+1)(2\cos 2\theta-1)\\
&= (4\cos^2 2\theta-1)\\
&= (2\times 2\cos^2 2\theta-1)\\
&= \{2(\cos 4\theta+1)-1\}\\
&= 2\cos 4\theta+1 = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(f) $\sqrt{2+\sqrt{2+2\cos 4\theta}} = 2\cos\theta$

⇨ Here,
$$\begin{aligned}
\text{LHS} &= \sqrt{2+\sqrt{2+2\cos 4\theta}}\\
&= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}\\
&= \sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}\\
&= \sqrt{2+2\cos 2\theta}\\
&= \sqrt{2(1+\cos 2\theta)}\\
&= \sqrt{2\times 2\cos^2\theta}\\
&= 2\cos\theta = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(g) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

⇨ Here,
$$\begin{aligned}
\text{LHS} &= \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}\\
&= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}\\
&= \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4\theta}}}\\
&= \sqrt{2+\sqrt{2+2\cos 4\theta}}\\
&= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}\\
&= \sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}\\
&= \sqrt{2+2\cos 2\theta}\\
&= \sqrt{2(1+\cos 2\theta)}\\
&= \sqrt{2\times 2\cos^2\theta}\\
&= 2\cos\theta = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(b) $\cos^2\theta+\sin^2\theta\cos 2\beta = \cos^2\beta+\sin^2\beta\cos 2\theta$

⇨ Here,
$$\begin{aligned}
\text{LHS} &= \cos^2\theta+\sin^2\theta\cos 2\beta\\
&= \cos^2\theta+\sin^2\theta(1-2\sin^2\beta)\\
&= \cos^2\theta+\sin^2\theta-2\sin^2\theta\sin^2\beta\\
&= 1-2\sin^2\theta\sin^2\beta\\
&= \sin^2\beta+\cos^2\beta-2\sin^2\theta\sin^2\beta\\
&= \cos^2\beta+\sin^2\beta-2\sin^2\theta\sin^2\beta\\
&= \cos^2\beta+\sin^2\beta(1-2\sin^2\theta)\\
&= \cos^2\beta+\sin^2\beta\cos 2\theta\\
&= \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(c) $(1+\cos 2\theta+\sin 2\theta)^2 = 4\cos^2\theta(1+\sin 2\theta)$

⇨ Here,

$$\begin{aligned}
\text{LHS} &= (1+\sin 2\theta+\cos 2\theta)^2\\
&= (1+2\sin\theta\cos\theta+2\cos^2\theta-1)^2\\
&= (2\cos\theta)^2(\sin\theta+\cos\theta)^2\\
&= 4\cos^2\theta(\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta)\\
&= 4\cos^2\theta(1+\sin 2\theta)\\
&= \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(e) $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1) = 2\cos 8\theta+1$

⇨ Here, LHS

$$\begin{aligned}
&= (2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= (4\cos^2\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= (2.2\cos^2\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= \{2(1+\cos 2\theta)-1\}(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= (2+2\cos 2\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= (2\cos 2\theta+1)(2\cos 2\theta-1)(2\cos 4\theta-1)\\
&= (4\cos^2 2\theta-1)(2\cos 4\theta-1)\\
&= (2.2\cos^2 2\theta-1)(2\cos 4\theta-1)\\
&= \{2.(1+\cos 4\theta)-1\}(2\cos 4\theta-1)\\
&= (2+2\cos 4\theta-1)(2\cos 4\theta-1)\\
&= (2\cos 4\theta+1)(2\cos 4\theta-1)\\
&= 4\cos^2 4\theta-1\\
&= 2.2\cos^2 4\theta-1\\
&= 2(1+\cos 8\theta)-1\\
&= 2+2\cos 8\theta-1\\
&= 2\cos 8\theta+1 = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(h) $\sin^2\theta\cos 2\beta-\sin^2\beta\cos 2\theta = \cos^2\beta-\cos^2\theta$

⇨ Here,

$$\begin{aligned}
\text{LHS} &= \sin^2\theta\cos 2\beta-\sin^2\beta\cos 2\theta\\
&= \sin^2\theta(1-2\sin^2\beta)-\sin^2\beta(1-2\sin^2\theta)\\
&= \sin^2\theta-2\sin^2\theta\sin^2\beta-\sin^2\beta+2\sin^2\theta\sin^2\beta\\
&= \sin^2\theta-\sin^2\beta\\
&= (1-\cos^2\theta)-(1-\cos^2\beta)\\
&= 1-\cos^2\theta-1+\cos^2\beta\\
&= \cos^2\beta-\cos^2\theta = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

(i) $\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A} = 1-\frac{1}{2}\sin 2A$

⇨ Here, LHS

$$\begin{aligned}
&= \frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}\\
&= \frac{(\cos A+\sin A)(\cos^2 A-\cos A\sin A+\sin^2 A)}{(\cos A+\sin A)}\\
&= \cos^2 A+\sin^2 A-\frac{1}{2}\times 2\sin A\cos A\\
&= 1-\frac{1}{2}\sin 2A = \text{RHS} \quad \textbf{Proved.}
\end{aligned}$$

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(j) $\cos^6 A + \sin^6 A = \frac{1}{4}(1 + 3\cos^2 2A)$

⇨ Here, LHS

$= \cos^6 A + \sin^6 A$

$= (\cos^2 A)^3 + (\sin^2 A)^3$

$= (\cos^2 A + \sin^2 A)^3 - 3\cos^2 A \sin^2 A (\cos^2 A + \sin^2 A)$

$= 1 - 3\cos^2 A \ \sin^2 A$

$= 1 - \frac{3}{4}\ (4\cos^2 A \ \sin^2 A)$

$= 1 - \frac{3}{4}\ (2\cos A \ \sin A)^2$

$= 1 - \frac{3}{4}\ (\sin 2A)^2$

$= 1 - \frac{3}{4}\ \sin^2 2A$

$= 1 - \frac{3}{4}\ (1 - \cos^2 2A)$

$= 1 - \frac{3}{4} + \frac{3}{4}\ \cos^2 2A$

$= \frac{1}{4} + \frac{3}{4}\ \cos^2 2A$

$= \frac{1}{4}(1 + 3\cos^2 2A) = \text{RHS}$ **Proved.**

(b) यदि $\cos\theta = \frac{1}{2}\left(b + \frac{1}{b}\right)$ भए प्रमाणित गर्नुहोस्

If $\cos\theta = \frac{1}{2}\left(b + \frac{1}{b}\right)$, prove that):

$\cos 3\theta = \frac{1}{2}\left(b^3 + \frac{1}{b^3}\right)$

⇨ Here, $\cos\theta = \frac{1}{2}\left(b + \frac{1}{b}\right)$

LHS $= \cos 3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$= 4\left[\frac{1}{2}\left(b + \frac{1}{b}\right)\right]^3 - 3 \times \frac{1}{2}\left(b + \frac{1}{b}\right)$

$= 4 \times \frac{1}{8}\left(b + \frac{1}{b}\right)^3 - 3 \times \frac{1}{2}\left(b + \frac{1}{b}\right)$

$= \frac{1}{2}\left(b + \frac{1}{b}\right)^3 - 3 \times \frac{1}{2}\left(b + \frac{1}{b}\right)$

$= \frac{1}{2}\left[\left(b + \frac{1}{b}\right)^3 - 3.b.\frac{1}{b}\left(b + \frac{1}{b}\right)\right]$

$= \frac{1}{2}\left(b^3 + \frac{1}{b^3}\right) = \text{RHS}$ **Proved.**

7. (a) यदि $\sin\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$ भए प्रमाणित गर्नुहोस् :

If $\sin\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$, prove that:

$\sin 3\theta = -\frac{1}{2}\left(p^3 + \frac{1}{p^3}\right)$

⇨ Here, $\sin\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

LHS

$= \sin 3\theta$

$= 3\sin\theta - 4\sin^3\theta$

$= 3 \times \frac{1}{2}\left(p + \frac{1}{p}\right) - 4\left\{\frac{1}{2}\left(p + \frac{1}{p}\right)\right\}^3$

$= 3 \times \frac{1}{2}\left(p + \frac{1}{p}\right) - 4 \times \frac{1}{8}\left(p + \frac{1}{p}\right)^3$

$= \frac{3}{2}\left(p + \frac{1}{p}\right) - \frac{1}{2}\left(p + \frac{1}{p}\right)^3$

$= \frac{1}{2}\left[3\left(p + \frac{1}{p}\right) - \left(p + \frac{1}{p}\right)^3\right]$

$= -\frac{1}{2}\left[\left(p + \frac{1}{p}\right)^3 - 3.p.\frac{1}{p}\left(p + \frac{1}{p}\right)\right]$

$= -\frac{1}{2}\left[p^3 + \left(\frac{1}{p}\right)^3\right]$

$= -\frac{1}{2}\left(p^3 + \frac{1}{p^3}\right) = \text{RHS}$ **Proved.**

8. प्रमाणित गर्नुहोस् (Prove that):

(a) $\cos^3 20° + \sin^3 10° = \frac{3}{4}(\cos 20° + \sin 10°)$

⇨ Here, LHS

$= \cos^3 20° + \sin^3 10°$

$= \frac{1}{4}\ 4(\cos^3 20° + \sin^3 10°)$

$= \frac{1}{4}\ (4\cos^3 20° + 4\sin^3 10°)$

$= \frac{1}{4}\ [\cos 3 \times 20° + 3\cos 20° + 3\sin 10° - \sin 3 \times 10°]$

$= \frac{1}{4}[\cos 60° + 3\cos 20° + 3\sin 10° - \sin 30°]$

$= \frac{1}{4}\left[\frac{1}{2} + 3(\cos 20° + \sin 10°) - \frac{1}{2}\right]$

$= \frac{1}{4} \times 3\ (\cos 20° + \sin 10°)$

$= \frac{3}{4}(\cos 20° + \sin 10°)$

$= \text{RHS}$ **Proved.**

(b) $\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

⇨ Here, LHS $= \cot 3A = \frac{1}{\tan 3A} = \frac{1}{\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}} = \frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A} = \frac{1 - \frac{3}{\cot^2 A}}{3\frac{1}{\cot A} - \frac{1}{\cot^3 A}} = \frac{\frac{\cot^2 A - 3}{\cot^2 A}}{\frac{3\cot^2 A - 1}{\cot^3 A}}$

$= \frac{\cot^2 A - 3}{\cot^2 A} \times \frac{\cot^3 A}{3\cot^2 A - 1} = \frac{(\cot^2 A - 3) \times \cot A}{3\cot^2 A - 1} = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1} = \text{RHS}$ **Proved.**

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(c) $\cos^3 A \cos 3A + \sin^3 A \sin 3A = \cos^3 2A$

⇨ Here, LHS

$= \cos^3 A \cos 3A + \sin^3 A \sin 3A$

$= \frac{1}{4}[4 \cos^3 A \cos 3A + 4 \sin^3 A \sin 3A]$

$= \frac{1}{4}[(3\cos A + \cos 3A) \cos 3A + (3\sin A - \sin 3A) \sin 3A]$

$= \frac{1}{4}[3 \cos A \cos 3A + \cos^2 3A + 3 \sin A \sin 3A - \sin^2 3A]$

$= \frac{1}{4}[3(\cos A \cos 3A + \sin A \sin 3A) + \cos^2 3A - \sin^2 3A]$

$= \frac{1}{4}[3 \cos(A - 3A) + \cos 2.3A]$

$= \frac{1}{4}(3\cos 2A + \cos 6A)$

$= \frac{1}{4}(3\cos 2A + \cos 3.2A)$

$= \frac{1}{4}(3\cos 2A + 4 \cos^3 2A - 3\cos 2A)$

$= \frac{1}{4} \times 4 \cos^3 2A$

$= \cos^3 2A = \text{RHS}$ **Proved.**

(d) $4(\cos^3 20° + \sin^3 50°) = 3(\cos 20° + \sin 50°)$

⇨ Here, LHS

$= 4(\cos^3 20° + \sin^3 50°)$

$= 4\cos^3 20° + 4\sin^3 50°$

$= \cos 3 \times 20° + 3 \cos 20° + 3 \sin 50° - \sin 3 \times 50°$

$= \cos 60° + 3(\cos 20° + \sin 50°) - \sin 150°$

$= \frac{1}{2} + 3 (\cos 20° + \sin 50°) - \frac{1}{2}$

$= 3 (\cos 20° + \sin 50°) = \text{RHS}$ **Proved.**

(e) $\tan A + \tan\left(\frac{\pi}{3} + A\right) + \tan\left(\frac{2\pi}{3} - A\right) = \tan A$

⇨ Here, LHS

$= \tan A + \tan\left(\frac{\pi}{3} + A\right) + \tan\left(\frac{2\pi}{3} - A\right)$

$= \tan A + \tan(60° + A) + \tan(120° - A)$

$= \tan A + \frac{\tan 60° + \tan A}{1 - \tan 60° \tan A} + \frac{\tan 120° - \tan A}{1 + \tan 120° \tan A}$

$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} + \frac{-\sqrt{3} - \tan A}{1 - \sqrt{3} \tan A}$

$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} - \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A}$

$= \tan A + 0$

$= \tan A$

$= \text{RHS}$ **Proved.**

CORRECTED QUESTION:

$\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

⇨ Here, LHS $= \tan A + \tan(60° + A) - \tan(60° - A)$

$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3} \tan A}$

$= \tan A + \frac{8\tan A}{1 - 3 \tan^2 A}$

$= \frac{9 \tan A - 3\tan^3 A}{1 - 3\tan^2 A}$

$= \frac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A}$

$= 3\tan 3A$

$= \text{RHS}$ **Proved.**

9. प्रमाणित गर्नुहोस् (Prove that):

(a) $\sin^4 A = \frac{1}{8}(3 - 4 \cos 2A + \cos 4A)$

⇨ Here,

LHS $= \sin^4 A$

$= \left(\frac{2\sin^2 A}{2}\right)^2$

$= \frac{1}{4}(2 \sin^2 A)^2$

$= \frac{1}{4}(1 - \cos 2A)^2$

$= \frac{1}{4}(1 - 2\cos 2A + \cos^2 2A)$

$= \frac{1}{4} \times \frac{1}{2}(2 - 4 \cos 2A + 2 \cos^2 2A)$

$= \frac{1}{8}(2 - 4\cos 2A + 1 + \cos 4A)$

$= \frac{1}{8}(3 - 4\cos 2A + \cos 4A)$

$= \text{RHS}$ **Proved.**

(b) $\cos^4 A = \frac{1}{8}(3 + 4 \cos 2A + \cos 4A)$

⇨ Here,

LHS $= \cos^4 A$

$= \left(\frac{2\cos^2 A}{2}\right)^2$

$= \frac{(1 + \cos 2A)^2}{4}$

$= \frac{1}{4}(1 + 2\cos 2A + \cos^2 2A)$

$= \frac{1}{4} \times \frac{1}{2}(2 + 4 \cos 2A + 2 \cos^2 2A)$

$= \frac{1}{8}(2 + 4\cos 2A + 1 + \cos 4A)$

$= \frac{1}{8}(3 + 4\cos 2A + \cos 4A)$

$= \text{RHS}$ **Proved.**

(c) $\cos 5A = 16 \cos^5 A - 20 \cos^3 A + 5 \cos A$

⇨ Here, LHS $= \cos 5A = \cos (3A + 2A)$

$= \cos 3A \cos 2A - \sin 3A \sin 2A$

$= (4 \cos^3 A - 3 \cos A) \cos 2A - (3\sin A - 4\sin^3 A) \sin 2A$

$= (4\cos^3 A - 3 \cos A) (2 \cos^2 A - 1) - (3\sin A - 4 \sin^3 A)\ 2\sin A \cos A$

$= 8 \cos^5 A - 4 \cos^3 A - 6 \cos^3 A + 3 \cos A - (3 - 4 \sin^2 A) 2 \sin^2 A \cos A$

$= (8 \cos^5 A - 10 \cos^3 A + 3 \cos A) - \{3 - 4(1 - \cos^2 A)\} 2(1 - \cos^2 A) \cos A$

$= 8 \cos^5 A - 10 \cos^3 A + 3 \cos A - (3 - 4 + 4 \cos^2 A) (2 \cos A - 2 \cos^3 A)$

$= 8 \cos^5 A - 10 \cos^3 A + 3 \cos A - (4\cos^2 A - 1) (2\cos A - 2 \cos^3 A)$

$= 8 \cos^5 A - 10 \cos^3 A + 3 \cos A - (8 \cos^3 A - 8 \cos^5 A - 2 \cos A + 2 \cos^3 A)$

$= 8 \cos^5 A - 10 \cos^3 A + 3 \cos A - (10 \cos^3 A - 8 \cos^5 A - 2 \cos A)$

$= 8 \cos^5 A - 10 \cos^3 A + 3 \cos A - 10 \cos^3 A + 8 \cos^5 A + 2 \cos A$

$= 16 \cos^5 A - 20 \cos^3 A + 5 \cos A = \text{RHS}$ **Proved.**

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(d) $\sin 5A = 16 \sin^5 A - 20 \sin^3 A + 5 \sin A$

⇨ Here, LHS $= \sin 5A$

$= \sin (3A + 2A)$

$= \sin 3A \cos 2A + \cos 3A \sin 2A$

$= (3 \sin A - 4 \sin^3 A) \cos 2A + (4 \cos^3 A - 3 \cos A) \sin 2A$

$= (3 \sin A - 4 \sin^3 A)\ (1 - 2 \sin^2 A) + (4 \cos^3 A - 3 \cos A)\ 2 \sin A \cos A$

$= (3 \sin A - 4 \sin^3 A) (1 - 2 \sin^2 A) + (4 \cos^2 A - 3)\ 2 \sin A \cos^2 A$

$= (3 \sin A - 4 \sin^3 A) (1 - 2 \sin^2 A) + \{4(1 - \sin^2 A) - 3\}\ 2 \sin A(1 - \sin^2 A)$

$= (3 \sin A - 4 \sin^3 A) (1 - 2 \sin^2 A) + (4 - 4 \sin^2 A - 3) (2 \sin A - 2 \sin^3 A)$

$= (3 \sin A - 4 \sin^3 A) (1 - 2 \sin^2 A) + (1 - 4 \sin^2 A) (2 \sin A - 2 \sin^3 A)$

$= 3 \sin A - 6 \sin^3 A - 4 \sin^3 A + 8 \sin^5 A + 2 \sin A - 2 \sin^3 A - 8 \sin^3 A + 8 \sin^5 A$

$= 16 \sin^5 A - 20 \sin^3 A + 5 \sin A =$ RHS **Proved.**

10. sine र cosine का अपवर्त्यकोणहरूका सम्बन्धहरू प्रयोग गरी sin 18°, sin 36° र sin 54° का मानहरू पत्ता लगाउनुहोस् । ती मानहरू प्रयोग गरी cos 18°, cos 36° र cos 54° तथा tan 18°, tan 36° र tan 54° का मानहरूसमेत पत्ता लगाई परीक्षण गरी एउटा प्रतिवेदन तयार गर्नुहोस् ।

Using the relations of multiple angles of sine and cosine, find the values of sin 18°, sin 36° and sin 54°. Find the values of cos 18°, cos 36° and cos 54° along with tan 18°, tan 36° and tan 54° using those relations and prepare a report on it.

⇨ Here,

(i) Let $\theta = 18°$ then $5\theta = 90°$

or, $2\theta + 3\theta = 90°$

or, $2\theta = 90° - 3\theta$

or, $\sin 2\theta = \sin (90° - 3\theta)$

or, $2 \sin \theta \cos \theta = \cos 3\theta$

or, $2 \sin \theta \cos \theta = 4 \cos^3\theta - 3 \cos \theta$

or, $2 \sin \theta \cos \theta - 4 \cos^3\theta + 3 \cos \theta = 0$

or, $\cos \theta\ (2 \sin \theta - 4 \cos^2 \theta + 3) = 0$

or, $2 \sin \theta - 4 \cos^2 \theta + 3 = 0$

$[\because \cos \theta = \cos 18° \neq 0]$

or, $2 \sin \theta - 4(1 - \sin^2 \theta) + 3 = 0$

or, $2 \sin \theta - 4 + 4 \sin^2 \theta + 3 = 0$

or, $4 \sin^2\theta + 2 \sin \theta - 1 = 0$

Comparing it with $ax^2 + bx + c = 0$ then,

$x = \sin \theta,\ a = 4,\ b = 2$ and $c = -1$

So, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

or, $\sin \theta = \dfrac{-2 \pm \sqrt{4 + 16}}{8} = \dfrac{-2 \pm \sqrt{20}}{8} = \dfrac{-2 \pm 2\sqrt{5}}{8} = \dfrac{\pm 2\sqrt{5} - 2}{8} = \dfrac{2\ (\pm \sqrt{5} - 1)}{8} = \dfrac{\pm \sqrt{5} - 1}{4}$

or, $\sin 18° = \dfrac{\sqrt{5} - 1}{4}$ $[\because \sin \theta = \sin 18° > 0]$

(ii) We have, $\cos^2 18° = 1 - \sin^2 18° = 1 - \left(\dfrac{\sqrt{5} - 1}{4}\right)^2 = 1 - \dfrac{(\sqrt{5} - 1)^2}{16} = \dfrac{16 - (5 - 2\sqrt{5} + 1)}{16} = \dfrac{16 - 5 + 2\sqrt{5} - 1}{16}$

or, $\cos^2 18° = \dfrac{10 + 2\sqrt{5}}{16}$ $\therefore\ \cos 18° = \dfrac{\sqrt{10 + 2\sqrt{5}}}{4}$

(iii) We have, $\cos 36° \cos 2.\ 18° = 1 - 2 \sin^2 18° = 1 - 2 \left(\dfrac{\sqrt{5} - 1}{4}\right)^2 = 1 - 2 \dfrac{(\sqrt{5} - 1)^2}{16} = 1 - \dfrac{(5 - 2\sqrt{5} + 1)}{8}$

$= 1 - \dfrac{6 - 2\sqrt{5}}{8} = \dfrac{8 - 6 + 2\sqrt{5}}{8} = \dfrac{2 + 2\sqrt{5}}{8}$ $\therefore\ \cos 36° = \dfrac{1 + \sqrt{5}}{4}$

(iv) We have, $\sin 36° = \sqrt{1 - \cos^2 36°} = \left\{1 - \left(\dfrac{\sqrt{5} + 1}{4}\right)^2\right\}^{1/2} = \left(1 - \dfrac{1 + 2\sqrt{5} + 5}{16}\right)^{1/2} = \left(1 - \dfrac{6 + 2\sqrt{5}}{16}\right)^{1/2}$

$= \left(\dfrac{16 - 6 - 2\sqrt{5}}{16}\right)^{1/2} = \left(\dfrac{10 - 2\sqrt{5}}{16}\right)^{1/2} = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$ $\therefore\ \sin 36° = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$

(v) $\sin 54° = \sin(90° - 36°) = \cos 36°$ $\therefore\ \sin 54° = \dfrac{\sqrt{5} + 1}{4}$

(vi) $\cos 54° = \cos (90° - 36°) = \sin 36°$ $\therefore\ \cos 54° = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$

(vii) $\tan 18° = \dfrac{\sin 18°}{\cos 18°} = \dfrac{\dfrac{\sqrt{5} - 1}{4}}{\dfrac{\sqrt{10 + 2\sqrt{5}}}{4}} = \dfrac{\sqrt{5} - 1}{\sqrt{10 + 2\sqrt{5}}}$

(viii) $\tan 36° = \dfrac{\sin 36°}{\cos 36°} = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4} \times \dfrac{4}{\sqrt{5} + 1} = \dfrac{\sqrt{10 - 2\sqrt{5}}}{\sqrt{5} + 1}$

(ix) $\tan 54° = \dfrac{\sin 54°}{\cos 54°} = \dfrac{\sqrt{5} + 1}{4} \div \dfrac{\sqrt{10 - 2\sqrt{5}}}{4} = \dfrac{\sqrt{5} + 1}{\sqrt{10 - 2\sqrt{5}}}$

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5.2 अपवर्तक कोणको त्रिकोणमितीय अनुपातहरू
TRIGONOMETRIC RATIOS OF SUB-MULTIPLE ANGLES

EXERCISE 5.2

1. (a) **अपवर्तक कोण भनेको के हो ? उदाहरणसहित लेख्नुहोस् । (What do you mean by sub-multiple angles? Write with examples.)**

⇨ Here, if A be a given angle then $\frac{A}{2}, \frac{A}{3}, \frac{A}{4}$ etc are called submultiple angles of the angle A.

Example:- If 72° be an angle then 36°, 24°, 18°, 9° etc are the submultiple angles of 72°.

(b) **sin A, cos A र tan A का अनुपातहरू $\frac{A}{2}$ को रूपमा लेख्नुहोस् । (Write the ratio of sin A, cos A and tan A in terms of $\frac{A}{2}$.)**

⇨ Here, (i) sin A

The ratios of sin A in terms of $\frac{A}{2}$ are as follows:

(a) $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$ (b) $\sin A = \dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$ (c) $\sin A = \dfrac{2\cot\frac{A}{2}}{1+\cot^2\frac{A}{2}}$

(ii) cos A

The ratios of cos A in terms of $\frac{A}{2}$ are as follows:

(a) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$ (b) $\cos A = 2\cos^2\frac{A}{2} - 1$ (c) $\cos A = 1 - 2\sin^2\frac{A}{2}$

(d) $\cos A = \dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$ (e) $\cos A = \dfrac{\cot^2\frac{A}{2}-1}{\cot^2\frac{A}{2}+1}$

(iii) tan A

The ratios of tan A interms of $\frac{A}{2}$ are as follows: $\tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

(c) **sin A, cos A र tan A का अनुपातहरू $\frac{A}{3}$ का रूपमा के के हुन्छन्, लेख्नुहोस् ।**

Write the ratio of sin A, cos A and tan A in terms of $\frac{A}{3}$.

⇨ Here, (i) $\sin A = 3\sin\frac{A}{3} - 4\sin^3\frac{A}{3}$ (ii) $\cos A = 4\cos^3\frac{A}{3} - 3\cos\frac{A}{3}$

(iii) $\tan A = \dfrac{3\tan\frac{A}{3} - \tan^3\frac{A}{3}}{1-3\tan^2\frac{A}{3}}$

2. (a) **यदि $\tan\frac{\theta}{2} = \frac{4}{3}$ भए sin θ, cos θ र tan θ को मान पत्ता लगाउनुहोस् । (If $\tan\frac{\theta}{2} = \frac{4}{3}$, find the value of sin θ, cos θ and tan θ.)**

⇨ Here, $\tan\frac{\theta}{2} = \frac{4}{3}$

$$\text{So, } \sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{2\times\frac{4}{3}}{1+\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1+\frac{16}{9}} = \frac{\frac{8}{3}}{\frac{25}{9}} = \frac{8}{3}\times\frac{9}{25} = \frac{24}{25}$$

$$\text{Again, } \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1-\left(\frac{4}{3}\right)^2}{1+\left(\frac{4}{3}\right)^2} = \frac{1-\frac{16}{9}}{1+\frac{16}{9}} = \frac{\frac{-7}{9}}{\frac{25}{9}} = -\frac{7}{25}$$

$$\text{Now, } \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2\times\frac{4}{3}}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} = -\frac{8}{3}\times\frac{9}{7} = -\frac{24}{7}$$

Thus, $\sin\theta = \frac{24}{25}$, $\cos\theta = -\frac{7}{25}$ and $\tan\theta = -\frac{24}{7}$.

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(b) यदि $\cos\frac{\theta}{2} = \frac{4}{5}$ भए $\cos\theta$, $\sin\theta$ र $\tan\theta$ को मान पत्ता लगाउनुहोस् ।(If $\cos\frac{\theta}{2} = \frac{4}{5}$, find the value of $\cos\theta$, $\sin\theta$ and $\tan\theta$.)

⇨ Here, $\cos\frac{\theta}{2} = \frac{4}{5}$

We have, $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 2\left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{32-25}{25} = \frac{7}{25}$

Again, $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{7}{25}\right)^2} = \sqrt{1-\frac{49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$

Now, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$

Thus, $\sin\theta = \frac{24}{25}$, $\cos\theta = \frac{7}{25}$ and $\tan\theta = \frac{24}{7}$.

(c) यदि $\sin\frac{\theta}{2} = \frac{1}{2}$ भए $\sin\theta$, $\cos\theta$ र $\tan\theta$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\theta}{2} = \frac{1}{2}$, find the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

⇨ Here, $\sin\frac{\theta}{2} = \frac{1}{2}$

or, $\sin\frac{\theta}{2} = \sin 30°$

or, $\frac{\theta}{2} = 30°$

∴ $\theta = 60°$

Now, $\sin\theta = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos\theta = \cos 60° = \frac{1}{2}$

$\tan\theta = \tan 60° = \sqrt{3}$

Thus, $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$ and $\tan\theta = \sqrt{3}$.

(e) यदि $\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$ भए $\cos\theta$, $\sin\theta$ र $\tan\theta$ को मान पत्ता लगाउनुहोस् ।

If $\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$, find the value of $\cos\theta$, $\sin\theta$ and $\tan\theta$.

⇨ Here, $\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$

or, $\cos\frac{\theta}{2} = \cos 30°$

or, $\frac{\theta}{2} = 30°$

∴ $\theta = 60°$

Now, $\cos\theta = \cos 60° = \frac{1}{2}$

$\sin\theta = \sin 60° = \frac{\sqrt{3}}{2}$

$\tan\theta = \tan 60° = \sqrt{3}$

Thus, $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = \sqrt{3}$.

(d) यदि $\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}$ भए $\sin\theta$ र $\cos\theta$ को मान पत्ता लगाउनुहोस् ।

If $\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}$, find the value of $\sin\theta$ and $\cos\theta$.

⇨ Here, $\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}$

or, $\sin\frac{\theta}{2} = \sin 45°$

or, $\frac{\theta}{2} = 45°$

∴ $\theta = 90°$

Now, $\sin\theta = \sin 90° = 1$

$\cos\theta = \cos 90° = 0$

Thus, $\sin\theta = 1$ and $\cos\theta = 0$.

(f) यदि $\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}$ भए $\sin\theta$, $\cos\theta$ र $\tan\theta$ को मान पत्ता लगाउनुहोस् ।

If $\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}$, find the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

⇨ Here, $\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}$

or, $\tan\frac{\theta}{2} = \tan 30°$

or, $\frac{\theta}{2} = 30°$

∴ $\theta = 60°$

Now, $\sin\theta = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos\theta = \cos 60° = \frac{1}{2}$

$\tan\theta = \tan 60° = \sqrt{3}$

Thus, $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$ and $\tan\theta = \sqrt{3}$.

3. (a) यदि $\sin\frac{\alpha}{3} = \frac{1}{2}$ भए $\sin\alpha$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{\alpha}{3} = \frac{1}{2}$ then find the value of $\sin\alpha$.)

⇨ Here, $\sin\frac{\alpha}{3} = \frac{1}{2}$

or, $\sin\frac{\alpha}{3} = \sin 30°$

or, $\frac{\alpha}{3} = 30°$

∴ $\alpha = 90°$

Now, $\sin\alpha = \sin 90° = 1$

Thus, the value of $\sin\alpha$ is 1.

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(b) **यदि $\cos\frac{\alpha}{3}=\frac{1}{\sqrt{2}}$ भए $\cos\alpha$ को मान पत्ता लगाउनुहोस् ।**

If $\cos\frac{\alpha}{3}=\frac{1}{\sqrt{2}}$ then find the value of $\cos\alpha$.

⇨ Here, $\cos\frac{\alpha}{3}=\frac{1}{\sqrt{2}}$

or, $\cos\frac{\alpha}{3}=\cos 45°$

or, $\frac{\alpha}{3}=45°$

$\therefore\ \alpha=135°$

$$\begin{aligned}\text{Now, } \cos\alpha &= \cos 135°\\ &= \cos(90°+45°)\\ &= -\sin 45° = -\frac{1}{\sqrt{2}}\end{aligned}$$

Thus, the value of $\cos\alpha$ is $-\frac{1}{\sqrt{2}}$.

(c) **यदि $\tan\frac{\theta}{3}=\frac{1}{5}$ भए $\tan\theta$ को मान पत्ता लगाउनुहोस् ।**

If $\tan\frac{\theta}{3}=\frac{1}{5}$ then find the value of $\tan\theta$.

⇨ Here, $\tan\frac{\theta}{3}=\frac{1}{5}$

We know that,

$$\tan\theta = \frac{3\tan\frac{\theta}{3}-\tan^3\frac{\theta}{3}}{1-3\tan^2\frac{\theta}{3}}$$

$$=\frac{3\times\frac{1}{5}-\left(\frac{1}{5}\right)^3}{1-3\times\left(\frac{1}{5}\right)^2}=\frac{\frac{3}{5}-\frac{1}{125}}{1-\frac{3}{25}}=\frac{\frac{74}{125}}{\frac{22}{25}}=\frac{37}{55}$$

Thus, the value of $\tan\theta$ is $\frac{37}{55}$.

(d) **यदि $\sin\frac{\theta}{3}=\frac{1}{2}\left(m+\frac{1}{m}\right)$ भए $\sin\theta$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{\theta}{3}=\frac{1}{2}\left(m+\frac{1}{m}\right)$ then find the value of $\sin\theta$.)**

⇨ Here, $\sin\frac{\theta}{3}=\frac{1}{2}\left(m+\frac{1}{m}\right)$

We know that, $\sin\theta = 3\sin\frac{\theta}{3}-4\sin^3\frac{\theta}{3}$

$$\sin\theta = 3\times\frac{1}{2}\left(m+\frac{1}{m}\right)-4\left\{\frac{1}{2}\left(m+\frac{1}{m}\right)\right\}^3 \qquad =\frac{3}{2}\left(m+\frac{1}{m}\right)-4\times\frac{1}{8}\left(m+\frac{1}{m}\right)^3$$

$$=\frac{3}{2}\left(m+\frac{1}{m}\right)-\frac{1}{2}\left(m+\frac{1}{m}\right)^3 \qquad =\frac{1}{2}\left[3\left(m+\frac{1}{m}\right)-\left(m+\frac{1}{m}\right)^3\right]$$

$$=\frac{1}{2}\left[3\left(m+\frac{1}{m}\right)-\left[m^3+\left(\frac{1}{m}\right)^3+3\,.\,m\,.\,\frac{1}{m}\left(m+\frac{1}{m}\right)\right]\right]$$

$$=\frac{1}{2}\left[3\left(m+\frac{1}{m}\right)-\left(m^3+\frac{1}{m^3}\right)-3\left(m+\frac{1}{m}\right)\right]$$

$$=\frac{1}{2}(-)\left(m^3+\frac{1}{m^3}\right)$$

$$\therefore\ \sin\theta=-\frac{1}{2}\left(m^3+\frac{1}{m^3}\right)$$

Thus, the value of $\sin\theta$ is $-\frac{1}{2}\left(m^3+\frac{1}{m^3}\right)$.

(e) **यदि $\cos\frac{\theta}{3}=\frac{1}{2}\left(p+\frac{1}{p}\right)$ भए $\cos\theta$ को मान पत्ता लगाउनुहोस् । (If $\cos\frac{\theta}{3}=\frac{1}{2}\left(p+\frac{1}{p}\right)$ then find the value of $\cos\theta$.)**

⇨ Here, $\cos\frac{\theta}{3}=\frac{1}{2}\left(p+\frac{1}{p}\right)$

$$\begin{aligned}\text{We have, } \cos\theta &= 4\cos^3\frac{\theta}{3}-3\cos\frac{\theta}{3}\\ &=4\left\{\frac{1}{2}\left(p+\frac{1}{p}\right)\right\}^3-3\times\frac{1}{2}\left(p+\frac{1}{p}\right)\\ &=4\times\frac{1}{8}\left(p+\frac{1}{p}\right)^3-\frac{3}{2}\left(p+\frac{1}{p}\right)\\ &=\frac{1}{2}\left(p+\frac{1}{p}\right)^3-\frac{3}{2}\left(p+\frac{1}{p}\right)\\ &=\frac{1}{2}\left[\left(p+\frac{1}{p}\right)^3-3\,.\,p\,.\,\frac{1}{p}\left(p+\frac{1}{p}\right)\right]\\ &=\frac{1}{2}\left[(p)^3+\left(\frac{1}{p}\right)^3\right]\end{aligned}$$

$$\therefore\ \cos\theta=\frac{1}{2}\left(p^3+\frac{1}{p^3}\right)$$

Thus, the value of $\cos\theta$ is $\frac{1}{2}\left(p^3+\frac{1}{p^3}\right)$.

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4. (a) यदि cos 30° = $\frac{\sqrt{3}}{2}$ भए प्रमाणित गर्नुहोस् (If cos 30° = $\frac{\sqrt{3}}{2}$ then prove that):

(i) $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

⇨ Here, cos 15°

We have,

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad 2\cos^2\frac{30°}{2} - 1 = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad 2\cos^2 15° = \frac{\sqrt{3}}{2} + 1$$

$$\text{or,}\quad 2\cos^2 15° = \frac{\sqrt{3}+2}{2}$$

$$\text{or,}\quad \cos^2 15° = \frac{\sqrt{3}+2}{4}$$

$$\text{or,}\quad \cos^2 15° = \frac{2\sqrt{3}+4}{4.2}$$

$$\text{or,}\quad \cos^2 15° = \frac{3+2\sqrt{3}+1}{8}$$

$$\text{or,}\quad \cos^2 15° = \frac{(\sqrt{3})^2 + 2\sqrt{3} + (1)^2}{(2\sqrt{2})^2}$$

$$\text{or,}\quad \cos^2 15° = \frac{(\sqrt{3}+1)^2}{(2\sqrt{2})^2}$$

$$\therefore\quad \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Proved.

(iii) $\tan 15° = 2 - \sqrt{3}$

⇨ Here, tan 15°

We have,

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad \frac{1-\tan^2\frac{30°}{2}}{1+\tan^2\frac{30°}{2}} = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad \frac{1-\tan^2 15°}{1+\tan^2 15°} = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad 2 - 2\tan^2 15° = \sqrt{3} + \sqrt{3}\tan^2 15°$$

$$\text{or,}\quad 2 - \sqrt{3} = 2\tan^2 15° + \sqrt{3}\tan^2 15°$$

$$\text{or,}\quad \tan^2 15°.(2+\sqrt{3}) = 2 - \sqrt{3}$$

$$\text{or,}\quad \tan^2 15° = \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

$$\text{or,}\quad \tan^2 15° = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\text{or,}\quad \tan^2 15° = \frac{(2-\sqrt{3})^2}{4-3}$$

$$\text{or,}\quad \tan^2 15° = \frac{(2-\sqrt{3})^2}{1}$$

$$\therefore\quad \tan 15° = 2 - \sqrt{3}$$

Proved.

(ii) $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

⇨ Here, sin 15°

We have,

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad 1 - 2\sin^2\frac{30}{2} = \frac{\sqrt{3}}{2}$$

$$\text{or,}\quad -2\sin^2 15° = \frac{\sqrt{3}}{2} - 1$$

$$\text{or,}\quad -2\sin^2 15° = \frac{\sqrt{3}-2}{2}$$

$$\text{or,}\quad -\sin^2 15° = \frac{\sqrt{3}-2}{4}$$

$$\text{or,}\quad -\sin^2 15° = \frac{2\sqrt{3}-4}{8}$$

$$\text{or,}\quad \sin^2 15° = \frac{4-2\sqrt{3}}{8}$$

$$\text{or,}\quad \sin^2 15° = \frac{3-2\sqrt{3}+1}{8}$$

$$\text{or,}\quad \sin^2 15° = \frac{(\sqrt{3}-1)^2}{(2\sqrt{2})^2}$$

$$\therefore\quad \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Proved.

(b) यदि cos 45° = $\frac{1}{\sqrt{2}}$ भए प्रमाणित गर्नुहोस्

If cos 45° = $\frac{1}{\sqrt{2}}$ then prove that :

(i) $\sin 22\frac{1}{2}° = \frac{1}{2}\sqrt{2-\sqrt{2}}$

⇨ Here, we have,

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\text{or,}\quad 1 - 2\sin^2\frac{45°}{2} = \frac{1}{\sqrt{2}}$$

$$\text{or,}\quad -2\sin^2\frac{45°}{2} = \frac{1}{\sqrt{2}} - 1$$

$$\text{or,}\quad 2\sin^2\frac{45°}{2} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\text{or,}\quad \sin^2\frac{45°}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

Multiplying numerator and denominator by $\sqrt{2}$.

$$\text{or,}\quad \sin^2\frac{45°}{2} = \frac{2-\sqrt{2}}{2.2}$$

$$\text{or,}\quad \sin\frac{45°}{2} = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

$$\therefore\quad \sin 22\frac{1°}{2} = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

Proved.

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(ii) $\cos 22\frac{1}{2}^\circ = \frac{1}{2}\sqrt{2+\sqrt{2}}$

⇨ Here, we have,

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

or, $2\cos^2\frac{45^\circ}{2} - 1 = \frac{1}{\sqrt{2}}$

or, $2\cos^2\frac{45^\circ}{2} = \frac{1}{\sqrt{2}} + 1$

or, $2\cos^2\frac{45^\circ}{2} = \frac{1+\sqrt{2}}{\sqrt{2}}$

Multiplying numerator and denominator by $\sqrt{2}$.

or, $2\cos^2\frac{45^\circ}{2} = \frac{\sqrt{2}+2}{2}$

or, $\cos^2\frac{45^\circ}{2} = \frac{2+\sqrt{2}}{4}$

or, $\cos\frac{45^\circ}{2} = \sqrt{\frac{2+\sqrt{2}}{4}}$

∴ $\cos 22\frac{1^\circ}{2} = \frac{1}{2}\sqrt{2+\sqrt{2}}$ **Proved.**

5. प्रमाणित गर्नुहोस् (Prove that):

(a) $\frac{\sin A}{1+\cos A} = \tan\frac{A}{2}$

⇨ Here,

$$\text{LHS} = \frac{\sin A}{1+\cos A} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{1+2\cos^2\frac{A}{2}-1} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2} = \text{RHS}$$

Proved.

(c) $1 + \sin A = \left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2$

⇨ Here, LHS $= 1 + \sin A$

$= \sin^2\frac{A}{2} + \cos^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}$

$= \left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2 = \text{RHS}$ **Proved.**

(d) $1 - \sin A = \left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)^2$

⇨ Here, LHS $= 1 - \sin A$

$= \sin^2\frac{A}{2} + \cos^2\frac{A}{2} - 2\sin\frac{A}{2}\cos\frac{A}{2}$

$= \left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)^2$

$=$ RHS **Proved.**

(iii) $\tan 22\frac{1^\circ}{2} = \sqrt{3-2\sqrt{2}}$

⇨ Here, we have,

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

or, $\frac{1-\tan^2\frac{45^\circ}{2}}{1+\tan^2\frac{45^\circ}{2}} = \frac{1}{\sqrt{2}}$

or, $\sqrt{2} - \sqrt{2}\tan^2\frac{45^\circ}{2} = 1 + \tan^2\frac{45^\circ}{2}$

or, $\sqrt{2} - 1 = \sqrt{2}\tan^2\frac{45^\circ}{2} + \tan^2\frac{45^\circ}{2}$

or, $\tan^2\frac{45^\circ}{2} \times (\sqrt{2}+1) = \sqrt{2} - 1$

or, $\tan^2\frac{45^\circ}{2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

or, $\tan^2\frac{45^\circ}{2} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

or, $\tan^2\frac{45^\circ}{2} = \frac{(\sqrt{2}-1)^2}{2-1}$

or, $\tan^2\frac{45^\circ}{2} = \frac{2-2\sqrt{2}+1}{2}$

or, $\tan^2\frac{45^\circ}{2} = 3 - 2\sqrt{2}$

or, $\tan\frac{45^\circ}{2} = \sqrt{3-2\sqrt{2}}$

∴ $\tan 22\frac{1^\circ}{2} = \sqrt{3-2\sqrt{2}}$ **Proved.**

5. (b) $\frac{\sin A}{1-\cos A} = \cot\frac{A}{2}$

⇨ Here, LHS $= \frac{\sin A}{1-\cos A}$

$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{1-\left(1-2\sin^2\frac{A}{2}\right)}$

$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{1-1+2\sin^2\frac{A}{2}}$

$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin\frac{A}{2}\sin\frac{A}{2}}$

$= \cot\frac{A}{2} = \text{RHS}$ **Proved.**

(e) $\frac{1-\cos\alpha}{1+\cos\alpha} = \tan^2\frac{\alpha}{2}$

⇨ Here, LHS $= \frac{1-\cos\alpha}{1+\cos\alpha}$

$= \frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}} = \tan^2\frac{\alpha}{2}$

$=$ RHS **Proved.**

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(f) $\dfrac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta}=\cot\dfrac{\theta}{2}$

⇨ Here,

$$\text{LHS}=\frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta}$$

$$=\frac{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}$$

$$=\cot\frac{\theta}{2}=\text{RHS}$$

Proved.

(h) $2\operatorname{cosec}\theta=\cot\dfrac{\theta}{2}+\tan\dfrac{\theta}{2}$

⇨ Here, LHS $=2\operatorname{cosec}\theta$

$$=2\times\frac{1}{\sin\theta}$$

$$=\frac{2}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{\sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}+\frac{\cos^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$=\tan\frac{\theta}{2}+\cot\frac{\theta}{2}=\text{RHS}$$

Proved.

(g) $\cot\dfrac{A}{2}-\tan\dfrac{A}{2}=2\cot A$

⇨ Here,

$$\text{LHS}=\cot\frac{A}{2}-\tan\frac{A}{2}$$

$$=\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}-\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}=\frac{\cos^2\frac{A}{2}-\sin^2\frac{A}{2}}{\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=\frac{\cos A}{\frac{1}{2}2\sin\frac{A}{2}\cos\frac{A}{2}}=\frac{\cos A}{\frac{1}{2}.\sin A}$$

$$=2.\frac{\cos A}{\sin A}$$

$$=2.\cot A=\text{RHS}$$

Proved.

(i) $\dfrac{\cos^3\frac{\theta}{2}-\sin^3\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}=1+\dfrac{1}{2}\sin\theta$

⇨ Here, LHS

$$=\frac{\cos^3\frac{\theta}{2}-\sin^3\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$=\frac{\left(\cos\frac{\theta}{2}\right)^3-\left(\sin\frac{\theta}{2}\right)^3}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$=\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}+\cos\frac{\theta}{2}\sin\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)}$$

$$=\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+\cos\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$=1+\frac{1}{2}\left(2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)$$

$$=1+\frac{1}{2}\sin\theta=\text{RHS}$$

Proved.

(j) $\dfrac{1-\sec\alpha}{\tan\alpha}=-\tan\dfrac{\alpha}{2}$

⇨ Here, LHS $=\dfrac{1-\sec\alpha}{\tan\alpha}$

$$=\frac{1-\frac{1}{\cos\alpha}}{\frac{\sin\alpha}{\cos\alpha}}=\frac{\frac{\cos\alpha-1}{\cos\alpha}}{\frac{\sin\alpha}{\cos\alpha}}$$

$$=\frac{\cos\alpha-1}{\sin\alpha}=\frac{-(1-\cos\alpha)}{\sin\alpha}$$

$$=-\frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=-\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$$=-\tan\frac{\alpha}{2}=\text{RHS}$$

Proved.

[Note: question is corrected.]

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6. प्रमाणित गर्नुहोस् (Prove that):

(a) $\tan\left(\frac{\pi}{4}+\frac{A}{2}\right)=\sqrt{\frac{1+\sin A}{1-\sin A}}$

⇨ Here,

$$\text{LHS} = \tan\left(\frac{\pi}{4}+\frac{A}{2}\right)=\frac{\sin\left(45°+\frac{A}{2}\right)}{\cos\left(45°+\frac{A}{2}\right)}$$

$$= \frac{\sin 45°\cos\frac{A}{2}+\cos 45°\sin\frac{A}{2}}{\cos 45°\cos\frac{A}{2}-\sin 45°\sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\cos\frac{A}{2}+\frac{1}{\sqrt{2}}\sin\frac{A}{2}}{\frac{1}{\sqrt{2}}\cos\frac{A}{2}-\frac{1}{\sqrt{2}}\sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)}$$

$$= \frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}$$

$$= \frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}\times\frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}}$$

$$= \frac{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)^2}{\cos^2\frac{A}{2}-\sin^2\frac{A}{2}}$$

$$= \frac{\cos^2\frac{A}{2}+\sin^2\frac{A}{2}+2\cos\frac{A}{2}\sin\frac{A}{2}}{\cos A}$$

$$= \frac{1+\sin A}{\cos A}=\frac{1+\sin A}{\sqrt{1-\sin^2 A}}$$

$$= \frac{\sqrt{1+\sin A}\,\sqrt{1+\sin A}}{\sqrt{(1+\sin A)}\,\sqrt{(1-\sin A)}}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$= \text{RHS}$ **Proved.**

(b) $\tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\sqrt{\frac{1-\sin A}{1+\sin A}}$

⇨ Here, LHS $= \tan\left(\frac{\pi}{4}-\frac{A}{2}\right)$

$$= \frac{\sin\left(45°-\frac{A}{2}\right)}{\cos\left(45°-\frac{A}{2}\right)}$$

$$= \frac{\sin 45°\cos\frac{A}{2}-\cos 45°\sin\frac{A}{2}}{\cos 45°\cos\frac{A}{2}+\sin 45°\sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\cos\frac{A}{2}-\frac{1}{\sqrt{2}}\sin\frac{A}{2}}{\frac{1}{\sqrt{2}}\cos\frac{A}{2}+\frac{1}{\sqrt{2}}\sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)}{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}$$

$$= \frac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}}\times\frac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}$$

$$= \frac{\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)^2}{\cos^2\frac{A}{2}-\sin 2\frac{A}{2}}$$

$$= \frac{\cos^2\frac{A}{2}+\sin^2\frac{A}{2}-2\cos\frac{A}{2}\sin\frac{A}{2}}{\cos A}$$

$$= \frac{1-\sin A}{\cos A}$$

$$= \frac{1-\sin A}{\sqrt{1-\sin^2 A}}$$

$$= \frac{\sqrt{1-\sin A}\,\sqrt{1-\sin A}}{\sqrt{1+\sin A}\,\sqrt{1-\sin A}}$$

$$= \sqrt{\frac{1-\sin A}{1+\sin A}}$$

$= \text{RHS}$ **Proved.**

(c) $\sec\left(\frac{\pi}{4}+\frac{A}{2}\right)\sec\left(\frac{\pi}{4}-\frac{A}{2}\right)=2\sec A$

⇨ Here,

$$\text{LHS} = \sec\left(\frac{\pi}{4}+\frac{A}{2}\right)\sec\left(\frac{\pi}{4}-\frac{A}{2}\right)=\frac{1}{\cos\left(45°+\frac{A}{2}\right)}\times\frac{1}{\cos\left(45°-\frac{A}{2}\right)}$$

$$= \frac{1}{\cos 45°\cos\frac{A}{2}-\sin 45°\sin\frac{A}{2}}\times\frac{1}{\cos 45°\cos\frac{A}{2}+\sin 45°\sin\frac{A}{2}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}}\cos\frac{A}{2}-\frac{1}{\sqrt{2}}\sin\frac{A}{2}}\times\frac{1}{\frac{1}{\sqrt{2}}\cos\frac{A}{2}+\frac{1}{\sqrt{2}}\sin\frac{A}{2}} = \frac{1}{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)}\times\frac{1}{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}$$

$$= \frac{1}{\frac{1}{2}\left(\cos^2\frac{A}{2}-\sin^2\frac{A}{2}\right)}=\frac{2}{\cos A}=2\sec A=\text{RHS}$$

Proved.

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(d) $\tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\frac{\cos A}{1+\sin A}$

⇨ Here, LHS

$$= \tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\frac{\sin\left(45^\circ-\frac{A}{2}\right)}{\cos\left(45^\circ-\frac{A}{2}\right)}$$

$$= \frac{\sin 45^\circ \cos\frac{A}{2}-\cos 45^\circ \sin\frac{A}{2}}{\cos 45^\circ \cos\frac{A}{2}+\sin 45^\circ \sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\cos\frac{A}{2}-\frac{1}{\sqrt{2}}\sin\frac{A}{2}}{\frac{1}{\sqrt{2}}\cos\frac{A}{2}+\frac{1}{\sqrt{2}}\sin\frac{A}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)}{\frac{1}{\sqrt{2}}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}$$

$$= \frac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}}\times\frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}}$$

$$= \frac{\cos^2\frac{A}{2}-\sin^2\frac{A}{2}}{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)^2}$$

$$= \frac{\cos A}{\cos^2\frac{A}{2}+\sin^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= \frac{\cos A}{1+\sin A} = \text{RHS}$$

Proved.

(e) $\cot\left(\frac{A}{2}+\frac{\pi}{4}\right)-\tan\left(\frac{A}{2}-\frac{\pi}{4}\right)=\frac{2\cos A}{1+\sin A}$

⇨ Here, LHS

$$= \cot\left(\frac{A}{2}+45^\circ\right)-\tan\left(\frac{A}{2}-45^\circ\right)$$

$$= \frac{\cos\left(\frac{A}{2}+45^\circ\right)}{\sin\left(\frac{A}{2}+45^\circ\right)}-\frac{\sin\left(\frac{A}{2}-45^\circ\right)}{\cos\left(\frac{A}{2}-45^\circ\right)}$$

$$= \frac{\cos\left(\frac{A}{2}+45^\circ\right)\cos\left(\frac{A}{2}-45^\circ\right)-\sin\left(\frac{A}{2}+45^\circ\right)\sin\left(\frac{A}{2}-45^\circ\right)}{\sin\left(\frac{A}{2}+45^\circ\right)\cos\left(\frac{A}{2}-45^\circ\right)}$$

$$= \frac{\cos\left(\frac{A}{2}+45^\circ+\frac{A}{2}-45^\circ\right)}{\left(\sin\frac{A}{2}\cos 45^\circ+\cos\frac{A}{2}\sin 45^\circ\right)\left(\cos\frac{A}{2}\cos 45^\circ+\sin\frac{A}{2}\sin 45^\circ\right)}$$

$$= \frac{\cos A}{\left(\sin\frac{A}{2}\times\frac{1}{\sqrt{2}}+\cos\frac{A}{2}\times\frac{1}{\sqrt{2}}\right)\left(\cos\frac{A}{2}\times\frac{1}{\sqrt{2}}+\sin\frac{A}{2}\times\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{\cos A}{\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)}$$

$$= \frac{\cos A}{\frac{1}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)^2}$$

$$= \frac{2\cos A}{\sin^2\frac{A}{2}+\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= \frac{2\cos A}{1+\sin A} = \text{RHS}$$

Proved.

2. गुणन र योगफलको रूपान्तरण

(Transformation of Product and Sum)

Important formulae

त्रिकोणमितीय सूत्रहरूको स्थानान्तरण (Transformation of Trigonometric Formulae)

गुणनमा स्थानान्तरण (Transformation in product)	योगफलमा स्थानान्तरण (Transformation in su)m
1. $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$	1. $2\sin A.\cos B=\sin(A+B)+\sin(A-B)$
2. $\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$	2. $2\cos A.\sin B=\sin(A+B)-\sin(A-B)$
3. $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$	3. $2\cos A.\cos B=\cos(A+B)+\cos(A-B)$
4. $\cos C-\cos D=2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$	4. $2\sin A.\sin B=\cos(A-B)-\cos(A+B)$

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QUESTIONS FROM SEE EXERCISE 2

A. VERY SHORT QUESTIONS

1. $\cos x - \cos y$ लाई sine को गुणनफलको रूपमा बदल्नुहोस् । (Convert $\cos x - \cos y$ in terms of product of sine.)

⇨ Here, the required relation is; $\cos x - \cos y = 2 \sin \left(\frac{x+y}{2}\right) \sin \left(\frac{y-x}{2}\right)$

2. sin 2P + sin 2Q लाई गुणनको रूपमा व्यक्त गर्नुहोस् . (Express sin 2P + sin 2Q into product form.)

⇨ Here, the required relation is; $\sin 2P + \sin 2Q = 2 \sin \left(\frac{2P+2Q}{2}\right) \cos \left(\frac{2P-2Q}{2}\right)$

$\therefore \quad \sin 2P + \sin 2Q = 2 \sin (P+Q) \cos (P-Q)$

3. sin 2P – sin 2Q लाई गुणनको रूपमा व्यक्त गर्नुहोस् । (Express sin 2P – sin 2Q into product form.)

⇨ Here, $\sin 2P - \sin 2Q = 2 \cos \left(\frac{2P+2Q}{2}\right) \sin \left(\frac{2P-2Q}{2}\right)$

$\therefore \quad \sin 2P - \sin 2Q = 2 \cos (P+Q) \sin (P-Q)$

4. 2 sin 2A cos 2B लाई A + B र A – B को रूपमा व्यक्त गर्नुहोस् ।

Express 2 sin 2A cos 2B interms of A + B and A – B.

⇨ Here, $2 \sin 2A \cos 2B = \sin (2A + 2B) + \sin (2A - 2B) = \sin 2(A+B) + \sin 2(A-B)$

5. sin 2C + sin 2D लाई $\frac{C+D}{2}$ र $\frac{C-D}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

Express sin 2C + sin 2D interms of $\frac{C+D}{2}$ and $\frac{C-D}{2}$.

⇨ Here, $\sin 2C + \sin 2D = 2\sin \left(\frac{2C+2D}{2}\right) \cos \left(\frac{2C-2D}{2}\right)$

$\therefore \quad \sin 2C + \sin 2D = 2 \sin 2. \left(\frac{C+D}{2}\right) . \cos 2 . \left(\frac{C-D}{2}\right)$

B. SHORT QUESTIONS

MODEL 1

1. प्रमाणित गर्नुहोस् (Prove that):

$\sin 35° + \cos 35° = \sqrt{2}\cos 10°$ [2075 R₂']

⇨ Here, LHS

$= \sin 35° + \cos 35°$

$= \sin (45° - 10°) + \cos (45° - 10°)$

$= \sin 45° \cos 10° - \cos 45° \sin 10° + \cos 45° \cos 10° + \sin 45° \sin 10°$

$= \frac{1}{\sqrt{2}} \cos 10° - \frac{1}{\sqrt{2}} \sin 10° + \frac{1}{\sqrt{2}} \cos 10° + \frac{1}{\sqrt{2}} \sin 10°$

$= \frac{2}{\sqrt{2}} \cos 10° = \sqrt{2} \cos 10° = \text{RHS}$ **Proved.**

2. प्रमाणित गर्नुहोस् (Prove that):

$\sin 110° - \sin 10° = \sin 50°$ [2074 S']

⇨ Here, LHS

$= \sin 110° - \sin 10°$

$= 2 \cos \left(\frac{110° + 10°}{2}\right) \sin \left(\frac{110° - 10°}{2}\right)$

$= 2 \cos 60° \sin 50°$

$= 2 \times \frac{1}{2} \sin 50°$

$= \sin 50° = \text{RHS}$ **Proved.**

3. प्रमाणित गर्नुहोस् (Prove that):

$\sin 80° - \cos 110° = \sqrt{3} \sin 50°$ [2074 R]

⇨ Here, LHS $= \sin 80° - \cos 110°$

$= \sin 80° - \cos (90° + 20°)$

$= \sin 80° + \sin 20°$

$= 2 \sin \left(\frac{80° + 20°}{2}\right) \cos \left(\frac{80° - 20°}{2}\right)$

$= 2 \sin 50° \cos 30°$

$= 2 \sin 50° \times \frac{\sqrt{3}}{2}$

$= \sqrt{3} \sin 50° = \text{RHS}.$

Thus, LHS = RHS completes the proof.

4. प्रमाणित गर्नुहोस् (Prove that):

$\frac{1}{2}(\sin 7\theta - \sin 3\theta) = \cos 5\theta \sin 2\theta$ [2073 S']

⇨ Here, LHS

$= \frac{1}{2}(\sin 7\theta - \sin 3\theta)$

$[\because \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}]$

$= \frac{1}{2} 2 . \cos \frac{(7\theta + 3\theta)}{2} \sin \left(\frac{7\theta - 3\theta}{2}\right)$

$= \cos 5\theta \sin 2\theta = \text{RHS}$ **Proved.**

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5. मान पत्ता लगाउनुहोस् (Find the value of):
sin 75° + sin 15° **[2065 R']**

⇨ Here, sin 75° + sin 15°

$= 2\sin\left(\frac{75° + 15°}{2}\right)\cos\left(\frac{75° - 15°}{2}\right)$

$= 2 \sin 45° \cos 30°$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \quad = \sqrt{\frac{3}{2}}$

7. मान पत्ता लगाउनुहोस् (Find the value of):
cos 15°– cos 75° **[2058 S]**

⇨ Here, given cos 15° – cos 75°

Here, 15° = 45° – 30° and 75° = 45° + 30°

Now, cos 15° – cos 75°

$= \cos(45° - 30°) - \cos(45° + 30°)$

$= \cos 45° . \cos 30° + \sin 45° . \sin 30°$
$\quad - (\cos 45° . \cos 30° - \sin 45° . \sin 30°)$

$= \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} . \frac{1}{2} - \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} . \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3} + 1 - \sqrt{3} + 1}{2\sqrt{2}}$

$= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ Thus, $\cos 15° - \cos 75° = \frac{1}{\sqrt{2}}$

9. प्रमाणित गर्नुहोस् (Prove that):
cos 70° + cos 40° = 2 cos 55°. cos 15° **[2060 R]**

⇨ Here, given LHS = cos 70° + cos 40°

Using formula,

$\cos C + \cos D = 2\cos\frac{C + D}{2} . \cos\frac{C - D}{2}$

Now, cos 70° + cos 40°

$= 2\cos\frac{70° + 40°}{2} . \cos\frac{70° - 40°}{2}$

$= 2\cos\frac{110°}{2} . \cos\frac{30°}{2}$

$= 2 \cos 55°. \cos 15° =$ RHS **Proved.**

11. प्रमाणित गर्नुहोस् (Prove that):
sin 50° + sin 70° = $\sqrt{3}$ cos10° **[2063S, 2068R']**

⇨ Here,

LHS $= \sin 50° + \sin 70°$

$= 2\sin\left(\frac{50° + 70°}{2}\right)\cos\left(\frac{50° - 70°}{2}\right)$

$= 2\sin 60° \cos 10°$

$= 2 \times \frac{\sqrt{3}}{2} \cos 10°$

$= \sqrt{3} \cos 10° =$ RHS **Proved.**

6. मान पत्ता लगाउनुहोस् (Find the value of):
sin 75°– sin 105° [2067 S]

⇨ Here, sin 75° – sin 105°

$= 2\cos\left(\frac{75° + 105°}{2}\right)\sin\left(\frac{75° - 105°}{2}\right)$

$= 2\cos 90° \sin(-15°)$

$= 2 \times 0 \times \sin(-15°)$

$= 0$

8. मान पत्ता लगाउनुहोस् (Find the value of):
sin 70° – cos 80° + cos 140° **[2061 R]**

⇨ Here, given sin 70° – cos 80° + cos 140°

$= \sin 70° + \cos 140° - \cos 80°$

$= \sin 70° - 2\sin\frac{140° + 80°}{2} . \sin\frac{140° - 80°}{2}$

$\left[\because \cos C - \cos D = -2\sin\frac{C + D}{2} . \sin\frac{C - D}{2}\right]$

$= \sin 70° - 2\sin\frac{220°}{2} . \sin\frac{60°}{2}$

$= \sin 70° - 2\sin 110°. \sin 30°$

$= \sin 70° - 2\sin(180° - 70°). \frac{1}{2}$

$\left[\because 110° = 180° - 70° \text{ and } \sin 30° = \frac{1}{2}\right]$

$= \sin 70° - \sin 70°$

$= 0$

10. प्रमाणित गर्नुहोस् (Prove that):

cos 75° + cos 15° = $\sqrt{\frac{3}{2}}$ **[2064 R']**

⇨ Here, LHS $= \cos 75° + \cos 15°$

$= 2\cos\left(\frac{75° + 15°}{2}\right)\cos\left(\frac{75° - 15°}{2}\right)$

$= 2\cos 45° \cos 30°$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}} =$ RHS **Proved.**

12. प्रमाणित गर्नुहोस् (Prove that):
cos 40° + sin 40° = $\sqrt{2}$ cos 5° **[2067 R]**

⇨ Here, LHS $= \cos 40° + \sin 40°$

$= \cos 40° + \sin(90° - 50°)$

$= \cos 40° + \cos 50°$

$= 2. \cos\frac{40° + 50°}{2}. \cos\frac{40° - 50°}{2}$

$= 2. \cos 45°. \cos 5°$

$= \sqrt{2}\sqrt{2}. \frac{1}{\sqrt{2}}. \cos 5°$

$= \sqrt{2} \cos 5° =$ RHS **Proved.**

13. प्रमाणित गर्नुहोस् (Prove that): sin 65° + cos 65° = $\sqrt{2}$ cos 20° **[2067 R']**

⇨ Here, LHS $= \sin 65° + \cos 65°$

$= \sin 65° + \cos(90° - 25°)$

$= \sin 65° + \sin 25°$

$= 2\sin\frac{65° + 25°}{2} . \cos\frac{65° - 25°}{2}$

$= 2\sin 45°. \cos 20° \qquad = 2. \frac{1}{\sqrt{2}}. \cos 20°$

$= \sqrt{2}. \sqrt{2}. \frac{1}{\sqrt{2}} \cos 20° \qquad = \sqrt{2} \cos 20°$

∴ LHS = RHS **Proved.**

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MODEL 2

14. sin 6A . cos 4A लाई sine वा cosine को योग वा अन्तरमा रूपान्तरण गर्नुहोस् ।
Convert sin 6A . cos 4A into sum or difference of sine or cosine. [SEE MODEL 2076]

⇨ Here, sin 6A . cos 4A

$$=\frac{1}{2}(2\sin 6A\,.\cos 4A)$$

$$=\frac{1}{2}[\sin(6A+4A)+\sin(6A-4A)]$$

$$=\frac{1}{2}(\sin 10A+\sin 2A)$$

Thus, $\sin 6A\,.\cos 4A=\frac{1}{2}(\sin 10A+\sin 2A)$

16. प्रमाणित गर्नुहोस् (Prove that):
$2\sin 7\theta\,.\cos 2\theta=\sin 9\theta+\sin 5\theta$ [2073 S]

⇨ Here, LHS = $2\sin 7\theta\,.\cos 2\theta$

We have,

$2\sin A\cos B=\sin(A+B)+\sin(A-B)$

So, LHS $=2\sin 7\theta\,.\cos 2\theta$

$=\sin(7\theta+2\theta)+\sin(7\theta-2\theta)$

$=\sin 9\theta+\sin 5\theta=$ RHS **Proved.**

18. मान पत्ता लगाउनुहोस् (Find the value of):
sin 75° sin 15° [2058 R, 2067 R]

⇨ Here, given sin 75° sin 15°

$$=\frac{1}{2}(2\sin 75^\circ\sin 15^\circ)$$

$$=\frac{1}{2}[\cos(75^\circ-15^\circ)-\cos(75^\circ+15^\circ)]$$

$$=\frac{1}{2}(\cos 60^\circ-\cos 90^\circ)$$

$$=\frac{1}{2}\left(\frac{1}{2}-0\right)\quad=\frac{1}{4}$$

19. मान पत्ता लगाउनुहोस् (Find the value of):
sin 105°. sin 15° [2059 S]

⇨ Here, sin 105°. sin 15°

$$=\frac{1}{2}(2\sin 105^\circ\sin 15^\circ)$$

$$=\frac{1}{2}[\cos(105^\circ-15^\circ)-\cos(105^\circ+15^\circ)]$$

$$=\frac{1}{2}[\cos 90^\circ-\cos 120^\circ]$$

$$=\frac{1}{2}\left[0-(-)\frac{1}{2}\right]$$

$$=\frac{1}{2}\times\frac{1}{2}\quad=\frac{1}{4}$$

21. मान पत्ता लगाउनुहोस् (Find the value of):
4 sin 105° sin 15° [2063 R']

⇨ Here, given 4sin 105°. sin 15°

$=\ 2.(2\sin 105^\circ.\ \sin 15^\circ)$

$=\ 2.\ [\cos(105^\circ-15^\circ)-\cos(105^\circ+15^\circ)]$

$=\ 2.\ [\cos 90^\circ-\cos 120^\circ]$

$$=\ 2\left[0-(-)\frac{1}{2}\right]$$

$$=\ 2\times\frac{1}{2}$$

$=\ 1$

15. प्रमाणित गर्नुहोस् (Prove that) :
2cos 70°. cos 20° = cos 50° [2073 R]

⇨ Here, LHS $=2\cos 70^\circ\cos 20^\circ$

$=\cos(70^\circ+20^\circ)+\cos(70^\circ-20^\circ)$

$=\cos 90^\circ+\cos 50^\circ$

$=0+\cos 50^\circ$

$=\cos 50^\circ$

= RHS **Proved.**

17. प्रमाणित गर्नुहोस् (Prove that) :
2 sin 50° . sin 40° = cos 10° [2070 R']

⇨ Here, LHS $=\ 2\sin 50^\circ\,.\sin 40^\circ$

$=\cos(50^\circ-40^\circ)-\cos(50^\circ+40^\circ)$

$=\cos 10^\circ-\cos 90^\circ$

$=\cos 10^\circ-0$

$=\cos 10^\circ$

= RHS **Proved.**

20. मान पत्ता लगाउनुहोस् (Find the value of):
cos 105° cos 15° [2069 R', 2059 R]

⇨ Here, given, cos 105° cos 15°

Here, 105° = 60° + 45° and 15° = 60° – 45°

$\therefore\ \cos 105^\circ\,.\cos 15^\circ$

$=\ \cos(60^\circ+45^\circ)\,.\cos(60^\circ-45^\circ)$

$=\ (\cos 60^\circ.\cos 45^\circ-\sin 60^\circ\,.\sin 45^\circ)\times(\cos 60^\circ\,.\cos 45^\circ+\sin 60^\circ\,.\sin 45^\circ)$

$$=\ \left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right)\times\left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right)$$

$$=\ \left(\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

$$=\ \frac{(1-\sqrt{3})}{2\sqrt{2}}\frac{(1+\sqrt{3})}{2\sqrt{2}}$$

$$=\frac{1-(\sqrt{3})^2}{4\times(\sqrt{2})^2}$$

$$=\frac{1-3}{4\times 2}$$

$$=\frac{-2}{8}$$

$$=\frac{-1}{4}$$

22. मान पत्ता लगाउनुहोस् (Find the value of):
4 cos 75° sin 105° [2064 R]

⇨ Here, 4cos 75°. sin 105°

$=\ 2.\ 2\cos 75^\circ\sin 105^\circ$

$=\ 2[\sin(75^\circ+105^\circ)-\sin(75^\circ-105^\circ)]$

$=\ 2[\sin 180^\circ-\sin(-30^\circ)]$

$=\ 2[0+\sin 30^\circ]$

$$=\ 2\left[0+\frac{1}{2}\right]$$

$=\ 1$

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23. मान पत्ता लगाउनुहोस् (Find the value of): $\cos 105° . \cos 15° = -\frac{1}{4}$ **[2068 R]**

⇨ Here, LHS $= \cos 105°. \cos 15°$

$$= \frac{1}{2}[\cos(105° + 15°) + \cos(105° - 15°)]$$

$$= \frac{1}{2}[\cos 120° + \cos 90°]$$

$$= \frac{1}{2} \times \left[\frac{1}{2} + 0\right] = -\frac{1}{4} = \text{RHS}$$

Proved.

MODEL 3

24. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A} = \tan 5A$$ **[2074 S]**

⇨ Here,

$$\text{LHS} = \frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A}$$

$$= \frac{2\sin\left(\frac{8A + 2A}{2}\right)\cos\left(\frac{8A - 2A}{2}\right)}{2\cos\left(\frac{8A + 2A}{2}\right)\cos\left(\frac{8A - 2A}{2}\right)}$$

$$= \frac{\sin 5A}{\cos 5A}$$

$$= \tan 5A$$

$$= \text{RHS}$$ **Proved.**

26. प्रमाणित गर्नुहोस् (Prove that) :

$$\frac{\cos A - \cos 5A}{\sin 5A - \sin A} = \tan 3A$$ **[2072 R]**

⇨ Here, LHS $= \frac{\cos A - \cos 5A}{\sin 5A - \sin A}$

$$= \frac{2\sin\left(\frac{A + 5A}{2}\right)\sin\left(\frac{5A - A}{2}\right)}{2\cos\left(\frac{5A + A}{2}\right)\sin\left(\frac{5A - A}{2}\right)}$$

$$= \frac{\sin 3A}{\cos 3A}$$

$$= \tan 3A = \text{RHS}$$ **Proved.**

27. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\cos 40° - \sin 30°}{\sin 60° - \cos 50°} = \tan 50°$$ **[2071 S]**

⇨ Here, LHS $= \frac{\cos 40° - \sin 30°}{\sin 60° - \cos 50°}$

$$= \frac{\cos (90° - 50°) - \sin 30°}{\sin(90° - 30°) - \cos 50°}$$

$$= \frac{\sin 50° - \sin 30°}{\cos 30° - \cos 50°}$$

$$= \frac{2\cos\left(\frac{50° + 30°}{2}\right)\sin\left(\frac{50° - 30°}{2}\right)}{2\sin\left(\frac{30° + 50°}{2}\right)\sin\left(\frac{50° - 30°}{2}\right)}$$

$$= \frac{\cos 40° \sin 10°}{\sin 40° \sin 10°}$$

$$= \cot 40°$$

$$= \cot (90° - 50°)$$

$$= \tan 50°$$

$$= \text{RHS}$$ **Proved.**

25. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \tan \frac{A + B}{2}$$ **[2074 R']**

⇨ Here, LHS $= \frac{\sin A + \sin B}{\cos A + \cos B}$

$$= \frac{2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)}{2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A + B}{2}\right)}{\cos\left(\frac{A + B}{2}\right)}$$

$$= \tan\left(\frac{A + B}{2}\right) = \text{RHS}$$

Thus, LHS = RHS completes the proof.

28. प्रमाणित गर्नुहोस् (Prove that):

$$\frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°} = \cot 35°$$ **[2072 S]**

⇨ Here,

RHS $= \cot 35°$

$$= \cot (45° - 10°)$$

$$= \frac{\cot 45° \cot 10° + 1}{\cot 10° - \cot 45°}$$

$$= \frac{1.\cot 10° + 1}{\cot 10° - 1} = \frac{\frac{\cos 10°}{\sin 10°} + 1}{\frac{\cos 10°}{\sin 10°} - 1}$$

$$= \frac{\frac{\cos 10° + \sin 10°}{\sin 10°}}{\frac{\cos 10° - \sin 10°}{\sin 10°}} = \frac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°}$$

$$= \frac{\cos (90° - 80°) + \sin 10°}{\cos 10° - \sin(90° - 80°)}$$

$$= \frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°} = \text{LHS}$$ **Proved.**

Alternative method:

LHS $= \frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°}$

$$= \frac{2\sin\left(\frac{80° + 10°}{2}\right)\cos\left(\frac{80° - 10°}{2}\right)}{2\sin\left(\frac{10° + 80°}{2}\right)\sin\left(\frac{80° - 10°}{2}\right)}$$

$$= \frac{2\sin 45° \cos 35°}{2\sin 45° \sin 35°}$$

$$= \frac{\cos 35°}{\sin 35°} = \cot 35° = \text{RHS}$$

Thus, LHS = RHS **Proved.**

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29. प्रमाणित गर्नुहोस् (Prove that) :

$\frac{1}{2}(\cos 2\theta - \cos 8\theta) = \sin 5\theta . \sin 3\theta$ **[2070 R]**

⇨ Here,

$$\text{LHS} = \frac{1}{2}(\cos 2\theta - \cos 8\theta)$$

$$= \frac{1}{2} \times 2 \sin\left(\frac{2\theta + 8\theta}{2}\right) \sin\left(\frac{8\theta - 2\theta}{2}\right)$$

$$= \sin 5\theta \sin 3\theta$$

$$= \text{RHS}$$

Proved.

30. प्रमाणित गर्नुहोस् (Prove that):

$2 \cos (45° + A) . \cos (45° - A) = \cos 2A$ **[2064 R]**

⇨ Here, LHS

$$= 2 \cos (45° + A) . \cos (45° - A)$$

$$= \cos \{(45° + A) + (45° - A)\} + \cos \{(45° + A) - (45° - A)\}$$

$$= \cos (45° + A + 45° - A) + \cos (45° + A - 45° + A)$$

$$= \cos 90° + \cos 2A$$

$$= 0 + \cos 2A$$

$$= \cos 2A = \text{RHS}$$

Proved.

31. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \frac{A+B}{2} \cot \frac{A-B}{2}$ **[2066 R]**

⇨ Here, $\text{LHS} = \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right) = \text{RHS}$ **Proved.**

C. LONG QUESTIONS

MODEL 1

1. प्रमाणित गर्नुहोस् (Prove that): **[2068 R, 2057 S, 2063 R]**

$\frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha \cos\alpha - \sin\beta \cos\beta} = \tan(\alpha + \beta)$

⇨ Here, LHS

$$= \frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha - \sin\beta\cos\beta}$$

$$= \frac{2(\sin\alpha - \sin\beta)(\sin\alpha + \sin\beta)}{2\sin\alpha\cos\alpha - 2\sin\beta\cos\beta}$$

$$= \frac{2 . 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} . 2\sin\frac{\alpha+\beta}{2} . \cos\frac{\alpha-\beta}{2}}{\sin 2\alpha - \sin 2\beta}$$

$$= \frac{2\left[2\sin\frac{\alpha+\beta}{2} . \cos\frac{\alpha+\beta}{2} . 2\sin\frac{\alpha-\beta}{2} . \cos\frac{\alpha-\beta}{2}\right]}{2\cos\frac{2\alpha+2\beta}{2} . \sin\frac{2\alpha-2\beta}{2}}$$

$$= \frac{\sin(\alpha+\beta) . \sin(\alpha-\beta)}{\cos(\alpha+\beta) . \sin(\alpha-\beta)}$$

$$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \tan(\alpha+\beta) = \text{RHS},$$

Proved.

3. प्रमाणित गर्नुहोस् (Prove that):

$\cos(60° + \theta)\cos(60° - \theta)\cos\theta = \frac{1}{4}\cos 3\theta$ **[2057 R]**

⇨ Here, LHS = $\cos (60° + \theta) \cos (60° - \theta) \cos \theta$

$$= (\cos 60° \cos\theta - \sin 60°\sin\theta)(\cos 60° . \cos\theta + \sin 60°\sin\theta)\cos\theta$$

$$= \left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)\cos\theta$$

$$= \left(\frac{1}{4}\cos^2\theta - \frac{3}{4}\sin^2\theta\right) . \cos\theta$$

$$= \frac{1}{4}\left[\cos^2\theta - 3(1 - \cos^2\theta)\right]\cos\theta$$

$$= \frac{1}{4}\left[\cos^2\theta - 3 + 3\cos^2\theta\right]\cos\theta$$

$$= \frac{1}{4}\left[4\cos^2\theta - 3\right]\cos\theta$$

$$= \frac{1}{4}\left[4\cos^3\theta - 3\cos\theta\right]$$

$$= \frac{1}{4}\cos 3\theta = \text{RHS}$$

Proved.

2. प्रमाणित गर्नुहोस् (Prove that):

$\frac{\cos^2\beta - \sin^2\alpha}{\sin\alpha\cos\alpha + \sin\beta\cos\beta} = \cot(\alpha + \beta)$ **[2064 R']**

⇨ Here, LHS $= \frac{\cos^2\beta - \sin^2\alpha}{\sin\alpha\cos\alpha + \sin\beta\cos\beta}$

$$= \frac{2\cos^2\beta - 2\sin^2\alpha}{2\sin\alpha\cos\alpha + 2\sin\beta\cos\beta}$$

$$= \frac{1 + \cos 2\beta - (1 - \cos 2\alpha)}{\sin 2\alpha + \sin 2\beta}$$

$$= \frac{1 + \cos 2\beta - 1 + \cos 2\alpha}{\sin 2\alpha + \sin 2\beta}$$

$$= \frac{\cos 2\alpha + \cos 2\beta}{\sin 2\alpha + \sin 2\beta}$$

$$= \frac{2\cos\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)}{2\sin\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)}$$

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \cot(\alpha+\beta) = \text{RHS}$$

Proved.

4. प्रमाणित गर्नुहोस् (Prove that):

$4\cos(60° + \theta).\sin(30° + \theta) . \cos\theta = \cos 3\theta$**[2064 R]**

⇨ Here, LHS

$$= 4\cos(60° + \theta).\sin(30° + \theta) . \cos\theta$$

$$= 2\cos\theta[2\cos(60° + \theta)\sin(30° + \theta)]$$

$$= 2\cos\theta[\sin(60° + \theta + 30° + \theta) - \sin(60° + \theta - 30° - \theta)]$$

$$= 2\cos\theta[\sin(90° + 2\theta) - \sin(30°)]$$

$$= 2\cos\theta[\cos 2\theta - \sin 30°]$$

$$= 2\cos\theta\cos 2\theta - 2\cos\theta \times \frac{1}{2}$$

$$= \cos(\theta + 2\theta) + \cos(\theta - 2\theta) - \cos\theta$$

$$= \cos 3\theta + \cos\theta - \cos\theta$$

$$= \cos 3\theta$$

$$= \text{RHS}$$

Proved.

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5. प्रमाणित गर्नुहोस् (Prove that): **[2063 S, 2060 S']**

$$4\sin\alpha\ \sin(60° - \alpha)\ \sin(60° + \alpha) = \sin 3\alpha$$

or, प्रमाणित गर्नुहोस् (Prove that): **[2069 R', 2067 S]**

$$\sin\theta.\ \sin(60° - \theta).\sin(60° + \theta) = \frac{1}{4}\sin 3\theta$$

⇨ Here, LHS

$= 4\sin\alpha\ \sin(60° - \alpha)\ \sin(60° + \alpha)$

$= 2\sin\alpha\ [2\sin(60° - \alpha)\ \sin(60° + \alpha)]$

$= 2\sin\alpha\ [\cos(60° - \alpha - 60° - \alpha) - \cos(60° - \alpha + 60° + \alpha)]$

$= 2\sin\alpha\ [\cos(-2\alpha) - \cos 120°]$

$= 2\sin\alpha \times \cos 2\alpha - 2\sin\alpha \times \left(-\frac{1}{2}\right)$

$= \sin(\alpha + 2\alpha) + \sin(\alpha - 2\alpha) + \sin\alpha$

$= \sin 3\alpha - \sin\alpha + \sin\alpha$

$= \sin 3\alpha$

$=$ RHS **Proved.**

7. प्रमाणित गर्नुहोस् (Prove that): **[2059 S, 2064 R']**

$$2\cos\frac{\pi^c}{13}.\cos\frac{9\pi^c}{13} + \cos\frac{3\pi^c}{13} + \cos\frac{5\pi^c}{13} = 0$$

⇨ Here, LHS

$= 2\cos\frac{\pi}{13}\ \cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= 2\cos\frac{\pi}{13}\ \cos\frac{9\pi}{13} + 2\cos\frac{1}{2}\left(\frac{3\pi}{13} + \frac{5\pi}{13}\right).\cos\frac{1}{2}\left(\frac{3\pi}{13} - \frac{5\pi}{13}\right)$

$\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}.\cos\frac{C-D}{2}\right]$

$= 2\cos\frac{\pi}{13}\ .\cos\frac{9\pi}{13} + 2\cos\frac{1}{2}\left(\frac{8\pi}{13}\right)\ \cos\frac{1}{2}\left(-\frac{2\pi}{13}\right)$

$= 2\cos\frac{\pi}{13}\ .\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}.\cos\frac{\pi}{13}$

$[\because \cos(-A) = \cos A]$

$= 2\cos\frac{\pi}{13}\ .\cos\left(\pi - \frac{4\pi}{13}\right) + 2\cos\frac{4\pi}{13}\ \cos\frac{\pi}{13}$

$\left[\because \frac{9\pi}{13} = \pi - \frac{4\pi}{13}\right]$

$= -2\cos\frac{\pi}{13}.\cos\frac{4\pi}{13} + 2\cos\frac{4\pi}{13}\ \cos\frac{\pi}{13}$

$= 0 =$ RHS **Proved.**

6. प्रमाणित गर्नुहोस् (Prove that):

$$\sec\left(\frac{\pi^c}{4} + \frac{\theta}{2}\right).\ \sec\left(\frac{\pi^c}{4} - \frac{\theta}{2}\right) = 2\sec\theta$$ **[2060 R]**

⇨ Here, LHS

$= \sec\left(\frac{\pi}{4} + \frac{\theta}{2}\right).\ \sec\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \dfrac{1}{\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right).\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$

$= \dfrac{2}{2\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right).\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$

$= \dfrac{2}{\cos\left(\frac{\pi}{4} + \frac{\theta}{2} + \frac{\pi}{4} - \frac{\theta}{2}\right) + \cos\left(\frac{\pi}{4} + \frac{\theta}{2} - \frac{\pi}{4} + \frac{\theta}{2}\right)}$

$= \dfrac{2}{\cos\frac{2\pi}{4} + \cos\frac{2\theta}{2}} = \dfrac{2}{\cos\frac{\pi}{2} + \cos\theta}$

$= \dfrac{2}{0 + \cos\theta} = \dfrac{2}{\cos\theta} = 2\sec\theta =$ RHS **Proved.**

8. प्रमाणित गर्नुहोस् (Prove that):

$$\cos^3 x\ \sin^2 x = \frac{1}{16}(2\cos x - \cos 3x - \cos 5x)$$ **[2065 R']**

⇨ Here, RHS

$= \frac{1}{16}(2\cos x - \cos 3x - \cos 5x)$

$= \frac{1}{16}\{2\cos x - (\cos 3x + \cos 5x)\}$

$= \frac{1}{16}\left\{2\cos x - 2\cos\left(\frac{3x + 5x}{2}\right)\cos\left(\frac{3x - 5x}{2}\right)\right\}$

$= \frac{1}{16}(2\cos x - 2\cos 4x\ \cos x)$

$= \frac{1}{16} \times 2\cos x(1 - \cos 4x)$

$= \frac{1}{8}\cos x.\ 2\sin^2 2x$

$= \frac{1}{4}\cos x(\sin 2x)^2$

$= \frac{1}{4}\cos x.\ (2\sin x\ \cos x)^2$

$= \frac{1}{4}\cos x.\ 4\sin^2 x\ \cos^2 x$

$= \cos^3 x\ \sin^2 x =$ LHS **Proved.**

MODEL 2

9. प्रमाणित गर्नुहोस् (Prove that): $\dfrac{\sin 40° - \sin 20°}{\cos 220° - \cos 200°} = \sqrt{3}$ **[2062 K]**

⇨ Here, LHS $= \dfrac{\sin 40° - \sin 20°}{\cos 220° - \cos 200°}$

$= \dfrac{\sin 40° - \sin 20°}{\cos(180° + 40°) - \cos(180° + 20°)}$

$= \dfrac{\sin 40° - \sin 20°}{-\cos 40° + \cos 20°}$

$= \dfrac{\sin 40° - \sin 20°}{\cos 20° - \cos 40°}$

$= \dfrac{2\cos\left(\frac{40° + 20°}{2}\right)\sin\left(\frac{40° - 20°}{2}\right)}{2\sin\left(\frac{40° - 20°}{2}\right)\sin\left(\frac{20° + 40°}{2}\right)}$

$= \dfrac{\cos 30°\ \sin 10°}{\sin 10°\ \sin 30°}$

$= \cot 30°$

$= \sqrt{3} =$ RHS **Proved.**

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10. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sin 2A + \sin 5A - \sin A}{\cos 2A + \cos 5A + \cos A} = \tan 2A$ **[2066 R']**

⇨ Here, LHS $= \frac{\sin 2A + \sin 5A - \sin A}{\cos 2A + \cos 5A + \cos A} = \frac{\sin 2A + 2\cos\left(\frac{5A+A}{2}\right)\sin\left(\frac{5A-A}{2}\right)}{\cos 2A + 2\cos\left(\frac{5A+A}{2}\right)\cos\left(\frac{5A-A}{2}\right)}$

$= \frac{\sin 2A + 2\cos 3A \sin 2A}{\cos 2A + 2\cos 3A \cos 2A} = \frac{\sin 2A(1 + 2\cos 3A)}{\cos 2A\ (1 + 2\cos 3A)}$

$= \frac{\sin 2A}{\cos 2A} = \tan 2A = \text{RHS}$ **Proved.**

MODEL 3

11. मान पत्ता लगाउनुहोस् (Find the value of):

sin 20° . sin 30° . sin 40° . sin 80° **[2076 M, 2072 R']**

⇨ Here, sin 20° . sin 30° . sin 40° . sin 80°

$= \sin 20° \times \frac{1}{2} \times \sin 40° \sin 80°$

$= \frac{1}{2} \sin 20° \sin 40° \sin 80°$

$= \frac{1}{2 \times 2} \sin 20° [2 \sin 40° \sin 80°]$

$= \frac{1}{4} \sin 20° [\cos (40° - 80°) - \cos (40° + 80°)]$

$= \frac{1}{4} \sin 20° [\cos (-40°) - \cos 120°]$

$= \frac{1}{4} \sin 20° [\cos 40° + \frac{1}{2}]$

$= \frac{1}{4} \sin 20° \cos 40° + \frac{1}{8} \sin 20°$

$= \frac{1}{8} (2 \sin 20° \cos 40°) + \frac{1}{8} \sin 20°$

$= \frac{1}{8} [\sin (20° + 40°) + \sin (20° - 40°)] + \frac{1}{8} \sin 20°$

$= \frac{1}{8} [\sin 60° + \sin (-20°)] + \frac{1}{8} \sin 20°$

$= \frac{1}{8} \times \sin 60° - \frac{1}{8} \sin 20° + \frac{1}{8} \sin 20°$

$= \frac{1}{8} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{16}$

Thus, sin 20° . sin 30° . sin 40° . sin 80° $= \frac{\sqrt{3}}{16}$.

13. प्रमाणित गर्नुहोस् (Prove that):

8 sin 20° . sin 40° . sin 80° = $\sqrt{3}$ **[2075 R_2']**

⇨ Here, LHS

$= 8 \sin 20° \sin 40° \sin 80°$

$= 4 \sin 20° (2 \sin 80° \sin 40°)$

$= 4 \sin 20° [\cos (80° - 40°) - \cos (80° + 40°)]$

$= 4 \sin 20° (\cos 40° - \cos 120°)$

$= 4 \sin 20° \left(\cos 40° + \frac{1}{2}\right)$

$= 4 \sin 20° \cos 40° + 2 \sin 20°$

$= 2 (2 \cos 40° \sin 20°) + 2 \sin 20°$

$= 2 [\sin (40° + 20°) - \sin (40° - 20°)] + 2 \sin 20°$

$= 2 [\sin 60° - \sin 20°] + 2 \sin 20°$

$= 2 \times \sin 60° - 2 \sin 20° + 2\ \text{siin}\ 20°$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} = \text{RHS}$ **Proved.**

12. क्याल्कुलेटर अथवा मान तालिकाको प्रयोग नगरीकन मान पत्ता लगाउनुहोस् : **[2075 R, 2075 R_2]**

Without using the calculator or table, find the value of : sin 100° . sin 120° . sin 140° . sin 160°

⇨ Here, sin 100° . sin 120° . sin 140° . sin 160°

$= \frac{\sqrt{3}}{2} \sin 100° . \sin 140° . \sin 160°$

$= \frac{\sqrt{3}}{2} \sin (180° - 80°) . \sin (180° - 40°) . \sin (180° - 20°)$

$= \frac{\sqrt{3}}{2} \sin 80° . \sin 40° . \sin 20°$

$= \frac{\sqrt{3}}{4} \sin 20° [2 \sin 80° . \sin 40°]$

$= \frac{\sqrt{3}}{4} \sin 20° [\cos (80° - 40°) - \cos (80° + 40°)]$

$= \frac{\sqrt{3}}{4} \sin 20° [\cos 40° - \cos 120°]$

$= \frac{\sqrt{3}}{4} \sin 20° [\cos 40° + \frac{1}{2}]$

$= \frac{\sqrt{3}}{8} . 2 \sin 20° \cos 40° + \frac{\sqrt{3}}{8} \sin 20°$

$= \frac{\sqrt{3}}{8} [\sin (20° + 40°) + \sin (20° - 40°)] + \frac{\sqrt{3}}{8} \sin 20°$

$= \frac{\sqrt{3}}{8} [\sin 60° - \sin 20°] + \frac{\sqrt{3}}{8} \sin 20°$

$= \frac{\sqrt{3}}{8} \times \sin 60° - \frac{\sqrt{3}}{8} \sin 20° + \frac{\sqrt{3}}{8} \sin 20°$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

Thus, the required value is $\frac{3}{16}$.

14. प्रमाणित गर्नुहोस् (Prove that) :

8cos 10° . cos 50° . cos 70° = $\sqrt{3}$ **[2070 R']**

⇨ Here, LHS

$= 8\cos 10° . \cos 50° . \cos 70°$

$= 4\cos 10° (2\cos 50° . \cos 70°)$

$= 4\cos 10° [\cos(50° + 70°) + \cos(50° - 70°)]$

$= 4\cos 10° [\cos 120° + \cos 20°]$

$= 4\cos 10° \left[-\frac{1}{2} + \cos 20°\right]$

$= -4 \times \frac{1}{2} \cos 10° + 4\cos 10° \cos 20°$

$= -2 \cos 10° + 2(2\cos 10° \cos 20°)$

$= -2\cos 10° + 2[\cos (10° + 20°) + \cos(10° - 20°)]$

$= -2\cos 10° + 2 \cos 30° + 2\cos 10°$

$= 2 \cos 30° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} = \text{RHS}$ **Proved.**

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QUESTIONS FROM CDC TEXTBOOK

5.3 त्रिकोणमितीय अनुपातहरूको रुपान्तरण (TRANSFORMATION OF TRIGONOMETRIC RATIOS)

EXERCISE 5.3

1. (a) **2 cos A cos B लाई cosine को योग वा अन्तरमा व्यक्त गर्नुहोस् ।**
Express 2 cos A cos B into the sum or difference of cosine.

⇨ Here, $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

(b) **2 sin α sin β लाई cosine को योग वा अन्तरमा व्यक्त गर्नुहोस् ।**
Express 2 sin α sin β into the sum or difference of cosine.

⇨ Here, $2 \sin \alpha \sin \beta = \cos (\alpha - \beta) - \cos (\alpha + \beta)$

(c) **2 sin *x* cos *y* लाई sine को योग वा अन्तरमा व्यक्त गर्नुहोस् ।**
Express 2 sin *x* cos *y* into the sum or difference of sine.

⇨ Here, $2 \sin x \cos y = \sin (x + y) + \sin (x - y)$

(d) **sin α + sin β लाई sine वा cosine को गुणनफलका रूपमा लेख्नुहोस् ।**
Write sin α + sin β in terms of product of sine or cosine.

⇨ Here, $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$

(e) **cos *x* – cos *y* लाई sine को गुणनफलका रूपमा लेख्नुहोस् ।**
Write cos *x* – cos *y* in terms of product of sine .

⇨ Here, $\cos x - \cos y = 2 \sin \left(\frac{x + y}{2}\right) \sin \left(\frac{y - x}{2}\right)$

(f) **cos A + cos B लाई cosine को गुणनफलका रूपमा लेख्नुहोस् ।**
Write cos A + cos B in terms of product of cosine.

⇨ Here, $\cos A + \cos B = 2 \cos \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right)$

(g) **sin *x* – sin *y* लाई sine वा cosine को गुणनफलका रूपमा व्यक्त गर्नुहोस् ।**
Write sin *x* – sin *y* in terms of product of sine or cosine.

⇨ Here, $\sin x - \sin y = 2 \cos \left(\frac{x + y}{2}\right) \sin \left(\frac{x - y}{2}\right)$

2. **तलका योग वा अन्तरलाई गुणनफलका रूपमा व्यक्त गर्नुहोस् । (Express the following sum or difference into product form.)**

(a) **sin 50° + sin 70°**

⇨ Here, $\sin 50° + \sin 70°$

$= \sin 70° + \sin 50°$

$= 2 \sin \frac{70° + 50°}{2} \cos \frac{70° - 50°}{2}$

$= 2 \sin \frac{120°}{2} \cos \frac{20°}{2}$

$= 2 \sin 60° \cos 10°$

$= 2 \times \frac{\sqrt{3}}{2} \cos 10° \quad = \sqrt{3} \cos 10°$

(c) **cos 70° + cos 40°**

⇨ Here, $\cos 70° + \cos 40°$

$= 2 \cos \left(\frac{70° + 40°}{2}\right) \cos \left(\frac{70° - 40°}{2}\right)$

$= 2 \cos \frac{110°}{2} \cos \frac{30°}{2}$

$= 2 \cos 55° \cos 15°$

(e) **sin 150° + sin 140°**

⇨ Here, $\sin 150° + \sin 140°$

$= \sin (180° - 30°) + \sin (180° - 40°)$

$= \sin 30° + \sin 40°$

$= \sin 40° + \sin 30°$

$= 2 \sin \left(\frac{40° + 30°}{2}\right) \cos \left(\frac{40° - 30°}{2}\right)$

$= 2 \sin \frac{70°}{2} \cos \frac{10°}{2}$

$= 2 \sin 35° \cos 5°$

(b) **cos 70° – cos 40°**

⇨ Here, $\cos 70° - \cos 40°$

$= 2 \sin \left(\frac{70° + 40°}{2}\right) \sin \left(\frac{40° - 70°}{2}\right)$

$= 2 \sin \frac{110°}{2} \sin \frac{-30°}{2}$

$= -2 \sin 55° \sin 15°$

(d) **sin 100° – sin 50°**

⇨ Here, $\sin 100° - \sin 50°$

$= 2 \cos \left(\frac{100° + 50°}{2}\right) \sin \left(\frac{100° - 50°}{2}\right)$

$= 2 \cos \frac{150°}{2} \sin \frac{50°}{2}$

$= 2 \cos 75° \sin 25°$

(f) **sin 46° – sin 20°**

⇨ Here, $\sin 46° - \sin 20°$

$= 2 \cos \left(\frac{46° + 20°}{2}\right) \sin \left(\frac{46° - 20°}{2}\right)$

$= 2 \cos \frac{66°}{2} \sin \frac{26°}{2}$

$= 2 \cos 33° \sin 13°$

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(g) $\sin 84° - \sin 116°$

⇨ Here, $\sin 84° - \sin 116°$

$= \sin 84° - \sin (180° - 64°)$

$= \sin 84° - \sin 64°$

$= 2 \cos \left(\frac{84° + 64°}{2}\right) \sin \left(\frac{84° - 64°}{2}\right)$

$= 2 \cos \frac{148°}{2} \sin \frac{20°}{2}$

$= 2 \cos 74° \sin 10°$

$= 2 \cos (90° - 16°) \sin (90° - 80°)$

$= 2 \sin 16° \cos 80°$

$= 2 \cos 80° \sin 16°$

(j) $\sin 5\theta - \sin 7\theta$

⇨ Here, $\sin 5\theta - \sin 7\theta$

$= 2 \cos \left(\frac{5\theta + 7\theta}{2}\right) \sin \left(\frac{5\theta - 7\theta}{2}\right)$

$= 2 \cos \frac{12\theta}{2} \sin \left(\frac{-2\theta}{2}\right)$

$= 2 \cos 6\theta \sin (-\theta)$

$= -2 \cos 6\theta \sin \theta$

(ℓ) $\cos A + \cos 7A$

⇨ Here, $\cos A + \cos 7A$

$= \cos 7A + \cos A$

$= 2 \cos \left(\frac{7A + A}{2}\right) \cos \left(\frac{7A - A}{2}\right)$

$= 2 \cos \frac{8A}{2} \cos \frac{6A}{2}$

$= 2 \cos 4A \cos 3A$

(n) $\sin 3\alpha - \sin \alpha$

⇨ Here, $\sin 3\alpha - \sin \alpha$

$= 2 \cos \left(\frac{3\alpha + \alpha}{2}\right) \sin \left(\frac{3\alpha - \alpha}{2}\right)$

$= 2 \cos 2\alpha \sin \alpha$

(h) $\cos 46° - \cos 20°$

⇨ Here, $\cos 46° - \cos 20°$

$= 2 \sin \left(\frac{46° + 20°}{2}\right) \sin \left(\frac{20° - 46°}{2}\right)$

$= 2 \sin 33° \sin (-13°)$

$= -2 \sin 33° \sin 13°$

(i) $\cos 110° + \cos 130°$

⇨ Here, $\cos 110° + \cos 130°$

$= 2 \cos \left(\frac{110° + 130°}{2}\right) \cos \left(\frac{110° - 130°}{2}\right)$

$= 2 \cos 120° \cos 10°$

$= 2 \times (-1) \frac{1}{2} \cos 10° = -\cos 10°$

(k) $\sin 5A + \sin 7A$

⇨ Here, $\sin 5A + \sin 7A$

$= \sin 7A + \sin 5A$

$= 2 \sin \left(\frac{7A + 5A}{2}\right) \cos \left(\frac{7A - 5A}{2}\right)$

$= 2 \sin \frac{12A}{2} \cos \frac{2A}{2}$

$= 2 \sin 6A \cos A$

(m) $\sin 5x - \sin 3x$

⇨ Here, $\sin 5x - \sin 3x$

$= 2 \cos \left(\frac{5x + 3x}{2}\right) \sin \left(\frac{5x - 3x}{2}\right)$

$= 2 \cos 4x \sin x$

(o) $\cos 5x + \cos 3x$

⇨ Here, $\cos 5x + \cos 3x$

$= 2 \cos \left(\frac{5x + 3x}{2}\right) \cos \left(\frac{5x - 3x}{2}\right)$

$= 2 \cos 4x \cos x$

(p) $\cos 7\theta - \cos 11\theta$ ⇨ Here, $\cos 7\theta - \cos 11\theta = 2 \sin \left(\frac{7\theta + 11\theta}{2}\right) \sin \left(\frac{11\theta - 7\theta}{2}\right) = 2 \sin 9\theta \sin 2\theta$

3. तल दिइएका गुणनफललाई sine वा cosine को योग वा अन्तरमा व्यक्त गर्नुहोस् ।
Express the following products into the sum or difference of sine or cosine.

(a) $\sin 50° \cos 32°$

⇨ Here, $\sin 50° \cos 32°$

$= \frac{1}{2} 2 \sin 50° \cos 32°$

$= \frac{1}{2} \{\sin (50° + 32°) + \sin (50° - 32°)\}$

$= \frac{1}{2} (\sin 82° + \sin 18°)$

(c) $\cos 61° \cos 39°$

⇨ Here, $\cos 61° \cos 39°$

$= \frac{1}{2} (2 \cos 61° \cos 39°)$

$= \frac{1}{2} \{\cos (61° + 39°) + \cos (61° - 39°)\}$

$= \frac{1}{2} \{\cos 100° + \cos 22°\}$

$= \frac{1}{2} \{\cos (180° - 80°) + \cos 22°\}$

$= \frac{1}{2} \{-\cos 80° + \cos 22°\}$

$= \frac{1}{2} (\cos 22° - \cos 80°)$

(b) $\cos 72° \sin 43°$

⇨ Here, $\cos 72° \sin 43°$

$= \frac{1}{2} 2 \cos 72° \sin 43°$

$= \frac{1}{2} \{\sin (72° + 43°) - \sin (72° - 43°)\}$

$= \frac{1}{2} (\sin 115° - \sin 29°)$

$= \frac{1}{2} \{\sin (180° - 65°) - \sin 29°\}$

$= \frac{1}{2} (\sin 65° - \sin 29°)$

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(d) sin 61° sin 39°

⇨ Here, $\sin 61° \sin 39°$

$= \frac{1}{2}(2 \sin 61° \sin 39°)$

$= \frac{1}{2}\{\cos(61° - 39°) - \cos(61° + 39°)\}$

$= \frac{1}{2}\{\cos 22° - \cos 100°\}$

$= \frac{1}{2}\{\cos 22° - \cos(180 - 80°)\}$

$= \frac{1}{2}(\cos 22° + \cos 80°)$

(f) sin 51° sin 10°

⇨ Here, $\sin 51° \sin 10°$

$= \frac{1}{2}(2 \sin 51° \sin 10°)$

$= \frac{1}{2}\{\cos(51° - 10°) - \cos(51° + 10°)\}$

$= \frac{1}{2}(\cos 41° - \cos 61°)$

(h) cos 140° sin 40°

⇨ Here, $\cos 140° \sin 40°$

$= \frac{1}{2}(2 \cos 140° \sin 40°)$

$= \frac{1}{2}\{\sin(140° + 40°) - \sin(140° - 40)\}$

$= \frac{1}{2}\{0 - \sin(180° - 80°)\}$

$= \frac{1}{2}\{-\sin 80°\} = -\frac{1}{2}\sin 80°$

(k) 2 sin 9θ cos 7θ

⇨ Here, $2 \sin 9\theta \cos 7\theta$

$= \sin(9\theta + 7\theta) + \sin(9\,\theta - 7\theta)$

$= \sin 16\theta + \sin 2\theta$

(ℓ) 2 cos 11θ cos 3θ

⇨ Here, $2 \cos 11\,\theta \cos 3\,\theta$

$= \cos(11\,\theta + 3\theta) + \cos(11\theta - 3\theta)$

$= \cos 14\,\theta + \cos 8\theta$

(m) 2 cos 9θ cos 5θ

⇨ Here, $2 \cos 9\theta \cos 5\theta$

$= \cos(9\,\theta + 5\theta) + \cos(9\theta - 5\theta)$

$= \cos 14\,\theta + \cos 4\theta$

(e) sin 36° sin 24°

⇨ Here, $\sin 36° \sin 24°$

$= \frac{1}{2}(2 \sin 36° \sin 24°)$

$= \frac{1}{2}\{\cos(36° - 24°) - \cos(36° + 24°)\}$

$= \frac{1}{2}(\cos 12° - \cos 60°)$

$= \frac{1}{2}\left(\cos 12° - \frac{1}{2}\right)$

(g) cos 22° sin 50°

⇨ Here, $\cos 22° \sin 50°$

$= \frac{1}{2}(2 \sin 50° \cos 22°)$

$= \frac{1}{2}\{\sin(50° + 22°) + \sin(50° - 22°)\}$

$= \frac{1}{2}(\sin 72° + \sin 28°)$

(i) 2 sin 5θ cos 2θ

⇨ Here, $2 \sin 5\theta \cos 2\theta$

$= \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)$

$= \sin 7\theta + \sin 3\theta$

(j) 2 sin 2x cos x

⇨ Here, $2 \sin 2x \cos x$

$= \sin(2x + x) + \sin(2x - x)$

$= \sin 3x + \sin x$

(n) cos 5α sin 3α

⇨ Here, $\cos 5\alpha \sin 3\alpha$

$= \frac{1}{2}(2 \cos 5\alpha \sin 3\alpha)$

$= \frac{1}{2}[\sin(5\alpha + 3\alpha) - \sin(5\alpha - 3\alpha)]$

$= \frac{1}{2}[\sin 8\alpha - \sin 2\alpha]$

4. प्रमाणित गर्नुहोस् (Prove that):

(a) $\sin 15° \sin 45° = \frac{\sqrt{3} - 1}{4}$

⇨ Here, LHS

$= \sin 15° \sin 45°$

$= \frac{1}{2}(2 \sin 15° \sin 45°)$

$= \frac{1}{2}[\cos(15° - 45°) - \cos(15° + 45°)]$

$= \frac{1}{2}[\cos 30° - \cos 60°]$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

$= \frac{1}{2} \times \frac{\sqrt{3} - 1}{2} = \frac{\sqrt{3} - 1}{4} = \text{RHS}$ **Proved.**

(b) $\cos 45° \cos 15° = \frac{\sqrt{3} + 1}{4}$

⇨ Here, LHS $= \cos 45° \cos 15°$

$= \frac{1}{2}(2 \cos 45° \cos 15°)$

$= \frac{1}{2}[\cos(45° + 15°) + \cos(45° - 15°)]$

$= \frac{1}{2}(\cos 60° + \cos 30°)$

$= \frac{1}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2} \times \frac{1 + \sqrt{3}}{2}$

$= \frac{1 + \sqrt{3}}{4} = \frac{\sqrt{3} + 1}{4} = \text{RHS}$ **Proved.**

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(c) $\sin 105° \cos 15° = \frac{\sqrt{3}+2}{4}$

⇨ Here, LHS

$= \sin 105° \cos 15°$

$= \frac{1}{2}(2 \sin 105° \cos 15°)$

$= \frac{1}{2}[\sin (105° + 15°) + \sin (105° - 15°)]$

$= \frac{1}{2}(\sin 120° + \sin 90°)$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} + 1\right)$

$= \frac{1}{2} \times \frac{\sqrt{3}+2}{2} = \frac{\sqrt{3}+2}{4} = \text{RHS}$ **Proved.**

(e) $\sin 15° \cos 105° = \frac{\sqrt{3}-2}{4}$

⇨ Here, LHS

$= \sin 15° \cos 105°$

$= \frac{1}{2}(2 \sin 15° \cos 105°)$

$= \frac{1}{2}[\sin (15° + 105°) + \sin (15° - 105°)]$

$= \frac{1}{2}[\sin 120° + \sin (-90°)]$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \sin 90°\right)$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - 1\right)$

$= \frac{1}{2} \times \frac{\sqrt{3}-2}{2}$

$= \frac{\sqrt{3}-2}{4} = \text{RHS}$ **Proved.**

(h) $\tan 55° - \tan 35° = 2 \tan 20°$

⇨ Here, LHS $= \tan 55° - \tan 35°$

$= \frac{\sin 55°}{\cos 55°} - \frac{\sin 35°}{\cos 35°}$

$= \frac{\cos 35° \sin 55° - \cos 55° \sin 35°}{\cos 55° \cos 35°}$

$= \frac{2 \sin (55° - 35°)}{2 \cos 55° \cos 25°}$

$= \frac{2 \sin 20°}{\cos (55° + 35°) + \cos (55° - 35°)}$

$= \frac{2 \sin 20°}{\cos 90° + \cos 20°}$

$= \frac{2 \sin 20°}{0 + \cos 20°}$

$= 2 \tan 20° = \text{RHS}$ **Proved.**

(d) $\cos 75° \cos 105° = \frac{\sqrt{3}-2}{4}$

⇨ Here, LHS

$= \cos 75° \cos 105°$

$= \frac{1}{2}(2 \cos 75° \cos 105°)$

$= \frac{1}{2}[\cos (75° + 105°) + \cos (105° - 75°)]$

$= \frac{1}{2}(\cos 180° + \cos 30°)$

$= \frac{1}{2}\left(-1 + \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}\left(\frac{-2+\sqrt{3}}{2}\right) = \frac{\sqrt{3}-2}{4} = \text{RHS}$ **Proved.**

(f) $2 \cos 105° \cos 15° = -\frac{1}{2}$

⇨ Here, LHS $= 2 \cos 105° \cos 15°$

$= \cos (105° + 15°) + \cos (105° - 15°)$

$= \cos 120° + \cos 90°$

$= -\frac{1}{2} + 0 = -\frac{1}{2} = \text{RHS}$ **Proved.**

(g) $2 \sin 75° \sin 45° = \frac{\sqrt{3}+1}{2}$

⇨ Here, LHS $= 2 \sin 75° \sin 45°$

$= \cos (75° - 45°) - \cos (75° + 45°)$

$= \cos 30° - \cos 120°$

$= \frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2} = \text{RHS}$ **Proved.**

(i) $\tan 50° - \tan 40° = 2 \tan 10°$

⇨ Here, LHS $= \tan 50° - \tan 40°$

$= \frac{\sin 50°}{\cos 50°} - \frac{\sin 40°}{\cos 40°}$

$= \frac{\sin 50° \cos 40° - \cos 50° \sin 40°}{\cos 50° \cos 40°}$

$= \frac{2 \sin (50° - 40°)}{2\cos 50° \cos 40°}$

$= \frac{2 \sin 10°}{\cos (50° + 40°) + \cos (50° - 40°)}$

$= \frac{2 \sin 10°}{\cos 90° + \cos 10°}$

$= \frac{2 \sin 10°}{\cos 10°}$

$= 2 \tan 10° = \text{RHS}$ **Proved.**

(j) $\tan 70° - \tan 20° = 2 \tan 50°$

⇨ Here, LHS $= \tan 70° - \tan 20°$

$= \frac{\sin 70°}{\cos 70°} - \frac{\sin 20°}{\cos 20°}$

$= \frac{\sin 70° \cos 20° - \sin 20° \cos 70°}{\cos 70° \cos 20°}$

$= \frac{2 \sin (70° - 20°)}{2 \cos 70° \cos 20°}$

$= \frac{2 \sin 50°}{\cos (70° + 20°) + \cos (70° - 20°)}$

$= \frac{2 \sin 50°}{\cos 90° + \cos 50°}$

$= \frac{2 \sin 50°}{0 + \cos 50°}$

$= \frac{2 \sin 50°}{\cos 50°} = 2 \tan 50° = \text{RHS}$ **Proved.**

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5. प्रमाणित गर्नुहोस् (Prove that):

(a) $\cos\theta \cos\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\cos 3\theta$

⇨ Here, LHS

$$= \cos\theta\cos\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right)$$

$$= \cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)$$

$$= \cos\theta\,(\cos 60^\circ\cos\theta+\sin 60^\circ\sin\theta)\,(\cos 60^\circ\cos\theta-\sin 60^\circ\sin\theta)$$

$$= \cos\theta\left(\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right)\left(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right)$$

$$= \cos\theta\left(\frac{1}{4}\cos^2\theta-\frac{3}{4}\sin^2\theta\right)$$

$$= \frac{1}{4}\cos\theta\,(\cos^2\theta-3\sin^2\theta)$$

$$= \frac{1}{4}\cos\theta\,(\cos^2\theta-3+\cos^2\theta)$$

$$= \frac{1}{4}\cos\theta\,(4\cos^2\theta-3)$$

$$= \frac{1}{4}(4\cos^3\theta-3\cos\theta)$$

$$= \frac{1}{4}\cos 3\theta = \text{RHS}$$

Proved.

(d) $\sec\left(\frac{\pi}{4}+\frac{A}{2}\right)\sec\left(\frac{\pi}{4}-\frac{A}{2}\right)=2\sec A$

⇨ Here, LHS

$$= \sec\left(\frac{\pi}{4}+\frac{A}{2}\right)\sec\left(\frac{\pi}{4}-\frac{A}{2}\right)$$

$$= \sec\left(45^\circ+\frac{A}{2}\right)\sec\left(45^\circ-\frac{A}{2}\right)$$

$$= \frac{1}{\cos\left(45^\circ+\frac{A}{2}\right)\cos\left(45^\circ-\frac{A}{2}\right)}$$

$$= \frac{2}{2\cos\left(45^\circ+\frac{A}{2}\right)\cos\left(45^\circ-\frac{A}{2}\right)}$$

$$= \frac{2}{\cos\left(45^\circ+\frac{A}{2}+45^\circ-\frac{A}{2}\right)+\cos\left(45^\circ+\frac{A}{2}-45^\circ+\frac{A}{2}\right)}$$

$$= \frac{2}{\cos 90^\circ+\cos A} = \frac{2}{0+\cos A} = \frac{2}{\cos A}$$

$$= 2\sec A = \text{RHS}$$

Proved.

(b) $2\sin\left(\frac{\pi}{4}+A\right)\sin\left(\frac{\pi}{4}-A\right)=\cos 2A$

⇨ Here, LHS $= 2\sin\left(\frac{\pi}{4}+A\right)\sin\left(\frac{\pi}{4}-A\right)$

$$= 2\sin(45^\circ+A)\sin(45^\circ-A)$$

$$= \cos\{(45^\circ+A)-(45^\circ-A)\}-\cos\{(45^\circ+A)+(45^\circ-A)\}$$

$$= \cos(2A)-\cos 90^\circ$$

$$= \cos 2A - 0$$

$$= \cos 2A = \text{RHS}$$

Proved.

(c) $\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-A\right)=\frac{1}{2}(1+\sin 2A)$

⇨ Here, LHS

$$= \sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-A\right)$$

$$= \sin(45^\circ+A)\cos(45^\circ-A)$$

$$= \frac{1}{2}\times 2\sin(45^\circ+A)\cos(45^\circ-A)$$

$$= \frac{1}{2}[\sin(45^\circ+A+45^\circ-A)+\sin(45^\circ+A-45^\circ+A)]$$

$$= \frac{1}{2}[\sin 90^\circ+\sin(0^\circ+2A)]$$

$$= \frac{1}{2}(1+\sin 2A) = \text{RHS}$$

Proved.

(e) $\operatorname{cosec}\left(\frac{\pi}{4}+\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}-\theta\right)=2\sec 2\theta$

⇨ Here, LHS

$$= \operatorname{cosec}\left(\frac{\pi}{4}+\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}-\theta\right)$$

$$= \operatorname{cosec}(45^\circ+\theta)\operatorname{cosec}(45^\circ-\theta)$$

$$= \frac{1}{\sin(45^\circ+\theta)\sin(45^\circ-\theta)}$$

$$= \frac{2}{2\sin(45^\circ+\theta)\sin(45^\circ-\theta)}$$

$$= \frac{2}{\cos(45^\circ+\theta-45^\circ+\theta)-\cos(45^\circ+\theta+45^\circ-\theta)}$$

$$= \frac{2}{\cos 2\theta-\cos 90^\circ}$$

$$= \frac{2}{\cos 2\theta-0}$$

$$= \frac{2}{\cos 2\theta}$$

$$= 2\sec 2\theta = \text{RHS}$$

Proved.

6. प्रमाणित गर्नुहोस् (Prove that): (a) $\sin 20^\circ\sin 40^\circ\sin 80^\circ=\frac{\sqrt{3}}{8}$

⇨ Here, LHS

$$= \sin 20^\circ\sin 40^\circ\sin 80^\circ$$

$$= \sin 20^\circ\,\frac{1}{2}(2\sin 80^\circ\sin 40^\circ)$$

$$= \frac{1}{2}\sin 20^\circ\,\{\cos(80^\circ-40^\circ)-\cos(80^\circ+40^\circ)\}$$

$$= \frac{1}{2}\sin 20^\circ\,\{\cos 40^\circ-\cos 120^\circ\}$$

$$= \frac{1}{2}\sin 20^\circ\left\{\cos 40^\circ+\frac{1}{2}\right\}$$

$$= \frac{1}{2}\cos 40^\circ\sin 20^\circ+\frac{1}{4}\sin 20^\circ$$

$$= \frac{1}{4}(2\cos 40^\circ\sin 20^\circ)+\frac{1}{4}\sin 20^\circ$$

$$= \frac{1}{4}\{\sin(40^\circ+20^\circ)-\sin(40^\circ-20^\circ)\}+\frac{1}{4}\sin 20^\circ$$

$$= \frac{1}{4}\{\sin 60^\circ-\sin 20^\circ\}+\frac{1}{4}\sin 20^\circ$$

$$= \frac{1}{4}\left\{\frac{\sqrt{3}}{2}-\sin 20^\circ\right\}+\frac{1}{4}\sin 20^\circ$$

$$= \frac{\sqrt{3}}{8}-\frac{1}{4}\sin 20^\circ+\frac{1}{4}\sin 20^\circ$$

$$= \frac{\sqrt{3}}{8} = \text{RHS}$$

Proved.

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(b) $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$

⇨ Here, LHS

$= \cos 20^\circ \cos 40^\circ \cos 80^\circ$

$= \cos 20^\circ \times \frac{1}{2} (2 \cos 80^\circ \cos 40^\circ)$

$= \frac{1}{2} \cos 20^\circ \{\cos (80^\circ + 40^\circ) + \cos (80^\circ - 40^\circ)\}$

$= \frac{1}{2} \cos 20^\circ \{\cos 120^\circ + \cos 40^\circ\}$

$= \frac{1}{2} \cos 20^\circ \left\{-\frac{1}{2} + \cos 40^\circ\right\}$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{2} \cos 40^\circ \cos 20^\circ$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{4} (2 \cos 40^\circ \cos 20^\circ)$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{4} \{\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)\}$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{4} \{\cos 60^\circ + \cos 20^\circ\}$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{4} \left\{\frac{1}{2} + \cos 20^\circ\right\}$

$= -\frac{1}{4} \cos 20^\circ + \frac{1}{8} + \frac{1}{4} \cos 20^\circ$

$= \frac{1}{8} = \text{RHS}$ **Proved.**

(d) $\sin 20^\circ \sin 30^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{16}$

⇨ Here, LHS

$= \sin 20^\circ \sin 30^\circ \sin 40^\circ \sin 80^\circ$

$= \sin 20^\circ \frac{1}{2} \sin 40^\circ \sin 80^\circ$

$= \sin 20^\circ \times \frac{1}{4} \times (2 \sin 80^\circ \sin 40^\circ)$

$= \frac{1}{4} \sin 20^\circ \{\cos (80^\circ - 40^\circ) - \cos (80^\circ + 40)\}$

$= \frac{1}{4} \sin 20^\circ \{ \cos 40^\circ - \cos 120^\circ\}$

$= \frac{1}{4} \sin 20^\circ \left\{\cos 40^\circ + \frac{1}{2}\right\}$

$= \frac{1}{4} \sin 20^\circ \cos 40^\circ + \frac{1}{8} \sin 20^\circ$

$= \frac{1}{8} 2 \cos 40^\circ \sin 20^\circ + \frac{1}{8} \sin 20^\circ$

$= \frac{1}{8} \{\sin (40^\circ + 20) - \sin (40^\circ - 20^\circ)\} + \frac{1}{8} \sin 20^\circ$

$= \frac{1}{8} \{\sin 60^\circ - \sin 20^\circ\} + \frac{1}{8} \sin 20^\circ$

$= \frac{1}{8} \left\{\frac{\sqrt{3}}{2} - \sin 20^\circ\right\} + \frac{1}{8} \sin 20^\circ$

$= \frac{\sqrt{3}}{16} - \frac{1}{8} \sin 20^\circ + \frac{1}{8} \sin 20^\circ$

$= \frac{\sqrt{3}}{16} = \text{RHS}$ **Proved.**

(c) $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}$

⇨ Here, LHS

$= \cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ$

$= \cos 20^\circ \frac{\sqrt{3}}{2} \cos 40^\circ \cos 80^\circ$

$= \cos 20^\circ \times \frac{\sqrt{3}}{4} (2 \cos 80^\circ \cos 40^\circ)$

$= \frac{\sqrt{3}}{4} \cos 20^\circ \{\cos (80^\circ + 40^\circ) + \cos (80^\circ - 40^\circ)\}$

$= \frac{\sqrt{3}}{4} \cos 20^\circ \{\cos 120^\circ + \cos 40^\circ\}$

$= \frac{\sqrt{3}}{4} \cos 20^\circ \left\{-\frac{1}{2} + \cos 40^\circ\right\}$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{4} \cos 40^\circ \cos 20^\circ$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{8} (2 \cos 40^\circ \cos 20^\circ)$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{8} \{\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)\}$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{8} \{\cos 60^\circ + \cos 20^\circ\}$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{8} \left\{\frac{1}{2} + \cos 20^\circ\right\}$

$= -\frac{\sqrt{3}}{8} \cos 20^\circ + \frac{\sqrt{3}}{16} + \frac{\sqrt{3}}{8} \cos 20^\circ$

$= \frac{\sqrt{3}}{16} = \text{RHS}$ **Proved.**

(e) $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$

⇨ Here, LHS

$= \sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ$

$= \sin 10^\circ \sin 50^\circ \times \frac{\sqrt{3}}{2} \times \sin 70^\circ$

$= \sin 10^\circ \times \frac{\sqrt{3}}{4} \times (2 \sin 70^\circ \sin 50^\circ)$

$= \frac{\sqrt{3}}{4} \sin 10^\circ \{\cos (70^\circ - 50^\circ) - \cos (70^\circ + 50^\circ)\}$

$= \frac{\sqrt{3}}{4} \sin 10^\circ \{\cos 20^\circ - \cos 120^\circ\}$

$= \frac{\sqrt{3}}{4} \sin 10^\circ \left\{\cos 20^\circ + \frac{1}{2}\right\}$

$= \frac{\sqrt{3}}{4} \cos 20^\circ \sin 10^\circ + \frac{\sqrt{3}}{8} \sin 10^\circ$

$= \frac{\sqrt{3}}{8} (2 \cos 20^\circ \sin 10^\circ) + \frac{\sqrt{3}}{8} \sin 10^\circ$

$= \frac{\sqrt{3}}{8} \{\sin (20^\circ + 10^\circ) - \sin (20^\circ - 10^\circ)\} + \frac{\sqrt{3}}{8} \sin 10^\circ$

$= \frac{\sqrt{3}}{8} \{\sin 30^\circ - \sin 10^\circ\} + \frac{\sqrt{3}}{8} \sin 10^\circ$

$= \frac{\sqrt{3}}{8} \times \frac{1}{2} - \frac{\sqrt{3}}{8} \sin 10^\circ + \frac{\sqrt{3}}{8} \sin 10^\circ$

$= \frac{\sqrt{3}}{16} = \text{RHS}$ **Proved.**

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(f) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

⇨ Here, LHS

$= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$= \cos 20^\circ \cos 40^\circ \times \frac{1}{2} \times \cos 80^\circ$

$= \frac{1}{2} \cos 20^\circ \times \frac{1}{2}\ (2 \cos 80^\circ \cos 40^\circ)$

$= \frac{1}{4} \cos 20^\circ\ \{\cos (80^\circ + 40^\circ) + \cos (80^\circ - 40^\circ)\}$

$= \frac{1}{4} \cos 20^\circ\ \{\cos 120^\circ + \cos 40^\circ\}$

$= \frac{1}{4} \cos 20^\circ \left\{-\frac{1}{2} + \cos 40^\circ\right\}$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{4} \cos 40^\circ \cos 20^\circ$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} (2 \cos 40^\circ \cos 20^\circ)$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \{\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)\}$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \{(\cos 60^\circ + \cos 20^\circ)\}$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \left\{\frac{1}{2} + \cos 20^\circ\right\}$

$= -\frac{1}{8} \cos 20^\circ + \frac{1}{16} + \frac{1}{8} \cos 20^\circ$

$= \frac{1}{16} =$ RHS **Proved.**

(h) $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

⇨ Here, LHS

$= \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$

$= \cos 10^\circ \times \frac{\sqrt{3}}{2} \times \cos 50^\circ \cos 70^\circ$

$= \frac{\sqrt{3}}{2} \cos 10^\circ \times \frac{1}{2}\ (2 \cos 50^\circ \cos 70^\circ)$

$= \frac{\sqrt{3}}{4} \cos 10^\circ\ [\cos (50^\circ + 70^\circ) + \cos (50^\circ - 70^\circ)]$

$= \frac{\sqrt{3}}{4} \cos 10^\circ\ [\cos 120^\circ + \cos 20^\circ]$

$= \frac{\sqrt{3}}{4} \cos 10^\circ\ [(-)\frac{1}{2} + \cos 20^\circ]$

$= -\frac{\sqrt{3}}{8} \cos 10^\circ + \frac{\sqrt{3}}{4} \cos 10^\circ \cos 20^\circ$

$= -\frac{\sqrt{3}}{8} \cos 10^\circ + \frac{\sqrt{3}}{8} (2 \cos 10^\circ \cos 20^\circ)$

$= -\frac{\sqrt{3}}{8} \cos 10^\circ + \frac{\sqrt{3}}{8} [\cos (10^\circ+20^\circ) + \cos (10^\circ - 20^\circ)]$

$= -\frac{\sqrt{3}}{8} \cos 10^\circ + \frac{\sqrt{3}}{8} (\cos 30^\circ + \cos 10^\circ)$

$= -\frac{\sqrt{3}}{8} \cos 10^\circ + \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8} \cos 10^\circ$

$= \frac{3}{16}$

= RHS **Proved.**

(g) $8 \cos 80^\circ \cos 140^\circ \cos 160^\circ = 1$

⇨ Here, LHS

$= 8 \cos 80^\circ \cos 140^\circ \cos 160^\circ$

$= 4(2 \cos 80^\circ \cos 140^\circ) \cos 160^\circ$

$= 4\ [\cos (80^\circ + 140^\circ) + \cos (80^\circ - 140^\circ)] \cos 160^\circ$

$= 4(\cos 220^\circ + \cos 60^\circ) \cos 160^\circ$

$= 4 \left(\cos 220^\circ + \frac{1}{2}\right) \cos 160^\circ$

$= 4 \cos 220^\circ \cos 160^\circ + 2 \cos 160^\circ$

$= 2(2 \cos 220^\circ \cos 160^\circ) + 2 \cos 160^\circ$

$= 2[\cos (220^\circ + 160^\circ) + \cos (220^\circ - 160^\circ)] + 2 \cos 160^\circ$

$= 2(\cos 380^\circ + \cos 60^\circ) + 2 \cos (180^\circ - 20^\circ)$

$= 2\ [\cos (360^\circ + 20^\circ) + \frac{1}{2}\ ] - 2 \cos 20^\circ$

$= 2 \cos 20^\circ + 1 - 2 \cos 20^\circ$

$= 1$ RHS **Proved.**

(i) $\sin 70^\circ \sin 130^\circ \sin 170^\circ = \frac{1}{8}$

⇨ Here, LHS

$= \sin 70^\circ \sin 130^\circ \sin 170^\circ$

$= \frac{1}{2} \sin 70^\circ\ (2 \sin 130^\circ \sin 170^\circ)$

$= \frac{1}{2} \sin 70^\circ\ [\cos (130^\circ - 170^\circ) - \cos (130^\circ + 170^\circ)]$

$= \frac{1}{2} \sin 70^\circ\ [\cos (-40^\circ) - \cos 300^\circ]$

$= \frac{1}{2} \sin 70^\circ\ [\cos 40^\circ - \cos (360^\circ - 60^\circ)]$

$= \frac{1}{2} \sin 70^\circ\ [\cos 40^\circ - \cos 60^\circ]$

$= \frac{1}{2} \sin 70^\circ \cos 40^\circ - \frac{1}{2} \sin 70^\circ \times \frac{1}{2}$

$= \frac{1}{4} (2 \sin 70^\circ \cos 40^\circ) - \frac{1}{4} \sin 70^\circ$

$= \frac{1}{4} [\sin (70^\circ + 40^\circ) + \sin (70^\circ - 40^\circ)] - \frac{1}{4} \sin 70^\circ$

$= \frac{1}{4} (\sin 110^\circ + \sin 30^\circ) - \frac{1}{4} \sin 70^\circ$

$= \frac{1}{4} \sin 110^\circ + \frac{1}{4} \sin 30^\circ - \frac{1}{4} \sin 70^\circ$

$= \frac{1}{4} \sin (90^\circ + 20^\circ) + \frac{1}{4} \times \frac{1}{2} - \frac{1}{4} \sin (90^\circ - 20^\circ)$

$= \frac{1}{4} \cos 20^\circ + \frac{1}{8} - \frac{1}{4} \cos 20^\circ$

$= \frac{1}{8} =$ RHS **Proved.**

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7. प्रमाणित गर्नुहोस् (Prove that):

(a) $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

⇨ Here, LHS

$$= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2 \sin \left(\frac{5x + 3x}{2}\right) \cos \left(\frac{5x - 3x}{2}\right)}{2 \cos \left(\frac{5x + 3x}{2}\right) \cos \left(\frac{5x - 3x}{2}\right)}$$

$$= \frac{\sin \frac{8x}{2} \cos \frac{2x}{2}}{\cos \frac{8x}{2} \cos \frac{2x}{2}}$$

$$= \frac{\sin 4x \cos x}{\cos 4x \cos x}$$

$= \tan 4x = \text{RHS}$ **Proved.**

(c) $\dfrac{\sin 2A + \sin 2B}{\cos 2A + \cos 2B} = \tan (A + B)$

⇨ Here, LHS

$$= \frac{\sin 2A + \sin 2B}{\cos 2A + \cos 2B}$$

$$= \frac{2 \sin \left(\frac{2A + 2B}{2}\right) \cos \left(\frac{2A - 2B}{2}\right)}{2 \cos \left(\frac{2A + 2B}{2}\right) \cos \left(\frac{2A - 2B}{2}\right)}$$

$$= \frac{\sin (A + B)}{\cos (A + B)}$$

$= \tan (A + B) = \text{RHS}$ **Proved.**

(e) $\dfrac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \sqrt{3}$

⇨ Here, LHS

$$= \frac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°}$$

$$= \frac{2 \cos \left(\frac{75° + 15°}{2}\right) \cos \left(\frac{75° - 15°}{2}\right)}{2 \cos \left(\frac{75° + 15°}{2}\right) \sin \left(\frac{75° - 15°}{2}\right)}$$

$$= \frac{\cos \frac{90°}{2} \cos \frac{60°}{2}}{\cos \frac{90°}{2} \sin \frac{60°}{2}}$$

$$= \frac{\cos 30°}{\sin 30°}$$

$= \cot 30° = \sqrt{3} = \text{RHS}$ **Proved.**

(b) $\dfrac{\sin 3A - \sin A}{\cos A - \cos 3A} = \cot 2A$

⇨ Here, LHS

$$= \frac{\sin 3A - \sin A}{\cos A - \cos 3A}$$

$$= \frac{2 \cos \left(\frac{3A + A}{2}\right) \sin \left(\frac{3A - A}{2}\right)}{-2 \sin \left(\frac{A + 3A}{2}\right) \sin \left(\frac{A - 3A}{2}\right)}$$

$$= \frac{\cos \frac{4A}{2} \sin \frac{2A}{2}}{-\sin \frac{4A}{2} \sin \frac{-2A}{2}}$$

$= \cot \dfrac{4A}{2} = \cot 2A = \text{RHS}$ **Proved.**

(d) $\dfrac{\cos 10° - \sin 10°}{\cos 10° + \sin 10°} = \tan 35°$

⇨ Here, LHS

$$= \frac{\cos 10° - \sin 10°}{\cos 10° + \sin 10°}$$

$$= \frac{\frac{1}{\sqrt{2}} \cos 10° - \frac{1}{\sqrt{2}} \sin 10°}{\frac{1}{\sqrt{2}} \cos 10° + \frac{1}{\sqrt{2}} \sin 10°}$$

$$= \frac{\sin 45° \cos 10° - \cos 45° \sin 10°}{\cos 45° \cos 10° + \sin 45° \sin 10°}$$

$$= \frac{\sin (45° - 10°)}{\cos (45° - 10°)} = \frac{\sin 35°}{\cos 35°}$$

$= \tan 35° = \text{RHS}$ **Proved.**

(f) $\dfrac{\cos 3A - \cos 2A + \cos A}{\sin 3A - \sin 2A + \sin A} = \cot 2A$

⇨ Here, LHS

$$= \frac{\cos 3A - \cos 2A + \cos A}{\sin 3A - \sin 2A + \sin A}$$

$$= \frac{\cos 3A + \cos A - \cos 2A}{\sin 3A + \sin A - \sin 2A}$$

$$= \frac{2 \cos \left(\frac{3A + A}{2}\right) \cos \left(\frac{3A - A}{2}\right) - \cos 2A}{2 \sin \left(\frac{3A + A}{2}\right) \cos \left(\frac{3A - A}{2}\right) - \sin 2A}$$

$$= \frac{2 \cos 2A \cos A - \cos 2A}{2 \sin 2A \cos A - \sin 2A}$$

$$= \frac{\cos 2A (2 \cos A - 1)}{\sin 2A (2 \cos A - 1)}$$

$= \dfrac{\cos 2A}{\sin 2A} = \cot 2A = \text{RHS}$ **Proved.**

(g) $\dfrac{\sin (A + B) - 2 \sin A + \sin (A - B)}{\cos (A + B) - 2 \cos A + \cos (A - B)} = \tan A$

⇨ Here, LHS $= \dfrac{\sin (A + B) - 2 \sin A + \sin (A - B)}{\cos (A + B) - 2 \cos A + \cos (A - B)} = \dfrac{\sin (A + B) + \sin (A - B) - 2 \sin A}{\cos (A + B) + \cos (A - B) - 2 \cos A}$

$$= \frac{\sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B - 2 \sin A}{\cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B - 2 \cos A}$$

$$= \frac{2 \sin A \cos B - 2 \sin A}{2 \cos A \cos B - 2 \cos A} = \frac{2 \sin A (\cos B - 1)}{2 \cos A (\cos B - 1)}$$

$= \dfrac{\sin A}{\cos A} = \tan A = \text{RHS}$ **proved.**

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(h) $\dfrac{\sin 2\theta + \sin 5\theta - \sin \theta}{\cos \theta + \cos 2\theta + \cos 5\theta} = \tan 2\theta$

⇨ Here, LHS

$$= \frac{\sin 2\theta + \sin 5\theta - \sin \theta}{\cos \theta + \cos 2\theta + \cos 5\theta}$$

$$= \frac{\sin 2\theta + (\sin 5\theta - \sin \theta)}{\cos 2\theta + (\cos \theta + \cos 5\theta)}$$

$$= \frac{\sin 2\theta + 2\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)}{\cos 2\theta + 2\cos\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{\theta-5\theta}{2}\right)}$$

$$= \frac{\sin 2\theta + 2\cos 3\theta \,.\, \sin 2\theta}{\cos 2\theta + 2\cos 3\theta \,.\, \cos 2\theta}$$

$$= \frac{\sin 2\theta\,(1 + 2\cos 3\theta)}{\cos 2\theta\,(1 + 2\cos 3\theta)}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta = \text{RHS}$$

Proved.

