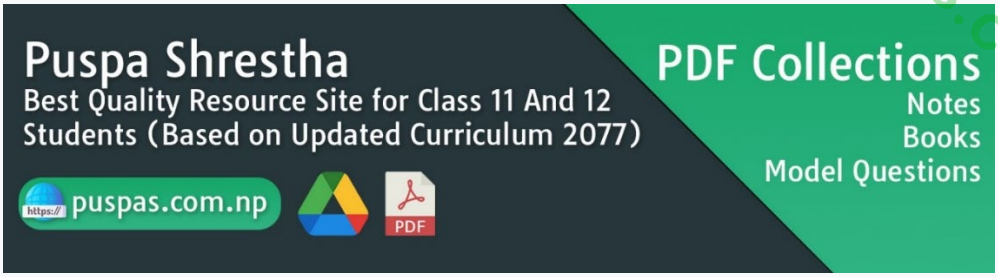
Puspa Shrestha
Best Quality Resource Site for Class 11 And 12 Students
(Based on Updated Curriculum 2077)



This PDF was downloaded from

puspas.com.np

Visit our website for more materials.

puspas.com.np

Follow us on:

AR Dinesh puspas.com.np Puspa Shrestha

www.drmathnp.com

17th EDITION

पाठ्यक्रम विकास केन्द्र (CDC) द्वारा स्वीकृत पाठ्यक्रमअनुसार
एस.ई.ई. परीक्षा २०७६ का लागि अङ्ग्रेजी र नेपाली दुवै माध्यममा

NEPAL'S BEST SELLING PRACTICE BOOK SERIES !

सातै प्रदेशका लागि
FOR SCORING A (OUTSTANDING) RESULT

OPTIONAL

New Model / New Syllabus

MATHEMATICS

for

SEE COMPETITORS

PRACTICE MATERIAL
FOR GRADE X & SEE EXAM

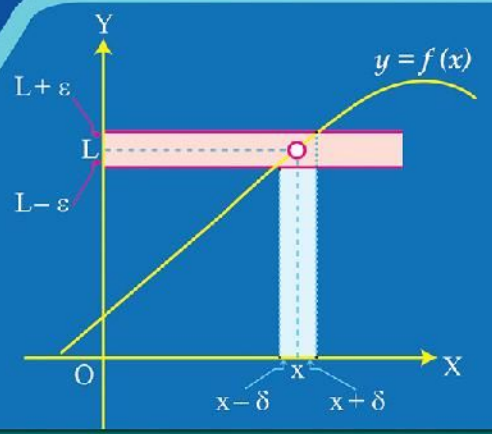
STEP 01 — TOPIC INTRODUCTION
SYLLABUS, DEFINITIONS, FORMULAE, KEY-POINTS & SEE REVIEW MESSAGE

STEP 02 — SEE EXERCISES
SEE QUESTIONS, MODEL QUESTIONS & BASIC QUESTIONS FOR SEE

STEP 03 — GENERAL EXERCISES
ADDITIONAL SHORT QUESTIONS, LONG AND VERY LONG QUESTIONS FOR SEE

STEP 04 — CHALLENGING EXERCISES
MORE DIFFICULT AND EXTRA TRICKY QUESTIONS FOR BRILLIANT STUDENTS

STEP 05 — QUESTIONS COLLECTION
SEE MODEL QUESTIONS, SEE EXAMINATION PAPERS & OTHERS



D. R. SIMKHADA

OPT. MATHEMATICS for SEE COMPETITORS D. R. SIMKHADA

pustaksansar.com.np

www.drmathnp.com

puspas.com.np

17th Edition

नेपाल सरकार, शिक्षा, विज्ञान तथा प्रविधि मन्त्रालय, पाठ्यक्रम विकास केन्द्र सानोठिमी, भक्तपुरद्वारा कक्षा १० का लागि स्वीकृत पाठ्यक्रम एवम् नयाँ विशिष्टीकरण तालिका तथा नमुना प्रश्नअनुसार

अङ्ग्रेजी र नेपाली दुवै माध्यममा

OPTIONAL MATHEMATICS FOR SEE COMPETITORS

(Class X & SEE Exam 2076)

BASED ON MODEL PATTERNS

By
D. R. Simkhada

Sweet 17th EDITION

UNIQUE FEATURES

- **SYLLABUS** and **SEE GRID** are divided into a number of small chapters.
- Each chapter starts with **SYLLABUS, GRID, FORMULAE & KEY POINTS.**
- **SEE / SLC REVIEW MESSAGES** have been provided.
- **SEE PROPOSED MODELS** have been provided.
- The varieties of the problems have been illustrated by **MODEL PATTERNS.**
- **HINTS AND SOLUTIONS** for the difficult problems have been presented.
- **TOPICWISE ASSIGNMENTS** for all the topics are given.
- **MODEL QUESTIONS & SEE QUESTIONS** have been incorporated.

Readmore

www.drmathnp.com

puspas.com.np

Page



शिक्षकहरूलाई कुनै कठिन समस्याहरू भएमा !

■ आफ्नो पूर्ण परिचय र फोन नम्बरसहित सम्पर्क गर्नुहोला । ■

Facebook Page

D. R. Simkhada

(Author)

E-mail: dayasimkhada@gmail.com वा dr_simkhada@yahoo.com
slcmath2@gmail.com वा slcmath@yahoo.com

Read to help others read !

We are pleased to inform that certain amount from each book will be allocated to **Charity Fund**. The fund will be provided to the deserving and needy students all over Nepal. The charity always welcomes cooperation and support from all the well wishers and agencies.

Paste your Photograph.

Name: .....

Roll No. : ..... *Section :* .....

Phone : .....

School : .....

Address : .....

OPTIONAL MATHEMATICS FOR SEE COMPETITORS

Book copyright © 2075 by D. R. Simkhada

Illustration copyright © 2075 by 24/7 Graphic Design

All rights reserved. No part of this book may be used or reproduced in any manner whatsoever without written permission.

1st Edition : 2060, Bhadra, 2nd Edition:2061, Bhadra, 3rd Edition :2063, Mangsir(Revised), 4th Edition :2064, Baishakh (Updated), 5th Edition :2065, Shrawan, 6th Edition : 2066, Baishakh, 7th Edition :2067, Jestha- 22, 8th Edition: 2068, Baishakh - 11, 9th Edition :2069 Baisakh - 4 (Revised), 10th Edition :2070 Jeth 10 (Restructured and Revised) 11th Edition :2071 Jeth 19 (Revised), 12th Edition :2071 Asar 1 (Revised) 14th Edition : 2073 Baisakh-25 (Revised), 15th Edition : 2074 Baisakh 1 (Revised),

16th Edition : 2075 Jeth 17 (Revised),

17th Edition: 2076 Asar 30 (Completely Revised)

Printed in Nepal & ISBN 9-789937-53135-1

Publishers and Distributors

Readmore

TU Road, Kuleshwor, P.O. Box 9240, Kathmandu, Nepal
Phone : 01-4672071, 01-4672073, 4036211 Fax: 01-4278050, 4036226
email : www.readmorenepal@gmail.com
www.readmorenepal.com

www.drmathnp.com

puspas.com.np

सत्रौँ संस्करणको बारेमा

सर्वप्रथम !

प्रस्तुत पुस्तक **OPTIONAL MATHEMATICS FOR SEE COMPETITORS** को यस अगाडिका संस्करणलाई आशातीत मन पराइदिनु भई यो संस्करण प्रकाशित गर्नमा उत्साह र उर्जा थपिदिनुहुने विद्यार्थीवर्ग र शिक्षक-शिक्षिकाज्यूहरूप्रति हार्दिक आभार तथा धन्यवाद दिँदै **SEE 2076** को लागि नेपाल सरकार, शिक्षा, विज्ञान तथा प्रविधि मन्त्रालय, पाठ्यक्रम विकास केन्द्र सानोठिमीद्वारा लागू गरिएको विशिष्टीकरण तालिका तथा नमुना प्रश्नपत्रअनुसार, तयार गरिएको यो परिमार्जित **सत्रौँ संस्करण** पाठकवर्ग-समक्ष प्रस्तुत गरिएको छ । विगतका वर्षहरूमा जस्तै यो वर्ष पनि यस पुस्तकले यहाँहरूको शैक्षिक परिलक्ष्य हासिल गर्न सहयोग पुऱ्यायो भने मेरो यो प्रयास सफल हुनेछ ।

पुस्तकमा !

SEE तहको अर्थपूर्ण सिकाइको निमित्त पाठ्यक्रमद्वारा निर्देशित उपलब्धिहरू हासिल गराउन र परिणाममुखी बनाउन कसरी, कहाँबाट, के-के पढाउने भन्ने मुख्य सन्दर्भ, यो पुस्तकको अघिल्ला संस्करणहरूका प्रयोगकर्ताहरूबाट प्राप्त सुझाउहरू, **SEE** परीक्षाका प्रश्न-संरचनाको प्रकृति तथा पाठ्यक्रम र विशिष्टीकरण तालिकालाई विचार गरी तयार गरिएको यो पुस्तकमा निम्नलिखित विशेषताहरू पाउनु हुनेछ:

- प्रत्येक पाठको सुरुमा आवश्यक सूत्रहरू र मुख्य बुँदाहरू समेटिएको ।
- अफ्ठ्यारा प्रश्नहरूका लागि इसारा (Hints) दिइएको ।
- प्रत्येक एकाइको सुरुमा सो एकाइको **Syllabus** र **SEE Specification Grid** समावेश गरिएको ।
- प्रत्येक पाठमा **ASIGNMENTS** प्रस्तुत गरिएको ।
- प्रत्येक पाठमा निम्नअनुसारका अभ्यासहरू राखिएको ।

SEE EXERCISE

SEE परीक्षाको प्रश्न-संरचनाको प्रकृति र कक्षा १० को पाठ्यपुस्तकलाई केन्द्रबिन्दु बनाई परिधिभित्र आधारभुत रूपमा आवश्यक हरेक किसिमका प्रश्नहरू क्रमश: सरलदेखि जटिलतिर **1A, 1B, 1C, ..., 2A, 2B, 2C, 2D,.....** का रूपमा राखिएका छन्, जसले गर्दा **'A'** शिक्षकबाट सिक्न र **B, C, D,.....** आफैँ प्रयास गर्न सहज हुने आशा गरेको छु ।

GENERAL EXERCISE

पाठ्यक्रम र **SEE** विशिष्टीकरण तालिका तथा **SEE** का प्रश्न संरचनाको प्रकृतिलाई लेखनको केन्द्रबिन्दु बनाई सोको परिधिभित्र **SEE** मा सोध्न सक्ने थप सम्भाव्य नमुना प्रश्नहरू (Possible Model Questions) लाई सकेसम्म **SEE EXERCISE** का प्रश्नहरूसँग पुनरावृत्ति (Repetition) नहुने गरी क्रमश: सरलदेखि जटिलतिर राखिएको छ । यसअन्तर्गत पनि, **1A, 1B, 1C, ..., 2A, 2B, 2C, 2D,.....** का रूपमा एउटै प्रकृतिका प्रश्नहरू राखिएका छन् । यसको उद्देश्य: **'A'** शिक्षकबाट सिक्ने र **B, C, D,.....** आफैँ प्रयास गर्ने रहेको छ । पाठ्यक्रम (Syllabus) ले निर्दिष्ट गरेको शत प्रतिशत विषय वस्तुहरू **SEE EXERCISE** र **GENERAL EXERCISE** मा पाउनु हुनेछ । तसर्थ, **SEE** परीक्षा तयारीको लागि यी अभ्यासहरू नै पर्याप्त छन् ।

www.drmathnp.com

puspas.com.np

CHALLENGING EXERCISE

केही उत्कृष्ट निजी तथा सरकारी शिक्षण संस्थाहरूमा शिक्षण गर्नुहुने शिक्षक-शिक्षिकाज्यूहरूको सुझाउअनुसार केवल मेधावी (Talented) विद्यार्थीहरूलाई ध्यानमा राखी केवल थप अभ्यासका लागि पाठ्यक्रमसँग सम्बन्धित तर उच्च कठिनाइ स्तर (High Difficulty Level) का प्रश्नहरू यस अन्तर्गत राखिएका छन् । सबै प्रयोगकर्तालाई **SEE EXERCISE** र **GENERAL EXERCISE** का सम्पूर्ण प्रश्नहरूको समाधान गर्न सक्ने भएपछि मात्र यो अभ्यासमा प्रवेश गर्नु, गराउनु उचित हुने धारणा अनुरोध गरिन्छ ।

अन्तमा !

पुस्तक प्रकाशन गरिदिने **Readmore Publishers & Distributors** तथा प्रकाशक भावा राजभण्डारीज्यूलाई धन्यवाद दिन चाहन्छु । अन्तमा कक्षा १० मा अध्ययनरत विद्यार्थीहरू तथा अध्यापनरत शिक्षकज्यूहरूको उच्चतम् सफलताको कामना एवम् विशेष धन्यवाद ज्ञापन गर्दै यहाँहरूसगँ रचनात्मक सुझावको अपेक्षा गर्दछु ।

काठमाडौँ

२०७६ असार - ३० लेखक

हरपल प्रेरणा, सल्लाह एवम् सुझाव दिनुहुने शिक्षकज्यूहरू

श्री लालबाबु यादव, श्री प्रकाश खड्का, श्री शम्भु प्रसाद अधिकारी, श्री ज्ञानेन्द्र न्यौपाने, श्री विनोदकुमार यादव, श्री महेशप्रसाद अधिकारी, श्री सुजन सिम्खडा, श्री भीमबहादुर खड्का, श्री टीकाराम खड्का, श्री विनोद भट्ट, श्री गोबिन्द प्रसाद पौडेल, श्रीमती बिमला धिताल, श्री बलराम घिमिरे, श्रीमती लक्ष्मी शर्मा, श्री दिवाकर आचार्य, श्री बालकृष्ण शाक्य, श्री कृष्णनारायण श्रेष्ठ, श्री योगेन्द्र पण्डित, श्री जगदिप राजबंशी, श्री शंकर के. सी., श्री बिष्णु पनेरु, श्री अजित पाण्डे, श्री हरि लामिछाने, श्री राजु लामिछाने, श्री श्रवण लाल कर्ण, श्री मनहरि खनाल, श्री निम भुजेल, श्री रमेश खनाल, श्री कृष्ण कँडेल, श्री सन्तोष सिम्खडा, श्री प्रकाश सिम्खडा, श्री जाहेद खाँ, श्री बिष्णु श्रेष्ठ, श्री कर्मा लामा, श्री प्रकाश भट्टराई, श्री खेमलाल दाहल, श्री त्रिबिक्रम सुबेदी, श्री महेन्द्र चौधरी, श्री हरि कँडेल, श्री प्रकाश ढकाल, श्री राजेन्द्र देवकोटा, श्री रामचन्द्र देवकोटा, श्री आई. आर. सिम्खडा, श्री टिका प्रसाद सिम्खडा, श्री मेदनाथ सापकोटा, श्री श्रीराम बन्जारा, श्री बेलिन्द्रबहादुर बन्जारा, श्री निर्मल पौडेल श्री श्यामकुमार थापा, श्री नारायण नेपाल, श्री सन्तोषराज अर्याल, श्री शैलेश स्थापित, श्रीमती संगीता श्रेष्ठ, श्री रुद्रप्रसाद सापकोटा, श्री शिरोमणि भट्टराई, श्री राजन वाग्ले, श्री शंकर भट्ट, श्री बिजय कुमार श्रेष्ठ, श्री सुवास इटनी, श्री सुनिल आचार्य, श्री ज्ञानेन्द्र गिरी, श्री युवराज मैनाली, श्री रेशमराज बराल, श्री विश्वप्रकाश बास्तोला, श्री भोलानाथ सापकोटा, श्री भानुप्रसाद उपाध्याय, श्री रमेशचन्द्र शर्मा, श्री गुरुप्रसाद पौडेल, श्री ओमकुमार क्षेत्री, श्री जी. एन. अर्याल, श्री पशुपती पौडेल, श्री किरण सुबेदी, श्री नारायण श्रेष्ठ, श्री रामहरि भण्डारी, श्री नारायण कँडेल, श्री विश्वेश्वर घिमिरे, श्री भिमप्रसाद पौडेल, श्री जगदीशचन्द्र शर्मा, श्री धर्मराज चालिसे, श्री मुक्तिराज अधिकारी, श्री उमराज आचार्य, श्री तुफान सिँह थापा, श्री महेन्द्र शर्मा, श्री देवी काफ्ले, श्री दिपेन्द्र गुप्ता, श्री प्रेम शाह, श्री अजय शाह, श्री शिवराज जोशी, श्री उपेन्द्र पण्डित, श्री गोकर्ण गिरी, श्री मदन अम्गाईं, श्री महाँप्रसाद अम्गाईं, श्री सुरेन्द्रप्रसाद शाह, श्री उत्तम भट्ट, श्री सुबिन्द्र बस्नेत, श्री नवराज रिजाल, श्री दीपक अधिकारी, श्री दीपक खतिवडा, श्री हरि लम्साल, श्री केशव पौडेल, श्री ध्रुव अधिकारी, श्री बद्री प्रसाद कोइराला, श्री भरतप्रसाद सिम्खडा, श्री शिवप्रसाद भट्ट, श्री दिपक थापामगर, श्री रूद्रप्रसाद सापकोटा, श्री त्रिलोचन शर्मा, श्री जीवराज पौडेल, श्री दामोदर पौडेल, श्री देबी प्रसाद पौडेल, श्री राजेश गुप्ता, श्री भरत शाही, श्री मणीराम लामीछाने, श्री छत्रबहादुर कटुवाल, श्री ऋषिराम अर्याल, श्री रामेश्वर प्रसाद यादव, श्री लक्ष्मी प्रसाद पाण्डे, श्री उपेन्द्र सुबेदी, श्री शिवराज ओली, श्री भक्तबहादुर शाही, श्री इश्वर अधिकारी श्री राम लखन राय, श्री लिला बहादुर थापा, श्री आश्विन पण्डित, श्री जित बहादुर खनाल दिपक श्रेष्ठ, श्री विनोद ठाकुर, श्री हृदेश कुमार चौरसिया, श्री राम कृष्ण पोखरेल, श्री चक्र पाल, श्री धनविर चौधरी, श्री पेसल घिमिरे, श्री जीवन चौधरी, श्री लालदेव यादव, श्री परमानन्द नाथ, श्री तपेन्द्र प्रसाद भट्ट, श्री नरेन्द्र कार्की, श्री केशव पौडेल, श्री राजु के.सी., श्री निश्चल लामा भाज्यु, श्री रवी पाण्डे, श्री नारायण अर्याल, श्री कृष्ण प्रताप महत, श्री विज्ञान पौडेल, श्री केश राज महतो, सुश्री मायाँ सापकोटा, श्री चुडामणी पौडेल, श्री विज्ञान पौडेल, श्री वेद प्रसाद शर्मा, श्री राम प्रसाद अर्याल, श्री रामेश लामीछाने, श्री पवन शर्मा, श्री बालकृष्ण श्रेष्ठ, लगायत सबैलाई

विशेष धन्यवाद ....

www.drmathnp.com

puspas.com.np

CONTENTS

एकाइ I बीज गणित (UNIT I ALGEBRA)

Marks for SEE 21

UNITS	Page	A	B	C	D
1. फलन (Function)	1	2 × 1	3 × 2	2 × 4	1 × 5
1.1. बीजीय फलन र त्रिकोणमितीय फलन Algebraic function & trigonometric function	2				
1.2. संयुक्त फलन (Composite Function)	8				
1.3. विपरीत फलन (Inverse Function)	18				
2. बहुपदीय (Polynomial)	32				
3. समानान्तरीय अनुक्रम (Arithmetic Sequence)	45				
4. गुणोत्तर अनुक्रम (Geometric Sequence)	64				
5. रेखीय योजना (Linear Programming)	83				
6. समीकरण र लेखाचित्र (Equation and Graph)	92				

एकाइ II निरन्तरता (UNIT II CONTINUITY)

Marks for SEE 5

UNITS	Page	A	B	C	D
1. सीमान्त मान र निरन्तरता (Limit and Continuity)	103	1 × 1	–	1 × 4	–

एकाइ III मेट्रिक्स (UNIT III MATRIX)

Marks for SEE 9

UNITS	Page	A	B	C	D
1. डिटरमिन्यान्ट, विपरीत मेट्रिक्स र समीकरण Determinant, Inverse Matrix and Equation)	126	1 × 1	2 × 2	1 × 4	-
2. क्रामरको नियम (Cramer's Rule)	139				

एकाइ IV निर्देशाङ्क ज्यामिति UNIT IV COORDINATE GEOMETRY

Marks for SEE 15

UNITS	Page	A	B	C	D
1. दुई सीधा रेखाहरूबिचको कोण Angle Between two Straight Lines	148	2 × 1	2 × 2	1 × 4	1 × 5
2. जोडा सीधारेखाहरू (Pair of Straight Lines)	167				
3. शाङ्कुक क्षेत्र (Conic Section)	181				
4. वृत्त (Circle)	185				

www.drmathnp.com

puspas.com.np

एकाइ V त्रिकोणमिति (UNIT V TRIGONOMETRY) — Marks for SEE 20

UNITS	Page	A	B	C	D
1. अपवर्त्य र अपवर्तक कोणहरू Multiple and Sub-Multiple Angles	206	2 × 1	3 × 2	3 × 4	-
2. गुणन र योगफलको रूपान्तरण Transformation of Product and Sum	229				
3. अनुबन्धित त्रिकोणमितीय सर्वसमिकाहरू Conditional Trigonometric Identities	238				
4. त्रिकोणमितीय समीकरणहरू Trigonometric Equations	249				
5. उचाइ र दूरी (Height and Distance)	258				

एकाइ VI भेक्टर (UNIT VI VECTOR) — Marks for SEE 10

UNITS	Page	A	B	C	D
1. स्केलर गुणनफल (Scalar Product)	276	1 × 1	2 × 2	-	1 × 5
2. भेक्टर ज्यामिति (Vector Geometry)	289				

एकाइ VII स्थानान्तरण (UNIT VII TRANSFORMATION) — Marks for SEE 10

UNITS	Page	A	B	C	D
1. संयुक्त स्थानान्तरण (Combined Transformation)	308	1 × 1	-	1 × 4	1 × 5
2. विपरीत स्थानान्तरण र विपरीत वृत्त Inversion Transformation and Inversion Circle	328				
3. मेट्रिक्स स्थानान्तरण (Matrix Transformation)	338				

एकाइ VIII तथ्याङ्कशास्त्र (UNIT VIII STATISTICS) — Marks for SEE 10

UNITS	Page	A	B	C	D
1. चतुर्थांशीय विचलन र यसको गुणाङ्क Quartile Deviation and its Coefficient	352	-	1 × 2	2 × 4	-
2. मध्यक भिन्नता र यसको गुणाङ्क Mean Deviation and its Coefficient	360				
3. स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क Standard Deviation and the Coefficient of Variation	366				

15 SETS NEW MODEL QUESTIONS 2076

SET 1 SEE MODEL QUESTIONS & MARKING SCHEME — ISSUED BY CDC — 376

SET 2 - 15 SEE MODEL QUESTIONS 2076 — 380

HINTS AND SOLUTION ON TRIGONOMETRY — 408

www.drmathnp.com

puspas.com.np

लेखकीय अनुरोध (AUTHOR'S REQUEST) !

STEP 1 : LOOK, STUDY & LEARN: CONCEPTS, DEFINITIONS & FORMULAE

STEP 2 : PRACTISE: SEE EXERCISE

STEP 3 : PRACTISE: GENERAL EXERCISE

STEP 4 : COMPLETE: ASSIGNMENTS

- सिकाई सरल (Simple) बाट जटिल (Complex) तिर केन्द्रित हुनुपर्दछ । त्यसैले माथि देखाएझै पहिले **SEE EXERCISE** अनि **GENERAL EXERCISE** अभ्यास गर्नु, गराउनु उचित हुन्छ ।
- SEE परीक्षाको प्रश्न-संरचनाको प्रकृतिलाई नियाल्दा एउटै प्रकृतिका अधिकांश प्रश्नहरू दोहोरिएको पाइन्छ । अघिल्ला वर्षहरूमा सोधिएका SEE का अधिकांश प्रश्नहरू दोहोरिएका छन् ।
- एसइई पाठ्यक्रम (SEE Curriculum) ले निर्दिष्ट गरिएको शत प्रतिशत (100%) विषयवस्तुहरू **SEE EXERCISE** र **GENERAL EXERCISE** मा पाउनु हुनेछ । तसर्थ SEE परीक्षा तयारीका लागि यी अभ्यासहरू नै पर्याप्त हुने धारणा अनुरोध गरिन्छ ।
- केवल मेधावी (Talented) विद्यार्थीहरूलाई ध्यानमा राखी थप अभ्यासका लागि प्रश्नहरू **CHALLENGING EXERCISE** अन्तर्गत राखिएका छन् । SEE परीक्षाको लागि **CHALLENGING EXERCISE** आवश्यक छैन ।
- कतिपय स्थानमा लिइने SEE Pre- Board Exam मा भने Challenging Exercise मा दिइएका समस्याहरू पनि सोधिने गरिएकोले मात्र Challenging Exercise यो पुस्तकमा राखिएको हो ।
- सबै प्रयोगकर्तालाई **SEE EXERCISE** र **GENERAL EXERCISE** का सम्पूर्ण प्रश्नहरू समाधान गर्न सक्ने भएपछि मात्र **CHALLENGING EXERCISE** मा प्रवेश गर्नु, गराउनु उचित हुने धारणा अनुरोध गरिन्छ ।
- यस पुस्तकमा केही प्रश्नहरूका उत्तरहरू छपाई सम्बन्धि त्रुटी वा अन्य कारणले गलत भएका पनि हुन सक्छन् त्यसैले शिक्षकको परामर्श लिनुहोला ।
- प्रयोगकर्ताहरूलाई यस पुस्तकमा दिइएका प्रश्नहरूको समाधान गर्न कठीन भए रिडमोर प्रकाशनले प्रकाशन गरेको SEE Manual हेर्नुहुन वा *dr_simkhada@yahoo.com, dayasimkhada@gmail.com, slcmath@yahoo.com, slcmath2@gmail.com* मध्ये कुनै एकमा आफ्नो पूर्ण परिचय सहित इमेल गर्नु भएमा यहाँहरूले समाधान पाउनुहुनेछ ।

एसईई / एसएलसी प्रश्नहरूको सङ्केत (THE NOTATION OF SEE / SLC QUESTIONS)

2075 R = 2075 सालको नियमित (RGULAR) परीक्षामा सोधिएको प्रश्न ।

2075 R_2 = 2075 सालको नियमित (RGULAR) पुनः परीक्षा प्रदेश नं २ मा सोधिएको प्रश्न ।

2075 R' = 2075 सालको संस्कृत मा. वि. तर्फको नियमित (RGULAR) परीक्षामा सोधिएको प्रश्न ।

2075 R'_2 = 2075 सालको संस्कृत मा. वि. तर्फको नियमित (RGULAR) पुनः परीक्षा प्रदेश नं २ मा सोधिएको प्रश्न ।

2074 S = 2074 सालको ग्रेड वृद्धि वा पुरक (SUPPLEMENTARY) परीक्षामा सोधिएको प्रश्न ।

2074 S' = 2074 सालको संस्कृत मा. वि. तर्फको ग्रेड वृद्धि वा पुरक (SUPPLEMENTARY) परीक्षामा सोधिएको प्रश्न ।

..... **अन्य सालहरूको पनि सोहीअनुसार सङ्केत गरिएको छ ।**

www.drmathnp.com

puspas.com.np

SEE SPECIFICATION GRID 2076
ISSUED BY CDC

SN	Contents	Topics	K Each of 1 Mark	U Each of 2 Marks	A Each of 4 Marks	HA Each of 5 Marks	TQ	TM
1.	बीज गणित Algebra	फलन (Function) बहुपदीयहरू (Polynomials) अनुक्रम र श्रेणी (Sequence and Series) वर्ग समीकरण र लेखाचित्र Quadratic Equation and Graph	2	3	2	1	8	21
2.	सीमान्त मान र निरन्तरता Limit and continuity	सङ्ख्याहरू र निरन्तरता (Numbers & Continuity) लेखाचित्रमा विच्छिन्नता (Discontinuity in Graph) निरन्तरताको साङ्केतिक प्रस्तुती Notational Representation of Continuity	1	-	1	-	2	5
3.	मेट्रिक्स Matrix	डिटरमिन्यान्ट र विपरित मेट्रिक्स Determinant and Inverse of Matrix समीकरणको मेट्रिक्स विधिबाट हल Solving Equations by Matrix Method क्रामरको नियम (Cramer's Rule)	1	2	1		4	9
4.	निर्देशाङ्क ज्यामिति Coordinate Geometry	दुई रेखाहरूबिचको कोण Angle between two lines जोडा सरल रेखाहरू (Pair of straight lines) शाङ्कि क क्षेत्रहरू (Conic Sections) वृत्त (Circle)	2	2	1	1	6	15
5.	त्रिकोणमिति Trigonometry	अपवर्त्य र अपवर्तक कोणहरू Multiple & sub-multiple angles त्रिकोणमितीय सर्वसमिकाहरूको रूपान्तरण Transformation of Trigonometric Identities अनुवन्धित त्रिकोणमितीय सर्वसमिकाहरू Conditional Trigonometric Identities त्रिकोणमितीय समीकरणहरू (Trigonometric Equations) उचाइ तथा दूरी (Height and Distance)	2	3	3	-	8	20
6.	भेक्टर Vectors	स्केलर गुणनफल (Scalar Product) भेक्टर ज्यामिति (Vector Geometry)	1	2	-	1	4	10
7.	स्थानान्तरण Transformation	संयुक्त स्थानान्तरण (Combined Transformation) विपरित स्थानान्तरण र विपरित वृत्त Inversion Transformation and Inversion Circle मेट्रिक्स स्थानान्तरण (Matrix Transformation)	1	-	1	1	3	10
8.	तथ्याङ्कशास्त्र Statistics	चतुर्थांशीय विचलन (Quartile Deviation) मध्यक भिन्नता (Mean Deviation) स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क Standard Deviation and Coefficient of Variation	-	1	2	-	3	10
		Total	10	13	11	4	38	100

INDEX

K = Knowledge | U = Understanding | A = Application | HA = Higher Ability | TQ = Total no. of Questions | TM = Total Marks

www.drmathnp.com

puspas.com.np

Unit I

बीजगणित
Algebra



Lejuene Dirichlet

A German mathematician who made significant contributions modern formal definition of function . By his definition, a way of expressing the dependence of one quantity on another quantity or quantities is called the function. In other words functions are specified as explicit rules or formulas that converted some input value (or values) into an output value.
He taught at the universities of Breslau (1827) and Berlin (1828 to1855) and in 1855 succeeded Carl at the University of G-ttingen but died of a heart attack only three years later.

SEE SPECIFICATION GRID **2 Very Short, 3 Short, 2 Long & 1 Very Long Questions**

S.N.	Contents	Topics	Knowledge	Understanding	Application	Higher Ability	Total no. of Questions	Total Marks
			Each of 1 Mark	Each of 2 Marks	Each of 4 Marks	Each of 5 Marks		
1.	बीज गणित (Algebra)	फलन (Function)	2	3	2	1	8	21
		बहुपदीयहरू (Polynomials)						
		अनुक्रम र श्रेणी (Sequence and Series)						
		वर्ग समीकरण र लेखाचित्र (Quadratic Equation and Graph)						

SYLLABUS ISSUED BY CDC

1. बीजगणित (Algebra) - 35 Periods

1. फलन (Function)

- बीजीय र त्रिकोणमितीय फलनहरू (निम्न अवस्थाका ग्राफसहित)
  Algebraic and trigonometric functions (Also, the graph of the following)
  ($y = mx + c$; $y = ax^2$, $a \neq 0$; $y = ax^3$, $a \neq 0$; $y = \sin A$; $y = \cos A$; $y = \tan A$, $(-2\pi \leq A \leq 2\pi)$
- संयुक्त फलन (दुई ओटाको मात्र) [Composite functions (Only two)]
- विपरीत फलन र चित्रात्मक (Arrow-diagram) रूपमा प्रस्तुत (Inverse function and representation in arrow diagram)

2. बहुपदीयहरू (Polynomials)

- बहुपदीयहरूको भाग (Division of Polynomials)
- सङ्क्षिप्त भाग विधि (Synthetic division Method)
- शेष साध्य र यसको प्रयोग (Remainder Theorem and its application)
- गुणन खण्ड साध्य र यसको प्रयोग (Factor Theorem and its application)
- शेष साध्य र गुणन खण्ड साध्यको प्रयोग (3 डिग्रीसम्मका समीकरण हल गर्न)
  Using Remainder Theorem and Factor Theorem (To solve Equation upto 3 degree)

3. अनुक्रम र श्रेणी (Sequence and Series)

- अङ्क गणितीय अनुक्रम र श्रेणी (परिचय, साधारण पद, मध्यमाहरू, योगफल र पहिला n ओटा प्राकृतिक सङ्ख्याहरूको योगफल (जोर बिजोरसमेत))
  Arithmetic Sequence and Series (Introduction, general terms, means, sum and sum of first n natural numbers (Also, odd and even)
- ज्यामितीय अनुक्रम र श्रेणी (परिचय, साधारण पद, मध्यमाहरू र सीमित पदहरूको योगफल)
  Geometric Sequence and Series (Introduction, general terms, means and sum of finite terms)

4. सरल रेखीय योजनासम्बन्धी समस्याहरू (Linear programming problems)

- परिचय (Introduction)
- रेखीय असमानताहरू (लेखाचित्रबाट असमानतासमेत पत्ता लगाउने)
  Linear inequalities (Also, finding the inequalities from the graph)
- रेखीय योजनाबाट अधिकतम र न्यूनतम मान निर्धारण
  Finding the maximum and minimum value from the linear programming

5. वर्गसमीकरण र लेखाचित्र (Quadratic Equation and Graph)

- वर्ग र घन फलनको लेखाचित्र (Graph of Quadratic function and cubic function)
- वर्गसमीकरणको हल (लेखा चित्रद्वारा) [Solving Quadratic equation (by graphical method)]
- रेखीय र वर्ग समीकरणको हल (लेखाचित्र र प्रतिस्थापन विधिद्वारा)
  Solving Linear and quadratic equation (Graphical and substitution method)

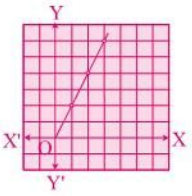
www.drmathnp.com

1.1 बीजीय फलन र त्रिकोणमितीय फलन
ALGEBRAIC FUNCTION AND TRIGONOMETRIC FUNCTION

1. बीजीय फलन (Algebraic Function)

दुईओटा चलहरू x र y को सम्बन्ध प्रस्तुत गर्ने र कुनै नियममा आधारीत फलनलाई बीजीय फलन भनिन्छ । जस्तै: $y = x + 2$

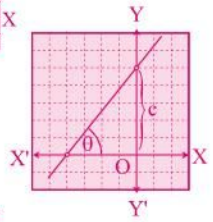
The function which describes correspondence between two variables x and y which are obtained by the finite rules is called an algebraic function. For example: $y = x + 2$



2. रेखीय फलन (Linear Function) [2059 S]

$f(x) = mx + c$ को रूपमा रहेको बीजीय फलनलाई रेखीय फलन भनिन्छ ।

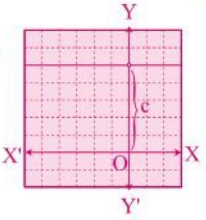
A function is called linear if it can be defined by an equation of the form $f(x) = mx + c$.



3. अचर फलन (Constant Function) [2059 S]

$f(x) = mx + c$ मा $m = 0$ भए यसलाई अचर फलन भनिन्छ ।

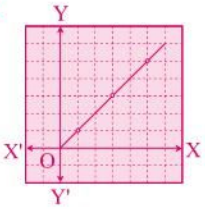
A function of the form $f(x) = mx + c$ where $m = 0$, is called a constant function.



4. एकात्मक फलन (Identity function)

$x \in A$ को लागि $f(x) = x$ को रूपमा व्यक्त गर्न सकिने फलन $f: A \to B$ लाई एकात्मक फलन भनिन्छ । यसलाई I ले जनाइन्छ ।

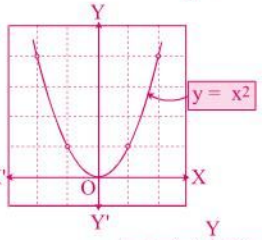
A function $f: A \to B$ is said to be an identity function if it can be expressed in the form of $f(x) = x$ for all $x \in A$. It is denoted by I.



5. वर्गघातीय फलन (Quadratic function)

$f(x) = ax^2 + bx + c;\ a \neq 0$ स्वरूपको फलनलाई वर्ग फलन भनिन्छ । जस्तै: $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ इत्यादि वर्ग फलनहरू हुन् ।

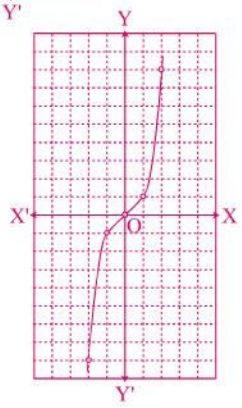
A function of the form $f(x) = ax^2 + bx + c;\ a \neq 0$ is called a quadratic function. e.g. $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ etc. are quadratic functions.



6. घनघातीय फलन (Cubic function)

$f(x) = ax^3 + bx^2 + cx + d\ ;\ a \neq 0$ स्वरूपको फलनलाई घन फलन भनिन्छ । जस्तै: $y = x^3$, $y = 2x^3 + 5$ इत्यादि घन फलनहरू हुन् ।

A function of the form $f(x) = ax^3 + bx^2 + cx + d;\ a \neq 0$ is called a cubic function. e.g. $y = x^3$, $y = 2x^3 + 5$ etc. are cubic functions.



7. अबीजीय फलन (Transcendental Function)

बीजीय फलनको जस्तो जोड, घटाऊ, गुणन र भागका गुणहरू सन्तुष्ट नगर्ने फलनलाई अबीजीय फलन भनिन्छ । जस्तै $f(x) = x^{\frac{1}{x}}$, $g(x) = \sin x$, $h(x) = x^x$ इत्यादि ।

A transcendental function is a function that does not satisfy the properties addition, subtraction, multiplication and division like an algebraic function. For example: $f(x) = x^{\frac{1}{x}}$, $g(x) = \sin x$, $h(x) = x^x$ etc.

8. त्रिकोणमितीय फलन (Trigonometric Function) [SEE MODEL 2076]

एउटा समकोण त्रिभुजको दुई भुजाहरूको अनुपातमा व्यक्त भएको कोण समावेश भएको फलनलाई त्रिकोणमितीय फलन भनिन्छ । जस्तै $f(\theta) = \sin\theta$, $g(\theta) = \cos 2\theta$, $h(\alpha) = \tan\alpha + 5$ इत्यादि । अर्को शब्दमा, त्रिकोणमितीय अनुपातहरू स्वतन्त्र चलको रूपमा समावेश भएको फलनलाई त्रिकोणमितीय फलन भनिन्छ ।

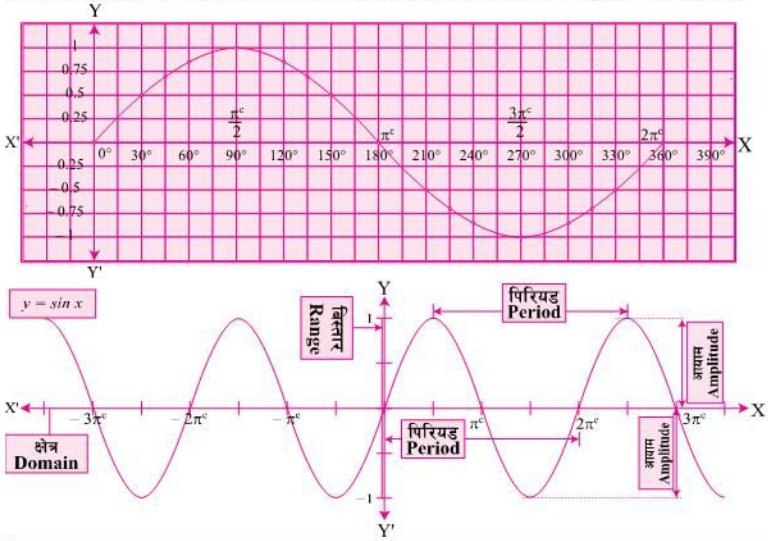
A function of an angle expressed as the ratio of two of the sides of a right angled triangle containing that angle, is called trigonometric function. For example: $f(\theta) = \sin\theta$, $g(\theta) = \cos 2\theta$, $h(\alpha) = \tan\alpha + 5$ etc. In other words, a function containing trigonometric ratio as an independent variable is called the trigonometric function.

puspas.com.np

www.drmathnp.com

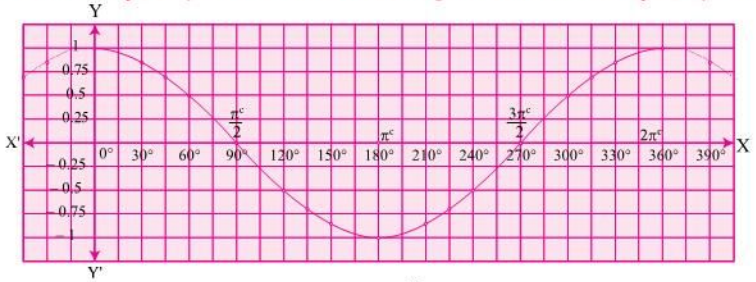
puspas.com.np

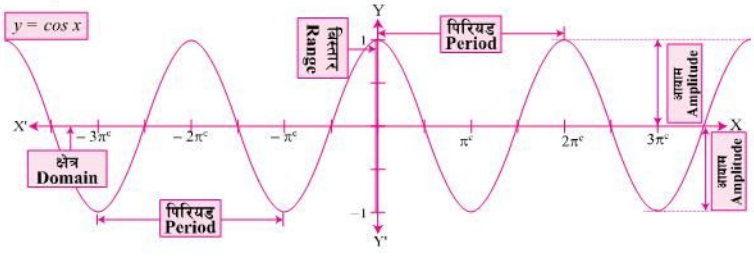
8.1 sine फलन $f(x) = y = \sin x$ को लेखाचित्र (Graph of sine function $f(x) = y = \sin x$)



sine फलनका विशेषताहरू (The characteristics of sine function)	
1.	क्षेत्र सबै वास्तविक सङ्ख्याहरू हुन्छन् । (The domain is the set of all real numbers.)
2.	विस्तार – 1 देखि 1 सम्मका वास्तविक सङ्ख्याहरू हुन्छन् । The range consists of all real numbers from – 1 to 1, inclusive.
3.	sine फलन अवधि (Period) $2\pi^c$ भएको आवधिक (Periodic) फलन हो । The sine function is periodic, with period $2\pi^c$.
4.	x-खण्डहरू ....., $-2\pi^c$, $-\pi^c$, 0, π^c, $2\pi^c$, $3\pi^c$, ...र y-खण्ड 0 हुन्छ । The x-intercepts are ....., $-2\pi^c$, $-\pi^c$, 0, π^c, $2\pi^c$, $3\pi^c$, ... and the y-intercept is 0.
5.	अधिकतम् मान 1 हुन्छ र यो x = ..., $-\frac{3\pi^c}{2}, \frac{\pi^c}{2}, \frac{5\pi^c}{2}, \frac{9\pi^c}{2}$, ... मा पर्दछ । The maximum value is 1 and occurs at x = ..., $-\frac{3\pi^c}{2}, \frac{\pi^c}{2}, \frac{5\pi^c}{2}, \frac{9\pi^c}{2}$, ...
6.	न्यूनतम् मान – 1 हुन्छ र यो x = ..., $-\frac{\pi^c}{2}, \frac{3\pi^c}{2}, \frac{7\pi^c}{2}, \frac{11\pi^c}{2}$, ... मा पर्दछ । The minimum value is – 1 and occurs at x = ..., $-\frac{\pi^c}{2}, \frac{3\pi^c}{2}, \frac{7\pi^c}{2}, \frac{11\pi^c}{2}$, ...

8.2 cosine फलन $f(x) = y = \cos x$ को लेखाचित्र (Graph of cosine function $f(x) = y = \cos x$)

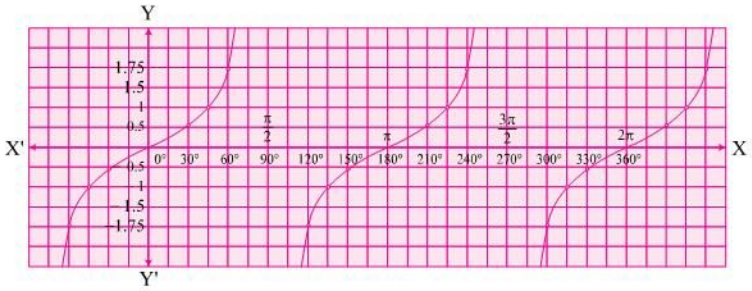


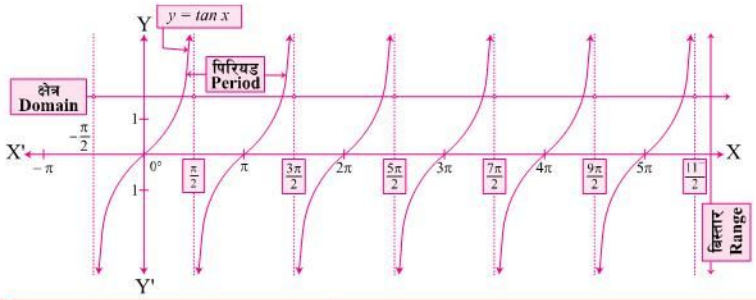


www.drmathnp.com

cosine फलनका विशेषताहरू (The characteristics of cosine function)	
1.	क्षेत्र सबै वास्तविक सङ्ख्याहरू हुन्छन् । (The domain is the set of all real numbers.)
2.	विस्तार – 1 देखि 1 सम्मका वास्तविक सङ्ख्याहरू हुन्छन् । The range consists of all real numbers from – 1 to 1, inclusive.
3.	cosine फलन अवधि (Period) $2\pi^c$ भएको आवधिक (Periodic) फलन हो । The cosine function is periodic, with period $2\pi^c$.
4.	x-खण्डहरू ....., $-\frac{3\pi^c}{2}, -\frac{\pi^c}{2}, \frac{\pi^c}{2}, \frac{3\pi^c}{2}, \frac{5\pi^c}{2}$, ...र y-खण्ड 1 हुन्छ । The x-intercepts are ....., $-\frac{3\pi^c}{2}, -\frac{\pi^c}{2}, \frac{\pi^c}{2}, \frac{3\pi^c}{2}, \frac{5\pi^c}{2}$, ... and the y-intercept is 1.
5.	अधिकतम् मान 1 हुन्छ र यो x = ..., $-2\pi^c$, 0, $2\pi^c$, $4\pi^c$, ... मा पर्दछ । The maximum value is 1 and occurs at x = ..., $-2\pi^c$, 0, $2\pi^c$, $4\pi^c$,....
6.	न्यूनतम् मान – 1 हुन्छ र यो x = ..., $-\pi^c$, π^c, $3\pi^c$, $5\pi^c$,.... मा पर्दछ । The minimum value is – 1 and occurs at x = ..., $-\pi^c$, π^c, $3\pi^c$, $5\pi^c$,....

8.3 tangent फलन $f(x) = y = \tan x$ को लेखाचित्र (Graph of tangent function $f(x) = y = \tan x$)





tangent फलनका विशेषताहरू (The characteristics of tangent function)	
1.	क्षेत्र $\frac{\pi^c}{2}$ र यसका विजोर गुणाङ्कहरू बाहेक सबै वास्तविक सङ्ख्याहरू हुन्छन् । The domain is the set of all real numbers, except odd multiples of $\frac{\pi^c}{2}$.
2.	विस्तारमा सबै वास्तविक सङ्ख्याहरू पर्दछन् । (The range consists of all real numbers.)
3.	tangent फलन अवधि (Period) π^c भएको आवधिक (Periodic) फलन हो । The tangent function is periodic, with period π^c.
4.	x-खण्डहरू ....., $-2\pi^c$, $-\pi^c$, 0, π^c, $2\pi^c$, $3\pi^c$, ...र y-खण्ड 0 हुन्छ । The x-intercepts are ....., $-2\pi^c$, $-\pi^c$, 0, π^c, $2\pi^c$, $3\pi^c$, ...and the y-intercept is 0.
5.	ठाडो रेखाहरू x = ..., $-\frac{3\pi^c}{2}, -\frac{\pi^c}{2}, \frac{\pi^c}{2}, \frac{3\pi^c}{2}$, ...मा पर्दछन् । ठाडो रेखाहरूलाई वक्रले कहिल्यै छुदैन तर नजिकै सम्म पुग्दछन् । The vertical line occur at x = ..., $-\frac{3\pi^c}{2}, -\frac{\pi^c}{2}, \frac{\pi^c}{2}, \frac{3\pi^c}{2}$,....The curve never touches the vertical line but approaches nearer.

puspas.com.np

www.drmathnp.com

puspas.com.np

8.4 अवधि र आयाम (Period and Amplitude)

यदि $b > 0$ भए $y = a \sin bx$ र $y = a \cos bx$ को आयाम (Amplitude) $= |a|$ र अवधि (Period) $= \frac{2\pi}{b}$ हुन्छ ।

If $b > 0$ then $y = a \sin bx$ and $y = a \cos bx$ have Amplitude $= |a|$ and Period $= \frac{2\pi}{b}$.

Example: $f(x) = 3 \sin 4x$ को आयाम (amplitude) र अवधि (period) पत्ता लगाउनुहोस् ।

Determine the amplitude and period of $f(x) = 3 \sin 4x$.

Solution: Here, $f(x) = 3 \sin 4x$

comparing it with $f(x) = a \sin bx$ then, $a = 3$ and $b = 4$

So, Amplitude $= |a| = |3| = 3$ And, Period $= \frac{2\pi^c}{b} = \frac{2 \times 180°}{4} = 90°$ or $\frac{\pi^c}{2}$

8.5 tangent फलनको आयाम (Amplitude) र अवधि (Period) (Amplitude and Period of tangent function)

एउटा tangent फलन $y = a \tan (bx)$ को अवधि (period) भन्नाले यसका ठाडो अनुवर्ती वक्रहरूको दूरी भन्ने बुझिन्छ ।
The period of a tangent function $y = a \tan (bx)$ is the distance between the vertical consecutive curves.

अवधि (Period) $= \frac{\pi}{|b|}$ हुन्छ । (Period is $\frac{\pi}{|b|}$ for tangent function)

tangent फलनको आयाम (Amplitude) हुदैन । (The tangent function does not have amplitude.)

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 3	2066 S	1

SEE EXERCISE 1.1

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. परिभाषा दिनुहोस् (Define):

A. बीजीय फलन (Algebraic Function) **B.** रेखीय फलन (Linear Function)

C. अचर फलन (Constant Function) **D.** एकात्मक फलन (Identity Function)

E. वर्ग फलन (Quadratic Function) **F.** घन फलन (Cubic Function)

2. परिभाषा दिनुहोस् (Define):

A. पूर्णकाल (Period) **B.** अबीजीय फलन (Transcendental function)

C. sine फलन (sine function) **D.** त्रिकोणमितीय फलन (Trigonometric Function)

3A. $y = f(x) = mx + c$ मा m को मान कति हुँदा यो अचर फलन हुन्छ ?
For what value of m in $y = f(x) = mx + c$ makes it a constant function?

3B. $y = f(x) = mx + c$ मा m र c का मानहरू कति कति हुँदा यो एकात्मक फलन हुन्छ ?
For what values of m and c in $y = f(x) = mx + c$ make it an identity function?

4A. $f(x) = ax^2 + bx + c, a \neq 0$ लाई लेखाचित्रमा प्रस्तुत गर्दा बन्ने वक्रलाई के भनिन्छ ?
What is the name of curve obtained by sketching the graph of $f(x) = ax^2 + bx + c, a \neq 0$?

4B. $f(x) = a(x - h)^2 + k, a \neq 0$ मा (h, k) ले के जनाउँदछ ?
What does (h, k) represent in the equation $f(x) = a(x - h)^2 + k, a \neq 0$?

4C. a को मान कति हुँदा $f(x) = a(x - h)^2 + k, a \neq 0$ ले दिने पाराबोलाको मुख माथितिर हुन्छ ?
For what value of a, the parabola given by $f(x) = a(x - h)^2 + k, a \neq 0$ has opening upward?

4D. a को मान कति हुँदा $f(x) = a(x - h)^2 + k, a \neq 0$ ले दिने पाराबोलाको मुख तलतिर हुन्छ ?
For what value of a, the parabola given by $f(x) = a(x - h)^2 + k, a \neq 0$ has opening downward?

5A. $f(x) = ax^2$ मा शीर्षबिन्दु र सममितीय रेखाको समीकरण लेख्नुहोस् ।
In $f(x) = ax^2$, write the vertex and the equation of line of symmetry.

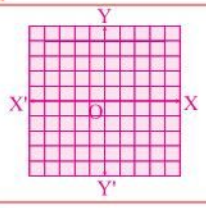
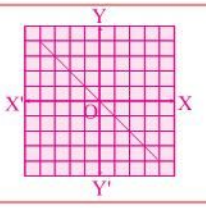
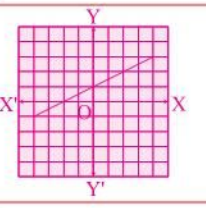
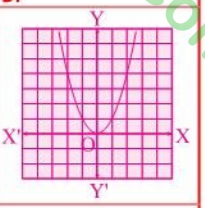
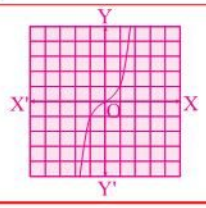
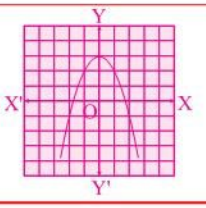
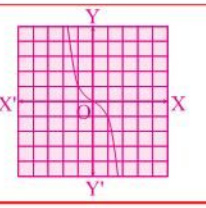
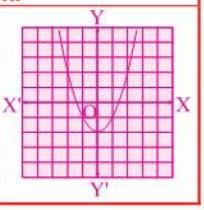
5B. $f(x) = ax^3$ मा शीर्षबिन्दु लेख्नुहोस् । (Write the vertex of $f(x) = ax^3$.)

www.drmathnp.com

puspas.com.np

Model 2 *Concept Recall*

6. लेखाचित्रमा दिइएको फलनको किसिम लेख्नुहोस् । **(Identify the type of function having following graph):**

A.	B.	C.	D.
			
E.	**F.**	**G.**	**H.**
			

7A. $y = \sin x$ को y-खण्ड कति हुन्छ ? (What is the y-intercept of $y = \sin x$?)

7B. $y = \cos x$ को y-खण्ड कति हुन्छ ? (What is the y-intercept of $y = \cos x$?)

7C. $y = \tan x$ को y-खण्ड कति हुन्छ ? (What is the y-intercept of $y = \tan x$?)

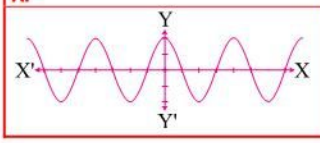
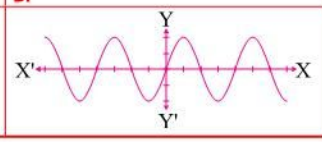
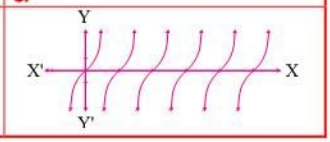
8A. $y = \sin x$ को अधिकतम मान कति हुन्छ ? (What is the maximum value of $y = \sin x$?)

8B. $y = \cos x$ को अधिकतम मान कति हुन्छ ? (What is the maximum value of $y = \cos x$?)

9A. $y = \sin x$ को न्यूनतम मान कति हुन्छ ? (What is the minimum value of $y = \sin x$?)

9B. $y = \cos x$ को न्यूनतम मान कति हुन्छ ? (What is the minimum value of $y = \cos x$?)

10. निम्नलिखित लेखाचित्रमा हुने फलनको प्रकार लेख्नुहोस् (Identify the type of function having following graph):

A.	B.	C.
		

11. निम्नलिखित फलनको क्षेत्र लेख्नुहोस् (Write the domain of following functions):

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = \tan x$

12. निम्नलिखित फलनको विस्तार लेख्नुहोस् (Write the range of following functions):

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = \tan x$

B. SHORT QUESTIONS

Model 1 *Problems based on trigonometric functions.*

1. तलका फलनहरूको लेखाचित्र नखिची आयाम (Amplitude) र अवधि (Period) पत्ता लगाउनुहोस् ।

Determine the amplitude and period of each function without graphing.

(a) $f(x) = 2 \sin x$ (b) $f(x) = 3 \cos x$ (c) $y = -4 \cos (2x)$

(d) $y = -\sin (3x)$ (e) $y = -3 \cos (3x)$ (f) $g(x) = -\sin\left(\frac{x}{2}\right)$

Model 2 *Problems based on algebraic function.*

2. तलका वर्गघातीय फलनहरूको शीर्षबिन्दु र सममिति अक्ष पत्ता लगाउनुहोस् ।

Find the vertex and line of symmetry of following quadratic functions.

(a) $f(x) = -x^2 + 6x - 8$ (b) $g(x) = x^2 + 10x + 19$ (c) $h(x) = 4x^2 + 5$

Model 3 *Problems based on finding domain, range, image, pre-image of simple algebraic function.*

3A. यदि फलन $f(x) = 4x - 5$ को विस्तार क्षेत्र $\{-1, 7\}$ भए, त्यसको क्षेत्र निकाल्नुहोस् ।

If the range of the function $f(x) = 4x - 5$ is $\{-1, 7\}$, find its domain. **[2066 S]**

www.drmathnp.com

puspas.com.np

3B. यदि एउटा फलन $f: x \to 3x + 5$ को विस्तार क्षेत्र $\{2, 8\}$ भए त्यसको क्षेत्र निकाल्नुहोस् ।
If the range of a function $f: x \to 3x + 5$ is $\{2, 8\}$, find its domain. [2059 R]

4A. फलन $f(x) = \frac{2x-3}{5}$ को फलन क्षेत्रमा हुने कुन सदस्यको प्रतिबिम्ब 7 हुन्छ ?
What element in the domain has image 7 under the function $f(x) = \frac{2x-3}{5}$? [2057 R]

4B. फलन $f(x) = 3x + 5$ को फलन क्षेत्रमा हुने कुन सदस्यको प्रतिबिम्ब 8 हुन्छ ?
What element in the domain has the image 8 under the function $f(x) = 3x + 5$? [2060 R]

5A. फलन $f(x) = \sqrt{x}$ मा 4 को पूर्व प्रतिबिम्ब कति हुन्छ ? (What will be the pre-image of 4 in the function $f(x) = \sqrt{x}$?) [2058 S, 2060 S]

5B. $f(x) = \sqrt[3]{x}$ मा 8 को पूर्व प्रतिबिम्ब कति होला ?(What will be the pre-image of 8 in $f(x) = \sqrt[3]{x}$?) [2062 R]

C. LONG QUESTIONS

Model 1 *Problems based on algebraic function.*

1. तलका फलनहरूको लेखाचित्र खिच्नुहोस् । (Draw the graph of the following functions.)

(a) $f(x) = x^2$ $[-3 \le x \le 3$ and $y = f(x)]$ (b) $f(x) = -x^2$ $[-4 \le x \le 4$ and $y = f(x)]$
(c) $f(x) = 2x^2$ $[-3 \le x \le 3$ and $y = f(x)]$ (d) $f(x) = -2x^2$ $[-3 \le x \le 3$ and $y = f(x)]$
(e) $f(x) = x^3$ $[-3 \le x \le 3$ and $y = f(x)]$ (f) $f(x) = -x^3$ $[-3 \le x \le 3$ and $y = f(x)]$
(g) $f(x) = 3x^2$ $[-3 \le x \le 3$ and $y = f(x)]$ (h) $f(x) = -3x^2$ $[-3 \le x \le 3$ and $y = f(x)]$

Model 2 *Problems based on trigonometric functions.*

2. तलका त्रिकोणमितीय फलनको लेखाचित्र खिच्नुहोस् । (Draw the graph of the following trigonometric functions.)

(a) $f(x) = \sin x$ $(-90° \le x \le 90°)$ (b) $f(x) = \cos x$ $(-90° \le x \le 90°)$
(c) $f(x) = \tan x$ $(-90° \le x \le 90°)$ (d) $f(x) = \sin x$ $(0° \le x \le \pi^c)$
(e) $f(x) = \cos x$ $(-\pi^c \le x \le \pi^c)$ (f) $f(x) = \sin 2x$ $(-90° \le x \le 90°)$
(g) $f(x) = \cos 2x$ $(-90° \le x \le 90°)$ (h) $f(x) = \tan 2x$ $(-90° \le x \le 90°)$
(i) $f(x) = 2\sin x$ $(-\pi c \le x \le \pi^c)$ (j) $f(x) = 2\cos x$ $(-\pi^c \le x \le \pi^c)$

ANSWERS

A. VERY SHORT QUESTIONS

3A. 0 **3B.** $m = 1, c = 0$ **4A.** Parabola **4B.** Vertex **4C.** $a > 0$ **4D.** $a < 0$ **5A.** (0, 0), y-axis
5B. (0, 0) **6A.** Constant function **6B.** Identity function **6C.** Linear function **6D.** Quadratic function **6E.** Cubic function
6F. Quadratic function **6G.** Cubic function **6H.** Quadratic function **7A.** 0 **7B.** 1
7C 0 **8A.** 1 **8B.** 1 **9A.** -1 **9B.** -1 **10A.** cosine function
10B. sine function **10C.** tan function **11A.** $(-\infty, +\infty)$ **11B.** $(-\infty, +\infty)$ **11C.** All real numbers except odd multiples of $\frac{\pi}{2}$
12A. $[-1, 1]$ **12B.** $[-1, 1]$ **12C.** $(-\infty, +\infty)$

B. SHORT QUESTIONS

1A. $a = 2 ; p = 2\pi$ **1B.** $a = 3 ; p = 2\pi$ **1C.** $a = 4 ; p = \pi$ **1D.** $a = 1 ; p = \frac{2\pi}{3}$ **1E.** $a = 3 ; p = \frac{2\pi}{3}$ **1F.** $a = 1 ; p = 4\pi$
2A. (3, 1); $x = 3$ **2B.** $(-5, -6)$; $x = -5$ **2C.** (0, 5); $x = 0$ **3A.** $\{1, 3\}$ **3B.** $\{-1, 1\}$ **4A.** 19
4B. 1 **5A.** 16 **5B.** 512

EXTRA KNOWLEDGE ! JUST FOR PRACTICE NOT FOR EXAM

दिइएको अवस्थामा sine फलन पत्ता लगाउनुहोस् । (In the given condition, find the sine function.)

(a) आयाम (Amplitude) = 3, अवधि (Period) = π
Hint: Let $f(x) = a \sin bx$ .....(i) be the required function
Then, Amplitude $|a| = 3$ i.e. $a = \pm 3$
And, Period $= \frac{2\pi}{b} = \pi$ or, $\frac{2}{b} = 1$ $\therefore$ $b = 2$
Substituting the values of a and b in (i) then, $f(x) = \pm 3 \sin 2x$.
(b) आयाम (Amplitude) = 2, अवधि (Period) = 4π
(c) आयाम (Amplitude) = 3, अवधि (Period) = 2

ANSWERS: 2B. $\pm 2 \sin \frac{x}{2}$ **2C.** $\pm 3 \sin \pi x$

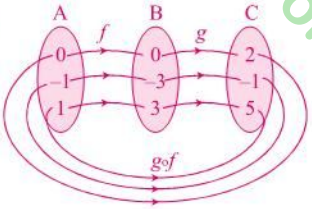
www.drmathnp.com

1.2 संयुक्त फलन COMPOSITE FUNCTION

1. संयुक्त फलन (Composite Function) **[SEE 2065 R', SEE 2074 R]**

यदि फलनहरू $f: A \to B$ र $g: B \to C$ भए A बाट C मा परिभाषित गरिएको फलनलाई f र g को संयुक्त फलन भनिन्छ । यसलाई $g \circ f$ ले जनाइन्छ ।

If $f: A \to B$ and $g: B \to C$ are two functions then the new function defined from A to C is called the composite function of f and g. It is denoted by $g \circ f$.



2. फलनहरूको संयोजन (Combination of functions)

फलनहरूको योग, अन्तर, गुणन र भागलाई ती फलनहरूको संयोजन भनिन्छ ।

The sum, difference, product or quotient of the functions is known as combination of the functions.

योगफल (Sum)	$(f+g)(x) = f(x) + g(x)$
अन्तर (Difference)	$(f-g)(x) = f(x) - g(x)$
गुणनफल (Product)	$(f \times g)(x) = f(x) \times g(x)$ or $f(x) \cdot g(x)$
भागफल (Quotient)	$(f/g)(x) = f(x)/g(x)$, as long as $g(x)$ isn't zero.

3. फलनहरूको वियोजन (Decomposition of functions)

संयुक्त फलनमा प्रयोग भएका फलनहरू छुट्टा छुट्टै पत्ता लगाउने प्रक्रियालाई फलनहरूको वियोजन भनिन्छ ।

The process of breaking apart a composite function into its separate components is called decomposition of functions.

Example: Write each function h as the composition of two functions f and g such that $h(x) = (f \circ g)(x)$

$h(x) = (f \circ g)(x)$	Outside $f(x)$	Inside $g(x)$	Notes
$(1-x)^3$	x^3	$1-x$	The big thing going on is cubing something, so the outside function is a cubing function. $(1-x)$ is what you're cubing, so it's the inside function.
$\sqrt{9-x}$	$\sqrt{x}$	$9-x$	The big thing going on is taking the square root (outside), $(9-x)$ is what you're taking the square root of (inside).
$\frac{4}{(5x^2+2)^2}$	$\frac{4}{x^2}$	$5x^2+2$	Looks like 4 over something squared.
	$\frac{4}{x}$	$(5x^2+2)^2$	An alternative, but correct answer.
$(x+2)^2 + 2(x+2) + 1$	$x^2 + 2x + 1$	$x+2$	$x+2$ is repeated, so that's a good choice for the inside function. Replace every occurrence of the pattern by x for the outside function.

FORMULAE AND KEY POINTS

1. $g \circ f(x) = (g \circ f)(x) = g(f(x))$
2. $f \circ g(x) = (f \circ g)(x) = f(g(x))$
3. $f^2(x) = f \circ f = f(f(x))$
4. $f^3(x) = f \circ f \circ f = f(f(f(x)))$
5. $h \circ (g \circ f) = h \circ (g \circ f)(x) = (h \circ g) \circ f(x) = h(g(f(x)))$
6. $(f \pm k)(x) = f(x) \pm k$; *where k is a scalar.*
7. $(kf)(x) = k(f(x))$; *where k is a scalar.*

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 S, 2065 M, 2065 R', 2066 R, 2067 S, 2069 R, 2069 R', 2074 R, 2074 S	9
Model 2	2065 R, 2066 R', 2067 R', 2068 S', 2070 R, R', S', 2071 R', 2072 S, SEE 2073 R, S, S', SEE 2074 R', S', SEE 2075 R', SEE 2075 R_2, SEE MODEL 2076	17
Model 3	2068 R, 2068 S	2
Model 4	2072 R	1

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2072 R'	1

www.drmathnp.com

puspas.com.np

SEE EXERCISE 1.2

A. VERY SHORT QUESTIONS

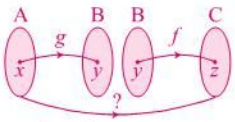
Model 1 *Definitions or formulae Recall.*

1. तलका पदहरू उदाहरण सहित परिभाषीत गर्नुहोस् । (Define the following terms with an example.)
A. संयुक्त फलन (Composite function) **B.** संयोजन फलन (Combination of functions)

2A. यदि $f : A \to B$ र $g : B \to C$ दुई फलनहरू हुन् भने संयुक्त फलन $A \to C$ लाई के ले जनाउँदछ ?
If $f : A \to B$ and $g : B \to C$ be the two functions then what will denote the composite function from $A \to C$?

2B. यदि $h : R \to S$ र $g : Q \to R$ दुई फलनहरू हुन् भने संयुक्त फलन $Q \to S$ लाई के ले जनाउँदछ ?
If $h : R \to S$ and $g : Q \to R$ be the two functions then what will denote the composite function from $Q \to S$?

2C. चित्रमा g : A ⟶ B र f: B ⟶ C छ । A बाट C सम्म परिभाषित फलनको नाम के हो ?
In figure, g : A ⟶ B and f: B ⟶ C. What is the name of function defined from A to C?



3A. $f \circ g(x)$ को विस्तार क्षेत्र f र g मध्ये कुन फलनको विस्तार क्षेत्रसँग बराबर हुन्छ ?
Which range of the functions between f and g is equal to the range of the functions $f \circ g(x)$?

3B. $g \circ f(x)$ को विस्तार क्षेत्र f र g मध्ये कुन फलनको विस्तार क्षेत्रसँग बराबर हुन्छ ?
Which range of the functions between f and g is equal to the range of the functions $g \circ f(x)$?

3C. $(f \circ g)(x)$ फलनको क्षेत्र f र g मध्ये कुन फलनको क्षेत्रसँग बराबर हुन्छ ?
Which domain of the functions between f and g is equal to the domain of the functions $f \circ g(x)$?

3D. $g \circ f(x)$ फलनको क्षेत्र f र g मध्ये कुन फलनको क्षेत्रसँग बराबर हुन्छ ?
Which domain of the functions between f and g is equal to the domain of the functions $g \circ f(x)$?

4A. यदि $f(x) = m$ र $g(x) = n$ भए $f \circ g(x)$ को मान कति होला ?
If $f(x) = m$ and $g(x) = n$ then what is the value of $f \circ g(x)$?

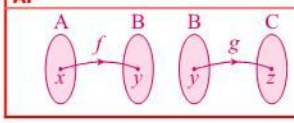
4B. यदि $f(x) = a$ र $g(x) = b$ भए $g \circ f(x)$ को मान कति होला ?
If $f(x) = a$ and $g(x) = b$ then what is the value of $g \circ f(x)$?

4C. कुन अवस्थामा '$f \circ g$' र '$g \circ f$' बराबर हुन्छन् ? (In what condition, '$f \circ g$' and '$g \circ f$' are equal?)

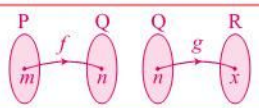
4D. दुईओटा फलनको संयुक्त फलन निकाल्नको लागि आवश्यक शर्त के हो ?
What is the necessary condition to find the composite function of two functions?

5. तलका अवस्थामा संयुक्त फलन $g \circ f$ पत्ता लगाउनुहोस् । (Find the composite function $g \circ f$ in each of the following conditions.)

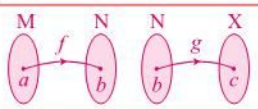
A.



B.



C.



Model 2 *Concept Recall*

6A. यदि फलन $f = \{(1, 2), (2, 3), (3, 4)\}$ र $g = \{(2, a), (4, c), (3, b)\}$ भए $g \circ f(1)$ पत्ता लगाउनुहोस् ।
If the function $f = \{(1, 2), (2, 3), (3, 4)\}$ and $g = \{(2, a), (4, c), (3, b)\}$ then find function $g \circ f(1)$.

6B. यदि फलन $h = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$, भए $g \circ h(3)$ पत्ता लगाउनुहोस् ।
If $h = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find $g \circ h(3)$.

6C. यदि फलन $f = \{(1, 3), (0, 0), (-1, -3)\}$ र $g = \{(0, 2), (-3, -1), (3, 5)\}$ भए $g \circ f(1)$ पत्ता लगाउनुहोस् ।
If $f = \{(1, 3), (0, 0), (-1, -3)\}$ and $g = \{(0, 2), (-3, -1), (3, 5)\}$ then find $g \circ f(1)$.

6D. यदि फलन $g = \{(1, 2), (2, 3), (3, 4)\}$ र $h = \{(2, 3), (3, 4), (4, 5)\}$, भए $h \circ g(3)$ पत्ता लगाउनुहोस् ।
If functions $g = \{(1, 2), (2, 3), (3, 4)\}$ and $h = \{(2, 3), (3, 4), (4, 5)\}$, find $h \circ g(3)$.

7A. यदि $f(x) = x + a$ र $g(x) = x$ भए $f \circ g(x)$ को मान कति हुन्छ ?
If $f(x) = x + a$ and $g(x) = x$ then what is the value of $f \circ g(x)$?

7B. यदि $f(x) = x$ र $g(x) = x + b$ भए $g(f(x))$ को मान कति हुन्छ ?
If $f(x) = x$ and $g(x) = x + b$ then what is the value of $g(f(x))$?

8A. यदि $f = \{(2, 5)\}$ र $g = \{(5, 7)\}$ भए $g \circ f$ को क्रमजोडा के हुन्छ ?
If $f = \{(2, 5)\}$ and $g = \{(5, 7)\}$, what is the order pair of $g \circ f$?

www.drmathnp.com
puspas.com.np

8B. यदि $g = \{(5, -7)\}$ र $f = \{(-7, 2)\}$ भए $f \circ g(5)$ को मान कति हुन्छ ?
If $g = \{(5, -7)\}$ and $f = \{(-7, 2)\}$, what is the value of $f \circ g(5)$?

8C. यदि $f(x) = x + 2$ र $g(x) = x$ भए $f \circ g(x)$ को मान कति हुन्छ ?
If $f(x) = x + 2$ and $g(x) = x$ then what is the value of $f \circ g(x)$?

9A. यदि $f = \{(1, 3), (2, 1)\}$ र $g = \{(1, 2), (3, 2)\}$ भए फलन $g \circ f$ को क्षेत्र कति हुन्छ ?
If $f = \{(1, 3), (2, 1)\}$ and $g = \{(1, 2), (3, 2)\}$ then what is the domain of $g \circ f$?

9B. यदि $f = \{(m, n), (a, b)\}$ र $g = \{(n, a), (b, c)\}$ भए फलन $g \circ f$ को विस्तार कति हुन्छ ?
If $f = \{(m, n), (a, b)\}$ and $g = \{(n, a), (b, c)\}$ then what is the range of $g \circ f$?

B. SHORT QUESTIONS

Model 1 *Problems based on composite functions in ordered pair form.*

1A. संयुक्त फलनको परिभाषा दिनुहोस् । यदि $f = \{(3, 4), (4, 5), (5, 6)\}$ र $g = \{(2, 3), (3, 4), (4, 5)\}$ भए $f \circ g$ लाई क्रमजोडाको रूपमा व्यक्त गर्नुहोस् ।

Define composite function. If $f = \{(3, 4), (4, 5), (5, 6)\}$ and $g = \{(2, 3), (3, 4), (4, 5)\}$, express $f \circ g$ in an ordered pair form. **[2074 R]**

1B. यदि $f = \{(1, 2), (2, 3), (3, 4)\}$ र $g = \{(2, a), (4, c), (3, b)\}$ भए संयुक्त फलन $g \circ f$ लाई मिलान चित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।

If the function $f = \{(1, 2), (2, 3), (3, 4)\}$ and $g = \{(2, a), (4, c), (3, b)\}$, then show the composite function $g \circ f$ in an arrow diagram and find it in ordered pair form. **[2069 R]**

1C. यदि $h = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$ भए $g \circ h$ लाई मिलान चित्र प्रयोग गरी क्रमजोडाको रूपमा लेख्नुहोस् ।

If $h = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, find $g \circ h$ using arrow diagram and write in ordered pair form. **[2069 R']**

1D. यदि $f = \{(1, 3), (0, 0), (-1, -3)\}$ र $g = \{(0, 2), (-3, -1), (3, 5)\}$ भए $g \circ f$ लाई मिलान चित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।

If $f = \{(1, 3), (0, 0), (-1, -3)\}$ and $g = \{(0, 2), (-3, -1), (3, 5)\}$ then show the function $g \circ f$ in the arrow diagram and find it in ordered pair form. **[2065 M]**

1E. संयुक्त फलनलाई परिभाषित गर्नुहोस् । यदि फलनहरू $g = \{(1, 2), (2, 3), (3, 4)\}$ र $h = \{(2, 3), (3, 4), (4, 5)\}$ भए संयुक्त फलन $h \circ g$ लाई मिलानचित्रमा देखाउनुहोस् ।

Define composite function. If functions $g = \{(1, 2), (2, 3), (3, 4)\}$ and $h = \{(2, 3), (3, 4), (4, 5)\}$, show the composite function $h \circ g$ in an arrow diagram. **[2065 R']**

1F. विपरीत फलनको परिभाषा दिनुहोस् । यदि $f = \{(1, 2), (3, 4), (5, 6)\}$ र $g = \{(2, 5), (4, 6), (6, 8)\}$ भए संयुक्त फलन $g \circ f$ पत्ता लगाउनुहोस् ।

Define inverse function. If $f = \{(1, 2), (3, 4), (5, 6)\}$ and $g = \{(2, 5), (4, 6), (6, 8)\}$, find the composite function $g \circ f$. **[2065 S]**

1G. यदि $f = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$ भए, $f \circ g$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।

If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ then show the function $f \circ g$ in an arrow diagram and find it in ordered-pair form. **[2066 R]**

1H. यदि फलनहरू $f = \{(0, 0), (1, 1), (2, 4), (3, 9)\}$ र $g = \{(1, 0), (2, 1), (3, 2), (4, 3)\}$ भए $f \circ g$ लाई मिलान चित्रमा देखाई क्रमजोडामा देखाउनुहोस् ।

If $f = \{(0, 0), (1, 1), (2, 4), (3, 9)\}$ and $g = \{(1, 0), (2, 1), (3, 2), (4, 3)\}$ then show the function $f \circ g$ in an arrow diagram and find it in ordered-pair form. **[2067 S]**

1I. यदि $f = \{(1, 3), (2, 4), (3, 5)\}$ र $g = \{(3, 1), (4, 2), (5, 3)\}$ भए $(f \circ g)$ लाई मिलान चित्रमा देखाउनुहोस् । $(f \circ g)$ का क्रमजोडाहरू लेख्नुहोस् ।

If $f = \{(1, 3), (2, 4), (3, 5)\}$ and $g = \{(3, 1), (4, 2), (5, 3)\}$, then show $(f \circ g)$ in an arrow-diagram. Write the ordered pairs of $(f \circ g)$. **[SEE 2074 S]**

www.drmathnp.com

puspas.com.np

Model 2 *Problems based on composite functions in algebraic form .*

2A. यदि $g(x) = 2x - 1$ र $f(x) = 4x$, भए $g \circ f(x)$ को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x - 1$ and $f(x) = 4x$, find the value of $g \circ f(x)$. [SEE MODEL 2076]

2B. यदि $f(x) = x + 1$ र $g(x) = 2x + 1$ भए $g \circ f(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = x + 1$ and $g(x) = 2x + 1$, find $g \circ f(x)$. [2072 S]

2C. यदि $h(x) = 3x - 2$ र $g(x) = 5x + 1$ भए $gh(x)$ पत्ता लगाउनुहोस् ।
If $h(x) = 3x - 2$ and $g(x) = 5x + 1$ then find $gh(x)$. [2068 S']

2D. यदि $f(x) = 3x + 2$ र $g(x) = 2x - 1$ भए, $g \circ f(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 2$ and $g(x) = 2x - 1$, find $g \circ f(x)$. [2065 R]

2E. यदि $f(x) = 2x + 3$ र $g(x) = 3x - 1$ भए $f \circ g(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$ and $g(x) = 3x - 1$, find $f \circ g(x)$. [2066 R']

2F. यदि $f(x) = 2x + 5$ र $g(x) = 3x - 1$ भए $g \circ f(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 5$ and $g(x) = 3x - 1$,find $g \circ f(x)$. [2071 R']

2G. यदि $g(x) = \frac{x+2}{5}$ र $h(x) = 3x - 2$ भए $gh(x)$ पत्ता लगाउनुहोस् ।
If $g(x) = \frac{x+2}{5}$ and $h(x) = 3x - 2$, find $gh(x)$. [2067 R']

3A. यदि $f(x) = 2x + 3$ भए $ff(2)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 2x + 3$ then find the value of $ff(2)$.) [SEE 2075 R_2]

3B. यदि $f(x) = 2x + 1$ भए $ff(2)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 1$ then find $ff(2)$.) [SEE 2075 R']

3C. यदि $f(x) = x - 2$ र $g(x) = 3x + 1$ भए $gof(x)$ र $g \circ f(2)$ पत्ता लगाउनुहोस् ।
If $f(x) = x - 2$ and $g(x) = 3x + 1$ then find $g \circ f(x)$ and $g \circ f(2)$. [SEE 2074 S']

4A. यदि $f(x) = x + 1$ र $g(x) = 2x + 1$ भए $g \circ f(2)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = x + 1$ and $g(x) = 2x + 1$ then find the value of $g \circ f(2)$. [SEE 2074 R']

4B. यदि $h(x) = 2x + 3$ र $g(x) = 3x - 2$ भए $g \circ h(5)$ को मान पत्ता लगाउनुहोस् ।
If $h(x) = 2x + 3$ and $g(x) = 3x - 2$, find the value of $g \circ h(5)$. [SEE 2073 S]

4C. यदि $f(x) = 3x + 15$ र $h(x) = 3x + 2$ भए $f \circ h(-2)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 15$ and $h(x) = 3x + 2$, find the value of $f \circ h(-2)$. [SEE 2073 S']

4D. यदि $h(x) = 2x + 3$ र $g(x) = 3x - 2$ भए $h \circ g(5)$ पत्ता लगाउनुहोस् ।
If $h(x) = 2x + 3$ and $g(x) = 3x - 2$, find the $h \circ g(5)$. [SEE 2073 R]

4E. यदि $f(x) = 2x - 3$ र $g(x) = x^2 + 1$ भए $f \circ g(3)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the value of $f \circ g(3)$. [2070 R']

5A. यदि $g(x) = \frac{x-2}{3}$ र $h(x) = 3x + 2$ भए, $gh(x)$ एउटा एकात्मक फलन हो भनी प्रमाणित गर्नुहोस् ।
If $g(x) = \frac{x-2}{3}$ and $h(x) = 3x + 2$, prove that $gh(x)$ is an identity function. [2070 R]

5B. यदि $f(x) = 2x + 5$ र $g(x) = \frac{x-5}{2}$ भए, $f \circ g(x)$ एउटा एकात्मक फलन हो भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$, prove that $f \circ g(x)$ is an identity function. [2070 S']

6A. यदि $f(x) = 4x + 5$ र $g(x) = 3x - 2$ भए, f र g को संयुक्त फलन पत्ता लगाउनुहोस् ।
If $f(x) = 4x + 5$ and $g(x) = 3x - 2$ then find the composite function of f and g.

6B. यदि $f(x) = 4x^2 - 3$ र $g(x) = 3 - \frac{1}{2}x^2$ भए, f र g को संयुक्त फलन पत्ता लगाउनुहोस् ।
If $f(x) = 4x^2 - 3$ and $g(x) = 3 - \frac{1}{2}x^2$ then find the composite function of f and g.

7A. यदि $f(x) = \frac{3}{x-1}$ र $g(x) = \frac{2}{x}$ भए, g र f को संयुक्त फलन पत्ता लगाउनुहोस् ।
If $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x}$ then find the composite function of g and f.

www.drmathnp.com

7B. यदि $f(x)=\frac{x}{x+3}$ र $g(x)=\frac{2}{x}$ भए, g र f को संयुक्त फलन पत्ता लगाउनुहोस् ।

If $f(x)=\frac{x}{x+3}$ and $g(x)=\frac{2}{x}$ then find the composite function of g and f.

Model 3 Problems based on finding value of letters.

8A. यदि $f: x \to 3x+b$ र $ff(2)=12$ भए b को मान पत्ता लगाउनुहोस् ।
If $f: x \to 3x+b$ and $ff(2)=12$, find the value of b. [2068 R]

8B. यदि $f(x)=3x$, $g(x)=x+2$ र $f\circ g(x)=18$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x)=3x$, $g(x)=x+2$ and $f\circ g(x)=18$, find the value of x. [2068 S]

8C. यदि $f(x)=4x$, $g(x)=x+1$ र $f\circ g(x)=20$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x)=4x$, $g(x)=x+1$ and $f\circ g(x)=20$, find the value of x.

9A. यदि $f(x)=\frac{1}{x+2}$ र $g(x)=\frac{4}{x-1}$ भए $f\circ g(x)=1$ मा क्षेत्र पत्ता लगाउनुहोस् ।

If $f(x)=\frac{1}{x+2}$ and $g(x)=\frac{4}{x-1}$ then find the domain in $f\circ g(x)=1$.

9B. यदि $f(x)=\frac{x}{x+3}$ र $g(x)=\frac{2}{x}$ भए $4f\circ g(x)=1$ मा क्षेत्र पत्ता लगाउनुहोस् ।

If $f(x)=\frac{x}{x+3}$ and $g(x)=\frac{2}{x}$ then find the domain in $4f\circ g(x)=1$.

Model 4 Problems based on finding missing function.

10A. यदि $f(x)=3x-2$ र $f\circ g(x)=6x-2$ भए $g(x)$ पत्ता लगाउनुहोस् । (If $f(x)=3x-2$ and $f\circ g(x)=6x-2$, find $g(x)$.) [2072 R]

10B. यदि $f(x)=4x+5$ र $f\circ g(x)=8x+13$ भए $g(x)$ पत्ता लगाउनुहोस् ।
If $f(x)=4x+5$ and $f\circ g(x)=8x+13$ then find $g(x)$.

11A. यदि $g(x)=2x$ र $f\circ g(x)=6x-2$ भए $f(x)$ पत्ता लगाउनुहोस् । (If $g(x)=2x$ and $f\circ g(x)=6x-2$, find $f(x)$.)

11B. यदि $g(x)=2x+2$ र $f\circ g(x)=8x+13$ भए $f(x)$ पत्ता लगाउनुहोस् ।
If $g(x)=2x+2$ and $f\circ g(x)=8x+13$ then find $f(x)$.

12A. यदि $f: \mathrm{R}\to\mathrm{R}: f(x)=\frac{1}{2}(x-3)$ र $(f\circ g): \mathrm{R}\to\mathrm{R}: (f\circ g(x))=x$ भए $g(x)$ पत्ता लगाउनुहोस् ।

If $f: \mathrm{R}\to\mathrm{R}: f(x)=\frac{1}{2}(x-3)$ and $(f\circ g): \mathrm{R}\to\mathrm{R}: (f\circ g(x))=x$ then find $g(x)$.

12B. यदि $g: \mathrm{R}\to\mathrm{R}: g(x)=4-x$ र $(f\circ g)(x)=11-2x$ भए $f(x)$ पत्ता लगाउनुहोस् ।
If $g: \mathrm{R}\to\mathrm{R}: g(x)=4-x$ and $(f\circ g)(x)=11-2x$ then find $f(x)$.

12C. यदि $f: \mathrm{N}\to\mathrm{R}: f(x)=x^2$ र $g: \mathrm{R}\to\mathrm{R}$ $(f\circ g)(x)=x^2-2x+1$ भए $g(x)$ पत्ता लगाउनुहोस् ।
If $f: \mathrm{N}\to\mathrm{R}: f(x)=x^2$ and $g: \mathrm{R}\to\mathrm{R}$ $(f\circ g)(x)=x^2-2x+1$ then find $g(x)$.

12D. यदि $f\circ g=2x+13$ र $g(x)=\frac{x+1}{2}$ भए $f(1)$ पत्ता लगाउनुहोस् । (If $f\circ g=2x+13$ and $g(x)=\frac{x+1}{2}$, find $f(1)$.)

Model 5 Problems based on decomposition of functions.

13A. यदि $h(x)=(2x-3)^5$ र $h(x)=(f\circ g)(x)$ भए $f(x)$ र $g(x)$ का कुनै दुईओटा सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।
If $h(x)=(2x-3)^5$ and $h(x)=(f\circ g)(x)$ then find any two possible formulas of $f(x)$ and $g(x)$.

13B. यदि $h(x)=(3x+2)^3$ र $h(x)=(f\circ g)(x)$ भए फलन $f(x)$ र $g(x)$ का सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।
If $h(x)=(3x+2)^3$ and $h(x)=(f\circ g)(x)$, find the possible formulas of $f(x)$ and $g(x)$.

14A. यदि $h(x)=\frac{1}{(x+3)^3}(x\neq -3)$ र $h(x)=(f\circ g)(x)$ भए फलन $f(x)$ र $g(x)$ का सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।

If $h(x)=\frac{1}{(x+3)^3}$ where $(x\neq -3)$ and $h(x)=(f\circ g)(x)$ then find the possible formulas of $f(x)$ and $g(x)$.

14B. यदि $p(x)=\frac{1}{(x+2)^3}(x\neq -2)$ र $p(x)=(f\circ g)(x)$ भए फलन $f(x)$ र $g(x)$ का सम्भावित सूत्रहरू पत्ता लगाउनुहोस् ।

If $p(x)=\frac{1}{(x+2)^3}$ where $x\neq -2$ and $p(x)=(f\circ g)(x)$, find the possible formulas of $f(x)$ and $g(x)$.

puspas.com.np

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS [For group C and D]

Model 1 Problems based on finding composite function.

1A. मानौं $f: R \to R : f(x) = 2x + 1$ र $g : R \to R : g(x) = (x^2 - 2)$ दिइएको छ भने पत्ता लगाउनुहोस् :
Let $f: R \to R : f(x) = 2x + 1$ and $g : R \to R : g(x) = (x^2 - 2)$ is given. Find:
(i) $(f \circ f)(x)$ (ii) $(g \circ f)(x)$ (iii) $(f \circ g)(x)$ (iv) $(g \circ g)(x)$

1B. यदि $f(x) = 3x - 4$, $h(x) = x + 3$ र $g(x) = -2x + 1$ भए पत्ता लगाउनुहोस् :
If $f(x) = 3x - 4$, $h(x) = x + 3$ and $g(x) = -2x + 1$ then find:
(i) $f \circ h$ (ii) $h \circ h$ (iii) $g \circ h$ (iv) $g \circ f(a)$

2A. यदि $f(x) = 2x$ र $g(x) = 3x^2 + 1$ भए पत्ता लगाउनुहोस् (If $f(x) = 2x$ and $g(x) = 3x^2 + 1$ then find):
(i) $f \circ g(4)$ (ii) $g \circ f(2)$ (iii) $f \circ f(1)$ (iv) $g \circ g(0)$

2B. यदि $f(x) = 3x + 2$ र $g(x) = 2x^2 - 1$ भए पत्ता लगाउनुहोस् (If $f(x) = 3x + 2$ and $g(x) = 2x^2 - 1$ then find):
(i) $f \circ g(4)$ (ii) $g \circ f(2)$ (iii) $f \circ f(1)$ (iv) $g \circ g(0)$

2C. यदि $f(x) = 2x^2$ र $g(x) = 1 - 3x^2$ भए पत्ता लगाउनुहोस् (If $f(x) = 2x^2$ and $g(x) = 1 - 3x^2$ then find):
(i) $f \circ g(4)$ (ii) $g \circ f(2)$ (iii) $f \circ f(1)$ (iv) $g \circ g(0)$

2D. यदि $f(x) = \frac{3}{x+1}$ र $g(x) = \sqrt[3]{x}$ भए पत्ता लगाउनुहोस् (If $f(x) = \frac{3}{x+1}$ and $g(x) = \sqrt[3]{x}$ then find):
(i) $f \circ g(4)$ (ii) $g \circ f(2)$ (iii) $f \circ f(1)$ (iv) $g \circ g(0)$

2E. यदि $f(x) = x^{\frac{3}{2}}$ र $g(x) = \frac{2}{x+2}$ भए पत्ता लगाउनुहोस् (If $f(x) = x^{\frac{3}{2}}$ and $g(x) = \frac{2}{x+2}$ then find):
(i) $f \circ g(4)$ (ii) $g \circ f(2)$ (iii) $f \circ f(1)$ (iv) $g \circ g(0)$

3A. यदि $f(x) = 3x + 4$ र $g(x) = 2(x + 1)$ भए $(f \circ g)(x) = (g \circ f)(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 3x + 4$ and $g(x) = 2(x + 1)$ then prove that $(f \circ g)(x) = (g \circ f)(x)$ [2072 R']

3B. यदि $f(x) = \frac{2x+5}{4}$ र $g(x) = \frac{4x+7}{5}$ भए $f \circ g(x) + \frac{9}{20} = g \circ f(x)$ हुन्छ भनी देखाउनुहोस् ।
If $f(x) = \frac{2x+5}{4}$ and $g(x) = \frac{4x+7}{5}$ then show that $f \circ g(x) + \frac{9}{20} = g \circ f(x)$.

4. $(f \circ g)(x) = (g \circ f)(x) = x$ हुन्छ भनी देखाउनुहोस् । (Show that $(f \circ g)(x) = (g \circ f)(x) = x$.)
A. $f(x) = 2x - 6$ and $2g(x) = x + 6$ **B.** $f(x) = 4 - 3x$ and $3g(x) = 4 - x$
C. $f(x) = ax + b$ and $a\,g(x) = x - b$ **D.** $f(x) = 3x - 4$ and $3g(x) = 4 + x$

5A. यदि $f(x) = x^2 + x + 2$ र $g(x) = 2x^2 - 7$ भए प्रमाणित गर्नुहोस् : $2\,f \circ g(2) = f(2)\,g(2)$
If $f(x) = x^2 + x + 2$ and $g(x) = 2x^2 - 7$ then prove that: $2\,f \circ g(2) = f(2)\,g(2)$.

5B. यदि $f(x) = 3x^2 + 5x$ र $g(x) = 4x^2 - 5$ भए प्रमाणित गर्नुहोस् : $f \circ g(4) = 3\,f(4)\,g(4) - 1298$
If $f(x) = 3x^2 + 5x$ and $g(x) = 4x^2 - 5$ then prove that: $f \circ g(4) = 3\,f(4)\,g(4) - 1298$.

Model 2 Problems based on finding the value of letters.

6A. यदि $f(x) = \frac{6}{x-2}$ $(x \neq 2, x \in R)$, $g(x) = ax^2 - 1$ र $(g \circ f)(5) = 7$ भए a को मान पत्ता लगाउनुहोस् ।
If $f(x) = \frac{6}{x-2}$ $(x \neq 2, x \in R)$, $g(x) = ax^2 - 1$ and $(g \circ f)(5) = 7$ then find the value of a.

6B. यदि $f: R \to R : f(x) = (ax + 5)$, $(f \circ g)(x) = 8x + 13$ र $(g \circ f)(5) = 8a$ भए a को मान पत्ता लगाउनुहोस् ।
If $f: R \to R : f(x) = (ax + 5)$, $(f \circ g)(x) = 8x + 13$ and $(g \circ f)(5) = 8a$ then find the value of a.

6C. यदि $f(x) = \frac{a}{3-x}$, $g(x) = 11 - bx^2$, $f \circ f(5) = \frac{4}{5}$ र $f \circ g(2) = \frac{1}{2}$ भए a र b का मानहरू पत्ता लगाउनुहोस् ।
If $f(x) = \frac{a}{3-x}$, $g(x) = 11 - bx^2$, $f \circ f(5) = \frac{4}{5}$ and $f \circ g(2) = \frac{1}{2}$, find the values of a and b.

6D. यदि $f(x) = 2x^3 - 3x^2 + 4x - 1$ र $g(x) = 2$ भए $f \circ g(x) + g \circ f(x) = x + 3$ हुने गरी x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x^3 - 3x^2 + 4x - 1$ and $g(x) = 2$, then find the value of x such that $f \circ g(x) + g \circ f(x) = x + 3$

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on composite function and missing function.*

7A. यदि $f(x) = 3x - 2$ र $f \circ g(x) = 6x - 2$ भए $g \circ f(x) = 8$ हुँदा x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x - 2$ and $f \circ g(x) = 6x - 2$ then find the value of x such that $g \circ f(x) = 8$.

7B. यदि $f(x) = 4x + 5$ र $f \circ g(x) = 8x + 13$ भए $g \circ f(x) = 28$ हुँदा x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 4x + 5$ and $f \circ g(x) = 8x + 13$, find the value of x such that $g \circ f(x) = 28$.

7C. यदि $f(x) = 2x + 1$, $f \circ g(x) = 2x + 11$ र $g \circ f(x) = 8$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 1$, $f \circ g(x) = 2x + 11$ and $g \circ f(x) = 8$ then find the value of x.

7D. यदि $f(x) = 4x + 5$, $f \circ g(x) = 8x + 17$ र $g \circ f(x) = 21$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 4x + 5$, $f \circ g(x) = 8x + 17$ and $g \circ f(x) = 21$ then find the value of x.

7E. यदि $f(x) = 4x + 2$ र $f \circ g(x) = 8x + 14$ भए $g \circ f(x) = 18$ हुने गरी x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 4x + 2$ and $f \circ g(x) = 8x + 14$, find the value of x such that $g \circ f(x) = 18$.

8A. यदि $g(x) = 2x$ र $f \circ g(x) = 6x - 2$ भए $g \circ f(x) = 10$ हुँदा x को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x$ and $f \circ g(x) = 6x - 2$ then find the value of x such that $g \circ f(x) = 10$.

8B. यदि $g(x) = 2x + 2$ र $f \circ g(x) = 8x + 13$ भए $g \circ f(x) = 20$ हुँदा x को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x + 2$ and $f \circ g(x) = 8x + 13$, find the value of x such that $g \circ f(x) = 20$.

8C. यदि $f(x) = 3x - 2$ र $f \circ g(x + 1) = 6x + 4$ भए $g \circ f(x) = 8$ हुँदा $2x$ को मान पत्ता लगाउनुहोस ।
If $f(x) = 3x - 2$ and $f \circ g(x + 1) = 6x + 4$, find the value of $2x$ such that $g \circ f(x) = 8$.

8D. यदि $f(x) = 3x + 5$ र $f \circ g(x + 2) = 12x + 17$ भए $g \circ f(x) = 88$ हुने गरी x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 5$ and $f \circ g(x + 2) = 12x + 17$, find the value of x such that $g \circ f(x) = 88$.

ANSWERS

A. VERY SHORT QUESTIONS

2A. $g \circ f(x)$ **2B.** $h \circ g$ **2C.** $f \circ g(x)$ **3A.** function f **3B.** function g **3C.** function g **3D.** function f
4A. m **4B.** b **5A.** $\{(x, z)\}$ **5B.** $\{(m, x)\}$ **5C.** $\{(a, c)\}$ **6A.** a **6B.** 1
6C. 5 **6D.** 5 **7A.** $x + a$ **7B.** $x + b$ **8A.** $\{(2, 7)\}$ **8B.** 2 **8C.** $x + 2$
9A. $\{1, 2\}$ **9B.** $\{a, c\}$

B. SHORT QUESTIONS

1A. $\{(2, 4), (3, 5), (4, 6)\}$ **1B.** $g \circ f = \{(1, a), (2, b), (3, c)\}$ **1C.** $g \circ h = \{(1, 3), (3, 1), (4, 3)\}$
1D. $g \circ f = \{(1, 5), (0, 2), (-1, -1)\}$ **1E.** $h \circ g = \{(1, 3), (2, 4), (3, 5)\}$ **1F.** $g \circ f = \{(1, 5), (3, 6), (5, 8)\}$
1G. $f \circ g = \{(1, 5), (2, 5), (5, 2)\}$ **1H.** $f \circ g = \{(1, 0), (2, 1), (3, 4), (4, 9)\}$ **1I.** $\{(3, 3), (4, 4), (5, 5)\}$

2A. $8x - 1$ **2B.** $2x + 3$ **2C.** $15x - 9$ **2D.** $6x + 3$ **2E.** $6x + 1$ **2F.** $6x + 14$ **2G.** $\frac{3x}{5}$ **3A.** 17 **3B.** 11
3C. $3x - 5, 1$ **4A.** 7 **4B.** 37 **4C.** 3 **4D.** 29 **4E.** 17 **6A.** $12x + 13$
6B. $-\frac{1}{2}(16x^4 - 24x^2 + 3)$ **7A.** $\frac{3x}{2 - x}$ **7B.** $\frac{2}{2 + 3x}$ **8A.** $-\frac{3}{2}$ **8B.** 4 **8C.** 4 **9A.** -3
9B. 2 **10A.** $2x$ **10B.** $2x + 2$ **11A.** $f(x) = 3x - 2$ **11B.** $f(x) = 4x + 5$ **12A.** $2x + 3$ **12B.** $2x + 3$
12C. $x - 1$ **12D.** 15 **13A.** possible answer ; $f(x) = x^5, g(x) = 2x - 3$ **13B.** possible answer ; $f(x) = x^3, g(x) = 3x + 2$
14A. possible answer ; $f(x) = \frac{1}{x^3}, g(x) = x + 3$ **14B.** possible answer ; $f(x) = \frac{1}{x^3}, g(x) = x + 2$

C. LONG QUESTIONS

1A. (i) $4x + 3$ (ii) $4x^2 + 4x - 1$ (iii) $2x^2 - 3$ (iv) $x^4 - 4x^2 + 2$ **1B.** (i) $3x + 5$ (ii) $x + 6$ (iii) $-2x - 5$ (iv) $9 - 6a$
2A. (i) 98 (ii) 49 (iii) 4 (iv) 4 **2B.** (i) 95 (ii) 127 (iii) 17 (iv) 1
2C. (i) 4418 (ii) -191 (iii) 8 (iv) -2 **2D.** (i) $\frac{3}{\sqrt[3]{4} + 1}$ (ii) 1 (iii) $\frac{6}{5}$ (iv) 0
2E. (i) $\frac{1}{3\sqrt{3}}$ or $\frac{1}{9}\sqrt{3}$ (ii) $\sqrt{2} - 1$ (iii) 1 (iv) $\frac{2}{3}$ **6A.** 2 **6B.** 6 or -1 **6C.** 4, 4 **6D.** 10 **7A.** 2
7B. 2 **7C.** 1 **7D.** 1 **7E.** $\frac{11}{8}$ **8A.** $\frac{7}{3}$ **8B.** $\frac{7}{2}$ **8C.** 4 **8D.** 6

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 1.2

A. SHORT QUESTIONS

Model 1 *Problems based on composite functions in ordered pair form.*

1A. यदि $f = \{(1, 4), (2, 4), (3, 5)\}$ र $g = \{(4, 2), (5, 2)\}$ भए $g \circ f$ लाई मिलान चित्रमा देखाई क्रम जोडाहरूको समूह बनाउनुहोस् ।
If $f = \{(1, 4), (2, 4), (3, 5)\}$ and $g = \{(4, 2), (5, 2)\}$, then show the function $g \circ f$ in an arrow diagram and find it in ordered pair form.

1B. $f: A \to B$ र $g: B \to C$ लाई निम्नअनुसार परिभाषित गरिएको छ । $g \circ f$ पत्ता लगाउनुहोस् ।
$f: A \to B$ and $g: B \to C$ are defined as follows. Find $g \circ f$.
$f = \{(a, x), (b, x), (c, y)\}$ and $g = \{(x, 1), (y, 2), (z, 2)\}$

1C. यदि $f(x) = 3x + 1$ र $g(x) = \{(9, 4), (11, 6), (13, 8)\}$ भए फलन $f \circ g$ लाई क्रम जोडाको रूपमा पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 1$ and $g(x) = \{(9, 4), (11, 6), (13, 8)\}$ then find the function $f \circ g$ in ordered pair form.

1D. यदि $f(x) = x + 1$ र $g(x) = \{(3, 4), (5, 6), (7, 8)\}$ भए फलन $f \circ g$ लाई क्रम जोडाको रूपमा पत्ता लगाउनुहोस् ।
If $f(x) = x + 1$ and $g(x) = \{(3, 4), (5, 6), (7, 8)\}$ then find the function $f \circ g$ in ordered pair form.

2A. यदि $f = \{(b, q), (a, p), (c, r)\}$ र $g \circ f = \{(a, 1), (c, 3), (b, 2)\}$भए फलन g लाई क्रम जोडाको रूपमा लेख्नुहोस् ।
If $f = \{(b, q), (a, p), (c, r)\}$ and $g \circ f = \{(a, 1), (c, 3), (b, 2)\}$, find the function g in terms of ordered pairs.

2B. यदि $g = \{(a, e), (b, f), (c, h), (d, g)\}$ र $f \circ g = \{(a, j), (b, \ell), (c, k), (d, i)\}$ भए फलन f लाई क्रमजोडाहरूको रूपमा लेखी f^{-1} पनि पत्ता लगाउनुहोस् ।
If $g = \{(a, e), (b, f), (c, h), (d, g)\}$ and $f \circ g = \{(a, j), (b, \ell), (c, k), (d, i)\}$ then find the function f in ordered pair form and also find f^{-1}.

3A. यदि $f = \{(2, 3), (1, 2), (3, 4)\}$ र $g = \{(3, 4), (4, 5), (2, 3)\}$, भए $g \circ f(1)$ र $f \circ g(2)$ पत्ता लगाउनुहोस् ।
If $f = \{(2, 3), (1, 2), (3, 4)\}$ and $g = \{(3, 4), (4, 5), (2, 3)\}$, find $g \circ f(1)$ and $f \circ g(2)$.

3B. यदि $f = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$ र $g = \{(3, 4), (6, 7), (9, 10)\}$ भए $g \circ f(3)$ र $f \circ g(3)$ पत्ता लगाउनुहोस् ।
If $f = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$ and $g = \{(3, 4), (6, 7), (9, 10)\}$ then find $g \circ f(3)$ and $f \circ g(3)$.

4A. यदि $f = \{(1, 2), (2, 3), (3, 4)\}$ र $g(x) = x + 1, x \in N$ भए संयुक्त फलन $g \circ f$ लाई क्रमजोडाहरूको रूपमा लेख्नुहोस् ।
If $f = \{(1, 2), (2, 3), (3, 4)\}$ and $g(x) = x + 1, x \in N$, write the composite function $g \circ f$ in ordered pair form.

4B. यदि f एउटा एकात्मक फलन र $g = \{(1, 3), (2, 4), (3, 5)\}$ भए संयुक्त फलन $g \circ f$ लाई क्रमजोडाहरूको रूपमा लेख्नुहोस् ।
If f is an identity function and $g = \{(1, 3), (2, 4), (3, 5)\}$, write the composite function $g \circ f$ in ordered pair form.

5A. यदि $g \circ f = \{(1, 2), (2, 3), (3, 4)\}$ र $f(x) = 2x - 1$ भए फलन g लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
If $g \circ f = \{(1, 2), (2, 3), (3,4)\}$ and $f(x) = 2x - 1$, find the function 'g' in order pair form.

5B. यदि $g \circ f = \{(2, 9), (3, 13), (4, 17)\}$ र $f(x) = 2x + 1$ भए फलन g लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
If $g \circ f = \{(2, 9), (3, 13), (4, 17)\}$ and $f(x) = 2x + 1$, find the function 'g' in order pair form.

5C. $f(x) = 2x - 3$ र $g = \{(1, 2), (4, 5), (6, -2)\}$ दुई फलनहरू हुन् । संयुक्त फलन $f \circ g$ लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
$f(x) = 2x - 3$ and $g = \{(1, 2), (4, 5), (6, -2)\}$ are two functions. Write down the composite function $f \circ g$ in order pair form.

5D. यदि $f \circ g$ एउटा एकात्मक फलन र $g = \{(b, s), (m, d), (\ell, a), (a, i)\}$ भए f लाई क्रमजोडाको रूपमा पत्ता लगाउनुहोस् ।
If $f \circ g$ is an identity function and $g = \{(b, s), (m, d), (\ell, a), (a, i)\}$ then find f in ordered pair form.

Model 2 *Problems based on composite functions.*

6A. यदि $f(x) = 2x + 3$ भए $f \circ f(-1)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 3$ then find $f \circ f(-1)$.)

6B. यदि $f(x) = \frac{1}{2x+1}$ भए $f \circ f(x)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = \frac{1}{2x+1}$ then find the value of $f \circ f(x)$.)

6C. यदि $f(x) = 2x - 1$ र $g(x) = x + 1$ भए $g \circ f(x)$ र $g \circ f(-2)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 1$ and $g(x) = x + 1$, find $g \circ f(x)$ and $g \circ f(-2)$.

6D. यदि $f(x) = x^2 + 5x + 7, x \in \Re$ र $g(x) = 5x - 3, x \in \Re$ भए $f \circ g(x)$ र $g \circ f(x)$ पत्ता लगाउनुहोस् ।
$f(x) = x^2 + 5x + 7, x \in \Re$ and $g(x) = 5x - 3, x \in \Re$, find $f \circ g(x)$ and $g \circ f(x)$.

www.drmathnp.com

6E. दुईओटा फलनहरूको संयुक्त फलन परिभाषित गर्नुहोस् । यदि $f(x) = 2x$ र $g(x) = x + 2$ भए f र g को संयुक्त फलन पत्ता लगाउनुहोस् ।
Define composite function of two functions. If $f(x) = 2x$ and $g(x) = x + 2$, find the composite of the functions f and g.

6F. यदि $f(a) = \frac{a}{m}$, $g(x) = 5x - 7$ र $f[g(x] = 15x - 21$ भए m को मान पत्ता लगाउनुहोस् ।

If $f(a) = \frac{a}{m}$, $g(x) = 5x - 7$ and $f[g(x] = 15x - 21$ then find the value of m.

7A. यदि $f(x) = \frac{1+x}{1-x}$ भए $f \circ f(\tan\theta)$ पत्ता लगाउनुहोस् । (If $f(x) = \frac{1+x}{1-x}$ then find $f \circ f(\tan\theta)$.)

7B. यदि $f(x) = \frac{x+1}{x-1}$ भए $f \circ f(\sin\theta)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = \frac{x+1}{x-1}$, find the value of $f \circ f(\sin\theta)$.)

8A. यदि $f(x) = \frac{x-1}{x+1}$ भए $f \circ f(x) = -\frac{1}{x}$ हुन्छ भनी देखाउनुहोस् । (If $f(x) = \frac{x-1}{x+1}$, show that: $f \circ f(x) = -\frac{1}{x}$.)

8B. यदि $f(x) = \sqrt[n]{a - x^n}$; $x \in \mathbf{N}$ भए $f \circ f(x) = x$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \sqrt[n]{a - x^n}$; $x \in \mathbf{N}$ then prove that: $f \circ f(x) = x$

9A. यदि $f(x) = x + 2$ र $g(x + 1) = x + 1$ भए $g \circ f(4)$ पत्ता लगाउनुहोस् ।
If $f(x) = x + 2$ and $g(x + 1) = x + 1$ then find $g \circ f(4)$.

9B. यदि $f(x) = x + 2$ र $g(x + 2) = x + 2$ भए $g \circ f(8)$ पत्ता लगाउनुहोस् ।
If $f(x) = x + 2$ and $g(x + 2) = x + 2$ then find $g \circ f(8)$.

10A. यदि $f(4x - 15) = 8x - 27$ भए $f \circ f(2)$ पत्ता लगाउनुहोस् । (If $f(4x - 15) = 8x - 27$, find $f \circ f(2)$.)

10B. यदि $g(2x - 5) = 8x - 19$ भए $g \circ g(2)$ पत्ता लगाउनुहोस् । (If $g(2x - 5) = 8x - 19$, find $g \circ g(2)$.)

11A. यदि $f(3x - 1) = -6x + 3$ भए $f \circ f(2)$ पत्ता लगाउनुहोस् । (If $f(3x - 1) = -6x + 3$, find $f \circ f(2)$.)

11B. यदि $f(2x + 5) = 6x + 19$ भए $f \circ f(2)$ पत्ता लगाउनुहोस् । (If $f(2x + 5) = 6x + 19$, find $f \circ f(2)$.)

Model 3 *Problems on finding value of x or a or r when the value of composite function is given.*

12A. यदि $k(x) = \frac{x}{r}$ र $g(x) = 2x + 5$ भए r को मान कति हुँदा $3[kg(x)] = 2x + 5$ हुन्छ, पत्ता लगाउनुहोस् ।

If $k(x) = \frac{x}{r}$ and $g(x) = 2x + 5$ then what value of r makes $3[kg(x)] = 2x + 5$?

12B. यदि $f(x) = ax + 3$ र $f \circ f(x) = a^2x$ भए a को मान पत्ता लगाउनुहोस् ।
If $f(x) = ax + 3$ and $f \circ f(x) = a^2x$ then find the value of a.

12C. यदि $f(x) = 2x + 5a - 1$ र $f \circ f(3) = 4$ भए 'a' को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 5a - 1$ and $f \circ f(3) = 4$, find the value of 'a'.

Model 4 *Problems based on identity function & constant function.*

13A. यदि $f(x) = \frac{1}{1-x}$ भए $f \circ f(x)$ एउटा एकात्मक फलन हुदैन भनी देखाउनुहोस् ।

If $f(x) = \frac{1}{1-x}$ then show that $f \circ f(x)$ is not an identity function.

13B. यदि $f(x) = 4x + 5$ र $g(x) = 15$ भए $f \circ g(x)$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 4x + 5$ and $g(x) = 15$ then prove that $f \circ g(x)$ is a constant function.

13C. यदि $f(x) = 3$ र $g(x) = 2x - 1$ भए $f \circ g(x)$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 3$ and $g(x) = 2x - 1$ then prove that $f \circ g(x)$ is a constant function.

13D. यदि एउटा फलन $f(x) = 5$ भए $f(4)$ र $f \circ f(-5)$ पत्ता लगाउनुहोस् ।
If a function $f(x) = 5$ then find $f(4)$ and $f \circ f(-5)$.

pustas.com.np

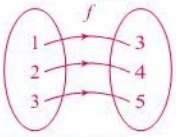
www.drmathnp.com

puspas.com.np

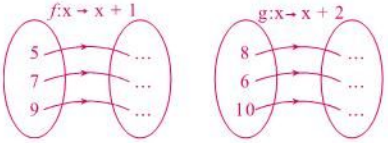
Model 5 *Problems based on composite function and inverse function (diagram form).*

14. संयुक्त फलन $g \circ f$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।
Show the function $g \circ f$ in the arrow diagram and find it in ordered pair form.

A.



B.



B. LONG QUESTIONS

Model 1 *Problems based on finding composite function of given algebraic functions.*

1A. यदि $f(x) = 2x + 3$ र $g \circ f(x) = 2x - 1$ भए $f \circ g(2)$ र $g \circ f(2)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$, $g \circ f(x) = 2x - 1$ then find $f \circ g(2)$ and $g \circ f(2)$.

1B. यदि $f(x) = x^2$ र $g \circ f(x) = x^2 - 3$ भए $f \circ g(2)$ र $g \circ f(2)$ पत्ता लगाउनुहोस् ।
If $f(x) = x^2$ and $g \circ f(x) = x^2 - 3$ then find $f \circ g(2)$ and $g \circ f(2)$.

1C. यदि $f, g : \Re \to \Re$ लाई क्रमशः $f(x) = x^3 + 2$ र $g(x) = 4x - 1$ द्वारा परिभाषित गरिए $f \circ g(x)$ र $g \circ f(x)$ पत्ता लगाउनुहोस् । साथै यी संयुक्त फलनहरू क्रम विनिमय हुँदैनन् भनी प्रमाणित गर्नुहोस् ।
If $f, g : \Re \to \Re$ defined by, $f(x) = x^3 + 2$ and $g(x) = 4x - 1$, find $f \circ g(x)$ and $g \circ f(x)$, and show that the composite functions are not commutative.
Hint: *$f \circ g(x) \neq g \circ f(x)$ shows that the composite functions are not commutative.*

Model 2 *Problems based on finding value of letters.*

2A. यदि $f(x) = 3x + k$, $g(x) = \dfrac{4x + m}{2x + 1}$, $f \circ f(2) = 30$ र $g \circ f(1) = 2$ भए m र k का मानहरू पत्ता लगाउनुहोस् ।
If $f(x) = 3x + k$, $g(x) = \dfrac{4x + m}{2x + 1}$, $f \circ f(2) = 30$ and $g \circ f(1) = 2$ then find the values of m and k.

2B. यदि $f(x) = kx + 3$, $g(x) = \dfrac{4x + 2}{2x + m}$, $f \circ f(5) = 57$ र $g \circ f(4) = 2$ भए m र k का मानहरू पत्ता लगाउनुहोस् ।
If $f(x) = kx + 3$, $g(x) = \dfrac{4x + 2}{2x + m}$, $f \circ f(5) = 57$ and $g \circ f(4) = 2$ then find the values of m and k.

3A. यदि $f(x) = 2x^2 + 5$ र $g(x) = 3x + a$ भए $f \circ g(x)$ को लेखाचित्र y-अक्षको 23 बाट पार हुने गरी a को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find 'a' so that the graph of $f \circ g(x)$ crosses the y-axis at 23.

3B. यदि $f(x) = 3x^2 - 7$ र $g(x) = 2x + a$ भए $f \circ g(x)$ को लेखाचित्र y-अक्षको 68 बाट पार हुने गरी a को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find 'a' so that the graph of $f \circ g(x)$ crosses the y-axis at 68.

4A. फलन $f(x + 2) = 3x + 5$ दिइएको छ । यदि $f[g(x) + 2] = x + 3$ भए $g(8)$ पत्ता लगाउनुहोस् ।
Function $f(x + 2) = 3x + 5$ is given. If $f[g(x) + 2] = x + 3$, find $g(8)$.

4B. फलन $f(x - 1) = 3x - 4$ दिइएको छ । यदि $f[g(x) + 5] = 3x + 2$ भए $g(8)$ पत्ता लगाउनुहोस् ।
Function $f(x - 1) = 3x - 4$ is given. If $f[g(x) + 5] = 3x + 2$, find $g(8)$.

ANSWERS

A. SHORT QUESTIONS

1A. $\{(1, 2), (2, 2), (3, 2)\}$ **1B.** $\{(a, 1), (b, 1), (c, 2)\}$ **1C.** $\{(9, 13), (11, 19), (13, 25)\}$
1D. $\{(3, 5), (5, 7), (7, 9)\}$ **2A.** $\{(p, 1), (q, 2), (r, 3)\}$ **2B.** $f = \{(e, j), (f, \ell), (h, k), (g, i)\}$, $f^{-1} = \{(j, e), (\ell, f), (k, h), (i, g)\}$
3A. 3, 4 **3B.** 10, 12 **4A.** $\{(1, 3), (2, 4), (3, 5)\}$ **4B.** $\{(1, 3), (2, 4), (3, 5)\}$
5A. $g = \{(1, 2), (3, 3), (5, 4)\}$ **5B.** $g = \{(5, 9), (7, 13), (9, 17)\}$ **5C.** $\{(1, 1), (4, 7), (6, -7)\}$ **5D.** $\{(s, b), (d, m), (a, \ell), (i, a)\}$
6A. 5 **6B.** $\dfrac{2x + 1}{2x + 3}$ **6C.** $2x, -4$ **6D.** $(25x^2 - 5x + 1), 5x^2 + 25x + 32$ **6E.** $2x + 2$ **6F.** $\dfrac{1}{3}$ **7A.** $-\cot\theta$
7B. $\sin\theta$ **9A.** 6 **9B.** 10 **10A.** 17 **10B.** 37 **11A.** 7 **11B.** 34 **12A.** 3
12B. 0 **12C.** $-1/3$ **13D.** 5, 5 **14A.** $\{(1, 7), (2, 8), (3, 9)\}$ **14B.** $\{(5, 8), (7, 10), (9, 12)\}$

B. LONG QUESTIONS

1A. $-1, 3$ **1B.** 1, 1 **1C.** $64x^3 - 48x^2 + 12x + 1$, $4x^3 + 7$ **2A.** 2, 3 **2B.** 1, 3 or -3.6
3A. ± 3 **3B.** ± 5 **4A.** 2 **4B.** 4

www.drmathnp.com

1.3 विपरीत फलन INVERSE FUNCTION

DEFINITIONS

1. एक एक सम्पूर्ण फलन (One to one onto function)

फलनमा सहक्षेत्र र विस्तार क्षेत्र बराबर छ र विस्तारका प्रत्येक सदस्यको एकल पूर्व प्रतिबिम्ब भएको फलनलाई एक एक सम्पूर्ण फलन भनिन्छ ।

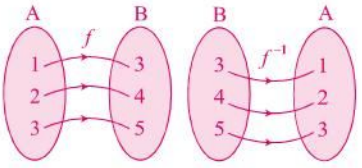
In a function, if the codomain is equal to range and each element of range has only one pre-image in domain then the function is known as one to one onto function.

2. विपरीत फलन (Inverse Function) [2065 S, 2067 R]

यदि $f: A \to B$ एक एक सम्पूर्ण फलन भए B बाट A मा परिभाषित गरिएको फलनलाई f को विपरीत फलन f^{-1} भनिन्छ ।

If f is a one to one onto function from A to B then the inverse function f^{-1} is defined as a function from B to A. For example:

If $f = \{(1, 3), (2, 4), (3, 5)\}$, $f^{-1} = \{(3, 1), (4, 2), (5, 3)\}$



3. एक-एक सम्पूर्ण फलनको विपरीत फलन पत्ता लगाउने तरिका

Method of finding inverse function of one to one onto function

Step 1	$y = f(x)$ मा x र y को स्थान बदल्नुहोस् । (Interchange the position of x and y in $y = f(x)$.)
Step 2	हल गरी y को मान पत्ता लगाउनुहोस् । (Find the value of y by solving.)
Step 3	$\therefore f^{-1}(x) = (y$ को मान) हुन्छ । ($\therefore f^{-1}(x) =$ (the value of y))

4. विपरीत फलन परीक्षण गर्ने (To check inverse function): $f^{-1}[f(x)] = x$ and $f[f^{-1}(x)] = x$

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 2	2067 R, 2068 R', 2069 S, 2070 S', 2071 R, 2071 S, 2072 R', 2073 R', SEE 2075 R, SEE 2075 R'$_2$, SEE MODEL 2076	10

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 2	2065 S, 2067 R', 2068 R, 2068 R', 2073 S, SEE 2075 R'$_2$	6
Model 3	2065 R, 2065 M, 2066 R, R', S, 2067 R, S, 2068 S, S', 2069 R, R', S, 2070 R', S, S', 2071 R', 2072 R, S, 2073 R, R', 2074 R', S, S', SEE 2075 R'	21
Model 4	2065 R', SEE 2074 R	2
Model 5	2070 R, 2071 R, 2071 S, SEE 2073 S', SEE 2075 R, SEE 2075 R$_2$	5

SEE EXERCISE 1.3

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरू उदाहरणसहित परिभाषीत गर्नुहोस् । (Define the following terms with an example.)

A. एक एक सम्पूर्ण फलन (One to one onto function) **B.** विपरीत फलन (Inverse Function)

2A. कस्तो अवस्थामा कुनै फलनको विपरीत फलन सम्भव हुन्छ ?
Under what condition, the inverse function of a function is possible?

2B. एकात्मक फलन र यसको विपरीतको सम्बन्ध के हुन्छ ?
What is the relation between an identity function and its inverse ?

3A. यदि $f = \{(a, b), (x, y)\}$ भए f को विपरीत फलन पत्ता लगाउनुहोस् ।
If $f = \{(a, b), (x, y)\}$ then find the inverse function of f.

3B. यदि $f^{-1} = \{(x, y), (c, d)\}$ भए फलन f पत्ता लगाउनुहोस् । (If $f^{-1} = \{(x, y), (c, d)\}$ then find the function f.)

Model 2 *Concept Recall*

4A. यदि $f: R \to R$ र $g: R \to R$ एक अर्काका विपरीत फलनहरू भए, प्रत्येक $x \in R$ को लागि $f \circ g(x)$ कति हुन्छ ?
If $f: R \to R$ and $g: R \to R$ are inverse to each other then for each $x \in R$, what is the value of $f \circ g(x)$?

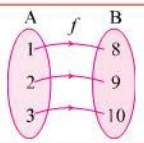
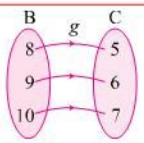
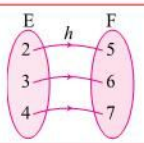
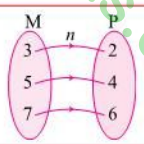
4B. यदि $f: R \to R$ र $g: R \to R$ एक अर्काका विपरीत फलनहरू भए, प्रत्येक $a \in R$ को लागि $g \circ f(a)$ कति हुन्छ ?
If $f: R \to R$ and $g: R \to R$ are inverse to each other then for each $a \in R$, what is the value of $g \circ f(a)$?

pustakalaya.com.np

www.drmathnp.com

puspas.com.np

5. तलका फलनहरूको विपरीत फलन पत्ता लगाउनुहोस् । (Find the inverse function of the following functions.)

A.	B.	C.	D.
			

6A. यदि $f = \{(2, -4)\}$, भए f^{-1} कति हुन्छ ? (If $f = \{(2, -4)\}$, what is f^{-1}?)

6B. यदि $g = \{(a, b)\}$, भए g^{-1} कति हुन्छ ? (If $g = \{(a, b)\}$, what is g^{-1}?)

7A. यदि $f(x) = x + a$, भए $f^{-1}(x)$ कति हुन्छ ? (If $f(x) = x + a$, what is $f^{-1}(x)$?)

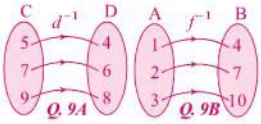
7B. यदि $p(x) = 2x$ भए $p^{-1}(x)$ कति हुन्छ ? (If $p(x) = 2x$, what is $p^{-1}(x)$?)

7C. यदि $f(x) = x + 2$, भए $f^{-1}(x)$ कति हुन्छ ? (If $f(x) = x + 2$, what is $f^{-1}(x)$?)

7D. यदि $g(x) = \frac{x}{2}$, भए $g^{-1}(x)$ कति हुन्छ ? (If $g(x) = \frac{x}{2}$, what is $g^{-1}(x)$?)

8. तलका फलनहरूको विपरीत फलन पत्ता लगाउनुहोस् । (Find the inverse functions of following functions.)
(a) $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$ (b) $g = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$

9A. चित्रमा एउटा विपरीत फलन d^{-1} लाई देखाइएको छ । d लाई क्रम जोडाहरूको रूपमा लेख्नुहोस् ।
In the figure, an inverse function d^{-1} is shown. Write down d as a set of ordered pairs.

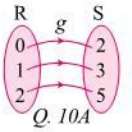


Q. 9A

Q. 9B

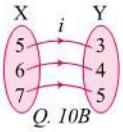
9B. चित्रमा एउटा विपरीत फलन f^{-1} लाई देखाइएको छ । f लाई क्रम जोडाहरूको रूपमा लेख्नुहोस् ।
In the figure, an inverse function f^{-1} is shown. Write down f as a set of ordered pairs.

10A. संगैको मिलान चित्रबाट $g^{-1}\{3, 5\}$ पत्ता लगाउनुहोस् ।
From the adjoining mapping diagram, find $g^{-1}\{3, 5\}$.



Q. 10A

10B. संगैको मिलान चित्रबाट $i^{-1}\{3\}$ पत्ता लगाउनुहोस् ।
From the adjoining mapping diagram, find $i^{-1}\{3\}$.



Q. 10B

B. SHORT QUESTIONS

Model 1 *Problems of inverse function based on ordered pairs .*

1A. फलन $f = \left\{\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right)\right\}$ को विस्तार क्षेत्र र विपरीत फलन लेख्नुहोस् ।

Write the range and inverse function of the function $f = \left\{\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right)\right\}$. [2061 R]

1B. यदि $f = \{(1, 2), (2, 3), (4, 5)\}$ भए f^{-1} पत्ता लगाई f^{-1} को क्षेत्र र विस्तार पत्ता लगाउनुहोस् ।
If $f = \{(1, 2), (2, 3), (4, 5)\}$, find f^{-1} and the domain and range of f^{-1}.

2A. यदि फलन $f(x) = 5x - 4$ को क्षेत्र $\{-1, 0, 1\}$ भए विस्तार क्षेत्र पत्ता लगाई विपरीत फलनलाई क्रमजोडामा देखाउनुहोस् ।
If $f(x) = 5x - 4$, where domain $= \{-1, 0, 1\}$ then find the range. Also find the inverse of $f(x)$ in ordered pair form.

2B. यदि $A = \{0, 1, 2, 3\}$, $B = \{10, 13, 16, 19\}$ र $f: A \to B$ एउटा फलन हो जहाँ $f(0) = 10$, $f(1) = 13$, $f(2) = 16$ र $f(3) = 19$ भए $f^{-1}: B \to A$ लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
If $A = \{0, 1, 2, 3\}$, $B = \{10, 13, 16, 19\}$ and $f: A \to B$ is a function such that $f(0) = 10$, $f(1) = 13$, $f(2) = 16$ and $f(3) = 19$, write down $f^{-1}: B \to A$ as the set of ordered pairs.

Model 2 *Problems of inverse function based on algebraic form.*

3A. यदि $f(x) = 4x + 5$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् । (Find $f^{-1}(x)$ if $f(x) = 4x + 5$.) [SEE MODEL 2076]

3B. $f: R \to R$ का लागि $f(x) = 2x - 1$ भए $f^{-1}(3)$ पत्ता लगाउनुहोस् । (For, $f: R \to R$, $f(x) = 2x - 1$, find $f^{-1}(3)$.)

3C. यदि $f(x) = 3x - 11$ भए, $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 3x - 11$, find the value of $f^{-1}(4)$.) [2068 R']

3D. यदि $g(x) = 4x - 2$ भए $g^{-1}(-1)$ को मान पत्ता लगाउनुहोस् । (If $g(x) = 4x - 2$, find the value of $g^{-1}(-1)$.) [2069S]

3E. यदि $f(x) = 2x + 3$ भए $f^{-1}(4)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 3$ then find $f^{-1}(4)$.) [2070 S']

3F. यदि $f(x) = 4x + 3$ भए $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् । (If $f(x) = 4x + 3$ then find the value of $f^{-1}(4)$.) [2072 R']

3G. विपरित फलनको परिभाषा दिनुहोस् । यदि $f(x) = 2x - 1$ भए $ff(-1)$ पत्ता लगाउनुहोस् ।
Define inverse function. If $f(x) = 2x - 1$, find $ff(-1)$. [SEE 2075 R, 2067 R]

www.drmathnp.com
puspas.com.np

4. तलका फलनहरू एक एक सम्पूर्ण फलनहरू हुन् । विपरीत फलन पत्ता लगाउनुहोस् ।
The following functions are one to one onto functions. Find the inverse function.

A. $f(x)=\frac{2}{3+x}$ **B.** $f(x)=\frac{4}{2-x}$ **C.** $f(x)=\frac{3x}{x+2}$ **D.** $f(x)=-\frac{2x}{x-1}$

E. $f(x)=\frac{2x}{3x-1}$ **F.** $f(x)=\frac{3x+1}{-x}$ **G.** $f(x)=\frac{3x+4}{2x-3}$ **H.** $f(x)=\frac{2x-3}{x+4}$

I. $f(x)=\frac{2x+3}{x+2}$ **J.** $f(x)=\frac{-3x-4}{x-2}$ **K.** $f(x)=\frac{x^2-4}{2x^2}, x>0$ **L.** $f(x)=\frac{x^2+3}{3x^2}, x>0$

5A. यदि $f(x)=\frac{2x+3}{2}$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् । (If $f(x)=\frac{2x+3}{2}$, then find the value of $f^{-1}(x)$.) **[SEE 2073 R']**

5B. यदि $f(x)=\frac{2x-3}{5}$ भए $f^{-1}\left(\frac{1}{5}\right)$ को मान पत्ता लगाउनुहोस् । (If $f(x)=\frac{2x-3}{5}$, find the value of $f^{-1}\left(\frac{1}{5}\right)$.) **[SEE 2075 R'$_2$]**

6A. यदि $f^{-1}(x)=\frac{x+3}{2}$ भए $f(x)$ को मान पत्ता लगाउनुहोस् । (If $f^{-1}(x)=\frac{x+3}{2}$, find $f(x)$.) **[2071 R]**

6B. यदि $f^{-1}(x)=2x-3$ भए $f(x)$ पत्ता लगाउनुहोस् । (If $f^{-1}(x)=2x-3$ then find $f(x)$.) **[2071 S]**

Model 3 ***Problems based on finding the value of x when inverse function is given.***

7A. यदि $f(x)=\frac{3x+4}{5}$ एउटा एक एक सम्पूर्ण फलन हो र $f^{-1}(x)=12$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x)=\frac{3x+4}{5}$ is one to one onto function and $f^{-1}(x)=12$ then find the value of x.

7B. यदि $g(x)=\frac{2x-7}{3}$ एउटा एक एक सम्पूर्ण फलन हो र $g^{-1}(x)=8$ भए x को मान पत्ता लगाउनुहोस् ।
If $g(x)=\frac{2x-7}{3}$ is one to one onto function and $g^{-1}(x)=8$ then find the value of x.

7C. यदि $g(x)=\frac{3-x}{x}$ एउटा एक एक सम्पूर्ण फलन हो र $g^{-1}(x)=1$ भए x को मान पत्ता लगाउनुहोस् ।
If $g(x)=\frac{3-x}{x}$ is one to one onto function and $g^{-1}(x)=1$ then find the value of x.

C. LONG QUESTIONS

Model 1 ***Problems based on inverse function.***

1A. फलन $f(x)=\frac{2x+1}{x-1}, x\neq 1$ को विपरीत फलन निकाल्नुहोस् । $f^{-1}[f(x)]=x$ र $f[f^{-1}(x)]=x$ परीक्षण गर्नुहोस् ।
Find the inverse of $f(x)=\frac{2x+1}{x-1}, x\neq 1$. Check the result $f^{-1}[f(x)]=x$ and $f[f^{-1}(x)]=x$.

1B. फलन $f(x)=\frac{2x+3}{x+2}, x\neq -2$ को विपरीत फलन निकाल्नुहोस् । $f^{-1}[f(x)]=x$ र $f[f^{-1}(x)]=x$ परीक्षण गर्नुहोस् ।
Find the inverse of $f(x)=\frac{2x+3}{x+2}, x\neq -2$. Check the result $f^{-1}[f(x)]=x$ and $f[f^{-1}(x)]=x$.

2. $f[g(x)]=x$ र $g[f(x)]=x$ देखाई f र g आपसमा विपरीत फलन हुन् भनी प्रमाणित गर्नुहोस् ।
Prove that f and g are inverse to each other by showing $f[g(x)]=x$ and $g[f(x)]=x$

A. $f(x)=\frac{2x+3}{x+4}$; $g(x)=\frac{4x-3}{2-x}$ **B.** $f(x)=\frac{x-5}{2x+3}$; $g(x)=\frac{3x+5}{1-2x}$

3A. यदि फलन $f:\mathrm{R}\to\mathrm{R}: f(x)=4x-3$ र $g:\mathrm{R}\to\mathrm{R}: g(x)=\frac{x+2}{5}$ भए $(f^{-1}\circ g^{-1})(2)$ को मान पत्ता लगाउनुहोस् ।
If function $f:\mathrm{R}\to\mathrm{R}: f(x)=4x-3$ and $g:\mathrm{R}\to\mathrm{R}: g(x)=\frac{x+2}{5}$ then find the value of $(f^{-1}\circ g^{-1})(2)$.

3B. यदि $f:\mathrm{R}\to\mathrm{R}: f(x)=\frac{x+2}{3}$ र $g:\mathrm{R}\to\mathrm{R}: g(x)=x-5$ भए $(f^{-1}\circ g^{-1}(1))$ को मान पत्ता लगाउनुहोस् ।
If function $f:\mathrm{R}\to\mathrm{R}: f(x)=\frac{x+2}{3}$ and $g:\mathrm{R}\to\mathrm{R}: g(x)=x-5$ then find the value of $(f^{-1}\circ g^{-1}(1))$.

www.drmathnp.com

puspas.com.np

Model 2 *Problems based on finding inverse function and composite function.*

4A. यदि $f(x) = 2x + 7$ भए $f(x+2)$ र $f^{-1}(x+2)$ पत्ता लगाउनुहोस् । (If $f(x) = 2x + 7$, find $f(x+2)$ and $f^{-1}(x+2)$.) [2067 R']

4B. यदि $f(x) = 8 - 3x$ भए $f^{-1}(-4)$ र $f \circ f(2)$ को मान निकाल्नुहोस् । (If $f(x) = 8 - 3x$, evaluate $f^{-1}(-4)$ and $f \circ f(2)$.) [2057 S]

4C. यदि $f(x) = 4x - 2$ र $g(x) = \frac{1}{x}$ भए $f^{-1}(6)$ र $g \circ f(2)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 4x - 2$ and $g(x) = \frac{1}{x}$, find the value of $f^{-1}(6)$ and $g \circ f(2)$. [2057 R]

4D. $f(x) = 2x - 3$ र $g(x) = x^2 + 2$ भए पत्ता लगाउनुहोस् : (i) $f^{-1}(-2)$ (ii) $f \circ g(x)$
If $f(x) = 2x - 3$ and $g(x) = x^2 + 2$, find : (i) $f^{-1}(-2)$ (ii) $f \circ g(x)$ [2060 S]

4E. यदि $f(x) = 2x - 5$ र $g(x) = 3x + 1$ दुई फलनहरू भए $f^{-1}(x)$ र $gf^{-1}(5)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 5$ and $g(x) = 3x + 1$ are two functions, find $f^{-1}(x)$ and $gf^{-1}(5)$. [2068 R']

4F. यदि $f(x) = 2x + 1$ र $g(x) = \frac{x-5}{2}$ भए $f^{-1}g^{-1}(3)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + 1$, $g(x) = \frac{x-5}{2}$ then find the value of $f^{-1}g^{-1}(3)$. [2065S]

4G. यदि फलन $f : R \to R$, $f(x) = 4x - 3$ र $g : R \to R$; $g(x) = \frac{x+2}{5}$ भए $(f^{-1} \circ g^{-1})(2)$ को मान पत्ता लगाउनुहोस् ।

If $f : R \to R$; $f(x) = 4x - 3$ and $g: R \to R$; $g(x) = \frac{x+2}{5}$ then find the value of $(f^{-1} \circ g^{-1})(2)$.

4H. यदि 3. $f(x) = 4x + 5$ र $g(x) = 5x - 4$ भए $f^{-1} \circ g^{-1}(1)$ को मान निकाल्नुहोस् ।
If 3. $f(x) = 4x + 5$ and $g(x) = 5x - 4$, find the value of $f^{-1} \circ g^{-1}(1)$. [2068R]

4I. यदि $f(x) = \frac{1}{x}, x \neq 0$ भए प्रमाणित गर्नुहोस् $f \circ f^{-1} = f^{-1} \circ f$. [If $f(x) = \frac{1}{x}, x \neq 0$ then prove that: $f \circ f^{-1} = f^{-1} \circ f$.] [2060S]

5A. यदि $f(x) = 1 + 2x$ र $g(x) = \frac{1}{1-x}$ भए $g^{-1}\left(\frac{1}{2}\right)$ र $f \circ g(-1)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 1 + 2x$ and $g(x) = \frac{1}{1-x}$, find the value of $g^{-1}\left(\frac{1}{2}\right)$ and $f \circ g(-1)$. [2058 R]

5B. यदि फलन $f(x) = \frac{2x+3}{x+2}$ र $g(x) = x - 2$ भए $f^{-1}(x)$, $f^{-1}(1)$, $f \circ g(x)$ र $f \circ g(1)$ को मान निकाल्नुहोस् ।

If function $f(x) = \frac{2x+3}{x+2}$ & $g(x) = x - 2$, find the values of $f^{-1}(x)$, $f^{-1}(1)$, $f \circ g(x)$ & $f \circ g(1)$. [2059 S]

6A. यदि $f(x) = \frac{x+3}{2}$ र $g(x) = 2x - 3$ भए $f^{-1} \circ f(x) = f \circ g(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{x+3}{2}$ and $g(x) = 2x - 3$, prove that $f^{-1} \circ f(x) = f \circ g(x)$. [2073 S]

6B. यदि $f(x) = 3x + 4$ र $g(x) = 2(x+1)$ भए प्रमाणित गर्नुहोस् $f \circ g = g \circ f$ र $f^{-1}(2)$ को मान निकाल्नुहोस् ।
If $f(x) = 3x + 4$ and $g(x) = 2(x + 1)$ then, prove that $f \circ g = g \circ f$ and find the value of $f^{-1}(2)$. [2060 R]

6C. यदि $f(x) = 2x - 3, x \in \Re$ भए $fof^{-1}(x) = f^{-1} \circ f(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 2x - 3, x \in \Re$, prove that $fof^{-1}(x) = f^{-1} \circ f(x)$. [2062 K]

6D. यदि $g(x) = \frac{3x-1}{x-1}, x \neq 1$ र $h(x) = \frac{2x-1}{x-3}, x \neq 3$ भए $(g \circ h)^{-1}x$ र $(h \circ g)^{-1}x$ पत्ता लगाउनुहोस् ।

If $g(x) = \frac{3x-1}{x-1}, x \neq 1$ and $h(x) = \frac{2x-1}{x-3}, x \neq 3$, find $(g \circ h)^{-1}x$ and $(h \circ g)^{-1}x$. [2075 R'$_2$]

Model 3 *On finding the value of letters (Inverse function and composite function).*

7A. फलन $f(x) = \frac{x}{2x-3}$ र $f(x) = f^{-1}(x)$ भए x को मान निकाल्नुहोस् ।

If $f(x) = \frac{x}{2x-3}$ and $f(x) = f^{-1}(x)$, find the value of x. [2065 M, 2068 S]

7B. यदि $f(x) = \frac{x}{3+2x}$ र $f(x) = f^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = \frac{x}{3+2x}$ and $f(x) = f^{-1}(x)$, find the value of x. [2069 R]

www.drmathnp.com

8A. यदि $f(x) = 12 - 3x$ र $f^{-1}(x) = 2$ भए x र $f \circ f^{-1}(P)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = 12 - 3x$ and $f^{-1}(x) = 2$, find the value of x and $f \circ f^{-1}(P)$. **[2066 S]**

8B. यदि $f(x) = 3x + 4$ र $g(x) = x + 1$ भए x को मान कति हुँदा $f^{-1}g(x) = 2$ हुन्छ ?
If $f(x) = 3x + 4$ and $g(x) = x + 1$, for what value of x, $f^{-1}g(x) = 2$? **[2067 R]**

9A. यदि $f(x) = 4x + 7$, $g(x) = 3x - 5$ र $f \circ g^{-1}(x) = 15$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 4x + 7$, $g(x) = 3x - 5$ and $f \circ g^{-1}(x) = 15$, find the value of x. **[2061 S]**

9B. यदि $f(x) = x^2 - 2x$, $g(x) = 2x + 3$ र $f \circ g^{-1}(x) = 3$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = x^2 - 2x$, $g(x) = 2x + 3$ and $f \circ g^{-1}(x) = 3$ then find the value of 'x'. **[2061 R, 2065 R, 2068 S']**

9C. फलनहरू $f(x) = 4x - 1$ र $g(x) = 2x - 3$ दिइएका छन् । यदि $g \circ f^{-1}(x) = x + 1$ भए x को मान निकाल्नुहोस् ।
It is given that the functions $f(x) = 4x - 1$ and $g(x) = 2x - 3$. If $g \circ f^{-1}(x) = x + 1$, find the value of x. **[SEE 2074 S']**

9D. यदि फलन $f(x) = 3x - 7$, र फलन $g(x) = \frac{5x+2}{3}$ दिइएको छ । यदि $g^{-1} f(x) = 8$ भए x को मान पत्ता लगाउनुहोस् ।
Give that the function $f(x) = 3x - 7$ and the function $g(x) = \frac{5x+2}{3}$. If $g^{-1} f(x) = 8$, find the value of x. **[2072 S]**

9E. यदि $f(x) = \frac{x+1}{2}$, $g(x) = \frac{x-5}{2}$ र $f \circ g^{-1}(x) = 6$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = \frac{x+1}{2}$, $g(x) = \frac{x-5}{2}$ and $f \circ g^{-1}(x) = 6$, find the value of x. **[SEE 2074 S, 2072 R]**

9F. यदि $f(x) = 3x - 7$, $g(x) = \frac{4x-2}{3}$ र $f^{-1}(x) = g(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x - 7$, $g(x) = \frac{4x-2}{3}$ and $f^{-1}(x) = g(x)$, find the value of x. **[2071 R']**

10A. यदि $f(x) = 2x - 5$, $g(x) = \frac{3x+5}{2}$ र $ff(x) = g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 5$, $g(x) = \frac{3x+5}{2}$ and $ff(x) = g^{-1}(x)$, then find the value of x. **[2075 R']**

10B. यदि $f = \{x, 5x - 13\}$, $g = \left\{x, \frac{2x+7}{3}\right\}$ र $g^{-1}(x) = f \circ f(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f = \{x, 5x - 13\}$, $g = \left\{x, \frac{2x+7}{3}\right\}$ र $g^{-1}(x) = f \circ f(x)$, find the value of x. **[SEE 2073 R]**

10C. फलनहरू $f(x) = \frac{4x-1}{2}$ र $g(x) = 3x - 1$ छन् । यदि $f \circ g(x) = f^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।
$f(x) = \frac{4x-1}{2}$ and $g(x) = 3x - 1$ are the functions. If $f \circ g(x) = f^{-1}(x)$, find the value of x. **[2070 S']**

10D. यदि $f(x) = 4x - 17$, $g(x) = \frac{2x+8}{5}$ र $f \circ f(x) = g^{-1}(x)$ भए, x को मान निकाल्नुहोस् ।
If $f(x) = 4x - 17$, $g(x) = \frac{2x+8}{5}$ and $f \circ f(x) = g^{-1}(x)$, find the value of x. **[2059 R]**

10E. फलन $f(x) = \frac{x-2}{2x+1}, x \neq -\frac{1}{2}$ र $g(x) = \frac{1}{x}, x \neq 0$ दिइएका छन् । यदि $f^{-1}(x) = g \circ f(x)$ भए x को मान पत्ता लगाउनुहोस् ।
Function $f(x) = \frac{x-2}{2x+1}, x \neq -\frac{1}{2}$ and $g(x) = \frac{1}{x}, x \neq 0$ are given. If $f^{-1}(x) = g \circ f(x)$, find the value of x. **[2062 S]**

10F. यदि $f(x) = 3x - 7$, $g(x) = \frac{x+2}{5}$ र $g^{-1} \circ f(x) = f(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x - 7$, $g(x) = \frac{x+2}{5}$ and $g^{-1} \circ f(x) = f(x)$, find the value of x. **[2066 R]**

10G. यदि $f(x) = 2x + 3$, $g(x) = \frac{x+5}{2}$ र $ff(x) = g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$, $g(x) = \frac{x+5}{2}$ and $ff(x) = g^{-1}(x)$, find the value of x. **[2070 R']**

10H. $f(x) = 2x + 1$ र $g(x) = \frac{3x+1}{2}$ दुई फलनहरू हुन् । यदि $ff(x) = g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।
$f(x) = 2x + 1$ and $g(x) = \frac{3x+1}{2}$ are two functions. If $ff(x) = g^{-1}(x)$, find the value of x. **[2069 S]**

puspas.com.np

www.drmathnp.com

puspas.com.np

10I. यदि $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ र $fg^{-1}(x) = f(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ and $fg^{-1}(x) = f(x)$ then find the value of x. [2074 R']

11A. यदि $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ र $g \circ f(4) = -8$ भए $f^{-1}(-2)$ र b का मानहरू निकाल्नुहोस् ।

If $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ and $g \circ f(4) = -8$, find the values of $f^{-1}(-2)$ & b. [2060 C]

11B. दुईओटा फलनहरू $h(x) = \frac{x}{2-x}$ र $g(x) = ax - 5$ छन् । यदि $g \circ h(4) = -13$ भए a र $g^{-1}(1)$ का मानहरू पत्ता लगाउनुहोस् ।

$h(x) = \frac{x}{2-x}$ and $g(x) = ax - 5$ are two functions. If $g \circ h(4) = -13$, find the values of a and $g^{-1}(1)$. [2070 S]

11C. यदि $f: x \to \frac{bx-3}{2}$, $h: x \to \frac{3}{x+2}$ $(x \neq -2)$ र $f \circ h^{-1}(1) = -\frac{1}{2}$ भए b को मान पत्ता लगाउनुहोस् ।

If $f: x \to \frac{bx-3}{2}$, $h: x \to \frac{3}{x+2}$ $(x \neq -2)$ and $f \circ h^{-1}(1) = -\frac{1}{2}$ then find the value of b.

11D. यदि $2f(x) = kx - 3$, $\frac{1}{3}g(x) = \frac{1}{x+2}$ र $f \circ g^{-1}(3) = -\frac{1}{4}$ भए k को मान पत्ता लगाउनुहोस् ।

If $2f(x) = kx - 3$, $\frac{1}{3}g(x) = \frac{1}{x+2}$ and $f \circ g^{-1}(3) = -\frac{1}{4}$, find the value of k. [SEE 2073 R']

12A. फलन $f(x) = \frac{3x+11}{x-3}$, $x \neq 3$ र $g(x) = \frac{x-3}{2}$ दिइएका छन् । $f^{-1}(x)$ पत्ता लगाउनुहोस् र $f(x) = g^{-1}(x)$ भए x का मानहरू पनि पत्ता लगाउनुहोस् । [2062 R]

Functions $f(x) = \frac{3x+11}{x-3}$, $x \neq 3$ and $g(x) = \frac{x-3}{2}$ are given. Find $f^{-1}(x)$. If $f(x) = g^{-1}(x)$, find the values of x.

12B. फलनहरू $h(x) = \frac{2x+3}{x+2}$ $(x \neq -2)$ र $g(x) = (x + 1)$ मा यदि $h \circ g^{-1}(x) = -2$ भए x को मान पत्ता लगाउनुहोस् ।

Functions $h(x) = \frac{2x+3}{x+2}$ $(x \neq -2)$ and $g(x) = (x + 1)$, $h \circ g^{-1}(x) = -2$, find the value of x. [2069 R']

13A. फलन $f(x) = 3x + a$ दिइएको छ । यदि $f \circ f(6) = 10$ भए a र $f^{-1}(4)$ को मान निकाल्नुहोस् ।

Given that function $f(x) = 3x + a$. If $f \circ f(6) = 10$, find the value of a and $f^{-1}(4)$. [2058 S]

13B. यदि $f(x) = 2x + k$ र $f \circ f(4) = 10$ भए k को मान र $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + k$ and $f \circ f(4) = 10$, find k and find the value of $f^{-1}(4)$. [2066 R']

13C. यदि $f(x) = 2x + k$ र $f^{-1}(4) + f(2) = 7$ भए k को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x + k$ and $f^{-1}(4) + f(2) = 7$, find the value of 'k'. [2067 S]

13D. यदि $f(x) = 3x + k$ र $f \circ f(3) = 16$ भए k को मान पत्ता लगाउनुहोस् । साथै $f \circ f(1)$ को मान पनि पत्ता लगाउनुहोस् ।

If $f(x) = 3x + k$ and $f \circ f(3) = 16$ then find the value of k. Also, find the value of $f \circ f(1)$.

Model 4 Problems based on an identity function.

14A. यदि फलन $f(x) = \frac{2x+1}{4}$ भए $f^{-1} \circ f(x)$ एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् र $f^{-1}(2)$ को मान पनि पत्ता लगाउनुहोस् ।

If function $f(x) = \frac{2x+1}{4}$, prove that $f^{-1} \circ f(x)$ is an identity function and find the value of $f^{-1}(2)$. [2065 R']

14B. यदि $f(x) = \frac{14-3x}{4x}$ र $f^{-1}(x) = 2$ भए x को मान पत्ता लगाउनुहोस् र $f \circ f^{-1}(A)$ एकात्मक फलन हो भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{14-3x}{4x}$ and $f^{-1}(x) = 2$, find the value of x and prove that $f \circ f^{-1}(A)$ is an identity function.

15A. दुई फलनहरू $f(x) = \frac{2x+5}{8}$ र $g(x) = 3x - 4$ छन् । यदि $(f \circ g)^{-1}(x)$ एउटा एकात्मक फलन हो भने x को मान पत्ता लगाउनुहोस् ।

Two functions are $f(x) = \frac{2x+5}{8}$ and $g(x)$ $3x - 4$. If $(f \circ g)^{-1}(x)$ is an identity function, find the value of x. [SEE 2074 R]

15B. $f(x) = 3x + 5$ र $g(x) = \frac{3x+2}{4}$ दिइएका छन् । x को कुन मानले $f \circ g^{-1}(x)$ लाई एकात्मक फलन बनाउँदछ ?

Given that $f(x) = 3x + 5$ and $g(x) = \frac{3x+2}{4}$. What value of x makes $f \circ g^{-1}(x)$ is an identity function?

www.drmathnp.com

15C. $f(x) = 4x + 2$ र $g(x) = \frac{4x+3}{4}$ दिइएका छन् । x को कुन मानले $f \circ g^{-1}(x)$ लाई एकात्मक फलन बनाउँदछ ?

Given that $f(x) = 4x + 2$ and $g(x) = \frac{4x+3}{4}$. What value of x makes $f \circ g^{-1}(x)$ is an identity function?

15D. $f(x) = x - 2$ र $g(x) = \frac{x+5}{2x+7}$ दिइएका छन् । x को कुन मानले $f \circ g^{-1}(x)$ लाई एकात्मक फलन बनाउँदछ ?

Given that $f(x) = x - 2$ and $g(x) = \frac{x+5}{2x+7}$. What value of x makes $f \circ g^{-1}(x)$ is an identity function?

Model 5 Problems based on composite functions based on missing function.

16A. यदि $g(x) = 2x - 3$ र $f \circ g(x) = 6x - 11$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् ।
If $g(x) = 2x - 3$ and $f \circ g(x) = 6x - 11$, then find $f^{-1}(x)$. [SEE 2075 R, 2075 R_2]

16B. फलनहरू $f(x) = 3x + 5$ र $f \circ g(x) = 12x - 7$ दिइएका छन् । यदि $g^{-1}(x) = 2$ भए x को मान निकाल्नुहोस् ।
It is given that the functions $f(x) = 3x + 5$ and $f \circ g(x) = 12x - 7$. If $g^{-1}(x) = 2$, find the value of x. [SEE 2073 S']

16C. यदि $f(x) = 3x - 4$ र $f^{-1}g(x) = \frac{3x+2}{3}$ भए $g(x)$ र $f^{-1}\left(\frac{1}{2}\right)$ निकाल्नुहोस् ।

If $f(x) = 3x - 4$ and $f^{-1}g(x) = \frac{3x+2}{3}$, find $g(x)$ and $f^{-1}\left(\frac{1}{2}\right)$. [2070 R]

16D. यदि $f(x) = 2x - 3$ र $f^{-1}g(x) = \frac{x+11}{4}$ भए $g(x)$ र $g^{-1}\left(\frac{1}{2}\right)$ निकाल्नुहोस् ।

If $f(x) = 2x - 3$ and $f^{-1}g(x) = \frac{x+11}{4}$, find $g(x)$ and $g^{-1}\left(\frac{1}{2}\right)$.

16E. यदि $f(x) = 3x - 1$, र $f \circ g(x) = 6x + 5$ भए $(g \circ f)^{-1}$ पत्ता लगाउनुहोस् ।
If $f(x) = 3x - 1$ and $f \circ g(x) = 6x + 5$, then find $(g \circ f)^{-1}$. [2071 R]

16F. यदि $f(x) = 2x - 7$ र $f \circ g(x) = 4x + 3$ भए $(g \circ f)^{-1}(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 7$ and $f \circ g(x) = 4x + 3$, find $(g \circ f)^{-1}(x)$. [2071 S]

ANSWERS

A. VERY SHORT QUESTIONS

3A. $f^{-1} = \{(b, a), (y, x)\}$ **3B.** $f^{-1} = \{(y, x), (d, c)\}$ **4A.** x **4B.** a **5A** $\{(8, 1), (9, 2), (10, 3)\}$
5B. $\{(5, 8), (6, 9), (7, 10)\}$ **5C.** $\{(5, 2), (6, 3), (7, 4)\}$ **5D.** $\{(2, 3), (4, 5), (6, 7)\}$ **6A.** $\{(-4, 2)\}$ **6B.** $\{(b, a)\}$
7A. $x - a$ **7B.** $\frac{x}{2}$ **7C.** $x - 2$ **7D.** $2x$ **8A.** $\{(5, 1), (6, 2), (7, 3), (8, 4)\}$ **8B.** $\{(1, a), (2, b), (3, c), (4, d)\}$
9A. $\{(4, 5), (6, 7), (8, 9)\}$ **9B.** $\{(4, 1), (7, 2), (10, 3)\}$ **10A.** $\{(1, 2)\}$ **10B.** $\{5\}$

B. SHORT QUESTIONS

1A. $R = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}, f^{-1} = \left\{\left(\frac{1}{2}, 2\right), \left(\frac{1}{3}, 3\right), \left(\frac{1}{4}, 4\right)\right\}$ **1B.** $f^{-1} = \{(2, 1), (3, 2), (5, 4)\}$, Dom. of $f^{-1} = \{2, 3, 5\}$, Range of $f^{-1} = \{1, 2, 4\}$
2A. $R = \{-9, -4, 1\}$, $f^{-1} = \{(-9, -1), (-4, 0), (1, 1)\}$ **2B.** $\{(10, 0), (13, 1), (16, 2), (19, 3)\}$ **3A.** $f^{-1}(x) = \frac{x-5}{4}$ **3B.** 2
3C. 5 **3D.** $\frac{1}{4}$ **3E.** $\frac{1}{2}$ **3F.** $\frac{1}{4}$ **3G.** -7 **4A.** $f^{-1}(x) = \frac{2-3x}{x}$ **4B.** $f^{-1}(x) = \frac{2x-4}{x}$ **4C.** $f^{-1}(x) = -\frac{2x}{x-3}$
4D. $f^{-1}(x) = \frac{x}{x+2}$ **4E.** $f^{-1}(x) = \frac{x}{3x-2}$ **4F.** $f^{-1}(x) = -\frac{1}{3+x}$ **4G.** $f^{-1}(x) = \frac{3x+4}{2x-3}$ **4H.** $f^{-1}(x) = \frac{4x+3}{2-x}$
4I. $f^{-1}(x) = -\frac{2x+3}{x-2}$ **4J.** $f^{-1}(x) = \frac{2x-4}{x+3}$ **4K.** $f^{-1}(x) = \frac{2}{\sqrt{1-2x}}$ **4L.** $f^{-1}(x) = \sqrt{\frac{3}{3x-1}}$ **5A.** $f^{-1}(x) = \frac{2x-3}{2}$
5B. 2 **6A.** $2x - 3$ **6B.** $\frac{x+3}{2}$ **7A.** 8 **7B.** 3 **7C.** 2

C. LONG QUESTIONS

1A. $f^{-1}(x) = \frac{x+1}{x-2}, x \neq 2$ **1B.** $f^{-1}(x) = \frac{3-2x}{x-2}, x \neq 2$ **3A.** $\frac{11}{4}$ **3B.** 16 **4A.** $2x + 11, \frac{x-5}{2}$ **4B.** 4, 2
4C. $2, \frac{1}{6}$ **4D.** $\frac{1}{2}, 2x^2 + 1$ **4E.** $\frac{x+5}{2}, 16$ **4F.** 5 **4G.** $\frac{11}{4}$ **4H.** $-\frac{1}{2}$ **5A.** $-1, 2$
5B. $\frac{3-2x}{x-2}, -1, \frac{2x-1}{x}, 1$ **6B.** $\frac{-2}{3}$ **6D.** $\frac{2x}{5-x}, \frac{2x+1}{5}$ **7A.** 0, 2 **7B.** $-1, 0$
8A. $x = 6$, $fof^{-1}(P) = P$ **8B.** 9 **9A.** 1 **9B.** 1, 9 **9C.** $x = -7$ **9D.** 7 **9E.** $x = 39$ **9F.** $x = 3$
10A. 4 **10B.** $3\frac{8}{47}$ or $\frac{149}{47}$ **10C.** $\frac{1}{2}$ **10D.** 6 **10E.** ± 1 **10F.** $\frac{5}{2}$ **10G.** -7 **10H.** -1 **10I.** 1
11A. 4, 3 **11B.** $4, \frac{3}{2}$ **11C.** 2 **11D.** $-\frac{5}{2}$ **12A.** $f^{-1}(x) = \frac{3x+11}{x-3}, x \neq 3$; $x = -2, 5$ **12B.** $-\frac{3}{4}$ **13A.** $-11, 5$
13B. $-2, 3$ **13C.** 2 **13D.** $-\frac{11}{4}, \frac{7}{4}$ **14A.** $\frac{7}{2}$ **14B.** 1 **15A.** $-\frac{3}{2}$ **15B.** $x = -1$ **15C.** $x = \frac{1}{3}$
15D. $\frac{-5 \pm \sqrt{39}}{2}$ **16A.** $\frac{x+2}{3}$ **16B.** 4 **16C.** $3x - 2, \frac{3}{2}$ **16D.** $f(x) \frac{x+5}{2}, -5$ **16E.** $\frac{x}{6}$ **16F.** $\frac{x+9}{4}$

puspas.com.np

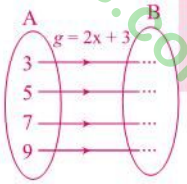
www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 1.3

A. SHORT QUESTIONS

Model 1 *Problems based on inverse function (diagram).*

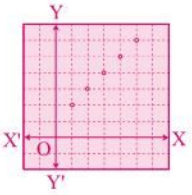


1A. चित्रमा एउटा फलन g लाई देखाईएको छ । (In the figure a function g is shown.)

(i) $g^{-1} : B \to A$ लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
Write down $g^{-1} : B \to A$ as a set of ordered pairs.

(ii) $g^{-1}\{9, 13\}$ पत्ता लगाउनुहोस् । (Find $g^{-1}\{9, 13\}$.)

(iii) फलनको क्षेत्र र विस्तार क्षेत्र लेख्नुहोस् । (Write the domain and range the fuction.)

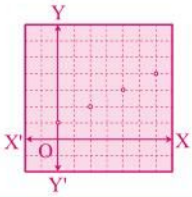


1B. दिइएको चित्रमा एउटा फलन f लाई देखाईएको छ ।
In the given figure, a function f is shown.

(i) $f^{-1} : B \to A$ लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
Write down $f^{-1} : B \to A$ as a set of ordered pairs.

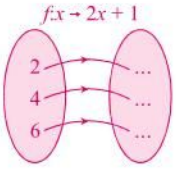
(ii) $f^{-1}\{3, 4\}$ पत्ता लगाउनुहोस् । (Find $f^{-1}\{3, 4\}$.)

(iii) फलनको क्षेत्र र विस्तार क्षेत्र लेख्नुहोस् । (Write the domain and range the fuction.)



1C. दिइएको लेखाचित्रले फलनको प्रतिनिधित्व गर्दछ । यसको विपरीत फलनको क्षेत्र र विस्तार लेख्नुहोस् ।
The given graph represents a function. Write the domain and range of its inverse fuction.

Model 2 *Problems based on inverse function and composite function in ordered pair form.*



2A. फलन f लाई $f(x) = 2x + 1$ द्वारा परिभाषित गरिएको छ । f^{-1} लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
The function f is defined by $f(x) = 2x + 1$. Write f^{-1} in ordered pair form.

2B. यदि $f = \{(1, 5), (2, 4), (3, 6)\}$ र $f \circ g = \{(5, 6), (4, 5), (6, 4)\}$ भए g^{-1} पत्ता लगाउनुहोस् ।
If $f = \{(1, 5), (2, 4), (3, 6)\}$ and $f \circ g = \{(5, 6), (4, 5), (6, 4)\}$ then find g^{-1}.

2C. यदि $f = \{(2, 3), (4, 5), (6, 7)\}$ र $g \circ f = \{(2, 4), (4, 6), (6, 8)\}$ भए g^{-1} पत्ता लगाउनुहोस् ।
If $f = \{(2, 3), (4, 5), (6, 7)\}$ and $g \circ f = \{(2, 4), (4, 6), (6, 8)\}$ then find g^{-1}.

3A. यदि $f = \{(1, 2), (3, 5), (4, 1)\}$ र $g = \{(2, 3), (5, 1), (1, 3)\}$ भए $f^{-1} \circ g^{-1}$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको रूपमा लेख्नुहोस् ।
If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ then show the function $f^{-1} \circ g^{-1}$ in a mapping diagram and find it in ordered pair form.

3B. यदि $f = \{(1, 3), (0, 0), (-1, -3)\}$ र $g = \{(0, 1), (-3, -1), (3, 0)\}$ भए $g^{-1} \circ f^{-1}$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको रूपमा लेख्नुहोस् ।
If $f = \{(1, 3), (0, 0), (-1, -3)\}$ and $g = \{(0, 1), (-3, -1), (3, 0)\}$ then show the function $g^{-1} \circ f^{-1}$ in a mapping diagram and find it in ordered pair form.

4A. यदि $f = \{(1, 2), (3, 4), (5, 6)\}$ र $g = \{(5, 2), (6, 4), (8, 6)\}$ भए संयुक्त फलन $g^{-1} \circ f$ पत्ता लगाउनुहोस् र मिलानचित्रमा देखाउनुहोस् ।
If $f = \{(1, 2), (3, 4), (5, 6)\}$ and $g = \{(5, 2), (6, 4), (8, 6)\}$ then find the composite function $g^{-1} \circ f$ and show in a mapping diagram.

4B. यदि $f = \{(2, 3), (4, 5), (5, 6)\}$ र $g = \{(4, 3), (2, 5), (5, 8)\}$ भए $g \circ f^{-1}$ लाई क्रमजोडाको रूपमा लेख्नुहोस् ।
If $f = \{(2, 3), (4, 5), (5, 6)\}$ and $g = \{(4, 3), (2, 5), (5, 8)\}$ then find $g \circ f^{-1}$ in ordered pair form.

5A. यदि $f^{-1} = \{(2, 1), (5, 3), (1, 4)\}$ र $g^{-1} = \{(3, 2), (1, 5), (3, 1)\}$ भए $f \circ g$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।
If $f^{-1} = \{(2, 1), (5, 3), (1, 4)\}$ and $g^{-1} = \{(3, 2), (1, 5), (3, 1)\}$ then show the function $f \circ g$ in arrow diagram and find it in ordered pair form.

5B. यदि $f^{-1} = \{(2, 1), (3, 2), (5, 3)\}$ र $g^{-1} = \{(2, 3), (4, 5), (6, 2)\}$ भए $g \circ f$ लाई मिलानचित्रमा देखाई क्रमजोडाहरूको समूह बनाउनुहोस् ।
If $f^{-1} = \{(2, 1), (3, 2), (5, 3)\}$ and $g^{-1} = \{(2, 3), (4, 5), (6, 2)\}$ then show the function $g \circ f$ in arrow diagram and find it in ordered pair form.

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on finding inverse function based on simple algebraic function.*

तलका फलनहरूको विपरीत फलन पत्ता लगाउनुहोस् (Find the inverse of following functions):

6A. $f = \{(x, 2x+1): x \in \Re\}$ **6B.** $f = \{(x, 3x+7): x \in \Re\}$ **6C.** $f(x) = \frac{ax+b}{cx+d}$

6D. $f(x) = x^3 + 5$ **6E.** $f(x) = \frac{1}{16}x^4$ **6F.** $f(x) = \frac{1}{27}x^3$

7A. यदि $f(x) = \frac{2x-1}{3}$ भए $f^{-1}(x)$ परिभाषित गर्ने सूत्र पत्ता लगाउनुहोस् ।

If $f(x) = \frac{2x-1}{3}$, find the formula that defines $f^{-1}(x)$.

7B. यदि $f(x) = \frac{2x+5}{x+2}, x \neq -2, x \in R$ भए $f^{-1}(-2)$ को मान पत्ता लगाउनुहोस् ।

If $f(x) = \frac{2x+5}{x+2}, x \neq -2, x \in R$, find the value of $f^{-1}(-2)$.

7C. यदि $f(x) = \frac{x+38}{10x}$ भए $f^{-1}(2)$ पत्ता लगाउनुहोस् । (If $f(x) = \frac{x+38}{10x}$ then find $f^{-1}(2)$.)

7D. यदि $f(x) = \frac{x-5}{8}$ भए $f^{-1}(2^{-1}) + f^{-1}(4^{-1})$ पत्ता लगाउनुहोस् । (If $f(x) = \frac{x-5}{8}$ then find $f^{-1}(2^{-1}) + f^{-1}(4^{-1})$.)

7E. यदि $f(x) = 3x - 2$ र $f^{-1}(x) = \frac{px+q}{r}$ भए p, q र r का मानहरू पत्ता लगाउनुहोस् ।

If $f(x) = 3x - 2$ and $f^{-1}(x) = \frac{px+q}{r}$ then find the values of p, q and r.

8A. यदि $f(4x+5) = 12x + 18$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् । (If $f(4x+5) = 12x + 18$ then find $f^{-1}(x)$.)

Hint: First Method

Here, $f(4x+5) = 12x + 18 = 3(4x+6)$

$= 3(4x+5+1)$

$\therefore \quad f(4x+5) = 3(4x+5) + 3$

$\Rightarrow \quad f(x) = 3x + 3$

Now, find $f^{-1}(x)$.

Second Method

Here, $f(4x+5) = 12x + 18$

or, $f\left(4 \times \frac{x}{4} + 5\right) = 12 \times \frac{x}{4} + 18 \quad \left[\text{As } x \to \frac{x}{4}\right]$

or, $f(x+5) = 3x + 18$

or, $f(x-5+5) = 3(x-5) + 18$ [As, $x \to x-5$]

$\therefore \quad f(x) = 3x + 3$

Now, find $f^{-1}(x)$.

Third Method : Here, $f(4x+5)$

Let, $4x + 5 = a$ then after simplification, $x = \frac{a-5}{4}$

so, $f(a) = 12\frac{(a-5)}{4} + 18$

or, $f(a) = 3a + 3 \quad \therefore \quad f(x) = 3x + 3$

or, $f(a) = 3(a-5) + 18$

Now, find $f^{-1}(x)$.

8B. यदि $f(x+3) = 3x + 5$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् । (If $f(x+3) = 3x + 5$ then find $f^{-1}(x)$.)

8C. यदि $f(x+3) = x + 6$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् । (If $f(x+3) = x + 6$, find $f^{-1}(x)$.)

9A. यदि $f(x) = 2x + 3$ र $f^{-1}(7) = k$ भए k को मान कति होला ?

If $f(x) = 2x + 3$ and $f^{-1}(7) = k$ then what is the value of k ?

9B. यदि $f(x) = 4x + k$ र $f^{-1}(11) = 2$ भए k को मान कति होला ?

If $f(x) = 4x + k$ and $f^{-1}(11) = 2$ then what is the value of k ?

9C. यदि $f(x) = 2x - 3$, $h(x) = x + 1$ र $f(x) = -h^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 2x - 3$, $h(x) = x + 1$ and $f(x) = -h^{-1}(x)$, then find the value of x.

9D. यदि $h = (x, 3x-1)$ र $h^{-1}(m) = 21 - 2m$ भए m को मान निकाल्नुहोस् ।

If $h = (x, 3x-1)$ and $h^{-1}(m) = 21 - 2m$, find the value of m.

10A. यदि $f^{-1}(x+2) = x$ भए $f(x)$ र $f^{-1}(4)$ पत्ता लगाउनुहोस् । (If $f^{-1}(x+2) = x$ then, find $f(x)$ and $f^{-1}(4)$.)

10B. यदि $g^{-1}(x+2) = \frac{x-2}{3}$ भए $g(x)$ को समीकरण पत्ता लगाउनुहोस् । (If $g^{-1}(x+2) = \frac{x-2}{3}$, find the equation of $g(x)$.)

10C. यदि $h^{-1}(x+1) = 2x - 1$ भए $h(x)$ पत्ता लगाउनुहोस् । (If $h^{-1}(x+1) = 2x-1$ then find $h(x)$.)

www.drmathnp.com

puspas.com.np

Model 4 *Problems based on identity function & constant function.*

11A. यदि $f(x) = 3x + 2$ भए $ff^{-1}(x)$ एउटा एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 3x + 2$ then prove that $ff^{-1}(x)$ is an identity function.

11B. यदि $f(x) = x^2 + 8$ भए $ff^{-1}(x)$ एउटा एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = x^2 + 8$ then prove that $ff^{-1}(x)$ is an identity function.

12A. यदि $f(x) = \frac{4x+5}{3}$ र $g(x) = 2$ भए $f^{-1}[g(x)]$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = \frac{4x+5}{3}$ and $g(x) = 2$ then prove that $f^{-1}[g(x)]$ is a constant function.

12B. यदि $f(x) = \frac{3x+1}{2}$ र $g(x) = 4$ भए $f^{-1}[g(x)]$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = \frac{3x+1}{2}$ and $g(x) = 4$ then prove that $f^{-1}[g(x)]$ is a constant function.

Model 5 *Problems based on inverse function & composite function.*

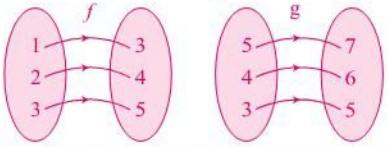
13A. यदि $h(x) = 5x$ र $g(x) = 7x - 1$, भए $(g\circ h)^{-1}(x)$ को मान निकाल्नुहोस् ।
If $h(x) = 5x$ and $g(x) = 7x - 1$, find $(g\circ h)^{-1}(x)$.

13B. यदि $f(x) = 2x + 3$ र $g(x) = x + 2$ भए $(f\circ g)^{-1}(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$ and $g(x) = x + 2$ then find $(f\circ g)^{-1}(x)$.

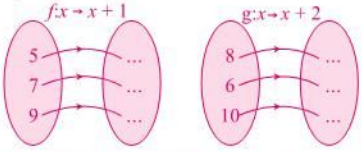
Model 6 *Problems based on composite function and inverse function (diagram form).*

14. तलका तीरचित्रहरू हेर्नुहोस् । $(g\circ f)^{-1}$ को क्रमजोडाहरूको समूह बनाउनुहोस् ।
Look at the following arrow diagrams. Find $(g\circ f)^{-1}$ in ordered pair form.

A.



B.



Model 7 *Problems on finding missing function when the composite function is given.*

15A. यदि $g(x) = 4x + 5$ र $f\circ g(x) = 8x + 10$ भए $f^{-1}(x)$ पत्ता लगाउनुहोस् ।
If $g(x) = 4x + 5$ and $f\circ g(x) = 8x + 10$ then find $f^{-1}(x)$.

15B. यदि $f(x) = 3x + 2$ र $f\circ g(x) = 5x + 4$ भए $g^{-1}(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 2$ and $f\circ g(x) = 5x + 4$ then find $g^{-1}(x)$.

B. LONG QUESTIONS

Model 1 *Problems based on inverse function and composite function.*

1A. फलन $k : x \to \frac{2x+5}{x+2}, x \neq -2, x \in \Re$ को विपरीत फलन र $k\circ k(2)$ पत्ता लगाउनुहोस् ।
Find the inverse function of $k : x \to \frac{2x+5}{x+2}, x \neq -2, x \in \Re$ and $k\circ k(2)$.

1B. फलन $f(x) = \frac{3x+1}{x-3}$ को विपरीत फलन र $f\circ f^{-1}(2)$ पत्ता लगाउनुहोस् ।
Find the inverse of the function $f(x) = \frac{3x+1}{x-3}$ and also find $f\circ f^{-1}(2)$.

1C. यदि $f(x) = \frac{3x-4}{x+1}$ भए $f\circ f^{-1}(2)$ निकाल्नुहोस् र $f(x) = 2$ हल गर्नुहोस् ।
If $f(x) = \frac{3x-4}{x+1}$, find $f\circ f^{-1}(2)$ and solve at $f(x) = 2$.

1D. यदि $f(x) = 2x + 3$ र $g(x) = \frac{2x-3}{5-4x}$ भए $g^{-1}\circ f(x)$ र $g^{-1}\circ f(2)$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$ and $g(x) = \frac{2x-3}{5-4x}$ then find $g^{-1}\circ f(x)$ and $g^{-1}\circ f(2)$.

www.drmathnp.com

1E. यदि $f(x)=\frac{2x-4}{3x+5}$ र $g(x)=\frac{6x+8}{7}$ भए $g\circ f^{-1}(x)$ र $f\circ g^{-1}(2)$ पत्ता लगाउनुहोस् ।

If $f(x)=\frac{2x-4}{3x+5}$ and $g(x)=\frac{6x+8}{7}$ then find the value of $g\circ f^{-1}(x)$ and $f\circ g^{-1}(2)$.

1F. यदि $f(x)=3x-4, x\in\Re$ र $g(x)=\frac{1-x}{2}, x\in\Re$ हरू दुईओटा फलनहरू भए $f^{-1}(x)$ र $g\circ f^{-1}(5)$ पत्ता लगाउनुहोस् ।

If $f(x)=3x-4, x\in\Re$ & $g(x)=\frac{1-x}{2}, x\in\Re$ be two functions, find $f^{-1}(x)$ & $g\circ f^{-1}(5)$.

2A. यदि $f(x)=x-1$ र $f\circ g(x)=2x+2$ भए $g^{-1}(-3)$ को मान पत्ता लगाउनुहोस् ।
If $f(x)=x-1$ and $f\circ g(x)=2x+2$ then find the value of $g^{-1}(-3)$.

2B. यदि $g(x)=2x+1$ र $g\circ f(x)=2x-1$ भए $f^{-1}(3)$ को मान पत्ता लगाउनुहोस् ।
If $g(x)=2x+1$ and $g\circ f(x)=2x-1$ then find the value of $f^{-1}(3)$.

2C. यदि $f(x)=2x+3$ र $fg(x)=2x+5$ भए $(g\circ f)^{-1}$ पत्ता लगाउनुहोस् ।
If $f(x)=2x+3$ and $fg(x)=2x+5$ then find the value of $(g\circ f)^{-1}$.

3A. यदि $f=\{(x,x^2+1)\}$ र $g=\{(x,x-1)\}$ भए पत्ता लगाउनुहोस् (If $f=\{(x,x^2+1)\}$ and $g=\{(x,x-1)\}$, find):
(i) $f\circ g(x)$ (ii) $g\circ f(a)$ (iii) $f^{-1}g(x)$ (iv) $g\circ f^{-1}(2)$

3B. यदि $f=\{(a,a+2)\}$ र $g=\{(a,2a^2)\}$ भए पत्ता लगाउनुहोस् (If $f=\{(a,a+2)\}$ and $g=\{(a,2a^2)\}$, find):
(i) $f\circ g(a)$ (ii) $g\circ f(x)$ (iii) $f^{-1}(g(a))$ (iv) $g\circ f^{-1}(a)$

4A. यदि $3f(x)=4x+5$ र $g(x)=5x-6$ भए $f\circ g^{-1}(1)$ पत्ता लगाउनुहोस् ।
If $3f(x)=4x+5$ and $g(x)=5x-6$, find $f\circ g^{-1}(1)$.

4B. यदि $f(x+5)=2x+13$ भए $fof^{-1}(6)+f^{-1}(7)$ पत्ता लगाउनुहोस् । (If $f(x+5)=2x+13$, find $fof^{-1}(6)+f^{-1}(7)$.)

4C. यदि $f(x)=3x+4$ र $g(x)=2(x+1)$ भए प्रमाणित गर्नुहोस् : $f\circ g=g\circ f$ साथै $g^{-1}(-3)$ को मान पनि पत्ता लगाउनुहोस् ।
If $f(x)=3x+4$ and $g(x)=2(x+1)$ then prove that: $f\circ g=g\circ f$. Also, find the value of $g^{-1}(-3)$.

Model 2 ***On finding the values of a, b, k or x (inverse function & composite function).***

5A. यदि $f(a)=2a-3, g(a)=a+1$ र $f\circ f(a)=-g^{-1}(a)$ भए a को मान पत्ता लगाउनुहोस् ।
If $f(a)=2a-3$, $g(a)=a+1$ and $f\circ f(a)=-g^{-1}(a)$, find the value of a.

5B. फलन $f(x)=ax+9$, $g(x)=3x+8$ र $f^{-1}(10)=g^{-1}(11)$ भए a को मान पत्ता लगाउनुहोस् ।
If $f(x)=ax+9$, $g(x)=3x+8$ and $f^{-1}(10)=g^{-1}(11)$, find the value of a.

5C. यदि $f(x)=ax+19, g(x)=13x+8$ र $f^{-1}(5)=g^{-1}(6)$ भए 'a' को मान पत्ता लगाउनुहोस् ।
If $f(x)=ax+19, g(x)=13x+8$ and $f^{-1}(5)=g^{-1}(6)$, find the value of a.

5D. फलन $f(x)=2x^2+b$, $g(x)=ax-2$ र $g^{-1}(2)=fg(1)=4$ भए a र b का मानहरू पत्ता लगाउनुहोस् ।
If $f(x)=2x^2+b$, $g(x)=ax-2$ and $g^{-1}(2)=fg(1)=4$, find the values of a and b.

5E. फलन $f(x)=ax-2$, $2g(x)=x+a$ र $g\,f(2)=2$ भए a र $fg^{-1}(2)$ का मानहरू पत्ता लगाउनुहोस् ।
If $f(x)=ax-2$, $2g(x)=x+a$ and $g\,f(2)=2$, find the values of a and $fg^{-1}(2)$.

6A. $f(x+2)=4x-9$ र $5g(x)=2x+8$ दिइएको छ । यदि $f\circ f(x)=g^{-1}(x)$ भए x को मान निकाल्नुहोस् ।
It is given that $f(x+2)=4x-9$ & $5g(x)=2x+8$. If $f\circ f(x)=g^{-1}(x)$, find the value of x.

6B. यदि $f(x+2)=4x-9$, $g(x)=\frac{2x+8}{5}$ र $f\circ f(x)=g^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x+2)=4x-9$, $g(x)=\frac{2x+8}{5}$ and $f\circ f(x)=g^{-1}(x)$ then find the value of x.

6C. $f(x+1)=x^2-1$ र $g(x+2)=2x+7$ दिइएका छन् । यदि $f\circ g^{-1}(x)=3$ भए x को मान पत्ता लगाउनुहोस् ।
It is given that $f(x+1)=x^2-1$ and $g(x+2)=2x+7$. If $f\circ g^{-1}(x)=3$, find the value of x.

6D. $f(x+2)=2x+7$ र $g(x+1)=x^2-1$ दिइएका छन् । यदि $g\circ f^{-1}(x)=3$ भए x को मान पत्ता लगाउनुहोस् ।
It is given that $f(x+2)=2x+7$ and $g(x+1)=x^2-1$. If $g\circ f^{-1}(x)=3$, find the value of x.

7A. यदि $f(x+3)=x^2+6x+11$ र $g(x)=\frac{x+3}{2}$ भए x को मान पत्ता लगाउनुहोस् जहाँ $f\circ g^{-1}(x)=83$ छ ।

If $f(x+3)=x^2+6x+11$ and $g(x)=\frac{x+3}{2}$, find the value of x if $f\circ g^{-1}(x)=83$.

7B. यदि $f(x-3)=x^2-6x+11$ र $g(x)=\frac{x+3}{2}$ भए x को मान पत्ता लगाउनुहोस् जहाँ $f\circ g^{-1}(x)=51$ छ ।

If $f(x-3)=x^2-6x+11$ and $g(x)=\frac{x+3}{2}$, find the value of x if $f\circ g^{-1}(x)=51$.

puspas.com.np

www.drmathnp.com

puspas.com.np

7C. यदि $f(x) = 4x - 2, g(x) = \frac{1}{x}$ र $\{f^{-1}(6)\} \times \{g \circ f(x)\} = 1$ भए x को मान पत्ता लगाउनुहोस् ।

If $f(x) = 4x - 2, g(x) = \frac{1}{x}$ and $\{f^{-1}(6)\} \times \{g \circ f(x)\} = 1$, find the value of x.

7D. यदि $g(p) = \frac{p-2}{2p+1}, p \neq \frac{1}{2}$ र $f(p) = \frac{1}{p}, p \neq 0$ भए $g^{-1}(p) = f \circ g(p)$ हुने गरी p को मान पत्ता लगाउनुहोस् ।

If $g(p) = \frac{p-2}{2p+1}, p \neq \frac{1}{2}$ and $f(p) = \frac{1}{p}, p \neq 0$ then find the value of p such that: $g^{-1}(p) = f \circ g(p)$

8A. $f = (x, 4x - 17)$ र $g = \left(x, \frac{2x+8}{5}\right)$ छन् । यदि $g^{-1}(x) = f \circ f(x)$ भए 'x' को मान निकाल्नुहोस् ।

Given that $f = (x, 4x - 17)$ & $g = \left(x, \frac{2x+8}{5}\right)$. If $g^{-1}(x) = f \circ f(x)$, find the value of 'x'.

8B. यदि $f = (x, 2x + 3), g = \left(x, \frac{x+5}{2}\right)$ र $g^{-1}(x) = f \circ f(x)$ भए x को मान पत्ता लगाउनुहोस् ।

If $f = (x, 2x + 3), g = \left(x, \frac{x+5}{2}\right)$ and $g^{-1}(x) = f \circ f(x)$, find the value of x.

8C. फलन $f\left(x, \frac{3x+10}{2}\right)$ र $h(x, 3x - 5)$ दिइएको छ । यदि $hh(x) = f^{-1}(x)$ भए x को मान पत्ता लगाउनुहोस् ।

Given that $f\left(x, \frac{3x+10}{2}\right)$ and $h(x, 3x - 5)$. If $hh(x) = f^{-1}(x)$, find the value of x.

Model 3 Problems based on inverse function and missing function.

9A. यदि $f(x) = x - 1$ र $f \circ g(x) = 2x + 2$ भए $g^{-1}(-3)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = x - 1$ and $f \circ g(x) = 2x + 2$, find the value of $g^{-1}(-3)$.

9B. यदि $g(x) = 2x + 1$ र $g \circ f(x) = 2x - 1$ भए $f^{-1}(-3)$ को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x + 1$ and $g \circ f(x) = 2x - 1$, find the value of $f^{-1}(-3)$.

9C. यदि $f(x) = 2x + 3$ र $f \circ g(x) = 2x + 5$ भए $(g \circ f)^{-1}$ पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 3$ and $f \circ g(x) = 2x + 5$, find $(g \circ f)^{-1}$.

Model 4 Problems based on identity, linear, quadratic and cubic functions.

10A. फलनहरू $f(x) = 3x + 1$ र $g(x) = \frac{1}{3}(x - 1)$ हरू आपसमा विपरीत छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the fuctions $f(x) = 3x + 1$ and $g(x) = \frac{1}{3}(x - 1)$ are inverse to each other.

10B. यदि $f(x) = \frac{3x-1}{x+2}$ भए $f^{-1} \circ f(x)$ एउटा एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{3x-1}{x+2}$ then prove that $f^{-1} \circ f(x)$ is an identity function.

10C. यदि $f(x) = \frac{4x+2}{5x-7}$ भए $f \circ f^{-1}(x)$ एउटा एकात्मक फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{4x+2}{5x-7}$ then prove that $f \circ f^{-1}(x)$ is an identity function.

10D. मानौं $f: \Re \to \Re$ लाई $f(x) = 2x - 4, x \in \Re$ द्वारा परिभाषीत गरिएको छ । f^{-1} पत्ता लगाउनुहोस र $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Let $f: \Re \to \Re$ be defined by $f(x) = 2x - 4, x \in \Re$. Find f^{-1} and prove that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

10E. मानौं $f: \Re \to \Re$ लाई $f(x) = ax + b, x \in \Re$ द्वारा परिभाषीत गरिएको छ, जहाँ a र b वास्तविक सङख्याहरू हुन् । f^{-1} पत्ता लगाउनुहोस र $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Let $f: \Re \to \Re$ be defined by $f(x) = ax + b, x \in \Re$, a and b are also real numbers. Find f^{-1} and show that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.

11A. यदि $f(x) = 4x + 5$ र $g(x) = 8x + 9$ भए $f^{-1} \circ g(x)$ एउटा रेखीय फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = 4x + 5$ and $g(x) = 8x + 9$ then prove that $f^{-1} \circ g(x)$ is a linear function.

11B. यदि $p(x) = 2x - 3$ र $q(x) = 7x + 9$ भए $q^{-1} \circ p^{-1}$ एउटा रेखीय फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $p(x) = 2x - 3$ and $q(x) = 7x + 9$ then prove that $q^{-1} \circ p^{-1}$ is a linear function.

www.drmathnp.com

12A. यदि $f(x)=\frac{3x+5}{x-2}$ र $g(x)=7$ दुई फलनहरू भए $f^{-1}\circ g(x)$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x)=\frac{3x+5}{x-2}$ and $g(x)=7$ are two functions then prove that $f^{-1}\circ g(x)$ is a constant function.

12B. यदि $h(x)=\frac{2x-3}{7}$ र $f(x)=3$ दुई फलनहरू भए $h^{-1}\circ f(x)$ एउटा अचर फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $h(x)=\frac{2x-3}{7}$ and $f(x)=3$ are two functions then prove that $h^{-1}\circ f(x)$ is a constant function.

13A. यदि $f(x)=4x+5$ र $g(x)=x^2+5x+6$ भए $f^{-1}\circ g(x)$ एउटा वर्ग फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x)=4x+5$ and $g(x)=x^2+5x+6$ then prove that $f^{-1}\circ g(x)$ is a quadratic function.

13B. यदि $f(x)=7x+9$ र $g(x)=5x^2+4x+3$ भए $f^{-1}\circ g(x)$ एउटा वर्ग फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x)=7x+9$ and $g(x)=5x^2+4x+3$ then prove that $f^{-1}\circ g(x)$ is a quadratic function.

14A. यदि $f(x)=x^3+x^2+1$ र $g(x)=2x+3$ भए $f\circ g^{-1}(x)$ एउटा घन फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x)=x^3+x^2+1$ and $g(x)=2x+3$ then prove that $f\circ g^{-1}(x)$ is a cubic function.

14B. यदि $f(x)=x^3+x+1$ र $g(x)=4x+5$ भए $f\circ g^{-1}(x)$ एउटा घन फलन हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x)=x^3+x+1$ and $g(x)=4x+5$ then prove that $f\circ g^{-1}(x)$ is a cubic function.

15A. Q ले आनुपातिक सङ्ख्याहरूको समूहलाई जनाउँछ । $f:Q\to Q: f(x)=2x+3, x\in Q$ कुनै एउटा फलन छ । के फलन f एक एक र सम्पूर्ण फलन हो ? यदि हो भने $f^{-1}(x)$ पत्ता लगाउनुहोस् ।
Q denotes the set of rational numbers. $f:Q\to Q: f(x)=2x+3, x\in Q$ is a function. Is the function f one to one onto? If so, find $f^{-1}(x)$.

Hint: Here, $f(x)=2x+3$ Let, for $x_1, x_2\in Q$, $f(x_1)=f(x_2)$
So, $2x_1+3=2x_2+3$ or, $x_1=x_2$
For every image, there is different pre-image or when two images are equal to each other then the pre-images are also equal.
So, the function f is one to one-function.

Again, let $y=f(x)$ or, $y=2x+3$ or, $y-3=2x$ or, $x=\frac{1}{2}(y-3)$

Here, For every $y\in Q$, $x\in Q$. So, f is a onto-function.
$f(x)$ is one to one and onto function so, $f^{-1}(x)$ is defined.

From the above, $f(x)=2x+3$ or, $y=2x+3$ or, $y-3=2x$ or, $\frac{1}{2}(y-3)=x$

or, $\frac{1}{2}(y-3)=f^{-1}(y)$ [When $f(x)=y, x=f^{-1}(y)$.] $\therefore\ f^{-1}(x)=\frac{1}{2}(x-3)$

15B. Q ले आनुपातिक सङ्ख्याहरूको समूहलाई जनाउँछ । $f:Q\to Q: f(x)=4x+5, x\in Q$ कुनै एउटा फलन छ । के फलन f एक एक र सम्पूर्ण फलन हो ? यदि हो भने $f^{-1}(x)$ पत्ता लगाउनुहोस् ।
Q denotes the set of rational numbers. $f:Q\to Q: f(x)=4x+5, x\in Q$ is a function. Is the function f one to one onto? If so, find $f^{-1}(x)$.

ANSWERS

A. SHORT QUESTIONS

1A. (i) $\{(9,3),(13,5),(17,7),(21,9)\}$ (ii) $\{3,5\}$ (iii) D = {3, 5, 7, 18}; R = {9, 13, 17, 21}
1B. (i) $\{(2,1),(3,2),(4,3),(5,4),(6,5)\}$ (ii) $\{2,3\}$ (iii) D = {1, 2, 3, 4, 5}; R = {2, 3, 4, 5, 6}
1C. Domain of inverse = {1, 2, 3, 4}, Range of inverse = {0, 2, 4, 6}
2A. $\{(5,2),(9,4),(13,6)\}$ **2B.** $g^{-1}=\{(3,5),(1,4),(2,6)\}$ **2C.** $\{(4,3),(6,5),(8,7)\}$ **3A.** $\{(3,1),(1,3),(3,4)\}$
3B. $\{(3,0),(0,3),(-3,-3)\}$ **4A.** $\{(1,5),(3,6),(5,8)\}$ **4B.** $\{(3,5),(5,3),(6,8)\}$ **5A.** $\{(1,5),(2,5),(5,2)\}$
5B. $\{(1,6),(2,2),(3,4)\}$ **6A.** $\{(x,\frac{x-1}{2}); x\in\Re\}$ **6B.** $f^{-1}=\left(x,\frac{x-7}{3}\right): x\in\Re$ **6C.** $\frac{b-dx}{cx-a}$ **6D.** $\sqrt[3]{x-5}$
6E. $2\sqrt[4]{x}$ **6F.** $3\sqrt[3]{x}$ **7A.** $\frac{3x+1}{2}$ **7B.** $-\frac{9}{4}$ **7C.** 2 **7D.** 16 **7E.** 1, 2, 3 **8A.** $\frac{x-3}{3}$
8B. $\frac{x+4}{3}$ **8C.** $x-3$ **9A.** 2 **9B.** 3 **9C.** $\frac{4}{3}$ **9D.** $\frac{62}{7}$ **10A.** $x+2$, 2 **10B.** $3x+4$
10C. $\frac{x+3}{2}$ **13A.** $\frac{x+1}{35}$ **13B.** $\frac{x-7}{2}$ **14A.** $\{(7,1),(8,2),(9,3)\}$ **14B.** $\{(8,5),(10,7),(12,9)\}$ **15A.** $\frac{x}{2}$ **15B.** $\frac{3x-2}{5}$

B. LONG QUESTIONS

1A. $\frac{-2x+5}{x-2}, x\neq 2; \frac{38}{17}$ **1B.** $\frac{3x+1}{x-3}$, 2 **1C.** 2, 6 **1D.** $\frac{5x+9}{4x+7}, \frac{19}{15}$ **1E.** $\frac{2(3x+20)}{7(2-3x)}; -\frac{1}{4}$ **1F.** $\frac{x+4}{3}, -1$
2A. -3 **2B.** 4 **2C.** $\frac{x-4}{2}$ **3A.** $x^2-2x+2, a^2, \sqrt{x-2}, 0$ **3B.** $2a^2+2, 2(x+2)^2, 2a^2-2, 2(a^2-4a+4)$
4A. $\frac{53}{15}$ **4B.** 8 **4C.** $\frac{-5}{2}$ **5A.** 2 **5B.** 1 **5C.** 91 **5D.** 1, 2 **5E.** 2,2
6A. 6 **6B.** 6 **6C.** 9, 1 **6D.** 1, 9 **7A.** $-3, 6$ **7B.** $-2, 5$ **7C.** 1 **7D.** ± 1
8A. 6 **8B.** -7 **8C.** 2 **9A.** -3 **9B.** -2 **9C.** $\frac{x-4}{2}$ **10D.** $\frac{x+4}{2}$ **10E.** $\frac{x-b}{a}$
15B. $\frac{x-5}{4}$

puspas.com.np

www.drmathnp.com

puspas.com.np

CHALLENGING EXERCISE 1

1. $f(x) = 3x - 4$ र $f \circ g^{-1}(x) = 5x + 7$ दिइएका छन् । यदि $g(x) = 5f(x) + 7$ भए x को मान पत्ता लगाउनुहोस् ।
$f(x) = 3x - 4$ and $f \circ g^{-1}(x) = 5x + 7$ are given. If $g(x) = 5f(x) + 7$, find the value of x.

2. मानौं $f(x) + 4 = 3x$ र $g(x) - 7 = 5\ f(x)$ छन् । A को मान कति हुँदा $5[f \circ g^{-1}(A)] = 6A - 32$ हुन्छ ?
Let $f(x) + 4 = 3x$ and $g(x) - 7 = 5\ f(x)$. For what value of A makes $5\ [f \circ g^{-1}(A)] = 6A - 32$?

3. फलन $(f \circ g)^{-1}(x) = 3x + 4$ र $f(x) = x + 2$ दिइएका छन् । यदि $g^{-1}(x) = 13$ भए x को मान पत्ता लगाउनुहोस् ।
Functions $(f \circ g)^{-1}(x) = 3x + 4$ and $f(x) = x + 2$ are given. If $g^{-1}(x) = 13$, find the value of x.

4. f र g का विपरीत फलनहरू क्रमशः $f^{-1}(x) = \frac{x-5}{4}$ र $g^{-1}(x) = \frac{x-2}{3}$ छन् । यदि $f^{-1}[g(x) + 3] = 3$ भए x को मान पत्ता लगाउनुहोस् ।
Inverse functions of f and g are $f^{-1}(x) = \frac{x-5}{4}$ and $g^{-1}(x) = \frac{x-2}{3}$ respectively. If $f^{-1}[g(x) + 3] = 3$, find the value of x.

5. यदि $f(x) = \frac{1}{ax+1}$ र $f \circ f(x) = \frac{2x+1}{2x+3}$ भए a को मान पत्ता लगाई $f^{-1}(1) = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $f(x) = \frac{1}{ax+1}$ and $f \circ f(x) = \frac{2x+1}{2x+3}$, find the value of a and prove that: $f^{-1}(1) = 0$.

6. यदि $2f(x) + f\left(\frac{1}{x}\right) = 4x + \frac{2}{x}$ भए $f^{-1}(4)$ को मान पत्ता लगाउनुहोस् ।
If $2f(x) + f\left(\frac{1}{x}\right) = 4x + \frac{2}{x}$ then find the value of $f^{-1}(4)$.

7. यदि $2\ f(x) - f\left(\frac{1}{x}\right) = \frac{6x^2-3}{x}$ भए $f \circ f(2)$ को मान पत्ता लगाउनुहोस् ।
If $2\ f(x) - f\left(\frac{1}{x}\right) = \frac{6x^2-3}{x}$ then find the value of $f \circ f(2)$.`

8. यदि $2\ f(x) + f\left(\frac{1}{x}\right) = 6x + \frac{3}{x}$ भए $f \circ f(2)$ को मान पत्ता लगाउनुहोस् ।
If $2\ f(x) + f\left(\frac{1}{x}\right) = 6x + \frac{3}{x}$ then find the value of $f \circ f(2)$.

9. यदि $2\ f(x) + 3\ f\left(\frac{1}{x}\right) = \frac{4x^2+6}{x}$ र $f^{-1}(x) = 1$ भए x को मान पत्ता लगाउनुहोस् ।
If $2\ f(x) + 3\ f\left(\frac{1}{x}\right) = \frac{4x^2+6}{x}$ and $f^{-1}(x) = 1$ then find the value of x.

10. यदि $f(x) = \frac{x}{2-x}\ (x \neq 2), g(x) = bx - a,\ g \circ f\ (4) = -8,\ g \circ f^{-1}(1) = 1$ र $g(x) + f(1) = 5$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = \frac{x}{2-x}\ (x \neq 2), g(x) = bx - a,\ g \circ f\ (4) = -8,\ g \circ f^{-1}(1) = 1$ and $g(x) + f(1) = 5$, find the value of x.

11. फलनहरू $f = \{(1, 2), (2, 4), (3, 6)\}$ र $g = \{(4, 9), (5, 11), (6, 13)\}$ छन् । यदि $(f \circ g)^{-1}(x) = \frac{3}{x+2}$ भए x को धनात्मक मान पत्ता लगाउनुहोस् ।
Functions $f = \{(1, 2), (2, 4), (3, 6)\}$ and $g = \{(4, 9), (5, 11), (6, 13)\}$ are given. If $(f \circ g)^{-1}(x) = \frac{3}{x+2}$ then find the positive value of x.

12. दुईओटा रेखीय फलनहरू f र g प्रत्येकमा 0 को प्रतिबिम्ब 0 छ । यदि $f(5) = 10$ र $g(4) = 12$ भए तिनीहरूको संयुक्त फलनको विपरीत फलन पत्ता लगाउनुहोस् ।
Two linear functions f and g having 0 image of 0 each are given. If $f(5) = 10$ and $g(4) = 12$, find the inverse function of linear composite function.

ANSWERS

1. $\frac{3}{4}$ **2.** 5 **3.** 1 **4.** 4 **5.** 2 **6.** 2 **7.** 18 **8.** 18
9. 2 **10.** 2 **11.** 4 **12.** $(gof)^{-1} = \frac{x}{6}$

www.drmathnp.com

2. बहुपदीय POLYNOMIAL

puspas.com.np

DEFINITIONS

1. बहुपदीय (Polynomial)

बहुपदीय भनेको एउटा अनुपातिक अभिव्यञ्जक हो जसका प्रत्येक पदहरूमा धनात्मक घात भएका चललाई अचरले गुणा गरिएको हुन्छ । यसको स्तरीय स्वरूपलाई निम्नानुसार व्यक्त गर्न सकिन्छ । $p(x) = a_0 + a_1 x^1 + a_2 x^2 + \ldots\ldots + a_n x^n$ जहाँ $a_0, a_1, \ldots, a_n$ हरू अचर हुन् भने x चल हो र 1, 2, 3, ……, n चलका घाताङ्कहरू हुन् ।

A polynomial is a rational expression each of whose terms consists a constant multiplied by a positive power of a variable. Its standard form is : $p(x) = a_0 + a_1 x^1 + a_2 x^2 + \ldots\ldots + a_n x^n$, where $a_0, a_1, a_2 \ldots\ldots, a_n$ are constants and x is variable, 1, 2, 3, ….., n are the powers of variable.

2. शेष साध्य (Remainder Theorem) [2057 R, 2060 S, 2066 S, 2068 R, 2069 R']

यदि $p(x)$ डिग्री n भएको बहुपदीय र $p(x)$ को एउटा भाजक $(x - a)$ भए $p(a)$ = शेष हुन्छ । जहाँ भागफल $q(x)$ को डिग्री $(n - 1)$ हुन्छ ।

If $p(x)$ is a polynomial of degree n and $(x - a)$ is a divisor of $p(x)$ then $p(a)$ is remainder, where the degree of quotient will be $(n - 1)$.

3. गुणनखण्ड साध्य (Factor Theorem) [2065 M, 2061 R, 2060 R, 2066 R, 2067 S, 2068 S]

यदि $p(x)$ डिग्री > 0 भएको एउटा बहुपदीय र $p(a) = 0$ भए $p(x)$ को एउटा गुणनखण्ड $(x - a)$ हुन्छ । विलोमको रूपमा, यदि $p(x)$ को एउटा गुणनखण्ड $(x - a)$ भए $p(a) = 0$ हुन्छ ।

If $p(x)$ be a polynomial (degree > 0) and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$. Conversely, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$

4. भागको नियम (Division Algorithm)

यदि बहुपदीय $P(x)$ लाई $D(x)$ ले भाग गर्दा भागफल $Q(x)$ र शेष R भए, $P(x) = D(x) \times Q(x) + R$ हुन्छ ।

If a polynomial $P(x)$ is divided by $D(x)$ so that the quotient is $Q(x)$ and remainder R then, $P(x) = D(x) \times Q(x) + R$.

$$D(x)\overline{)\,P(x)\quad(Q(x)}$$
$$\underline{-\,D(x)\times Q(x)}$$
$$R$$
$$P(x) = D(x) \times Q(x) + R$$

5. सङ्क्षिप्त भाग विधि (Synthetic Division Method)

डिग्री 1 भन्दा ठूलो भएको बहुपदीयका चलहरूको गुणाङ्कहरू मात्र लेखेर डिग्री 1 भएको द्विपदीयको अचर पदको चिन्ह बदलेर गरिने सरलीकृत भागलाई सङ्क्षिप्त भाग विधि भनिन्छ ।

A simplified method for dividing a polynomial of degree > 1 by another polynomial of the first degree by writing down only the coefficients of the several powers of the variable and changing the sign of the constant term, is known as synthetic division method.

6. बहुपदीय समीकरण (Polynomial equation)

चल र अचलहरू मिली बनेका बहुपदहरू भएको समीकरणलाई बहुपदीय समीकरण भनिन्छ ।

A polynomial equation is an equation that has multiple terms made up of numbers and variables.

7. बहुपदीयको मूलहरू वा शुन्यहरू (Roots or zeros of a polynomial)

बहुपदीयको मान शुन्य ल्याउन प्रयोग हुने चलको मानहरूलाई बहुपदीयको मूलहरू वा शुन्यहरू भनिन्छ ।

The roots or zeros of a polynomial are those values of the variable that cause the polynomial to evaluate to zero.

FORMULAE AND KEY POINTS

➢ **शेषसाध्य (Remainder Theorem):**

यदि $p(x)$ लाई $(x - a)$ ले भाग गरियो भने शेष $= p(a)$ हुन्छ । (If $p(x)$ is divided by $(x - a)$, the remainder $= p(a)$.)

भाजक (Divisor)	शेष (Remainder)
$x + a$ i.e. $x - (-a)$	$p(-a)$
$ax - b$ i.e. $a\left(x - \frac{b}{a}\right)$	$p\left(\frac{b}{a}\right)$
$ax + b$ i.e. $a\left(x + \frac{b}{a}\right)$	$p\left(-\frac{b}{a}\right)$
$b - ax$ i.e. $-a\left(x - \frac{b}{a}\right)$	$p\left(\frac{b}{a}\right)$

➢ **गुणनखण्ड साध्य (Factor Theorem)**

(i) यदि $p(x)$ को एउटा गुणनखण्ड $(x - a)$ भए $p(a) = 0$ हुन्छ । (If $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$.)

(ii) यदि $p(a) = 0$ भए $p(x)$ को एउटा गुणनखण्ड $(x - a)$ हुन्छ । (If $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$.)

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 2	2071 R, 2071 S	2
Model 3	2068 R, 2069 R', 2070 S, 2072 R, SEE 2074 R, SEE 2074 S	6
Model 4	2065 R, R', 2068 S', 2070 R, SEE 2073 R, S, S', SEE 2075 R'	8
Model 5	2065 M, 2066 R, S, 2067 R', S, 2068 S, 2070 S', 2071 R', 2074 S'	9
Model 6	2065 S, 2066 R', 2067 R, 2068 R', 2069 R, S, 2070 R', 2072 R', S, 2073 R', SEE 2074 R', SEE 2075 R, R_2, R'_2	14

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, R, S, 2066 R, R', S, 2067 R, R', S, 2068 R, R', S, S', 2069 R, S, 2070 R', S', 2071 R', 2072 R, R', S, 2073 R' SEE 2073 S, 2073 S', 2074 R, 2074 R', SEE 2074 S, S', SEE 2075 R, SEE 2075 R'$_2$, 2075 R', SEE MODEL 2076	22
Model 2	2065 R', 2069 R', 2070 R, 2070 S, 2071 R, 2071 S, 2073 R, SEE 2075 R_2	8

SEE EXERCISE 2

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् । (Define the following terms.)

- **A.** बहुपदीय (Polynomial)
- **B.** बहुपदीयको डिग्री (Degree of polynomial)
- **C.** सङ्क्षिप्त भाग विधि (Synthetic Division method)
- **D.** रेखीय बहुपदीय (Linear Polynomial)
- **E.** भागको सिद्धान्त (Division Algorithm)
- **F.** अगाडिका गुणाङ्कहरू (Leading Coefficients)
- **G.** बहुपदीय समीकरण (Polynomial equation)
- **H.** बहुपदीयको मूल वा शुन्य (Root or zero of a polynomial)

2A. यदि $f(x)$ को डिग्री '$4m$' र $g(x)$ को डिग्री 'm' भए $f(x) \div g(x)$ को डिग्री कति होला ?
If the degree of $f(x)$ is '$4m$' and degree of $g(x)$ is 'm,' what is the degree of $f(x) \div g(x)$?

2B. यदि $f(x)$ को डिग्री 'm' र $g(x)$ को डिग्री 'n', $m > n$ भए $f(x) \div g(x)$ को डिग्री कति होला ?
If the degree of $f(x)$ is 'm' and degree of $g(x)$ is 'n', $m > n$ then what is the degree of $f(x) \div g(x)$?

2C. यदि $f(x)$ र $g(x)$ दुईओटा बहुपदीयहरू भए कस्तो अवस्थामा $f(x) \div g(x)$ अपरिभाषीत हुन्छ ?
If $f(x)$ and $g(x)$ be the two polynomials, under what condition $f(x) \div g(x)$ will be undefined?

3A. यदि भागफल $q(x)$, शेष $r(x)$, र भाजक $d(x)$ भए बहुपदीय $f(x)$ लाई $q(x)$, $r(x)$ र $d(x)$ को रूपमा लेख्नुहोस् ।
If the quotient $q(x)$, remainder $r(x)$, and divisor $d(x)$ then write the polynomial $f(x)$ in terms of $q(x)$, $r(x)$ and $d(x)$.

3B. यदि भागफल Q, शेष R, र भाजक D भए बहुपदीय $f(x)$ लाई Q, R र D को रूपमा लेख्नुहोस् ।
If the quotient Q, remainder R, and divisor D then write the polynomial $f(x)$ in terms of Q, R and D.

3C. $f(x) = q(x) \times d(x) + r(x)$ मा हरेक पदको अर्थ लेख्नुहोस् ।
In $f(x) = q(x) \times d(x) + r(x)$, write the meaning of each term.

4A. सङ्क्षिप्त भाग विधिमा भाज्य र भागफलको डिग्रीहरूको अन्तर कति हुनुपर्छ ?
What should be the difference of degree of dividend and quotient in the Synthetic division method ?

4B. सङ्क्षिप्त भाग विधिमा भाजक र भागफलको डिग्रीको योगफल कति हुन्छ ?
What should be the sum of degree of divisor and quotient in the Synthetic division method ?

4C. सङ्क्षिप्त भाग विधिमा भाजक (divisor) को डिग्री कति हुन्छ ?
What is the degree of divisor in synthetic division method?

5A. शेष साध्य उल्लेख गर्नुहोस् । (State Remainder Theorem.)

5B. यदि एउटा बहुपदीय $f(x)$ लाई $(x - a)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(x - a)$, what will be its remainder?

5C. यदि एउटा बहुपदीय $f(x)$ लाई $(x + b)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(x + b)$, what will be its remainder?

6A. यदि एउटा बहुपदीय $q(x)$ लाई $(ax - b)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $q(x)$ is divided by $(ax - b)$, what will be its remainder?

6B. यदि एउटा बहुपदीय $f(x)$ लाई $(px + q)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(px + q)$, what is its remainder?

www.drmathnp.com

7A. गुणनखण्ड साध्य उल्लेख गर्नुहोस् । (State the Factor Theorem.)

7B. जब एउटा बहुपदीय $q(x)$ लाई शेष $q(-a)=0$ हुने गरी अर्को बहुपदीयले भाग गरिन्छ भने एउटा गुणनखण्ड के होला ?
When a polynomial $q(x)$ is divided by another polynomial such that remainder $q(-a)=0$, what will be its one of the factor?

7C. यदि बहुपदीय $p(x)$ को एउटा गुणनखण्ड $(x-m)$ भए $p(m)$ को मान कति होला ?
If $(x-m)$ is a factor of polynomial $p(x)$, what is the value of $p(m)$?

8A. एउटा बहुपदीय $f(x)$ मा $f\left(\frac{b}{a}\right)=0$ भए यसका गुणनखण्डहरू मध्य एउटा गुणनखण्ड पत्ता लगाउनुहोस् ।

In a polynomial $f(x)$, $f\left(\frac{b}{a}\right)=0$, what will be its one of the factors?

8B. $f\left(-\frac{m}{n}\right)=0$ हुनेगरी एउटा बहुपदीय $f(x)$ छ । यसका गुणखनण्डहरू मध्य एउटा गुणनखण्ड पत्ता लगाउनुहोस् ।

$f(x)$ is a polynomial such that $f\left(-\frac{m}{n}\right)=0$. Find its one of the factors.

9A. $f(x)=(x-c)\times q(x)+r(x)$ मा $f(x)$ को एउटा गुणनखण्ड $x-c$ भए शेष $r(x)$ कति हुन्छ ?
In $f(x)=(x-c)\times q(x)+r(x)$, $x-c$ is a factor of $f(x)$ then what will be the remainder $r(x)$?

9B. $f(x)$, $d(x)$ र $q(x)$ मा $f(x)$ ले n डिग्रीको बहुपदीय, $d(x)$ ले भाजक र $q(x)$ ले भागफललाई जनाउँछ भने यिनीहरू बिचको सम्बन्ध लेख्नुहोस्, जहाँ $d(x)$ ले $f(x)$ को गुणनखण्डलाई जनाउँछ ।
In $f(x)$, $d(x)$ and $q(x)$, $f(x)$ represents a polynomial of degree n, $d(x)$ is divisor and $q(x)$ is quotient then write the relation between them where $d(x)$ represents a factor of $f(x)$.

10A. बहुपदीय $f(x)=x^2-a^2=0$ का मूलहरू के के हुन् ? (What are the roots of the polynomial $f(x)=x^2-a^2=0$?)

10B. बहुपदीय $p(x)=x^2-m^2$ का शुन्यहरू के के हुन् ? (What are the zeros of the polynomial $p(x)=x^2-m^2$?)

Model 2 *Concept Recall*

11A. बहुपदीय x^3+x^2+2x+5 को डिग्री कति हुन्छ ? (What is the degree of polynomial x^3+x^2+2x+5?)

11B. बहुपदीय $f(x)=2x^3+4x^2+6x+7$ को डिग्री कति हुन्छ ? (What is the degree of polynomial $f(x)=2x^3+4x^2+6x+7$?)

12A. यदि $f(x)=x^5-2x^3+7$ र $g(x)=x^2-3$ भए $f(x)\div g(x)$ को भागफलको डिग्री पत्ता लगाउनुहोस् ।
If $f(x)=x^5-2x^3+7$ and $g(x)=x^2-3$ then what is the degree of quotient in $f(x)\div g(x)$?

12B. यदि $p(x)=x^3-8$ र $q(x)=x-2$ भए $p(x)\div q(x)$, को भागफलको डिग्री पत्ता लगाउनुहोस् ।
If $p(x)=x^3-8$ and $q(x)=x-2$ then what is the degree of quotient when $p(x)\div q(x)$?

13A. शेष कति होला जब एउटा बहुपदीय $f(x)$ लाई $(2x+3)$ ले भाग गरिएको छ ?
What is the remainder when a polynomial $f(x)$ is divided by $(2x+3)$?

13B. शेष कति होला जब एउटा बहुपदीय x^3-1 लाई $(x-2)$ ले भाग गरिएको छ ?
What is the remainder when a polynomial x^3-1 is divided by $(x-2)$?

13C. शेष कति होला जब एउटा बहुपदीय x^3-4 लाई $(x+1)$ ले भाग गरिएको छ ?
What is the remainder when a polynomial x^3-4 is divided by $(x+1)$?

14A. यदि $f(x)=x^2-4$ को एउटा गुणनखण्ड $(x-k)$ भए k को मान कति हुन्छ ?
If $(x-k)$ is a factor of $f(x)=x^2-4$, what is the value of k ?

14B. यदि बहुपदीय $p(x)=8x^3-8$ को एउटा गुणनखण्ड $(x-1)$ भए $p(1)$ को मान कति हुन्छ ?
If $(x-1)$ is a factor of polynomial $p(x)=8x^3-8$, what is the value of $p(1)$?

14C. यदि बहुपदीय $p(x)=5x^3+5$ को एउटा गुणनखण्ड $(x-1)$ भए $p(1)$ को मान कति हुन्छ ?
If $(x-1)$ is a factor of polynomial $p(x)=5x^3+5$, what is the value of $p(1)$?

15A. $f(x)=x^2-16$ का शुन्यहरू अनुमान गर्नुहोस् । (Guess the zeros of the polynomial $f(x)=x^2-16$.)

15B. बहुपदीय $p(x)=(x+3)(x-4)$ का मूलहरू पत्ता लगाउनुहोस् ।
Find the roots of the polynomial $p(x)=(x+3)(x-4)$.

B. SHORT QUESTIONS

Model 1 *Problems based on long division of polynomial.*

1. भाग गर्नुहोस् (Divide):

A. $(x^3-27)\div(x-3)$ **B.** $x^4-16\div x-2$

C. $(x^3+6x^2+12x+8)\div(x+2)$ **D.** $(6x^3+x^2-19x+6)\div(x-2)$

www.drmathnp.com

puspas.com.np

2. भागफल q(x), शेष r(x), र भाजक d(x) दिइएको अवस्थामा बहुपदीय $f(x)$ पत्ता लगाउनुहोस् :
Find the polynomial $f(x)$ when quotient q(x), remainder r(x) and divisor d(x) are given.

A. d(x) = ($x-1$); q(x) = $4x+5$; r(x) = 7
B. d(x) = ($2x-3$); q(x) = $2x+3$; r(x) = 5
C. d(x) = ($7-x$); q(x) = x^2+x+1; r(x) = 7
D. d(x) = (x^2+3); q(x) = $3x-5$; r(x) = $1-x$

Model 2 Problems based on synthetic division.

3. सङ्क्षिप्त भाग विधिबाट भागफल र शेष पत्ता लगाउनुहोस् :
Find the quotient and remainder using synthetic division method.

A. $(y^3+y^2-20) \div (y+2)$
B. $(2x^3-7x^2-4x+5) \div (x-3)$
C. $(x^3-7x^2+13x+3) \div (x-2)$
D. $(x^3-3x+10) \div (x+1)$
E. $(5x^4-2x+5) \div (x+3)$
F. $(x^7+x^6-x^5-2x^4+2) \div (x+1)$

4. सङ्क्षिप्त भाग विधिबाट भागफल र शेष पत्ता लगाउनुहोस् ।
Find the quotient and remainder using synthetic division method.

A. $(2x^4-3x^2-1) \div (2x-1)$
B. $(4x^4-3x^2+2) \div (4x-1)$
C. $(3x^4-7x^2+6x-2) \div (3x+2)$
D. $(4x^4-3x^2+7x+8) \div (2x+3)$

5A. यदि $2x^3-7x^2+x+10 = (x-1)\,Q(x) + R$ भए शेष R र बहुपदीय $Q(x)$ पत्ता लगाउनुहोस् ।
If $2x^3-7x^2+x+10 = (x-1)\,Q(x) + R$, find the remainder R and polynomial $Q(x)$. [2058 S]

5B. यदि $x^3-19x-30 = (x+2).Q(x)$ भए सङ्क्षिप्त भाग विधिको प्रयोग गरेर $Q(x)$ पत्ता लगाउनुहोस् ।
If $x^3-19x-30 = (x+2).Q(x)$, find $Q(x)$ by using synthetic division method. [2071 R]

5C. यदि $x^3-21x-20 = (x+1).Q(x)$ भए सङ्क्षिप्त भाग विधि प्रयोग गरेर $Q(x)$ पत्ता लगाउनुहोस् ।
If $x^3-21x-20 = (x+1).Q(x)$, find $Q(x)$ by using synthetic division method. [2071 S]

Model 3 Problems based on Remainder theorem.

6. शेष साध्यको परिभाषा लेख्नुहोस् र सो साध्य प्रयोग गरी शेष पत्ता लगाउनुहोस् ।
State the remainder theorem and use it to find the remainder.

A. $x^6-1 \div (x+1)$ [2068 R]
B. $x^3+5x^2-6x-16 \div (x+2)$ [2070 S]
C. $2x^3-7x^2+5x+94 \div (x-3)$ [2069 R']
D. $2x^3-7x^2+5x+4 \div (x-3)$ [2060 S, SEE 2074 R]
E. $3x^3-5x^2+2x-3 \div (x-2)$ [SEE 2074 S, 2057 R]
F. $8x^3-4x^2+2x-5 \div (2x-1)$ [2072 R]

7A. यदि $f(x) = x^3-x^2+1$ र $g(x) = x+1$ भए $f(x)$ लाई $g(x)$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
If $f(x) = x^3-x^2+1$ and $g(x) = x+1$ then find the remainder when $f(x)$ is divided by $g(x)$.

7B. शेष साध्य प्रयोग गरी x^3-x^2+1 लाई $x-2$ ले भाग गर्दा आउने शेष पत्ता लगाउनुहोस् ।
Find the remainder when x^3-x^2+1 is divided by $x-2$, using remainder theorem.

8. तल दिइएको अवस्थामा शेष साध्य प्रयोग गरी शेष पत्ता लगाउनुहोस् :
Find the remainder using remainder theorem in the following conditions.

A. $(4x^2+6x+8) \div (2x-1)$
B. $(6x^3+4x^2+3x+4) \div (3x-4)$
C. $8x^4+5x^3-2x^2+7x-1) \div (2x+6)$
D. $(5x^4-6x^3+8x^2-10x+12) \div (3x+9)$

Model 4 Problems based on finding value of k or p or a when the remainder is given.

9. k, p वा a को मान पत्ता लगाउनुहोस् । (Find the value of k, p or a.)

	बहुपदीय (Polynomial)	भाजक (Divisor)	शेष (Remainder)		
A.	$f(x) = 4x^3-3x^2+3x-k$	$(x-2)$	12	k = ?	[SEE 2075 R']
B.	$2x^3-4x^2+kx+10$	$(x+2)$	4	k = ?	[SEE 2073 R]
C.	$x^3-kx^2-13x+10$	$(x+2)$	4	k = ?	[SEE 2073 S]
D.	$2x^3+3x^2-px+4$	$(x+3)$	10	p = ?	[SEE 2073 S']
E.	x^3+ax^2-x+7	$(x-3)$	4	a = ?	[2062K]
F.	$x^3+6x^2+kx+10$	$(x+2)$	4	k = ?	[2070 R]
G.	$f(x) = x^3-px^2-(2-p)x+32$	$(x+2)$	10	p = ?	[2068 S']
H.	$f(x) = x^3-(p-2)x^2-px+28$	$(x+3)$	10	p = ?	[2065 R]
I.	$2x^3+3x^2-kx+4$	$(x-2)$	$2k$	k = ?	[2065 R']

www.drmathnp.com

J. फलन $f(x) = 2x^4 - 3x^2 + 6x + k$ दिइएको छ । यदि $f(1) = 0$ भए k को मान निकाल्नुहोस् ।
Given that the function $f(x) = 2x^4 - 3x^2 + 6x + k$. If $f(1) = 0$, find the value of k. **[2060 R]**

10A. यदि बहुपदीय $x^4 + 5x^3 - kx^2 + 7x + 10$ लाई $(x + 1)$ ले भाग गर्दा शेष 12 रहन्छ भने k को मान पत्ता लगाउनुहोस् ।
If polynomial $x^4 + 5x^3 - kx^2 + 7x + 10$ is divided by $(x + 1)$, the remainder is 12. Find the value of k.

10B. यदि बहुपदीय $f(x) = x^3 - 9x^2 + (k + 1)x - 7$ को एउटा गुणनखण्ड $(x - 7)$ भए k को मान पत्ता लगाउनुहोस् ।
If $(x - 7)$ is a factor of polynomial $f(x) = x^3 - 9x^2 + (k + 1)x - 7$, find the value of k.

Model 5 *Problems based on factor theorem.*

11. गुणनखण्ड साध्यको कथन लेख्नुहोस् र सो साध्य प्रयोग गरी बहुपदीयको भाजक गुणनखण्ड हो कि होइन यकिन गर्नुहोस् ।
State factor theorem. Use factor theorem to determine whether the divisor is a factor of the polynomial or not.

	बहुपदीय (Polynomial)	भाजक (Divisor)	
A.	$x^6 - 64$	$(x - 2)$	
B.	$x^3 - 8x + 3$	$(x + 3)$	**[2071 R']**
C.	$3x^3 - 2x^2 + 5x - 14$	$(x - 2)$	**[2066 S]**
D.	$x^3 - 15x + 4$	$x + 4$	**[2068 S]**
E.	$3x^3 + 2x - 5$	$(x - 1)$	**[2065 M]**
F.	$f(x) = x^3 - 3x^2 - 4x + 12$	$(x + 2)$	**[2066 R]**
G.	$3x^3 - 15x + 6$	$(x - 2)$	**[2067 R']**
H.	$f(x) = x^3 + 6x^2 + 7x + 2$	$(x + 1)$	**[2067S]**
I.	$2y^3 + 4y^2 + 3y + 6$	$(y + 2)$	**[2070 S']**
J.	$x^3 - 9x^2 + 24x - 20$	$(x - 2)$	**[2074 S']**

Model 6 *Problems based on finding the value of letters when one factor is given.*

12. बहुपदीय र सोको एउटा गुणनखण्ड दिइएको अवस्थामा वर्णहरूको मान पत्ता लगाउनुहोस् ।
Find the value of letters when a polynomial and its one factor is given.

	बहुपदीय (Polynomial)	गुणनखण्ड (Factor)	पत्ता लगाउनुपर्ने (To find)	
A.	$p(x) = 2x^3 - 6x^2 - 5m - 2$	$(x - 2)$	m	**[SEE 2075 R]**
B.	$2x^3 + 3x^2 - mx + 4$	$(x + 2)$	m	**[SEE 2075 R_2]**
C.	$f(x) = x^3 - 3x^2 - 4x + k$	$(x - 2)$	k	**[SEE 2075 R'_2]**
D.	$x^3 - kx^2 + x - 6 = 0$	$(x - 2)$	'k'	**[2069 R]**
E.	$2x^3 - x^2 + 10x - k$	$(x - 3)$	k	**[SEE 2073 R']**
F.	$x^3 + px^2 + 4x + 5$	$(x - 5)$	p	**[2072 S]**
G.	$2x^3 - kx^2 - 8x + 5$	$(x + 1)$	k	**[2061 R, 2068 R']**
H.	$x^3 + kx^2 - 4x + 12$	$(x + 2)$	k	**[2057 S, SEE 2074 R']**
I.	$P(x) = 2x^3 + 3x^2 - kx + 4$	$(x + 3)$	k	**[2065 S]**
J.	$2x^3 - 7px + (p - 12)$	$(x - 5)$	p	**[2072 R']**
K.	$x^3 - (k - 1)x^2 + kx + 54$	$(x + 3)$	k	**[2058 R]**
L.	$x^3 + 4x^2 + kx - 30$	$(x - 3)$	k	**[2061 S]**
M.	$P(x) = 2x^3 + 3x^2 + kx + 4$	$(x - 3)$	k	**[2066 R']**
N.	$2x^3 + x^2 - kx - 1 = 0$	$(x - 1)$	k	**[2060 C]**
O.	$3x^3 + Kx^2 - 2x - 8$	$(x + 2)$	K	**[2059 S, 2067 R]**
P.	$a^4 - 3a^3 - 2a^2 + a + P$	$(a + 1)$	P	**[2060 S']**
Q.	$x^3 - 19x - p$	$(x + 2)$	p	**[2070 R']**
R.	$x^3 - mx^2 - 4x + m + 12$	$(x - m)$	m	**[2069 S]**

puspas.com.np

www.drmathnp.com

puspas.com.np

13A. $2x^3 + ax^2 + x + 2$ को एउटा गुणनखण्ड $2x + 1$ भए, a को मान पत्ता लगाउनुहोस् ।
If $2x + 1$ is a factor of $2x^3 + ax^2 + x + 2$, find the value of a. [2059 R]

13B. यदि $6x^3 - (k + 6)x^2 + 2kx - 25$ को एउटा गुणनखण्ड $2x - 5$ भए k को मान निकाल्नुहोस् ।
If $2x - 5$ is a factor of $6x^3 - (k + 6)x^2 + 2kx - 25$, find the value of k. [2062 R]

C. LONG QUESTIONS

हल गर्नुहोस् (Solve):

Model 1 Problems based on solving cubic equations (non-zero coefficients.

1. **A.** $x^3 - 3x^2 - 4x + 12 = 0$ [SEE MODEL 2076] **B.** $x^3 - 7x^2 + 7x + 15 = 0$ [2057 R, 2066R']
C. $x^3 - 4x^2 - 7x + 10 = 0$ [2057 S] **D.** $y^3 - 8y^2 + 19y - 12 = 0$ [2065 S, 2066 R]
E. $x^3 - 3x^2 - 10x + 24 = 0$ [2065 R, 2067 R', 2072 R'] **F.** $x^3 - 6x^2 + 11x - 6 = 0$ [2075R', 66 S, 67 S, 68 S', 73 R']
G. $x^3 - 4x^2 + x + 6 = 0$ [2070 R'] **H.** $x^2(x - 1) = 2(7x - 12)$ [2073 S']
I. $x(x^2 + 3) - 2(3x^2 - 5) = 0$ [2075 R'$_2$] **J.** $x(x^2 - 9x + 23) = 15$

2. **A.** $2x^3 - 9x^2 + 7x + 6 = 0$ [SEE 2075 R] **B.** $2x^3 + 9x^2 + 7x - 6 = 0$ [SEE 2074 R]
C. $2x^3 + 3x^2 - 11x - 6 = 0$ [2059R,69S,71R',2074 R'] **D.** $x^2(2x + 3) - (11x + 6) = 0$ [SEE 2073 S]
E. $2x^3 - 3x^2 - 11x + 6 = 0$ [2062 R, 2070S'] **F.** $2x^3 + x^2 - 2x - 1 = 0$ [2060 R]
G. $2x^3 - 3x^2 - 3x + 2 = 0$ [2074 S', 2060 S, 2065 M] **H.** $2x^3 + 5x^2 - 4x - 3 = 0$ [2061 S]
I. $2x^3 - 5x^2 - 6x + 9 = 0$ [2060 C] **J.** $2x^3 - 4x^2 - 7x + 14 = 0$ [2068 S]
K. $2x^3 - 3x^2 - 18x - 8 = 0$ [2069 R] **L.** $2x^3 + 6 = 3x^2 + 11x$ [2068 R, 2072 R]
M. $2x^3 + x^2 - 5x + 2 = 0$ [SEE 2074 S] **N.** $2(x^3 - 3) + x = 3x(4 - x)$

3. **A.** $6x^3 + 7x^2 - x - 2 = 0$ [2058 S] **B.** $6x^3 - 7x^2 - 7x + 6 = 0$ [2059 S]
C. $6x^3 - 13x^2 + x + 2 = 0$ [2062 S] **D.** $8x^3 - 2x^2 - 5x - 1 = 0$ [2058 R]
E. $6x^3 - 5x^2 - 3x + 2 = 0$ [2067 R] **F.** $3y^3 - 14y^2 - 7y + 10 = 0$ [2068 R']
G. $6x^3 + x^2 - 19x + 6 = 0$ [2072 S] **H.** $3x^2(x - 1) = (x - 1)(4 - x)$

Model 2 Problems based on solving cubic equations (one-zero coefficient)

4. **A.** $x^3 - 3x - 2 = 0$ [2060 S'] **B.** $z^3 - 19z - 30 = 0$ [2065 R']

5. **A.** $2x^3 + 13x^2 = 36$ [2075 R$_2$] **B.** $3x^3 - 13x^2 + 16 = 0$ [2061 R, 2069 R']
C. $x^3 - 21x - 20 = 0$ [2070 R] **D.** $6x^3 = 4 - 13x^2$ [2071 R]
E. $2x^3 = 13x^2 - 36$ [2070 S] **F.** $x^3 = 7x^2 - 36$ [2071 S]
G. $3x^3 = 7x^2 - 4$ [SEE 2073 R] **H.** $2(x^3 + 1) = 3x^2 + 1$

Model 3 Problems based on solving cubic equation and a linear equation.

6. **A.** $y = x^3 - 4x^2 + x + 8$ and $y = 2$ [2062K] **B.** $y = 7$ and $y = x^3 + 7x^2 - 21x - 20$

Model 4 Problems based on Remainder theorem and Factor theorem.

7A. $(x + 1)$, $(x - 4)$ र $(2x - 3)$ हरू $2x^3 - 9x^2 + x + 12$ का गुणनखण्डहरू हुन् भनी देखाउनुहोस् ।
Show that $2x^3 - 9x^2 + x + 12$ has the factors $(x + 1)$, $(x - 4)$ and $(2x - 3)$.

7B. $x^3 + 7x(x - 3) - 27$ का गुणनखण्डहरू $(x + 1)$, $(x - 3)$ र $(x + 9)$ हुन् भनी प्रमाणित गर्नुहोस् ।
Prove that $(x + 1)$, $(x - 3)$ and $(x + 9)$ are the factors of $x^3 + 7x(x - 3) - 27$.

7C. गुणनखण्ड साध्यको प्रयोग गरी $f(x) = 6x^3 + 11x^2 - 26x - 15$ का गुणनखण्डहरू $(x + 3)$, $(2x + 1)$, $(3x + 5)$ मध्ये कुन कुन हुन् ? पत्ता लगाउनुहोस् ।
Out of $(x + 3)$, $(2x + 1)$ and $(3x + 5)$, which are the factors of $f(x) = 6x^3 + 11x^2 - 26x - 15$? Find by using factor theorem.

8A. यदि $2x^2 - 5x + a$ र $x^3 - x^2 + ax + 5$ दुवैलाई $(x + 2)$ ले भाग गर्दा बराबर शेष आउँछ भने a को मान पत्ता लगाउनुहोस् ।
If $2x^2 - 5x + a$ and $x^3 - x^2 + ax + 5$ leave equal remainder when divided by $(x + 2)$, find the value of a.

8B. यदि $x^3 - ax^2 + 8x + 11$ र $2x^3 - ax^2 + 7ax + 13$ दुवैलाई $(x - 1)$ ले भाग गर्दा बराबर शेष आउँछ भने a को मान पत्ता लगाउनुहोस् ।
If $x^3 - ax^2 + 8x + 11$ and $2x^3 - ax^2 + 7ax + 13$ leave equal remainder when divided by $(x - 1)$, find the value of a.

www.drmathnp.com

ANSWERS

A. VERY SHORT QUESTIONS

2A. $3m$ **2B.** $m-n$ **3A.** $f(x)=q(x)\times d(x)+r(x)$ **3B.** $f(x)=Q\times D+R$ **4A.** >1

4B. degree of dividend **4C.** degree of (dividend – quotient) **5B.** $f(a)$ **5C.** $f(-b)$ **6A.** $q\left(\frac{b}{a}\right)$

6B. $f\left(-\frac{q}{p}\right)$ **7B.** $(x+a)$ **7C.** 0 **8A.** $x-\frac{b}{a}$ **8B.** $\left(x+\frac{m}{n}\right)$ **9A.** $f(c)$ **10A.** $\pm a$

10B. $\pm m$ **11A.** 3 **11B.** 3 **12A.** 3 **12B.** 2 **13A.** $f\left(-\frac{3}{2}\right)$ **13B.** 7

13C. -5 **14A.** ± 2 **14B.** 0 **14C.** 10 **15A.** ± 4 **15B.** $-3, 4$

B. SHORT QUESTIONS

1A. x^2+3x+9 **1B.** x^3+2x^2+4x+8 **1C.** x^2+4x+4 **1D.** $6x^2+13x+7$ and Remainder $=20$ **2A.** $4x^2+x+2$

2B. $4x^2-4$ **2C.** $-x^3+6x^2+6x+14$ **2D.** $3x^3-5x^2+8x-14$ **3A.** $y^2-y+2;-24$ **3B.** $2x^2-x-7;-16$

3C. $x^2-5x+3;9$ **3D.** $x^2-x-2;12$ **3E.** $5x^3-15x^2+45x-137;416$ **3F.** $x^6-x^4-x^3+x^2-x+1;1$

4A. $x^3+\frac{x^2}{2}-\frac{5x}{4}-\frac{5}{8};-\frac{13}{8}$ **4B.** $x^3+\frac{x^2}{4}-\frac{11x}{16}-\frac{11}{64};\frac{117}{64}$ **4C.** $x^3-\frac{2}{3}x^2-\frac{17}{9}x+\frac{88}{27};-\frac{230}{27}$ **4D.** $2x^3-3x^2+3x-1;11$

5A. $R=6, Q(x)=2x^2-5x-4$ **5B.** $x^2-2x-15$ **5C.** x^2-x-20 **6A.** 0 **6B.** 8 **6C.** 100

6D. 10 **6E.** 5 **6F.** -4 **7A.** -1 **7B.** 5 **8A.** 12 **8B.** $\frac{88}{3}$ **8C.** 473 **8D.** 681

9A. $k=14$ **9B.** -13 **9C.** 6 **9D.** 11 **9E.** -3 **9F.** 11 **9G.** 3 **9H.** $\frac{3}{2}$ **9I.** 8 **9J.** -5

10A. -13 **10B.** 14 **11A.** *Yes* **11B.** *Yes* **11C.** *Not* **11D.** *Yes* **11E.-11J.** *Yes* **12A.** $m=-2$ **12B.** 0

12C. $k=12$ **12D.** $k=1$ **12E.** 75 **12F.** -6 **12G.** 11 **12H.** -3 **12I.** $\frac{23}{3}$ **12J.** 7 **12K.** 3

12L. -11 **12M.** $-28\frac{1}{3}$ **12N.** 2 **12O.** 7 **12P.** -1 **12Q.** 30 **12R.** 4 **13A.** -5 **13B.** 25

C. LONG QUESTIONS

1A. $-2, 2, 3$ **1B.** $-1, 3, 5$ **1C.** $1, -2, 5$ **1D.** $1, 3, 4$ **1E.** $2, -3, 4$ **1F.** $1, 3, 2$ **1G.** $3, -1, 2$

1H. $-4, 3, 2$ **1I.** $-1, 2, 5$ **1J.** $1, 3, 5$ **2A.** $\frac{-1}{2}, 2, 3$ **2B.** $-2, -3, \frac{1}{2}$ **2C.** $2, -3, -\frac{1}{2}$ **2D.** $2, -3, -\frac{1}{2}$

2E. $-2, \frac{1}{2}, 3$ **2F.** $1, -1, \frac{-1}{2}$ **2G.** $-1, 2, \frac{1}{2}$ **2H.** $1, -3, -\frac{1}{2}$ **2I.** $1, 3, -\frac{3}{2}$ **2J.** $2, \pm\sqrt{\frac{7}{2}}$ **2K.** $4, -\frac{1}{2}, -2$

2L. $-2, \frac{1}{2}, 3$ **2M.** $1, -2, \frac{1}{2}$ **2N.** $2, -3, -\frac{1}{2}$ **3A.** $-1, \frac{1}{2}, -\frac{2}{3}$ **3B.** $-1, \frac{2}{3}, \frac{3}{2}$ **3C.** $2, \frac{1}{2}, -\frac{1}{3}$ **3D.** $1, -\frac{1}{4}, -\frac{1}{2}$

3E. $1, -\frac{2}{3}, \frac{1}{2}$ **3F.** $-1, 5, \frac{2}{3}$ **3G.** $-1, \frac{3}{2}, \frac{1}{3}$ **3H.** $1, 1, -\frac{4}{3}$ **4A.** $2, -1, -1$ **4B.** $5, -3, -2$

5A. $-2, -6, \frac{3}{2}$ **5B.** $4, -1, \frac{4}{3}$ **5C.** $-4, -1, 5$ **5D.** $-2, \frac{-2}{3}, \frac{1}{2}$ **5E.** $-\frac{3}{2}, 6, 2$ **5F.** $-2, 3, 6$ **5G.** $1, 2, -\frac{2}{3}$

5H. $1, 1, -\frac{1}{2}$ **6A.** $-1, 2, 3$ **6B.** $-1, 3, -9$ **7C.** $(x+3)$ and $(2x+1)$ **8A.** $-\frac{25}{3}$ **8B.** $\frac{5}{7}$

GENERAL EXERCISE 2

A. SHORT QUESTIONS

Model 1 *On finding quotient, divisor & remainder (remainder theorem & synthetic division).*

1A. यदि $x^4-5x^3+2x-5=(x-5)\,Q(x)+R$ भए $Q(x)$ र R पत्ता लगाउनुहोस् ।
If $x^4-5x^3+2x-5=(x-5)\,Q(x)+R$, find $Q(x)$ and R.

1B. यदि $2x^3-11x^2+19x-10=(x-5)\,Q(x)+R$ भए $Q(x)$ र R पत्ता लगाउनुहोस् ।
If $2x^3-11x^2+19x-10=(x-5)\,Q(x)+R$, find $Q(x)$ and R.

1C. यदि $x^3+x^2-x+1=D(x)\,(x^2+2x+1)+2$ भए $D(x)$ पत्ता लगाउनुहोस् ।
If $x^3+x^2-x+1=D(x)\,(x^2+2x+1)+2$ then find $D(x)$.

Model 2 *Problems based on finding value of a or k or p or m when remainder is given.*

2A. यदि एउटा बहुपदीय $2x^3+3x^2+kx+4$ लाई $(x-2)$ ले भाग गर्दा शेष $2(4-k)$ हुने भए k को मान पत्ता लगाउनुहोस् ।
If a polynomial $2x^3+3x^2+kx+4$ leaves a remainder $2(4-k)$ when divided by $(x-2)$, find the value of k.

2B. यदि एउटा बहुपदीय $x^3-ax^2-2x+2a+6$ लाई $(x-a)$ ले भाग गर्दा शेष $(a+2)$ हुने भए a को मान पत्ता लगाउनुहोस् ।
If a polynomial $x^3-ax^2-2x+2a+6$ leaves a remainder $(a+2)$ when divided by $(x-a)$, find the value of a.

2C. यदि एउटा बहुपदीय $p(y)=(y+3)^9+(3y+m)^3$ लाई $(y+3)$ ले भाग गरियो भने -8 शेष रहन्छ, m को मान पत्ता लगाउनुहोस् ।
If a polynomial $p(y)=(y+3)^9+(3y+m)^3$ is divided by $(y+3)$ leaves the remainder -8, find the value of m.

3A. यदि ax^3+2x^2-3 र x^2-ax+4 दुवैलाई $x-2$ ले भाग गरियो भने बराबर शेष रहन्छ भने a को मान पत्ता लगाउनुहोस् ।
If both ax^3+2x^2-3 and x^2-ax+4 leaves the same remainder when divided by $x-2$, find a.

3B. यदि px^3-3x^2+9 र x^2-px+4 दुवैलाई $x+1$ ले भाग गरियो भने बराबर शेष रहन्छ भने p को मान पत्ता लगाउनुहोस् ।
If both px^3-3x^2+9 and x^2-px+4 leaves the same remainder when divided by $x+1$, find p.

puspas.com.np

www.drmathnp.com

pustas.com.np

3C. यदि एउटा बहुपदीय $4x^4 - (p+1)x^2 + 8x + 14$ लाई $2x + 1$ ले भाग गर्दा शेष 8.5 आयो भने p को मान शेष साध्य प्रयोग गरी पत्ता लगाउनुहोस् ।
If a polynomial $4x^4 - (p+1)x^2 + 8x + 14$ is divided by $2x + 1$, leaves the remainder 8.5. Find the value of p by using remainder theorem.

3D. बहुपदीय $x^3 - 19x - \frac{3}{m}$ लाई $(x+3)$ ले भाग गर्दा आउने शेष निकाल्नुहोस् । साथै शेष 9 भएको अवस्थामा 'm' को मान कति होला ?
Find the remainder when a polynomial $x^3 - 19x - \frac{3}{m}$ is divided by $(x + 3)$. Find the value of 'm' if the remainder is 9.

3E. यदि $f(x) = x^3 + 4x^2 + kx + 6$ लाई $(2x - 1)$ ले नि:शेष भाग जान्छ भने k को मान निकाल्नुहोस् ।
Find the value of k if $f(x) = x^3 + 4x^2 + kx + 6$ is exactly divisible by $(2x - 1)$.

Model 3 Problems based on proving a factor by using factor theorem.

4A. $p(x) = x(x-1)(x-2)(x-3) - 120$ को एउटा गुणनखण्ड $(x - 5)$ हो भनी प्रमाणित गर्नुहोस् ।
Prove that the polynomial $p(x) = x(x-1)(x-2)(x-3) - 120$ has a factor $(x - 5)$.

4B. बहुपदीय $p(x) = (2x+1)(x-2)(2x-3) - 3$ को एउटा गुणनखण्ड $(x - 1)$ हो भनी प्रमाणित गर्नुहोस् ।
Prove that the polynomial $p(x) = (2x+1)(x-2)(2x-3) - 3$ has a factor $(x - 1)$.

5A. $6x^3 - x^2 - 5x + 2$ को एउटा गुणनखण्ड $2x - 1$ हो भनी देखाउनुहोस् ।
Show that $2x - 1$ is a factor of $6x^3 - x^2 - 5x + 2$.

5B. $3x^3 - 7x^2 - 7x + 3$ को एउटा गुणनखण्ड $1 - 3x$ हो भनी देखाउनुहोस् ।
Show that $1 - 3x$ is a factor of $3x^3 - 7x^2 - 7x + 3$.

5C. यदि n एउटा विजोर सङ्ख्या भए $x^n + a^n$ को एउटा गुणनखण्ड $(x + a)$ हो भनी देखाउनुहोस् ।
Show that $x^n + a^n$ has a factor $(x + a)$ if n is an odd number.

5D. यदि n जोर भए $x^n - a^n$ को एउटा गुणनखण्ड $(x - a)$ हुन्छ भनी देखाउनुहोस् ।
If n is even then show that $(x - a)$ is a factor of $x^n - a^n$.

5E. $x^n - a^n$ को एउटा गुणनखण्ड $x - a$ हो भनी देखाउनुहोस् । (Show that $x^n - a^n$ has a factor $x - a$.)

6A. बहुपदीयहरू $x^2 + px + q$ र $x^2 + mx + n$ को साझा गुणनखण्ड $(x + a)$ भए $a(m - p) = (n - q)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If a common factor of polynomial $x^2 + px + q$ and $x^2 + mx + n$ is $(x + a)$ then prove that: $a(m - p) = (n - q)$

6B. यदि $x^2 - ax + b$ र $x^2 - cx + d$ दुवैको एउटा गुणनखण्ड $(x - m)$ भए $m = \frac{d-b}{c-a}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $x^2 - ax + b$ and $x^2 - cx + d$ both have a factor $x - m$ then prove that $m = \frac{d-b}{c-a}$.

7A. यदि $x^2 + kx + 1$ को एउटा गुणनखण्ड $x - 1$ भए $x^2 + 3x + 2$ को एउटा गुणनखण्ड $x - k$ हुन्छ भनी देखाउनुहोस् ।
If $x - 1$ is a factor of $x^2 + kx + 1$ then show that $x - k$ is a factor of $x^2 + 3x + 2$.

7B. $x^2 - kx + 6$ को एउटा गुणनखण्ड $(x - 3)$ छ भने $x^2 - 7x + 10$ को एउटा गुणनखण्ड $(x - k)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
One factor of $x^2 - kx + 6$ is $(x - 3)$. Prove that $(x - k)$ is a factor of $x^2 - 7x + 10$.

Model 4 On finding value of k or m or p when a factor or a root is given.

8A. यदि $2x^k - 3x^{k-1} + 4x - 39$ को एउटा गुणनखण्ड $x - 3$ भए k को मान पत्ता लगाउनुहोस् ।
If $x - 3$ is a factor of $2x^k - 3x^{k-1} + 4x - 39$, find the value of k.

8B. यदि $x^k - 3x^{k-1} + 4x - 4$ को एउटा गुणनखण्ड $x - 2$ भए k को मान पत्ता लगाउनुहोस् ।
If $x - 2$ is a factor of $x^k - 3x^{k-1} + 4x - 4$, find the value of k.

8C. बहुपदीय $2x^m - 4x^{m-2} - 128$ को एउटा गुणनखण्ड $(x - 2)$ भए m को मान पत्ता लगाउनुहोस् ।
If $(x - 2)$ is a factor of the polynomial $2x^m - 4x^{m-2} - 128$, then find the value of m.

8D. यदि $p(x) = 4x^m - 27x^{m-3} - 243$ को एउटा गुणनखण्ड $x - 3$ भए m को मान पत्ता लगाउनुहोस् ।
If $x - 3$ is a factor of $p(x) = 4x^m - 27x^{m-3} - 243$ then find the value of m.

8E. यदि $x^k + 2x^{k-1} - 4x + 4$ को एउटा गुणनखण्ड $(x - 2)$ भए k को मान पत्ता लगाउनुहोस् ।
If $(x - 2)$ is a factor of $x^k + 2x^{k-1} - 4x + 4$ then find the value of k.

8F. यदि $f(x) = x^3 + ax^2 + bx - 6$ को एउटा गुणनखण्ड $(x - 1)$ भए $a + b = 5$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $(x - 1)$ is a factor of $f(x) = x^3 + ax^2 + bx - 6$ then prove that $a + b = 5$.

8G. यदि $p(x) = (x+2)^7 + (2x+k)^3$ को एउटा गुणनखण्ड $(x + 2)$ भए k को मान पत्ता लगाउनुहोस् ।
If one factor of $p(x) = (x+2)^7 + (2x+k)^3$ is $(x + 2)$, find the value of k.

www.drmathnp.com
puspas.com.np

8H. यदि $f(x) = x^3 - px^2 - 2x + p + 4$ को एउटा गुणनखण्ड $(x - p)$ भए p को मान पत्ता लगाउनुहोस् ।
If one factor of $f(x) = x^3 - px^2 - 2x + p + 4$ is $(x - p)$, find the value of p.

9A. $2x^3 + 4x^2 + kx + 6 = 0$ को एउटा मूल -2 भए k को मान पत्ता लगाउनुहोस् ।
Find the value of k so that the equation $2x^3 + 4x^2 + kx + 6 = 0$ has one root -2.

9B. k को मान कति हुँदा $f(x) = 3x^3 + 3x + k$ र $f(-1) = 0$ हुन्छ ?
For what value of k, $f(x) = 3x^3 + 3x + k$ and $f(-1) = 0$?

9C. यदि $4x^3 - 3mx + 5$ लाई $(x - 1)$ ले निःशेष भाग जान्छ भने m को मान पत्ता लगाउनुहोस् ।
If $4x^3 - 3mx + 5$ is exactly divisible by $(x - 1)$, find the value of m.

Model 5 ***On finding polynomial when factors or divisor, quotient & remainder are given.***

10A. एउटा बहुपदीय बनाउनुहोस् जसका गुणनखण्डहरू $(x - 2)$ र $(x + 3)$ हुन् ।
Form a polynomial whose factors are $(x - 2)$ and $(x + 3)$.

10B. एउटा बहुपदीय बनाउनुहोस् जसका गुणनखण्डहरू $(x - 1)$, $(x - 2)$ र $(x - 3)$ हुन् ।
Form a polynomial whose factors are $(x - 1)$, $(x - 2)$ and $(x - 3)$.

11A. बहुपदीय $f(x)$ लाई $(x - a)$ ले भाग गर्दा शेष a^2 र भागफल $(5x - 2)$ रहन्छ भने $f(x)$ पत्ता लगाउनुहोस् ।
A polynomial $f(x)$ is divided by $(x - a)$ to get remainder a^2 & quotient $(5x - 2)$. Find $f(x)$.

11B. बहुपदीय $p(x)$ लाई $4x + 3$ ले भाग गर्दा भागफल $2x^2 - 3x + 1$ र शेष 4 रह्यो भने सो बहुपदीय $p(x)$ पत्ता लगाउनुहोस् ।
A polynomial $p(x)$ is divided by $4x + 3$ to get the quotient $2x^2 - 3x + 1$ and remainder 4, find the $p(x)$.

11C. बहुपदीय $f(x)$ लाई $4x + 2$ ले भाग गर्दा भागफल $2x^3 - 4x^2 + 8$ र शेष 12 भए सो बहुपदीय $f(x)$ पत्ता लगाउनुहोस् ।
A polynomial $f(x)$ is divided by a linear polynomial $4x + 2$ to get the quotient $2x^3 - 4x^2 + 8$ and remainder 12, find the polynomial $f(x)$.

Model 6 ***On making a polynomial by addition or subtraction with given factor or divisor & remainder.***

12A. $x^3 - 6x^2 + 11x - 8$ मा कति जोड्दा एउटा गुणनखण्ड $(x - 3)$ भएको बहुपदीय बनाउन सकिन्छ ?
What must be added to $x^3 - 6x^2 + 11x - 8$ to make a polynomial having a factor $(x - 3)$?

12B. $2x^3 + x^2 - 2x + 5$ मा कति घटाउदा एउटा गुणनखण्ड $(x + 1)$ भएको बहुपदीय बनाउन सकिन्छ ?
What must be subtracted from $2x^3 + x^2 - 2x + 5$ to make it having a factor $x + 1$?

12C. $x^3 - 3x^2 + 4x - 15$ मा कति जोड्यो भने त्यसलाई $(x - 3)$ ले निःशेष भाग जान्छ ?
What must be added to $x^3 - 3x^2 + 4x - 15$ to make it exactly divisible by $(x - 3)$?

12D. $x^3 + 6x^2 - x + 5$ मा कति जोड्यो भने त्यसलाई $(x + 3)$ ले निःशेष भाग जाला ?
What must be added to $x^3 + 6x^2 - x + 5$ to make it exactly divisible by $(x + 3)$?

13A. $3x^2 + 7x$ मा कति जोडियो भने सो बहुपदीयलाई $x - 3$ ले भाग गर्दा शेष 2 हुन्छ ?
What number should be added to $3x^2 + 7x$ so that when the resulting polynomial is divided by $x - 3$ and the remainder is 2?

13B. बहुपदीय $4x^3 + 5x + 3$ बाट कति घटाइयो भने $2x + 5$ ले भाग गर्दा -80 शेष रहला ?
What number should be subtracted from $4x^3 + 5x + 3$ so that the resulting polynomial leaves remainder -80 when divided by $2x + 5$?

13C. $3x^2 + 5x$ बाट कति घटाउँदा सो बहुपदीयलाई $2x + 5$ ले भाग गर्दा शेष 1 हुन्छ ?
What number should be subtracted from $3x^2 + 5x$ so that the resulting polynomial leaves the remainder 1 when divided by $2x + 5$?

13D. बहुपदीय $p(x) = 2x^3 - 7x^2 + 8x + 7$ मा कति जोडियो भने $x + 3$ ले भाग गर्दा शेष 10 रहन्छ ?
What must be added to polynomial $p(x) = 2x^3 - 7x^2 + 8x + 7$ to get the remainder 10 dividing by $x + 3$?

Model 7 ***Problems based on finding a root or factorization of the given polynomial.***

14. तलका बहुपदीयको एउटा मूल पत्ता लगाउनुहोस् (Find a root of following polynomials):
A. $p(y) = y^3 - 8y^2 + 19y - 12$ **B.** $p(x) = x^3 + 2x^2 - 1$ **C.** $p(m) = m^3 - 5m + 4$

15. खण्डीकरण गर्नुहोस् (Factorize) :
A. $x^3 + x^2 + x + 1$ **B.** $x^3 - x^2 - x + 1$ **C.** $x^3 + 2x^2 - x - 2$
D. $x^3 - 2x^2 + 1$ **E.** $abx^2 + (b^2 - ac)\,x - bc$ **F.** $4\sqrt{3}\,x^2 + 5x + 2\sqrt{3}$

16A. यदि $3x^2 + px - 2$ को एउटा गुणनखण्ड $(x - 2)$ भए अर्को गुणनखण्ड पत्ता लगाउनुहोस् ।
If $(x - 2)$ is a factor of $3x^2 + px - 2$, find the other factor.

16B. बहुपदीय $px^2 - 11x - 6$ को एउटा गुणनखण्ड $5x + 2$ छ । अर्को गुणनखण्ड पत्ता लगाउनुहोस् ।
The polynomial $px^2 - 11x - 6$ has a factor $5x + 2$. Find the other factor.

www.drmathnp.com

puspas.com.np

Model 8 *Problems based on multiplication and division of polynomials.*

17A. यदि $p(x) = x^2 - 3x + 5$ र $q(x) = x^3 - 5x^2 + x + 2$ भए $p(x).\ q(x)$ र यसको डिग्री पनि पत्ता लगाउनुहोस् ।
If $p(x) = x^2 - 3x + 5$ and $q(x) = x^3 - 5x^2 + x + 2$, find $p(x).\ q(x)$ and its degree .

17B. यदि $p(x) = 2x^4 + 3x^3 - 4x^2 + 2x - 3$ र $d(x) = x - 1$ भए $p(x) \div d(x)$ र यसको डिग्री पनि पत्ता लगाउनुहोस् ।
If $p(x) = 2x^4 + 3x^3 - 4x^2 + 2x - 3$ and $d(x) = x - 1$, find $p(x) \div d(x)$. Also, find the degree of it.

B. LONG QUESTIONS

Model 1 *Problems based on solving cubic equations.*

मूलहरू पत्ता लगाउनुहोस् (Find the roots of): **(1 to 4)**

1. **A.** $x^3 - 13x + 12 = 0$ **B.** $x^3 = 3(x - 1) + 5$ **C.** $x^3 - 13x - 12 = 0$
D. $x^3 = 21x + 20$ **E.** $x^3 - 31x - 30 = 0$ **F.** $x^3 - 7x - 6 = 0$
G. $x^3 = 19x + 30$ **H.** $x^3 - 37x - 84 = 0$ **I.** $x^3 - 7x + 6 = 0$
J. $w^3 - 5w - 12 = 0$ **K.** $f(x) = x(x^2 - 7) + 6 = 0$ **L.** $f(x) = x(x^2 - 19) - 30 = 0$

2. **A.** $2x^3 - 7x^2 + 9 = 0$ **B.** $x^3 - 3x^2 + 4 = 0$ **C.** $3x^3 - 7x^2 + 4 = 0$
D. $x^3 + 3x^2 - 4 = 0$ **E.** $4x^3 - 13x^2 + 9 = 0$ **F.** $3x^3 + 7x^2 - 4 = 0$
G. $2m^3 + 13m^2 - 36 = 0$ **H.** $2n^3 - 3n^2 + 1 = 0$ **I.** $v^3 - 7v^2 + 36 = 0$

3. **A.** $x^3 = 4x^2 - x - 6$ **B.** $3x^3 + 4x^2 - 5x - 2 = 0$ **C.** $6x^3 + 17x^2 - 5x - 6 = 0$
D. $x^3 + 5x^2 - 2x - 24 = 0$ **E.** $s^3 + 7s^2 - 21s - 27 = 0$ **F.** $y^3 + 10y^2 + 7y - 18 = 0$
G. $z^3 - 6z^2 + 11z - 6 = 0$ **H.** $t^3 + 5t^2 - 2t - 24 = 0$ **I.** $x^2(x + 1) = (x + 1)(x - 6)$

4. **A.** $(x - 1)(2x^2 + 15x + 15) - 21 = 0$ **B.** $(x + 2)(x^2 - 6x + 13) - 20 = 0$
C. $(x + 1)(x^2 - 5x + 10) - 12 = 0$ **D.** $(e - 2)(e^2 - 9e + 13) + 5 = 0$

5. बहुपदीय $p(x)$ लाई खण्डीकरण गर्नुहोस् र समीकरण $p(x) = 0$ हल गर्नुहोस् :
Factorize the polynomial $p(x)$ and solve the equation $p(x) = 0$:
A. $p(x) = x^2(x - 6) + 3x + 10$ **B.** $p(x) + 9x^2 = 2x^3 + x + 12$
C. $p(x) + 18 = x^3 + 10x^2 + 7x$ **D.** $p(x) = x(x^2 + 3) - 2(3x^2 - 5)$

6A. यदि पहिलो बहुपदीय $f(x) = x^2(x - 9)$ र दोस्रो बहुपदीय $g(x) = 23x - 15$ भए x को लागि हल गर्नुहोस् जब $f(x) + g(x) = 0$ छ ।
If the first polynomial $f(x) = x^2(x - 9)$ and the second polynomial $g(x) = 23x - 15$, solve for x when $f(x) + g(x) = 0$.

6B. यदि पहिलो बहुपदीय $f(x) = 4x^3 - 8x^2$ र दोस्रो बहुपदीय $g(x) = 3x^3 - 2x^2 - 11x + 6$ भए x को लागि हल गर्नुहोस् जब $f(x) - g(x) = 0$ छ ।
If the first polynomial $f(x) = 4x^3 - 8x^2$ and the second polynomial $g(x) = 3x^3 - 2x^2 - 11x + 6$, solve for x when $f(x) - g(x) = 0$.

Model 2 *Problems based on solving cubic equation and a linear equation.*

7. हल गर्नुहोस् (Solve):
A. $y = x^3 - 7x^2 + 7x + 18$ and $y = 3$ **B.** $y = x^3 - 4x^2 - 7x + 15$ and $y = 5$
C. $y = x^3 - 8x^2 + 20x - 8$ and $y = x + 4$ **D.** $y = x^3 - 3x^2 - 9x + 28$ and $y = x + 4$
E. $y = 2x^3 + 3x^2 - 10x - 7$ and $x = y + 1$ **F.** $y = 2x^3 + x^2 - 3x + 1$ and $x = 2 - y$

Model 3 *Problems based on the factorization of cubic equation.*

8. खण्डीकरण गर्नुहोस् (Factorize):
A. $p(x) = x^3 - 4x^2 + x + 6$ **B.** $p(x) = x^3 - 7x^2 + 7x + 15$
C. $p(x) = x^3 - 4x^2 - 7x + 10$ **D.** $p(x) = 8x^3 - 2x^2 - 5x - 1$
E. $p(x) = 6x^3 + 7x^2 - x - 2$ **F.** $p(x) = 2x^3 + 3x^2 - 11x - 6$
G. $P(x) = 6x^3 - 7x^2 - 7x + 6$ **H.** $p(x) = 2x^3 + x^2 - 2x - 1$
I. $p(a) = 2a^3 - 3a^2 - 3a + 2$ **J.** $p(a) = 3a^3 - 13a^2 + 16$

www.drmathnp.com

K. $p(w) = w^3 - 13w - 12$ **L.** $p(u) = 3u^3 - 7u^2 + 4$

M. $y^3 - 19y - 30$ **N.** $Q^3 - 5Q + 4$

9A. $f(x) = x^3 - x^2 - 4x + 4$ को मान $x = -3, -2, -1, 0, 1, 2, 3$ हुँदा पत्ता लगाउनुहोस् र $f(x)$ का रेखीय गुणनखण्डहरू पनि पत्ता लगाउनुहोस् ।
Find the value of $f(x) = x^3 - x^2 - 4x + 4$ at $x = -3, -2, -1, 0, 1, 2, 3$ and find the linear factors of $f(x)$.

9B. $f(x) = x^3 - 7x^2 + 7x + 15$ को मान $x = -3, -2, -1, 0, 1, 2, 3$ हुँदा पत्ता लगाउनुहोस् र $f(x)$ का रेखीय गुणनखण्डहरू पनि पत्ता लगाउनुहोस् ।
Find the value of $f(x) = x^3 - 7x^2 + 7x + 15$ at $x = -3, -2, -1, 0, 1, 2, 3$ & find the linear factors of $f(x)$.

Model 4 ***Problems on finding the value of letters when factors or divisor & remainder are given.***

10A. यदि $(x-1)$ र $(x-2)$ हरू $x^3 + ax^2 + bx - 6$ का गुणनखण्डहरू भए a र b का मानहरू पत्ता लगाउनुहोस् ।
Find the values of 'a' and 'b' If $(x-1)$ and $(x-2)$ are factors of $x^3 + ax^2 + bx - 6$.

10B. यदि $x^3 + 10x^2 + ax + b$ लाई $x-1$ र $x+2$ ले निःशेष भाग जान्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।
Find the values of a and b so that the polynomial $x^3 + 10x^2 + ax + b$ is exactly divisible by $x-1$ as well as $x+2$.

10C. यदि $x^3 + ax^2 + bx + 6$ को गुणनखण्ड $x-2$ छ र $x-3$ ले भाग गर्दा शेष 3 रहन्छ भने a र b पत्ता लगाउनुहोस् ।
If $x^3 + ax^2 + bx + 6$ has $x-2$ as a factor and leaves a remainder 3 when divided by $x-3$, find a and b.

10D. अभिव्यञ्जक $ax^2 + bx + 1$ लाई $(x-1)$ ले भाग गर्दा शेष 2 छ र जब यसलाई $x+1$ ले भाग गरिन्छ तब शेष 4 रहन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।
The expression $ax^2 + bx + 1$ has the remainder 2 when it is divided by $(x-1)$ and when it is divided by $(x+1)$, the remainder is 4. Find the values of a and b.

10E. जब एउटा बहुपदीय $y^2 + my + n$ लाई $y-1$ र $y+2$ ले भाग गरिन्छ तब शेषहरू क्रमशः 4 र 5 हुन्छन् । m र n का मानहरू पत्ता लगाउनुहोस् ।
When a polynomial $y^2 + my + n$ is divided by $y-1$ and $y+2$, the remainders are 4 and 5 respectively. Find the values of m and n.

10F. अभिव्यञ्जक $x^2 + bx + c$ लाई $(x+1)$ ले भाग गर्दा शेष 1 रहन्छ र जब यसलाई $x-2$ ले भाग गरिन्छ तब शेष 13 रहन्छ भने b र c का मानहरू पत्ता लगाउनुहोस् ।
The expression $x^2 + bx + c$ has the remainder 1 when it is divided by $(x+1)$ and when it is divided by $(x-2)$, the remainder is 13. Find the values of b and c.

10G. बहुपदीय $p(x) = x^3 + ax^2 + bx + 30$ को एउटा गुणनखण्ड $x-5$ छ । यदि $p(x)$ लाई $x+6$ ले भाग गरियो भने शेष -396 हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।
A polynomial $p(x) = x^3 + ax^2 + bx + 30$ has a factor $x-5$. If $p(x)$ is divided by $x+6$ then the remainder is -396 then find the values of a and b.

10H. यदि बहुपदीय $2x^3 + ax^2 + bx - 2$ को एउटा गुणनखण्ड $x-2$ र $2x-3$ ले भाग गर्दा शेष 7 रहे a र b का मानहरू पत्ता लगाउनुहोस् ।
If the polynomial $2x^3 + ax^2 + bx - 2$ has a factor $x-2$ and leaves remainder 7 when divided by $2x-3$, find the values of a and b.

10I. यदि $f(x) = 2x^3 + ax^2 - 11x + b$ को एउटा गुणनखण्ड $x-2$ र $x-3$ ले $f(x)$ लाई भाग गर्दा शेष 42 आउँछ भने a र b का मानहरू पत्ता लगाउनुहोस् ।
Find the values of a and b if $f(x) = 2x^3 + ax^2 - 11x + b$ has a factor $x-2$ and leaves remainder 42 when divided by $x-3$.

10J. यदि $f(x) = mx^3 - 3x^2 - 11x + n$ को एउटा गुणनखण्ड $x-3$ छ र यसलाई $x-1$ ले भाग गर्दा शेष -6 आउँछ भने m र n का मानहरू पत्ता लगाउनुहोस् ।
Find the values of m and n if $f(x) = mx^3 - 3x^2 - 11x + n$ has a factor $x-3$ and leaves remainder -6 when divided by $x-1$.

10K. यदि बहुपदीय $x^3 + mx^2 + nx + 6$ को एउटा गुणनखण्ड $x-2$ हो र उक्त बहुपदीयलाई $x-3$ ले भाग गर्दा शेष 3 आउँछ भने m र n का मानहरू पत्ता लगाउनुहोस् ।
If $x^3 + mx^2 + nx + 6$ has $x-2$ as a factor and leaves a remainder 3 when divided by $x-3$, find the values of m and n

11A. यदि बहुपदीयहरू $x^3 + 4x^2 - 2x + 1$ र $x^3 + 3x^2 - x + 7$ लाई $x-a$ ले भाग गर्दा बराबर शेष आउँछ भने a का सम्भावित मानहरू निकाल्नुहोस् ।
If the polynomials $x^3 + 4x^2 - 2x + 1$ and $x^3 + 3x^2 - x + 7$ are divided by $x-a$ leaves the equal remainder, find the possible values of 'a'.

puspas.com.np

www.drmathnp.com

11B. यदि $x^2 + ax - 6 = 0$ र $x^2 - 9x + b = 0$ को एउटा साझा गुणनखण्ड $x - 2$ भए a र b का मानहरू पत्ता लगाउनुहोस् ।
If $x - 2$ is a common factor of $x^2 + ax - 6 = 0$ & $x^2 - 9x + b = 0$, find the values of a and b.

11C. यदि $x^3 - 2x^2 + px + 6 = 0$ र $x^2 - 5x + q = 0$ को एउटा साझा गुणनखण्ड $x - 3$ भए $6p + 5q$ को मान पत्ता लगाउनुहोस् ।
If $x - 3$ is a common factor of $x^3 - 2x^2 + px + 6 = 0$ and $x^2 - 5x + q = 0$ then find the value of $6p + 5q$.

Model 5 *Problems based on finding other factors when a factor of a polynomial is given.*

12A. यदि $p(x) = x^3 + 5x^2 + 8x + 4$ को एउटा गुणनखण्ड $x + 2$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
If the one factor of $p(x) = x^3 + 5x^2 + 8x + 4$ is $(x + 2)$, find the other factors.

12B. बहुपदीय $x^3 - 4x^2 + x + 6$ को एउटा गुणनखण्ड $x - 3$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
A polynomial $x^3 - 4x^2 + x + 6$ has a factor $x - 3$. Find the other factors.

12C. यदि $4x^3 + x^2 - 36x - 9$ को एउटा गुणनखण्ड $(4x + 1)$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
If $4x^3 + x^2 - 36x - 9$ has a factor $(4x + 1)$ then find the other factors.

12D. यदि $p(x) = x^3 - 2ax^2 + ax - 2$ को एउटा गुणनखण्ड $(x - 2)$ भए a को मान पत्ता लगाई $p(x)$ को खण्डीकरण गर्नुहोस् ।
If $(x - 2)$ is a factor of $p(x) = x^3 - 2ax^2 + ax - 2$, find the value of a and hence factorise the polynomial $p(x)$.

12E. यदि $p(x) = 6x^3 + ax^2 - 10x - 3$ को एउटा गुणनखण्ड $x + 1$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
If $p(x) = 6x^3 + ax^2 - 10x - 3$ has a factor $x + 1$, find the other factors.

12F. यदि $p(x) = x^3 - ax^2 - 2x + a + 4$ को एउटा गुणनखण्ड $x - a$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
If $p(x) = x^3 - ax^2 - 2x + a + 4$ has a factor $x - a$, find the other factors.

12G. $P(x) = x^3 + ax^2 - 7x + 6$ को एउटा गुणनखण्ड $(x + 3)$ हो भने a को मान निकाली $P(x)$ लाई पूर्णरूपले खण्डीकरण गर्नुहोस् ।
If $x + 3$ is a factor of $P(x) = x^3 + ax^2 - 7x + 6$, find the value of a and then factorize $P(x)$ completely.

ANSWERS

A. SHORT QUESTIONS

1A. $Q(x) = x^3 + 2$; R = 5 **1B.** $2x^2 - x + 14$; 60 **1C.** $(x - 1)$ **2A.** -6 **2B.** 4
2C. 7 **3A.** $\frac{3}{10}$ **3B.** $\frac{1}{2}$ **3C.** 4 **3D.** $30 - \frac{3}{m}, \frac{1}{7}$ **3E.** -14
8A. 3 **8B.** 3 **8C.** $m = 7$ **8D.** 4 **8E.** 1
8G. 4 **8H.** 4 **9A.** 3 **9B.** 6 **9C.** 3
10A. $x^2 + x - 6$ **10B.** $x^3 - 6x^2 + 11x - 6$ **11A.** $5x^2 - (5a + 2)x + 2a + a^2$ **11B.** $p(x) = 8x^3 - 6x^2 - 5x + 7$
11C. $8x^4 - 12x^3 + 32x + 28$ **12A.** 2 **12B.** 6 **12C.** 3 **12D.** -35
13A. -46 **13B.** -152 **13C.** $\frac{21}{4}$ **13D.** 164 **14A.** 1, 3, 4
14B. $\frac{-1 + \sqrt{5}}{2}$ or -1 **14C.** 1 or $\frac{-1 \pm \sqrt{17}}{2}$ **15A.** $(x + 1)(x^2 + 1)$ **15B.** $(x + 1)(x - 1)(x - 1)$
15C. $(x + 2)(x + 1)(x - 1)$ **15D.** $(x - 1)(x^2 - x - 1)$ **15E.** $(ax + b)(bx - c)$ **15F.** $(\sqrt{3}x + 2)(4x - \sqrt{3})$
16A. $3x + 1$ **16B.** $2x - 3$ **17A.** $x^5 - 8x^4 + 21x^3 - 26x^2 - x + 10$; 5 **17B.** $2x^3 + 5x^2 + x + 3$; 3

B. LONG QUESTIONS

1A. 1, 3, -4 **1B.** 2, -1, -1 **1C.** -1, -3, 4 **1D.** -1, -4, 5 **1E.** 6, -5, -1 **1F.** -1, -2, 3
1G. 5, -2, -3 **1H.** 7, -3, -4 **1I.** -2, 3, 6 **1J.** $3, \frac{-3 + \sqrt{-7}}{2}$ **1K.** $x = 1, 2, -3$ **1L.** -2, 5, -3
2A. $-1, 3, \frac{3}{2}$ **2B.** 2, 2, -1 **2C.** $-1, -2, \frac{2}{3}$ **2D.** $-2, -2, 1$ **2E.** $1, 3, -\frac{3}{4}$ **2F.** $1, 2, -\frac{2}{3}$
2G. $-2, -6, \frac{3}{2}$ **2H.** 1, 1, $-1/2$ **2I.** -2, 3, 6 **3A.** $-1, 2, 3$ **3B.** $1, -2, -\frac{1}{3}$ **3C.** $\frac{2}{3}, -3, -\frac{1}{2}$
3D. 2, -4, -3 **3E.** 1, 3, -9 **3F.** 1, -9, -2 **3G.** 1, 2, 3 **3H.** -4, -3, 2 **3I.** $-1, 2, -3$
4A. $-2, -6, \frac{3}{2}$ **4B.** $-1, 2, 3$ **4C.** 1, 1, 2 **4D.** 1, 3, 7 **5A.** $(x - 2)(x - 5)(x + 1)$; $x = 2$ or 5 or -1
5B. $(x + 1)(x - 4)(2x - 3)$; -1, 4, 3/2 **5C.** $(x - 1)(x + 2)(x + 9)$; $x = 1, -2, -9$ **5D.** $(x + 1)(x - 2)(x - 5)$; $x = -1$ or 2 or 5
6A. 1, 5, 3 **6B.** 1, 2, 3 **7A.** $x = -1, 3, 5$ and $y = 3$ **7B.** $x = 1, -2, 5$ and $y = 5$
7C. $x = 1, 3, 4$ and $y = 5, 7, 8$ **7D.** $x = 2, -3, 4$ and $y = 6, 1, 8$ **7E.** $x = 2, -3, -\frac{1}{2}$ and $y = 1, -4, -\frac{3}{2}$
7F. $x = 1, -1, \frac{-1}{2}$ & $y = -1, -3, -\frac{5}{2}$ **8A.** $(x - 2)(x + 1)(x - 3)$ **8B.** $(x - 5)(x + 1)(x - 3)$
8C. $(x - 5)(x + 2)(x - 1)$ **8D.** $(x - 1)(2x + 1)(4x + 1)$ **8E.** $(2x - 1)(x + 1)(3x + 2)$
8F. $(x - 2)(x + 3)(2x + 1)$ **8G.** $(2x - 3)(x + 1)(3x - 2)$ **8H.** $(x - 1)(x + 1)(2x + 1)$
8I. $(a - 2)(a + 1)(2a - 1)$ **8J.** $(a - 4)(a + 1)(3a - 4)$ **8K.** $(w - 4)(w + 3)(w + 1)$
8L. $(u - 2)(3u + 2)(u - 1)$ **8M.** $(y - 5)(y + 3)(y + 2)$ **8N.** $(Q - 1)(Q^2 + Q - 4)$
9A. Factors are $(x - 1)$, $(x - 2)$ and $(x + 2)$ **9B.** factors are $(x + 1)$, $(x - 3)$ and $(x - 5)$
10A. -6, 11 **10B.** 7, -18 **10C.** $-3, -1$ **10D.** 2, -1 **10E.** -2, 5 **10F.** $b = c = 3$ **10G.** $-6, -1$
10H. -17, 27 **10I.** 3, -6 **10J.** 2, 6 **10K.** $-3, -1$ **11A.** 3, -2 **11B.** 1, 14 **11C.** 0
12A. $(x + 2)$, $(x + 1)$ **12B.** $(x + 1)$, $(x - 2)$ **12C.** $(x + 3)(x - 3)$ **12D.** $a = 1$, $(x - 2)(x^2 + 1)$ **12E.** $(3x + 1)(2x - 3)$
12F. 4, $\pm\sqrt{2}$ **12G.** $a = 0$, $(x - 1)(x - 2)(x + 3)$

www.drmathnp.com

CHALLENGING EXERCISE 2

1. बहुपदीयहरू $p(x) = x^3 + 2x^2 - 5ax - 7$ र $q(x) = x^3 + ax^2 - 12x + 6$ लाई क्रमशः $(x + 1)$ र $(x - 2)$ ले भाग गर्दा R_1 र R_2 शेष आँउदछ । यदि $2R_1 + R_2 = 6$ भए a को मान पत्ता लगाउनुहोस् ।
Let R_1 and R_2 are the remainders when the polynomials $p(x) = x^3 + 2x^2 - 5ax - 7$ and $q(x) = x^3 + ax^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$ respectively. If $2R_1 + R_2 = 6$ then find the value of a.

2. बहुपदीय $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ को एउटा गुणनखण्ड $(x^2 - 1)$ भए $a + c + e = b + d = 0$ हुन्छ भनी देखाउनुहोस् ।
If $(x^2 - 1)$ is a factor of $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, show that : $a + c + e = b + d = 0$

3. $f(x) = ax^2 + bx - 6$ र $p(x) = bx^2 + ax$ दिइएको छ । यदि $x - 2$ ले $f(x)$ लाई निःशेष भाग जान्छ तर $p(x)$ लाई भाग गर्दा -12 शेष रहन्छ भने $f(x) + p(x) + x^3 + x^2 - 2x + 4$ लाई खण्डीकरण गर्नुहोस् ।
$f(x) = ax^2 + bx - 6$ and $p(x) = bx^2 + ax$ are given. If $f(x)$ is divided by $x - 2$ exactly but leaves the remainder -12 when it divides $p(x)$ then factorize $f(x) + p(x) + x^3 + x^2 - 2x + 4$.

4. $p(x) = x^3 + x^2 - ax - a$ को एउटा गुणनखण्ड $x - 2$ छ । यदि $f(x) = 2x^3 - x^2 - px - 2$ को पनि एउटा गुणनखण्ड $x - 2$ भए $p(x) = f(x) + 5x^2 + x - 6$ हल गर्नुहोस् ।
One factor of $p(x) = x^3 + x^2 - ax - a$ is $x - 2$. If one factor of $f(x) = 2x^3 - x^2 - px - 2$ is also $x - 2$ then solve $p(x) = f(x) + 5x^2 + x - 6$.

5. यदि $x^2 - bx + c = (x + p)(x - q)$ भए $x^2 - bxy + cy^2$ लाई खण्डीकरण गर्नुहोस् ।
If $x^2 - bx + c = (x + p)(x - q)$ then factorize $x^2 - bxy + cy^2$.

6. यदि $x^4 + px^3 + 2x^3 - 3x + q$ का दुईओटा गुणनखण्डहरू $(x + 1)$ र $(x - 1)$ भए अन्य गुणनखण्ड पत्ता लगाउनुहोस् ।
If the two factors of $x^4 + px^3 + 2x^3 - 3x + q$ are $(x + 1)$ and $(x - 1)$, find the other factor.

7. यदि $p(x) = x^3 - kx^2 + x + 6$ को एउटा गुणनखण्ड $x - 2$ भए $f(x) = p(x) + x^3 + 7x^2 - 12x - 12$ का गुणनखण्डहरू पत्ता लगाउनुहोस् र $f(x) = 0$ हल गर्नुहोस् ।
If $x - 2$ is a factor of $p(x) = x^3 - kx^2 + x + 6$ and $f(x) = p(x) + x^3 + 7x^2 - 12x - 12$, find the factors of $f(x)$ and solve at $f(x) = 0$.

8. बहुदपीय $x^4 - 6x^3 + 16x^2 - 25x + 10$ लाई अर्को बहुपदीय $x^2 - 2x + k$ ले भाग गर्दा शेष $x + a$ हुन्छ भने k र a को मान पत्ता लगाउनुहोस् ।
If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder is $x + a$, find the value of k and a.

9. $a^2b^2(a - b) + b^2c^2(b - c) + c^2a^2(c - a)$ लाई $(a - b)$, $(b - c)$ र $(c - a)$ ले निःशेष भाग जान्छ भनी देखाउनुहोस् ।
Show that $a^2b^2(a - b) + b^2c^2(b - c) + c^2a^2(c - a)$ is exactly divisible by $(a - b)$, $(b - c)$ & $(c - a)$.

10. $a^4 - a^3 + a^2 + 2$ को मान निकाल्नुहोस् जब $a^2 + 2 = 2a$ छ । (Find the value of $a^4 - a^3 + a^2 + 2$ when $a^2 + 2 = 2a$.)

11. खण्डीकरण गर्नुहोस् (Factorize) :

A. $2x^4 - 7x^3 + 4x^2 + 7x - 6$

B. $x^4 - x^3 - 19x^2 + 49x - 30$

C. $(x - 1)(x - 2)(x + 3)(x + 4) + 4$

D. $(x + 1)(x + 2)(x + 3)(x + 4) - 8$

E. $(x + 1)(x + 2)(x + 3)(x + 4) - 120$

F. $(2x - 1)(2x - 3)(2x - 5)(2x - 7) - 9$

G. $x(x - 1)(x - 2)(x - 3) + 1$

12. x को लागि हल गर्नुहोस् (Solve for x): $24x^3 - 26x^2 + 9x - 1 = 0$ $(0 < x < 1)$

ANSWERS

1. 2
3. $(x - 2)(x + 1)(x + 1)$
4. $-2, -2, 1$
5. $(x - qy)(x + py)$
6. $(x^2 + 3x + 1)$
7. $(x - 2)(x + 3)(2x + 1)$: $2, -3, -1/2$
8. $k = 5, a = -5$
10. 0
11A. $(x + 1)(x - 1)(x - 2)(2x - 3)$
11B. $(x - 1)(x - 2)(x - 3)(x + 5)$
11C. $(x^2 + 2x - 4)(x^2 + 2x - 7)$
11D. $(x^2 + 5x + 2)(x^2 + 5x + 8)$
11E. $(x^2 + 5x + 16)(x + 6)(x - 1)$
11F. $8(2x^2 - 8x + 3)(x - 2)^2$
11G. $(x^2 - 3x + 1)(x^2 - 3x + 1)$
12. $x = 1/2, 1/3, 1/4$

puspas.com.np

www.drmathnp.com

puspa.com.np

3. समानान्तरीय अनुक्रम ARITHMETIC SEQUENCE

DEFINITIONS

❋ **अनुक्रम (Sequence)** [2063 M, 2064 R', SEE 2073 R]

कुनै निश्चित नियममा आधारित सङ्ख्याहरूको समूहलाई अनुक्रम भनिन्छ । जस्तै: 3, 5, 7.....

A list of numbers in definite order is called sequence. e.g. 3,5, 7.....

❋ **श्रेणी (Series)** [2063 M, 2064 R', SEE 2073 R]

यदि अनुक्रमका पदहरूलाई योगफलको रूपमा व्यक्त गरिएमा त्यसलाई उक्त अनुक्रमसँग सम्बन्धित श्रेणी भनिन्छ । जस्तै: 2 + 4 + 6 + 8 ..... एउटा श्रेणी हो ।

When the terms of sequence are connected by addition or subtraction signs: e.g. 2 + 4 + 6 + 8 .... which is a series.

❋ **अङ्कगणितीय या समानान्तरीय अनुक्रम र श्रेणी (Arithmetic Sequence and Series)** [2063 R, 2064 R, 2063S, SEE 2073 S]

यदि कुनै अनुक्रमको प्रत्येक पद अघिल्लो पदभन्दा कुनै निश्चित सङ्ख्याले बढिरहेका वा घटिरहेका हुन्छन् भने त्यस्तो अनुक्रमलाई समानान्तरीय अनुक्रम भनिन्छ । यो अनुक्रमसँग सम्बन्धित श्रेणीलाई समानान्तरीय श्रेणी भनिन्छ । जस्तै: 3, 7, 11, 15, ... एउटा समानान्तरीय अनुक्रम भए 3 + 7 + 11 + 15 + ... एउटा समानान्तरीय श्रेणी हो ।

A sequence is said to be an arithmetic sequence when its terms increase or decrease continually by a common difference. The corresponding series of this sequence is called the arithmetic series. e.g. if 3, 7, 11, 15, ...is an arithmetic sequence then 3 + 7 + 11 + 15 + ... is an arithmetic series.

❋ **अङ्कगणितीय मध्यमा (Arithmetic Mean)**

समानान्तरीय अनुक्रमका पहिलो पद र अन्तिम पद बिचको पद वा पदहरूलाई समानान्तरीय मध्यमा भनिन्छ । जस्तै: 3, 7, 11, 15, 19, 23 एउटा समानान्तरीय अनुक्रम भए 7, 11, 15, 19 समानान्तरीय मध्यमाहरू हुन् ।

In arithmetic sequence, the term/terms between the first term and the last term is/are called arithmetic mean/means, e.g. if 3, 7, 11, 15, 19, 23 are in arithmetic sequence then 7, 11, 15, 19 are arithmetic means .

FORMULAE AND KEY POINTS

समानान्तरीय अनुक्रम वा श्रेणी (Arithmetic Sequence or Series): AS		सङ्केत (Index)
साधारण पद General term	$t_n = a + (n-1)d$	a = पहिलो पद (First term) n = पदहरूको सङ्ख्या (No. of terms) d = समान अन्तर (Common difference) b = अन्तिम पद (Last term) $m_n = n$ औं मध्यमा (n^{th} mean)
मध्यमा Mean	$AM = \frac{a+b}{2}$	
मध्यमाहरू Means	$d = \frac{b-a}{n+1}$; $m_1 = a + d, m_2 = a + 2d, ..., m_n = a + nd$	
योगफल (Sum)	$S_n = \frac{n}{2}\{2a + (n-1)d\}$ **or** $S_n = \frac{n}{2}(a + b)$	

यदि समानान्तरीय मध्यमाहरू (AMs) को सङ्ख्याहरू = m भए (If number of arithmetic means (AMs) = m): $d = \frac{b-a}{m+1}$

- पहिला n ओटा प्राकृतिक सङ्ख्याहरूको योगफल (The sum of first n-natural numbers) = $\frac{n}{2}(n+1)$
- पहिला n ओटा बिजोर सङ्ख्याहरूको योगफल (The sum of first n-odd numbers) = n^2
- पहिला n ओटा जोर सङ्ख्याहरूको योगफल (The sum of first n-even numbers) = $n(n+1)$

AS का पदहरूलाई सङ्केत गर्ने सजिलो तरिका (The easy method to denote the terms of AS):

पदहरूको सङ्ख्या (Number of terms)	3	4	5
पदहरू (Terms)	$a-d, a, a+d$	$a-3d, a-d, a+d, a+3d$	$a-2d, a-d, a, a+d, a+2d$
समान अन्तर (Common Difference)	d	$2d$	d

- यदि n ओटा पदहरूको योगफल S_n भए साधारण पद $(t_n) = S_n - S_{n-1}$ हुन्छ ।

  If the sum of n terms is S_n then the general $(t_n) = S_n - S_{n-1}$.

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

A. VERT SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, 2066 R', 2066 S, 2067 S, 2068 R', 2071 S, SEE 2073 S, SEE 2074 R', SEE 2074 R, 2074 S, SEE 2075 R'_2	11
Model 2	2065 M, 2069 R', 2074 S'	3

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2068 S'	2
Model 2	2065 R', 2070 R, 2070 S	3
Model 3	2065 S, 2065 E, 2067 R, 2067 R', 2069 S, 2070 R', 2070 S', 2071 R', 2073 S', 2075 R', 2075 R'_2	10
Model 4	2073 R'	1
Model 5	2069 R	1
Model 6	2068 S, 2071 R, 2072 R'	3
Model 8	SEE MODEL 2076	1

SEE EXERCISE 3

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
A. अनुक्रम (Sequence) **B.** श्रेणी (Series)
C. समानान्तरीय अनुक्रम र श्रेणी (Arithmetic sequence and series)
D. समान अन्तर (Common difference) **E.** समानान्तरीय मध्यमा (Arithmetic mean)

2A. एउटा AS को n औं पद के हुन्छ, जसको पहिलो पद 'a' र समान अन्तर 'd' छ ?
What is the n^{th} term of an AS whose first term is 'a' and common difference is 'd'?

2B. सूत्र $t_n = a + (n-1)d$ मा प्रत्येक पदको अर्थ उल्लेख गर्नुहोस् ।
State the meaning of each term in the formula: $t_n = a + (n-1)d$.

3A. दुई सङ्ख्याहरू a र b बिचको अङ्कगणितीय मध्यक के हुन्छ ?
What is the arithmetic mean between two numbers a and b? [SEE MODEL 2076]

3B. पहिलो पद 'a' र समान अन्तर 'd' भएको AS को n औं समानान्तरीय मध्यमा के हुन्छ ?
What is the n^{th} arithmetic mean of an AS whose first term is 'a' and common difference is 'd'?

3C. पहिलो पद 'a' र समान अन्तर 'd' भएको AS को छैटौं समानान्तरीय मध्यमा के हुन्छ ?
What is the 6^{th} arithmetic mean of an AS whose first term is 'a' and common difference is 'd'?

3D. n ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 'a' र अन्तिम पद 'b' छ भने समान अन्तर कति होला ?
What is the common difference of an AS whose first term is 'a' and last term is 'b' and having 'n' arithmetic means?

4A. पहिलो n ओटा प्राकृतिक सङ्ख्याहरूको योगफल कति हुन्छ ? (What is the sum of first n natural numbers?)

4B. पहिलो n ओटा जोर प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of first n even natural numbers.

4C. पहिलो n ओटा बिजोर प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।
Write the sum of first n odd natural numbers.

5A. पहिलो पद 'a' र अन्तिम पद 'b' भएको समानान्तरीय अनुक्रमको n ओटा पदहरूको योगफल कति हुन्छ ?
What is the sum of n terms of arithmetic sequence whose first term is 'a' and last term is 'b'?

5B. पहिलो पद 'a' र समान अन्तर 'd' भएको समानान्तरीय अनुक्रम को n ओटा पदहरूको योगफल लेख्नुहोस् ।
Write the sum of n terms of an AS having first term 'a' and common difference 'd'.

6A. दिइएको सूत्रमा हरेक पदको अर्थ लेख्नुहोस् । (Write the meaning of each term in the formula): $S_n = \frac{n}{2}\{2a + (n-1)d\}$.

6B. दिइएको सूत्रमा हरेक पदको अर्थ लेख्नुहोस् । (Write the meaning of each term in the formula): $S_n = \frac{n}{2}(a + \ell)$.

www.drmathnp.com

puspas.com.np

Model 2 *Concept Recall*

7. तलका अनुक्रम मध्य कुन समानान्तरीय अनुक्रम हो ?
Which of the following sequence are arithmetic sequence?

A. 1, 2, 3, 4, 5, 6, 7 **B.** 4, 6, 8, 10, 12, 14, 16 **C.** 18, 15, 12, 9, 6, 3

D. 100, 87, 74, 61, 48 **E.** 22, 32, 42,52, 62, 72 **F.** 2, 4, 8, 16, 32, 64, 128

8. तलका प्रत्येक अनुक्रममा समान अन्तर पत्ता लगाउनुहोस् ।
Find the common difference in each of the following sequence.

A. 3, 4, 5, 6, 7 **B.** 10, 12, 14, 16 **C.** 180, 150, 120, 90 **D.** 113, 100, 87, 74, 61

9A. एउटा AP को छैटौं पद 19 र समान अन्तर 3 छ । यसको सातौं पद पत्ता लगाउनुहोस् ।
6^{th} term of an AP is 19 and the common difference is 3. Find its 7^{th} term.

9B. एउटा AP को आठौं पद 44 र समान अन्तर 4 छ । यसको सातौं पद पत्ता लगाउनुहोस् ।
8^{th} term of an AP is 44 and the common difference is 4. Find its 7^{th} term.

10. समानान्तरीय मध्यमा पत्ता लगाउनुहोस् **(Find the arithmetic mean between):**
A. 19 and 25 **B.** 22 and 8 **C.** 63 and 87

11A. एउटा AS को पहिलो पद र तेस्रो पद क्रमशः 84 र 72 भए दोस्रो पद पत्ता लगाउनुहोस् ।
Find the second term of an AS whose first and third terms are 84 and 72 respectively.

11B. एउटा AS को पहिलो पद र तेस्रो पद क्रमशः 90 र 80 भए दोस्रो पद पत्ता लगाउनुहोस् ।
Find the second term of an AS whose first and third terms are 90 and 80 respectively.

12. तल तोकिएको पद पत्ता लगाउनुहोस् (Calculate the terms indicated below):
A. $t_3 = 3$, $t_5 = 5$ and $t_4 = ?$ **B.** $t_7 = 16$, $t_9 = 20$ and $t_8 = ?$
C. $t_9 = 38$, $t_{11} = 46$ and $t_{10} = ?$ **D.** $t_6 = 18$, $t_8 = 24$ and $t_7 = ?$

13A. एउटा AS को पहिलो पद 8 र समान अन्तर 2 भए $8^{औं}$ मध्यमा कति हुन्छ ?
What is the 8^{th} mean of an AS whose 1^{st} term is 8 and common difference is 2?

13B. एउटा AS मा पहिलो पद = समान अन्तर = 5 भए $5^{औं}$ मध्यमा कति होला ?
In an AS, first term = common difference = 5 then what is the 5^{th} mean?

14A. 6 ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 5 र अन्तिम पद 40 भए समान अन्तर कति होला ?
What is the common difference of an AS whose first term is 5 and last term is 40 and having 6 arithmetic means?

14B. 7 ओटा समानान्तरीय मध्यमा भएको AS को पहिलो पद 4 र अन्तिम पद 44 भए समान अन्तर कति होला ?
What is the common difference of an AS whose first term is 4 and last term is 44 and having 7 arithmetic means?

15A. पहिलो पद 2 र अन्तिम पद 20 भएको समानान्तरीय अनुक्रमको पहिलो 10 ओटा पदहरूको योगफल कति हुन्छ ?
What is the sum of first 10 terms of an arithmetic sequence whose first term is 2 and last term is 20?

15B. पहिलो पद '3' र समान अन्तर '2' भएको AS को पहिलो 10 ओटा पदहरूको योगफल कति हुन्छ ?
Write the sum of first 10 terms of an AS having first term '3' and common difference '2'.

16A. पहिला 10 ओटा प्राकृतिक सङ्ख्याहरूको योगफल कति हुन्छ ? (What is the sum of first 10 natural numbers?)

16B. पहिला 11 ओटा जोर प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of first 11 even natural numbers.

16C. पहिला 15 ओटा बिजोर प्राकृतिक सङ्ख्याहरूको योगफल कति हुन्छ ? (Write the sum of first 15 odd natural numbers.)

B. SHORT QUESTIONS

Model 1 *Problems based on finding indicated term of an AS.*

1A. समानान्तर श्रेणीको एउटा उदाहरणसहित परिभाषा दिनुहोस् ।
Define arithmetic series with an example. **[2073 S]**

1B. अनुक्रम 3, 6, 9, 12,.... को $15^{औं}$ पद पत्ता लगाउनुहोस् ।
Find the 15^{th} term of the sequence 3, 6, 9, 12,.... **[2068 R']**

1C. अनुक्रम 10, 15, 20, 25,.... को $10^{औं}$ पद पत्ता लगाउनुहोस् ।

www.drmathnp.com

Find the 10^{th} term of the sequence 10, 15, 20, 25,....

2A. समानान्तरीय अनुक्रम $10, 12\frac{1}{2}, 15, 17\frac{1}{2}$ ..... को कति औं पदको मान 30 हुन्छ ?

Which term of the arithmetic sequence $10, 12\frac{1}{2}, 15, 17\frac{1}{2}$ .....is 30? [SEE 2075 R']

2B. श्रेणी $5 + 9 + 13 + ... + 77$ मा कतिओटा पदहरू छन्, पत्ता लगाउनुहोस् ।
How many terms are there in the series $5 + 9 + 13 + ... + 77$? Find. [2065 R]

2C. श्रेणी $3 + 6 + 9 + 12 + ...$ को कति औं पद 96 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series $3 + 6 + 9 + 12 + ...$ is 96 ? Find it. [2066 R']

2D. श्रेणी $24 + 28 + 32 + ...$ को कुन पद 52 हुन्छ ? (Which term is 52 in a series $24 + 28 + 32 + ...$?) [2067 S]

2E. श्रेणी $5 + 9 + 13 + ...$ को कुनचाहिँ पद 85 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series $5 + 9 + 13 + ...$ is 85? Find it. [2071 S]

3A. यदि श्रेणी $80 + 70 + 60 + ...$ को $n^{\text{औं}}$ पद शून्य भए n को मान निकाल्नुहोस् ।
If the n^{th} term of the series $80 + 70 + 60 + ...$ is zero, find the value of n. [2074 R]

3B. यदि श्रेणी $84 + 78 + 72 + ...$ को $n^{\text{औं}}$ पद 0 भए n को मान पत्ता लगाउनुहोस् ।
If the n^{th} term of series $84 + 78 + 72 + ...$ is 0, find the value of n. [2074 S]

Or, श्रेणी $84 + 78 + 72 + ...$ को कुन पद 0 हुन्छ ? (Which term of the series $84 + 78 + 72 + ...$ is 0 ?)

4A. यदि $25, m + 1, 35$ एउटा समानान्तरीय अनुक्रम भए m को मान पत्ता लगाउनुहोस् ।
If $25, m + 1, 35$ is an arithmetic sequence then find the value of m. [2074 R']

4B. यदि $6, p, q$ र 18 अङ्कगणितीय अनुक्रममा छन् भने p र q का मानहरू पत्ता लगाउनुहोस् ।
If $6, p, q$ and 18 are in an arithmetic sequence, find the values of p and q. [2064 S]

4C. $5, x, y, 11$ एउटा समानान्तरीय अनुक्रम हो भने x र y का मान पत्ता लगाउनुहोस् ।
$5, x, y, 11$ is an arithmetic sequence. Find the values of x and y. [2066 S]

Model 2 ***Problems based on finding the sum of given number of terms of an AS.***

5A. $2 + 4 + 6 + ... 20$ ओटा पदहरू भएको श्रेणीको योगफल निकाल्नुहोस् ।
Find the sum of the series $2 + 4 + 6 + ... 20$ terms. [2065 M]

5B. $5 + 11 + 17 + ...$सातौं पदसम्मको योगफल निकाल्नुहोस् ।
Find the sum of $5 + 11 + 17 + ...$ seventh term. [2064 R]

5C. AP मा रहेका $2, 6, 10, ...$ को पहिला एघार पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of first eleven terms of the AP $2, 6, 10, ...$

5D. श्रेणी $2 + 6 + 10 + ...$ को पहिलो 10 ओटा पदहरुको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 10 terms of the series $2 + 6 + 10 + ...$ [2074 S']

6A. पहिलो 'n' ओटा प्राकृतिक सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 'n' natural numbers. [2069 R']

6B. पहिलो 20 ओटा "विजोर" प्राकृतिक सङ्ख्याको योगफल निकाल्नुहोस् ।
Find the sum of the first twenty 'odd natural numbers'.

6C. पहिलो 10 ओटा "जोर प्राकृतिक सङ्ख्याहरू" को योगफल निकाल्नुहोस् ।
Find the sum of first 10 even natural numbers.

7A. समानान्तरीय श्रेणी $\sum_{n=4}^{7}(3n-2)$ को योगफल निकाल्नुहोस् । (Find the sum of the arithmetic series $\sum_{n=4}^{7}(3n-2)$.) [2057R]

7B. समानान्तरीय श्रेणी $\sum_{n=1}^{5}(4n+5)$ को योगफल निकाल्नुहोस् । (Find the sum of the arithmetic series $\sum_{n=1}^{5}(4n+5)$.)

C. LONG QUESTIONS

Model 1 ***Problems based on finding indicated term of an AP when other two terms are given.***

1A. यदि एउटा समानान्तरीय श्रेणीको पाचौँ पद र बाह्रौँ पदहरू क्रमशः -16 र 5 भए यसको तीसौँ पद पत्ता लगाउनुहोस् ।
If 5^{th} and 12^{th} term of an arithmetic series are -16 and 5 respectively, find its 30^{th} term. [2068S']

puspas.com.np

www.drmathnp.com

puspas.com.np

1B. एउटा अङ्कगणितीय अनुक्रमको तेस्रो पद – 40 र तेह्रौं पद 0 भए अठ्ठाइसौं पद कति हुन्छ ?
The 3rd term and the 13th term of an arithmetic sequence are – 40 and 0 respectively, what is the 28th term of the sequence ? [2065 M]

1C. यदि एउटा समानान्तरीय श्रेणीको पाँचौं र दशौं पदहरू क्रमशः 17 र 42 भए यसको बीसौं पद पत्ता लगाउनुहोस् ।
If 5th and 10th term of an arithmetic series are 17 and 42 respectively, find its 20th term. [2060 S']

1D. यदि एउटा AP को पाँचौं पद $\frac{1}{8}$ र आठौं पद $\frac{1}{5}$ भए सो AP को 40 औं पद पत्ता लगाउनुहोस् ।
If the fifth term of an AP is $\frac{1}{8}$ and eighth term is $\frac{1}{5}$, find the 40th term of the AP.

2A. कुनै समानान्तरीय श्रेणीको छैटौं पद 23 र यसको नवौं पद 35 भए कुन पद 67 होला ?
If the 6th term of an arithmetic series is 23 and its 9th term 35, which term will be 67? [2063 R']

2B. कुनै समानान्तरीय श्रेणीको छैटौं पद 30 र यसको नवौं पद 45 भए कुन पद 15 होला ?
If the 6th term of an arithmetic series is 30 and its 9th term 45, which term will be 15?

2C. कुनै समानान्तरीय श्रेणीको पाँचौं पद 19 र आठौं पद 31 भए कुन चाहिँ पद 67 होला ?
If the 5th term of an arithmetic series is 19 and 8th term is 31, which term is 67 ?

Model 2 *Problems based on finding indicated term of an AP.*

3A. यदि कुनै समानान्तरीय श्रेणीको चौथो पद 1 र पहिलो आठौं पदसम्मको योगफल 18 भए सो श्रेणीको दसौं पद पत्ता लगाउनुहोस् ।
If the fourth term of an AP is 1 and the sum of its first eight terms is 18, find the tenth term of the series. [2060 S]

3B. एउटा समानान्तरीय श्रेणीको पहिला तीन पदहरूको योगफल 42 र पहिलो पाँच पदहरूको योगफल 80 भए त्यस श्रेणीको बिसौं पद निकाल्नुहोस् ।
If the sum of the first three terms of an arithmetic series is 42 and that of the first five terms is 80, find the twentieth term of the series. [2062 S]

3C. यदि कुनै समानान्तरीय श्रेणीको पहिलो सातौं पदहरूको योगफल 14 र पहिलो दस पदहरूको योगफल 125 भए चौथो पद पत्ता लगाउनुहोस् ।
If the sum of the first seven terms of an arithmetic series is 14 and the sum of the first ten terms is 125 then find the fourth term of the series. [2063 R]

4A. समानान्तरीय अनुक्रममा भएका तीनओटा सङ्ख्याहरूको योगफल 51 र तिनीहरूको वर्गको योगफल 939 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् । [2070 S]
The sum of three numbers in an arithmetic progression is 51 and the sum of their square is 939. Find the numbers.

4B. समानान्तरीय अनुक्रममा रहेका तीन पदहरूको योगफल 21 र गुणनफल 315 भए, ती पदहरू निकाल्नुहोस् ।
Three terms in an arithmetic progression have sum 21 and product 315. Find the terms. [2070 R]

4C. समानान्तरीय श्रेणीमा रहेका तीनओटा पदहरूको योग 36 र तिनीहरूको गुणनफल 1140 छ भने ती पदहरू पत्ता लगाउनुहोस् ।
The sum of three terms in an arithmetic series is 36 and their product is 1140. Find these terms. [2065 R']

4D. एउटा समानान्तरीय श्रेणीको पहिला तीन पदहरूको योग 21 छ । यदि तेस्रो पदबाट पहिलो दुई पदहरूको योग घटाइयो भने 9 हुन्छ भने सो श्रेणीका तीन पदहरू निकाल्नुहोस् ।
The sum of the first three terms of an AS is 21. If the sum of the first two terms is subtracted from the third term then it would be 9. Find the three terms of the series. [2059S]

Model 3 *On finding the sum of AS when the terms or sum of certain terms are given.*

5A. यदि एउटा समानान्तरीय श्रेणीको 4 औं र 15 औं पदहरू क्रमशः 11 र 44 भए यसका प्रथम 20 पदहरूको योग पत्ता लगाउनुहोस् ।
If the 4th and 15th terms of an arithmetic series are 11 and 44 respectively then find the sum of its first 20 terms. [SEE 2075 R']

5B. कुनै एउटा समानान्तरीय श्रेणीको तेस्रो र सातौं पदहरू क्रमशः 18 र 30 भए प्रथम 20 पदहरूको योगफल पत्ता लगाउनुहोस् । [2071 R']
The 3rd and 7th terms of an arithmetic series are 18 and 30 respectively. Find the sum of the first 20 terms.

5C. एउटा समानान्तरीय श्रेणीको नवौं पद र उन्नाइसौं पदहरू क्रमशः 40 र 60 छन् भने सो श्रेणीको पहिलो 25 पदहरूको योगफल निकाल्नुहोस् ।
The ninth and nineteenth terms of an Arithmetic Series are 40 and 60 respectively. Find the sum of the first 25 terms of the series. [2070 R']

5D. यदि कुनै समानान्तरीय श्रेणीको तेस्रो पद 1 र पाचौं पद 7 भए सो श्रेणीका पहिला दस पदहरूको योगफल निकाल्नुहोस् । [2061 S]

www.drmathnp.com

If the third term of an arithmetic series is 1 and fifth term is 7, find the sum of first ten terms of the series.

5E. यदि एउटा समानान्तरीय श्रेणीको दसौँ पद 39 र सत्रौ पद 67 छन् भने पहिलो 26 पदहरूको योगफल निकाल्नुहोस् ।
If the 10^{th} term of an arithmetic series is 39 and the 17^{th} term is 67, find the sum of the first 26 terms. **[2065S, 2073 S']**

5F. यदि कुनै समानान्तरीय श्रेणीको तेस्रो र एघारौँ पदहरू क्रमशः 8 र – 8 भए सो श्रेणीको पहिला सात ओटा पदहरूको योगफल निकाल्नुहोस् ।
If the third and eleventh terms of an arithmetic series are 8 and – 8 respectively, find the sum of first seven terms of the series. **[2067 R']**

5G. 20 ओटा पदहरू भएको एउटा समानान्तरीय श्रेणीको अन्तिम पद 195 र समान अन्तर 5 भए त्यो श्रेणीको योगफल निकाल्नुहोस् ।
The last term of an arithmetic series of 20 terms is 195 and the common difference is 5. Calculate the sum of the series. **[2058 R]**

5H. यदि एउटा समानान्तरीय श्रेणीको पहिलो 15 ओटा पदहरूको योगफल 465 र 12 औँ पद 47 छ भने सो श्रेणीको पहिलो 22 ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the sum of the first 15 terms of an arithmetic series is 465 and its 12^{th} term is 47, find the sum of the first 22 terms of the series. **[2070 S']**

6A. एउटा समानान्तरीय श्रेणीको चौथो पद पहिलो पदको तेब्बर छ र सातौँ पद तेस्रो पदको दोब्बर भन्दा पनि 1 ले बढी छ । सो श्रेणीका पहिला दश पदहरूको योगफल निकाल्नुहोस् ।
The fourth term of an arithmetic series is three times the first term and seventh term exceeds twice the third term by 1. Find the sum of the first ten terms of the series. **[2075 R'$_2$]**

6B. एउटा समानान्तरीय अनुक्रममा छैँटौँ पद चौथो पदको तीनगुणा र पहिला तीन पदहरूको योगफल – 12 भए, पहिला दसओटा पदहरूको योगफल निकाल्नुहोस् ।
In an Arithmetic sequence, the sixth term is equal to three times the fourth term and the sum of the first three terms is – 12. Find the sum of the first ten terms. **[2069 S]**

6C. एउटा समानान्तरीय श्रेणीको चौधौँ पद 2 र पहिला दस पदहरूको योगफल – 150 भए सो श्रेणीको पहिला पच्चिस पदहरूको योगफल निकाल्नुहोस् ।
Fourteenth term of an arithmetic series is 2 and the sum of its first ten terms is – 150. Find the sum of the first twenty five terms of the series. **[2060 C, 2065 E]**

6D. एउटा समानान्तरीय श्रेणीको पहिलो दस पदहरूको योगफल 50 र पाँचौँ पद दोस्रो पदको तेब्बर भए पहिलो पद र पहिलो बिस पदहरूको योगफल निकाल्नुहोस् ।
The sum of the first ten terms of an arithmetic progression is 50 and its fifth term is treble of the second term. Calculate the first term and the sum of the first twenty terms. **[2064 S, 2067 R]**

Model 4 *Problems based on finding the sum when the sums are given.*

7A. यदि एउटा समानान्तरीय श्रेणीमा प्रथम 9 पदहरूको योगफल 72 छ र प्रथम 17 पदहरूको योगफल 289 भए प्रथम 25 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The sum of first 9 terms of an arithmetic series is 72 and the sum of first 17 terms is 289. Find the sum of first 25 terms. **[SEE 2073 R']**

7B. यदि एउटा समानान्तरीय श्रेणीमा प्रथम 7 पदहरूको योगफल 14 छ र प्रथम 11 पदहरूको योगफल 66 भए प्रथम 25 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The sum of first 7 terms of an arithmetic series is 14 and the sum of first 11 terms is 66. Find the sum of first 25 terms.

7C. एउटा समानान्तरीय श्रेणीको 9 औँ पद 19 र प्रथम 12 पदहरूको योग 168 भए प्रथम 8 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of first eight terms of an arithmetic series whose 9^{th} term is 19 and sum of first 12 terms is 168.

Model 5 *Problems based on finding common difference and terms of an AS.*

8A. एउटा समानान्तरीय श्रेणीको पहिलो पद र अन्तिम पद क्रमशः – 24 र 72 छन् । यदि सबै पदहरूको योगफल 600 भए सो श्रेणीको पदहरूको सङ्ख्या र समान अन्तर निकाल्नुहोस् ।
The first and last terms of an arithmetic series are – 24 and 72 respectively. If sum of all terms of the series is 600, find the number of terms and the common difference of the series. **[2059 R]**

8B. एउटा समानान्तरीय श्रेणीका पहिला दस पदहरूको योगफल 520 छ । यदि यसको सातौँ पद तेस्रो पदको दोब्बर भए सो श्रेणीको पहिलो पद र समान अन्तर पत्ता लगाउनुहोस् ।
In an AS, the sum of the first ten terms is 520. If its seventh term is double of its third term, calculate the first term and the common difference of the series. **[2061 R]**

pustas.com.np

www.drmathnp.com

pustas.com.np

8C. एउटा समानान्तरीय श्रेणीको पहिलो आठ पदहरूको योग 180 र पाँचौ पद पहिलो पदको पाँचगुणा भए त्यस श्रेणीको पहिलो पद र समान अन्तर पत्ता लगाउनुहोस् ।
The sum of first eight terms of an arithmetic series is 180 and its fifth term is five times of the first term then find the first term and common difference. **[2069 R]**

Model 6 Problems based on AM of AP.

9A. 5 र 25 को बिचमा चारओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert four arithmetic means between 5 and 25.) **[2072 R']**

9B. – 7 र 17 को बिचमा पाँचओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 5 arithmetic means between – 7 and 17.)

9C. – 3 र 32 को बिचमा 6 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 6 arithmetic means between – 3 and 32.)

9D. 5 र 65 को बिचमा 3 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 3 arithmetic means between 5 and 65.)

9E. 5 र 25 को बिचमा 3 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 3 arithmetic means between 5 and 25.)

9F. 24 र 39 को बिचमा 4 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 4 arithmetic means between 24 and 39.)

10A. 15 र 45 को बिचमा m ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा 30 भए m को मान निकाल्नुहोस् ।
There are m arithmetic means between 15 and 45. If the third mean is 30, find the value of m. **[2068 S]**

10B. 3 र 21 को बिचमा m ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा 12 भए m को मान निकाल्नुहोस् ।
There are m arithmetic means between 3 and 21. If the third mean is 12, find the value of m.

11A. 4 र 24 को बिचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा र अन्तिम मध्यमाको अनुपात 4 : 5 भए उक्त श्रेणीमा जम्मा कतिओटा पदहरू छन् ? पत्ता लगाउनुहोस् ।
There are n arithmetic means between 4 and 24. If the ratio of third mean to the last mean is 4 : 5 then find the number of terms in the series. **[2071 R]**

11B. 2 र 14 का बिचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा र अन्तिम मध्यमाको अनुपात 2 : 3 भए उक्त श्रेणीमा जम्मा कतिओटा पदहरू छन् ? पत्ता लगाउनुहोस् ।
There are n arithmetic means between 2 and 14. If the ratio of third mean to the last mean is 2 : 3 then find the number of terms in the series.

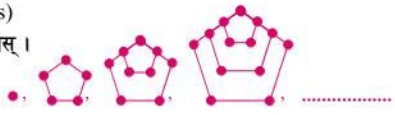
11C. 3 र 39 को बिचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा : अन्तिम मध्यमा = 3 : 7 भए n को मान पत्ता लगाउनुहोस् ।
There are n arithmetic means between 3 and 39. Find the value of n so that third mean : last mean = 3 : 7.

Hint: $a = 3,\ b = 39,\ m_3 : m_n = 3 : 7$ Now, $d = \frac{b-a}{n+1} = \frac{39-3}{n+1} = \frac{36}{n+1}$

And, $\frac{m_3}{m_n} = \frac{a+3d}{a+nd}$ or, $\frac{3}{7} = \frac{a+3.\frac{36}{n+1}}{a+n.\frac{36}{n+1}}$ Simplify it then $n = 8$.

Model 7 Problems based on patterns of numbers and diagrams.

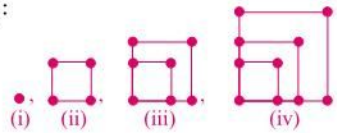
12A. दिइएका सङ्ख्याहरूको ढाँचामा (In the following pattern of numbers)



(i) अनुक्रमको त्यही क्रममा ठीक पछि आउने थप एउटा चित्र बनाउनुहोस् ।
Add one more figure in the same pattern.

(i) यो अनुक्रमको $n^{\text{औं}}$ पदको सूत्र पत्ता लगाउनुहोस् ।
Find the formula for the n^{th} term of the sequence. **[SLC 2063 M]**

12B. तल दिइएका चित्रहरूका ढाँचा अध्ययन गरी सोधिएका प्रश्नहरूको उत्तर दिनुहोस् :
Study the following pattern of the given figures:



(i) अनुक्रमको त्यही क्रममा ठीक पछि आउने थप एउटा चित्र बनाउनुहोस् ।
Add one more figures in the same pattern.

(ii) यो अनुक्रमको $n^{\text{औं}}$ पदको सूत्र पत्ता लगाउनुहोस् ।
Find the formula of the n^{th} term of the sequence.

(iii) यो अनुक्रमको $10^{\text{औं}}$ पद पत्ता लगाउनुहोस् । (Find the 10^{th} term of the same sequence.) **[SLC 2064 R]**

www.drmathnp.com

puspas.com.np

Model 8 *Problems based on application of AS.*

13A. 10 वर्ष पहिलेको जनसङ्ख्या 18395 भएको एउटा सानो सहरले हरेक वर्ष 270 वासिन्दा गुमाउँदछ । सो सहरको अहिलेको जनसङ्ख्या कति होला ?
Small town where population was 18395, ten years ago, has lost 270 inhabitants each year since then. What is the present population of small town?

13B. 10 वर्ष पहिलेको जनसङ्ख्या 12317 भएको एउटा सानो सहरले हरेक वर्ष 250 वासिन्दा गुमाउँदछ । सो सहरको अहिलेको जनसङ्ख्या कति होला ?
A small town whose population was 12317 ten years ago, has lost 250 inhabitants each year since then. What is the present population of small town?

14A. एकजना मानिसको मासिक तलब रु. 65000 छ । यदि उसले हरेक वर्ष रु. 150 को वृद्धि पाउँदछ भने $10^{औं}$ वर्षमा उसले कति तलब पाउला ?
A man has a monthly salary of Rs 65000. If he gets an increment of Rs 150 every year, how much salary does he receive in 10^{th} year?

14B. एकजना मानिसको मासिक तलब रु. 10000 छ । यदि उसले हरेक वर्ष रु. 300 को वृद्धि पाउँदछ भने $10^{औं}$ वर्षमा उसले कति तलब पाउला ?
A man has a monthly salary of Rs 10000. If he gets an annual increment of Rs 300, how much salary does he receive in 10^{th} year?

15A. एउटा कम्पनीले पहिलो वर्ष 100 क्याल्कुलेटर उत्पादन गऱ्यो । पाँच वर्षको अन्तसम्ममा जम्मा 4500 क्याल्कुलेटर उत्पादन गऱ्यो । हरेक वर्षको उत्पादन समान रूपमा बढेको छ भनी मान्नुहोस् । हरेक वर्ष उत्पादनमा भएको वृद्धि पत्ता लगाउनुहोस् ।
A firm produced 100 calculator sets during its first year. The total number of calculator sets produced at the end of five years is 4500. Assume that the production increases uniformly each year. Estimate the increase in production each year.

15B. एउटा कम्पनीले पहिलो वर्ष 1000 रेडियो उत्पादन गऱ्यो । दस वर्षको अन्तसम्ममा जम्मा 14500 रेडियो उत्पादन गऱ्यो । हरेक वर्षको उत्पादन समान रूपमा बढेको छ भनी मान्नुहोस् । हरेक वर्ष उत्पादनमा भएको वृद्धि पत्ता लगाउनुहोस् ।
A firm produced 1000 radio sets during its first year. The total number of radio sets produced at the end of 10 years is 14500. Assume that the production increases uniformly each year. Estimate the increase in production each year.

16A. एकजना अभिभावकले बैंकको बचत खातामा उसको शिशुको पहिलो जन्मदिनमा रु. 500, दोस्रो जन्मदिनमा रु. 1000 र तेस्रो जन्मदिनमा रु. 1500 गरी हरेक जन्मदिनमा रु. 500 बढाएर जम्मा गर्दछ । शिशुको $16^{औं}$ जन्मदिन पुग्दा उसले कति रकम जम्मा गर्दछ ?
A parent places in the saving bank Rs 500 on his child's first birthday, Rs 1000 on his second, Rs 1500 on his third and so on increasing the amount by Rs 500 on each birthday. How much will he save up when the child reaches the sixteenth birthday, the latter inclusive?

16B. एकजना अभिभावकले बैंकको बचत खातामा उसको शिशुको पहिलो जन्मदिनमा रु. 800, दोस्रो जन्मदिनमा रु. 1200 र तेस्रो जन्मदिनमा रु. 1600 गरी हरेक जन्मदिनमा रु. 400 बढाएर जम्मा गर्दछ । शिशुको $15^{औं}$ जन्मदिन पुग्दा उसले कति रकम जम्मा गर्दछ ?
A parent places in the saving bank Rs 800 on his child's first birthday, Rs 1200 on his second, Rs 1600 on his third and so on increasing the amount by Rs 400 on each birthday. How much will he save up when the child reaches the fifteenth birthday, the latter inclusive?

16C. एकजना ठेकेदारले कुनै काम निश्चित समय भन्दा ढिलो गरे वापत पहिलो दिनको रु. 200, दोस्रो दिनको रु 250 र तेस्रो दिनको रु. 300 गर्दै प्रत्येक पछिल्लो दिनमा पहिलो दिनको भन्दा रु 50 बढि जरिवाना तिर्नुपर्छ । यदि उक्त ठेकेदारले कुनै काम 30 दिन ढिला गरेमा जम्मा कति जरिवाना तिर्नुपर्ला ?
A contractor on construction job specifies a penalty for delay of completion beyond a certain date as: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day and so on. The penalty for each succeding day being Rs 50 more than that of the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days? **[SEE MODEL 2076]**

16D. एकजना ठेकेदारले कुनै काम निश्चित समय भन्दा ढिलो गरे वापत पहिलो दिनको रु. 100, दोस्रो दिनको रु 150 र तेस्रो दिनको रु. 200 गर्दैगर्दै प्रत्येक पछिल्लो दिनमा पहिलो दिनको भन्दा रु 50 बढि जरिवाना तिर्नुपर्छ । यदि उक्त ठेकेदारले कुनै काम 20 दिन ढिला गरेमा जम्मा कति जरिवाना तिर्नुपर्ला ?
A contractor on construction job specifies a penalty for delay of completion beyond a certain date as: Rs 100 for the first day, Rs 150 for the second day, Rs 200 for the third day and so on. The penalty for each succeding day being Rs 50 more than that of the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 20 days?

www.drmathnp.com

pustas.com.np

17A. एकजना मानिसको सुरूको तलब रु. 20000 छ र हरेक वर्ष रु. 2000 का दरले बढ्छ । $9^{औं}$ वर्षको तलब र 9 वर्ष सम्मको जम्मा तलब पत्ता लगाउनुहोस् ।
The starting salary of a man is Rs 20,000 and it increases by Rs 2000 every year. Find the salary of 9^{th} year and total salary for 9 years.

17B. एकजना मानिसको सुरूको तलब रु. 15000 छ र हरेक वर्ष रु. 1500 का दरले बढ्छ । $10^{औं}$ वर्षको तलब र 10 वर्ष सम्मको जम्मा तलब पत्ता लगाउनुहोस् ।
The starting salary of a man is Rs 15000 and it increases by Rs 1500 every year. Find the salary of 10^{th} year and total salary for 10 years.

18A. मोहनले सोहनसँग विना ब्याजमा रु. 14500 ऋण लिन्छ र 10 किस्तामा तिर्ने सहमती गर्दछ । उसले रु. 1000 पहिलो किस्तामा तिर्छ र हरेक किस्तामा अघिल्लो किस्तामा बराबरी रकम बढ्ने गरी तिर्छ । उसको अन्तिम किस्ता कति थियो ?
Mohan secures an interest free loan of Rs 14500 from Sohan and agrees to repay it in 10 installments. He pays Rs 1000 as first installment and then increases each installment by equal amount over the preceding installment. What will be his last installment?

18B. रामले श्यामसँग विना ब्याजमा रु. 2,90,000 ऋण लिन्छ र 20 किस्तामा तिर्ने सहमती गर्दछ । उसले रु. 5000 पहिलो किस्तामा तिर्छ र हरेक किस्तामा अघिल्लो किस्तामा बराबरी रकम बढ्ने गरी तिर्छ । उसको अन्तिम किस्ता कति थियो ?
Ram secures an interest free loan of Rs 2,90,000 from Shyam and agrees to repay it in 20 installments. He pays Rs 5000 as first installment and then increases each installment by equal amount over the preceding installment. What will be his last installment?

19A. एउटा मानिसले रु. 975 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 5 ले कम छ । पहिलो किस्ता वापतको रकम रु. 100 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 975 in monthly installments, each installment being less than the former by Rs 5. The amount of first installment is Rs 100. In how many installments will the entire amount be paid ? Give reason.

19B. एउटा मानिसले रु. 6200 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 20 ले कम छ । पहिलो किस्ता वापतको रकम रु. 500 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 6200 in monthly installments, each installment being less than the former by Rs 20. The amount of first installment is Rs 500. In how many installments will the entire amount be paid ? Give reason.

20A. एउटा मानिसले रु. 45750 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 50 ले बढी छ । पहिलो किस्ता वापतको रकम रु. 800 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 45750 in monthly installments, each installment being more than the former by Rs 50. The amount of first installment is Rs 800. In how many installments will the entire amount be paid ? Give reason.

20B. एउटा मानिसले रु. 51200 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 40 ले बढी छ । पहिलो किस्ता वापतको रकम रु. 500 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 51200 in monthly installments, each installment being more than the former by Rs 40. The amount of first installment is Rs 500. In how many installments will the entire amount be paid ? Give reason.

ANSWERS

A. VERY SHORT QUESTIONS

8A. 1 **8B.** 2 **8C.** -30 **8D.** -13 **9A.** 22 **9B.** 40 **10A.** 22 **10B.** 15 **10C.** 75
11A. 78 **11B.** 85 **12A.** 4 **12B.** 18 **12C.** 42 **12D.** 21 **13A.** 24 **13B.** 30 **14A.** 5
14B. 5 **15A.** 110 **15B.** 120 **16A.** 55 **16B.** 132 **16C.** 225

B. SHORT QUESTIONS

1B. 45 **1C.** 55 **2A.** 9^{th} term **2B.** 19 **2C.** 32 **2D.** 8 **2E.** 21^{st} term **3A.** 9 **3B.** 15
4A. 29 **4B.** 10, 14 **4C.** 7, 9 **5A.** 420 **5B.** 161 **5C.** 242 **5D.** 200 **6A.** $\frac{n(n+1)}{2}$
6B. 400 **6C.** 110 **7A.** 58 **7B.** 85

C. LONG QUESTIONS

1A. 59 **1B.** 60 **1C.** 92 **1D.** 1 **2A.** 17 **2B.** 3^{rd} **2C.** 17 **3A.** 16
3B. 50 **3C.** 2 **4A.** 11, 17, 23 or 23, 17, 11 **4B.** 9, 7, 5 or 5, 7, 9 **4C.** 5, 12, 19 **4D.** $-1, 7, 15$
5A. 610 **5B.** 810 **5C.** 1200 **5D.** 85 **5E.** 1378 **5F.** 42 **5G.** 2950 **5H.** 990
6A. 120 **6B.** 100 **6C.** 0 **6D.** 1/2 , 200 **7A.** 650 **7B.** 500 **7C.** 80 **8A.** 25, 4
8B. 16, 8 **8C.** $a = d = 5$ **9A.** 9, 13, 17, 21 **9B.** $-3, 1, 5, 9, 13$ **9C.** 2, 7, 12, 17, 22, 27
9D. 20, 35, 50 **9E.** 10, 15, 20 **9F.** 27, 30, 33, 36 **10A.** $m = 5$ **10B.** $m = 5$ **11A.** 6
11B. 7 **11C.** 8 **12A.** $4n - 3$ **12B.** $3n - 2$, 28 **13A.** 15965 **13B.** 10067 **14A.** Rs 66350
14B. Rs 12700 **15A.** 400 **15B.** 100 **16A.** Rs 68, 000 **16B.** Rs 54000 **16C.** Rs 27750
16D. Rs 11500 **17A.** Rs 36,000; Rs 252,000 **17B.** Rs 28500, Rs 217500 **18A.** Rs 1900 **18B.** Rs 24000
19A. 15 **19B.** 20 **20A.** 30 **20B.** 40

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 3

A. SHORT QUESTIONS

Model 1 *Problems based on finding indicated term of an AS.*

1A. समानान्तरीय अनुक्रम 3, 7, 11, ..., 407 को पछाडिबाट बीसौँ पद पत्ता लगाउनुहोस् ।
Find the 20^{th} term from the end of an arithmetic sequence 3, 7, 11, ..., 407.

1B. समानान्तरीय अनुक्रम 3, 5, 7, 9, ..., 201 को पछाडिबाट बाह्रौँ पद पत्ता लगाउनुहोस् ।
Find the 12^{th} term from the end of an arithmetic sequence 3, 5, 7, 9, ..., 201.

1C. अङ्कगणितीय अनुक्रम 4, K, 10, …… को दसौँ पद पत्ता लगाउनुहोस् ।
Find the tenth term of the arithmetic sequence: 4, K, 10, ……

1D. अङ्कगणितीय अनुक्रम 5, k, 11, …… को दसौँ पद पत्ता लगाउनुहोस् ।
Find the tenth term of the arithmetic sequence: 5, k, 11, ……

2A. अनुक्रम 2, 4, 6, 8……को $n^{\text{औँ}}$ पद पत्ता लगाउनुहोस् । (Find n^{th} term of the sequence 2, 4, 6, 8……)

2B. 20, 15, 10, 5…… को $n^{\text{औँ}}$ पद पत्ता लगाउनुहोस् । (Find n^{th} term of the sequence 20, 15, 10, 5……..)

3A. एउटा समानान्तरीय श्रेणीका पहिलो र तेस्रो पदहरू क्रमश $(a+b)^2$ र $(a-b)^2$ भए दोस्रो पद कति होला ?
If the first and the third terms of an arithmetic series are $(a+b)^2$ and $(a-b)^2$ respectively, what is its second term ?

3B. एउटा समानान्तरीय श्रेणीको पहिलो र तेस्रो पदहरू क्रमश (a^3+b^3) र (a^3-b^3) भए पाँचौँ पद कति होला ?
If the first and the third terms of an arithmetic series are (a^3+b^3) and (a^3-b^3) respectively, what is its fifth term ?

3C. एउटा समानान्तरीय अनुक्रमको सातौँ पद (x^2+y^2) र नवौँ पद (x^2-y^2) भए आठौँ पद पत्ता लगाउनुहोस् ।
Find the 8^{th} term of an arithmetic sequence whose 7^{th} term is (x^2+y^2) and 9^{th} term is (x^2-y^2).

4A. एउटा अनुक्रमको $n^{\text{औँ}}$ पद $(2n+1)$ छ । के यो अनुक्रम समानान्तरीय अनुक्रम हो ? यदि भए, यसको $12^{\text{औँ}}$ पद पत्ता लगाउनुहोस् ।
The n^{th} term of a sequence is $(2n+1)$. Is the sequence, so formed, is arithmetic sequence ? If so, find its 12^{th} term.

4B. $\{a_n\} = \{3n+5\}$ एउटा समानान्तरीय अनुक्रम हो भनी देखाउनुहोस् र $10^{\text{औँ}}$ पद पत्ता लगाउनुहोस् ।
Show that $\{a_n\} = \{3n+5\}$ is an arithmetic sequence and find 10^{th} term.

4C. यदि $x-1, x+3$ र $2x+1$ हरू समानान्तरीय अनुक्रममा भए सो श्रेणीको दोस्रो पद पत्ता लगाउनुहोस् ।
If $x-1, x+3$ and $2x+1$ are in AS, find the 2^{nd} term of the sequence.

Model 2 *Problems based on finding the position of indicated term of an AP.*

5A. अनुक्रम $34, 32\frac{1}{2}, 31, 29\frac{1}{2}$ ..... को कुनचाहिँ पद 16 हुन्छ ?

Which term of the sequence $34, 32\frac{1}{2}, 31, 29\frac{1}{2}$ ..... is 16 ?

5B. के समानान्तरीय श्रेणी $25 + 22\frac{1}{2} + 20$.....मा –10 पर्दछ ?

Does –10 belong to the arithmetic series $25 + 22\frac{1}{2} + 20$..... ?

5C. यदि दुई ओटा अङ्कगणितीय अनुक्रमहरू $-19-12-5+2+$ ..... र $1+6+11+$ … का $n^{\text{औँ}}$ पदहरू बराबर छन् भने n को मान पत्ता लगाउनुहोस् ।
The n^{th} terms of two AP's $-19-12-5+2+$ ..... and $1+6+11+$ ….. are equal then find the value of n.

5D. अनुक्रम 3, 8, 13, ..., 58 मा कति पदहरू छन् ? (How many terms are there in a sequence 3, 8, 13, ... 58?)

5E. यदि 25, 50, 75, …….. को $k^{\text{औँ}}$ पद 1000 भए k को मान निकाल्नुहोस् ।
If the k^{th} term of 25, 50, 75 …… is 1000, find the value of k.

Model 2 *Problems based on finding the sum of given number of terms of an AS.*

6A. एउटा समानान्तरीय अनुक्रम 3, 7, 11, ..., 407 को पछाडिका 20 पदहरूको योगफल पत्ता लगाउनुहोस् ।
What is the sum of last 20 terms of an arithmetic sequence 3, 7, 11, ..., 407 ?

6B. एउटा समानान्तरीय अनुक्रम 3, 5, 7, 9,...., 201 को पछाडिका 12 पदहरूको योगफल पत्ता लगाउनुहोस् ।
What is the sum of last 12 terms of an arithmetic sequence 3, 5, 7, 9, ..., 201 ?

www.drmathnp.com
puspas.com.np

7A. एउटा समानान्तरीय श्रेणीको नवौं पद 4 भए सो श्रेणीको पहिलो सत्र पदसम्मको योगफल निकाल्नुहोस् ।
The ninth term of an arithmetic series is 4. Find the sum of the first seventeen terms.

7B. एउटा समानान्तरीय श्रेणीको एघारौं पद 60 भए सो श्रेणीको पहिलो एक्काइस पदसम्मको योगफल निकाल्नुहोस् ।
The eleventh term of an arithmetic series is 60. Find the sum of the first twenty first terms.

8A. एउटा समानान्तरीय श्रेणीको पहिलो पद 8 छ । यदि बिसौं पद 150 भए पहिलो पद 20 पदसम्मको योगफल पत्ता लगाउनुहोस् ।
The first term of an arithmetic series is 8. If 20^{th} term is 150 then find the sum of first twentith term.

8B. एउटा समानान्तरीय श्रेणीको पहिलो पद 12 छ । यदि चौबिसौं पद 240 भए पहिलो पद 24 पदसम्मको योगफल पत्ता लगाउनुहोस् ।
The first term of an arithmetic series is 12. If 24^{th} term is 240 then find the sum of first twenty-fourth term.

Model 4 *Problems based on arithmetic mean/means between two numbers.*

9A. यदि a, m र b एउटा AP मा भए $m = \frac{a+b}{2}$ हुन्छ भनी देखाउनुहोस् ।

If a, m and b are in an AP then show that $m = \frac{a+b}{2}$.

9B. यदि x र $\frac{2}{3}x$ को अङ्कगणितीय मध्यमा 20 भए x को मान निकाल्नुहोस् ।

If arithmetic mean between x and $\frac{2}{3}x$ is 20, find the value of x.

9C. एउटा सङ्ख्या अर्को सङ्ख्याको व्युत्क्रम छ र तिनीहरूको समानान्तरीय मध्यक $\frac{5}{4}$ छ भने ती सङ्ख्याहरू निकाल्नुहोस् ।

One number is reciprocal to another and the arithmetic mean between them is $\frac{5}{4}$. Find those numbers.

9D. दुईओटा सङ्ख्याहरू एक आपसमा व्युतक्रमानुपातिक छन् र तिनीहरूको समानान्तरीय मध्यक $\frac{13}{5}$ भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

The arithmetic mean of two numbers which are reciprocal to each other is $\frac{13}{5}$. Find the numbers.

10A. दुई सङ्ख्याहरू 5 : 3 को अनुपातमा छन् । यदि तिनीहरूको अङ्कगणितीय मध्यमा 40 भए उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Two numbers are in the ratio of 5 : 3. If their arithmetic mean is 40, find the numbers.

10B. यदि दुई सङ्ख्याहरूबिचको अङ्कगणितीय मध्यमा 20 र पहिलो सङ्ख्या 24 भए दोस्रो सङ्ख्या पत्ता लगाउनुहोस् ।
If arithmetic mean between two numbers is 20 and the first number is 24, find the second number.

11A. 4 र 10 को बिचमा 2 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 2 arithmetic means between 4 and 10.)

11B. 5 र 20 को बिचमा 2 ओटा समानान्तरीय मध्यमाहरू भर्नुहोस् । (Insert 2 arithmetic means between 5 and 20.)

Model 5 *Problems based on finding value of k or P when terms of AP are given.*

12A. यदि 48, $24k$ र $15k$ हरू समानान्तरीय अनुक्रममा भए k को मान पत्ता लगाउनुहोस् ।
If 48, $24k$ and $15k$ are in arithmetic sequence, find the value of k.

12B. यदि 40, 2P र 3P हरू AP मा भए P को मान पत्ता लगाउनुहोस् । (If 40, 2P & 3P are in AP, find the value of P.)

12C. एउटा AP को पहिला तीन पदहरू $(k+2)$, $(4k-6)$ र $(3k-2)$ भए k को मान पत्ता लगाउनुहोस् ।
Find the value of k if $(k+2)$, $(4k-6)$ and $(3k-2)$ are first three terms of an AP.

12D. यदि $(P+1)$, 3P र $(4P+2)$ एउटा AP का 3 ओटा अनुवर्ती पदहरू भए P को मान पत्ता लगाउनुहोस् ।
If $(P+1)$, 3P and $(4P+2)$ be any 3 consecutive terms of an AP, find the value of P.

12E. यदि k, k^2+1 र $k+6$ हरू एउटा AP मा भए k को मान पत्ता लगाउनुहोस ।
If k, k^2+1 and $k+6$ are in an AP, find the value of k.

12F. यदि $\frac{8}{P}, \frac{12}{P}$, 4 एउटा समानान्तरीय अनुक्रममा भए P को मान पत्ता लगाउनुहोस् ।

If $\frac{8}{P}, \frac{12}{P}$, 4 are in an arithmetic sequence then find the value of P.

Model 6 *Problems based on finding the first term of an AP.*

13A. एउटा समानान्तरीय अनुक्रमको सातौं पद 14 र दोस्रो पद 4 भए पहिलो पद पत्ता लगाउनुहोस् ।
An arithmetic sequence has 7^{th} term 14 and 2^{nd} term 4, find the first term.

13B. एउटा समानान्तरीय अनुक्रमको सातौं पद 17 र दोस्रो पद 7 भए पहिलो पद पत्ता लगाउनुहोस् ।
An arithmetic sequence has 7^{th} term 17 and 2^{nd} term 7, find the first term.

www.drmathnp.com

13C. एउटा समानान्तरीय अनुक्रमको सातौँ पद 26 छ । यदि पहिलो पदको दोब्बर समान अन्तर भए पहिलो पद पत्ता लगाउनुहोस् ।
An arithmetic sequence has 7^{th} term 26. If the common difference is double of the first term, find the first term.

13D. एउटा समानान्तरीय अनुक्रमको पाचौँ पद 39 छ । यदि पहिलो पद समान अन्तरको $33\frac{1}{3}\%$ भए पहिलो पद पत्ता लगाउनुहोस् ।
An arithmetic sequence has 5^{th} term 39. If the first term is $33\frac{1}{3}\%$ of common difference, find the first term.

14A. एउटा समानान्तरीय श्रेणीको बिसौँ पद 150 र पहिला 20 पदहरू सम्मको योगफल 1580 भए पहिलो पद पत्ता लगाउनुहोस् ।
In an arithmetic series, 20^{th} term is 150 and the sum of first twentith term is 1580. Find the first term.

14B. एउटा समानान्तरीय श्रेणीको चौबिसौँ पद 240 र पहिला 24 पदहरू सम्मको योगफल 3600 भए पहिलो पद पत्ता लगाउनुहोस् ।
In an arithmetic series, 24^{th} term is 240 and the sum of first twenty-fourth term is 3600. Find the first term.

Model 7 *Problems based on finding common difference of AP.*

15A. चौथो पद 9 र पहिलो पद 21 भएको समानान्तरीय श्रेणीमा समान अन्तर कति हुन्छ ?
If 4^{th} and 1^{st} terms of an arithmetic series are 9 and 21 respectively, what is the common difference?

15B. यदि एउटा समानान्तरीय श्रेणीको पहिलो पद 6 र आठौँ पद 30 भए समान अन्तर पत्ता लगाउनुहोस् ।
If the first term of an arithmetic sequence is 6 and eighth term is 30, find the common difference.

16A. यदि कुनै अङ्कगणितीय श्रेणीमा पहिलो पाँच पदहरूको योगफल 120 र प्रथम पद 16 भए समान अन्तर पत्ता लगाउनुहोस् ।
If the sum of first five terms of an arithmetic series with 1^{st} term 16 is 120, find the common difference.

16B. यदि कुनै अङ्कगणितीय श्रेणीमा पहिलो पाँच पदहरूको योगफल 30 र प्रथम पद 2 भए समान अन्तर पत्ता लगाउनुहोस् ।
If the sum of first five terms of an arithmetic series with 1^{st} term 2 is 30, find the common difference.

17A. 4 र 36 को बिचमा 6 ओटा समानान्तरीय मध्यमा भए समान अन्तर निकाल्नुहोस् ।
If there are 6 AM between 4 and 36, find common difference.

17B. 3 र 18 को बिचमा 4 ओटा समानान्तरीय मध्यमा भए समान अन्तर निकाल्नुहोस् ।
If there are 4 AM between 3 and 18, find common difference.

17C. एउटा AP को पहिला n ओटा पदहरूको योगफल $3n^2 + 2n$ भए सो AP को समान अन्तर पत्ता लगाउनुहोस् ।
The sum of first n terms of an AP is $3n^2 + 2n$. Find the common difference of the AP.

Model 8 *Problems based on finding number of terms of an AP.*

18A. एउटा समानान्तरीय अनुक्रमको पहिलो पद 4 र समान अन्तर 5 छ । योगफल 265 बनाउन कतिओटा पदहरू लिनुपर्छ ?
The first term of an arithmetic series is 4 and the common difference is 5. How many terms must be taken so that their sum may be 265 ?

18B. एउटा समानान्तरीय अनुक्रमको पहिलो पद 5 र समान अन्तर 2 छ । योगफल 140 बनाउन कतिओटा पदहरू लिनुपर्छ ?
The first term of an arithmetic series is 5 and the common difference is 2. How many terms must be taken so that their sum may be 140 ?

18C. श्रेणी $24 + 48 + 72 + \ldots$ मा कति ओटा पदहरूसम्म जम्मा गर्दा योगफल 504 हुन्छ ?
How many terms of the series $24 + 48 + 72 + \ldots$ will add up to 504 ?

19A. पहिलो पद 8 र अन्तिम पद 150 भएको एउटा समानान्तरीय श्रेणीको योगफल 1580 भए पदहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
The sum of arithmetic series having first term 8 and the last term 150 is 1580. Find the number of terms.

19B. पहिलो पद 12 र अन्तिम पद 240 भएको एउटा समानान्तरीय श्रेणीको योगफल 3024 भए पदहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
The sum of arithmetic series having first term 12 and the last term 240 is 3024. Find the number of terms.

Model 9 *Problems based on finding indicated term of an AS when sum is given & vice-versa.*

20A. एउटा AP को $n^{\text{औँ}}$ पदसम्मको योगफल $3n^2 + 5n$ र $m^{\text{औँ}}$ पद 164 भए m को मान पत्ता लगाउनुहोस् ।
If the sum of n terms of an AP is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.

20B. एउटा AP को $n^{\text{औँ}}$ पदसम्मको योगफल $4n^2 + 3n$ र $P^{\text{औँ}}$ पद 63 भए P को मान पत्ता लगाउनुहोस् ।
If the sum of n terms of an AP is $4n^2 + 3n$ and its P^{th} term is 63, find the value of P.

21A. कुनै त्रिभुजको भुजाहरूका लम्बाइ AS मा छन् र यसको परिमिति 27 cm छ । यदि यसको सबैभन्दा छोटो भुजा 5 cm भए बाँकी भुजाका लम्बाइहरू पत्ता लगाउनुहोस् ।
The length of sides of a triangle form an AS. If the perimeter of the triangle is 27 cm and the shortest side is 5 cm long, find the length of other sides.

21B. कुनै चतुर्भुजको भुजाहरूका लम्बाइहरू AS मा छन् र यसको परिमिति 28 cm छ । यदि यसको सबैभन्दा छोटो भुजा 4 cm भए बाँकी भुजाका लम्बाइहरू पत्ता लगाउनुहोस् ।
The lengths of sides of a quadrilateral form an AS. If the perimeter of the quadrilateral is 28 cm and the shortest side is 4 cm long, find the length of other sides.

www.drmathnp.com

puspas.com.np

22A. एउटा AP को पहिला 11 ओटा पदहरूको योगफल 198 छ । सो श्रेणीको छैटौं पद पत्ता लगाउनुहोस् ।
The sum of first 11 terms of an AP is 198. Find the sixth term of the series.

22B. एउटा AP को पहिला 23 ओटा पदहरूको योगफल 1035 छ । सो श्रेणीको बाह्रौं पद पत्ता लगाउनुहोस् ।
The sum of first 23 terms of an AP is 1035. Find the twelfth term of the series.

22C. एउटा AP को पहिला 5 पदहरूको योगफल 65 भए सो AP को तेस्रो पद पत्ता लगाउनुहोस् ।
The sum of first five terms of an AP is 65, find the third term.

22D. एउटा AP को एघारौं पद 528 भए पहिला एक्काइस पदहरूको योग पत्ता लगाउनुहोस् ।
Eleventh term of an AP is 528. Find the sum of first twenty one terms.

23A. एउटा AS मा पहिला 20 पदसम्मको योगफल 1580 छ । यदि पहिलो पद 8 भए $20^{\text{औं}}$ पद कति होला ?
In an AS, the sum of first 20 terms is 1580. If the first term is 8, what is the 20^{th} term ?

23B. एउटा AS मा 20 पदसम्मको योगफल 3300 छ । यदि पहिलो पद 12 भए $20^{\text{औं}}$ पद कति होला ?
In an AS, the sum of first 20 terms is 3300. If the first term is 12, what is the 20^{th} term ?

24A. अनुक्रमको योगफल पत्ता लगाउनुहोस् जसको साधारण पद $t_n = 2n + 4$ हो जहाँ $2 \le n \le 5$ छ ।
Find the sum of the sequence whose general term $t_n = 2n + 4$ for $2 \le n \le 5$.

24B. एउटा अनुक्रममा $t_n = 5n - 2$ भए S_n पत्ता लगाउनुहोस् । (In a sequence, $t_n = 5n - 2$, find S_n.)

24C. एउटा अनुक्रममा $t_n = 5n + 8$ भए S_n पत्ता लगाउनुहोस् । (If in a sequence, $t_n = 5n + 8$, find S_n.)

Model 10 ***Problems based on sigma notation of an AP.***

25. दिइएको समानान्तरीय श्रेणीमा k को मान कति होला ? (In the given arithmetic series, what is the value of k ?)

A. $\sum_{n=1}^{3}(3n + k) = 36$ **B.** $\sum_{n=4}^{7}(3n - k) = 58$

26. दिइएको समानान्तरीय श्रेणीमा आठौं पद पत्ता लगाउनुहोस् । (In the given arithmetic series, find the eighth term.)

A. $\sum_{n=3}^{10}(kn + 6) = 100$ **B.** $\sum_{n=1}^{9}(5n + p) = 225$

B. LONG QUESTIONS

Model 1 ***Problems based on finding indicated term of an AS.***

1A. कुनै समानान्तरीय श्रेणीका $11^{\text{औं}}$ र $25^{\text{औं}}$ पदहरू क्रमशः 20 र 41 छन् । त्यो श्रेणीका पहिला तीन पदहरू पत्ता लगाउनुहोस् ।
Find the first three terms of an AP whose 11^{th} and 25^{th} terms are 20 and 41 respectively.

1B. कुनै समानान्तरीय श्रेणीको एघारौं पद 22 र पच्चीसौं पद 43 भए 33 औं पद कति होला ?
If the 11^{th} term of an arithmetic series is 22 and 25^{th} term is 43, what is the 33^{rd} term?

2A. एउटा समानान्तरीय श्रेणीको तेस्रो र सातौं पदको योगफल 8 छ र तेह्रौं पद पाँचौं पदको चार गुणा छ भने श्रेणीको एघारौं पद पत्ता लगाउनुहोस् ।
If the sum of 3^{rd} and 7^{th} terms of an arithmetic series is 8 and 13^{th} term is 4 times the fifth term then find the 11^{th} term of the series.

2B. एउटा समानान्तरीय श्रेणीको छैटौं पद पहिलो पदको पाँचगुणा छ र एघारौं पद पाँचौं पदको दोब्बरभन्दा 3 ले धेरै छ । सो श्रेणीको आठौं पद पत्ता लगाउनुहोस् ।
The sixth term of an arithmetic series is 5 times the first term and the eleventh term exceeds twice the fifth term by 3. Find the 8^{th} term.

2C. एउटा समानान्तरीय श्रेणीमा पाँचौं पद तेस्रो पदको दोब्बर भए सातौं पदको दोब्बर तेह्रौं पद हुन्छ भनी देखाउनुहोस् ।
In an arithmetic series, fifth term is double of the third term. Prove that thirteenth term is double of the seventh term.

2D. एउटा समानान्तरीय श्रेणीको पाचौं पदको पाँच गुणा र छैटौं पदको छ गुणा बराबर भए एघारौं पद पत्ता लगाउनुहोस् ।
In an arithmetic series, five times of the fifth term is equal to six times of the sixth term. Find the eleventh term.

2E. एउटा समानान्तरीय श्रेणीमा $7\,t_7 = 11\,t_{11}$ भए t_{18} को मान पत्ता लगाउनुहोस् ।
In an arithmetic series $7\,t_7 = 11\,t_{11}$. Find the value of t_{18}.

Model 2 ***Problems based on finding sum of arithmetic sequence/series.***

3A. एउटा समानान्तरीय अनुक्रममा पहिला 20 ओटा पद र 30 ओटा पदहरूको योगफल बराबर छन् भने पहिला 50 ओटा पदहरूको योगफल कति होला ?
In an arithmetic sequence, the sum of first 20 terms is equal to that of 30 terms. What is the sum of first 50 terms?

www.drmathnp.com
pusras.com.np

3B. एउटा समानान्तरीय अनुक्रममा पहिला 15 ओटा पद र 25 ओटा पदहरूको योगफल बराबर छन् भने पहिला 40 ओटा पदहरूको योगफल कति होला ?
In an arithmetic sequence, the sum of first 15 terms is equal to that of 25 terms. What is the sum of first 40 terms?

4A. एउटा अङ्कगणितीय अनुक्रमको पहिलो तीनओटा पदहरूको योग 12 छ । यदि यसको छैटौँ पद 16 भए यसको पहिलो दुई पदहरूको योगफल पत्ता लगाउनुहोस् ।
The sum of first three terms of an AP is 12. If its 6^{th} term is 16, find the sum of its first two terms.

4B. एउटा समानान्तरीय श्रेणीको चौथो पद पहिलो पदको तीन गुणा छ । यदि सो श्रेणीको सातौँ पद 30 भए प्रथम 15 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The fourth term of an arithmetic series is equal to 3 times the first term. If the 7^{th} term of the arithmetic series is 30, find the sum of first 15 terms.

4C. एउटा समानान्तरीय अनुक्रममा पहिला 4 ओटा पदहरूको योगफल 48 छ र त्यसपछिका 4 पदहरूको योगफल 144 छ । पहिला 25 पदहरूको योगफल पत्ता लगाउनुहोस् ।
In an AP, the sum of the first 4 terms is 48 and the sum of next four terms is 144. Find the sum of first 25 terms.

4D. एउटा समानान्तरीय अनुक्रममा पहिला 4 ओटा पदहरूको योगफल 50 छ र त्यसपछिका 4 पदहरूको योगफल 130 छ । पहिला 25 पदहरूको योगफल पत्ता लगाउनुहोस् ।
In an AP, the sum of the first 4 terms is 50 and sum of next four terms is 130. Find the sum of first 25 terms.

4E. एउटा समानान्तरीय अनुक्रममा पहिला 5 ओटा पदहरूको योगफल 30 र त्यसपछिका 5 ओटा पदहरूको योगफल 80 छ । पहिला 15 ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
If an AP, the sum of first 5 terms is 30 and the sum of next five terms is 80, find the sum of first 15 terms.

5A. के 33 श्रेणी $3 + 6 + 9 + \ldots + 42$ को एउटा पद हो ? सो श्रेणीको योगफल पनि पत्ता लगाउनुहोस् ।
Is 33 a term of the series $3 + 6 + 9 + \ldots + 42$? Find the sum of the series.

5B. अनुक्रम 13, 16, 19, 22 ....., 39 मा कति ओटा पदहरू होलान् ? योगफल पनि पत्ता लगाउनुहोस् ।
How many terms are there in the sequence 13, 16, 19, 22, … , 39? Also find the sum.

6A. एउटा समानान्तरीय श्रेणीको छैटौँ पद यसको दोस्रो पदको चारगुणा छ । यदि पहिला 24 पदको योगफल 1704 भए पहिला 48 पदहरूको योगफल निकाल्नुहोस् ।
The 6^{th} term of an AP is four times the 2^{nd} term and the sum of first 24 terms is 1704. Find the sum of the first 48 terms.

6B. एउटा समानान्तरीय श्रेणीको चौथो पद पहिलो पदको तेब्बर छ र सातौँ पद तेस्रो पदको दोब्बरभन्दा पनि 1 ले बढी छ । सो श्रेणीको पहिला 15 ओटा पदहरूको योगफल निकाल्नुहोस् ।
The fourth term of an A.S. is three times the 1^{st} term and the 7^{th} term exceeds twice the third term by 1. Find the sum of first 15 terms of the series.

6C. 18 ओटा पदहरू भएको एउटा समानान्तरीय श्रेणीको अन्तिम पद 75 र समान अन्तर 4 भए त्यो श्रेणीको योगफल निकाल्नुहोस् ।
The last term of an arithmetic series of 18 terms is 75 and the common difference is 4. Calculate the sum of the series.

6D. समानान्तरीय श्रेणी हुने गरी 30 दिनसम्म रकम जम्मा गर्दा पाँचौँ दिनमा रु. 22 र दशौँ दिनमा रु. 47 जम्मा गर्नुपर्छ भने 30 दिनसम्म कति रकम जम्मा होला ?
It needs Rs 22 at 5^{th} day and Rs 47 at 10^{th} day to deposit some amount to form an AP till 30 days. How much amount will be deposited in 30 days?

7A. 200 र 500 को बिचमा पर्ने 9 ले निःशेष भाग लाग्ने सङ्ख्याहरूको योगफल निकाल्नुहोस् ।
Find the sum of all the numbers between 200 and 500 which are divisible by 9.

7B. 6 ले भाग जाने 100 देखि 200 को बिचमा पर्ने सबै सङ्ख्याहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of all numbers from 100 to 200 which are divisible by 6.

8A. समानान्तरीय श्रेणी 52, 48, 44 ..... मा कतिओटा पदहरू भए तिनीहरूको योगफल 360 होला ? साथै $S_{12} = S_{15}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
How many terms of the AP 52, 48, 44 ..... must be taken so that the sum may be 360 ? Also prove that: $S_{12} = S_{15}$

8B. समानान्तरीय श्रेणी 50, 45, 40, ... मा कतिओटा पदहरू भए तिनीहरूको योगफल 270 हुन्छ ? साथै $S_6 = S_{15}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
How many terms of the AP 50, 45, 40,..... must be taken so that the sum may be 270 ? Also prove that $S_6 = S_{15}$

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on finding indicated term of an AS when sum is given.*

9A. एउटा समानान्तरीय श्रेणीका पहिला तीन पदहरूको योग 30 छ । यदि तेस्रो पदबाट पहिला दुई पदहरूको योग घटाइयो भने – 6 हुन्छ भने सो श्रेणीका 3 पदहरू पत्ता लगाउनुहोस् ।
The sum of first three terms of an arithmetic series is 30. If the sum of first two terms is subtracted from the third term then it would be – 6, find the three terms of the series.

9B. एउटा समानान्तरीय श्रेणीका पहिला तीन पदहरूको योग 18 छ । यदि तेस्रो पदबाट पहिला दुई पदहरूको योग घटाइयो भने 0 हुन्छ । सो श्रेणीका 3 पदहरू पत्ता लगाउनुहोस् ।
The sum of first three terms of an AP is 18. If the sum of first two terms is subtracted from the third term then it would be 0, find the three terms of the series.

9C. यदि एउटा AP को $9^{\text{औं}}$ पद 19 र यसका पहिला 12 पदहरूको योगफल 168 भए सो अनुक्रमको $15^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
If the 9^{th} term of an AP is 19 and the sum of its first 12 terms is 168, find the 15^{th} term of the progression.

9D. एउटा AP को चौथो र आठौं पदहरूको योगफल 24 छ । छैटौं र दशौं पदको योगफल 34 छ । $26^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
The sum of the 4^{th} and 8^{th} term of an AP is 24. The sum of the 6^{th} and 10^{th} term is 34. Find the 26^{th} term.

9E. एउटा AP को चौथो र आठौं पदहरूको योगफल 24 छ । छैटौं र दशौं पदको योगफल 32 छ । $26^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
The sum of the 4^{th} and 8^{th} term of an AP is 24. The sum of the 6^{th} and 10^{th} term is 32. Find the 26^{th} term.

Model 4 *On finding common difference, first term and number of terms of an AP.*

10A. एउटा समानान्तरीय श्रेणीको पहिलो पद र अन्तिम पद क्रमशः 5 र 50 छन् । यदि सबै पदहरूको योगफल 275 भए सो श्रेणीको पदसङ्ख्या र समान अन्तर पत्ता लगाउनुहोस् ।
The first and last term of an arithmetic series are 5 and 50 respectively. If the sum of all terms of the series is 275, find the number of terms and the common difference of the series.

10B. एउटा समानान्तरीय श्रेणीको पहिलो पद र अन्तिम पद क्रमशः 4 र 40 छन् । यदि सबै पदहरूको योगफल 220 भए सो श्रेणीको पदसङ्ख्या र समान अन्तर पत्ता लगाउनुहोस् ।
The first and last term of an arithmetic series are 4 and 40 respectively. If the sum of all terms of the series is 220, find the number of terms and the common difference of the series.

11A. 15 ओटा पदहरू भएको एउटा समानान्तरीय श्रेणीको तेस्रो पद 14 छ र योगफल 585 छ भने यसको पहिलो पद र समान अन्तर पत्ता लगाउनुहोस् ।
An AP has 15 terms whose third term is 14 and sum is 585. Find its first term and the common difference.

11B. 16 ओटा पदहरू भएको एउटा समानान्तरीय श्रेणीको तेस्रो पद 11 छ र योगफल 528 छ भने यसको पहिलो पद र समान अन्तर पत्ता लगाउनुहोस् ।
An AP has 16 terms whose third term is 11 and sum is 528. Find its first term and the common difference.

11C. एउटा समानान्तरीय श्रेणीको दोस्रो पद र आठौं पदको योगफल 10 तथा पहिलो 6 ओटा पदहरूको योगफल 3 छ भने श्रेणीको प्रथम पद र समान अन्तर निकाल्नुहोस् ।
The sum of 2^{nd} and 8^{th} term of an arithmetic sequence is 10 and the sum of first six terms of the series is 3. Find the first term and common difference.

Model 5 *Problems based on selection of terms in AP (Dividing into 3 or 4 parts).*

12A. 69 लाई समानान्तरीय श्रेणीमा हुने गरी 3 भागमा यसरी विभाजन गर्नुहोस् जहाँ साना 2 भागहरूको गुणनफल 483 होस् ।
Split 69 into three parts such that they are in arithmetic series and the product of two smaller parts is 483.

12B. 15 लाई समानान्तरीय श्रेणीमा हुने गरी तीन भागमा विभाजन गर्नुहोस् जसको गुणनफल 120 हुन्छ ।
Divide 15 into three parts which are in arithmetic series and whose product is 120.

13A. एउटा समानान्तरीय श्रेणीको तीनओटा पदहरूको योगफल 27 छ र तिनीहरूको वर्गको योगफल 293 भए ती पदहरू पत्ता लगाउनुहोस् ।
The sum of three terms of an arithmetic series is 27 and the sum of their squares is 293. Find the numbers.

13B. एउटा समानन्तरीय श्रेणीको तीनओटा पदको योगफल 27 छ र तिनीहरूको वर्गको योगफल 315 भए ती पदहरू पत्ता लगाउनुहोस् ।
The sum of three terms of an AP is 27 and the sum of their square is 315. Find the numbers.

13C. एउटा AP मा $S_3 = 21$ र $t_1^2 + t_2^2 + t_3^2 = 179$ भए t_1, t_2 र t_3 का मानहरू पत्ता लगाउनुहोस् ।
In an AP, $S_3 = 21$ and $t_1^2 + t_2^2 + t_3^2 = 179$. Find the values of t_1, t_2 and t_3.

13D. समानान्तरीय श्रेणीमा रहेका तीन सङ्ख्याहरूको योगफल 12 छ र तिनीहरूको वर्गको योगफल 56 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of 3 numbers in an AP is 12 & the sum of whose squares is 56. Find the numbers.

14A. एउटा समानान्तरीय श्रेणीको तीनओटा पदहरूको योगफल – 3 छ र तिनीहरूको गुणनफल 8 भए ती पदहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms of an arithmetic series is – 3 & their product is 8. Find the numbers.

www.drmathnp.com

14B. एउटा समानान्तरीय श्रेणीको तीनओटा पदहरूको योगफल 27 छ र तिनीहरूको गुणनफल 648 भए ती पदहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms of an arithmetic series is 27 and their product is 648. Find the numbers.

14C. एउटा समानान्तरीय श्रेणीको तीनओटा पदहरूको योगफल 24 छ र तिनीहरूको गुणनफल 440 भए ती पदहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms of an arithmetic series is 24 and their product is 440. Find the numbers.

14D. समानान्तरीय श्रेणीमा रहेका तीनओटा पदहरूको योगफल 36 र तिनीहरुको गुणनफल 1140 छ भने, ती पदहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms in an arithmetic series is 36 & their product is 1140. Find these terms.

14E. समानान्तरीय श्रेणीमा रहेका तीनओटा सङ्ख्याहरूको योगफल 12 छ र तिनीहरूको घनहरूको योग 288 छ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers in an AP is 12 & the sum their cubes is 288. Find the numbers.

14F. समानान्तरीय श्रेणीमा रहेका तीनओटा सङ्ख्याहरूको योगफल 21 छ र पहिलो र तेस्रो पदको गुणनफल दोस्रो पदभन्दा 6 ले बढी छ । ती तीन पदहरू पत्ता लगाउनुहोस् ।
The sum of three terms of an AP is 21 and the product of the first and the third term exceeds the second term by 6. Find the three terms.

15A. एउटा AP को 4 ओटा पदहरू पत्ता लगाउनुहोस् जसको योगफल 20 छ र वर्गहरूको योगफल 120 छ ।
Find four numbers in AP whose sum is 20 and the sum of whose squares is 120.
Hint: Let the numbers be $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$ and the common difference is $2d$.

15B. समानान्तरीय श्रेणीमा रहेका पहिलो चार पदहरू पत्ता लगाउनुहोस्, जसको योगफल 20 र तिनीहरूका वर्गहरूको योगफल 180 छ ।
Find the first four terms in an arithmetic series whose sum is 20 and the sum of whose squares is 180.

15C. 32 लाई छेउका पदहरूको गुणनफल र मध्यमाहरूको गुणनफलको अनुपात 7 : 15 हुनेगरी एउटा AP को 4 भागमा विभाजन गर्नुहोस् ।
Divide 32 into 4 parts which are in AP such that the product of extremes is to the product of means is 7 : 15.

15D. एउटा AP मा रहेका 4 ओटा सङ्ख्याहरू पत्ता लगाउनुहोस् जसको योगफल 50 छ र सबैभन्दा ठूलो पद सबैभन्दा सानो पदको 4 गुणा छ ।
Find the four numbers in AP, whose sum is 50 and in which the greatest number is 4 times the least.

15E. तीनओटा सङ्ख्याहरू 1 : 2 : 4 को अनुपातमा छन् । यदि पहिलो सङ्ख्यामा 3 जोडियो र तेस्रो सङ्ख्याबाट 8 घटाइयो भने नयाँ सङ्ख्याहरू पहिलो पद र तेस्रो पद भई दोस्रो सङ्ख्या दोस्रो पद भएको एउटा AP बन्दछ । सुरूका सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in the ratio of 1 : 2 : 4. If 3 is added to the first and 8 is subtracted from the third, the new numbers will be the first and third terms of an A.P., whose second term is the second number. Find the original numbers.

15F. तीनओटा सङ्ख्याहरू 1 : 2 : 4 को अनुपातमा छन् । यदि पहिलो सङ्ख्यामा 2 जोडियो र तेस्रो सङ्ख्याबाट 6 घटाइयो भने नयाँ सङ्ख्याहरू पहिलो पद र तेस्रो पद भई दोस्रो सङ्ख्या दोस्रो पद भएको एउटा AP बन्दछ । सुरूका सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in the ratio of 1 : 2 : 4. If 2 is added to the first and 6 is subtracted from the third, the new numbers will be the first and third terms of an A.P., whose second term is the second number. Find the original numbers.

Model 6 *Problems based on finding value of variables when terms of AP are given.*

16A. एउटा समानान्तरीय श्रेणीका पहिला तीन पदहरू $p+2$, $2p-1$ र $p+6$ भए p र पाचौं पद सम्मको योगफल निकाल्नुहोस् ।
If the first 3 terms of an arithmetic series are $p+2$, $2p-1$ and $p+6$ then find p and sum of its first five terms.

16B. $2(k-1)$, $k+2$ र $3k$ एउटा AP का तीनओटा पदहरू भए k को मान र पहिलो 5 ओटा पदहरूको योगफल निकाल्नुहोस् ।
$2(k-1)$, $k+2$ and $3k$ are the three terms of an AP. Find the value of k and sum of first 5 terms.

16C. यदि 10, $m+5$,..., $3m$, 35 हरू AP का पदहरू भए m को मान र चौथो पद पत्ता लगाउनुहोस् ।
If 10, $m+5$,... , $3m$, 35 are the terms of AP, find the value of m and the fourth term.

16D. एउटा समानान्तरीय अनुक्रमका तीन अनुवर्ती पदहरू $2x+1$, x^2+x+1 र $3x^2-3x+3$ भए x को मान पत्ता लगाउनुहोस् ।
Find the value of x so that $2x+1$, x^2+x+1 and $3x^2-3x+3$ are the three consecutive terms of an AP.

16E. a र b का मानहरू पत्ता लगाउनुहोस् जब 12, $a+b$, $2a$, b हरू एउटा AP मा छन् ।
Find the values of a and b such that 12, $a+b$, $2a$, b are in an AP.

16F. $2p$, $2p+q$, $p+4q$ र 35 एउटा AP मा भए p र q का मानहरू पत्ता लगाउनुहोस् ।
Find the values of p and q such that $2p$, $2p+q$, $p+4q$, 35 are in an AP

17A. 13, p, q, r, 29 एउटा समानान्तरीय अनुक्रममा भए p, q र r का मानहरू निकाल्नुहोस् ।
If 13, p, q, r, 29 are in an arithmetic sequence, find the values of p, q and r.

17B. 15, p, q, r, 25 एउटा समानान्तरीय अनुक्रममा भए p, q र r का मानहरू पत्ता लगाउनुहोस् ।
Find the values of p, q and r if 15, p, q, r, 25 are in an arithmetic sequence.

pusmas.com.np

www.drmathnp.com

puspas.com.np

17C. 25, x, y, z, 45 हरू एउटा AP मा भए x, y र z का मानहरू पत्ता लगाउनुहोस् ।
If 25, x, y, z, 45 are in an AP, find the values of x, y and z.

17D. a, 30, b, 50, c हरू एउटा AP मा भए a, b र c का मानहरू पत्ता लगाउनुहोस् ।
If a, 30, b, 50, c are in an AP, find the values of a, b and c.

Model 7 Problems based on arithmetic mean/means between two numbers.

18A. तेस्रो मध्यमा र अन्तिम मध्यमा क्रमशः 8 र 18 हुने गरी a र b का बिचमा 8 ओटा समानान्तरीय मध्यमाहरू भरिएका छन् । a र b का मानहरू पत्ता लगाउनुहोस् ।
8 arithmetic means are inserted between a and b such that the third and last means are 8 and 18 respectively. Find the values of a and b.

18B. दोस्रो र अन्तिम मध्यमा 1 : 4 को अनुपातमा छन् भने 5 र 35 को बिचमा कतिओटा समानान्तरीय मध्यमाहरू होलान् ?
How many arithmetic means are between 5 and 35 such that the ratio of second mean and last mean is equal to 1 : 4 ?

18C. 1 र 19 को बिचमा भएका समानान्तरीय मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् । जहाँ पहिलो र अन्तिम मध्यमाको अनुपात 1: 4 छ ।
Find the number of arithmetic means between 1 and 19 where the first mean is to the last as 1: 4.

18D. दुई सङ्ख्याहरू a र b को बिचमा तीनओटा समानान्तरीय मध्यमाहरू छन् । यदि पहिलो र तेस्रो मध्यमाहरू क्रमशः 16 र 34 भए, दोस्रो मध्यमा a र b का मानहरू पत्ता लगाउनुहोस् ।
Three AM's are inserted between two numbers a and b. If the first and the third means are 16 and 34, find the second mean and also the values of a and b.

18E. 1 र 31 को बिचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि सातौँ मध्यमा र $(n-1)$ औँ मध्यमा 5 : 9 को अनुपातमा भए n को मान पत्ता लगाउनुहोस् ।
There are n arithmetic means between 1 and 31. If the ratio of seventh mean to $(n-1)^{th}$ mean is 5 : 9, find the value of n.

19A. 2 र b को बिचमा 6 ओटा समानान्तरीय मध्यमाहरू छन् र चौथो मध्यमा शून्य भए बाँकी मध्यमाहरू र b को मान पत्ता लगाउनुहोस् ।
If 6 arithmetic means are inserted between 2 and b and the fourth mean is zero, find the other means and value of b.

19B. 3 र p को बिचमा 6 ओटा समानान्तरीय मध्यमाहरू छन् र चौथो मध्यमा 15 भए बाँकी मध्यमाहरू र p को मान पत्ता लगाउनुहोस् ।
If 6 arithmetic means are inserted between 3 and p and the fourth mean is 15, find the other means and value of p.

20A. दुईओटा सङ्ख्याहरूको गुणनफल 45 छ र तिनीहरूको समानान्तरीय मध्यमा 7 छ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The product of two numbers is 45 and their arithmetic mean is 7. Find the numbers.

20B. दुईओटा सङ्ख्याहरूको गुणनफल 48 छ र तिनीहरूको समानान्तरीय मध्यमा 7 छ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The product of two numbers is 48 and their arithmetic mean is 7. Find the numbers.

Model 8 Problems based on proving the algebraic conditions based on AP.

21A. यदि a, b र c एउटा AP मा भए $(a-c)^2 = 4(b^2 - ac)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If a, b and c are in an AP, prove that: $(a-c)^2 = 4(b^2 - ac)$

21B. एउटा AP मा तीनओटा सङ्ख्याहरू a, b र c छन् । यदि $y^2 = xz$ भए $x^{b-c} \cdot y^{c-a} = z^{b-a}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
The three numbers a, b and c are in an AP. If $y^2 = xz$, prove that: $x^{b-c} \cdot y^{c-a} = z^{b-a}$

21C. एउटा समानान्तरीय अनुक्रममा $p^{औँ}$ पद q र $q^{औँ}$ पद p भए $n^{औँ}$ पद $(p+q-n)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In an arithmetic sequence, p^{th} term is q and the q^{th} term is p. Show that n^{th} term is $(p+q-n)$.

21D. यदि एउटा AS का $p^{औँ}$, $q^{औँ}$ र $r^{औँ}$ पदहरू क्रमशः a, b र c भए प्रमाणित गर्नुहोस् : $a(q-r) + b(r-p) + c(p-q) = 0$.
If p^{th}, q^{th} and r^{th} terms of an AS are a, b and c respectively, prove that $a(q-r) + b(r-p) + c(p-q) = 0$.

21E. यदि एउटा AS का $p^{औँ}$, $q^{औँ}$ र $r^{औँ}$ पदहरू क्रमशः a, b र c भए प्रमाणित गर्नुहोस् : $p(b-c) + q(c-a) + r(a-b) = 0$.
If p^{th}, q^{th} and r^{th} terms of an AS are a, b and c respectively, prove that $p(b-c) + q(c-a) + r(a-b) = 0$.

Model 9 Problems based on application of arithmetic sequence.

22A. अश्मिनाले प्रति महिनाको $200 मा वार्षिक वृद्धिदर $50 सहित एउटा रोजगार पाउँछिन् । उनीले 10 वर्षमा कति कमाउँछिन् ? उनको एघारौँ वर्षको तलब पत्ता लगाउनुहोस् ।
Ashmina gets an employment of $ 200 in a month with an annual increment of $ 50. How much does she earn in 10 years? Find her salary at the 11^{th} year.

www.drmathnp.com

pusnas.com.np

22B. एउटी केटीले एक महिना भरिमा रु. 32 बचत गर्दछिन् । अर्को महिनामा रु. 36 र तेस्रो महिनामा रु. 40 बचत गर्दछिन् । यदि उनले यहि क्रममा बचत गर्दै जान्छिन् भने कति महिनामा रु. 2000 बचत गर्दछिन् ?
A girl saves Rs 32 during one month, Rs 36 in the next month, Rs 40 in the third month. If she continues her savings in this sequence, in how many months will she save Rs 2000 ?

22C. एकजना मानिसले रु. 3250 को ऋण पहिलो महिना रु. 20 र त्यस पछाडि हरेक महिना रु. 15 को दरले बढाएर तिर्छ । ऋण चुक्ता गर्न उसलाई कति समय लाग्ला ?
A man repays a loan of Rs 3250 by paying Rs 20 in the first month and then increases the payment by Rs 15 every month. How long will it take him to clear the loan ?

22D. एउटा कक्षामा रहेका केटाहरूको उमेर AP मा छ । समान अन्तर 4 महिना छ । यदि सबैभन्दा कान्छो केटाको उमेर 8 वर्ष मात्र र सबै केटाहरूको उमेरको योगफल 168 वर्ष भए केटाहरूको सङ्ख्या र सबैभन्दा जेठो केटाको उमेर पत्ता लगाउनुहोस् ।
A class consists of a number of boys whose age are in AP. The common difference is 4 months. If the youngest boy of the class be only 8 years and the sum of the ages of all boys be 168 years, find the number of boys and age of the oldest boy.

Hint: Here, $a = 8$ yrs $\quad d = 4 \text{ months} = \frac{4}{12} = \frac{1}{3}$ yr $\quad S_n = 168$ years

We know, $S_n = \frac{n}{2}[2a + (n-1)d]$ or, $168 = \frac{n}{2}[2 \times 8 + (n-1) \times \frac{1}{3}]$

or, $168 = \frac{n}{2}\left(\frac{48 + n - 1}{3}\right)$ or, $168 \times 6 = 47n + n^2$ or, $n^2 + 47n - 1008 = 0$

or, $n^2 + 63n - 16n - 1008 = 0$ or, $n(n + 63) - 16(n + 63) = 0$ or, $n = -63, 16$

$\therefore$ number of boys $(n) = 16$

The age of the oldest boy $(t_{16}) = a + (n-1)d = 8 + (16-1) \times \frac{1}{3} = 8 + 5 = 13$ yrs.

22E. एउटा समुदायमा भएका मानिसहरूको उमेर समानान्तर अनुक्रममा छ । उनीहरूको उमेरको समान अन्तर 3 महिना छ । यदि सबैभन्दा कम उमेरको मानिस 10 वर्षको र सबै मानिसहरूको उमेरको योगफल 341 वर्ष 3 महिना भए सबैजना मानिसहरूको सङ्ख्या र सबैभन्दा जेठो मानिसको उमेर पत्ता लगाउनुहोस् ।
A community consists of a number of people whose ages are in AP. The common difference of their ages is 3 months. The youngest people is of 10 years and the sum of their ages is 341 years 3 months. Find the total number of people and the age of eldest people.

22F. एउटा क्रिकेट टिममा भएका खेलाडीहरूको उमेर समानान्तर अनुक्रममा छ । उनीहरूको उमेरको समान अन्तर 4 महिना छ । यदि सबैभन्दा कम उमेरको खेलाडी 18 वर्षको र सबै खेलाडीहरूको उमेरको योगफल 550 वर्ष भए सबैजना खेलाडीहरूको सङ्ख्या र सबैभन्दा जेठो खेलाडीको उमेर पत्ता लगाउनुहोस् ।
A cricket team consists of a number of players whose ages are in AP. The common difference of their ages is 4 months. The youngest player is of 18 years and the sum of their ages is 550 years. Find the total number of players and the age of eldest player.

22G. एउटा वित्तीय संस्थाको एकजना अधिकृतले पाउने तलब सम्बन्धमा यस्तो सूचना जारी गरेको रहेछ: "सुरुको तलब प्रति महिना रु. 6000 र त्यसपछि प्रत्येक 6/6 महिनामा रु. 400 का दरले 5 वर्षसम्म थप्दै जाने" । अब यही दरमा उसको अन्तिम तलब दर (पाँचवर्ष पछिको) प्रति महिना कति रुपैयाँ हुने रहेछ ? पत्ता लगाउनुहोस् ।
The salary of an officer in a finance company is noticed as: "The starting salary per month is Rs 6000 and increase Rs 400 per six month for 5 years". In this way, find the rate of last salary per month after 5 years.

Hint: $a = 6000, d = 400, n = (5 \times 2 + 1) = 11$

$t_n = a + (n-1)d = a + 10d = 10000$

22H. एउटा जुत्ता कारखानाले पहिलो वर्ष 10000 जोर र दोस्रो वर्ष 12000 जोर जुत्ता उत्पादन गरेछ । यदि कारखानाले प्रत्येक वर्ष उत्पादन वृद्धि गर्ने जुत्ताको सङ्ख्या समान रहेछ भने, सो कारखानाले 8 वर्षमा जम्मा कति जोर जुत्ता उत्पादन गर्दो रहेछ ?
A shoe factory produces 10000 pairs of shoes in first year and 12000 pairs of shoes in second year. If the number of shoes produced in each year is increased by the same rate, find the total number of pair of shoes produced in 8 years.

22I. कृषि कार्यका लागि एउटा फाइनान्स कम्पनीबाट लिइएको ऋणको सावाँ र ब्याज गरेर जम्मा रु. 29000 तिर्नुपर्नेमा मासिक किस्ताबन्दिका दरले तिर्दै जाँदा यो रकम 20 महिनामा चुक्ता हुने रहेछ । यदि प्रत्येक किस्तामा रु. 100 बढी रकम तिर्दै जानुपर्ने सर्त भए, पहिलो किस्ताको रकम कति होला ?
To pay a sum of money amounts of Rs 29000 to farming from a Finance Company, it takes 20 months to pay per month in installment system. If in each installment Rs 100 more is paid, find the amount that should be paid in first installment.

www.drmathnp.com

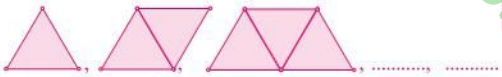
puspas.com.np

Model 10 Problems of patterns of figures based on arithmetic sequence

23A. तल दिइएका सङ्ख्याहरूको ढाँचा अध्ययन गर्नुहोस् (Study the following patterns of numbers):

(i) अनुक्रममा यसपछि आउने दुईओटा ढाँचा थप्नुहोस् । (Add two more patterns in the sequence.)

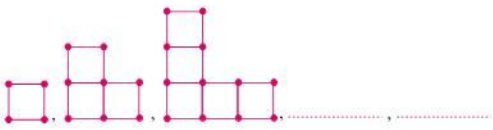
(ii) $n^{\text{औं}}$ पदमा हुने थोप्लाको सूत्र पत्ता लगाउनुहोस् ।
Find the formula for the n^{th} dots in the term of the sequence.



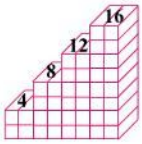
23B. तल दिइएका सङ्ख्याहरूको ढाँचा अध्ययन गर्नुहोस्:
Study the following patterns of numbers:

(i) अनुक्रममा यसपछि आउने दुईओटा ढाँचा थप्नुहोस् ।
Add two more patterns in the sequence.

(ii) $n^{\text{औं}}$ पदको सूत्र पत्ता लगाउनुहोस् ।
Find the formula for the n^{th} term of the sequence.



23C. सँगैको चित्र हेर्नुहोस् । भऱ्याङको पहिलो खुट्किलामा 4 ओटा ब्लकहरू छन् र चौथो खुट्किलामा 16 ओटा ब्लकहरू छन् । दसौँ खुट्किलामा कति ओटा ब्लकहरू होलान् ? 1 देखि $10^{\text{औं}}$ खुट्किलामा प्रयोग भएका जम्मा ब्लकहरू पत्ता लगाउनुहोस् ।
Look at the adjoining figure. The first step of ladder has 4 blocks and fourth steps of the ladder has 16 blocks. How many blocks are there in 10^{th} steps ? Find the total number of blocks from first steps to tenths steps.



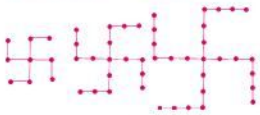
23D. दिइएको सङ्ख्याहरूको ढाँचाबाट $n^{\text{औं}}$ पद र $10^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
From the given patterns of numbers, find the n^{th} term and 10^{th} term.



23E. दिइएको सङ्ख्याहरुको ढाँचामा (In the given pattern of numbers):

(i) एउटा थप चित्र बनाउनुहोस् । (Add one more figure.)

(ii) यस्तै दशओटा चित्रहरूमा जम्मा थोप्लाहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
Find the sum of the dots present in such 10 figures.



23F. दिइएको सङ्ख्याहरूको ढाँचाबाट $n^{\text{औं}}$ पद र $10^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
From the given patterns of numbers, find the n^{th} term and 10^{th} term.



ANSWERS

A. SHORT QUESTIONS

1A. 331 **1B.** 179 **1C.** 31 **1D.** 32 **2A.** $2n$ **2B.** $25 - 5n$ **3A.** $a^2 + b^2$
3B. $a^3 - 3b^3$ **3C.** x^2 **4A.** Yes, 25 **4B.** 35 **4C.** 6, 9 **5A.** 13
5B. yes **5C.** 11 **5D.** 15 **5E.** 40 **6A.** 7380 **6B.** 2280 **7A.** 68
7B. 1260 **8A.** 1580 **8B.** 3024 **9B.** 24 **9C.** $2, \frac{1}{2}$ **9D.** $5, \frac{1}{5}$ **10A.** 50, 30
10B. 16 **11A.** 6, 8 **11B.** 10, 15 **12A.** $\frac{16}{11}$ **12B.** 40 **12C.** 3 **12D.** 3
12E. –1, 2 **12F.** 4 **13A.** 2 **13B.** 5 **13C.** 2 **13D.** 3 **14A.** 8
14B. 35 **15A.** – 4 **15B.** $\frac{24}{7}$ **16A.** 4 **16B.** 2 **17A.** $\frac{32}{7}$ **17B.** 3
17C. 6 **18A.** 10 **18B.** 10 **18C.** 6 **19A.** 20 **19B.** 24 **20A.** 27
20B. 8 **21A.** 9 cm, 13 cm **21B.** 6 cm, 8 cm, 10cm **22A.** 18 **22B.** 45 **22C.** 13
22D. 11088 **23A.** 150 **23B.** 318 **24A.** 44 **24B.** $\frac{n}{2}(5n + 1)$ **24C.** $\frac{n}{2}(5n + 21)$ **25A.** 6
25B. 2 **26A.** 16 **26B.** 40

B. LONG QUESTIONS

1A. 5, 6½, 8 **1B.** 55 **2A.** 13 **2B.** 33 **2D.** 0 **2E.** 0 **3A.** 0
3B. 0 **4A.** 5 **4B.** 510 **4C.** 1875 **4D.** 1625 **4E.** 240 **5A.** Yes, 315
5B. 43, 3268 **6A.** 6864 **6B.** 255 **6C.** 738 **6D.** Rs 2235 **7A.** 11583 **7B.** 2550
8A. 12 or 15 **8B.** 9, 12 **9A.** 8, 10, 12 **9B.** 3, 6, 9 **9C.** 31 **9D.** 69.5 **9E.** 52
10A. 10, 5 **10B.** 10, 4 **11A.** 4 and 5 **11B.** 3 and 4 **11C.** –7, 3 **12A.** 21, 23, 25 **12B.** 4, 5, 6
13A. 4, 9, 14 or 14, 9, 4 **13B.** 3, 9, 15 **13C.** 3,7,11 or 11,7,3 **13D.** 2, 4, 6 or 6, 4, 2 **14A.** – 4, – 1, 2 or, 2, – 1, – 4
14B. 6, 9, 12 **14C.** 5, 8, 11 **14D.** 5, 12, 19 ;19, 12, 5 **14E.** 2, 4, 6 or, 6, 4, 2 **14F.** 1, 7, 13 **15A.** 2, 4, 6, 8
15B. –1, 3, 7, 11 **15C.** 2, 6, 10, 14 **15D.** 5, 10, 15, 20 **15E** 5, 10, 20 **15F.** 4, 8, 16 **16A.** 5, 55 **16B.** 2, 30
16C. 10, 25 **16D.** 1 or 2 **16E.** 4 and 6 **16F.** 10, 5 **17A.** 17, 21, 25 **17B.** 17½, 20, 22½
17C. 30, 35, 40 **17D.** 20, 40, 60 **18A.** 2, 20 **18B.** 17 **18C.** 5 **18D.** 25, 7, 43 **18E.** 14
19A. $1\frac{1}{2}, 1, \frac{1}{2}, -\frac{1}{2} - 1, -1\frac{1}{2}$ **19B.** 6, 9, 12, 18, 21, 24 **20A.** 5 and 9 or 9 and 5 **20B.** 6 and 8 or 8 and 6
22A. \$ 4250, \$ 700 **22B.** 25 months **22C.** 20 months **22D.** 16, 13 yrs **22E.** 26, 16 yrs, 3 month **22F.** 25, 26 yrs
22G. Rs 10,000 **22H.** 136000 **22I.** Rs 500 **23A.** n + 2 **23B.** 4n
23C. 40, 220 **23D.** 2n + 2, 22 **23E.** 450 **23F.** 2n + 3, 23

www.drmathnp.com

4. गुणोत्तर अनुक्रम GEOMETRIC SEQUENCE

DEFINITIONS

❁ **गुणोत्तर अनुक्रम (Geometric Sequence)** [2063 R']

यदि अनुक्रममा क्रमिक पदहरूको अनुपात एउटै भए यस्तो अनुक्रमलाई गुणोत्तर अनुक्रम भनिन्छ र यो अनुक्रमसँग सम्बन्धित श्रेणीलाई गुणोत्तर श्रेणी भनिन्छ । जस्तै: 1, 2, 4, 8.....र 16, 8, 4 ..... इत्यादि गुणोत्तर अनुक्रममा छन् ।

Quantities are said to be in geometric sequence when they increase or decrease by a constant factor. The constant factor is called the common ratio.

e.g. 1, 2, 4, 8.....and 16, 8, 4 ..... etc. are in geometric sequence.

❁ **गुणोत्तर श्रेणी (Geometric Series)** [SEE 2073 S']

लगातार पदहरूको अनुपात सधैँ बराबर हुने श्रेणीलाई गुणोत्तर श्रेणी भनिन्छ । जस्तै: 2 + 4 + 8 + 16 + 32 ... एउटा गुणोत्तर श्रेणी हो । अर्को शब्दमा गुणोत्तर अनुक्रमसँग सम्बन्धित श्रेणीलाई गुणोत्तर श्रेणी भनिन्छ ।

A geometric series is a series with a constant ratio between successive terms. For example: 2 + 4 + 8 + 16 + 32 + ... is a geometric series. In other words, the corresponding series of a geometric sequence is called the geometric series.

❁ **गुणोत्तर मध्यमा (Geometric Mean)** [2065 R']

गुणोत्तर अनुक्रममा पहिलो पद र अन्तिम पदहरूबिचको पद या पदहरूलाई गुणोत्तर मध्यमा भनिन्छ ।

In geometric sequence the term/terms between the first term and the last term is/are called the geometric mean/means.

FORMULAE AND KEY POINTS

गुणोत्तर अनुक्रम वा श्रेणी (Geometric Sequence or Series): GS		सङ्केत Index
साधारण पद (General term)	$t_n = ar^{n-1}$	a = पहिलो पद (First term) n = पदहरूको सङ्ख्या (No. of terms) r = समान अनुपात (Common ratio) b = अन्तिम पद (Last term) $m_n = n$ औँ मध्यमा (n^{th} mean)
मध्यमा (Mean)	$GM = \sqrt{ab}$	
मध्यमाहरू (Means)	$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$; $m_1 = ar, m_2 = ar^2, \ldots, m_n = ar^n$	
योगफल (Sum)	$S_n = \frac{a(r^n - 1)}{r - 1}$ **Or** $S_n = \frac{br - a}{r - 1}$	

❁ यदि गुणोत्तर मध्यमाहरू (GMs) को सङ्ख्याहरू = m भए $r = \left(\frac{b}{a}\right)^{\frac{1}{m+1}}$ हुन्छ ।

If the number of geometric means (GMs) = m then $r = \left(\frac{b}{a}\right)^{\frac{1}{m+1}}$.

❁ यदि दुई सङ्ख्याहरू a र b बिचको समानान्तरीय मध्यक = AM र गुणोत्तर मध्यक = GM भए,

पहिलो सङ्ख्या $(a) = AM - \sqrt{AM^2 - GM^2}$ र दोस्रो सङ्ख्या $(b) = AM + \sqrt{AM^2 - GM^2}$ हुन्छ ।

If a and b are two numbers and their arithmetic mean = AM and geometric mean = GM then,

First number $(a) = AM - \sqrt{AM^2 - GM^2}$ and Second number $(b) = AM + \sqrt{AM^2 - GM^2}$

❁ **GS का पदहरूलाई सङ्केत गर्ने सजिलो तरिका (The easy method to denote the terms of GS):**

पदहरूको सङ्ख्या (Number of terms)	3	4	5	6
पदहरू (Terms)	$\frac{a}{r}, a, ar$	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$
समान अनुपात (Common ratio)	r	r^2	r	r^2

puspa.com.np

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2067 R, 2068 R, 2068 S, 2070 R, 2072 R', 2073 R', SEE 2073 S'	7
Model 2	2065 R', 2065 E, 2069 S, 2070 R', 2070 S', 2071 R, 2071 R', 2072 R, 2072 S, SEE 2075 R, R', R_2	11
Model 3	2065 S, 2066 R, 2067 R', 2068 S', 2070 S	5
Model 4	2069 R	1

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2069 R', SEE 2074 S	2
Model 2	2066 R, 2068 R, 2073 R	3
Model 3	2065 E, 2065 R, 2071 S, 2072 R, 2072 S, 2074 R', SEE 2074 S'	7
Model 4	2067 S	1
Model 5	2066 R', 2066 S, 2068 R', SEE 2073 S	3
Model 6	SEE 2074 R	1

SEE EXERCISE 4

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् । (Define the following terms.)

A. गुणोत्तर अनुक्रम/श्रेणी (Geometric sequence/series)

B. समान अनुपात (Common ratio) **C.** गुणोत्तर मध्यमा (Geometric mean)

2A. पहिलो पद 'a' र समान अनुपात 'r' भएको गुणोत्तर अनुक्रमको साधारण पद लेख्नुहोस् ।
Write the general term of a geometric sequence having first term 'a' and common ratio 'r'.

2B. पहिलो पद 'a', अन्तिम पद 'b' र पदहरूको सङ्ख्या 'n' भएको GS को समान अनुपात कति हुन्छ ?
What is the common ratio of a GS having first term 'a', last term 'b' and number of terms 'n'?

3A. दुईओटा धनात्मक सङ्ख्याहरू a र b बिचको गुणोत्तर मध्यमा कति हुन्छ ?
What is the geometric mean between two positive numbers a and b?

3B. यदि पहिलो पद 'a' र समान अनुपात 'r' भए गुणोत्तर अनुक्रमको $n^{\text{औं}}$ मध्यमा कति हुन्छ ?
If 'a' is first term and 'r' is common ratio then what is n^{th} mean of geometric sequence?

3C. यदि 'a' र 'b' दुई धनात्मक सङ्ख्याहरू भए तिनीहरूका AM र GM बीचको सम्बन्ध लेख्नुहोस् ।
If 'a' and 'b' are the two positive numbers, write the relation between their AM and GM.

3D. यदि 'a' र 'b' बीचको गुणोत्तर मध्यमा 'G' भए a, b र G को सम्बन्ध पत्ता लगाउनुहोस् ।
If 'G' is the geometric mean between 'a' and 'b', find the relation of a, b and G.

3E. a र b को बीचमा n ओटा मध्यमाहरू भएको GS को समान अनुपात के हुन्छ ?
What is the common ratio of GS having n means between 'a' and 'b'?

4A. पहिलो पद 'a' र समान अनुपात 'r' भएको गुणोत्तर श्रेणीमा 'n' ओटा पदहरू भए अन्तिम पद (b) कति होला ?
What is the last term 'b' of a GP of n terms having first term 'a' and common ratio 'r'?

4B. पहिलो पद 'a' र समान अनुपात 'r' भएको गुणोत्तर श्रेणीको 'n' ओटा पदहरूको योगफल 's' भए s, a, r र n को सम्बन्ध पत्ता लगाउनुहोस् ।
If 's' is the sum of n terms of a GS with first term 'a' and common ratio 'r' then find the relation between s, a, r and n.

4C. पहिलो पद 'a', अन्तिम पद 'ℓ' र समान अनुपात 'r' भएको GS को योगफल कति हुन्छ ?
What is the sum of GS having first term 'a', last term 'ℓ' and common ratio 'r'?

4D. सूत्र $S_n = \frac{a(r^n - 1)}{r - 1}$ मा प्रत्येक पदहरूको अर्थ लेख्नुहोस् ।

Write the meaning of each term in the formula: $S_n = \frac{a(r^n - 1)}{r - 1}$

www.drmathnp.com

Model 2 *Concept Recall*

5. तलका मध्ये कुन चाहिँ गुणोत्तर अनुक्रम हो ? (Which of the following sequences are geometric sequence ?)

A. 1, 2, 4, 8, 16, 32 **B.** 2, 4, 6, 8, 10, 12, 14 **C.** $3, 3^2, 3^3, 3^4, 3^5$

D. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4$ **E.** 64, 32, 16, 8, 4, 2 **F.** $5, -25, 125, -625$

6A. श्रेणी 4, 8, 16, 32, ...को समान अनुपात कति हुन्छ ?
What is the common ratio of the series 4, 8, 16, 32, ...?

6B. श्रेणी 81, 27, 9, 3, ...को समान अनुपात कति हुन्छ ?
What is the common ratio of the series 81, 27, 9, 3, ...?

7A. चौथो पद 16 र समान अनुपात 2 भएको GS को पाचौँ पद पत्ता लगाउनुहोस् ।
Find the fifth term of GS whose fourth term is 16 and common ratio is 2.

7B. नवौँ पद 27 र समान अनुपात 3 भएको GS को दसौँ पद पत्ता लगाउनुहोस् ।
Find the tenth term of GS whose ninth term is 27 and common ratio is 3.

7C. तेह्रौ पद 625 र समान अनुपात 5 भएको GS को 12 औँ पद पत्ता लगाउनुहोस् ।
Find the 12^{th} term of GS whose 13^{th} term is 625 and common ratio is 5.

8A. पहिलो पद '5' र समान अनुपात '2' भएको गुणोत्तर अनुक्रमको पाचौँ पद लेख्नुहोस् ।
Write the 5^{th} term of geometric sequence having first term '5' and common ratio '2'.

8B. पहिलो पद '2' र समान अनुपात '3' भएको गुणोत्तर अनुक्रमको चौथो पद लेख्नुहोस् ।
Write the 4^{th} term of geometric sequence having first term '2' and common ratio '3'.

9A. दुई सङ्ख्याहरू 1 र 4 को गुणोत्तर मध्यमा पत्ता लगाउनुहोस् ।
Find the geometric mean of two numbers 1 and 4.

9B. दुई सङ्ख्याहरू 1 र 9 को गुणोत्तर मध्यमा पत्ता लगाउनुहोस् ।
Find the geometric mean of two numbers 1 and 9.

10A. यदि एउटा GS को पहिलो पद 4 र समान अनुपात 2 भए दोस्रो मध्यमा पत्ता लगाउनुहोस् ।
If a GS has first term 4 and common ratio 2 then find the 2^{nd} mean.

10B. यदि एउटा GS को पहिलो पद 3 र समान अनुपात 5 भए दोस्रो मध्यमा पत्ता लगाउनुहोस् ।
If a GS has first term 3 and common ratio 5 then find the 2^{nd} mean.

11A. एउटा गुणोत्तर श्रेणीको पहिला 5 पदहरूको योगफल 124 र पहिला 4 पदहरूको योगफल 60 छ । पाचौँ पद पत्ता लगाउनुहोस् ।
The sum of first 5 terms of a geometric series is 124 and the sum of first 4 terms of the geometric series is 60. Find the 5^{th} term.

11B. एउटा गुणोत्तर श्रेणीको n पदहरूको योगफल 3577 र $(n-1)$ पदहरूको योगफल 1785 छ । $n^{औँ}$ पद पत्ता लगाउनुहोस् ।
The sum of n terms of a geometric series is 3577 and the sum of $(n-1)$ terms of the geometric series is 1785. Find the n^{th} term.

11C. एउटा गुणोत्तर श्रेणीको पहिला 10 पदहरूको योगफल 2046 र $10^{औँ}$ पद 1024 छ । सो गुणोत्तर श्रेणीको पहिला 9 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The sum of first 10 terms of a geometric series is 2046 and the 10^{th} term is 1024. Find the sum of first 9 terms of the geometric series.

B. SHORT QUESTIONS

Model 1 *Problems based on general terms of GS.*

1A. गुणोत्तर श्रेणीको परिभाषा एउटा उदाहरणसहित दिनुहोस् । (Define Geometric series with an example.) **[SEE 2073 S']**

1B. गुणोत्तर अनुक्रम 2, 4, 8, 16 ..... को पन्ध्रौँ पद पत्ता लगाउनुहोस् ।
Find the fifteen (15^{th}) term of a geometric sequence 2, 4, 8, 16, ... **[2068 S]**

1C. गुणोत्तर अनुक्रमको पहिलो र दोस्रो पद क्रमशः 32 र 8 भए छैटौँ पद कति होला ? **[2064 R]**
First and second terms of a geometrical sequence are 32 and 8 respectively. What will be the sixth term?

2A. यदि 4, a र 16 गुणोत्तरीय अनुक्रममा भए a को मान पत्ता लगाउनुहोस् ।
If 4, a and 16 are in the geometric sequence, find the value of a. **[SEE 2073 R']**

2B. यदि a, $a-2$, $a+1$ गुणोत्तरीय अनुक्रममा भए a को मान पत्ता लगाउनुहोस् ।
If a, $a-2$, $a+1$ are in geometric sequence, find the value of a. **[2068 R]**

puspas.com.np

www.drmathnp.com

puspas.com.np

2C. यदि $x+4, x-2$ र $x-6$ ज्यामितीय अनुक्रममा भए x को मान निकाल्नुहोस् ।
If $x+4, x-2$ and $x-6$ are in geometric sequence, find the value of x. [2067 R]

2D. यदि $m+2, m+8$ र $17+m$ गुणोत्तर अनुक्रममा भए m को मान पत्ता लगाउनुहोस् ।
If $m+2, m+8$ and $17+m$ are in geometric sequence, find the value of m. [2072 R']

3A. यदि गुणोत्तर श्रेणीको तेस्रो पद 3 छ भने यसका प्रथम पाँच पदहरूको गुणनफल निकाल्नुहोस् ।
If the third term of a geometric series is 3, find the product of its first five terms. [2070 R]

3B. यदि गुणोत्तर श्रेणीको तेस्रो पद 8 छ भने यसका प्रथम पाँच पदहरूको गुणनफल निकाल्नुहोस् ।
If the third term of a geometric series is 8, find the product of its first five terms.

Model 2 Problems based on understanding mean/means.

4A. यदि 4 र x को बिचमा पर्ने गुणोत्तर मध्यमा 8 भए x को मान पत्ता लगाउनुहोस् ।
If geometric mean is 8 between of 4 and x, find the value of x. [2065 R']

4B. यदि x र 27 को बिचमा पर्ने गुणोत्तर मध्यमा 9 भए x को मान पत्ता लगाउनुहोस् ।
If geometric mean is 9 between of x and 27, find the value of x.

4C. यदि 2 र x को गुणोत्तर मध्यमा 4 भए तिनीहरूको समानान्तरीय मध्यमा पत्ता लगाउनुहोस् ।
If the geometric mean of 2 and x is 4, find their arithmetic mean. [2072 R]

4D. यदि 4 र x को समानान्तरीय मध्यमा 34 भए तिनीहरूको गुणोत्तर मध्यमा निकाल्नुहोस् ।
If the arithmetic mean between 4 and x is 34, find their geometric mean. [2072 S]

4E. यदि x र 8 को समानान्तरीय मध्यमा 5 भए गुणोत्तर मध्यमाको मान निकाल्नुहोस् ।
If 5 is the arithmetic mean of x and 8, find value of the geometric mean.

5A. कुनै गुणोत्तर अनुक्रमको पहिलो पद 8 र अन्तिम पद 1/2 भए सो अनुक्रमको तीनओटा गुणोत्तर मध्यमाहरू पत्ता लगाउनुहोस् ।
If the first term is 8 and the last term is 1/2 of a geometrical sequence, find three geometrical means. [2065 E]

5B. कुनै गुणोत्तर अनुक्रमको पहिलो पद 2 र अन्तिम पद 32 भए सो अनुक्रमको तीनओटा गुणोत्तर मध्यमाहरू पत्ता लगाउनुहोस् ।
If the first term is 2 and the last term is 32 of a geometrical sequence, find three geometrical means.

6A. यदि 2, 4, m एउटा गुणोत्तर अनुक्रम भए m को मान पत्ता लगाउनुहोस् ।
If 2, 4, m is a geometric sequence then find the value of m. [SEE 2075 R']

6B. दिइएको गुणोत्तर अनुक्रममा x, y र z का मानहरू पत्ता लगाउनुहोस्: $\frac{1}{8}, x, y, z, 2$

Find the values of x, y and z from the given geometrical sequence: $\frac{1}{8}, x, y, z, 2$ [2060 C]

6C. यदि $\frac{1}{3}, p, q, 9$ गुणोत्तरीय अनुक्रममा भए p र q का मानहरू पत्ता लगाउनुहोस् ।

If $\frac{1}{3}, p, q, 9$ are in the geometric sequence, find the values of p and q. [2069 S]

7A. श्रेणी $3+6+12+\ldots$ को कुनचाहिँ पद 192 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series $3+6+12+\ldots\ldots$ is 192 ? Find it. [2071 R]

7B. दिइएको श्रेणीको पदसङ्ख्या पत्ता लगाउनुहोस् । (Find the number of terms of the given series.) [2071 R']
$2-2\sqrt{2}+4-4\sqrt{2}+\ldots+16$.

8A. 4 र 16 बिचको समानान्तरीय मध्यमा र गुणोत्तर मध्यमा निकाल्नुहोस् ।
Calculate the arithmetic mean and geometric mean between 4 and 16. [SEE 2075 R]

8B. 3 र 27 को समानान्तरीय मध्यमा र गुणोत्तर मध्यमा पत्ता लगाउनुहोस् ।
Find the Arithmetic mean and Geometric mean of 3 and 27. [SEE 2075 R_2, 2070 R']

8C. यदि दुईओटा धनात्मक सङ्ख्याहरूको योगफल 20 छ र ती सङ्ख्याहरूको गुणनफल 36 भए ती सङ्ख्याहरू बिचको अङ्कगणितीय मध्यक (A.M.) र ज्यामितीय मध्यक (G.M.) पत्ता लगाउनुहोस् ।
If the sum of two positive numbers is 20 and their product is 36, find the arithmetic mean (A.M.) and geometric mean (G.M.) between these numbers. [2070 S']

Model 3 Problems based on finding the sum of a geometric sequence.

9A. $81+27+9+\ldots\ldots\ldots$ को पहिलो पाँचौँ पदसम्मको योगफल निकाल्नुहोस् ।
Find the sum of first five terms of $81+27+9+\ldots\ldots\ldots$ [2063 R]

9B. दिइएको श्रेणीको योगफल निकाल्नुहोस् : $2+4+8+\ldots\ldots$ 8 ओटा पदहरू सम्म
Find the sum of the given series : $2+4+8+\ldots\ldots$ upto 8 terms. [2066 R]

www.drmathnp.com

9C. गुणोत्तर अनुक्रम 48, 24, 12, ... को पहिलो पाँच पदहरूको योगफल निकाल्नुहोस् ।
Find the sum of the first five terms of the geometric sequence 48, 24, 12, ... [2065 S]

9D. दिइएको गुणोत्तर श्रेणीको योगफल निकाल्नुहोस् : $3 + 6 + 12 + \ldots\ldots\ldots + 768$
Find the sum of the given geometric series : $3 + 6 + 12 + \ldots\ldots\ldots + 768$ [2067 R']

9E. श्रेणी $9 + 3 + 1 + \ldots + \frac{1}{27}$ को योगफल निकाल्नुहोस् । (Find the sum of the series): $9 + 3 + 1 + \ldots + \frac{1}{27}$ [2070 S]

9F. एउटा गुणोत्तर श्रेणीको पहिलो पद 1 र दोस्रो पद 3 भए सो श्रेणीको पहिला 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the first term is 1 and the second term is 3 of a geometric series, find the sum of the first six terms. [2068S']

Model 4 *Problems based on finding the common ratio of GP.*

10A. पहिलो पद 2 र तेस्रो पद 242 भएको गुणोत्तर अनुक्रमको समान अनुपात पत्ता लगाउनुहोस् ।
Find the common ratio of a geometric sequence, whose first term is 2 and the third term is 242. [2069 R]

10B. पहिलो पद 3 र तेस्रो पद 12 भएको गुणोत्तर अनुक्रमको समान अनुपात पत्ता लगाउनुहोस् ।
Find the common ratio of a GS, whose first term is 3 and the third term is 12.