(i) $\dfrac{\cos 80° + \cos 20°}{\sin 80° - \sin 20°} = \sqrt{3}$

⇨ Here, LHS

$$= \frac{\cos 80° + \cos 20°}{\sin 80° - \sin 20°}$$

$$= \frac{2\cos\left(\frac{80°+20°}{2}\right)\cos\left(\frac{80°-20°}{2}\right)}{2\cos\left(\frac{80°+20°}{2}\right)\sin\left(\frac{80°-20°}{2}\right)}$$

$$= \frac{\cos 50° \cos 30°}{\cos 50° \sin 30°}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \sqrt{3}$$

$$= \text{RHS}$$

Proved.

(j) $\dfrac{\sin 75° + \sin 15°}{\cos 75° + \cos 15°} = 1$

⇨ Here, LHS

$$= \frac{\sin 75° + \sin 15°}{\cos 75° + \cos 15°}$$

$$= \frac{2\sin\left(\frac{75°+15°}{2}\right)\cos\left(\frac{75°-15°}{2}\right)}{2\cos\left(\frac{75°+15°}{2}\right)\cos\left(\frac{75°-15°}{2}\right)}$$

$$= \frac{\sin 45° \cos 30°}{\cos 45° \cos 30°}$$

$$= \frac{\sin 45°}{\cos 45°}$$

$$= \tan 45°$$

$$= 1$$

$$= \text{RHS}$$

Proved.

8. प्रमाणित गर्नुहोस् (Prove that):

(a) $\dfrac{\sin \theta - \sin 3\theta + \sin 5\theta - \sin 7\theta}{\cos \theta - \cos 3\theta - \cos 5\theta + \cos 7\theta} = \cot 2\theta$

⇨ Here, LHS

$$= \frac{\sin \theta - \sin 3\theta + \sin 5\theta - \sin 7\theta}{\cos \theta - \cos 3\theta - \cos 5\theta + \cos 7\theta}$$

$$= \frac{(\sin \theta - \sin 7\theta) + (\sin 5\theta - \sin 3\theta)}{(\cos \theta + \cos 7\theta) - (\cos 5\theta + \cos 3\theta)}$$

$$= \frac{2\cos\left(\frac{\theta+7\theta}{2}\right)\sin\left(\frac{\theta-7\theta}{2}\right) + 2\cos\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)}{2\cos\left(\frac{\theta+7\theta}{2}\right)\cos\left(\frac{\theta-7\theta}{2}\right) - 2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}$$

$$= \frac{2\cos 4\theta \sin(-3\theta) + 2\cos 4\theta \sin \theta}{2\cos 4\theta \cos 3\theta - 2\cos 4\theta \cos \theta}$$

$$= \frac{2\cos 4\theta\,(-\sin 3\theta + \sin \theta)}{2\cos 4\theta\,(\cos 3\theta - \cos \theta)} = \frac{\sin \theta - \sin 3\theta}{\cos 3\theta - \cos \theta}$$

$$= \frac{2\cos\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{\theta-3\theta}{2}\right)}{2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{\theta-3\theta}{2}\right)}$$

$$= \frac{\cos 2\theta}{\sin 2\theta} = \cot 2\theta = \text{RHS}$$

Proved.

(b) $\dfrac{\sin 5° - \sin 15° + \sin 25° - \sin 35°}{\cos 5° - \cos 15° - \cos 25° + \cos 35°} = \cot 10°$

⇨ Here, LHS $= \dfrac{\sin 5° - \sin 15° + \sin 25° - \sin 35°}{\cos 5° - \cos 15° - \cos 25° + \cos 35°}$

$$= \frac{(\sin 5° - \sin 35°) + (\sin 25° - \sin 15°)}{(\cos 5° + \cos 35°) - (\cos 15° + \cos 25°)}$$

$$= \frac{2\cos\left(\frac{5°+35°}{2}\right)\sin\left(\frac{5°-35°}{2}\right) + 2\cos\left(\frac{15°+25°}{2}\right)\sin\left(\frac{25°-15°}{2}\right)}{2\cos\left(\frac{5°+35°}{2}\right)\cos\left(\frac{5°-35°}{2}\right) - 2\cos\left(\frac{15°+25°}{2}\right)\cos\left(\frac{15°-25°}{2}\right)}$$

$$= \frac{2\cos 20° \sin(-15°) + 2\cos 20° \sin 5°}{2\cos 20° \cos 15° - 2\cos 20° \cos 5°} = \frac{2\cos 20°\,(-\sin 15° + \sin 5°)}{2\cos 20°\,(\cos 15° - \cos 5°)}$$

$$= \frac{\sin 5° - \sin 15°}{\cos 15° - \cos 5°} = \frac{2\cos\left(\frac{5°+15°}{2}\right)\sin\left(\frac{5°-15°}{2}\right)}{2\sin\left(\frac{15°+5°}{2}\right)\sin\left(\frac{5°-15°}{2}\right)} = \frac{\cos 10°}{\sin 10°}$$

$$= \cot 10° = \text{RHS}$$

Proved.

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(c) $\dfrac{1 - \cos 10° + \cos 40° - \cos 50°}{1 + \cos 10° - \cos 40° - \cos 50°} = \tan 5° \cot 20°$

⇨ Here, LHS

$$= \frac{1 - \cos 10° + \cos 40° - \cos 50°}{1 + \cos 10° - \cos 40° - \cos 50°}$$

$$= \frac{1 - (\cos 10° - \cos 40°) - \cos 50°}{1 + (\cos 10° - \cos 40°) - \cos 50°}$$

$$= \frac{1 - 2\sin\left(\frac{10° + 40°}{2}\right)\sin\left(\frac{40° - 10°}{2}\right) - \cos 50°}{1 + 2\sin\left(\frac{10° + 40°}{2}\right)\sin\left(\frac{40° - 10°}{2}\right) - \cos 50°}$$

$$= \frac{1 - 2\sin 25° \sin 15° - \cos 50°}{1 + 2\sin 25° \sin 15° - \cos 50°}$$

$$= \frac{(1 - \cos 50°) - 2\sin 25° \sin 15°}{(1 - \cos 50°) + 2\sin 25° \sin 15°}$$

$$= \frac{2\sin^2 25° - 2\sin 25° \sin 15°}{2\sin^2 25° + 2\sin 25° \sin 15°}$$

$$= \frac{2\sin 25°(\sin 25° - \sin 15°)}{2\sin 25°(\sin 25° + \sin 15°)}$$

$$= \frac{\sin 25° - \sin 15°}{\sin 25° + \sin 15°}$$

$$= \frac{2\cos\left(\frac{25° + 15°}{2}\right)\sin\left(\frac{25° - 15°}{2}\right)}{2\sin\left(\frac{25° + 15°}{2}\right)\cos\left(\frac{25° - 15°}{2}\right)}$$

$$= \frac{\cos 20° \sin 5°}{\sin 20° \cos 5°} = \cot 20° \tan 5°$$

$= \tan 5° \cot 20° = \text{RHS}$ **Proved.**

9. प्रमाणित गर्नुहोस् (Prove that:

(a) $\dfrac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan (A + B)$

⇨ Here, LHS

$$= \frac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B}$$

$$= \frac{2(\sin A - \sin B)(\sin A + \sin B)}{2\sin A\cos A - 2\sin B\cos B}$$

$$= \frac{2 \,.\, 2\cos\frac{A + B}{2}\sin\frac{A - B}{2} \,.\, 2\sin\frac{A + B}{2} \,.\, \cos\frac{A - B}{2}}{\sin 2A - \sin 2B}$$

$$= \frac{2\left[2\sin\frac{A+B}{2} \,.\, \cos\frac{A+B}{2} \,.\, 2\sin\frac{A-B}{2} \,.\, \cos\frac{A-B}{2}\right]}{2\cos\frac{2A+2B}{2} \,.\, \sin\frac{2A-2B}{2}}$$

$$= \frac{\sin(A + B) \,.\, \sin(A - B)}{\cos(A + B) \,.\, \sin(A - B)}$$

$$= \frac{\sin(A + B)}{\cos(A + B)}$$

$= \tan(A + B) = \text{RHS}$ **Proved.**

(d) $\dfrac{1 - \cos A + \cos B - \cos (A + B)}{1 + \cos A - \cos B - \cos (A - B)} = \tan\dfrac{A}{2}\cot\dfrac{B}{2}$

⇨ Here, LHS

$$= \frac{1 - \cos A + \cos B - \cos(A + B)}{1 + \cos A - \cos B - \cos(A - B)}$$

$$= \frac{1 - \cos(A + B) - \cos A + \cos B}{1 - \cos(A + B) + \cos A - \cos B}$$

$$= \frac{2\sin^2\left(\frac{A + B}{2}\right) - (\cos A - \cos B)}{2\sin^2\left(\frac{A + B}{2}\right) + (\cos A - \cos B)}$$

$$= \frac{2\sin^2\left(\frac{A + B}{2}\right) - 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{B - A}{2}\right)}{2\sin^2\left(\frac{A + B}{2}\right) + 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{B - A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{A + B}{2}\right)\left[\sin\left(\frac{A + B}{2}\right) - \sin\left(\frac{B - A}{2}\right)\right]}{2\sin\left(\frac{A + B}{2}\right)\left[\sin\left(\frac{A + B}{2}\right) + \sin\left(\frac{B - A}{2}\right)\right]}$$

$$= \frac{\sin\left(\frac{A}{2} + \frac{B}{2}\right) + \sin\left(\frac{A}{2} - \frac{B}{2}\right)}{\sin\left(\frac{A}{2} + \frac{B}{2}\right) - \sin\left(\frac{A}{2} - \frac{B}{2}\right)}$$

$$= \frac{\sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2} + \sin\frac{A}{2}\cos\frac{B}{2} - \cos\frac{A}{2}\sin\frac{B}{2}}{\sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2} - \sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2}}$$

$$= \frac{2\sin\frac{A}{2}\cos\frac{B}{2}}{2\cos\frac{A}{2}\sin\frac{B}{2}} = \tan\frac{A}{2}\cot\frac{B}{2} = \text{RHS}$$ **Proved.**

(b) $\dfrac{\cos^2 A - \sin^2 B}{\sin A \cos A + \sin B \cos B} = \cot (A + B)$

⇨ Here, LHS

$$= \frac{\cos^2 A - \sin^2 B}{\sin B \cos B + \sin A \cos A}$$

$$= \frac{2\cos^2 A - 2\sin^2 B}{2\sin B\cos B + 2\sin A\cos A}$$

$$= \frac{1 + \cos 2A - (1 - \cos 2B)}{\sin 2B + \sin 2A}$$

$$= \frac{1 + \cos 2A - 1 + \cos 2B}{\sin 2B + \sin 2A}$$

$$= \frac{\cos 2B + \cos 2A}{\sin 2B + \sin 2A}$$

$$= \frac{2\cos\left(\frac{2B + 2A}{2}\right)\cos\left(\frac{2B - 2A}{2}\right)}{2\sin\left(\frac{2B + 2A}{2}\right)\cos\left(\frac{2B - 2A}{2}\right)}$$

$$= \frac{\cos(B + A)}{\sin(B + A)}$$

$= \cot(B + A) = \cot(A + B) = \text{RHS}$ **Proved.**

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3. अनुबन्धित त्रिकोणमितीय सर्वसमीकाहरू
Conditional Trigonometric Identities

Formulae and Key Points

1. π^c = त्रिभुजका तीनकोणहरूको योग (Sum of three angles of a triangle) = 180°
2. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए $A + B = \pi^c - C$ and $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$ हुन्छ ।

   If $A + B + C = \pi^c$ then $A + B = \pi^c - C$ and $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$

   (i) $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$ (ii) $\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \sin\frac{C}{2}$

   (iii) $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \cot\frac{C}{2}$
3. यदि $A + B + C = \pi^c$ भए $A + B = 180° - C$ हुन्छ । (If $A + B + C = \pi^c$, then $A + B = 180° - C$.)

   (i) $\cos(A + B) = \cos(180° - C) = -\cos C$ (ii) $\sin(A + B) = \sin(180° - C) = \sin C$

   (iii) $\tan(A + B) = \tan(180° - C) = -\tan C$

QUESTIONS FROM SEE EXERCISE 3

A. VERY SHORT QUESTIONS

1. यदि $A + B + C = \pi^c$ भए $\cos(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(A + B)$ in terms of angle C.

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
or, $\cos(A + B) = \cos(\pi^c - C)$
∴ $\cos(A + B) = -\cos C$

2. यदि $A + B + C = \pi^c$ भए $\cos\left(\frac{A}{2} + \frac{B}{2}\right)$ लाई कोण $\frac{C}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

If $A + B + C = \pi^c$, express $\cos\left(\frac{A}{2} + \frac{B}{2}\right)$ in terms of angle $\frac{C}{2}$.

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
or, $\frac{A + B}{2} = \frac{\pi^c - C}{2}$
or, $\cos\left(\frac{A + B}{2}\right) = \cos\left(90° - \frac{C}{2}\right)$
∴ $\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\frac{C}{2}$

3. यदि $A + B + C = \pi^c$ भए $\cos(2A + 2B)$ लाई कोण 2C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(2A + 2B)$ in terms of angle 2C.

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
or, $2A + 2B = 2\pi^c - 2C$
or, $\cos(2A + 2B) = \cos(360° - 2C)$
∴ $\cos(2A + 2B) = \cos 2C$

4. यदि $A + B + C = \pi^c$ भए $\cos(4A + 4B)$ लाई कोण 4C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(4A + 4B)$ in terms of angle 4C.

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
or, $4A + 4B = 4\pi^c - 4C$
or, $\cos(4A + 4B) = \cos(4\pi^c - 4C)$
∴ $\cos(4A + 4B) = \cos 4C$

B. SHORT QUESTIONS

1. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस्
If $A + B + C = \pi^c$ then prove that:
$\tan A + \tan B = -\tan C(1 - \tan A \tan B)$

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
Taking tan on both the sides;
$\tan(A + B) = \tan(\pi^c - C)$
or, $\frac{\tan A + \tan B}{1 - \tan A + \tan B} = -\frac{\tan C}{1}$
∴ $\tan A + \tan B = -\tan C(1 - \tan A \tan B)$

Proved.

2. यदि $A + B + C = \pi^c$ र $\cos A = \cos B \cos C$ भए, प्रमाणित गर्नुहोस्: $\tan A = \tan B + \tan C$
If $A + B + C = \pi^c$ and $\cos A = \cos B \cos C$, prove that: $\tan A = \tan B + \tan C$

⇨ Here, $A + B + C = \pi^c$ and $\cos A = \cos B \cos C$
Now, $B + C = \pi^c - A$
or, $\sin(B + C) = \sin(\pi^c - A) = \sin A$
or, $\sin B \cos C + \cos B \sin C = \sin A$
or, $\frac{\sin B \cos C}{\cos A} + \frac{\cos B \sin C}{\cos A} = \frac{\sin A}{\cos A}$
[∵ Dividing both sides by cosA]
or, $\frac{\sin B \cos C}{\cos B \cos C} + \frac{\cos B \sin C}{\cos B \cos C} = \tan A$
Thus, $\tan B + \tan C = \tan A$ **Proved.**

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3. यदि A + B + C = π^c भए प्रमाणित गर्नुहोस् :
If A + B + C = π^c then prove that:
$\sin A + \sin B = 2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

or, $\frac{A+B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$

Taking sin on both the sides;

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90° - \frac{C}{2}\right) = \cos\frac{C}{2}$$

Now, LHS $= \sin A + \sin B$

$= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= 2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)$

$=$ RHS **Proved.**

4. यदि A + B + C = π^c भए प्रमाणित गर्नुहोस्
If A + B + C = π^c then prove that:
$\cos 2A + \cos 2B = -2\cos C\cos(A-B)$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

Taking cos on both the sides,

$$\cos(A + B) = \cos(\pi^c - C) = -\cos C$$

Now, LHS $= \cos 2A + \cos 2B$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)$

$= 2\cos(A+B)\cos(A-B)$

$= 2(-)\cos C\cos(A-B)$

$= -2\cos C\cos(A-B)$

$=$ RHS **Proved.**

5. यदि A + B + C = π^c भए प्रमाणित गर्नुहोस्
If A + B + C = π^c then prove that:
$\cos^2 A + \cos^2 B = 1 - \cos C\cos(A-B)$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

Taking cos on both the sides;

$\cos(A + B) = \cos(\pi^c - C)$

∴ $\cos(A + B) = -\cos C$

Now,

LHS $= \cos^2 A + \cos^2 B$

$= \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2}$

$= \frac{1}{2} + \frac{\cos 2A}{2} + \frac{1}{2}\frac{\cos 2B}{2}$

$= 1 + \frac{1}{2}(\cos 2A + \cos 2B)$

$= 1 + \frac{1}{2}.2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)$

$= 1 + \cos(A+B)\cos(A-B)$

$= 1 - \cos C\cos(A-B)$

$=$ RHS **Proved.**

6. यदि A + B + C = π^c भए प्रमाणित गर्नुहोस् :
If A + B + C = π^c then prove that :
$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} = 1 - \sin\frac{C}{2}\cos\left(\frac{A}{2} - \frac{B}{2}\right)$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

or, $\frac{A+B}{2} = \frac{\pi^c - C}{2}$

Taking cos on both the sides then,

$$\cos\left(\frac{A+B}{2}\right) = \cos\left(90° - \frac{C}{2}\right) = \sin\frac{C}{2}$$

Now, LHS $= \sin^2\frac{A}{2} + \sin^2\frac{B}{2}$

$= \frac{1-\cos A}{2} + \frac{1-\cos B}{2}$

$= \frac{1}{2} - \frac{\cos A}{2} + \frac{1}{2} - \frac{\cos B}{2}$

$= 1 - \frac{1}{2}(\cos A + \cos B)$

$= 1 - \frac{1}{2}2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= 1 - \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)$

$= 1 - \sin\frac{C}{2}\cos\left(\frac{A}{2} - \frac{B}{2}\right)$

$=$ RHS **Proved.**

C. LONG QUESTIONS

MODEL 1

1. यदि ∡A + ∡B + ∡C = 180° भए, प्रमाणित गर्नुहोस् (If ∡A + ∡B + ∡C = 180°, prove that): $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$
[2057 M]

⇨ Here, $A + B + C = \pi^c$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi^C}{2}$ or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

Taking tan on both sides, we get,

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

or, $\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\frac{C}{2}$

or, $\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}.\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$

or, $\tan\frac{A}{2}.\tan\frac{C}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} = 1 - \tan\frac{A}{2} + \tan\frac{B}{2}$

or, $\tan\frac{A}{2}.\tan\frac{C}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{A}{2}.\tan\frac{B}{2} = 1$

∴ LHS = RHS **Proved.**

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2. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस् If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that): $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$

[2075 R', 2058 R, 2066 R]

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi^c}{2}$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

Taking cot on both sides, we get

$$\cot\left(\frac{A}{2} + \frac{B}{2}\right) = \cot\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

or, $\dfrac{\cot\frac{A}{2}\cot\frac{B}{2} - 1}{\cot\frac{A}{2} + \cot\frac{B}{2}} = \tan\frac{C}{2}$

or, $\dfrac{\cot\frac{A}{2}\cot\frac{B}{2} - 1}{\cot\frac{A}{2} + \cot\frac{B}{2}} = \dfrac{1}{\cot\frac{C}{2}}$

or, $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} - \cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{B}{2}$

$\therefore$ $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$

$\therefore$ LHS = RHS **Proved.**

MODEL 2

3. यदि $A + B + C = 180°$ भए प्रमाणित गर्नुहोस् :

If $A + B + C = 180°$, Prove that:

$\sin A - \sin B + \sin C = 4\sin\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$ [2072 S]

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$ or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\therefore$ $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$ and $\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\frac{C}{2}$

LHS

$= \sin A - \sin B + \sin C$

$= 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} + \sin C$

$= 2\sin\frac{C}{2}\sin\frac{A-B}{2} + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin\frac{C}{2}\left\{\sin\frac{A-B}{2} + \cos\frac{C}{2}\right\}$

$= 2\sin\frac{C}{2}\left\{\sin\frac{A-B}{2} + \sin\frac{A+B}{2}\right\}$

$= 2\sin\frac{C}{2} \times 2\sin\left(\dfrac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\right)\cos\left(\dfrac{\frac{A-B}{2} - \frac{A+B}{2}}{2}\right)$

$= 4\sin\frac{C}{2}\sin\frac{A-B+A+B}{4}\cos\frac{A-B-A-B}{4}$

$= 4\sin\frac{C}{2}\sin\frac{A}{2}\cos\frac{B}{2}$

$= 4\sin\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

$=$ RHS **Proved.**

4. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

[2074 R', 2072 R', 2057 R, 2064 R']

⇨ Here, $A + B + C = 180°$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{180°}{2}$

or, $\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$

or, $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\left(90° - \frac{C}{2}\right)$

$\therefore$ $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$

RHS

$= \sin A + \sin B + \sin C$

$= 2\sin\frac{A+B}{2} . \cos\frac{A-B}{2} + 2\sin\frac{C}{2} . \cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} + \sin\frac{C}{2}\right)$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right)$

$= 2\cos\frac{C}{2} . 2\cos\dfrac{\left(\frac{A-B}{2} + \frac{A+B}{2}\right)}{2} . \cos\dfrac{\left(\frac{A-B}{2} - \frac{A+B}{2}\right)}{2}$

$= 4\cos\frac{C}{2}\cos\frac{A}{2}\cos\left(-\frac{B}{2}\right)$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} =$ RHS **Proved.**

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5. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$

[2058 S, 2064 R, 2070 R']

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$

or, $\sin\left(\frac{A}{2}+\frac{B}{2}\right) = \sin\left(\frac{\pi^c}{2}-\frac{C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A}{2}+\frac{B}{2}\right) = \cos\left(\frac{\pi^c}{2}-\frac{C}{2}\right) = \sin\frac{C}{2}$

LHS

$= \cos A + \cos B + \cos C$

$= 2\cos\frac{A+B}{2}.\cos\frac{A-B}{2} + 1 - 2\sin^{2}\frac{C}{2}$

$= 2\sin\frac{C}{2}.\cos\frac{A-B}{2} - 2\sin^{2}\frac{C}{2} + 1$

$= 1 + 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} - \sin\frac{C}{2}\right]$

$= 1 + 2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$

$= 1 + 2\sin\frac{C}{2}.2\sin\frac{1}{2}\left(\frac{A-B}{2}+\frac{A+B}{2}\right).\sin\frac{1}{2}\left(\frac{A+B}{2}-\frac{A-B}{2}\right)$

$= 1 + 4\sin\frac{C}{2}.\sin\frac{1}{2}\left(\frac{A-B+A+B}{2}\right).\sin\frac{1}{2}\left(\frac{A+B-A+B}{2}\right)$

$= 1 + 4\sin\frac{C}{2}.\sin\frac{A}{2}.\sin\frac{B}{2}$

$= 1 + 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} = \text{RHS}$ **Proved.**

6. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\cos B + \cos C - \cos A = 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1$ [2057 S]

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

or, $\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$

or, $\sin\left(\frac{A+B}{2}\right) = \sin\left(90° - \frac{C}{2}\right) = \cos\frac{C}{2}$

LHS

$= \cos B + \cos C - \cos A$

$= 2\cos\frac{B+C}{2}.\cos\frac{B-C}{2} - \left(1 - 2\sin^{2}\frac{A}{2}\right)$

$= 2\sin\frac{A}{2}.\cos\frac{B-C}{2} - 1 + 2\sin^{2}\frac{A}{2}$

$= -1 + 2\sin\frac{A}{2}.\cos\frac{B-C}{2} + 2\sin^{2}\frac{A}{2}$

$= -1 + 2\sin\frac{A}{2}\left(\cos\frac{B-C}{2} + \sin\frac{A}{2}\right)$

$= -1 + 2\sin\frac{A}{2}\left(\cos\frac{B-C}{2} + \cos\frac{B+C}{2}\right)$

$= -1 + 2\sin\frac{A}{2}\left[2\cos\frac{1}{2}\left(\frac{B-C}{2}+\frac{B+C}{2}\right)\cos\frac{1}{2}\left(\frac{B-C}{2}-\frac{B+C}{2}\right)\right]$

$= -1 + 2\sin\frac{A}{2}\left[2\cos\frac{B}{2}.\cos\left(\frac{-C}{2}\right)\right]$

$= -1 + 4\sin\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$

$= 4\sin\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2} - 1 = \text{RHS}$ **Proved.**

7. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\sin A + \sin B - \sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

⇨ Here, A, B, C are the angels of ΔABC

Then, $A + B + C = \pi^c$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi^c}{2}$ or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$

$\sin\left(\frac{A}{2}+\frac{B}{2}\right) = \sin\left(\frac{\pi^c}{2}-\frac{C}{2}\right) = \cos\frac{C}{2}$

or, $\cos\left(\frac{A}{2}+\frac{B}{2}\right) = \cos\left(\frac{\pi^c}{2}-\frac{C}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$= \sin A + \sin B - \sin C$

$= 2\sin\frac{A+B}{2}.\cos\frac{A-B}{2} - 2\sin\frac{C}{2}.\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}.\cos\frac{A-B}{2} - 2\sin\frac{C}{2}.\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right)$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A}{2} + \frac{B}{2}\right)$

$= 2\cos\frac{C}{2}.2\sin\frac{\left(\frac{A-B}{2}+\frac{A+B}{2}\right)}{2}.\sin\frac{\left(\frac{A+B}{2}-\frac{A-B}{2}\right)}{2}$

$= 4\cos\frac{C}{2}.\sin\frac{A}{2}.\sin\frac{B}{2}$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} = \text{RHS}$ **Proved.**

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8. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस् (If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that):

$\sin B + \sin C - \sin A = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ [2061 R]

⇨ Here, given $A + B + C = \pi^c$

or, $B + C = \pi^c - A$

or, $\frac{B}{2} + \frac{C}{2} = \frac{\pi^c}{2} - \frac{A}{2}$

or, $\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi^c}{2} - \frac{A}{2}\right) = \cos\frac{A}{2}$ and

$\cos\left(\frac{B+C}{2}\right) = \cos\left(\frac{\pi^c}{2} - \frac{A}{2}\right) = \sin\frac{A}{2}$

Now, LHS

$= \sin B + \sin C - \sin A$

$= 2\sin\frac{B+C}{2}\,.\,\cos\frac{B-C}{2} - 2\sin\frac{A}{2}\,.\,\cos\frac{A}{2}$

$= 2\cos\frac{A}{2}\,.\,\cos\frac{B-C}{2} - 2\sin\frac{A}{2}\,.\,\cos\frac{A}{2}$

$= 2\cos\frac{A}{2}\left[\cos\frac{B-C}{2} - \sin\frac{A}{2}\right]$

$= 2\cos\frac{A}{2}\left[\cos\frac{B-C}{2} - \cos\frac{B+C}{2}\right]$

$= 2\cos\frac{A}{2}\left[2\sin\frac{1}{2}\left(\frac{B-C}{2} + \frac{B+C}{2}\right).\sin\frac{1}{2}\left(\frac{B+C}{2} - \frac{B-C}{2}\right)\right]$

$= 2\cos\frac{A}{2}\left[2\sin\frac{B}{2}.\sin\frac{C}{2}\right]$

$= 4\cos\frac{A}{2}\,.\,\sin\frac{B}{2}\,.\,\sin\frac{C}{2} = \text{RHS}$

MODEL 3

9. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्
If $A + B + C = \pi^c$ prove that: [2075 R_2', 2073 R']
$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

⇨ Here, $A + B + C = \pi$

or, $A + B = \pi - C$

$\sin(A + B) = \sin C$

$\cos(A + B) = -\cos C$

LHS

$= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + \sin 2C$

$= 2\sin C\cos(A - B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A - B) + \cos C\}$

$= 2\sin C\{\cos(A - B) - \cos(A + B)\}$

$= 2\sin C.\,2\sin\left(\frac{A-B+A+B}{2}\right)\sin\left(\frac{A+B-A+B}{2}\right)$

$= 4\sin C\sin A\sin B$

$= 4\sin A\sin B\sin C = \text{RHS}$ **Proved.**

10. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस्
If $A + B + C = \pi^c$ then prove that: [2074 S']
$\cos 2A - \cos 2B + \cos 2C = 1 - 4\sin A.\cos B.\sin C$

⇨ Here, $A + B + C = \pi^c$

$A + B = \pi^c - C$

Taking sin and cos on both sides then,

$\sin(A + B) = \sin(\pi^c - C) = \sin C$

$\cos(A + B) = \cos(\pi^c - C) = -\cos C$

LHS

$= \cos 2A - \cos 2B + \cos 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2B-2A}{2}\right) + \cos 2C$

$= 2\sin(A + B)\sin(B - A) + \cos 2C$

$= 2\,.\,\sin C\sin(B - A) + 1 - 2\sin^2 C$

$= 2\sin C\sin(B - A) - 2\sin^2 C + 1$

$= 2\sin C[\sin(B - A) - \sin C] + 1$

$= 2\sin C[\sin(B - A) - \sin(A + B)] + 1$

$= 2\sin C[-\sin(A - B) - \sin(A + B)] + 1$

$= 2\sin C(-)[\sin(A + B) + \sin(A - B)] + 1$

$= -2\sin C\,2\sin A\cos B + 1$

$= 1 - 4\sin A\cos B\sin C$

$= \text{RHS}$ **Proved.**

11. यदि $A + B + C = \frac{\pi^c}{2}$ भए प्रमाणित गर्नुहोस्

If $A + B + C = \frac{\pi^c}{2}$ then prove that: [2074 S]

$\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A.\sin B.\sin C$

⇨ Here, $A + B + C = \frac{\pi^c}{2} = 90°$

or, $A + B = 90° - C$

Taking sin and cos on both the sides then,

$\sin(A + B) = \sin(90° - C) = \cos C$

$\cos(A + B) = \cos(90° - C) = \sin C$

LHS

$= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + \cos 2C$

$= 2\sin C\cos(A - B) + 1 - 2\sin^2 C$

$= 2\sin C\cos(A - B) - 2\sin^2 C + 1$

$= 2\sin C[\cos(A - B) - \sin C] + 1$

$= 2\sin C[\cos(A - B) - \cos(A + B)] + 1$

$= 2\sin C\,2\sin A\sin B + 1$

$= 1 + 4\sin A\sin B\sin C = \text{RHS}$ **Proved.**

12. यदि $A + B + C = 90°$ भए प्रमाणित गर्नुहोस्
If $A + B + C = 90°$ then prove that:
$\sin 2A + \sin 2B + \sin 2C = 4\cos A\cos B\cos C$ [2074 R]

⇨ Here, $A + B + C = 90°$

or, $A + B = 90° - C$

Taking sin and cos on the both the sides,

$\sin(A + B) = \sin(90° - C) = \cos C$

$\cos(A + B) = \cos(90° - C) = \sin C$

Now,

LHS $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right) + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + 2\sin C\cos C$

$= 2\cos C\cos(A - B) + 2\sin C\cos C$

$= 2\cos C[\cos(A - B) + \sin C]$

$= 2\cos C[\cos(A - B) + \cos(A + B)]$

$= 2\cos C\,.\,2\cos A\cos B$

$= 4\cos A\cos B\cos C = \text{RHS}$ **Proved.**

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13. यदि $P + Q + R = 180°$ भए प्रमाणित गर्नुहोस्:

If $P + Q + R = 180°$ then prove that: [2073 S]

$\cos(Q + R - P) + \cos(R + P - Q) + \cos(P + Q - R) = 1 + 4\cos P.\cos Q.\cos R$

⇨ Here, $P + Q + R = \pi^c$

or, $P + Q = \pi^c - R$

$\sin(P + Q) = \sin(\pi^c - R) = \sin R$

$\cos(P + Q) = \cos(\pi^c - R) = -\cos R$

LHS

$= \cos(Q + R - P) + \cos(R + P - Q) + \cos(P + Q - R)$

$= \cos(180° - 2P) + \cos(180° - 2Q) + \cos(180° - 2R)$

$= -\cos 2P - \cos 2Q - \cos 2R$

$= -(\cos 2P + \cos 2Q) - \cos 2R$

$= -2\cos(P + Q)\cos(P - Q) - \cos 2R$

$= 2\cos R\cos(P - Q) - 2\cos^2 R + 1$

$= 1 + 2\cos R[\cos(P - Q) - \cos R]$

$= 1 + 2\cos R[\cos(P - Q) + \cos(P + Q)]$

$= 1 + 2\cos R(2\cos P\cos Q)$

$= 1 + 4\cos P\cos Q\cos R = \text{RHS}$ **Proved.**

14. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\cos 2A + \cos 2B + \cos 2C = -(4\cos A.\cos B.\cos C + 1)$

[2065 M, 2065 S, 2060 S']

⇨ Here, $A + B + C = \pi^c$,

$\sin(A + B) = \sin(\pi - C) = \sin C$

$\cos(A + B) = \cos(\pi - C) = -\cos C$

LHS $= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right) + 2\cos^2 C - 1$

$= 2\cos(A + B).\cos(A - B) + 2\cos^2 C - 1$

$= -2\cos C.\cos(A - B) + 2\cos^2 C - 1$

$= -2\cos C[\cos(A - B) - \cos C] - 1$

$= -2\cos C[\cos(A - B) + \cos(A + B)] - 1$

$= -2\cos C(2\cos A.\cos B) - 1$

$= -(4\cos A\cos B\cos C + 1) = \text{RHS}$ **Proved.**

16. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A.\sin B.\cos C$

⇨ Here, given $A + B + C = 180°$

or, $A + B = 180° - C$

or, $\cos(A + B) = \cos(180° - C) = -\cos C$

$\therefore$ $\sin(A + B) = \sin(180° - C) = \sin C$

Now, LHS

$= \cos 2A + \cos 2B - \cos 2C$

$= 2\cos\left(\frac{2A + 2B}{2}\right).\cos\left(\frac{2A - 2B}{2}\right) - (2\cos^2 C - 1)$

$= 2\cos(A + B).\cos(A - B) - 2\cos^2 C + 1$

$= 1 - 2\cos C.\cos(A - B) - 2\cos^2 C$

$= 1 - 2\cos C[(\cos(A - B) + \cos C]$

$= 1 - 2\cos C[\cos(A - B) - \cos(A + B)]$

$= 1 - 2\cos C\left[2\sin\left(\frac{A-B+A+B}{2}\right).\sin\left(\frac{A+B-A+B}{2}\right)\right]$

$= 1 - 2\cos C\left[2\sin\frac{2A}{2}.\sin\frac{2B}{2}\right]$

$= 1 - 4\sin A.\sin B.\sin C = \text{RHS}$ **Proved.**

15. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$, prove that: [2070 R]

$$\frac{\sin 2A + \sin 2B + \sin 2C}{4\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}} = 8\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$$

⇨ Here, $A + B + C = \pi^c$

$A + B = \pi^c - C$

Taking sin and cos on both the sides then,

$\sin(A + B) = \sin(\pi^c - C) = \sin C$

$\cos(A + B) = \cos(\pi^c - C) = -\cos C$

Taking $\sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right) + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + 2\sin C\cos C$

$= 2\sin C\cos(A - B) + 2\sin C\cos C$

$= 2\sin C[\cos(A - B) + \cos C]$

$= 2\sin C[\cos(A - B) - \cos(A + B)$

$= 2\sin C\, 2\sin A\sin B$

$= 4\sin A\sin B\sin C$

Now, LHS

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{4\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}}$$

$$= \frac{4\sin A\sin B\sin C}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$= \frac{4 \times 2\sin\frac{A}{2}\cos\frac{A}{2} \times 2\sin\frac{B}{2}\cos\frac{B}{2} \times 2\sin\frac{C}{2}\cos\frac{C}{2}}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$= 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$ **Proved.**

17. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्

If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:

$\cos 2A - \cos 2B - \cos 2C = 4\cos A\sin B\sin C - 1$

⇨ Here, given $A + B + C = 180°$

or, $A + B = 180° - C$

Putting sin on both sides

$\sin(A + B) = \sin(180° - C) = \sin C$

Putting cos on both sides

$\cos(A + B) = \cos(180° - C) = -\cos C$

LHS

$= \cos 2A - \cos 2B - \cos 2C$

$= -2\sin\left(\frac{2A + 2B}{2}\right)\sin\left(\frac{2A - 2B}{2}\right) - (1 - 2\sin^2 C)$

$= -2\sin(A + B).\sin(A - B) - 1 + 2\sin^2 C$

$= -2\sin C[\sin(A - B) - \sin C] - 1$

$= -2\sin C[\sin(A - B) - \sin(A + B)] - 1$

$= -2\sin C\left[2\cos\left(\frac{A-B+A+B}{2}\right).\sin\left(\frac{A-B-A-B}{2}\right)\right] - 1$

$= -2\sin C\left[2\cos\frac{2A}{2}.\sin\left(\frac{-2B}{2}\right)\right] - 1$

$= -2\sin C[-2\cos A\sin B] - 1$

$= 4\cos A\sin B\sin C - 1 = \text{RHS}$ **Proved.**

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18. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए, प्रमाणित गर्नुहोस्
If $\measuredangle A + \measuredangle B + \measuredangle C = 180°$, prove that:
$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$
[2060CP, 2066 R', 2067 R']

⇨ Here, $A + B + C = 180°$
or, $A + B = 180° - C$
Taking sin and cos on the both the sides
$\sin (A + B) = \sin (180° - C) = \sin C$
$\cos (A + B) = \cos (180° - C) = -\cos C$
LHS $= \sin 2A + \sin 2B - \sin 2C$

$$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) - \sin 2C$$
$= 2\sin (A + B) \cos (A - B) - 2\sin C \cos C$
$= 2\sin C \cos (A - B) - 2\sin C \cos C$
$= 2\sin C [\cos (A - B) - \cos C]$
$= 2\sin C [\cos (A - B) + \cos (A + B)]$
$= 2\sin C [2\cos A \cos B]$
$= 4\cos A \cos B \sin C =$ RHS **Proved.**

19. यदि $\measuredangle X + \measuredangle Y + \measuredangle Z = 180°$ भए, प्रमाणित गर्नुहोस्
If $\measuredangle X + \measuredangle Y + \measuredangle Z = 180°$, prove that: [2065 R']
$\cos 4X + \cos 4Y + \cos 4Z = -1 + 4 \cos 2X \cos 2Y \cos 2Z$

⇨ Here, $X + Y + Z = \pi^c$
or, $2X + 2Y + 2Z = 2\pi^c$
or, $2X + 2Y = 2\pi^c - 2Z$
Taking sin and cos on both the sides,
$\sin(2X + 2Y) = \sin(2\pi^c - 2Z) = -\sin 2Z$
$\cos(2X + 2Y) = \cos(2\pi^c - 2Z) = \cos 2Z$
Now, LHS $= \cos 4X + \cos 4Y + \cos 4Z$

$$= 2\cos\left(\frac{4X+4Y}{2}\right)\cos\left(\frac{4X-4Y}{2}\right) + \cos 4Z$$
$= 2\cos (2X + 2Y) \cos (2X - 2Y) + 2\cos^2 2Z - 1$
$= 2 \cos 2Z \cos(2X - 2Y) + 2\cos^2 2Z - 1$
$= 2\cos 2Z [\cos(2X - 2Y) + \cos 2Z] - 1$
$= 2\cos 2Z [\cos(2X - 2Y) + \cos (2X + 2Y)] - 1$
$= 2\cos 2Z\, 2\cos 2X \cos 2Y - 1$
$= -1 + 4 \cos 2X \cos 2Y \cos 2Z =$ RHS **Proved.**

20. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्
If $A + B + C = \pi^c$ prove that: [2066 S]
$\sin (B + C - A) + \sin (C + A - B) + \sin (A + B - C)$
$= 4\sin A \sin B \sin C$

⇨ Here, $A + B + C = \pi^c$
Then, $A + B = \pi^c - C$
Taking sin and cos on both the sides then,
$\sin(A + B) = \sin(\pi^c - C) = \sin C$
$\cos(A + B) = \cos(\pi^c - C) = -\cos C$
LHS
$= \sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C)$
$= \sin(180° - A - A) + \sin(180° - B - B) + \sin(180 - C - C)$
$= \sin(180° - 2A) + \sin(180° - 2B) + \sin(180° - 2C)$
$= \sin 2A + \sin 2B + \sin 2C$

$$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C \cos C$$
$= 2\sin(A + B) \cos(A - B) + 2\sin C \cos C$
$= 2\sin C \cos (A - B) + 2\sin C \cos C$
$= 2\sin C [\cos (A - B) + \cos C]$
$= 2\sin C [\cos (A - B) - \cos (A + B)]$
$= 2\sin C\, 2\sin A \sin B$
$= 4\sin A \sin B \sin C =$ RHS **Proved.**

21. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस् :
If $A + B + C = \pi^c$ prove that: [2073 R, 2068 R]
$\cos (B + C - A) + \cos (C + A - B) + \cos (A + B - C)$
$= 4\cos A \cos B \cos C + 1$

⇨ Here, $A + B + C = \pi^c$
or, $A + B = \pi^c - C$
$\sin (A + B) = \sin (\pi^c - C) = \sin C$
$\cos (A + B) = \cos (\pi^c - C) = -\cos C$
LHS
$= \cos (B + C - A) + \cos (C + A - B) + \cos (A + B - C)$
$= \cos (180° - 2A) + \cos (180° - 2B) + \cos (180° - 2C)$
$= -\cos 2A - \cos 2B - \cos 2C$
$= -(\cos 2A + \cos 2B) - \cos 2C$
$= -2\cos (A + B) \cos (A - B) - \cos 2C$
$= 2\cos C \cos (A - B) - 2\cos^2 C + 1$
$= 1 + 2\cos C [\cos (A - B) - \cos C]$
$= 1 + 2\cos C [\cos (A - B) + \cos (A + B)]$
$= 1 + 2\cos C (2\cos A \cos B)$
$= 1 + 4 \cos A \cos B \cos C$
$=$ RHS **Proved.**

MODEL 4

22. यदि $A + B + C = 180°$ भए प्रमाणित गर्नुहोस् । **(If $A + B + C = 180°$ then prove that): [2072 R]**
$\cos^2 A + \cos^2 B + 2\cos A \cos B \cos C = \sin^2 C$

⇨ Here, $A + B = 180° - C$
or, $\sin(A + B) = \sin (180° - C) = \sin C$ or, $\cos (A + B) = \cos (180° - C) = -\cos C$
LHS $= \cos^2 A + \cos^2 B + 2\cos A \cos B \cos C$

$$= \frac{1 + \cos 2A}{2} + \frac{1 + \cos 2B}{2} + 2\cos A \cos B \cos C$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2A + \frac{1}{2} + \frac{1}{2}\cos 2B + 2\cos A \cos B \cos C$$

$$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) + 2\cos A \cos B \cos C$$

$$= 1 + \frac{1}{2}2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\cos A \cos B \cos C$$
$= 1 + \cos (A + B) \cos (A - B) + 2\cos A \cos B \cos C$
$= 1 - \cos C \cos (A - B) + 2\cos A \cos B \cos C$
$= 1 - \cos C [\cos (A - B) - 2\cos A \cos B]$
$= 1 - \cos C[\cos A \cos B + \sin A \sin B - 2\cos A \cos B]$
$= 1 + \cos C [\cos A \cos B - \sin A \sin B]$
$= 1 + \cos C \cos (A + B)$
$= 1 + \cos C (-) \cos C = 1 - \cos^2 C = \sin^2 C =$ RHS **Proved.**

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23. यदि $A + B + C = 180°$ भए प्रमाणित गर्नुहोस्

If $A + B + C = 180°$, prove that: [2071 R', 2069 S]

$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A. \cos B. \cos C$

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

Taking sin and cos on both the sides

$\sin (A + B) = \sin (180° - C) = \sin C$

$\cos (A + B) = \cos (180° - C) = - \cos C$

LHS $= \sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + \sin^2 C$

$= \frac{1}{2} - \frac{1}{2} \cos 2A + \frac{1}{2} - \frac{1}{2} \cos 2B + \sin^2 C$

$= 1 - \frac{1}{2} (\cos 2A + \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2} 2\cos \frac{2A + 2B}{2} \cos \frac{2A - 2B}{2} + \sin^2 C$

$= 1 - \cos (A + B) \cos (A - B) + \sin^2 C$

$= 1 + \cos C \cos (A - B) + 1 - \cos^2 C$

$= 2 + \cos C [\cos (A - B) - \cos C]$

$= 2 + \cos C [\cos (A - B) + \cos (A + B)]$

$= 2 + 2\cos A \cos B \cos C =$ RHS **Proved.**

25. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$ prove that: [2064 S]

$\cos^2 A - \cos^2 B - \cos^2 C = 2 \cos A. \sin B. \sin C - 1$

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

Taking sin and cos on both the sides,

$\sin (A + B) = \sin (180° - C) = \sin C$

$\cos (A + B) = \cos (180° - C) = - \cos C$

LHS

$= \cos^2 A - \cos^2 B - \cos^2 C$

$= \frac{1 + \cos 2A}{2} - \frac{1 + \cos 2B}{2} - \cos^2 C$

$= \frac{1}{2} + \frac{1}{2} \cos 2A - \frac{1}{2} - \frac{1}{2} \cos 2B - \cos^2 C$

$= \frac{1}{2}[\cos 2A - \cos 2B] - \cos^2 C$

$= \frac{1}{2} (-) 2\sin \left(\frac{2A + 2B}{2}\right) \sin \left(\frac{2A - 2B}{2}\right) - \cos^2 C$

$= - \sin (A + B) \sin (A - B) - \cos^2 C$

$= - \sin C \sin (A - B) - (1 - \sin^2 C)$

$= - \sin C \sin (A - B) - 1 + \sin^2 C$

$= - \sin C \sin (A - B) + \sin^2 C - 1$

$= - \sin C [\sin (A - B) - \sin C] - 1$

$= - \sin C [\sin (A - B) - \sin (A + B)] - 1$

$= \sin C [\sin (A + B) - \sin (A - B)] - 1$

$= \sin C\, 2\cos A \sin B - 1$

$= 2\cos A \sin B \sin C - 1$ RHS **Proved.**

24. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$ prove that: [2063 S, 2062 K]

$\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

Taking sin and cos on both the sides,

$\sin (A + B) = \sin (180° - C) = \sin C$

$\cos (A + B) = \cos (180° - C) = - \cos C$

LHS

$= \cos^2 A + \cos^2 B + \cos^2 C$

$= \frac{1 + \cos 2A}{2} + \frac{1 + \cos 2B}{2} + \cos^2 C$

$= \frac{1}{2} + \frac{1}{2} \cos 2A + \frac{1}{2} + \frac{1}{2} \cos 2B + \cos^2 C$

$= 1 + \frac{1}{2} [\cos 2A + \cos 2B] + \cos^2 C$

$= 1 + \frac{1}{2} 2\cos \left(\frac{2A + 2B}{2}\right) \cos \left(\frac{2A - 2B}{2}\right) + \cos^2 C$

$= 1 + \cos (A + B) \cos (A - B) + \cos^2 C$

$= 1 - \cos C \cos (A - B) + \cos^2 C$

$= 1 - \cos C[\cos (A - B) - \cos C]$

$= 1 - \cos C[\cos (A - B) + \cos (A + B)]$

$= 1 - \cos C\, 2\cos A \cos B$

$= 1 - 2 \cos A \cos B \cos C =$ RHS **Proved.**

26. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$ prove that:

$\sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A . \cos B . \sin C$

[SEE 2076 M, 2059 S, 2065 R]

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

or, $\sin(A + B) = \sin(180° - C) = \sin C$

or, $\cos(A + B) = \cos(180° - C) = - \cos C$

LHS

$= \sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1 - \cos 2A}{2} - \frac{1 - \cos 2B}{2} + \sin^2 C$

$= \frac{\cos 2B - \cos 2A}{2} + \sin^2 C$

$= \frac{1}{2}\left(2\sin \frac{2B + 2A}{2} . \sin \frac{2A - 2B}{2}\right) + \sin^2 C$

$= \sin (A + B). \sin (A - B) + \sin^2 C$

$= \sin C. \sin (A - B) + \sin^2 C$ [$\because$ $\sin (A + B) = \sin C$]

$= \sin C [\sin (A - B) + \sin C]$

$= \sin C [\sin (A - B) + \sin (A + B)]$

$= \sin C. 2\sin \frac{A - B + A + B}{2} . \cos \frac{A - B - A - B}{2}$

$= 2\sin C.\sin \frac{2A}{2} . \cos \left(-\frac{2B}{2}\right)$

$= 2\sin C. \sin A. \cos B$

$= 2\sin A. \cos B. \sin C =$ RHS **Proved.**

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MODEL 5

27. यदि P, Q र R एउटा ΔPQR का कोणहरू हुन् भने प्रमाणित गर्नुहोस् (If P, Q and R are the angles of a ΔPQR, prove that):

$$2\left(\sin\frac{Q}{2}+\sin\frac{R}{2}\right)\left(\sin\frac{Q}{2}-\sin\frac{R}{2}\right)-\cos P=1-4\cos\frac{P}{2}.\cos\frac{Q}{2}.\sin\frac{R}{2}$$

[2075 R, 2075 R_2]

⇨ Here, $P+Q+R=\pi^c$ $\Rightarrow Q+R=\pi^c-P$

$\sin\left(\frac{R+Q}{2}\right)=\sin\left(\frac{\pi^c}{2}-\frac{P}{2}\right)=\cos\frac{P}{2}$ $\qquad$ $\cos\left(\frac{R+Q}{2}\right)=\cos\left(\frac{\pi^c}{2}-\frac{P}{2}\right)=\sin\frac{P}{2}$

Now, LHS $=2\left(\sin\frac{Q}{2}+\sin\frac{R}{2}\right)\left(\sin\frac{Q}{2}-\sin\frac{R}{2}\right)-\cos P=2\left(\sin^2\frac{Q}{2}-\sin^2\frac{R}{2}\right)-\cos P$

$=2\left[\left(\frac{1-\cos Q}{2}\right)-\left(\frac{1-\cos R}{2}\right)\right]-\cos P=1-\cos Q-1+\cos R-\cos P$

$=\cos R-\cos Q-\cos P=2\sin\frac{Q+R}{2}.\sin\frac{Q-R}{2}-\cos P$

$=2\cos\frac{P}{2}.\sin\frac{Q-R}{2}-2\cos^2\frac{P}{2}+1=2\cos\frac{P}{2}\left[\sin\frac{Q-R}{2}-\cos\frac{P}{2}\right]+1$

$=2\cos\frac{P}{2}\left[\sin\frac{Q-R}{2}-\sin\frac{Q+R}{2}\right]+1$

$=2\cos\frac{P}{2}\left[2\cos\frac{\left(\frac{Q-R}{2}\right)+\left(\frac{Q+R}{2}\right)}{2}.\sin\frac{\left(\frac{Q-R}{2}\right)-\left(\frac{Q+R}{2}\right)}{2}\right]+1$

$=2\cos\frac{P}{2}\left[2\cos\frac{Q}{2}.\left(-\sin\frac{R}{2}\right)\right]+1=1-4\cos\frac{P}{2}.\cos\frac{Q}{2}.\sin\frac{R}{2}=$ RHS **Proved.**

28. यदि A + B + C = π^c भए, प्रमाणित गर्नुहोस्

If A + B + C = π^c prove that:

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

[2069 R, 2063 M]

⇨ Here, We have, $A+B+C=\pi^c$

or, $A+B=\pi^c-C$

or, $\frac{A}{2}+\frac{B}{2}=\frac{\pi^c}{2}-\frac{C}{2}$

$\therefore\ \cos\left(\frac{A}{2}+\frac{B}{2}\right)=\cos\left(\frac{\pi^c}{2}-\frac{C}{2}\right)=\sin\frac{C}{2}$

Now, LHS

$=\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}$

$=\frac{1-\cos A}{2}+\frac{1-\cos B}{2}+\sin^2\frac{C}{2}$

$=1-\frac{1}{2}[\cos A+\cos B]+\sin^2\frac{C}{2}$

$=1-\frac{1}{2}\left[2\cos\left(\frac{A}{2}+\frac{B}{2}\right).\cos\left(\frac{A}{2}-\frac{B}{2}\right)\right]-\sin^2\frac{C}{2}$

$=1-\left[\sin\frac{C}{2}.\cos\left(\frac{A}{2}-\frac{B}{2}\right)\right]-\sin^2\frac{C}{2}$

$=1-\sin\frac{C}{2}\left[\cos\left(\frac{A}{2}-\frac{B}{2}\right)-\sin\frac{C}{2}\right]$

$=1-\sin\frac{C}{2}\left[\cos\left(\frac{A}{2}-\frac{B}{2}\right)-\cos\left(\frac{A}{2}+\frac{B}{2}\right)\right]$

$=1-\sin\frac{C}{2}.2\sin\frac{A}{2}.\sin\frac{B}{2}$

$=1-2\sin\frac{A}{2}.\sin\frac{B}{2}\sin\frac{C}{2}=$ RHS **Proved.**

29. यदि A + B + C = π^c भए, प्रमाणित गर्नुहोस्

If A + B + C = π^c prove that:

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

[2070 S', 2063 R', 2067 R]

⇨ Here, $A+B+C=180°$

or, $A+B=180°-C$

or, $\frac{A+B}{2}=90°-\frac{C}{2}$

Taking sin and cos on both the sides,

$\sin\left(\frac{A+B}{2}\right)=\sin\left(90°-\frac{C}{2}\right)=\cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)=\cos\left(90°-\frac{C}{2}\right)=\sin\frac{C}{2}$

LHS $=\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}$

$=\frac{1-\cos A}{2}+\frac{1-\cos B}{2}-\sin^2\frac{C}{2}$

$=\frac{1}{2}-\frac{\cos A}{2}+\frac{1}{2}-\frac{\cos B}{2}-\sin^2\frac{C}{2}$

$=1-\frac{1}{2}[\cos A+\cos B]-\sin^2\frac{C}{2}$

$=1-\frac{1}{2}2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)-\sin^2\frac{C}{2}$

$=1-\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-\sin^2\frac{C}{2}$

$=1-\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right]$

$=1-\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right]$

$=1-\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$

$=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=$ RHS **Proved.**

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MODEL 6

30. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$ prove that: **[2065 E]**

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$$

⇨ Here, $A + B + C = \pi^c$

or, $\frac{A+B}{2} = \sin\frac{\pi^c - C}{2} = \cos\frac{C}{2}$

or, $\cos\frac{A+B}{2} = \cos\frac{\pi^C - C}{2} = \sin\frac{C}{2}$

Now, LHS

$= \cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2}$

$= \frac{1+\cos A}{2} + \frac{1+\cos B}{2} - \cos^2\frac{C}{2}$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}(\cos A + \cos B) - \cos^2\frac{C}{2}$

$= 1 + \frac{1}{2}.2\cos\frac{A+B}{2}.\cos\frac{A-B}{2} - \cos^2\frac{C}{2}$

$= 1 + \sin\frac{C}{2}.\cos\frac{A-B}{2} - 1 + \sin^2\frac{C}{2}$

$= \sin\frac{C}{2}\left(\cos\frac{A-B}{2} + \sin\frac{C}{2}\right)$

$= \sin\frac{C}{2}\left\{\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right\}$

$= \sin\frac{C}{2}.2\cos\frac{A}{2}.\cos\frac{B}{2}$

$= 2\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$

$\therefore$ LHS = RHS **Proved.**

31. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस्

If $A + B + C = \pi^c$ prove that: **[2060 R]**

$$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{\pi^c - A}{4}.\sin\frac{\pi^c - B}{4}.\sin\frac{\pi^c - C}{4}$$

⇨ Here, LHS $= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$

$= 2\sin\frac{1}{2}\left(\frac{A}{2} + \frac{B}{2}\right).\cos\frac{1}{2}\left(\frac{A}{2} - \frac{B}{2}\right) + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$= 2\sin\left(\frac{A+B}{4}\right).\cos\left(\frac{A-B}{4}\right) + \cos 2\left(\frac{\pi - C}{4}\right)$

$= 2\sin\left(\frac{\pi - C}{4}\right).\cos\left(\frac{A-B}{4}\right) + 1 - 2\sin^2\left(\frac{\pi - C}{4}\right)$

$= 1 + 2\sin\left(\frac{\pi - C}{4}\right)\left(\cos\frac{A-B}{4} - \sin\frac{\pi - C}{4}\right)$

$= 1 + 2\sin\left(\frac{\pi - C}{4}\right)\left(\cos\frac{A-B}{4} - \cos\left(\frac{\pi}{2} - \frac{\pi - C}{4}\right)\right)$

$= 1 + 2\sin\left(\frac{\pi - C}{4}\right)\left(\cos\frac{A-B}{4} - \cos\frac{\pi + C}{4}\right)$

$= 1 + 2\sin\frac{\pi - C}{4}\left(2\sin\frac{1}{2}\left(\frac{A-B+\pi+C}{4}\right).\sin\frac{1}{2}\left(\frac{\pi+C-A+B}{4}\right)\right)$

$= 1 + 2\sin\frac{\pi - C}{4}\left(2\sin\frac{(A+C-B+\pi)}{8}.\sin\frac{(C+B-A+\pi)}{8}\right)$

$= 1 + 2\sin\frac{\pi - C}{4}\left(2\sin\frac{(\pi-B-B+\pi)}{8}.\sin\frac{(\pi-A-A+\pi)}{8}\right)$

$= 1 + 4\sin\frac{\pi - C}{4}.\sin\frac{2\pi - 2B}{8}.\sin\frac{2\pi - 2A}{8}$

$= 1 + 4\sin\frac{\pi - A}{4}\sin\frac{\pi - B}{4}\sin\frac{\pi - C}{4}$ = RHS **Proved.**

QUESTIONS FROM CDC TEXTBOOK

5.4. अनुबन्धित त्रिकोणमितीय सर्वसमिका (CONDITIONAL TRIGONOMETRIC IDENTITIES)

EXERCISE 5.4

1. अनुबन्धित त्रिकोणमितीय सर्वसमिका भनेको के हो ? उदाहरणहित लेख्नुहोस् ।

What do you mean by conditional trigonometric identities? Write with examples.

⇨ Here, the trigonometric identities which are true for certain given conditions and identities are known as conditional trigonometric identities. Examples of conditional identities:

(i) If $A + B + C = \pi^c$ then $\sin(A + B) = \sin C$

(ii) If $A + B + C = \pi^c$ then $\tan(A + B) = -\tan C$

(iii) If $A + B = 90°$ then $\sin A = \cos B$

2. यदि $A + B + C = \pi$ भए प्रमाणित गर्नुहोस् **(If $A + B + C = \pi$ then Prove that):**

(a) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

⇨ Here, $A + B + C = \pi$ or, $A + B = \pi - C$

$\sin(A + B) = \sin C$

$\cos(A + B) = -\cos C$

LHS $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right) + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + \sin 2C$

$= 2\sin C\cos(A - B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A - B) + \cos C\}$

$= 2\sin C\{\cos(A - B) - \cos(A + B)\}$

$= 2\sin C.2\sin\left(\frac{A - B + A + B}{2}\right)\sin\left(\frac{A + B - A + B}{2}\right)$

$= 4\sin C\sin A\sin B$

$= 4\sin A\sin B\sin C$ = RHS **Proved.**

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(b) $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$

⇨ Here, $A + B + C = \pi$

or, $A + B = \pi - C$

$\therefore \sin (A + B) = \sin C$

$\therefore \cos (A + B) = -\cos C$

LHS

$= \sin 2A + \sin 2B - \sin 2C$

$= 2 \sin \left(\frac{2A + 2B}{2}\right) \cos \left(\frac{2A - 2B}{2}\right) - \sin 2C$

$= 2 \sin (A + B) \cos (A - B) - \sin 2C$

$= 2 \sin C \cos (A - B) - 2 \sin C \cos C$

$= 2 \sin C \{\cos (A - B) - \cos C\}$

$= 2 \sin C \{\cos (A - B) + \cos (A + B)\}$

$= 2\sin C\, 2\cos \left(\frac{A - B + A + B}{2}\right) \cos \left(\frac{A - B - A - B}{2}\right)$

$= 4 \sin C \cos A \cos B$

$= 4 \cos A \cos B \sin C =$ RHS. **Proved**

(d) $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$

⇨ Here, $A + B + C = \pi$

or, $A + B = \pi - C$

$\therefore \sin (A + B) = \sin C$

$\therefore \cos (A + B) = -\cos C$

LHS

$= \cos 2A + \cos 2B - \cos 2C$

$= 2 \cos \frac{2A + 2B}{2} \cos \frac{2A - 2B}{2} - \cos 2C$

$= 2 \cos (A + B) \cos (A - B) - \cos 2C$

$= -2 \cos C \cos (A - B) - 2 \cos^2 C + 1$

$= 1 - 2 \cos C \{\cos (A - B) + \cos C\}$

$= 1 - 2 \cos C \{\cos (A - B) - \cos (A + B)\}$

$= 1 - 2 \cos C . 2 \sin \frac{A - B + A + B}{2} \sin \frac{A + B - A + B}{2}$

$= 1 - 4 \cos C \sin A \sin B$

$= 1 - 4 \sin A \sin B \cos C =$ RHS **Proved.**

(f) $\cos 2A - \cos 2B - \cos 2C = 4 \cos A \sin B \sin C - 1$

⇨ Here, given $A + B + C = 180°$

or, $A + B = 180° - C$

Taking sin on both sides,

$\sin (A + B) = \sin (180° - C) = \sin C$

Taking cos on both sides,

$\cos (A + B) = \cos (180° - C) = -\cos C$

LHS

$= \cos 2A - \cos 2B - \cos 2C$

$= -2\sin \left(\frac{2A + 2B}{2}\right) \sin \left(\frac{2A - 2B}{2}\right) - (1 - 2\sin^2 C)$

$= -2\sin (A + B). \sin (A - B) - 1 + 2\sin^2 C$

$= -2\sin C [\sin (A - B) - \sin C] - 1$

$= -2\sin C [\sin (A - B) - \sin (A + B)] - 1$

$= -2\sin C \left[2\cos \left(\frac{A-B+A+B}{2}\right). \sin \left(\frac{A-B-A-B}{2}\right)\right] - 1$

$= -2\sin C \left[2\cos \frac{2A}{2}. \sin \left(\frac{-2B}{2}\right)\right] - 1$

$= -2\sin C [-2\cos A \sin B] - 1$

$= 4\cos A \sin B \sin C - 1 =$ RHS **Proved.**

(c) $\cos 2A - \cos 2B + \cos 2C = 1 - 4 \sin A \cos B \sin C$

⇨ Here, $A + B + C = \pi$

or, $A + B = \pi - C$

$\therefore \sin (A + B) = \sin C$

$\therefore \cos (A + B) = -\cos C$

LHS

$= \cos 2A - \cos 2B + \cos 2C$

$= 2 \sin \left(\frac{2A + 2B}{2}\right) \sin \left(\frac{2B - 2A}{2}\right) + \cos 2C$

$= 2 \sin (A + B) \sin (B - A) + \cos 2C$

$= 2 \sin C \sin (B - A) + 1 - 2 \sin^2 C$

$= 1 + 2 \sin C \{\sin (B - A) - \sin C\}$

$= 1 + 2 \sin C \{\sin (B - A) - \sin (A + B)\}$

$= 1 + 2\sin C 2\cos \left(\frac{B - A + A + B}{2}\right) \sin \left(\frac{B - A - A - B}{2}\right)$

$= 1 - 4 \sin C \cos B \sin A$

$= 1 - 4 \sin A \cos B \sin C =$ RHS **Proved**

(e) $\sin 2A - \sin 2B - \sin 2C = -4 \sin A \cos B \cos C$

⇨ Here, $A + B + C = \pi^c$ or, $A + B = \pi^c - C$

Taking sin and cos on both the sides then,

$\sin (A + B) = \sin (\pi^c - C) = \sin C$

$\cos (A + B) = \cos (\pi^c - C) = -\cos C$

So, LHS

$= \sin 2A - \sin 2B - \sin 2C$

$= 2 \cos \left(\frac{2A + 2B}{2}\right) \sin \left(\frac{2A - 2B}{2}\right) - \sin 2C$

$= 2 \cos (A + B) \sin (A - B) - 2 \sin C \cos C$

$= -2 \cos C \sin (A - B) - 2 \sin C \cos C$

$= -2 \cos C [\sin (A - B) + \sin C]$

$= -2 \cos C [\sin (A - B) + \sin (A + B)]$

$= -2 \cos C [\sin A \cos B - \cos A \sin B$
$+ \sin A \cos B + \cos A \sin B]$

$= -2 \cos C . 2 \sin A \cos B$

$= -4 \sin A \cos B \cos C =$ RHS **Proved.**

3. यदि A + B + C = π भए प्रमाणित गर्नुहोस्
If A + B + C = π then Prove that:

(a) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

⇨ Here, $A + B + C = 180°$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{180°}{2}$ or, $\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$

or, $\sin \left(\frac{A}{2} + \frac{B}{2}\right) = \sin \left(90° - \frac{C}{2}\right)$

$\therefore \sin \left(\frac{A}{2} + \frac{B}{2}\right) = \cos \frac{C}{2}$

RHS

$= \sin A + \sin B + \sin C$

$= 2\sin \frac{A + B}{2} . \cos \frac{A - B}{2} + 2\sin \frac{C}{2} . \cos \frac{C}{2}$

$= 2 \cos \frac{C}{2} \left(\cos \frac{A - B}{2} + \sin \frac{C}{2}\right)$

$= 2 \cos \frac{C}{2} \left(\cos \frac{A - B}{2} + \cos \frac{A + B}{2}\right)$

$= 2\cos\frac{C}{2}\, 2\cos \frac{\left(\frac{A - B}{2} + \frac{A + B}{2}\right)}{2} \cos \frac{\left(\frac{A - B}{2} - \frac{A + B}{2}\right)}{2}$

$= 4\cos \frac{C}{2} \cos \frac{A}{2} \cos \left(-\frac{B}{2}\right)$

$= 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} =$ RHS **Proved.**

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(b) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\therefore \quad \sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$= \cos A + \cos B + \cos C$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + \cos C$

$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2}$

$= 1 + 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} - \sin\frac{C}{2}\right\}$

$= 1 + 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right\}$

$= 1 + 2\sin\frac{C}{2}\,2\left\{\sin\left(\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\right)\sin\left(\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2}\right)\right\}$

$= 1 + 4\sin\frac{C}{2}\sin\left(\frac{A-B+A+B}{4}\right)\sin\left(\frac{A+B-A+B}{4}\right)$

$= 1 + 4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$

$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$ **Proved.**

(c) $\cos A + \cos B - \cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1$

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\therefore \quad \sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$= \cos A + \cos B - \cos C$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - \cos C$

$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} - \left(1 - 2\sin^2\frac{C}{2}\right)$

$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 2\sin^2\frac{C}{2} - 1$

$= 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \sin\frac{C}{2}\right\} - 1$

$= 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right\} - 1$

$= 2\sin\frac{C}{2}\,2\cos\left(\frac{A-B+A+B}{4}\right)\cos\left(\frac{A-B-A-B}{4}\right) - 1$

$= 4\sin\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2} - 1$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1 = \text{RHS}$ **Proved**

(d) $\cos B + \cos C - \cos A = 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1$

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\frac{C}{2}$

Now, LHS $= \cos B - \cos A + \cos C$

$= 2\sin\frac{B+A}{2}\,.\,\sin\frac{A-B}{2} + \cos C$

$= 2\cos\frac{C}{2}\,.\,\sin\frac{A-B}{2} + 2\cos^2\frac{C}{2} - 1$

$= 2\cos\frac{C}{2}\left\{\sin\frac{A-B}{2} + \cos\frac{C}{2}\right\} - 1$

$= 2\cos\frac{C}{2}\left\{\sin\frac{A-B}{2} + \sin\frac{A+B}{2}\right\} - 1$

$= 2\cos\frac{C}{2}\,2\sin\left(\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\right)\cos\left(\frac{\frac{A-B}{2} - \frac{A+B}{2}}{2}\right) - 1$

$= 4\cos\frac{C}{2}\sin\left(\frac{A-B+A+B}{4}\right)\cos\left(\frac{A-B-A-B}{4}\right) - 1$

$= 4\cos\frac{C}{2}\sin\frac{A}{2}\cos\frac{B}{2} - 1$

$= 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1 = \text{RHS}$ **Proved**

(e) $\sin A - \sin B + \sin C = 4\sin\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$ or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\therefore \quad \sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$

$\therefore \quad \cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$= \sin A - \sin B + \sin C$

$= 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} + \sin C$

$= 2\sin\frac{C}{2}\sin\frac{A-B}{2} + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin\frac{C}{2}\left\{\sin\frac{A-B}{2} + \cos\frac{C}{2}\right\}$

$= 2\sin\frac{C}{2}\left\{\sin\frac{A-B}{2} + \sin\frac{A+B}{2}\right\}$

$= 2\sin\frac{C}{2} \times 2\sin\frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}\cos\frac{\frac{A-B}{2} - \frac{A+B}{2}}{2}$

$= 4\sin\frac{C}{2}\sin\frac{A-B+A+B}{4}\cos\frac{A-B-A-B}{4}$

$= 4\sin\frac{C}{2}\sin\frac{A}{2}\cos\frac{B}{2}$

$= 4\sin\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$ **Proved**

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(f) **$\sin A - \sin B - \sin C = -4 \cos \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$**

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

or, $\frac{A+B}{2} = \frac{\pi^c - C}{2}$

Taking sin and cos on both the sides then,

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90° - \frac{C}{2}\right) = \cos\frac{C}{2}$$

$$\cos\left(\frac{A+B}{2}\right) = \cos\left(90° - \frac{C}{2}\right) = \sin\frac{C}{2}$$

LHS

$= \sin A - \sin B - \sin C$

$= 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) - \sin C$

$= 2\sin\frac{C}{2} \sin\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right) - \cos\frac{C}{2}\right]$

$= 2\sin\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right) - \sin\left(\frac{A+B}{2}\right)\right]$

$= -2\sin\frac{C}{2}\left[\sin\left(\frac{A+B}{2}\right) - \sin\left(\frac{A-B}{2}\right)\right]$

$= -2\sin\frac{C}{2}\ 2\cos\frac{A}{2}\ \sin\frac{B}{2}$

$= -4\cos\frac{A}{2}\ \sin\frac{B}{2}\ \sin\frac{C}{2}$ = RHS **Proved.**

(b) **$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$**

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

$\therefore$ $\sin(A + B) = \sin C$

$\therefore$ $\cos(A + B) = -\cos C$

Now, LHS

$= \sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1}{2}(2\sin^2 A + 2\sin^2 B) + \sin^2 C$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2B) + \sin^2 C$

$= \frac{1}{2}(2 - \cos 2A - \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2} 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + \sin^2 C$

$= 1 - \cos(A + B)\cos(A - B) + \sin^2 C$

$= 1 + \cos C\cos(A - B) + \sin^2 C$

$= 1 + \cos C\cos(A - B) + 1 - \cos^2 C$

$= 2 + \cos C\{\cos(A - B) - \cos C\}$

$= 2 + \cos C\{\cos(A - B) + \cos(A + B)\}$

$= 2 + \cos C\ 2\cos\left(\frac{A-B+A+B}{2}\right)\cos\left(\frac{A-B-A-B}{2}\right)$

$= 2 + 2\cos C\cos A\cos B$

$= 2 + 2\cos A\cos B\cos C$ = RHS **Proved**

4. **यदि A + B + C = π भए प्रमाणित गर्नुहोस्**

If A + B + C = π then prove that:

(a) **$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \sin B \cos C$**

⇨ Here, $A + B + C = 180°$

or, $(A + B) = (180° - C)$

$\therefore$ $\sin(A + B) = \sin(180° - C) = \sin C$

$\therefore$ $\cos(A + B) = \cos(180° - C) = -\cos C$

Now, LHS

$= \cos^2 A + \cos^2 B - \cos^2 C$

$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) - \cos^2 C$

$= \frac{1}{2}(\cos 2A + 1 + \cos 2B + 1) - \cos^2 C$

$= \frac{1}{2}(2 + \cos 2A + \cos 2B) - \cos^2 C$

$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) - \cos^2 C$

$= 1 + \frac{1}{2} \times 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) - \cos^2 C$

$= 1 + \cos(A + B)\cos(A - B) - \cos^2 C$

$= 1 - \cos C\cos(A - B) - \cos^2 C$

$= 1 - \cos C\{\cos(A - B) + \cos C\}$

$= 1 - \cos C\{\cos(A - B) - \cos(A + B)\}$

$= 1 - \cos C\ 2\sin\left(\frac{A-B+A+B}{2}\right)\sin\left(\frac{A+B-A+B}{2}\right)$

$= 1 - 2\cos C\sin A\sin B$

$= 1 - 2\sin A\sin B\cos C$ = RHS **Proved.**

(c) **$\sin^2 A - \sin^2 B + \sin^2 C = 2\sin A \cos B \sin C$**

⇨ Here, $A + B + C = 180°$

or, $A + B = 180° - C$

$\therefore$ $\sin(A + B) = \sin C$

$\therefore$ $\cos(A + B) = -\cos C$

Now, LHS

$= \sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1}{2}(2\sin^2 A - 2\sin^2 B) + \sin^2 C$

$= \frac{1}{2}(1 - \cos 2A - 1 + \cos 2B) + \sin 2C$

$= \frac{1}{2}(\cos 2B - \cos 2A) + \sin 2C$

$= \frac{1}{2} \times 2\sin\left(\frac{2B+2A}{2}\right)\sin\left(\frac{2A-2B}{2}\right) + \sin^2 C$

$= \sin(B + A)\sin(A - B) + \sin^2 C$

$= \sin C\sin(A - B) + \sin^2 C$

$= \sin C\{\sin(A - B) + \sin C\}$

$= \sin C\{\sin(A - B) + \sin(A + B)\}$

$= \sin C \times 2\sin\left(\frac{A-B+A+B}{2}\right)\cos\left(\frac{A-B-A-B}{2}\right)$

$= 2\sin C\sin A\cos B$

$= 2\sin A\cos B\sin C$ = RHS **Proved.**

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(d) $\cos^2 A + \cos^2 B - \sin^2 C = -2\cos A\cos B\cos C$

⇨ Here, $A + B = 180° - C$

or, $\sin(A + B) = \sin(180° - C) = \sin C$

or, $\cos(A + B) = \cos(180° - C) = -\cos C$

Now, LHS

$$= \cos^2 A + \cos^2 B - \sin^2 C$$

$$= \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} - \sin^2 C$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2A + \frac{1}{2} + \frac{1}{2}\cos 2B - \sin^2 C$$

$$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) - \sin^2 C$$

$$= 1 + \frac{1}{2} 2\cos\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) - \sin^2 C$$

$$= 1 + \cos(A + B)\cos(A - B) - (1 - \cos^2 C)$$

$$= 1 - \cos C\cos(A - B) + \cos^2 C - 1$$

$$= -\cos C[\cos(A - B) - \cos C]$$

$$= -\cos C[\cos(A - B) + \cos(A + B)]$$

$$= -\cos C\, 2\cos A\ \cos B$$

$$= -2\cos A\cos B\cos C = \text{RHS}$$ **Proved.**

5. **यदि $A + B + C = \pi$ भए प्रमाणित गर्नुहोस्**
If $A + B + C = \pi$ then prove that:

(a) $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A + B + C = 180°$

or, $\frac{A}{2} + \frac{B}{2} = \frac{180°}{2} - \frac{C}{2}$

$\therefore \sin\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\left(\frac{180°}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$

$\therefore \cos\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\left(\frac{180°}{2} - \frac{C}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$$= \sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2}$$

$$= \frac{1}{2}\left(2\sin^2\frac{A}{2} + 2\sin^2\frac{B}{2}\right) - \sin^2\frac{C}{2}$$

$$= \frac{1}{2}(1 - \cos A + 1 - \cos B) - \sin^2\frac{C}{2}$$

$$= \frac{1}{2}(2 - \cos A - \cos B) - \sin^2\frac{C}{2}$$

$$= 1 - \frac{1}{2}(\cos A + \cos B) - \sin^2\frac{C}{2}$$

$$= 1 - \frac{1}{2} 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$$

$$= 1 - \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$$

$$= 1 - \sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \sin\frac{C}{2}\right\}$$

$$= 1 - \sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right\}$$

$$= 1 - \sin\frac{C}{2} \times 2\cos\frac{\left(\frac{A-B}{2} + \frac{A+B}{2}\right)}{2}\cos\frac{\left(\frac{A+B}{2} - \frac{A-B}{2}\right)}{2}$$

$$= 1 - 2\sin\frac{C}{2}\cos\left(\frac{A-B+A+B}{4}\right)\cos\left(\frac{A+B-A+B}{4}\right)$$

$$= 1 - 2\sin\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}$$

$$= 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$$ **Proved**

(e) $\sin^2 A - \sin^2 B - \sin^2 C = -2\cos A\sin B\sin C$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

Taking sin and cos on both sides then,

$\sin(A + B) = \sin(\pi^c - C) = \sin C$

$\cos(A + B) = \cos(\pi^c - C) = -\cos C$

Now, LHS

$$= \sin^2 A - \sin^2 B - \sin^2 C$$

$$= \frac{1-\cos 2A}{2} - \frac{1-\cos 2B}{2} - \sin^2 C$$

$$= \frac{1}{2} - \frac{\cos 2A}{2} - \frac{1}{2} + \frac{\cos 2B}{2} - \sin^2 C$$

$$= -\frac{1}{2}[\cos 2A - \cos 2B] - \sin^2 C$$

$$= -\frac{1}{2} 2\sin\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2B-2A}{2}\right) - \sin^2 C$$

$$= -\sin(A + B)\sin(B - A) - \sin^2 C$$

$$= \sin C\sin(A - B) - \sin^2 C$$

$$= \sin C[\sin(A - B) - \sin C]$$

$$= \sin C[\sin(A - B) - \sin(A + B)]$$

$$= -\sin C[\sin(A + B) - \sin(A - B)]$$

$$= -\sin C\, 2\cos A\sin B = -2\cos A\sin B\sin C$$

$$= \text{RHS}$$ **Proved.**

(b) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A + B + C = \pi^c$

or, $A + B = \pi^c - C$

or, $\frac{A+B}{2} = \frac{\pi^c - C}{2}$

Taking sin and cos on both sides then,

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi^c - C}{2}\right) = \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi^c - C}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$$= \cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$$

$$= \frac{1+\cos A}{2} + \frac{1+\cos B}{2} + \cos^2\frac{C}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\cos A + \frac{1}{2} + \frac{1}{2}\cos B + \cos^2\frac{C}{2}$$

$$= 1 + \frac{1}{2}(\cos A + \cos B) + \cos^2\frac{C}{2}$$

$$= 1 + \frac{1}{2} 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos^2\frac{C}{2}$$

$$= 1 + \cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos^2\frac{C}{2}$$

$$= 1 + \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 1 - \sin^2\frac{C}{2}$$

$$= 2 + \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$$

$$= 2 + \sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$$

$$= 2 + \sin\frac{C}{2} 2\sin\frac{A}{2}\sin\frac{B}{2}$$

$$= 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$$ **Proved.**

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(c) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A+B+C=\pi^c$

or, $\sin\frac{A+B}{2}=\sin\frac{\pi^c-C}{2}=\cos\frac{C}{2}$

or, $\cos\frac{A+B}{2}=\cos\frac{\pi^c-C}{2}=\sin\frac{C}{2}$

Now, LHS

$$= \cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}$$

$$= \frac{1+\cos A}{2}+\frac{1+\cos B}{2}-\cos^2\frac{C}{2}$$

$$= \frac{1}{2}+\frac{1}{2}+\frac{1}{2}(\cos A+\cos B)-\cos^2\frac{C}{2}$$

$$= 1+\frac{1}{2}.\ 2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}-\cos^2\frac{C}{2}$$

$$= 1+\sin\frac{C}{2}.\cos\frac{A-B}{2}-1+\sin^2\frac{C}{2}$$

$$= \sin\frac{C}{2}\left(\cos\frac{A-B}{2}+\sin\frac{C}{2}\right)$$

$$= \sin\frac{C}{2}\left\{\cos\left(\frac{A}{2}-\frac{B}{2}\right)+\cos\left(\frac{A}{2}+\frac{B}{2}\right)\right\}$$

$$= \sin\frac{C}{2}.\ 2\cos\frac{A}{2}.\cos\frac{B}{2}$$

$$= 2\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$$

= RHS **Proved.**

(e) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A+B+C=180°$ or, $A+B=180°-C$

or, $\frac{A+B}{2}=90°-\frac{C}{2}$

Taking sin and cos on both the sides, $\sin\left(\frac{A+B}{2}\right)$

$=\sin\left(90°-\frac{C}{2}\right)=\cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)=\cos\left(90°-\frac{C}{2}\right)=\sin\frac{C}{2}$

LHS

$$= \sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}$$

$$= \frac{1-\cos A}{2}+\frac{1-\cos B}{2}-\sin^2\frac{C}{2}$$

$$= \frac{1}{2}-\frac{\cos A}{2}+\frac{1}{2}-\frac{\cos B}{2}-\sin^2\frac{C}{2}$$

$$= 1-\frac{1}{2}[\cos A+\cos B]-\sin^2\frac{C}{2}$$

$$= 1-\frac{1}{2}2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)-\sin^2\frac{C}{2}$$

$$= 1-\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-\sin^2\frac{C}{2}$$

$$= 1-\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right]$$

$$= 1-\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right]$$

$$= 1-\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$$

$$= 1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=\text{RHS}$$

Proved.

(d) $\sin^2\frac{A}{2}-\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

⇨ Here, $A+B+C=\pi^c$ or, $A+B=\pi^c-C$

or, $\frac{A+B}{2}=\frac{\pi^c-C}{2}$

Taking sin and cos on both sides then,

$\sin\left(\frac{A+B}{2}\right)=\sin\left(\frac{\pi^c-C}{2}\right)=\cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)=\cos\left(\frac{\pi^c-C}{2}\right)=\sin\frac{C}{2}$

LHS

$$= \sin^2\frac{A}{2}-\sin^2\frac{B}{2}+\sin^2\frac{C}{2}$$

$$= \frac{1-\cos A}{2}-\frac{1-\cos B}{2}+\sin^2\frac{C}{2}$$

$$= \frac{1}{2}-\frac{1}{2}\cos A-\frac{1}{2}+\frac{1}{2}\cos B+\sin^2\frac{C}{2}$$

$$= -\frac{1}{2}\cos A+\frac{1}{2}\cos B+\sin^2\frac{C}{2}$$

$$= -\frac{1}{2}(\cos A-\cos B)+\sin^2\frac{C}{2}$$

$$= -\frac{1}{2}2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)+\sin^2\frac{C}{2}$$

$$= -\cos\frac{C}{2}\sin\left(\frac{B-A}{2}\right)+1-\cos^2\frac{C}{2}$$

$$= 1+\cos\frac{C}{2}\sin\left(\frac{A-B}{2}\right)-\cos^2\frac{C}{2}$$

$$= 1+\cos\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right)-\cos\frac{C}{2}\right]$$

$$= 1+\cos\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right)-\sin\left(\frac{A+B}{2}\right)\right]$$

$$= 1-\cos\frac{C}{2}\left[\sin\left(\frac{A+B}{2}\right)-\sin\left(\frac{A-B}{2}\right)\right]$$

$$= 1-\cos\frac{C}{2}\ 2\cos\frac{A}{2}\ \sin\frac{B}{2}$$

$$= 1-2\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}\ =\text{RHS}$$

Proved.

(f) $\cos^2\frac{A}{2}-\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

⇨ Here, $A+B+C=\pi^c$

or, $A+B=\pi^c-C$ or, $\frac{A+B}{2}=\frac{\pi^c-C}{2}$

Taking sin and cos on both sides then,

$\sin\left(\frac{A+B}{2}\right)=\sin\left(\frac{\pi^c-C}{2}\right)=\cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)=\cos\left(\frac{\pi^c-C}{2}\right)=\sin\frac{C}{2}$

LHS

$$= \cos^2\frac{A}{2}-\cos^2\frac{B}{2}+\cos^2\frac{C}{2}$$

$$= \frac{1+\cos A}{2}-\frac{1+\cos B}{2}+\cos^2\frac{C}{2}$$

$$= \frac{1}{2}+\frac{1}{2}\cos A-\frac{1}{2}-\frac{1}{2}\cos B+\cos^2\frac{C}{2}$$

$$= \frac{1}{2}(\cos A-\cos B)+\cos^2\frac{C}{2}$$

$$= \frac{1}{2}2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)+\cos^2\frac{C}{2}$$

$$= \cos\frac{C}{2}\sin\left(\frac{B-A}{2}\right)+\cos^2\frac{C}{2}$$

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$$= \cos\frac{C}{2}\left[\sin\left(\frac{B-A}{2}\right)+\cos\frac{C}{2}\right]$$

$$= \cos\frac{C}{2}\left[\sin\left(\frac{B-A}{2}\right)+\sin\left(\frac{A+B}{2}\right)\right]$$

$$= \cos\frac{C}{2}\left[\sin\left(\frac{A+B}{2}\right)-\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \cos\frac{C}{2}\,2\cos\frac{A}{2}\sin\frac{B}{2}$$

$$= 2\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} = \text{RHS}$$

Proved.

6. यदि A + B + C = 180° भए प्रमाणित गर्नुहोस् (If A + B + C = 180° then Prove that):

(a) $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} = \cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}$

⇨ Here, $A + B + C = \pi$

or, $\frac{A}{2}+\frac{B}{2}+\frac{C}{2} = \frac{\pi^C}{2}$

or, $\frac{A}{2}+\frac{B}{2} = \frac{\pi}{2}-\frac{C}{2}$

Taking cot on both sides, we get

$$\cot\left(\frac{A}{2}+\frac{B}{2}\right) = \cot\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

or, $\dfrac{\cot\frac{A}{2}\cot\frac{B}{2}-1}{\cot\frac{A}{2}+\cot\frac{B}{2}} = \tan\frac{C}{2}$

or, $\dfrac{\cot\frac{A}{2}\cot\frac{B}{2}-1}{\cot\frac{A}{2}+\cot\frac{B}{2}} = \dfrac{1}{\cot\frac{C}{2}}$

or, $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} - \cot\frac{C}{2} = \cot\frac{A}{2}+\cot\frac{B}{2}$

$\therefore$ $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} = \cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}$

$\therefore$ LHS = RHS **Proved**

(b) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

⇨ Here, $A + B + C = \pi$ or, $A + B = \pi - C$

Taking tan on both sides, we get

or, $\tan(A + B) = \tan(\pi - C)$

or, $\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

or, $\tan A + \tan B = -\tan C + \tan A \tan B \tan C$

$\therefore$ $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

Proved.

(c) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

⇨ Here, $A + B + C = \pi$ or, $A + B = \pi - C$

Taking cot on both sides, we get

or, $\cot(A + B) = \cot(\pi - C)$

or, $\dfrac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$

or, $\cot A \cot B - 1 = -\cot C \cot A - \cot B \cot C$

$\therefore$ $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

Proved

(d) $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

⇨ Here, $A + B + C = \pi$

or, $2A + 2B + 2C = 2\pi$

or, $2A + 2B = 2\pi - 2C$

Taking tan on both sides, we get

or, $\tan(2A + 2B) = \tan(2\pi - 2C)$

or, $\dfrac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$

or, $\tan 2A + \tan 2B = -\tan 2C + \tan 2A \tan 2B \tan 2C$

$\therefore$ $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

7. यदि A + B + C = π भए प्रमाणित गर्नुहोस् (If A + B + C = π then prove that):

(a) $\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$

⇨ Here, LHS $= \sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}$

$$= 2\sin\frac{A+B}{4}\cos\frac{A-B}{4}+\cos\left(\frac{\pi^c}{2}-\frac{C}{2}\right)$$

$$= 2\sin\frac{\pi^c-C}{4}\cos\frac{A-B}{4}+\cos\left(\frac{\pi^c-C}{2}\right)$$

$$= 2\sin\frac{\pi^c-C}{4}\cos\frac{A-B}{4}+1-2\sin^2\frac{\pi^c-C}{4}$$

$$= 1+2\sin\frac{\pi^c-C}{4}\left(\cos\frac{A-B}{4}-\sin\frac{\pi^c-C}{4}\right)$$

$$= 1+2\sin\frac{\pi^c-C}{4}\left[\cos\frac{A-B}{4}-\cos\left(\frac{\pi^c}{2}-\frac{\pi^c-C}{4}\right)\right]$$

$$= 1+2\sin\frac{\pi^c-C}{4}\left[\cos\frac{A-B}{4}-\cos\frac{\pi^c+C}{4}\right]$$

$$= 1+2\sin\frac{\pi^c-C}{4}\left[2\sin\frac{A-B+\pi^c+C}{8}\sin\frac{\pi^c+C-A+B}{8}\right]$$

$$= 1+2\sin\frac{\pi^c-C}{4}\left[2\sin\frac{2\pi^c-2B}{8}\sin\frac{2\pi^c-2A}{8}\right]$$

$$= 1+4\sin\frac{\pi^c-A}{4}\sin\frac{\pi^c-B}{4}\sin\frac{\pi^c-C}{4} = \text{RHS}$$

Proved.

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(b) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{B+C}{4}\right)\sin\left(\frac{C+A}{4}\right)$

⇨ Here, LHS

$$= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$$

$$= 2\sin\frac{A+B}{4}\cos\frac{A-B}{4} + \cos\left(\frac{\pi^c}{2} - \frac{C}{2}\right)$$

$$= 2\sin\frac{\pi^c - C}{4}\cos\frac{A-B}{4} + \cos\left(\frac{\pi^c - C}{2}\right)$$

$$= 2\sin\frac{\pi^c - C}{4}\cos\frac{A-B}{4} + 1 - 2\sin^2\frac{\pi^c - C}{4}$$

$$= 1 + 2\sin\frac{\pi^c - C}{4}\left(\cos\frac{A-B}{4} - \sin\frac{\pi^c - C}{4}\right)$$

$$= 1 + 2\sin\frac{\pi^c - C}{4}\left[\cos\frac{A-B}{4} - \cos\left(\frac{\pi^c}{2} - \frac{\pi^c - C}{4}\right)\right]$$

$$= 1 + 2\sin\frac{\pi^c - C}{4}\left[\cos\frac{A-B}{4} - \cos\frac{\pi^c + C}{4}\right]$$

$$= 1 + 2\sin\frac{\pi^c - C}{4}\left[2\sin\frac{A-B+\pi^c+C}{8}\sin\frac{\pi^c+C-A+B}{8}\right]$$

$$= 1 + 2\sin\frac{\pi^c - C}{4}\left[2\sin\frac{2\pi^c - 2B}{8}\sin\frac{2\pi^c - 2A}{8}\right]$$

$$= 1 + 4\sin\frac{\pi^c - A}{4}\sin\frac{\pi^c - B}{4}\sin\frac{\pi^c - C}{4}$$

We have, $A + B + C = \pi^c$

or, $A + B = \pi^c - C \quad \therefore \quad \frac{A+B}{4} = \frac{\pi^c - C}{4}$

Similarly, $\frac{B+C}{4} = \frac{\pi^c - A}{4}$ and $\frac{C+A}{4} = \frac{\pi^c - B}{4}$

So, $1 + 4\sin\left(\frac{\pi^c - A}{4}\right)\sin\left(\frac{\pi^c - B}{4}\right)\sin\left(\frac{\pi^c - C}{4}\right)$

$$= 1 + 4\sin\left(\frac{B+C}{4}\right)\sin\left(\frac{C+A}{4}\right)\sin\left(\frac{A+B}{4}\right)$$

= RHS **Proved.**

(c) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$

⇨ Here, LHS

$$= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$$

$$= 2\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right) + \sin\left(\frac{\pi^c}{2} - \frac{C}{2}\right)$$

$$= 2\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right) + \sin 2\left(\frac{\pi^c - C}{4}\right)$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right)\cos\left(\frac{A-B}{4}\right) + 2\sin\left(\frac{\pi^c - C}{4}\right)\cos\left(\frac{\pi^c - C}{4}\right)$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) + \sin\left(\frac{\pi^c - C}{4}\right)\right]$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) + \cos\left(\frac{\pi^c}{2} - \frac{\pi^c - C}{4}\right)\right]$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) + \cos\left(\frac{\pi^c + C}{4}\right)\right]$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right)2\cos\left(\frac{A-B+\pi^c+C}{8}\right)\cos\left(\frac{A-B-\pi^c-C}{8}\right)$$

$$= 2\cos\left(\frac{\pi^c - C}{4}\right).2\cos\left(\frac{2\pi^c - 2B}{8}\right)\cos\left(\frac{2A - 2\pi^c}{8}\right)$$

$$= 4\cos\left(\frac{\pi^c - C}{4}\right)\cos\left(\frac{\pi^c - B}{4}\right)\cos\left(\frac{\pi^c - A}{4}\right)$$

$$= 4\cos\left(\frac{\pi^c - A}{4}\right)\cos\left(\frac{\pi^c - B}{4}\right)\cos\left(\frac{\pi^c - C}{4}\right)$$

= RHS **Proved.**

8. यदि $A + B + C = \pi$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi$ then prove that):

(a) $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

⇨ Here, $A + B + C = \pi$ Again, $A + B + C = \pi$ $A + B = \pi - C$ or, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

or, $\sin(A + B) = \sin(\pi - C) = \sin C$ $\therefore \cos(A + B) = \cos(\pi - C) = -\cos C$

$\therefore \sin\left(\frac{A}{2} + \frac{B}{2}\right) = \left(\sin 90° - \frac{C}{2}\right) = \cos\frac{C}{2}$ $\therefore \cos\left(\frac{A}{2} + \frac{B}{2}\right) = \left(\cos 90° - \frac{C}{2}\right) = \sin\frac{C}{2}$

Now, LHS

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C}$$

$$= \frac{2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + \sin 2C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C}$$

$$= \frac{2\sin C\cos(A - B) + \sin 2C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C}$$

$$= \frac{2\sin C\cos(A - B) + 2\sin C\cos C}{2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 2\sin\frac{C}{2}\cos\frac{C}{2}}$$

$$= \frac{2\sin C\{\cos(A - B) + \cos C\}}{2\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \sin\frac{C}{2}\right\}}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}\{\cos(A - B) - \cos(A + B)\}}{\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right\}}$$

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$$= \frac{2\sin\frac{C}{2}\left\{2\sin\frac{A-B+A+B}{2}\sin\frac{A+B-A+B}{2}\right\}}{2\cos\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\cos\frac{\frac{A-B}{2}-\frac{A+B}{2}}{2}} = \frac{\sin\frac{C}{2}\,2\sin A\sin B}{\cos\frac{A-B+A+B}{4}\cos\frac{A-B-A-B}{4}}$$

$$= \frac{\sin\frac{C}{2}\,2\times 2\sin\frac{A}{2}\cos\frac{A}{2}\,2\sin\frac{B}{2}\cos\frac{B}{2}}{\cos\frac{2A}{4}\cos\frac{-2B}{4}} = \frac{8\sin\frac{C}{2}\sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}$$

$$= 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$$ **Proved**

(b) $\dfrac{\sin B+\sin C-\sin A}{\sin A+\sin B+\sin C} = \tan\dfrac{B}{2}\tan\dfrac{C}{2}$

⇨ Here, given $A+B+C=\pi^c$

or, $B+C=\pi^c-A$

or, $\frac{B}{2}+\frac{C}{2}=\frac{\pi^c}{2}-\frac{A}{2}$

or, $\sin\left(\frac{B+C}{2}\right)=\sin\left(\frac{\pi^c}{2}-\frac{A}{2}\right)=\cos\frac{A}{2}$

and $\cos\left(\frac{B+C}{2}\right)=\cos\left(\frac{\pi^c}{2}-\frac{A}{2}\right)=\sin\frac{A}{2}$

Now, $\sin B+\sin C-\sin A$

$=2\sin\frac{B+C}{2}.\cos\frac{B-C}{2}-2\sin\frac{A}{2}.\cos\frac{A}{2}$

$=2\cos\frac{A}{2}.\cos\frac{B-C}{2}-2\sin\frac{A}{2}.\cos\frac{A}{2}$

$=2\cos\frac{A}{2}\left[\cos\frac{B-C}{2}-\sin\frac{A}{2}\right]$

$=2\cos\frac{A}{2}\left[\cos\frac{B-C}{2}-\cos\frac{B+C}{2}\right]$

$=2\cos\frac{A}{2}\left[2\sin\frac{1}{2}\left(\frac{B-C}{2}+\frac{B+C}{2}\right).\sin\frac{1}{2}\left(\frac{B+C}{2}-\frac{B-C}{2}\right)\right]$

$=2\cos\frac{A}{2}\left[2\sin\frac{B}{2}.\sin\frac{C}{2}\right]$

$=4\cos\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$ ....(i)

Again, $A+B+C=180°$

or, $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{180°}{2}$

or, $\frac{A}{2}+\frac{B}{2}=90°-\frac{C}{2}$

or, $\sin\left(\frac{A}{2}+\frac{B}{2}\right)=\sin\left(90°-\frac{C}{2}\right)$

$\therefore$ $\sin\left(\frac{A}{2}+\frac{B}{2}\right)=\cos\frac{C}{2}$

Now, $\sin A+\sin B+\sin C$

$=2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}+2\sin\frac{C}{2}.\cos\frac{C}{2}$

$=2\cos\frac{C}{2}\left(\cos\frac{A-B}{2}+\sin\frac{C}{2}\right)$

$=2\cos\frac{C}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)$

$=2\cos\frac{C}{2}\,2\cos\frac{\left(\frac{A-B}{2}+\frac{A+B}{2}\right)}{2}.\cos\frac{\left(\frac{A-B}{2}-\frac{A+B}{2}\right)}{2}$

$=4\cos\frac{C}{2}\cos\frac{A}{2}\cos\left(-\frac{B}{2}\right)$

$=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ ....(ii)

Now, LHS

$=\dfrac{\sin B+\sin C-\sin A}{\sin A+\sin B+\sin C}$

$=\dfrac{4\cos\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

$=\dfrac{\sin\frac{B}{2}}{\cos\frac{B}{2}}.\dfrac{\sin\frac{C}{2}}{\cos\frac{C}{2}}$

$=\tan\frac{B}{2}\tan\frac{C}{2}=\text{RHS}$ Proved.

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4. त्रिकोणमितीय समीकरणहरू
Trigonometric Equations

Formulae and Key Points

क्र.सं. S.N.	समीकरण Equation	अवस्था Condition	चतुर्थांश Quadrant	मानहरू Values
1.	$\sin\theta = k$	+ve & ≤ 1	1st & 2nd	θ, $(180° - \theta)$, $(360° + \theta)$
2.	$\sin\theta = -k$	$-$ve ≥ -1	3rd & 4th	$(180° + \theta)$, $(360° - \theta)$
3.	$\sin\theta = k$	θ is non standard		$\theta = \sin^{-1} k$
4.	$\sin\theta = k$	$k > 1$ or $k < 1$		no solution
5.	$\cos\theta = k$	+ve & ≤ 1	1st & 4th	θ, $(360° - \theta)$, $(360° + \theta)$
6.	$\cos\theta = -k$	$-$ve ≥ -1	2nd & 3rd	$(180° - \theta)$, $(180° + \theta)$
7.	$\cos\theta = k$	θ is non standard		$\theta = \cos^{-1} k$
8.	$\cos\theta = k$	$k > 1$ or k < 1		no solution
9.	$\tan\theta = k$	+ve	1st & 3rd	θ, $(180° + \theta)$, $(360° + \theta)$
10.	$\tan\theta = -k$	$-$ve	2nd & 4th	$(180° - \theta)$, $(360° - \theta)$
11.	$\tan\theta = k$	θ is non standard		$\theta = \tan^{-1} k$

QUESTIONS FROM SEE EXERCISE 4

A. VERY SHORT QUESTIONS

1. (a) त्रिकोणमितीय समीकरणको परिभाषा दिनुहोस् । (Define trigonometric equation.)

⇨ Here, the equation involving trigonometric ratios is called trigonometric equation. For example: $y = \sin x$, $y = 2\tan x$, $2\cos^2\theta + 1 = 0$ etc.

2. (b) सर्वसमिका र समीकरणमा के फरक छ ? (What is the difference between identity and equation?)

⇨ Here, the difference between identity and equation is ;

सर्वसमिका (Identity)	समीकरण (Equation)
If the relation of an equality is satisfied by every value of the variable, then the relation is called identity. For example: In the relation $\sin^2\theta + \cos^2\theta = 1$, every value of the θ satisfies this relation.	If the relation of equality is satisfied only by the fixed value of the variable, then this relation is called equation. For example: $y = \sin x$, $y = 2x + 3$ etc.

3. यदि $x \le \sin\theta \le y$ भए x र y का मानहरू लेख्नुहोस् । (If $x \le \sin\theta \le y$ then write the values of x and y.)

⇨ Here, the values of x and y are -1 and 1 respectively.

4. यदि $\sin\theta = \cos\theta$ भए θ को न्यूनकोणीय मान कति हुन्छ ? (If $\sin\theta = \cos\theta$, what is the acute value of θ?)

⇨ Here, the value of θ is 45°.

5. यदि $\tan(180° + \theta) = x$ भए x कति हुन्छ ? (If $\tan(180° + \theta) = x$ then what is x?)

⇨ Here, $x = \tan(180° + \theta)$

$\therefore\ x = \tan\theta$

6. यदि $\cos\theta = -1$ भए θ को मान 90° देखि 180° सम्म कति हुन्छ ?
If $\cos\theta = -1$ then what is the value of θ from 90° to180°?

⇨ Here, $\cos\theta = -1$

or, $\cos\theta = \cos 180°$

$\therefore\ \theta = 180°$

7. यदि $\sin\theta = \frac{\sqrt{3}}{2}$, $0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ? (If $\sin\theta = \frac{\sqrt{3}}{2}$, $0° \le \theta \le 180°$, what is the value of θ.)

⇨ Here, $\sin\theta = \frac{\sqrt{3}}{2}$

or, $\sin\theta = \sin 60°$ or, $\sin 120°$

$\therefore\ \theta = 60°$ or $120°$

8. $\sin^2\theta + \cos^2\theta = 1$ मा θ का मानहरू के के हुन्छन् ? (What are the values of θ in $\sin^2\theta + \cos^2\theta = 1$?)

⇨ Here, all values of θ satisfies the given identity $\sin^2\theta + \cos^2\theta = 1$

So, the values of θ is all the real numbers.

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B. SHORT QUESTIONS

MODEL 1

1. यदि 2sin 2θ = $\sqrt{3}$ भए θ को मान पत्ता लगाउनुहोस् ।
If 2sin 2θ = $\sqrt{3}$, find the value of θ. (0° ≤ θ ≤ 180°)
[2076 Model]

⇨ Here, $2\sin 2\theta = \sqrt{3}$

or, $\sin 2\theta = \dfrac{\sqrt{3}}{2}$

or, $\sin 2\theta = \sin 60°, \sin 120°, \sin 420°$

or, $2\theta = 60°, 120°, 420°$

∴ $\theta = 30°, 60°, 210°$

Thus, the required values of θ are 30° and 60°.

3. हल गर्नुहोस् (Solve): cosec A = 2sin A [0° ≤ A ≤ 90°]
[2075 R₂]

⇨ Here, cosec A = 2 sin A

or, $\dfrac{1}{\sin A} = 2\sin A$

or, $\sin^2 A = \dfrac{1}{2}$

or, $\sin A = \dfrac{1}{\sqrt{2}} = \sin 45°$

∴ $A = 45°$

Thus, the value of A is 45°.

5. हल गर्नुहोस् (Solve): cosec θ = sec θ [0° ≤ θ ≤ 90°]
[2074 R]

⇨ Here, cosec θ = sec θ

or, $\dfrac{1}{\sin\theta} = \dfrac{1}{\cos\theta}$

or, $\dfrac{\cos\theta}{\sin\theta} = 1$

or, $\cot\theta = 1$

or, $\cot\theta = \cot 45°$

∴ $\theta = 45°$

Thus, the value of θ is 45°.

7. हल गर्नुहोस् (Solve): sin θ cos θ = $\frac{1}{2}$ [0° ≤ θ ≤ 90°]
[2074 S]

⇨ Here, $\sin\theta\cos\theta = \dfrac{1}{2}$

or, $2\sin\theta\cos\theta = 1$

or, $\sin 2\theta = 1$

or, $\sin 2\theta = 90°$

or, $2\,\theta = 90°$

∴ $\theta = 45°$

Thus, the value of θ is 45°.

9. हल गर्नुहोस् (Solve) :
$1 - \tan^2\theta = -2$ [0° ≤ θ ≤ 90°] [2072 R]

⇨ Here, $1 - \tan^2\theta = -2$

or, $3 = \tan^2\theta$

or, $\tan^2\theta = (\pm\sqrt{3})^2$

∴ $\tan\theta = \pm\sqrt{3}$ [$\because \theta \le 90°$]

or, $\tan\theta = \tan 60°$

∴ $\theta = 60°$

Thus, the value of θ is 60°.

2. हल गर्नुहोस् (Solve): 2cos θ = sec θ [0° ≤ θ ≤ 90°]
[2075 R]

⇨ Here, 2 cos θ = sec θ

or, $2\cos\theta = \dfrac{1}{\cos\theta}$

or, $\cos^2\theta = \dfrac{1}{2}$

or, $\cos\theta = \pm\dfrac{1}{\sqrt{2}} = \cos 45°$

∴ $\theta = 45°$

Thus, the value of θ is 45°.

4. हल गर्नुहोस् (Solve): 2cos θ – $\sqrt{2}$ = 0 [0° ≤ θ ≤ 90°]
[2075 R']

⇨ Here, $2\cos\theta - \sqrt{2} = 0$

or, $2\cos\theta = \sqrt{2}$

or, $\cos\theta = \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

or, $\cos\theta = \cos 45°$

∴ $\theta = 45°$

Thus, the value of θ is 45°.

6. हल गर्नुहोस् (Solve): 2 sin θ – $\sqrt{2}$ = 0 [0° ≤ θ ≤ 90°]
[2074 R']

⇨ Here, $2\sin\theta - \sqrt{2} = 0$

or, $2\sin\theta = \sqrt{2}$

or, $\sin\theta = \dfrac{\sqrt{2}}{2}$

or, $\sin\theta = \dfrac{1}{\sqrt{2}}$

or, $\sin\theta = 45°$

∴ $\theta = 45°$

Thus, the solution is θ = 45°.

8. हल गर्नुहोस् (Solve) :
$\sqrt{3}$ tan θ + 3 = 0 [0° ≤ θ ≤ 180°] [2073 R']

⇨ Here, $\sqrt{3}\tan\theta + 3 = 0$

or, $\sqrt{3}\tan\theta = -3$

or, $\tan\theta = -\sqrt{3}$

or, $\tan\theta = \tan(180° - 60°)$ or $(360° - 60°)$

∴ $\theta = 120°$ or $300°$

Thus, the value of θ is 120° or 300°.

10. हल गर्नुहोस् (Solve) :
$\sqrt{3}$ tan A + 1 = 0 (0° ≤ A ≤ 180°) [2072 R']

⇨ Here, $\sqrt{3}\tan A + 1 = 0$

or, $\sqrt{3}\tan A = -1$

or, $\tan A = -\dfrac{1}{\sqrt{3}}$

or, $\tan A = \tan(180° - 30°)$

∴ $A = 150°$

Thus, the value of A is 150°.

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11. हल गर्नुहोस् (Solve):

$4 - 3\sec^2\theta = 0$ $(0° \le \theta \le 90°)$ [2072 S]

⇨ Here, $4 - 3\sec^2\theta = 0$

or, $4 = 3\sec^2\theta$

or, $\frac{4}{3} = \sec^2\theta$

or, $\sec\theta = \frac{2}{\sqrt{3}}$

or, $\sec\theta = \sec 30°$ $[\because \theta \le 90°]$

$\therefore$ $\theta = 30°$

Thus, value of θ is 30°.

13. हल गर्नुहोस् (Solve) :

$\sqrt{3}\tan\theta - 3 = 0$ $[0° \le \theta \le \pi^c]$ [2070 R]

⇨ Here, $\sqrt{3}\tan\theta = 3$

or, $\tan\theta = \sqrt{3}$

or, $\tan\theta = \tan 60°$ $[\because \theta \le 180°]$

$\therefore$ $\theta = 60°$

Thus, the value of θ is 60°.

15. हल गर्नुहोस् (Solve):

$3\tan\theta = \sqrt{3}$ $[0° \le \theta \le 180°]$ [2063 R', 2066 S]

⇨ Here, $3\tan\theta = \sqrt{3}$

or, $\tan\theta = \frac{\sqrt{3}}{3}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 30°$

$\therefore$ $\theta = 30°$

Thus, value of θ is 30°.

17. हल गर्नुहोस् (Solve):

$\cot^2 x = 3$ $(0° \le x \le \pi^c)$ [2065 R]

⇨ Here, $\cot^2 x = 3$

or, $\cot x = \pm\sqrt{3} = \cot 30°$ or 150°

Thus, x = 30° or 150°.

18. हल गर्नुहोस् (Solve):

$4\sin\alpha = 3\,\text{cosec}\,\alpha$ $(0° \le \alpha \le 180°)$ [2065 R']

⇨ Here, $4\sin\alpha = 3\,\text{cosec}\,\alpha$

or, $4\sin\alpha = 3\frac{1}{\sin\alpha}$

or, $\sin^2\alpha = \frac{3}{4}$

or, $\sin^2\alpha = \left(\pm\frac{\sqrt{3}}{2}\right)^2$

$\therefore$ $\sin\alpha = \pm\frac{\sqrt{3}}{2}$

or, $\sin\alpha = \sin 60°$ or $\sin(180° - 60°)$

$\therefore$ $\alpha = 60°$ or 120°

Thus, value of α is 60° or 120°.

12. हल गर्नुहोस् (Solve):

$3\tan\theta - \sqrt{3} = 0$ $(0° \le \theta \le 90°)$ [2071 R']

⇨ Here, $3\tan\theta - \sqrt{3} = 0$

or, $3\tan\theta = \sqrt{3}$

or, $\tan\theta = \frac{\sqrt{3}}{3}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 30°$ [Since $0° \le \theta \le 90°$]

$\therefore$ $\theta = 30°$

Thus $\theta = 30°$ is the solution.

14. हल गर्नुहोस् (Solve):

$2\cos\theta = 1$ $[0° \le \theta \le 180°]$ [2063 M]

⇨ Here, we have, $2\cos\theta = 1$

$\therefore$ $\cos\theta = \frac{1}{2}$ when $\theta = 60°$

But $\cos\theta$ is positive in the fourth quadrant also.

So, $\theta = 360° - 60°$

$\therefore$ $\theta = 300°$

Thus, the value of θ is 60° or 300°.

16. हल गर्नुहोस् (Solve):

$2\cos^2\theta - 1 = 0$ $[0° \le \theta \le 180°]$ [2063 S, 2067 R]

⇨ Here, $2\cos^2\theta - 1 = 0$

or, $\cos^2\theta = \frac{1}{2}$ $\therefore$ $\cos\theta = \pm\frac{1}{\sqrt{2}}$

Taking (+)ve sign, $\cos\theta = \frac{1}{\sqrt{2}} = \cos 45°$

$\therefore$ $\theta = 45°$

Taking (–) ve sign $\cos\theta = -\frac{1}{\sqrt{2}}$

or, $\cos\theta = \cos(180° - 45°)$

$\therefore$ $\theta = 135°$

Thus, $\theta = 45°$ or 135°.

19. हल गर्नुहोस् (Solve):

$\sin\theta - \tan\theta = 0$ $[0° \le \theta \le 90°]$ [2062 K]

⇨ Here, $\sin\theta - \tan\theta = 0$

or, $\sin\theta - \frac{\sin\theta}{\cos\theta} = 0$

or, $\sin\theta\left(1 - \frac{1}{\cos\theta}\right) = 0$

Either, $\sin\theta = 0$

or, $1 - \frac{1}{\cos\theta} = 0$

or, $\frac{1}{\cos\theta} = 1$

or, $\sin\theta = \sin 0°$ or 180°

or, $\cos\theta = 1$

$\therefore$ $\theta = 0°$ or 180°

or, $\cos\theta = \cos 0°$ or 360°

$\therefore$ $\theta = 0°$ or 360°

Thus, $\theta = 0°$ is the required solution.

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20. हल गर्नुहोस् (Solve):

$\frac{\sqrt{2}}{\cos\theta} + 2 = 0\ (0° \le \theta \le 180°)$ [2066 R]

⇨ Here, $\frac{\sqrt{2}}{\cos\theta} + 2 = 0$

or, $\frac{\sqrt{2}}{\cos\theta} = -\frac{2}{1}$

or, $-2\cos\theta = \sqrt{2}$

or, $\cos\theta = \frac{\sqrt{2}}{-2}$

or, $\cos\theta = \frac{\sqrt{2}}{-\sqrt{2}\times\sqrt{2}}$

or, $\cos\theta = -\frac{1}{\sqrt{2}} = \cos(180° - 45°)$

or, $\cos\theta = \cos 135°$

$\therefore\ \theta = 135°$

Thus, the value of θ is 135°.

21. यदि $2\sin\theta - \sqrt{3} = 0$ भए θ को न्यूनकोणीय मान पत्ता लगाउनुहोस् ।

If $2\sin\theta - \sqrt{3} = 0$, find the acute value of θ. [2068 R]

⇨ Here, $2\sin\theta - \sqrt{3} = 0$

or, $2\sin\ = \sqrt{3}$

or, $\sin\ = \frac{\sqrt{3}}{2}$

or, $\sin\theta\ = \sin 60°$

$\therefore\ \theta = 60°$

Thus, the acute value of θ is 60°.

MODEL 2

22. हल गर्नुहोस् (Solve):

$\sin x - \sin 2x = 0\ [0° \le x \le \frac{\pi^c}{2}]$ [2071 S]

⇨ Here, $\sin x - \sin 2x = 0$

or, $\sin x - 2\sin x\cos x = 0$

or, $\sin x(1 - 2\cos x) = 0$

Either, $\sin x = 0$

or, $1 - 2\cos x = 0$

or, $\sin x = \sin 0°$

or, $2\cos x = 1$

$\therefore\ x = 0°$

or, $\cos x = \frac{1}{2}$

or, $\cos x = \cos 60°$

$\therefore\ x = 60°$

Thus, x = 0° or 60° is the solution.

24. हल गर्नुहोस् (Solve):

$\sin\theta\ =\ \cos\theta\ [0° \le \theta \le 180°]$ [2063 R, 2065 S]

⇨ Here, given, $\sin\theta = \cos\theta\quad [0° \le \theta \le 180°]$

or, $\sin\theta = \sqrt{1 - \sin^2\theta}$

Squaring on both sides we get, $\sin^2\theta = 1 - \sin^2\theta$

or, $\sin^2\theta + \sin^2\theta = 1$

or, $2\sin^2\theta = 1$

or, $\sin^2\theta = \frac{1}{2}$

or, $\sin\theta = \pm\frac{1}{\sqrt{2}}$ Either, $\sin\theta = \frac{1}{\sqrt{2}}$

or, $\sin\theta = \sin 45°, \sin 135°$

$\therefore\ \theta = 45°, 135°$

or, $\sin\theta = -\frac{1}{\sqrt{2}}$

or, $\sin\theta = \sin(180° + 45°), \sin(360° - 45°)$

or, $\sin\theta = \sin 225°, \sin 315°$

or, $\theta = 225°, 315°$ (As the range is $0° \le \theta \le 180°$, it is negleted)

Thus, θ = 45°, 135°.

23. हल गर्नुहोस् (Solve) :

$\sin 2\theta + \cos\theta = 0\ [0° \le \theta \le 90°]$ [2071 R]

⇨ Here, $\sin 2\theta + \cos\theta = 0$

or, $2\sin\theta.\cos\theta + \cos\theta = 0$

or, $\cos\theta(2\sin\theta + 1) = 0$

Either, $\cos\theta = 0$

or, $\cos\theta = \cos 90°$

$\therefore\ \theta = 90°$

or, $2\sin\theta = -1$

or, $\sin\theta = -\frac{1}{2}$

or, $\sin\theta = \sin(180° + 30°)$ or $(360° - 30°)$

$\therefore\ \theta = 210°$ or $330°$ (reject)

Thus, the value of θ is 90°.

25. यदि $\sin\theta - \cos\theta = 0$ भए θ को न्युनकोणी मान पत्ता लगाउनुहोस् ।

If $\sin\theta - \cos\theta = 0$, find the acute angled value of θ.

[2075 R_2', 2074 S', 2065 M]

⇨ Here, $\sin\theta - \cos\theta = 0$

or, $\sin\theta = \cos\theta$

or, $\cos(90° - \theta) = \cos\theta$

or, $90° - \theta = \theta$

or, $90° = 2\theta$

$\therefore\ \theta\ = 45°$

Thus, the vlaue of θ is 45°.

26. यदि $\tan\alpha = \cot\alpha$ भए α को मान पत्ता लगाउनुहोस् जहाँ $0° \le \alpha \le 90°$ छ ।

If $\tan\alpha = \cot\alpha$, find the value of α where $0° \le \alpha \le 90°$. [2070 R', 2064 S]

⇨ Here, $\tan\alpha = \cot\alpha$

or, $\cot(90° - \alpha)$ or $(270° - \alpha) = \cot\alpha$

or, $(90° - \alpha)$ or $(270° - \alpha) = \alpha$

$\therefore\ 90° - \alpha = \alpha$ and $270° - \alpha = \alpha$

or, $2\alpha = 90°$ and $270° = 2\alpha$

$\therefore\ \alpha = 45°$ and $\alpha = 135°$ ($\alpha = 45°$ for acute angle.)

Thus, the value of α is 45°.

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27. हल गर्नुहोस् (Solve): $\cos 2x = \sin x\ [0° \le \theta \le 180°]$ **[2064 R']**

⇨ Here, $\cos 2x = \sin x\ [0° \le \theta \le 180°]$

or, $\sin(90° - 2x)$ or $(90° + 2x) = \sin x$

or, $90° - 2x = x$ and $90° + 2x = x$

or, $90° = 3x$ and $90° = -x$

$\therefore\ x = 30°$ and $x = 270°$

Thus, the value of x is 30°.

MODEL 3

28. हल गर्नुहोस् (Solve):

$\cos^2\theta - \cos\theta + \frac{1}{4} = 0\ (0° \le \theta \le 90°)$ **[2073 S]**

⇨ Here, $\cos^2\theta - \cos\theta + \frac{1}{4} = 0$

or, $4\cos^2\theta - 4\cos\theta + 1 = 0$

or, $(2\cos\theta)^2 - 2.\ 2\cos\theta\ .\ 1 + 1^2 = 0$

or, $(2\cos\theta - 1)^2 = 0$

or, $2\cos\theta - 1 = 0$

or, $2\cos\theta = 1$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos 60°$

$\therefore\ \theta = 60°$

Thus, $\theta = 60°$ is the solution.

30. हल गर्नुहोस् (Solve) :

$\sin^2\theta - \sin\theta + \frac{1}{4} = 0\ [0° \le \theta \le 90°]$ **[2073 R]**

⇨ Here, $\sin^2\theta - \sin\theta + \frac{1}{4} = 0$

or, $4\sin^2\theta - 4\sin\theta + 1 = 0$

or, $(2\sin\theta - 1)^2 = 0$

or, $2\sin\theta - 1 = 0$

or, $2\sin\theta = 1$

or, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin 30°\ [\because \theta \le 90°]$

$\therefore\ \theta = 30°$

Thus, the value of θ is 30°.

32. हल गर्नुहोस् (Solve):

$2\cos^2\theta - \sqrt{3}\cos\theta = 0\ [0° \le \theta \le 90°]$ **[2064 S]**

⇨ Here, $2\cos^2\theta - \sqrt{3}\cos\theta = 0$

or, $\cos\theta\ (2\cos\theta - \sqrt{3}) = 0$

Either, $\cos\theta = 0$

or, $2\cos\theta - \sqrt{3} = 0$

or, $\cos\theta = \cos 90°$ or $(360° - 90°)$

or, $2\cos\theta = \sqrt{3}$

$\therefore\ \theta = 90°$ or $270°$

or, $\cos\theta = \frac{\sqrt{3}}{2}$

or, $\cos\theta = \cos 30°$ or $(360° - 30°)$

$\therefore\ \theta = 30°$ or $330°$

Thus, the values of θ are 30° and 90°.

29. हल गर्नुहोस् (Solve):

$\sin^2\frac{\theta}{2} - \sin\frac{\theta}{2} + \frac{1}{4} = 0\ [0° \le \theta \le 90°]$ **[2073 S']**

⇨ Here, $\sin^2\frac{\theta}{2} - \sin\frac{\theta}{2} + \frac{1}{4} = 0$

or, $\dfrac{4\sin^2\frac{\theta}{2} - 4\sin\frac{\theta}{2} + 1}{4} = 0$

or, $4\sin^2\frac{\theta}{2} - 4\sin\frac{\theta}{2} + 1 = 0$

or, $\left(2\sin\frac{\theta}{2} - 1\right)^2 = 0$ or, $2\sin\frac{\theta}{2} - 1 = 0$

or, $\sin\frac{\theta}{2} = \frac{1}{2}$ or, $\sin\frac{\theta}{2} = \sin 30°$

or, $\frac{\theta}{2} = 30°$ $\therefore\ \theta = 60°$

Thus, the value of θ is 60°.

31. हल गर्नुहोस् (Solve): $2\cos^2\theta = -\sqrt{3}\cos\theta\ \ [0° \le \theta \le 180°]$
[2067S, 2059 R, 2068R']

⇨ Here, given equation, $2\cos^2\theta = -\sqrt{3}\cos\theta$

or, $2\cos^2\theta + \sqrt{3}\cos\theta = 0$

or, $\cos\theta\ (2\cos\theta + \sqrt{3}) = 0$

Here, either $\cos\theta = 0$ .... (i)

or, $2\cos\theta + \sqrt{3} = 0$ .... (ii)

From (i), $\cos\theta = 0 = \cos 90°$

$\therefore\ \theta = 90°$

From (ii), $2\cos\theta + \sqrt{3} = 0$

or, $\cos\theta = \frac{-\sqrt{3}}{2} = \cos 150°$

$\therefore\ \theta = 150°$

$\therefore\ \theta = 90°, 150°$

Thus, required value of θ is 90° or 150°.

33. हल गर्नुहोस् (Solve):

$2\cos^2\theta = 3\sin\theta\ \ (0° \le \theta \le \frac{\pi^c}{2})$ **[2066 R']**

⇨ Here, $2\cos^2\theta = 3\sin\theta$

or, $2(1 - \sin^2\theta) = 3\sin\theta$

or, $2 - 2\sin^2\theta = 3\sin\theta$

or, $2\sin^2\theta + 3\sin\theta - 2 = 0$

or, $2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$

or, $2\sin\theta\ (\sin\theta + 2) - 1\ (\sin\theta + 2) = 0$

or, $(\sin\theta + 2)\ (2\sin\theta - 1) = 0$

Either, $\sin\theta + 2 = 0$

or, $2\sin\theta - 1 = 0$

$\therefore\ \sin\theta = -2$ (impossible)

or, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin 30°$

$\therefore\ \theta = 30°$

Thus, the value of θ is 30°

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34. हल गर्नुहोस् (Solve):

$\operatorname{cosec} x - 2\sin x = 1 \ (0° \le x \le \frac{\pi^c}{2})$ **[2067 R']**

⇨ Here, $\operatorname{cosec} x - 2\sin x = 1$

or, $\frac{1}{\sin x} - 2\sin x = 1$ or, $\frac{1 - 2\sin^2 x}{\sin x} = 1$

or, $1 - 2\sin^2 x = \sin x$

or, $2\sin^2 x + \sin x - 1 = 0$

or, $2\sin^2 x + 2\sin x - \sin x - 1 = 0$

or, $2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

or, $(\sin x + 1)(2\sin x - 1) = 0$

Either, $\sin x + 1 = 0$

or, $\sin x = -1$

or, $\sin x = \sin 270°$

$\therefore$ $x = 270°$

or, $2\sin x - 1 = 0$

or, $2\sin x = 1$

or, $\sin x = \frac{1}{2}$

or, $\sin x = \sin 30°$

or, $\sin 150°$

$\therefore$ $x = 30°$ or $150°$

Thus, required value of $x = 30°$ $[\because 0 \le x \le 90°]$

35. हल गर्नुहोस् (Solve):

$\cos^2\frac{\theta}{2} - \cos\frac{\theta}{2} + \frac{1}{4} = 0 \ [0° \le \theta \le 180°]$ **[2060 R]**

⇨ Here, given $\cos^2\frac{\theta}{2} - \cos\frac{\theta}{2} + \frac{1}{4} = 0$

where $0° \le \theta \le 180°$

or, $\cos^2\frac{\theta}{2} - 2.\cos\frac{\theta}{2}.\frac{1}{2} + \frac{1}{4} = 0$

or, $\left(\cos\frac{\theta}{2} - \frac{1}{2}\right)^2 = 0$

or, $\cos\frac{\theta}{2} - \frac{1}{2} = 0$

or, $\cos\frac{\theta}{2} = \frac{1}{2} = \cos 60°$

or, $\frac{\theta}{2} = 60°$

$\therefore$ $\theta = 60° \times 2 = 120°$

Thus, required value of θ is 120°.

C. LONG QUESTIONS

MODEL 1

1. हल गर्नुहोस् (Solve):

$4\sin^2\theta - 8\cos\theta - 4 = 0 \ (0° \le \theta \le 360°)$ **[2075 R_2']**

⇨ Here, given equation is $4\sin^2\theta - 8\cos\theta - 4 = 0$

or, $4(1 - \cos^2\theta) - 8\cos\theta - 4 = 0$

or, $4 - 4\cos^2\theta - 8\cos\theta - 4 = 0$

or, $4\cos^2\theta + 8\cos\theta = 0$

or, $4\cos\theta(\cos\theta + 2) = 0$

Either $\cos\theta = 0$

$\Rightarrow$ $\theta = 90°, 270°$

Either $\cos\theta + 2 = 0$

$\Rightarrow$ $\cos\theta = -2$ (not valid)

$\therefore$ $\theta = 90°, 270°$

Thus, $\theta = 90°$ or $270°$ is the required solution.

2. हल गर्नुहोस् (Solve):

$\cos^2\theta + \sin\theta = \frac{1}{4} \ (0° \le \theta \le 360°)$ **[2074 S']**

⇨ Here, $\cos^2\theta + \sin\theta = \frac{1}{4}$

or, $1 - \sin^2\theta + \sin\theta = \frac{1}{4}$

or, $4 - 4\sin^2\theta + 4\sin\theta = 1$

or, $3 - 4\sin^2\theta + 4\sin\theta = 0$

or, $4\sin^2\theta - 4\sin\theta - 3 = 0$

or, $4\sin^2\theta - 6\sin\theta + 2\sin\theta - 3 = 0$

or, $2\sin\theta(2\sin\theta - 3) + 1(2\sin\theta - 3) = 0$

or, $(2\sin\theta - 3)(2\sin\theta + 1) = 0$

Either, $2\sin\theta - 3 = 0$ .....(i)

or, $2\sin\theta + 1 = 0$ ......(ii)

From (i), $2\sin\theta = 3$

$\therefore$ $\sin\theta = \frac{3}{2} = 1.5$ (impossible)

From (ii), $2\sin\theta + 1 = 0$ or, $2\sin\theta = -1$ or, $\sin\theta = -\frac{1}{2}$

or, $\sin\theta = \sin(180° + 30°)$ or $(360° - 30°)$ or, $\theta = 210°$ or $330°$

Thus, $\theta = 210°$ or $330°$ is the solution.

3. हल गर्नुहोस् (Solve):

$\tan^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\tan\theta = -1 \ (0° \le \theta \le 360°)$[2074 R]

⇨ Here, $\tan^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\tan\theta = -1$

or, $\tan^2\theta + \sqrt{3}\tan\theta + \frac{1}{\sqrt{3}}\tan\theta + 1 = 0$

or, $\tan\theta(\tan\theta + \sqrt{3}) + \frac{1}{\sqrt{3}}(\tan\theta + \sqrt{3}) = 0$

or, $(\tan\theta + \sqrt{3})\left(\tan\theta + \frac{1}{\sqrt{3}}\right) = 0$

Either, $\tan\theta + \sqrt{3} = 0$ ....(i)

or, $\tan\theta + \frac{1}{\sqrt{3}} = 0$ ....(ii)

From equation (i);

$\tan\theta = -\sqrt{3}$

or, $\tan\theta = \tan(180° - 60°)$ or $(360° - 60°)$

or, $\tan\theta = \tan 120°$ or $300°$

$\therefore$ $\theta = 120°$ or $300°$

From equation (ii);

$\tan\theta = -\frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan(180° - 30°)$ or $(360° - 30°)$

or, $\tan\theta = \tan 150°$ or $330°$

$\therefore$ $\theta = 150°$ or $330°$

Thus, $\theta = 120°, 150°, 300°, 330°$ is the solution.

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4. हल गर्नुहोस् (Solve): [2073 S, 2073 S', 2070 R']

$2\sqrt{3}\sin^2\theta = \cos\theta$ $(0° \le \theta \le 360°)$

⇨ Here, $2\sqrt{3}\sin^2\theta = \cos\theta$
or, $2\sqrt{3}(1 - \cos^2\theta) = \cos\theta$
or, $2\sqrt{3} - 2\sqrt{3}\cos^2\theta = \cos\theta$
or, $2\sqrt{3}\cos^2\theta + \cos\theta - 2\sqrt{3} = 0$
or, $2\sqrt{3}\cos^2\theta + 4\cos\theta - 3\cos\theta - 2\sqrt{3} = 0$
or, $2\cos\theta(\sqrt{3}\cos\theta + 2) - \sqrt{3}(\sqrt{3}\cos\theta + 2) = 0$
or, $(2\cos\theta - \sqrt{3})(\sqrt{3}\cos\theta + 2) = 0$
Either, $2\cos\theta - \sqrt{3} = 0$ or, $\sqrt{3}\cos\theta + 2 = 0$
or, $2\cos\theta = \sqrt{3}$ or, $\cos\theta = -\frac{2}{\sqrt{3}}$ (Impossible)
or, $\cos\theta = \frac{\sqrt{3}}{2}$
or, $\cos\theta = \cos 30°$ or $(360° - 30°)$
∴ $\theta = 30°$ or $330°$
Thus, the value of θ is 30° or 330°.

5. हल गर्नुहोस् (Solve): [2070 R]

$(1 - \sqrt{3})\tan\theta + 1 + \sqrt{3} = \sqrt{3}\sec^2\theta$ $(0° \le \theta \le 360°)$

⇨ Here, $(1 - \sqrt{3})\tan\theta + 1 + \sqrt{3} = \sqrt{3}\sec^2\theta$
or, $(1 - \sqrt{3})\tan\theta + 1 + \sqrt{3} = \sqrt{3}(1 + \tan^2\theta)$
or, $(1 - \sqrt{3})\tan\theta + 1 + \sqrt{3} = \sqrt{3} + \sqrt{3}\tan^2\theta$
or, $\sqrt{3}\tan^2\theta + (\sqrt{3} - 1)\tan\theta - 1 = 0$
or, $\sqrt{3}\tan^2\theta + \sqrt{3}\tan\theta - \tan\theta - 1 = 0$
or, $\sqrt{3}\tan\theta(\tan\theta + 1) - 1(\tan\theta + 1) = 0$
or, $(\tan\theta + 1)(\sqrt{3}\tan\theta - 1) = 0$
Either, $\tan\theta + 1 = 0$ or, $\sqrt{3}\tan\theta - 1 = 0$
or, $\tan\theta = -1$.....(i) or, $\tan\theta = \frac{1}{\sqrt{3}}$.........(ii)
From (i);
or, $\tan\theta = \tan(180° - 45°)$ or $(360° - 45°)$
∴ $\theta = 135°$ or $315°$
From (ii);
or, $\tan\theta = \tan 30°$ or $(180° + 30°)$
∴ $\theta = 30°$ or $210°$
Thus, the values of θ are; 30°, 135°, 210°, 315°.

6. हल गर्नुहोस् (Solve):

$\cos^2\theta - \sin\theta - \frac{1}{4} = 0$ $(0^0 \le \theta \le 360°)$[2068 R, 2061 R]

⇨ Here, $\cos^2\theta - \sin\theta = \frac{1}{4}$
or, $4\cos^2\theta - 4\sin\theta - 1 = 0$
or, $4(1 - \sin^2\theta) - 4\sin\theta - 1 = 0$
or, $4 - 4\sin^2\theta - 4\sin\theta - 1 = 0$
or, $-4\sin^2\theta - 4\sin\theta + 3 = 0$
or, $4\sin^2\theta + 4\sin\theta - 3 = 0$
or, $4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$
or, $2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$
or, $(2\sin\theta - 1)(2\sin\theta + 3) = 0$
Either, $2\sin\theta - 1 = 0$ or, $2\sin\theta + 3 = 0$
or, $2\sin\theta = 1$ or, $2\sin\theta = -3$
or, $\sin\theta = \frac{1}{2}$ or, $\sin\theta = -\frac{3}{2}$ (Impossible)
or, $\sin\theta = \sin 30°$ or $(180° - 30°)$
∴ $\theta = 30°$ or $150°$
Thus, the values of θ are 30° and 150°.

7. हल गर्नुहोस् (Solve):

$2\sqrt{3}\cos^2\theta = \sin\theta$ $(0° \le \theta \le 360°)$ [2073 R]

⇨ Here, $2\sqrt{3}\cos^2\theta = \sin\theta$
or, $2\sqrt{3}(1 - \sin^2\theta) = \sin\theta$
or, $2\sqrt{3} - 2\sqrt{3}\sin^2\theta - \sin\theta = 0$
or, $2\sqrt{3}\sin^2\theta + \sin\theta - 2\sqrt{3} = 0$
or, $2\sqrt{3}\sin^2\theta + 4\sin\theta - 3\sin\theta - 2\sqrt{3} = 0$
or, $2\sin\theta(\sqrt{3}\sin\theta + 2) - \sqrt{3}(\sqrt{3}\sin\theta + 2) = 0$
or, $(\sqrt{3}\sin\theta + 2)(2\sin\theta - \sqrt{3}) = 0$
Either, $\sqrt{3}\sin\theta + 2 = 0$ ..... (i)
or, $2\sin\theta - \sqrt{3} = 0$ .... (ii)
From, (i) $\sqrt{3}\sin\theta + 2 = 0$
or, $\sqrt{3}\sin\theta = -2$
or, $\sin\theta = -\frac{2}{\sqrt{3}} = -1.154$ (Impossible)
From (ii); $2\sin\theta = \sqrt{3}$
or, $\sin\theta = \frac{\sqrt{3}}{2}$
or, $\sin\theta = \sin 60°$ or $(180° - 60°)$
∴ $\theta = 60°$ or $120°$
Thus, $\theta = 60°$ or $120°$ is the solution.

8. हल गर्नुहोस् (Solve):

$2\cos^2\theta = 3\sin\theta$ $(0° \le \theta \le 180°)$ [2057 S]

⇨ Here, given equation, $2\cos^2\theta = 3\sin\theta$
or, $2(1 - \sin^2\theta) - 3\sin\theta = 0$
or, $2 - 2\sin^2\theta - 3\sin\theta = 0$
or, $2 - 3\sin\theta - 2\sin^2\theta = 0$
or, $2 + \sin\theta - 4\sin\theta - 2\sin^2\theta = 0$
or, $1(2 + \sin\theta) - 2\sin\theta(2 + \sin\theta) = 0$
or, $(2 + \sin\theta)(1 - 2\sin\theta) = 0$
Here, $2 + \sin\theta = 0$ .... (i)
or, $1 - 2\sin\theta = 0$ .... (ii)
From equation (i) $2 + \sin\theta = 0$
or, $\sin\theta = -2$
Here, solution of θ for $\sin\theta = -2$ is impossible.
Again, from equation (ii), $1 - 2\sin\theta = 0$
or, $1 = 2\sin\theta$
or, $\sin\theta = \frac{1}{2} = \sin 30°$ or $\sin 150°$
Thus, $\theta = 30°, 150°$ are the required solutions.

9. हल गर्नुहोस् (Solve):

$2\cos^2\theta + \sin\theta = 2$ $(0° \le \theta \le 180°)$ [2061 S]

⇨ Here, given equation, $2\cos^2\theta + \sin\theta = 2$
or, $2(1 - \sin^2\theta) + \sin\theta - 2 = 0$
or, $2 - 2\sin^2\theta + \sin\theta - 2 = 0$
or, $\sin\theta - 2\sin^2\theta = 0$
or, $\sin\theta(1 - 2\sin\theta) = 0$
Either, $\sin\theta = 0$ .....(i) or, $1 - 2\sin\theta = 0$......(ii)
From (i);
or, $\sin\theta = 0 = \sin 0°$ or, $\sin 180°$
∴ $\theta = 0°$ or, $180°$
From (ii);
Again, $1 - 2\sin\theta = 0$
or, $1 = 2\sin\theta$
or, $\sin\theta = \frac{1}{2} = \sin 30°$
or, $\sin 150°$
∴ $\theta = 30°$ or, $150°$
Thus, the values of θ are 0°, 30°, 150°, 180°.

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10. हल गर्नुहोस् (Solve):

$1 + \cos\theta = 2\sin^2\theta$ [0° ≤ θ ≤ 180°] [2058 S]

⇨ Here, given, $1 + \cos\theta = 2\sin^2\theta$

or, $1 + \cos\theta = 2(1 - \cos^2\theta)$

or, $1 + \cos\theta = 2 - 2\cos^2\theta$

or, $2\cos^2\theta + \cos\theta - 1 = 0$

or, $2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$

or, $2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$

or, $(2\cos\theta - 1)(\cos\theta + 1) = 0$

Hence, $2\cos\theta - 1 = 0$ .... (i)

or, $\cos\theta + 1 = 0$ .... (ii)

From equation (i), $2\cos\theta - 1 = 0$

or, $2\cos\theta = 1$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos 60°$

∴ $\theta = 60°$

Again, from equation (ii), $\cos\theta + 1 = 0$

or, $\cos\theta = -1$

or, $\cos\theta = \cos 180°$

or, $\theta = 180°$

Thus, the values of θ are 60° & 180°.

12. हल गर्नुहोस् (Solve):

$7\sin^2\theta + 3\cos^2\theta = 4$ (0° ≤ θ ≤ 360°) [2065 E]

⇨ Here, $7\sin^2\theta + 3\cos^2\theta = 4$

or, $7(1 - \cos^2\theta) + 3\cos^2\theta = 4$

or, $7 - 7\cos^2\theta + 3\cos^2\theta = 4$

or, $7 - 4\cos^2\theta = 4$

or, $3 - 4\cos^2\theta = 0$

or, $3 = 4\cos^2\theta$

or, $\frac{3}{4} = \cos^2\theta$

or, $\cos\theta = \pm\sqrt{\frac{3}{4}}$

or, $\cos\theta = \pm\frac{\sqrt{3}}{2}$

or, $\cos\theta = \cos 30°$

or, cos 330°

or, $\theta = 30°$ or 330°

or, $\cos\theta = \cos 150°$ or cos 210°

Thus, required value of θ are 30° or 330° or, 150° or 210°.

or, $\theta = 150°$ or 210°

11. हल गर्नुहोस् (Solve):

$\cos^2 x = 3\sin^2 x + 4\cos x$ (0° ≤ x ≤ 360°) [2062 R]

⇨ Here, given $\cos^2 x = 3\sin^2 x + 4\cos x$

or, $3\sin^2 x - \cos^2 x + 4\cos x = 0$

or, $3(1 - \cos^2 x) - \cos^2 x + 4\cos x = 0$

or, $3 - 3\cos^2 x - \cos^2 x + 4\cos x = 0$

or, $3 + 4\cos x - 4\cos^2 x = 0$

or, $3 + 6\cos x - 2\cos x - 4\cos^2 x = 0$

or, $3(1 + 2\cos x) - 2\cos x(1 + 2\cos x) = 0$

or, $(1 + 2\cos x)(3 - 2\cos x) = 0$

Either, $1 + 2\cos x = 0$ .... (i)

or, $3 - 2\cos x = 0$ .... (ii)

From equation (i) $2\cos x = 1$

or, $\cos x = -\frac{1}{2} = \cos 120°$

∴ $x = 120°$

Again from equation (ii) $2\cos x = 3$

∴ $\cos x = \frac{3}{2}$

since the value of cos x lies –1 and 1,

So in $\cos x = \frac{3}{2}$, the solution of x is impossible.

Thus, x = 120° is the required value.

13. हल गर्नुहोस् (Solve):

$6\sin^2 X + \cos X = 5$ (0° ≤ X ≤ 360°) [2067 S, 2067 R']

⇨ Here, $6\sin^2 X + \cos X = 5$

or, $6(1 - \cos^2 X) + \cos X - 5 = 0$

or, $6 - 6\cos^2 X + \cos X - 5 = 0$

or, $-6\cos^2 X + \cos X + 1 = 0$

or, $6\cos^2 X - \cos X - 1 = 0$

or, $6\cos^2 X - 3\cos X + 2\cos X - 1 = 0$

or, $3\cos X(2\cos X - 1) + 1(2\cos X - 1) = 0$

or, $(2\cos X - 1)(3\cos X + 1) = 0$

Either, $2\cos X - 1 = 0 \Rightarrow \cos X = \frac{1}{2}$ .... (i)

or, $3\cos X + 1 = 0$ $\Rightarrow \cos X = -\frac{1}{3}$

$\Rightarrow X = \cos^{-1}\left(-\frac{1}{3}\right)$

From (i); $\cos X = \frac{1}{2}$

or, $\cos X = \cos 60°$ or (360° – 60°)

∴ X = 60° or 300°

Thus, X = 60° or 300° or $\cos^{-1}\left(-\frac{1}{3}\right)$ are the solution of given equation.

14. हल गर्नुहोस् (Solve): $3\cos 2x - 1 = 2\sin^2 x$ (0° ≤ x ≤ 180°) [2064 R']

⇨ Here, $3\cos 2x - 1 = 2\sin^2 x$

or, $3(1 - 2\sin^2 x) - 1 = 2\sin^2 x$

or, $3 - 6\sin^2 x - 1 - 2\sin^2 x = 0$

or, $2 - 8\sin^2 x = 0$

or, $8\sin^2 x = 2$

or, $\sin^2 x = \frac{1}{4}$

or, $\sin^2 x = \left(\pm\frac{1}{2}\right)^2$

∴ $\sin x = \pm\frac{1}{2}$

Taking (+) ve sign $\sin x = \frac{1}{2}$

or, $\sin x = \sin 30°$ or (180° – 30°)

∴ x = 30° or 150°

Taking (–) ve sign $\sin x = -\frac{1}{2}$

or, $\sin x = \sin(180° + 30°)$ or (360° – 30°)

∴ x = 210° or 330°

Thus, x = 30° or 150° are the required solution.

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15. हल गर्नुहोस् (Solve):
$\cos 2\theta = \sin\theta \quad (0° \le \theta \le 360°)$ [2058 R]

⇨ Here, given $\cos 2\theta = \sin\theta$
or, $1 - 2\sin^2\theta = \sin\theta$ [$\because \cos 2\theta = 1 - 2\sin^2\theta$]
or, $2\sin^2\theta + \sin\theta - 1 = 0$
or, $2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$
or, $2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$
or, $(\sin\theta + 1)(2\sin\theta - 1) = 0$
Either, $2\sin\theta - 1 = 0$ .... (i)
or, $\sin\theta + 1 = 0$ .... (ii)
From (i),
$2\sin\theta = 1$
or, $\sin\theta = \frac{1}{2} = \sin 30°$ or $\sin 150°$
$\therefore \theta = 30°$ or $150°$
Again from (ii),
$\sin\theta = -1 = \sin 270°$
$\therefore \theta = 270°$
Thus, $\theta = 30°, 150°, 270°$ are the required values of θ.

16. हल गर्नुहोस् (Solve):
$\sec\theta . \tan\theta = \sqrt{2} \quad (0° \le \theta \le 180°)$ [2060 R]

⇨ Here, $\sec\theta . \tan\theta = \sqrt{2}$
or, $\frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta} = \sqrt{2}$
or, $\frac{\sin\theta}{\cos^2\theta} = \sqrt{2}$
or, $\sin\theta = \sqrt{2}\cos^2\theta$
or, $\sin\theta = \sqrt{2}(1 - \sin^2\theta)$
or, $\sin\theta = \sqrt{2} - \sqrt{2}\sin^2\theta$
or, $\sqrt{2}\sin^2\theta + \sin\theta - \sqrt{2} = 0$
or, $\sqrt{2}\sin^2\theta + 2\sin\theta - \sin\theta - \sqrt{2} = 0$
or, $\sqrt{2}\sin\theta(\sin\theta + \sqrt{2}) - 1(\sin\theta + \sqrt{2}) = 0$
or, $(\sin\theta + \sqrt{2})(\sqrt{2}\sin\theta - 1) = 0$
Either, $\sin\theta + \sqrt{2} = 0$ or, $\sqrt{2}\sin\theta - 1 = 0$
or, $\sin\theta = -\sqrt{2}$ (Impossible)
or, $\sqrt{2}\sin\theta = 1$
or, $\sin\theta = \frac{1}{\sqrt{2}}$
or, $\sin\theta = \sin 45°$ or $(180° - 45°)$
$\therefore \theta = 45°$ or $135°$
Thus, the values of θ are $45°$ and $135°$.

17. हल गर्नुहोस् (Solve): $\cot^2 x + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot x = -1 \quad (0° \le x \le 360°)$ [2068 R', 2065 R']

⇨ Here, $\cot^2 x + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot x = -1$
or, $\cot^2 x + \left(\frac{3+1}{\sqrt{3}}\right)\cot x = -1$
or, $\cot^2 x + \frac{4}{\sqrt{3}}\cot x = -1$
or, $\sqrt{3}\cot^2 x + 4\cot x = -\sqrt{3}$
or, $\sqrt{3}\cot^2 x + 4\cot x + \sqrt{3} = 0$
or, $\sqrt{3}\cot^2 x + 3\cot x + \cot x + \sqrt{3} = 0$
or, $\sqrt{3}\cot x(\cot x + \sqrt{3}) + 1(\cot x + \sqrt{3}) = 0$
or, $(\cot x + \sqrt{3})(\sqrt{3}\cot x + 1) = 0$
Either, $\cot x + \sqrt{3} = 0$ .... (i)
or, $\sqrt{3}\cot x + 1 = 0$ .... (ii)
From the equation (i); $\cot x + \sqrt{3} = 0$
or, $\cot x = -\sqrt{3}$
or, $\cot x = \cot(180° - 30°)$ or $(360° - 30°)$
$\therefore x = 150°$ or $330°$
From the equation (ii); $\sqrt{3}\cot x = -1$
or, $\cot x = -\frac{1}{\sqrt{3}}$
or, $\cot x = \cot(180° - 60°)$ or $(360° - 60°)$
$\therefore x = 120°$ or $300°$
Thus, $x = 120°, 150°, 300°, 330°$ are the required values.

MODEL 2

18. हल गर्नुहोस् (Solve):
$\sin x + \cos x = 1 \quad (0° \le x \le 360°)$ [2075 R', 2073 R']

⇨ Here, $\sin x + \cos x = 1$
Dividing both the sides by,
$\sqrt{(\text{coeff . sin x})^2 + (\text{coeff . cos x})^2}$
i.e. $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$ then,
$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$
or, $\sin x\cos 45° + \cos x\sin 45° = \frac{1}{\sqrt{2}}$
or, $\sin(x + 45°) = \frac{1}{\sqrt{2}}$
or, $\sin(x + 45°) = \sin 45°, \sin 135°, \sin(360° + 45°)$
So, $x + 45° = 45°$and $x + 45° = 135°$
or, $x + 45° = 405°$
$\therefore x = 0°$
$\therefore x = 90°$
$\therefore x = 360°$
Thus, $x = 0°$ or $90°$ or $360°$.

19. हल गर्नुहोस् (Solve): [2075 R, 2075 R$_2$]
$\sqrt{3}\tan\theta = \sqrt{3}\sec\theta - 1 \quad (0° \le \theta \le 360°)$

⇨ Here, $\sqrt{3}\tan\theta = \sqrt{3}\sec\theta - 1$
or, $\sqrt{3}\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{\cos\theta} - 1$
or, $\sqrt{3}\sin\theta = \sqrt{3} - \cos\theta$
or, $\sqrt{3}\sin\theta + \cos\theta = \sqrt{3}$
Dividing both the sides by $\sqrt{(\sqrt{3})^2 + 1} = 2$, we get
$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{\sqrt{3}}{2}$
or, $\sin\theta\cos 30° + \cos\theta\sin 30° = \frac{\sqrt{3}}{2}$
or, $\sin(\theta + 30°) = \sin 60°, \sin(180° - 60°)$
Either, $\theta + 30° = 60°$
or, $\theta + 30° = 120°$
$\therefore \theta = 30°$
$\therefore \theta = 90°$
Thus, $\theta = 30°$ or $90°$ is the solution.

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20. हल गर्नुहोस् (Solve): **[2074 S, 2072 R]**

$1+\sqrt{3}\tan\alpha-\sec\alpha=0\ (0°\le\alpha\le 360°)$

⇨ Here, $1+\sqrt{3}\ \tan\alpha-\sec\alpha=0$

or, $1+\sqrt{3}\ \tan\alpha=\sec\alpha$

or, $(1+\sqrt{3}\ \tan\alpha)^2=\sec^2\alpha$

or, $1+2\sqrt{3}\ \tan\alpha+3\tan^2\alpha=1+\tan^2\alpha$

or, $2\tan^2\alpha+2\sqrt{3}\ \tan\alpha=0$

or, $2\tan\alpha\ (\tan\alpha+\sqrt{3}\ )=0$

Either, $2\tan\alpha=0$

or, $\tan\alpha+\sqrt{3}\ =0$

or, $\tan\alpha=0$

or, $\tan\alpha=-\sqrt{3}$

or, $\tan\alpha=\tan 0°, 360°$

or, $\tan\alpha=\tan(180°-60°)$

$\therefore\ \alpha=0°$ or $360°$

or, $\tan\alpha=\tan 120°$

$\therefore\ \alpha=120°$

Thus, $\alpha=0°$ or $120°$ or $360°$ is the solution.

Alternative method,

Here, $1+\sqrt{3}\tan\alpha-\sec\alpha=0$

or, $1+\sqrt{3}\dfrac{\sin\alpha}{\cos\alpha}-\dfrac{1}{\cos\alpha}=0$

or, $\dfrac{\cos\alpha+\sqrt{3}\sin\alpha-1}{\cos\alpha}=0$

or, $\cos\alpha+\sqrt{3}\sin\alpha-1=0$

or, $\cos\alpha+\sqrt{3}\sin\alpha=1$ .....(i)

Constant term

$=\sqrt{(\text{coeff. of }\cos\alpha)^2+(\text{coeff. of }\sin\alpha)^2}$

$=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{1+3}=2$

Dividing equation (i) on both the sides by 2,

$\dfrac{1}{2}\cos\alpha+\dfrac{\sqrt{3}}{2}\sin\alpha=\dfrac{1}{2}$

or, $\sin 30°\cos\alpha+\cos 30°\sin\alpha=\dfrac{1}{2}$

or, $\sin(30°+\alpha)=\dfrac{1}{2}$

or, $\sin(30°+\alpha)=\sin 30°$ or $\sin(180°-30°)$ or $\sin(360°+30°)$

or, $30°+\alpha=30°$ or $150°$ or $390°$

$\therefore\ \alpha=0°$ or $120°$ or $360°$

Thus, the value of α is $0°$ or $120°$ or $360°$.

21. हल गर्नुहोस् (Solve):

$\sqrt{3}\sin x-\cos x=\sqrt{2}\quad(0°\le x\le 360°)$

[2071 R, 2069 R', 2057 R, 2066 R']

⇨ Here, given, $\sqrt{3}\ \sin x-\cos x=\sqrt{2}$

Dividing both sides by;

$\sqrt{(\text{Coeff. of }\sin x)^2+(\text{Coeff. of }\cos x)^2}$

$=\sqrt{3+1}\ =2$ we get,

$\dfrac{\sqrt{3}}{2}\ \sin x-\dfrac{1}{2}\ \cos x=\dfrac{\sqrt{2}}{2}$

or, $\cos 30°\ \sin x-\sin 30°\cos x\ =\dfrac{1}{\sqrt{2}}$

or, $\sin x\cos 30°-\cos x\sin 30°=\dfrac{1}{\sqrt{2}}$

or, $\sin(x-30°)=\dfrac{1}{\sqrt{2}}\ =\sin 45$ or $\sin 135°$

$\therefore\ x-30°=45°$ or, $x-30°=135°$

Thus, $x\ =75°$ or $165°$ are the required solution.

22. हल गर्नुहोस् (Solve): **[2074 R', 2071 R', 2065 S]**

$\sqrt{3}\sin\theta+\cos\theta=1\quad(0°\le\theta\le 360°)$

⇨ Here, $\sqrt{3}\sin\theta+\cos\theta=1$

Constant term $=$

$\sqrt{(\text{coeff. of }\sin\theta)^2+(\text{coeff. of }\cos\theta)^2}$

$=\sqrt{(\sqrt{3})^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2$

Dividing both the sides by 2

$\dfrac{\sqrt{3}}{2}\sin\theta+\dfrac{1}{2}\cos\theta=\dfrac{1}{2}$

or, $\sin\theta\cos 30°+\cos\theta\sin 30°=\dfrac{1}{2}$

or, $\sin(\theta+30°)=\sin 30°$ or $\sin(180°-30°)$ or $\sin(360°+30°)$

$\theta+30°=30°$ or $150°$ or $390°$

or, $\theta=0°$ or $120°$ or $360°$

Thus, $\theta=0°$ or $120°$ or $360°$ are the required solution.

23. हल गर्नुहोस् (Solve): **[2071 S, 2065 M]**

$\sin A=\sqrt{3}(1-\cos A)\quad(0°\le A\le 360°)$

⇨ Here, $\sin A+\sqrt{3}\cos A=\sqrt{3}$

Constant term $=\sqrt{(1)^2+(\sqrt{3})^2}=\sqrt{1+3}=2$

Dividing both sides by 2

$\dfrac{1}{2}\sin A+\dfrac{\sqrt{3}}{2}\cos A=\dfrac{\sqrt{3}}{2}$

or, $\sin A.\cos 60°+\cos A.\sin 60°=\dfrac{\sqrt{3}}{2}$

or, $\sin(A+60°)=\sin 60°, \sin 120°, \sin 420°$

or, $A+60°=60°, 120°, 420°$

$\therefore\ A=0°, 60°, 360°$

Thus, $A=0°$ or $60°$ or $360°$ are the required solutions.

24. हल गर्नुहोस् (Solve):

$\sqrt{3}\sin\alpha+\cos\alpha=1\quad(0°\le\alpha\le 180°)$ **[2057 R]**

⇨ Here, $\sqrt{3}\sin\alpha+\cos\alpha=1$

Constant term $=$

$\sqrt{(\text{coeff. of }\sin\alpha)^2+(\text{coeff. of }\cos\alpha)^2}$

$=\sqrt{(\sqrt{3})^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2$

Dividing both the sides by 2

$\dfrac{\sqrt{3}}{2}\sin\alpha+\dfrac{1}{2}\cos\alpha=\dfrac{1}{2}$

or, $\sin\alpha\cos 30°+\cos\theta\sin 30°=\dfrac{1}{2}$

or, $\sin(\alpha+30°)=\sin 30°$ or $\sin(180°-30°)$

or, $\alpha+30°=30°$ or $150°$

or, $\alpha=0°$ or $120°$

Thus, $\alpha=0°$ or $120°$ are the required solution.

25. हल गर्नुहोस् (Solve): **[2072 S, 2066 R]**

$\sqrt{3}\ \sin\theta+\cos\theta=\sqrt{2}\quad(0°\le\theta\le 2\pi^c)$

⇨ Here, $\sqrt{3}\sin\theta+\cos\theta=\sqrt{2}$

Constant term $=$

$\sqrt{(\text{coeff. of }\sin\theta)^2+(\text{coeff. of }\cos\theta)^2}$

$=\sqrt{(\sqrt{3})^2+(1)^2}=\sqrt{3+1}=2$

Dividing the given equation by 2

$\dfrac{\sqrt{3}}{2}\sin\theta+\dfrac{1}{2}\cos\theta=\dfrac{\sqrt{2}}{2}$

or, $\sin\theta\cos 30°+\cos\theta.\sin 30°=\dfrac{1}{\sqrt{2}}$

or, $\sin(\theta+30°)=\sin 45°$ or $(180°-45°)$

or, $\theta+30°=45°$ or $135°$

$\therefore\ \theta=15°$ or $105°$

Thus, the values of θ are $15°$ or $105°$.

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26. हल गर्नुहोस् (Solve): [2069 R, 2066 S, 2060S']

$\sqrt{3}\cos x + \sin x = \sqrt{3}$ $\quad (0 \le x \le 2\pi)$

⇨ Here, $\sqrt{3}\cos x + \sin x = \sqrt{3}$

$\sin x + \sqrt{3}\cos x = \sqrt{3}$

Constant term

$= \sqrt{(\text{coefficient of sin x})^2 + (\text{coefficient of cos x})^2}$

$= \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

Dividing both sides by 2,

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{3}}{2}$

or, $\sin x.\cos 60° + \cos x.\sin 60° = \frac{\sqrt{3}}{2}$

or, $\sin(x + 60°) = \sin 60°, \sin 120°, \sin 420°$

or, $x + 60° = 60°, 120°, 420°$

Thus, x = 0°, 60°, 360° are the required solution.

28. हल गर्नुहोस् (Solve): [2070 S', 2064 R]

$\sqrt{3}\cot A = \frac{\sqrt{3}}{\sin A} - 1$ $\quad (0° \le A \le 360°)$

⇨ Here, $\sqrt{3}\cot A = \frac{\sqrt{3}}{\sin A} - 1$

or, $\sqrt{3}\cot A + 1 = \frac{\sqrt{3}}{\sin A}$

or, $\sqrt{3}\cot A \times \sin A + 1 \times \sin A = \frac{\sqrt{3}}{\sin A} \times \sin A$

or, $\sqrt{3}\frac{\cos A}{\sin A} \times \sin A + \sin A = \sqrt{3}$

or, $\sqrt{3}\cos A + \sin A = \sqrt{3}$

C $= \sqrt{(\text{coeff of cos A})^2 + (\text{coeff of sin A})^2}$

$= \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

Dividing the both sides by 2 then,

$\frac{\sqrt{3}}{2}\cos A + \frac{1}{2}\sin A = \frac{\sqrt{3}}{2}$

or, $\sin 60° \cos A + \cos 60° \sin A = \frac{\sqrt{3}}{2}$

or, $\sin(A + 60°) = \sin 60°$ or $(180° - 60°)$ or, $(360° + 60°)$

or, $\sin(A + 60°) = \sin 60°$ or $120°$ or $420°$

or, $A + 60° = 60°$ or $120°$ or $420°$

∴ $A = 0°$ or $60°$ or, $360°$

Thus, the values of A or 0°, 60° and 360°.

30. हल गर्नुहोस् (Solve):

$\sin 3x + \sin x = 2\sin x$ $\quad (0° \le x \le 180°)$ [2063 R']

⇨ Here, $\sin 3x + \sin x = 2\sin x$

or, $2\sin\left(\frac{3x + x}{2}\right)\cos\left(\frac{3x - x}{2}\right) = 2\sin x$

or, $2\sin 2x \cos x = 2\sin x$

or, $2.2\sin x.\cos x \cos x - 2\sin x = 0$

or, $2\sin x(2\sin x \cos x - 1) = 0$

or, $2\sin x(\sin 2x - 1) = 0$

Either, $2\sin x = 0$ $\qquad$ or, $\sin 2x - 1 = 0$

or, $\sin x = 0$ $\qquad$ or, $\sin 2x = 1$

or, $\sin x = \sin 0°$ or $360°$ $\qquad$ or, $\sin 2x = \sin 90°$

∴ $x = 0°$ or $360°$ $\qquad$ or, $2x = 90°$

∴ $x = 45°$

Thus, the values of x are 0° and 45°.

27. हल गर्नुहोस् (Solve):

$\sqrt{3}\tan\theta = \sec\theta - 1$ $\quad (0° \le \theta \le 360°)$ [2065 R]

⇨ Here, $\sqrt{3}\tan\theta = \sec\theta - 1$

or, $\sqrt{3}\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - 1$

or, $\frac{\sqrt{3}\sin\theta - 1}{\cos\theta} = -1$

or, $\sqrt{3}\sin\theta - 1 = -\cos\theta$

or, $\sqrt{3}\sin\theta + \cos\theta = 1$ .... (i)

Constant term $= \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4}$

Dividing equation (i) by 2, $\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{1}{2}$

or, $\sin\theta\cos 30° + \cos\theta\sin 30° = \frac{1}{2}$

or, $\sin(\theta + 30°) = \sin 30°$ or $150°$ or $(360° + 30°)$

or, $\theta + 30° = 30°$ or $150°$ or $390°$

Thus, θ = 0° or, 120° or, 360° are the required solution.

MODEL 3

29. हल गर्नुहोस् (Solve):

$\sin 3\theta + \sin\theta = \sin 2\theta$ $\quad (0° \le \theta \le 180°)$

[2072 R', 2061 R]

⇨ Here, given $\sin 3\theta + \sin\theta = \sin 2\theta$

or, $2\sin\frac{3\theta + \theta}{2}.\cos\frac{3\theta - \theta}{2} - \sin 2\theta = 0$

or, $2\sin\frac{4\theta}{2}.\cos\frac{2\theta}{2} - \sin 2\theta = 0$

or, $2\sin 2\theta.\cos\theta - \sin 2\theta = 0$

or, $\sin 2\theta(2\cos\theta - 1) = 0$

or, $\sin 2\theta = 0$ or $2\cos\theta - 1 = 0$

or, $\sin 2\theta = \sin 0°$ or $2\cos\theta = 1$

or, $2\theta = 0°$ or $\cos\theta = \frac{1}{2} = \cos 60°$

or, $\theta = 0°$ or $\theta = 60°$

Thus, θ = 0°, 60° are the required solution.

31. हल गर्नुहोस् (Solve):

$\cos 3x + \cos x = 2\cos x$ $\quad (0° \le x \le 360°)$ [2063 M]

⇨ Here, We have given; $\cos 3x + \cos x = 2\cos x$.

or, $2\cos\frac{3x + x}{2}.\cos\frac{3x - x}{2} = 2\cos x$

or, $2\cos 2x.\cos x = 2\cos x$

or, $2\cos 2x.\cos x - 2\cos x = 0$

or, $2(\cos 2x - 1)\cos x = 0$

or, $(\cos 2x - 1)\cos x = 0$

Either $\cos 2x - 1 = 0$.....(i) $\qquad$ or, $\cos x = 0$.....(ii)

From (i); $\cos 2x = 1$

or, $\cos 2x = \cos 0°$ or $\cos 360°$

or, $2x = 0°$ or $360°$

∴ $x = 0°$ or $180°$

From (ii); $\cos x = 0$

∴ $x = 90°$ or $270°$

Thus, x = 0°, 90°, 270°, 180°.

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32. हल गर्नुहोस् (Solve):

$\cos 3\theta + \cos\theta = \cos 2\theta$ ($0° \le \theta \le 180°$) [2064 S]

⇨ Here, $\cos 3\theta + \cos\theta = \cos 2\theta$

or, $2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) = \cos 2\theta$

or, $2\cos 2\theta \cos\theta - \cos 2\theta = 0$

or, $2\cos\theta - 1 = 0$

or, $\cos 2\theta = \cos 90°$ or $(360° - 90°)$

or, $2\cos\theta = 1$

or, $2\theta = 90°$ or $270°$

or, $\cos\theta = \frac{1}{2}$

$\therefore$ $\theta = 45°$ or $135°$

or, $\cos\theta = \cos 60°$ or $(360° - 60°)$

$\therefore$ $\theta = 60°$ or $330°$

Thus, the values of θ are 45°, 60° and 135°.

33. हल गर्नुहोस् (Solve):

$\sin 4x + \sin 2x = \cos x$ ($0° \le x \le 360°$) [2063 S]

⇨ Here, $\sin 4x + \sin 2x = \cos x$

or, $2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) = \cos x$

or, $2\sin 3x \cos x - \cos x = 0$

or, $\cos x(2\sin 3x - 1) = 0$

Either $\cos x = 0$

or, $\cos x = \cos 90°$ or $(360° - 90°)$

or, $2\sin 3x = 1$

$\therefore$ $x = 90°$ or $270°$

or, $2\sin 3x - 1 = 0$

or, $\sin 3x = \frac{1}{2}$

or, $\sin 3x = \sin 30°$ or $(180° - 30°)$

or, $3x = 30°$ or $150°$

$\therefore$ $x = 10°$ or $50°$

Thus, the values of x are 10°, 50°, 90° and 270°

QUESTIONS FROM CDC TEXTBOOK

5.5 त्रिकोणमितीय समीकरण (TRIGONOMETRIC EQUATION)

EXERCISE 5.5

1. हल गर्नुहोस् (Solve) : ($0 \le \theta \le 90°$)

(a) $\sin\theta = \frac{1}{2}$

⇨ Here, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin 30°$

$\therefore$ $\theta = 30°$

(e) $\text{cosec}\,\theta = \sqrt{2}$

⇨ Here, $\text{cosec}\,\theta = \sqrt{2}$

or, $\text{cosec}\,\theta = \text{cosec}\,45°$

$\therefore$ $\theta = 45°$

(i) $\cos\theta = 1$

⇨ Here, $\cos\theta = 1$

or, $\cos\theta = \cos 0°$

$\therefore$ $\theta = 0°$

(b) $\cos\theta = \frac{\sqrt{3}}{2}$

⇨ Here, $\cos\theta = \frac{\sqrt{3}}{2}$

or, $\cos\theta = \cos 30°$

$\therefore$ $\theta = 30°$

(f) $\cot\theta = \sqrt{3}$

⇨ Here, $\cot\theta = \sqrt{3}$

or, $\cot\theta = \cot 30°$

$\therefore$ $\theta = 30°$

(j) $\sin\theta = 1$

⇨ Here, $\sin\theta = 1$

or, $\sin\theta = \sin 90°$

$\therefore$ $\theta = 90°$

(c) $\sqrt{3}\tan\theta = 1$

⇨ Here, $\sqrt{3}\tan\theta = 1$

or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan 30°$

$\therefore$ $\theta = 30°$

(g) $2\sin\theta - \sqrt{3} = 0$

⇨ Here, $2\sin\theta - \sqrt{3} = 0$

or, $2\sin\theta = \sqrt{3}$

or, $\sin\theta = \frac{\sqrt{3}}{2}$

or, $\sin\theta = \sin 60°$

$\therefore$ $\theta = 60°$

(d) $\sec\theta = 2$

⇨ Here, $\sec\theta = 2$

or, $\sec\theta = \sec 60°$

$\therefore$ $\theta = 60°$

(h) $\cos\theta = 0$

⇨ Here, $\cos\theta = 0$

or, $\cos\theta = \cos 90°$

$\therefore$ $\theta = 90°$

2. हल गर्नुहोस् (Solve) : ($0 \le \theta \le 360°$)

(a) $2\cos\theta - 1 = 0$

⇨ Here, $2\cos\theta - 1 = 0$

or, $2\cos\theta = 1$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos 60°$ or, $(360° - 60°)$

[cos is (+ve) in 1st and 4th quadrant]

$\therefore$ $\theta = 60°$ or $300°$

Thus, θ = 60° or 300° is the solution.

(b) $\sqrt{3}\tan\theta + 1 = 0$

⇨ Here, $\sqrt{3}\tan\theta + 1 = 0$

or, $\sqrt{3}\tan\theta = -1$

or, $\tan\theta = -\frac{1}{\sqrt{3}}$

The value of tan θ is negative in 2nd & 4th quadrants .

In second quadrant ,	In fourth quadrant,
$\tan\theta = \tan(180° - 30°)$	$\tan\theta = \tan(360° - 30°)$
or, $\tan\theta = \tan 150°$	or, $\tan\theta = \tan 330°$
$\therefore$ $\theta = 150°$	$\therefore$ $\theta = 330°$

Thus, θ = 150° or 330° is the solution.

(c) $\sqrt{2}\sec\theta + 2 = 0$

⇨ Here, $\sqrt{2}\sec\theta + 2 = 0$

or, $\sqrt{2}\sec\theta = -2$

or, $\sec\theta = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

or $\frac{1}{\cos\theta} = -\sqrt{2}$

or, $\cos\theta = -\frac{1}{\sqrt{2}}$

The value of cos θ is negative in second & third quadrants.

In second quadrant ,	In third quadrant,
$\cos\theta = \cos(180° - 45°)$	$\cos\theta = \cos(180° + 45°)$
or, $\cos\theta = \cos 135°$	or, $\cos\theta = \cos 225°$
$\therefore$ $\theta = 135°$	$\therefore$ $\theta = 225°$

Thus, θ = 135° or 225° is the solution.

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(d) $\sin\theta + \frac{\sqrt{3}}{2} = 0$

⇨ Here, $\sin\theta + \frac{\sqrt{3}}{2} = 0$

or, $\sin\theta = -\frac{\sqrt{3}}{2}$

Value of sin θ is negative in third & fourth quadrants.
In third quadrant ,
$\sin\theta = \sin(180° + 60°)$
or, $\sin\theta = \sin 240°$
$\therefore\ \theta = 240°$
In fourth quadrant
$\sin\theta = \sin(360° - 60°)$
or, $\sin\theta = \sin 300°$
$\therefore\ \theta = 300°$

Thus, θ = 240° or 300° is the solution.

(f) $\tan\theta - \sqrt{3} = 0$

⇨ Here, $\tan\theta - \sqrt{3} = 0$
or, $\tan\theta = \sqrt{3}$
or, $\tan\theta = \tan 60°$ or $\tan(180° + 60°)$
or, $\tan\theta = \tan 60°$ or $\tan 240°$
$\therefore\ \theta = 60°$ or $240°$
Thus, θ = 60° or 240° is the soultion.

(h) $3\cot\theta - \sqrt{3} = 0$

⇨ Here, $3\cot\theta - \sqrt{3} = 0$
or, $3\cot\theta = \sqrt{3}$

or, $\cot\theta = \frac{\sqrt{3}}{3}$

$\therefore\ \cot\theta = \frac{1}{\sqrt{3}}$

or, $\cot\theta = \cot 60°$ or $\cot(180° + 60°)$
$\therefore\ \theta = 60°$ or $240°$
Thus, θ = 60° or 240° is the solution.

(j) $2\cos\theta + 1 = 0$

⇨ Here, $2\cos\theta + 1 = 0$

or, $2\cos\theta = -1$

or, $\cos\theta = -\frac{1}{2}$

The values of cos θ is negative in second & third quadrant .

In second quadrant ,

$\cos\theta = \cos(180° - 60°)$

or, $\cos\theta = \cos 120°$

$\therefore\ \theta = 120°$

In third quadrant,

$\cos\theta = \cos(180° + 60°)$

or, $\cos\theta = \cos 240°$

$\therefore\ \theta = 240°$

Thus, θ = 120° or 240° is the solution.

(e) $\cos\theta - \frac{\sqrt{3}}{2} = 0$

⇨ Here, $\cos\theta - \frac{\sqrt{3}}{2} = 0$

or, $\cos\theta = \frac{\sqrt{3}}{2}$

Value of cos θ is positive in first & fourth quadrant,
In first quadrant ,
$\cos\theta = \cos 30°$
$\therefore\ \theta = 30°$
In fourth quadrant,
$\cos\theta = \cos(360° - 30°)$
or, $\cos\theta = \cos 330°$
$\therefore\ \theta = 330°$
Thus θ = 30° or 330° is the solution.

(g) $\sqrt{3}\ \mathrm{cosec}\,\theta + 2 = 0$

⇨ Here, $\sqrt{3}\ \mathrm{cosec}\,\theta + 2 = 0$
or, $\sqrt{3}\ \mathrm{cosec}\,\theta = -2$

or, $\mathrm{cosec}\,\theta = -\frac{2}{\sqrt{3}}$

or, $\mathrm{cosec}\,\theta = -\mathrm{cosec}\,60°$
or, $\mathrm{cosec}\,\theta = \mathrm{cosec}(180° + 60°)$ or $\mathrm{cosec}(360° - 60°)$
$\therefore\ \theta = 240°$ or $300°$
Thus, θ = 240° or 300° is the solution.

(i) $2\sin\theta + 1 = 0$

⇨ Here, $2\sin\theta + 1 = 0$
or, $2\sin\theta = -1$

or, $\sin\theta = -\frac{1}{2}$

or, $\sin\theta = -\sin 30°$
or, $\sin\theta = \sin(180° + 30°)$ or $\sin(360° - 30°)$
or, $\sin\theta = \sin 210°$ or $\sin 330°$
$\therefore\ \theta = 210°$ or $330°$
Thus, θ = 210° or 330° is the solution.

3. हल गर्नुहोस् (Solve) : $(0 \le x \le 180°)$

(a) $\tan x - \sin x = 0$

⇨ Here, $\tan x - \sin x = 0$
or, $\tan x = \sin x$

or, $\frac{\sin x}{\cos x} = \sin x$

or, $\sin x = \sin x \cos x$
or, $\sin x - \sin x \cos x = 0$
or, $\sin x(1 - \cos x) = 0$
Either $\sin x = 0$ ...... (1)
or, $1 - \cos x = 0$ ....(2)
from equation (1)
or, $\sin x = \sin 0°, \sin 180°, \sin 360°$
$\therefore\ x = 0°, 180°, 360°$
From equation (2)
$\cos x = 1$
value of cos θ is positive in first & fourth quadrants ,
In first quadrant ,
$\cos x = \cos 0°$
$\therefore\ x = 0°$
In fourth quadrant
$\cos x = \cos(360° - 0°)$
$\therefore\ x = \cos 360°$
Thus, x = 0° is the solution.

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(b) $\tan x + \cot x = 2$

⇨ Here, $\tan x + \cot x = 2$

or, $\tan x + \dfrac{1}{\tan x} = 2$

or, $\dfrac{\tan^2 x + 1}{\tan x} = 2$

or, $\tan^2 x + 1 = 2\tan x$

or, $\tan^2 x - 2\tan x + 1 = 0$

or, $(\tan x - 1)^2 = 0$

or, $\tan x - 1 = 0$

or, $\tan x = 1$

Value of tan x is positive in first & third quadrants,

In first quadrant,

$\tan x = \tan 45°$

$\therefore$ $x = 45°$

In third quadrant ,

$\tan x = \tan(180° + 45°)$

or, $\tan x = \tan 225°$

$\therefore$ $x = 225°$

Thus, x = 45° is the solution.

(d) $7 \sin^2 x + 3 \cos^2 x - 4 = 0$

⇨ Here, $7 \sin^2 x + 3\cos^2 x - 4 = 0$

or, $7\sin^2 x + 3(1 - \sin^2 x) - 4 = 0$

or, $7\sin^2 x + 3 - 3 \sin^2 x - 4 = 0$

or, $4\sin^2 x - 1 = 0$

or, $4\sin^2 x = 1$

or, $\sin^2 x = \dfrac{1}{4}$

$\therefore$ $\sin x = \pm \dfrac{1}{2}$

sin x is positive in first & second quadrants,
but negative in third & fourth quadrants,

In first quadrant,

$\sin x = \dfrac{1}{2}$

or, $\sin x = \sin 30°$

$\therefore$ $x = 30°$

In second quadrant,

$\sin x = \dfrac{1}{2}$

or, $\sin x = \sin(180° - 30°)$

or, $\sin x = \sin 150°$

$\therefore$ $x = 150°$

In third quadrant,

$\sin x = -\dfrac{1}{2}$

or, $\sin x = \sin (180° + 30°)$

or, $\sin x = \sin 210°$

$\therefore$ $x = 210°$

In fourth quadrant ,

$\sin x = -\dfrac{1}{2}$

or, $\sin x = \sin (360° - 30°)$

or, $\sin x = \sin 330°$

$\therefore$ $x = 330°$

Thus, x = 30° or 150° is the solution.

(c) $6 \sin^2 x + \cos x = 5$

⇨ Here, $6 \sin^2 x + \cos x = 5$

or, $6 (1 - \cos^2 x) + \cos x - 5 = 0$

or, $6 - 6\cos^2 x + \cos x - 5 = 0$

or, $1 - 6 \cos^2 x + \cos x = 0$

or, $6\cos^2 x - \cos x - 1 = 0$

or, $6 \cos^2 x - 3 \cos x + 2 \cos x - 1 = 0$

or, $3\cos x (2 \cos x - 1) + 1(2 \cos x - 1) = 0$

or, $(2\cos x - 1) (3\cos x + 1) = 0$

Either, $2\cos x - 1 = 0$ ....(1)

or, $3\cos x + 1 = 0$ ...... (2)

From equation (1); $2 \cos x = 1$

or, $\cos x = \dfrac{1}{2}$

The value of cos x is positive in first & fourth quadrants.

In first quadrant, $\cos x = \cos 60°$

$\therefore$ $x = 60°$

In fourth quadrant, $\cos x = \cos (360° - 60°)$

or, $\cos x = \cos 300°$

$\therefore$ $x = 300°$

From equation (2) ; $3\cos x = -1$

or, $\cos x = -\dfrac{1}{3}$

Value of cos x is negative in second & third quadrants.

In second quadrant, $\cos x = \cos (180° - 70.52°)$

or, $\cos x = \cos 109.48°$

$\therefore$ $x = 109.48°$

In third quadrant, $\cos x = \cos (180° + 70.52°)$

or, $\cos x = \cos 250.52°$

$\therefore$ $x = 250.52°$

Thus, x = 60° or 70.52° is the solution.

(e) $2 \cos^2 x - 2 \sin x = \dfrac{1}{2}$

⇨ Here, $2\cos^2 x - 2\sin x = \dfrac{1}{2}$

or, $4\cos^2 x - 4\sin x = 1$

or, $4(1 - \sin^2 x) - 4\sin x = 1$

or, $4 - 4\sin^2 x - 4\sin x = 1$

or, $- 4\sin^2 x - 4\sin x + 3 = 0$

or, $4\sin^2 x + 4\sin x - 3 = 0$

or, $4\sin^2 x + 6\sin x - 2\sin x - 3 = 0$

or, $2\sin x (2\sin x + 3) - 1 (2\sin x + 3) = 0$

or, $(2\sin x + 3) (2\sin x - 1) = 0$

Either $2\sin x - 1 = 0$ ....(1)

or, $2\sin x + 3 = 0$............(2)

From equation (1)

$2\sin x = 1$

or, $\sin x = \dfrac{1}{2}$

Value of sin x is positive in first & second quadrants.

In first quadrant,

or, $\sin x = \sin 30°$

$\therefore$ $x = 30°$

In second quadrant,

$\sin x = \sin (180° - 30°)$

or, $\sin x = \sin 150°$

$\therefore$ $x = 150°$

From equation (2),

$2\sin x = - 3$

or, $\sin x = -\dfrac{3}{2}$

$\therefore$ $x = \sin^{-1} (-\dfrac{3}{2})$ (Imposible)

Thus, x = 30° or 150° is the solution.

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(f) $6\sin^2 x + 4\cos^2 x = 5$

⇨ Here, $6\sin^2 x + 4\cos^2 x = 5$
or, $6\sin^2 x + 4\cos^2 x - 5 = 0$
or, $6\sin^2 x + 4(1 - \sin^2 x) - 5 = 0$
or, $6\sin^2 x + 4 - 4\sin^2 x - 5 = 0$
or, $2\sin^2 x - 1 = 0$
or, $2\sin^2 x = 1$
or, $\sin^2 x = \frac{1}{2}$
or, $\sin^2 x = \left(\pm\frac{1}{\sqrt{2}}\right)^2$
$\therefore$ $\sin x = \pm\frac{1}{\sqrt{2}}$
Taking (+)ve sign,
$\sin x = \frac{1}{\sqrt{2}}$
or, $\sin x = \sin 45°$ or $(180° - 45°)$
$\therefore$ $x = 45°$ or $135°$
Taking (–ve) sign,
$\sin x = -\frac{1}{\sqrt{2}}$
or, $\sin x = -\sin 45°$
or, $\sin x = \sin(180° + 45°)$ or $\sin(360° - 45°)$
$\therefore$ $x = 225°$ or $315°$
Thus, $x = 45°$ or $135°$ is the solution.

(h) $\operatorname{cosec} x - 2\sin x = 1$

⇨ Here, $\operatorname{cosec} x - 2\sin x = 1$
or, $\frac{1}{\sin x} - 2\sin x = 1$
or, $\frac{1 - 2\sin^2 x}{\sin x} = 1$
or, $1 - 2\sin^2 x = \sin x$
or, $-2\sin^2 x - \sin x + 1 = 0$
or, $2\sin^2 x + \sin x - 1 = 0$
or, $2\sin^2 x + 2\sin x - \sin x - 1 = 0$
or, $2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
or, $(\sin x + 1)(2\sin x - 1) = 0$
Either, $\sin x + 1 = 0$ ....(1)
or, $2\sin x - 1 = 0$ ....(2)
From equation (1); $\sin x = -1$
Values of sin x is negative in third & fourth quadrants.
In third quadrant,
$\sin x = \sin(180° + 90°)$
or, $\sin x = \sin 270°$
$\therefore$ $x = 270°$
In fourth quadrant,
$\sin x = \sin(360° - 90°)$
or, $\sin x = \sin 270°$
$\therefore$ $x = 270°$
From equation (2),
$2\sin x = 1$
or, $\sin x = \frac{1}{2}$
Value of sin x is positive in first & second quadrants.
In first quadrant,
$\sin x = \sin 30°$
$\therefore$ $x = 30°$
In second quadrant
$\sin x = \sin(180° - 30°)$
or, $\sin x = \sin 150°$
$\therefore$ $x = 150°$
Thus, $x = 30°$ or $150°$ is the solution.

(g) $4\sec^2 x - 7\tan^2 x = 3$

⇨ Here, $4\sec^2 x - 7\tan^2 x = 3$
or, $4(1 + \tan^2 x) - 7\tan^2 x - 3 = 0$
or, $4 + 4\tan^2 x - 7\tan^2 x - 3 = 0$
or, $-3\tan^2 x + 1 = 0$
or $3\tan^2 x = 1$
or, $\tan^2 x = \frac{1}{3}$
$\therefore$ $\tan x = \pm\frac{1}{\sqrt{3}}$
Value of tan x is positive in first & third quadrants,
& negative in second & fourth quadrants,
In first quadrant,
$\tan x = \tan 30°$
$\therefore$ $x = 30°$
In second quadrant,
$\tan x = \tan(180° - 30°)$
or, $\tan x = \tan 150°$
$\therefore$ $x = 150°$
In third quadrant,
$\tan x = \tan(180° + 30°)$
or, $\tan x = \tan 210°$
$\therefore$ $x = 210°$
In fourth quadrant,
$\tan x = \tan(360° - 30°)$
or, $\tan x = \tan(360° - 30°)$
or, $\tan x = \tan 330°$
$\therefore$ $x = 330°$
Thus, $x = 30°$ or $150°$ is the solution.

(i) $\tan^2 x - 3\sec x + 3 = 0$

⇨ Here, $\tan^2 x - 3\sec x + 3 = 0$
or, $\sec^2 x - 1 - 3\sec x + 3 = 0$
or, $\sec^2 x - 3\sec x + 2 = 0$
or, $\sec^2 x - \sec x - 2\sec x + 2 = 0$
or, $\sec x(\sec x - 1) - 2(\sec x - 1) = 0$
or, $(\sec x - 1)(\sec x - 2) = 0$
Either, $\sec x - 1 = 0$.......(i)
or, $\sec x - 2 = 0$...........(i)
From (i)
or, $\sec x = 1$
or, $\sec x = \sec 0°$ or $360°$
$\therefore$ $x = 0°$ or $360°$
From (ii)
or, $\sec x = 2$
or, $\sec x = \sec 60°$ or $\sec(360° - 60°)$
$\therefore$ $x = 60°$ or $300°$
Thus, $x = 0°$ or $60°$ is the solution.

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(j) $\sin 3x + \cos 3x = \frac{1}{2}$

⇨ Here, $\sin 3x + \cos 3x = \frac{1}{2}$

or, $\sin 3x + \sin (90° - 3x) = \frac{1}{2}$

or, $2 \sin \left(\frac{3x + 90° - 3x}{2}\right) \cos \left(\frac{3x - 90° + 3x}{2}\right) = \frac{1}{2}$

or, $2 \sin 45° \cos \left(\frac{6x - 90°}{2}\right) = \frac{1}{2}$

or, $2 \times \frac{1}{\sqrt{2}} . \cos (3x - 45°) = \frac{1}{2}$

or, $\cos (3x - 45°) = \frac{1}{2} \times \frac{\sqrt{2}}{2}$

or, $\cos (3x - 45°) = \frac{1}{2\sqrt{2}}$

or, $\cos (3x - 45°) = \cos 69.30°$ or $\cos (360° - 69.30°)$

or, $(3x - 45°) = 69.30°$ or $290.70°$

or, $3x = 69.30° + 45°$ or $290.70° + 45°$

or, $3x = 114.30°$ or $335.70°$

∴ $x = 38.1°$ or $111.9°$

Thus, $x = 38.1°$ or $111.9°$ is the solution.

(b) $\sin x + \cos x = \frac{1}{\sqrt{2}}$

⇨ Here, $\sin x + \cos x = \frac{1}{\sqrt{2}}$

Constant term

$= \sqrt{(\text{coefficient of } \sin x)^2 + (\text{coefficient of } \cos x)^2}$

$= \sqrt{1^2 + 1^2} = \sqrt{2}$

Dividing the given equation by $\sqrt{2}$ then,

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2} \times \sqrt{2}}$$

or, $\sin x \cos 45° + \cos x \sin 45° = \frac{1}{2}$

or, $\sin (x + 45°) = \sin 30°$ or $(180° - 30°)$

or, $x + 45° = 30°$ or $150°$

or, $x = - 15°$ or $105°$

or, $x = (360° - 15°)$, or $105°$

∴ $x = 345°$ or $105°$

Thus, $x = 105°$ or $345°$ is the solution.

(d) $\sqrt{3} \cos x + \sin x = \sqrt{3}$

⇨ Here, $\sqrt{3} \cos x + \sin x = \sqrt{3}$

or, $\sin x + \sqrt{3} \cos x = \sqrt{3}$

Constant term

$= \sqrt{(\text{coefficient of } \sin x)^2 + (\text{coefficient of } \cos x)^2}$

$= \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$

Dividing both sides by 2, $\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x = \frac{\sqrt{3}}{2}$

or, $\sin x. \cos 60° + \cos x. \sin 60° = \frac{\sqrt{3}}{2}$

or, $\sin (x + 60°) = \sin 60°, \sin 120°, \sin 420°$

or, $x + 60° = 60°, 120°, 420°$

Thus, $x = 0°, 60°, 360°$ are the required solution.

4. **हल गर्नुहोस् (Solve) : $(0 \le x \le 360°)$**

(a) $\sin x + \cos x = \sqrt{2}$

⇨ Here, $\sin x + \cos x = \sqrt{2}$

Constant term

$= \sqrt{(\text{coefficient of } \sin x)^2 + (\text{coefficient of } \cos x)^2}$

$= \sqrt{1^2 + 1^2} = \sqrt{2}$

Dividing the given equation by $\sqrt{2}$ then,

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

or, $\sin x \cos 45° + \cos x \sin 45° = 1$

or, $\sin (x + 45°) = \sin 90°$

or, $x + 45° = 90°$

∴ $x = 90° - 45° = 45°$

Thus, $x = 45°$ is the solution.

(c) $\sin x + \cos x = 1$

⇨ Here, $\sin x + \cos x = 1$

Dividing both the sides by,

$\sqrt{(\text{coeff . } \sin x)^2 + (\text{coeff . } \cos x)^2}$

i.e. $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$ then,

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$$

or, $\sin x \cos 45° + \cos x \sin 45° = \frac{1}{\sqrt{2}}$

or, $\sin (x + 45°) = \frac{1}{\sqrt{2}}$

or, $\sin (x + 45°) = \sin 45°, \sin 135°, \sin (360° + 45°)$

So, $x + 45° = 45°$ ∴ $x = 0°$

and $x + 45° = 135°$ ∴ $x = 90°$

or, $x + 45° = 405°$ ∴ $x = 360°$

Thus, $x = 0°$ or $90°$ or $360°$ is the solution.

(e) $\cos x + \frac{1}{\sqrt{3}} \sin x = 1$

⇨ Here, multiplying both sides by $\sqrt{3}$, we get,

$\sqrt{3} \cos x + \sin x = \sqrt{3}$

Now, refer to Q No. (d) above.

(f) $\cos x - \sqrt{3} \sin x = 1$

⇨ Here, $\cos x - \sqrt{3} \sin x = 1$

Constant term

$= \sqrt{(\text{coeff . of } \cos x)^2 + (\text{coeff . of } \sin x)^2}$

$= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$

Dividing given equation by 2 then,

$$\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \frac{1}{2}$$

or, $\sin 30° \cos x - \cos 30° \sin x = \frac{1}{2}$

or, $\sin (30° - x) = \sin 30°$ or $(180° - 30°)$

or, $30° - x = 30°$ or $150°$

or, $- x° = 0°$ or $120°$

or, $x = 0°$ or $- 120°$

or, $x = 0°$ or $(360° - 120°)$

∴ $x = 0°$ or $240°$

Thus, $x = 0°$ or $240°$ is the solution.

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(g) $\sin x + \sqrt{3}\cos x = \sqrt{2}$

⇨ Here, $\sin x + \sqrt{3}\cos x = \sqrt{2}$

Constant term

$$= \sqrt{(\text{coeff. of } \sin x)^2 + (\text{coeff. of } \cos x)^2}$$

$$= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

Dividing given equation by 2 then,

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

or, $\sin x \cos 60° + \cos x \sin 60° = \frac{1}{\sqrt{2}}$

or, $\sin(x + 60°) = \sin 45°$ or $(180° - 45°)$

or, $x + 60° = 45°$ or $135°$

or, $x = -15°$ or $75°$

or, $x = (360° - 15°)$ or $75°$

$\therefore$ $x = 345°$ or $75°$

Thus, $x = 345°$ or $75°$ is the solution.

(i) $\sqrt{3}\sin x + \cos x = 1$

⇨ Here, $\sqrt{3}\sin x + \cos x = 1$

Constant term

$$= \sqrt{(\text{coeff. of } \sin x)^2 + (\text{coeff. of } \cos x)^2}$$

$$= \sqrt{(\sqrt{3})^2 + 1} = \sqrt{3+1} = \sqrt{4} = 2$$

Dividing given equation by 2 then,

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

or, $\sin x \cos 30° + \cos x \sin 30° = \sin 30°$ or $\sin(180° - 30°)$ or $\sin(360° + 30°)$

or, $\sin(x + 30°) = \sin 30°$ or $\sin 150°$ or $\sin 390°$

or, $x + 30° = 30°$ or $150°$ or $390°$

$\therefore$ $x = 0°$ or $120°$ or $360°$

Thus, $x = 0°$ or $120°$ or $360°$ is the solution.

5. हल गर्नुहोस् (Solve) : $(0 \le \theta \le 360°)$

(a) $\sin 3\theta + \sin 2\theta - \sin\theta = 0$

⇨ Here, $\sin 3\theta + \sin 2\theta - \sin\theta = 0$

or, $\sin 3\theta - \sin\theta + \sin 2\theta = 0$

or, $2\cos\left(\frac{3\theta + \theta}{2}\right)\sin\left(\frac{3\theta - \theta}{2}\right) + \sin 2\theta = 0$

or, $2\cos 2\theta \sin\theta + \sin 2\theta = 0$

or, $2\cos 2\theta \sin\theta + 2\sin\theta\cos\theta = 0$

or, $2\sin\theta(\cos 2\theta + \cos\theta) = 0$

Either, $2\sin\theta = 0$.................(i)

or, $\sin\theta = 0$

or, $\sin\theta = \sin 0°$ or $(180° - 0°)$ or $(360° + 0°)$

or, $\theta = 0°$ or $180°$ or $360°$

OR, $\cos 2\theta + \cos\theta = 0$..............(ii)

or, $\cos 2\theta = -\cos\theta$

or, $\cos 2\theta = \cos(180° - \theta)$ or $(180° + \theta)$

or, $2\theta = (180° - \theta)$ or $(180° + \theta)$

or, $2\theta = 180° - \theta$ or, $2\theta = 180 + \theta$

or, $3\theta = 180°$ or, $\theta = 180°$

$\therefore$ $\theta = 60°$ $\therefore$ $\theta = 180°$

Thus, $\theta = 0°$ or $60°$ or $180°$ or $360°$ is the solution.

(h) $\cos x + \sqrt{3}\sin x = 2$

⇨ Here, $\cos x + \sqrt{3}\sin x = 2$

Constant term

$$= \sqrt{(\text{coeff. of } \cos x)^2 + (\text{coeff. of } \sin x)^2}$$

$$= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

Dividing given equation by 2 then,

$$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{2}{2}$$

or, $\sin 30° \cos x + \cos 30° \sin x = 1$

or, $\sin(30° + x) = \sin 90°$

or, $30° + x = 90°$

$\therefore$ $x = 60°$

Thus, $x = 60°$ is the solution.

(j) $\sqrt{3}\sin x - \cos x = \sqrt{2}$

⇨ Here, $\sqrt{3}\sin x - \cos x = \sqrt{2}$

Now,

$$\sqrt{(\text{coefficient of } 1^{st} \text{ term})^2 + (\text{coefficient of } 2^{nd} \text{term})^2}$$

$$= \sqrt{\sqrt{3}^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Dividing above equation by 2,

$$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$$

or, $\cos 30° \sin x - \sin 30° \cos x = \frac{1}{\sqrt{2}}$

or, $\sin x \cos 30° - \cos x \sin 30° = \frac{1}{\sqrt{2}}$

or, $\sin(x - 30°) = \frac{1}{\sqrt{2}}$

Value of sin x is positive in first & second quadrants,

In first quadrant,

$\sin(x - 30°) = \sin 45°$

or, $(x - 30°) = 45°$

$\therefore$ $x = 45° + 30° = 75°$

In second quadrant,

$\sin(x° - 30°) = \sin(180° - 45°)$

or, $\sin(x° - 30°) = \sin 135°$

or, $x - 30° = 135°$

$\therefore$ $x = 135° + 30° = 165°$

Thus, $x = 75°$ or $165°$ is the solution.

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(b) $\sin 4\theta + \sin 2\theta = 0$

⇨ Here, $\sin 4\theta + \sin 2\theta = 0$

or, $2 \sin \left(\frac{4\theta + 2\theta}{2}\right) \cos \left(\frac{4\theta - 2\theta}{2}\right) = 0$

or, $2 \sin 3\theta \cos \theta = 0$

or, $\sin 3\theta \cos \theta = 0$

Either, $\cos \theta = 0$

or, $\cos \theta = \cos 90°$ or $\cos (360° - 90°)$

$\therefore\ \theta = 90°$ or $270°$

OR, $\sin 3\theta = 0$

or, $3 \sin \theta - 4\sin^3 \theta = 0$

or, $\sin \theta (3 - 4 \sin^2 \theta) = 0$

Either, $\sin \theta = 0$ .... (i)

or, $3 - 4 \sin^2 \theta = 0$ .... (ii)

From (i) $\sin \theta = 0$

or, $\sin \theta = \sin 0°, \sin 180°, \sin 360°$

$\therefore\ \theta = 0°, 180°, 360°$

From (ii) $3 - 4 \sin^2\theta = 0$

or, $4 \sin^2 \theta = 3$

or, $\sin^2\theta = \frac{3}{4}$

or, $\sin \theta = \pm \frac{\sqrt{3}}{2}$

Taking (+) ve sign,

or, $\sin \theta = \frac{\sqrt{3}}{2}$

or, $\sin \theta = \sin 60°$ or $\sin (180° - 60°)$

$\therefore\ \theta = 60°$ or $120°$

Taking (–) ve sign,

or, $\sin \theta = -\frac{\sqrt{3}}{2}$

or, $\sin \theta = -\sin 60°$

or, $\sin \theta = \sin (180° + 60°)$ or $\sin (360° - 60°)$

$\therefore\ \theta = 240°$ or $300°$

Thus, $\theta = 0°, 60°, 90°, 120°, 180°, 240°, 270°, 300°, 360°$ is the solution.

(d) $\cos \theta - \cos 3\theta = \sin 2\theta$

⇨ Here, $\cos \theta - \cos 3\theta = \sin 2\theta$

or, $2 \sin\left(\frac{3\theta + \theta}{2}\right) \sin \left(\frac{3\theta - \theta}{2}\right) = \sin 2\theta$

or, $2 \sin 2\theta \sin \theta = \sin 2\theta$

or, $2 \sin 2\theta \sin \theta - \sin 2\theta = 0$

or, $\sin 2\theta (2 \sin \theta - 1) = 0$

Either, $\sin 2\theta = 0$ .... (i)

or, $2 \sin \theta - 1 = 0$ .... (ii)

From (i), $\sin 2\theta = 0$

or, $\sin 2\theta = \sin 0°$ or $\sin 180°$ or $\sin 360°$ or $\sin 540°$ or $\sin 720°$

or, $2\theta = 0°, 180°, 360°, 540°, 720°$

$\therefore\ \theta = 0°, 90°, 180°, 270°, 360°$

From (ii), $2 \sin \theta - 1 = 0$

or, $2 \sin \theta = 1$

or, $\sin \theta = \frac{1}{2}$

or , $\sin \theta = \sin 30°$ or $\sin (180° - 30°)$

$\therefore\ \theta = 30°$ or $150°$

Thus, $\theta = 0°, 30°, 90°, 150°, 180°, 270°, 360°$ is the solution.

(c) $\cos 4\theta + \cos 2\theta = 0$

⇨ Here, $\cos 4\theta + \cos 2\theta = 0$

or, $2 \cos \left(\frac{4\theta + 2\theta}{2}\right) \cos \left(\frac{4\theta - 2\theta}{2}\right) = 0$

or, $2 \cos 3\theta \cos \theta = 0$

$\therefore\ \cos 3\theta \cos \theta = 0$

Either, $\cos 3\theta = 0$ .... (i)

or, $\cos \theta = 0$ .... (ii)

From (i), $\cos 3\theta = 0$

or, $4 \cos^3\theta - 3 \cos \theta = 0$

or, $\cos \theta (4 \cos^2\theta - 3) = 0$

Either, $\cos \theta = 0$ .... (iii)

or, $4 \cos^2\theta - 3 = 0$ .... (iv)

From (iii), $\cos \theta = 0$

or, $\cos \theta = \cos 90°, \cos (360° - 90°)$

$\therefore\ \theta = 90°$ or $270°$

From (iv), $4 \cos^2\theta - 3 = 0$

or, $4 \cos^2 \theta = 3$

or, $\cos^2 \theta = \frac{3}{4}$

$\therefore\ \cos \theta = \pm \frac{\sqrt{3}}{2}$

Taking (+) ve sign,

or, $\cos \theta = \frac{\sqrt{3}}{2}$

or, $\cos \theta = \cos 30°$ or $\cos (360° - 30°)$

$\therefore\ \theta = 30°$ or $330°$

Taking (–) ve sign,

or, $\cos \theta = -\frac{\sqrt{3}}{2}$

or, $\cos \theta = -\cos 30°$

or, $\cos \theta = \cos (180° - 30°)$ or $\cos (180° + 30°)$

$\therefore\ \theta = 150°, 210°$

Thus, $\theta = 30°, 90°, 150°, 210°, 270°, 330°$ is the solution.

(e) $\cos \theta + \cos 3\theta = 2 \cos 2\theta$

⇨ Here, $\cos \theta + \cos 3\theta = 2 \cos 2\theta$

or, $2 \cos\left(\frac{\theta + 3\theta}{2}\right) \cos \left(\frac{\theta - 3\theta}{2}\right) = 2 \cos 2\theta$

or, $2 \cos 2\theta \cos (-\theta) - 2 \cos 2\theta = 0$

or, $2 \cos 2\theta (\cos \theta - 1) = 0$

Either, $2 \cos 2\theta = 0$

OR, $\cos \theta - 1 = 0$

or, $\cos 2\theta = 0$

or, $\cos 2\theta = \cos 90°, \cos (360° - 90°), \cos (360° + 90°) \cos (720° - 90°)$

or, $2\theta = 90°, 270°, 450°, 630°$

$\therefore\ 45°, 135°, 225°, 315°$

Again, $\cos \theta - 1 = 0$

or, $\cos \theta = 1$

or, $\cos \theta = \cos 0°$ or $360°$

Thus, $\theta = 0°, 45°, 135°, 225°, 315°, 360°$ is the solution.

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(f) $\cos\theta + \cos 3\theta = -\cos 5\theta$

⇨ Here, $\cos\theta + \cos 3\theta = -\cos 5\theta$

or, $\cos\theta + \cos 5\theta + \cos 3\theta = 0$

or, $2\cos\left(\frac{\theta + 5\theta}{2}\right)\cos\left(\frac{\theta - 5\theta}{2}\right) + \cos 3\theta = 0$

or, $2\cos 3\theta \cos 2\theta + \cos 3\theta = 0$

or, $\cos 3\theta\,(2\cos 2\theta + 1) = 0$

Either $\cos 3\theta = 0$ ....(i)

or, $2\cos 2\theta + 1 = 0$ ....(ii)

From (i), $\cos 3\theta = 0$

or, $\cos 3\theta = \cos 90°, \cos(360° - 90°), \cos(360° + 90°), \cos(720° - 90°)$
$\cos(720° + 90°), \cos(1080 - 90°), \cos(1080 + 90°)$

or, $3\theta = 90°, 270°, 450°, 630°, 810°, 990°, 1170°$

$\therefore\ \theta = 30°, 90°, 150°, 210°, 270°, 330°, 390°$

Again, from (ii), $2\cos 2\theta = -1$

or, $\cos 2\theta = -\frac{1}{2}$

or, $\cos 2\theta = -\cos 60°$

or, $\cos 2\theta = \cos(180° - 60°), \cos(180° + 60°), \cos(540° - 60°), \cos(540° + 60°)$

or, $2\theta = 120°, 240°, 480°, 600°$

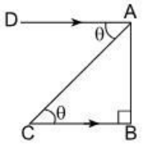
$\therefore\ \theta = 60°, 120°, 240°, 300°$

Thus, $\theta = 30°, 60°, 90°, 120°, 150°, 210°, 240°, 270°, 300°, 330°$ is the solution.

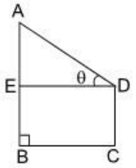
5. उचाइ र दूरी
(Height and Distance)

Key Points

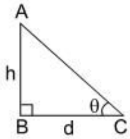
1. सँगैको चित्रमा (In the adjoining figure),
   उन्नतांशकोण = अवनतिकोण हुन्छ ।
   Angle of elevation = Angle of Depression.
   ($\measuredangle ACB = \measuredangle DAC = \theta$)



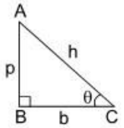
2. सँगैको चित्रमा (In the adjoining figure),
   (i) ED = BC
   (ii) AE = AB – EB = AB – CD



3. सँगैको चित्रमा (In the adjoining figure);
   (i) $h = d\tan\theta$
   (ii) $d = h\cot\theta$



4. सँगैको चित्रमा (In the adjoining figure);
   (i) $p = b\tan\theta$
   (ii) $b = \frac{p}{\tan\theta} = p\cot\theta$
   (iii) $h = \frac{p}{\sin\theta}$ or $\frac{b}{\cos\theta}$
   (iv) $p = h\sin\theta$ and $b = h\cos\theta$



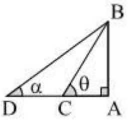
QUESTIONS FROM SEE EXERCISE 5

A. VERY SHORT QUESTIONS

1. उन्नतांश कोणको परिभाषा लेख्नुहोस् । (Define angle of elevation.) **[SEE MODEL 2076]**

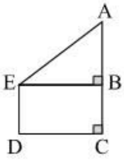
⇨ Here, the angle made by the line of sight with the horizontal line through the position of an observer's eye is called an angle of elevation. In this case the object to be observed lies above the position of the eye.

2. चित्रमा α र θ मध्य कुन ठूलो हुन्छ ?
In the figure, which is greater between α and θ?



⇨ Here, θ is greater than α .

3. दिइएको चित्रबाट DE लाई AC र AB को रूपमा लेख्नुहोस् ।
From the given figure, write DE in terms of AC and AB.



⇨ Here, DE = BC = AC – AB

4. यदि एउटा 'x' m अग्लो रुखले 'x' m लामो छाँया बनाउँदछ भने सूर्यको उचाइ पत्ता लगाउनुहोस् ।
If a 'x' m high tree casts 'x' m long shadow, what is the altitude of the sun?

⇨ Here, height of tree and length of shadow are equal.
So, altitude of sun = 45°

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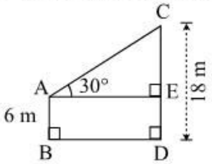
5. 12 m अग्लो खम्बामा 13 m लामो भऱ्याङ अड्याइएको छ । तिनीहरूको फेदको दूरी पत्ता लगाउनुहोस् ।
A 13 m long ladder rests at the top of 12 m high pole. Find the distance between their feet.

⇨ Here, it forms a right angled triangle.
So, $b = \sqrt{h^2 - p^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$
Thus, distance between their feet is 5 m.

B. SHORT QUESTIONS

1. चित्रमा AB = 6 m, CD = 18 m र ∡CAE = 30° छ भने BD को नाप पत्ता लगाउनुहोस् ।
In the figure, AB = 6 m, CD = 18 m and ∡CAE = 30° then find the measure of BD.



⇨ Here, AB = 6 m, CD = 18 m and ∡CAE = 30°
We know that;

$$\tan\theta = \frac{CE}{AE}$$

$$\text{or, } \tan 30° = \frac{12}{AE}$$

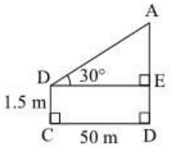
$$\text{or, } \frac{1}{\sqrt{3}} = \frac{12}{BD}$$

$$\text{or, } BD = 12\sqrt{3}\text{ m}$$

$$\therefore\ BD = 20.78\text{ m}$$

Thus, the measure of BD is 20.78 m.

3. चित्रमा CD = 1.5 m, BC = 50 m र ∡ADE = 30° छ भने AB को नाप पत्ता लगाउनुहोस् ।
In the figure, CD = 1.5 m, BC = 50 m and ∡ADE = 30° then find the measure of AB.



⇨ Here, CD = 1.5 m, BC = 50 m and ∡ADE = 30°
In right angled triangle ADE,

$$\tan \measuredangle ADE = \frac{AE}{DE}$$

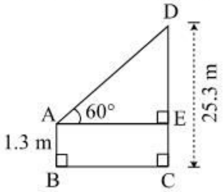
$$\text{or, } \tan 30° = \frac{AE}{BC} \quad [\because DE = CB = 50\text{ m}]$$

$$\therefore\ AE = BC \times \tan 30° = 50 \times \frac{1}{\sqrt{3}} = 28.87\text{ m}$$

$$\therefore\ AB = AE + BE = 28.87 + 1.5 = 30.37\text{ m.}$$

Thus, the measure of AB is 30.37 m.

2. चित्रमा AB = 1.3 m, CD = 25.3 m र ∡CAE = 60° छ भने BD को नाप पत्ता लगाउनुहोस् ।
In the figure, AB = 1.3 m, CD = 25.3 m and ∡CAE = 60° then find the measure of BD.



⇨ Here, let, AB = 1.3 m, CD = 25.3 m and ∡CAE = 60°
Then, CE = CD – AB = 25.3 m – 1.3 m = 24 m

$$\text{Now, } \tan\theta = \frac{p}{b}$$

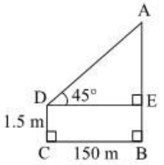
$$\text{or, } \tan 60° = \frac{CE}{AE}$$

$$\text{or, } \sqrt{3} = \frac{24}{AE}$$

$$\therefore\ AE = \frac{24}{\sqrt{3}} = 8\sqrt{3}\text{ m}$$

Thus, the measure of BD is $8\sqrt{3}$ m.

4. चित्रमा CD = 1.5 m, BC = 150 m र ∡ADE = 45° छ भने AB को नाप पत्ता लगाउनुहोस् ।
In the figure, CD = 1.5 m, BC = 150 m and ∡ADE = 45° then find the measure of AB.



⇨ Here, CD = 1.5 m, BC = 150 m and ∡ADE = 45°

From rt. angled ΔADE, $\tan \measuredangle ADE = \dfrac{AE}{DE}$

$$\text{or, } \tan 45° = \frac{AE}{CB} \qquad \text{or, } 1 = \frac{AE}{150\text{ m}}$$

$$\therefore\ AE = 150\text{ m}$$

So, AB = AE + BE = 150 m + CD = 150 m + 1.5 m
= 151.5 m
Thus, the measure of AB is 151.5 m.

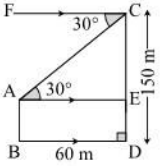
5. चित्रमा CD = 150 m, BD = 60 m र ∡FCA = ∡CAE = 30° छ भने AB को नाप पत्ता लगाउनुहोस् ।
In the figure, CD = 150 m, BD = 60 m and ∡FCA = ∡CAE = 30° then find the measure of AB.

⇨ Here, CD = 150 m, BD = 60 m and ∡FCA = ∡CAE = 30°
In rt. angled triangle AEC,

$$\tan 30° = \frac{CE}{AE} \qquad \text{or, } \frac{1}{\sqrt{3}} = \frac{CE}{60\text{ m}}$$

$$\therefore\ CE = 60 \times \frac{1}{\sqrt{3}} = 34.64\text{ m}$$

So, AB = ED = CD – CE = 150 m – 34.64 m = 115.36 m.
Thus, the measure of AB is 115.36 m.



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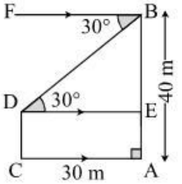
6. चित्रमा AB = 40 m, AC = 30 m र ∡FBD = ∡BDE = 30° छ भने CD को नाप पत्ता लगाउनुहोस् ।
In the figure, AB = 40 m, AC = 30 m and ∡FBD = ∡BDE = 30° then find the measure of CD.



⇨ Here, AB = 40 m, AC = 30 m and ∡FBD = ∡BDE = 30°

∴ ∡BDE = 30°, AB = 40 m and AC = 30 m

In ΔBED,

$$\tan 30^\circ = \frac{BE}{ED}$$

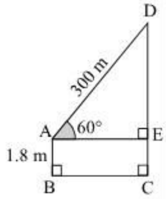
or, $\frac{1}{\sqrt{3}} = \frac{BE}{AC}$ [∵ DE = AC]

or, $BE = \frac{AC}{\sqrt{3}} = \frac{30\text{ m}}{\sqrt{3}} = 17.3$ m

Now, CD = AB – BE = 40 m – 17.3 m = 22.7 m

Thus, the measure of CD is 22.70 m.

9. चित्रमा AB = 1.8 m, AC = 300 m र ∡DAE = 60° छ भने CD को नाप पत्ता लगाउनुहोस् ।
In the figure, AB = 1.8 m, AC = 300 m and ∡DAE = 60° then find the measure of CD.



⇨ Here, AB = 1.8 m, AC = 300 m and ∡DAE = 60°.

Then, we have, $\sin\theta = \frac{p}{h}$

or, $\sin 60^\circ = \frac{DE}{AD}$

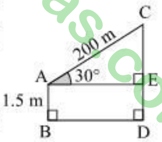
or, $\frac{\sqrt{3}}{2} = \frac{DE}{300}$

∴ $DE = 150\sqrt{3}$ m

Now, $CD = DE + CE = 150\sqrt{3}$ m + 1.8 m = 261.60 m

Thus, the measure of CD is 261.50 m.

7. चित्रमा AB = 1.5 m, AC = 200 m र ∡CAE = 30° छ भने CD को नाप पत्ता लगाउनुहोस् ।
In the figure, AB = 1.5 m, AC = 200 m and ∡CAE = 30° then find the measure of CD.



⇨ Here, AB = 1.5 m, AC = 200 m and ∡CAE = 30°.

From the figure, $\sin\theta = \frac{p}{h}$

or, $\sin 30^\circ = \frac{CE}{AC}$

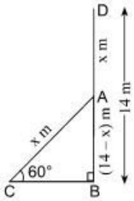
or, $\frac{1}{2} = \frac{CE}{200}$

∴ CE = 100 m

Now, CD = CE + ED = 100 m + 1.5 m = 101.50 m

Thus, the measure of CD is 101.50 m.

8. चित्रमा BD = 14 m, AC = AD = x m, ∡ACB = 60° र ∡ABC = 90° छ भने AC को नाप पत्ता लगाउनुहोस् ।
In the figure, BD = 14 m, AC = AD = x m, ∡ACB = 60° and ∡ABC = 90° then find the measure of AC.



⇨ Here, BD = 14 m, AC = AD = x m, ∡ACB = 60° and ∡ABC = 90°

If AC = x then AB = (14 – x) m.

In rt. angled Δ ACB, $\sin\measuredangle ACB = \frac{AB}{AC}$

or, $\sin 60^\circ = \frac{14 - x}{x}$

or, $\frac{\sqrt{3}}{2} = \frac{14 - x}{x}$

or, $\sqrt{3}x = 28 - 2x$

or, $\sqrt{3}x + 2x = 28$

or, $x\left(\sqrt{3} + 2\right) = 28$

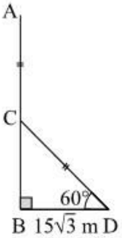
or, $x \times (1.73 + 2) = 28$

or, $x \times 3.73 = 28$

∴ $x = \frac{28}{3.73} = 7.5$ m.

Thus, the measure of AC is 7.50 m.

10. चित्रमा BD = 15$\sqrt{3}$ m, AC = CD, ∡CDB = 60° र ∡ABD = 90° छ भने AB को नाप पत्ता लगाउनुहोस् ।
In the figure, BD = 15$\sqrt{3}$ m, AC = CD, ∡CDB = 60° and ∡ABD = 90° then find the measure of AB.



⇨ Here, BD = 15$\sqrt{3}$ m, AC = CD, ∡CDB = 60° and ∡ABD = 90°

From right angled triangle,

$$\cos\measuredangle D = \frac{BD}{CD}$$

or, $\cos 60^\circ = \frac{15\sqrt{3}}{CD}$

or, $\frac{1}{2} = \frac{15\sqrt{3}}{CD}$

∴ $CD = 30\sqrt{3}$ m

Again, $\tan\theta = \frac{p}{b}$

or, $\tan 60^\circ = \frac{BC}{BD}$

or, $\sqrt{3} = \frac{BC}{15\sqrt{3}}$

∴ BC = 45 m

Now, AB = AC + BC = CD + BC

$= 30\sqrt{3} + 45 = 96.96$ m

Thus, the measure of AB is 96.96 m.

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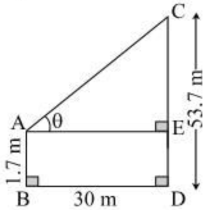
11. चित्रमा AB = 1.7 m, CD = 53.6 m, AE = BD = 30 m र ∡AEC = 90° छ भने ∡CAE को नाप पत्ता लगाउनुहोस् ।

In the figure, AB = 1.7 m, CD = 53.6 m, AE = BD = 30 m and ∡AEC = 90° then find the measure of ∡CAE.



⇨ Here, let, AB = 1.7 m, CD = 53.6 m, AE = BD = 30 m and ∡CAE = θ

From the figure alongside,

$$\tan\theta = \frac{p}{b} = \frac{CE}{AE}$$

or, $\tan\theta = \frac{CD - ED}{30} = \frac{53.6 - 1.7}{30} = \frac{51.9}{30}$

or, $\tan\theta = 1.73$

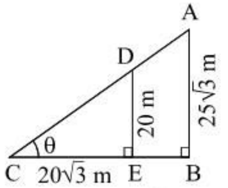
or, $\tan\theta = \tan 60°$

$\therefore\ \theta = 60°$

Thus, the measure of ∡CAE is 60°.

13. चित्रमा DE = 20 m, CE = 20$\sqrt{3}$ m, AB = 25$\sqrt{3}$ m र ∡DEC = ∡ABC = 90° छ भने BC को नाप पत्ता लगाउनुहोस् ।

In the figure, DE = 20 m, CE = 20$\sqrt{3}$ m, AB = 25$\sqrt{3}$ m and ∡DEC = ∡ABC = 90° then find the measure of BC.



⇨ Here, DE = 20 m, CE = 20$\sqrt{3}$ m, AB = 25$\sqrt{3}$ m and ∡DEC = ∡ABC = 90°

Let, ∡DCE = θ

Now, From right angled ΔDEC,

$$\tan\theta = \frac{DE}{CE} = \frac{20m}{20\sqrt{3}m} = \frac{1}{\sqrt{3}}$$

or, $\tan\theta = \tan 30°$

$\therefore\ \theta = 30°$

Again, from right angled ΔABC,

$$\tan\theta = \frac{AB}{CB}$$

or, $\tan 30° = \frac{25\sqrt{3}\text{ m}}{CB}$

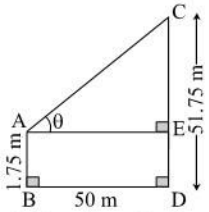
or, $\frac{1}{\sqrt{3}} = \frac{25\sqrt{3}\text{ m}}{CB}$

$\therefore\ CB = 75$ m.

Thus, the measure of BC is 75 m.

12. चित्रमा AB = 1.75 m, CD = 51.75 m, AE = BD = 50 m र ∡AEC = 90° छ भने ∡CAE को नाप पत्ता लगाउनुहोस् ।

In the figure, AB = 1.75 m, CD = 51.75 m, AE = BD = 50 m and ∡AEC = 90° then find the measure of ∡CAE.



⇨ Here, AB = 1.75 m, CD = 51.75 m, AE = BD = 50 m and ∡AEC = 90°.

Here, CE = (CD – ED) = (CD – AB) = (51.75 – 1.75) = 50 m

In the right angled ΔAED,

$\tan \measuredangle CAE = \frac{CE}{AE} = \frac{50\text{ m}}{50\text{ m}} = 1 = \tan 45°$ [∵ AE = DB = 50 m]

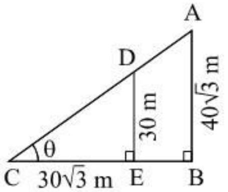
or, $\tan \measuredangle CAE = \tan 45°$

$\therefore\ \measuredangle CAE = 45°$

Thus, the measure of ∡CAE is 45°.

14. चित्रमा DE = 30 m, CE = 30$\sqrt{3}$ m, AB = 40$\sqrt{3}$ m र ∡DEC = ∡ABC = 90° छ भने BC को नाप पत्ता लगाउनुहोस् ।

In the figure, DE = 30 m, CE = 30$\sqrt{3}$ m, AB = 40$\sqrt{3}$ m and ∡DEC = ∡ABC = 90° then find the measure of BC.



⇨ Here, DE = 30 m, CE = 30$\sqrt{3}$ m, AB = 40$\sqrt{3}$ m and ∡DEC = ∡ABC = 90°

Let, ∡DCE = θ

Now, from right angled ΔDEC,

$$\tan\theta = \frac{DE}{CE} = \frac{30\text{ m}}{30\sqrt{3}\text{ m}} = \frac{1}{\sqrt{3}}$$

or, $\tan\theta = \tan 30°$

$\therefore\ \theta = 30°$

Again, from right angled ΔABC,

$$\tan\theta = \frac{AB}{CB}$$

or, $\tan 30° = \frac{40\sqrt{3}\text{ m}}{CB}$

or, $\frac{1}{\sqrt{3}} = \frac{40\sqrt{3}\text{ m}}{CB}$

$\therefore\ CB = 120$ m.

Thus, the measure of BC is 120 m.

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C. LONG QUESTIONS

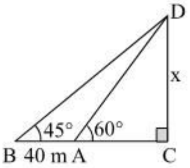
MODEL 1

1. **जमिनमा रहेको कुनै एउटा बिन्दुबाट एउटा रूखको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइयो । सो बिन्दुबाट 40 मिटर अभ्क पर गएर हेर्दा उन्नतांश कोण 45° पाइयो भने सो रूखको उचाइ पत्ता लगाउनुहोस् ।**

   The angle of elevation of the top of a tree as observed from a point on the ground is found to be 60°. On walking 40 meter away from the point, the angle of the elevation was found to be 45°. Find the height of the tree. [2074 S']

⇨ Here, let CD = x m be the height of tree.

Let $\measuredangle DBC = 45°$ and $\measuredangle DAC = 60°$ are the angles of elevation. Let AB = 40 m be the distance between two points of obsevrations.



From right angled ΔDAC,

$$\tan 60° = \frac{CD}{AC}$$

or, $\sqrt{3} = \frac{x}{AC}$

$\therefore$ $AC = \frac{x}{\sqrt{3}}$

From right angled ΔDBC,

$$\tan 45° = \frac{CD}{BC}$$

or, $1 = \frac{x}{40 + AC}$

or, $40 + AC = x$

or, $40 + \frac{x}{\sqrt{3}} = x$

or, $40 = x - \frac{x}{\sqrt{3}}$

or, $40 = x\left(1 - \frac{1}{\sqrt{3}}\right)$

$\therefore$ $x = 94.64$ m

Thus, the height of the tree is 94.64 m.

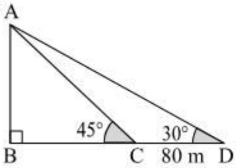
2. **कुनै एउटा बिन्दुबाट एक स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो र सो बिन्दुबाट 80 मिटर टाढा जाँदा सो कोण 30° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

   The angles of elevation of the top of a tower from a point was observed to be 45° and walking 80 m away from that point, it was found to be 30°. Find the height of the tower. [2074 R']

⇨ Here, let AB be the height of a tower. Let C and D be the points of observer and CD = 80 m.

Let $\measuredangle ADB = 30°$ and $\measuredangle ACB = 45°$ be the angles of elevation of the top of tower.

From right angled ΔABC,



$$\tan \theta = \frac{AB}{BC}$$

or, $\tan 45° = \frac{AB}{BC}$

or, $1 = \frac{AB}{BC}$

$\therefore$ AB = BC

From right angled ΔADB,

$$\tan \theta = \frac{AB}{DB}$$

or, $\tan 30° = \frac{AB}{DC + BC}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{80 + AB}$

or, $\sqrt{3}\,AB = 80 + AB$

or, $AB(\sqrt{3} - 1) = 80$

or, $AB = \frac{80}{\sqrt{3} - 1} = 109.28$

Thus, the height of the tower is 109.28 m

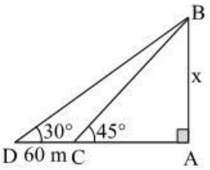
3. **कुनै एउटा बिन्दुबाट स्तम्भको टुप्पोको उन्नतांशकोण 45° अवलोकन गरियो र सो बिन्दुबाट 60 मिटर टाढा जाँदा सो कोण 30° पाइयो । स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

   The angle of elevation of the top of a tower from a point was observed to be 45° and on walking 60 meter away from that point it was found to be 30°. Find the height of the tower. [2071 R']

⇨ Here, let AB = x m be the height of a tower.

Let $\measuredangle ADB = 30°$ and $\measuredangle ACB = 45°$ are the angles of elevation.

Let CD = 60 m be the distance between two points C & D.



From the right angled ΔABC,

$$\tan \theta = \frac{p}{b} = \frac{AB}{AC}$$

or, $\tan 45° = \frac{x}{AC}$

or, $1 = \frac{x}{AC}$

$\therefore$ AC = x m

Again, from right angled ΔABD,

$$\tan 30° = \frac{p}{b} = \frac{AB}{AD}$$

or, $\frac{1}{\sqrt{3}} = \frac{x}{CD + AC}$

or, $\sqrt{3}\,x = CD + AC$

or, $\sqrt{3}\,x = 60 + x$

or, $x(\sqrt{3} - 1) = 60$

$\therefore$ $x = \frac{60}{\sqrt{3} - 1} = 81.96$

Thus, the height of the tower is 81.96 m.

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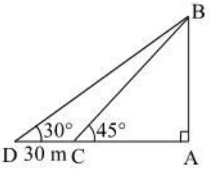
4. **कुनै एउटा बिन्दुबाट स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो र सो बिन्दुबाट 30 मिटर टाढा जाँदा सो कोण 30° पाइयो । यदि स्तम्भ र ती दुई बिन्दुहरू एकै रेखामा भए सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of the top of a tower from a point was observed to be 45°. On walking 30 m away from that point it was found to be 30°. Find the height of the tower. [2057 R]

⇨ Here, In the fig, let AB be the height of the tower. C and D are the two points lying on the same line. Now the angle of elevations from the two points C and D on the top of the tower are $\measuredangle ACB = 45°$ and $\measuredangle ADB = 30°$ respectively.



The distance between C & D = CD = 30 m.

The height of the tower, AB = ?

Now from the right angled ΔACB,

$$\tan 45° = \frac{AB}{AC}$$

or, $1 = \frac{AB}{AC}$

$\therefore$ AC = AB .... (i)

Again from rt. angled Δ ADB, $\tan 30° = \frac{AB}{AD}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{AC + CD}$

or, $AC + CD = AB\sqrt{3}$

or, $30 + AB = AB\sqrt{3}$ ($\because$ AC = AB)

or, $30 = AB\sqrt{3} - AB$

or, $30 = AB(\sqrt{3} - 1)$

or, $30 = AB(1.732 - 1)$

or, $30 = AB \times 0.732$

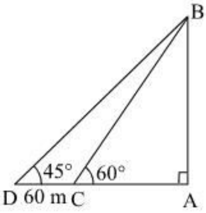
or, $AB = \frac{30}{0.732} = 40.98$ m.

Thus, the height of the tower (AB) = 40.98 m.

5. **कुनै मैदानमा रहेको एउटा बिन्दुबाट स्तम्भको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइएछ । सो बिन्दुबाट 60 मिटर अझ पर गएर हेर्दा उन्नतांश कोण 45° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of the top of a tower as observed from a point on the ground is found to be 60°. On walking 60 m away from the point, the angle of the elevation was found to be 45°. Find the height of the tower. [2073 S', 2060 S]

⇨ Here, let AB be a tower. Let C be any point on the same level from which an angle of elevation of the top of the tower is $\measuredangle ACB = 60°$. Also the angle of elevation of the point B observed from the point D which is 60 m from the point C is $\measuredangle ADB = 45°$.



Here CD = 60 m.

Now, from rt. angled ΔABC, $\tan 60° = \frac{AB}{AC}$

or, $\sqrt{3} = \frac{AB}{AC}$ $\therefore$ $AC = \frac{AB}{\sqrt{3}}$ ...... (i)

Again from rt. angled ΔABD, $\tan 45° = \frac{AB}{AD}$

or, $1 = \frac{AB}{AC + CD} = \frac{AB}{AC + 60}$

$\therefore$ AC+ 60 = AB .... (ii)

From equation (i) and (ii),

$\frac{AB}{\sqrt{3}} + 60 = AB$

or, $AB + 60\sqrt{3} = AB\sqrt{3}$

or, $60\sqrt{3} = AB\sqrt{3} - AB$

or, $60\sqrt{3} = AB(\sqrt{3} - 1)$

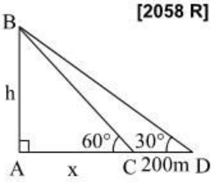
or, $AB = \frac{60\sqrt{3}}{\sqrt{3} - 1} = \frac{60 \times 1.732}{1.732 - 1} = \frac{103.920}{0.732} = 141.96$ m.

Thus, height of the tower (AB) = 141.96 m.

6. **कुनै बिन्दुबाट धरहराको टुप्पाको उन्नतांश कोण 60° अवलोकन गरियो र सो बिन्दुबाट 200 मिटर टाढा जाँदा सो कोण 30° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of a tower was observed to be 60° from a point. On walking 200 m away from the point it was found to be 30°. Find the height of the tower. [2058 R]

⇨ Here, let, AB be the height of a tower. The angle of elevation observed from a point C to the top of the tower B is $\measuredangle ACB = 60°$ and from the point D is $\measuredangle ADB = 30°$, where DC = 200 m.



Now, from rt. angled ΔABC

$$\tan 60° = \frac{AB}{AC}$$

or, $\sqrt{3} = \frac{AB}{AC}$

$\therefore$ $AC = \frac{AB}{\sqrt{3}}$ .... (i)

Again, from rt. angled ΔABD

$$\tan 30° = \frac{AB}{AD}$$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{AC+DC}$

or, $AC + 200 = AB\sqrt{3}$

or, $\frac{AB}{\sqrt{3}} + 200 = AB\sqrt{3}$

or, $AB + 200\sqrt{3} = AB.3$

or, $200\sqrt{3} = 3AB - AB$

or, $2\,AB = 200\sqrt{3}$

or, $AB = \frac{200\sqrt{3}}{2}$

or, $AB = 100 \times 1.732 = 173.2$

Thus, the height of the tower is AB = 173.2 m.

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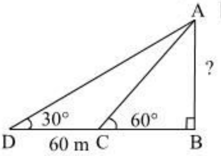
7. **कुनै स्थानबाट एउटा घरको टुप्पोमा हेर्दा उन्नतांश कोण 60° अवलोकन गरियो र सोही स्थानबाट 60 m टाढा गएर अवलोकन गर्दा उन्नतांशकोण 30° को पाइयो । यदि घर र ती दुई स्थानहरू एउटै सतहको रेखामा भएका रहेछन् भने घरको उचाइ कति हुन्छ होला ?**
The angle of elevation of the top of a house from a point on the ground was observed to be 60°. On walking 60 m away from that point it was found to be 30°. If house and these points are in the same line of the same plain, find the height of the house. **[2066 S]**



⇨ Here, let, AB be the height of a house, $\measuredangle ADB = 30°$ and $\measuredangle ACB = 60°$ are the angles of elevation of the top of the house. CD = 60 m be the distance between point of observation.

From right angled triangle ABC

$$\tan\theta = \frac{AB}{BC}$$

or, $\tan 60° = \frac{AB}{BC}$

or, $\sqrt{3} = \frac{AB}{BC}$

$\therefore$ $AB = \sqrt{3}\,BC$

From right angled triangle ADB,

$$\tan\theta = \frac{AB}{BD}$$

or, $\tan 30° = \frac{\sqrt{3}\,BC}{BC + 60}$

or, $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,BC}{BC + 60}$

or, $BC + 60 = 3BC$

or, $2BC = 60$

$\therefore$ $BC = 30$

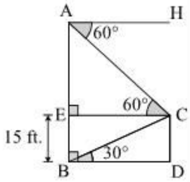
Now, height of house $= AB = \sqrt{3}BC = 30\sqrt{3}$ m

Thus, height of the house is $30\sqrt{3}$ m.

MODEL 2

8. **एउटा घरको छत र फेदबाट एउटा रुखको टुप्पोको अवनति कोण र उन्नतांश कोणहरू क्रमशः 60° र 30° छन् । यदि रुखको उचाइ 15 फिट भए घरको उचाइ पत्ता लगाउनुहोस् ।**
From the roof and foot of a house, the angles of depression and elevation of the top of a tree are 60° and 30° respectively. If the height of the tree is 15 ft, find the height of the house. **[2075 R]**

⇨ Here, let AB be the height of house and EB = CD = 15 ft be the height of tree where the angle of depression i.e. $\measuredangle HAC = \measuredangle ACE = 60°$ and angle of elevation is $\measuredangle DBC = 30°$.



From right angled ΔBDC,

$\tan 30° = \frac{DC}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{15}{BD} \Rightarrow BD = 15\sqrt{3}$ ft

Again, from right angled ΔAEC,

$\tan 60° = \frac{AE}{EC}$

or, $\sqrt{3} = \frac{AE}{BD} = \frac{AE}{15\sqrt{3}}$

$\Rightarrow AE = 15\sqrt{3} \times \sqrt{3} = 45$ ft

Now, height of house AB

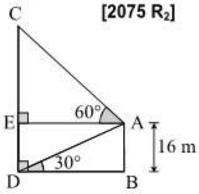
= AE + EB

= 45 ft + 15 ft

= 60 ft

Thus, height of the house is 60 ft.

9. **16 मिटर अग्लो एउटा भवनको छतबाट ठीक अगाडि रहेको एउटा बत्तीको खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा उन्नतांश कोण र अवनति कोण क्रमशः 60° र 30° पाइयो भने सो खम्बाको उचाइ पत्ता लगाउनुहोस् ।**
From the roof of a building 16 meter high, the angles of elevation and depression of the top and the foot of an electric pole are observed to be 60° and 30° respectively. Find the height of the pole. **[2075 R$_2$]**



⇨ Here, CD is the pole whose height is to be determined. AB = ED = 16 m is the height of building, $\measuredangle CAE = 60°$ is the angle of elevation and $\measuredangle ADB = 30°$ is the angle of depression.

AE = BD is the distance between the building and pole.

From right angled triangle ADB,

$\tan 30° = \frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{16}{BD}$

$\therefore$ $BD = 16\sqrt{3}$ m

Again from right angled ΔACE,

$\tan 60° = \frac{CE}{AE}$

or, $\sqrt{3} = \frac{CE}{BD}$

or, $\sqrt{3} \times 16\sqrt{3} = CE$

$\therefore$ CE = 48 m

Now, height of pole = CD = CE + ED

= CE + AB

= 48 m + 16 m

= 64 m

Thus, the height of the pole is 64 m.

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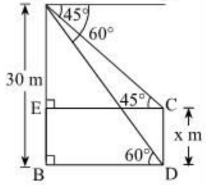
10. 30 मिटर अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदको अवनति कोण क्रमशः 45° र 60° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

From the top of 30 m high cliff, the angles of depression of the top and the foot of a tower are observed to be 45° and 60° respectively. Find the height of the tower. [2074 S]



⇨ Here, let AB = 30 m be the height of cliff. Let CD = x m be the height of tower.

$\measuredangle FAC = 45°$ and $\measuredangle FAD = 60°$ are the angles of depressions.

From the right angled triangle ABD,

$\tan 60° = \frac{AB}{BD}$

or, $\sqrt{3} = \frac{30}{BD}$

or, $BD = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

Again, from right angled ΔAEC,

$\tan 45° = \frac{AE}{EC}$

or, $1 = \frac{AE}{EC}$

or, AE = EC

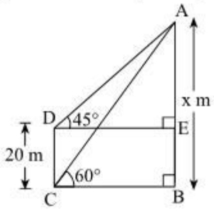
or, $AE = BD = 10\sqrt{3}$ m.

Now, $CD = EB = AB - AE = 30 \text{ m} - 10\sqrt{3}$ m
$= 12.68$ m

Thus, the height of the tower is 12.68 m.

12. 20 m अग्लो एउटा घरको छत र फेदबाट एउटा स्तम्भको टुप्पोको उन्नतांश कोणहरू क्रमशः 45° र 60° छन् भने उक्त स्तम्भको उचाइ पत्ता लगाउनुहोस् ।

From the roof and basement of a house 20 m high, the angles of elevation of the top of a tower are 45° and 60° respectively. Find the height of the tower.[2071 R]



⇨ Here, let CD = 20 m be the height of a house and AB = x m be the height of the tower.

Let $\measuredangle ADE = 45°$ and $\measuredangle ACB = 60°$ are the angles of elevation of the top of tower from top and bottom of the house respectively.

From right angled ΔAED, $\tan 45° = \frac{AE}{DE}$

or, $1 = \frac{AE}{DE}$

or, DE = AE

or, $BC = (x - 20)$

Again, from right angled ΔABC, $\tan 60° = \frac{AB}{BC}$

or, $\sqrt{3} = \frac{x}{x - 20}$

or, $\sqrt{3}x - 20\sqrt{3} = x$

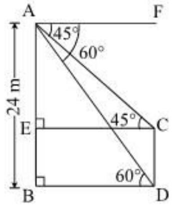
or, $x(\sqrt{3} - 1) = 20\sqrt{3}$

$\therefore \quad x = \frac{20\sqrt{3}}{\sqrt{3} - 1} = 47.32$

Thus, the height of the tower is 47.32 m.

11. 24 मिटर अग्लो एउटा स्तम्भको टुप्पोबाट एउटा खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा अवनति कोणहरू क्रमशः 45° र 60° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।

From the top of a tower 24 meter high, the angles of depression of the top and the foot of a pole are observed to be 45° and 60° respectively. Find the height of the pole. [2074 R]



⇨ Here, let AB = 24 m be the height of a tower and CD be the height of a pole.

Let $\measuredangle FAC = 45°$ and $\measuredangle FAD = 60°$ be the angles of depression.

We have,

$\measuredangle FAC = \measuredangle ACE = 45°$ and $\measuredangle FAD = \measuredangle ADB = 60°$

From right angled ΔAEC, $\tan\theta = \frac{AE}{EC}$

or, $\tan 45° = \frac{AE}{EC}$

or, $1 = \frac{AE}{EC}$

$\therefore$ AE = EC

From right angled ΔABD, $\tan\theta = \frac{AB}{BD}$

or, $\tan 60° = \frac{24}{BD}$ or, $\sqrt{3} = \frac{24}{BD}$

or, $BD = \frac{24}{\sqrt{3}} = 8\sqrt{3}$

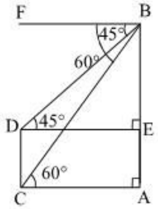
Now, $CD = EB = AB - AE = 24 - EC = 24 - BD$
$= 24 - 8\sqrt{3}$

$\therefore$ CD = 10.14 m

Thus, the height of the pole is 10.14 m.

13. 21 मिटर अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदको अवनति कोण क्रमशः 45° र 60° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

From the top of 21 m. high cliff, the angles of depression of the top and the bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower. [2071 S, 2057 S]



⇨ Here, let, AB be a high cliff where height of cliff AB = 21 m. CD be a tower. The angle of depressions from the top of the cliff to the top and bottom of the tower are $\measuredangle FBD = 45°$ and $\measuredangle FBC = 60°$.

Here, AC, ED and BF are parallel so,

$\measuredangle FBD = \measuredangle BDE = 45°$ and $\measuredangle FBC = \measuredangle BCA = 60°$

Now, from rt. angled ΔDEB; $\tan 45° = \frac{BE}{ED}$

or, $1 = \frac{BE}{ED}$ or, BE = ED .... (i)

Again, from rt. angled ΔBAC; $\tan 60° = \frac{AB}{AC}$

or, $\sqrt{3} = \frac{21}{AC}$ or, $AC = \frac{21}{\sqrt{3}}$

But from (i) BE = ED.

So, $BE = AC = \frac{21}{\sqrt{3}} = \frac{21}{1.732} = 12.12$

Thus, the height of the tower (CD) = AE
= AB – BE = 21 – 12.12 = 8.88 m.

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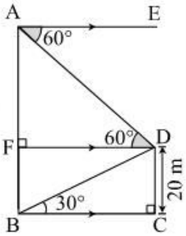
14. एउटा धरहराको टुप्पो र फेदबाट त्यसको ठीक अगाडि रहेको 20 मिटर अग्लो एउटा मन्दिरको गजुरमा हेर्दा अवनति कोण र उन्नतांश कोण क्रमशः 60° र 30° पाइएछन् भने सो धरहराको उचाइ पत्ता लगाउनुहोस् ।

The angles of depression and elevation of the pinnacle of a temple 20 meter high are found to be 60° and 30° from the top and foot of a tower respectively. Find the height of the tower. [2070 R']



⇨ Here, let AB the height of a tower and CD = 20 m be the height of temple.

Let $\measuredangle EAD = 60°$ and $\measuredangle DBC = 30°$ be the angle of depression and angle of elevation of the pinnacle of the temple from the top of the tower and bottom of the tower.

From the figure,

$\measuredangle EAD = \measuredangle ADF = 60°$

From right angled triangle BCD,

$$\tan\theta = \frac{p}{b} = \frac{CD}{BC}$$

or, $\tan 30° = \dfrac{20}{BC}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{20}{BC}$

$\therefore$ $BC = 20\sqrt{3}$ m

From the right angled triangle AFD, $\tan\theta = \dfrac{P}{b} = \dfrac{AF}{FD}$

or, $\tan 60° = \dfrac{AF}{BC}$

or, $\sqrt{3} = \dfrac{AF}{20\sqrt{3}}$

$\therefore$ $AF = 60$

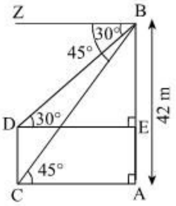
Now, $AB = AF + BF = 60 + 20$

$\therefore$ $AB = 80$ m

Thus, the height of tower is 80 m.

15. 42 मि. अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदका अवनति कोणहरू क्रमशः 30° र 45° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

From the top of 42 m high cliff, the angle of depression of the top and bottom of a tower are observed to be 30° and 45° respectively. Find the height of the tower.[2061S]



⇨ Here, let, AB be a high cliff where height of the cliff AB = 42 m. Let CD be a tower. The angles of depression observed from the top B of the cliff to the top D and bottom C of the tower are $\measuredangle ZBD = 30°$ and $\measuredangle ZBC = 45°$ respectively.

Here from the figure, since CA parallel to DE and BZ, so $\measuredangle ZBD = \measuredangle BDE = 30°$ and $\measuredangle ZBC = \measuredangle BCA = 45°$.

Now from rt. angle ΔABC, $\tan 45° = \dfrac{AB}{AC}$

or, $1 = \dfrac{AB}{AC}$

$\therefore$ $AC = AB = 42$ m ...... (i)

Again from rt. angled ΔBDE, $\tan 30° = \dfrac{BE}{ED}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{AB - AE}{AC}$ [$\because$ BE = AB – AE & AC = ED]

or, $\dfrac{1}{\sqrt{3}} = \dfrac{42 - AE}{42}$

or, $\dfrac{42}{\sqrt{3}} = 42 - CD$ [$\because$ AE = CD]

or, $CD = 42 - \dfrac{42}{\sqrt{3}} = 42\left(1 - \dfrac{1}{\sqrt{3}}\right)$

$= 42\left(\dfrac{\sqrt{3}-1}{\sqrt{3}}\right) = 17.750577$

Thus, height of the tower (CD) = 17.75 m.

16. एउटा 25 मिटर अग्लो खम्बाको टुप्पोको अवनति कोण र उन्नतांश कोण एउटा स्तम्भको टुप्पो र फेदबाट क्रमशः 60° र 30° छन् भने सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।

The angle of depression and elevation of the top of a pole 25 m high from the top and bottom of a tower are 60° and 30° respectively. Find the height of the tower. [2065 S, 2064 R]

⇨ Here, let, AB = 25 m be the height of a pole and CD be the tower. Let $\measuredangle ACB = 30°$ and $\measuredangle EDA = 60°$ are the angle of elevation and the angle of depression.

From the figure, FC = AB = 25 m, $\measuredangle DAF = \measuredangle EDA = 60°$

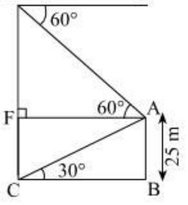
From the right angled ΔABC,

$\tan\theta = \dfrac{p}{b}$ i.e. $\tan 30° = \dfrac{AB}{BC}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{25}{BC}$

$\therefore$ $BC = 25\sqrt{3}$ m

$\therefore$ $AF = BC = 25\sqrt{3}$ m



From the right angled ΔDAF,

$\tan\theta = \dfrac{p}{b}$

i.e. $\tan 60° = \dfrac{DF}{AF}$

or, $\sqrt{3} = \dfrac{DF}{25\sqrt{3}}$

$\therefore$ $DF = 75$ m

Now, Height of the tower = CF + DF = 25 m + 75 m = 100 m

Thus, the height of the tower is 100 m.

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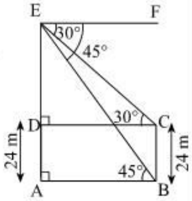
17. **एउटा स्तम्भको टुप्पोबाट 24 मिटर अग्लो घरको टुप्पोको अवनति कोण 30° र घरको फेदबाट स्तम्भको टुप्पोको उन्नतांश कोण 45° छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of depression of the top of building of 24 m height from the top of a tower is 30° and the angle of elevation of the top of the tower from the foot of building is 45°. Find the height of the tower. [2063 R]

⇨ Here, let, BC be a building of 24 m height.

$\therefore$ BC = 24 m

AE be a tower. The angle of depression from top of the tower to the top of the building is $\measuredangle FEC = 30°$ and angle of elevation from the foot of the building to the top of the tower is $\measuredangle ABE = 45°$.



since EF // CD // AB, so $\measuredangle FEC = \measuredangle ECD = 30°$, $\measuredangle FEB = \measuredangle EBA = 45°$, CB = AD = 24 m

From rt. angled ΔEAB, $\tan 45° = \frac{AE}{AB}$

or, $1 = \frac{AE}{AB}$

or, AB = AE

or, AB = 24 + DE .... (i) [AE = AD + DE]

Again, from rt. angled ΔEDC, $\tan 30° = \frac{ED}{DC}$

or, $\frac{1}{\sqrt{3}} = \frac{ED}{DC}$

or, $DC = \sqrt{3}\ ED$

or, $AB = \sqrt{3}\ ED$

or, $24 + ED = \sqrt{3}\ ED$

or, $24 = \sqrt{3}\ ED - ED$

or, $24 = ED(\sqrt{3} - 1)$

or, $ED = \frac{24}{\sqrt{3} - 1} = \frac{24}{1.732 - 1} = \frac{24}{0.732}$

or, ED = 32.78 m

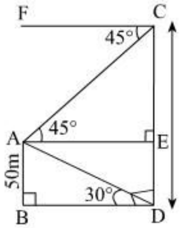
$\therefore$ AE = AD + ED = (24 + 32.78) m = 56.78 m

Thus, the required height of the tower is 56.78 m.

18. **50 मिटर अग्लो एउटा टावरको टुप्पोमा दोस्रो टावरको टुप्पो र फेदबाट हेर्दा बनेका अवनति र उन्नतांश कोणहरू क्रमश: 45° र 30° छन् । दोस्रो टावरको उचाइ पत्ता लगाउनुहोस् ।**

The angles of depression and elevation of the top of a tower 50 meter high from the top and bottom of a second tower are 45° and 30° respectively. Find the height of the second tower. [2066 R']

⇨ Here, let, AB = 50 m be a tower and CD be the next tower. Let, $\measuredangle FCA = 45°$ be the angle of depression and $\measuredangle ADB = 30°$ be the angle of elevation.



We know that,

In right angled ΔBAD,

$\tan \theta = \frac{p}{b}$

or, $\tan 30° = \frac{50}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{50}{BD}$

$\therefore$ $BD = 50\sqrt{3}$ m

And $\measuredangle FCA = \measuredangle CAE = 45°$

[$\because$ Being alternate angles in // lines]

In right angled ΔCAE,

$\tan \theta = \frac{p}{b}$

or, $\tan 45° = \frac{CE}{AE}$

or, $1 = \frac{CE}{BD}$ [$\because$ AE = BD]

or, $1 = \frac{CE}{50\sqrt{3}}$

$\therefore$ $CE = 50\sqrt{3}$ m

Now, the height of the second tower

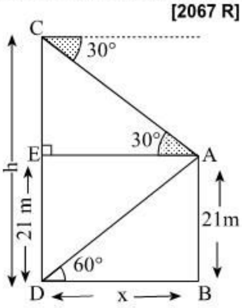
$= CE + ED = 50\sqrt{3} + 50 = 136.60$ m

Thus, the height of second tower is 136.60 m.

19. **एउटा घरको अगाडिको एउटा रूखको टुप्पोमा एउटा चरा बसिरहेको छ । सो घरको आधार सतह र छतबाट चरालाई हेर्दा बन्ने उन्नतांश कोण र अवनति कोण क्रमश: 60° र 30° छन् । यदि रूखको उचाइ 21 मि. भए घरको उचाइ पत्ता लगाउनुहोस् ।**

A bird is sitting on the top of a tree which is in front of a house. The angle of elevation and angle of depression of the bird from the bottom and top of the house are 60° and 30° respectively. If the height of the tree is 21 m, find the height of the house. [2067 R]

⇨ Here, AB = 21 m be height of a tree, CD = h be a height of a house BD = AE = x be a distance between tree & house. CE = (h – 21) where $\measuredangle ADB = 60°$, EAC = 30°.



Now, from right angled ΔABD

$\tan 60° = \frac{21}{x}$

or, $\sqrt{3} = \frac{21}{x}$

or, $x = \frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} = 7\sqrt{3}$ m

From right angled ΔCEA, $\tan 30° = \frac{h - 21}{x}$

or, $\frac{1}{\sqrt{3}} = \frac{h - 21}{7\sqrt{3}}$

or, 7 = h – 21

or, 7 + 21 = h

or, h = 28 m

Thus, the required height of a house is 28 m.

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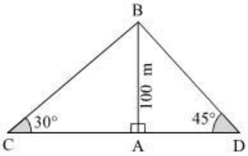
MODEL 3

20. **दुईजना मानिसहरू 100 मिटर अग्लो स्तम्भको विपरित दिशातिर छन् । यदि तिनीहरूले स्तम्भको टुप्पोलाई अवलोकन गर्दा उन्नतांश कोणहरू क्रमश: 30° र 45° पाएछन् भने ती मानिसहरू बिचको दूरी पत्ता लगाउनुहोस् ।**

Two men are on the opposite direction of a tower 100 m high. If they observed the top of the tower, the angles of elevation are found to be 30° and 45° respectively. Find the distance between the men.



⇨ Here, from the figure, height of the tower (AB) = 100 m

Two men are at C and D where the angles of elevation of the top of tower are ∡ACB = 30° and ∡ADB = 45° respectively.

CD = Distance between two men = ?

Now, from rt. ∡ed ΔACB,

$$\tan 30° = \frac{AB}{AC}$$

or, $\frac{1}{\sqrt{3}} = \frac{100}{AC}$

∴ $AC = 100\sqrt{3}$ m.

Again, from rt. ∡ed ΔADB,

$$\tan 45° = \frac{AB}{AD}$$

or, $1 = \frac{100}{AD}$

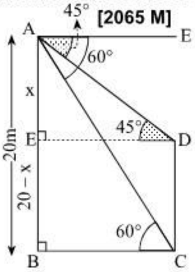
∴ AD = 100 m

Now, $CD = AC + AD = (100\sqrt{3} + 100)$ m
$= 273.20$ m

Thus, the distance between two men is 273.20 m.

21. **20 मिटर अग्लो भवनबाट एउटा स्तम्भको टुप्पो र फेद हेर्दा अवनति कोणहरू क्रमश:45° र 60° पाइयो भने स्तम्भको उचाइ र भवन र स्तम्भबीचको दूरी पत्ता लगाउनुहोस् ।**

From the top of a building of 20 m high, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole and the distance between the house and the pole. **[2065 M]**



⇨ Here, let , AB be the building and CD be a pole, ∡FAD = 45° angle of depression of D and ∡FAC = 60° angle of depression of C. AB = 20 m height of the building.

CD = height of the pole.

From the figure,

When AE = x then

BE = 20 – x and BE = CD.

∡FAD = ∡ADE = 45°, ∡FAC = ∡ACB = 60°

From the right angled ΔAED, $\tan\theta = \frac{AE}{ED}$

or, $\tan 45° = \frac{x}{ED}$

or, $1 = \frac{x}{ED}$

∴ ED = x

So, BC = x

From the right angled ΔABC, $\tan 60° = \frac{AB}{BC} = \frac{20}{x}$

or, $\sqrt{3} = \frac{20}{x}$ ∴ $x = \frac{20}{\sqrt{3}} = 11.55$ m

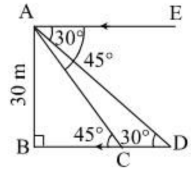
Now, CD = BE = 20 – x = 20 – 11.55 = 8.45 m

Thus, the height of the pole = 8.45 m and distance between the house and the pole = 11.55 m

22. **एउटा 30 m अग्लो भवनबाट एक जना मानिसले भवनको पूर्वपट्टिको समतल मैदानमा बसिरहेको दुई जना मानिसलाई हेर्दा अवनति कोणहरू 45° र 30° बनाउँदछ भने ती दुई मानिसहरूबीच कति दूरी रहेछ ?**

From the top of a building 30 m high a man observes two persons sitting on the ground, both due east on the same line at angles of depression of 45° and 30°. How far apart are the two persons? **[2063 M]**

⇨ Here, In the figure, AB = 30 m (height of the house) C and D are the positions of two persons due east of AB, so that ∡EAC = 45° is the angle of depression of C from A and ∡EAD = 30° is the angle of depression of D from A.



Now, In the triangle ACB; ∡ACB = 45°

And $\tan 45° = \frac{AB}{BC}$

or, $1 = \frac{30\text{ m}}{BC}$

∴ BC = 30 m

And in triangle ABD, ∡ADB = 30°

So, $\tan 30° = \frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{30\text{ m}}{BD}$

∴ $BD = 30\sqrt{3}$ m

Then, $CD = BD - BC = (30\sqrt{3} - 30)$ m
$= 30(\sqrt{3} - 1)$ m
$= 30 \times (1.732 - 1)$ m
$= 30 \times 0.732$ m
$= 21.96$ m

Thus, the distance between two persons = 21.96 m

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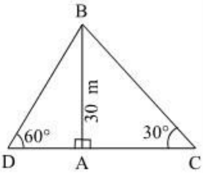
23. दुई जना मानिसहरू कुनै 30 मिटर अग्लो स्तम्भको विपरीत दिशामा छन् । तिनीहरूले सो स्तम्भको उन्नतांश कोणहरू अवलोकन गर्दा 30° र 60° पाएछन् भने तिनीहरूबीचको दूरी पत्ता लगाउनुहोस् ।

Two men are on the opposite side of a tower of 30 m high. They observed the angles of elevation of the top of the tower and found to be 30° and 60°. Find the distance between them. [2059 R]

⇨ Here, from the figure,
Height of the tower (AB) = 30 m
C and D are the two sides of the tower from which angles of elevation of the top of tower are $\measuredangle ADB = 60^\circ$ and $\measuredangle ACB = 30^\circ$.



Distance between two points, CD = ?

Now, from rt. angled ΔACB, $\tan 30^\circ = \frac{AB}{AC}$

or, $\frac{1}{\sqrt{3}} = \frac{30}{AC}$

$\therefore \quad AC = 30\sqrt{3}$

Again, from rt. angled ΔADB

$\tan 60^\circ = \frac{AB}{AD}$

or, $\sqrt{3} = \frac{30}{AD}$

$\therefore \quad AD = \frac{30}{\sqrt{3}}$

Now, distance between two points

$$CD = AC + AD = 30\sqrt{3} + \frac{30}{\sqrt{3}}$$

$$= \frac{30 \times 3 + 30}{\sqrt{3}}$$

$$= \frac{90 + 30}{\sqrt{3}} = \frac{120}{\sqrt{3}}$$

$$= \frac{40 \times 3}{\sqrt{3}}$$

$$= 40 \times \sqrt{3}$$

$$= 40 \times 1.732 = 69.280$$

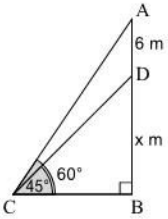
Thus, the required distance (CD) = 69.28 m.

MODEL 4

24. एउटा धरहराको ठीक अगाडि जमिनको सतहमा रहेको एक बिन्दुबाट धरहराको माथि ठड्याइएको 6 m अग्लो ध्वजदण्डको टुप्पो र फेदका उन्नतांश कोणहरू क्रमशः 60° र 45° पाइयो । धरहराको उचाइ र धरहराको फेदबाट सो बिन्दुसम्मको दूरी पत्ता लगाउनुहोस् ।

From a point at the ground level infront of a tower, the angle of elevations of the top and bottom of flagstaff 6 m high situated at the top of a tower are observed 60° and 45° respectively. Find the height of the tower and the distance between the base of the tower and point of observation. [2076 Model]

⇨ Here, let AD = 6 m be the height of flagstaff.
Let BD = x m be the height of the tower
$\measuredangle DCB = 45^\circ$ and $\measuredangle ACB = 60^\circ$ are the angles of the elevations of the bottom and top of the flagstaff respectively.



From the right angled ΔBCD,

$\tan \theta = \frac{p}{b}$

i.e. $\tan 45^\circ = \frac{BD}{BC}$

or, $1 = \frac{x}{BC}$

$\therefore \quad BC = x$

Again, from the right angled ΔACB, $\tan\theta = \frac{p}{b}$

i.e. $\tan 60^\circ = \frac{AB}{BC} = \frac{AD + BD}{BC}$

or, $\sqrt{3} = \frac{6 + x}{x}$

or, $\sqrt{3}\,x = 6 + x$

or, $\sqrt{3}\,x - x = 6$

or, $x(\sqrt{3} - 1) = 6$

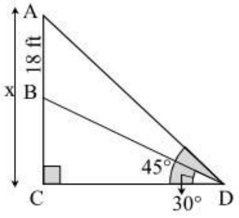
$\therefore \quad x = 8.19$ m

Thus, the height of the tower is 8.19 m and the distance between the base of the tower and point of observation is also 8.19 m.

25. एक ठाउँबाट एउटा घरको छत र एउटा झ्यालको उन्नतांश कोणहरू क्रमशः 45° र 30° छन् । यदि उक्त झ्याल घरको छतदेखि 18 फिट तल रहेको छ भने सो घरको उचाइ पत्ता लगाउनुहोस् ।

The angles of elevations of the roof of a house and window are 45° and 30° respectively. If the window is 18 feet below the roof of the house, find the height of the house. [2073 R']

⇨ Here, let, AC = x ft be the height of roof of a house. Let AB = 18 ft, B is the point on AC. Let $\measuredangle BDC = 30^\circ$ and $\measuredangle ADC = 45^\circ$ are the angles of elevations.
From the figure, BC = (x – 18) ft



From right angled ΔBDC, $\tan 30^\circ = \frac{BC}{DC} = \frac{x - 18}{DC}$

or, $\frac{1}{\sqrt{3}} = \frac{x - 18}{DC}$ $\qquad \therefore \quad DC = \sqrt{3}(x - 18)$

Again, from right angled ΔADC, $\tan 45^\circ = \frac{AC}{DC} = \frac{x}{\sqrt{3}(x - 18)}$

or, $1 = \frac{x}{\sqrt{3}(x - 18)}$ $\qquad$ or, $\sqrt{3}x - 18\sqrt{3} = x$

or, $x(\sqrt{3} - 1) = 18\sqrt{3}$ $\qquad$ or, $x = \frac{18\sqrt{3}}{\sqrt{3} - 1} = 42.59$ ft

Thus, the height of house is 42.59 ft.

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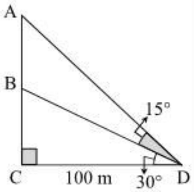
26. स्तम्भ र स्तम्भमाथि खडा गरिराखेको ध्वजदण्डले स्तम्भको फेददेखि 100 मिटर पर रहेको स्थानमा क्रमश: 30° र 15° को कोण बनाउँछन् भने ध्वजदण्डको उचाइ पत्ता लगाउनुहोस् ।
A tower and flagstaff on its top subtend angles of 30° and 15° respectively at a point 100 meter away from the foot of the tower, find the height of flagstaff. [2072 R']



⇨ Here, let, AB be the height of flagstaff and BC be a tower.

Let ∡BDC and ∡ADB be the angles whose values are 30° and 15° respectively.

From right angled ΔBCD,

$$\tan\theta = \frac{p}{b} = \frac{BC}{CD}$$

or, $\tan 30° = \frac{BC}{100}$

or, $\frac{1}{\sqrt{3}} = \frac{BC}{100}$

∴ $BC = \frac{100}{\sqrt{3}} = 57.73$

Again, ∡ADC = 30° + 15° = 45°

From the right angled ΔACD

$$\tan\theta = \frac{p}{b} = \frac{AC}{100}$$

or, $\tan 45° = \frac{AC}{100}$

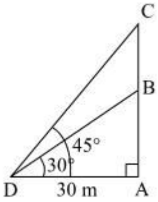
or, $1 = \frac{AC}{100}$

∴ AC = 100

Now, AB = AC – BC = 100 – 57.73 = 42.26

Thus, the height of flagstaff is 42.26 m.

27. एउटा खम्बामाथि एउटा झन्डा खडा गरिएको छ। खम्बाको फेददेखि 30 मिटर टाढाको बिन्दुबाट झन्डाको टुप्पो र फेदमा अवलोकन गर्दा उन्नतांश कोणहरू क्रमश: 45° र 30° पाइयो भने झन्डाको उचाइ पत्ता लगाउनुहोस् ।
A pole is surmounted on its top by a flagstaff. The angles of elevation of the top and the bottom of the flagstaff as observed from a point 30 meters away from the top of the pole are found to be 45° and 30° respectively. Find the height of the flagstaff.[2058 S]



⇨ Here, let, AB be a pole and BC be a flagstaff surmounted on its top. The angles of elevation on the top and bottom of flagstaff observed from the point D are ∡ADC = 45° and ∡ADB = 30° where AD = 30 m.

Now, from rt. angled ΔADB,

$$\tan 30° = \frac{AB}{DA}$$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{30}$

∴ $AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 10 \times 1.732 = 17.32$ m ..... (i)

Again, from rt. angled ΔADC,

$$\tan 45° = \frac{AC}{AD}$$

or, $1 = \frac{AB + BC}{30}$

or, 30 = AB + BC

or, BC = 30 – AB = 30 – 17.32 = 12.68

Thus, the height of flagstaff (BC) = 12.68 m.

28. एउटा धरहरामाथि 7 m उचाइ भएको एउटा झन्डा खडा गरिएको छ । जमिनको कुनै एक बिन्दुमा धरहरा र झन्डाले क्रमश: 45° र 15° का कोणहरू बनाउँदछन् भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
A flagstaff of height 7 meter stands on the top of a tower. The angles subtended by the tower and the flagstaff to a point on the ground are 45° and 15° respectively. Find the height of the tower. [2068 R, 2060 R]

⇨ Here, let, AB be the height of the tower and BC the flagstaff where BC = 7 m. The angles made by the top of the tower and flagstaff on the point D on the ground are 45° and 15° respectively.

or, ∡ADB = 45° and ∡BDC = 15°

so that ∡ADC = 45° + 15° = 60°

The height of the tower (AB) = ?

We have, from rt. angled ΔADB, $\tan 45° = \frac{AB}{AD}$

or, $1 = \frac{AB}{AD}$

∴ AD = AB ..... (i)

Again, from rt. angled ΔADC

$$\tan 60° = \frac{AC}{AD}$$

or, $\sqrt{3} = \frac{AB + BC}{AD}$

or, $\sqrt{3}$ AD = AB + 7 (∵ BC = 7)

or, $\sqrt{3}$ AB = AB + 7

or, $\sqrt{3}$ AB – AB = 7

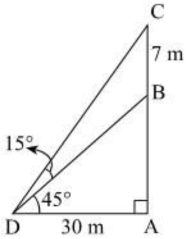
or, AB $(\sqrt{3} - 1) = 7$

or, AB (1.732 – 1) = 7

or, AB × 0.732 = 7

or, $AB = \frac{7}{0.732} = \frac{7000}{732} = 9.56$

Thus, the height of the tower (AB) = 9.56 m.



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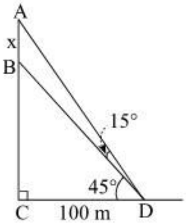
29. **एउटा शालिक स्तम्भको टुप्पोमा उभ्याइएको छ । यदि स्तम्भको फेददेखि 100 मि. परको बिन्दुमा स्तम्भ र शालिकले क्रमशः 45° र 15° को कोण बनाउँछन् भने शालिकको उचाइ पत्ता लगाउनुहोस् ।**

A statue stands on the top of a column. The angles subtend by the column and statue at a point 100 m away from the foot of the column are 45° and 15° respectively. Find the height of the statue. [2065 R]

⇨ Here, let, AB = x m be the height of a statue. Let BC be the column. ∡BDC = 45° and ∡ADC = (45° + 15°) = 60° are the angles of elevations CD = 100 m be the distance between column and the point.



From right angled ΔBCD

$$\tan 45° = \frac{BC}{CD}$$

or, $1 = \frac{BC}{100\text{ m}}$

∴ BC = 100 m

Again, from the right angled ΔACD

$$\tan (45° + 15°) = \frac{AC}{CD}$$

or, $\tan 60° = \frac{x + BC}{100\text{ m}}$

or, $\sqrt{3} = \frac{x + 100\text{ m}}{100\text{ m}}$

or, $x + 100\text{ m} = 100\sqrt{3}\text{ m}$

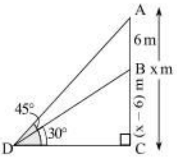
or, $x = 73.21\text{ m}$

Thus, the height of the statue is 73.21 m.

30. **कुनै एउटा ठाउँबाट ठीक अगाडि रहेको घरको छत र सो घरको छतदेखि 6 मि. तल रहेको तल रहेको झ्यालमा हेर्दा क्रमशः 45° र 30° को उन्नतांश कोणहरू बन्दछन् भने सो घरको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation observed from a place to the roof of house standing infront of the place and to a window 6 m below its roof are found to be 45° and 30° respectively. Find the height of the house. [2067 S]

⇨ Here, let, AC = x m be the height of roof of a house. Let AB = 6 m be the point on AC. Let ∡BDC = 30° and ∡ADC = 45° are the angles of elevations.



From the figure, BC = (x – 6) m

From right angled ΔBDC

$$\tan 30° = \frac{BC}{DC} = \frac{x - 6}{DC}$$

or, $\frac{1}{\sqrt{3}} = \frac{x - 6}{DC}$

∴ $DC = \sqrt{3}(x - 6)$

Again, from right angled ΔADC,

$$\tan 45° = \frac{AC}{DC} = \frac{x}{\sqrt{3}(x - 6)}$$

or, $1 = \frac{x}{\sqrt{3}(x - 6)}$

or, $\sqrt{3}x - 6\sqrt{3} = x$

or, $x(\sqrt{3} - 1) = 6\sqrt{3}$

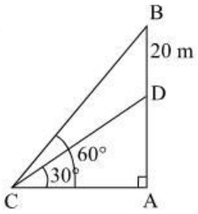
or, $x = \frac{6\sqrt{3}}{\sqrt{3} - 1} = 14.19\text{ m}$

Thus, the height of house is 14.19 m.

31. **समतलमा रहेको कुनै बिन्दुबाट सोही समतलमा रहेको एउटा खम्बाको टुप्पोको उन्नतांशकोण अवलोकन गर्दा 60° पाइयो र सो खम्बाको टुप्पोबाट 20 मि. तलको बिन्दुको उन्नतांश कोण 30° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।**

From a point on the horizontal plane, the angle of elevation of the top of a pillar standing on the same plane was observed and found to be 60° and the angle of elevation of a point 20 m below the top of the pillar was found to be 30°. Find the height of the pillar. [2062 R, 2063 R']

⇨ Here, let AB be the pillar standing on a plane and C be a point on the same plane from which angle of elevation of the top of the pillar is ∡ACB = 60° and the angle of elevation 20m. below the top of the pillar is ∡ACD = 30°. Where BD = 20 m.



To find, height of the pillar (AB) = ?

Now, from rt.angled ΔACD, $\tan 30° = \frac{AD}{AC}$

or, $\frac{1}{\sqrt{3}} = \frac{AD}{AC}$

∴ $AC = \sqrt{3}\ AD$ .... (i)

Again from rt. angled ΔACB, $\tan 60° = \frac{AB}{AC}$

or, $\sqrt{3} = \frac{AD + DB}{AC}$

∴ $AC = \frac{AD + 20}{\sqrt{3}}$ .... (ii)

From (i) and (ii); $\sqrt{3}\ AD = \frac{AD + 20}{\sqrt{3}}$

or, 3AD = AD + 20

or, 3AD – AD = 20

or, 2AD = 20

or, $AD = \frac{20}{2} = 10$

∴ AD = 10 m.

Thus, the height of the pillar (AB) = AD + DB = 10 + 20 = 30 m

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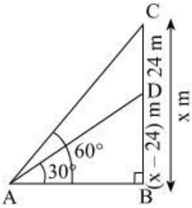
32. समतलमा रहेको कुनै बिन्दुबाट सोही समतलमा रहेको एउटा धरहराका टुप्पोको उन्नतांश कोण अवलोकन गर्दा 60° पाइयो र सो धरहराको टुप्पोबाट 24 मिटर तलको बिन्दुको उन्नतांश कोण सोही बिन्दुबाट 30° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

From a point on the horizontal plane, the angle of elevation of the top of a tower standing on the same plane was observed and found to be 60°. The angle of elevation of a point 24 meter below the top of the tower was found to be 30° from the same point. Find the height of the tower. [2064 S]

⇨ Here, let BC = x m be the height of the tower. $\measuredangle CAB = 60°$ and $\measuredangle DAB = 30°$ are the angles of elevation of the top of the tower and a point 24 m below the top.



Then from the figure,

BD = (x – 24) m

From the right angled ΔABD,

$\tan\theta = \frac{p}{b}$ i.e. $\tan 30° = \frac{x-24}{AB}$

or, $\frac{1}{\sqrt{3}} = \frac{x-24}{AB}$

$\therefore$ $AB = (x-24)\sqrt{3}$

Again, from the right angled ΔABC, $\tan\theta = \frac{p}{b}$

or, $\tan 60° = \frac{x}{AB}$

or, $\sqrt{3} = \frac{x}{(x-24)\sqrt{3}}$

or, $3(x-24) = x$

or, $3x - 72 = x$

or, $3x - x = 72$

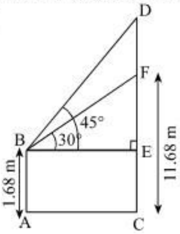
or, $2x = 72$

$\therefore$ $x = 36$

Thus, the height of the tower is 36 m.

33. 1.68 मिटर अग्लो मानिसले एउटा घर र त्यसको झ्यालको टुप्पो कुनै स्थानबाट अवलोकन गर्दा उन्नतांश कोणहरू क्रमशः 45° र 30° पाएछ । यदि झ्यालको उचाइ जमिनबाट 11.68 मिटर भए घरको उचाइ पत्ता लगाउनुहोस् ।

A man of 1.68 m high observed the angles of elevation of the top of a house and its window from a place and found to be 45° and 30° respectively. If the height of the window from the ground is 11.68 m, calculate the height of the house. [2061 R]



⇨ Here, let AB be the height of a man where AB = 1.68 m. CD be the height of the house and F be the position of the window where CF = 11.68m.

Here, AC // BE,

So, AB = CE = 1.68 m.

Also, $\measuredangle EBD = 45°$ and $\measuredangle EBF = 30°$

Here, EF = CF – CE

= 11.68 – 1.68 = 10 m

Now from rt. angled ΔBEF,

$\tan 30° = \frac{EF}{BE}$

or, $\frac{1}{\sqrt{3}} = \frac{10}{BE}$

$\therefore$ $BE = 10\sqrt{3}$ m.

Again from rt. angled ΔBED,

$\tan 45° = \frac{ED}{BE}$

or, $1 = \frac{ED}{BE}$

or, BE = ED

or, $10\sqrt{3} = ED$

$\therefore$ $ED = 10\sqrt{3} = 10 \times 1.732 = 17.32$ m

Thus, the height of the house (CD) = 17.32 + 1.68

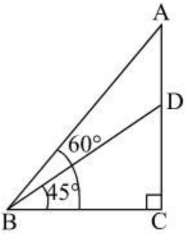
= 19 m.

34. 9 मिटर लामो भऱ्याङ कुनै एउटा भवनको छतबाट केही तलको बिन्दुसम्म पुगेको छ । भऱ्याङको फेदबाट भवनको छतको उन्नतांशकोण 60° छ भने भवनको उचाइ पत्ता लगाउनुहोस् । यहाँ भऱ्याङले जमिनसँग 45° कोण बनाएको छ ।

A ladder 9 m long reaches to a point below the top of building. From the foot of the ladder, the angle of elevation of the building is 60°. Find the height of the building. Here the ladder makes the angle 45° with ground. [2060 S']

⇨ Here, AC be the height of the building and BC be the distance between the bottom of the building & ladder. BD = 9 m be a length of ladder.

$\measuredangle DBC = 45°$ & $\measuredangle ABC = 60°$



Now, from right angled ΔDCB,

$\cos 45° = \frac{BC}{BD}$

or, $\frac{1}{\sqrt{2}} = \frac{BC}{9}$

or, $BC = \frac{9}{\sqrt{2}}$

$\therefore$ BC = 6.36 m

Then, from right angled ΔACB,

$\tan 60° = \frac{AC}{BC}$

or, $\sqrt{3} = \frac{AC}{6.36}$

or, $AC = \sqrt{3} \times 6.36 = 11.01$ m

Thus, the required height of the building is 11.01 m.

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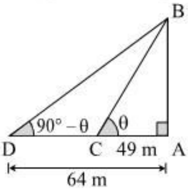
MODEL 5

35. **एउटा स्तम्भको फेददेखि 49 मिटर र 64 मिटरको दूरीबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angles of elevation of the top of a tower as observed from the distance of 49 metre and 64 metre from the foot of the tower are found to be complementary. Find the height of the tower. [SEE 2075 R'$_2$]

⇨ Here, AB is the tower whose height is to be determined. Let C and D be two points which are 49 m and 64 m far from the foot of tower respectively. Since the angles of elevation observed from C and D to the top of tower are complementary,



So, let $\measuredangle ACB = \theta$ then $\measuredangle ADB = 90° - \theta$.

Now, from the figure, AC = 49 m, AD = 64 m,

From rt. $\measuredangle$ed ΔABC, $\tan\theta = \frac{AB}{AC}$

$\therefore \quad \tan\theta = \frac{AB}{49}$ .... (i)

Again from rt. $\measuredangle$ed ΔADB, $\tan(90° - \theta) = \frac{AB}{AD}$

or, $\cot\theta = \frac{AB}{64}$ $\qquad \therefore \quad \tan\theta = \frac{64}{AB}$ .... (ii)

From (i) and (ii), we get, $\frac{AB}{49} = \frac{64}{AB}$

or, $AB^2 = 49 \times 64$

$\therefore \quad AB = 7 \times 8 = 56$ m

Thus, the height of the tower is 56 m.

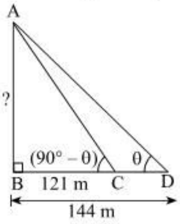
36. **एउटा स्तम्भको फेददेखि 144 मिटर र 121 मिटरको दूरीमा रहेका दुई बिन्दुहरूबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angles of elevation of the top o a tower as observed from the points at the distances of 144 meter and 121 meter from the foot of the tower are found to be complementary. Find the height of the tower. [2070 R]

⇨ Here, let AB be a tower whose height is required to find.

Let, $\measuredangle ACB = (90° - \theta)$ and $\measuredangle ADB = \theta$ are the angle of elevations.