C. LONG QUESTIONS

Model 1 *Problems based on finding indicated term of a GS.*

1A. एउटा ज्यामितीय श्रेणीको पहिला तीन पदहरू $x + 6, x$ र $x - 3$ भए x र पाँचौँ पदको मान निकाल्नुहोस् ।
$x + 6, x$ and $x - 3$ are the first three terms of a geometric series. Find the value of x and its fifth term. [2058S]

1B. एउटा ज्यामितीय श्रेणीको पहिला तीन पदहरू $2p, 2p + 3$ र $2p + 9$ भए p र पाँचौँ पदको मान निकाल्नुहोस् ।
$2p, 2p + 3$ and $2p + 9$ are the first three terms of a geometric series. Find the value of p and its fifth term.

1C. तीनओटा सङ्ख्याहरू 1 : 4 : 12 को अनुपातमा छन् । यदि पहिलो सङ्ख्यामा 1 जोडियो भने सो सङ्ख्या र बाँकी दुई सङ्ख्याहरूले एउटा GP बनाउँदछन् । सुरूका सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in the ratio 1 : 4 : 12. If one is added to the first number, the resulting number together with other two numbers form a G.P. Find the original numbers.

1D. तीनओटा सङ्ख्याहरू 1 : 5 : 20 को अनुपातमा छन् । यदि पहिलो सङ्ख्यामा 1 जोडियो भने सो सङ्ख्या र बाँकी दुई सङ्ख्याहरूले एउटा GP बनाउँदछन् । सुरूका सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in the ratio 1 : 5 : 20. If one is added to the first number, the resulting number together with other two numbers form a G.P. Find the original numbers.

2A. गुणोत्तर श्रेणीको लगातार आउने तीनओटा पदहरूको योगफल 62 र गुणनफल 1000 भए, ती पदहरू पत्ता लगाउनुहोस् ।
The sum of three consecutive terms in GP is 62 and their product is 1000, find the terms. [2063 R]

2B. गुणोत्तर श्रेणीमा भएका तीनओटा पदहरूको योगफल 14 र गुणनफल 64 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
If the sum of three numbers in geometric series is 14 and their product is 64, find the numbers. [2069 R']

2C. गुणोत्तर श्रेणीमा भएका तीनओटा धनात्मक सङ्ख्याहरूको योगफल 104 छ र त्यसका पहिलो र तेस्रो सङ्ख्याहरूको गुणनफल 576 भए उक्त संख्याहरु निकाल्नुहोस् ।
The sum of three positive numbers in geometric series is 104 and the product of the first and third numbers is 576, find the numbers. [SEE 2074 S]

Model 2 *Problems based on geometric mean/means between two numbers.*

3A. $\frac{1}{9}$ र 9 को बिचमा 3 ओटा गुणोत्तर मध्यमाहरू निकाल्नुहोस् । (Insert 3 geometric means between $\frac{1}{9}$ and 9.) [2057S]

3B. $\frac{2}{3}$ र 162 को बिचमा 4 ओटा गुणोत्तर मध्यमाहरू भर्नुहोस् । (Insert 4 GMs between $\frac{2}{3}$ and 162.) [2066 R]

3C. गुणोत्तर श्रेणीमा हुने गरी 4 र 8^{-1} को बिचमा 4 ओटा सङ्ख्याहरू भर्नुहोस् ।
Insert 4 numbers between 4 and 8^{-1} so that they form a geometric series.

3D. 35 र 2240 को बिचमा 5 ओटा गुणोत्तर मध्यमाहरू भर्नुहोस् । (Insert 5 geometric means between 35 and 2240.)

3E. 3 र 48 को बिचमा पर्ने 3 ओटा गुणोत्तर मध्यमाहरू भर्नुहोस् । (Insert 3 geometric means between 3 and 48.)

4A. 5 र 80 को बिचमा पर्ने केही ज्यामितीय मध्यमाहरू छन् । यदि दोस्रो मध्यमा 20 भए ती दुई सङ्ख्याभित्र पर्ने मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् र अन्तिम मध्यमा पनि पत्ता लगाउनुहोस् ।
There are some geometric means between 5 and 80. If the second mean be 20, find the number of means between the two numbers. Also, find the last mean. [2062 R]

4B. 5 र 80 को बिचमा केही गुणोत्तर मध्यमाहरू छन् । यदि दोस्रो मध्यमा 20 भए मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
Some geometric means are inserted between 5 and 80. Find the number of means between the numbers if second mean is 20.

puspas.com.np

www.drmathnp.com

pustas.com.np

4C. 2 र 32 को बिचमा केही गुणोत्तर मध्यमाहरू छन् । यदि तिनीहरू बिचको दोस्रो मध्यमा 8 भए मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
Some geometric means are inserted between 2 and 32. Find the number of means between the numbers if second mean is 8.

4D. $\frac{1}{2}$ र 16 को बिचमा पर्ने केही ज्यामितीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा 4 भए ती दुई सङ्ख्याहरूभित्र पर्ने मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् र अन्तिम मध्यमा पनि पत्ता लगाउनुहोस् ।
There are some geometric means between $\frac{1}{2}$ and 16. If the third mean be 4, find the number of means between two numbers. Also, find the last mean. **[2063 R']**

5A. 1 र 64 का बिचमा पर्ने गुणोत्तर मध्यमाहरूको सङ्ख्या निकाल्नुहोस् जसको पहिलो र अन्तिम मध्यमाको अनुपात 1 : 16 छ ।
Find the number of geometric means inserted between 1 and 64 in which the ratio of first mean to the last mean is 1 : 16. **[2068 R]**

5B. 2^{-1} र 64 को बिचमा भरिएका गुणोत्तर मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् जहाँ पहिलो मध्यमा र अन्तिम मध्यमा 1 : 32 छ ।
Find the number of GM's inserted between 2^{-1} and 64 when the first mean to the last mean as 1 : 32.

5C. 2 र 128 को बिचमा भरिएका गुणोत्तर मध्यमाहरूको सङ्ख्या पत्ता लगाउनुहोस् जहाँ पहिलो मध्यमा र अन्तिम मध्यमा 1 : 16 छ ।
Find the number of GM's inserted between 2 and 128 where the first mean to the last mean as 1 : 16.

5D. $\frac{16}{27}$ र $\frac{243}{16}$ को बीचमा n ओटा गुणोतर मध्यमाहरू छन् र $(n-1)^{th}$ मध्यमा : 4^{th} मध्यमा = 9 : 4 भए n पत्ता लगाउनुहोस् ।
n geometric means are inserted between $\frac{16}{27}$ and $\frac{243}{16}$ such that $(n-1)^{th}$ mean : 4^{th} mean = 9 : 4, find n.

6A. यदि a र b को बिचमा पाँचओटा गुणोत्तर मध्यमाहरू छन् । यदि दोस्रो र अन्तिम मध्यमा क्रमशः 2 र 16 छन् भने a र b का मानहरू पत्ता लगाउनुहोस् ।
There are five geometric means between a and b. If second and last mean are 2 and 16 respectively, find the values of a and b. **[SEE 2073 R]**

6B. a र b को बिचमा पाँचओटा गुणोत्तर मध्यमाहरू छन् । यदि दोस्रो र अन्तिम मध्यमाहरू क्रमशः 63 र 1701 छन् भने a र b का मानहरू पत्ता लगाउनुहोस् ।
There are 5 geometric means between a and b. If the second and last means are 63 and 1701 respectively, find the values of a and b.

Model 3 Problems based on finding the sum of a geometric sequence.

7A. एउटा गुणोत्तर श्रेणीको चौथो पद 24 र समान अनुपात 2 भए उक्त श्रेणीको पहिलो 9 ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 9 terms of a geometric series whose fourth term is 24 and common ratio is 2. **[2074 S']**

7B. यदि एउटा गुणोत्तर श्रेणीको दोस्रो पद 48 र पाँचौ पद 6 छ भने प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् । **[2074 R']**
If the second term of a geometric series is 48 and its fifth term is 6 then find the sum of the first 6 terms.

7C. एउटा गुणोत्तर श्रेणीको चौथो पद 54 र समान अनुपात 3 भए उक्त श्रेणीको प्रथम 8 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 8 terms of a geometric series whose fourth term is 54 and common ratio is 3. **[2071 S]**

7D. एउटा गुणोत्तर श्रेणीको चौथो पद 81 र समान अनुपात 3 भए उक्त श्रेणीको प्रथम 8 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 8 terms of a geometric series whose fourth term is 81 and common ratio is 3.

7E. दोस्रो पद 6 र पाँचौं पद 162 भएको गुणोत्तर श्रेणीको पहिलो पाँच पदको योगफल निकाल्नुहोस् ।
The second term of GP is 6 and 5^{th} term is 162. Find the sum of its first 5 terms.

8A. एउटा गुणोत्तर श्रेणीको तेस्रो पद र सातौं पद क्रमशः 20 र 320 भए उक्त श्रेणीको प्रथम 9 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 9 terms of a geometric series whose third term and seventh term are 20 and 320 respectively. **[2072 R]**

8B. तेस्रो पद 12 र सातौं पद 192 भएको कुनै गुणोत्तर श्रेणीको पहिलो दसौं पदसम्मको योगफल निकाल्नुहोस् ।
The third term is 12 and seventh term is 192 of a geometric series, find the sum of its first ten terms. **[2064S]**

8C. कुनै GP को तेस्रो पद 12 र छैटौं पद 96 भए पहिलो आठौं पदसम्मको योगफल निकाल्नुहोस् ।
If the third term of a GP is 12 and sixth term is 96, find the sum of the first eight terms. **[2065 E]**

8D. एउटा गुणोत्तर श्रेणीको दोस्रो पद र पाचौं पद क्रमशः 15 र 405 भए उक्त श्रेणीको प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the second term and fifth term of a geometric series are 15 and 405 respectively, find the sum of the series from first to 6 terms. **[2072 S]**

9A. $3 + 6 + 12 + \ldots + 768$ दिइएको श्रेणी छ । ($3 + 6 + 12 + \ldots + 768$ is the given series.)
(i) यो श्रेणीको पद सङ्ख्या पत्ता लगाउनुहोस् । (Find the number of terms of this series.)
(ii) यो श्रेणीको योगफल निकाल्नुहोस् । (Find the sum of the series.) **[2063 M]**

www.drmathnp.com
puspas.com.np

9B. $4 + 8 + 16 + \ldots + 4096$ दिइएको श्रेणी छ । ($4 + 8 + 16 + \ldots + 4096$ is the given series.)
(i) यो श्रेणीको पद सङ्ख्या पत्ता लगाउनुहोस् । (Find the number of terms of this series.)
(ii) यो श्रेणीको योगफल निकाल्नुहोस् । (Find the sum of the series.)

10A. धनात्मक समान अनुपात भएको एउटा गुणोत्तर श्रेणीको पहिला चार पदहरूको योगफल 40 र पहिला दुई पदहरूको योगफल 4 छ भने सो श्रेणीको पहिला आठ पदहरूको योगफल निकाल्नुहोस् ।
The sum of first four terms is 40 and the sum of first two terms is 4 of a geometric series whose common ratio is positive. Find the sum of first eight terms. **[2060 R, 2065 R]**

10B. धनात्मक समानुपात भएको एउटा गुणोत्तर श्रेणीको पहिलो 4 पदहरूको योगफल 30 र पहिलो 2 पदहरूको योगफल 6 छ भने सो श्रेणीको पहिलो 10 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The sum of first four terms is 30 and the sum of first two terms is 6 of a geometric series whose common ratio is positive. Find the sum of first ten terms.

Model 4 *Problems based on finding common ratio and first term of a gemetric sequence.*

11A. यदि एउटा गुणोत्तर श्रेणीको पहिलो तीन पदहरूको योगफल 1 र यसको पहिलो 6 पदहरूको योगफल 28 भए सो श्रेणीको समानुपात र पहिलो पद पत्ता लगाउनुहोस् ।
In a geometric series the sum of the first three terms is 1 and sum of its first 6 terms is 28, find the common ratio and the first term. **[2067 S, 2064 R']**

11B. यदि एउटा गुणोत्तर श्रेणीको पहिला दुई पदहरूको योगफल 4 र यसको पहिलो चार पदहरूको योगफल 20 भए उक्त श्रेणीको समान अनुपात र पहिलो पद पत्ता लगाउनुहोस् ।
In a geometric series, the sum of first two terms is 4 and sum of its first four terms is 20, find the common ratio and the first term.

12A. एउटा समानान्तरीय श्रेणीका दोस्रो, चौथो र नवौं पदहरू गुणोत्तर श्रेणीमा भए सो गुणोत्तर श्रेणीको समानुपात निकाल्नुहोस् ।
The second, fourth and ninth terms of an arithmetic progression are in geometric progression. Calculate the common ratio of the geometric progression. **[2064 R]**

12B. एउटा AP को दोस्रो, छैटौं र अठारौं पदहरू एउटा GP का अनुवर्ती पदहरू भए समान अनुपात पत्ता लगाउनुहोस् ।
If second, sixth and eighteenth term of an AP are consecutive terms of a GP, find the common ratio.

13A. एउटा गुणोत्तर श्रेणीको छैटौं पद यसको दोस्रो पदको 16 गुणा र पहिला सात पदहरूको योगफल $\frac{127}{4}$ छ भने यो श्रेणीको धनात्मक समानुपात र पहिलो पद पत्ता लगाउनुहोस् ।
In a geometric series, if the sixth term is 16 times the second term and the sum of first seven terms is $\frac{127}{4}$; then find positive common ratio and the first term of the series. **[2063 S]**

13B. एउटा गुणोत्तर श्रेणीको पाचौं पद यसको दोस्रो पदको 8 गुणा र पहिला चार पदहरूको योगफल 60 छ भने यो श्रेणीको धनात्मक समानुपात र पहिलो पद पत्ता लगाउनुहोस् ।
In a geometric series, if the fifth term is 8 times the second term and the sum of first four terms is 60; then find positive common ratio and the first term of the series.

Model 5 *Problems based on finding the two numbers when their AM and GM are given.*

14A. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 25 र तिनीहरूको गुणोत्तर मध्यमा 20 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The arithmetic mean of two numbers is 25 and their geometric mean is 20, find the numbers. **[SEE 2073S]**

14B. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 5 र गुणोत्तर मध्यमा 4 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Find two numbers whose arithmetic mean is 5 and geometric mean is 4. **[2057 R]**

14C. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 10 र गुणोत्तर मध्यमा 8 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Find two numbers whose arithmetic mean is 10 and geometric mean is 8. **[2062 K, 2068R']**

14D. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 50 र गुणोत्तर मध्यमा 40 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Find two numbers whose arithmetic mean is 50 and geometric mean is 40. **[2066 R', 2066S]**

Model 6 *Mixed problems based on AP and GP.*

15A. एउटा समानान्तरीय श्रेणीमा तीनओटा पदहरूको योगफल 24 छ । यदि ती पदहरूमा क्रमशः 1, 6 र 18 जोड्दा परिणाम गुणोत्तर श्रेणीमा हुन्छ भने ती पदहरू निकाल्नुहोस् ।
The sum of three terms in an arithmetic series is 24. If 1, 6 and 18 are added to them respectively, the results are in geometrical series, find the terms. **[SEE 2074 R]**

15B. एउटा GP को तीनओटा पदहरूको योगफल 56 छ । यदि 1, 7 र 21 क्रमशः ती पदहरूबाट घटाइयो भने AP बन्छ । GP का ती पदहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms of a GP is 56. If 1, 7 and 21 are subtracted from the terms respectively, then it forms an AP. Find the terms of GP.

www.drmathnp.com

puspas.com.np

15C. AP मा रहेका तीन सङ्ख्याहरूको योगफल 45 छ । यदि 17, 11 र 3 तिनीहरूबाट क्रमशः घटाईयो भने प्राप्त हुने सङ्ख्याहरू GP मा हुनेछन् । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers in A.P. is 45. If 17, 11 and 3 are subtracted from them respectively, the resulting numbers will be in G.P. Find the numbers.

15D. एउटा GP मा तीन सङ्ख्याहरूको योग 14 छ । यदि पहिलो दुई सङ्ख्याहरू प्रत्येक 1 ले बढ्यो र तेस्रो पद 1 ले घट्यो भने ती सङ्ख्याहरू AP मा हुन्छन् । उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of 3 numbers in GP is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in AP. Find the numbers.

16A. एउटा गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 243 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद एउटा समानान्तर श्रेणीको दशौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको पहिला 19 पदहरूको योगफल पत्ता लगाउनुहोस् । [SEE 2075 R, 2075 R_2]
The product of the first five terms of a geometric series is 243. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of the first 19 terms of the arithmetic series.

16B. एउटा गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 32 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद एउटा समानान्तर श्रेणीको दशौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको पहिला 19 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of the first five terms of a geometric series is 32. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of the first 19 terms of the arithmetic series.

16C. एउटा गुणोत्तर श्रेणीमा पाचौँ पद, सातौँ पद र नवौँ पदको गुणनफल 27 छ । यदि उक्त गुणोत्तर श्रेणीको सातौँ पद एउटा समानान्तर श्रेणीको आठौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको पन्ध्रौँ पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of fifth term, seventh term and ninth term of a GS is 27. If the seventh term of the GS is equal to the eighth term of an AS, find the sum of first 15 terms of the AS.

16D. एउटा गुणोत्तर श्रेणीको $30^{\text{औँ}}$ पद, $40^{\text{औँ}}$ पद र $50^{\text{औँ}}$ पदहरूको गुणनफल 512 छ । यदि उक्त गुणोत्तर श्रेणीको $40^{\text{औँ}}$ पद एउटा समानान्तर श्रेणीको $20^{\text{औँ}}$ पदसँग बराबर भए उक्त समानान्तर श्रेणीको उनान्चालिसौँ पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of 30^{th}, 40^{th} and 50^{th} terms of a GS is 512. If the 20^{th} term of an AS is equal to the 40^{th} term of the GS, find the sum of first 39 terms of the AS.

Model 7 *Application of geometric sequence.*

17A. सुधिरले रु. 4368 वार्षिक 6 ओटा किस्ताहरूमा, हरेक किस्ता अघिल्लो किस्ताको तेब्बर हुने गरी तिर्ने शर्तमा ऋणमा लियो । पहिलो किस्ता र अन्तिम किस्ता पत्ता लगाउनुहोस् ।
Sudhir borrows Rs 4368 which he promises to pay in 6 annual installments, each installment being treble of the preceding one. Find the first and the last installment.

17B. सुधाले रु. 19682 वार्षिक 9 ओटा किस्ताहरूमा, हरेक किस्ता अघिल्लो किस्ताको तेब्बर हुने गरी तिर्ने शर्तमा ऋणमा लिइन् । पहिलो किस्ता र अन्तिम किस्ता पत्ता लगाउनुहोस् ।
Sudha borrows Rs 19682 which she promises to pay in 9 annual installments, each installment being treble of the preceding one. Find the first and the last installment.

ANSWERS

A. VERY SHORT QUESTIONS

6A. 2 **6B.** $\frac{1}{3}$ **7A.** 32 **7B.** 81 **7C.** 125 **8A.** 80 **8B.** 54
9A. 2 **9B.** 3 **10A.** 16 **10B.** 75 **11A.** 64 **11B.** 1792 **11C.** 1022

B. SHORT QUESTIONS

1B. 32768 **1C.** $\frac{1}{32}$ **2A.** 8 **2B.** $\frac{4}{5}$ **2C.** 14 **2D.** 10 **3A.** 243
3B. 32768 **4A.** 16 **4B.** 3 **4C.** 5 **4D.** 16 **4E.** 4 **5A.** 4, 2, 1
5B. 4, 8, 16 **6A.** $m = 1$ **6B.** $\frac{1}{4}, \frac{1}{2}$ & 1 **6C.** 1, 3 **7A.** 7^{th} term **7B.** 7 **8A.** 10, 8
8B. 15, 9 **8C.** 10, 6 **9A.** 121 **9B.** 510 **9C.** 93 **9D.** 1533 **9E.** $\frac{364}{27}$
9F. 364 **10A.** ±11 **10B.** ± 2

C. LONG QUESTIONS

1A. $6, \frac{3}{4}$ **1B.** $\frac{3}{2}$, 48 **1C.** 3, 12, 36 **1D.** 4, 20, 80 **2A.** 2, 10, 50 or 50, 10, 2
2B. 8, 4, 2 or 2, 4, 8 **2C.** 72, 24, 8, or 8, 24, 72 **3A.** $\frac{1}{3}$, 1, 3 or $-\frac{1}{3}$, 1, –3 **3B.** 2, 6, 18, 54 **3C.** 2, 1, $\frac{1}{2}, \frac{1}{4}$
3D. 70, 140, 280, 560, 1120, or –70, 140, –280, 260, – 120 **3E.** 6, 12, 24 **4A.** 3, 40 **4B.** 3 **4C.** 3
4D. 4, 8 **5A.** 5 **5B.** 6 **5C.** 5 **5D.** 7 **6A.** $\frac{1}{2}$, 32 **6B.** 7, 5103
7A. 15337 **7B.** 189 **7C.** 6560 **7D.** 10083 **7E.** 242 **8A.** 2555 **8B.** 3069 or –1023
8C. 765 **8D.** 1820 **9A.** 9, 1533 **9B.** 11, 8188 **10A.** 3280 **10B.** 2046 **11A.** 3, $\frac{1}{13}$
11B. $r = 2$ or -2, $a = \frac{4}{3}$ or -4 **12A.** $\frac{5}{2}$ **12B.** 3 **13A.** 2, $\frac{1}{4}$ **13B.** 4, 2
14A. 10 and 40 or 40 and 10 **14B.** 8, 2 or 2, 8 **14C.** 4, 16 or 16, 4 **14D.** 20, 80 **15A.** 27, 8, – 11 or 6, 8, 10
15B. 32, 16, 8 **15C.** 25, 15 and 5 or 19 or 15 or 11 **15D.** 2, 4, 8, or 8, 4, 2 **16A.** 57 **16B.** 38
16C. 45 **16D.** 312 **17A.** Rs 12, Rs 2916 **17B.** Rs 2, Rs 13122

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 4

A. SHORT QUESTIONS

Model 1 *Problems based on finding indicated term of a GS.*

1A. गुणोत्तर अनुक्रम 32, 16, 8, ... को नवौं पद पत्ता लगाउनुहोस् ।
Find the 9^{th} term of the geometric sequence : 32, 16, 8, ...

1B. गुणोत्तर श्रेणी $\frac{2}{3} - \frac{1}{6} + \frac{1}{24} - .....$ को 5 औं पद पत्ता लगाउनुहोस् ।
Find the 5^{th} term of geometric series $\frac{2}{3} - \frac{1}{6} + \frac{1}{24} - .....$

2A. यदि एउटा GP को पहिलो पद 7 र समान अनुपात 2 छ भने तेस्रो पद र आठौं पद निकाल्नुहोस् ।
If the first term of a GP is 7 and common ratio is 2, find the 3^{rd} term & the 8^{th} term.

2B. यदि एउटा GP को पहिलो पद 8 र समान अनुपात 3 छ भने तेस्रो पद र आठौं पद निकाल्नुहोस् ।
If the first term of a GP is 8 and common ratio is 3, find third term and eight term.

3A. कुनै गुणोत्तर श्रेणीको पहिलो तीन पदहरूको गुणनफल – 8 भए दोस्रो पद पत्ता लगाउनुहोस् ।
The product of the first three terms of a geometric series is – 8. Find its second term.

3B. कुनै गुणोत्तर श्रेणीको पहिलो पाँच पदहरूको गुणनफल 32 भए तेस्रो पद पत्ता लगाउनुहोस् ।
The product of the first five terms of a geometric series is 32. Find its third term.

3C. कुनै गुणोत्तर श्रेणीको तेश्रो पद 2 भए यसका पहिला पाँच पदहरूको गुणनफल पत्ता लगाउनुहोस् ।
The third term of a geometric series is 2. Find the product of its first five terms .

3D. एउटा GP मा पाचौं पद, सातौं पद र नवौं पदहरूको गुणनफल 27 भए सातौं पद पत्ता लगाउनुहोस् ।
The product of fifth term, seventh term and ninth term of a geometric sequence is 27. Find the seventh term.

3E. एउटा गुणोत्तर अनुक्रममा दोस्रो पद, चौथो पद, छैठौं पद र आठौं पदहरूको गुणनफल 625 भए पाचौं पद पत्ता लगाउनुहोस् ।
The product of second term, fourth term, sixth term and eighth term of a geometric sequence is 625. Find the fifth term.

3F. एउटा GS मा $30^{औं}$ पद, $40^{औं}$ पद र $50^{औं}$ पदहरूको गुणनफल 512 भए $40^{औं}$ पद पत्ता लगाउनुहोस् ।
The product of 30^{th}, 40^{th} and 50^{th} terms of a GS is 512. Find the fortieth term.

3G. यदि एउटा GS मा $15^{औं}$, $25^{औं}$ र $35^{औं}$ पदहरूको गुणनफल 512 भए यसको $25^{औं}$ पद पत्ता लगाउनुहोस् ।
If the product of 15^{th}, 25^{th} and 35^{th} terms of a GS is 512. Find its 25^{th} term.

4A. 10, 20, 40, ... को $n^{औं}$ पद पत्ता लगाउनुहोस् । (Find the n^{th} term of 10, 20, 40,.....)

4B. 96, 48, 24, ... को $n^{औं}$ पद पत्ता लगाउनुहोस् । (Find the n^{th} term of 96, 48, 24,.....)

Model 2 *Problems based on finding the position of a given term in a given GP.*

5A. श्रेणी 128 + 64 + 32 ... को कुन पद 0.25 हुन्छ ? (Which term of the series 128 + 64 + 32 .... is 0.25 ?)

5B. श्रेणी $\frac{3}{2} + \frac{9}{4} + ...$ को कुन चाहिँ पद $\frac{729}{64}$ हुन्छ ? (Which term of the series $\frac{3}{2} + \frac{9}{4} + ...$ is $\frac{729}{64}$?)

6A. समानान्तरीय अनुक्रम 23, 17, 11, ... को कुनचाहिँ पद पहिलो ऋणात्मक पद हुन्छ ?
Which term of the AP 23, 17, 11, ... is the first negative term ?

6B. समानान्तरीय अनुक्रम 55, 50, 45, 40, ... को कुनचाहिँ पद पहिलो ऋणात्मक पद हुन्छ ?
Which term of the AP 55, 50, 45, 40, ... is the first negative term ?

6C. समानान्तरीय अनुक्रम – 75, – 70, – 65, ... को कनुचाहिँ पद पहिलो धनात्मक पद हुन्छ ?
Which term of AP – 75, – 70, – 65, ... is the first positive term ?

6D. समानान्तरीय अनुक्रम – 101, – 91, – 81, ... को कनुचाहिँ पद पहिलो धनात्मक पद हुन्छ ?
Which term of AP – 101, – 91, – 81, ... is the first positive term ?

Model 3 *Problems based on geometric mean/means between two numbers.*

7A. यदि x र x^3 को गुणोत्तर मध्यमा 16 भए x को मान निकाल्नुहोस् ।
If 16 is the geometric mean of x and x^3, find the value of x.

7B. यदि a, m र b एउटा गुणोत्तर श्रेणीमा भए $m = \sqrt{ab}$ हुन्छ भनी देखाउनुहोस् ।
If a, m and b are in geometric series, show that: $m = \sqrt{ab}$

www.drmathnp.com

puspas.com.np

7C. यदि $(p+2)$ र $9(p+2)$ को GM $= (p+10)$ भए p को मान पत्ता लगाउनुहोस् ।
If $(p+10)$ is GM of $(p+2)$ and $9(p+2)$, find the value of p.

7D. यदि $(k+4)$ र $25(k+4)$ को गुणोत्तर मध्यक 30 भए समानान्तरीय मध्यक पत्ता लगाउनुहोस् ।
If the geometric mean of $(k+4)$ and $25(k+4)$ is 30, find the arithmetic mean.

8A. एउटा गुणोत्तर श्रेणीको दोस्रो र छैटौं मध्यमाहरू क्रमशः 8 र 128 भए पाचौं पद पत्ता लगाउनुहोस् ।
If the second and sixth means of a geometric series are 8 and 128 respectively then find the value of fifth term.

8B. एउटा गुणोत्तर श्रेणीको पाचौं र नवौं मध्यमाहरू क्रमशः 243 र 19683 भए आठौं पद पत्ता लगाउनुहोस् ।
If the fifth and ninth means of a geometric series are 243 and 19683 respectively then find the eighth term.

Model 4 Problems based on finding the sum of a geometric sequence.

9A. श्रेणी $81 - 27 + 9 - \ldots\ldots\ldots$ का 8 पदहरूको योगफल निकाल्नुहोस् ।
Find the sum of $81 - 27 + 9 - \ldots\ldots\ldots$ to 8 terms.

9B. 4 को 4^1 बाट सुरु हुने बढ्दो आठओटा घाताङ्कहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of first eight powers of 4 starting from 4^1.

9C. योगफल पत्ता लगाउनुहोस् (Find the sum of): $8 + 2 + \frac{1}{2} + \ldots\ldots\ldots + \frac{1}{128}$

Model 5 Problems based on finding values of letters when terms of GP are given.

10A. यदि $x+9, x-6$ र 4 एउटा गुणोत्तर श्रेणीमा भए x को मान पत्ता लगाउनुहोस् ।
If $x+9, x-6$ and 4 are in a geometric series, find the value of x.

10B. यदि $x, x+3, x+9$ एउटा गुणोत्तर श्रेणीमा भए x को मान पत्ता लगाउनुहोस् ।
If $x, x+3, x+9$ are in a geometric series, find the value of x.

10C. यदि $4, 2x-4, \ldots, 64, 21x+2$ एउटा गुणोत्तर अनुक्रमको पदहरू भए चौथो पद पत्ता लगाउनुहोस् ।
If $4, 2x-4, \ldots, 64, 21x+2$ are the terms of geometric sequence, find the fourth term.

10D. यदि $\frac{1}{3}, x+1, y+2, z+3, 27$ एउटा GP का पदहरू भए x, y र z को मान पत्ता लगाउनुहोस् ।
If $\frac{1}{3}, x+1, y+2, z+3, 27$ are in GP, find the values of x, y and z.

10E. यदि $2, 4, 8, \ldots\ldots$ को k^{th} पद 128 भए k को मान निकाल्नुहोस् ।
If the k^{th} term of $2, 4, 8, \ldots\ldots$ is 128, find the value of k.

10F. यदि $81, 27, 9, \ldots\ldots$ को k^{th} पद 1/9 भए k को मान निकाल्नुहोस् ।
If the k^{th} term of $81, 27, 9, \ldots\ldots$ is 1/9, find the value of k.

10G. यदि ज्यामितीय अनुक्रमका पहिला तीन पदहरू $4\sqrt{2k}, 3\sqrt{2}+k$ र $2\sqrt{2k}$ भए k को धनात्मक मान पत्ता लगाउनुहोस् ।
If $4\sqrt{2k}, 3\sqrt{2}+k$ and $2\sqrt{2k}$ are the first three terms of a geometric sequence, find the positive value of k.

10H. यदि ज्यामितीय अनुक्रमका पहिला तीन पदहरू $2\sqrt{2k}, 6+\sqrt{2k}$ र $8\sqrt{2k}$ भए k को धनात्मक मान पत्ता लगाउनुहोस् ।
If $2\sqrt{2k}, 6+\sqrt{2k}$ and $8\sqrt{2k}$ are the first three terms of a geometric sequence, find the positive value of k.

11 तलका समीकरणहरू हल गर्नुहोस् (Solve the following equations):

A. $6 + 18 + 54 + \ldots\ldots + k = 2184$ **B.** $5 + 15 + 45 + \ldots\ldots + k = 5465$

Model 6 On finding number of terms, common ratio and first term of a GS.

12A. एउटा GS को प्रथम र तेस्रो पदहरू क्रमशः 2 र 8^{-1} छन् भने त्यो श्रेणीको समान अनुपात निकाल्नुहोस् ।
If the first and third terms of a GS are 2 and 8^{-1}, find the common ratio of the series.

12B. गुणोत्तर श्रेणीको प्रथम र पाँचौ पदहरू क्रमशः 625 र 1 छन् भने सो श्रेणीको समान अनुपात निकाल्नुहोस् ।
If the 1st and 5th terms of a GS are 625 and 1, find the common ratio of the series.

13. श्रेणीको खाली ठाउँमा कति पदहरू थप्न सकिन्छ ? (How many terms can be inserted in the gap of the series?)

A. $\frac{1}{5} + 1 + 5 + \ldots\ldots\ldots\ldots = \frac{781}{5}$ **B.** $64 + 16 + 4 + \ldots\ldots\ldots\ldots = \frac{5461}{64}$

13C. श्रेणी $32 + 48 + 72 + \ldots\ldots\ldots\ldots$ देखि 546.75 सम्म कतिओटा पदहरू छन् ?
How many terms of the series $32 + 48 + 72 + \ldots\ldots\ldots\ldots$ are upto 546.75 ?

www.drmathnp.com
puspas.com.np

14A. एउटा गुणोत्तर श्रेणीको दोस्रो पद र पाचौँ पद क्रमशः 9 र 243 भए पहिलो पद पत्ता लगाउनुहोस् ।
A GS has second term and fifth term 9 and 243 respectively, find the first term.

14B. एउटा गुणोत्तर श्रेणीको तेस्रो पद र छैटौँ पद क्रमशः 27 र 729 भए पहिलो पद पत्ता लगाउनुहोस् ।
A GS has third term and sixth term 27 and 729 respectively, find the first term.

15A. एउटा गुणोत्तर श्रेणीको समान अनुपात पहिलो पदको दोब्बर छ । यदि सो गुणोत्तर श्रेणीको प्रथम 2 पदहरूको योगफल 10 भए पहिलो पद पत्ता लगाउनुहोस् ।
The common ratio of a geometric series is double of the first term. If the sum of first 2 terms of the geometric series is 10, find the first term.

15B. एउटा GS को पहिला 3 पदहरूको योगफलको 9 गुणा पहिला 6 पदहरूको योगफल भए समान अनुपात पत्ता लगाउनुहोस् ।
The sum of first 6 terms of a GS is 9 times the sum of the first 3 terms. Find the common ratio.

15C. एउटा गुणोत्तर श्रेणीको पहिला 3 पदहरूको योगफलको 28 गुणा पहिला 6 पदहरूको योगफल भए समान अनुपात पत्ता लगाउनुहोस् ।
The sum of first 6 terms of a GS is 28 times the sum of the first 3 terms. Find the common ratio.

16A. एउटा GP को केही पदहरूको योगफल 315 छ । यदि पहिलो पद र समान अनुपात क्रमशः 5 र 2 भए पदहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
The sum of terms of a GP is 315. If the first term and the common ratio are 5 and 2 respectively, find the number of terms.

16B. अनुक्रम $\sqrt{3}, 3, 3\sqrt{3}, ...$ को कतिओटा पदहरूको योगफल $39 + 13\sqrt{3}$ होला ?
How many terms of the sequences $\sqrt{3}, 3, 3\sqrt{3}, ...$ has the sum $39 + 13\sqrt{3}$?

Model 7 Problems based on AM and GM between two numbers.

17A. 4 र 9 बिचको AM र GM पत्ता लगाउनुहोस् र AM > GM हुन्छ भनी देखाउनुहोस् ।
Find AM and GM between 4 and 9 and show that: AM > GM

17B. 16 र 25 बिचको AM र GM पत्ता लगाउनुहोस् र AM > GM हुन्छ भनी देखाउनुहोस् ।
Find AM and GM between 16 and 25 and show that AM > GM

17C. अङ्कगणितीय मध्यमा र गुणोत्तर मध्यमा AM र GM भए प्रमाणित गर्नुहोस् : $AM \geq GM$
If AM and GM are arithmetic and Geometric mean, prove that: $AM \geq GM$

18A. a र 8 को समानान्तरीय मध्यमा 5 भए गुणोत्तर मध्यमा कति होला ?
The arithmetic mean of a and 8 is 5. What is the geometric mean ?

18B. P र 27 को गुणोत्तर मध्यमा 9 भए समानान्तरीय मध्यमा कति होला ?
The geometric mean of P and 27 is 9. What is its arithmetic mean ?

Model 8 Problems based on sigma notation of a GP.

19. तलका श्रेणीहरू गुणोत्तर श्रेणी हुन् भनी देखाउनुहोस् । (Show that the following series are the geometric series.)

A. $\sum_{n=1}^{20} 3(4)^{n-2}$ **B.** $\sum_{k=1}^{10} 3^k$ **C.** $\sum_{n=1}^{5} 5 \times 4^{n+1}$

20. गुणोत्तर श्रेणीहरूको योगफल पत्ता लगाउनुहोस् (Find the sum of the geometic series):

A. $\sum_{n=2}^{6} 3(4)^{n-2}$ **B.** $\sum_{k=3}^{8} 2(-2)^k$ **C.** $\sum_{n=1}^{5} 3^n$

21. तलका गुणोत्तर श्रेणीमा k को मान पत्ता लगाउनुहोस् । (In the following geometric series, find the value of k.)

A. $\sum_{n=1}^{5} k \cdot 3^n = 1815$ **B.** $\sum_{n=1}^{5} k\,(4)^{n-1} = 1023$

Model 9 Problems based on application of geometric sequence/series.

22A. एउटा कल्चरमा 100 ब्याक्टेरिया छन् । एक दिन पछि त्यहाँ 300 ब्याक्टेरिया भए र दुई दिनपछि 900 ब्याक्टेरिया भए भने आठौँ दिनपछि त्यहाँ कति ब्याक्टेरिया होलान् ?
A culture contains 100 bacteria. After one day, there are 300 bacteria present and after two days 900 bacteria present. How many bacteria will be there after eight days?

22B. एउटी केटीले एक महिनामा रु. 400 बचत गर्दछिन् । अर्को महिनामा रु. 800 र तेस्रो महिनामा रु.1600 बचत गर्दछिन् । यदि उनले यही क्रममा बचत गर्दै जान्छिन् भने 8 महिनामा कति बचत गर्दछिन् ?
A girl saves Rs 400 during one month, Rs 800 in the next month; Rs 1600 in the third month. If she continues her savings in this sequence, find her saving in 8 months .

www.drmathnp.com

puspas.com.np

B. LONG QUESTIONS

Model 1 Problems based on finding indicated term of a GS.

1A. दोस्रो र पाँचौं पदहरू क्रमशः 6 र 162 भएको गुणोत्तर श्रेणीको चौथो पद पत्ता लगाउनुहोस् ।
Find the 4^{th} term of a GS whose 2^{nd} term and 5^{th} terms are 6 and 162 respectively.

1B. एउटा GS को छैटौं र दशौं पदहरू क्रमशः 64 र 1024 भए यसको आठौं पद निकाल्नुहोस् ।
The 6^{th} and 10^{th} terms of a GS are 64 and 1024 respectively. Find the 8^{th} term.

2A. एउटा गुणोत्तर श्रेणीको तेस्रो र छैटौं पदहरू 1 : 8 को अनुपातमा छन् । यदि आठौं पद 512 भए सो श्रेणीको पाचौं पद पत्ता लगाउनुहोस् ।
The third and sixth terms of a geometric series are in the ratio of 1 : 8. If eighth term of the series is 512 then find the fifth term.

2B. एउटा गुणोत्तर श्रेणीको तेस्रो र छैटौं पदहरू 1 : 8 को अनुपातमा छन् । यदि आठौं पद 384 भए सो श्रेणीको पाचौं पद पत्ता लगाउनुहोस् ।
The third and sixth terms of a geometric series are in the ratio of 1 : 8. If eighth term of the series is 384 then find the fifth term.

2C. एउटा GP को पहिलो पद तेस्रो पद भन्दा 9 ले धेरै र दोस्रो पद, चौथो पद भन्दा 18 ले धेरै छ । सो GP को पाचौं पद पत्ता लगाउनुहोस् ।
In a GP, the first term is 9 more than the third term and the second term is 18 more than fourth term. Find the fifth term of the GP.

2D. एउटा समानान्तरीय अनुक्रमको पाचौं पद दोस्रो पदको दोब्बर भन्दा दुईले धेरै छ र सो अनुक्रमको सातौं पद तेस्रो पदको तेब्बर भन्दा 1 ले कमी छ । उक्त अनुक्रमको 20 औं पद पत्ता लगाउनुहोस् ।
The fifth term of an AP is two more than double of second term and seventh term of the AP is one less than thrice the third term. Find the 20^{th} term of the AP.

3A. यदि $4, 2x-4, \ldots\ldots, 64, 21x+2$ एउटा गुणोत्तर अनुक्रमको पदहरू भए x को मान र चौथो पद पत्ता लगाउनुहोस् ।
If $4, 2x-4, \ldots\ldots, 64, 21x+2$ are the terms of geometric sequence, find the value of x and the fourth term.

3B. $k+9, k-6$ र 4 एउटा गुणोत्तर अनुक्रमका तीन पदहरू भए k को मानहरू पत्ता लगाउनुहोस र प्रत्येक अवस्थामा चौथो पद पनि पत्ता लगाउनुहोस् ।
Find the values of k for which $k+9$, $k-6$ and 4 are the three terms of a geometric progression and calculate the fourth term in each case.

Model 2 Problems based on finding the position of a given term in a given GP.

4A. एउटा गुणोत्तर अनुक्रमको दोस्रो पद 1 र चौथो पद $\frac{1}{4}$ भए कुन चाहिँ पद $\frac{1}{128}$ होला ?
If second term and the fourth term of a geometric sequence are 1 and $\frac{1}{4}$ respectively, which term is $\frac{1}{128}$?

4B. एउटा गुणोत्तर अनुक्रमको दोस्रो पद 20 र पाचौं पद 160 भए कुन चाहिँ पद 5120 होला ?
If second term and the fifth term of a geometric sequence are 20 and 160 respectively, which term is 5120 ?

Model 3 Problems based on geometric means between two numbers.

5A. यदि $\frac{2}{3}, x, y, -\frac{1}{96}$ गुणोत्तर श्रेणीमा छन् भने x र y का मानहरू पत्ता लगाउनुहोस् ।
If $\frac{2}{3}, x, y, -\frac{1}{96}$ are in geometric series, find the values of x and y.

5B. $64, x, y$ र 8 गुणोत्तर श्रेणीमा छन् भने x र y का मानहरू पत्ता लगाउनुहोस् ।
Find the values of x and y if $64, x, y$ and 8 are in geometric series.

5C. यदि $a, -3, 9, b$ गुणोत्तर श्रेणीमा छन् भने a र b का मानहरू निकाल्नुहोस् ।
If $a, -3, 9, b$ are in geometric series, find the values of a and b.

5D. यदि $1024, a, b, c, d, 1$ एउटा GP मा भए a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।
If $1024, a, b, c, d, 1$ are in a GP then find the values of a, b, c and d.

Model 4 Problems based on finding number of geometric means and the value of x.

6A. 8^{-1} र x को बिचमा 5 ओटा गुणोत्तर मध्यमाहरू छन् । यदि तेस्रो मध्यमा 1 भए अन्य मध्यमाहरू पत्ता लगाई x को मान पत्ता लगाउनुहोस् ।
5 GM's are inserted between 8^{-1} and x. If the third mean is 1, find out the other means and the value of x.

www.drmathnp.com

pusnas.com.np

6B. 4 र x को बिचमा 4 ओटा गुणोत्तर मध्यमाहरू छन् । यदि तेस्रो मध्यमा 32 भए अन्य मध्यमाहरू पत्ता लगाई x को मान पत्ता लगाउनुहोस् ।
4 GM's are inserted between 4 and x. If the third mean is 32, find out the other means and the value of x.

6C. 5 र x को बिचमा 4 ओटा गुणोत्तर मध्यमाहरू छन् । यदि तेस्रो मध्यमा 40 भए अन्य मध्यमाहरू पत्ता लगाई x को मान पत्ता लगाउनुहोस् ।
4 GM's are inserted between 5 and x. If the third mean is 40, find out the other means and the value of x.

6D. x र 729 को बिचमा 4 ओटा गुणोत्तर मध्यमाहरू छन् । यदि तेस्रो मध्यमा 81 भए अन्य मध्यमाहरू पत्ता लगाई x को मान पत्ता लगाउनुहोस् ।
4 GM's are inserted between x and 729. If the third mean is 81, find out the other means and the value of x.

Model 5 *Problems based on terms and the sum of a geometric sequence/series.*

7A. यदि एउटा GP को सातौँ पद 1 र एघारौँ पदको 16 गुणा पनि 1 छ भने पहिला 7 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the 7^{th} term of a GP is 1 and 16 times its 11^{th} term is also 1, find the sum of its first 7 terms.

7B. एउटा गुणोत्तर अनुक्रमको छैटौँ पद 1 र दोस्रो पदको 16 गुणा पनि 1 छ । पहिला एघार पदहरूको योगफल पत्ता लगाउनुहोस् ।
In a geometric sequence, the sixth term is 1 and the 16 times of second term is also 1. Find the sum of first eleven terms.

7C. एउटा गुणोत्तर श्रेणीका पहिलो 2 ओटा पदहरूको योगफल 3 र पहिलो 4 ओटा पदहरूको योगफल 15 छ । यदि समान अनुपात धनात्मक भए पहिलो 6 ओटा पदहरूको योगफल निकाल्नुहोस् ।
In a geometric series, the sum of first two terms is 3 and the sum of first four terms is 15. If the common ratio is +ve, find the sum of first six terms.

7D. एउटा धनात्मक समान अनुपात भएको गुणोत्तर श्रेणीका पहिला चार पदहरूको योगफल 40 र पहिला दुई पदहरूको योगफल 4 छ भने सो श्रेणीका पहिला आठ पदहरूको योगफल निकाल्नुहोस् ।
In a geometric series having +ve value of common ratio, the sum of first four terms is 40 and the sum of first two terms is 4. Find the sum of first eight terms of the series.

8A. दिइएको श्रेणी $9 + 3 + 1 + \ldots\ldots\ldots + 9^{-1}$ छ । यो श्रेणीमा कतिओटा पदहरू छन् ? श्रेणीको योगफल निकाल्नुहोस् ।
Given series is $9 + 3 + 1 + \ldots\ldots\ldots + 9^{-1}$. How many terms are there in the series ? Calculate the sum.

8B. दिइएको श्रेणी $4 + 8 + 16 + \ldots\ldots + 256$ छ । यो श्रेणीमा कतिओटा पदहरू छन् ? श्रेणीको योगफल निकाल्नुहोस् ।
Given series is $4 + 8 + 16 + \ldots\ldots + 256$. How many terms are there in the series ? Calculate the sum.

8C. अनुक्रमहरू 2, 4, 8, 16, 32 र 128, 32, 8, 2, 2^{-1} का संगती पदहरूको गुणनफलहरूको योगफल क्याल्कुलेटरको प्रयोग नगरी पत्ता लगाउनुहोस् ।
Find the sum of products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, 2^{-1} without using calculator.

Model 6 *On finding number of terms, common ratio and first term of a GS.*

9A. धनात्मक समानुपात भएको एउटा GP मा जम्मा 8 ओटा पदहरू छन् । यदि पहिला चार पदहरूको योग 45 र अन्तिम चार पदहरूको योग 720 भए प्रथम पद र समान अनुपात पत्ता लगाउनुहोस् ।
A GP has 8 terms whose common ratio is positive. If the sum of the first 4 terms is 45 and the sum of the last 4 terms is 720, find the common ratio and the first term.

9B. एउटा GP का प्रथम चारओटा पदहरूको योगफल 30 र त्यसका अन्तिम चारओटा पदहरूको योगफल 960 छ । यदि प्रथम र अन्तिम पदहरू क्रमश: 2 र 512 छन् भने समान अनुपात निकाल्नुहोस् ।
The sum of the first four terms of a GP is 30 and that of the last four terms is 960. If the first and the last term of the GP are 2 and 512 respectively, find the common ratio.

9C. एउटा गुणोत्तर श्रेणीको पहिलो तीन पदहरूको योगफल 16 छ र त्यसपछिका तीन पदहरूको योगफल 128 छ भने पहिलो पद पत्ता लगाउनुहोस् ।
The sum of first three terms of a GP is 16 and the sum of the next three terms is 128. Find the first term.

9D. आठ ओटा पदहरू भएको गुणोत्तर श्रेणीको प्रथम तीन पदहरूको योग 13/9 र अन्तिम तीन पदहरूको योग 351 भए प्रथम पद र समान अनुपात पत्ता लगाउनुहोस् ।
A geometric series has 8 terms whose sum of the first three terms is 13/9 and the sum of last three terms is 351. Find the first term and the common ratio of the series.

9E. आठ ओटा पदहरू भएको गुणोत्तर श्रेणीको प्रथम तीन पदहरूको योग 7 र अन्तिम तीन पदहरूको योग 224 भए प्रथम पद र समान अनुपात पत्ता लगाउनुहोस् ।
A geometric series has 8 terms whose sum of the first three terms is 7 and the sum of last three terms is 224. Find the first term and the common ratio of the series.

www.drmathnp.com
puspas.com.np

10A. यदि एउटा गुणोत्तर श्रेणीको S_3 र S_6 क्रमशः 14 र 126 भए समान अनुपात पत्ता लगाउनुहोस् ।
If S_3 and S_6 of a geometric series are 14 and 126 respectively, find common ratio.

10B. एउटा गुणोत्तर श्रेणीको S_3 र S_6 क्रमश : 7 र 63 भए समान अनुपात पत्ता लगाउनुहोस् ।
If S_3 and S_6 of a GS are 7 and 63 respectively, find the common ratio.

10C. एउटा गुणोत्तर श्रेणीमा यदि $S_6 = 28$ र $S_3 = 1$ भए r र a को मान पत्ता लगाउनुहोस् ।
In a GP, if $S_6 = 28$ and $S_3 = 1$, find r and a.

11A. एउटा GP को चौथो पद, सातौँ पद र अन्तिम पद क्रमशः 10, 80 र 2560 भए पहिलो पद र पदहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
The fourth, seventh and the last term of a GP are 10, 80 and 2560 respectively. Find the first term and the number of terms in the GP.

11B. यदि गुणोत्तर अनुक्रमहरू 5, 10, 20, ... र 1280, 640, 320, ... का $n^{\text{औँ}}$ पदहरू बराबर भए n को मान पत्ता लगाउनुहोस् ।
If the GP's 5, 10, 20, ... and 1280, 640, 320, ... have their n^{th} terms equal, find the value of n.

12A. एउटा गुणोत्तर श्रेणीको पहिलो पद 1 छ । यदि यसको तेस्रो पद र पाचौँ पदको योगफल 90 भए समान अन्तर पत्ता लगाउनुहोस् ।
The first term of a GP is 1. If the sum of the third term and fifth term is 90 then find the common ratio of the GP.

12B. एउटा गुणोत्तर श्रेणीको पहिलो पद 2 छ । यदि यसको तेस्रो पद र पाचौँ पदको योगफल 40 भए समान अन्तर पत्ता लगाउनुहोस् ।
The first term of a GP is 2. If the sum of the third term and fifth term is 40 then find the common ratio of the GP.

Model 7 *Problems based on finding geometric sequence/series.*

13A. पाचौँ पद 81 र आठौँ पद 2187 भएको एउटा गुणोत्तर श्रेणी पत्ता लगाउनुहोस् ।
Find a geometric series whose 5^{th} and 8^{th} terms are 81 and 2187 respectively.

13B. दोस्रो पद 6 र पाचौँ पद 48 भएको एउटा गुणोत्तर अनुक्रम पत्ता लगाउनुहोस् ।
Find a geometrical sequence whose 2^{nd} term is 6 and 5^{th} term is 48.

14A. एउटा गुणोत्तर श्रेणीको छैठौँ पद तेस्रो पदको 8 गुणा र पहिला 2 पदहरूको योग 6 भए सो गुणोत्तर श्रेणी पत्ता लगाउनुहोस् ।
In a geometric series, 6^{th} term is 8 times of the third term and the sum of the first two terms is 6. Find the geometric series.

14B. एउटा गुणोत्तर श्रेणीको पाँचौ पद दोस्रो पदको 27 गुणा र पहिला 2 पदहरूको योग 4 भए सो गुणोत्तर श्रेणी पत्ता लगाउनुहोस् ।
In a geometric series, the fifth term is 27 times the second term and sum of the first two terms is 4. Find the geometric series.

14C. एउटा गुणोत्तर श्रेणी पत्ता लगाउनुहोस् जसको पहिला दुई पदहरूको योगफल – 4 छ र पाचौँ पद तेस्रो पदको 4 गुणा छ ।
Find the GP for which the sum of first two terms is – 4 and fifth term is 4 times the third term.

14D. एउटा GP का पहिला चार पदहरू पत्ता लगाउनुहोस् जहाँ तेस्रो सङ्ख्या पहिलो भन्दा 9 ले धेरै छ र दोस्रो सङ्ख्या चौथो भन्दा 18 ले धेरै छ ।
Find first four numbers in GP in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18.

Model 8 *Problems based on sum and product of selected terms in GP.*

15A. एउटा GP मा पाचौँ पद 2 भए पहिलो पाँचओटा अनुवर्ती विजोर पदहरूको गुणनफल पत्ता लगाउनुहोस् ।
In a GP, fifth term is 2. Find the product of first five consecutive odd terms.

15B. एउटा GP मा छैटौँ पद 3 भए पहिलो पाँचओटा अनुवर्ती जोर पदहरूको गुणनफल पत्ता लगाउनुहोस् ।
In a GP, sixth term is 3. Find the product of first five consecutive even terms.

16A. गुणोत्तर श्रेणीमा रहेका तीन सङ्ख्याहरूको योगफल 38 र तिनीहरूको गुणनफल 1728 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers in a geometric series is 38 and their product is 1728. Find the numbers.

16B. गुणोत्तर श्रेणीमा रहेका तीन सङ्ख्याहरूको योगफल $\frac{13}{3}$ र तिनीहरूको गुणनफल 1 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

The sum of three numbers in a geometric series is $\frac{13}{3}$ and their product is 1. Find the numbers.

16C. गुणोत्तर श्रेणीमा रहेका तीन सङ्ख्याहरूको योगफल 7 र तिनीहरूको गुणनफल 8 छ भने ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers in a geometric series is 7 and their product is 8. Find the numbers.

16D. गुणोत्तर श्रेणीमा रहेका तीन सङ्ख्याहरूको गुणनफल 64 र तिनीहरूको योगफल 21 छ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The product of three numbers in geometric series is 64 and their sum is 21. Find the numbers.

16E. गुणोत्तर अनुक्रममा भएका तीनओटा सङ्ख्याहरूको गुणनफल 512 र पहिला दुई सङ्ख्याहरूको योगफल 10 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
If the product of three numbers in geometric sequence is 512 and sum of the first two numbers is 10, find the numbers.

www.drmathnp.com

pusmas.com.np

16F. गुणोत्तर श्रेणीमा भएका तीनओटा धनात्मक सङ्ख्याहरूको योगफल 104 छ र त्यसका प्रथम र तृतीय पदहरूको गुणनफल 576 भए उक्त सङ्ख्याहरू निकाल्नुहोस् ।
The sum of three positive numbers in GP is 104 and the product of their extreme terms is 576. Find the numbers.

16G. एउटा गुणोत्तर श्रेणीका तीन पदहरूको गुणनफल 216 र ती सङ्ख्याहरूको जोडा सङ्ख्याको गुणनफलको योग 156 हुन्छ भने ती सङ्ख्याहरू निकाल्नुहोस् ।
The product of three numbers in GP is 216 and the sum of their product in pairs is 156. Find the numbers.

16H. एउटा गुणोत्तर श्रेणीका तीन पदहरूको गुणनफल 216 र ती सङ्ख्याहरूको जोडा सङ्ख्याको गुणनफलको योग 126 हुन्छ भने ती सङ्ख्याहरू निकाल्नुहोस् ।
The product of three numbers in GP is 216 and the sum of their product in pairs is 126. Find the numbers.

Model 9 Problems based on AM and GM between two numbers.

17A. दुई सङ्ख्याको समानान्तरीय मध्यमा 50 र गुणोत्तर मध्यमा समानान्तरीय मध्यमाको 80% छ भने ती दुई सङ्ख्याहरूबिचको चार ओटा समानान्तरीय मध्यमा पत्ता लगाउनुहोस् ।
The arithmetic mean of two numbers is 50 and geometric mean is 80% of arithmetic mean, find out four arithmetic mean between these two numbers.

17B. कुनै दुई सङ्ख्याहरूबिचका समानान्तरीय मध्यमा र गुणोत्तर मध्यमाको अनुपात 5: 3 छ भने ती दुई सङ्ख्याहरूको अनुपात पत्ता लगाउनुहोस् ।
Find the ratio of two numbers when the ratio of their arithmetic mean and geometric mean is 5: 3.

17C. दुईओटा सङ्ख्याहरूको समानान्तरीय मध्यमाको चारगुणा गुणोत्तर मध्यमाको पाँचगुणासँग बराबर छ । यदि समानान्तरीय मध्यमा र गुणोत्तर मध्यमाको योगफल 18 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The four times of the arithmetic mean is five times of the geometric mean of two numbers. If the sum of arithmetic mean & geometric mean is 18, find the two numbers.

17D. दुईओटा सङ्ख्याहरू पत्ता लगाउनुहोस् जसको AM + GM = 9 र AM : GM = 5 : 4 छ ।
Find the two numbers whose AM + GM = 9 and AM : GM = 5 : 4.

17E. दुईओटा सङ्ख्याहरू पत्ता लगाउनुहोस् जसको AM + GM = 25 र AM : GM = 5 : 3 छ ।
Find the two numbers whose AM + GM = 25 and AM : GM = 5 : 3.

Model 10 Problems based on AP and GP (Mixed).

18A. एउटा समानान्तरीय श्रेणीको 3 ओटा पदहरूको योग 15 र पहिलो पद र तेस्रो पदको गुणोत्तर मध्यमा 4 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms in arithmetic series is 15. The geometric mean of the first term and the third terms is 4. Find the numbers.

18B. एउटा समानान्तरीय श्रेणीका 3 ओटा पदहरूको योग 13 र पहिलो पद र तेस्रो पदको गुणोत्तर मध्यमा 4 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of 3 terms in arithmetic series is 13. The geometric mean of the first term and the third term is 4. Find the numbers.

19A. तीनओटा सङ्ख्याहरू GP मा छन् जसको योग 70 छ । यदि छेउका प्रत्येक पदलाई 4 ले गुणन गरी मध्यमालाई 5 ले गुणन गरियो भने तीनीहरू AP मा हुन्छन् । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in GP whose sum is 70. If the extremes be each multiplied by 4 and the mean by 5, they will be in AP. Find the numbers.

19B. एउटा GP को पहिलो तीन पदहरूको गुणनफल 1000 छ । यदि दोस्रो पदमा 6 र तेस्रो पदमा 7 जोडियो भने ती पदहरू AP मा हुन्छन् । सो GP पत्ता लगाउनुहोस् ।
The product of first three terms of a GP is 1000. If 6 is added to its second term and 7 added to its third term, the terms are in AP. Find the GP.

19C. GP मा रहेका तीनओटा सङ्ख्याहरूको गुणनफल 216 छ । यदि तीनीहरूमा 2, 8, 6 क्रमशः जोड्यो भने AP बन्छ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The product of three numbers in GP is 216. If 2, 8, 6 be added to them respectively, the results are in AP. Find the numbers.

20A. एउटा AP मा रहेका तीन पदहरूको योगफल 12 छ । यदि तेस्रो पदमा 2 जोडियो भने ती सङ्ख्याहरू GP मा हुन्छन् । उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of the three numbers in AP is 12. If 2 is added to the third term then the resulting numbers are in GP. Find the numbers.

20B. एउटा AP मा रहेका तीन पदहरूको योगफल 9 छ । यदि तेस्रो पदमा 4 जोडियो भने ती सङ्ख्याहरू GP मा हुन्छन् । उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of the three numbers in AP is 9. If 4 is added to the third term then the resulting numbers are in GP. Find the numbers.

www.drmathnp.com

pustaks.com.np

20C. एउटा AP मा रहेका तीन पदहरूको योगफल 15 छ । यदि दोस्रो पदबाट 2 घटायो भने ती सङ्ख्याहरू GP मा हुन्छन् । उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of the three numbers in AP is 15. If 2 is subtracted from the second term then the resulting numbers are in GP. Find the numbers.

20D. एउटा AP मा रहेको तीन पदहरूको योगफल 12 छ । यदि 2 र 7 क्रमशः दोस्रो र तेस्रो पदमा जोडियो भने ती सङ्ख्याहरू GP मा हुन्छन् । उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of the three numbers in AP is 12. If 2 and 7 are added to the second and third term respectively then the resulting numbers are in GP. Find the numbers.

20E. एउटा AP का तीनओटा पदहरूको योगफल 21 छ । यदि दोस्रो सङ्ख्या 1 ले घटाइयो र तेस्रो सङ्ख्या 1 ले बढाइयो भने एउटा GP बन्दछ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers of an AP is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain the GP. Find the numbers.

20F. समानान्तर अनुक्रममा रहेका तीन पदहरूको योगफल 15 छ । यदि ती पदहरूमा क्रमशः 1, 3, 9 जोड्दा गुणोत्तर अनुक्रम बन्दछ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers are in an arithmetic sequence and their sum is 15. If 1, 3 and 9 be added to them respectively, they form a geometric sequence. Find the numbers.

20G. समानान्तरीय श्रेणीमा रहेका तीनओटा सङ्ख्याहरूको योगफल 21 छ । यदि ती सङ्ख्याहरूमा क्रमशः 1, 3 र 10 जोड्ने हो भने ती सङ्ख्याहरू गुणोत्तर श्रेणीमा हुन्छन् । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Three numbers whose sum is 21 are in AP. If 1, 3 and 10 are added to them respectively, then the numbers are in GP. Determine the numbers.

21A. कुनै गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 32 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद कुनै समानान्तर श्रेणीको दशौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको प्रथम 19 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of the first five terms of a geometric series is 32. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of first 19 terms of the arithmetic series.

21B. कुनै गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 243 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद कुनै समानान्तर श्रेणीको दशौँ पदसँग बराबर भए उक्त समानान्तर श्रेणीको प्रथम 19 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of the first five terms of a geometric series is 243. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of first 19 terms of the arithmetic series.

Model 11 Problems based on application of geometric sequence/series.

22A. एउटा समबाहु त्रिभुजको एउटा भुजा 96 cm छ । यसका भुजाहरूको मध्यबिन्दुहरू जोडेर अर्को त्रिभुज बनाइएको छ । यो प्रक्रिया बढाउँदै जाँदा सुरुदेखि पाँचौँ त्रिभुजसम्मको परिमितिको योगफल पत्ता लगाउनुहोस् ।
One side of an equilateral triangle is 96 cm. The mid-points of its sides are joined to form another triangle whose mid-points, in turn, are joined to form still another triangle. The process is continued. Find the sum of the perimeter of all the five triangles from the beginning.

22B. एउटा समबाहु त्रिभुजको एउटा भुजा 64 cm छ । यसका भुजाहरूको मध्यबिन्दुहरू जोडेर अर्को त्रिभुज बनाइएको छ । यो प्रक्रिया बढाउँदै जाँदा सुरुदेखि पाँचौँ त्रिभुजसम्मको परिमितिको योगफल पत्ता लगाउनुहोस् ।
One side of an equilateral triangle is 64 cm. The mid-points of its sides are joined to form another triangle whose mid-points, in turn, are joined to form still another triangle. The process is continued. Find the sum of the perimeter of all the five triangles from the beginning.

23A. 8 ओटा टोकरीहरूमा फूलहरू राखिएको छ । प्रत्येक टोकरीहरूमा राखिएका फूलहरूको सङ्ख्याबाट एउटा गुणोत्तर श्रेणी बन्दछ । यदि चौथो र छैठौँ टोकरीहरूमा क्रमशः 24 र 96 फूलहरू भए पहिलो र अन्तिम टोकरीमा भएका फूलहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
There are 8 baskets full of flowers. The numbers of flowers in each basket form a GP. If the 4^{th} and 6^{th} baskets contain 24 and 96 flowers respectively, find the number of the flowers in the 1^{st} and last basket.

23B. एउटा चिडियाखानामा 8 थरी बाँदरहरू छन् । हरेक थरी बाँदरहरूले एउटा गूणोत्तर श्रेणी बनाँउदछ । यदि चौथो र छैटौँ थरी बाँदरहरू क्रमशः 54 र 486 भए पहिलो र अन्तिम थरी बाँदरहरूको सङ्ख्या पत्ता लगाउनुहोस् । साथै उक्त चिडियाखानामा भएका जम्मा बाँदरहरूको सङ्ख्या पनि पत्ता लगाउनुहोस् ।
There are 8 varieties of monkeys in a zoo. The number of each variety forms a GP. If the 4^{th} and 6^{th} variety consist of 54 and 486 monkeys respectively, find the number in the first and the last variety. Also find the total number of monkeys in zoo.

23C. एउटा चिडियाखानामा 10 थरी चराहरू छन् । हरेक थरीका चराहरू अर्को थरीका चराहरूको दोब्बर छन् । यदि पहिलो थरीका चराहरूको सङ्ख्या 2 भए अन्तिम थरीका चराहरूको सङ्ख्या र सो चिडियाखानामा भएका जम्मा चराहरूको सङ्ख्या पत्ता लगाउनुहोस् ।
There are 10 varieties of birds in a zoo. The number of each variety being double of the number of another variety. If the number in the first variety is 2, find the number in the last variety and also the total number of all the varieties of birds in the zoo.

www.drmathnp.com

24. तलका सङ्ख्याहरूको ढाँचाबाट $n^{\text{औं}}$ पद र पहिला n ओटा पदहरूको योगफल पत्ता लगाउनुहोस् ।
From the following patterns of numbers, find the n^{th} term and the sum of first n terms.

A.	1, 4, 16, ____
B.	____, ____
C.	____, ____

ANSWERS

A. SHORT QUESTIONS

1A. $\frac{1}{8}$ **1B.** $\frac{1}{384}$ **2A.** 28, 896 **2B.** 72, 17496 **3A.** -2 **3B.** 2 **3C.** 32

3D. 3 **3E.** 5 **3F.** 8 **3G.** 8 **4A.** $10\,(2)^{n-1}$ **4B.** $96\left(\frac{1}{2}\right)^{n-1}$ **5A.** 10

5B. 6 **6A.** -1 **6B.** 13 **6C.** 17 **6D.** 12 **7A.** ± 4 **7C.** 2 or -4

7D. 78 **8A.** 32 **8B.** 2187 **9A.** $60\frac{20}{27}$ or 60.74 **9B.** 87380 **9C.** $10\frac{127}{128}$ **10A.** 0, 16

10B. 3 **10C.** 32 **10D.** 0, 1, 6 **10E.** 7 **10F.** 7 **10G.** $\sqrt{2}$ **10H.** $\sqrt{2}$

11A. 1458 **11B.** 3645 **12A.** $\frac{1}{4}$ **12B.** $\frac{1}{5}$ **13A.** 2 **13B.** 4 **13C.** 8

14A. 3 **14B.** 3 **15A.** $2, -\frac{5}{2}$ **15B.** 2 **15C.** 3 **16A.** 6 **16B.** 6

17A. 6.5, 6 **17B.** 20.5, 20 **18A.** 4 **18B.** 15 **20A.** 1023 **20B.** 336 **20C.** 363

21A. 5 **21B.** 3 **22A.** 218700 **22B.** Rs 51200

B. LONG QUESTIONS

1A. 54 **1B.** 256 **2A.** 64 **2B.** 48 **2C.** -48 **2D.** $\frac{97}{4}$ **3A.** 6, 32

3B. 0 or 16, $\frac{-8}{3}$ or $\frac{8}{5}$ **4A.** 9 **4B.** 10 **5A.** $-\frac{1}{6}, \frac{1}{24}$ **5B.** 32, 16 **5C.** 1, -27

5D. 256, 64, 16, 4 **6A.** $\frac{1}{4}, \frac{1}{2}$, 2, 4, 8 **6B.** 8, 16, 32, 64;128 **6C.** 10, 20, 40, 80; 160

6D. 9, 27, 81, 243 **7A.** 127 **7B.** $63\frac{31}{32}$ **7C.** 63 **7D.** 3280 **8A.** $5, \frac{121}{9}$ **8B.** 7, 508

8C. 496 **9A.** 3, 2 **9B.** 2 **9C.** $\frac{16}{7}$ **9D.** $a = \frac{1}{9}, r = 3$ **9E.** $a = 1, r = 2$ **10A.** 2

10B. 2 **10C.** $3, \frac{1}{13}$ **11A.** $\frac{5}{4}$, 12 **11B.** 5 **12A.** ± 3 **12B.** ± 2

13A. 1, 3, 9, 27,..... **13B.** 3, 6, 12, 24, 48, ... **14A.** 2, 4, 8, ... **14B.** 1, 3, 9, 27, ... **14C.** $-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}$ or 4, -8, 16

14D. 3, -6, 12, -24 **15A.** 32 **15B.** 243 **16A.** 8, 12, 18 or 18, 12, 8 **16B.** $\frac{1}{3}$, 1, 3

16C. 1, 2, 4 or 4, 2, 1 **16D.** 1, 4, 16 **16E.** 2, 8, 32 **16F.** 8, 24, 72; 72, 24, 8 **16G.** 18, 6, 2, **or** 2, 6, 18

16H. 3, 6, 12 **17A.** 32, 44, 56, 68 **17B.** 9 : 1 or 1 : 9 **17C.** 16, 4 **17D.** 2 & 8 or 8 & 2 **17E.** 3 or 27; 27 or 3

18A. 2, 5, 8 **18B.** $\frac{8}{3}, \frac{13}{3}$, 6 or 6, $\frac{13}{3}, \frac{8}{3}$ **19A.** 10, 20, 40 or 40, 20, 10 **19B.** 5, 10, 20, ... or 20, 10, 5, ...

19C. 18, 6, 2 or 2, 6, 18 **20A.** 2, 4, 6, or 8, 4, 0 **20B.** 1, 3, 5 or 9, 3, -3 **20C.** 1, 5, 9 or, 9, 5, 1

20D. 3, 4, 5, or 12, 4, -4 **20E.** 12, 7, 2 or 3, 7, 11 **20F.** 3, 5, 7 or 15, 5, -5

20G. 4, 7, 10 or 19, 7, -5 **21A.** 38 **21B.** 57 **22A.** 558 cm **22B.** 372 cm

23A. 3, 384 **23B.** 2,4374, 6560 **23C.** 1024, 2046 **24A.** $t_n = 4^{n-1}$, $S_n = \frac{1}{3}(4^n - 1)$ **24B.** $t_n = 2^n$, $S_n = 2^{n+1} - 2$

24C. $t_n = 3^n$, $S_n = \frac{3}{2}(3^n - 1)$

CHALLENGING EXERCISE 3

1. एउटा गुणोत्तर अनुक्रमको $(m+n)^{\text{औं}}$ पद र $(m-n)^{\text{औं}}$ पद क्रमशः p र q भए यसको $m^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।
If $(m+n)^{\text{th}}$ term and $(m-n)^{\text{th}}$ term of a geometric sequence are p and q respectively, find the m^{th} term.

2. एउटा अनुक्रमको $n^{\text{औं}}$ पद, n मा आधारित रेखीय अभिव्यञ्जक भए सो अनुक्रम एउटा समानान्तरीय अनुक्रम हुन्छ र यो समानान्तरीय अनुक्रमको समान अन्तर n को गुणाङ्कसँग बराबर हुन्छ भनी प्रमाणित गर्नुहोस् ।
If n^{th} term of a sequence is a linear expression in n then, prove that, the sequence, so formed, is an arithmetic sequence and the common difference of this arithmetic sequence is equal to the coefficient of n .

3. एउटा AP को दोस्रो, $29^{\text{औं}}$ र अन्तिम पदहरू क्रमशः 7.75 , 1 र -6.5 छन् । सो श्रेणीमा कति ओटा पदहरू होलान् ?
The 2^{nd}, 29^{th} and last terms of an arithmetic series are 7.75, 1 and -6.5 respectively. How many terms are in the arithmetic sequence ?

puspas.com.np

www.drmathnp.com

pustaks.com.np

4. अङ्कहरू AP मा भएका तीन अङ्कको एउटा धनात्मक सङ्ख्यामा अङ्कहरूको योग 15 छ । अङ्कहरूलाई उल्टयाउदा बन्ने सङ्ख्या सुरुको सङ्ख्या भन्दा 594 कम भए सो सङ्ख्या पत्ता लगाउनुहोस् ।
The digits of positive integer, having three digits, are in AP and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

5. यदि x, y र z एउटा गुणोत्तर अनुक्रमका $p^{\text{औ}}$, $q^{\text{औ}}$ र $r^{\text{औ}}$ पदहरू हुन् भने $x^{q-r}.y^{r-p}.z^{p-q}=1$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If x, y and z be the p^{th}, q^{th} and r^{th} terms of a geometric sequence, prove that: $x^{q-r}.y^{r-p}.z^{p-q}=1$

6. a, b र c समानान्तरीय अनुक्रममा र x, y र z एउटा गुणोत्तर अनुक्रममा छन् भने प्रमाणित गर्नुहोस् : $x^{b-c}y^{c-a}z^{a-b}=1$
a , b and c are in an arithmetic sequence x , y , z are in a geometric sequence. Prove that: $x^{b-c}y^{c-a}z^{a-b}=1$

7. यदि a, b र c गुणोत्तर अनुक्रममा छन् र $a^{1/x}=b^{1/y}=c^{1/z}$ भए $2y=x+z$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If a, b, c are in geometric sequence and $a^{1/x}=b^{1/y}=c^{1/z}$ then show that : $2y=x+z$.

8. यदि एउटा GS को $4^{\text{औं}}$, $7^{\text{औं}}$ र $10^{\text{औं}}$ पदहरू क्रमशः x, y र z भए $y^2=zx$ हुन्छ भनी देखाउनुहोस् ।
If 4^{th}, 7^{th} and 10^{th} terms of a GS are x, y and z respectively, show that: $y^2=zx$

9. यदि a, b, c र d एउटा GP मा भए प्रमाणित गर्नुहोस् (If a, b, c and d are in a GP then show that):
$(b-c)^2+(c-a)^2+(d-b)^2=(a-d)^2$

10. यदि a, b र c एउटा GP मा भए प्रमाणित गर्नुहोस् (If a, b and c are in a GP then prove that):
$(a+2b+2c)(a-2b+2c)=a^2+4c^2$

11. एउटा गुणोत्तर अनुक्रममा रहेका 4 ओटा अनुवर्ती पदहरूको योग 30 छ । यदि पहिलो पद र अन्तिम पदको समानान्तरीय मध्यमा 9 भए समान अनुपात पत्ता लगाउनुहोस् ।
The sum of the four consecutive terms of a geometric sequence is 30. If the AM of the first term and last term is 9, find the common ratio.

12. एउटा गुणोत्तर अनुक्रममा 4 ओटा अनुवर्ती पदहरूको योग 15 र पहिलो पद र अन्तिम पदको समानान्तरीय मध्यमा 4.5 भए समान अनुपात पत्ता लगाउनुहोस् ।
The sum of four consecutive terms of a geometric sequence is 15 and the arithmetic mean of the first term and the last term is 4.5. Find the common ratio.

13. एउटा गुणोत्तर अनुक्रममा पहिला 6 ओटा पदहरूको योगफल पहिला तीन पदहरूको योगफलको 9 गुणा छ । समान अनुपात पत्ता लगाउनुहोस् ।
The sum of first 6 terms of a geometric sequence is 9 times the sum of first 3 terms. Find the common ratio.

14. दुई धनात्मक सङ्ख्याहरूको AM र GM को योगफल र अन्तर क्रमशः 80 र 20 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum and difference of AM and GM between two positive numbers are respectively 80 and 20. Find the numbers.

15. दुईओटा धनात्मक वास्तविक सङ्ख्याहरूको अनुपात $p+\sqrt{p^2-q^2}:p-\sqrt{p^2-q^2}$ भए तिनीहरूको AM र GM को अनुपात पत्ता लागउनुहोस् ।
The ratio of two positive real numbers is $p+\sqrt{p^2-q^2}:p-\sqrt{p^2-q^2}$ then find their raito of AM and GM.

16. दुईओटा AP का समान अन्तरहरू बराबर छन् । तिनीहरूको $100^{\text{औं}}$ पदहरूको अन्तर 100 छ । $1000^{\text{औं}}$ पदहरूको अन्तर कति होला ?
Two APs have the same common difference. The difference between their 100^{th} term is 100. What is the difference between their 1000^{th} terms ?

17. यदि एउटा AP को पहिलो पद 2 छ र पहिला 5 पदहरूको योगफल त्यसपछिका 5 पदहरूको योगफलको एक चौथाइ छ भने पहिला 30 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the first term of an AP is 2 and the sum of first five terms is equal to one fourth of the sum of the next five terms, find the sum of first 30 terms.

18. यदि दुईओटा सङ्ख्या a र b बिचको समानान्तरीय मध्यमा A र दुईओटा गुणोत्तर मध्यमाहरू p र q भए $\frac{p^2}{q}+\frac{q^2}{p}=2A$ हुन्छ भनी देखाउनुहोस् ।
If single arithmetic mean A and two geometric means p and q be inserted between two numbers a and b then show that: $\frac{p^2}{q}+\frac{q^2}{p}=2A$

19. दुईओटा समानान्तरीय अनुक्रमका n ओटा पदहरूको योगफलहरूको अनुपात $(7n+1):(4n+27)$ छन् । तिनीहरूको 11 औं पदहरूको अनुपात पत्ता लगाउनुहोस् ।
The ratio between the sum of n terms of two arithmetical progressions is $(7n+1):(4n+27)$. Find the ratio of their 11^{th} term.

Hint: $S_n=\frac{n}{2}[2a_1+(n-1)d_1]$ .....(i) and $S'_n=\frac{n}{2}[2a_2+(n-1)d_2]$ .....(ii)

$\therefore \frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}}=\frac{7n+1}{4n+27}$ (given) $\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}=\frac{7n+1}{4n+27}$ .....(iii)

www.drmathnp.com

Ratio of 11^{th} terms $= \frac{a_1+10d_1}{a_2+10d_2} = \frac{2a_1+20d_1}{2a_2+20d_2} = \frac{2a_1+(21-1)d_1}{2a_2+(21-1)d_2} = \frac{7\times21+1}{4\times21+27} = \frac{148}{111}$ [∵ Putting $n = 21$ in (iii)]

20. यदि a र b बिचको समानान्तरीय मध्यमा $\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$ भए n को मान पत्ता लगाउनुहोस् ।

If the arithmetic mean between a and b is $\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$ then find the value of n.

21. दुईओटा धनात्मक सङ्ख्याहरू a र b जहाँ $a > b$ को समानान्तरीय मध्यमा गुणोत्तर मध्यमाको दोब्बर भए $a : b = (2+\sqrt{3}) : (2-\sqrt{3})$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

The arithmetic mean between two positive numbers a and b where $a > b$ is twice their geometric mean. Prove that: $a : b = (2+\sqrt{3}) : (2-\sqrt{3})$

Hint: $AM = \frac{1}{2}(a+b)$ and $GM = \sqrt{ab}$ So, $\frac{1}{2}(a+b) = 2\sqrt{ab}$

or, $\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$ or, $\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3}{1}$ [By componendo and dividendo]

or, $\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \left(\frac{\sqrt{3}}{1}\right)^2$ or, $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1}$ or, $\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ [By componendo & dividendo]

or, $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ or, $\frac{a}{b} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

22. दुईओटा सङ्ख्याहरूको योगफल तिनीहरूको गुणोत्तर मध्यमाको 6 गुणा छ । ती सङ्ख्याहरू $(3+2\sqrt{2}) : (3-2\sqrt{2})$ को अनुपातमा छन् भनी प्रमाणित गर्नुहोस् ।

The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio of $(3+2\sqrt{2}) : (3-2\sqrt{2})$.

23. एउटा समानान्तरीय श्रेणीको $m^{औं}$ पद n^{-1} र $n^{औं}$ पद m^{-1} छन् भने उक्त श्रेणीको पहिलो $(mn)^{औं}$ पदहरूको योगफल $\frac{1}{2}(mn+1)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If m^{th} term is n^{-1} and n^{th} term is m^{-1} in an AP, prove that the sum of first (mn) terms of the series is $\frac{1}{2}(mn+1)$.

24. कुनै अङ्कगणितीय र गुणोत्तर श्रेणी दुवैका पहिला पदहरू 1 छन् र तिनीहरूका दोस्रा पदहरू पनि बराबर छन् । यदि अङ्कगणितीय श्रेणीको चौधौं पद गुणोत्तर श्रेणीको तेस्रो पदको तीन गुणा भए, अङ्कगणितीय श्रेणीको समान अन्तर निकाल्नुहोस् ।

In arithmetic and geometric series both have their first term 1 and their second terms are also equal. If the 14^{th} term of the arithmetic series is three times the 3^{rd} term of the geometric series, find the common difference of the arithmetic series.

25. दुईओटा सङ्ख्याहरू बिचको एउटा गुणोत्तर मध्यमा GM छ । यदि m र n दुईओटा समानान्तरीय मध्यमाहरू भए $GM^2 = (2m-1)(2n-m)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

GM be the geometric mean between two numbers. If m and n be the two arithmetic means between two numbers then prove that: $GM^2 = (2m-n)(2n-m)$

26. एउटा गुणोत्तर श्रेणीको n, $2n$ र $3n$ ओटा पदहरूको योग क्रमशः S_1, S_2 र S_3 भए $S_1(S_3-S_2) = (S_2-S_1)^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If S_1, S_2 and S_3 be respectively the sum of n, $2n$ and $3n$ terms of a GP, prove that $S_1(S_3-S_2) = (S_2-S_1)^2$.

Hint: Let a be the first term and r be the common ratio of the given GP. Then,

$S_1(S_3-S_2) = \frac{a(1-r^n)}{(1-r)} \cdot \left\{\frac{a(1-r^{3n})}{(1-r)} - \frac{a(1-r^{2n})}{(1-r)}\right\} = \frac{a(1-r^n)}{(1-r)} \cdot \frac{(a-ar^{3n}-a+ar^{2n})}{(1-r)} = \frac{a(1-r^n)}{(1-r)} \cdot \frac{ar^{2n}(1-r^n)}{(1-r)} = \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$

and $(S_2-S_1)^2 = \left\{\frac{a(1-r^{2n})}{(1-r)} - \frac{a(1-r^n)}{(1-r)}\right\}^2 = \frac{(a-ar^{2n}-a+ar^n)^2}{(1-r)^2} = \frac{\{ar^n(1-r^n)\}^2}{(1-r)^2} = \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$

Thus, $S_1(S_3-S_2) = (S_2-S_1)^2$ Proved.

27. एउटा AP का पहिला n, $2n$ र $3n$ पदहरूको योगफल क्रमशः S_1, S_2 र S_3 भए $S_3 = 3(S_2-S_1)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the sum of first n, $2n$, $3n$ terms of an AP be S_1, S_2, S_3 respectively then prove that: $S_3 = 3(S_2-S_1)$

28. एउटा गुणोत्तर श्रेणीमा चौथो पद र दसौं पदको गुणनफलसँग सातौं पदको वर्ग बराबर हुन्छ भनी प्रमाणित गर्नुहोस् ।

In a geometric series, prove that the square of seventh term is equal to the product of fourth term and tenth term.

ANSWERS

1. $\sqrt{pq}$	**3.** 59	**4.** 852	**11.** $2, \frac{1}{2}$	**12.** 2 or, $\frac{1}{2}$	**13.** 2	**14.** 90, 10
15. p : q	**16.** 100	**17.** – 2550	**19.** $\frac{148}{111}$	**20.** 0	**24.** 2 or $\frac{1}{3}$	

puspas.com.np

www.drmathnp.com

pusposa.com.np

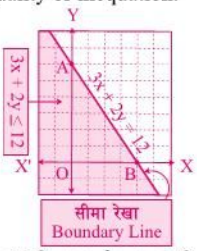
5. रेखीय योजना LINEAR PROGRAMMING

DEFINITIONS

1. **समीकरण (Equation):** बराबर (=) चिन्ह प्रयोग भएको गणितीय वाक्यलाई समीकरण भनिन्छ ।
A mathematical statement containing the equal sign (=) is called an equation.

2. **असमानता (Inequality):** चिन्हहरू ($<, >, \leq$ वा $\geq$) मध्ये कुनै एक चिन्ह प्रयोग भएको गणितीय वाक्यलाई असमानता भनिन्छ ।
A mathematical statement containing one of the signs ($<, >, \leq$ or $\geq$) is called an inequality or inequation.

3. **सीमा रेखा (Boundary line)**
समतल सतहलाई 2 भागमा विभाजन गर्ने तथा दिइएको असमानताको सङ्गति समीकरण जनाउने रेखालाई सीमा रेखा भनिन्छ ।
The line which is the corresponding equation of given inequality and it divides the co-ordinate graph (plane) into two half planes is called the boundary line.



4. **रेखीय असमानता (Linear inequality)**
रेखीय असमानता भन्नाले रेखीय सम्बन्धको स्वरूप $ax + by + c \geq 0$, $ax + by + c \leq 0$, $ax + by + c > 0$, $ax + by + c < 0$, (i.e. $ax + by + c \neq 0$) भन्ने बुझिन्छ ।
Linear inequality is a relation represented by $ax + by + c \geq 0$, $ax + by + c \leq 0$, $ax + by + c > 0$, $ax + by + c < 0$, (i.e. $ax + by + c \neq 0$).

5. **सर्तहरू (Constraints)**
उद्देश्य फलनमा भएका चलहरूद्वारा मान्य हुने अवस्थाहरूको समूहलाई सर्तहरू भनिन्छ । यी असमानतासँग सम्बन्धित अवस्थाहरू हुन् ।
The conditions to be satisfied by the variables of the objective function are said to be constraints. They are the conditions related to inequalities.

6. **हल समूह (Solution Set)**
दिइएको असमानतालाई मान्य हुने गरी उक्त असमानताले बनाएको अर्धसमतलमा पर्ने बिन्दुहरूको समूहलाई हल समूह भनिन्छ ।
The set of points lying on the half-plane determined by the given inequality is called solution set.

7. **हल क्षेत्र (Feasible region or convex polygonal region)**
दिइएका असमानताहरूसँग सम्बन्धित सीमा रेखाहरूद्वारा प्रतिच्छेदन गरी बनेको सरल बन्द बहुभुजलाई हल क्षेत्र भनिन्छ ।
A closed plane region bounded by the intersection of the finite number of boundary lines is called feasible region.

8. **रेखीय योजना (Linear Programming)**
रेखीय सर्तहरूको आधारमा रेखीय फलनको अधिकतम वा न्यूनतम मान पत्ता लगाउने गणितीय तरिकालाई रेखीय योजना भनिन्छ ।
Linear programming is the mathematical technique for finding the maximum or minimum value of a linear function subject to the set of linear constraints.

9. **उद्देश्य फलन (Objective Function)**
कुनै रेखीय फलनको मान निश्चित सर्तहरू मान्य हुने गरी अधिकतम वा न्यूनतम गरिन्छ । यसरी अधिकतम वा न्यूनतम मान निकाल्नुपर्ने रेखीय फलनलाई उद्देश्य फलन भनिन्छ ।
The linear function which is to be maximized or minimized with respect to given constraints is called objective function.

FORMULAE AND KEY POINTS

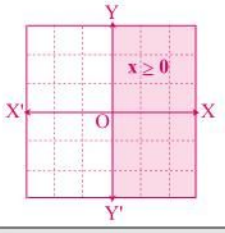
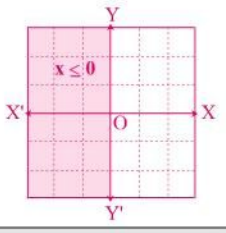
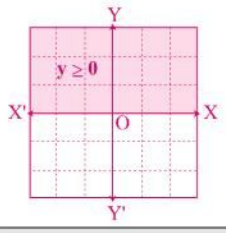
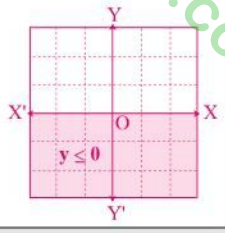
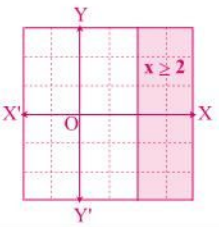
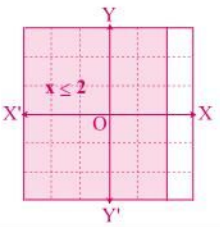
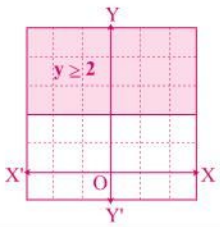
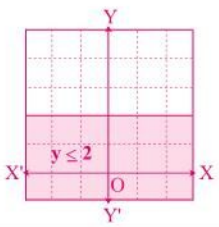
1. $ax + by \leq c$ र $ax + by \geq c$ को सीमारेखाको समीकरण $ax + by = c$ हुन्छ ।
The equation of boundary line of $ax + by \leq c$ and $ax + by \geq c$ is $ax + by = c$.

2. $ax + by \leq c$ र $ax + by \geq c$ को हल क्षेत्रहरू परीक्षण बिन्दु लिएर पहिचान गर्नुपर्दछ ।
The solution sets of $ax + by \leq c$ and $ax + by \geq c$ should be identified by taking a test point.

असमानताहरू (Inequalities)	हल क्षेत्र (Solution Set)
$x \geq 0$ and $y \geq 0$	पहिलो चतुर्थांश (First quadrant)
$x \leq 0$ and $y \geq 0$	दोस्रो चतुर्थांश (Second quadrant)
$x \leq 0$ and $y \leq 0$	तेस्रो चतुर्थांश (Third quadrant)
$x \geq 0$ and $y \leq 0$	चौथो चतुर्थांश (Fourth quadrant)
$x \geq k$	रेखा $x = k$ देखि धनात्मक दिशा (Positive direction from the line $x = k$)
$y \geq k$	रेखा $y = k$ देखि धनात्मक दिशा (Positive direction from the line $y = k$)
$x \leq k$	रेखा $x = k$ देखि ऋणात्मक दिशा (Negative direction from the line $x = k$)
$y \leq k$	रेखा $y = k$ देखि ऋणात्मक दिशा (Negative direction from the line $y = k$)

www.drmathnp.com

3. हल क्षेत्र (Solution Set)

$x \geq 0$ र $x \leq 0$ को लेखाचित्र (Graph of $x \geq 0$ and $x \leq 0$)		$y \geq 0$ र $y \leq 0$ को लेखाचित्र (Graph of $y \geq 0$ and $y \leq 0$)	
			
$x \geq 2$ र $x \leq 2$ को लेखाचित्र (Graph of $x \geq 2$ and $x \leq 2$)		$y \geq 2$ र $y \leq 2$ को लेखाचित्र (Graph of $y \geq 2$ and $y \leq 2$)	
			

5. उदाहरण (Example)

$Z = 3x + 4y$ को निम्नलिखित अवस्थामा हुने अधिकतम मान निकाल्नुहोस् : $x - y \leq 6, x - y \leq 4, x \geq 0$ र $y \geq 0$

Find the maximum value of $Z = 3x + 4y$ under the following constraints: $x - y \leq 6, x - y \leq 4, x \geq 0$ and $y \geq 0$

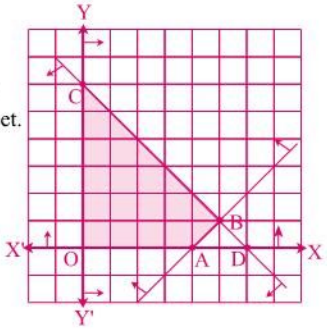
Solution: The boundary line of $x - y \leq 6$ is $x - y = 6$.

x	6	0
y	0	6

Let, (0, 0) be a test point. Then, $x + y \leq 6$ becomes $0 \leq 6$(True)
So, the half-plane containing the origin represents the solution set.
The boundary line of $x - y \leq 4$ is $x - y = 4$.

x	4	5
y	0	1

Let, (0, 0) be a test point. Then, $x - y \leq 4$ becomes $0 \leq 4$(True)
So, the half-plane containing the origin represents the solution set.
$x \geq 0$ and $y \geq 0$ represents the first quadrant.
OABC is the shaded and feasible region.



Vertices	Value of Z	Remarks
O(0, 0)	$3 \times 0 + 4 \times 0 = 0$	
A(4, 0)	$3 \times 4 + 4 \times 0 = 12$	
B(5, 1)	$3 \times 5 + 4 \times 1 = 19$	
C(0, 6)	$3 \times 0 + 4 \times 6 = 24$	Maximum

∴ The maximun value of Z is 24 at (0, 6).

SEE / SLC REVIEW MESSAGE

LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2065 E, 65 R', 2066 R, 2066 S, 2067 R, 2067 R', 2069 R, 2069 S, 2070 R', 2072 R, S, SEE 2074 R, SEE 2074 S, SEE 2075 R'_2, SEE MODEL 2076	14

SEE EXERCISE 5

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
   - **A.** समीकरण (Equation)
   - **B.** असमानता (Inequality)
   - **C.** सीमा रेखा (Boundary line)
   - **D.** बहुभुजीय सम्भाव्य क्षेत्र (Polygonal feasible region)
   - **E.** सर्तहरू (Constraints)
   - **F.** रेखीय योजना (Linear programming)
   - **G.** उद्देश्य फलन (Objective function)
   - **H.** रेखीय असमानता प्रणाली (System of linear inequalities)

puspas.com.np

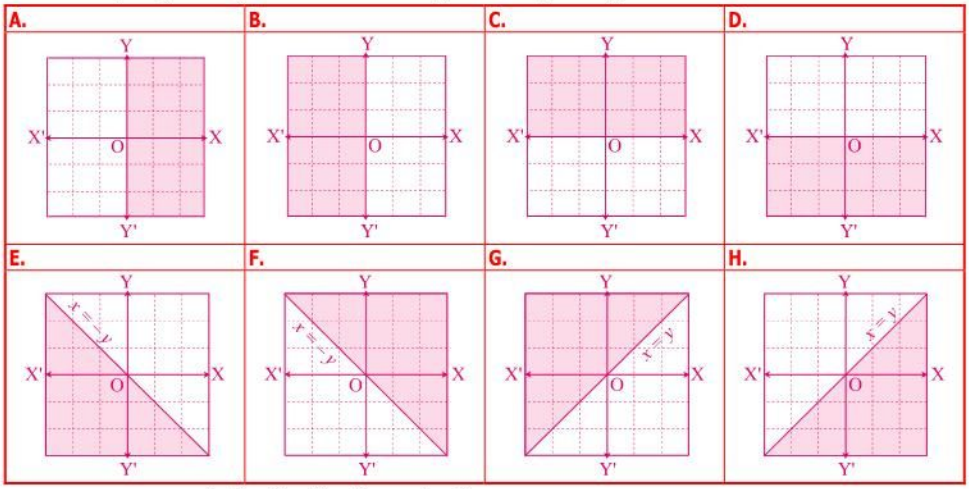
www.drmathnp.com

puspas.com.np

2A. लेखाचित्रमा असमानता $x \geq 0$ ले कुन कुन चतुर्थांशहरू जनाउँदछ ?
Which quadrants do the inequality $x \geq 0$ represent in a graph?

2B. लेखाचित्रमा असमानता $y \geq 0$ ले कुन कुन चतुर्थांशहरू जनाउँदछ ?
Which quadrants do the inequality $y \geq 0$ represent in a graph?

3. दिइएको चित्रमा छाँया परेको भागले जनाउने असमानता पत्ता लगाउनुहोस् ।
Find the inequality that represents the shaded region in the given figure.

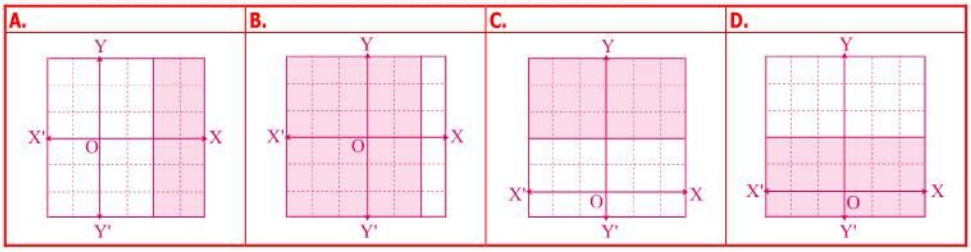


4A. असमानता $ax + by \leq c$ को सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $ax + by \leq c$.

4B. असमानता $ax + by \geq c$ को सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $ax + by \geq c$.

Model 2 *Concept Recall*

5. दिइएको चित्रमा छाँया परेको भागले जनाउने असमानता पत्ता लगाउनुहोस् ।
Find the inequality that represents the shaded region in the given figure.



6A. असमानता $2x + 3y \leq 6$ को सङ्गति सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $2x + 3y \leq 6$.

6B. असमानता $2x + y > 10$ को सङ्गति सीमा रेखाको समीकरण लेख्नुहोस् ।
Write the equation of boundary line of the inequality $2x + y > 10$.

7A. $x \geq 2$ को हल क्षेत्र रेखा $x = 2$ बाट कता पट्टी पर्छ ? (In which side from the line $x = 2$ is the solution region of $x \geq 2$ lie?)

7B. $y \leq 5$ को हल क्षेत्र रेखा $y = 5$ बाट कता पट्टी पर्छ ? (In which side from the line $y = 5$ is the solution region of $y \leq 5$ lie?)

7C. $x \leq 3$ को हल क्षेत्र रेखा $x = 3$ बाट कता पट्टी पर्छ ? (In which side from the line $x = 3$ is the solution region of $x \leq 3$ lie?)

8A. के असमानता $2x + 3y \geq 6$ को हल समूहमा (2, 3) पर्दछ ? (Does (2, 3) lie on the solution set of inequality $2x + 3y \geq 6$?)

8B. के असमानता $3x + 2y \geq 8$ को हल समूहमा (0, 0) पर्दछ ? (Does (0, 0) lie on the solution set of inequality $3x + 2y \geq 8$?)

8C. के असमानता $4x + 5y \leq 20$ को हल समूहमा (1, 0) पर्दछ ? (Does (1, 0) lie on the solution set of inequality $4x + 5y \leq 20$?)

9A. असमानता $4x + 3y \geq 12$ मा बिन्दु (4, 5) पर्छ कि पर्दैन, परीक्षण गर्नुहोस् ।
Test whether the point (4, 5) lies in the solution set of the inequality $4x + 3y \geq 12$ or not.

9B. असमानता $4x + 5y \geq 20$ मा बिन्दु (7, 4) पर्छ कि पर्दैन, परीक्षण गर्नुहोस् ।
Test whether the point (7, 4) lies in the solution set of the inequality $4x + 5y \geq 20$ or not.

www.drmathnp.com

9C. असमानता $2x + 3y \leq 10$ मा बिन्दु (1, 3) पर्छ कि पर्दैन, परीक्षण गर्नुहोस् ।
Test whether the point (1, 3) lies in the solution set of the inequality $2x + 3y \leq 10$ or not.

10A. सीमा रेखाको समीकरण $2x + 3y = 6$ र हल समूहमा बिन्दु (0, 0) भएको असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $2x + 3y = 6$ and the solution set contains (0, 0).

10B. सीमा रेखाको समीकरण $7x + 5y = 20$ र हल समूहमा बिन्दु (0, 0) नहुने असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $7x + 5y = 20$ and the solution set does not contain (0, 0).

10C. सीमा रेखाको समीकरण $4x + 3y = 12$ र हल समूहमा बिन्दु (1, 0) भएको असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $4x + 3y = 12$ and the solution set contains (1, 0).

10D. सीमा रेखाको समीकरण $4x + 3y = 12$ र हल समूहमा बिन्दु (1, 2) नहुने असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $4x + 3y = 12$ and the solution set does not contain (1, 2).

11A. रेखा $x = 1$ सहित दाँयातिर हल समूह पर्ने असमानता लेख्नुहोस् ।
Write the inequality whose solution is to the right side of the line $x = 1$ including itself.

11B. रेखा $x = 2$ सहित बाँयातिर हल समूह पर्ने असमानता लेख्नुहोस् ।
Write the inequality whose solution is to the left side of the line $x = 2$ including itself.

11C. रेखा $y = 4$ सहित तलतिर हल समूह पर्ने असमानता लेख्नुहोस् ।
Write the inequality whose solution is downward from the line $y = 4$ including itself.

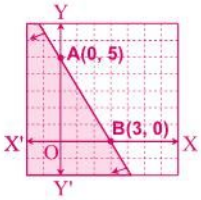
11D. रेखा $y + 3 = 0$ सहित माथितिर हल समूह पर्ने असमानता लेख्नुहोस् ।
Write the inequality whose solution is upward from the line $y + 3 = 0$ including itself.

B. SHORT QUESTIONS

Model 1 *Finding inequalities from the graph.*

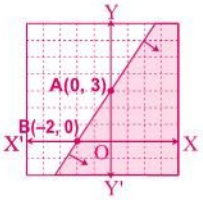
1. दिइएको चित्रमा छाया पारेको भाग जनाउने असमानता पत्ता लगाउनुहोस्:
Find the inequality that represents the shaded region in the given figure:

A.



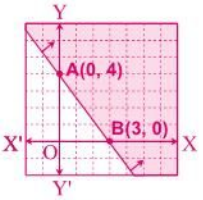
[SLC 2063 M]

B.



[SLC 2063 R]

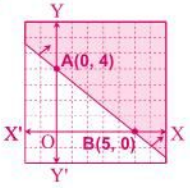
C.



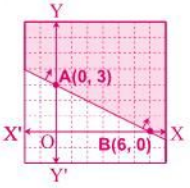
[SLC 2063 R']

2. तलका लेखाचित्रहरूमा छाया पारिएको भागले प्रतिनिधित्व गरेको सम्बन्धित असमानताहरू लेख्नुहोस् ।
Write down the corresponding inequalities represented by the shaded region in the following graphs.

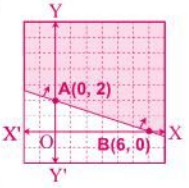
A.



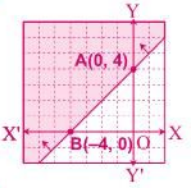
B.



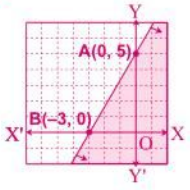
C.



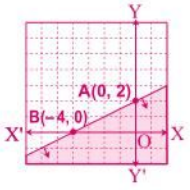
D.



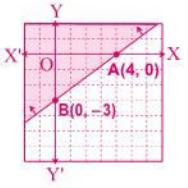
E.



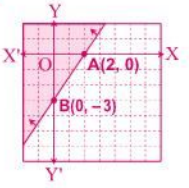
F.



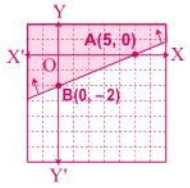
G.



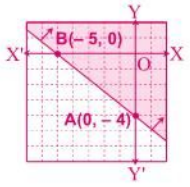
H.



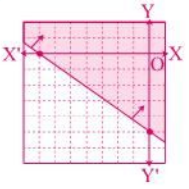
I.



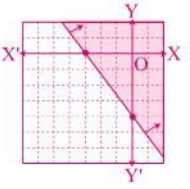
J.



K.



L.



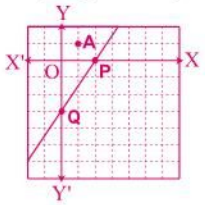
puspas.com.np

www.drmathnp.com

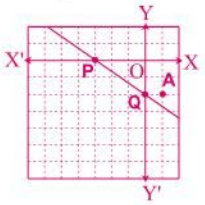
puspas.com.np

3. रेखा PQ लाई प्रसङ्गको रेखामानी बिन्दु A सन्तुलन गर्ने अर्धसमतलको असमानता पत्ता लगाउनुहोस्:
Find the inequality of half-plane that satisfies the point A with respect to the line PQ:

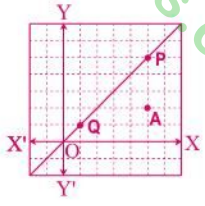
A.



B.

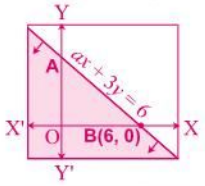


C.

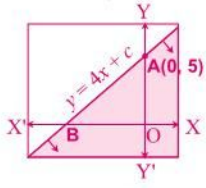


4. चित्रमा छाया पारिएको भागले जनाउने असमानता लेख्नुहोस्:
Write down the inequality represented by the shaded region in the figure:

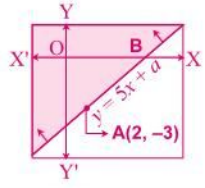
A.



B.



C.



Model 2 *Problems based on drawing graph.*

5. तलका असमानताहरूलाई लेखाचित्रमा देखाउनुहोस् । (Show the following inequalities on a graph.)

A.	$-x+y\le 4$	**B.**	$-5x+3y\le 15$	**C.**	$-x+2y\le 4$
D.	$3x-4y\le 12$	**E.**	$-3x+2y\ge 6$	**F.**	$2x-5y\le 10$
G.	$4x+5y\ge -20$	**H.**	$5x+7y\ge -35$	**I.**	$4x+3y\ge -12$

6. तलका असमानताहरूलाई लेखाचित्रमा देखाउनुहोस् । (Show the following inequalities in a graph.)

A.	$4(x-2)+5(y-3)\le 15$	**B.**	$5x-11\le 7x-5<9$	**C.**	$\frac{5x-8}{3}\ge\frac{4x-7}{2}$
D.	$(2x-1)\ge\frac{1}{2}(x+1)$	**E.**	$\frac{2x+1}{3}\ge\frac{3x-1}{2}$	**F.**	$\frac{5x}{4}-\frac{4x-1}{3}>1$
G.	$\frac{1}{4}\left(\frac{2}{3}x+1\right)\ge\frac{1}{3}(x-2)$	**H.**	$3x-2>x+\frac{4-x}{3}$	**I.**	$\frac{1}{3}(x+y)-\frac{1}{2}x\le 1$
J.	$\frac{1}{2}(x-y)+\frac{1}{4}y\le 16$	**K.**	$\frac{1}{4}(x+y)+\frac{1}{3}x\le 12$	**L.**	$\frac{4}{5}(x+y)+\frac{1}{5}(x-y)\le 2$

Model 3 *Problems based on finding value of letters.*

7A. असमानता $ax+2y\le 6$ ले बिन्दु (4, 5) लाई सन्तुलन गर्दछ भने a को सम्भाव्य मान पत्ता लगाउनुहोस् ।
If the inequality $ax+2y\le 6$ satisfies the point (4, 5) then find the possible value of a.

7B. असमानता $px+5y\ge 25$ को हल समूहमा बिन्दु (2, 3) पर्दछ भने p को सम्भाव्य मान पत्ता लगाउनुहोस् ।
If the point (2, 3) lies in the solution set of $px+5y\ge 25$, find the possible value of p.

7C. असमानता $4x+my<16$ को हल समूहमा बिन्दु (– 2, 3) पर्दछ भने m को सम्भाव्य मान पत्ता लगाउनुहोस् ।
If the point (– 2, 3) lies in the solution set of $4x+my<16$, find the possible value of m.

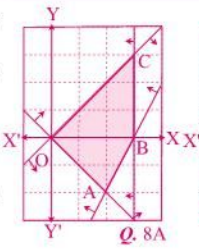
7D. असमानता $\frac{x}{4}+\frac{y}{k}>4$ को हल समूहमा बिन्दु (3, 4) पर्दछ भने k को सम्भाव्य मान पत्ता लगाउनुहोस् ।

If the point (3, 4) lies in the solution set of $\frac{x}{4}+\frac{y}{k}>4$, find the possible value of k.

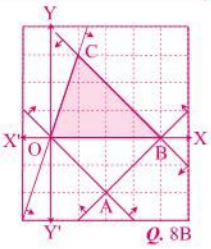
Model 4 *Problems based on finding maximum and minimum value.*

8A. रङ्ग्याइएको बहुभुजीय क्षेत्रको आधारमा उद्देश्य फलन P = $2x+3y$ को अधिकतम् र न्यूनतम् मानहरू पत्ता लगाउनुहोस्:
Find the maximum and minimum values of the objective function P = $2x+3y$ over the given shaded polygonal region.

8B. उद्देश्य फलन P = $3x-y+2$ को अधिकतम् मान छाया पारिएको क्षेत्रबाट निकाल्नुहोस् ।
Find the maximum value of the objective function P = $3x-y+2$ over the shaded region.



Q. 8A



Q. 8B

www.drmathnp.com

9A. उद्देश्य फलन $P = 2x + 3y$ को बहुभुजीय सम्भाव्य क्षेत्रका शीर्षहरू A(2, 0), O(0, 0) र C(3, 0) का आधारमा अधिकतम् मान पत्ता लगाउनुहोस् ।
Find the maximum value of the objective function $P = 2x + 3y$ when the vertices of a polygonal feasible region are A(2, 0), O(0, 0) and C(3, 0).

9B. उद्देश्य फलन $P = 4x + 5y$ को बहुभुजीय सम्भाव्य क्षेत्रका शीर्षहरू A(5, 3), B(4, 5) र O(0, 0) का आधारमा अधिकतम् मान पत्ता लगाउनुहोस् ।
Find the maximum value of the objective function $P = 4x + 5y$ when the vertices of a polygonal feasible region are A(5, 3), B(4, 5) and O(0, 0).

9C. उद्देश्य फलन $L = 2x + 4y$ को बहुभुजीय सम्भाव्य क्षेत्रका शीर्षहरू S(0, 3), L(4, 0) र O(0, 0) का आधारमा न्यूनतम् मान पत्ता लगाउनुहोस् ।
Find the minimum value of the objective function $L = 2x + 4y$ when the vertices of a polygonal feasible region are S(0, 3), L(4, 0) and O(0, 0).

9D. उद्देश्य फलन $Z = 7x + 3y$ को बहुभुजीय सम्भाव्य क्षेत्रका शीर्षहरू R(7, 0), A(0, 7) र O(0, 0) का आधारमा न्यूनतम् मान पत्ता लगाउनुहोस् ।
Find the minimum value of the objective function $Z = 7x + 3y$ when the vertices of a polygonal feasible region are R(7, 0), A(0, 7) and O(0, 0).

C. LONG QUESTIONS

Model 1 *On finding the maximum value of objective function under given constraints .*

उद्देश्य फलनको निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।
Maximize the objective function under the following constraints.

	शर्तहरू (Constraints)		उद्देश्य फलन (Objective function)	
1A.	$x + y \le 6, x - y \ge 2$	$x \ge 0, y \ge 0$	$P = 3x + 5y$	[2072 S]
1B.	$x + y \le 6, x - y \ge -2,$	$x \ge 0, y \ge 0$	$P = 6x + 5y$	[2057 R, 2065 E]
1C.	$x + y \le 7, x - y \ge -1,$	$x \ge 0, y \ge 0$	$P = 3x + 4y$	[2059 S]
1D.	$x + y \le 6, x - y \le 4,$	$x \ge 0, y \ge 0$	$P(x, y) = 3x + 5y$	[2062 K]
1E.	$x + y \le 6, x - y \le 4,$	$x \ge 0, y \ge 0$	$Z = 3x + 4y$	[2066 R]
1F.	$x + y \ge 6, x - y \ge 4,$	$x \ge 0, y \ge 0$	$P = 3x + 2y$	[2074 S]
1G.	$x + y \le 6, x - y \le 4$	$x \ge 0, y \ge 0$	$Z = 10x + 12y$	[2075 R'$_2$]
2A.	$x + y \le 7, x + 2y \le 10$	$x \ge 0, y \ge 0$	$P = 7x + 3y$	[2072 R]
2B.	$x + 2y \le 7, x - y \le 4,$	$x \ge 0, y \ge 0$	$P = 9x + 7y$	[2069 S]
2C.	$x + 2y \le 7, x - y \ge 4,$	$x \ge 0, y \ge 0$	$P = 9x + 7y$	[2057 S]
2D.	$x - 2y \le 1, x + y \le 4,$	$x \ge 0, y \ge 0$	$F = 3x + 5y$	[2067 R]
2E.	$x + 2y \ge 1, x + y \le 5,$	$x \ge 0, y \ge 0$	$P = 3x + 2y + 5$	[2064 R]
2F.	$x + y \le 7, x + 2y \le 10,$	$x \ge 0, y \ge 0$	$Z = 8x + 3y$	[2063 S]
2G.	$2y \ge x - 1, \quad x + y \le 4;$	$x \ge 0, y \ge 0$	$P = 3x + y$	[2058 R]
2H.	$2x + y \le 5, y - x \ge 1,$	$x \ge 0, y \ge 0$	$P = 2x + 5y$	[2060 C]
3A.	$x + 2y \le 10, 2x + y \le 14,$	$x \ge 0, y \ge 0$	$P = 2x + 3y$	[2063 R]
3B.	$2x + y \le 8, 2x + 3y \le 12,$	$x \ge 0, y \ge 0$	$P = x + 2y$	[2063 M]
3C.	$2x + y \le 20, 2x + 3y \le 24,$	$x \ge 0, y \ge 0$	$P = 5x + 3y$	[2065 M]
3D.	$2x + 5y \le 16, 2x + y \le 8,$	$x \ge 0, y \ge 0$	$P = 5x + 4y$	[2066 S]
3E.	$2x + y \ge 5, 3x + y \le 7$	$x \ge 0, y \ge 0$	$Q = 5x + 4y$	[2067 R']
3F.	$3x + 2y \le 12, 7x + 5y \le 28,$	$x \ge 0, y \ge 0$	$P = 14x + 16y$	[2069 R
3G.	$x + 2y \le 6, 3x + 2y \le 12,$	$x \ge 0, y \ge 0$	$P = 5x + 6y$	[2061 S]
3H.	$x + 2y \le 12, 3x + 2y \le 24,$	$x \ge 0, y \ge 0$	$P = 10x + 12y$	[2060 R]
3I.	$2x + y \le 8, 2x + 4y \le 14$	$x \ge 0, y \ge 0$	$Z = 6x + 5y$	[2070 R']
4A.	$x + y \ge 0, x - y \le 0,$	$y \le 2, x \ge -1$	$P = 3x + 2y$	[2060 S', SEE 2074 R]

pusnas.com.np

www.drmathnp.com

puspas.com.np

4B. $2x + 3y \geq 6, 2x - 3y \leq 6,$ $x \geq 0, y \leq 2$ $P = 4x + y$ [2063 R']

4C. $x + 2y \leq 6, x - 2y \leq 2,$ $x \geq -2$ $Z = 5x + 3y$ [2065 R']

4D. $5x + 2y \leq 45, 4x + 5y \leq 53,$ $x \geq 2, y \geq 0$ $Z = -2x + 5y$ [2064 R']

Model 2 *On finding the minimum value of objective function under given constraints .*

5A. $Z = 2x - 3y$ को निम्नलिखित अवस्थामा हुने न्यूनतम मान निकाल्नुहोस् : $x + y \geq 0, x - y \leq 0, x \geq -1$ and $y \leq 2$
Minimize $Z = 2x - 3y$, under the following constraints: $x + y \geq 0, x - y \leq 0, x \geq -1$ and $y \leq 2$ [2064 S]

5B. $Z = 4x + 5y + 20$ को निम्नलिखित अवस्थामा हुने न्यूनतम मान निकाल्नुहोस् : $4 \geq y - x \geq 1$ र $1 \leq x \leq 6$
Minimize $Z = 4x + 5y + 20$, under the following constraints: $4 \geq y - x \geq 1$ and $1 \leq x \leq 6$

Model 3 *On finding the maximum/minimum value of objective function.*

तलका शर्तहरूको आधारमा उद्देश्य फलन P को अधिकतम र न्यूनतम मानहरू पत्ता लगाउनुहोस् :
Find the maximum and minimum values of the objective function P under the constraints:

6A. $x - 2y \leq 1, x + y \leq 4, x \geq 0, y \geq 0$ $P = 5x + 4y$ [SEE MODEL 2076]

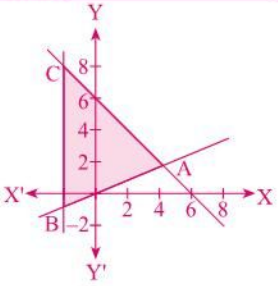
6B. $3x + 5y \leq 24, 0, \leq y \leq 4$ and $0 \leq x \leq 7$ $P(x, y) = 16x - 2y$

6C. $2x + 3y \geq 6, 2x - 3y \leq 6$ and $y \leq 2$ $P(x, y) = 4x - y$

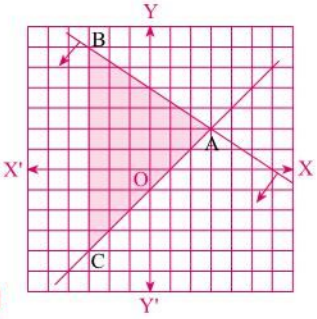
6D. $x + y \geq 6, x - y \geq 4$ and $x \leq 6$ $P(x, y) = 3y + 2x$

Model 4 *On finding inequalities and maximum/minimum value from the given graph .*

7A. दिइएको चित्रमा B र C को निर्देशाङ्क क्रमशः (–2, –1) र (–2, 8) छन् । त्रिभुज ABC भित्र छाया परेको भाग तीनओटा असमान समीकरणले जनाएको छ । तिनीहरूमध्ये एउटा $x + y \leq 6$ छ भने A को निर्देशाङ्क र अरू दुई असमान समीकरण पत्ता लगाउनुहोस् । तीनओटै असमान समीकरणलाई मान्य हुने मानबाट $x + 2y$ को अधिकतम मान निकाल्नुहोस् ।
In the given diagram, the coordinates of B and C are (–2, –1) and (–2, 8) respectively. The shaded region inside the triangle ABC is represented by three inequalities, one of these is $x + y \leq 6$. Write down the co-ordinates of A and other two inequalities. Also calculate the maximum value of $x + 2y$ from the values which satisfy all three inequalities. [2059 R]



7B. दिइएको चित्रमा A, B र C का निर्देशाङ्कहरू क्रमशः (3, 2), (–3, 6) र (– 3, – 4) छन् । ABC भित्र छाया परेको भाग तीनओटा असमान समीकरणहरूले जनाएको छ । ती तीन असमान समीकरणहरू पत्ता लगाउनुहोस् र ती तीन असमान समीकरणहरूलाई मान्य हुने मानबाट $4x - 5y$ को न्यूनतम मान पनि निकाल्नुहोस् ।
In the given diagram, the co-ordinates of A, B and C are (3, 2), (– 3, 6) and (– 3, – 4) respectively. The shaded region inside the ΔABC is represented by three inequalities. Write down the equations of these three inequalities and also calculate the minimum value of $4x - 5y$ from the values which satisfy all the three inequalities. [2062 R]



ANSWERS

A. VERY SHORT QUESTIONS

2A. 1st and 4th **2B.** 1st and 2nd **3A.** $x \geq 0$ **3B.** $x \leq 0$ **3C.** $y \geq 0$ **3D.** $y \leq 0$ **3E.** $x \leq -y$
3F. $x \geq -y$ **3G.** $x \leq y$ **3H.** $x \geq y$ **4A.** $ax + by = c$ **4B.** $ax + by = c$ **5A.** $x \geq 2$ **5B.** $x \leq 2$
5C. $y \geq 2$ **5D.** $y \leq 2$ **6A.** $2x + 3y = 6$ **6B.** $2x + y = 10$ **7A.** right side **7B.** downward
7C. Left side **8A** Yes **8B.** No **8C.** Yes **9A.** Yes **9B.** Yes **9C.** No
10A. $2x + 3y \leq 6$ **10B.** $7x + 5y \geq 20$ **10C.** $4x + 3y \leq 12$ **10D.** $4x + 3y \geq 12$ **11A.** $x \geq 1$ **11B.** $x \leq 2$ **11C.** $y \leq 4$
11D. $y \geq -3$

B. SHORT QUESTIONS

1A. $5x + 3y \leq 15$ **1B.** $2y - 3x \leq 6$ **1C.** $4x + 3y \geq 12$ **2A.** $4x + 5y \geq 20$ **2B.** $x + 2y \geq 6$ **2C.** $x + 3y \geq 6$ **2D.** $x - y \leq -4$
2E. $5x - 3y \geq -15$ **2F.** $x - 2y \geq -4$ **2G.** $3x - 4y \leq 12$ **2H.** $3x - 2y \leq 6$ **2I.** $2x - 5y \leq 10$ **2J.** $4x + 5y \geq -20$
2K. $5x + 7y \geq -35$ **2L.** $4x + 3y \geq -12$ **3A.** $3x - 2y \leq 6$ **3B.** $2x + 3y \geq -6$ **3C.** $x \geq y$
4A. $x + 3y \leq 6$ **4B.** $y \leq 4x + 5$ **4C.** $y \geq 5x - 13$ **7A.** $a \leq -1$ **7B.** $p \geq 5$ **7C.** $m < 8$ **7D.** $k < \frac{16}{13}$
8A. 15, –2 **8B.** 14 at (4, 0) **9A.** 6 **9B.** 41 **9C.** 0 **9D.** 0

www.drmathnp.com

C. LONG QUESTIONS

1B. 22 at (4, 2) **1C.** 36 at (6, 0) **1D.** 25 at (3, 4) **1E.** 30 at (0, 6) **1F.** 24 at (0, 6)
1G. $P_{max} = 18$ at (6, 0) **1H.** 72 at (0, 6) **2A.** 49 at (7, 0) **2B.** 52 at (5, 1) **2C.** 63 at (7, 0)
2D. 20 at (0, 4) **2E.** 20 at B(5, 0) **2F.** 35 at (7, 0) **2G.** 10 at (3, 1)
2H. 25 at (0, 5) **3A.** 18 at (6, 2) **3B.** 8 at (0, 4) **3C.** 51 at (9, 2) **3D.** 23 at (3, 2)
3E. 28 at (0, 7) **3F.** 89.6 at (0, 5.6) **3G.** 24 at $\left(3, 1\frac{1}{2}\right)$ **3H.** 96 at (6, 3) **3I.** 28 at (3, 2)
4A. 10 at (2, 2) **4B.** 26 at (6, 2) **4C.** 23 at (4, 1) **4D.** 41 at (2, 9) **5A.** -8 at $(-1, 2)$
5B. 34 at (1, 2) **6A.** Max^m.19 at (3, 1), Min^m. 0 at (0, 0) **6B.** Max^m. 112 at (7, 0), Min^m. -8 at (0, 4)
6C. Max^m. 22 at (6, 2), Min^m.-2 at (0, 2) **6D.** Max^m. 18 at (6, 2), Min^m. 0 at (0, 0)
7A. A(4, 2), $x \geq -2$, $x \leq 2y$; 14 at $(-2, 8)$ **7B.** $2x + 3y \leq 12$, $x - y \leq 1$, $x \geq -3$; -42 at $(-3, 6)$

GENERAL EXERCISE 5

LONG QUESTIONS

Model 1 *On finding the maximum value of objective function under given constraints .*

उद्देश्य फलन 'P' को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् :
Maximize the objective function 'P' under the following constraints :

1A. $3x + 4y \leq 24$, $0 \leq y \leq 3$ and $0 \leq x \leq 4$; $P(x, y) = 8x - y + 20$

1B. $3x + 4y \leq 24$, $0 \leq y \leq 4$ and $0 \leq x \leq 5$; $P(x, y) = 16x - 2y + 40$

1C. $2x + y \geq 8$, $y \leq 2x$ and $x \leq 4$; $P(x, y) = 5x + 3y$

1D. $x + 2y \geq 5$, $y \leq 2x$ and $x \leq 5$; $P(x, y) = 6x + 2y$

1E. $2x - y \geq -4$, $x + y \geq -2$ and $4x + y \leq 4$; $P(x, y) = 7x + y$

1F. $x + y \leq 6$, $x - y \leq 4$, $x \geq 6$ and $y \geq 0$ $P = 10x + 12y$

1G. $2x - 3y \leq 4$, $2x + 3y \geq 6$ and $y \leq 2$ $P = 5x - 3y$

2A. $2x \leq (16 - 5y) = (8 - y)$, $x \geq 0$ and $y \geq 0$; $P = 5x + 4y$

2B. $x \leq (7 - 2y) = (y + 4)$, $x \geq 0$ and $y \geq 0$; $P = 9x + 7y$

2C. $1 \leq y - x \leq 4$ and $1 \leq x \leq 6$; $P = 4x + 5y$

Model 2 *On finding the minimum value of objective function under given constraints .*

उद्देश्य फलन 'Z' को निम्नलिखित अवस्थामा न्यूनतम मान निकाल्नुहोस् :
Minimize the objective function 'Z' under the following constraints :

3A. $x + y \leq 4$, $x \geq 0$ and $y \geq 0$; $Z = x + y + 2$

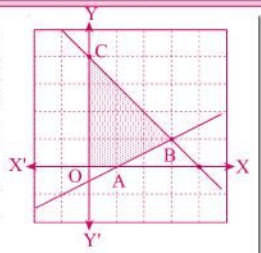
3B. $x + y \geq 6$, $x - y \geq 4$, $x \geq 6$, $x \geq 0$ and $y \geq 0$; $Z = 2x + y$

3C. $x + y \leq 6$, $x - y \leq 4$, $x \leq 6$, $x \geq 0$ and $y \geq 0$; $Z = 2x + 3y - 7$

3D. $y - 2x \leq 0$, $2y \geq 5$ and $x \leq 5$; $Z = 3x + 5y$

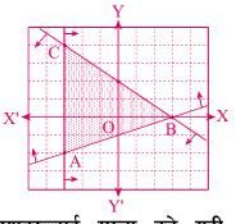
Model 3 *On finding inequalities and maximum/minimum value from the given graph .*

4A. संगैको लेखाचित्रमा A, B र C कानिर्देशाङ्कहरू क्रमशः (1, 0), (3, 1) र (0, 4) छन् । यदि छाया पारिएको भागलाई जनाउने एउटा असमानता $x + y \leq 4$ भए बाँकी अन्य असमानताहरू पत्ता लगाउनुहोस् र $P = 2x + 5y$ को अधिकतम् मान समेत पत्ता लगाउनुहोस् ।



In the adjoining graph, the co-ordinates of A, B and C are (1, 0), (3, 1) and (0, 4) respectively. If one of the inequalities represented by shaded region is $x + y \leq 4$, find the other remaining inequalities. Calculate the maximum value of $P = 2x + 5y$.

4B. दिइएको चित्रमा A, B र C का निर्देशाङ्कहरू क्रमशः (–3, –2), (3, 0) र (– 3, 4) छन् । छाया पारिएको भागलाई जनाउने तीनओटा असमानताहरू पत्ता लगाउनुहोस् र ती तीन असमान समीकरणहरूलाई मान्य हुने गरी $2x + 3y$ को अधिकतम् मान पनि पत्ता लगाउनुहोस् ।



In the figure, the co-ordinates of A, B and C are (– 3, – 2), (3, 0) and (– 3, 4) respectively. Find the three inequalities represented by the shaded region and also calculate the maximum value of $2x + 3y$ which satisfy all three inequalities.

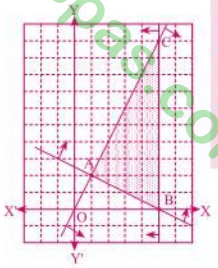
puspas.com.np

www.drmathnp.com

puspas.com.np

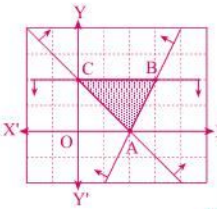
4C. ΔABC भित्र छाया पारिएको भागले तीनओटा असमानताहरू जनाउँदछ । ती असमानताहरू लेख्नुहोस् । साथै तीनओटै असमानताहरूलाई सन्तुलन गर्ने मानहरूबाट $3x + 5y$ को अधिकतम मान पनि पत्ता लगाउनुहोस् ।

The shaded region inside the ΔABC is represented by three inequalities. Write down the inequalities. Also calculate the maximum value of $3x + 5y$ from the values which satisfy all three inequalities.



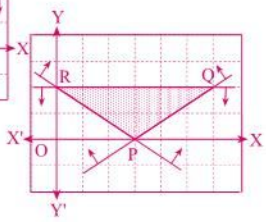
4D. सँगैको चित्रबाट छाया पारिएको भागले जनाउने तीनओटा असमानताहरू र $4x + 9y$ को अधिकतम् मान पत्ता लगाउनुहोस् ।

From the given figure, find the inequalities represented by the shaded region and calculate the maximum value of $4x + 9y$.



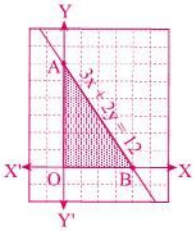
4E. चित्रको छायाङ्कित क्षेत्र जनाउने असमानता (सर्त) हरू लेख्नुहोस् र $P = 4x - y$ को अधिकतम मान निकाल्नुहोस् ।

Write down the system of inequalities that describes the shaded region and find the maximum value of $P = 4x - y$.



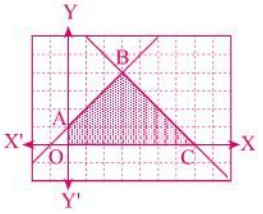
4F. सँगैको चित्रमा बहुभुजीय क्षेत्र हल क्षेत्र हुने असमानताहरू पत्ता लगाउनुहोस् र $P = 10x + 8y$ को अधिकतम मान पत्ता लगाउनुहोस् ।

In the figure, find the inequalities which represent the shaded region as solution set and find the maximum value of $P = 10x + 8y$.



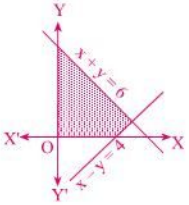
4G. सँगैको चित्रमा बहुभुजीय क्षेत्र हल क्षेत्र हुने असमानताहरू पत्ता लगाउनुहोस् ।

In the figure, find the inequalities which represent the shaded region as solution set.



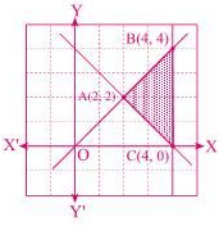
4H. चित्रको छायाङ्कित क्षेत्र जनाउने असमानताहरू लेख्नुहोस् र $P = 2x + y - 3$ को अधिकतम मान निकाल्नुहोस् ।

Write down the system of inequalities that describes the shaded region and find the maximum value of $P = 2x + y - 3$.



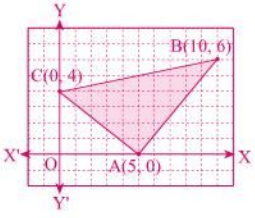
4I. लेखाचित्रमा छाया पारिएको भाग जनाउने असमानताहरू लेख्नुहोस् र $F = 3x + 7y$ को अधिकतम मान पत्ता लगाउनुहोस् ।

Write down the system of inequalities that describe the shaded region in the graph and find the minimum value of $F = 3x + 7y$.



4J. चित्रमा छाया पारिएको भागले घेरिएका रेखाहरूले जनाउने असमानताहरू पत्ता लगाउनुहोस् । साथै उद्देश्यफलन $F = x + 3y$ को दिइएका शीर्षहरूमा न्यूनतम मान पत्ता लगाउनुहोस् ।

Find the inequalities of the boundary lines of the shaded region. Also find the minimum value of the objective function $F = x + 3y$ at the given vertices.



ANSWERS

1A. 52 at (4, 0) **1B.** A(5, 0), 120 **1C.** 44 at (4, 8) **1D.** 50 at (5, 0) **1E.** 10 at (2, – 4)

1F. 72 at (0, 6) **1G.** 11.5 at $\left(\frac{5}{2}, \frac{1}{3}\right)$ **2A.** 23 at (3, 2) **2B.** 63 at (7, 0) **2C.** 74 at (6, 0)

3A. 2 at (0, 0) **3B.** 11 at (5, 1) **3C.** – 7 at (0, 0) **3D.** 17 at (1.5, 2.5)

4A. $x \le 2y + 1,\ x \ge 0, y \ge 0$; 20 at (0, 4)

4B. $2x + 3y \le 6, x - 3y \le 3, x \ge -3$; 6

4C. $x + 2y \ge 5, y \le 2x, x \le 5$; 65 at (5, 10)

4D. $y \le 2, 2x - y \le 4, x + y \ge 2$; 30 at (3, 2)

4E. 22 at (6, 2); $2x + 3y \ge 6, 2x - 3y \le 6, y \le 2$

4F. $3x + 2y \le 12, x \ge 0, y \ge 0$; 48 at (0, 6)

4G. $x + y \le 7, x - y \ge -1, x \ge 0, y \ge 0$

4H. $x \ge 0, y \ge 0, x + y \le 6, x - y \le 4$; 8 at (–2, 5)

4I. $x \le y, x + y \le 4, x \ge 4$; 40 at (4, 4)

4J. $4x + 5y \ge 20, 6x - 5y \le 30, x - 5y \ge -20$, 5 at (5, 0)

www.drmathnp.com

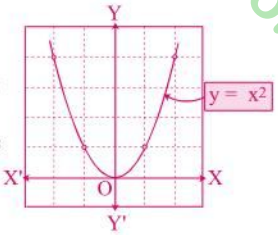
6. समीकरण र लेखाचित्र
EQUATION AND GRAPH

DEFINITIONS

1. वर्गघातीय फलन (Quadratic function)

$f(x) = ax^2 + bx + c$; $a \neq 0$ स्वरूपको फलनलाई वर्ग फलन भनिन्छ । जस्तै: $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ इत्यादि वर्ग फलनहरू हुन् ।

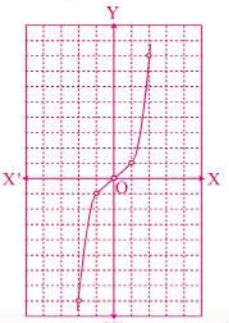
A function of the form $f(x) = ax^2 + bx + c$; $a \neq 0$ is called a quadratic function. e.g. $f(x) = 2x^2 + 3x + 5$, $f(x) = x^2$ etc. are quadratic functions.



2. पाराबोला (Parabola)

वर्ग समीकरणलाई लेखाचित्रमा प्रस्तुत गर्दा प्राप्त हुने वक्रलाई पाराबोला भनिन्छ ।

The smooth curve obtained by the graph of a quadratic equation is called the parabola.



3. पाराबोलाको शीर्ष (Vertex of a parabola)

पाराबोला घुमेको बिन्दुलाई पाराबोलाको शीर्ष भनिन्छ ।

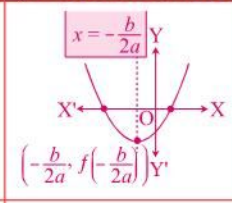
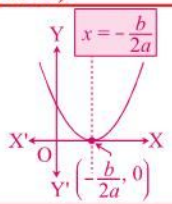
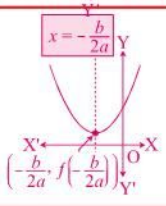
The turning point of the parabola is called its vertex.

4. घन घातीय फलन (Cubic function)

$f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$ स्वरूपको फलनलाई घन फलन भनिन्छ । जस्तै: $y = x^3$, $y = 2x^3 + 5$ इत्यादि घन फलनहरू हुन् ।

A function of the form $f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$ is called a cubic function. e.g. $y = x^3$, $y = 2x^3 + 5$ etc. are cubic functions.

5. पाराबोलाका विभिन्न अवस्थाहरू (Different conditions of parabola):

चित्रहरू Figures			
सममिति अक्ष Axis of Symmetry	$x = -\frac{b}{2a}$	$x = -\frac{b}{2a}$	$x = -\frac{b}{2a}$
शीर्षबिन्दु Vertex	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	$\left(-\frac{b}{2a}, 0\right)$	$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
विभेदक Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
x-खण्डहरू x-intercepts	दुईओटा x -खण्डहरू Two x-intercepts	एउटा x - खण्ड One x-intercept	x - खण्ड हुँदैन No x-intercept

एउटा वर्ग समीकरण $f(x) = ax^2 + bx + c$, $a \neq 0$ को लेखाचित्र ।

Graph of a quadratic equation $f(x) = ax^2 + bx + c$, $a \neq 0$.

(i) $a > 0$ भए माथितिर खुलेको हुन्छ ।
If $a > 0$ then it opens up.

(ii) $a < 0$ भए तलतिर खुलेको हुन्छ ।
If $a < 0$ then it opens down.

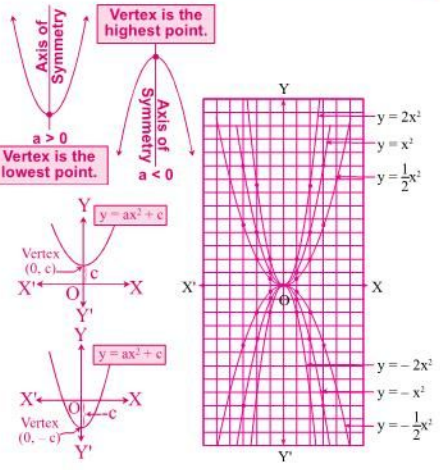
(iii) a को मान ठूलो हुँदा खुलेको भाग साँघुरो हुन्छ ।
The larger value of 'a' makes narrower opening.

(iv) a को मान सानो हुँदा खुलेको भाग फराकिलो हुन्छ ।
The smaller value of a makes the wider opening.

(v) समीकरण $y = ax^2 + c$ मा (In the equation $y = ax^2 + c$)

(a) $c > 0$ भए शीर्ष उद्गम बिन्दुबाट c एकाइ माथि हुन्छ ।
If $c > 0$ then the vertex is c units above the origin.

(b) $c < 0$ भए शीर्ष उद्गम बिन्दुबाट c एकाइ तल हुन्छ ।
If $c < 0$ then the vertex is c units below the origin.

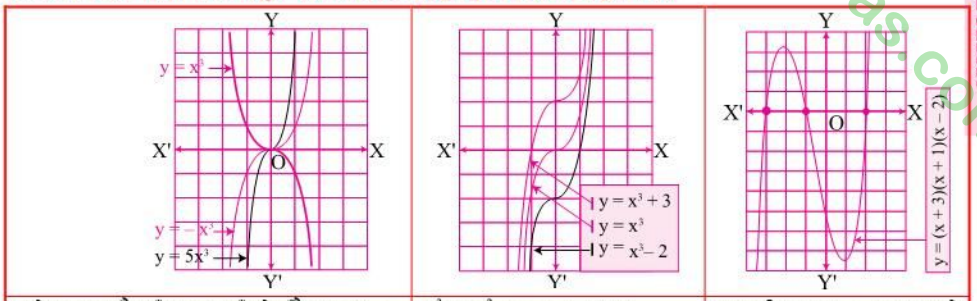


puspas.com.np

www.drmathnp.com

puspas.com.np

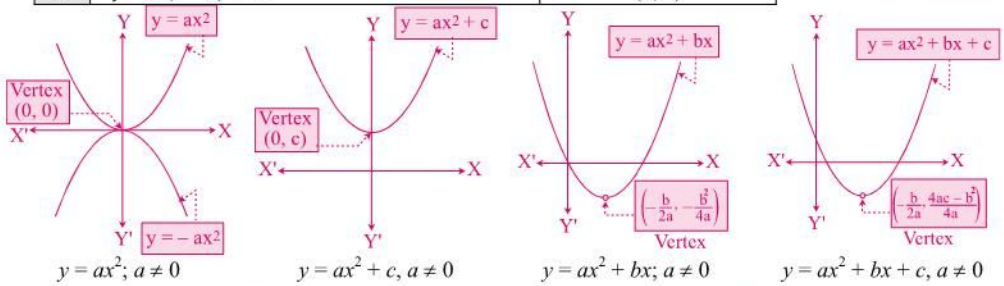
6. घन समीकरणका लेखाचित्रको प्रकृति (Nature of the graph of cubic equation)



a को मान बढ्दै जाँदा वक्र साँगुरो हुँदै जान्छ । a को मान घट्दै जाँदा वक्र फराकिलो हुँदै जान्छ । When the value of a increases then the curve will be narrower. When the value of 'a' decreases then the curve will be wider.	$ax^3 + bx^2 + cx + d = 0$ मा अचल पद d ले y-खण्ड जनाउँदछ । The constant term d in the equation $ax^3 + bx^2 + cx + d = 0$ represents the y- intercept.	घन समीकरणका x - खण्डहरूले मूलहरू जनाउँदछन् । The x - intercepts of cubic equation represent the roots.

FORMULAE AND KEY POINTS

S.N.	पाराबोलाको समीकरण (Equation of Parabola)	शीर्षबिन्दु (Vertex)
1.	$y = ax^2 + bx + c, a \neq 0$	$\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$
2.	$y = ax^2 + c, a \neq 0$	$(0, c)$
3.	$y = ax^2 + bx; a \neq 0$	$\left(\frac{-b}{2a}, \frac{-b^2}{4a}\right)$
4.	$y = ax^2; a \neq 0$	$(0, 0)$
5.	$y = a(x - h)^2 + k$	(h, k)



$y = ax^2; a \neq 0$ $y = ax^2 + c, a \neq 0$ $y = ax^2 + bx; a \neq 0$ $y = ax^2 + bx + c, a \neq 0$

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 5	SEE MODEL 2076	1

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, S, 2065 E, 2066 R', 2067 S, 2068 R, R', S', 2069 R', 2070 R, S, 2071 R', 2072 R', 2071 S, 2073 R, 2073 R', SEE 2074 S', SEE 2075 R, R', R'₂	14
Model 2	2068 S, 2070 S', 2071 R	3

SEE EXERCISE 6

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):

A. वर्गघातीय समीकरण (Quadratic equation)
B. पाराबोला वा अनुवृत्त (Parabola)
C. पाराबोलाको शीर्ष (Vertex of parabola)
D. घनघातीय समीकरण (Cubic equation)

www.drmathnp.com
puspas.com.np

2A. अचलहरू p, q र r प्रयोग भएको एउटा पूर्ण वर्ग समीकरण लेख्नुहोस् ।
Write a complete quadratic equation having constants p, q, and r.

2B. अचलहरू a, b, c र d को प्रयोग गरी एउटा पूर्ण घन फलनको उदाहरण दिनुहोस् ।
Using the constants a, b, c and d, give an example of a complete cubic equation.

3A. समीकरण $y = ax^2 + bx + c, a \neq 0$ भएको पाराबोलाको शीर्षको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2 + bx + c, a \neq 0$?

3B. समीकरण $y = ax^2 + c = 0$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2 + c = 0$?

3C. समीकरण $y = ax^2$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = ax^2$?

3D. समीकरण $y = a(x - h)^2 + k$ भएको पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क के हुन्छ ?
What is the coordinates of vertex of parabola whose equation is $y = a(x - h)^2 + k$?

4A. वर्ग फलन $y = ax^2$ मा a को मान धनात्मक वा ऋणात्मक के हुँदा शीर्षबिन्दु न्यूनतम हुन्छ ?
Which value of a, positive or negative, in the quadratic equation $y = ax^2$ is the vertex point minimum?

4B. वर्ग फलन $y = ax^2$ मा a को मान धनात्मक वा ऋणात्मक के हुँदा शीर्षबिन्दु अधिकतम हुन्छ ?
Which value of a, positive or negative, in the quadratic equation $y = ax^2$ is the vertex point maximum?

5A. वर्ग फलन $y = ax^2$ मा $a > 0$ हुँदा पाराबोलाको खुलेको भाग माथि फर्कन्छ वा तल फर्कन्छ ?
In which side, above or below, the opening of the parabola $y = ax^2$ faces when $a > 0$?

5B. वर्ग फलन $y = ax^2$ मा $a < 0$ हुँदा पाराबोलाको खुलेको भाग माथि फर्कन्छ वा तल फर्कन्छ ?
In which side, above or below, the opening of the parabola $y = ax^2$ faces when $a < 0$?

6. तलका अवस्थामा वर्ग फलन $y = a(x - h)^2 + k$ को पाराबोलाको शीर्ष बिन्दु कहाँ पर्छ ? लेख्नुहोस् ।
Where does the vertex of parabola $y = a(x - h)^2 + k$ lie in the following condition ?

A. $h = 0$ **B.** $h > 0$ **C.** $h < 0$ **D.** $k = 0$ **E.** $k > 0$ **F.** $k < 0$

7A. समीकरण $ax^2 + bx + c = 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $ax^2 + bx + c = 0$?

7B. समीकरण $y = ax^2 + c, a \neq 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $y = ax^2 + c, a \neq 0$?

7C. समीकरण $y = ax^2 + bx, a \neq 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $y = ax^2 + bx, a \neq 0$?

7D. समीकरण $y = ax^2, a \neq 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $y = ax^2, a \neq 0$?

8A. वर्ग फलन $y = ax^2$ मा a को धनात्मक मान बढ्दै जाँदा पाराबोलाको मुख (खुलेको भाग) साँगुरो वा फराकिलो के हुँदै जान्छ ?
What will be the opening of parabola, wider or narrower, when the positive value of a increases in $y = ax^2$?

8B. वर्ग फलन $y = ax^2$ मा a को धनात्मक मान घट्दै जाँदा पाराबोलाको मुख (खुलेको भाग) साँगुरो वा फराकिलो के हुँदै जान्छ ?
What will be the opening of parabola, wider or narrower, when the positive value of a decreases in $y = ax^2$?

9A. वर्ग समीकरण $y = ax^2 + c$ मा c ले के जनाउँदछ ?
What does c represent in the quadratic equation $y = ax^2 + c$?

9B. घन समीकरण $ax^3 + bx^2 + cx + d = 0$ मा 'd' ले के जनाउँदछ ?
What does 'd' represent in the cubic equation $ax^3 + bx^2 + cx + d = 0$?

Model 2 Concept Recall

10A. समीकरणहरू $y = 2x^2$ र $y = 3x^2$ मध्य कुनको वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = 2x^2$ and $y = 3x^2$ has the wider curve?

10B. समीकरणहरू $y = -3x^2$ र $y = -4x^2$ मध्य कुनको वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = -3x^2$ and $y = -4x^2$ has the wider curve ?

10C. समीकरणहरू $y = 5x^2$ र $y = 3x^2$ मध्य कुनको वक्र बढी साँगुरो हुन्छ ?
Which of the equations $y = 5x^2$ and $y = 3x^2$ has the skinner curve?

11A. समीकरण $y = 2x^2$ को न्यूनतम मान कति होला ? (What is the minimum value of $y = 2x^2$?)

11B. समीकरण $y = -3x^2$ को अधिकतम मान कति होला ? (What is the maximum value of $y = -3x^2$?)

12A. समीकरण $y = x^2 + 5$ हुने पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the coordinates of vertex of parabola whose equation is $y = x^2 + 5$.

www.drmathnp.com

puspas.com.np

12B. समीकरण $y = 2x^2 - 3$ हुने पाराबोलाको शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the coordinates of vertex of parabola whose equation is $y = 2x^2 - 3$.

13A. समीकरण $y = x^2 - 2x$ भएको पाराबोलाको सममिति रेखाको समीकरण के होला ?
What is the equation of line of symmetry of parabola whose equation is $y = x^2 - 2x$?

13B. समीकरण $y = 2x^2 + 4x + 2$ भएको पाराबोलाको सममिति रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of line of symmetry of parabola whose equation is $y = 2x^2 + 4x + 2$.

14A. समीकरण $y = -2x^2 + 3x + 4$ भएको पाराबोला कुन दिशामा खुल्छ ?
In which direction does the parabola of equation $y = -2x^2 + 3x + 4$ open?

14B. समीकरण $y = 4x^2 + 5$ भएको पाराबोला कुन दिशामा खुल्छ ?
In which direction does the parabola of equation $y = 4x^2 + 5$ open?

15A. समीकरणहरू $y = 3x^3$ र $y = 4x^3$ मध्ये कुन को वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = 3x^3$ and $y = 4x^3$ has the wider curve?

15B. समीकरणहरू $y = -3x^3$ र $y = -2x^3$ मध्ये कुन को वक्र बढी फराकिलो हुन्छ ?
Which of the equations $y = -3x^3$ and $y = -2x^3$ has the wider curve?

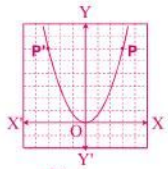
16. तलका वर्ग समीकरणहरूलाई वर्ग समीकरण र रेखीय समीकरणको रूपमा व्यक्त गर्नुहोस् ।
Express the following quadratic equations in terms of a quadratic equation and a linear equation.

A. $x^2 - 2x + 1 = 0$ **B.** $x^2 - 3x + 2 = 0$ **C.** $x^2 - 5x + 6 = 0$

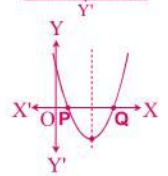
B. SHORT QUESTIONS

Model 1 *Problems based on reading graph.*

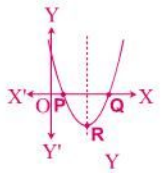
1A. सँगै दिइएको लेखाचित्रबाट आवश्यक समीकरण लेख्नुहोस् र तोकिएका दुई बिन्दुहरू P र P' का निर्देशाङ्कहरू समेत उल्लेख गर्नुहोस् ।
Write the required equation from the given graph and determine the co-ordinates of two points P and P'. **[SLC 2063 R]**



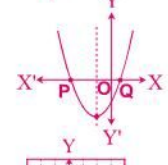
1B. सँगैको चित्रमा $y = ax^2 + bx + c$ को लेखाचित्र देखाइएको छ । लेखाचित्रले X-अक्षलाई P र Q मा काटेको छ । P र Q का निर्देशाङ्कहरू के-के होलान् र यी निर्देशाङ्कका X-निर्देशाङ्कहरूले के-के जनाउँदछन् ?
Figure given alongside represents the sketch of the graph $y = ax^2 + bx + c$. The graph intersects X-axis at P and Q. What are the co-ordinates of P and Q and what do their X-co-ordinates represent ? **[SLC 2063 M]**



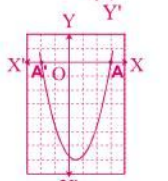
1C. सँगैको चित्रमा बिन्दु रेखाले $y = ax^2 + bx + c$ को लेखाचित्रले दिने प्याराबोलाको अक्षलाई जनाउँदछ र R त्यस्तै शीर्षबिन्दु हो भने बिन्दु रेखाको समीकरण र शीर्षबिन्दु R को निर्देशाङ्क पत्ता लगाउनुहोस् ।
If the dotted line in the adjoining figure represents the axis of the parabola given by the graph of $y = ax^2 + bx + c$ and R is its vertex; find the equation of the dotted line and the co-ordinates of the vertex R. **[SLC 2063 S]**



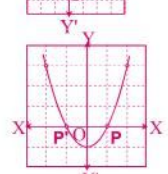
1D. सँगैको चित्रमा $y = x^2 + 4x - 5$ को लेखाचित्र देखाइएको छ । यसले X-अक्षको बिन्दुहरू P र Q मा काटेको छ । P र Q का निर्देशाङ्कहरू के-के हुन् ? यी निर्देशाङ्कको X-निर्देशाङ्कहरूले के जनाउँदछन् ?
Figure along the side represents the sketch of the graph $y = x^2 + 4x - 5$. It cuts the X-axis at P and Q. What are the co-ordinates of P and Q? What do these X-co-ordinates denote? **[SLC 2063 R']**



1E. सँगै दिइएको लेखाचित्रबाट A र A' का निर्देशाङ्कहरू पत्ता लगाई सो लेखाचित्र जनाउने समीकरण लेख्नुहोस् ।
Find the co-ordinates of A and A' from the given adjoining graph and write the equation to represent the given graph. **[SLC 2064R]**



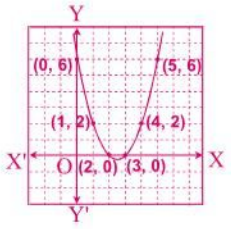
1F. सँगै दिइएको लेखाचित्रबाट P र P' का निर्देशाङ्कहरू पत्ता लगाई सो लेखाचित्र जनाउने समीकरण लेख्नुहोस् ।
Find the co-ordinates of P and P' from the given adjoining graph and write the equation to represent the given graph.



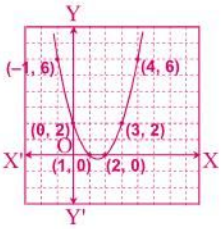
www.drmathnp.com
puspas.com.np

2. दिइएको लेखाचित्रबाट पाराबोलाको Y-खण्ड, X-खण्ड र सममिति अक्षको समीकरण पत्ता लगाउनुहोस्:
Find the Y-intercept, X-intercept and equation of line of symmetry in the given graph:

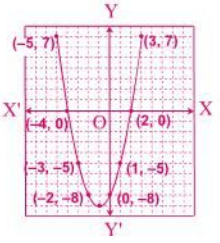
A.



B.

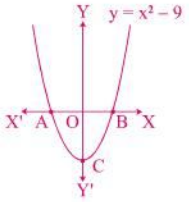


C.

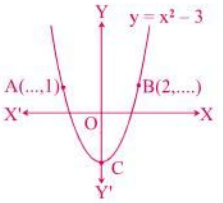


3. तलको चित्रमा देखाइएको जानकारीबाट बिन्दुहरू A, B र C का निर्देशाङ्कहरू पत्ता लगाउनुहोस्:
From the given information in the figure, find the co-ordinates of points A, B and C:

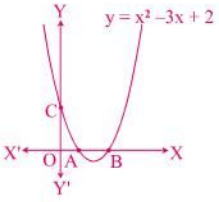
A.



B.



C.



Model 2 Problems based on finding vertex.

4A. पाराबोलाको मानक समीकरण लेखी $y = x^2 - 2x + 1$ को शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Write the standard equation of a parabola and find the vertex of $y = x^2 - 2x + 1$.

4B. $y = 4x^2 - 3$ बाट बन्ने पाराबोलाको शीर्षबिन्दु पत्ता लगाउनुहोस् ।
Find the vertex of parabola, which is formed from $y = 4x^2 - 3$. **[SLC 2063 R']**

4C. $f(x) = 5x^2 + 6$ बाट बन्ने पाराबोलाको शीर्षबिन्दु पत्ता लगाउनुहोस् ।
Find the vertex of parabola, which is formed from $f(x) = 5x^2 + 6$.

5. लेखाचित्र नखिचिकन दिइएका पाराबोलाको शीर्षबिन्दु पत्ता लगाउनुहोस् :
Without drawing the graph, find the vertex of given parabola:

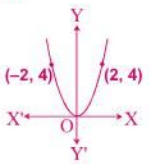
A. $x^2 - 2x - 3 = 0$ **B.** $4x - x^2 = 0$ **C.** $x^2 + 2x - 3 = 0$

D. $y = -x^2 - 2x + 3$ **E.** $y = -x^2 + 2x + 2$ **F.** $y = x^2 - 4x + 3$

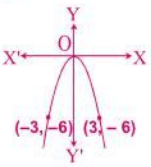
Model 3 Problems based on finding the quadratic equation.

6. तलका प्रत्येक चित्रमा देखाइएको पाराबोलाको समीकरण पत्ता लगाउनुहोस्:
Find the equation of the parabola shown in the following graph:

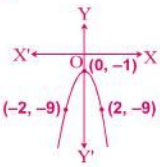
A.



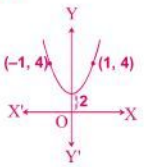
B.



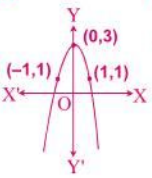
C.



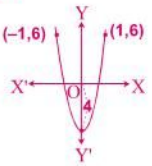
D.



E.



F.



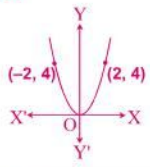
7A. उद्गमबिन्दुमा शीर्ष भएको एउटा पाराबोला बिन्दु (4, 32) भएर जान्छ भने सो पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the parabola which pass through the point (4, 32) and vertex lies at the origin.

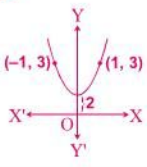
7B. उद्गमबिन्दुमा शीर्ष भएको एउटा पाराबोला बिन्दु (2, 8) भएर जान्छ भने सो पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the parabola which pass through the point (2, 8) and vertex lies at the origin.

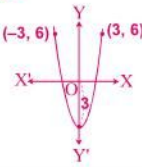
www.drmathnp.com

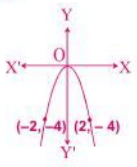
puspas.com.np

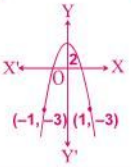
8. तलका लेखाचित्रमा देखाइएको पाराबोलाको समीकरण र Y-खण्ड पत्ता लगाउनुहोस्:
Find the equation and Y-intercept of the parabola shown in the following graph:

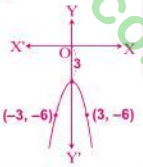
A.


B.


C.


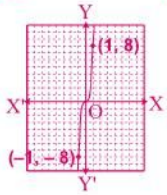
D.


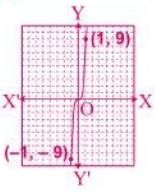
E.


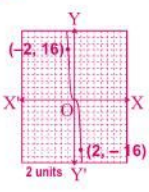
F.


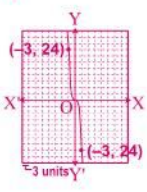
Model 4 Problems based on finding the cubic equation.

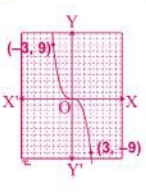
9. तलका लेखाचित्रसँग सम्बन्धित घन समीकरण पत्ता लगाउनुहोस् (Find the cubic equation related to given graph):

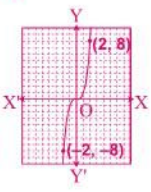
A.


B.


C.


D.


E.


F.


10A. उद्गमबिन्दु भएर जाने एउटा घन समीकरणको वक्रमा एउटा बिन्दु (2, 32) पर्दछ भने सो घन समीकरण पत्ता लगाउनुहोस् ।
The curve of a cubic equation passes through the origin contains the point (2, 32). Find the cubic equation.

10B. उद्गमबिन्दु भएर जाने एउटा घन समीकरणको वक्रमा एउटा बिन्दु (3, 24) पर्दछ भने सो घन समीकरण पत्ता लगाउनुहोस् ।
The curve of a cubic equation passes through the origin contains the point (3, 24). Find the cubic equation.

10C. उद्गमबिन्दु भएर जाने एउटा घन समीकरणको वक्रमा एउटा बिन्दु (– 4, 32) पर्दछ भने सो घन समीकरण पत्ता लगाउनुहोस् ।
The curve of a cubic equation passes through the origin contains the point (– 4, 32). Find the cubic equation.

10D. उद्गमबिन्दु भएर जाने एउटा घन समीकरणको वक्रमा एउटा बिन्दु (– 3, 81) पर्दछ भने सो घन समीकरण पत्ता लगाउनुहोस् ।
The curve of a cubic equation passes through the origin contains the point (– 3, 81). Find the cubic equation.

Model 5 Problems based on finding intercepts of parabola and point of intersection.

11A. $f(x) = x^2 - 1$ र $f(x) = 3$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।
What will be the points of intersection of the curve $f(x) = x^2 - 1$ and $f(x) = 3$? **[SEE MODEL 2076]**

11B. $f(x) = x^2 - 5$ र $f(x) = 4$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।
What will be the points of intersection of the curve $f(x) = x^2 - 5$ and $f(x) = 4$?

11C. $f(x) = x^2 - 5x$ र $f(x) = -6$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।
What will be the points of intersection of the curve $f(x) = x^2 - 5x$ and $f(x) = -6$?

12. प्रत्येक फलनका ग्राफ नखिचिकन तिनीहरूका X-अक्षमा रहेका बिन्दुहरू पत्ता लगाउनुहोस्:
Without drawing the graph of each of function, determine their points on X-axis:

A. $f(x) = x^2 + 4x$ **B.** $f(x) = 2x^2 + 6x + 5$ **C.** $y = x^2 + 2x - 3$ **D.** $y = x^2 - x - 2$

E. $y = x^2 - 3x - 4$ **F.** $y = x^2 - 5x + 6$ **G.** $y = x^2 - 7x + 12$

[Hint: To find the points at X-axis, we have $f(x) = 0$. **]**

13. दिइएका वर्ग समीकरणले दिने पाराबोलाको X-खण्ड र Y-खण्ड पत्ता लगाउनुहोस्:
Find the X-intercept and Y-intercept of the parabola given by the equation:

A. $y = x^2 - 4$ **B.** $y = x^2 - 9$ **C.** $y = x^2 - 16$ **D.** $y = x^2 + 4x$

E. $y = x^2 - 2x$ **F.** $y = x^2 + 9x$

[Hint: For x–intercept put $y = 0$, For y–intercept put $x = 0$]

Model 6 Problems based on graphing equation.

14. तलका वर्ग समीकरणहरूको लेखाचित्र खिच्नुहोस् (Draw the graph of the following quadratic equations):

A. $y = 2x^2$ **B.** $y = 3x^2$ **C.** $y = -2x^2$ **D.** $y = x^2 - 5$

E. $y = x^2 + 2$ **F.** $y = 2x^2 + 4$ **G.** $y = 4x^2 - 2$ **H.** $y = 4x^3$

I. $y = -3x^3$ **J.** $y = x^3 + 3$ **K.** $y = x^3 + 3$

www.drmathnp.com

C. LONG QUESTIONS

Model 1 *Problems based on solving equation of the form $ax^2 + bx + c = 0$.*

1. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically):

A. $x^2 - 3x = 10$ [SEE 2075 R]

B. $x^2 + 2x - 3 = 0$ [2075R$_2$, 2074S', 2065E, 2066R', 2063S,2058S]

C. $x^2 - 2x - 3 = 0$ [2075 R, 2069 R', 2073 R']

D. $x^2 - x - 2 = 0$ [2061 R, 2063 M, 2068 R', SEE 2074 R']

E. $x^2 + x - 2 = 0$ [2071 S, SEE 2073 S]

F. $x^2 - 3x + 2 = 0$ [2063 R', 2065 S, 2071 R', 2073 R']

G. $x^2 - 5x + 6 = 0$ [2064 S, 2072 R']

H. $x^2 - x - 12 = 0$ [2065 R]

I. $x^2 - 4x + 3 = 0$ [2067 S]

J. $x^2 - 3x - 4 = 0$ [SEE 2073 S]

K. $x^2 - 2x - 15 = 0$ [2068 R]

L. $x^2 + 7x + 12 = 0$ [2070 R]

M. $3x^2 + 5x + 2 = 0$ [2068 S']

N. $x^2 - 2x - 8 = 0$ [2070 S]

2A. वर्ग समीकरण $x^2 - 3x - 10 = 0$ लाई लेखाचित्रद्वारा हल गर्नुहोस्, साथै पाराबोला र सरलरेखा खण्ड प्रतिच्छेदित भएका बिन्दुहरू पनि पत्ता लगाउनुहोस् ।

Solve graphically the quadratic equation $x^2 - 3x - 10 = 0$. Also find the intersecting points of parabola and straight line. [2064 R]

2B. वर्ग समीकरण $x^2 + 12 = 7x$ लाई लेखाचित्रद्वारा हल गर्नुहोस्, साथै पाराबोला र सरलरेखा खण्ड प्रतिच्छेदित भएका बिन्दुहरू पनि पत्ता लगाउनुहोस् ।

Solve graphically the quadratic equation $x^2 + 12 = 7x$. Also find the intersecting points of parabola and straight line.

Model 2 *Problems based on solving the equations of the form $y = ax^2$ and $y = mx + c$.*

3. दिइएको समीकरणहरूलाई लेखाचित्रद्वारा हल गर्नुहोस् र प्रतिच्छेदित बिन्दुहरू समेत देखाउनुहोस्:

Solve the following equations graphically and also indicate the point of intersection:

A. $y = x^2$ and $y = 3 - 2x$ [2060 S

B. $y = x^2$ and $y = 2x - 1$ [2064 R']

C. $x^2 = 2y$ and $y = x$ [2068 S]

D. $y = x^2$ and $y = 2 - x$ [2071 R]

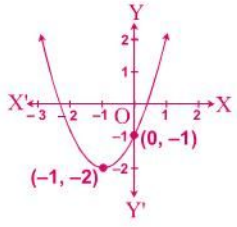
E. $y = x^2$ and $y = 2x + 8$ [2070 S']

F. $y = x^2$ and $y = x + 2$

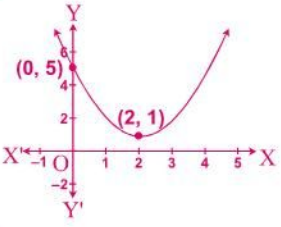
Model 3 *Problems based on finding quadratic equation.*

4. तलका पाराबोलाहरूको समीकरण पत्ता लगाउनुहोस् । (Find the equation of following parabolas.)

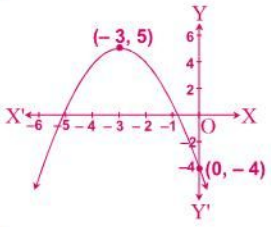
A.



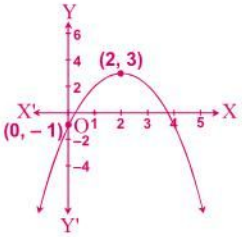
B.



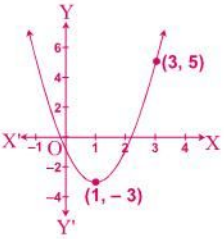
C.



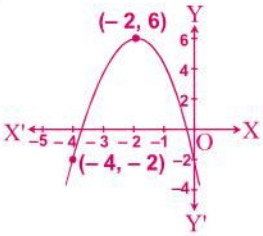
D.



E.



F.



puspas.com.np

www.drmathnp.com

puspas.com.np

5A. फलन $f(x) = ax^2 + bx + c$ को लेखाचित्रमा शीर्षबिन्दु (0, 2) छ र बिन्दु (1, 8) भएर जान्छ। a, b र c को मानहरू पत्ता लगाउनुहोस्।
The graph of the function $f(x) = ax^2 + bx + c$ has vertex at (0, 2) and passes through the point (1, 8). Find a, b and c.

5B. फलन $f(x) = ax^2 + bx + c$ को लेखाचित्रमा शीर्षबिन्दु (1, 4) छ र बिन्दु (– 1, – 8) भएर जान्छ। a, b र c का मानहरू पत्ता लगाउनुहोस्।
The graph of the function $f(x) = ax^2 + bx + c$ has vertex at (1, 4) and passes through the point (– 1, – 8). Find a, b and c.

5C. शीर्षबिन्दु (– 1, 2) र बिन्दु (1, – 4) भएर जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस्।
Find the equation of parabola with vertex at (– 1, 2) and passing through the point (1, – 4).
Hint: Use the formula : $y = a(x – h)^2 + k$; where (h, k) is vertex

5D. शीर्षबिन्दु (1, – 1) र बिन्दु (2, 3) भएर जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस्।
Find the equation of parabola with vertex at (1, – 1) and passing through the point (2, 3).

6A. बिन्दुहरू (0, 8), (1, 3) र (2, 0) बाट जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस्।
Find the equation of the parabola passing through the points (0, 8), (1, 3) and (2, 0).

6B. बिन्दुहरू (0, – 2), (1, 0) र (2, 4) बाट जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस्।
Find the equation of the parabola passing through the points (0, – 2), (1, 0) and (2, 4).

ANSWERS

A. VERY SHORT QUESTIONS

2A. $px^2 + qx + r = 0$ **2B.** $ax^3 + bx^2 + cx + d = 0$ **3A.** $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$ **3B.** (0, c)
3C. (0, 0) **3D.** (h, k) **4A.** Positive **4B.** Negative **5A.** above **5B.** below **6A.** Y - axis
6B. Right of y -axis by 'h' units **6C.** Left of y - axis by 'h' units **6D.** x - axis **6E.** above x - axis by k units
6F. below x - axis by k units **7A.** $x = -\frac{b}{2a}$ **7B.** $x = 0$ **7C.** $x = -\frac{b}{2a}$ **7D.** $x = 0$ **8A.** Narrower
8B. Wider **9A.** y intercept **9B.** y-intercept **10A.** $y = 2x^2$ **10B.** $y = -4x^2$ **10C.** $y = 5x^2$ **11A.** 0
11B. 0 **12A.** (0, 5) **12B.** (0, – 3) **13A.** $x = 1$ **13B.** $x = -1$ **14A.** downwards **14B.** upwards
15A. $y = 3x^3$ **15B.** $y = -3x^3$ **16A.** $y = x^2$ and $y = 2x - 1$ **16B.** $y = x^2$ and $y = 3x - 2$
16C. $y = x^2$ and $y = 5x - 6$

B. SHORT QUESTIONS

1A. (3, 6) & (– 3, 6)) ; $y = \frac{2}{3}x^2$ **1B.** $P = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $Q = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$; roots of equation
1C. $x = \frac{-b}{2a}$; $R\left[\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right]$ **1D.** (– 5, 0), (1, 0) ; roots of equation **1E.** (3, 0), (– 2, 0); $x^2 - x - 6 = 0$
1F. (1, 0) (– 1, 0); $y = x^2 - 1$ **2A.** 6, 2 and 3, $2x = 5$ **2B.** 2, 1 and 2, $2x = 3$
2C. – 8, – 4 and 2, $x + 1 = 0$ **3A.** A(– 3, 0), B(3, 0), C(0, – 9) **3B.** A(– 2, 1), B(2, 1), C(0, – 3)
3C. A(1, 0), B(2, 0), C(0, 2) **4A.** (1, 0) **4B.** (0, – 3) **4C.** (0, 6) **5A.** (1, – 4) **5B.** (2, 4)
5C. (– 1, – 4) **5D.** (– 1, 4) **5E.** (1, 3) **5F.** (2, – 1) **6A.** $y = x^2$ **6B.** $y = -\frac{2}{3}x^2$
6C. $y = -2x^2 - 1$ **6D.** $y = 2x^2 + 2$ **6E.** $y = -2x^2 + 3$ **6F.** $y = 2x^2 + 4$ **7A.** $y = 2x^2$ **7B.** $y = 2x^2$ **8A.** $y = x^2$, 0
8B. $y = x^2 + 2$, 2 **8C.** $y = x^2 - 3$, – 3 **8D.** $y = -x^2$, 0 **8E.** $y = -5x^2 + 2$, 2 **8F.** $y = -\frac{1}{3}x^2 - 3$, – 3 **9A.** $y = 8x^3$ **9B.** $y = 9x^3$
9C. $y = -2x^3$ **9D.** $y = -\frac{8}{9}x^3$ **9E.** $y = -\frac{1}{3}x^3$ **9F.** $y = x^3$ **10A.** $y = 4x^3$ **10B.** $y = \frac{8}{9}x^3$ **10C.** $y = -\frac{1}{2}x^3$
10D. $y = 3x^3$ **11A.** (– 2, 3) and (2, 3) **11B.** (– 3, 4) and (3, 4) **11C.** (2, – 6) and (3, – 6)
12A. (0, 0), (– 4, 0) **12B.** None **12C.** (1, 0), (– 3, 0) **12D.** (2, 0), (– 1, 0) **12E.** (4, 0), (– 1, 0) **12F.** (3, 0), (2, 0)
12G. (3, 0), (4, 0) **13A.** ± 2; – 4 **13B.** ± 3; – 9 **13C.** ± 4; – 16 **13D.** 0, – 4 ; 0 **13E.** 0, 2 ; 0 **13F.** 0, – 9 ; 0

C. LONG QUESTIONS

1A. – 2 or 5 **1B.** 1, –3 **1C.** 3, – 1 **1D.** 2, – 1 **1E.** 1, – 2 **1F.** 2, 1
1G. 3, 2 **1H.** 4, – 3 **1I.** 1, 3 **1J.** 4, – 1 **1K.** – 3, 5 **1L.** –3, –4
1M. $-1, -\frac{2}{3}$ **1N.** 4, – 2 **2A.** – 2 or 5; (–2, 4), (5, 25) **2B.** 4 or 3; (4, 16), (3, 9) **3A.** (1, 1), (–3, 9)
3B. (1, 1) **3C.** $x = (0, 2), y = (0, 2)$ **3D.** (1, 1) and (– 2, 4) **3E.** (– 2, 4) and (4, 16) **3F.** (2, 4) and (– 1, 1)
4A. $f(x) = (x + 1)^2 - 2 = x^2 + 2x - 1$ **4B.** $f(x) = (x - 2)^2 + 1 = x^2 - 4x + 5$ **4C.** $f(x) = -(x + 3)^2 + 5 = -x^2 - 6x - 4$
4D. $f(x) = -(x - 2)^2 + 3 = -x^2 + 4x - 1$ **4E.** $f(x) = 2(x - 1)^2 - 3 = 2x^2 - 4x - 1$ **4F.** $f(x) = -2(x + 2)^2 + 6 = -2x^2 - 8x - 2$
5A. $a = 6, b = 0, c = 2$ **5B.** $a = -3, b = 6, c = 1$ **5C.** $y = -\frac{3}{2}(x + 1)^2 + 2$
5D. $y = 4(x - 1)^2 - 1$ or $y = 4x^2 - 8x + 3$ **6A.** $y = x^2 - 6x + 8$ **6B.** $y = x^2 + x - 2$

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 6

LONG QUESTIONS

Model 1 *Graph of equation of the form $y = ax^2 + bx + c$ & finding the line of symmetry.*

1. तल दिइएका वर्ग समीकरणहरूको ग्राफ खिच्नुहोस् र तिनीहरूको सममितिको अक्ष पनि पत्ता लगाउनुहोस् ।
Draw the graph of the following quadratic equations and find their line of symmetry.

A. $y = x^2 + 2x - 8$ **B.** $y = x^2 - 2x + 3$ **C.** $y = -x^2 - 2x + 8$

D. $y = x^2 - 4x - 5$ **E.** $f(x) = x^2 - 6x + 9$ **F.** $f(x) = x^2 - 3x + 3$

2A. पाराबोला $f(x) = -3(x-1)^2 - 5$ को लागी निम्नलिखित उल्लेख गर्नुहोस् साथै लेखाचित्रमा पनि देखाउनुहोस् ।
For the parabola $f(x) = -3(x-1)^2 - 5$, state the following and also show in a graph.

(i) पाराबोलाको खुलेको भागको दिशा (Direction of opening of the parabola)

(ii) शीर्षको अवस्थिति (Location of the vertex) (iii) सममिति रेखाको समीकरण (Equation of line of symmetry)

(iv) पाराबोलाको Y-खण्ड (Y-intercept of the parabola)

2B. $f(x) = x^2 - 2x - 5$ मा निम्नानुसार पत्ता लगाउनुहोस् । (For $f(x) = x^2 - 2x - 5$, find the following.)

(i) शीर्ष (Vertex) (ii) पाराबोलाको खुलेको भागको दिशा (Direction of opening of the parabola)

(iii) सममिति रेखाको समीकरण (Equation of line of symmetry)

(iv) पाराबोलाको Y-अक्षमा रहेको बिन्दु (Point at Y-axis)

Model 2 *Problems based on solving equation of the form $ax^2 + bx + c = 0$.*

3. तलका समीकरणहरूलाई लेखाचित्रद्वारा हल गर्नुहोस् (Solve the following equations by graphical method):

A. $x^2 - 4x + 4 = 0$ **B.** $x^2 - 2x + 1 = 0$ **C.** $x^2 - x - 6 = 0$

D. $x^2 + x - 6 = 0$ **E.** $3x^2 + 5x + 2 = 0$ **F.** $2x^2 + x - 6 = 0$

G. $x^2 + 4x - 5 = 0$ **H.** $x^2 - 4x + 3 = 0$ **I.** $x^2 - 6x + 9 = 0$

Model 3 *Problems based on solving quadratic equation and linear equation.*

4. लेखाचित्रविधि र प्रतिस्थापन विधिद्वारा हल गर्नुहोस् (Solve by graphical method & method of substitution):

A. $y = x^2 - x - 3$ and $x = y$ **B.** $y = x^2 - 2x$ and $y = x$ **C.** $y = x^2$ and $2x + y = 3$

D. $y = x^2$ and $y = 5x - 6$ **E.** $y = x^2 + 8x - 6$ and $x + y = 4$ **F.** $y = x^2 - 2x$ and $y = x - 1$

G. $y = x^2 - 5$ and $y = 2x + 3$ **H.** $y = -x^2$ and $x - y = 2$ **I.** $y = x^2$ and $y = 2x - 1$

5A. वर्ग समीकरण $y = x^2 + 2$ ले दिने पाराबोला र बिन्दुहरू $(-2, 6)$ र $(2, 6)$ भएर जाने रेखाको प्रतिच्छेदन बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the point of intersections of the parabola of a quadratic equation $y = x^2 + 2$ and the line passing through the points $(-2, 6)$ and $(2, 6)$.

5B. वर्ग समीकरण $y = x^2 + 3$ ले दिने पाराबोला र बिन्दुहरू $(-3, 6)$ र $(0, 9)$ भएर जाने रेखाको प्रतिच्छेदन बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the point of intersections of the parabola of a quadratic equation $y = x^2 + 3$ and the line passing through the points $(-3, 6)$ and $(0, 9)$.

5C. समीकरणहरू $y = x^2 - 5x + 6$ र $y = 2$ को लेखाचित्र खिच्नुहोस् र प्रतिच्छेदित बिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
Draw the graph of the equations $y = x^2 - 5x + 6$ and $y = 2$ and also find the co-ordinates of the point of their intersection.

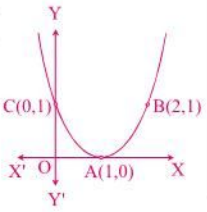
Model 4 *Problems based on drawing the graph of cubic equation.*

6. ग्राफ खिच्नुहोस् (Draw the graph of):

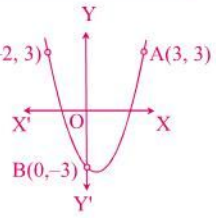
A. $y = x^3$ and $y = -x^3$ **B.** $y = -2x^3$ and $y = 2x^3$ **C.** $y = 3x^3$ and $y = -3x^3$

Model 5 *Problems based on finding the equation of parabola.*

7A. सँगैको चित्रमा दिइएको जानकारी अनुसार पाराबोलाको समीकरण प्रतिपादन गर्नुहोस् ।
Derive the equation of parabola given in the adjoining figure.



7B. सँगैको चित्रमा दिइएको जानकारी अनुसार पाराबोलाको समीकरण प्रतिपादन गर्नुहोस् ।
Derive the equation of parabola given in the adjoining figure.



www.drmathnp.com

puspas.com.np

Model 6 *Problems based on cubic equation and linear equation.*

8. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically):

A. $y = x^3$ and $x + y = 10$ **B.** $y = x^3$ and $x + y = 2$ **C.** $y = x^3$ and $x = y$

D. $y = x^3$ and $x = -y$

ANSWERS

1A. $x + 1 = 0$ **1B.** $x - 1 = 0$ **1C.** $x + 1 = 0$ **1D.** $x + 2 = 0$ **1E.** $x = 3$ **1F.** $2x - 3 = 0$

2A. Downward, $(1, -5)$, $x = 1, -8$ **2B.** $(1, -6)$, Upward, $x = 1$, $(0, -5)$ **3A.** 2, 2 **3B.** 1 **3C.** $3, -2$

3D. $2, -3$ **3E.** $-1, -\frac{2}{3}$ **3F.** $-2, \frac{3}{2}$ **3G.** $(1, -5)$ **3H.** $(1, 3)$ **3I.** 3, 3

4A. $(3, 3)$, $(-1, -1)$ **4B.** $(0, 0)$, $(3, 3)$ **4C.** $(1, 1)$, $(-3, 9)$ **4D.** $(2, 4)$, $(3, 9)$ **4E.** $(-10, 14)$, $(1, 3)$ **4F.** $(2.6, 1.6)$, $(0.4, -0.6)$

4G. $(4,11)$, $(-2, -1)$ **4H.** $(1, -1)$, $(-2, -4)$ **4I.** $(1, 1)$ **5A.** $(-2, 6)$, $(2, 6)$ **5B.** $(-2, 7)$, $(3, 12)$ **5C.** $(1, 2)$, $(4, 2)$

7A. $y = x^2 - 2x + 1$ **7B.** $y = x^2 - x - 3$ **8A.** $(2, 8)$ **8B.** $(1, 1)$ **8C.** $(1, 1)$, $(-1, -1)$, $(0, 0)$

8D. $(0, 0)$

CHALLENGING EXERCISE 4

1. लेखाचित्रविधिद्वारा हल गर्नुहोस् (Solve by graphical method):

A. $y = x^3$ and $y = x^2 + x - 1$ **B.** $y = x^3$ and $y = 2x^2 + 4x - 8$

2. एउटा वर्ग समीकरणले दिने पाराबोला निम्नलिखित बिन्दुहरू भएर जान्छ भने सो वर्ग समीकरण पत्ता लगाउनुहोस् ।

The parabola of a quadratic equation passes through following points. Find the quadratic equation.

A. $(1, 0)$, $(2, 0)$ and $(4, 6)$ **B.** $(0, 2)$, $(2, 0)$ and $(3, 2)$

3. X-खण्ड र Y-खण्ड दुवै 1 एकाइ बनाई बिन्दु $(2, 1)$ भएर जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of parabola passing through the point $(2, 1)$ and making X-intercept and Y-intercept both 1.

4. X-खण्ड र Y-खण्ड दुवै क्रमशः 1 र -3 बनाई बिन्दु $(2, 5)$ भएर जाने पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of parabola passing through the point $(2, 5)$ and making X-intercept and Y-intercept 1 and -3 respectively.

5. एउटा कम्पनीले निम्नानुसारको आवश्यकताहरू बुझी न्यूनतम मात्रामा कति कति ब्राण्ड A र ब्राण्ड B उत्पादन गर्न उपयुक्त होला ?

At what minimum amount the brand A and brand B must be produced by a company according to the following demands.

	ब्राण्ड A (Brand A): x	ब्राण्ड B (Brand B): y	आवश्यकता (Demand)
Vitamin A	10 mg	15 mg	2100 mg
Vitamin B	5 mg	15 mg	1500 mg
प्रति चक्की मूल्य (Price per piece)	Rs 6	Rs 8	

Hint: Objective function (P) $= 6x + 8y$ and Constraints are $10x + 15y \le 2100$; $5x + 15y \le 1500$; $x, y \ge 0$

6. एउटा केकलाई 150 ग्राम पिठो र 50 ग्राम चिल्लो पदार्थ चाहिन्छ र अर्को थरी केकलाई 75 ग्राम पिठो र 75 ग्राम चिल्लो पदार्थ चाहिन्छ । अमराले उनीसँग 1.5 kg पिठो र 0.6 kg चिल्लो पदार्थ हुँदा सम्भव भएसम्म धेरै केकहरू बनाउन चाहिन्छन्, उनीले प्रत्येक केकहरू कति कति बनाउनु पर्ला ?

One cake requires 150 gm flour and 50 gm fat and another requires 75 g flour and 75 gm fat. Amara wants to make as many cakes as possible when 1.5 kg flour and 0.6 kg fat are available. How many of each kind should she make?

Hint: Let x be the number of cakes of the first kind and y be the number of 2nd kind.

First case, $150x + 75y \le 1500$ $\therefore$ $2x + y \le 20$ **Second case,** $50x + 75y \le 600$ $\therefore$ $2x + 3y \le 24$

$\because x$ and y will be (+)ve, i.e. $x \ge 0$ and $y \ge 0$

She has to maximize $F = x + y$ over the constraints; $2x + y \le 20$, $2x + 3y \le 24$, $x \ge 0$, $y \ge 0$.

7. शर्तहरू $x + y \le 6$, $2x - y \le 3$, $x \ge 0$ र $y \ge 0$ को आधारमा उद्देश्य फलन $F = 4x + 3y + a$; $a > 0$ को अधिकतम मान 26 भए a को मान कति होला ?

If the maximum value of $F = 4x + 3y + a$; $a > 0$ under the constraints $x + y \le 6$, $2x - y \le 3$, $x \ge 0$ and $y \ge 0$ is 26, what is the value of a ?

8. शर्तहरू $x - 2y \le 2$, $x + y \le 5$, $x \ge 0$ र $y \ge 0$ को आधारमा उद्देश्य फलन $R = 5x - 4y + p$; $p > 0$ को न्यूनतम मान 2 भए p को मान कति होला ?

If the minimum value of $R = 5x - 4y + p$; $p > 0$ under the constraints $x - 2y \le 2$, $x + y \le 5$ $x \ge 0$ and $y \ge 0$ is 2, what is the value of p ?

ANSWERS

1A. $(1, 1)$, $(-1, -1)$ **1B.** $(2, 2)$, $(-2, -2)$ **2A.** $y = x^2 - 3x + 2$ **2B.** $y = x^2 - 3x + 2$ **3.** $y = x^2 - 2x + 1$

4. $y = x^2 + 2x - 3$ **5.** 1120, Brand A = 0, Brand B = 140 **6.** 9 and 2 **7.** 5 **8.** 22

www.drmathnp.com

puspas.com.np

ASSIGNMENT ON ALGEBRA

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' 4 × 1 = 4

1. विपरीत फलनको परिभाषा दिनुहोस् । (Define Inverse function.)
2. 2 र 8 बीचको गुणोत्तर मध्यमा पत्ता लगाउनुहोस् । (Find the geometric mean between 2 and 8.)
3. पाराबोला $y = x^2 - 4x + 3$ को शीर्षबिन्दु पत्ता लगाउनुहोस् ।

   Find the coordinates of vertex of parabola of $y = x^2 - 4x + 3$.
4. पहिलो पद 'a' र अन्तिम पद 'ℓ' भएको AP को n ओटा पदहरूको योगफल कति हुन्छ ?

   What is the sum of n terms of AP whose first term is 'a' and last term is 'ℓ'?

Group 'B' 2 × 2 = 4

5. यदि $f = \{(1, 4), (2, 6), (3, 8), (4, 5)\}$ र $g = \{(8, 7), (6, 5), (4, 3), (5, 4)\}$ भए फलन $g \circ f$ लाई मिलान चित्रमा देखाई यसलाई क्रम जोडाको रूपमा लेख्नुहोस् ।

   If $f = \{(1, 4), (2, 6), (3, 8), (4, 5)\}$ and $g = \{(8, 7), (6, 5), (4, 3), (5, 4)\}$ then show the function $g \circ f$ in the arrow diagram and find it in ordered pair form.
6. गुणनखण्ड साध्य उल्लेख गर्नुहोस् । $3x^3 + 2x - k(3k^2 + 2)$ को एउटा गुणनखण्ड $(x - k)$ हो कि होइन पत्ता लगाउन सोको प्रयोग गर्नुहोस् ।

   State the factor theorem. Use it to find whether $(x - k)$ is a factor of $3x^3 + 2x - k(3k^2 + 2)$.

Group 'C' 3 × 4 = 12

7. एउटा फलनको विपरीतलाई $f^{-1}(x) = \frac{3x}{2x-1}$ द्वारा परिभाषीत गरिएको छ । यदि $f^{-1}(f(x)) = 5$ भए x को मान पत्ता लगाउनुहोस् ।

   The inverse of a function $f(x)$ is defined by $f^{-1}(x) = \frac{3x}{2x-1}$. If $f^{-1}(f(x)) = 5$, find the value of x.
8. p को लागि हल गर्नुहोस् (Solve for p): $\frac{2}{p+1} = \frac{3p}{p^3+1}$
9. एउटा AP को छैटौं पद पहिलो पदको पाँचगुणा र एघारौं पद पाँचौं पदको दोब्बर भन्दा 3 ले धेरै छ । $8^{\text{औं}}$ पद पत्ता लगाउनुहोस् ।

   The sixth term of an AP is 5 times the first term and the 11^{th} term exceeds twice the fifth term by 3. Find the 8^{th} term.

Group 'D' 1 × 5 = 5

10. तलका शर्तहरूको आधारमा $P = 5x + 3y$ को अधिकतम मान पत्ता लगाउनुहोस् ।

    Find the maximum value of $P = 5x + 3y$ under the following constraints:

    $2x + y \leq 20$, $2x + 3y \leq 24$, $x \geq 0$ and $y \geq 0$

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

2. 4 **3.** $(2, -1)$ **4.** $\frac{n}{2}(a + \ell)$ **5.** $gof = \{(1, 3), (2, 5), (3, 7), (4, 4)\}$

6. Yes **7.** 5 **8.** $-1, 2, \frac{1}{2}$ **9.** 33 **10.** 51 at (9, 2)

www.drmathnp.com

puspas.com.np

Unit II

निरन्तरता
Continuity



WEIERSTRASS

In the development of calculus of the 17th and 18th centuries, the modern idea of the limit of a function goes back to Bolzano who, in 1817, introduced the basics of the epsilon-delta technique to define continuous functions. However, his work was not known during his lifetime (Felscher 2000). Cauchy discussed variable quantities, infinitesimals, and limits and defined continuity of $y = f(x)$ by saying that an infinitesimal change in x necessarily produces an infinitesimal change in y in his 1821 book Cours d'analyse, while (Grabiner 1983) claims that he only gave a verbal definition. Weierstrass first introduced the epsilon-delta definition of limit in the form it is usually written today. He also introduced the notations lim and lim $x \to x_0$ (Burton 1997).

SEE SPECIFICATION GRID **1 Very Short Question and 1 Long Question**

S. N.	Contents	Topics	Knowledge Each of 1 Mark	Understanding Each of 2 Marks	Application Each of 4 Marks	Higher Ability Each of 5 Marks	Total no. of Questions	Total Marks
2.	सीमान्त मान र निरन्तरता Limit and continuity	सङ्ख्याहरू र निरन्तरता Numbers and Continuity लेखाचित्रमा विच्छिन्नता Discontinuity in Graph निरन्तरताको सङ्केत Notational Representation of Continuity	1	-	1	-	2	5

SYLLABUS ISSUED BY CDC

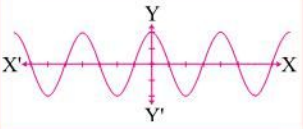
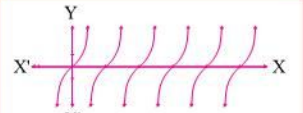
2. सीमान्त मान र निरन्तरता (Limit and continuity) - 10 Periods

निरन्तरताको सामान्य अवधारणा (Simple concept of Continuity)

- सङ्ख्याहरूको समूह (प्राकृतिक सङ्ख्या, पूर्ण सङ्ख्या, अनुपातिक सङ्ख्या) मा निरन्तरता वा अविच्छिन्नता खोजी
  Investigation of continuity on set of numbers (Natural numbers, whole numbers, rational numbers)
- वास्तविक सङ्ख्याको समूहमा अविच्छिन्नता वा निरन्तरता (continuity) को खोजी
  Investigation of continuity on set of real numbers
- विभिन्न सङ्ख्याहरूको समूहमा निरन्तरता र विच्छिन्नताको खोजी (सङ्ख्यारेखा र रचनाबाट)
  Investigation of continuity and discontinuity from the number line and construction on the set of different numbers
- लेखाचित्रबाट फलनको अविच्छिन्नताको खोजी र साङ्केतिक प्रस्तुति
  Investigation of continuity of function from the graph and Notational representation

DEFINITIONS AND KEY POINTS

2.1 निरन्तरताको धारणा (Concept of continuity)

निरन्तरता Continuity				
विच्छिन्नता Discontinuity				

यदि कुनै फलनको लेखाचित्र खिच्दा सिसाकलमलाई नउचालीकन लगातार खिच्न सकिन्छ भने यसलाई निरन्तरता भनिन्छ ।
If the graph of a function can be drawn without lifting the pencil from the paper then it is called continuity.

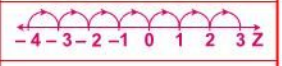
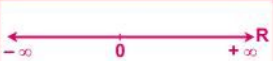
www.drmathnp.com

puspas.com.np

निरन्तरताका उदाहरणहरू (Examples of continuity)

- विरुवाको वृद्धि (Growth of plant)
- पानीको बहाव (Water flow)
- चलिरहेको गाडी (Bus movement)
- वास्तविक सङ्ख्याहरूको सङ्ख्यारेखा (Number line of real numbers) etc.

2.2 सङ्ख्याहरूको समूहको क्रममा निरन्तरता (Continuity in the order of set of numbers)

सङ्ख्याहरू (Numbers)	सङ्ख्यारेखा (Number line)
प्राकृतिक सङ्ख्याहरू (Natural numbers): {1, 2, 3, 4, 5, ...}	
पूर्ण सङ्ख्याहरू (Whole numbers): {0, 1, 2, 3, 4, 5, ...}	
पूर्णाङ्कहरू (Integers): {..., – 3, – 2, – 1, 0, 1, 2, 3, ...}	
वास्तविक सङ्ख्याहरू (Real Numbers): प्राकृतिक सङ्ख्याहरू (Natural numbers), पूर्ण सङ्ख्याहरू (Whole numbers), पूर्णाङ्कहरू (Integers), आनुपातिक सङ्ख्याहरू (Rational Numbers), अनानुपातिक सङ्ख्याहरू (Irrational numbers)	

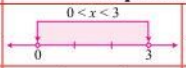
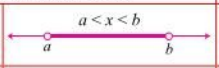
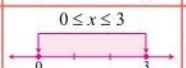
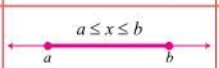
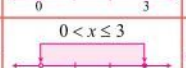
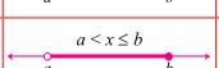
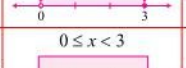
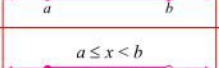
(a) प्राकृतिक सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन ।
The number line of natural numbers does not show continuity.

(b) पूर्ण सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन । (The number line of whole numbers does not show continuity.)

(c) पूर्णाङ्कहरूको सङ्ख्या रेखाले निरन्तरता देखाउँदैन । (The number line of integers does not show continuity.)

(d) वास्तविक सङ्ख्याहरूको सङ्ख्या रेखाले निरन्तरताको प्रतिनिधित्व गर्दछ । किनकी दुईओटा वास्तविक सङ्ख्याहरूको बीचमा अनगिन्ती वास्तविक सङ्ख्याहरू हुन्छन् ।
The number line of real numbers shows the continuity because there are many real numbers between any two real numbers.

2.3 अन्तराल (Interval)

अन्तराल सङ्केत Interval Notation		समूह सङ्केत Set Notation	लेखाचित्र वा सङ्ख्या रेखा Graph or Number Line		अन्तरालको नाम Interval name
(a)	(a, b)	$\{x : a < x < b\}$			खुल्ला अन्तराल Open Interval
(b)	$[a, b]$	$\{x : a \le x \le b\}$			बन्द अन्तराल Closed Interval
(c)	$(a, b]$	$\{x : a < x \le b\}$			बायाँ खुल्ला अन्तराल Left open Interval
(d)	$[a, b)$	$\{x : a \le x < b\}$			दायाँ खुल्ला अन्तराल Right open Interval

(a) खुल्ला अन्तराल : अन्तिम बिन्दुहरू a र b दुवै समावेश नहुने अन्तराललाई खुल्ला अन्तराल भनिन्छ । यसलाई (a, b) ले जनाइन्छ ।
Open Interval: An interval not containing both the end points a and b is known as an open interval. It is denoted by (a, b).

(b) बन्द अन्तराल: अन्तिम बिन्दुहरू a र b दुवै समावेश हुने अन्तराललाई बन्द अन्तराल भनिन्छ । यसलाई $[a, b]$ ले जनाइन्छ ।
Closed Interval: An interval containing both the end points a and b is known as a closed interval. It is denoted by $[a, b]$.

(c) बायाँ खुल्ला अन्तराल: अन्तिम बिन्दु a नपर्ने तर अन्तिम बिन्दु b पर्ने अन्तराललाई बायाँ खुल्ला अन्तराल भनिन्छ । यसलाई $(a, b]$ ले जनाइन्छ ।
Left open Interval: An interval not containing the end point a but containing the end point b is known as the left open interval. It is denoted by $(a, b]$.

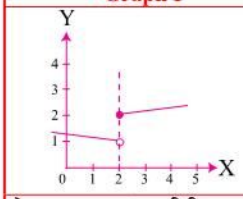
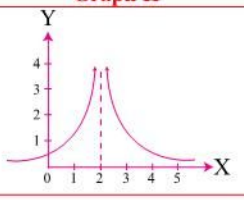
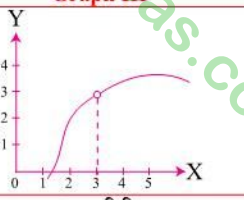
(d) दायाँ खुल्ला अन्तराल : अन्तिम बिन्दु a पर्ने तर अन्तिम बिन्दु b नपर्ने अन्तराललाई दायाँ खुल्ला अन्तराल भनिन्छ । यसलाई $[a, b)$ ले जनाइन्छ ।
Right open Interval: An interval containing the end point a but not containing the end point b is known as the right open interval. It is denoted by $[a, b)$.

2.4 लेखाचित्रमा विच्छिन्नताको खोजी (Investigation of discontinuity in graph)

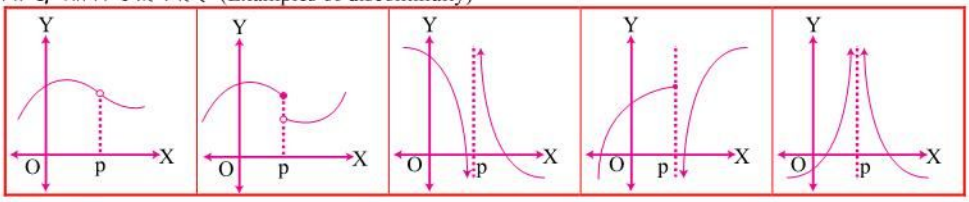
यदि कुनै पनि फलनको लेखाचित्रको कुनै बिन्दुमा छलाङ (Jump), छिद्र (hole), रिक्त (gap), उछाल (cusp) र टुटेको (break) छ भने सो बिन्दुमा विच्छिन्नता छ भनिन्छ ।
If the graph of any function has 'jump', hole, gap, cusp and break in a point then the function is said to be discontinuous at that point.

www.drmathnp.com

pushas.com.np

Graph I	Graph II	Graph III
		
यो फलन $x = 2$ मा विच्छिन्न छ । यसलाई छलाङ (Jump) विच्छिन्नता भनिन्छ । This function is discontinuous at $x = 2$. This is jump discontinuity	यो फलन $x = 2$ मा विच्छिन्न छ । यस्तो विच्छिन्नतालाई असीमित विच्छिन्नता भनिन्छ । This function is discontinuous at $x = 2$. This type of discontinuity is called infinite discontinuity.	यो फलन $x = 3$ मा विच्छिन्न छ । यस्तो विच्छिन्नतालाई निष्कासन विच्छिन्नता भनिन्छ । This function is discontinuous at $x = 3$. This type of discontinuity is called removable discontinuity.

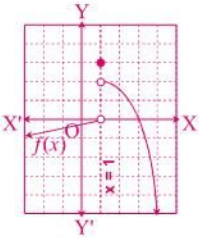
विच्छिन्नताका उदाहरणहरू (Examples of discontinuity)



2.5 बायाँबाट सीमान्त मान (Left hand limit)

जब x बायाँबाट बिन्दु a को नजिक पुग्दछ, यसलाइ $x \to a^-$ अथवा $x \to a - 0$ ले जनाइन्छ र यस्तो अवस्थामा फलन $f(x)$ का लागि बायाँबाट बिन्दु a मा सीमान्त मानलाई $\lim\limits_{x \to a^-} f(x)$ अथवा $\lim\limits_{x \to a-0} f(x)$ द्वारा परिभाषित गरिन्छ ।

When x approaches to a point a from the left, then it is denoted by $x \to a^-$ or $x \to a - 0$ and the left hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^-} f(x)$ or $\lim\limits_{x \to a-0} f(x)$.



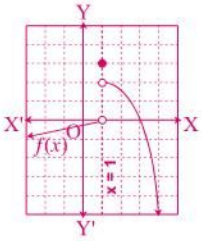
चित्रमा फलन $f(x)$ का लागि बायाँबाट बिन्दु $x = 1$ मा सीमान्त मान $= \lim\limits_{x \to 1^-} f(x) = 0$ हुन्छ ।

In the figure, the left hand limit of a function $f(x)$ at a point $x = 1$ is $= \lim\limits_{x \to 1^-} f(x) = 0$.

2.6 दायाँबाट सीमान्त मान (Right hand limit)

जब x दायाँबाट बिन्दु a को नजिक पुग्दछ, यसलाइ $x \to a^+$ अथवा $x \to a + 0$ ले जनाइन्छ र यस्तो अवस्थामा फलन $f(x)$ का लागि दायाँबाट बिन्दु a मा सीमान्त मानलाई $\lim\limits_{x \to a^+} f(x)$ अथवा $\lim\limits_{x \to a+0} f(x)$ द्वारा परिभाषित गरिन्छ ।

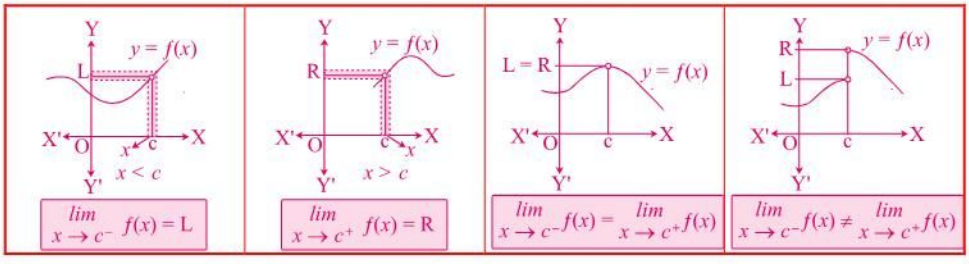
When x approaches to a point 'a' from the right, then it is denoted by $x \to a^+$ or $x \to a + 0$ and the right hand limit of a function $f(x)$ at a point $x = a$ is defined by $\lim\limits_{x \to a^+} f(x)$ or $\lim\limits_{x \to a+0} f(x)$.



चित्रमा फलन $f(x)$ का लागि दायाँबाट बिन्दु $x = 1$ मा सीमान्त मान $= \lim\limits_{x \to 1^+} f(x) = 2$ हुन्छ ।

In the figure, the right hand limit of a function $f(x)$ at a point $x = 1$ is $= \lim\limits_{x \to 1^+} f(x) = 2$.