BC = 121 m and BD = 144 m



From the right angled ΔABC,

$\tan(90° - \theta) = \frac{AB}{BC}$

or, $\cot\theta = \frac{AB}{121}$ ...... (i)

From right angled ΔBAD,

$\tan\theta = \frac{AB}{BD} = \frac{AB}{144}$

$\therefore \quad \cot\theta = \frac{144}{AB}$ .... (ii)

From (i) and (ii) then, $\frac{AB}{121} = \frac{144}{AB}$

or, $AB^2 = 144 \times 121$

$\therefore \quad AB = 12 \times 11 = 132$

Thus, the height of the tower is 132 m.

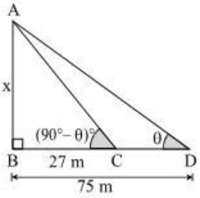
37. **एउटा धरहराको टुप्पोलाई यसको फेदबाट 27 m र 75 m दूरीमा एकैतिरबाट अवलोकन गर्दा उन्नतांशकोणहरू समपूरक पाइयो भने उक्त धरहराको उचाइ पत्ता लगाउनुहोस् ।**

The angles of elevation of the top of a tower observed from 27 m and 75 m away from its foot on the same side are found to be complementary. Find the height of the tower. [2072 S]

⇨ Here, let, AB be the height of a tower. Let, $\measuredangle ACB$ and $\measuredangle ADB$ are two complementary angles.

So, $\measuredangle ADB = \theta$ and

$\measuredangle ACB = 90° - \theta$



From the right angled triangle ABC

$\tan(90° - \theta) = \frac{p}{b} = \frac{AB}{BC}$

or, $\cot\theta = \frac{AB}{27}$ .... (i)

From the right angled ΔABD

$\cot\theta = \frac{b}{p} = \frac{BD}{AB} = \frac{75}{AB}$ .... (ii)

Equating (i) and (ii) then,

$\frac{AB}{27} = \frac{75}{AB}$

or, $AB^2 = 2025$

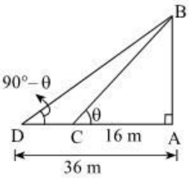
$\therefore \quad AB = 45$

Thus the height of the tower is 45 m.

38. **एउटा स्तम्भको फेददेखि 36 मिटर र 16 मिटरको दूरीबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of the top of a tower as observed from the distances of 36 m & 16 m from the foot of the tower are found to be complementary. Find the height of the tower. [2059 S]

⇨ Here, let AB be the tower. Let C and D be two points which are 16 m and 36 m far from the bottom of the tower respectively. Since the angles of elevation observed from C and D to the top of the tower are complementary,



So, let $\measuredangle ACB = \theta$, then $\measuredangle ADB = 90° - \theta$.

Now, from figure AB = ?, AC = 16 m, AD = 36 m

In right angled ΔABC, $\tan\theta = \frac{AB}{AC}$

$\therefore \quad \tan\theta = \frac{AB}{16}$ ...... (i)

From rt. angled ΔADB,

$\tan(90° - \theta) = \frac{AB}{AD}$

or, $\cot\theta = \frac{AB}{36}$

or, $\tan\theta = \frac{36}{AB}$ ..... (ii)

From equation (i) and (ii),

We get, $\frac{AB}{16} = \frac{36}{AB}$

or, $AB^2 = 36 \times 16 = 576$

$\therefore \quad AB = 24$ m

Thus, the height of the tower (AB) = 24 m.

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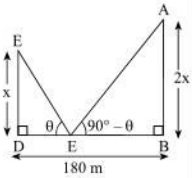
39. **180 मिटर दूरीमा रहेका दुई खम्बाहरू छन् । एउटा खम्बा अर्कोभन्दा दोब्बर अग्लो छ । खम्बाहरूको फेद जोड्ने रेखाहरूको मध्यबिन्दुबाट टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक रहेछ भने लामो खम्बाको उचाइ पत्ता लगाउनुहोस् ।**

Two posts are 180 meter apart and the height of one is double that of the other. From the mid point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, find the height of longer post. [2065 R']



⇨ Here, let AB = 2ED = 2x be the height of two posts, BD = 180 m be the distance between two posts.

$BC = CD = \frac{180 \text{ m}}{2} = 90 \text{ m}$

∡ACB and ∡ECD are the angles of elevations of top of two posts from the point C.

From the question,

If ∡ECD = θ then ∡ACB = 90° – θ

Now, from right angled ΔABC, $\tan(90° - \theta) = \frac{p}{b}$

or, $\cot\theta = \frac{AB}{BC} = \frac{2x}{90} = \frac{x}{45}$ .... (i)

Again, from right angled ΔECD,

$\cot\theta = \frac{b}{p} = \frac{CD}{ED} = \frac{90}{x}$ .... (ii)

From (i) and (ii), $\frac{x}{45} = \frac{90}{x}$

or, $x^2 = 45 \times 90$

or, $x = \sqrt{45 \times 90} = 45\sqrt{2}$

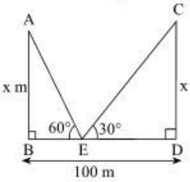
So, height of longer post = 2x $= 2 \times 45\sqrt{2} = 90\sqrt{2}$ m.

Thus, the height of longer post is $90\sqrt{2}$ m.

MODEL 6

40. **बराबर उचाइ भएको दुई खम्बाहरू 100 मिटर फराकिलो बाटोको दुई छेउमा रहेका छन् । खम्बाहरूबीचको कुनै बिन्दुबाट खम्बाको टुप्पोहरूमा हेर्दा उन्नतांश कोणहरू 60° र 30° का छन् भने खम्बाहरूको उचाइ कति होला ?**

Two pillars of the same height are situated on the road 100 m apart. From any point between the pillars, the angles of elevation as observed to the top of pillars are 60° and 30°. What will be the height of the pillars ? [2064 R']



⇨ Here, let, AB = CD = x m be the height of two pillars. Let BD = 100 m be the distance between the pillars. ∡AEB = 60° and ∡CED = 30° are the angle of elevation of the pillars.

From the right angled ΔABE,

$\tan\theta = \frac{p}{b}$ i.e. $\tan 60° = \frac{x}{BE}$

or, $\sqrt{3} = \frac{x}{BE}$ ∴ $BE = \frac{x}{\sqrt{3}}$

Again, from the right angled ΔCED, $\tan\theta = \frac{p}{b}$

or, $\tan 30° = \frac{x}{ED}$ or, $\frac{1}{\sqrt{3}} = \frac{x}{ED}$

∴ $ED = \sqrt{3}x$

Now, BD = BE + ED or, $100 = \frac{x}{\sqrt{3}} + \sqrt{3}x$

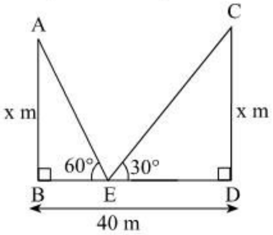
or, $100 = \frac{x + 3x}{\sqrt{3}}$ or, $100\sqrt{3} = 4x$

∴ $x = 25\sqrt{3}$

Thus, the height of the pillars is $25\sqrt{3}$ m each.

41. **40 m फराकिलो सडकको दुवैतिर किनारामा बराबर उचाइ भएका दुईओटा स्तम्भहरू छन् । सो स्तम्भहरू रहेको सडकमा पर्ने कुनै एउटा बिन्दुबाट स्तम्भहरूको टुप्पाको उन्नतांशकोणहरू 60° र 30° छन् भने तिनीहरूको उचाइ र सो बिन्दुको स्थिति पत्ता लगाउनुहोस् ।**

Two pillars of equal height stand on either side of a road way which is 40 m wide. At a point on the road way between the pillars, the angle of elevations of the tops of the pillars are 60° and 30°, find the height and the position of the point.



⇨ Here, let, AB = CD = x m be the height of two pillars. Let BD = 40 m be the distance between the pillars. ∡AEB = 60° and ∡CED = 30° are the angle of elevation of the pillars.

From the right angled ΔABE,

$\tan\theta = \frac{p}{b}$ i.e. $\tan 60° = \frac{x}{BE}$

or, $\sqrt{3} = \frac{x}{BE}$ ∴ $BE = \frac{x}{\sqrt{3}}$ ....(i)

Again, from the right angled ΔCED, $\tan\theta = \frac{p}{b}$

or, $\tan 30° = \frac{x}{ED}$ or, $\frac{1}{\sqrt{3}} = \frac{x}{ED}$

∴ $ED = \sqrt{3}x$ .....(ii)

Now, BD = BE + ED

or, $40 = \frac{x}{\sqrt{3}} + \sqrt{3}x$ or, $40 = \frac{x + 3x}{\sqrt{3}}$

or, $40\sqrt{3} = 4x$ ∴ $x = 10\sqrt{3}$

Putting the value of x in (i), $BE = \frac{10\sqrt{3}}{\sqrt{3}} = 10$ m

Putting the value of x in (ii), $ED = \sqrt{3} \times 10\sqrt{3} = 30$ m

Thus, the height of each pillar is $10\sqrt{3}$ m and the point is 10 m and 30 m away from the 1st and 2nd pillar respectively.

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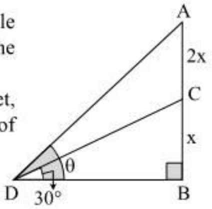
MODEL 7

42. एउटा ठाडो खम्बा AB लाई बिन्दु C ले AC : CB = 2 : 1 हुने गरी विभाजन गरिएको छ । जमिनको कुनै एउटा बिन्दुबाट C को उन्नतांश कोण 30° छ भने सोही बिन्दुबाट A को उन्नतांश कोण कति हुन्छ ? पत्ता लगाउनुहोस् जहाँ B खम्बाको फेद हो ।

A vertical pole AB is divided by a point C such that AC : CB = 2 : 1. The angle of elevation of C from a point on the ground is 30°. What is the angle of elevation of A from the same point? Find it where B is the foot of the pole. [2072 R]



⇨ Here, let AB be a vertical pole which is divided by C in the ratio of 2 : 1.

So, AC = 2x and BC = x. Let, $\measuredangle CDB = 30°$ be an angle of elevation.

We know that, $\tan\theta = \frac{p}{b}$

$$\tan 30° = \frac{x}{BD}$$

or, $\frac{1}{\sqrt{3}} = \frac{x}{BD}$

∴ $BD = \sqrt{3}\,x$

Again, from right angled ΔADB, $\tan\theta = \frac{p}{b} = \frac{AB}{BD}$

or, $\tan\theta = \frac{3x}{BD}$

or, $\tan\theta = \frac{3x}{\sqrt{3}\,x}$

or, $\tan\theta = \sqrt{3}$

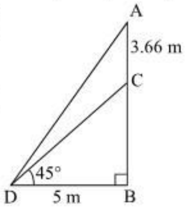
or, $\tan\theta = \tan 60°$

∴ $\theta = 60°$

Thus, the value of θ is 60°.

43. एउटा भित्तामा टाँसिएको एउटा पोस्टरको उचाइ 3.66 मि. रहेछ । भित्ताको फेददेखि 5 मि. टाढा एउटै समतलमा परेको बिन्दुबाट पोष्टरको तल्लो किनारा हेर्दा उन्नतांश कोण 45° पाइयो भने त्यही बिन्दुबाट हेर्दा पोष्टरको माथिल्लो किनाराको उन्नतांश कोण कति होला ?

A poster hanging on a wall has a vertical height 3.66 m. From a point 5 m away from the wall on the same plane the angle of elevation of the bottom edge of the poster was found to be 45°. What will be the angle of elevation of the top edge of the poster if it is observed from the same point on the horizontal plane.[2063 S]



⇨ Here, let AC = 3.66 m be the height of the poster and AB be the height of the wall.

Let BD = 5 m be the distance between B and D. $\measuredangle CDB = 45°$ be the angle of elevation.

From the right angled ΔBCD,

$$\tan\theta = \frac{p}{b}$$

or, $\tan 45° = \frac{BC}{BD}$

or, $1 = \frac{BC}{5}$

∴ BC = 5 m

Again, from the right angled ΔABD,

$\tan\theta = \frac{p}{b}$ i.e. $\tan \measuredangle ADB = \frac{AB}{BD} = \frac{3.66 + 5}{5}$

or, $\tan \measuredangle ADB = \frac{8.66}{5}$

or, $\tan \measuredangle ADB = 1.732$

or, $\tan \measuredangle ADB = \tan 60°$

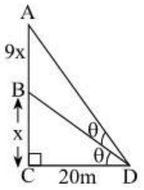
∴ $\measuredangle ADB = 60°$

Thus, the required angle of elevation is 60°.

MODEL 8

44. एउटा ठाडो खम्बालाई 9 : 1 हुने गरी कुनै बिन्दुले विभाजन गरेको छ । यदि उक्त खम्बाको दुवै भागहरूले खम्बाको फेददेखि 20 मिटरको दूरीमा रहेको कुनै एक बिन्दुमा आपसमा बराबर कोणहरू बनाउँदछन् भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।

A vertical pole is divided by any point in the ratio 9 : 1. If both the segments of a pole subtend equal angles to each other at a distance of 20 m away from the foot of the pole, find the height of the pole. [2065 E]



⇨ Here, AC be the height of the pole, CD = 20 m be the distance between bottom of pole & observation point. B divides the line AC in the ratio of 9 : 1.

So, AB = 9x & BC = x and $\measuredangle CDB = \measuredangle BDA = \theta$.

Now, from right angled ΔBCD, $\tan\theta = \frac{x}{20}$

Again, from right angled ΔACD, $\tan 2\theta = \frac{10x}{20}$

or, $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{x}{2}$ or, $\frac{2.\frac{x}{20}}{1-\left(\frac{x}{20}\right)^2} = \frac{x}{2}$

or, $\frac{\frac{x}{10}}{\frac{400-x^2}{400}} = \frac{x}{2}$ or, $\frac{40x}{400-x^2} = \frac{x}{2}$

or, $80 = 400 - x^2$ or, $x^2 = 320$

∴ $x = 17.88$

So, $10x = 10 \times 17.88 = 178.8$

Thus, the required height of the pole is 178.8 m.

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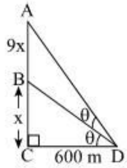
45. **एउटा रेडियो प्रशारण केन्द्रको खम्बा टुप्पोबाट 9 : 1 को अनुपातमा विभक्त छ । सो खम्बाका दुवै भागहरूले जमिनदेखि सीधा 600 मि. टाढा बराबर कोणहरू बनाउँछन् भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।**

A tower of a radio station is divided by a point in the ratio of 9 : 1. From the top if both the parts subtends equal angles at a point on ground level 600 m away from its bottom, find the height of the tower.

⇨ Here, AC be the height of the pole, CD = 600 m be the distance between bottom of pole & observation point. B divides the line AC in the ratio of 9 : 1. So, AB = 9x & BC = x and ∡CDB = ∡BDA = θ.



Now, from right angled ΔBCD, $\tan\theta = \frac{x}{600}$

Again, from right angled ΔACD

$$\tan 2\theta = \frac{AC}{CD} = \frac{AB+BC}{600} = \frac{9x+x}{600} = \frac{10x}{600} = \frac{x}{60}$$

$$\text{or, } \frac{2\tan\theta}{1-\tan^2\theta} = \frac{x}{60} \qquad \text{or, } \frac{2.\frac{x}{600}}{1-\left(\frac{x}{600}\right)^2} = \frac{x}{60}$$

$$\text{or, } \frac{\frac{x}{300}}{\frac{360000-x^2}{360000}} = \frac{x}{60}$$

$$\text{or, } \frac{x}{300} \times \frac{360000}{360000-x^2} = \frac{x}{60}$$

or, $1200 \times 60 = 360000 - x^2$

or, $x^2 = 360000 - 72000$

or, $x = \sqrt{288000} = 536.656$

Now, height of tower = AC = AB + BC

= 9x + x = 10x

= 10 × 536.656

= 5366.56

Thus, the required height of tower is 5366.56 m.

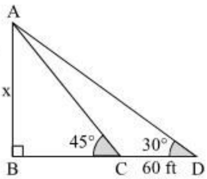
MODEL 9

46. **समतल सतहमा ठड्याइएको एउटा स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 45° भएको समयमा भन्दा सूर्यको उचाइ 30° भएको समयमा 60 फिट बढी लामो पाइयो भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।**

The shadow of a tower standing on a plain is found to be 60 feet longer when the sun's altitude is 30° than when it is 45°. What will be the height of the tower ? Find it. [2073 R]

⇨ Here, let AB be the height of the towers and BC and BD are the length of shadows.

Let ∡ACB = 45° and ∡ADB = 30° be the angle of elevation or heights of sun at two points.



From the right angled ΔABC, $\tan\theta = \frac{p}{b}$

or, $\tan 45° = \frac{AB}{BC}$

or, $1 = \frac{AB}{BC}$

∴ AB = BC

Again, from the right angled triangle ABD, $\tan\theta = \frac{p}{b}$

or, $\tan 30° = \frac{AB}{BD}$ or, $\frac{1}{\sqrt{3}} = \frac{AB}{BC+60}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{AB+60}$

or, $AB + 60 = AB\sqrt{3}$

or, $AB(1-\sqrt{3}) = -60$

or, $AB = \frac{60}{\sqrt{3}-1}$

∴ AB = 81.96

Thus, the height of the tower is 81.96 feet.

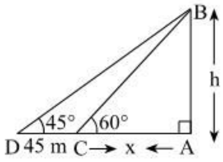
47. **एउटा चउरमा रहेको कुनै स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 60° भएको समयमा भन्दा सूर्यको उचाइ 45° भएको समयमा 45 मि. बढी लामो हुन्छ भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।**

The shadow of a tower on the ground is found to be 45 m longer when the sun's altitude is 45° than when it is 60°. What will be the height of the tower? Find it. [2073 S]

OR कुनै स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 60° भएको समयभन्दा सूर्यको उचाइ 45° भएको समयमा 45 m बढी लामो हुन्छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।

The shadow of a tower on the ground is found to be 45 m longer when sun's altitude is 45° than when it is 60°. Find the height of the tower. [2068 R']

⇨ Here, AB be a height of a tower, ∡ADB = 45° & ∡ACB = 60° are the angles made by the altitude of the sun, suppose CD = 45 m, CA = x m, AB = h m.



Now, from right angled ΔABC where

$\tan 60° = \frac{h}{x}$

or, $\sqrt{3} = \frac{h}{x}$

or, $x = \frac{h}{\sqrt{3}}$

From right angled ΔABD

$\tan 45° = \frac{h}{x+45}$

or, $1 = \frac{h}{x+45}$

or, $\frac{h}{\sqrt{3}} + 45 = h \quad \left[\because x = \frac{h}{\sqrt{3}}\right]$

or, $h + 45\sqrt{3} = \sqrt{3}h$

or, $h(\sqrt{3}-1) = 45\sqrt{3}$

or, $h = \frac{45\sqrt{3}}{\sqrt{3}-1} = \frac{77.94}{0.732} = 106.47$ m

Thus, the required height of the tower is 106.47 m.

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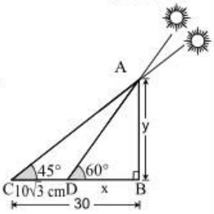
48. यदि सूर्यको उचाइ 60° बाट 45° हुँदा एउटा स्तम्भको छाया 10$\sqrt{3}$ m बढ्छ भने स्तम्भको उचाइ र सूर्यको उचाइ 60° हुँदाको अवस्थामा स्तम्भको छायाको लम्बाइ पत्ता लगाउनुहोस् ।

The length of the shadow of a vertical column increases by 10$\sqrt{3}$ m when the altitude of the sun becomes 45° from 60°. Find the height of the column and the length of the shadow when the sun's altitude was 60°. [2066 R]



⇨ Here, let ∡ADB = 45°, ∡ACB = 60° be the angles made by altitude of sun and AB = y m be the height of column. Let, BC = x m and BD = (x + 10$\sqrt{3}$)m be the length of shadows then, from the right angled triangle ABC,

$$\tan\theta = \frac{p}{b}$$

or, $\tan 60° = \frac{y}{x}$

or, $\sqrt{3} = \frac{y}{x}$

∴ $y = \sqrt{3}\,x$

From the right angled triangle ABD, $\tan\theta = \frac{p}{b}$

or, $\tan 45° = \frac{y}{x + 10\sqrt{3}}$

or, $1 = \frac{\sqrt{3}\,x}{x + 10\sqrt{3}}$

or, $\sqrt{3}x = x + 10\sqrt{3}$

or, $\sqrt{3}x - x = 10\sqrt{3}$

or, $x(\sqrt{3} - 1) = 10\sqrt{3}$

∴ $x = \frac{10\sqrt{3}}{\sqrt{3} - 1}$

The height of column = $y = \sqrt{3}x$

$= \frac{\sqrt{3} \times 10\sqrt{3}}{\sqrt{3} - 1}$

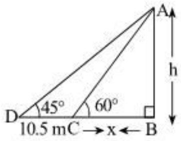
$= \frac{30}{\sqrt{3} - 1} = 40.98$ m

The length of shadow = $x = \frac{10\sqrt{3}}{\sqrt{3} - 1} = 23.66$ m

Thus, the height of column and length of shadow are 40.98 m and 23.66 m respectively.

49. कुनै धरहराको छाया सूर्यको उचाइ 60° भएको समयभन्दा सूर्यको उचाइ 45° भएको समयमा 10.5 m बढी लामो हुन्छ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

The shadow of a tower on the ground is found to be 10.5 m longer when sun's altitude is 45° than it is 60°. Find the height of the tower. [2067 R']



⇨ Here, AB be a height of a tower. ∡ADB = 45° and ∡ACB = 60° are the angles made by the altitude of sun. Where DC = 10.5 m, CB = x & AB = h.

Now, from right angled ΔABC

$\tan 60° = \frac{h}{x}$

or, $\sqrt{3} = \frac{h}{x}$

or, $x = \frac{h}{\sqrt{3}}$

Then, from right angled ΔABD,

$\tan 45° = \frac{h}{x + 10.5\text{ m}}$

or, $1 = \frac{h}{\left(\frac{h}{\sqrt{3}} + 10.5\right)}$

or, $\left(\frac{h}{\sqrt{3}} + 10.5\right) = h$

or, $\frac{h + \sqrt{3} \times 10.5}{\sqrt{3}} = h$

or, $h + 18.18 = \sqrt{3}h$

or, $18.18 = \sqrt{3}h - h$

or, $h(\sqrt{3} - 1) = 18.18$

or, $h = \frac{18.18}{0.732} = 24.84$ m

Thus, required height of a tower is 24.84 m.

QUESTIONS FROM CDC TEXTBOOK

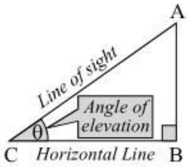
5.6 उचाइ र दुरी (HEIGHT AND DISTANCE)

EXERCISE 5.6

1. (a) उन्नतांश कोण भनेको के हो ? चित्रसहित प्रष्ट पार्नुहोस् ।

What do you mean by angle of elevation? Clarify with figures.

⇨ The angle made by the line of sight with the horizontal line through the position of an observer's eye is called an angle of elevation. In this case, the object to be observed lies above the position of the eye. In the figure alongside, A is the position of an object, AC is the line of sight, BC is the horizontal line and ∡ACB is an angle of elevation.



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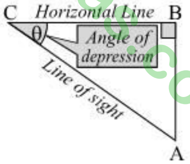
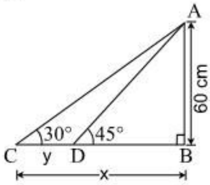
(b) **अवनति कोण भनेको के हो ? चित्रसहित प्रष्ट पार्नुहोस् ।**
What do you mean by angle of depression? Clarify with figures.



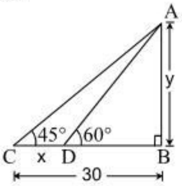
⇨ The angle made by the line of sight with the horizontal line through the position of an observer's eye is called an angle of depression. In this case, the object to be observed lies below the position of an eye. In the figure alongside, A is the position of an object, AC is the line of sight, BC is the horizontal line and ∡ACB is an angle of depression.

2. **दिइएका चित्रबाट x र y का मानहरू पत्ता लगाउनुहोस् । (From the given figures , find the values of x and y.)**

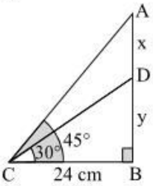
(a)



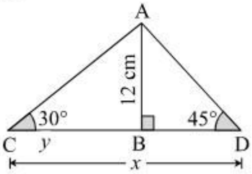
(b)



(c)



(d)



⇨ (a) Here, from right angled triangle ABD,

$$\tan\theta = \frac{AB}{BD}$$

or, $\tan 45° = \frac{60 \text{ cm}}{BD}$

or, $1 = \frac{60 \text{ cm}}{BD}$

$\therefore$ BD = 60 cm

Again, from right angled triangle ACB, $\tan\theta = \frac{p}{b}$

or, $\tan 30° = \frac{AB}{BC}$

or, $\frac{1}{\sqrt{3}} = \frac{60 \text{ cm}}{x}$

$\therefore$ $x = 60\sqrt{3}$ cm

Now, y = BC – BD

$= 60\sqrt{3}$ cm – 60 cm

$= (60\sqrt{3} - 60)$ cm

= 43.92 cm

Thus, $x = 60\sqrt{3}$ cm and y = 43.92 cm.

⇨ (c) Here, from right angled ΔCBD,

$$\tan 30° = \frac{BD}{BC}$$

or, $\frac{1}{\sqrt{3}} = \frac{y}{24}$ cm

or, $y = \frac{24 \text{ cm}}{\sqrt{3}} = 8\sqrt{3}$ cm

Again, from right angled ΔCBA,

$$\tan 45° = \frac{AB}{BC}$$

or, $1 = \frac{x+y}{24}$

or, $x + y = 24$

or, $x + 8\sqrt{3} = 24$

or, $x = 24 - 8\sqrt{3} = 10.14$ cm

Thus, x = 10.14 cm and $y = 8\sqrt{3}$ cm.

⇨ (b) Here, from right angled ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

or, $1 = \frac{y}{30}$

$\therefore$ y = 30

Again, from right angled ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

or, $\sqrt{3} = \frac{y}{BD}$

or, $\sqrt{3} = \frac{30}{BD}$

or, $BD = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

Now, $x = BC - BD = 30 - 10\sqrt{3} = 12.68$

Thus, x = 12.68 and y = 30.

⇨ (b) Here, from right angled ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

or, $1 = \frac{y}{30}$

$\therefore$ y = 30

Again, from right angled ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

or, $\sqrt{3} = \frac{y}{BD}$

or, $\sqrt{3} = \frac{30}{BD}$

or, $BD = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

Now, $x = BC - BD = 30 - 10\sqrt{3} = 12.68$

Thus, x = 12.68 and y = 30.

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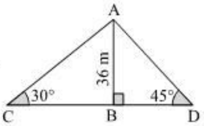
3. (a)

एउटा 36 m अग्लो खम्बाको टुप्पोलाई सोही समतलमा दुई विपरीत दिशामा रहेका बिन्दुहरूबाट हेर्दा 30° र 45° का उन्नतांश कोणहरू बन्छन् भने ती दुई बिन्दुहरूबिचको दुरी पत्ता लगाउनुहोस् ।

The angle of elevations of the top of a pole 36 m high from the opposite sides of the pole are found to be 30° and 45° respectively. Find the distance between the points.



⇨ Here, let AB = 36 m be the height of the pole. $\measuredangle ACB = 30°$ and $\measuredangle ADB = 45°$ are the angles of elevation of top of the pole from the points C and D. CD is the distance between the points which is to be calculated.

From the right angled triangle ABC, $\tan 30° = \frac{AB}{BC}$

or, $\frac{1}{\sqrt{3}} = \frac{36\text{ m}}{BC}$

$\therefore$ $BC = 36\sqrt{3}$ m

Again, from the right angled ΔABD, $\tan 45° = \frac{AB}{BD}$

or, $1 = \frac{36\text{ m}}{BD}$

$\therefore$ BD = 36 m

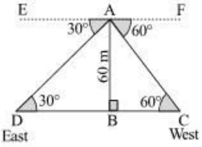
Now, CD = CB + BD

$= 36\sqrt{3}\text{ m} + 36\text{ m} = 98.35$ m

Thus, the distance between the points is 98.35 m.

(c) एउटा 60 m अग्लो घरको छतबाट पूर्व र पश्चिमतिर रहेका दुई स्थानको अवनति कोणहरू क्रमशः 30° र 60° पाइयो भने ती दुई स्थानबिचको दुरी पत्ता लगाउनुहोस् ।

The angle of depressions of two places due east and west from the top of a 60 m high house are found to be 30° and 60° respectively. Find the distance between two places.



⇨ Here, let AB = 60 m be the height of a house. $\measuredangle EAD = 30°$ and $\measuredangle FAC = 60°$ are the angles of depressions of two places due east and west respectively.

$\measuredangle ADB = EAD = 30°$ and $\measuredangle FAC = \measuredangle ACB = 60°$

From right angled ΔABD,

$\tan 30° = \frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{60\text{ m}}{BD}$

$\therefore$ $BD = 60\sqrt{3}$ m

Again, from right angled triangle ABC,

$\tan 60° = \frac{AB}{BC}$

or, $\sqrt{3} = \frac{60}{BC}$

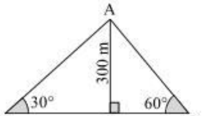
or, $BC = \frac{60}{\sqrt{3}} = \frac{20\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$ m

Now, $CD = BC + BD = 20\sqrt{3} + 60\sqrt{3} = 80\sqrt{3}$

Thus, the distance between the points is $80\sqrt{3}$ m.

(b) एउटा 300 m अग्लो स्तम्भको टुप्पोलाई एउटै समतलमा रहेका र विपरीत दिशामा पर्ने स्थानहरूबाट अवलोकन गर्दा उन्नतांश कोणहरू क्रमशः 60° र 30° पाइयो । ती दुई स्थानहरूबिचको दुरी पत्ता लगाउनुहोस् ।

The angle of elevations of the top of a tower 300 m high from the two places which are in the opposite sides of the tower are found to be 60° and 30° respectively. Find the distance between the two places.



⇨ Here, let AB = 300 m be the height of a tower. $\measuredangle ACB = 30°$ and $\measuredangle ADB = 60°$ are the angle of elevations of the top of tower.

From the right angled ΔABC, $\tan 30° = \frac{AB}{BC}$

or, $\frac{1}{\sqrt{3}} = \frac{300\text{ m}}{BC}$

$\therefore$ $BC = 300\sqrt{3}$ m

Again, from the right angled ΔABD, $\tan 60° = \frac{AB}{BD}$

or, $\sqrt{3} = \frac{300}{BD}$

or, $BD = \frac{300}{\sqrt{3}} = 100\sqrt{3}$ m

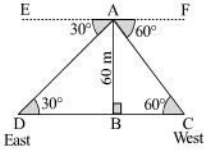
Now, CD $= BC + BD = 300\sqrt{3}\text{ m} + 100\sqrt{3}$ m

$= 400\sqrt{3}\text{ m} = 692.82$ m

Thus, the required distance is 692.82 m.

(d) एउटा पहाडको टुप्पोलाई पहाडको उत्तरतिरको स्थानबाट हेर्दा उन्नतांश कोण 55° र पहाडको दक्षिणतिर रहेको स्थानबाट हेर्दा उन्नतांश कोण 35° छ । यदि ती स्थानहरू बिचको दुरी 1800 m रहेछ भने पहाडको उचाइ पत्ता लगाउनुहोस् ।

The angle of elevation of the top of a mountain observed from a place due north is found to be 55° & that from a place due south is 35°. If the places lie on the same level a distance of 1800 m, find the height of the mountain.



⇨ Here, let, AB is the mountain. C is a point of observation in the north, D is a point of observation in the south. CD = 1800 m.

$\measuredangle ACB = 55°$ and $\measuredangle ADB = 35°$, AB = ?

In right angled ΔBAC,

$\tan 55° = \frac{AB}{AC}$

or, $1.43 = \frac{AB}{AC}$

or, $AC = \frac{AB}{1.43}$ ..... (i)

In right angled ΔBAD,

$\tan 35° = \frac{AB}{AD}$

or, $0.70 = \frac{AB}{AD}$

or, $AD = \frac{AB}{0.70}$ ..... (ii)

Now, CD = AC + AD .... (iii)

From (i), (ii) & (iii), $1800\text{ m} = \frac{AB}{1.43} + \frac{AB}{0.70}$

or, $1800\text{ m} = \frac{0.70AB + 1.43AB}{1.43 \times 0.70}$

or, $1800 \times 1.43 \times 0.70\text{ m} = 2.13AB$

$\therefore$ $AB = \frac{1801.8}{2.13}\text{ m} = 845.92$ m

Thus, height of mountain is 845.92 m.

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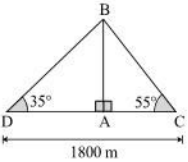
(d) **एउटा पहाडको टुप्पोलाई पहाडको उत्तरतिरको स्थानबाट हेर्दा उन्नतांश कोण 55° र पहाडको दक्षिणतिर रहेको स्थानबाट हेर्दा उन्नतांश कोण 35° छ । यदि ती स्थानहरू बिचको दुरी 1800 m रहेछ भने पहाडको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of the top of a mountain observed from a place due north is found to be 55° & that from a place due south is 35°. If the places lie on the same level a distance of 1800 m, find the height of the mountain.



⇨ Here, let, AB is the mountain. C is a point of observation in the north, D is a point of observation in the south.

CD = 1800 m.

$\measuredangle ACB = 55°$ and $\measuredangle ADB = 35°$, AB = ?

In right angled ΔBAC,

$\tan 55° = \frac{AB}{AC}$

or, $1.43 = \frac{AB}{AC}$

or, $AC = \frac{AB}{1.43}$ ..... (i)

In right angled ΔBAD,

$\tan 35° = \frac{AB}{AD}$

or, $0.70 = \frac{AB}{AD}$

or, $AD = \frac{AB}{0.70}$ ..... (ii)

Now, CD = AC + AD .... (iii)

From (i) , (ii) & (iii), $1800 \text{ m} = \frac{AB}{1.43} + \frac{AB}{0.70}$

or, $1800 \text{ m} = \frac{0.70AB + 1.43AB}{1.43 \times 0.70}$

or, $1800 \times 1.43 \times 0.70 \text{ m} = 2.13AB$

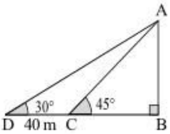
or, $AB = \frac{1801.8}{2.13} \text{ m}$

$\therefore$ AB = 845.92 m

Thus, height of mountain is 845.92 m.

(b) **कुनै एउटा निश्चित बिन्दुबाट ठिक अगाडि स्तम्भको टुप्पो हेर्दा 30° को उन्नतांश कोण पाइएको स्तम्भतिर 40 m अगाडि बढेर फेरि स्तम्भको टुप्पो हेर्दा उन्नतांश कोण 45° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of elevation of a tower as observed from a fixed point is 30°. On walking 40 m towards the tower, the angle of elevation was found to be 45°. Find the height of the tower.



⇨ Here, let AB be the height of a tower. $\measuredangle ADC = 30°$ and $\measuredangle ACB = 45°$ be the angles of elevation. CD = 40 m be the distance between the points.

From the right angled ΔACB, $\tan 45° = \frac{AB}{BC}$

or, $1 = \frac{AB}{BC}$ $\therefore$ AB = BC

From the right angled ΔABD, $\tan 30° = \frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{CD + BC}$ or, $\frac{1}{\sqrt{3}} = \frac{AB}{40 + AB}$

or, $\sqrt{3}\,AB = 40 + AB$

or, $\sqrt{3}\,AB - AB = 40$

or, $AB(\sqrt{3} - 1) = 40$

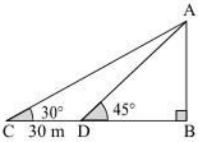
or, $AB = \frac{40}{\sqrt{3} - 1} = 54.64$

Thus, the height of the tower is 54.64 m.

4.

(a) **एउटा धरहराको टुप्पोमा सोही समतलमा धरहराबाट एकैतिर पर्ने दुईओटा बिन्दुहरूबाट हेर्दा उन्नतांश कोणहरू क्रमश: 30° र 45° पाइएछन् । यदि ती स्थानहरूबिचको दुरी 30 m रहेछ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।**

The top of a tower is observed from two points on the same sides, on the same line and on the same level of the tower and the angle of the elevation are found to be 30° and 45° respectively. If the distance between the places is 30 m then find the height of the tower.



⇨ Here, let AB be the height of the tower. $\measuredangle ACB = 30°$ and $\measuredangle ADB = 45°$ are the angle of elevations. CD = 30 m be the distance between the points.

From the right angled ΔADB, $\tan 45° = \frac{AB}{BD}$

or, $1 = \frac{AB}{BD}$ $\therefore$ AB = BD

Again, from the right angled ΔABC, $\tan 30° = \frac{AB}{BC}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{CD + BD}$ or, $\frac{1}{\sqrt{3}} = \frac{AB}{30 + AB}$

or, $30 + AB = \sqrt{3}\,AB$

or, $AB - \sqrt{3}\,AB = -30$

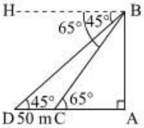
or, $AB(1 - \sqrt{3}) = -30$

or, $AB = \frac{30}{\sqrt{3} - 1}$ $\therefore$ AB = 40.98 m

Thus, the height of the tower is 40.98 m.

(c) **एउटा स्तम्भदेखि एकैतिर पर्ने 50 m को दुरीमा दुई बिन्दुहरूमा स्तम्भको टुप्पोबाट हेर्दा अवनति कोणहरू क्रमश: 45° र 65° छन् भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

How high is the tower if the angles of depression of two places 50 m apart when observed from the top of the tower are 45° and 65° ?



⇨ Here, let, AB be the tower . C and D are two places which are observed by the observer from the tower . HB be the horizon .

$\measuredangle HBD = \measuredangle ADB = 45°$

$\measuredangle HBC = \measuredangle BCA = 65°$, CD = 50 m, AB = ?

In right angled ΔABC, $\tan 65° = \frac{AB}{AC}$

or, $2.14 = \frac{AB}{AC}$ or, $AC = \frac{AB}{2.14}$ .... (i)

In right angled ΔABD, $\tan 45° = \frac{AB}{AD}$

$1 = \frac{AB}{AD}$ or, AD = AB .... (ii)

Now, AD = AC + CD .... (iii)

From (i), (ii) & (iii) $AB = \frac{AB}{2.14} + 50 \text{ m}$

or, $AB = \frac{AB + 107\text{m}}{2.14}$

or, $2.14AB - AB = 107 \text{ m}$

or, $1.14AB = 107 \text{ m}$

or, $AB = \frac{107\text{m}}{1.14} = 93.85 \text{ m}$

Thus, height of the tower is 93.85 m.

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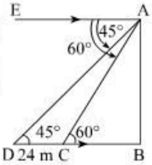
(d) **धरहराको टुप्पोबाट एउटै दिशातर्फ 24 m को दुरीमा रहेका कुनै दुई स्थान हेर्दा अवनति कोणहरू क्रमशः 60° र 45° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।**

From the top of a tower, the measures of angles of depression of two points which are at the distance of 24 m in the same direction are found to be 60° and 45° respectively. Find the height of the tower.

⇨ Here, let AB be the height of a tower. $\measuredangle EAD = 45°$ and $\measuredangle EAC = 60°$ are the angles of depressions. CD = 24 m be the distance between two points.



From the figure,

$\measuredangle ADB = \measuredangle EAD = 45°$ and

$\measuredangle ACB = \measuredangle EAC = 60°$

From the right angled triangle ABC, $\tan 60° = \dfrac{AB}{BC}$

or, $\sqrt{3} = \dfrac{AB}{BC}$ $\quad\therefore\quad AB = \sqrt{3}\,BC$

From the right angled triangle ABD; $\tan 45° = \dfrac{AB}{BD}$

or, $1 = \dfrac{AB}{BC + CD}$ $\quad$ or, $BC + CD = AB$

or, $BC + 24 = \sqrt{3}\,BC$ $\quad$ or, $24 = \sqrt{3}\,BC - BC$

or, $BC(\sqrt{3} - 1) = 24$

$\therefore\quad BC = \dfrac{24}{\sqrt{3} - 1} = 32.78$

Now, $AB = \sqrt{3} \times BC = \sqrt{3} \times 32.78 = 56.78$

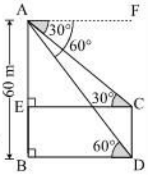
Thus the height of the tower is 56.78 m.

(b) **एउटा 60 m अग्लो घरको छतबाट घरको सिधा अगाडि रहेको बत्तीको खम्बाको टुप्पो र फेदमा हेर्दा अवनति कोणहरू क्रमश : 30° र 60° रहेका छन् भने खम्बाको उचाइ र खम्बादेखि घरसम्मको दुरी पत्ता लगाउनुहोस् ।**

From the top of a 60 m high house, the angles of depressions of the top and the bottom of an electric pole are 30° and 60° respectively. Find the height of the pole and distance between the pole and house.

⇨ Here, let AB = 60 m be the height of a house.

$\measuredangle FAC = 30°$ and $\measuredangle FAD = 60°$ are the angles of depressions of top and bottom of electric pole CD respectively.



From figure, $\measuredangle ACE = \measuredangle FAC = 30°$

and $\measuredangle ADB = \measuredangle FAD = 60°$

From the right angled ΔABD, $\tan 60° = \dfrac{AB}{BD}$

or, $\sqrt{3} = \dfrac{60°}{BD}$ $\quad$ or, $BD = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ m

From the right angled ΔAEC, $\tan 30° = \dfrac{AE}{EC}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{AE}{BD}$ $\quad$ or, $\dfrac{1}{\sqrt{3}} = \dfrac{AE}{20\sqrt{3}}$

$\therefore\quad AE = 20$ m

Now, CD = EB = AB – AE = 60 m – 20 m

$\therefore\quad CD = 40$ m

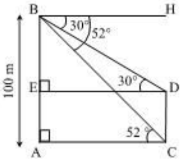
Thus, distance between pole and the house is $20\sqrt{3}$ m and the height of the pole is 40 m.

5.

(a) **कुनै 100 m अग्लो चट्टानको शिखरबाट कुनै रुखको टुप्पो र फेदमा हेर्दा अवनति कोणहरू क्रमशः 30° र 52° छन् भने रुखको उचाइ पत्ता लगाउनुहोस् ।**

From the top of a 100 m high cliff, the angles of depression of the top and the foot of a tree are found to be 30° and 52° respectively. How high is the tree?

⇨ Here, let, AB be the cliff and CD is a tree infront of AB. BH is the horizon.



AB = 100 m

$\measuredangle HBC = \measuredangle BCA = 52°$

$\measuredangle HBD = \measuredangle BDE = 30°$

In right angled ΔABC,

$\tan 52° = \dfrac{AB}{AC}$

or, $1.28 = \dfrac{100\text{ m}}{AC}$

or, $AC = \dfrac{100}{1.28}\text{ m} = 78.125$ m

In right angled ΔBDE, $\tan 30° = \dfrac{BE}{ED}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{BE}{AC}$ $\quad$ or, $\dfrac{1}{1.73} = \dfrac{BE}{78.125\text{ m}}$

or, $BE = \dfrac{78.125\text{ m}}{1.73}$ $\quad\therefore\quad BE = 45.16$ m

Now,

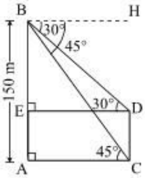
CD = AE = AB – BE = 100 m – 45.16 m = 54.84 m

Thus, height of tree is 54.84 m

(c) **कुनै 150 m अग्लो धरहरामा बसेर एउटा घरको धुरी र फेदमा हेर्दा अवनति कोणहरू क्रमशः 30° र 45° छन् भने घरको उचाइ र घर तथा धरहराबिचको दुरी पत्ता लगाउनुहोस् ।**

What is the height of a house if the angles of depression of it's top and foot are respectively 30° and 45° when observed from the top of a tower 150 m high ? Also find the distance between house and tower.

⇨ Here, let, AB be the tower and CD be the house. BH is horizon.



AB = 150 m, $\measuredangle HBD = \measuredangle BDE = 30°$

$\measuredangle HBC = \measuredangle ACB = 45°$

In right angled ΔABC, $\tan 45° = \dfrac{AB}{AC}$

or, $1 = \dfrac{AB}{AC}$

or, $1 = \dfrac{150\text{ m}}{AC}$

$\therefore\quad AC = 150$ m

In right angled ΔBDE, $\tan 30° = \dfrac{BE}{ED}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{BE}{AC}$

or, $BE = \dfrac{150\text{ m}}{\sqrt{3}}$

or, $BE = \dfrac{150\text{ m}}{1.732} = 86.60$ m

Now, CD = AE = AB – BE = 150 m – 86.60 m = 63.40 m

Thus, the height of the house is 63.40 m and distance between house and tower is 150 m.

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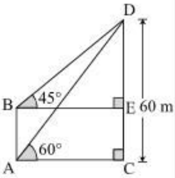
(d) **घरको छत र भुईंबाट ठिक अगाडि रहेको मन्दिरको टुप्पोमा हेर्दा क्रमशः 45° र 60° का उन्नतांश कोणहरू बनेका छन् । यदि मन्दिरको उचाइ 60 m छ भने घरको उचाइ तथा घर र मन्दिरबिचको दुरी पत्ता लगाउनुहोस् ।**

From the roof and foot of a house, the angles of elevations of the top of a temple are 45° and 60° respectively. If the height of the temple is 60 m, find the height of the house and the distance between the house and temple.



⇨ Here, let AB be the height of the house and CD = 60 m be the height of the temple.

$\measuredangle$DBE = 45° and $\measuredangle$DAC = 60° are the angles of elevation.

From the right angled ΔADC,

$$\tan 60° = \frac{CD}{AC}$$

$$\text{or, } \sqrt{3} = \frac{60}{AC}$$

$$\text{or, } AC = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$$

From the right angled ΔBDE,

$$\tan 45° = \frac{DE}{BE}$$

$$\text{or, } 1 = \frac{DE}{AC}$$

$$\text{or, } 1 = \frac{DE}{20\sqrt{3}}$$

$$\therefore \quad DE = 20\sqrt{3} \text{ m}$$

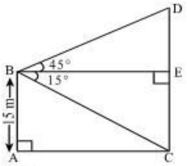
Now, $AB = EC = CD - DE = 60 \text{ m} - 20\sqrt{3} \text{ m}$

$= 25.35$ m

Thus, the height of the house is 25.35 m and the distance between the house and temple is $20\sqrt{3}$ m.

(b) **एउटा 15 m अग्लो घरको छतबाट टेलिभिजन टावरको टुप्पो र फेद हेर्दा क्रमशः उन्नतांश कोण 45° र अवनति कोण 15° छ भने टावरको उचाइ पत्ता लगाउनुहोस् ।**

From the top of a 15 m high house, an observer finds the angle of elevation of the top of the telecom reflector antenna to be 45° and the angle of depression of the foot of the antenna to be 15°. Find the height of the antenna.



⇨ Here, let, AB be the house and an observer is sitting at point B.CD is the telecom reflector antenna.

AB = CE = 15 m,

$\measuredangle$DBE = 45° & $\measuredangle$CBE = 15°

In right angled ΔBCE, $\tan 15° = \frac{CE}{BE}$

$$\text{or, } 0.267 = \frac{AB}{BE} \quad (\because CE = AB)$$

$$\text{or, } 0.267 = \frac{15\text{m}}{BE}$$

$$\therefore \quad BE = \frac{15 \text{ m}}{0.267} = 56.18 \text{ m}$$

In right angled ΔBDE, $\tan 45° = \frac{DE}{BE}$

$$\text{or, } 1 = \frac{DE}{56.18} \text{ m} \qquad \therefore \quad DE = 56.18 \text{ m}$$

Now , CD = CE + DE = AB + 56.18 m

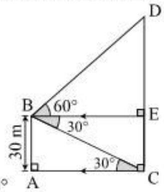
= 15 m + 56.18 m = 71.18 m

Thus, the height of the antenna is 71.18 m.

6.

(a) **एउटा 30 m अग्लो घरको छतबाट ठिक अगाडि रहेको स्तम्भको टुप्पो र फेदमा हेर्दा क्रमशः उन्नतांश कोण 60° र अवनति कोण 30° पाइयो भने स्तम्भको उचाइ र घर तथा स्तम्भबिचको दुरी पत्ता लगाउनुहोस् ।**

From the top of a 30 m high house, the angle of elevation and angle of depression of the top and the bottom of a tower are 60° and 30° respectively. Find the height of the tower and the distance between the tower and the house.



⇨ Here, let AB = 30 m be the height of a house. $\measuredangle$DBE = 60° be the angle of elevation and $\measuredangle$EBC = 30° be the angle of depression of top and bottom of the tower respectively.

From the figure, $\measuredangle$ACB = $\measuredangle$EBC = 30°

From the right angled ΔABC,

$$\tan 30° = \frac{AB}{AC}$$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{30}{AC}$$

$$\therefore \quad AC = 30\sqrt{3} \text{ m}$$

From the right angled ΔBDE, $\tan 60° = \frac{DE}{BE}$

$$\text{or, } \sqrt{3} = \frac{DE}{AC}$$

$$\text{or, } \sqrt{3} = \frac{DE}{30\sqrt{3}}$$

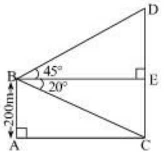
$$\therefore \quad DE = 90 \text{ m}$$

Now, CD = DE + EC = 90 m + 30 m = 120 m

Thus, height of the tower is 120 m and the distance between the tower and the house is $30\sqrt{3}$ m.

(c) **एउटा 200 m अग्लो टापुमा बसेर एउटा पहाडको टुप्पोमा हेर्दा उन्नतांश कोण 45° र फेदमा हेर्दा अवनति कोण 20° छ भने पहाडको उचाइ पत्ता लगाउनुहोस् ।**

From the top of a 200 m high island, the angle of elevation of the top of a mountain is found to be 45° and the angle of it's foot is 20°. Find the altitude of the mountain.



⇨ Here, let, AB be the island and CD is the mountain infront of AB .

AB = 200 m, $\measuredangle$DBE = 45° and $\measuredangle$CBE = 20°, CD = ?

In right angled ΔCBE,

$$\tan 20° = \frac{CE}{BE}$$

$$\text{or, } 0.36 = \frac{AB}{BE}$$

$$\text{or, } BE = \frac{200 \text{ m}}{0.36} = 555.56 \text{ m}$$

In right angled ΔDBE, $\tan 45° = \frac{DE}{BE}$

$$\text{or, } 1 = \frac{DE}{55.56 \text{ m.}}$$

$$\therefore \quad DE = 555.56 \text{ m}$$

Now, CD = CE + DE

= AB + DE

= 200 m + 555.56m

= 755.56 m

Thus, height of the mountain is 755.56 m.

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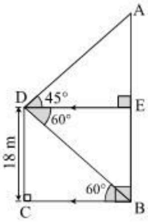
(d) **एउटा खम्बाको फेदबाट ठिक अगाडि रहेको घरको छतमा हेर्दा 60° को उन्नतांश कोण बन्छ र घरको छतबाट खम्बाको टुप्पोमा हेर्दा 45° को उन्नतांश कोण बन्छ । यदि घरको उचाइ 18 m भए खम्बाको उचाइ र घर तथा खम्बाबिचको दुरी पत्ता लगाउनुहोस् ।**

From the foot of a pole, the angle of elevation of the roof of a house which is infront of the pole is 60°. From the roof of the house, the angle of elevation of the top of the pole is 45°. If the height of the house is 18 m, find the height of the pole and the distance between the pole and the house.



⇨ Here, let CD = 18 m be the height of a house.

$\measuredangle DBC = 60°$ and $\measuredangle ADE = 45°$ are the angles of elevations.

From the right angled ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\text{or, } \sqrt{3} = \frac{18 \text{ m}}{CB}$$

$$\text{or, } BC = \frac{18 \text{ m}}{\sqrt{3}} = 6\sqrt{3} \text{ m}$$

From the right angled ΔADE,

$$\tan 45° = \frac{AE}{DE}$$

$$\text{or, } 1 = \frac{AE}{BC}$$

$$\text{or, } 1 = \frac{AE}{6\sqrt{3}}$$

$$\therefore \ AE = 6\sqrt{3} \text{ m}$$

Now, $AB = AE + EB = 6\sqrt{3} + CD = 6\sqrt{3} + 18$

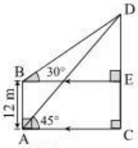
$\therefore \ AB = 28.39$ m

Thus, the height of the pole is 28.39 m and the distance between pole and house is $6\sqrt{3}$ m.

7.

(a) **एउटा 12 m अग्लो घरको छत र भुइँबाट घरको ठिक अगाडि रहेको रुखको टुप्पोमा हेर्दा उन्नतांश कोणहरू क्रमश: 30° र 45° का बन्दछन् भने रुखको उचाइ र घर र रुखबिचको दुरी पत्ता लगाउनुहोस् ।**

From the roof and floor of a 12 m high house, the angle of elevations of the top of a tree which is infront of the house are 30° and 45° respectively. Find the height of the tree and the distance between the house and tree.



⇨ Here, let AB = 12 m be the height of the house and CD be the height of the tree. $\measuredangle DBE = 30°$ and $\measuredangle DAC = 45°$ are the angle of elevations.

From the right angled ΔACD, $\tan 45° = \dfrac{CD}{AC}$

$$\text{or, } 1 = \frac{CD}{AC} \qquad \therefore \ AC = CD \text{ ....(i)}$$

From the right angled ΔBED, $\tan 30° = \dfrac{DE}{BE}$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{DE}{BE} \qquad \text{or, } \frac{1}{\sqrt{3}} = \frac{DE}{AC}$$

$AC = \sqrt{3}\, DE$ ....(ii)

From (i) and (ii); $\sqrt{3}\, DE = CD$

or, $\sqrt{3}\,(CD - EC) = CD$

or, $\sqrt{3}\, CD - \sqrt{3}\, EC = CD$

or, $\sqrt{3}\, CD - CD = \sqrt{3}\, EC$

or, $CD\,(\sqrt{3} - 1) = \sqrt{3}\, AB$

$$\text{or, } CD = \frac{\sqrt{3} \times 12}{\sqrt{3} - 1} \qquad \therefore \ CD = 28.39 \text{ m}$$

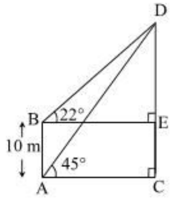
Now, from (i); AC = CD

$\therefore \ AC = 28.39$ m

Thus, height of the tree is 28.39 m and distance between the tree and house is 28.39 m.

(b) **कुनै 10 m अग्लो खम्बाको फेद र टुप्पोबाट खम्बादेखि ठिक अगाडि रहेको घण्टाघरको टुप्पोको उन्नतांश कोणहरू क्रमश: 45° र 22° छन् भने घण्टाघरको उचाइ र खम्बादेखि घण्टाघरको दुरी पत्ता लगाउनुहोस् ।**

A pole is 10 m high. From the foot and the top of the pole, the angles of elevation of a clock tower situated infront of the pole are found to be 45° and 22° respectively. Find the height of the clock tower and its distance from the pole.



⇨ Here, let AB be the pole and CD be the clock-tower situated infront of the pole. AB = 10 m, $\measuredangle DBE = 22°$ and $\measuredangle DAC = 45°$, CD = ?

In right angled ΔDAC, $\tan 45° = \dfrac{CD}{AC}$

$$\text{or, } 1 = \frac{CD}{AC}$$

or, CD = AC

In right angled ΔDBE,

$$\tan 22° = \frac{DE}{BE}$$

$$\text{or, } 0.40 = \frac{DE}{AC}$$

or, DE = 0.40 AC

Now, CD = CE + DE

or, AC = AB + 0.40AC

or, AC – 0.40AC = 10 m

or, 0.60 AC = 10 m

$$\text{or, } AC = \frac{10}{0.60} \text{ m} = 16.67 \text{ m}$$

$\therefore \ CD = AC = 16.67$ m

Thus, height of the clock -tower is 16.67 m and their separation is 16.67 m.

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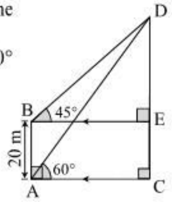
(c) **एउटा खम्बा 20 m अग्लो छ। खम्बाको ठिक अगाडि रहेको घरको छतमा खम्बाको टुप्पोबाट 45° र फेदबाट 60° को उन्नतांश कोण बन्दछ भने घरको उचाइ र खम्बादेखि घरसम्मको दुरी पत्ता लगाउनुहोस् ।**

A post is 20 m high. The angle of elevation of the top of a house which is infront of the post is 45° from its top and 60° from its foot. Find the height of the house and distance between house and the post.

⇨ Here, AB = 20 m be the height of the post. CD be the height of the house.
$\measuredangle DBE = 45°$ and $\measuredangle DAC = 60°$ are the angles of elevation.



From the right angled ΔACD,

$$\tan 60° = \frac{CD}{AC}$$

or, $\sqrt{3} = \frac{CD}{AC}$

$\therefore$ $CD = \sqrt{3}\,AC$ ....(i)

From right angled ΔBDE, $\tan 45° = \frac{DE}{BE}$

or, $1 = \frac{DE}{AC}$ $\qquad \therefore$ $AC = DE$ ....(ii)

From (i) & (ii) $CD = \sqrt{3}\,AC$

or, $DE + EC = \sqrt{3}\,AC$

or, $AC + AB = \sqrt{3}\,AC$

or, $AC + 20 = \sqrt{3}\,AC$

or, $20 = \sqrt{3}\,AC - AC$

or, $20 = (\sqrt{3} - 1)\,AC$

or, $AC = \frac{20}{\sqrt{3} - 1} = 27.32$ m

Now, $CD = \sqrt{3}\,AC = \sqrt{3} \times 27.32$

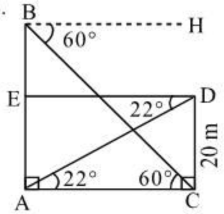
$\therefore$ $CD = 47.32$

Thus, the height of the house is 47.32 m and distance between the house and post is 27.32 m.

(d) **एउटा धरहराको टुप्पोबाट 20 m अग्लो स्तम्भको फेदमा हेर्दा अवनति कोण 60° र स्तम्भको टुप्पोबाट धरहराको फेदमा हेर्दा अवनति कोण 22° पाएछ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।**

From the top of a tower, the angle of depression of the foot of a 20 m high column on the ground is 60° and the angle of depression of the foot of the tower observed from the top of the column is 22°. What is the height of the tower ?

⇨ Here, let AB be the tower. CD be the column infront of AB.



CD = 20m

$\measuredangle HBC = \measuredangle BCA = 60°$

$\measuredangle EDA = \measuredangle DAC = 22°$

AB = ?

In right angled ΔACD,

$$\tan 22° = \frac{CD}{AC}$$

or, $0.40 = \frac{20\text{ m}}{AC}$

or, $AC = \frac{20\text{ m}}{0.40}$

$\therefore$ AC = 50 m

In right angled ΔACB,

$$\tan 60° = \frac{AB}{AC}$$

or, $\sqrt{3} = \frac{AB}{50\text{ m}}$

or, $AB = \sqrt{3} \times 50$ m

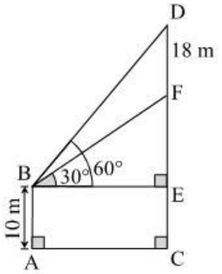
or, $AB = 1.732 \times 50\text{ m} = 86.60$ m

Thus, height of the tower is 86.60 m.

8. **(a)** **एउटा 10 m अग्लो घरको छतबाट आफ्नो अगाडि रहेको स्तम्भको टुप्पो हेर्दा 60° को उन्नतांश कोण र सो टुप्पोदेखि 18 m तल हेर्दा 30° को उन्नतांश कोण बन्छ भने स्तम्भको उचाइ र घरदेखि स्तम्भबिचको दुरी पत्ता लगाउनुहोस् ।**

The angle of elevation of the top of a tower from the roof of a building 10 m high is found to be 60°. From the same place, the angle of elevation at the point 18 m below the top of the tower is observed to be 30°. Find the height of the tower and the distance between the tower and building.

⇨ Here, AB = 10 m be the height of a house.
$\measuredangle DBE = 60°$ and $\measuredangle FBE = 30°$ are the angles of elevation.
DF = 18 m



From the right angled ΔBEF, $\tan 30° = \frac{EF}{BE}$

or, $\frac{1}{\sqrt{3}} = \frac{EF}{BE}$ $\qquad \therefore$ $BE = \sqrt{3}\,EF$ ....(i)

From right angled ΔBED, $\tan 60° = \frac{ED}{BE}$

or, $\sqrt{3} = \frac{ED}{BE}$ $\qquad$ or, $\sqrt{3} = \frac{DF + EF}{BE}$

or, $\sqrt{3} = \frac{18 + EF}{BE}$ $\qquad$ or, $\sqrt{3} = \frac{18 + EF}{\sqrt{3}\,EF}$

or, $3EF = 18 + EF$ $\qquad$ or, $2\,EF = 18$

$\therefore$ EF = 9 m

Now, height of tower (CD) = DF + EF + EC
= 18 m + 9 m + AB
= 18 m + 9 m + 10 m
= 37 m

Distance between tower and house = $BE = AC = \sqrt{3}\,EF = \sqrt{3} \times 9\text{ m} = 9\sqrt{3}$ m

Thus, the height of tower is 37 m and distance between tower and house is $9\sqrt{3}$ m.

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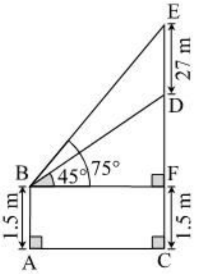
(b) **एकजना 1.5 m अग्लो मानिसले अगाडि रहेको एउटा भवनको छतमा र छतमाथि राखिएको टावरको टुप्पोमा हेर्दा 45° र 75° का दुई उन्नतांश कोणहरू बनाउँछ । यदि टावरको उचाइ 27 m भए भवनको उचाइ कति होला ? पत्ता लगाउनुहोस् ।**

A man 1.5 m tall observed the angle of elevations of the roof of a building and the top of the tower on the roof of the building 45° and the 75°. If the height of the tower is 27 m then find the height of the building.

⇨ Here, let AB = 1.5 m be the height of a man and CD be the height of building.

ED = 27 m be the height of the tower. $\measuredangle DBF = 45°$ and $\measuredangle EBF = 75°$ are the angles of elevation.



From the right angled ΔBDF,

$$\tan 45° = \frac{DF}{BF}$$

or, $1 = \frac{DF}{BF}$

∴ BF = DF

From right angled ΔEBF,

$$\tan 75° = \frac{EF}{BF}$$

or, $3.732 = \frac{ED + DF}{DF}$

or, $3.732 \times DF = 27 + DF$

or, $3.732 \times DF - DF = 27$

or, $DF (3.732 - 1) = 27$

∴ $DF = \frac{27}{2.732} = 9.88$ m

Now, height of the building = CD = DF + FC

$= 9.88 + 1.5 = 11.38$ m

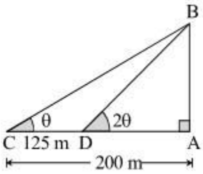
Thus, the height of the building is 11.38 m.

(c) **कुनै एउटा ठाउँबाट 200 m टाढा रहेको कुनै एउटा स्तम्भको टुप्पो हेर्दा जति डिग्रीको कोण बन्छ ठिक 125 m नजिक गएर सोही टुप्पोमा हेर्दा दोब्बरकोण बन्दछ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।**

The angle of the top of a tower observed from 125 m far is double of the angle observed from 200 m far from the tower. What is the height of the tower?

⇨ Here, let AB be the height of a tower. Let C and D are the points of observations.

So, AC = 200 m and CD = 125 m. $\measuredangle BCD = \theta$ and $\measuredangle BDA = 2\theta$ be the angles of elevation.



From the right angled ΔABC,

$$\tan \theta = \frac{AB}{AC}$$

or, $\tan \theta = \frac{AB}{200}$ ....(i)

From the right angled ΔABD,

$$\tan 2\theta = \frac{AB}{AD}$$

or, $\tan 2\theta = \frac{AB}{AC - CD}$

or, $\tan 2\theta = \frac{AB}{200 - 125}$

or, $\frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{AB}{75}$

or, $\frac{2 \times \frac{AB}{200}}{1 - \left(\frac{AB}{200}\right)^2} = \frac{AB}{75}$

or, $\frac{\frac{AB}{100}}{1 - \frac{AB^2}{40000}} = \frac{AB}{75}$

or, $\frac{AB}{100} \times \frac{40000}{40000 - AB^2} = \frac{AB}{75}$

or, $\frac{AB \times 400}{40000 - AB^2} = \frac{AB}{75}$

or, $40000 - AB^2 = 75 \times 400$

or, $40000 - 30000 = AB^2$

or, $10000 = AB^2$

∴ $AB = 100$

Thus, the height of the tower is 100 m.

9. **(a)** **आफ्नो विद्यालयको अग्लो भवनको एकैतिर दुई स्थानबाट भवनको छतको उन्नतांश कोण क्लिनोमिटरको प्रयोग गरी पत्ता लगाउनुहोस । दुई स्थानबिचको दुरी मिटरमा लिएर भवनको उचाइ पत्ता लगाउनुहोस् ।**

⇨ Here, show to your teacher.

(b) **आफ्नो टोल गाउँघरमा भएको कुनै अग्लो स्थान, स्तम्भ, टावर, धरहरा वा अग्लो अपार्टमेन्ट वा अन्य केही प्राकृतिक उचाइ भएका स्थानको उचाइ पत्ता लगाउने (सिधै ननापी) तरिका पत्ता लगाई र सोको प्रतिवेदन तयार गर्नुहोस् ।**

⇨ Here, show to your teacher.

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भेक्टर (Vector)

1. स्केलार गुणनफल
Vector Operation

FORMULAE

1. $\vec{a}$ र $\vec{b}$ कुनै दुई भेक्टरहरूका लागि (For any two vector $\vec{a}$ and $\vec{b}$):

   (i) $(\vec{a}+\vec{b})^2 = a^2 + b^2 + 2\vec{a}.\vec{b}$ (ii) $(\vec{a}-\vec{b})^2 = a^2 + b^2 - 2\vec{a}.\vec{b}$

   (iii) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = a^2 - b^2$ (iv) $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (v) $(\vec{a})^2 = a^2 = |\vec{a}|.|\vec{a}| = \vec{a}.\vec{a}$

2. यदि $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ भए (If $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ then), $\vec{a}\,.\,\vec{b} = (a_1.\,a_2 + b_1\,.\,b_2)$

or यदि $\vec{a} = (a_1, a_2)$ र $\vec{b} = (b_1, b_2)$ भए $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$ हुन्छ ।

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$, then $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$.

QUESTIONS FROM SEE EXERCISE 1

A. VERY SHORT QUESTIONS

1. **दुई भेक्टरहरूको स्केलर गुणनफलमा समानान्तर हुने अवस्था लेख्नुहोस् ।**
   State the condition of parallelism in scalar product of two vectors.

⇨ Here, two vectors $\vec{a}$ and $\vec{b}$ are parallel if $\vec{a} = m\vec{b}$; where m is any scalar.

2. **यदि $\vec{a}$ र $\vec{b}$ बिचको कोण θ भए $\vec{a}$ र $\vec{b}$ को स्केलर गुणनफल के हुन्छ ?**
   What is the scalar product of two vectors $\vec{a}$ and $\vec{b}$ if the angle between them is θ ? **[SEE MODEL 2076]**

⇨ Here, the scalar product between $\vec{a}$ and $\vec{b}$ is ; $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$.

3. **स्केलर गुणनफल $(\vec{a}\,.\,\vec{b})$ पत्ता लगाउने सूत्र लेख्नुहोस् । जहाँ $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ छन् ।**

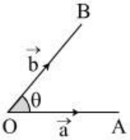
   Write down the formula to calculate scalar product $(\vec{a}\,.\,\vec{b})$, where $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

⇨ Here, $\vec{a}\,.\,\vec{b} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}.\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = a_1\,.\,a_2 + b_1\,.\,b_2$

4. **यदि x - अक्षतिरको एकाइ भेक्टर $\vec{i}$ भए $\vec{i}^{\,2}$ को मान कति हुन्छ ?**
   If $\vec{i}$ is unit vector along X-axis, what is the value of $\vec{i}^{\,2}$?

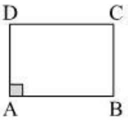
⇨ Here, $\vec{i}$ is unit vector along X-axis. So, the value of $\vec{i}^{\,2}$ is 1.

5. **चित्रमा दिइएको जानकारीबाट $\vec{a}.\vec{b}$ पत्ता लगाउने सूत्र लेख्नुहोस् ।**
   From the information given in the figure, write the formula to calculate $\vec{a}.\vec{b}$.



⇨ Here, the required formula is; $\vec{a}.\vec{b} = |\vec{a}|\,.\,|\vec{b}|\cos\theta$

6. **चित्रमा ABCD एउटा आयत हो । $\vec{AB}.\,\vec{BC}$ को मान कति होला ?**
   In the figure ABCD is a rectangle. What is the value of $\vec{AB}.\,\vec{BC}$?



⇨ Here, the value of $\vec{AB}.\,\vec{BC}$ is 0 because the angle between two vectors is 90°.

7. **यदि $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ र $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ भए $\vec{i}\,.\,\vec{j}$ पत्ता लगाउनुहोस् (If $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ then find $\vec{i}\,.\,\vec{j}$):**

⇨ Here, $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ So, $\vec{i}\,.\,\vec{j} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1 \times 0 + 0 \times 1 = 0$

8. **यदि $\vec{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् । (If $\vec{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ then find $\vec{a}.\vec{b}$.)**

⇨ Here, $\vec{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ So, $\vec{a}\,.\,\vec{b} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \end{pmatrix} = 2.3 + 0.0 = 6$

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9. यदि $|\vec{a}| = 4$ भए $|\vec{a}|^2$ पत्ता लगाउनुहोस् । (If $|\vec{a}| = 4$ then find $|\vec{a}|^2$.)

⇨ Here, $|\vec{a}| = 4$ So, $|\vec{a}|^2 = |\vec{a}| . |\vec{a}| = 4 . 4 = 16$

10. यदि $\vec{a} = 4\vec{i}$ भए a^2 पत्ता लगाउनुहोस् । (If $\vec{a} = 4\vec{i}$ then find a^2.)

⇨ Here, $\vec{a} = 4\vec{i}$ so, $a^2 = (\vec{a})^2 = (4\vec{i})^2 = 16$ $(\because i . i = i^2 = 1)$

11. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू हुन् र $\vec{a}.\vec{b} = \frac{1}{2}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}.\vec{b} = \frac{1}{2}$ then find the angle between $\vec{a}$ and $\vec{b}$.

⇨ Here, $\vec{a}$ and $\vec{b}$ are unit vectors so, $|\vec{a}| = |\vec{b}| = 1$

$\vec{a} . \vec{b} = |\vec{a}| \, |\vec{b}| \cos\theta$ or, $\frac{1}{2} = 1 . 1 \cos\theta$

or, $\cos\theta = \cos 60°$ $\therefore$ $\theta = 60°$

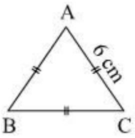
Thus, angle between $\vec{a}$ and $\vec{b}$ is 60°.

B. SHORT QUESTIONS

MODEL 1

1. दिइएको $\triangle ABC$ मा $AB = BC = CA = 6$ से.मि. छन् भने $\overrightarrow{BA} . \overrightarrow{BC}$ को मान निकाल्नुहोस् । **[2074 S]**

In the given $\triangle ABC$, $AB = BC = CA = 6$ cm. Find the value of $\overrightarrow{BA} . \overrightarrow{BC}$.



⇨ Here, $AB = BC = CA = 6$ cm & $\measuredangle ABC = 60°$

We know that,

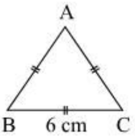
$$\overrightarrow{BA} . \overrightarrow{BC} = |\overrightarrow{BA}| . |\overrightarrow{BC}| \cos \measuredangle ABC$$
$$= 6 . 6 . \cos 60°$$
$$= 36 . \frac{1}{2}$$

$\therefore$ $\overrightarrow{BA} . \overrightarrow{BC} = 18$

Thus, the value of $\overrightarrow{BA} . \overrightarrow{BC}$ is 18.

2. दिइएको ΔABC मा $AB = BC = CA = 6$ cm छन् भने $\overrightarrow{AB} . \overrightarrow{AC}$ को मान निकाल्नुहोस् । **[2074 R]**

In the given ΔABC, $AB = BC = CA = 6$ cm. Find the value of $\overrightarrow{AB} . \overrightarrow{AC}$.



⇨ Here, $AB = BC = CA = 6$ cm & $\measuredangle BAC = 60°$

We know that,

$$\overrightarrow{AB} . \overrightarrow{AC} = |\overrightarrow{AB}| . |\overrightarrow{AC}| \cos \measuredangle BAC$$
$$= 6 . 6 . \cos 60°$$
$$= 36 . \frac{1}{2}$$

$\therefore$ $\overrightarrow{AB} . \overrightarrow{AC} = 18$

Thus, the value of $\overrightarrow{AB} . \overrightarrow{AC}$ is 18.

3. यदि $|\vec{a}| = 3\sqrt{3}$, $|\vec{b}| = 4$ र $\vec{a}$ र $\vec{b}$ बीचको कोण $(\theta) = 60°$ भए $\vec{a} . \vec{b}$ को मान पत्ता लगाउनुहोस् ।

If $|\vec{a}| = 3\sqrt{3}$, $|\vec{b}| = 4$ and angle between $\vec{a}$ and $\vec{b}$, $(\theta) = 60°$ then find the value of $\vec{a} . \vec{b}$. **[2073 R']**

⇨ Here, $|\vec{a}| = 3\sqrt{3}$, $|b| = 4$ and $\theta = 60°$.

We know that,

$$\text{scalar product } (\vec{a} . \vec{b}) = |a| \, |b| \cos\theta$$
$$= 3\sqrt{3} \times 4 \times \cos 60°$$
$$= 3\sqrt{3} \times 4 \times \frac{1}{2} = 6\sqrt{3}$$

Thus, the scalar product is $6\sqrt{3}$.

4. यदि $|\vec{a}| = 5\sqrt{3}$, $|\vec{b}| = 6$ र $\theta = 30°$ भए $\vec{a}.\vec{b}$ को मान पत्ता लगाउनुहोस् ।

If $|\vec{a}| = 5\sqrt{3}$, $|\vec{b}| = 6$ and $\theta = 30°$, find the value of $\vec{a}.\vec{b}$ **[2070 R']**

⇨ Here, $|\vec{a}| = 5\sqrt{3}$, $|\vec{b}| = 6$ and $\theta = 30°$

We know that, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

or, $\cos 30° = \dfrac{\vec{a}.\vec{b}}{5\sqrt{3} \times 6}$

or, $\dfrac{\sqrt{3}}{2} = \dfrac{\vec{a}.\vec{b}}{5\sqrt{3} \times 6}$

or, $\vec{a}.\vec{b} = 45$ Thus, the value of $\vec{a}.\vec{b}$ is 45.

5. यदि $\vec{i}$ र $\vec{j}$, x-अक्ष र y-अक्षतिरका एकाइ भेक्टरहरू भए $\vec{i}.\vec{i} = 1$ र $\vec{i}.\vec{j} = 0$ हुन्छन् भनी प्रमाणित गर्नुहोस् । **[2065 R]**

If $\vec{i}$ and $\vec{j}$ are unit vectors along the x-axis and y-axis respectively, prove that $\vec{i}.\vec{i} = 1$ and $\vec{i}.\vec{j} = 0$.

⇨ Here, we have, $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Now, $\vec{i} . \vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1.1 + 0.0$ $[\because \text{Scalar product} = x_1x_2 + y_1y_2]$ $\therefore$ $\vec{i} . \vec{i} = 1$ **Proved.**

Again, $\vec{i} . \vec{j} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1.0 + 0.1 = 0 + 0$ $\therefore$ $\vec{i} . \vec{j} = 0$ **Proved.**

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6. **यदि $\vec{a} = (a_1, a_2)$ र $\vec{b} = (b_1, b_2)$ भए $\vec{a}.\vec{b} = a_1b_1 + a_2b_2$ हुन्छ भनी सिद्ध गर्नुहोस् ।**

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$, prove that $\vec{a}.\vec{b} = a_1b_1 + a_2b_2$. [2067 R]

⇨ Here, $\vec{a} = (a_1, a_2) = (a_1\vec{i} + a_2\vec{j})$ and

$\vec{b} = (b_1, b_2) = (b_1\vec{i} + b_2\vec{j})$

Now,
$$\begin{aligned}\vec{a}.\vec{b} &= (a_1\vec{i} + a_2\vec{j})(b_1\vec{i} + b_2\vec{j})\\ &= a_1b_1\vec{i}^2 + a_1b_2\vec{i}.\vec{j} + a_2b_1\vec{j}.\vec{i} + a_2b_2\vec{j}^2\\ &= a_1b_1.1 + 0 + 0 + a_2b_2.1\\ &\qquad [\vec{i}^2 = 1, \vec{j}^2 = 1, \vec{i}.\vec{j} = 0, \vec{j}.\vec{i} = 0]\\ &= a_1b_1 + a_2b_2\end{aligned}$$

Thus, $\vec{a}.\vec{b} = a_1b_1 + a_2b_2$ completes the proof.

8. **यदि $|\vec{a}| = 4\sqrt{2}, |\vec{b}| = 6$ र $\theta = 45°$ भए $\vec{a}.\vec{b}$ को मान पत्ता लगाउनुहोस् ।**

If $|\vec{a}| = 4\sqrt{2}, |\vec{b}| = 6$ र $\theta = 45°$ find the value of $\vec{a}.\vec{b}$. [2068 R]

⇨ Here, $|\vec{a}| = 4\sqrt{2}, |\vec{b}| = 6$ and $\theta = 45°$

We know that, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

or, $\cos 45° = \dfrac{\vec{a}.\vec{b}}{4\sqrt{2}\times 6}$

or, $\dfrac{1}{\sqrt{2}} = \dfrac{\vec{a}.\vec{b}}{4\sqrt{2}\times 6}$

$\therefore \quad \vec{a}.\vec{b} = 24$

Thus, the value of $\vec{a}.\vec{b}$ is 24.

10. **यदि $|\vec{a}| = 10, |\vec{b}| = 3$ र $\vec{a}.\vec{b} = 15\sqrt{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।**

If $|\vec{a}| = 10, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 15\sqrt{2}$ then find the angle between $\vec{a}$ and $\vec{b}$. [SEE 2075 R']

⇨ Here, $|\vec{a}| = 10, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 15\sqrt{2}$

We know that,

Angle between two vectors is given by,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$$

or, $\cos\theta = \dfrac{15\sqrt{2}}{10\times 3} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

or, $\cos\theta = \cos 45°$

$\therefore \quad \theta = 45°$

Thus, angle between $\vec{a}$ and $\vec{b}$ is 45°.

7. **यदि $\vec{a} = \begin{pmatrix}2\\3\end{pmatrix}$ र $\vec{b} = \begin{pmatrix}3\\-1\end{pmatrix}$ भए a^2 र b^2 पत्ता लगाउनुहोस् ।**

If $\vec{a} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}3\\-1\end{pmatrix}$ then find a^2 and b^2. [2066 S]

⇨ Here, $\vec{a} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}3\\-1\end{pmatrix}$

So,
$$\begin{aligned}a^2 &= \vec{a}.\vec{a} = \begin{pmatrix}2\\3\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix}\\ &= 2\times 2 + 3\times 3 = 13\\ b^2 &= \vec{b}.\vec{b} = \begin{pmatrix}3\\-1\end{pmatrix}\begin{pmatrix}3\\-1\end{pmatrix}\\ &= 3\times 3 + (-1)\times(-1) = 10\end{aligned}$$

Thus, $a^2 = 13$ and $b^2 = 10$

MODEL 2

9. **यदि $|\vec{a}| = 2, |\vec{b}| = 12$ र $\vec{a}.\vec{b} = 12$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।**

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 2, |\vec{b}| = 12$ and $\vec{a}.\vec{b} = 12$. [SEE MODEL 2076]

⇨ Here, $|\vec{a}| = 2, |\vec{b}| = 12$ and $\vec{a}.\vec{b} = 12$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Then, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{12}{2\times 12}$

or, $\cos\theta = \dfrac{1}{2}$

or, $\cos\theta = \cos 60°$

$\therefore \quad \theta = 60°$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is 60°.

11. **यदि $|\vec{p}| = 4, |\vec{q}| = 3$ र $\vec{p}.\vec{q} = 6$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।**

If $|\vec{p}| = 4, |\vec{q}| = 3$ and $\vec{p}.\vec{q} = 6$ then find the angle between $\vec{p}$ and $\vec{q}$. [SEE 2075 R'$_2$]

⇨ Here, $|\vec{p}| = 4, |\vec{q}| = 3$ and $\vec{p}.\vec{q} = 6$

We know that, angle between $\vec{p}$ and $\vec{q}$ is given by,

$$\cos\theta = \frac{\vec{p}.\vec{q}}{|\vec{p}|.|\vec{q}|} = \frac{6}{4\times 3} = \frac{1}{2}$$

or, $\cos\theta = \dfrac{1}{2} = \cos 60°$

$\therefore \quad \theta = 60°$

Thus, angle between $\vec{p}$ and $\vec{q}$ is 60°.

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12. यदि $|\vec{a}| = 4, |\vec{b}| = 5$ र $\vec{a}.\vec{b} = 10$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $|\vec{a}| = 4, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10$ then find the angle between $\vec{a}$ and $\vec{b}$. [2073 R]

⇨ Here, $|\vec{a}| = 4, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10$

Let θ be angle between $\vec{a}$ and $\vec{b}$ then,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{10}{4\times5} = \frac{1}{2}$$

or, $\cos\theta = \cos 60°$

$\therefore\ \theta = 60°$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is 60°.

14. यदि $\vec{a} = 4\vec{i} + 2\vec{j}$ र $\vec{b} = -\vec{i} + 2\vec{j}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$, then find the angle between $\vec{a}$ and $\vec{b}$. [2072 R]

⇨ Here, $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$

So, $\vec{a}.\vec{b} = (4\vec{i} + 2\vec{j}).(-\vec{i} + 2\vec{j})$

$= -4 + 4$

$\therefore\ \vec{a}.\vec{b} = 0$

Since $\vec{a}.\vec{b} = 0$ So, the given vectors are perpendicular vectors.
Thus, the angle between the vectors is 90°.

15. यदि $\vec{p}.\vec{q} = 18\sqrt{3}, |\vec{p}| = 6$ र $|\vec{q}| = 6$ भए $\vec{p}$ र $\vec{q}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{p}.\vec{q} = 18\sqrt{3}, |\vec{p}| = 6$ and $|\vec{q}| = 6$, find the angle between $\vec{p}$ and $\vec{q}$. [2071 S]

⇨ Here, $\vec{p}.\vec{q} = 18\sqrt{3}, |\vec{p}| = 6$ and $|\vec{q}| = 6$

Let θ be angle between $\vec{p}$ and $\vec{q}$ then,

$$\cos\theta = \frac{\vec{p}.\vec{q}}{|\vec{p}||\vec{q}|} = \frac{18\sqrt{3}}{6\times6} = \frac{\sqrt{3}}{2}$$

or, $\cos\theta = \cos 30°$

$\therefore\ \theta = 30°$

Thus, the angle between $\vec{p}$ and $\vec{q}$ is 30°.

13. यदि $|\vec{a}| = 4, |\vec{b}| = 5$ र $\vec{a}.\vec{b} = 10\sqrt{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।

If $|\vec{a}| = 4, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10\sqrt{2}$, then find the angle between $\vec{a}$ and $\vec{b}$. [2073 S']

⇨ Here, $|\vec{a}| = 4, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10\sqrt{2}$

Let, θ be angle between $\vec{a}$ and $\vec{b}$ then,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{10\sqrt{2}}{4\times5} = \frac{10\sqrt{2}}{20} = \frac{1}{\sqrt{2}}$$

or, $\cos\theta = \cos 45°$

$\therefore\ \theta = 45°$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is 45°.

16. यदि $\vec{p} + \vec{q} + \vec{r} = 0, |\vec{p}| = 6, |\vec{q}| = 7$ र $|\vec{r}| = \sqrt{127}$ भए $\vec{p}$ र $\vec{q}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{p} + \vec{q} + \vec{r} = 0, |\vec{p}| = 6, |\vec{q}| = 7$ and $|\vec{r}| = \sqrt{127}$, find the angle between $\vec{p}$ and $\vec{q}$. [2071 R]

⇨ Here, $\vec{p} + \vec{q} + \vec{r} = 0, |\vec{p}| = 6, |\vec{q}| = 7$ and $|\vec{r}| = \sqrt{127}$

$\vec{p} + \vec{q} = -\vec{r}$

or, $(\vec{p} + \vec{q})^2 = (-\vec{r})^2$

or, $p^2 + q^2 + 2\vec{p}.\vec{q} = r^2$

or, $6^2 + 7^2 + 2\vec{p}.\vec{q} = 127$

or, $2\vec{p}.\vec{q} = 42$

$\therefore\ \vec{p}.\vec{q} = 21$

Let θ be the angle $\vec{p}$ and $\vec{q}$ then

$$\cos\theta = \frac{\vec{p}.\vec{q}}{|\vec{p}||\vec{q}|}$$

or, $\cos\theta = \dfrac{21}{6.7}$

or, $\cos\theta = \dfrac{1}{2}$

or, $\cos\theta = \cos 60°$

$\therefore\ \theta = 60°$

Thus, the angle between $\vec{p}$ and $\vec{q}$ is 60°.

17. यदि $\vec{a} = 10\vec{i} - 7\vec{j}$ र $\vec{b} = 7\vec{i} + 10\vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a} = 10\vec{i} - 7\vec{j}$ and $\vec{b} = 7\vec{i} + 10\vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. [2061 S]

⇨ Here, given vectors, $\vec{a} = 10\vec{i} - 7\vec{j}$ and $\vec{b} = 7\vec{i} + 10\vec{j}$

If θ be angle between two vectors $\vec{a}$ and $\vec{b}$, then θ = ?

We have, $\vec{a}.\vec{b} = (10\vec{i} - 7\vec{j}).(7\vec{i} + 10\vec{j})$

$= 70\,\vec{i}.\vec{i} + 100\,\vec{i}.\vec{j} - 49\,\vec{j}.\vec{i} - 70\,\vec{j}.\vec{j}$

$= 70 \times 1 - 70 \times 1$ $\quad [\because \vec{i}.\vec{i} = \vec{j}.\vec{j} = 1, \vec{i}.\vec{j} = \vec{j}.\vec{i} = 0]$

$= 70 - 70$

$= 0$

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Again, $|\vec{a}| = |10\vec{i} - 7\vec{j}|$
$= \sqrt{10^2 + (-7)^2}$
$= \sqrt{100 + 49}$
$= \sqrt{149}$

$|\vec{b}| = |7\vec{i} + 10\vec{j}|$
$= \sqrt{7^2 + 10^2}$
$= \sqrt{49 + 100}$
$= \sqrt{149}$

We know that, $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|.|\vec{b}|}$

So, $\cos\theta = \dfrac{0}{\sqrt{149}.\sqrt{149}} = 0 = \cos 90°$

$\therefore\ \theta = 90°$

Thus, the required angle between two vectors is $(\theta) = 90°$.

18. यदि $\vec{a} = \vec{i} + 3\vec{j}$ र $\vec{b} = 2\vec{i} + \vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बीचमा हुने कोण पत्ता लगाउनुहोस् ।

If $\vec{a} = \vec{i} + 3\vec{j}$ and $\vec{b} = 2\vec{i} + \vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. [2060 R]

⇨ Here, given vectors $\vec{a} = \vec{i} + 3\vec{j} = (1, 3)$

$\vec{b} = 2\vec{i} + \vec{j} = (2, 1)$

If θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

So, $\vec{a}\cdot\vec{b} = (1, 3).(2, 1) = 1.2 + 3.1 = 2 + 3 = 5$

and $|\vec{a}| = |(1, 3)| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$|\vec{b}| = |(2, 1)| = \sqrt{2^2 + 1^2} = \sqrt{5}$

Now, $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{5}{\sqrt{10}\sqrt{5}} = \dfrac{5}{\sqrt{50}}$

$= \dfrac{5}{\sqrt{2\times 25}} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$= \cos 45°$

$\therefore\ \theta = 45°$

Thus, the angle between $\vec{a}$ and $\vec{b}$ $(\theta) = 45°$

19. यदि $\vec{a} = 2\vec{i} - \vec{j}$ र $\vec{b} = \vec{i} - 3\vec{j}$ भए भेक्टरहरू बीचको कोण पत्ता लगाउनुहोस् ।

Find the angle between the vectors $\vec{a} = 2\vec{i} - \vec{j}$ and $\vec{b} = \vec{i} - 3\vec{j}$. [2068 R']

⇨ Here, $\vec{a} = 2\vec{i} - \vec{j} = (2, -1)$, $\vec{b} = \vec{i} - 3\vec{j} = (1, -3)$

Let, θ be an angle between $\vec{a}$ & $\vec{b}$

Now, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

or, $\cos\theta = \dfrac{(2, -1).(1, -3)}{\sqrt{(2)^2 + (-1)^2}.\sqrt{(1)^2 + (-3)^2}}$

or, $\cos\theta = \dfrac{2 + 3}{\sqrt{4 + 1}\sqrt{1 + 9}}$

or, $\cos\theta = \dfrac{5}{\sqrt{5}\sqrt{10}}$

or, $\cos\theta = \dfrac{5}{\sqrt{5}\times\sqrt{5}\times\sqrt{2}}$

or, $\cos\theta = \dfrac{5}{5\sqrt{2}}$

or, $\cos\theta = \cos 45°$

or, $\theta = 45°$

Thus, the required angle between $\vec{a}$ & $\vec{b}$ is 45°.

20. यदि $\vec{a} = 3\vec{i} + 4\vec{j}$ र $\vec{b} = 8\vec{i} - 6\vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बीचमा हुने कोण पत्ता लगाउनुहोस् ।

If $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 8\vec{i} - 6\vec{j}$, find the angle between the vectors $\vec{a}$ and $\vec{b}$. [2058 S]

⇨ Here, given vectors are

$\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 8\vec{i} - 6\vec{j}$

Now, $\vec{a}.\vec{b} = (3\vec{i} + 4\vec{j}).(8\vec{i} - 6\vec{j})$

$= 24\vec{i}^2 - 18\vec{i}.\vec{j} + 32\vec{j}.\vec{i} - 24\vec{j}^2$

$= 24 - 24$

Again,

$|\vec{a}| = |3\vec{i} + 4\vec{j}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$|\vec{b}| = |8\vec{i} - 6\vec{j}| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$

If θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then,

Using formula, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

or, $\cos\theta = \dfrac{0}{5\times 10} = \dfrac{0}{50} = 0 = \cos 90°$

Thus, θ = 90° is the required angle.

21. यदि $|\vec{OA}| = 6$, $|\vec{OB}| = 7$ र $\vec{OA}.\vec{OB} = 21$ भए ∡AOB को मान पत्ता लगाउनुहोस् ।

If $|\vec{OA}| = 6$, $|\vec{OB}| = 7$ and $\vec{OA}.\vec{OB} = 21$, find the value of the ∡AOB. [2062 S]

⇨ Here, $|\vec{OA}| = 6$, $|\vec{OB}| = 7$ and $\vec{OA}.\vec{OB} = 21$

Let, θ be the angle between $\vec{OA}$ and $\vec{OB}$ then,

$\cos\theta = \pm\dfrac{\vec{OA}.\vec{OB}}{|\vec{OA}||\vec{OB}|}$

or, $\cos\theta = \pm\dfrac{21}{6\times 7}$

or, $\cos\theta = \pm\dfrac{1}{2}$

or, $\cos\theta = \cos 60°$ or $120°$

Thus, the value of ∡AOB is 60° or 120°.

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22. यदि $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\3\sqrt{3}\end{pmatrix}$ भए कोण AOB को मान निकाल्नुहोस् ।

If $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\3\sqrt{3}\end{pmatrix}$, calculate the angle AOB. [2061 R]

⇨ Here, given vectors,

$$\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} = \sqrt{3}\,\vec{i} + 1\,\vec{j} \text{ and}$$

$$\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\3\sqrt{3}\end{pmatrix} = \sqrt{3}\,\vec{i} + 3\sqrt{3}\,\vec{j}$$

So, $\overrightarrow{OA}\,.\overrightarrow{OB}$

$= (\sqrt{3}\,\vec{i} + \vec{j}).(\sqrt{3}\,\vec{i} + 3\sqrt{3}\,\vec{j})$

$= 3\,\vec{i}\,.\vec{i} + 3\times3\,\vec{i}\,.\vec{j} + \sqrt{3}\,\vec{j}\,.\vec{i} + 3\sqrt{3}\,\vec{j}\,.\vec{j}$

$= 3 + 3\sqrt{3}$

$= 3(1 + \sqrt{3})$

Again, $|\overrightarrow{OA}| = |\sqrt{3}\,\vec{i} + \vec{j}|$

$= \sqrt{(\sqrt{3})^2 + 1^2}$

$= \sqrt{3 + 1}$

$= 2$

$|\overrightarrow{OB}| = |\sqrt{3}\,\vec{i} + 3\sqrt{3}\,\vec{j}|$

$= \sqrt{(\sqrt{3})^2 + (3\sqrt{3})^2)}$

$= \sqrt{3 + 9\times3}$

$= \sqrt{30}$

If $\measuredangle AOB = \theta$,

Then, using formula $\cos\theta = \dfrac{\overrightarrow{OA}\,.\overrightarrow{OB}}{|\overrightarrow{OA}|\,|\overrightarrow{OB}|}$

or, $\cos\theta = \dfrac{3\times(1+\sqrt{3})}{2\times\sqrt{30}}$

$= 0.748$

$= \cos 41.56°$

Thus, the value of $\measuredangle$AOB is 41.56°

23. यदि $\overrightarrow{OA} = 7\vec{i} - 5\vec{j}$ र $\overrightarrow{OB} = 5\vec{i} - 7\vec{j}$ भए $\measuredangle$AOB को मान पत्ता लगाउनुहोस् ।

If $\overrightarrow{OA} = 7\vec{i} - 5\vec{j}$ and $\overrightarrow{OB} = 5\vec{i} - 7\vec{j}$, find the value of $\measuredangle$AOB. [2057 S]

⇨ Here, given vectors;

$\overrightarrow{OA} = 7\,\vec{i} - 5\,\vec{j}$ and $\overrightarrow{OB} = 5\,\vec{i} - 7\,\vec{j}$

So,

$|\overrightarrow{OA}| = |7\,\vec{i} - 5\,\vec{j}\,| = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$

$|\overrightarrow{OB}| = |5\,\vec{i} - 7\,\vec{j}\,| = \sqrt{5^2 + (-7)^2} = \sqrt{25 + 49} = \sqrt{74}$

Again,

$\overrightarrow{OA}\,.\,\overrightarrow{OB} = (7\,\vec{i} - 5\,\vec{j}\,).(5\,\vec{i} - 7\,\vec{j}\,)$

$= 35\,\vec{i}^{\,2} - 49\,\vec{i}\,.\,\vec{j} - 25\,\vec{j}\,.\,\vec{i} + 35\,\vec{j}^{\,2}$

$= 35 + 35 = 70$

If θ is the angle between two given vectors, then

Using formula, $\cos\theta = \dfrac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}|\,|\overrightarrow{OB}|}$

$\therefore\ \cos\theta = \dfrac{70}{\sqrt{74}.\sqrt{74}} = \dfrac{70}{74} = 0.946$

$= \cos 18.92°$

Thus, the value of $\measuredangle$AOB is 18.92°

24. यदि $\overrightarrow{OA} = 4\,\vec{i} + 2\,\vec{j}$ र $\overrightarrow{OB} = -\vec{i} + 2\,\vec{j}$ भए $\measuredangle$AOB पत्ता लगाउनुहोस् ।

If $\overrightarrow{OA} = 4\,\vec{i} + 2\,\vec{j}$ and $\overrightarrow{OB} = -\,\vec{i} + 2\,\vec{j}$, find the $\measuredangle$AOB. [2066 R]

⇨ Here, $\overrightarrow{OA} = 4\,\vec{i} + 2\,\vec{j}$ and $\overrightarrow{OB} = -\,\vec{i} + 2\,\vec{j}$

$\overrightarrow{OA}\,.\,\overrightarrow{OB} = (4\,\vec{i} + 2\,\vec{j}\,).\ (-\,\vec{i} + 2\,\vec{j}\,)$

$= 4(-1) + 2.2$

$= -4 + 4 = 0$

$\therefore\ \overrightarrow{OA}\,.\,\overrightarrow{OB} = 0$ shows that $\overrightarrow{OA} \perp \overrightarrow{OB}$.

Thus, the value of $\measuredangle$AOB is 90°.

MODEL 3

25. यदि $\vec{p}\,.\vec{q} = 6$, $|\vec{q}\,| = 2\sqrt{3}$ एकाइ र $\vec{p}$ र $\vec{q}$ बिचको कोण 30° भए $\vec{p}$ को लम्बाइ पत्ता लगाउनुहोस् ।

If $\vec{p}\,.\vec{q} = 6$, $|\vec{q}\,| = 2\sqrt{3}$ unit and the angle between $\vec{p}$ and $\vec{q}$ is 30°, find the length of $\vec{p}$.

[SEE 2075 R, 2075 R$_2$]

⇨ Here, $\vec{p}\,.\vec{q} = 6$, $|\vec{q}\,| = 2\sqrt{3}$ unit and angle (θ) = 30°

We know that, $\vec{p}\,.\vec{q} = |\vec{p}\,|\,.\,|\vec{q}\,|\cos\theta$

or, $6 = |\vec{p}\,|\,.\,2\sqrt{3}\times\cos 30°$

or, $|\vec{p}\,| = \dfrac{6}{2\sqrt{3}\times\dfrac{\sqrt{3}}{2}} = \dfrac{6}{3} = 2$

Thus, the length of $\vec{p}$ is 2 unit.

26. यदि $|\vec{b}\,| = 6$, $\vec{a}\,.\,\vec{b} = 12$ र $\vec{a}$ र $\vec{b}$ बीचको कोण (θ) = 60° भए $|\vec{a}\,|$ को मान पत्ता लगाउनुहोस् ।

If $|\vec{b}\,| = 6$, $\vec{a}\,.\,\vec{b} = 12$ and angle between $\vec{a}$ and $\vec{b}$ (θ) = 60°, find the value of $|\vec{a}\,|$. [2072 R']

⇨ Here, $|\vec{b}\,| = 6$, $\vec{a}\,.\,\vec{b} = 12$ and θ = 60°

We know that, $\cos\theta = \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}\,|\,|\vec{b}\,|}$

or, $\cos 60° = \dfrac{12}{|\vec{a}\,|\times 6}$

or, $\dfrac{1}{2} = \dfrac{2}{|\vec{a}\,|}$

Thus, the value of $|\vec{a}\,|$ is 4.

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MODEL 4

27. यदि $\vec{a} = 6\vec{i} - 8\vec{j}$ र $\vec{b} = 4\vec{i} + 3\vec{j}$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ आपसमा लम्ब छन् भनी प्रमाणित गर्नुहोस् ।

If $\vec{a} = 6\vec{i} - 8\vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$, prove that the vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other. [SEE 2074 S', 2060 S]

⇨ Here, $\vec{a} = 6\vec{i} - 8\vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$

Now, $\vec{a}.\vec{b} = (6\vec{i} - 8\vec{j}).(4\vec{i} + 3\vec{j})$

$= 24\vec{i}^{\,2} + 18\vec{i}.\vec{j} - 32\vec{j}.\vec{i} - 24\vec{j}^{\,2}$

$= 24 - 24$

$= 0$

Since, $\vec{a}.\vec{b} = 0$, so the given vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

28. यदि $(\vec{x} + \vec{y})^2 = (\vec{x} - \vec{y})^2$ भए $\vec{x}$ र $\vec{y}$ आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

If $(\vec{x} + \vec{y})^2 = (\vec{x} - \vec{y})^2$ prove that $\vec{x}$ and $\vec{y}$ are perpendicular each other. [2065 R']

⇨ Here, $(\vec{x} + \vec{y})^2 = (\vec{x} - \vec{y})^2$

or, $(\vec{x})^2 + 2.\vec{x}.\vec{y} + (\vec{y})^2 = (\vec{x})^2 - 2.\vec{x}.\vec{y} + (\vec{y})^2$

or, $x^2 + 2.\vec{x}.\vec{y} + \vec{y}^{\,2} = x^2 - 2.\vec{x}.\vec{y} + y^2$

or, $4\vec{x}.\vec{y} = 0$

or, $\vec{x}.\vec{y} = 0$

Thus, $\vec{x}$ and $\vec{y}$ are perpendicular to each other.

MODEL 5

29. यदि $\vec{a} = 3\vec{i} + m\vec{j}$ र $\vec{b} = 6\vec{i} - 2\vec{j}$ एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।

If $\vec{a} = 3\vec{i} + m\vec{j}$ and $\vec{b} = 6\vec{i} - 2\vec{j}$ are perpendicular to each other then find the value of m. [2074 R']

⇨ Here, $\vec{a} = 3\vec{i} + m\vec{j}$ and $\vec{b} = 6\vec{i} - 2\vec{j}$

Since, $\vec{a} \perp \vec{b}$ So, $\vec{a}.\vec{b} = 0$

i.e. $(3\vec{i} + m\vec{j}).(6\vec{i} - 2\vec{j}) = 0$

or, $3\vec{i}.(6\vec{i} - 2\vec{j}) + m\vec{j}.(6\vec{i} - 2\vec{j}) = 0$

or, $18(\vec{i})^2 - 6\vec{i}.\vec{j} + 6m\vec{i}.\vec{j} - 2m(\vec{j})^2 = 0$

or, $18 \times 1 - 6\vec{i}.\vec{j} + 6m\vec{i}.\vec{j} - 2m \times 1 = 0$

or, $18 - 2m = 0$ $[\because \vec{i}.\vec{j} = 0]$

or, $2m = 18$

$\therefore$ $m = 9$

Thus, the value of m is 9.

30. यदि $\vec{a} = -5\vec{i} + 3\vec{j}$ र $\vec{b} = p\vec{i} + (p + 2)\vec{j}$ एक आपसमा लम्ब भए p को मान पत्ता लगाउनुहोस् ।

If $\vec{a} = -5\vec{i} + 3\vec{j}$ and $\vec{b} = p\vec{i} + (p + 2)\vec{j}$ are perpendicular to each other, find the value of p. [2071 R']

⇨ Here, $\vec{a} = -5\vec{i} + 3\vec{j}$ and $\vec{b} = p\vec{i} + (p + 2)\vec{j}$

Since, $\vec{a} \perp \vec{b}$ so $\vec{a}.\vec{b} = 0$

i.e. $(-5\vec{i} + 3\vec{j}).[p\vec{i} + (p + 2)\vec{j}] = 0$

or, $-5 \times p + 3(p + 2) = 0$

or, $-5p + 3p + 6 = 0$

or, $-2p = -6$

$\therefore$ $p = 3$

Thus, the value of p is 3.

31. यदि $10\vec{i} - 7\vec{j}$ र $a\vec{i} + 10\vec{j}$ आपसमा लम्ब भए a को मान पत्ता लगाउनुहोस् ।

If $10\vec{i} - 7\vec{j}$ and $a\vec{i} + 10\vec{j}$ are perpendicular to each other find the value of a. [2072 S]

⇨ Here, given perpendicular vectors are:

$(10\vec{i} - 7\vec{j})$ and $(a\vec{i} + 10\vec{j})$.

Using the condition of perpendicularity;

$(10\vec{i} - 7\vec{j}).(a\vec{i} + 10\vec{j}) = 0$

or, $10a - 70 = 0$

or, $10a = 70$

$\therefore$ $a = 7$

Thus, the value of a is 7.

32. x को मान कति हुँदा भेक्टरहरू $3\vec{i} - 2\vec{j}$ र $x\vec{i} + 3\vec{j}$ आपसमा लम्ब हुन्छन् ?

For what value of x are the two vectors $3\vec{i} - 2\vec{j}$ & $x\vec{i} + 3\vec{j}$ perpendicular ? [2065 M]

⇨ Here, let, $\vec{a} = 3\vec{i} - 2\vec{j}$ and $\vec{b} = x\vec{i} + 3\vec{j}$ are the given vectors

Now, by using the formula;

$x_1y_1 + x_2y_2 = 0$

or, $3 \times x + (-2)3 = 0$

or, $3x - 6 = 0$

$\therefore$ $x = 2$

Thus, required value of x is 2.

33. x को मान कति हुँदा भेक्टरहरू $2\vec{i} - 3\vec{j}$ र $x\vec{i} - 2\vec{j}$ एक आपसमा लम्ब हुन्छन् ?

For what value of x vectors $2\vec{i} - 3\vec{j}$ and $x\vec{i} - 2\vec{j}$ are perpendicular to each other ? [2066 R']

⇨ Here, $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = x\vec{i} - 2\vec{j}$

Since, $\vec{a} \perp \vec{b}$ so $\vec{a}.\vec{b} = 0$

or, $(2\vec{i} - 3\vec{j}).(x\vec{i} - 2\vec{j}) = 0$

or, $2x + 6 = 0$

or, $2x = -6$

$\therefore$ $x = -3$

Thus, the value of x is – 3.

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34. यदि $\vec{x} = 3\vec{i} + m\vec{j}$ र $\vec{y} = 4\vec{i} - 2\vec{j}$ एक आपसमा लम्ब भए m को मान कति हुन्छ ?

If $\vec{x} = 3\vec{i} + m\vec{j}$ & $\vec{y} = 4\vec{i} - 2\vec{j}$ are perpendicular, what is the value of m ? [2065 S]

⇨ Here, $\vec{x} = 3\vec{i} + m\vec{j}$ and $\vec{y} = 4\vec{i} - 2\vec{j}$

Since $\vec{x}$ and $\vec{y}$ are perpendicular to each other, $\vec{x} . \vec{y} = 0$

or, $(3\vec{i} + m\vec{j}).(4\vec{i} - 2\vec{j}) = 0$

or, $3.4 + m(-2) = 0$ [∵Scalar product = $x_1x_2 + y_1y_2$]

or, $12 - 2m = 0$

or, $2m = 12$

∴ $m = 6$

Thus, the value of m is 6.

35. यदि $\vec{a} = 3\vec{i} + 4\vec{j}$ र $\vec{b} = 4\vec{i} + p\vec{j}$ परस्पर लम्ब भए p को मान निकाल्नुहोस् ।

If $\vec{a} = 3\vec{i} + 4\vec{j}$ & $\vec{b} = 4\vec{i} + p\vec{j}$ are perpendicular to each other, find the value of p. [2067 S]

⇨ Here, $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 4\vec{i} + p\vec{j}$

Since, $\vec{a} \perp \vec{b}$

So, $\vec{a} . \vec{b} = 0$

i.e. $(3\vec{i} + 4\vec{j}).(4\vec{i} + p\vec{j}) = 0$

or, $3.4 + 4.p = 0$

or, $12 + 4p = 0$

or, $4p = -12$

∴ $p = -3$

Thus, the value of p is – 3.

36. यदि $\vec{a} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} p \\ p+2 \end{pmatrix}$ आपसमा लम्ब भए p को मान निकाल्नुहोस् ।

If $\vec{a} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ & $\vec{b} = \begin{pmatrix} p \\ p+2 \end{pmatrix}$ are perpendicular, find the value of p. [2059 R, 2067R']

⇨ Here, given vectors, $\vec{a} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} = (-5, 3)$ and $\vec{b} = \begin{pmatrix} p \\ p+2 \end{pmatrix} = (p, p+2)$

For the vectors $\vec{a}$ & $\vec{b}$ to be perpendicular, $\vec{a} . \vec{b} = 0$

So, $(-5,3) . (p, p+2) = 0$

or, $-5p + 3(p+2) = 0$

or, $-5p + 3p + 6 = 0$

or, $-2p = -6$

∴ $p = 3$ Thus, required value of p is 3.

MODEL 6

37. यदि $\vec{a} + 2\vec{b}$ र $5\vec{a} - 4\vec{b}$ आपसमा लम्ब तथा $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other and $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$. [2070 R]

⇨ Here, $\vec{a}$ and $\vec{b}$ are unit vectors so $|\vec{a}| = 1$ and $|\vec{b}| = 1$

Since $(\vec{a} + 2\vec{b})$ and $(5\vec{a} - 4\vec{b})$ are perpendicular to each other.

So, $(\vec{a} + 2\vec{b}) . (5\vec{a} - 4\vec{b}) = 0$

or, $5a^2 - 4\vec{a}.\vec{b} + 10\vec{a}.\vec{b} - 8b^2 = 0$

or, $5 \times 1 + 6\vec{a}.\vec{b} - 8 \times 1 = 0$

or, $6\vec{a}.\vec{b} = 3$ ∴ $\vec{a}.\vec{b} = \frac{1}{2}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$ then,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1}{2 \times 1 \times 1} = \frac{1}{2}$$

or, $\cos\theta = \cos 60°$

∴ $\theta = 60°$

Thus, the angle between the vectors is 60°.

38. यदि $\vec{m} = 2\vec{a} - 3\vec{b}$, $\vec{n} = 3\vec{a} - 2\vec{b}$ र $\vec{m}.\vec{n} = 12$ भए एकाइ भेक्टरहरू $\vec{a}$ र $\vec{b}$ एकआपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

If $\vec{m} = 2\vec{a} - 3\vec{b}$, $\vec{n} = 3\vec{a} - 2\vec{b}$ and $\vec{m}.\vec{n} = 12$, prove that the unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

⇨ Here, $\vec{a}$ and $\vec{b}$ are unit vectors.

So, $|\vec{a}| = |\vec{b}| = 1$ i.e. $a^2 = b^2 = 1$

We have given, $\vec{m}.\vec{n} = 12$

or, $(2\vec{a} - 3\vec{b}).(3\vec{a} - 2\vec{b}) = 12$

or, $6a^2 - 4\vec{a}.\vec{b} - 9\vec{a}.\vec{b} + 6b^2 = 12$

or, $6 \times 1 - 13\vec{a}.\vec{b} + 6 \times 1 = 12$

or, $13\vec{a}.\vec{b} = 12 - 12 = 0$

∴ $\vec{a}.\vec{b} = 0$

Since the dot product of $\vec{a}$ and $\vec{b}$ is zero 0 i.e. $\vec{a}.\vec{b} = 0$

So, the vectors are perpendicular to each other. Proved.

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QUESTIONS FROM CDC TEXTBOOK

6.1 दुईओटा भेक्टरहरूको स्केलर गुणनफल (SCALAR OR DOT PRODUCT OF TWO VECTORS)

EXERCISE 6.1

1. (a) **स्केलर गुणनको परिभाषा लेख्नुहोस् ।**

Define scalar product.

⇨ Here, the product of magnitude of two vectors with cosine angle between them is called the scalar product. In other words, if $\vec{a}$ and $\vec{b}$ are two vectors then the scalar product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\,.\,\vec{b}$ and defined by;

$\vec{a}\,.\,\vec{b} = |\vec{a}|\,.\,|\vec{b}|\cos\theta$

(c) **$\vec{a}\,.\,\vec{b}$ को अधिकतम मान प्राप्त गर्न $\vec{a}$ र $\vec{b}$ बिचको कोण कति हुन्छ ?**

To obtain the maximum value of $\vec{a}\,.\,\vec{b}$, what is the angle between $\vec{a}$ and $\vec{b}$?

⇨ Here, if $\theta = 0°$ then the value of $\vec{a}\,.\,\vec{b}$ will be maximum.

2. (a) **यदि $\vec{a} = \begin{pmatrix}-4\\2\end{pmatrix}$ र $\vec{b} = \begin{pmatrix}1\\2\end{pmatrix}$ भए $\vec{a}\,.\,\vec{b}$ पत्ता लगाउनुहोस् ।**

If $\vec{a} = \begin{pmatrix}-4\\2\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}1\\2\end{pmatrix}$ then find $\vec{a}\,.\,\vec{b}$.

⇨ Here, $\vec{a} = \begin{pmatrix}-4\\2\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}1\\2\end{pmatrix}$

$$\begin{aligned}\text{So, } \vec{a}\,.\,\vec{b} &= \begin{pmatrix}-4\\2\end{pmatrix}.\begin{pmatrix}1\\2\end{pmatrix}\\ &= -4\times1+2\times2\\ &= -4+4\\ \therefore\ \vec{a}\,.\,\vec{b} &= 0\end{aligned}$$

(c) **यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू भए $\vec{a}\,.\,\vec{b}$ पत्ता लगाउनुहोस् जहाँ $|\vec{a}+\vec{b}| = \sqrt{2}$ छ ।**

If $\vec{a}$ and $\vec{b}$ are unit vectors then find $\vec{a}\,.\,\vec{b}$ where $|\vec{a}+\vec{b}| = \sqrt{2}$.

⇨ Here, $\vec{a}$ and $\vec{b}$ are unit vectors.

So, $|\vec{a}| = 1$ and $|\vec{b}| = 1$, $|\vec{a}+\vec{b}| = \sqrt{2}$

Now, $(|\vec{a}+\vec{b}|)^2 = (\sqrt{2})^2$

or, $(\vec{a})^2 + 2\,\vec{a}\,.\,\vec{b} + (\vec{b})^2 = 2$

or, $a^2 + 2\,\vec{a}\,.\,\vec{b} + b^2 = 2$

or, $1^2 + 2\,\vec{a}\,.\,\vec{b} + 1^2 = 2$

or, $2\,\vec{a}\,.\,\vec{b} = 0$

$\therefore\ \vec{a}\,.\,\vec{b} = 0$

(b) **$\overrightarrow{AB}\,.\,\overrightarrow{BD}$ पत्ता लगाउनुहोस् । (Find $\overrightarrow{AB}\,.\,\overrightarrow{BD}$.)**

⇨
$$\begin{aligned}\text{Here, } \overrightarrow{AB}\,.\,\overrightarrow{BD} &= \begin{pmatrix}1\\-4\end{pmatrix}.\begin{pmatrix}3\\5\end{pmatrix}\\ &= 1\times3+(-4)\times5\\ &= 3-20\\ \therefore\ \overrightarrow{AB}\,.\,\overrightarrow{BD} &= -17\end{aligned}$$

(b) **यदि $\vec{a}\,.\,\vec{b} = 0$ भए $\vec{a}$ र $\vec{b}$ बीचको सम्बन्ध के हुन्छ ?**

If $\vec{a}\,.\,\vec{b} = 0$ then what is the relation between $\vec{a}$ and $\vec{b}$?

⇨ Here, if $\vec{a}\,.\,\vec{b} = 0$ then $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

(d) **यदि $\vec{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix}$ र $\vec{b} = \begin{pmatrix}b_1\\b_2\end{pmatrix}$ भए $\vec{a}\,.\,\vec{b}$ कति हुन्छ ?**

If $\vec{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}b_1\\b_2\end{pmatrix}$ then find $\vec{a}\,.\,\vec{b}$.

⇨ Here, $\vec{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}b_1\\b_2\end{pmatrix}$ then,

$\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$

(b) **यदि $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\-3\end{pmatrix}$ भए $\overrightarrow{OA}\,.\,\overrightarrow{OB}$ पत्ता लगाउनुहोस् ।**

If $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\-3\end{pmatrix}$ then find $\overrightarrow{OA}\,.\,\overrightarrow{OB}$.

⇨ Here, $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\-3\end{pmatrix}$

$$\begin{aligned}\text{So, } \overrightarrow{OA}\,.\,\overrightarrow{OB} &= \begin{pmatrix}\sqrt{3}\\1\end{pmatrix}.\begin{pmatrix}\sqrt{3}\\-3\end{pmatrix}\\ &= \sqrt{3}\times\sqrt{3}+1\times(-3)\\ &= 3-3\\ \therefore\ \overrightarrow{OA}\,.\,\overrightarrow{OB} &= 0\end{aligned}$$

3. **यदि A(– 2, 1), B(– 1, – 3), C(3, – 2) र D(2, 2) भए,**

If A(– 2, 1), B(– 1, – 3), C(3, – 2) and D(2, 2) then,

(a) **$\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ र $\overrightarrow{BD}$ पत्ता लगाउनुहोस् ।**

Find $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{BD}$.

⇨ Here, A (– 2, 1), B(– 1, – 3), C(3, – 2) and D(2, 2) are given.

$$\begin{aligned}\text{So, } \overrightarrow{AB} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}-1+2\\-3-1\end{pmatrix} = \begin{pmatrix}1\\-4\end{pmatrix}\\ \overrightarrow{BC} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}3+1\\-2+3\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}\\ \overrightarrow{CD} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}2-3\\2+2\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}\\ \overrightarrow{AC} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}3+2\\-2-1\end{pmatrix} = \begin{pmatrix}5\\-3\end{pmatrix}\\ \overrightarrow{AD} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}2+2\\2-1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}\\ \overrightarrow{BD} &= \begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix} = \begin{pmatrix}2+1\\2+3\end{pmatrix} = \begin{pmatrix}3\\5\end{pmatrix}\end{aligned}$$

(c) **$\overrightarrow{AC}\,.\,\overrightarrow{BD}$ बीचको कोण पत्ता लगाउनुहोस् ।**

Find the angle between $\overrightarrow{AC}\,.\,\overrightarrow{BD}$.

⇨
$$\begin{aligned}\text{Here, } \overrightarrow{AC}\,.\,\overrightarrow{BD} &= \begin{pmatrix}5\\-3\end{pmatrix}.\begin{pmatrix}3\\5\end{pmatrix}\\ &= 5\times3+(-3)\,5\\ &= 15-15 = 0\end{aligned}$$

Since $\overrightarrow{AC}\,.\,\overrightarrow{BD} = 0$ so angle between $\overrightarrow{AC}$ & $\overrightarrow{BD}$ is 90°.

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(d) $\overrightarrow{AC}^2$ र $\overrightarrow{CD}^2$ पत्ता लगाउनुहोस् । (Find $\overrightarrow{AC}^2$ and $\overrightarrow{CD}^2$.)

⇨ Here, $\overrightarrow{AC}^2 = \overrightarrow{AC}\,.\,\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}.\begin{pmatrix}5\\-3\end{pmatrix} = 25 + 9 = 34$

$\overrightarrow{CD}^2 = \overrightarrow{CD}\,.\,\overrightarrow{CD} = \begin{pmatrix}-1\\4\end{pmatrix}.\begin{pmatrix}-1\\4\end{pmatrix} = (-1)(-1) + 4 \times 4 = 1 + 16 = 17$

4. (a) यदि $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}a\\-2\end{pmatrix}$, $\measuredangle AOB = 90°$ भए a को मान पत्ता लगाउनुहोस् ।

If $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}a\\-2\end{pmatrix}$, $\measuredangle AOB = 90°$ then find the value of a.

⇨ Here, $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix}a\\-2\end{pmatrix}$ and $\measuredangle AOB = 90°$

So, $\overrightarrow{AB}\,.\,\overrightarrow{OB} = 0$

or, $\begin{pmatrix}2\\3\end{pmatrix}.\begin{pmatrix}a\\-2\end{pmatrix} = 0$

or, $2 \times a + 3(-2) = 0$

or, $2a = 6$

$\therefore$ $a = 3$

Thus, the value of a is 3.

(c) यदि $\vec{a} = \begin{pmatrix}2+k\\4-k\end{pmatrix}$ र $\vec{b} = \begin{pmatrix}2\\3\end{pmatrix}$ एकआपसमा लम्ब भए k को मान पत्ता लगाउनुहोस् ।

If $\vec{a} = \begin{pmatrix}2+k\\4-k\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}2\\3\end{pmatrix}$ are perpendicular to each other, find the value of k.

⇨ Here, $\vec{a} = \begin{pmatrix}2+k\\4-k\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}2\\3\end{pmatrix}$

Since, $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

So, $\vec{a}\,.\,\vec{b} = 0$

or, $\begin{pmatrix}2+k\\4-k\end{pmatrix}.\begin{pmatrix}2\\3\end{pmatrix} = 0$

or, $2(2 + k) + 3(4 - k) = 0$

or, $4 + 2k + 12 - 3k = 0$

or, $-k = -16$

$\therefore$ $k = 16$

Thus, the value of k is 16.

(b) यदि $\vec{p} = \begin{pmatrix}2\\a\end{pmatrix}$ र $\vec{q} = \begin{pmatrix}3\\-1\end{pmatrix}$ एकआपसमा लम्ब भए a को मान पत्ता लगाउनुहोस् ।

If $\vec{p} = \begin{pmatrix}2\\a\end{pmatrix}$ and $\vec{q} = \begin{pmatrix}3\\-1\end{pmatrix}$ are perpendicular to each other, find the value of a.

⇨ Here, $\vec{p} = \begin{pmatrix}2\\a\end{pmatrix}$ and $\vec{q} = \begin{pmatrix}3\\-1\end{pmatrix}$

Since, $\vec{p}$ and $\vec{q}$ are perpendicular to each other.

So, $\vec{p}\,.\,\vec{q} = 0$

or, $\begin{pmatrix}2\\a\end{pmatrix}.\begin{pmatrix}3\\-1\end{pmatrix} = 0$

or, $2 \times 3 + a(-1) = 0$

or, $6 - a = 0$

or, $-a = -6$

$\therefore$ $a = 6$

Thus, the value of a is 6.

5. (a) यदि $|\overrightarrow{OP}| = 6$, $\overrightarrow{OP}\,.\,\overrightarrow{OQ} = 24$ र $\measuredangle POQ = 60°$ भए $|\overrightarrow{OQ}|$ को मान पत्ता लगाउनुहोस् ।

If $|\overrightarrow{OP}| = 6$, $\overrightarrow{OP}\,.\,\overrightarrow{OQ} = 24$ and $\measuredangle POQ = 60°$ then find the value of $|\overrightarrow{OQ}|$.

⇨ Here, $|\overrightarrow{OP}| = 6$, $\overrightarrow{OP}\,.\,\overrightarrow{OQ} = 24$ and $\measuredangle POQ = 60°$

Let θ be the angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ then,

$$\cos\theta = \frac{\overrightarrow{OP}\,.\,\overrightarrow{OQ}}{|\overrightarrow{OP}|\,|\overrightarrow{OQ}|}$$

or, $\cos \measuredangle POQ = \dfrac{24}{6\,|\overrightarrow{OQ}|}$

or, $\cos 60° = \dfrac{4}{|\overrightarrow{OQ}|}$

or, $\dfrac{1}{2} = \dfrac{4}{|\overrightarrow{OQ}|}$ $\qquad \therefore \ |\overrightarrow{OQ}| = 8$

Thus, the value of $|\overrightarrow{OQ}|$ is 8.

(b) यदि $\overrightarrow{OA}\,.\,\overrightarrow{OB} = 30$, $\measuredangle AOB = 45°$ र $|\overrightarrow{OB}| = 4\sqrt{2}$ भए $|\overrightarrow{OA}|$ को मान पत्ता लगाउनुहोस् ।

If $\overrightarrow{OA}\,.\,\overrightarrow{OB} = 30$, $\measuredangle AOB = 45°$ and $|\overrightarrow{OB}| = 4\sqrt{2}$ then find the value of $|\overrightarrow{OA}|$.

⇨ Here, $\overrightarrow{OA}\,.\,\overrightarrow{OB} = 30$, $\measuredangle AOB = 45°$ & $|\overrightarrow{OB}| = 4\sqrt{2}$

Let θ be the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ then,

$$\cos\theta = \frac{\overrightarrow{OA}\,.\,\overrightarrow{OB}}{|\overrightarrow{OA}|\,|\overrightarrow{OB}|}$$

or, $\cos \measuredangle AOB = \dfrac{30}{|\overrightarrow{OA}|\,.\,4\sqrt{2}}$

or, $\cos 45° = \dfrac{30}{|\overrightarrow{OA}|\,4\sqrt{2}}$

or, $\dfrac{1}{\sqrt{2}} = \dfrac{30}{|\overrightarrow{OA}|\,4\sqrt{2}}$

or, $1 = \dfrac{15}{|\overrightarrow{OA}|\,.\,2}$

or, $|\overrightarrow{OA}| = \dfrac{15}{2}$

Thus, the value of $|\overrightarrow{OA}|$ is $\dfrac{15}{2}$.

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(c) यदि $\overrightarrow{OC}\,.\,\overrightarrow{OD} = -14\sqrt{3}$, $\measuredangle OCD = 150°$ र $|\overrightarrow{OC}| = 4$ भए $|\overrightarrow{OD}|$ को मान पत्ता लगाउनुहोस् ।

If $\overrightarrow{OC}\,.\,\overrightarrow{OD} = -14\sqrt{3}$, $\measuredangle OCD = 150°$ and $|\overrightarrow{OC}| = 4$ then find the value of $|\overrightarrow{OD}|$.

⇨ Here, $\overrightarrow{OC}\,.\,\overrightarrow{OD} = -14\sqrt{3}$, $\measuredangle OCD = 150°$, $|\overrightarrow{OC}| = 4$

Let θ be angle between $\overrightarrow{OC}$ and $\overrightarrow{OD}$ then,

$$\cos\theta = \frac{\overrightarrow{OC}\,.\,\overrightarrow{OD}}{|\overrightarrow{OC}|\,|\overrightarrow{OD}|}$$

or, $\cos \measuredangle OCD = \dfrac{-14\sqrt{3}}{4 \times |\overrightarrow{OD}|}$

or, $\cos 150° = \dfrac{-7\sqrt{3}}{2\,|\overrightarrow{OD}|}$

or, $-\dfrac{\sqrt{3}}{2} = -\dfrac{7\sqrt{3}}{2\,|\overrightarrow{OD}|}$

∴ $|\overrightarrow{OD}| = 7$

Thus, the value of $|\overrightarrow{OD}|$ is 7.

(b) यदि $\vec{p} + \vec{q} + \vec{r} = (0, 0)$, $|\vec{p}| = 6$, $|\vec{q}| = 10$ र $\vec{p}\,.\,\vec{q} = 30$ भए $|\vec{r}|$ पत्ता लगाउनुहोस् ।

If $\vec{p} + \vec{q} + \vec{r} = (0, 0)$, $|\vec{p}| = 6$, $|\vec{q}| = 10$ and $\vec{p}\,.\,\vec{q} = 30$ then find the value of $|\vec{r}|$.

⇨ Here, $\vec{p} + \vec{q} + \vec{r} = 0$, $|\vec{p}| = 6$, $|\vec{q}| = 10$, $\vec{p}\,.\,\vec{q} = 30$ and $|\vec{r}| = ?$

Taking $\vec{p} + \vec{q} + \vec{r} = 0$

or, $\vec{p} + \vec{q} = -\vec{r}$

Squaring on the both sides then,

$$(\vec{p} + \vec{q})^2 = (-\vec{r})^2$$

or, $(\vec{p})^2 + 2\,.\,\vec{p}\,.\,\vec{q} + (\vec{q})^2 = \vec{r}^{\,2}$

or, $6^2 + 2\,.\,30 + 10^2 = \vec{r}^{\,2}$

or, $36 + 2 \times 30 + 100 = \vec{r}^{\,2}$

or, $196 = \vec{r}^{\,2}$

or, $(\pm 14)^2 = \vec{r}^{\,2}$

∴ $|\vec{r}| = \pm 14$

Thus the value of $|\vec{r}|$ is ± 14.

6. (a) यदि $\vec{a} + \vec{b} + \vec{c} = (0, 0)$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ र $|\vec{c}| = 4$ भए $\vec{a}\,.\,\vec{c}$ को मान पत्ता लगाउनुहोस् ।

If $\vec{a} + \vec{b} + \vec{c} = (0, 0)$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 4$ then find the value of $\vec{a}\,.\,\vec{c}$.

⇨ Here, $\vec{a} + \vec{b} + \vec{c} = (0, 0)$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 4$

Taking, $\vec{a} + \vec{b} + \vec{c} = (0, 0)$

or, $\vec{a} + \vec{c} = -\vec{b}$

Squaring on the both sides then,

$$(\vec{a} + \vec{c})^2 = (-\vec{b})^2$$

or, $(\vec{a})^2 + 2\vec{a}\,.\,\vec{c} + (\vec{c})^2 = 5^2$

or, $3^2 + 2\vec{a}\,.\,\vec{c} + 4^2 = 5^2$

or, $9 + 16 + 2\,\vec{a}\,.\,\vec{c} = 25$

or, $2\vec{a}\,.\,\vec{c} = 0$

or, $\vec{a}\,.\,\vec{c} = 0$

Thus, $\vec{a}\,.\,\vec{c} = 0$.

7. (a) कुनै भेक्टरहरू $\vec{a}$ र $\vec{b}$ का लागि $(\vec{a} + \vec{b})\,.\,(\vec{a} - \vec{b}) = 0$ भए $|\vec{a}| = |\vec{b}|$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

For any two vectors $\vec{a}$ and $\vec{b}$, if $(\vec{a} + \vec{b})\,.\,(\vec{a} - \vec{b}) = 0$ then prove that $|\vec{a}| = |\vec{b}|$.

⇨ Here, $(\vec{a} + \vec{b})\,(\vec{a} - \vec{b}) = 0$

or, $(\vec{a})^2 - (\vec{b})^2 = 0$

or, $(|\vec{a}|)^2 - (|\vec{b}|)^2 = 0$

or, $(|\vec{a}|)^2 = (|\vec{b}|)^2$

∴ $|\vec{a}| = |\vec{b}|$

(b) यदि $\vec{a} \perp \vec{b}$ भए $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{a} \perp \vec{b}$ then prove that $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$

⇨ Here, $\vec{a} \perp \vec{b}$ so, $\vec{a}\,.\,\vec{b} = 0$

LHS $= (\vec{a} + \vec{b})^2$

$= (\vec{a})^2 + 2\,.\,\vec{a}\,.\,\vec{b} + (\vec{b})^2$

$= (\vec{a})^2 + 2 \times 0 + (\vec{b})^2$

$= (\vec{a})^2 + 0 + (\vec{b})^2$

$= (\vec{a})^2 - 0 + (\vec{b})^2$

$= (\vec{a})^2 - 2\,\vec{a}\,.\,\vec{b} + (\vec{b})^2$

$= (\vec{a} - \vec{b})^2 =$ RHS **Proved.**

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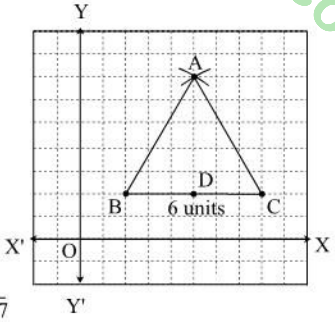
8. **लेखाचित्रमा समबाहु त्रिभुज ABC खिच्नुहोस् र (Draw an equilateral ΔABC on a graph and)**

(a) **$\overrightarrow{AB}\,.\,\overrightarrow{AC}$, $\overrightarrow{BC}\,.\,\overrightarrow{BA}$ र $\overrightarrow{CA}\,.\,\overrightarrow{CB}$ पत्ता लगाउनुहोस् । (Find $\overrightarrow{AB}\,.\,\overrightarrow{AC}$, $\overrightarrow{BC}\,.\,\overrightarrow{BA}$ and $\overrightarrow{CA}\,.\,\overrightarrow{CB}$.)**

⇨ Here, ΔABC is an equilateral Δ where BC = 6 units



For, $\overrightarrow{AB}\,.\,\overrightarrow{AC}$, $|\overrightarrow{AB}| = 6$ units, $|\overrightarrow{AC}| = 6$ units, $\measuredangle BAC = 60°$

Now, $\overrightarrow{AB}\,.\,\overrightarrow{AC} = |\overrightarrow{AB}|\,|\overrightarrow{AC}| \cos \measuredangle BAC = 6 \times 6 \times \cos 60° = 36 \times \frac{1}{2}$

$\therefore \quad \overrightarrow{AB}\,.\,\overrightarrow{AC} \quad = 18$

Similarly, $\overrightarrow{BC}\,.\,\overrightarrow{BA} = 18$ and $\overrightarrow{CA}\,.\,\overrightarrow{CB} = 18$

(b) **$\overrightarrow{BC}$ को मध्य बिन्दु D पत्ता लगाउनुहोस् । (Find the mid point D of $\overrightarrow{BC}$.)**

⇨ Here, The midpoint of D BC = (5, 2).

(c) **$\overrightarrow{BC}\,.\,\overrightarrow{AD}$ कति हुन्छ ? पत्ता लगाउनुहोस् । (Find $\overrightarrow{BC}\,.\,\overrightarrow{AD}$.)**

⇨ Here, From the figure; $AD = \sqrt{AB^2 - BD^2} = \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27}$

$\therefore \quad AD = 3\sqrt{3}$ $\qquad$ i.e. $|\overrightarrow{AD}| = 3\sqrt{3}$ units.

Now, $\overrightarrow{BC}\,.\,\overrightarrow{AD} = |\overrightarrow{BC}|\,|\overrightarrow{AD}| \cos \measuredangle ADB = 6 \times 3\sqrt{3} \cos 90° = 6 \times 3\sqrt{3} \cos \times 0$

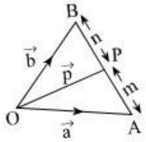
$\therefore \quad \overrightarrow{BC}\,.\,\overrightarrow{AD} \quad = 0$

Thus, the value of $\overrightarrow{BC}\,.\,\overrightarrow{AD}$ is 0.

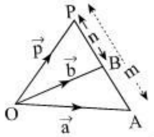
2. भेक्टर ज्यामिति
Vector Geometry

Formulae and Key Points

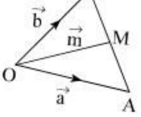
1. भित्री विभाजन सूत्र (Internal section formula): $\vec{p} = \dfrac{n\vec{a} + m\vec{b}}{m + n}$



2. बाहिरी विभाजन सूत्र (External section formula): $\vec{p} = \dfrac{m\vec{b} - n\vec{a}}{m - n}$

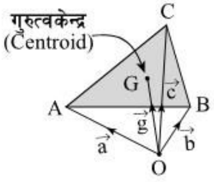


3. मध्यबिन्दु सूत्र (Mid–point formula): $\vec{m} = \frac{1}{2}(\vec{a} + \vec{b})$



4. यदि $\vec{a}$, $\vec{b}$ र $\vec{c}$ हरू ΔABC का शीर्षहरूको स्थिति भेक्टरहरू भए यसको गुरुत्वकेन्द्र $(\overrightarrow{OG}) = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ हुन्छ ।

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of vertices of ΔABC then the position k centroid $(\overrightarrow{OG}) = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.



QUESTIONS FROM SEE EXERCISE 2

A. VERY SHORT QUESTIONS

1. **भेक्टर जोडको त्रिभुज नियम (Triangle law of vector addition)**

⇨ Here, the sum of vectors represented by two sides in an order is equal to the vector represented by remaining side taken in opposite order. In the figure, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ is called triangle law of vector.

2. **खण्ड सूत्र $\overrightarrow{OP} = \dfrac{m\,\overrightarrow{OB} + n\,\overrightarrow{OA}}{m + n}$ मा m र n ले के के जनाउँदछन् ?**

What do m and n represent in the section formula $\overrightarrow{OP} = \dfrac{m\,\overrightarrow{OB} + n\,\overrightarrow{OA}}{m + n}$?

⇨ Here, m and n represent the ratios in which $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are divided.

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3. **यदि O उद्गम बिन्दु र A र B जोड्ने रेखा खण्डको मध्यबिन्दु M भए $\overrightarrow{OA}, \overrightarrow{OB}$ र $\overrightarrow{OM}$ को सम्बन्ध लेख्नुहोस् ।**

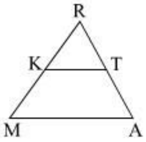
If M is the mid point of line segment joining A and B and O is origin, write the relation between $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OM}$.

⇨ Here, $\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$ is the required relation.

4. **$\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ हुनेगरी ∆ABC एउटा त्रिभुज छ । बिन्दु G को विशिष्ट नाम उल्लेख गर्नुहोस्, जहाँ O उद्गम बिन्दु हो ।**

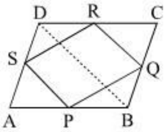
ABC is a triangle such that $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ and O be the origin, write the special name of the point G.

⇨ Here, G is the centroid point of triangle ABC.



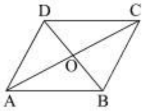
5. **सँगैको चित्रमा AR र RM का मध्यबिन्दुहरू क्रमशः T र K छन् । KT र MA को सम्बन्ध लेख्नुहोस् ।**
In the adjoining figure, T and K are the midpoints of AR and RM respectively. Write the relation between KT and MA.

⇨ Here, $KT = \frac{1}{2}$ MA and KT // MA are the required relations.



6. **चित्रमा AB, BC, CD र AD का मध्यबिन्दुहरू क्रमशः P, Q, R र S छन् । PS र BD को सम्बन्ध लेख्नुहोस् ।**
In the figure, P, Q, R and S are the midpoints of AB, BC, CD and AD respectively. Write the relation between PS and BD.

⇨ Here, $PS = \frac{1}{2}$ BD and PS // BD are the required relations.



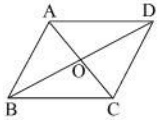
7. **दिइएको चित्रमा ABCD एउटा समबाहु चतुर्भुज हो । यदि $\overrightarrow{BD} . \overrightarrow{AC} = 0$ भए ∡AOB को नाप कति होला ?**

In the given figure, ABCD is a rhombus. If $\overrightarrow{BD} . \overrightarrow{AC} = 0$, what is the value of ∡AOB?

⇨ Here, $\overrightarrow{BD} . \overrightarrow{AC} = 0$ shows that the angle between two vectors is 90°. So, ∡AOB is 90°.