बायाँबाट सीमान्त मान र दायाँबाट सीमान्त मानका उदाहरणहरू (Examples of left hand limit and right hand limit)

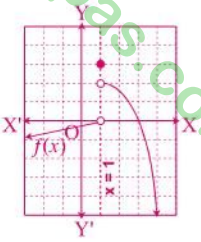


www.drmathnp.com

puspas.com.np

2.7 फलनको मान (Value of function)

यदि f एउटा फलन भए यसको क्षेत्रका हरेक सङ्ख्या x को सङ्गति प्रतिबिम्ब विस्तारमा $f(x)$ हुन्छ र 'f of x' अथवा 'f at x' भनेर पढिन्छ। $f(x)$ लाई फलन f को x मा हुने मान भनिन्छ।
If f is a function, then for each number x in its domain, the corresponding image in the range is designated by the symbol $f(x)$, read as 'f of x' or as 'f at x'. We refer to $f(x)$ as the value of function f at the number x.



चित्रमा फलन $f(x)$ का लागि बिन्दु $x = 1$ मा फलनको मान = $f(1) = 3$ हुन्छ।
In the figure, the value of the function $f(x)$ at a point $x = 1$ is $f(1) = 3$

2.8 निरन्तरता (Continuity)

कुनै फलन $x = a$ मा निरन्तर हुनको लागि निम्न अवस्थाहरू मान्य हुनु पर्दछ।
For a function to be continuous at $x = a$, the following conditions must be satisfied.

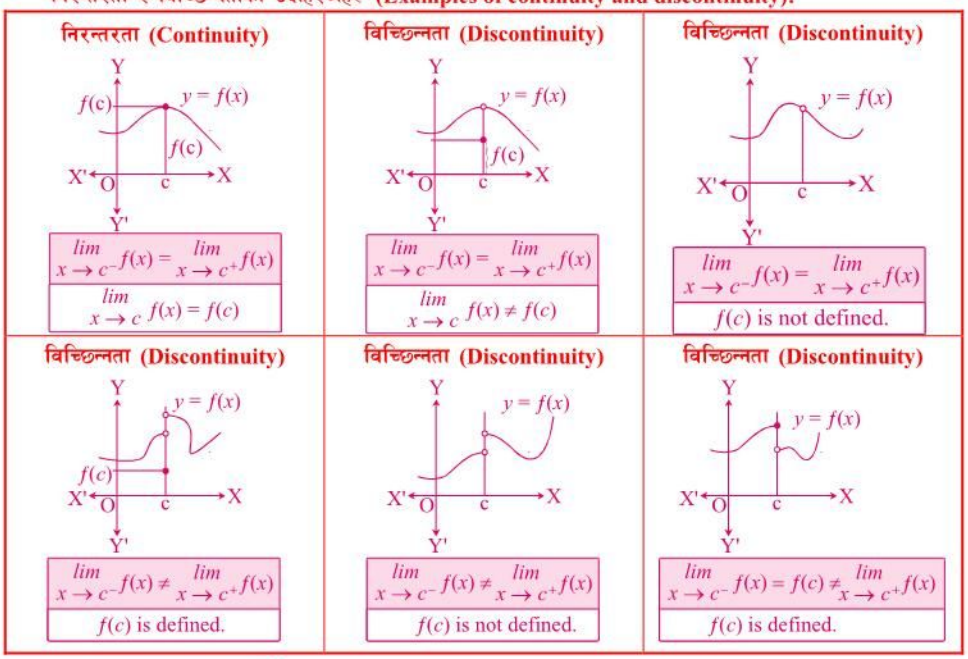
(i) $f(a)$ को अस्तित्व हुनुपर्छ वा $f(a)$ परिभाषीत हुनुपर्छ। ($f(a)$ must exists.)

(ii) बायाँ पक्षबाट सीमान्त मान = दायाँ पक्षबाट सीमान्त मान (Left hand limit = Right hand limit)

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

(iii) $\lim_{x \to a} f(x) = f(a)$ अर्को शब्दमा, कुनै बिन्दु 'a' मा फलनको मान र फलनको सीमान्त मान बराबर भए सो फलनलाई बिन्दु 'a' मा निरन्तर छ भनिन्छ।
In other words , if value of function and the limit value of the function at a point a are equal then the function is continuous at 'a'.

निरन्तरता र विच्छिन्नताका उदाहरणहरू (Examples of continuity and discontinuity):



SOLVED EXAMPLES

Example 1 : एउटा बिरुवा 44 cm अग्लो छ। यदि बिरुवाको उचाइ हरेक दिन 8 cm बढ्दै जान्छ भने 3 दिन पछि बिरुवाको उचाइ कति होला ? के यो वृद्धिले निरन्तरताको प्रतिनिधित्व गर्दछ ? उत्तरको पुष्टि गर्नुहोस्।
A plant is 44 cm tall. If the height of the plant is increasing 8 cm each day, what will be the height of the plant after 3 days? Does this increment represent continuity? Justify your answer.

Solution: Here, the height of the plant = 44 cm,
Rate of increment in each day = 8 cm
Now, the height of the plant after 3 days
= 44 cm + 8 cm + 8 cm + 8 cm = 68 cm
Since the plant is living thing, so it increases continuously with the same rate in the certain time.
Thus, the increasing height of plant is continuity.



www.drmathnp.com

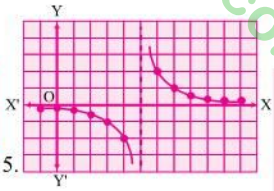
puspas.com.np

Example 2 : $f(x) = \frac{2}{x-5}$ को लेखाचित्र खिच्नुहोस् । कुन बिन्दुमा सो फलन विच्छिन्न हुन्छ ? पत्ता लगाउनुहोस् ।

Draw the graph of $f(x) = \frac{2}{x-5}$ and find, in which point the function is discontinuous?

Solution: Here, $f(x) = \frac{2}{x-5}$ i.e. $y = \frac{2}{x-5}$

x	-1	0	1	2	3	4	5	6	7	9
y	$-0.33\overline{}$	-0.4	-0.5	$-0.66\overline{}$	-1	-2	∞	2	1	0.5



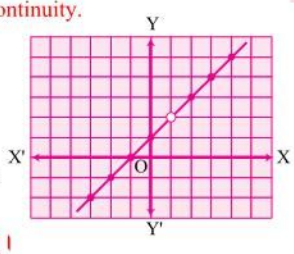
From the graph, we can say that the function is discontinuous at $x = 5$.

Example 3 : $g(x) = \frac{x^2-1}{x-1}$ को लेखाचित्र खिच्नुहोस् र विच्छिन्नताको बिन्दु खोजी गर्नुहोस् ।

Draw the graph of $g(x) = \frac{x^2-1}{x-1}$ and investigate the point of discontinuity.

Solution: Here, $g(x) = \frac{x^2-1}{x-1}$, i.e. $y = \frac{x^2-1}{x-1}$

x	-3	-2	-1	0	1	2	3	4
y	-2	-1	0	1	∞	3	4	5

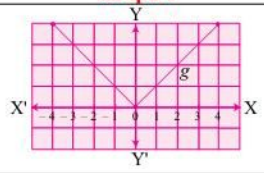


From the figure, we can say that the function $g(x)$ is discontinuous at $x = 1$.

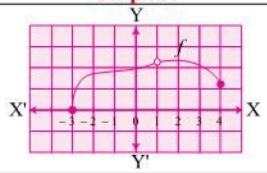
Example 4 : तलका लेखाचित्रमा फनलहरूको निरन्तरता वा विच्छिन्नताको खोजी गर्नुहोस् ।

Investigate continuity or discontinuity of the curve in the following function.

Graph I



Graph II



Solution:

(a) Here, the graph is drawn from $x = -4$ to $x = 4$.
There are no jumps or holes in the graph.
We say that the function g is continuous over $[-4, 4]$.
That is, this graph can be traced without lifting the pencil from the paper.

(b) Here, the graph is drawn from $x = -3$ to $x = 4$. There is a hole in the graph at $x = 1$.
We say that the function f is discontinuous at $x = 1$.
Similarly, the function f is continuous at $[-3, 1)$ and $(1, 4]$.
In other words, f is continuous at $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 2$, $x = 3$ etc.

Example 5 : सँगैको लेखाचित्र प्रयोग गरी तलका प्रश्नहरूको उत्तर दिनुहोस् ।

Use the adjoining graph and answer the following questions:

(a) पत्ता लगाउनुहोस् (Find) : $\lim_{x \to 1^+} f(x)$, $\lim_{x \to 1^-} f(x)$ & $\lim_{x \to 1} f(x)$

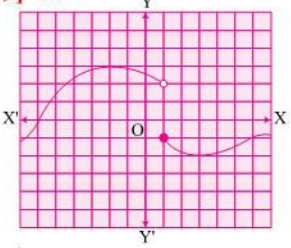
(b) $f(1)$ पत्ता लगाउनुहोस् । (Find $f(1)$.)

(c) के $x = 1$ मा f निरन्तर छ ? (Is f continuous at $x = 1$?)

(d) $\lim_{x \to -2} f(x)$ पत्ता लगाउनुहोस् । (Find $\lim_{x \to -2} f(x)$.)

(e) $f(-2)$ पत्ता लगाउनुहोस् । (Find $f(-2)$.)

(f) के $x = -2$ मा f निरन्तर छ ? (Is f continuous at $x = -2$?)



Solution :

(a) For $\lim_{x \to 1^+} f(x)$ This is the right hand limit.

So observe the graph from the right side then, $\lim_{x \to 1^+} f(x) = -1$

Again, for $\lim_{x \to 1^-} f(x)$

This is the left hand limit. So observe the graph from the left side then, $\lim_{x \to 1^-} f(x) = 2$

Now, $\lim_{x \to 1} f(x)$ does not exist.

That is, $f(x)$ is discontinuous at $x = 1$.

www.drmathnp.com

puspas.com.np

(b) From the graph, $f(1) = -1$

(c) $f(x)$ is not continuous at $x = 1$

(d) Here, $\lim_{x \to -2} f(x)$

From the graph, $\lim_{x \to -2} f(x) = 3$

(e) From the graph, $f(-2) = 3$

(f) From the graph, f is continuous at $x = -2$

Example 6 : फलन $f(x) = x^2$ को $x = 4$ मा निरन्तरता वा विच्छिन्नता पत्ता लगाउनुहोस् ।

Find the continuity or discontinuity of the function $f(x) = x^2$ at $x = 4$.

Solution : First method;

Here, $f(x) = x^2$ Functional value at $x = 4$ is ; $f(4) = 4^2 = 16$

For left hand limit of $f(x)$ at $x = 4$	For right hand limit of $f(x)$ at $x = 4$
Choose $x = 3.9, 3.99, 3.999, \ldots\ldots\ldots\ldots$ So, $f(3.9) = 3.9^2 = 15.21$ $f(3.99) = 3.99^2 = 15.92$ $f(3.999) = 3.999^2 = 15.99$ $\ldots\ldots\ldots\ldots$ $\therefore \lim_{x \to 4^-} f(x) = 16$	Choose $x = 4.1, 4.01, 4.001, \ldots\ldots\ldots\ldots$ So, $f(4.1) = 4.1^2 = 16.81$ $f(4.01) = 4.01^2 = 16.08$ $f(4.001) = 4.001^2 = 16.008$ $\ldots\ldots\ldots\ldots$ $\therefore \lim_{x \to 4^+} f(x) = 16$

This shows that, functional value = left hand limit = right hand limit.

Thus, the given function is continuous at $x = 4$.

Alternative Method:

Here, $f(x) = x^2$. We have to test the continuity at $x = 4$.

For left hand limit of $f(x)$ at $x = 4$	For right hand limit of $f(x)$ at $x = 4$
$\lim_{x \to 4^-} f(x)$ $= \lim_{x \to 4^-} x^2 = (4^-)^2 = 16$ $\therefore \lim_{x \to 4^-} f(x) = 16$	$\lim_{x \to 4^+} f(x)$ $= \lim_{x \to 4^+} x^2 = (4^+)^2 = 16$ $\therefore \lim_{x \to 4^+} f(x) = 16$

Again, functional value at $x = 4$ is given by $f(4) = 4^2 = 16$

$\therefore$ Left hand limit = right hand limit = functional value at $x = 4$

Thus, $f(x)$ is continuous at $x = 4$.

Example 7 : $f(x) = 2 - 3x^2$ को $x = 0$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Test the continuity or discontinuity of $f(x) = 2 - 3x^2$ at $x = 0$

Solution : First method;

Here, $f(x) = 2 - 3x^2$ Functional value at $x = 0$ is ; $f(0) = 2 - 3 \times 0^2 = 2 - 0 = 2$

For left hand limit of $f(x)$ at $x = 0$	For right hand limit of $f(x)$ at $x = 0$
Choose $x = -0.1, -0.01, -0.001, \ldots\ldots\ldots\ldots$ So, $f(-0.1) = 2 - 3(-0.1)^2 = 1.97$ $f(-0.01) = 2 - 3(-0.01)^2 = 1.999$ $f(-0.001) = 2 - 3(-0.001)^2 = 1.999$ $\ldots\ldots\ldots\ldots$ $\therefore \lim_{x \to 0^-} f(x) = 2$	Choose $x = 0.1, 0.01, 0.001, \ldots\ldots\ldots\ldots$ So, $f(0.1) = 2 - 3(0.1)^2 = 1.97$ $f(0.01) = 2 - 3(0.01)^2 = 1.999$ $f(0.001) = 2 - 3(0.001)^2 = 1.999$ $\ldots\ldots\ldots\ldots$ $\therefore \lim_{x \to 0^+} f(x) = 2$

This shows that, functional value = left hand limit = right hand limit.

Thus, the given function is continuous at $x = 0$.

Alternative method;

Here, $f(x) = 2 - 3x^2$

For left hand limit of $f(x)$ at $x = 0$	For right hand limit of $f(x)$ at $x = 0$
$\lim_{x \to 0^-} f(x)$ $= \lim_{x \to 0^-} (2 - 3x^2)$ $= 2 - 3 \times (0^-)^2$ $= 2$ $\therefore \lim_{x \to 0^-} f(x) = 2$	$\lim_{x \to 0^+} f(x)$ $= \lim_{x \to 0^+} (2 - 3x^2)$ $= 2 - 3(0^+)^2$ $= 2$ $\therefore \lim_{x \to 0^+} f(x) = 2$

Again, functional value at $x = 0$ is given by: $f(0) = 2 - 3 \times 0 = 2$

$\therefore$ Left-hand limit = right-hand limit = functional value at $x = 0$

Thus, $f(x)$ is continuous at $x = 0$.

www.drmathnp.com

puspas.com.np

Example 8 : $f(x)=\frac{1}{x-2}$ को $x \neq 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Test the continuity or discontinuity of $f(x)=\frac{1}{x-2}$ at $x \neq 2$.

Solution : **First method;**

Here, $f(x)=\frac{1}{x-2}$ and $x \neq 0$

Choose any one number other than 2, say 3.

Functional value at $x = 3$,

$f(3)=\frac{1}{3-2}=\frac{1}{1}=1$

For left hand limit of $f(x)$ at $x = 3$	For right hand limit of $f(x)$ at $x = 3$
Choose $x = 2.9, 2.99, 2.999, \ldots\ldots$	Choose $x = 3.01, 3.001, 3.0001, \ldots\ldots$
So, $f(2.9) = \frac{1}{2.9-2} = \frac{1}{0.9} = 1.11$	So, $f(3.01) = \frac{1}{3.01-2} = \frac{1}{1.01} = 0.990$
$f(2.99) = \frac{1}{2.99-2} = \frac{1}{0.99} = 1.01$	$f(3.001) = \frac{1}{3.001-2} = \frac{1}{1.001} = 0.999$
$f(2.999) = \frac{1}{2.999-2} = \frac{1}{0.999} = 1.001$	$f(3.0001) = \frac{1}{3.0001-2} = \frac{1}{1.0001} = 0.999$
$\ldots\ldots$	$\ldots\ldots$
$\therefore \lim_{x \to 3^-} f(x) = 1$	$\therefore \lim_{x \to 3^+} f(x) = 1$

This shows that, functional value = left hand limit = right hand limit.
Thus, the given function is continuous at $x = 3$.

Alternative method: Here, $f(x)=\frac{1}{x-2}$ except $x = 2$. So we take $x = a$, $a \neq 2$.

Functional value $= f(a)=\frac{1}{a-2}$

For left hand limit of $f(x)$ at $x = a$	For right hand limit of $f(x)$ at $x = a$
$\lim_{x \to a^-} \frac{1}{x-2} = \frac{1}{(a^-)-2} = \frac{1}{a-2}$	$\lim_{x \to a^+} \frac{1}{x-2} = \frac{1}{(a^+)-2} = \frac{1}{a-2}$
$\therefore \lim_{x \to a^-} f(x) = \frac{1}{a-2}$	$\therefore \lim_{x \to a^+} f(x) = \frac{1}{a-2}$

$\therefore$ Left hand limit = Right hand limit $= f(a)$, at $x = a \neq 2$. So, $f(x)$ is continuous at $x \neq 2$.

Example 9 : $f(x)=\frac{1}{1-x}$ को $x = 1$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Test the continuity or discontinuity of $f(x)=\frac{1}{1-x}$ at $x = 1$.

Solution : Here, $f(x)=\frac{1}{1-x}$

Functional value at $x = 1$,

Now, $f(1)=\frac{1}{1-1}=\frac{1}{0}=\infty$

Functional value is not defined.

Thus, $f(x)=\frac{1}{1-x}$ is discontinuous at $x = 1$.

Example 10 : $f(x)=\frac{x^2-9}{x-3}$ को $x = 3$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Test the continuity or discontinuity of $f(x)=\frac{x^2-9}{x-3}$ at $x = 3$.

Solution : Here, $f(x)=\frac{x^2-9}{x-3}$

Functional value at $x = 3$,

Now, $f(3)=\frac{3^2-9}{3-3}=\frac{0}{0}$ which is indeterminate form.

So, the given function is discontinuous at $x = 3$.

Example 11 : $f(x) \begin{matrix} = 2-x^2, x \leq 2 \\ = x-4, x > 2 \end{matrix}\Big\}$ को $x = 2$ मा निरन्तरता परीक्षण गर्नुहोस् ।

Examine the continuity of function $f(x) \begin{matrix} = 2-x^2, x \leq 2 \\ = x-4, x > 2 \end{matrix}\Big\}$ at $x = 2$

Solution: **First Method:**

For $x = 2$, the function is $f(x) = 2 - x^2$

Here, functional value at $x = 2$ is; $f(2) = 2 - 2^2 = 2 - 4 = -2$

www.drmathnp.com

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
Choose the numbers $x = 1.9, 1.99, 1.999, ...$ For the numbers less than 2, the function is $f(x) = 2 - x^2$ So, $f(1.9) = 2 - (1.9)^2 = -1.61$ $f(1.99) = 2 - (1.99)^2 = -1.9601$ $f(1.999) = 2 - (1.999)^2 = -1.996$ ..... $\therefore \lim_{x \to 2^-} f(x) = -2$	Choose the numbers $x = 2.01, 2.001, 2.0001, ...$ For the numbers greater than 2, the function is $f(x) = x - 4$ So, $f(2.01) = 2.01 - 4 = -1.99$ $f(2.001) = 2.001 - 4 = -1.999$ $f(2.0001) = 2.0001 - 4 = -1.9999$ ..... $\therefore \lim_{x \to 2^+} f(x) = -2$

This shows that, functional value = left hand limit = right hand limit
Thus, the given function is continuous at $x = 2$.

Alternative Method: Here, $f(x) \begin{matrix} = 2 - x^2, & x \le 2 \\ = x - 4, & x > 2 \end{matrix}$ } at $x = 2$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
$= \lim_{x \to 2^-} f(x)$ $= \lim_{x \to 2^-} (2 - x^2)$ $= [2 - (2^-)^2] = 2 - 4 = -2$ $\therefore \lim_{x \to 2^-} f(x) = -2$	$= \lim_{x \to 2^+} f(x)$ $= \lim_{x \to 2^+} (x - 4)$ $= (2^+) - 4 = -2$ $\therefore \lim_{x \to 2^+} f(x) = -2$

Again, the functional value of the function at $x = 2$ is, $f(2) = 2 - 2^2 = -2$

As left-hand limit = right hand limit = functional value
So the given function is continuous at point $x = 2$.

Example 12 : फलन $f(x) \begin{matrix} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{matrix}$ } को $x = 5$ मा निरन्तरता परीक्षण गर्नुहोस् ।

Examine the continuity of function $f(x) \begin{matrix} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{matrix}$ } at $x = 5$

Solution: **First Method**

Here, for $x = 5$, the function is $f(x) = x^2 + 2$
So, functional value at $x = 5$ is; $f(5) = 5^2 + 2 = 25 + 2 = 27$

For left hand limit of $f(x)$ at $x = 5$	For right hand limit of $f(x)$ at $x = 5$
Choose the numbers $x = 4.9, 4.99, 4.999, ...$ For the numbers less than 5, the function is $f(x) = x^2 + 2$ So, $f(4.9) = 4.9^2 + 2 = 26.01$ $f(4.99) = 4.99^2 + 2 = 26.90$ $f(4.999) = 4.999^2 + 2 = 26.99$ ..... $\therefore \lim_{x \to 5^-} f(x) = 27$	Choose the numbers $x = 5.01, 5.001, 5.0001, ...$ For the numbers greater than 5, the function is $f(x) = 3x + 12$ So, $f(5.01) = 3 \times 5.01 + 12 = 27.03$ $f(5.001) = 3 \times 5.001 + 12 = 27.003$ $f(5.0001) = 3 \times 5.0001 + 12 = 27.0003$ ..... $\therefore \lim_{x \to 5^+} f(x) = 27$

This shows that, functional value = left hand limit = right hand limit.
Thus, the given function is continuous at $x = 5$.

Alternative Method: Here, $f(x) = \begin{matrix} = x^2 + 2, & x \le 5 \\ = 3x + 12, & x > 5 \end{matrix}$ } at $x = 5$

For left hand limit of $f(x)$ at $x = 5$	For right hand limit of $f(x)$ at $x = 5$
$= \lim_{x \to 5^-} f(x)$ $= \lim_{x \to 5^-} (x^2 + 2)$ $= \{(5^-)^2 + 2\} = 25 + 2 = 27$ $\therefore \lim_{x \to 5^-} f(x) = 27$	$= \lim_{x \to 5^+} f(x)$ $= \lim_{x \to 5^+} (3x + 12)$ $= 3 \times (5^+) + 12 = 27$ $\therefore \lim_{x \to 5^+} f(x) = 27$

Again, functional value of the function $f(x)$ at $x = 5$ is; $f(5) = 5^2 + 2 = 25 + 2 = 27$
As the left-hand limit = right hand limit = value of the function
So the function $f(x)$ is continuous at point $x = 5$.

pusmas.com.np

www.drmathnp.com

puspas.com.np

Example 13 : फलन $f(x), x = 2$ मा निरन्तर हुन्छ भनी देखाउनुहोस् । **(Show that the function $f(x)$ is continuous at $x = 2$.)**

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ 3, & x = 2 \\ x + 1, & x > 2 \end{cases}$$

Solution: Here,

For functional value at $x = 2$, $f(2) = 3$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
Choose $x = 1.9, 1.99, 1.999, \ldots\ldots$ For the numbers less than 2, the function is $f(x) = 2x - 1$ So, $f(1.9) = 2 \times 1.9 - 1 = 2.8$ $f(1.99) = 2 \times 1.99 - 1 = 2.98$ $f(1.999) = 2 \times 1.999 - 1 = 2.998$ $\ldots\ldots$ $\therefore \lim_{x \to 2^-} f(x) = 3$	Choose $x = 2.01, 2.001, 2.0001, \ldots\ldots$ For the numbers greater than 2, the function is; $f(x) = x + 1$ So, $f(2.01) = 2.01 + 1 = 3.01$ $f(2.001) = 2.001 + 1 = 3.001$ $f(2.0001) = 2.0001 + 1 = 3.0001$ $\ldots\ldots$ $\therefore \lim_{x \to 2^+} f(x) = 3$

This shows that, functional value = LHL = RHL Proved.

Thus, the function is continuous at $x = 2$.

Alternative method

Functional value at $x = 2$, $f(2) = 3$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x - 1)$ $= 2 \times 2 - 1 = 3$ $\therefore \lim_{x \to 2^-} f(x) = 3$	$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x + 1)$ $= 2 + 1 = 3$ $\therefore \lim_{x \to 2^+} f(x) = 3$

So, functional value = LHL = RHL Proved.

Thus, the function is continuous at $x = 2$.

Example 14 : एउटा फलन $f(x)$ लाई निम्नानुसार परिभाषीत गरिएको छ **(A function $f(x)$ is defined as follows):**

$$f(x) = \begin{cases} 2x^2 + 1, & x < 3 \\ 5, & x = 3 \\ 6x + 1, & x > 3 \end{cases}$$

उल्लेख गरिएको बिन्दुमा माथिको फलनको निरन्तरता परीक्षण गर्नुहोस् । यदि $f(x)$ निरन्तर छैन भने कसरी निरन्तर बनाउन सकिन्छ ?

Investigate the continuity of the above function at the point mentioned. If $f(x)$ is not continuous, how can it be made continuous?

Solution: Here,

For functional value at $x = 3$, $f(3) = 5$

For left hand limit of $f(x)$ at $x = 3$	For right hand limit of $f(x)$ at $x = 3$
Choose $x = 2.9, 2.99, 2.999, \ldots\ldots$ For 'x' less than 3, the function is $f(x) = 2x^2 + 1$ So, $f(2.9) = 2 \times (2.9)^2 + 1 = 17.82$ $f(2.99) = 2 \times (2.99)^2 + 1 = 18.88$ $f(2.999) = 2 \times (2.999)^2 + 1 = 18.98$ $\ldots\ldots$ $\therefore \lim_{x \to 3^-} f(x) = 19$	Choose $x = 3.01, 3.001, 3.0001, \ldots\ldots$ For 'x' greater than 3, the function is $f(x) = 6x + 1$ So, $f(3.01) = 6 \times 3.01 + 1 = 19.06$ $f(3.001) = 6 \times 3.001 + 1 = 19.006$ $f(3.0001) = 6 \times 3.0001 + 1 = 19.0006$ $\ldots\ldots$ $\therefore \lim_{x \to 3^+} f(x) = 19$

$\therefore$ LHL = RHL So the limit exists. So, $\lim_{x \to 3} f(x) = 19$ but $f(3) = 5$

Hence the given function is discontinuous at $x = 3$.

Thus, to make above function continuous, $f(3)$ must be 19. i.e. $f(3) = 19$.

Alternative method:

For left hand limit of $f(x)$ at $x = 3$	For right hand limit of $f(x)$ at $x = 3$
$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (2x^2 + 1)$ $= 2 \times 3^2 + 1 = 19$ $\therefore \lim_{x \to 3^-} f(x) = 19$	$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 1)$ $= 6 \times 3 + 1 = 19$ $\therefore \lim_{x \to 3^+} f(x) = 19$

www.drmathnp.com

puspas.com.np

Since left hand limit = right hand limit, so the limit exists.

$\therefore \lim_{x \to 3} f(x) = 19$. But $\lim_{x \to 3} f(x) \neq f(3)$

Hence the given function $f(x)$ is discontinuous at $x = 3$.

Thus, to make above function continuous, $f(3)$ must be 19. i.e. $f(3) = 19$.

Example 15 : यदि दिइएको फलन $x = 2$ मा निरन्तर भए k को मान पत्ता लगाउनुहोस् ।

If the given function is continuous at $x = 2$ then find the value of k. $f(x) = \begin{cases} kx - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

Solution: Here, $f(x) = \begin{cases} kx - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

The functional value at $x = 2$; $\therefore \quad f(2) = 2 \times 2 - 3 = 1$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
Choose $x = 1.9, 1.99, 1.999, ...$ For x less than 2, the function is $f(x) = kx - 1$ So, $f(1.9) = 1.9k - 1$ $f(1.99) = 1.99k - 1$ $f(1.999) = 1.999k - 1$ ..... $\therefore \lim_{x \to 2^-} f(x) = 2k - 1$	Choose $x = 2.01, 2.001, 2.0001, ...$ For 'x' greater than 2, the function is $f(x) = 2x - 3$ So, $f(2.01) = 2 \times 2.01 - 3 = 1.02$ $f(2.001) = 2 \times 2.001 - 3 = 1.002$ $f(2.0001) = 2 \times 2.0001 - 3 = 1.0002$ ..... $\therefore \lim_{x \to 2^+} f(x) = 1$

For the function $f(x)$ to be continuous at $x = 2$,

We have, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

or, $2k - 1 = 1 = 1$ or, $2k = 2$ $\therefore \quad k = 1$

Thus, the value of k is 1.

Alternative method

Functional value at $x = 2$, $f(2) = 2 \times 2 - 3 = 1$

For left hand limit of $f(x)$ at $x = 2$	For right hand limit of $f(x)$ at $x = 2$
$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (kx - 1)$ $= 2k - 1$ $\therefore \lim_{x \to 2^-} f(x) = 2k - 1$	$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x - 3)$ $= 2 \times 2 - 3 = 1$ $\therefore \lim_{x \to 2^+} f(x) = 1$

If the function is continuous then, LHL = RHL = functional value

i.e. $2k - 1 = 1 = 1$ or, $2k = 2$ $\therefore \quad k = 1$

Thus, the value of k is 1.

SEE REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 3	SEE MODEL 2076	1

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):

A. निरन्तरता (Continuity)

B. विच्छिन्नता (Discontinuity)

C. खुल्ला अन्तराल (Open interval)

D. बन्द अन्तराल (Closed interval)

E. बायाँ पक्षबाट सीमान्त मान (Left hand limit)

F. दायाँ पक्षबाट सीमान्त मान (Right hand limit)

www.drmathnp.com
puspas.com.np

G. फलनको मान (Functional value)

2. निम्नलिखित निरन्तरता वा विच्छिन्नता के हो, निर्धारण गर्नुहोस् ।
Determine following whether as an example of continuity or discontinuity:

A. एउटा बिरुवाको वृद्धि (Growth of a plant)

B. सडकमा साइकल चलाउँदा चक्काले बनाउने बिन्दुपथ (Locus of the wheel while cycling in the street)

C. ड्रम बजाउँदा निस्कने आवाज (Sound from drum beating)

D. वास्तविक सङ्ख्याहरूको सङ्ख्या रेखा (Number line of Real numbers)

E. प्राकृतिक सङ्ख्याहरूको सङ्ख्या रेखा (Number line of natural numbers)

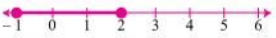
F. पूर्णाङ्कहरूको सङ्ख्या रेखा (Number line of integers)

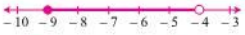
3A. अन्तराल (interval) $[a, b]$ को अर्थ के हुन्छ ? (What is the meaning of the interval $[a, b]$?)

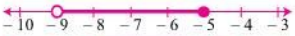
3B. अन्तराल (interval) (a, b) को अर्थ के हुन्छ ? (What do you meant by the interval (a, b)?)

4. तलका सङ्ख्या रेखाहरूका लागि अन्तराल सङ्केत लेख्नुहोस् । (Write the interval notation for the number line.)

A.


B.


C.


D.


E.


F.


G.


H.


5. तलका लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् । (Write the interval notation for the following.)

A. $-3 \le x \le 3$ **B.** $-4 < x < 4$ **C.** $\{x : -14 \le x < -11\}$ **D.** $\{x : 6 < x \le 20\}$

6A. बायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् । (Write down the notational representation of left hand limit.)

6B. दायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् । (Write down the notational representation of right hand limit.)

7A. $\lim\limits_{x \to a^-} f(x)$ लाई वाक्यमा लेख्नुहोस् । (Write $\lim\limits_{x \to a^-} f(x)$ in sentence.)

7B. $\lim\limits_{x \to a^+} f(x)$ लाई वाक्यमा लेख्नुहोस् । (Write $\lim\limits_{x \to a^+} f(x)$ in sentence.)

7C. $\lim\limits_{x \to a} f(x)$ लाई वाक्यमा लेख्नुहोस् । (Write $\lim\limits_{x \to a} f(x)$ in sentence.)

8A. फलन निरन्तर हुनको लागि आवश्यक शर्त के हो ?(What is the necessary condition for a function to be continuous?)

8B. बिन्दु $x = a$ मा फलन $f(x)$ को निरन्तरता हुने अवस्थालाई सङ्केतमा लेख्नुहोस् ।
Write the condition in notation of continuity of $f(x)$ at $x = a$.

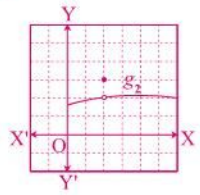
9A. सङ्ख्या रेखामा अविच्छिन्नता हुने सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers which is continuous in number line. **[SEE MODEL 2076]**

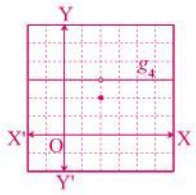
9B. सङ्ख्या रेखामा प्राकृतिक सङ्ख्याहरूको समूह र वास्तविक सङ्ख्याहरूको समूह मध्ये कुन समूहमा निरन्तरता हुन्छ ?
In which set of numbers, do you find continuity between set of natural numbers and set of real numbers in number line?

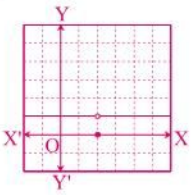
9C. सङ्ख्या रेखामा विच्छिन्नता जनाउने कुनै एक सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers whose number line is discontinuous.

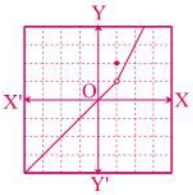
Model 2 *Concept Recall*

10. तलका फलनहरू कुन बिन्दुमा विच्छिन्नता भएका छन् ? (In which points, the following function are discontinuous?)

A.


B.


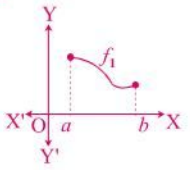
C.


D.


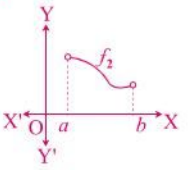
www.drmathnp.com

11. तलका फलनहरूको अन्तराल लेख्नुहोस् । (Find the interval of the following functions.)

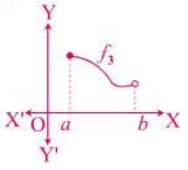
A.



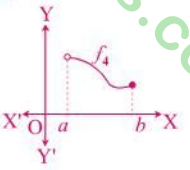
B.



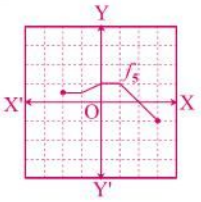
C.



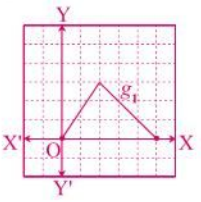
D.



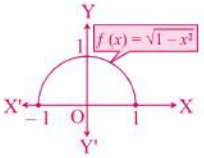
E.



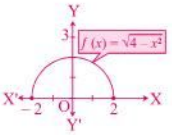
F.



G.



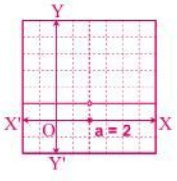
H.



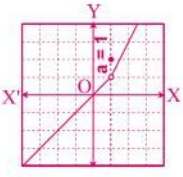
12. तलका लेखाचित्रहरूबाट फलनको $x = a$ मा बायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।

From the following graph, find the left hand limit of the function at $x = a$.

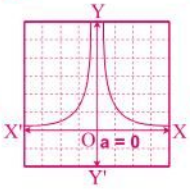
A.



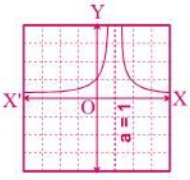
B.



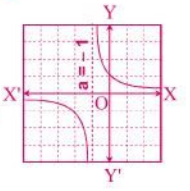
C.



D.



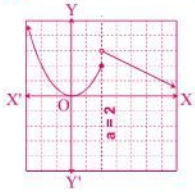
E.



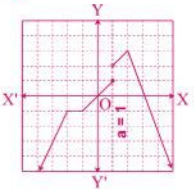
13. तलका लेखाचित्रहरूबाट फलनको $x = a$ मा दायाँ तिरको सीमान्त मान पत्ता लगाउनुहोस् ।

From the following graph, find the right hand limit of the function at $x = a$.

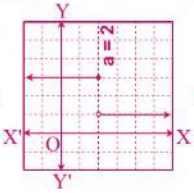
A.



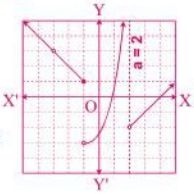
B.



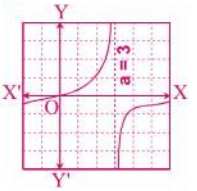
C.



D.



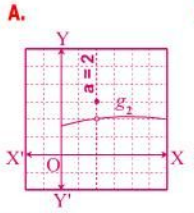
E.



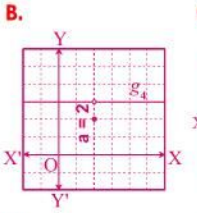
14. तलका लेखाचित्रहरूबाट फलनको $x = a$ मा मान पत्ता लगाउनुहोस् ।

Find the functional value of the function at $x = a$, from the following graph.)

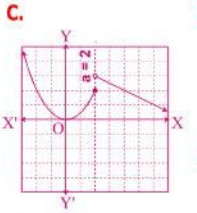
A.



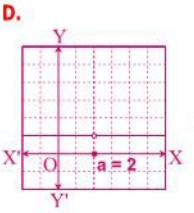
B.



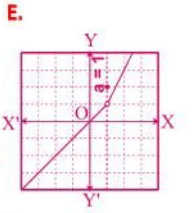
C.



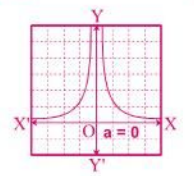
D.



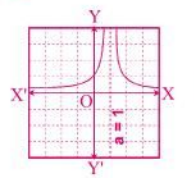
E.



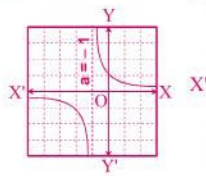
F.



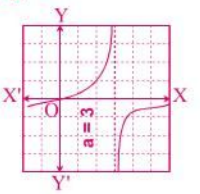
G.



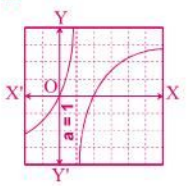
H.



I.



J.



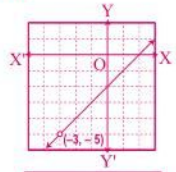
puspas.com.np

www.drmathnp.com

puspas.com.np

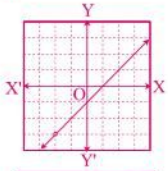
15. तलका लेखाचित्रहरूबाट निरन्तरता अन्तराल लेख्नुहोस् । (From the following graph, find the continuity interval.)

A.



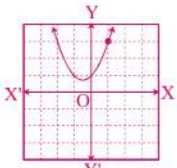
$f(x) = \frac{x^2 + x - 6}{x + 3}$

B.



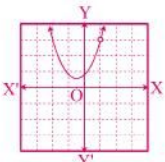
$f(x) = \frac{x^2 + x - 2}{x - 2}$

C.



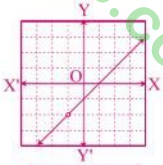
$g(x) = x^2 + x + 1$

D.



$f(x) = \frac{x^3 - 1}{x - 1}$

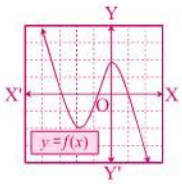
E.



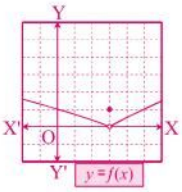
$f(x) = \frac{x^2 - 1}{x + 1}$

16. तलका लेखाचित्रबाट फलनहरू निरन्तर वा विच्छिन्न के छन् ? पत्ता लगाउनुहोस् ।
From the following graph, find whether the functions are continuous or discontinuous.

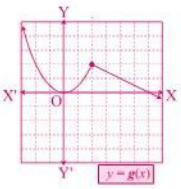
A.



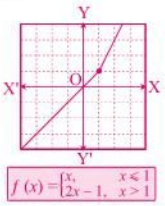
B.



C.

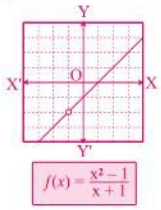


D.



$f(x) = \begin{cases} x, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$

E.



$f(x) = \frac{x^2 - 1}{x + 1}$

17. तलका फलनहरूको विच्छिन्नताको बिन्दु लेख्नुहोस् । (Write the point of discontinuity of following functions.)

(a) $f(x) = \frac{1}{x + 1}$ (b) $g(x) = \frac{1}{x - 2}$ (c) $h(x) = \frac{1}{x + 5}$

18. तलका अन्तरालहरूले के जनाउँदछन् ? शब्दमा लेख्नुहोस् । (What do the following intervals denote? Write in words.)

(a) [2, 3] (b) [2, 3) (c) (2, 3] (d) (2, 3)

19. तलका भनाइहरूलाई अन्तराल सङ्केतमा लेख्नुहोस् । (Write the following statements into interval notation.)

(a) 2 देखि 5 सम्मका सबै सङ्ख्याहरूको निरन्तर समावेश (All numbers from 2 to 5 are included continuously.)

(b) 2 देखि 5 भन्दा साना सबै सङ्ख्याहरू (All numbers from 2 and less than 5.)

(c) 2 भन्दा ठूला र 5 भन्दा साना सबै सङ्ख्याहरू (All numbers greater than 2 and less than 5.)

(d) 2 भन्दा ठूला 5 सम्मका सबै सङ्ख्याहरू (All numbers greater than 2 upto 5.)

B. LONG QUESTIONS

Model 1 *Problems on continuity based on value table.*

1. तालिका प्रयोग गरी निम्नलिखित मानहरू पत्ता लगाई $x = a$ मा फलनको निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Use table to find the following and examine continuity or discontinuity at $x = a$.

A. $f(x) = 3x - 1$

$a = 6$

x	5.8	5.9	5.99	5.999	.....	6.001	6.01	6.1	6.8
$f(x) = 3x - 1$	.....	.....	.....	.....	.....	.....	.....	.....	.....

?

(i) $\lim_{x \to 6^-} f(x)$ (ii) $\lim_{x \to 6^+} f(x)$ (iii) $\lim_{x \to 6} f(x)$ (iv) $f(6)$

B. $f(x) = x^3 - 2x - 2$

$a = 8$

x	7.8	7.9	7.99	7.999	.....	8.001	8.01	8.1	8.4
$f(x) = x^3 - 2x - 2$	.....	.....	.....	.....	.....	.....	.....	.....	.....

?

(i) $\lim_{x \to 8^-} f(x)$ (ii) $\lim_{x \to 8^+} f(x)$ (iii) $\lim_{x \to 8} f(x)$ (iv) $f(8)$

www.drmathnp.com

puspas.com.np

C. $f(x) = \dfrac{4x^2 - 1}{2x - 1}$

$a = 0.5$

x	0.49	0.499	0.4999	0.49999	.....	0.51	0.511	0.5111	0.51111
$f(x) = \dfrac{4x^2 - 1}{2x - 1}$	.....	.....	.....	.....	.....	.....	.....	.....	.....

?

(i) $\lim\limits_{x \to 0.5^-} f(x)$ (ii) $\lim\limits_{x \to 0.5^+} f(x)$ (iii) $\lim\limits_{x \to 0.5} f(x)$ (iv) $f(0.5)$

D. $f(x) = \dfrac{x^3 - 1}{x - 1}$

$a = 1$

x	0.9	0.99	0.999	0.9999	.....	1.00001	1.0001	1.001	1.01
$f(x) = \dfrac{x^3 - 1}{x - 1}$	.....	.....	.....	.....	.....	.....	.....	.....	.....

?

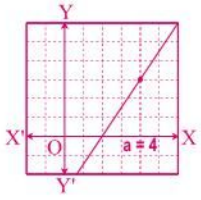
(i) $\lim\limits_{x \to 1^-} f(x)$ (ii) $\lim\limits_{x \to 1^+} f(x)$ (iii) $\lim\limits_{x \to 1} f(x)$ (iv) $f(1)$

Model 2 *Problems based on functional value, LHL, RHL and limit value.*

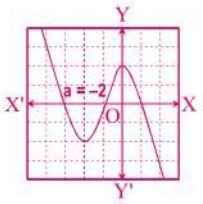
2. लेखाचित्र प्रयोग गरी दिइएको बिन्दुमा फलनको निरन्तरता परीक्षण गर्नुहोस् ।
Use graph to examine the continuity at the given point.

(i) $\lim\limits_{x \to a^-} f(x)$ (ii) $\lim\limits_{x \to a^+} f(x)$ (iii) $\lim\limits_{x \to a} f(x)$ (iv) $f(a)$

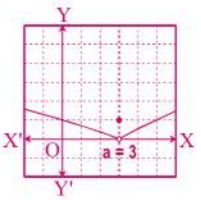
A.



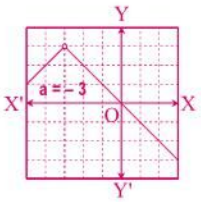
B.



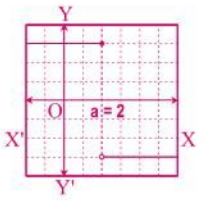
C.



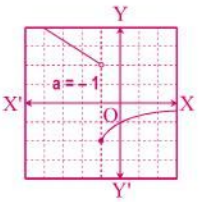
D.



E.

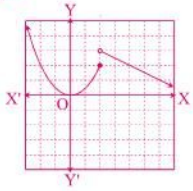


F.

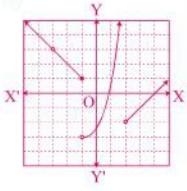


3. लेखाचित्र प्रयोग गरी (i) निरन्तरताको अन्तराल (ii) विच्छिन्नताका बिन्दुहरू र (iii) $x = 0$ मा निरन्तरता वा विच्छिन्नता पत्ता लगाउनुहोस् ।
Use the graph to determine (i) continuity interval (ii) points of discontinuities and (iii) examine continuity or discontinuity at $x = 0$.

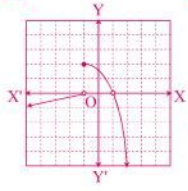
A.



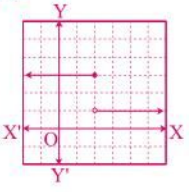
B.



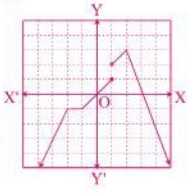
C.



D.



E.



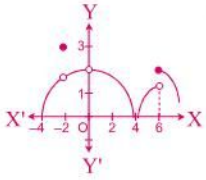
www.drmathnp.com

pushas.com.np

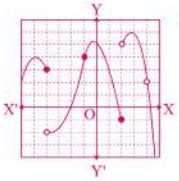
4. तलका चित्रमा आनुपातिक फलनहरूका लेखाचित्र दिइएको छ । उक्त लेखाचित्र कुनै एउटा कम्पनिको नाफा/नोक्सानसँग सम्बन्धित छ । उक्त वक्र कुन-कुन बिन्दुमा विच्छिन्न छ र किन ? कारण लेख्नुहोस् ।

In the following figure, the graph of rational function is shown. The graph represent the profit/loss of a company. At which points does the function is discontinuous and why ? Give reason.

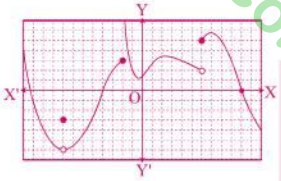
A.



B.



C.

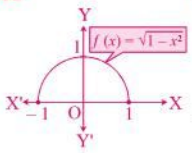


5. लेखाचित्र प्रयोग गरी $x = 0$ मा निम्नलिखित पत्ता लगाउनुहोस् । साथै, निरन्तरताको अन्तराल पनि पत्ता लगाउनुहोस् ।

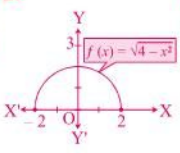
Using graph, find the following at $x = 0$. Also, find the continuity interval.

(i) $\lim_{x \to 0^-} f(x)$ (ii) $\lim_{x \to 0^+} f(x)$ (iii) $\lim_{x \to 0} f(x)$ (iv) $f(0)$

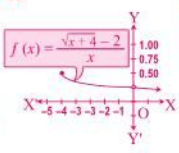
A.



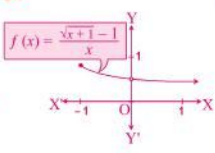
B.



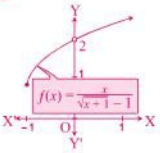
C.



D.



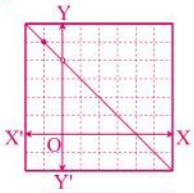
E.



6. लेखाचित्र प्रयोग गरी निम्नलिखित पत्ता लगाउनुहोस् । (Use the graph to find the following.)

(i) $\lim_{x \to a^-} f(x)$ (ii) $\lim_{x \to a^+} f(x)$ (iii) $\lim_{x \to a} f(x)$ (iv) $f(a)$

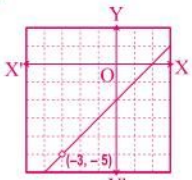
A.



$f(x) = \frac{4x - x^2}{x}$

$a = 0$

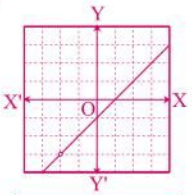
B.



$f(x) = \frac{x^2 + x - 6}{x + 3}$

$a = -3$

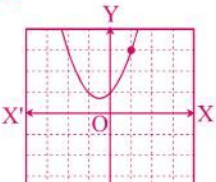
C.



$f(x) = \frac{x^2 + x - 2}{x - 2}$

$a = -2$

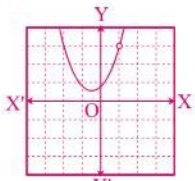
D.



$g(x) = x^2 + x + 1$

$a = 1$

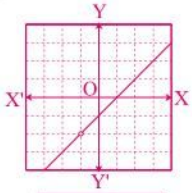
E.



$f(x) = \frac{x^3 - 1}{x - 1}$

$a = 1$

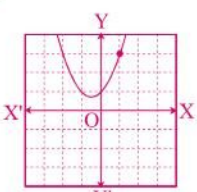
F.



$f(x) = \frac{x^2 - 1}{x + 1}$

$a = 1$

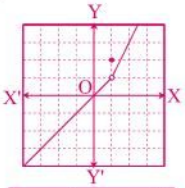
G.



$g(x) = x^2 + x + 1$

$a = 2$

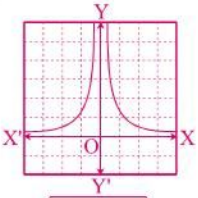
H.



$f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

$a = 1$

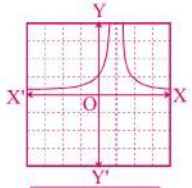
I.



$f(x) = \frac{1}{x^2}$

$a = 0$

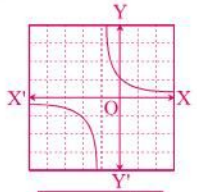
J.



$f(x) = \frac{1}{(x - 1)^2}$

$a = 1$

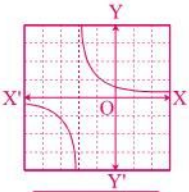
K.



$f(x) = \frac{1}{2x - 1}$

$a = -1$

L.



$f(x) = \frac{1}{x + 2}$

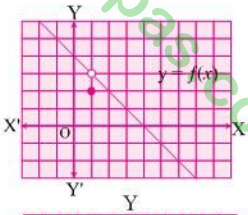
$a = -2$

www.drmathnp.com

puspas.com.np

7. यदि $f(x) = \begin{cases} 4-x, & x \neq 1 \\ 2, & x = 1 \end{cases}$ भए निम्नलिखित पत्ता लगाउनुहोस् ।

If $f(x) = \begin{cases} 4-x, & x \neq 1 \\ 2, & x = 1 \end{cases}$ then find the following.

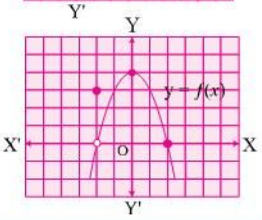


(i) $\lim_{x \to 1^+} f(x)$ (ii) $\lim_{x \to 1^-} f(x)$ (iii) $f(1)$

(iv) के $x = 1$ मा f निरन्तर छ ? (Is f continuous at $x = 1$?)

8. यदि $f(x) = \begin{cases} 4-x^2, & x \neq -2 \\ 3, & x = -2 \end{cases}$ भए निम्नलिखित पत्ता लगाउनुहोस् ।

If $f(x) = \begin{cases} 4-x^2, & x \neq -2 \\ 3, & x = -2 \end{cases}$ then find the following.



(i) $\lim_{x \to -2^+} f(x)$ (ii) $\lim_{x \to -2^-} f(x)$ (iii) $f(-2)$

(iv) के $x = -2$ मा f निरन्तर छ ? (Is f continuous at $x = -2$?)

Model 3 *Problems based on testing continuity.*

9A. यदि $f(x) = 2x + 1$ र $x \in$ R भए, (If $f(x) = 2x + 1$ and $x \in$ R then,)

(i) $f(0.99)$, $f(0.999)$ र $f(0.9999)$ पत्ता लगाउनुहोस् । (Find $f(0.99)$, $f(0.999)$ and $f(0.9999)$.)

(ii) $f(1.001)$, $f(1.0001)$ र $f(1.00001)$ पत्ता लगाउनुहोस् । (Find $f(1.001)$, $f(1.0001)$ and $f(1.00001)$.)

(iii) $\lim_{x \to 1} f(x)$ र $f(1)$ पत्ता लगाउनुहोस् । (Find $\lim_{x \to 1} f(x)$ and $f(1)$.)

(iv) के निष्कर्ष निकाल्न सकिन्छ ? (What conclusion can be drawn?)

9B. यदि $f(x) = 1 - 2x$ र $x \in$ R भए, (If $f(x) = 1 - 2x$ and $x \in$ R then,)

(i) $f(2.001)$, $f(2.0001)$ र $f(2.00001)$ पत्ता लगाउनुहोस् । (Find $f(2.001)$, $f(2.0001)$ and $f(2.00001)$.)

(ii) $f(1.999)$, $f(1.9999)$ र $f(1.99999)$ पत्ता लगाउनुहोस् । (Find $f(1.999)$, $f(1.9999)$ and $f(1.99999)$.)

(iii) $\lim_{x \to 2} f(x)$ र $f(2)$ पत्ता लगाउनुहोस् । (Find $\lim_{x \to 2} f(x)$ and $f(2)$.)

(iv) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? (Is $f(x)$ continuous at $x = 2$?)

9C. यदि $f(x) = 2x - 3$ र $x \in$ R भए, (If $f(x) = 2x - 3$ and $x \in$ R then,)

(i) $f(1.99)$, $f(1.999)$ र $f(1.9999)$ पत्ता लगाउनुहोस् । (Find $f(1.99)$, $f(1.999)$ and $f(1.9999)$.)

(ii) $f(2.001)$, $f(2.0001)$ र $f(2.00001)$ पत्ता लगाउनुहोस् । (Find $f(2.001)$, $f(2.0001)$ and $f(2.00001)$.)

(iii) $\lim_{x \to 2} f(x)$ र $f(2)$ पत्ता लगाउनुहोस् । (Find $\lim_{x \to 2} f(x)$ and $f(2)$.)

(iv) के निष्कर्ष निकाल्न सकिन्छ ? (What conclusion can be drawn?)

9D. वास्तविक फलन $f(x) = 2x + 3$ को लागि, (For a real valued function $f(x) = 2x + 3$,) **[SEE MODEL 2076]**

(i) $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ र $f(3)$ का मानहरू पत्ता लगाउनुहोस् ।
Find the values of $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ and $f(3)$.

(ii) के यो फलन $x = 3$ मा अविच्छिन्न हुन्छ ? (Is this function continuous at $x = 3$?)

10A. यदि $f(x) = 4x + 5$ एउटा वास्तविक मान भएको फलन भए, (If $f(x) = 4x + 5$ is a real valued function,)

(i) $x = 1.99$ र 1.999 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 1.99$ and 1.999?)

(ii) $x = 2.01$ र 2.001 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 2.01$ and 2.001?)

(iii) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन् ? (What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?)

(iv) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? (Is the function continuous at $x = 2$?)

10B. यदि $f(x) = 3x + 2$ एउटा वास्तविक मान भएको फलन भए, (If $f(x) = 3x + 2$ is a real valued function,)

(i) $x = 1.99$ र 1.999 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 1.99$ and 1.999?)

(ii) $x = 2.01$ र 2.001 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 2.01$ and 2.001?)

(iii) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन् ? (What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?)

(iv) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? (Is the function continuous at $x = 2$?)

www.drmathnp.com

pushpas.com.np

10C. यदि $f(x) = 4x + 7$ एउटा वास्तविक मान भएको फलन भए, (If $f(x) = 4x + 7$ is a real valued function,)

(i) $x = 1.99$ र 1.999 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 1.99$ and 1.999?)

(ii) $x = 2.01$ र 2.001 मा $f(x)$ का मानहरू के के हुन्छन् ? (What are the values of $f(x)$ at $x = 2.01$ and 2.001?)

(iii) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन् ? (What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?)

(iv) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? (Is the function continuous at $x = 2$?)

10D. यदि $f(x) = \begin{cases} x+2, & 1 \le x \le 2 \\ 4x-4, & x > 2 \end{cases}$ भए, (If $f(x) = \begin{cases} x+2, & 1 \le x \le 2 \\ 4x-4, & x > 2 \end{cases}$ then,)

(a) $x = 1.9999$ मा $f(x)$ को मान पत्ता लगाउनुहोस् । (For $x = 1.9999$, find the value of $f(x)$.)

(b) $x = 2.0001$ मा $f(x)$ को मान पत्ता लगाउनुहोस् । (For $x = 2.0001$, find the value of $f(x)$.)

(iii) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन् ? (What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?)

(d) के फलन $x = 2$ मा निरन्तर छ ? (Is the function continuous at $x = 2$?)

Model 4 Problems based on testing continuity or discontinuity.

11A. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3, & x > 0 \end{cases}$ को $x = 0$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3, & x > 0 \end{cases}$ at $x = 0$ by calculating left hand limit, right hand limit and functional value.

11B. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 4x-1, & x < 1 \\ 7x, & x \ge 1 \end{cases}$ को $x = 1$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of $f(x) = \begin{cases} 4x-1, & x < 1 \\ 7x, & x \ge 1 \end{cases}$ at $x = 1$ by calculating left hand limit, right hand limit and functional value.

11C. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 4x+3, & x \le 2 \\ 5x+1, & x > 2 \end{cases}$ को $x = 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of $f(x) = \begin{cases} 4x+3, & x \le 2 \\ 5x+1, & x > 2 \end{cases}$ at $x = 2$ by calculating left hand limit, right hand limit and functional value.

11D. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} x^2+1, & x \le 3 \\ 2x+4, & x > 3 \end{cases}$ को $x = 3$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

Examine the continuity or discontinuity of $f(x) = \begin{cases} x^2+1, & x \le 3 \\ 2x+4, & x > 3 \end{cases}$ at $x = 3$ by calculating left hand limit, right hand limit and functional value.

12A. यदि $x \le 2$ हुँदा $f(x) = 2x - 1$ र $x > 2$ हुँदा $f(x) = x + 1$ भए $x = 2$ मा फलनको निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

If $f(x) = 2x - 1$ when $x \le 2$ and $f(x) = x + 1$ when $x > 2$ then examine the continuity or discontinuity of the function at $x = 2$.

12B. यदि $x \le 2$ हुँदा $f(x) = 5x + 2$ र $x > 2$ हुँदा $f(x) = 6x$ भए $x = 2$ मा फलनको निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

If $f(x) = 5x + 2$ when $x \le 2$ and $f(x) = 6x$ when $x > 2$ then examine the continuity or discontinuity of the function at $x = 2$.

Model 5 Problems based on testing discontinuity.

13A. फलन $f(x) = \dfrac{x^2 + 3x}{x - 1}$; $x \in$ R, $x = 1$ मा विच्छिन्न छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x) = \dfrac{x^2 + 3x}{x - 1}$; $x \in$ R, is discontinuous at $x = 1$.

www.drmathnp.com

puspas.com.np

13B. फलन $f(x)=\frac{4x-x^2}{x}$; $x \in$ R, $x=0$ मा विच्छिन्न छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\frac{4x-x^2}{x}$; $x \in$ R, is discontinuous at $x=0$.

13C. फलन $f(x)=\frac{x^2+x-2}{x-2}$; $x \in$ R, $x=2$ मा विच्छिन्न छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\frac{x^2+x-2}{x-2}$; $x \in$ R, is discontinuous at $x=2$.

13D. फलन $f(x)=\frac{x^2+x-6}{x+3}$ बिन्दु $x=-3$ मा निरन्तर छैन भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\frac{x^2+x-6}{x+3}$ is not continuous at a point $x=-3$.

ANSWERS

A. VERY SHORT QUESTIONS

2A. continuity **2B.** continuity **2C.** discontinuity **2D.** continuity **2E.** discontinuity **2F.** discontinuity
4A. $(0, 5)$ **4B.** $[-1, 2]$ **4C.** $[-9, -4)$ **4D.** $(-9, -5]$ **4E.** $[x, x+h]$
4F. $(x, x+h]$ **4G.** $(-\infty, b)$ **4H.** $(-\infty, a]$ **5A.** $[-3, 3]$ **5B.** $(-4, 4)$ **5C.** $[-14, -11)$
5D. $(6, 20]$ **6A.** $\lim_{x \to a^-} f(x)$ **6B.** $\lim_{x \to a^+} f(x)$ **8A.** $\lim_{x \to a} f(x) = f(a)$ **8B.** $\lim_{x \to a} f(x) = f(a)$
9A. Real number **9B.** Real number **10A.** $x=2$ **10B.** $x=2$ **10C.** $x=2$ **10D.** $x=1$ **11A** $[a, b]$ **11B.** (a, b)
11C. $[a, b)$ **11D.** $(a, b]$ **11E.** $[-2, 3]$ **11F.** $[0, 5]$ **11G.** $[-1, 1]$ **11H.** $[-2, 2]$ **12A.** 1
12B. 1 **12C.** ∞ **12D.** ∞ **12E.** $-\infty$ **13A.** 3 **13B.** 2 **13C.** 1 **13D.** -2
13E. $-\infty$ **14A.** 3 **14B.** 2 **14C.** 2 **14D.** 0 **14E** 2 **14F-14J** DNE
15A. $(-\infty, -3), (-3, \infty)$ **15B.** $(-\infty, -2), (-2, \infty)$ **15C.** $(-\infty, \infty)$ **15D.** $(-\infty, 1), (1, \infty)$ **15E.** $(-\infty, -1), (-1, \infty)$
16A. continuous **16B.** discontinuous **16C.** continuous **16D.** continuous **16E.** discontinuous **17A.** $x=-1$
17B. $x=2$ **17C.** $x=-5$ **19A.** $[2, 5]$ **19B.** $[2, 5)$ **19C.** $(2, 5)$ **19D.** $(2, 5]$

B. LONG QUESTIONS

1A. (i) 17 (ii) 17 (iii) 17 (iv) 17; continuity at $x=6$.
1B. (i) 494 (ii) 494 (iii) 494 (iv) 494 ; continuity at $x=8$
1C. (i) 2 (ii) 2 (iii) 2 (iv) Undefined; Discontinuity at $x=0.5$
1D. (i) 3 (ii) 3 (iii) 3 (iv) undefined; Discontinuity at $x=1$
2A. (i) 3 (ii) 3 (iii) 3 (iv) 3 ; continuous
2B. (i) -2 (ii) -2 (iii) -2 (iv) -2 ; continuous
2C. (i) 0 (ii) 0 (iii) 0 (iv) 1 ; Discontinuous
2D. (i) 3 (ii) 3 (iii) 3 (iv) DNE ; Discontinuous
2E. (i) 3 (ii) -3 (iii) DNE (iv) 3 ; Discontinuous
2F. (i) 2 (ii) -2 (iii) DNE (iv) -2 ; Discontinuous
3A. (i) $(-\infty, 2], (2, \infty)$ (ii) $x=2$ (iii) continuous
3B. (i) $(-\infty, -3), (-3, -1], (-1, 2), (2, \infty)$ (ii) $x=-3, x=-1, x=2$ (iii) continuous
3C. (i) $(-\infty, -1), [-1, 1), (1, \infty)$ (ii) $x=-1, x=1$ (iii) continuous
3D. (i) $(-\infty, 2], (-2, \infty)$ (ii) $x=2$ (iii) continuous
3E. (i) $(-\infty, 1], (1, \infty)$ (ii) $x=1$ (iii) continuous
4A. $x=-2, x=0, x=4$ and $x=6$ **4B.** $x=-4, x=2$ and $x=4$ **4C.** $x=-8, x=-2$ and $x=6$
5A. (i) 1 (ii) 1 (iii) 1 (iv) 1 ; $[-1, 1]$
5B. (i) 2 (ii) 2 (iii) 2 (iv) 2 ; $[-2, 2]$
5C. (i) 0.25 (ii) 0.25 (iii) 0.25 (iv) DNE ; $[-4, 0), (0, \infty)$
5D. (i) 0.5 (ii) 0.5 (iii) 0.5 (iv) DNE; $[-1, 0), (0, \infty)$
5E. (i) 2 (ii) 2 (iii) 2 (iv) DNE ; $[-1, 0), (0, \infty)$
6A. (i) 4 (ii) 4 (iii) 4 (iv) DNE **6B.** (i) -5 (ii) -5 (iii) -5 (iv) DNE
6C. (i) -3 (ii) -3 (iii) -3 (iv) DNE **6D.** (i) 3 (ii) 3 (iii) 3 (iv) 3
6E. (i) 3 (ii) 3 (iii) 3 (iv) DNE **6F.** (i) 0 (ii) 0 (iii) 0 (iv) 0
6G. (i) 1 (ii) 1 (iii) 1 (iv) 0 **6H.** (i) 1 (ii) 1 (iii) 1 (iv) 2
6I. (i) ∞ (ii) ∞ (iii) ∞ (iv) DNE **6J.** (i) ∞ (ii) ∞ (iii) ∞ (iv) DNE
6K. (i) $-\infty$ (ii) ∞ (iii) DNE (iv) DNE **6L.** (i) $-\infty$ (ii) ∞ (iii) DNE (iv) DNE
7. (i) 3 (ii) 3 (iii) 2 (iv) No **8.** (i) 0 (ii) 0 (iii) 3 (iv) No
9A. (i) 2.98, 2.998, 2.9998 (ii) 3.002, 3.0002, 3.00002 (iii) 3, 3 (iv) continuous at $x=1$
9B. (i) $-3.002, -3.0002, -3.00002$ (ii) $-2.998, -2.9998, -2.99998$ (iii) $-3, -3$ (iv) Yes, continuous at $x=2$
9C. (i) 0.98, 0.998, 0.9998 (ii) 1.002, 1.0002, 1.00002 (iii) 1, 1 (iv) $f(x)$ is continuous at $x=2$
9D. (i) 8.9 , 8.98, 9.02, 9.1 and $f(3)=9$ (ii) $f(x)$ is continuous at $x=3$
10A. (i) 12.96, 12.996 (ii) 13.04, 13.004 (iii) 13, 13 (iv) Yes
10B. (i) 7.97, 7.997 (ii) 8.03, 8.003 (iii) 8, 8 (iv) Yes
10C. (i) 14.96, 14.996 (ii) 15.04, 15.004 (iii) 15, 15 (iv) Yes
10D. (i) 3.9999 (ii) 4.0004 (iii) 4, 4 (iv) Yes
11A. Continuous **11B.** Discontinuous **11C.** Continuous **11D.** Continuous
12A. Continuous at $x=2$ **12B.** Continuous at $x=2$

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 1

LONG QUESTIONS

Model 1 *Problems based on testing continuity or discontinuity.*

1. सीमान्त मान र फलनको मान पत्ता लगाउनुहोस् । साथै, तोकिएको बिन्दुमा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Find the limit value and functional value. Also, examine the continuity or discontinuity at the point mentioned.

A. $f(x) = 3x^2 - 8x + 6$ at $x = 1$ **B.** $f(x) = 3x^2 + 2x - 1$ at $x = 2$

C. $f(x) = 4x^2 + 2x - 9$ at $x = \frac{1}{2}$ **D.** $f(x) = 4x^2 - 4x + 1$ at $x = \frac{1}{2}$

2. सीमान्त मान र फलनको मान पत्ता लगाउनुहोस् । साथै, तोकिएको बिन्दुमा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Find the limit value and functional value. Also, examine the continuity or discontinuity at the point mentioned.

A. $f(x) = \frac{x^2-4}{x-2}$ at $x = 2$ **B.** $f(x) = \frac{x^2-16}{x-4}$ at $x = 4$ **C.** $f(x) = \frac{x^2-4}{x+2}$ at $x = 2$

D. $f(x) = \frac{x^2-9}{x+3}$ at $x = 3$ **E.** $f(x) = \frac{x^2-1}{x-1}$ at $x = 1$ **F.** $f(x) = \frac{x^2-25}{x-5}$ at $x = 5$

G. $f(x) = \frac{3x+2}{2x+3}$ at $x = -1$ **H.** $f(x) = \frac{x+1}{2x+5}$ at $x = 2$ **I.** $f(x) = \frac{3x+2}{2x-1}$ at $x = 1$

3. तलका फलनहरूको निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of the following functions.

A. $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$ at $x = 3$ **B.** $f(x) = \begin{cases} \frac{x^2-4x}{x-4}, & x \neq 4 \\ 4, & x = 4 \end{cases}$ at $x = 4$

C. $f(x) = \begin{cases} \frac{x^2-5x}{x-5}, & x \neq 5 \\ 5, & x = 5 \end{cases}$ at $x = 5$ **D.** $f(x) = \begin{cases} \frac{x^2-25}{x-5}, & x \neq 5 \\ 10, & x = 5 \end{cases}$ at $x = 5$

E. $f(x) = \begin{cases} \frac{4x^2-16}{2x-4}, & x \neq 2 \\ 8, & x = 2 \end{cases}$ at $x = 2$ **F.** $f(x) = \begin{cases} \frac{x^2-3x+2}{x-2}, & x \neq 2 \\ 1, & x = 2 \end{cases}$ at $x = 2$

G. $f(x) = \begin{cases} \frac{x^2-3x}{x-3}, & x \neq 3 \\ 3, & x = 3 \end{cases}$ at $x = 3$ **H.** $f(x) = \begin{cases} \frac{x^2-100}{x-10}, & x \neq 10 \\ 20, & x = 10 \end{cases}$ at $x = 10$

I. $f(x) = \begin{cases} \frac{x^2-7x}{x-7}, & x \neq 7 \\ 7, & x = 7 \end{cases}$ at $x = 7$ **J.** $f(x) = \begin{cases} \frac{x^2-2x}{x-2}, & x \neq 2 \\ 2, & x = 2 \end{cases}$ at $x = 2$

4A. फलन $f(x) = \begin{cases} 2x-1, & x < 2 \\ 3, & x = 2 \\ x+1, & x > 2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x) = \begin{cases} 2x-1, & x < 2 \\ 3, & x = 2 \\ x+1, & x > 2 \end{cases}$ is continuous at a point $x = 2$.

4B. फलन $f(x) = \begin{cases} 3x-1, & x < 2 \\ 5, & x = 2 \\ 2x+1, & x > 2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x) = \begin{cases} 3x-1, & x < 2 \\ 5, & x = 2 \\ 2x+1, & x > 2 \end{cases}$ is continuous at a point $x = 2$.

4C. फलन $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x-1, & x > 1 \end{cases}$ बिन्दु $x = 1$ मा निरन्तर छैन भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x-1, & x > 1 \end{cases}$ is not continuous at a point $x = 1$.

4D. फलन $f(x) = \begin{cases} 2x+1, & x < 1 \\ 3, & x = 1 \\ 4x-1, & x > 1 \end{cases}$ बिन्दु $x = 1$ मा निरन्तर छ भनी प्रमाणित गर्नुहोस् ।

www.drmathnp.com

puspas.com.np

Prove that the function $f(x)=\begin{cases}2x+1, & x<1\\ 3, & x=1\\ 4x-1, & x>1\end{cases}$ is continuous at a point $x=1$.

4E. फलन $f(x)=\begin{cases}-x, & x<0\\ x, & 0<x<1\\ 2-x, & x\geq 1\end{cases}$ बिन्दु $x=1$ मा निरन्तर छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\begin{cases}-x, & x<0\\ x, & 0<x<1\\ 2-x, & x\geq 1\end{cases}$ is continuous at a point $x=1$.

4F. फलन $f(x)=\begin{cases}2x+1, & x<1\\ 2x, & x=1\\ 3x, & x>1\end{cases}$ बिन्दु $x=1$ मा विच्छिन्न छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\begin{cases}2x+1, & x<1\\ 2x, & x=1\\ 3x, & x>1\end{cases}$ is discontinuous at a point $x=1$.

4G. फलन $f(x)=\begin{cases}2x^2+1, & x<2\\ 8, & x=2\\ 4x+1, & x>2\end{cases}$ बिन्दु $x=2$ मा विच्छिन्न छ भनी प्रमाणित गर्नुहोस् ।

Prove that the function $f(x)=\begin{cases}2x^2+1, & x<2\\ 8, & x=2\\ 4x+1, & x>2\end{cases}$ is discontinuous at a point $x=2$.

Model 2 Problems based on finding value of letters.

5A. यदि फलन $f(x)=\begin{cases}3-x, & x\leq 0\\ k, & x>0\end{cases}$ बिन्दु $x=0$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}3-x, & x\leq 0\\ k, & x>0\end{cases}$ is continuous at a point $x=0$ then find the value of k.

5B. यदि फलन $f(x)=\begin{cases}4x-k, & x<1\\ 7x, & x\geq 1\end{cases}$ बिन्दु $x=1$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}4x-k, & x<1\\ 7x, & x\geq 1\end{cases}$ is continuous at a point $x=1$ then find the value of k.

5C. यदि फलन $f(x)=\begin{cases}5x+k, & x\leq 2\\ 6x, & x>2\end{cases}$ बिन्दु $x=2$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}5x+k, & x\leq 2\\ 6x, & x>2\end{cases}$ is continuous at a point $x=2$ then find the value of k.

5D. यदि फलन $f(x)=\begin{cases}\dfrac{2x^2-50}{x-5}, & x\neq 5\\ k, & x=5\end{cases}$ बिन्दु $x=5$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}\dfrac{2x^2-50}{x-5}, & x\neq 5\\ k, & x=5\end{cases}$ is continuous at a point $x=5$ then find the value of k.

5E. यदि फलन $f(x)=\begin{cases}\dfrac{x^2-2x}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$ बिन्दु $x=2$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}\dfrac{x^2-2x}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$ is continuous at a point $x=2$ then find the value of k.

5F. यदि फलन $f(x)=\begin{cases}kx-1, & x<2\\ 2x-3, & x\geq 2\end{cases}$ बिन्दु $x=2$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x)=\begin{cases}kx-1, & x<2\\ 2x-3, & x\geq 2\end{cases}$ is continuous at a point $x=2$ then find the value of k.

ANSWERS

1A. (i) 1 (ii) 1 (iii) continuous at $x=1$ **1B.** (i) 15 (ii) 15 (iii) continuous at $x=2$

1C. (i) -7 (ii) -7 (iii) continuous at $x=\frac{1}{2}$ **1D.** (i) 0 (ii) 0 (iii) continuous at $x=\frac{1}{2}$

2A. (i) 4 (ii) undefined (iii) discontinuous at $x=2$

www.drmathnp.com

puspas.com.np

2B. (i) 8 (ii) indeterminant form (iii) discontinuous at $x = 4$
2C. (i) 0 (ii) 0 (iii) continuous at $x = 2$
2D. (i) 0 (ii) 0 (iii) continuous at $x = 3$
2E. (i) 2 (ii) indeterminant form (iii) discontinuous at $x = 1$
2F. (i) 10 (ii) indeterminant form (iii) discontinuous at $x = 5$
2G. (i) -1 (ii) -1 (iii) continuous at $x = -1$ **2H.** (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$ (iii) continuous at $x = 2$
2I. (i) 5 (ii) 5 (iii) continuous at $x = 1$ **3A.** continuous at $x = 3$ **3B.** continuous at $x = 4$ **3C.** continuous at $x = 5$
3D. continuous at $x = 5$ **3E.** continuous at $x = 2$ **3F.** continuous at $x = 2$ **3G.** continuous at $x = 3$ **3H.** continuous at $x = 10$
3I. continuous at $x = 7$ **3J.** continuous at $x = 2$ **5A.** $k = 3$ **5B.** $k = -3$ **5C.** $k = 2$ **5D.** $k = 20$ **5E.** $k = 2$
5F. $k = 1$

CHALLENGING EXERCISE 1

1A. यदि फलन $f(x) = \begin{cases} 2x-1, & x<2 \\ k+1, & x=2 \\ x+1, & x>2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x) = \begin{cases} 2x-1, & x<2 \\ k+1, & x=2 \\ x+1, & x>2 \end{cases}$ is continuous at a point $x = 2$ then find the value of k.

1B. यदि फलन $f(x) = \begin{cases} 3+x, & x<2 \\ x+k, & x=2 \\ 4x-3, & x>2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर भए k को मान कति होला ?

If the function $f(x) = \begin{cases} 3+x, & x<2 \\ x+k, & x=2 \\ 4x-3, & x>2 \end{cases}$ is continuous at a point $x = 2$ then find the value of k.

2A. फलन $f(x) = \begin{cases} 2x^2+1, & x<2 \\ 10, & x=2 \\ 4x+1, & x>2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर बनाउन के गर्नुपर्ला ?

How can $f(x) = \begin{cases} 2x^2+1, & x<2 \\ 10, & x=2 \\ 4x+1, & x>2 \end{cases}$ be made continuous at $x = 2$?

2B. फलन $f(x) = \begin{cases} 2x+1, & x<1 \\ 15, & x=1 \\ 3x, & x>1 \end{cases}$ बिन्दु $x = 1$ मा निरन्तर बनाउन के गर्नुपर्ला ?

How can $f(x) = \begin{cases} 2x+1, & x<1 \\ 15, & x=1 \\ 3x, & x>1 \end{cases}$ be made continuous at $x = 1$?

2C. फलन $f(x) = \begin{cases} 4x^2-1, & x<2 \\ 4, & x=2 \\ 7x+1, & x>2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर बनाउन के गर्नुपर्ला ?

How can $f(x) = \begin{cases} 4x^2-1, & x<2 \\ 4, & x=2 \\ 7x+1, & x>2 \end{cases}$ be made continuous at $x = 2$?

3A. यदि $x < 2$ हुँदा $f(x) = 5$, $x = 2$ हुँदा $f(x) = 2x + 1$ र $x > 2$ हुँदा $f(x) = 5$ भए $x = 2$ हुँदा फलन $f(x)$ निरन्तर हुन्छ भनी देखाउनुहोस् ।

If $f(x) = 5$ for $x < 2$, $f(x) = 2x + 1$ for $x = 2$ and $f(x) = 5$ for $x > 2$ then show that the function $f(x)$ is continuous at $x = 2$.

3B. यदि $x < 3$ हुँदा $f(x) = 2x - 1$, $x = 3$ हुँदा $f(x) = 5$ र $x > 3$ हुँदा $f(x) = 8 - x$ हुन्छ भने $x = 3$ मा फलन $f(x)$ निरन्तर हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = 2x - 1$ for $x < 3$, $f(x) = 5$ for $x = 3$ and $f(x) = 8 - x$ for $x > 3$ then prove that the function $f(x)$ is continuous at $x = 3$.

3C. यदि $x > 0$ हुँदा $f(x) = 1$, $x = 0$ हुँदा $f(x) = 0$ र $x < 0$ हुँदा $f(x) = -1$ भए फलन $f(x)$ बिन्दु $x = 0$ मा विच्छिन्न हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = 1$ for $x > 0$, $f(x) = 0$ for $x = 0$ and $f(x) = -1$ for $x < 0$, prove that the function $f(x)$ is discontinuous at $x = 0$.

ANSWERS

1A. $k = 2$ **1B.** $k = 3$ **2A.** $f(2)$ must be 9 **2B.** $f(1)$ must be 3 **2C.** $f(2)$ must be 15.

www.drmathnp.com

puspas.com.np

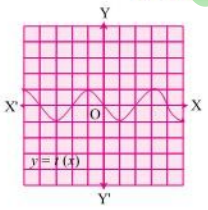
ASSIGNMENT ON CONTINUITY

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' 5 × 1 = 5

1. निरन्तरताको परिभाषा दिनुहोस् । (Define continuity.)
2. विच्छिन्नताको परिभाषा दिनुहोस् । (Define discontinuity.)
3. लेखाचित्रमा दिइएको वक्र निरन्तरता वा विच्छिन्नता के हो पहिचान गर्नुहोस् ।
   Identify whether the curve given in graph is an example of continuity or discontinuity.



4. दायाँ पक्षबाट लिइने सीमान्त मानको साङ्केतिक प्रस्तुती लेख्नुहोस् । (Write the notational representation of right hand limit.)
5. $\lim_{x \to a^-} f(x) = b$ लाई शब्दमा लेख्नुहोस् । (Write in words : $\lim_{x \to a^-} f(x) = b$.)

Group 'C' 5 × 4 = 20

6. $g(x) = \frac{x^2 - 1}{x - 1}$ को लेखाचित्र खिच्नुहोस् र विच्छिन्नताको बिन्दु खोजी गर्नुहोस् ।

   Draw the graph of $g(x) = \frac{x^2 - 1}{x - 1}$ and investigate the point of discontinuity.

7. $h(x) = \begin{cases} 3x + 2, & x < 1 \\ x - 2, & x \geq 1 \end{cases}$ को लेखाचित्र खिच्नुहोस् र विच्छिन्नताको बिन्दु खोजी गर्नुहोस् ।

   Draw the graph of $h(x) = \begin{cases} 3x + 2, & x < 1 \\ x - 2, & x \geq 1 \end{cases}$ and investigate the point of discontinuity.

8. फलन $f(x) = 2x^2 - 3x + 4$ को $x = 3$ मा निरन्तरता वा विच्छिन्नता परिक्षण गर्नुहोस् ।
   Test continuity or discontinuity of the function: $f(x) = 2x^2 - 3x + 4$ at $x = 3$

9. फलन $g(x) \begin{matrix} = 2x - 3, & x < 1 \\ = x + 2, & 1 \leq x \leq 5 \end{matrix}$ को $x = 1$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।

   Test continuity or discontinuity of the function: $g(x) \begin{matrix} = 2x - 3, & x < 1 \\ = x + 2, & 1 \leq x \leq 5 \end{matrix}$ at $x = 1$

10. $f(x) = 2x + 3$ वास्तविक मान भएको एउटा फलन छ । ($f(x) = 2x + 3$ is a real valued function.)
    (a) $x = 3.9, 3.99$ र 3.999 मा $f(x)$ पत्ता लगाउनुहोस् । (Find $f(x)$ at $x = 3.9, 3.99$ and 3.999.)
    (b) $x = 4.1, 4.01$ र 4.001 मा $f(x)$ पत्ता लगाउनुहोस् । (Find $f(x)$ at $x = 4.1, 4.01$ and 4.001.)
    (c) $f(4)$ को मान कति हुन्छ ? (What is the value of $f(4)$?)
    (d) $\lim_{x \to 4^-} f(x)$ र $\lim_{x \to 4^+} f(x)$ का मानहरू पत्ता लगाउनुहोस् । (What are the values of $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$?)
    (e) के $x = 4$ मा $f(x)$ निरन्तर छ ? (Does $f(x)$ continuous at $x = 4$?)

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

3. Continuity **4.** $\lim_{x \to a^+} f(x)$ **5.** As x approaches to a^- then $f(x)$ approaches to b. (Left hand limit of a function $f(x)$ is b.)

6. at $x = 1$ **7.** $x = 1$ **8.** Continuous **9.** Discontinuity **10.** (a) 10.8, 10.98, 10.998

(b) 11.2, 11.02, 11.002 (c) 11 (d) 11 and 11 (e) Yes

www.drmathnp.com

puspas.com.np

Unit III

मेट्रिक्स
Matrix



SYLVESTER

"Matrix" is the Latin word for womb. 'Mould', 'form' 'block' are three other words carrying almost identical meaning. In Europe, Sylvester (1851) first used the term matrix informally. Its use in modern sense is due to his pupil (1855).

A matrix is a concise and useful way of uniquely representing and working with . In particular, every can be represented by a matrix, and every matrix corresponds to a unique . The matrix, and its close relative the , are extremely important concepts in , and were first formulated by Sylvester (1851) and Cayley.

SEE SPECIFICATION GRID **1 Very short Question, 2 Short Questions, 1 Long Question**

Contents	Topics	Knowledge Each of 1 Mark	Understanding Each of 2 Marks	Application Each of 4 Marks	Higher Ability Each of 5 Marks	Total no. of Questions	Total Marks
मेट्रिक्स Matrix	डिटरमिन्यान्ट र विपरित मेट्रिक्स Determinant and Inverse of Matrix	1	2	1		4	9
	समीकरणको मेट्रिक्स विधिबाट हल Solving Equations by Matrix Method						
	क्रामरको नियम (Cramer's Rule)						

SYLLABUS ISSUED BY CDC

3. मेट्रिक्स (Matrix) **15 Periods**

- 2 × 2 मेट्रिक्सको डिटरमिनान्ट (Determinant of a 2 × 2 matrix)
- 2 × 2 मेट्रिक्सको विपरीत मेट्रिक्स (Inverse of a 2 × 2 matrix)
- मेट्रिक्स विधिबाट दुई चलयुक्त युगपत रेखीय समीकरणहरूको हल
  Solution of simultaneous equations in two variables by the matrix Method
- Cramer's rule र यसको प्रयोग (2 × 2) मेट्रिक्ससम्म मात्र)
  Cramer's rule and its application (up to 2 × 2 matrix only)

FORMULAE AND KEY POINTS

➢ **मेट्रिक्सका किसिमहरू (Types of Matrixes)**

$A=\begin{bmatrix}1\\0\\1\end{bmatrix}$ लहर मेट्रिक्स **(Column matrix)**	$A=[3\ \ 3]$, $B=[1\ \ 0\ \ 2]$ पङ्क्ति मेट्रिक्स **(Row matrix)**	$I=\begin{bmatrix}1&2\\3&4\end{bmatrix}$, $B=\begin{bmatrix}1&2&3\\1&0&3\\3&0&0\end{bmatrix}$ वर्गाकार मेट्रिक्स **(Square matrix)**
$A=\begin{bmatrix}2\\3\\4\end{bmatrix}$, $B=\begin{bmatrix}4&7\\5&8\\6&9\end{bmatrix}$ आयताकार मेट्रिक्स **Rectangular Matrix**	$O=\begin{bmatrix}0&0\\0&0\end{bmatrix}$, $O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ शून्य मेट्रिक्स **(Null matrix)**	$A=\begin{bmatrix}1&0\\0&2\end{bmatrix}$, $B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ विकर्ण मेट्रिक्स **(Diagonal matrix)**
$A=\begin{bmatrix}a&0\\0&a\end{bmatrix}$, $B=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$ स्केलर मेट्रिक्स **(Scalar matrix)**	$I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ एकाइ मेट्रिक्स **Identity or unit matrix**	$\begin{pmatrix}1&2\\0&3\end{pmatrix}$, $\begin{pmatrix}a&b&c\\0&d&e\\0&0&f\end{pmatrix}$ त्रिभुजाकार मेट्रिक्स Triangular matrix.

www.drmathnp.com

puspas.com.np

➢ **मेट्रिक्सहरूको गुणनफल (Product of matrices)**

यदि मेट्रिक्सहरू A र B क्रमशः क्रमहरू $m \times n$ र $n \times p$ भएका मेट्रिक्सहरू भए तिनीहरूको गुणनफल भन्नाले क्रम $m \times p$ भएको मेट्रिक्स AB भन्ने बुझिन्छ ।

If A and B are two matrices of order $m \times n$ and $n \times p$ respectively, then their product AB is a matrix of order $m \times p$.

$$A_{m \times n} \quad B_{n \times p} \quad = \quad AB_{m \times p}$$

➢ **मेट्रिक्सको डिटरमिनान्ट (Determinant of a matrix)**

मानौं $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ एउटा 2×2 मेट्रिक्स भए वास्तविक सङ्ख्या $(ad - bc)$ लाई मेट्रिक्स A को डिटरमिनान्ट भनिन्छ र यसलाई $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ले जनाइन्छ ।

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 matrix then its determinant is the real number $(ad - bc)$.

We write, Det. $A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Note: यदि $A = [a]$ भए $|A| = a$ हुन्छ । (If $A = [a]$ then $|A| = a$.)

➢ **विपरीत मेट्रिक्सहरू (Inverse matrices)** **[SLC 2065 M, 2071 R']**

यदि A र B समान क्रमको वर्गाकार मेट्रिक्सहरू, I सोही क्रमको एकाइ मेट्रिक्स र $AB = BA = I$ भए मेट्रिक्सहरू A र B लाई आपसमा विपरीत मेट्रिक्सहरू भनिन्छ । A को विपरीत मेट्रिक्सलाई A^{-1} ले जनाइन्छ । त्यसैले $A^{-1} = B$ हुन्छ ।

If A and B are two square matrices of same order, I is an identity matrix of the same order and $AB = BA = I$ then A and B are said to be inverse to each other. The inverse of A is denoted by A^{-1}. So $A^{-1} = B$.

➢ **एकल मेट्रिक्स र स्वामित्वहिन एकल मेट्रिक्स (Singular Matrix and Non singular matrix)**

एउटा वर्गाकार मेट्रिक्स A मा $|A| = 0$ भए सो मेट्रिक्सलाई एकल मेट्रिक्स भनिन्छ । यदि $|A| \neq 0$ भए A लाई स्वामित्वहिन एकल मेट्रिक्स भनिन्छ ।

A square matrix A is called a singular matrix if $|A| = 0$. If $|A| \neq 0$ then A is non-singular matrix.

➢ **आसन्न मेट्रिक्स (Adjoint matrix)**

2×2 क्रम भएको वर्ग मेट्रिक्समा मुख्य विकर्णका सदस्यहरूको स्थान परिवर्तन र सहायक विकर्णका सदस्यहरूको चिन्ह परिवर्तन गर्दा प्राप्त हुने नयाँ मेट्रिक्सलाई सो मेट्रिक्सको आसन्न मेट्रिक्स भनिन्छ ।

A new matrix, formed by interchanging the position of the elements of the principal diagonal and changing the sign of the elements of the secondary diagonal of a square matrix of order 2×2, is called an adjoint matrix. For example: If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then adj.$(A) = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

FORMULAE AND KEY POINTS

1. यदि $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ भए $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$ हुन्छ । (If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$.)

2. यदि $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ भए $\text{Adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ हुन्छ । (If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\text{Adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.)

3. $A^{-1} = \frac{1}{|A|} \text{Adj}(A)$ 4. $AX = B \Rightarrow X = A^{-1}B$ 5. $XA = B \Rightarrow X = BA^{-1}$

1. डिटरमिन्यान्ट, विपरीत मेट्रिक्स र समीकरण
DETERMINANT, INVERSE MATRIX AND EQUATION

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2066 R, 2066 R', 2067 R, 2067 S, 2068 R', 2068 S, 2068 S', SEE 2074 S'	8

www.drmathnp.com

Model 2	2065 R', 2065 E, 2069 R', 2069 S, 2070 R, 2070 R', 2070 S', 2072 R', 2073 R, R', SEE 2073 S, 2073S', 2074 R', 2075 R', R_2'	15
Model 3	2065 S, 2065 E, 2065 M, 2066 S, 2068 R', 2070 S, 2071 R, 2071 R', 2071 S, 2072 R, 2073 R', SEE 2074 R, SEE 2074 R', SEE 2074 S, SEE 2075 R, R_2, SEE MODEL 2076	16
Model 4	2067 R, 2067 R', 2068 R, 2069 R, 2072 S, SEE 2074 S'	6

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, R' S, M, E, 2066 R, R' S, 2067 R, R' S, 2068 R, R' S, S', 2069 R, R' S, 2070 R, R' S, S', 2071 R, R', 2071 S, 2072 R', 2072 R, 2072 S, 2073 R, 2073 R', 2073 S, 2073 S', 2074 R, 2074 R', 2074 S, S', 2075 R, R', R_2, R_2', SEE MODEL 2076	36

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको उदाहरण सहित परिभाषा दिनुहोस् । (Define the following terms with example.)

A. एकल मेट्रिक्स (Singular matrix) **B.** स्वामित्वहिन एकल मेट्रिक्स (Non singular matrix)

C. विपरीत मेट्रिक्स (Inverse matrix)

2A. कस्तो अवस्थामा युगपथ रेखीय समीकरणहरूको एकल समाधान हुँदैन ?
Under what condition, the system of simultaneous equation has no unique solution?

2B. मेट्रिक्स समीकरणको साधारण रूप कस्तो हुन्छ ? (What is the general form of matrix equation?)

3A. कस्तो अवस्थामा एउटा मेट्रिक्सको विपरीत सम्भव हुन्छ ?
Under what condition, the inverse of a matrix is possible?

3B. एकाइ मेट्रिक्सको विपरीत मेट्रिक्स के हुन्छ ? (What is the inverse of an identity matrix?)

4A. A, B र X तीनओटा मेट्रिक्सहरू छन् । यदि $AX = B$ र $|A| \neq 0$ भए X को मान पत्ता लगाउनुहोस् ।
A, B and X are three matrices. If $AX = B$ and $|A| \neq 0$, what is the value of X?

4B. A, B र X तीनओटा मेट्रिक्सहरू छन् । यदि $XA = B$ र $|A| \neq 0$ भए X को मान पत्ता लगाउनुहोस् ।
A, B and X are three matrices. If $XA = B$ and $|A| \neq 0$, what is the value of X?

5A. यदि $A = [m]$ भए $|A|$ कति हुन्छ ? (If $A = [m]$ then what is the value of $|A|$?)

5B. यदि $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ भए $|A|$ कति हुन्छ ? (If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ then what is the value of $|A|$?

5C. मेट्रिक्स $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ भए $|A|$ को मान कति हुन्छ ? (If matrix $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, what is the value of $|A|$?)

[SEE MODEL 2076]

5D. एउटा मेट्रिक्समा मुख्य विकर्णका सदस्यहरूको गुणनफल m र सहायक विकर्णका सदस्यहरूको गुणनफल n भए सो मेट्रिक्सको डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।
In a matrix, the product of elements of leading diagonal is m and the product of elements of secondary diagonal is n. Find the determinant of the matrix.

6A. यदि $AB = BA = I$ भए A र B कस्तो मेट्रिक्सहरू हुन् ? (If $AB = BA = I$ then what types of matrices are A and B?)

6B. यदि $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ भए $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ले के जनाउँदछ ?

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then what does $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ denote?

6C. $(A^{-1})^{-1}$ को मान कति हुन्छ ? (What is the value of $(A^{-1})^{-1}$?)

7A. मेट्रिक्स $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ का मुख्य विकर्णका सदस्यहरू के के हुन् ?

What are the elements of principal / leading / major diagonal of a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

7B. मेट्रिक्स $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ का सहायक विकर्णका सदस्यहरू के के हुन् ?

What are the elements of secondary diagonal of a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

pushpas.com.np

www.drmathnp.com

puspas.com.np

8A. समीकरणहरू $ax + by = c$ र $mx + ny = p$ लाई मेट्रिक्सको रूपमा व्यक्त गर्नुहोस् ।
Express the equations $ax + by = c$ and $mx + ny = p$ into matrix form.

8B. समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ लाई मेट्रिक्सको रूपमा व्यक्त गर्नुहोस् ।
Express the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ into matrix form.

9A. 2×2 एकाइ मेट्रिक्सको डिटरमिन्यान्ट के हुन्छ ? (What is the determinant of a 2×2 identity matrix?)

9B. 2×2 शुन्य मेट्रिक्सको डिटरमिन्यान्ट के हुन्छ ? (What is the determinant of a 2×2 Null matrix?)

10A. यदि $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ भए $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ले के जनाउँदछ ? (If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then what does $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ denote?)

10B. $A = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$ को आसन्न (Adjoint) मेट्रिक्स पत्ता लगाउनुहोस् । (Find the adjoint matrix of $A = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$.)

Model 2 *Concept Recall*

11A. यदि $\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 3$ भए k को मान पत्ता लगाउनुहोस् । (If $\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 3$, find the value of k.)

11B. यदि $\begin{bmatrix} a & 1 \\ 1 & 1 \end{bmatrix} = 1$ भए a को मान पत्ता लगाउनुहोस् । (If $\begin{bmatrix} a & 1 \\ 1 & 1 \end{bmatrix} = 1$ then find value of a.)

12A. यदि $A = \begin{bmatrix} 2 & 0 \\ b & x \end{bmatrix}$ र A एकल मेट्रिक्स भए x को मान पत्ता लगाउनुहोस् ।
If $A = \begin{bmatrix} 2 & 0 \\ b & x \end{bmatrix}$ and A is a singular matrix then find the value of x.

12B. यदि $\begin{bmatrix} 2 & 5 \\ 6 & x \end{bmatrix}$ एउटा एकल मेट्रिक्स भए x को मान पत्ता लगाउनुहोस् । (If $\begin{bmatrix} 2 & 5 \\ 6 & x \end{bmatrix}$ is a singular matrix, find the value of x.)

13A. यदि $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ भए X को मान पत्ता लगाउनुहोस् । (If $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find the value of X.)

13B. यदि $X\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ भए X को मान पत्ता लगाउनुहोस् । (If $X\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$, find the value of X.)

14A. यदि $A = [4]$ भए $|A|$ पत्ता लगाउनुहोस् । (If $A = [4]$ then find $|A|$.)

14B. यदि $A = [5]$ भए $|A|$ पत्ता लगाउनुहोस् । (If $A = [5]$ then find $|A|$.)

15A. यदि $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ भए Adj. (A) पत्ता लगाउनुहोस् । (If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$ then find Adj. (A).)

15B. यदि $P = \begin{bmatrix} 4 & 4 \\ 2 & 3 \end{bmatrix}$ भए Adj. (A) पत्ता लगाउनुहोस् । (If $A = \begin{bmatrix} 4 & 4 \\ 2 & 3 \end{bmatrix}$ then find Adj. (A).)

16A. समीकरणहरू $2x + 3y = 5$ र $3x + y = 4$ लाई मेट्रिक्स रूपमा लेख्नुहोस् ।
Write the equations $2x + 3y = 5$ and $3x + y = 4$ in matrix form.

16B. समीकरणहरू $x + 2y = 8$ र $2x + 3y = 11$ लाई मेट्रिक्स रूपमा लेख्नुहोस् ।
Write the equations $x + 2y = 8$ and $2x + 3y = 11$ in matrix form.

B. SHORT QUESTIONS

Model 1 *Problems based on finding the determinant of matrix.*

1A. यदि $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ र $B = \begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix}$ भए $A - 3B$ को डिटरमिन्यान्ट निकाल्नुहोस् ।
If $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -5 \\ 6 & 2 \end{pmatrix}$, find the determinant of $A - 3B$. **[2066 R']**

1B. यदि $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ र $B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ भए $3A - 4B$ को डिटरमिन्यान्ट निकाल्नुहोस् ।
If $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$, find the determinant of $3A - 4B$. **[2057 R, 2068 R']**

www.drmathnp.com

puspas.com.np

1C. यदि $P=\begin{pmatrix}1 & 2\\ 3 & -4\end{pmatrix}$ र $Q=\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$ भए $2P-3Q$ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}1 & 2\\ 3 & -4\end{pmatrix}$ and $Q=\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$, find the determinant of $2P-3Q$. [2057 S]

1D. यदि $P=\begin{pmatrix}3 & -4\\ 2 & 1\end{pmatrix}$ र $Q=\begin{pmatrix}1 & -2\\ 3 & -1\end{pmatrix}$ भए $3P-2Q$ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}3 & -4\\ 2 & 1\end{pmatrix}$ and $Q=\begin{pmatrix}1 & -2\\ 3 & -1\end{pmatrix}$, find the determinant of $3P-2Q$. [2061 S]

2A. यदि $A=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$ भए A^2 को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।

Find the value of determinant of A^2 if $A=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$.

2B. यदि $A=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$ र $B=\begin{bmatrix}1 & 0\\ 2 & -1\end{bmatrix}$ भए $|AB|$ पत्ता लगाउनुहोस् ।

If $A=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & -1\end{bmatrix}$, find $|AB|$. [SEE 2074 S']

2C. यदि $P=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ र $Q=\begin{pmatrix}2 & 3\\ 1 & 4\end{pmatrix}$ भए PQ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ and $Q=\begin{pmatrix}2 & 3\\ 1 & 4\end{pmatrix}$, find the determinant of PQ. [2058 S]

2D. यदि $P=\begin{pmatrix}3 & 2\\ 4 & -1\end{pmatrix}$ र $Q=\begin{pmatrix}2 & -1\\ 3 & 2\end{pmatrix}$ भए PQ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}3 & 2\\ 4 & -1\end{pmatrix}$ and $Q=\begin{pmatrix}2 & -1\\ 3 & 2\end{pmatrix}$, find the determinant of PQ. [2066 R]

2E. यदि $M=\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}$ र $N=\begin{pmatrix}-4 & -6\\ 3 & 2\end{pmatrix}$ भए MN को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $M=\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}$ and $N=\begin{pmatrix}-4 & -6\\ 3 & 2\end{pmatrix}$, find the determinant of MN. [2067 R]

2F. यदि $A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$ र $B=\begin{bmatrix}2 & -1\\ 2 & -3\end{bmatrix}$ भए AB को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1\\ 2 & -3\end{bmatrix}$, find the determinant of AB. [2067 S]

2G. यदि $A=\begin{pmatrix}3 & 1\\ 4 & 2\end{pmatrix}$ र $B=\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}$ भए AB को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $A=\begin{pmatrix}3 & 1\\ 4 & 2\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}$, find the determinant of AB. [2068 S]

2H. यदि $A=\begin{bmatrix}2 & -1\\ -4 & 6\end{bmatrix}$ र $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ भए AI को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $A=\begin{bmatrix}2 & -1\\ -4 & 6\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find the determinant of AI. [2068 S']

3A. यदि $A=\begin{pmatrix}3 & 5\\ -6 & 0\end{pmatrix}$ र $B=\begin{pmatrix}18 & -6\\ 12 & 3\end{pmatrix}$ भए $3A-\frac{1}{3}B$ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $A=\begin{pmatrix}3 & 5\\ -6 & 0\end{pmatrix}$ and $B=\begin{pmatrix}18 & -6\\ 12 & 3\end{pmatrix}$, find the determinant of $3A-\frac{1}{3}B$. [2059 S]

3B. यदि $P=\begin{pmatrix}2 & 0\\ -1 & 3\end{pmatrix}$ र $Q=\begin{pmatrix}2 & 4\\ -6 & 2\end{pmatrix}$ भए $5P-\frac{1}{2}Q+2I$ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}2 & 0\\ -1 & 3\end{pmatrix}$ and $Q=\begin{pmatrix}2 & 4\\ -6 & 2\end{pmatrix}$, find the determinant of $5P-\frac{1}{2}Q+2I$. [2059 R]

www.drmathnp.com

puspas.com.np

3C. यदि $P=\begin{pmatrix}1 & -2\\ 3 & 4\end{pmatrix}$ र $Q=\begin{pmatrix}-3 & 0\\ 1 & -2\end{pmatrix}$ भए $5P-2Q-3I$ को डिटरमिन्यान्ट निकाल्नुहोस् ।

If $P=\begin{pmatrix}1 & -2\\ 3 & 4\end{pmatrix}$ and $Q=\begin{pmatrix}-3 & 0\\ 1 & -2\end{pmatrix}$, find the determinant of $5P-2Q-3I$. [2061 R]

Model 2 ***Problems based on finding the value of unknown letters when determinant is given.***

4A. यदि $\begin{vmatrix}3x & -6\\ 4 & 2\end{vmatrix}=0$ भए x को मान पत्ता लगाउनुहोस् । (If $\begin{vmatrix}3x & -6\\ 4 & 2\end{vmatrix}=0$, find the value of x.) [SEE 2073 R]

4B. यदि $\begin{vmatrix}x & -2x\\ 3x & 4x\end{vmatrix}=100$ भए x को मान पत्ता लगाउनुहोस् । (If $\begin{vmatrix}x & -2x\\ 3x & 4x\end{vmatrix}=100$, find the value of x.) [2065 E]

4C. यदि $\begin{vmatrix}x & 6\\ 2 & 3\end{vmatrix}=0$ भए x को मान निकाल्नुहोस् । (If $\begin{vmatrix}x & 6\\ 2 & 3\end{vmatrix}=0$, find the value of x.) [2070 S']

4D. यदि मेट्रिक्स $P=\begin{bmatrix}2x & -6\\ 3 & 4\end{bmatrix}$ र P को डिटरमिन्यान्ट 10 भए x को मान पत्ता लगाउनुहोस् ।

If matrix $P=\begin{bmatrix}2x & -6\\ 3 & 4\end{bmatrix}$ and determinant of P is 10 then find the value of x. [SEE 2074 R']

4E. यदि $\begin{vmatrix}2p & 7\\ -5 & 9\end{vmatrix}=-1$ भए p को मान निकाल्नुहोस् । (If $\begin{vmatrix}2p & 7\\ -5 & 9\end{vmatrix}=-1$, find the value of p.) [SEE 2073 S]

4F. यदि $\begin{vmatrix}3p & 2\\ -4 & 3\end{vmatrix}=8$ भए p को मान निकाल्नुहोस् । (If $\begin{vmatrix}3p & 2\\ -4 & 3\end{vmatrix}=8$, find the value of p.) [SEE 2073 S']

4G. यदि मेट्रिक्स $A=\begin{pmatrix}3p & -2\\ 4 & 3\end{pmatrix}$ को डिटरमिन्यान्ट (determinant) 70 भए, p को मान पत्ता लगाउनुहोस् ।

If the determinant of matrix $A=\begin{pmatrix}3p & -2\\ 4 & 3\end{pmatrix}$ is 70, find the value of p. [2060 CP]

4H. यदि मेट्रिक्स $A=\begin{pmatrix}3 & 1\\ -2 & x\end{pmatrix}$ को डिटरमिन्यान्ट (determinant) 14 भए, x को मान पत्ता लगाउनुहोस् ।

If the determinant of matrix $A=\begin{pmatrix}3 & 1\\ -2 & x\end{pmatrix}$ is 14, find the value of x. [2063 S]

4I. यदि मेट्रिक्स $A=\begin{pmatrix}2 & m\\ -1 & 6\end{pmatrix}$ को डिटरमिन्यान्ट (determinant) 20 भए, m को मान पत्ता लगाउनुहोस् ।

If the determinant of matrix $A=\begin{pmatrix}2 & m\\ -1 & 6\end{pmatrix}$ is 20, find the value of m. [2069 R']

4J. यदि मेट्रिक्स $M=\begin{bmatrix}-5 & -8\\ p & 12\end{bmatrix}$ को डिटरमिन्यान्ट (determinant) 20 भए, p को मान पत्ता लगाउनुहोस् ।

If the determinant of matrix $M=\begin{bmatrix}-5 & -8\\ p & 12\end{bmatrix}$ is 20, find the value of p. [2070 R]

4K. यदि $K=\begin{bmatrix}2 & -3\\ 5 & m\end{bmatrix}$ र $|K|=23$ भए, m को मान पत्ता लगाउनुहोस् ।

If $K=\begin{bmatrix}2 & -3\\ 5 & m\end{bmatrix}$ and $|K|=23$ then find the value of m. [SEE 2073 R']

4L. यदि $E=\begin{bmatrix}-3 & 4\\ 2 & -2a\end{bmatrix}$ र $|E|=22$ भए a को मान पत्ता लगाउनुहोस् ।

If $E=\begin{bmatrix}-3 & 4\\ 2 & -2a\end{bmatrix}$ and $|E|=22$, find the value of a. [2069 S]

4M. यदि $M=\begin{bmatrix}5 & 3\\ 6 & K\end{bmatrix}$ र $|M|=27$ भए K को मान पत्ता लगाउनुहोस् ।

If $M=\begin{bmatrix}5 & 3\\ 6 & K\end{bmatrix}$ and $|M|=27$ then find the value of K. [2072 R']

www.drmathnp.com

puspas.com.np

4N. यदि मेट्रिक्स $A = \begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ र A को डिटरमिन्यान्ट 4 भए x को मान पत्ता लगाउनुहोस् ।

If the matrix $A = \begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ and determinant of A is 4 then, find the value of x. [SEE 2075 R']

4O. यदि $P = \begin{pmatrix} a & 5 \\ 3 & a \end{pmatrix}$ र $|P| = 2a$, भए 'a' को मान निकाल्नुहोस् । (If $P = \begin{pmatrix} a & 5 \\ 3 & a \end{pmatrix}$ and $|P| = 2a$, find the value of 'a'.)

5A. यदि $M = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, $N = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ र $|MN| = 10$ भए P को मान पत्ता लगाउनुहोस् ।

If $M = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, $N = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ and $|MN| = 10$, find the value of P. [SEE 2075 R'$_2$]

5B. यदि $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$ र $|AB| = 5$ भए x को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & x \end{pmatrix}$ and $|AB| = 5$, find the value of x. [2065 R']

5C. यदि $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}$ र $|BA| = -5$ भए m को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & m \end{pmatrix}$ and $|BA| = -5$, find the value of m. [2070 R']

5D. यदि $A = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix}$ र $|A - B - 5I| = 14$ भए x को मान पत्ता लगाउनुहोस् ।

If $A = \begin{pmatrix} -4 & 5 \\ 7 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 6 \\ x & 3 \end{pmatrix}$ and $|A - B - 5I| = 14$, find the value of x. [2062 R]

Model 3 Problems based on inverse of matrix.

6A. विपरीत मेट्रिक्सको परिभाषा लेख्नुहोस् । कुन अवस्थामा विपरीत मेट्रिक्स परिभाषित हुँदैन ? लेख्नुहोस् । [SEE 2075 R, 2075 R$_2$]
Write the definition of inverse matrix. In which condition the inverse matrix cannot be defined? Write it.

6B. x को मान कति हुँदा मेट्रिक्स $\begin{pmatrix} 4 & -3 \\ x & 3 \end{pmatrix}$ को विपरीत मेट्रिक्स परिभाषित गर्न सकिँदैन ? पत्ता लगाउनुहोस् ।

For what value of x, the inverse of matrix $\begin{pmatrix} 4 & -3 \\ x & 3 \end{pmatrix}$ cannot be defined? Find it. [SEE 2074 R]

6C. a को मान कति हुँदा $A = \begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$ को विपरित मेट्रिक्स परिभाषित हुँदैन ? पत्ता लगाउनुहोस् ।

For what value of a, the inverse of $A = \begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$ is not defined? Find it. [SEE 2074 S]

6D. यदि मेट्रिक्स $\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$ को विपरित मेट्रिक्स परिभाषित गर्न सकिँदैन भने x को मान पत्ता लगाउनुहोस् ।

If the inverse of the matrix $\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$ can not be defined, find the value of x. [2072 R]

6E. यदि मेट्रिक्स $\begin{pmatrix} x & 8 \\ 4 & 5 \end{pmatrix}$ को विपरीत मेट्रिक्स नभएमा x को मान कति हुन्छ ? पत्ता लगाउनुहोस् ।

For what value of x, the matrix $\begin{pmatrix} x & 8 \\ 4 & 5 \end{pmatrix}$ has no inverse matrix ? Find it. [2070 S]

6F. यदि मेट्रिक्स $\begin{pmatrix} x+2 & x \\ 3 & 5 \end{pmatrix}$ को विपरीत मेट्रिक्स नभएमा x को मान कति हुन्छ ? पत्ता लगाउनुहोस् ।

For what value of x, the matrix $\begin{pmatrix} x+2 & x \\ 3 & 5 \end{pmatrix}$ has no inverse matrix? Find it.

6G. यदि $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ भए $|A|$ पत्ता लगाई A^{-1} परिभाषित हुन्छ, हुँदैन लेख्नुहोस् ।

If $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find $|A|$ and write A^{-1} is defined or not. [SEE MODEL 2076]

www.drmathnp.com

7. विपरीत मेट्रिक्स पत्ता लगाउनुहोस् । (Find the inverse matrix.)

A. $A=\begin{pmatrix}2&1\\3&4\end{pmatrix}$ [2065 M] **B.** $A=\begin{pmatrix}7&4\\3&2\end{pmatrix}$ [2060 R] **C.** $P=\begin{pmatrix}2&0\\3&1\end{pmatrix}$ [2065 S]

D. $P=\begin{pmatrix}3&0\\0&2\end{pmatrix}$ [2066 S] **E.** $P=\begin{pmatrix}2&3\\1&2\end{pmatrix}$ [2068 R'] **F.** $A=\begin{pmatrix}1&2\\4&5\end{pmatrix}$ [2071 R']

G. $A=\begin{bmatrix}5&3\\7&4\end{bmatrix}$ [SEE 2073 R'] **H.** $A=\begin{bmatrix}2&3\\-3&-5\end{bmatrix}$ [SEE 2074 R'] **I.** $A=\begin{bmatrix}2&4\\1&3\end{bmatrix}$

8A. यदि मेट्रिक्स A को विपरित मेट्रिक्स $A^{-1}=\begin{bmatrix}1&3\\2&4\end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।

If the inverse matrix of a matrix A is $A^{-1}=\begin{bmatrix}1&3\\2&4\end{bmatrix}$, find the matrix A. [2071 S]

8B. यदि A को विपरित मेट्रिक्स $A^{-1}=\begin{bmatrix}3&2\\5&4\end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।

If the inverse of matrix A is $A^{-1}=\begin{bmatrix}3&2\\5&4\end{bmatrix}$, find the matrix A. [2071 R]

8C. यदि A को विपरित मेट्रिक्स $A^{-1}=\begin{bmatrix}2&-3\\-1&-2\end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।

If the inverse of matrix A is $A^{-1}=\begin{bmatrix}2&-3\\-1&-2\end{bmatrix}$, find the matrix A.

8D. यदि विपरीत मेट्रिक्स $A^{-1}=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ भए मेट्रिक्स A को मान निकाल्नुहोस् ।

If inverse matrix $A^{-1}=\begin{pmatrix}1&2\\3&4\end{pmatrix}$, find the value of matrix A. [2065 E]

8E. यदि विपरीत मेट्रिक्स $A^{-1}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ भए मेट्रिक्स A को मान निकाल्नुहोस् ।

If inverse matrix $A^{-1}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, find the value of matrix A.

Model 4 ***Problems on finding the value of m & n or x & y when pair of inverse matrices are given.***

9. तल दिइएका मेट्रिक्सहरू एक अर्काका विपरीत मेट्रिक्सहरू भए x, y वा m, n का मानहरू पत्ता लगाउनुहोस् ।
If the matrices given below are inverse to each other, find the values of x, y or m, n.

A. $\begin{pmatrix}2m&7\\5&9\end{pmatrix}$ and $\begin{pmatrix}9&n\\-5&4\end{pmatrix}$ [2061 R] **B.** $A=\begin{bmatrix}1&-2\\0&x\end{bmatrix}$ and $B=\begin{bmatrix}1&4\\y&2\end{bmatrix}$ [59 R, 2068R]

C. $\begin{pmatrix}x&2x-9\\-y&3\end{pmatrix}$ and $\begin{pmatrix}3&5\\y&x\end{pmatrix}$ [2062R] **D.** $\begin{pmatrix}x&3\\-1&-1\end{pmatrix}$ and $\begin{pmatrix}-1&-3\\1&2\end{pmatrix}$ [2067R']

E. $\begin{pmatrix}2x&7\\5&9\end{pmatrix}$ and $\begin{pmatrix}9&y\\-5&4\end{pmatrix}$ [2069R] **F.** $\begin{pmatrix}m&4\\-1&3\end{pmatrix}$ and $\begin{pmatrix}3&-4\\1&m\end{pmatrix}$ [2067R]

G. मेट्रिक्स $\begin{bmatrix}m&2\\7&3\end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix}3&-2\\-7&m\end{bmatrix}$ भए m को मान निकाल्नुहोस् ।

If the inverse of the matrix $\begin{bmatrix}m&2\\7&3\end{bmatrix}$ is the matrix $\begin{bmatrix}3&-2\\-7&m\end{bmatrix}$, find the value of m. [2072 S]

H. यदि मेट्रिक्सहरू $\begin{pmatrix}4&2\\5&3\end{pmatrix}$ र $\begin{pmatrix}x&-1\\-\frac{5}{2}&2\end{pmatrix}$ आपसमा एक अर्काका विपरीत मेट्रिक्स हुन् भने x को मान निकाल्नुहोस् ।

If the matrices $\begin{pmatrix}4&2\\5&3\end{pmatrix}$ and $\begin{pmatrix}x&-1\\-\frac{5}{2}&2\end{pmatrix}$ are inverse matrix to each other, calculate the value of x.

[SEE 2074 S']

puspas.com.np

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS

Model 1 *Problems based on solving simultaneous equation by matrix method.*

1. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $3x + 5y = 11, 2x - 3y = 1$ [SEE MODEL 2076]

B. $2x + 3y = 3, x + 2y = 1$ [SEE 2075 R', 2073 R']

C. $2x + 3y = 3, x - 4y = 7$ [2074 S']

D. $5x - 2y = 10, 4x - 3y = -6$ [2075 R'$_2$]

E. $5x + 3y = 9; 4y + 7x = 13$ [2072 R]

F. $x - 5y = 8, 6x + 11y = 7$ [2072 R']

G. $3x - 5y = 3, \ 4x + 3y = 4$ [2059 R]

H. $4x - 3y = 11, 3x + 7y = -1$ [2060R, 2058R]

I. $3x + 5y = 21, 2x + 3y = 13$ [2057 R]

J. $3x - 2y = 5, x + y = 5$ [2057 S]

K. $2x + y = 3, 3x + 2y = 2$ [2058 S]

L. $x + 2y = 8, 2x + 3y = 11$ [2061 S]

M. $2x + 3y = 5, 5x - 2y = 3$ [2060 S]

N. $3x + y = 51, 4x - 3y = 3$ [2061 R]

O. $x + 3y = 12, 4x - 3y = 3$ [2062 K]

P. $3x + 2y = 1, \ 7x + 5y = 4$ [2062 S]

Q. $4x - 5y = 2, 3x + 4y = 48$ [2064 R']

R. $2x - y = 5, x - 2y = 1$ [2065 R]

S. $x - 2y = -7, 3x + 7y = 5$ [2065 E]

T. $5x - 3y = 13, x - 5y = 7$ [2065 S]

U. $7x - 2y = 3, \ x + 4y = 9$ [2066 R']

V. $3x - y = 2, \ x + 2y = 3$ [2067 R']

W. $2x - 5y = 1, 7x + 3y = 24$ [2060 S']

X. $2x + 3y = 10, 5x + 3y = 16$ [2068 R']

Y. $3x + 5y = 24, \ 5x - 2y = 9$ [2068 S]

Z. $7x + 4y = 37, x + 3y = 15$ [2068 S']

AA. $9x - 8y = 12, 2x + 3y = 17$ [2071 R', SEE 2074 R']

AB. $4x + 5y = 2, 2x + 3y = 0$ [2070 S']

2. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $2x - 3y - 1 = 0, 3x + 4y - 10 = 0$ [59 R, 69 S]

B. $2x - 3y - 7 = 0, 4y - 3x = -10$ [2067 R, 2074 R]

C. $3x = 5y + 3, 4x - 4 = -3y$ [2067 S]

D. $3x + 2y = 3, 4x - y + 7 = 0$ [2069 R']

E. $2x + 3y - 18 = 0, 3x - 2y - 1 = 0$ [2059 S]

F. $5x + 2y - 3 = 0, x + 2y = 5$ [2060 CP]

G. $2x + 3y + 4 = 0, -5x + 4y + 13 = 0$ [2062 R]

H. $5x + 7y = 1, x + 4y + 5 = 0$ [2066 R]

I. $4x + 3y = 5, y - 3x = -7$ [2064 R]

J. $x - 3y = 2, \ 2y + x = 7$ [2063 S]

K. $3x + 5y = 24, 5x = 2y + 9$ [2071 R]

L. $5x - 9y = 47; 2y + 3x = 6$ [2072 S]

3. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $\frac{3}{y} = \frac{2}{x} + 2, \frac{6}{x} - \frac{2}{y} = 1$ [SEE 2075 R]

B $x = \frac{2}{3}y, \ 4x - 3y = 1$ [2063 M, 2071 S]

C. $\frac{3}{2}x + 2y = 1, \ \frac{x}{3} - \frac{y}{3} = 1$ [2065 M]

D. $\frac{5}{2}x - y = 5, \ \frac{4}{3}x - y = -2$ [2065 R']

E. $\frac{5}{x} + 3y = 7, 7y - \frac{10}{x} = 12$ [SEE 2073 R, 2074 S]

F. $x - \frac{2}{y} = 4; \ 2x + \frac{3}{y} = 1$ [2070 R]

G. $3x + \frac{4}{y} = 7, x + \frac{1}{y} = 3$ [2063 R']

H. $\frac{4}{x} + \frac{3}{y} = 1, \frac{3}{x} - \frac{2}{y} = \frac{1}{24}$ [2064 S, 2075 R$_2$]

I. $\frac{4}{x} + \frac{3}{y} = 7, \ \frac{3}{x} + \frac{2}{y} = 4$ [2070 S]

J. $\frac{4}{x} + \frac{1}{y} = 3, \frac{2}{x} - \frac{5}{y} = -4$ [SEE 2073 S, SEE 2073 S']

K. $\frac{6}{x} + \frac{3}{y} = 4, \ \frac{4}{x} - \frac{3}{y} = 1$ [2066 S]

L. $\frac{10}{x} + \frac{12}{y} = 6, \ \frac{25}{x} - \frac{2}{y} = 2$ [2070 R']

M. $\frac{6}{x} - \frac{2}{y} = 1$ and $2y - 3x = 2xy$

N. $\frac{7}{x} + 1 = \frac{2y}{x}, \ \frac{3x}{y} + 7 = \frac{5}{y}$

4. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $\frac{3x + 5y}{8} = \frac{5x - 2y}{3} = 3$ [2068 R]

B. $\frac{3x + 5y}{4} = \frac{7x + 3y}{5} = 4$ [2069 R]

www.drmathnp.com

puspas.com.np

Model 2 *Problems based on finding determinant of matrix.*

5A. यदि $A=\begin{pmatrix}3&5\\1&2\end{pmatrix}$ भए $A^2+5A^{-1}-14I$ को डिटरमिन्यान्ट निकाल्नुहोस्, जसमा I एउटा 2×2 एकाइ मेट्रिक्स हो ।

If $A=\begin{pmatrix}3&5\\1&2\end{pmatrix}$, find the determinant of $A^2+5A^{-1}-14I$, where I is a 2×2 unit matrix. **[2063 R']**

5B. यदि $M=\begin{pmatrix}3&4\\-2&5\end{pmatrix}$ र $N=\begin{pmatrix}2&1\\-1&3\end{pmatrix}$ भए $(2M+3N)^T$ को मेट्रिक्स पत्ता लगाई त्यसको डिटरमिन्यान्ट निकाल्नुहोस् ।

If $M=\begin{pmatrix}3&4\\-2&5\end{pmatrix}$ and $N=\begin{pmatrix}2&1\\-1&3\end{pmatrix}$, find the matrix of $(2M+3N)^T$ and then find its determinant. **[2064 R']**

ANSWERS

A. VERY SHORT QUESTIONS

2B. $AX=B$ **3B.** Itself **4A.** $X=A^{-1}B$ **4B.** $X=BA^{-1}$ **5A.** m **5B.** a^2-b^2 **5C.** $ps-qr$
5D. $m-n$ **6A.** Inverse to each other **6B.** Inverse of A **6C.** A **7A.** a, d **7B.** c, b
8A. $\begin{pmatrix}a&b\\m&n\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}c\\p\end{pmatrix}$ **8B.** $\begin{pmatrix}a_1&b_1\\a_2&b_2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}c_1\\c_2\end{pmatrix}$ **9A.** 1 **9B.** 0 **10A.** Adjoint of A
10B. $\begin{bmatrix}b&-a\\-b&a\end{bmatrix}$ **11A.** 3 **11B.** 2 **12A.** 0 **12B.** 15 **13A.** $\begin{bmatrix}2\\3\end{bmatrix}$ **13B.** $\begin{bmatrix}5\\-6\end{bmatrix}$
14A. 4 **14B.** 5 **15A.** $\begin{pmatrix}4&-2\\-3&4\end{pmatrix}$ **15B.** $\begin{pmatrix}3&-4\\-2&4\end{pmatrix}$ **16A.** $\begin{pmatrix}2&3\\3&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}5\\4\end{pmatrix}$
16B. $\begin{pmatrix}1&2\\2&3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}8\\11\end{pmatrix}$

B. SHORT QUESTIONS

1A. 289 **1B.** 16 **1C.** 5 **1D.** 35 **2A.** 25 **2B.** -11 **2C.** 0
2D. -77 **2E.** 10 **2F.** 40 **2G.** -2 **2H.** 8 **3A.** 371 **3B.** 172
3C. 298
4A. -4 **4B.** $\sqrt{10}$ **4C.** 4 **4D.** -1 **4E.** -2 **4F.** 0 **4G.** $\frac{62}{9}$
4H. 4 **4I.** 8 **4J.** 10 **4K.** 4 **4L.** 5 **4M.** 9 **4N.** 14
4O. $-3, 5$ **5A.** 4 **5B.** 1 **5C.** 1 **5D.** -7 **6B.** -4 **6C.** 4
6D. $x=6$ **6E.** $\frac{32}{5}$ **6F.** -5 **6G.** $5, |A|\neq0$, so A^{-1} exists **7A.** $\frac{1}{5}\begin{pmatrix}4&-1\\-3&2\end{pmatrix}$
7B. $\begin{pmatrix}1&-2\\-3/2&7/2\end{pmatrix}$ **7C.** $\frac{1}{2}\begin{pmatrix}1&0\\-3&2\end{pmatrix}$ **7D.** $\begin{pmatrix}\frac{1}{3}&0\\0&\frac{1}{2}\end{pmatrix}$ **7E.** $\begin{pmatrix}2&-3\\-1&2\end{pmatrix}$ **7F.** $\frac{-1}{3}\begin{pmatrix}5&-2\\-4&1\end{pmatrix}$ **7G.** $\begin{bmatrix}-4&3\\7&-5\end{bmatrix}$
7H. $\begin{pmatrix}5&3\\-3&-2\end{pmatrix}$ **7I.** $\frac{1}{2}\begin{pmatrix}3&-4\\-1&2\end{pmatrix}$ **8A.** $\begin{bmatrix}-2&3/2\\1&-1/2\end{bmatrix}$ **8B.** $\begin{bmatrix}2&-1\\\frac{-5}{2}&\frac{3}{2}\end{bmatrix}$ **8C.** $\begin{pmatrix}2/7&-3/7\\-1/7&-2/7\end{pmatrix}$
8D. $\begin{pmatrix}-2&1\\\frac{3}{2}&-\frac{1}{2}\end{pmatrix}$ **8E.** $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ **9A.** $2, -7$ **9B.** $\frac{1}{2}, 0$ **9C.** $0, 2; -\frac{1}{5}, 1$ **9D.** 2
9E. $x=2, y=-7$ **9F.** $m=-1$ **9G.** 5 **9H.** $\frac{3}{2}$

C. LONG QUESTIONS

1A. $x=2, y=1$ **1B.** $x=3, y=-1$ **1C.** $x=3, y=-1$ **1D.** $x=6, y=10$ **1E.** $3, -2$ **1F.** $3, -1$ **1G.** 1, 0
1H. $2, -1$ **1I.** 2, 3 **1J.** 3, 2 **1K.** $4, -5$ **1L.** $-2, 5$ **1M.** 1, 1 **1N.** 12, 15
1O. 3, 3 **1P.** $-3, 5$ **1Q.** 8, 6 **1R.** 3, 1 **1S.** $-3, 2$ **1T.** $2, -1$ **1U.** 1, 2
1V. 1, 1 **1W.** 3, 1 **1X.** 2, 2 **1Y.** 3, 3 **1Z.** 3, 4 **AA.** 4, 3 **1AB.** $(3, -2)$
2A. 2, 1 **2B.** $2, -1$ **2C.** 1, 0 **2D.** $-1, 3$ **2E.** 3, 4 **2F.** $\frac{-1}{2}, \frac{11}{4}$ **2G.** $1, -2$
2H. $3, -2$ **2I.** $2, -1$ **2J.** 5, 1 **2K.** 3, 3 **2L.** $4, -3$ **3A.** $x=2, y=1$ **3B.** $-2, -3$
3C. $2, -1$ **3D.** 6, 10 **3E.** 5, 2 **3F.** $2, -1$ **3G.** $5, -\frac{1}{2}$ **3H.** 8, 6 **3I.** $-\frac{1}{2}, \frac{1}{5}$
3J. 2, 1 **3K.** 2, 3 **3L.** $\frac{80}{9}, \frac{32}{13}$ **3M.** $-14, -\frac{7}{5}$ **3N.** $-3, 2$ **4A.** 3, 3
4B. 2, 2 **5A.** 100 **5B.** $\begin{pmatrix}12&-7\\11&19\end{pmatrix}$, 305

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 1

A. SHORT QUESTIONS

Model 1 *Problems based on finding determinant of matrix.*

डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् (Find the value of determinant):

1A. $A=\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ and $B=\begin{pmatrix}3 & 1\\ 5 & 3\end{pmatrix}$; $|A+B+I|$

1B. $A=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$ and $B=\begin{pmatrix}4 & 5\\ 6 & 3\end{pmatrix}$; $|A+B-2I|$

2A. $A=\begin{pmatrix}1 & 4\\ 2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-3 & 2\\ 4 & 0\end{pmatrix}$; $|A^2+B|$

2B. $B=\begin{pmatrix}-3 & 2\\ 4 & 0\end{pmatrix}$ and $C=\begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix}$; $|BC+C|$

3A. $P=\begin{pmatrix}2 & 4\\ 5 & 3\end{pmatrix}$ and $Q=\begin{pmatrix}3 & 6\\ 2 & 8\end{pmatrix}$; $|(PQ)^T|$

3B. $A=\begin{pmatrix}5 & -2\\ 3 & -3\end{pmatrix}$; $|AA^T|$

4A. $P=\begin{pmatrix}1/2 & 4\\ -2 & 0\end{pmatrix}$ and $Q=\begin{pmatrix}2 & -1/3\\ 1 & -5\end{pmatrix}$; $|(PQ)^T|$

4B. $M=\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix}$ and $N=\begin{pmatrix}0 & -2\\ 5 & -1\end{pmatrix}$; $|M^TN|$

5A. एउटा 2×2 मेट्रिक्स A को डिटरमिन्यान्ट पत्ता लगाउनुहोस् जसका सदस्यहरू $a_{ij}=3i+2j$ छन् ।
Find the determinant of a 2×2 matrix A whose elements are given by $a_{ij}=3i+2j$.

5B. एउटा 2×2 मेट्रिक्स A को डिटरमिन्यान्ट पत्ता लगाउनुहोस् जसका सदस्यहरू $a_{ij}=2i+4j$ छन् ।
Find the determinant of a 2×2 matrix A whose elements are given by $a_{ij}=2i+4j$.

6. मान निकाल्नुहोस् (Evaluate):

A. $\begin{vmatrix}x^2+x+1 & x^2\\ x+1 & x-1\end{vmatrix}$

B. $\begin{vmatrix}x^2+xy+y^2 & x^2-y^2\\ x^2+y^2 & x^2-xy+y^2\end{vmatrix}$

C. $\begin{vmatrix}x^2-x+1 & x-1\\ x+1 & x+1\end{vmatrix}$

D. $\begin{vmatrix}\cos 25^\circ & \sin 25^\circ\\ \sin 65^\circ & \cos 65^\circ\end{vmatrix}$

7A. यदि $A=\begin{pmatrix}4 & -2\\ 5 & -3\end{pmatrix}$, भए सिद्ध गर्नुहोस् $|A'|=|A|$. (If $A=\begin{pmatrix}4 & -2\\ 5 & -3\end{pmatrix}$, show that $|A'|=|A|$.)

7B. यदि $A=\begin{pmatrix}0.8 & 0.6\\ -0.6 & 0.8\end{pmatrix}$ भए $|A^3|$ पत्ता लगाउनुहोस् । (If $A=\begin{pmatrix}0.8 & 0.6\\ -0.6 & 0.8\end{pmatrix}$ then find $|A^3|$.)

Model 2 *Problems based on finding the value of x or m or k when determinant is given.*

8. x को मान पत्ता लगाउनुहोस् (Find the value of x):

A. $\begin{vmatrix}x & 3\\ 5 & 2x\end{vmatrix}=\begin{vmatrix}5 & -4\\ 5 & 3\end{vmatrix}$

B. $\begin{vmatrix}2x+3 & 2\\ x-1 & 4\end{vmatrix}=2$

C. $\begin{vmatrix}x & 5\\ x & x\end{vmatrix}=-6$

D. $\begin{vmatrix}x & x\\ 4 & 2x\end{vmatrix}=\begin{vmatrix}4 & 2\\ 8 & 4\end{vmatrix}$

E. $\begin{vmatrix}\cot A & x\\ -2 & \tan A\end{vmatrix}=15$

F. $\begin{vmatrix}\sec A & 2x\sin A\\ \operatorname{cosec} A & \cos A\end{vmatrix}=-15$

G. यदि $A=\begin{pmatrix}x & \cos^2\theta\\ -1 & \sin\theta\end{pmatrix}$ र $|A|=1$ भए x को मान पत्ता लगाउनुहोस् ।
If $A=\begin{pmatrix}x & \cos^2\theta\\ -1 & \sin\theta\end{pmatrix}$ and $|A|=1$ then find the value of x.

H. यदि $A=\begin{pmatrix}x & \sin^2\theta\\ -1 & \cos\theta\end{pmatrix}$ र $|A|=1$ भए x को मान पत्ता लगाउनुहोस् ।
If $A=\begin{pmatrix}x & \sin^2\theta\\ -1 & \cos\theta\end{pmatrix}$ and $|A|=1$ then find the value of x.

9. m को मान पत्ता लगाउनुहोस् (Find the value of m.):

A. $A=\begin{pmatrix}1 & 2\\ 3 & m\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}$ and $|AB|=24$

B. $P=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $Q=\begin{pmatrix}2 & 3\\ 1 & m\end{pmatrix}$ and $|PQ|=5$

C. $A=\begin{bmatrix}2 & 0\\ 3 & 1\end{bmatrix}$, $B=\begin{bmatrix}4 & 5\\ 6 & m\end{bmatrix}$ and $|A+B+I|=25$

D. $P=\begin{pmatrix}0 & 2\\ 3 & m\end{pmatrix}$, $Q=\begin{pmatrix}3 & 2\\ -1 & 2\end{pmatrix}$ & $|2P+Q|=-12$

E. $A=\begin{bmatrix}0 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}3 & 2\\ -1 & m\end{bmatrix}$ and $|2A+B|=-12$

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on proving determinant .*

10. सिद्ध गर्नुहोस् (Show that):

A. $\begin{vmatrix} 1+a & 1 \\ 1 & 1+b \end{vmatrix} = ab\left(1+\frac{1}{a}+\frac{1}{b}\right)$

B. $\begin{vmatrix} a-b & 2a \\ -2b & a-b \end{vmatrix} = (a+b)^2$

C. $\begin{vmatrix} a+b & 2a \\ 2b & a+b \end{vmatrix} = (a-b)^2$

D. $\begin{vmatrix} x+y & -y \\ y & x-y \end{vmatrix} = x^2$

E. $\begin{vmatrix} 2+x & x \\ y & 2+y \end{vmatrix} = 2(2+x+y)$

F. $\begin{vmatrix} b+c & a \\ c+a & c \end{vmatrix} = \begin{vmatrix} c+a & a-b \\ c & c-a \end{vmatrix}$

G. $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix} = \begin{vmatrix} 5 & 7 \\ 2 & 3 \end{vmatrix}$

H. $\begin{vmatrix} \sin 40° & \cos 140° \\ \sin 50° & \cos 50° \end{vmatrix} = 1$

Model 4 *Problems based on finding inverse of given matrix or proving inverse matrices.*

11A. मेट्रिक्स A = [2] भए मेट्रिक्स A को डिटरमिन्यान्ट पत्ता लगाउनुहोस् । साथै A^2 पनि पत्ता लगाउनुहोस् ।
If matrix A = [2] then find the determinant of matrix A. Also find A^2.

11B. मेट्रिक्स A = [3] भए मेट्रिक्स A को डिटरमिन्यान्ट पत्ता लगाउनुहोस् । साथै A^2 पनि पत्ता लगाउनुहोस् ।
If matrix A = [3] then find the determinant of matrix A. Also find A^2.

11C. यदि $P=\begin{pmatrix} 1 & 3 \\ 0 & 3 \end{pmatrix}$ र $Q=\begin{pmatrix} 1 & 3 \\ 4 & 6 \end{pmatrix}$ भए $(QP)^{-1}$ पत्ता लगाउनुहोस् । (If $P=\begin{pmatrix} 1 & 3 \\ 0 & 3 \end{pmatrix}$ and $Q=\begin{pmatrix} 1 & 3 \\ 4 & 6 \end{pmatrix}$, find $(QP)^{-1}$.)

12. दिइएका मेट्रिक्सहरू विपरीत मेट्रिक्सहरू हुन् भनी देखाउनुहोस् । (Show that the given matrices are inverse to each other.)

A. $A=\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ and $B=\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$

B. $A=\begin{pmatrix} 5 & 2 \\ -8 & -3 \end{pmatrix}$ and $B=\begin{pmatrix} -3 & -2 \\ 8 & 5 \end{pmatrix}$

13A. यदि $A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ भए A को विपरीत A नै हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, show that inverse of A is A itself.

13B. यदि $A=\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ भए A को विपरीतको 9 गुणा A नै हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $A=\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, show that the nine times the inverse of A is A itself.

13C. यदि $P=\begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$ भए $P.P^{-1}$ एउटा एकात्मक मेट्रिक्स हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $P=\begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$ then prove that $P.P^{-1}$ is an identity matrix.

Model 5 *Problems based on finding value of k or m or p or x based on inverse matrices.*

14. x को मान कति हुँदा, A को आफ्नै विपरीतको अस्तित्व हुँदैन ? (For what value of x, A does not have its own inverse ?)

A. $A=\begin{pmatrix} x-3 & x-1 \\ x & 2x-2 \end{pmatrix}$

B. $A=\begin{pmatrix} 2x & x \\ 3x & x+1 \end{pmatrix}$

C. $A=\begin{pmatrix} 2x & x+2 \\ x+2 & x+2 \end{pmatrix}$

15A. $A=\begin{pmatrix} 3 & 4 \\ -6 & 1-a^2 \end{pmatrix}$ एउटा वर्गाकार मेट्रिक्स हो । यदि A^T एकल मेट्रिक्स भए a को मान पत्ता लगाउनुहोस् ।
$A=\begin{pmatrix} 3 & 4 \\ -6 & 1-a^2 \end{pmatrix}$ is a square matrix. Find the value of a if A^T is singular.

15B. $P=\begin{pmatrix} 8 & 2-2m^2 \\ 6 & -12 \end{pmatrix}$ एउटा वर्गाकार मेट्रिक्स हो । यदि P^T एकल मेट्रिक्स भए m को मान पत्ता लगाउनुहोस् ।
$P=\begin{pmatrix} 8 & 2-2m^2 \\ 6 & -12 \end{pmatrix}$ is a square matrix. Find the value of m if P^T is singular.

www.drmathnp.com

puspas.com.np

15C. यदि $P=\begin{pmatrix}4 & 1-m^2\\ 3 & -6\end{pmatrix}$ एउटा वर्गाकार मेट्रिक्स भए m को मान पत्ता लगाउनुहोस् जहाँ P^T एउटा एकल मेट्रिक्स हो ।

If $P=\begin{pmatrix}4 & 1-m^2\\ 3 & -6\end{pmatrix}$ is a square matrix, find the value of m if P^T is singular matrix.

16A. यदि $A=\begin{pmatrix}p & 3\\ 2 & 4\end{pmatrix}$ र $B=\begin{pmatrix}4 & 3\\ 5 & 2\end{pmatrix}$ भए p को मान कति हुँदा AB को विपरीत परिभाषित गर्न सकिँदैन ।

If $A=\begin{pmatrix}p & 3\\ 2 & 4\end{pmatrix}$ and $B=\begin{pmatrix}4 & 3\\ 5 & 2\end{pmatrix}$ then for what value of p, AB does not have its inverse?

16B. यदि $A=\begin{pmatrix}p & 3\\ 2 & 1\end{pmatrix}$ र $B=\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}$ भए p को मान कति हुँदा AB को विपरीत परिभाषित गर्न सकिँदैन ?

If $A=\begin{pmatrix}p & 3\\ 2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}$ for what value of p, AB does not have its inverse ?

16C. x को मान कति हुँदा मेट्रिक्स $A=\begin{pmatrix}x+1 & 2x^2-11\\ 2 & 2x+3\end{pmatrix}$ एउटा एकल (singular) मेट्रिक्स हुन्छ ।

For what value of x, the matrix $A=\begin{pmatrix}x+1 & 2x^2-11\\ 2 & 2x+3\end{pmatrix}$ is a singular matrix?

16D. x को मान कति हुँदा मेट्रिक्स $A=\begin{pmatrix}3x+4 & 2x+1\\ 2x-1 & 3x-4\end{pmatrix}$ एउटा एकल (singular) मेट्रिक्स हुन्छ ।

For what value of x, the matrix $A=\begin{pmatrix}3x+4 & 2x+1\\ 2x-1 & 3x-4\end{pmatrix}$ is a singular matrix?

B. LONG QUESTIONS

Model 1 *Problems based on solving matrix equations.*

1. तलका मेट्रिक्स समीकरणहरू मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve the following matrix equations by matrix method):

A. $\begin{pmatrix}2 & 3\\ -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}7\\ -2\end{pmatrix}$ **B.** $\begin{pmatrix}4 & 5\\ 4 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}17\\ 14\end{pmatrix}$

C. $\begin{pmatrix}4 & -3\\ 5 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}9\\ 18\end{pmatrix}$ **D.** $\begin{pmatrix}26 & -16\\ 29 & -33\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}46\\ 21\end{pmatrix}$

Model 2 *Problems based on solving linear simultaneous equations by matrix method.*

2. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $\frac{x+y}{5}=1=y-x$ **B.** $\frac{x+2y}{8}=1=y-x$ **C.** $\frac{x+2y}{8}=1=2y+1-x$

D. $\frac{2x+4}{5}=y=\frac{40-3x}{4}$ **E.** $\frac{3(x-1)}{5}=y=\frac{4(1-x)}{3}$ **F.** $x=\frac{2y+10}{5},\frac{4x+6}{3}=y$

G. $\frac{x}{2}+\frac{y}{3}=2,\frac{x}{2}+\frac{2y}{3}=3$ **H.** $\frac{3}{2}x+2y=1,\frac{y}{3}=1+\frac{x}{3}$ **I.** $\frac{x}{4}-\frac{y}{3}=-1,x+2y=6$

J. $\frac{5y}{8}+\frac{3x}{8}=3=\frac{5x}{3}-\frac{2y}{3}$ **K.** $\frac{5y}{8}+\frac{3x}{8}=2=\frac{5x}{3}-\frac{2y}{3}$ **L.** $\frac{5y}{8}+\frac{3x}{8}=4=\frac{5x}{3}-\frac{2y}{3}$

3. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $4x-\frac{9}{y}+11=0,\frac{6}{y}-3x=8$ **B.** $4x+\frac{3}{y}=9,3x+\frac{6}{y}=8$ **C.** $4x+\frac{6}{y}=10,4x-\frac{9}{y}=5$

D. $xy+1=3y,3x+\frac{4}{y}=7$ **E.** $\frac{6}{x}-\frac{2}{y}=1,3x-2y=2xy$ **F.** $\frac{4}{x}+\frac{3}{y}=1,\frac{3}{x}-\frac{2}{y}=\frac{1}{24}$

G. $x+\frac{6}{y}-7=0,2xy+4=10y$ **H.** $2x+\frac{4}{y}-10=0,xy+6=9y$ **I.** $\frac{2}{x}-2y+7=0,7x-3xy=4$

J. $\frac{10}{x}+y=21,2y-\frac{30}{x}=37$ **K.** $\frac{3}{x}+\frac{2}{y}=4,3x+4y=7xy$ **L.** $\frac{15}{y}+x=7,3x-\frac{20}{y}=8$

M. $\frac{3}{y}-\frac{5}{x}=\frac{3}{xy},\frac{4}{y}+\frac{3}{x}=\frac{4}{xy}$ **N.** $\frac{1}{y}+\frac{1}{x}=\frac{5}{xy},\frac{2}{y}-\frac{1}{x}=\frac{1}{xy}$ **O.** $\frac{1}{y}+\frac{1}{x}=\frac{9}{xy},\frac{2}{y}-\frac{1}{x}=\frac{3}{xy}$

www.drmathnp.com

puspas.com.np

4. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $2(x+1)=3(y-1),\ 3(x+2)=3(y+1)$ **B.** $2x=(y+1),\ y=3(x-1)$

C. $2(3u-v)=5uv,\ 2(u+3v)=5uv$ **D.** $2(3u-v)=4uv,\ 2(u+3v)=8uv$

5. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):

A. $32=2^{x+y}$ and $1=7^{2x-3y}$ **B.** $81=3^{x+y},\ 1=9^{x-3y}$

Model 3 Problems based on determinant of matrices.

6A. यदि $A=\begin{pmatrix}5&-3\\2&-4\end{pmatrix}$ र $B=\begin{pmatrix}4&2\\-3&5\end{pmatrix}$ भए प्रमाणित गर्नुहोस् : $|AB|=|A|\,|B|$

If $A=\begin{pmatrix}5&-3\\2&-4\end{pmatrix}$ and $B=\begin{pmatrix}4&2\\-3&5\end{pmatrix}$, verify that: $|AB|=|A|\,|B|$

6B. यदि $A=\begin{pmatrix}1&2\\2&-1\end{pmatrix}$ र $B=\begin{pmatrix}1&0\\2&1\end{pmatrix}$ भए $|A^2-B^2|$ पत्ता लगाउनुहोस् ।

If $A=\begin{pmatrix}1&2\\2&-1\end{pmatrix}$ and $B=\begin{pmatrix}1&0\\2&1\end{pmatrix}$ then, find the value of $|A^2-B^2|$.

6C. यदि $A=\begin{pmatrix}2&1\\3&5\end{pmatrix}$ र $B=\begin{pmatrix}9&0\\6&8\end{pmatrix}$ भए $|(A-B)^t|=|A^t-B^t|$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $A=\begin{pmatrix}2&1\\3&5\end{pmatrix}$ and $B=\begin{pmatrix}9&0\\6&8\end{pmatrix}$ then prove that: $|(A-B)^t|=|A^t-B^t|$

6D. यदि $A=\begin{pmatrix}1&4\\2&1\end{pmatrix}$, $B=\begin{pmatrix}-3&2\\4&0\end{pmatrix}$ र $C=\begin{pmatrix}1&0\\0&2\end{pmatrix}$ भए $(A^2+BC)^t$ को डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।

If $A=\begin{pmatrix}1&4\\2&1\end{pmatrix}$, $B=\begin{pmatrix}-3&2\\4&0\end{pmatrix}$ and $C=\begin{pmatrix}1&0\\0&2\end{pmatrix}$, find the determinant of $(A^2+BC)^t$.

7A. यदि $A=\begin{pmatrix}a&4\\1&2\end{pmatrix}$, $B=\begin{pmatrix}2\\-1\end{pmatrix}$ र $C=\begin{pmatrix}4\\0\end{pmatrix}$ तथा $AB=C$ भए $|A|$ पत्ता लगाउनुहोस् ।

If $A=\begin{pmatrix}a&4\\1&2\end{pmatrix}$, $B=\begin{pmatrix}2\\-1\end{pmatrix}$ and $C=\begin{pmatrix}4\\0\end{pmatrix}$ and $AB=C$ then find $|A|$.

7B. यदि $A=\begin{pmatrix}1&x\\0&1\end{pmatrix}$, $B=\begin{pmatrix}3&2\\-5&4\end{pmatrix}$ र A^2-B^T को डिटरमिनान्ट 8 भए x को मान पत्ता लगाउनुहोस् ।

If $A=\begin{pmatrix}1&x\\0&1\end{pmatrix}$, $B=\begin{pmatrix}3&2\\-5&4\end{pmatrix}$ and the determinant of A^2-B^T is 8, find the value of x.

7C. यदि $A=\begin{pmatrix}2&a\\0&2\end{pmatrix}$, $B=\begin{pmatrix}1&3\\2&4\end{pmatrix}$ र $|(A^2-B^T)|=18$ भए a को मान पत्ता लगाउनुहोस् ।

If $A=\begin{pmatrix}2&a\\0&2\end{pmatrix}$, $B=\begin{pmatrix}1&3\\2&4\end{pmatrix}$ and $|(A^2-B^T)|=18$ then find the value of a.

Model 4 Problems based on inverse matrices.

8A. यदि $A=\begin{pmatrix}3&2\\6&8\end{pmatrix}$ र $B=\begin{pmatrix}3&0\\0&2\end{pmatrix}$ भए प्रमाणित गर्नुहोस् : $(AB)^{-1}=B^{-1}A^{-1}$

If $A=\begin{pmatrix}3&2\\6&8\end{pmatrix}$ and $B=\begin{pmatrix}3&0\\0&2\end{pmatrix}$, verify that: $(AB)^{-1}=B^{-1}A^{-1}$

8B. यदि $A=\begin{pmatrix}4&5\\2&1\end{pmatrix}$ भए $A-3I=2(I+3A^{-1})$ हुन्छ भनी देखाउनुहोस् ।

If $A=\begin{pmatrix}4&5\\2&1\end{pmatrix}$, then show that $A-3I=2(I+3A^{-1})$.

9A. यदि $\begin{pmatrix}5&-3\\5&0\end{pmatrix}+2A=\begin{pmatrix}-1&1\\1&0\end{pmatrix}$ भए A^{-1} को मान पत्ता लगाउनुहोस् ।

If $\begin{pmatrix}5&-3\\5&0\end{pmatrix}+2A=\begin{pmatrix}-1&1\\1&0\end{pmatrix}$ then find the value of A^{-1}.

www.drmathnp.com

puspas.com.np

9B. यदि $P=\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$ $Q=\begin{pmatrix}0 & p\\ q & r\end{pmatrix}$ र $QP=P^{-1}$ छ भने p, q र r को मान निकाल्नुहोस् ।

If $P=\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$, $Q=\begin{pmatrix}0 & p\\ q & r\end{pmatrix}$ and $QP=P^{-1}$, find the value of p, q and r.

Model 5 *Problems based on application of matrix equations.*

10A. 17 के.जी. चिनी र 4 के.जी. चियाको मूल्य रु. 1110 र 8 के.जी. चिनी र 2 के.जी. चियाको मूल्य रु. 540 पर्दछ । प्रति के.जी. चिनी र चियाको मूल्य मेट्रिक्स विधिबाट पत्ता लगाउनुहोस् ।

17 kg of sugar and 4 kg of tea cost Rs 1110 and 8 kg of sugar and 2 kg of tea cost Rs 540. Find the cost per kg of sugar and tea using matrix method.

10B. दुई सङ्ख्याहरूको फरकको 5 भागको 1 भाग बराबर 3 र तिनीहरूको योगको एक तिहाई 7 भए मेट्रिक्स विधिको प्रयोग गरी ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।

One fifth of the difference of two numbers is 3 and one-third of their sum is 7. Find the numbers using matrix method.

ANSWERS

A. SHORT QUESTIONS

1A. 40 **1B.** -39 **2A.** -26 **2B.** -20 **3A.** -168 **3B.** 81 **4A.** $-\frac{232}{3}$ **4B.** 20 **5A.** -6

5B. -8 **6A.** $-x^2-1$ **6B.** $y^2(x^2+2y^2)$ **6C.** x^3-x^2+2 **6D.** 0 **7B.** 1 **8A.** ±5 **8B.** -2 **8C.** 3, 2

8D. 0, 2 **8E.** 7 **8F.** 8 **8G.** $\sin\theta$ **8H.** $\cos\theta$ **9A.** $\frac{30}{31}$ **9B.** 1 **9C.** $\frac{59}{7}$ **9D.** 2

9E. -2 **11A.** 2, [4] **11B.** 3, [9] **11C.** $-\frac{1}{18}\begin{pmatrix}30 & -12\\ -4 & 1\end{pmatrix}$ **14A.** 1 or 6 **14B.** 0 or 2 **14C.** ±2

15A. ±3 **15B.** ±3 **15C.** ±3 **16A.** $\frac{3}{2}$ **16B.** 6 **16C.** $x=5, -5/2$ **16D.** $\pm\sqrt{3}$

B. LONG QUESTIONS

1A. 2, 1 **1B.** 3, 1 **1C.** 3, 1 **1D.** 3, 2 **2A.** 2, 3 **2B.** 2, 3 **2C.** 4, 2 **2D.** 8, 4 **2E.** 1, 0

2F. 6, 10 **2G.** 2, 3 **2H.** $-\frac{10}{7}, \frac{11}{7}$ **2I.** 0, 3 **2J.** 3, 3 **2K.** 2, 2 **2L.** 4, 4 **3A.** $-2, 3$ **3B.** 2, 3

3C. 2, 3 **3D.** $5, -\frac{1}{2}$ **3E.** 2, 1 **3F.** 8, 6 **3G.** 4, 2 **3H.** 3, 1 **3I.** $-2, 3$ **3J.** 10, 20 **3K.** $-\frac{1}{2}, \frac{1}{5}$

3L. 4, 5 **3M.** 1, 0 **3N.** 2, 3 **3O.** 4, 5 **4A.** 2, 1 **4B.** 8, 6 **4C.** 2, 1 **4D.** 1, 1 **5A.** 3, 2

5B. 3, 1 **6B.** 16 **6D.** -42 **7A.** 4 **7B.** -2 **7C.** 2 **9A.** $\frac{1}{4}\begin{pmatrix}0 & -2\\ 2 & -3\end{pmatrix}$ **9B.** $0, 0, \frac{1}{2}$

10A. Rs 30, Rs 150 **10B.** 18, 3

2. क्रामरको नियम CRAMER'S RULE

2.1 क्रामरको परिचय (Introduction to Cramer)



The name Gabriel Cramer is associated with Cramer's rule, as well as with the introduction of the concept of utility to mathematics.

He was born in Geneva on July 31, 1704, Cramer was the son of Jean, a physician, and Anne Mallet Cramer. By the age of 18, Cramer had earned his doctorate with a dissertation on the qualities of sound, and two years later was appointed co-chair of Mathematics at the Académie de la Rive.

In 1750 he published Cramer's rule, This rule gave a general formula for the solution for any unknown in a linear equation system having a unique solution, in terms of determinants implied by the system. This rule is still standard.

2.2 क्रामरको नियम (Cramer's rule)

रेखीय समीकरणहरू $a_1x+b_1y=c_1$ र $a_2x+b_2y=c_2$ मा $D=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$, $D_x=\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$ र $D_y=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$ भए सुत्रहरू $x=\frac{D_x}{D}$ र $y=\frac{D_y}{D}$ लाई क्रामरको नियम भनिन्छ ।

In the linear equations $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ if $D=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$, $D_x=\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$ and $D_y=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$ then the formulae $x=\frac{D_x}{D}$ and $y=\frac{D_y}{D}$ is known as Cramer's rule.

www.drmathnp.com

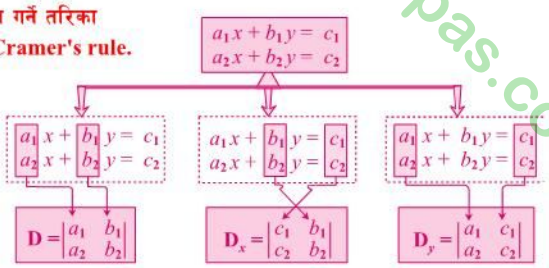
puspas.com.np

2.3 क्रामरको नियम प्रयोग गरी रेखीय समीकरणहरू हल गर्ने तरिका
Method of solving linear equations using Cramer's rule.

यदि $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ भए,

If $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ then,

Then $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.



Note: D_x and D_y can also be represented by D_1 and D_2 respectively.

SOLVED EXAMPLES

Example 1: क्रामरको नियम प्रयोग गरी समीकरणहरू हल गर्नुहोस् (Use Cramer's rule to solve the equations): $x + 2y = 7$ and $2x - y = 4$

Solution: Here, $x + 2y = 7$ .....(i) and $2x - y = 4$ .....(ii)

Coefficient of x	Coefficient of y	Constant term
1	2	7
2	– 1	4

Now, $D = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1 \times (-1) - 2 \times 2 = -5$

$D_x = \begin{vmatrix} 7 & 2 \\ 4 & -1 \end{vmatrix} = 7 \times (-1) - 4 \times 2 = -15$ and $D_y = \begin{vmatrix} 1 & 7 \\ 2 & 4 \end{vmatrix} = 1 \times 4 - 7 \times 2 = -10$

Using Cramer's rule, $x = \frac{D_x}{D} = \frac{-15}{-5} = 3$ and $y = \frac{D_y}{D} = \frac{-10}{-5} = 2$

Thus, $x = 3$ and $y = 2$.

Example 2: क्रामरको नियम प्रयोग गरी समीकरणहरूको पद्धतीलाई हल गर्नुहोस् ।
Solve the system of equations using Cramer's rule:
$\frac{5}{y} + \frac{4}{x} = 58$ and $\frac{7}{x} = 67 - \frac{3}{y}$

Solution: Here, $\frac{4}{x} + \frac{5}{y} = 58$ .....(i) and $\frac{7}{x} + \frac{3}{y} = 67$.....(ii)

Coefficient of $\frac{1}{x}$	Coefficient of $\frac{1}{y}$	Constant term
4	5	58
7	3	67

Now, $D = \begin{vmatrix} 4 & 5 \\ 7 & 3 \end{vmatrix} = 4 \times 3 - 7 \times 5 = 12 - 35 = -23$

$D_x = \begin{vmatrix} 58 & 5 \\ 67 & 3 \end{vmatrix} = 58 \times 3 - 67 \times 5 = 174 - 335 = -161$

$D_y = \begin{vmatrix} 4 & 58 \\ 7 & 67 \end{vmatrix} = 4 \times 67 - 7 \times 58 = 268 - 406 = -138$

Using Cramer's rule,

$\frac{1}{x} = \frac{D_x}{D} = \frac{-161}{-23} = 7$ $\quad \therefore \quad x = \frac{1}{7}$

$\frac{1}{y} = \frac{D_y}{D} = \frac{-138}{-23} = 6$ $\quad \therefore \quad y = \frac{1}{6}$

Thus, $x = \frac{1}{7}$ and $y = \frac{1}{6}$.

www.drmathnp.com

puspas.com.np

Example 3: यदि $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ र $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ भए $x = \frac{D_1}{D}$ र $y = \frac{D_2}{D}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In $a_1x + b_1y = c_1$ **and** $a_2x + b_2y = c_2$, **if** $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ **and** $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ **then prove that** $x = \frac{D_1}{D}$ **and** $y = \frac{D_2}{D}$.

Solution: Here, $a_1x + b_1y = c_1$ .....(1) and $a_2x + b_2y = c_2$ .....(2).

Multiplying equation (1) by b_2 and equation (2) by b_1 then,

$b_2 \times [a_1x + b_1y = c_1] \Rightarrow a_1b_2x + b_1b_2y = b_2c_1$ .....(i)

$b_1 \times [a_2x + b_2y = c_2] \Rightarrow a_2b_1x + b_1b_2y = b_1c_2$ .....(ii)

Subtracting equation (ii) from (i), $a_1b_2x - a_2b_1x = b_2c_1 - b_1c_2$

or, $(a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2$

or, $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ or, $Dx = D_1$ $\therefore$ $x = \frac{D_1}{D}$

Again, multiplying equation (1) by a_2 and equation (2) by a_1 then,

$a_2 \times [a_1x + b_1y = c_1] \Rightarrow a_1a_2x + b_1a_2y = a_2c_1$ .....(iii)

$a_1 \times [a_2x + b_2y = c_2] \Rightarrow a_1a_2x + a_1b_2y = a_1c_2$ .....(iv)

Subtracting equation (iv) from (iii) then, $b_1a_2y - a_1b_2y = a_2c_1 - a_1c_2$

or, $(b_1a_2 - a_1b_2)y = a_2c_1 - a_1c_2$

or, $(a_1b_2 - b_1a_2)y = a_1c_2 - a_2c_1$

or, $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ or, $Dy = D_2$ $\therefore$ $y = \frac{D_2}{D}$

Thus, $x = \frac{D_1}{D}$ and $y = \frac{D_2}{D}$ **Proved.**

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE Examination History (2065 to 2075)	n(Q)
Model 2	SEE MODEL 2076	1

SEE EXERCISE 2

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1A. क्रामरको नियम प्रयोग गरी कस्ता समीकरणहरू हल गर्न सकिन्छ ?
What type of equations can be solved by using Cramer's rule?

1B. क्रामरको नियम अनुसार D, D_x र D_y दिइएको बेला x को मान निकाल्ने सूत्र के हुन्छ ?
What should be the formula to calculate x using Cramer's rule when D, D_x and D_y are given?

1C. क्रामरको नियम अनुसार D, D_x र D_y दिइएको बेला y को मान निकाल्ने सूत्र के हुन्छ ?
Write the formula to calculate y using Cramer's rule when D, D_x and D_y are given.

2A. समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा क्रामरको नियम अनुसार D को मान लेख्नुहोस् ।
According to the Cramer's rule, write the value of D in the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

2B. समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा क्रामरको नियम अनुसार D_x को मान लेख्नुहोस् ।
According to the Cramer's rule, write the value of D_x in the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

2C. समीकरणहरू $a_1x + b_1y = c_1$ र $a_2x + b_2y = c_2$ मा क्रामरको नियम अनुसार D_y को मान लेख्नुहोस् ।
According to the Cramer's rule, write the value of D_y in the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

2D. क्रामरको नियम अनुसार D को मान कति हुँदा समीकरणहरूको समाधान परिभाषीत हुदैन ?
What value of D makes the solution of equations undefined according to the Cramer's rule?

Model 2 *Concept Recall*

3A. क्रामरको नियम अनुसार समीकरणहरू $2x + 3y = 5$ र $x + y = 2$ बाट D को मान पत्ता लगाउनुहोस् ।
Find the value of D using Cramer's rule from the equations: $2x + 3y = 5$ and $x + y = 2$

www.drmathnp.com

puspas.com.np

3B. क्रामरको नियम अनुसार समीकरणहरू $4x + 5y = 8$ र $x + y = 2$ बाट D_x को मान पत्ता लगाउनुहोस् ।
Find the value of D_x using Cramer's rule from the equations: $4x + 5y = 8$ and $x + y = 2$

3C. क्रामरको नियम अनुसार समीकरणहरू $3x + 2y = 5$ र $x + 2y = 4$ बाट D_y को मान पत्ता लगाउनुहोस् ।
Find the value of D_y using Cramer's rule from the equations: $3x + 2y = 5$ and $x + 2y = 4$

4A. यदि क्रामरको नियम अनुसार x र y का गुणाङ्कहरूको डिटरमिन्यान्ट (D) = 18, अचर पद र y को गुणाङ्कहरूको डिटरमिन्यान्ट (D_x) = – 54 भए x को मान पत्ता लगाउनुहोस् ।
If the determinant of coefficient of x and y i.e. (D) = 18, determinant of constant term and coefficient of y (D_x) = – 54 then using Cramer's rule, find the value of x.

4B. यदि क्रामरको नियम अनुसार x र y का गुणाङ्कहरूको डिटरमिन्यान्ट (D) = 18, x को गुणाङ्कहरू र अचर पदहरूको डिटरमिन्यान्ट (D_y) = 36 भए y को मान पत्ता लगाउनुहोस् ।
If the determinant of coefficient of x and y i.e. (D) = 18, determinant of coefficient of x and constant term (D_y) = 36 then using Cramer's rule, find the value of y.

5A. क्रामरको नियम अनुसार, $D = 4$, $D_x = 12$ र $D_y = 4$ भए x को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, $D = 4$, $D_x = 12$ and $D_y = 4$ then find the value of x.

5B. क्रामरको नियम अनुसार, $D = 2$, $D_x = 8$ र $D_y = 6$ भए y को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, $D = 2$, $D_x = 8$ and $D_y = 6$ then find the value of y.

B. SHORT QUESTIONS

Model 1 ***Problems based on D, D_x and D_y .***

1A. यदि $D = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$, $D_x = \begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix}$ र $D_y = \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}$ भए x को मान पत्ता लगाउनुहोस् ।
If $D = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$, $D_x = \begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix}$ and $D_y = \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}$ then, find the value of x.

1B. यदि $D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$, $D_x = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$ र $D_y = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$ भए y को मान पत्ता लगाउनुहोस् ।
If $D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$, $D_x = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$ and $D_y = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$ then, find the value of y.

2A. यदि $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix}$ र $D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix}$ भए क्रामरको नियम अनुसार x र y का मानहरू पत्ता लगाउनुहोस् ।
If $D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix}$ and $D_y = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix}$ then find the values of x and y using Cramer's rule.

2B. यदि $D = \begin{vmatrix} 5 & -3 \\ 1 & -5 \end{vmatrix}$, $D_x = \begin{vmatrix} 13 & -3 \\ 7 & -5 \end{vmatrix}$ र $D_y = \begin{vmatrix} 5 & 13 \\ 1 & 7 \end{vmatrix}$ भए क्रामरको नियम अनुसार x र y का मानहरू पत्ता लगाउनुहोस् ।
If $D = \begin{vmatrix} 5 & -3 \\ 1 & -5 \end{vmatrix}$, $D_x = \begin{vmatrix} 13 & -3 \\ 7 & -5 \end{vmatrix}$ and $D_y = \begin{vmatrix} 5 & 13 \\ 1 & 7 \end{vmatrix}$ then find the values of x and y using Cramer's rule.

Model 2 ***Problems based on finding D, D_x and D_y from the equations.***

3A. क्रामरको नियम अनुसार $ax + by = c$ र $px + qy = r$ मा D_1 र D_2 को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, find the values of D_1 and D_2 for $ax + by = c$ and $px + qy = r$. [SEE MODEL 2076]

3B. क्रामरको नियम अनुसार $ax + by = c$ र $bx + ay = d$ मा D र D_x को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, find the values of D and D_x for $ax + by = c$ and $bx + ay = d$.

3C. समीकरणहरू $ax + by = c$ र $bx + ay = d$ मा क्रामरको नियम अनुसार $D_1 = D_2$ भए $(c - d)(a + b) = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
According to the Cramer's rule, if $D_1 = D_2$ in the equations $ax + by = c$ and $bx + ay = d$ then prove that $(c - d)(a + b) = 0$.

3D. समीकरणहरू $ax + by = 1$ र $bx + ay = 1$ मा क्रामरको नियम अनुसार $\frac{D_x}{D}$ पत्ता लगाउनुहोस् ।
According to the Cramer's rule, find $\frac{D_x}{D}$ in the equations $ax + by = 1$ and $bx + ay = 1$.

www.drmathnp.com
puspas.com.np

3E. समीकरणहरू $mx + ny = 2$ र $nx + my = 2$ मा क्रामरको नियम अनुसार $\frac{D}{D_y}$ पत्ता लगाउनुहोस् ।

According to the Cramer's rule, find $\frac{D}{D_y}$ in the equations $mx + ny = 2$ and $nx + my = 2$.

4A. समीकरणहरू $x + 3y = 12$ र $4x - 3y = 3$ मा क्रामरको नियम अनुसार D र D_x का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D and D_x using Cramer's rule from the equations $x + 3y = 12$ and $4x - 3y = 3$.

4B. समीकरणहरू $3x + y = 51$ र $4x - 3y = 3$ मा क्रामरको नियम अनुसार D र D_x का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D and D_x using Cramer's rule from the equations $3x + y = 51$ and $4x - 3y = 3$.

5A. समीकरणहरू $x + 3y = 12$ र $4x - 3y = 3$ मा क्रामरको नियम अनुसार D र D_y का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D and D_y using Cramer's rule from the equations $x + 3y = 12$ and $4x - 3y = 3$.

5B. समीकरणहरू $3x + 2y = 1$ र $7x + 5y = 4$ मा क्रामरको नियम अनुसार D र D_y का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D and D_y using Cramer's rule from the equations $3x + 2y = 1$ and $7x + 5y = 4$.

6A. समीकरणहरू $4x - 5y = 2$ र $3x + 4y = 48$ मा क्रामरको नियम प्रयोग गरी D_x र D_y का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D_x and D_y using Cramer's rule from the equations $4x - 5y = 2$ and $3x + 4y = 48$.

6B. समीकरणहरू $2x - y = 5$ र $x - 2y = 1$ मा क्रामरको नियम प्रयोग गरी D_x र D_y का मानहरू पत्ता लगाउनुहोस् ।
Find the values of D_x and D_y using Cramer's rule from the equations $2x - y = 5$ and $x - 2y = 1$.

7A. समीकरणहरू $3x + 5y - 21 = 0$ र $2x + 3y = 13$ मा क्रामरको नियम प्रयोग गर्दा $x = \frac{D_1}{D} = \frac{D_1}{(-1)}$ र $y = \frac{D_2}{D} = \frac{D_2}{(-1)}$ भए D_1 र D_2 का मानहरू पत्ता लगाउनुहोस् ।

In Cramer's rule for solving equations $3x + 5y - 21 = 0$ and $2x + 3y = 13$, $x = \frac{D_1}{D} = \frac{D_1}{(-1)}$ and $y = \frac{D_2}{D} = \frac{D_2}{(-1)}$.
Find the values of D_1 and D_2.

7B. समीकरणहरू $3x - 5y = 14$ र $6x - 7y = 25$ मा क्रामरको नियम प्रयोग गर्दा $x = \frac{D_1}{D} = \frac{D_1}{9}$ र $y = \frac{D_2}{D} = \frac{D_2}{9}$ भए D_1 र D_2 का मानहरू पत्ता लगाउनुहोस् ।

In Cramer's rule for solving equations $3x - 5y = 14$ and $6x - 7y = 25$, $x = \frac{D_1}{D} = \frac{D_1}{9}$ and $y = \frac{D_2}{D} = \frac{D_2}{9}$. Find the values of D_1 and D_2.

Model 3 *Problems based on making equations from D, D_x and D_y .*

8A. यदि $D = \begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} 9 & 3 \\ 13 & 7 \end{vmatrix}$ र $D_y = \begin{vmatrix} 5 & 9 \\ 4 & 13 \end{vmatrix}$ भए यी मानहरूसँग सम्बन्धित समीकरणहरू पत्ता लगाउनुहोस् ।

If $D = \begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix}$, $D_x = \begin{vmatrix} 9 & 3 \\ 13 & 7 \end{vmatrix}$ and $D_y = \begin{vmatrix} 5 & 9 \\ 4 & 13 \end{vmatrix}$ then write the corresponding equations to these values.

8B. यदि $D = \begin{vmatrix} 1 & -5 \\ 6 & 11 \end{vmatrix}$, $D_x = \begin{vmatrix} 8 & -5 \\ 7 & 11 \end{vmatrix}$ र $D_y = \begin{vmatrix} 1 & 8 \\ 6 & 7 \end{vmatrix}$ भए यी मानहरूसँग सम्बन्धित समीकरणहरू पत्ता लगाउनुहोस् ।

If $D = \begin{vmatrix} 1 & -5 \\ 6 & 11 \end{vmatrix}$, $D_x = \begin{vmatrix} 8 & -5 \\ 7 & 11 \end{vmatrix}$ and $D_y = \begin{vmatrix} 1 & 8 \\ 6 & 7 \end{vmatrix}$ then write the corresponding equations to these values.

Model 4 *Problems based on finding value of letters.*

9A. यदि समीकरणहरू $ax - 5y = 3$ र $4x + 3y = 4$ मा $D = 29$ भए क्रामरको नियम प्रयोग गरी a को मान पत्ता लगाउनुहोस् ।
If the equations $ax - 5y = 3$ and $4x + 3y = 4$ have $D = 29$ then find the value of a by using Cramer's rule.

9B. यदि समीकरणहरू $4x - 3y = 11$ र $px + 7y = -1$ मा $D = 37$ भए क्रामरको नियम प्रयोग गरी p को मान पत्ता लगाउनुहोस् ।
If the equations $4x - 3y = 11$ and $px + 7y = -1$ have $D = 37$ then find the value of p by using Cramer's rule.

10A. यदि समीकरणहरू $mx + 5y = 21$ र $2x + 3y = 13$ मा $D_y = -3$ भए क्रामरको नियम प्रयोग गरी m को मान पत्ता लगाउनुहोस् ।
If the equations $mx + 5y = 21$ and $2x + 3y = 13$ have $D_y = -3$ then find the value of m by using Cramer's rule.

10B. k को मान कति हुँदा समीकरणहरू $3x - 2y = 5$ र $x + y = k + 2$ मा $D_x = 15$ हुन्छ ?
For what value of k, the equations $3x - 2y = 5$ and $x + y = k + 2$ have $D_x = 15$?

11A. k को मान कति हुँदा समीकरणहरू $2x + y = k$ र $kx + 2y = 2$ मा $D_y + 5 = 0$ हुन्छ ?
For what value of k, the equations $2x + y = k$ and $kx + 2y = 2$ have $D_y + 5 = 0$?

11B. k को मान कति हुँदा समीकरणहरू $x + 2y = 8$ र $2x + ky = 4k - 1$ मा $D_x = 5 - k$ हुन्छ ?
For what value of k, the equations $x + 2y = 8$ and $2x + ky = 4k - 1$ have $D_x = 5 - k$?

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS

Model 1 *Solving simple simultaneous equations by the Cramer's rule.*

1. क्रामरको नियम प्रयोग गरी तलका समीकरणहरू हल गर्नुहोस् ।
Solve the following equations by using Cramer's rule.

A. $2x + y = 3, 3x + 2y = 2$ **B.** $x + 2y = 8, 2x + 3y = 11$

C. $2x + 3y = 5, 5x - 2y = 3$ **D.** $3x + y = 51, 4x - 3y = 3$

E. $x + 3y = 12, 4x - 3y = 3$ **F.** $3x + 2y = 1, \ 7x + 5y = 4$

G. $4x - 5y = 2, 3x + 4y = 48$ **H.** $2x - y = 5, x - 2y = 1$

I. $x - 2y = -7, 3x + 7y = 5$ **J.** $5x - 3y = 13, x - 5y = 7$

K. $7x - 2y = 3, \ x + 4y = 9$ **L.** $3x - 2 = y, \ x + 2y = 3$

M. $2x - 5y = 1, 7x + 3y = 24$ **N.** $2x + 3y = 10, 5x + 3y = 16$

2. तलका समीकरणहरू क्रामरको नियम प्रयोग गरी हल गर्नुहोस् । (Solve the following equations using Cramer's rule.)

A. $3x = 5y + 3, 4x - 4 = -3y$ **B.** $3x + 2y = 3, 4x - y + 7 = 0$

C. $2x + 3y - 18 = 0, 3x - 2y - 1 = 0$ **D.** $5x + 2y - 3 = 0, x + 2y = 5$

E. $2x + 3y + 4 = 0, -5x + 4y + 13 = 0$ **F.** $5x + 7y = 1, x + 4y + 5 = 0$

G. $2x - y = 1, 2y + x = 3$ **H.** $4x + 3y = 5, y - 3x = -7$

I. $x - 3y = 2, \ 2y + x = 7$ **J.** $3x + 5y = 24, 5x = 2y + 9$

Model 2 *Problems based on solving fraction involving simultaneous equations by the Cramer's rule.*

3. तलका समीकरणहरू क्रामरको नियम प्रयोग गरी हल गर्नुहोस् । (Solve the following equations using Cramer's rule.)

A. $x - \frac{2}{y} = 4; \ 2x + \frac{3}{y} = 1$ **B.** $3x + \frac{4}{y} = 7, x + \frac{1}{y} = 3$

C. $\frac{4}{x} + \frac{3}{y} = 1, \frac{3}{x} - \frac{2}{y} = \frac{1}{24}$ **D.** $\frac{6}{x} + \frac{3}{y} = 4, \ \frac{4}{x} - \frac{3}{y} = 1$

E. $\frac{10}{x} + \frac{12}{y} = 6, \frac{25}{x} - \frac{2}{y} = 2$ **F.** $\frac{4}{x} + \frac{3}{y} = 7, \ \frac{3}{x} + \frac{2}{y} = 4$

4. तलका समीकरणहरू क्रामरको नियम प्रयोग गरी हल गर्नुहोस् । (Solve the following equations using Cramer's rule.)

A. $x = \frac{2}{3}y, \ 4x - 3y = 1$ **B.** $\frac{3}{2}x + 2y = 1, \ \frac{x}{3} - \frac{y}{3} = 1$

C. $\frac{5}{2}x - y = 5, \ \frac{4}{3}x - y = -2$ **D.** $\frac{3x + 5y}{8} = \frac{5x - 2y}{3} = 3$

E. $\frac{3x + 5y}{4} = \frac{7x + 3y}{5} = 4$ **F.** $\frac{3x + 2y}{3} = \frac{x + 2y}{2} = 4$

ANSWERS

A. VERY SHORT QUESTIONS

1B. $\frac{D_x}{D}$ **1C.** $\frac{D_y}{D}$ **2A.** $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ **2B.** $\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ **2C.** $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ **2D.** 0 **3A.** -1

3B. -2 **3C.** 7 **4A.** -3 **4B.** 2 **5A.** 3 **5B.** 3

B. SHORT QUESTIONS

1A. 2 **1B.** 1 **2A.** $x = -3, y = 2$ **2B.** $x = 2, y = -1$ **3A.** $D_1 = cq - br, D_2 = ar - pc$ **3B.** $(a^2 - b^2), (ca - bd)$

3D. $\frac{1}{a+b}$ **3E.** $\frac{m+n}{2}$ **4A.** $-15, -45$ **4B.** $-13, -156$ **5A.** $-15, -45$ **5B.** 1, 5 **6A.** 248, 186

6B. $-9, -3$ **7A.** $D_1 = -2, D_2 = -3$ **7B.** $D_1 = 27, D_2 = -9$ **8A.** $5x + 3y = 9$ and $4x + 7y = 13$

8B. $x - 5y = 8, 6x + 11y = 7$ **9A.** $a = 3$ **9B.** $p = 3$ **10A.** $m = 3$ **10B.** $k = 3$ **11A.** $k = \pm 3$ **11B.** $k = 3$

C. LONG QUESTIONS

1A. 4, −5 **1B.** −2, 5 **1C.** 1, 1 **1D.** 12, 15 **1E.** 3, 3 **1F.** −3, 5 **1G.** 8, 6

1H. 3, 1 **1I.** −3, 2 **1J.** 2, −1 **1K.** 1, 2 **1L.** 1, 1 **1M.** 3, 1 **1N.** 2, 2

2A. 1, 0 **2B.** −1, 3 **2C.** 3, 4 **2D.** $\frac{-1}{2}, \frac{11}{4}$ **2E.** 1, −2 **2F.** 3, −2 **2G.** 1, 1

2H. 2, −1 **2I.** 5, 1 **2J.** 3, 3 **3A.** 2, −1 **3B.** $5, -\frac{1}{2}$ **3C.** 8, 6 **3D.** 2, 3

3E. $\frac{80}{9}, \frac{32}{13}$ **3F.** $-\frac{1}{2}, \frac{1}{5}$ **4A.** −2, −3 **4B.** 2, −1 **4C.** 6, 10 **4D.** 3, 3 **4E.** 2, 2 **4F.** 2, 3

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 2

LONG QUESTIONS

Model 1 *Problems based on simple simultaneous linear equations.*

1. तलका समीकरणहरू क्रामरको नियमबाट हल गर्नुहोस् । (Solve the following equations by the Cramer's rule.)

A. $3x - 4 = 4y;\ x - 6y = -36$

B. $5x - 3y = 8$ and $2x + 5y = 59$

C. $2x - 3y = 3,\ 4x - y = 1$

D. $x + y = \frac{7}{6},\ 2x - 3y = -1$

E. $3x + 2y + 9 = 0,\ 2x - 3y = -6$

F. $2(x - 1) = y,\ 3(x - 1) = -4y$

G. $8x + 11 = 3y - 20,\ 6y - 15 = -2x + 11$

H. $3x - 2y = 1,\ -x + 4y = 3$

Model 2 *Problems based on solving fraction involving simultaneous equations by the Cramer's rule.*

2. तलका समीकरणहरू क्रामरको नियमबाट हल गर्नुहोस् । (Solve the following equations by the Cramer's rule.)

A. $\frac{4}{x} + \frac{5}{y} = 58,\ \frac{7}{x} + \frac{3}{y} = 67$

B. $\frac{x}{7} - \frac{2y}{7} = -1,\ \frac{3x}{5} + \frac{7y}{5} = 1$

C. $\frac{x+1}{8} = \frac{y+3}{5} = \frac{x-y}{4}$

D. $\frac{2}{3}x + y = 1,\ \frac{1}{2}x + y = \frac{1}{2}$

Model 3 *Word problems based on simultaneous equations.*

3. क्रामरको नियम प्रयोग गरी तलका समस्याहरू हल गर्नुहोस् । (Using Cramer's rule, solve the following equations.)

A. यदि दुई सङ्ख्याहरूको योग 10 र तिनीहरूको अन्तर 2 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
If the sum of two numbers is 10 and their difference is 2, find the numbers.

B. यदि दुई सङ्ख्याहरूको योगफल 24 र तिनीहरूको अन्तर 4 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
If the sum of two numbers is 24 and their difference is 4 then find the numbers.

C. यदि दुई सङ्ख्याहरूको योग 17 र अन्तर 3 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
If the sum of two numbers is 17 and their difference is 3, find the numbers.

D. दुईओटा धनात्मक सङ्ख्याहरूको योगफल र अन्तर क्रमशः 15 र 5 भए ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum and difference of two positive numbers are 15 and 5 respectively. Find the numbers.

ANSWERS

1A. $x = 12, y = 8$ **1B.** 7, 9 **1C.** 0, −1 **1D.** $x = \frac{1}{2}$ and $y = \frac{2}{3}$ **1E.** −3, 0 **1F.** 1, 0 **1G.** −2, 5 **1H.** 1, 1

2A. $x = \frac{1}{7}$ and $y = \frac{1}{6}$ **2B.** −3, 2 **2C.** 15, 7 **2D.** 3, −1

3A. 6, 4 **3B.** 14, 10 **3C.** 10, 7 **3D.** 10, 5

CHALLENGING EXERCISE 1

1. मेट्रिक्स विधिद्वारा हल गर्नुहोस् (Solve by matrix method):

A. $3y + 4x = 2xy,\ \frac{9}{x} - \frac{5}{2} = \frac{2}{y}$

B. $2ax - 2by = -a - 4b,\ 2bx + 2ay = 4a - b$

C. $\sqrt{2}x + \sqrt{3}y = 5,\ \sqrt{8}x = \sqrt{3}y + 1$

D. $3(2u + v) = 7uv,\ 3(u + 3v) = 11uv$

2. क्रामरको नियम प्रयोग गरी तलका समीकरणहरू हल गर्नुहोस् । (Solve the following equations using Cramer's rule.)

A. $0.4x + 0.3y = 1.7,\ 0.7x - 0.2y = 0.8$

B. $0.5x + 0.7y = 0.74,\ 0.3x + 0.5y = 0.5$

C. $\sqrt{2}x - \sqrt{3}y = 0,\ \sqrt{5}x + \sqrt{2}y = 0$

D. $217x + 131y = 913,\ 131x + 217y = 827$

3. मेट्रिक्स विधिद्वारा हल गर्नुहोस् (Solve by matrix method):

A. $\frac{x}{2} + y = 0.8,\ \frac{7}{x + \frac{y}{2}} = 10$

B. $3x - \frac{y+7}{11} + 2 = 10,\ 2y + \frac{x+11}{7} = 10$

C. $\frac{1}{7x} + \frac{1}{6y} = 3,\ \frac{1}{2x} - \frac{1}{3y} = 5$

D. $\frac{1}{2x} - \frac{1}{y} = -1,\ \frac{1}{x} + \frac{1}{2y} = 8$

E. $\frac{2x}{a} + \frac{y}{b} = 2$ and $\frac{x}{a} - \frac{y}{b} = 4$

F. $\frac{x+y}{xy} = 2,\ \frac{x-y}{xy} = 6$

www.drmathnp.com

puspas.com.np

G. $\frac{2x+y}{2} = x - \frac{5y}{3} = 1$ **H.** $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}; x \neq 0, y \neq 0$

I. $\frac{xy}{x+y} = \frac{6}{5}$ and $\frac{xy}{y-x} = 6; x \neq 0, y \neq 0$ **J.** $x + y = 2xy, \frac{x-y}{xy} = 6; x \neq 0, y \neq 0$

ANSWERS

1A. 3, 4 **1B.** $-\frac{1}{2}, 2$ **1C.** $\sqrt{2}, \sqrt{3}$ **1D.** $1, \frac{3}{2}$ **2A.** 2, 3 **2B.** 0.5, 0.7 **2C.** 0, 0

2D. 3, 2 **3A.** 0.4, 0.6 **3B.** 3, 4 **3C.** $\frac{1}{14}, \frac{1}{6}$ **3D.** $\frac{1}{6}, \frac{1}{4}$ **3E.** $2a, -2b$ **3F.** $-\frac{1}{2}, \frac{1}{4}$

3G. 1, 0 **3H.** 1, 3 **3I.** 2, 3 **3J.** $-\frac{1}{2}, \frac{1}{4}$

ASSIGNMENT ON MATRIX

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' 3 × 1 = 3

1. एकल मेट्रिक्सको परिभाषा दिनुहोस् । (Define singular matrix.)
2. मेट्रिक्स $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ को विपरीत निकाल्ने सूत्र लेख्नुहोस् । (Write the formula to calculate inverse of matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$?)
3. यदि $\begin{vmatrix} x & 2 \\ 3 & -1 \end{vmatrix} = 0$ भए x को मान पत्ता लगाउनुहोस् । (If $\begin{vmatrix} x & 2 \\ 3 & -1 \end{vmatrix} = 0$, find the value of x.)

Group 'B' 3 × 2 = 6

4. यदि $\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ भए मेट्रिक्स $\begin{pmatrix} x \\ y \end{pmatrix}$ पत्ता लगाउनुहोस् ।
   Find the matrix $\begin{pmatrix} x \\ y \end{pmatrix}$ if $\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$
5. यदि $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ र $B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ भए (3A – 4B) को डिटरमिन्यान्टको मान पत्ता लगाउनुहोस् ।
   If $A = \begin{pmatrix} 3 & 4 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$, find the value of the determinant of (3A – 4B).
6. यदि मेट्रिक्स $\begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}$ को विपरीत मेट्रिक्स $\begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix}$ भए m र n का मानहरू पत्ता लगाउनुहोस् ।
   If the inverse of the matrix $\begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}$ is the matrix $\begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix}$, calculate the values of m and n.

Group 'C' 4 × 4 = 16

7. क्रामरको नियम प्रयोग गरी हल गर्नुहोस् (Solve by using Cramer's rule): $3x - 5y = 3$ and $4x + 3y = 4$
8. क्रामरको नियम प्रयोग गरी हल गर्नुहोस् (Solve by using Cramer's rule): $3x + \frac{4}{y} = 7$ and $x + \frac{1}{y} = 3$
9. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $2x - y = 1$ and $4x + 3y = 17$
10. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method): $\frac{4}{x} + \frac{3}{y} = 1$ and $\frac{3}{x} - \frac{2}{y} = \frac{1}{24}$

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

3. -6 **4.** $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ **5.** 24 **6.** 2, – 7 **7.** (1, 0) **8.** $(5, -\frac{1}{2})$

9. (2, 3) **10.** (8, 6)

www.drmathnp.com

puspas.com.np

Unit IV

निर्देशाङ्क ज्यामिती
Co-ordiante Geometry



René Descartes

René Descartes was a French philosopher, mathematician, physicist, and writer . He has been dubbed the "Father of Modern Philosophy", and much of subsequent Western philosophy is a response to his writings, which continue to be studied closely to this day. Descartes' influence in mathematics is also apparent, the Cartesian coordinate system: allowing geometric shapes to be expressed in algebraic equations; being named for him. He is credited as the father of co-ordinate geometry.

SEE SPECIFICATION GRID **2 Very Short & 2 Short, 1 Long & 1 Very Long Questions**

Contents	Topics	Knowledge	Understanding	Application	Higher Ability	Total no. of Questions	Total Marks
		Each of 1 Mark	Each of 2 Marks	Each of 4 Marks	Each of 5 Marks		
निर्देशाङ्क ज्यामिति Coordinate Geometry	दुई रेखाहरूबिचको कोण Angle between two lines	2	2	1	1	6	15
	जोडा सरल रेखाहरू Pair of straight lines						
	शाङ्कि क क्षेत्रहरू Conic Sections						
	वृत्त (Circle)						

SYLLABUS ISSUED BY CDC

4. निर्देशाङ्क ज्यामिति (Coordinate Geometry) **- 30 Periods**

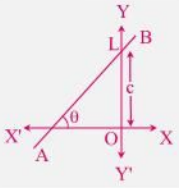
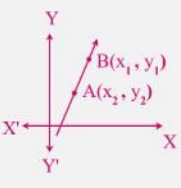
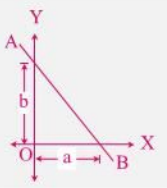
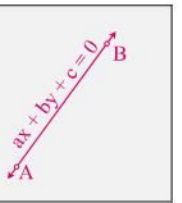
- दुई सरल रेखाहरू बिचका कोणहरू (समानान्तर र लम्ब हुने अवस्थासहित)
  Angle between two straight lines (with the conditions of being parallel and perpendicular)
- जोडा सरल रेखाहरूको समीकरण (Equation of a pair of straight lines)
  - समघातीय वर्ग समीकरणले दिने रेखाहरू र तिनीहरू बिचको कोण (लम्ब हुने र सम्पाती हुने अवस्थासमेत)
    The lines given by homogeneous equation of 2nd degree and angle between the lines (Also, the conditions of perpendicularity and coincident)
- शाङ्कि क क्षेत्रहरू (Conic sections)
  - परिचय र यसका प्रकारहरू (सोली र समतलीय सतहको प्रतिच्छेदनबाट मात्र)
    Introduction and its types (From the intersection of cone and plane surface only)
- वृत्त (Circle)
  - वृत्तको परिभाषा (सोली र समतलीय सतहको प्रतिच्छेदनका आधारमा)
    Definition of circle (On the basis of the intersection of plane and cone)
  - वृत्तको समीकरणहरू $x^2 + y^2 = r^2$, $(x - h)^2 + (y - k)^2 = r^2$, $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ र $x^2 + y^2 + 2gx + 2fy + c = 0$ स्वरूप र तत्सम्बन्धी समस्याहरू
    Equations of circle in the form of $x^2 + y^2 = r^2$, $(x - h)^2 + (y - k)^2 = r^2$, $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ and $x^2 + y^2 + 2gx + 2fy + c = 0$ and related problems.

www.drmathnp.com

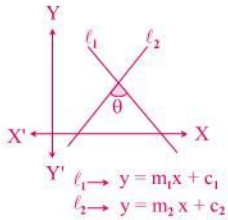
puspas.com.np

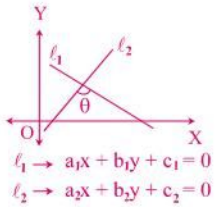
1. दुई सीधा रेखाहरूबिचको कोण
ANGLE BETWEEN TWO STRAIGHT LINES

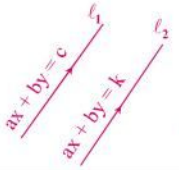
FORMULAE AND KEY POINTS

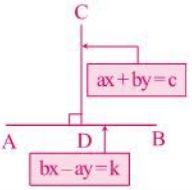
रेखा AB को Line AB				
भुकाव (Slope)	$m = \tan\theta$	$m = \frac{y_2 - y_1}{x_2 - x_1}$	$m = -\frac{b}{a}$	$m = -\frac{a}{b} = -\frac{x\text{-को गुणांक}}{y\text{-को गुणांक}}$
समीकरण Equation	$y = mx + c$	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$	$\frac{x}{a} + \frac{y}{b} = 1$	$ax + by + c = 0$

	रेखाहरूको समीकरण Equation of Lines	रेखाहरू बिचको कोण Angle between Lines	रेखाहरूको अवस्था (Condition of Lines)	
			समानान्तर हुने being parallel	लम्ब हुने being perpendicular
1.	$y = m_1x + c_1$ $y = m_2x + c_2$	$\theta = \tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$	$m_1 = m_2$	$m_1 . m_2 = -1$
2.	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\theta = \tan^{-1}\left(\pm\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right)$	$a_1b_2 = a_2b_1$	$a_1a_2 + b_1b_2 = 0$









रेखाको समीकरण Equation of Line		भुकाव slope	समानान्तर हुने रेखाको समीकरण Equation of parallel line	लम्ब हुने रेखाको समीकरण Equation of perpendicular line
1.	$ax + by = c$	$m = -\frac{a}{b}$	$ax + by = k$	$bx - ay = k$
eg	$6x + 3y = 8$	$m = -\frac{6}{3} = -2$	$6x + 3y = k$	$3x - 6y = k$
2.	$y = mx + c$	m	$y = mx + k$	$y = -\frac{1}{m}x + k$
eg	$y = 5x + 4$	5	$y = 5x + k$	$y = -\frac{1}{5}x + k$

Note : जहाँ k कुनै अचल सङ्ख्या हो । (Where k, is any constant.)

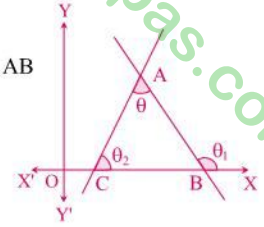
Proof of Important theorem

1. दिइएका दुई सीधा रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ बिचको कोण पत्ता लगाउनुहोस् । साथै यी रेखाहरू लम्ब हुँदा र समानान्तर हुँदा हुने सम्बन्ध पनि पत्ता लगाउनुहोस् ।
Find the angle between two given straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Also derive the condition of the lines being perpendicular and parallel. **[2063 R, 2064 R']**

www.drmathnp.com

puspas.com.np

Solution



Take two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

Now, to find angle between these lines, draw lines AB and CD show that AB and AC meet X-axis at B and C respectively.

Let, $\measuredangle XBA = \theta_1$ and $\measuredangle XCA = \theta_2$.

So, $\tan\theta_1 = m_1$ and $\tan\theta_2 = m_2$

Let $\measuredangle CAB = \theta$ then $\theta = \measuredangle XBA - \measuredangle XCA$ [$\because \theta_1 = \theta + \theta_2$]

$\therefore \quad \theta = \theta_1 - \theta_2$

Now, $\tan\theta = \tan(\theta_1 - \theta_2) = \dfrac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 . \tan\theta_2} = \dfrac{m_1 - m_2}{1 + m_1.m_2}$ $\quad \therefore \quad \tan\theta = \left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$ ..... (1)

$(180° - \theta)$ is also angle between them so $\tan(180° - \theta) = \left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$.

or, $-\tan\theta = \left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$ $\quad \therefore \quad \tan\theta = -\left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$ ..... (2)

Combining equation (i) and (ii) ; $\tan\theta = \pm\left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$

$\therefore \quad \theta = \tan^{-1}\left(\dfrac{m_1 - m_2}{1 + m_1.m_2}\right)$ is the required angle between the lines.

Condition of two lines being Parallel

If the lines are parallel then,

$\tan\theta = \tan 0° = 0.$

So, $\tan 0° = \pm\dfrac{m_1 - m_2}{1 + m_1.m_2}$

or, $\dfrac{m_1 - m_2}{1 + m_1.m_2} = 0$

or, $m_1 - m_2 = 0$

$\therefore \quad m_1 = m_2$

Thus, the slopes of lines are to be equal.

Condition of lines being Perpendicular

If the lines are perpendicular to each other then,

$\theta = 90°$ and $\cot\theta = \cot 90° = 0.$

So, $\cot\theta = \pm\dfrac{1 + m_1m_2}{m_1 - m_2}$

or, $\cot 90° = \pm\dfrac{1 + m_1m_2}{m_1 - m_2}$

or, $0 = \pm\dfrac{1 + m_1m_2}{m_1 - m_2}$

or, $1 + m_1m_2 = 0 \quad \therefore \quad m_1m_2 = -1$

Thus, the product of the slopes of the lines is -1.

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, 2066 S, 2071 R, 2071 S, SEE 2074 S', SEE 2075 R'	6
Model 2	2065 M, 2066 R', 2067 R', 2068 R', 2069 S, 2070 S, 2070 S', 2071 R', 2073 R, 2073 R', 2074 R', SEE MODEL 2076	12
Model 3	2067 S, 2072 R	2
Model 4	2065 S, 2066 R, 2068 R, 2068 S, 2068 S', 2069 R, 2069 R', 2070 R, 2070 R', 2072 R', S, SEE 2073 S, SEE 2073 S', SEE 2074 R, S', SEE 2075 R, SEE 2075 R_2, SEE 2075 R'_2	17
Model 5	2067 R, SEE 2075 R'_2	2

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 S, 2068 R', 2068 S, 2068 S', 2070 R, 2070 R', 2072 R, 2072 R'	8
Model 2	2066 R', 2067 S, 2069 R, 2072 S, SEE 2075 R'	5
Model 3	2065 R, 2065 R', 2065 M, 2067 R, 2067 R', 2069 S, 2071 R', 2071 S, 2073 R', SEE 2074R', SEE 2074 S'	11
Model 4	2070 S, 2070 S', SEE 2074 R, SEE 2074 S	4
Model 5	2065 E, 2066 R, 2066 S, 2069 R', 2071 R, SEE 2073 S', SEE 2075 R, R_2	7
Model 6	2068 R, 2073 R, SEE 2073 S	3

www.drmathnp.com

puspas.com.np

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. भुकावको आधारमा तलका पदहरूको परिभाषा दिनुहोस् । (Define the following terms on the basis of slope.)
A. समानान्तर रेखाहरू (Parallel lines) **B.** लम्ब रेखाहरू (Perpendicular lines)

2A. $y = m_1x + c_1$ र $y = m_2x + c_2$ रेखाहरू बीचको कोण निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

2B. $a_1x + b_1y + c_1 = 0$ र $a_2x + b_2y + c_2 = 0$ रेखाहरू बीचको कोण निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate angle between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

2C. यदि दुई सिधा रेखाहरूका झुकावहरू क्रमशः m_1 र m_2 छन् र तिनीहरू बिचको कोण θ भए $\tan\theta$ को मान पत्ता लगाउने सुत्र लेख्नुहोस् ।
If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for $\tan\theta$. **[SEE MODEL 2076]**

3A. यदि दुई रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ आपसमा समानान्तर भए यिनीहरूको झुकावको सम्बन्ध लेख्नुहोस् ।
If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel to each other, write the relation between their slopes.

3B. यदि दुई रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ आपसमा लम्ब भए यिनीहरूको झुकावको सम्बन्ध लेख्नुहोस् ।
If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular to each other, write the relation between their slopes.

4A. $ax + by + c = 0$ सँग समानान्तर हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight line parallel to $ax + by + c = 0$.

4B. $ax + by + c = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
What is the equation of straight line perpendicular to $ax + by + c = 0$?

5A. झुकाव $-\frac{1}{m}$ भएको रेखासँग लम्ब हुने रेखाको झुकाव कति हुन्छ ?
What is the slope of a line which is perpendicular to the line having slope $-\frac{1}{m}$?

5B. रेखा $y = -\frac{1}{m}x + c$ सँग लम्ब हुने रेखाको समीकरण के हुन्छ ?
What will be the equation of a line perpendicular to the line $y = -\frac{1}{m}x + c$?

Model 2 *Concept Recall*

6A. रेखाहरू $x = 0$ र $y = 0$ बीचको कोण कति हुन्छ ? (What is the angle between the lines $x = 0$ and $y = 0$?)

6B. रेखाहरू $x = y$ र $x = -y$ बीचको कोण कति हुन्छ ? (What is the angle between the lines $x = y$ and $x = -y$?)

7A. एउटा रेखा (ℓ_1) को झुकाव $\frac{3}{4}$ छ । रेखा (ℓ_1) सँग समानान्तर हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
The slope of a line (ℓ_1) is $\frac{3}{4}$. Find the slope of another line parallel to the line (ℓ_1).

7B. रेखा $15x - 3y = 7$ को झुकाव 5 छ । यो रेखासँग समानान्तर हुने रेखाको झुकाव कति हुन्छ ?
What is the slope of line parallel to $15x - 3y = 7$ if its slope is 5 ?

8A. रेखा $2x - 3y = 5$ को झुकाव $\frac{2}{3}$ छ । रेखा $2x - 3y = 5$ सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
The slope of a line $2x - 3y = 5$ is $\frac{2}{3}$. Find the slope of line perpendicular to $2x - 3y = 5$.

8B. रेखा $x - 5y = 2$ को झुकाव $\frac{1}{5}$ छ । यो रेखासँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of line perpendicular to $x - 5y = 2$ if its slope is $\frac{1}{5}$.

www.drmathnp.com

puspas.com.np

B. SHORT QUESTIONS

Model 1 *Problems based on finding angle between two lines.*

1A. यदि दुई सीधा रेखाहरूको झुकावहरू क्रमशः $\sqrt{3}$ र $\frac{1}{\sqrt{3}}$ भए तिनीहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् । [SEE 2075 R']

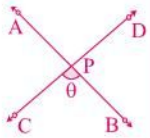
If the slopes of two straight lines are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ respectively then find the acute angle between them.

1B. रेखाहरू MN र PQ का झुकावहरू क्रमशः 2 र – 3 छन् भने ती रेखाहरूबीचको अधिककोण पत्ता लगाउनुहोस् । [SEE 2074 S']
The slopes of the lines MN and PQ are 2 and – 3 respectively. Find the obtuse angle between the lines.

1C. झुकावहरू क्रमशः $-\sqrt{3}$ र $\sqrt{3}$ भएका रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines having slopes $-\sqrt{3}$ and $\sqrt{3}$ respectively. [2071 S]

1D. यदि रेखाहरू AB र CD का झुकावहरू क्रमशः 3 र – 2 भए तिनीहरू बिचको न्यूनकोण पत्ता लगाउनुहोस् । [2071R]
If the slopes of the lines AB and CD are 3 and – 2 respectively, find the acute angle between them.