8. **दिइएको चित्रमा, ABCD एउटा समानान्तर चतुर्भुज हो । यदि $2\overrightarrow{BO} = \overrightarrow{BD}$ भए $\overrightarrow{BO}$ र $\overrightarrow{OD}$ को सम्बन्ध लेख्नुहोस् ।**

In the given figure, ABCD is a parallelogram. If $2\overrightarrow{BO} = \overrightarrow{BD}$, write the relation between $\overrightarrow{BO}$ and $\overrightarrow{OD}$.



⇨ Here, $2\overrightarrow{BO} = \overrightarrow{BD}$

or, $2\overrightarrow{BO} = \overrightarrow{BO} + \overrightarrow{OD}$

∴ $\overrightarrow{BO} = \overrightarrow{OD}$ is the required relation.

9. **यदि O उद्गम बिन्दु र $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ भए AB को मध्यबिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If O be the origin and $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ then find the position vector of M which is the mid point ofAB.

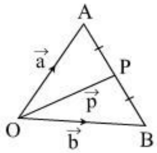
⇨ Here, $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) = \frac{1}{2}(8\vec{i} + 12\vec{j}) = 4\vec{i} + 6\vec{j}$.

B. SHORT QUESTIONS

MODEL 1

1. **दिइएको चित्रमा $\overrightarrow{AP}$ पत्ता लगाई $\vec{p}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा व्यक्त गर्नुहोस् ।**

From the given figure, find $\overrightarrow{AP}$ and express $\vec{p}$ in terms of $\vec{a}$ and $\vec{b}$. **[SEE MODEL 2076]**



⇨ Here, we know,

(i) $\overrightarrow{AO} + \overrightarrow{OP} = \overrightarrow{AP}$ [∵ triangle law of vector addition]

or, $-\overrightarrow{OA} + \overrightarrow{OP} = \overrightarrow{AP}$

or, $-\vec{a} + \vec{p} = \overrightarrow{AP}$

∴ $\overrightarrow{AP} = \vec{p} - \vec{a}$

(ii) Since P is the mid point of AB so, using mid point formula, $\overrightarrow{OP} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$

∴ $\vec{p} = \dfrac{\vec{a} + \vec{b}}{2}$

Thus, the required relation is $\vec{p} = \dfrac{\vec{a} + \vec{b}}{2}$.

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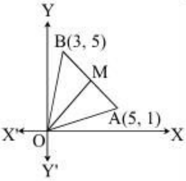
2. **सँगैको चित्रमा बिन्दुहरू A र B का निर्देशाङ्कहरू क्रमशः (5, 1) र (3, 5) छन् । यदि बिन्दु M रेखाखण्ड AB को मध्यबिन्दु हो भने बिन्दु M को स्थिति भेक्टर ($\overrightarrow{OM}$) पत्ता लगाउनुहोस् ।**



In the figure alongside, the co-ordinates of the points A and B are (5, 1) and (3, 5) respectively. If the point M be the midpoint of the line segment AB, find the position vector ($\overrightarrow{OM}$) of the point M. [SEE 2075 R'₂]

⇨ Here, let O be the origin then,

$\overrightarrow{OA} = 5\vec{i} + \vec{j}$ and $\overrightarrow{OB} = 3\vec{i} + 5\vec{j}$

By using mid point formula,

$$\overrightarrow{OM} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$$

$$= \frac{5\vec{i} + \vec{j} + 3\vec{i} + 5\vec{j}}{2}$$

$$= \frac{8\vec{i} + 6\vec{j}}{2} = 4\vec{i} + 3\vec{j}$$

Thus, the position vector of the point M is $4\vec{i} + 3\vec{j}$.

3. **बिन्दुहरू A र P का स्थिति भेक्टरहरू क्रमशः $3\vec{i} + 5\vec{j}$ र $\vec{i} + 4\vec{j}$ छन् । यदि AB को मध्यबिन्दु P भए B को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

The position vectors of the points A and P are $3\vec{i} + 5\vec{j}$ and $\vec{i} + 4\vec{j}$ respectively. If P is the middle point of AB then find the position vector of B. [2074 R']

⇨ Here, let O be the origin then,

$\overrightarrow{OA} = 3\vec{i} + 5\vec{j}$ and $\overrightarrow{OP} = \vec{i} + 4\vec{j}$

We have,

$$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$$

$$\text{or, } \vec{i} + 4\vec{j} = \frac{1}{2}(3\vec{i} + 5\vec{j} + \overrightarrow{OB})$$

$$\text{or, } 2\vec{i} + 8\vec{j} = 3\vec{i} + 5\vec{j} + \overrightarrow{OB}$$

$$\text{or, } -\vec{i} + 3\vec{j} = \overrightarrow{OB}$$

$$\therefore \overrightarrow{OB} = -\vec{i} + 3\vec{j}$$

Thus, the position vector of B is $-\vec{i} + 3\vec{j}$.

4. **A र B का स्थिति भेक्टरहरू क्रमशः $7\vec{i} + 2\vec{j}$ र $3\vec{i} - 4\vec{j}$ छन् । यदि बिन्दु P रेखा AB को मध्यबिन्दु हो भने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

The position vectors of A and B are $7\vec{i} + 2\vec{j}$ and $3\vec{i} - 4\vec{j}$ respectively. If point P is the mid-point of line AB, find the position vector of the point P. [2074 S']

⇨ Here, let O be the origin then,

$\overrightarrow{OA} = 7\vec{i} + 2\vec{j}$, $\overrightarrow{OB} = 3\vec{i} - 4\vec{j}$ and

P is the mid point of AB.

We know that, $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

$$= \frac{1}{2}(7\vec{i} + 2\vec{j} + 3\vec{i} - 4\vec{j})$$

$$= \frac{1}{2}(10\vec{i} - 2\vec{j})$$

$$= 5\vec{i} - \vec{j}$$

Thus, the position vector of P is $5\vec{i} - \vec{j}$.

5. **यदि दुई बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $\begin{pmatrix}3\\2\end{pmatrix}$ र $\begin{pmatrix}-5\\4\end{pmatrix}$ भए AB को मध्यबिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If the position vectors of two points A and B are $\begin{pmatrix}3\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\4\end{pmatrix}$ respectively then find position vector of the mid-point C of AB. [2073 R']

⇨ Here, let O be the origin then,

$\overrightarrow{OA} = \begin{pmatrix}3\\2\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}-5\\4\end{pmatrix}$

Let C be the mid-point of AB.

$$\text{Then, } \overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) = \frac{1}{2}\left[\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}-5\\4\end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix}3-5\\2+4\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-2\\6\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$

Thus, the position vector of C is $\begin{pmatrix}-1\\3\end{pmatrix}$

6. **बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $(8\vec{i} + 6\vec{j})$ र $(3\vec{i} + \vec{j})$ छन् । AB लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If the position vectors of the vertices A and B are $(8\vec{i} + 6\vec{j})$ and $(3\vec{i} + \vec{j})$ respectivley. Find the position vector of the point C which divides AB in the ratio 2 : 3 internally. [2073 S']

⇨ Here, let O be the origin then, $\overrightarrow{OA} = 8\vec{i} + 6\vec{j}$, $\overrightarrow{OB} = 3\vec{i} + \vec{j}$ and ratio = m : n = 2 : 3

We have,

$$\overrightarrow{OC} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n} = \frac{2\overrightarrow{OB} + 3\overrightarrow{OA}}{2+3} = \frac{2(3\vec{i} + \vec{j}) + 3(8\vec{i} + 6\vec{j})}{5} = \frac{1}{5}(6\vec{i} + 2\vec{j} + 24\vec{i} + 18\vec{j}) = \frac{1}{5}(30\vec{i} + 20\vec{j})$$

$$\therefore \overrightarrow{OC} = 6\vec{i} + 4\vec{j}$$

Thus, the position vector of C is $6\vec{i} + 4\vec{j}$.

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7. A र B का स्थिति भेक्टरहरू क्रमश: $9\vec{i}+7\vec{j}$ र $\vec{i}-3\vec{j}$ छन् । यदि AB को मध्यबिन्दु M भए M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $9\vec{i}+7\vec{j}$ and $\vec{i}-3\vec{j}$ respectively. If M is the mid-point of AB, find the position vector of M. [2072 R]

⇨ Here, let O be the origin then,

$\vec{OA} = 9\vec{i}+7\vec{j}$ and $\vec{OB} = \vec{i}-3\vec{j}$

$$\text{We know that, } \vec{OM} = \frac{\vec{OA}+\vec{OB}}{2}$$

$$= \frac{9\vec{i}+7\vec{j}+\vec{i}-3\vec{j}}{2}$$

$$= \frac{10\vec{i}+4\vec{j}}{2}$$

$$= 5\vec{i}+2\vec{j}$$

Thus, the position of M is $5\vec{i}+2\vec{j}$.

8. यदि बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $\begin{pmatrix}3\\4\end{pmatrix}$ र $\begin{pmatrix}4\\5\end{pmatrix}$ भए AB को मध्यबिन्दु K को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of two points A and B are $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ respectively then find the position vector of the mid-point K of AB. [2072 R']

⇨ Here, let O be the origin then,

$\vec{OA} = \begin{pmatrix}3\\4\end{pmatrix}$ and $\vec{OB} = \begin{pmatrix}4\\5\end{pmatrix}$

We know that,

$$\vec{OK} = \frac{\vec{OA}+\vec{OB}}{2} = \frac{1}{2}\left[\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}4\\5\end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix}3+4\\4+5\end{pmatrix} = \frac{1}{2}\begin{pmatrix}7\\9\end{pmatrix} = \begin{pmatrix}\frac{7}{2}\\\frac{9}{2}\end{pmatrix}$$

Thus, the position vector of K is $\frac{7}{2}\vec{i}+\frac{9}{2}\vec{j}$.

9. यदि M र N का स्थिति भेक्टरहरू क्रमश: $9\vec{i}+3\vec{j}$ र $3\vec{i}+5\vec{j}$ भए $\vec{MP}=\vec{PN}$ हुने गरी एउटा बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of M and N are $9\vec{i}+3\vec{j}$ and $3\vec{i}+5\vec{j}$ respectively, find the position vector of a point P such that $\vec{MP}=\vec{PN}$. [2071 R]

⇨ Here, let O be the centre $\vec{OM} = 9\vec{i}+3\vec{j}$ and $\vec{ON} = 3\vec{i}+5\vec{j}$

We have given, $\vec{MP} = \vec{PN}$

or, $\vec{OP}-\vec{OM} = \vec{ON}-\vec{OP}$

or, $2\vec{OP} = \vec{OM}+\vec{ON}$

$$= 9\vec{i}+3\vec{j}+3\vec{i}+5\vec{j}$$

$$= 12\vec{i}+8\vec{j}$$

$\therefore \vec{OP} = 6\vec{i}+4\vec{j}$

Thus, the position vector of P is $6\vec{i}+4\vec{j}$.

10. रेखाखण्ड EF को मध्यबिन्दुको स्थिति भेक्टर $4\vec{i}+7\vec{j}$ र बिन्दु E को स्थिति भेक्टर $-3\vec{i}-4\vec{j}$ छन् भने बिन्दु F को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vector of the midpoint of the line segment EF is $4\vec{i}+7\vec{j}$ and the position vector of the point E is $-3\vec{i}-4\vec{j}$, find the position vector of the point F. [2071 S]

⇨ Here, let O be the origin and P be the mid-point of EF. So, $\vec{OP} = 4\vec{i}+7\vec{j}$, $\vec{OE} = -3\vec{i}-4\vec{j}$

Since P is the mid-point of EF.

$$\text{So, } \vec{OP} = \frac{\vec{OE}+\vec{OF}}{2}$$

$$\text{or, } 4\vec{i}+7\vec{j} = \frac{-3\vec{i}-4\vec{j}+\vec{OF}}{2}$$

or, $8\vec{i}+14\vec{j} = -3\vec{i}-4\vec{j}+\vec{OF}$

or, $11\vec{i}+18\vec{j} = \vec{OF}$

Thus, the position vector of F

i.e. $\vec{OF} = 11\vec{i}+18\vec{j}$.

11. यदि A र B को क्रमश: स्थिति भेक्टर $3\vec{i}+4\vec{j}$ र $7\vec{i}+8\vec{j}$ भए A र B जोड्ने रेखाको मध्यबिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

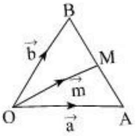
If the position vectors of A and B are $3\vec{i}+4\vec{j}$ and $7\vec{i}+8\vec{j}$ respectively, find the position vector of the mid-point of the line joining A and B. [2058 R]

⇨ Here, given that, the position vector of A be $\vec{a} = 3\vec{i}+4\vec{j}$

The position vector of B be $\vec{b} = 7\vec{i}+8\vec{j}$

Let $\vec{m}$ be the position vector of the line joining A and B.

Then by using formula,



$$\text{Position vector of mid-point } (\vec{m}) = \frac{\vec{a}+\vec{b}}{2}$$

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∴ The position vector of $\vec{m}$ $= \dfrac{\left(3\vec{i}+4\vec{j}\right)+\left(7\vec{i}+8\vec{j}\right)}{2} = \dfrac{3\vec{i}+4\vec{j}+7\vec{i}+8\vec{j}}{2}$

$$= \frac{10\vec{i}+12\vec{j}}{2} = \frac{2\left(5\vec{i}+6\vec{j}\right)}{2} = 5\vec{i}+6\vec{j}$$

Thus, position vector of mid point of AB is $5\vec{i}+6\vec{j}$.

12. स्थिति भेक्टरको परिभाषा दिनुहोस् । यदि बिन्दु A(3, 4) र B(– 1, 2) जोड्ने सीधा रेखाको मध्यबिन्दु P भए, P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

Define position vector. If P is the mid-point of the straight line joining the points A(3, 4) and B(– 1, 2), find the position vector of P. **[2060 S']**

⇨ Here, any vector which is started from origin (0, 0) is known as position vector.

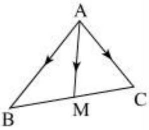
Here, A(3, 4) & B(– 1, 2) are given points P(x, y) be the mid-points of AB.

Now, $\overrightarrow{OP} = \dfrac{\overrightarrow{OA}+\overrightarrow{OB}}{2} = \dfrac{(3, 4)+(-1, 2)}{2} = \dfrac{(3-1, 4+2)}{2} = \dfrac{(2, 6)}{2} = (1, 3)$

Thus, position vector of P is (1, 3).

13. दिइएको चित्रमा, ΔABC को मध्यिका AM छ । प्रमाणित गर्नुहोस्:

In the given figure, AM is the median of ΔABC. Prove that:



$$\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$$

⇨ Here,

(i) $\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$ [Triangle law of vector addition]

$= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

(ii) $\overrightarrow{AM} = \overrightarrow{AC} + \overrightarrow{CM}$ [Triangle law of vector addition]

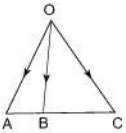
$= \overrightarrow{AC} - \frac{1}{2}\overrightarrow{BC}$

(iii) $2\overrightarrow{AM} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \overrightarrow{AC} - \frac{1}{2}\overrightarrow{BC}$ [From (i) & (ii)]

or, $2\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{AC}$

Thus, $\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$ **Proved.**

OR दिइएको चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र $\overrightarrow{OM} = \vec{m}$ र M रेखाखण्ड AB को मध्यबिन्दु भए प्रमाणित गर्नुहोस् : $\vec{m} = \dfrac{\vec{a}+\vec{b}}{2}$



In the figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ and M is the mid-point of $\overrightarrow{AB}$. Prove that: $\vec{m} = \dfrac{\vec{a}+\vec{b}}{2}$

⇨ Here, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ & $\overrightarrow{OM} = \vec{m}$

In ΔAOB and M be a mid-point of AB,

So, $\overrightarrow{AM} = \overrightarrow{MB}$

Now, by using triangular law,

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$ ……….. (i) (From ΔOAM)

$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$ ……….. (ii) (From ΔOBM)

Adding equation (i) & (ii) we get,

$2\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{OB} + \overrightarrow{BM}$

or, $\overrightarrow{OM} = \dfrac{\vec{a} + \overrightarrow{AM} + \vec{b} - \overrightarrow{MB}}{2}$

or, $\overrightarrow{OM} = \dfrac{\vec{a} + \vec{b} + \overrightarrow{AM} - \overrightarrow{AM}}{2}$ [∵ $\overrightarrow{AM} = \overrightarrow{MB}$]

∴ $\vec{m} = \dfrac{\vec{a}+\vec{b}}{2}$ **Proved.**

MODEL 2

14. यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ र M ले BA लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस् : $\vec{m} = \frac{1}{5}(3\vec{a} + 2\vec{b})$

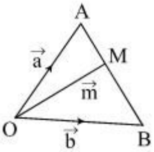
If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ and M divides BA internally in the ratio of 3 : 2 then, prove that: $\vec{m} = \frac{1}{5}(3\vec{a} + 2\vec{b})$

[SEE 2075 R']

⇨ Here, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ and ratio = 3 : 2

By section formula of internal division, $\overrightarrow{OM} = \dfrac{3\,\overrightarrow{OA} + 2\,\overrightarrow{OB}}{3+2}$

or, $\vec{m} = \dfrac{3\,\vec{a} + 2\,\vec{b}}{5}$ ∴ $\vec{m} = \frac{1}{5}(3\vec{a} + 2\vec{b})$ Proved.



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15. बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $5\vec{i} + 3\vec{j}$ र $2\vec{i} - 3\vec{j}$ छन् भने AB लाई 2 : 1 को अनुपातमा विभाजन गर्ने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $5\vec{i} + 3\vec{j}$ and $2\vec{i} - 3\vec{j}$ respectively. Find the position vector of the point P which divides AB in the ratio of 2 : 1. [2074 R]

⇨ Here, let O be the origin then, $\overrightarrow{OA} = 5\vec{i} + 3\vec{j}$ and

$\overrightarrow{OB} = 2\vec{i} - 3\vec{j}$

Division ratio = m : n = 2 : 1

We have the section formula of internal division,

$$\overrightarrow{OP} = \frac{m\ \overrightarrow{OB} + n\ \overrightarrow{OA}}{m+n}$$

$$= \frac{2(2\vec{i} - 3\vec{j}) + 1(5\vec{i} + 3\vec{j})}{2+1}$$

$$= \frac{4\vec{i} - 6\vec{j} + 5\vec{i} + 3\vec{j}}{3}$$

$$= \frac{9\vec{i} - 3\vec{j}}{3}$$

$$\therefore \quad \overrightarrow{OP} = 3\vec{i} - \vec{j}$$

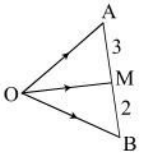
Thus, the position vector of P is $3\vec{i} - \vec{j}$.

16. बिन्दुहरू A र B का स्थिती भेक्टरहरू क्रमशः $3\vec{i} - \vec{j}$ र $4\vec{i} + 5\vec{j}$ छन् भने BA लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $3\vec{i} - \vec{j}$ and $4\vec{i} + 5\vec{j}$ respectively. Find the position vector of the point M which divides BA internally in the ratio of 2 : 3.

[2074 S]

⇨ Here, let O be the origin then,



$\overrightarrow{OA} = 3\vec{i} - \vec{j}$ and $\overrightarrow{OB} = 4\vec{i} + 5\vec{j}$

Ratio for AB = m : n = 3 : 2

We have,

$$\overrightarrow{OM} = \frac{m\ \vec{b} + n\ \vec{a}}{m+n}$$

$$= \frac{3(4\vec{i} + 5\vec{j}) + 2(3\vec{i} - \vec{j})}{3+2}$$

$$= \frac{12\vec{i} + 15\vec{j} + 6\vec{i} - 2\vec{j}}{5}$$

$$= \frac{18\vec{i} + 13\vec{j}}{5}$$

$$= \frac{18}{5}\vec{i} + \frac{13}{5}\vec{j}$$

Thus, the position vector of M is $\frac{18}{5}\vec{i} + \frac{13}{5}\vec{j}$.

17. A र B का स्थिति भेक्टरकहरू $2\vec{i} + 7\vec{j}$ र $7\vec{i} - 3\vec{j}$ छन् । AB लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $2\vec{i} + 7\vec{j}$ and $7\vec{i} - 3\vec{j}$. Find the position vector of a point which divides AB internally in the ratio of 2 : 3. [2071 R']

⇨ Here, let O be the origin then $\overrightarrow{OA} = 2\vec{i} + 7\vec{j}$ and $\overrightarrow{OB} = 7\vec{i} - 3\vec{j}$ and the given ratio = m : n = 2 : 3

We know that,

Position vector of dividing point P is

$$(\overrightarrow{OP}) = \frac{m\vec{b} + n\vec{a}}{m+n} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$$

$$= \frac{2(7\vec{i} - 3\vec{j}) + 3(2\vec{i} + 7\vec{j})}{2+3}$$

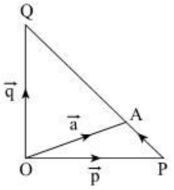
$$= \frac{14\vec{i} - 6\vec{j} + 6\vec{i} + 21\vec{j}}{5}$$

$$= \frac{20\vec{i} + 15\vec{j}}{5} = 4\vec{i} + 3\vec{j}$$

Thus, the position vector of the dividing point is $4\vec{i} + 3\vec{j}$.

18. चित्रमा, यदि $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$ छ भने प्रमाणित गर्नुहोस्:

$$\vec{a} = \frac{1}{4}\left(3\vec{p} + \vec{q}\right)$$



In the figure, if $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$, prove that:

$$\vec{a} = \frac{1}{4}\left(3\vec{p} + \vec{q}\right)$$

⇨ Here, $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$

By the triangle law of vector subtraction,

or, $\overrightarrow{OA} - \overrightarrow{OP} = \frac{1}{4}(\overrightarrow{OQ} - \overrightarrow{OP})$

or, $\vec{a} - \vec{p} = \frac{1}{4}(\vec{q} - \vec{p})$

or, $4\vec{a} - 4\vec{p} = \vec{q} - \vec{p}$

or, $4\vec{a} = \vec{q} + 3\vec{p}$

Thus, $\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$ Proved.

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19. **यदि बिन्दुहरू P र Q का स्थिति भेक्टरहरू क्रमश: $3\vec{i} + 6\vec{j}$ र $5\vec{i} + 2\vec{j}$ छन् भने रेखा PQ लाई भित्रबाट 3 : 2 मा विभाजन गर्ने बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If $3\vec{i} + 6\vec{j}$ and $5\vec{i} + 2\vec{j}$ are the position vectors of the points P and Q respectively, find the position vectors of the point M which divides the line PQ internally in the ratio of 3 : 2. [2070 R']

⇨ Here, $\overrightarrow{OP} = 3\vec{i} + 6\vec{j}$ and $\overrightarrow{OQ} = 5\vec{i} + 2\vec{j}$

Ratio = 3 : 2 = m : n

We have, $\overrightarrow{OM} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

$$= \frac{3\,\overrightarrow{OQ} + 2\,\overrightarrow{OP}}{3+2}$$

$$= \frac{3(5\vec{i} + 2\vec{j}) + 2(3\vec{i} + 6\vec{j})}{5}$$

$$= \frac{1}{5}(15\vec{i} + 6\vec{j} + 6\vec{i} + 12\vec{j})$$

$$= \frac{1}{5}(21\vec{i} + 18\vec{j})$$

$$= \frac{21}{5}\vec{i} + \frac{18}{5}\vec{j}$$

Thus, the position vector of M is $\frac{21}{5}\vec{i} + \frac{18}{5}\vec{j}$.

20. **बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $5\vec{i} + 2\vec{j}$ र $3\vec{i} + 6\vec{j}$ छन् भने AB लाई भित्रबाट 2 : 3 मा विभाजन गर्ने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If $5\vec{i} + 2\vec{j}$ & $3\vec{i} + 6\vec{j}$ are the position vectors of points A and B respectively. Then find the position vector of point 'P' which divides AB internally in the ratio of 2 : 3.

⇨ Here, let O be the origin then

$\vec{a} = \overrightarrow{OA} = 5\vec{i} + 2\vec{j}$, $\vec{b} = \overrightarrow{OB} = 3\vec{i} + 6\vec{j}$ and m : n = 2 : 3

We know that,

$$\overrightarrow{OP} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

$$= \frac{2(\overrightarrow{OB}) + 3(\overrightarrow{OA})}{2+3}$$

$$= \frac{2(3\vec{i} + 6\vec{j}) + 3(5\vec{i} + 2\vec{j})}{5}$$

$$= \frac{6\vec{i} + 12\vec{j} + 15\vec{i} + 6\vec{j}}{5}$$

$$= \frac{21\vec{i} + 18\vec{j}}{5}$$

$$= \frac{21}{5}\vec{i} + \frac{18}{5}\vec{j}$$

Thus, the position vector of P is $\frac{21}{5}\vec{i} + \frac{18}{5}\vec{j}$.

21. **रेखा AB लाई बिन्दु C ले 3 : 1 को अनुपातमा भित्रपट्टि विभाजन गरेको छ । यदि A र B को स्थिति भेक्टर क्रमश: $\vec{i} - 3\vec{j}$ र $2\vec{i} + 5\vec{j}$ भए $|\overrightarrow{AB}|$ र C बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

Point C divides the line AB internally in the ratio of 3 : 1. If the position vectors of A and B are $\vec{i} - 3\vec{j}$ and $2\vec{i} + 5\vec{j}$ respectively, find $|\overrightarrow{AB}|$ and the position vector of point C. [2059 S]

⇨ Here, given

Position vector of point A is $\overrightarrow{OA} = \vec{i} - 3\vec{j}$ and

Position vector of point B is $\overrightarrow{OB} = 2\vec{i} + 5\vec{j}$

Hence, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= (2\vec{i} + 5\vec{j}) - (\vec{i} - 3\vec{j})$$

$$= 2\vec{i} + 5\vec{j} - \vec{i} + 3\vec{j}$$

$$= \vec{i} + 8\vec{j}$$

Again $|\overrightarrow{AB}| = |\vec{i} + 8\vec{j}|$

$$= \sqrt{1^2 + 8^2}$$

$$= \sqrt{1 + 64}$$

$$= \sqrt{65}$$

Since point C divides AB in the ratio 3 : 1, so the positive vector of point C is

$$\overrightarrow{OC} = \frac{n.\overrightarrow{OA} + m.\overrightarrow{OB}}{m+n} \quad \text{where } m : n = 3 : 1$$

$$\therefore \overrightarrow{OC} = \frac{1.(\vec{i} - 3\vec{j}) + 3(2\vec{i} + 5\vec{j})}{3+1}$$

$$= \frac{\vec{i} - 3\vec{j} + 6\vec{i} + 15\vec{j}}{4}$$

$$= \frac{7\vec{i} + 12\vec{j}}{4}$$

$$= \frac{7}{4}\vec{i} + 3\vec{j}$$

Thus, the position vector of C is;

$\left(\frac{7}{4}\vec{i} + 3\vec{j}\right)$ and $|\overrightarrow{AB}| = \sqrt{65}$.

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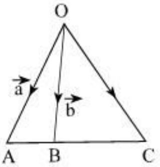
22. दिइएको चित्रमा $\vec{OA} = \vec{a}$ र $\vec{OB} = \vec{b}$ छ । यदि $\vec{AC} = 3\vec{AB}$ भए $\vec{OC}$ पत्ता लगाउनुहोस् ।



In the given figure, $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$. If $\vec{AC} = 3\vec{AB}$, find $\vec{OC}$.
[2058 R, 2067 S, 2069 R]

⇨ Here, In the figure,

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{AC} = 3\vec{AB}$.

To find $\vec{OC} = ?$

Here, from ΔOAB

By using triangle law of vector,

$\vec{AB} + \vec{BO} = \vec{AO}$

or, $\vec{AB} - \vec{OB} = -\vec{OA}$

or, $\vec{AB} = \vec{OB} - \vec{OA}$

∴ $\vec{AB} = \vec{b} - \vec{a}$

Now, $\vec{AC} = 3\vec{AB}$

∴ $\vec{AC} = 3\left(\vec{b} - \vec{a}\right) = 3\vec{b} - 3\vec{a}$

Again from ΔOAC, by using triangle law of vector

$\vec{AC} + \vec{CO} = \vec{AO}$

or, $\vec{AC} - \vec{AO} = -\vec{CO}$

or, $\vec{AC} + \vec{OA} = \vec{OC}$

or, $\vec{OC} = 3\vec{b} - 3\vec{a} + \vec{a}$

Thus, $\vec{OC} = 3\vec{b} - 2\vec{a}$

25. यदि ΔABC का शीर्षबिन्दुहरू A, B र C का स्थिति भेक्टरहरू क्रमश: $(3\vec{i} + 5\vec{j})$, $(5\vec{i} - \vec{j})$ र $(\vec{i} + 8\vec{j})$ छन् भने त्रिभुजको गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of the vertices A, B and C of ΔABC are $(3\vec{i} + 5\vec{j})$, $(5\vec{i} - \vec{j})$ and $(\vec{i} + 8\vec{j})$ respectively, find the position vectors of the centroid G of the triangle. **[2073 R]**

⇨ Here, let O be the origin then, $\vec{OA} = 3\vec{i} + 5\vec{j}$, $\vec{OB} = 5\vec{i} - \vec{j}$ and $\vec{OC} = \vec{i} + 8\vec{j}$

Let, G be the centroid then,

$$\vec{OG} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$$

$$= \frac{1}{3}(3\vec{i} + 5\vec{j} + 5\vec{i} - \vec{j} + \vec{i} + 8\vec{j})$$

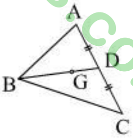
$$= \frac{1}{3}(9\vec{i} + 12\vec{j})$$

or, $\vec{OG} = 3\vec{i} + 4\vec{j}$

Thus, the position vector of G is $3\vec{i} + 4\vec{j}$.

MODEL 3

23. दिइएको चित्रमा AD = DC र बिन्दु G त्रिभुज ABC को गुरुत्वकेन्द्र हो । यदि बिन्दुहरू B र D का स्थिति भेक्टरहरू क्रमश : $3\vec{i} + 7\vec{j}$ र $3\vec{i} - 2\vec{j}$ छन् भने G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।



In the given figure, AD = DC and the point G is the centroid of the triangle ABC. If the position vectors of the points B and D are $3\vec{i} + 7\vec{j}$ and $3\vec{i} - 2\vec{j}$ respectively, find the position vector of G.
[SEE 2075 R, 2075 R$_2$]

⇨ Here, let O be the origin then,

$\vec{OB} = 3\vec{i} + 7\vec{j}$ and $\vec{OD} = 3\vec{i} - 2\vec{j}$

Since AD = DC so, BD is the median of ΔABC,

The ratio in which G divides BD = 2 : 1 then,

$$\text{Position vector of G} = \frac{m\vec{OD} + n\vec{OB}}{m+n}$$

$$= \frac{2(3\vec{i} - 2\vec{j}) + 1(3\vec{i} + 7\vec{j})}{2+1}$$

$$= \frac{6\vec{i} - 4\vec{j} + 3\vec{i} + 7\vec{j}}{3}$$

$$= \frac{9\vec{i} + 3\vec{j}}{3}$$

$$= 3\vec{i} + \vec{j}$$

Thus, position vector of centroid G is $3\vec{i} + \vec{j}$.

24. यदि ΔABC का शिर्षबिन्दुहरू A, B र C का स्थिति भेक्टरहरू क्रमश: $(3\vec{i} + 4\vec{j})$, $(4\vec{i} + 5\vec{j})$ र $(5\vec{i} + 6\vec{j})$ छन् भने सो त्रिभुजको गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of the vertices A, B, and C of ΔABC are $(3\vec{i} + 4\vec{j})$, $(4\vec{i} + 5\vec{j})$ and $(5\vec{i} + 6\vec{j})$ respectivley, find the position vector of the centroid G of the triangle. **[2073 S]**

⇨ Here, let O be the origin then,

$\vec{OA} = 3\vec{i} + 4\vec{j}$, $\vec{OB} = 4\vec{i} + 5\vec{j}$ and

$\vec{OC} = 5\vec{i} + 6\vec{j}$

We have,

position vector of centroid $(\vec{OG})$

$$= \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$$

$$= \frac{1}{3}(3\vec{i} + 4\vec{j} + 4\vec{i} + 5\vec{j} + 5\vec{i} + 6\vec{j})$$

$$= \frac{1}{3}(12\vec{i} + 15\vec{j})$$

∴ $\vec{OG} = 4\vec{i} + 5\vec{j}$

Thus, the position vector of centroid G is $4\vec{i} + 5\vec{j}$.

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26. यदि A(3, 5), B(5, – 1) र C(2, 4) ΔABC का शीर्षबिन्दुहरू र ΔABC को गुरुत्वकेन्द्र G भए G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
If A(3, 5), B(5, – 1) and C(2, 4) are the vertices of ΔABC and G is the centroid of the ΔABC. Find the position vector of G.
[2065 S]

⇨ Here, given vertices of ΔABC are; A(3, 5), B(5, – 1) and C(2, 4)

Let, O be the origin then, $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

We know that, The position vector of centroid of ΔABC; $(\overrightarrow{OG}) = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$

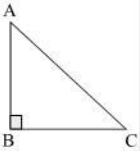
So, $\overrightarrow{OG} = \frac{1}{3}\left[\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix} 3+5+2 \\ 5-1+4 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 10 \\ 8 \end{pmatrix}$

$\therefore \overrightarrow{OG} = \begin{pmatrix} \frac{10}{3} \\ \frac{8}{3} \end{pmatrix}$

Thus, the position vector of centroid G is $\frac{10}{3}\vec{i} + \frac{8}{3}\vec{j}$.

MODEL 4

27. दिइएको चित्रमा ∠ABC = 90° छ भने $\overrightarrow{AC}^2 = \overrightarrow{AB}^2 + \overrightarrow{BC}^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In the given figure ∠ABC = 90°.
Prove that : $\overrightarrow{AC}^2 = \overrightarrow{AB}^2 + \overrightarrow{BC}^2$. **[2068 R]**



⇨ Here, ∠ABC = 90° so $\overrightarrow{AB}.\overrightarrow{BC} = 0$
By the triangle law of vector addition,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

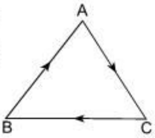
or, $\overrightarrow{AC}^2 = (\overrightarrow{AB} + \overrightarrow{BC})^2$

or, $\overrightarrow{AC}^2 = \overrightarrow{AB}^2 + 2\overrightarrow{AB}.\overrightarrow{BC} + \overrightarrow{BC}^2$

or, $\overrightarrow{AC}^2 = \overrightarrow{AB}^2 + 2.0 + \overrightarrow{BC}^2$

Thus, $\overrightarrow{AC}^2 = \overrightarrow{AB}^2 + \overrightarrow{BC}^2$ **Proved.**

28. दिइएको त्रिभुज ABC मा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$ in given triangle ABC.



⇨ Here, $\vec{AB}$, $\vec{BC}$ and $\vec{CA}$ are the vectors representing the sides of given ΔABC.
Now, by using 'Triangular law of vector addition"
We have,

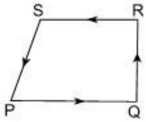
$\vec{BC} + \vec{CA} = \vec{BA}$

or, $-\vec{BA} + \vec{BC} + \vec{CA} = 0$

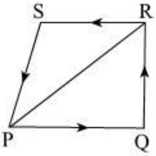
Thus, $\vec{AB} + \vec{BC} + \vec{CA} = 0$, where - $\vec{BA} = \vec{AB}$.

Proved.

29. दिइएको चतुर्भुज PQRS मा $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SP} = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SP} = 0$ in the given quadrilateral PQRS. **[2059 R]**



⇨ Here, in the given figure, join the points P and R so that the two triangles PQR and RSP are formed.
Now, from ΔPQR by using triangle law of vector,



$\vec{PQ} + \vec{QR} = \vec{PR}$ .... (i)

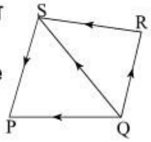
Again, from ΔRSP, $\vec{RS} + \vec{SP} = \vec{RP}$ .... (ii)
Adding equations (i) & (ii), we get,

$\vec{PQ} + \vec{QR} + \vec{RS} + \vec{SP} = \vec{PR} + \vec{RP}$

or, $\vec{PQ} + \vec{QR} + \vec{RS} + \vec{SP} = \vec{PR} - \vec{PR}$

Thus, $\vec{PQ} + \vec{QR} + \vec{RS} + \vec{SP} = 0$ **Proved.**

30. यदि PQRS एउटा चतुर्भुज भए प्रमाणित गर्नुहोस्
If PQRS is a quadrilateral, prove that:
$\overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SP} = \overrightarrow{QP}$



⇨ Here, LHS $= \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{SP}$

$= \overrightarrow{QS} + \overrightarrow{SP}$

[∵ From ΔQRS triangular law]

$= \overrightarrow{QP}$ [∵ From ΔQSP triangular law]

Thus, LHS = RHS **Proved.**

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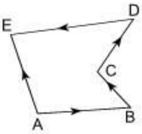
31. चित्रमा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given diagram, prove that: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$.



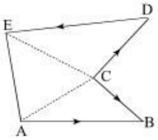
⇨ Here, in the given figure, to prove :

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$

Here in the given figure,

Using triangle law of vector addition,

In ΔABC



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ ...... (i)

Again, from ΔACD

$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$

$\therefore \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ ...... (ii)

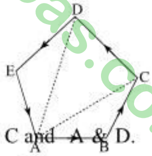
Again, from ΔADE, $\overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$

or, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = -\overrightarrow{EA}$

Thus, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$

32. चित्रमा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given figure, prove that: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$ [2062R]



⇨ Here, in the given figure, join A & C and A & D.

To Show : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$

Now using triangle law of vector addition

LHS $= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

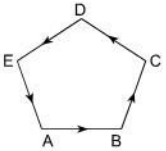
$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE}$

$= \overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$

Thus, LHS = RHS

33. दिइएको पञ्चभुज ABCDE मा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

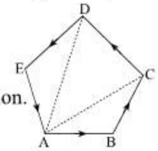
In the given pentagon ABCDE, prove that: [2060 CP] $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$



⇨ Here, ABCDE is a pentagon.

Joining AC and AD.

By the triangle law of vector addition.



(i) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

(ii) $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$

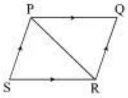
(iii) $\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE}$ [From (ii)]

or, $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$ [From (i)]

Thus, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$ **Proved.**

MODEL 5

34. दिइएको समानान्तर चतुर्भुजमा $\overrightarrow{SR} = 4\overrightarrow{i} - 2\overrightarrow{j}$ र $\overrightarrow{PR} = 6\overrightarrow{i} + 5\overrightarrow{j}$ भए $\overrightarrow{QR}$ पत्ता लगाउनुहोस् ।



In the given parallelogram $\overrightarrow{SR} = 4\overrightarrow{i} - 2\overrightarrow{j}$ and $\overrightarrow{PR} = 6\overrightarrow{i} + 5\overrightarrow{j}$, find $\overrightarrow{QR}$.

⇨ Here, $\overrightarrow{SR} = 4\overrightarrow{i} - 2\overrightarrow{j}$ and $\overrightarrow{PR} = 6\overrightarrow{i} - 5\overrightarrow{j}$

We know that,

$\overrightarrow{QR} = \overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS}$

$= 6\overrightarrow{i} + 5\overrightarrow{j} - \overrightarrow{SR}$

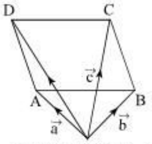
$= 6\overrightarrow{i} + 5\overrightarrow{j} - (4\overrightarrow{i} - 2\overrightarrow{j})$

$= (6 - 4)\overrightarrow{i} + (5 + 2)\overrightarrow{j}$

$\therefore \overrightarrow{QR} = 2\overrightarrow{i} + 7\overrightarrow{j}$

Thus, $\overrightarrow{QR}$ is $2\overrightarrow{i} + 7\overrightarrow{j}$.

35. दिइएको चित्रमा ABCD एउटा समानान्तर चतुर्भुज हो । $\overrightarrow{OA} = \overrightarrow{a}$, $\overrightarrow{OB} = \overrightarrow{b}$ र $\overrightarrow{OC} = \overrightarrow{c}$ भए $\overrightarrow{OD}$ निकाल्नुहोस् ।



In the given figure, ABCD is a parallelogram. If $\overrightarrow{OA} = \overrightarrow{a}$, $\overrightarrow{OB} = \overrightarrow{b}$ and $\overrightarrow{OC} = \overrightarrow{c}$, find $\overrightarrow{OD}$.

⇨ Here, given that ABCD is a parallelogram where,

$\overrightarrow{OA} = \overrightarrow{a}$, $\overrightarrow{OB} = \overrightarrow{b}$, $\overrightarrow{OC} = \overrightarrow{c}$, $\overrightarrow{OD} = ?$

Here in ΔOAB using triangle law of vector addition,

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$

or, $\overrightarrow{AB} = -\overrightarrow{a} + \overrightarrow{b}$

or, $\overrightarrow{BA} = -(-\overrightarrow{a} + \overrightarrow{b})$

$\therefore \overrightarrow{BA} = \overrightarrow{a} - \overrightarrow{b}$ .... (i)

Again, in ΔOCD,

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{c} + \overrightarrow{BA}$ [$\because$ $\overrightarrow{CD} = \overrightarrow{BA}$]

$= \overrightarrow{c} + \overrightarrow{a} - \overrightarrow{b}$ [$\because$ from (i)]

Thus, $\overrightarrow{OD} = \overrightarrow{a} - \overrightarrow{b} + \overrightarrow{c}$

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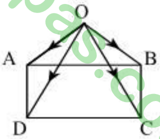
36. दिइएको चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ र $\overrightarrow{AD} = \overrightarrow{BC}$ भए $\overrightarrow{OD}$ पत्ता लगाउनुहोस् ।

In the given diagram, find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ and $\overrightarrow{AD} = \overrightarrow{BC}$. [2067 R]



⇨ Here, from the given figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$, $\overrightarrow{AD} = \overrightarrow{BC}$

Now, $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ [From ΔAOD]

or, $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ [Given]

or, $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BO} + \overrightarrow{OC}$ [From ΔBOC]

or, $\overrightarrow{OD} = \overrightarrow{OA} - \overrightarrow{OB} + \overrightarrow{OC}$

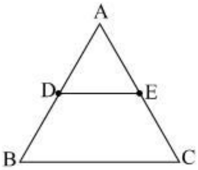
or, $\overrightarrow{OD} = \vec{a} - \vec{b} + \vec{c}$

Thus, the required value of $\overrightarrow{OD}$ is $(\vec{a} - \vec{b} + \vec{c})$.

MODEL 6

37. ΔABC मा AB र AC लाई क्रमश: D र E ले 1 : 2 को अनुपातमा विभाजन गर्दछन् भने $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In ΔABC, D and E divides AB and AC in the ratio of 1 : 2 respectively. Prove that: $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$ [2066 S]



⇨ Here, $\dfrac{\overrightarrow{AD}}{\overrightarrow{DB}} = \dfrac{\overrightarrow{AE}}{\overrightarrow{EC}} = \dfrac{1}{2}$ $\quad \therefore \overrightarrow{DB} = 2\overrightarrow{AD}$ and $\overrightarrow{EC} = 2\overrightarrow{AE}$

Now, $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ [By triangular law of vector addition]

$= \overrightarrow{BD} + \overrightarrow{DA} + \overrightarrow{AE} + \overrightarrow{EC}$

$= -2\overrightarrow{AD} + \overrightarrow{DA} + \overrightarrow{AE} + 2\overrightarrow{AE}$

$= 3\overrightarrow{DA} + 3\overrightarrow{AE}$

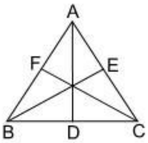
$= 3(\overrightarrow{DA} + \overrightarrow{AE})$

$= 3\overrightarrow{DE}$

Thus, $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$ **Proved.**

38. ΔABC मा D, E, F हरू क्रमश: भुजाहरू BC, CA र AB का मध्यबिन्दुहरू भए $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$ प्रमाणित गर्नुहोस् ।

In ΔABC, D, E, F are the mid points of sides BC, CA and AB respectively. Prove that: $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$ [2058 R]



⇨ Here, In ΔABC, points D, E & F are the mid points of BC, CA & AB respectively.

Now, LHS $= \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$

$= \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2} + \dfrac{\overrightarrow{BC} + \overrightarrow{BA}}{2} + \dfrac{\overrightarrow{CB} + \overrightarrow{CA}}{2}$

$= \dfrac{\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BC} + \overrightarrow{BA} + \overrightarrow{CB} + \overrightarrow{CA}}{2}$

$= \dfrac{\overrightarrow{AB} - \overrightarrow{CA} + \overrightarrow{BC} - \overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{CA}}{2}$

$= \dfrac{0}{2}$

$= 0$

Thus, LHS = RHS **Proved.**

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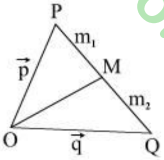
C. LONG QUESTIONS

MODEL 1



1. **यदि $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ र रेखाखण्ड PQ लाई भित्रपट्टिबाट बिन्दु M ले $m_1 : m_2$ को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस्: $\overrightarrow{OM} = \dfrac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$**

If $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ and the point M divides the line segment PQ internally in the ratio of $m_1 : m_2$ then prove that: $\overrightarrow{OM} = \dfrac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$ **[2072 R']**

⇨ Here, let O be the origin. The PQ is such that: $\frac{PM}{MQ} = \frac{m_1}{m_2}$

$$\text{or, } \frac{\overrightarrow{PM}}{\overrightarrow{MQ}} = \frac{m_1}{m_2}$$

$$\text{or, } m_2\overrightarrow{PM} = m_1\overrightarrow{MQ}$$

$$\text{or, } m_2(\overrightarrow{OM} - \overrightarrow{OP}) = m_1(\overrightarrow{OQ} - \overrightarrow{OM})$$

$$\text{or, } m_2\overrightarrow{OM} - m_2\vec{p} = m_1\vec{q} - m_1\overrightarrow{OM}$$

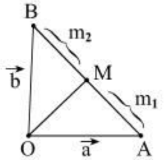
$$\text{or, } (m_2 + m_1)\overrightarrow{OM} = m_1\vec{q} + m_2\vec{p}$$

$$\therefore \overrightarrow{OM} = \frac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$$

Proved.

2. **यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र रेखाखण्ड AB लाई भित्रपट्टीबाट बिन्दु M ले $m_1 : m_2$ को अनुपातमा विभाजन गर्दछ भने, प्रमाणित गर्नुहोस् :**

If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and the point M divides the line segment AB internally in the ratio of $m_1 : m_2$, then prove that: $\overrightarrow{OM} = \dfrac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$ **[2071 R]**



⇨ Here, let O be the origin $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and M be a point on AB which divides AB in the ratio of $m_1 : m_2$.

From the figure, $\frac{AM}{MB} = \frac{m_1}{m_2}$

$$\text{or, } \frac{\overrightarrow{AM}}{\overrightarrow{MB}} = \frac{m_1}{m_2}$$

$$\text{or, } m_2\overrightarrow{AM} = m_1\overrightarrow{MB}$$

$$\text{or, } m_2(\overrightarrow{OM} - \overrightarrow{OA}) = m_1(\overrightarrow{OB} - \overrightarrow{OM})$$

$$\text{or, } m_2(\overrightarrow{OM} - \vec{a}) = m_1(\vec{b} - \overrightarrow{OM})$$

$$\text{or, } m_2\overrightarrow{OM} - m_2\vec{a} = m_1\vec{b} - m_1\overrightarrow{OM}$$

$$\text{or, } \overrightarrow{OM}(m_1 + m_2) = m_2\vec{a} + m_1\vec{b}$$

$$\text{or, } \overrightarrow{OM} = \frac{m_2\vec{a} + m_1\vec{b}}{m_1 + m_2} = \frac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$$

Thus, $\overrightarrow{OM} = \dfrac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$ Proved.

Thus, $\overrightarrow{OM} = \dfrac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$

Proved.

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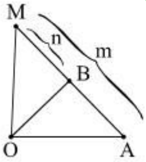
3. यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र रेखाखण्ड AB लाई बाहिरपट्टीबाट बिन्दु M ले m:n को अनुपातमा विभाजन गर्दछ भने, प्रमाणित गर्नुहोस् :

If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and the point M divides the line segment AB externally in the ratio of m:n then prove that $\overrightarrow{OM} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$. [2071 S]



⇨ Here, let O be the origin and M divides the line segment AB externally in the ratio of m : n.

$$\frac{AM}{BM} = \frac{m}{n}$$

or, $m\,BM = n\,AM$

or, $m\,\overrightarrow{BM} = n\,\overrightarrow{AM}$

or, $m(\overrightarrow{OM} - \overrightarrow{OB}) = n(\overrightarrow{OM} - \overrightarrow{OA})$

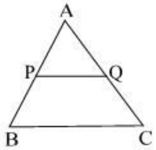
or, $m\,\overrightarrow{OM} - m\,\vec{b} = n\,\overrightarrow{OM} - n\,\vec{a}$

or, $m\,\overrightarrow{OM} - n\,\overrightarrow{OM} = m\,\vec{b} - n\,\vec{a}$

or, $\overrightarrow{OM}\,(m-n) = m\,\vec{b} - n\,\vec{a}$

Thus, $\overrightarrow{OM} = \dfrac{m\,\vec{b} - n\,\vec{a}}{m-n}$ **Proved.**

MODEL 2



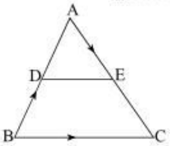
4. दिइएको चित्रमा P र Q त्रिभुज ABC को क्रमशः भुजा AB र AC का मध्य बिन्दुहरू भए PQ // BC र BC = 2PQ हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

In the figure, P and Q are the middle points of AB & AC respectively of the ΔABC. Prove vectorically that PQ // BC & BC = 2PQ [SEE 2075 R', 2059 S]

Or त्रिभुज ABC मा D र E क्रमशः भुजाहरू AB र AC का मध्यबिन्दुहरू हुन् । भेक्टर विधिबाट प्रमाणित गर्नुहोस्: $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$

In the Δ ABC, D and E are the mid-points of the sides AB and AC respectively. Prove by vector method that: $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$

[2066 R']



Or त्रिभुजको कुनै दुई भुजाहरूको मध्यबिन्दुहरू जोड्ने रेखाखण्ड तेस्रो भुजासँग समानान्तर भइ त्यसको आधा हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

Prove by vector method that a line segment joining the mid points of any two sides of a triangle is parallel to the third side and to half of it. [2060 S]

⇨ Here, let ABC be a triangle with P and Q be the mid-points of AB and AC respectively.

(i) $\overrightarrow{BA} = 2\overrightarrow{PA}$ [∵ P is a mid point of AB]

(ii) $\overrightarrow{AC} = 2\,\overrightarrow{AQ}$ [∵ Q is a mid point of AC]

(iii) $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ [Triangle law of vector addition]

or, $2\overrightarrow{PA} + 2\,\overrightarrow{AQ} = \overrightarrow{BC}$

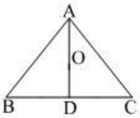
or, $2\,(\overrightarrow{PA} + \overrightarrow{AQ}) = \overrightarrow{BC}$

(iv) $\overrightarrow{PA} + \overrightarrow{AQ} = \overrightarrow{PQ}$ [Triangle law of vector addition]

(v) ∴ $2\overrightarrow{PQ} = \overrightarrow{BC}$ [From (iii) and (iv)]

Thus, $\overrightarrow{PQ} // \overrightarrow{BC}$ and $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{BC}$. **Proved.**

5. यदि $\vec{a}$, $\vec{b}$ र $\vec{c}$ ΔABC को शीर्ष बिन्दुहरूका स्थिति भेक्टरहरू भए ΔABC को भारकेन्द्र G को स्थिति भेक्टर $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।



If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the vertices of a ΔABC. Prove by vector method that the position vector of the centroid G of the triangle ΔABC is $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$. [2066 R]

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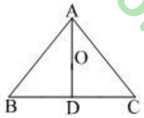
Or, त्रिभुजको गुरुत्वकेन्द्र (भारकेन्द्र) को स्थिति भेक्टर यसको शीर्षहरूका स्थिति भेक्टरहरूको योगफलको एकतीहाइ हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

Using vector method, prove that the position vector of the centroid of a triangle is one third the sum of the position vectors of its vertices. **[2065 M]**

⇨ Here, let, O be the origin then, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OG} = \vec{g}$



(i) Position vector of D, $(\overrightarrow{OD}) = \frac{1}{2}(\vec{b} + \vec{c})$ [∵ Mid-point theorem]

(ii) Since centroid divides the median AD in the ratio of 2 : 1 = m : n.

From ΔOAD,

$$\overrightarrow{OG} = \frac{m\vec{b} + n\vec{a}}{m+n} = \frac{2.\overrightarrow{OD} + 1.\overrightarrow{OA}}{2+1} = \frac{2\left\{\frac{1}{2}(\vec{b} + \vec{c})\right\} + 1.\vec{a}}{2+1} = \frac{\vec{b} + \vec{c} + \vec{a}}{3}$$

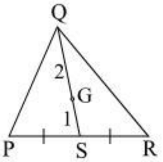
$$\therefore \quad \vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$$

Thus, the position vector of the point G is $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ **Proved.**

Or, ΔPQR मा मध्यरेखा QS र गुरुत्वकेन्द्र G छन् । यदि O उद्गम बिन्दु हो भने, प्रमाणित गर्नुहोस् : $\overrightarrow{OG} = \frac{1}{3}\left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$

In ΔPQR, QS is a median and G is the centroid. If O is the origin then prove that: $\overrightarrow{OG} = \frac{1}{3}\left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$ [2074 R]

⇨ Here, let O be the origin and G be the centroid of ΔPQR.
In ΔPQR; QS is a median.
So, S is the mid point of PR.



(i) $\overrightarrow{OS} = \frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OR})$

Since, centroid (G) divides median (QS) in the ratio of 2 : 1 = m : n
So, using section formula

(ii) $$\overrightarrow{OG} = \frac{m\,\overrightarrow{OS} + n\,\overrightarrow{OQ}}{m+n} = \frac{2\,\overrightarrow{OS} + 1\,\overrightarrow{OQ}}{2+1} = \frac{2 \times \frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OR}) + \overrightarrow{OQ}}{3} \text{ [from (i)]}$$

$$= \frac{\overrightarrow{OP} + \overrightarrow{OR} + \overrightarrow{OQ}}{3}$$

$$\therefore \quad \overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR})$$

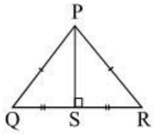
Thus, the position vector of centroid i.e. $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR})$ **Proved.**

6. समद्विबाहु त्रिभुजको मध्यिका आधारमा लम्ब हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।

Prove vectorically that the median of an isosceles triangle is perpendicular to the base. **[2072 S, 2065 M]**

OR दिइएको चित्रमा समद्विबाहु ΔPQR मा PQ = PR र QS = SR भए रेखा PS आधार QR मा लम्ब हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।

In the given figure, in isosceles ΔPQR, if PQ = PR and QS = SR then prove by vector method that the line PS is perpendicular to base QR.[2073 R, 2069 R', 2069 S, 2067 R]



⇨ Here, let PQR be an isosceles triangle with PQ = QR and PS is median of ΔPQR.

(i) $\overrightarrow{PS} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{PR})$ [Mid point theorem]

(ii) $\overrightarrow{QR} = (\overrightarrow{PR} - \overrightarrow{PQ})$ [Triangle law of vector subtraction]

(iii) $$\overrightarrow{PS}.\overrightarrow{QR} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{PR})(\overrightarrow{PR} - \overrightarrow{PQ})$$

$$= \frac{1}{2}[(\overrightarrow{PR})^2 - (\overrightarrow{PQ})^2]$$

$$= \frac{1}{2}(PR^2 - PQ^2)$$

$$= \frac{1}{2} \times 0 \text{ [Since PR = PQ]}$$

(iv) $\overrightarrow{PS} \perp \overrightarrow{QR}$ [∵ $\overrightarrow{PS}.\overrightarrow{QR} = 0$] **Proved.**

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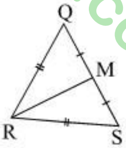
OR दिइएको चित्रमा RQ = RS र QM = SM भए भेक्टर विधिबाट प्रमाणित गर्नुहोस् : RM ⊥ QS
In the given figure, RQ = RS and QM = SM then prove by vector method: RM ⊥ QS **[2074 S, 2073 S, 2073 S']**



⇨ Given: In the given figure, RQ = RS and QM = SM
To Prove: RM ⊥ QS

(i) $\overrightarrow{RM} = \frac{1}{2}(\overrightarrow{RQ} + \overrightarrow{RS})$ [Using mid point theorem]

(ii) $\overrightarrow{SQ} = \overrightarrow{SR} + \overrightarrow{RQ}$ [Using triangle law of vector addition]

(iii) $\overrightarrow{RM} \cdot \overrightarrow{SQ} = \frac{1}{2}(\overrightarrow{RQ} + \overrightarrow{RS}) \cdot (\overrightarrow{SR} + \overrightarrow{RQ})$

$= \frac{1}{2}(\overrightarrow{RQ} + \overrightarrow{RS}) \cdot (\overrightarrow{RQ} - \overrightarrow{RS})$

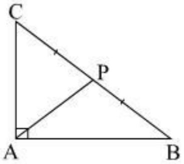
$= \frac{1}{2}[(\overrightarrow{RQ})^2 - (\overrightarrow{RS})^2]$

$= \frac{1}{2}(RQ^2 - RS^2)$

$= \frac{1}{2} \times 0$ [$\because$ RQ = RS] = 0

(iv) RM ⊥ SQ [From (iii) $\overrightarrow{RM} \cdot \overrightarrow{SQ} = 0$] **Proved.**

7. समकोणी त्रिभुजमा कर्णको मध्यबिन्दु यसका शीर्षहरूबाट समदुरीमा पर्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the middle point of hypotenuse of a right angled triangle is equidistant from its vertices. **[2067 S, 2065 M]**



⇨ Here, let, ABC be a right angled triangle whose right angle is at A.
Let, P be the middle point of the hypotenuse BC. Let, A be the origin then,

(i) $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$ (Triangle law of vector addition)

(ii) $\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC}$ (Triangle law of vector addition)

(iii) $\overrightarrow{AB} \perp \overrightarrow{AC}$ [$\because \measuredangle A = 90°$]

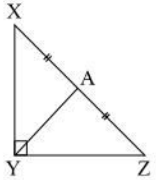
or, $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$ [$\because$ AB ⊥ AC]

or, $(\overrightarrow{AP} + \overrightarrow{PB}) \cdot (\overrightarrow{AP} + \overrightarrow{PC}) = 0$ [$\because$ From (i) and (ii)]

or, $(\overrightarrow{AP} + \overrightarrow{CP}) \cdot (\overrightarrow{AP} - \overrightarrow{CP}) = 0$ [$\because \overrightarrow{CP} = \overrightarrow{PB}$ and $\overrightarrow{PC} = -\overrightarrow{CP}$]
or, $(AP)^2 - (CP)^2 = 0$
or, $AP^2 = CP^2$

$\therefore$ AP = CP = PB [$\therefore$ CP = PB] **Proved.**

OR दिइएको त्रिभुज XYZ मा ∡XYZ = 90° र XZ को मध्यबिन्दु A छ भने XA = YA = ZA हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In the given triangle XYZ, ∡XYZ = 90° and A is the middle point of XZ. Prove by vector method that: XA = YA = ZA. **[2070 R, 2073 R', 2070 S]**



⇨ Here, in the given triangle XYZ, ∡XYZ = 90° and A is the middle point of XZ.
To Prove: XA = YA = ZA

(i) $\overrightarrow{YZ} = \overrightarrow{YA} + \overrightarrow{AZ}$ [By the triangle law of vector addition]

(ii) $\overrightarrow{XY} = \overrightarrow{YA} + \overrightarrow{AY}$ [By the triangle law of vector addition]

(iii) $\overrightarrow{XY} \cdot \overrightarrow{YZ} = 0$ [Since ∡XYZ = 90°]

or, $(\overrightarrow{XA} + \overrightarrow{AY}) \cdot (\overrightarrow{YA} + \overrightarrow{AZ}) = 0$

or, $(\overrightarrow{XA} + \overrightarrow{AY}) \cdot (-\overrightarrow{AY} + \overrightarrow{XA}) = 0$

or, $(\overrightarrow{XA} + \overrightarrow{AY}) \cdot (\overrightarrow{XA} - \overrightarrow{AY}) = 0$

or, $(\overrightarrow{XA})^2 - (\overrightarrow{AY})^2 = 0$
or, $XA^2 - AY^2 = 0$
or, $XA^2 = AY^2$
$\therefore$ XA = AY

(iv) XA = AY = AZ [$\because$ XA = AZ and from (iii)] **Proved.**

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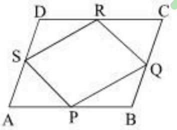
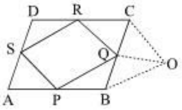
MODEL 3

8. **चतुर्भुजका भुजाहरूको मध्यबिन्दुहरू क्रमश: जोड्दै जाँदा बन्ने चित्र समानान्तर चतुर्भुज हुन्छ भनी प्रमाणित गर्नुहोस् ।**
Prove that the lines joining the middle points of the sides of a quadrilateral taken in order is a parallelogram.
[SEE MODEL 2076, 2065 M]

Or **दिइएको चित्रमा P, Q, R र S क्रमश: AB, BC, CD र DA का मध्यबिन्दुहरू भए PQRS स.च. हो भनी प्रमाणित गर्नुहोस् ।**
In the given figure P, Q, R and S are the mid points of AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram. **[2059 R, 2062 K, 2065 S]**



⇨ Here, In the figure alongside, let ABCD be a quadrilateral and PQRS be the resulting quadrilateral by joining the middle points of sides AB, BC, CD and DA. Let O be the origin of vectors then by mid point theorem,



(i) $\overrightarrow{OP} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$; $\overrightarrow{OQ} = \dfrac{\overrightarrow{OB} + \overrightarrow{OC}}{2}$; $\overrightarrow{OR} = \dfrac{\overrightarrow{OC} + \overrightarrow{OD}}{2}$; $\overrightarrow{OS} = \dfrac{\overrightarrow{OD} + \overrightarrow{OA}}{2}$

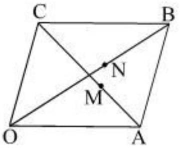
(ii) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \dfrac{\overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA} - \overrightarrow{OB}}{2} = \dfrac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

(iii) $\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS} = \dfrac{\overrightarrow{OC} + \overrightarrow{OD} - \overrightarrow{OD} - \overrightarrow{OA}}{2} = \dfrac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

Therefore $\overrightarrow{PQ} = \overrightarrow{SR}$ i.e. PQ = SR and PR // SR
Similarly SP = RQ and SP // RQ Thus, PQRS is a parallelogram. **Proved.**

9. **समानान्तर चतुर्भुजका विकर्णहरू आपसमा समद्विभाजन हुन्छन् भनी प्रमाणित गर्नुहोस् ।**
Prove that the diagonals of a parallelogram bisect each other. **[2075 R'$_2$, 2057S,2061S,2065 M,2067 R',2068 R]**

⇨ Here, let, OABC be a parallelogram and AC and OB are the diagonals.
Let O be the origin , M and N are mid points of AC and OB in parallelogram OABC.

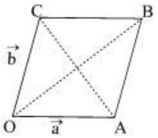


(a) $\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ [Mid point theorem]

(b) $\overrightarrow{ON} = \dfrac{\overrightarrow{OB}}{2} = \dfrac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ [Parallelogram Law of vector addition]

$\therefore$ $\overrightarrow{OM} = \overrightarrow{ON}$ so M and N are coincide. Thus, diagonals of parallelogram bisects to each other.

10. **समबाहु चतुर्भुजको विकर्णहरू समकोण हुने गरी समद्विभाजन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।**
Prove that the vector method, the diagonals of rhombus bisects each to other at right angle.
[2070 S', 2065 R']



⇨ Here, let, OABC be a rhombus and OB and AC be its diagonals.

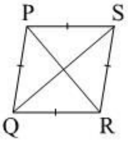
Let O be the origin, $\overrightarrow{AB} = \overrightarrow{OC} = \vec{b}$ and $\overrightarrow{OA} = \overrightarrow{CB} = \vec{a}$
From the triangle law of vector and parallelogram law of vector,

(i) $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = -\vec{b} + \vec{a} = \vec{a} - \vec{b}$ and $\overrightarrow{OB} = \vec{a} + \vec{b}$

(ii) $\overrightarrow{CA} \,.\, \overrightarrow{OB} = (\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = (\vec{a})^2 - (\vec{b})^2$
$= a^2 - b^2$ [$\because$ the square of any vector is a scalar]
$= a^2 - a^2$ [$b^2 = a^2$ since the sides of a rhombus]

$\therefore$ $\overrightarrow{CA} \,.\, \overrightarrow{OB} = 0$ where, the dot product of the diagonals of the rhombus is 0. Thus, the diagonals are perpendicular to each other.

Or **दिइएको चित्रमा** PQ = QR = RS = SP **भए भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् :** PR ⊥ QS
In the given figure, PQ = QR = RS = SP. Prove by vector method: PR ⊥ QS. **[2072 R]**



⇨ Here, Given PQ = QR = RS = SP To Prove: PR ⊥ QS
Proof:

1. $\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{SR}$ [$\because$ Triangle law of vector addition.]

2. $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$ [$\because$ Triangle law of vector addition.]

3. $\overrightarrow{PR}.\overrightarrow{QS} = (\overrightarrow{PS} + \overrightarrow{SR}).(\overrightarrow{QP} + \overrightarrow{PS})$ [From 1 & 2]
$= (\overrightarrow{PS} + \overrightarrow{SR}).(\overrightarrow{PS} - \overrightarrow{PQ})$
$= (\overrightarrow{PS} + \overrightarrow{SR}).(\overrightarrow{PS} - \overrightarrow{SR})$ [$\because \overrightarrow{PQ} = \overrightarrow{SR}$]
$= PS^2 - SR^2 = 0$

4. PR ⊥ QS [Since $\overrightarrow{PR}.\overrightarrow{QS} = 0$ in (3)]

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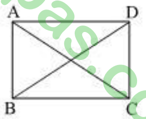
11. **आयतका विकर्णहरू बराबर हुन्छन् भनी भेक्टरविधिद्वारा प्रमाणित गर्नुहोस् ।**
Prove vectorically that the diagonals of a rectangle are equal to each other.
[2071 R', 2069 R, 2068 R', 2065 M, SLC 2066 S]



⇨ Here, let, ABCD be a rectangle and AC and BD are its diagonals.

(i) $AC^2 = (\overrightarrow{AC})^2 = (\overrightarrow{AB} + \overrightarrow{BC})^2$

$= (\overrightarrow{DC} + \overrightarrow{BC})^2$ [∵ AB = DC]

$= \overrightarrow{DC}^2 + 2\overrightarrow{DC}.\overrightarrow{BC} + \overrightarrow{BC}^2$

$= \overrightarrow{CD}^2 + 2 \times 0 + \overrightarrow{BC}^2$ [$\because \overrightarrow{DC} \perp \overrightarrow{BC}$]

$\therefore AC^2 = \overrightarrow{CD}^2 + \overrightarrow{BC}^2$

(ii) $BD^2 = \overrightarrow{BD}^2 = (\overrightarrow{BC} + \overrightarrow{CD})^2$

$= \overrightarrow{BC}^2 + 2\overrightarrow{BC}.\overrightarrow{CD} + \overrightarrow{CD}^2$

$= \overrightarrow{BC}^2 + \overrightarrow{CD}^2$ [$\because \overrightarrow{BC} \perp \overrightarrow{CD}$]

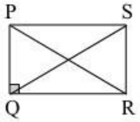
$\therefore \overrightarrow{BD}^2 = \overrightarrow{CD}^2 + \overrightarrow{BC}^2$

(iii) $AC^2 = BD^2$ [From (i) and (ii)] $\therefore AC = BD$

Thus, diagonals of a rectangle are equal to each other **Proved.**

Thus, the height of the tower is 109.28 m

Or **दिइएको चित्रमा यदि PQRS एउटा आयत भए भेक्टर विधिबाट प्रमाणित गर्नुहोस् : PR = QS.**
In the given figure, if PQRS is a rectangle then prove by vector method: PR = QS. [2074 R']



⇨ Here, let, PQRS be a rectangle and PR and QS are its diagonals.

(i) $PR^2 = (\overrightarrow{PR})^2 = (\overrightarrow{PQ} + \overrightarrow{QR})^2$

$= (\overrightarrow{SR} + \overrightarrow{QR})^2$ [∵ PQ = SR]

$= \overrightarrow{SR}^2 + 2\overrightarrow{SR}.\overrightarrow{QR} + \overrightarrow{QR}^2$

$= \overrightarrow{RS}^2 + 2 \times 0 + \overrightarrow{QR}^2$ [$\because \overrightarrow{SR} \perp \overrightarrow{QR}$]

$\therefore PR^2 = \overrightarrow{RS}^2 + \overrightarrow{QR}^2$

(ii) $QS^2 = \overrightarrow{QS}^2 = (\overrightarrow{QR} + \overrightarrow{RS})^2$

$= \overrightarrow{QR}^2 + 2\overrightarrow{QR}.\overrightarrow{RS} + \overrightarrow{RS}^2$

$= \overrightarrow{QR}^2 + \overrightarrow{RS}^2$ [$\because \overrightarrow{QR} \perp \overrightarrow{RS}$]

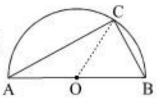
$\therefore \overrightarrow{QS}^2 = \overrightarrow{RS}^2 + \overrightarrow{QR}^2$

(iii) $PR^2 = QS^2$ [From (i) and (ii)]

$\therefore PR = QS$ **Proved.**

MODEL 4

12. **अर्धवृत्त (वृत्तार्ध) मा बन्ने परिधिकोण समकोण हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।**
Prove vectorically that the angle at the circumference in a semi circle is a right angle. [2074 S', 2057 R]



⇨ Here, let, ACB be a semi-circle with centre O. Let O be the origin. AB is diameter of semicircle. Join O and C

(i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (Triangle law of vector)

(ii) $\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$ (Triangle law of vector)

(iii) $\overrightarrow{BC}.\overrightarrow{CA} = (\overrightarrow{OC} - \overrightarrow{OB}).(\overrightarrow{OA} - \overrightarrow{OC})$

$= (\overrightarrow{OC} + \overrightarrow{OA}).(\overrightarrow{OA} - \overrightarrow{OC})$ [$\overrightarrow{OA} = -\overrightarrow{OB}$ being radii and in opposite direction.]

$= (\overrightarrow{OA})^2 - (\overrightarrow{OC})^2 = OA^2 - OC^2$

(iv) $\overrightarrow{BC}.\overrightarrow{CA} = OC^2 - OC^2 = 0$ [∵ OA = OC = radii of circle]

$\therefore \measuredangle BCA = 90°$ **Proved.**

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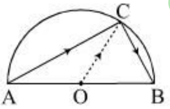
Or **व्यास AB भएको एउटा अर्धवृत्तको परिधिकोण ACB एक समकोण हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**
Prove by vector method that the circumference angle ACB of a semi-circle with a diameter AB is a right-angle.
[SEE 2075 R, 2075 R_2]

⇨ Here, Let ACB is a semi-circle having diameter AB, centre O and ∡ACB is the angle at circumference.
To Prove: ∡ACB = 90°
Proof:



(i) In ΔAOC, $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

(ii) In ΔBOC, $\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB}$

(iii) $\overrightarrow{AC}\,.\,\overrightarrow{CB} = (\overrightarrow{AO} + \overrightarrow{OC})\,.\,(\overrightarrow{CO} + \overrightarrow{OB})$

$= (\overrightarrow{AO} + \overrightarrow{OC})\,.\,(-\overrightarrow{OC} + \overrightarrow{AO})$ [∵ $\overrightarrow{CO} = -\overrightarrow{OC}$ and $\overrightarrow{OB} = \overrightarrow{AO}$ = radius]

(iv) $\overrightarrow{AC}\,.\,\overrightarrow{CB} = \overrightarrow{AO}^2 - \overrightarrow{OC}^2 = 0$ [∵ $\overrightarrow{AO} = \overrightarrow{OC}$ = radius]

(v) $\overrightarrow{AC} \perp \overrightarrow{CB}$ [∵ $\overrightarrow{AC}\,.\,\overrightarrow{CB} = 0$]
∴ ∡ACB = 90° **Proved.**

QUESTIONS FROM CDC TEXTBOOK

6.2.1 मध्यबिन्दु साध्य (MID-POINT THEOREM)

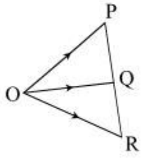
EXERCISE 6.2.1

1. (a) **भेक्टरको मध्य बिन्दु साध्यको कथन लेख्नुहोस् । (Write the mid point theorem of vector.)**

⇨ Here, the mid point theorem of vector states that, if $\vec{a}$ be the position vector of point A, $\vec{b}$ be the position vector of point B and M be the mid point of AB, then the position vector of M is; $\vec{m} = \frac{1}{2}\left(\vec{a} + \vec{b}\right)$.

(b) **बिन्दु C ले AB लाई $m_1 : m_2$ को अनुपातमा भित्री रूपमा विभाजन गर्छ भने C को स्थिति भेक्टर A र B को स्थिति भेक्टरका पदमा लेख्नुहोस् ।**
If point C divides AB internally in $m_1 : m_2$ ratio then write the position vector of C in terms of position vectors of A and B.

⇨ Here, the position vector of C ($\overrightarrow{OC}$) = $\dfrac{m_1\,\overrightarrow{OB} + m_2\,\overrightarrow{OA}}{m_1 + m_2}$



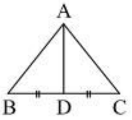
(c) **चित्रमा PR : PQ = m : n भए $\overrightarrow{OP}$ लाई $\overrightarrow{OQ}$ र $\overrightarrow{OR}$ को पदमा लेख्नुहोस् ।**
In figure, if PR : PQ = m : n then write $\overrightarrow{OP}$ in terms of $\overrightarrow{OQ}$ and $\overrightarrow{OR}$.

⇨ Here, $\overrightarrow{OP} = \dfrac{m\,\overrightarrow{OR} - n\,\overrightarrow{OQ}}{m - n}$

2. (a) **यदि बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $2\vec{i} + 5\vec{j}$ र $4\vec{i} - 3\vec{j}$ भए AB को मध्यबिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**
If the position vectors of point A and B are $2\vec{i} + 5\vec{j}$ and $4\vec{i} - 3\vec{j}$ respectively then find the position vector of C, the mid point of AB.

⇨ Here, $\overrightarrow{OC} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2} = \frac{1}{2}(2\vec{i} + 5\vec{j} + 4\vec{i} - 3\vec{j}) = \frac{1}{2}(6\vec{i} + 2\vec{j}) = 3\vec{i} + \vec{j}$

Thus, the position vector of C is $3\vec{i} + \vec{j}$.



(b) **चित्रमा BD = DC, $\overrightarrow{AB} = 3\vec{i} + 5\vec{j}$ र $\overrightarrow{AC} = 7\vec{i} + 9\vec{j}$ भए $\overrightarrow{AD}$ पत्ता लगाउनुहोस् ।**
In the figure, if BD = DC, $\overrightarrow{AB} = 3\vec{i} + 5\vec{j}$ and $\overrightarrow{AC} = 7\vec{i} + 9\vec{j}$ then find $\overrightarrow{AD}$.

⇨ Here, $\overrightarrow{AB} = 3\vec{i} + 5\vec{j}$ and $\overrightarrow{AC} = 7\vec{i} + 9\vec{j}$
D is the midpoint of BC.
So, $\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC}) = \frac{1}{2}(3\vec{i} + 5\vec{j} + 7\vec{i} + 9\vec{j}) = \frac{1}{2}(10\vec{i} + 14\vec{j})$

∴ $\overrightarrow{AD} = 5\vec{i} + 7\vec{j}$

Thus, the value of $\overrightarrow{AD}$ is $5\vec{i} + 7\vec{j}$.

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(c) **यदि, $\overrightarrow{OA} = 7\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = 2\vec{j} - \vec{i}$ र $\overrightarrow{AC} = \overrightarrow{CB}$ भए $\overrightarrow{OC}$ पत्ता लगाउनुहोस् ।**

If $\overrightarrow{OA} = 7\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = 2\vec{j} - \vec{i}$ and $\overrightarrow{AC} = \overrightarrow{CB}$ then find $\overrightarrow{OC}$.

⇨ Here, $\overrightarrow{OA} = 7\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = 2\vec{j} - \vec{i}$, $\overrightarrow{AC} = \overrightarrow{CB}$
We have,

$$\overrightarrow{AC} = \overrightarrow{CB}$$

or, $\overrightarrow{OC} - \overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{OC}$

or, $\overrightarrow{OC} - (7\vec{i} + 3\vec{j}) = (2\vec{j} - \vec{i}) - \overrightarrow{OC}$

or, $2\overrightarrow{OC} = 2\vec{j} - \vec{i} + 7\vec{i} + 3\vec{j}$

or, $2\overrightarrow{OC} = 5\vec{j} + 6\vec{i}$

or, $\overrightarrow{OC} = \frac{5}{2}\vec{j} + 3\vec{i}$

$\therefore$ $\overrightarrow{OC} = 3\vec{i} + \frac{5}{2}\vec{j}$

Thus, the position vector of C is $3\vec{i} + \frac{5}{2}\vec{j}$.

3. **(a)** **A(2, − 3) र B(3, 2) रेखाखण्ड AB को छेउका बिन्दुहरू हुन् । M ले AB लाई 2 : 3 को अनुपातमा भित्रबाट विभाजन गरेको छ । M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**
A(2, − 3) and B(3, 2) are the end points of line segment AB. M divides AB internally in the ratio of 2 : 3. Find the position vector of M.

⇨ Here, let O be the origin then,

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Ratio = m : n = 2 : 3
We know that, the section formula of internal division.

$$\overrightarrow{OM} = \frac{m\vec{b} + n\vec{a}}{m+n} = \frac{2 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -3 \end{pmatrix}}{2+3}$$

$$= \frac{1}{5}\left[\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ -9 \end{pmatrix}\right] = \frac{1}{5}\begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$$\therefore \quad \overrightarrow{OM} = \begin{pmatrix} \frac{12}{5} \\ -1 \end{pmatrix}$$

Thus, the position vector of M is $\frac{12}{5}\vec{i} - \vec{j}$.

(b) **यदि P र Q का स्थिति भेक्टरहरू क्रमश : $2\vec{i} - 3\vec{j}$ र $3\vec{i} - 2\vec{j}$ छन् भने PQ लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्ने बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If the position vectors of P and Q are $2\vec{i} - 3\vec{j}$ and $3\vec{i} - 2\vec{j}$ respectively, then find the position vector of M which divides PQ internally in the ratio of 3 : 2.

⇨ Here, let O be the origin.

Then $\overrightarrow{OP} = 2\vec{i} - 3\vec{j}$ and $\overrightarrow{OQ} = 3\vec{i} - 2\vec{j}$
Division ratio = m : n = 3 : 2
We have, internal section formula,

$$\overrightarrow{OM} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

$$= \frac{3(3\vec{i} - 2\vec{j}) + 2(2\vec{i} - 3\vec{j})}{3+2}$$

$$= \frac{9\vec{i} - 6\vec{j} + 4\vec{i} - 6\vec{j}}{5}$$

$$= \frac{13\vec{i} - 12\vec{j}}{5}$$

Thus, the position vector of M is $\frac{13}{5}\vec{i} - \frac{12}{5}\vec{j}$.

(c) **यदि P र Q का स्थिति भेक्टरहरू क्रमश : $2\vec{i} - \vec{j}$ र $5\vec{i} + 2\vec{j}$ छन् भने PQ लाई बाहिरबाट 1 : 2 को अनुपातमा विभाजन गर्ने बिन्दु R को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If the position vectors of P and Q are $2\vec{i} - \vec{j}$ and $5\vec{i} + 2\vec{j}$ respectively, find the position vector of a point R which divides PQ in the ratio of 1 : 2 externally.

⇨ Here, let O be the origin.

Then $\overrightarrow{OP} = 2\vec{i} - \vec{j}$ and $\overrightarrow{OQ} = 5\vec{i} + 2\vec{j}$
External division ratio = m : n = 1 : 2
We know that, the external section formula,

$$\overrightarrow{OR} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

$$= \frac{1(5\vec{i} + 2\vec{j}) - 2(2\vec{i} - \vec{j})}{1-2}$$

$$= \frac{5\vec{i} + 2\vec{j} - 4\vec{i} + 2\vec{j}}{-1}$$

$$= \frac{\vec{i} + 4\vec{j}}{-1} = -\vec{i} - 4\vec{j}$$

Thus, the position vector of R is $-\vec{i} - 4\vec{j}$.

(d) **यदि C र D का स्थिति भेक्टरहरू क्रमश : $6\vec{i} + 2\vec{j}$ र $7\vec{i} - 3\vec{j}$ छन् भने CD लाई बाहिरबाट 4 : 5 को अनुपातमा विभाजन गर्ने बिन्दु R को स्थिति भेक्टर पत्ता लगाउनुहोस् ।**

If the position vectors of C and D are $6\vec{i} + 2\vec{j}$ and $7\vec{i} - 3\vec{j}$ respectively, find the position vector of a point R which divides CD externally in the ratio of 4 : 5.

⇨ Here, let O be the origin. Then, $\overrightarrow{OC} = 6\vec{i} + 2\vec{j}$ and $\overrightarrow{OD} = 7\vec{i} - 3\vec{j}$
External division ratio = m : n = 4 : 5

We know that, the external section formula, $\overrightarrow{OR} = \frac{m\vec{b} - n\vec{a}}{m-n}$

$$= \frac{4(7\vec{i} - 3\vec{j}) - 5(6\vec{i} + 2\vec{j})}{4-5} = \frac{28\vec{i} - 12\vec{j} - 30\vec{i} - 10\vec{j}}{-1} = \frac{-2\vec{i} - 22\vec{j}}{-1} = 2\vec{i} + 22\vec{j}$$

Thus, the position vector of R is $2\vec{i} + 22\vec{j}$.

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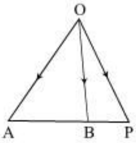
4. (a) चित्रमा बिन्दु P ले AB लाई 3 : 4 को अनुपातमा विभाजन गरेको छ । $\overrightarrow{OP}$ लाई $\overrightarrow{OA}$ र $\overrightarrow{OB}$ का पदमा व्यक्त गर्नुहोस् ।

In the figure, point P divides AB in the ratio of 3 : 4. Express $\overrightarrow{OP}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

⇨ Here, P divides AB in the ratio of 3 : 4.



So, $\frac{AP}{BP} = \frac{3}{4}$

or, $4\,AP = 3BP$

or, $4\,\overrightarrow{AP} = 3\,\overrightarrow{BP}$

By the triangle law of vector subtraction;

or, $4(\overrightarrow{OP} - \overrightarrow{OA}) = 3(\overrightarrow{OP} - \overrightarrow{OB})$

or, $4\,\overrightarrow{OP} - 4\,\overrightarrow{OA} = 3\,\overrightarrow{OP} - 3\,\overrightarrow{OB}$

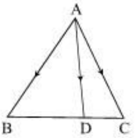
or, $4\,\overrightarrow{OP} - 3\,\overrightarrow{OP} = 4\,\overrightarrow{OA} - 3\,\overrightarrow{OB}$

or, $\overrightarrow{OP} = 4\,\overrightarrow{OA} - 3\,\overrightarrow{OB}$

Thus, $\overrightarrow{OP}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $(4\,\overrightarrow{OA} - 3\,\overrightarrow{OB})$.

(c) चित्रमा $\overrightarrow{CD} = \frac{1}{4}\overrightarrow{BC}$ भए $\overrightarrow{BA} = 5\,\overrightarrow{CA} + 4\,\overrightarrow{AD}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the figure, if $\overrightarrow{CD} = \frac{1}{4}\overrightarrow{BC}$ then prove that $\overrightarrow{BA} = 5\,\overrightarrow{CA} + 4\,\overrightarrow{AD}$.



⇨ Here, $\overrightarrow{CD} = \frac{1}{4}\,\overrightarrow{BC}$

or, $4\,\overrightarrow{CD} = \overrightarrow{BC}$

or, $4(\overrightarrow{CA} + \overrightarrow{AD}) = \overrightarrow{BA} + \overrightarrow{AC}$

or, $4\,\overrightarrow{CA} + 4\,\overrightarrow{AD} = \overrightarrow{BA} + \overrightarrow{AC}$

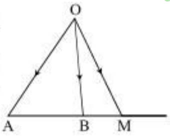
or, $4\,\overrightarrow{CA} - \overrightarrow{AC} + 4\,\overrightarrow{AD} = \overrightarrow{BA}$

or, $4\,\overrightarrow{CA} + \overrightarrow{CA} + 4\,\overrightarrow{AD} = \overrightarrow{BA}$

$\therefore\ \overrightarrow{BA} = 5\,\overrightarrow{CA} + 4\,\overrightarrow{AD}$ **Proved.**

(b) चित्रमा $\overrightarrow{OA} = 2\vec{a} + 3\vec{b}$, $\overrightarrow{OB} = -\vec{a} + 2\vec{b}$ र M ले AB लाई बाहिरबाट 5 : 2 को अनुपातमा विभाजन गरेको छ भने $\overrightarrow{OM} = \frac{1}{3}(4\vec{b} - 9\vec{a})$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the figure, $\overrightarrow{OA} = 2\vec{a} + 3\vec{b}$, $\overrightarrow{OB} = -\vec{a} + 2\vec{b}$ and M divides AB externally in the ratio of 5 : 2. Prove that $\overrightarrow{OM} = \frac{1}{3}(4\vec{b} - 9\vec{a})$.



⇨ Here, $\overrightarrow{OA} = 2\,\vec{a} + 3\,\vec{b}$ and $\overrightarrow{OB} = -\,\vec{a} + 2\,\vec{b}$

M divides AB in the ratio of 5 : 2.

So, $\frac{AM}{BM} = \frac{5}{2}$

or, $2AM = 5BM$

or, $2\,\overrightarrow{AM} = 5\,\overrightarrow{BM}$

or, $2(\overrightarrow{OM} - \overrightarrow{OA}) = 5(\overrightarrow{OM} - \overrightarrow{OB})$

or, $2\overrightarrow{OM} - 2\,\overrightarrow{OA} = 5\overrightarrow{OM} - 5\,\overrightarrow{OB}$

or, $5\,\overrightarrow{OB} - 2\,\overrightarrow{OA} = 3\overrightarrow{OM}$

or, $\overrightarrow{OM} = \frac{5}{3}\,\overrightarrow{OB} - \frac{2}{3}\,\overrightarrow{OA}$

or, $\overrightarrow{OM} = \frac{5}{3}(-\,\vec{a} + 2\,\vec{b}) - \frac{2}{3}(2\,\vec{a} + 3\,\vec{b})$

$= -\frac{5}{3}\,\vec{a} + \frac{10}{3}\,\vec{b} - \frac{4}{3}\,\vec{a} - 2\,\vec{b}$

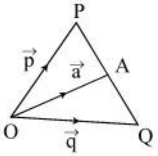
$= \left(-\frac{5}{3} - \frac{4}{3}\right)\vec{a} + \left(\frac{10}{3} - 2\right)\vec{b}$

$= -3\,\vec{a} + \frac{4}{3}\,\vec{b}$

$= \frac{4}{3}\,\vec{b} - 3\,\vec{a}$

$\therefore\ \overrightarrow{OM} = \frac{1}{3}(4\,\vec{b} - 9\,\vec{a})$ **Proved.**

5. चित्रमा यदि $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$ भए प्रमाणित गर्नुहोस् : $\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$ **(In figure, if $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$ then prove that $\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$)**



⇨ Here, $\overrightarrow{PA} = \frac{1}{4}\,\overrightarrow{PQ}$

or, $4\,\overrightarrow{PA} = \overrightarrow{PQ}$

or, $4(\overrightarrow{OA} - \overrightarrow{OP}) = (\overrightarrow{OQ} - \overrightarrow{OP})$

or, $4\,\overrightarrow{OA} - 4\,\overrightarrow{OP} = \overrightarrow{OQ} - \overrightarrow{OP}$

or, $4\,\vec{a} - 4\,\vec{p} = \vec{q} - \vec{p}$

or, $4\,\vec{a} = 4\,\vec{p} - \vec{p} + \vec{q}$

or, $4\,\vec{a} = 3\,\vec{p} + \vec{q}$

$\therefore\ \vec{a} = \frac{1}{4}(3\,\vec{p} + \vec{q})$ **Proved.**

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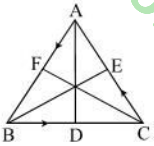
6. **चित्रमा ΔABC का भुजाहरू AB, BC र AC का मध्य बिन्दुहरू क्रमशः F, D र E छन् । $\overrightarrow{AD}$ + $\overrightarrow{BE}$ + $\overrightarrow{CF}$ = (0, 0) हुन्छ भनी प्रमाणित गर्नुहोस् ।**

In the figure, the mid points of sides AB, BC, and AC of ΔABC are F, D and E respectively. Prove that $\overrightarrow{AD}$ + $\overrightarrow{BE}$ + $\overrightarrow{CF}$ = (0, 0).



⇨ Here, using mid point formula;

(i) $\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

(ii) $\overrightarrow{BE} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$

(iii) $\overrightarrow{CF} = \frac{1}{2}(\overrightarrow{CA} + \overrightarrow{CB})$

Adding all of them then,

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{CB}) = \frac{1}{2} \times 0$$

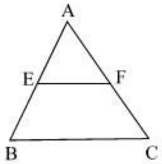
$\therefore\ \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$ **Proved.**

6.2.2 त्रिभुज सम्बन्धी साध्यहरू (THEOREMS RELATED TO TRIANGLE)

EXERCISE 6.2.2

1. (a) **चित्रमा ΔABC का भुजाहरू AB र AC का मध्य बिन्दुहरू क्रमशः E र F छन् । EF र BC को सम्बन्ध लेख्नुहोस् ।**

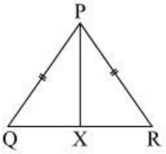
In the figure, the mid point of sides AB and AC of ΔABC are E and F respectively. Write the relation of EF and BC.



⇨ Here, the required relation is EF = $\frac{1}{2}$ BC.

(b) **चित्रमा ΔPQR को मध्यिका PX हो । PQ = PR छ । PX र QR को सम्बन्ध लेख्नुहोस् ।**

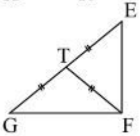
In the figure, PX is the median of ΔPQR and PQ = PR. Write the relation of PX and QR.



⇨ Here, the required relation is PX ⊥ QR.

(c) **चित्रमा ΔEFG दिइएको छ, जहाँ GT = ET = FT छ । EF र GF को सम्बन्ध लेख्नुहोस् ।**

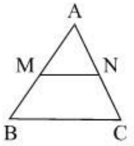
In the figure, ΔEFG is given where GT = ET = FT. Write the relation of EF and GF.



⇨ Here, the required relation is EF ⊥ GF.

2. (a) **चित्रमा ΔABC का भुजाहरू AB र AC का मध्य बिन्दुहरू क्रमशः M र N छन् । MN = $\frac{1}{2}$ BC र MN // BC हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

In the figure, the mid point of sides AB and AC of ΔABC are M and N respectively. Prove by vector method that: MN = $\frac{1}{2}$ BC and MN // BC.



⇨ Here,

Let ABC be a triangle with M and N be the mid-points of AB and AC respectively.

(i) $\overrightarrow{BA} = 2\overrightarrow{MA}$ [∵ M is the mid point of AB]

(ii) $\overrightarrow{AC} = 2\overrightarrow{AN}$ [∵ N is the mid point of AC]

(iii) $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ [Triangle law of vector addition]

or, $2\overrightarrow{MA} + 2\overrightarrow{AN} = \overrightarrow{BC}$ or, $2(\overrightarrow{MA} + \overrightarrow{AN}) = \overrightarrow{BC}$

(iv) $\overrightarrow{MA} + \overrightarrow{AN} = \overrightarrow{MN}$ [Triangle law of vector addition]

(v) $\therefore\ 2\overrightarrow{MN} = \overrightarrow{BC}$ [From (iii) and (iv)] [∵ If $\vec{a} = m\vec{b}$ then $\vec{a} // \vec{b}$]

Thus, $\overrightarrow{MN} // \overrightarrow{BC}$ and $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}$.

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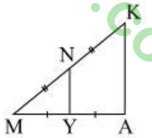
(b) चित्रमा ΔKAM का भुजाहरू KM र AM का मध्य बिन्दुहरू जोड्ने रेखाखण्ड NY छ। भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् :
In the figure, NY is the line segment joining the mid points of sides KM and MA of ΔKAM. Prove by vector method.

(i) $\overrightarrow{YN} // \overrightarrow{AK}$ **(ii)** $\overrightarrow{YN} = \frac{1}{2}\overrightarrow{AK}$



⇨ Here,
Let KAM be a triangle with Y and N be the mid-points of MA and MK respectively.

(i) $\overrightarrow{AM} = 2\overrightarrow{YM}$ [∵ Y is the mid point of MA]

(ii) $\overrightarrow{MK} = 2\,\overrightarrow{MN}$ [∵ N is the mid point of MK]

(iii) $\overrightarrow{AM} + \overrightarrow{MK} = \overrightarrow{AK}$ [Triangle law of vector addition]

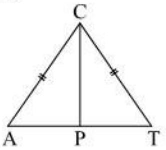
or, $2\overrightarrow{YM} + 2\,\overrightarrow{MN} = \overrightarrow{AK}$ or, $2\,(\overrightarrow{YM} + \overrightarrow{MN}) = \overrightarrow{AK}$

(iv) $\overrightarrow{YM} + \overrightarrow{MN} = \overrightarrow{YN}$ [Triangle law of vector addition]

(v) ∴ $2\overrightarrow{YN} = \overrightarrow{AK}$ [From (iii) and (iv)] [∴ If $\vec{a} = m\vec{b}$ then $\vec{a} // \vec{b}$]

Thus, $\overrightarrow{YN} // \overrightarrow{AK}$ and $\overrightarrow{YN} = \frac{1}{2}\overrightarrow{AK}$.` **Proved.**

3. **(a) चित्रमा त्रिभुज CAT मा CA = CT छ। भुजा AT मा मध्यिका CP छ। भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : CP ⊥ AT**
In the figure, in ΔCAT, CA = CT. Median CP lies on side AT. Prove by vector method: CP ⊥ AT.



⇨ Here,
Let CAT be an isosceles triangle with CA = CT and CP is median of ΔCAT.

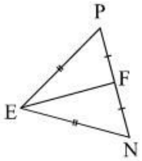
(i) $\overrightarrow{CP} = \frac{1}{2}(\overrightarrow{CA} + \overrightarrow{CT})$ [Mid point theorem]

(ii) $\overrightarrow{AT} = (\overrightarrow{CT} - \overrightarrow{CA})$ [Triangle law of vector subtraction]

(iii) $\overrightarrow{CP}.\overrightarrow{AT} = \frac{1}{2}(\overrightarrow{CA} + \overrightarrow{CT})(\overrightarrow{CT} - \overrightarrow{CA}) = \frac{1}{2}[(\overrightarrow{CT})^2 - (\overrightarrow{CA})^2] = \frac{1}{2}(CT^2 - CA^2) = \frac{1}{2} \times 0$ [Since CT = CA]

(iv) $\overrightarrow{CP} \perp \overrightarrow{AT}$ [∵ $\overrightarrow{CP}.\overrightarrow{AT} = 0$] **Proved.**

(b) चित्रमा EP = EN र PF = FN छ। भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : EF⊥ PN
In the figure, EP = EN and PF = FN. Prove by vector method: EF⊥ PN



⇨ Here,
Let ENP be an isosceles triangle with EN = EP and EF is median of ΔENP.

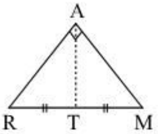
(i) $\overrightarrow{EF} = \frac{1}{2}(\overrightarrow{EN} + \overrightarrow{EP})$ [Mid point theorem]

(ii) $\overrightarrow{PN} = (\overrightarrow{EN} - \overrightarrow{EP})$ [Triangle law of vector subtraction]

(iii) $\overrightarrow{EF}.\overrightarrow{PN} = \frac{1}{2}(\overrightarrow{EN} + \overrightarrow{EP})(\overrightarrow{EN} - \overrightarrow{EP}) = \frac{1}{2}[(\overrightarrow{EN})^2 - (\overrightarrow{EP})^2] = \frac{1}{2}(EN^2 - EP^2) = \frac{1}{2} \times 0$ [Since EP = EN]

(iv) $\overrightarrow{EF} \perp \overrightarrow{PN}$ [∵ $\overrightarrow{EF}.\overrightarrow{PN} = 0$] **Proved.**

4. **(a) चित्रमा समकोणी त्रिभुज RAM को कर्ण RM को मध्य बिन्दु T छ। भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : RT = AT = MT**
In the given right angled ΔRAM, the mid point of hypotenuse RM is T. prove by vector method: RT = AT = MT.



⇨ Here, let, RAM be a right angled triangle whose right angle is at A.
Let, T be the middle point of the hypotenuse RM. Let, A be the origin then,

(i) $\overrightarrow{AM} = \overrightarrow{AT} + \overrightarrow{TM}$ (Triangle law of vector addition)

(ii) $\overrightarrow{AR} = \overrightarrow{AT} + \overrightarrow{TR}$ (Triangle law of vector addition)

(iii) $\overrightarrow{AM} \perp \overrightarrow{AR}$ [∵ ∡A = 90°] or, $\overrightarrow{AM}.\overrightarrow{AR} = 0$ [∵ AM ⊥ AR]

or, $(\overrightarrow{AT} + \overrightarrow{TM}).(\overrightarrow{AT} + \overrightarrow{TR}) = 0$ [∵ From (i) and (ii)]

or, $(\overrightarrow{AT} + \overrightarrow{RT}).(\overrightarrow{AT} - \overrightarrow{RT}) = 0$ [∵ $\overrightarrow{RT} = \overrightarrow{TM}$ and $\overrightarrow{TR} = -\overrightarrow{RT}$]

or, $(AT)^2 - (RT)^2 = 0$ or, $AT^2 = RT^2$ or, AT = RT

∴ AT = RT = TM [∴ RT = TM] **Proved.**

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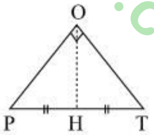
(b) **चित्रमा POT एउटा समकोणी त्रिभुज र PT को मध्य बिन्दु H छ । भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : OH = PH = TH**
In the figure, ΔPOT is a right angled triangle and H is the mid point of PT. Prove by vector method: OH = PH = TH.



⇨ Here, let, OPT be a right angled triangle whose right angle is at O.
Let, H be the middle point of the hypotenuse PT and O be the origin then,

(i) $\overrightarrow{OP} = \overrightarrow{OH} + \overrightarrow{HP}$ (Triangle law of vector addition)

(ii) $\overrightarrow{OT} = \overrightarrow{OH} + \overrightarrow{HT}$ (Triangle law of vector addition)

(iii) $\overrightarrow{OP} \perp \overrightarrow{OT}$ [$\because \measuredangle O = 90°$] or, $\overrightarrow{OP} . \overrightarrow{OT} = 0$ [$\because$ OP ⊥ OT]

or, $(\overrightarrow{OH} + \overrightarrow{HP}) . (\overrightarrow{OH} + \overrightarrow{HT}) = 0$ [$\because$ From (i) and (ii)]

or, $(\overrightarrow{OH} + \overrightarrow{TH}). (\overrightarrow{OH} - \overrightarrow{TH}) = 0$ [$\because \overrightarrow{TH} = \overrightarrow{HP}$ and $\overrightarrow{HT} = -\overrightarrow{TH}$]

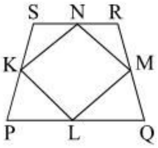
or, $(OH)^2 - (TH)^2 = 0$ or, $OH^2 = TH^2$ or, OH = TH

∴ OH = TH = HP [∵ TH = HP] **Proved.**

6.2.3 चतुर्भुज तथा अर्धवृत सम्बन्धी साध्यहरू (THEOREMS ON QUADRILATERAL AND SEMI-CIRCLE)

EXERCISE 6.2.3

1. (a) **चित्रमा, बिन्दुहरू K, L, M र N क्रमशः चतुर्भुज PQRS का भुजाहरू PS, PQ, QR र RS का मध्य बिन्दुहरू हुन् । KLMN कस्तो प्रकारको चतुर्भुज हो, लेख्नुहोस् ।**
In the figure, points K, L, M and N are the middle points of sides PS, PQ, QR and RS respectively of quadrilateral PQRS. What type of quadrilateral is KLMN? Write.



⇨ Here, KLMN is a parallelogram.

(b) **अर्धवृत्तमा बनेको परिधि कोण कति हुन्छ ? (What is the angle at the circumference of a semi-circle?)**

⇨ Here, angle at the circumference of semi-circle is 90°.

(c) **समबाहु चतुर्भुज ABCD मा विकर्ण AC र BD छन् । $\overrightarrow{AC} . \overrightarrow{BD}$ कति हुन्छ ? लेख्नुहोस् ।**

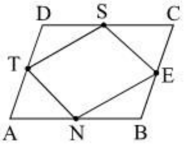
A rhombus ABCD has diagonals AC and BD. Find $\overrightarrow{AC} . \overrightarrow{BD}$.

⇨ Here, angle between the diagonals of a rhombus is 90°.

So, $\overrightarrow{AC} . \overrightarrow{BD} = 0$

2. (a) **चतुर्भुज ABCD का भुजाहरू AB, BC, CD र AD का मध्य बिन्दुहरू क्रमशः N, E, S र T छन् । NEST एउटा समानान्तर चतुर्भुज हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**
N, E, S and T are the mid points of sides AB, BC, CD and DA respectively of a quadrilateral ABCD. Prove by the vector method that NEST is a parallelogram.

⇨ Here,
In the figure alongside, let ABCD be a quadrilateral and NEST be the resulting quadrilateral formed by joining the middle points of sides AB, BC, CD and DA respectively. Let O be the origin of vectors then by mid point theorem,



(i) $\overrightarrow{ON} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$; $\overrightarrow{OE} = \dfrac{\overrightarrow{OB} + \overrightarrow{OC}}{2}$; $\overrightarrow{OS} = \dfrac{\overrightarrow{OC} + \overrightarrow{OD}}{2}$; $\overrightarrow{OT} = \dfrac{\overrightarrow{OD} + \overrightarrow{OA}}{2}$

(ii) $\overrightarrow{NE} = \overrightarrow{OE} - \overrightarrow{ON} = \dfrac{\overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA} - \overrightarrow{OB}}{2} = \dfrac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

(iii) $\overrightarrow{TS} = \overrightarrow{OS} - \overrightarrow{OT} = \dfrac{\overrightarrow{OC} + \overrightarrow{OD} - \overrightarrow{OD} - \overrightarrow{OA}}{2} = \dfrac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

Therefore $\overrightarrow{NE} = \overrightarrow{TS}$
i.e. NE = TS and NE // TS similarly, TN = SE and TN // SE
Thus, NEST is a parallelogram. **Proved.**

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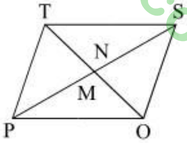
(b) **समानान्तर चतुर्भुज POST का विकर्णहरू समद्विभाजित हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

Prove by vector method that the diagonals of a parallelogram POST bisect to each other.

⇨ Here,

Let POST be a parallelogram and OT and PS are the diagonals.

Let O be the origin , M and N are mid points of OT and PS in parallelogram POST.



(i) $\overrightarrow{PM} = \dfrac{\overrightarrow{PO} + \overrightarrow{PT}}{2}$ [Mid point theorem]

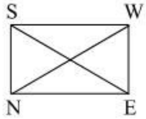
(ii) $\overrightarrow{PN} = \dfrac{\overrightarrow{PS}}{2} = \dfrac{\overrightarrow{PO} + \overrightarrow{PT}}{2}$ [Parallelogram Law of vector addition]

$\therefore$ $\overrightarrow{PM} = \overrightarrow{PN}$ so M and N are coincide.

Thus, diagonals of parallelogram POST bisects to each other. **Proved.**

3. **(a)** **आयत NEWS का विकर्णहरू बराबर हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

Prove by the vector method that the diagonals of rectangle NEWS are equal.



⇨ Here,

Let, NEWS be a rectangle and NW and ES are its diagonals.

(i) $NW^2 = (\overrightarrow{NW})^2 = (\overrightarrow{NE} + \overrightarrow{EW})^2 = (\overrightarrow{SW} + \overrightarrow{EW})^2 = \overrightarrow{SW}^2 + 2\overrightarrow{SW}.\overrightarrow{EW} + \overrightarrow{EW}^2 = \overrightarrow{WS}^2 + 2 \times 0 + \overrightarrow{EW}^2$ [$\because \overrightarrow{SW} \perp \overrightarrow{EW}$]

$\therefore$ $NW^2 = \overrightarrow{WS}^2 + \overrightarrow{EW}^2$

(ii) $ES^2 = \overrightarrow{ES}^2 = (\overrightarrow{EW} + \overrightarrow{WS})^2 = \overrightarrow{EW}^2 + 2\overrightarrow{EW}.\overrightarrow{WS} + \overrightarrow{WS}^2 =$ $\overrightarrow{EW}^2 + \overrightarrow{WS}^2$ [$\because \overrightarrow{EW} \perp \overrightarrow{WS}$]

$\therefore$ $\overrightarrow{ES}^2 = \overrightarrow{WS}^2 + \overrightarrow{EW}^2$

(iii) $NW^2 = ES^2$ [From (i) and (ii)] $\therefore$ $NW = ES$ **Proved.**

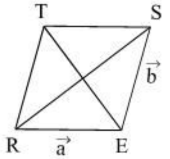
Thus, the diagonals of a rectangle NEWS are equal to each other.

(b) **समबाहु चतुर्भुज REST का विकर्णहरू समकोण हुने गरी समद्विभाजित हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

Prove vectorically that the diagonals of rhombus REST are intersected at right angle.

⇨ Here,

Let REST be a rhombus and RS and ET be its diagonals.



Let R be the origin, $\overrightarrow{ES} = \overrightarrow{RT} = \vec{b}$ and $\overrightarrow{RE} = \overrightarrow{TS} = \vec{a}$

From the triangle law of vector and parallelogram law of vector,

(i) $\overrightarrow{TE} = \vec{a} - \vec{b}$ and $\overrightarrow{RS} = \vec{a} + \vec{b}$

(ii) $\overrightarrow{TE} \,.\, \overrightarrow{RS} = (\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = (\vec{a})^2 - (\vec{b})^2 = a^2 - b^2$ [$\because$the square of any vector is a scalar]

$= a^2 - a^2$ [$b^2 = a^2$ since the sides of a rhombus]

$\therefore$ $\overrightarrow{TE} \,.\, \overrightarrow{RS} = 0$

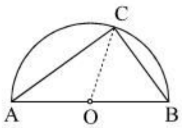
Since, the dot product of the diagonals of the rhombus REST is 0. So, the diagonals are perpendicular to each other. **Proved.**

4. **(a)** **अर्धवृत्तमा बनेको कोण एक समकोण हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

Prove vectorically that the angle at the circumference of a semi circle is a right angle.

⇨ Here,

Let ACB be a semi-circle with centre O. Let O be the origin. AB is diameter of semicircle. Join O and C.



(i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (Triangle law of vector)

(ii) $\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$ (Triangle law of vector)

(iii) $\overrightarrow{BC}.\,\overrightarrow{CA} = (\overrightarrow{OC} - \overrightarrow{OB})\,.\,(\overrightarrow{OA} - \overrightarrow{OC})$

$= (\overrightarrow{OC} + \overrightarrow{OA})\,.\,(\overrightarrow{OA} - \overrightarrow{OC})$ [$\overrightarrow{OA} = -\,\overrightarrow{OB}$ [Being radii and in opposite direction.]

$= (\overrightarrow{OA})^2 - (\overrightarrow{OC})^2 = OA^2 - OC^2$

(iv) $\overrightarrow{BC}.\,\overrightarrow{CA} = OC^2 - OC^2 = 0$ [$\because$ OA = OC = radii of circle] $\therefore$ $\measuredangle BCA = 90°$

Thus, the angle at the circumference of a semicircle is a right angle. **Proved.**

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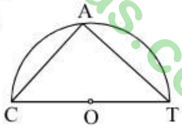
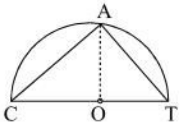
(b) **चित्रमा O अर्धवृत्तको केन्द्र हो । ∡CAT एक समकोण हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।**

In the figure, O is the centre of semi-circle. Prove vectorically that ∡CAT is a right angle.



⇨ Here,

Let CAT be a semi-circle with centre O. Let O be the origin. CT is diameter of semicircle. Join O and A.



(i) $\overrightarrow{TA} = \overrightarrow{OA} - \overrightarrow{OT}$ (Triangle law of vector)

(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ (Triangle law of vector)

(iii) $\overrightarrow{TA} . \overrightarrow{AC} = (\overrightarrow{OA} - \overrightarrow{OT}) . (\overrightarrow{OC} - \overrightarrow{OA})$

$= (\overrightarrow{OA} + \overrightarrow{OC}) . (\overrightarrow{OC} - \overrightarrow{OA})$ [$\overrightarrow{OC} = - \overrightarrow{OT}$ [Being radii and in opposite direction.]

$= (\overrightarrow{OC})^2 - (\overrightarrow{OA})^2 = OC^2 - OA^2$

(iv) $\overrightarrow{TA} . \overrightarrow{AC} = OA^2 - OA^2 = 0$ [∵ OC = OA = radii of circle]

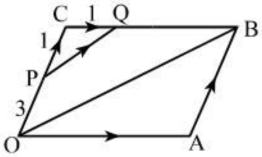
∴ ∡TAC = 90° **Proved.**

5. **भेक्टर ज्यामिति र ज्यामितिमा प्रमाणित गर्ने साध्यहरूमा के फरक छ ? उदाहरणसहित छोटो प्रतिवेदन तयार गर्नुहोस् । उक्त प्रतिवेदनलाई कक्षाकोठामा प्रस्तुत गर्नुहोस् ।**

OTHER IMPORTANT QUESTION

1. **OABC एउटा समानान्तर चतुर्भुज हो । बिन्दु P र Q ले OC र BC लाई CP : PO = CQ : QB = 1 : 3 हुने गरी विभाजन गर्दछन् । यदि $\overrightarrow{OA} = \vec{a}$ र $\overrightarrow{OC} = \vec{c}$ छन् भने $\overrightarrow{PQ}$ जनाउने भेक्टर पत्ता लगाउनुहोस् र प्रमाणित गर्नुहोस् $\overrightarrow{PQ}$ र $\overrightarrow{OB}$ समानान्तर छन् ।**

OABC is parallelogram, the point P and Q divide OC and BC such that CP : PO = CQ : QB = 1 : 3. If $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$. Find the vector which represents $\overrightarrow{PQ}$ and prove that $\overrightarrow{PQ}$ and $\overrightarrow{OB}$ are parallel.



⇨ Here, CP : PO = CQ : QB = 1 : 3,

$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$

$\overrightarrow{PC} = \frac{1}{4}\overrightarrow{OC} = \frac{1}{4}\vec{c}$

$\overrightarrow{CQ} = \frac{1}{4}\overrightarrow{CB} = \frac{1}{4}\overrightarrow{OA} = \frac{1}{4}\vec{a}$

In Δ PCQ,

$\overrightarrow{PQ} = \overrightarrow{PC} + \overrightarrow{CQ}$ (By triangle law)

$= \frac{1}{4}\vec{c} + \frac{1}{4}\vec{a}$

$= \frac{1}{4}(\vec{c} + \vec{a})$

In Δ OCB, $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$= \overrightarrow{OC} + \overrightarrow{OA}$

$= \vec{c} + \vec{a}$

∴ $\overrightarrow{PQ} = \frac{1}{4}\overrightarrow{OB}$

Thus, $\overrightarrow{PQ}$ and $\overrightarrow{OB}$ are parallel.

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स्थानान्तरण (Transformation)

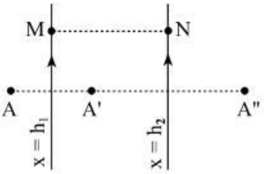
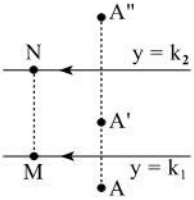
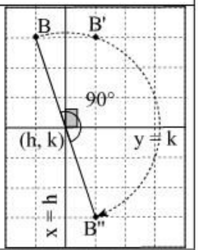
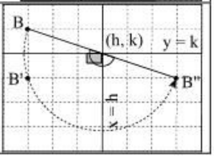
1. संयुक्त स्थानान्तरण
Combined Transformation

FORMULAE

- परावर्तन (Reflection)

	परावर्तनको अक्ष **Axis of Reflection**	वस्तु **Object**	आकृति **Image**
(i)	X-axis or $y = 0$	$P(x, y)$	$\longrightarrow P'(x, -y)$
(ii)	Y-axis or $x = 0$	$P(x, y)$	$\longrightarrow P'(-x, y)$
(iii)	$y = x$ or $x = y$	$P(x, y)$	$\longrightarrow P'(y, x)$
(iv)	$y = -x$ or $x = -y$	$P(x, y)$	$\longrightarrow P'(-y, -x)$
(v)	$x = h$	$P(x, y)$	$\longrightarrow P'(2h - x, y)$
(vi)	$y = k$	$P(x, y)$	$\longrightarrow P'(x, 2k - y)$

- संयुक्त परावर्तन (Combined Reflection)

S.N.	परावर्तनका अक्षहरू Axes of Reflections	एकल स्थानान्तरण Single transformation	चित्र Figure
	समानान्तर रेखाहरू Parallel Lines	विस्थापन Translation	
1.	पहिले $x = h_1$ र पछि $x = h_2$ मा परावर्तन गरिएमा If reflection $x = h_1$ is followed by reflection in $x = h_2$	विस्थापन भेक्टर $\begin{pmatrix} 2(h_2 - h_1) \\ 0 \end{pmatrix}$ वा $\begin{pmatrix} 2\overrightarrow{MN} \\ 0 \end{pmatrix}$ द्वारा हुने विस्थापन The translation by the translation vector $\begin{pmatrix} 2(h_2 - h_1) \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2\overrightarrow{MN} \\ 0 \end{pmatrix}$	
2.	पहिले $y = k_1$ र पछि $y = k_2$ मा परावर्तन गरिएमा (समानान्तर अक्षमा) If reflection $y = k_1$ is followed by reflection in $y = k_2$ (parallel axis)	विस्थापन भेक्टर $\begin{pmatrix} 0 \\ 2(k_2 - k_1) \end{pmatrix}$ वा $\begin{pmatrix} 0 \\ 2\overrightarrow{MN} \end{pmatrix}$ द्वारा हुने विस्थापन The translation by the translation vector $\begin{pmatrix} 0 \\ 2(k_2 - k_1) \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 2\overrightarrow{MN} \end{pmatrix}$	
	प्रतिच्छेदित रेखाहरू Intersecting Lines	परीक्रमण Rotation	
3.	पहिले $x = h$ र पछि $y = k$ मा परावर्तन गरिएमा If reflection $x = h$ is followed by reflection in $y = k$	बिन्दु (h, k) को वरिपरि हुने ऋणात्मक अर्ध परिक्रमण Negative half turn rotation about the the point (h, k).	
4.	पहिले $y = k$ र पछि $x = h$ मा परावर्तन गरिएमा If reflection $y = k$ is followed by reflection in $x = h$	बिन्दु (h, k) को वरिपरि हुने धनात्मक अर्ध परिक्रमण Positive half turn rotation about the point (h, k).	

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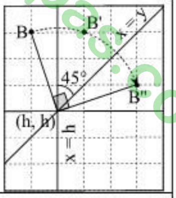
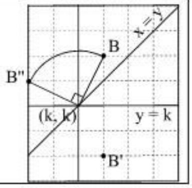
5.	पहिले $x = h$ र पछि $x = y$ मा परावर्तन गरिएमा If reflection $x = h$ is followed by reflection in $x = y$	बिन्दु (h, h) को वरिपरि हुने ऋणात्मक चौथाइ परिक्रमण Negative quarter turn rotation about the point (h, h).	
6.	पहिले $y = k$ र पछि $x = y$ मा परावर्तन गरिएमा If reflection $y = k$ is followed by reflection in $x = y$	बिन्दु (k, k) को वरिपरि हुने ऋणात्मक चौथाइ परिक्रमण Negative quarter turn rotation about the point (k, k).	

➢ **परिक्रमण (Rotation)**

	परिक्रमणको कोण Angle of rotation	परिक्रमणको केन्द्र Centre of rotation	वस्तु Object	प्रतिबिम्ब Image
1.	+90° or – 270°	(0, 0)	P(x,y)	⟶ P'(–y,x)
2.	–90° or + 270°	(0, 0)	P(x,y)	⟶ P'(y, –x)
3.	± 180°	(0, 0)	P(x, y)	⟶ P'(–x, –y)

4. यदि R_1 र R_2 हरू दुई एउटै केन्द्रमा आधारीत परिक्रमणहरू भए संयुक्त परिक्रमण :
If R_1 and R_2 are two rotations about the same centre then combined rotation:
$(R_1 \circ R_2) = (R_2 \circ R_1) = R_1 + R_2$

अर्थात् यदि $R_1[(0, 0), \theta_1]$ र $R_2 = [(0, 0), \theta_2]$ दुईओटा परीक्रमणहरू भए संयुक्त परीक्रमण:
$R_2 \circ R_1 = R_1 \circ R_2 = R[(0, 0), (\theta_1 + \theta_2)]$ हुन्छ ।
If $R_1[(0, 0), \theta_1]$ and $R_2 = [(0, 0), \theta_2]$ are two rotations then the combined rotation:
$R_2 \circ R_1 = R_1 \circ R_2 = R[(0, 0), (\theta_1 + \theta_2)]$

➢ **विस्थापन (Translation)**

1. यदि $T = \begin{pmatrix} a \\ b \end{pmatrix}$ विस्थापन भेक्टर र P(x, y) वस्तु भए आकृति P'(x + a, y + b) हुन्छ ।

If $T = \begin{pmatrix} a \\ b \end{pmatrix}$ is translation vector and P(x, y) be an object then image is P'(x + a, y + b).

2. यदि T_1 र T_2 हरू दुई विस्थापनहरू भए संयुक्त विस्थापन: $(T_1 \circ T_2) = (T_2 \circ T_1) = T_1 + T_2$
If T_1 & T_2 are 2 translations then combined translation: $(T_1 \text{ o } T_2) = (T_2 \circ T_1) = T_1 + T_2$

➢ **विस्तारीकरण (Enlargement)**

	विस्तारीकरणको केन्द्र Centre of enlargement	नापो Scale factor	वस्तु Object	आकृति Image
1.	(0, 0)	k	(x, y)	⟶ (kx, ky)
2.	(a, b)	k	(x, y)	⟶ (k(x – a) + a, k(y – b) + b)

3. यदि $E_1[(a, b), k_1]$ र $E_2 = [(a, b), k_2]$ दुईओटा विस्तारीकरणहरू भए संयुक्त विस्तारीकरण :
$E_2 \circ E_1 = E_1 \circ E_2 = E[(a, b), k_1 \times k_2]$ हुन्छ ।
If $E_1[(a, b), k_1]$ and $E_2 = [(a, b), k_2]$ are two enlargements then the combined enlargement:
$E_2 \circ E_1 = E_1 \circ E_2 = E[(a, b), k_1 \times k_2]$

4. यदि F ले एउटा स्थानान्तरण र G ले अर्को स्थानान्तरण जनाउँछन् भने,

(i) संयुक्त स्थानान्तरण (G∘F) The combined transformation (G∘F)
= पहिले F द्वारा हुने स्थानान्तरण र पछि सोही प्रतिबिम्बलाई G द्वारा हुने स्थानान्तरण जनाउँछ ।
= First the transformation by F and then the transformation of the image by G

अर्थात् (Or)

G∘F = पहिले F र पछि G

G∘F = F is followed by G

(ii) संयुक्त स्थानान्तरण (F∘G) The combined transformation (F∘G)
= पहिले G द्वारा हुने स्थानान्तरण र पछि सोही प्रतिबिम्बलाई F द्वारा हुने स्थानान्तरण जनाउँछ ।
= First the transformation by G and then the transformation of the first image by F

अर्थात् (Or)

F∘G = पहिले G र पछि F

F∘G = G is followed by F

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QUESTIONS FROM SEE EXERCISE 1

A. VERY SHORT QUESTIONS

1. संयुक्त स्थानान्तरण भन्नाले के बुझिन्छ ? (What do you meant by combined transformation?)

⇨ Here, when an object has been transformed, its image can again be transformed to form a new image. Such transformation is called combined transformations.

2. x-अक्षको परावर्तन पछि y-अक्षमा परावर्तन गर्दा हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् ।
State the single transformation when the reflection in X-axis is followed by the reflection in Y-axis.

⇨ Here, $P(x, y) \xrightarrow{\text{Reflection in x-axis}} P'(x, -y) \xrightarrow{\text{Reflection in y-axis}} P''(-x, -y)$
Thus, the single transformation of reflection in x-axis followed by reflection in y-axis is the half turn rotation about origin.

3. स्थानान्तरण r_1 ले एउटा बिन्दु A लाई A' मा र स्थानान्तरण r_2 ले A' लाई A" मा स्थानान्तरण गर्दछ भने $r_2 \circ r_1$ ले के गर्छ ?
Transformation r_1 transforms a point A to A' and r_2 transforms point A' to A". What does $r_2 \circ r_1$ do?

⇨ Here, $r_2 \circ r_1$ transforms first A to A' (i.e. r_1) and then A' to A" (i.e. r_2).

4. यदि F ले रेखा $y = x$ मा हुने परावर्तन र G ले रेखा $x = 0$ मा हुने परावर्तनलाई जनाउँछन् भने G∘F ले कुन स्थानान्तरणलाई प्रतिनिधित्व गर्छ ?
If F be the reflection on line $y = x$ and G be the reflection on line $x = 0$ then state what does G∘F represent ?

⇨ Here, $P(x, y) \xrightarrow{F} P'(y, x) \xrightarrow{G} P''(-y, x)$
Thus, G∘F represent + 90° rotation about origin.

5. बिन्दु (x, y) लाई पहिले रेखा $x = 0$ मा परावर्तन गरेपछि प्राप्त प्रतिबिम्बलाई $y = k$ मा परावर्तन गर्दा अन्तिम प्रतिबिम्ब पत्ता लगाउनुहोस् ।
Point (x, y) is reflected in the line $x = 0$ at first and then the image so formed is reflected in the line $y = k$. Find the final image.

⇨ Here, $(x, y) \xrightarrow{\text{Reflection in } x = 0} (-x, y) \xrightarrow{\text{Reflection in } y = k} (-x, 2k - y)$
Thus, the final image of point (x, y) is (– x, 2k – y).

6. F : $(x, y) \rightarrow (2x, y)$ र T : $(x, y) \rightarrow (x, y - 1)$ दुईओटा स्थानान्तरण हुन् । उक्त स्थानान्तरणको संयुक्त स्थानान्तरण F∘T पत्ता लगाउनुहोस् ।
F : $(x, y) \rightarrow (2x, y)$ and T : $(x, y) \rightarrow (x, y - 1)$ are any two transformation. Find the combined transformation F∘T.

⇨ Here, F∘T = F(T) = F(x, y – 1) = 2x, y – 1
Thus, the combined transformation F∘T is (2x, y – 1).

7. एउटा परिक्रमण $R_1[(0, 0); x°]$ पछि $R_2[(0, 0); y°]$ गरिन्छ । संयुक्त परिक्रमण के हुन्छ ?
A rotation $R_1[(0, 0); x°]$ is followed by $R_2[(0, 0); y°]$. What is the combined rotation?

⇨ Here, the combined rotation is; R[(0, 0); x° + y°].

8. दुईओटा लगातार विस्तारीकरणहरू $E_1[O, K_1]$ र $E_2[O, K_2]$ द्वारा परिभाषित गरिएको भए संयुक्त विस्तारीकरण के होला ?
What will be the combined enlargement of two successive enlargements defined by $E_1[O, K_1]$ and $E_2[O, K_2]$?

⇨ Here, the combined enlargement of $E_1[O, K_1]$ and $E_2[O, K_2]$ is ; $E[O, K_1K_2]$.

9. बिन्दु P(a, b) लाई रेखा $y = x$ मा परावर्तन गरेपछि उद्गम बिन्दुको वरिपरि – 90° मा घुमाउँदा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the image of point P(a, b) after reflection on the line $y = x$ followed by the rotation through – 90° about the origin.

⇨ Here, $P(a, b) \xrightarrow{\text{Reflection in } y = x} P'(b, a) \xrightarrow{\text{Rotation } [(0,\ 0);\ -90°]} P''(a, -b)$
Thus, the final image of point P is P"(a, – b)

10. बिन्दु (a, b) लाई उद्गमबिन्दु O बाट + 90° घुमाउँदा बन्ने प्रतिविम्ब Y-अक्षमा परावर्तन गराउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Point (a, b) is rotated about the origin O through + 90° and the image so obtained is relfected on the Y-axis. Find the co-ordinates of the image.

⇨ Here, $(a, b) \xrightarrow{\text{Rotation } [90°;\ (0,\ 0)]} (-b, a) \xrightarrow{\text{Reflection in y-axis}} (b, a)$
Thus, the coordinates of image is (b, a).

11. यदि r_1 ले X-अक्षद्वारा हुने परावर्तन र r_2 ले उद्गम बिन्दुबाट + 90° को परिक्रमण जनाउँछ भने बिन्दु A(x, y) को संयुक्त स्थानान्तरण $r_1 \circ r_2$ द्वारा हुने प्रतिबिम्बित बिन्दु पत्ता लगाउनुहोस् ।
If r_1 is the reflection about the X-axis and r_2 is the rotation about + 90° through the origin, find the image of the point A(x, y) under the combined transformation of $r_1 \circ r_2$.

⇨ Here, $A(x, y) \xrightarrow{r_2} A'(-y, x) \xrightarrow{r_1} A''(-y, -x)$
Thus, the image of point A(x, y) is A"(– y, – x) under $r_1 \circ r_2$.

12. बिन्दु (a, b) लाई पहिले X-अक्षमा परावर्तन गरी $\begin{pmatrix} m \\ n \end{pmatrix}$ द्वारा स्थानान्तरण गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

Find the co-ordinates of image of point (a, b) when it is first reflected on the X-axis and then translated by $\begin{pmatrix} m \\ n \end{pmatrix}$.

⇨ Here, $(a, b) \xrightarrow{\text{Reflection in x-axis}} (a, -b) \xrightarrow{\text{Translation by } \begin{pmatrix} m \\ n \end{pmatrix}} (a + m, -b + n)$
Thus, the image of (a, b) is (a + m, n – b).

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13. बिन्दु (*x*, *y*) लाई पहिले उद्गमबिन्दु O बाट – 90° घुमाउँदा हुने प्रतिबिम्बलाई $\begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा विस्थापन गर्दा हुने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the image of a point (*x*, *y*) when it is first rotated about the origin O through – 90° and then translated by $\begin{pmatrix} a \\ b \end{pmatrix}$

⇨ Here, $(x, y) \xrightarrow{Rotation\ [-90°;\ (0,\ 0)]} (y, -x) \xrightarrow{Translation\ by \begin{pmatrix} a \\ b \end{pmatrix}} (y + a, -x + b)$

Thus, the coordinates of image of (x, y) is (a + y, b – x).

14. बिन्दु P(*x*, *y*) लाई *x* = *h* रेखामा परावर्तन गर्नुहोस् । प्राप्त प्रतिबिम्बलाई E[(0, 0), *k*] ले विस्तारीकरण गर्नुहोस् । अन्तिम प्रतिबिम्बको निर्देशाङ्कहरू लेख्नुहोस् ।
Reflect a point P(*x*, *y*) on the line *x* = *h*. Enlarge the image so obtained by E[(0, 0), *k*]. Write the coordinates of the final image.

⇨ Here, $P(x, y) \xrightarrow{Reflection\ in\ x = h} P'(2h - x, y) \xrightarrow{Enlarged\ by\ [(0,\ 0);\ k]} P''(2hk - kx, ky)$

Thus, the coordinates of final image is P"(2hk – kx, ky).

15. बिन्दु A(*x*, *y*) लाई भेक्टर $\begin{pmatrix} a \\ b \end{pmatrix}$ ले स्थानान्तरण गरी आउने प्रतिबिम्बलाई E[(0, 0), *k*] ले विस्तारीकरण गर्नुहोस् र अन्तिम प्रतिबिम्बको निर्देशाङ्कहरू लेख्नुहोस् ।
The point A(*x*, *y*) is translated by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ and enlarge the image so obtained by E[(0, 0), *k*]. Write the co-ordinates of the final image.

⇨ Here, $A(x, y) \xrightarrow{Translation\ by \begin{pmatrix} a \\ b \end{pmatrix}} A'(x + a, y + b) \xrightarrow{Enlarged\ by\ [(0,\ 0);\ k]} A''[k(x + a), k(y + b)]$

Thus, the co-ordinates of final image is A"[k(x + a), k(y + b)].

16. यदि $T_1 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ र $T_2 = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ भए $T_2 \circ T_1$ भेक्टर पत्ता लगाउनुहोस् । **(If $T_1 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ find the vector for $T_2 \circ T_1$.)**

⇨ Here, $T_2 \circ T_1 = \begin{pmatrix} 6 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 - 3 \\ -3 + 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

B. LONG QUESTIONS

MODEL 1

1. X- अक्ष र Y- अक्षमा क्रमशः हुने संयुक्त परावर्तनहरूसँग समतुल्य हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् । सोही एकल स्थानान्तरण प्रयोग गरेर A(2, 3), B(3, – 4) र C(1, – 2) शीर्षबिन्दुहरू भएको ΔABC को प्रतिबिम्बको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेख्नुहोस् । साथै वस्तु र प्रतिबिम्बलाई एउटै लेखाचित्रमा खिच्नुहोस् ।
State the single transformation equivalent to the combination of reflections on the X-axis and Y-axis respectively. Using this single transformation find the coordinates of the vertices of the image of ΔABC having vertices A(2, 3), B(3, – 4) and C(1, – 2). Also, draw the object and image on the same graph. [2071 R]

⇨ Here, A(2, 3), B(3, – 4) and C(1, – 2)

We know that,

$$\begin{array}{ccc} \text{Object} & & \text{Image} \\ (x, y) & \xrightarrow{Reflection\ (x\text{-}axis)} & (x, -y) \end{array}$$

$$\begin{array}{ccc} \text{Object} & & \text{Image} \\ (x, y) & \xrightarrow{Reflection\ (y\text{-}axis)} & (-x, y) \end{array}$$

$$\therefore\ (x, -y) \xrightarrow{Combined\ Reflection} (-x, -y)$$

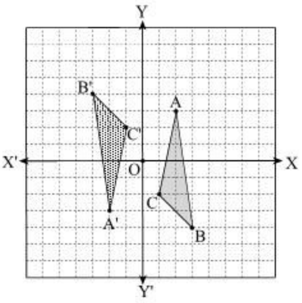
Which shows that the combined transformation is equivalent to the rotation of 180° through the origin.

We have,

$$\begin{array}{ccc} \text{Object} & & \text{Image} \\ (x, y) & \xrightarrow{Rotation\ through\ [180°,\ (0,\ 0)]} & (-x, -y) \\ A(2, 3) & \xrightarrow{Rotation\ through\ [180°,\ (0,\ 0)]} & A'(-2, -3) \\ B(3, -4) & \xrightarrow{Rotation\ through\ [180°,\ (0,\ 0)]} & B'(-3, 4) \\ C(1, -2) & \xrightarrow{Rotation\ through\ [180°,\ (0,\ 0)]} & C'(-1, 2) \end{array}$$

Thus, the required images are A'(– 2, – 3), B'(– 3, 4) and C'(– 1, 2).

Now showing the object and image in the same graph.



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2. **शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको ΔABC लाई रेखा y = x र X-अक्षमा क्रमशः परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्कहरू लेखी लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटा स्थानान्तरणले जनाउन सकिन्छ ? उल्लेख गर्नुहोस् ।**

A ΔABC with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the line y = x and the X-axis. Find the stating co-ordinates and graphically represent the image under these transformations. State also the single transformation given by the combinations of these transformations. **[2071 R']**

⇨ Here, A(1, 2), B(4, – 1) and C(2, 5)

We know that,

Object Image

$$(x, y) \xrightarrow{\text{Reflection in } y = x} (y, x)$$

$$A(1, 2) \xrightarrow{\text{Reflection in } y = x} A'(2, 1)$$

$$B(4, -1) \xrightarrow{\text{Reflection in } y = x} B'(-1, 4)$$

$$C(2, 5) \xrightarrow{\text{Reflection in } y = x} C'(5, 2)$$

Again,

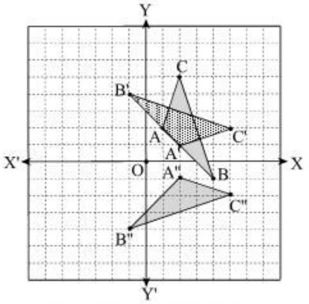
Object Image

$$(x, y) \xrightarrow{\text{Reflection in x-axis}} (x, -y)$$

$$A'(2, 1) \xrightarrow{\text{Reflection in x-axis}} A''(2, -1)$$

$$B'(-1, 4) \xrightarrow{\text{Reflection in x-axis}} B''(-1, -4)$$

$$C'(5, 2) \xrightarrow{\text{Reflection in x-axis}} C''(5, -2)$$



Taking object and final image,

Object Final image

$$A(1, 2) \longrightarrow A''(2, -1)$$

$$(x, y) \longrightarrow (y, -x)$$

Which shows that the combination of transformation is equivalent to – 90° rotation about the origin.

3. **X–अक्ष र Y–अक्षमा क्रमशः हुने संयुक्त परावर्तनहरूसँग समतुल्य हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् । सोही एकल स्थानान्तरण प्रयोग गरेर P(4, 3), Q(1, 1) र R(5, – 1) शीर्षबिन्दुहरू भएको ΔPQR को प्रतिबिम्बको शीर्षबिन्दुहरूका निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै वस्तु र प्रतिबिम्बलाई एउटै लेखाचित्रमा खिच्नुहोस् ।**

State the single transformation equivalent to the combination of reflections on the X-axis and Y-axis respectively. Using this single transformation find the coordinates of the vertices of the image of ΔPQR having vertices P(4, 3), Q(1, 1) and R(5, – 1). Also draw the object and image on the same graph. **[2071 S]**

⇨ Here, P(4, 3), Q(1, 1) and R(5, – 1)

We have,

Object Image

$$(x, y) \xrightarrow{\text{Reflection in x-axis}} (x, -y)$$

Again,

Object Image

$$(x, y) \xrightarrow{\text{Reflection in y-axis}} (-x, y)$$

$$\therefore\ (x, -y) \xrightarrow{\text{Reflection in y-axis}} (-x, -y)$$

Thus, the combination of transformation is 180° rotation about the origin. Which is the single transformation.

Now,

Object Image

$$(x, y) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} (-x, -y)$$

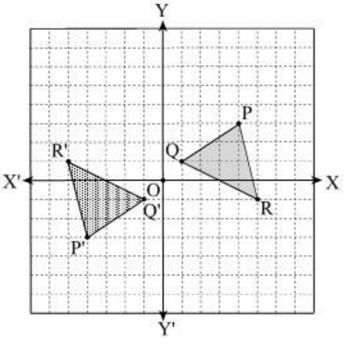
$$P(4, 3) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} P'(-4, -3)$$

$$Q(1, 1) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} Q'(-1, -1)$$

$$R(5, -1) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} R'(-5, 1)$$

Thus, the co-ordinates of images are; P'(– 4, – 3), Q'(– 1, – 1) and R'(– 5, 1).

Drawing ΔABC and ΔA'B'C' in graph.



4. **शीर्षबिन्दुहरू A(6, 2), B(10, 2), C(11, 4) र D(5, 4) भएको चतुर्भुज ABCD लाई x-अक्षमा र त्यसपछि y-अक्षमा परावर्तन गराउदा संयुक्त स्थानान्तरण बन्ने प्रतिबिम्बको निर्देशाङ्कहरू निकाल्नुहोस् ।**

Find the co-ordinates of a image of a quadrilateral ABCD with vertices A(6, 2), B(10, 2), C(11, 4) and D(5, 4) under the combined transformations of the reflection on x-axis then on the y-axis. **[2068 R']**

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⇨ Here, A(6, 2), B(10, 2), C(11, 4) & D(5, 4) are the co-ordinates of a quadrilateral ABCD.