1E. चित्रमा रेखा AB को झुकाव $\frac{2}{3}$ र CD को झुकाव 3 भए न्यूनकोण $\measuredangle$CPB को नाप पत्ता लगाउनुहोस् ।



In the figure, the slope of line AB is $\frac{2}{3}$ and the slope of CD is 3. Find the measure of an acute $\measuredangle$CPB.

2A. रेखाहरू $2x - y + 4 = 0$ र $3x + y + 3 = 0$ बिचको अधिककोण निकाल्नुहोस् ।
Find the obtuse angle between two lines $2x - y + 4 = 0$ and $3x + y + 3 = 0$. [2065 R]

2B. समीकरणहरू $\sqrt{3}x - y + 6 = 0$ र $y + 3 = 0$ भएका सीधारेखाहरू बिचको न्यूनकोण पत्ता लगाउनुहोस् । [2066S]
Find the acute angle between two straight lines having equations $\sqrt{3}x - y + 6 = 0$ and $y + 3 = 0$.

Model 2 *Problems based on condition of parallel lines.*

3A. रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ बिचको कोणको सूत्र लेख्नुहोस् । यी दुई रेखाहरू समानान्तर हुने अवस्था समेत लेख्नुहोस् ।
Write the formula of the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Also write the condition of the parallelism of these two lines. [2070 S']

3B. यदि सीधा रेखाहरू $\ell x + my + c = 0$ र $px + qy + r = 0$ एक आपसमा समानान्तर भए प्रमाणित गर्नुहोस् : $\ell q = mp$. [2074 R']
If the straight lines $\ell x + my + c = 0$ and $px + qy + r = 0$ are parallel to each other then prove that: $\ell q = mp$.

3C. $2x - 8y + 6 = 0$ र $3x - 12y - 4 = 0$ रेखाहरू समानान्तर छन् भनी देखाउनुहोस् ।
Show that the lines $2x - 8y + 6 = 0$ and $3x - 12y - 4 = 0$ are parallel. [2066 R']

3D. समीकरणहरू $3x + 5y = 4$ र $6x + 10y = 3$ भएका रेखाहरू समानान्तर हुन्छन् भनी देखाउनुहोस् ।
Prove that the lines with equations $3x + 5y = 4$ and $6x + 10y = 3$ are parallel to each other. [2067 R']

3E. सरल रेखाहरू $3x + 4y + 5 = 0$ र $6x + 8y + 7 = 0$ का झुकाव पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् । [SEE MODEL 2076]
Find the slopes of two straight lines $3x + 4y + 5 = 0$ and $6x + 8y + 7 = 0$ and write the relationship between them.

3F. सरल रेखाहरू $4x - 5y = 7$ र $8x - 10y = 20$ का झुकावहरू पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् ।
Find the slopes of straight lines $4x - 5y = 7$ and $8x - 10y = 20$ and write the relationship between them.

3G. बिन्दुहरू (3, – 4) र (– 2, 6) भएर जाने सीधारेखा र समीकरण $2x + y + 3 = 0$ भएको सीधारेखा समानान्तर हुन्छन् भनी प्रमाणित गर्नुहोस् ।
Prove that the straight line passing through the points (3, – 4) and (– 2, 6) and the straight line having equation $2x + y + 3 = 0$ are parallel. [SEE 2073 R]

4A. दुई सरल रेखाहरू $Px + 3y - 12 = 0$ र $4y - 3x + 7 = 0$ समानान्तर भए P को मान पत्ता लगाउनुहोस् । [2071 R']
If the straight lines $Px + 3y - 12 = 0$ and $4y - 3x + 7 = 0$ are parallel to each other, find the value of P.

4B. यदि रेखाहरू $2x - 8y + 6 = 0$ र $px - 12y - 4 = 0$ एक आपसमा समानान्तर भए, p को मान पत्ता लगाउनुहोस् ।
If the lines $2x - 8y + 6 = 0$ and $px - 12y - 4 = 0$ are parallel to each other, find the value of p. [2070 S]

4C. यदि रेखाहरू $3x + my = 5$ र $\frac{x}{2} + \frac{y}{3} = 1$ एक आपसमा समानान्तर छन् भने m को मान पत्ता लगाउनुहोस् ।

If the lines $3x + my = 5$ and $\frac{x}{2} + \frac{y}{3} = 1$ are parallel to each other, find the value of m. [SEE 2073 R']

5A. यदि बिन्दुहरू (3, – 4) र (– 2, a) भएर जाने रेखा समीकरण $y + 2x + 3 = 0$ भएको रेखासँग समानान्तर हुन्छ भने a को मान पत्ता लगाउनुहोस् ।
If the line passing through (3, – 4) and (– 2, a) is parallel to the line given by the equation $y + 2x + 3 = 0$, find the value of a . [2065 M]

www.drmathnp.com

puspas.com.np

5B. बिन्दुहरू $(a, 4)$ र $(2, -5)$ जोड्ने सीधा रेखा $2x - 3y = 12$ समीकरण हुने सीधा रेखासँग समानान्तर भए a को मान पत्ता लगाउनुहोस् ।
If the line joining the points $(a, 4)$ and $(2, -5)$ is parallel to the line with given equation $2x - 3y = 12$, find the value of a. **[2068 R']**

5C. यदि बिन्दुहरू $(4, -p)$ र $(-2, 6)$ भएर जाने रेखा समीकरण $2y + 3x = -3$ भएको रेखासँग समानान्तर छ भने p को मान पत्ता लगाउनुहोस् ।
If the line passing through the points $(4, -p)$ and $(-2, 6)$ is parallel to the line having an equation $2y + 3x = -3$, find the value of p. **[2069 S]**

Model 3 *Problems based on finding slope of line.*

6A. सीधारेखा $3x - 4y = 10$ सँग लम्ब हुने रेखाको झुकाव निकाल्नुहोस् ।
Find the slope of the line perpendicular to the straight line $3x - 4y = 10$. **[2072 R]**

6B. रेखा $3x + 2y + 4 = 0$ सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of the line which is perpendicular to the line $3x + 2y + 4 = 0$. **[2067 S]**

6C. सीधा रेखा $4x + 3y = 12$ सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of the straight line perpendicular to $4x + 3y = 12$. **[2064 R']**

Model 4 *Problems based on condition of perpendicular lines.*

7A. रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ बिचको कोण पत्ता लगाउने सूत्र लेख्नुहोस् । साथै ती रेखाहरू आपसमा लम्ब हुने अवस्था पनि लेख्नुहोस् ।
Write down the formula to find the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Also write the condition of perpendicularity of these lines. **[SEE 2075 R, 2075 R$_2$]**

7B. यदि रेखाहरू $l_1x + m_1y + n_1 = 0$ र $l_2x + m_2y + n_2 = 0$ एक आपसमा लम्ब भए $l_1l_2 + m_1m_2 = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ are perpendicular to each other, prove that $l_1l_2 + m_1m_2 = 0$. **[2074 R]**

7C. रेखाहरू $ax + by + c = 0$ र $bx - ay + c = 0$ एक आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the line $ax + by + c = 0$ and $bx - ay + c = 0$ are perpendicular to each other. **[2068 S]**

7D. यदि दुईओटा सरल रेखाहरू $px + qy + r = 0$ र $\ell x - my + n = 0$ एक आपसमा लम्ब भए, $p\ell = qm$ हुन्छ भनी देखाउनुहोस् ।
If two straight lines $px + qy + r = 0$ and $\ell x - my + n = 0$ are perpendicular to each other, show that $p\ell = qm$. **[2070 R]**

7E. समीकरण $px + qy + r = 0$ र $qx - py + s = 0$ भएका सीधा रेखाहरू आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् । **[2073 S]**
Prove that the straight lines having equations $px + qy + r = 0$ and $qx - py + s = 0$ are perpendicular to each other.

7F. रेखाहरू $x - 3y - 5 = 0$ र $3x + y - 15 = 0$ एक आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the lines $x - 3y - 5 = 0$ and $3x + y - 15 = 0$ are perpendicular to each other. **[SEE 2075 R'$_2$]**

7G. समीकरणहरू $y + \sqrt{3}x + 4 = 0$ र $x - \sqrt{3}y = 5$ भएका दुई सीधा रेखाहरू एक आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the lines having equations $y + \sqrt{3}x + 4 = 0$ and $x - \sqrt{3}y = 5$ are perpendicular to each other. **[2069 R]**

7H. समीकरणहरू $(2 + \sqrt{3})x - y + 1 = 0$ र $(2 - \sqrt{3})x + y = 0$ भएका दुई सीधा रेखाहरू आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।
Prove that the two straight lines having equations $(2 + \sqrt{3})x - y + 1 = 0$ and $(2 - \sqrt{3})x + y = 0$ are perpendicular to each other. **[2073 S']**

7I. दुई सरल रेखाहरू $(2 - \sqrt{3})x - y - 1 = 0$ र $(\sqrt{3} + 2)x + y = -5$ एक आपसमा लम्ब हुन्छ भनी प्रमाणित गर्नुहोस् । **[2069 R']**
Prove that two straight lines $(2 - \sqrt{3})x - y - 1 = 0$ and $(\sqrt{3} + 2)x + y = -5$ are perpendicular to each other.

7J. रेखाहरू $3y - 2x - 1 = 0$ र $3x + 2y + 5 = 0$ आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।
Prove that the lines $3y - 2x - 1 = 0$ and $3x + 2y + 5 = 0$ are perpendicular to each other. **[2068 S']**

7K. दुई सरल रेखाहरू $3x + 4y = 8$ र $4x - 3y = 10$ का झुकावहरू पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् ।
Find the slopes of two straight lines $3x + 4y = 8$ and $4x - 3y = 10$, write the relationship between them.

7L. दुई सरल रेखाहरू $2x + 5y = 8$ र $10x - 4y = 25$ का झुकाव पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् ।
Find the slopes of two straight lines $2x + 5y = 8$ and $10x - 4y = 25$, write the relationship between them.

8A. समीकरणहरू $3x + 4y = 12$ र $2x - py = 5$ भएका सीधा रेखाहरू लम्ब छन् भने p को मान पत्ता लगाउनुहोस् ।
The straight lines having equations $3x + 4y = 12$ and $2x - py = 5$ are perpendicular to each other, find the value of p. **[2072 S]**

8B. यदि सीधा रेखाहरू $2x + 3y + 6 = 0$ र $ax - 5y + 20 = 0$ एक आपसमा लम्ब छन् भने a को मान पत्ता लगाउनुहोस् ।
If the straight lines $2x + 3y + 6 = 0$ and $ax - 5y + 20 = 0$ are perpendicular to each other, find the value of a. **[2068 R]**

www.drmathnp.com

puspas.com.np

8C. यदि समीकरणहरू $3x-4y+7=0$ र $ax+3y-5=0$ ले दिने रेखाहरू आपसमा लम्ब हुन्छन् भने a को मान पत्ता लगाउनुहोस् ।
Find the value of a if the lines given by the equations $3x - 4y + 7 = 0$ and $ax + 3y - 5 = 0$ are perpendicular to each other. [2063 S]

8D. $kx-3y-31=0$ र $3x+4y+5=0$ आपसमा लम्बवत भए k को मान निकाल्नुहोस् ।
If $kx - 3y - 31 = 0$ and $3x + 4y + 5 = 0$ are perpendicular to each other, find the value of k. [2066 R]

8E. दुई सरल रेखाहरू $px+3y-12=0$ र $4y-3x+7=0$ एक आपसमा लम्ब भए p को मान निकाल्नुहोस् । [2070 R']
Two straight lines $px + 3y - 12 = 0$ and $4y - 3x + 7 = 0$ are perpendicular to each other, find the value of p.

8F. समीकरणहरू $2x-y+1=0$ र $x+my-7=0$ भएका दुई रेखाहरू आपसमा लम्ब छन् भने m को मान पत्ता लगाउनुहोस् ।
The two straight lines having equations $2x - y + 1 = 0$ and $x + my - 7 = 0$ are perpendicular to each other. Find the value of m. [SEE 2074 S']

9A. यदि बिन्दुहरू $(3,-4)$ र $(-2,k)$ भएर जाने रेखा समीकरण $5x+y=-3$ भएको रेखासँग लम्ब हुन्छ भने k को मान पत्ता लगाउनुहोस् ।
If the line passing through the points $(3, -4)$ and $(-2, k)$ is perpendicular to the line having equation $5x + y = -3$, find the value of k. [2072 R']

9B. रेखा $kx-3y+6=0$ र बिन्दुहरू $(4,3)$ र $(5,-3)$ जोड्ने रेखा आपसमा लम्ब छन् भने k को मान कति हुन्छ ? [2064 R]
For what value of k, the line $kx - 3y + 6 = 0$ is perpendicular to the line joining the points $(4, 3)$ and $(5, -3)$?

9C. रेखा $ax+3y+5=0$ र बिन्दुहरू $(4,3)$ र $(6,-3)$ जोड्ने रेखा आपसमा लम्ब छन् भने a को मान पत्ता लगाउनुहोस् ।
The line $ax + 3y + 5 = 0$ is perpendicular to the line joining the points $(4, 3)$ and $(6, -3)$, find the value of a. [2065 S]

Model 5 *Problem based on finding the value of m or k when angle between the lines is given.*

10A. सरल रेखा $3x-my=19$ ले सरल रेखा $3x+5y=7$ सँग $45°$ को कोण बनाउँछ भने m को मान निकाल्नुहोस् ।
Calculate the value of m if the line $3x - my = 19$ makes an angle of $45°$ with the line $3x + 5y = 7$. [2067 R]

10B. दुईओटा सरल रेखाहरू $2x+6=y$ र $3x+ky=-4$ बिचको न्यूनकोण $45°$ छ भने k को मान पत्ता लगाउनुहोस् ।
Find the value of k if the acute angle between $2x + 6 = y$ and $3x + ky = -4$ is $45°$.

10C. सरल रेखा $3x-ky=8$ ले सरल रेखा $3x+5y=17$ सँग $45°$ को कोण बनाउँदछ भने k को मान पत्ता लगाउनुहोस् ।
Find the value of k if a line $3x - ky = 8$ makes an angle of $45°$ with the line $3x + 5y = 17$. [SEE 2075 R'$_2$]

10D. सरल रेखा $kx+y=2$ र $(2-\sqrt{3})x-y=1-2\sqrt{3}$ बिचको कोण $60°$ छ भने k को मान पत्ता लगाउनुहोस् ।
Find the value of k if the angle between the lines $kx + y = 2$ and $(2-\sqrt{3})x - y = 1 - 2\sqrt{3}$ is $60°$.

C. LONG QUESTIONS

Model 1 *Problems based on finding equation of parallel line to the given line.*

1A. समीकरण $x-2y-4=0$ भएको रेखासँग समानान्तर हुने र बिन्दु $(3,2)$ बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line passing through the point $(3, 2)$ and parallel to the line having equation $x - 2y - 4 = 0$. [2072 R']

1B. बिन्दु $(-2,-3)$ बाट जाने र रेखा $5x+7y=14$ सँग समानान्तर हुने सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line which is parallel to the line with equation $5x + 7y = 14$ and passes through the point $(-2, -3)$. [2068 R']

1C. बिन्दु $(3,2)$ भएर जाने र रेखा $5x+3y=13$ सँग समानान्तर हुने सरल रेखाको समीकरण पत्ता लगाउनुहोस् । [2068 S']
Find the equation of a straight line which passes through the point $(3, 2)$ and is parallel to the line $5x + 3y = 13$.

2A. समीकरण $2x+y-4=0$ भएको रेखासँग समानान्तर हुने र y-अक्षमा बनाएको खण्डको लम्बाइ 2 एकाइ भएको सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line which is parallel to the line having equation $2x + y - 4 = 0$ and making an intercept of length 2 units along y-axis. [2072 R]

2B. रेखा $5x+4y=9$ सँग समानान्तर हुने र X-खण्ड -5 बनाउने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line parallel to the line $5x + 4y = 9$ and making an intercept -5 on the X-axis.

2C. X-अक्षमा उद्गमबिन्दुबाट 4 एकाइको खण्ड बनाउने र रेखा $2x-3y+6=0$ सँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line which cuts off an intercept 4 units on X-axis from the origin and is parallel to the line $2x - 3y + 6 = 0$.

3A. बिन्दुहरू $(2,3)$ र $(3,-1)$ जोड्ने रेखासँग समानान्तर भई बिन्दु $(2,1)$ भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line which passes through the point $(2, 1)$ and is parallel to the line joining the points $(2, 3)$ and $(3, -1)$. [2065 S]

3B. बिन्दुहरू $(2,3)$ र $(4,5)$ जोड्ने रेखासँग समानान्तर भई बिन्दु $(3,8)$ भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line which passes through the point $(3, 8)$ and is parallel to the line joining the points $(2, 3)$ and $(4, 5)$.

www.drmathnp.com

puspas.com.np

4A. रेखा $3x - 2y = 5$ सँग समानान्तर भई बिन्दुहरू $(-4, 2)$ र $(2, 4)$ जोड्ने रेखाखण्डको मध्यबिन्दु भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line parallel to the line $3x - 2y = 5$ and passing through the midpoint of the line segment joining the points $(-4, 2)$ and $(2, 4)$. [2068 S]

4B. रेखा $3x - 2y = 4$ सँग समानान्तर भई बिन्दुहरू $(2, -4)$ र $(2, 4)$ जोड्ने रेखाखण्डको मध्यबिन्दु भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a line parallel to to the line $3x - 2y = 4$ and passing through the mid-point of the line segment connecting $(2, -4)$ and $(2, 4)$.

4C. रेखा $4x - y + 7 = 0$ सँग समानान्तर भई बिन्दुहरू $(-2, 3)$ र $(4, -7)$ जोड्ने रेखाखण्डको मध्यबिन्दु भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a line parallel to to the line $4x - y + 7 = 0$ and passing through the mid-point of the line segment connecting $(-2, 3)$ and $(4, -7)$.

5A. A(3, 2), B(1, – 1) र C(5, – 5) त्रिभुज ABC का शीर्षबिन्दुहरू हुन् । ΔABC को गुरुत्वकेन्द्रबाट जाने र BC भुजासँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
A(3, 2), B(1, – 1) and C(5, – 5) are the vertices of a triangle ABC. Find the equation of a straight line passing through the centroid of ΔABC and parallel to the side BC. [2070 R']

5B. शीर्षबिन्दुहरू P(3, 3), Q(– 2, – 6) र R(5, – 3) भएको ΔPQR को भारकेन्द्र भएर जाने र QR रेखासँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line passing through the centroid of ΔPQR with vertices P(3, 3), Q(– 2, – 6) and R(5, – 3) and parallel to the line QR. [2070 R]

Model 2 ***Problems based on finding equation of perpendicular line passing through given point.***

6A. बिन्दु (3, 2) भएर जाने र रेखा $4x - 3y - 10 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line passing through the point (3, 2) and perpendicular to the line $4x - 3y - 10 = 0$. [2075 R']

6B. समीकरण $7x - 5y - 6 = 0$ भएको रेखासँग लम्ब हुने र बिन्दु $(-1, -2)$ बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line which is perpendicular to the line $7x - 5y - 6 = 0$ and passing through the point $(-1, -2)$. [2072 S]

6C. बिन्दु (2, 3) बाट जाने र रेखा $4x - 3y = 10$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2057 R]
Find the equation of straight line passing through the point (2, 3) and perpendicular to the line $4x - 3y = 10$.

6D. बिन्दु (2, – 3) बाट जाने र रेखा $3x + 4y + 18 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2057S]
Find the equation of straight line passing through the point (2, – 3) & perpendicular to the line $3x + 4y + 18 = 0$.

6E. बिन्दु (4, 6) बाट जाने र रेखा $x - 2y - 2 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2058 R]
Find the equation of straight line passing through the point (4, 6) and perpendicular to the line $x - 2y - 2 = 0$.

6F. बिन्दु $(-2, -3)$ बाट जाने र रेखा $5x + 7y = 14$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight line passing through the point $(-2, -3)$ and perpendicular to the line $5x + 7y = 14$. [2058 S, 2066 R']

6G. बिन्दु $(-6, 4)$ बाट जाने र रेखा $3x - 4y + 9 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2059 S]
Find the equation of straight line passing through the point $(-6, 4)$ and perpendicular to the line $3x - 4y + 9 = 0$.

6H. बिन्दु (2, 3) बाट जाने र रेखा $x - 3y - 2 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2060 R]
Find the equation of straight line passing through the point (2, 3) and perpendicular to the line $x - 3y - 2 = 0$.

6I. बिन्दु (2, – 3) बाट जाने र रेखा $5x - 4y + 19 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2061S]
Find the equation of straight line passing through the point (2, – 3) and perpendicular to the line $5x - 4y + 19 = 0$.

6J. बिन्दु (7, 1) बाट जाने र रेखा $5x + 7y + 12 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् । [2063 R']
Find the equation of straight line passing through the point (7, 1) and perpendicular to the line $5x + 7y + 12 = 0$.

6K. बिन्दु (2, 3) बाट जाने र रेखा $x - 2y - 5 = 0$ सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् । [2063 S]
Find the equation of straight line passing through the point (2, 3) and perpendicular to the line $x - 2y - 5 = 0$.

7A. बिन्दु P (– 2, 4) बाट रेखा $7x - 24y + 10 = 0$ मा लम्ब PQ खिचिएको छ भने PQ रेखाको समीकरण पत्ता लगाउनुहोस् र PQ को लम्बाइ पनि पत्ता लगाउनुहोस् ।
From the point P(– 2, 4), if PQ is drawn perpendicular to the line $7x - 24y + 10 = 0$, find the equation of the line PQ. Also, determine the length of PQ. [2061 R]

www.drmathnp.com

puspas.com.np

7B. बिन्दु P(3, 5) बाट रेखा $2x = 2y + 4$ मा लम्ब PQ खिचिएको छ भने PQ रेखाको समीकरण पत्ता लगाउनुहोस् र PQ को लम्बाइ पनि पत्ता लगाउनुहोस् ।
From the point P(3, 5), if PQ is drawn perpendicular to the line $2x = 2y + 4$, find the equation of the line PQ. Also, determine the length of PQ.

8A. M(4, 7) र N(5, – 2) दुईओटा बिन्दुहरू हुन् । MN मा लम्ब हुने र बिन्दु (2, 3) भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
M(4, 7) and N(5, – 2) are two points. Find the equation of the straight line perpendicular to MN and passing through the point (2, 3). **[2067 S]**

8B. बिन्दु (– 6, 2) भएर जाने र बिन्दुहरू (– 3, 2) र (5, – 3) जोड्ने रेखामा लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight line passing through the point (– 6, 2) and perpendicular to the line joining the points (– 3, 2) and (5, –3). **[2069 R]**

8C. बिन्दुहरू (3, 4) र (– 1, – 6) जोड्ने रेखाको मध्यबिन्दुबाट जाने र रेखा $3x - 8y + 7 = 0$ सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Determine the equation of the line passing through the mid-points of the line joining the points (3, 4) and (– 1, – 6) and perpendicular to the line $3x - 8y + 7 = 0$. **[2062 R]**

Model 3 ***On finding equation of perpendicular bisector of line segment joining given points.***

9. दिइएका बिन्दुहरू जोड्ने रेखाखण्डसँग लम्बार्धक रेखाको समीकरण पत्ता लगाउनुहोस्:
Find the equation of the perpendicular bisector of the line segment joining given points:

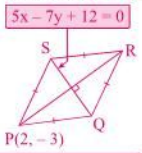
A.	(2, 4) and (8, 10).	**[2074 R']**	**B.**	(3, – 7) and (– 5, 3)	**[2060 S]**
C.	(3, – 1) and (7, 1)	**[2059 R, 2065 R]**	**D.**	(3, 5) and (9, 3)	**[2061 S, 2063 R]**
E.	(4, – 2) and (6, 2)	**[2060 CP]**	**F.**	(3, 2) and (7, 6)	**[2067 R']**
G.	(4, – 5) and (–8, 9)	**[2069 S]**	**H.**	A(3, – 7) and B(– 5, 3).	**[2071 R']**
I.	(4, – 3) and (8, 5)	**[2064 S]**	**J.**	(3, 5) and (– 7, 3).	**[SEE 2073 R']**
K.	M(2, 5) and N(8, – 1)	**[2074 S']**			

10A. एउटा वर्गको विकर्णका अन्तिम बिन्दुहरू (2, 3) र (– 6, 5) हुन् भने अर्को विकर्णको समीकरण पत्ता लगाउनुहोस् । **[2065 M]**
If (2, 3) and (– 6, 5) are the end points of one of the diagonal of a square, find the equation of the other diagonal.

10B. समबाहु चतुर्भुज ABCD का विपरीत शीर्षबिन्दुहरू A(2, 4) र C(8, 10) भए पत्ता लगाउनुहोस्:
(i) AC विकर्णको मध्यबिन्दु (ii) AC भुजाको झुकाव (iii) BD विकर्णको समीकरण
A(2, 4) and C(8, 10) are the opposite vertices of a rhombus ABCD, find:
(i) the mid point of diagonal AC. (ii) slope of AC. (iii) the equation of the diagonal BD. **[2065 R']**

10C. समबाहु चतुर्भुज ABCD को दुई विपरीत शीर्ष बिन्दुहरू A(3, 5) र C(7, 9) भए विकर्ण BD को समीकरण पत्ता लगाउनुहोस् ।
A(3, 5) and C(7, 9) are the opposite vertices of a rhombus ABCD. Find the equation of the diagonal BD. **[2067 R]**

10D. दिइएको चित्रमा PQRS एउटा समबाहु चतुर्भुज हो । यदि विकर्ण QS को समीकरण $5x - 7y + 12 = 0$ छ र बिन्दु P को निर्देशाङ्कहरू (2, – 3) भए विकर्ण PR को समीकरण पत्ता लगाउनुहोस् ।
In the given figure, PQRS is a rhombus. If the equation of a diagonal QS is $5x - 7y + 12 = 0$ and the coordinates of the point P are (2, – 3), find the equation of the diagonal PR. **[2071 S]**



Model 4 ***Problems based on finding equation of height of a triangle.***

11A. बिन्दुहरू M(3, 4), N(– 1, 1) र P(5, – 1) एउटा त्रिभुज MNP का शीर्षबिन्दुहरू हुन् । बिन्दु N(– 1, 1) बाट खिचिएको त्रिभुज MNP को उचाइको समीकरण पत्ता लगाउनुहोस् ।
The points M(3, 4), N(– 1, 1) and P(5, – 1) are the vertices of a triangle MNP. Find the equation of the altitude of the triangle MNP drawn from the point N(–1, 1). **[2074 R]**

11B. बिन्दुहरू P(2, 3), Q(– 1, 1) र R(5, – 1) त्रिभुज PQR का शीर्षबिन्दुहरू हुन् । बिन्दु Q(– 1, 1) बाट खिचिएको त्रिभुज PQR को उचाइको समीकरण पत्ता लगाउनुहोस् ।
The points P(2, 3), Q(– 1, 1) and R(5, – 1) are the vertices of a triangle PQR. Find the equation of the altitude of the triangle PQR drawn from the point Q(– 1, 1). **[2074 S]**

11C. एउटा त्रिभुजका शीर्षबिन्दुहरू A(– 2, 1), B(2, 2) र C(6, – 2) छन् । A बाट BC मा खिचिएको उचाइ AD को समीकरण पत्ता लगाउनुहोस् ।
The vertices of a triangle are A(– 2, 1), B(2, 2) and C(6, – 2). Find the equation of the altitude AD drawn from A to BC. **[2070 S']**

11D. A(2, 3), B(– 4, 1) र C(2, 0) शीर्षबिन्दुहरू भएको त्रिभुज ABC को शीर्षबिन्दु A(2, 3) बाट खिचिएको उचाइको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the altitude of triangle ABC with vertices A(2, 3), B(– 4, 1) and C(2, 0) drawn from the vertex A(2, 3). **[2062 K]**

www.drmathnp.com

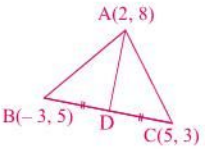
puspas.com.np

11E. ΔABC का शीर्षबिन्दुहरू A(3, 4), B(– 2, 2) र C(3, – 3) छन् । शीर्षबिन्दु A बाट सम्मुख भुजा BC मा AD लम्ब खिचिएको छ । AD को समीकरण पत्ता लगाउनुहोस् ।

The vertices of a ΔABC are A(3, 4), B(– 2, 2) and C(3, – 3). AD is perpendicular drawn from the vertex A on the opposite side BC. Find the equation of AD. **[2064 R]**

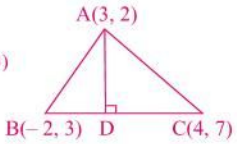
11F. दिइएको चित्रमा BC को मध्यबिन्दु D भए AD ⊥ BC हुन्छ भनी प्रमाणित गर्नुहोस् । AD को समीकरण पनि पत्ता लगाउनुहोस् ।

In the given figure, D is the mid-point of BC. Prove that AD ⊥ BC. Find the equation of AD also. **[2060S']**



11G. दिइएको चित्रबाट रेखा AD को समीकरण पत्ता लगाउनुहोस् ।

Find the equation of line AD from the given figure.



12A. बिन्दु C ले बिन्दुहरू A(2, 3) र B(– 4, 1) जोड्ने रेखाखण्ड AB लाई 2 : 1 को अनुपातमा विभाजन गर्दछ । बिन्दु C भएर जाने र AB सँग लम्ब हुने रेखाको समीकरण निकाल्नुहोस् ।

The point C divides the line segment AB joining the points A(2, 3) and B(– 4, 1) in the ratio 2 : 1. Find the equation of the line passing through the point C and perpendicular to AB. **[2070 S]**

12B. A(– 2, 6) र B(3, – 4) जोड्ने रेखालाई 2: 3 को अनुपातमा विभाजन गर्ने बिन्दु P भएर जाने र सीधा रेखा $2x + 3y = 6$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Write down the equation of the straight line perpendicular to $2x + 3y = 6$ and which passes through P, where P divides the line joining A(– 2, 6) and B(3, – 4) in the ratio of 2 : 3.

***Model 5** On finding equation of the straight lines, passing through (x, y) & making θ° with given line.*

13A. बिन्दु (4, – 1) भएर जाने र रेखा $2x - 3y = 5$ सँग 45° को कोण बनाउने कुनै एउटा सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of any one straight line passing through the point (4, – 1) and making an angle of 45° with the line $2x - 3y = 5$. **[SEE 2075 R, 2075 R$_2$]**

13B. बिन्दु (2, – 1) भएर जाने र समीकरण $6x + 5y - 2 = 0$ भएको रेखासँग 45° को कोण बनाउने एउटा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through a point (2, – 1) and making an angle of 45° with the straight line having equation $6x + 5y - 2 = 0$. **[SEE 2073 S']**

13C. रेखा $x - 3y = 2$ सँग 45° को कोण बनाउने र बिन्दु (2, 3) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the straight lines passing through the point (2, 3) and making an angle of 45° with the line $x - 3y = 2$. **[2063 M, 2069 R']**

13D. समीकरण $4x - 5y + 9 = 0$ भएको रेखासँग 45° को कोण बनाउने र बिन्दु (5, 0) बाट जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the straight lines passing through the point (5, 0) & making an angle of 45° with the line $4x - 5y + 9 = 0$. **[2066 R]**

13E. समीकरण $x - \sqrt{3}y = 4$ भएको रेखासँग 30° को कोण बनाउने र बिन्दु (1, 0) बाट जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

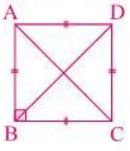
Find the equation of the straight lines passing through the point (1, 0) & inclined at an angle of 30° with the line $x - \sqrt{3}y = 4$. **[2065 E]**

13F. बिन्दु (1, – 4) बाट जाने र रेखा $2x + 3y + 5 = 0$ सँग 45° को कोण बनाउने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the straight lines passing through (1, – 4) and making an angle of 45° with the line $2x + 3y + 5 = 0$. **[2066 S]**

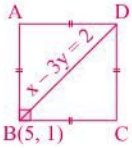
14A. दिइएको वर्ग ABCD मा विकर्ण AC को समीकरण $3x - 4y + 10 = 0$ र शीर्षबिन्दु B को निर्देशाङ्कहरू (4, – 5) भए विकर्ण BD को समीकरण पत्ता लगाउनुहोस् ।

The equation of a diagonal AC of given square ABCD is $3x - 4y + 10 = 0$ and the coordinates of vertex B are (4, – 5). Find the equation of diagonal BD. **[2071 R]**



14B. ABCD एउटा वर्ग हो । यसको विकर्ण BD को समीकरण $x - 3y = 2$ भए AB र BC का समीकरणहरू पत्ता लगाउनुहोस् ।

ABCD is a square. If the equation of its diagonal BD is $x - 3y = 2$, find the equations of AB and BC.



COORDINATES

www.drmathnp.com

puspas.com.np

Model 6 *Miscellaneous problems on angle between two lines.*

15A. रेखा $2x - y = 3$ सँग समानान्तर भई रेखाहरू $3x + y = 7$ र $3y = 4x - 5$ को प्रतिच्छेदन बिन्दुबाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the line passing through the point of intersection of the lines $3x + y = 7$ and $3y = 4x - 5$ and parallel to the line $2x - y = 3$. **[SEE 2073 S]**

15B. रेखाहरू $x + y = 5$ र $x - y = 1$ को प्रतिच्छेदन बिन्दु भएर जाने र रेखा $2x + 3y = 5$ सँग समानान्तर हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the line passing through the point of intersection of the lines $x + y = 5$ and $x - y = 1$ and parallel to the line $2x + 3y = 5$.

15C. सीधारेखाहरू $3x + 4y - 11 = 0$ र $2x - 3y + 4 = 0$ को प्रतिच्छेदन बिन्दु भएर जाने र सीधारेखा $2x + 3y = 5$ सँग समानान्तर हुने एउटा सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through the point of intersection of the lines $3x + 4y - 11 = 0$ and $2x - 3y + 4 = 0$ and parallel to the straight line $2x + 3y = 5$.

15D. सीधारेखाहरू $3x + 4y = 7$ र $5x - 2y = 3$ को प्रतिच्छेदन बिन्दु भएर जाने र सीधारेखा $2x + 3y = 5$ सँग लम्ब हुने एउटा सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line passing through the point of intersection of the straight lines $3x + 4y = 7$ and $5x - 2y = 3$ and perpendicular to the straight line $2x + 3y = 5$. **[SEE 2073 R]**

15E. रेखाहरू $x + 2y = 4$ र $x - 2y = 0$ को प्रतिच्छेदन बिन्दु भएर जाने र रेखा $6x - 7y + 8 = 0$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the line passing through the point of intersection of the lines $x + 2y = 4$ and $x - 2y = 0$ and perpendicular to the line $6x - 7y + 8 = 0$.

16A. यदि सरल रेखा $\frac{x}{a} + \frac{y}{b} = 1$ रेखाहरू $x + y = 3$ र $2x - 3y = 1$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $y = x - 6$ सँग समानान्तर हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।

If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point of intersection of the lines $x + y = 3$ and $2x - 3y = 1$ and is parallel to the line $y = x - 6$, then find the values of a and b. **[2068 R]**

16B. यदि सरल रेखा $\frac{x}{a} + \frac{y}{b} = 1$ रेखाहरू $x = 3$ र $2x + 3y = 12$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $2x + 3y = 8$ सँग लम्ब हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।

If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point of intersection of the lines $x = 3$ and $2x + 3y = 12$ and is perpendicular to the line $2x + 3y = 8$, then find the values of a and b.

ANSWERS

A. VERY SHORT QUESTIONS

5A. m **5B.** $y = mx + c$ **6A.** 90° **6B.** 90° **7A.** $\frac{3}{4}$ **7B.** 5 **8A.** $-\frac{3}{2}$ **8B.** -5

B. SHORT QUESTIONS

1A. 30° **1B.** 135° **1C.** 60° **1D.** 45° **1E.** 37.87° **2A.** 135° **2B.** 60°

3F. $-\frac{3}{4}$, Parallel **3G.** $\frac{4}{5}$, Parallel **4A.** $-\frac{9}{4}$ **4B.** 3 **4C.** 2 **5A.** 6 **5B.** $\frac{31}{2}$

5C. 3 **6A.** $-\frac{4}{3}$ **6B.** $\frac{2}{3}$ **6C.** $\frac{3}{4}$ **8A.** $\frac{3}{2}$ **8B.** $\frac{15}{2}$ **8C.** $a = 4$ **8D.** 4 **8E.** 4

8F. 2 **9A.** -5 **9B.** $\frac{1}{2}$ **9C.** -1 **10A.** 12 **10B.** -9 **10C.** 12 or $-\frac{3}{4}$ **10D.** $2 + \sqrt{3}$

C. LONG QUESTIONS

1A. $x - 2y + 1 = 0$ **1B.** $5x + 7y + 31 = 0$ **1C.** $5x + 3y = 21$ **2A.** $2x + y = 2$ **2B.** $5x + 4y + 25 = 0$

2C. $2x - 3y = 8$ **3A.** $4x + y = 9$ **3B.** $x - y + 5 = 0$ **4A.** $3x - 2y + 9 = 0$ **4B.** $3x - 2y = 6$

4C. $4x - y = 6$ **5A.** $3x + 3y - 5 = 0$ **5B.** $3x - 7y - 20 = 0$ **6A.** $3x + 4y - 17 = 0$ **6B.** $5x + 7y + 19 = 0$

6C. $3x + 4y = 18$ **6D.** $4x - 3y - 17 = 0$ **6E.** $2x + y = 14$ **6F.** $7x - 5y - 1 = 0$ **6G.** $4x + 3y + 12 = 0$

6H. $3x + y = 9$ **6I.** $4x + 5y + 7 = 0$ **6J.** $7x - 5y = 44$ **6K.** $2x + y - 7 = 0$ **7A.** $24x + 7y + 20 = 0$; 4 units

7B. $x + y = 8$; $2\sqrt{2}$ units **8A.** $x - 9y + 25 = 0$ **8B.** $8x - 5y + 58 = 0$ **8C.** $8x + 3y = 5$ **9A.** $x + y = 12$

9B. $4x - 5y - 6 = 0$ **9C.** $2x + y = 10$ **9D.** $3x - y = 14$ **9E.** $x + 2y = 5$ **9F.** $x + y = 9$

9G. $7y = 6x + 26$ **9H.** $4x - 5y = 6$ **9I.** $x + 2y = 8$ **9J.** $5x + y + 6 = 0$ **9K.** $x - y - 3 = 0$

10A. $4x - y + 12 = 0$ **10B.** (5, 7), $1, x + y = 12$ **10C.** $x + y = 12$ **10D.** $7x + 5y + 1 = 0$ **11A.** $2x - 5y + 7 = 0$

11B. $3x - 4y + 7 = 0$ **11C.** $x - y + 3 = 0$ **11D.** $6x - y = 9$ **11E.** $x - y + 1 = 0$ **11F.** $y = 4x$

11G. $3x + 2y - 13 = 0$ **12A.** $9x + 3y + 13 = 0$ **12B.** $3x - 2y + 4 = 0$ **13A.** $5x - y = 21$ or $x + 5y + 1 = 0$

13B. $11x - y = 23$, $x + 11y + 9 = 0$ **13C.** $2x - y = 1$, $x + 2y = 8$ **13D.** $9x - y = 45$, $x + 9y = 5$ **13E.** $y = 0$ and $\sqrt{3}x - y = \sqrt{3}$

13F. $5x + y = 1$ and $x - 5y = 21$ **14A.** $4x + 3y - 1 = 0$ **14B.** $x + 2y = 7$, $2x - y = 9$ **15A.** $2x - y = 3$

15B. $2x + 3y = 12$ **15C.** $2x + 3y = 8$ **15D.** $3x - 2y = 1$ **15E.** $7x + 6y = 20$ **16A.** 1, -1

16B. 5/3, $-5/2$

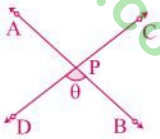
www.drmathnp.com

puspas.com.np

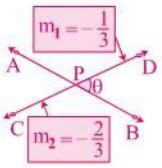
GENERAL EXERCISE 1

A. SHORT QUESTIONS

Model 1 *Problems based on finding angle between two lines.*



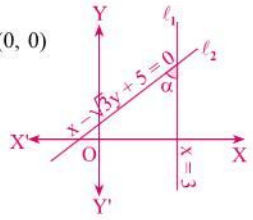
1A. सँगैको चित्रमा रेखा AB को झुकाव $-\sqrt{3}$ र CD को झुकाव $\sqrt{3}$ भए $\measuredangle$BPD को नाप पत्ता लगाउनुहोस् ।
In the adjoining figure, the slope of line AB is $-\sqrt{3}$ and the slope of CD is $\sqrt{3}$. Find the measure of $\measuredangle$BPD.



1B. चित्रमा दिइएको जानकारीबाट $\tan\theta$ को मान पत्ता लगाउनुहोस् ।
From the given information in figure, find the value of $\tan\theta$.

1C. झुकाव 3 भएको रेखाले $4x - y = 0$ सँग बनाउने न्यूनकोण पत्ता लगाउनुहोस् ।
Find an actue angle between the line whose slope is 3 and the line $4x - y = 0$.

1D. रेखा PQ को झुकाव 0.2 छ र रेखा RS बिन्दुहरू (2, 3) र (0, 0) भएर जान्छ भने यी रेखाहरूको बिच बन्ने न्यूनकोण पत्ता लगाउनुहोस् ।
Slope of a straight line PQ is 0.2 and the line RS passes through the points (0, 0) and (2, 3). Find acute angle between the lines.



1E. दिइएको चित्रबाट ℓ_1 र ℓ_2 बिचको कोण पत्ता लगाउनुहोस् ।
Find the angle between ℓ_1 and ℓ_2 from the figure.

2. दिइएका समीकरणहरू भएका रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस्:
Find the acute angle between the lines having given equations:

A. $y = 3x + 7, 3y - 2x = 8$ **B.** $2x - y = 0, x - 3y = 0$

3. दिइएका समीकरणहरू भएका रेखाहरूबिचको अधिक कोण पत्ता लगाउनुहोस्:
Find the obtuse angle between the lines having given equations:

A. $x + \sqrt{3}y = 2, \sqrt{3}x + y = 7$ **B.** $2x - y = 0, x - 3y = 0$

C. $y - (2 - \sqrt{3})x - 4 = 0, y = (2 + \sqrt{3})x - 5$ **D.** $\sqrt{3}x - y + 4 = 0, \sqrt{3}y = x - 5$

Model 2 *Problems based on slope of a line and angle between two lines.*

4A. $2(x + y) = 3(x - y) + 8$ सँग समानान्तर हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of the line parallel to $2(x + y) = 3(x - y) + 8$.

4B. रेखा $x\cos\alpha + y\sin\alpha = p$ सँग लम्ब हुने रेखाको झुकाव निकाल्नुहोस् ।
Find the slope of the line which is perpendicular to the line $x\cos\alpha + y\sin\alpha = p$.

4C. रेखा $x\cos 30° + y\sin 30° = 4$ सँग लम्ब हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of the line which is perpendicular to the line $x\cos 30° + y\sin 30° = 4$.

5A. सीधा रेखा $\sqrt{3}x + y = 2$ सँग 60° को कोण बनाउने सीधा रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of a straight line which makes an angle 60° with the line $\sqrt{3}x + y = 2$.

5B. सीधा रेखा $\sqrt{3}x + y = 2$ सँग 30° को कोण बनाउने सीधा रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of a straight line which makes an angle 30° with the line $\sqrt{3}x + y = 2$.

6A. बिन्दुहरू (– 1, – 1) र (5, 1) बाट जाने रेखासँग 45° को कोण बनाउने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of a line which makes an angle of 45° with the line passing through (– 1, – 1) and (5, 1).

6B. बिन्दुहरू (1, – 1) र (6, –7) बाट जाने रेखासँग 45° को कोण बनाउने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of a line which makes an angle of 45° with the line passing through (1, – 1) and (6, – 7).

6C. बिन्दुहरू (2, 3) र (0, 2) बाट जाने रेखासँग 30°को कोण बनाउने रेखाको झुकाव पत्ता लगाउनुहोस् ।
Find the slope of a straight line which makes an angle 30° with line passing through (2, 3) and (0, 2).

7A. ΔABC का शीर्षबिन्दुहरू A(10, 4), B(– 4, 3) र C(– 2, 1) छन् । उचाइ AD को झुकाव निकाल्नुहोस् ।
The vertices of Δ ABC are A(10, 4), B(– 4, 3) and C(– 2, 1). Find the slope of altitude AD.

7B. ΔABC मा शीर्षबिन्दुहरू A(5, 3), B(10, 2) र C(7, 5) छन् भने उचाइ BD को झुकाव निकाल्नुहोस् ।
The vertices of ΔABC are A(5, 3), B(10, 2) and C(7, 5). Find the slope of altitude BD.

COORDINATES

www.drmathnp.com

puspas.com.np

Model 3 Problems based on condition of parallel lines.

8A. समीकरण $y + 2x + 3 = 0$ भएको रेखा र बिन्दुहरू $(3, -4)$ र $(-2, 6)$ जोड्ने रेखा एक आपसमा समानान्तर हुन्छन् भनी देखाउनुहोस् ।
Show that the line joining the points $(3, -4)$ and $(-2, 6)$ is parallel to the line $y + 2x + 3 = 0$.

8B. बिन्दुहरू $(2, 4)$ र $(3, 6)$ जोड्ने रेखा र बिन्दुहरू $(4, 8)$ र $(6, 12)$ जोड्ने रेखा एक आपसमा समानान्तर हुन्छन् भनी देखाउनुहोस् ।
Show that the line joining the points $(2, 4)$ and $(3, 6)$ and the line joining the points $(4, 8)$ and $(6, 12)$ are parallel to each other.

9A. बिन्दुहरू $(2, P)$ र $(3, 6)$ जोड्ने रेखा र बिन्दुहरू $(4, 8)$ र $(6, 12)$ जोड्ने रेखा आपसमा समानान्तर भए P को मान निकाल्नुहोस् ।
The line joining the points $(2, P)$ and $(3, 6)$ & line joining the points $(4, 8)$ and $(6, 12)$ are parallel to each other then find the value of P.

9B. रेखा $2x + 4y = 9$ र बिन्दुहरू $(a, 2)$ र $(4, 3)$ जोड्ने रेखा आपसमा समानान्तर भए a को मान निकाल्नुहोस् ।
The line $2x + 4y = 9$ and the line joining the points $(a, 2)$ and $(4, 3)$ are parallel to each other then find the value of a.

9C. रेखा $4x + 5y = 20$ र बिन्दुहरू $(0, P)$ र $(10, 0)$ जोड्ने रेखा आपसमा समानान्तर भए P को मान निकाल्नुहोस् ।
The line $4x + 5y = 20$ and the line joining the points $(0, P)$ and $(10, 0)$ are parallel to each other then find the value of P.

10. दिइएका समीकरणहरू भएका रेखाहरू एक आपसमा समानान्तर भए k को मान निकाल्नुहोस्:
If the lines having the following equations are parallel to each other, find the value of k:

A. $(k + 3)x + y = 0$, $2x - ky + 7 = 0$

B. $2kx - y - 19 = 0$, $(k - 1)x + 3y = 0$

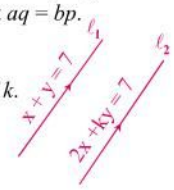
C. $3x + 2ky = 6$ and $8y = 6x - 5$

D. $kx - 5y + 2 = 0$ and $2x + y + 1 = 0$

11A. यदि रेखाहरू $ax + by + 7 = 0$ र $4x - 3y + 2 = 0$ परस्पर समानान्तर छन् भने a र b को सम्बन्ध निकाल्नुहोस् ।
If the lines $ax + by + 7 = 0$ and $4x - 3y + 2 = 0$ are parallel to each other, find the relation between a and b.

11B. यदि दुई रेखाहरू $ax + by + c = 0$ र $px + qy + r = 0$ आपसमा समानान्तर भए $aq = bp$ हुन्छ भनी देखाउनुहोस् ।
If two lines $ax + by + c = 0$ and $px + qy + r = 0$ are parallel to each other then show that $aq = bp$.

11C. चित्रमा दिइएका रेखाहरू ℓ_1 र ℓ_2 आपसमा समानान्तर भए k को मान पत्ता लगाउनुहोस् ।
In the given figure, the given lines ℓ_1 and ℓ_2 are parallel to each other. Find the value of k.



11D. यदि रेखाहरू $\frac{x}{3} - \frac{y}{4} = 1$ र $ax + y = 5$ समानान्तर छन् भने a को मान पत्ता लगाउनुहोस् ।

If the lines $\frac{x}{3} - \frac{y}{4} = 1$ and $ax + y = 5$ are parallel then find the value of a.

Model 4 Problems based on perpendicular lines.

12A. समीकरण $x - 2y + 3 = 0$ भएको रेखा र बिन्दुहरू $(6, -8)$ र $(-4, 12)$ जोड्ने रेखा आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the line joining the points $(6, -8)$ and $(-4, 12)$ and the line $x - 2y + 3 = 0$ are perpendicular.

12B. बिन्दुहरू $(2, 2)$ र $(6, 6)$ जोड्ने रेखा र बिन्दुहरू $(4, 4)$ र $(2, 6)$ जोड्ने रेखा आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the line joining the points $(2, 2)$ and $(6, 6)$ and the line joining the points $(4, 4)$ and $(2, 6)$ are perpendicular.

13A. बिन्दुहरू $(2, A)$ र $(6, 6)$ जोड्ने रेखा र बिन्दुहरू $(4, 4)$ र $(2, 6)$ जोड्ने रेखा एक आपसमा लम्ब भए A को मान निकाल्नुहोस् ।
The line joining the points $(2, A)$ & $(6, 6)$ & the line joining the points $(4, 4)$ & $(2, 6)$ are perpendicular to each other. Find the value of A.

13B. समीकरणहरू $\frac{3}{y} - \frac{4}{x} + \frac{7}{xy} = 0$ र $4x - my - 5 = 0$ ले दिने रेखाहरू एक आपसमा लम्ब भए m को मान निकाल्नुहोस् ।

The lines given by the equations $\frac{3}{y} - \frac{4}{x} + \frac{7}{xy} = 0$ and $4x - my - 5 = 0$ are perpendicular to each other. Find the value of m.

13C. यदि $\frac{x}{a} + \frac{y}{4} = 1$ र $3x + 4y = 1$ आपसमा लम्ब भए a को मान पत्ता लगाउनुहोस् ।

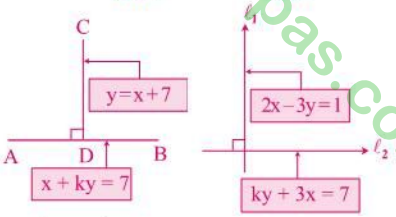
If $\frac{x}{a} + \frac{y}{4} = 1$ and $3x + 4y = 1$ are perpendicular to each other then find the value of a.

www.drmathnp.com

puspas.com.np

14A. चित्रमा रेखा AB को समीकरण $x + ky = 7$ र रेखा CD को समीकरण $y = x + 7$ छ । यदि CD ⊥ AB भए k को मान पत्ता लगाउनुहोस् ।
In the adjoining figure, equation of line AB is $x + ky = 7$ and line CD is $y = x + 7$. If CD ⊥ AB then find the value of k.

14B. सँगैको चित्रबाट $\ell_1 \perp \ell_2$ भए k को मान पत्ता लगाउनुहोस् ।
In the adjoining figure $\ell_1 \perp \ell_2$, find the value of k.



14C. यदि दुई रेखाहरू $ax + by + c = 0$ र $mx + qy + t = 0$ आपसमा लम्ब भए प्रमाणित गर्नुहोस्: $am + bq = 0$
If two lines $ax + by + c = 0$ and $mx + qy + t = 0$ are orthogonal then prove that: $am + bq = 0$

14D. k को मान कति हुँदा सरल रेखाहरू $(k + 1)x - 5y + 4 = 0$ र $(1 - k)x - 7y + 2 = 0$ एक आपसमा लम्ब हुन्छन् ?
For what value of k, the straight lines $(k + 1)x - 5y + 4 = 0$ and $(1 - k)x - 7y + 2 = 0$ are perpendicular to each other?

14E. यदि दुई रेखाहरू $(a^2 - b^2)x = (p + q)y$ र $(p^2 - q^2)x + (a + b)y = 0$ आपसमा लम्ब भए प्रमाणित गर्नुहोस्: $(a - b)(p - q) = 1$
If two lines $(a^2 - b^2)x = (p + q)y$ and $(p^2 - q^2)x + (a + b)y = 0$ are orthogonal then prove that: $(a - b)(p - q) = 1$

Model 5 *On finding the value of k when two lines and angle between them are given.*

15A. सरल रेखाहरू $6x + 2y - 7 = 0$ र $kx + 6y + 5 = 0$ बिचको अधिककोण 135° छ भने k को मान पत्ता लगाउनुहोस् ।
Find the value of k if the obtuse angle between the lines $6x + 2y - 7 = 0$ & $kx + 6y + 5 = 0$ is 135°.

15B. $2x + y = 5$ र $3x - ky + 5 = 0$ बिचको न्यूनकोण $\tan^{-1}\left(\frac{11}{2}\right)$ छ भने k को मान पत्ता लगाउनुहोस् ।

Find the value of k if an acute angle between $2x + y = 5$ and $3x - ky + 5 = 0$ is $\tan^{-1}\left(\frac{11}{2}\right)$.

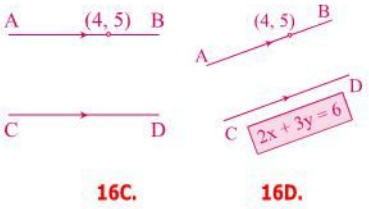
Model 6 *Problems based on finding equation of parallel lines.*

16A. झुकाव 5 हुने रेखासँग समानान्तर भई बिन्दु (4, 5) भएर जाने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight line passing through (4, 5) and parallel to the line having slope 5.

16B. झुकाव 6 भएको रेखासँग समानान्तर भई बिन्दु (2, 3) भएर जाने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight line passing through (2, 3) and parallel to the line having slope 6.

16C. सँगैको चित्रमा AB // CD छ । यदि CD को झुकाव 2/3 भए रेखा AB को समीकरण पत्ता लगाउनुहोस् ।
In the adjoining figure, AB // CD. If slope of CD is 2/3 then find the equation of line AB.

16D. चित्रमा रेखा AB बिन्दु (4, 5) भएर गएको छ । यदि AB // CD भए AB को समीकरण पत्ता लगाउनुहोस् ।
In the given figure, the line AB is passing through the point (4, 5). If AB // CD then find the equation of AB.



Model 7 *Problems based on finding equation of perpendicular lines.*

17A. झुकाव $-\frac{1}{5}$ भएको रेखासँग लम्ब भई बिन्दु (3, 4) भएर जाने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line perpendicular to the line having slope $-\frac{1}{5}$ and passing through (3, 4).

17B. झुकाव $-\frac{1}{4}$ भएको रेखासँग लम्ब भई बिन्दु (5, 7) भएर जाने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a straight line perpendicular to the line having slope $-\frac{1}{4}$ and passing through (5, 7).

17C. झुकाव $-\frac{4}{3}$ भएको एउटा रेखासँग लम्ब भई बिन्दु (–1, 3) भएर जाने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a line which passes through the point (–1, 3) and perpendicular to the line having slope $-\frac{4}{3}$.

www.drmathnp.com

puspas.com.np

B. LONG QUESTIONS

Model 1 *Problems based on equation of parallel line.*

1A. रेखा $x - 3y = 4$ सँग समानान्तर भई शीर्षबिन्दुहरू A(3, – 4), B(– 2, 1) र C(5, 0) भएको ΔABC को गुरुत्वकेन्द्र भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a line parallel to $x - 3y = 4$ and passing through the centroid of the ΔABC whose vertices are A(3, – 4), B(– 2, 1) and C(5, 0).

1B. रेखा $x - y + 1 = 0$ सँग समानान्तर भई शीर्षबिन्दुहरू A(1, 8), B(3, 7) र C(5, 9) भएको ΔABC को गुरुत्वकेन्द्र भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।

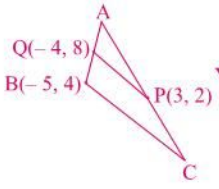
Find the equation of a line parallel to $x - y + 1 = 0$ and passing through the centroid of the ΔABC whose vertices are A(1, 8), B(3, 7) and C(5, 9).

2A. रेखा BC सँग समानान्तर भई बिन्दु A भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् । जहाँ A(– 5, 6), B(– 1, – 4) र C(3, 2) छन् ।

Find the equation of a line parallel to BC and passing through the point A where; A(– 5, 6), B(– 1, – 4) and C(3, 2).

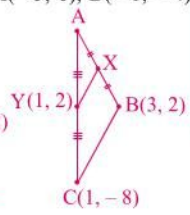
2B. चित्रमा P र Q क्रमशः AC र AB का मध्यबिन्दुहरू हुन् । यदि P(3, 2), Q(– 4, 8) र B(– 5, 4) भए BC को समीकरण पत्ता लगाउनुहोस् ।

In the figure, P and Q are the mid–points of AC and AB respectively. If P(3, 2), Q(– 4, 8) and B(– 5, 4), find the equation of BC.



2C. सँगैको चित्रबाट रेखा XY को समीकरण पत्ता लगाउनुहोस् ।

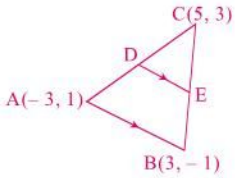
From the adjoining figure, find the equation of line XY.



2B. **2C.**

2D. सँगैको चित्रमा AC को मध्यबिन्दु D हो र DE // AB छ । DE को समीकारण निकाल्नुहोस् र E का निर्देशाङ्कहरू निकाल्नुहोस् ।

In the adjoining diagram, D is the midpoint of AC and DE // AB. Find the equation of DE and then find also the coordinates of E.



3A. यदि सरल रेखा $\frac{x}{a}+\frac{y}{b}=1$ रेखाहरू $x + y = 5$ र $2x - 3y = 0$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $2x + 3y = 5$ सँग समानान्तर हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।

If the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the point of intersection of the lines $x + y = 5$ and $2x - 3y = 0$ and is parallel to the line $2x + 3y = 5$, then find the values of a and b.

3B. यदि सरल रेखा $\frac{x}{a}+\frac{y}{b}=1$ रेखाहरू $x + 2y = 7$ र $2x + 3y = 12$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $2x + 3y = 8$ सँग समानान्तर हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।

If the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the point of intersection of the lines $x + 2y = 7$ and $2x + 3y = 12$ and is parallel to the line $2x + 3y = 8$, then find the values of a and b.

3C. यदि रेखा $bx + ay = ab$ रेखाहरू $x + y = 3$ र $2x - 3y = 1$ को प्रतिच्छेदन बिन्दु भएर गई रेखा $y = x - 6$ सँग समानान्तर हुन्छ भने a र b का मानहरू पत्ता लगाउनुहोस् ।

If the line $bx + ay = ab$ passes through the point of intersection of the lines $x + y = 3$ and $2x - 3y = 1$ and is parallel to the line $y = x - 6$, then find the values of a and b.

4A. यदि सरल रेखा $y = mx + c$ रेखाहरू $x - 2y = - 1$ र $x + 3y = 9$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $y = 4x + 8$ सँग समानान्तर हुन्छ भने m र c का मानहरू पत्ता लगाउनुहोस् ।

If the line $y = mx + c$ passes through the point of intersection of the lines $x - 2y = - 1$ and $x + 3y = 9$ and is parallel to the line $y = 4x + 8$, then find the values of m and c.

4B. यदि सरल रेखा $y = mx + c$ रेखाहरू $x + 2y = 7$ र $4x - 3y = 6$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $y = 2x + 3$ सँग समानान्तर हुन्छ भने m र c का मानहरू पत्ता लगाउनुहोस् ।

If the line $y = mx + c$ passes through the point of intersection of the lines $x + 2y = 7$ and $4x - 3y = 6$ and is parallel to the line $y = 2x + 3$, then find the values of m and c.

www.drmathnp.com

puspas.com.np

4C. यदि रेखा $my + x = cm$ रेखाहरू $x - 23 = -4y$ र $7x = 3y + 6$ का प्रतिच्छेदन बिन्दु भएर जान्छ र $5x - 4y = 6$ सँग समानान्तर हुन्छ भने m र c का मानहरू पत्ता लगाउनुहोस् ।
If the line $my + x = cm$ passes through the point of intersection of the lines $x - 23 = -4y$ and $7x = 3y + 6$ and parallel to the line $5x - 4y = 6$, then find the values of m and c.

5A. यदि सरल रेखा $Ax + By = C$ रेखाहरू $x + y = 5$ र $x - y = -1$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $4x + 5y = 20$ सँग समानान्तर हुन्छ भने A = 4 हुँदा B र C का मानहरू पत्ता लगाउनुहोस् ।
If the line $Ax + By = C$ passes through the point of intersection of the lines $x + y = 5$ and $x - y = -1$ and is parallel to the line $4x + 5y = 20$, find the values of B and C when A = 4.

5B. यदि सरल रेखा $x \cos\alpha + y \sin\alpha = p$ रेखाहरू $x = 2$ र $y = 3$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $\sqrt{3}x + y = 2$ सँग समानान्तर हुन्छ भने न्यूनकोण α र p का मानहरू पत्ता लगाउनुहोस् ।
If the line $x \cos\alpha + y \sin\alpha = p$ passes through the point of intersection of the lines $x = 2$ and $y = 3$ and is parallel to the line $\sqrt{3}x + y = 2$, find the values of acute angle α and p.

5C. यदि सरल रेखा $x \cos\alpha + y \sin\alpha = p$ रेखाहरू $x = \sqrt{3}$ र $y = 2$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $\sqrt{3}x + y = 2$ सँग समानान्तर हुन्छ भने न्यूनकोण α र p का मानहरू पत्ता लगाउनुहोस् ।
If the line $x \cos\alpha + y \sin\alpha = p$ passes through the point of intersection of the lines $x = \sqrt{3}$ and $y = 2$ and is parallel to the line $\sqrt{3}x + y = 2$, find the values of acute angle α and p.

5D. यदि रेखा $x \cos\alpha + y \sin\alpha = 3$ बिन्दु (2, 3) भएर जान्छ र रेखा $\frac{x}{2} - \frac{y}{6} = 1$ सँग समानान्तर हुन्छ भने $\sin\alpha$ र $\cos\alpha$ का मानहरू पत्ता लगाउनुहोस् ।
If the line $x \cos\alpha + y \sin\alpha = 3$ passes through the point (2, 3) and it is parallel to the line $\frac{x}{2} - \frac{y}{6} = 1$, find the values of $\sin\alpha$ and $\cos\alpha$.

Model 2 *Problems based on equation of perpendicular line to the given line.*

6A. $2x + 3y + 7 = 0$ सँग लम्ब भई X-खण्ड 5 बनाउने सीधारेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight line orthogonal to $2x + 3y + 7 = 0$ and making X-intercept 5.

6B. $7x + 5y + 12 = 0$ सँग लम्ब भई Y-खण्ड 7 बनाउने सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line perpendicular to $7x + 5y + 12 = 0$ and making Y- intercept 7.

6C. बिन्दुहरू (3, 4) र (– 1, – 6) जोड्ने रेखाको मध्यबिन्दुबाट जाने र रेखा $3x - 8y + 7 = 0$ सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Determine the equation of the line passing through the mid-points of the line joining the points (3, 4) and (– 1, – 6) and perpendicular to the line $3x - 8y + 7 = 0$.

6D. बिन्दुहरू (2, 4) र (2, – 4) जोड्ने रेखाको मध्यबिन्दु भएर जाने र रेखा $3x - 2y = 5$ सँग लम्ब हुने सरल रेखाको समीकरण निकाल्नुहोस् ।
Find the equation of straight line which passes through the mid-point of the line segment joining (2, 4) and (2, – 4) and perpendicular to the line $3x - 2y = 5$.

6E. शीर्षबिन्दुहरू (2, 5), (1, 0) र (– 3, 4) भएको त्रिभुजको भार केन्द्र भई जाने र सरल रेखा $3x - 5y = 10$ सँग लम्ब हुने सीधा रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a straight line passing through the centroid of the triangle whose vertices are (2, 5), (1, 0) and (– 3, 4) and perpendicular to the line $3x - 5y = 10$.

7A. $3x - 4y + 5 = 0$ सँग लम्ब भई दुई सरल रेखाहरू $2x + y = 5$ र $x - y = 1$ को प्रतिच्छेदन बिन्दु भएर जाने सरल रेखाको समीकरण निकाल्नुहोस् ।
Find the equation of straight line which is perpendicular to the line $3x - 4y + 5 = 0$ and passes through the point of intersection of the lines $2x + y = 5$ and $x - y = 1$.

7B. यदि सरल रेखा $y = mx + c$ रेखाहरू $x - 2y = -1$ र $y = 2$ को प्रतिच्छेदन बिन्दु भएर जान्छ र रेखा $y = 4x + 8$ सँग लम्ब हुन्छ भने m र c का मानहरू पत्ता लगाउनुहोस् ।
If the line $y = mx + c$ passes through the point of intersection of the lines $x - 2y = -1$ and $y = 2$ and is perpendicular to the line $y = 4x + 8$, then find the values of m and c.

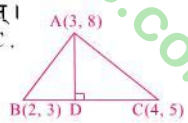
www.drmathnp.com

puspas.com.np

Model 3 Problems based on finding equation of height or median of a triangle.

8A. यदि AP ⊥ BC र A(– 2, 1), B(2, 2), C(6, – 2) भए BC को झुकाव र AP को समीकरण पत्ता लगाउनुहोस् ।
A (– 2, 1) B(2, 2) and C(6, – 2). Find the gradient of BC and equation of AP if AP ⊥ BC .

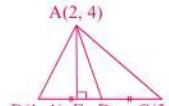
8B. दिइएको चित्रबाट AD को समीकरण पत्ता लगाउनुहोस् ।
From the given figure, find the equation of AD.



8C. A(2, 3), B(– 4, 1) र C(2, 0) शीर्षबिन्दुहरू भएको ΔABC मा शीर्षबिन्दु A बाट खिचिएको उचाइको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the altitude from the vertex A of ΔABC, where A(2, 3), B(– 4, 1) & C(2, 0) are the vertices of ΔABC.

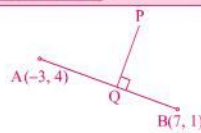
9A. A(0, 6) र B(10, 0) दुई बिन्दुहरू र O उद्गमबिन्दु छ भने ΔAOB को मध्यिका OM र उचाइ OD को समीकरण पत्ता लगाउनुहोस् ।
A(0, 6) and B(10, 0) are the two points and O is the origin. Find the equation of median OM and altitude OD of triangle AOB.

9B. ΔABC का शीर्षबिन्दुहरू A(2, 4), B(1, 1) र C(5, 1) छन् । ΔABC को मध्यिका AD र उचाइ AE का समीकरणहरू पत्ता लगाउनुहोस् ।
The veritices of ΔABC are A(2, 4), B(1, 1) and C(5, 1). Find the equations of median AD and height AE.



Model 4 On finding equation of perpendicular or parallel line which divide the given line in m : n.

10A. चित्रमा बिन्दु Q ले रेखाखण्ड AB लाई 3 : 2 को अनुपातमा विभाजन गर्दछ र PQ ⊥ AB भए PQ को समीकरण पत्ता लगाउनुहोस् ।
In the figure, point Q divides the line segment AB in the ratio of 3 : 2 and PQ ⊥ AB. Find the equation of PQ.



10B. बिन्दुहरू A र B का निर्देशाङ्कहरू क्रमशः (5, 4) र (– 3, 2) छन् । AB लाई 2 : 1 को अनुपातमा विभाजन गर्ने बिन्दु C भई जाने र AB सँग लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
The points A and B have co-ordinates (5, 4) and (– 3, 2) respectively. Find the equation of the line passing through C which divides AB in the ratio of 2 : 1 and perpendicular to AB.

10C. बिन्दुहरू A(3, 6) र B(– 6, 9) जोड्ने रेखासँग लम्ब हुने र AB रेखालाई 2 : 1 को अनुपातमा विभाजन गर्ने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line perpendicular to the line joining the points A(3, 6) and B(– 6, 9) which divides the line AB in the ratio of 2 : 1.

Model 5 Problems based on finding angle between two lines.

11A. यदि P(1, 1), Q(4, 1) र R(4, – 2) हरू ΔPQR का शीर्षहरू भए ∠QPR को नाप पत्ता लगाउनुहोस् ।
If P(1, 1), Q(4, 1) and R(4, – 2) are the vertices of ΔPQR, find the measure of ∠QPR.

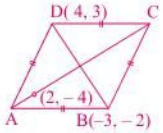
11B. यदि P(1, 3), Q(– 1, 1) र R(3, 1) हरू ΔPQR का शीर्षहरू भए ∠PQR को नाप पत्ता लगाउनुहोस् ।
If P(1, 3), Q(– 1, 1) and R(3, 1) are the vertices of ΔPQR, find the measure of ∠PQR.

Model 6 Problems based on finding equation of perpendicular bisector.

12A. बिन्दुहरू (3, 2) र (5, – 6) जोड्ने रेखाखण्डको लम्वार्धकको समीकरण निकाल्नुहोस् ।
Find the equation of the perpendicular bisector of the line segment joining (3, 2) and (5, – 6).

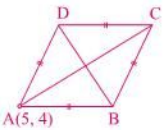
12B. यदि समबाहु चतुर्भुजको एउटा विकर्णको अन्तिम बिन्दुहरू (3, 7) र (– 2, – 5) भए अर्को विकर्णको समीकरण पत्ता लगाउनुहोस् ।
If (3, 7) and (– 2, – 5) are the end points of a diagonal of a rhombus, find the equation of its other diagonal.

12C. दिइएको चित्रमा ABCD एउटा समबाहु चतुर्भुज हो । बिन्दु (2, – 4) भएर जाने विकर्ण AC को समीकरण पत्ता लगाउनुहोस् ।
In the given figure, ABCD is a rhombus. Find the equation of diagonal AC passing through the point (2, – 4).



12D. सँगैको चित्रमा BD को समीकरण $4x - 3y = 10$ भए AC को समीकरण पत्ता लगाउनुहोस् ।
In the figure, equation of BD is $4x - 3y = 10$. Find the equation of AC.

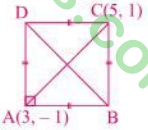
12E. A(– 3, 2) र C(1, 5) वर्ग ABCD का दुई विपरीत शीर्षबिन्दुहरू हुन् भने विकर्ण BD को समीकरण पत्ता लगाउनुहोस् ।
A(– 3, 2) and C(1, 5) are the opposite vertices of a square ABCD. Find the equation of the diagonal BD.



www.drmathnp.com

12F. एउटा वर्गको विकर्णका अन्तिम बिन्दुहरू (3, 4) र (1, –1) हुन् भने अर्को विकर्णको समीकरण पत्ता लगाउनुहोस् ।
If (3, 4) and (1, –1) are the end points of one diagonal of a square, find the equation of the other diagonal.

12G. वर्ग ABCD को विकर्ण BD को समीकरण पत्ता लगाउनुहोस् जहाँ A र C का निर्देशाङ्कहरू क्रमशः A(3, – 1) र C(5, 1) छन् ।
Find the equation of the diagonal BD of square ABCD where the co-ordinates of A and C are A(3, – 1) and C(5, 1).



12H. A(1, 5), B(3, – 7) र C(9, – 4) भएको एउटा समानान्तर चतुर्भुज छ । CD को समीकरण पत्ता लगाउनुहोस् ।
ABCD is a parallelogram with A(1, 5), B(3, – 7) and C(9, – 4). Find the equation of CD.

Model 7 ***On finding equation of the straight lines, passing through (x, y) & making θ° with given line.***

13A. रेखा $2x - y + 7 = 0$ सँग 45° को कोण बनाउने र बिन्दु (4, 5) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (4, 5) & making an angle of 45° with the line $2x - y + 7 = 0$.

13B. रेखा $3x + y - 5 = 0$ सँग 45° को कोण बनाउने र बिन्दु (2, 3) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (2, 3) & making an angle of 45° with the line $3x + y - 5 = 0$.

13C. सीधारेखा $2x + 3y + 7 = 0$ सँग 45° को कोण बनाई बिन्दु (1, – 4) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through (1, – 4) that make an angle of 45° with the straight line $2x + 3y + 7 = 0$.

13D. सीधारेखा $2x + 3y + 5 = 0$ सँग 45° को कोण बनाई बिन्दु (3, 2) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (3, 2) & make an angle of 45° with the line $2x + 3y + 5 = 0$.

13E. रेखा $x - y + 1 = 0$ सँग 45° को कोण बनाउने र बिन्दु (– 1, 2) भएर जाने सिधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (– 1, 2) and making an angle of 45° with the line $x - y + 1 = 0$.

13F. रेखा $3x + 4y - 5 = 0$ सँग 45° को कोण बनाउने र बिन्दु (2, 3) भएर जाने सीधारेखाहरूको समीकरण पत्ता लगाउनुहोस ।
Find the equation of straight lines passing through the point (2, 3) and making an angle of 45^0 with the line $3x + 4y - 5 = 0$.

13G. समीकरण $2x + y - 7 = 0$ भएको रेखासँग 45° को कोण बनाउने र बिन्दु (7, 7) बाट जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of straight lines passing through (7, 7) and making an angle of 45° with $2x + y - 7 = 0$.

13H. सीधा रेखा $x + 3y - 2 = 0$ सँग 45° को कोण बनाइ बिन्दु (2, – 1) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the lines which pass through the point (2, – 1) and are inclined at 45° to the straight line $x + 3y - 2 = 0$.

13I. सीधा रेखा $3x + y - 8 = 0$ सँग 45° को कोण बनाई बिन्दु (2, 3) भएर जाने सीधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (2, 3) and making an angle of 45° with the line $3x + y - 8 = 0$.

14A. रेखा $\sqrt{3}x + y = 1$ सँग 60° को कोण बनाउने र बिन्दु (3, – 2) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (3, – 2) & making an angle of 60° with the line $\sqrt{3}x + y = 1$.

14B. बिन्दु (4, – 5) भएर जाने र रेखा $\sqrt{3}x + y = 5$ सँग 60° को कोण बनाउने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (4, – 5) & inclined at an angle of 60° to the line $\sqrt{3}x + y = 5$.

14C. बिन्दु $\left(\frac{16}{\sqrt{3}}, -3\right)$ बाट जाने र रेखा $\sqrt{3}x - y + 6 = 0$ सँग 60° को कोण बनाउने सरल रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight line passing through the point $\left(\frac{16}{\sqrt{3}}, -3\right)$ and making an angle of 60° with the line $\sqrt{3}x - y + 6 = 0$.

14D. बिन्दु (3, – 2) बाट जाने $\sqrt{3}x + y = 1$ ले प्रतिनिधित्व गर्ने रेखासँग 60° को कोण बनाउने सीधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equations of straight lines which pass through (3, – 2) and making an angle of 60° with the line $\sqrt{3}x + y = 1$.

COORDINATES

www.drmathnp.com

puspas.com.np

15A. बिन्दु (2, –1) बाट खिचिएका दुईओटा सिधा रेखाहरू प्रत्येकले सीधा रेखा $6x + 5y - 2 = 0$ सँग 135° को कोण बनाउँदछन् भने ती दुई सीधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Two straight lines are drawn through (2, – 1) each inclined at 135° to the straight line $6x + 5y - 2 = 0$. Find the equations of the two straight lines.

15B. एउटा वर्ग PQRS को एउटा शीर्षबिन्दु P(2, 3) र विकर्ण PR को समीकरण $x - 3y = 2$ छ । शीर्षबिन्दु P बाट जाने दुई भुजाहरूको समीकरण पत्ता लगाउनुहोस् ।
A square PQRS has a vertex P(2, 3) and the equation of the diagonal PR is $x - 3y = 2$. Find the equation of two sides of the square through P.

Model 8 *Problems based on right angled triangle.*

16. दिइएका बिन्दुहरू समकोण त्रिभुजका शीर्षबिन्दुहरू भए a को मान पत्ता लगाउनुहोस् ।
The given points are the vertices of a right angled triangle. Find the value of a.

A. A(8, – 10), B(7, – 3), C(0, a) and $\angle B = 90°$
B. P(5, 0), Q(a, 5), R(0, 5) and $\angle Q = 90°$.
C. X(a, 1), Y(9, 4), Z(6, 4) and $\angle Y = 90°$
D. X(a, 5), Y(2, 1), Z(6, 1) and $\angle Y = 90°$.

ANSWERS

A. SHORT QUESTIONS

1A. 60° **1B.** $\frac{3}{11}$ **1C.** 4.4° **1D.** 45° **1E.** 30° **2A.** 37. 9° **2B.** 45° **3A.** 150° **3B.** 135°
3C. 120° **3D.** 150° **4A.** $\frac{1}{5}$ **4B.** $\tan\alpha$ **4C.** $\frac{1}{\sqrt{3}}$ **5A.** $0, \sqrt{3}$ **5B.** $-\frac{1}{\sqrt{3}}$ **6A.** $2, -\frac{1}{2}$ **6B.** $11, -\frac{1}{11}$
6C. $\frac{\sqrt{3}-1}{1+2\sqrt{3}}$ **7A.** 1 **7B.** – 1 **9A.** 4 **9B.** 6 **9C.** 8 **10A.** –1 or –2 **10B.** $\frac{1}{7}$ **10C.** – 2
10D. – 10 **11A.** $3a + 4b = 0$ **11C.** 2 **11D.** $-\frac{4}{3}$ **13A.** 2 **13B.** – 3 **13C.** – 3 **14A.** 1
14B. 2 **14D.** $k = \pm 6$ **15A.** 3 or – 12 **15B.** $k = 4$ **16A.** $5x - y = 15$ **16B.** $6x - y = 9$
16C. $3y = 2x + 7$ **16D.** $2x + 3y = 23$ **17A.** $5x - y = 11$ **17B.** $4x - y = 13$ **17C.** $3x - 4y = -15$

B. LONG QUESTIONS

1A. $x - 3y = 5$ **1B.** $x - y + 5 = 0$ **2A.** $3x - 2y + 27 = 0$ **2B.** $6x + 7y + 2 = 0$ **2C.** $5x - y = 3$
2D. $x + 3y = 7$, (4, 1) **3A.** 6, 4 **3B.** 6, 4 **3C.** 1, – 1 **4A.** 4, –10
4B. 2, – 4 **4C.** $\frac{4}{5}$, – 1 **5A.** 5, 23 **5B.** 30°, $\frac{5}{2}$ **5C.** $\frac{3+2\sqrt{3}}{2}$
5D. 3 and – 1 **6A.** $3x - 2y = 15$ **6B.** $5x - 7y + 49 = 0$ **6C.** $8x + 3y = 5$ **6D.** $2x + 3y = 4$
6E. $5x + 3y - 9 = 0$ **7A.** $4x + 3y = 11$ **7B.** 4, – 11 **8A.** $m = -1, x - y + 3 = 0$ **8B.** $x + y = 11$
8C. $6x - y = 9$ **9A.** $3x = 5y, 5x = 3y$ **9B.** $3x + y = 10, x = 2$ **10A.** $50x - 15y = 117$
10B. $12x + 3y = 4$ **10C.** $y = 3x + 17$ **11A.** 45° or 135° **11B.** 45° or 135° **12A.** $x - 4y = 12$
12B. $10x + 24y = 29$ **12C.** $7x + 5y + 6 = 0$ **12D.** $3x + 4y = 31$ **12E.** $8x + 6y = 13$ **12F.** $4x + 10y = 23$
12G. $x + y = 4$ **12H.** $6x + y = 50$ **13A.** $x - 3y + 11 = 0, 3x + y = 17$
13B. $x + 2y = 8, y = 2x - 1$ **13C.** $5x + y = 1, x - 5y = 21$ **13D.** $5x + y = 17, x - 5y + 7 = 0$ **13E.** $y = 2$ and $x = -1$
13F. $7x - y - 11 = 0, 5x - 7y + 11 = 0$ **13G.** $x + 3y - 28 = 0, 3x - y = 14$ **13H.** $x - 2y - 4 = 0, 2x + y - 3 = 0$
13I. $x + 2y - 8 = 0, 2x - y - 1 = 0$ **14A.** $y + 2 = 0, \sqrt{3}x - y = 3\sqrt{3} + 2$ **14B.** $y + 5 = 0, \sqrt{3}x - y = 5 + 4\sqrt{3}$
14C. $\sqrt{3}x + y = 13, y + 3 = 0$ **14D.** $y = (2 + \sqrt{3})x - 3\sqrt{3} - 8$ and $y = (\sqrt{3} - 2)x - 3\sqrt{3} + 4$
15A. $x + 11y + 9 = 0, 11x - y - 23 = 0$ **15B.** $2x - y = 1, x + 2y = 8$ **16A.** – 4 **16B.** 5 **16C.** 9 **16D.** 2

CHALLENGING EXERCISE 1

1. बिन्दु (4, 4) बाट रेखा $x + y = 4$ मा खिचेको लम्बले काट्ने बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the foot of the perpendicular from the point (4, 4) on the line $x + y = 4$.

2. बिन्दु (3, 4) बाट रेखा $x + 2y = 6$ मा खिचेको लम्बले काट्ने बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the coordinates of the foot of the perpendicular from the point (3, 4) on the line $x + 2y = 6$.

3. बिन्दुहरू (2, – 1) र (6, 1) जोड्ने रेखाखण्डलाई बिन्दु (4, 1) बाट खिचिएको लम्बले कुन अनुपातमा विभाजन गर्दछ, पत्ता लगाउनुहोस् ।
Find the ratio in which the perpendicular from (4, 1) to the line segment joining (2, – 1) and (6, 1) divides the segment.

4. बिन्दु P $(2, \sqrt{3})$ भएर जाने रेखाले X-अक्षसँग 60° को कोण बनाउँदछ र रेखा $x + \sqrt{3}y = 12$ लाई बिन्दु Q मा काट्छ भने PQ को लम्बाइ पत्ता लगाउनुहोस् ।
A straight line passes through the point P$(2, \sqrt{3})$ and makes an angle of 60° with X-axis. If this line intersects another line having equation $x + \sqrt{3}y = 12$ at Q, find the length of PQ.

5. समीकरण $2x + 3y = 11$ भएको रेखासँग समानान्तर हुने र अक्षहरूमा बनाएको खण्डहरूको योगफल 15 हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight line which is parallel to $2x + 3y = 11$ and such that the sum of the intercepts on axes is 15.

www.drmathnp.com

puspas.com.np

6. समीकरण $2x + 3y = 11$ भएको रेखासँग लम्ब हुने र अक्षहरूमा बनाएको खण्डहरूको योगफल 15 हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight line which is perpendicular to $2x + 3y = 11$ and such that the sum of the intercepts on axes is 15.

7. बिन्दु (3, 2) बाट रेखा $2x - 3y = 1$ मा खिचेको लम्बको प्रतिच्छेदन बिन्दु र (4, 1) बाट जाने सीधा रेखाको समीकरण निकाल्नुहोस् ।
Find the equation of the straight line joining the point (4, 1) to the foot of the perpendicular drawn from the point (3, 2) on the line $2x - 3y = 1$.

8. रेखा $x \sec\theta + y \operatorname{cosec}\theta = a$ सँग लम्ब भई बिन्दु $(a\cos^3\theta, a\sin^3\theta)$ भएर जाने रेखाको समीकरण $x\cos\theta - y\sin\theta = a\cos 2\theta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the equation of the line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$ is $x\cos\theta - y\sin\theta = a\cos 2\theta$.

9. रेखा $x \operatorname{cosec}\theta - y \sec\theta = a$ सँग समानान्तर भइ बिन्दु $(a\cos^3\theta, a\sin^3\theta)$ भएर जाने रेखाको समिकरण $x\cos\theta - y\sin\theta = a\cos 2\theta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the equation of straight line which passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and is parallel to the straight line $x \operatorname{cosec}\theta - y \sec\theta = a$ is $x\cos\theta - y\sin\theta = a\cos 2\theta$.

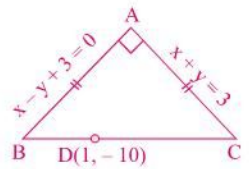
10. बिन्दु (a, b) भएर जाने र रेखा $yb = 2p(x + a)$ सँग लम्ब हुने रेखाको समीकरण $2p(y - b) + b(x - a) = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Show that the equation of the straight line which passes through the point (a, b) and is perpendicular to the straight line $yb = 2p(x + a)$ is $2p(y - b) + b(x - a) = 0$.

11. एउटा सीधा रेखा $2x - y = 5$ सोही रेखामा पर्ने x-सदस्य र y-सदस्य बराबर भएको एउटा बिन्दुको वरिपरि 45° घडीको विपरीत दिशामा घुम्छ भने सो रेखाको नयाँ अवस्थाको समीकरण पत्ता लगाउनुहोस् ।
The line $2x - y = 5$ turns about the point on it, whose ordinate and abscissae are equal, through an angle of 45°, in anticlockwise direction. Find the equation of the line in new position.

12. रेखा $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ को निम्नलिखित अवस्थामा k को मान पत्ता लगानुहोस् ।
Find the value of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is
(i) x-अक्षसँग समानान्तर हुँदा (Parallel to x-axis.) (ii) y-अक्षसँग समानान्तर हुँदा (Parallel to y-axis.)
(iii) उदगम् बिन्दु भएर जाँदा (Passing through the origin.)

13. एउटा समकोण समद्विबाहु त्रिभुजको कर्णका अन्तिम छेउका बिन्दुहरू (1, 3) र (– 4, 1) भए सो त्रिभुजको समकोण बनाउने भुजाहरूको समीकरणहरू पत्ता लगाउनुहोस् ।
The hypotenuse of an isosceles right angled triangle has its ends at the points (1, 3) and (– 4, 1). Find the equation of the legs (perpendicular sides) of the triangle.

14. एउटा समद्विबाहु ΔABC(AB = AC) का भुजाहरू AB र AC का समीकरणहरू क्रमशः $x - y + 3 = 0$ र $x + y = 3$ छन् । बाँकी भुजा BC को समीकरण पत्ता लगाउनुहोस् । जुनचाहिँ बिन्दु P(1, – 10) भएर जान्छ ।
The equation of AB and AC of an isosceles ΔABC (AB = AC) are $x - y + 3 = 0$ and $x + y = 3$ respectively. Find the equation of the remaining side BC, which passes through the point P(1, – 10).



15. $3x - 4y + 18 = 0$ सँग एउटा रेखा 'ℓ' लम्ब छ र रेखा 'ℓ' ले निर्देशाङ्क अक्षहरूसँग घेरिएको एउटा त्रिभुजको क्षेत्रफल 6 वर्ग एकाइ भए रेखा 'ℓ' को समीकरण पत्ता लगाउनुहोस् ।
A line 'ℓ' is perpendicular to $3x - 4y + 18 = 0$ and area of a triangle bounded by the line 'ℓ' with coordinate axes is 6 sq. units, find equation of the line 'ℓ'.

16. $3x - 2y + 4 = 0$ सँग एउटा रेखा 'ℓ' लम्ब छ र रेखा 'ℓ' ले निर्देशाङ्क अक्षहरूसँग घेरिएको एउटा त्रिभुजको क्षेत्रफल 27 वर्ग एकाइ भए रेखा 'ℓ' को समीकरण पत्ता लगाउनुहोस् ।
A line 'ℓ'. is perpendicular to $3x - 2y + 4 = 0$ and area of a triangle bounded by the line 'ℓ' with coordinate axes is 27 sq. units, find equation of the line 'ℓ'.

ANSWERS

1. (2, 2) **2.** (2, 2) **3.** 3:2 **4.** $\frac{7}{2}$ untis **5.** $2x + 3y = 18$ **6.** $2y = 3x + 90$
7. $10x + 11y = 51$ **11.** $3x + y = 20$ **12.** (i) 3 (ii) ± 2 (iii) 1, 6 **13.** $7x - 3y + 31 = 0, 3x + 7y = 24$
14. $y + 10 = 0$ **15.** $4x + 3y = \pm 12$ **16.** $2x + 3y = \pm 18$

www.drmathnp.com

2. जोडा सीधारेखाहरू PAIR OF STRAIGHT LINES

puspas.com.np

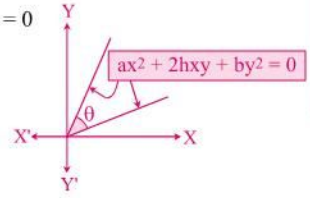
FORMULAE AND KEY POINTS

	रेखाहरूको समीकरण Equation of Lines	रेखाहरू बिचको कोण Angle between Lines	रेखाहरूको अवस्था (Condition of Lines)	
			सम्पाती हुने being coincident	लम्ब हुने being perpendicular
1.	$ax^2 + 2hxy + by^2 = 0$	$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$	$h^2 = ab$	$a + b = 0$
2.	$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$	$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$	$h^2 = ab$	$a + b = 0$

3. $ax^2 + 2hxy + by^2 = 0$ ले जनाउने सीधारेखाहरू (The straight lines represented by $ax^2 + 2hxy + by^2 = 0$ are):

(a) $ax + hy + y\sqrt{h^2 - ab} = 0$ (b) $ax + hy - y\sqrt{h^2 - ab} = 0$

4. $ax^2 + 2hxy + by^2 = 0$ ले जनाउने रेखाहरूसँग लम्ब भई उद्गम बिन्दु भएर जाने रेखाहरूको एकल समीकरण $bx^2 - 2hxy + ay^2 = 0$ हुन्छ ।
The sinhgle equation of the lines through origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.



Proof of Important theorem

1. डिग्री 2 भएको जोडा रेखाको साधारण समीकरण पत्ता लगाउनुहोस् ।
Find the general equation of the second degree of pair of lines.

Solution :

Here, let two general equations of straight lines be $a_1x + b_1y + c_1 = 0$……(i) and $a_2x + b_2y + c_2 = 0$……(ii)
Combining equation (i) and (ii), $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$……(iii)
Simplifying the equation (iii), we get;

$$a_1a_2x^2 + (a_1b_2 + a_2b_1)xy + b_1b_2y^2 + (b_1c_2 + b_2c_1)y + (a_1c_2 + a_2c_1)x + c_1c_2 = 0$$

Substituting: $a_1a_2 = a$, $b_1b_2 = b$ and $c_1c_2 = c$, $a_1b_2 + a_2b_1 = 2h$, $a_1c_2 + a_2c_1 = 2g$ and $b_1c_2 + b_2c_1 = 2f$ then,
The equation becomes, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

2. 2 डिग्रीको वर्ग समघातीय समीकरण प्रतिपादन गर्नुहोस् । (Derive homogeneous equation of second degree.)

Solution :

Here, let two equations of straight lines passing through origin, $a_1x + b_1y = 0$…(i) and $a_2x + b_2y = 0$……..(ii)
Combining equations (i) and (ii), we get, $(a_1x + b_1y)(a_2x + b_2y) = 0$
From multiplication, $a_1a_2x^2 + a_1b_2xy + b_1a_2xy + b_1b_2y^2 = 0$
or, $a_1a_2x^2 + (a_1b_2 + a_2b_1)xy + b_1b_2y^2 = 0$ ……(iii)
Substituting $a_1a_2 = a$; $a_1b_2 + a_2b_1 = 2h$ and $b_1b_2 = b$. Then the equation (iii) becomes, $ax^2 + 2hxy + by^2 = 0$.
This is the homogenous equation of second degree.

3. सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ बिचको कोण पत्ता लगाउनुहोस् । साथै यी रेखाहरू लम्ब हुँदा र समानान्तर हुँदा हुने सम्बन्ध पनि पत्ता लगाउनुहोस् ।
Find the angle between the straight lines $ax^2 + 2hxy + by^2 = 0$. Also derive the condition of the lines being perpendicular and parallel. [2063 M, 2064 S, 2065 E, 2065 S]

Solution

Here, $ax^2 + 2hxy + by^2 = 0$ ..... (i) denotes two straight lines passing through origin.
Let the two lines are: $y = m_1x$ and $y = m_2x$ i.e., $y - m_1x = 0$ and $y - m_2x = 0$
Combining these two equations, $(y - m_1x)(y - m_2x) = 0$
or, $y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$ ..... (ii)

Comparing the coefficient of equations (i) and (ii) then, $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

If angle between the lines is θ then,

$$\tan\theta = \pm\frac{m_1 - m_1}{1 + m_1m_2} = \pm\frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2} = \pm\frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{1 + \frac{a}{b}} = \pm\frac{2\sqrt{(h^2 - ab)}}{a + b}$$

$$\therefore \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

www.drmathnp.com

Condition of Perpendicularity:

If the pairs of lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular to each other then, $\theta = 90°$ & $\cot\theta = 0$.

Now, $\frac{a+b}{2\sqrt{h^2-ab}} = \cot 90°$ or, $\frac{a+b}{2\sqrt{h^2-ab}} = 0$ $\therefore$ $a + b = 0$

Thus, the lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular to each other.

Condition of Coincident:

If the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ are coincident to each other then, $\theta = 0°$ and $\tan\theta = \tan 0° = 0$.

$\tan\theta = \left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$ or, $\tan 0° = \left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$

or, $0 = \left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$ or, $2\sqrt{h^2-ab} = 0$

or, $\sqrt{h^2-ab} = 0$ or, $h^2 - ab = 0$

$\therefore$ $h^2 = ab$ Thus, $h^2 = ab$ is the required result .

4. वर्ग समघातीय समीकरण $ax^2 + 2hxy + by^2 = 0$ ले सँधै उद्गमबिन्दु भएर जाने सरल रेखाको प्रतिनिधित्व गर्दछ भनी प्रमाणित गर्नुहोस् ।

Prove that the homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines through the origin. [2063 S, 2063 R']

Solution

The given equation is $ax^2 + 2hxy + by^2 = 0$ Multiplying it by a we get, $a^2x^2 + 2ahxy + aby^2 = 0$

or, $a^2x^2 + 2hxy + h^2y^2 - h^2y^2 + aby^2 = 0$ or, $(ax)^2 + 2.\,ax.\,hy + (hy)^2 - (h^2 - ab)y^2 = 0$

or, $(ax + hy)^2 - \left(\sqrt{h^2-ab}.y\right)^2 = 0$ or, $\left(ax + hy - \sqrt{h^2-ab}.y\right)\left(ax + hy + \sqrt{h^2-ab}.y\right) = 0$

Either, $ax + hy - \sqrt{h^2-ab}\,y = 0$ .....(i) or, $ax + hy + \sqrt{h^2-ab}\,y = 0$ .....(ii)

Equation from (i) and (ii) are lines through origin.

Thus, the homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines through the origin.

5. $ax^2 + 2hxy + by^2 = 0$ ले जनाउने रेखाहरूसँग लम्ब भई उद्गम बिन्दु भएर जाने रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।

Find the single equation of the lines through origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$

Solution

Here, let $y = m_1x$ ..... (i) and $y = m_2x$ .... (ii) be the separate equations of the given pair of lines.

Combining (i) and (ii) we get, $(m_1x - y)(m_2x - y) = 0$ or, $m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0$ ..... (iii)

Again, $ax^2 + 2hxy + by^2 = 0$ $\therefore$ $\frac{a}{b}x^2 + \frac{2hxy}{b} + y^2 = 0$ ..... (iv)

Since, (iii) and (iv) represent the same lines, $\therefore$ $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

Now, Equations of the lines through the origin and perpendicular to (i) and (ii) are;

$m_1y + x = 0$ ..... (v) and $m_2y + x = 0$ ..... (vi)

The combined equation of (v) and (vi) is;

$(m_1y + x)(m_2y + x) = 0$ or, $m_1m_2y^2 + (m_1 + m_2)xy + x^2 = 0$

Putting the values of $(m_1 + m_2)$ and m_1m_2;

$\frac{a}{b}y^2 - \frac{2h}{b}xy + x^2 = 0$ $\therefore$ $ay^2 - 2hxy + bx^2 = 0$

Thus, $bx^2 - 2hxy + ay^2 = 0$ is the single equation of pair of lines perpendicular to $ax^2 + 2hxy + by^2 = 0$ and passing through origin.

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2066 R', 2066 S, 2067 R', 2070 S, 2071 R', 2072 R', SEE 2074 R	6
Model 2	2070 S', 2072 R, SEE MODEL 2076	3
Model 3	2065 E, 2067 R, 2068 R, 2069 R, 2069 R', 2073 R', SEE 2075 R', SEE 2075 R'_2	8
Model 4	2065 R, R', S, 2066 R, R', 2067 S, 2068 R', S, S', 2069 S, 2070 R', 2071 R, 2071 S, 2072 S, 2073 R, R', S, S', SEE 2074 R', SEE 2074 S, S', SEE 2075 R, R_2, R'	22

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2066 S, 2067 S, 2068 R, 2068 R', 2068 S, R', 2071 R, 2071 R', 2072 R', 2072 S, 2074 S', 2075 R'_2	11
Model 2	2065 R', 2067 R', 2068 S', 2070 S', 2073 R, SEE 2073 S, SEE 2073 S', SEE 2074 R'	4
Model 3	2065 M, 2065 R, 2066 R, 2067 R, 2069 R', 2069 S, 2070 R, 2070 S, SEE MODEL 2076	9
Model 4	2071 S, 2072 R, SEE 2075 R, R_2	3
Model 5	2068 S', SEE 2074 R, SEE 2074 S	3

puspas.com.np

www.drmathnp.com

puspas.com.np

SEE EXERCISE 2

A. VERY SHORT QUESTIONS

Model 1 *Definition and formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
- **A.** समघातीय वर्ग समीकरण (Homogeneous equation)
- **B.** दुई डिग्रीको साधारण समीकरण (General equation of second degree)

2A. कस्तो अवस्थामा जोडा सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ लम्ब हुन्छन् ?
Under what condition, the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will be perpendicular?

2B. जोडा सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ समपाती हुने अवस्था उल्लेख गर्नुहोस् ।
State the condition under which the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will be coincident.

2C. $ax^2 + 2hxy + by^2 = 0$ ले के लाई जनाउँदछ ? (What does $ax^2 + 2hxy + by^2 = 0$ represent?)

3A. x र y मा रहेको उद्गम बिन्दु भएर जाने समघातीय वर्ग समीकरण के हुन्छ ?
What is the homogeneous equation of second degree in x and y representing a pair of straight lines through origin?

3B. x र y मा रहेको दुई डिग्रीको साधारण समीकरण लेख्नुहोस् ।
Write the general equation of second degree in x and y representing a pair of straight lines.

4A. $ax^2 + 2hxy + by^2 = 0$ सँग लम्ब भई उद्गम बिन्दु भएर जाने जोडा रेखाहरूको एकल समीकरण के हुन्छ ?
What is the single equation of pair of lines perpendicular to $ax^2 + 2hxy + by^2 = 0$ and passing through origin?

4B. जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ बीचको कोण पत्ता लगाउने सूत्र लेख्नुहोस् ।
Write the formula to find angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$.

5A. यदि जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ को ढुकावहरू m_1 र m_2 भए $(m_1 + m_2)$ को मान लेख्नुहोस् ।
If m_1 and m_2 are the slopes of two lines $ax^2 + 2hxy + by^2 = 0$ then write the value of $(m_1 + m_2)$

5B. यदि जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ को ढुकावहरू m_1 र m_2 भए $(m_1 \times m_2)$ को मान लेख्नुहोस् ।
If m_1 and m_2 are the slopes of two lines $ax^2 + 2hxy + by^2 = 0$ then write the value of $(m_1 \times m_2)$

Model 2 *Concept Recall*

6A. जोडा रेखाहरू $x - 2 = 0$ र $x + 2 = 0$ ले जनाउने एकल समीकरण के हुन्छ ?
What is the single equation to represent the pair of lines $x - 2 = 0$ and $x + 2 = 0$?

6B. जोडा रेखाहरू $x + a = 0$ र $x - a = 0$ ले जनाउने एकल समीकरण के हुन्छ ?
What is the single equation to represent the pair of lines $x + a = 0$ and $x - a = 0$?

6C. रेखाहरू $x = 0$ र $y = 0$ ले जनाउने एकल समीकरण के हुन्छ ?
What is the single equation to represent the lines $x = 0$ and $y = 0$?

7A. जोडा रेखाको एकल समीकरण $(x - y)(x + y) = 0$ भए बेग्ला बेग्लै समीकरण पत्ता लगाउनुहोस् ।
The single equation of pair of lines is $(x - y)(x + y) = 0$. Find the separate equations of the lines.

7B. जोडा रेखाको एकल समीकरण $(x - 2y)(x + 2y) = 0$ भए बेग्ला बेग्लै समीकरण पत्ता लगाउनुहोस् ।
The single equation of pair of lines is $(x - 2y)(x + 2y) = 0$. Find the separate equations of the lines.

B. SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका समीकरणहरूले प्रतिनिधित्व गर्ने जोडा रेखाबाट बेग्लाबेग्लै रेखाको समीकरण पत्ता लगाउनुहोस्:
Find the separate equation of the lines represented by the given equations:

A.	$2x^2 + 7xy + 3y^2 = 0$	[2057 R]	**B.**	$6x^2 + 5xy - 6y^2 = 0$	[2060 S, SEE 2074 R]
C.	$3x^2 - 8xy - 3y^2 = 0$	[2062 R]	**D.**	$2x^2 - 5xy + 3y^2 = 0$	[2064 R']
E.	$2x^2 + xy - 6y^2 = 0$	[2063 S]	**F.**	$x^2 - 3xy + 2y^2 = 0$	[2065 M]
G.	$3x^2 - 4xy + y^2 = 0$	[2067 R']			

2. तलका समीकरणहरूले प्रतिनिधित्व गर्ने जोडा रेखाबाट बेग्लाबेग्लै रेखाको समीकरण पत्ता लगाउनुहोस्:
Find the separate equation of the lines represented by the given equations:

A.	$x^2 + x - y - y^2 = 0$	[2066 S]	**B.**	$x^2 - y^2 - 2x + 2y = 0$	[2061 R, 2071 R', 2070S, 2072 R']
C.	$x^2 - y^2 + 3x - 3y = 0$	[2061 S]	**D.**	$3x^2 - 5xy - 2y^2 - x + 2y = 0$	[2066 R']

www.drmathnp.com

puspas.com.np

Model 2 Problems based on finding single equation of lines.

3A. जोडा रेखाहरू $3x + 2y = 0$ र $2x - 3y = 0$ लाई जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation for the pair of lines represented by $3x + 2y = 0$ and $2x - 3y = 0$. [SEE MODEL 2076]

3B. जोडा समीकरणहरू $ax = by$ र $bx + ay = 0$ लाई प्रतिनिधित्व गर्ने एउटै समीकरण लेख्नुहोस् ।
Write the single equation representing the pair of equations $ax = by$ and $bx + ay = 0$.

3C. एक जोडा सरल रेखाहरू $x = 3y$ र $3x = y$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation which represents a pair of straight lines $x = 3y$ and $3x = y$. [2072 R]

3D. दुई जोडा रेखाहरू $x = 2$ र $y = 3$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation which represents the pair of straight lines $x = 2$ and $y = 3$. [2070 S']

3E. कुनै दुई जोडा रेखाहरू $x = 2y$ र $2x = y$ द्वारा बन्ने वर्ग समघातीय समीकरण पत्ता लगाउनुहोस् ।
Find the homogeneous equation of 2nd degree from the pair of lines $x = 2y$ & $2x = y$. [2063 R]

Model 3 Problems based on finding angle between lines.

4. जोडा रेखाको बिचको कोणहरू पत्ता लगाउनुहोस् (Find the angles between the pair of lines):

A. $6x^2 - xy - y^2 = 0$ [2058 S] **B.** $3x^2 + 2y^2 - 5xy = 0$ [2060 S']

C. $3x^2 + 7xy + 2y^2 = 0$ [2069 R] **D.** $x^2 - 3xy - 4y^2 = 0$ [2069 R']

E. $x^2 - 2xy \operatorname{cosec} \alpha + y^2 = 0$ [2067 R] **F.** $x^2 + 4xy + y^2 = 0$ [SEE 2075 R', SEE 2073 R']

5. जोडा रेखाबिचको अधिककोण पत्ता लगाउनुहोस् (Find the obtuse angle between the pair of lines):

A. $x^2 - 4xy + y^2 = 0$ [2059 S] **B.** $12x^2 - 23xy + 5y^2 = 0$ [2060 R]

C. $3x^2 - 7xy + 2y^2 = 0$ [2063 R']

6. जोडा रेखाबिचको न्यूनकोण पत्ता लगाउनुहोस् (Find the acute angle between the pair of lines):

A. $2x^2 - 7xy + 3y^2 = 0$ [SEE 2075 R'$_2$] **B.** $x^2 - 4xy + y^2 = 0$ [2062 K, 2068 R]

C. $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2 = 0$ [2062 CP, 2065 E] **D.** $6x^2 + xy - y^2 = 0$ [2062 S]

Model 4 Problems on condition of coincident and perpendicular lines.

7. दिइएका समीकरणले प्रतिनिधित्व गर्ने रेखाहरू बिचको कोण एक समकोण हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the angle between the lines represented by given equation is a right angle.

A. $5x^2 - 6xy - 5y^2 = 0$ [2071 S] **B.** $6x^2 - 5xy - 6y^2 = 0$ [2065 R']

C. $6x^2 + 5xy - 6y^2 = 0$ [2067 S] **D.** $2x^2 - 3xy - 2y^2 = 0$ [2068 S']

8A. m को मान कति भए $3x^2 + 8xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने दुई रेखाहरू आपसमा लम्ब हुन्छन् ? पत्ता लगाउनुहोस् ।
For what value of m, the two lines represented by $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other? Find it. [SEE 2075 R']

8B. यदि $3x^2 - 5xy + py^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबीचको कोण एक समकोण भए p को मान निकाल्नुहोस् ।
If the angle between the lines represented by $3x^2 - 5xy + py^2 = 0$ is a right angle, find the value of p. [SEE 2074 S]

8C. यदि समीकरण $mx^2 - 5xy - 6y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।
If the pair of lines represented by an equation $mx^2 - 5xy - 6y^2 = 0$ are perpendicular to each other, find the value of m. [2069 S]

8D. समीकरण $5x^2 + 24xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा रेखाहरू आपसमा लम्ब छन् भने m को मान पत्ता लगाउनुहोस् ।
An equation $5x^2 + 24xy + my^2 = 0$ represents a pair of straight lines which are perpendicular to each other. Find the value of m. [SEE 2074 S']

8E. यदि समीकरण $7x^2 - 8xy - 5y^2 + mx^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।
If the pair of lines represented by an equation $7x^2 - 8xy - 5y^2 + mx^2 = 0$ are perpendicular to each other, find the value of m.

www.drmathnp.com

puspas.com.np

9. दिइएका समीकरणले प्रतिनिधित्व गर्ने रेखाहरू सम्पाती हुन्छन भनी प्रमाणित गर्नुहोस् ।
Prove that the lines represented by given equation are coincident.

A. $16x^2 - 24xy + 9y^2 = 0$ [2071 R] **B.** $9x^2 - 24xy + 16y^2 = 0$ [2065 R, SEE 2074 R']

C. $16x^2 + 24xy + 9y^2 = 0$ [2068 S]

10. तलका समीकरणहरूले जनाउँने रेखाहरू सम्पाती हुन्छन् भने m वा k को मान पत्ता लगाउनुहोस्:
Find the value of m or k if the lines represented by the following equations are coincide:

A. $16x^2 - kxy + 9y^2 = 0$ [SEE 2075 R] **B.** $kx^2 - 24xy + 16y^2 = 0$ [SEE 2075 R_2]

C. $x^2 + 2kxy + 4y^2 = 0$ [SEE 2073 R'] **D.** $2x^2 - 3xy + my^2 = 0$ [2068 R']

E. $5x^2 - 8xy + my^2 = 0$ [2065 S, 2066 R'] **F.** $3x^2 - 12xy + my^2 = 0$ [2066 R]

G. $kx^2 - 12xy + 3y^2 = 0$ [2070 R'] **H.** $(m-6)x^2 - 8xy + my^2 = 0$ [2070 R]

I. $3x^2 - 6xy + (k+4)y^2 = 0$ [2072 S] **J.** $2x^2 + 8xy - (m+1)y^2 = 0$ [SEE 2073 R]

K. $9x^2 + 12xy + (k+1)y^2 = 0$ [SEE 2073 S] **L.** $9x^2 + (k+1)xy + 4y^2 = 0$ [SEE 2073 S']

C. LONG QUESTIONS

Model 1 *Problems based on finding seprate equation and angle between them (algebraic).*

1. दिइएका समीकरणले प्रतिनिधित्व गर्ने दुई रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरूबिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the equations of two lines represented by the given equation. Also, find the angle between them.

A. $3x^2 - 8xy - 3y^2 = 0$ [SEE 2075R'$_2$] **B.** $6x^2 + xy - y^2 = 0$ [SEE 2074 S']

C. $2x^2 + 3xy + y^2 = 0$ [2072 R'] **D.** $2x^2 + 7xy + 3y^2 = 0$ [2058 R, 2068 R']

E. $2x^2 - 7xy + 3y^2 = 0$ [2068 S] **F.** $2x^2 + 5xy + 3y^2 = 0$ [2064 R]

G. $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$ [2067 S] **H.** $6x^2 - xy - y^2 = 0$ [2072 S]

2A. $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू पत्ता लगाउनुहोस् ।
Determine the lines represented by the equation $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$. [2068 R]

2B. समीकरण $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$ ले जनाउने एक जोडी सरल रेखाहरूको समीकरण निकाल्नुहोस् । [2066 S]
Find the equation of a pair of straight lines represented by the given equation $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$.

3A. $5x^2 - 8xy + py^2 = 0$ ले दुईओटा परस्पर लम्ब रेखाहरूलाई जनाउँछ भने p को मान पत्ता लगाउनुहोस् । साथै $5x^2 - 8xy + (p+8)y^2 = 0$ ले जनाउँने सिधा रेखाहरूको समीकरण पनि पत्ता लगाउनुहोस् ।
If the straight lines represented by $5x^2 - 8xy + py^2 = 0$ are perpendicular to each other, then find the value of p. Also, find the equations of straight lines represented by $5x^2 - 8xy + (p+8)y^2 = 0$. [2071 R]

3B. यदि समीकरण $3x^2 + 8xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने दुई सरल रेखाहरू एक आपसमा लम्ब छन् भने ती दुई रेखाहरूको बेग्लाबेग्लै समीकरण पत्ता लगाउनुहोस् ।
If two straight lines represented by an equation $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other, find the separate equation of two lines. [2070 R']

Model 2 *Problems based on separate equations and angle between the lines (trigonometric).*

4. दिइएका समीकरणले प्रतिनिधित्व गर्ने दुई रेखाहरूको समीकरण पत्ता लगाउनुहोस् र ती रेखाहरूबिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the equations of two lines represented by the given equation. Also, find the angle between them.

A. $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ [2059 R] **B.** $x^2 - 2xy \cot 2\alpha - y^2 = 0$ [2065 R', 2068 S']

5A. समीकरण $x^2 + 2xy \sec\theta + y^2 = 0$ ले जनाउने रेखाहरू बिचको न्यूनकोण α भए $\alpha = \theta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If α be the acute angle made by the straight lines represented by the equation $x^2 + 2xy \sec\theta + y^2 = 0$, prove that $\alpha = \theta$. [2067 R', 2070 S']

5B. यदि $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा सरल रेखाहरूबिचको न्यूनकोण α भए प्रमाणित गर्नुहोस्: $\alpha = 90° - \theta$
If the acute angle between the pair of straight lines represented by $x^2 - 2xy \operatorname{cosec}\theta + y^2 = 0$ is α then prove that: $\alpha = 90° - \theta$. [SEE 2073 R]

5C. यदि $x^2 + 2xy \operatorname{cosec}\theta + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको न्यूनकोण α भए, प्रमाणित गर्नुहोस् : $\alpha = 90° - \theta$
If the acute angle between the pair of lines represented by $x^2 + 2xy \operatorname{cosec}\theta + y^2 = 0$ is α then prove that: $\alpha = 90° - \theta$. [SEE 2073 S, SEE 2073 S', SEE 2074 R']

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on finding equations of perpendicular lines to given lines.*

6A. उद्गम बिन्दुबाट जाने र $2x^2 - 5xy + 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$. **[SEE MODEL 2076]**

6B. समीकरण $2x^2 - 3xy - 5y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दुबाट जाने एक जोडा रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the origin and perpendicular to the lines represented by the equation $2x^2 - 3xy - 5y^2 = 0$. **[2070 R]**

6C. समीकरण $2x^2 - 3xy + y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दु भएर जाने एक जोडा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of the pair of straight lines passing through the origin and perpendicular to the pair of lines represented by an equation $2x^2 - 3xy + y^2 = 0$. **[2069 S]**

6D. रेखाहरू $x^2 + 5xy + 6y^2 = 0$ सँग लम्ब हुने र उद्गम बिन्दुबाट जाने रेखाको समीकरण निकाल्नुहोस् ।
Find the equation of the pair of straight lines which pass through the origin and perpendicular to the lines represented by $x^2 + 5xy + 6y^2 = 0$. **[2069 R']**

6E. $x^2 - 5xy + 4y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भई उद्गम बिन्दुबाट जाने रेखाहरूको एउटै समीकरण के हुन्छ ?
What is the single equation of straight lines through the origin and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$? **[2064 R', 2065 R]**

6F. समीकरण $x^2 - xy - 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दु भएर जाने एक जोडा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines through the origin and perpendicular to the lines pairs represented by $x^2 - xy - 2y^2 = 0$. **[2065 M]**

6G. समीकरण $2x^2 - xy - 3y^2 = 0$ ले जनाउने एक जोडी रेखासँग लम्ब हुने र उद्गम बिन्दुबाट जाने एक जोडी रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a pair of straight lines passing through the origin and perpendicular to the pair of lines represented by $2x^2 - xy - 3y^2 = 0$. **[2066 R]**

6H. समीकरण $2x^2 - 7xy + 5y^2 = 0$ ले जनाउने रेखाहरूसँग लम्ब भई उद्गम बिन्दु भएर जाने जोडा रेखाको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the origin and perpendicular to the lines represented by the equation $2x^2 - 7xy + 5y^2 = 0$. **[2067 R, 2070 S]**

6I. समीकरण $ax^2 + 2hxy + by^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने र उद्गम बिन्दु भएर जाने एक जोडा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the origin and perpendicular to the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$.

7A. $x^2 - 5xy + 4y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भइ बिन्दु $(1, 2)$ बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of lines which pass through the point $(1, 2)$ and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$. **[2071 R']**

7B. $x^2 - xy - 2y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भइ बिन्दु $(3, -1)$ बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of lines which pass through the point $(3, -1)$ and perpendicular to the lines represented by $x^2 - xy - 2y^2 = 0$.

7C. $x^2 - xy - 2y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग लम्ब भइ बिन्दु $(1, 0)$ बाट जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of lines which pass through the point $(1, 0)$ and perpendicular to the lines represented by $x^2 - xy - 2y^2 = 0$.

7A. $(x + y - 3)(4x + y - 6) = 0$ **7B.** $(2x + y - 5)(x - y - 4) = 0$
7C. $(2x + y - 2)(x - y - 1) = 0$

4. समीकरण $x^2 - xy - 2y^2 = 0$ ले जनाउने सीधा रेखाहरूसँग लम्ब भई बिन्दु $(3, -1)$ भएर जाने सीधा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of the pair of the lines passing through point $(3, -1)$ and perpendicular to the lines represented by the equation $x^2 - xy - 2y^2 = 0$.

4. $2x^2 - xy - y^2 - 13x + y + 20 = 0$

3. समीकरण $ax^2 + 2hxy + by^2 = 0$ ले जनाउने सीधा रेखाहरूसँग लम्ब भई उद्गमबिन्दु भएर जाने सीधा रेखाहरूको एकल समीकरण $bx^2 - 2hxy + ay^2 = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the single equation of two lines which pass through the origin and perpendicular to the pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.

www.drmathnp.com

puspas.com.np

Model 4 *Problems based on finding value of k or p when angle between lines is given.*

8A. यदि समीकरण $2x^2 + kxy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको कोण 45° भए k को धनात्मक मान पत्ता लगाई रेखाहरूका बेग्लाबेग्लै समीकरणहरू पनि पत्ता लगाउनुहोस् ।
If an angle between the pair of lines represented by the equation $2x^2 + kxy + 3y^2 = 0$ is 45°, then find the positive value of k and also find the separate equations of the lines. **[SEE 2075 R, 2075 R$_2$]**

8B. यदि समीकरण $2x^2 + kxy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबिचको कोण 45° भए k को मान पत्ता लगाउनुहोस् ।
If the angle between the lines represented by $2x^2 + kxy + 3y^2 = 0$ is 45°, find the value of k. **[2071 S]**

8C. समीकरण $x^2 + xy - Ky^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा सरल रेखाहरूबिचको कोण 45° छ भने K को मान पत्ता लगाउनुहोस् ।
The angle between a pair of straight lines represented by the equation $x^2 + xy - Ky^2 = 0$ is 45°. Find the value of K. **[2072 R]**

8D. समीकरण $x^2 + 2hxy + 12y^2 = 0$ ले जनाउने रेखाहरूबिचको न्यून कोण $\tan^{-1}\left(\frac{1}{13}\right)$ भए ती रेखाहरूको छुट्टाछुट्टै समीकरणहरू पत्ता लगाउनुहोस् ।
If an acute angle between the lines represented by the equation $x^2 + 2hxy + 12y^2 = 0$ is $\tan^{-1}\left(\frac{1}{13}\right)$, find the separate equations of the lines.

Model 6 *Problems based on finding equation of parallel lines to given lines.*

9A. समीकरण $3x^2 + xy - 10y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग समानान्तर हुने र बिन्दु (1, 0) भएर जाने एक जोडी रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a pair of lines passing through the point (1, 0) and parallel to the lines represented by the equation $3x^2 + xy - 10y^2 = 0$. **[2074 R]**

9B. समीकरण $x^2 + 3xy - 4y^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा सरल रेखाहरूसँग समानान्तर हुने र बिन्दु (0, 1) भएर जाने एकजोडा सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the point (0, 1) and parallel to the pair of straight lines represented by the equation $x^2 + 3xy - 4y^2 = 0$. **[SEE 2074 S]**

9C. समीकरण $2x^2 - 7xy - 9y^2 = 0$ ले जनाउने सीधा रेखाहरूसँग समानान्तर भई बिन्दु (2, – 1) भएर जाने सीधा रेखाहरूको एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of two straight lines which pass through the point (2, – 1) and are parallel to the pair of lines represented by the equation $2x^2 - 7xy - 9y^2 = 0$.

9D. बिन्दु (1, 4) भएर जाने जोडा रेखाहरूको समीकरण निकाल्नुहोस्, जुन रेखाहरू समीकरण $3x^2 + 8xy + 4y^2 = 0$ भएका रेखाहरूसँग समानान्तर हुन्छन् ।
Find the equation of pair of straight lines that pass through the point (1, 4) and parallel to the lines represented by $3x^2 + 8xy + 4y^2 = 0$.

9E. सीधा रेखाहरू $a_1x + b_1y + c_1 = 0$ र $a_2x + b_2y + c_2 = 0$ सँग समानान्तर भई उद्गम बिन्दु भएर जाने जोडा सीधा रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of pair of straight lines passing through origin and parallel to the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. **[2068 S']**

ANSWERS

A. VERY SHORT QUESTIONS

(2A - 4B); Show to your teacher. **5A.** $-\frac{2h}{b}$ **5B.** $\frac{a}{b}$ **6A.** $x^2 - 4 = 0$
6B. $x^2 - a^2 = 0$ **6C.** $xy = 0$ **7A.** $x - y = 0$ and $x + y = 0$ **7B.** $x - 2y = 0$ and $x + 2y = 0$

B. SHORT QUESTIONS

1A. $2x + y = 0, x + 3y = 0$ **1B.** $2x + 3y = 0, 3x - 2y = 0$ **1C.** $x - 3y = 0, 3x + y = 0$ **1D.** $x - y = 0, 2x - 3y = 0$
1E. $2x - 3y = 0, x + 2y = 0$ **1F.** $x = y, x = 2y$ **1G.** $3x - y = 0, x - y = 0$ **2A.** $x - y = 0, x + y + 1 = 0$
2B. $x - y = 0, x + y - 2 = 0$ **2C.** $x - y = 0, x + y + 3 = 0$ **2D.** $x = 2y, 3x + y = 1$ **3A.** $6x^2 - 5xy - 6y^2 = 0$
3B. $abx^2 + (a^2 - b^2)xy - aby^2 = 0$ **3C.** $3x^2 - 10xy + 3y^2 = 0$ **3D.** $xy - 3x - 2y + 6 = 0$ **3E.** $2x^2 - 5xy + 2y^2 = 0$

4A. 45°, 135° **4B.** $\tan^{-1}\left(\pm\frac{1}{5}\right)$ **4C.** 45°, 135° **4D.** $\theta = \tan^{-1}\left(\pm\frac{5}{3}\right)$ or 59.04°, 120.96°
4E. $90° - \alpha$ **4F.** 60°, 120° **5A.** 120° **5B.** 135° **5C.** 135° **6A.** 45°
6B. 60° **6C.** 60° **6D.** 45° **8A.** $m = -3$ **8B.** $p = -3$ **8C.** 6
8D. – 5 **8E.** – 2 **10A.** $k = \pm 24$ **10B.** 9 **10C.** ± 2 **10D.** $\frac{9}{8}$
10E. $\frac{16}{5}$ **10F.** 12 **10G.** 12 **10H.** – 2 or 8 **10I.** – 1 **10J.** – 9
10K. 3 **10L.** 11 or – 13

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS

1A. $(x-3y)=0;\ (3x+y)=0;\ 90°$ **1B.** $(3x-y)(2x+y)=0$; 45° or 135°
1C. $x+y=0$ and $2x+y=0$; 18.43° or 161.57° **1D.** $2x+y=0$ and $x+3y=0$; 45° or 135°
1E. $2x-y=0, x-3y=0$, 45° or 135° **1F.** $x+y=0, 2x+3y=0$; 11.31°, 168.69°
1G. $ax-by=0, bx+ay=0$, 90° **1H.** $2x-y=0$ and $3x+y=0, \theta = 45°$ or 135°
2A. $x+y+3=0, x+y-5=0$ **2B.** $x-3y+4=0, 2x+y=5$ **3A.** $p=-5, x-y=0, 5x-3y=0$
3B. $x+3y=0, 3x-y=0$ **4A.** $x=(\text{cosec}\,\theta \pm \cot\theta)\,y;\ \tan^{-1}(\pm\cot\theta)$
4B. $x-y(\cot 2\alpha + \text{cosec}\,2\alpha)=0,\ x-y(\cot 2\alpha - \text{cosec}\,2\alpha)=0$, 90° **6A.** $2x^2+5xy+2y^2=0$ **6B.** $5x^2-3xy-2y^2=0$
6C. $x^2+3xy+2y^2=0$ **6D.** $6x^2-5xy+y^2=0$ **6E.** $4x^2+5xy+y^2=0$ **6F.** $2x^2-xy-y^2=0$
6G. $3x^2-xy-2y^2=0$ **6H.** $5x^2+7xy+2y^2=0$ **6I.** $bx^2-2hxy+ay^2=0$
7A. $(x+y-3)(4x+y-6)=0$ **7B.** $(2x+y-5)(x-y-4)=0$
7C. $(2x+y-2)(x-y-1)=0$ **8A.** $k=7, 2x+y=0$ and $x+3y=0$ **8B.** ± 7
8C. 0, 6 **8D.** $x+3y=0, x+4y=0$ or, $x-3y=0$ and $x-4y=0$ **9A.** $3x^2+xy-10y^2-6x-y+3=0$
9B. $x^2+3xy-4y^2-3x+8y-4=0$ **9C.** $2x^2-7xy-9y^2-15x-4y+13=0$ **9D.** $3x^2+8xy+4y^2-38x-40y+99=0$
9E. $ax^2+2hxy+by^2=0$

GENERAL EXERCISE 2

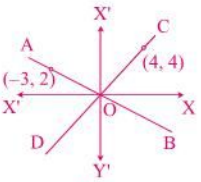
A. SHORT QUESTIONS

Model 1 *Problems based on finding single equation.*

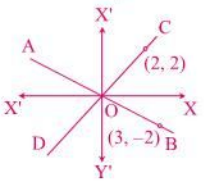
1. तलका जोडा रेखाहरूलाई प्रतिनिधित्व गर्ने एउटै समीकरण पत्ता लगाउनुहोस्:
Find the single equation represented by the following pair of lines:

A. $x+3y=0$ and $2x+y=0$ **B.** $2x+3y=0$ and $3x=2y$
C. $2x+3y-1=0$ and $x-2y=0$ **D.** $5x=2y$ and $2x-3y=7$
E. $ax+hy+y\sqrt{h^2-ab}=0$ and $ax+hy-y\sqrt{h^2-ab}=0$

2A. चित्रमा AB र CD उदगम्बिन्दु भएर गएका छन् । यदि बिन्दुहरू (– 3, 2) र (4, 4) क्रमशः रेखाहरू AB र CD मा पर्दछन् भने AB र CD दुवै रेखाको संयुक्त समीकरण पत्ता लगाउनुहोस् ।
In the figure, AB and CD are passing through origin. If the points (– 3, 2) and (4, 4) are in the lines AB and CD respectively then find the joint equation of AB and CD.



2B. चित्रमा दिइएको जानकारीबाट AB र CD को संयुक्त समीकरण पत्ता लगाउनुहोस् ।
From the given figure, find the joint equation of AB and CD.



2C. रेखाहरू $x=-2$ र $y=x$ लाई जनाउने एक समीकरण निकाल्नुहोस् ।
Find the single equation of the lines $x=-2$ and $y=x$.

2D. $y=x\sin\theta$ र $y+x\sin\theta=0$ लाई जनाउने एउटै समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of the lines representing $y=x\sin\theta$ and $y+x\sin\theta=0$.

2E. y-अक्ष र $x+2y=5$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation to represent $x+2y=5$ and y-axis.

2F. x-अक्ष र $x+2y=5$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation to represent $x+2y=5$ and x-axis.

Model 2 *Problems based on finding separate equation.*

3. तलका समीकरणहरूले प्रतिनिधित्व गरेको बेग्ला बेग्लै समीकरणहरू पत्ता लगाउनुहोस् :
Find the separate equations of the line represented by the following equations:

A. $x^2-y^2-3x+3y=0$ **B.** $x^2-4y^2-2x+4y=0$ **C.** $x(x-5y)+4y^2=0$
D. $x(x+y-c)-yc=0$ **E.** $x(x-1)-y(y-1)=0$ **F.** $ab(x^2+y^2)-xy(a^2+b^2)=0$
G. $ab(x^2-y^2)+(a^2-b^2)xy=0$ **H.** $2a(x^2-y^2)+(a^2-4)xy=0$ **I.** $9b(x^2-y^2)+(81-b^2)xy=0$
J. $ab\,x^2+a^2x+b^2x+ab=0$ **K.** $(ax+by)^2+(bx-ay)^2$ **L.** $x(x-y)+2(x-1)-2y+2=0$

Model 3 *Problems based on angle between lines.*

4. तलका समीकरणहरूले प्रतिनिधित्व गरेका सरल रेखाहरू बिचको कोण पत्ता लगाउनुहोस्:
Find the angle between the lines, represented by the following equations:

A. $x^2-3xy+2y^2=0$ **B.** $33x^2-44xy+11y^2=0$ **C.** $x^2-3xy-4y^2=0$

www.drmathnp.com

puspas.com.np

D. $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ **E.** $x^2 - 2pxy + y^2 = 0$ **F.** $x^2 - 2\sqrt{10}\,xy + y^2 = 0$

G. $(\cos^2 A - \sin^2 B)\,x^2 - 2xy \sin A \cos B + (\sin^2 A - \cos^2 B)\,y^2 = 0$ **H.** $x^2 - 2\sqrt{17}\,xy + y^2 = 0$

5. तलका समीकरणहरूले प्रतिनिधित्व गरेका सरल रेखाहरू बिचको न्यूनकोण कोण पत्ता लगाउनुहोस्:
Find an acute angle between the lines represented by the following equations:

A. $2x^2 - 3xy + y^2 = 0$ **B.** $2x^2 - 7xy + 3y^2 = 0$ **C.** $x^2 + 3xy + 2y^2 = 0$

D. $12x^2 - 23xy + 5y^2 = 0$ **E.** $x^2 - 2xy \sec A + y^2 = 0$ **F.** $x^2 - 2xy \sec 10° + y^2 = 0$

6. तलका समीकरणहरूले प्रतिनिधित्व गरेका सरल रेखाहरू बिचको अधिक कोण पत्ता लगाउनुहोस्:
Find the obtuse angle between the lines represented by the following equations:

A. $12x^2 - 23xy + 5y^2 = 0$ **B.** $2x^2 - 7xy + 3y^2 = 0$ **C.** $x^2 + 3xy + 2y^2 = 0$

D. $x^2 - xy = 0$ **E.** $x^2 - 2xy \sec 30° + y^2 = 0$ **F.** $x^2 - 2xy \sec 60° + y^2 = 0$

7A. समीकरण $2x^2 + 3xy - 2y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडी रेखाहरूको समीकरण पत्ता लगाउनुहोस् । ती दुई रेखाहरूबिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the pair of lines represented by the equation $2x^2 + 3xy - 2y^2 = 0$. Also, find the angle between them.

7B. समीकरण $3x^2 + 8xy - 3y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडी रेखाहरूको समीकरण पत्ता लगाउनुहोस् । ती दुई रेखाहरूको बिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the pair of lines represented by the equation $3x^2 + 8xy - 3y^2 = 0$. Also, find the angle between them.

8A. समीकरण $(p^2 - 3q^2)\,x^2 + 8pq\,xy + (q^2 - 3p^2)\,y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूको बिचको न्यूनकोण $60°$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that an acute angle between the straight lines represented by the equation $(p^2 - 3q^2)\,x^2 + 8pq\,xy + (q^2 - 3p^2)\,y^2 = 0$ is $60°$.

8B. यदि समीकरण $x^2 + 2mxy + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडी रेखाहरूबिचको कोण $60°$ भए m को मान निकाल्नुहोस् ।
If the angle between the lines represented by the equation $x^2 + 2mxy + y^2 = 0$ is $60°$, find the value of m.

Model 4 Problems based on conditions of perpendicularity.

9. तलका समीकरणहरूले जनाउने रेखाहरू आपसमा लम्ब भए p को मान निकाल्नुहोस् :
Find the value of p if the lines represented by the following equations are perpendicular:

A. $\frac{5}{6}x^2 + \frac{18}{13}xy + py^2 = 0$ **B.** $(p+5)\,x^2 - 5xy - (3p-1)\,y^2 = 0$

C. $(p^2+7)\,x^2 - xy - (3p+5)y^2 = 0$ **D.** $\frac{5}{6}x^2 + \frac{18}{13}xy + \left(p + \frac{25}{6}\right)y^2 = 0$

E. $(p+2)x^2 = 4xy - (2p+5)y^2$ **F.** $(p+2)x^2 = (p+222)xy + (3p-6)y^2$

10A. यदि $(a+b)x^2 + 5xy - (2a-b)y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू परस्पर लम्ब छन् भने a र b को सम्बन्ध लेख्नुहोस् ।
If $(a+b)x^2 + 5xy - (2a-b)y^2 = 0$ represents two straight lines perpendicular to each other, write the relationship between a and b.

10B. यदि $(m+n)x^2 + 7xy - 2(m-n)y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू परस्पर लम्ब छन् भने m र n को सम्बन्ध लेख्नुहोस् ।
If $(m+n)x^2 + 7xy - 2(m-n)y^2 = 0$ represents two straight lines perpendicular to each other, write the relationship between m and n.

Model 5 Problems based on condition of coincident.

11. तलका समीकरणहरूले प्रतिनिधित्व गरेका सरल रेखाहरू सम्पाती हुन्छन् भनी देखाउनुहोस्:
Show that the straight lines represented by the following equations are coincident:

A. $2x^2 - 8xy + 8y^2 = 0$ **B.** $\frac{27}{2}x^2 - 24xy + \frac{32}{3}y^2 = 0$

C. $9x^2 + 12xy + 4y^2 = 0$ **D.** $x^2 - y^2 - 2xy \tan\alpha + y^2 \sec^2\alpha = 0$

12. तलका समीकरणहरूले जनाउँने रेखाहरू सम्पाती हुन्छन् भने m वा k को मान पत्ता लगाउनुहोस्:
Find the values of m or k if the lines represented by the following equations are coincident:

A. $mx^2 - 8xy + 8y^2 = 0$ **B.** $5x^2 + 6xy - my^2 = 0$

www.drmathnp.com

puspas.com.np

C. $kx^2 - y^2 - 2xy \tan\theta + y^2\sec^2\theta = 0$ **D.** $4x^2 + 5xy + (m+2)y^2 = 0$

E. $3x^2 - 6xy + (m+4)y^2 = 0$ **F.** $(m-9)x^2 = (2m-10)xy - my^2$

G. $(10m-1)x^2 + (5m+3)xy + (m-1)y^2 = 0$ **H.** $(m+3)^2 + 2(m-3)xy + (m-3)y^2 = 0$

I. $(1-k)x^2 + (5k+3)xy - (k-1)y^2 = 0$

13. यदि $(a+1)x^2 + 4bxy + 5y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू परस्पर सम्पाती छन् भने a र b बिचको सम्बन्ध लेख्नुहोस् ।
If $(a+1)x^2 + 4bxy + 5y^2 = 0$ represents two parallel straight lines and they are coincident, write the relationship between a and b.

14. यदि $(a+b)x^2 + 5xy - (2a-b)y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू परस्पर लम्ब छन् भने a र b को सम्बन्ध लेख्नुहोस् ।
If $(a+b)x^2 + 5xy - (2a-b)y^2 = 0$ represents two straight lines perpendicular to each other, write the relationship between a and b.

B. LONG QUESTIONS

Model 1 *Problems based on finding separate equations and angle between the lines (algebraic).*

1. तलका समीकरणहरूले प्रतिनिधित्व गरेको बेग्ला बेग्लै समीकरणहरू पत्ता लगाउनुहोस् । साथै सो रेखाहरू बिचको कोण पत्ता लगाउनुहोस्:
Find the separate equations of the line represented by the following equations. Also, find the angle between the lines:

A. $x^2 - y^2 + x - y = 0$ **B.** $x^2 + 4xy + y^2 = 0$ **C.** $y^2 - xy - 6x^2 = 0$

D. $3x^2 - 8xy - 3y^2 = 0$ **E.** $10x^2 - 11xy - 6y^2 = 0$ **F.** $3x^2 + 5xy - 2y^2 = 0$

2. तलका समीकरणहरूले प्रतिनिधित्व गरेको बेग्लाबेग्लै समीकरणहरू पत्ता लगाउनुहोस् । साथै सो रेखाहरू बिचको कोण पत्ता लगाउनुहोस्:
Find the separate equations of the line represented by the following equations. Also, find the angle between the lines:

A. $2x^2 - 6y^2 + xy - 16x + 24y = 0$ **B.** $3x^2 - 7xy + 2y^2 + 3x - y = 0$

C. $2x^2 - xy - 6y^2 + 3x + 22y - 20 = 0$ **D.** $6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0$

E. $3x^2 - 2xy - y^2 - 5x + y + 2 = 0$ **F.** $2x^2 - 3xy + y^2 + 2x - y = 0$

Model 2 *Problems based on finding separate equations and angle between the lines (trigonometric).*

3. तलका समीकरणहरूले प्रतिनिधित्व गरेको बेग्लाबेग्लै समीकरणहरू पत्ता लगाउनुहोस् । साथै सो रेखाहरू बिचको कोण पत्ता लगाउनुहोस्:
Find the separate equations of the line represented by the following equations. Also, find the angle between the lines:

A. $x^2 + 2xy\sec\theta + y^2 = 0$

Hint: Here, $x^2 + 2xy\sec\theta + y^2 = 0$ or, $x^2 + 2x(y\sec\theta) + (y\sec\theta)^2 - (y\sec\theta)^2 + y^2 = 0$

or, $(x + y\sec\theta)^2 - y^2(\sec^2\theta - 1) = 0$ or, $(x + y\sec\theta)^2 - y^2\tan^2\theta = 0$

or, $(x + y\sec\theta + y\tan\theta)(x + y\sec\theta - y\tan\theta) = 0$

$\therefore$ $x + y\sec\theta + y\tan\theta = 0$.....(i) and $x + y\sec\theta - y\tan\theta = 0$ .....(ii)

Equations (i) and (ii) are the required equations of the lines.

Again, $x^2 + 2xy\sec\theta + y^2 = 0$

Let α be the angle between them then; $\tan\alpha = \pm\dfrac{2\sqrt{h^2 - ab}}{a+b} = \pm\dfrac{2\sqrt{\sec^2\theta - 1 \times 1}}{1+1} = \pm\tan\theta$ $\therefore$ $\alpha = \pm\theta$

B. $x^2 - 2xy\sec A + y^2 = 0$ **C.** $x^2 - 2xy\cot A - y^2 = 0$

D. $x^2 + 2xy\cot 2\alpha - y^2 = 0$ **E.** $x^2 - y^2 - 2xy\tan\alpha + y^2\sec^2\alpha = 0$

F. $x^2 + 2xy\cot 40° - y^2 = 0$ **G.** $x^2 - 2xy\operatorname{cosec} 20° + y^2 = 0$

H. $x^2 - 2xy\cot 40° - y^2 = 0$

3I. $x^2 + 2xy\sec 15° + y^2 = 0$ ले जनाउने रेखाहरू बिचको न्यूनकोण α भए $\alpha = 15°$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If α be the acute angle made by the straight lines represented by the equation $x^2 + 2xy\sec 15° + y^2 = 0$, prove that $\alpha = 15°$.

www.drmathnp.com

puspas.com.np

4A. समीकरणहरू $(a^2 - 3b^2)\,x^2 + 8abxy + (b^2 - 3a^2)\,y^2 = 0$ ले जनाउने रेखाहरू बिचको अधिककोण 120° हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove the an obtuse angle between the straight lines represented by the equation $(a^2 - 3b^2)\,x^2 + 8abxy + (b^2 - 3a^2)\,y^2 = 0$ is 120°.

4B. समीकरणहरू $(3b^2 - a^2)x^2 - 8abxy + (3a^2 - b^2)\,y^2 = 0$ ले जनाउने रेखाहरू बिचको न्यूनकोण 60° हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove the an acute angle between the straight lines represented by the equation $(3b^2 - a^2)x^2 - 8abxy + (3a^2 - b^2)\,y^2 = 0$ is 60°.

4C. समीकरण $ab(x^2 - y^2) + (a^2 - b^2)\,xy = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूको समीकरण पत्ता लगाउनुहोस् । साथै ती रेखाहरू बिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the equation of a pair of lines represented by the equation $ab(x^2 - y^2) + (a^2 - b^2)xy = 0$. Also find the angle between them.

Model 3 Problems based on finding homogeneous equations and angle between the lines.

5. तलका जोडा रेखाहरूद्वारा बन्ने वर्ग समघातीय समीकरण पत्ता लगाउनुहोस् । साथै तिनीहरू बिचको कोण पनि पत्ता लगाउनुहोस् ।
Find the homogeneous equation of 2nd degree from the given pair of lines. Also find angle between them.

A. $x + y + 3 = 0,\ 2x + y = 1$ **B.** $x + y = 0,\ x - y + 1 = 0$

C. $x + y = 0,\ x - 2y = 1$ **D.** $x = (\text{cosec}\,\theta + \cot\theta)\,y,\ x = (\text{cosec}\,\theta - \cot\theta)\,y$

E. $x - y(\cot 2\alpha + \text{cosec}\,2\alpha) = 0$ and $x - y\,(\cot 2\alpha - \text{cosec}\,2\alpha) = 0$

Model 4 Problems based on finding separate equations and their point of intersection.

6A. समीकरण $6x^2 + xy - 25x - y^2 + 25 = 0$ ले जनाउने रेखाहरूको समीकरणहरू पत्ता लगाउनुहोस् र ती रेखाको प्रतिच्छेदन बिन्दु पत्ता लगाउनुहोस् ।
Find the separate equations of the lines represented by $6x^2 + xy - 25x - y^2 + 25 = 0$ and find point of intersections.

6B. समीकरण $x^2 + 3xy + 2y^2 + 5y + 4x + 3 = 0$ जनाउने रेखाहरूको समीकरणहरू पत्ता लगाउनुहोस् र ती रेखाको प्रतिच्छेदन बिन्दु पत्ता लगाउनुहोस् ।
Find the separate equations of the lines represented by the equation $x^2 + 3xy + 2y^2 + 5y + 4x + 3 = 0$ and find point of intersections.

6C. समीकरण $x^2 - y^2 - 2x + 6y - 8 = 0$ ले जनाउने सीधारेखाहरू र तिनीहरूको प्रतिच्छेदन बिन्दु पत्ता लगाउनुहोस् ।
Find the straight lines represented by the equation $x^2 - y^2 - 2x + 6y - 8 = 0$ and their point of intersection.

Model 5 On finding separate equations and proving angle between them is given angle.

7A. $2x^2 - xy - y^2 + 5x + y + 2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू $\tan^{-1}(3)$ को कोणमा प्रतिच्छेदन हुन्छन् भनी देखाउनुहोस् । साथै ती जोडा रेखाहरूको समीकरण पनि पत्ता लगाउनुहोस् ।
Show that $2x^2 - xy - y^2 + 5x + y + 2 = 0$ represents two straight lines intersecting at an angle of $\tan^{-1}(3)$. Also, find the separate equation of the lines.

7B. $x^2 - y^2 - x + 3y - 2 = 0$ ले दुई सीधारेखाहरूको प्रतिनिधित्व गर्दछ भनी देखाउनुहोस् । ती रेखाहरूको समीकरणहरू पत्ता लगाउनुहोस् र तिनीहरू समकोण हुने गरी प्रतिच्छेदन हुन्छन् भनी देखाउनुहोस् ।
Show that $x^2 - y^2 - x + 3y - 2 = 0$ represents a pair of straight lines. Find the equations of these lines and show that they are intersected at right angle.

7C. $x^2 - xy - 6y^2 - 3x + 14y - 4 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू 45° को कोणमा प्रतिच्छेदन हुन्छन् भनी देखाउनुहोस् । साथै ती जोडा रेखाहरूको समीकरण पनि पत्ता लगाउनुहोस् ।
Show that $x^2 - xy - 6y^2 - 3x + 14y - 4 = 0$ represents a pair of lines inclined at 45° to each other. Also, find the separate equation of the lines.

7D. समीकरण $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबिचको कोण $\tan^{-1}\left(\frac{2}{11}\right)$ र ती रेखाहरू बिन्दु $(1, -2)$ मा प्रतिच्छेदन हुन्छन् भनी प्रमाणित गर्नुहोस् ।
Prove that the angle between the lines represented by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ is $\tan^{-1}\left(\frac{2}{11}\right)$ and these lines intersect at the point of $(1, -2)$.

Model 6 On finding separate equations when the condition of perpendicular or coincident is given.

8A. $4x^2 - 2kxy + 9y^2 = 0$ ले जनाउने छुट्टाछुट्टै रेखाहरू सम्पाती भए ती रेखाका समीकरणहरू पत्ता लगाउनुहोस् ।
Find the separate equations of lines represented by $4x^2 - 2kxy + 9y^2 = 0$ if the lines are coincident.

www.drmathnp.com

puspas.com.np

8B. समीकरण $kx^2 + 3xy - 2(k-1)y^2 = 0$ ले जनाउने रेखाहरू आपसमा लम्ब भए ती रेखाका समीकरणहरू पत्ता लगाउनुहोस् ।
Find the separate equations of the lines represented by $kx^2 + 3xy - 2(k-1)y^2 = 0$ if the lines are perpendicular to each other.

8C. समीकरण $(a+2)x^2 + 8xy + (1-4a)y^2 = 0$ ले जनाउने रेखाहरू आपसमा लम्ब भए ती रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
If the lines represented by $(a+2)x^2 + 8xy + (1-4a)y^2 = 0$ are perpendicular to each other, find the equations of the lines.

8D. समीकरण $px^2 - 8xy - 3y^2 = 0$ ले जनाउने दुई रेखाहरू एक आपसमा लम्ब छन् । ती लम्ब रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
The lines represented by $px^2 - 8xy - 3y^2 = 0$ are perpendicular. Find the equation of these perpendicular lines.

8E. समीकरण $(P+1)x^2 + (1-2P)y^2 - 3x + 3y = 0$ ले जनाउने रेखाहरू कुन बिन्दुमा लम्ब हुने गरी काटिएलान् ?
In which point do the lines represented by the equation $(P+1)x^2 + (1-2P)y^2 - 3x + 3y = 0$ intersect perpendicularly ?

COORDINATES

Model 7 Problems based on finding value of letters when angle between them is given.

9A. यदि समीकरण $2x^2 - 7xy + ky^2 = 0$ ले जनाउने रेखाहरूबिचको कोण 45° भए k को मान कति होला ?
If an angle between the lines represented by $2x^2 - 7xy + ky^2 = 0$ is 45° then what is the value of k ?

9B. समीकरण $(5m-4)x^2 - mxy - 6my^2 = 0$ ले जनाउने रेखाहरूबिचको न्यूनकोण 45° भए m को धनात्मक मान पत्ता लगाउनुहोस् । साथै जोडा रेखाहरूको छुट्टाछुट्टै समीकरणहरू पनि पत्ता लगाउनुहोस् ।
If the acute angle between the lines represented by the equation $(5m-4)x^2 - mxy - 6my^2 = 0$ is 45°, find the positive value of 'm'. Also, find the separate equations of the lines.

Model 8 Problems based on finding equations of perpendicular/parallel lines to given lines.

10. तलका रेखाहरूसँग लम्ब हुने र उद्गम बिन्दु भएर जाने सीधा रेखाहरूको समीकरण पत्ता लगाउनुहोस्:
Find the equation of straight lines through the origin and perpendicular to the given lines:

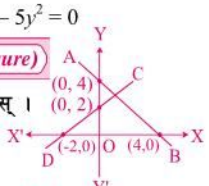
A. $x^2 + 3xy + 2y^2 = 0$ **B.** $2x^2 - xy - 3y^2 = 0$ **C.** $3x^2 + 8xy - 3y^2 = 0$
D. $2x^2 - 7xy + 5y^2 = 0$ **E.** $2x^2 + 6xy + 4y^2 = 0$ **F.** $3x^2 + 8xy + 5y^2 = 0$

11. तलका रेखाहरूसँग समानान्तर हुने र बिन्दु (2, 3) भएर जाने सीधा रेखाहरूको समीकरण पत्ता लगाउनुहोस्:
Find the equation of straight lines through (2, 3) and parallel to the given lines:

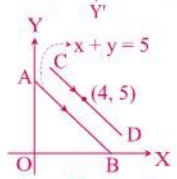
A. $x^2 + xy = 0$ **B.** $x^2 - 6xy + 8y^2 = 0$ **C.** $4x^2 + 8xy - 5y^2 = 0$

Model 9 Problems based on finding equations and angle between them (on the basis of figure)

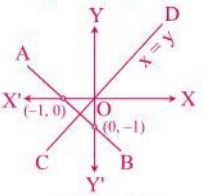
12A. सँगैको चित्रमा दिइएको जानकारीबाट रेखाहरू AB र CD जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
From the given figure, find the single equation of lines which represents the lines AB and CD.



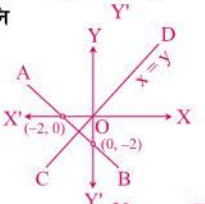
12B. सँगैको चित्रमा रेखा AB को समीकरण $x + y = 5$ छ । यदि रेखा AB सँग समानान्तर हुने रेखा CD बिन्दु (4, 5) भएर गएको छ भने रेखाहरू AB र CD दुवैलाई जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
In the adjoining figure, the equation of line AB is $x + y = 5$. If the line CD which is parallel to AB is passing through the point (4, 5), find the single equation represented by both AB and CD.



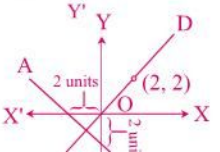
12C. चित्रमा रेखा AB बिन्दुहरू (– 1, 0) र (0, – 1) भएर गएको छ । रेखा CD बिन्दु (4, 4) भएर गएको छ । यी दुवै रेखाहरू जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
In the figure, line AB passes through the points (– 1, 0) & (0, – 1). Line CD passes through the point (4, 4). Find the single equation to represent these both lines.



13A. चित्रमा रेखा AB बिन्दुहरू (– 2, 0) र (0, – 2) भएर गएको छ । यदि CD को समीकरण $x = y$ भए रेखाहरू AB र CD दुवै जनाउने एकल समीकरण पत्ता लगाई प्रतिच्छेदन बिन्दु पनि पत्ता लगाउनुहोस् ।
In the adjoining figure, line AB passes through the points (– 2, 0) and (0, – 2). If the equation of CD is $x = y$, find the single equation representing the lines AB and CD and the point of intersection of the lines.



13B. चित्रमा दिइएको जानकारीबाट रेखाहरू AB र CD को संयुक्त समीकरण र तिनीहरूबिचको कोण पत्ता लगाउनुहोस् ।



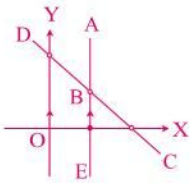
www.drmathnp.com

puspas.com.np

From the given information in the figure, find the joint equation of AB and CD and the angle between them.

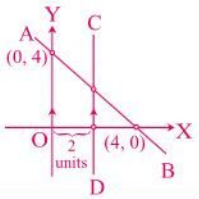
13C. चित्रमा रेखाहरू AB र CD को संयुक्त समीकरण $x^2 + xy - 6x - 2y + 8 = 0$ भए रेखा CD को समीकरण पहिचान गरी CD ले X-अक्षसँग बनाएको कोण पत्ता लगाउनुहोस् ।

In the figure, the joint equation of lines AB & CD is $x^2 + xy - 6x - 2y + 8 = 0$. Identify the equation of line CD and the angle made by CD with X-axis.



13D. चित्रबाट रेखाहरू AB र CD को संयुक्त समीकरण र तिनीहरूबिचको कोण पत्ता लगाउनुहोस् ।

From the figure, find the joint equation of AB and CD and angle between them.



Model 10 Miscellaneous problems.

14A. अक्षहरूमा खण्डहरू 4 र 5 बनाउने रेखा र बिन्दुहरू (2, 3) र (4, 5) भएर जाने रेखा दुवै जनाउने सीधारेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।

Find the single equation of straight lines which of one is making intercepts 4 and 5 on axes and the other is passing through the points (2, 3) and (4, 5).

14B. अक्षहरूमा खण्डहरू 8 र 10 बनाउने रेखा र बिन्दुहरू (4, 6) र (8, 10) भएर जाने रेखा दुवै जनाउने सीधारेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।

Find the single equation of straight lines which of one is making intercepts 8 and 10 on axes and the other is passing through the points (4, 6) and (8, 10).

ANSWERS

A. SHORT QUESTIONS

1A. $2x^2 + 7xy + 3y^2 = 0$ **1B.** $6x^2 + 5xy - 6y^2 = 0$ **1C.** $2x^2 - xy - 6y^2 - x + 2y = 0$ **1D.** $10x^2 - 19xy + 6y^2 - 35x + 14y = 0$

1E. $ax^2 + 2hxy + by^2 = 0$ **2A.** $2x^2 + xy - 3y^2 = 0$ **2B.** $2x^2 + xy - 3y^2 = 0$ **2C.** $x^2 - xy + 2x - 2y = 0$

2D. $y^2 - x^2 \sin^2\theta = 0$ **2E.** $x^2 + 2xy - 5x = 0$ **2F.** $2y^2 + xy - 5y = 0$ **3A.** $x = y, x + y = 3$

3B. $x - 2y = 0, x + 2y - 2 = 0$ **3C.** $x - 4y = 0, x - y = 0$ **3D.** $x - c = 0, x + y = 0$ **3E.** $x - y = 0, x + y = 1$

3F. $bx - ay = 0, ax - by = 0$ **3G.** $ax = by, bx + ay = 0$ **3H.** $ax = 2y, 2x + ay = 0$ **3I.** $9x = by, bx + 9y = 0$

3J. $bx + a = 0, ax + b = 0$ **3K.** $a^2 + b^2 = 0$, $x^2 + y^2 = 0$ **3L.** $x = -2, y = x$ **4A.** 18° or 162° **4B.** 26° or 154°

4C. $\tan^{-1}\left(\pm\frac{5}{3}\right)$ **4D.** 30°, 150° **4E.** $\tan^{-1}(\pm\sqrt{p^2-1})$ **4F.** 60°, 120° **4G.** 90° **4H.** $\tan^{-1}(\pm 4)$ **5A.** 18°

5B. 45° **5C.** 18.4° **5D.** 45° **5E.** A° **5F.** 10° **6A.** 135° **6B.** 135°

6C. 161.6° **6D.** 135° **6E.** 150° **6F.** 120° **7A.** $x + 2y = 0, 2x - y = 0$; 90°

7B. $3x - y = 0, x + 3y = 0$; 90° **8B.** $m = \pm 2$ **9A.** $-\frac{5}{6}$ **9B.** $p = 3$ **9C.** $p = 2, 1$ **9D.** -5

9E. $-\frac{7}{3}$ **9F.** 4 **10A.** $a = 2b$ **10B.** $3n = m$ **12A.** 2 **12B.** $-\frac{9}{5}$ **12C.** 1

12D. $-\frac{7}{16}$ **12E.** -1 **12F.** 25 **12G.** 5 **12H.** 3 **12I.** $-\frac{1}{7}, -\frac{5}{3}$

13. $4b^2 = 5(a + 1)$ **14.** $a = 2b$

B. LONG QUESTIONS

1A. $x - y = 0, x + y + 1 = 0$, 90° **1B.** $x + (2 + \sqrt{3})y = 0, x + (2 - \sqrt{3})y = 0$; 60°, 120° **1C.** $y = 3x, y + 2x = 0$; 135° or 45°

1D. $x - 3y = 0, 3x + y = 0$, 90° **1E.** $2x - 3y = 0, 5x + 2y = 0$, 78.11°, 101.89° **1F.** $x + 2y = 0, 3x - y = 0$, 45°, 135°

2A. $2x - 3y = 0, x + 2y = 8, \tan^{-1}\left(\frac{7}{4}\right)$ **2B.** $x - 2y + 1 = 0, 3x - y = 0$; 45° or 135° **2C.** $2x + 3y = 5, x - 2y + 4 = 0$; 60°,120°

2D. $2x - y + 3 = 0, 3x + 4y - 1 = 0$; 80° or 100° **2E.** $x = y + 1, 3x + y = 2$; 63° or 117° **2F.** $2x = y, x = y - 1$; 18° or 162°

3B. $x = (\sec A + \tan A)\, y$ and $x = (\sec A - \tan A)\, y$; $\pm A$ **3C.** $x = (\cot A + \text{cosec } A)\, y$ and $x = (\cot A - \text{cosec } A)\, y$; 90°

3D. $x - (\text{cosec } 2\alpha - \cot 2\alpha)\, y = 0$ and $x + (\text{cosec } 2\alpha + \cot 2\alpha)$, 90° **3E.** $x = y \tan\alpha, x = y \tan\alpha$; 0°, 180°

3F. $x - (\text{cosec } 40° - \cot 40°)\, y = 0$ and $x + (\text{cosec } 40° + \cot 40°)$, 90° **3G.** $x = (\text{cosec } 20° \pm \cot 20°)\, y$; tan–1(± cot 20°)

3H. $x - y(\cot 40° + \text{cosec } 40°) = 0, x - y(\cot 40° - \text{cosec } 40°) = 0$, 90° **4C.** $ax - by = 0, bx + ay = 0$; 90°

5A. $2x^2 + 3xy + y^2 + 5x + 2y = 3, \tan^{-1}\left(\pm\frac{1}{3}\right)$ **5B.** $x^2 - y^2 + x + y$, 90° **5C.** $x^2 - xy - 2y^2 - x - y, \tan^{-1}(\pm 3)$

5D. $x^2 - 2xy \text{ cosec } \theta + y^2 = 0, \tan^{-1}(\pm \cot\theta)$ **5E.** $x^2 - 2xy \cot 2A - y^2 = 0$, 0, 90° **6A.** $2x + y - 5 = 0, 3x - y - 5 = 0$; (2, 1)

6B. $x + y + 1 = 0, x + 2y + 3 = 0$, (1, –2) **6C.** $x - y + 2 = 0, x + y = 4$; (1, 3) **7A.** $x - y + 2 = 0, 2x + y + 1 = 0$

7B. $x - y + 1 = 0, x + y = 2$ **7C.** $3y = x + 1, x + 2y = 2$ **8A.** $k = \pm 6, 2x - 3y = 0, 2x - 3y = 0, 2x + 3y = 0, 2x + 3y = 0$

8B. $2, 2x = y, x + 2y = 0$ **8C.** $a = 1, x + 3y = 0, 3x - y = 0$ **8D.** $x - 3y = 0$ and $3x + y = 0$ **8E.** $p = 2, \left(\frac{1}{2}, \frac{1}{2}\right)$

9A. 3 or –15 **9B.** $x - 3y = 0, x + 2y = 0$ **10A.** $2x^2 - 3xy + y^2 = 0$ **10B.** $3x^2 - xy - 2y^2 = 0$ **10C.** $3x^2 + 8xy - 3y^2 = 0$

10D. $5x^2 + 7xy + 2y^2 = 0$ **10E.** $4x^2 - 6xy + 2y^2 = 0$ **10F.** $5x^2 - 8xy + 3y^2 = 0$ **11A.** $(x - 2)(x + y - 5) = 0$

www.drmathnp.com

11B. $(x-2y+4)(x-4y+10)=0$ **11C.** $(2x+5y-19)(2x-y-1)=0$ **12A.** $x^2-y^2-2x+6y-8=0$
12B. $x^2+2xy+y^2-14x-14y+45=0$ **12C.** $x^2-y^2+x-y=0$ **13A.** $x^2-y^2+2x-2y=0$; $(-1,-1)$
13B. $x^2-y^2+2x-2y=0$; 90° **13C.** $x+y=4$; 45° or 135° **13D.** $x^2+xy-6x-2y+8=0$; 45° or 135°
14A. $5x^2-xy-4y^2-15x+24y-20=0$ **14B.** $5x^2-xy-4y^2-30x+48y-80=0$

CHALLENGING EXERCISE 2

1. समीकरण $16x^2+24xy+9y^2+24x+18y-5=0$ ले जोडा समानान्तर रेखाहरूको प्रतिनिधित्व गर्दछ भनी देखाउनुहोस् र तिनीहरूबिचको दुरी पनि पत्ता लगाउनुहोस् ।
Show that $16x^2+24xy+9y^2+24x+18y-5=0$ represents a pair of parallel straight lines and find the distance between them.

2. समीकरण $x^2+6xy+9y^2+4x+12y-5=0$ ले जोडा समानान्तर रेखाहरूको प्रतिनिधित्व गर्दछ भनी देखाउनुहोस् र तिनीहरूबिचको दुरी पनि पत्ता लगाउनुहोस् ।
Show that $x^2+6xy+9y^2+4x+12y-5=0$ represents a pair of parallel straight lines and find the distance between them.

6. जोडा रेखा $2x^2-xy-6y^2+4x+6y=0$ सँग लम्ब हुने र बिन्दु (2, 3) भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of pair of lines passing through the point (2, 3) and perpendicular to the line pair $2x^2-xy-6y^2+4x+6y=0$.

7. समीकरण $(x^2+y^2)\sin^2\alpha=(x\cos\theta-y\sin\theta)^2$ ले प्रतिनिधित्व गरेको रेखाहरूबिचको कोण पत्ता लगाउनुहोस् ।
Find the angle between the lines represented by the equation: $(x^2+y^2)\sin^2\alpha=(x\cos\theta-y\sin\theta)^2$

Hint : Here, the given equation of two lines is; $(x^2+y^2)\sin^2\alpha=(x\cos\theta-y\sin\theta)^2$
i.e. $x^2\sin^2\alpha+y^2\sin^2\alpha=x^2\cos^2\theta+y^2\sin^2\theta-2xy\sin\theta.\cos\theta$
or, $x^2(\sin^2\alpha-\cos^2\theta)+y^2(\sin^2\alpha-\sin^2\theta)+2xy\sin\theta.\cos\theta=0$
or, $(\sin^2\alpha-\cos^2\theta)x^2+2\sin\theta.\cos\theta\,xy+y^2(\sin^2\alpha-\sin^2\theta)=0$ ....(i)
Comparing the equation (i) with $ax^2+2hxy+by^2=0$,
$a=\sin^2\alpha-\cos^2\theta$, $h=\sin\theta.\cos\theta$, $b=\sin^2\alpha-\sin^2\theta$
If β be the angle between the two lines then, we know,

$$\tan\beta=\frac{2\sqrt{h^2-ab}}{a+b}=\frac{2\sqrt{\sin^2\theta\cos^2\theta-(\sin^2\alpha-\cos^2\theta)(\sin^2\alpha-\sin^2\theta)}}{\sin^2\alpha-\cos^2\theta+\sin^2\alpha-\sin^2\theta}$$

$$=\frac{2\sqrt{\sin^2\theta.\cos^2\theta-\sin^4\alpha+\sin^2\alpha\sin^2\theta+\sin^2\alpha\cos^2\theta-\sin^2\theta.\cos^2\theta}}{\sin^2\alpha-1+\sin^2\theta+\sin^2\alpha-\sin^2\theta}$$

$$=\frac{2\sin\alpha\sqrt{-\sin^2\alpha+\sin^2\theta+\cos^2\theta}}{2\sin^2\alpha-1}=\frac{2\sin\alpha\sqrt{1-\sin^2\alpha}}{\cos2\alpha}=\frac{2\sin\alpha.\cos\alpha}{\cos2\alpha}=\frac{\sin2\alpha}{\cos2\alpha}=\tan2\alpha$$

$\therefore \tan\beta=\tan2\alpha$
Thus, the angle between the lines represented by the given equation is 2α.

8. समीकरण $x^2+y^2=c^2\left(\frac{bx+ay}{ab}\right)^2$ ले प्रतिनिधित्व गरेको रेखाहरूले एउटा समकोण बनाउने भए $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{3}{c^2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the lines represented by the equation $x^2+y^2=c^2\left(\frac{bx+ay}{ab}\right)^2$ form a right angle then prove that : $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{3}{c^2}$

9. $x^2+y^2=a^2\left(\frac{y-mx}{c}\right)^2$ ले जनाउने रेखाहरू आपसमा लम्ब भए $\frac{c^2}{a^2}=\frac{1+m^2}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the two lines represented by the equation $x^2+y^2=a^2\left(\frac{y-mx}{c}\right)^2$ are perpendicular to each other then prove that: $\frac{c^2}{a^2}=\frac{1+m^2}{2}$

10. समीकरण $q^2(x^2+y^2)=p^2(y-mx)^2$ ले जनाउने रेखाहरू आपसमा सम्पाती भए $\frac{p^2}{q^2}=\frac{1}{1+m^2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the lines represented by the equation $q^2(x^2+y^2)=p^2(y-mx)^2$ are coincident; prove that: $\frac{p^2}{q^2}=\frac{1}{1+m^2}$

11. रेखाहरू $2kx^2+4xy+y^2=0$ बिचको कोण 60° भए सीधा रेखाहरूको छुट्टाछुट्टै समीकरणहरू पत्ता लगाउनुहोस् ।
If the angle between the lines $2kx^2+4xy+y^2=0$ is 60°, find the separate equation of the straight lines.

ANSWERS

puspas.com.np

www.drmathnp.com

puspas.com.np

1. $\frac{2\sqrt{14}}{5}$ units **2.** $\frac{3\sqrt{10}}{5}$ units

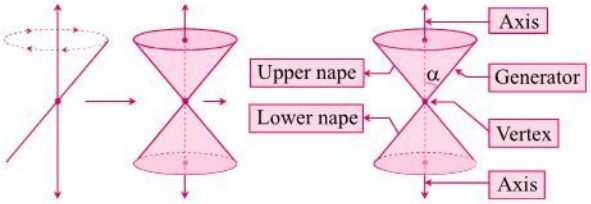
6. $6x^2 - xy - 2y^2 - 21x + 14y = 0$ **11.** $x = y(-2 \pm 3)$ or $x = (\pm\sqrt{3} - 2)\,y$ and $13x = (6 \pm 5\sqrt{3})y$

www.drmathnp.com

puspas.com.np

3. शाङ्कि क्षेत्र CONIC SECTION

1. सोली (Cone) :



वृत्ताकार आधारलाई कुनै बिन्दुमा वक्र सतहद्वारा जोडिएको तीन आयामिक वस्तुलाई सोली भनिन्छ ।
अर्को शब्दमा, समकोण त्रिभुजलाई यसैको एउटा छोटो भुजाको वरीपरी छिटो भन्दा छिटो घुमाउदा देखिने ठोसलाई सोली भनिन्छ ।

A 3 - dimensional solid object that has a circular base joined to a point by a curved surface, is called cone. In other words, rotating a right angled triangle around one of its shortest sides as fast as possible will produce a cone.

2. सोलीको अक्ष (Axis of cone) :

सोलीको शिर्षबिन्दु र वृत्ताकार आधारको केन्द्रबिन्दु जोड्ने रेखालाई सोलीको अक्ष भनिन्छ ।
The line joining vertex and the centre of the circular base is called the axis of cone.

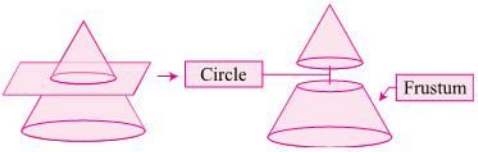
3. सोलीको जेनेरेटर (Generator of cone) :

सोलीको शिर्षबिन्दु र वृत्ताकार आधारको परिधि जोड्ने रेखा वा छड्के उचाइलाई सोलीको जेनेरेटर भनिन्छ ।
The line joining vertex and the circumference of the circular base or slant height of cone is called the generator of cone.

4. शाङ्कि क्षेत्र (Conic section)

सोलीको सतहलाई समतल सतहले प्रतिच्छेदन गर्दा प्राप्त हुने वक्रलाई शाङ्कि क्षेत्र भनिन्छ ।
A conic section is a curve obtained by the intersection of the surface of a cone with a plane.

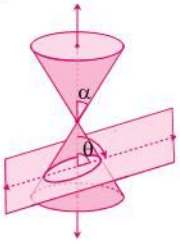
5. वृत्त (Circle)



कुनै समतलीय सतहले सोलीको अक्षसँग 90° बनाई वा सोलीको आधारसँग समानान्तर भई सोलीलाई प्रतिच्छेदित गर्दा बन्ने आकृति वा समतलीय वक्र नै वृत्त हो ।
अर्को शब्दमा, समतल सतहले सोलीको अक्षसँग लम्ब हुने गरी प्रतिच्छेदन गर्दा बन्ने वक्र वा क्षेत्रलाई वृत्त भनिन्छ ।

The plane curve formed by intersecting a cone with a plane surface making an angle of 90° with the axis of cone or parallel to the base of the cone is called circle.
In other words, if a plane cuts the cone perpendicularly to its axis, then the section formed by the intersection of the plane and cone is a circle.

6. दीर्घवृत्त (Ellipse)



कुनै समतलीय सतहले सोलीको एउटा भागलाई काट्दा उक्त सतहले सोलीको अक्षसँग बनाएको कोण θ छ र उक्त कोण θ को मान यदि semi vertical angle (α) भन्दा ठूलो र 90° भन्दा सानो अर्थात् $\alpha < \theta < 90°$ छ भने सो अवस्थामा सोली र सतहको प्रतिच्छेदनबाट बनेको समतलीय वक्र नै दीर्घवृत्त (Ellipse) हो ।
अर्को शब्दमा, समतल सतहले अर्धशिर्षकोण भन्दा ठूलो कोण अक्षसँग बनाई प्रतिच्छेदन गर्दा बन्ने बन्द वक्र वा क्षेत्रलाई दीर्घवृत्त (Ellipse) भनिन्छ ।

The plane curve formed by intersecting a cone with a plane surface making an angle of θ with the axis of cone in such a way that the value of θ is greater than the semi vertical angle (α) and less than 90° ($\alpha < \theta < 90°$) is called an ellipse.
In other words, if a plane cuts the cone such that the angle made by the plane with the axis is greater than the semi-vertical angle, then the closed curve or the section formed is an ellipse.

www.drmathnp.com

puspas.com.np

7. अनुवृत्त (Parabola)



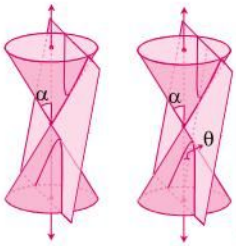
यदि समतल सतहले सोलीलाई प्रतिच्छेदन गर्दा सो समतलीय सतहले बनाएको कोण θ र semi vertical angle (α) बराबर भएको अवस्थामा (θ = α) अथवा जेनेरेटर (generator) सँग समतलीय सतह समानान्तर छ भने समतलीय सतह र सोलीको प्रतिच्छेदनबाट बन्न गएको वक्रलाई अनुवृत्त (Parabola) भनिन्छ ।

अर्को शब्दमा, समतल सतहले सोलीको जेनेरेटरसँग समानान्तर हुने गरी प्रतिच्छेदन गर्दा प्राप्त हुने खुल्ला वक्रलाई अनुवृत्त (Parabola) भनिन्छ ।

The plane curve formed by intersecting a cone with a plane surface in such a way that the plane surface is parallel to the generator and the angle made by the plane surface with the axis of cone (θ) is equal to the semi-vertical angle α (θ = α) is called parabola.

In other words, if a plane cuts the cone such that it is parallel to the generator of the cone, then the open curve or the section formed will be a parabola.

8. अति परावलय (Hyperbola):



समतलीय सतहले सोलीको दुवै भागलाई काटेको अवस्थामा समतलीय सतहले सोलीको अक्षसँग बनाएको कोण θ छ भने θ < α (semi-vertical angle) हुन्छ । सोलीको दुवै भाग र समतलीय सतहको प्रतिच्छेदनबाट बन्न गएको समतलीय वक्रलाई अति परावलय (Hyperbola) भनिन्छ ।

अर्को शब्दमा, समतल सतहले एउटै शीर्षबिन्दु भएको जोडी सोलीको अक्षसँग अर्धशीर्षकोण भन्दा सानो कोण बनाई प्रतिच्छेदन गर्दा बन्ने खुल्ला वक्र वा क्षेत्रलाई अति परावलय (Hyperbola) भनिन्छ ।

A plane curve formed by intersecting a double napped cone with a plane in such a way that the angle made by the plane surface with the axis of cone (θ) is less than the semi-vertical angle α (θ < α) is called hyperbola.

In other words, if a plane cuts the double right cone such that the angle between the axis and the plane is less than the semi-vertical angle, then their intersection forms a curve or a section which is a hyperbola.

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTION

Model	SEE Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

SEE EXERCISE 3

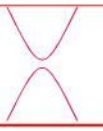
A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. तलका पदहरू परिभाषीत गर्नुहोस् (Define the following terms):

A. सोली (Cone) **B.** शाङ्कि क्षेत्र (Conic section) **C.** पाराबोला (Parabola)
D. दीर्घ वृत्त (Ellipse) **E.** अतिपरावलय (Hyperbola) **F.** सोलीको अक्ष (Axis of cone)
G. वृत्त (Circle)

2. तलका वक्रहरूको नामाकरण गर्नुहोस् (Name the following curves):

(a)	(b)	(c)	(d)
			

3A. एउटा सोलीलाई समतलीय सतहले आधारको समानान्तर हुनेगरि प्रतिच्छेदन गर्दा कुन ज्यामितीय चित्र बन्दछ ?
Which geometrical figure will be formed if a plane intersects a cone parallel to its base? **[SEE MODEL 2076]**

3B. यदि प्रतिच्छेदित सतह सोलीको जेनेरेटरसँग समानान्तर भए कुन शाङ्कि भाग बन्दछ ?
If the intersection plane is parallel to the generator of cone then what conic does it form?

3C. यदि प्रतिच्छेदित सतह सोलीको अक्षसँग समानान्तर भए कुन शाङ्कि भाग बन्दछ ?
If the intersection plane is parallel to the axis of cone then what conic does it form?

www.drmathnp.com

puspas.com.np

4A. एउटा सतहले सोलीको अक्षसँग लम्ब हुने गरी र सोलीको शीर्षबिन्दुबाट नजाने गरी प्रतिच्छेदित गर्दछ । यसरी बन्ने शाङ्कि‍क भागको नाम लेख्नुहोस् ।
A plane perpendicularly intersects a cone not passing through the vertex. Name the section so formed.

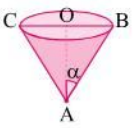
4B. एउटा सतहले समकोणी सोलीलाई शीर्षबिन्दुबाट नजाने गरी र सोलीको अक्षसँग सो सतहले बनाएको कोण भन्दा अर्धशीर्षकोण (Semi vertical angle) α सानो हुने गरी प्रतिच्छेदन गर्दछ । यसरी बन्ने शाङ्कि‍क भागको नाम लेख्नुहोस् ।
A plane cuts the right cone not passing through its vertex and semi vertical angle α is less than angle made by the plane to the axis of cone. Name the section so formed.

5A. सोलीको अर्धशीर्षकोण α र सोलीको अक्षसँग समतल सतहले बनाएको कोण θ छ । यदि $\theta = \alpha$ भए समतलीय सतह र सोलीको प्रतिच्छेदनबाट बन्ने शाङ्कि‍कलाई के भनिन्छ ?
The semi vertical angle of a cone is α and the angle made by the plane with the axis of cone is θ . If $\theta = \alpha$, what is the name of the conic section formed by the intersection of a plane surface and cone?

5B. सोलीको अर्धशीर्षकोण α र सोलीको अक्षसँग समतल सतहले बनाएको कोण θ छ । यदि $\alpha < \theta < 90°$ भए समतलीय सतह र सोलीको प्रतिच्छेदनबाट बन्ने शाङ्कि‍कलाई के भनिन्छ ?
The semi vertical angle of a cone is α and the angle made by the plane with the axis of cone is θ . If $\alpha < \theta < 90°$, what is the name of the conic section formed by the intersection of a plane surface and cone?

5C. सोलीको अर्धशीर्षकोण α र सोलीको अक्षसँग समतल सतहले बनाएको कोण θ छ । यदि $\theta < \alpha$ भए समतलीय सतह र सोलीको प्रतिच्छेदनबाट बन्ने शाङ्कि‍कलाई के भनिन्छ ?
The semi vertical angle of a cone is α and the angle made by the plane with the axis of cone is θ . If $\theta < \alpha$, what is the name of the conic section generated by the intersection of a plane surface and cone?

6. सँगैको चित्रमा तलका प्रत्येकले के जनाउँदछ ?
What does each of the following represent in the figure?



A. AO **B.** AB **C.** α **D.** A

7A. कस्तो अवस्थामा दीर्घ वृत्त बन्दछ ? (In which condition, ellipse is formed?)

7B. कस्तो अवस्थामा पाराबोला बन्दछ ? (In which condition, parabola is formed?)

7C. कस्तो अवस्थामा अति परावलय बन्दछ ? (In which condition, hyperbola is formed?)

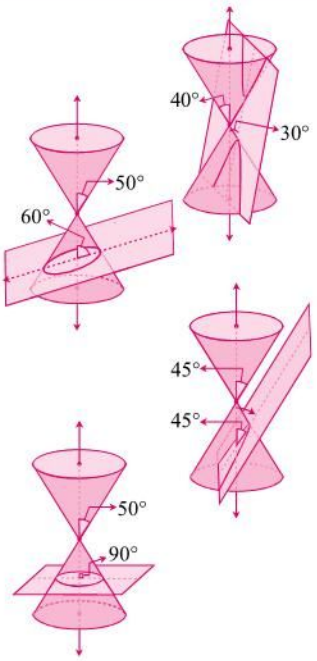
Model 2 *Concept Recall*

8A. एउटा सोलीको अर्ध शीर्षकोण 40° छ । समतल सतहले सोलीको अक्षसँग 30° को कोण बनाएर सो सोलीलाई प्रतिच्छेदन गर्दा बन्ने शाङ्कि‍कको नाम लेख्नुहोस् ।
The semi vertical angle of a cone is 40°. If the plane surface intersects the cone making an angle of 30° with the axis of the cone, write the name of the conic section so formed.

8B. एउटा सोलीको अर्ध शीर्षकोण 50° छ । समतल सतहले सोलीको अक्षसँग 60° को कोण बनाएर सो सोलीलाई प्रतिच्छेदन गर्दा बन्ने शाङ्कि‍कको नाम लेख्नुहोस् ।
The semi vertical angle of a cone is 50°. If the plane surface intersects the cone making an angle of 60° with the axis of the cone, write the name of the conic section so formed.

8C. एउटा सोलीको अर्ध शीर्षकोण 45° छ । समतल सतहले सोलीको अक्षसँग 45° को कोण बनाएर सो सोलीलाई प्रतिच्छेदन गर्दा बन्ने शाङ्कि‍कको नाम लेख्नुहोस् ।
The semi vertical angle of a cone is 45°. If the plane surface intersects the cone making an angle of 45° with the axis of the cone, write the name of the conic section so formed.

8D. एउटा सोलीको अर्ध शीर्षकोण 50° छ । समतल सतहले सोलीको अक्षसँग 90° को कोण बनाएर सो सोलीलाई प्रतिच्छेदन गर्दा बन्ने शाङ्कि‍कको नाम लेख्नुहोस् ।
The semi vertical angle of a cone is 50°. If the plane surface intersects the cone making an angle of 90° with the axis of the cone, write the name of the conic section so formed.



www.drmathnp.com

puspas.com.np

B. SHORT QUESTIONS

Model 1 *Problems based on vertical angle and angle with plane.*

1A. एउटा सोलीको शीर्षकोण 80° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा पाराबोला बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोण पत्ता लगाउनुहोस् ।
The vertical angle of a cone is 80°. If the cone is intersected by a plane so that a parabola is formed, find the angle between the axis of cone and the plane surface.

1B. एउटा सोलीको शीर्षकोण 70° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा पाराबोला बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोण पत्ता लगाउनुहोस् ।
The vertical angle of a cone is 70°. If the cone is intersected by a plane so that a parabola is formed, find the angle between the axis of cone and the plane surface.

2A. एउटा सोलीको शीर्षकोण 90° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा दीर्घ वृत्त बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?
The vertical angle of a cone is 90°. If the cone is intersected by a plane so that an ellipse is formed, what will be the possible value of angle between axis of cone and the plane?

2B. एउटा सोलीको शीर्षकोण 84° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा दीर्घ वृत्त बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?
The vertical angle of a cone is 84°. If the cone is intersected by a plane so that an ellipse is formed, what will be the possible value of angle between axis of cone and the plane?

3A. एउटा सोलीको शीर्षकोण 70° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा अति परावलय बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?
The vertical angle of a cone is 70°. If the cone is intersected by a plane so that an hyperbola is formed, what will be the possible value of angle between axis of cone and the plane?

3B. एउटा सोलीको शीर्षकोण 60° छ । सो सोलीलाई एउटा समतल सतहले प्रतिच्छेदन गर्दा एउटा अति परावलय बन्यो भने सोलीको अक्ष र समतल सतह बीचको कोणको सम्भाव्य मान कति होला ?
The vertical angle of a cone is 60°. If the cone is intersected by a plane so that an hyperbola is formed, what will be the possible value of angle between axis of cone and the plane?

Model 2 *Problems based on definitions.*

4A. वृत्त र दीर्घ वृत्तमा फरक लेख्नुहोस् । (Write the differences between circle and ellipse.)

4B. पाराबोला र हाइपरबोला (अति परावलय) मा फरक लेख्नुहोस् ।
Write the differences between parabola and hyperbola.

5A. सोलीको सचित्र परिभाषा दिनुहोस् । (Define cone with figure.)

5B. सोलीका विभिन्न भागहरू सचित्र देखाउनुहोस् । (Show the different parts of cone with figure.)

C. LONG QUESTIONS

1. शाङ्कुिक क्षेत्रको बारेमा लेखी यसका प्रकारहरू लेख्नुहोस् । (Write about conic section and its types.)

2. दीर्घ वृत्त, पाराबोला र हाइपरबोला बन्ने अवस्थाहरू उल्लेख गर्नुहोस् ।
Mention the conditions of formation of ellipse, parabola and hyperbola.

ANSWERS

A. VERY SHORT QUESTIONS

3A. Circle	**3B.** Parabola	**3C.** hyperbola	**4A.** circle	**4B.** Ellipse	**5A.** Parabola
5B. Ellipse	**5C.** Hyperbola	**6A.** axis of cone	**6B.** generator of cone	**6C.** semi vertical angle	**6D.** vertex
7A. $\alpha < \theta < 90°$	**7B.** $\theta = \alpha$	**7C.** $\theta < \alpha$	**8A.** Hyperbola	**8B.** Ellipse	**8C.** Parabola
8D. Circle					

A. SHORT QUESTIONS

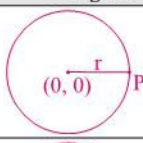
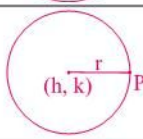
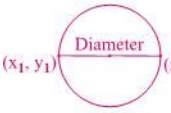
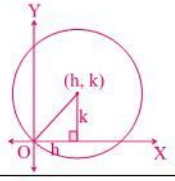
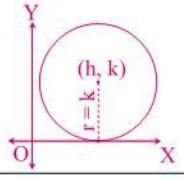
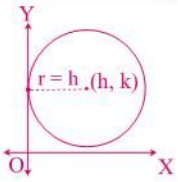
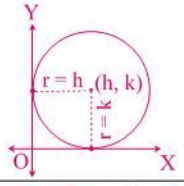
1A. 40°	**1B.** 35°	**2A.** $> 45°$	**2B.** $> 42°$	**3A.** $< 35°$	**3B.** $< 30°$

www.drmathnp.com

puspas.com.np

4. वृत्त CIRCLE

FORMULAE AND KEY POINTS

क्र.स. S.N.	चित्र Figure	वृत्तको अवस्था Condition of Circle	वृत्तको समीकरण Equation of Circle
1.		वृत्तको केन्द्र उद्‌गम् बिन्दुमा हुँदा । The centre of circle is at origin.	$x^2 + y^2 = r^2$
2.		वृत्तको केन्द्र बिन्दु (h, k) हुँदा । The centre of circle is (h, k).	$(x-h)^2 + (y-k)^2 = r^2$
3.		व्यासका दुई छेउका बिन्दुहरू दिँदा । Two ends of diameter are given.	$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$
4.		उद्‌गम् बिन्दु भएर जाने । Passing through origin $(r^2 = h^2 + k^2)$.	$(x-h)^2 + (y-k)^2 = h^2 + k^2$ $(r^2 = h^2 + k^2)$
5.		X-अक्षलाई छुँदा । Circle touches X-axis $(r = k)$.	$(x-h)^2 + (y-k)^2 = k^2$ $(r = k)$
6.		Y-अक्षलाई छुँदा । Circle touches Y-axis $(r = h)$.	$(x-h)^2 + (y-k)^2 = h^2$ $(r = h)$
7.		दुवै अक्षलाई छुँदा । Circle touches both the axes $(r = h = k)$.	$(x-h)^2 + (y-h)^2 = h^2$ **or,** $(x-k)^2 + (y-k)^2 = k^2$ **or,** $(x-r)^2 + (y-r)^2 = r^2$ $(r = h = k)$

8. वृत्तको समीकरण $x^2 + y^2 + 2gx + 2fy + c = 0$ को रूपमा भए (If the equation a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$):

(i) केन्द्र = $(-g, -f)$ or, $(-\frac{1}{2}x$ को गुणाङ्क, $-\frac{1}{2}y$ को गुणाङ्क $)$

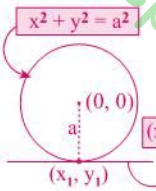
Centre = $(-g, -f)$ or, $(-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of $y)$

(ii) अर्धव्यास $= \sqrt{g^2 + f^2 - c}$ or, $\frac{1}{2}\sqrt{(x$ को गुणाङ्क $)^2 + (y$ को गुणाङ्क $)^2 - 4c}$

Radius $= \sqrt{g^2 + f^2 - c}$ or, $\frac{1}{2}\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2 - 4c}$

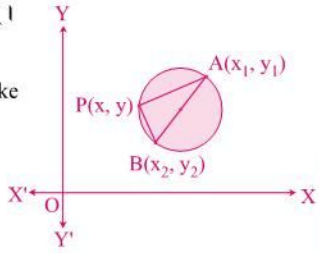
www.drmathnp.com

9. वृत्त $x^2 + y^2 = a^2$ को बिन्दु (x_1, y_1) मा खिचिएको स्पर्शरेखाको समीकरण $xx_1 + yy_1 = a^2$ हुन्छ ।
The equation of tangent in a circle $x^2 + y^2 = a^2$ at a point (x_1, y_1) is $xx_1 + yy_1 = a^2$.



Proof of Important theorem

➢ व्यासका छेउ छेउका बिन्दुहरू दिइएको अवस्थामा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle, whose ends of diameter are given.



Solution

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the ends of diameter of the circle. Take another point $P(x, y)$ at the circumference.

Now, slope of AP $(m_1) = \frac{y - y_1}{x - x_1}$ and slope of BP $(m_2) = \frac{y - y_2}{x - x_2}$

We have; $m_1 \times m_2 = -1$

[$\because \measuredangle APB$ = is the angle at the circumference so AP $\perp$ BP]

or, i.e. $\left(\frac{y - y_1}{x - x_1}\right) \times \left(\frac{y - y_2}{x - x_2}\right) = -1$ or, $\frac{(y - y_1)(y - y_2)}{(x - x_1)(x - x_2)} = -1$

or, $(y - y_1)(y - y_2) = -(x - x_1)(x - x_2)$ or, $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

This is the required equation of the circle.

SEE / SLC REVIEW MESSAGE

A. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2066 R', 2067 R, R', 2068 R, 2069 S, 2070 S, S', 2071 R, S, 2072 R, 2072 S, 2073 R', 2075 R'$_2$	12
Model 2	2065 M, 2065 S, 2066 R, 2068 S, 2070 R, 2070 R', SEE 2074 R', SEE 2074 S'	6
Model 3	2066 S, 2068 R', SEE 2074 R, SEE 2074 S, SEE 2075 R'	5
Model 4	2065 E, 2067 S	2
Model 5	2065 R, 2065 R', 2069 R, 2069 R', 2071 R' 2072 R', 2073 R, SEE 2073 S, SEE 2073 S', SEE 2075 R, R', R$_2$	12

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2065 R, 2068 S, 2069 R', 2070 R', SEE 2074 R', SEE 2074 S', SEE 2075 R'$_2$	7
Model 2	2067 R', 2073 R, SEE 2073 S, SEE 2073 S', SEE 2074 S	4
Model 3	2070 S', 2072 R, 2072 S	3
Model 4	2065 E, 2065 S, 2066 R, 2067 R, 2068 R, 2068 S'	5
Model 5	2067 S, 2069 S, 2071 R, 2071 S, 2073 R'	5
Model 6	2066 S	1
Model 7	2065 R', 2069 R, SEE MODEL 2076	3
Model 8	-	-
Model 9	2068 R', 2070 R, 2070 S, 2072 R'	4
Model 10	SEE 2074 R	1
Model 11	2071 R', SEE 2075 R, R$_2$	3

SEE EXERCISE 4

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1A. शाङ्कि क्षेत्रको आधारमा वृत्तको परिभाषा दिनुहोस् । (Define circle with respect to the conic section.)

1B. एकै केन्द्रित वृत्तको परिभाषा दिनुहोस् । (Define concentric circle.)

2A. समीकरण $x^2 + y^2 = r^2$ ले जनाउने बिन्दुपथको नाम लेख्नुहोस् ।
Write the name of the locus represented by the equation $x^2 + y^2 = r^2$.

2B. समीकरण $(x - h)^2 + (y - k)^2 = r^2$ ले जनाउने बिन्दुपथको नाम लेख्नुहोस् ।
Write the name of the locus represented by the equation $(x - h)^2 + (y - k)^2 = r^2$.

3A. केन्द्र $(0, 0)$ र अर्धव्यास 'r' भएको वृत्तको समीकरण के हुन्छ ?
What is the equation of circle whose centre is $(0, 0)$ and radius 'r'?

3B. केन्द्र (a, b) र अर्धव्यास 'r' भएको वृत्तको समीकरण लेख्नुहोस् ।
State the equation of circle whose centre is (a, b) and radius 'r'.

puspas.com.np

www.drmathnp.com
puspas.com.np

3C. केन्द्र (h, k) हुने र बिन्दु $(0, 0)$ भएर जाने वृत्तको समीकरण के हुन्छ ?
What is the equation of circle which passes through $(0, 0)$ and has its centre at (h, k)?

4A. वृत्तको व्यास रूपको समीकरण लेख्नुहोस् । (State the diameter form of equation of circle.)

4B. एउटा वृत्तको साधारण रूपको समीकरण के हुन्छ ? (What is the general equation of a circle?)

4C. वृत्तको साधारण रूपको समीकरण दिइएको बेला केन्द्रबिन्दु र अर्धव्यास निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate centre and radius of a circle when its general equation is given.

4D. एउटा वृत्तको समीकरण $x^2 + y^2 + 2gx + 2fy + c = 0$ मा $\sqrt{g^2 + f^2 - c}$ र $(-g, -f)$ ले के के जनाउँदछन् ?
What do $\sqrt{g^2 + f^2 - c}$ and $(-g, -f)$ represent in the equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$?

4E. समीकरण $x^2 + y^2 + 2ax + 2by + c = 0$ ले वृत्तको समीकरणलाई जनाउँदछ भने उक्त वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
If the equation $x^2 + y^2 + 2ax + 2by + c = 0$ represents equation of circle then find the centre of the circle.

4F. समीकरण $x^2 + y^2 + 2ax + 2by + c = 0$ ले वृत्तको समीकरणलाई जनाउँदछ भने उक्त वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the equation $x^2 + y^2 + 2ax + 2by + c = 0$ represents equation of circle then find the radius of the circle.

5A. केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' एकाइ भएको वृत्तमा कुन अवस्थामा $r = k$ हुन्छ ?
Under what condition will $r = k$ of a circle with centre (h, k) and radius 'r' units?

5B. केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' एकाइ भएको वृत्तमा कुन अवस्थामा $r = h$ हुन्छ ?
Under what condition will $r = h$ of a circle with centre (h, k) and radius 'r' units?

5C. केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' एकाइ भएको वृत्तमा कुन अवस्थामा $r = h = k$ हुन्छ ?
Under what condition will $r = h = k$ of a circle with centre (h, k) and radius 'r' units?

6A. यदि P र Q केन्द्रबिन्दु भएका दुई वृत्तहरू बाहिरपट्टी बिन्दु A मा स्पर्श भएका छन् भने PA, QA र PQ को सम्बन्ध लेख्नुहोस् ।
If two circles with centres P and Q touch externally at a point A then write the relation among PA, QA and PQ.

6B. यदि P र Q केन्द्रबिन्दु भएका दुई वृत्तहरू भित्रपट्टी बिन्दु A मा स्पर्श भएका छन् भने PA, QA र PQ को सम्बन्ध लेख्नुहोस् ।
If two circles with centres P and Q touch internally at a point A then write the relation among PA, QA and PQ.

Model 2 Concept Recall

7A. एउटा वृत्तको केन्द्र $(0, 0)$ र अर्धव्यास 3 एकाइ छ । यसको समीकरण लेख्नुहोस् ।
A circle has its centre at $(0, 0)$ and radius 3 units. Write its equation.

7B. एउटा वृत्तको केन्द्र $(-4, -5)$ छ र यसले x - अक्षलाई छुन्छ । यसको अर्धव्यास कति हुन्छ ?
A circle has centre at $(-4, -5)$ and touches x-axis. What is its radius?

7C. एउटा वृत्तको केन्द्र $(-4, -5)$ छ र यसले y - अक्षलाई छुन्छ । यसको अर्धव्यास कति हुन्छ ?
A circle has centre at $(-4, -5)$ and touches y-axis. What is its radius?

7D. एउटा वृत्तको अर्धव्यास 4 एकाइ छ र दुवै अक्षलाई छुने गरी चौथो चतुर्थांशमा पर्दछ । केन्द्रबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
A circle has radius 4 units and lies on fourth quadrant touching both axes. Write the coordinates of centre.

8A. वृत्त $(x-1)^2 + (y-2)^2 = 2^2$ ले कुन अक्षलाई छुन्छ, किन ?
Which axis does the circle $(x-1)^2 + (y-2)^2 = 2^2$ touch, why?

8B. वृत्त $(x-4)^2 + y^2 = 4^2$ ले कुन अक्षलाई छुन्छ, किन ?
Which axis does the circle $(x-4)^2 + y^2 = 4^2$ touch, why?

8C. वृत्त $(x-2)^2 + (y-2)^2 = 2^2$ ले कुन कुन अक्षहरूलाई छुन्छ, किन ?
Which axes do the circle $(x-2)^2 + (y-2)^2 = 2^2$ touch, why?

9A. वृत्त $(x-2)^2 + (y-3)^2 = 5^2$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of circle $(x-2)^2 + (y-3)^2 = 5^2$.

9B. वृत्त $(x-4)^2 + y^2 = 3^2$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of circle $(x-4)^2 + y^2 = 3^2$.

9C. वृत्त $x^2 + (y-5)^2 = 6^2$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of circle $x^2 + (y-5)^2 = 6^2$.

9D. वृत्त $(x+a)^2 + (y+b)^2 = c^2$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of circle $(x+a)^2 + (y+b)^2 = c^2$.

www.drmathnp.com

puspas.com.np

B. SHORT QUESTIONS

Model 1 *Problems based on centre / radius / diameter / circumference of a circle.*

1. वृत्तको केन्द्रबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । (Find the co-ordinates of the centre of the circle.)

A. $x^2 + y^2 - 20y + 75 = 0$ [2063 R'] **B.** $x^2 + y^2 + 2x - 1 = 0$ [2063 M]

C. $x^2 + y^2 - 4x + 3 = 0$ [2064 R'] **D.** $x^2 + y^2 - 6x + 2y + 1 = 0$ [2068 R]

E. $x^2 + y^2 + 4x - 5 = 0$ [2069 S, 2070 S']

2. वृत्तको अर्धव्यासको लम्बाइ पत्ता लगाउनुहोस् । (Find the length of the radius of the circle.)

A. $x^2 + y^2 + 4x - 6y + 4 = 0$ [2063 R] **B.** $x^2 + y^2 - 4x + 10y - 7 = 0$ [2064 R]

3A. यदि एउटा वृत्तको समीकरण $(x + 5)^2 + y^2 = 64$ भए त्यसको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
If the equation of a circle is $(x + 5)^2 + y^2 = 64$, find its centre and radius. [2075 R'$_2$]

3B. एउटा वृत्तको समीकरण $x^2 + y^2 - 2y = 24$ भए त्यसको अर्धव्यास र केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
If the equation of a circle is $x^2 + y^2 - 2y = 24$, find its radius and centre. [2058 R, 2066 R']

3C. समीकरण $x^2 + (y + 2)^2 = 49$ ले प्रतिनिधित्व गर्ने वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle represented by $x^2 + (y + 2)^2 = 49$. [2067 R']

3D. समीकरण $x^2 + y^2 - 10x - 4y = 7$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of a circle having the equation $x^2 + y^2 - 10x - 4y = 7$. [2072 S]

4A. एउटा वृत्तको समीकरण $(x + 5)^2 + y^2 = 121$ भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र व्यास निकाल्नुहोस् । [2059 R]
If the equation of a circle is $(x + 5)^2 + y^2 = 121$, find the co-ordinates of the centre of the circle and its diameter.

4B. समीकरण $x^2 + y^2 - 2y - 48 = 0$ भएको एउटा वृत्तको परिधिको लम्बाइ पत्ता लगाउनुहोस् ।
Find the length of the circumference of a circle having the equation $x^2 + y^2 - 2y - 48 = 0$. [2067 R]

5A. यदि वृत्त $x^2 + y^2 - ax - by - 12 = 0$ को केन्द्र $(2, 3)$ भए a र b का मानहरू निकाल्नुहोस् ।
If the centre of the circle $x^2 + y^2 - ax - by - 12 = 0$ is $(2, 3)$, find the values of a and b. [2071 R]

5B. समीकरण $x^2 + y^2 - mx + 6y - 12 = 0$ भएको वृत्तको केन्द्रबिन्दु $(-2, -3)$ भए m को मान पत्ता लगाउनुहोस् । [2070S]
Find the value of m when the centre of a circle with equation $x^2 + y^2 - mx + 6y - 12 = 0$ is $(-2, -3)$.

6A. यदि वृत्त $x^2 + y^2 - 4x - 6y - k = 0$ को अर्धव्यास 4 एकाइ भए k को मान पत्ता लगाउनुहोस् ।
If the radius of the circle $x^2 + y^2 - 4x - 6y - k = 0$ is 4 units, find the value of k. [2072 R, 2071 S]

6B. यदि वृत्त $x^2 + y^2 - 6x - 4y + k = 0$ को अर्धव्यास 2 एकाइ भए k को मान पत्ता लगाउनुहोस् ।
If the radius of the circle $x^2 + y^2 - 6x - 4y + k = 0$ is 2 units, find the value of k.

6C. यदि समीकरण $x^2 + y^2 + 4x - 6y + h = 0$ र अर्धव्यास 3 एकाइ भएको एउटा वृत्त छ भने, h को मान निकाल्नुहोस् ।
If the equation of circle is $x^2 + y^2 + 4x - 6y + h = 0$ and its radius is 3, find the value of h.

7A. $x + y = 5$ र $2x - y = 1$ वृत्तका दुई व्यासहरूका समीकरणहरू भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
Find the co-ordinates of the centre of a circle having equations of two diameters $x + y = 5$ and $2x - y = 1$. [2073 R']

7B. $x - y = -1$ र $2x + y = 13$ वृत्तका दुई व्यासहरूका समीकरणहरू भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
Find the co-ordinates of the centre of a circle having equations of two diameters $x - y = -1$ and $2x + y = 13$.

Model 2 *Problems based on finding equation of circle when centre and radius are given.*

8A. केन्द्रबिन्दु $(0, 0)$ र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre $(0, 0)$ and radius 3 units. [2063 M]

8B. केन्द्रबिन्दु $(3, 0)$ र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having the centre $(3, 0)$ and radius 5 units. [2065 M]

8C. केन्द्रबिन्दु $(0, 2)$ र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre $(0, 2)$ and radius is 3 units. [2064 S]

8D. केन्द्रबिन्दु $(2, -1)$ र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle with centre $(2, -1)$ and radius 3 units. [2065 S]

8E. केन्द्रबिन्दु $(-4, 1)$ र अर्धव्यास 4 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre at $(-4, 1)$ and radius 4 units. [2066 R]

8F. केन्द्रबिन्दु $(1, 5)$ र अर्धव्यास 4 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle with centre $(1, 5)$ and radius is 4 units. [2068 S]

www.drmathnp.com

puspas.com.np

8G. केन्द्र $(1, -2)$ र व्यास $2\sqrt{5}$ एकाइ भएको एउटा वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of a circle having centre $(1, -2)$ and the diameter $2\sqrt{5}$ units. [2070 R]

8H. केन्द्रबिन्दु $(2, 3)$ र अर्धव्यास 5 एकाइ भएको एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre $(2, 3)$ and radius 5 units. [SEE 2074 S', 2070 R', SEE 2074 R']

Model 3 *On finding equation of circle when centre & a point at the circumference are given.*

9A. केन्द्रबिन्दु $(0, 0)$ भएको र बिन्दु $(2, -3)$ भएर जाने एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre $(0, 0)$ and passes through the point $(2, -3)$. [SEE 2074 R]

9B. केन्द्रबिन्दु $(2, 3)$ भएको र बिन्दु $(-2, 0)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre $(2, 3)$ and passes through the point $(-2, 0)$. [2066S]

9C. केन्द्रबिन्दु $(2, 3)$ भएको र बिन्दु $(6, 0)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the centre $(2, 3)$ and passes through the point $(6, 0)$. [2068 R']

9D. केन्द्रबिन्दु $(3, 4)$ भएको र बिन्दु $(7, 7)$ भएर जाने एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre $(3, 4)$ and passes through the point $(7, 7)$. [SEE 2074 S]

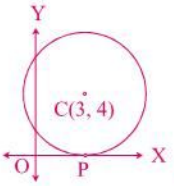
9E. केन्द्रबिन्दु $(2, 3)$ र बिन्दु $(5, 6)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre at $(2, 3)$ and passing through the point $(5, 6)$. [SEE 2075 R']

Model 4 *Problems based on centre and touching point to an axis.*

10A. P(3, 4) केन्द्रबिन्दु भएको वृत्तले X-अक्षमा बिन्दु (3, 0) मा छोएर जान्छ भने यो वृत्तको अर्धव्यासको नाप कति हुन्छ ? [2063 S]
What is the length of the radius of the circle having centre at P(3, 4) and that touches the X-axis at (3, 0) ?

10B. P(3, 4) केन्द्रबिन्दु भएको वृत्तले Y-अक्षमा बिन्दु (0, 4) मा छोएर जान्छ भने यो वृत्तको अर्धव्यासको नाप कति होला ? [2065 E]
What is the length of radius of the circle having centre at P(3, 4) and that touches the Y-axis at (0, 4) ?

10C. सँगैको चित्रमा C(3, 4) वृत्तको केन्द्रबिन्दु हो । सो वृत्तले X-अक्षको P बिन्दुमा छोएको छ भने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
In the adjoining figure, C(3, 4) is the centre of the circle. The circle touches X-axis at P. Find the equation of the circle. [2060 CP]



10D. केन्द्र (3, 6) भएको र X-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre (3, 6) and touching the X-axis. [2067 S]

10E. Y-अक्षलाई स्पर्श गर्ने र केन्द्रबिन्दु (5, 4) भएको वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle having centre at (5, 4) and that touches the Y-axis. [2063 M]

Model 5 *Problem based on finding equation of circle when end points of a diameter are given.*

11. दिइएका बिन्दुहरू कुनै वृत्तको व्यासको छेउछेउका बिन्दुहरू हुन् भने सो वृत्तको समीकरण निकाल्नुहोस् ।
Given points are the end points of a diameter. Find the equation of the circle.

A.	(0, 6) and (8, 0)	[SEE 2075 R]	**B.**	(3, 0) and (0, 4)	[2075 R_2]
C.	(3, 4) and (– 3, – 4)	[SEE 2075 R']	**D.**	(1, 3) and (2, – 5)	[SEE 2073 R]
E.	A(5, 6) and B(3, 4)	[2071 R']	**F.**	A(3, 4) and B(2, –7).	[2069 R]
G.	M(6, 7) and N(2, 3).	[2069 R']	**H.**	(3, 2) and (– 1, 6).	[2065 R]
I.	(– 1, 5) and (3, – 3).	[2065R']	**J.**	(1, 2) and (3, 6)	[2058 S]
K.	(– 1, 0) and (7, 4).	[2063 S]	**L.**	(4, 1) and (6, 5)	[2072 R']
M.	(2, 3) and (4, 0)	[SEE 2073 S]	**N.**	(– 1, 2) and (0, – 4)	[SEE 2073 S']

C. LONG QUESTIONS

Model 1 *Problems based on finding centre, radius and diameter.*

1. वृत्तको केन्द्रको निर्देशाङ्क र अर्धव्यास पत्ता लगाउनुहोस् (Find the co-ordinates of centre and radius of the circles):

A.	$x^2 + y^2 - 2x - 6y + 1 = 0$	[2059 S]	**B.**	$x^2 + y^2 - 10x - 14y + 49 = 0$	[2061S]
C.	$x^2 + y^2 - 10x + 4y + 13 = 0$	[2069R', SEE 2074 R']	**D.**	$x^2 + y^2 + 6x - 4y - 3 = 0$	[SEE 2074 S']
E.	$2x - 6y - x^2 - y^2 = 1$	[2057S]	**F.**	$2x^2 + 2y^2 - 2x + 6y = 45$	[2057 R]
G.	$2x^2 + 2y^2 - 8x - 12y + 1 = 0$	[2065M]	**H.**	$2x^2 + 2y^2 - 20x - 28y + 98 = 0$	[2075 R'_2]
I.	$4x^2 + 4y^2 - 8x + 24y + 32 = 0$	[2068 S]	**J.**	$9x^2 + 9y^2 - 36x + 6y = 107$	[2062R, 2065R]

www.drmathnp.com

puspas.com.np

2A. एउटा वृत्तको समीकरण $4x^2 + 4y^2 - 24x - 20y - 3 = 0$ भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र व्यास पत्ता लगाउनुहोस् ।
If the equation of a circle is $4x^2 + 4y^2 - 24x - 20y - 3 = 0$, find the co-ordinates of its centre and the diameter of the circle. **[2062S, 2070 R']**

2B. एउटा वृत्तको समीकरण $2x^2 + 2y^2 + 20 = 4x + 20y$ भए सो वृत्तको केन्द्रबिन्दुको निर्देशाङ्क र व्यास पत्ता लगाउनुहोस् ।
If the equation of a circle is $2x^2 + 2y^2 + 20 = 4x + 20y$, find the co-ordinates of its centre and the diameter of the circle.

Model 2 Problems based on finding co-ordinates of other end when the one end is given.

3A. यदि समीकरण $x^2 + y^2 + 4x - 6y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्कहरू $(0, 2)$ भए उक्त व्यासको अर्को छेउको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 + 4x - 6y + 8 = 0$ is $(0, 2)$ then find the co-ordinates of the other end of the diameter. **[SEE 2074 S, SEE 2073 S]**

3B. यदि समीकरण $x^2 + y^2 - 6x + 4y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्कहरू $(1, -1)$ भए उक्त व्यासको अर्को छेउको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 - 6x + 4y + 8 = 0$ is $(1, -1)$ then find the co-ordinates of the other end of the diameter. **[SEE 2073 S']**

3C. यदि समीकरण $x^2 + y^2 + 4x - 6y + 8 = 0$ भएको वृत्तको एउटा व्यासको एक छेउको निर्देशाङ्क $(-4, 4)$ भए उक्त व्यासको अर्को छेउको निर्देशाङ्क पत्ता लगाउनुहोस् ।
If the coordinates of one end of a diameter of the circle having equation $x^2 + y^2 + 4x - 6y + 8 = 0$ is $(-4, 4)$, then find the co-ordinates of the other end of the diameter. **[SEE 2073 R]**

3D. वृत्त $x^2 + y^2 - 2x - 2y = 8$ को व्यासको एउटा छेउको निर्देशाङ्क $(2, 4)$ भए अर्को छेउको निर्देशाङ्क पत्ता लगाउनुहोस् ।
The co-ordinates of an end point of the diameter of circle $x^2 + y^2 - 2x - 2y = 8$ is $(2, 4)$, find the coordinate of other end. **[2060 S']**

3E. समीकरण $x^2 + y^2 - 4x - 6y + 11 = 0$ भएको वृत्तको एउटा व्यासको एक छेउ $(1, 2)$ भए अर्को छेउ पत्ता लगाउनुहोस् । **[2067 R']**
If $(1, 2)$ is one end of a diameter of the circle with equation $x^2 + y^2 - 4x - 6y + 11 = 0$, find the other end.

Model 3 Problems based on finding equation of circle when passing point and centre are given.

4A. बिन्दुहरू $(-1, 3)$ र $(3, 1)$ जोड्ने रेखाको मध्य बिन्दुबाट जाने र केन्द्रबिन्दु $(4, 6)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre $(4, 6)$ and passing through the midpoint of the line joining the points $(-1, 3)$ and $(3, 1)$. **[2072 S]**

4B. बिन्दुहरू $(4, 1)$ र $(-8, -1)$ जोड्ने रेखाखण्डको मध्य बिन्दुबाट जाने र केन्द्रबिन्दु $(2, 3)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre $(2, 3)$ and passing through the midpoint of the line segment joining the points $(4, 1)$ and $(-8, -1)$.

4C. बिन्दुहरू $(4, -2)$ र $(10, 4)$ जोड्ने रेखाखण्डलाई $1 : 2$ को अनुपातमा भित्रपट्टिबाट विभाजन गर्ने बिन्दुबाट जाने र केन्द्रबिन्दु $(2, 3)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre $(2, 3)$ and passing through the point which divides the line segment joining the points $(4, -2)$ and $(10, 4)$ in the ratio of $1 : 2$ internally.

5A. केन्द्र बिन्दु $(1, 2)$ भएको र रेखाहरू $x + 2y = 3$ र $3x + y = 4$ को प्रतिच्छेदन बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre $(1, 2)$ and passing through the point of intersection of the lines $x + 2y = 3$ and $3x + y = 4$. **[2072 R]**

5B. केन्द्र $(3, 4)$ भएको एउटा वृत्त रेखाहरू $3x - y = 14$ र $2x - y = 7$ को प्रतिच्छेदन बिन्दु भएर जान्छ । सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
A circle having centre $(3, 4)$ passes through the point of intersection of the lines $3x - y = 14$ and $2x - y = 7$. Find the equation of the circle. **[2070 S']**

Model 4 Problems based on finding equation of circle when equation of diameters are given.

6A. बिन्दु $(0, 2)$ बाट जाने एउटा वृत्तको दुई व्यासहरूका समीकरणहरू $4x - 2y = 5$ र $8x + 6y = -5$ भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle passing through the points $(0, 2)$ are $4x - 2y = 5$ and $8x + 6y = -5$. Find the equation of the circle. **[2068 S']**

6B. रेखाहरू $x - y = 4$ र $2x + 3y + 7 = 0$ को प्रतिच्छेदन बिन्दु केन्द्रबिन्दु भएको र बिन्दु $(2, 4)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having the centre as the point of intersection of the lines $x - y = 4$ and $2x + 3y + 7 = 0$ and passing through the point $(2, 4)$. **[2068 R]**

www.drmathnp.com

puspas.com.np

6C. बिन्दु (3, 4) बाट जाने एउटा वृत्तको दुई व्यासहरूका समीकरणहरू $x + y = 14$ र $2x - y = 4$ भए वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle passing through the point (3, 4) are $x + y = 14$ and $2x - y = 4$. Find the equation of the circle. **[2061 R]**

6D. सरल रेखाहरू $2x + y = 4$ र $2y - x = 3$ का प्रतिच्छेदन बिन्दुमा केन्द्रबिन्दु पर्ने तथा बिन्दु (4, 6) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle whose centre is at the point of intersection of $2x + y = 4$ and $2y - x = 3$ and passing through the point (4, 6). **[2065 S, 2066 R]**

6E. वृत्तको व्यासहरू $3x - 4y - 1 = 0$ र $2x + y - 8 = 0$ हुने र बिन्दु (1, 4) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through the point (1, 4) and the diameters of the circle are $3x - 4y - 1 = 0$ and $2x + y - 8 = 0$. **[2064 S]**

6F. वृत्तको दुई व्यासका समीकरणहरू क्रमशः $2x + y = 5$ र $x - y = 1$ र बिन्दु (1, 4) भएर जाने वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle which passes through the point (1, 4) and two diameters whose equations are $2x + y = 5$ and $x - y = 1$. **[2065 E]**

6G. बिन्दु (5, 1) बाट जाने एउटा वृत्तका दुई व्यासहरूका समीकरणहरू क्रमशः $x - y = 3$ र $2x + y = 21$ भए सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle passing through the point (5, 1) are $x - y = 3$ and $2x + y = 21$ respectively. Find the equation of the circle. **[2067 R]**

6H. एउटा वृत्तको अर्धव्यास 5 एकाइ छ र दुई व्यासहरूका समीकरणहरू $2x - y = 5$ र $x - 3y + 5 = 0$ छन् । सो वृत्तको समीकरण निकाल्नुहोस् र उक्त वृत्त उद्गमबिन्दुबाट जान्छ भनी प्रमाणित गर्नुहोस् ।
A circle has radius 5 units & the equations of its 2 diameters are $2x - y = 5$ & $x - 3y + 5 = 0$. Find the equation of the circle and prove that it passes through the origin. **[2060 R]**

Model 5 *On finding equation of circle when equation of a diameter & two passing points are given.*

7A. बिन्दुहरू (2, 3) र (5, 4) भएर जाने तथा केन्द्रबिन्दु रेखा $2x + 3y - 7 = 0$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle passing through the points (2, 3) and (5, 4) and centre lies on the line $2x + 3y - 7 = 0$. **[SEE 2073 R']**

7B. बिन्दुहरू (3, 2) र (5, 4) भएर जाने र केन्द्र $3x - 2y - 1 = 0$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the points (3, 2) and (5, 4) and its centre lies on the line $3x - 2y - 1 = 0$. **[2071 S]**

7C. एउटा वृत्तको केन्द्रबिन्दु, रेखा $x - 4y = 1$ मा पर्दछ । सो वृत्त बिन्दुहरू (3, 7) र (5, 5) भएर जान्छ भने उक्त वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle whose centre lies on the line $x - 4y = 1$ and which passes through the points (3, 7) and (5, 5). **[2071 R]**

7D. बिन्दुहरू (4, 1) र (6, 5) भएर जाने र केन्द्रबिन्दु रेखा $4x + y = 16$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through the points (4, 1) and (6, 5) having its centre on the line $4x + y = 16$. **[2069 S]**

7E. बिन्दुहरू (2, 3) र (– 1, 2) भएर जाने र वृत्तको केन्द्रबिन्दु रेखा $2x - 3y + 1 = 0$ मा पर्दछ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle passing through the points (2, 3) and (– 1, 2) and centre lies on the line $2x - 3y + 1 = 0$. **[2067 S]**

7F. बिन्दुहरू (– 2, 0) र (0, – 2) भएर जाने र केन्द्रबिन्दु सीधारेखा $2x - 3y + 1 = 0$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the points (– 2, 0) and (0, – 2) and its centre lies on the straight line $2x - 3y + 1 = 0$. **[2063 R']**

7G. उद्गम बिन्दु र बिन्दु (4, 2) बाट जाने वृत्तको केन्द्रबिन्दु रेखा $x + y = 1$ मा पर्दछ भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle which passes through the origin and the point (4, 2) lies on the line $x + y = 1$. Find the equation of the circle. **[2064 R]**

Model 6 *On finding equation of circle when touching point to the axis and passing points are given.*

8A. X-अक्षको बिन्दु (3, 0) मा छोएर र (1, 2) बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् । **[2064 R']**
Find the equation of the circle which touches the X-axis at a point (3, 0) and passing through point (1, 2).

8B. केन्द्रबिन्दु (b, 4) तथा X-अक्षको (– 2, 0) र (4, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having the centre at (b, 4) and passing through the points (– 2, 0) and (4, 0) of X-axis. **[2066 S]**

www.drmathnp.com

puspas.com.np

Model 7 *On finding centre or radius or equation of circle when three passing points are given.*

9A. एउटा पाङ्ग्राको परिधिमा रहेका तीनओटा बिन्दुहरू (5, 7) , (– 1, 7) र (5, – 1) एउटा निश्चित बिन्दुबाट बराबर दुरीमा पर्दछन् । उक्त निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै उक्त तीन बिन्दुहरू पर्ने गरी बिन्दुपथ पत्ता लगाउनुहोस् ।
On a wheel, there are three points (5, 7) , (– 1, 7) and (5, – 1) located such that the distance from a fixed point to these points is always equal. Find the coordinate of the fixed point and then derive the equation representing the locus that contains all three points. **[SEE MODEL 2076]**

9B. तीन जना विद्यार्थीहरू एउटा खेल मैदानमा वृत्ताकार आकृत्ति बन्ने गरी परिधिमा बसेका छन् । उनीहरूको बसेको ठाउँको निर्देशाङ्कहरू (1, 2), (3, 4) र (3, 2) छन् । तिनीहरूबाट बराबर दूरीमा पर्ने निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । उक्त वृत्ताकार पथको बिन्दु पथको समीकरण पत्ता लगाउनुहोस् ।
Three students are sitting in a playground to make a circular path. They are sitting on the circumference, the co-ordinates of their position on the circle are (1, 2), (3, 4) and (3, 2). Find the co-ordinates of the point equidistant from their position. Also, find the equation of the locus.

9C. बिन्दुहरू P(5, 7), Q(6, 6) र R(2, – 2) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the points P(5, 7), Q(6, 6) and R(2, – 2). **[2069 R]**

10A. बिन्दुहरू A(– 4, – 2), B(2, 6) र C(2, – 2) भएर जाने वृत्तको अर्धव्यास निकाल्नुहोस् ।
Find the radius of the circle which passes through the points A(– 4, – 2), B(2, 6) and C(2, – 2). **[2060 S]**

10B. बिन्दुहरू (1, 2), (3, – 4) र (5, – 6) भएर जाने वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
Find the radius of the circle passing through the points (1, 2), (3, – 4) and (5, – 6). **[2063 R]**

10C. बिन्दुहरू (– 6, 5), (– 3, – 4) र (2, 1) बाट जाने वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
Find the centre point of a circle which passes through the points (– 6, 5), (– 3, – 4) and (2, 1). **[2065 R']**

11. तलका बिन्दुहरू एउटै वृत्तमा पर्छन् भनी प्रमाणित गर्नुहोस् । (Prove that the following points lie on a same circle.)

A. (3, 3), (6, 4), (7, 1) and (4, 6) **B.** (1, 0), (2, – 7), (8, 1) and (9, – 6)

Model 8 *On finding point of intersection of a circle and a line and distance between the points.*

12A. यदि रेखा $x + y = 1$ ले वृत्त $x^2 + y^2 = 1$ लाई दुई बिन्दुहरूमा काट्दछ भने उक्त बिन्दुहरूबिचको दूरी पत्ता लगाउनुहोस् ।
If the line $x + y = 1$ cuts a circle $x^2 + y^2 = 1$ at two points, find the distance between the two points. **[2062 K]**

12B. केन्द्रबिन्दु (0, 0) र अर्धव्यास 2 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् । उक्त वृत्त र रेखा $x + y = 2$ को प्रतिच्छेदन बिन्दुहरूको निर्देशाङ्कहरू पनि पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre (0, 0) whose radius is 2 units. Also, find the co-ordinates of the points of intersection of the circle and the line $x + y = 2$.

Model 9 *Problems based on finding equation of concentric cricle.*

13A. समीकरण $x^2 + y^2 - 6x + y = 1$ भएको वृत्तसँग केन्द्रित हुने र बिन्दु (4, – 2) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + y = 1$ and passing through the point (4, – 2). **[2072 R']**

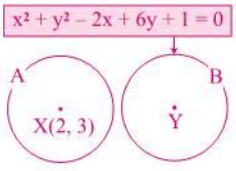
13B. वृत्त $2x^2 + 2y^2 + 4x - 2y + 1 = 0$ सँग समकेन्द्रीत हुने र बिन्दु (4, – 2) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle concentric with the circle $2x^2 + 2y^2 + 4x - 2y + 1 = 0$ and passing through the point (4, – 2). **[2070 S]**

13C. समीकरण $x^2 + y^2 + 6x - 8y - 11 = 0$ भएको एउटा वृत्तको केन्द्रमा केन्द्रीत हुने र बिन्दु (4, 3) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through the point (4, 3) and concentric with the circle having equation $x^2 + y^2 + 6x - 8y - 11 = 0$. **[2070 R]**

13D. बिन्दु (5, 4) बाट जाने र वृत्त $x^2 + y^2 - 8x + 12y + 15 = 0$ सँग एक केन्द्रित हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which is concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through the point (5, 4). **[2068 R']**

Model 10 *Problems based on equal circles.*

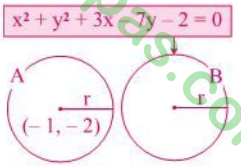
14A. दिइएको चित्रमा बराबर वृत्तहरू A र B का केन्द्रबिन्दुहरू क्रमशः X र Y छन् । यदि X को निर्देशाङ्क (2, 3) र वृत्त B को समीकरण $x^2 + y^2 - 2x + 6y + 1 = 0$ छ भने वृत्त A को समीकरण निकाल्नुहोस् ।
In the given figure, the centres of two equal circles A and B are X and Y respectively. If the coordinates of X are (2, 3) and the equation of circle B is $x^2 + y^2 - 2x + 6y + 1 = 0$ then find the equation of the circle A. **[SEE 2074 R]**



www.drmathnp.com

puspas.com.np

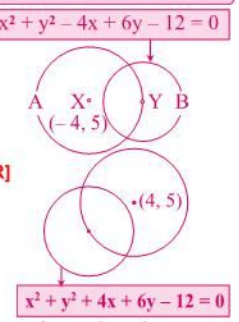
14B. अर्धव्यास वृत्त $x^2 + y^2 + 3x - 7y - 2 = 0$ को अर्धव्याससँग बराबर हुने र बिन्दु $(-1, -2)$ केन्द्र भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle whose centre is $(-1, -2)$ and which has the same radius as the circle $x^2 + y^2 + 3x - 7y - 2 = 0$.



14C. अर्धव्यास वृत्त $x^2 + y^2 - 2x - 6y + 1 = 0$ को अर्धव्याससँग बराबर हुने र $(2, 3)$ केन्द्र भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle whose centre is $(2, 3)$ and which has the same radius as the circle $x^2 + y^2 - 2x - 6y + 1 = 0$.

Model 11 ***Problems based on circle passing through centre of given circle.***

15A. दिइएको चित्रमा केन्द्रबिन्दु X भएको वृत्त A वृत्त B को केन्द्रबिन्दु Y भएर गएको छ । यदि वृत्त B को समीकरण $x^2 + y^2 - 4x + 6y - 12 = 0$ र X को निर्देशाङ्कहरू $(-4, 5)$ छन् भने वृत्त A को समीकरण पत्ता लगाउनुहोस् ।
In the figure, the circle A with centre X passes through the centre Y of the circle B. If the equation of circle B is $x^2 + y^2 - 4x + 6y - 12 = 0$ and the co-ordinates of X are $(-4, 5)$, then find the equation of the circle A. **[SEE 2075 R]**



15B. वृत्त $x^2 + y^2 + 4x + 6y - 12 = 0$ को केन्द्र भएर जाने र केन्द्रबिन्दु $(4, 5)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle whose centre is $(4, 5)$ and it passes through the centre of circle $x^2 + y^2 + 4x + 6y - 12 = 0$

15C. केन्द्र $(-1, 2)$ भएको र समीकरण $x^2 + y^2 - 6x - 10y - 2 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre $(-1, 2)$ and passing through the centre of the circle having equation $x^2 + y^2 - 6x - 10y - 2 = 0$. **[SEE 2075 R_2]**

15D. वृत्त $x^2 + y^2 - 10x + 4y + 13 = 0$ को केन्द्र भएर जाने र केन्द्रबिन्दु $(2, 2)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle whose centre is $(2, 2)$ and it passes through the centre of circle $x^2 + y^2 - 10x + 4y + 13 = 0$.

15E. केन्द्रबिन्दु $(2, -1)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस्, जुन समीकरण $x^2 + y^2 + 4x - 2y - 4 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जान्छ ।
Find the equation of the circle with centre $(2, -1)$ which passes through the centre of the circle $x^2 + y^2 + 4x - 2y - 4 = 0$.

15F. केन्द्रबिन्दु $(2, 3)$ भएको र समीकरण $x^2 + y^2 - 6x + 2y + 6 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre $(2, 3)$ and passing through the centre of the circle $x^2 + y^2 - 6x + 2y + 6 = 0$.

15G. केन्द्रबिन्दु $(3, 2)$ भएको र समीकरण $x^2 + y^2 - 2x + 4y - 4 = 0$ भएको वृत्तको केन्द्रबिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre $(3, 2)$ and passing through the centre of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$. **[2071 R']**

ANSWERS

A. VERY SHORT QUESTIONS

(2A - 5C) ; Show to your teacher. **6A.** PA + QA = PQ **6B.** PA = PQ + AQ **7A.** $x^2 + y^2 = 9$
7B. 5 units **7C.** 4 Units **7D.** $(4, -4)$ **8A.** x-axis, $k = r$
8B. y-axis, $h = r$ **8C.** both, $r = h = k$ **9A.** (2, 3), 5 units **9B.** (4, 0), 3 units
9C. (0, 5), 6 units **9D.** $(-a, -b)$, c units

B. SHORT QUESTIONS

1A. (0, 10) **1B.** (–1, 0) **1C.** (2, 0) **1D.** (3, –1)
1E. (– 2, 0) **2A.** 3 units **2B.** 6 **3A.** (– 5, 0); 8 units
3B. (0, 1), 5 units **3C.** (0, – 2), 7 units **3D.** (5, 2),6 units **4A.** (–5, 0), 22 units
4B. 44 units **5A.** a = 4, b = 6 **5B.** – 4 **6A.** 3
6B. 9 **6C.** 4 **7A.** (2, 3) **7B.** (4, 5)
8A. $x^2 + y^2 = 9$ **8B.** $x^2 + y^2 - 6x = 16$ **8C.** $x^2 + y^2 - 4y - 5 = 0$ **8D.** $x^2 + y^2 - 4x + 2y = 4$
8E. $x^2 + y^2 + 8x - 2y + 1 = 0$ **8F.** $x^2 + y^2 - 2x - 10y + 10 = 0$ **8G.** $x^2 + y^2 - 2x + 4y = 0$ **8H.** $x^2 + y^2 - 4x - 6y - 12 = 0$
9A. $x^2 + y^2 = 13$ **9B.** $x^2 + y^2 - 4x - 6y = 12$ **9C.** $x^2 + y^2 - 4x - 6y = 12$ **9D.** $x^2 + y^2 - 6x - 8y = 0$
9E. $x^2 + y^2 - 4x - 6y - 5 = 0$ **10A.** 4 units **10B.** 3 units **10C.** $x^2 + y^2 - 6x - 8y + 9 = 0$
10D. $x^2 + y^2 - 6x - 12y + 9 = 0$ **10E.** $x^2 + y^2 - 10x - 8y + 16 = 0$ **11A.** $x^2 + y^2 - 8x - 6y = 0$ **11B.** $x^2 + y^2 - 3x - 4y = 0$
11C. $x^2 + y^2 = 25$ **11D.** $x^2 + y^2 - 3x + 2y - 13 = 0$ **11E.** $x^2 + y^2 - 8x - 10y + 39 = 0$ **11F.** $x^2 + y^2 - 5x + 3y - 22 = 0$
11G. $x^2 + y^2 - 8x - 10y + 33 = 0$ **11H.** $x^2 + y^2 - 2x - 8y + 9 = 0$ **11I.** $x^2 + y^2 - 2x - 2y - 18 = 0$ **11J.** $x^2 + y^2 - 4x - 8y + 15 = 0$
11K. $x^2 + y^2 - 6x - 4y - 7 = 0$ **11L.** $x^2 + y^2 - 10x - 6y + 29 = 0$ **11M.** $x^2 + y^2 - 6x - 3y + 8 = 0$ **11N.** $x^2 + y^2 + x + 2y - 8 = 0$

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS

1A. (1, 3), 3 units **1B.** (5, 7), 5 units **1C.** (5, – 2), 4 units **1D.** (– 3, 2) ; 8 units

1E. (1, –3), 3 units **1 F.** $\left(\frac{1}{2}, -\frac{3}{2}\right)$, 5 units **1G.** (2, 3), $\frac{5}{\sqrt{2}}$ units **1H.** (5, 7) ; 5 units

1I. (1, – 3), $\sqrt{2}$ units **1J.** $\left(2, \frac{-1}{3}\right)$, 4 units **2A.** $\left(3, \frac{5}{2}\right)$, 8 units **2B.** (1, 5), 8 units

3A. (– 4, 4) **3B.** (5, –3) **3C.** (0, 2) **3D.** (0, – 2)

3E. (3, 4) **4A.** $x^2 + y^2 - 8x - 12y + 27 = 0$ **4B.** $x^2 + y^2 - 4x - 6y = 12$ **4C.** $x^2 + y^2 - 4x - 6y = 12$

5A. $x^2 + y^2 - 2x - 4y + 4 = 0$ **5B.** $x^2 + y^2 - 6x - 8y = 0$ **6A.** $x^2 + y^2 - x + 3y - 10 = 0$ **6B.** $x^2 + y^2 - 2x + 6y - 40 = 0$

6C. $x^2 + y^2 - 12x - 16y + 75 = 0$ **6D.** $x^2 + y^2 - 2x - 4y = 20$ **6E.** $x^2 + y^2 - 6x - 4y + 5 = 0$ **6F.** $x^2 + y^2 - 4x - 2y = 5$

6G. $x^2 + y^2 - 16x - 10y + 64 = 0$ **6H.** $x^2 + y^2 - 8x - 6y = 0$ **7A.** $x^2 + y^2 - 10x + 2y + 1 = 0$ **7B.** $x^2 + y^2 - 6x - 8y + 21 = 0$

7C. $x^2 + y^2 + 6x + 2y - 90 = 0$ **7D.** $x^2 + y^2 - 6x - 8y + 15 = 0$ **7E.** $x^2 + y^2 - 2x - 2y - 3 = 0$ **7F.** $x^2 + y^2 - 2x - 2y - 8 = 0$

7G. $x^2 + y^2 - 8x + 6y = 0$ **8A.** $x^2 + y^2 - 6x - 4y + 9 = 0$ **8B.** $x^2 + y^2 - 2x - 8y - 8 = 0$ **9A.** $x^2 + y^2 - 4x - 6y - 12 = 0$

9B. $x^2 + y^2 - 4x - 6y + 11 = 0$ **9C.** $x^2 + y^2 - 4x - 6y - 12 = 0$ **10A.** 5 units **10B.** 10 units

10C. (– 3, 1) **12A.** $\sqrt{2}$ units **12B.** $x^2 + y^2 = 4$, (2, 0), (0, 2) **13A.** $x^2 + y^2 - 6x + y + 6 = 0$

13B. $x^2 + y^2 + 2x - y = 30$ **13C.** $x^2 + y^2 + 6x - 8y - 25 = 0$ **13D.** $x^2 + y^2 - 8x + 12y = 49$ **14A.** $x^2 + y^2 - 4x - 6y + 4 = 0$

14B. $2x^2 + 2y^2 + 4x + 8y - 23 = 0$ **14C.** $x^2 + y^2 - 4x - 6y + 4 = 0$ **15A.** $x^2 + y^2 + 8x - 10y - 59 = 0$

15B. $x^2 + y^2 - 8x - 10y - 59 = 0$ **15C.** $x^2 + y^2 + 2x - 4y - 20 = 0$ **15D.** $x^2 + y^2 - 4x - 4y = 17$

15E. $x^2 + y^2 - 4x + 2y - 15 = 0$ **15F.** $x^2 + y^2 - 4x - 6y - 4 = 0$ **15G.** $x^2 + y^2 - 6x - 4y = 7$

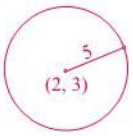
GENERAL EXERCISE 4

A. SHORT QUESTIONS

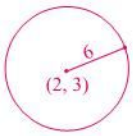
Model 1 *Problems based on equation of circle from the given figures.*

1. दिइएको वृत्तहरूको समीकरण पत्ता लगाउनुहोस् (Find the equation of given circles):

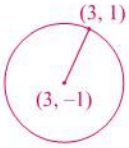
A.



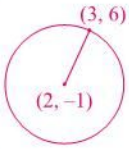
B.



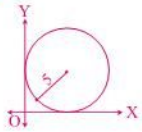
C.



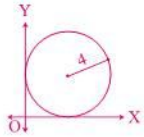
D.



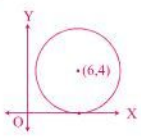
E.



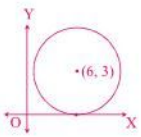
F.



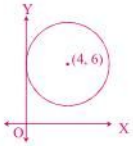
G.



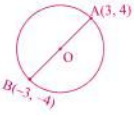
H.



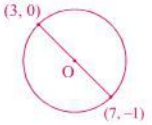
I.



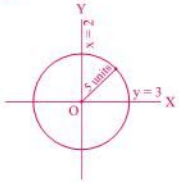
J.



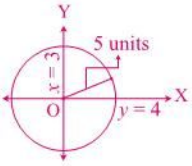
K.



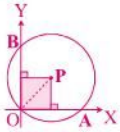
L.



M.

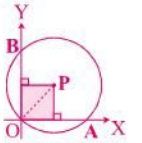


N.



OB = 8 units
OA = 6 units

O.



OB = 30 units
OA = 16 units

Model 2 *Problems based on finding equation of circle when centre and a passing point are given.*

2A. केन्द्र (2, 3) र (6, 6) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre (2, 3) and passing through (6, 6).

2B. केन्द्र $(-p, -q)$ र अर्धव्यास $(p + q)$ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with centre $(-p, -q)$ and having radius $(p + q)$.

Model 3 *Problems based on finding equation of circle when it touches X-axis.*

3A. केन्द्रबिन्दु (– 2, 3) र X- अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with center (– 2, 3) and touching the X-axis.

3B. केन्द्रबिन्दु (5, 4) र X- अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with center (5, 4) and touching the X-axis.

3C. केन्द्रबिन्दु (5, 4) भई X-अक्षको बिन्दु (5, 0) मा छोएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Obtain the equation of the circle having centre at (5, 4) and that touches the x-axis at (5, 0).

www.drmathnp.com

puspas.com.np

3D. अर्धव्यास 6 एकाइ भएको वृत्तको केन्द्र पहिलो चतुर्थांशमा पर्दछ । यदि सो वृत्तले X-अक्षलाई (5, 0) मा छुन्छ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle having radius 6 units lies in the first quadrant. If the circle touches the X-axis at (5, 0), find the equation of the circle.

3E. अर्धव्यास 3 एकाइ भएको वृत्तको केन्द्र तेस्रो चतुर्थांशमा पर्दछ । यदि सो वृत्तले X-अक्षलाई (– 4, 0) मा छुन्छ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle having radius 3 units lies in the third quadrant. If the circle touches the X-axis at (– 4, 0), find the equation of the circle.

Model 4 *Problems based on finding equation of circle when it touches Y-axis.*

4A. Y-अक्षलाई छुने र केन्द्र (4, – 3) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle with centre (4, – 3) and touching on Y-axis.

4B. Y-अक्षलाई छुने र केन्द्र (5, 6) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle with centre (5, 6) and touching on Y-axis.

4C. अर्धव्यास 4 एकाइ भएको वृत्तको केन्द्र दोस्रो चतुर्थांशमा पर्दछ । यदि सो वृत्तले Y-अक्षलाई (0, 3) मा छुन्छ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle having radius 4 units lies in the second quadrant. If the circle touches the y-axis at (0, 3), find the equation of the circle.

4D. अर्धव्यास 5 एकाइ भएको वृत्तको केन्द्र तेस्रो चतुर्थांशमा पर्दछ । यदि सो वृत्तले Y-अक्षलाई (0, – 4) मा छुन्छ भने उक्त वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The centre of a circle having radius 5 units lies in the third quadrant. If the circle touches the y-axis at (0, – 4), find the equation of the circle.

Model 5 *On finding equation of circle when it touches both the axes.*

5A. दुवै धनात्मक अक्षलाई छुने र अर्धव्यास 6 भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having radius 6 and touching both the axes.

5B. धनात्मक X-अक्ष र Y-अक्षलाई छुने र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which touches the positive X and Y-axes and whose radius is 5 units.

5C. दुवै अक्षलाई छुने, पहिलो चतुर्थांशमा पर्ने र अर्धव्यास 3 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching both the axes, lies in first quadrant and having radius 3 units.

5D. दुवै अक्षलाई छुने, दोस्रो चतुर्थांशमा पर्ने र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching both the axes, lies in second quadrant and having radius 5 units.

5E. दुवै अक्षलाई छुने, तेस्रो चतुर्थांशमा पर्ने र अर्धव्यास 4 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching both the axes, lies in third quadrant and having radius 4 units.

5F. दुवै अक्षलाई छुने, चौथो चतुर्थांशमा पर्ने र अर्धव्यास 2 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching both the axes, lies in fourth quadrant and having radius 2 units.

5G. अर्धव्यास 4 एकाइ भएर ऋणात्मक x-अक्ष र धनात्मक y-अक्षलाई छोएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with the radius 4 units and touching negative x-axis and positive y-axis.

5H. दुवै अक्षलाई छुने र केन्द्रबिन्दु सरल रेखा $x - 2y = 3$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which touches both the axes and has its centre on the line $x - 2y = 3$.

5I. केन्द्रबिन्दु $2x + y = 6$ मा पर्ने र दुवै धनात्मक अक्षहरूलाई छोएर जाने वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of a circle whose centre lies on the line $2x + y = 6$ and touches both the positive axes.

Model 6 *Problems based on finding centre, radius or diameter or circumference of circle.*

6 वृतहरूको केन्द्र र अर्धव्यास पत्ता लगाउनुहोस (Find the centre and radius of the circle):

A. $x^2 + y^2 = 2x + 3$ **B.** $(x + 3)^2 + y^2 = 4$

7A. एउटा वृत्तको समीकरण $x^2 + y^2 - 6x - 4y + 3 = 0$ भए वृत्तको व्यास र केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
The equation of a circle is $x^2 + y^2 - 6x - 4y + 3 = 0$, find the diameter and the centre .

7B. समीकरण $x^2 + y^2 + 2gx + 2fy + c = 0$ भएको वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle having equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

7C. वृत्त $(x - 2)^2 + (y - 3)^2 = 49$ को केन्द्रबिन्दु र परिधि पत्ता लगाउनुहोस् ।
Find the centre and circumference of circle $(x - 2)^2 + (y - 3)^2 = 49$.

www.drmathnp.com

puspas.com.np

7D. एउटा वृत्तको समीकरण $(2x-4)^2+(2y+6)^2=100$ भए यसको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Equation of a circle is $(2x-4)^2+(2y+6)^2=100$. Find its centre and radius.

7E. दिइएको वृत्तको समीकरण $(2x-1)^2+(2y+3)^2=4$ बाट उक्त वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of the circle having given equation $(2x-1)^2+(2y+3)^2=4$.

7F. वृत्त $4x-6y-x^2-y^2+8=0$ को परिधिको लम्बाइ निकाल्नुहोस् ।
Calcualte the length of the circumference of the circle $4x-6y-x^2-y^2+8=0$.

Model 7 Problems based on finding point of intersection of circle or a line.

8A. समीकरण $x^2+y^2-5x-7y+6=0$ भएको वृत्तले Y-अक्षलाई कुन-कुन बिन्दुमा काट्छ, पत्ता लगाउनुहोस् ।
Find the points on Y-axis where the circle having equation $x^2+y^2-5x-7y+6=0$ intersect.

8B. समीकरण $x^2+y^2-8x-8y-9=0$ भएको वृत्तले X-अक्षलाई कुन-कुन बिन्दुमा काट्छ, पत्ता लगाउनुहोस् ।
Find the points on X-axis where the circle with equation $x^2+y^2-8x-8y-9=0$ intersect.

8C. समीकरण $2x^2+2y^2-5x+7y-36=0$ भएको वृत्तले सीधा रेखा $y=3$ लाई कुन-कुन बिन्दुमा काट्छ, पत्ता लगाउनुहोस् ।
Find the points on the line $y=3$, where the circle with equation $2x^2+2y^2-5x+7y-36=0$ intersect.

Model 8 Problems based on finding length of chord or distance of the chord from centre.

9A. समीकरण $(x+2)^2+y^2=169$ भएको वृत्तको केन्द्रबाट 5 एकाइ टाढा पर्ने जीवा कति लामो होला ?
How long is the chord of a circle with equation $(x+2)^2+y^2=169$ which is at a distance of 5 units from its centre?

9B. समीकरण $(x-2)^2+(y-3)^2=25$ भएको वृत्तको केन्द्रबाट 4 एकाइ टाढा पर्ने जीवा कति लामो होला ?
How long is the chord of a circle with equation $(x-2)^2+(y-3)^2=25$ which is at a distance of 4 units from its centre?

9C. समीकरण $x^2+y^2=100$ भएको वृत्तको एउटा जीवाको लम्बाइ 16 एकाइ छ भने केन्द्रबाट जीवा कति टाढा होला ?
If the length of a chord of a circle with equation $x^2+y^2=100$ is 16 units, how far is the chord from the centre ?

10A. समीकरण $x^2+y^2=25$ भएको वृत्तको एउटा जीवाको लम्बाइ 8 एकाइ छ भने केन्द्रबाट जीवा कति टाढा होला ?
If the length of a chord of a circle with equation $x^2+y^2=25$ is 8 units, how far is the chord from the centre ?

10B. समीकरण $(x-2)^2+(y-3)^2=25$ भएको वृत्तको एउटा जीवाको लम्बाइ 6 एकाइ छ भने केन्द्रबाट जीवा कति टाढा होला ?
If the length of a chord of a circle with equation $(x-2)^2+(y-3)^2=25$ is 6 units, how far is the chord from the centre ?

Model 9 On finding value of unknown letters & centre when passing point and equation of circle are given.

11A. यदि वृत्त $x^2+y^2-6x-by+9=0$ बिन्दु $(3, 4)$ भएर जान्छ भने वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
If the circle $x^2+y^2-6x-by+9=0$ passes through the point $(3, 4)$, find the center of the circle.

11B. यदि वृत्त $x^2+y^2+ax+2y-4=0$ बिन्दु $(2, 2)$ भएर जान्छ भने वृत्तको केन्द्रबिन्दु पत्ता लगाउनुहोस् ।
If circle $x^2+y^2+ax+2y-4=0$ passes through the point $(2, 2)$, find the centre of the circle.

11C. यदि वृत्त $4x^2+4y^2+px-20y-3=0$ को व्यास 8 एकाइ भए को मान पत्ता लगाउनुहोस् ।
If the diameter of the circle $4x^2+4y^2+px-20y-3=0$ is 8 units, find the value of.

11D. यदि वृत्त $x^2+y^2-x+4y+7=0$ को व्यास 2 एकाइ भए p को मान पत्ता लगाउनुहोस् ।
If the diameter of the circle $x^2+y^2-x+4y+7=0$ is 2 units, find the value of.

11E. समीकरण $x^2+6y+y^2-Rx-12=0$ भएको वृत्तको केन्द्रबिन्दु $(-2, -3)$ भए R को मान पत्ता लगाउनुहोस् ।
Find the value of R when the centre of a circle with equation $x^2+6y+y^2-Rx-12=0$ is $(-2, -3)$.

Model 10 Problems based on equation of circle and tangent.

12A. केन्द्रबिन्दु $(2, -1)$ भएको र सरल रेखा $3x-4y+1=0$ लाई छोएर जाने वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle whose centre is $(2, -1)$ and which just touches the line $3x-4y+1=0$.

12B. केन्द्रबिन्दु $(4, 3)$ भएको र सरलरेखा $6x-8y+20=0$ मा स्पर्श हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle whose center is $(4, 3)$ and which touches the straight line $6x-8y+20=0$.

www.drmathnp.com

puspas.com.np

B. LONG QUESTIONS

Model 1 *Problems based on finding centre and radius when equation of circle is given.*

1. तलका वृत्तहरूको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् (Find the centre and radius of the following circles):

A. $5x^2 + 5y^2 - 50x + 20y + 65 = 0$ **B.** $2x^2 + 2y^2 + 8x - 8y - 2 = 0$

C. $2x^2 + 2y^2 + 6x - 2y - 13 = 0$ **D.** $2x^2 + 2y^2 + 4x - 2y + 1 = 0$

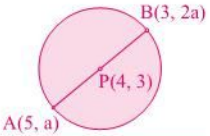
Model 2 *On finding equation of circle when end points of a diameter are given.*

2. वृत्तको समीकरण, अर्धव्यास र केन्द्र पत्ता लगाउनुहोस् जसको व्यासको छेउहरू निम्न छन् :
Find the equation, radius and centre of circle whose ends of diameter are:

A. (0, 1) and (1, 1) **B.** (1, 2) and (3, 4)

3A. एउटा वृत्त बिन्दुहरू P(3, 6), Q(10, 7) र R(7, 3) भएर जान्छ । $\angle PRQ = 90^\circ$ छ भनी प्रमाणित गरी यस वृत्तको समीकरण निकाल्नुहोस् ।
A circle passes through the points P(3, 6), Q(10, 7) and R(7, 3). Prove that $\angle PRQ = 90^\circ$ and hence find equation of the circle.

3B. दिइएको चित्रमा P(4, 3) वृत्तको केन्द्रबिन्दु र AB व्यास भए सो वृत्तको समीकरण निकाल्नुहोस् ।
In the given diagram P(4, 3) is the centre of the circle and AB is a diameter. Find the equation of the circle.



Model 3 *Problems based on finding one end of diameter or length of diameter.*

4A. समीकरण $x^2 + y^2 - 4x - 6y - 12 = 0$ भएको वृत्तको एउटा व्यासको एक छेउ $(2, -2)$ भए अर्को छेउको निर्देशाङ्कहरू र सो व्यासको लम्बाइ पत्ता लगाउनुहोस् ।
If one end of a diameter of a circle $x^2 + y^2 - 4x - 6y - 12 = 0$ is $(2, -2)$, find the coordinates of the other end and the length of the diameter.

4B. यदि वृत्त $x^2 + y^2 - 10x - 12y + 51 = 0$ को व्यासको एक छेउ (8, 7) भए अर्को छेउको निर्देशाङ्क पत्ता लगाउनुहोस् ।
If one end of a diameter of the circle $x^2 + y^2 - 10x - 12y + 51 = 0$ is (8, 7), find the coordinates of the other end.

Model 4 *On finding equation of circle when radius, passing point and position of centre are given.*

5A. अर्धव्यास 5 एकाइ तथा केन्द्र X-अक्षमा पर्ने र बिन्दु (2, 3) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle with radius 5 units and the centre lies on the X-axis and passing through the point (2, 3).

5B. अर्धव्यास 5 एकाइ तथा केन्द्र Y-अक्षमा पर्ने र बिन्दु (4, 3) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle with radius 5 units and the centre lies on the Y-axis and passing through the point (4, 3).

Model 5 *On finding equation of circle when centre & passing two points are given.*

6A. केन्द्रबिन्दु $(6, a)$ तथा Y-अक्षको (0, – 2) र (0, 14) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having the centre $(6, a)$ and passing through the points (0, – 2) and (0, 14) of Y-axis.

6B. केन्द्रबिन्दु (– 8, P) तथा Y-अक्षको (0, –12) र (0, 18) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having centre (– 8, P) and passing through the points (0, – 12) and (0, 18) of Y-axis.

6C. केन्द्रबिन्दु $(a + 1, 4)$ तथा X-अक्षको (2, 0) र (8, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle having the centre at $(a + 1, 4)$ and passing through the points (2, 0) and (8, 0) of X-axis.

6D. केन्द्रबिन्दु (R, – 3) तथा X-अक्षको (– 2, 0) र (6, 0) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle having the centre at (R, – 3) and passing through the points (– 2, 0) and (6, 0) of X-axis.

Model 6 *Problems based on finding equation of concentric circle.*

7A. बिन्दु (4, – 2) भएर जाने र केन्द्र वृत्त $x^2 + y^2 - 6x + 4y - 1 = 0$ को केन्द्रसँग बराबर हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
What is the equation of the circle through (4, – 2) and having the same centre as the circle $x^2 + y^2 - 6x + 4y - 1 = 0$?

7B. वृत्त $x^2 + y^2 - 4x + 6y + 4 = 0$ सँग एकै केन्द्रित हुने र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle concentric with the circle $x^2 + y^2 - 4x + 6y + 4 = 0$ and whose radius is 5 units.

7C. वृत्त $x^2 + y^2 - 2x - 6y + 1 = 0$ सँग एकै केन्द्रित हुने र बिन्दु (4, – 1) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle concentric with the circle $x^2 + y^2 - 2x - 6y + 1 = 0$ and passing through the point (4, – 1).

www.drmathnp.com

puspas.com.np

7D. वृत्त $25x^2 + 25y^2 - 200x - 250y + 989 = 0$ सँग एकै केन्द्रित भई व्यासार्ध दिइएको वृत्तको आधा भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle concentric with circle $25x^2 + 25y^2 - 200x - 250y + 989 = 0$ and having radius half of the radius of given circle.

7E. केन्द्रबिन्दु $(1, -2)$ भएको र अर्धव्यास वृत्त $x^2+y^2-4x+2y+1=0$ को व्याससँग बराबर भएको वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of a circle having centre at $(1, -2)$ and radius equal to the diameter of the circle $x^2 + y^2 - 4x + 2y + 1 = 0$.

7F. $x^2 + y^2 - 4x - 6y - 3 = 0$ सँग एकै केन्द्रित हुने र y-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle concentric with $x^2 + y^2 - 4x - 6y - 3 = 0$ and which touches the y-axis.

Model 7 Problems based on finding equation of circle when radius & two passing points are given.

8A. अर्धव्यास 10 एकाइ र बिन्दुहरू $(3, -2)$ र $(-1, 6)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the points $(3, -2)$ and $(-1, 6)$ and its radius is 10 units.

8B. अर्धव्यास 1 एकाइ र बिन्दुहरू $(1, 1)$ र $(2, 2)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the points $(1, 1)$ and $(2, 2)$ and its radius is 1 unit.

Model 8 On finding equation of circle when equation of diameters are given.

9A. व्यासहरू $2x + y = 5$ र $x - y = 1$ हुने र Y-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching the Y-axis and whose diameters are $2x + y = 5$ and $x - y = 1$.

9B. व्यासहरू $x + y = 2$ र $2x + y = -2$ हुने र X-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle touching the X-axis and whose diameters are $x + y = 2$ and $2x + y = -2$.

9C. व्यासहरू $x + y = 3$ र $2x + y = 5$ हुने र रेखाहरू $x = 1$ र $y = 4$ को प्रतिच्छेदन बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through the point of intersection of lines $x = 1$ and $y = 4$ and whose diameters are $x + y = 3$ and $2x + y = 5$.

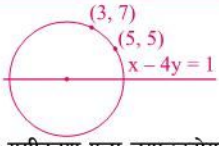
9D. बिन्दु $(4, 1)$ भएर जाने र केन्द्रबिन्दु रेखाहरू $x + 2y - 1 = 0$ र $2x - y - 7 = 0$ को प्रतिच्छेदन बिन्दु हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle whose centre is the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y - 7 = 0$ and passes through a point $(4, 1)$.

9E. दुईओटा व्यासहरूको समीकरण $x - y = -2$ र $5x + y = 38$ भएको र बिन्दु $(3, 4)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The equations of two diameters of a circle are $x - y = -2$ and $5x + y = 38$ and the circle passes through $(3, 4)$. Find the equation of the circle.

9F. एउटा वृत्तको व्यास 10 एकाइ छ । यदि यसका दुईओटा व्यासहरूको समीकरणहरू $x - y = 1$ र $x + 2y = 13$ भए, वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The diameter of a circle is 10 units. If the equations of its two diameters are $x - y = 1$ and $x + 2y = 13$, find the equation of the circle.

Model 9 On finding equation of circle when line through centre & two passing points are given.

10A. केन्द्रबिन्दु रेखा $x - 4y = 1$ मा रहने र बिन्दुहरू $(3, 7)$ र $(5, 5)$ भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the points $(3, 7)$ and $(5, 5)$ and having its centre on the line $x - 4y = 1$.



10B. बिन्दुहरू A(5, 4) र B(3, 2) भएर जाने वृत्तको केन्द्र रेखा $3x - 2y = 1$ मा पर्दछ भने, वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle passing through the points (5, 4) and (3, 2) whose centre lies on the line $3x - 2y = 1$.

10C. केन्द्र चौथो चतुर्थांशमा र रेखा $2x + y - 1 = 0$ मा पर्ने र बिन्दु $(4, 3)$ भएर जाने वृत्तको क्षेत्रफल 25π वर्ग एकाइ भए सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Derive the equation of the circle whose centre lies in the fourth quadrant and on the line $2x + y - 1 = 0$, area 25π sq. units and the circle passes through $(4, 3)$.

Model 10 Problems based on finding equation of diameter or radius.

11A. उद्गमबिन्दु भएर जाने वृत्त $x^2 + y^2 - 4x - 6y - 12 = 0$ को व्यासको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the diameter of circle $x^2 + y^2 - 4x - 6y - 12 = 0$ passing through origin.

11B. बिन्दु $(4, 5)$ भएर जाने वृत्त $x^2 - 4x - 6y + y^2 = 23$ को अर्धव्यासको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the radius of circle $x^2 - 4x - 6y + y^2 = 23$ passing through $(4, 5)$.

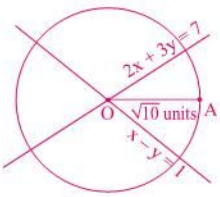
www.drmathnp.com

puspas.com.np

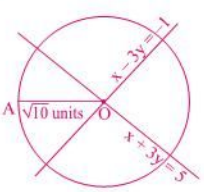
Model 11 *Problems based on finding equation of circle from the given figure.*

12. तलका प्रत्येक चित्रहरूको सहायताबाट वृत्तको समीकरण पत्ता लगाउनुहोस् ।
With the help of each of the following figures, find the equation of circle.

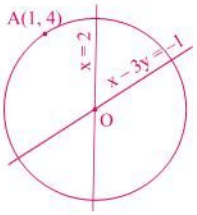
A.



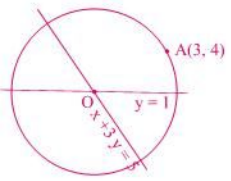
B.



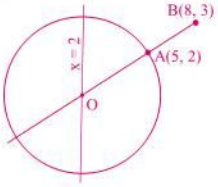
C.



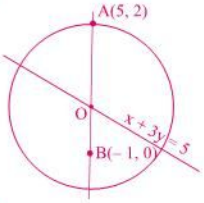
D.



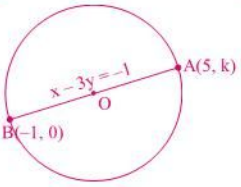
E.



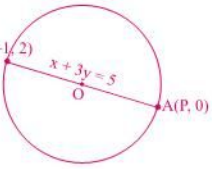
F.



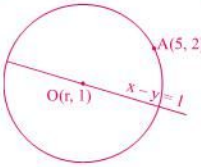
G.



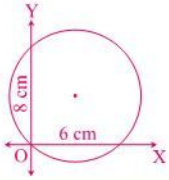
H.



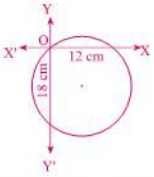
I.



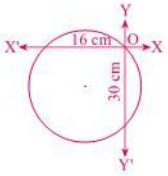
J.



K.



L.



Model 12 *Problems based on finding value of letters and radius or centre from the given equation of the circle.*

13A. समीकरण $x^2 + y^2 - 10x + py + 13 = 0$ भएको वृत्तको केन्द्रबिन्दु $(5, -2)$ भए p को मान पत्ता लगाई अर्धव्यास पनि पत्ता लगाउनुहोस् ।
The centre of a circle having equation $x^2 + y^2 - 10x + py + 13 = 0$ is $(5, -2)$. Find the value of p and also the radius.

13B. समीकरण $x^2 + y^2 - 4x - ky + 11 = 0$ भएको वृत्तको केन्द्रबिन्दु $(2, 3)$ भए k को मान पत्ता लगाई वृत्तको अर्धव्यास पनि पत्ता लगाउनुहोस् ।
The centre of a circle having equation $x^2 + y^2 - 4x - ky + 11 = 0$ is $(2, 3)$. Find the value of k and also the radius of the circle.

13C. समीकरण $x^2 + y^2 - ax - by - 12 = 0$ भएको वृत्तको केन्द्रबिन्दु $(2, 3)$ भए a र b को मान पत्ता लगाई वृत्तको अर्धव्यास पनि पत्ता लगाउनुहोस् ।
The centre of a circle having equation $x^2 + y^2 - ax - by - 12 = 0$ is $(2, 3)$. Find the value of a and b and also the radius of the circle.

13D. वृत्त $x^2 + y^2 - px = 0$ को अर्धव्यास 5 एकाइ भए p को मान र केन्द्रबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
The radius of the circle $x^2 + y^2 - px = 0$ is 5 units. Find the value of p and the co-ordinates of the centre.

13E. वृत्त $x^2 + y^2 - 4x - by - 5 = 0$ को अर्धव्यास $\sqrt{10}$ एकाइ भए b को मान र केन्द्रबिन्दुको निर्देशाङ्क पत्ता लगानुहोस् ।
The radius of the circle $x^2 + y^2 - 4x - by - 5 = 0$ is $\sqrt{10}$ units. Find the value of b and the co-ordinates of the centre.

13F. वृत्त $x^2 + y^2 - 2x - 2y = p$ को अर्धव्यास $\sqrt{5}$ एकाइ भए p को मान र केन्द्रबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
The radius of circle $x^2 + y^2 - 2x - 2y = p$ is $\sqrt{5}$ unit. Find the value of p and the co-ordinates of centre.

13G. यदि समीकरण $4x^2 + 4y^2 + px - 20y - 3 = 0$ भएको वृत्तको परिधि 8π एकाइ भए 'p' को मान निकाल्नुहोस् ।
If the circumference of the circle with equation $4x^2 + 4y^2 + px - 20y - 3 = 0$ is 8π units, find the value of 'p'.

14A. समीकरण $x^2 + y^2 - ax + 6y - 12 = 0$ भएको वृत्तको केन्द्रबिन्दुबाट सीधारेखा $3x - 4y + 2 = 0$ सम्मको लम्बदूरी 4 भए a को मान पत्ता लगाउनुहोस् ।
If the perpendicular distance of the line $3x - 4y + 2 = 0$ from the center of the circle $x^2 + y^2 - ax + 6y - 12 = 0$ is 4, find the value of a.

www.drmathnp.com

14B. समीकरण $x^2 + y^2 - ax - 2y - 5 = 0$ भएको वृत्तको केन्द्रबिन्दुबाट सीधारेखा $3x + 4y + 10 = 0$ सम्मको लम्बदूरी 4 भए a को मान पत्ता लगाउनुहोस् ।
If the perpendicular distance of the line $3x + 4y + 10 = 0$ from the center of the circle $x^2 + y^2 - ax - 2y - 5 = 0$ is 4, find the value of a.

Model 13 Problems based on proving touching axis.

15A. वृत्त $x^2 + y^2 - 4x - 4y + 4 = 0$ ले दुवै अक्षहरूलाई छुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ touches both the axes.

15B. वृत्त $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ ले दुवै अक्षहरूलाई छुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ touches both the axes.

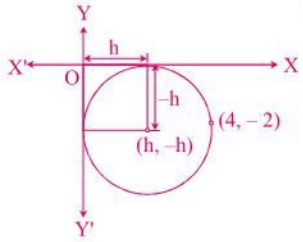
15C. वृत्त $x^2 + y^2 + 8x - 12y + 16 = 0$ ले X-अक्ष र रेखा $y = 12$ लाई छुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the circle $x^2 + y^2 + 8x - 12y + 16 = 0$ touches X-axis and line $y = 12$.

15D. वृत्त $x^2 + y^2 + 8x - 12y + 16 = 0$ ले रेखाहरू $x = 2$ र $x + 10 = 0$ लाई छुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the circle $x^2 + y^2 + 8x - 12y + 16 = 0$ touches the lines $x = 2$ and $x + 10 = 0$.

Model 14 Problems based on finding equation of circle based on touching axis.

16A. बिन्दु $(4, -2)$ भएर जाने र दुवै अक्षहरूलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् जुन वृत्त पूरै चौथो चतुर्थांशमा पर्दछ ।
Find the equation of the circle through the point $(4, -2)$ which touches both the axes such that the whole of the circle lies in the fourth quadrant.

Hints: Here the circle touches both the axes & lies in 4th quadrant.
Centre = $(h, -h)$ and radius = h
Using formula, $(h-4)^2 + (-h+2)^2 = h^2$
or, $h^2 - 8h + 16 + (2-h)^2 = h^2$
or, $h^2 - 8h + 16 + 4 - 4h + h^2 = h^2$
or, $h^2 - 12h + 20 = 0$
or, $h^2 - 10h - 2h + 20 = 0$
or, $h(h-10) - 2(h-10) = 0$
or, $(h-10)(h-2) = 0$
$\therefore \quad h = 10$ or 2



16B. बिन्दु $(4, -8)$ भएर जाने र दुवै अक्षहरूलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् जुन वृत्त पूरै चौथो चतुर्थांशमा पर्दछ ।
Find the equation of the circle through the point $(4, -8)$ which touches both the axes such that the whole of the circle lies in the fourth quadrant.

Model 15 Problems based on intercepts on circle.

17A. एउटा वृत्तको व्यासको दुई छेउका निर्देशाङ्कहरू $(-1, 4)$ र $(3, 0)$ छन् । वृत्तले X-अक्षमा कति लम्बाइको खण्ड बनाउला ?
The extremities of a diameter of a circle have co-ordinates $(-1, 4)$ and $(3, 0)$. What length does the circle intercept on the X-axis ?

17B. एउटा वृत्तको व्यासको दुई छेउका निर्देशाङ्कहरू $(6, -2)$ र $(6, 8)$ छन् । वृत्तले X-अक्षमा कति लम्बाइको खण्ड बनाउला ?
The extremities of a diameter of a circle have co-ordinates $(6, -2)$ and $(6, 8)$. What length does the circle intercept on the X-axis ?

18A. वृत्त $x^2 + y^2 - 2x - 3 = 0$ मा सीधा रेखा $x + y = 3$ ले बनाउने खण्डको लम्बाइ पत्ता लगाउनुहोस् ।
Find the length of intercept made by the straight line $x + y = 3$ with circle $x^2 + y^2 - 2x - 3 = 0$.

18B. यदि रेखा $x + y = 2$ ले एउटा वृत्त $x^2 + y^2 = 4$ लाई 2 ओटा बिन्दुहरूमा काट्दछ भने उक्त बिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The line $x + y = 2$ cuts a circle $x^2 + y^2 = 4$ at two points. Find the co-ordinates of the points.

19A. उद्गम बिन्दुबाट जाने र X र Y अक्षमा बनाउने खण्डको लम्बाइ क्रमशः 6 र 8 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the origin and makes intercepts of length 6 and 8 units on X and Y axis respectively.

19B. उद्गम बिन्दुबाट जाने र X र Y अक्षमा बनाउने खण्डको लम्बाइ क्रमशः 10 र 24 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which passes through the origin and makes intercepts of length 10 and 24 units on X and Y axis respectively.

19C. उद्गम बिन्दु भएर जाने र X-अक्ष र Y-अक्षको धनात्मक दिशामा क्रमशः 4 एकाइ र 6 एकाइ लामा जीवाहरू बनाउने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the origin and cuts off chords of lengths 4 and 6 units on the positive side of the X-axis and Y-axis respectively.

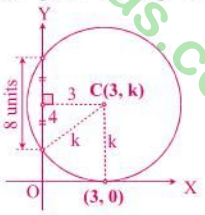
puspas.com.np

www.drmathnp.com

puspas.com.np

19D. उद्गमबिन्दुबाट 3 एकाइ टाढा X-अक्षलाई छुने र Y-खण्ड 8 हुने गरी Y-अक्षलाई काट्ने वृत्तको केन्द्रबिन्दु पहिलो चतुर्थांशमा पर्दछ भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle whose centre lies in first quadrant, touches the axis of X at a distance 3 units from the origin and cuts off an intercept 8 from the axes of Y.



Hint : Radius of the circle is k.

or, $\sqrt{3^2+4^2} = k$ $\quad\therefore\quad r = 5$ units

From figure, Centre = (3, 5) = (h, k)

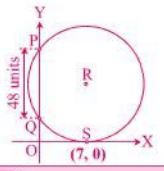
$\therefore$ Equation of the circle is; $(x-h)^2 + (y-k)^2 = r^2$

19E. उद्गमबिन्दुबाट 4 एकाइ टाढा Y-अक्षलाई छुने र X-खण्ड 6 हुने गरी काट्ने वृत्तको केन्द्रबिन्दु पहिलो चतुर्थांशमा पर्दछ भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the circle whose centre lies in first quadrant, touches the axis of y at a distance 4 units from the origin and cuts intercept 6 from the axis of x.

19F. सँगैको चित्रमा वृत्तको केन्द्रबिन्दु R छ । वृत्तले x-अक्षको S(7, 0) मा छोएको छ र y-अक्षमा PQ = 48 एकाइको खण्ड बनाएको छ । यो जानकारीबाट वृत्तको समीकरण पत्ता लगाउनुहोस् ।

In the figure alongside, R is the centre of the circle. The circle touches x-axis at S(7, 0) and intercepts y-axis with PQ = 48 units. From this information, find the equation of the circle.



COORDINATES

Model 16 On finding centre, radius & equation of circle when three passing points are given.

20. तल दिइएका बिन्दुहरू भएर जाने वृत्तको केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस्:

Find the centre and radius of the circle passing through the following given points:

A. (2, – 2), (6, 6), (7, 3) **B.** (3, 3), (5, 5), (3, 7)

C. (3, 1), (1, – 1), (3, – 3) **D.** (1, 2), (3, 4), (5, 2)

21. दिइएका बिन्दुहरू भएर जाने वृत्तको केन्द्र, अर्धव्यास र समीकरण पत्ता लगाउनुहोस्:

Find the centre, radius and equation of a circle passing through given points:

A. (0, 0) , (2, 0) and (0, 4) **B.** (1, 2) , (2, 1) and (3, 2)

C. (2, – 1), (6, 1) and (6, 3) **D.** (2, – 2), (6, 6) and (5, 7)

Model 17 Problems based on finding equation of image circle.

22A. वृत्त $x^2 + y^2 + 8x - 16y + 64 = 0$ लाई ऐना रेखा $x = 0$ मा हुने प्रतिबिम्बको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the image of the circle $x^2 + y^2 + 8x - 16y + 64 = 0$ in the mirror line $x = 0$.

22B. वृत्त $x^2 + y^2 + 8x - 16y + 64 = 0$ लाई ऐना रेखा $y = 0$ मा हुने प्रतिबिम्बको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the image of the circle $x^2 + y^2 + 8x - 16y + 64 = 0$ in the mirror line $y = 0$.

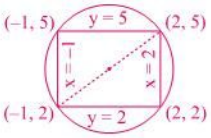
22C. वृत्त $x^2 + y^2 - 4x - 6y - 12 = 0$ लाई ऐना रेखा $x = y$ मा हुने प्रतिबिम्बको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of the image of the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ in the mirror line $x = y$.

Model 18 Problems based on finding equation of circle when cyclic rectangle or square is given.

23A. एउटा आयतका भुजाहरूका समीकरणहरू $x = 2$, $x = -1$, $y = 5$ र $y = 2$ छन् । उक्त आयतमा भएको परिगत वृत्तको समीकरण पत्ता लगाउनुहोस् ।

The sides of a rectangle are $x = 2$, $x = -1$, $y = 5$ and $y = 2$. Find the equation of the circle circumscribing the rectangle.



Hint: From the figure the vertices of a rectangle are; (– 1, 2), (– 1, 5), (2, 5) and (2, 2). So, the end points of a diagonal are (– 1, 2) and (2, 5) i.e. the end points of a diameter.

23B. एउटा आयतका भुजाहरूका समीकरणहरू $x = 1$, $x = 5$, $y = 1$ र $y = 3$ छन् । उक्त आयतमा भएको परिगत वृत्तको समीकरण पत्ता लगाउनुहोस् ।

The sides of a rectangle are $x = 1$, $x = 5$, $y = 1$ and $y = 3$. Find the equation of the circle circumscribing the rectangle.

23C. एउटा वर्गका भुजाहरूका समीकरणहरू $x = 1$, $x = 3$, $y = 1$ र $y = 3$ छन् । उक्त वर्गमा भएको परिगत वृत्तको समीकरण पत्ता लगाउनुहोस् ।

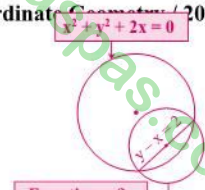
The sides of a square are $x = 1$, $x = 3$, $y = 1$ and $y = 3$. Find the equation of the circle circumscribing the square.

23D. एउटा आयतका शीर्षबिन्दुहरू क्रमशः (1, 1), (5, – 1), (9, 7) र (5, 8) छन् । उक्त आयतमा भएको परिगत वृत्तको समीकरण पत्ता लगाउनुहोस् ।

The vertices of a rectangle are (1, 1), (5, – 1), (9, 7) and (5, 8). Find the equation of the circle circumscribing the rectangle.

www.drmathnp.com

pucpas.com.np



Model 19 Miscellaneous problems on circle.

24A. वृत्त $x^2 + y^2 + 2x = 0$ मा एउटा जीवाको समीकरण $y - x = 2$ छ । यो जीवा व्यास हुने अर्को वृत्तको समीकरण पत्ता लगाउनुहोस् ।
If $y - x = 2$ is the equation of a chord of the circle $x^2 + y^2 + 2x = 0$, find the equation of the circle of which this chord is a diameter.

24B. वृत्त $x^2 + y^2 + 2y = 0$ मा एउटा जीवाको समीकरण $x - y = 2$ छ । यो जीवा व्यास हुने अर्को वृत्तको समीकरण पत्ता लगाउनुहोस् ।
If $x - y = 2$ is the equation of a chord of the circle $x^2 + y^2 + 2y = 0$, find the equation of the circle of which this chord is a diameter.

ANSWERS

A. SHORT QUESTIONS

1A. $x^2 + y^2 - 4x - 6y = 12$ **1B.** $x^2 + y^2 - 4x - 6y = 23$ **1C.** $x^2 + y^2 - 6x + 2y + 6 = 0$
1D. $x^2 + y^2 - 4x + 2y = 45$ **1E.** $x^2 + y^2 - 10x - 10y + 25 = 0$ **1F.** $x^2 + y^2 - 8x - 8y + 16 = 0$
1G. $x^2 + y^2 - 12x - 8y + 36 = 0$ **1H.** $x^2 + y^2 - 12x - 6y + 36 = 0$ **1I.** $x^2 + y^2 - 8x - 12y + 36 = 0$
1J. $x^2 + y^2 = 25$ **1K.** $x^2 + y^2 - 10x + y + 21 = 0$ **1L.** $x^2 + y^2 - 4x - 6y = 12$
1M. $x^2 + y^2 - 6x - 8y = 0$ **1N.** $x^2 + y^2 - 6x - 8y = 0$ **1O.** $x^2 + y^2 - 16x - 30y = 0$
2A. $x^2 - 4x - 6y + y^2 = 12$ **2B.** $x^2 + y^2 + 2px + 2qy = 2pq$ **3A.** $x^2 + y^2 + 4x - 6y + 4 = 0$
3B. $x^2 + y^2 - 10x - 8y + 25 = 0$ **3C.** $x^2 + y^2 - 10x - 8y + 25 = 0$ **3D.** $x^2 + y^2 - 10x - 12y + 25 = 0$
3E. $x^2 + y^2 + 8x - 6y + 16 = 0$ **4A.** $x^2 + y^2 - 8x + 6y + 9 = 0$ **4B.** $x^2 + y^2 - 10x - 12y + 36 = 0$
4C. $x^2 + y^2 + 8x - 6y + 9 = 0$ **4D.** $x^2 + y^2 + 10x + 8y + 16 = 0$ **5A.** $x^2 + y^2 - 12x - 12y + 36 = 0$
5B. $x^2 + y^2 - 10x - 10y + 25 = 0$ **5C.** $x^2 + y^2 - 6x - 6y + 9 = 0$ **5D.** $x^2 + y^2 + 10x - 10y + 25 = 0$
5E. $x^2 + y^2 + 8x + 8y + 16 = 0$ **5F.** $x^2 + y^2 - 4x + 4y + 4 = 0$ **5G.** $x^2 + y^2 + 8x - 8y + 16 = 0$
5H. $x^2 + y^2 + 6x + 6y + 9 = 0$ **5I.** $x^2 + y^2 - 4x - 4y + 4 = 0$ **6A.** (1, 0), 2 units
6B. (– 3, 0), 2 units **7A.** $2\sqrt{10}$ units; (3, 2) **7B.** $(-g, -f), \sqrt{g^2 + f^2 - c}$
7C. (2, 3); 44 units **7D.** (2, – 3), 5 units **7E.** $\left(\frac{1}{2}, -\frac{3}{2}\right)$, 1 unit
7F. 28. 81 units **8A.** (0, 1), (0, 6) **8B.** (– 1, 0), (9, 0)
8C. (1, 3), $\left(\frac{3}{2}, 3\right)$ **9A.** 24 units **9B.** 9 units **9C.** 6 units **10A.** 3
10B. 4 units **11A.** (3, 2) **11B.** (2, – 1) **11C.** ± 24 **11D.** 4 **11E.** – 4
12A. $25x^2 + 25y^2 - 100x + 50y + 4 = 0$ **12B.** $x^2 + y^2 - 8x - 6y + 21 = 0$

B. LONG QUESTIONS

1A. (5, – 2), 4 units **1B.** (–2, 2), 3 units **1C.** $\left(-\frac{3}{2}, \frac{1}{2}\right)$, 3 units
1D. $\left(-1, \frac{1}{2}\right), \frac{\sqrt{3}}{2}$ units **2A.** $x^2 + y^2 - x - 2y + 1 = 0$; $\left(\frac{1}{2}, 1\right)$, 0.5
2B. $x^2 + y^2 - 4x - 6y + 11 = 0$; (2, 3), $\sqrt{2}$ **3A.** $x^2 + y^2 - 13x - 13y + 72 = 0$ **3B.** $x^2 + y^2 - 8x - 6y + 23 = 0$
4A. (2, 8), 10 units **4B.** (2, 5)
5A. $x^2 + y^2 - 12x + 11 = 0$ or $x^2 + y^2 + 4x - 21 = 0$ **5B.** $x^2 + y^2 = 25$, $x^2 + y^2 - 12y + 11 = 0$
6A. $x^2 + y^2 - 12x - 12y = 28$ **6B.** $x^2 + y^2 + 16x - 6y - 216 = 0$ **6C.** $x^2 + y^2 - 10x - 8y + 16 = 0$
6D. $x^2 + y^2 - 4x + 6y = 12$ **7A.** $x^2 + y^2 - 6x + 4y + 12 = 0$ **7B.** $x^2 + y^2 - 4x + 6y - 12 = 0$
7C. $x^2 + y^2 - 2x - 6y - 15 = 0$ **7D.** $25x^2 + 25y^2 - 200x - 250y + 1016 = 0$
7E. $x^2 + y^2 - 2x + 4y - 11 = 0$ **7F.** $x^2 + y^2 - 4x - 6y + 9 = 0$
8A. $x^2 + y^2 - 18x - 12y + 17 = 0$, $x^2 + y^2 + 14x + 4y - 47 = 0$ **8B.** $(x - 1)^2 + (y - 2)^2 = 1$ or $(x - 2)^2 + (y - 1)^2 = 1$
9A. $x^2 + y^2 - 4x - 2y + 1 = 0$ **9B.** $x^2 + y^2 + 8x - 12y + 16 = 0$ **9C.** $x^2 + y^2 - 4x - 2y - 5 = 0$
9D. $x^2 + y^2 - 6x + 2y + 5 = 0$ **9E.** $x^2 + y^2 - 12x - 16y + 75 = 0$ **9F.** $x^2 + y^2 - 10x - 8y + 16 = 0$
10A. $x^2 + y^2 + 6x + 2y - 90 = 0$ **10B.** $x^2 + y^2 - 6x - 8y + 21 = 0$ **10C.** $x^2 + y^2 - 2x + 2y = 23$
11A. $3x - 2y = 0$ **11B.** $x - y + 1 = 0$
12A. to I. Answers of all questions : $x^2 + y^2 - 4x - 2y - 5 = 0$ **12J.** $x^2 + y^2 - 6x - 8y = 0$
12K. $x^2 + y^2 - 12x + 18y = 0$ **12L.** $x^2 + y^2 + 16x + 30y = 0$ **13A.** 4, 4 units
13B. 6, $\sqrt{2}$ units **13C.** 4, 6; 5 units
13D. 10, (5, 0) **13E.** 2, (2, 1) **13F.** 3, (1, 1) **13G.** 24
14A. 4 **14B.** 4
16A. $x^2 + y^2 - 20x + 20y + 100 = 0$, $x^2 + y^2 - 4x + 4y + 4 = 0$ **16B.** $x^2 + y^2 - 40x + 40y + 400 = 0$, $x^2 + y^2 - 8x + 8y + 16 = 0$
17A. 4 units **17B.** 8 units **18A.** $2\sqrt{2}$ units **18B.** (0, 2) or (2, 0)
19A. $x^2 + y^2 - 6x - 8y = 0$ **19B.** $x^2 + y^2 - 10x - 24y = 0$ **19C.** $x^2 + y^2 - 4x - 6y = 0$

www.drmathnp.com

puspas.com.np

19D. $x^2 + y^2 - 6x - 10y + 9 = 0$ **19E.** $x^2 + y^2 - 10x - 8y + 16 = 0$ **19F.** $x^2 + y^2 - 14x - 50y + 49 = 0$

20A. (2,3), 5 **20B.** (3, 5), 2 **20C.** (3, –1); 2 units **20D.** (3, 2); 2 units

21A. (1, 2), $\sqrt{5}$, $x^2 + y^2 - 2x - 4y = 0$ **21B.** (2, 2), 1 unit, $x^2 + y^2 - 4x - 4y + 7 = 0$

21C. (3, 2), $\sqrt{10}$, $x^2 + y^2 - 6x - 4y + 3 = 0$ **21D.** (2, 3) ; 5 units, $x^2 + y^2 - 4x - 6y = 12$

22A. $x^2 + y^2 - 8x - 16y + 64 = 0$ **22B.** $x^2 + y^2 + 8x + 16y + 64 = 0$ **22C.** $x^2 + y^2 - 6x - 4y - 12 = 0$

23A. $x^2 + y^2 - x - 7y + 8 = 0$ **23B.** $x^2 + y^2 - 6x - 4y + 8 = 0$ **23C.** $x^2 + y^2 - 4x - 4y + 6 = 0$

23D. $x^2 + y^2 - 10x - 8y + 16 = 0$ **24A.** $x^2 + y^2 + 3x - y + 2 = 0$ **24B.** $x^2 + y^2 - x + 3y + 2 = 0$

CHALLENGING EXERCISE 4

1. केन्द्र रेखा $x - 2y = 3$ मा रहेको र दुवै ऋणात्मक अक्षहरूलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which touches both the negative axes and has its centre on the line $x - 2y = 3$.

2. केन्द्र रेखा $x + 1 + 2(y + 1) = 6$ मा रहेको र दुवै धनात्मक अक्षहरूलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle which touches both the positive axes and has its center on the line $x + 1 + 2(y + 1) = 6$.

3. केन्द्रबिन्दु चौथो चतुर्थांश तथा रेखा $2x + y = 1$ मा रहेको, अर्धव्यास 5 एकाइ हुने र बिन्दु (4, 3) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle with centre on the fourth quadrant and on the line $2x + y - 1 = 0$, radius 5 units and passing through the point (4, 3).

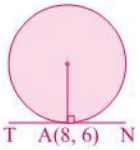
4. बिन्दुहरू (1, 6), (8, 1), (13, 6) र (8, 11) चक्रीय बिन्दुहरू हुन् भनी प्रमाणित गर्नुहोस् ।
Show that the points (1, 6), (8, 1), (13, 6) and (8, 11) are concyclic points.

5. रेखा $x - y + 5 = 0$ ले वृत्त $x^2 + y^2 - 2x - 10y + 1 = 0$ लाई बिन्दुहरू A र B मा काट्दछ । व्यास AB हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The line $x - y + 5 = 0$ cuts the circle $x^2 + y^2 - 2x - 10y + 1 = 0$ at points A and B. Find the equation of the circle whose diameter is AB.

6. वृत्त $x^2 + y^2 = 25$ को बिन्दु (3, – 4) मा खिचिएको स्पर्श रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, – 4).

7. वृत्त $x^2 + y^2 - 2x + 4y + 3 = 0$ को बिन्दु (2, 3) मा खिचिएको स्पर्श रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the eq^n of the tangent to the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ at the point (2, 3).

8. संगैको चित्रमा दिइएको वृत्तको समीकरण $x^2 + y^2 - 10x - 6y + 16 = 0$ भए स्पर्शरेखा TAN को समीकरण पत्ता लगाउनुहोस् ।
In the figure, the equation of adjoining circle is $x^2 + y^2 - 10x - 6y + 16 = 0$. Find the equation of tangent TAN.



9. रेखा $3x - 4y = 25$ वृत्त $x^2 + y^2 = 25$ को स्पर्शरेखा हुन्छ भनी प्रमाणित गर्नुहोस् । फेरि स्पर्शबिन्दु पनि पत्ता लगाउनुहोस् ।
Prove that $3x - 4y = 25$ is a tangent to the circle $x^2 + y^2 = 25$. Also, find the point of contact.

10. $x^2 + y^2 - 4x - 6y - 3 = 0$ सँग एकै केन्द्रित हुने र Y-अक्षलाई छुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle concentric with $x^2 + y^2 - 4x - 6y - 3 = 0$ and which touches the Y-axis.

11. वृत्त $x^2 + y^2 - 6x + 12y + 15 = 0$ सँग एकै केन्द्रित हुने र यसको क्षेत्रफलको दोब्बर क्षेत्रफल हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and of double its area.

12. व्यासका समीकरणहरू $2x - 3y + 12 = 0$ र $x + 4y = 5$ तथा क्षेत्रफल 154 वर्ग एकाइ भएको एउटा वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle whose diameters are $2x - 3y + 12 = 0$ and $x + 4y = 5$ and area is 154 sq. units.

13. उद्गम बिन्दुबाट 4 एकाइ दूरीमा रहेका X-अक्षमा पर्ने दुईओटा बिन्दुहरू भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस्, जसको अर्धव्यास 5 एकाइ छ ।
Find the equation of the circle which passes through two points on the X -axis which are at distances 4 units from the origin and whose radius is 5 units.

14. अक्षहरूको धनात्मक भागमा क्रमश: 3 र 4 खण्डहरू बनाई उद्गम बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the origin and cuts off intercepts 3 and 4 from the positive parts of the axes respectively..

www.drmathnp.com

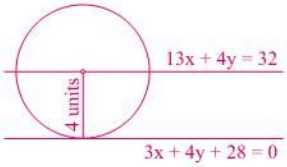
puspas.com.np

15. वृत्त $x^2 + y^2 + 14x + 10y - 26 = 0$ को भित्र पर्ने र बिन्दु $(-1, 3)$ मा दिइएको वृत्तलाई छुने तथा अर्धव्यास 5 एकाइ भएको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle of radius 5 units which lies within the circle $x^2 + y^2 + 14x + 10y - 26 = 0$ and which touches the given circle at the point $(-1, 3)$.

16. निर्देशाङ्क अक्षहरूमा $2x + 3y = 6$ ले बनाउने खण्डलाई व्यास मानी बनाइएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle drawn on the intercept made by the the $2x + 3y = 6$ between the co-ordinate axes as diameter.

17. वृत्त $x^2 + y^2 - 10x = 0$ मा एउटा जीवाको समीकरण $y = 2x$ छ । यो जीवा व्यास हुने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as a diameter.

18. एउटा वृत्तको अर्धव्यास 4 एकाइ छ । त्यस वृत्तको केन्द्र समीकरण $13x + 4y = 32$ भएको सीधा रेखामा पर्दछ र सो वृत्तले $3x + 4y + 28 = 0$ समीकरण भएको सीधा रेखालाई छुन्छ भने त्यस वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with radius 4 units, whose centre lies on the line $13x + 4y = 32$ and which touches the line $3x + 4y + 28 = 0$.



Hint: Here, radius $(r) = 4$ units
Let the centre of circle be (h, k). Centre (h, k) lies on $13x + 4y = 32$.
So, $13h + 4k = 32$ .....(i)
Distance between (h, k) and tangent is 4 units.

So, Using formula, $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} = |P|$

or, $\frac{3h + 4k + 28}{\sqrt{3^2 + 4^2}} = 4$

or, $3h + 4k + 28 = 4 \times 5$

or, $3h + 4k = -8$ .....(ii)

Solving (i) and (ii) then,

$$\begin{array}{r} 13h + 4k = 32 \\ 3h + 4k = -8 \\ - \quad - \quad + \\ \hline 10h = 40 \end{array}$$

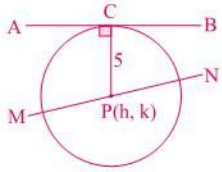
$\therefore \quad h = 4$

From equation (ii); $3h + 4k = -8$
or, $3 \times 4 + 4k = -8$
or, $4k = -20$
$\therefore \quad k = -5$

Now, centre $(h, k) = (4, -5)$ and radius $(r) = 4$ units.
We know that,
Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$
or, $(x - 4)^2 + (y + 5)^2 = 4^2$
or, $x^2 - 8x + 16 + y^2 + 10y + 25 = 16$
$\therefore \quad x^2 + y^2 - 8x + 10y + 25 = 0$

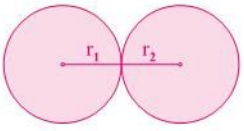
Thus, the required equation of circle is $x^2 + y^2 - 8x + 10y + 25 = 0$.

19. चित्रमा वृत्तको बिन्दु C मा AB स्पर्श रेखा छ र केन्द्र (h, k) भएर MN गएको छ । यदि AB र MN का समीकरणहरू क्रमशः $3x + 4y + 7 = 0$ र $2x + y = 7$ छन् र वृत्तको अर्धव्यास 5 एकाइ छ भने वृत्तको समीकरण पत्ता लगाउनुहोस ।
In the figure, AB is tangent to the circle at C and MN passes through the centre P(h, k). If the equations of AB and MN are $3x + 4y + 7 = 0$ & $2x + y = 7$ respectively and radius of the circle 5 units, find the equation of the circle.



20. अर्धव्यास 4 एकाइ हुने, रेखा $4x + 13y = 32$ मा केन्द्र पर्ने र रेखा $4x + 3y + 28 = 0$ लाई छोएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle with radius 4 units, whose centre lies on the line $4x + 13y = 32$ and which touches the line $4x + 3y + 28 = 0$.

21. वृत्तहरू $x^2 + y^2 + 2ax + c^2 = 0$ र $x^2 + y^2 + 2by + c^2 = 0$ बाहिरपट्टीबाट छुदाँ $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^4}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch externally, prove that: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^4}$.



ANSWERS

1. $x^2 + y^2 + 6x + 6y + 9 = 0$
2. $(x - 1)^2 + (y - 1)^2 = 1$
3. $(x - 1)^2 + (y + 1)^2 = 25$
5. $x^2 + y^2 - x + 11y + 6 = 0$
6. $3x - 4y = 25$
7. $x + 5y - 17 = 0$
8. $x + y = 14$
9. $(3, -4)$
10. $(x - 2)^2 + (y - 3)^2 = 4$
11. $(x - 3)^2 + (y + 6)^2 = 60$
12. $(x + 3)^2 + (y - 2)^2 = 49$
13. $x^2 + y^2 \pm 6y = 16$
14. $4x^2 + 4y^2 - 12x - 16y = 12$
15. $(x + 4)^2 + (y + 1)^2 = 5^2$
16. $x^2 + y^2 - 3x - 2y = 0$
17. $x^2 + y^2 - 2x - 4y = 0$
19. $x^2 + y^2 - 4x - 6y - 12 = 0$
20. $x^2 + y^2 + 10x - 8y + 25 = 0$

www.drmathnp.com

puspas.com.np

ASSIGNMENT ON COORDINATES

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' 2 × 1 = 2

1. कुन अवस्थामा जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ सम्पाती हुन्छन् ?
Under what condition, the pair of lines $ax^2 + 2hxy + by^2 = 0$ are coincident?

2. सोली र समतल सतहको आधारमा दीर्घ वृत्तको परिभाषा दिनुहोस् । (Define ellipse on the basis of cone and plane.)

Group 'B' 3 × 2 = 6

3. बिन्दुहरू (4, 8) र (6, 12) जोड्ने रेखाखण्ड र बिन्दुहरू (2, p) र (3, 6) भएर जाने रेखाखण्ड समानान्तर भए p को मान पत्ता लगाउनुहोस् ।
If the line segment joining the points (2, p) and (3, 6) is parallel to the line segment joining the points (4, 8) and (6, 12), find the value of p.

4. $6x^2 - xy - y^2 = 0$ ले जनाउने जोडा रेखाहरू बीचका कोणहरू पत्ता लगाउनुहोस् ।
Find the angles between the lines represented by $6x^2 - xy - y^2 = 0$.

5. वृत्त $x^2 + y^2 + 4x - 6y + 4 = 0$ को अर्धव्यासको लम्बाइ पत्ता लगाउनुहोस् ।
Calculate the length of radius of the circle $x^2 + y^2 + 4x - 6y + 4 = 0$.

Group 'C' 3 × 4 = 12

6. बिन्दु (2, 3) भएर जाने र रेखा $x - 2y = 5$ सँग लम्ब हुने एउटा रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a line through the point (2, 3) and perpendicular to the line $x - 2y = 5$.

7. बिन्दु (2, 3) भएर जाने र रेखा $x - 3y = 2$ सँग 45° को कोण बनाउने सिधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (2, 3) and making an angle of 45° with the line $x - 3y = 2$.

8. $x^2 - 5xy + 4y^2 = 0$ ले जनाउने रेखाहरूसँग लम्ब भइ उद्गम बिन्दु भएर जाने सीधा रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
What is the single equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 - 5xy + 4y^2 = 0$?

Group 'D' 1 × 5 = 5

9. बिन्दु (3, 4) भएर जाने वृत्तको व्यासहरूका समीकरणहरू $x + y = 14$ र $2x - y = 4$ छन् । सो वृत्तको समीकरण पत्ता लगाउनुहोस् । साथै सो वृत्तलाई लेखाचित्रमा पनि देखाउनुहोस् ।
The equations of two diameters of a circle passing through the point (3, 4) are $x + y = 14$ and $2x - y = 4$. Find the equation of the circle. Also, show the circle in a graph.

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

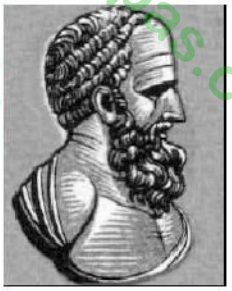
1. $h^2 - ab = 0$ 3. 4 4. 45°, 135° 5. 3 6. $2x + y = 7$
7. $x + 2y = 8, 2x - y = 1$ 8. $4x^2 + 5xy + y^2 = 0$ 9. $x^2 + y^2 - 12x - 16y + 75 = 0$

www.drmathnp.com

puspas.com.np

Unit V

त्रिकोणमिति
Trigonometry



HIPPARCHUS

*Trigonometry (from Greek trigonon "triangle" + metron "measure") sounds almost like Trikonamiti (त्रिकोणमिति, in Sanskrit) means "measurement of triangle". Very little is said about how and when the name "Trigonometry" originated. Traces of certain aspects of what we today call trigonometry are found in most ancient civilizations by Babylonian Plimpton, Aryabhatta (6th century C.E..) and Bhaskara I.(7th century C.E.), (9th century) and Guo Shoujing 13th century C.E.). **Hipparchus** was a careful and cautious scientist who prepared the way for those who followed him by establishing a high standard of observational astronomy, by devising trigonometrical methods of solving problems in mathematical astronomy.*

SEE SPECIFICATION GRID **2 Very Short, 3 Short Questions & 3 Long Questions**

Contents	Topics	Knowledge Each of 1 Mark	Understanding Each of 2 Marks	Application Each of 4 Marks	Higher Ability Each of 5 Marks	Total no. of Questions	Total Marks
त्रिकोणमिति Trigonometry	अपवर्त्य र अपवर्तक कोणहरू Multiple & sub-multiple angles	2	3	3	-	8	20
	त्रिकोणमितीय सर्वसमीकाहरूको रूपान्तरण Transformation of Trigonometric Identities						
	अनुबन्धीत सर्वसमीकाहरू Conditional Identities						
	त्रिकोणमितीय समीकरणहरू Trigonometric Equations						
	उचाइ तथा दुरी (Height and Distance)						

TRIGONOMETRY

SYLLABUS ISSUED BY CDC

5. **त्रिकोणमिति (Trigonometry)** - 35 Periods

- त्रिकोणमितीय सर्वसमीकाहरूको रूपान्तरण (Conversion of trigonometric identities)
  - अपवर्त्य (Multiple) र अपवर्तक (Sub-multiple) कोणहरूको त्रिकोणमितीय अनुपात (sin, cos र tan मात्र)
    Trigonometric ratios of Multiple and Submultiple angles (sin, cos and tan only)
  - जोड, अन्तर र गुणन (sine र cosine मात्र) का रूपमा व्यक्त
    Transformation of sum and product (sine and cosine only)
- दिइएको अनुबन्ध $A + B + C = \pi^c$ को आधारमा त्रिकोणमितीय सर्वसमिकाहरू प्रमाणित
  Proving the conditional trigonometric identities on the basis of condition $A + B + C = \pi^c$
- त्रिकोणमितीय समीकरणको हल (दुई घात सम्म) $(0 \le \theta \le 2\pi^c)$
  Solving trigonometric equation (upto power 2) $(0 \le \theta \le 2\pi^c)$
- उचाइ र दुरीका शाब्दिक समस्याहरू (दुइ ओटा उन्नतांश र अवनति कोण समावेश भएका)
  Word problems on height and distance (Including two angle of elevations and two angle of depressions.)

1. अपवर्त्य र अपवर्तक कोणहरू
MULTIPLE AND SUB-MULTIPLE ANGLES

DEFINITIONS

A. अपवर्त्य कोण (Multiple Angle)

यदि A कुनै कोण भए 2A, 3A, 4A, ... इत्यादिलाई A का अपवर्त्य कोणहरू भनिन्छ ।
If A be any angle then 2A, 3A, 4A, ... etc are called the multiple angles of A.

B. अपवर्त्यक कोण (Submultiple Angle)

यदि A कुनै कोण भए $\frac{A}{2}, \frac{A}{3}$, ... इत्यादिलाई A का अपवर्तक कोणहरू भनिन्छ ।

If A be any angle then $\frac{A}{2}, \frac{A}{3}$, ... etc are called the submultiple angles of A.

www.drmathnp.com

pupas.com.np

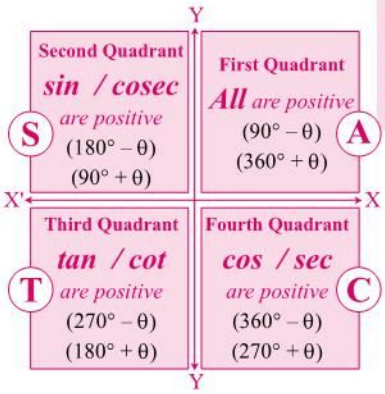
FORMULAE AND KEY POINTS

A. त्रिकोणमितीय अनुपातहरू (Trigonometric Ratios)

1. $\sin\theta = \frac{p}{h}$
2. $\operatorname{cosec}\theta = \frac{h}{p}$
3. $\cos\theta = \frac{b}{h}$
4. $\sec\theta = \frac{h}{b}$
5. $\tan\theta = \frac{p}{b}$
6. $\cot\theta = \frac{b}{p}$
7. $\sin A \times \operatorname{cosec} A = 1$
8. $\cos A \times \sec A = 1$
9. $\cot A \times \tan A = 1$
10. $\cos^2\theta + \sin^2\theta = 1$
11. $\cot A = \frac{\cos A}{\sin A}$
12. $\tan A = \frac{\sin A}{\cos A}$
13. $\sec^2\theta - \tan^2\theta = 1$
14. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
15. $\tan A = \frac{\sec A}{\operatorname{cosec} A}$
16. $\sin A = \sqrt{1 - \cos^2 A}$
17. $\cos A = \sqrt{1 - \sin^2 A}$
18. $\sec A = \sqrt{1 + \tan^2 A}$
19. $\tan A = \sqrt{\sec^2 A - 1}$
20. $\operatorname{cosec} A = \sqrt{1 + \cot^2 A}$
21. $\cot A = \sqrt{\operatorname{cosec}^2 A - 1}$
22. $\operatorname{cosec} A = \frac{1}{\sin A}$
23. $\sec A = \frac{1}{\cos A}$
24. $\tan A = \frac{1}{\cot A}$

B. त्रिकोणमितीय अनुपातहरूका मान (Value of Trigonometric Ratios)

1. 0°, 180° र 360° को लागि सबै अनुपातहरू बदलिँदैन । (For 0°, 180° and 360° all ratios are unchanged.)
2. 90° र 270° को लागि सबै अनुपातहरू निम्नअनुसार बदलिन्छन् ।
   For 90° and 270° all ratios are changed according as:
   cos ⟷ sin, tan ⟷ cot, cosec ⟷ sec
3. चिह्नहरू (i.e. +ve –ve) "CAST" नियमअनुसार बदलिन्छन् । बदलिँदा प्रश्नमा दिइएको अनुपात बदलिन्छ ।
   signs (i.e. + ve –ve) are assigned according to "CAST" formula for the question (not for the answers).



Example:

$\sin(90° - \theta) = \cos\theta$ $\quad$ $\operatorname{cosec}(90° - \theta) = \sec\theta$

$\cos(180° - \theta) = -\cos\theta$ $\quad$ $\sec(90° - \theta) = \operatorname{cosec}\theta$

$\tan(270° - \theta) = \cot\theta$ $\quad$ $\cot(180° - \theta) = -\cot\theta$

C. मान तालिका (Value Table)

Ratios / Values	sin	cos	tan	cosec	sec	cot
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	∞	1	∞	0
120°	$\frac{\sqrt{3}}{2}$	$\frac{-1}{2}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	–2	$-\frac{1}{\sqrt{3}}$
135°	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	–1	$\sqrt{2}$	$-\sqrt{2}$	–1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	2	$-\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
180°	0	–1	0	∞	–1	∞

www.drmathnp.com

puspas.com.np

D. मिश्रित कोणहरू (Compound Angles)

	Sum - Formulae	Difference - Formulae
1.	$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$	$\sin(A-B) = \sin A.\cos B - \cos A.\sin B$
2.	$\cos(A+B) = \cos A.\cos B - \sin A.\sin B$	$\cos(A-B) = \cos A.\cos B + \sin A.\sin B$
3.	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$
4.	$\cot(A+B) = \frac{\cot A.\cot B - 1}{\cot B + \cot A}$	$\cot(A-B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}$

5. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

6. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

E. यदि $\frac{a}{b} = \frac{c}{d}$ भए $\left(\text{If } \frac{a}{b} = \frac{c}{d} \text{ then}\right)$,

इन्भरटेण्डो Invertendo	अल्टरनेण्डो Alternendo	कम्पोनेण्डो Componendo	डेभिडेण्डो Devidendo	कम्पोनेण्डो र डेभिडेण्डो Componendo and Devidendo
$\frac{b}{a} = \frac{d}{c}$	$\frac{a}{c} = \frac{b}{d}$	$\frac{a+b}{b} = \frac{c+d}{d}$	$\frac{a-b}{b} = \frac{c-d}{d}$	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

TRIGONOMETRY

DOUBLE ANGLES FORMULAE

	अपवर्त्य कोणहरू (Multiple Angles)		अपवर्तक कोणहरू (Submultiple Angles)
1.	$\sin 2A = 2\sin A\cos A$	1.	$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
2.	$\cos 2A = \cos^2 A - \sin^2 A$	2.	$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$
3.	$\cos 2A = 2\cos^2 A - 1$	3.	$\cos A = 2\cos^2\frac{A}{2} - 1$
4.	$\cos 2A = 1 - 2\sin^2 A$	4.	$\cos A = 1 - 2\sin^2\frac{A}{2}$
5.	$1 + \cos 2A = 2\cos^2 A$	5.	$1 + \cos A = 2\cos^2\frac{A}{2}$
6.	$1 - \cos 2A = 2\sin^2 A$	6.	$1 - \cos A = 2\sin^2\frac{A}{2}$
7.	$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$	7.	$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$
8.	$\cos A = \pm\sqrt{\frac{1+\cos 2A}{2}}$	8.	$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$
9.	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$	9.	$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
10.	$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$	10.	$\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
11.	$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$	11.	$\cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
12.	$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$	12.	$\tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A}$
13.	$\tan A = \pm\sqrt{\frac{1-\cos 2A}{1+\cos 2A}}$	13.	$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$
14.	$\cot^2 A = \frac{1+\cos 2A}{1-\cos 2A}$	14.	$\cot^2\frac{A}{2} = \frac{1+\cos A}{1-\cos A}$
15.	$\cot A = \pm\sqrt{\frac{1+\cos 2A}{1-\cos 2A}}$	15.	$\cot\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{1-\cos A}}$

www.drmathnp.com

TRIPLE ANGLES FORMULAE

16.	$\sin 3A = 3\sin A - 4\sin^3 A$	16.	$\sin A = 3\sin\frac{A}{3} - 4\sin^3\frac{A}{3}$
17.	$4\sin^3 A = 3\sin A - \sin 3A$	17.	$4\sin^3\frac{A}{3} = 3\sin\frac{A}{3} - 3\sin A$
18.	$\sin^3 A = \frac{3\sin A - \sin 3A}{4}$	18.	$\sin^3\frac{A}{3} = \frac{3\sin\frac{A}{3} - \sin A}{4}$
19.	$\cos 3A = 4\cos^3 A - 3\cos A$	19.	$\cos A = 4\cos^3\frac{A}{3} - 3\cos\frac{A}{3}$
20.	$4\cos^3 A = 3\cos A + \cos 3A$	20.	$4\cos^3\frac{A}{3} = 3\cos\frac{A}{3} + \cos A$
21.	$\cos^3 A = \frac{3\cos A + \cos 3A}{4}$	21.	$\cos^3\frac{A}{3} = \frac{3\cos\frac{A}{3} + \cos A}{4}$
22.	$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$	22.	$\tan A = \frac{3\tan\frac{A}{3} - \tan^3\frac{A}{3}}{1 - 3\tan^2\frac{A}{3}}$
23.	$\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$	23.	$\cot A = \frac{\cot^3\frac{A}{3} - 3\cot\frac{A}{3}}{3\cot^2\frac{A}{3} - 1}$

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2068 R, 2068 S', 2070 S', 2071 R'	4
Model 2	2069 R, 2072 R'	1
Model 3	2066 S, 2067 R, 2068 R', 2069 S, 2073 R, SEE 2073 S', SEE 2074 S, S', SEE 2075 R'$_2$	9
Model 4	2065 E, 2066 R, 2066 R', 2069 R', 2070 R, 2073 R', SEE 2073 S, SEE 2074 R	5
Model 5	2065 R, 2065 S, 2066 R', 2066 S, 2067 R', 2068 R', 2068 S, 2069 R', 2069 S, 2070 R', 2070 S, 2072 S, SEE 2075 R, R', R'$_2$	14
Model 6	2065 M, 2065 R, 2067 S, 2068 S', 2069 R, 2071 R, 2071 R', 2071 S, 2072 R, R', S, 2073 R', SEE 2074 R', SEE 2074 S', SEE 2075 R, R$_2$, R', SEE MODEL 2076	13

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2065 E, 2065 R, 2065 S, 2066 R, 2066 S, 2067 R, 2068 S', 2070 S', 2071 R, 2071 R', 2071 S, 2072 R, 2073 R, 2073 R', SEE 2073 S, SEE 2073 S', SEE 2074 R, SEE 2074 R', SEE 2074 S, SEE 2075 R'	12
Model 2	2065 E, 2065 R'	2
Model 3	2070 R	1

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

अपवर्त्य कोणहरू (Multiple Angles)	अपवर्तक कोणहरू (Submultiple Angles)
1A. अपवर्त्य कोण भनेको के हो ? (What is multiple angle?)	**1B.** अपवर्तक कोणको परिभाषा दिनुहोस्। (Define submultiple angle.)
2A. sin 2A लाई sin A र cos A को रूपमा लेख्नुहोस् । Write sin 2A interms of sin A and cos A.	**2B.** sin A लाई $\sin\frac{A}{2}$ र $\cos\frac{A}{2}$ को रूपमा लेख्नुहोस् । Write sin A interms of $\sin\frac{A}{2}$ and $\cos\frac{A}{2}$.
3A. sin 2A लाई tan A को रूपमा लेख्नुहोस् । Write sin 2A interms of tan A. [SEE MODEL 2076]	**3B.** sin A लाई $\tan\frac{A}{2}$ को रूपमा लेख्नुहोस् । Write sin A interms of $\tan\frac{A}{2}$.
4A. cos 2A लाई cos A को रूपमा लेख्नुहोस् । Write cos 2A interms of cos A.	**4B.** cos A लाई $\cos\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । Express cos A interms of $\cos\frac{A}{2}$.

pueplas.com.np

www.drmathnp.com

puspas.com.np

5A. cos 2A र sin A बीचको सम्बन्ध के हुन्छ ? What is the relation between cos 2A and sin A?	**5B.** cos A लाई $\sin\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । Express cos A interms of $\sin\frac{A}{2}$.
6A. cos 2A लाई tan A को रूपमा व्यक्त गर्नुहोस् । Express cos 2A interms of tan A.	**6B.** cos A लाई $\tan\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । Express cos A interms of $\tan\frac{A}{2}$.
7A. tan 2A लाई tan A को रूपमा लेख्नुहोस् । Write tan 2A interms of tan A.	**7B.** tan A लाई $\tan\frac{A}{2}$ को रूपमा लेख्नुहोस् । Write tan A interms of $\tan\frac{A}{2}$.
8A. cos 3A र cos A बीचको सम्बन्ध लेख्नुहोस् । Write the relation between cos 3A and cos A.	**8B.** cos A र $\cos\frac{A}{3}$ बीचको सम्बन्ध लेख्नुहोस् । Write the relation between cos A and $\cos\frac{A}{3}$.
9A. sin 3A र sin A बीचको सम्बन्ध लेख्नुहोस् । Write the relation between sin 3A and sin A.	**9B.** sin A र $\sin\frac{A}{3}$ बीचको सम्बन्ध लेख्नुहोस् । Write the relation between sin A and $\sin\frac{A}{3}$.
10A. tan 3A र tan A बीचको सम्बन्ध लेख्नुहोस् । Write the relation between tan 3A and tan A.	**10B.** tan A र $\tan\frac{A}{3}$ बीचको सम्बन्ध लेख्नुहोस् । Write the relation between tan A and $\tan\frac{A}{3}$.

TRIGONOMETRY

Model 2 Concept Recall

11A. $\sin(A+B) = \sin A\cos B + \cos A\sin B$, मा A = B भए कुन सूत्र बन्छ ?
In $\sin(A+B) = \sin A\cos B + \cos A\sin B$, if A = B then which formula will be formed?

11B. $\cos(A+B) = \cos A\cos B - \sin A\sin B$, मा A = B भए कुन सूत्र बन्छ ?
In $\cos(A+B) = \cos A\cos B - \sin A\sin B$, if A = B then which formula will be formed?

11C. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$, मा A = B भए कुन सूत्र बन्छ ?
In $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$, if A = B then which formula will be formed?

12A. यदि $\sin A = x$ भए cos 2A कति हुन्छ ? (If $\sin A = x$ then what is cos 2A?)

12B. यदि $\cos A = m$ भए cos 2A कति हुन्छ ? (If $\cos A = m$ then what is cos 2A?)

12C. यदि $\tan A = p$ भए cos 2A कति हुन्छ ? (If $\tan A = p$ then what is cos 2A?)

13A. यदि $\sin A = m$ र $\cos A = n$ भए sin 2A कति हुन्छ ? (If $\sin A = m$ and $\cos A = n$ then what is sin 2A?)

13B. यदि $\tan A = y$ भए sin 2A कति हुन्छ ? (If $\tan A = y$ then what is sin 2A?)

13C. यदि $\tan A = z$ भए tan 2A कति हुन्छ ? (If $\tan A = z$ then what is tan 2A?)

14A. sin 4A लाई 2A को रूपमा व्यक्त गर्नुहोस् । (Express sin 4A interms of 2A.)

14B. cos 4A लाई cos 2A को रूपमा व्यक्त गर्नुहोस् । (Express cos 4A interms of cos 2A.)

14C. cos 4A लाई sin 2A को रूपमा व्यक्त गर्नुहोस् । (Express cos 4A interms of sin 2A.)

14D. cos 4A लाई tan 2A को रूपमा व्यक्त गर्नुहोस् । (Express cos 4A interms of tan 2A.)

14E. tan 4A लाई tan 2A को रूपमा व्यक्त गर्नुहोस् । (Express tan 4A interms of tan 2A.)

14F. sin 6A लाई tan 3A को रूपमा व्यक्त गर्नुहोस् । (Express sin 6A interms of tan 3A.)

15A. यदि $\sin\frac{A}{2} = m$ भए cos A कति हुन्छ ? (If $\sin\frac{A}{2} = m$ then what is cos A?)

15B. यदि $\cos\frac{A}{2} = n$ भए cos A कति हुन्छ ? (If $\cos\frac{A}{2} = n$ then what is cos A?)

15C. यदि $\tan\frac{A}{2} = x$ भए tan A कति हुन्छ ? (If $\tan\frac{A}{2} = x$ then what is tan A?)

www.drmathnp.com

puepas.com.np

15D. यदि $\tan\frac{A}{2} = y$ भए sin A कति हुन्छ ? (If $\tan\frac{A}{2} = y$ then what is sin A?)

15E. यदि $\tan\frac{A}{2} = p$ भए cos A कति हुन्छ ? (If $\tan\frac{A}{2} = p$ then what is cos A?)

16A. $\sin\frac{A}{2}$ लाई $\frac{A}{4}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\sin\frac{A}{2}$ interms of $\frac{A}{4}$.)

16B. $\cos\frac{A}{2}$ लाई $\sin\frac{A}{4}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\cos\frac{A}{2}$ interms of $\sin\frac{A}{4}$.)

16C. $\cos\frac{A}{2}$ लाई $\cos\frac{A}{4}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\cos\frac{A}{2}$ interms of $\cos\frac{A}{4}$.)

16D. $\cos\frac{A}{2}$ लाई $\tan\frac{A}{4}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\cos\frac{A}{2}$ interms of $\tan\frac{A}{4}$.)

16E. $\tan\frac{A}{4}$ लाई $\tan\frac{A}{8}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\tan\frac{A}{4}$ interms of $\tan\frac{A}{8}$.)

16F. $\sin\frac{A}{8}$ लाई $\tan\frac{A}{16}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\sin\frac{A}{8}$ interms of $\tan\frac{A}{16}$.)

17. दिइएका त्रिकोणमितीय अनुपातसँग बराबर हुने त्रिकोणमितीय अनुपात लेख्नुहोस् ।
Write the trigonometric ratio is equal to the given trigonometric ratio.

A. $\cos^2\frac{A}{2} - \sin^2\frac{A}{2}$ **B.** $2\sin\frac{A}{2}\cos\frac{A}{2}$ **C.** $2\cos^2\frac{A}{2} - 1$

D. $\dfrac{2\tan\frac{A}{4}}{1-\tan^2\frac{A}{4}}$ **E.** $\dfrac{3\tan 2A - \tan^3 2A}{1 - 3\tan^2 2A}$ **F.** $\dfrac{3\tan\frac{A}{2} - \tan^3\frac{A}{2}}{1 - 3\tan^2\frac{A}{2}}$

G. $4\cos^3 2A - 3\cos 2A$ **H.** $3\sin 3A - 4\sin^3 3A$ **I.** $\dfrac{1-\tan^2\frac{A}{4}}{1+\tan^2\frac{A}{4}}$

18A. यदि $\sin A = \frac{1}{2}$ भए cos 2A को मान पत्ता लगाउनुहोस् । (If $\sin A = \frac{1}{2}$, find the value of cos 2A.)

18B. यदि $\cos A = \frac{1}{2}$ भए cos 2A को मान पत्ता लगाउनुहोस् । (If $\cos A = \frac{1}{2}$, find the value of cos 2A.)

18C. यदि $\tan A = \frac{1}{2}$ भए cos 2A को मान पत्ता लगाउनुहोस् । (If $\tan A = \frac{1}{2}$, find the value of cos 2A.)

18D. यदि $\sin A = \frac{1}{2}$ भए sin 3A को मान पत्ता लगाउनुहोस् । (If $\sin A = \frac{1}{2}$, find the value of sin 3A.)

19A. यदि $\cos\frac{A}{3} = \frac{\sqrt{3}}{2}$ भए cos A को मान पत्ता लगाउनुहोस् । (If $\cos\frac{A}{3} = \frac{\sqrt{3}}{2}$, find the value of cos A.)

19B. यदि $\sin\frac{A}{3} = \frac{1}{2}$ भए sin A को मान पत्ता लगाउनुहोस् । (If $\sin\frac{A}{3} = \frac{1}{2}$, find the value of sin A.)

19C. यदि $\sin\frac{A}{2} = \frac{\sqrt{3}}{2}$ भए cos A को मान पत्ता लगाउनुहोस् । (If $\sin\frac{A}{2} = \frac{\sqrt{3}}{2}$, find the value of cos A.)

B. SHORT QUESTIONS

Model 1 *Problems based on finding value of trigonometric ratios of 2A.*

1A. यदि $\sin A = \frac{3}{4}$ भए cos 2A को मान निकाल्नुहोस् । (If $\sin A = \frac{3}{4}$, find the value of cos 2A.) **[2068R]**

1B. यदि $\sin A = \frac{3}{5}$ भए cos 2A को मान निकाल्नुहोस् । (If $\sin A = \frac{3}{5}$, find the value of cos 2A.) **[2064 R', 2071 R']**

1C. यदि $5\sin\theta = 4$ भए cos 2θ को मान निकाल्नुहोस् । (If $5\sin\theta = 4$, find the value of cos 2θ.) **[2063M]**

www.drmathnp.com

1D. यदि $\sin\theta = \frac{3}{5}$ भए $\sin 2\theta$ को मान निकाल्नुहोस् । (If $\sin\theta = \frac{3}{5}$, find the value of $\sin 2\theta$.) [2063R']

1E. यदि $\cos\theta = \frac{3}{4}$ भए $\sin 2\theta$ को मान निकाल्नुहोस् । (If $\cos\theta = \frac{3}{4}$, find the value of $\sin 2\theta$.) [2063S]

1F. यदि $\tan A = \frac{3}{4}$ भए $\sin 2A$ को मान निकाल्नुहोस् । (If $\tan A = \frac{3}{4}$, find the value of $\sin 2A$.) [2063R, 2068S']

1G. यदि $\cos\theta = \frac{12}{13}$ र $\sin\alpha = \frac{4}{5}$ भए $\sin 2\theta$ and $\cos 2\alpha$ को मान निकाल्नुहोस् ।

If $\cos\theta = \frac{12}{13}$ and $\sin\alpha = \frac{4}{5}$, find the value of $\sin 2\theta$ and $\cos 2\alpha$. [2059S]

2A. यदि $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ भए $\cos 2\theta = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$ हुन्छ भनी सिद्ध गर्नुहोस् ।

If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, prove that: $\cos 2\theta = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$ [2070 S']

2B. यदि $\sin\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ भए $\cos 2\theta + \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then prove that: $\cos 2\theta + \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right) = 0$.

TRIGONOMETRY

Model 2 Problems based on finding value of trigonometric ratios of 3A.

3A. यदि $\sin A = \frac{1}{2}$ भए $\sin 3A$ को मान पत्ता लगाउनुहोस् । (If $\sin A = \frac{1}{2}$, find the value of $\sin 3A$.) [SEE MODEL 2076, 2072 R']

3B. यदि $\cos\theta = \frac{4}{5}$ भए $\cos 3\theta$ को मान पत्ता लगाउनुहोस् । (If $\cos\theta = \frac{4}{5}$, find the value of $\cos 3\theta$.) [2069 R]

3C. यदि $\cos\theta = \frac{3}{5}$ भए $\cos 3\theta$ को मान पत्ता लगाउनुहोस् । (If $\cos\theta = \frac{3}{5}$, find the value of $\cos 3\theta$.) [2064 S]

3D. यदि $\sin\alpha = \frac{3}{4}$ भए $\cos 2\alpha$ र $\sin 3\alpha$ को मान पत्ता लगाउनुहोस् । (If $\sin\alpha = \frac{3}{4}$, find the value of $\cos 2\alpha$ and $\sin 3\alpha$) [2059 R]

Model 3 On finding value of trigonometric ratios A when A/2 is given or vice-versa.

4A. यदि $\tan\frac{A}{2} = \frac{3}{4}$ भए $\sin A$ को मान निकाल्नुहोस् । (If $\tan\frac{A}{2} = \frac{3}{4}$ then find the value of $\sin A$.) [SEE 2073 R]

4B. यदि $\tan\frac{A}{2} = \frac{3}{4}$ भए $\tan A$ को सङ्ख्यात्मक मान पत्ता लगाउनुहोस् । (If $\tan\frac{A}{2} = \frac{3}{4}$, find the numerical value of $\tan A$.) [2064 R]

4C. यदि $\sin\frac{\theta}{2} = \frac{1}{2}$ भए $\cos\theta$ को मान निकाल्नुहोस् । (If $\sin\frac{\theta}{2} = \frac{1}{2}$, find the value of $\cos\theta$.) [SEE 2074 S]

4D. यदि $\sin\frac{\alpha}{2} = \frac{3}{5}$ भए $\sin\alpha$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{\alpha}{2} = \frac{3}{5}$, find the value of $\sin\alpha$.) [2062 S]

4E. यदि $\cos\frac{A}{2} = \frac{3}{5}$ भए $\cos A$ को मान पत्ता लगाउनुहोस् । (If $\cos\frac{A}{2} = \frac{3}{5}$, find the value of $\cos A$.) [2068 R']

5A. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ हुन्छ भनी देखाउनुहोस् । (If $\cos 30° = \frac{\sqrt{3}}{2}$, show that $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$) [2075 R'$_2$]

Hint: Here, we know that, $\cos 2A = 2\cos^2 A - 1$ $\quad\therefore\quad \cos A = \sqrt{\frac{1+\cos 2A}{2}}$

For $A = 15°$; $\cos 15° = \sqrt{\frac{1+\cos 2\,.\,15°}{2}} = \sqrt{\frac{1+\cos 30°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{\frac{2+\sqrt{3}}{2}}}{2} = \sqrt{\frac{2+\sqrt{3}}{4}} = \sqrt{\frac{(2+\sqrt{3})\times 2}{4\times 2}}$

$= \sqrt{\frac{4+2\sqrt{3}}{8}} = \sqrt{\frac{3+1+2\sqrt{3}}{8}} = \sqrt{\frac{(\sqrt{3})^2 + 2\,.\,\sqrt{3}\,.\,1 + 1^2}{8}} = \sqrt{\frac{(\sqrt{3}+1)^2}{8}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \text{RHS}$ Proved.

puspas.com.np

www.drmathnp.com

5B. यदि $\cos 330° = \frac{\sqrt{3}}{2}$ भए प्रमाणित गर्नुहोस् (If $\cos 330° = \frac{\sqrt{3}}{2}$, prove that) : $\sin 165° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ [2060 C]

5C. यदि $\sin 405° = \frac{1}{\sqrt{2}}$ भए प्रमाणित गर्नुहोस् (If $\sin 405° = \frac{1}{\sqrt{2}}$, prove that) : $\sin 375° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

5D. यदि $\cos 210° = -\frac{\sqrt{3}}{2}$ भए $\tan 105°$ को मान पत्ता लगाउनुहोस् । (If $\cos 210° = -\frac{\sqrt{3}}{2}$, find the value of $\tan 105°$.)

5E. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए, $\sin 15°$ को मान निकाल्नुहोस् । (Find the value of $\sin 15°$ if $\cos 30° = \frac{\sqrt{3}}{2}$.) [2066 S]

5F. यदि $\cos 45° = \frac{1}{\sqrt{2}}$ भए प्रमाणित गर्नुहोस् (If $\cos 45° = \frac{1}{\sqrt{2}}$, prove that) : $\sin\left(22\frac{1}{2}\right)^\circ = \frac{\sqrt{2-\sqrt{2}}}{2}$ [2069 S]

5G. यदि $\cos 45° = \frac{1}{\sqrt{2}}$ भए, प्रमाणित गर्नुहोस् (If $\cos 45° = \frac{1}{\sqrt{2}}$, prove that): $\tan 22\frac{1^\circ}{2} = \sqrt{2} - 1$ [SEE 2073 S']

5H. यदि $\cos 45° = \frac{1}{\sqrt{2}}$ भए प्रमाणित गर्नुहोस् (If $\cos 45° = \frac{1}{\sqrt{2}}$, prove that): $\cos 22\frac{1}{2}° = \frac{1}{2}\sqrt{2+\sqrt{2}}$

5I. यदि $\cos 45° = \frac{1}{\sqrt{2}}$ भए प्रमाणित गर्नुहोस् (If $\cos 45° = \frac{1}{\sqrt{2}}$, prove that): $\tan 22\frac{1^\circ}{2} = \sqrt{2-\sqrt{2}}$

6A. $\cos A$ लाई $\cot\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\cos A$ interms of $\cot\frac{A}{2}$.) [2067 R]

6B. $\sin A$ लाई $\cot\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\sin A$ interms of $\cot\frac{A}{2}$.)

7A. क्याल्कुलेटर वा तालिका प्रयोग नगरीकन $\cos 15°$ को मान पत्ता लगाउनुहोस् ।
Calculate the value of $\cos 15°$ without using a calculator or table. [SEE 2074 S']

7B. क्याल्कुलेटर वा तालिका प्रयोग नगरीकन $\sin 15°$ को मान पत्ता लगाउनुहोस् ।
Calculate the value of $\sin 15°$ without using a calculator or table.

7C. क्याल्कुलेटर वा तालिका प्रयोग नगरीकन $\tan 15°$ को मान पत्ता लगाउनुहोस् ।
Calculate the value of $\tan 15°$ without using a calculator or table.

Model 4 *Problems based on finding value of trigonometric ratios of A when A/3 is given.*

8A. यदि $\sin\frac{A}{3} = \frac{4}{5}$ भए $\sin A$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{A}{3} = \frac{4}{5}$ then find the value of $\sin A$.) [SEE 2073 R', 2065 E]

8B. यदि $\cos\frac{\alpha}{3} = \frac{1}{2}$ भए $\sin\alpha$ को मान निकाल्नुहोस् । (If $\cos\frac{\alpha}{3} = \frac{1}{2}$, find the value of $\sin\alpha$.) [2070 R]

8C. यदि $\cos\frac{\alpha}{3} = \frac{1}{2}$ भए $\cos\alpha$ को मान पत्ता लगाउनुहोस् । (If $\cos\frac{\alpha}{3} = \frac{1}{2}$, find the value of $\cos\alpha$.) [2069 R']

8D. यदि $\cos\frac{\theta}{3} = \frac{3}{5}$ भए $\cos\theta$ को मान पत्ता लगाउनुहोस् । (If $\cos\frac{\theta}{3} = \frac{3}{5}$, find the value of $\cos\theta$.) [2058 S]

8E. यदि $\sin\frac{\alpha}{3} = \frac{3}{5}$ भए $\sin\alpha$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{\alpha}{3} = \frac{3}{5}$, find the value of $\sin\alpha$.) [2060R, 2066R', SEE 2073 S, SEE 2074 R]

8F. यदि $\sin\frac{\theta}{3} = \frac{1}{2}$ भए $\sin\theta$ को मान पत्ता लगाउनुहोस् । (If $\sin\frac{\theta}{3} = \frac{1}{2}$, find the value of $\sin\theta$.) [2061S, 2065S, 2066R]

Model 5 *Problems based on proving trigonometric identities based on 2A.*

9. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos^2(45° + A) + \cos^2(45° - A) = 1$ [2069 S]

B. $\sin^2(45° - \theta) + \sin^2(45° + \theta) = 1$ [60C, 63R]

C. $1 - \sin 2A = 2\sin^2(45° - A)$ [63M, 65R]

D. $1 + \cos 2A = 2\sin^2(90° - A)$ [2064 R']

E. $2\cos^2(45° - \theta) = 1 + \sin 2\theta$ [2066 R']

F. $\frac{1 + \tan^2(45° - \beta)}{1 - \tan^2(45° - \beta)} = \text{cosec } 2\beta$ [2067 R']

G. $\frac{1 - \tan^2(45° - A)}{1 + \tan^2(45° - A)} = \sin 2A$ [2069 R']

H. $\tan\left(\frac{\pi^c}{4} + B\right) = \frac{\cos 2B}{1 - \sin 2B}$

pueplas.com.np

www.drmathnp.com

10. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{1-\cos 2A}{\sin 2A} = \tan A$ [2068 R'] **B.** $\frac{\sin 2A}{1+\cos 2A} = \tan A$ [2068 S]

C. $\frac{\sin 2A - \sin A}{1 - \cos A + \cos 2A} = \tan A$ [2064 S] **D.** $\frac{\sin\alpha + \sin 2\alpha}{1+\cos\alpha+\cos 2\alpha} = \tan\alpha$ [66 S, 60 S', 2072S]

E. $\frac{1+\cos A+\cos 2A}{\sin A+\sin 2A} = \cot A$ [2075 R'$_2$] **F.** $\sin A.\cos 2A = \frac{1}{4}\sin 4A.\sec A$ [2065 S]

G. $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$ [2070 R'] **H.** $\frac{1-2\sin A.\cos A}{2} = \sin^2\left(\frac{\pi^c}{4} - A\right)$ [2070 S]

I. $\sin^4 B + \cos^4 B = 1 - \frac{1}{2}\sin^2 2B$ [SEE 2075 R] OR, $\cos^4 A + \sin^4 A = 1 - \frac{1}{2}\sin^2 2A$ [2075 R']

11A. $\frac{1+\cos 2A}{\sin 2A}$ लाई cot A को रूपमा व्यक्त गर्नुहोस् । (Express $\frac{1+\cos 2A}{\sin 2A}$ interms of cot A.)

11B. $\frac{\cos 2A}{1-\sin 2A}$ लाई tan A को रूपमा व्यक्त गर्नुहोस् । (Express $\frac{\cos 2A}{1-\sin 2A}$ interms of tan A.)

11C. $\sqrt{2+\sqrt{2+2\cos 4x}}$ लाई cos x को रूपमा व्यक्त गर्नुहोस् । (Express $\sqrt{2+\sqrt{2+2\cos 4x}}$ interms of cos x.)

11D. सरल गर्नुहोस् (Simplify): $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A}$

TRIGONOMETRY

Model 6 *Problems based on proving trigonometric identities based on A/2.*

12. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{2\sin\beta + \sin 2\beta}{2\sin\beta - \sin 2\beta} = \cot^2\frac{\beta}{2}$ [2059 R] **B.** $\frac{\cos\alpha}{1-\sin\alpha} = \frac{1+\tan\alpha/2}{1-\tan\alpha/2}$ [2059 S]

C. $\cot A = \frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$ [2060 R] **D.** $\cos^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right) - \sin^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right) = \sin\frac{\theta}{2}$ [2061R]

E. $\frac{1-\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)}{1+\tan^2\left(\frac{\pi}{4}-\frac{\theta}{4}\right)} = \sin\frac{\theta}{2}$ [2064 R] **F.** $\frac{\sin\frac{\theta}{2} - \sqrt{1+\sin\theta}}{\cos\frac{\theta}{2} - \sqrt{1+\sin\theta}} = \cot\frac{\theta}{2}$ [2063 R']

G. $\frac{\sin\frac{A}{2} + \sin A}{1+\cos\frac{A}{2}+\cos A} = \tan\frac{A}{2}$ [2065 M, 2072 R'] **H.** $\tan\frac{\theta}{2} = \frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$ [67 S, 2072 R, S, 2073 R']

I. $\frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} = \cot\frac{\theta}{2}$ [2075 R', 2069 R, 2071 R'] OR, $\frac{1+\sin A+\cos A}{1+\sin A-\cos A} = \cot\frac{A}{2}$ [SEE 2074 S']

J. $\frac{1+\cos A+\sin A}{1-\cos A+\sin A} = \frac{1}{\tan\frac{A}{2}}$ [2068 S'] **K.** $\frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$ [2065 R]

L. $\frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$ [SEE 2074 S'] **M.** $\cot\frac{A}{2} - \tan\frac{A}{2} = 2\cot A$ [2071 R]

N. $\operatorname{cosec}\theta - \cot\theta = \tan\frac{\theta}{2}$ [2071 S] **O.** $\operatorname{cosec} 20° - \cot 20° = \tan 10°$

13A. $\frac{\sin A}{1+\cos A}$ लाई tangent अनुपातको अपवर्तकको रूपमा व्यक्त गर्नुहोस् ।

Express $\frac{\sin A}{1+\cos A}$ in terms of sub-multiple angle of tangent. [SEE MODEL 2076]

13B. $\frac{1-\cos A}{\sin A}$ लाई $\tan\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । (Express $\frac{1-\cos A}{\sin A}$ in terms of $\tan\frac{A}{2}$.)

14A. यदि $\sin\frac{\theta}{2} = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए प्रमाणित गर्नुहोस् (If $\sin\frac{\theta}{2} = \frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that): $\cos\theta = -\frac{1}{2}\left(a^2+\frac{1}{a^2}\right)$ [2074 R']

puspas.com.np

www.drmathnp.com

14B. यदि $\cos\frac{\alpha}{2}=\frac{1}{2}\left(\frac{k^2+1}{k}\right)$ भए प्रमाणित गर्नुहोस् (If $\cos\frac{\alpha}{2}=\frac{1}{2}\left(\frac{k^2+1}{k}\right)$ then prove that): $\cos\alpha=\frac{1}{2}\left(\frac{k^4+1}{k^2}\right)$

14C. यदि $\sin\frac{\alpha}{2}=\frac{1}{2}\left(\frac{k^2+1}{k}\right)$ भए प्रमाणित गर्नुहोस् (If $\sin\frac{\alpha}{2}=\frac{1}{2}\left(\frac{k^2+1}{k}\right)$ then prove that): $\cos\alpha=-\frac{1}{2}\left(\frac{k^4+1}{k^2}\right)$

15A. यदि $\sin\frac{A}{3}=\frac{1}{2}\left(a+\frac{1}{a}\right)$ भए प्रमाणित गुर्नहोस् । (If $\sin\frac{A}{3}=\frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that): $\sin A=-\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ [SEE 2075 R]

15B. यदि $\cos\frac{B}{3}=\frac{1}{2}\left(m+\frac{1}{m}\right)$ भए प्रमाणित गुर्नहोस् (If $\cos\frac{B}{3}=\frac{1}{2}\left(m+\frac{1}{m}\right)$ then, prove that): $\cos B=\frac{1}{2}\left(m^3+\frac{1}{m^3}\right)$ [SEE 2075 R_2]

C. LONG QUESTIONS

Model 1 *Problems based on proving trigonometric identities based on 2A.*

प्रमाणित गर्नुहोस् (Prove that):

1. A. $\frac{1}{\sin 10^\circ}-\frac{\sqrt{3}}{\cos 10^\circ}=4$ OR $\operatorname{cosec}10^\circ-\sqrt{3}\sec 10^\circ=4$ [SEE 2075 R', 2065 M, 2059 S, 2063 M]

B. $\frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}=4$ OR $\sqrt{3}\operatorname{cosec}20^\circ-\sec 20^\circ=4$ [2061 R, 2065 E, 2065 S, 2067 R]

C. $\sec 20^\circ-\sqrt{3}\operatorname{cosec}20^\circ=-4$ [2068 S']

D. $\operatorname{cosec}40^\circ-\sqrt{3}\sec 40^\circ=4\cot 100^\circ$

2. A. $\cos^6\theta-\sin^6\theta=\cos 2\theta\left(1-\frac{1}{4}\sin^2 2\theta\right)$ [2058 S, 2062R]

B. $\cos^6\theta+\sin^6\theta=\frac{1}{8}(5+3\cos 4\theta)$ [2063 R']

C. $8(\sin^6 p+\cos^6 p)=5+3\cos 4p$ [SEE 2073 R']

D. $\cos^8\theta+\sin^8\theta=1-\sin^2 2\theta+\frac{1}{8}\sin^4 2\theta$ [2064 R]

3. A. $\operatorname{cosec}2A+\operatorname{cosec}4A=\cot A-\cot 4A$ [2074 S, SEE 2074 R]

B. $\frac{1}{\sin 2A}+\frac{\cos 4A}{\sin 4A}=\cot A-\operatorname{cosec}4A$ [2060 CP]

C. $\operatorname{cosec}2A+\cot 4A=\cot A-\operatorname{cosec}4A$ [2063 R]

D. $\operatorname{cosec}20^\circ+\cot 40^\circ=\cot 10^\circ-\operatorname{cosec}40^\circ$

E. $\operatorname{cosec}40^\circ-\cot 80^\circ=\tan 20^\circ+\operatorname{cosec}80^\circ$

F. $\operatorname{cosec}16^\circ-\cot 32^\circ=\tan 8^\circ+\operatorname{cosec}32^\circ$

4. A. $4\operatorname{cosec}2A.\cot 2A=\operatorname{cosec}^2 A-\sec^2 A$ [2061 S]

B. $(1+\sin 2A+\cos 2A)^2=4\cos^2 A(1+\sin 2A)$ [2064 S]

5. A. $\cos^2\alpha+\sin^2\alpha.\cos 2\beta=\cos^2\beta+\sin^2\beta.\cos 2\alpha$ [2066 R, 2071 R']

HINT: LHS $=\cos^2\alpha+\sin^2\alpha\cos 2\beta$ $=\cos^2\alpha+\sin^2\alpha(1-2\sin^2\beta)$
$=\cos^2\alpha+\sin^2\alpha-2\sin^2\alpha\sin^2\beta$ $=1-2\sin^2\alpha\sin^2\beta$
$=1-(1-\cos 2\alpha)\sin^2\beta$ $=1-\sin^2\beta+\cos 2\alpha\sin^2\beta$ $=\cos^2\beta+\cos 2\alpha\sin^2\beta$

B. $\sin^2 A-\cos^2 A.\cos 2B=\sin^2 B-\cos^2 B.\cos 2A$ [SEE 2074 R']

C. $\cos^3 A.\cos 3A+\sin^3 A.\sin 3A=\cos^3 2A$ [2071 R, 2070 S', 2071 S]

D. $\sin^2 A-\cos^2 A\cos 2B=\sin^2 B-\cos^2 B\cos 2A$

6. A. $\tan\theta+2\tan 2\theta+4\cot 4\theta=\cot\theta$ [2072 R]

B. $\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta=\cot\theta$

HINT: (b)

FIRST METHOD

LHS $=\tan\theta+2\tan 2\theta+4\tan 4\theta+8.\frac{1-\tan^2 4\theta}{2\tan 4\theta}$

$=\tan\theta+2\tan 2\theta+\frac{8\tan^2 4\theta+8-8\tan^2 4\theta}{2\tan 4\theta}$

$=\tan\theta+2\tan 2\theta+\frac{4}{\tan 4\theta}$

$=\tan\theta+\frac{4\tan^2 2\theta+4-4\tan^2 2\theta}{2\tan 2\theta}$

$=\tan\theta+\frac{2}{\tan 2\theta}$ simplify it.

SECOND METHOD

$\cot\theta-\tan\theta=\frac{1}{\tan\theta}-\tan\theta=\frac{1-\tan^2\theta}{\tan\theta}$

$=\frac{2(1-\tan^2\theta)}{2\tan\theta}=\frac{2}{\tan 2\theta}=2\cot 2\theta$

$\therefore \tan\theta=\cot\theta-2\cot 2\theta$

LHS $=\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta$
$=\cot\theta-2\cot 2\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta$
$=\cot\theta-2(\cot 2\theta-\tan 2\theta)+4\tan 4\theta+8\cot 8\theta$
$=\cot\theta-2\times 2\cot 4\theta+4\tan 4\theta+8\cot 8\theta$
$=\cot\theta-4(\cot 4\theta-\tan 4\theta)+8\cot 8\theta$
$=\cot\theta-4\times 2\cot 8\theta+8\cot 8\theta$
$=\cot\theta=$ RHS

TRIGONOMETRY

www.drmathnp.com

7. प्रमाणित गर्नुहोस् (Prove that):

A. $\sin^4 x = \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$ [2066 S] **B.** $\sin^4 2A = \frac{1}{8}(3 - 4\cos 4A + \cos 8A)$

C. $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$ **D.** $\frac{\sec 4\theta - 1}{\sec 2\theta - 1} = \tan 4\theta \cot\theta$ [SEE 2073 R, SEE 2073 S, SEE 2073 S*]

8. A. यदि $2\tan\alpha = 3\tan\beta$ भए $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $2\tan\alpha = 3\tan\beta$, prove that: $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$ [2065 R]

B. यदि $2\tan\alpha = 3\tan\beta$ भए $\tan(\alpha + \beta) = \frac{5\sin 2\beta}{5\cos 2\beta - 1}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $2\tan\alpha = 3\tan\beta$ then prove that: $\tan(\alpha + \beta) = \frac{5\sin 2\beta}{5\cos 2\beta - 1}$

Model 2 Problems based on proving trigonometric identities based on A/2.

प्रमाणित गर्नुहोस् (Prove that):

9. A. $\frac{\sin 2A}{1 + \cos 2A} \cdot \frac{\cos A}{1 + \cos A} = \tan\frac{A}{2}$ [2060 S] **B.** $\frac{1 + \cos 2\theta}{\sin 2\theta} \cdot \frac{1 + \cos\theta}{\cos\theta} = \cot\frac{\theta}{2}$ [2065 R']

10. A. $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\left(\frac{A - B}{2}\right)$ [2065 E]

B. $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\frac{\alpha - \beta}{2}\right)$

Model 3 Problems based on proving the trigonometric identities by substituting values.

11. प्रमाणित गर्नुहोस् (Prove that) :

A. $8\left(1 + \sin\frac{\pi^c}{8}\right)\left(1 + \sin\frac{3\pi^c}{8}\right)\left(1 - \sin\frac{5\pi^c}{8}\right)\left(1 - \sin\frac{7\pi^c}{8}\right) = 1$ [2070 R]

B. $\left(1 - \cos\frac{\pi^c}{8}\right)\left(1 - \cos\frac{3\pi^c}{8}\right)\left(1 - \cos\frac{5\pi^c}{8}\right)\left(1 - \cos\frac{7\pi^c}{8}\right) = \frac{1}{8}$

C. $\left(1 + \cos\frac{\pi^c}{8}\right)\left(1 + \cos\frac{3\pi^c}{8}\right)\left(1 + \cos\frac{5\pi^c}{8}\right)\left(1 + \cos\frac{7\pi^c}{8}\right) = \frac{1}{8}$

D. $\left(1 + \cos\frac{\pi^c}{12}\right)\left(1 + \cos\frac{5\pi^c}{12}\right)\left(1 + \cos\frac{7\pi^c}{12}\right)\left(1 + \cos\frac{11\pi^c}{12}\right) = \frac{1}{16}$

E. $\left(1 - \cos\frac{\pi^c}{12}\right)\left(1 - \cos\frac{5\pi^c}{12}\right)\left(1 - \cos\frac{7\pi^c}{12}\right)\left(1 - \cos\frac{11\pi^c}{12}\right) = \frac{1}{16}$

F. $\sin^4\frac{\pi^c}{8} + \sin^4\frac{3\pi^c}{8} + \sin^4\frac{5\pi^c}{8} + \sin^4\frac{7\pi^c}{8} = \frac{3}{2}$

G. $\cos^4\frac{\pi^c}{8} + \cos^4\frac{3\pi^c}{8} + \cos^4\frac{5\pi^c}{8} + \cos^4\frac{7\pi^c}{8} = \frac{3}{2}$

H. $\sin^6\frac{\pi^c}{8} + \sin^6\frac{3\pi^c}{8} + \sin^6\frac{5\pi^c}{8} + \sin^6\frac{7\pi^c}{8} = \frac{5}{4}$

ANSWERS

A. VERY SHORT QUESTIONS

(2A-17I) Show to your teacher **18A.** $\frac{1}{2}$ **18B.** $-\frac{1}{2}$ **18C.** $\frac{3}{5}$ **18D.** 1 **19A.** 0 **19B.** 1 **19C.** $-\frac{1}{2}$

B. SHORT QUESTIONS

1A. $-\frac{1}{8}$ **1B.** $\frac{7}{25}$ **1C.** $-\frac{7}{25}$ **1D.** $\frac{24}{25}$ **1E.** $\frac{3\sqrt{7}}{8}$ **1F.** $\frac{24}{25}$ **1G.** $\frac{120}{169}, \frac{-7}{25}$ **3A.** 1 **3B.** $-\frac{44}{125}$

3C. $\frac{-117}{125}$ **3D.** $-\frac{1}{8}, \frac{9}{16}$ **4A.** $\frac{24}{25}$ **4B.** $\frac{24}{7}$ **4C.** $\frac{1}{2}$ **4D.** $\frac{24}{25}$ **4E.** $-\frac{7}{25}$ **5D.** $2 + \sqrt{3}$

5E. $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ **6A.** $\frac{\cot^2\frac{A}{2} - 1}{\cot^2\frac{A}{2} + 1}$ **6B.** $\frac{2\cot\frac{A}{2}}{1 + \cot^2\frac{A}{2}}$ **7A.** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{2 + \sqrt{3}}}{2}$ **7B.** $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{2 - \sqrt{3}}}{2}$ **7C.** $2 - \sqrt{3}$

8A. $\frac{44}{125}$ **8B.** 0 **8C.** -1 **8D.** $-\frac{117}{125}$ **8E.** $\frac{117}{125}$ **8F.** 1 **11A.** $\cot A$ **11B.** $\frac{1 + \tan A}{1 - \tan A}$

11C. $2\cos x$ **11D.** 2 **13A.** $\tan\frac{A}{2}$ **13B.** $\tan\frac{A}{2}$

puspas.com.np

www.drmathnp.com

pupas.com.np

GENERAL EXERCISE 1

A. SHORT QUESTIONS

Model 1 *Problems based on expressing one interms of other.*

1A. sin 2A लाई cot A को रूपमा व्यक्त गर्नुहोस् । (Express sin 2A interms of cot A.)

1B. cos 2A लाई cot A को रूपमा व्यक्त गर्नुहोस् । (Express cos 2A interms of cot A.)

1C. sin A लाई $\cot\frac{A}{2}$ को रूपमा व्यक्त गर्नुहोस् । (Express sin A interms of $\cot\frac{A}{2}$.)

1D. cos 4A लाई cos A को रूपमा व्यक्त गर्नुहोस् । (Express cos 4A interms of cos A.)

1E. (sin 2A + cos 2A) लाई tan A को रूपमा व्यक्त गर्नुहोस् । (Express (sin 2A + cos 2A) interms of tan A.)

Model 2 *Problems based on finding value of trigonometric ratios of 2A.*

2. तलको अवस्थामा sin 2A को मान पत्ता लगाउनुहोस् (In the following conditions, find the value of sin 2A):

A. $\sin A = \frac{3}{5}$ **B.** $\cos A = \frac{4}{5}$ **C.** $\tan A = \frac{3}{4}$

3. तलका अवस्थामा cos 2A पत्ता लगाउनुहोस् (In the following condition, find the value of cos 2A):

A. $\cos A = \frac{15}{17}$ **B.** $\sin A = \frac{4}{5}$ **C.** $\tan A = \frac{5}{12}$

4. यदि 3tan A = 2 भए मान पत्ता लगाउनुहोस् (If 3tan A = 2, find the value of):

A. cos 2A **B.** sin 2A **C.** tan 2A

5A. यदि $\cos\theta = \frac{1}{2}\left(m+\frac{1}{m}\right)$ भए $\cos 2\theta = \frac{1}{2}\left(m^2+\frac{1}{m^2}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos\theta = \frac{1}{2}\left(m+\frac{1}{m}\right)$ then prove that: $\cos 2\theta = \frac{1}{2}\left(m^2+\frac{1}{m^2}\right)$.

5B. यदि $\sin\theta = \frac{1}{2}\left(m+\frac{1}{m}\right)$ भए $\cos 2\theta = -\frac{1}{2}\left(m^2+\frac{1}{m^2}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin\theta = \frac{1}{2}\left(m+\frac{1}{m}\right)$ then prove that: $\cos 2\theta = -\frac{1}{2}\left(m^2+\frac{1}{m^2}\right)$.

6A. यदि $\tan\theta = \frac{b}{a}$ भए $a\cos 2\theta + b\sin 2\theta = a$ हुन्छ भनी प्रमाणित गर्नुहोस् । (If $\tan\theta = \frac{b}{a}$, prove that $a\cos 2\theta + b\sin 2\theta = a$)

6B. यदि $\tan\frac{\theta}{2} = \frac{a}{b}$ भए sin θ + cos θ को मान पत्ता लगाउनुहोस् । (If $\tan\frac{\theta}{2} = \frac{a}{b}$, find the value of sin θ + cos θ.)

7. यदि 5sin A = 4 र A < 90° भए sin 4A को मान पत्ता लगाउनुहोस् । (If 5sin A = 4 and A < 90°, find the value of sin 4A.)

8A. यदि sec 2α = 3 भए tan α को मान निकाल्नुहोस् । (If sec 2α = 3, find the value of tan α.)

8B. यदि $\sec\alpha = \frac{5}{4}$, भए tan 2α को मान पत्ता लगाउनुहोस् । (If $\sec\alpha = \frac{5}{4}$, find the value of tan 2α.)

8C. यदि $\cos 2A = \frac{13}{36}$ भए $\cos A = \pm\frac{7}{6\sqrt{2}}$ हुन्छ भनी प्रमाणित गर्नुहोस् । (If $\cos 2A = \frac{13}{36}$, prove that: $\cos A = \pm\frac{7}{6\sqrt{2}}$)

8D. यदि $\cos 8y = \frac{13}{36}$ भए cos 4y को मान पत्ता लगाउनुहोस् । (If $\cos 8y = \frac{13}{36}$ then find the value of cos 4y.)

8E. यदि $\cos 40A = \frac{1}{9}$ भए sin 20A को मान पत्ता लगाउनुहोस् । (If $\cos 40A = \frac{1}{9}$ then find the value of sin 20A.)

Model 3 *Problems based on finding value of trigonometric ratios of 3A.*

9A. यदि $\sin\theta = 5\times10^{-1}$ भए sin 3θ र cos 3θ का मानहरू पत्ता लगाउनुहोस् ।
If $\sin\theta = 5\times10^{-1}$ then find the values of sin 3θ and cos 3θ.

9B. यदि $\cos A = 6\times10^{-1}$ भए sin 3A र cos 3A का मानहरू पत्ता लगाउनुहोस् ।
If $\cos A = 6\times10^{-1}$ then find the values of sin 3 A and cos 3A.

9C. यदि 3 tan (A + B) = 4 tan (A – B) = 12 भए tan 2A को मान पत्ता लगाउनुहोस् ।
If 3 tan (A + B) = 4 tan (A – B) = 12 then find the value of tan 2A.

www.drmathnp.com

puspas.com.np

10A. यदि $\sin\alpha = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए $\sin 3\alpha = -\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin\alpha = \frac{1}{2}\left(a+\frac{1}{a}\right)$ then prove that $\sin 3\alpha = -\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

10B. यदि $\cos\alpha = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए $\cos 3\alpha = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos\alpha = \frac{1}{2}\left(a+\frac{1}{a}\right)$ then prove that $\cos 3\alpha = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

11. $8\cos^3\frac{\pi^c}{9} - 6\cos\frac{\pi^c}{9}$ को मान पत्ता लगाउनुहोस् । (Find the value of $8\cos^3\frac{\pi^c}{9} - 6\cos\frac{\pi^c}{9}$.)

Model 4 *On finding value of trigonometric ratios of θ when the value of θ/2 is given.*

12. तलका प्रत्येक अवस्थामा $\sin\theta$, $\cos\theta$ र $\tan\theta$ का मानहरू पत्ता लगाउनुहोस् :
In each of the following conditions, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$:

A. $\sin\frac{\theta}{2} = \frac{3}{5}$ **B.** $\cos\frac{\theta}{2} = \frac{3}{5}$ **C.** $\tan\frac{\theta}{2} = \frac{5}{12}$

13. तलको प्रत्येक अवस्थामा $\sin\theta$, $\cos\theta$ र $\tan\theta$ का मानहरू पत्ता लगाउनुहोस् :
In each of the following conditions, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$:

A. $\sin\frac{\theta}{2} = \frac{3}{5}$ **B.** $\tan\frac{\theta}{2} = \frac{3}{4}$ **C.** $\tan\frac{\theta}{2} = \frac{1}{2}$

Model 5 *On finding value of trigonometric ratios of A/2 when value of trigonometric ratios of A is known.*

14. तलका त्रिकोणमितीय अनुपातहरूको क्याल्कुलेटरको प्रयोग नगरी मान पत्ता लगाउनुहोस् :
Find the value of the following trigonometrical ratios without using calculator:

A. $\cos 22°30'$ **B.** $\sin 22\frac{1}{2}°$ **C.** $\sin\frac{3\pi^c}{8}$ **D.** $\cos\frac{11\pi^c}{12}$

15A. यदि $\cos 330° = \frac{\sqrt{3}}{2}$, भए सिद्ध गर्नुहोस् (If $\cos 330° = \frac{\sqrt{3}}{2}$ then show that) : $\sin^2 165° = \frac{2-\sqrt{3}}{4}$

15B. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए $\sin 15°$ को मान पत्ता लगाउनुहोस् । (If $\cos 30° = \frac{\sqrt{3}}{2}$, find the value of $\sin 15°$.)

15C. यदि $\cos 30° = \frac{\sqrt{3}}{2}$ भए, सिद्ध गर्नुहोस् (If $\cos 30° = \frac{\sqrt{3}}{2}$, show that $\cos 15°$): $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

15D. यदि $\sin 30° = \frac{1}{2}$ हो भने प्रमाणित गर्नुहोस् (If $\sin 30° = \frac{1}{2}$, then prove that): $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

15E. यदि $5\cos\alpha = 3$ भए $\cos\frac{\alpha}{2}$ को धनात्मक मान निकाल्नुहोस् । (If $5\cos\alpha = 3$, find the positive value of $\cos\frac{\alpha}{2}$.)

15F. यदि $5\cos A = 4$ भए $\cos\frac{A}{2}$ को धनात्मक मान निकाल्नुहोस् । (If $5\cos A = 4$, find the positive value of $\cos\frac{A}{2}$.)

16. प्रमाणित गर्नुहोस् (Prove that):

A. $\tan 15° - \cot 15° = -2\sqrt{3}$ **B.** $\cot 22\frac{1}{2}° - \tan 22\frac{1}{2}° = 2$ **C.** $\tan 75° + \cot 75° = 4$

Model 6 *On finding value of trigonometric ratios of A when value of trigonometric ratios of A/3 is given.*

17A. यदि $\cos\frac{\theta}{3} = \frac{3}{5}$ भए $\sin\theta = \frac{44}{125}$ प्रमाणित गर्नुहोस् । (If $\cos\frac{\theta}{3} = \frac{3}{5}$, prove that: $\sin\theta = \frac{44}{125}$.)

17B. यदि $\cos\frac{\theta}{3} = \frac{2}{3}$ भए $\cos\theta = -\frac{22}{27}$ र $\sin^2\theta = \frac{245}{729}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos\frac{\theta}{3} = \frac{2}{3}$ then prove that: $\cos\theta = -\frac{22}{27}$ and $\sin^2\theta = \frac{245}{729}$

18A. यदि $\sin\frac{\alpha}{3} = \frac{4}{5}$ भए $\sin\alpha = \frac{44}{125}$ र $\cos\alpha = -\frac{117}{125}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin\frac{\alpha}{3} = \frac{4}{5}$ then prove that $\sin\alpha = \frac{44}{125}$ and $\cos\alpha = -\frac{117}{125}$.

www.drmathnp.com

pueas.com.np

18B. यदि $\sin\frac{\alpha}{3}=\frac{3}{5}$ प्रमाणित गर्नुहोस् (If $\sin\frac{\alpha}{3}=\frac{3}{5}$ then prove that) : $\cos\alpha = -\frac{44}{125}$

18C. यदि $\sin\frac{\alpha}{3}=\frac{1}{2}$ भए $\sin\alpha$ को मान निकाल्नुहोस् । (If $\sin\frac{\alpha}{3}=\frac{1}{2}$, find the value of $\sin\alpha$.)

19A. यदि $\sin\frac{\alpha}{6}=\frac{1}{2}\left(p+\frac{1}{p}\right)$ भए प्रमाणित गर्नुहोस्: $\sin\frac{\alpha}{2}=-\frac{1}{2}\left(p^3+\frac{1}{p^3}\right)$

If $\sin\frac{\alpha}{6}=\frac{1}{2}\left(p+\frac{1}{p}\right)$ then prove that: $\sin\frac{\alpha}{2}=-\frac{1}{2}\left(p^3+\frac{1}{p^3}\right)$

19B. यदि $\cos\frac{\beta}{6}=\frac{1}{2}\left(a+\frac{1}{a}\right)$ भए प्रमाणित गर्नुहोस् (If $\cos\frac{\beta}{6}=\frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that) : $\cos\frac{\beta}{2}=\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

20. प्रमाणित गर्नुहोस् (Prove that):

A. $8\cos^3\frac{\pi^c}{9}-6\cos\frac{\pi}{9}=1$ **B.** $8\cos^3\frac{\pi^c}{12}-6\cos\frac{\pi}{12}=\sqrt{2}$

C. $6\sin\frac{\pi}{18}-8\sin^3\frac{\pi}{18}=1$ **D.** $12\sin\frac{\pi}{9}-16\sin^3\frac{\pi}{9}=2\sqrt{3}$

Model 7 Problems based on proving trigonometric identities based on 2A.

21. प्रमाणित गर्नुहोस् (Prove that):

A. $(\sin\theta+\cos\theta)^2=1+\sin 2\theta$ **B.** $(\sin\theta-\cos\theta)^2=1-\sin 2\theta$ **C.** $\frac{\sin 2A}{1-\cos 2A}=\cot A$

D. $\frac{\sin 2A}{1+\cos 2A}=\tan A$ **E.** $\frac{1+\cos 2A}{\sin 2A}=\cot A$ **F.** $\frac{1-\sin 2A}{\cos 2A}=\frac{1-\tan A}{1+\tan A}$

G. $\frac{1-\cos 2A}{2\cos A}=\sin A\tan A$ **H.** $\frac{\cos 2A}{1+\sin 2A}=\frac{1-\tan A}{1+\tan A}$ **I.** $\frac{1+\cos 2\alpha}{1-\cos 2\alpha}=\cot^2\alpha$

J. $\frac{1-\cos 2A}{1+\cos 2A}=\tan^2 A$ **K.** $2\cot 2A=\cot A-\tan A$ **L.** $\frac{1}{\tan A}-\frac{1}{\tan 2A}=\operatorname{cosec} 2A$

M. $\cos^4 A-\sin^4 A=\cos 2A$ **N.** $\cos^2\alpha=\frac{\tan 2\alpha+\sin 2\alpha}{2\tan 2\alpha}$ **O.** $\cos^2\alpha-\sin^2\alpha=\frac{\sin 4\alpha}{2\sin 2\alpha}$

P. $\tan(45°+\theta)=\sec 2\theta+\tan 2\theta$ **Q.** $\tan(45°-\theta)=\sec 2\theta-\tan 2\theta$ **R.** $\tan(225°+A)=\frac{\cos 2A}{1-\sin 2A}$

S. $\frac{\sin^3\alpha+\cos^3\alpha}{\sin\alpha+\cos\alpha}=1-\frac{1}{2}\sin 2\alpha$ **T.** $\operatorname{cosec} 2\theta+\cot 2\theta=\cot\theta$ **U.** $\operatorname{cosec} 2\theta-\cot 2\theta=\tan\theta$

V. $\cos 4A=1-8\sin^2 A+8\sin^4 A$ **W.** $\frac{1-\tan^2(45°-\theta)}{1+\tan^2(45°-\theta)}=\sin 2\theta$ **X.** $\frac{\sin 2A-\cos A}{1-\sin A-\cos 2A}=\cot A$

Y. $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}=\frac{\sec A}{\operatorname{cosec} A}$ **Z.** $\frac{1+\sin 4A}{\cos 4A}=\frac{1+\tan 2A}{1-\tan 2A}$ **ZA.** $\frac{1+\cos 4A}{1-\cos 4A}=\cot^2 2A$

ZB. $2\cos^2\theta+\cos^2 2\theta-2\cos^2\theta\cos 2\theta=1$ **ZC.** $2\cos^2 2\theta+\cos^2 4\theta-2\cos^2 2\theta\cos 4\theta=1$

Model 8 Problems based on proving trigonometric identities based on A/2.

22. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{1-\cos A}{\sin A}=\tan\frac{A}{2}$ **B.** $\frac{\sin A}{1+\cos A}=\tan\frac{A}{2}$ **C.** $\frac{1-\sin A}{\cos A}=\frac{1-\tan A/2}{1+\tan A/2}$

D. $\frac{1-\cos A}{2\cos A/2}=\sin\frac{A}{2}\tan\frac{A}{2}$ **E.** $\frac{\cos A}{1+\sin A}=\frac{1-\tan A/2}{1+\tan A/2}$ **F.** $\frac{1+\cos\alpha}{1-\cos\alpha}=\cot^2\frac{\alpha}{2}$

G. $\frac{1-\cos A}{1+\cos A}=\tan^2\frac{A}{2}$ **H.** $2\cot A=\cot\frac{A}{2}-\tan\frac{A}{2}$ **I.** $\frac{\sin\frac{\alpha}{2}+\sin\alpha}{1+\cos\frac{\alpha}{2}+\cos\alpha}=\tan\frac{\alpha}{2}$

J. $\cos^4\frac{A}{2}-\sin^4\frac{A}{2}=\cos A$ **K.** $\cos^2\frac{\alpha}{2}=\frac{\tan\alpha+\sin\alpha}{2\tan\alpha}$ **L.** $\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}=\frac{\sin 2\alpha}{2\sin\alpha}$

www.drmathnp.com

M. $\frac{\sin^3\frac{\alpha}{2}+\cos^3\frac{\alpha}{2}}{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}} = 1-\frac{1}{2}\sin\alpha$ **N.** $\frac{\sin^3\frac{\alpha}{2}-\cos^3\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}} = 1+\frac{1}{2}\sin\alpha$ **O.** $\sec A - \tan A = \frac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}}$

P. $\tan(45°+\frac{\theta}{2}) = \sec\theta + \tan\theta$ **Q.** $\frac{\tan x+\sin x}{2\tan x} = \cos^2\frac{x}{2}$ **R.** $\cos A\cos 2A\cos 4A = \frac{\sin 8A}{8\sin A}$

S. $\frac{\cos\frac{\theta}{2}-\sqrt{1+\sin\theta}}{\sin\frac{\theta}{2}-\sqrt{1+\sin\theta}} = \tan\frac{\theta}{2}$ **T.** $\text{cosec}\,\theta - \cot\theta = \tan\frac{\theta}{2}$ **U.** $\tan\frac{\theta}{2}+\cot\frac{\theta}{2} = 2\text{cosec}\,\theta$

Model 9 Miscellaneous Problems.

23. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{\sin 20°-\sin 10°}{1-\cos 10°+\cos 20°} = \tan 10°$ **B.** $\frac{\sin 25°+\sin 50°}{1+\cos 25°+\cos 50°} = \tan 25°$

C. $4\sin 12°.\cos 24° = \sin 48.\sec 12°$ **D.** $\frac{2\sin 8°+\sin 16°}{2\sin 8°-\sin 16°} = \cot^2 4°$

E. $\frac{\cos 20°}{1-\sin 20°} = \frac{1+\tan 10°}{1-\tan 10°}$ **F.** $2\cot 20° = (\cot 10° - \tan 10°)$

G. $\frac{\sin 20°}{1+\cos 20°}\times\frac{\cos 10°}{1-\cos 10°} = \cot 5°$ **H.** $\frac{1-\cos 20°+\sin 20°}{1+\cos 20°+\sin 20°} = \tan 10°$

24. प्रमाणित गर्नुहोस् (Prove that):

A. $\tan A - \cot A = -2\cot 2A$

HINT: LHS $= \frac{\sin A}{\cos A}-\frac{\cos A}{\sin A} = \frac{\sin^2 A-\cos^2 A}{\sin A\cos A} = \frac{2(\cos^2 A-\sin^2 A)}{2\sin A\cos A} = -\frac{2\cos 2A}{\sin 2A} = -2\cot 2A$ = RHS

B. $\cot 20° - \tan 20° = 2\cot 40°$ **C.** $\tan 75° + \cot 75° = 4$

25. मान पत्ता लगाउनुहोस् (Find the value of):

A. $\tan 15° - \cot 15°$ **B.** $\tan 75° - \cot 75°$

C. $\cot 22\frac{1°}{2} - \tan 22\frac{1°}{2}$ **D.** $\cot 67\frac{1°}{2} - \tan 67\frac{1°}{2}$

B. LONG QUESTIONS

Model 1 Problems based on proving trigonometric identities of 2A.

प्रमाणित गर्नुहोस् (Prove that):

1. A. $(\cos 2\alpha - \cos 2\beta)^2 + (\sin 2\alpha + \sin 2\beta)^2 = 4\sin^2(\alpha+\beta)$

B. $(\cos 2\alpha - \cos 2\beta)^2 + (\sin 2\alpha - \sin 2\beta)^2 = 4\sin^2(\alpha-\beta)$

C. $(\cos 2A + \cos 2B)^2 + (\sin 2A + \sin 2B)^2 = 4\cos^2(A-B)$

D. $(\cos 2A + \cos 2B)^2 + (\sin 2A - \sin 2B)^2 = 4\cos^2(A+B)$

2. A. $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1) = 2\cos 4\theta + 1$

B. $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1) = 2\cos 8\theta + 1$

3. A. $\frac{4(\cos^4 A+\sin^4 A)}{\cos^4 A-\sin^4 A} = (3+\cos 4A)\sec 2A$ **B.** $\cos^6\theta+\sin^6\theta = \frac{1}{4}(1+3\cos^2 2\theta)$

C. $\cos^6 A - \sin^6 A = \frac{1}{4}(\cos^3 2A + 3\cos 2A)$ **D.** $\cos^6 A - \sin^6 A = \cos 2A\left(1-\frac{1}{4}\sin^2 2A\right)$

E. $4(\cos^6\theta-\sin^6\theta) = \cos^3 2\theta + 3\cos 2\theta$ **F.** $2(\sin^6 x+\cos^6 x) - 3(\sin^4 x+\cos^4 x) + 1 = 0$

G. $3(\sin x-\cos x)^4 + 6(\sin x+\cos x)^2 + 4(\sin^6 x+\cos^6 x) = 13$

4. A. $\frac{1}{\cos 40°}+\frac{\sqrt{3}}{\sin 40°} = 4$ **B.** $\frac{1}{\sin 50°}+\frac{\sqrt{3}}{\cos 50°} = 4$ **C.** $\frac{1}{\sin 70°}+\frac{\sqrt{3}}{\cos 70°} = \frac{4\cos 10°}{\cos 50°}$

D. $\frac{1}{\cos 50°}+\frac{\sqrt{3}}{\sin 50°} = \frac{4\cos 20°}{\cos 10°}$ **E.** $\frac{1}{\cos 80°}-\frac{\sqrt{3}}{\sin 80°} = 4$ **F.** $\frac{\sqrt{3}}{\text{cosec}\,75°}-\frac{1}{\sec 75°} = \sqrt{2}$

G. $\text{cosec}\,20° + \sqrt{3}\sec 20° = 4\cot 40°$ **H.** $\tan 45°\sec 40° + \tan 60°\,\text{cosec}\,40° = 4$

puspas.com.np

www.drmathnp.com

pueplas.com.np

5. **A.** $\frac{1-\sin 2\theta}{\cos 2\theta}=\frac{1-\tan\theta}{1+\tan\theta}=\tan(45^\circ-\theta)$ **B.** $\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}}=\frac{\cos 2\theta}{1+\sin 2\theta}=\tan(45^\circ-\theta)$

C. $\frac{1+\sin 2\theta}{1-\sin 2\theta}=\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2=\left(\frac{\cot\theta+1}{\cot\theta-1}\right)^2$ **D.** $\frac{\cos 2A}{1+\sin 2A}=\frac{\cos A-\sin A}{\cos A+\sin A}=\frac{1-\tan A}{1+\tan A}$

6. **A.** $\cos(36^\circ-\theta)\cos(36^\circ+\theta)+\cos(54^\circ-\theta)\cos(54^\circ+\theta)=\cos 2\theta$

B. $\cos(35^\circ-\alpha)\cos(35^\circ+\alpha)+\cos(55^\circ-\alpha)\cos(55^\circ+\alpha)=\cos 2\alpha$

C. $\cos(56^\circ+\theta).\cos(56^\circ-\theta)+\cos(34^\circ+\theta).\cos(34^\circ-\theta)=\cos 2\theta$

D. $\cos(54^\circ+\theta).\cos(54^\circ-\theta)+\cos(36^\circ+\theta).\cos(36^\circ-\theta)=\cos 2\theta$

7. **A.** $\frac{\sin 3A}{\sin A}-\frac{\cos 3A}{\cos A}=2$ **B.** $\frac{\sin 5x}{\sin x}-\frac{\cos 5x}{\cos x}=4\cos 2x$

C. $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\tan\theta$ **D.** $\cot x=\frac{1+\sin 2x+\cos 2x}{1+\sin 2x-\cos 2x}$

8. **A.** $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}-\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=2\tan 2\theta$ **B.** $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=2\sec 2\theta$

C. $\frac{\cos A}{\cos A-\sin A}-\frac{\cos A}{\cos A+\sin A}=\tan 2A$ **D.** $\frac{\tan^2\theta}{\tan^2\theta-1}+\frac{\operatorname{cosec}^2\theta}{\sec^2\theta-\operatorname{cosec}^2\theta}=-\sec 2\theta$

E. $\frac{1}{\sin^2 A}-\frac{1}{\cos^2 A}=\frac{4}{\sin 2A\tan 2A}$ **F.** $\frac{1-\cos 2A}{\sin 2A}=\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A}$

9. **A.** $\frac{1}{\sin 4A}+\frac{\cos 8A}{\sin 8A}=\cot 2A-\operatorname{cosec} 8A$ **B.** $\frac{1}{\sin 12A}+\frac{\cos 24A}{\sin 24A}=\cot 6A-\operatorname{cosec} 24A$

C. $\operatorname{cosec} 8A+\cot 16A=\cot 4A-\operatorname{cosec} 16A$ **D.** $\operatorname{cosec} 4A+\operatorname{cosec} 8A=\cot 2A-\cot 8A$

E. $\operatorname{cosec} 2A-\cot 4A=\tan A+\operatorname{cosec} 4A$ **F.** $\operatorname{cosec} 4A-\cot 8A=\tan 2A+\operatorname{cosec} 8A$

10. **A.** $\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta}=\cos 2\theta$ **B.** $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}=\tan 2\theta+\sec 2\theta$

C. $\frac{\sec 12A-1}{\sec 6A-1}=\frac{\tan 12A}{\tan 3A}$ **D.** $\frac{\sec 16A-1}{\sec 8A-1}=\frac{\tan 16A}{\tan 4A}$

11. **A.** $\operatorname{cosec} 2A+\operatorname{cosec} 4A+\operatorname{cosec} 8A=\cot A-\cot 8A$

B. $\operatorname{cosec} 4A+\operatorname{cosec} 8A+\operatorname{cosec} 16A=\cot 2A-\cot 16A$

12. **A.** $\sqrt{2+\sqrt{2+2\cos 4\theta}}=2\cos\theta$ **B.** $2\cos\theta=\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

C. $2\cos A=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 16A}}}}$

13. **A.** $\frac{\cos 2\theta}{1+\sin 2\theta}=\tan\left(\frac{\pi^c}{4}-\theta\right)$ **B.** $\frac{\cos 2\theta}{1-\sin 2\theta}=\tan\left(\frac{\pi^c}{4}+\theta\right)$

C. $\tan(\theta+45^\circ)-\tan(\theta-45^\circ)=2\sec 2\theta$ **D.** $\tan(\theta+45^\circ)+\tan(\theta-45^\circ)=2\tan 2\theta$

E. $\cot\left(A+\frac{\pi^c}{4}\right)-\tan\left(A-\frac{\pi^c}{4}\right)=\frac{2\cos 2A}{1+\sin 2A}$ **F.** $\operatorname{cosec} 2\theta=\frac{\tan(45^\circ+\theta)+\tan(45^\circ-\theta)}{\tan(45^\circ+\theta)-\tan(45^\circ-\theta)}$

G. $\tan(60^\circ+A).\tan(60^\circ-A)=\frac{2\cos 2A+1}{2\cos 2A-1}$ **H.** $\cos^2\left(\frac{\pi}{4}-B\right)-\sin^2\left(\frac{\pi}{4}-B\right)=\sin 2B$

I. $\tan\left(C+\frac{\pi}{4}\right)+\tan\left(C-\frac{\pi}{4}\right)=2\tan 2C$ **J.** $2\sin^2\left(\frac{\pi}{4}-A\right)=\frac{(1-\tan A)^2}{1+\tan^2 A}$

14. **A.** $\frac{\tan 25^\circ+\tan 15^\circ}{\tan 25^\circ-\tan 15^\circ}=4\cos 10^\circ.\cos 20^\circ$ **B.** $\frac{\tan 70^\circ+\tan 20^\circ}{\tan 70^\circ-\tan 20^\circ}=\operatorname{cosec} 50^\circ$

C. $\frac{\tan 20^\circ+\tan 10^\circ}{\tan 20^\circ-\tan 10^\circ}=\frac{1}{2}\operatorname{cosec} 10^\circ$ **D.** $\frac{\cot 24^\circ+\cot 8^\circ}{\cot 24^\circ-\cot 8^\circ}=-2\cos 16^\circ$

www.drmathnp.com

15. A. $\frac{2\sin x}{\cos 3x} + \frac{2\sin 3x}{\cos 9x} + \frac{2\sin 9x}{\cos 27x} = \tan 27x - \tan x$

HINT: LHS $= \left\{\frac{2\sin x \cos x}{\cos 3x \cos x} + \frac{2\sin 3x \cos 3x}{\cos 9x \cos 3x} + \frac{2\sin 9x \cos 9x}{\cos 27x \cos 9x}\right\} = \left\{\frac{\sin 2x}{\cos 3x \cos x} + \frac{\sin 6x}{\cos 9x \cos 3x} + \frac{\sin 18x}{\cos 27x \cos 9x}\right\}$

$= \left\{\frac{\sin(3x-x)}{\cos 3x \cos x} + \frac{\sin(9x-3x)}{\cos 9x \cos 3x} + \frac{\sin(27x-9x)}{\cos 27x \cos 9x}\right\} = (\tan 3x - \tan x) + (\tan 9x - \tan 3x) + (\tan 27x - \tan 9x)$

$= (\tan 27x - \tan x)$

B. $\frac{2\sin 2x}{\cos 6x} + \frac{2\sin 6x}{\cos 18x} + \frac{2\sin 18x}{\cos 54x} = \tan 54x - \tan 2x$

C. $(2\cos\theta - 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1) = \frac{2\cos 8\theta + 1}{2\cos\theta + 1}$

HINT: LHS $= (2\cos\theta - 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1) = \frac{1}{2\cos\theta + 1}(4\cos^2\theta - 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1)$

$= \frac{1}{2\cos\theta + 1}[2(1 + \cos 2\theta) - 1](2\cos 2\theta - 1)(2\cos 4\theta - 1) = \frac{1}{2\cos\theta + 1}[(2\cos 2\theta + 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1)$

$= \frac{1}{2\cos\theta + 1}[(4\cos^2 2\theta - 1)(2\cos 4\theta - 1) = \frac{1}{2\cos\theta + 1}[(2(\cos 4\theta + 1) - 1](2\cos 4\theta - 1)$

$= \frac{1}{2\cos\theta + 1}[(2\cos 4\theta + 1)(2\cos 4\theta - 1) = \frac{1}{2\cos\theta + 1}(4\cos^2 4\theta - 1)$

$= \frac{1}{2\cos\theta + 1}[2(1 + \cos 8\theta) - 1] = \frac{2\cos 8\theta + 1}{2\cos\theta + 1}$

D. $(2\cos\theta - 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1)(2\cos 8\theta - 1) = \frac{2\cos 16\theta + 1}{2\cos\theta + 1}$

E. $\cos 4\theta - \cos 4\alpha = 8(\cos\theta - \cos\alpha)(\cos\theta + \cos\alpha)(\cos\theta - \sin\alpha)(\cos\theta + \sin\alpha)$

F. $\sin 128\theta = 128\sin\theta \cos\theta \cos 2\theta \cos 4\theta \cos 8\theta \cos 16\theta \cos 32\theta \cos 64\theta$

TRIGONOMETRY

Model 2 Problems based on proving trigonometric identities of 3A.

16. प्रमाणित गर्नुहोस् (Prove that):

A. $4(\cos^3 10 + \sin^3 20°) = 3(\cos 10° + \sin 20°)$

HINT: Use the relation: $4\cos^3 A = 3\cos A + \cos 3A$ and $4\sin^3 A = 3\sin A - \sin 3A$

B. $4(\cos^3 20° + \sin^3 10°) = 3(\cos 20° + \sin 10°)$

C. $4(\cos^3 20° + \sin^3 50°) = 3(\cos 20° + \sin 50°)$

17. A. $\frac{\cos^3\theta - \cos 3\theta}{\cos\theta} + \frac{\sin^3\theta + \sin 3\theta}{\sin\theta} = 3$ **B.** $\sin^3 A.\cos 3A + \cos^3 A.\sin 3A = \frac{3}{4}\sin 4A$

Or, $4\sin^3\theta.\cos 3\theta + 4\cos^3\theta.\sin 3\theta = 3\sin 4\theta$

HINT: LHS $= 4\sin^3\theta.\cos 3\theta + 4\cos^3\theta.\sin 3\theta = (3\sin\theta - \sin 3\theta)\cos 3\theta + (3\cos\theta + \cos 3\theta)\sin 3\theta$

$= 3\sin\theta\cos 3\theta - \sin 3\theta\cos 3\theta + 3\cos\theta\sin 3\theta + \cos 3\theta\sin 3\theta$

$= 3\sin\theta\cos 3\theta + 3\cos\theta\sin 3\theta = 3\sin(\theta + 3\theta) = 3\sin 4\theta$

C. $\sin^3 A\sin 3A + \cos^3 A\cos 3A = \cos^3 2A$

HINT: Use the formulae: $\sin^3 A = \frac{3\sin A - \sin 3A}{4}$ and $\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$

D. $\cos^3\theta\frac{\sin 3\theta}{3} + \sin^3\theta\frac{\cos 3\theta}{3} = \frac{\sin 4\theta}{4}$

18. A. $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ **B.** $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

C. $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$ **D.** $\cos^4 A = \frac{3}{8} + \frac{1}{2}\cos 2A + \frac{1}{8}\cos 4A$

19. A. $\cot(B + 45°) - \tan(B - 45°) = \frac{2\cos 2B}{1 + \sin 2B}$ **B.** $\tan(A + 30°) + \cot(A - 30°) = \frac{1}{\sin 2A - \sin 60°}$

Model 3 Problems based on proving trigonometric identities based on A/2.

20. प्रमाणित गर्नुहोस् (Prove that):

A. $(\cos\alpha - \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\sin^2\left(\frac{\alpha + \beta}{2}\right)$

B. $(\cos A + \cos B)^2 + (\sin A - \sin B)^2 = 4\cos^2\left(\frac{A + B}{2}\right)$

puspas.com.np

www.drmathnp.com

puepas.com.np

C. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\left(45° + \frac{\theta}{2}\right)$

D. $\tan\left(\frac{\pi}{4} - \frac{A}{2}\right) = \sqrt{\frac{1-\sin A}{1+\sin A}}$

E. $\sec\left(\frac{\pi}{4} + \frac{A}{2}\right)\sec\left(\frac{\pi}{4} + \frac{A}{2}\right) = 2\sec A$

F. $\frac{\cos\theta}{1+\sin\theta} = \tan\left(\frac{\pi^c}{4} - \frac{\theta}{2}\right)$

G. $\frac{\cos\theta}{1-\sin\theta} = \tan\left(\frac{\pi^c}{4} + \frac{\theta}{2}\right)$

H. $\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\frac{\theta}{2}$

I. $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} = \cot\frac{\theta}{2}$

J. $\frac{\cos\theta}{1+\sin\theta} = \tan\left(\frac{\pi^c}{4} - \frac{\theta}{2}\right)$

K. $\frac{\cos\theta}{1-\sin\theta} = \tan\left(\frac{\pi^c}{4} + \frac{\theta}{2}\right)$

L. $\frac{\sec\frac{\theta}{4}-1}{\sec\frac{\theta}{8}-1} = \tan\frac{\theta}{4}\cot\frac{\theta}{16}$

21. A. $\tan\left(\frac{\pi^c}{4} + \frac{A}{2}\right) = \sqrt{\frac{1+\sin A}{1-\sin A}} = \frac{1+\sin A}{\cos A}$

B. $\cot\left(\frac{A}{2} + \frac{\pi^c}{4}\right) - \tan\left(\frac{A}{2} - \frac{\pi^c}{4}\right) = \frac{2\cos A}{1+\sin A}$

C. $\cot\left(\frac{B}{2} + 45°\right) - \tan\left(\frac{B}{2} - 45°\right) = \frac{2\cos B}{1+\sin B}$

D. $\tan\left(\frac{A}{2} + 30°\right) + \cot\left(\frac{A}{2} - 30°\right) = \frac{1}{\sin A - \sin 60°}$

22. A. $\left(2\cos\frac{\theta}{2}+1\right)\left(2\cos\frac{\theta}{2}-1\right)(2\cos\theta - 1) = 2\cos 2\theta + 1$

B. $\left(2\cos\frac{\theta}{2}+1\right)\left(2\cos\frac{\theta}{2}-1\right)(2\cos\theta - 1)(2\cos 2\theta - 1) = 2\cos 4\theta + 1$

C. $(2\cos\theta + 1) = \left(2\cos\frac{\theta}{2}-1\right)\left(2\cos\frac{\theta}{4}-1\right)\left(2\cos\frac{\theta}{8}-1\right)\left(2\cos\frac{\theta}{8}+1\right)$

23. A. $\cos^6\frac{\theta}{4} - \sin^6\frac{\theta}{4} = \cos\frac{\theta}{2}\left(1 - \frac{1}{4}\sin^2\frac{\theta}{2}\right)$

B. $\cos^6\frac{\theta}{4} + \sin^6\frac{\theta}{4} = \frac{1}{8}(5 + 3\cos\theta)$

C. $\cos^8\frac{\theta}{4} + \sin^8\frac{\theta}{4} = 1 - \sin^2\frac{\theta}{2} + \frac{1}{8}\sin^4\frac{\theta}{2}$

D. $\frac{4\left(\cos^4\frac{A}{4} + \sin^4\frac{A}{4}\right)}{\cos^4\frac{A}{4} - \sin^4\frac{A}{4}} = (3 + \cos A)\sec\frac{A}{2}$

E. $\cos^6\frac{\theta}{4} + \sin^6\frac{\theta}{4} = \frac{1}{4}\left(1 + 3\cos^2\frac{\theta}{2}\right)$

F. $\cos^6\frac{A}{2} - \sin^6\frac{A}{2} = \frac{1}{4}(\cos^3 A + 3\cos A)$

G. $\cos^6\frac{A}{2} - \sin^6\frac{A}{2} = \cos A\left(1 - \frac{1}{4}\sin^2 A\right)$

H. $4\left(\cos^6\frac{\theta}{2} - \sin^6\frac{\theta}{2}\right) = \cos^3\theta + 3\cos\theta$

I. $2\left(\sin^6\frac{x}{2} + \cos^6\frac{x}{2}\right) - 3\left(\sin^4\frac{x}{2} + \cos^4\frac{x}{2}\right) + 1 = 0$

J. $8\left(\cos^6\frac{\theta}{2} - \sin^6\frac{\theta}{2}\right) = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}(7 + \cos 2\theta)$

K. $3\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^4 + 6\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 + 4\left(\sin^6\frac{x}{2} + \cos^6\frac{x}{2}\right) = 13$

Model 4 *Problems based on proving the trigonometric identities by substituting values.*

प्रमाणित गर्नुहोस् (Prove that):

24. A. $\cos^2 A + \cos^2\left(A + \frac{2\pi^c}{3}\right) + \cos^2\left(A - \frac{2\pi^c}{3}\right) = \frac{3}{2}$

B. $\cos^2 A + \cos^2\left(A + \frac{\pi^c}{3}\right) + \cos^2\left(A - \frac{\pi^c}{3}\right) = \frac{3}{2}$

C. $\frac{4\left(\cos^4\frac{\pi^c}{8} + \cos^4\frac{3\pi^c}{8}\right)}{\cos^4\frac{7\pi^c}{8} - \cos^4\frac{11\pi^c}{8}} = 2\sqrt{2}\sin\frac{\pi^c}{3}\tan\frac{\pi^c}{3}$

www.drmathnp.com

puspas.com.np

Model 5 *Problems based on finding the value of trigonometric ratios on the basis of given value.*

25. यदि $\cos\alpha = \frac{3}{5}$ र $\sin\beta = \frac{12}{13}$ तथा α र β दुवै धनात्मक न्यूनकोणहरू भए मान पत्ता लगाउनुहोस् :

If $\cos\alpha = \frac{3}{5}$ and $\sin\beta = \frac{12}{13}$, α and β being (+) ve acute angles, find the value of:

A. $\sin^2\left(\frac{\alpha-\beta}{2}\right)$ **B.** $\cos^2\left(\frac{\alpha+\beta}{2}\right)$ **C.** $\cos\left(\frac{\alpha-\beta}{2}\right)$

26. यदि $\cos\alpha = \frac{3}{5}$ र $\cos\beta = \frac{4}{5}$ भए $\cos\left(\frac{\alpha-\beta}{2}\right)$ को मान पत्ता लगाउनुहोस् ।

If $\cos\alpha = \frac{3}{5}$ and $\cos\beta = \frac{4}{5}$ then find the value of $\cos\left(\frac{\alpha-\beta}{2}\right)$. **HINT:** $\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}[1-\cos(\alpha-\beta)]$

Model 6 *Trigonometric identities of form cos A cos 2A cos 4A cos 8A .... cos nA = sin 2n A/2n sin A.*

27. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos A\cos 2A\cos 4A\cos 8A = \frac{\sin 16A}{16\sin A}$ **B.** $8\sin A\cos A\cos 2A\cos 4A = \sin 8A$

C. $16\sin A\cos A\cos 2A\cos 4A\cos 8A = \sin 16A$ **D.** $32\sin A\cos A\cos 2A\cos 4A\cos 8A\cos 16A = \sin 32A$

E. $16\cos 36^\circ\cos 72^\circ\cos 108^\circ\cos 144^\circ = 1$

HINT: LHS $= 16\cos 36^\circ\cos 72^\circ\cos 108^\circ\cos 144^\circ = 8\frac{(2\sin 36^\circ\cos 36^\circ)\cos 72^\circ\cos 108^\circ\cos 144^\circ}{\sin 36^\circ}$

$= 4\frac{2\sin 72^\circ\cos 72^\circ\cos 108^\circ\cos 144^\circ}{\sin 36^\circ} = 2\frac{(2\sin 144^\circ\cos 144^\circ)\cos 108^\circ}{\sin 36^\circ}$

$= 2\frac{\sin 288^\circ\cos 108^\circ}{8\sin 36^\circ} = 2\frac{\sin(270^\circ+18^\circ)\cos(90^\circ+18^\circ)}{\sin 36^\circ} = \frac{2\sin 18^\circ\cos 18^\circ}{\sin 36^\circ} = \frac{\sin 36^\circ}{\sin 36^\circ} = 1 = \text{RHS}$

F. $16\cos 24^\circ\cos 48^\circ\cos 96^\circ\cos 168^\circ = 1$

Model 7 *Problems based on conditional identities.*

28. यदि $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$ भए प्रमाणित गर्नुहोस् (If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$, prove that):

A. $\cos 2\theta = \frac{1}{2}\left(a^2+\frac{1}{a^2}\right)$ **B.** $\cos 3\theta = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

29A. यदि $\sin\frac{\theta}{3} = \frac{1}{2}\left(x+\frac{1}{x}\right)$ भए सिद्ध गर्नुहोस् (If $\sin\frac{\theta}{3} = \frac{1}{2}\left(x+\frac{1}{x}\right)$, prove that): $\sin\theta + \frac{1}{2}\left(x^3+\frac{1}{x^3}\right) = 0$

29B. यदि $\cos\frac{\theta}{3} = \frac{1}{2}\left(p+\frac{1}{p}\right)$ भए प्रमाणित गर्नुहोस् (If $\cos\frac{\theta}{3} = \frac{1}{2}\left(p+\frac{1}{p}\right)$ then prove that): $\cos\theta = \frac{1}{2}\left(p^3+\frac{1}{p^3}\right)$

30. यदि $2\tan\alpha = 3\tan\beta$ भए प्रमाणित गर्नुहोस् (If $2\tan\alpha = 3\tan\beta$ then prove that):

A. $\cot(\alpha-\beta) = \frac{5-\cos 2\beta}{\sin 2\beta}$ **B.** $\cos 2\alpha = \frac{13\cos 2\beta - 5}{13 - 5\cos 2\beta}$ **C.** $\sin 2\alpha = \frac{12\tan\beta}{4+9\tan^2\beta}$

31. यदि $2\tan\alpha = \tan\beta$ भए प्रमाणित गर्नुहोस् (If $2\tan\alpha = \tan\beta$ then prove that): $\cos 2\alpha = \frac{3+5\cos 2\beta}{5+3\cos 2\beta}$

32. यदि $2\tan\frac{\alpha}{2} = 3\tan\frac{\beta}{2}$ भए प्रमाणित गर्नुहोस् (If $2\tan\frac{\alpha}{2} = 3\tan\frac{\beta}{2}$ then prove that): $\cos\alpha = \frac{13\cos\beta - 5}{13 - 5\cos\beta}$

33. यदि $2\tan\frac{\alpha}{2} = \tan\frac{\beta}{2}$ भए प्रमाणित गर्नुहोस् (If $2\tan\frac{\alpha}{2} = \tan\frac{\beta}{2}$ then prove that): $\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$

34. यदि $\tan^2\alpha = 1 + 2\tan^2\beta$ भए प्रमाणित गर्नुहोस् (If $\tan^2\alpha = 1 + 2\tan^2\beta$, prove that) : $\cos 2\beta = 1 + 2\cos 2\alpha$

35A. यदि $7\tan\alpha = 1$ र $3\tan\beta = 1$ भए $(\alpha+2\beta) = 45^\circ$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $7\tan\alpha = 1$ and $3\tan\beta = 1$ then prove that: $(\alpha+2\beta) = 45^\circ$

35B. यदि $7\tan\alpha = 1$ र $3\tan\beta = 1$ भए $\cos 2\alpha = \sin 4\beta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $7\tan\alpha = 1$ and $3\tan\beta = 1$ then prove that: $\cos 2\alpha = \sin 4\beta$

TRIGONOMETRY

www.drmathnp.com

35C. यदि $\sin\alpha = \frac{4}{5}$ र $\cos\beta = \frac{5}{13}$ भए $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

ANSWERS

A. SHORT QUESTIONS

1A. $\frac{2\cot A}{1+\cot^2 A}$ **1B.** $\frac{\cot^2 A - 1}{\cot^2 A + 1}$ **1C.** $\frac{2\cot\frac{A}{2}}{1+\cot^2\frac{A}{2}}$ **1D.** $1 - 8\cos^2 A + 8\cos^4 A$ **1E.** $\frac{1+2\tan A - \tan^2 A}{1+\tan^2 A}$

2A. $\frac{24}{25}$ **2B.** $\frac{24}{25}$ **2C.** $\frac{24}{25}$ **3A.** $\frac{161}{289}$ **3B.** $\frac{-7}{25}$ **3C.** $\frac{7}{25}$ **4A.** $\frac{5}{13}$ **4B.** $\frac{12}{13}$

4C. $\frac{12}{5}$ **6B.** $\frac{2ab + b^2 - a^2}{a^2 + b^2}$ **7.** $-\frac{336}{625}$ **8A.** $\frac{1}{\sqrt{2}}$ **8B.** $\frac{24}{7}$ **8D.** $\frac{7\sqrt{2}}{12}$ **8E.** $\frac{2}{3}$

9A. 1, 0 **9B.** $\frac{44}{125}, -\frac{117}{125}$ **9C.** $-\frac{7}{11}$ **11.** 1 **12A.** $\frac{24}{25}, \frac{-7}{25}, \frac{-24}{7}$ **12B.** $\frac{24}{25}, -\frac{7}{25}, -\frac{24}{7}$

12C. $\frac{120}{169}, \frac{119}{169}, \frac{120}{119}$ **13A.** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ **13B.** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ **13C.** $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ **14A.** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **14B.** $\frac{\sqrt{2+\sqrt{2}}}{2}$

14C. $\frac{\sqrt{2+\sqrt{2}}}{2}$ **14D.** $-\frac{\sqrt{2+\sqrt{3}}}{2}$ **15B.** $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **15E.** $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **15F.** $\sqrt{\frac{4}{5}}$ **18C.** 1

25A. $-2\sqrt{3}$ **25B.** $2\sqrt{3}$ **25C.** 2 **25D.** -2

B. LONG QUESTIONS

25A. $\frac{1}{65}$ **25B.** $\frac{16}{65}$ **25C.** $\frac{8}{\sqrt{65}}$ **26.** $\frac{7\sqrt{2}}{10}$

CHALLENGING EXERCISE 1

1A. त्रिकोणमितीय तालिकाको प्रयोग नगरी $\sin 18°$ को मान पत्ता लगाउनुहोस् ।
Find the value of $\sin 18°$ without using trigonometric table.

HINT: Let, $A = 18$ then $5A = 90°$
or, $\cos 3A = \cos(90° - 2A)$
or, $\cos A(4\cos^2 A - 3) = 2\sin A.\cos A$
or, $4(1 - \sin^2 A) - 3 = 2\sin A$
$\therefore \sin A = \frac{-2 \pm \sqrt{4 - 4.4(-1)}}{2\times 4} = \frac{-2 \pm \sqrt{20}}{8} = \frac{2(-1 \pm \sqrt{5})}{8}$
or, $3A = 90° - 2A$
or, $4\cos^3 A - 3\cos A = \sin 2A$
or, $4\cos^2 A - 3 = 2\sin A$
or, $4\sin^2 A + 2\sin A - 1 = 0$
$\therefore \sin 18° = \frac{-1+\sqrt{5}}{4}$

1B. त्रिकोणमितीय तालिकाको प्रयोग नगरी $\cos 18°$ को मान पत्ता लगाउनुहोस् ।
Find the value of $\cos 18°$ without using trigonometric table.

1C. यदि $\sin 18° = \frac{\sqrt{5}-1}{4}$ र $\cos 36° = \frac{\sqrt{5}+1}{4}$ भए प्रमाणित गर्नुहोस् ।

If $\sin 18° = \frac{\sqrt{5}-1}{4}$ and $\cos 36° = \frac{\sqrt{5}+1}{4}$, prove that:

$$\left(1+\cos\frac{\pi^c}{10}\right)\left(1+\cos\frac{3\pi^c}{10}\right)\left(1+\cos\frac{7\pi^c}{10}\right)\left(1+\cos\frac{9\pi^c}{10}\right) = \frac{1}{16}$$

2A. यदि $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ भए $\tan 7\frac{1°}{2} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ then prove that: $\tan 7\frac{1°}{2} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$

HINT: Here, $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\sin 15° = \sqrt{1 - \cos^2 15°} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

We have, $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$ or, $\tan\frac{15°}{2} = \frac{1-\cos 15°}{\sin 15°} = \frac{1 - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3}-1}$

$= \frac{2\sqrt{6} + 2\sqrt{2} - 3 - \sqrt{3} - \sqrt{3} - 1}{3-1} = \frac{2\sqrt{6} - 2\sqrt{3} + 2\sqrt{2} - 4}{2} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$

www.drmathnp.com

2B. यदि $\tan 15° = 2 - \sqrt{3}$ भए $\tan 7\frac{1°}{2}$ को मान निकाल्नुहोस् । (If $\tan 15° = 2 - \sqrt{3}$, find the value of $\tan 7\frac{1°}{2}$.)

HINT: Here, $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$ or, $\tan 15° = \frac{2\tan\frac{15°}{2}}{1-\tan^2\frac{15°}{2}}$ or, $(2-\sqrt{3}) = \frac{2\tan\frac{15°}{2}}{1-\tan^2\frac{15°}{2}}$

or, $(2-\sqrt{3})\tan^2\frac{15°}{2} + 2\tan\frac{15°}{2} - (2-\sqrt{3}) = 0$ Using quadratic formula and simplification gives the result.

2C. प्रमाणित गर्नुहोस् (Prove that) : $\cot 7\frac{1°}{2} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

HINT: Here, $\cot 7\frac{1°}{2} = \frac{1+\cos 15°}{\sin 15°} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1} = \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{2}$

$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2} = \sqrt{6}+\sqrt{2}+\sqrt{3}+2 = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

2D. प्रमाणित गर्नुहोस् (Prove that) : $\sin 7\frac{1°}{2} = \frac{1}{4}\sqrt{8-2\sqrt{6}-2\sqrt{2}}$

HINT: $\sin 7\frac{1°}{2} = \sqrt{\frac{1}{2}(1-\cos 15°)} = \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} = \sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}} \times \frac{\sqrt{8}}{\sqrt{8}}$

2E. प्रमाणित गर्नुहोस् (Prove that) : $\tan 7°\,30' = \sqrt{6}-\sqrt{3}+\sqrt{2}-2$

HINT: Here, $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} \Rightarrow \tan\theta - \tan^2\frac{\theta}{2}.\tan\theta = 2\tan\frac{\theta}{2} \Rightarrow \tan^2\frac{\theta}{2}\tan\theta + 2\tan\frac{\theta}{2} - \tan\theta = 0$

It is quadratic in $\tan\frac{\theta}{2}$. So, $\tan\frac{\theta}{2} = \frac{-2+\sqrt{4+4.\tan\theta.\tan\theta}}{2.\tan\theta} = \frac{-1+\sqrt{1+\tan^2\theta}}{\tan\theta}$

Now, $\tan 7°\,30' = \frac{-1+\sqrt{1-(2-\sqrt{3})^2}}{2-\sqrt{3}}$ [$\because \tan 15° = 2-\sqrt{3}$] $= \frac{-1+\sqrt{8-4\sqrt{3}}}{2-\sqrt{3}}$

$= \frac{-1+\sqrt{(\sqrt{6}-\sqrt{2})^2}}{2-\sqrt{3}} = \frac{1+\sqrt{6}-\sqrt{2}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{-(2+\sqrt{3})+(\sqrt{6}-\sqrt{2})(2+\sqrt{3})}{4-3} = \frac{-2-\sqrt{3}+2\sqrt{6}+3\sqrt{2}-2\sqrt{2}-\sqrt{6}}{1}$

$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2$

2F. प्रमाणित गर्नुहोस् (Prove that) : $\cos 7.5° = \frac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}$

HINT: $\cos 7.5° = \sqrt{\frac{1+\cos 15°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{2}} = \sqrt{\frac{2\sqrt{2}+\sqrt{3}+1}{4\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}}$

2G. यदि $\tan\left(22\frac{1}{2}\right)° = \sqrt{2}-1$ भए $\tan 11\frac{1°}{4} = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\tan\left(22\frac{1}{2}\right)° = \sqrt{2}-1$, prove that $\tan 11\frac{1°}{4} = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$.

3. यदि $\sin\theta + \sin\alpha = x$ र $\cos\theta + \cos\alpha = y$ भए प्रमाणित गर्नुहोस : $\tan\left(\frac{\theta-\alpha}{2}\right) = \pm\sqrt{\frac{4-x^2-y^2}{x^2+y^2}}$

If $\sin\theta + \sin\alpha = x$ and $\cos\theta + \cos\alpha = y$ then prove that: $\tan\left(\frac{\theta-\alpha}{2}\right) = \pm\sqrt{\frac{4-x^2-y^2}{x^2+y^2}}$

HINT: $\sin\theta + \sin\alpha = x$ .....(i) $\quad \cos\theta + \cos\alpha = y$.....(ii)

Squaring and adding two equations; $\quad 2 + 2(\sin\theta\sin\alpha + \cos\theta\cos\alpha) = x^2 + y^2$

or, $\cos(\theta-\alpha) = \frac{x^2+y^2-2}{2}$ or, $\frac{1-\tan^2\left(\frac{\theta-\alpha}{2}\right)}{1+\tan^2\left(\frac{\theta-\alpha}{2}\right)} = \frac{x^2+y^2-2}{2}$

or, $\left(\frac{x^2+y^2-2}{2}+1\right)\tan^2\left(\frac{\theta-\alpha}{2}\right) = 1 - \frac{x^2+y^{-2}-2}{2}$ Simplify it then you will get the result.

TRIGONOMETRY

pusparas.com.np

www.drmathnp.com

4. यदि $\tan\frac{A}{2}=\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$ भए प्रमाणित गर्नुहोस् (If $\tan\frac{A}{2}=\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$, prove that) : $\cos\theta=\frac{\cos A-e}{1-e\cos A}$

HINT: Here, $\cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\frac{1-\left(\frac{1+e}{1-e}\right)\tan^2\frac{A}{2}}{1+\left(\frac{1+e}{1-e}\right)\tan^2\frac{A}{2}}=\frac{1-e-(1+e)\tan^2\frac{A}{2}}{1-e+(1+e)\tan^2\frac{A}{2}}=\frac{1-\tan^2\frac{A}{2}-e\left(1+\tan^2\frac{A}{2}\right)}{1+\tan^2\frac{A}{2}-e\left(1-\tan^2\frac{A}{2}\right)}=\frac{\left(\frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}\right)-e}{1-e\left(\frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}\right)}$

5. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos\frac{\pi^c}{7}\cos\frac{2\pi^c}{7}\cos\frac{3\pi^c}{7}=\frac{1}{8}$

HINT : LHS $=\cos\frac{\pi^c}{7}\cos\frac{2\pi^c}{7}\cos\frac{3\pi^c}{7}=\frac{2\sin\frac{\pi^c}{7}\cos\frac{\pi^c}{7}\cos\frac{2\pi^c}{7}\cos\frac{3\pi^c}{7}}{2\sin\frac{\pi^c}{7}}=\frac{\sin\frac{4\pi^c}{7}\cos\frac{3\pi^c}{7}}{4\sin\frac{\pi^c}{7}}$

$=\frac{\sin\left(\pi^c-\frac{3\pi^c}{7}\right)\cos\frac{3\pi^c}{7}}{4\sin\frac{\pi^c}{7}}=\frac{2\sin\frac{3\pi^c}{7}\cos\frac{3\pi^c}{7}}{8\sin\frac{\pi^c}{7}}=\frac{\sin\frac{6\pi^c}{7}}{8\sin\frac{\pi^c}{7}}=\frac{\sin\left(\pi^c-\frac{\pi^c}{7}\right)}{8\sin\frac{\pi^c}{7}}=\frac{1}{8}$

B. $\cos\frac{\pi^c}{7}\cos\frac{2\pi^c}{7}\cos\frac{4\pi^c}{7}=-\frac{1}{8}$

C. $\sin\frac{\pi^c}{18}.\sin\frac{5\pi^c}{18}.\sin\frac{7\pi^c}{18}=\frac{1}{8}$

D. $\cos\frac{2\pi^c}{15}\cos\frac{4\pi^c}{15}\cos\frac{8\pi^c}{15}\cos\frac{14\pi^c}{15}=\frac{1}{16}$

E. $8\cos\frac{\pi^c}{7}\sin\frac{3\pi^c}{14}\sin\frac{\pi^c}{14}=1$

HINT E.: $=8\cos\frac{\pi}{7}\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right).\sin\frac{\pi}{14}=8\cos\frac{\pi}{7}\cos\frac{2\pi}{7}.\sin\frac{\pi}{14}=\frac{4\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.2\sin\frac{\pi}{14}.\cos\frac{\pi}{14}}{\cos\frac{\pi}{14}}$

$=\frac{4\cos\frac{\pi}{7}.\sin\frac{\pi}{7}.\cos\frac{2\pi}{7}}{\cos\frac{\pi}{14}}=\frac{2\sin\frac{2\pi}{7}.\cos\frac{2\pi}{7}}{\cos\frac{\pi}{14}}=\frac{\sin\frac{4\pi}{7}}{\cos\frac{\pi}{14}}$

$=\frac{\cos\left(\frac{\pi}{2}-\frac{4\pi}{7}\right)}{\cos\frac{\pi}{14}}=\frac{\cos\left(-\frac{\pi}{14}\right)}{\cos\frac{\pi}{14}}=\frac{\cos\frac{\pi}{14}}{\cos\frac{\pi}{14}}=1$

F. $\sin 6°.\cos 12°.\cos 24°.\cos 48°=\frac{1}{16}$

G. $(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)=\tan 8\theta.\cot\theta$

HINT: LHS $=(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)=\frac{(1+\cos 2\theta)(1+\cos 4\theta)(1+\cos 8\theta)}{\cos 2\theta\cos 4\theta\cos 8\theta}$

$=\frac{(2\cos^2\theta)(2\cos^2 2\theta)(2\cos^2 4\theta)}{\cos 2\theta\cos 4\theta\cos 8\theta}=\frac{8\cos\theta}{\cos 8\theta}[\cos\theta\cos 2\theta\cos 4\theta]$

$=\frac{4\cos\theta}{\sin\theta\cos 8\theta}[2\sin\theta\cos\theta\cos 2\theta\cos 4\theta]=\frac{2\cos\theta}{\sin\theta\cos 8\theta}[2\sin 2\theta\cos 2\theta\cos 4\theta]$

$=\frac{\cos\theta}{\sin\theta\cos 8\theta}[2\sin 4\theta\cos 4\theta]=\frac{\cos\theta}{\sin\theta\cos 8\theta}\times\sin 8\theta=\cot\theta\tan\theta=\text{RHS}$

H. $(1+\sec 20°)(1+\sec 40°)(1+\sec 80°)=\cot^2 10°$

I. $(\sec^2 10°-1)(1+\sec 20°)(1+\sec 40°)(1+\sec 80°)=1$

6A. यदि $9A=\pi^c$ भए प्रमाणित गर्नुहोस् (If $9A=\pi^c$, prove that): $\cos A\cos 2A\cos 3A\cos 4A=\frac{1}{16}$

6B. यदि $11\theta=\pi^c$ भए प्रमाणित गर्नुहोस् (If $11\theta=\pi^c$ then prove that): $\sec\theta\sec 2\theta\sec 3\theta\sec 4\theta\sec 5\theta=32$

HINT: LHS $=\frac{1}{\cos\theta\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta}=\frac{2\sin\theta}{2\sin\theta\cos\theta\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta}$

$=\frac{4\sin\theta}{2\sin 2\theta\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta}=\frac{8\sin\theta}{2\sin 4\theta\cos 4\theta\cos 3\theta\cos 5\theta}$

$=\frac{16\sin\theta}{2\sin 8\theta\cos(11\theta-8\theta)\cos 5\theta}=\frac{16\sin\theta}{-2\sin 8\theta\cos 8\theta\cos 5\theta}=\frac{16\sin\theta}{-\sin 16\theta\cos 5\theta}$

$=\frac{32\sin\theta}{-2\sin(11\theta+5\theta)\cos 5\theta}=\frac{32\sin\theta}{+2\sin 5\theta\cos 5\theta}=\frac{32\sin\theta}{\sin 10\theta}=\frac{32\sin\theta}{\sin(11\theta-\theta)}=32$

TRIGONOMETRY

pueps.com.np

www.drmathnp.com

6C. यदि $13A = \pi^c$ भए प्रमाणित गर्नुहोस् (If $13A = \pi^c$, prove that): $\cos A \cos 2A \cos 3A \cos 4A \cos 5A \cos 6A = \frac{1}{64}$

6D. यदि $13A = \pi^c$ भए प्रमाणित गर्नुहोस् (If $13A = \pi^c$, prove that) : $\sec A \sec 2A \sec 3A \sec 4A \sec 5A \sec 6A = 64$

6E. यदि $\frac{\pi^c}{11} = \alpha$ भए प्रमाणित गर्नुहोस् (If $\frac{\pi^c}{11} = \alpha$ then prove that): $\cos\alpha \cos 2\alpha \cos 3\alpha \cos 4\alpha \cos 5\alpha = \frac{1}{32}$

6F. यदि $14\alpha = \pi^c$ भए प्रमाणित गर्नुहोस् (If $14\alpha = \pi^c$, prove that):
$\sin\alpha \sin 3\alpha \sin 5\alpha \sin 7\alpha \sin 9\alpha \sin 11\alpha \sin 13\alpha = \frac{1}{64}$

7. यदि 'α' र 'β' न्यूनकोणहरू र $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$ भए $\tan\alpha = \sqrt{2}\tan\beta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If 'α' and 'β' are acute angles and $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$ then prove that: $\tan\alpha = \sqrt{2}\tan\beta$

HINT: Here, $\frac{\cos 2\alpha}{1} = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$

or, $\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{3 - \cos 2\beta - 3\cos 2\beta + 1}{3 - \cos 2\beta + 3\cos 2\beta - 1}$

or, $\tan^2\alpha = 2\frac{\sin^2\beta}{\cos^2\beta}$

By Componendo - dividendo,

or, $\frac{2\sin^2\alpha}{2\cos^2\alpha} = \frac{4 - 4\cos 2\beta}{2 + 2\cos 2\beta}$

$\therefore \tan\alpha = \sqrt{2}\tan\beta$

8. प्रमाणित गर्नुहोस् (Prove that): $\cos^3 A + \cos^3(120° + A) + \cos^3(240° + A) = \frac{3}{4}\cos 3A$

HINT: We know that, $\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow \cos^3 A = \frac{1}{4}(\cos 3A + 3\cos A)$

$$\begin{aligned}
\text{LHS} &= \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos(360° + 3A) + 3\cos(120° + A)\} + \frac{1}{4}\{\cos(720° + 3A) + 3\cos(240° + A)\} \\
&= \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos 3A + 3\cos(120° + A)\} + \frac{1}{4}\{\cos 3A + 3\cos(240° + A)\} \\
&= \frac{3}{4}\cos 3A + \frac{3}{4}[\cos A + \cos\{180° - (60° - A)\} + \cos\{180° + (60° + A)\}] \\
&= \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A - \cos(60° - A) - \cos(60° + A)\} \\
&= \frac{3}{4}\cos 3A + \frac{3}{4}\left[\cos A - \left(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A + \frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right)\right] \\
&= \frac{3}{4}\cos 3A + \frac{3}{4}(\cos A - \cos A) = \frac{3}{4}\cos 3A = \text{RHS}
\end{aligned}$$

9. प्रमाणित गर्नुहोस् (Prove that): $\sin^3 A + \sin^3(60° + A) + \sin^3(240° + A) = \sin^3 A$

10. प्रमाणित गर्नुहोस् (Prove that) : $\sin^3 A + \sin^3(120° + A) + \sin^3(240° + A) = -\frac{3}{4}\sin 3A$

11. प्रमाणित गर्नुहोस् (Prove that): $\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

HINT: $\text{LHS} = \tan A + \tan(60° + A) - \tan(60° - A) = \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$

$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A} = \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A} = \frac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A} = 3\tan 3A = \text{RHS}$

12. यदि $q\tan A = p$ जहाँ, $A = 6B$ र A एउटा न्यूनकोण भए $p\,\text{cosec}\,2B - q\sec 2B = 2\sqrt{p^2 + q^2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $q\tan A = p$ where $A = 6B$ and A is an acute angle then prove that $p\,\text{cosec}\,2B - q\sec 2B = 2\sqrt{p^2 + q^2}$.