Now,

Object $\qquad\qquad\qquad\qquad$ Image

$P(x, y) \xrightarrow{\text{Reflection in x-axis}} P'(x, -y)$

$A(6, 2) \xrightarrow{\text{Reflection in x-axis}} A'(6, -2)$

$B(10, 2) \xrightarrow{\text{Reflection in x-axis}} B'(10, -2)$

$C(11, 4) \xrightarrow{\text{Reflection in x-axis}} C'(11, -4)$

$D(5, 4) \xrightarrow{\text{Reflection in x-axis}} D'(5, -4)$

Again,

Object $\qquad\qquad\qquad\qquad$ Image

$P(x, y) \xrightarrow{\text{Reflection in y-axis}} P'(-x, y)$

$A'(6, -2) \xrightarrow{\text{Reflection in y-axis}} A''(-6, -2)$

$B'(10, -2) \xrightarrow{\text{Reflection in y-axis}} B''(-10, -2)$

$C'(11, -4) \xrightarrow{\text{Reflection in y-axis}} C''(-11, -4)$

$D'(5, -4) \xrightarrow{\text{Reflection in y-axis}} D''(-5, -4)$

Thus, required image of the given object are A'(6, – 2), B'(10, – 2), C'(11, – 4), D'(5, – 4) and A"(– 6, – 2), B"(– 10, – 2), C"(– 11, – 4), D"(– 5, – 4).

5. लेखाचित्रमा A(5, 4), B(2, 2) र C(5, 2) शीर्षबिन्दुहरू भएको त्रिभुज ABC खिच्नुहोस् । ΔABC लाई X-अक्षमा र त्यसपछि प्राप्त प्रतिबिम्बलाई रेखा y = x मा लगातार परावर्तन गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्क लेखाचित्रमा देखाउनुहोस् । [2063 M]

On a graph paper, draw a triangle ABC having the vertices A(5, 4), B(2, 2) and C(5, 2). Find the image of ΔABC by stating co-ordinates and graphing them after successive reflection in X-axis followed by a reflection in the line y = x.

⇨ Here, we have given, A(5, 4), B(2, 2) and C(5, 2)

Let, reflection in X-axis is P and reflection in the line x = y is Q

Then; combined transformation = Q∘P

$\therefore \quad Q \circ P\,(x, y) \quad = Q[(P(x, y)] = Q(x, -y)$

$\therefore \quad Q \circ P\,(x, y) \quad = (-y, x)$

Now,

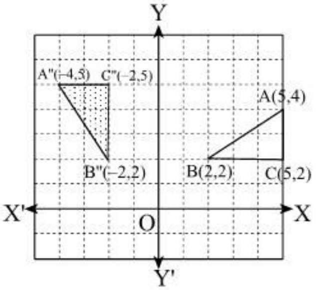
Object $\qquad\qquad\qquad\qquad$ Image

$(x, y) \xrightarrow{\text{Combined reflection } Q \circ P} (-y, x)$

$A(5, 4) \xrightarrow{\text{Combined reflection } Q \circ P} A''(-4, 5)$

$B(2, 2) \xrightarrow{\text{Combined reflection } Q \circ P} B''(-2, 2)$

$C(5, 2) \xrightarrow{\text{Combined reflection } Q \circ P} C''(-2, 5)$



Thus, the co-ordinates of final image of ΔABC are; A"(– 4, 5), B" (– 2, 2), C" (–2, 5) which is shown in the graph by shading.

6. लेखाचित्रमा P(1, 4), Q(4, 1) र R(7, 5) शीर्षबिन्दुहरू भएको त्रिभुज PQR खिच्नुहोस् । ΔPQR लाई Y-अक्षमा र त्यसपछि प्राप्त प्रतिबिम्बलाई रेखा x = y मा लगातार परावर्तन गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाई प्रतिबिम्बित त्रिभुजहरू लेखाचित्रमा देखाउनुहोस् । [2063 R]

On a graph paper, draw a triangle PQR having the vertices P(1, 4), Q(4, 1) and R(7, 5). Find the image of ΔPQR by stating co-ordinates and graphing them after successive reflection in Y-axis followed by a reflection in the line x = y.

⇨ Here, we have,

Object $\qquad\qquad\qquad\qquad$ Image

$(x, y) \xrightarrow{\text{Reflection in y-axis}} (-x, y)$

$P(1, 4) \xrightarrow{\text{Reflection in y-axis}} P'(-1, 4)$

$Q(4, 1) \xrightarrow{\text{Reflection in y-axis}} Q'(-4, 1)$

$R(7, 5) \xrightarrow{\text{Reflection in y-axis}} R'(-7, 5)$

Again,

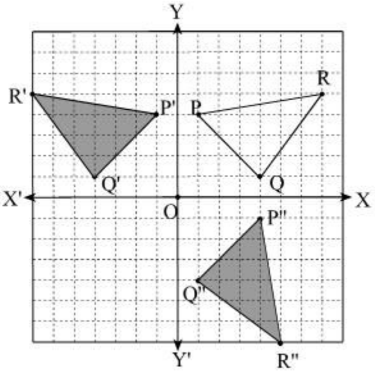
Object $\qquad\qquad\qquad\qquad$ Image

$(x, y) \xrightarrow{\text{Reflection in } x = y} (y, x)$

$P'(-1, 4) \xrightarrow{\text{Reflection in } x = y} P''(4, -1)$

$Q'(-4, 1) \xrightarrow{\text{Reflection in } x = y} Q''(1, -4)$

$R'(-7, 5) \xrightarrow{\text{Reflection in } x = y} R''(5, -7)$



The graph is as shown above:

Thus, co-ordinates of ΔP'Q'R' are (– 1, 4), (– 4, 1) and (– 7, 5) and co-ordinates of ΔP"Q"R" are (4, – 1), (1, – 4) and (5, – 7) respectively.

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7. **शीर्षबिन्दुहरू A (1, 2), B (4, – 1) र C (2, 5) भएको त्रिभुजलाई रेखाहरू x = 5 र y = – 2 मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।**

A triangle with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the lines x = 5 and y = – 2. Find by stating coordinates and graphically represent the images under these transformations. State also the single transformation given by the combinations of these transformations. **[2065 M]**

⇨ Here,

we know that, reflection in x = 5 gives;

Object Image

$(x, y) \xrightarrow{\text{Reflection in } x = 5} (2 \times 5 - x, y) = (10 - x, y)$

$A(1, 2) \xrightarrow{\text{Reflection in } x = 5} A'(9, 2)$

$B(4, -1) \xrightarrow{\text{Reflection in } x = 5} B'(6, -1)$

$C(2, 5) \xrightarrow{\text{Reflection in } x = 5} C'(8, 5)$

Again, Reflection in y = – 2 gives;

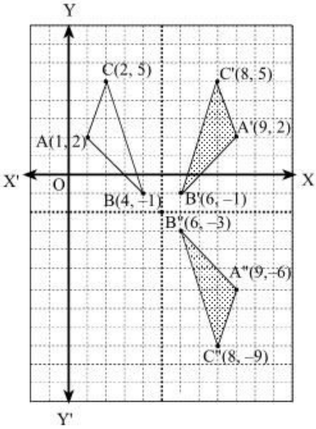
Object Image

$(x, y) \xrightarrow{\text{Reflection in } y = -2} (x, 2 \times (-2) - y) = (x, -4 - y)$

$A'(9, 2) \xrightarrow{\text{Reflection in } y = -2} A''(9, -6)$

$B'(6, -1) \xrightarrow{\text{Reflection in } y = -2} B''(6, -3)$

$C'(8, 5) \xrightarrow{\text{Reflection in } y = -2} C''(8, -9)$



Thus, the graph of the combined transformation is the single transformation as half turn about (5, – 2).

8. **शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको त्रिभुजलाई रेखा x = – 1 र y = 2 मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।**

A triangle with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the line x = – 1 and y = 2. Find the stating co-ordinates and represent the images graphically under these transformations. State also the single transformation given by the combinations of these transformations. **[2069 R, 2065 R]**

⇨ Here, A(1, 2), B(4, – 1) and C(2, 5)

We have,

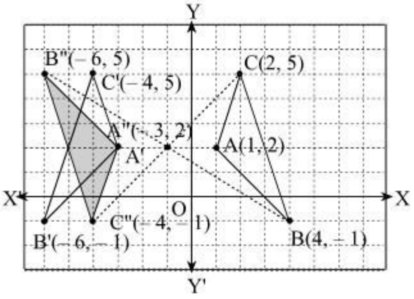
So, Object Image

$(x, y) \xrightarrow{\text{Reflection in } x = -1} (-2 - x, y)$

$A(1, 2) \xrightarrow{\text{Reflection in } x = -1} A'(-3, 2)$

$B(4, -1) \xrightarrow{\text{Reflection in } x = -1} B'(-6, -1)$

$C(2, 5) \xrightarrow{\text{Reflection in } x = -1} C'(-4, 5)$



Again,

Object Image

$(x, y) \xrightarrow{\text{Reflection in } y = 2} (x, 4 - y)$

$A'(-3, 2) \xrightarrow{\text{Reflection in } y = 2} A''(-3, 2)$

$B'(-6, -1) \xrightarrow{\text{Reflection in } y = 2} B''(-6, 5)$

$C'(-4, 5) \xrightarrow{\text{Reflection in } y = 2} C''(-4, -1)$

Graphical representation;

Thus, from the graph the single transformation is enlargement with centre (– 1, 2) and scale factor – 1 or the half turn rotation about (–1, 2).

9. **शीर्षबिन्दुहरू A (1, 2), B (4, – 1) र C (2, 5) भएको त्रिभुजलाई रेखाहरू x = 4 र y = – 3 मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशांक लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।**

A triangle with vertices A (1, 2), B (4, – 1) and C (2, 5) is reflected successively in the lines x = 4 and y = – 3. Find by stating co-ordinates and graphically represents the images under these transformations given by the combination of these transformations. **[2066 R]**

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⇨ Here, A(1, 2), B(4, – 1) and C(2, 5)
We know that,

Object $\qquad\qquad\qquad\qquad$ Image

(x, y) $\xrightarrow{\textit{Reflection in } x = h = 4}$ (2h – x, y) = (8 – x, y)

A(1, 2) $\xrightarrow{\textit{Reflection in } x = h = 4}$ A'(7, 2)

B(4, – 1) $\xrightarrow{\textit{Reflection in } x = h = 4}$ B'(4, – 1)

C(2, 5) $\xrightarrow{\textit{Reflection in } x = h = 4}$ C'(6, 5)

We have,

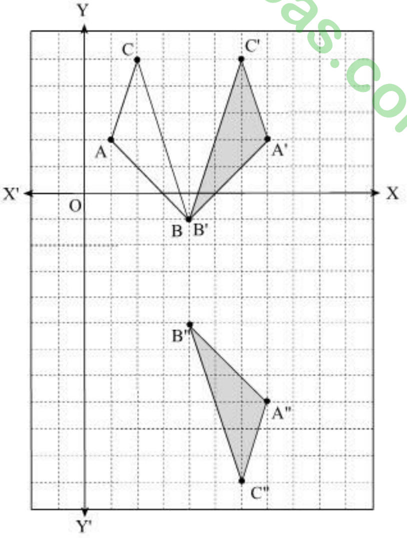
Object $\qquad\qquad\qquad\qquad$ Image

(x, y) $\xrightarrow{\textit{Reflection in } y = k = -3}$ (x, 2k – y) = (x, – 6 – y)

A'(7, 2) $\xrightarrow{\textit{Reflection in } y = k = -3}$ A"(7, – 8)

B'(4, – 1) $\xrightarrow{\textit{Reflection in } y = k = -3}$ B"(4, – 5)

C'(6, 5) $\xrightarrow{\textit{Reflection in } y = k = -3}$ C"(6, – 11)



Graphing the object and image.
Thus, the single transformation is the half turn rotation about the point (4, – 3).

MODEL 2

10. बिन्दुहरू H(4, – 2), R(2, 1) र T(5, 2) एउटा त्रिभुज HRT का तीनओटा शीर्षबिन्दुहरू हुन् । यी बिन्दुहरूलाई परिक्रमण [(0, 0), 180°] र उही दिशामा परिक्रमण [(0, 0), 90°] बाट हुने संयुक्त स्थानान्तरणले स्थानान्तरण गराउँदा बन्ने प्रतिबिम्बहरूको निर्देशाङ्क पत्ता लगाउनुहोस् । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।

Points H(4, – 2), R(2, 1) and T(5, 2) are the vertices of a triangle HRT. These points are transformed by a single transformation obtained by a rotation [(0, 0), 180°] and on the same direction a rotation [(0, 0), 90°]. Find the co-ordinates of the images of the points. Draw both the triangles on the same graphpaper. [2066 S]

⇨ Here, H(4, – 2), R(2, 1) and T(5, 2); R_1 = [(0, 0), 180°] and R_2 [(0, 0), 90°]
Combined rotation = $R_1 + R_2$ = [(0, 0); 270°]
Now, we have

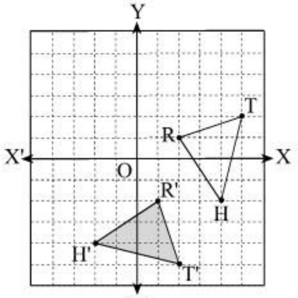
Object $\qquad\qquad\qquad\qquad$ **Image**

(*x*, *y*) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ (*y*, – *x*)

H(4, – 2) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ H'(– 2, – 4)

R(2, 1) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ R'(1, – 2)

T(5, 2) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ T'(2, – 5)



Both the triangles on the same graph paper are shown alongside:
Thus, the co-ordinates of final images are; H'(– 2, – 4), R'(1, – 2) and T'(2, – 5).

11. बिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) एउटा त्रिभुज ABC का तिनओटा शीर्षबिन्दुहरू हुन् । यी बिन्दुहरूलाई परिक्रमण [(0, 0), 90°] र उही दिशामा परिक्रमण [(0, 0), 180°] बाट हुने संयुक्त स्थानान्तरणले स्थानान्नतरण गराउँदा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।

Points A(1, 2), B(4, – 1) and C(2, 5) are the vertices of a triangle ΔABC. These points are transformed by a single transformation obtained by a rotation [(0, 0), 90°] and on the same direction a rotation [(0, 0), 180°]. Find the co-ordinates of the images of the points. Draw both the triangles on the same graph paper.

⇨ Here, A(1, 2), B(4, – 1) and C(2, 5) are the vertices of ΔABC.
By the question, R_1 = [(0, 0) ; 90°] and R_2 = [(0, 0) ; 180°]
So, combined rotation = $R_1 + R_2$ = [(0, 0) ; 90°] + [(0, 0) ; 180°]
∴ $R_1 + R_2$ = = [(0, 0) ; 270°]
Now, we have,

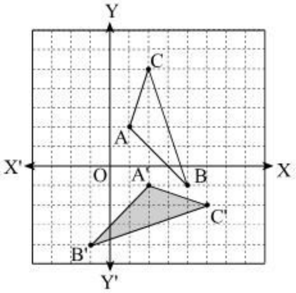
Object $\qquad\qquad\qquad\qquad$ **Image**

(*x*, *y*) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ (*y*, – *x*)

So, A(1, 2) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ A'(2, – 1)

A(4, – 1) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ B'(– 1, – 4)

C(2, 5) $\xrightarrow{\textit{Rotation through } [(0,\, 0);\, 270°]}$ C'(5, – 2)



ΔABC and ΔA'B'C' are shown on the graph.
Thus, the co-ordinates of vertices of image are A'(2, – 1), B'(– 1, – 4) and C'(5, – 2).

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MODEL 3

12. **ΔABC का शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(6, 2) छन् । यदि $E_1 = [(0, 0), 2]$ र $E_2 = [(0, 2), 2]$ भए ΔABC लाई $E_2 \circ E_1$ संयुक्त विस्तारीकरण गरी प्रतिबिम्बहरूको निर्देशाङ्क पत्ता लगाउनुहोस् । दुवै त्रिभुजलाई उही ग्राफमा खिच्नुहोस् ।**

The vertices of ΔABC are A(2, 3), B(4, 5) and C(6, 2). If $E_1 = [(0, 0), 2]$ and $E_2 = [(0, 2), 2]$, then find the co-ordinates of image of ΔABC under enlargement $E_2 \circ E_1$. Draw both triangles on the same graph. **[2064 R']**

⇨ Here, A(2, 3), B(4, 5) and C(6, 2) and $E_1 = [(0, 0); 2]$ and $E_2 = [(0, 2); 2]$

We have,

Object $\qquad\qquad$ Image

$$P(x, y) \xrightarrow{\text{Enlarged by } E_1[(0,\ 0),\ ;k]} P'(kx, ky)$$

$$(x, y) \xrightarrow{\text{Enlarged by } E_1[(0,\ 0),\ ;2]} (2x, 2y)$$

Again,

Object $\qquad\qquad$ Image

$$(x, y) \xrightarrow{\text{Enlarged by } E[(a,\ b),\ ;k]} (k(x - a) + a, k(y - b) + b)$$

$$(x, y) \xrightarrow{\text{Enlarged by } E_2[(0,\ 2),\ ;2]} (2(x - 0) + 0, 2(y - 2) + 2)$$
$$= (2x, 2y - 2)$$

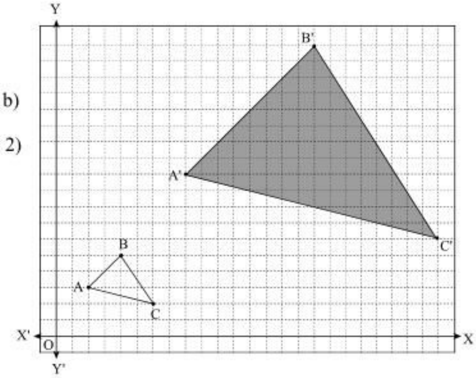
$$(2x, 2y) \xrightarrow{\text{Enlarged by } E_2[(0,\ 2),\ ;2]} (2 \times 2x, 2 \times 2y - 2)$$

$$\text{So, } (x, y) \xrightarrow{\text{Enlarged by } E_2 \circ E_1} (4x, 4y - 2)$$

$$A(2, 3) \xrightarrow{\text{Enlarged by } E_2 \circ E_1} A'(8, 10)$$

$$B(4, 5) \xrightarrow{\text{Enlarged by } E_2 \circ E_1} B'(16, 18)$$

$$C(6, 2) \xrightarrow{\text{Enlarged by } E_2 \circ E_1} C'(24, 6)$$



Now, showing both triangle on the same graph.

Thus, the coordinates of image of ΔABC are A'(8, 10), B'(16, 18) and C'(24, 6).

13. **ΔABC का शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) छन् । यदि $E_1 = [(0, 0), 4]$ र $E_2 = [(0, 0), 0.5]$ भए ΔABC लाई $E_2 \circ E_1$ संयुक्त विस्तारीकरण गरी प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुवै त्रिभुजलाई उही ग्राफमा खिच्नुहोस् ।**

The vertices of ΔABC are A(1, 2), B(4, – 1) and C(2, 5). If $E_1 = [(0, 0), 4]$ and $E_2 = [(0, 0), 0.5]$, then find the co-ordinates of image of ΔABC under enlargement $E_2 \circ E_1$. Draw both triangle on the same graph.

⇨ Here, A(1, 2), B(4, – 1) , C(2, 5), $E_1 = [(0, 0); 4]$ and $E_2 = [(0, 0); 0.5]$

Now, $E_2 \circ E_1 = [(0, 0); 0.5] \circ [(0, 0); 4] = E[(0, 0); 2]$

We know that,

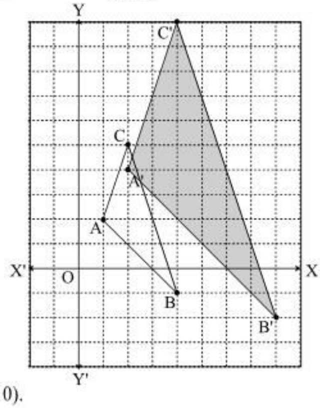
Object $\qquad\qquad$ Image

$$P(x, y) \xrightarrow{\text{Enlarged by } E[(0,\ 0);\ k]} P'(kx, ky)$$

$$\text{So, } A(1, 2) \xrightarrow{\text{Enlarged by } E[(0,\ 0);\ 2]} A'(2, 4)$$

$$B(4, -1) \xrightarrow{\text{Enlarged by } E[(0,\ 0);\ 2]} B'(8, -2)$$

$$C(2, 5) \xrightarrow{\text{Enlarged by } E[(0,\ 0);\ 2]} C'(4, 10)$$



ΔABC and ΔA'B'C' are shown on the graph.

Thus, the co-ordinates of vertices of image triangle are A'(2, 4), B'(8, – 2) and C'(4, 10).

MODEL 4

14. **शिर्षबिन्दुहरू X(1, 2), Y(– 1, – 2) र Z(5, 0) भएको त्रिभुज XYZ लाई भेक्टर $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई X-अक्षमा परावर्तन गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शिर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔXYZ र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

A triangle XYZ with vertices X(1, 2), Y(– 1, – 2) and Z(5, 0) is translated by a vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. The image so formed is reflected on the X-axis. Write the co-ordinates of the vertices of the images thus obtained and represent the ΔXYZ and its images on the same graph paper. **[2073 S]**

⇨ Here, X(1, 2), Y(– 1, – 2) and Z(5, 0); Translation vector (TV) = $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ First, the ΔXYZ is translated by $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

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We have,

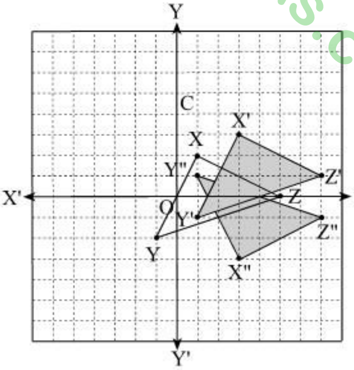
Object **Image**

$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix}} (x + a, y + b)$

$X(1, 2) \xrightarrow{\text{Translation by} \begin{pmatrix} 2 \\ 1 \end{pmatrix}} X'(3, 3)$

$Y(-1, -2) \xrightarrow{\text{Translation by} \begin{pmatrix} 2 \\ 1 \end{pmatrix}} Y'(1, -1)$

$Z(5, 0) \xrightarrow{\text{Translation by} \begin{pmatrix} 2 \\ 1 \end{pmatrix}} Z'(7, 1)$



Again, ΔX'Y'Z' is reflected in x-axis as below

Object Image

$(x, y) \xrightarrow{\text{Reflection in x-axis}} (x, -y)$

$X'(3, 3) \xrightarrow{\text{Reflection in x-axis}} X''(3, -3)$

$Y'(1, -1) \xrightarrow{\text{Reflection in x-axis}} Y''(1, 1)$

$Z'(7, 1) \xrightarrow{\text{Reflection in x-axis}} Z''(7, -1)$

Now, the graph of ΔXYZ, ΔX'Y'Z' and ΔX"Y"Z" is shown alongside.
Thus, the vertices of images are; X'(3, 3), Y'(1, – 1), Z'(7, 1), X"(3, – 3), Y"(1, 1) and Z"(7, – 1).

15. शीर्षबिन्दुहरू P(2, 1), Q(4, 3) र R(6, – 1) भएको त्रिभुज PQR लाई X-अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई भेक्टर $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ले विस्थापन गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle PQR with vertices P(2, 1), Q(4, 3) and R(6, – 1) is reflected on the X-axis. The image so formed is translated by the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ Write down the co-ordinates of the vertices of the images thus obtained and represent the ΔPQR and its images on the same graph paper. [2073 S']

⇨ Here, P(2, 1), Q(4, 3) and R(6, – 1); Translation Vector (TV) = $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

First, we have to reflect ΔPQR in x axis.
We have,

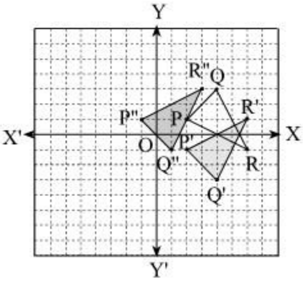
Object Image

$(x, y) \xrightarrow{\text{Reflection in x-axis}} (x, -y)$

$P(2, 1) \xrightarrow{\text{Reflection in x-axis}} P'(2, -1)$

$Q(4, 3) \xrightarrow{\text{Reflection in x-axis}} Q'(4, -3)$

$R(6, -1) \xrightarrow{\text{Reflection in x-axis}} R'(6, 1)$



Again, we have to translate ΔP'Q'R' by the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

Object **Image**

$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix}} (x + a, y + b)$

$P'(2, -1) \xrightarrow{\text{Translation by} \begin{pmatrix} -3 \\ 2 \end{pmatrix}} P''(-1, 1)$

$Q'(4, -3) \xrightarrow{\text{Translation by} \begin{pmatrix} -3 \\ 2 \end{pmatrix}} Q''(1, -1)$

$R'(6, 1) \xrightarrow{\text{Translation by} \begin{pmatrix} -3 \\ 2 \end{pmatrix}} R''(3, 3)$

Now, the graph of ΔPQR, ΔP'Q'R' and ΔP"Q"R" is shown alongside.
Thus, the co-ordinates of images are; P'(2, – 1), Q'(4, – 3), R'(6, 1), P"(– 1, 1), Q"(1, – 1), R"(3, 3).

16. शीर्षबिन्दुहरू P(2, 4), Q(–1, 2) र R(5, –1) भएको ΔPQR लाई विस्थापन भेक्टर T = $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ले विस्थापन गर्नुहोस् र त्यसको प्रतिबिम्बलाई रेखा x + y = 0 मा परावर्तन गर्दा प्राप्त हुने प्रतिविम्बहरूका शीर्षबिन्दुहरूका निर्देशांकहरू लेखेर वस्तु र प्रतिविम्बहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।

Translate the ΔPQR with vertices P(2, 4), Q(–1, 2) and R(5, –1) by translation vector T = $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$and then reflect the image so obtained in the line x + y = 0. Write the co-ordinates of the vertices of the image and draw the object and images on the same graph paper. [2070 R']

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⇨ Here, vertices of ΔPQR are P(2, 4), Q(– 1, 2) and R(5, 1)

We know that,

Object Image

$$(x, y) \xrightarrow{\text{Translation by } \begin{pmatrix}-2\\3\end{pmatrix}} (x-2, y+3)$$

$$P(2, 4) \xrightarrow{\text{Translation by } \begin{pmatrix}-2\\3\end{pmatrix}} P'(0, 7)$$

$$Q(-1, 2) \xrightarrow{\text{Translation by } \begin{pmatrix}-2\\3\end{pmatrix}} Q'(-3, 5)$$

$$R(5, -1) \xrightarrow{\text{Translation by } \begin{pmatrix}-2\\3\end{pmatrix}} R'(3, 2)$$

We have,

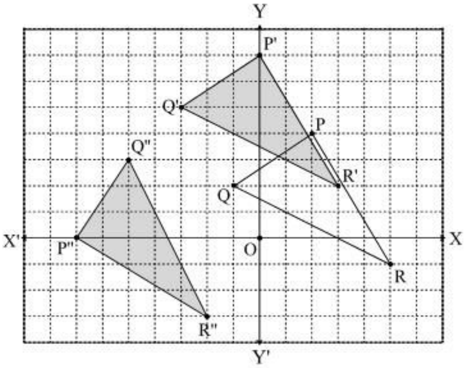
Object Image

$$(x, y) \xrightarrow{\text{Reflection in the line } x+y=0} (-y, -x)$$

$$P'(0, 7) \xrightarrow{\text{Reflection in the line } x+y=0} P''(-7, 0)$$

$$Q'(-3, 5) \xrightarrow{\text{Reflection in the line } x+y=0} Q''(-5, 3)$$

$$R'(3, 2) \xrightarrow{\text{Reflection in the line } x+y=0} R''(-2, -3)$$



Thus, the co-ordinates of images are: P'(0, 7), Q'(– 3, 5), R'(3, 2), P''(– 7, 0), Q''(– 5, 3) and R''(– 2, – 3).

17. **शीर्षबिन्दुहरू O(0, 0), A(2, 0), B(3, 2) र C(1, 2) भएको चतुर्भुज OABC लाई विस्थापन भेक्टर $\begin{pmatrix}0\\2\end{pmatrix}$ले विस्थापन गरी प्राप्त प्रतिबिम्बलाई x = 3 मा परावर्तन गर्नुहोस् । यसरी प्राप्त प्रतिबिम्बहरू र वस्तुलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

Translate the quadrilateral OABC having vertices O(0, 0), A(2, 0), B(3, 2) and C(1, 2) by translation vector $\begin{pmatrix}0\\2\end{pmatrix}$. Reflect the image so formed on the line x = 3. Represent the images and object in the same graph. [2068 R]

⇨ Here, O(0, 0), A(2, 0), B(3, 2) and C(,1 2)

We know that,

Object **Image**

$$(x, y) \xrightarrow{\text{Translation by } \begin{pmatrix}a\\b\end{pmatrix}} (x+a, y+b)$$

$$O(0, 0) \xrightarrow{\text{Translation by } \begin{pmatrix}0\\2\end{pmatrix}} O'(0, 2)$$

$$A(2, 0) \xrightarrow{\text{Translation by } \begin{pmatrix}0\\2\end{pmatrix}} A'(2, 2)$$

$$B(3, 2) \xrightarrow{\text{Translation by } \begin{pmatrix}0\\2\end{pmatrix}} B'(3, 4)$$

$$C(1, 2) \xrightarrow{\text{Translation by } \begin{pmatrix}0\\2\end{pmatrix}} C'(1, 4)$$

Again,

We have,

Object Image

$$(x, y) \xrightarrow{\text{Reflection in } x = h} (2h - x, y)$$

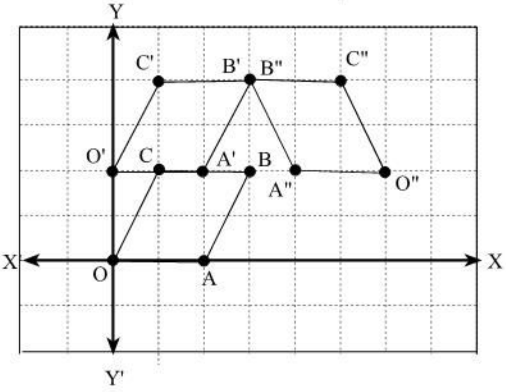
$$O'(0, 2) \xrightarrow{\text{Reflection in } x = 3} O''(6, 2)$$

$$A'(2, 2) \xrightarrow{\text{Reflection in } x = 3} A''(4, 2)$$

$$B'(3, 4) \xrightarrow{\text{Reflection in } x = 3} B''(3, 4)$$

$$C'(1, 4) \xrightarrow{\text{Reflection in } x = 3} C''(5, 4)$$

Thus, the images are: O'(0, 2), A'(2, 2), B'(3, 4), C'(1, 4) and O''(6, 2), A''(4, 2), B''(3, 4), C''(5, 4).



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18. O(0, 0), A(2, 0), B(3, 1), र C(1, 1) शीर्षबिन्दु भएको चित्र लेखाचित्रमा खिची यसलाई $\begin{pmatrix}0\\2\end{pmatrix}$ द्वारा स्थानान्तरण गरी सोही लेखाचित्रमा O'A'B'C' खिच्नुहोस् । त्यसपछि चित्र O'A'B'C' लाई x = 3 रेखामा परावर्तन गरी त्यसै लेखाचित्रमा O"A"B"C" चित्र खिच्नुहोस् र O", A", B" र C" का निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

Draw a figure having the vertices O(0, 0), A(2, 0), B(3, 1) and C(1, 1) on a graph paper. It is translated by $\begin{pmatrix}0\\2\end{pmatrix}$ and plot the figure O'A'B'C' on the same graph paper then the figure O'A'B'C' is reflected on the line x = 3 to form the figure O"A"B"C" on the same graph. Determine the vertices of O", A", B" and C". [2057 R]

⇨ Here, O(0, 0), A(2, 0), B(3, 1) and C(1, 1)

We have,

Object **Image**

$$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix}0\\2\end{pmatrix}} (x, y+2)$$

$$O(0, 0) \xrightarrow{\text{Translation by} \begin{pmatrix}0\\2\end{pmatrix}} O'(0, 2)$$

$$A(2, 0) \xrightarrow{\text{Translation by} \begin{pmatrix}0\\2\end{pmatrix}} A'(2, 2)$$

$$B(3, 1) \xrightarrow{\text{Translation by} \begin{pmatrix}0\\2\end{pmatrix}} B'(3, 3)$$

$$C(1, 1) \xrightarrow{\text{Translation by} \begin{pmatrix}0\\2\end{pmatrix}} C'(1, 3)$$

Again,

Object Image

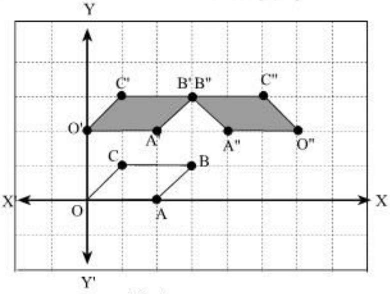
$$(x, y) \xrightarrow{\text{Reflection in } x = 3} (6 - x, y)$$

$$\text{So, } O'(0, 2) \xrightarrow{\text{Reflection in } x = 3} O''(6, 2)$$

$$A'(2, 2) \xrightarrow{\text{Reflection in } x = 3} A''(4, 2)$$

$$B'(3, 3) \xrightarrow{\text{Reflection in } x = 3} B''(3, 3)$$

$$C'(1, 3) \xrightarrow{\text{Reflection in } x = 3} C''(5, 3)$$



Thus, from the graph, the coordinates of the points O", A", B" and C" are respectively, O"(6, 2), A"(4, 2) , B"(3, 3) and C"(5, 3).

19. त्रिभुज ABC का शीर्षबिन्दुहरू क्रमशः (– 2, – 4), (– 4, – 2) र (– 6, – 4) छन् त्रिभुज ABC लाई $\begin{pmatrix}2\\3\end{pmatrix}$ मा विस्थापन गर्नुहोस् र प्राप्त प्रतिबिम्बलाई x = 0 मा परावर्तन गरी संयुक्त स्थानान्तरणका प्रतिबिम्बका शीर्षबिन्दुका निर्देशाङ्क लेख्नुहोस् र सबै त्रिभुजहरूलाई लेखाचित्रमा देखाउनुहोस् ।

The vertices of ΔABC are (– 2, – 4), (– 4, – 2) and (– 6, – 4) respectively. Translate ΔABC by the vector $\begin{pmatrix}2\\3\end{pmatrix}$ and reflect the image on the line x = 0. Find the co-ordinates of the vertices of the image under compound transformation and draw all the triangles in the graph. [2065 E]

⇨ Here, A(– 2, – 4), B(– 4, – 2) & C(– 6, – 4) are the vertices of the given triangle ABC.

We have,

Object **Image**

$$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix}a\\b\end{pmatrix}} (x + a, y + b)$$

$$A(-2, -4) \xrightarrow{\text{Translation by} \begin{pmatrix}2\\3\end{pmatrix}} A'(0, -1)$$

$$B(-4, -2) \xrightarrow{\text{Translation by} \begin{pmatrix}2\\3\end{pmatrix}} B'(-2, 1)$$

$$C(-6, -4) \xrightarrow{\text{Translation by} \begin{pmatrix}2\\3\end{pmatrix}} C'(-4, -1)$$

Again,

Object Image

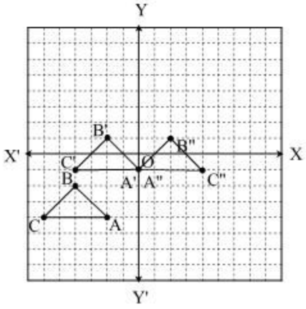
$$P(x, y) \xrightarrow{\text{Reflection}[x = 0 \text{ or } y\text{-axis}]} P'(-x, y)$$

$$A'(0, -1) \xrightarrow{\text{Reflection}[x = 0 \text{ or } y\text{-axis}]} A''(0, -1)$$

$$B'(-2, 1) \xrightarrow{\text{Reflection}[x = 0 \text{ or } y\text{-axis}]} B''(2, 1)$$

$$C'(-4, -1) \xrightarrow{\text{Reflection}[x = 0 \text{ or } y\text{-axis}]} C''(4, -1)$$

Thus, required vertices of the image of the given triangle are A'(0, – 1), B'(– 2, 1), C'(– 4, – 1) and A"(0, – 1), B"(2, 1), C"(4, – 1).



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20. **शीर्षबिन्दुहरू A(1, 0), B(2, 1) र C(3, – 1) भएको त्रिभुजलाई T$\begin{pmatrix}1\\2\end{pmatrix}$मा विस्थापन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क लेखेर लेखाचित्रमा देखाउनुहोस् । उक्त प्रतिबिम्बलाई फेरि x = 2 रेखामा परावर्तन गरी सोही लेखाचित्रमा देखाउनुहोस् ।**

A triangle with vertices A(1, 0), B(2, 1) and C(3, – 1) is translated by T$\begin{pmatrix}1\\2\end{pmatrix}$and draw its image in graph. Again its image is reflected in the line x = 2 and draw its image in same graph. **[2069 S, 2065 S]**

⇨ Here, the given vertices of ΔABC are, A(1, 0), B(2, 1) and C(3, – 1).

It is translated by T.V. = $\begin{pmatrix}1\\2\end{pmatrix}$

So, we have,

Object **Image**

$$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\2\end{pmatrix}} (x + 1, y + 2)$$

$$A(1, 0) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\2\end{pmatrix}} A'(2, 2)$$

$$B(2, 1) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\2\end{pmatrix}} B'(3, 3)$$

$$C(3, -1) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\2\end{pmatrix}} C'(4, 1)$$

Now, the image are shown in the graph paper.

Thus, the vertices of image triangle are A'(2, 2) B'(3, 3), C'(4, 1),A"(2, 2), B"(1, 3) and C"(0, 1).

Again, ΔA'B'C' is reflected in x = 2

So, we know that,

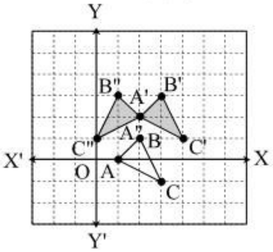
Object Image

$$(x, y) \xrightarrow{\text{Reflection in the line } x = 2} (4 - x, y)$$

$$A'(2, 2) \xrightarrow{\text{Reflection in the line } x = 2} A''(2, 2)$$

$$B'(3, 3) \xrightarrow{\text{Reflection in the line } x = 2} B''(1, 3)$$

$$C'(4, 1) \xrightarrow{\text{Reflection in the line } x = 2} C''(0, 1)$$



MODEL 5

21. **शीर्षबिन्दुहरू P(2, 1), Q(2, 4) र R(5, 2) भएको ΔPQR लाई भेक्टर$\begin{bmatrix}1\\-2\end{bmatrix}$ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरि धनात्मक दिशातिर 90° मा परीक्रमण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको निर्देशाङ्कहरू लेखी ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

ΔPQR with vertices P(2, 1), Q(2, 4) and R(5, 2) is translated by the vector $\begin{bmatrix}1\\-2\end{bmatrix}$ and then the image so formed is rotated about the origin through 90° in positive direction. Write down the co-ordinates of the vertices of the images thus obtained and present ΔPQR and its images in the same graph paper. **[SEE 2075 R']**

⇨ Here, the given vertices of ΔPQR are: P(2, 1), Q(2, 4) and R(5, 2)

We know that,

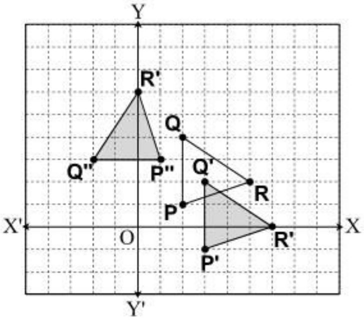
Object Image

$$A(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix}a\\b\end{pmatrix}} A'(x + a, y + b)$$

$$P(2, 1) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\-2\end{pmatrix}} P'(3, -1)$$

$$Q(2, 4) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\-2\end{pmatrix}} Q'(3, 2)$$

$$R(5, 2) \xrightarrow{\text{Translation by} \begin{pmatrix}1\\-2\end{pmatrix}} R'(6, 0)$$



Again,

Object Image

$$(x, y) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} (-y, x)$$

$$P'(3, -1) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} P''(1, 3)$$

$$Q'(3, 2) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} Q''(-2, 3)$$

$$R'(6, 0) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} R''(0, 6)$$

ΔPQR and its both images are shown in the graph.

Thus, the vertices of final image of ΔPQR are P"(1, 3), Q"(– 2, 3) and R" (0, 6).

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22. A (2, 3), B (0, 3) र O(0, 0) भएको ΔAOB लाई $\overrightarrow{AB}$ द्वारा विस्थापन र O को वरिपरि + 90° परिक्रमण गरिएको छ । विस्थापनलाई T र परिक्रमणलाई R ले जनाइएको छ । संयुक्त स्थानान्तरण T∘R अनुसार प्रतिविम्ब पत्ता लगाउनुहोस् ।

ΔABO having A(2, 3), B(0, 3) and O(0, 0) is transformed by the translation vector $\overrightarrow{AB}$ and rotated through 90° (+) ve rotation about O. Translation is denoted by T and the rotation is denoted by R. Find the image under the combined transformation T∘R.

⇨ Here, A(2, 3), B(0, 3) and O(0, 0) are the vertices of ΔAOB.

Now, combined transformation T∘R suggests that the first transformation through rotation R and second through translation T.

We know,

Object **Image**

$$P(x, y) \xrightarrow{\text{Rotation through } [(0,\ 0);\ 90°]} P'(-y, x)$$

$$\text{So, } A(2, 3) \xrightarrow{\text{Rotation through } [(0,\ 0);\ 90°]} A'(-3, 2)$$

$$B(0, 3) \xrightarrow{\text{Rotation through } [(0,\ 0);\ 90°]} B'(-3, 0)$$

$$O(0, 0) \xrightarrow{\text{Rotation through } [(0,\ 0);\ 90°]} O'(0, 0)$$

We have, translation (T) = $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (0, 3) - (2, 3)$

$\therefore$ T = (– 2, 0)

Now,

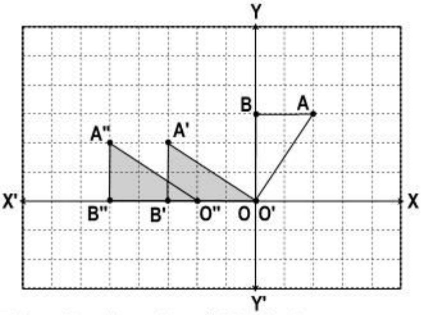
Object **Image**

$$P(x, y) \xrightarrow{\text{Translation by } \begin{pmatrix} a \\ b \end{pmatrix}} P'(x + a, y + b)$$

$$\text{So, } A'(-3, 2) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 0 \end{pmatrix}} A''(-5, 2)$$

$$B'(-3, 0) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 0 \end{pmatrix}} B''(-5, 0)$$

$$O'(0, 0) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 0 \end{pmatrix}} O''(-2, 0)$$



ΔAOB and its both images are shown in the graph.

Thus, the images under T∘R are A'(– 3, 2), B'(– 3, 0), O'(0, 0), A"(– 5, 2), B"(– 5, 0) and O"(– 2, 0).

MODEL 6

23. चतुर्भुज ABCD का शीर्षबिन्दुका निर्देशाङ्कहरू A(1, 1), B(2, 3), C(4, 2) र D(3, – 2) छन् । उक्त चतुर्भुजलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गर्नुहोस् । प्रतिबिम्ब चतुर्भुजलाई $y = -x$ मा परावर्तन गर्नुहोस् । माथिका दुई स्थानान्तरणको संयुक्त स्थानान्तरणलाई जनाउने स्थानान्तरण लेख्नुहोस् ।

The coordinates of vertices of a quadrilateral ABCD are A(1, 1), B(2, 3), C(4, 2) and D(3, – 2). Rotate this quadrilateral about origin through 180°. Reflect this image of quadrilateral about $y = -x$. Write the name of transformation which denotes the combined transformation of above two transformations. **[SEE MODEL 2076]**

⇨ Here, A(1, 1), B(2, 3), C(4, 2) and D(3, – 2) are the vertices of quadrilateral ABCD.

Now, rotating four points about the origin through 180° by formula, we get,

Object Image

$$P(x, y) \xrightarrow{\text{Rotation through } [180°;\ (0,\ 0)]} P'(-x, -y)$$

$$A(1, 1) \xrightarrow{\text{Rotation through } [180°;\ (0,\ 0)]} A'(-1, -1)$$

$$B(2, 3) \xrightarrow{\text{Rotation through } [180°;\ (0,\ 0)]} B'(-2, -3)$$

$$C(4, 2) \xrightarrow{\text{Rotation through } [180°;\ (0,\ 0)]} C'(-4, -2)$$

$$D(3, -2) \xrightarrow{\text{Rotation through } [180°;\ (0,\ 0)]} D'(-3, 2)$$

Again, we know,

Object **Image**

$$P(x, y) \xrightarrow{\text{Reflection in } y = -x} P'(-y, -x)$$

$$A'(-1, -1) \xrightarrow{\text{Reflection in } y = x} A''(1, 1)$$

$$B'(-2, -3) \xrightarrow{\text{Reflection in } y = x} B''(3, 2)$$

$$C'(-4, -2) \xrightarrow{\text{Reflection in } y = x} C''(2, 4)$$

$$D'(-3, 2) \xrightarrow{\text{Reflection in } y = x} D''(-2, 3)$$

Thus, the combined transformation is reflection about the line $y = x$.

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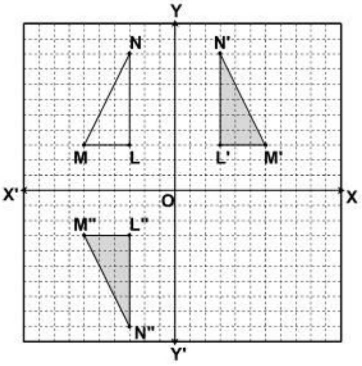
24. शीर्षबिन्दुहरू क्रमशः L(– 3, 3), M(– 6, 3) र N(– 3, 9) भएको त्रिभुज LMN लाई y - अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरी 180° मा परीक्रमण गरिएको छ। यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी त्रिभुज LMN र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस्।

A triangle LMN with vertices L(–3, 3), M(– 6, 3) and N(– 3, 9) respectively is reflected in y-axis and then image formed is rotated about the origin through 180°. Write the co-ordinates of the vertices of the images thus obtained and present the triangle LMN and its images in the same graph paper. [SEE 2075 R'$_2$]

⇨ Here, the coordinates of vertices of ΔLMN are L (– 3, 3), M (– 6, 3) & N (– 3, 9).

We know that,



Object Image

$$P(x, y) \xrightarrow{\textit{Reflection in y-axis}} P"(-x, y)$$

$$\text{So, } L(-3, 3) \xrightarrow{\textit{Reflection in y-axis}} L'(3, 3)$$

$$M(-6, 3) \xrightarrow{\textit{Reflection in y-axis}} M'(6, 3)$$

$$N(-3, 9) \xrightarrow{\textit{Reflection in y-axis}} N'(3, 9)$$

Again, we know,

Object Image

$$P(x, y) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} P'(-x, -y)$$

$$\text{So, } L'(3, 3) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} L"(-3, -3)$$

$$M'(6, 3) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} M"(-6, -3)$$

$$N'(3, 9) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} N"(-3, -9)$$

ΔLMN and its both images are shown in the graph.

Thus, the final coordinates of ΔLMN are L" (– 3, – 3), M" (– 6, – 3) and N" (– 3, – 9).

25. शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, 4) भएको ΔABC लाई x - अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गरिएको छ। यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस्।

A ΔABC with vertices A(2, 3), B(4, 5) and C(1, 4) is reflected in x-axis and then image so formed is rotated about the origin through 180°. Write the coordinates of the vertices of the images thus obtained and present the ΔABC and its images in the same graph paper. [2074 R']

⇨ Here, given vertices of ΔABC are; A(2, 3), B(4, 5) and C(1, 4)

ΔABC is reflected in x - axis:

We have,

Object **Image**

$$(x, y) \xrightarrow{\textit{Reflection (x-axis)}} (x, -y)$$

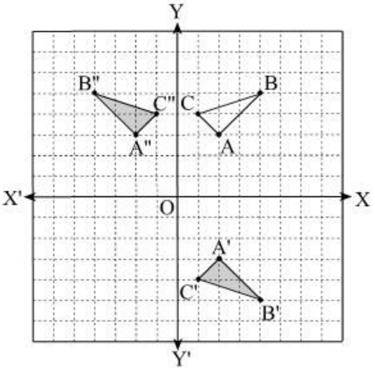
$$A(2, 3) \xrightarrow{\textit{Reflection (x-axis)}} A'(2, -3)$$

$$B(4, 5) \xrightarrow{\textit{Reflection (x-axis)}} B'(4, -5)$$

$$C(1, 4) \xrightarrow{\textit{Reflection (x-axis)}} C'(1, -4)$$

Again ΔA'B'C' is rotated through 180° about the origin.

So,



Object Image

$$P(x, y) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} P'(-x, -y)$$

$$A'(2, -3) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} A"(-2, 3)$$

$$B'(4, -5) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} B"(-4, 5)$$

$$C'(1, -4) \xrightarrow{\textit{Rotation through [180°; (0, 0)]}} C"(-1, 4)$$

Now, both the images of ΔABC are shown in the graph alongside,

Thus, the co-ordinates of images are; A'(2, – 3), B'(4, – 5), C'(1, – 4) and A"(– 2, 3), B"(– 4, 5), C"(– 1, 4).

26. शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) भएको ΔABC लाई लेखाचित्रमा खिची यसलाई उद्गमबिन्दु O बाट – 90° परिक्रमण गरी सोही लेखाचित्रमा ΔA'B'C' खिच्नुहोस्। त्यसपछि ΔA'B'C' लाई रेखा x – y = 0 मा परावर्तन गरी त्यसै लेखाचित्रमा ΔA'B'C' खिच्नुहोस्। ΔA"B"C" को शीर्षबिन्दुहरूको निर्देशाङ्क लेख्नुहोस्।

Draw ΔABC having the vertices A(2, 0), B(3, 1) and C(1, 1) on a graph paper. It is rotated about the origin O through – 90° and present ΔA'B'C' on the same graph paper. Then the ΔA'B'C' is reflected on the line x – y = 0 and plot the ΔA"B"C" on the same graph paper. Write down the co-ordinates of vertices of the ΔA"B"C". [2062 K]

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⇨ Here, A(2, 0), B(3, 1) and C(1, 1)

We have,

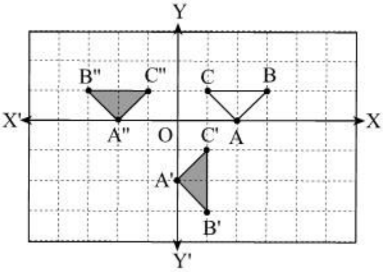
Object **Image**

$P(x, y) \xrightarrow{\text{Rotation through } [-90^\circ\ ;\ (0,\ 0)]} P'(y, -x)$

$A(2, 0) \xrightarrow{\text{Rotation through } [-90^\circ\ ;\ (0,\ 0)]} A'(0, -2)$

$B(3, 1) \xrightarrow{\text{Rotation through } [-90^\circ\ ;\ (0,\ 0)]} B'(1, -3)$

$C(1, 1) \xrightarrow{\text{Rotation through } [-90^\circ\ ;\ (0,\ 0)]} C'(1, -1)$



Again,

Object Image

$(x, y) \xrightarrow{\text{Reflection } [x - y = 0]} (y, x)$

$A'(0, -2) \xrightarrow{\text{Reflection } [x - y = 0]} A''(-2, 0)$

$B'(1, -3) \xrightarrow{\text{Reflection } [x - y = 0]} B''(-3, 1)$

$C'(1, -1) \xrightarrow{\text{Reflection } [x - y = 0]} C''(-1, 1)$

The graphical representation of the transformation is shown above.

Thus, the required vertices of the image triangle are A"(– 2, 0), B"(– 3, 1) and C"(– 1, 1).

27. शीर्षबिन्दुहरू A(2, 5), B(– 1, 3) र C(4, 1) भएको त्रिभुजलाई उद्गमबिन्दुको वरिपरि + 90° परिक्रमण गराउँदा प्राप्त प्रतिबिम्बलाई रेखा x = 0 मा परावर्तन गराइन्छ । यसरी प्राप्त प्रतिबिम्बका शीर्षबिन्दुका निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुवै प्रतिबिम्बीत त्रिभुजहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् र उक्त दुवै स्थानान्तरणलाई जनाउने एकल स्थानान्तरण पनि लेख्नुहोस् ।

A triangle having vertices A(2, 5), B(– 1, 3) and C(4, 1) is rotated about origin through + 90°. The image so obtained is reflected on the line x = 0. Find the vertices of image triangles. Show all the triangles in the same graph paper and also write the single transformation to represent these two transformations. [2067 S]

⇨ Here, A(2, 5), B(– 1, 3) and C(4, 1)

We know that,

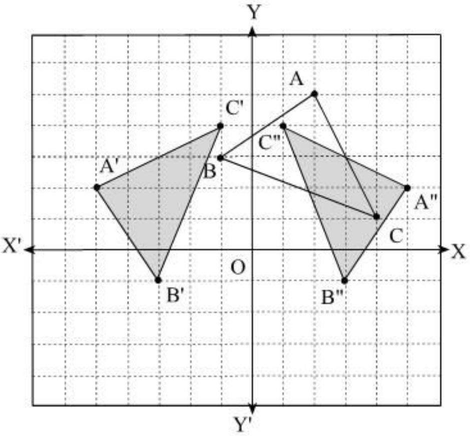
Object **Image**

$P(x, y) \xrightarrow{\text{Rotation through } [90^\circ\ ;\ (0,\ 0)]} P'(-y, x)$

$A(2, 5) \xrightarrow{\text{Rotation through } [90^\circ\ ;\ (0,\ 0)]} A'(-5, 2)$

$B(-1, 3) \xrightarrow{\text{Rotation through } [90^\circ\ ;\ (0,\ 0)]} B'(-3, -1)$

$C(4, 1) \xrightarrow{\text{Rotation through } [90^\circ\ ;\ (0,\ 0)]} C'(-1, 4)$



Again,

We have,

Object Image

$(x, y) \xrightarrow{\text{Reflection in } x = 0} (-x, y)$

So, $A'(-5, 2) \xrightarrow{\text{Reflection in } x = 0} A''(5, 2)$

$B'(-3, -1) \xrightarrow{\text{Reflection in } x = 0} B''(3, -1)$

$C'(-1, 4) \xrightarrow{\text{Reflection in } x = 0} C''(1, 4)$

Now both the triangles on same graph is shown alongside.

Thus, the single transformation to represent these two transformations is a reflection in the line x = y.

MODEL 7

28. शीर्षबिन्दुहरू A(2, 3), B(2, 6) र C(3, 4) भएको त्रिभुज ABC लाई T = $\begin{bmatrix}-2\\3\end{bmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका दुवै प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle ABC with vertices A(2, 3), B(2, 6) and C(3, 4) is translated by T = $\begin{bmatrix}-2\\3\end{bmatrix}$ and the image so obtained is enlarged by E[(0, 0), 2]. Write the co-ordinates of the vertices of the images so formed and represent ΔABC and its both images in the same graph. [SEE 2075 R]

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⇨ Here, A(2, 3), B(2, 6) and C(3, 4) are the vertices of ΔABC.
We know that,

Object **Image**

$$P(x, y) \xrightarrow{\text{Translation by } \begin{pmatrix} a \\ b \end{pmatrix}} P'(x + a, y + b)$$

$$A(2, 3) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} A'(0, 6)$$

$$B(2, 6) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} B'(0, 9)$$

$$C(3, 4) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} C'(1, 7) \quad -$$

Again, we know,

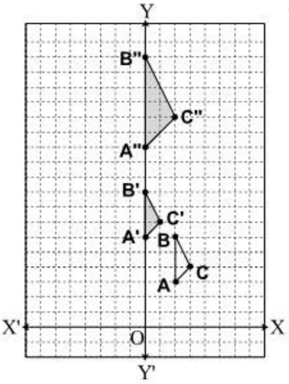
Object **Image**

$$P(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); k]} P'(kx, ky)$$

$$A'(0, 6) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} A''(0, 12)$$

$$B'(0, 9) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} B''(0, 18)$$

$$C'(1, 7) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} C''(2, 14)$$



The triangle ABC and its both images are shown in graph.
Thus, the final images of vertices of ΔABC are A''(0, 12), B''(0, 18), C''(2, 14).

29. P(3, 4), Q(2, 1) र R(4, 2) शीर्षबिन्दुहरू भएको ΔPQR लाई T = $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

ΔPQR having the vertices P(3, 4), Q(2, 1) and R(4, 2) is translated by T = $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ The image so formed is enlarged by E[(0, 0), 2]. Writing the co-ordinates of the vertices of images thus obtained, represent the ΔPQR and its images in the same graph paper. **[2074 R]**

⇨ Here, given vertices of ΔPQR are, P(3, 4), Q(2, 1) and R(4, 2). It is translated first.
So, we have

Object **Image**

$$(x, y) \xrightarrow{\text{Translation by } \begin{pmatrix} a \\ b \end{pmatrix}} (x + a, y + b)$$

$$P(3, 4) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} P'(1, 7)$$

$$Q(2, 1) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} Q'(0, 4)$$

$$R(4, 2) \xrightarrow{\text{Translation by } \begin{pmatrix} -2 \\ 3 \end{pmatrix}} R'(2, 5)$$

Again, the image is enlarged by E[(0,0), 2].
So, we have,

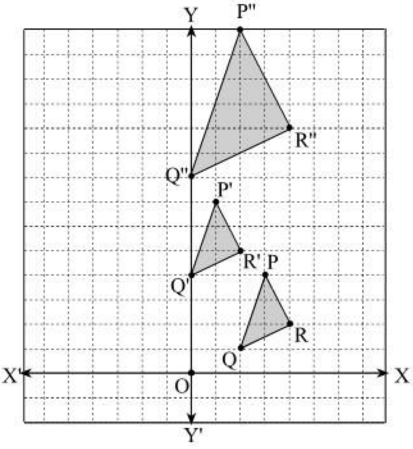
Object Image

$$A(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); k]} A'(kx, ky)$$

$$P'(1, 7) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} P''(2, 14)$$

$$Q'(0, 4) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} Q''(0, 8)$$

$$R'(2, 5) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} R''(4, 10)$$



Now, the graph of ΔPQR, ΔP'Q'R', ΔP''Q''R'' are shown in the graph alongside.
Thus, the co-ordinates of image are: P'(1, 7), Q'(0, 4), R'(2, 5) and P''(2, 14), Q''(0, 8), R''(4, 10).

30. शीर्षबिन्दुहरू A(3, 2), B(– 1, 0) र C(4, – 1) भएको त्रिभुज ABC लाई भेक्टर $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ले विस्थापन गरी बन्ने प्रतिबिम्बलाई केन्द्रबिन्दु (0, 0) र नापो 3 को आधारमा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

The image of the triangle ABC with vertices A(3, 2), B(– 1, 0) and C(4, – 1) formed by translating it by the vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$is enlarged on the basis of the centre (0, 0) and scale factor 3. Write the coordinates of the vertices of the images thus obtained and present ΔABC and its images on the same graph paper. **[2074 S]**

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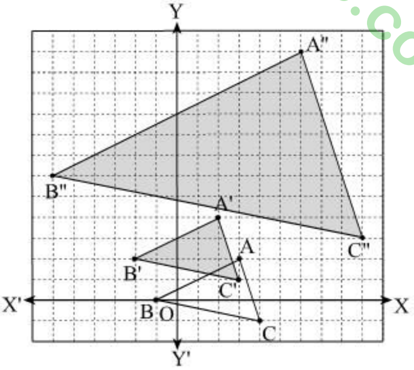
⇨ Here, A(3, 2), B(– 1, 0) and C(4, – 1).

We have,



Object **Image**

$$P(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix}} P'(x + a, y + b)$$

$$A(3, 2) \xrightarrow{\text{Translation by} \begin{pmatrix} -1 \\ 2 \end{pmatrix}} A'(2, 4)$$

$$B(-1, 0) \xrightarrow{\text{Translation by} \begin{pmatrix} -1 \\ 2 \end{pmatrix}} B'(-2, 2)$$

$$C(4, -1) \xrightarrow{\text{Translation by} \begin{pmatrix} -1 \\ 2 \end{pmatrix}} C'(3, 1)$$

Again,

Object Image

$$P(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); k]} P'(kx, ky)$$

$$A'(2, 4) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} A''(6, 12)$$

$$B'(-2, 2) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} B''(-6, 6)$$

$$C'(3, 1) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} C''(9, 3)$$

Now, the graph of both the images are shown above.

Thus, the coordinates of images are:

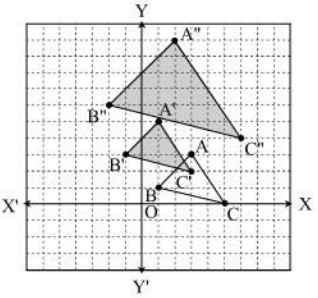
A'(2, 4), B'(– 2, 2), C'(3, 1), A''(6, 12), B''(– 6, 6) and C''(9, 3).

31. शीर्षबिन्दुहरू A(3, 3), B(1, 1) र C(5, 0) भएको त्रिभुज ABC लाई भेक्टर $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई केन्द्रबिन्दु (0, 0) र नापो 2 को आधारमा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle ABC with vertices A(3, 3), B(1, 1) and C(5, 0) is translated by a vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ The image so formed is enlarged with the centre (0, 0) and scale factor 2. Write the coordinates of the vertices of the images thus obtained and present ΔABC and its images on the same graph paper. **[2074 S']**

⇨ Here, A(3, 3), B(1, 1) and C(5, 0); TV = $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and E[(0, 0), 2].

We know that,



Object **Image**

$$(x, y) \xrightarrow{\text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix}} (x + a, y + b)$$

$$A(3, 3) \xrightarrow{\text{Translation by} \begin{pmatrix} -2 \\ 2 \end{pmatrix}} A'(1, 5)$$

$$B(1, 1) \xrightarrow{\text{Translation by} \begin{pmatrix} -2 \\ 2 \end{pmatrix}} B'(-1, 3)$$

$$C(5, 0) \xrightarrow{\text{Translation by} \begin{pmatrix} -2 \\ 2 \end{pmatrix}} C'(3, 2)$$

Again,

Object Image

$$P(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); k]} P'(kx, ky)$$

$$A'(1, 5) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} A''(2, 10)$$

$$B'(-1, 3) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} B''(-2, 6)$$

$$C'(3, 2) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} C''(6, 4)$$

Now, the graph is given alongside;

Thus, the co-ordinates of images are as follows:

A'(1, 5), B'(– 1, 3), C'(3, 2) and A''(2, 10), B''(– 2, 6), C''(6, 4).

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32. ΔABC का शीर्ष बिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) छन् । ΔABC लाई भेक्टर $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] द्वारा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्ष बिन्दुहरूका निर्देशाङ्कहरू लेखेर वस्तु र प्रतिबिम्बहरूलाई एउटै लेखचित्रमा प्रस्तुत गर्नुहोस् ।

The vertices of ΔABC are A(2, 0), B(3, 1) and C(1, 1). ΔABC is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ The image so formed is enlarged by E[(0, 0), 2]. Write the coordinates of the vertices of images thus obtained and represent the object and the images on the same graph paper. **[2072 R]**

⇨ Here, given vertices of ΔABC are; A(2, 0), B(3, 1) and C(1, 1).

We know that,

Object Image

$$(x, y) \xrightarrow{\textit{Translation by} \begin{pmatrix} 2 \\ -3 \end{pmatrix}} (x + 2, y - 3)$$

$$A(2, 0) \xrightarrow{\textit{Translation by} \begin{pmatrix} 2 \\ -3 \end{pmatrix}} A'(4, -3)$$

$$B(3, 1) \xrightarrow{\textit{Translation by} \begin{pmatrix} 2 \\ -3 \end{pmatrix}} B'(5, -2)$$

$$C(1, 1) \xrightarrow{\textit{Translation by} \begin{pmatrix} 2 \\ -3 \end{pmatrix}} C'(3, -2)$$

Again,

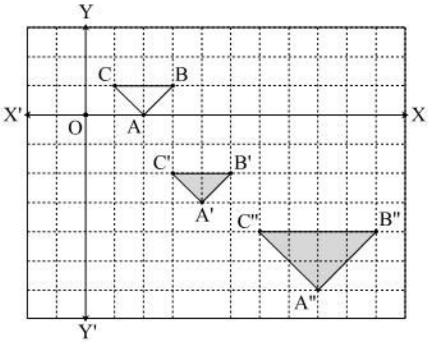
Object Image

$$P(x, y) \xrightarrow{\textit{Enlarged by } E[(0, 0); 2]} P'(kx, ky)$$

$$A'(4, -3) \xrightarrow{\textit{Enlarged by } E[(0, 0); 2]} A''(8, -6)$$

$$B'(5, -2) \xrightarrow{\textit{Enlarged by } E[(0, 0); 2]} B''(10, -4)$$

$$C'(3, -2) \xrightarrow{\textit{Enlarged by } E[(0, 0); 2]} C''(6, -4)$$



Now showing the object and image in the same graph.

Thus, the required images are: A'(4, – 3), B'(5, – 2) and C'(3, – 2), A''(8, – 6), B''(10, – 4) and C''(6, – 4).

MODEL 8

33. E ले केन्द्रबिन्दु (3, 1) र नापो 2 भएको विस्तारीकरण र R ले $y = x$ मा हुने परावर्तन जनाउँदछन् । शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, – 2) भएको ΔABC को संयुक्त स्थानान्तरण E∘R द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै लेखाचित्रमा खिच्नुहोस् ।

E denotes the enlargement about the centre (3, 1) with a scale factor of 2 and R denotes a reflection on the line $y = x$. Find the image of ΔABC having the vertices A(2, 3), B(4, 5) and C(1, – 2) under the combined transformation E∘R. Draw both figures on the same graph paper. **[SEE 2075 R_2]**

⇨ Here, given vertices of ΔABC are: A(2, 3), B(4, 5) and C(1, – 2).

We know that,

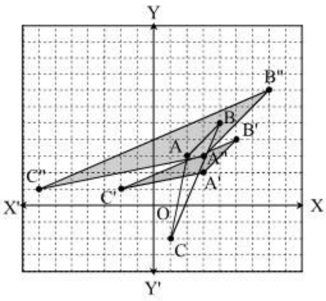
Object Image

$$P(x, y) \xrightarrow{\textit{Reflection in } y = x} P'(y, x)$$

$$\text{So, } A(2, 3) \xrightarrow{\textit{Reflection in } y = x} A'(3, 2)$$

$$B(4, 5) \xrightarrow{\textit{Reflection in } y = x} B'(5, 4)$$

$$C(1, -2) \xrightarrow{\textit{Reflection in } y = x} C'(-2, 1)$$



Again we know,

Object Image

$$P(x, y) \xrightarrow{\textit{Enlarged by } E[(a, b), k]} P'[k(x - a) + a, k(y - b) + b]$$

$$\text{So, } A'(3, 2) \xrightarrow{\textit{Enlarged by } E[(3, 1), 2]} A''([2(3 - 3) + 3, 2(2 - 1) + 1] = A''(3, 3)$$

$$B'(5, 4) \xrightarrow{\textit{Enlarged by } E[(3, 1), 2]} B''[2(5 - 3) + 3, 2(4 - 1) + 1] = B''(7, 7)$$

$$C'(-2, 1) \xrightarrow{\textit{Enlarged by } E[(3, 1), 2]} C''[2(-2 - 3) + 3, 2(1 - 1) + 1] = C''(-7, 1)$$

ΔABC and its both images are drawn on the graph paper.

Thus, the vertices of final image of ΔABC are A''(3, 3), B''(7, 7) and C''(– 7, 1).

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34. **शीर्षबिन्दुहरू P(2, 1), Q(5, 3) र R(7, – 1) भएको त्रिभुज PQR लाई Y-अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] द्वारा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

Triangle PQR having vertices P(2, 1), Q(5, 3) and R(7, – 1) is reflected on the y-axis. the image so formed is enlarged by E[(0, 0), 2]. Write the co-ordinates of the vertices of the images thus obtained and present the ΔPQR and its images in the same graph paper. **[2073 R]**

⇨ Here, P(2, 1), Q(5, 3) and R(7, – 1)

We know that,

Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\textit{Reflection in y-axis}} (-x, y)$$

$$P(2, 1) \xrightarrow{\textit{Reflection in y-axis}} P'(-2, 1)$$

$$Q(5, 3) \xrightarrow{\textit{Reflection in y-axis}} Q'(-5, 3)$$

$$R(7, -1) \xrightarrow{\textit{Reflection in y-axis}} R'(-7, -1)$$

Again,

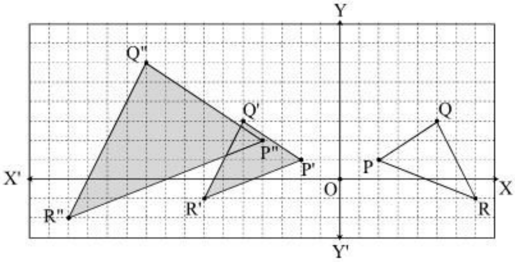
Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} (2x, 2y)$$

$$P'(-2, 1) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} P''(-4, 2)$$

$$Q'(-5, 3) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} Q''(-10, 6)$$

$$R'(-7, -1) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} R''(-14, -2)$$



Thus, co-ordinates of images are; P'(– 2, 1), Q'(– 5, 3), R'(– 7, – 1) and P"(– 4, 2), Q"(– 10, 6), R"(– 14, – 2).

35. **ΔPQR को शीर्षबिन्दुहरू P(1, 2), Q(2, 1) र R(4, 3) छन् । ΔPQR लाई y = x मा परावर्तन गरेपछि E[(0, 0), 2] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । ΔPQR र यसको प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

P(1, 2), Q(2, 1) and R(4, 3) are the vertices of the triangle PQR. Find the coordinates of the images of the triangle PQR under the reflection on the line y = x followed by the enlargement E[(0, 0), 2]. Present the ΔPQR and its images on the same graph. **[2072 S]**

⇨ Here, the vertices of ΔPQR are: P(1, 2), Q(2, 1) and R(4, 3).

We know that,

Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\textit{Reflection in } y = x} (y, x)$$

$$P(1, 2) \xrightarrow{\textit{Reflection in } y = x} P'(2, 1)$$

$$Q(2, 1) \xrightarrow{\textit{Reflection in } y = x} Q'(1, 2)$$

$$R(4, 3) \xrightarrow{\textit{Reflection in } y = x} R'(3, 4)$$

Again we have,

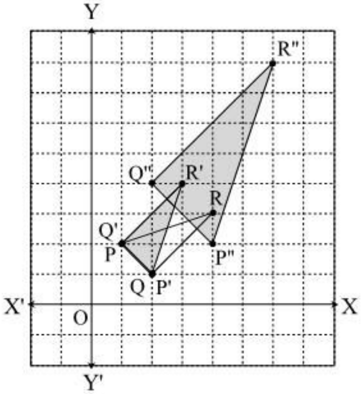
Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} (2x, 2y)$$

$$P'(2, 1) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} P''(4, 2)$$

$$Q'(1, 2) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} Q''(2, 4)$$

$$R'(3, 4) \xrightarrow{\textit{Enlarged by E[(0, 0); 2]}} R''(6, 8)$$



Thus, the coordinates of images are:
P'(2, 1), Q'(1, 2), R'(3, 4),
P"(4, 2), Q"(2, 4) and R"(6, 8).

36. **r_1 ले केन्द्रबिन्दु (– 3, – 4) बाट हुने नापो 3 भएको विस्तारीकरण जनाउँछ र r_2 ले y = – x मा परावर्तन हुने स्थानान्तरण जनाउँछ । यदि शीर्षबिन्दुहरू A(– 4, 6), B(– 6, –10) र C(12, – 8) भएको ΔABC लाई $r_1 o r_2$ द्वारा स्थानान्तरण गर्दा ΔABC को प्रतिबिम्ब ΔA'B'C' भए A', B' र C' को निर्देशाङ्क पत्ता लगाउनुहोस् ।**

r_1 denotes enlargement about the centre (– 3, – 4) with a scale factor of 3 and r_2 denotes a reflection in the line y = – x. ΔABC having the vertices A(– 4, 6), B(– 6, – 10) and C(12, – 8) is mapped onto the ΔA'B'C' under the transformation $r_1 o r_2$. Find the co-ordinates of A', B' and C'. **[2063 R']**

⇨ Here, r_1 = Enlargement about the centre (– 3, – 4) with scale factor 3.

r_2 = Reflection in the line y = – x and A(– 4, 6), B(– 6, – 10) and C(12, – 8).

Since $r_1 o r_2$ represents first reflection then enlargement so, we know that,

Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\textit{Reflection in } y = -x} (-y, -x)$$

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And also,

Object Image

$P(x, y) \xrightarrow{\text{Enlarged by } E[(a, b); k]} P'[k(x - a) + a, k(y - b) + b]$

So, $(- y, - x) \xrightarrow{\text{Enlarged by } E[(-3, -4); 3]} [3(- y + 3) - 3, 3(- x + 4) - 4]$
$= (- 3y + 6, - 3x + 8)$

Thus, the co-ordinates of combined transformation $r_1 \circ r_2$ is $(- 3y + 6, - 3x + 8)$

Now,

Object **Image**

$A(- 4, 6) \xrightarrow{\text{Combined transformation } r_1 \circ r_2} A'[(- 3) \times 6 + 6, (- 3) \times (- 4) + 8] = (- 12, 20)$

$B(- 6, - 10) \xrightarrow{\text{Combined transformation } r_1 \circ r_2} B'[(- 3) \times (- 10) + 6, (- 3) \times (- 6) + 8] = (36, 26)$

$C(12, - 8) \xrightarrow{\text{Combined transformation } r_1 \circ r_2} C'[(- 3) \times (- 8) + 6, (- 3) \times 12 + 8] = (30, - 28)$

Thus, the co-ordinates of A', B' and C' are (– 12, 20), (36, 26) and (30, – 28) respectively.

37. E ले केन्द्रबिन्दु (– 3, – 4) बाट हुने नापो 2 भएको विस्तारीकरण र R ले y = 0 मा परावर्तन हुने जनाउँछ । यदि शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) भएको ΔABC लाई संयुक्त स्थानान्तरण E∘R द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै ग्राफमा खिच्नुहोस् ।

E denotes enlargement about the centre (– 3, – 4) with a scale factor of 2 and R denotes a reflection on the line y = 0. ΔABC having the vertices A(2, 0), B(3, 1) and C(1, 1) is mapped under the combined transformation E∘R. Find the image of ΔABC and draw two figure on the same graph paper. [2065 R']

⇨ Here, E = Enlargement [(– 3, – 4); 2] and R = Reflection in y = 0

Given vertices are; A(2, 0), B(3, 1) and C(1, 1)

E∘R represents first reflection then the enlargement

So, Object Image

$(x, y) \xrightarrow{\text{Reflection in } y = 0} (x, - y)$

$A(2, 0) \xrightarrow{\text{Reflection in } y = 0} A'(2, 0)$

$B(3, 1) \xrightarrow{\text{Reflection in } y = 0} B'(3, - 1)$

$C(1, 1) \xrightarrow{\text{Reflection in } y = 0} C'(1, - 1)$

Again,

Object Image

$(x, y) \xrightarrow{\text{Enlarged by } E[(a, b); k]} [k(x - a) + a, k(y - b) + b]$

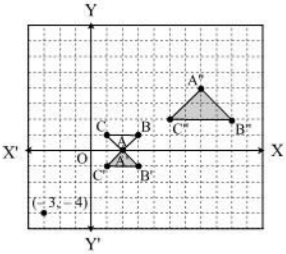
$\therefore (x, y) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} [2(x + 3) - 3, 2(y + 4) - 4]$
$= (2x + 3, 2y + 4)$

So, $A'(2, 0) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} A''(2 \times 2 + 3, 2 \times 0 + 4) = A''(7, 4)$

$B'(3, - 1) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} B''(2 \times 3 + 3, 2 \times (- 1) + 4 = B''(9, 2)$

$C'(1, - 1) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} C''(2 \times 1 + 3, 2 \times (- 1) + 4 = C''(5, 2)$

Now, the two figures on the same graph is shown alongside:



Thus, the co-ordinates of final images are; A"(7, 4), B"(9, 2) and C"(5, 2).

38. त्रिभुज ABC का शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) छन् । त्रिभुज ABC लाई रेखा x = y मा परावर्तन परेपछि E[(– 3, – 4), 2] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । त्रिभुज ABC र त्यसका प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A(2, 0), B(3, 1) and C(1, 1) are the vertices of a triangle ABC. Find the co-ordinates of the vertices of the triangle ABC under the reflection on the line x = y followed by the enlargement E[(– 3, – 4), 2]. Present the triangle ABC and its images on the same graph paper. [2067 R]

⇨ Here, A(2, 0), B(3, 1) & C(1, 1) are the given vertices of ΔABC.

Now,

Object Image

$P(x, y) \xrightarrow{\text{Reflection in } x = y} P'(y, x)$

$A(2, 0) \xrightarrow{\text{Reflection in } x = y} A'(0, 2)$

$B(3, 1) \xrightarrow{\text{Reflection in } x = y} B'(1, 3)$

$C(1, 1) \xrightarrow{\text{Reflection in } x = y} C'(1, 1)$

Again,

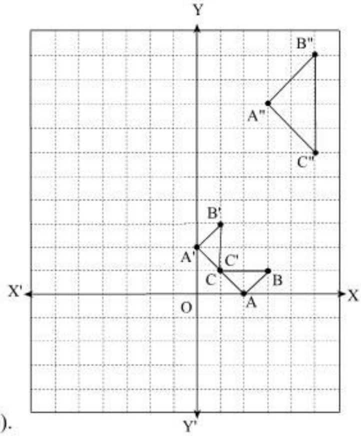
Object Image

$P(x, y) \xrightarrow{\text{Enlarged by } E[(a, b); k]} P'[k(x - a) + a, k(y - b) + b]$

$A'(0, 2) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} A''[2(0 + 3) - 3, 2(2 + 4) - 4] = A''(3, 8)$

$B'(1, 3) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} B''[2(1 + 3) - 3, 2(3 + 4) - 4] = B''(5, 10)$

$C'(1, 1) \xrightarrow{\text{Enlarged by } E[(-3, -4); 2]} C''[2(1 + 3) - 3, 2(1 + 4) - 4] = C''(5, 6)$



ΔABC and its both images are shown in graph.

Thus, the co-ordinates of final image are A"(3, 8), B"(5, 10) and C"(5, 6).

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39. त्रिभुज EFG का शीर्षबिन्दुहरू क्रमशः E(1, 0), F(2, 3) र G(1, 3) छन् । ΔEFG लाई रेखा x = y मा परावर्तन गरेपछि E[(0, 0), 3] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्कहरू लेख्नुहोस । वस्तु र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

The co-ordinates of the vertices of triangle EFG are E(1, 0), F(2, 3) and G(1, 3) respectively. Find the co-ordinates of vertices of images of ΔEFG under the reflection in the line x = y followed by enlargement E[(0, 0), 3]. Represent object and its images in same graph. **[2066 R']**

⇨ Here, E(1, 0), F(2, 3) and G(1, 3)

We know that,

$$\text{Object } (x, y) \xrightarrow{\textit{Reflection in the line } x = y} \text{Image } (y, x)$$

$$\text{So, } E(1, 0) \xrightarrow{\textit{Reflection in the line } x = y} E'(0, 1)$$

$$F(2, 3) \xrightarrow{\textit{Reflection in the line } x = y} F'(3, 2)$$

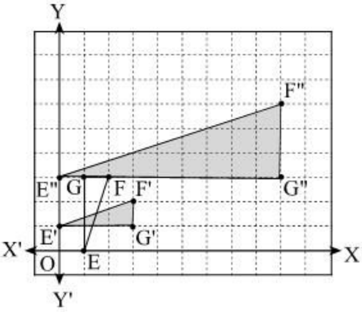
$$G(1, 3) \xrightarrow{\textit{Reflection in the line } x = y} G'(3, 1)$$

$$\text{Object } (x, y) \xrightarrow{\textit{Enlarged by } E[(0, 0); 3]} \text{Image } (3x, 3y)$$

$$\text{SO, } E'(0, 1) \xrightarrow{\textit{Enlarged by } E[(0, 0); 3]} E''(0, 3)$$

$$F'(3, 2) \xrightarrow{\textit{Enlarged by } E[(0, 0); 3]} F''(9, 6)$$

$$G'(3, 1) \xrightarrow{\textit{Enlarged by } E[(0, 0); 3]} G''(9, 3)$$



Now, the object and image on the same graph is shown alongside.

Thus, vertices of final image are E"(0, 3), F"(9, 6) and G"(9, 3).

MODEL 9

40. R₁ ले केन्द्रबिन्दु (3, 4) बाट हुने नापो 2 भएको विस्तारीकरण जनाउँछ र R₂ ले रेखा y = x मा परावर्तन हुने स्थानान्तरण जनाउँछ । यदि शीर्षबिन्दुहरू P(– 3, 5), Q(7, 4) र R(6, – 2) भएको ΔPQR लाई R₂oR₁ द्वारा स्थानान्तरण गर्दा ΔPQR को प्रतिबिम्ब ΔP₁Q₁R₁ भए P₁, Q₁ र R₁ को निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

R_1 denotes an enlargement about the center (3, 4) with a scale factor of 2 and R_2 denotes a reflection on the line y = x. ΔPQR with the vertices P(– 3, 5), Q(7, 4) and R(6, – 2) is mapped on to the $\Delta P_1Q_1R_1$ under the transformation $R_2 \circ R_1$. Find the coordinates of P_1, Q_1 and R_1. **[2064 R]**

⇨ Here, R_1 = Enlargement about (3, 4) with scale factor 2

R_2 = Reflection on the line y = x

We have,

$$\text{Object } (x, y) \xrightarrow{\textit{Enlarged by } E[(a, b); k]} \text{Image } [k(x-a)+a,\ k(y-b)+b]$$

$$\therefore\ (x, y) \xrightarrow{\textit{Enlarged by } E[(3, 4); 2]} [2(x-3)+3,\ 2(y-4)+4]$$
$$= (2x-3,\ 2y-4)$$

Again,

$$\text{Object } (x, y) \xrightarrow{\textit{Reflection in } y = x} \text{Image } (y, x)$$

$$\text{So, } (2x-3,\ 2y-4) \xrightarrow{\textit{Reflection in } y = x} (2y-4,\ 2x-3)$$

Now,

$$\text{Object } (x, y) \xrightarrow{\textit{Combined transformation of } R_2 o R_1} \text{Image } (2y-4,\ 2x-3)$$

$$P(-3, 5) \xrightarrow{\textit{Combined transformation of } R_2 o R_1} P_1(6, -9)$$

$$Q(7, 4) \xrightarrow{\textit{Combined transformation of } R_2 o R_1} Q_1(4, 11)$$

$$R(6, -2) \xrightarrow{\textit{Combined transformation of } R_2 o R_1} R_1(-8, 9)$$

Thus, the co-ordinates of P_1, Q_1 and R_1 are (6, – 9), (4, 11) and (– 8, 9) respectively.

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41. **शीर्षबिन्दुहरू A(1, 0), B(0, 2) र C(2, 1) भएको ΔABC लाई लेखाचित्रमा खिच्नुहोस् । ΔABC लाई पहिले E(O, 2) द्वारा विस्तार गरी प्राप्त प्रतिबिम्बित ΔA'B'C' लाई Y-अक्षमा परावर्तन गर्नुहोस् र सबै त्रिभुजहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

Sketch the graph of ΔABC having vertices A(1, 0), B(0, 2) and C(2, 1) in a graph paper. Enlarge the ΔABC by E(O, 2) first and then reflect the image ΔA'B'C' in the Y-axis. Show all triangles on same graph. **[2064 S]**

⇨ Here, A(1, 0), B(0, 2) and C(2, 1)

We have,

Object Image

$(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} (2x, 2y)$

So, $A(1, 0) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} A'(2, 0)$

$B(0, 2) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} B'(0, 4)$

$C(2, 1) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} C'(4, 2)$

Again,

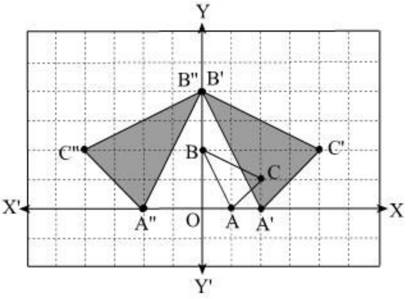
Object Image

$(x, y) \xrightarrow{\text{Reflection in y-axis}} (-x, y)$

$A'(2, 0) \xrightarrow{\text{Reflection in y-axis}} A''(-2, 0)$

$B'(0, 4) \xrightarrow{\text{Reflection in y-axis}} B''(0, 4)$

$C'(4, 2) \xrightarrow{\text{Reflection in y-axis}} C''(-4, 2)$



Now, the object and image are shown in the graph alongside.

Thus, the final images are A''(– 2, 0), B''(0, 4) and C''(– 4, 2).

MODEL 10

42. **एउटा समानान्तर चतुर्भुज** LOVE **का शीर्षबिन्दुहरू** L(2, 2), O(6, 2), V(7, 4) **र** E(3, 4) **छन् ।** ▱LOVE **लाई उद्गम बिन्दुको वरिपरि** 90° **ले धनात्मक दिशामा परिक्रमण गरी फेरि** E[(0, 0), 3] **मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै वस्तु प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

L(2, 2), O(6, 2), V(7, 4) and E(3, 4) are the vertices of a parallelogram LOVE. Find the co-ordinates of the vertices of the images of ▱LOVE under the rotation of positive 90° about origin followed by enlargement E[(0, 0), 3]. Represent the object and images on the same graph paper. **[2073 R']**

⇨ Here, L(2, 2), O(6, 2), V(7, 4), E(3, 4)

We know that,

Object Image

$(x, y) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} (-y, x)$

$L(2, 2) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} L'(-2, 2)$

$O(6, 2) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} O'(-2, 6)$

$V(7, 4) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} V'(-4, 7)$

$E(3, 4) \xrightarrow{\text{Rotation through } [90°; (0, 0)]} E'(-4, 3)$

Again,

Object Image

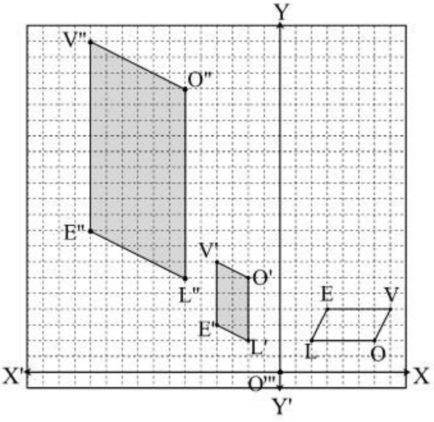
$(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} (3x, 3y)$

$L'(-2, 2) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} L''(-6, 6)$

$O'(-2, 6) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} O''(-6, 18)$

$V'(-4, 7) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} V''(-12, 21)$

$E'(-4, 3) \xrightarrow{\text{Enlarged by } E[(0, 0); 3]} E''(-12, 9)$



Thus, the co-ordinates of images are; L'(– 2, 2), O'(– 2, 6), V'(– 4, 7), E'(– 4, 3), L''(– 6, 6), O''(– 6, 18), V''(– 12, 21), E''(– 12, 9).

43. **त्रिभुज** CAT **का शीर्षबिन्दुहरू C(2, 5), A(– 1, 3) र T(4, 1) छन् । त्रिभुज CAT लाई उद्गम बिन्दुको वरिपरि 90° ले धनात्मक दिशामा परिक्रमण गरी फेरि E[(0, 0), 2] मा विस्तृतीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै वस्तु र प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

C(2, 5), A(– 1, 3) and T(4, 1) are the vertices of a triangle CAT. Find the coordinates of the vertices of the image of ΔCAT under the rotation of positive 90° about origin followed by enlargement E[(0, 0), 2]. Represent the object and images on the same graph paper. **[2072 R']**

⇨ Here, vertices of ΔCAT are; C(2, 5), A(– 1, 3) and T(4, 1)

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We know that,

Object **Image**

$$P(x, y) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} P'(-y, x)$$

$$C(2, 5) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} C'(-5, 2)$$

$$A(-1, 3) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} A'(-3, -1)$$

$$T(4, 1) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} T'(-1, 4)$$

We have,

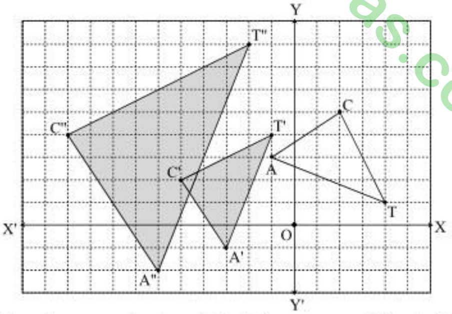
Object Image

$$P(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} P'(2x, 2y)$$

$$C'(-5, 2) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} C''(-10, 4)$$

$$A'(-3, -1) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} A''(-6, -2)$$

$$T'(-1, 4) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} T''(-2, 8)$$



Thus, the co-ordinates of final images are; C'(– 5, 2), A'(– 3, – 1), T'(–1, 4), C''(– 10, 4), A''(– 6, – 2) & T''(– 2, 8).

44. **त्रिभुज KLM को शीर्षबिन्दुहरू K(2, 5), L(–1, 3) र M(4, 1) छन् । ΔKLM लाई उद्गम बिन्दुको वरिपरि 90° ऋणात्मक परिक्रमण गरी फेरि E[(0, 0), 2] मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू निकाल्नुहोस् । साथै वस्तु र प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

K(2, 5), L(–1, 3) and M(4, 1) are the vertices of a triangle KLM. Find the co-ordinates of the vertices of the images of ΔKLM under the rotation of negative 90° about the origin followed by the enlargement E[(0, 0), 2]. Present object and its images on the same graph paper. **[2070 R]**

⇨ Here, vertices of ΔKLM are K(2, 5), L(– 1, 3) and M(4, 1).

We know that,

Object Image

$$(x, y) \xrightarrow{\text{Rotation through } [-90^\circ ; (0, 0)]} (y, -x)$$

$$K(2, 5) \xrightarrow{\text{Rotation through } [-90^\circ ; (0, 0)]} K'(5, -2)$$

$$L(-1, 3) \xrightarrow{\text{Rotation through } [-90^\circ ; (0, 0)]} L'(3, 1)$$

$$M(4, 1) \xrightarrow{\text{Rotation through } [-90^\circ ; (0, 0)]} M'(1, -4)$$

Again,

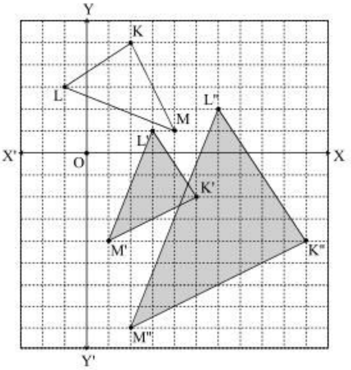
Object Image

$$(x, y) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} (2x, 2y)$$

$$K'(5, -2) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} K''(10, -4)$$

$$L'(3, 1) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} L''(6, 2)$$

$$M'(1, -4) \xrightarrow{\text{Enlarged by } E[(0, 0); 2]} M''(2, -8)$$



Thus, the coordinates of images are: K'(5, – 2), L'(3, 1), M(1, – 4), K''(10, – 4), L''(6, 2) and M''(2, – 8).

45. **त्रिभुज PQR का शीर्षबिन्दुहरू P(2, 0), Q(– 1, 3) र R(2, 4) छन् । त्रिभुज PQR लाई बिन्दु (0, 0) को वरिपरी + 90° परिक्रमण गरी फेरी केन्द्र (2, 1) र नापो – 2 को आधारमा विस्तारीकरण गर्दा बन्ने प्रतिबिम्ब पत्ता लगाउनुहोस् ।**

P(2, 0), Q(– 1, 3) and R(2, 4) are the vertices of Δ PQR. Find the image of ΔPQR under the rotation of + 90° about the point (0, 0) followed by the enlargement about the centre (2, 1) and scale factor –2. **[2067 R']**

⇨ Here, P(2, 0), Q(– 1, 3) & R(2, 4) are the vertices of triangle PQR.

Now,

Object **Image**

$$A(x, y) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} A'(-y, x)$$

$$P(2, 0) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} P'(0, 2)$$

$$Q(-1, 3) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} Q'(-3, -1)$$

$$R(2, 4) \xrightarrow{\text{Rotation through } [90^\circ ; (0, 0)]} R'(-4, 2)$$

Again,

Object Image

$$A(x, y) \xrightarrow{\text{Enlarged by } E[(a, b); k]} A'[k(x - a) + a, k(y - b) + b]$$

$$P'(0, 2) \xrightarrow{\text{Enlarged by } E[(2, 1); -2]} P''[-2(0 - 2) + 2, -2(2 - 1) + 1]$$
$$= P''(4 + 2, -2 + 1) = P''(6, -1)$$

$$Q'(-3, -1) \xrightarrow{\text{Enlarged by } E[(2, 1); -2]} Q''[-2(-3 - 2) + 2, -2(-1 - 1) + 1]$$
$$= Q''(-2 \times -5 + 2, -2 \times -2 + 1) = Q''(12, 5)$$

$$R'(-4, 2) \xrightarrow{\text{Enlarged by } E[(2, 1); -2]} R''[-2(-4 - 2) + 2, -2(2 - 1) + 1]$$
$$= R''(-2 \times (-6) + 2, -2 \times 1 + 1)$$
$$= R''(12 + 2, -2 + 1) = R''(14, -1)$$

Thus, required images of the given object are P'(0, 2), Q'(– 3, – 1), R'(– 4, 2), P''(6, – 1), Q''(12, 5) & R''(14, – 1).

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QUESTIONS FROM CDC TEXTBOOK

7.1 संयुक्त स्थानान्तरण (COMPOSITION OF TRANSFORMATION/COMBINED TRANSFORMATION)

EXERCISE 7.1

1. (a) रेखाहरू $x = 3$ र $y = 5$ मा हुने परावर्तनको संयुक्त स्थानान्तरणले कुन स्थानान्तरण दिन्छ ?
What is the transformation represented by reflection in x = 3 followed by y = 5 ?

⇨ Here, the single transformation is the halfturn about the point (3, 5).

(b) परिक्रमण $R_1[(0, 0), 80°]$ र R_2 [(0, 0), 100°] ले दिने संयुक्त स्थानान्तरण $R_1 \circ R_2$ के हुन्छ ?
What is the combined transformation, $R_1 \circ R_2$ given by the rotations $R_1[(0, 0), 80°]$ and R_2 [(0, 0), 100°]?

⇨ Here, the combined transformation is;

$$R_1 \circ R_2 = R_1 + R_2 = [(0, 0), 80°] + [(0, 0), 100°] = [(0, 0), 180°]$$

Thus, the combined transformation is [(0, 0); 180°].

(c) विस्तार $E_1[(a, b), k_1]$ र $E_2[(a, b), k_2]$ को संयुक्त स्थानान्तरण $E_1 \circ E_2$ के हुन्छ ?
What is the combined transformation $E_1 \circ E_2$, given by the enlargements $E_1[(a, b), k_1]$ and $E_2[(a, b), k_2]$

⇨ Here, the combined transformation is;

$$E_1 \circ E_2 = [(a, b), k_1] \circ [(a, b), k_2] = [(a, b), k_1 \times k_2]$$

Thus, the combined transformation is $E[(a, b), k_1 \times k_2]$.

(d) यदि विस्थापनहरू $T_1\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ र $T_2\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ भए $T_1 \circ T_2$ कति हुन्छ ?

If translations $T_1\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $T_2\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ then what is $T_1 \circ T_2$?

⇨ Here, $T_1 \circ T_2 = T_1 + T_2 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \end{pmatrix}$

Thus, the value of $T_1 \circ T_2$ is $\begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \end{pmatrix}$.

2. तल दिइएको तालिकामा स्थानान्तरणहरूको विवरण दिइएको छ । (In the following table, the transformations are given.)

R_1	x-अक्षमा परावर्तन (Reflection in x-axis)
R_2	y-अक्षमा परावर्तन (Reflection in y-axis)
R_3	$x = 3$ मा हुने परावर्तन (Reflection in $x = 3$)
R_4	$y = -2$ मा हुने परावर्तन (Reflection in $y = -2$)
r_1	उद्गम बिन्दु वरिपरि 90° मा हुने धनात्मक परिक्रमण (Rotation through 90° positive about the origin)
r_2	उद्गम बिन्दु वरिपरि 270° मा हुने धनात्मक परिक्रमण (Rotation through 270° positive about the origin)
r_3	उद्गम बिन्दु वरिपरि 180° मा हुने धनात्मक परिक्रमण (Rotation through 180° positive about the origin)
T_1	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ को विस्थापन (Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$)
T_2	$\begin{pmatrix} -4 \\ -5 \end{pmatrix}$ को विस्थापन (Translation of $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$)
E_1	केन्द्र (0, 0) र नापो – 2 भएको विस्तार (Enlargement of centre at (0, 0) and scale factor – 2)
E_2	केन्द्र (0, 0) र नापो $\frac{3}{2}$ भएको विस्तार (Enlargement of centre at (0, 0) and scale factor $\frac{3}{2}$)
E_3	केन्द्र (2, 3) र नापो 3 भएको विस्तार (Enlargement of centre at (2, 3) and scale factor 3)
E_4	केन्द्र (2, 3) र नापो $\frac{2}{3}$ भएको विस्तार (Enlargement of centre at (2, 3) and scale factor $\frac{2}{3}$)

तालिकामा दिइएको विवरणका आधारमा निम्न स्थानान्तरणहरू पत्ता लगाउनुहोस् ।
On the basis of given information, find the following transformations.

(a) $R_1 \circ R_2(4, 6)$

⇨ Here, $R_1 \circ R_2(4, 6) = R_1[R_2(4, 6)] = R_1(-4, 6)$

$$[\because (x, y) \xrightarrow{R_2 = \text{Reflection in y-axis}} (-x, y)]$$

$\therefore$ $R_1 \circ R_2(4, 6) = (-4, -6)$

$$[\because (x, y) \xrightarrow{R_1 = \text{Reflection in x-axis}} (x, -y)]$$

Thus, $R_1 \circ R_2(4, 6) = (-4, -6)$.

(b) $R_2 \circ R_1(4, 6)$

⇨ Here, $R_2 \circ R_1(4, 6) = R_2[R_1(4, 6)] = R_2(4, -6)$

$$[\because (x, y) \xrightarrow{R_1 = \text{Reflection in x-axis}} (x, -y)]$$

$\therefore$ $R_2 \circ R_1(4, 6) = (-4, -6)$

$$[\because (x, y) \xrightarrow{R_2 = \text{Reflection in y-axis}} (-x, y)]$$

Thus, $R_2 \circ R_1(4, 6) = (-4, -6)$.

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(c) $R_1\circ R_4(-2, 3)$

⇨ Here, $R_1\circ R_4(-2, 3) = R_1[R_4(-2, 3)]$

$= R_1[-2, 2\times(-2) - 3]$

$[\because (x, y) \xrightarrow{R_4 = \text{Reflection in } y = -2 = k} (x, 2k - y)]$

$= R_1(-2, -7)$

$= (-2, 7)$

$[\because (x, y) \xrightarrow{R_1 = \text{Reflection in x-axis}} (x, -y)]$

Thus, $R_1\circ R_4(-2, 3) = (-2, 7)$.

(e) $r_1\circ r_3(2, -3)$

⇨ Here, $r_1\circ r_3(2, -3)$

We have, $r_1\circ r_3$

$= r_1 + r_3$

$= 90°\ (+)\text{ve} + 180°\ (+)\text{ve}$

$= 270°\ (+)\text{ve}$

$= 90°\ (-)\text{ve}$

So, Object $\qquad$ Image

$(x, y) \xrightarrow{\text{Rotation through } [-90°; (0, 0)]} (y, -x)$

$\therefore (2, -3) \xrightarrow{\text{Rotation through } [-90°; (0, 0)]} (-3, -2)$

Thus, $r_1\circ r_3\ (2, -3)$ is $(-3, -2)$.

(g) $r_1\circ r_2(-3, 5)$

⇨ Here, $r_1\circ r_2(-3, 5)$

We have,

$r_1\circ r_2 = r_1 + r_2$

$= \text{Rotation of } [90°; (0, 0)] + \text{Rotation of } [270°; (0, 0)]$

$= \text{Rotation of } [(90° + 270°); (0, 0)]$

$= \text{Rotation of } [360°; (0, 0)]$

Now, Object $\qquad$ Image

$(x, y) \xrightarrow{r_1\circ r_2 = \text{Rotation of } [360°; (0, 0)]} (x, y)$

$(-3, 5) \xrightarrow{r_1\circ r_2 = \text{Rotation of } [360°; (0, 0)]} (-3, 5)$

Thus, $r_1\circ r_2\ (-3, 5)$ is $(-3, 5)$.

(d) $R_3\circ R_2(-3, -4)$

⇨ Here, $R_3\circ R_2(-3, -4) = R_3[R_2(-3, -4)]$

$= R_3(3, -4)$

$[\because (x, y) \xrightarrow{R_2 = \text{Reflection in y-axis}} (-x, y)]$

$= (2\times 3 - 3, -4)$

$[\because (x, y) \xrightarrow{R_3 = \text{Reflection in } x = 3 = h} (2h - x, y)]$

$= (3, -4)$

Thus, $R_3\circ R_2(-3, -4) = (3, -4)$.

(f) $r_2\circ r_3(2, 4)$

⇨ Here, $r_2\circ r_3(2, 4)$

We have, $r_2\circ r_3 = r_2 + r_3$

$= 270°$ rotation about origin + $180°$ rotation about origin

$= (270° + 180°)$ rotation about origin

$= 450°$ rotation about origin

$= (450° - 360°)$ rotation about origin

$\therefore r_2\circ r_3 = 90°$ rotation about the origin.

Now, Object $\qquad$ Image

$(x, y) \xrightarrow{r_2\circ r_3 = \text{Rotation of } [90°; (0, 0)]} (-y, x)$

$\therefore (2, 4) \xrightarrow{r_2\circ r_3 = \text{Rotation of } [90°; (0, 0)]} (-4, 2)$

Thus, $r_2\circ r_3\ (2, 4)$ is $(-4, 2)$.

(h) $T_1\circ T_2(3, 4)$

⇨ Here, $T_1\circ T_2(3, 4)$

We have,

$$T_1\circ T_2 = T_1 + T_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 2-4 \\ 3-5 \end{pmatrix}$$

$$\therefore T_1\circ T_2 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

Now, Object $\qquad$ Image

$(x, y) \xrightarrow{T_1\circ T_2 = \text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}} (x + a, y + b)$

$\therefore (3, 4) \xrightarrow{T_1\circ T_2 = \text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}} (3 - 2, 4 - 2) = (1, 2)$

Thus, $T_1\circ T_2(3, 4)$ is $(1, 2)$.

(i) $T_2\circ T_1(-4, -8)$

⇨ Here, $T_2\circ T_1(-4, -8)$

We have $T_2\circ T_1 = T_2 + T_1 = \begin{pmatrix} -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -4+2 \\ -5+3 \end{pmatrix}$ $\qquad \therefore T_2\circ T_1 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Now, Object $\qquad$ Image

$(x, y) \xrightarrow{T_2\circ T_1 = \text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}} (x + a, y + b)$

$\therefore (-4, -8) \xrightarrow{T_1\circ T_2 = \text{Translation by} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}} (-4 - 2, -8 - 2)$

$= (-6, -10)$

Thus, $T_2\circ T_1\ (-4, -8)$ is $(-6, -10)$

(j) $E_1\circ E_2(5, 6)$

⇨ Here, $E_1\circ E_2(5, 6)$

We have, $E_1\circ E_2 = E_1(E_2) = [(0, 0); -2]\ [(0, 0); \frac{3}{2}] = [(0, 0); -2\times\frac{3}{2}]$ $\qquad \therefore E_1\circ E_2 = [(0, 0); -3]$

Now, Object $\qquad$ Image

$(x, y) \xrightarrow{E_1\circ E_2 = \text{Enlarged by } [(0, 0); k] = [(0, 0); -3]} (kx, ky)$

$\therefore (5, 6) \xrightarrow{E_1\circ E_2 = \text{Enlarged by } [(0, 0); k] = [(0, 0); -3]} (-3\times 5, -3\times 6)$

$= (-15, -18)$

Thus, $E_1\circ E_2(5, 6)$ is $(-15, -18)$.

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(k) $E_2 \circ E_1(-2, 3)$

⇨ Here, $E_2 \circ E_1(-2, 3)$

We have, $E_2 \circ E_1 = E_2(E_1) = [(0, 0); \frac{3}{2}]\ [(0, 0); -2] = [(0, 0); \frac{3}{2} \times (-2)]$

$\therefore\ E_2 \circ E_1 = [(0, 0); -3]$

Now, Object $\qquad\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{E_2 \circ E_1 = \text{Enlarged by } [(0, 0); k] = [(0, 0); -3]} (kx, ky)$$

$$\therefore\ (-2, 3) \xrightarrow{E_2 \circ E_1 = \text{Enlarged by } [(0, 0); k] = [(0, 0); -3]} (-3 \times (-2), -3 \times 3) = (6, -9)$$

Thus, $E_2 \circ E_1(-2, 3)$ is $(6, -9)$.

(ℓ) $E_4 \circ E_3(-1, 5)$

⇨ Here, $E_4 \circ E_3(-1, 5)$

We have, $E_4 \circ E_3 = E_4(E_3) = [(2, 3); \frac{2}{3}]\ [(2, 3); 3] = [(2, 3); \frac{2}{3} \times 3]$

$\therefore\ E_4 \circ E_3 = [(2, 3); 2]$

Now, Object $\qquad\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{E_4 \circ E_3 = \text{Enlarged by } [(a, b); k] = [(2, 3); 2]} [k(x - a) + a, k(y - b) + b]$$

$$\therefore\ (-1, 5) \xrightarrow{E_4 \circ E_3 = \text{Enlarged by } [(a, b); k] = [(2, 3); 2]} [2(-1 - 2) + 2, 2(5 - 3) + 3] = (-6 + 2, 4 + 3) = (-4, 7)$$

Thus, $E_4 \circ E_3(-1, 5)$ is $(-4, 7)$.

3. (a) **शीर्षबिन्दुहरू A(2, – 1), B(2, 1) र C(4, – 1) भएको ΔABC लाई पहिले रेखा $y - x = 0$ मा परावर्तन गर्नुहोस् । प्राप्त प्रतिबिम्बलाई पुनः उद्गम बिन्दुको वरिपरि अर्धपरिक्रमण गराउनुहोस् । प्राप्त अन्तिम प्रतिबिम्ब ΔA"B"C" र ΔABC लाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

Reflect ΔABC having vertices A(2, – 1), B(2, 1) and C(4, – 1) in the line y – x = 0 first. Rotate the image so obtained through half turn about the origin. Represent the final image ΔA"B"C" and ΔABC in a same graph.

⇨ Here, A(2, – 1), B(2, 1) and C(4, – 1) are given vertices.

We have,

Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\text{Reflection in } y - x = 0} (y, x)$$

$$A(2, -1) \xrightarrow{\text{Reflection in } y - x = 0} A'(-1, 2)$$

$$B(2, 1) \xrightarrow{\text{Reflection in } y - x = 0} B'(1, 2)$$

$$C(4, -1) \xrightarrow{\text{Reflection in } y - x = 0} C'(-1, 4)$$

Again,

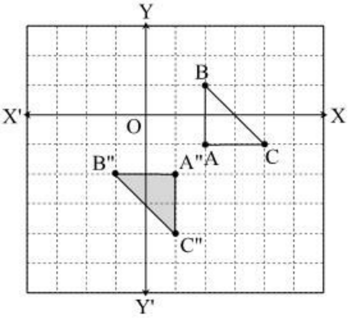
Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\text{Half turn rotation about origin}} (-x, -y)$$

$$A'(-1, 2) \xrightarrow{\text{Half turn rotation about origin}} A''(1, -2)$$

$$B'(1, 2) \xrightarrow{\text{Half turn rotation about origin}} B''(-1, -2)$$

$$C'(-1, 4) \xrightarrow{\text{Half turn rotation about origin}} C''(1, -4)$$



Now, the representation of ΔA"B"C" and ΔABC in the same graph.

(b) **शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको त्रिभुजलाई क्रमशः रेखा $x = -3$ र $y = 4$ मा परावर्तन गरिएको छ । उक्त संयुक्त स्थानान्तरणबाट प्राप्त प्रतिबिम्ब ΔA"B"C" र ΔABC लाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।**

A triangle having vertices A(1, 2), B(4, – 1) and C(2, 5) is successively reflected in the lines x = – 3 and y = 4. Represent the image ΔA"B"C" and ΔABC so obtained in the same graph.

⇨ Here, the vertices of ΔABC are; A(1, 2), B(4, – 1) and C(2, 5).

We have,

Object $\qquad\qquad\qquad$ Image

$$(x, y) \xrightarrow{\text{Reflection in } x = h = -3} (2h - x, y) = (-6 - x, y)$$

$$A(1, 2) \xrightarrow{\text{Reflection in } x = h = -3} A'(-7, 2)$$

$$B(4, -1) \xrightarrow{\text{Reflection in } x = h = -3} B'(-10, -1)$$

$$C(2, 5) \xrightarrow{\text{Reflection in } x = h = -3} C'(-8, 5)$$

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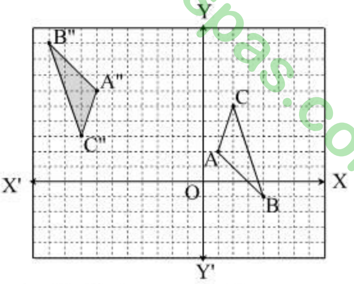
Again,

$$\text{Object } (x, y) \xrightarrow{\text{Reflection in } y = k = 4} \text{Image } (x, 2k - y) = (x, 8 - y)$$

$$A'(-7, 2) \xrightarrow{\text{Reflection in } y = k = 4} A''(-7, 6)$$

$$B'(-10, -1) \xrightarrow{\text{Reflection in } y = k = 4} B''(-10, 9)$$

$$C'(-8, 5) \xrightarrow{\text{Reflection in } y = k = 4} C''(-8, 3)$$



Now, the representation of ΔA"B"C" and ΔABC in the same graph.

(c) ΔMNP का शीर्षबिन्दुहरू M(1, 1), N(3, 1) र P(2, 3) को प्रतिबिम्ब उद्‌गम बिन्दुको वरिपरि 90° धनात्मक परिक्रमणअनुसार पत्ता लगाउनुहोस् । प्राप्त प्रतिबिम्बलाई पुनः (– 1, 2) केन्द्र र नापो 2 भएको विस्तारद्वारा विस्तारीकरण गर्नुहोस् । अन्तिम प्रतिबिम्ब ΔM"N"P" र वस्तु ΔMNP लाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Find the image of ΔMNP having vertices M(1, 1), N(3, 1) and P(2, 3) through 90° positive rotation about the origin. Enlarge the image so obtained by enlargement of centre at (– 1, 2) and scale factor 2. Represent the final image ΔM"N"P" and object ΔMNP in the same graph.

⇨ Here, the vertices of ΔMNP are; M(1, 1), N(3, 1) and P(2, 3).

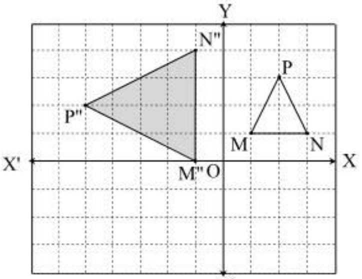
We have,

$$\text{Object } (x, y) \xrightarrow{\text{Rotation through } [90^\circ; (0, 0)]} \text{Image } (-y, x)$$

$$M(1, 1) \xrightarrow{\text{Rotation through } [90^\circ; (0, 0)]} M'(-1, 1)$$

$$N(3, 1) \xrightarrow{\text{Rotation through } [90^\circ; (0, 0)]} N'(-1, 3)$$

$$P(2, 3) \xrightarrow{\text{Rotation through } [90^\circ; (0, 0)]} P'(-3, 2)$$



Again,

$$\text{Object } (x, y) \xrightarrow{\text{Enlarged by } E[(a, b); k] = E[(-1, 2); 2]} \text{Image } [k(x - a) + a, k(y - b) + b]$$

$$= (2(x + 1) - 1, 2(y - 2) + 2)$$

$$= (2x + 2 - 1, 2y - 4 + 2)$$

$$= (2x + 1, 2y - 2)$$

$$\therefore \quad M'(-1, 1) \xrightarrow{\text{Enlarged by } E[(a, b); k] = E[(-1, 2); 2]} M''(2 \times (-1) + 1, 2 \times 1 - 2) = (-1, 0)$$

$$N'(-1, 3) \xrightarrow{\text{Enlarged by } E[(a, b); k] = E[(-1, 2); 2]} N''(2 \times (-1) + 1, 2 \times 3 - 2) = (-1, 4)$$

$$P'(-3, 2) \xrightarrow{\text{Enlarged by } E[(a, b); k] = E[(-1, 2); 2]} P''(2 \times (-3) + 1, 2 \times 2 - 2) = (-5, 2)$$

Now, the representation of ΔM"N"P" and ΔMNP in the same graph...

4. (a) शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको त्रिभुजलाई रेखाहरू r_1(x = 4) र r_2(x = – 1) मा लगातार परावर्तन गरिएको छ । दुवै स्थानान्तरणहरूले जनाउने संयुक्त स्थानानतरण $r_2 \circ r_1$ पत्ता लगाउनुहोस् । $r_2 \circ r_1$ द्वारा प्राप्त प्रतिबिम्ब ΔA'B'C' र वस्तु ΔABC लाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् । A triangle having vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the lines r_1(x = 4) and r_2(x = – 1). Find the combined transformation $r_2 \circ r_1$. Represent the image ΔA'B'C' so obtained by $r_2 \circ r_1$ and the object ΔABC in the same graph.

⇨ Here, A(1, 2), B(4, – 1) and C(2, 5). r_1 and r_2 are parallel lines. So, the combined transformation $r_2 \circ r_1$ is the translation. $r_1 = x = 4 = h_1$ and $r_2 = x = -1 = h_2$

$$\therefore \quad r_2 \circ r_1 = TV = 2\begin{pmatrix} h_2 - h_1 \\ 0 \end{pmatrix} = 2\begin{pmatrix} -1-4 \\ 0 \end{pmatrix} = 2\begin{pmatrix} -5 \\ 0 \end{pmatrix} = 2\begin{pmatrix} -10 \\ 0 \end{pmatrix}$$

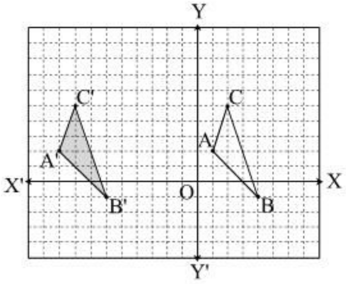
We have,

$$\text{Object } (x, y) \xrightarrow{r_2 o r_1 = \text{Translation by} \begin{pmatrix} -10 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}} \text{Image } (x + a, y + b) = (x - 10, y)$$

$$A(1, 2) \xrightarrow{r_2 o r_1 = \text{Translation by} \begin{pmatrix} -10 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}} A'(-9, 2)$$

$$B(4, -1) \xrightarrow{r_2 o r_1 = \text{Translation by} \begin{pmatrix} -10 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}} B'(-6, -1)$$

$$C(2, 5) \xrightarrow{r_2 o r_1 = \text{Translation by} \begin{pmatrix} -10 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}} C'(-8, 5)$$



Now, representation of ΔA'B'C' and ΔABC in the same graph.

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(b) शीर्षबिन्दुहरू A(2, 2), B(6, 2), C(6, 6) र D(2, 6) भएको चतुर्भुज ABCD लाई पहिले x-अक्षमा र त्यसपछि y-अक्षमा परावर्तन गराउँदा संयुक्त स्थानान्तरणद्वारा बन्ने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् । वस्तु र प्रतिबिम्बलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् । साथै संयुक्त स्थानान्तरण के हुन्छ, लेख्नुहोस् ।

Find the co-ordinates of image of a quadrilateral ABCD having vertices A(2, 2), B(6, 2), C(6, 6) and D(2, 6) which is first reflected in x-axis and then in y-axis under the combined transformation. Represent the object and image in the same graph. Also, write the single transformation.

⇨ Here, the vertices of a quad. ABCD are; A(2, 2), B(6, 2), C(6, 6) and D(2, 6).

We have, the combined reflection of x-axis followed by y-axis is the half turn rotation about the origin.

We have,

Object Image

$$(x, y) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} (-x, -y)$$

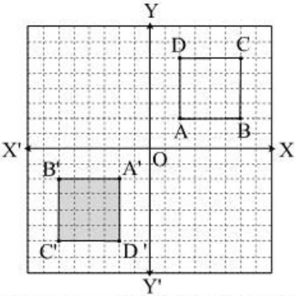
$$A(2, 2) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} A'(-2, -2)$$

$$B(6, 2) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} B'(-6, -2)$$

$$C(6, 6) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} C'(-6, -6)$$

$$D(2, 6) \xrightarrow{\text{Rotation through } [180°, (0, 0)]} D'(-2, -6)$$

Now, the graphical representation of object and the image.



(c) शीर्षबिन्दुहरू A(3, 0), B(4, 2), C(2, 4) र D(1, 2) भएको चतुर्भुज ABCD दिइएको छ । यसलाई उद्गम बिन्दुको वरिपरि + 180° मा परिक्रमण गरेपछि प्राप्त प्रतिबिम्बलाई पुनः उद्गम बिन्दुको वरिपरि 90° ले धनात्मक दिशामा परिक्रमण गरिएको छ । संयुक्त स्थानान्तरणद्वारा प्राप्त प्रतिबिम्ब चतुर्भुज A'B'C'D' र ABCD लाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A quadrilateral ABCD having vertices A(3, 0), B(4, 2), C(2, 4) and D(1, 2) is given. It is rotated through 180° about the origin and the image so obtained is rotated through 90° about the origin in positive direction. Represent the image quadrilateral A'B'C'D' obtained under combined transformation and the quadrilateral ABCD in the same graph.

⇨ Here, A(3, 0), B(4, 2), C(2, 4) and D(1, 2) are given vertices.

Combined rotation is = (180° + 90°) about (0, 0)
= 270° about (0, 0)
= – 90° about (0, 0)

We have,

Object Image

$$(x, y) \xrightarrow{\text{Rotation through } [-90° ; (0, 0)]} (y, -x)$$

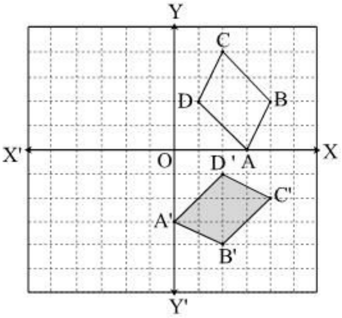
$$A(3, 0) \xrightarrow{\text{Rotation through } [-90° ; (0, 0)]} A'(0, -3)$$

$$B(4, 2) \xrightarrow{\text{Rotation through } [-90° ; (0, 0)]} B'(2, -4)$$

$$C(2, 4) \xrightarrow{\text{Rotation through } [-90° ; (0, 0)]} C'(4, -2)$$

$$D(1, 2) \xrightarrow{\text{Rotation through } [-90° ; (0, 0)]} D'(2, -1)$$

Now, the graphical representation of quadrilateral ABCD and its image A'B'C'D' under combined transformation is shown below:



5. हाम्रो दैनिक जीवनमा परावर्तन, परिक्रमण विस्थापन र विस्तार प्रयोग भएका दुई दुई ओटा उदाहरणहरू खोजी गर्नुहोस् । एकै पटक दुईओटा अथवा एकपछि अर्को प्रयोग भएको उदाहरण पनि खोजी गरी प्राप्त नतिजाका बारेमा छोटकरीमा लेख्नुहोस् ।

⇨ Show to your teacher.

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2. विपरीत स्थानान्तरण र विपरीत वृत्त
Inversion Transformation and Inversion Circle

FORMULAE AND KEY POINTS

2.1. विपरीत स्थानान्तरण (Inversion Transformation)

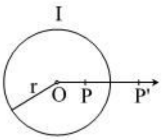
कुनै वृत्तमा आधारीत स्थानान्तरणलाई विपरीत स्थानान्तरण भनिन्छ ।

A transformation with respect to a circle is called inversion transformation or simply inversion.

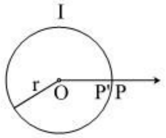
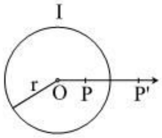
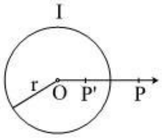
2.2 उत्क्रम वृत्त वा विपरीत वृत्त र उत्क्रम बिन्दु (Inversion circle or circle inversion and inversion point)

केन्द्रबिन्दु O र अर्धव्यास r भएको वृत्तको केन्द्रबिन्दु हुँदै गएको रेखामा केन्द्रबिन्दु O बाहेकको कुनै बिन्दु P को प्रतिबिम्ब OP × OP' = r^2 हुनेगरी P' हुन्छ भने सो वृत्तलाई उत्क्रम वृत्त र P' लाई उत्क्रम बिन्दु भनिन्छ ।

In a circle with centre O and radius r, if P be a point other than origin then the inverse of P with respect to the circle is a point P' on line OP such that OP × OP' = r^2. The image P' is the inversion point of P and the circle is inversion circle.



2.3 उत्क्रम बिन्दु र उत्क्रम वृत्तका विभिन्न अवस्थाहरू (Different conditions of inversion point and inversion circle)

वृत्तको परिधिमा बिन्दु हुँदा **Point is at the circumference**	वृत्त भित्र बिन्दु हुँदा **Point is inside the circle**	वृत्त बाहिर बिन्दु हुँदा **Point is outside the circle**
		
वस्तु र प्रतिविम्ब एउटै हुन्छ । Object and image are at the same position.	वस्तु र प्रतिविम्ब भित्र र बाहिर हुन्छन् । Object and image are inside and outside the circle.	वस्तु र प्रतिविम्ब बाहिर र भित्र हुन्छन् । Object and image are outside and inside the circle.
OP = OP' = r	**OP < r & OP' > r**	**OP > r & OP' < r**

Note: (i) (P')' = P (ii) P' को उत्क्रम बिन्दु P हुन्छ । (Inversion point of P' is P.)

2.4. दिइएको बिन्दुको उत्क्रम बिन्दु पत्ता लगाउने सूत्रहरू । (Formulae of finding the inversion point of given point.)

(i) केन्द्र (0, 0) भएको वृत्तको आधारमा बिन्दु P(x, y) को उत्क्रम बिन्दु P'(x', y') को निर्देशाङ्क: $(x', y') = \left(\frac{r^2x}{x^2+y^2}, \frac{r^2y}{x^2+y^2}\right)$ हुन्छ ।

The coordinates of the inverse point P'(x', y') of the point P(x, y) with respect to the circle having centre (0, 0) is; $(x', y') = \left(\frac{r^2x}{x^2+y^2}, \frac{r^2y}{x^2+y^2}\right)$

(ii) केन्द्र (h, k) भएको वृत्तको आधारमा बिन्दु P(x, y) को उत्क्रम बिन्दु P'(x', y') को निर्देशाङ्क:

$\left(h + \frac{r^2\,(x-h)}{(x-h)^2+(y-k)^2}, k + \frac{r^2\,(y-k)}{(x-h)^2+(y-k)^2}\right)$ हुन्छ ।

The coordinates of the inverse point P'(x', y') of the point P(x, y) with respect to the circle having centre (h, k) is; $\left(h + \frac{r^2\,(x-h)}{(x-h)^2+(y-k)^2}, k + \frac{r^2\,(y-k)}{(x-h)^2+(y-k)^2}\right)$

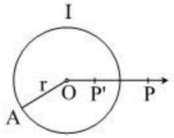
QUESTIONS FROM SEE EXERCISE 2

A. VERY SHORT QUESTIONS

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms) :

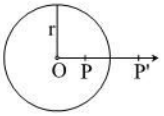
A. विपरीत स्थानान्तरण (Inversion transformation)

⇨ Here, a transformation with respect to a circle is called inversion transformation or simply inversion.



B. विपरीत वृत्त (Inversion circle)

⇨ Here, in a circle with centre O and radius r, if P be a point other than origin then the inverse of P with respect to the circle is a point P' on line OP such that OP × OP' = r^2. The image P' is the inversion point of P and the circle is inversion circle.



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2. **यदि उत्क्रम वृत्तको बाहिर बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?**
Where is the inverse of P if point P is outside the circle of inversion?

⇨ Here, if P is outside the circle then its inversion point P' is inside the circle.

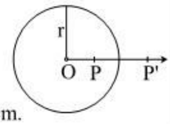
3. **उत्क्रम वृत्तको केन्द्रबाट जाने रेखाको विपरीत के हुन्छ ? (What is the inverse of a line through the centre of inversion circle?)**

⇨ Here, the inverse of a point lying at the centre is undefined. So the inverse of a line through centre is also undefined.

4. **केन्द्रबिन्दु उद्गम बिन्दुमा पर्ने वृत्तमा P(*x*, *y*) को उत्क्रम बिन्दु P'(*x*', *y*') भए P'(*x*', *y*') निकाल्ने सूत्र लेख्नुहोस् ।**
Write the formula to find P'(*x*', *y*') if P'(*x*', *y*') is the inversion point of P(*x*, *y*) in a circle with centre at origin.

⇨ Here, the formula is; $x' = \frac{r^2x}{x^2+y^2}$ and $y' = \frac{r^2y}{x^2+y^2}$

5. **सँगैको चित्रमा P को विपरीत P' छ । यदि OP × OP' = 25 cm² भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।**
In the adjoining figure, P' is the inverse of P. If OP × OP' = 25 cm² then find the radius of the inversion circle.



⇨ Here, $OP \times OP' = 25\ cm^2$
So, $r^2 = OP \times OP'$
or, $r^2 = 25\ cm^2$
∴ $r = 5$ cm
Thus, the radius of the circle is 5 cm.

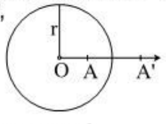
7. **केन्द्रबिन्दु O र अर्धव्यास *r* भएको वृत्तमा P को उत्क्रम बिन्दु P' छ । यदि OP = 5 cm र OP' = 20 cm भए *r* को मान पत्ता लगाउनुहोस् ।**
In a circle with centre O and radius *r*, P' is the inversion point of P. If OP = 5 cm and OP' = 20 cm, find the value of *r*.

⇨ Here, OP = 5 cm, OP' = 20 cm & r = ?
We have, $r^2 = OP \times OP'$
or, $r^2 = 5\ cm \times 20\ cm$
or, $r^2 = 100\ cm^2$
or, $r^2 = (10\ cm)^2$
∴ $r = 10$ cm

8. **चित्रमा OA = 6 cm र OP = 4 cm छन् । PP' को नाप कति होला ?**
In the figure, OA = 6 cm and OP = 4 cm. What will be the measure of PP'?

⇨ Here, OA = 6 cm, OP = 4 cm
By the definition, $OP \times OP' = r^2$
or, $4 \times OP' = 6^2$
∴ OP' = 9 cm
Now, PP' = OP' – OP = 9 cm – 4 cm = 5 cm.
Thus, the measure of PP' is 5 cm.

6. **उत्क्रम वृत्तको चित्रमा A को विपरीत A' छ । यदि *r* = 8 cm भए OA × OA' को नाप पत्ता लगाउनुहोस् ।**
In the figure of inversion circle, A' is the inverse of A. If *r* = 8 cm then find the measure of OA × OA',

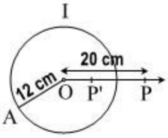


⇨ Here, r = 8 cm,
We have, $OA \times OA' = r^2$
or, $OA \times OA' = (8\ cm)^2$
∴ $OA \times OA' = 64\ cm^2$.
Thus, the measure of (OA × OA') is 64 cm².

9. **केन्द्रबिन्दु 'O' भएको वृत्तमा अर्धव्यास '*r*' छ । P को उत्क्रम बिन्दु P' छ । यदि *r* = 4 cm र OP' = 2 cm भए OP को नाप पत्ता लगाउनुहोस् ।**
In a circle with centre 'O' and radius '*r*', the image of P is P'. If *r* = 4 cm and OP' = 2 cm, find the measure of OP.

⇨ Here, r = 4 cm, OP' = 2 cm & OP = ?
We have, $OP \times OP' = r^2$
or, $OP \times 2\ cm = (4\ cm)^2$
or, $OP \times 2\ cm = 16\ cm^2$
∴ OP = 8 cm

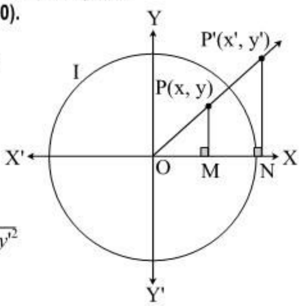
10. **चित्रमा O वृत्तको केन्द्रबिन्दु हो, OA = 12 cm र OP = 20 cm छन् । यदि P को उत्क्रम बिन्दु P' भए PP' को नाप पत्ता लगाउनुहोस् ।**
In the figure, O is the centre of circle, OA = 12 cm and OP = 20 cm. If P' is the inversion image of P then find the measure of PP'.



⇨ Here, radius (OA) = 12 cm and OP = 20 cm, PP' = ?
We know that, $OP \times OP' = r^2$
or, $20\ cm \times OP' = (12\ cm)^2$
or, $20\ cm \times OP' = 144\ cm^2$
∴ OP' = 7.2 cm
Now, PP' = OP – OP' = 20 cm – 7.2 cm
∴ PP' = 12.8 cm
Thus, the measure of PP' is 12.8 cm.

B. LONG QUESTIONS

1. **ज्यामितीय विधिबाट केन्द्र (0, 0) भएको वृत्तको आधारमा बिन्दु P(x, y) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।**
Geometrically find the inversion point of point P(x, y) in a circle having centre (0, 0).



⇨ When the centre of inversion circle is origin (0, 0).
In the figure, P'(*x*', *y*') is the inverse of P(*x*, *y*) with respect to the inversion circle I having equation $x^2 + y^2 = r^2$.
Let O be the origin and *r* be the radius of the circle.
Then, $OP \times OP' = r^2$
Draw PM ⊥ OX and P'N ⊥ OX.
Then, OM = *x*, ON = *x*', PM = *y* and P'N = *y*'
Using Pythagorean theorem we get,

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} \text{ and } OP' = \sqrt{(x'-0)^2 + (y'-0)^2} = \sqrt{x'^2 + y'^2}$$

We have, ΔP'NO and ΔPMO are similar by AA test of similarity of triangles.

Then we know that, $\frac{ON}{OM} = \frac{P'N}{PM} = \frac{OP'}{OP}$

or, $\frac{x'}{x} = \frac{y'}{y} = \frac{OP'}{OP} \times \frac{OP}{OP}$

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or, $\frac{x'}{x} = \frac{y'}{y} = \frac{r^2}{OP^2}$ [$\because$ OP × OP' = r^2]

or, $\frac{x'}{x} = \frac{y'}{y} = \frac{r^2}{x^2 + y^2}$

Taking first and third ratios then,	Taking second and third ratios then,
$\frac{x'}{x} = \frac{r^2}{x^2 + y^2}$ $\therefore \quad x' = \frac{r^2x}{x^2 + y^2}$	$\frac{y'}{y} = \frac{r^2}{x^2 + y^2}$ $\therefore \quad y' = \frac{r^2y}{x^2 + y^2}$

Thus, the co-ordinates of the inverse P'(x', y') of P(x, y) with respect to the circle I is $(x', y') = \left(\frac{r^2x}{x^2 + y^2}, \frac{r^2y}{x^2 + y^2}\right)$

Note:

(i) If $x^2 + y^2 = 0$ for the centre of inversion and x' and y' do not have real values then the inverse of the centre of inversion is a point at infinity.

(ii) For any point on the inversion circle $x^2 + y^2 = r^2$, the inverse point P'(x' , y') of P(x, y) is given by $x' = x$ and $y' = y$. i.e. P'(x', y') = P(x, y)

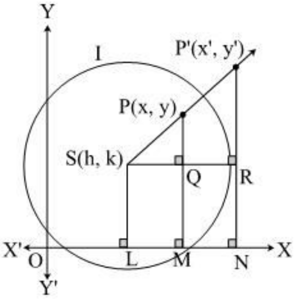
(iii) The inverse of the inverse point is given by $x = \frac{r^2x'}{x'^2 + y'^2}$ and $y = \frac{r^2y'}{x'^2 + y'^2}$

2. ज्यामितीय विधिबाट केन्द्र (h, k) भएको वृत्तको आधारमा बिन्दु P(x, y) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।

Geometrically find the inversion point of point P(x, y) in a circle having centre (h, k).

⇨ When the centre of inversion is other than origin i.e. (h, k).

In the figure, P'(x', y') is the inverse of a point P(x, y) with respect to the circle I whose equation is $(x - h)^2 + (y - k)^2 = r^2$. The centre of the circle is (h, k) and the radius of the circle is r.



Draw SL ⊥ OX, P'N ⊥ OX, PM ⊥ OX, SR ⊥ P'N.

Then, OL = h, OM = x, ON = x', SL = k, PM = y, P'N = y'

We have, SP = $\sqrt{(x - h)^2 + (y - k)^2}$ and SP' = $\sqrt{(x' - h)^2 + (y' - k)^2}$

Now, ΔSRP' and ΔSQP are similar by AA similarity.

Then, we have, $\frac{SR}{SQ} = \frac{P'R}{PQ} = \frac{SP'}{SP}$

or, $\frac{LN}{LM} = \frac{P'N - NR}{PM - QM} = \frac{SP' \times SP}{SP^2}$

or, $\frac{ON - OL}{OM - OL} = \frac{P'N - SL}{PM - SL} = \frac{SP' \times SP}{SP^2}$

or, $\frac{x' - h}{x - h} = \frac{y' - k}{y - k} = \frac{r^2}{(x - h)^2 + (y - k)^2}$ [$\because$ SP' × SP = r^2]

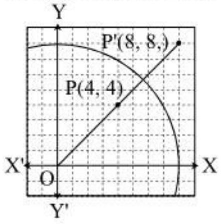
Taking first and third ratios then,	Similarly taking second and third ratios then,
$\frac{x' - h}{x - h} = \frac{r^2}{(x - h)^2 + (y - k)^2}$ or, $x' - h = \frac{r^2\,(x - h)}{(x - h)^2 + (y - k)^2}$ $\therefore \quad x' = h + \frac{r^2\,(x - h)}{(x - h)^2 + (y - k)^2}$	$\frac{y' - k}{y - k} = \frac{r^2}{(x - h)^2 + (y - k)^2}$ or, $y' - k = \frac{r^2\,(y - k)}{(x - h)^2 + (y - k)^2}$ $\therefore \quad y' = k + \frac{r^2\,(x - k)}{(x - h)^2 + (y - k)^2}$

So, the coordinates of the inverse point P'(x', y') of the point P(x, y) with respect to the circle I is;

$$\left(h + \frac{r^2\,(x - h)}{(x - h)^2 + (y - k)^2}, k + \frac{r^2\,(y - k)}{(x - h)^2 + (y - k)^2}\right)$$

3. वृत्त $x^2 + y^2 = 64$ को आधारमा बिन्दु (4, 4) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै सो बिन्दु र सोको उत्क्रम बिन्दुलाई एउटै लेखाचित्रमा पनि देखाउनुहोस् ।

Find the inverse of the point (4, 4) with respect to the circle $x^2 + y^2 = 64$. Also, show the inversion point in the same graph.



⇨ Here, given equation of circle is $x^2 + y^2 = 64$

or, $x^2 + y^2 = 8^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 8

Also, given point; (4, 4) = (x, y)

We know that,

$$P(x, y) \xrightarrow{Inversion\ [(0,\ 0);\ r]} P\left[\frac{r^2x}{x^2 + y^2}, \frac{r^2y}{x^2 + y^2}\right]$$

$$P(4, 4) \xrightarrow{Inversion\ [(0,\ 0);\ 8]} P\left[\frac{8^2 \times 4}{4^2 + 4^2}, \frac{8^2 \times 4}{4^2 + 4^2}\right] = P'(8, 8)$$

Thus, the inverse of given point (4, 4) is (8, 8)

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4. **दिइएको बिन्दु (4, 5) को वृत्त $x^2 + y^2 - 4x - 6y - 3 = 0$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै सोको उत्क्रम बिन्दुलाई लेखाचित्रमा देखाउनुहोस् ।**
Find the inversion point of the given point (4, 5) with respect to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$. Also show the inversion point in the graph.

⇨ Here, the given equation of circle is; $x^2 + y^2 - 4x - 6y - 3 = 0$
or, $x^2 - 4x + y^2 - 6y - 3 = 0$
or, $x^2 - 2 . x . 2 + 2^2 + y^2 - 2 . y . 3 + 3^2 = 3 + 2^2 + 3^2$
or, $(x - 2)^2 + (y - 3)^2 = 16$
∴ $(x - 2)^2 + (y - 3)^2 = 4^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,
centre $(h, k) = (2, 3)$ and radius $(r) = 4$
Given point P(4, 5) = P(x, y)
Let, the point of inverse be P' (x', y')

We have, $x' = \frac{r^2 (x - h)}{(x - h)^2 + (y - k)^2} + h$ and $y' = \frac{r^2 (y - k)}{(x - h)^2 + (y - k)^2} + k$

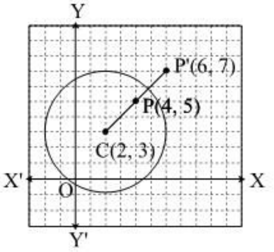
or, $x' = \frac{4^2 (4 - 2)}{(4 - 2)^2 + (5 - 3)^2} + 2$ and $y' = \frac{4^2 (5 - 3)}{(4 - 2)^2 + (5 - 3)^2} + 3$

or, $x' = \frac{16 \times 2}{4 + 4} + 2$ and $y' = \frac{16 \times 2}{4 + 4} + 3$

or, $x' = 4 + 2$ and $y' = 4 + 3$

∴ $x' = 6$ and $y' = 7$

Thus, the inversion point is (6, 7).
Now, the graphical representation is shown alongside.