HINT: $\tan A = \frac{p}{q}$ So, $\sin A = \frac{p}{\sqrt{p^2 + q^2}}$ and $\cos A = \frac{q}{\sqrt{p^2 + q^2}}$

$$\begin{aligned}
\text{LHS} &= p\,\text{cosec}\,2B - q\sec 2B = \sqrt{p^2 + q^2}\left(\frac{p}{\sqrt{p^2 + q^2}}\text{cosec}\,2B - \frac{q}{\sqrt{p^2 + q^2}}\sec 2B\right) \\
&= \sqrt{p^2 + q^2}(\sin A.\,\text{cosec}\,2B - \cos A\sec 2B) = \sqrt{p^2 + q^2}\left(\frac{\sin A}{\sin 2B} - \frac{\cos A}{\cos 2B}\right) \\
&= \sqrt{p^2 + q^2}\,\frac{(\sin A\cos 2B - \cos A\sin 2B)}{\sin 2B\cos 2B} = \sqrt{p^2 + q^2}\,\frac{\sin(A - 2B)}{\sin 2B\cos 2B} \\
&= 2\sqrt{p^2 + q^2}\cdot\frac{\sin(6B - 2B)}{2\sin 2B\cos 2B}\ [\because A = 6B] = 2\sqrt{p^2 + q^2}\cdot\frac{\sin 4B}{\sin 4B} = 2\sqrt{p^2 + q^2} = \text{RHS}
\end{aligned}$$

13A. प्रमाणित गर्नुहोस् (Prove that): $\text{cosec}\,\frac{\alpha}{4} + \text{cosec}\,\frac{\alpha}{2} + \text{cosec}\,\alpha = \cot\frac{\alpha}{8} - \cot\alpha$

13B. प्रमाणित गर्नुहोस् (Prove that): $\text{cosec}\,\frac{\alpha}{8} + \text{cosec}\,\frac{\alpha}{4} + \text{cosec}\,\frac{\alpha}{2} = \cot\frac{\alpha}{16} - \cot\frac{\alpha}{2}$

TRIGONOMETRY

puspas.com.np

www.drmathnp.com

2. गुणन र योगफलको रूपान्तरण
TRANSFORMATION OF PRODUCT AND SUM

IMPORTANT FORMULAE

त्रिकोणमितीय सूत्रहरूको स्थानान्तरण (Transformation of Trigonometric Formulae)

गुणनमा स्थानान्तरण (Transformation in product)	योगफलमा स्थानान्तरण (Transformation in sum)
1. $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$	1. $2 \sin A . \cos B = \sin(A + B) + \sin(A - B)$
2. $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)$	2. $2\cos A . \sin B = \sin(A + B) - \sin(A - B)$
3. $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$	3. $2\cos A . \cos B = \cos(A + B) + \cos(A - B)$
4. $\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) . \sin\left(\frac{D-C}{2}\right)$	4. $2\sin A . \sin B = \cos(A - B) - \cos(A + B)$

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R', 2067 R, 2067 R', 2067 S, 2068 R', 2069 R, 2070 S, SEE 2073 S', SEE 2074 R, SEE 2074 S', SEE 2075 R'_2	11
Model 2	2067 R, 2068 R, 2069 R', 2070 R', 2073 R, SEE 2073 S, SEE MODEL 2076	7
Model 3	2066 R, 2070 R, 2070 S', 2071 S, 2072 R, 2072 S, SEE 2074 R', SEE 2074 S	8

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R', 2067 S, 2068 R, 2069 R'	4
Model 2	2066 R'	1
Model 3	2069 R, 2070 R', 2072 R', SEE 2075 R, SEE 2075 R_2, R'_2, SEE MODEL 2076	5
Model 4	2070 S	1

SEE EXERCISE 2

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1. योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् । (Express in the sum or difference form.)
A. $2 \cos A \cos B$ **B.** $2 \sin \alpha \sin \beta$ **C.** $2 \sin x \cos y$ **D.** $2 \cos \theta \sin \beta$

2A. $\sin \alpha + \sin \beta$ लाई sine वा cosine को गुणनफलको रूपमा बदल्नुहोस् ।
Convert $\sin \alpha + \sin \beta$ in terms of product of sine or cosine.

2B. $\cos x - \cos y$ लाई sine को गुणनफलको रूपमा बदल्नुहोस् ।
Convert $\cos x - \cos y$ in terms of product of sine.

2C. $\cos A + \cos B$ लाई cosine को गुणनफलको रूपमा व्यक्त गर्नुहोस् ।
Express $\cos A + \cos B$ in terms of product of cosine.

2D. $\sin x - \sin y$ लाई sine वा cosine को गुणनफलको रूपमा व्यक्त गर्नुहोस् ।
Express $\sin x - \sin y$ in terms of product of sine or cosine.

Model 2 *Concept Recall*

3A. $\sin 2P + \sin 2Q$ लाई गुणनको रूपमा व्यक्त गर्नुहोस् । (Express $\sin 2P + \sin 2Q$ into product form.)

3B. $\cos 2M + \cos 2N$ लाई गुणनको रूपमा व्यक्त गर्नुहोस् । (Express $\cos 2M + \cos 2N$ into product form.)

4A. $\sin 2P - \sin 2Q$ लाई गुणनको रूपमा व्यक्त गर्नुहोस् । (Express $\sin 2P - \sin 2Q$ into product form.)

4B. $\cos 4M - \cos 4N$ लाई गुणनको रूपमा व्यक्त गर्नुहोस् । (Express $\cos 4M - \cos 4N$ into product form.)

pueppss.com.np

www.drmathnp.com

5. $A + B$ र $A - B$ को रूपमा व्यक्त गर्नुहोस् । (Express interms of $A + B$ and $A - B$.)

A. $2 \sin 2A \cos 2B$ **B.** $2 \cos 2A \sin 2B$ **C.** $2 \cos 4A \cos 4B$ **D.** $2 \sin 3A \sin 3B$

6. $\frac{C+D}{2}$ र $\frac{C-D}{2}$ को रूपमा व्यक्त गर्नुहोस् । (Express interms of $\frac{C+D}{2}$ and $\frac{C-D}{2}$.)

A. $\sin 2C + \sin 2D$ **B.** $\sin 4C - \sin 4D$ **C.** $\cos 6C + \cos 6D$ **D.** $\cos 8C - \cos 8D$

B. SHORT QUESTIONS

Model 1 *Problems based on finding values or proving using the formula to transform the product.*

1. मान पत्ता लगाउनुहोस् (Find the value of):

A. $\sin 75° + \sin 15°$ [2065 R'] **1B.** $\sin 75° - \sin 105°$ [2067 S]

C. $\cos 15° - \cos 75°$ [2058 S] **1D.** $\sin 70° - \cos 80° + \cos 140°$ [2061 R]

2. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{1}{2}(\sin 7\theta - \sin 3\theta) = \cos 5\theta \sin 2\theta$ [SEE 2073 S'] **B.** $\cos 105° + \cos 15° = \frac{1}{\sqrt{2}}$ [2069 R]

C. $\cos 70° + \cos 40° = 2 \cos 55°. \cos 15°$ [2060 R] **D.** $\sin 80° - \cos 110° = \sqrt{3} \sin 50°$ [SEE 2074 R]

E. $\sin 110° - \sin 10° = \sin 50°$ [SEE 2074 S'] **F.** $\cos 75° + \cos 15° = \sqrt{\frac{3}{2}}$ [2064 R']

G. $\sin 50° + \sin 70° = \sqrt{3} \cos 10°$ [63S, 68R'] **H.** $\cos 40° + \sin 40° = \sqrt{2} \cos 5°$ [2067 R]

I. $\sin 65° + \cos 65° = \sqrt{2} \cos 20°$ [2067 R'] **J.** $\cos 55° + \sin 55° = \sqrt{2} \cos 10°$ [2070 S]

K. $\cos 20° + \cos 140° + \cos 100° = 0$ [2070 S] **L.** $\sin 35° + \cos 35° = \sqrt{2} \cos 10°$ [SEE 2075 R'$_2$]

Model 2 *Problems based on finding values using the formula to transform the sum.*

3. $\sin 6A . \cos 4A$ लाई sine वा cosine को योग वा अन्तरमा रूपान्तरण गर्नुहोस् ।
Convert $\sin 6A . \cos 4A$ into sum or difference of sine or cosine. [SEE MODEL 2076]

4. योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् । (Express in the sum or difference form.)

A. $\cos 8A \cos 6A$ **B.** $\sin 20\alpha \sin 28\alpha$ **C.** $\sin 8x \cos 10x$ **D.** $\cos 30\theta \sin 20\theta$

5. मान पत्ता लगाउनुहोस् (Find the value of):

A. $\sin 75° \sin 15°$ [2058 R, 2067 R] **B.** $\cos 105° \cos 15°$ [2059 R]

C. $\sin 105°. \sin 15°$ [2059 S, 2069 R'] **D.** $4 \sin 105° \sin 15°$ [2063 R']

E. $4 \cos 75° \sin 105°$ [2064 R] **F.** $4 \sin 15° \cos 15°$

6. प्रमाणित गर्नुहोस् (Prove that) :

A. $2\cos 70°. \cos 20° = \cos 50°$ [SEE 2073 R] **B.** $\cos 105° . \cos 15° = -\frac{1}{4}$ [2068 R]

C. $2 \sin 50° . \sin 40° = \cos 10°$ [2070R'] **D.** $2 \sin 7\theta . \cos 2\theta = \sin 9\theta + \sin 5\theta$ [SEE 2073 S]

Model 3 *On proving identities using formula to transform to product into sum or difference.*

प्रमाणित गर्नुहोस् (Prove that):

7. A. $2 \cos (45° + A).\cos (45° - A) = \cos 2A$ [2064 R] **B.** $2\cos (45° + A).\sin (45° + A) = \cos 2A$ [2070 S']

C. $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \frac{A+B}{2} \cot \frac{A-B}{2}$ [2066R] **D.** $\frac{\sin A + \sin B}{\cos A + \cos B} = \tan \frac{A+B}{2}$ [SEE 2074 R']

E. $\frac{1}{2}(\cos 2\theta - \cos 8\theta) = \sin 5\theta . \sin 3\theta$ [2070 R] **F.** $\frac{1}{2}(\cos 10° - \cos 40°) = \sin 25°.\sin 15°$

8. A. $\frac{\cos A - \cos 5A}{\sin 5A - \sin A} = \tan 3A$ [2072 R] **B.** $\frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A} = \tan 5A$ [SEE 2074 S]

9. A. $\frac{\cos 40° - \sin 30°}{\sin 60° - \cos 50°} = \tan 50°$ [2071 S] **B.** $\frac{\sin 25° + \sin 15°}{\cos 25° + \sin 75°} = \tan 20°$

C. $\frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°} = \cot 35°$ [2072 S] **D.** $\frac{\cos 10° + \cos 80°}{\sin 80° - \sin 10°} = \tan 55°$

puspas.com.np

www.drmathnp.com

pupas.com.np

C. LONG QUESTIONS

Model 1 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

1. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha - \sin\beta\cos\beta} = \tan(\alpha+\beta)$ [2068 R, 2057 S, 2063 R]

B. $\frac{\cos^2\beta - \sin^2\alpha}{\sin\alpha\cos\alpha + \sin\beta\cos\beta} = \cot(\alpha+\beta)$ [2064 R']

C. $\frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha + \sin\beta\cos\beta} = \tan(\alpha-\beta)$

D. $\frac{\cos^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha + \sin\beta\cos\beta} = \cot(\alpha+\beta)$

2. **A.** $\cos(60^0+\theta)\cos(60^0-\theta)\cos\theta = \frac{1}{4}\cos 3\theta$ [2057 R]

B. $4\cos(60°+\theta).\sin(30°+\theta).\cos\theta = \cos 3\theta$ [2064 R]

C. $4\sin\alpha\sin(60°-\alpha)\sin(60°+\alpha) = \sin 3\alpha$ [2063 S, 2060 S']

OR, $\sin\theta.\sin(60°-\theta).\sin(60°+\theta) = \frac{1}{4}\sin 3\theta$ [2067 S, 2069 R']

3. **A.** $\sec\left(\frac{\pi^C}{4}+\frac{\theta}{2}\right).\sec\left(\frac{\pi^C}{4}-\frac{\theta}{2}\right) = 2\sec\theta$ [2060 R]

B. $\operatorname{cosec}\left(\frac{\pi^c}{4}+\frac{\theta}{2}\right)\sec\left(\frac{\pi^c}{4}+\frac{\theta}{2}\right) = 2\sec\theta$

4. **A.** $2\cos\frac{\pi^c}{13}.\cos\frac{9\pi^c}{13}+\cos\frac{3\pi^c}{13}+\cos\frac{5\pi^c}{13} = 0$ [59 S, 64R']

B. $2\cos\frac{12\pi^c}{13}\cos\frac{4\pi^c}{13}-\cos\frac{10\pi^c}{13}-\cos\frac{8\pi^c}{13} = 0$

C. $2\cos\frac{13\pi^c}{26}\cos\frac{7\pi^c}{26}+\cos\frac{8\pi^c}{13}+\cos\frac{5\pi^c}{13} = 0$

D. $2\sin\frac{11\pi^c}{26}\sin\frac{31\pi^c}{26}+\sin\frac{7\pi^c}{26}+\sin\frac{3\pi^c}{26} = 0$

5. **A.** $\cos^3 x\sin^2 x = \frac{1}{16}(2\cos x - \cos 3x - \cos 5x)$ [2065 R']

B. $\cos^2 x\sin 4x = \frac{1}{4}(\sin 2x + 2\sin 4x + \sin 6x)$

Model 2 *On proving trigonometric identities using formula to transform sum or difference into product.*

6. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{\sin 40°-\sin 20°}{\cos 220°-\cos 200°} = \sqrt{3}$ [2062 K]

B. $\frac{\cos 285°+\cos 345°}{\sin 435°-\sin 375°} = \sqrt{3}$

C. $\frac{\cos 340°-\cos 290°}{\sin 430°-\sin 380°} = 1$

D. $\frac{\sin 130°-\sin 140°}{\cos 310°+\cos 220°} = -1$

E. $\frac{\cos 320°+\cos 120°}{\sin 120°+\sin 220°} = \tan 50°$

F. $\frac{\cos 50°-\cos 70°}{\cos 580°-\cos 560°} = \sqrt{3}$

7. **A.** $\frac{\sin 2A+\sin 5A-\sin A}{\cos 2A+\cos 5A+\cos A} = \tan 2A$ [2066 R']

B. $\frac{\sin 2A+\sin 3A+\sin 4A}{\cos 2A+\cos 3A+\cos 4A} = \tan 3A$

C. $\frac{\sin\theta+\sin 3\theta+\sin 5\theta}{\cos\theta+\cos 3\theta+\cos 5\theta} = \tan 3\theta$

D. $\frac{\cos 3A-\cos 2A+\cos A}{\sin 3A-\sin 2A+\sin A} = \cot 2A$

E. $\frac{\cos 2A+\cos 5A+\cos A}{\sin 2A+\sin 5A-\sin A} = \cot 2A$

F. $\frac{\cos 20°+\cos 50°+\cos 10°}{\sin 20°+\sin 50°-\sin 10°} = \cot 20°$

Model 3 *Problems based on finding value of trigonometric ratios in product form.*

8. प्रमाणित गर्नुहोस् (Prove that) :

A. $16\sin 20°.\sin 40°.\sin 60°.\sin 80° = 3$ [2069 R]

B. $8\cos 10°.\cos 50°.\cos 70° = \sqrt{3}$ [SEE 2070 R']

C. $8\sin 20°.\sin 40°.\sin 80° = \sqrt{3}$ [2075 R'$_2$]

D. $\sin 20°\sin 30°\sin 40°\sin 80° = \frac{\sqrt{3}}{16}$ [2072 R']

E. $\sin 10°\sin 50°\sin 70° = \frac{1}{8}$

F. $\cos 80°\cos 40°\cos 20°\sin 150° = \frac{1}{16}$

9 क्याल्कुलेटरको प्रयोग नगरीकन मान पत्ता लगाउनुहोस् (Without using the calculator, find the value of):

A. $\sin 100°.\sin 120°.\sin 140°.\sin 160°$ [SEE 2075 R, 2075 R$_2$]

B. $\cos 40°.\cos 100°.\cos 160°.\cos 300°$

C. $\sin 20°\sin 30°\sin 40°\sin 80°$ [SEE MODEL 2076]

www.drmathnp.com

puspas.com.np

Model 4 *Conditional problems on transformation of sum and product.*

10A. यदि $\frac{1}{\sin A}+\frac{1}{\cos A}=\frac{1}{\sin B}+\frac{1}{\cos B}$ भए प्रमाणित गर्नुहोस् : $\cot\left(\frac{A+B}{2}\right)=\tan A.\tan B$

If $\frac{1}{\sin A}+\frac{1}{\cos A}=\frac{1}{\sin B}+\frac{1}{\cos B}$, prove that : $\cot\left(\frac{A+B}{2}\right)=\tan A.\tan B$ [2070 S]

Hint: Here, $\frac{1}{\sin A}+\frac{1}{\cos A}=\frac{1}{\sin B}+\frac{1}{\cos B}$ or, $\frac{1}{\sin A}-\frac{1}{\sin B}=\frac{1}{\cos B}-\frac{1}{\cos A}$

or, $\frac{\sin B-\sin A}{\sin A\sin B}=\frac{\cos A-\cos B}{\cos B\cos A}$ or, $\frac{\sin B-\sin A}{\cos A-\cos B}=\frac{\sin A\sin B}{\cos B\cos A}$

or, $\frac{2\cos\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}=\tan A.\tan B$ or, $\cot\left(\frac{B+A}{2}\right)=\tan A\tan B$ Proved.

10B. यदि $\frac{1}{\sin A}-\frac{1}{\cos A}=\frac{1}{\sin B}-\frac{1}{\cos B}$ भए प्रमाणित गर्नुहोस् : $\tan A.\tan B+\cot\left(\frac{A+B}{2}\right)=0$

If $\frac{1}{\sin A}-\frac{1}{\cos A}=\frac{1}{\sin B}-\frac{1}{\cos B}$, prove that : $\tan A.\tan B+\cot\left(\frac{A+B}{2}\right)=0$

ANSWERS

A. VERY SHORT QUESTIONS : Show to your teacher.

B. SHORT QUESTIONS

1A. $\sqrt{\frac{3}{2}}$ **1B.** 0 **1C.** $\frac{1}{\sqrt{2}}$ **1D.** 0 **3.** $\frac{1}{2}[\sin 10A+\sin 2A]$ **4A.** $\frac{1}{2}[\cos 14A+\cos 2A]$

4B. $\frac{1}{2}[\cos 8\alpha-\cos 48\alpha]$ **4C.** $\frac{1}{2}[\sin 18x-\sin 2x]$ **4D.** $\frac{1}{2}[\sin 50\theta-\sin 10\theta]$ **5A.** $\frac{1}{4}$ **5B.** $-\frac{1}{4}$ **5C.** $\frac{1}{4}$

5D. 1 **5E.** 1 **5F.** 1

C. LONG QUESTIONS : **9A.** $\frac{3}{16}$ **9B.** $\frac{1}{16}$ **9C.** $\frac{\sqrt{3}}{16}$

TRIGONOMETRY

GENERAL EXERCISE 2

A. SHORT QUESTIONS

Model 1 *Problems based on finding values or proving using the formula to transform the product.*

1. क्याल्कुलेटरको प्रयोग नगरी मान पत्ता लगाउनुहोस् (Find the value without using calculator):

A. $\cos 15°+\cos 75°$ **B.** $\sin 75°-\sin 105°$ **C.** $\sin 75°+\sin 105°$

D. $\sin 105°-\cos 15°$ **E.** $\cos 105°+\cos 15°$ **F.** $\sin 75°-\sin 15°$

2. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos 105°+\sin 105°=\cos 45°$ **B.** $\sin 42°+\cos 72°=\cos 12°$ **C.** $\cos 50°+\cos 70°=\cos 10°$

D. $\sin 50°-\sin 70°+\sin 10°=0$ **E.** $\cos 40°+\cos 80°=\cos 20°$ **F.** $\sin 51°+\cos 81°-\cos 21°=0$

G. $\sin 35°+\cos 35°=\sqrt{2}\cos 10°$ **H.** $\cos 56°+\sin 86°=\sqrt{3}\cos 26°$ **I.** $\cos 40°-\sin 40°=\sqrt{2}\sin 5°$

J. $\cos 18°-\sin 18°=\sqrt{2}\sin 27°$ **K.** $\cos 5\theta+\cos 3\theta=2\cos 4\theta\cos\theta$

L. $\cos(45°+\theta)+\cos(45°-\theta)=\sqrt{2}\cos\theta$ **M.** $\cos 70°+\cos 40°=2\cos 55°\cos 15°$

3. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos 52°+\cos 68°+\cos 172°=0$ **B.** $\cos 40°+\sin 40°-\sqrt{2}\cos 5°=0$

C. $\sin 51°+\cos 81°-\cos 21°=0$ **D.** $\cos 28°-\sin 58°-\sin 2°=0$

E. $\cos 10°+\cos 110°+\cos 130°=0$ **F.** $\sin 20°+\sin 140°+\cos 170°=0$

G. $\sin 50°-\sin 70°+\sin 10°=0$ **H.** $\cos 80°+\cos 40°=\cos 20°$

I. $\cos 20°-\sin 10°+\cos 140°=0$ **J.** $\sin 50°+\sin 70°-\sqrt{3}\sin 80°=0$

Model 2 *Problems based on finding values using the formula to transform the sum.*

4. क्याल्कुलेटरको प्रयोग नगरी मान पत्ता लगाउनुहोस् (Find the value without using calculator):

A. $\sin 15°\sin 105°$ **B.** $\cos 15°\cos 75°$

www.drmathnp.com

pueplas.com.np

Model 3 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

5. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{\cos A + \cos B}{\sin A + \sin B} = \cot\left(\frac{A+B}{2}\right)$ **B.** $\frac{\sin 7A + \sin 3A}{\cos 7A + \cos 3A} = \tan 5A$ **C.** $\frac{\sin 3A - \sin A}{\cos A - \cos 3A} = \cot 2A$

D. $\frac{\cos 2x - \cos 2y}{\sin 2x + \sin 2y} = \tan(y - x)$ **E.** $\frac{\sin A - \sin B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right)$ **F.** $\frac{\sin 7A - \sin 5A}{\cos 7A + \cos 5A} = \tan A$

G. $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot\frac{A+C}{2}$ **H.** $\frac{\sin 5A - \sin 3A}{\cos 3A - \cos 5A} = \cot 4A$ **I.** $\frac{\sin 5A - \sin 3A}{\cos 3A + \cos 5A} = \tan A$

J. $\frac{\sin 7\theta + \sin 3\theta}{\cos 7\theta - \cos 3\theta} = -\cot 2\theta$ **K.** $\frac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \sqrt{3}$ **L.** $\frac{\cos 20° - \cos 70°}{\sin 70° - \sin 20°} = 1$

M. $\frac{\sin 50° - \sin 40°}{\cos 50° - \cos 40°} = -1$ **N.** $\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\left(\frac{\alpha+\beta}{2}\right)$ **O.** $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

P. $\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \tan\theta$ **Q.** $\frac{\sin 3A - \sin A}{\cos A - \cos 3A} = \cot 2A$ **R.** $\frac{\sin 30° - \sin 10°}{\cos 10° - \cos 30°} = \cot 20°$

R. $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$ **S.** $\frac{\cos B - \cos A}{\cos A + \cos B} = \tan\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right)$

6. A. $\cos(45° + \theta)\sin(45° - \theta) = \frac{1}{2}(1 - \sin 2\theta)$ **B.** $\sin(45° + A)\sin(45° - A) = \frac{1}{2}\cos 2A$

C. $2\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{3\pi}{4} + \theta\right) = 1 - 2\sin 2\theta$ **D.** $2\sin(45° + A)\cos(45° - A) = 1 + \sin 2A$

E. $\sin 16° \sin 32° + \cos 8° \cos 56° = \cos 24° \cos 40°$ **F.** $\cos 4\theta \sin 2\theta + \sin 3\theta \cos\theta = \sin 5\theta \cos\theta$

G. $\cos 40° \sin 20° + \sin 30° \cos 10° = \sin 50° \cos 10°$ **H.** $\cos 16° \sin 8° + \sin 12° \cos 4° = \sin 20° \cos 4°$

B. LONG QUESTIONS

Model 1 *On proving trigonometric identities using formula to transform sum or difference into product.*

1. प्रमाणित गर्नुहोस् (Prove that):

A. $\frac{\cos^2\alpha - \cos^2\beta}{\sin\alpha\cos\alpha - \sin\beta\cos\beta} = -\tan(\alpha - \beta)$ **B.** $\frac{\cos^2\beta - \cos^2\alpha}{\sin\alpha\cos\alpha - \sin\beta\cos\beta} = \tan(\alpha + \beta)$

C. $\frac{2\sin^2 A\cos^2 A - 2\sin^2 B\cos^2 B}{\sin^2 A - \sin^2 B} = \cos 2A + \cos 2B$

Model 2 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

2. A. $\frac{\cos 7A + \cos A + \cos 5A + \cos 3A}{\sin 7A + \sin A + \sin 5A + \sin 3A} = \cot 4A$ **B.** $\frac{\sin A + \sin 2A + \sin 4A + \sin 5A}{\cos A + \cos 2A + \cos 4A + \cos 5A} = \tan 3A$

C. $\frac{\sin\theta - \sin 3\theta + \sin 5\theta - \sin 7\theta}{\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta} = \cot 2\theta$ **D.** $\frac{\sin 5\theta - \sin 7\theta + \sin 8\theta - \sin 4\theta}{\cos 4\theta - \cos 5\theta - \cos 8\theta + \cos 7\theta} = \cot 6\theta$

E. $\frac{\sin 10° + \sin 30° + \sin 50° + \sin 70°}{\cos 10° + \cos 30° + \cos 50° + \cos 70°} = \tan 40°$ **F.** $\frac{\sin 5° + \sin 10° + \sin 20° + \sin 25°}{\cos 5° + \cos 10° + \cos 20° + \cos 25°} = \tan 15°$

G. $\frac{\sin 20° - \sin 28° + \sin 32° - \sin 16°}{\cos 16° - \cos 20° - \cos 32° + \cos 28°} = \cot 24°$ **H.** $\frac{\sin 5° - \sin 15° + \sin 25° - \sin 35°}{\cos 5° - \cos 15° - \cos 25° + \cos 35°} = \cot 10°$

I. $(\sin 3\theta + \sin\theta)\sin\theta + (\cos 3\theta - \cos\theta)\cos\theta = 0$ **J.** $\frac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$

3. A. $\frac{\sin(\alpha+\beta) - 2\sin\alpha + \sin(\alpha-\beta)}{\cos(\alpha+\beta) - 2\cos\alpha + \cos(\alpha-\beta)} = \tan\alpha$ **B.** $\frac{\sin(A-C) + 2\sin A + \sin(A+C)}{\sin(B-C) + 2\sin B + \sin(B+C)} = \frac{\sin A}{\sin B}$

C. $\frac{1 - \cos A + \cos B - \cos(A+B)}{1 + \cos A - \cos B - \cos(A+B)} = \tan\frac{A}{2}\cot\frac{B}{2}$ **D.** $\frac{1 - \cos 10 + \cos 40 - \cos 50}{1 + \cos 10 - \cos 40 - \cos 50} = \tan 5° \cot 20°$

4. A. $\frac{\sin A\sin 2A + \sin 3A\sin 6A}{\sin A\cos 2A + \sin 3A\cos 6A} = \tan 5A$ **B.** $\frac{\sin 11A\sin A + \sin 7A\sin 3A}{\cos 11A\sin A + \cos 7A\sin 3A} = \tan 8^a$

TRIGONOMETRY

www.drmathnp.com

puspas.com.np

C. $\frac{\cos 5A \sin 2A - \cos 4A \sin 3A}{\sin 5A \sin 2A - \cos 4A \cos 3A} = \cot 2A$

D. $\frac{\sin 8\theta \cos \theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos \theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$

E. $\frac{\sin 3\theta \cos 4\theta - \sin \theta \cos 2\theta}{\sin 4\theta \sin \theta + \cos \theta \cos 6\theta} = \tan 2\theta$

F. $\frac{\sin 32° \cos 4° - \sin 24° \cos 12°}{\cos 8° \cos 4° - \sin 12° \sin 16°} = \tan 8°$

G. $\frac{\sin 18° \cos 24° - \sin 6° \cos 12°}{\sin 24° \sin 6° + \cos 6° \cos 36°} = \tan 12°$

H. $\frac{\sin 5° \sin 10° + \sin 15° \sin 30°}{\sin 5° \cos 10° + \sin 15° \cos 30°} = \tan 25°$

I. $\frac{\cos 2A \cos 3A - \cos 2A \cos 7A}{\sin 4A \sin 3A - \sin 2A \sin 5A} = \frac{\sin 7A + \sin 3A}{\sin A}$

Model 3 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

5. A. $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$

B. $\cos 10° \cos 50° \cos 70° = \frac{\sqrt{3}}{8}$

C. $\cos 10° \cos 50° \cos 60° \cos 70° = \frac{\sqrt{3}}{16}$

D. $\sin 20° \sin 40° \sin 60° \sin 80° = \frac{3}{16}$

E. $\cos 40° \cos 80° \cos 160° = \frac{-1}{8}$

F. $\cos 40° . \cos 100° . \cos 160° = \frac{1}{8}$

G. $\cos 10° \cos 30° \cos 50° \cos 70° = \frac{3}{16}$

H. $8 \cos 20° . \cos 40° . \cos 80° = 1$

I. $\sin 6° . \sin 42° . \sin 66° . \sin 78° = \frac{1}{16}$

J. $8 \sin 26° . \sin 34° . \sin 60° . \sin 86° = \sqrt{3} \sin 78°$

Model 4 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

6. A. $\frac{\sin (p+2)\theta - \sin p\theta}{\cos p\theta - \cos (p+2)\theta} = \cot (p+1)\theta$

B. $\frac{\sin p\theta + \sin (p+1)\theta}{\cos p\theta + \cos (p+1)\theta} = \tan \left(\frac{2p+1}{2}\right)\theta$

C. $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

D. $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Model 5 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

7. A. $\cos^2 \alpha \sin^3 \alpha = \frac{1}{16}(2\sin \alpha + \sin 3\alpha - \sin 5\alpha)$

B. $\cos^2 A \sin 4A = \frac{1}{4}(\sin 2A + 2 \sin 4A + \sin 6A)$

C. $\sin^2 A \sin 3A = \frac{1}{4}(-\sin 5A + 2 \sin 3A - \sin A)$

D. $\cos^4 A \sin^2 A = \frac{1}{32}(-\cos 6A - 2 \cos 4A + \cos 2A + 2)$

E. $\cos^2 A . \sin^4 A = \frac{1}{32}(\cos 6A - 2\cos 4A - \cos 2A + 2)$

F. $16 \cos^3 A = 16\cos^5 A + 2\cos A - \cos 3A - \cos 5A$

HINT: RHS $= 16 \cos^5 A + 2 \cos A - \cos 3A - \cos 5A = 16\cos^5 A + 2\cos A - 2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)$

$= 16 \cos^5 A + 2\cos A - 2\cos 4A \cos A = 2\cos A [8\cos^4 A + 1 - \cos 4A]$

$= 2\cos A [8\cos^4 A + 2\sin^2 2A] = 2\cos A [8\cos^4 A + 2 \times 4 \sin^2 A \cos^2 A]$

$= 2\cos A \times 8\cos^2 A[\cos^2 A + \sin^2 A] = 16\cos^3 A = LHS$

G. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Model 6 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

8. A. $\sin A + \sin 3A + \sin 5A + \sin 7A = 4 \cos A \cos 2A \sin 4A$

B. $\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$

C. $\cos A + \cos 3A + \cos 5A + \cos 7A = 4 \cos A \cos 2A \cos 4A$

D. $1 + \cos 2A + \cos 4A + \cos 6A = 4 \cos A \cos 2A \cos 3A$

E. $(\sin A + \sin 3A) \sin A + (\cos 3A - \cos A) \cos A = 0$

9. A. $\cos 2\theta . \cos \frac{\theta}{2} - \cos 3\theta . \cos \frac{9\theta}{2} = \sin 5\theta \sin \frac{5\theta}{2}$

B. $\sin \frac{\theta}{4} . \sin \frac{11\theta}{4} + \sin \frac{3\theta}{4} . \sin \frac{7\theta}{4} = \sin 2\theta \sin \theta$

C. $\sin \frac{\theta}{2} \sin \theta + \cos \frac{\theta}{4} \cos \frac{7\theta}{4} = \cos \frac{3\theta}{4} \cos \frac{5\theta}{4}$

D. $\sin \frac{\theta}{2} . \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} . \sin \frac{11\theta}{2} = \sin 2\theta \sin 5\theta$

TRIGONOMETRY

www.drmathnp.com

pueplus.com.np

Model 7 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

10. A. $\sin\theta \sin(120° - \theta) \sin(120° + \theta) = \frac{1}{4}\sin 3\theta$ **B.** $\sin\theta \sin(420° - \theta) \sin(420° + \theta) = \frac{1}{4}\sin 3\theta$

C. $4\sin\alpha \sin\left(\alpha + \frac{\pi^c}{3}\right) \sin\left(\alpha + \frac{2\pi^c}{3}\right) = \sin 3\alpha$ **D.** $\cos 2\theta \operatorname{cosec}(45° + \theta) \operatorname{cosec}(45° - \theta) = 2$

E. $\cos 2\theta \sec(45° + \theta) \sec(45° - \theta) = 2$ **F.** $\cot\theta \cos(30° - \theta) - \sin(30° - \theta) = \frac{\sqrt{3}}{2}\operatorname{cosec}\theta$

G. $\sin A + \sin\left(A + \frac{2\pi^c}{3}\right) + \sin\left(A + \frac{4\pi^c}{3}\right) = 0$ **H.** $\cos A + \cos(120° - A) + \cos(120° + A) = 0$

11. A. $\operatorname{cosec}\left(\frac{\pi^c}{4} + \theta\right) \operatorname{cosec}\left(\frac{\pi^c}{4} - \theta\right) = 2\sec 2\theta$ **B.** $2\sin\left(\frac{\pi^c}{6} + \theta\right) \cos\left(\frac{\pi^c}{6} - \theta\right) = \frac{\sqrt{3}}{2} + \sin 2\theta$

C. $2\sin\left(\frac{\pi^c}{4} - \theta\right) \sin\left(\frac{3\pi^c}{4} + \theta\right) = (\sin\theta - \cos\theta)^2$ **D.** $\sin^2\left(\frac{\pi^c}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi^c}{8} - \frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\sin\theta$

Model 8 *On proving identities using formula to transform to product into sum or difference and vice-versa.*

12. A. $\cos\frac{\pi^c}{9} + \cos\frac{5\pi^c}{9} + \cos\frac{7\pi^c}{9} = 0$ **HINT:** LHS = $\cos 20° + (\cos 100° + \cos 140°)$

B. $\sin\frac{5\pi^c}{18} - \sin\frac{7\pi^c}{18} + \sin\frac{\pi^c}{18} = 0$ **C.** $\sin\frac{\pi^c}{9} \cdot \sin\frac{2\pi^c}{9} \cdot \sin\frac{4\pi^c}{9} = \frac{\sqrt{3}}{8}$

13. A. $4(\cos^3 20° + \sin^3 50°) = 3(\cos 20° + \sin 50°) = 3\sqrt{3}\cos 10°$

B. $4(\cos^3 10 + \sin^3 20°) = 3(\cos 10° + \sin 20°) = 3\sqrt{3}\cos 40°$

C. $4(\cos^3 20° + \sin^3 10°) = 3(\cos 20° + \sin 10°) = 3\sqrt{3}\cos 50°$

Model 9 *The conditional problems based on the transformation of sum formula.*

14A. यदि $x\cos\theta + y\sin\theta = x\cos\beta + y\sin\beta$ भए $\frac{x}{y} = \cot\left(\frac{\theta+\beta}{2}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $x\cos\theta + y\sin\theta = x\cos\beta + y\sin\beta$ then prove that: $\frac{x}{y} = \cot\left(\frac{\theta+\beta}{2}\right)$

14B. यदि $\sin A + \sin B = x$ र $\cos A + \cos B = y$ भए $\cos\left(\frac{A+B}{2}\right)$ को मान कति होला ?

If $\sin A + \sin B = x$ and $\cos A + \cos B = y$ then what is the value of $\cos\left(\frac{A+B}{2}\right)$?

14C. यदि $\frac{4}{\sec\theta} + \frac{2}{\operatorname{cosec}\theta} = \frac{4}{\sec\alpha} + \frac{2}{\operatorname{cosec}\alpha}$ भए $\cot\left(\frac{\theta+\alpha}{2}\right) = 2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\frac{4}{\sec\theta} + \frac{2}{\operatorname{cosec}\theta} = \frac{4}{\sec\alpha} + \frac{2}{\operatorname{cosec}\alpha}$ then prove that: $\cot\left(\frac{\theta+\alpha}{2}\right) = 2$

14D. यदि $\frac{1}{\sin 2A} + \frac{1}{\cos 2A} = \frac{1}{\sin 2B} + \frac{1}{\cos 2B}$ भए प्रमाणित गर्नुहोस्: $\cot(A + B) = \tan 2A.\tan 2B$

If $\frac{1}{\sin 2A} + \frac{1}{\cos 2A} = \frac{1}{\sin 2B} + \frac{1}{\cos 2B}$ then prove that: $\cot(A + B) = \tan 2A.\tan 2B$

14E. यदि $\frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}\alpha} = \frac{1}{\sec\theta} + \frac{1}{\sec\alpha}$ भए $\sec\left(\frac{\theta+\alpha}{2}\right) = \sqrt{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}\alpha} = \frac{1}{\sec\theta} + \frac{1}{\sec\alpha}$ then prove that: $\sec\left(\frac{\theta+\alpha}{2}\right) = \sqrt{2}$

14F. यदि $\frac{3}{\sec x} + \frac{3}{\sec y} = \frac{4}{\operatorname{cosec} x} + \frac{4}{\operatorname{cosec} y}$ भए $\cot\left(\frac{x+y}{2}\right)$ को मान पत्ता लगाउनुहोस् ।

If $\frac{3}{\sec x} + \frac{3}{\sec y} = \frac{4}{\operatorname{cosec} x} + \frac{4}{\operatorname{cosec} y}$ then find the value of $\cot\left(\frac{x+y}{2}\right)$.

www.drmathnp.com

14G. यदि $\frac{4}{\text{cosec A}}-\frac{3}{\sec B}=\frac{3}{\sec A}-\frac{4}{\text{cosec B}}$ भए $4\sec\left(\frac{A+B}{2}\right)=5$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\frac{4}{\text{cosec A}}-\frac{3}{\sec B}=\frac{3}{\sec A}-\frac{4}{\text{cosec B}}$ then prove that: $4\sec\left(\frac{A+B}{2}\right)=5$

14H. यदि $\frac{3}{\sec A}+\frac{3}{\sec B}=\frac{4}{\text{cosec A}}+\frac{4}{\text{cosec B}}$ भए प्रमाणित गर्नुहोस्:

If $\frac{3}{\sec A}+\frac{3}{\sec B}=\frac{4}{\text{cosec A}}+\frac{4}{\text{cosec B}}$ then prove that:

(i) $\tan\left(\frac{A+B}{2}\right)=\frac{3}{4}$ (ii) $\cos\left(\frac{A+B}{2}\right)=\frac{4}{5}$ (iii) $\sin\left(\frac{A+B}{2}\right)=\frac{3}{5}$

14I. यदि $\frac{3}{\sec A}+\frac{3}{\sec B}=\frac{4}{\text{cosec A}}-\frac{4}{\text{cosec B}}$ भए प्रमाणित गर्नुहोस् (If $\frac{3}{\sec A}+\frac{3}{\sec B}=\frac{4}{\text{cosec A}}-\frac{4}{\text{cosec B}}$ then prove that):

(i) $\tan\left(\frac{A-B}{2}\right)=\frac{3}{4}$ (ii) $\cos\left(\frac{A-B}{2}\right)=\frac{4}{5}$ (iii) $\sin\left(\frac{A-B}{2}\right)=\frac{3}{5}$

14J. यदि $\frac{2}{\sec x}-\frac{4}{\text{cosec } x}=\frac{4}{\text{cosec } y}-\frac{2}{\sec y}$ भए $\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\frac{2}{\sec x}-\frac{4}{\text{cosec } x}=\frac{4}{\text{cosec } y}-\frac{2}{\sec y}$ then prove that $\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$

14K. यदि $\frac{2}{\text{cosec A}}-\frac{3}{\sec B}=\frac{3}{\sec A}-\frac{2}{\text{cosec B}}$ भए $\tan\left(\frac{A+B}{2}\right)=\frac{3}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\frac{2}{\text{cosec A}}-\frac{3}{\sec B}=\frac{3}{\sec A}-\frac{2}{\text{cosec B}}$ then prove that: $\tan\left(\frac{A+B}{2}\right)=\frac{3}{2}$.

14L. यदि $2(\cos A+\cos B)=1$ र $4(\sin A+\sin B)=1$ भए $\tan\left(\frac{A+B}{2}\right)$ र $\cos(A+B)$ का मानहरू पत्ता लगाउनुहोस् ।

If $2(\cos A+\cos B)=1$ and $4(\sin A+\sin B)=1$ then find the values of $\tan\left(\frac{A+B}{2}\right)$ and $\cos(A+B)$.

15A. यदि $\cos A+\cos B=\frac{1}{3}$ र $\sin A+\sin B=\frac{1}{4}$ भए $\tan\left(\frac{A+B}{2}\right)=\frac{3}{4}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos A+\cos B=\frac{1}{3}$ and $\sin A+\sin B=\frac{1}{4}$ then prove that: $\tan\left(\frac{A+B}{2}\right)=\frac{3}{4}$

15B. यदि $\cos x+\cos y=\frac{1}{2}$ र $\sin x+\sin y=\frac{1}{4}$ भए $\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\cos x+\cos y=\frac{1}{2}$ and $\sin x+\sin y=\frac{1}{4}$ then prove that: $\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$

15C. यदि $\sin x+\sin y=a$ र $\cos x+\cos y=b$ भए $\tan\left(\frac{x+y}{2}\right)=\frac{a}{b}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\sin x+\sin y=a$ and $\cos x+\cos y=b$ then prove that: $\tan\left(\frac{x+y}{2}\right)=\frac{a}{b}$

15D. यदि $\sin\theta+\sin\phi=m$ र $\cos\theta+\cos\phi=n$ भए प्रमाणित गर्नुहोस्: $\sec\left(\frac{\theta+\phi}{2}\right)=\frac{\sqrt{m^2+n^2}}{n}$

If $\sin\theta+\sin\phi=m$ and $\cos\theta+\cos\phi=n$, prove that: $\sec\left(\frac{\theta+\phi}{2}\right)=\frac{\sqrt{m^2+n^2}}{n}$

15E. यदि $\text{cosec A}+\sec A=\text{cosec B}+\sec B$ भए प्रमाणित गर्नुहोस्: $\tan A\times\tan B=\cot\frac{A+B}{2}$

If $\text{cosec A}+\sec A=\text{cosec B}+\sec B$ then prove that: $\tan A\times\tan B=\cot\frac{A+B}{2}$

ANSWERS

A. SHORT QUESTIONS

1A. $\sqrt{\frac{3}{2}}$ 1B. 0 1C. $\frac{1+\sqrt{3}}{\sqrt{2}}$ 1D. 0 1E. $\frac{1}{\sqrt{2}}$ 1F. $\frac{1}{\sqrt{2}}$ 4A. $\frac{1}{4}$ 4B. $\frac{1}{4}$

B. LONG QUESTIONS : 14B. $\frac{y}{\sqrt{x^2+y^2}}$ 14F. $\frac{4}{3}$ 14L. $\frac{1}{2}$ and $\frac{3}{5}$

puspas.com.np

www.drmathnp.com

pueas.com.np

CHALLENGING EXERCISE 2

1. प्रमाणित गर्नुहोस् (Prove that):

A. $\cos A + \cos B + \cos C + \cos(A + B + C) = 4\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$

B. $\sin A + \sin B + \sin C - \sin(A + B + C) = 4\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B+C}{2}\right)\sin\left(\frac{C+A}{2}\right)$

C. $\cos A \sin(B - C) + \cos B \sin(C - A) + \cos C \sin(A - B) = 0$

D. $\sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D) = 0$

E. $\cos(A + B + C) + \cos(-A + B + C) + \cos(A - B + C) + \cos(A + B - C) = 4\cos A \cos B \cos C$

F. $\frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) + \sin(A-B+C) - \sin(A+B-C)} = \cot C$

G. $\frac{\cos 2A \cos 3A - \cos 2A \cos 7A + \cos A \cos 10A}{\sin 4A \sin 3A - \sin 2A \sin 5A + \sin 4A \sin 7A} = \cot 6A \cot 5A$

H. $\frac{\sin A \cos 3A \cos 2A - \sin A \cos 7A \cos 2A}{\sin 4A \sin 3A - \sin 2A \sin 5A} = \sin 7A + \sin 3A$

2. प्रमाणित गर्नुहोस् (Prove that): $\cos\frac{2\pi^c}{7} + \cos\frac{4\pi^c}{7} + \cos\frac{8\pi^c}{7} = -\frac{1}{2}$

HINT: LHS $= \cos\frac{2\pi^c}{7} + \cos\frac{4\pi^c}{7} + \cos\frac{8\pi^c}{7} = 2\cos\frac{3\pi^c}{7}\cos\frac{\pi^c}{7} + 2\cos^2\frac{4\pi^c}{7} - 1 = 2\cos\left(\pi^c - \frac{4\pi^c}{7}\right)\cos\frac{\pi^c}{7} + 2\cos^2\frac{4\pi^c}{7} - 1$

$= -2\cos\frac{4\pi^c}{7}\left[\cos\frac{\pi^c}{7} - 2\cos\frac{4\pi^c}{7}\right] - 1 = -2\cos\frac{4\pi^c}{7}\left[\cos\frac{\pi^c}{7} - \cos\left(\pi^c - \frac{3\pi^c}{7}\right)\right] - 1 = -2\cos\frac{4\pi^c}{7}\left[2\cos\frac{\pi^c}{7} + \cos\frac{3\pi^c}{7}\right] - 1$

$= -2\cos\frac{4\pi^c}{7}\left[2\cos\frac{2\pi^c}{7} \times \cos\frac{\pi^c}{7}\right] - 1 = -4\cos\frac{4\pi^c}{7}\cos\frac{2\pi^c}{7}\cos\frac{\pi^c}{7} - 1 = \frac{2\sin\frac{\pi^c}{7}\cos\frac{\pi^c}{7} \times 2\cos\frac{2\pi^c}{7}\cos\frac{4\pi^c}{7}}{\sin\frac{\pi^c}{7}} - 1$

$= \frac{\sin\frac{2\pi^c}{7} \times 2\cos\frac{2\pi^c}{7}\cos\frac{4\pi^c}{7}}{\sin\frac{\pi^c}{7}} - 1 = -\frac{\sin\frac{4\pi^c}{7}\cos\frac{4\pi^c}{7}}{\sin\frac{\pi^c}{7}} - 1 = -\frac{1}{2}\frac{\sin\frac{8\pi^c}{7}}{\sin\frac{\pi^c}{7}} - 1 = -\frac{1}{2}\frac{\sin\left(\pi^c + \frac{\pi^c}{7}\right)}{\sin\frac{\pi^c}{7}} - 1 = -\frac{1}{2}\frac{-\sin\frac{\pi^c}{7}}{\sin\frac{\pi^c}{7}} - 1 = -\frac{1}{2}$

3A. यदि $4\sin B = 3\sin(2A + B)$ भए प्रमाणित गर्नुहोस् (If $4\sin B = 3\sin(2A + B)$, prove that) : $7\cot(A + B) = \cot A$

3B. यदि $\sin(A + B) = 4\sin(A - B)$ भए प्रमाणित गर्नुहोस् (If $\sin(A + B) = 4\sin(A - B)$, prove that): $3\tan A = 5\tan B$

4A. यदि $\sin x = k \sin y$ भए प्रमाणित गर्नुहोस् (If $\sin x = k \sin y$ then prove that): $\tan\left(\frac{x-y}{2}\right) = \frac{k-1}{k+1}\tan\left(\frac{x+y}{2}\right)$

4B. यदि $b \sin B = a \sin(2A + B)$ भए प्रमाणित गर्नुहोस्: $\cot(A + B) = \frac{b-a}{b+a}\cot A$

If $b \sin B = a \sin(2A + B)$ then prove that: $\cot(A + B) = \frac{b-a}{b+a}\cot A$

4C. यदि $\sin(A + B) = k \sin(A - B)$ भए प्रमाणित गर्नुहोस्: $\tan A = \frac{k+1}{k-1}\tan B$

If $\sin(A + B) = k \sin(A - B)$ then prove that: $\tan A = \frac{k+1}{k-1}\tan B$

5. यदि $\cos\theta = \cos A \cos B$ भए प्रमाणित गर्नुहोस् (If $\cos\theta = \cos A \cos B$, prove that): $\tan\frac{\theta + A}{2}\tan\frac{\theta - A}{2} = \tan^2\frac{B}{2}$

HINT: Here, $\cos B = \frac{\cos\theta}{\cos A}$ or, $\frac{1 - \tan^2\frac{B}{2}}{1 + \tan^2\frac{B}{2}} = \frac{\cos\theta}{\cos A}$ or, $\frac{\left(1 - \tan^2\frac{B}{2}\right) + \left(1 + \tan^2\frac{B}{2}\right)}{\left(1 - \tan^2\frac{B}{2}\right) - \left(1 + \tan^2\frac{B}{2}\right)} = \frac{\cos\theta + \cos A}{\cos\theta - \cos A}$

Using componendo and devidendo;

or, $\frac{2}{-2\tan^2\frac{B}{2}} = \frac{2\cos\left(\frac{\theta+A}{2}\right)\cos\left(\frac{\theta-A}{2}\right)}{-2\sin\left(\frac{\theta+A}{2}\right)\sin\left(\frac{\theta-A}{2}\right)} = \frac{1}{\tan\left(\frac{\theta+A}{2}\right)\tan\left(\frac{\theta-A}{2}\right)}$ $\therefore \tan^2\frac{B}{2} = \tan\left(\frac{\theta+A}{2}\right)\tan\left(\frac{\theta-A}{2}\right)$

6. प्रमाणित गर्नुहोस् (Prove that):

A. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$ **B.** $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

C. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$ **D.** $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

www.drmathnp.com

puspas.com.np

3. अनुबन्धित त्रिकोणमितीय सर्वसमिकाहरू
CONDITIONAL TRIGONOMETRIC IDENTITIES

DEFINITIONS

अनुबन्धित त्रिकोणमितीय सर्वसमिकाहरू (Conditional trigonometric identities)

निश्चितसर्तका आधारमा मात्र सत्य हुने त्रिकोणमितीय सर्वसमिकाहरूलाई नै अनुबन्धित त्रिकोणमितीय सर्वसमिकाहरू भनिन्छ ।
Some trigonometric identities, which are true only for certain given condition are known as conditional identities.

FORMULAE AND KEY POINTS

1. π^c = त्रिभुजका तिनकोणहरूको योग (Sum of three angles of a triangle) = 180°
2. यदि $\measuredangle A + \measuredangle B + \measuredangle C = 180°$ भए $A + B = \pi^c - C$ and $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$ हुन्छ ।

   If $A + B + C = \pi^c$ then $A + B = \pi^c - C$ and $\frac{A}{2} + \frac{B}{2} = \frac{\pi^c}{2} - \frac{C}{2}$

   (i) $\sin\left(\frac{A}{2} + \frac{B}{2}\right) = \sin\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$ (ii) $\cos\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \sin\frac{C}{2}$

   (iii) $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi^c}{2} - \frac{C}{2}\right) = \cot\frac{C}{2}$
3. यदि $A + B + C = \pi^c$ भए $A + B = 180° - C$ हुन्छ । (If $A + B + C = \pi^c$, then $A + B = 180° - C$.)

   (i) $\cos(A + B) = \cos(180° - C) = -\cos C$ (ii) $\sin(A + B) = \sin(180° - C) = \sin C$

   (iii) $\tan(A + B) = \tan(180° - C) = -\tan C$

TRIGONOMETRY

SEE / SLC REVIEW MESSAGE

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2066 R, 2069 R', SEE 2075 R'	2
Model 2	2070 R', 2072 R', 2072 S, SEE 2074 R'	4
Model 3	2065 M, R', S, 2066 R', S, 2067 R', S, 2068 R, R', S, S', 2070 R, S, 2073 R, R', SEE 2073 S, 2074 R, SEE 2074 S, S', SEE 2075 R'_2	14
Model 4	2065 R, 2069 S, 2071 R', 2072 R, SEE MODEL 2076	4
Model 5	2065 E, 2067 R, 2069 R, 2070 S', SEE 2075 R, R_2	4
Model 6	2071 R, 2071 S	1

SEE EXERCISE 3

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1A. अनुबन्धित त्रिकोणमितीय सर्वसमिकाको परिभाषा दिनुहोस् । (Define Conditional trigonometric identities.)

1B. त्रिकोणमितीय सर्वसमिका र अनुबन्धित त्रिकोणमितीय सर्वसमिकामा के फरक छ ?
What is the difference between trigonometric identity and conditional trigonometric identity?

2A. यदि $A + B + C = \pi^c$ भए $\cos(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(A + B)$ in terms of angle C.

2B. यदि $A + B + C = \pi^c$ भए $\sin(B + A)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\sin(B + A)$ in terms of angle C.

2C. यदि $A + B + C = \pi^c$ भए $\tan(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\tan(A + B)$ in terms of angle C.

2D. यदि $A + B + C = \pi^c$ भए $\cot(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cot(A + B)$ in terms of angle C.

www.drmathnp.com

pupas.com.np

Model 2 *Concept Recall*

3A. यदि $A + B + C = \pi^c$ भए $\cos\left(\frac{A}{2} + \frac{B}{2}\right)$ लाई कोण $\frac{C}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

If $A + B + C = \pi^c$, express $\cos\left(\frac{A}{2} + \frac{B}{2}\right)$ in terms of angle $\frac{C}{2}$.

3B. यदि $A + B + C = \pi^c$ भए $\sin\left(\frac{A}{2} + \frac{B}{2}\right)$ लाई कोण $\frac{C}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

If $A + B + C = \pi^c$, express $\sin\left(\frac{A}{2} + \frac{B}{2}\right)$ in terms of angle $\frac{C}{2}$.

3C. यदि $A + B + C = \pi^c$ भए $\tan\left(\frac{A}{2} + \frac{B}{2}\right)$ लाई कोण $\frac{C}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

If $A + B + C = \pi^c$, express $\tan\left(\frac{A}{2} + \frac{B}{2}\right)$ in terms of angle $\frac{C}{2}$.

3D. यदि $A + B + C = \pi^c$ भए $\cot\left(\frac{A}{2} + \frac{B}{2}\right)$ लाई कोण $\frac{C}{2}$ को रूपमा व्यक्त गर्नुहोस् ।

If $A + B + C = \pi^c$, express $\cot\left(\frac{A}{2} + \frac{B}{2}\right)$ in terms of angle $\frac{C}{2}$.

4A. यदि $A + B + C = \pi^c$ भए $\cos(2A + 2B)$ लाई कोण $2C$ को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(2A + 2B)$ in terms of angle $2C$.

4B. यदि $A + B + C = \pi^c$ भए $\sin(2A + 2B)$ लाई कोण $2C$ को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\sin(2A + 2B)$ in terms of angle $2C$.

5A. यदि $A + B + C = \pi^c$ भए $\cos(4A + 4B)$ लाई कोण $4C$ को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(4A + 4B)$ in terms of angle $4C$.

5B. यदि $A + B + C = \pi^c$ भए $\sin(6A + 6B)$ लाई कोण $6C$ को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\sin(6A + 6B)$ in terms of angle $6C$.

B. SHORT QUESTIONS

Model 1 *Problems based on tangent and cotangent.*

1. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\tan A + \tan B = -\tan C(1 - \tan A \tan B)$

B. $\cot A \cot B - 1 = -\cot C(\cot A + \cot B)$

C. $\tan\frac{A}{2} + \tan\frac{B}{2} = \cot\frac{C}{2}\left(1 - \tan\frac{A}{2}\tan\frac{B}{2}\right)$

D. $\cot\frac{A}{2}\cot\frac{B}{2} - 1 = \tan\frac{C}{2}\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right)$

2A. यदि $A + B + C = \pi^c$ र $\cos A = \cos B \cos C$ भए, प्रमाणित गर्नुहोस्: $\tan A = \tan B + \tan C$
If $A + B + C = \pi^c$ and $\cos A = \cos B \cos C$, prove that: $\tan A = \tan B + \tan C$ **[2065 R']**

2B. यदि $A + B + C = \pi^c$ र $\cos B = \cos A . \cos C$ भए $\tan B = \tan A + \tan C$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $A + B + C = \pi^c$ and $\cos B = \cos A . \cos C$, prove that $\tan B = \tan A + \tan C$.

Model 2 *Problems based on sine and cosine.*

3. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\sin A + \sin B = 2\cos\frac{C}{2}\cos\left(\frac{A - B}{2}\right)$

B. $\cos A + \cos B = 2\sin\frac{C}{2}\cos\left(\frac{A - B}{2}\right)$

C. $\cos B + \cos C = 2\sin\frac{A}{2}\cos\left(\frac{B - C}{2}\right)$

D. $\sin B + \sin C = 2\cos\frac{A}{2}\cos\left(\frac{B - C}{2}\right)$

E. $\sin A - \sin B = 2\sin\frac{C}{2}\sin\left(\frac{A - B}{2}\right)$

F. $\cos B - \cos A = 2\cos\frac{C}{2}\sin\left(\frac{A - B}{2}\right)$

www.drmathnp.com

puspas.com.np

4. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\cos 2A + \cos 2B = -2 \cos C \cos (A - B)$ **B.** $\cos 2A - \cos 2B = -2 \sin C \sin (A - B)$

C. $\sin 2A + \sin 2B = 2 \sin C \cos (A - B)$ **D.** $\sin 2A - \sin 2B = -2 \cos C \sin (A - B)$

E. $\cos 4A + \cos 4B = 2 \cos 2C \cos (2A - 2B)$ **F.** $\sin 4A + \sin 4B = -2 \sin 2C \cos (2A - 2B)$

5. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\cos^2 A + \cos^2 B = 1 - \cos C \cos (A - B)$ **B.** $\cos^2 A - \cos^2 B = -\sin C \sin (A - B)$

C. $\sin^2 A - \sin^2 B = \sin C \sin (A - B)$ **D.** $\sin^2 A + \sin^2 B = 1 + \cos C \cos (A - B)$

6. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} = 1 - \sin \frac{C}{2} \cos \left(\frac{A}{2} - \frac{B}{2}\right)$ **B.** $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} = 1 + \sin \frac{C}{2} \cos \left(\frac{A - B}{2}\right)$

C. LONG QUESTIONS

Model 1 *Problems based on tangent and cotangent.*

1. यदि $\angle A + \angle B + \angle C = 180°$ भए, प्रमाणित गर्नुहोस् (If $\angle A + \angle B + \angle C = 180^\circ$, prove that):

A. $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$ [2057 M]

B. $\cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} = \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$ [SEE 2075 R', 2058 R, 2066 R]

C. $\cot 2A \cot 2B + \cot 2B \cot 2C + \cot 2C \cot 2A = 1$ [2069 R']

Model 2 *Problems based on conversion of sum/difference into product form.*

2. यदि $\angle A + \angle B + \angle C = 180°$ भए, प्रमाणित गर्नुहोस् (If $\angle A + \angle B + \angle C = 180^\circ$, prove that):

A. $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$ [2057 R, 2064 R', 2072 R']

B. $\sin A - \sin B + \sin C = 4 \sin \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$ [2072 S]

C. $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$ [2059 R]

D. $\sin B + \sin C - \sin A = 4 \cos \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ [2061 R]

E. $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} . \sin \frac{B}{2} . \sin \frac{C}{2}$ [2058 S, 2064 R, 2070 R']

F. $\cos B + \cos C - \cos A = 4 \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} - 1$ [2057 S]

3. यदि $P + Q + R = 180°$ भए प्रमाणित गर्नुहोस् (If $P + Q + R = 180°$ then prove that):

$\sin P + \sin Q + \sin R = 4 \cos \frac{P}{2} . \cos \frac{Q}{2} . \cos \frac{R}{2}$ [SEE 2074 R']

Model 3 *Problems based on conversion of sum/difference of multiple angles into product form.*

4. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

A. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A . \sin B . \sin C$ [SEE 2075 R'2, SEE 2073 R']

B. $\cos 2A + \cos 2B + \cos 2C = -(4 \cos A . \cos B . \cos C + 1)$ [2065 M, 2065 S, 2060 S']

C. $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A . \sin B . \cos C$ [2067 S, 2061 S, 2068 S, 2068 S']

D. $\cos 2A - \cos 2B - \cos 2C = 4 \cos A \sin B \sin C - 1$ [2068 R', 2063 R, 2060 S]

E. $\cos 2A - \cos 2B + \cos 2C = 1 - 4 \sin A . \cos B . \sin C$ [SEE 2074 S']

F. $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$ [2060CP, 2066 R', 2067 R']

5. यदि $A + B + C = 90°$ भए प्रमाणित गर्नुहोस् (If $A + B + C = 90°$ then prove that):

$\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$ [SEE 2074 R]

www.drmathnp.com

pueas.com.np

6. यदि $A + B + C = \frac{\pi^c}{2}$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \frac{\pi^c}{2}$ then prove that):

$\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A . \sin B . \sin C$ [SEE 2074 S]

7. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

A. $\frac{\sin 2A + \sin 2B + \sin 2C}{4 \cos \frac{A}{2} . \cos \frac{B}{2} . \cos \frac{C}{2}} = 8 \sin \frac{A}{2} . \sin \frac{B}{2} . \sin \frac{C}{2}$ [2070 R]

B. $\frac{\sin 2A + \sin 2B - \sin 2C}{4 \sin \frac{C}{2} \cos \frac{C}{2}} = 2\cos A \cos B$

8. यदि $\angle X + \angle Y + \angle Z = 180°$ भए प्रमाणित गर्नुहोस् (If $\angle X + \angle Y + \angle Z = 180°$ then prove that):

A. $\cos 4X + \cos 4Y + \cos 4Z = -1 + 4 \cos 2X \cos 2Y \cos 2Z$ [2065 R']

B. $\sin 6X + \sin 6Y + \sin 6Z = 4 \sin 3X \sin 3Y \sin 3Z$

9. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, then prove that):

A. $\sin (B + C - A) + \sin (C + A - B) + \sin (A + B - C) = 4\sin A \sin B \sin C$ [2066 S]

HINT: Here, $A + B + C = \pi^c$ Then, $A + B = \pi^c - C$

So, $\sin (A + B) = \sin (\pi^c - C) = \sin C$ and $\cos (A + B) = \cos (\pi^c - C) = -\cos C$

LHS $= \sin (B + C - A) + \sin (C + A - B) + \sin(A + B - C)$

$= \sin (180° - 2A) + \sin (180° - 2B) + \sin (180° - 2C)$

$= \sin 2A + \sin 2B + \sin 2C = 2\sin\left(\frac{2A + 2B}{2}\right) \cos\left(\frac{2A - 2B}{2}\right) + 2\sin C \cos C$

$= 2\sin (A + B) \cos (A - B) + 2\sin C \cos C = 2\sin C \cos (A - B) + 2\sin C \cos C$

$= 2\sin C [\cos (A - B) + \cos C] = 2\sin C[\cos (A - B) - \cos (A + B)]$

$= 2\sin C \, 2\sin A \sin B = 4\sin A \sin B \sin C = RHS$

B. $\cos (B + C - A) + \cos (C + A - B) + \cos (A + B - C) = 4\cos A \cos B \cos C + 1$ [2068 R, SEE 2073 R]

C. यदि $P + Q + R = 180°$ भए प्रमाणित गर्नुहोस् (If $P + Q + R = 180°$, then prove that):

$\cos(Q + R - P) + \cos (R + P - Q) + \cos (P + Q - R) = 1 + 4\cos P . \cos Q . \cos R$ [SEE 2073 S]

D. यदि $\alpha + \beta + \theta = \pi^c$ भए प्रमाणित गर्नुहोस् (If $\alpha + \beta + \theta = \pi^c$, prove that): [2070 S]

$\sin (\alpha + \beta - \theta) + \sin (\beta + \theta - \alpha) + \sin (\theta + \alpha - \beta) = 4 \sin \alpha . \sin \beta . \sin \theta$

Model 4 *Problems based on conversion of square of sum/difference of angles into product form.*

10. यदि $A + B + C = \pi^c$ भए, प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

A. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$ [2063 S]

HINT: We have, $A + B + C = \pi^c$ $\therefore A + B = \pi^c - C$

$\cos(A + B) = \cos(\pi^c - C)$ [Taking cos on both sides.]

or, $\cos A . \cos B - \sin A . \sin B = -\cos C$

or, $\cos A . \cos B + \cos C = \sin A . \sin B$

or, $(\cos A . \cos B + \cos C)^2 = \sin^2 A . \sin^2 B$ [Squaring both sides.]

or, $\cos^2 A . \cos^2 B + 2\cos A . \cos B . \cos C + \cos^2 C = (1 - \cos^2 A)(1 - \cos^2 B)$

or, $\cos^2 A . \cos^2 B + 2\cos A . \cos B . \cos C + \cos^2 C = 1 - \cos^2 A - \cos^2 B + \cos^2 A . \cos^2 B$

$\therefore \cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A . \cos B . \cos C$

B. $\cos^2 A - \cos^2 B - \cos^2 C = 2 \cos A . \sin B . \sin C - 1$ [2064 S]

C. $\cos^2 A + \cos^2 B + 2\cos A \cos B \cos C = \sin^2 C.$ [2072 R]

D. $1 - 2 \cos A . \cos B . \cos C = \cos^2 A + \cos^2 B + \cos^2 C$ [2062 K]

E. $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A.\cos B.\cos C$ [2069 S, 2071 R']

F. $\sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A . \cos B . \sin C$ [SEE MODEL 2076, 2059 S, 2065 R]

www.drmathnp.com

puspas.com.np

Model 5 *Problems based on conversion of square of sum/difference of half angles into product form.*

11. यदि $\angle A + \angle B + \angle C = 180°$ भए, प्रमाणित गर्नुहोस् (If $\angle A + \angle B + \angle C = 180°$, prove that):

A. $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ [2063 M, 2069 R]

B. $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$ [2063 R', 2067 R, 2070 S']

C. $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$ [2065 E]

D. यदि P, Q र R एउटा ΔPQR का कोणहरू हुन् भने प्रमाणित गर्नुहोस् :
If P, Q and R are the angles of a ΔPQR, prove that:

$$2\left(\sin\frac{Q}{2} + \sin\frac{R}{2}\right)\left(\sin\frac{Q}{2} - \sin\frac{R}{2}\right) - \cos P = 1 - 4\cos\frac{P}{2}.\cos\frac{Q}{2}.\sin\frac{R}{2}$$ [2075 R, 2075 R_2]

E. $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{\pi^c - A}{4}.\sin\frac{\pi^c - B}{4}.\sin\frac{\pi^c - C}{4}$ [2060 R]

HINT: LHS $= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 2\sin\frac{A+B}{4}\cos\frac{A-B}{4} + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$= 2\sin\frac{\pi - C}{4}\cos\frac{A-B}{4} + \cos\left(\frac{\pi - C}{2}\right) = 2\sin\frac{\pi - C}{4}\cos\frac{A-B}{4} + 1 - 2\sin^2\frac{\pi - C}{4}$

$= 1 + 2\sin\frac{\pi - C}{4}\left(\cos\frac{A-B}{4} - \sin\frac{\pi - C}{4}\right) = 1 + 2\sin\frac{\pi - C}{4}\left[\cos\frac{A-B}{4} - \cos\left(\frac{\pi}{2} - \frac{\pi - C}{4}\right)\right]$

$= 1 + 2\sin\frac{\pi - C}{4}\left[\cos\frac{A-B}{4} - \cos\frac{\pi + C}{4}\right]$

$= 1 + 2\sin\frac{\pi - C}{4}\left[2\sin\frac{A - B + \pi + C}{8}\sin\frac{\pi + C - A + B}{8}\right]$

$= 1 + 2\sin\frac{\pi - C}{4}\left[2\sin\frac{2\pi - 2B}{8}\sin\frac{2\pi - 2A}{8}\right] = 1 + 4\sin\frac{\pi - A}{4}\sin\frac{\pi - B}{4}\sin\frac{\pi - C}{4} = \text{RHS}$

Model 6 *Problems based on conversion of sum of pair of product into continued product form.*

12. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\sin A\cos B\cos C + \sin B\cos C\cos A + \sin C\cos A\cos B = \sin A\sin B\sin C$ [2062R]

HINT: $\sin(A + B + C) = \sin 180°$
or, $\sin(A + B).\cos C + \cos(A + B)\sin C = 0$
or, $(\sin A\cos B + \cos A\sin B).\cos C + (\cos A\cos B - \sin A\sin B)\sin C = 0$
or, $\sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C = 0$
$\therefore$ $\sin A\cos B\cos C + \sin B\cos C\cos A + \sin C\cos A\cos B = \sin A\sin B\sin C$

B. $\cos A\sin B\sin C + \cos B\sin C\sin A + \cos C\sin A\sin B = 1 + \cos A\cos B\cos C$

HINT: Here, $A + B + C = \pi^c$ or, $\cos(A + B + C) = \cos\pi^c$
or, $\cos(A + B)\cos C - \sin(A + B)\sin C = -1$
or, $(\cos A\cos B - \sin A\sin B)\cos C - (\sin A\cos B + \cos A\sin B)\sin C = -1$
or, $\cos A\cos B\cos C - \sin A\sin B\cos C - \sin A\cos B\sin C - \cos A\sin B\sin C = -1$
$\therefore$ $\cos A\sin B\sin C + \cos B\sin C\sin A + \cos C\sin A\sin B = 1 + \cos A\cos B\cos C$

13A. यदि $P + Q + R = 180°$ भए, प्रमाणित गर्नुहोस् (If $P + Q + R = 180°$, then prove that):

$$\frac{\cos P}{\sin Q\,.\,\sin R} + \frac{\cos Q}{\sin R\,.\,\sin P} + \frac{\cos R}{\sin P\,.\,\sin Q} = 2$$ [2071 R, 2071 S]

13B. यदि $A + B + C = 180°$ भए, प्रमाणित गर्नुहोस् (If $A + B + C = 180°$, then prove that):

$$\frac{\sin A}{\cos B\cos C} + \frac{\sin B}{\cos C\cos A} + \frac{\sin C}{\cos A\cos B} = 2\tan A\tan B\tan C$$

TRIGONOMETRY

www.drmathnp.com

pupas.com.np

GENERAL EXERCISE 3

LONG QUESTIONS

Model 1 *Problems based on tangent and cotangent.*

1. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

B. $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

C. $\cot 2A \cot 2B + \cot 2B \cot 2C + \cot 2A \cot 2C = 1$

D. $\tan\frac{3A}{2}.\tan\frac{3B}{2} + \tan\frac{3B}{2}.\tan\frac{3C}{2} + \tan\frac{3C}{2}.\tan\frac{3A}{2} = 1$

E. $\tan\frac{5A}{2}\tan\frac{5B}{2} + \tan\frac{5B}{2}\tan\frac{5C}{2} + \tan\frac{5A}{2}\tan\frac{5C}{2} = 1$

F. $\cot\frac{3A}{2} + \cot\frac{3B}{2} + \cot\frac{3C}{2} = \cot\frac{3A}{2}\cot\frac{3B}{2}\cot\frac{3C}{2}$

G. $\cot\frac{5A}{2} + \cot\frac{5B}{2} + \cot\frac{5C}{2} = \cot\frac{5A}{2}\cot\frac{5B}{2}\cot\frac{5C}{2}$

H. $\tan(B + C - A) + \tan(C + A - B) + \tan(A + B - C) = \tan(B + C - A)\tan(C + A - B)\tan(A + B - C)$

I. $\cot(B + C - A)\cot(C + A - B) + \cot(C + A - B)\cot(A + B - C) + \cot(A + B - C)\cot(B + C - A) = 1$

Model 2 *Problems based on conversion of sum/difference into product form.*

2. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\cos A + \cos B - \cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1$

B. $\cos A - \cos B + \cos C = -1 + 4\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

C. $-\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

D. $\cos B + \cos C - \cos A + 1 = 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

E. $\cos A + \cos B + \cos C = 4\cos\left(\frac{\pi^c - A}{2}\right)\cos\left(\frac{\pi^c - B}{2}\right)\cos\left(\frac{\pi^c - C}{2}\right) + 1$

F. $\sin A + \sin B + \sin C = 4\sin\frac{B + C}{2}.\sin\frac{C + A}{2}.\sin\frac{A + B}{2}$

HINT: Here, $A + B + C = 180°$ or, $\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$ $\therefore \sin\left(\frac{A}{2} + \frac{B}{2}\right) = \cos\frac{C}{2}$

$$\text{RHS} = \sin A + \sin B + \sin C = 2\sin\frac{A + B}{2}.\cos\frac{A - B}{2} + 2\sin\frac{C}{2}.\cos\frac{C}{2}$$

$$= 2\cos\frac{C}{2}\left(\cos\frac{A - B}{2} + \sin\frac{C}{2}\right) = 2\cos\frac{C}{2}\left(\cos\frac{A - B}{2} + \cos\frac{A + B}{2}\right)$$

$$= 2\cos\frac{C}{2}.2\cos\frac{\left(\frac{A - B}{2} + \frac{A + B}{2}\right)}{2}.\cos\frac{\left(\frac{A - B}{2} - \frac{A + B}{2}\right)}{2}$$

$$= 4\cos\frac{C}{2}\cos\frac{A}{2}\cos\left(-\frac{B}{2}\right) = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$= 4\cos\left(\frac{\pi^c}{2} - \frac{B + C}{2}\right)\cos\left(\frac{\pi^c}{2} - \frac{C + A}{2}\right)\cos\left(\frac{\pi^c}{2} - \frac{A + B}{2}\right)$$

$$= 4\sin\left(\frac{B + C}{2}\right)\sin\left(\frac{C + A}{2}\right)\sin\left(\frac{A + B}{2}\right)$$

G. $\frac{2(\sin A + \sin B + \sin C)}{\sin A \sin B \sin C} = \operatorname{cosec}\frac{A}{2}\operatorname{cosec}\frac{B}{2}\operatorname{cosec}\frac{C}{2}$ **H.** $\frac{(\sin A + \sin B - \sin C)}{\sin A \sin B} = \sec\frac{A}{2}\sec\frac{B}{2}\cos\frac{C}{2}$

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on conversion of sum/difference of multiple angles into product form.*

3. यदि $A+B+C=\pi^c$ भए प्रमाणित गर्नुहोस् (If $A+B+C=\pi^c$ then prove that):

A. $\sin 2A-\sin 2B+\sin 2C=4\cos A\sin B\cos C$

B. $-\sin 2A+\sin 2B+\sin 2C=4\sin A\cos B\cos C$

C. $-\cos 2A+\cos 2B+\cos 2C=1-4\cos A\sin B\sin C$

D. $\dfrac{\sin 2A-\sin 2B+\sin 2C}{4\sin\frac{B}{2}\cos\frac{B}{2}}=2\cos A\cos C$

E. $\dfrac{-\sin 2A+\sin 2B+\sin 2C}{4\sin\frac{A}{2}\cos\frac{A}{2}}=2\cos B\cos C$

F. $\dfrac{\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C)}{\sin A\sin B\sin C}=4$

4. यदि $A+B+C=\pi^c$ भए प्रमाणित गर्नुहोस् (If $A+B+C=\pi^c$ then prove that):

A. $\sin 3A+\sin 3B+\sin 3C=-4\cos\dfrac{3A}{2}\cos\dfrac{3B}{2}\cos\dfrac{3C}{2}$

B. $\sin 5A+\sin 5B+\sin 5C=4\cos\dfrac{5A}{2}\cos\dfrac{5B}{2}\cos\dfrac{5C}{2}$

Model 4 *Problems based on conversion of square of sum/difference of angles into product form.*

5. यदि $A+B+C=\pi^c$ भए प्रमाणित गर्नुहोस् (If $A+B+C=\pi^c$ then prove that):

A. $\cos^2A+\cos^2B-\cos^2C=1-2\sin A\sin B\cos C$

B. $\sin^2A+\sin^2B-\sin^2C=2\sin A\sin B\cos C$

C. $-\sin^2A+\sin^2B+\sin^2C=2\cos A\sin B\sin C$

D. $\cos^2A-\cos^2B+\cos^2C=1-2\sin A\cos B\sin C$

E. $-\cos^2A+\cos^2B+\cos^2C=1-2\cos A\sin B\sin C$

F. $\cos^2A+\cos^2B-\sin^2C=-2\cos A\cos B\sin C$

G. $1-2\cos B\cos C\cos A-\cos^2A=\cos^2B+\cos^2C$

H. $\cos^2A+\cos^2B+\cos^2C+2\cos A.\cos B.\cos C=1$

I. $\sin^2 2A+\sin^2 2B+\sin^2 2C=2-2\cos 2A\cos 2B\cos 2C$

J. $\dfrac{\sin^2A-\sin^2B+\sin^2C}{\sin 2A\sin 2C}=\dfrac{\cos B}{2\cos A\cos C}$

K. $\dfrac{\sin^2A+\sin^2B-\sin^2C}{\sin A\sin B\sin C}=2\cot C$

L. $\dfrac{-\sin^2A+\sin^2B+\sin^2C}{\sin A\sin B\sin C}=2\cot A$

M. $\dfrac{\sin^2A-\sin^2B+\sin^2C}{2\sin A\sin B\sin C}=\cot B$

Model 5 *Problems based on conversion of square of sum/difference of half angles into product form.*

6. यदि $A+B+C=\pi^c$ भए प्रमाणित गर्नुहोस् (If $A+B+C=\pi^c$ then prove that):

A. $\cos^2\dfrac{A}{2}+\cos^2\dfrac{B}{2}+\cos^2\dfrac{C}{2}=2\left(1+\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right)$

B. $\sin^2\dfrac{A}{2}-\sin^2\dfrac{B}{2}+\sin^2\dfrac{C}{2}=1-2\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$

C. $-\sin^2\dfrac{A}{2}+\sin^2\dfrac{B}{2}+\sin^2\dfrac{C}{2}=1-2\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

D. $\cos^2\dfrac{A}{2}-\cos^2\dfrac{B}{2}+\cos^2\dfrac{C}{2}=2\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$

E. $-\cos^2\dfrac{A}{2}+\cos^2\dfrac{B}{2}+\cos^2\dfrac{C}{2}=2\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

F. $\sin^2\dfrac{A}{2}+\sin^2\dfrac{B}{2}-\sin^2\dfrac{C}{2}=1-2\sin\dfrac{B+C}{2}\sin\dfrac{A+C}{2}\cos\dfrac{A+B}{2}$

G. $\cos^2\dfrac{A}{2}+\cos^2\dfrac{B}{2}-\cos^2\dfrac{C}{2}=2\sin\dfrac{B+C}{2}\sin\dfrac{A+C}{2}\cos\dfrac{A+B}{2}$

7. यदि $A+B+C=\pi^c$ भए प्रमाणित गर्नुहोस् (If $A+B+C=\pi^c$ then prove that):

A. $\sin\dfrac{A}{2}+\sin\dfrac{B}{2}+\sin\dfrac{C}{2}=1+4\sin\left(\dfrac{B+C}{4}\right)\sin\left(\dfrac{C+A}{4}\right)\sin\left(\dfrac{A+B}{4}\right)$

HINT: RHS $=1+4\sin\left(\dfrac{B+C}{4}\right)\sin\left(\dfrac{C+A}{4}\right)\sin\left(\dfrac{A+B}{4}\right)=1+4\sin\left(\dfrac{\pi^c-A}{4}\right)\sin\left(\dfrac{\pi^c-B}{4}\right)\sin\left(\dfrac{\pi^c-C}{4}\right)$

Refer SEE Review Exercise Q.No. 11E.

www.drmathnp.com

pueplas.com.np

B. $\cos\frac{A}{2}-\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\frac{\pi^c+A}{4}\cos\frac{\pi^c-B}{4}\cos\frac{\pi^c+C}{4}$

HINT: LHS $=\cos\frac{A}{2}-\cos\frac{B}{2}+\cos\frac{C}{2}=2\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{B-A}{4}\right)+\sin\left(\frac{\pi^c}{2}-\frac{C}{2}\right)$

$=2\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{B-A}{4}\right)+\sin 2\left(\frac{\pi^c-C}{4}\right)$

$=2\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{B-A}{4}\right)+2\sin\left(\frac{\pi^c-C}{4}\right)\cos\left(\frac{\pi^c-C}{4}\right)$

$=2\sin\left(\frac{A+B}{4}\right)\left[\sin\left(\frac{B-A}{4}\right)+\cos\left(\frac{\pi^c-C}{4}\right)\right]$

$=2\sin\left(\frac{A+B}{4}\right)\left[\sin\left(\frac{B-A}{4}\right)+\sin\left(\frac{\pi^c}{2}-\frac{\pi^c-C}{4}\right)\right]$

$=2\sin\left(\frac{A+B}{4}\right)\left[\sin\left(\frac{B-A}{4}\right)+\sin\left(\frac{\pi^c+C}{4}\right)\right]$

$=2\sin\left(\frac{A+B}{4}\right)\left[2\sin\left(\frac{B-A+\pi^c+C}{8}\right)\times\cos\left(\frac{B-A-\pi^c-C}{8}\right)\right]$

$=4\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{B+C}{4}\right)\times\cos\left(\frac{C+A}{4}\right)$

$=4\cos\left(\frac{\pi^c}{2}-\frac{A+B}{4}\right)\cos\left(\frac{\pi^c}{2}-\frac{B+C}{4}\right)\cos\left(\frac{\pi^c-B}{4}\right)$

$=4\cos\left(\frac{2\pi^c-A-B}{4}\right)\cos\left(\frac{2\pi^c-B-C}{4}\right)\cos\left(\frac{\pi^c-B}{4}\right)$

$=4\cos\left(\frac{\pi^c+C}{4}\right)\cos\left(\frac{\pi^c+A}{4}\right)\cos\left(\frac{\pi^c-B}{4}\right)$

C. $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\left(\frac{\pi^c-A}{4}\right)\cos\left(\frac{\pi^c-B}{4}\right)\cos\left(\frac{\pi^c-C}{4}\right)$

HINT: LHS $=\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}$

$=2\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right)+\sin\left(\frac{\pi^c}{2}-\frac{C}{2}\right)$

$=2\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right)+\sin 2\left(\frac{\pi^c-C}{4}\right)$

$=2\cos\left(\frac{\pi^c-C}{4}\right)\cos\left(\frac{A-B}{4}\right)+2\sin\left(\frac{\pi^c-C}{4}\right)\cos\left(\frac{\pi^c-C}{4}\right)$

$=2\cos\left(\frac{\pi^c-C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right)+\sin\left(\frac{\pi^c-C}{4}\right)\right]$

$=2\cos\left(\frac{\pi^c-C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right)+\cos\left(\frac{\pi^c}{2}-\frac{\pi^c-C}{4}\right)\right]$

$=2\cos\left(\frac{\pi^c-C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right)+\cos\left(\frac{\pi^c+C}{4}\right)\right]$

$=2\cos\left(\frac{\pi^c-C}{4}\right)2\cos\left(\frac{A-B+\pi^c+C}{8}\right)\cos\left(\frac{A-B-\pi^c-C}{8}\right)$

$=2\cos\left(\frac{\pi^c-C}{4}\right).2\cos\left(\frac{2\pi^c-2B}{8}\right)\cos\left(\frac{2A-2\pi^c}{8}\right)$

$=4\cos\left(\frac{\pi^c-C}{4}\right)\cos\left(\frac{\pi^c-B}{4}\right)\cos\left(\frac{\pi^c-A}{4}\right)$

D. $\cos\frac{A}{2}+\cos\frac{B}{2}-\cos\frac{C}{2}=4\cos\frac{\pi^c-A}{4}\cos\frac{\pi^c-B}{4}\sin\frac{\pi^c-C}{4}$

www.drmathnp.com

puspas.com.np

Model 6 *Miscellaneous problems based on conditional identities.*

8A. यदि $A + B + C = 270°$ भए प्रमाणित गर्नुहोस् (If $A + B + C = 270°$ then prove that):
$\sin A \cos B \sin C + \cos A \sin B \sin C + \sin A \sin B \cos C = \cos A \cos B \cos C$

HINT: Here, $A + B + C = 270°$
So, $\cos(A + B + C) = \cos 270°$
or, $\cos(A + B + C) = \cos 270°$
or, $\cos(A + B)\cos C - \sin(A + B)\sin C = 0$

8B. यदि $A + B + C = 270°$ भए प्रमाणित गर्नुहोस् (If $A + B + C = 270°$ then prove that):
$\sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C + 1 = \sin A \sin B \sin C$

HINT: Here, $A + B + C = 270°$
So, $\sin(A + B + C) = \sin 270°$
or, $\sin(A + B + C) = \sin 270°$
or, $\sin(A + B)\cos C + \cos(A + B)\sin C = -1$

9. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that):

A. $\tan\frac{A}{2} + \tan\left(\frac{B+C}{2}\right) = \sec\frac{A}{2}\sec\left(\frac{B+C}{2}\right)$

HINT: $A + B + C = \pi^c$

$\therefore \cos\left(\frac{B+C}{2}\right) = \cos\left(\frac{\pi^c - A}{2}\right) = \sin\frac{A}{2}$ $\quad \therefore \sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi^c - A}{2}\right) = \cos\frac{A}{2}$

$$\text{LHS} = \tan\frac{A}{2} + \tan\left(\frac{B+C}{2}\right) = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} + \frac{\sin\left(\frac{B+C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} + \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin^2\frac{A}{2} + \cos^2\frac{A}{2}}{\cos\frac{A}{2}\sin\frac{A}{2}}$$

$$= \frac{1}{\cos\frac{A}{2}\sin\frac{A}{2}} = \frac{1}{\cos\frac{A}{2}\cos\left(\frac{B+C}{2}\right)} = \sec\frac{A}{2}\sec\left(\frac{B+C}{2}\right)$$

B. $\cot\frac{A}{2} + \cot\left(\frac{B+C}{2}\right) = \operatorname{cosec}\frac{A}{2}\operatorname{cosec}\left(\frac{B+C}{2}\right)$

C. $\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2$

HINT: $\text{LHS} = \frac{\sin A \cos A + \sin B \cos B + \sin C \cos C}{\sin A \sin B \sin C} = \frac{\sin 2A + \sin 2B + \sin 2C}{2 \sin A \sin B \sin C}$

The numerator is same as 3(a) so we get, $\frac{4 \sin A \sin B \sin C}{2 \sin A \sin B \sin C} = 2 = \text{RHS}$

10A. यदि $A + B = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B = \pi^c$, prove that) : $2(1 - \sin A \sin B) = \cos^2 A + \cos^2 B$

10B. यदि $B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $B + C = \pi^c$, prove that) : $2(1 - \sin B \sin C) = \cos^2 C + \cos^2 B$

CHALLENGING EXERCISE 3

1. यदि $\cos A + \cos B + \cos C = 0$ भए प्रमाणित गर्नुहोस् (If $\cos A + \cos B + \cos C = 0$, prove that):
$\cos 3A + \cos 3B + \cos 3C = 12 \cos A \cos B \cos C$.

2. यदि $A + B = C$ भए प्रमाणित गर्नुहोस् (If $A + B = C$ then prove that) :
$\cos^2 A + \cos^2 B + \cos^2 C = 1 + 2 \cos A . \cos B . \cos C$

3. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that) :
$\sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B) = 4 \sin\left(\frac{B-C}{2}\right)\sin\left(\frac{C-A}{2}\right)\sin\left(\frac{A-B}{2}\right)$

4. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):
$\sin(A + 2B) + \sin(B + 2C) - \sin(C + 2A) = 4 \sin\frac{A-B}{2} . \cos\frac{B-C}{2} . \cos\frac{C-A}{2}$

www.drmathnp.com

pueas.com.np

5. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

$$\sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B) = 4\sin\left(\frac{B - C}{2}\right)\sin\left(\frac{C - A}{2}\right)\sin\left(\frac{A - B}{2}\right)$$

6. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, Prove that):

$$\cos(B + 2C) + \cos(C + 2A) + \cos(A + 2B) = 1 - 4\cos\frac{B - C}{2}\cos\frac{C - A}{2}\cos\frac{A - B}{2}$$

7. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

$\sin A \sin B \sin C (\cot A + \cot B + \cot C) = 1 + \cos A \cos B \cos C$

8. यदि $A + B + C = 0$ भए प्रमाणित गर्नुहोस् (If $A + B + C = 0$, prove that):

$$\cos A + \cos B + \cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1$$

9. यदि $A + B + C + D = 2\pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C + D = 2\pi^c$ then prove that):

$$\cos A + \cos B + \cos C + \cos D = 4\cos\frac{A + B}{2}\cos\frac{A + C}{2}\cos\frac{B + C}{2}$$

HINT: Here, $\cos A + \cos B + \cos C + \cos D$

$$= (\cos A + \cos B) + \{\cos C + \cos(A + B + C)\}$$

$$= 2\cos\frac{A + B}{2}\cos\frac{A - B}{2} + 2\cos\frac{A + B + 2C}{2}\cos\frac{A + B}{2}$$

$$= 2\cos\frac{A + B}{2}\left[\cos\frac{A - B}{2} + \cos\frac{A + B + 2C}{2}\right]$$

$$= 2\cos\frac{A + B}{2}\left[2\cos\frac{A - B + A + B + 2C}{4}\cos\frac{A - B - A - B - 2C}{4}\right]$$

$$= 4\cos\frac{A + B}{2}\cos\frac{A + C}{2}\cos\frac{B + C}{2}$$

10. यदि $A + B + C + D = 2\pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C + D = 2\pi^c$ then prove that):

$$\cos A - \cos B + \cos C - \cos D = 4\sin\frac{A + B}{2}\cos\frac{A + C}{2}\sin\frac{B + C}{2}$$

11. ABCD एउटा चर्तुभुज भए प्रमाणित गर्नुहोस् (ABCD is a quadrilateral. Prove that):

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} + \cos^2\frac{D}{2} = 2 - 2\cos\frac{A + B}{2}\cdot\cos\frac{A + C}{2}\cdot\cos\frac{A + D}{2}$$

12. यदि $A + B + C = \pi^c$ भए (If $A + B + C = \pi^c$, prove that): $\dfrac{\cot B + \cot C}{\tan B + \tan C} + \dfrac{\cot C + \cot A}{\tan C + \tan A} + \dfrac{\cot A + \cot B}{\tan A + \tan B} = 1$

13. यदि $A + B + C = \pi^c$ भए (If $A + B + C = \pi^c$ then prove that):

$$\frac{\tan A}{\tan B} + \frac{\tan B}{\tan C} + \frac{\tan C}{\tan A} + \frac{\tan B}{\tan A} + \frac{\tan C}{\tan B} + \frac{\tan A}{\tan C} = \cos A \sec B \sec C + \cos B \sec C \sec A + \cos C \sec A \sec B$$

14. यदि $A + B + C = 2S$ भए (If $A + B + C = 2S$, prove that): $\sin(S - A)\sin(S - B) + \sin S \sin(S - C) = \sin A \sin B$

Hint: LHS $= \sin(S - A)\sin(S - B) + \sin S \sin(S - C)$

$$= \frac{1}{2}[2\sin(S - A)\sin(S - B) + 2\sin S \sin(S - C)]$$

$$= \frac{1}{2}[\cos(A - B) - \cos(2S - A - B) + \cos C - \cos(2S - C)]$$

$$= \frac{1}{2}[\cos(A - B) - \cos C + \cos C - \cos(A + B)] \quad [\because 2S = A + B + C]$$

$$= \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$= \sin A \sin B = \text{RHS}$$

www.drmathnp.com

15. यदि $A + B + C = \pi^c$ र $\sin\left(A + \frac{C}{2}\right) = p \sin\frac{C}{2}$ भए प्रमाणित गर्नुहोस्: $\cot\frac{A}{2}\cot\frac{B}{2} = \frac{p+1}{p-1}$

If $A + B + C = \pi^c$ and $\sin\left(A + \frac{C}{2}\right) = p \sin\frac{C}{2}$ then prove that: $\cot\frac{A}{2}\cot\frac{B}{2} = \frac{p+1}{p-1}$

Hint: We have, $\sin\left(A + \frac{C}{2}\right) = p \sin\frac{C}{2}$ or, $\frac{p}{1} = \frac{\sin\left(A + \frac{C}{2}\right)}{\sin\frac{C}{2}}$

Applying componendo and dividendo, we have, $\frac{p+1}{p-1} = \frac{\sin\left(A + \frac{C}{2}\right) + \sin\frac{C}{2}}{\sin\left(A + \frac{C}{2}\right) - \sin\frac{C}{2}}$

or, $\frac{p+1}{p-1} = \frac{2\sin\left(\frac{A + \frac{C}{2} + \frac{C}{2}}{2}\right)\cos\left(\frac{A + \frac{C}{2} - \frac{C}{2}}{2}\right)}{2\cos\left(\frac{A + \frac{C}{2} + \frac{C}{2}}{2}\right)\sin\left(\frac{A + \frac{C}{2} - \frac{C}{2}}{2}\right)} = \frac{2\sin\left(\frac{A+C}{2}\right).\cos\frac{A}{2}}{2\cos\left(\frac{A+C}{2}\right).\sin\frac{A}{2}}$

$= \tan\left(\frac{A+C}{2}\right).\cot\frac{A}{2} = \tan\left(90° - \frac{B}{2}\right).\cot\frac{A}{2} = \cot\frac{B}{2}\cot\frac{A}{2} = \text{RHS}$

TRIGONOMETRY

16. यदि $A + B + C = \frac{\pi^c}{2}$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \frac{\pi^c}{2}$ then prove that):

A. $\tan A - \tan B = \sqrt{1 + \tan^2 C}\,(\sec A \cos B - \cos A \sec B)$

B. $\tan(A + B)[\tan A - \tan B] = \sqrt{1 + \cot^2 C}\,(\sec A \cos B - \cos A \sec B)$

17. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$, prove that):

A. $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin 2A + \sin 2B - \sin 2C} = \tan A \tan B$

B. $\frac{\sin 2A + \sin 2B + \sin 2C}{\cos A + \cos B + \cos C - 1} = 8\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

C. $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

D. $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A - \sin B + \sin C} = 8\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

E. $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B - \sin C} = 8\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

F. $\frac{\sin 2A + \sin 2B + \sin 2C}{-\sin A + \sin B + \sin C} = 8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

G. $\frac{\sin B + \sin C - \sin A}{\sin A + \sin B + \sin C} = \tan\frac{A}{2}\tan\frac{C}{2}$

H. $\frac{\sin A + \sin B + \sin C}{\sin A - \sin B + \sin C} = \tan\frac{A}{2}\tan\frac{C}{2}$

I. $\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \tan\frac{A}{2}\tan\frac{B}{2}$

18. यदि $A + B + C = \frac{\pi^c}{2}$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \frac{\pi^c}{2}$, prove that): $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin 2A + \sin 2B - \sin 2C} = \cot A \cot B$

19. ΔABC मा प्रमाणित गर्नुहोस् (In ΔABC, prove that): $\frac{\cot A + \cot B + \cot C}{\cot A \,.\, \cot B \,.\, \cot C} - \frac{1}{\cos A \,.\, \cos B \,.\, \cos C} = 1$

20. यदि $A + B + C + D = 2\pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C + D = 2\pi^c$ then prove that):

$\cos A + \cos B + \cos C + \cos D = 4\cos\frac{A+B}{2}\cos\frac{A+C}{2}\cos\frac{B+C}{2}$

Hint: Here, $\cos A + \cos B + \cos C + \cos D$

$= (\cos A + \cos B) + \{\cos C + \cos(A + B + C)\}$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + 2\cos\frac{A+B+2C}{2}\cos\frac{A+B}{2}$

$= 2\cos\frac{A+B}{2}\left[\cos\frac{A-B}{2} + \cos\frac{A+B+2C}{2}\right]$

$= 2\cos\frac{A+B}{2}\left[2\cos\frac{A-B+A+B+2C}{4}\cos\frac{A-B-A-B-2C}{4}\right]$

$= 4\cos\frac{A+B}{2}\cos\frac{A+C}{2}\cos\frac{B+C}{2} = \text{RHS}$

puspas.com.np

www.drmathnp.com

4. त्रिकोणमितीय समीकरणहरू TRIGONOMETRIC EQUATIONS

DEFINITIONS

A. सर्वसमिका (Identity):

बराबर चिह्न समावेश भएको कुनै पनि सम्बन्धमा त्यहाँ भएको चलको जुनसुकै मानले उक्त सम्बन्ध मान्य हुन्छ भने, उक्त सम्बन्धलाई सर्वसमिका भनिन्छ । जस्तै: $\sin^2\theta + \cos^2\theta = 1$ मा θ को जुनसुकै मान पनि मान्य हुन्छ । त्यसैले यो सर्वसमिका हो ।

If the relation of an equality is satisfied by every value of the variable, then the relation is called identity. For example: In the relation $\sin^2\theta + \cos^2\theta = 1$, every value of the θ satisfies this relation.

B. समीकरण (Equation):

बराबर चिह्न समावेश भएको कुनै पनि सम्बन्धमा त्यहाँ भएको चलको निश्चित मानले मात्र उक्त सम्बन्ध मान्य हुन्छ भने, उक्त सम्बन्धलाई समीकरण भनिन्छ । जस्तै: $y = \sin x$, $y = 2x + 3$ इत्यादि ।

If the relation of equality is satisfied only by the fixed value of the variable, then this relation is called equation. For example: $y = \sin x$, $y = 2x + 3$ etc.

C. त्रिकोणमितीय समीकरण (Trigonometric equation):

त्रिकोणमितीय अनुपातहरू समावेश भएको समीकरणलाई त्रिकोणमितीय समीकरण भनिन्छ । जस्तै:
$y = \sin x$, $y = 2\tan x$, $2\cos^2\theta + 1 = 0$ इत्यादि ।

The equation involving trigonometric ratios is called trigonometric equation. For example:
$y = \sin x$, $y = 2\tan x$, $2\cos^2\theta + 1 = 0$ etc.

D. समीकरणको हल (Solution of the equation):

त्रिकोणमितीय समीकरणमा भएको कोणको मान पत्ता लगाउनुलाई त्रिकोणमितीय समीकरणको हल भनिन्छ । यसरी त्रिकोणमितीय समीकरणको हल गर्दा उक्त समीकरणमा उल्लेखित कोणका मानहरू अनन्त हुन सक्छन् । तर यहाँ $0°$ देखि $360°$ ($0° \le \theta \le 2\pi$) को क्षेत्रभित्रका कोणहरू मात्र पत्ता लगाइन्छ ।

To find the value of the angle of trigonometric ratios of the trigonometric equation is the solution of the equation. There are infinite number of solutions of the trigonometric equation. Here we discuss the solution within $0°$ to $360°$ ($0° \le \theta \le 2\pi$).

FORMULAE AND KEY POINTS

मान तालिका (Value Table)

Values	sin	cos	tan	cosec	sec	cot
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	∞	1	∞	0
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2	$-\frac{1}{\sqrt{3}}$
135°	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	2	$-\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
180°	0	-1	0	∞	-1	∞
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	-2	$-\frac{2}{\sqrt{3}}$	$\sqrt{3}$
225°	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2	$\frac{1}{\sqrt{3}}$
270°	-1	0	∞	-1	∞	0

www.drmathnp.com

300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
315°	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
360°	0	1	0	∞	1	∞

क्र.सं. S.N.	समीकरण Equation	अवस्था Condition	चतुर्थांश Quadrant	मानहरू Values
1.	$\sin\theta = k$	+ve & ≤ 1	1^{st} & 2^{nd}	$\theta, (180° - \theta), (360° + \theta)$
2.	$\sin\theta = -k$	–ve ≥ -1	3^{rd} & 4^{th}	$(180° + \theta), (360° - \theta)$
3.	$\sin\theta = k$	θ is non standard		$\theta = \sin^{-1} k$
4.	$\sin\theta = k$	$k > 1$ or $k < 1$		no solution
5.	$\cos\theta = k$	+ve & ≤ 1	1^{st} & 4^{th}	$\theta, (360° - \theta), (360° + \theta)$
6.	$\cos\theta = -k$	–ve ≥ -1	2^{nd} & 3^{rd}	$(180° - \theta), (180° + \theta)$
7.	$\cos\theta = k$	θ is non standard		$\theta = \cos^{-1} k$
8.	$\cos\theta = k$	$k > 1$ or $k < 1$		no solution
9.	$\tan\theta = k$	+ve	1^{st} & 3^{rd}	$\theta, (180° + \theta), (360° + \theta)$
10.	$\tan\theta = -k$	–ve	2^{nd} & 4^{th}	$(180° - \theta), (360° - \theta)$
11.	$\tan\theta = k$	θ is non standard		$\theta = \tan^{-1} k$

SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, R', 2066 R, 2067 R, 2068 R, S, 2069 R, R', S, 2070 R, S, S', 2071 R', 2072 R, R', S, 2073 R', SEE 2074R, SEE 2074R', SEE 2074 S, SEE 2075 R, R', R_2, SEE MODEL 2076	22
Model 2	2065 M, 2065 S, 2068 S', 2070 R', 2071 R, 2071 S, SEE 2074 S', SEE 2075 R'_2	6
Model 3	2066 R', 2067 R', S, 2068 R', 2073 R, SEE 2073 S, SEE 2073 S'	6

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 E, 2065 R', 2067 R', S, 2068 R, S, S', 2069 S, 2070 R, 2070 R', 2073 R, SEE 2073 S, SEE 2073 S', SEE 2074 R, SEE 2074 S', SEE 2075 R'_2	13
Model 2	2065 M, R, S, 2066 R, R', S, 2067 R, 2069 R, R', 2070 S', 2071 R, 2071 R', 2071 S, 2072 S, R, 2073 R', SEE 2074 R', S, SEE 2075 R, R', R_2	12
Model 3	2070 S, 2072 R'	2

SEE EXERCISE 4

A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1A. त्रिकोणमितीय समीकरणको परिभाषा दिनुहोस् । (Define trigonometric equation.)

1B. सर्वसमिका र समीकरणमा के फरक छ ? (What is the difference between identity and equation?)

2A. यदि $x \leq \sin\theta \leq y$ भए x र y का मानहरू लेख्नुहोस् । (If $x \leq \sin\theta \leq y$ then write the values of x and y.)

2B. यदि $x \leq \cos\theta \leq y$ भए x र y का मानहरू लेख्नुहोस् । (If $x \leq \cos\theta \leq y$ then write the values of x and y.)

3A. यदि $\sin\theta = \cos\theta$ भए θ को न्यूनकोणीय मान कति हुन्छ ? (If $\sin\theta = \cos\theta$, what is the acute value of θ?)

3B. यदि sec A = cosec A भए A को न्यूनकोणीय मान कति हुन्छ ? (If sec A = cosec A, what is the acute value of A?)

3C. यदि cot X = tan X भए X को न्यूनकोणीय मान कति हुन्छ ? (If cot X = tan X, what is the acute value of X?)

puspas.com.np

www.drmathnp.com

pueas.com.np

Model 2 *Concept Recall*

4A. यदि $\tan(180° + \theta) = x$ भए x कति हुन्छ ? (If $\tan(180° + \theta) = x$ then what is x?)

4B. यदि $\sin(90° - \theta) = x$ भए x कति हुन्छ ? (If $\sin(90° - \theta) = x$ then what is x?)

4C. यदि $\cos(360° - \theta) = x$ भए x कति हुन्छ ? (If $\cos(360° - \theta) = x$ then what is x?)

5A. यदि $\cos\theta = -1$ भए θ को मान $90°$ देखि $180°$ सम्म कति हुन्छ ?
If $\cos\theta = -1$ then what is the value of θ from $90°$ to$180°$?

5B. यदि $\sin\theta = \frac{1}{2}$ भए θ को मान $90°$ र $180°$ को बिचमा कति हुन्छ ?

If $\sin\theta = \frac{1}{2}$ then what is the value of θ between $90°$ and $180°$?

5C. यदि $\tan\theta = -\sqrt{3}$ भए θ को मान $90°$ र $180°$ को बिचमा कति हुन्छ ?
If $\tan\theta = -\sqrt{3}$ then what is the value of θ between $90°$ and $180°$?

6A. यदि $\sin\theta = \frac{\sqrt{3}}{2}, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ?(If $\sin\theta = \frac{\sqrt{3}}{2}, 0° \le \theta \le 180°$, what is the value of θ.)

6B. यदि $\cos A = \frac{\sqrt{3}}{2}, 0° \le A \le 180°$ भए A को मान कति हुन्छ ? (If $\cos A = \frac{\sqrt{3}}{2}, 0° \le A \le 180°$, what is the value of A.)

6C. यदि $\cos\theta = \frac{1}{2}, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ?(If $\cos\theta = \frac{1}{2}, 0° \le \theta \le 180°$, what is the value of θ.)

6D. यदि $\tan\theta = \frac{1}{\sqrt{3}}, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ? (If $\tan\theta = \frac{1}{\sqrt{3}}, 0° \le \theta \le 180°$, what is the value of θ.)

6E. यदि $\sin\theta + \frac{\sqrt{3}}{2} = 0, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ? (If $\sin\theta + \frac{\sqrt{3}}{2} = 0, 0° \le \theta \le 180°$, what is the value of θ.)

6F. यदि $\cos\theta + \frac{\sqrt{3}}{2} = 0, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ? (If $\cos\theta + \frac{\sqrt{3}}{2} = 0, 0° \le \theta \le 180°$, what is the value of θ.)

6G. यदि $\tan\theta + \sqrt{3} = 0, 0° \le \theta \le 180°$ भए θ को मान कति हुन्छ ? (If $\tan\theta + \sqrt{3} = 0, 0° \le \theta \le 180°$, what is the value of θ.)

7A. $\sin^2\theta + \cos^2\theta = 1$ मा θ का मानहरू के के हुन्छन् ? (What are the values of θ in $\sin^2\theta + \cos^2\theta = 1$?)

7B. $\sec^2\theta - \tan^2\theta = 1$ मा θ का मानहरू के के हुन्छन् ? (What are the values of θ in $\sec^2\theta - \tan^2\theta = 1$?)

B. SHORT QUESTIONS

हल गर्नुहोस् (Solve) [Range of value is upto $180°$ if not indicated]

Model 1 *The trigonometric equations reducible to the form: $\sin\theta = k$, $\cos\theta = k$ or $\tan\theta = k$.*

1. A. $2\cos\theta = 1$ [2063 M] **B.** $2\sin\theta - \sqrt{3} = 0$ [2068 R]

C. $2\sin\theta - \sqrt{2} = 0$ [SEE 2074 R'] **D.** $2\cos\theta - \sqrt{2} = 0 [0° \le \theta \le 90°]$ [SEE 2075 R']

E. $\frac{\sqrt{2}}{\cos\theta} + 2 = 0$ [2066 R] **F.** $2\cos^2\theta - 1 = 0$ [2063S, 2067R]

G. $4\cos^2 A = 3$ [2070 S'] **H.** $2\sin^2\alpha - 1 = 0$ [2068 S]

I. $\sin\theta\cos\theta = \frac{1}{2}(0° \le \theta \le 90°)$ [SEE 2074 S] **J.** $\sin\theta\cos\theta = \frac{\sqrt{3}}{4}$

2. A. $3\tan\theta = \sqrt{3}$ [2063 R'] **B.** $3\tan\theta - \sqrt{3} = 0 \quad (0° \le \theta \le 90°)$ [2071 R']

C. $\sqrt{3}\tan\theta - 3 = 0$ [2070 R] **D.** $\sqrt{3}\tan\theta + 3 = 0$ [SEE 2073 R']

E. $\tan^2 A - 1 = 2$ [2069 S] **F.** $1 - \tan^2\theta = -2 (0° \le \theta \le 90°)$ [2072 R]

G. $\sqrt{3}\tan A + 1 = 0$ [2072 R'] **H.** $\cot^2 x = 3$ [2065 R]

I. $4 - 3\sec^2\theta = 0 \quad (0° \le \theta \le 90°)$ [2072 S] **J.** $\sqrt{2}\sec\theta + 2 = 0$ [2069 R, 2069 R']

www.drmathnp.com

3. **A.** $4 \sin \alpha = 3 \operatorname{cosec} \alpha$ [2065R'] **B.** $\sin \theta - \tan \theta = 0$ [2062 K]

C. $3 \sec \theta = 4 \cos \theta \ [0° \le \theta \le 90°]$ [2070 S] **D.** $\operatorname{cosec} \theta = \sec \theta$ [SEE 2074 R]

E. $2\cos \theta = \sec \theta \ [0° \le \theta \le 90°]$ [SEE 2075 R] **F.** $\operatorname{cosec} A = 2\sin A \ [0° \le A \le 90°]$ [SEE 2075 R_2]

4. θ को मान पत्ता लगाउनुहोस् (Find the value of θ): $[0° \le \theta \le 180°]$

A. $2 \sin 2\theta = \sqrt{3}$ [SEE MODEL 2076] **B.** $2 \cos 2\theta = 1$ **C.** $2 \sin 2\theta - \sqrt{2} = 0$

D. $2 \cos 2\theta - \sqrt{2} = 0$ **E.** $3 \tan 2\theta = \sqrt{3}$ **F.** $\sqrt{3} \tan 2\theta + 3 = 0$

G. $2 \sin 3\theta = \sqrt{3}$ **H.** $2 \cos 3\theta = 1$ **I.** $\sqrt{3} \tan 3\theta - 3 = 0$

Model 2 *Problems based on the trigonometric equations of the form: sin θ = cos θ or tan θ = cot θ .*

5. **A.** $\sin \theta = \cos \theta$ [2063R, 2065S] **B.** $\sin \theta - \cos \theta = 0$ [SEE 2075 R'$_2$, 2074 S', 2065 M]

C. $\tan \alpha = \cot \alpha$ [2064 S, 2070 R'] **D.** $\cos 2x = \sin x$ [2064 R']

E. $\sin 2\theta + \cos \theta = 0 \ [0° \le \theta \le 90°]$ [2071 R] **F.** $\sin x - \sin 2x = 0 \ [0° \le x \le \frac{\pi^C}{2}]$ [2071 S]

6A. यदि $\sin x - \sqrt{3} \cos x = 0$ भए x को न्यूनकोणी मान पत्ता लगाउनुहोस् ।
If $\sin x - \sqrt{3} \cos x = 0$ then find the acute value of x. [2068 S']

6B. यदि $\sqrt{3} \sin \theta = 3\cos \theta$ भए θ को न्यूनकोणी मान पत्ता लगाउनुहोस् ।
If $\sqrt{3}\sin \theta = 3 \cos \theta$ then find the acute value of θ.

Model 3 *Problems based on the trigonometric equations (Quadratic) form.*

7. **A.** $2\cos^2 \theta = -\sqrt{3} \cos \theta$ [2067S, 2059 R, 2068R'] **B.** $2\cos^2 \theta - \sqrt{3} \cos \theta = 0$ [2064 S]

C. $2\cos^2 \theta = 3 \sin \theta$ [2066 R'] **D.** $\operatorname{cosec} x - 2\sin x = 1 \ (0° \le x \le \frac{\pi^c}{2})$ [2067 R']

8. **A.** $\sin^2 \theta - \sin \theta + \frac{1}{4} = 0 \ [0° \le \theta \le 90°]$ [2073 R] **B.** $\cos^2 \theta - \cos \theta + \frac{1}{4} = 0 \ (0° \le \theta \le 90°)$ [2073 S]

9. **A.** $\cos^2 \frac{\theta}{2} - \cos \frac{\theta}{2} + \frac{1}{4} = 0$ [2060 R] **B.** $2\cos^2 \frac{\theta}{2} - \sqrt{3} \cos \frac{\theta}{2} = 0$

C. $\sin^2 \frac{\theta}{2} - \sin \frac{\theta}{2} + \frac{1}{4} = 0 \ [0° \le \theta \le 90°]$ [SEE 2073 S'] **D.** $\sin \frac{x}{2} - \sqrt{3} \cos \frac{x}{2} = 0$

C. LONG QUESTIONS

हल गर्नुहोस् (Solve):

Model 1 *Problems based on the trigonometric equations (Quadratic) in sine/cosine/tangent/co-tangent .*

1. **A.** $2 \cos^2 \theta = 3 \sin \theta$ $(0° \le \theta \le 180°)$ [2057 S]

B. $\cos^2 \theta - \sin \theta - \frac{1}{4} = 0$ $(0^0 \le \theta \le 360°)$ [2068 R, 2061 R]

C. $\cos^2 \theta + \sin \theta = \frac{1}{4}$ $(0° \le \theta \le 360°)$ [SEE 2074 S']

D. $2 \cos^2 \theta + \sin \theta = 2$ $(0° \le \theta \le 180°)$ [2061 S]

E. $1 + \cos \theta = 2 \sin^2 \theta$ $(0° \le \theta \le 180°)$ [2058 S]

F. $2 \sin^2 \theta - \cos \theta = 1$ $(0° \le \theta \le 360°)$ [2062 R, 2068 S, 2068 S']

G. $\cos^2 x = 3 \sin^2 x + 4 \cos x$ $(0° \le x \le 360°)$ [2062 R]

H. $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ $(0° \le \theta \le 360°)$ [2065 E]

I. $6 \sin^2 X + \cos X = 5$ $(0° \le X \le 360°)$ [2067 S, 2067 R']

J. $4\sin^2 \theta - 8\cos \theta - 4 = 0$ $(0° \le \theta \le 360°)$ [SEE 2075 R'$_2$]

K. $3\sin^2 x + 4\cos x = 4$ $(0° \le x \le 360°)$ [2069 S]

L. $2\sqrt{3} \sin^2 \theta = \cos \theta$ $(0° \le \theta \le 360°)$ [2070 R', SEE 2073 S', SEE 2073 S]

M. $2\sqrt{3} \cos^2 \theta = \sin \theta$ $(0° \le \theta \le 360°)$ [SEE 2073 R]

puspas.com.np

www.drmathnp.com

pueplas.com.np

2. **A.** $3\cos 2x - 1 = 2\sin^2 x$ $(0° \le x \le 180°)$ **[2064 R']**

B. $\cos 2\theta = \sin\theta$ $(0° \le \theta \le 360°)$ **[2058 R]**

C. $\sec\theta . \tan\theta = \sqrt{2}$ $(0° \le \theta \le 180°)$ **[2060 R]**

3. **A.** $(1-\sqrt{3})\tan\theta + 1 + \sqrt{3} = \sqrt{3}\sec^2\theta$ $(0° \le \theta \le 360°)$ **[2070 R]**

B. $\cot^2 x + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot x = -1$ $(0° \le x \le 360°)$ **[2068 R', 2065 R']**

C. $\tan^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\tan\theta = -1$ $(0° \le \theta \le 360°)$ **[SEE 2074 R]**

Model 2 *Problems based on the trigonometric equations of the form a cos θ + b sin θ = 0 or similar.*

4. **A.** $\sin\theta + \cos\theta = 1$ $(0° \le \theta \le 360°)$ **[SEE 2075 R', SEE 2073 R']**

B. $\sqrt{3}\sin\theta + \cos\theta = 1$ $(0° \le \theta \le 360°)$ **[2065 S, 2071 R', SEE 2074 R']**

C. $\sin A = \sqrt{3}(1 - \cos A)$ $(0° \le A \le 360°)$ **[2065 M, 2071 S]**

OR, $\cos\theta + \frac{1}{\sqrt{3}}\sin\theta = 1$ $(0° \le \theta \le 360°)$ **[2067 R]**

D. $\sqrt{3}\sin\alpha - \cos\alpha = \sqrt{2}$ $(0^0 \le \alpha \le 180°)$ **[2057 R, 2069 R', 2071 R, 2066 R']**

E. $\sqrt{3}\sin\theta + \cos\theta = \sqrt{2}$ $(0° \le \theta \le 2\pi^c)$ **[2066 R, 2072 S]**

F. $\sqrt{3}\cos x + \sin x = \sqrt{3}$ $(0 \le x \le 2\pi)$ **[2066 S, 2060S', 2069 R]**

G. $\sqrt{3}\tan\theta = \sqrt{3}\sec\theta - 1$ $(0° \le \theta \le 360°)$ **[SEE 2075 R, 2075 R_2]**

H. $\sqrt{3}\tan\theta = \sec\theta - 1$ $(0° \le \theta \le 360°)$ **[2065 R]**

I. $1 + \sqrt{3}\tan\alpha - \sec\alpha = 0$ $(0° \le \alpha \le 360°)$ **[SEE 2074 S, SEE 2072 R]**

J. $\sqrt{3}\cot A = \frac{\sqrt{3}}{\sin A} - 1$ $(0° \le A \le 360°)$ **[2064 R, 2070 S']**

Model 3 *The trigonometric equations of the form sin C + sin D = sin E or similar or in cosine.*

5. **A.** $\cos\theta + \cos 2\theta + \cos 3\theta = 0$ $(0° \le \theta \le \pi^c)$ **[2070 S]**

B. $\sin 3\theta + \sin\theta = \sin 2\theta$ $(0° \le \theta \le 180°)$ **[2061 R, 2072 R']**

C. $\sin 3x + \sin x = 2\sin x$ $(0° \le x \le 180°)$ **[2063 R']**

D. $\cos 3x + \cos x = 2\cos x$ $(0° \le x \le 360°)$ **[2063 M]**

E. $\cos 3\theta + \cos\theta = \cos 2\theta$ $(0° \le \theta \le 180°)$ **[2064 S]**

F. $\sin 4x + \sin 2x = \cos x$ $(0° \le x \le 360°)$ **[2063 S]**

ANSWERS

A. VERY SHORT QUESTIONS

2A. $-1, 1$ **2B.** $-1, 1$ **3A.** 45° **3B.** 45° **3C.** 45° **4A.** $x = \tan\theta$ **4B.** $x = \cos\theta$
4C. $x = \cos\theta$ **5A.** 180° **5B.** 30°, 150° **5C.** 120° **6A.** 60°, 120° **6B.** 30° **6C.** 60°
6D. 30° **6E.** No solution **6F.** 150° **6G.** 120° **7A.** For all values **7B.** For all values

B. SHORT QUESTIONS

1A. 60° **1B.** 60°, 120° **1C.** 45° **1D.** 45° **1E.** 135° **1F.** 45°, 135° **1G.** 30°, 150°
1H. 45°, 135° **1I.** 45° **1J.** 30°, 60° **2A.** 30° **2B.** 30° **2C.** 60° **2D.** 120°
2E. 60°, 120° **2F.** 60° **2G.** 150° **2H.** 30°, 150° **2I.** 30° **2J.** 135°, 225° **3A.** 60°, 120°
3B. 0°, 180° **3C.** 30° **3D.** 45° **3E.** 45° **3F.** 45°
4A. 30°, 60° **4B.** 30°, 150° **4C.** 22.5°, 67.5° **4D.** 22.5°, 157.5° **4E.** 15°, 105° **4F.** 60°, 150°
4G. 20°, 40°, 140° **4H.** 20°, 100°, 140° **4I.** 20°, 80°, 140° **5A.** 45° **5B.** 45° **5C.** 45°, 135° **5D.** 30°
5E. 90° **5F.** 0°, 60° **6A.** 60° **6B.** 60° **7A.** 150°, 90° **7B.** 90°, 30° **7C.** 30°
7D. 30° **8A.** 30° **8B.** 60° **9A.** 120° **9B.** 180°, 60° **9C.** 60° **9D.** 120°

C. LONG QUESTIONS

1A. 30° 150° **1B.** 30°, 150° **1C.** 210°, 330° **1D.** 0°, 30°, 150°, 180° **1E.** 0°, 60° **1F.** 60°, 180°, 300°
1G. 120°, 240° **1H.** 30°, 150°, 210°, 330° **1I.** 60°, 300° , $\cos^{-1}(1/3)$ **1J.** 90°, 270°
1K. 0°, 70.5°, 289.5°, 360° Or, 0°, $\cos^{-1}(1/3)$, 360° **1L.** 30°, 330° **1M.** 60°, 120° **2A.** 30°, 150°
2B. 30°, 150°, 270° **2C.** 45°, 135° , 180° **3A.** 30°, 135°, 210°, 315° **3B.** 120°, 150°, 300°, 330°
3C. 120°, 150°, 300°, 330° **4A.** 0°, 90°, 360° **4B.** 0°, 120°, 360° **4C.** 0°, 60°, 360° **4D.** 75°, 165°
4E. 15°, 105° **4F.** 0°, 60°, 360° **4G.** 30°, 90° **4H.** 0°, 120°, 360° **4I.** 0°, 120°, or 360° **4J.** 0°, 60°, 360°
5A. 45°, 120°, 135° **5B.** 0°, 60°, 90°, 180° **5C.** 0°, 45°, 135° **5D.** 0°, 90°, 180°, 270°, 360°
5E. 45°, 60°, 135°, 300° **5F.** 10°,50°,90°,130°, 250°, 270°, 290°

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 4

A. SHORT QUESTIONS

Model 1 *The trigonometric equations reducible to the form: sin θ = k, cos θ = k or tan θ = k .*

1. हल गर्नुहोस् (Solve): $[0° \le \theta \le 90°]$

A. $\tan\theta = 2\sin 60°$ **B.** $\sec\theta = \frac{1}{\sin 30°}$ **C.** $\sin 2\theta = \frac{1}{\operatorname{cosec} 30°}$

D. $2\sin\theta - 1 = \cos 90°$ **E.** $4\sin^2\theta = \tan^2 60°$ **F.** $4\cos^2\theta - 1 = \tan 0°$

G. $2\cos^2\theta - 1 = \tan 0°$ **H.** $4\cos^2\theta - 3 = 0$ **I.** $4\cos^2\theta - 3 = 0$

J. $\operatorname{cosec}\theta = \sec 45°\tan 45°$ **K.** $\sqrt{3}\cot\theta = \sin 30°\sec 60°$ **L.** $\cos 3\theta = \frac{1}{\sec 45°}$

M. $2\sin 2\theta = \frac{1}{\tan 30°}$ **N.** $\tan 2\theta = \frac{1}{\tan 60°}$ **O.** $\sqrt{2}\sec\theta - 2 = 0$

2. हल गर्नुहोस् (Solve): $[0° \le \theta \le 180°]$

A. $\sin\theta : \operatorname{cosec}\theta = 3 : 4$ **B.** $\cos\theta : \sec\theta = 3 : 4$ **C.** $\tan\theta : \cot\theta = 3 : 1$

D. $\sec\theta : \operatorname{cosec}\theta = 1 : \sqrt{3}$ **E.** $\operatorname{cosec}\theta : \sec\theta = 1 : \sqrt{3}$ **F.** $\tan\theta : \sin\theta = 2 : 1$

G. $\cot\theta : \cos\theta = 2 : \sqrt{3}$ **H.** $\cos\theta : \cot\theta = 1 : 2$

Model 2 *Problems based on solving the trigonometric equations by making separate equations.*

3. हल गर्नुहोस् (Solve):

A. $\cos\theta(2\cos\theta + \sqrt{3}) = 0$ $[0° \le \theta \le 180°]$ **B.** $\cos\theta(2\sin\theta - 1) = 0$ $[0° \le \theta \le 360°]$

C. $(1 - 2\cos x)(1 + 3\cos x) = 0$ $[0° \le x \le 360°]$ **D.** $(\sin\theta + 1)(2\sin\theta - 1) = 0$ $[0° \le \theta \le 360°]$

E. $\sin\theta(1 - 2\cos\theta) = 0$ $[0° \le \theta \le 360°]$ **F.** $(2\sin\theta - 1)(\tan\theta - \sqrt{3}) = 0$ $[0° \le \theta \le 90°]$

G. $\cos 2\theta(2\cos\theta - 1) = 0$ $[0° \le \theta \le 90°]$ **H.** $\sin 2A = \sin A$ $[0° \le A \le 180°]$

Model 3 *Problems based on the trigonometric equations reducible to the quadratic form.*

4. तलका समीकरणहरू हल गर्नुहोस् (Solve the following equations) :

A. $\frac{\cos\theta}{1 - \sin\theta} + \frac{\cos\theta}{1 + \sin\theta} = 4$ $[0° \le \theta < 90°]$ **B.** $\frac{\cos^2\theta - 3\cos\theta + 2}{1 - \cos^2\theta} = 1$ $[0° \le \theta < 90°]$

C. $\sin^2\theta + \cos^2\theta = \sin 2\theta$ $[0° \le \theta < 90°]$ **D.** $\operatorname{cosec}^2 A + \cot^2 A = 3$ $[0° \le A \le 2\pi^C]$

E. $\sin^2\theta + \sin\theta = 1$ $[0° \le \theta \le 180°]$ **F.** $\sin^2\theta - \sin\theta = 0$ $[0° \le \theta \le 360°]$

G. $\tan\theta + \cot\theta = 2$ $[0° \le \theta \le 360°]$ **H.** $2\cos\theta + 2\sec\theta = 5$ $[0° \le \theta \le 90°]$

Model 4 *Problems based on finding the value of trigonometric terms sin mθ or cos mθ or tan mθ.*

5A. यदि $4\cos^2 x - 1 = 0$ and $0° \le x \le 90°$ भए $\tan x$ को मान पत्ता लगाउनुहोस् ।
If $4\cos^2 x - 1 = 0$ and $0° \le x \le 90°$ then find the value of $\tan x$.

5B. यदि $\tan\theta = \cot\theta$ र $0° \le \theta \le 90°$ भए $\sin\theta$ को मान पत्ता लगाउनुहोस् ।
If $\tan\theta = \cot\theta$ and $0° \le \theta \le 90°$ then find the value of $\sin\theta$.

6A. यदि $\sin 2\theta = \cos 3\theta$ र $0° \le \theta \le 90°$ भए $\sin 5\theta$ को मान पत्ता लगाउनुहोस् ।
If $\sin 2\theta = \cos 3\theta$ and $0° \le \theta \le 90°$ then find the value of $\sin 5\theta$.

6B. यदि $\sin 2\theta = \cos 4\theta$ र $0° \le \theta \le 90°$ भए $\tan 3\theta$ को मान पत्ता लगाउनुहोस् ।
If $\sin 2\theta = \cos 4\theta$ and $0° \le \theta \le 90°$ then find the value of $\tan 3\theta$.

Model 5 *Problems based on the trigonometric equations of the form: sin C = cos D or tan C = cot D.*

7. हल गर्नुहोस् (Solve): $[0° \le x \le 90°]$

A. $\sin x = \cos 2x$ **B.** $\sin 3x = \cos 7x$ **C.** $\cos 2x = \sin 3x$

D. $\tan x = \cot 5x$ **E.** $\sin x - \sin 2x = 0$ **F.** $\sec x = \operatorname{cosec} x$

www.drmathnp.com

pueas.com.np

8A. यदि $\cos 4A = \sin 2A$ र A पहिलो चतुर्थांशमा पर्ने भए A को मान पत्ता लगाउनुहोस् ।
If $\cos 4A = \sin 2A$ and A lies in the first quadrant, find the value of A.

8B. यदि $\sin 3\theta = \cos(\theta - 6°)$ भए θ को मान 0° देखि 90° को बिचमा पत्ता लगाउनुहोस् ।
If $\sin 3\theta = \cos(\theta - 6°)$ then, find the value of θ between 0° to 90°.

8C. यदि $\sin(\theta + 24°) = \cos\theta$ भए θ को मान 0° देखि 90° को बिचमा पत्ता लगाउनुहोस् ।
If $\sin(\theta + 24°) = \cos\theta$ then, find the value of θ between 0° to 90°.

Model 6 ***Problems based on the trigonometric equations reducible to: sin (A ± B) or cos (A ± B).***

9. हल गर्नुहोस् (Solve):

A. $\cos 45° \sin\theta + \sin 45° \cos\theta = \frac{1}{\sqrt{2}}$ $[0° \le \theta \le 90°]$

B. $\cos 45° \sin x + \sin 45° \cos x = 1$ $[0° \le x \le 180°]$

C. $\sin 60° \cos x + \cos 60° \sin x = \frac{\sqrt{3}}{2}$ $[0° \le x \le 90°]$

D. $\sin\alpha \cos 30° - \cos\alpha \sin 30° = \frac{1}{\sqrt{2}}$ $[0° \le \alpha \le 180°]$

E. $\sin\theta \cos 30° + \cos\theta \sin 30° = \frac{1}{2}$ $[0° \le \theta \le 180°]$

F. $\sin\theta \cos 45° + \cos\theta \sin 45° = 1$ $[0° \le \theta \le 180°]$

Model 7 ***Problems based on the trigonometric equations having sub-multiple angle.***

10. हल गर्नुहोस् (Solve):

A. $1 = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$ $[0° \le \theta \le 180°]$

B. $4 \cos^2\frac{\theta}{2} + 4 \cos\frac{\theta}{2} - 3 = 0$ $[0° \le \theta \le 180°]$

C. $2 \sin\theta \cos\frac{\theta}{2} - \sin\theta = 0$ $[0° \le \theta \le 180°]$

D. $\cos^2\frac{\theta}{3} - \cos\frac{\theta}{3} + \frac{1}{4} = 0$ $[0 \le \theta \le 180°]$

E. $\sin\frac{\theta}{2} - \cos\frac{\theta}{2} = 0$ $(0° \le \theta \le 90°)$

F. $\tan\frac{\theta}{2} - \cot\frac{\theta}{2} = 0$ $(0° \le \theta \le 90°)$

TRIGONOMETRY

B. LONG QUESTIONS

हल गर्नुहोस् (Solve):

Model 1 ***The trigonometric equations (Quadratic) in sine/cosine/tangent/co-tangent.***

1. A. $5\cos\theta - 2\cos^2\theta = 2$ $[0° \le \theta \le 360°]$

B. $2\cos^2 A + 4\sin^2 A = 3$ $[0° \le A \le 180°]$

C. $2\sin^2 x + 3\cos x = 3$ $[0° \le x \le 90°]$

D. $6\cos^2 x + 4\sin^2 x = 5$ $[0° \le x \le 360°]$

E. $6\sin^2\theta + 4 = 11\sin\theta$ $[0° \le \theta \le 90°]$

F. $4\cos^2 x + 4\sin x = 5$ $[0° \le x \le 90°]$

G. $2\sin^2 x - 3\sin x + 1 = 0$ $[0° \le x \le 180°]$

H. $2\sqrt{3}\cos^2 x = \sin x$ $[0° \le x \le 180°]$

I. $\sqrt{3} + 4\cos^2 x = 2(\sqrt{3} + 1)\cos x$ $[0° \le x \le 360°]$

J. $\sqrt{2}\cos^2 x + \cos x = 0$ $[0° \le x \le 360°]$

K. $5\sin^2 x + 3\cos^2 x = 4$ $[0° \le x \le 180°]$

L. $2\cos^2\theta + \sin\theta = 2$ $[0° \le \theta \le 180°]$

M. $4\sin^2\theta - 8\cos\theta = 4$ $[0° \le \theta \le 360°]$

N. $\cos^2\theta + 3\sin^2\theta + \sin\theta = 2$ $[0° \le \theta \le 360°]$

O. $2\cos^2\theta - \cos\theta = 1$ $[0° \le \theta \le 2\pi^c]$

P. $6\sin^2 x + 11\cos x + 1 = 0$ $[0° \le x \le 360°]$

2. A. $2\sin\theta = \tan\theta$ $[0° \le \theta \le 360°]$

B. $\operatorname{cosec} x - 2\sin x = 1$ $[0° \le x \le 180°]$

C. $3\cot x - \tan x = 2$ $[0° \le x \le 180°]$

D. $2\cos x + 2\sec x = 5$ $[0° \le x \le 180°]$

E. $\sqrt{2}\sin\theta = \tan\theta$ $[0° \le \theta \le 360°]$

F. $\operatorname{cosec} x . \cot x = \sqrt{2}$ $[0° \le x \le 360°]$

3 A. $3\tan^2\theta - 4\sec\theta - 1 = 0$ $[0° \le \theta \le 360°]$

B. $\tan^2 x - 3\sec x + 3 = 0$ $[0° \le x \le 2\pi°]$

C. $4\sec^2 A - 7\tan^2 A = 3$ $[0° \le A \le 180°]$

D. $\cot^2\theta + \operatorname{cosec}^2\theta = 3$ $[0° \le \theta \le 90°]$

E. $\tan\theta = \sqrt{3}\sec\theta - 1$ $[0° \le \theta \le 360°]$

F. $\tan^2\theta = \sec\theta + 1$ $[0° \le \theta \le 360°]$

G. $\tan^2 A - (1 + \sqrt{3})\tan A + \sqrt{3} = 0$ $[0° \le A \le 360°]$

H. $\tan^2\theta + (1 - \sqrt{3})\tan\theta - \sqrt{3} = 0$ $[0° \le \theta \le 360°]$

4. A. $2\sin 2\theta + \sin\theta = 0$ $[0° \le \theta \le 360°]$

B. $\cos 2\theta + \cos\theta = 0$ $[0° \le \theta \le 360°]$

C. $\sin 2\theta + \cos\theta = 1 + 2\sin\theta$ $[0° \le \theta \le 360°]$

5. θ को मान 0° देखि 180° मा पत्ता लगाउनुहोस् जसले $1 - \cos\theta = 2\sin^2\theta$ लाई सन्तुलन गर्नेछ ।
Find the values of θ in 0° to 180° which will satisfy the equation $1 - \cos\theta = 2\sin^2\theta$.

www.drmathnp.com

6. हल गर्नुहोस् (Solve): $[0° \le \theta \le 90°]$

A. $\frac{\cos\theta}{\text{cosec}\,\theta + 1} + \frac{\cos\theta}{\text{cosec}\,\theta - 1} = 2$ **B.** $\frac{\cos\theta}{1 - \sin\theta} + \frac{\cos\theta}{1 + \sin\theta} = 4$ **C.** $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

Model 2 The trigonometric equations reducible to a cos θ + b sin θ = 0.

7. हल गर्नुहोस् (Solve):

A. $\cos\theta + \sin\theta = \frac{1}{\sqrt{2}}$ $[0° \le \theta \le 360°]$ **B.** $\sqrt{3}\cos\beta + \sin\beta = 2$ $[0° \le \beta \le 360°]$

C. $\sin A + \cos A = 1$ $[0° \le A \le 180°]$ **D.** $\cos x + \sqrt{3}\sin x = 2$ $[0° \le x \le 360°]$

E. $\sin\theta + \cos\theta = \sqrt{2}$ $[0° \le \theta \le 180°]$ **F.** $\sqrt{3} = \sqrt{3}\cos x + \sin x$ $[0° \le x \le 180°]$

G. $\cos x - \sqrt{3}\sin x = 1$ $[0° \le x \le 360°]$ **H.** $\sin\theta - \cos\theta = \frac{1}{\sqrt{2}}$ $[0° \le \theta \le 360°]$

I. $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$ $[0° \le \theta \le 360°]$ **J.** $\cos\theta - \sqrt{3}\sin\theta = -1$ $[0° \le \theta \le 360°]$

K. $\sqrt{3}\tan x - \sqrt{2}\sec x = 1$ $[0° \le x \le 360°]$ **L.** $\sqrt{3}\tan\alpha = \sec\alpha - 1$ $[0° \le \alpha \le 360°]$

M. $\sqrt{3}(\cot\theta - \text{cosec}\,\theta) + 1 = 0$ $[0° \le \theta \le 360°]$

Model 3 The trigonometric equations of the form sin C + sin D = sine or in cosine.

8 **A.** $\cos 3A + \cos 2A + \cos A = 0$ $[0° \le A \le 180°]$ **B.** $\cos A + \sin A = \cos 2A + \sin 2A$ $[0° \le A \le 90°]$

C. $\sin\theta + \sin 2\theta + \sin 3\theta = 0$ $[0° \le \theta \le 360°]$ **D.** $\sin 2x + \sin 4x = \cos x + \cos 3x$ $[0° \le x \le 90°]$

E. $\sin\theta + \sin 5\theta = \sin 3\theta$ $[0° \le \theta \le 360°]$ **F.** $\sin 3\theta + \sin\theta = \sin 2\theta$ $[0° \le \theta \le 360°]$

G. $\cos 7\theta = \cos\theta - \sin 4\theta$ $[0° \le \theta \le 90°]$

Model 4 On finding value of (A + B) or (A – B) or A or B from the given pair of equations.

9. यदि $\tan A + \cot B = 2$ र $2\cos A \sin B = \frac{1}{\sqrt{2}}$ भए (A – B) को मान पत्ता लगाउनुहोस् ।

If $\tan A + \cot B = 2$ and $2\cos A \sin B = \frac{1}{\sqrt{2}}$ then find the value of (A – B).

10. (A + B) को मान निकाल्नुहोस् (Find the value of (A + B)):

A. $2\sin A.\sin B = \frac{\sqrt{3}}{2}$ & $\cot A + \cot B = 2$ **B.** $\tan A + \tan B = 2$ & $2\cos A.\cos B = 1$

11. न्यूनकोणहरू A र B का मानहरू पत्ता लगाउनुहोस् (Find the values of acute angles A and B):

A. $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$ **B.** $\sin(A + B) = \frac{\sqrt{3}}{2}$ and $\cos(A - B) = \frac{1}{\sqrt{2}}$

C. $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$ **D.** $\cos(A + B) = 0$ and $\sin(A - B) = \frac{1}{2}$

E. $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$

Model 5 Miscellaneous problems on trigonometricequations.

12. हल गर्नुहोस (Solve): $(0 \le x \le 360°)$

A. $\cos(45° + x) + \cos(45° - x) = 1$ **B.** $\cos(135° + x) - \cos(135° - x) = -1$

C. $\frac{\cos 6x + \cos 4x}{\sin 6x - \sin 4x} = \sqrt{3}$ **D.** $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \frac{1}{\sqrt{3}}$ **E.** $\frac{\sin 3x - \sin x}{\cos 2x} = 1$

F. $\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} = 1$ **G.** $\frac{\sin 2x}{1 - \cos 2x} = \sqrt{3}$ **H.** $\frac{1 - \cos 2x}{1 + \cos 2x} = 3$

I. $\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x} = \sqrt{3}$ **J.** $\text{cosec}\,2x + \cot 2x = \sqrt{3}$ **K.** $\frac{1}{\tan 2x - \tan x} - \frac{1}{\cot 2x - \cot x} = 1$

L. $\frac{1}{\tan 3x - \tan 2x} - \frac{1}{\cot 3x - \cot 2x} = \sqrt{3}$ **M.** $\sqrt{2 + \sqrt{2 + 2\cos 4x}} = 1$

ANSWERS

A. SHORT QUESTIONS

1A. 60° **1B.** 60° **1C.** 15° **1D.** 30° **1E.** 60° **1F.** 60° **1G.** 45° **1H.** 30°
1I. 30° **1J.** 45° **1K.** 60° **1L.** 15° **1M.** 30° **1N.** 45° **1O.** 45° **2A.** 60°, 120°

puspas.com.np

www.drmathnp.com

2B. 30°, 150° **2C.** 60°, 120° **2D.** 30° **2E.** 60° **2F.** 60° **2G.** 60°, 120° **2H.** 30°, 150° **3A.** 90° or 150°
3B. 30°, 90°, 150°, 270° **3C.** 60°, 109°, 251°, 300° **3D.** 30°, 150°, 270° **3E.** 0°, 60°, 180°, 300°, 360°
3F. 30° or 60° **3G.** 45° or 60° **3H.** 0°, 60° **4A.** 60° **4B.** 0°, 60° **4C.** 45° **4D.** 45°, 135°, 225°, 300°
4E. 38.17°, 141.83° **4F.** 0°,90°,180°,360° **4G.** 45° **4H.** 60° **5A.** $\sqrt{3}$ **5B.** $\frac{1}{\sqrt{2}}$
6A. 1 **6B.** 1 **7A.** 30° **7B.** 9° **7C.** 18° **7D.** 15° **7E.** 0°, 60° **7F.** 45°
8A. 45° **8B.** 24° **8C.** 33° **9A.** 0°, 90° **9B.** 45° **9C.** 0°, 60° **9D.** 75° or 165° **9E.** 0° or 120°
9E. 45° **10A.** 90° **10B.** 120° **10C.** 0°, 120° **10D.** 180° **10E.** 90° **10F.** 90°

B. LONG QUESTIONS

1A. 60°, 300° **1B.** 45°, 135° **1C.** 0°, 60° **1D.** 45°, 135°, 225°, 315° **1E.** 30° **1F.** 30° **1G.** 30°, 90°, 150°
1H. 60°, 120° **1I.** 30°, 60°, 300°, 330° **1J.** 90°, 135°, 225° **1K.** 45°, 135° **1L.** 0°, 30°, 150°, 180°
1M. 90°, 270° **1N.** 30°, 150°, 270° **1O.** 0°, 120°, 240°, 360° **1P.** 60°, 300°
2A. 0°, 60°,180°,300°,360° **2B.** 30°, 150°,270° **2C.** 45° **2D.** 60° **2E.** 0°, 45°, 180°,315°,360°
2F. 45°, 315° **3A.** 60°, 300° **3B.** 0°,60°,300°,360° **3C.** 30°, 150° **3D.** 45° **3E.** no solution
3F. 60°, 180°, 300° **3G.** 45°, 60°, 225°, 240° **3H.** 60°, 135°, 240°, 315° **4A.** 0°, 104.5°, 180°, 255.5°, 360°
4B. 60°, 180°,300° **4C.** 0°, 210°, 330°, 360° **5.** 0° or 120° **6A.** 45° **6B.** 60° **6C.** 60°
7A. 105°, 345° **7B.** 30° **7C.** 0°, 90° **7D.** 60° **7E.** 45° **7F.** 60°, 0° **7G.** 0°, 240°
7H. 75°, 195° **7I.** 75°, 345° **7J.** 60°, 180° **7K.** 75°, 165° **7L.** 0°, 120°, 360° **7M.** 0°, 60°, 360°
8A. 45°, 135°, 120°, 240° **8B.** 0°, 30° **8C.** 0°, 120°, 180°, 240°, 360° **8D.** 18°, 90°, 270°
8E. 0°,30°, 120° ,150° **8F.** 0°,60°, 90°, 180°, 270°, 300°, 360° **8G.** 0°, 10°, 50°, 130°, 170°, 360°
9. 45°,315° **10A.** 60° **10B.** 90° **11A.** 60°, 30° **11B.** 52.5°, 7.5° **11C.** 45°, 15°
11D. 60°, 30° **11E.** 45°, 15° **12A.** 45° or 315° **12B.** 45° or 135° **12C.** 30° or 210° **12D.** 30° or 210°
12E. 30°, 150° **12F.** 45° or 225° **12G.** 30° or 210° **12H.** 30°, 150°, 210°, 330° **12I.** 60° or 240°
12J. 30°, 210° **12K.** 45°, 225° **12L.** 30°, 210° **12M.** 60°, 300°

CHALLENGING EXERCISE 4

हल गर्नुहोस् (Solve):

1. $\frac{\sqrt{3}}{\sin 2\theta} + \frac{1}{\cos 2\theta} = 4$ $[0° \le \theta \le 90°]$

2. $4\cos^2\theta \cdot \sin\theta - 2\sin^2\theta = 3\sin\theta$ $[0° \le \theta \le 360°]$

3. $\sin 2\theta = 3\tan\theta.\cos 2\theta$ $[0° \le \theta \le 180°]$

4. $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$ $[0° \le \theta \le 360°]$

5. $2\sin^2 2A + \sin 2A - 1 = 0$ $[0° \le A \le 360°]$

6. $2\sin^2 x + \sin^2 2x = 2$ $[0° \le x \le 360°]$

7. $\cos 10A + 7 = 8\cos 5A$ $[0° \le A \le 360°]$

HINT: Here, $\cos 10A - 8\cos 5A + 7 = 0$ or, $1 + \cos 10A - 8\cos 5A + 6 = 0$
or, $2\cos^2 5A - 8\cos 5A + 6 = 0$ or, $\cos^2 5A - 4\cos 5A + 3 = 0$
Let, $\cos 5A = t$ then, $t^2 - 4t + 3 = 0$ or, $(t-1)(t-3) = 0$
Either, $t - 1 = 0$ ..... (i) or, $t - 3 = 0$ ....(ii)
From (i), $\cos 5A = 1$ or, $\cos 5A = \cos 0°, \cos 360°, \cos 720°, \cos 1080°, \cos 1440°, \cos 1800°$
or, $5A = 0°, 360°, 720°, 1080°, 1440°, 1800°$ or, $A = 0°, 72°, 144°, 216°, 288°, 360°$
From (ii); $\cos 5A = 3$ Since, 3 is out of range[– 1 to 1] so value of A does not exit.
Thus, values of A are 0°, 72°, 144°, 216°, 288° or 360°.

8. $3\sin 2A + 4\cos 2A = 2$ $[0° \le A \le 360°]$

9. $2\cos^3 A + 2\sin^2 A\cos A = 5\sin A.\cos^2 A$ $[0° \le A \le 360°]$

10. $2\tan 3x\cos 2x + 1 = \tan 3x + 2\cos 2x$ $(0° \le x \le 180°)$

11. $2\cot 3\theta\sin 2\theta + 1 = \cot 3\theta + 2\sin 2\theta$ $(0° \le \theta \le 180°)$

12. $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ $[0° \le \theta \le 90°]$

13. $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$ $[0° \le x \le 360°]$

HINT: $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$ or, $\sin 3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$
or, $3\sin\alpha - 4\sin^3\alpha = 4\sin^2 x\sin\alpha - 4\sin^3\alpha$ or, $3\sin\alpha = 4\sin^2 x\sin\alpha$
or, $\sin^2 x = \frac{3}{4} = \pm\left(\frac{\sqrt{3}}{2}\right)^2$ or, $\sin x = \pm\frac{\sqrt{3}}{2}$
or, $\sin x = \sin 60°$ or $120°$ or $240°$ or $300°$ $\therefore$ $x = 60°$ or $120°$ or $240°$ or $300°$

14. $4\sin x\sin 2x\sin 4x = \sin 3x$ $[0° \le x \le 90°]$

HINT: Here, $4\sin x\sin 2x\sin 4x = \sin 3x$ or, $4\sin x\sin(3x-x).\sin(3x+x) = \sin 3x$
or, $4[\sin x(\sin^2 3x - \sin^2 x)] = 3\sin x - 4\sin^3 x$ or, $4\sin x\sin^2 3x - 4\sin^3 x = 3\sin x - 4\sin^3 x$
or, $4\sin x\sin^2 3x = 3\sin x$ or, $\sin x(4\sin^2 3x - 3) = 0$
or, $\sin x = 0$ or, $\sin^2 3x = \frac{3}{4}$ or, $\sin 3x = \pm\left(\frac{\sqrt{3}}{2}\right)^2$
or, $\sin x = \sin 0°$ or $\sin 360°$ or, $\sin 3x = \sin 60°$ or $120°$ or $240°$ or $300°$
$\therefore$ $x = 0°$ or $360°$ or, $3x = 60°$ or $120°$ or $240°$ or $300°$
$\therefore$ $x = 20°, 40°, 80°, 100°$
$\therefore$ $x = 0°, 20°, 40°, 80°$

15. यदि $2\cos x\cos y = 1$ र $\tan x + \tan y = 2$ भए x र y का मानहरू पत्ता लगाउनुहोस् ।
If $2\cos x\cos y = 1$ and $\tan x + \tan y = 2$ then find the values of x and y.

ANSWERS

1. 30° **2.** 0°, 18°, 180°, 270°, 360° **3.** 0°, 30°, 150°, 180° **4.** 20° 80°
5. 15° or 315° **6.** $x = 45°, 135°, 225°, 315°$ **7.** 0°, 72°, 144°, 216°, 288°, or 360°
8. 51.75°, 53°, 165.25°, 345.25° **9.** 90°, 26.6°, 63.4°, 206.6°, 243.4° **10.** $x = 15°, 30°, 75°, 150°$ **11.** 15°, 75°
12. 60°, 35° **15.** 0°, 20°, 40°, 80°, 100°

pustas.com.np

www.drmathnp.com

5. उचाइ र दूरी HEIGHT AND DISTANCE

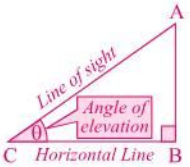
pustaka.com.np

DEFINITIONS

A. उन्नतांशकोण (Angle of elevation): [SEE MODEL 2076]

अवलोकनकर्ताले क्षितिजरेखामाथि परेको बिन्दुलाई हेर्दा उक्त क्षितिजरेखासँग बनाएको कोणलाई उन्नतांशकोण (Angle of elevation) भनिन्छ । चित्रमा BC क्षितिजरेखा, AC दृष्टि रेखा, A वस्तुको स्थिति भए, ∠ACB एक उन्नतांश कोण हो ।

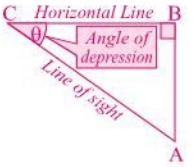
The angle made by the line of sight with the horizontal line through the position of an observer's eye is called an angle of elevation. In this case the object to be observed lies above the position of the eye. In the figure alongside, A is the position of an object, AC is the line of sight, BC is the horizontal line and ∠ACB is an angle of elevation.



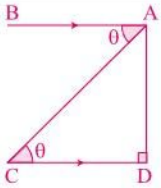
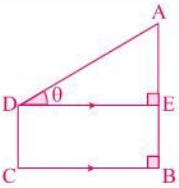
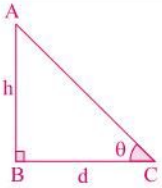
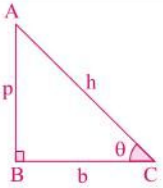
B. अवनतिकोण (Angle of depression):

अवलोकनकर्ताले क्षितिजरेखा भन्दा तल परेको बिन्दुलाई हेर्दा उक्त क्षितिजरेखासँग बनाएको कोणलाई अवनतिकोण (Angle of depression) भनिन्छ । चित्रमा BC क्षितिज रेखा, AC दृष्टि रेखा, A वस्तुको स्थिति भए, ∠ACB एक अवनतिकोण हो ।

The angle made by the line of sight with the horizontal line through the position of an observer's eye is called an angle of depression. In this case the object to be observed lies below the position of an eye. In the figure alongside, A is the position of an object, AC is the line of sight, BC is the horizontal line and ∠ACB is an angle of depression.



FORMULAE AND KEY POINTS

			
उन्नतांशकोण = अवनतिकोण Angle of elevation = Angle of Depression ∠DCA = ∠CAB = θ	(i) ED = BC (ii) AE = AB – EB (iii) AE = AB – CD	(i) $h = d \tan\theta$ (ii) $d = h \cot\theta$	(i) $p = b \tan\theta$ (ii) $b = \frac{p}{\tan\theta}$ (iii) $h = \frac{p}{\sin\theta}$ (iv) $h = \frac{b}{\cos\theta}$ (v) $p = h \sin\theta$ and $b = h \cos\theta$

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2066 S, 2070 S', 2071 R', SEE 2073 S, SEE 2074 R', SEE 2074 S'	6
Model 2	2065 S, 2066 R', 2067 R, 2068 S', 2069 R, 2069 R', 2070R', 2071 R, 2071 S, SEE 2074 R, SEE 2074 S, SEE 2075 R, R_2	13
Model 3	2065 M, 2068 S', SEE 2075 R'	3
Model 4	2065 R, 2067 S, 2068 R, 2069 S, 2072 R', 2073 R', SEE MODEL 2076	7
Model 5	2065 R', 2070 R, 2070 S, 2072 S, SEE 2075 R'_2	5
Model 6	-	-
Model 7	2072 R	1
Model 8	2065 E	1
Model 9	2066 R, 2067 R', 2068 R', 2073 R, SEE 2073 S'	5

www.drmathnp.com

pupas.com.np

SEE EXERCISE 5

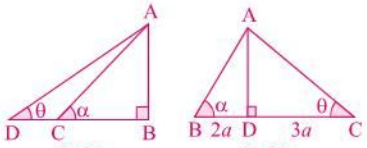
A. VERY SHORT QUESTIONS

Model 1 *Definitions or formulae Recall.*

1A. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
(a) उन्नतांश कोण (Angle of elevation) [SEE MODEL 2076]
(b) अवनति कोण (Angle of depression)
(c) सूर्यको उचाइ (Altitude of the sun)

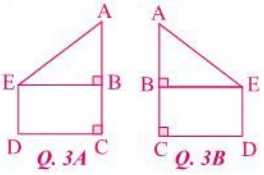
2A. चित्रमा α र θ मध्य कुन ठूलो हुन्छ ?
In the figure, which is greater between α and θ?

2B. चित्रमा α र θ मध्य कुन ठूलो हुन्छ ?
In the figure, which is greater between α and θ?



3A. दिइएको चित्रबाट DE लाई AC र BC को रूपमा लेख्नुहोस् ।
From the given figure, write DE in terms of AC and BC.

3B. दिइएको चित्रबाट AB लाई AC र ED को रूपमा लेख्नुहोस् ।
From the given figure, write AB interms of AC and ED.



Model 2 *Concept Recall*

4A. यदि एउटा 'x' m अग्लो रुखले 'x' m लामो छाँया बनाउँदछ भने सूर्यको उचाइ पत्ता लगाउनुहोस् ।
If a 'x' m high tree casts 'x' m long shadow, what is the altitude of the sun?

4B. यदि एउटा 'x' m अग्लो रुखले 4 : 15 pm मा '$2x$' m लामो छाँया बनाउँदछ भने '$2x$' m अग्लो रुखले सोही समयमा कति लामो छाँया बनाउला ?
If a 'x' m tall tree casts its shadow '$2x$' m at 4 : 15 pm, what will be the length of shadow of a tree of height '$2x$' m at the same time?

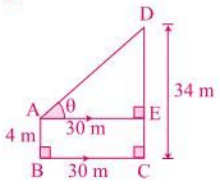
5A. 12 m अग्लो खम्बामा 13 m लामो भऱ्याङ अड्याइएको छ । तिनीहरूको फेदको दूरी पत्ता लगाउनुहोस् ।
A 13 m long ladder rests at the top of 12 m high pole. Find the distance between their feet.

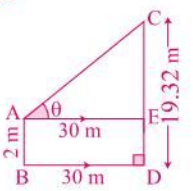
5B. सूर्यको उचाइ 45° भएको समयमा 10 m अग्लो रुखको छाँयाको लम्बाइ पत्ता लगाउनुहोस् ।
Find the length of shadow of 10 m tall tree when altitude of sun is 45°.

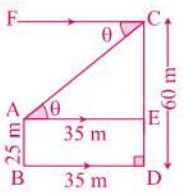
B. SHORT QUESTIONS

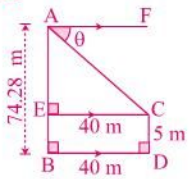
Model 1 *Problems based on height of distance related to figures.*

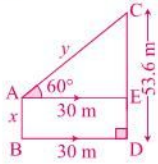
1. दिइएका चित्रहरूबाट θ, x वा y का मानहरू पत्ता लगाउनुहोस् । (From the given figures, find the values of θ, x or y.)

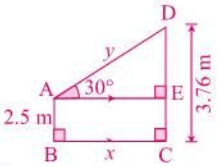
A.


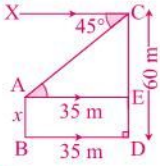
B.


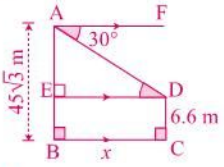
C.


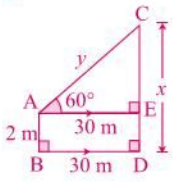
D.


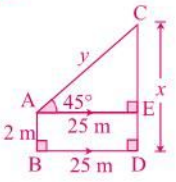
E.


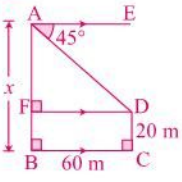
F.


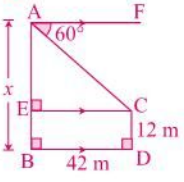
G.


H.


I.


J.


K.


L.


www.drmathnp.com

C. LONG QUESTIONS

Model 1 *On finding height of tower if angles of the top from 2 points with their distance is given.*

1A. जमिनमा रहेको कुनै एउटा बिन्दुबाट एउटा रूखको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइयो । सो बिन्दुबाट 40 मिटर अझ पर गएर हेर्दा उन्नतांश कोण 45° पाइयो भने सो रूखको उचाइ पत्ता लगाउनुहोस् । [SEE 2074 S']
The angle of elevation of the top of a tree as observed from a point on the ground is found to be 60°. On walking 40 meter away from the point, the angle of the elevation was found to be 45°. Find the height of the tree.

1B. कुनै एउटा बिन्दुबाट एक स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो र सो बिन्दुबाट 80 मिटर टाढा जाँदा सो कोण 30° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower from a point was observed to be 45° and on walking 80 m away from that point, it was found to be 30°. Find the height of the tower. [SEE 2074 R']

1C. कुनै मैदानमा रहेको एउटा बिन्दुबाट एउटा स्तम्भको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइएछ । सो बिन्दुबाट 60 मि. अझ पर गएर हेर्दा उन्नतांश कोण 45° पाइयो भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower as observed from a point on the ground is found to be 60°, on walking 60 meter away from the point, the angle of the elevation was found to be 45°. What will be the height of the tower? Find it. [SEE 2073 S']

1D. कुनै एउटा बिन्दुबाट स्तम्भको टुप्पोको उन्नतांशकोण 45° अवलोकन गरियो र सो बिन्दुबाट 60 मिटर टाढा जाँदा सो कोण 30° पाइयो । स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower from a point was observed to be 45° and on walking 60 metre away from that point, it was found to be 30°. Find the height of the tower. [2071 R']

1E. कुनै मैदानमा रहेको एउटा बिन्दुबाट एक स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो । सो बिन्दुबाट 60 मिटर टाढा गएर हेर्दा उन्नतांश कोण 30° पाइयो । यदि स्तम्भ र अवलोकन बिन्दुहरू एउटै सरलरेखामा पर्दछन् भने स्तम्भको उचाइ $30(\sqrt{3}+1)$ मिटर हुन्छ भनी प्रमाणित गर्नुहोस् । [2070 S']
The angle of elevation of the top of a tower was observed to be 45° from a point on the ground. On walking 60 meter away from that point, the angle of elevation was found to be 30°. If the tower and the points of observation lie on the same straight line, prove that the height of the tower is $30(\sqrt{3}+1)$ meter.

1F. कुनै एउटा बिन्दुबाट स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो र सो बिन्दुबाट 30 मिटर टाढा जाँदा सो कोण 30° पाइयो । यदि स्तम्भ र ती दुई बिन्दुहरू एकै रेखामा भए सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower from a point was observed to be 45°. On walking 30 m away from that point, it was found to be 30°. If the tower and the points of observation lie on the same straight line, find the height of the tower. [2057 R]

1G. कुनै मैदानमा रहेको एउटा बिन्दुबाट स्तम्भको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइएछ । सो बिन्दुबाट 60 मिटर अझ पर गएर हेर्दा उन्नतांश कोण 45° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् । [2060 S]
The angle of elevation of the top of a tower as observed from a point on the ground is found to be 60°. On walking 60 m away from the point, the angle of the elevation was found to be 45°. Find the height of the tower.

1H. कुनै बिन्दुबाट धरहराको टुप्पाको उन्नतांश कोण 60° अवलोकन गरियो र सो बिन्दुबाट 200 मिटर टाढा जाँदा सो कोण 30° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of a tower was observed to be 60° from a point. On walking 200 m away from the point, it was found to be 30°. Find the height of the tower. [2058 R]

1I. कुनै स्थानबाट एउटा घरको टुप्पोमा हेर्दा उन्नतांशकोण 60° अवलोकन गरियो र सोही स्थानबाट 60 m टाढा गएर अवलोकन गर्दा उन्नतांशकोण 30° को पाइयो । यदि घर र ती दुई स्थानहरू एउटै सतहको रेखामा भएका रहेछन् भने घरको उचाइ कति हुन्छ होला ?
The angle of elevation of the top of a house from a point on the ground was observed to be 60°. On walking 60 m away from that point, it was found to be 30°. If house and these points are in the same line of the same plain, find the height of the house. [2066 S]

Model 2 *On finding height of a tower if next tower & 2 angles from the top & bottom are given.*

2A. एउटा घरको छत र फेदबाट एउटा रुखको टुप्पोको अवनति कोण र उन्नतांश कोणहरू क्रमशः 60° र 30° छन् । यदि रुखको उचाइ 15 फिट भए घरको उचाइ पत्ता लगाउनुहोस् ।
From the roof and foot of a house, the angles of depression and elevation of the top of a tree are 60° and 30° respectively. If the height of the tree is 15 ft, find the height of the house. [SEE 2075 R]

2B. 16 मिटर अग्लो एउटा भवनको छतबाट ठीक अगाडि रहेको एउटा बत्तीको खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा उन्नतांश कोण र अवनति कोण क्रमशः 60° र 30° पाइयो भने सो खम्बाको उचाइ पत्ता लगाउनुहोस् ।
From the roof of a building 16 meter high, the angles of elevation and depression of the top and the foot of an electric pole are observed to be 60° and 30° respectively. Find the height of the pole. [SEE 2075 R_2]

puspas.com.np

www.drmathnp.com

2C. 30 मिटर अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदको अवनति कोण क्रमश: 45° र 60° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the top of 30 m high cliff, the angles of depression of the top and the foot of a tower are observed to be 45° and 60° respectively. Find the height of the tower. **[SEE 2074 S]**

2D. 24 मिटर अग्लो एउटा स्तम्भको टुप्पोबाट एउटा खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा अवनति कोणहरू क्रमश: 45° र 60° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।
From the top of a tower 24 meter high, the angles of depression of the top and the foot of a pole are observed to be 45° and 60° respectively. Find the height of the pole. **[SEE 2074 R]**

2E. 21 मिटर अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदको अवनति कोणहरू क्रमश: 45° र 60° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the top of 21 m high cliff, the angles of depression of the top and the bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower. **[2057 S, 2071 S]**

2F. 42 मि. अग्लो चट्टानको टुप्पोबाट एउटा धरहराको टुप्पो र फेदका अवनति कोणहरू क्रमश: 30° र 45° अवलोकन गरियो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the top of 42 m high cliff, the angle of depression of the top and bottom of a tower are observed to be 30° and 45° respectively. Find the height of the tower. **[2061S]**

2G. एउटा घरको छतबाट त्यसको ठीक अगाडि रहेको 5 m अग्लो खम्बाको टुप्पोमा हेर्दा अवनति कोण 60° र त्यस घरको फेदबाट खम्बाको टुप्पो हेर्दा उन्नतांश कोण 30° छन् भने घरको उचाइ पत्ता लगाउनुहोस् ।
The angle of depression of the top of a 5 m high pole observed from the top of a house is 60° and the angle of elevation of the top of the pole from the foot of the house is 30° then find the height of the house. **[2069R]**

2H. 20 m अग्लो एउटा घरको छत र फेदबाट एउटा स्तम्भको टुप्पोको उन्नतांश कोणहरू क्रमश: 45° र 60° छन् भने उक्त स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
From the roof and basement of a house 20 m high, the angles of elevation of the top of a tower are 45° and 60° respectively. Find the height of the tower. **[2071 R]**

2I. एउटा 25 मिटर अग्लो खम्बाको टुप्पोको अवनति कोण र उन्नतांश कोण एउटा स्तम्भको टुप्पो र फेदबाट क्रमश: 60° र 30° छन् भने सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of depression and elevation of the top of a pole 25 m high from the top and bottom of a tower are 60° and 30° respectively. Find the height of the tower. **[2064 R]**

2J. एउटा 30 मिटर अग्लो घरको धुरीबाट एउटा स्तम्भको टुप्पोको उन्नतांश कोण 30° र फेदको अवनति कोण 45° छन् भने सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower is 30° and the angle of depression of the bottom is 45° from the top of the 30 m high building. Find the height of the tower. **[2068 S']**

2K. 1.75 m अग्लो मानिसले एउटा घरको टुप्पोमा हेर्दा उन्नतांश कोण 45° र फेदमा हेर्दा अवनति कोण 10° पाएछ भने सो घरको उचाइ पत्ता लगाउनुहोस् ।
A man 1.75 m tall observed the angle of elevation of the top and the angle of depression of the bottom of a house to be 45° and 10° respectively. Find the height of the house. [tan 10° = 0.176] **[2062K]**

2L. 50 मिटर अग्लो एउटा टावरको टुप्पोमा दोस्रो टावरको टुप्पो र फेदबाट हेर्दा बनेका अवनति र उन्नतांश कोणहरू क्रमश: 45° र 30° छन् । दोस्रो टावरको उचाइ पत्ता लगाउनुहोस् ।
The angles of depression and elevation of the top of a tower 50 meter high from the top and bottom of a second tower are 45° and 30° respectively. Find the height of the second tower. **[2066 R']**

2M. एउटा 30 मिटर अग्लो घरको छानोबाट धरहराको टुप्पोको उन्नतांश कोण 45° र फेदको अवनतीकोण 30° पाइयो । उक्त धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the roof of a house 30 meter high, the angle of elevation of the top of the tower is 45° and the angle of depression of its foot is 30°. Find the height of the tower. **[2069 R']**

3A. एउटा स्तम्भको टुप्पोबाट 24 मिटर अग्लो घरको टुप्पोको अवनति कोण 30° र घरको फेदबाट स्तम्भको टुप्पोको उन्नतांश कोण 45° छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of depression of the top of building of 24 m height from the top of a tower is 30° and the angle of elevation of the top of the tower from the foot of building is 45°. Find the height of the tower. **[2063 R]**

3B. एउटा धरहराको टुप्पो र फेदबाट त्यसको ठीक अगाडि रहेको 25 m अग्लो खम्बाको टुप्पोमा हेर्दा अवनति कोण र उन्नतांश कोण क्रमश: 60° र 30° छन् भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
If the angles of depression and elevation of the top of a pole 25 m high from the top and bottom of a tower are 60° and 30° respectively, find the height of the tower. **[2065 S]**

3C. एउटा घरको अगाडिको एउटा रूखको टुप्पोमा एउटा चरा बसिरहेको छ । सो घरको आधार सतह र छतबाट चरालाई हेर्दा बन्ने उन्नतांश कोण र अवनति कोण क्रमश: 60° र 30° छन् । यदि रूखको उचाइ 21 मि. भए घरको उचाइ पत्ता लगाउनुहोस् ।
A bird is sitting on the top of a tree which is in front of a house. The angle of elevation and angle of depression of the bird from the bottom and top of the house are 60° and 30° respectively. If the height of the tree is 21m, find the height of the house. **[2067 R]**

www.drmathnp.com

3D. एउटा धरहराको टुप्पो र फेदबाट त्यसको ठीक अगाडि रहेको 20 मिटर अग्लो एउटा मन्दिरको गजुरमा हेर्दा अवनति कोण र उन्नतांश कोण क्रमशः 60° र 30° पाइएछन् भने सो धरहराको उचाइ पत्ता लगाउनुहोस् ।
The angles of depression and elevation of the pinnacle of a temple 20 meter high are found to be 60° and 30° from the top and foot of a tower respectively. Find the height of the tower. **[2070 R']**

3E. एउटा 120 मिटर अग्लो मिनारको टुप्पोबाट त्यसको अगाडि रहेको रुखको टुप्पो र फेदको अवलोकन गर्दा अवनति कोणहरू क्रमशः 30° र 60° पाइए । रुखको उचाइ र मिनार तथा रुखबीचको दूरी पत्ता लगाउनुहोस् ।
If the top and bottom of a tree infront of a tower 120 m high is observed from the top of the tower, the angle of depressions of the top and bottom are found to be 30° and 60° respectively. Find the height of the tree and distance between the tower and the tree.

Model 3 ***Problems based on finding distance between two posts when angles & height of one is given.***

4A. दुईजना मानिसहरू 100 मिटर अग्लो स्तम्भको विपरित दिशातिर छन् । यदि तिनीहरूले स्तम्भको टुप्पोलाई अवलोकन गर्दा उन्नतांश कोणहरू क्रमशः 30° र 45° पाएछन् भने ती मानिसहरू बिचको दूरी पत्ता लगाउनुहोस् ।
Two men are on the opposite direction of a tower 100 m high. If they observed the top of the tower, the angles of elevation are found to be 30° and 45° respectively. Find the distance between the men. **[SEE 2075 R']**

4B. दुई जना मानिसहरू कुनै 30 मिटर अग्लो स्तम्भको विपरीत दिशामा छन् । तिनीहरूले सो स्तम्भको उन्नतांश कोणहरू अवलोकन गर्दा 30° र 60° पाएछन् भने तिनीहरूबिचको दूरी पत्ता लगाउनुहोस् ।
Two men are on the opposite side of a tower of 30 m high. They observed the angles of elevation of the top of the tower and found to be 30° and 60°. Find the distance between them. **[2059 R]**

4C. 60 m अग्लो स्तम्भको टुप्पोलाई सो स्तम्भको फेददेखि केही परको बिन्दुबाट हेर्दा उन्नतांशकोण 45° पाइयो र सो बिन्दुदेखि अझ परको अर्को बिन्दुबाट हेर्दा उन्नतांशकोण 30° पाइयो भने ती दुई स्थानहरूबिचको दूरी पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower 60 m high is observed from a point on the ground to be 45°. On walking away from the point, the angle of the elevation was found to be 30°. Find the distance between the two places. **[2068 S']**

4D. एउटा 30 m अग्लो भवनबाट एक जना मानिसले भवनको पूर्वतिरको समतल मैदानमा बसिरहेको दुई जना मानिसलाई हेर्दा अवनति कोणहरू 45° र 30° बनाउँदछ भने ती दुई मानिसहरूबिच कति दूरी रहेछ ?
From the top of a building 30 m high, a man observes two persons sitting on the ground, both due east on the same line at angles of depression of 45° and 30°. How far apart are the two persons? **[2063 M]**

4E. 20 मिटर अग्लो भवनबाट एउटा स्तम्भको टुप्पो र फेदमा हेर्दा अवनति कोणहरू क्रमशः 45° र 60° पाइए भने स्तम्भको उचाइ र भवन र स्तम्भबिचको दूरी पत्ता लगाउनुहोस् ।
From the top of a building of 20 m high, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole and the distance between the house and the pole. **[2065 M]**

Model 4 ***On finding height of flagstaff /statue /tower when the angles from the same point are given.***

5A. स्तम्भ र स्तम्भमाथि खडा गरिराखेको ध्वजदण्डले स्तम्भको फेददेखि 100 मिटर पर रहेको स्थानमा क्रमशः 30° र 15° को कोण बनाउँछन् भने ध्वजदण्डको उचाइ पत्ता लगाउनुहोस् ।
A tower and flagstaff on its top subtend angles of 30° and 15° respectively at a point 100 meter away from the foot of the tower. Find the height of flagstaff. **[2072 R']**

5B. एउटा धरहरामाथि 7 m उचाइ भएको एउटा झन्डा खडा गरिएको छ । जमिनको कुनै एक बिन्दुमा धरहरा र झन्डाले क्रमशः 45° र 15° का कोणहरू बनाउँदछन् भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
A flagstaff of height 7 metres stands on the top of a tower. The angles subtended by the tower and the flagstaff to a point on the ground are 45° and 15° respectively. Find the height of the tower. **[2068 R, 2060 R]**

5C. एउटा शालिक स्तम्भको टुप्पोमा उभ्याइएको छ । यदि स्तम्भको फेददेखि 100 मि. परको बिन्दुमा स्तम्भ र शालिकले क्रमशः 45° र 15° को कोण बनाउँछन् भने शालिकको उचाइ पत्ता लगाउनुहोस् ।
A statue stands on the top of a column. The angles subtended by the column and statue at a point 100 m away from the foot of the column are 45° and 15° respectively. Find the height of the statue. **[2065 R]**

6A. एउटा खम्बामाथि एउटा झन्डा खडा गरिएको छ । खम्बाको फेददेखि 30 मिटर टाढाको बिन्दुबाट झन्डाको टुप्पो र फेदमा अवलोकन गर्दा उन्नतांश कोणहरू क्रमशः 45° र 30° पाइए भने झन्डाको उचाइ पत्ता लगाउनुहोस् ।
A pole is surmounted on its top by a flagstaff. The angles of elevation of the top and the bottom of the flagstaff as observed from a point 30 meters away from the top of the pole are found to be 45° and 30° respectively. Find the height of the flagstaff. **[2058 S]**

6B. कुनै एउटा ठाउँबाट ठीक अगाडि रहेको घरको छत र सो घरको छतदेखि 6 मि. तल रहेको झ्यालमा हेर्दा क्रमशः 45° र 30° को उन्नतांश कोणहरू बन्दछन् भने सो घरको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation observed from a place to the roof of house standing infront of the place and to a window 6 m below its roof are found to be 45° and 30° respectively. Find the height of the house. **[2067 S]**

pusmas.com.np

www.drmathnp.com
pustakas.com.np

6C. एक ठाउँबाट एउटा घरको छत र एउटा झ्यालको उन्नतांशकोणहरू क्रमशः 45° र 30° छन् । यदि उक्त झ्याल घरको छतदेखि 18 फिट तल रहेको छ भने सो घरको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevations of the root of a house and a window are 45° and 30° respectively. If the window is 18 feet below the roof of the house, find the height of the house. **[SEE 2073 R']**

7A. धरहराको ठीक अगाडि जमिनको सतहमा रहेको एक बिन्दुबाट धरहराको माथि ठड्याइएको 6 m अग्लो ध्वजदण्डको टुप्पो र फेदका उन्नतांश कोणहरू क्रमशः 60° र 45° पाइए । धरहराको उचाइ र धरहराको फेदबाट सो बिन्दुसम्मको दूरी पत्ता लगाउनुहोस् ।
From a point at the ground level infront of a tower, the angle of elevations of the top and bottom of flagstaff 6 m high situated at the top of a tower are observed 60° and 45° respectively. Find the height of the tower and the distance between the base of the tower and point of observation. **[SEE MODEL 2076]**

7B. 10 m अग्लो घरको छतबाट अगाडि रहेको कुनै एउटा स्तम्भको टुप्पोमा हेर्दा उन्नतांश कोण 60° पाइयो । स्तम्भको टुप्पोदेखि 18 m तलको कुनै बिन्दुमा सोही स्थानबाट हेर्दा उन्नतांश कोण 30° बन्दछ । स्तम्भको उचाइ तथा स्तम्भ र घरबिचको दूरी पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower from the roof of a building 10 m high is found to be 60°. From the same place, the angle of elevation at the point 18 m below the top of the tower is observed to be 30°. Find the height of the tower and the distance between the tower and building. **[2069 S]**

7C. समतलमा रहेको कुनै बिन्दुबाट सोही समतलमा रहेको एउटा खम्बाको टुप्पोको उन्नतांशकोण अवलोकन गर्दा 60° पाइयो र सो खम्बाको टुप्पोबाट 20 मि. तलको बिन्दुको उन्नतांश कोण 30° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।
From a point on the horizontal plane, the angle of elevation of the top of a pillar standing on the same plane was observed and found to be 60° and the angle of elevation of a point 20 m below the top of the pillar was found to be 30°. Find the height of the pillar. **[2062 R, 2063 R']**

7D. समतलमा रहेको कुनै बिन्दुबाट सोही समतलमा रहेको एउटा धरहराको टुप्पोको उन्नतांश कोण अवलोकन गर्दा 60° पाइयो र सो धरहराको टुप्पोबाट 24 मिटर तलको बिन्दुको उन्नतांश कोण सोही बिन्दुबाट 30° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
From a point on the horizontal plane, the angle of elevation of the top of a tower standing on the same plane was observed and found to be 60°. The angle of elevation of a point 24 meter below the top of the tower was found to be 30° from the same point. Find the height of the tower. **[2064 S]**

7E. 1.68 मिटर अग्लो मानिसले एउटा घर र त्यसको झ्यालको टुप्पो कुनै स्थानबाट अवलोकन गर्दा उन्नतांश कोणहरू क्रमशः 45° र 30° पाएछ । यदि झ्यालको उचाइ जमिनबाट 11.68 मिटर भए घरको उचाइ पत्ता लगाउनुहोस् ।
A man of 1.68 m high observed the angles of elevation of the top of a house and its window from a place and found to be 45° and 30° respectively. If the height of the window from the ground is 11.68 m, calculate the height of the house. **[2061 R]**

7F. 9 मिटर लामो भ्याङ कुनै एउटा भवनको छतबाट केही तलको बिन्दुसम्म पुगेको छ । भ्याङको फेदबाट भवनको छतको उन्नतांशकोण 60° छ भने भवनको उचाइ पत्ता लगाउनुहोस् । यहाँ भ्याङले जमिनसँग 45° कोण बनाएको छ ।
A ladder 9 m long reaches to a point below the top of building. From the foot of the ladder, the angle of elevation of the building is 60°. Find the height of the building. Here the ladder makes the angle 45° with ground. **[2060 S']**

Model 5 ***On finding height of tower or post when the angles of the elevation are given complementary.***

8A. एउटा स्तम्भको फेददेखि 49 मिटर र 64 मिटरको दूरीबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of a tower as observed from the distance of 49 metre and 64 metre from the foot of the tower are found to be complementary. Find the height of the tower. **[SEE 2075 R'$_2$]**

8B. एउटा धरहराको टुप्पोलाई यसको फेदबाट 27 m र 75 m दूरीमा एकैतिरबाट अवलोकन गर्दा उन्नतांशकोणहरू समपूरक पाइयो भने उक्त धरहराको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of a tower observed from 27 m and 75 m away from its foot on the same side are found to be complementary. Find the height of the tower. **[2072 S]**

8C. समतल स्थानमा भएका कुनै दुईओटा बिन्दुहरूबाट एउटा टावरको टुप्पोमा हेर्दा बन्ने उन्नतांश कोणहरू समपूरक पाइयो । यदि ती बिन्दुहरू उक्त टावरको फेदबाट 20 मिटर र 45 मिटरको दूरीमा छन् भने, सो टावरको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of a tower observed from two points on the same plane are found to be complementary. If the points are at a distance of 20 m and 45 m from the foot of the tower, find the height of the tower. **[2070 S]**

8D. एउटा स्तम्भको फेददेखि 36 मिटर र 16 मिटरको दूरीबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower as observed from the distances of 36 m & 16 m from the foot of the tower are found to be complementary. Find the height of the tower. **[2059 S]**

8E. एउटा स्तम्भको फेददेखि 144 मिटर र 121 मिटरको दूरीमा रहेका दुई बिन्दुहरूबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् । **[2070 R]**
The angles of elevation of the top of a tower as observed from the points at the distances of 144 metre and 121 metre from the foot of the tower are found to be complementary. Find the height of the tower.

www.drmathnp.com

8F. 180 मिटर दूरीमा रहेका दुई खम्बाहरू छन् । एउटा खम्बा अर्कोभन्दा दोब्बर अग्लो छ । खम्बाहरूको फेद जोड्ने रेखाहरूको मध्यबिन्दुबाट टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक रहेछ भने लामो खम्बाको उचाइ पत्ता लगाउनुहोस् ।
Two posts are 180 meter apart and the height of one is double than that of the other. From the mid point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, find the height of longer post. **[2065 R']**

Model 6 *On finding height of equal pillars when elevation angles from a point & their distance are given.*

9A. बराबर उचाइ भएको दुई खम्बाहरू 100 मिटर फराकिलो बाटोको दुई छेउमा रहेका छन् । खम्बाहरूबिचको कुनै बिन्दुबाट खम्बाको टुप्पोहरूमा हेर्दा उन्नतांश कोणहरू 60° र 30° का छन् भने खम्बाहरूको उचाइ कति होला ?
Two pillars of the same height are situated on the road 100 m apart. From any point between the pillars, the angles of elevation as observed to the top of pillars are 60° and 30°. What will be the height of the pillars ? **[2064 R']**

9B. 40 m फराकिलो सडकको दुवैतिर किनारामा बराबर उचाइ भएका दुईओटा स्तम्भहरू छन् । सो स्तम्भहरू रहेको सडकमा पर्ने कुनै एउटा बिन्दुबाट स्तम्भहरूको टुप्पाको उन्नतांशकोणहरू 60° र 30° छन् भने तिनीहरूको उचाइ र सो बिन्दुको स्थिति पत्ता लगाउनुहोस् ।
Two pillars of equal height stand on either side of a road way which is 40 m wide. At a point on the road way between the pillars, the angle of elevations of the tops of the pillars are 60° and 30°, find the height and the position of the point.

Model 7 *On finding elevation angle from a point when position of the point & another angle is given.*

10A. एउटा भित्तामा टाँसिएको एउटा पोस्टरको उचाइ 3.66 मि. रहेछ । भित्ताको फेददेखि 5 मि. टाढा एउटै समतलमा परेको बिन्दुबाट पोष्टरको तल्लो किनारा हेर्दा उन्नतांश कोण 45° पाइयो भने त्यही बिन्दुबाट हेर्दा पोष्टरको माथिल्लो किनाराको उन्नतांश कोण कति होला ?
A poster hanging on a wall has a vertical height 3.66 m. From a point 5 m away from the wall on the same plane, the angle of elevation of the bottom edge of the poster was found to be 45°. What will be the angle of elevation of the top edge of the poster if it is observed from the same point on the horizontal plane. **[2063 S]**

10B. एउटा पोलमा टाँगिएको झन्डाको लम्बाइ 4.22 मि. रहेछ । पोलको फेददेखि 10 मि. टाढा एउटै समतलमा परेको बिन्दुबाट झन्डाको तल्लो किनारा हेर्दा उन्नतांश कोण 30° पाइयो भने त्यही बिन्दुबाट हेर्दा झन्डाको माथिल्लो किनाराको उन्नतांश कोण कति होला ?
A flagstaff having height 4.22 m is on the top of a vertical pole. From a point 10 m away from the pole on the same plane, the angle of elevation of the bottom edge of the pole was found to be 30°. What will be the angle of elevation of the top edge of the flagstaff if it is observed from the same point on the horizontal plane.

10C. एउटा ठाडो खम्बा AB लाई बिन्दु C ले AC : CB = 2 : 1 हुने गरी विभाजन गरेको छ । जमिनको कुनै एउटा बिन्दुबाट C को उन्नतांश कोण 30° छ भने सोही बिन्दुबाट A को उन्नतांश कोण कति हुन्छ ? पत्ता लगाउनुहोस् जहाँ B खम्बाको फेद हो ।
A vertical pole AB is divided by a point C such that AC : CB = 2 : 1. The angle of elevation of C from a point on the ground is 30°. What is the angle of elevation of A from the same point? Find it, where B is the foot of the pole. **[2072 R]**

Model 8 *On finding height of pole when the equal angles from the point divide the pole in a ratio.*

11A. एउटा ठाडो खम्बालाई 9 : 1 हुने गरी कुनै बिन्दुले विभाजन गरेको छ । यदि उक्त खम्बाको दुवै भागहरूले खम्बाको फेददेखि 20 मिटरको दूरीमा रहेको कुनै एक बिन्दुमा आपसमा बराबर कोणहरू बनाउँदछन् भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।
A vertical pole is divided by any point in the ratio of 9 : 1. If both the segments of a pole subtend equal angles to each other at a distance of 20 m away from the foot of the pole, find the height of the pole. **[2065E]**

11B. एउटा रेडियो प्रशारण केन्द्रको खम्बा टुप्पोबाट 9 : 1 को अनुपातमा विभक्त छ । सो खम्बाका दुवै भागहरूले जमिनदेखि सीधा 600 मि. टाढा बराबर कोणहरू बनाउँछन् भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।
A tower of a radio station is divided by a point in the ratio of 9 : 1. From the top if both the parts subtends equal angles at a point on ground level 600 m away from its bottom, find the height of the tower.

Model 9 *Problems based on height of sun and shadow of a column.*

12A. समतल सतहमा ठड्याइएको एउटा स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 45° भएको समयमा भन्दा सूर्यको उचाइ 30° भएको समयमा 60 फिट बढी लामो पाइयो भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।
The shadow of a tower standing on a plain is found to be 60 feet longer when the sun's altitude is 30° than when it is 45°. What will be the height of the tower ? Find it. **[SEE 2073 R]**

12B. यदि सूर्यको उचाइ 60° बाट 45° हुँदा एउटा स्तम्भको छाया $10\sqrt{3}$ m बढ्छ भने स्तम्भको उचाइ र सूर्यको उचाइ 60° हुँदाको अवस्थामा स्तम्भको छायाको लम्बाइ पत्ता लगाउनुहोस् । **[2066 R]**
The length of the shadow of a vertical column increases by $10\sqrt{3}$ m when the altitude of the sun becomes 45° from 60°. Find the height of the column and the length of the shadow when the sun's altitude was 60°.

www.drmathnp.com

12C. कुनै धरहराको छाया सूर्यको उचाइ 60° भएको समयभन्दा सूर्यको उचाइ 45° भएको समयमा 10.5 m बढी लामो हुन्छ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
The shadow of a tower on the ground is found to be 10.5 m longer when sun's altitude is 45° than when it is 60°. Find the height of the tower. **[2067 R']**

12D. कुनै स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 60° भएको समयभन्दा सूर्यको उचाइ 45° भएको समयमा 45 m बढी लामो हुन्छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The shadow of a tower on the ground is found to be 45 m longer when sun's altitude is 45° than when it is 60°. Find the height of the tower. **[2068 R']**

OR, एउटा चउरमा रहेको कुनै स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 60° भएको समयमा भन्दा सूर्यको उचाइ 45° भएको समयमा 45 मि. बढी लामो हुन्छ भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।
The shadow of a tower on the ground is found to be 45 m longer when the sun's altitude is 45° than when it is 60°. What will be the height of the tower? Find it. **[SEE 2073 S]**

ANSWERS

A. VERY SHORT QUESTIONS

4A. 45° **4B.** 4x m **5A.** 5 m **5B.** 10 m

B. SHORT QUESTIONS

1A. 45° **1B** 30° **1C.** 45° **1D.** 60° **1E.** 1.64 m, 60 m **1F.** 2.18 m, 2.52 m **1G.** 25 m
1H. 123.56 m **1I.** 53.96 m, 60 m **1J.** 27 m, 35.35 m **1K.** 80 m **1L.** 84.74 m

C. LONG QUESTIONS

1A. 94.64 m **1B.** 109.28 m **1C.** 141.96 m **1D.** 81.97 m **1F.** 40.98 m **1G.** 141.96 m **1H.** 173.2 m
1I. $30\sqrt{3}$ m **2A.** 60 ft **2B.** 64 m **2C.** 12.68 m **2D.** 10.15 m **2E.** 8.87 m **2F.** 17·75 m
2G. 20 m **2H.** 47.32 m **2I.** 100 m **2J.** 47.32 m **2K.** 11.69 **2L.** 136.60 m **2M.** 81.96 m
3A. 56.79 m **3B.** 100 m **3C.** 28 m **3D.** 80 m **3E.** 80 m, 69.28 m **4A.** 273.20 m **4B.** 69.28 m
4C. 43.92 m **4D.** 21.96 m **4E.** 8.45 m, 11.55 m **5A.** 42.26m **5B.** 9.56 m **5C.** 73.20 m **6A.** 12.68 m
6B. 14.196 m **6C.** 42.58 **7A.** 8.19 m, 8.19 m **7B.** $9\sqrt{3}$ m, 37 m **7C.** 30 m **7D.** 36 m **7E.** 19 m
7F. 11.02 m **8A.** 56 m **8B.** 45 m **8C.** 30 m **8D.** 24 m **8E.** 132 m **8F.** 127.28 m
9A. $25\sqrt{3}$ m **9B.** $10\sqrt{3}$ m, 10m, 30m from 1st, 2nd pole **10A.** 60° **10B.** 45° **10C.** 60°
11A. 178.8 m **11B.** 5366.6 m **12A.** 81.96 ft. **12B.** 40.98 m, 23.66m **12C.** 24.84 m **12D.** 106.47 m

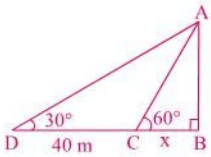
GENERAL EXERCISE 5

LONG QUESTIONS

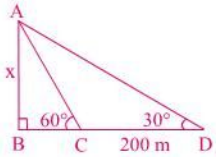
Model 1 *Problems based on finding value of x from the given figure.*

1. तलका चित्रहरूमा x को मान पत्ता लगाउनुहोस् (Find the value of x in the following figures):

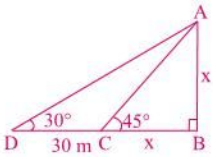
A.



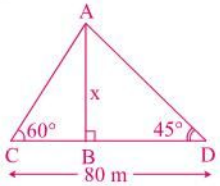
B.



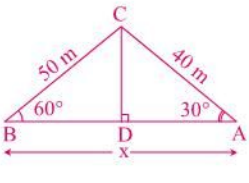
C.



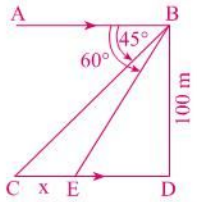
D.



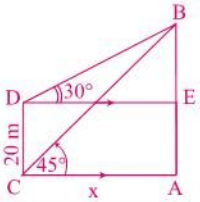
E.



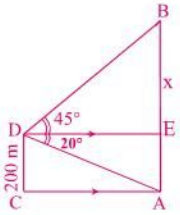
F.



G.



H.



Model 2 *On finding height of tower or post when height of sun and the shadows are given.*

2A. सूर्यको उचाइ 45° बाट घटेर 30° हुँदा कुनै सीधा खम्बाको छायाको लम्बाइ 60 m बढी हुन्छ भने खम्बाको उचाइ 30 $(\sqrt{3}+1)$ मिटर हुन्छ भनी देखाउनुहोस् ।
The shadow of a column standing on a level plane is found to be 60 m longer when the sun's altitude is 30° than when it is 45°. Prove that the height of the column is 30 $(\sqrt{3}+1)$ m.

2B. एउटा 2 m अग्लो मानिस बत्तीको खम्बादेखि 4 m टाढा उभिएको अवस्थामा उसको छायाको लम्बाइ 4 m हुन्छ भने उक्त खम्बाको उचाइ पत्ता लगाउनुहोस् ।
A man who is 2 meter tall stands at a distance of 4 meter from a lamp post and it is observed that his shadow is 4 meter long. Find the height of the lamp post.

www.drmathnp.com

2C. 1.6 मि उचाइ भएको एक व्यक्ति विजुलीको खम्बाबाट 4.68 m दूरीमा छ र यदि त्यस व्यक्तिको छाया 2.88 m लामो छ भने बिजुलीको खम्बाको उचाइ पत्ता लगाउनुहोस् ।
A man of height 1.6 m stands at a distance of 4.68 m from a lamp post. If it is observed that his shadow is 2.88 m long, find the height of the lamp post.

Model 3 Problems related to flagstafff when the angles from the same point are given.

3A. कुनै खम्बाको टुप्पामा ध्वजदण्ड खडा गरी राखिएको छ । खम्बाको फेददेखि 80 m परबाट खम्बा र ध्वजदण्डको टुप्पोमा हेर्दा उन्नतांश कोणहरू क्रमश: 30° र 45° बनेका छन् भने ध्वजदण्डको उचाइ पत्ता लगाउनुहोस् ।
A flagstaff stands on the top of a pole. The angles subtended by the top of the pole and the top of the flagstaff at a point 80 m away from the foot of the pole are 30° and 45° respectively. Find the height of the flagstaff.

3B. एउटा खम्बा र यसमाथि खडा गरिएको ध्वजदण्डले जमिनको कुनै एउटा बिन्दुमा क्रमश: 30° र 15° का कोणहरू बनाउँदछन् । यदि ध्वजदण्डको लम्बाइ 15 फिट भए खम्बाबाट सो बिन्दुसम्मको दूरी तथा खम्बाको उचाइ पत्ता लगाउनुहोस् ।
A tower and a flagstaff on the top of the tower subtend angles of 30° and 15° respectively at a point on the ground. If the length of flagstaff is 15ft, find the distance of the point on the ground from the foot of the tower and height of the tower.

Model 4 Problems based on finding distance between two points.

4A. कुनै पहाडको पूर्वतिर पर्ने बिन्दु P बाट पहाडको उन्नतांश कोण 45° छ र पहाडको ठीक पश्चिम पर्ने बिन्दुबाट पहाडको उन्नतांश कोण 30° छ । यदि पहाडको उचाइ $200\sqrt{3}$ m छ भने बिन्दु P र M बिचको दूरी पत्ता लगाउनुहोस ।
The elevation of a hill from a place P due east of it is 45°, at a place M due west of the hill, the elevation is 30°. If the height of the hill is $200\sqrt{3}$ m, find the distance between P and M.

4B. कुनै पहाडको पूर्वतिर पर्ने बिन्दु P बाट पहाडको उन्नतांश कोण 60° छ र पहाडको ठीक पश्चिम पर्ने बिन्दुबाट पहाडको उन्नतांश कोण 30° छ । यदि पहाडको उचाइ $300\sqrt{3}$ m छ भने बिन्दु P र M बिचको दूरी पत्ता लगाउनुहोस ।
The elevation of a hill from a place P due east of it is 60°, at a place M due west of the hill, the elevation is 30^0. If the height of the hill is $300\sqrt{3}$ m, find the distance between P and M.

4C. कुनै पहाडको उत्तरतिर पर्ने बिन्दु P बाट पहाडको उन्नतांश कोण 60° छ र पहाडको ठीक दक्षिण पर्ने बिन्दुबाट पहाडको उन्नतांश कोण 30° छ । यदि पहाडको उचाइ $100\sqrt{3}$ m छ भने बिन्दु P र M बिचको दूरी पत्ता लगाउनुहोस ।
The elevation of a hill from a place P due north of it is 60°, at a place M due south of the hill, the elevation is 30°. If the height of the hill is $100\sqrt{3}$ m, find the distance between P and M.

4D. कुनै पहाडको उत्तरतिर पर्ने बिन्दु P बाट पहाडको उन्नतांश कोण 60° छ र पहाडको ठीक दक्षिण पर्ने बिन्दुबाट पहाडको उन्नतांश कोण 45° छ । यदि पहाडको उचाइ 150 m छ भने बिन्दु P र M बिचको दूरी पत्ता लगाउनुहोस ।
The elevation of a hill from a place P due north of it is 60°, at a place M due south of the hill, the elevation is 45°. If the height of the hill is 150 m, find the distance between P and M.

Model 5 On finding distance between two posts & height of one when angles & height of next is given.

5A. एउटा खम्बा 20 मि. अग्लो छ । यस खम्बाको ठीक अगाडि रहेको एउटा घरको टुप्पोको उन्नतांश कोण खम्बाको टुप्पोबाट 30° र फेदबाट 45° छ भने: (क) घरको उचाइ कति होला ? (ख) खम्बादेखि घरको दूरी कति होला ?
A pillar is 20 m high. The angle of elevation of the top of a house just infront of it, from the top of pillar is 30° and from the foot is 45°, (a) Find the height of the house. (b) Find the distance between the pillar & the house.

5B. 50 मि. अग्लो रूखको फेदबाट एउटा स्तम्भको उन्नतांश कोण 60° छ र टुप्पाबाट उन्नतांश कोण 30° छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
From the foot of a tree 50 m high, the angle of elevation of a top of tower is 60° and from the top, the angle of elevation is 30°. Find the height of the tower.

5C. वनस्थलीमा भएको एउटा ठाडो खम्बामा P र Q दुईओटा बिन्दुहरू छन् । जसको सामुन्ने नार्गाजुन पहाड छ । P बाट सो पहाडको टुप्पाको उन्नतांशकोण 60° र Q बाट सोही पहाडको टुप्पाको उन्नतांशकोण 45° छ । यदि PQ = 400 m भए सो नार्गाजुन पहाडको बिन्दु P को सतह भन्दा माथिको उचाइ पत्ता लगाउनुहोस् ।
P and Q are two points on a vertical column at Vanasthali in front of which there is Nagarjun-hill. From P the angle of elevation of the top of the hill is 60° and from Q, the angle of elevation of the top of the same hill is 45°. Find the height of the Nagarjun-hill above the level of P if PQ = 400 m.

5D. कुनै 20 मिटर अग्लो रूखको ठीक अगाडि रहेको खम्बाको टुप्पाको उन्नतांश कोणहरू रूखको टुप्पाबाट 45° छ र यसको फेदबाट 60° छ भने खम्बाको उचाइ र रूखको फेदबाट यसको दूरी निकाल्नुहोस् ।
From the top of the tree 20 m high, the elevation of a pillar just infront of it is observed to be 45° and that from the base is 60°. Find the height of the pillar and distance between them.

pusnas.com.np

www.drmathnp.com
pustas.com.np

5E. एउटा 30 मि अग्लो घरको छानोबाट एउटा धरहराको टुप्पाको उन्नतांशकोण 45° र फेदको अवनतीकोण 30° पाइयो । उक्त धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the roof of a house 30 m high, the angle of elevation of the top of a tower is 45° and the angle of depression of its foot is 30°. Find the height of the tower.

5F. 50 मिटर अग्लो एउटा टावरको माथिबाट एउटा रुखको टुप्पो र फेदको अवनति कोणहरू 30° र 60° पाइयो भने रुखको उचाइ पत्ता लगाउनुहोस् ।
From the top of a tower of 50 m high, the angles of depression of the top and bottom of a tree are found to be 30^0 and 60^0 .Find the height of the tree.

5G. 30 मि. अग्लो स्तम्भको टुप्पोबाट अर्को स्तम्भवको टुप्पो र फेदमा हेर्दा अवनति कोणहरू क्रमशः 30° र 60° पाइयो भने दोस्रो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
From the top of a tower, 30 m high, the angles of depression of the top and bottom of a second tower are observed to be 30° and 60° respectively. Find the height of the second tower.

5H. 12 फिट उचाइ भएको एउटा पानी ट्याङ्की एउटा भवनको छानामाथि राखिएको छ । उक्त भवन र ट्याङ्कीले समतलमा रहेको एउटा बिन्दुमा क्रमश : 45° र 15° का कोणहरू बनाउँदछन् भने भवनको उचाइ पत्ता लगाउनुहोस् ।
A water tank of height 12ft is fixed on the roof of a building. The angles subtended by the building and the tank to a point on the ground are 45° and 15° respectively. Find the height of the building.

6A. एउटा धरहराको टुप्पोबाट 70 m अग्लो घरको छतमा हेर्दा 45° को अवनति कोण बन्यो । यदि घरको फेदबाट धरहराको टुप्पोको उन्नतांश कोण 60° छ भने धरहराको उचाइ कति होला ?
The angle of depression of the top of 70 m high building from the top of a tower is 45° and the angle of elevation of the top of the tower from foot of building is 60°. Find the height of the tower.

6B. एउटा धरहराको टुप्पोबाट 60 m अग्लो घरको छतमा हेर्दा 30° को अवनति कोण बन्यो । यदि घरको फेदबाट धरहराको टुप्पोको उन्नतांश कोण 60° छ भने धरहराको उचाइ कति होला ?
The angle of depression of the top of 60 m high building from the top of a tower is 30°. If the angle of elevation of the top of the tower from foot of building is 60°, find the height of the tower.

Model 6 ***On finding distance of two points when height of tower & depression angles are given.***

7A. 80 मिटर अग्लो धरहराको पूर्व अवस्थित दुई वस्तुहरूको अवनति कोणहरू क्रमशः 45° र 60° देखिन्छन् भने ती दुई वस्तुबिचको दूरी निकाल्नुहोस् ।
From the top of the tower, 80 m high, the measures of the angles of depression of two objects due east of the tower are found to be 45° and 60° respectively. Find the distance between the objects.

7B. 100 मिटर अग्लो धरहराबाट पूर्व अवस्थित दुईओटा वस्तुहरूको अवनति कोणहरू नाप्दा 45° र 60° छन् भने ती वस्तुहरू बिचको दूरी पत्ता लगाउनुहोस् ।
From the top of the tower 100 m high, the measures of depression of two objects due east of the tower are found to be 45° and 60°. Find the distance between the objects.