5. **यदि केन्द्रबिन्दु O भएको वृत्तको आधारमा बिन्दु A(3, 4) को उत्क्रम बिन्दु A' (12, 16) छ भने सो वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।**
If the inversion of a point A(3, 4) with respect to a circle having centre as origin is A'(12, 16), find the radius of the circle.

⇨ Here, centre of inversion = (0, 0) Given point = A(3, 4) = A(x, y)
Inversion point = A'(12, 16) = A'(x', y') Radius of the circle (r) = ?

We know that, $x' = \frac{r^2 x}{x^2 + y^2}$ and $y' = \frac{r^2 y}{x^2 + y^2}$

or, $12 = \frac{r^2 \times 3}{3^2 + 4^2}$ or, $16 = \frac{r^2 \times 4}{3^2 + 4^2}$

or, $12 = \frac{r^2 \times 3}{25}$ or, $16 = \frac{r^2 \times 4}{25}$

or, $\frac{12 \times 25}{3} = r^2$ or, $\frac{16 \times 25}{4} = r^2$

or, $r^2 = 100$ or, $100 = r^2$

∴ $r = 10$ ∴ $r = 10$

Thus, the radius of the inversion circle is 10 units.

6. **बिन्दुहरू P(4, 5) र Q(4, 3) जोड्ने रेखाखण्डको केन्द्र C(2, 3) र अर्धव्यास 4 एकाइ भएको वृत्तको आधारमा उत्क्रम रेखाखण्ड पत्ता लगाउनुहोस् । सो उत्क्रमलाई एउटै लेखाचित्रमा देखाउनुहोस् ।**
Find the inverse line segment joining the points P(4, 5) and Q(4, 3) with respect to a circle with centre C(2, 3) and radius 4 units. Show the inversion on the same graph.

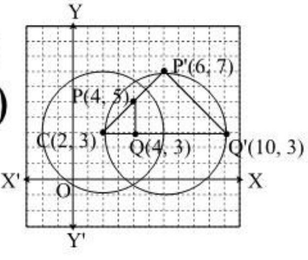
⇨ Here, in the circle, centre C(2, 3) = C(h, k) and Radius (r) = 4 units
Let, the inversion point of P(4, 5) or P(x, y) is P'(x', y')

$$\begin{aligned} \text{We know that, } P'(x', y') &= P'\left(\frac{r^2(x - h)}{(x - h)^2 + (y - k)^2} + h, \frac{r^2(y - k)}{(x - h)^2 + (y - k)^2} + k\right) \\ &= P'\left(\frac{4^2(4 - 2)}{(4 - 2)^2 + (5 - 3)^2} + 2, \frac{4^2(5 - 3)}{(4 - 2)^2 + (5 - 3)^2} + 3\right) \\ &= P'\left(\frac{16 \times 2}{4 + 4} + 2, \frac{16 \times 2}{4 + 4} + 3\right) \\ &= P'(4 + 2, 4 + 3) \end{aligned}$$

∴ P'(x', y') = P'(6, 7)
Let, the inversion point of Q(4, 3) or Q(x, y) is Q'(x', y')

$$\begin{aligned} \text{We know that, } Q'(x', y') &= Q'\left(\frac{r^2(x - h)}{(x - h)^2 + (y - k)^2} + h, \frac{r^2(y - k)}{(x - h)^2 + (y - k)^2} + k\right) \\ &= Q'\left(\frac{4^2(4 - 2)}{(4 - 2)^2 + (3 - 3)^2} + 2, \frac{4^2(3 - 3)}{(4 - 2)^2 + (3 - 3)^2} + 3\right) \\ &= Q'\left(\frac{16 \times 2}{4 + 0} + 2, \frac{16 \times 0}{4 + 0} + 3\right) \\ &= Q'(8 + 2, 0 + 3) = Q'(10, 3) \end{aligned}$$

Thus, the inversion line segment has the points P'(6, 7) and Q'(10, 3).
Now, representation on the graph;



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7. अर्धव्यास 4 एकाइ भएको एउटा वृत्तको केन्द्र बिन्दु O बाट क्रमशः 2, 4, 8 एकाइ टाढा रहेका बिन्दुहरू P, Q र R का उत्क्रम बिन्दुहरूको वृत्तको केन्दबाट दूरी पत्ता लगाउनुहोस् ।
Find the distance from the centre of inverse of points P, Q and R which are at the distances of 2, 4, 8 units respectively from the centre O of the given circle with radius 4 units.

⇨ Here, OP = 2 units, OQ = 4 units, OR = 8 units

Let, P', Q', and R' be the inversion points of P, Q and R respectively.

So, inverse of P is P'.
i.e $OP \times OP' = r^2$
or, $2 \times OP' = 4^2$
or, $2 \times OP' = 16$
∴ $OP' = 8$ units

Similarly Q' is inverse point of Q.
i.e $OQ \times OQ' = r^2$
or, $4 \times OQ' = 4^2$
or, $4 \times OQ' = 16$
∴ $OQ' = 4$ units

Again, R' is the inverse point of R.
i.e $OR \times OR' = r^2$
or, $OR \times 8 = 4^2$
or, $OR \times 8 = 16$
∴ $OR = 2$ units

Thus, P' is at 8 units, Q' is at 4 units and R' is at 2 units from O.

8. बिन्दु (2, 2) को वृत्त $x^2 + y^2 = 2(2x + y + 2)$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inverse point of (2, 2) with respect to the circle $x^2 + y^2 = 2(2x + y + 2)$.

⇨ Here, given equation of the circle is;
$x^2 + y^2 = 4x + 2y + 4$
or, $x^2 + y^2 - 4x - 2y - 4 = 0$
or, $x^2 - 4x + y^2 - 2y - 4 = 0$
or, $x^2 - 4x + 4 + y^2 - 2y + 1 = 4 + 4 + 1$
or, $(x - 2)^2 + (y - 1)^2 = 9$
or, $(x - 2)^2 + (y - 1)^2 = 3^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we get,
Centre = (h, k) = (2, 1) and
radius (r) = 3
∴ h = 2, k = 1 and r = 3

Let, (x', y') be the inverse point of (2, 2) then,

$$x' = h + \frac{(x-h)\,r^2}{(x-h)^2 + (y-k)^2}$$
$$= 2 + \frac{(2-2) \times 3^2}{(2-2)^2 + (2-1)^2} = 2 + \frac{0 \times 3^2}{0^2 + 1^2}$$
∴ $x' = 2$

$$\text{Again, } y' = k + \frac{(y-k)\,r^2}{(x-h)^2 + (y-k)^2}$$
$$= 1 + \frac{(2-1) \times 3^2}{(2-2)^2 + (2-1)^2} = 1 + \frac{9}{0^2 + 1^2}$$
∴ $y' = 10$

Thus, (2, 10) is the inverse point of (2, 2).

9. एउटा वृत्त $x^2 + y^2 - 4x - 6y - 23 = 0$ को आधारमा (1, 6) र (5, 2) को मध्यबिन्दुको उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inverse point of midpoint of (1, 6) and (5, 2) with respect to the circle $x^2 + y^2 - 4x - 6y - 23 = 0$.

⇨ Here, given equation of circle is; $x^2 + y^2 - 4x - 6y - 23 = 0$
or, $x^2 - 4x + y^2 - 6y - 23 = 0$
or, $x^2 - 2 \,.\, x \,.\, 2 + 2^2 + y^2 - 2 \,.\, y \,.\, 3 + 3^2 = 2^2 + 3^2 + 23$
or, $(x - 2)^2 + (y - 3)^2 = 36$
or, $(x - 2)^2 + (y - 3)^2 = 6^2$
Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then, centre (h, k) = (2, 3) and radius (r) = 6

We have, midpoint of (1, 6) and (5, 2) is given by; $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+5}{2}, \frac{6+2}{2}\right) = (3, 4)$

Let the point of inverse be (x', y').
We have,

$x' = h + \frac{r^2\,(x-h)}{(x-h)^2 + (y-k)^2}$ and $y' = k + \frac{r^2\,(y-k)}{(x-h)^2 + (y-k)^2}$
or, $x' = 2 + \frac{6^2\,(3-2)}{(3-2)^2 + (4-3)^2}$ and $y' = 3 + \frac{6^2\,(4-3)}{(3-2)^2 + (4-3)^2}$
or, $x' = 2 + \frac{36 \times 1}{1+1}$ and $y' = 3 + \frac{36 \times 1}{1+1}$
or, $x' = 2 + 18$ and $y' = 3 + 18$
∴ $x' = 20$ and ∴ $y' = 21$

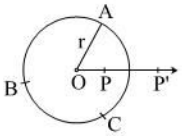
Thus, the inversion point is (20, 21).

QUESTIONS FROM CDC TEXTBOOK

7.2 विपरीत स्थानान्तरण र विपरीत वृत्त (INVERSION TRANSFORMATOIN AND INVERSION CIRCLE)

EXERCISE 7.2

1. दिइएको चित्रका आधारमा तल दिइएका अवधारणाहरू व्याख्या गर्नुहोस् ।
On the basis of given figure, explain the following concepts.



(a) उत्क्रम वा विपरित वृत्त (Inversion circle)
⇨ Here, the inversion circle is ABC.

(b) उत्क्रम अर्धव्यास (Inversion radius)
⇨ Here, the inversion radius is OA.

(c) बिन्दु P को उत्क्रम (Inversion) बिन्दु (The inversion point of point P)
⇨ Here, the inversion point of P is P'.

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(d) **बिन्दु P' को उत्क्रम (Inversion) बिन्दु** (The inversion point of point P')

⇨ Here, the inversion point of P' is P.

(e) **OP, OP' र OA बिचको सम्बन्ध** (Relation among OP, OP' and OA)

⇨ Here, the relation among OP, OP' and OA is OP × OP' = OA^2.

2. **तल दिइएको जानकारीका आधारमा उत्क्रम (Inversion) बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।**
From the information given below, find the co-ordinates of inversion points.

	बिन्दु (Point)	उत्क्रम वृत्तको समीकरण (Equation of inversion circle)	उत्क्रम बिन्दु (Inversion point)
(a)	A(3, 4)	$x^2 + y^2 = 1$	A' = ?
(b)	B(4, 0)	$x^2 + y^2 = 4$	B' = ?
(c)	C(– 7, 0)	$x^2 + y^2 = 49$	C' = ?
(d)	D(0, 45)	$x^2 + y^2 = 25$	D' = ?
(e)	$E\left(\frac{1}{2}, \frac{1}{4}\right)$	$x^2 + y^2 = 1$	E' = ?
(f)	$F\left(\frac{4}{5}, \frac{3}{5}\right)$	$x^2 + y^2 = 16$	F' = ?
(g)	G(– 4, – 5)	$x^2 + y^2 = \sqrt{10}$	G' = ?

⇨ (a) Here, Given equation of inversion circle is; $x^2 + y^2 = 1$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 1

Also, given point: A(3, 4) = (x, y)

Let the image point be (x', y').

We have, $x' = \frac{r^2x}{x^2 + y^2} = \frac{1^2 \times 3}{3^2 + 4^2} = \frac{3}{25}$ and $y' = \frac{r^2y}{x^2 + y^2} = \frac{1^2 \times 4}{3^2 + 4^2} = \frac{4}{25}$

$\therefore \quad (x', y') = \left(\frac{3}{25}, \frac{4}{25}\right)$

Thus, the inverse of point A(3, 4) is $A'\left(\frac{3}{25}, \frac{4}{25}\right)$.

⇨ (b) Here, Given equation of inversion circle is; $x^2 + y^2 = 4$

$\therefore \quad x^2 + y^2 = 2^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 2

Also, given point; B(4, 0) = (x, y)

Let the image point be (x', y').

We know that, $x' = \frac{r^2x}{x^2 + y^2} = \frac{2^2 \times 4}{4^2 + 0^2} = \frac{16}{16} = 1$ and $y' = \frac{r^2y}{x^2 + y^2} = \frac{2^2 \times 0}{4^2 + 0^2} = \frac{0}{16} = 0$

$\therefore \quad (x', y') = (1, 0)$

Thus, the inverse of given point B(4, 0) is B'(1, 0).

⇨ (c) Here, Given equation of inversion circle is; $x^2 + y^2 = 49$

$\therefore \quad x^2 + y^2 = 7^2$

Comparing it with $x^2 + y^2 = r^2$ then,

Centre = (0, 0) and radius (r) = 7

Also, given point: C(– 7, 0) = (x, y)

Let (x', y') be the inversion point of (x, y).

We have, $x' = \frac{r^2x}{x^2 + y^2} = \frac{7^2 \times (-7)}{(-7)^2 + 0^2} = -7$ and $y' = \frac{r^2y}{x^2 + y^2} = \frac{7^2 \times 0}{(-7)^2 + 0^2} = \frac{0}{49} = 0$

$\therefore \quad (x', y') = (-7, 0)$

Thus, the inverse of given point C(– 7, 0) is C'(– 7, 0).

⇨ (d) Here, Given equation of inversion circle is; $x^2 + y^2 = 25$

$\therefore \quad x^2 + y^2 = 5^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 5

Also, given point; D(0, 45) = (x, y)

Let (x', y') be the inversion point of (x, y).

We have, $x' = \frac{r^2x}{x^2 + y^2} = \frac{5^2 \times 0}{0 + 45^2} = \frac{0}{45^2} = 0$ and $y' = \frac{r^2y}{x^2 + y^2} = \frac{5^2 \times 45}{0 + 45^2} = \frac{25}{45} = \frac{5}{9}$

$\therefore \quad (x', y') = \left(0, \frac{5}{9}\right)$

Thus, the inverse of given point D(0, 45) is $D'\left(0, \frac{5}{9}\right)$.

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⇨ (e) Here, Given equation of inversion circle is; $x^2 + y^2 = 1$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 1

Also, given point; $E\left(\frac{1}{2}, \frac{1}{4}\right) = (x, y)$

Let, (x', y') be the inversion point of (x, y).

We have, $$x' = \frac{r^2 x}{x^2 + y^2} = \frac{1^2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{\frac{1}{2}}{\frac{1}{4} + \frac{1}{16}} = \frac{\frac{1}{2}}{\frac{5}{16}} = \frac{8}{5}$$

$$y' = \frac{r^2 y}{x^2 + y^2} = \frac{1^2 \times \frac{1}{4}}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{16}} = \frac{\frac{1}{4}}{\frac{5}{16}} = \frac{4}{5}$$

$\therefore \quad (x', y') = \left(\frac{8}{5}, \frac{4}{5}\right)$

Thus, the inverse of given point $E\left(\frac{1}{2}, \frac{1}{4}\right)$ is $E'\left(\frac{8}{5}, \frac{4}{5}\right)$.

⇨ (f) Here, Given equation of inversion circle is; $x^2 + y^2 = 16$ $\quad\therefore \quad x^2 + y^2 = 4^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = 4.

Also, given point; $F\left(\frac{4}{5}, \frac{3}{5}\right) = (x, y)$

Let (x', y') be the inversion point of (x, y).

We have, $$x' = \frac{r^2 x}{x^2 + y^2} = \frac{4^2 \times \frac{4}{5}}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \frac{\frac{64}{5}}{\frac{16}{25} + \frac{9}{25}} = \frac{\frac{64}{5}}{1} = \frac{64}{5}$$

$$y' = \frac{r^2 y}{x^2 + y^2} = \frac{4^2 \times \frac{3}{5}}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \frac{\frac{48}{5}}{\frac{16}{25} + \frac{9}{25}} = \frac{\frac{48}{5}}{1} = \frac{48}{5}$$

$\therefore \quad (x', y') = \left(\frac{64}{5}, \frac{48}{5}\right)$

Thus, the inverse of given point $F\left(\frac{4}{5}, \frac{3}{5}\right)$ is $F'\left(\frac{64}{5}, \frac{48}{5}\right)$.

⇨ (g) Here, Given equation of inversion circle is; $x^2 + y^2 = 10$ $\quad\therefore \quad x^2 + y^2 = (\sqrt{10})^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius (r) = $\sqrt{10}$

Also, given point; G(– 4, – 5) = (x, y).

Let (x', y') be the inversion point of (x, y).

We have, $$x' = \frac{r^2 x}{x^2 + y^2} = \frac{10 \times (-4)}{(-4)^2 + (-5)^2} = \frac{-40}{16 + 25} = -\frac{40}{41}$$

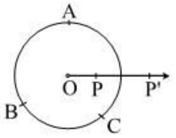
$$y' = \frac{r^2 y}{x^2 + y^2} = \frac{10 \times (-5)}{(-4)^2 + (-5)^2} = \frac{-50}{16 + 25} = -\frac{50}{41}$$

$\therefore \quad (x', y') = \left(-\frac{40}{41}, -\frac{50}{41}\right)$

Thus, the inverse of given point G(– 4, – 5) is $G'\left(-\frac{40}{41}, -\frac{50}{41}\right)$.

3. (a) चित्रमा उत्क्रम (Inversion) वृत्त ABC को केन्द्र O र P को उत्क्रम (Inversion) बिन्दु P' छ । यदि वृत्तको समीकरण $x^2 + y^2 = 36$ र OP = 4 एकाइ भए OP' पत्ता लगाउनुहोस् ।

In the figure, P' is the inversion point of P and O is the centre of an inversion circle ABC. If the equation of the cirlce is $x^2 + y^2 = 36$ and OP = 4 units, find OP'.



⇨ Here, O is the centre of inversion circle ABC.

Equation of circle is; $x^2 + y^2 = 36$ $\quad$ i.e. $x^2 + y^2 = 6^2$

Comparing it with $x^2 + y^2 = r^2$ then, radius (r) = 6 units

By the definition of inversion circle;

$OP \times OP' = r^2$

or, $4 \times OP' = 6^2$

or, $4 \times OP' = 36$

$\therefore \quad OP' = 9$ units

Thus, the measure of OP' is 9 units.

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(b) एउटा उत्क्रम (Inversion) वृत्तको केन्द्र C(2, 3) र परिधिमा पर्ने बिन्दु A(6, 7) छ । बिन्दु Q' को उत्क्रम (Inversion) बिन्दु Q छ । यदि OQ = 8 एकाइ भए OQ' पत्ता लगाउनुहोस् ।
The centre of inversion circle is C(2, 3) and the point at the circumference is A(6, 7). Q' is the inversion point of Q. If OQ = 8 units then find OQ'.

⇨ Here, Centre = (h, k) = (2, 3)
Point at the circumference = A(6, 7) and OQ = 8 units, OQ' = ?
We have,
radius = distance between (2, 3) and (6, 7) = $\sqrt{(6-2)^2+(7-3)^2} = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32}$
$\therefore$ $r = 4\sqrt{2}$ units.
Now, by the definition of inversion circle; $OQ \times OQ' = r^2$
or, $8 \times OQ' = (4\sqrt{2})^2$
or, $8 \times OQ' = 32$
$\therefore$ OQ = 4 units.
Thus, the measure of OQ is 4 units.

4. तल दिइएको अवस्थामा प्रत्येक बिन्दुको उत्क्रम बिन्दु (Inversion Point) पत्ता लगाउनुहोस् ।
In the given condition, find the inversion point of each of the following points.

	बिन्दु (Point)	उत्क्रम वृत्तको समीकरण (Equation of inversion circle)	उत्क्रम बिन्दु (Inversion Point)
(a)	M(0, 4)	$(x-1)^2+(y-3)^2=16$	M' = ?
(b)	N(3, 4)	$x^2+y^2-4x-6y-23=0$	N' = ?
(c)	P(– 1, – 3)	$x^2+y^2+6x-8y-11=0$	O' = ?

⇨ (a) Here, given equation of circle is; $(x-1)^2+(y-3)^2=16$
Comparing it with $(x-h)^2+(y-k)^2=r^2$ then, centre (h, k) = (1, 3) and radius (r) = 4
Given point M(0, 4) = M(x, y).
Let the inverse point be M'(x', y').
We have,
$x' = h + \frac{r^2(x-h)}{(x-h)^2+(y-k)^2}$ and $y' = k + \frac{r^2(y-k)}{(x-h)^2+(y-k)^2}$
or, $x' = 1 + \frac{4^2(0-1)}{(0-1)^2+(4-3)^2}$ and $y' = 3 + \frac{4^2(4-3)}{(0-1)^2+(4-3)^2}$
or, $x' = 1 + \frac{16(-1)}{1+1}$ and $y' = 3 + \frac{16 \times 1}{1+1}$
or, $x' = 1 - 8$ and $y' = 3 + 8$
$\therefore$ $x' = -7$ $\therefore$ $y' = 11$
Thus, the inversion point is M'(– 7, 11).

⇨ (b) Here, given equation of circle is; $x^2+y^2-4x-6y-23=0$
or, $x^2-4x+y^2-6y-23=0$
or, $x^2-2.x.2+2^2+y^2-2.y.3+3^2=2^2+3^2+23$
or, $(x-2)^2+(y-3)^2=36$
or, $(x-2)^2+(y-3)^2=6^2$
Comparing it with $(x-h)^2+(y-k)^2=r^2$ then, centre (h, k) = (2, 3) and radius (r) = 6
Given point N(3, 4) = N(x, y)
Let the point of inverse be N' (x', y').
We have,
$x' = h + \frac{r^2(x-h)}{(x-h)^2+(y-k)^2}$ and $y' = k + \frac{r^2(y-k)}{(x-h)^2+(y-k)^2}$
or, $x' = 2 + \frac{6^2(3-2)}{(3-2)^2+(4-3)^2}$ and $y' = 3 + \frac{6^2(4-3)}{(3-2)^2+(4-3)^2}$
or, $x' = 2 + \frac{36 \times 1}{1+1}$ and $y' = 3 + \frac{36 \times 1}{1+1}$
or, $x' = 2 + 18$ and $y' = 3 + 18$
$\therefore$ $x' = 20$ and $\therefore$ $y' = 21$
Thus, the inversion point is N'(20, 21).

⇨ (c) Here, given equation of circle is; $x^2+y^2+6x-8y-11=0$
or, $x^2+6x+y^2-8y-11=0$
or, $x^2+2.x.3+3^2+y^2-2.y.4+4^2=11+3^2+4^2$
or, $(x+3)^2+(y-4)^2=36$
or, $(x+3)^2+(y-4)^2=6^2$
Comparing it with $(x-h)^2+(y-k)^2=r^2$ then, centre (h, k) = (– 3, 4) and radius (r) = 6

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Given point P(– 1, – 3) = P(x, y)
Let the point of inverse be P'(x', y')
We have,

$$x' = h + \frac{r^2 (x-h)}{(x-h)^2 + (y-k)^2} \quad \text{and} \quad y' = k + \frac{r^2 (y-k)}{(x-h)^2 + (y-k)^2}$$

$$\text{or, } x' = -3 + \frac{6^2 [-1-(-3)]}{[-1-(-3)]^2 + (-3-4)^2} \quad \text{and} \quad y' = 4 + \frac{6^2 (-3-4)}{[-1-(-3)]^2 + (-3-4)^2}$$

$$\text{or, } x' = -3 + \frac{36 \times 2}{4+49} \quad \text{and} \quad y' = 4 + \frac{36 \times (-7)}{4+49}$$

$$\text{or, } x' = -3 + \frac{72}{53} \quad \text{and} \quad y' = 4 - \frac{252}{53}$$

$$\text{or, } x' = \frac{(-3) \times 53 + 72}{53} \quad \text{and} \quad y' = \frac{4 \times 53 - 252}{53}$$

$$\text{or, } x' = \frac{-159 + 72}{53} \quad \text{and} \quad y' = \frac{212 - 252}{53}$$

$$\therefore \ x' = \frac{-87}{53} \quad \text{and} \quad y' = \frac{-40}{53}$$

Thus, the inversion point is $P'\left(\frac{-87}{53}, \frac{-40}{53}\right)$.

3. मेट्रिक्स स्थानान्तरण
Matrix Transformation

Formulae and Key points

(i)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	X- अक्षमा परावर्तन Reflection in x–axis	P(x, y) → P'(x, – y)
(ii)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Y- अक्षमा परावर्तन Reflection in y-axis	P(x, y) → P'(– x, y)
(iii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	y = x मा परावर्तन Reflection in y = x	P(x, y) → P'(y, x)
(iv)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	y = – x मा परावर्तन Reflection in y = – x	P(x, y) → P'(– y, – x)
(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	R_o [O, + 90°]	P(x, y) → P'(– y, x)
(vi)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	R_o [O, – 90°]	P(x, y) → P'(y, – x)
(vii)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	R_o [O, ±180°]	P(x, y) → P'(– x, – y)
(viii)	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	E[O, k]	P(x, y) → P' (kx, ky)
(ix)	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x-a \\ y-b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$	E[(a, b), k]	P(x, y) → P' (k(x – a) + a, k(y – b) + b)

स्थानान्तरण मेट्रिक्स पत्ता लगाउने तरिका:

जस्तै: X-**अक्षमा परावर्तन हुँदा वस्तु** = (x, y) **भए प्रतिबिम्ब** = (x, – y) **हुन्छ ।**

अब, प्रतिबिम्बका x-**सदस्य र** y-**सदस्यलाई समीकरणको रूपमा व्यक्त गर्दा,**

$$x = 1 \times x + 0 \times y \rightarrow 1 \times x + 0 \times y = x$$

$$-y = 0 \times x + (-1) \times y \rightarrow 0 \times x + (-1) \times y = -y$$

यसलाई मेट्रिक्सको रूपमा लेख्दा,

$$\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\boxed{\text{प्रतिविम्व}} = \boxed{\text{स्थानान्तरण मेट्रिक्स}} \quad \boxed{\text{वस्तु}}$$

If an object (x, y) is reflected on X -axis then the image will be (x, –y).
Now, expressing the x-component and y-component of this image in matrix form then.

$$x = 1 \times x + 0 \times y \rightarrow 1 \times x + 0 \times y = x$$

$$-y = 0 \times x + (-1) \times y \rightarrow 0 \times x + (-1) \times y = -y$$

Writing this equations in matrix form then,

$$\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\boxed{\text{Object}} = \boxed{\text{TM}} \quad \boxed{\text{Image}}$$

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QUESTIONS FROM SEE EXERCISE 3

A. VERY SHORT QUESTIONS

1. मेट्रिक्स स्थानान्तरण (Matrix transformation)

⇨ Here, the transformation by matrix is called matrix transformation. If $\begin{pmatrix} a \\ b \end{pmatrix}$ be the transformation on matrix then p(x, y) is transformed into P'(x + a, y + b).

2. मेट्रिक्स $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् । (Find the transformation represented by the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.)

⇨ Here, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Let (x, y) be any point. So, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.x + 1.y \\ 1.x + 0.y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$

Thus, the given matrix represents the reflection in y = x.

3. मेट्रिक्स $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् । (Find the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$)

⇨ Here, $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Let (x, y) be any point. So, $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.x - 1.y \\ 1.x + 0.y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

Thus, the given matrix represents + 90° rotation about origin.

4. मेट्रिक्स $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् । (Find the transformation represented by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.)

⇨ Here, $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. Let (x, y) be any point. So, $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + 0.y \\ 0.x + 3y \end{pmatrix} = \begin{pmatrix} 3x \\ 3y \end{pmatrix}$

Thus, the required transformation is the enlargement about origin with scale factor 3.

5. बिन्दु A(*x*, *y*) लाई बिन्दु A(– *x*, *y*) मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2 × 2 matrix which transforms A(*x*, *y*) to A(– *x*, *y*).

⇨ Here, image matrix = $\begin{pmatrix} -x \\ y \end{pmatrix} = \begin{pmatrix} -x.1 + 0.y \\ x.0 + 1.y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Thus, required transformation matrix is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

6. बिन्दु (4, 5) लाई 2 × 1 स्थानान्तरण मेट्रिक्स $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ले स्थानान्तरण गर्नुहोस् ।

Transform a point (4, 5) by a 2 × 1 transformation matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

⇨ Here, given point is (4, 5) and Transformation Matrix = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

We have, Image = TM + Object = $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

7. मेट्रिक्स $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ले दिने स्थानान्तरण के हो पत्ता लगाउनुहोस् ।

Find the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

⇨ Here, transformation matrix = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ represents the reflection in X-axis.

8. मेट्रिक्स $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ले कुन स्थानान्तरण जनाउँछ ? (Which transformation is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$?)

⇨ Here, the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ represents reflection in X-axis

9. मेट्रिक्स $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ले कुन स्थानान्तरण जनाउँदछ ? (What transformation does the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ denote?)

⇨ Here, the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ denotes the reflection on the line y = – x.

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10. मेट्रिक्स $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ले कुन स्थानान्तरण जनाउँछ ? (To what transformation is the matrix $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$associated ?)

⇨ Here, the matrix $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$is associated to rotation about the origin through – 90° or + 270°.

11. मेट्रिक्स $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$ले कुन स्थानान्तरण जनाउँदछ ? (Which transformation is represented by the matrix $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$?)

⇨ Here, Reflection on the line $y = -x$ is represented by the matrix $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

12. मेट्रिक्स $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ले कुन स्थानान्तरणलाई जनाउँछ ? (What transformation does $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$denote ?)

⇨ Here, $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$denotes the transformation reflection in Y-axis.

13. मेट्रिक्स $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ले प्रतिनिधित्व गर्ने स्थानान्तरण कुन हो ? (Which type of transformation does $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$represent ?)

⇨ Here, the matrix $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$represents the reflection in Y-axis.

14. मेट्रिक्स $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ ले कुन स्थानान्तरण जनाउँदछ ? (To what transformation is the matrix $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ associated ?)

⇨ Here, The matrix $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$indicates positive quarter turn about origin.

15. मेट्रिक्स $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ले जनाउने स्थानान्तरण पत्ता लगाउनुहोस् ।(Find the transformation which is denoted by the matrix $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$)

[2067 R]

⇨ Here, $(x, y) \rightarrow (-y, x)$ is rotation above (+ 90°) at centre (0, 0).

16. मेट्रिक्स $\begin{bmatrix}3&0\\0&3\end{bmatrix}$ले कुन स्थानान्तरणलाई जनाउँछ ? (Which transformation is represented by the matrix $\begin{bmatrix}3&0\\0&3\end{bmatrix}$?)

⇨ Here, $\begin{bmatrix}3&0\\0&3\end{bmatrix}=\begin{bmatrix}k&0\\0&k\end{bmatrix}$represents enlargement by scale factor $k = 3$ with the centre at origin.

17. मेट्रिक्स $\begin{pmatrix}2&0\\0&2\end{pmatrix}$ ले कुन स्थानान्तरण जनाउँछ ? (To what transformation is the matrix $\begin{pmatrix}2&0\\0&2\end{pmatrix}$associated ?)

⇨ Here the matrix is $\begin{pmatrix}2&0\\0&2\end{pmatrix}$associated to the enlargement of scale factor 2 and centre is at origin.

18. मेट्रिक्स $\begin{pmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{pmatrix}$ले दिने स्थानान्तरण के हो ? पत्ता लगाउनुहोस् । (Find the transformation represented by matrix $\begin{pmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{pmatrix}$)

⇨ Here, $\begin{pmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{pmatrix}$is the matrix similar to $\begin{pmatrix}k&0\\0&k\end{pmatrix}$

So, it represents enlargement of centre (0, 0) and scale factor $\frac{1}{2}$.

19. बिन्दु A(– 3, 1) लाई बिन्दु A(3, 1) मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2 × 2 matrix which transforms A(– 3, 1) to A(3, 1).

⇨ Here, It seems that (x, y) has image (– x, y). Thus, the transformation matrix is $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$

20. कुनै एउटा 2 × 2 स्थानान्तरण मेट्रिक्सद्वारा (x, y) यदि (y, x) मा स्थानान्तरण हुन्छ भने उक्त स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
If a point (x, y) is transformed into (y, x) by a 2 × 2 transformation matrix, then find the matrix.

⇨ Here, $\begin{pmatrix}x\\y\end{pmatrix}$be an object & $\begin{pmatrix}y\\x\end{pmatrix}$be an image. Thus, $\begin{pmatrix}0&1\\1&0\end{pmatrix}$be a require matrix.

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21. कुनै एउटा 2 × 2 स्थानान्तरण मेट्रिक्सद्वारा (a, b) यदि (b, – a) मा स्थानान्तरण हुन्छ भने उक्त स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
If a point (a, b) is transformed into (b, – a) by a 2 × 2 transformation matrix, then find the matrix.

⇨ Here, object P(a, b) and image P'(b, – a) $\therefore \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is a require 2 × 2 transformation matrix.

B. LONG QUESTIONS

MODEL 1

1. समानान्तर चतुर्भुज QRST का शीर्षबिन्दुहरू Q(– 1, 1), R(– 2, – 1), S(2, – 1) र T(3, 1) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरी हुने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The parallelogram QRST has the vertices Q(–1, 1), R(– 2, – 1), S(2, – 1) and T(3, 1). Transform the given parallelogram under the matrix $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ and find the coordinates of vertices of its image. **[2071 S]**

⇨ Here,

Object in matrix form $= \begin{array}{c} \begin{matrix} Q & R & S & T \end{matrix} \\ \begin{bmatrix} -1 & -2 & 2 & 3 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array}$ Transformation matrix $= \begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$

We know that,

$$\begin{aligned} \text{Image} &= \text{TM} \times \text{Object} \\ &= \begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & -2 & 2 & 3 \\ 1 & -1 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \times (-1) + 3 \times 1 & 4 \times (-2) + 3 \times (-1) & 4 \times 2 + 3\,(-1) & 4 \times 3 + 3 \times 1 \\ 2 \times (-1) + 0 \times 1 & 2 \times (-2) + (0 \times (-1)) & 2 \times 2 + 0 \times (-1) & 2 \times 3 + 0 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} -4+3 & -8-3 & 8-3 & 12+3 \\ -2+0 & -4+0 & 4+0 & 6+0 \end{bmatrix} \\ &= \begin{array}{c} \begin{matrix} Q' & R' & S' & T' \end{matrix} \\ \begin{bmatrix} -1 & -11 & 5 & 15 \\ -2 & -4 & 4 & 6 \end{bmatrix} \end{array} \end{aligned}$$

Thus, Q'(– 1, – 2), R'(– 11, – 4), S'(5, 4) and T'(15, 6) are the required vertices of given parallelogram, QRST.

2. वर्ग WXYZ का शीर्षबिन्दुहरू W(0, 3), X(1, 1), Y(3, 2) र Z(2, 4) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरी हुने प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।
The square WXYZ has the vertices W(0, 3), X(1, 1), Y(3, 2) and Z(2, 4). Transform the given square WXYZ under the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ and find the coordinates of the vertices of its image. **[2071 R]**

⇨ Here, W(0, 3), X(1, 1), Y(3, 2) and Z(2, 4). The object in matrix form $= \begin{bmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

Transformation matrix (TM) $= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

We know that,

$$\begin{aligned} \text{Image} &= \text{TM} \times \text{Object} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 0 - 1 \times 3 & 0 \times 1 - 1 \times 1 & 0 \times 3 - 1 \times 2 & 0 \times 2 - 1 \times 4 \\ -1 \times 0 + 0 \times 3 & -1 \times 1 + 0 \times 1 & -1 \times 3 + 0 \times 2 & -1 \times 2 + 0 \times 4 \end{bmatrix} \\ \therefore \text{Image} &= \begin{bmatrix} -3 & -1 & -2 & -4 \\ 0 & -1 & -3 & -2 \end{bmatrix} \end{aligned}$$

Thus, the co-ordinates of images are; W'(– 3, 0), X'(– 1, – 1), Y'(– 2, – 3) and Z'(– 4, – 2).

3. A (1, – 1), B (3, 2) र C (0, 2) भएको ΔABC लेखाचित्रमा खिच्नुहोस् । सो ΔABC लाई मेट्रिक्स $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ ले स्थानान्तरण गर्दा हुन आउने प्रतिबिम्ब सोही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
Draw a ΔABC where vertices are A (1, – 1), B (3, 2) and C (0, 2) on a graph paper. Draw the image of the ΔABC on the same graph under the transformation given by the matrix $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$. **[2057 S]**

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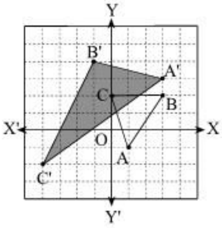
⇨ Here, given, vertices of ΔABC are A(1, – 1), B(3, 2) and C(0, 2).

Given matrix $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

Matrix form of the vertices of ΔABC is $\begin{array}{ccc} A & B & C \\ \end{array}$ $\begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & 2 \end{pmatrix}$

Now, translation by the matrix is; $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & 2 \end{pmatrix} \end{array}$

$$= \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 1\times1-2\times(-1) & 1\times3-2\times2 & 1\times0-2\times2 \\ 2\times1-1\times(-1) & 2\times3-1\times2 & 2\times0-1\times2 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 3 & -1 & -4 \\ 3 & 4 & -2 \end{bmatrix} \end{array}$$



Thus, the vertices of image ΔA'B'C' of the ΔABC are A'(3, 3), B'(– 1, 4) and C'(– 4, – 2). The graph of ΔABC and its image ΔA'B'C' is as shown alongside.

4. **शीर्षबिन्दुहरू A(3, 1), B(– 2, – 1) र C(4, 2) भएको त्रिभुजलाई स्थानान्तरण मेट्रिक्स $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ द्वारा स्थानान्तरण गराउँदा बन्ने प्रतिबिम्ब ΔA'B'C' का शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।**

Find the co-ordinates of the vertices of image ΔA'B'C' which is transformed by the transformation of ΔABC having vertices A(3, 1), B(– 2, – 1) and C(4, 2) under the transformation by a matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. [2066 R']

⇨ Here, A(3, 1), B(– 2, – 1) and C(4, 2)

TM = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and Object in matrix form = $\begin{pmatrix} 3 & -2 & 4 \\ 1 & -1 & 2 \end{pmatrix}$

We know that; image = TM × object $= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & -2 & 4 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 6+0 & -4+0 & 8+0 \\ 0+2 & 0-2 & 0+4 \end{pmatrix}$

∴ Image = $\begin{pmatrix} 6 & -4 & 8 \\ 2 & -2 & 4 \end{pmatrix}$

Thus, the co-ordinates of image are; A'(6, 2), B'(– 4, – 2) and C'(8, 4).

5. **एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ मेट्रिक्सले स्थानान्तरण गराई आउने प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क लेख्नुहोस् ।**

Transform a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ under the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and write the co-ordinates of the vertices of the images. [2065 R]

⇨ Here, object = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ Transformation matrix = $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

We have, Image = TM × Object $= \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+0 & 3+0 & 3+2 & 0+2 \\ 0+0 & 1+0 & 1+1 & 0+1 \end{pmatrix}$

∴ Image = $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

Thus, the co-ordinates of images are (0, 0), (3, 1), (5, 2) and (2, 1).

6. **एकाइ वर्गलाई मेट्रिक्स $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ ले स्थानान्तरण गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।**

Find the co-ordinates of the vertices of the image of the unit square transformed by the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$. [2067 S]

⇨ Here, TM = $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ & object = Unit square = $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

We know that,

Image = TM × Object $= \begin{pmatrix} 4 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+0 & 4+0 & 4+2 & 0+2 \\ 0+0 & 1+0 & 1+2 & 0+2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

Thus, the co-ordinates of the vertices of the image are; (0, 0), (4, 1), (6, 3) and (2, 2).

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MODEL 2

7. शीर्षबिन्दुहरू M(0, 0), N(1, 0), O(1, 1) र P(0, 1) भएको एकाइ वर्ग MNOP लाई *y* = – *x* अक्षमा स्थानान्तरण गर्ने मेट्रिक्सद्वारा स्थानान्तरण गर्दा हुने प्रतिबिम्ब चतुर्भुज M'N'O'P' का शीर्षबिन्दुहरू लेख्नुहोस् ।
A unit square MNOP having vertices M(0, 0), N(1, 0), O(1, 1) and P(0, 1) is transformed under the matrix transformation through *y* = – *x* and write the vertices of the images quadrilateral M'N'O'P' so formed. [2069 R]

⇨ Here, M(0, 0), N(1, 0), O(1, 1) and P(0, 1) are the vertices of unit square.

∴ Object in matrix form = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

In the reflection in y = – x, the image of point (x, y) is (– y, – x)

i.e. $x = 0 . x + (-1) . y$

$y = (-1) . x + 0 . y$

∴ TM = $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

We know that, Image = TM × Object = $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+0 & 0+0 & 0-1 & 0-1 \\ 0+0 & -1+0 & -1+0 & 0+0 \end{pmatrix}$

∴ Image = $\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$

Thus, the vertices of image are M'(0, 0), N'(0, – 1), O'(– 1, – 1) and P'(– 1, 0).

8. शीर्षबिन्दुहरू P(– 3, 2), Q(– 1, 4) र R(2, 0) भएको ΔPQR लाई X-अक्षमा परावर्तन हुने प्रतिबिम्ब मेट्रिक्स विधि प्रयोग गरी यसका आकृतिको प्रतिबिम्बहरू पत्ता लगाउनुहोस् ।
P(– 3, 2), Q(– 1, 4) and R(2, 0) are the vertices of ΔPQR. Find the vertices of the image triangle under the reflection on X-axis by using matrix method. [2064 R']

⇨ Here, P(– 3, 2), Q(– 1, 4) and R(2, 0) are the vertices of ΔPQR.

Object in matrix form = $\begin{pmatrix} -3 & -1 & 2 \\ 2 & 4 & 0 \end{pmatrix}$

In the reflection in X-axis the image is (x, – y).

∴ $x = 1.x + 0.y$ and $-y = 0.x + (-1).y$ ∴ TM = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Now, Image = TM × Object = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -3 & -1 & 2 \\ 2 & 4 & 0 \end{pmatrix} = \begin{pmatrix} -3+0 & -1+0 & 2+0 \\ 0-2 & 0-4 & 0+0 \end{pmatrix}$

∴ Image = $\begin{pmatrix} -3 & -1 & 2 \\ -2 & -4 & 0 \end{pmatrix}$

Thus, the vertices of image are P'(– 3, – 2), Q'(– 1, – 4) and R'(2, 0) .

MODEL 3

9. बिन्दुहरू A(4, 1) र B(7, 5) जोड्ने रेखाखण्ड AB लाई स्थानान्तरण गर्दा A'(– 4, 1) र B'(– 7, 5) बिन्दुहरू जोड्ने रेखाखण्ड A'B' बन्दछ । यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
A line segment AB joining the points A(4, 1) and B(7, 5) is transformed to the line segment A'B' joining the points A'(– 4, 1) and B'(– 7, 5). Find the 2 × 2 matrix that represents this transformation. [SEE 2075 R, 2075 R₂]

⇨ Here, let the 2 × 2 transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

We know that, TM × Object = Image

or, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 4 & 7 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} -4 & -7 \\ 1 & 5 \end{pmatrix}$

or, $\begin{pmatrix} 4a+b & 7a+5b \\ 4c+d & 7c+5d \end{pmatrix} = \begin{pmatrix} -4 & -7 \\ 1 & 5 \end{pmatrix}$

Equating the corresponding elements, we get,

$4a + b = -4$ ....(A) and

$7a + 5b = -7$ ....(B)

$4c + d = 1$ ....(C) and

$7c + 5d = 5$ ....(D)

Solving (A) and (B), we get,

$(4a + b = -4) \times 5$

$7a + 5b = -7$

$\underline{- \quad - \quad +}$

$13a = -13$ ∴ $a = -1$

And, $4 \times -1 + b = -4$

∴ $b = 0$

Solving (C) and (D), we get,

$(4c + d = 1) \times 5$

$7c + 5d = 5$

$\underline{- \quad - \quad -}$

$13c = 0$

∴ $c = 0$ and $4c + d = 1$

∴ $d = 1$

Thus, the required 2 × 2 transformation matrix is,

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

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10. **बिन्दु P(5, 1) र Q(8, 6) भएको रेखा PQ लाई स्थानान्तरण गर्दा PQ को प्रतिबिम्ब $P_1(-5, 1)$ र $Q_1(-8, 6)$ भएको रेखा P_1Q_1 बन्दछ भने कुन एउटै स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।**
Line PQ having P(5, 1) and Q(8, 6) maps onto the line P_1Q_1 having $P_1(-5, 1)$ and $Q_1(-8, 6)$ so that the image P_1Q_1 of PQ is formed. Which is the single transformation for this mapping ? Also find 2 × 2 matrix that represents the transformation. **[2059 R]**

⇨ Here, the given image of line PQ is P_1Q_1 where $P(5, 1) \rightarrow P_1(-5, 1)$ and $Q(8, 6) \rightarrow Q_1(-8, 6)$
Since, the point P(x, y) reflected on y-axis gives the image $P'(-x, y)$
i.e. $P(x, y) \rightarrow P'(-x, y)$ under the reflection along y-axis
So, the single transformation that line PQ maps to line P_1Q_1 is "reflection about y-axis".

Again, let the required matrix that maps PQ to P_1Q_1 is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

So that,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{array}{c} \begin{array}{cc} P & Q \end{array} \\ \begin{pmatrix} 5 & 8 \\ 1 & 6 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} P_1 & Q_1 \end{array} \\ \begin{pmatrix} -5 & -8 \\ 1 & 6 \end{pmatrix} \end{array}$$

or, $\begin{pmatrix} 5a+b & 8a+6b \\ 5c+d & 8c+d \end{pmatrix} = \begin{pmatrix} -5 & -8 \\ 1 & 6 \end{pmatrix}$

or, $5a + b = -5$ .... (i) and $8a + 6b = -8$ .... (ii)
$5c + d = 1$ .... (iii) and $8c + 6d = 6$ .... (iv)
From equation (i), putting $b = -5 - 5a$, in (ii) we get,
$8a + 6(-5 - 5a) = -8$
or, $8a - 30 - 30a = -8$
or, $-22a = 30 - 8$
or, $-22a = 22$
∴ $a = -1$ and $b = -5 - 5 \times (-1) = -5 + 5 = 0$
Again from equation (iii), putting $d = 1 - 5c$ in (iv) we get,
$8c + 6(1 - 5c) = 6$
or, $8c + 6 - 30c = 6$
or, $-22c = 6 - 6$
∴ $c = 0$ and $d = 1 - 5 \times 0 = 1$

Thus, the required matrix $= \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

11. **बिन्दु P(2, 8) र Q(7, 1) भएको रेखा PQ लाई स्थानान्तरण गर्दा PQ को प्रतिबिम्ब $P_1(-8, -2)$ र $Q_1(-1, -7)$ भएको रेखा P_1Q_1 बन्दछ भने कुन एउटै स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।**
Line PQ with P(2, 8) and Q(7, 1) maps onto the line P_1Q_1 having $P_1(-8, -2)$ and $Q_1(-1, -7)$ so that the image P_1Q_1 of PQ is formed. Which is the single transformation for this mapping? Also find 2 × 2 matrix which represents this transformation. **[2061 R]**

⇨ Here, given the images of the points; $P(2, 8) \rightarrow P_1(-8, -2)$ and $Q(7, 1) \rightarrow Q_1(-1, -7)$
We know that the image of a point P(a, b) under reflection on the line $y = -x$ is $P(a, b) \rightarrow P'(-b, -a)$.
Hence the single transformation for the mappings
$P(2, 8) \rightarrow P_1(-8, -2)$ and $Q(7, 1) \rightarrow Q_1(-1, -7)$ is a reflection on the line $y = -x$.

Again let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2 × 2 matrix which transforms $P \rightarrow P_1$ and $Q \rightarrow Q_1$ then,

we have,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{array}{c} \begin{array}{cc} P & Q \end{array} \\ \begin{pmatrix} 2 & 7 \\ 8 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} P_1 & Q_1 \end{array} \\ \begin{pmatrix} -8 & -1 \\ -2 & -7 \end{pmatrix} \end{array}$$

or, $\begin{pmatrix} 2a+8b & 7a+b \\ 2c+8d & 7c+d \end{pmatrix} = \begin{pmatrix} -8 & -1 \\ -2 & -7 \end{pmatrix}$

or, $2a + 8b = -8$ ...... (i) and $7a + b = -1$ .... (ii)
$2c + 8d = -2$ .... (iii) and $7c + d = -7$ .... (iv)
From equation (ii); putting $b = -1 - 7a$ in equation (i), we get, $2a + 8(-1 - 7a) = -8$
or, $2a - 8 - 56a = -8$
or, $-54a = -8 + 8$
or, $-54a = 0$
∴ $a = 0$ and $b = -1 - 7a = -1 - 7 \times 0 = -1$
Again, from equation (iv), putting $d = -7 - 7c$ in equation (iii) we get,
$2c + 8(-7 - 7c) = -2$
or, $2c - 56 - 56c = -2$
or, $-54c = -2 + 56$
∴ $c = \frac{-54}{54} = -1$ and $d = -7 - 7 \times -1 = -7 + 7 = 0$

Thus, required matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

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MODEL 4

12. **P(1, 2), Q(4, 1) र R(2, 5) शीर्षबिन्दुहरू भएको त्रिभुज PQR लाई एउटा 2 × 2 मेट्रिक्सले स्थानान्तरण गर्दा बन्ने प्रतिबिम्बको शीर्षबिन्दुहरूका निर्देशाङ्क P'(5, 2), Q'(6, 1) र R'(12, 5) भए 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।**
Triangle PQR having the vertices P(1, 2), Q(4, 1) and R(2, 5) is transformed by a 2 × 2 matrix so that the co-ordinates of the vertices of its image are P'(5, 2), Q'(6, 1) and R'(12, 5). Find the 2 × 2 matrix. **[SEE 2075 R'$_2$]**

⇨ Here, let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the required 2 × 2 transformation matrix.

We have, TM × object = Image

or, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 12 \\ 2 & 1 & 5 \end{pmatrix}$

or, $\begin{pmatrix} a+2b & 4a+b & 2a+5b \\ c+2d & 4c+d & 2c+5d \end{pmatrix} = \begin{pmatrix} 5 & 6 & 12 \\ 2 & 1 & 5 \end{pmatrix}$

Now, equating the corresponding elements then

$a + 2b = 5$ .... (i) and $4a + b = 6$ .... (ii)

$c + 2d = 2$ .... (iii) and $4c + d = 1$ .... (iv)

from (i) and (ii),

$$\begin{array}{l} (a + 2b = 5) \times 4 \\ 4a + b = 6 \\ \underline{-\quad - \quad -} \\ 7b = 14 \end{array}$$

∴ $b = 2$ and putting $b = 2$ in (i), $a + 2 \times 2 = 5$

∴ $a = 1$

from (iii) and (iv)

$$\begin{array}{l} (c + 2d = 2) \times 4 \\ 4c + d = 1 \\ \underline{-\quad - \quad -} \\ 7d = 7 \end{array}$$

∴ $d = 1$ and putting $d = 1$ in (iv), $4c + 1 = 1$

∴ $c = 0$

So, $a = 1, b = 2, c = 0$ and $d = 1$

Thus, the required 2 × 2 transformation matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

13. **शीर्षबिन्दुहरू P(4, 3), Q(6, 4) र R(8, 1) भएको ΔPQR लाई शीर्षबिन्दुहरू P'(– 3, – 4), Q'(– 4, – 6) र R'(– 1, – 8) भएको ΔP'Q'R' मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।**
Find a 2 × 2 matrix which transforms a ΔPQR with vertices P(4, 3), Q(6, 4) and R(8, 1) into the ΔP'Q'R' with vertices P'(– 3, – 4), Q'(– 4, – 6) and R'(–1 , – 8). **[2072 R', 2060 CP]**

⇨ Here, given vertices of ΔPQR; P(4, 3), Q(6, 4) and R(8, 1)

Vertices of ΔP'Q'R'; P'(– 3, – 4), Q'(– 4, – 6) and R'(– 1, – 8).

Object in matrix form = $\begin{bmatrix} 4 & 6 & 8 \\ 3 & 4 & 1 \end{bmatrix}$ Image in the matrix form = $\begin{bmatrix} -3 & -4 & -1 \\ -4 & -6 & -8 \end{bmatrix}$

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2 × 2 transformation matrix then, Image = TM × Object

or, $\begin{bmatrix} -3 & -4 & -1 \\ -4 & -6 & -8 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 4 & 6 & 8 \\ 3 & 4 & 1 \end{bmatrix}$

∴ $\begin{bmatrix} -3 & -4 & -1 \\ -4 & -6 & -8 \end{bmatrix} = \begin{bmatrix} 4a+3b & 6a+4b & 8a+b \\ 4c+3d & 6c+4d & 8c+d \end{bmatrix}$

Equating the corresponding elements;

$4a + 3b = -3$ .... (i)

$8a + b = -1$ .... (ii)

$4c + 3d = -4$ .... (iii)

$8c + d = -8$ .... (iv)

Solving equation (i) and (ii) × 3 then,

$$\begin{array}{l} 4a + 3b = -3 \\ 24a + 3b = -3 \\ \underline{-\quad -\qquad +} \\ -2a \quad = 0 \qquad \therefore a = 0 \end{array}$$

Solving equation (iii) × 2 and (iv)

$$\begin{array}{l} 8c + 6d = -8 \\ 8c + d = -8 \\ \underline{-\quad -\qquad +} \\ 5d \quad = 0 \qquad \therefore d = 0 \end{array}$$

From (ii)

$8a + b = -1$

or, $8 \times 0 + b = -1$

∴ $b = -1$

From (iv);

$8c + d = -8$

or, $8c + 0 = -8$

∴ $c = -1$

Thus, the required TM is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

14. **शीर्षबिन्दुहरू A(3, 6), B(5, – 3) र C(– 4, 2) भएको ΔABC लाई एउटा 2 × 2 मेट्रिक्सले स्थानान्तरण गर्दा बन्ने प्रतिबिम्बको शीर्षबिन्दुहरूका निर्देशाङ्कहरू A'(– 3, – 6), B'(– 5, 3) र C'(4, – 2) भए 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।**
ΔABC having the vertices A(3, 6), B(5, – 3) and C(– 4, 2) is transformed by a 2 × 2 matrix so that the co-ordinates of the vertices of its image are A'(– 3, – 6), B'(–5, 3) and C'(4, – 2). Find the 2 × 2 matrix. **[2070 R]**

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⇨ Here, co-ordinates of ΔABC are: A(3, 6), B(5, – 3) and C(– 4, 2)

So, object in matrix from $= \begin{bmatrix} 3 & 5 & -4 \\ 6 & -3 & 2 \end{bmatrix}$

Co-ordinates of image are; A(– 3, – 6), B'(– 5, 3), C'(4, – 2)

Image in matrix form $= \begin{bmatrix} -3 & -5 & 4 \\ -6 & 3 & -2 \end{bmatrix}$

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2 × 2 transformation matrix.

Then, Image = TM × Object

or, $\begin{bmatrix} -3 & -5 & 4 \\ -6 & 3 & -2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 5 & -4 \\ 6 & -3 & 2 \end{bmatrix}$

or, $\begin{bmatrix} -3 & -5 & 4 \\ -6 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 3a+6b & 5a-3b & -4a+2b \\ 3c+6d & 5c-3d & -4c+2d \end{bmatrix}$

Equating the corresponding elements;

$3a + 6b = -3 \Rightarrow a + 2b = -1$ ……… (i)

$-4a + 2b = 4 \Rightarrow 2a - b = -2$ …….. (ii)

$3c + 6d = -6 \Rightarrow c + 2d = -2$ ………..(iii)

$-4c + 2d = -2 \Rightarrow 2c - d = 1$ ………..(iv)

Solving equation (iii) and (iv) × 2 then,

$c + 2d = -2$

$\underline{4c - 2d = 2}$

$5c = 0 \quad \therefore c = 0$

Now, TM $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Solving equation (i) × 2 and (ii)

$2a + 4b = -2$

$2a - b = -2$

$\underline{-\quad +\quad +}$

$5b = 0 \quad \therefore b = 0$

Putting b = 0 in equation (i) then,

$a + 2b = -1$

or, $a + 2 \times 0 = -1$

$\therefore \ a = -1$

Putting the value of c in (ii),

$0 + 2d = -2$

$\therefore \ d = -1$

Thus, the required TM is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

15. शीर्षबिन्दु P(3, 6), Q(4, 2), र R(2, 2) भएको ΔPQR लाई स्थानान्तरण गर्दा P_1(– 6, 3), Q_1 (– 2, 4) र R_1 (– 2, 2) शीर्षबिन्दु भएको प्रतिबिम्ब $\Delta P_1Q_1R_1$ बन्दछ भने कुन एउटै स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2×2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।

ΔPQR whose vertices are P(3, 6), Q(4, 2) and R(2, 2) maps onto the $\Delta P_1Q_1R_1$ with the vertices P_1 (– 6, 3), Q_1(– 2, 4) and R_1(– 2, 2). Which is the single transformation for this mapping? Also find the 2 × 2 matrix that represents this transformation. **[2058 R]**

⇨ Here, given in the transformation of ΔPQR onto $\Delta P_1Q_1R_1$

P (3, 6) → P_1(– 6, 3), Q (4, 2) → Q_1(– 2, 4) and R (2, 2) → R_1(– 2, 2)

We know that,

the rotation of positive quarter (+ 90°) turn about the origin gives the transformation of P (a, b) → P'(– b, a). Since, the given transformations are of the form P (a, b) → P'(– b, a). So, the required single transformation for the given mapping is the "rotation" which is the positive quarter turn (+ 90) through the origin.

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a required 2 × 2 matrix.

Then, When P is transformed to P_1 then

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$$

or, $\begin{pmatrix} 3a+6b \\ 3c+6d \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

$\therefore \ 3a + 6b = -6$ . . . . (i) and

$3c + 6d = 3$ . . . . .(ii)

When Q is transformed to Q_1, then,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

or, $\begin{pmatrix} 4a+2b \\ 4c+2d \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$\therefore \ 4a + 2b = -2$ . . . . (iii) and

$4c + 2d = 4$ . . . . (iv)

Similarly, when R is transformed to R_1, then,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

or, $\begin{pmatrix} 2a+2b \\ 2c+2d \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$\therefore \ 2a + 2b = -2$ .... (v) and

$2c + 2d = 2$ .... (vi)

Now, from equation (i), putting a = – 2 – 2b in equation (iii), we get

$4(-2 - 2b) + 2b = -2$

or, $-8 - 8b + 2b = -2$

or, $-6b = -2 + 8$

$\therefore \ b = \frac{6}{-6} = -1$

Putting the value of b in equation (i), we get

$a = -2 - 2 \times (-1) = -2 + 2 = 0$

Again, from equation (ii), putting c = 1 – 2d, in equation (iv), we get,

$4(1 - 2d) + 2d = 4$

or, $4 - 8d + 2d = 4$

or, $-6d = 4 - 4$

$\therefore \ d = 0$

Again putting the value of d in equation (ii), we get

$c = 1 - 2 \times 0 = 1$

Thus, the required 2 × 2 matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

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16. A(2, 7), B(2, 9), र C(6, 7) शीर्षबिन्दु भएको ΔABC लाई स्थानान्तरण गर्दा A'(7, 2), B'(9, 2) र C'(7, 6) शीर्षबिन्दु भएको ΔA'B'C' बन्दछ भने, कुन स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? र यस स्थानान्तरणलाई जनाउने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।
ΔABC with the vertices A(2, 7), B(2, 9) and C(6, 7) is mapped on to ΔA'B'C', whose vertices are A'(7, 2), B'(9, 2) and C'(7, 6). Which is the single transformation for the mapping? Also find the 2 × 2 matrix which represents this transformation. **[2058 S]**

⇨ Here, given vertices of ΔABC are;
A (2, 7), B (2, 9) and C (6, 7).
Vertices of image ΔA'B'C' are;
A' (7, 2), B' (9, 2) and C' (7, 6).

A (2, 7) → A' (7, 2),
B (2, 9) → B' (9, 2) and
C (6, 7) → C' (7, 6)

Since the given transformation is of the form P (a, b) → P' (b, a) which is the reflection in line y = x.

Hence, the given transformation is the reflection on the line y = x.

Now matrix form of vertices of ΔABC is $\begin{pmatrix} 2 & 2 & 6 \\ 7 & 9 & 7 \end{pmatrix}$

Also matrix form of image ΔA'B'C' is $\begin{pmatrix} 7 & 9 & 7 \\ 2 & 2 & 6 \end{pmatrix}$

Let, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a required matrix which transforms ΔABC → ΔA'B'C',

$$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 2 & 2 & 6 \\ 7 & 9 & 7 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{pmatrix} 7 & 9 & 7 \\ 2 & 2 & 6 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{pmatrix} 2a + 7b & 2a + 9b & 6a + 7b \\ 2c + 7d & 2c + 9d & 6c + 7d \end{pmatrix} = \begin{pmatrix} 7 & 9 & 7 \\ 2 & 2 & 6 \end{pmatrix}$$

Since two matrices are equal. So,
$2a + 7b = 7$ .... (i)
$2a + 9b = 9$ .... (ii)
$6a + 7b = 7$ .... (iii)
$2c + 7d = 2$ .... (iv)
$2c + 9d = 2$ .... (v)
$6c + 7d = 6$ .... (vi)

Now, from equation (i), putting $a = \frac{7 - 7b}{2}$ in (ii) we get,

$2\left(\frac{7 - 7b}{2}\right) + 9b = 9$
or, $7 - 7b + 9b = 9$
or, $2b = 9 - 7$
$\therefore$ $b = 1$

Putting b = 1 in (i), we get, $a = \frac{7 - 7b}{2} = \frac{7 - 7.1}{2} = \frac{0}{2} = 0$

Again from equation (iv), putting $c = \frac{2 - 7d}{2}$ in (v) we get, $2\left(\frac{2 - 7d}{2}\right) + 9d = 2$
or, $2 - 7d + 9d = 2$
or, $2d = 2 - 2$
$\therefore$ $d = 0$

Putting d = 0 in equation (iv) we get, $c = \frac{2 - 7.0}{2} = \frac{2}{2} = 1$

$\therefore$ The required 2 × 2 matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

17. शीर्षबिन्दु P(1, 2), Q(5, 2) र R(1, 5) भएको ΔPQR लाई स्थानान्तरण गर्दा P' (1, – 2), Q'(5, – 2), र R'(1, – 5) शीर्षबिन्दु भएको प्रतिबिम्ब ΔP'Q'R' बन्दछ भने कुन स्थानान्तरणले सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।
ΔPQR whose vertices are P(1, 2), Q(5, 2) and R(1, 5) maps on to the ΔP'Q'R' with the vertices P'(1, – 2), Q'(5, – 2), and R'(1, – 5). Which is the single transformation for this mapping ? Also find the 2 × 2 matrix which represents this transformation. **[2059 S]**

⇨ Here, given vertices of ΔPQR are P(1, 2), Q(5, 2) and R(1, 5)
The vertices of image ΔP'Q'R' are P'(1, – 2), Q'(5, – 2) and R'(1, – 5)
Where, P(1, 2) → P'(1, – 2), Q(5, 2) → Q'(5, – 2) and R(1, 5) → R'(1, – 5)
We know that the reflection of a point A (a, b) about x-axis is A'(a, – b) or, A(a, b) → A'(a, – b). Hence, the given transformation is the reflection on x-axis.

Let the 2 × 2 matrix representing the given transformation is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Then we have,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{array}{c} P \\ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{array} = \begin{array}{c} P' \\ \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{pmatrix} a + 2b \\ c + 2d \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$\therefore$ $a + 2b = 1$ .... (i) and $c + 2d = -2$ .... (ii)

$$\text{Again, } \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{array}{c} Q \\ \begin{pmatrix} 5 \\ 2 \end{pmatrix} \end{array} = \begin{array}{c} Q' \\ \begin{pmatrix} 5 \\ -2 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{pmatrix} 5a + 2b \\ 5c + 2d \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$\therefore$ $5a + 2b = 5$ ..... (iii) and $5c + 2d = -2$ ...... (iv)

$$\text{And, } \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{array}{c} R \\ \begin{pmatrix} 1 \\ 5 \end{pmatrix} \end{array} = \begin{array}{c} R' \\ \begin{pmatrix} 1 \\ -5 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{pmatrix} a + 5b \\ c + 5d \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$\therefore$ $a + 5b = 1$ .... (v) and $c + 5d = -5$ ...... (vi)

Putting (i) a = 1 – 2b from (i) in equation (iii), we get,
$5(1 - 2b) + 2b = 5$
or, $5 - 10b + 2b = 5$
or, $-8b = 5 - 5 = 0$ $\therefore$ $b = 0$
Again, putting the value of b in equation (i), we get,
$a + 2 \times 0 = 1$
$\therefore$ $a = 1$
Also from equation (ii) putting $c = -2 - 2d$ in equation (iv), we get,
$5(-2 - 2d) + 2d = -2$
or, $-10 - 10d + 2d = -2$
or, $-8d = 10 - 2 = 8$
$\therefore$ $d = -1$
Now putting the value of d in equation (iv), we get,
$5c + 2 \times (-1) = -2$
or, $5c - 2 = -2$
or, $5c = -2 + 2$
$\therefore$ $c = 0$
Thus, the required 2 × 2 matrix is;
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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18. शीर्षबिन्दुहरू P(– 2, 4), Q(6, 2) र R(1, – 1) भएको ΔPQR लाई स्थानान्तरण गर्दा P₁(4, 2), Q₁(2, – 6) र R₁ (– 1, – 1) शीर्ष बिन्दुहरू भएको ΔP₁ Q₁ R₁ बन्दछ भने कुन स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई जनाउने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।

ΔPQR with the vertices P(– 2, 4), Q(6, 2) and R(1, – 1) is mapped onto the ΔP₁ Q₁ R₁ whose vertices are P₁(4, 2), Q₁(2, – 6) and R₁ (– 1, – 1). Which is the single transformation for this mapping? Also find 2 × 2 matrix which represents this transformation. **[2062R]**

⇨ Here, given vertices of ΔPQR are $P(-2, 4)$, $Q(6, 2)$ and $R(1, -1)$

The vertices of image $\Delta P_1Q_1R_1$ are $P_1(4, 2)$, $Q_1(2, -6)$ and $R_1(-1, -1)$

or, $P(-2, 4) \rightarrow P_1(4, 2)$, $Q(6, 2) \rightarrow Q_1(2, -6)$ and $R(1, -1) \rightarrow R_1(-1, -1)$

Since the given transformation is of the form $A(a, b) \rightarrow A'(b, -a)$ which is the rotation about origin through $-90°$.

So the required single transformation for the given mapping is the rotation about origin through $-90°$. (negative quarter turn).

Again, Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a required 2 × 2 matrix.

Then, the matrix form of vertices of ΔPQR and the image $\Delta P_1Q_1R_1$ is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -2 & 6 & 1 \\ 4 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 2 & -1 \\ 2 & -6 & -1 \end{pmatrix}$$

or, $\begin{pmatrix} -2a+4b & 6a+2b & a-b \\ -2c+4d & 6c+2d & c-d \end{pmatrix} = \begin{pmatrix} 4 & 2 & -1 \\ 2 & -6 & -1 \end{pmatrix}$

or, $-2a + 4b = 4$ .... (i)

$6a + 2b = 2$ .... (ii)

$a - b = -1$ .... (iii)

$-2c + 4d = 2$ .... (iv)

$6c + 2d = -6$ .... (v) and

$c - d = -1$ .... (vi)

From equation (iii) putting $a = b - 1$ in equation (i), we get,

$-2(b - 1) + 4b = 4$

or, $-2b + 2 + 4b = 4$

or, $2b = 2$

$\therefore$ $b = 1$

Putting $b = 1$ in (iii) we get,

$a = b - 1 = 1 - 1 = 0$

$\therefore$ $a = 0$

Again from (vi), putting $c = d - 1$ in (iv), we get,

$-2(d - 1) + 4d = 2$

or, $-2d + 2 + 4d = 2$

or, $2d = 0$

$\therefore$ $d = 0$

Putting $d = 0$ in (vi) we get,

$c - 0 = -1$

$\therefore$ $c = -1$

Thus, the required 2 × 2 matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

19. P(– 3, – 4), Q(– 4, – 6) र R(– 1, – 8) शीर्षबिन्दु भएको ΔPQR लाई स्थानान्तरण गर्दा P₁(4, 3), Q₁(6, 4) र R₁(8, 1) शीर्षबिन्दुहरू भएको ΔP₁Q₁R₁ बन्दछ भने कुन स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई जनाउने 2 × 2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।

ΔPQR with the vertices P(– 3, –4), Q(– 4, – 6) and R(– 1, – 8) is mapped onto the ΔP₁Q₁R₁ whose vertices are P₁(4, 3), Q₁ (6, 4) and R₁(8, 1). Which is the single transformation for this mapping ? And also find the 2 × 2 matrix which represents this transformation. **[2062 S]**

⇨ Here, $P(-3, -4)$, $Q(-4, -6)$, $R(-1, -8)$ and $P_1(4, 3)$, $Q_1(6, 4)$, $R_1(8, 1)$

Given condition shows that $(x, y) \rightarrow (-y, -x)$.

So the single transformation for this mapping is reflection on the line $x = -y$.

$\therefore$ Object in matrix form $= \begin{pmatrix} -3 & -4 & -1 \\ -4 & -6 & -8 \end{pmatrix}$ $\therefore$ Image in matrix form $= \begin{pmatrix} 4 & 6 & 8 \\ 3 & 4 & 1 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the required transformation matrix.

Then, Image = TM × Object

or, $\begin{pmatrix} 4 & 6 & 8 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -3 & -4 & -1 \\ -4 & -6 & -8 \end{pmatrix}$

or, $\begin{pmatrix} 4 & 6 & 8 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} -3a-4b & -4a-6b & -a-8b \\ -3c-4d & -4c-6d & -c-8d \end{pmatrix}$

Equating the corresponding elements from equal matrices,

$-3a - 4b = 4$ ……. (i)

$-3c - 4d = 3$ ………… (iii)

$-a - 8b = 8$ ……….. (ii)

$-c - 8d = 1$ ………. (iv)

Solving (i) and (ii) × 3,

$$\begin{array}{l} -3a - 4b = 4 \\ -3a - 24b = 24 \\ \underline{+ \quad + \qquad -} \end{array}$$

or, $20b = -20$ $\therefore$ $b = -1$

From (i)

$-3a - 4(-1) = 4$

or, $-3a = 4 - 4 = 0$

$\therefore$ $a = 0$

Solving (iii) and (iv) × 3,

$$\begin{array}{l} -3c - 4d = 3 \\ -3c - 24d = 3 \\ \underline{+ \quad + \qquad -} \\ \quad 20\,d = 0 \end{array}$$

$\therefore$ $d = 0$

From (iii)

$-3c - 4 \times 0 = 3$

or, $-3c = 3$

$\therefore$ $c = -1$

So, the required transformation matrix;

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

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20. शीर्षबिन्दुहरू P(– 3, 5), Q(7, 4) र R(6, – 2) भएको ΔPQR लाई स्थानान्तरण गर्दा P_1(5, 3), Q_1(4, – 7) र R_1(– 2, – 6) शीर्षबिन्दुहरू भएको $\Delta P_1Q_1R_1$ बन्दछ भने यस स्थानान्तरणलाई जनाउने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

A ΔPQR with the vertices P(– 3, 5), Q(7, 4) and R(6, – 2) is mapped onto the $\Delta P_1Q_1R_1$ where vertices are P_1(5, 3), Q_1(4, – 7) and R_1(– 2, – 6). Find 2 × 2 matrix which represents this transformation. [2063 R']

⇨ Here, P(– 3, 5), Q(7, 4) and R(6, – 2)

$P_1(5, 3)$, $Q_1(4, – 7)$ and $R_1(– 2, – 6)$

Object in matrix form = $\begin{pmatrix} -3 & 7 & 6 \\ 5 & 4 & -2 \end{pmatrix}$

Image in the matrix form = $\begin{pmatrix} 5 & 4 & -2 \\ 3 & -7 & -6 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the transformation matrix.

Then, Image = TM × Object

or, $\begin{pmatrix} 5 & 4 & -2 \\ 3 & -7 & -6 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -3 & 7 & 6 \\ 5 & 4 & -2 \end{pmatrix}$

or, $\begin{pmatrix} 5 & 4 & -2 \\ 3 & -7 & -6 \end{pmatrix} = \begin{pmatrix} -3a+5b & 7a+4b & 6a-2b \\ -3c+5d & 7c+4d & 6c-2d \end{pmatrix}$

Equating the corresponding elements,

$-3a + 5b = 5$ .... (i)

$-3c + 5d = 3$ .... (iii)

$6a - 2b = -2$ .... (ii)

$6c - 2d = -6$ .... (iv)

Solving (i) × 2 and (ii)

$$\begin{aligned} -6a + 10b &= 10 \\ \underline{6a - 2b} &\underline{= -2} \\ 8b &= 8 \end{aligned}$$

$\therefore\ b = 1$

From (i)

$-3a + 5 = 5$

$\therefore\ a = 0$

Solving (iii) × 2 and (iv)

$$\begin{aligned} -6c + 10d &= 6 \\ \underline{6c - 2d} &\underline{= -6} \\ 8d &= 0 \end{aligned}$$

$\therefore\ d = 0$

From (iii)

$-3c + 0 = 3$

$\therefore\ c = -1$

Thus, the required transformation matrix;

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

MODEL 5

21. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2× 2 म्याट्रिक्स पत्ता लगाउनुहोस् ।

Find the 2× 2 matrix which transforms a unit square to a parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ [SEE MODEL 2076, 2074 S', 2065 E]

⇨ Here, Object = unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and Image = $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the 2 × 2 transformation matrix.

We know that, Image = TM × object

i.e. $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix}$

$\therefore\ \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

Equating the corresponding elements. Then, $a = 3$, $b = 1$, $c = 0$, $d = 1$ So, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$

Thus, the required transformation matrix is $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$

22. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।

Find the 2 × 2 transformation matrix which transforms the unit square into a parallelogram $\begin{bmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

[SEE 2074 S, 2073 R, 2067 R]

⇨ Here, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a required 2 × 2 matrix and $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ is a unit square matrix.

Now, according to question, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ or, $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

Taking the relation of corresponding elements a = 4, b = 2, c = 1 and d = 2

Thus, $\begin{pmatrix} 4 & 2 \\ 1 & 2 \end{pmatrix}$ is the required transformation matrix.

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23. $\begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$ लाई $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find 2 × 2 matrix, which transforms $\begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$ into $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$. **[2073 R']**

⇨ Here, object $= \begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$

Image $= \begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$

Let, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2 × 2 transformation matrix.

We know that, Image = TM × Object

or, $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$

or, $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 0+0 & 2a+3b & 3a+5b & 2a+b \\ 0+0 & 2c+3d & 3c+5d & 2c+d \end{bmatrix}$

or, $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2a+3b & 3a+5b & 2a+b \\ 0 & 2c+3d & 3c+5d & 2c+d \end{bmatrix}$

Equating the corresponding elements then,

$2a + 3b = 3$ .... (i)

$2c + 3d = 2$ .... (iii)

$2a + b = 1$ .... (ii)

$2c + d = 2$ .... (iv)

Solving equations (i) and (ii),

$$\begin{array}{l} 2a + 3b = 3 \\ 2a + b = 1 \\ \underline{-\quad -\quad -} \\ \quad 2b = 2 \end{array}$$

$\therefore$ b = 1

Putting b = 1 in (i) $2a + 3 \times 1 = 3$

or, $2a = 0$ $\quad\therefore$ a = 0

Solving equations (iii) and (iv) then,

$$\begin{array}{l} 2c + 3d = 2 \\ 2c + \ \ d = 2 \\ \underline{-\quad -\quad -} \\ \quad 2d \ \ = 0 \end{array}$$

$\therefore$ d = 0

From (iii); $2c + 3 \times 0 = 2$

or, $2c = 2$

$\therefore$ c = 1

So, TM $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Thus, the transformation matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

24. शीर्षबिन्दुहरू A(2, 0), B(5, 1), C(4, 4) र D(1, 3) भएको एउटा वर्ग ABCD लाई 2×2 मेट्रिक्सले समानान्तर चतुर्भुज A'B'C'D' मा स्थानान्तरण गर्दा समानान्तर चतुर्भुजका शीर्षबिन्दुहरू A'(2, 2), B'(7, 3), C'(12, – 4) र D'(7, – 5) हुन्छन् भने 2×2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।

A square ABCD with vertices A(2, 0), B(5, 1), C(4, 4) and D(1, 3) is mapped onto a parallelogram A'B'C'D' by a 2×2 matrix so that the vertices of the parallelogram are A'(2, 2), B'(7, 3), C'(12, – 4) and D'(7, – 5). Find the 2 × 2 transformation matrix. **[2072 R, 2066 R]**

⇨ Here, A(2, 0), B(5, 1), C(4, 4) and D(1, 3)

The object in matrix form $= \begin{bmatrix} 2 & 5 & 4 & 1 \\ 0 & 1 & 4 & 3 \end{bmatrix}$

Image: A'(2, 2), B'(7, 3), C'(12, – 4), D'(7, – 5)

Image in matrix form $= \begin{bmatrix} 2 & 7 & 12 & 7 \\ 2 & 3 & -4 & -5 \end{bmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be required TM.

We know that, Image = TM × object

or, $\begin{bmatrix} 2 & 7 & 12 & 7 \\ 2 & 3 & -4 & -5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 5 & 4 & 1 \\ 0 & 1 & 4 & 3 \end{bmatrix}$

or, $\begin{bmatrix} 2 & 7 & 12 & 7 \\ 2 & 3 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 2a+0 & 5a+b & 4a+4b & a+3b \\ 2c+0 & 5c+d & 4c+4d & c+3d \end{bmatrix}$

or, $\begin{bmatrix} 2 & 7 & 12 & 7 \\ 2 & 3 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 2a & 5a+b & 4a+4b & a+3b \\ 2c & 5c+d & 4c+4d & c+3d \end{bmatrix}$

Equating the corresponding elements;

$2a = 2 \qquad \Rightarrow a = 1$

$2c = 2 \qquad \Rightarrow c = 1$

$a + 3b = 7 \qquad \Rightarrow 1 + 3b = 7$

$\Rightarrow 3b = 6 \qquad \therefore b = 2$

$c + 3d = -5 \qquad \Rightarrow 1 + 3d = -5$

$\Rightarrow 3d = -6 \qquad \therefore d = -2$

Thus, the required TM is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$.

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25. **कुन 2 × 2 मेट्रिक्सले एकाई वर्गलाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix}$ मा विस्थापन गर्छ ?**

Which 2 × 2 matrix maps a unit square to parallelogram $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix}$? **[2071 R']**

⇨ Here, Object = unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and Image = $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be 2 × 2 transformation matrix.

We know that, Image = TM × Object

$$\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & a+0 & a+b & 0+b \\ 0 & c+0 & c+d & 0+d \end{pmatrix}$$

$$\therefore \begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$$

Equating the corresponding elements then, $a = 3$, $b = 2$, $c = -1$ and $d = -1$

$$\therefore \quad TM = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -1 & -1 \end{pmatrix}$$

Thus, $\begin{pmatrix} 3 & 2 \\ -1 & -1 \end{pmatrix}$ is the required transformation matrix.

26. **शीर्षबिन्दुहरू P(– 2, 0), Q(– 5, 1), R(– 4, 4) र S(– 1, 3) भएको एउटा वर्ग PQRS लाई 2 × 2 मेट्रिक्सले समानान्तर चतुर्भुज P'Q'R'S' मा स्थानान्तरण गर्दा समानान्तर चतुर्भुजका शीर्षबिन्दुहरू P'(– 2, – 2), Q'(– 3, – 7), R'(4, – 12) र S'(5, – 7) हुन्छन् भने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।**

A square PQRS with vertices P(– 2, 0), Q(– 5, 1), R(– 4, 4) and S(– 1, 3) is mapped onto a parallelogram P'Q'R'S' by a 2 × 2 matrix so that the vertices of the parallelogram are P'(– 2, – 2), Q'(– 3, – 7), R'(4, – 12) and S'(5, – 7). Find the 2 × 2 transformation matrix. **[2070 R']**

⇨ Here, P(– 2, 0), Q(– 5, 1), R(– 4, 4) and S(– 1, 3) So, the object in the matrix form = $\begin{bmatrix} -2 & -5 & -4 & -1 \\ 0 & 1 & 4 & 3 \end{bmatrix}$

P'(– 2, – 2), Q'(– 3, – 7), R'(4, – 12), S'(5, – 7) So, the image in matrix form = $\begin{bmatrix} -2 & -3 & 4 & 5 \\ -2 & -7 & -12 & -7 \end{bmatrix}$

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2 × 2 transformation matrix.

We know that, Image = TM × Object

$$\text{or, } \begin{bmatrix} -2 & -3 & 4 & 5 \\ -2 & -7 & -12 & -7 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -2 & -5 & -4 & -1 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} -2 & -3 & 4 & 5 \\ -2 & -7 & -12 & -7 \end{bmatrix} = \begin{bmatrix} -2a & -5a+b & -4a+4b & -a+3b \\ -2c & -5c+d & -4c+4c & -c+3d \end{bmatrix}$$

Equating the corresponding elements then,

$-2a = -2 \quad \Rightarrow a = 1$

$-2c = -2 \quad \Rightarrow c = 1$

$-a + 3b = 5 \quad \Rightarrow -1 + 3b = 5 \quad \Rightarrow b = \frac{6}{3} = 2$

$-c + 3d = -7 \quad \Rightarrow -1 + 3d = -7 \quad \Rightarrow d = \frac{-6}{3} = -2$

$$\text{So, } TM = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

Thus, the transformation matrix is $\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$.

27. **A(0, 3), B(1, 1), C(3, 2) र D(2, 4) भएको वर्ग ABCD लाई 2 × 2 मेट्रिक्सद्वारा स्थानान्तरण गर्दा A'(6, – 6), B'(3, – 1), C'(7, – 1) र D'(10, – 6) भएको समानान्तर चतुर्भुज A'B'C'D' हुन्छ भने सो 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।**

A square ABCD whose vertices are A(0, 3), B(1, 1), C(3, 2) and D(2, 4) is mapped to the parallelogram A'B'C'D' by a 2 × 2 matrix so that the vertices of the parallelogram are A'(6, – 6), B(3, –1), C'(7, –1) and D'(10, – 6). Find the 2 × 2 matrix. **[2057 R, 2060 S, 2064 S, 2065 R']**

⇨ Here, the matrix formed by the vertices of the square ABCD is: $\begin{pmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{pmatrix}$

The matrix formed by the vertices of parallelogram A'B'C'D' is. $\begin{pmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{pmatrix}$

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Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the square matrix which maps the square ABCD to the parallelogram A'B'C'D'.

Then,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{pmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{pmatrix} a\times 0+b\times 3 & a\times 1+b\times 1 & a\times 3+b\times 2 & a\times 2+b\times 4 \\ c\times 0+d\times 3 & c\times 1+d\times 1 & c\times 3+d\times 2 & c\times 2+d\times 4 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{pmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{pmatrix} \end{array}$$

$$\text{or, } \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{pmatrix} 3b & a+b & 3a+2b & 2a+4b \\ 3d & c+d & 3c+2d & 2c+4d \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{pmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{pmatrix} \end{array}$$

The matrices on LHS and RHS will be equal only when the corresponding elements on both are the same.

Therefore, equating the corresponding coordinates of A' we get,

$3b = 6 \qquad \therefore\ b = 2 \qquad$ and $3d = -6 \qquad \therefore\ d = -2$

Again equating the entries of B', we get, $a + b = 3$ .... (i) and $c + d = -1$ .... (ii)

Putting $b = 2$ in (i), we get, $a + 2 = 3 \therefore\ a = 3 - 2 = 1$

and putting $d = -2$ in (ii), we get, $c - 2 = -1 \therefore\ c = 1$

Thus, the required square matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$

28. A(0, 0), B(1, 0), C(1, 1) र D(0, 1) भएको एउटा वर्ग ABCD लाई 2 × 2 मेट्रिक्स द्वारा स्थानान्तरण गर्दा A'(0, 0), B'(3, 0), C'(4, 1) र D'(1, 1) भएको समानान्तर चतुर्भुज A' B' C' D' हुन्छ भने सो मेट्रिक्स पत्ता लगाउनुहोस् ।

A square ABCD having the vertices A (0, 0), B (1, 0), C (1, 1) and D (0, 1) is mapped to the parallelogram A' B' C' D' by a 2 × 2 matrix so that the vertices of the parallelogram are A'(0, 0), B'(3, 0), C'(4, 1) and D'(1, 1). Find the matrix. [2057S, 2060S']

⇨ Here, vertices of square ABCD are A(0, 0), B(1, 0), C(1, 1) and D(0, 1) and the vertices of image parallelogram A'B'C'D' of square ABCD are A'(0, 0), B'(3, 0), C'(4, 1) and D'(1, 1).

Here, matrix form of vertices of square ABCD is; $\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$ and

matrix form of parallelogram A'B'C'D' is; $\begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2 × 2 matrix which transforms square ABCD to parallelogram A'B'C'D'. So that,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\text{or, } \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} a\times 0+b\times 0 & a\times 1+b\times 0 & a\times 1+b\times 1 & a\times 0+b\times 1 \\ c\times 0+d\times 0 & c\times 1+d\times 0 & c\times 1+d\times 1 & c\times 0+d\times 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\text{or, } \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Since, two matrices are equal. So $a = 3$, $c = 0$, $b = 1$ and $d = 1$

Thus, the required 2 × 2 matrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$.

29. P(0, 3), Q(1, 1), R(3, 2) र S(2, 4) भएको वर्ग PQRS लाई 2 × 2 मेट्रिक्सद्वारा परिवर्तन गर्दा P'(3, 0), Q'(1, – 1), R'(2, – 3) र S'(4, – 2) भएको समानान्तर चतुर्भुज P'Q'R'S' हुन्छ भने सो 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

A square PQRS with the vertices P(0, 3), Q(1, 1), R(3, 2) and S(2, 4) is mapped onto the parallelogram P'Q'R'S' by 2 × 2 matrix so that the vertices of the parallelogram are P'(3, 0), Q'(1, – 1), R'(2, – 3) and S'(4, – 2). Find the 2 × 2 matrix.

[2069 S, 2061 S]

⇨ Here, given vertices of square PQRS are P(0, 3), Q(1, 1), R(3, 2) and S(2, 4).

Vertices of parallelogram P'Q'R'S' when transformed by 2 × 2 matrix are P'(3, 0), Q'(1, – 1), R'(2, – 3) and S' (4, – 2).

Here matrix form of vertices of square PQRS is $\begin{pmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{pmatrix}$

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Matrix form of vertices of parallelogram P'Q'R'S' is $\begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & -1 & -3 & -2 \end{pmatrix}$

If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2 × 2 matrix which transforms square PQRS to the parm P'Q'R'S',

So, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 3 & 2 \\ 3 & 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & -1 & -3 & -2 \end{pmatrix}$

or, $\begin{pmatrix} a\times 0 + b\times 3 & a\times 1 + b\times 1 & a\times 3 + b\times 2 & a\times 2 + b\times 4 \\ c\times 0 + d\times 3 & c\times 1 + d\times 1 & c\times 3 + d\times 2 & c\times 2 + d\times 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & -1 & -3 & -2 \end{pmatrix}$

or, $\begin{pmatrix} 3b & a+b & 3a+2b & 2a+4b \\ 3d & c+d & 3c+2d & 2c+4d \end{pmatrix} = \begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & -1 & -3 & -2 \end{pmatrix}$

Equating the corresponding terms,

or, $3b = 3,\ a + b = 1,\ 3a + 2b = 2,\ 2a + 4b = 4$

$3d = 0,\ c + d = -1,\ 3c + 2d = -3,\ 2c + 4d = -2$

Here, $3b = 3$ $\quad\therefore\ b = 1,$

$a + b = 1$ $\quad\therefore\ a = 1 - b = 1 - 1 = 0$

$3d = 0$ $\quad\therefore\ d = 0$

And, $c + d = -1$ and $c = -1 - d = -1 - 0 = -1$

Thus, required matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

30. A(0, 0), B(1, 0), C(1, 1) र D(0, 1) शीर्षबिन्दु भएको एउटा एकाइ वर्गलाई शीर्षबिन्दुहरू A'(0, 0), B'(2, 0), C'(2, 1) र D'(0, 1) हुने समानान्तर चतुर्भुजमा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 matrix that maps the unit square ABCD having vertices A(0,0), B(1, 0), C(1, 1) and D(0, 1) to a parallelogram A'B'C'D' having vertices A'(0, 0), B'(2, 0), C'(2, 1) and D'(0, 1). [2063 M]

⇨ Here, Object = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and Image = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

TM = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the required matrix.

We have, TM × Object = Image

or, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Comparing corresponding terms, $a = 2,\ b = 0,\ c = 0$ and $d = 1$

Thus, the required transformation matrix = $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

31. एउटा 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् जसले शीर्षबिन्दुहरू O(0, 0), A(2, 0), B(2, 2) र C(0, 2) भएको वर्गलाई शीर्षबिन्दुहरू O'(0, 0), A'(4, 0), B'(10, 4) र C'(6, 4) भएको समानान्तर चतुर्भुजमा मापन गर्दछ ।

Find a 2 × 2 matrix that maps a square having vertices O(0, 0), A(2, 0), B(2, 2) and C(0, 2) to a parallelogram having vertices O'(0, 0), A'(4, 0), B'(10, 4) and C'(6, 4). [2063 S]

⇨ Here, Object = Square = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ and Image = Parallelogram = $\begin{pmatrix} 0 & 4 & 10 & 6 \\ 0 & 0 & 4 & 4 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the transformation matrix then,

Image = TM × Object

or, $\begin{pmatrix} 0 & 4 & 10 & 6 \\ 0 & 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 4 & 10 & 6 \\ 0 & 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 0+0 & 2a+0 & 2a+2b & 0+2b \\ 0+0 & 2c+0 & 2c+2d & 0+2d \end{pmatrix} = \begin{pmatrix} 0 & 2a & 2a+2b & 2b \\ 0 & 2c & 2c+2d & 2d \end{pmatrix}$

Equating the corresponding elements; $2a = 4,\ 2c = 0,\ 2b = 6,\ 2d = 4$

$\therefore\ a = 2,\ c = 0,\ b = 3,\ d = 2$

Thus, the required transformation matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$.

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32. A(0, 0), B(0, 1), C(1, 1), D(1, 0) भएको वर्ग ABCD लाई 2 × 2 मेट्रिक्सद्वारा स्थानान्तरण गर्दा A'(0, 0), B'(0, – 2), C'(2, – 2) र D'(2, 0) भएको वर्ग A' B' C' D' हुन्छ भने सो 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

A square ABCD whose vertices are A(0, 0), B(0, 1), C(1, 1), D(1, 0) is mapped onto the square A' B' C' D' by a 2 × 2 matrix so that the vertices of the square A' B' C' D' are A'(0, 0), B'(0, – 2), C'(2, – 2) and D'(2, 0). Find the 2 × 2 matrix. [2060 R]

⇨ Here, given vertices of square ABCD are A (0, 0), B (0, 1), C(1, 1), D (1, 0) and vertices of square A'B'C'D' are A' (0, 0), B' (0, – 2), C' (2, – 2) and D' (2, 0).

Here, vertices of square ABCD in the matrix form = $\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

And the vertices of square A'B'C'D' in matrix form = $\begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & -2 & -2 & 0 \end{pmatrix}$

If M = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix which translates square ABCD to A'B'C'D'

So, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & -2 & -2 & 0 \end{pmatrix}$

or, $\begin{pmatrix} a\times 0+b\times 0 & a\times 0+b\times 1 & a\times 1+b\times 1 & a\times 1+b\times 0 \\ c\times 0+d\times 0 & c\times 0+d\times 1 & c\times 1+d\times 1 & c\times 1+d\times 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & -2 & -2 & 0 \end{pmatrix}$

or, $\begin{pmatrix} 0 & b & a+b & a \\ 0 & d & c+d & c \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & -2 & -2 & 0 \end{pmatrix}$

Equating the corresponding elements.

or, $b = 0, a + b = 2,$

$\therefore$ $a = 2$

$d = -2, c + d = -2,$ $\therefore$ $c = 0$

Hence, $b = 0, a = 2, c = 0,$ $\therefore$ $d = -2$

Thus, required matrix M = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

33. बिन्दुहरू P(0, 0), Q(2, 0), R(2, 1) र S(0, 1) लाई क्रमशः P'(0, 0), Q'(2, 4), R'(2, 5) र S'(0, 1) मा स्थानान्तरण गर्ने एउटा 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Points P(0, 0), Q(2, 0), R(2, 1) and S(0, 1) are transformed by a 2 × 2 matrix to P'(0, 0), Q'(2, 4), R'(2, 5) and S'(0, 1) respectively. Find the matrix. [2066 S]

⇨ Here, P(0, 0), Q(2, 0), R(2, 1) and S(0, 1)

P'(0, 0), Q'(2, 4), R'(2, 5) and S'(0, 1)

Object in matrix form = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and Image in matrix form = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 4 & 5 & 1 \end{pmatrix}$

Let, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the transformation matrix.

Then, TM × Object = Image

or, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 4 & 5 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0+0 & 2a+0 & 2a+b & 0+b \\ 0+0 & 2c+0 & 2c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 4 & 5 & 1 \end{pmatrix}$

Equating the corresponding elements,

$2a = 2 \Rightarrow a = 1,$ $2c = 4 \Rightarrow c = 2,$ $b = 0,$ $d = 1$

$\therefore$ T.M. = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

Thus, the required transformation matrix is $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

34. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 transformation matrix which transforms a unit square to the parallelogram $\begin{bmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{bmatrix}$. [2068 R]

⇨ Here, let the transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Then, TM = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Object = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ Image = $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}$

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We know that, Image = TM × Object

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix} = \begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix}$

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

Equating the corresponding elements, a = 6, b = 2, c = 2 and d = 4

Thus, the TM = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 2 & 4 \end{pmatrix}$

35. **एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।**

Find the 2 × 2 matrix which the unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ transform to a parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

[2074 R', 2073 S', 2072 S, 2070 S', 2063 R]

⇨ Here, given unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and parallelogram = $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

Let the required 2 × 2 transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

We have, transformation matrix × object = image

So, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

or, $\begin{bmatrix} a\times 0+b\times 0 & a\times 1+b\times 0 & a\times 1+b\times 1 & a\times 0+b\times 1 \\ c\times 0+d\times 0 & c\times 1+d\times 0 & c\times 1+d\times 1 & c\times 0+d\times 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

or, $\begin{bmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{bmatrix} = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

or, $\begin{bmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{bmatrix} = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

Comparing the corresponding elements of equal matrices we get,

a = 3, b = 1, c = 1 and d = 2

Thus, the required 2 × 2 matrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.

36. **एकाइ वर्गलाई स.च. $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।**

Find 2 × 2 transformation matrix in which a unit square is transformed into a parallelogram $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ [2067 R']

⇨ Here, let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a required transformation matrix.

$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ be a unit square matrix.

Now, according to question, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$

Taking the relation of corresponding elements from equal matrices then, a = 2, b = 0, c = 0 and d = 2.

Thus, $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ is the required 2 × 2 matrix.

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37. एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 transformation matrix which transforms a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$to the parallelogram $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

[2075 R', 2073 S, 2068 R', 2065 M]

⇨ Here, let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$be a required transformation matrix.

Now, according to question, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

Taking the relation of corresponding elements from equal matrices,

$a = 3, b = 2, c = 1, d = 1$

Thus, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$is the required 2 × 2 matrix.

38. मेट्रिक्स $\begin{pmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$लाई समानान्तर चतुर्भुज $\begin{pmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$मा स्थानान्तरण गर्ने एउटा 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 transformation matrix in which a matrix $\begin{pmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ is transformed into a parallelogram $\begin{pmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$.

[2065 S]

⇨ Here, object = $\begin{pmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ Image = $\begin{bmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the transformation matrix.

Then we have, Image = TM × Object

or, $\begin{bmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2a+0 & 4a+0 & 4a+2b & 2a+2b \\ 2c+0 & 4c+0 & 4c+2d & 2c+2d \end{bmatrix}$

$\therefore \begin{bmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2a & 4a & 4a+2b & 2a+2b \\ 2c & 4c & 4c+2d & 2c+2d \end{bmatrix}$

Equating the corresponding elements then,

$2a = -2 \qquad \Rightarrow a = -1$

$2c = 0 \qquad \Rightarrow c = 0$

$2a + 2b = -2 \qquad \Rightarrow 2 \times (-1) + 2b = -2 \qquad \therefore\ b = 0$

$2c + 2d = 2 \qquad \Rightarrow 2 \times 0 + 2d = 2 \qquad \therefore\ d = 1$

Now, the transformation matrix = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Thus, $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is the required transformation matrix.

39. मेट्रिक्स $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ले आयत $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$लाई आयत$\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ मा स्थानान्तरण गर्दछ भने सो मेट्रिक्स $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ को मान निकाल्नुहोस् । कुन स्थानान्तरले गर्दा सो प्रतिबिम्ब बन्दछ ?

The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$maps the rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$onto the rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ find the value of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Which is the transformation for this mapping.

[2062 K]

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⇨ Here, TM = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, Image = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ and Object = $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

We know that, Image = TM × Object

or, $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 0+0 & 2a+0 & 2a+b & 0+b \\ 0+0 & 2c+0 & 2c+d & 0+d \end{pmatrix} = \begin{pmatrix} 0 & 2a & 2a+b & b \\ 0 & c & 2c+d & d \end{pmatrix}$

Equating the corresponding elements then,

$2a = 2$ $\quad \therefore \quad a = 1$ $\quad b = 0$

$2c = 0$ $\quad \therefore \quad c = 0$ $\quad d = -1$

Thus, the required transformation = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ represents the reflection in x-axis.

MODEL 6

40. **यदि मेट्रिक्स $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ ले एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$ मा स्थानान्तरण गर्दछ भने a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।**

If the matrix $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ transforms a unit square to the parallelogram $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$, find the values of a, b, c and d.

SEE 2074 R]

⇨ Here, Transformation matrix (TM) = $\begin{pmatrix} a & 2 \\ b & 2 \end{pmatrix}$ Object = Unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Image = Parallelogram = $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$

We know that, Image = TM × Object

or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} a & 2 \\ b & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} 0 & a+0 & a+2 & 0+2 \\ 0 & b+0 & b+2 & 0+2 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} 0 & a & a+2 & 2 \\ 0 & b & b+2 & 2 \end{pmatrix}$

Equating the corresponding elements then, $a = 4$, $b = 1$, $c = a + 2 = 4 + 2 = 6$, $d = 2$.

Thus, the values of a, b, c and d are 4 ,1, 6 and 2 respectively.

41. **यदि मेट्रिक्स $\begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}$ ले एकाइ वर्ग लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$ मा स्थानान्तरण गर्दछ भने *a, b, c* र *d* का मानहरू पत्ता लगाउनुहोस् ।**

If a matrix $\begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}$ transform a unit square to the parallelogram $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$ find the values of *a, b, c* and *d*.

⇨ Here, Transformation matrix (TM) = $\begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}$ Object = Unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Image = Parallelogram = $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$

We know that, Image = TM × Object

or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} 0 & a+1+0 & a+1+2 & 0+2 \\ 0 & b-1+0 & b-1+2 & 0+2 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix} = \begin{pmatrix} 0 & a+1 & a+3 & 2 \\ 0 & b-1 & b+1 & 2 \end{pmatrix}$

Equating the corresponding elements then,

$a + 1 = 4$ $\quad \therefore \quad a = 3$

$c = a + 3 = 3 + 3 = 6$ $\quad \therefore \quad c = 6$

$b - 1 = 1$ $\quad \therefore \quad b = 2$

$\therefore \quad d = 2$

Thus, the values of a, b, c and d are 3, 2, 6 and 2 respectively.

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QUESTIONS FROM CDC TEXTBOOK

7.3 मेट्रिक्सको प्रयोग गरी स्थानान्तरण (TRANSFORMATION USING MATRIX)

EXERCISE 7.3

1. (a) कुनै वस्तुलाई x-अक्षमा परावर्तन गर्ने 2 × 2 मेट्रिक्स लेख्नुहोस् । (Write the 2 × 2 matrix which reflects an object in x - axis.)

⇨ Here, the matrix which reflects in x - axis is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) कुनै वस्तुको [+ 90°, (0, 0)] परिक्रमणसँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write the matrix associated to the rotation [+90°, (0, 0)] for an object.

⇨ Here, the matrix associated to rotation [+ 90°, (0, 0)] is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(c) कुनै (2 × 2) क्रमको मेट्रिक्सलाई अर्को (2 × 2) क्रमको मेट्रिक्सले गुणन गर्दा कुन क्रमको मेट्रिक्स प्राप्त हुन्छ ?
What order's matrix can be obtained when a (2 × 2) matrix is multiplied by another (2 × 2) matrix?

⇨ Here, when a (2 × 2) matrix is multiplied by another (2 × 2) matrix then the order of the product will also be (2 × 2).

(d) मेट्रिक्स $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ ले कुन स्थानान्तरणलाई जनाउँछ ? (What transformation does the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ denote?)

⇨ Here, the transformation denoted by $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is reflection in the line x = – y.

2. बिन्दु A(– 4, 6) लाई तल दिइएका मेट्रिक्सहरूद्वारा स्थानान्तरण गर्नुहोस् । (Transform a point A(– 4, 6) by the following given matrices.)

(a) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ **(b)** $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(d)** $\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}$

⇨ (a) Here, A(– 4, 6) is transformed by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

So, Object in matrix form = $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Transformation matrix = $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

We have,

Image = TM + object

$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0-4 \\ 4+6 \end{pmatrix}$$

$\therefore$ Image = $\begin{pmatrix} -4 \\ 10 \end{pmatrix}$

Thus, the image is A' (– 4, 10).

⇨ (c) Here, A(– 4, 6) is transformed by $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

So, object in matrix form = $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Transformation matrix = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

We have,

Image = TM + Object

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2-4 \\ 3+6 \end{pmatrix}$$

$\therefore$ Image = $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

Thus, the image is A' (– 2, 9).

⇨ (b) Here, A(– 4, 6) is transformed by $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

So, object in matrix form = $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Transformation matrix = $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

We have,

Image = TM + Object

$$= \begin{pmatrix} -5 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -5-4 \\ 0+6 \end{pmatrix}$$

$\therefore$ Image = $\begin{pmatrix} -9 \\ 6 \end{pmatrix}$

Thus, the image is A' (– 9, 6).

⇨ (d) Here, A(– 4, 6) is transformed by $\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}$

So, object in matrix form = $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Image in matrix form = $\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}$

We have,

Image = Object + TM

$$= \begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -4+\frac{1}{2} \\ 6+\frac{3}{2} \end{pmatrix}$$

$\therefore$ Image = $\begin{pmatrix} -\frac{7}{2} \\ \frac{15}{2} \end{pmatrix}$

Thus, the image is A'$\left(-\frac{7}{2}, \frac{15}{2}\right)$.

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3. (a) **रेखाखण्ड PQ का छेउका निर्देशाङ्कहरू P(3, 4) र Q(8, 4) छन् । PQ लाई मेट्रिक्स $\begin{pmatrix}3&2\\1&5\end{pmatrix}$ले स्थानान्तरण गर्दा प्राप्त प्रतिबिम्बका निर्देशाङ्कहरू लेख्नुहोस् ।**

The co-ordinates of end points of a line segment PQ are P(3, 4) and Q(8, 4). Transform PQ by a matrix $\begin{pmatrix}3&2\\1&5\end{pmatrix}$ and write the co-ordinates of image.

⇨ Here, P(3, 4) and Q(8, 4)

So, object in matrix form $= \begin{pmatrix}3&8\\4&4\end{pmatrix}$ Given 2 × 2 transformation matrix $= \begin{pmatrix}3&2\\1&5\end{pmatrix}$

We have,

$$\text{Image} = \text{TM} \times \text{Object} = \begin{pmatrix}3&2\\1&5\end{pmatrix} \times \begin{pmatrix}3&8\\4&4\end{pmatrix} = \begin{pmatrix}3\times3+2\times4 & 3\times8+2\times4\\1\times3+5\times4 & 1\times8+5\times4\end{pmatrix} = \begin{pmatrix}9+8 & 24+8\\3+20 & 8+20\end{pmatrix}$$

$$\therefore \quad \text{Image} = \begin{pmatrix}17&32\\23&28\end{pmatrix}$$

Thus, the co-ordinates of images are P'(17, 23) and Q'(32, 28).

(b) **A(2, 3), B(2, 6), C(5, 2) र D(5, 6) शीर्षबिन्दुहरू भएको आयतलाई मेट्रिक्स $\begin{pmatrix}1&0\\2&1\end{pmatrix}$ले स्थानान्तरण गर्दा चतुर्भुज A'B'C'D' प्राप्त हुन्छ । A', B', C' र D' का निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।**

A rectangle having vertices A(2, 3), B(2, 6), C(5, 2) and D(5, 6) is transformed by a matrix $\begin{pmatrix}1&0\\2&1\end{pmatrix}$ so that the image is quadrilateral A' B' C' D'. Find the co-ordinates of A', B', C' and D'.

⇨ Here, A(2, 3), B(2, 6), C(5, 2) & D(5, 6) are the vertices of rectangle.

The object in matrix form $= \begin{pmatrix}2&2&5&5\\3&6&2&6\end{pmatrix}$ Transformation matrix $= \begin{pmatrix}1&0\\2&1\end{pmatrix}$

We have,

$$\text{Image} = \text{TM} \times \text{Object} = \begin{pmatrix}1&0\\2&1\end{pmatrix} \times \begin{pmatrix}2&2&5&5\\3&6&2&6\end{pmatrix} = \begin{pmatrix}1\times2+0\times3 & 1\times2+0\times6 & 1\times5+0\times2 & 1\times5+0\times6\\2\times2+1\times3 & 2\times2+1\times6 & 2\times5+1\times2 & 2\times5+1\times6\end{pmatrix}$$

$$\therefore \quad \text{Image} = \begin{pmatrix}2&2&5&5\\7&10&12&16\end{pmatrix}$$

Thus, the co-ordinates of images are A'(2, 7), B'(2, 10), C'(5, 12) and D'(5, 16).

(c) **एकाइ वर्ग OABC लाई मेट्रिक्स $\begin{pmatrix}3&2\\1&1\end{pmatrix}$ले स्थानान्तरण गर्दा चतुर्भुज O' A' B' C' प्राप्त हुन्छ । O', 'A, B', र C' का निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।**

A unit square OABC is transformed by $\begin{pmatrix}3&2\\1&1\end{pmatrix}$ then a quadrilateral O' A' B' C' is obtained. Find the co-ordinates of O', A', B' and C'.

⇨ Here, unit square in matrix form $\begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix}$ Given transformation matrix $= \begin{pmatrix}3&2\\1&1\end{pmatrix}$

We have,

$$\text{Image} = \text{TM} \times \text{Object} = \begin{pmatrix}3&2\\1&1\end{pmatrix} \times \begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix} = \begin{pmatrix}3\times0+2\times0 & 3\times1+2\times0 & 3\times1+2\times1 & 3\times0+2\times1\\1\times0+1\times0 & 1\times1+1\times0 & 1\times1+1\times1 & 1\times0+1\times1\end{pmatrix}$$

$$\therefore \quad \text{Image} = \begin{pmatrix}0&3&5&2\\0&1&2&1\end{pmatrix}$$

Thus, the co-ordinates of images are O'(0, 0), A'(3, 1), B'(5, 2) and C'(2, 1).

4. (a) **समानान्तर चतुर्भुज $\begin{pmatrix}0&3&4&1\\0&0&1&1\end{pmatrix}$लाई एकाइ वर्गमा स्थानान्तरण गर्ने मेट्रिक्स पत्ता लगाउनुहोस् ।**

Find a transformation matrix which transforms a parallelogram $\begin{pmatrix}0&3&4&1\\0&0&1&1\end{pmatrix}$ into a unit square.

⇨ Here, object in matrix form $= \begin{pmatrix}0&3&4&1\\0&0&1&1\end{pmatrix}$ Image in matrix form $= \begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix}$

Let $\begin{pmatrix}a&b\\c&d\end{pmatrix}$ be the required 2 × 2 matrix.

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We know that, Image = TM × Object

or, $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a\times 0+b\times 0 & a\times 3+b\times 0 & a\times 4+b\times 1 & a\times 1+b\times 1 \\ c\times 0+d\times 0 & c\times 3+d\times 0 & c\times 4+d\times 1 & c\times 1+d\times 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3a & 4a+b & a+b \\ 0 & 3c & 4c+d & c+d \end{pmatrix}$

Equating the corresponding elements then,

$3a = 1$ $\quad$ $3c = 0$ $\quad$ $a + b = 0$ $\quad$ $c + d = 1$

$\therefore\ a = \frac{1}{3}$ $\quad$ $\therefore\ c = 0$ $\quad$ or, $\frac{1}{3} + b = 0$ $\quad$ or, $0 + d = 1$

$\therefore\ b = -\frac{1}{3}$ $\quad$ $\therefore\ d = 1$

Thus, the required 2 × 2 transformation matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 \end{pmatrix}$

(b) एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 transformation matrix which transforms a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ into a parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

⇨ Here, let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2 × 2 transformation matrix.

Then, object in matrix form = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ $\qquad$ Image in matrix form = $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

We have, Image = TM × object

or, $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} a\times 0+b\times 0 & a\times 1+b\times 0 & a\times 1+b\times 1 & a\times 0+b\times 1 \\ c\times 0+d\times 0 & c\times 1+d\times 0 & c\times 1+d\times 1 & c\times 0+d\times 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

Equating the corresponding elements, $a = 3$, $c = 0$, $b = 1$, $d = 1$

Thus, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ is the required 2 × 2 matrix.

(c) एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2 × 2 matrix which transforms a unit square into a parallelogram $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix}$

⇨ Here, object = unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ $\qquad$ Image = parallelogram = $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix}$

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the 2 × 2 transformation matrix.

Then, Image = TM × Object

or, $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} a\times 0+b\times 0 & a\times 1+b\times 0 & a\times 1+b\times 1 & a\times 0+b\times 1 \\ c\times 0+d\times 0 & c\times 1+d\times 0 & c\times 1+d\times 1 & c\times 0+d\times 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

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Equating the corresponding elements,
Then a = 2, c = 3, b = 3, d = 1

Thus, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$ is the required transformation matrix.

5. (a) **उद्गम बिन्दु वरिपरि – 90° मा परिक्रमण गरी रेखा $x = 0$ मा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण $y = -x$ मा हुने परावर्तन सँग समतुल्य हुन्छ भनी मेट्रिक्स स्थानान्तरणद्वारा प्रमाणित गर्नुहोस् ।**
Verify by the matrix transformation method that the combined transformation of the rotation through – 90° about the origin followed by reflection in the line x = 0 is equivalent to the reflection in y = – x.

⇨ Here, R_1 = Rotation [– 90°, (0, 0)] and R_2 = Reflection in x = 0

$\therefore \quad R_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $R_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\text{So, } R_2 \circ R_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \times 0 + 0 \times (-1) & -1 \times 1 + 0 \times 0 \\ 0 \times 0 + 1 \times (-1) & 0 \times 1 + 1 \times 0 \end{pmatrix}$$

$$\therefore \quad R_2 \circ R_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

= Which is associated to the reflection in y = – x.

Thus, R[– 90°; (0, 0)] followed by reflection in x = 0 is equivalent to reflection in y = – x. **Proved.**

(b) **मेट्रिक्स स्थानान्तरण प्रयोग गरी उद्गम बिन्दुको वरिपरि + 90° धनात्मक दिशामा परिक्रमण गरी y-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।**
Verify by matrix transformation method that the combined transformation of rotation through + 90° about the origin followed by the reflection in y –axis is equivalent to the reflection in y = x.

⇨ Here, let R_1 = Rotation [+ 90°; (0, 0)]
R_2 = Reflection in y-axis

$\therefore \quad R_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $R_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\text{So, } R_2 \circ R_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \times 0 + 0 \times 1 & (-1) \times (-1) + 0 \times 0 \\ 0 \times 0 + 1 \times 1 & 0 \times (-1) + 1 \times 0 \end{pmatrix}$$

$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; Which is associated to the reflection in y = x.

Thus, R[+ 90°; (0, 0)] followed by reflection in y-axis is equivalent to reflection in y = x. **Proved.**

6. (a) **बिन्दु (x, y) लाई $(2x - y, 3x + 4y)$ मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।**
Find the 2 × 2 matrix which transforms a point (x, y) into (2x – y, 3x + 4y).

⇨ Here, $x \to 2x - y$ and $y \to 3x + 4y$
Expressing these mapping into equations then,
$x \to 2 . x - 1 . y$
$y \to 3 . x + 4 . y$
So, Image = TM × Object

$$= \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Thus, the 2 × 2 transformation matrix is $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$

(b) **बिन्दु (x, y) लाई $(x - 2y, 2x - 3y)$ मा स्थानान्तरण गर्ने मेट्रिक्स पत्ता लगाउनुहोस् ।**
Find the transformation matrix which transforms a point (x, y) into (x – 2y, 2x – 3y).

⇨ Here, $x \to x - 2y$ and $y \to 2x - 3y$
Expressing these mapping into equations then,
$x \to 1 . x - 2 . y$
$y \to 2 . x - 3 . y$
So, Image = TM × Object

$$= \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Thus, the 2 × 2 transformation matrix is $\begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

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तथ्याङ्कशास्त्र (Statistics)

1. चतुर्थांशीय विचलन
Quartile Deviation

FORMULAE

<table>
<tr><td colspan="4">अविच्छिन्न श्रेणीमा चतुर्थांशीय भिन्नता (Quartile Deviation in Continuous Series)</td></tr>
<tr><td>पहिलो चतुर्थांश (First Quartile): Q_1</td><td colspan="2">दोस्रो चतुर्थांश (Second Quartile): Q_2</td><td>तेस्रो चतुर्थांश (Third Quartile): Q_3</td></tr>
<tr><td>Q_1 श्रेणी (Q_1 Class) = $\left(\frac{N}{4}\right)^{th}$ item</td><td colspan="2">Q_2 श्रेणी (Q_2 Class) = $\left(\frac{N}{2}\right)^{th}$ item</td><td>Q_3 श्रेणी (Q_3 Class) = $\left(\frac{3N}{4}\right)^{th}$ item</td></tr>
<tr><td>$Q_1 = L + \frac{i}{f}\left(\frac{N}{4} - c.f.\right)$</td><td colspan="2">$Q_2 = L + \frac{i}{f}\left(\frac{N}{2} - c.f.\right)$</td><td>$Q_3 = L + \frac{i}{f}\left(\frac{3N}{4} - c.f.\right)$</td></tr>
<tr><td colspan="2">चतुर्थांशीय भिन्नता (Quartile Deviation):
$Q.D. = \frac{Q_3 - Q_1}{2}$</td><td colspan="2">Q.D. को गुणाङ्क (Coefficient of Q.D.) = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$</td></tr>
<tr><td colspan="4">अविच्छिन्न श्रेणीमा मध्यक भिन्नता (Mean Deviation in Continuous Series)</td></tr>
<tr><td colspan="2">मध्यकबाट मध्यक भिन्नता (Mean Deviation from mean):
$M.D.=\frac{\Sigma f|m - \overline{X}|}{N}$;
जहाँ, m = मध्यमान (mid value) र $\overline{X}$ = मध्यक (Mean)</td><td colspan="2">मध्यिकाबाट मध्यक भिन्नता (Mean Deviation from median):
$M.D.=\frac{\Sigma f|m - Med|}{N}$
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<tr><td colspan="2">M.D. को गुणाङ्क (Coefficient of M.D.) = $\frac{M.D.}{\text{मध्यक (Mean)}}$</td><td colspan="2">M.D. को गुणाङ्क (Coefficient of M.D.) = $\frac{M.D.}{\text{मध्यिका (Median)}}$</td></tr>
<tr><td colspan="4">M.D. को गुणाङ्क (Coefficient of M.D.) = (M.D.) ÷ ($\overline{X}$) OR (M.D.) ÷ (Med.)</td></tr>
<tr><td colspan="4">अविच्छिन्न श्रेणीमा स्तरीय भिन्नता (Standard Deviation in Continuous Series)</td></tr>
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Direct method</td><td>वास्तविक मध्यक बिधि
Actual mean method</td><td>अनुमानित मध्यक बिधि
Assumed mean method</td><td>पद बिचलन बिधि
Step deviation method</td></tr>
<tr><td>$\sigma = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$</td><td>$\sigma = \sqrt{\frac{\Sigma f(m - \overline{X})^2}{N}}$</td><td>$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$</td><td>$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times i$</td></tr>
<tr><td colspan="4">Where, $\overline{X}$ = मध्यक (Mean), d = X – अनुमानित मध्यक (assumed mean), $d' = \frac{X - \text{अनुमानित मध्यक (assumed mean)}}{\text{वर्गान्तर (class size)}}$
N = बारम्बारता (frequency), m = मध्यमान (mid value), S.D. = σ,</td></tr>
<tr><td colspan="4">बिचरणशीलताको गुणाङ्क (Coefficient of Variation) = $\frac{\sigma}{\overline{X}} \times 100\%$</td></tr>
</table>

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QUESTIONS FROM SEE EXERCISE 1

A. SHORT QUESTIONS

1. चतुर्थांशीय विचलनको परिभाषा दिनुहोस् ।

Define quartile deviation.

⇨ Here, the difference between the upper quartile (Q_3) and lower quartile (Q_1) is known as inter-quartile range and half of inter-quartile range is known as quartile deviation. It is given by, $QD = \frac{1}{2}(Q_3 - Q_1)$

2. चतुर्थांशीय विचलनको गुणाङ्क भन्नाले के बुझिन्छ ?

What do you mean by coefficient of quartile deviation?

⇨ Here, since quartile deviation is the absolute measure of dispersion, so the relative measure of dispersion depending upon the quartiles is known as coefficient of quartile deviation. It is given by, $\text{Coeff. of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

3. यदि $Q_1 = 34$ र $Q_3 = 57.5$ भए चतुर्थांशीय विस्तार र चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।

If $Q_1 = 34$ and $Q_3 = 57.5$, find the inter-quartile range and quartile deviation.

⇨ Here given, $Q_1 = 34$, $Q_3 = 57.5$

We have,

$\text{Inter-quartile range} = Q_3 - Q_1 = 57.5 - 34 = 23.5$

$\text{Quartile deviation (QD)} = \frac{Q_3 - Q_1}{2} = \frac{23.5}{2} = 11.75$

Thus, inter-quartile range is 23.5 and QD is 11.75.

4. यदि $Q_1 = 15.5$ र $Q_3 = 35.5$ भए चतुर्थांशीय विचलन र त्यसको गुणाङ्क पत्ता लगाउनुहोस् ।

If $Q_1 = 15.5$ and $Q_3 = 35.5$, find the quartile deviation and its coefficient.

⇨ Here given, $Q_1 = 15.5$, $Q_3 = 35.5$

We have, $QD = \frac{Q_3 - Q_1}{2} = \frac{35.5 - 15.5}{2} = \frac{20}{2} = 10$

$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{35.5 - 15.5}{35.5 + 15.5} = \frac{20}{51} = 0.392$

Thus, QD and its coefficient are 10 and 0.392 respectively.

5. कुनै श्रेणीको माथिल्लो चतुर्थांश र तल्लो चतुर्थांशको योग र अन्तर क्रमश : 31.25 र 10.25 छन् । माथिल्लो चतुर्थांश र तल्लो चतुर्थांश पत्ता लगाउनुहोस् ।

If the sum and difference of upper and lower quartiles of a series are 31.25 and 10.25 respectively, find the upper and lower quartiles.

⇨ Here given, $Q_3 + Q_1 = 31.25$ ....(i)

And, $Q_3 - Q_1 = 10.25$ ....(ii)

Where Q_3 = upper quartile and Q_1 = lower quartile

Adding equation (i) and equation (ii), we get,

$2Q_3 = 31.25 + 10.25 = 41.50$

$\therefore \quad Q_3 = \frac{41.50}{2} = 20.75$

Putting the value of Q_3 in equation (i) we get,

$Q_1 = 31.25 - Q_3 = 31.25 - 20.75 = 10.5$

Thus, $Q_1 = 10.5$ and $Q_3 = 20.75$.

6. कुनै श्रेणीको चतुर्थांशीय विचलनको गुणाङ्क 0.52 र चतुर्थांशीय विस्तार 12.5 छ । माथिल्लो र तल्लो चतुर्थांशहरूको योगफल पत्ता लगाउनुहोस् ।

The coefficient of Q.D. of a series is 0.52 and inter-quartile range is 12.5. Find the sum of upper and lower quartiles.

⇨ Here given, Coeff. of QD = 0.52,

Inter-quartile range $(Q_3 - Q_1) = 12.5$

We have, $\text{Coeff. of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

or, $0.52 = \frac{12.5}{Q_3 + Q_1}$

$\therefore \quad Q_3 + Q_1 = \frac{12.5}{0.52} = 24.04$

Thus, required sum is 24.04.

7. यदि कुनै तथ्याङ्कमा न्यूनतम 25% को अधिकतम मान 5.25 र अधिकतम 25% को न्यूनतम मान 25.75 भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।

If the maximum value of minimum 25% of a data is 5.25 and the minimum value of maximum 25% is 25.75, find the quartile deviation.

⇨ Here, first quartile (Q_1) is the maximum value of minimum 25% and upper quartile (Q_3) is the minimum value of maximum 25% of the data.

So, $Q_1 = 5.25$ and $Q_3 = 25.75$

We have,

$QD = \frac{Q_3 - Q_1}{2} = \frac{25.75 - 5.25}{2} = \frac{20.50}{2} = 10.25$

Thus, QD is 10.25.

8. यदि एउटा अविच्छिन्न तथ्याङ्कमा माथिल्लो चतुर्थांश र तल्लो चतुर्थांश क्रमश: 28 र 15 भए उक्त तथ्याङ्कको चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।

If the upper quartile and lower quartile of a continuous data are 28 and 15 respectively, find quartile deviation and its coefficient.

⇨ Here, upper quartile (Q_3) = 28 and

lower quartile (Q_1) = 15

$\therefore \quad \text{Quartile deviation (QD)} = \frac{Q_3 - Q_1}{2} = \frac{28 - 15}{2} = 6.5$

Coefficient of quartile deviation

$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{28 - 15}{28 + 15} = \frac{13}{43} = 0.302$

Thus, the quartile deviation and its coefficient are 6.5 and 0.302 respectively.

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9. यदि तल्लो चतुर्थांश र चतुर्थांशीय विचलन क्रमश: 13.5 र 9.5 भए चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
If the lower quartile and quartile deviation are 13.5 and 9.5 respectively, find coefficient of quartile deviation.

⇨ Here, lower quartile $(Q_1) = 13.5$ and
Quartile deviation (QD) = 9.5

We have, $\quad Q.D. = \dfrac{Q_3 - Q_1}{2}$

or, $9.5 = \dfrac{Q_3 - 13.5}{2}$

or, $19 = Q_3 - 13.5$

$\therefore \quad Q_3 = 32.5$

Again, coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{32.5 - 13.5}{32.5 + 13.5} = \frac{19}{46} = 0.41$$

Thus, the coefficient of quartile deviation is 0.41

11. एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको दुई गुणा तेस्रो चतुर्थांश छ । यदि सो तथ्याङ्कको मध्यिकाको मान 45 भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In a continuous series, third quartile is two times of first quartile. If the median of the data is 45, find the quartile deviation.

⇨ Here, median = 45 and $Q_3 = 2Q_1$

i.e. $\dfrac{Q_1 + Q_3}{2} = 45$

$\therefore \quad Q_1 + Q_3 = 90$

Now, $Q_1 + 2Q_1 = 90 \quad [\because\ Q_3 = 2Q_1]$

or, $3Q_1 = 90$

$\therefore \quad Q_1 = 30$ and

$Q_3 = 2Q_1 = 2 \times 30 = 60$

Now, quartile deviation $= \dfrac{Q_3 - Q_1}{2} = \dfrac{60 - 30}{2} = 15$

Thus, quartile deviation is 15.

13. एउटा निरन्तर तथ्याङ्कमा चतुर्थांशीय विस्तार 20 र Q_1 = 10 छन् । चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
The interquartile range of a continuous data is 20 and Q_1 = 10. What is the value of quartile deviation and its coefficient?

⇨ Here, interquartile range = 20 and $Q_1 = 10$

i.e. $Q_3 - Q_1 = 20$

or, $Q_3 - 10 = 20$ $\quad \therefore$

We have, Quartile Deviation (QD)

$$= \frac{Q_3 - Q_1}{2} = \frac{30 - 10}{2} = 10$$

Coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{30 - 10}{30 + 10} = \frac{20}{40} = 0.5$$

Thus, QD is 10 and it's coefficient is 0.5.

10. एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको स्थान 15$^{\text{औं}}$ पद छ । बारम्बारताहरूको योगफल र तेस्रो चतुर्थांशको स्थान पत्ता लगाउनुहोस् ।
In a continuous data, the position of first quartile is 15th term. Find the sum of frequencies of the data and the position of the third quartile.

⇨ Here, position of $Q_1 = 15^{th}$ term

i.e. $\dfrac{N}{4} = 15 \qquad \therefore \quad N = 60$

Now, position of $Q_3 = \left(\dfrac{3N}{4}\right)^{th}$ term

$= \left(\dfrac{3 \times 60}{4}\right)^{th}$ term

$= 45^{th}$ term

Thus, sum of frequencies (N) is 60 and position of Q_3 is 45^{th} term.

12. एउटा तथ्याङ्कको चतुर्थांशीय विचलन र पहिलो चतुर्थांश क्रमश: 20 र 35 छन् । सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, quartile deviation and the first quartile are 20 and 35 respectively. Find the third quartile and the coefficient of quartile deviation.

⇨ Here, quartile deviation = 20 and $Q_1 = 35$

i.e. $\dfrac{Q_3 - Q_1}{2} = 20$

or, $Q_3 - 35 = 40$

$\therefore \quad Q_3 = 75$

Coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{75 - 35}{75 + 35} = \frac{40}{110} = 0.36$$

Thus, Q_3 is 75 and coefficient of QD is 0.36.

14. एउटा तथ्याङ्कमा Q_3 र Q_1 को अन्तर 20 छ र तिनीहरूको योग 40 छ । QD र QD को गुणाङ्क पत्ता लगाउनुहोस् ।
The difference of Q_3 and Q_1 in a data is 20 and their sum is 40. Find QD and the coefficient of QD.

⇨ Here, $\quad Q_3 - Q_1 = 20$ .... (i)
$\qquad\qquad Q_3 + Q_1 = 40$ .... (ii)

Adding (i) and (ii) then $2Q_3 = 60$

$\therefore \quad Q_3 = 30$

From (i), $Q_3 - Q_1 = 20$

or, $30 - Q_1 = 20$

Now, Quartile Deviation (QD)

$$= \frac{Q_3 - Q_1}{2} = \frac{30 - 10}{2} = 10$$

Coefficient of QD $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{30 - 10}{30 + 10} = \dfrac{20}{40} = 0.5$

Thus, QD is 10 and it's coefficient is 0.5.

15. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमश: 15 र $\frac{3}{7}$ छन् । पहिलो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data, the quartile deviation and its coefficient are 15 and $\frac{3}{7}$ respectively. Find the first quartile.

⇨ Here, QD = 15 and coefficient of QD $= \dfrac{3}{7}$ $\qquad$ So, $\dfrac{Q_3 - Q_1}{2} = 15$ $\qquad \therefore \quad Q_3 - Q_1 = 30$ .... (i)

Again, $\dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{3}{7}$ $\qquad$ or, $\dfrac{30}{Q_3 + Q_1} = \dfrac{3}{7}$ $\qquad$ or, $Q_3 + Q_1 = \dfrac{30 \times 7}{3}$

$\therefore \quad Q_3 + Q_1 = 70$ .... (ii)

Subtracting (i) from (ii) then, $Q_3 + Q_1 - Q_3 + Q_1 = 70 - 30$

or, $2Q_1 = 40$ $\qquad \therefore \quad Q_1 = 20$ $\qquad$ Thus, first quartile (Q_1) is 20.

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16. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमश: 14 र $\frac{7}{22}$ छन् । तेस्रो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data the quartile deviation and its coefficient are 14 and $\frac{7}{22}$ respectively. Find the third quartile.

⇨ Here, QD = 14 and coefficient of QD = $\frac{7}{22}$

So, $\frac{Q_3 - Q_1}{2} = 14$

$\therefore\ Q_3 - Q_1 = 28$ .... (i)

Again, $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{7}{22}$

or, $\frac{28}{Q_3 + Q_1} = \frac{7}{22}$

or, $Q_3 + Q_1 = \frac{28 \times 22}{7}$

$\therefore\ Q_3 + Q_1 = 88$ .... (ii)

Adding (i) and (ii) then, $2Q_3 = 88 + 28$

$\therefore\ \frac{116}{2} = 58$

Thus, third quartile (Q_3) is 58.

17. एउटा तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय भिन्नता क्रमश: 58 र 28 छन् । चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

The third quartile and inter-quartile range of a data are 58 and 28 respectively. Find the coefficient of the quartile deviation.

⇨ Here, inter-quartile range = 28 and $Q_3 = 58$

So, $Q_3 - Q_1 = 28$

or, $58 - Q_1 = 28$

$\therefore\ Q_1 = 30$

We have, coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{58 - 30}{58 + 30} = \frac{28}{88} = \frac{7}{22}$$

Thus, the coefficient of QD is $\frac{7}{22}$.

18. एउटा तथ्याङ्कको तेस्रो चतुर्थांश 15 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{1}{14}$ भए सो तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The third quartile of a data is 15. If the coefficient of quartile deviation is $\frac{1}{14}$, find the first quartile and the inter-quartile range of the data.

⇨ Here, third quartile (Q_3) = 15 and

coefficient of QD = $\frac{1}{14}$

So, $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{14}$

or, $\frac{15 - Q_1}{15 + Q_1} = \frac{1}{14}$

or, $210 - 14Q_1 = 15 + Q_1$

or, $15\,Q_1 = 195$

$\therefore\ Q_1 = \frac{195}{15} = 13$

We have inter-quartile range $= Q_3 - Q_1$

$= 15 - 13 = 2$

Thus, first quartile is 13 and inter-quartile range is 2.

19. एउटा तथ्याङ्कको पहिलो चतुर्थांश 30 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{7}{22}$ भए सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The first quartile of a data is 30. If the coefficient of quartile deviation is $\frac{7}{22}$, find the third quartile and the inter-quartile range of the data.

⇨ Here, first quartile (Q_1) = 30 and

coefficient of QD = $\frac{7}{22}$

So, $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{7}{22}$

or, $\frac{Q_3 - 30}{Q_3 + 30} = \frac{7}{22}$

or, $22\,Q_3 - 660 = 7Q_3 + 210$

or, $15\,Q_3 = 870$

$\therefore\ Q_3 = \frac{870}{15} = 58$

We have, inter-quartile range $= Q_3 - Q_1 = 58 - 30 = 28$

Thus, third quartile is 58 and inter-quartile range is 28.

20. कुनै श्रेणीको पहिलो चतुर्थांशीय मान *x* र चतुर्थांशीय विचलन (*x*) भए त्यसको तेस्रो चतुर्थांशीय मान तथा चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

In a data, value of first quartile is '*x*' and quartile deviation is also (*x*). Find the third quartile and the coefficient of quartile deviation.

⇨ Here, first quartile (Q_1) = x and quartile deviation = x

So, $\frac{Q_3 - Q_1}{2} = x$

or, $Q_3 - x = 2x$

or, $Q_3 = x + 2x = 3x$

$\therefore\ Q_3 = 3x$

Again, coefficient of quartile deviation is; $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{3x - x}{3x + x} = \frac{2x}{4x} = 0.5$

Thus, third quartile is 3x and coefficient of QD is 0.5

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B. LONG QUESTIONS

1. **तलको तथ्याङ्कको चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।**
Calculate quartile deviation and its coefficient of the following data.

Items	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	10	20	10	5

⇨ Here, calculation of QD

Items	Frequency	c.f
10 - 20	5	5
20 - 30	10	15
30 - 40	20	35
40 - 50	10	45
50 - 60	5	50
	N = 50	

So, $N = \Sigma f = 50$

Now, $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{50}{4}\right)^{th} = 12.5^{th}$ item

The c.f. just greater than 12.5 is 15 whose corresponding class is 20 - 30.

$\therefore$ Exact $Q_1 = \ell + \frac{\frac{N}{4} - c.f.}{f} \times i = 20 + \frac{12.5 - 5}{10} \times 10 = 20 + 7.5 = 27.5$

Again, $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 12.5^{th} = 37.5^{th}$ item

The, c.f. just greater than 37.5 is 45 whose corresponding class is 40 - 50.

$\therefore$ Exact $Q_3 = \ell + \frac{\frac{3N}{4} - c.f.}{f} \times h = 40 + \frac{37.5 - 35}{10} \times 10 = 40 + 2.5 = 42.5$

Now, $QD = \frac{Q_3 - Q_1}{2} = \frac{42.5 - 27.5}{2} = \frac{15}{2} = 7.5$ Also, coeff. of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{42.5 - 27.5}{42.5 + 27.5} = \frac{15}{70} = 0.214$

Thus, QD is 7.5 and it's coefficient is 0.214.

2. **तल दिइएको तथ्याङ्कको चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।**
Find the quartile deviation and its coefficient of the data given below:

C.I	2 - 4	8 - 10	6 - 8	4 - 6	10 - 12
f	5	10	12	8	7

⇨ Here, arranging the given data into ascending order to calculate QD, we have

C.I	f	c.f
2 -4	5	5
4 - 6	8	13
6 - 8	12	25
8 - 10	10	35
10 - 12	7	42
	N = 42	

For Q_1: $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{42}{4}\right)^{th}$ item $= 10.5^{th}$ item

The, c.f. just greater that 10.5 is 13 whose corresponding class is 4 - 6.

Now, $Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 4 + \frac{10.5 - 5}{8} \times 2 = 4 + \frac{5.5 \times 2}{8} = 4 + \frac{11}{8} = 4 + 1.375$

$\therefore$ $Q_1 = 5.375$

For Q_3: $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 10.5^{th}$ item $= 31.5^{th}$ item

The, c.f. just greater than 31.5 is 35 whose corresponding class is 8 - 10.

Now, exact $Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 8 + \frac{31.5 - 25}{10} \times 2 = 8 + \frac{6.5}{5} = 8 + 1.3 = 9.3$ $\therefore$ $Q_3 = 9.3$

We have, $QD = \frac{Q_3 - Q_1}{2} = \frac{9.3 - 5.375}{2} = \frac{3.925}{2} = 1.9625$

Also, coeff. of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{9.3 - 5.375}{9.3 + 5.375} = \frac{3.925}{14.675} = 0.267$

Thus, QD is 1.9625 and its coefficient is 0.267.

3. **तलको तथ्याङ्कको चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।**
Find the Q.D. and its coefficient of the following data:

Age (In year)	5 - 10	5 - 15	5 - 20	5 - 25	5 - 30
No. of persons	4	16	32	38	40

⇨ Here, the given data is in cumulative frequency form, so changing into ordinary form and calculating QD,

Age	f	c.f
5 - 10	4	4
10 - 15	12	16
15 - 20	16	32
20 - 25	6	38
25 - 30	2	40
	N = 40	

For Q_1: Q_1 = value of $\left(\frac{N}{4}\right)^{th}$ item = value of $\left(\frac{40}{4}\right)^{th}$ item $= 10^{th}$ item

The c.f. just greater than 10 is 16 whose corresponding class is 10 - 15.

So, Q_1 class = 10 - 15

Now, exact $Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 10 + \frac{10 - 4}{12} \times 5 = 10 + \frac{6 \times 5}{12} = 10 + 2.5 = 12.5$

$\therefore$ $Q_1 = 12.5$

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For Q_3: $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 10^{th} = 30^{th}$ item

The c.f. just greater than 30 is 32 whose corresponding class is 15 - 20.

Now, Exact $Q_3 = L + \dfrac{\frac{3N}{4} - c.f.}{f} \times h = 15 + \dfrac{30-16}{16} \times 5 = 15 + \dfrac{14 \times 5}{16} = 15 + \dfrac{35}{8} = 15 + 4.375 = 19.375$

Now, $QD = \dfrac{Q_3 - Q_1}{2} = \dfrac{19.375 - 12.5}{2} = \dfrac{6.875}{2} = 3.4375$

Also, coeff. of $QD = \dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{19.375 - 12.5}{19.375 + 12.5} = \dfrac{6.875}{31.875} = 0.216$

Thus, QD is 3.4375 and it's coefficient is 0.216.

4. **तल दिइएको तथ्याङ्कको चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।**
Find the quartile deviation and its coefficient of the data given below:

Class Interval	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69
Frequency	8	16	4	4	4

⇨ Here, the given data is in inclusive type, so correction factor is $C_f = \dfrac{30-29}{2} = \dfrac{1}{2} = 0.5$

Then making the above data exclusive by subtracting 0.5 from each lower limit and adding 0.5 to each upper limit and calculating QD we have:

C.I	Frequency	c.f
19.5 - 29.5	8	8
29.5 - 39.5	16	24
39.5 - 49.5	4	28
49.5 - 59.5	4	32
59.5 - 69.5	4	36
	N = 36	

Now, for Q_1: $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{36}{4}\right)^{th}$ item $= 9^{th}$ term

The c.f. just greater than 9 is 24 whose corresponding class is (29.5 - 39.5).

Now, $Q_1 = L + \dfrac{\frac{N}{4} - c.f.}{f} \times h = 29.5 + \dfrac{9-8}{16} \times 10 = 29.5 + \dfrac{10}{16} = 29.5 + 0.625 = 30.125$

Q_3 = value of $\left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 9^{th}$ item $= 27^{th}$ item

The c.f. just greater than 27 is 28 whose corresponding class is 39.5 - 49.5

Now, exact $Q_3 = L + \dfrac{\frac{3N}{4} - c.f.}{f} \times h = 39.5 + \dfrac{27-24}{4} \times 10 = 39.5 + \dfrac{30}{4} = 39.5 + 7.5 = 47$

Now, quartile deviation $(QD) = \dfrac{Q_3 - Q_1}{2} = \dfrac{47 - 30.125}{2} = \dfrac{16.875}{2} = 8.4375$

Also, coeff. of $QD = \dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{47 - 30.125}{47 + 30.125} = \dfrac{16.875}{77.125} = 0.2188$

Thus, QD is 8.4375 and it's coefficient is 0.2188.

5. **एउटा कक्षाका 30 जना विद्यार्थीहरूले प्राप्त गरेको अङ्कहरू निम्नानुसार दिइएका छन् ।**
The marks obtained by 30 students in a class are given below:

20	41	65	60	73	70	30	55	39	40	26	34	43	49	50	53	57	52	56	45	77	78	61	52	50	56	44	37

20 - 30 पहिलो वर्गान्तर लिएर एउटा बारम्बारता तालिका बनाई माथिको तथ्याङ्कको चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
Prepare the frequency distribution of above data with 20 - 30 as the first class-interval and find the quartile deviation.

⇨ Here, Calculation of frequency table taking (20 - 30) as the first class interval and QD.

Marks	Tallies	f	c.f
20 - 30	|||	3	3
30 - 40	||||	4	7
40 - 50	卌 |	6	13
50 - 60	卌 ||||	9	22
60 - 70	||||	4	26
70 - 80	||||	4	30
		N = 30	

Now, Q_1 = value of $\left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{30}{4}\right)^{th}$ item $= 7.5^{th}$ item

The c.f. just greater than 7.5 is 13 whose corresponding class is (40 - 50).

Now, exact $Q_1 = L + \dfrac{\frac{N}{4} - c.f.}{f} \times h, = 40 + \dfrac{7.5 - 7}{6} \times 10 = 40 + \dfrac{0.5 \times 10}{6} = 40 + \dfrac{5}{6}$

$= 40 + 0.83 = 40.83$

$\therefore \quad Q_1 = 40.83$

Q_3 = value of $\left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 7.5^{th}$ item $= 22.5^{th}$ item.

The c.f. just greater than 22.5 is 26 whose corresponding class is (60 - 70).

Now, exact $Q_3 = L + \dfrac{\frac{3N}{4} - c.f.}{f} \times h = 60 + \dfrac{22.5 - 22}{4} \times 10 = 60 + \dfrac{0.5 \times 10}{4} = 60 + \dfrac{5}{4} = 60 + 1.25 = 61.25$

$\therefore \quad Q_3 = 61.25$

We have, $QD = \dfrac{Q_3 - Q_1}{2} = \dfrac{61.25 - 40.83}{2} = 10.21$

Thus, required quartile deviation is 10.21.

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6. तल दिइएको तथ्याङ्कबाट चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।
Find the quartile deviation and its coefficient from the given data.

Ages (in Years)	**10 - 12**	**12 - 14**	**14 - 16**	**16 - 18**	**18 - 20**
No. of Students	**14**	**18**	**23**	**18**	**7**

⇨ Here, making table for calculating Q_1 and Q_3:

Age	*f*	cf
10 - 12	14	14
12 - 14	18	32
14 - 16	23	55
16 - 18	18	73
18 - 20	7	80
	N = 80	

Q_1 class $= \left(\frac{N}{4}\right)^{th}$ class $= \left(\frac{80}{4}\right)^{th}$ class = 20th class = (12 - 14)

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times i = 12 + \frac{\frac{80}{4} - 14}{18} \times 2 = 12 + \frac{20 - 14}{9} = 12 + 0.67 = 12.67$$

Q_3 class $= \left(\frac{3N}{4}\right)^{th}$ class $= 3 \times 20^{th}$ class $= 60^{th}$ class = (16 - 18)

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times i = 16 + \frac{\frac{3 \times 80}{4} - 55}{18} \times 2 = 16 + \frac{60 - 55}{9} = 16 + 0.56 = 16.56$$

Now, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{16.56 - 12.67}{2} = \frac{3.89}{2} = 1.945$

And, Coeff. of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{16.56 - 12.67}{16.56 + 12.67} = \frac{3.89}{29.23} = 0.133$

Thus, QD is 1.945 and its coefficient is 0.133.

7. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन पत्ता लगाउनुहोस् । **(Calculate the quartile deviation from the following data.)**

Marks	**0 - 10**	**0 - 20**	**0 - 30**	**0 - 40**	**0 - 50**	**0 - 60**	**0 - 70**
No. of Students	**3**	**5**	**10**	**14**	**16**	**19**	**20**

⇨ Here, making cumulative frequency table,

Marks	**0 - 10**	**10 - 20**	**20 - 30**	**30 - 40**	**40 - 50**	**50 - 60**	**60 - 70**
No. of Students	**3**	**2**	**5**	**4**	**2**	**3**	**1**
cf	**3**	**5**	**10**	**14**	**16**	**19**	**20**

Now, we have, Q_1 class $= \left(\frac{N}{4}\right)^{th}$ class $= \left(\frac{20}{4}\right)^{th}$ class = 5th class = 10 - 20,

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times i = 10 + \frac{5 - 3}{2} \times 10 = 10 + 10 = 20$$

Q_3 class $= \left(\frac{3N}{4}\right)^{th}$ class $= \left(\frac{3 \times 20}{4}\right)^{th}$ class = 15th class = 40 - 50,

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times i = 40 + \frac{15 - 14}{2} \times 10 = 40 + 5 = 45$$

We know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{45 - 20}{2} = \frac{25}{2} = 12.5$

Thus, required quartile deviation is 12.5.

8. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।
Find the quartile deviation and its coefficient from the following data.

Class	**20 - 30**	**40 - 50**	**60 - 70**	**30 - 40**	**50 - 60**
Frequency	**5**	**4**	**6**	**3**	**2**

⇨ Here, making table for calculating Q_1 and Q_3:

Class	f	*cf*
20 - 30	5	5
30 - 40	3	8
40 - 50	4	12
50 - 60	2	14
60 - 70	6	20
	N = 20	

The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item$= \left(\frac{20}{4}\right)^{th}$ item = 5th item = (20 - 30)

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - c.f}{f} \times i = 20 + \frac{\frac{20}{4} - 0}{5} \times 10 = 20 + 10 = 30$$

Again, the position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 15th item = (60 - 70)

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - c.f}{f} \times i = 60 + \frac{15 - 14}{6} \times 10 = 60 + 1.6 = 61.6$$

We know that, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{61.6 - 30}{2} = 15.8$

Again, coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{61.6 - 30}{61.6 + 30} = \frac{31.6}{91.6} = 0.34$

Thus, the quartile deviation and its coefficient are 15.8 and 0.34 respectively.

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9. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।

Calculate Q.D. and its coefficient from the following data.

Value	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59
Frequency	3	6	8	5	2

⇨ Here, given classes are inclusive classes. So, we have to change inclusive classes into exclusive classes before calculating QD. For this, we first need to find correction factor by using following formula.

$$\text{Correction factor} = \frac{\text{Lower limit of succeeding class} - \text{upper limit of preceding class}}{2} = \frac{20-19}{2} = \frac{30-29}{2} = \frac{1}{2} = 0.5$$

Calculation of QD and its coefficient:

Inclusive classes	frequency (f)	Exclusive classes	c.f.
10 - 19	3	9.5 - 19.5	3
20 - 29	6	19.5 - 29.5	9
30 - 39	8	29.5 - 39.5	17
40 - 49	5	39.5 - 49.5	22
50 - 59	2	49.5 - 59.5	24
	N = 24		

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{24}{4}\right)^{th}$ item = 6^{th} item = (19.5 - 29.5).

We know, $Q_1 = \ell + \frac{\frac{N}{4} - c.f.}{f} \times h = 19.5 + \frac{6-3}{6} \times 10 = 19.5 + \frac{30}{6} = 19.5 + 5 = 24.5$

For Q_3: Again, the position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item $= 3 \times 6^{\,th}$ item $= 18^{\,th}$ item = (39.5 - 49.5)

We know, $Q_3 = \ell + \frac{\frac{3N}{4} - cf.}{f} \times h = 39.5 + \frac{18-17}{5} \times 10 = 39.5 + 2 = 41.5$

Also, we know, $QD = \frac{Q_3 - Q_1}{2} = \frac{41.5 - 24.5}{2} = \frac{17}{2} = 8.5$

and coefficient of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{41.5 - 24.5}{41.5 + 24.5} = \frac{17}{66} = 0.258$

Thus, QD and it's coefficient are 8.5 and 0.258 respectively.

10. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र सोको गुणाङ्क पत्ता लगाउनुहोस् ।

Calculate the quartile deviation and its coefficient from the following data.

Weight (in kg) below	2	4	6	8	10	12
No. of cats	3	8	18	30	36	40

⇨ Here, calculation of quartile deviation and its coefficient:

Weight	f	c.f.
below 2	3	3
2 - 4	5	8
4 - 6	10	18
6 - 8	12	30
8 - 10	6	36
10 - 12	4	40
	N = 40	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{40}{4}\right)^{th}$ item = 10^{th} item = (4 - 6).

We know, $Q_1 = \ell + \frac{\frac{N}{4} - c.f.}{f} \times h = 4 + \frac{10-8}{10} \times 2 = 4 + \frac{4}{10} = 4 + 0.4 = 4.4$ kg

For Q_3: Again, the position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item $= 3 \times 10^{\,th}$ item $= 30^{\,th}$ item = (6 - 8)

We know, $Q_3 = \ell + \frac{\frac{3N}{4} - c.f.}{f} \times h = 6 + \frac{30-18}{12} \times 2 = 8$ kg

$\therefore \quad QD = \frac{Q_3 - Q_1}{2} = \frac{8 - 4.4}{2} = \frac{3.6}{2} = 1.8$ kg and coefficient of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{8 - 4.4}{8 + 4.4} = \frac{3.6}{12.4} = 0.29$

Thus, QD and its coefficient are 1.8 kg and 0.29 respectively.

11. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र सोको गुणाङ्क पत्ता लगाउनुहोस् ।

Calculate the quartile deviation and its coefficient from the following data.

Class interval	10 - 20	10 - 30	10 - 40	10 - 50	10 - 60
Frequency	1	3	6	8	10

⇨ Here, as the lower limit of each class interval is same, the given frequency is cumulative frequency. So, we first convert it into simple frequency distribution.

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Calculation of quartile deviation and its coefficient.

CI	f	cf
10 - 20	1	1
20 - 30	2	3
30 - 40	3	6
40 - 50	2	8
50 - 60	2	10
	N = 10	

For Q₁: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item= $\left(\frac{10}{4}\right)^{th}$ item = 2.5th item = (20 - 30).

We know, $Q_1 = \ell + \frac{\frac{N}{4} - \text{c.f.}}{f} \times h = 20 + \frac{2.5 - 1}{2} \times 10 = 20 + \frac{15}{2} = 20 + 7.5 = 27.5$

For Q₃: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item= 3 × 2.5th item = 7.5th item = (40 - 50).

We know, $Q_3 = \ell + \frac{\frac{3N}{4} - \text{c.f.}}{f} \times h = 40 + \frac{7.5 - 6}{2} \times 10 = 40 + \frac{15}{2} = 40 + 7.5 = 47.5$

$\therefore$ $QD = \frac{Q_3 - Q_1}{2} = \frac{47.5 - 27.5}{2} = \frac{20}{2} = 10$ and coefficient of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{47.5 - 27.5}{47.5 + 27.5} = \frac{20}{75} = 0.267$

Thus, QD and its coefficient are 10 and 0.267 respectively.

12. 24 जना विद्यार्थीहरूको प्राप्ताङ्कहरू तल दिइएका छन् । यदि एउटा वर्गान्तर 15 - 20 भए बारम्बारता तालिका बनाउनुहोस् । साथै, चतुर्थांशीय विचलन र सो को गुणाङ्क पनि पत्ता लगाउनुहोस् ।

The marks of 24 students are given below. Construct frequency table if one of class interval is 15 - 20. Also find the quartile deviation and its coefficient.

18	9	4	6	11	27	22	16	8	14	16	29	3	19	24	26	2	25	10	18	21	23	14	23

⇨ Here, frequency table for calculation of quartile deviation.

Class	Tallies	f	cf
0 - 5	∣∣∣	3	3
5 - 10	∣∣∣	3	6
10 - 15	∣∣∣∣	4	10
15 - 20	卌	5	15
20 - 25	卌	5	20
25 - 30	∣∣∣∣	4	24
		N = 24	

The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item = $\left(\frac{24}{4}\right)^{th}$ item = 6th item = (5 - 10)

So, $Q_1 = L + \frac{\frac{N}{4} - \text{c.f.}}{f} \times i = 5 + \frac{\frac{24}{4} - 3}{3} \times 5 = 5 + 5 = 10$

The position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item = 3 × 6th item = 18th item = (20 - 25)

So, $Q_3 = L + \frac{\frac{3N}{4} - \text{c.f.}}{f} \times i = 20 + \frac{18 - 15}{5} \times 5 = 23$

Now, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{23 - 10}{2} = 6.5$

Again, coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{23 - 10}{23 + 10} = \frac{13}{33} = 0.39$

Thus, the quartile deviation and its coefficient are 6.5 and 0.39 respectively.

13. तलको तथ्याङ्कको पहिलो वर्गान्तर (10 - 20) लिएर बारम्बारता तालिका बनाई चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।

Taking class interval of (10 - 20) as first class, prepare a frequency distribution table. Also, find the Q.D. and its coefficient from the following data:

17,	51,	35,	28,	25,	56,	46,	40,	12,	63,	54,	64,	36,	76,	65,	60,	22,	45,	58,	15,
29,	70,	23,	30,	83,	38,	75,	13,	48,	20,	42,	37,	41,	49,	39,	59,	46,	35,	47,	88

⇨ Here, calculation of a frequency distribution table, Q.D. and its coefficient.

Class	Tally bars	f	c.f.
10 - 20	∣∣∣∣	4	4
20 - 30	卌 ∣	6	10
30 - 40	卌 ∣∣	7	17
40 - 50	卌 ∣∣∣∣	9	26
50 - 60	卌	5	31
60 - 70	∣∣∣∣	4	35
70 - 80	∣∣∣	3	38
80 - 90	∣∣	2	40
		N = 40	

For Q₁: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item = $\left(\frac{40}{4}\right)^{th}$ item = 10th item = (20 - 30)

We know, $Q_1 = \ell + \frac{\frac{N}{4} - \text{c.f.}}{f} \times h = 20 + \frac{10 - 4}{6} \times 10 = 30$

For Q₃: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3 × 10th item = 310th item = (50 - 60)

We know, $Q_3 = \ell + \frac{\frac{3N}{4} - \text{c.f.}}{f} \times h = 50 + \frac{30 - 26}{5} \times 10 = 50 + \frac{40}{5} = 58$

Also, we know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{58 - 30}{2} = \frac{28}{2} = 14$

and coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{58 - 30}{58 + 30} = \frac{28}{88} = 0.32$

Thus, QD and its coefficient are 14 and 0.32 respectively.

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14. कुनै कक्षाका 30 जना विद्यार्थीहरूले पाएको प्राप्ताङ्क निम्नअनुसार छ :
The marks obtained by 30 students of a class are given below:

40	42	60	65	70	72	30	54	38	20	25	35	42	48	50
58	53	59	60	46	78	77	62	50	51	54	42	35	28	54

पहिलो वर्गान्तर 20 - 30 हुने गरी माथिको तथ्याङ्कलाई वर्गान्तरमा मिलाई चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Classify the above data with 20 - 30 as the first class-interval and find the quartile deviation and its co-efficient.

⇨ Here, calculation table:

Marks	f	cf
20 - 30	3	3
30 - 40	4	7
40 - 50	6	13
50 - 60	9	22
60 - 70	4	26
70 - 80	4	30
	N = 30	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{30}{4}\right)^{th}$ item $= 7.5^{th}$ item = (40 - 50)

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 40 + \frac{7.5 - 7}{6} \times 10 = 40.83$$

For Q_3: The position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= \frac{3 \times 30}{4} = 22.5^{th}$ item.= (60-70)

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 60 + \frac{22.5 - 22}{4} \times 10 = 61.25$$

We know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{61.25 - 40.83}{2} = 10.21$

Again, co-efficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{61.25 - 40.83}{61.25 + 40.83} = 0.2$

15. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the quartile deviation and the coefficient of quartile deviation from the following data.

Daily wage (in Rs)	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250
No. of labourers	7	20	15	10	8

⇨ Here, calculation table:

Daily wage	f	cf
0 - 50	7	7
50 - 100	20	27
100 - 150	15	42
150 - 200	10	52
200 - 250	8	60
	N = 60	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{60}{4}\right)^{th}$ item $= 15^{th}$ item = (50 - 100)

We know, $Q_1 = \ell + \frac{\frac{N}{4} - c.f.}{f} \times h = 50 + \frac{15 - 7}{20} \times 50 = 70$

For Q_3: The position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 15^{th}$ item $= 45^{th}$ item. = 150-200

We know, $Q_3 = \ell + \frac{\frac{3N}{4} - c.f.}{f} \times h = 150 + \frac{45 - 42}{10} \times 50 = 165$

Now, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{165 - 70}{2} = 47.5$ and coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{165 - 70}{165 + 70} = 0.404$

Thus, QD and it's coefficient are 47.5 and 0.404 respectively.

16. तल 50 जना विद्यार्थीहरूको गणित विषयको प्राप्ताङ्क विवरण दिइएको छ ।
Following is the distribution of marks of mathematics obtained by 50 students.

प्राप्ताङ्क Marks	0 भन्दा बढी More than 0	10 भन्दा बढी More than 10	20 भन्दा बढी More than 20	30 भन्दा बढी More than 30	40 भन्दा बढी More than 40	50 भन्दा बढी More than 50
विद्यार्थी सङ्ख्या No. of students	50	46	40	20	10	3

चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् । **(Find the quartile deviation and its coefficient.)**

⇨ Here, the given distribution is the cumulative frequency distribution. So we have to change into simple frequency distribution. The frequency are the cumulative frequencies. So, simple frequencies are to be calculated by subtracting a frequency with the preceding one.

Marks	f	cf
0 - 10	50 – 46 = 4	4
10 - 20	46 – 40 = 6	10
20 - 30	40 – 20 = 20	30
30 - 40	20 – 10 = 10	40
40 - 50	10 – 3 = 7	47
50 and above	3	50
	N = 50	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{50}{4}\right)^{th}$ item $= 12.5^{th}$ item = (20 - 30)

We know, $Q_1 = \ell + \frac{\frac{N}{4} - c.f.}{f} \times h = 20 + \frac{12.5 - 10}{20} \times 10 = 21.25$

For Q_3: The position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item $= 3 \times 12.5^{th}$ item $= 37.5^{th}$ item. = 30-40

We know , $Q_3 = \ell + \frac{\frac{3N}{4} - c.f.}{f} \times h = 30 + \frac{37.5 - 30}{10} \times 10 = 37.5$

Now, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{37.5 - 21.25}{2} = 8.125$ and coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{37.5 - 21.25}{37.5 + 21.25} = 0.276$

Thus, QD and it's coefficient are 8.125 and 0.276 respectively.

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17. कुनै निश्चित समुदायमा 40 परिवारले प्रयोग गर्ने पानीको आयतन (लिटरमा) तल दिइएको छ । यसबाट चतुर्थांशीय विचलन र त्यसको गुणाङ्क पत्ता लगाउनुहोस् ।
The volume of water (in liters) used by 40 families in a certain locality is given below. Find the quartile deviation and its coefficient.

Volume of water (in litre)	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
No. of families	4	12	16	6	2

⇨ Here, calculation table:

Volume	f	cf
5 - 10	4	4
10 - 15	12	16
15 - 20	16	32
20 - 25	6	38
25 - 30	2	40
	N = 40	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{40}{4}\right)^{th}$ item = 10th item = (10 - 15)

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 10 + \frac{10 - 4}{12} \times 5 = 10 + \frac{6}{12} \times 5 = 12.5$$

For Q_3: The position of $Q_3 = \left(\frac{3N}{4}\right)^{th}$ item = 3 ×10th item = 30th item. = 15-20

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 15 + \frac{30 - 16}{16} \times 5 = 15 + \frac{14 \times 5}{16} = 19.375$$

We know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{19.375 - 12.5}{2} = 3.4375$

Again, co-efficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{19.375 - 12.5}{19.375 + 12.5} = \frac{6.875}{31.875} = 0.2157$

Thus, QD and it's coefficient are 3.4375 and 0.2157 respectively.

18. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Compute the quartile deviation and its coefficient from the following distribution.

वर्गान्तर (Class Interval)	20 - 30	50 - 60	40 - 50	30 - 40	60 - 70
बारम्बारता (Frequency)	8	4	4	16	4

⇨ Here, calculation table:

CI	f	cf
20 - 30	8	8
30 - 40	16	24
40 - 50	4	28
50 - 60	4	32
60 - 70	4	36
	N = 36	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{36}{4}\right)^{th}$ item = 9th item = (30 - 40)

$$\therefore \quad Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 30 + \frac{9 - 8}{16} \times 10 = 30 + \frac{1}{16} \times 10 = 30.625$$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3 × 9th item = = 27th item = (40 - 50)

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 40 + \frac{27 - 24}{4} \times 10 = 40 + \frac{3}{4} \times 10 = 47.5$$

We know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{47.5 - 30.625}{2} = \frac{16.875}{2} = 8.437$

Again, co-efficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{47.5 - 30.625}{47.5 + 30.625} = \frac{16.875}{78.125} = 0.216$

Thus, QD and it's coefficient are 8.437 and 0.216 respectively.

19. तलको तथ्याङ्कबाट चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Calculate the quartile deviation and its coefficient from the following data.

x	0 - 10	0 - 20	0 - 30	0 - 40	0 - 50	0 - 60	0 - 70	0 - 80	0 - 90
f	11	29	54	82	112	145	167	182	204

⇨ Here, calculation table:

x	f	c.f.
0 - 10	11	11
10 - 20	29 – 11 = 18	29
20 - 30	54 – 29 = 25	54
30 - 40	82 – 54 = 28	82
40 - 50	112 – 82 = 30	112
50 - 60	145 – 112 = 33	145
60 - 70	167 – 145 = 22	167
70 - 80	182 – 167 = 15	182
80 - 90	204 – 182 = 22	204
	N = 204	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{204}{4}\right)^{th}$ item = 51th item = (20 - 30)

We know, $Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 20 + \frac{51 - 29}{25} \times 10 = 28.8$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3 × 51th item = = 153th item = (60 - 70)

$$\therefore \quad Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 60 + \frac{153 - 145}{22} \times 10 = 63.64$$

We know, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{63.64 - 28.8}{2} = 17.42$

Again, co-efficient of Quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{63.64 - 28.8}{63.64 + 28.8} = 0.37$

Thus, QD and it's coefficient are 17.42 and 0.37 respectively.

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QUESTIONS FROM CDC TEXTBOOK

8.1 चतुर्थांशीय विचलन (QUARTILE DEVIATION)

EXERCISE 8.1

1. (a) **विचरणशीलता भनेको के हो ? उदाहरणसहित व्याख्या गर्नुहोस् । (What is dispersion ? Explain with examples.)**

⇨ A measure of dispersion is the method of finding information regarding the amount of variability or spread or deviation or scatterness present in the data. For example

Data 1	2, 4, 6, 8, 10, 12
Data 2	10, 30, 50, 70, 90

Comparing the above data, then we get

(i) The expansion of the data (1) is less than that of data (2) so the dispersion of data (1) is less than the dispersion of data (2).

(ii) If the items of the data are indentical then there is no dispersion.

(b) **विचरणशीलता मापनका विधिहरूको सूची बनाउनुहोस् । (List the ways to measure dispersion.)**

⇨ The ways to measure dispersion are as follows:

(i) Range (ii) Quartile deviation (iii) Mean deviation (iv) Standard deviation

(c) **चतुर्थांशीय विचलनको परिभाषा दिनुहोस् । (Define quartile deviation.)**

⇨ The difference between the first quartile (Q_1) and the third quartile (Q_3) is known as inter-quartile range. The half of the inter-quartile range is called Semi-interquartile Range or Quartile Deviation.

(d) **चतुर्थांशीय विचलनको गुणाङ्कको परिभाषा दिनुहोस् । (Define coefficient of quartile deviation.)**

⇨ The relative measure based on lower and upper quartiles is known as coefficient of quartile deviation.

(e) **चतुर्थांशीय विचलनका गुण र दोषहरू लेख्नुहोस् । (Write the merits and demerits of quartile deviation.)**

⇨ The merits and demerits of quartile deviation are as follows:

Merits	Demerits
1. QD is simple to understand and easy to calculate.	1. QD is not based on all observations.
2. It is not affected by extreme terms because 25% of upper and 25% of lower terms are left out.	2. It is too much affected by the fluctuations of samples in the calculation.
3. QD can be calculated even for open-end interval.	3. Arrangement of data are necessary.
4. MD and SD can be calculated using: 6QD = 5MD = 4SD	4. If the values are irregular then result is affected badly.

(f) **चतुर्थांशीय विचलन र चतुर्थांशीय विचलनको गुणाङ्कबिच भिन्नता उल्लेख गर्नुहोस् ।**
State the difference between quartile deviation and coefficient of quartile deviation.

⇨ Quartile deviation is the measure of dispersion depending upon the lower and upper quartile whereas coefficient of quartile deviation is the relative measure based on upper and lower quartiles.

2. (a) **यदि एउटा तथ्याङ्कको पहिलो चतुर्थांश 35 र तेस्रो चतुर्थांश 75 छ भने, चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
If the first quartile of a data is 35 and third quartile is 75, find the quartile deviation and it's coefficient.
[SEE MODEL 2076]

⇨ Here, first quartile (Q_1) = 35 and third quartile (Q_3) = 75

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{75 - 35}{2} = 20$

And, Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{75 - 35}{75 + 35} = \frac{40}{110} = 0.36$

Thus, QD is 20 and it's coefficient is 0.36.

(b) **यदि एउटा तथ्याङ्कको पहिलो चतुर्थांश 45 र तेस्रो चतुर्थांश 55 छ भने, चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
If the first quartile of a data is 45 and third quartile is 55, find the quartile deviation and it's coefficient.

⇨ Here, first quartile (Q_1) = 45 and third quartile (Q_3) = 55

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{55 - 45}{2} = 5$

And, coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{55 - 45}{55 + 45} = \frac{10}{100} = 0.1$

Thus, QD is 5 and it's coefficient is 0.1.

3. **तल दिइएको तथ्याङ्कबाट चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :**
Find the quartile deviation and its coefficient from the given data.

(a)

Mark obtained	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	15	10	8	6	2

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⇨ Here, calculation of quartile deviation and it's coefficient.

Marks (x)	f	cf
20 - 30	5	5
30 - 40	15	20
40 - 50	10	30
50 - 60	8	38
60 - 70	6	44
70 - 80	2	46
	N = 46	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{46}{4}\right)^{th}$ item = 11.5th item = (30 - 40)

$$Q_1 = \ell + \left(\frac{\frac{N}{4} - cf}{f}\right) \times h = 30 + \frac{11.5 - 5}{15} \times 10 = 30 + 4.33 = 34.33$$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3×11.5th item = 34.5th item = (50 - 60)

Now, $Q_3 = \ell + \left(\frac{\frac{3N}{4} - cf}{f}\right) \times h = 50 + \frac{34.5 - 30}{8} \times 10 = 50 + 5.625 = 55.625$ $\qquad \therefore \quad Q_3 = 55.625$

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{55.625 - 34.33}{2} = \frac{21.295}{2} = 10.64$

Again, Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{55.625 - 34.33}{55.625 + 34.33} = \frac{21.295}{89.955} = 0.236$

Thus, QD is 10.64 and it's coefficient is 0.236.

(b)

Age of bulb (in hours)	0-250	250-500	500-750	750-1000	1000-1250	1250-1500	1500-1750	1750-2000
No. of bulbs	1	3	7	12	25	39	11	2

⇨ Here, calculation of quartile deviation and it's coefficient.

Age	f	cf
0 - 250	1	1
250 - 500	3	4
500 - 750	7	11
750 - 1000	12	23
1000 - 1250	25	48
1250 - 1500	39	87
1500 - 1750	11	98
1750 - 2000	2	100
	N = 100	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{100}{4}\right)^{th}$ item = 25th item = (1000 - 1250)

Now, $Q_1 = \ell + \left(\frac{\frac{N}{4} - cf}{f}\right) \times h = 1000 + \frac{25 - 23}{25} \times 250 = 1000 + 20 = 1020$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3 × 25th item = 75th item = (1250 - 1500)

Now, $Q_3 = \ell + \left(\frac{\frac{3N}{4} - cf}{f}\right) \times h = 1250 + \frac{75 - 48}{39} \times 250 = 1250 + 173.076 = 1423.076$

$\therefore \quad Q_3 = 1423.076$

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{1423.076 - 1020}{2} = \frac{403.076}{2} = 201.538$

Again, Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1423.076 - 1020}{1423.076 + 1020} = \frac{403.076}{2443.076} = 0.165$

Thus, QD is 201.538 and it's coefficient is 0.165.

(c)

Age (in yrs)	0-5	5-10	10-15	15-20	20-25	25-30
Number	23	22	17	13	13	12

⇨ Here, calculation of quartile deviation and it's coefficient.

Age	f	cf
0 - 5	23	23
5 - 10	22	45
10 - 15	17	62
15 - 20	13	75
20 - 25	13	88
25 - 30	12	100
	N = 100	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{100}{4}\right)^{th}$ item = 25th item = (5 - 10)

Now, $Q_1 = \ell + \left(\frac{\frac{N}{4} - cf}{f}\right) \times h = 5 + \frac{25 - 23}{22} \times 5 = 5 + 0.45 = 5.45$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item = 3 × 25th item = 75th item = (15 - 20)

Now, $Q_3 = \ell + \left(\frac{\frac{3N}{4} - cf}{f}\right) \times h = 15 + \frac{75 - 62}{13} \times 5 = 15 + 5 = 20$ $\qquad \therefore \quad Q_3 = 20$

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{20 - 5.45}{2} = \frac{14.55}{2} = 7.275$

Again, Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{20 - 5.45}{20 + 5.45} = \frac{14.55}{25.45} = 0.57$

Thus, QD is 7.275 and it's coefficient is 0.57.

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4. कक्षा 8 मा अध्ययनरत 28 जना विद्यार्थीले गणित विषयमा प्राप्त गरेको प्राप्ताङ्क निम्न लिखित छ। उक्त तथ्याङ्कका आधारमा (10-20) को पहिलो वर्गान्तर लिएर बारम्बारता तालिका बनाई चतुर्थांशीय भिन्नता र यसको गुणाङ्क निकाल्नुहोस् :
The marks obtained in mathematics by 28 students studying in class 8 are given below. Taking (10 - 20) as first class interval, make a frequency table and find the quartile deviation and it's coefficient on the basis of given data.
48, 50, 34, 29, 56, 40, 14, 62, 28, 70, 22, 30, 38, 74, 13, 47, 20, 53, 64, 34, 75, 66, 60, 21, 45, 57, 15, 41

⇨ Here, construction of frequency table and calculation of quartiles.

CI	Tallies	f	cf
10 - 20	𝍷𝍷𝍷	3	3
20 - 30	𝍸	5	8
30 - 40	𝍷𝍷𝍷𝍷	4	12
40 - 50	𝍸	5	17
50 - 60	𝍷𝍷𝍷𝍷	4	21
60 - 70	𝍷𝍷𝍷𝍷	4	25
70 - 80	𝍷𝍷𝍷	3	28
		N = 28	

For Q_1: The position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \left(\frac{28}{4}\right)^{th}$ item = 7^{th} item = (20 - 30)

Now, $Q_1 = \ell + \left(\frac{\frac{N}{4} - cf}{f}\right) \times h = 20 + \frac{7-3}{5} \times 10 = 20 + 8 = 28$

For Q_3: The position of $Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item $= 3 \times 7^{th}$ item = = 21^{th} item = (50 - 60)

Now, $Q_3 = \ell + \left(\frac{\frac{3N}{4} - cf}{f}\right) \times h = 50 + \frac{21-17}{4} \times 10 = 50 + 10 = 60$ $\qquad \therefore \quad Q_3 = 60$

We have, Quartile Deviation (QD) $= \frac{Q_3 - Q_1}{2} = \frac{60-28}{2} = \frac{32}{2} = 16$

Again, Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{60-28}{60+28} = \frac{32}{88} = 0.36$

Thus, QD is 16 and it's coefficient is 0.36.

2. मध्यक भिन्नता
Mean Deviation

QUESTIONS FROM SEE EXERCISE 2

A. SHORT QUESTIONS

1. एउटा निरन्तर श्रेणीमा $\sum fm = 120$, N = 10 र $\sum f|m - \bar{x}| = 34$ छ। सो तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्।
A continuous series has $\sum fm = 120$, N = 10 and $\sum f|m - \bar{x}| = 34$. Find mean deviation and its coefficient from mean.

⇨ Here, $\sum fm = 120$, N = 10 and $\sum f|m - \bar{x}| = 34$ $\qquad$ Now, mean $(\bar{x}) = \frac{\sum fm}{N} = \frac{120}{10} = 12$

We have, mean deviation from mean $= \frac{\sum f|m - \bar{x}|}{N} = \frac{34}{10} = 3.4$

And, coefficient of mean deviation $= \frac{\text{MD from mean}}{\text{mean}} = \frac{3.4}{12} = 0.283$

Thus, mean deviation is 3.4 and it's coefficient is 0.283.

2. एउटा निरन्तर श्रेणीमा $\sum fm = 2400$, N = 100 र $\sum f(m - \bar{x}) = 448$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्।
In a continuous series, $\sum fm = 2400$, N = 100 and $\sum f(m - \bar{x}) = 448$ then find mean deviation from mean and its coefficient.

⇨ Here, $\sum fm = 2400$, N = 100 and $\sum f(m - \bar{x}) = 448$ $\qquad$ Now, mean $(\bar{x}) = \frac{\sum fm}{N} = \frac{2400}{100} = 24$

We have, mean deviation from mean $= \frac{\sum f(m - \bar{x})}{N} = \frac{448}{100} = 4.48$

Again, coefficient of mean deviation is $= \frac{\text{MD from mean}}{\text{mean}} = \frac{4.48}{24} = 0.187$

Thus, mean deviation from mean is 4.48 and it's coefficient is 0.187.

3. एउटा निरन्तर श्रेणीमा मध्यिका 50, N = 8 र $\sum f(m - md) = 59$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्।
In a continuous series, median = 50, N = 8 and $\sum f(m - md) = 59$. Find the mean deviation from median and its coefficient.

⇨ Here, median = 50, N = 8 and $\sum f(m - md) = 59$

We know that, mean deviation from median $= \frac{\sum f(m - md)}{N} = \frac{59}{8} = 7.375$

Again, coefficient of mean deviation is $= \frac{\text{MD from median}}{\text{median}} = \frac{7.375}{50} = 0.1475$

Thus, the mean deviation from median is 7.375 and it's coefficient is 0.1475.

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4. **एउटा निरन्तर श्रेणीमा मध्यिका 12, N = 100 र $\sum f(m - md) = 224$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, median = 12, N = 100 and $\sum f(m - md) = 224$. Find mean deviation from median and its coefficient.

⇨ Here, median = 12, N = 100 and $\sum f(m - md) = 224$

We know that, mean deviation from median $= \frac{\sum f(m - md)}{N} = \frac{224}{100} = 2.24$

Again, coefficient of mean deviation is $= \frac{\text{MD from median}}{\text{median}} = \frac{2.24}{12} = 0.187$

Thus, the mean deviation from median is 2.24 and it's coefficient is 0.187.

B. LONG QUESTIONS

MODEL 1

1. **तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।**
Find the mean deviation from median of the data given below: **[2074 R]**

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

⇨ Here, Calculation of mean deviation from median

Class interval	x	f	c.f	$\lvert D\rvert = \lvert x - \text{Md}\rvert$	$f\lvert D\rvert$
0 - 10	5	5	5	23	115
10 - 20	15	8	13	13	104
20 - 30	25	15	28	3	45
30 - 40	35	16	44	7	112
40 - 50	45	6	50	17	102
		N = 50			$\sum f\lvert D\rvert = 478$

For median, $\left(\frac{N}{2}\right)^{th}$ item $= \left(\frac{50}{2}\right)^{th}$ item $= 25^{th}$ item $= 20 - 30$ class interval

We have, Median $= L + \frac{i}{f}\left(\frac{N}{2} - cf\right) = 20 + \frac{10}{15}(25 - 13) = 20 + \frac{2 \times 12}{3} = 28$ $\qquad \therefore$ Median = 28

Now, mean deviation from median: MD $= f(\sum f\lvert D\rvert, N) = \frac{478}{50} = 9.56$

Thus, the mean deviation from median is 9.56.

2. **तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।**
Find the mean deviation from median of the data given below: **[2074 S]**

X	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	4	5	6

⇨ Here, Calculation of mean deviation :

X	f	cf	M	$\lvert M - \text{Md.}\rvert$	$f\lvert M - \text{Md.}\rvert$
0 - 10	2	2	5	27	54
10 - 20	3	5	15	17	51
20 - 30	4	9	25	7	28
30 - 40	5	14	35	3	15
40 - 50	6	20	45	13	78
	N = 20				$\sum f\lvert M - \text{Md.}\rvert = 226$

We have, Median class $= \left(\frac{N}{2}\right)^{th}$ item $= \left(\frac{20}{2}\right)^{th}$ item $= 10^{th}$ item $= 30 - 40$ class interval

We have, Median $= L + \frac{i}{f}\left(\frac{N}{2} - cf\right) = 30 + \frac{10}{5}\left(\frac{20}{2} - 9\right) = 30 + 2 \times 1 = 32$ $\qquad \therefore$ Median = 32

Now, Mean deviation $= \frac{\sum f\lvert M - \text{Md.}\rvert}{N} = \frac{226}{20} = 11.30$

Thus, the mean deviation is 11.30.

3. **दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।**
Find the mean deviation from median of the following data. **[2074 S']**

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	7	5	10	12	6

⇨ Here, calculation of mean deviation from median.

CI	f	c.f.	M	$\lvert D\rvert = \lvert M - \text{Md.}\rvert$	$f\lvert D\rvert$
0-10	7	7	5	23	161
10-20	5	12	15	13	65
20-30	10	22	25	3	30
30-40	12	34	35	7	84
40-50	6	40	45	17	102
	N = 40				$\sum f\lvert D\rvert = 442$

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We know, that, Median class $= \left(\frac{N}{2}\right)^{th}$ item $= \left(\frac{40}{2}\right)^{th}$ item $= 20^{th}$ item ∴ Median class = 20 - 30

Again, median $= L + \frac{i}{f}\left(\frac{N}{2} - cf\right) = 20 + \frac{10}{10}(20 - 12) = 20 + 8 = 28$

Now, Mean deviation $= \frac{\Sigma f|D|}{N} = \frac{442}{40} = 11.05$

Thus, the mean deviation is 11.05.

4. **तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नताको गणना गर्नुहोस् :**

Calculate the mean deviation of the data given below from median: **[2072 R]**

Marks obtained	10-20	20-30	30-40	40-50	50-60
No. of students	5	4	5	4	2

⇨ Here, calculation of mean deviation from median:

Marks	*f*	*cf*	m	|d| = |m – Md|	*f*|D|
10-20	5	5	15	17	85
20-30	4	9	25	7	28
30-40	5	14	35	3	15
40-50	4	18	45	13	52
50-60	2	20	55	23	46
	N = 20				Σf|D| = 226

Median class $= \left(\frac{N}{2}\right)^{th}$ item $= \left(\frac{20}{2}\right)^{th}$ item $= 10^{th}$ item = 30-40

We know that, Median (Md) $= L + \frac{i}{f}\left(\frac{N}{2} - cf\right) = 30 + \frac{10}{5}(10 - 9) = 30 + 2 = 32$

We have, Mean deviation $= \frac{\Sigma f|D|}{N} = \frac{226}{20} = 11.30$

Thus, mean deviation from median is 11.30

5. **तलको तथ्याङ्कअनुसार मध्यिकाबाट मध्यक भिन्नता निकाल्नुहोस्:**

Find the mean deviation from the median of the following data: **[2062R]**

Age in year	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of People	5	7	8	6	4

⇨ Here, from the given data, to calculate median:

Age in year	f	cf	Mid-value (m)	D = m – Md	|D|	*f*|D|
20 - 30	5	5	25	–18.75	18.75	93.75
30 - 40	7	12	35	–8.75	8.75	61.25
40 - 50	8	20	45	1.25	1.25	10
50 - 60	6	26	55	11.25	11.25	67.5
60 - 70	4	30	65	21.25	21.25	85
	N = 30					Σf|D| = 317.5

Here, $\Sigma f = N = 30$, $\Sigma f|D| = 317.5$

Here, Median class $= \left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{30}{2}\right)^{th}$ term $= 15^{th}$ term = 40 - 50

$$\therefore \text{ Exact median (Md)} = L + \frac{\frac{N}{2} - c.f.}{f} \times i = 40 + \frac{15 - 12}{8} \times 10 = 40 + \frac{3}{8} \times 10 = 40 + \frac{15}{4} = 40 + 3.75 = 43.75$$

Thus, mean deviation from median (M.D.) $= \frac{\Sigma f|D|}{N} = \frac{317.5}{30} = 10.583$

MODEL 2

6. **तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता निकाल्नुहोस् :**

Compute the mean deviation from mean of the data given below: **[2060CP]**

Age (in yrs)	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
No. of boys	12	8	10	6	4

⇨ Here, calculation of mean deviation from mean

Age	No. of boys (f)	Mid-value (m)	fm	|D| = |m – $\bar{x}$|	f|D|
0-4	12	2	24	6.2	74.4
4-8	8	6	48	2.2	17.6
8-12	10	10	100	1.8	18
12-16	6	14	84	5.8	34.8
16-20	4	18	72	9.8	39.2
	N = 40		$\Sigma fm = 328$		Σf |D| = 184

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We know that, mean $(\overline{x}) = \frac{\sum fm}{N} = \frac{328}{40} = 8.2$ Now, mean deviation $= \frac{\sum f|D|}{N} = \frac{184}{40} = 4.6$

Thus, mean deviation from mean is 4.6.

7. तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता निकाल्नुहोस् :

Find the average deviation from the given data: **[2065 R']**

CI	5 - 15	5 - 25	5 - 35	5 - 45	5 - 55
f	2	5	11	16	20

⇨ Here, calculation of average deviation,

CI	f	m	fm	$\lvert D\rvert = \lvert m - \overline{x}\rvert$	f|D|
5-15	2	10	20	23	46
15-25	3	20	60	13	39
25-35	6	30	180	3	18
35-45	5	40	200	7	35
45-55	4	50	200	17	68
	N = 20		$\sum fm = 660$		$\sum f\lvert D\rvert = 206$

We have, mean $(\overline{x}) = \frac{\sum fm}{N} = \frac{660}{20} = 33$ We know that, Average deviation $= \frac{\sum f|D|}{N} = \frac{206}{20} = 10.3$

Thus, average deviation of the given data is 10.3.

8. तलको आँकडाको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :

From the data given, find the mean deviation from the mean and its coefficient : **[SEE MODEL 2076, 2066 R]**

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	2	3	6	5	4

⇨ Here, calculation of mean deviation and its coefficient

Marks	f	m	fm	$\lvert D\rvert = \lvert m - \overline{x}\rvert$	f|D|
0-10	2	5	10	23	46
10-20	3	15	45	13	39
20-30	6	25	150	3	18
30-40	5	35	175	7	35
40-50	4	45	180	17	68
	N = 20		$\sum fm = 560$		$\sum f\lvert D\rvert = 206$

We have, mean $(\overline{x}) = \frac{\sum fm}{N} = \frac{560}{20} = 28$ We know that, M.D. $= \frac{\sum f|D|}{N} = \frac{206}{20} = 10.3$

Now, coefficient of MD $= \frac{MD}{\overline{x}} = \frac{10.3}{28} = 0.37$

Thus, the mean deviation and coefficient of mean deviation of given data are 10.3 and 0.37 respectively.

9. निम्न प्राप्ताङ्कको मध्यकबाट मध्यक भिन्नता पत्ता लगाउनुहोस् । यसको गुणाङ्क पनि पत्ता लगाउनुहोस्:

Find the mean deviation from mean of the following data. Also find its coefficient: **[2066S]**

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

⇨ Here, calculation of mean deviation:

Marks obtained	f	m	fm	$D = \lvert m - \overline{x}\rvert$	f|D|
0-10	5	5	25	22	110
10-20	8	15	120	12	96
20-30	15	25	375	2	30
30-40	16	35	560	8	128
40-50	6	45	270	18	108
	N = 50		$\sum fm = 1350$		$\sum f\lvert D\rvert = 472$

We have, Mean $(\overline{x}) = \frac{\sum fm}{N} = \frac{1350}{50} = 27$ We know that, Mean deviation $= \frac{\sum f|D|}{N} = \frac{472}{50} = 9.44$

The coefficient of M.D. $= \frac{MD}{\overline{x}} = \frac{9.44}{27} = 0.35$ Thus, M.D. = 9.44 and coefficient of M.D is 0.35.

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MODEL 3

10. **तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् :**

Compute the mean deviation from the mean and its coefficient from the data given below:

Mid Value	1	2	3	4	5
f	2	5	6	5	2

⇨ Here, from the given data:

Score (m)	Frequency (f)	fm	$\lvert D\rvert = \lvert x - \bar{x}\rvert$	f|D|
1	2	2	2	4
2	5	10	1	5
3	6	18	0	0
4	5	20	1	5
5	2	10	2	4
Total	$\Sigma f = 20$	$\Sigma fm = 60$		$\Sigma f\lvert D\rvert = 18$

Here, Arithmetic mean $(\bar{x}) = \frac{\Sigma fm}{\Sigma f} = \frac{60}{20} = 3$

∴ Mean deviation from mean (M.D.) $= \frac{\Sigma f\lvert D\rvert}{\Sigma f} = \frac{18}{20} = 0.9$

Also, the coefficient of mean deviation from mean $= \frac{\text{M.D. from mean}}{\text{Mean}} = \frac{0.9}{3} = 0.3$

Thus, MD and its coefficient are 0.9 and 0.3 respectively.

11. **तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् :**

Compute the mean deviation from the mean and its coefficient from the data given below:

Mid Value	10	15	20	25	30
f	2	4	6	8	5

⇨ Here, from the given data:

m	f	f.m	$D = m - \bar{x}$	|D|	f|D|
10	2	20	– 12	12	24
15	4	60	– 7	7	28
20	6	120	– 2	2	12
25	8	200	3	3	24
30	5	150	8	8	40
Total	25	550			128

Here, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{550}{25} = 22.$

We have, mean deviation from mean (M.D.) $= \frac{\Sigma f\lvert D\rvert}{N}$

∴ Mean deviation (M.D.) $= \frac{128}{25} = 5.12$

Then, coefficient of M.D. $= \frac{\text{M.D.}}{\bar{x}} = \frac{5.12}{22} = 0.23$

Thus, MD and its coefficient are 5.12 and 0.23 respectively.

12. **तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् :**

Compute the mean deviation from the mean and its coefficient from the data given below:

Mid Value	10	15	20	25	30
f	4	5	7	9	5

⇨ Here, calculation of mean deviation and its coefficient

m	f	fm	$\lvert D\rvert = \lvert x - \bar{x}\rvert$	f |D|
10	4	40	11	44
15	5	75	6	30
20	7	140	1	7
25	9	225	4	36
30	5	150	9	45
	N = 30	$\Sigma fm = 630$		$\Sigma f\lvert D\rvert = 162$

We know that, Mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{630}{30} = 21$

We have, mean deviation $= \frac{\Sigma f\lvert D\rvert}{N} = \frac{162}{30} = 5.4$

Now, the coefficient of mean deviation $= \frac{\text{M.D.}}{\bar{x}} = \frac{5.4}{21} = 0.257$

Thus, the mean deviation and the coefficient of the mean deviation are 5.4 and 0.257 respectively.

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QUESTIONS FROM CDC TEXTBOOK

8.2 मध्यक भिन्नता (MEAN DEVIATION)

EXERCISE 8.2

1. (a) **मध्यक भिन्नताको परिचय दिनुहोस् । (Introduce mean deviation.)**

⇨ The mean of the deviation of the item from the mean or median or mode of a numerical data is known as mean deviation.

In other words, mean deviation is the average of the absolute deviation taken from central value (mean, median or mode). Mean deviation is also called average deviation.

(b) **मध्यक भिन्नताको गुणाङ्क भनेको के हो ? यसको प्रयोग उदाहरणसहित व्याख्या गर्नुहोस् ।**
What is coefficient of mean deviation? Explain it's uses with examples.

⇨ The relative measure of dispersion based on mean deviation is called coefficient of mean deviation.
It is used to find out the scatterness, variability, deviation or spreadness of data. If the data are identical, there is no dispersion.

(c) **मध्यक भिन्नताका गुण र दोषहरू उल्लेख गर्नुहोस् । (List the merits and demerits of mean deviation.)**

⇨ The merits and demerits of mean deviation are as follows:

Merits	Demerits
1. It is simple to understand and easy to compute.	1. If average is in fraction or in decimal then it is difficult to calculate.
2. It is based on each and every data.	2. Signs are ignored while taking the deviation.
3. If calculated from median, it is less effected by extreme term.	3. This method will not give us very accurate result.

(d) **मध्यक भिन्नता र यसको गुणाङ्क गणना गर्दा के के को सापेक्षमा गर्न सकिन्छ ? कुन विधि बढी उपयुक्त होला ? कारणसहित व्याख्या गर्नुहोस् ।**

On what respect the calculation of mean deviation and its coefficient can be done ? Which method is more appropriate? Explain with reason.

⇨ The mean deviation and its coefficient can be calculated by using mean and median. The method of calculation by using mean is more appropriate because all the items are involved in the calculation.

2. (a) **तल दिइएको तथ्याङ्कका आधारमा मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् ।**
Calculate the mean deviation from median and it's coefficent from the following data.

C.I	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	6	8	11	14	8	3

⇨ Here, calculation of mean deviation and it's coefficient from median.

CI	f	cf	Mid value (m)	|m – Med|	f |m – Med|
10 - 20	6	6	15	25	150
20 - 30	8	14	25	15	120
30 - 40	11	25	35	5	55
40 - 50	14	39	45	5	70
50 - 60	8	47	55	15	120
60 - 70	3	50	65	25	75
	N = 50				Σf |m – Med| = 590

For median; $\left(\frac{N}{2}\right)^{th}$ value = $\left(\frac{50}{2}\right)^{th}$ value = 25th value ∴ Median class = 30 - 40

Now, Median (Med) = $\ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 30 + \frac{25 - 14}{11} \times 10 = 30 + 10 = 40$ ∴ Median (Med) = 40

We have, mean deviation from median $= \frac{\Sigma f\,|m - Med|}{N} = \frac{590}{50} = 11.8$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{median}} = \frac{11.8}{40} = 0.295$

Thus, mean deviation is 11.8 and it's coefficient is 0.295.

(b) **कक्षा 10 का 50 जना विद्यार्थीहरूको उचाइ तल तालिकामा दिइएको छ । दिइएको तथ्याङ्कका आधारमा मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् ।**
The height of 50 students of class 10 is given in the table below. Calculate the mean deviation from median and it's coefficient on the basis of given data.

Height (cm)	95-105	105-115	115-125	125-135	135-145	145-155
No. of students	6	8	11	14	8	3

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⇨ Here, calculation of mean deviation from median and it's coefficient.

CI	f	Mid value (m)	cf	|m – Med|	f |m – Med|
95 - 105	6	100	6	25	150
105 - 115	8	110	14	15	120
115 - 125	11	120	25	5	55
125 - 135	14	130	39	5	70
135 - 145	8	140	47	15	120
145 - 155	3	150	50	25	75
	N = 50				Σf |m – Med| = 590

For median, $\left(\frac{N}{2}\right)^{th}$ value $= \left(\frac{50}{2}\right)^{th}$ value = 25th value ∴ Median class = 115 - 125

Now, Median $= \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 115 + \frac{25 - 14}{11} \times 10 = 115 + 10 = 125$ ∴ Median (Med) = 125

We have, mean deviation from median $= \frac{\Sigma f\,|m - Med|}{N} = \frac{590}{50} = 11.8$

Again, coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{Median}} = \frac{11.8}{125} = 0.094$

Thus, mean deviation from median is 11.8 and it's coefficient is 0.094.

(c) तल दिइएको तथ्याङ्कका आधारमा मध्यिकाबाट मध्यक भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
Calculate the mean deviation from median and it's coefficient on the basis of following data.

Height	100-200	200-300	300-400	400-500	500-600	600-700	700-800
No. of students	4	6	10	20	10	6	4

⇨ Here, calculation of mean deviation and it's coefficient from median.

Height	f	cf	Mid value (m)	|m – Med|	f |m – Med|
100 - 200	4	4	150	300	1200
200 - 300	6	10	250	200	1200
300 - 400	10	20	350	100	1000
400 - 500	20	40	450	0	0
500 - 600	10	50	550	100	1000
600 - 700	6	56	650	200	1200
700 - 800	4	60	750	300	1200
	N = 60				Σf |m – Med| = 6800

For median; $\left(\frac{N}{2}\right)^{th}$ value $= \left(\frac{60}{2}\right)^{th}$ value = 30th value ∴ Median class = 400 - 500

We have, Median (Med)$= \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 400 + \frac{30 - 20}{20} \times 100 = 400 + 50 = 450$

We have, mean deviation from median $= \frac{\Sigma f\,|m - Med|}{N} = \frac{6800}{60} = 113.33$

Again, coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{Median}} = \frac{113.33}{450} = 0.25$

Thus, mean deviation from median is 113.33 and it's coefficient is 0.25.

3. तल दिइएको तथ्याङ्कका आधारमा मध्यकबाट मध्यक भिन्नता र मध्यक भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
Calculate the mean deviation from mean and it's coefficient on the basis of following data.

(a)

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	18	24	20	19	5

⇨ Here, calculation of mean deviation and it's coefficient from mean.

Marks	frequency (f)	Mid value (m)	fm	$\|D\| = \|m - \bar{x}\|$	f |D|
0 - 10	2	5	10	25.8	51.6
10 - 20	18	15	270	15.8	284.4
20 - 30	24	25	600	5.8	139.2
30 - 40	20	35	700	4.2	84
40 - 50	19	45	855	14.2	269.8
50 - 60	5	55	275	24.2	121
	N = 88		Σfm = 2710		Σf|D| = 950

We have, Mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{2710}{88} = 30.8$ We know that, mean deviation $= \frac{\Sigma f\,|D|}{N} = \frac{950}{88} = 10.8$

Again, Coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{mean }(\bar{x})} = \frac{10.8}{30.8} = 0.35$

Thus, mean deviation is 10.8 and it's coefficient is 0.35.

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(b)

Weight	20-25	25-30	30-35	35-40	40-45	45-50
No. of men	7	3	6	4	8	2

⇨ Here, calculation of mean deviation and it's coefficient from mean.

Weight (kg)	frequency (f)	Mid value (m)	fm	$\lvert D\rvert = \lvert m - \bar{x}\rvert$	f $\lvert D\rvert$
20 - 25	7	22.5	157.5	11.5	80.5
25 - 30	3	27.5	82.5	6.5	19.5
30 - 35	6	32.5	195	1.5	9
35 - 40	4	37.5	150	3.5	14
40 - 45	8	42.5	340	8.5	68
45-50	2	47.5	95	13.5	27
	N = 30		Σfm = 1020		Σf $\lvert D\rvert$ = 218

We have, mean ($\bar{x}$) $= \frac{\Sigma fm}{N} = \frac{1020}{30} = 34$ We know that, mean deviation $= \frac{\Sigma f|D|}{N} = \frac{218}{30} = 7.27$

Again, Coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{mean } (\bar{x})} = \frac{7.27}{34} = 0.213$

Thus, mean deviation is 7.27 and it's coefficient is 0.213.

(c)

Marks	$5 \le x < 10$	$10 \le x < 15$	$15 \le x < 20$	$20 \le x < 25$	$25 \le x < 30$
No. of students	7	4	5	6	3

⇨ Here, calculation of mean deviation and it's coefficient.

CI	frequency (f)	Mid value (m)	fm	$\lvert D\rvert = \lvert m - \bar{x}\rvert$	f $\lvert D\rvert$
5 - 10	7	7.5	52.5	8.8	61.6
10 - 15	4	12.5	50	3.8	15.2
15 - 20	5	17.5	87.5	1.2	6
20 - 25	6	22.5	135	6.2	37.2
25 - 30	3	27.5	82.5	11.2	33.6
	N = 25		Σfm = 407.5		Σf $\lvert D\rvert$ = 153.6

We have, mean ($\bar{x}$) $= \frac{\Sigma fm}{N} = \frac{407.5}{25} = 16.3$

We know that, mean deviation $= \frac{\Sigma f|D|}{N} = \frac{153.6}{25} = 6.144$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean } (\bar{x})} = \frac{6.144}{16.3} = 0.376$

Thus, mean deviation is 6.144 and it's coefficient is 0.376.

4. (a) **तल दिइएको तथ्याङ्कका आधारमा मध्यक र मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस् । 20 वर्ष र 20 वर्षभन्दा माथिकाले मात्र सर्वेक्षणमा भाग लिएका छन् ।**

Calculate the mean deviation and its coefficient from mean and median on the basis of following data. People having age 20 years or above 20 years are only participated in the survey.

Age	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90
No. of men	3	64	196	349	489	540	542

⇨ (i) Here, calculation of mean deviation and it's coefficient from mean.

CI	frequency (f)	Mid value (m)	fm	$\lvert D\rvert = \lvert m - \bar{x}\rvert$	f $\lvert D\rvert$
20 - 30	3	25	75	29.72	89.16
30 - 40	64 – 3 = 61	35	2135	19.72	1202.92
40 - 50	196 – 64 = 132	45	5940	9.72	1283.04
50 - 60	349 – 196 = 153	55	8415	0.28	42.84
60 - 70	489 – 349 = 140	65	9100	10.28	1439.2
70 - 80	540 – 489 = 51	75	3825	20.28	1034.28
80 - 90	542 – 540 = 2	85	170	30.28	60.56
	N = 542		Σfm = 29660		Σf $\lvert D\rvert$ = 5152

We have, mean ($\bar{x}$) $= \frac{\Sigma fm}{N} = \frac{29660}{542} = 54.72$

We know that, mean deviation $= \frac{\Sigma f|D|}{N} = \frac{5152}{542} = 9.50$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean } (\bar{x})} = \frac{9.50}{54.72} = 0.173$

Thus, mean deviation and it's coefficient from mean are 9.50 and 0.173 respectively.

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⇨ (ii) Here, calculation of mean deviation and it's coefficient from median.

CI	**cf**	**f**	**Mid value (m)**	**|m – Med|**	**f |m – Med|**
20 - 30	3	3	25	29.9	89.7
30 - 40	64	64 – 3 = 61	35	19.9	1213.9
40 - 50	196	196 – 64 = 132	45	9.9	1306.8
50 - 60	349	349 – 196 = 153	55	0.1	15.3
60 - 70	489	489 – 349 = 140	65	10.1	1414
70 - 80	540	540 – 489 = 51	75	20.1	1025.1
80 - 90	542	542 – 540 = 2	85	30.1	60.2
		N = 542			Σf |m – Med| = 5125

For median, $\left(\frac{N}{2}\right)^{th}$ value = $\left(\frac{542}{2}\right)^{th}$ value = 271st value ∴ Median class = 50 - 60

Now, Median (Med) $= \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 50 + \frac{271 - 196}{153} \times 10 = 54.90$ ∴ Median (Med) = 54.90

We know that, Mean deviation from median $= \frac{\Sigma f\,|m - Med|}{N} = \frac{5125}{542} = 9.46$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{median}} = \frac{9.46}{54.90} = 0.172$

Thus, mean deviation and it's coefficient from median are 9.46 and 0.172 respectively.

(b) **एउटा बगैँचाका 50 ओटा विरुवाको उचाइ विवरण निम्नअनुसार छ। जम्मा 48 cm सम्म उचाइ भएका विरुवाहरू मात्र सर्वेक्षणमा छन्।**
The height of 50 plants of a garden is as below. Only the plants having height upto 48 cm are in the survey.

Height	**above 0 cm**	**above 8 cm**	**above 16 cm**	**above 24 cm**	**above 32 cm**	**above 40 cm**
No. of plants	**50**	**42**	**35**	**30**	**18**	**6**

माथिको तथ्याङ्कको आधारमा (On the basis of above data),

(i) **मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस्। (Calculate the mean deviation from mean and its coefficient.)**

⇨ Here, calculation of mean deviation and it's coefficient from mean.

CI	**f**	**m**	**fm**	**$\|D\| = \|m - \bar{x}\|$**	**f |D|**
0 - 8	8	4	32	20.96	167.68
8 - 16	7	12	84	12.96	90.72
16 - 24	5	20	100	4.96	24.8
24 - 32	12	28	336	3.04	36.48
32 - 40	12	36	432	11.04	132.48
40 - 48	6	44	264	19.04	114.24
	N = 50		Σfm = 1248		Σf |D| = 566.4

We have, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{1248}{50} = 24.96$ So, Mean deviation from mean $= \frac{\Sigma f\,|D|}{N} = \frac{566.4}{50} = 11.328$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean}} = \frac{11.328}{24.96} = 0.453$

Thus, mean deviation and it's coefficient from mean are 11.328 and 0.453 respectively.

(ii) **मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क निकाल्नुहोस्। (Calculate the mean deviation and its coefficient from median.)**

⇨ Here, calculation of mean deviation and it's coefficient from median.

CI	**f**	**cf**	**mid value (m)**	**|m – Med|**	**f |m – Med|**
0 - 8	8	8	4	23.33	186.64
8 - 16	7	15	12	15.33	107.31
16 - 24	5	20	20	7.33	36.65
24 - 32	12	32	28	0.67	8.04
32 - 40	12	44	36	8.67	104.04
40 - 48	6	50	44	16.67	100.02
	N = 50				Σf |m – med| = 542.7

For median; $\left(\frac{N}{2}\right)^{th}$ value = $\left(\frac{50}{2}\right)^{th}$ value = 25th value ∴ median class = 24 - 32

Now, Median $= \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 24 + \frac{25 - 20}{12} \times 8 = 27.33$ ∴ Median (Med) = 27.33

We know that,Mean deviation from median $= \frac{\Sigma f\,|m - med|}{N} = \frac{542.7}{50} = 10.854$

Again, coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{median}} = \frac{10.854}{27.33} = 0.397$

Thus, mean deviation and it's coefficient from median are 10.854 and 0.397 respectively.

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5. **एउटा सर्वेक्षणमा 20 जनाको समूहमा रहेका व्यक्तिहरूको तौल (कि.ग्रा.मा) निम्नअनुसार पाइयो । यो तथ्याङ्कलाई 10 को वर्गान्तर तालिका निर्माण गरी मध्यक र मध्यिकाबाट मध्यक भिन्नता एवम् मध्यक भिन्नताको गुणाङ्क निकाल्नुहोस् ।**
In a survey, the weight (in kg) of a group of 20 persons is found as below. Taking class interval of size 10, find the mean deviation and it's coefficient from mean and median.
59, 71, 45, 44, 35, 21, 29, 42, 37, 49, 58, 69, 55, 39, 79, 50, 65, 52, 60, 64

⇨ (i) Here, calculation of mean deviation and it's coefficient from mean.

CI	f	m	fm	$\lvert m-\bar{x}\rvert$	$f\lvert m-\bar{x}\rvert$
20 - 30	2	25	50	26	52
30 - 40	3	35	105	16	48
40 - 50	4	45	180	6	24
50 - 60	5	55	275	4	20
60 - 70	4	65	260	14	56
70 - 80	2	75	150	24	48
	N = 20		$\Sigma fm = 1020$		$\Sigma f\lvert m-\bar{x}\rvert = 248$

We have, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{1020}{20} = 51$ We know that, Mean deviation $= \frac{\Sigma f\lvert m-\bar{x}\rvert}{N} = \frac{248}{20} = 12.4$

Again, coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{mean}} = \frac{12.4}{51} = 0.24$

Thus, mean deviation and it's coefficient from mean are 12.4 and 0.24 respectively.

⇨ (ii) Here, calculation of mean deviation and it's coefficient from median.

CI	f	cf	m	$\lvert m-\text{Med}\rvert$	$f\lvert m-\text{Med}\rvert$
20 - 30	2	2	25	27	54
30 - 40	3	5	35	17	51
40 - 50	4	9	45	7	28
50 - 60	5	14	55	3	15
60 - 70	4	18	65	13	52
70 - 80	2	20	75	23	46
	N = 20				$\Sigma f\lvert x-\text{Med}\rvert = 246$

For median; $\left(\frac{N}{2}\right)^{th}$ value $= \left(\frac{20}{2}\right)^{th}$ value = 10th value ∴ median class = 50 - 60

Now, Median $= \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 50 + \frac{10-9}{5} \times 10 = 52$ ∴ Median (Med) = 52

We know that, Mean deviation from median $= \frac{\Sigma f\lvert m-\text{Med}\rvert}{N} = \frac{246}{20} = 12.3$

Again, Coefficient of mean deviation $= \frac{\text{mean deviation}}{\text{median}} = \frac{12.3}{52} = 0.23$

Thus, mean deviation and it's coefficient from median are 12.3 and 0.23 respectively.

3. स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क
Standard Deviation and Coefficient of Variation

QUESTIONS FROM SEE EXERCISE 3

A. SHORT QUESTIONS

1. **एउटा निरन्तर श्रेणीमा $\Sigma fm = 1775, \Sigma fm^2 = 639375$ र N = 5 भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $\Sigma fm = 1775, \Sigma fm^2 = 639375$ and N = 5 then find standard deviation and its coefficient.

⇨ Here, $\Sigma fm = 1775, \Sigma fm^2 = 639375$ and N = 5

Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{1775}{5} = 355$

We have, standard deviation $(\sigma) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2} = \sqrt{\frac{639375}{5} - \left(\frac{1775}{5}\right)^2} = \sqrt{127875 - 126025}$

$= \sqrt{1850} = 43.01$

Again, coefficient of standard deviation $= \frac{\sigma}{\text{mean}} = \frac{43.01}{355} = 0.12$

Thus, SD is 43.01 and it's coefficient is 0.12

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2. **एउटा निरन्तर श्रेणीमा $N = 47, \Sigma fm = 770$ र $\Sigma fm^2 = 16450$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $N = 47, \Sigma fm = 770$ and $\Sigma fm^2 = 16450$ then find the standard deviation and its coefficient.

⇨ Here, $N = 47, \Sigma fm = 770, \Sigma fm^2 = 16450$ Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{770}{47} = 16.38$

We have standard deviation $(\sigma) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2} = \sqrt{\frac{16450}{47} - \left(\frac{770}{47}\right)^2} = \sqrt{350 - 268.40} = \sqrt{81.6} = 9.03$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{9.03}{16.38} = 0.55$

Thus, standard deviation is 9.03 and it's coefficient is 0.55

3. **एउटा निरन्तर श्रेणीमा $\Sigma fd = -14, \Sigma fd^2 = 460, d = m - A, A = 46$ र $N = 7$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $\Sigma fd = -14, \Sigma fd^2 = 460, d = m - A, A = 46$ and $N = 7$ then find standard deviation and its coefficient.

⇨ Here, $\Sigma fd = -14, \Sigma fd^2 = 460, d = m - A, A = 46$ and $N = 7$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd}{N} = 46 - \frac{14}{7} = 46 - 2 = 44$

We have, Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{460}{7} - \left(\frac{-14}{7}\right)^2} = \sqrt{65.714 - 4} = \sqrt{61.714} = 7.85$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{7.85}{44} = 0.17$

Thus, standard deviation is 7.85 and it's coefficient is 0.17

4. **एउटा निरन्तर श्रेणीमा $\Sigma fd = 0, \Sigma fd^2 = 848, N = 100, d = m - A$ र $A = 12$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $\Sigma fd = 0, \Sigma fd^2 = 848, N = 100, d = m - A$ and $A = 12$ then find the standard deviation and its coefficient.

⇨ Here, $\Sigma fd = 0, \Sigma fd^2 = 848, N = 100, d = m - A$ and $A = 12$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd}{N} = 12 + 0 = 12$ $\therefore \bar{x} = 12$

We know that, Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{848}{100} - 0} = \sqrt{8.48} = 2.91$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{2.91}{12} = 0.24$

Thus, standard deviation is 2.91 and it's coefficient is 0.24

5. **एउटा निरन्तर श्रेणीमा $\bar{x} = 7, \Sigma fm = 35, \Sigma fd^2 = 10$ र $d = m - \bar{x}$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $\bar{x} = 7, \Sigma fm = 35, \Sigma fd^2 = 10$ and $d = m - \bar{x}$ then find standard deviation and its coefficient.

⇨ Here, $\bar{x} = 7, \Sigma fm = 35, \Sigma fd^2 = 10, d = m - \bar{x}$

We have, mean $(\bar{x}) = \frac{\Sigma fm}{N}$ or, $7 = \frac{35}{N}$ $\therefore N = \frac{35}{7} = 5$

Now, standard deviation by actual mean method is; $\sigma = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{10}{5}} = \sqrt{2} = 1.414$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{1.414}{7} = 0.202$

Thus, standard deviation is 1.414 and it's coefficient is 0.202

6. **एउटा निरन्तर श्रेणीमा $N = 30, \Sigma fm = 870$ र $\Sigma f(m - \bar{x})^2 = 1070$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $N = 30, \Sigma fm = 870$ and $\Sigma f(m - \bar{x})^2 = 1070$ then find the standard deviation and its coefficient.

⇨ Here, $N = 30, \Sigma fm = 870$ and $\Sigma f(m - \bar{x})^2 = 1070$ Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{870}{30} = 29$

We have, standard deviation by actual mean method is; $\sigma \sqrt{\frac{\Sigma f(m - \bar{x})^2}{N}} = \sqrt{\frac{1070}{30}} = \sqrt{35.667} = 5.97$

Again coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{5.97}{29} = 0.206$

Thus, standard deviation is 5.97 and it's coefficient is 0.206

7. **एउटा निरन्तर श्रेणीमा $\Sigma fm = 114, \bar{x} = 19$ र $\Sigma f(m - \bar{x})^2 = 232$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
In a continuous series, $\Sigma fm = 114, \bar{x} = 19$ and $\Sigma f(m - \bar{x})^2 = 232$ then find standard deviation and its coefficient.

⇨ Here, $\Sigma fm = 114, \bar{x} = 19, \Sigma f(m - \bar{x})^2 = 232$

We have, $\bar{x} = \frac{\Sigma fm}{N}$ or, $19 = \frac{114}{N}$ $\therefore N = \frac{114}{19} = 6$

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Now, standard deviation by actual mean method is; $\sigma = \sqrt{\frac{\Sigma f(m-\bar{x})^2}{N}} = \sqrt{\frac{232}{6}} = \sqrt{38.667} = 6.22$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{6.22}{19} = 0.33$

Thus, standard deviation is 6.22 and it's coefficient is 0.33

8. एउटा निरन्तर श्रेणीमा $N = 10, \Sigma fm = 40, \Sigma fm^2 = 192$ र $\bar{x} = 4$ भए सो श्रेणीको विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $N = 10, \Sigma fm = 40, \Sigma fm^2 = 192$ and $\bar{x} = 4$ then find the coefficient of variation.

⇨ Here, $N = 10, \Sigma fm = 40, \Sigma fm^2 = 192$ and $\bar{x} = 4$

We have, standard deviation (σ) $= \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2} = \sqrt{\frac{192}{10} - \left(\frac{40}{10}\right)^2} = \sqrt{19.2 - 16} = \sqrt{3.2} = 1.7888$

Now, we know Coefficient of variation (CV) $= \frac{\sigma}{\bar{x}} \times 100\% = \frac{1.7888}{4} \times 100\% = 44.72\%$

Thus, required coefficient of variation is 44.72%

9. एउटा अविच्छिन्न श्रेणीमा $N = 28, \Sigma fd' = -2, \Sigma fd'^2 = 36$ र $10d' = m - a$ भए स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
In a continuous data, $N = 28, \Sigma fd' = -2, \Sigma fd'^2 = 36$ and $10d' = m - a$ then find the standard deviation.

⇨ Here, $N = 28, \Sigma fd' = -2, \Sigma fd'^2 = 36, 10d' = m - a \therefore d' = \frac{m-a}{10}$

We know that, standard deviation by step deviation method is given by;

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c = \sqrt{\frac{36}{28} - \left(\frac{-2}{28}\right)^2} \times 10 = \sqrt{1.2857 - 0.0051} \times 10 = \sqrt{1.2806} \times 10 = 11.31$$

Thus, standard deviation is 11.31

10. तलको जानकारीबाट S.D. र यसको गुणाङ्क पत्ता लगाउनुहोस् (From the following information, find S.D. and its coefficient) :
$N = 7, \Sigma fd = 28, \Sigma fd^2 = 224, d = m - A$ & $A = 10$

⇨ Here, $N = 7, \Sigma fd = 28, \Sigma fd^2 = 224, d = m - A$ and $A = 10$

We know, mean ($\bar{x}$) $= A + \frac{\Sigma fd}{N} = 10 + \frac{28}{7} = 10 + 4 = 14$

Now, standard deviation (σ) $= \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{224}{7} - \left(\frac{28}{7}\right)^2} = \sqrt{32 - 16} = \sqrt{16} = 4$

Again, coefficient of standard deviation is; $= \frac{\sigma}{\bar{x}} = \frac{4}{14} = 0.28$

Thus, standard deviation is 4 and it's coefficient is 0.28

11. कुनै तथ्याङ्कको स्तरीय भिन्नता 0.25 भए विचरणशीलता कति हुन्छ ? पत्ता लगाउनुहोस् ।
If the standard deviation of a set of data is 0.25, find its variance. **[SEE MODEL 2076]**

⇨ Here, standard deviation (σ) = 0.25,
We know that, Variance = (standard deviation)$^2 = \sigma^2 = (0.25)^2 = 0.0625$
Thus, variance is 0.0625.

B. LONG QUESTIONS

MODEL 1

1. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् । (Find the standard deviation of the data given below.)
[2075 R', 2073 S']

Class-interval	0 - 10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

⇨ Here, calculation of standard deviation.

x	f	m	fm	$d = \lvert m - \bar{x} \rvert$	d^2	fd^2
0 - 10	5	5	25	22	484	2420
10 - 20	8	15	120	12	144	1152
20 - 30	15	25	375	2	4	60
30 - 40	16	35	560	8	64	1024
40 - 50	6	45	270	18	324	1944
	N = 50		$\Sigma fm = 1350$			$\Sigma fd^2 = 6600$

We know, mean ($\bar{x}$) $= \frac{\Sigma fm}{N} = \frac{1350}{50} = 27$

Now, standard deviation (σ) $= \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{6600}{50}} = \sqrt{132} = 11.49$

Thus, standard deviation of given data is 11.49

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2. **तल दिइएको आँकडाबाट स्तरीय भिन्नता पत्ता लगाउनुहोस् (Calculate the standard deviation of the data given below): [2072 S]**

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	6	5	4	3

⇨ Calculation of standard deviation.

CI	f	m	fm	fm²
0-10	2	5	10	50
10-20	6	15	90	1350
20-30	5	25	125	3125
30-40	4	35	140	4900
40-50	3	45	135	6075
	N = 20		$\Sigma f\text{m} = 500$	$\Sigma f\text{m}^2 = 15500$

We know that,

$$\text{Standard deviation (SD)} = \sqrt{\frac{\Sigma f\text{m}^2}{\text{N}} - \left(\frac{\Sigma f\text{m}}{\text{N}}\right)^2}$$

$$= \sqrt{\frac{15500}{20} - \left(\frac{500}{20}\right)^2}$$

$$= \sqrt{775 - 25^2} = \sqrt{775 - 625}$$

$$= 12.24$$

Thus, the standard deviation is 12.24.

3. **दिइएको तथ्याङ्कबाट स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation from the given data): [2072 R']**

Marks obtained	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	5	4

⇨ Here, calculation of standard deviation:

Marks	f	m	fm	fm²
30-40	2	35	70	2450
40-50	3	45	135	6075
50-60	6	55	330	18150
60-70	5	65	325	21125
70-80	4	75	300	22500
	N = 20		$\Sigma f\text{m} = 1160$	$\Sigma f\text{m}^2 = 70300$

We know that,

$$\text{S.D.} = \sqrt{\frac{\Sigma f\text{m}^2}{\text{N}} - \left(\frac{\Sigma f\text{m}}{\text{N}}\right)^2}$$

$$= \sqrt{\frac{70300}{20} - \left(\frac{1160}{20}\right)^2}$$

$$= \sqrt{3515 - 3364}$$

$$= \sqrt{151} = 12.29$$

Thus, the standard deviation is 12.29.

4. **तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation of the data given below): [2071 R]**

Class interval	5-15	15-25	25-35	35-45	45-55
Frequency	7	3	6	4	5

⇨ Here, calculation of standard deviation: Let assumed mean (A) be 30.

CI	f	mid-value	d = M – A	fd	fd²
5-15	7	10	– 20	– 140	2800
15-25	3	20	– 10	– 30	300
25-35	6	30	0	0	0
35-45	4	40	10	40	400
45-55	5	50	20	100	2000
	N = 25			$\Sigma f\text{d} = -30$	$\Sigma f\text{d}^2 = 5500$

We know that,

Standard deviation (SD)

$$= \sqrt{\frac{\Sigma f\text{d}^2}{\text{N}} - \left(\frac{\Sigma f\text{d}}{\text{N}}\right)^2}$$

$$= \sqrt{\frac{5500}{25} - \left(\frac{-30}{25}\right)^2}$$

$$= \sqrt{220 - 1.44} = \sqrt{218.56} = 14.78$$

Thus, the standard deviation is 14.78.

5. **तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation of the data give below): [2071 S]**

Class interval	25-35	35-45	45-55	55-65	65-75
Frequency	5	4	6	7	3

⇨ Here, calculation of standard deviation. Let the assumed mean be 50.

CI	Mid-value	f	d = x – A	fd	fd²
25-35	30	5	– 20	– 100	2000
35-45	40	4	– 10	– 40	400
45-55	50	6	0	0	0
55-65	60	7	10	70	700
65-75	70	3	20	60	1200
		N = 25		$\Sigma f\text{d} = -10$	$\Sigma f\text{d}^2 = 4300$

We know that,

Standard deviation (SD)

$$= \sqrt{\frac{\Sigma f\text{d}^2}{\text{N}} - \left(\frac{\Sigma f\text{d}}{\text{N}}\right)^2}$$

$$= \sqrt{\frac{4300}{25} - \left(\frac{-10}{25}\right)^2}$$

$$= \sqrt{172 - 0.16} = 13.11$$

Thus, the standard deviation is 13.11

6. **तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।**

Find the standard deviation of the data given below. [2070 R]

Class interval	0-4	4-8	8-12	12-16	16-20
Frequency	15	12	10	8	5

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⇨ Here, calculation of standard deviation,

class	f	m	fm	fm^2
0-4	15	2	30	60
4-8	12	6	72	432
8-12	10	10	100	1000
12-16	8	14	112	1568
16-20	5	18	90	1620
	N = 50		$\sum fm = 404$	$\sum fm^2 = 4680$

$$\text{S.D. }(\sigma) = \sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$$

$$= \sqrt{\frac{4680}{50} - \left(\frac{404}{50}\right)^2}$$

$$= \sqrt{93.60 - 65.28} = \sqrt{28.32} = 5.32$$

Thus, the standard deviation is 5.32.

7. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation): [2068 R]

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	5	3	2

⇨ Here, calculation of standard deviation

CI	f	m	fm	fm²
0-10	4	5	20	100
10-20	6	15	90	1350
20-30	5	25	125	3125
30-40	3	35	105	3675
40-50	2	45	90	4050
	N = 20		$\sum fm = 430$	$\sum fm^2 = 12300$

We know that,

$$\text{Standard deviation }(\sigma) = \sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$$

$$= \sqrt{\frac{12300}{20} - \left(\frac{430}{20}\right)^2}$$

$$= \sqrt{152.75}$$

Thus, SD is 12.36.

8. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation): [2067 S]

CI	0-10	10-20	20-30	30-40	40-50
f	8	10	12	8	2

⇨ Here, calculation of standard deviation Let, a = 25 be the assumed mean.

CI	f	m	d = m – a	fd	fd²
0-10	8	5	– 20	– 160	3200
10-20	10	15	– 10	– 100	1000
20-30	12	25 = a	0	0	0
30-40	8	35	10	80	800
40-50	2	45	20	40	800
	N = 40			$\sum fd = -140$	$\sum fd^2 = 5800$

We know that,

$$\text{Standard deviation }(\sigma) = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= \sqrt{\frac{5800}{40} - \left(\frac{-140}{40}\right)^2}$$

$$= \sqrt{145 - 12.25}$$

$$= \sqrt{132.75} = 11.52$$

Thus, the standard deviation of given data is 11.52.

9. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation of the data given below): [2074 R', 2073 S, 2059 R]

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	2	9	10	7	1

⇨ Here, from the given data:

CI	m	f	fm	$d = m - \bar{x}$	d²	fd^2
0 - 10	5	2	10	–18.6	345.96	691.92
10 - 20	15	9	135	–8.6	73.96	665.64
20 - 30	25	10	250	1.4	1.96	19.6
30 - 40	35	7	245	11.4	129.96	909.72
40 - 50	45	1	45	21.4	457.96	457.96
		N = 29	$\sum fm = 685$			$\sum fd^2 = 2744.84$

Where, $N = 29, \sum fm = 685$

$$\therefore \quad \bar{x} = \frac{\sum fm}{N} = \frac{685}{29} = 23.6$$

Again, $\sum fd^2 = 2744.84$ and $N = 29$

∴ Standard deviation (σ)

$$= \sqrt{\frac{\sum fd^2}{N}} = \sqrt{\frac{2744.84}{29}}$$

$$= \sqrt{94.64} = 9.72$$

10. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation): [2065 M]

x	0-10	10-20	20-30	30-40	40-50
f	5	4	4	6	1

⇨ Calculation of standard deviation

x	f	m	d = m – a	fd	fd²
0-10	5	5	– 20	– 100	2000
10-20	4	15	– 10	– 40	400
20-30	4	25 = a	0	0	0
30-40	6	35	10	60	600
40-50	1	45	20	20	400
	N = 20			$\sum fd = -60$	$\sum fd^2 = 3400$

We have,

$$\text{Standard deviation }(\sigma) = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= \sqrt{\frac{3400}{20} - \left(\frac{-60}{20}\right)^2}$$

$$= \sqrt{(170 - 9)} \quad = 12.68.$$

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11. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् **(Find the standard deviation):** **[2065 S]**

Marks obtained)	0-10	10-20	20-30	30-40	40-50
No. of students	1	4	7	3	5

⇨ Here, the calculation of the standard deviation:

Marks	f	m	fm	fm²
0-10	1	5	5	25
10-20	4	15	60	900
20-30	7	25	175	4375
30-40	3	35	105	3675
40-50	5	45	225	10125
	N = 20		Σfm = 570	Σfm² = 19100

We know that,

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{19100}{20} - \left(\frac{570}{20}\right)^2}$$

$$= \sqrt{955 - 812.25} \quad = \sqrt{142.75} = 11.95$$

Thus, the standard deviation of given data is 11.95.

MODEL 2

12. तलको तथ्याङ्कबाट स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।

Find the standard deviation and its coefficient from the following date. **[2071 R']**

Marks Obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	12	20	40	12	8

⇨ Here, calculation of standard deviation and its coefficient. Let assumed mean (A) = 25

Marks	*f*	m	d = m – A	*fd*	*fd*²
0-10	8	5	– 20	– 160	3200
10-20	12	15	– 10	– 120	1200
20-30	20	25 = A	0	0	0
30-40	40	35	10	400	4000
40-50	12	45	20	240	4800
50-60	8	55	30	240	7200
	N = 100			Σfd = 600	Σfd^2 = 20400

We know that,

Standard deviation (SD)

$$= \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{20400}{100} - \left(\frac{600}{100}\right)^2}$$

$$= \sqrt{204 - 36} = \sqrt{168} = 12.96$$

Now, mean $(\overline{x}) = A + \frac{\Sigma fd}{N} = 25 + \frac{600}{100} = 25 + 6 = 31$

We have, coefficient of SD $= \frac{SD}{Mean} = \frac{12.96}{31} = 0.42$

Thus, SD and its coefficient are 12.96 and 0.42 respectively.

13. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र त्यसको गुणाङ्क पत्ता लगाउनुहोस्:

Find the standard deviation and its coefficient from the given data: **[2069 R']**

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	15	25	35	45

⇨ Here, calculation of SD and its coefficient:

C.I	*f*	m	*f*m	d = $\lvert m - \overline{x}$ (33)$\rvert$	d²	*f*d²
0-10	5	5	25	28	784	3920
10-20	15	15	225	18	324	4860
20-30	25	25	625	8	64	1600
30-40	35	35	1225	2	4	140
40-50	45	45	2025	12	144	6480
Total	**Σf = 125**		Σfm = **4125**			Σfd² = **17000**

We know, mean $\overline{X} = \frac{\Sigma fm}{\Sigma f} = \frac{4125}{125} = 33 \qquad \therefore \ \overline{X} = 33$

Now, standard deviation $= \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{17000}{125}} = 11.66 \qquad \therefore \ \sigma = 11.66$

Again, coefficient of standard deviation $= \frac{\sigma}{\overline{X}} = \frac{11.66}{33} = 0.35$

Thus, standard deviation is 11.66 and its coefficient is 0.35.

MODEL 3

14. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।

Find the standard deviation and coefficient of variation from given data. **[SEE MODEL 2076]**

Age	0-4	4-8	8-12	12-16	16-20	20-24
No. of students	7	7	10	15	7	6

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⇨ Here, calculation of SD and coefficient of variation.

Age	f	x	$d = x - 10$	fd	fd^2
0-4	7	2	– 8	– 56	448
4-8	7	6	– 4	– 28	112
8-12	10	10	0	0	0
12-16	15	14	4	60	240
16-20	7	18	8	56	448
20-27	6	22	12	72	864
Total	**N = 52**			$\Sigma fd =$ **104**	$\Sigma fd^2 =$ **2112**

Let, assumed mean (a) be 10.

We know that, $\overline{X} = a + \frac{\Sigma fd}{N} = 10 + \frac{104}{52} = 10 + 2$ $\therefore$ $\overline{X} = 12$

Again, SD $= \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{2112}{52} - \left(\frac{104}{52}\right)^2} = \sqrt{40.61 - 4} = \sqrt{36.61}$ $\therefore$ SD = 6.05

Now, coefficient of variation $= \frac{SD}{\overline{X}} \times 100\% = \frac{6.05}{12} \times 100\% = 50.42\%$

Thus, SD and coefficient of variation are 6.05 and 50.42% respectively.

15. **तल दिइएको तथ्याङ्कबाट विचरणशीलताको गुणाङ्क गणना गर्नुहोस् ।**

Calculate the coefficient of variation from the data given below. **[SEE 2075 R]**

Class-interval	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequency	**5**	**8**	**15**	**16**	**6**

⇨ Here, calculation of coefficient of variation

x	f	m	fm	$d = m - \overline{x}$ (27)	d^2	fd^2
0 - 10	5	5	25	– 22	484	2420
10 - 20	8	15	120	– 12	144	1152
20 - 30	15	25	375	– 2	4	60
30 - 40	16	35	560	8	64	1024
40 - 50	6	45	270	18	324	1944
	N = 50		Σfm = 1350			$\Sigma fd^2 = 6600$

We know that, mean $(\overline{x}) = \frac{\Sigma fm}{N} = \frac{1350}{50} = 27$

Now, standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{6600}{50}} = 11.49$

We know, coefficient of variation $= \frac{\sigma}{\overline{x}} \times 100\% = \frac{11.49}{27} \times 100\% = 42.55\%$

Thus, the coefficient of variation is 42.55%.

16. **निम्न तथ्याङ्कबाट विचरणशीलताको गुणाङ्क गणना गर्नुहोस् ।**

Calculate the coefficient of variation from the data given below. **[SEE 2075 R$_2$]**

Class-interval	**0 - 20**	**20 - 40**	**40 - 60**	**60 - 80**	**80 - 100**
Frequency	**2**	**3**	**4**	**5**	**6**

⇨ Here, calculation of coefficient of variation

x	f	m	fm	$d = m - \overline{x}$ (60)	d^2	fd^2
0 - 20	2	10	20	– 50	2500	5000
20 - 40	3	30	90	– 30	900	2700
40 - 60	4	50	200	– 10	100	400
60 - 80	5	70	350	10	100	500
80 - 100	6	90	540	30	900	5400
	N = 20		Σfm = 1200			$\Sigma fd^2 = 14000$

We know that, mean $(\overline{x}) = \frac{\Sigma fm}{N} = \frac{1200}{20} = 60$

Now, standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{14000}{20}} = 26.46$

Coefficient of variation $= \frac{\sigma}{\overline{x}} \times 100\% = \frac{26.46}{60} \times 100\% = 44.1\%$

Thus, the coefficient of variation of given data is 44.1%.

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MODEL 4

17. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation from the following data):

Mid value marks	5	10	15	20	25	30
No. of students	2	3	5	6	3	1

⇨ Here, from the given data

m	f	fm	$d = m - \bar{x}$	d^2	$f.d^2$
5	2	10	– 12	144	288
10	3	30	– 7	49	147
15	5	75	– 2	4	20
20	6	120	3	9	54
25	3	75	8	64	192
30	1	30	13	169	169
Total	N = 20	340			870

$\therefore$ Mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{340}{20} = 17$

Now, Standard deviation (σ)

$= \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{870}{20}}$

$= \sqrt{43.5} = 6.595$

18. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation from the following data):

Mid value	12	13	14	15	16	17
f	2	3	6	4	2	1

⇨ Here, from the given data :

m	*f*	*fm*	$d = m - \bar{x}$	d^2	fd^2
12	2	24	-2.2	4.84	9.68
13	3	39	-1.2	1.44	4.32
14	6	84	-0.2	0.04	0.24
15	4	60	0.8	0.64	2.56
16	2	32	1.8	3.24	6.48
17	1	17	2.8	7.84	7.84
Total	$\Sigma f = 18$	$\Sigma fm = 256$			$\Sigma fd^2 = 31.12$

Here, Mean $(\bar{x}) = \frac{\Sigma fm}{\Sigma f} = \frac{256}{18} = 14.2$

$\therefore$ Standard deviation (σ)

$= \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{31.12}{18}}$

$= \sqrt{1.7289} = 1.3149$

Thus, SD = 1.3149.

19. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation from the following data):

Mid value of height class	10	20	30	40	50
Number of Plants	8	12	15	9	6

⇨ Here, calculation of standard deviation:

m	f	d = m – a	fd	fd^2
10	8	– 20	– 160	3200
20	12	– 10	– 120	1200
30 = a	15	0	0	0
40	9	10	90	900
50	6	20	120	2400
	N = 50		$\Sigma fd = 70$	$\Sigma fd^2 = 7700$

We have,

Standard deviation $= \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{7700}{50} - \left(\frac{-70}{50}\right)^2} = \sqrt{154 - 1.96}$

Thus, Standard deviation (σ) = 12.33

20. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation from the following data):

Mid value	5	10	15	20	25	30
f	12	8	4	7	10	6

⇨ Here, Calculation of SD

m	f	fm	fm^2
5	12	60	300
10	8	80	800
15	4	60	900
20	7	140	2800
25	10	250	6250
30	6	180	5400
	N = 47	$\Sigma fm = 770$	$\Sigma fm^2 = 16450$

We have,

Standard deviation (S.D.) $= \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2} = \sqrt{\frac{16450}{47} - \left(\frac{770}{47}\right)^2} = 9.03$

Thus, the standard deviation is 9.03.

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QUESTIONS FROM CDC TEXTBOOK

8.3 स्तरीय भिन्नता (STANDARD DEVIATION)

EXERCISE 8.3

1. (a) **परिभाषा दिनुहोस् (Define):**

(i) **स्तरीय भिन्नता (Standard deviation)**

⇨ The square root of the average of squares of the deviation taken from the mean is called the standard deviation. It is denoted by σ.

(ii) **स्तरीय भिन्नताको गुणाङ्क (Coefficient of standard deviation)**

⇨ The relative measure of dispersion based on the standard deviation is known as the coefficient of standard deviation.

(iii) **विचरणशीलताको गुणाङ्क (Coefficient of variation)**

⇨ If the coefficient of standard deviation is expressed as percentage then it is known as coefficient of variation.

(b) **स्तरीय भिन्नताका गुण र दोषहरू उल्लेख गर्नुहोस् । (List the merits and demerits of standard deviation.)**

⇨ The merits and demerits of standard deviation are as follows:

Merits	Demerits
1. It is based on all the observations.	1. As compared to other measures, it is difficult to compute.
2. It is the best measure of dispersion.	2. It can not be calculated for open end classes.
3. Standard deviation is rigidly defined and its value is always definite.	3. It gives more weightage to extreme values.

(c) **मध्यक भिन्नता र स्तरीय भिन्नताबिच फरक पत्ता लगाउनुहोस् ।**
Find the difference between mean deviation and standard deviation.

⇨ Mean deviation is the average of the absolute deviations taken from central value (mean, median, mode) whereas standard deviation is the positive square root of the average of squares of the deviation taken from mean or median.

(d) **स्तरीय भिन्नताको गुणाङ्क र विचरणशीलताको गुणाङ्कबिच फरक पत्ता लगाउनुहोस् ।**
Find the difference between coefficient of standard deviation and coefficient of variation.

⇨ Coefficient of standard deviation is expressed in the form of integer whereas coefficient of variation is expressed in percentage.

2. **तल दिइएको तथ्याङ्कको आधारमा स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।**
Find the standard deviation and it's coefficient on the basis of following data.

(a)

Age (yrs.)	10-15	15-20	20-25	25-30	30-35	35-40
No. of men	16	23	28	18	10	5

⇨ Here, calculation of standard deviation and it's coefficient.

CI	f	m	fm	m^2	fm^2
10 - 15	16	12.5	200	156.25	2500
15 - 20	23	17.5	402.5	306.25	7043.75
20 - 25	28	22.5	630	506.25	14175
25 - 30	18	27.5	495	756.25	13612.5
30 - 35	10	32.5	325	1056.25	10562.5
35 - 40	5	37.5	187.5	1406.25	7031.25
	N = 100		Σfm = 2240		$\Sigma fm^2 = 54925$

Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{2240}{100} = 22.4$

We know that,
Standard deviation (σ)

$$= \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{54925}{100} - (22.4)^2}$$

$$= \sqrt{549.25 - 501.76} = \sqrt{47.49} = 6.89$$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{6.89}{22.4} = 0.3$

Thus, SD and it's coefficients are 6.89 and 0.3 respectively.

(b)

Daily wage (Rs)	100-125	125-150	150-175	175-200	200-225
No. of workers	75	57	81	19	12

⇨ Here, calculation of standard deviation and it's coefficient. Let assumed mean (A) = 145.

CI	f	m	d = m – 145	$d' = \frac{d}{10}$	d'^2	fd'	fd'^2
100 - 125	75	112.5	– 32.5	– 3.25	10.5625	– 243.75	792.18
125 - 150	57	137.5	– 7.5	– 0.75	0.5625	– 42.75	32.06
150 - 175	81	162.5	17.5	1.75	3.0625	141.75	248.06
175 - 200	19	187.5	42.5	4.25	18.0625	80.75	343.18
200 - 225	12	212.5	67.5	6.75	45.5625	81	546.75
	N = 244					Σfd' = 17	$\Sigma fd'^2 = 1962.23$

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Now, mean $(\bar{x}) = A + \frac{\Sigma fd'}{N} \times c = 145 + \frac{17}{244} \times 10 = 145.69$

We know that,

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c = \sqrt{\frac{1962.23}{244} - \left(\frac{17}{244}\right)^2} \times 10 = \sqrt{8.041 - 0.004} \times 10$

$= 28.35$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{28.35}{145.69} = 0.194$

Thus, SD and it's coefficient are 28.35 and 0.194 respectively.

(c)

Class interval	**0-4**	**4-8**	**8-12**	**12-16**	**16-20**	**20-24**
Frequency	**7**	**7**	**10**	**15**	**7**	**6**

⇨ Here, calculation of standard deviation and it's coefficient. Let assumed mean (A) = 10.

CI	f	m	d = m – 10	d^2	fd	fd^2
0 - 4	7	2	– 8	64	– 56	448
4 - 8	7	6	– 4	16	– 28	112
8 - 12	10	10	0	0	0	0
12 - 16	15	14	4	16	60	240
16 - 20	7	18	8	64	56	448
20 - 24	6	22	12	144	72	864
	N = 52				Σfd = 104	$\Sigma fd^2 = 2112$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd}{N} = 10 + \frac{104}{52} = 10 + 2 = 12$

We know that,

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{2112}{52} - \left(\frac{104}{52}\right)^2} = \sqrt{40.615 - 4} = 6.05$

Again, coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{6.05}{12} = 0.504$

Thus, standard deviation and it's coefficient are 6.05 and 0.504 respectively.

3. (a) कक्षा दशको एकाइ परीक्षामा 40 जना विद्यार्थीहरूले पाएको अङ्क निम्न तालिकामा दिइएको छ । उक्त तथ्याङ्कबाट स्तरीय भिन्नताको गुणाङ्क र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।

The marks obtained by 40 students in the unit test of class 10 is given in the table below. Find the coefficient of standard deviation and coefficient of variation from the data.

Marks obtained (*x*)	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**
No. of students (f)	**4**	**8**	**10**	**16**	**6**	**6**

⇨ Here, calculation of coefficient of standard deviation and coefficient of variance.

CI	f	m	fm	$d = m - \bar{x}$	d^2	fd^2
30 - 40	4	35	140	– 26	676	2704
40 - 50	8	45	360	– 16	256	2048
50 - 60	10	55	550	– 6	36	360
60 - 70	16	65	1040	4	16	256
70 - 80	6	75	450	14	196	1176
80 - 90	6	85	510	24	576	3456
	N = 50		Σfm = 3050			$\Sigma fd^2 = 10000$

Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{3050}{50} = 61$

We know that, Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{10000}{50}} = \sqrt{200} = 14.14$

Coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{14.14}{61} = 0.231$

Again, coefficient of variance $= \frac{\sigma}{\bar{x}} \times 100\ \% = 0.231 \times 100\ \% = 23.1\ \%$

Thus, coefficient of SD is 0.231 and CV is 23.1 %.

(b) तल दिइएको तथ्याङ्कको आधारमा स्तरीय भिन्नताको गुणाङ्क र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।

Find the coefficient of standard deviation and coefficient of variation on the basis of given data.

Class interval	**25-29**	**30-34**	**35-39**	**40-44**	**45-49**	**50-54**
Frequency	**2**	**3**	**4**	**7**	**2**	**1**

⇨ Here, calculation of coefficient of standard deviation and coefficient of variance by making the given data

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continuous. Let assumed mean (A) = 35

CI	f	m	d = m – A (35)	d^2	fd	fd^2
24.5 - 29.5	2	27	– 8	64	– 16	128
29.5 - 34.5	3	32	– 3	9	– 9	27
34.5 - 39.5	4	37	2	4	8	16
39.5 - 44.5	7	42	7	49	49	343
44.5 - 49.5	2	47	12	144	24	288
49.5 - 54.5	1	52	17	289	17	289
	N = 19				Σfd = 73	$\Sigma fd^2 = 1091$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd}{N} = 35 + \frac{73}{19} = 38.84$

Standard deviation (σ) $= \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{1091}{19} - \left(\frac{73}{19}\right)^2} = \sqrt{57.421 - 14.761} = \sqrt{42.66} = 6.53$

Coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{6.53}{38.84} = 0.168$

Again, coefficient of variance $= \frac{\sigma}{\bar{x}} \times 100\ \% = 0.168 \times 100\ \% = 16.8\ \%$

Thus, coefficient of SD is 0.168 and CV is 16.8 %.

(c) एउटा कार्यालयमा काम गर्ने 100 जना कर्मचारीको खाजा खर्च निम्नअनुसार छ।
The snack expenditure of 100 civil servants working in an office is as below.

Expenditure (Rs)	60-63	63-66	66-69	69-72	72-75
Servants civil	5	18	42	27	8

उक्त तथ्याङ्कबाट स्तरीय भिन्नताको गुणाङ्क र विचरणशीलताको गुणाङ्क निकाल्नुहोस्।
From the above data, find the coefficient of standard deviation and coefficient of variation.

⇨ Here, calculation of coefficient of standard deviation and coefficient of variance.

CI	f	m	fm	$d = m - \bar{x}$	d^2	fd^2
60 - 63	5	61.5	307.5	– 6.45	41.6025	208.0125
63 - 66	18	64.5	1161	– 3.45	11.9025	214.245
66 - 69	42	67.5	2835	– 0.45	0.2025	8.505
69 - 72	27	70.5	1903.5	2.55	6.5025	175.5675
72 - 75	8	73.5	588	5.55	30.8025	246.42
	N = 100		Σfm = 6795			$\Sigma fd^2 = 852.75$

Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{6795}{100} = 67.95$

We know that, Standard deviation (σ) $= \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{852.75}{100}} = 2.92$

Coefficient of standard deviation $= \frac{\sigma}{\bar{x}} = \frac{2.92}{67.95} = 0.0429$

Again, coefficient of variance $= \frac{\sigma}{\bar{x}} \times 100\ \% = 0.0429 \times 100\ \% = 4.29\ \%$

Thus, coefficient of SD is 0.0429 and CV is 4.29 %.

4. तल दिइएको तथ्याङ्कबाट निम्न विधि प्रयोग गरी स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क (C.V.) पत्ता लगाउनुहोस्।
Find the standard deviation and coefficient of variation (C.V) from the given data by using the given methods.
(a) प्रत्यक्ष विधि (Direct method) (b) अनुमानित मध्यक विधि (Assumed mean method) (c) पद विचलन विधि (Step deviation method)

Class interval	0-20	20-40	40-60	60-80	80-100
Frequency	15	13	18	16	10

⇨ Here, calculation of standard deviation and coefficient of variation by direct method.

CI	f	m	fm	m^2	fm^2
0 - 20	15	10	150	100	1500
20 - 40	13	30	390	900	11700
40 - 60	18	50	900	2500	45000
60 - 80	16	70	1120	4900	78400
80 - 100	10	90	900	8100	81000
	N = 72		Σfm = 3460		$\Sigma fd^2 = 217600$

Now, mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{3460}{72} = 48.055$

We know that,

Standard deviation (σ) $= \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2} = \sqrt{\frac{217600}{72} - \left(\frac{3460}{72}\right)^2} = \sqrt{3022.22 - 2309.33} = \sqrt{712.89} = 26.7$

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Again, coefficient of variance (CV) $= \frac{\sigma}{\bar{x}} \times 100\,\% = \frac{26.7}{48.055} \times 100\,\% = 55.56\,\%$

Thus, standard deviation is 26.7 and CV is 55.56 %.

(b)

⇨ Here, calculation of standard deviation and coefficient of variance by assumed mean method. Let assumed mean (A) = 45.

CI	**f**	**m**	**d = m – A (45)**	**d^2**	**fd**	**fd^2**
0 - 20	15	10	– 35	1225	– 525	18375
20 - 40	13	30	– 15	225	– 195	2925
40 - 60	18	50	5	25	90	450
60 - 80	16	70	25	625	400	10000
80 - 100	10	90	45	2025	450	20250
	N = 72				Σfd = 220	$\Sigma fd^2 = 52000$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd}{N} = 45 + \frac{220}{72} = 48.055$

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = \sqrt{\frac{52000}{72} - \left(\frac{220}{72}\right)^2} = \sqrt{722.22 - 9.336} = \sqrt{712.884} = 26.7$

Again, coefficient of variance (CV) $= \frac{\sigma}{\bar{x}} \times 100\,\% = \frac{26.7}{48.055} \times 100\,\% = 55.56\,\%$

Thus, standard deviation 26.7 or CV is 55.56 %.

(c)

⇨ Here, calculation of standard deviation and coefficient of variance by step deviation method. Let assumed mean (A) = 45.

CI	**f**	**m**	**d = m – A (45)**	**$d' = \frac{m-A}{c(5)}$**	**d'^2**	**fd'**	**fd'^2**
0 - 20	15	10	– 35	– 7	49	– 105	735
20 - 40	13	30	– 15	– 3	9	– 39	117
40 - 60	18	50	5	1	1	18	18
60 - 80	16	70	25	5	25	80	400
80 - 100	10	90	45	9	81	90	810
	N = 72					Σfd' = 44	$\Sigma fd'^2 = 2080$

Now, mean $(\bar{x}) = A + \frac{\Sigma fd'}{N} \times c = 45 + \frac{44}{72} \times 5 = 48.05$

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c = \sqrt{\frac{2080}{72} - \left(\frac{44}{72}\right)^2} \times 5 = \sqrt{28.88 - 0.373} \times 5 = 26.7$

Again, coefficient of variance (CV) $= \frac{\sigma}{\bar{x}} \times 100\,\% = \frac{26.7}{48.05} \times 100\,\% = 55.56\,\%$

Thus, standard deviation is 26.7 and CV is 55.56 %.

5. एउटा सुपर मार्केटमा काम गर्ने 114 जना कर्मचारीहरूको दैनिक ज्याला निम्नअनुसार छ ।
The daily wages of 114 employees working in a super market are as below.

Wage (Rs)	**1200-1250**	**1250-1300**	**1300-1350**	**1350-1400**	**1400-1450**
No. of workers	**20**	**26**	**32**	**21**	**15**

(a) कर्मचारीहरूको औसत ज्याला कति रहेछ ? (What was the average wage of the employees?)

(b) उक्त तथ्याङ्कको स्तरीय भिन्नता निकाल्नुहोस् । (Calculate the standard deviation of the above data.)

(c) सो तथ्याङ्कको विचरणशीलताको गुणाङ्क कति हुन्छ ? (What will be the coefficient of variation of the above data?)

⇨ Here, calculation of mean, SD and CV of given data. Let assumed mean (A) = 1315.

CI	**f**	**m**	**d = m – A**	**d' = d ÷ 10**	**fd'**	**fd'^2**
1200 - 1250	20	1225	– 90	– 9	– 180	1620
1250 - 1300	26	1275	– 40	– 4	– 104	416
1300 - 1350	32	1325	10	1	32	32
1350 - 1400	21	1375	60	6	126	756
1400 - 1450	15	1425	110	11	165	1815
	N = 114				Σfd' = 39	$\Sigma fd'^2 = 4639$

(a) Here, mean wages $= A + \frac{\Sigma fd'}{N} \times c = 1315 + \frac{39}{114} \times 10 = 1318.421$

Thus, the mean wages is Rs 1318.421.

(b) Here, standard deviation $(\sigma) = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c = \sqrt{\frac{4639}{114} - \left(\frac{39}{114}\right)^2} \times 10$

$= \sqrt{40.629 - 0.117} \times 10 = 63.69$

Thus, standard deviation of data is 63.69.

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(c) Here, coefficient of variation (CV) = $\frac{\sigma}{\overline{x}} \times 100\ \% = \frac{63.69}{1318.421} \times 100\ \% = 4.83\ \%$

Thus, the coefficient variation of given data is 4.83 %.

6. (a) एउटा गाउँमा भएका विभिन्न उमेर समूहका मानिसहरूको विवरण यस प्रकार छ।
The distribution of people having different age group of a locality is as below.

Age (years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of males	18	45	31	28	22	16	9	4	2
No. of females	10	42	30	20	27	9	14	2	4

यस तथ्याङ्कका आधारमा निम्न प्रश्नहरूको उत्तर दिनुहोस्। Answer the following questions on the basis of this data.

(i) महिला र पुरुषको औसत उमेर कति कति छ ? What is the average age of females and males ?

(ii) महिला र पुरुषको उमेरको स्तरीय विचलन कति कति छ ? What is the standard deviation of age of females and males ?

(iii) महिला र पुरुषमध्ये कसको उमेर बढी स्थिर छ ? Whose age is more stable among males and females?

⇨ Here, Calculation of mean and variance. Let total number of males be x and total number of females be y.

CI	x	y	m	xm	ym	m^2	xm^2	ym^2
0 - 10	18	10	5	90	50	25	450	250
10 - 20	45	42	15	675	630	225	10125	9450
20 - 30	31	30	25	775	750	625	19375	18750
30 - 40	28	20	35	980	700	1225	34300	24500
40 - 50	22	27	45	990	1215	2025	44550	54675
50 - 60	16	9	55	880	495	3025	48400	27225
60 - 70	9	14	65	585	910	4225	38025	59150
70 - 80	4	2	75	300	150	5625	22500	11250
80 - 90	2	4	85	170	340	7225	14450	28900
	$\Sigma x = 175$	$\Sigma y = 158$		$\Sigma xm = 5445$	$\Sigma ym = 5240$		$\Sigma xm^2 = 232175$	$\Sigma ym^2 = 234150$

(i) Mean age of females $(\overline{y}) = \frac{\Sigma ym}{\Sigma y} = \frac{5240}{158} = 33.164$ $\therefore\ \overline{y} = 33.164$

Mean age of males $(\overline{x}) = \frac{\Sigma xm}{\Sigma x} = \frac{5445}{175} = 31.114$ $\therefore\ \overline{x} = 31.114$

Thus, the mean age of females is 33.164 years and that of males is 31.114 years.

(ii) Here, we know that,

$$\text{Standard deviation for females} = \sqrt{\frac{\Sigma ym^2}{\Sigma y} - \left(\frac{\Sigma ym}{\Sigma y}\right)^2}$$
$$= \sqrt{\frac{234150}{158} - \left(\frac{5240}{158}\right)^2}$$
$$= \sqrt{1481.962 - 1099.887}$$
$$= \sqrt{382.075} = 19.546$$
$$\therefore\ \sigma_y = 19.546$$

$$\text{Similarly, standard deviation for males} = \sqrt{\frac{\Sigma xm^2}{\Sigma x} - \left(\frac{\Sigma xm}{\Sigma x}\right)^2}$$
$$= \sqrt{\frac{232175}{175} - \left(\frac{5445}{175}\right)^2}$$
$$= \sqrt{1326.714 - 968.098}$$
$$= \sqrt{358.616}$$
$$= 18.937$$
$$\therefore\ \sigma_x = 18.937$$

Thus, standard deviation for females is 19.546 and that for males is 18.937.

(iii) Here, coefficient of variation for females = $\frac{\sigma_y}{\overline{y}} \times 100\ \% = \frac{19.546}{33.164} \times 100\ \% = 58.937\ \%$

$\therefore\ (CV)_y = 58.937\ \%$

Similarly, coefficient of variation for males = $\frac{\sigma_x}{\overline{x}} \times 100\ \% = \frac{18.937}{31.114} \times 100\ \% = 60.863\ \%$

$\therefore\ (CV)_x = 60.863\ \%$

Since, $(CV)_x > (CV)_y$ so age of females is more stable than that of males.

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(b) दुईओटा नयाँ मोडेल A र B का फ्रिजहरूको आयु यस प्रकार रहेको छ :

The life of two new model fridges A and B are as follows:

समय (वर्षमा) Time (in years)	फ्रिजको सङ्ख्या (No. of fridges)	
	मोडेल A (Model A)	मोडेल B (Model B)
0-2	5	2
2-4	16	7
4-6	13	12
6-8	7	19
8-10	5	9
10-12	4	1

उक्त तथ्याङ्कका आधारमा पत्ता लगाउनुहोस् (According to the above data find the following):

(i) प्रत्येक मोडेलको फ्रिजको औसत आयु कति कति छ ? (What is the average life of each of the above fridges?)

(ii) दुईमध्ये कुन मोडेलको आयुमा एकरूपता छ ? (Which of the above fridges has the life uniformity?)

(iii) कुन मोडेलको फ्रिज किन्दा उपयुक्त हुन्छ, किन ? (Which model's fridge is appropriate to buy, why?)

⇨ Here, Calculation of average life of fridge and the uniformity.

A					B				
X	f	m	fm	fm^2	X	f	m	fm	fm^2
0-2	5	1	5	5	0-2	2	1	2	1
2-4	16	3	48	144	2-4	7	3	21	63
4-6	13	5	65	325	4-6	12	5	60	300
6-8	7	7	49	343	6-8	19	7	133	931
8-10	5	9	45	405	8-10	9	9	81	729
10-12	4	11	44	484	10-12	1	11	11	121
	N = 50		**$\Sigma fm = 256$**	**$\Sigma fm^2 = 1706$**		**N = 50**		**$\Sigma fm = 308$**	**$\Sigma fm^2 = 2146$**

The average life of fridge A

$$\overline{x_1} = \frac{\Sigma fm}{N} = \frac{256}{50}$$

∴ $\overline{x_1} = 5.12$ years

Again,

$$\text{Standard deviation } (\sigma_1) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{1706}{50} - \left(\frac{256}{50}\right)^2}$$

$$= \sqrt{34.12 - 26.21}$$

$$= \sqrt{7.9056}$$

$$\therefore \quad \sigma \quad = 2.812$$

Now, coefficient of variation

$$(CV_1) \quad = \frac{\sigma_1}{\overline{x_1}} \times 100\%$$

$$= \frac{2.812}{5.12} \times 100\%$$

$$= 54.915\%$$

The average life of fridge B

$$\overline{x_2} = \frac{\Sigma fm}{N} = \frac{308}{50} = 6.16$$

∴ $\overline{x_2} = 6.16$ years

Again,

$$\text{Standard deviation } (\sigma_2) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{2146}{50} - \left(\frac{308}{50}\right)^2}$$

$$= \sqrt{42.92 - (6.16)^2}$$

$$= \sqrt{4.9744}$$

$$\therefore \quad \sigma \quad = 2.23$$

Now, coefficient of variation

$$(CV_2) \quad = \frac{\sigma_2}{\overline{x_2}} \times 100\%$$

$$= \frac{2.2303}{6.16} \times 100\%$$

$$= 36.206\%$$

It shows that model B has the life more uniform because CV of model B is less than the CV of model A.

Model B is appropriate to buy because the CV of model B is less than life CV of model A.

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7. **कक्षा 10 मा अध्ययनरत 30 जना विद्यार्थीहरूको एकाई परीक्षाको प्राप्ताङ्क निम्नअनुसार छ।**

10, 11, 18, 20, 18, 18, 17, 16, 14, 12, 10, 22, 24, 28, 23, 14, 16, 20, 23, 26, 28, 29, 9, 4, 8, 12, 5, 9, 8, 10,

The marks obtained by 30 students of grade 10 in a unit test of full marks 30 are as follows:

दिइएको तथ्याङ्कलाई 5 को वर्गान्तरमा बारम्बारता तालिका बनाई स्तरीय भिन्नता, यसको गुणाङ्क र विचरणशीलताको गुणाङ्क समेत निकाल्नुहोस्।

Make the frequency table of above data taking the interval 5 and find the standard deviation, its coefficient and also, the coefficient of variation.

⇨ Here, calculation of frequency table, standard deviation, its coefficient and the coefficient of variation.

Marks (x)	No. of students (f)	mid value (m)	fm	fm^2
0-5	1	2.5	2.5	6.25
5-10	5	7.5	37.5	281.25
10-15	8	12.5	100	1250
15-20	6	17.5	105	1837.50
20-25	6	22.5	135	3037.5
25-30	4	27.5	110	3025
	N = 30		$\Sigma fm = 490$	$\Sigma fm^2 = 9437.5$

We know that, Mean $(\bar{x}) = \frac{\Sigma fm}{N} = \frac{490}{30} = 16.33$

Again, Standard deviation $(\sigma) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$

$$= \sqrt{\frac{9437.5}{30} - \left(\frac{490}{30}\right)^2}$$

$$= \sqrt{314.5833 - (16.33)^2}$$

$$= \sqrt{47.805522}$$

$$= 6.914$$

Now, coefficient of SD $= \frac{\sigma}{\bar{x}} = \frac{6.914}{16.33} = 0.423$

And, coefficient of variation (CV) $= \frac{\sigma}{\bar{x}} \times 100\% = \frac{6.914}{16.33} \times 100\% = 42.3\%$

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पाठ्यक्रम विकास केन्द्र, सानोठिमी भक्तपुर

नमूना प्रश्न पत्र - २०७६

विषय : ऐच्छिक (अतिरिक्त) गणित **पूर्णाङ्क : १००**

विषय कोड : ४४१ **समय : ३ घण्टा**

GROUP A 10 × 1 = 10

1. (a) त्रिकोणमीतीय फलनको परिभाषा लेख्नुहोस् ।
Define trigonometric function.

(b) दुई सङ्ख्याहरू a र b बिचको अङ्कगणितीय मध्यक के हुन्छ ?
What is the arithmetic mean between two numbers a and b?

2. (a) सङ्ख्या रेखामा अविच्छिन्नता हुने सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers which is continuous in number line.

(b) मेट्रिक्स $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ को डिटरमिन्यान्ट के हुन्छ ?
What is the determinant of matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$?

3. (a) यदि दुई सिधा रेखाहरूका झुकावहरू क्रमश: m_1 र m_2 छन् र तिनीहरू बिचको कोण θ भए tan θ को मान पत्ता लगाउने सुत्र लेख्नुहोस् ।
If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for tan θ.

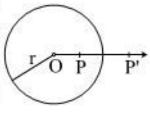
(b) एउटा सोलीलाई समतलीय सतहले आधारको समानान्तर हुनेगरि प्रतिच्छेदन गर्दा के बन्दछ ?
What will be formed if a plane intersects a cone parallel to its base?

4. (a) sin 2A लाई tan A को रूपमा व्यक्त गर्नुहोस् ।
Express sin 2A in terms of tan A.

(b) उन्नतांश कोणको परिभाषा लेख्नुहोस् ।
Define angle of elevation.

5. (a) यदि $\vec{a}$ र $\vec{b}$ बिचको कोण θ भए $\vec{a}$ र $\vec{b}$ को स्केलर गुणनफल के हुन्छ ?
What is the scalar product of two vectors $\vec{a}$ and $\vec{b}$ if the angle between them is θ ?

(b) एउटा इन्भर्सन स्थानान्तरणमा P को प्रतिबिम्ब P' र केन्द्र O भएको इन्भर्सन वृत्तको अर्धव्यास r भए OP.OP' र r को सम्बन्ध लेख्नुहोस् ।



In an inversion transformation if P' is image of P and r is radius of inversion circle with centre O, write the relation of OP.OP' and r.

GROUP B 13 × 2 = 26

6. (a) यदि $f(x) = 4x + 5$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् ।
Find $f^{-1}(x)$ if $f(x) = 4x + 5$.

(b) यदि $g(x) = 2x - 1$ र $f(x) = 4x$, भए $g_o f(x)$ को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x - 1$ and $f(x) = 4x$, find the value of $g_o f(x)$.

(c) $f(x) = x^2 - 1$ र $f(x) = 3$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।
What will be the points of intersection of the curve $f(x) = x^2 - 1$ and $f(x) = 3$?

7. (a) यदि $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ भए A^2 को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।
Find the value of determinant of A^2 if $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

(b) क्रामरको नियम अनुसार $ax + by = c$ र $px + qy = r$ मा D_1 र D_2 को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, find the values of D_1 and D_2 for $ax + by = c$ and $px + qy = r$.

8. (a) झुकावहरू क्रमश: $\sqrt{3}$ र $\frac{1}{\sqrt{3}}$ भएका दुई सरल रेखाहरूबीचको न्यून कोण कति हुन्छ ?
What will be the acute angle between two straight lines if their slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ respectively ?

(b) जोडा रेखाहरू $3x + 2y = 0$ र $2x - 3y = 0$ लाई जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation for the pair of lines represented by $3x + 2y = 0$ and $2x - 3y = 0$.

9. (a) यदि $\sin A = \frac{1}{2}$ भए sin 3A को मान पत्ता लगाउनुहोस् ।
Find the value of sin 3A if $\sin A = \frac{1}{2}$.

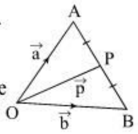
(b) $\frac{\sin A}{1 + \cos A}$ लाई $\tan\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् ।
Express $\frac{\sin A}{1 + \cos A}$ in terms of $\tan\frac{A}{2}$.

(c) यदि $2\sin 2\theta = \sqrt{3}$ भए θ को मान पत्ता लगाउनुहोस् । $(0° \le \theta \le 180°)$
If $2\sin 2\theta = \sqrt{3}$, find the value of θ. $(0° \le \theta \le 180°)$

10. (a) यदि $|\vec{a}| = 2$, $|\vec{b}| = 12$ र $\vec{a}.\vec{b} = 12$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।
Find the angle between two vectors $\vec{a}$ and $\vec{b}$

if $|\vec{a}| = 2$, $|\vec{b}| = 12$ and $\vec{a}.\vec{b} = 12$.

(b) दिइएको चित्रमा $\vec{a}$, $\vec{b}$ र $\vec{p}$ को सम्बन्ध के हुन्छ ? पत्ता लगाउनुहोस् ।
From the given figure, find the relationship among $\vec{a}$, $\vec{b}$ and $\vec{p}$.



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(c) एउटा तथ्याङ्कको पहिलो चतुर्थांश 35 र तेस्रो चतुर्थांश 75 छ । चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, first quartile is 35 and the third quartile is 75. Find the quartile deviation and its coefficient.

GROUP C 11 × 4 = 44

11. हल गर्नुहोस् (Solve) : $x^3 - 3x^2 - 4x + 12 = 0$

12. $x - 2y \leq 1, x + y \leq 4, x \geq 0, y \geq 0$ को आधारमा $P = 5x + 4y$ को अधिकतम र न्यूनतम मान पत्ता लगाउनुहोस् ।
Optimize $P = 5x + 4y$ under the given constraints: $x - 2y \leq 1, x + y \leq 4, x \geq 0, y \geq 0$

13. वास्तविक फलन $f(x) = 2x + 3$ को लागि
(a) $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ र $f(3)$ का मानहरू पत्ता लगाउनुहोस् ।
(b) के यो फलन $x = 3$ मा अविच्छिन्न हुन्छ ?
For a real valued function $f(x) = 2x + 3$
(a) Find the values of $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ and $f(3)$.
(b) Is this function continuous at $x = 3$?

14. म्याट्रिक्स विधिको प्रयोग गरी तल दिइएका समीकरणहरू हल गर्नुहोस् : $3x + 5y = 11, 2x - 3y = 1$
By using matrix method, solve the following system of equations: $3x + 5y = 11, 2x - 3y = 1$

15. उद्गम बिन्दुबाट जाने र $2x^2 - 5xy + 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$.

16. प्रमाणित गर्नुहोस् (Prove that):

$$\sin 20^\circ . \sin 30^\circ . \sin 40^\circ . \sin 80^\circ = \frac{\sqrt{3}}{16}$$

17. यदि $A + B + C = \pi$ भए प्रमाणित गर्नुहोस् :
If $A + B + C = \pi$, prove that:
$\sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A \cos B \sin C$

18. धरहराको ठीक अगाडि जमिनको सतहमा रहेको एक बिन्दुबाट धरहराको माथि ठड्याइएको 6 m अग्लो ध्वजदण्डको टुप्पो र फेदका उन्नतांश कोणहरू क्रमशः 60° र 45° पाईयो । धरहराको उचाइ र धरहराको फेदबाट सो बिन्दुसम्मको दूरी पत्ता लगाउनुहोस् ।
From a point at the ground level infront of a tower, the angle of elevations of the top and bottom of flagstaff 6 m high situated at the top of a tower are observed 60° and 45° respectively. Find the height of the tower and the distance between the base of the tower and point of observation.

19. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2×2 म्याट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2×2 matrix which transforms a unit square to a parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

20. दिइएको तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the mean deviation from mean and its coefficient from given data.

प्राप्ताङ्क Marks obtained	0-10	10-20	20-30	30-40	40-50
विद्यार्थी सङ्ख्या No. of students	2	3	6	5	4

21. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and coefficient of variation from given data.

उमेर(Age)	0-4	4-8	8-12	12-16	16-20	20-24
विद्यार्थी सङ्ख्या No. of students	7	7	10	15	7	6

GROUP D 4 × 5 = 20

22. एकजना ठेकेदारले कुनै काम निश्चित समय भन्दा ढिलो गरे वापत पहिलो दिनको रु. 200, दोस्रो दिनको रु 250 र तस्रो दिनको रु 300 गर्दै प्रत्येक पछिल्लो दिनमा पहिलो दिनको भन्दा रु 50 बढि जरिवाना तिर्नुपर्छ । यदि उक्त ठेकेदारले कुनै काम 30 दिन ढिला गरेमा जम्मा कति जरिवाना तिर्नुपर्ला ?
A contractor on construction job specifies a penalty for delay of completion beyond a certain date as: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day and so on. The penalty for each succeding day being Rs 50 more than that of the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

23. एउटा पाङ्ग्राको परिधिमा रहेका तीनओटा बिन्दुहरू (5, 7) , (– 1, 7) र (5, – 1) एउटा निश्चित बिन्दुबाट बराबर दुरीमा पर्दछन् । उक्त निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै उक्त तीन बिन्दुहरू पर्ने गरी बिन्दुपथ पत्ता लगाउनुहोस् ।
On a wheel, there are three points (5, 7) , (– 1, 7) and (5, – 1) located such that the distance from a fixed point to these points is always equal. Find the coordinate of the fixed point and then derive the equation representing the locus that contains all three points.

24. भेक्टर विधिको प्रयोग गरी कुनैपनि चतुर्भुजको भुजाहरूका मध्यबिन्दुहरू क्रमशः जोड्दै जाँदा बन्ने चतुर्भुज समानान्तर चतुर्भुज हुन्छ, भनी प्रमाणित गर्नुहोस् ।
By using vector method, prove that the quadrilateral formed by joining the midpoints of adjacent sides of a quadrilateral is a parallelogram.

25. मानौं A(1, – 1), B(5, 2) र C(3, 5) त्रिभुज ABC का शिर्षबिन्दुका निर्देशाङ्कहरू हुन् । उक्त त्रिभुजलाई लेखाचित्रमा प्रस्तुत गरी $E_1[O(0, 0) ; 2]$ द्वारा बिस्तार गर्नुहोस् । पुनः विस्तारीत प्रतिबिम्बलाई Y-अक्षमा परावर्तन गरी लेखाचित्रमा प्रस्तुत गर्नुहोस् । त्रिभुज ABC र स्थानान्तरण पछिका प्रतिबिम्बहरूको प्रकृती लेख्नुहोस् ।
Let A(1, – 1), B(5, 2) and C(3, 5) are the coordinates of vertices of a triangle ABC. Plot this triangle in graph. Enlarge this triangle with $E_1[O(0, 0) ; 2]$ and then reflect the enlarged image about Y-axis in that graph. Write the nature of object triangle and its images after transformations.

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माध्यमिक शिक्षा उत्तिर्ण परीक्षा
उत्तर कुञ्जिका - २०७६

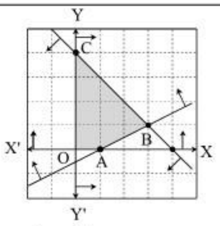
GROUP A 10 × 1 = 10

1.	**(a)**	The function with trigonometric ratios as the independent variables..........	1
	(b)	$\frac{a+b}{2}$	1
2.	**(a)**	Set of real numbers	1
	(b)	$ps - qr$..........	1
3.	**(a)**	$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1.m_2}$	1
	(b)	circle	1
4.	**(a)**	$\frac{2\tan A}{1+\tan^2 A}$	1
	(b)	Correct definition of angle of elevation.......	1
5.	**(a)**	$\vec{a}\,.\,\vec{b} = \lvert\vec{a}\rvert\,\lvert\vec{b}\rvert\cos\theta$	1
	(b)	$OP.OP' = r^2$..........	1

GROUP B 13 × 2 = 26

6.	**(a)**	Let, $y = 4x + 5$ (i) $x = 4y + 5$ .... (ii) $f^{-1}(x) = \frac{x-5}{4}$	 1 1
	(b)	(i) $g_o f(x) = g(4x)$ .... (ii) $g(4x) = 2(4x) - 1 = 8x - 1$	1 1
	(c)	(i) $x^2 - 1 = 3$ $x = \pm 2$.......... (ii) ∴ The points are $(- 2, 3)$ and $(2, 3)$.....	 1 1
7.	**(a)**	(i) $A^2 = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ $= \begin{pmatrix} 1 & -3 \\ 9 & -2 \end{pmatrix}$ .... (ii) $\lvert A^2\rvert = - 2 + 27 = 25$	 1 1
	(b)	(i) $D_1 = \begin{vmatrix} c & b \\ r & q \end{vmatrix} = cq - br$.......... (ii) $D_2 = \begin{vmatrix} a & c \\ p & r \end{vmatrix} = ar - pc$..........	1 1
8.	**(a)**	(i) $\tan\theta = \pm\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\,.\frac{1}{\sqrt{3}}}$ .... (ii) ∴ $\theta = 30°$	1 1
	(b)	(i) $(3x + 2y)(2x - 3y) = 0$ .... (ii) $6x^2 - 5xy - 6y^2 = 0$....	1 1
9.	**(a)**	(i) $= 3\,.\,\frac{1}{2} - 4\left(\frac{1}{2}\right)^3$ .... (ii) $= 1$..........	1 1
	(b)	(i) $\frac{\sin A}{1 + \cos A} = \frac{\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}}{1 + \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}}$ .... (ii) $= \tan\frac{A}{2}$	1 1
	(c)	(i) $2\theta = 60°, 120°$ .... (ii) ∴ $\theta = 30°, 60°$	1 1
10.	**(a)**	(i) $\cos\theta = \frac{12}{2 \times 12}$ .... (ii) ∴ $\theta = 60°$	1 1
	(b)	(i) $\vec{p} - \vec{b} = \vec{a} - \vec{p}$ .... (ii) $\vec{p} = \frac{\vec{a} + \vec{b}}{2}$	1 1
	(c)	(i) Q.D. $= \frac{75-35}{2} = 20$.......... (ii) Coefficient of Q.D $= \frac{75-35}{75+35} = 0.36$.......	1 1

GROUP C 11 × 4 = 44

11.	Let $f(x) = x^3 - 3x^2 - 4x + 12$ (i) $f(2) = 0$, ∴ $x - 2$ is a factor of $f(x)$ .... (ii) $(x - 2)(x^2 - x - 6) = 0$.......... (iii) $(x - 2)(x + 2)(x - 3) = 0$.......... (iv) ∴ $x = - 2 , 2 , 3$	 1 1 1 1
12.	 (i) For correct graph .... (ii) Finding values of at (0, 0), (1, 0), (3, 1) and (0, 4).......... (iii) Maximum value of P is 19 at (3, 1) And minimum value is 0 at (0, 0)	 1+1 1 1
13.	(i) $f(2.95) = 8.9$, $f(2.99) = 8.98$, $f(3.01) = 9.02$, $f(3.05) = 9.1$, $f(3) = 9$ .... (ii) $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = f(3)$.......... (iii) ∴ $f(x)$ is continuous at $x = 3$..........	1+1 1 1
14.	(i) $\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$ .... (ii) $A^{-1} = \frac{1}{-19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}$ .... (iii) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ 1 \end{pmatrix}$......... (iv) ∴ $x = 2$ and $y = 1$..........	1 1 1 1
15.	(i) Finding separate equations $x - 2y = 0$ and $2x - y = 0$ .... Separate equations perpendicular to above equations and passing through origin are (ii) $2x + y = 0$ and $x + 2y = 0$.......... (iii) Equation of pair of lines is $(2x + y)(x + 2y) = 0$.......... (iv) ∴ $2x^2 + 5xy + 2y^2 = 0$ is the required equation of pair of line	1 1 1 1
16.	(i) $= \frac{1}{4}\,.\,\sin 20°.\,(2\sin 80°\,.\,\sin 40°)$ $= \frac{1}{4}.\sin 20°.[\cos(80° - 40°) - \cos(80° + 40°)]$	 1

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(ii) $= \frac{1}{4} . \sin 20° . \cos 40° + \frac{1}{8} . \sin 20°$ 1

(iii) $= \frac{1}{8} [\sin (40°+20°) - \sin (40°-20°)] + \frac{1}{8} \sin 20°$ 1

(iv) $= \frac{\sqrt{3}}{16}$ 1

17. (i) $\sin^2 A - \sin^2 B + \sin^2 C$

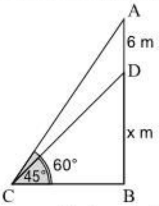
$= \frac{1}{2}(\cos 2B - \cos 2A) + \sin^2 C$ 1

(ii) $= \sin C . \sin (A - B) + \sin^2 C$ 1

(iii) $= \sin C . [\sin (A - B) + \sin (A + B)]$........... 1

(iv) $= 2 \sin A \cos B \sin C$.................................. 1

18.



(i) Correct figure with description.................. 1

let, $BD = x$ m, $\tan 45° = \frac{x}{CB}$, $CB = x$ m

(ii) $\tan 60° = \frac{x+6}{CB}$, $CB = \frac{x+6}{\sqrt{3}}$ m 1

(iii) $x = \frac{x+6}{\sqrt{3}}$, $x = 8.19$ m 1

(iv) ∴ Height = 8.19 m and distance = 8.19 m 1

19. (i) let, the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$....... 1

(ii) $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$...... 1

(iii) $a = 3, b = 1, c = 0$ and $d = 1$....................... 1

(iv) ∴ The matrix is $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ 1

20.

Marks	f	x	fx	$\lvert D\rvert = \lvert x - \bar{x}\rvert$	$f\lvert D\rvert$
0-10	2	5	10	23	46
10-20	3	15	45	13	39
20-30	6	25	150	3	18
30-40	5	35	175	7	35
40-50	4	45	180	17	68
Total	N = 20		*560*		*206*

(i) For correct table... 1

(ii) $\bar{x} = \frac{560}{20} = 28$.. 1

(iii) M.D. $= \frac{206}{20} = 10.3$.. 1

(iv) Coefficient of M.D $= \frac{10.3}{28} = 0.3678$ 1

21.

Age	f	x	$d = x - 10$	fd	fd^2
0-4	7	2	– 8	– 56	448
4-8	7	6	– 4	– 28	112
8-12	10	10	0	0	0
12-16	15	14	4	60	240
16-20	7	18	8	56	448
20-27	6	22	12	72	864
Total	**N = 52**			**104**	**2112**

(i) For correct table 1

Let assumed mean (A) = 10

(ii) $\bar{x} = 10 + \frac{104}{52} = 12$ 1

(iii) $\sigma = \sqrt{\frac{2112}{52} - \left(\frac{104}{52}\right)^2} = 6.05$ 1

(iv) Coefficient of variation $= \frac{6.05}{12} \times 10\%$

$= 50.42\%$ 1

GROUP D 4 × 5 = 20

22. (i) Writing the series 200, 250, 300, ... 1

(ii) Writing $a = 200$, $d = 50$ and $n = 30$ 1

(iii) Formula for $S_n = \frac{n}{2} [2a + (n - 1) d]$........ 1

(iv) $\frac{30}{2}[2 \times 200 + 29 \times 50]$ 1

(v) $15[400 + 1450] = 27750$....................... 1

23. suppose the fixed point is $P(x, y)$

The three points are A(5, 7) , B(–1, 7) and C(5, – 1)

(i) PA = PB and PA = PC........................ 1

(ii) finding $x = 2$ from PA= PB.................. 1

(iii) finding $y = 3$ from PA = PC 1

(iv) Finding Distance PA = 5 by

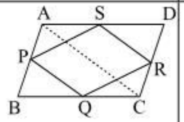
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

The locus of a point (X, Y) with center (2, 3) and radius 5 as

$\sqrt{(X - 2)^2 + (Y - 3)^2} = 5$

(iv) $x^2 + y^2 - 4x - 6y - 12 = 0$.................. 1

24. (i) For correct figure. 1



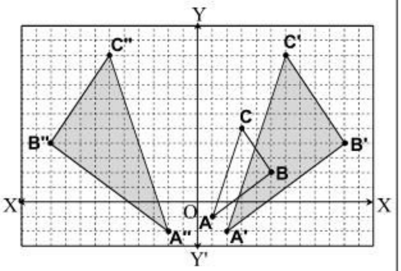
(ii) $\overrightarrow{PQ} = \frac{1}{2} \overrightarrow{AC}$ 1

(iii) $\overrightarrow{PQ} = K \overrightarrow{AC}$ where $k = \frac{1}{2}$ ∴ $\overrightarrow{PQ} // \overrightarrow{AC}$ 1

(iv) $\overrightarrow{SR} = \frac{1}{2} \overrightarrow{AC}$ and $\overrightarrow{SR} // \overrightarrow{AC}$................. 1

(v) ∴ PQRS is a parallelogram 1

25.



(i) Plotting ΔABC 1

(ii) Proper enlargement by origin 1

(iii) Plotting the enlarged image................. 1

(iv) Reflection in y-axis 1

(v) ΔABC, ΔA'B"C' and ΔA"B"C" are similar. ΔA'B"C' and ΔA"B"C" are congruent 1