7C. एउटा h m अग्लो स्तम्भको टुप्पोबाट एउटै समतलमा रहेका 2 ओटा बिन्दुहरूलाई हेर्दा अवनति कोणहरू क्रमशः 60° र 30° पाइएछ भने ती दुई वस्तुबिचको दूरी $(2h \div \sqrt{3})$ m हुन्छ भनी प्रमाणित गर्नुहोस् ।
From the top of the cliff h m, high it is found that the angle of depression of two points on the same plane is 60° & 30°. Prove that the distance between them is $(2h \div \sqrt{3})$ m.

Model 7 ***On finding height of tower or post when the angles of the elevation are given complementary.***

8A. एउटा स्तम्भको टुप्पाको उन्नतांश कोणहरू सो स्तम्भबाट 242 m र 288 m टाढा एकैतिर र एउटै सीधा रेखामा हेर्दा समपूरक बनेका छन् भने सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of tower from two points 242 m and 288 m away from it on the same side of a straight line, are found to be complementary. Find the height of the tower.

8B. एउटा स्तम्भको टुप्पाको उन्नतांश कोणहरू सो स्तम्भबाट 169 m र 196 m टाढा एकैतिर र एउटै सीधारेखामा हेर्दा समपूरक बनेका छन् भने सो स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of tower from two points 169 m and 196 m away from it on the same side of a straight line, are found to be complementary. Find the height of the tower.

8C. एउटा स्तम्भको फेददेखि 64 मिटर र 49 मिटरको दूरीबाट स्तम्भको टुप्पोमा अवलोकन गर्दा उन्नतांश कोणहरू समपूरक भएको पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
The angle of elevation of the top of a tower as observed from the distances of 64 m and 49 m from the foot of the tower are found to be complementary. Find the height of the tower.

www.drmathnp.com

pusnas.com.np

8D. एउटा स्तम्भबाट 'a' मिटर र 'b' मिटर टाढा रहेका बिन्दुहरूको स्तम्भको टुप्पोमा हेर्दा बनेका उन्नतांशकोणहरू समपूरक भए सो स्तम्भको उचाइ $\sqrt{ab}$ मिटर हुन्छ भनी प्रमाणित गर्नुहोस् ।

The angles of elevation of the top of a tower from two points 'a' m and 'b' m from the base in the same straight line with it, are complementary. Prove that the height of the tower is $\sqrt{ab}$.

8E. एउटै समतल मैदानमा रहेको एउटा रूखको उचाइ एउटा घरको भन्दा दुई गुणा छ । घरको फेद र धुरीबाट उक्त रूखको टुप्पोमा हेर्दा बनेका उन्नतांश कोणहरू समपूरक छन् । यदि रूख र घरबिचको दूरी 10 m भए रूखको उचाइ पत्ता लगाउनुहोस् ।

The height of a tree is twice than that of a house, both of which lie on the same horizontal ground. The angles of elevation of the top of the tree from the bottom and the top of the house are complementary. If the distance between the tree and the house is 10 m, find the height of the tree.

8F. 120 फिट फरकमा रहेका दुईओटा खम्बाहरूमा एउटाको उचाइ अर्कोको भन्दा दोब्बर छ । ती खम्बाको फेदलाई जोड्ने रेखाको ठीक बीचबाट दुवै खम्बाको टुप्पो हेर्दा उन्नताशं कोणहरू समपूरक पाइयो भने ती खम्बाका उचाइ पत्ता लगाउनुहोस् ।

Two posts are 120 feet apart and the height of one is double than that of the other. From the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary. Find the height of the posts.

8G. दुईओटा स्तम्भहरू 40 मि. को दुरीमा रहेका छन् । सो स्तम्भहरूको ठीक बिचको बिन्दुबाट स्तम्भहरूको टुप्पोको उन्नतांश कोणहरू समपूरक पाइयो । यदि एउटा स्तम्भको उचाइ अर्कोको भन्दा दोब्बर भए सो स्तम्भहरूको उचाइहरू पत्ता लगाउनुहोस् ।

Two pillars are at a distance of 40 m. The angles of elevation of the tops of the pillars from the point mid way between them are found to be complementary. If height of one pillar is double of the other, find their heights.

Model 8 ***On finding height or distance when value of sine or tangent or co-tangent are given.***

9A. 96 ft. अग्लो स्तम्भको टुप्पाबाट सो स्तम्भको फेदको सतहमा रहेका दुईओटा सवारी साधनहरूलाई हेर्दा अवनति कोणहरू क्रमश $x°$ र $y°$ भयो भने सो सवारी साधनहरूबिचको दूरी पत्ता लगाउनुहोस् । जहाँ $\tan x° = \frac{3}{4}$ र $\tan y° = \frac{1}{3}$ छ ।

From the top of the tower 96 ft high, the angles of depression of two vehicles on a road at the same level as the base of the tower and on the same side of it, are $x°$ and $y°$ respectively where $\tan x° = \frac{3}{4}$ and $\tan y° = \frac{1}{3}$. Calculate the distance between the vehicles.

9B. 150 मि. अग्लो स्तम्भको टुप्पाबाट सो स्तम्भको फेदको सतहमा रहेका दुईओटा सवारी साधनहरूलाई हेर्दा अवनति कोणहरू क्रमश $x°$ र $y°$ भयो भने सो सवारी साधनहरूबिचको दूरी पत्ता लगाउनुहोस् । जहाँ $3\tan x° = 1$ र $7\tan y° = 5$ छ ।

From the top of the tower 150 m high, the angles of depression, of two vehicles on a road at the same level as the base of the tower and on the same side of it, are $x°$ and $y°$ where $3\tan x° = 1$ and $7\tan y° = 5$. Calculate the distance between the vehicles.

9C. कुनै बिन्दुबाट एउटा धरहराको उन्नतांशकोणको sine अनुपात $\frac{3}{\sqrt{34}}$ छ । धरहरातिर 48 m हिँडेपछि सो धरहराको उन्नतांशकोणको sine अनुपात $\frac{5}{\sqrt{34}}$ पाउँछ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।

At a certain point, the angle of elevation of a tower is found to be such that its sine is $\frac{3}{\sqrt{34}}$. On walking 48 m directly towards the tower, the angle of elevation is an angle whose sine is $\frac{5}{\sqrt{34}}$. Find the height of the tower.

9D. क्षितिजतलमा रहेको एउटा धरहराको टुप्पो P र फेद Q छ । सो समतल सतहमा दुई बिन्दुहरू A र B छन् जहाँ AB = 32 m र $\angle PQA$ एक समकोण छ । यदि $\cot PAQ = \frac{2}{5}$ र $\cot PBQ = \frac{3}{5}$ भए धरहराको उचाइ पत्ता लगाउनुहोस् ।

P is the top and Q is the foot of a tower standing on a horizontal plane. A and B are two points on this plane such that AB = 32 meters and $\angle PQA$ is a right angle. If $\cot PAQ = \frac{2}{5}$ and $\cot PBQ = \frac{3}{5}$, find the height of the tower.

www.drmathnp.com

pustas.com.np

Model 9 *On finding height of flagstaff or pole when an object & a point x m below it is given .*

10A. एउटा 9 m लामो भऱ्याङ ध्वजदण्डको टुप्पो भन्दा 9 m तल छ र भऱ्याङको फेदबाट ध्वजदण्डको टुप्पाको उन्नतांशकोण 60° रहेको छ । ध्वजदण्डको उचाइ निकाल्नुहोस् ।
A ladder 9 m long reaches a point 9 m below the top of a vertical flagstaff. From the foot of the ladder, the angle of elevation of the flagstaff is 60°. Find the height of the flagstaff.

10B. एउटा 12 m लामो भऱ्याङ ध्वजदण्डको टुप्पो भन्दा 12 m तल छ र भऱ्याङको फेदबाट ध्वजदण्डको टुप्पाको उन्नतांशकोण 60° रहेको छ । ध्वजदण्डको उचाइ निकाल्नुहोस् ।
A ladder 12 m long reaches a point 12 m below the top of a vertical flagstaff. From the foot of the ladder the angle of elevation of the flagstaff is 60°. Find the height of the flagstaff.

10C. 6 m लम्बाइ भएको भऱ्याङले एउटा खम्बालाई टुप्पोबाट 6 m तल छोएको छ । यदि भऱ्याङको फेदले खम्बाको टुप्पोसित बनाएको उन्नतांश कोण 60° भए खम्बाको उचाइ निकाल्नुहोस् ।
A ladder 6 m long touches a point 6 m below the top of a vertical pole. From the foot of the ladder, the angle of elevation of the pole is 60°. Find the height of the pole.

10D. एउटा हवाइजहाज 600 m उचाइमा रहँदा अर्को हवाईजहाजको ठीक माथितिर पर्दछ । अवलोकन बिन्दुबाट ती दुई जहाजहरूको उन्नतांशकोणहरू क्रमश: 60° र 45° भए पहिलो भन्दा कति मिटर मुनिबाट दोस्रो उडिरहेको होला ?
An aeroplane when 600 meters high, passes vertically above another at instant when the angles of elevation at the same observing point are 60° and 45° respectively. How many metres lower is the one than the other?

Model 10 *Problems related to broken parts of tree by the wind when an angle at a point is given.*

11A. आँधीले गर्दा 12 मिटर उचाइ भएको एउटा रूख भाँचिएछ र यसको टुप्पाले जमिनसँग 60° को कोण बनाएछ भने आधारबाट कति उचाइमाथि आँधीद्वारा रूख भाँचिएको होला ? पत्ता लगाउनुहोस् ।
A tree, 12 m high, is broken by the wind in such a way that its top touches the ground and makes an angle of 60° with the ground. At what height from the bottom of the tree is broken by the wind ?

11B. 40 m अग्लो एउटा सीधा रूख हावाले भाँचिदा यसको टुप्पाले जमिनसँग 30° को कोण बनाएको छ । रूखको फेददेखि रूखको टुप्पाले जमिनमा छोएको बिन्दुसम्मको दूरी पत्ता लगाउनुहोस् । रूखको भाँचिएको भागसम्मको उचाइ पनि पत्ता लगाउनुहोस् ।
A straight 40 m high tree is broken by the wind and its upper part meets the ground at an angle of 30°. Find the distance of the foot of the tree from the point where the top of the tree meets the ground. Find also the height of the tree at which it was broken.

11C. हुरीले गर्दा सीधा रूख भाँचिएर रूखको टुप्पोले छेउबाट 7 m टाढाको समतल सतहको बिन्दुमा छुँदा 45° को कोण बनाउँदछ भने त्यो रूख भाँचिनुभन्दा पहिलेको उचाइ पत्ता लगाउनुहोस् ।
The upper part of straight tree broken by the wind makes an angle of 45° with plain surface at point 7 m from the foot of the tree. Find the height of the tree before it was broken.

11D. हुरीले गर्दा सीधा रूख भाँचिएर रूखको टुप्पोले छेउबाट 9 m टाढाको समतल सतहको बिन्दुमा छुँदा 45° को कोण बनाउँदछ भने त्यो रूख भाँचिनुभन्दा पहिलेको उचाइ पत्ता लगाउनुहोस् ।
The upper part of a straight tree broken by the wind makes an angle of 45° with plain surface at point 9 m from the foot of the tree. Find the height of the tree before it was broken.

Model 11 *Problems based on equal post when the angle from a point & distance is given.*

12A. बराबर उचाइ भएका दुईओटा बिजुलीका खम्बा छन् । ती खम्बाबिच उभिएको एक केटाले प्रत्येक खम्बाको टुप्पाका उन्नतांश कोण 30° पाएछ र कुनै एक खम्बाको 15 मिटर नजिक जाँदा उसले त्यस खम्बाको उन्नतांश कोण 45° पाएछ भने ती दुई खम्बाको उचाइ र ती दुई खम्बाहरू बिचको दूरी पनि पत्ता लगाउनुहोस् ।
Two lamp posts are of equal height. A boy standing midway between them observes the angle of elevation of either posts to be 30°. After walking 15 m towards one of them, he observes its elevation to be 45°. Find the height of the posts and the distance between them.

12B. बराबर उचाइ भएका दुईओटा बिजुलीका खम्बा छन् । ती खम्बाबिच उभिएको एक केटाले प्रत्येक खम्बाको टुप्पाका उन्नतांश कोण 45° पाएछ र कुनै एक खम्बाको 15 मिटर नजिक जाँदा उसले त्यस खम्बाको उन्नतांश कोण 60° पाएछ भने ती दुई खम्बाको उचाइ र ती दुई खम्बाहरू बिचको दूरी पनि पत्ता लगाउनुहोस् ।
Two lamp posts are of equal height. A boy standing midway between them observes the angle of elevation of either posts to be 45°. After walking 15 m towards one of them, he observes its elevation to be 60°. Find the height of the posts and the distance between them.

12C. बराबर उचाइ भएका दुईओटा बिजुलीका खम्बाहरू छन् । दुबै खम्बाको बिचमा उभिएको एक जना मानिसले प्रत्येक खम्बामा हेर्दा बनाएको उन्नतांश कोण 30° पाएछ र एक खम्बालाई 20 m नजिक गएपछि फेरि हेर्दा उन्नतांश कोण 45° पाएछ भने खम्बाहरूको उचाइ र खम्बाहरूबिचको दूरी पत्ता लगाउनुहोस् ।
Two lamp posts are of equal height. A man standing mid-way between them observes the angle of elevation of either post to be 30°. After walking 20 m towards one of them, he observes its elevation to be 45°. Find the height of the posts and the distance between the posts.

www.drmathnp.com

puspas.com.np

Model 12 *Finding height or distance when equal angle making line divides the pole in given ratio.*

13A. एउटा ठाडो खम्बा AB लाई AD : DB = 4 : 3 हुने गरी बिन्दु D ले विभाजन गरेको छ । यदि उक्त खम्बाको दुवै भागहरूले खम्बाको फेद B देखि 50 m को दूरीमा रहेको कुनै एक बिन्दुमा आपसमा बराबर कोणहरू बनाउँदछन् भने खम्बाको उचाइ कति होला ?
A vertical pole AB is divided by a point D such that AD : DB = 4 : 3. If both the segments of a pole subtend equal angles to each other at a distance of 50 m away from the foot of the pole B, find the height of the pole.

13B. एउटा ठाडो खम्बा AB लाई AC : CB = 9 : 1 हुने गरी बिन्दु C ले विभाजन गरेको छ । यदि उक्त खम्बाको दुवै भागहरूले खम्बाको फेद B देखि 20 m को दूरीमा रहेको कुनै एक बिन्दुमा आपसमा बराबर कोणहरू बनाउँदछन् भने खम्बाको उचाइ कति होला ?
A vertical pole AB is divided by a point C such that AC : CB = 9 : 1. If both the segments of a pole subtend equal angles to each other at a distance of 20 m away from the foot of the pole B, find the height of the pole.

13C. न्यूयोर्क शहरको कुनै एउटा अग्लो भवन एक बिन्दुमा 1 : 9 को अनुपातमा विभक्त छ । यदि भवनको दुवै खण्डले भवनको फेदबाट 60 m परेको एउटा बिन्दुमा बराबर कोण अन्तरित गर्दछ भने भवनको उचाइ पत्ता लगाउनुहोस् ।
A high building of New York city is divided by a point in the ratio of 1 : 9. If both the parts of the building subtend equal angles at a point on ground level 60 m away from its bottom, find the height of the the building.

13D. $10\sqrt{3}$ मि. अग्लो एउटा खम्बाले सो खम्बाको फेदबाट 30 मि. टाढाको कुनै बिन्दुमा θ कोण बनाएको छ । पहिलो बिन्दु र खम्बाको फेदबिचमा पर्ने अर्को कुनै बिन्दुमा उक्त खम्बाले 2θ कोण बनाउँदछ । दोस्रो बिन्दु र खम्बाको फेदबिचको दूरी पत्ता लगाउनुहोस् र साथै दुई बिन्दुहरूबिचको दूरी पत्ता लगाउनुहोस् ।
A pole $10\sqrt{3}$ m high, subtends an angle θ at a point which is 30 m away from the foot of the pole. At another point between the first point and the foot of the pole, the pole subtends an angle 2θ. Find the distance of the second point from the foot of the pole and also the distance between the two points.

13E. कुनै एउटा खम्बालाई खम्बाबाट 10 m टाढाको कुनै बिन्दुबाट हेर्दा बराबर कोणद्वारा 2 : 3 को अनुपातमा विभाजन गर्दछ भने खम्बाको उचाइ कति मिटर होला ?
A pole is observed 10 m away from the pole and the pole is divided in the ratio of 2 : 3 by the equal angles. What is height of the pole ?

13F. 25 फिट अग्लो स्तम्भ AB को फेदबाट 50 फिट दूरीको बिन्दुमा स्तम्भले र स्तम्भमाथि खडा गरिराखेको ध्वजदण्ड BC ले बराबर कोण अन्तरित गर्दछ भने ध्वजदण्डको उचाइ पत्ता लगाउनुहोस् ।
At a distance of 50 feet from the foot of a tower AB of height 25 feet, a flagstaff BC and the tower subtends equal angles. Find the height of the flagstaff.

Model 13 *Proving the height of the second post when height of the first post & their angles are given.*

14A. दुई खम्बाहरूमध्ये पहिलो खम्बाको फेदबाट दोस्रो खम्बाको टुप्पाको उन्नतांश कोण 60° र दोस्रो खम्बाको फेदबाट पहिलो खम्बाको टुप्पाको उन्नतांशकोण 30° छ । यदि पहिलो खम्बाको उचाइ $10\sqrt{3}$ m भए दोस्रो खम्बाको उचाइ $30\sqrt{3}$ m हुन्छ भनी प्रमाणित गर्नुहोस् ।
There are two posts, the angle of elevation of the top of the second post from the foot of the first is 60° and that of the top of the first post from the foot of the second is 30°. If the height of the first post is $10\sqrt{3}$ m, prove that the height of the second post is $30\sqrt{3}$ m.

14B. क्षितिजमा रहेका दुई स्तम्भहरूमध्ये पहिलो स्तम्भको फेदबाट दोस्रो स्तम्भको टुप्पाको उन्नतांश कोण 60° र दोस्रो स्तम्भको फेदबाट पहिलो स्तम्भको टुप्पाको उन्नतांश कोण 30° छ भने दोस्रो स्तम्भको उचाइ पहिलोको तिन गुणा छ भनी प्रमाणित गर्नुहोस् ।
There are two towers on a horizontal plane. When observed from the foot of the first, the angle of elevation of the top of the other is 60° and when observed from the foot of the second, the angle of elevation of the top of the first is 30°. Prove that the second tower is three times as high as the first.

14C. एउटा अन्वेषणकर्ताले स्तम्भको h m उचाइमा रहेको झ्यालको सीधा तलको बिन्दुबाट सीधा रेखामा पर्ने क्षितिज तलका दुई बिन्दुहरू A र B मा हेर्दा 30° र 15° को अवनति कोणहरू पाउँछ भने AB = 2h m हुन्छ भनी प्रमाणित गर्नुहोस् ।
A spectator ascends a tower to a small window at a height h m and observes the angles of depression of two objects A and B on the horizontal plane to be 30° and 15°. If A and B are in a straight line with a point vertically below the window, prove that AB = 2h m.

Model 14 *Problems based on finding the speed or time of an aeroplane or a boat or a girl.*

15A. एउटा हवाईजहाज जमिनदेखि 750 m माथि क्षितिजसँग समानान्तर हुनेगरी उडिरहको बेला उन्नतांशकोण 60° छ । 5 सेकेण्डपछि जहाजको उन्नतांशकोण घटेर 30° भए जहाजको गति किलोमिटर प्रतिघण्टामा निकाल्नुहोस् ।
A aeroplane flying horizontally 750 m above the ground is observed at an elevation of 60°. If after 5 seconds the elevation is observed to be 30°, find the speed of the aeroplane in km per hour.

www.drmathnp.com

15B. एउटा चट्टानको शिखरबाट एकजना केटाले यसैतर्फ आइरहेकी एउटी केटीलाई हेर्दा अवनति कोण 30° पाउँछ र 20 मिनेटपछि अवनतिकोण 45° पाउँछ । यदि चट्टान 2 km अग्लो छ भने केटीको गति पत्ता लगाउनुहोस् ।
From the top of a cliff, a boy finds the angle of depression of a girl comming towards it to be 30°. After 20 minutes, the angle of depression is observed to be 45°. If the height of the cliff is 2 km, find the speed of the girl.

15C. एउटा डुङ्गा 150 m अग्लो चट्टानबाट परतिर खियाइरहेको छ । दुई मिनेटमा चट्टानको टुप्पोबाट डुङ्गाको अवनति कोण 60° बाट 45° पुग्दछ भने डुङ्गाको गति पत्ता लगाउनुहोस् ।
A boat is being rowed away from a cliff 150 metres high. At the top of the cliff, the angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

15D. एउटा ढिस्कोको टुप्पोमा बसेर एकजना मानिसले डुङ्गालाई हेर्दा अवनती कोण 30° पाएछ । जुन डुङ्गा समान गतिमा नदी किनारतिर अवलोकनकर्ताको सिधैं तलतिर आइरहेको छ । चार मिनेट पछि उक्त डुङ्गाको अवनति कोण 60° पाइएछ । उक्त डुङ्गालाई नदी किनारमा पुग्न लाग्ने समय पत्ता लगाउनुहोस् ।
A man on a cliff observes a boat at an angle of depression of 30°, which is approaching the shore to the point immediately beneath the observer with a uniform speed. Four minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.

15E. समुन्द्री तटमा रहेको एउटा स्तम्भको टुप्पोबाट एउटा मानिसले एउटा डुङ्गा हेर्दा अवनति कोण 30° बाट 60° मा परिवर्तन हुँदा सो डुङ्गालाई 10 मिनेट लाग्दछ भने समुन्द्री तटसम्म सो डुङ्गा आउन कति समय लाग्ला ?
A man on the top of the tower, standing in the seashore finds that a boat coming towards him makes 10 minutes for the angle of depression to changes from 30° to 60°. How soon will the boat reach the seashore?

Model 15 *On finding height of pole when one angle of top of pole is double of another angle.*

16A. बाटो AB खम्बाको फेद बिन्दु C तिर अगाडि बढेको छ । बिन्दुहरू B र C बिन्दु A देखि क्रमशः $8\sqrt{3}$ m र $12\sqrt{3}$ m टाढा छन् । यदि बिन्दु B बाट खम्बाको टुप्पोमा अवलोकन गर्दा बन्ने उन्नतांश कोण बिन्दु A बाट बन्ने उन्नतांश कोणभन्दा दोब्बर हुन्छ भने खम्बाको उचाइ निकाल्नुहोस् ।
AB is a straight road leading to C, the foot of the pole; B and C are being $8\sqrt{3}$ m and $12\sqrt{3}$ m respectively far from the point A. If the angle of elevation of the top of pole at B is double of the angle of elevation of A, find the height of the pole.

16B. कुनै एउटा ठाउँबाट 200 मिटर टाढारहेको कुनै एउटा स्तम्भको टुप्पोलाई हेर्दा जति डिग्रीको कोण बन्दछ ठीक 125 मिटर नजिक गएर सोही टुप्पोमा हेर्दा दोब्बर कोण बन्दछ भने सो स्तम्भको उचाइ कति होला, निकाल्नुहोस् ।
The angle of the top of a tower observed from 125 m far is double of the angle observed from 200 m far from the tower. What is the height of the tower ?

16C. AB एउटा सीधा सडक हो जुन एउटा धरहराको फेद C भएर जान्छ । बिन्दु A धरहराको फेद C बाट 120 m टाढा छ र बिन्दु B बिन्दु A बाट 75 m नजिक छ । यदि B बाट लिइएको धरहराको उन्नतांश कोण बिन्दु A बाट लिइएको उन्नतांशकोणको दोब्बर छ भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
AB is a straight road leading to C, the foot of a tower. A being at a distance 120 metres from C and B being 75 meters nearer. If the angle of elevation of the tower at B be the double of the angle of elevation of the tower at A, find the height of the tower.

16D. PQ एउटा सीधा सडक हो जुन एउटा स्तम्भको फेद R भएर जान्छ । बिन्दु P स्तम्भको फेदबाट 200 m टाढा छ र बिन्दु Q चाहिँ बिन्दु P भन्दा 125 m नजिक छ । यदि बिन्दु Q बाट लिइएको स्तम्भको उन्नतांशकोण बिन्दु P बाट लिइएको उन्नतांशकोणको दोब्बर छ भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
PQ is a straight road leading to R, the foot of a tower. P being at a distance of 200 m from R and Q being 125 m nearer. If the angle of the elevation of the tower at Q be the double of the angle of the elevation at P, find the height of the tower.

Model 16 *Miscellaneous problems*

17A. गोलाकार पोखरीमा गाडिएको बाँसको पानी माथिको भाग 30 m छ र टुप्पाले किनारको कुनै स्थानमा 30° को कोण बनाएको छ । त्यो स्थानबाट विपरीत भागमा रहेको किनाराको स्थानमा 60° को कोण बनाएको छ । पोखरीको व्यास निकाल्नुहोस् ।
A 30 m long bamboo standing in a circular pond subtends an angle of 30° at a point in the circumference and angle of 60° at another point directly opposite to the first point in the circumference. Find the diameter of the pond.

17B. एउटा चटके 9 मिटर उचाइ भएका दुई खम्बाहरूको टुप्पामा बाँधिएको लचकदार डोरीमाथि हिँडिरहेको थियो । जब ऊ जमिन सतहभन्दा 2.7 मिटर माथि थियो, डोरीको दुई अंशहरूले जमिनसँग समानान्तर क्षितिज रेखासँग 30° र 60° का कोणहरू बनाएको पाइयो भने डोरीको लम्बाइ कति होला ?
A rope dancer was walking on a loose rope tied to the tops of two equal post of height 9 m. When he was 2.7 m above the ground, it was found that the stretched pieces of the rope made angles of 30° and 60° with horizontal line parallel to the ground. Find the length of the rope.

www.drmathnp.com

17C. एउटा चटके 9 m उचाइ भएका दुईओटा खम्बाहरूको टुप्पामा बाँधिएको लचकदार डोरीमाथि हिँडिरहेको थियो । जब ऊ जमिनको सतहभन्दा 3 m माथि थियो, डोरीका दुई अंशहरूले जमिनसँग समानान्तर क्षितिज रेखासँग क्रमश: 30° र 60° को कोणहरू बनाएको पाइयो भने डोरीको लम्बाइ कति होला ?

A rope dancer was walking on a loose rope tied to the top of two equal posts of height 9 m. When he was 3 m above the ground, it was found that the stretched pieces of the rope made angles of 30° and 60° with the horizontal line parallel to the ground. Find the length of the rope.

17D. एउटा अपूर्ण स्तम्भको टुप्पाको आधारदेखि 100 m टाढाको बिन्दुबाट हेर्दा उन्नतांशकोण 45° को बनेको छ । सो उन्नतांशकोण 60° मा परिवर्तन गराउन स्तम्भको उचाइ कति बढाउनुपर्ला ?

The angle of elevation to the top of an incomplete tower from a distance of 100 m from its base is found to be 45°. What height should it be raised so that the elevation may change to 60° ?

17E. एउटा भवनको छतको उन्तांश कोण सोही भवनको सतहको 10 मि. टाढाको बिन्दुबाट 45° को छ । नयाँ छतको उन्नतांश कोण 60° हुन भवनलाई कति अग्लो बनाउनुपर्ला ?

The angle of elevation of the top of a building from 10 m away in the same level is 45°. By what height should the building be raised to get the angle of elevation of the new top of the building to be 60°?

17F. एउटा अपूर्ण घरको टुप्पाको उन्नतांशकोण जमिनको एउटा बिन्दुबाट 30° पाइयो । सो अपूर्ण घरको उचाइ 30 m छ । अब घरको उचाइ कति बढाउँदा त्यही बिन्दुबाट घरको टुप्पामा 45° को उन्नतांशकोण बन्छ ?

The angle of elevation of the top of an incomplete house as observed from the point on the ground is 30°. If the height of the incomplete house is 30 m, how high the house must be raised so that the angle of elevation of the top of the complete house as observed from the same pont on the ground is 45°?

17G. एउटा धरहराको उचाइ यसमाथि रहेको ध्वजदण्डको उचाइको आधा छ । यदि जमिनको कुनै बिन्दुबाट धरहराको टुप्पाको उन्नतांश कोण 30° भए ध्वजदण्डको टुप्पाको उन्नतांशकोण पत्ता लगाउनुहोस् ।

The height of a tower is half the height of the flagstaff at its top. The angle of elevation of the top of the tower as seen from any point on the ground is 30°. Find the angle of elevation of the top of the flagstaff from the same point.

17H. एउटा 1.5 m उचाइ भएको केटा 30 m अग्लो भवनको अगाडि उभिएको छ । त्यसबेला उसको आँखाले भवनको छानाको उन्नतांश कोण 30° बनाउँछ । ऊ भवनतिर केही दूरी हिंडिसकेपछि उक्त छानाको उन्नतांश कोण 60° पाउँछ भने उक्त केटाले हिंडेको दूरी पत्ता लगाउनुहोस् ।

A 1.5 m tall boy is standing in front of a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

TRIGONOMETRY

ANSWERS

1A. 20m **1B.** $100\sqrt{3}$ m **1C.** 40.98 m **1D.** 50.70 m **1E.** 59.64 m **1F.** 42.27 m **1G.** 47.32 m

1H. 549.49m **2B.** 4 m **2C.** 4.19 m **3A.** 33. 81 m **3B.** 35.49 ft **4A.** 964.41 m **4B.** 1200 m

4C. 400 m **4D.** 236.60 m **5A.** 47.32m, 47.32 m **5B.** 75 m **5C.** 546.41 m **5D.** 47.32 m, 27.32 m

5E. 81.96 m **5F.** 33.33 m **5G.** 20 m **5H.** 16.93 ft. **6A.** 165.62 m **6B.** 111.96 m **7A.** 33.81 m

7B. 42.26 m **8A.** 264 m **8B.** 182 m **8C.** 56 m **8E.** $10\sqrt{2}$m **8F.** $30\sqrt{2}$ feet , $60\sqrt{2}$ feet

8G. $10\sqrt{2}$ m, $20\sqrt{2}$ m **9A.** 160ft **9B.** 240 m **9C.** 45 m **9D.** 160 m **10A.** 13.5 m

10B. 18 m **10C.** 9 m **10D.** 253.58 m **11A.** 5.57 m **11B.** 26.7m,13.3m **11C.** 16.89 m **11D.** 21.72 m

12A. 70.98 m, 20.49m **12B.** 35.49m, 70. 98m **12C.** 27.32m, 94.64m **13A.** 44.09 m

13B. 178.9m **13C.** 536.6 m **13D.** 10 m, 20 m **13E.** 11.18 m **13F.** $41\frac{2}{3}$m **14A.** $30\sqrt{3}$ m **15A.** 623.50 km/hr

15B. 73.2m/min **15C.** 0.528 m/s **15D.** 2 min **15E.** 5 minutes **16A.** 12 m **16B.** 100 m

16C. 60 m **16D.** 100 m **17A.** 69.28m **17B.** 19.87 m **17C.** 18.92 m **17D.** 73.20 m **17E.** 7.32 m

17F. 21.9 m **17G.** 60° **17H.** 32.91 m

pusmas.com.np

www.drmathnp.com

pupas.com.np

CHALLENGING EXERCISE 5

1. $10\sqrt{3}$ मिटर अग्लो खम्बाले खम्बाको फेदबाट 30 मिटर टाढाको बिन्दुमा θ कोण अन्तरित गर्दछ र बिचको कुनै बिन्दुमा 2θ कोण अन्तरित गर्दछ । खम्बाको फेददेखि दोस्रो बिन्दुसम्मको दूरी र दूई बिन्दुहरूबिचको दूरी पत्ता लगाउनुहोस् ।

A pole $10\sqrt{3}$ m high subtends an angle θ at the point 30 m away from the foot of the pole, at another point in between the pole subtends an angle 2θ. Find how far the second point is from the foot of the pole and also the distance between two points.

2. कुनै पहाडको पूर्वतिर पर्ने बिन्दु A बाट पहाडको उन्नतांश कोण 45° छ । A को ठीक दक्षिणतिर पर्ने B बिन्दुबाट पहाडको उन्नतांश कोण 30° छ । यदि पहाडको उचाइ $200\sqrt{2}$ मिटर छ भने बिन्दुहरू A र B बिचको दूरी पत्ता लगाउनुहोस् ।

The elevation of a hill from a place A due east of it is 45°. At a place B due south of A, the elevation is 30°. If the height of hill is $200\sqrt{2}$ m, find the distance between A & B.

3. एउटा मानिसले कुनै धरहराको ठीक दक्षिणतिरको एक बिन्दुबाट यसको टुप्पोको उन्नतांश कोण 60° पाउँछ । त्यसपछि ऊ ठीक पश्चिमतिर 60 m मिटर हिँडेपछि उन्नतांश कोण 30° पाउँछ भने धरहराको उचाइ र यसबाट उसको प्रारम्भिक दूरी पत्ता लगाउनुहोस् ।

From a point due south of a tower, a man observes the angle of elevation of the top of the tower to be 60°. He then walks 60 m due west on the horizontal plane and observes the angle of elevation to be 30°. Find the height of the tower and his initial distance from the tower.

4. 40 मि × 30 मि. को फुटबल खेल्ने चौरको एक कुनामा राखिएको खम्बालाई क्रमागत नभएको त्यस्तै अर्को कुनाबाट हेर्दा 30° को कोण बनेछ भने खम्बाको उचाइ कति होला ?

The pole in one corner of a football ground of 40 m × 30 m is observed from the another corner which is opposite , non co-interior to the pole is 30°. Find the height of the pole.

5. कुनै स्तम्भको फेदबाट एउटा धरहराको टुप्पोको उन्नतांश कोण 45° छ र स्तम्भको टुप्पाबाट धरहराको फेदको अवनति कोण 30° छ । एक मानिस स्तम्भको फेदबाट धरहरातिर 10 मिटर हिँडेपछि धरहराको टुप्पाको उन्नतांशकोण 60° पाउँछ भने स्तम्भ र धरहराको उचाइ पत्ता लगाउनुहोस् ।

From the foot of a column, the angle of elevation of the top of a tower is 45° and from the top of the column, the angle of depression of the bottom of the tower is 30°. A man walks 10 metres from the bottom of the column towards the tower. He notices the angle of elevation of its top to be 60°. Find the heights of the column and the tower.

6. एउटा ठाडो स्तम्भमाथि h उचाइ भएको ध्वजदण्ड खडा गरिराखेको छ । जमिनको कुनै बिन्दुबाट ध्वजदण्डको फेदको उन्नतांशकोण α र टुप्पाको उन्नतांशकोण β छन् भने स्तम्भको उचाइ $\frac{h \tan\alpha}{\tan\beta - \tan\alpha}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β, prove that the height of the tower is $\frac{h \tan\alpha}{\tan\beta - \tan\alpha}$.

7. पोखरीको सतहभन्दा 400 फिट माथिको कुनै बिन्दुबाट एउटा चराको उन्नतांशकोण 30° छ र सोही बिन्दुबाट पोखरीभित्र चराको छायाको अवनति कोण 60° छ भने चरा पोखरीको सतहबाट कति माथि होला ?

The angle of elevation of a bird from a point 400 feet above a pond is 30° and the angle of depression on its image in the pond is found to be 60°. Find the height of the bird from the surface of water.

HINT: Let CG = x denotes the height of a bird from the surface of water.

From right angled ΔEFG,

$$\tan 30° = \frac{GF}{EF}$$

or, $\frac{1}{\sqrt{3}} = \frac{x - 400}{EF}$

$\therefore$ $EF = \sqrt{3}(x - 400)$

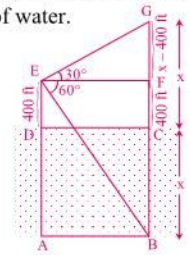
Again, from right angled ΔEFB,

$$\tan 60° = \frac{x + 400}{\sqrt{3}(x + 400)}$$

or, $\sqrt{3} = \frac{x + 400}{\sqrt{3}(x - 400)}$

$\therefore$ $x = 800$ ft.

Hence, the height of the bird from the surface of water is 800 ft.



www.drmathnp.com

8. पहाडको फेदबाट टुप्पोको उन्नतांश कोण 45° को छ र उक्त पहाड 30° कोणको झुकावमा 1 km उक्लिदा पहाडको टुप्पाको उन्नतांश कोण परिवर्तन भई 60° को हुन्छ भने पहाडको उचाइ पत्ता लगाउनुहोस् ।
At the foot of a mountain, the angle of elevation of its summit is 45°. After ascending 1 km forwards at an inclination of 30°, the elevation changes to 60°. Find the height of the mountain.

9. मैदानको एउटा बिन्दुबाट एउटा हवाइजहाजको उन्नतांश कोण 45° छ । 30 सेकेण्डको उडानपछि यो 30° मा परिवर्तन हुन्छ । यदि सो हवाइजहाज मैदानदेखि 6000 m को स्थिर उचाइमा उडिरहेको छ भने हवाइजहाजको गति पत्ता लगाउनुहोस् ।
The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight for 30 seconds, the elevation changes to 30°. If the aeroplane is flying at a constant height of 6000 m, find the speed of the aeroplane.

ANSWERS

1. 10 m, 20 m **2.** 400 m **3.** 36.74 m, 63.63 m **4.** $\frac{50}{\sqrt{3}}$ or, 28.86 m
5. 13.66m, 23. 66 m **7.** 800 ft. **8.** 1.366 km **9.** 146.41 m/s

ASSIGNMENT ON TRIGONOMETRY

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

TRIGONOMETRY

Group 'A' **3 × 1 = 3**

1. 2 cos A cos B लाई cosine को योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् ।
Express 2 cos A cos B in the sum or difference of cosine.
2. cos A र $\tan\frac{A}{2}$ को सम्बन्ध लेख्नुहोस् । (Write the relation between cos A and $\tan\frac{A}{2}$.)
3. sin 2A र tan A को सम्बन्ध लेख्नुहोस् । (Write the relation between sin 2A and tan A.)

Group B **3 × 2 = 6**

4. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$
5. क्याल्कुलेटर वा तालिका प्रयोग नगरी sin 75° . sin 15° को मान पत्ता लगाउनुहोस् ।
Without using calculator or a table, find the value of sin 75° . sin 15°.
6. न्यूनकोण θ का लागि हल गर्नुहोस् (Solve for acute angle θ): sin θ = tan θ

Group C **4 × 4 = 16**

7. प्रमाणित गर्नुहोस् (Prove that): cosec 10° − $\sqrt{3}$sec 10° = 4 (without using calculator)
8. यदि A + B + C = π^c भए प्रमाणित गर्नुहोस् (If A + B + C = π^c, prove that)
cos 2A + cos 2B + cos 2C = − (1 + 4 cos A cos B cos C)
9. हल गर्नुहोस् (Solve): sin A = $\sqrt{3}$ (1 − cos A) (0° ≤ A ≤ 360°)
10. 20 m अग्लो भवनको छतबाट एउटा खम्बाको टुप्पो र फेदको अवनती कोणहरू क्रमशः 45° र 60° छन् । सो खम्बाको उचाइ र घर र खम्बा बीचको दूरी पत्ता लगाउनुहोस् ।
From the top of a building of 20 m high, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole and distance between the house and the pole.

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

5. $\frac{1}{4}$ **6.** 0° **9.** 0°, 60°, 360° **10.** 8.45 m, 11.55 m

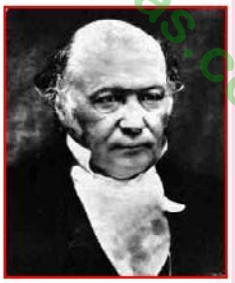
pusmas.com.np

www.drmathnp.com

puspas.com.np

Unit VI

भेक्टर
Vector



William R. Hamilton

The word vector has been derived from a Latin word vectus, which means "to carry". The germinal ideas of modern vector theory date from around 1800 when Caspar Wessel (1745-1818) and Jean Robert Argand (1768-1822) described that how a complex number a + ib could be given a geometric interpretation with the help of directed line segment in co-ordinate plane. William Rowen Hamilton (1805-1865) and Irish mathematician was the first to use the term vector of a directed line segment in his book Lectures on Quaternions (1853).

SEE SPECIFICATION GRID **1 Very Short, 2 Short & 1 Very Long Questions**

Contents	Topics	Knowledge	Understanding	Application	Higher Ability	Total no. of Questions	Total Marks
		Each of 1 Mark	Each of 2 Marks	Each of 4 Marks	Each of 5 Marks		
भेक्टर Vectors	स्केलर गुणनफल Scalar Product	1	2	-	1	4	10
	भेक्टर ज्यामिति Vector Geometry						

SYLLABUS ISSUED BY CDC

6. भेक्टर (Vector) **- 18 Periods**

- भेक्टरहरूको स्केलर गुणनफल (भेक्टरहरू लम्ब हुने अवस्था)
  Scalar (dot) product of vectors (condition of perpendicularity)
- भेक्टर ज्यामिति (Vector Geometry)
  - मध्यबिन्दु साध्य र खण्ड सूत्र साध्यहरू (Midpoint theorem and section formulae)
- साध्यहरू (Theorems)
  - त्रिभुजका दुई ओटा भुजाको मध्यबिन्दु जोड्ने रेखा तेस्रो भुजासँग समानान्तर भई आधा हुन्छ ।
    A line segment joining the mid points of any two sides of a triangle is parallel to the third side and is half of it.
  - समद्विबाहु त्रिभुजको शीर्षबिन्दु र आधारको मध्यबिन्दु जोड्ने रेखा आधारमा लम्ब हुन्छ ।
    The line joining vertex and the midpoint of base of an isosceles triangle is perpendicular to the base.
  - चतुर्भुजको भुजाहरूका मध्यबिन्दुहरू क्रमशः जोड्दै जाँदा बन्ने चतुर्भुज समानान्तर चतुर्भुज हुन्छ ।
    The lines joining the middle points of the sides of a quadrilateral taken in order is a parallelogram.
  - समानान्तर चतुर्भुजका विकर्णहरू परस्पर समद्विभाजन हुन्छन् ।
    The diagonals of a parallelogram bisect to each other.
  - आयतका विकर्णहरू बराबर हुन्छन् । (The diagonals of a rectangle are equal to each other.)
  - समबाहु चतुर्भुज (rhombus) का विकर्णहरू समकोण हुनेगरी प्रतिच्छेदन हुन्छन् ।
    The diagonals of a rhombus intersect at a right angle.
  - वृत्तार्धको कोण एक समकोण हुन्छ । (The angle at the circumference in a semi circle is a right angle.)
  - समकोणी त्रिभुजको कर्णको मध्यबिन्दु शीर्षबिन्दुबाट समदुरीमा पर्दछ ।
    The middle point of hypotenuse of a right angled triangle is equidistant from it's vertices.

www.drmathnp.com

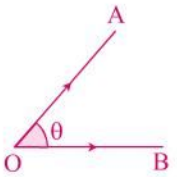
puspas.com.np

1. स्केलर गुणनफल SCALAR PRODUCT

➢ बिन्दु गुणनफल या स्केलर गुणनफल (Dot product or scalar product)

दुईओटा भेक्टरहरूको परिमाणहरू र तिनीहरूबिचको cosine कोणको गुणनफललाई स्केलर गुणनफल भनिन्छ । अर्को शब्दमा, यदि $\vec{a}$ र $\vec{b}$ दुईओटा भेक्टरहरू भए $\vec{a}$ र $\vec{b}$ को बिन्दु गुणनफललाई $\vec{a}.\vec{b}$ ले जनाइन्छ र $\vec{a}\,.\,\vec{b} = |\vec{a}|\,.\,|\vec{b}|\cos\theta$ द्वारा परिभाषित गरिन्छ ।

The product of magnitude of two vectors with cosine angle between them is called the scalar product. In other words, if $\vec{a}$ and $\vec{b}$ are two vectors then the scalar product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\,.\,\vec{b}$ and defined by $\vec{a}\,.\,\vec{b} = |\vec{a}|\,.\,|\vec{b}|\cos\theta$.



FORMULAE AND KEY POINTS

1. $\vec{a}$ र $\vec{b}$ कुनै दुई भेक्टरहरूका लागि (For any two vector $\vec{a}$ and $\vec{b}$):

   (i) $(\vec{a}+\vec{b})^2 = a^2 + b^2 + 2\vec{a}.\vec{b}$ (ii) $(\vec{a}-\vec{b})^2 = a^2 + b^2 - 2\vec{a}.\vec{b}$

   (iii) $(\vec{a}+\vec{b})\,.\,(\vec{a}-\vec{b}) = a^2 - b^2$ (iv) $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

   (v) $(\vec{a})^2 = a^2 = |\vec{a}|.|\vec{a}| = \vec{a}.\vec{a}$

2. यदि $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ भए (If $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ then), $\vec{a}\,.\,\vec{b} = (a_1.a_2 + b_1\,.\,b_2)$

or यदि $\vec{a} = (a_1, a_2)$ र $\vec{b} = (b_1, b_2)$ भए $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$ हुन्छ भनी सिद्ध गर्नुहोस् ।

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$, prove that $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$. [SLC 2067 R]

Proof

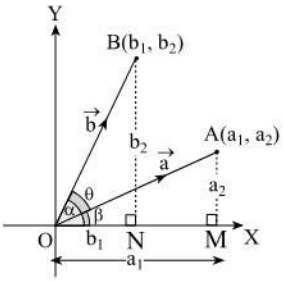
Here, let, $\vec{i}$ and $\vec{j}$ be the standard unit vectors along X-axis and Y-axis respectively.

So, $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ can be written in the form of : $\vec{a} = a_1\vec{i} + a_2\vec{j}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$

$$\text{Now, } \vec{a}\,.\,\vec{b} = (a_1\vec{i} + a_2\vec{j})\,.\,(b_1\vec{i} + b_2\vec{j}) = a_1b_1\vec{i}\,.\vec{i} + a_1b_2\vec{i}\,.\vec{j} + a_2b_1\vec{j}\,.\vec{i} + a_2b_2\vec{j}\,.\vec{j}$$
$$= a_1b_1(1) + a_1b_2(0) + a_2b_1(0) + a_2b_2(1) = a_1b_1 + 0 + 0 + a_2b_2$$
$$= a_1b_1 + a_2b_2$$

Thus $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2$

3. $\vec{a}$ र $\vec{b}$ बिचको कोण θ भए (If θ be angle between $\vec{a}$ and $\vec{b}$) : $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|}$



Proof

Let O be the origin and A(a_1, a_2) and B(b_1, b_2) be two points.

Then, $\overrightarrow{OA} = \vec{a} = (a_1, a_2)$ and $\overrightarrow{OB} = \vec{b} = (b_1, b_2)$

So, $OA = a = |\vec{a}|$ and $OB = b = |\vec{b}|$

If $\angle XOA = \beta$ & $\angle XOB = \alpha$ then $\angle AOB = \alpha - \beta = \theta$ (suppose)

From A and B, draw AM and BN perpendicular to OX.

From the right angled triangle OMA,

$$\cos\beta = \frac{OM}{OA} = \frac{a_1}{a} \qquad \therefore\ a_1 = a\cos\beta$$

$$\sin\beta = \frac{MA}{OA} = \frac{a_2}{a} \qquad \therefore\ a_2 = a\sin\beta$$

Similarly, $b_1 = b\cos\alpha$ and $b_2 = b\sin\alpha$

$$\text{Now, } a_1b_1 + a_2b_2 = a\cos\beta\, b\cos\alpha + a\sin\beta\, b\sin\alpha = ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$
$$= ab\{\cos(\alpha - \beta)\}$$

$$\therefore\quad a_1b_1 + a_2b_2 = ab\cos\theta$$

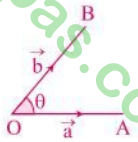
$$\text{or, } \cos\theta = \frac{a_1b_1 + a_2b_2}{ab} \qquad \therefore\quad \cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Proved.

VECTOR

www.drmathnp.com

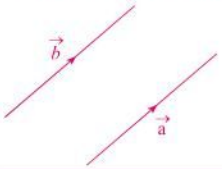
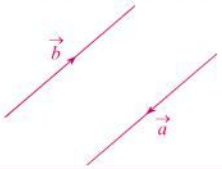
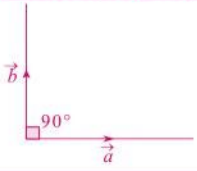
4. $\vec{a}$ र $\vec{b}$ को स्केलर गुणनफल : $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$.
Dot product of $\vec{a}$ and $\vec{b}$: $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$.



5. यदि $\vec{i}$ र $\vec{j}$ क्रमशः X-अक्ष र Y-अक्ष तिरका एकाइ भेक्टरहरू भए :
If $\vec{i}$ and $\vec{j}$ are unit vectors along X-axis and Y-axis :
$\vec{i}.\vec{i} = \vec{j}.\vec{j} = 1$ and $\vec{i}.\vec{j} = \vec{j}.\vec{i} = 0$

6. बिन्दु गुणनफल या स्केलर गुणनफलका विभिन्न अवस्थाहरू (Different cases of dot product or scalar product):

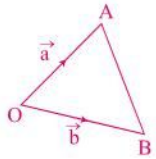
7. यदि $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ भए (If $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ then),

पहिलो अवस्था (Case I)	दोस्रो अवस्था (Case II)	तेस्रो अवस्था (Case III)
		
समान र समानान्तर भेक्टरहरू Like and parallel vectors	असमान र समानान्तर भेक्टरहरू Unlike and parallel vectors	लम्ब भेक्टरहरू Perpendicular vectors
Angle between them is 0°. So, $\theta = 0°$	Angle between them is 180°. So, $\theta = 180°$	Angle between them is 90°. So, $\theta = 90°$
By definition $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos 0°$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert.1$ $\therefore \vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert$	By definition $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos 180°$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert(-1)$ $\therefore \vec{a}.\vec{b} = -\lvert\vec{a}\rvert\lvert\vec{b}\rvert$	By definition $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos 90°$ or, $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert 0$ $\therefore \vec{a}.\vec{b} = 0$

(a) लम्ब हुने अवस्था (Condition of perpendicularity): $\vec{a}.\vec{b} = 0$

(b) समानान्तर हुने अवस्था (Condition for being parallel): $\vec{a} = m\vec{b}$; where, m is a scalar.

8. $\overrightarrow{AB}$ = बिन्दु B को स्थिति भेक्टर – बिन्दु A को स्थिति भेक्टर
अथवा, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $\quad\therefore\quad \overrightarrow{AB} = \vec{b} - \vec{a}$



$\overrightarrow{AB}$ = Position vector of point B – Position vector of point A
i.e. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $\quad\therefore\quad \overrightarrow{AB} = \vec{b} - \vec{a}$

SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, 2066 S, 2067 R, 2068 R, 2069 S, 2070 R', 2073 R', SEE 2074 R, SEE 2074 S	9
Model 2	2066 R, 2068 R', 2068 S, 2070 S, 2070 S', 2071 R, 2071 S, 2072 R, 2073 R, 2073 S', SEE MODEL 2076	12
Model 3	2072 R', SEE 2075 R', SEE 2075 R'_2,	2
Model 4	2065 R', 2069 R', SEE 2074 S'	3
Model 5	2065 M, 2065 S, 2066 R', 2067 R', 2067 S, 2068 S', 2069 R, 2071 R', 2072 S, SEE 2074 R'	9
Model 6	2070 R	1

puspas.com.np

www.drmathnp.com

puspas.com.np

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions and formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
A. स्केलर गुणनफल (Scalar product) **B.** समानान्तर भेक्टरहरू (Parallel vectors)
C. लम्ब भेक्टरहरू (Perpendicular vectors)

2A. दुई भेक्टरहरूको स्केलर गुणनफलमा समानान्तर हुने अवस्था लेख्नुहोस् ।
State the condition of parallelism in scalar product of two vectors.

2B. दुई भेक्टरहरूको स्केलर गुणनफलमा लम्ब हुने अवस्था लेख्नुहोस् ।
State the condition of perpendicularity in scalar product of two vectors.

3A. यदि $\vec{a}$ र $\vec{b}$ बिचको कोण θ भए $\vec{a}$ र $\vec{b}$ को स्केलर गुणनफल के हुन्छ ?
What is the scalar product of two vectors $\vec{a}$ and $\vec{b}$ if the angle between them is θ ? [SEE MODEL 2076]

3B. यदि $\vec{a}\cdot\vec{b}=0$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुन्छ ? (If $\vec{a}\cdot\vec{b}=0$, what is the angle between $\vec{a}$ and $\vec{b}$?)

3C. यदि $\vec{a}=k\vec{b}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुन्छ ? जहाँ k एउटा स्केलर परिमाण हो ।
If $\vec{a}=k\vec{b}$, what is the angle between $\vec{a}$ and $\vec{b}$ where k is a scalar quantity?

3D. $\vec{a}\cdot\vec{b}$ को अधिकतम मान प्राप्त गर्न $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुनुपर्छ ?
What should be the angle between $\vec{a}$ and $\vec{b}$ to get the maximum value of $\vec{a}\cdot\vec{b}$?

3E. $\vec{a}\cdot\vec{b}$ को न्यूनतम मान प्राप्त गर्न $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुनुपर्छ ?
What should be the angle between $\vec{a}$ and $\vec{b}$ to get the minimum value of $\vec{a}\cdot\vec{b}$?

4A. स्केलर गुणनफल $(\vec{a}\cdot\vec{b})$ पत्ता लगाउने सूत्र लेख्नुहोस् । जहाँ $\vec{a}=\begin{pmatrix}a_1\\b_1\end{pmatrix}$ र $\vec{b}=\begin{pmatrix}a_2\\b_2\end{pmatrix}$ छन् ।
Write down the formula to calculate scalar product $(\vec{a}\cdot\vec{b})$, where $\vec{a}=\begin{pmatrix}a_1\\b_1\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}a_2\\b_2\end{pmatrix}$.

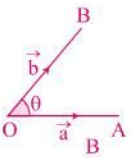
4B. यदि $|\vec{a}|=m, |\vec{b}|=n, \vec{a}\cdot\vec{b}=p$ र $\vec{a}$ र $\vec{b}$ बीचको कोण $60°$ भए m, n र p को सम्बन्ध लेख्नुहोस् ।
If $|\vec{a}|=m, |\vec{b}|=n, \vec{a}\cdot\vec{b}=p$ and the angle between $\vec{a}$ and $\vec{b}$ is $60°$ then, write the relation among m, n and p.

5A. यदि x - अक्षतिरको एकाइ भेक्टर $\vec{i}$ भए $\vec{i}^{\,2}$ को मान कति हुन्छ ?
If $\vec{i}$ is unit vector along X-axis, what is the value of $\vec{i}^{\,2}$?

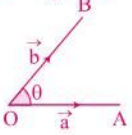
5B. यदि y - अक्षतिरको एकाइ भेक्टर $\vec{j}$ भए $\vec{j}\cdot\vec{j}$ को मान कति हुन्छ ?
If $\vec{j}$ is unit vector along Y-axis, what is the value of $\vec{j}\cdot\vec{j}$?

5C. $\vec{i}\cdot\vec{j}$ को मान कति हुन्छ ? (What is the value of $\vec{i}\cdot\vec{j}$?)

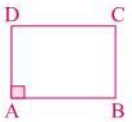
6A. चित्रमा दिइएको जानकारीबाट $\vec{a}.\vec{b}$ पत्ता लगाउने सूत्र लेख्नुहोस् ।
From the information given in the figure, write the formula to calculate $\vec{a}.\vec{b}$.



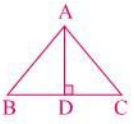
6B. चित्रमा दिइएको जानकारीबाट $\cos\theta$ पत्ता लगाउने सूत्र लेख्नुहोस् ।
From the information given in the figure, write the formula to calculate $\cos\theta$.



7A. चित्रमा ABCD एउटा आयत हो । $\overrightarrow{AB}.\overrightarrow{BC}$ को मान कति होला ?
In the figure ABCD is a rectangle. What is the value of $\overrightarrow{AB}.\overrightarrow{BC}$?



7B. चित्रमा AD ⊥ BC छ भने $\overrightarrow{AD}.\overrightarrow{BC}$ कति होला ?
In the figure AD ⊥ BC then what is the value of $\overrightarrow{AD}.\overrightarrow{BC}$?



www.drmathnp.com

puspas.com.np

Model 2 *Concept Recall*

8. यदि $\vec{i}=\begin{pmatrix}1\\0\end{pmatrix}$ र $\vec{j}=\begin{pmatrix}0\\1\end{pmatrix}$ भए पत्ता लगाउनुहोस् (If $\vec{i}=\begin{pmatrix}1\\0\end{pmatrix}$ and $\vec{j}=\begin{pmatrix}0\\1\end{pmatrix}$):

A. $\vec{i}\,.\,\vec{i}$ **B.** $\vec{j}\,.\,\vec{j}$ **C.** $\vec{i}\,.\vec{j}$ **D.** $\vec{j}\,.\,\vec{i}$

9A. यदि $\vec{a}=\begin{pmatrix}2\\0\end{pmatrix}$ र $\vec{b}=\begin{pmatrix}3\\0\end{pmatrix}$ भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् । (If $\vec{a}=\begin{pmatrix}2\\0\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}3\\0\end{pmatrix}$ then find $\vec{a}.\vec{b}$.)

9B. यदि $\vec{p}=\begin{pmatrix}0\\3\end{pmatrix}$ र $\vec{q}=\begin{pmatrix}0\\4\end{pmatrix}$ भए $\vec{p}.\vec{q}$ पत्ता लगाउनुहोस् । (If $\vec{p}=\begin{pmatrix}0\\3\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}0\\4\end{pmatrix}$ then find $\vec{p}.\vec{q}$.)

10A. यदि $|\vec{a}|=4$ भए $|\vec{a}|^2$ पत्ता लगाउनुहोस् । (If $|\vec{a}|=4$ then find $|\vec{a}|^2$.)

10B. यदि AB = 6 cm भए $(\vec{AB})^2$ पत्ता लगाउनुहोस् । (If AB = 6 cm then find $(\vec{AB})^2$.)

11A. यदि $\vec{a}=4\vec{i}$ भए a^2 पत्ता लगाउनुहोस् । (If $\vec{a}=4\vec{i}$ then find a^2.)

11B. यदि $\vec{a}=\vec{i}+\vec{j}$ भए a^2 पत्ता लगाउनुहोस् । (If $\vec{a}=\vec{i}+\vec{j}$ then find a^2.)

12A. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू हुन् र $\vec{a}.\vec{b}=\frac{1}{2}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}.\vec{b}=\frac{1}{2}$ then find the angle between $\vec{a}$ and $\vec{b}$.

12B. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू हुन् र $\vec{a}.\vec{b}=\frac{\sqrt{3}}{2}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}.\vec{b}=\frac{\sqrt{3}}{2}$ then find the angle between $\vec{a}$ and $\vec{b}$.

12C. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू हुन् र $\sqrt{2}\,\vec{a}.\vec{b}=1$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a}$ and $\vec{b}$ are unit vectors and $\sqrt{2}\,\vec{a}.\vec{b}=1$ then find the angle between $\vec{a}$ and $\vec{b}$.

B. SHORT QUESTIONS

Model 1 *Problems on scalar products.*

1A. यदि $\vec{i}$ र $\vec{j}$, x-अक्ष र y-अक्षतिरका एकाइ भेक्टरहरू भए $\vec{i}\,.\,\vec{i}=1$ र $\vec{i}\,.\vec{j}=0$ हुन्छन् भनी प्रमाणित गर्नुहोस् । **[2065 R]**

If $\vec{i}$ and $\vec{j}$ are unit vectors along the x-axis and y-axis respectively, prove that $\vec{i}\,.\,\vec{i}=1$ and $\vec{i}\,.\vec{j}=0$.

1B. यदि $\vec{a}=(a_1, a_2)$ र $\vec{b}=(b_1, b_2)$ भए $\vec{a}.\vec{b}=a_1b_1+a_2b_2$ हुन्छ भनी सिद्ध गर्नुहोस् ।

If $\vec{a}=(a_1, a_2)$ and $\vec{b}=(b_1, b_2)$, prove that $\vec{a}.\vec{b}=a_1b_1+a_2b_2$. **[2067 R]**

1C. यदि $\vec{a}=\begin{pmatrix}2\\3\end{pmatrix}$ र $\vec{b}=\begin{pmatrix}3\\-1\end{pmatrix}$ भए a^2 र b^2 पत्ता लगाउनुहोस् । (If $\vec{a}=\begin{pmatrix}2\\3\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}3\\-1\end{pmatrix}$ then find a^2 and b^2.) **[2066 S]**

1D. यदि $\vec{a}=\begin{pmatrix}2\\1\end{pmatrix}$ र $\vec{b}=\begin{pmatrix}2\\-3\end{pmatrix}$ भए $\vec{a}\,.\,\vec{b}$ पत्ता लगाउनुहोस् । (If $\vec{a}=\begin{pmatrix}2\\1\end{pmatrix}$ & $\vec{b}=\begin{pmatrix}2\\-3\end{pmatrix}$ then find $\vec{a}\,.\,\vec{b}$.)

1E. यदि $\vec{a}=\begin{pmatrix}3\\4\end{pmatrix}$ र $\vec{b}=\begin{pmatrix}1\\1\end{pmatrix}$ भए $\vec{a}\,.\,\vec{b}$ पत्ता लगाउनुहोस् । (If $\vec{a}=\begin{pmatrix}3\\4\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}1\\1\end{pmatrix}$ then find $\vec{a}\,.\,\vec{b}$.)

1F. यदि $|\vec{a}|=3\sqrt{3}, |\vec{b}|=4$ र $\vec{a}$ र $\vec{b}$ बिचको कोण $(\theta)=60°$ भए $\vec{a}.\vec{b}$ को मान पत्ता लगाउनुहोस् ।

If $|\vec{a}|=3\sqrt{3}, |\vec{b}|=4$ and angle between $\vec{a}$ and $\vec{b}$ $(\theta)=60°$ then find the value of $\vec{a}.\vec{b}$. **[SEE 2073 R']**

1G. यदि $|\vec{a}|=4\sqrt{2}, |\vec{b}|=6$ र $\theta=45°$ भए $\vec{a}.\vec{b}$को मान पत्ता लगाउनुहोस् ।

If $|\vec{a}|=4\sqrt{2}, |\vec{b}|=6$ and $\theta=45°$, find the value of $\vec{a}.\vec{b}$. **[2068 R]**

1H. यदि $|\vec{a}|=5\sqrt{3}, |\vec{b}|=6$ र $\theta=30°$ भए $\vec{a}.\vec{b}$ को मान पत्ता लगाउनुहोस् ।

If $|\vec{a}|=5\sqrt{3}, |\vec{b}|=6$ and $\theta=30°$, find the value of $\vec{a}.\vec{b}$. **[2070 R']**

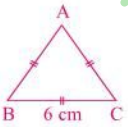
www.drmathnp.com

puspas.com.np

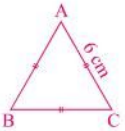
2A. यदि $\vec{p} = 2\vec{i} + 3\vec{j}$, $\vec{q} = -a\vec{i} + 4\vec{j}$ र $\vec{p}.\vec{q} = 0$ भए a को मान पत्ता लगाउनुहोस् ।
If $\vec{p} = 2\vec{i} + 3\vec{j}$, $\vec{q} = -a\vec{i} + 4\vec{j}$ and $\vec{p}.\vec{q} = 0$, find the value of a. [2069 S]

2B. यदि $\vec{a} = 10\vec{i} - 7\vec{j}$, $\vec{b} = 7\vec{i} + m\vec{j}$ र $\vec{a}.\vec{b} = 0$ भए m को मान पत्ता लगाउनुहोस् ।
If $\vec{a} = 10\vec{i} - 7\vec{j}$, $\vec{b} = 7\vec{i} + m\vec{j}$ and $\vec{a}.\vec{b} = 0$, find the value of m.

3A. दिइएको ΔABC मा AB = BC = CA = 6 cm छन् भने $\overrightarrow{AB}.\overrightarrow{AC}$ को मान निकाल्नुहोस् ।
In the given ΔABC, AB = BC = CA = 6 cm. Find the value of $\overrightarrow{AB}.\overrightarrow{AC}$. [SEE 2074 R]

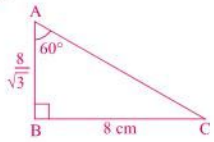


3B. दिइएको ΔABC मा AB = BC = CA = 6 से.मि. छन् भने $\overrightarrow{BA}.\overrightarrow{BC}$ को मान निकाल्नुहोस् ।
In the given ΔABC, AB = BC = CA = 6 cm. Find the value of $\overrightarrow{BA}.\overrightarrow{BC}$. [SEE 2074 S]

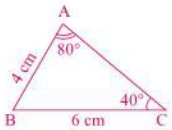


4. $\overrightarrow{AB}.\overrightarrow{BC}$ पत्ता लगाउनुहोस् । (Find $\overrightarrow{AB}.\overrightarrow{BC}$.)

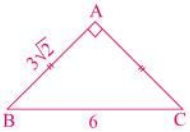
A.



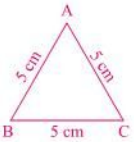
B.



C.



D.



5A. यदि $\vec{a} + \vec{b} + \vec{c} = (0, 0)$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ र $|\vec{c}| = 4$ भए $\vec{a} \cdot \vec{c}$ को मान पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = (0, 0)$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 4$ then find the value of $\vec{a} \cdot \vec{c}$.

5B. यदि $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 8$, $|\vec{b}| = 5$ र $|\vec{c}| = \sqrt{129}$ भए $\vec{a} \cdot \vec{b}$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 8$, $|\vec{b}| = 5$ and $|\vec{c}| = \sqrt{129}$ then find the value of $\vec{a} \cdot \vec{b}$.

5C. यदि $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 5\sqrt{3}$, $|\vec{b}| = 5$ र $|\vec{c}| = 10$ भए $\vec{a} \cdot \vec{c}$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 5\sqrt{3}$, $|\vec{b}| = 5$ and $|\vec{c}| = 10$ then find the value of $\vec{a} \cdot \vec{c}$.

5D. यदि $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 10\sqrt{3}$, $|\vec{b}| = 10$ र $|\vec{c}| = 20$ भए $\vec{a} \cdot \vec{b}$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 10\sqrt{3}$, $|\vec{b}| = 10$ and $|\vec{c}| = 20$ then find the value of $\vec{a} \cdot \vec{b}$.

VECTOR

Model 2 Problems based on angle between two vectors.

6A. यदि $|\vec{a}| = 2$, $|\vec{b}| = 12$ र $\vec{a}.\vec{b} = 12$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।
Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if 2, $|\vec{b}| = 12$ and $\vec{a}.\vec{b} = 12$. [SEE MODEL 2076]

6B. यदि $|\vec{a}| = 10$, $|\vec{b}| = 3$ र $\vec{a}.\vec{b} = 15\sqrt{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 10$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 15\sqrt{2}$ then find the angle between $\vec{a}$ and $\vec{b}$. [SEE 2075 R']

6C. यदि $|\vec{p}| = 4$, $|\vec{q}| = 3$ र $\vec{p}.\vec{q} = 6$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{p}| = 4$, $|\vec{q}| = 3$ and $\vec{p}.\vec{q} = 6$ then find the angle between $\vec{p}$ and $\vec{q}$. [SEE 2075 R'$_2$]

6D. यदि $|\vec{a}| = 4$, $|\vec{b}| = 5$ र $\vec{a}.\vec{b} = 10$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 4$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10$ then find the angle between $\vec{a}$ and $\vec{b}$. [SEE 2073 R]

6E. यदि $\vec{p} \cdot \vec{q} = 18\sqrt{3}$, $|\vec{p}| = 6$ र $|\vec{q}| = 6$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{p} \cdot \vec{q} = 18\sqrt{3}$, $|\vec{p}| = 6$ and $|\vec{q}| = 6$, find the angle between $\vec{p}$ and $\vec{q}$. [2071 S]

6F. यदि $|\vec{a}| = 3$, $|\vec{b}| = 5$ र $\vec{a}.\vec{b} = \frac{15}{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = \frac{15}{2}$, find the angle between $\vec{a}$ and $\vec{b}$. [2070 S']

6G. यदि $|\overrightarrow{OA}| = 6$, $|\overrightarrow{OB}| = 7$ र $\overrightarrow{OA} \cdot \overrightarrow{OB} = 21$ भए $\measuredangle AOB$ को मान पत्ता लगाउनुहोस् ।
If $|\overrightarrow{OA}| = 6$, $|\overrightarrow{OB}| = 7$ and $\overrightarrow{OA} \cdot \overrightarrow{OB} = 21$, find the value of the $\measuredangle AOB$. [2062 S]

www.drmathnp.com

puspas.com.np

6H. यदि $|\vec{a}| = 4, |\vec{b}| = 5$ र $\vec{a}.\vec{b} = 10\sqrt{2}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 4, |\vec{b}| = 5$ and $\vec{a}.\vec{b} = 10\sqrt{2}$, then find the angle between $\vec{a}$ and $\vec{b}$. [SEE 2073 S']

7A. एकाइ भेक्टर $\vec{i}$ र $\vec{a} = \sqrt{3}\vec{i} + \vec{j}$ बिचको कोण निकाल्नुहोस् ।
Find the angle between unit vector $\vec{i}$ and $\vec{a} = \sqrt{3}\vec{i} + \vec{j}$. [2070 S]

7B. यदि $\vec{a} = 10\vec{i} - 7\vec{j}$ र $\vec{b} = 7\vec{i} + 10\vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = 10\vec{i} - 7\vec{j}$ and $\vec{b} = 7\vec{i} + 10\vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. [2061 S]

7C. यदि $\vec{a} = \vec{i} + 3\vec{j}$ र $\vec{b} = 2\vec{i} + \vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बिचमा हुने कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = \vec{i} + 3\vec{j}$ and $\vec{b} = 2\vec{i} + \vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. [2060 R]

7D. यदि $\vec{a} = 3\vec{i} + 4\vec{j}$ र $\vec{b} = 8\vec{i} - 6\vec{j}$ भए $\vec{a}$ र $\vec{b}$ को बिचमा हुने कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 8\vec{i} - 6\vec{j}$, find the angle between the vectors $\vec{a}$ and $\vec{b}$. [2058 S]

7E. यदि $\vec{a} = 2\vec{i} - \vec{j}$ र $\vec{b} = \vec{i} - 3\vec{j}$ भए भेक्टरहरू बिचको कोण पत्ता लगाउनुहोस् ।
Find the angle between the vectors $\vec{a} = 2\vec{i} - \vec{j}$ and $\vec{b} = \vec{i} - 3\vec{j}$. [2068 R']

7F. यदि $\vec{a} = 4\vec{i} + 2\vec{j}$ र $\vec{b} = -\vec{i} + 2\vec{j}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$, then find the angle between $\vec{a}$ and $\vec{b}$. [2072 R]

8A. यदि $\overrightarrow{OA} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$ भए कोण AOB को मान निकाल्नुहोस् ।
If $\overrightarrow{OA} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$, calculate the angle AOB. [2061 R]

8B. यदि $\overrightarrow{OA} = 7\vec{i} - 5\vec{j}$ र $\overrightarrow{OB} = 5\vec{i} - 7\vec{j}$ भए $\angle AOB$ को मान पत्ता लगाउनुहोस् ।
If $\overrightarrow{OA} = 7\vec{i} - 5\vec{j}$ and $\overrightarrow{OB} = 5\vec{i} - 7\vec{j}$, find the value of $\angle AOB$. [2057 S]

8C. यदि $\overrightarrow{OA} = 4\vec{i} + 2\vec{j}$ र $\overrightarrow{OB} = -\vec{i} + 2\vec{j}$ भए $\angle AOB$ पत्ता लगाउनुहोस् ।
If $\overrightarrow{OA} = 4\vec{i} + 2\vec{j}$ and $\overrightarrow{OB} = -\vec{i} + 2\vec{j}$, find $\angle AOB$. [2066 R]

8D. यदि $\overrightarrow{OA}$ र $\overrightarrow{OB}$ दुईओटा भेक्टरहरू हुन् र A को स्थिति भेक्टर (1, 2) र B को स्थिति भेक्टर (3, 4) छन् भने यी दुई भेक्टरहरू बिचको कोण निकाल्नुहोस् ।
There are two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ such that the position vector of A is (1, 2) and that of B is (3, 4). Find the angle between two given vectors. [2068 S]

9A. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू छन् र $|\vec{a} + \vec{b}| = 2$ भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् ।
If $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{a} + \vec{b}| = 2$ then find $\vec{a}.\vec{b}$.

9B. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू छन् र $|\vec{a} - \vec{b}| = \sqrt{2}$ भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् ।
If $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{a} - \vec{b}| = \sqrt{2}$ then find $\vec{a}.\vec{b}$.

9C. यदि $|\vec{a}| = 3, |\vec{b}| = 4$ र $|\vec{a} + \vec{b}| = 5$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$ then find the angle between $\vec{a}$ and $\vec{b}$.

9D. यदि $|\vec{a}| = 9, |\vec{b}| = 12$ र $|\vec{a} + \vec{b}| = 15$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 9, |\vec{b}| = 12$ and $|\vec{a} + \vec{b}| = 15$ then find the angle between $\vec{a}$ and $\vec{b}$.

9E. यदि $|\vec{a}| = 4, |\vec{b}| = 4\sqrt{2}$ र $|\vec{a} - \vec{b}| = 4$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 4, |\vec{b}| = 4\sqrt{2}$ and $|\vec{a} - \vec{b}| = 4$ then find the angle between $\vec{a}$ and $\vec{b}$.

9F. यदि $|\vec{a}| = 5, |\vec{b}| = 5\sqrt{2}$ र $|\vec{a} - \vec{b}| = 5$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 5, |\vec{b}| = 5\sqrt{2}$ and $|\vec{a} - \vec{b}| = 5$ then find the angle between $\vec{a}$ and $\vec{b}$.

www.drmathnp.com

puspas.com.np

9G. यदि $|\vec{a}| = 6, |\vec{b}| = 4\sqrt{3}$ र $|\vec{a} - \vec{b}| = 2\sqrt{3}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 6, |\vec{b}| = 4\sqrt{3}$ and $|\vec{a} - \vec{b}| = 2\sqrt{3}$ then find the angle between $\vec{a}$ and $\vec{b}$.

9H. यदि $|\vec{a}| = 5, |\vec{b}| = 10$ र $|\vec{a} + \vec{b}| = 5\sqrt{3}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 5, |\vec{b}| = 10$ and $|\vec{a} + \vec{b}| = 5\sqrt{3}$ then find the angle between $\vec{a}$ and $\vec{b}$.

9I. यदि $|\vec{a}| = 10, |\vec{b}| = 20$ र $|\vec{a} - \vec{b}| = 10\sqrt{3}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 10, |\vec{b}| = 20$ and $|\vec{a} - \vec{b}| = 10\sqrt{3}$ then find the angle between $\vec{a}$ and $\vec{b}$.

10A. यदि $\vec{p} + \vec{q} + \vec{r} = 0, |\vec{p}| = 6, |\vec{q}| = 7$ र $|\vec{r}| = \sqrt{127}$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{p} + \vec{q} + \vec{r} = 0, |\vec{p}| = 6, |\vec{q}| = 7$ and $|\vec{r}| = \sqrt{127}$, find the angle between $\vec{p}$ and $\vec{q}$. **[2071 R]**

10B. यदि $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 6, |\vec{b}| = 10$ र $|\vec{c}| = 14$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 6, |\vec{b}| = 10$ and $|\vec{c}| = 14$, find the angle between $\vec{a}$ and $\vec{b}$.

Model 3 *Problems based on finding magnitude when angle is given.*

11A. यदि $\overrightarrow{OA} \cdot \overrightarrow{OB} = 12, \angle AOB = 60°$ र $\overrightarrow{OA} = 4$ भए $|\overrightarrow{OB}|$ पत्ता लगाउनुहोस् ।
If $\overrightarrow{OA} \cdot \overrightarrow{OB} = 12, \angle AOB = 60°$ and $\overrightarrow{OA} = 4$ find $|\overrightarrow{OB}|$.

11B. यदि $|\overrightarrow{OP}| = 6, \overrightarrow{OP} \cdot \overrightarrow{OQ} = 24$ र $\angle POQ = 60°$ भए $|\overrightarrow{OQ}|$ को मान पत्ता लगाउनुहोस् ।
If $|\overrightarrow{OP}| = 6, \overrightarrow{OP} \cdot \overrightarrow{OQ} = 24$ and $\angle POQ = 60°$ then find the value of $|\overrightarrow{OQ}|$.

11C. यदि $\overrightarrow{OA} \cdot \overrightarrow{OB} = 30, \angle AOB = 45°$ र $|\overrightarrow{OB}| = 4\sqrt{2}$ भए $|\overrightarrow{OA}|$ को मान पत्ता लगाउनुहोस् ।
If $\overrightarrow{OA} \cdot \overrightarrow{OB} = 30, \angle AOB = 45°$ and $|\overrightarrow{OB}| = 4\sqrt{2}$ then find the value of $|\overrightarrow{OA}|$.

11D. यदि $\overrightarrow{OC} \cdot \overrightarrow{OD} = -14\sqrt{3}, \angle OCD = 150°$ र $|\overrightarrow{OC}| = 4$ भए $|\overrightarrow{OD}|$ को मान पत्ता लगाउनुहोस् ।
If $\overrightarrow{OC} \cdot \overrightarrow{OD} = -14\sqrt{3}, \angle OCD = 150°$ and $|\overrightarrow{OC}| = 4$ then find the value of $|\overrightarrow{OD}|$.

VECTOR

11E. यदि $|\vec{b}| = 6, \vec{a} \cdot \vec{b} = 12$ र $\vec{a}$ र $\vec{b}$ बिचको कोण $(\theta) = 60°$ भए $|\vec{a}|$ को मान पत्ता लगाउनुहोस् ।
If $|\vec{b}| = 6, \vec{a} \cdot \vec{b} = 12$ and angle between $\vec{a}$ and $\vec{b}$ $(\theta) = 60°$, find the value of $|\vec{a}|$. **[2072 R']**

11F. यदि $\overrightarrow{AB} \cdot \overrightarrow{BC} = 7, \angle ABC = 30°$ र $|\overrightarrow{AB}| = \sqrt{13}$ एकाइ भए $|\overrightarrow{BC}|$ पत्ता लगाउनुहोस् ।
If $\overrightarrow{AB} \cdot \overrightarrow{BC} = 7, \angle ABC = 30°$ and $|\overrightarrow{AB}| = \sqrt{13}$ units, find $|\overrightarrow{BC}|$.

11G. यदि $\vec{a} \cdot \vec{b} = 5, |\vec{a}| = \sqrt{5}$ तथा $\vec{a}$ र $\vec{b}$ बिचको कोण 45° भए $\vec{b}$ को लम्बाइ पत्ता लगाउनुहोस् ।
If $\vec{a} \cdot \vec{b} = 5, |\vec{a}| = \sqrt{5}$ and the angle between $\vec{a}$ and $\vec{b}$ is 45°, find the length of $\vec{b}$.

11H. यदि $\vec{p} \cdot \vec{q} = 6, |\vec{q}| = 2\sqrt{3}$ एकाइ र $\vec{p}$ र $\vec{q}$ बिचको कोण 30° भए $\vec{p}$ को लम्बाइ पत्ता लगाउनुहोस् ।
If $\vec{p} \cdot \vec{q} = 6, |\vec{q}| = 2\sqrt{3}$ unit and the angle between $\vec{p}$ and $\vec{q}$ is 30°, find the length of $\vec{p}$. **[SEE 2075 R, 2075 R$_2$]**

12A. यदि $\vec{p} + \vec{q} + \vec{r} = (0, 0), |\vec{p}| = 6, |\vec{q}| = 10$ र $\vec{p} \cdot \vec{q} = 30$ भए $|\vec{r}|$ पत्ता लगाउनुहोस् ।
If $\vec{p} + \vec{q} + \vec{r} = (0, 0), |\vec{p}| = 6, |\vec{q}| = 10$ and $\vec{p} \cdot \vec{q} = 30$ then find the value of $|\vec{r}|$.

12B. यदि $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 10, |\vec{b}| = 5$ र $\vec{a} \cdot \vec{b} = 50$ भए $|\vec{c}|$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 10, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 50$ then find $|\vec{c}|$.

12C. यदि $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 2\sqrt{2}, |\vec{b}| = 2\sqrt{2}$ र $\vec{a} \cdot \vec{b} = 0$ भए $|\vec{c}|$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 2\sqrt{2}, |\vec{b}| = 2\sqrt{2}$ and $\vec{a} \cdot \vec{b} = 0$ then find the value of $|\vec{c}|$.

12D. यदि $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 2, |\vec{b}| = 3$ र $\vec{a} \cdot \vec{b} = 6$ भए $|\vec{c}|$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 6$ then find $|\vec{c}|$.

12E. यदि $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{b}| = 5, |\vec{c}| = 6$ र $\vec{b} \cdot \vec{c} = -18$ भए $|\vec{a}|$ पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{b}| = 5, |\vec{c}| = 6$ and $\vec{b} \cdot \vec{c} = -18$ then find $|\vec{a}|$.

www.drmathnp.com

puspas.com.np

Model 4 *Problems based on proving perpendicular vectors.*

13A. यदि $\vec{a} = 6\vec{i} - 8\vec{j}$ र $\vec{b} = 4\vec{i} + 3\vec{j}$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ आपसमा लम्ब छन् भनी प्रमाणित गर्नुहोस् ।

If $\vec{a} = 6\vec{i} - 8\vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$, prove that the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

[SEE 2074 S', 2060S]

13B. $\vec{a} = 2\vec{i} - 3\vec{j}$ र $\vec{b} = 9\vec{i} + 6\vec{j}$ भेक्टरहरू आपसमा लम्ब छन् भनी प्रमाणित गर्नुहोस् ।

Prove that the vectors $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = 9\vec{i} + 6\vec{j}$ are perpendicular to each other. [2069 R']

14A. यदि $(\vec{x} + \vec{y})^2 = (\vec{x} - \vec{y})^2$ भए $\vec{x}$ र $\vec{y}$ आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

If $(\vec{x} + \vec{y})^2 = (\vec{x} - \vec{y})^2$, prove that $\vec{x}$ and $\vec{y}$ are perpendicular to each other. [2065 R']

14B. यदि $\vec{a}$ र $\vec{b}$ आपसमा लम्ब भए $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then prove that: $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$

14C. यदि $\vec{p}$ र $\vec{q}$ आपसमा लम्ब भए $(\vec{p} + 2\vec{q})^2 = (\vec{p} - 2\vec{q})^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{p}$ and $\vec{q}$ are perpendicular to each other then prove that: $(\vec{p} + 2\vec{q})^2 = (\vec{p} - 2\vec{q})^2$

14D. कुनै भेक्टरहरू $\vec{a}$ र $\vec{b}$ का लागि $(\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = 0$ भए $|\vec{a}| = |\vec{b}|$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

For any two vector $\vec{a}$ and $\vec{b}$, if $(\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = 0$ then prove that $|\vec{a}| = |\vec{b}|$.

Model 5 *Problems based on finding the values of unknown letters from perpendicular vectors.*

15A. यदि $\vec{a} = 3\vec{i} + m\vec{j}$ र $\vec{b} = 6\vec{i} - 2\vec{j}$ एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।

If $\vec{a} = 3\vec{i} + m\vec{j}$ and $\vec{b} = 6\vec{i} - 2\vec{j}$ are perpendicular to each other then find the value of m. [SEE 2074 R']

15B. यदि $10\vec{i} - 7\vec{j}$ र $a\vec{i} + 10\vec{j}$ आपसमा लम्ब भए a को मान पत्ता लगाउनुहोस् ।

If $10\vec{i} - 7\vec{j}$ and $a\vec{i} + 10\vec{j}$ are perpendicular to each other, find the value of a. [2072 S]

15C. यदि $\vec{a} = -5\vec{i} + 3\vec{j}$ र $\vec{b} = p\vec{i} + (p+2)\vec{j}$ एक आपसमा लम्ब भए p को मान पत्ता लगाउनुहोस् । [2071 R']

If $\vec{a} = -5\vec{i} + 3\vec{j}$ and $\vec{b} = p\vec{i} + (p+2)\vec{j}$ are perpendicular to each other, find the value of p.

15D. यदि $\vec{a} = 3\vec{i} - m\vec{j}$ र $\vec{b} = 10\vec{i} + 6\vec{j}$ एक आपसमा लम्ब भए m को मान कति हुन्छ ? [2069 R, 2068S']

If $\vec{a} = 3\vec{i} - m\vec{j}$ and $\vec{b} = 10\vec{i} + 6\vec{j}$ are perpendicular to each other, what is the value of m ?

15E. x को मान कति हुँदा भेक्टरहरू $3\vec{i} - 2\vec{j}$ र $x\vec{i} + 3\vec{j}$ एक आपसमा लम्ब हुन्छन् ?

For what value of x, are the two vectors $3\vec{i} - 2\vec{j}$ and $x\vec{i} + 3\vec{j}$ perpendicular ? [2065 M]

15F. x को मान कति हुँदा भेक्टरहरू $2\vec{i} - 3\vec{j}$ र $x\vec{i} - 2\vec{j}$ एक आपसमा लम्ब हुन्छन् ?

For what value of x, vectors $2\vec{i} - 3\vec{j}$ and $x\vec{i} - 2\vec{j}$ are perpendicular to each other ? [2066 R']

15G. यदि $\vec{x} = 3\vec{i} + m\vec{j}$ र $\vec{y} = 4\vec{i} - 2\vec{j}$ एक आपसमा लम्ब भए m को मान कति हुन्छ ?

If $\vec{x} = 3\vec{i} + m\vec{j}$ and $\vec{y} = 4\vec{i} - 2\vec{j}$ are perpendicular, what is the value of m ? [2065 S]

15H. यदि $\vec{a} = 3\vec{i} + 4\vec{j}$ र $\vec{b} = 4\vec{i} + p\vec{j}$ परस्पर लम्ब भए p को मान निकाल्नुहोस् ।

If $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = 4\vec{i} + p\vec{j}$ are perpendicular to each other, find the value of p. [2067 S]

15I. यदि $\vec{a} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} p \\ p+2 \end{pmatrix}$ एक आपसमा लम्ब भए p को मान निकाल्नुहोस् ।

If $\vec{a} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ & $\vec{b} = \begin{pmatrix} p \\ p+2 \end{pmatrix}$ are perpendicular, find the value of p. [2059 R, 2067R']

www.drmathnp.com

puspas.com.np

Model 6 *Problems based on the condition of perpendicularity of given two vectors.*

16A. यदि $\vec{a}+2\vec{b}$ र $5\vec{a}-4\vec{b}$ आपसमा लम्ब तथा $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।

If $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other and $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$. [2070 R]

16B. $(2\vec{a}+4\vec{b})$ र $(10\vec{a}+8\vec{b})$ भेक्टरहरू एक आपसमा लम्ब हुनेगरी $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू हुन् भने $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।

Vectors $(2\vec{a}+4\vec{b})$ and $(10\vec{a}+8\vec{b})$ are orthogonal to each other where $\vec{a}$ and $\vec{b}$ are unit vectors. Find the angle between $\vec{a}$ and $\vec{b}$.

16C. यदि $\vec{m}=2\vec{a}-3\vec{b}, \vec{n}=3\vec{a}-2\vec{b}$ र $\vec{m}.\vec{n}=12$ भए एकाइ भेक्टरहरू $\vec{a}$ र $\vec{b}$ एकआपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।

If $\vec{m}=2\vec{a}-3\vec{b}$, $\vec{n}=3\vec{a}-2\vec{b}$ and $\vec{m}.\vec{n}=12$, prove that the unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

ANSWERS

A. VERY SHORT QUESTIONS

3A. $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$ **3B.** 90° **3C.** 0° or 180° **3D.** 0° **3E.** 180° **4A.** $a_1a_2+b_1b_2$
4B. $2p=mn$ **5A.** 1 **5B.** 1 **5C.** 0 **8A.** 1 **8B.** 1 **8C.** 0 **8D.** 0
9A. 6 **9B.** 12 **10A.** 16 **10B.** 36 cm^2 **11A.** 16 **11B.** 2 **12A.** 60° **12B.** 30° **12C.** 45°

B. SHORT QUESTIONS

1C. 13, 10 **1D.** 1 **1E.** 7 **1F.** $6\sqrt{3}$ **1G.** 24 **1H.** 45 **2A.** 6 **2B.** 10
3A. 18 **3B.** 18 **4A.** 0 **4B.** 12 **4C.** 18 **4D.** 12.5 **5A.** 0 **5B.** 20
5C. -75 **5D.** 0 **6A.** 60° **6B.** 45° **6C.** 60° **6D.** 60° **6E.** 30° **6F.** 60°
6G 60° **6H.** 45° **7A.** 30° **7B.** 90° **7C.** 45° **7D.** 90° **7E.** 45° **7F.** 90°
8A. 41.56° **8B.** 18.92° **8C.** 90° **8D.** $\cos^{-1}\left(\frac{11}{5\sqrt{5}}\right)$ **9A.** 1 **9B.** 0 **9C.** 90°
9D. 90° **9E.** 45° **9F.** 45° **9G.** 30° **9H.** 120° **9I.** 60° **10A.** 60° **10B.** 60°
11A. 6 **11B.** 8 **11C.** $\frac{15}{2}$ **11D.** 7 **11E.** 4 **11F.** 2.24 units **11G.** $\sqrt{10}$ **11H.** 2 **12A.** 14
12B. 15 **12C.** 4 **12D.** 5 **12E.** 5 **15A.** 9 **15B.** 7 **15C.** p = 3
15D. $m=5$ **15E.** 2 **15F.** -3 **15G.** 6 **15H.** -3 **15I.** 3 **16A.** 60° **16B.** $\cos^{-1}\left(\frac{-13}{14}\right)$

VECTOR

GENERAL EXERCISE 1

A. SHORT QUESTIONS

Model 1 *Problems based on position vector.*

1A. यदि $\overrightarrow{OA}=\begin{pmatrix}3\\4\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}9\\12\end{pmatrix}$ भए $\overrightarrow{AB}$ लाई $x\vec{i}+y\vec{j}$ मा व्यक्त गर्नुहोस् ।

If $\overrightarrow{OA}=\begin{pmatrix}3\\4\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}9\\12\end{pmatrix}$, express $\overrightarrow{AB}$ in the form $x\vec{i}+y\vec{j}$.

1B. यदि $\overrightarrow{OA}=\begin{pmatrix}2\\1\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}8\\9\end{pmatrix}$ भए $\overrightarrow{AB}$ लाई $x\vec{i}+y\vec{j}$ मा व्यक्त गर्नुहोस् ।

If $\overrightarrow{OA}=\begin{pmatrix}2\\1\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}8\\9\end{pmatrix}$, express $\overrightarrow{AB}$ in the form $x\vec{i}+y\vec{j}$.

Model 2 *Problems based on finding the scalar product of two vectors.*

2A. यदि $|\vec{a}|=10$, $|\vec{b}|=12$ र $\vec{a}$ र $\vec{b}$ बिचको न्यूनकोण 60° भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् ।

If $|\vec{a}|=10$, $|\vec{b}|=12$ and an acute angle between $\vec{a}$ and $\vec{b}$ is 60° then find $\vec{a}.\vec{b}$.

2B. यदि $|\vec{a}|=\sqrt{5}$, $|\vec{b}|=\sqrt{13}$, $\vec{a}$ र $\vec{b}$ बिचको न्यूनकोण $\cos^{-1}\left(\frac{1}{\sqrt{65}}\right)$ छ भए $\vec{a}.\vec{b}$ पत्ता लगाउनुहोस् ।

If $|\vec{a}|=\sqrt{5}$, $|\vec{b}|=\sqrt{13}$ and an acute angle between $\vec{a}$ and $\vec{b}$ is $\cos^{-1}\left(\frac{1}{\sqrt{65}}\right)$ then find $\vec{a}.\vec{b}$.

www.drmathnp.com

puspas.com.np

3. $\overrightarrow{AB}\,.\,\overrightarrow{AC}$ पत्ता लगाउनुहोस् (Find $\overrightarrow{AB}.\,\overrightarrow{AC}$.) :

A. A(2, 6), B(– 3, 3) and C(5, 1) **B.** A(4, 12), B(– 6, 6) and C(10, 2)

4. $\vec{b}\,.\vec{c}$ पत्ता लगाउनुहोस् (Find $\vec{b}\,.\vec{c}$.):

A. $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=5, |\vec{b}|=5$ and $\vec{a}.\vec{b}=-7$

B. $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=13, |\vec{b}|=13$ and $\vec{a}.\vec{b}=119$

5. $\vec{a}=(a_1, a_2), \vec{b}=(b_1, b_2)$ र $\vec{c}=(c_1, c_2)$ भए सिद्ध गर्नुहोस् (If $\vec{a}=(a_1, a_2), \vec{b}=(b_1, b_2)$ and $\vec{c}=(c_1, c_2)$, prove that):

A. $(\vec{a}+\vec{b})^2=a^2+b^2+2\vec{a}.\vec{b}$ **B.** $(\vec{a}-\vec{b})^2=a^2+b^2-2\vec{a}.\vec{b}$ **C.** $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=a^2-b^2$

D. $\vec{a}.\vec{b}=\vec{b}.\vec{a}$ **E.** $(\vec{a})^2=a^2=|\vec{a}|\,|\vec{a}|=\vec{a}.\vec{a}$ **F.** $(\vec{a}+\vec{b}).\vec{c}=\vec{c}.(\vec{a}+\vec{b})$

G. $(\vec{a}+\vec{b}).\vec{c}=\vec{c}.\vec{a}+\vec{c}.\vec{b}$ **H.** $(\vec{a}+\vec{b}).2\vec{c}=2\vec{c}.\vec{a}+2\vec{c}.\vec{b}$

Model 3 *Problems based on finding value of x or p.... when scalar product of two vector is given.*

6. p को मान पत्ता लगाउनुहोस् । (Find the value of p.)

A. $\vec{a}=p\vec{i}+3\vec{j}, \vec{b}=5\vec{i}-\vec{j}$ & $\vec{a}.\vec{b}=7$ **B.** $\vec{a}=2\vec{i}+p\vec{j}, \vec{b}=2\vec{i}-3\vec{j}$ & $\vec{a}.\vec{b}=1$

C. $\vec{a}=\begin{pmatrix}-2\\ p+1\end{pmatrix}, \vec{b}=\begin{pmatrix}4\\ -5\end{pmatrix}$ & $\vec{a}.\vec{b}=-23$ **D.** $\vec{p}=\begin{pmatrix}p+2\\ 1\end{pmatrix}, \vec{q}=\begin{pmatrix}p+1\\ 4\end{pmatrix}$ & $\vec{p}.\vec{q}=16$

E. $\vec{a}=\begin{pmatrix}p\\ p+2\end{pmatrix}, \vec{b}=\begin{pmatrix}5\\ p-2\end{pmatrix}$ & $\vec{a}.\vec{b}=20$ **F.** $\vec{a}=(p+1)\vec{i}+3\vec{j}, \vec{b}=(p+3)\vec{i}-\vec{j}$ & $\vec{a}.\vec{b}=5$

G. $\vec{a}=p\vec{i}-(p-3)\vec{j}, \vec{b}=(p-1)\vec{i}+(p-5)\vec{j}$ and $\vec{a}.\vec{b}=5$

7A. ΔABC मा A(2, 0), B(6, s – 2), C(10, 4) र $\overrightarrow{AB}\,.\,\overrightarrow{BC}=16$ भए s को मान कति होला ?

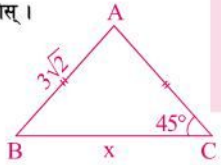
In ΔABC, A(2, 0), B(6, s – 2) ,C(10, 4) and $\overrightarrow{AB}\,.\,\overrightarrow{BC}=16$, what is the value of s?

7B. ΔXYZ मा X(2,2), Y(r + 2, 4), Z(10, 2) र $\overrightarrow{XY}\,.\,\overrightarrow{YZ}=12$ भए r को मान पत्ता लगाउनुहोस् ।

In ΔXYZ, X(2, 2), Y(r + 2, 4), Z(10, 2) and $\overrightarrow{XY}\,.\,\overrightarrow{YZ}=12$, find the value of r.

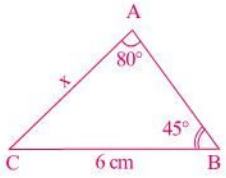
8A. चित्रमा $\overrightarrow{AB}\,.\,\overrightarrow{CB}=18$ भए x को मान पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{AB}\,.\,\overrightarrow{CB}=18$, find the value of x.



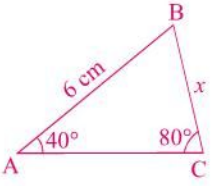
8B. चित्रमा $\overrightarrow{CB}\,.\,\overrightarrow{CA}=12$ भए x को मान पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{CB}\,.\,\overrightarrow{CA}=12$, find the value of x.



8C. चित्रमा $\overrightarrow{BA}\,.\,\overrightarrow{BC}=12$ भए x को मान पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{BA}\,.\,\overrightarrow{BC}=12$ then find the value of x.



Model 4 *Problems based on finding angle between given two vectors.*

9. $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् । (Find the angle between $\vec{a}$ and $\vec{b}$.)

A. $\vec{a}\,.\,\vec{b}=10, |\vec{a}|=5$ and $|\vec{b}|=2$ **B.** $\vec{a}\,.\,\vec{b}=5, |\vec{a}|=\sqrt{5}, |\vec{b}|=\sqrt{15}$

C. $\vec{a}$ र $\vec{b}$ दुई एकाइ भेक्टरहरू र तिनीहरूको स्केलर गुणनफल $\frac{1}{2}$ छ ।

$\vec{a}$ and $\vec{b}$ are two unit vectors and their dot product is $\frac{1}{2}$.

D. $\vec{a}$ र $\vec{b}$ दुई एकाइ भेक्टरहरू र तिनीहरूको स्केलर गुणनफल $\frac{\sqrt{3}}{2}$ छ ।

$\vec{a}$ and $\vec{b}$ are two unit vectors and their dot product is $\frac{\sqrt{3}}{2}$.

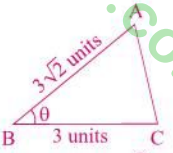
www.drmathnp.com

E. $\vec{a} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ **F.** $\vec{a} = \begin{pmatrix} 0 \\ a \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

G. $\vec{a} = m\vec{i}$ and $\vec{b} = 4\vec{i} + 4\vec{j}$ **H.** $\vec{a} = n\vec{j}$ and $\vec{b} = 5\vec{i} + 5\vec{j}$

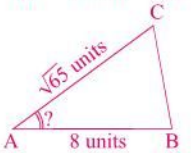
10A. चित्रमा $\overrightarrow{BA} \cdot \overrightarrow{BC} = 9$ भए $\angle B$ को नाप पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{BA} \cdot \overrightarrow{BC} = 9$ then find the measure of $\angle B$.



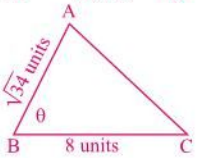
10B. चित्रमा $|\overrightarrow{AC}| = \sqrt{65}$ एकाइ, $|\overrightarrow{AB}| = 8$ एकाइ र $\overrightarrow{AC} \cdot \overrightarrow{AB} = 4\sqrt{195}$ भए $\angle A$ को नाप पत्ता लगाउनुहोस् ।

In the figure, $|\overrightarrow{AC}| = \sqrt{65}$ units, $|\overrightarrow{AB}| = 8$ units and $\overrightarrow{AC} \cdot \overrightarrow{AB} = 4\sqrt{195}$, find the measure of $\angle A$.



10C. चित्रमा $\overrightarrow{AB} \cdot \overrightarrow{BC} = 4\sqrt{34}$, $|\overrightarrow{BA}| = \sqrt{34}$ एकाइ र $|\overrightarrow{BC}| = 8$ एकाइ भए θ को मान पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{AB} \cdot \overrightarrow{BC} = 4\sqrt{34}$, $|\overrightarrow{BA}| = \sqrt{34}$ units and $|\overrightarrow{BC}| = 8$ units. Find the value of θ.



11. $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् । (Find the angle between $\vec{a}$ and $\vec{b}$.)

A. $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$

B. $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 4, |\vec{b}| = 3$ and $|\vec{c}| = \sqrt{37}$

C. $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 6, |\vec{b}| = 7$ and $|\vec{c}| = \sqrt{106}$

D. $\vec{a}, \vec{b}, \vec{c}$ भेक्टरहरू एकाइ भेक्टरहरू हुन र $\vec{a} - \vec{b} + \vec{c} = 0$ छ ।
$\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a} - \vec{b} + \vec{c} = 0$.

12A. यदि A(2, 3), B(4, 5) र C(7, 2) भए $\angle ABC$ को नाप पत्ता लगाउनुहोस् ।
If A(2, 3), B(4, 5) and C(7, 2) then find the measure of $\angle ABC$.

12B. यदि A(3, 1), B(1, – 1) र C(3, – 1) भए $\angle BAC$ को नाप पत्ता लगाउनुहोस् ।
If A(3, 1), B(1, – 1) and C(3, – 1) then find the measure of $\angle BAC$.

12C. यदि $\overrightarrow{OA} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = -2\vec{i} + 5\vec{j}$ र $\overrightarrow{OC} = 6\vec{i} + 11\vec{j}$ भए $\angle ACB$ पत्ता लगाउनुहोस् ।

If $\overrightarrow{OA} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = -2\vec{i} + 5\vec{j}$ and $\overrightarrow{OC} = 6\vec{i} + 11\vec{j}$, find $\angle ACB$.

13A. यदि $\vec{a} = 2\vec{i} - p\vec{j}$ र $\vec{b} = \vec{i} - 3\vec{j}$ बिचको कोण 45° भए p को मान कति होला ?

If angle between $\vec{a} = 2\vec{i} - p\vec{j}$ and $\vec{b} = \vec{i} - 3\vec{j}$ is 45° then what is the value of p ?

13B. यदि $\vec{a} = 2\vec{i} + 2\vec{j}$ र $\vec{b} = 4\vec{i} + p\vec{j}$ बिचको कोण 45° भए p को मान कति होला ?

If angle between $\vec{a} = 2\vec{i} + 2\vec{j}$ and $\vec{b} = 4\vec{i} + p\vec{j}$ is 45° then what is the value of p ?

Model 5 *Problems based on the condition of perpendicularity of given two vectors.*

14A. यदि A(2, 4), B(2, 2) र C(4, 2) भए $AB \perp BC$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If A(2, 4), B(2, 2) and C(4, 2) then prove that: $AB \perp BC$

14B. यदि A(2, 5), B(0, 3) र C(2, 1) भए $AB \perp BC$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If A(2, 5), B(0, 3) and C(2, 1) then prove that: $AB \perp BC$

15A. यदि $\vec{a} = 2\vec{i} + 3\vec{j}$ र $\vec{a} \perp \vec{b}$ भए $\vec{b}$ पत्ता लगाउनुहोस् जसको X-अक्षतिरको लम्बाइ 6 एकाइ छ ।
If $\vec{a} = 2\vec{i} + 3\vec{j}$ and $\vec{a} \perp \vec{b}$, find $\vec{b}$ whose length along X-axis is 6 units.

15B. भेक्टर $\vec{a} = 2\vec{i} + 4\vec{j}$ सँग लम्ब भई Y-अक्षतिर 5 एकाइ लम्बाइ हुने भेक्टर पत्ता लगाउनुहोस् ।
Find a vector perpendicular to $\vec{a} = 2\vec{i} + 4\vec{j}$ whose length along Y-axis is 5 units.

15C. यदि $\vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ भए $\vec{a}$ सँग लम्ब भई X-अक्षतिर 6 एकाइ लम्बाइ हुने भेक्टर पत्ता लगाउनुहोस् ।

If $\vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, find a vector perpendicular to $\vec{a}$ whose length along X-axis is 6 units.

puspas.com.np

www.drmathnp.com

puspas.com.np

15D. m को मान कति हुँदा $\vec{a} = \begin{pmatrix} m \\ m+2 \end{pmatrix}$ र $\vec{b} = (2, -3m)$ एकआपसमा लम्ब हुन्छन् ?

For what value of m are the vectors $\vec{a} = \begin{pmatrix} m \\ m+2 \end{pmatrix}$ and $\vec{b} = (2, -3m)$ perpendicular to each other?

16A. यदि बिन्दुहरू A(2, 6), B(4, 4), C(2, 2) र D(6, 6) भए AB ⊥ CD हुन्छ भनी प्रमाणित गर्नुहोस् ।
If the points are A(2, 6), B(4, 4), C(2, 2) and D(6, 6) then prove that AB ⊥ CD.

16B. यदि बिन्दुहरू A(2, 6), B(4, 4), C(6, 2) र D(6, 6) भए AC ⊥ BD हुन्छ भनी प्रमाणित गर्नुहोस् ।
If the points are A(2, 6), B(4, 4), C(6, 2) and D(6, 6) then prove that: AC ⊥ BD.

16C. शीर्षबिन्दुहरू A(1, 1), B(1, 3) र C(5, 1) भएको त्रिभुज एउटा समकोण त्रिभुज हो भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
If the vertices triangle are A(1, 1), B(1, 3) and C(5, 1) then prove that it is a right angled triangle.

16D. यदि $\overrightarrow{OA} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = -2\vec{i} + 5\vec{j}$ र $\overrightarrow{OC} = 6\vec{i} + 11\vec{j}$, भए प्रमाणित गर्नुहोस्: $\overrightarrow{AB} \perp \overrightarrow{AC}$
If $\overrightarrow{OA} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{OB} = -2\vec{i} + 5\vec{j}$ and $\overrightarrow{OC} = 6\vec{i} + 11\vec{j}$, prove that: $\overrightarrow{AB} \perp \overrightarrow{AC}$

17A. यदि $(\vec{a} + 2\vec{b})^2 = (\vec{a} - 2\vec{b})^2$ भए $\vec{a}$ र $\vec{b}$ आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।
If $(\vec{a} + 2\vec{b})^2 = (\vec{a} - 2\vec{b})^2$ then prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

17B. यदि $|4\vec{a} + \vec{b}| = |4\vec{a} - \vec{b}|$ भए $\vec{a}$ र $\vec{b}$ आपसमा लम्ब हुन्छन् भनी प्रमाणित गर्नुहोस् ।
If $|4\vec{a} + \vec{b}| = |4\vec{a} - \vec{b}|$ then prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Model 6 Problems based on parallel vectors.

18A. यदि $2\vec{i} + 3\vec{j}$ र $(k+1)\vec{i} + 2k\vec{j}$ समानान्तर भेक्टरहरू भए k को मान पत्ता लगाउनुहोस् ।
If $2\vec{i} + 3\vec{j}$ and $(k+1)\vec{i} + 2k\vec{j}$ are parallel vectors, find the value of k.

18B. यदि $(k+1)\vec{i} + (k+2)\vec{j}$ र $4\vec{i} + 6\vec{j}$ समानान्तर भेक्टरहरू भए k को मान पत्ता लगाउनुहोस् ।
If $(k+1)\vec{i} + (k+2)\vec{j}$ and $4\vec{i} + 6\vec{j}$ are parallel vectors, find the value of k.

18C. यदि $k\vec{i} + (k+1)\vec{j}$ र $12\vec{i} + 15\vec{j}$ समानान्तर भेक्टरहरू भए k को मान पत्ता लगाउनुहोस् ।
If $k\vec{i} + (k+1)\vec{j}$ and $12\vec{i} + 15\vec{j}$ are parallel vectors, find the value of k.

18D. यदि $(k+2)\vec{i} + 3\vec{j}$ र $8\vec{i} + (k+4)\vec{j}$ समानान्तर भेक्टरहरू भए k को मान पत्ता लगाउनुहोस् ।
If $(k+2)\vec{i} + 3\vec{j}$ and $8\vec{i} + (k+4)\vec{j}$ are parallel vectors, find the value of k.

18E. यदि $4\vec{i} + (k+1)\vec{j}$ र $(k+4)\vec{i} + 10\vec{j}$ समानान्तर भेक्टरहरू भए k को मान पत्ता लगाउनुहोस् ।
If $4\vec{i} + (k+1)\vec{j}$ and $(k+4)\vec{i} + 10\vec{j}$ are parallel vectors, find the value of k.

Model 7 Problems based on finding the magnitude of a vector.

19A. स्केलार गुणनफलको प्रयोग गरी बिन्दुहरू A(2, 2) र B(5, 6) बिचको दूरी पत्ता लगाउनुहोस् ।
Find the distance between the points A(2, 2) and B(5, 6) by using scalar product.

19B. स्केलार गुणनफलको प्रयोग गरी बिन्दुहरू A(2, 2) र B(11, 14) बिचको दूरी पत्ता लगाउनुहोस् ।
Find the distance between the points A(2, 2) and B(11, 14) by using scalar product.

Model 8 Problems based on square of a vector.

20A. यदि $\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ भए $a^2 - b^2$ पत्ता लगाउनुहोस् । (If $\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, find $a^2 - b^2$.)

20B. यदि $\vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ भए $a^2 + b^2$ पत्ता लगाउनुहोस् । (If $\vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ find $a^2 + b^2$.)

21. यदि $\vec{a}$ र $\vec{b}$ दुई एकाइ भेक्टरहरू भए प्रमाणित गर्नुहोस् (If $\vec{a}$ and $\vec{b}$ are two unit vectors then prove that):

A. $(\vec{a} - \vec{b})^2 = \left(\frac{\vec{a} - \vec{b}}{ab}\right)^2$

B. $\left(\frac{\vec{a}}{a^2} + \frac{\vec{b}}{b^2}\right)^2 = \left(\frac{\vec{a} + \vec{b}}{ab}\right)^2$

www.drmathnp.com

puspas.com.np

22A. यदि $\vec{m}$ र $\vec{n}$ एकाइ भेक्टरहरू तथा तिनीहरू बिचको कोण θ भए $\frac{1}{2}|\vec{m}+\vec{n}| = \cos\frac{\theta}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{m}$ and $\vec{n}$ are unit vectors and θ be the angle between them then prove that: $\frac{1}{2}|\vec{m}+\vec{n}| = \cos\frac{\theta}{2}$.

Hint: Since $\vec{m}$ and $\vec{n}$ are unit vectors, $|\vec{m}| = 1$ and $|\vec{n}| = 1$

So, $\vec{m}.\vec{n} = |\vec{m}||\vec{n}|\cos\theta$ $\quad\therefore\ \vec{m}.\vec{n} = \cos\theta$ .....(i)

Now, LHS $= \frac{1}{2}|\vec{m}+\vec{n}| = \frac{1}{2}\sqrt{(\vec{m}+\vec{n})^2} = \frac{1}{2}\sqrt{m^2+2\vec{m}.\vec{n}+n^2}$

$= \frac{1}{2}\sqrt{1+2.\cos\theta+1} = \frac{1}{2}\sqrt{2(1+\cos\theta)} = \frac{1}{2}\sqrt{2\times 2\cos^2\frac{\theta}{2}} = \frac{1}{2}.2\cos\frac{\theta}{2} = \cos\frac{\theta}{2} = \text{RHS}$

22B. यदि $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू र तिनीहरूबिचको कोण θ भए $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ be the angle between them, show that: $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$.

22C. यदि $\vec{p}$ र $\vec{q}$ दुईओटा एकाइ भेक्टरहरू तथा तिनीहरूबिचको कोण θ भए $\left|\frac{\vec{p}-\vec{q}}{\vec{p}+\vec{q}}\right| = \tan\frac{\theta}{2}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $\vec{p}$ and $\vec{q}$ are two unit vectors and angle between them is θ then prove that: $\left|\frac{\vec{p}-\vec{q}}{\vec{p}+\vec{q}}\right| = \tan\frac{\theta}{2}$

B. LONG QUESTIONS

Model 1 *Problems based on proving the given vertices are the vertices of a rhombus or a rectangle.*

1A. $\vec{i}+2\vec{j}, 4\vec{i}+\vec{j}, 5\vec{i}+4\vec{j}$ र $2\vec{i}+5\vec{j}$ समबाहु चतुर्भुजका शीर्षहरूको स्थिति भेक्टरहरू हुन् भनी देखाउनुहोस् ।

Show that the vectors $\vec{i}+2\vec{j}, 4\vec{i}+\vec{j}, 5\vec{i}+4\vec{j}$ and $2\vec{i}+5\vec{j}$ are the position vectors of vertices of a rhombus.

1B. स्थिति भेक्टरहरू $\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}, \begin{pmatrix}3\\4\end{pmatrix}$ र $\begin{pmatrix}2\\4\end{pmatrix}$ भएका चार शीर्षबिन्दुहरू आयतका शीर्षहरू हुन् भनी देखाउनुहोस् ।

Show that the four points whose position vectors are $\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}, \begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix}$ form the sides of a rectangle.

Model 2 *Problems based on finding angles by vector method.*

2A. ΔABC मा A(2, 6), B(6, 2) र C(6, 6) छन् । $\angle CAB$ र $\angle CBA$ का मानहरू पत्ता लगाउनुहोस् ।
In ΔABC, A(2, 6), B(6, 2) and C(6, 6). Find the values of $\angle CAB$ and $\angle CBA$.

2B. ΔABC मा A(4, 4), B(2, 2) र C(6, 2) भए $\angle BAC$ र $\angle ABC$ का मानहरू पत्ता लगाउनुहोस् ।
In ΔABC, A(4, 4), B(2, 2) and C(6, 2) then find the values of $\angle BAC$ and $\angle ABC$.

2C. ΔABC मा A(2, 0), B(6, 4) र C(2, 4) छन् । $2\angle CAB = \angle ACB$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In ΔABC, A(2, 0), B(6, 4) and C(2, 4) then prove that: $2\angle CAB = \angle ACB$.

ANSWERS

A. SHORT QUESTIONS

1A. $6\vec{i}+8\vec{j}$ **1B.** $6\vec{i}+8\vec{j}$ **2A.** 60 **2B.** 1 **3A.** 0
3B. 0 **4A.** -18 **4B.** -288 **6A.** 2 **6B.** 1 **6C.** 2 **6D.** 2 or -5
6E. 3, -8 **6F.** 1, -5 **6G.** $\frac{20}{7}$ **7A.** 6 or 2 **7B.** 4 **8A.** 6 units **8B.** 3.49 units
8C. 4 cm **9A.** 0° **9B.** 55° **9C.** 60° **9D.** 30° **9E.** 45° **9F.** 53.13°
9G. 45° **9H.** 45° **10A.** 45° **10B.** 30° **10C.** 60° **11A.** 60° **11B.** 60°
11C. 75.52° **11D.** 60° **12A.** 90° **12B.** 45° **12C.** $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ **13A.** 1 or -4 **13B.** 0
15A. $\begin{pmatrix}6\\-4\end{pmatrix}$ **15B.** $\begin{pmatrix}-10\\5\end{pmatrix}$ **15C.** $\begin{pmatrix}6\\-8\end{pmatrix}$ **15D.** $0, -\frac{3}{4}$ **18A.** 3 **18B.** 1 **18C.** 4
18D. $-8, 2$ **18E.** $-9, 4$ **19A.** 5 units **19B.** 15 units **20A.** -8 **20B.** 45

B. LONG QUESTIONS

2A. 45°, 45° **2B.** 90°, 45°

VECTOR

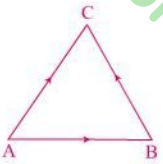
www.drmathnp.com

2. भेक्टर ज्यामिति VECTOR GEOMETRY

DEFINITIONS

1. भेक्टरको त्रिभुज नियम (Triangle law of vector)

त्रिभुजमा क्रमअनुसार लिइएका दुई भुजाहरूले जनाउने भेक्टरहरूको जोड बाँकी भुजाको विपरीत क्रमले प्रतिनिधित्व गरेको भेक्टरसँग बराबर हुन्छ ।

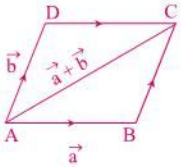


चित्रमा $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ लाई भेक्टर जोडसम्बन्धी त्रिभुज नियम भनिन्छ ।

The sum of vectors represented by two sides in an order is equal to the vector represented by remaining side taken in opposite order. In the figure, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ is called triangle law of vector.

2. भेक्टर जोडको समानान्तर चतुर्भुज नियम (Parallelogram law of vector addition)

समानान्तर चतुर्भुजमा एउटै बिन्दुबाट सुरू भएका आसन्न भुजाहरूले जनाउने भेक्टरहरूको जोड सोही बिन्दुबाट सुरू भएको विकर्णले जनाउने भेक्टरसँग बराबर हुन्छ । यस नियमलाई भेक्टर जोडको समानान्तर चतुर्भुज नियम भनिन्छ ।

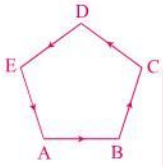


चित्रमा: $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$ लाई भेक्टर जोडसम्बन्धी स.च. नियम भनिन्छ ।

The sum of two co-initial vectors represented by the adjacent sides of a parallelogram is equal to the vector represented by co-initial diagonal. This method of vector addition is called the parallelogram law of vector addition. In the figure, $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$ is called the parallelogram law of vector addition.

3. भेक्टर जोडको बहुभुज नियम (Polygon law of vector addition)

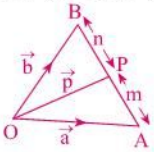
एउटा बहुभुजको क्रममा लिइएका भुजाहरूले जनाउने भेक्टरहरूको जोड अन्तिम भुजाको विपरीत क्रमले जनाउने भेक्टरसँग बराबर हुन्छ । जस्तै : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$



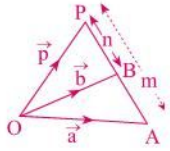
The polygon law of vector addition states that if vectors represented by the sides of a polygon taken in order, then the resultant vector is the vector represented by the closing side of the polygon taken in opposite order, e.g. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$

FORMULAE AND KEY POINTS

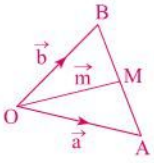
1. भित्री विभाजन सूत्र (Internal section formula): $\vec{p} = \frac{n\vec{a} + m\vec{b}}{m+n}$



2. बाहिरी विभाजन सूत्र (External section formula): $\vec{p} = \frac{m\vec{b} - n\vec{a}}{m-n}$

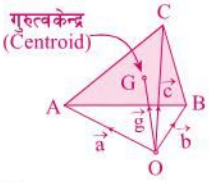


3. मध्यबिन्दु सूत्र (Mid-point formula): $\vec{m} = \frac{1}{2}(\vec{a} + \vec{b})$



4. यदि $\vec{a}$, $\vec{b}$ र $\vec{c}$ हरू ΔABC का शीर्षहरूको स्थिति भेक्टरहरू भए यसको गुरुत्वकेन्द्र $(\overrightarrow{OG}) = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ हुन्छ ।

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of vertices of ΔABC then the position vector of the centroid $(\overrightarrow{OG}) = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.



SEE / SLC REVIEW MESSAGE

B. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2067 R', 2068 R', S, 2071 R, S, 2072 R, 2072 R', SEE 2073 R', SEE 2073 S' , SEE 2074 R', SEE 2074 S', SEE 2075 R'_2, SEE MODEL 2076	13
Model 2	2066 R, 2067 S, 2068 S', 2069 R, 2070 R, 2070 R', 2071 R', SEE 2074 R, SEE 2074 S, SEE 2075 R'	9
Model 3	2065 S, 2070 S, 2073 R, SEE 2073 S, SEE 2075 R, R_2	5
Model 4	2068 R, 2069 R', 2070 S'	3
Model 5	2067 R, 2072 S	2
Model 6	2066 S, 2069 S	2

puspas.com.np

www.drmathnp.com

C. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2071 R, 2071 S, 2072 R'	3
Model 2	2066 R, 2066 R', 2067 R, 2067 S, 2069 R', 2069 S, 2070 R, 2070 S, 2072 S, 2073 R, R', S, S', 2074 R, S, SEE 2075 R'	16
Model 3	2065 M, R', S, 2066 S, 2067 R', 2068 R, R', 2069 R, 2070 R', S', 2071 R', 2072 R, 2074 R', 2075R'$_2$, SEE MODEL 2076	15
Model 4	SEE 2074 S', SEE 2075 R, R$_2$	3

SEE EXERCISE 2

A. VERY SHORT QUESTIONS

Model 1 *Definitions and formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):

A. स्थिति भेक्टर (Position vector) **B.** एउटा त्रिभुजको गुरूत्व केन्द्र (Centroid of a triangle)

C. भेक्टर जोडको त्रिभुज नियम (Triangle law of vector addition)

D. भेक्टर घटाउ सम्बन्धी त्रिभुज नियम (Triangle law of vector subtraction)

2A. खण्ड सूत्र $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$ मा m र n ले के के जनाउँदछन् ?

What do m and n represent in the section formula $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$?

2B. भित्री विभाजन सम्बन्धी खण्ड सूत्र लेख्नुहोस् । (Write the section formula of internal division.)

3A. यदि O उद्गम बिन्दु र A र B जोड्ने रेखा खण्डको मध्यबिन्दु M भए $\overrightarrow{OA}$, $\overrightarrow{OB}$ र $\overrightarrow{OM}$ को सम्बन्ध लेख्नुहोस् ।

If M is the mid point of line segment joining A and B and O is origin, write the relation between $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OM}$.

3B. यदि रेखाखण्ड AB लाई P ले $m : n$ को अनुपातमा विभाजन गर्दछ भने A, B र P को स्थिति भेक्टरहरू को सम्बन्ध लेख्नुहोस् ।
If P divides line segment AB internally in the ratio $m : n$, write the relation between the position vector of A, B and P.

3C. बाहिरी बिभाजन सम्बन्धी खण्ड सूत्र उल्लेख गर्नुहोस् । (State the section formula of external division.)

3D. भेक्टर ज्यामितिको मध्यबिन्दु सूत्र उल्लेख गर्नुहोस् । (State the mid point formula of vector geometry.)

4A. $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ हुनेगरी ΔABC एउटा त्रिभुज छ । बिन्दु G को विशिष्ट नाम उल्लेख गर्नुहोस्, जहाँ O उद्गम बिन्दु हो ।

ABC is a triangle such that $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ and O be the origin, write the special name of the point G.

4B. यदि एउटा त्रिभुजको शीर्षबिन्दुहरूका स्थिति भेक्टरहरूको योगफल $6\vec{i} + 9\vec{j}$ भए यसको गुरूत्व केन्द्रको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the sum of position vectors of vertices of a triangle is $6\vec{i} + 9\vec{j}$ then find the position vector of its centroid.

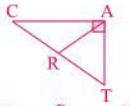
5A. सँगैको चित्रमा AR र RM का मध्यबिन्दुहरू क्रमशः T र K छन् । KT र MA को सम्बन्ध लेख्नुहोस् ।
In the adjoining figure, T and K are the midpoints of AR and RM respectively. Write the relation between KT and MA.



5B. सँगैको चित्रमा PN = PE र ER = RN छन् । PR र EN को सम्बन्ध उल्लेख गर्नुहोस् ।
In the adjoining figure, PN = PE and ER = RN. State the relation between PR and EN.

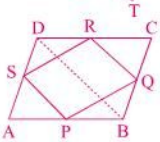


5C. सँगैको चित्रमा CT को मध्यबिन्दु R छ । CR र AR बीचको सम्बन्ध के हुन्छ ?
In the adjoining figure, R is the mid point of CT. What is the relation between CR and AR?



6. चित्रमा AB, BC, CD र AD का मध्यबिन्दुहरू क्रमशः P, Q, R र S छन् । निम्न सम्बन्ध लेख्नुहोस् :
In the figure, P, Q, R and S are the midpoints of AB, BC, CD and AD respectively. Write the relation between:

A. PQ and BD **B.** RS and BD

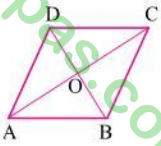


puspas.com.np

www.drmathnp.com

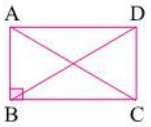
7A. दिइएको चित्रमा ABCD एउटा समवाहु चतुर्भुज हो । यदि $\overrightarrow{BD} \cdot \overrightarrow{AC} = 0$ भए $\angle AOB$ को नाप कति होला ?

In the given figure, ABCD is a rhombus. If $\overrightarrow{BD} \cdot \overrightarrow{AC} = 0$, what is the value of $\angle AOB$?



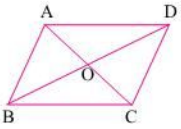
7B. दिइएको चित्रमा ABCD एउटा आयत हो । $\overrightarrow{AD} \cdot \overrightarrow{CD}$ को मान कति हुन्छ ?

In the given figure, ABCD is a rectangle. What will be the value of $\overrightarrow{AD} \cdot \overrightarrow{CD}$?



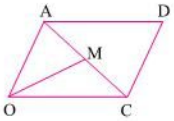
8A. दिइएको चित्रमा, ABCD एउटा समानान्तर चतुर्भुज हो । यदि $2\overrightarrow{BO} = \overrightarrow{BD}$ भए $\overrightarrow{BO}$ र $\overrightarrow{OD}$ को सम्बन्ध लेख्नुहोस् ।

In the given figure, ABCD is a parallelogram. If $2\overrightarrow{BO} = \overrightarrow{BD}$, write the relation between $\overrightarrow{BO}$ and $\overrightarrow{OD}$.



8B. दिइएको चित्रमा, AOCD एउटा समानान्तर चतुर्भुज हो । यदि AC को मध्यबिन्दु M भए $\overrightarrow{OM}$ लाई $\overrightarrow{OA}$ र $\overrightarrow{OC}$ को रूपमा लेख्नुहोस् ।

In the given figure, AOCD is a parallelogram. If M is midpoint of AC then express $\overrightarrow{OM}$ interms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$.



Model 2 *Concept Recall*

9A. यदि O उद्गम बिन्दु र $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ भए AB को मध्यबिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If O be the origin and $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ then find the position vector of M which is the mid point of AB.

9B. यदि O उद्गम बिन्दु र $\overrightarrow{OA} + \overrightarrow{OB} = 6\vec{i} + 8\vec{j}$ भए AB को मध्यबिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If O be the origin and $\overrightarrow{OA} + \overrightarrow{OB} = 6\vec{i} + 8\vec{j}$ then find the position vector of M which is the mid point of AB.

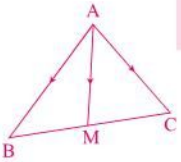
B. SHORT QUESTIONS

Model 1 *Problems based on mid-point formula.*

1A. दिइएको चित्रमा, ΔABC को मध्यिका AM छ । प्रमाणित गर्नुहोस्: $\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

In the figure, AM is the median of ΔABC. Prove that: $\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

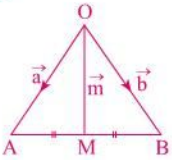
[2065 M]



1B. दिइएको चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र $\overrightarrow{OM} = \vec{m}$ र M रेखाखण्ड AB को मध्यबिन्दु भए प्रमाणित गर्नुहोस् : $\vec{m} = \frac{\vec{a} + \vec{b}}{2}$

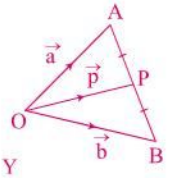
In the figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ and M is the mid-point of $\overrightarrow{AB}$. Prove that: $\vec{m} = \frac{\vec{a} + \vec{b}}{2}$

[2067 R', 2068 R']



1C. दिइएको चित्रमा $\overrightarrow{AP}$ पत्ता लगाई $\vec{p}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा व्यक्त गर्नुहोस् ।

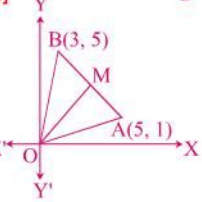
From the given figure find $\overrightarrow{AP}$ and express $\vec{p}$ in terms of $\vec{a}$ and $\vec{b}$. [SEE MODEL 2076]



2A. संगैको चित्रमा बिन्दुहरू A र B का निर्देशाङ्कहरू क्रमशः (5, 1) र (3, 5) छन् । यदि बिन्दु M रेखाखण्ड AB को मध्यबिन्दु हो भने बिन्दु M को स्थिति भेक्टर ($\overrightarrow{OM}$) पत्ता लगाउनुहोस् ।

In the figure alongside, the co-ordinates of the points A and B are (5, 1) and (3, 5) respectively. If the point M be the midpoint of the line segment AB, find the position vector ($\overrightarrow{OM}$) of the point M.

[SEE 2075 R'$_2$]

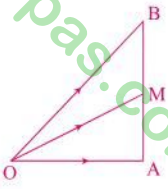


puspas.com.np

www.drmathnp.com

puspas.com.np

2B. कुनै रेखामा रहेका बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $\begin{pmatrix}3\\5\end{pmatrix}$ र $\begin{pmatrix}-1\\-1\end{pmatrix}$ छन् भने AB को मध्यबिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।



In a line, there are two points A and B whose position vectors are $\begin{pmatrix}3\\5\end{pmatrix}$ and $\begin{pmatrix}-1\\-1\end{pmatrix}$ respectively. Find the position vectors of M which is the mid-point of AB. **[2068 S]**

2C. यदि बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $\begin{pmatrix}3\\4\end{pmatrix}$ र $\begin{pmatrix}4\\5\end{pmatrix}$ भए AB को मध्यबिन्दु K को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of two points A and B are $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ respectively then find the position vector of the mid-point K of AB. **[2072 R']**

2D. यदि दुई बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $\begin{pmatrix}3\\2\end{pmatrix}$ र $\begin{pmatrix}-5\\4\end{pmatrix}$ भए AB को मध्यबिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of two points A and B are $\begin{pmatrix}3\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\4\end{pmatrix}$ respectively then find position vector of the mid-point C of AB. **[SEE 2073 R']**

3A. A र B का स्थिति भेक्टरहरू क्रमश: $7\vec{i}+2\vec{j}$ र $3\vec{i}-4\vec{j}$ छन् । यदि बिन्दु P रेखा AB को मध्यबिन्दु हो भने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $7\vec{i}+2\vec{j}$ and $3\vec{i}-4\vec{j}$ respectively. If point P is the mid-point of line AB, find the position vector of the point P. **[SEE 2074 S']**

3B. बिन्दुहरू A र P का स्थिति भेक्टरहरू क्रमश: $3\vec{i}+5\vec{j}$ र $\vec{i}+4\vec{j}$ छन् । यदि AB को मध्यबिन्दु P भए B को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of the points A and P are $3\vec{i}+5\vec{j}$ and $\vec{i}+4\vec{j}$ respectively. If P is the middle point of AB then find the position vector of B. **[SEE 2074 R']**

3C. बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमश: $(8\vec{i}+6\vec{j})$ र $(3\vec{i}+\vec{j})$ छन् । AB लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of the vertices A and B are $(8\vec{i}+6\vec{j})$ and $(3\vec{i}+\vec{j})$ respectivley. Find the position vector of the point C, which divides AB in the ratio of 2 : 3 internally. **[SEE 2073 S']**

3D. A र B का स्थिति भेक्टरहरू क्रमश: $9\vec{i}+7\vec{j}$ र $\vec{i}-3\vec{j}$ छन् । यदि AB को मध्यबिन्दु M भए M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $9\vec{i}+7\vec{j}$ and $\vec{i}-3\vec{j}$ respectively. If M is the mid-point of AB, find the position vecotr of M. **[2072 R]**

3E. यदि M र N का स्थिति भेक्टरहरू क्रमश: $9\vec{i}+3\vec{j}$ र $3\vec{i}+5\vec{j}$ भए $\overrightarrow{MP}=\overrightarrow{PN}$ हुने गरी एउटा बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of M and N are $9\vec{i}+3\vec{j}$ and $3\vec{i}+5\vec{j}$ respectively, find the position vector of a point P such that $\overrightarrow{MP}=\overrightarrow{PN}$. **[2071 R]**

3F. यदि A र B का स्थिति भेक्टर क्रमश: $3\vec{i}+4\vec{j}$ र $7\vec{i}+8\vec{j}$ भए A र B जोड्ने रेखाको मध्यबिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of A and B are $3\vec{i}+4\vec{j}$ and $7\vec{i}+8\vec{j}$ respectively, find the position vector of the mid-point of the line joining A and B. **[2058 R]**

3G. स्थिति भेक्टरको परिभाषा दिनुहोस् । यदि बिन्दु A(3, 4) र B(– 1, 2) जोड्ने सीधा रेखाको मध्यबिन्दु P भए, P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

Define position vector. If P is the mid-point of the straight line joining the points A(3, 4) and B(– 1, 2), find the position vector of P. **[2060 S']**

3H. रेखाखण्ड EF को मध्यबिन्दुको स्थिति भेक्टर $4\vec{i}+7\vec{j}$ र बिन्दु E को स्थिति भेक्टर $-3\vec{i}-4\vec{j}$ छन् भने बिन्दु F को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

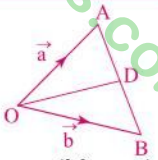
If the position vector of the midpoint of the line segment EF is $4\vec{i}+7\vec{j}$ and the position vector of the point E is $-3\vec{i}-4\vec{j}$, find the position vector of the point F. **[2071 S]**

www.drmathnp.com

puspas.com.np

Model 2 *Problems based on section formulae.*

4A. बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $\vec{a}$ र $\vec{b}$ छन् भने AB लाई भित्रबाट 2 : 3 मा विभाजन गर्ने बिन्दु D को स्थिति भेक्टर पत्ता लगाउनुहोस् ।



If $\vec{a}$ and $\vec{b}$ are the position vectors of points A and B respectively, then find the position vector of point D which divides AB internally in the ratio of 2 : 3. **[2068 S']**

4B. बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $5\vec{i}+3\vec{j}$ र $2\vec{i}-3\vec{j}$ छन् भने AB लाई 2 : 1 को अनुपातमा विभाजन गर्ने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $5\vec{i}+3\vec{j}$ and $2\vec{i}-3\vec{j}$ respectively. Find the position vector of the point P which divides AB in the ratio of 2 : 1. **[SEE 2074 R]**

4C. बिन्दुहरू A र B का स्थिती भेक्टरहरू क्रमशः $3\vec{i}-\vec{j}$ र $4\vec{i}+5\vec{j}$ छन् भने BA लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $3\vec{i}-\vec{j}$ and $4\vec{i}+5\vec{j}$ respectively. Find the position vector of the point M which divides BA internally in the ratio 2 : 3. **[SEE 2074 S]**

4D. A र B का स्थिति भेक्टरकहरू क्रमशः $2\vec{i}+7\vec{j}$ र $7\vec{i}-3\vec{j}$ छन् । AB लाई भित्रबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $2\vec{i}+7\vec{j}$ and $7\vec{i}-3\vec{j}$ respectively. Find the position vector of a point which divides AB internally in the ratio of 2 : 3. **[2071 R']**

4E. बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $5\vec{i}+2\vec{j}$ र $3\vec{i}+6\vec{j}$ छन् भने AB लाई भित्रबाट 2 : 3 मा विभाजन गर्ने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If $5\vec{i}+2\vec{j}$ and $3\vec{i}+6\vec{j}$ are the position vectors of points A and B respectively, find the position vector of point 'P' which divides AB internally in the ratio of 2 : 3. **[2066 R]**

4F. यदि बिन्दुहरू P र Q का स्थिति भेक्टरहरू क्रमशः $3\vec{i}+6\vec{j}$ र $5\vec{i}+2\vec{j}$ छन् भने रेखा PQ लाई भित्रबाट 3 : 2 मा विभाजन गर्ने बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If $3\vec{i}+6\vec{j}$ and $5\vec{i}+2\vec{j}$ are the position vectors of the points P and Q respectively, find the position vectors of the point M which divides the line PQ internally in the ratio of 3 : 2. **[2070 R']**

4G. रेखा AB लाई बिन्दु C ले 3 : 1 को अनुपातमा भित्रपट्टि विभाजन गरेको छ । यदि A र B का स्थिति भेक्टरहरू क्रमशः $\vec{i}-3\vec{j}$ र $2\vec{i}+5\vec{j}$ भए $|\overrightarrow{AB}|$ र C बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

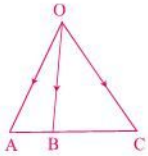
Point C divides the line AB internally in the ratio of 3 : 1. If the position vectors of A and B are $\vec{i}-3\vec{j}$ and $2\vec{i}+5\vec{j}$ respectively, find $|\overrightarrow{AB}|$ and the position vector of point C. **[2059 S]**

5A. यदि $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OM}=\vec{m}$ र M ले BA लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस् : $\vec{m}=\frac{1}{5}(3\vec{a}+2\vec{b})$

If $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OM}=\vec{m}$ and M divides BA internally in the ratio of 3 : 2 then, prove that: $\vec{m}=\frac{1}{5}(3\vec{a}+2\vec{b})$ **[SEE 2075 R']**

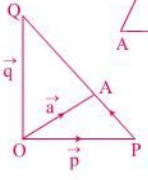
5B. यदि $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OM}=\vec{m}$ र M ले BA लाई 4 : 5 को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस्: If $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OM}=\vec{m}$ and M divides BA in the ratio of 4 : 5, prove that: $\vec{m}=\frac{1}{9}(4\vec{a}+5\vec{b})$

6A. दिइएको चित्रमा $\overrightarrow{OA}=\vec{a}$ र $\overrightarrow{OB}=\vec{b}$ छ । यदि $\overrightarrow{AC}=3\overrightarrow{AB}$ भए $\overrightarrow{OC}$ पत्ता लगाउनुहोस् ।



In the given figure, $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$. If $\overrightarrow{AC}=3\overrightarrow{AB}$, find $\overrightarrow{OC}$. **[2058 R, 2067 S, 2069 R]**

6B. दिइएको चित्रमा, यदि $\overrightarrow{PA}=\frac{1}{4}\overrightarrow{PQ}$ छ भने प्रमाणित गर्नुहोस्: $\vec{a}=\frac{1}{4}(3\vec{p}+\vec{q})$



In the given figure, if $\overrightarrow{PA}=\frac{1}{4}\overrightarrow{PQ}$, prove that : $\vec{a}=\frac{1}{4}(3\vec{p}+\vec{q})$ **[2070 R]**

VECTOR

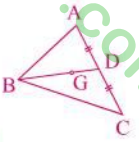
www.drmathnp.com

puspas.com.np

Model 3 *Problems based on centroid formula.*

7A. दिइएको चित्रमा AD = DC र बिन्दु G त्रिभुज ABC को गुरुत्वकेन्द्र हो । यदि बिन्दुहरू B र D का स्थिति भेक्टरहरू क्रमश : $3\vec{i}+7\vec{j}$ र $3\vec{i}-2\vec{j}$ छन् भने G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In the given figure, AD = DC and the point G is the centroid of the triangle ABC. If the position vectors of the points B and D are $3\vec{i}+7\vec{j}$ and $3\vec{i}-2\vec{j}$ respectively, find the position vector of G. **[SEE 2075 R, SEE 2075 R_2]**



7B. यदि ΔABC का शिर्षबिन्दुहरू A, B र C का स्थिति भेक्टरहरू क्रमश: $(3\vec{i}+4\vec{j})$, $(4\vec{i}+5\vec{j})$ र $(5\vec{i}+6\vec{j})$ छन् भने सो त्रिभुजको गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of the vertices A, B, and C of ΔABC are $(3\vec{i}+4\vec{j})$, $(4\vec{i}+5\vec{j})$ and $(5\vec{i}+6\vec{j})$ respectivley, find the position vector of the centroid G of the triangle. **[SEE 2073 S]**

7C. यदि ΔABC का शीर्षबिन्दुहरू A, B र C का स्थिति भेक्टरहरू क्रमश: $(3\vec{i}+5\vec{j})$, $(5\vec{i}-\vec{j})$ र $(\vec{i}+8\vec{j})$ छन् भने त्रिभुजको गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the position vectors of the vertices A, B and C of ΔABC are $(3\vec{i}+5\vec{j})$, $(5\vec{i}-\vec{j})$ and $(\vec{i}+8\vec{j})$ respectively, find the position vectors of the centroid G of the triangle. **[SEE 2073 R]**

7D. यदि A(3, 5), B(5, – 1) र C(2, 4) ΔABC का शीर्षबिन्दुहरू र ΔABC को गुरुत्वकेन्द्र G भए G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If A(3, 5), B(5, – 1) and C(2, 4) are the vertices of ΔABC and G is the centroid of the ΔABC, find the position vector of G. **[2065 S]**

7E. ΔABC को भुजा BC को मध्यबिन्दु D छ । यदि बिन्दुहरू A र D का स्थिति भेक्टरहरू क्रमश: $3\vec{i}+5\vec{j}$ र $3\vec{i}-4\vec{j}$ भए ΔABC को भार केन्द्र (G) को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

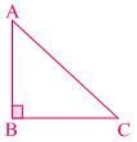
D is the mid-point of the side BC of ΔABC. If the position vectors of the points A and D are $3\vec{i}+5\vec{j}$ and $3\vec{i}-4\vec{j}$ respectively, find the position vector of the centroid (G) of ΔABC. **[2070 S]**

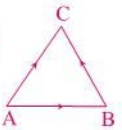
Model 4 *Problems based on sum of vectors represented by sides of polygon.*

VECTOR

8A. दिइएको चित्रमा $\angle ABC = 90°$ छ भने $\vec{AC}^2 = \vec{AB}^2 + \vec{BC}^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the given figure $\angle ABC = 90°$. Prove that : $\vec{AC}^2 = \vec{AB}^2 + \vec{BC}^2$. **[2068 R]**



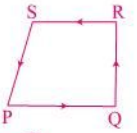


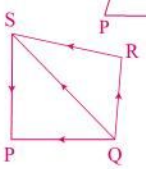
8B. दिइएको त्रिभुज ABC मा $\vec{AB}+\vec{BC}+\vec{CA}=0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

Prove that $\vec{AB}+\vec{BC}+\vec{CA}=0$ in given triangle ABC. **[2060 R]**

9A. दिइएको चतुर्भुज PQRS मा $\vec{PQ}+\vec{QR}+\vec{RS}+\vec{SP}=0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

Prove that $\vec{PQ}+\vec{QR}+\vec{RS}+\vec{SP}=0$ in the given quadrilateral PQRS. **[2059 R]**



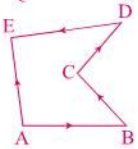


9B. यदि PQRS एउटा चतुर्भुज भए प्रमाणित गर्नुहोस् : $\vec{QR}+\vec{RS}+\vec{SP}=\vec{QP}$

If PQRS is a quadrilateral, prove that: $\vec{QR}+\vec{RS}+\vec{SP}=\vec{QP}$ **[2060 S']**

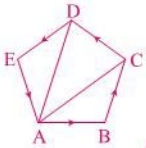
9C. दिइएको चित्रमा $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}=\vec{AE}$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given figure, prove that: $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}=\vec{AE}$ **[2062R]**



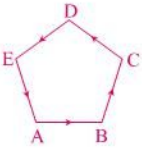
10A. दिइएको चित्रमा $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EA}=0$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given diagram, prove that: $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EA}=0$. **[2059 S]**



10B. दिइएको पञ्चभुज ABCDE मा $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EA}=0$ हुन्छ भनी प्रमाणित गर्नुहोस् । **[2060 CP]**

In the given pentagon ABCDE, prove that: $\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EA}=0$



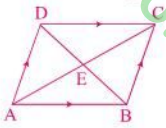
www.drmathnp.com

11A. दिइएको चित्रमा OABC एउटा समानान्तर चतुर्भुज हो । प्रमाणित गर्नुहोस् :
In the given figure, OABC is a parallelogram, prove that:

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$ [2070 S']

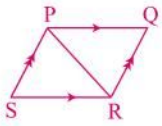


11B. दिइएको चित्रमा ABCD एउटा समानान्तर चतुर्भुज हो । यदि AC र BD विकर्णहरू भए प्रमाणित गर्नुहोस्:
In the given figure ABCD is a parallelogram. If AC and BD are diagonals, prove that: $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$ [2069 R']

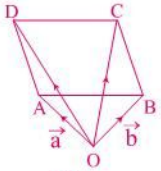


Model 5 Problems based on finding a vector interms of given vector.

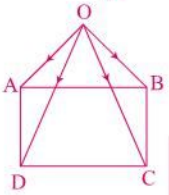
12A. दिइएको समानान्तर चतुर्भुजमा $\overrightarrow{SR} = 4\vec{i} - 2\vec{j}$ र $\overrightarrow{PR} = 6\vec{i} + 5\vec{j}$ भए $\overrightarrow{QR}$ पत्ता लगाउनुहोस् ।
In the given parallelogram, $\overrightarrow{SR} = 4\vec{i} - 2\vec{j}$ and $\overrightarrow{PR} = 6\vec{i} + 5\vec{j}$, find $\overrightarrow{QR}$. [2072 S]



12B. दिइएको चित्रमा ABCD एउटा समानान्तर चतुर्भुज हो । $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र $\overrightarrow{OC} = \vec{c}$ भए $\overrightarrow{OD}$ निकाल्नुहोस् ।
In the given figure, ABCD is a parallelogram. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$, find $\overrightarrow{OD}$. [2058 S]



12C. दिइएको चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ र $\overrightarrow{AD} = \overrightarrow{BC}$ भए $\overrightarrow{OD}$ पत्ता लगाउनुहोस् ।
In the given diagram, find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ and $\overrightarrow{AD} = \overrightarrow{BC}$. [2067R]



Model 6 Problems based on application of vector geometry.

13A. ΔABC मा AB र AC लाई क्रमशः D र E ले 1 : 2 को अनुपातमा विभाजन गर्दछन् भने $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In ΔABC, D and E divides AB and AC in the ratio of 1 : 2 respectively. Prove that: $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$

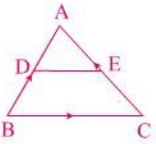
HINT: Here $\overrightarrow{DB} = 2\overrightarrow{AD}$ and $\overrightarrow{EC} = 2\overrightarrow{AE}$ [2066 S]

Now, $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ [By triangle law of vector addition]

$= \overrightarrow{BD} + \overrightarrow{DA} + \overrightarrow{AE} + \overrightarrow{EC} = -2\overrightarrow{AD} + \overrightarrow{DA} + \overrightarrow{AE} + 2\overrightarrow{AE}$

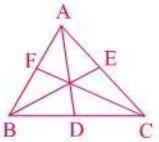
$= 3\overrightarrow{DA} + 3\overrightarrow{AE} = 3(\overrightarrow{DA} + \overrightarrow{AE}) = 3\overrightarrow{DE}$

$\therefore \overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$ Proved.

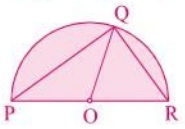


13B. ΔABC मा AB र AC लाई क्रमशः D र E ले 1 : 3 को अनुपातमा विभाजन गर्दछन् भने $\overrightarrow{DE} = \frac{1}{4}\overrightarrow{BC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In ΔABC, D and E divides AB and AC in the ratio of 1 : 3 respectively. Prove that: $\overrightarrow{DE} = \frac{1}{4}\overrightarrow{BC}$

14A. ΔABC मा D, E, F हरू क्रमशः भुजाहरू BC, CA र AB का मध्यबिन्दुहरू भए $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
In ΔABC, D, E, F are the mid points of sides BC, CA and AB respectively. Prove that: $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$ [2058 R]



14B. दिइएको चित्र केन्द्र बिन्दु O भएको अर्धवृत्त हो । प्रमाणित गर्नुहोस्:
The given figure is semi circle with the centre O. Prove that :
$(\overrightarrow{QO} + \overrightarrow{OP}).(\overrightarrow{QO} + \overrightarrow{OR}) = 0$ [2069 S]



puspas.com.np

www.drmathnp.com

puspas.com.np

C. LONG QUESTIONS

यस अन्तर्गत **SLC/SEE** मा सोधिएका र कक्षा 10 को नयाँ पाठ्यक्रममा भएका साध्यहरू मात्र समावेश छन् । **SLC** मा सोधिएका अन्य साध्यहरू नयाँ पाठ्यक्रममा नभएकाले हटाइएको छ ।

Model 1 Problems based on section formula.

1A. यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र रेखाखण्ड AB लाई भित्रपट्टीबाट बिन्दु M ले $m_1 : m_2$ को अनुपातमा विभाजन गर्दछ भने, प्रमाणित गर्नुहोस् : $\overrightarrow{OM} = \dfrac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$

If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and the point M divides the line segment AB internally in the ratio of $m_1 : m_2$, then prove that: $\overrightarrow{OM} = \dfrac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$ **[2071 R]**

OR यदि $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ र रेखाखण्ड PQ लाई भित्रपट्टिबाट बिन्दु M ले $m_1 : m_2$ को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस्: $\overrightarrow{OM} = \dfrac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$

If $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ and the point M divides the line segment PQ internally in the ratio of $m_1 : m_2$ then prove that: $\overrightarrow{OM} = \dfrac{m_1\vec{q} + m_2\vec{p}}{m_1 + m_2}$ **[2072 R']**

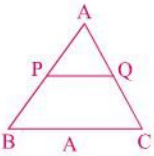
1B. यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र रेखाखण्ड AB लाई बाहिरपट्टीबाट बिन्दु M ले $m : n$ को अनुपातमा विभाजन गर्दछ भने, प्रमाणित गर्नुहोस्:

If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and the point M divides the line segment AB externally in the ratio of $m : n$ then prove that: $\overrightarrow{OM} = \dfrac{m\vec{b} - n\vec{a}}{m - n}$ **[2071 S]**

Model 2 Theorems related to triangle.

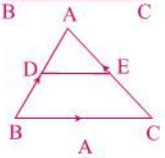
2. दिइएको चित्रमा P र Q त्रिभुज ABC को क्रमशः भुजा AB र AC का मध्य बिन्दुहरू भए PQ // BC र BC= 2PQ हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

In the figure, P and Q are the middle points of AB & AC respectively of the ΔABC. Prove vectorically that PQ // BC & BC = 2PQ **[2059 S]**



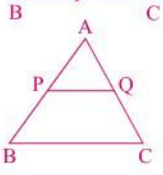
OR त्रिभुज ABC मा D र E क्रमशः भुजाहरू AB र AC का मध्यबिन्दुहरू हुन् । भेक्टर विधिबाट प्रमाणित गर्नुहोस्: $\overrightarrow{DE} = \dfrac{1}{2}\overrightarrow{BC}$

In the triangle ABC, D and E are the mid-points of the sides AB and AC respectively. Prove by vector method that: $\overrightarrow{DE} = \dfrac{1}{2}\overrightarrow{BC}$ **[2066 R']**



OR त्रिभुजको कुनै दुई भुजाहरूको मध्यबिन्दुहरू जोड्ने रेखाखण्ड तेस्रो भुजासँग समानान्तर भइ त्यसको आधा हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

Prove by vector method that a line segment joining the mid points of any two sides of a triangle is parallel to the third side and is half of it. **[2060 S, SEE 2075 R']**



HINT: Let ABC be a triangle with P and Q be the mid-points of AB and AC respectively.

(i) $\overrightarrow{BA} = 2\overrightarrow{PA}$ [$\because$ P is a mid point of AB] (ii) $\overrightarrow{AC} = 2\overrightarrow{AQ}$ [$\because$ Q is a mid point of AC]

(iii) $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ [Triangle law of vector addition]

or, $2\overrightarrow{PA} + 2\overrightarrow{AQ} = \overrightarrow{BC}$ or, $2(\overrightarrow{PA} + \overrightarrow{AQ}) = \overrightarrow{BC}$

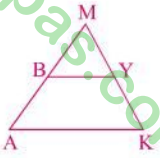
(iv) $\overrightarrow{PA} + \overrightarrow{AQ} = \overrightarrow{PQ}$ [Triangle law of vector addition]

(v) $\therefore 2\overrightarrow{PQ} = \overrightarrow{BC}$ [From (iii) and (iv)] [$\because$ If $\vec{a} = m\vec{b}$ then $\vec{a} \parallel \vec{b}$]

Hence, $\overrightarrow{PQ} \parallel \overrightarrow{BC}$ and $\overrightarrow{PQ} = \dfrac{1}{2}\overrightarrow{BC}$.

www.drmathnp.com
puspas.com.np

OR चित्रमा ΔKAM का भुजाहरू KM र AM का मध्य विन्दुहरू जोड्ने रेखाखण्ड BY छ । भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् :
In the figure, BY is the segment joining the mid points of sides KM and MA of ΔKAM. Prove by vector method :

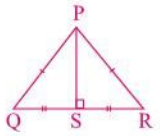


(i) $BY \parallel AK$ (ii) $BY = \frac{1}{2} AK$

3. समद्विबाहु त्रिभुजको मध्यिका आधारमा लम्ब हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove vectorically that the median of an isosceles triangle is perpendicular to the base. **[2072 S]**

OR समद्विबाहु त्रिभुजको शीर्षविन्दुबाट आधारको मध्यविन्दु जोड्ने रेखा उक्त आधारमा लम्ब हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the line segment joining the vertex and mid-point of the base of an isosceles triangle is perpendicular to the base. **[2069 R', 2069 S, SEE 2073 R]**

OR दिइएको चित्रमा समद्विबाहु ΔPQR मा PQ = PR र QS = SR भए रेखा PS आधार QR मा लम्ब हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
In the given figure, in isosceles ΔPQR, if PQ = PR and QS = SR then prove by vector method that the line PS is perpendicular to base QR. **[2067 R]**



HINT: Let PQR be an isosceles triangle with PQ = QR and PS is median of ΔPQR.

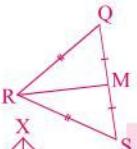
(i) $\overrightarrow{PS} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{PR})$ [Mid point theorem]

(ii) $\overrightarrow{QR} = (\overrightarrow{PR} - \overrightarrow{PQ})$ [Triangle law of vector subtraction]

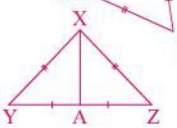
(iii) $\overrightarrow{PS} \cdot \overrightarrow{QR} = \frac{1}{2}(\overrightarrow{PQ} + \overrightarrow{PR})(\overrightarrow{PR} - \overrightarrow{PQ}) = \frac{1}{2}[(\overrightarrow{PR})^2 - (\overrightarrow{PQ})^2] = \frac{1}{2}(PR^2 - PQ^2) = \frac{1}{2} \times 0$ [Since PR = PQ]

(iv) $\overrightarrow{PS} \perp \overrightarrow{QR}$ [$\because \overrightarrow{PS} \cdot \overrightarrow{QR} = 0$]

OR दिइएको चित्रमा RQ = RS र QM = SM भए भेक्टर विधिबाट प्रमाणित गर्नुहोस् : $RM \perp QS$
In the given figure, RQ = RS and QM = SM then prove by vector method: $RM \perp QS$.
[SEE 2074 S, 2073 S]



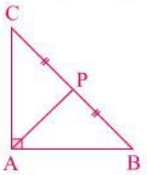
OR दिइएको चित्रमा XY = XZ र AY = AZ भए भेक्टर विधिबाट प्रमाणित गर्नुहोस् : $XA \perp YZ$
In the given figure, XY = XZ and AY = AZ then prove by vector method: $XA \perp YZ$
[SEE 2073 S']



OR एउटा त्रिभुज CAT मा CA = CT छ । भुजा AT मा मध्यिका CP छ । भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : $CP \perp AT$
In ΔCAT, CA = CT. median CP lies on side AT. prove by vector method: $CP \perp AT$.

4. समकोणी त्रिभुजमा कर्णको मध्यविन्दु यसका शीर्षहरूबाट समदुरीमा पर्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the middle point of hypotenuse of a right angled triangle is equidistant from its vertices.
[2067 S]

HINT: Let, ABC be a right angled triangle whose right angle is at A.
Let, P be the middle point of the hypotenuse BC. Let, A be the origin then,



(i) $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$ (Triangle law of vector addition)

(ii) $\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC}$ (Triangle law of vector addition)

(iii) $\overrightarrow{AB} \perp \overrightarrow{AC}$ [$\because \angle A = 90°$] or, $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$ [$\because AB \perp AC$]

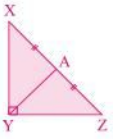
or, $(\overrightarrow{AP} + \overrightarrow{PB}) \cdot (\overrightarrow{AP} + \overrightarrow{PC}) = 0$ [$\because$ From (i) and (ii)]

or, $(\overrightarrow{AP} + \overrightarrow{CP}) \cdot (\overrightarrow{AP} - \overrightarrow{CP}) = 0$ [$\because \overrightarrow{CP} = \overrightarrow{PB}$ and $\overrightarrow{PC} = -\overrightarrow{CP}$]

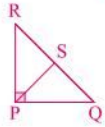
or, $(AP)^2 - (CP)^2 = 0$ or, $AP^2 = CP^2$

$\therefore$ $AP = CP = PB$ [$\because CP = PB$]

OR दिइएको त्रिभुज XYZ मा $\angle XYZ = 90°$ र XZ को मध्यविन्दु A छ भने XA = YA = ZA हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In the given triangle XYZ, $\angle XYZ = 90°$ and A is the middle point of XZ. Prove by vector method that : XA = YA = ZA. **[2070 R, 2070 S]**



OR दिइएको समकोण त्रिभुज PQR मा $\angle RPQ = 90°$ र RQ को मध्यविन्दु S छ भने भेक्टर विधिबाट प्रमाणित गर्नुहोस्: PS = QS = RS
In the given figure, $\angle RPQ = 90°$ and S is the mid-point of RQ. Prove by vector method that: PS = QS = RS. **[SEE 2073 R']**



www.drmathnp.com

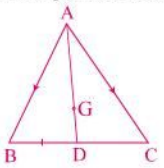
5. यदि $\vec{a}$, $\vec{b}$ र $\vec{c}$, ΔABC को शीर्ष बिन्दुहरूका स्थिति भेक्टरहरू भए ΔABC को भारकेन्द्र G को स्थिति भेक्टर $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् । [2066 R]

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the vertices of a ΔABC, prove by vector method that the position vector of the centroid G of the triangle ΔABC is $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.

OR त्रिभुजको गुरुत्वकेन्द्र (भारकेन्द्र) को स्थिति भेक्टर यसको शीर्षहरूका स्थिति भेक्टरहरूको योगफलको एकतीहाइ हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।

Using vector method, prove that the position vector of the centroid of a triangle is one third the sum of the position vectors of its vertices.

HINT: Let O be the origin and G be the centroid of ΔABC. In ΔABC, AD is median. So D is the mid point of BC.



Let, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$.

(i) $\overrightarrow{OD} = \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC})$ [From mid point theorem]

(ii) $\overrightarrow{OG} = \frac{m\overrightarrow{OD} + n\overrightarrow{OA}}{m+n}$ (Formula)

Centroid divides median in the ratio of 2 : 1. So, the ratio is; $m : n = 2 : 1$.

$$\overrightarrow{OG} = \frac{2.\frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC}) + 1.\overrightarrow{OA}}{2+1} = \frac{\overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OA}}{3} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$$

$\therefore \overrightarrow{OG} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ which is the required result.

OR ΔPQR मा मध्यरेखा QS र गुरुत्वकेन्द्र G छन् । यदि O उद्गम बिन्दु हो भने, प्रमाणित गर्नुहोस् :

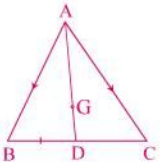
$$\overrightarrow{OG} = \frac{1}{3}\left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$$

In ΔPQR, QS is a median and G is the centroid. If O is the origin then prove that:

$$\overrightarrow{OG} = \frac{1}{3}\left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$$ [SEE 2074 R, 2073 R']

OR चित्रमा, ΔABC को मध्यिका AD लाई बिन्दु G ले AG : GD = 2 : 1 को अनुपातमा विभाजन गरेको छ, प्रमाणित गर्नुहोस् :

In the figure, point G divides median AD of ΔABC in the ratio AG : GD = 2 : 1. Prove that:



G को स्थिति भेक्टर = $\frac{1}{3}$ (A, B र C को स्थिति भेक्टरको योगफल)

Position vector of G = $\frac{1}{3}$ [sum of position vectors of A, B and C]

or, $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$

⇨ Here, AG : GD = 2 : 1

or, $\frac{AG}{GD} = \frac{2}{1}$

or, $\overrightarrow{AG} = 2\overrightarrow{GD}$

or, $\overrightarrow{OG} - \overrightarrow{OA} = 2(\overrightarrow{OD} - \overrightarrow{OG})$

or, $\overrightarrow{OG} - \overrightarrow{OA} = 2\overrightarrow{OD} - 2\overrightarrow{OG}$

or, $\overrightarrow{OG} + 2\overrightarrow{OG} = 2\overrightarrow{OD} - \overrightarrow{OA}$

or, $3\overrightarrow{OG} = 2 \times \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC}) + \overrightarrow{OA}$ [By mid point theorem]

or, $3\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OA}$

or, $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ Proved.

VECTOR

puspas.com.np

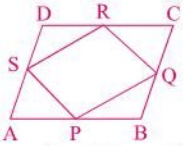
www.drmathnp.com

puspas.com.np

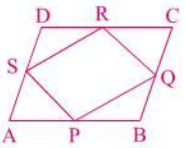
Model 3 *Theorems related to quadrilaterals.*

6. चतुर्भुजका भुजाहरूको मध्यबिन्दुहरू क्रमशः जोड्दै जाँदा बन्ने चित्र समानान्तर चतुर्भुज हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the lines joining the middle points of the sides of a quadrilateral taken in order is a parallelogram. [SEE MODEL 2076]

OR दिइएको चित्रमा P, Q, R र S क्रमशः भुजाहरू AB, BC, CD र DA का मध्यबिन्दुहरू भए PQRS समानान्तर चतुर्भुज हो भनी प्रमाणित गर्नुहोस् ।
In the given figure P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram. [2059 R, 2062 K, 2065 S]



HINT: In the figure alongside, let ABCD be a quadrilateral and PQRS be the resulting quadrilateral formed by joining the middle points of sides AB, BC, CD and DA respectively. Let O be the origin of vectors then by mid point theorem,



(i) $\overrightarrow{OP} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$; $\overrightarrow{OQ} = \frac{\overrightarrow{OB} + \overrightarrow{OC}}{2}$; $\overrightarrow{OR} = \frac{\overrightarrow{OC} + \overrightarrow{OD}}{2}$; $\overrightarrow{OS} = \frac{\overrightarrow{OD} + \overrightarrow{OA}}{2}$

(ii) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \frac{\overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA} - \overrightarrow{OB}}{2} = \frac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

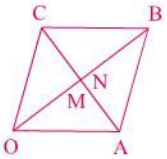
(iii) $\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS} = \frac{\overrightarrow{OC} + \overrightarrow{OD} - \overrightarrow{OD} - \overrightarrow{OA}}{2} = \frac{\overrightarrow{OC} - \overrightarrow{OA}}{2}$

Therefore $\overrightarrow{PQ} = \overrightarrow{SR}$ i.e. PQ = SR and PR // SR Similarly, SP = RQ and SP // RQ
Thus, PQRS is a parallelogram.

OR चतुर्भुज ABCD का भुजाहरू AB, BC, CD र AD का मध्य विन्दुहरू क्रमशः N, E, S र T छन् । NEST एउटा समानान्तर चतुर्भुज हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
N, E, S and T are the mid points of sides AB, BC, CD and DA respectively of a quadrilateral ABCD. Prove by the vector method that NEST is a parallelogram.

7. समानान्तर चतुर्भुजका विकर्णहरू आपसमा समद्विभाजन हुन्छन् भनी प्रमाणित गर्नुहोस् । [2068 R]
Prove that the diagonals of a parallelogram bisect to each other. [2057S, 2061S, 2065 M, 2067 R']

HINT: Let OABC be a parallelogram and AC and OB are the diagonals.
Let O be the origin , M and N are mid points of AC and OB in parallelogram OABC.



(i) $\overrightarrow{OM} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ [Mid point theorem]

(ii) $\overrightarrow{ON} = \frac{\overrightarrow{OB}}{2} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ [Parallelogram Law of vector addition]

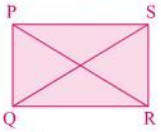
$\therefore \overrightarrow{OM} = \overrightarrow{ON}$ so M and N are coincide.
Thus, diagonals of parallelogram bisects to each other.

OR समानान्तर चतुर्भुज PQRS का विकर्णहरू आपसमा समद्विभाजन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals of a parallelogram PQRS bisect to each other. [SEE 2075 R'$_2$]

OR समानान्तर चतुर्भुज POST का विकर्णहरू समद्विभाजित हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals of a parallelogram POST bisect to each other.

8. आयतका विकर्णहरू बराबर हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् । [2068 R', SLC 2066 S, 2070 R', 2071 R']
Prove vectorically that the diagonals of a rectangle are equal to each other.

OR चित्रमा PQRS एउटा आयत हो । PR = QS हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
In the given figure, PQRS is a rectangle. Prove vectorically that PR = QS. [2069 R, SEE 2074 R']



HINT: Let, PQRS be a rectangle and PR and QS are its diagonals.

(i) $PR^2 = \overrightarrow{(PR)}^2 = (\overrightarrow{PQ} + \overrightarrow{QR})^2 = (\overrightarrow{SR} + \overrightarrow{QR})^2 = \overrightarrow{SR}^2 + 2\overrightarrow{SR}.\overrightarrow{QR} + \overrightarrow{QR}^2 = \overrightarrow{RS}^2 + 2\times 0 + \overrightarrow{QR}^2$ [$\because \overrightarrow{SR} \perp \overrightarrow{QR}$]

$\therefore PR^2 = \overrightarrow{RS}^2 + \overrightarrow{QR}^2$

(ii) $QS^2 = \overrightarrow{QS}^2 = (\overrightarrow{QR} + \overrightarrow{RS})^2 = \overrightarrow{QR}^2 + 2\overrightarrow{QR}.\overrightarrow{RS} + \overrightarrow{RS}^2 = \overrightarrow{QR}^2 + \overrightarrow{RS}^2$ [$\because \overrightarrow{QR} \perp \overrightarrow{RS}$]

$\therefore \overrightarrow{QS}^2 = \overrightarrow{RS}^2 + \overrightarrow{QR}^2$

(iii) $PR^2 = QS^2$ [From (i) and (ii)] $\therefore PR = QS$

Thus, the diagonals of a rectangle are equal to each other.

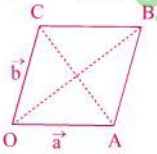
VECTOR

www.drmathnp.com

OR आयत NEWS का विकर्णहरू बराबर हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove by the vector method that the diagonals of rectangle NEWS are equal.

9. समबाहु चतुर्भुजका विकर्णहरू समकोण हुने गरी प्रतिच्छेदन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals of rhombus intersect to each other at a right angle. **[2065 R']**

HINT: Let OABC be a rhombus and OB and AC be its diagonals.



Let O be the origin, $\overrightarrow{AB} = \overrightarrow{OC} = \vec{b}$ and $\overrightarrow{OA} = \overrightarrow{CB} = \vec{a}$
From the triangle law of vector and parallelogram law of vector,

(i) $\overrightarrow{CA} = \vec{a} - \vec{b}$ and $\overrightarrow{OB} = \vec{a} + \vec{b}$

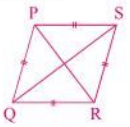
(ii) $\overrightarrow{CA} \cdot \overrightarrow{OB} = (\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = (\vec{a})^2 - (\vec{b})^2 = a^2 - b^2$ [$\because$ the square of any vector is a scalar]
$= a^2 - a^2$ [$b^2 = a^2$ since the sides of a rhombus]

$\therefore \quad \overrightarrow{CA} \cdot \overrightarrow{OB} = 0$

Hence, the dot product of the diagonals of the rhombus is 0. So, the diagonals are perpendicular to each other.

OR समबाहु चतुर्भुज PQRS का विकर्णहरू PR र QS समकोणमा प्रतिच्छेदन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals PR and QS of the rhombus PQRS intersect at right angle. **[2070 S']**

OR दिइएको चित्रमा PQ = QR = RS = SP भए भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : PR ⊥ QS
In the given figure, PQ = QR = RS = SP. Prove by vector method: PR ⊥ QS. **[2072 R]**



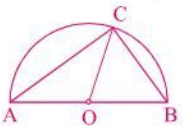
OR समबाहु चतुर्भुज REST का विकर्णहरू एक आपसमा समकोण भई प्रतिच्छेदित हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् । के उक्त अवस्था सबै प्रकारका समानान्तर चतुर्भुजहरूमा हुन्छ ? लेख्नुहोस् ।
Prove vectorically that the diagonals of rhombus REST are intersected at right angle. Is this condition satisfies in each type of parallelogram ? Write it.

Model 4 *Theorems based on semicircle.*

10. अर्धवृत्त (वृत्तार्ध) मा बन्ने परिधिकोण समकोण हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove vectorically that the angle at the circumference of a semi circle is a right angle. **[SEE 2074 S', 2057 R]**

OR व्यास AB भएको एउटा अर्धवृत्तको परिधिकोण ACB एक समकोण हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove by vector method that the circumference angle ACB of a semi-circle with a diameter AB is a right-angle. **[SEE 2075 R, 2075 R_2]**

HINT: Let ACB be a semi-circle with centre O. Let O be the origin. AB is diameter of semicircle. Join O and C.



(i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (Triangle law of vector)

(ii) $\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$ (Triangle law of vector)

(iii) $\overrightarrow{BC} \cdot \overrightarrow{CA} = (\overrightarrow{OC} - \overrightarrow{OB}) \cdot (\overrightarrow{OA} - \overrightarrow{OC})$
$= (\overrightarrow{OC} + \overrightarrow{OA}) \cdot (\overrightarrow{OA} - \overrightarrow{OC})$ [$\overrightarrow{OA} = -\overrightarrow{OB}$ [Being radii and in opposite direction.]
$= (\overrightarrow{OA})^2 - (\overrightarrow{OC})^2 = OA^2 - OC^2$

(iv) $\overrightarrow{BC} \cdot \overrightarrow{CA} = OC^2 - OC^2 = 0$ [$\because$ OA = OC = radii of circle] $\therefore \angle BCA = 90°$

OR अर्धवृत्त CAT मा बनेको परिधि कोण ∠CAT एक समकोण हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove vectorically that the angle at the circumference ∠CAT of a semicircle CAT is a right angle.

ANSWERS

A. VERY SHORT QUESTIONS

4A. Centroid **4B.** $2\vec{i} + 3\vec{j}$ **7A.** 90° **7B.** 0 **8A.** $\overrightarrow{BO} = \overrightarrow{OD}$ **8B.** $\overrightarrow{OM} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ **9A.** $4\vec{i} + 6\vec{j}$ **9B.** $3\vec{i} + 4\vec{j}$

B. SHORT QUESTIONS

1C. $\overrightarrow{AC} = \vec{p} - \vec{a}, \vec{p} = \frac{\vec{a} + \vec{b}}{2}$ **2A.** $4\vec{i} + 3\vec{j}$ **2B.** $\overrightarrow{OM} = \vec{i} + 2\vec{j}$ **2C.** $\frac{7}{2}\vec{i} + \frac{9}{2}\vec{j}$ **2D.** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **3A.** $5\vec{i} - \vec{j}$

3B. $-\vec{i} + 3\vec{j}$ **3C.** $6\vec{i} + 4\vec{j}$ **3D.** $5\vec{i} + 2\vec{j}$ **3E.** $6\vec{i} + 4\vec{j}$ **3F.** $5\vec{i} + 6\vec{j}$ **3G.** $\vec{i} + 3\vec{j}$

3H. $11\vec{i} + 18\vec{j}$ **4A.** $\overrightarrow{OD} = \frac{2\vec{b} + 3\vec{a}}{5}$ **4B.** $\vec{p} = 3\vec{i} - \vec{j}$ **4C.** $\frac{18}{5}\vec{i} + \frac{13}{5}\vec{j}$ **4D.** $4\vec{i} + 3\vec{j}$ **4E.** $\frac{1}{5}(21\vec{i} + 18\vec{j})$

4F. $\frac{1}{5}(21\vec{i} + 18\vec{j})$ **4G.** $\sqrt{65}, \frac{7}{4}\vec{i} + 3\vec{j}$ **6A.** $-2\vec{a} + 3\vec{b}$ **7A.** $3\vec{i} + \vec{j}$ **7B.** $4\vec{i} + 5\vec{j}$ **7C.** $3\vec{i} + 4\vec{j}$

7D. $\frac{10}{3}\vec{i} + \frac{8}{3}\vec{j}$ **7E.** $3\vec{i} - \vec{j}$ **12A.** $2\vec{i} + 7\vec{j}$ **12B.** $\vec{a} - \vec{b} + \vec{c}$ **12C.** $\vec{a} - \vec{b} + \vec{c}$

puspas.com.np

www.drmathnp.com

puspas.com.np

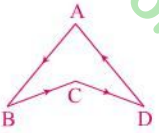
GENERAL EXERCISE 2

A. SHORT QUESTIONS

Model 1 *Problems based on sum of vectors represented by sides of polygon.*

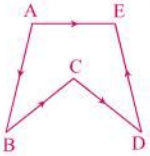
1A. संगैको चित्रमा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the adjoining figure, prove that: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$



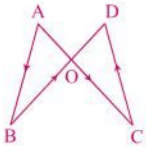
1B. दिइएको चित्रमा $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given figure, prove that: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$



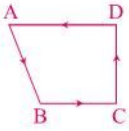
1C. दिइएको चित्रमा $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{BD}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the given figure, prove that: $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{BD}$.



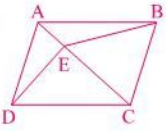
1D. दिइएको चित्रमा $\overrightarrow{AD} = 2\overrightarrow{BC}$ भए $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{BC}$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the figure, if $\overrightarrow{AD} = 2\overrightarrow{BC}$ then prove that: $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{BC}$



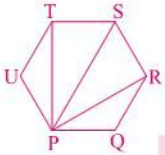
1E. दिइएको चित्रमा, ABCD एउटा समानान्तर चतुर्भुज हो । यदि विकर्ण AC मा कुनै बिन्दु E भए $\overrightarrow{BE} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{CE}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the figure, ABCD is a parallelogram. If E is any point on the diagonal AC then prove that: $\overrightarrow{BE} = \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{CE}$



1F. यदि PQRSTU एउटा नियमित षड्भुज भए प्रमाणित गर्नुहोस्: $\overrightarrow{PR} + \overrightarrow{PS} + \overrightarrow{PT} = 5\overrightarrow{QR}$

If PQRSTU is regular hexagon then prove that: $\overrightarrow{PR} + \overrightarrow{PS} + \overrightarrow{PT} = 5\overrightarrow{QR}$



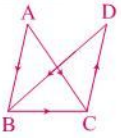
HINT: $\overrightarrow{PR} + \overrightarrow{PS} + \overrightarrow{PT} = (\overrightarrow{PQ} + \overrightarrow{QR}) + 2\overrightarrow{QR} + (\overrightarrow{PS} + \overrightarrow{ST})$

$= 3\overrightarrow{QR} + \overrightarrow{PQ} + (2\overrightarrow{QR} + \overrightarrow{QP}) = 5\overrightarrow{QR} + \overrightarrow{PQ} + \overrightarrow{QP} = 5\overrightarrow{QR}$

Model 2 *Problems based on triangle law of vector addition and subtraction.*

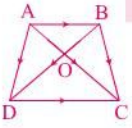
2A. दिइएको चित्रमा $\overrightarrow{AB} - \overrightarrow{CD} - \overrightarrow{AC} = \overrightarrow{DB}$ हुन्छ भनी सिद्ध गर्नुहोस् ।

In the given figure, prove that: $\overrightarrow{AB} - \overrightarrow{CD} - \overrightarrow{AC} = \overrightarrow{DB}$



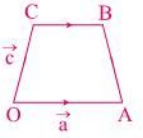
2B. दिइएको चित्रमा $\overrightarrow{AO} - \overrightarrow{BO} + \overrightarrow{BC} = \overrightarrow{AC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the given figure, prove that: $\overrightarrow{AO} - \overrightarrow{BO} + \overrightarrow{BC} = \overrightarrow{AC}$



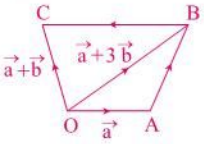
2C. चित्रमा $\overrightarrow{OA} = 2\overrightarrow{CB} = \vec{a}$ र $\overrightarrow{OC} = \vec{c}$ भए $\overrightarrow{OB}$ र $\overrightarrow{BA}$ लाई $\vec{a}$ र $\vec{c}$ को रूपमा पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{OA} = 2\overrightarrow{CB} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$. Find $\overrightarrow{OB}$ and $\overrightarrow{BA}$ in terms of $\vec{a}$ and $\vec{c}$.



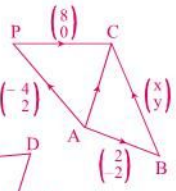
2D. चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{a} + \vec{b}$ र $\overrightarrow{OB} = \vec{a} + 3\vec{b}$ भए $\overrightarrow{BC}$ र $\overrightarrow{AB}$ को मान $\vec{a}$ र $\vec{b}$ को रूपमा पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{a} + \vec{b}$ and $\overrightarrow{OB} = \vec{a} + 3\vec{b}$ then find $\overrightarrow{BC}$ and $\overrightarrow{AB}$ interms of $\vec{a}$ and $\vec{b}$.



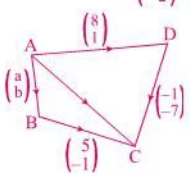
2E. चित्रमा दिइएको जानकारीबाट x र y का मानहरू पत्ता लगाउनुहोस् ।

From the given information in the figure, find the values of x and y.



2F. चित्रमा दिइएको जानकारीबाट a र b का मानहरू पत्ता लगाउनुहोस् ।

From the given information in the figure, find the values of a and b.

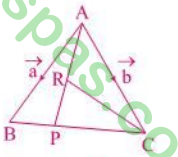


www.drmathnp.com

puspas.com.np

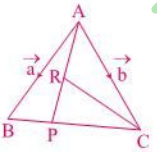
2G. सँगैको चित्रमा BC को एउटा बिन्दु P छ जहाँ 3PB = 2PC र AP को मध्यबिन्दु R छ। यदि $\overrightarrow{AB} = \vec{a}$ र $\overrightarrow{AC} = \vec{b}$ भए $\overrightarrow{CR}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा पत्ता लगाउनुहोस्।

In the diagram, P is a point on BC such that 3PB = 2PC and R is the mid-point of AP. If $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$, find $\overrightarrow{CR}$ in terms of $\vec{a}$ and $\vec{b}$.



2H. ΔABC मा 5BP = 2PC हुने गरी BC मा एउटा बिन्दु P छ र AP को मध्यबिन्दु R हो। यदि $\overrightarrow{AB} = \vec{a}$ र $\overrightarrow{AC} = \vec{b}$ भए $\overrightarrow{CR}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा पत्ता लगाउनुहोस्।

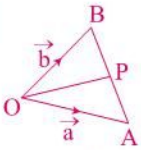
In ΔABC, P is a point on BC such that 5 BP = 2 PC and R is the mid-point of AP. If $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AC} = \vec{b}$, find $\overrightarrow{CR}$ in terms of $\vec{a}$ and $\vec{b}$.



Model 3 Problems based on mid-point theorem.

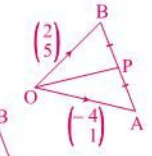
3A. चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र $\overrightarrow{AB} = 2\overrightarrow{AP}$ भए $\overrightarrow{OP}$ को मान $\vec{a}$ र $\vec{b}$ को रूपमा पत्ता लगाउनुहोस्।

In the figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{AB} = 2\overrightarrow{AP}$, find the value of $\overrightarrow{OP}$ in terms of $\vec{a}$ and $\vec{b}$.



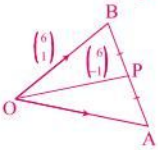
3B. सँगैको चित्रबाट बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस्।

From the adjoining figure, find the position vector of the point P.



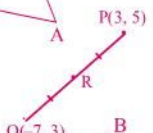
3C. सँगैको चित्रबाट बिन्दु A को स्थिति भेक्टर पत्ता लगाउनुहोस्।

From the adjoining figure, find the position vector of the point A.



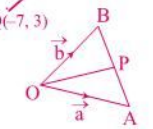
3D. चित्रमा बिन्दुहरू P र Q का निर्देशाङ्कहरू क्रमशः (3, 5) र (– 7, 3) छन्। रेखा PQ को मध्यबिन्दु R को निर्देशाङ्क पत्ता लगाउनुहोस्।

In the figure, co-ordinates of points P and Q are (3, 5) and (– 7, 3) respectively. Find the co-ordinates of midpoint R of PQ.



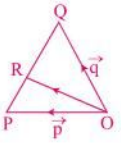
3E. चित्रमा AB को मध्यबिन्दु P भए प्रमाणित गर्नुहोस्: $\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OP}$

In the given figure, P is mid point of AB then prove that: $\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OP}$



3F. यदि $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ र बिन्दु R ले PQ लाई 2PR = PQ हुनेगरी काट्छ भने $\overrightarrow{OR}$ लाई $\vec{p}$ र $\vec{q}$ को रूपमा निकाल्नुहोस्।

If $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ and the point R divides PQ internally such that 2PR = PQ, then find $\overrightarrow{OR}$ interms of $\vec{p}$ and $\vec{q}$.



4A. A र B का स्थिति भेक्टरहरू क्रमशः $9\vec{i} + 2\vec{j}$ र $1\vec{i} + 3\vec{j}$ छन्। $\overrightarrow{AB} = \overrightarrow{BP}$ हुने गरी बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस्।

The position vector of A is $9\vec{i} + 2\vec{j}$ and the position vector of B is $1\vec{i} + 3\vec{j}$. Find the position vector of a point P such that $\overrightarrow{AB} = \overrightarrow{BP}$.

4B. यदि $\overrightarrow{OA} = 2\vec{i} - 3\vec{j}$ र $\overrightarrow{OB} = 3\vec{i} - 2\vec{j}$ भए $\overrightarrow{AB}$ को मध्यबिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस्।

If $\overrightarrow{OA} = 2\vec{i} - 3\vec{j}$ and $\overrightarrow{OB} = 3\vec{i} - 2\vec{j}$ then find the position vector of the mid-point of $\overrightarrow{AB}$.

Model 4 Problems based on section formula of internal and external division.

5A. P र Q का स्थिति भेक्टरहरू क्रमशः $\begin{pmatrix}1\\6\end{pmatrix}$ र $\begin{pmatrix}6\\1\end{pmatrix}$ छन्। PQ लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्ने बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस्।

The position vector of P and Q are $\begin{pmatrix}1\\6\end{pmatrix}$ and $\begin{pmatrix}6\\1\end{pmatrix}$ respectively. Find the position vector of a point which divides PQ internally in the ratio of 3 : 2.

www.drmathnp.com

5B. A र B का स्थिति भेक्टरहरू क्रमशः $2\vec{i}+7\vec{j}$ र $4\vec{i}-3\vec{j}$ छन् । AB लाई भित्रपट्टिबाट 2 : 3 को अनुपातमा विभाजन गर्ने बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vector of A and B are $2\vec{i}+7\vec{j}$ and $4\vec{i}-3\vec{j}$ respectively. Find the position vector of a point which divides AB internally in the ratio of 2 : 3.

5C. R ले PQ लाई 3 : 5 को अनुपातमा विभाजन गरेको छ । यदि बिन्दुहरू P र Q का स्थिति भेक्टरहरू क्रमशः $6\vec{i}+4\vec{j}$ र $2\vec{i}-8\vec{j}$ भए R को स्थिति भेक्टर निकाल्नुहोस् ।

R divides PQ in the ratio of 3 : 5. If the position vectors of points P and Q are $6\vec{i}+4\vec{j}$ and $2\vec{i}-8\vec{j}$ respectively, find the position vector of R.

5D. A र B का स्थिति भेक्टरहरू क्रमशः $\vec{a}=3\vec{i}+8\vec{j}$ र $\vec{b}=3\vec{i}-2\vec{j}$ छन् । C ले AB लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्दछ । C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vectors of A and B are $\vec{a}=3\vec{i}+8\vec{j}$ and $\vec{b}=3\vec{i}-2\vec{j}$ respectively. C divides AB in the ratio of 3 : 2 internally. Find the position vector of C.

5E. यदि $\overrightarrow{OP}=\vec{p}$, $\overrightarrow{OQ}=\vec{q}$ र M ले PQ लाई भित्रपट्टिबाट 2PM = PQ हुने गरी काट्छ भने $\overrightarrow{OM}$ लाई $\vec{p}$ र $\vec{q}$ को रूपमा व्यक्त गर्नुहोस् ।

If $\overrightarrow{OP}=\vec{p}$, $\overrightarrow{OQ}=\vec{q}$ and M divides PQ internally so that 2PM = PQ, find $\overrightarrow{OM}$ in term of $\vec{p}$ & $\vec{q}$.

5F. बिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् जसले A(4, 6) र B(– 1, 6) जोड्ने रेखाखण्डलाई 2 : 3 को अनुपातमा भित्रपट्टिबाट विभाजन गर्दछ ।

Find the position vector of a point M which divides the joint of A(4, 6) and B(– 1, 6) in the ratio of 2 : 3 internally.

6A. रेखा AB लाई बाहिरपट्टीबाट 2 : 5 को अनुपातमा विभाजन गर्ने बिन्दुको स्थिति भेक्टर पत्ता लगाउनुहोस् जहाँ $\overrightarrow{OA}=\begin{pmatrix}2\\4\end{pmatrix}$ र $\overrightarrow{OB}=\begin{pmatrix}5\\1\end{pmatrix}$ छन् ।

Find the position vector of a point which divides the line AB externally in the ratio of 2 : 5 where $\overrightarrow{OA}=\begin{pmatrix}2\\4\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}5\\1\end{pmatrix}$.

6B. यदि $\overrightarrow{OM}=-\vec{i}+5\vec{j}$, $\overrightarrow{ON}=3\vec{i}-2\vec{j}$ र P ले MN लाई बाहिरपट्टिबाट 3 : 1 को अनुपातमा विभाजन गर्दछ भने $\overrightarrow{OP}$ पत्ता लगाउनुहोस् ।

If $\overrightarrow{OM}=-\vec{i}+5\vec{j}$, $\overrightarrow{ON}=3\vec{i}-2\vec{j}$ and P divides MN externally in the ratio of 3 : 1, find $\overrightarrow{OP}$.

6C. ΔABC मा D र E ले क्रमशः AB र AC लाई 1 : 3 को अनुपातमा विभाजन गर्दछ भने $\overrightarrow{DE}=\frac{1}{4}\overrightarrow{BC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In ΔABC, D and E divide AB and AC in the ratio of 1 : 3 respectively, prove that $\overrightarrow{DE}=\frac{1}{4}\overrightarrow{BC}$.

6D. ΔABC मा D र E ले क्रमशः AB र AC लाई 1 : 4 को अनुपातमा विभाजन गर्दछ भने $\overrightarrow{DE}=\frac{1}{5}\overrightarrow{BC}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In ΔABC, D and E divide AB and AC in the ratio of 1 : 4 respectively, prove that $\overrightarrow{DE}=\frac{1}{5}\overrightarrow{BC}$.

7A. AB रेखाखण्डलाई P ले कुन अनुपातमा विभाजन गर्दछ ? जहाँ $\overrightarrow{OA}=4\vec{i}+4\vec{j}$, $\overrightarrow{OB}=7\vec{i}+7\vec{j}$ र $\overrightarrow{OP}=-\vec{i}-\vec{j}$ छन् ।

In what ratio does P divide the line segment AB where, $\overrightarrow{OA}=4\vec{i}+4\vec{j}$, $\overrightarrow{OB}=7\vec{i}+7\vec{j}$ and $\overrightarrow{OP}=-\vec{i}-\vec{j}$.

7B. AB रेखाखण्डलाई P ले कुन अनुपातमा विभाजन गर्दछ ? जहाँ $\overrightarrow{OA}=\vec{i}-3\vec{j}$, $\overrightarrow{OB}=2\vec{i}+5\vec{j}$ र $\overrightarrow{OP}=\frac{7}{4}\vec{i}+3\vec{j}$ छन् ।

In what ratio does P divide the line segment AB where, $\overrightarrow{OA}=\vec{i}-3\vec{j}$, $\overrightarrow{OB}=2\vec{i}+5\vec{j}$ and $\overrightarrow{OP}=\frac{7}{4}\vec{i}+3\vec{j}$.

puspas.com.np

www.drmathnp.com

puspas.com.np

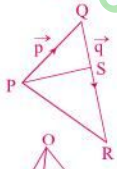
8A. चित्रमा $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र $\overrightarrow{AB} = 3\overrightarrow{AP}$ भए $\overrightarrow{OP}$ को मान $\vec{a}$ र $\vec{b}$ को रूपमा पत्ता लगाउनुहोस् ।

In the figure, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{AB} = 3\overrightarrow{AP}$, find the value of $\overrightarrow{OP}$ in terms of $\vec{a}$ and $\vec{b}$.



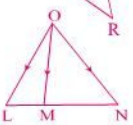
8B. चित्रमा $\overrightarrow{PQ} = \vec{p}$ र $\overrightarrow{QS} = \vec{q}$ दिइएको छ । यदि $\frac{QS}{SR} = \frac{1}{3}$ भए $\overrightarrow{PR}$ लाई $\vec{p}$ र $\vec{q}$ को रूपमा व्यक्त गर्नुहोस् ।

In the diagram, $\overrightarrow{PQ} = \vec{p}$ & $\overrightarrow{QS} = \vec{q}$. If $\frac{QS}{SR} = \frac{1}{3}$, express $\overrightarrow{PR}$ in terms of $\vec{p}$ and $\vec{q}$.



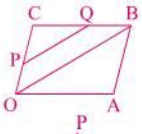
8C. चित्रमा $\overrightarrow{OL} = \vec{\ell}$, $\overrightarrow{OM} = \vec{m}$ र $\overrightarrow{ON} = 3\vec{m} - 2\vec{\ell}$ भए $\overrightarrow{LN} = 3\overrightarrow{LM}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the figure, $\overrightarrow{OL} = \vec{\ell}$, $\overrightarrow{OM} = \vec{m}$ and $\overrightarrow{ON} = 3\vec{m} - 2\vec{\ell}$ then prove that: $\overrightarrow{LN} = 3\overrightarrow{LM}$



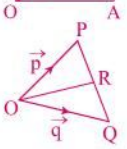
8D. चित्रमा $\frac{OP}{PC} = \frac{CQ}{QP} = \frac{1}{3}$ भए प्रमाणित गर्नुहोस् $\overrightarrow{PQ} \parallel \overrightarrow{OB}$.

In the figure, $\frac{OP}{PC} = \frac{CQ}{QP} = \frac{1}{3}$ then prove that $\overrightarrow{PQ} \parallel \overrightarrow{OB}$.



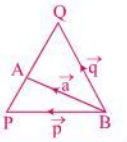
8E. चित्रमा $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ र $\overrightarrow{PQ} = 4\overrightarrow{RQ}$ भए $\overrightarrow{OR}$ लाई $\vec{p}$ र $\vec{q}$ को रूपमा व्यक्त गर्नुहोस् ।

In the given diagram, if $\overrightarrow{OP} = \vec{p}$, $\overrightarrow{OQ} = \vec{q}$ and $\overrightarrow{PQ} = 4\overrightarrow{RQ}$ then find $\overrightarrow{OR}$ in terms of $\vec{p}$ and $\vec{q}$.



8F. दिइएको चित्रमा $PA = \frac{1}{4}PQ$ भए $\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If the given figure, if $PA = \frac{1}{4}PQ$, prove that $\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$



VECTOR

Model 5 Problems based on parallelogram law of vector addition.

9A. स. च. OABC मा $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ भए बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In a parm. OABC, $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ Find the position vector of the point C.

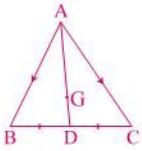
9B. स. च. OABC मा $\overrightarrow{OA} = 3\vec{i} + \vec{j}$ र $\overrightarrow{OB} = 4\vec{i} + 5\vec{j}$ भए बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In the parm. OABC, $\overrightarrow{OA} = 3\vec{i} + \vec{j}$ and $\overrightarrow{OB} = 4\vec{i} + 5\vec{j}$. Find the position vector of the point C.

Model 6 Problems based on the position vector of centroid of a triangle.

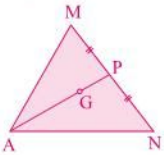
10A. चित्रमा AD मध्यिका हो जहाँ A र D का स्थिति भेक्टरहरू क्रमशः $3\vec{i} - 2\vec{j}$ र $-3\vec{i} - 4\vec{j}$ छन् । गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In the figure, AD is median where the position vectors of A and D are $3\vec{i} - 2\vec{j}$ and $-3\vec{i} - 4\vec{j}$ respectively. Find the position vector of centroid G.



10B. सँगैको चित्रमा AP, ΔAMN को मध्यिका हो । A र P का स्थिति भेक्टरहरू क्रमशः $3\vec{i} - 5\vec{j}$ र $-2\vec{i} - 4\vec{j}$ छन् भने गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In the adjoining figure, AP is a median of ΔAMN. The position vectors of A and P are $3\vec{i} - 5\vec{j}$ and $-2\vec{i} - 4\vec{j}$ respectively. Find the position vector of the centroid G.



10C. यदि A(3, 2) र B(8, 6) एउटा त्रिभुजको मध्यिकाको अन्तिम बिन्दुहरू हुन् भने सो त्रिभुजको गुरुत्वकेन्द्रको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If A(3, 2) and B(8, 6) are the end points of a median of a triangle, find the position vector of the centroid of the triangle.

10D. यदि बिन्दुहरू A(–1, –1), B(5, 1) र C(2, 6) हरू ΔABC का शीर्षहरू भए यसको गुरुत्वकेन्द्रको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

If the points A(–1, –1), B(5, 1) and C(2, 6) are the vertices of triangle ABC, find the position vector of its centroid.

www.drmathnp.com

puspas.com.np

10E. ΔABC मा $\overrightarrow{OA} = 5\vec{i} + 4\vec{j}$, $\overrightarrow{OB} = 6\vec{i} - 2\vec{j}$ र $\overrightarrow{OC} = -\vec{i} + 3\vec{j}$ छन् । ΔABC को गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In ΔABC, $\overrightarrow{OA} = 5\vec{i} + 4\vec{j}$, $\overrightarrow{OB} = 6\vec{i} - 2\vec{j}$ and $\overrightarrow{OC} = -\vec{i} + 3\vec{j}$. Find the position vector of centroid G of ΔABC.

10F. ΔABC मा $\overrightarrow{OA} = 6\vec{i} - 2\vec{j}$, $\overrightarrow{OB} = -\vec{i} + 3\vec{j}$ र $\overrightarrow{OC} = 5\vec{i} + 4\vec{j}$ छन् । ΔABC को गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

In ΔABC, $\overrightarrow{OA} = 6\vec{i} - 2\vec{j}$, $\overrightarrow{OB} = -\vec{i} + 3\vec{j}$ and $\overrightarrow{OC} = 5\vec{i} + 4\vec{j}$. Find the position vector of centroid G of ΔABC.

10G. ΔABC का दुई शीर्षहरूको स्थिति भेक्टरहरू $\overrightarrow{OA} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ छन् । यदि गुरुत्वकेन्द्रको स्थिति भेक्टर $\overrightarrow{OG} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ भए बाँकी शीर्षको स्थिति भेक्टर पत्ता लगाउनुहोस् ।

The position vector of two vertices of ΔABC are $\overrightarrow{OA} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$. If the position vector of centroid be $\overrightarrow{OG} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, find the position vector of remaining vertex.

10H. ΔABC मा $\overrightarrow{OA} = 3\vec{i} - 5\vec{j}$, $\overrightarrow{OB} = -7\vec{i} + 4\vec{j}$ र गुरुत्व केन्द्र G को स्थिति भेक्टर $\overrightarrow{OC} = 2\vec{i} + \vec{j}$ भए $\overrightarrow{OC}$ पत्ता लगाउनुहोस् ।

In triangle ABC, $\overrightarrow{OA} = 3\vec{i} - 5\vec{j}$, $\overrightarrow{OB} = -7\vec{i} + 4\vec{j}$ and the position vector of centroid G is $2\vec{i} + \vec{j}$. Find $\overrightarrow{OC}$.

10I. ΔABC मा $\overrightarrow{OA} = 4\vec{i} - 5\vec{j}$, $\overrightarrow{OB} = 6\vec{i} + 4\vec{j}$ र गुरुत्वकेन्द्र G को स्थिति भेक्टर $\overrightarrow{OG} = 2\vec{i} + \vec{j}$ भए $\overrightarrow{OC}$ पत्ता लगाउनुहोस् ।

In ΔABC, $\overrightarrow{OA} = 4\vec{i} - 5\vec{j}$, $\overrightarrow{OB} = 6\vec{i} + 4\vec{j}$ and the position vector of the centroid G is $\overrightarrow{OG} = 2\vec{i} + \vec{j}$. Find $\overrightarrow{OC}$.

VECTOR

Model 7 ***Problems based on application of vector geometry.***

11A. भेक्टरहरू $\vec{a} = 4\vec{i} + 6\vec{j}$ र $\vec{b} = 8\vec{i} + 12\vec{j}$ समानान्तर भेक्टरहरू हुन् भनी प्रमाणित गर्नुहोस् ।

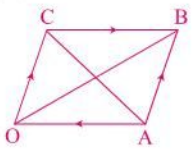
Prove that the vectors $\vec{a} = 4\vec{i} + 6\vec{j}$ and $\vec{b} = 8\vec{i} + 12\vec{j}$ are parallel vectors.

11B. भेक्टरहरू $\vec{a} = 2\vec{i} - 3\vec{j}$ र $\vec{b} = -4\vec{i} + 6\vec{j}$ समानान्तर र असमान भेक्टरहरू हुन् भनी प्रमाणित गर्नुहोस् ।

Prove that the vectors $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = -4\vec{i} + 6\vec{j}$ are parallel and unlike vectors.

12A. यदि OABC एउटा समानान्तर चतुर्भुज भए $\overrightarrow{AC} - \overrightarrow{OB} = 2\overrightarrow{AO}$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If OABC is a parallelogram then prove that: $\overrightarrow{AC} - \overrightarrow{OB} = 2\overrightarrow{AO}$

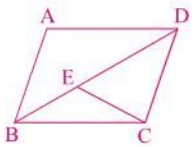


12B. यदि AC र BD समानान्तर चतुर्भुजका विकर्णहरू भए सिद्ध गर्नुहोस् : $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$.

If AC and BD are the diagonals of a parallelogram ABCD, show that $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$.

12C. सर्गैंको चित्रमा ABCD एउटा समानान्तर चतुर्भुज हो । यदि $\overrightarrow{BA} = \vec{a}$, $\overrightarrow{EB} = \frac{1}{2}\vec{b}$ र DE : EB = 2 : 1 भए $\overrightarrow{CE}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा व्यक्त गर्नुहोस् ।

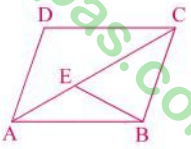
In the adjoining figure, ABCD is a parallelogram. If $\overrightarrow{BA} = \vec{a}$, $\overrightarrow{EB} = \frac{1}{2}\vec{b}$ and DE : EB = 2 : 1 then express $\overrightarrow{CE}$ in terms of $\vec{a}$ and $\vec{b}$.



www.drmathnp.com

12D. सर्गैको चित्रमा ABCD एक समानान्तर चतुर्भुज हो । यदि $\overrightarrow{DC} = \vec{b} - \vec{a}$, $\overrightarrow{EC} = 3\vec{x}$ र AE : EC = 1 : 3 भए $\overrightarrow{BE}$ लाई $\vec{a}$, $\vec{b}$ र $\vec{x}$ का रूपमा व्यक्त गर्नुहोस् ।

In the adjoining figure, ABCD is a parallelogram. If $\overrightarrow{DC} = \vec{b} - \vec{a}$, $\overrightarrow{EC} = 3\vec{x}$ and AE : EC = 1 : 3, express $\overrightarrow{BE}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{x}$.



B. LONG QUESTIONS

1. मध्यबिन्दु सूत्र उल्लेख गरी भेक्टर विधिबाट प्रतिपादन गर्नुहोस् । (State and prove mid-point formula by the vector method.)
2. भित्री विभाजन सूत्र उल्लेख गरी भेक्टर विधिबाट प्रतिपादन गर्नुहोस् ।
   State and prove Internal section formula by the vector method.
3. बाहिरी विभाजन सूत्र उल्लेख गरी भेक्टर विधिबाट प्रतिपादन गर्नुहोस् ।
   State and prove external section formula by the vector method.

ANSWERS

A. SHORT QUESTIONS

2C. $\overrightarrow{OB} = \vec{c} + \frac{1}{2}\vec{a}$, $\overrightarrow{BA} = \frac{\vec{a}}{2} - \vec{c}$ **2D.** $\overrightarrow{BC} = -2\vec{b}$, $\overrightarrow{AB} = 3\vec{b}$ **2E.** $x = 2, y = 4$ **2F.** $2, -5$

2G. $\frac{1}{10}(3\vec{a} - 8\vec{b})$ **2H.** $\overrightarrow{CR} = \frac{1}{14}(5\vec{a} - 12\vec{b})$ **3A.** $\frac{1}{2}(\vec{a} + \vec{b})$ **3B.** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **3C.** $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

3D. $(-2, 4)$ **3F.** $\frac{\vec{p} + \vec{q}}{2}$ **4A.** $-7\vec{i} + 4\vec{j}$ **4B.** $\frac{5}{2}\vec{i} - \frac{5}{2}\vec{j}$ **5A.** $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **5B.** $\frac{14}{5}\vec{i} + 3\vec{j}$

5C. $\frac{9\vec{i} - \vec{j}}{2}$ **5D.** $3\vec{i} + 2\vec{j}$ **5E.** $\frac{1}{2}(\vec{p} + \vec{q})$ **5F.** $2\vec{i} + 6\vec{j}$ **6A.** $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$ **6B.** $5\vec{i} - \frac{11}{2}\vec{j}$

7A. 5: 8 externally **7B.** 3 : 1 **8A.** $\frac{1}{3}(2\vec{a} + \vec{b})$ **8B.** $\vec{p} + 4\vec{q}$ **8E.** $\overrightarrow{OR} = \frac{1}{4}(\vec{p} + 3\vec{q})$ **9A.** $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

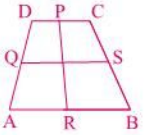
9B. $\overrightarrow{OC} = \vec{i} + 4\vec{j}$ **10A.** $-\vec{i} - \frac{10}{3}\vec{j}$ **10B.** $-\frac{1}{3}\vec{i} - \frac{13}{3}\vec{j}$ **10C.** $\frac{19}{3}\vec{i} + \frac{14}{3}\vec{j}$ **10D.** $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **10E.** $\frac{10}{3}\vec{i} + \frac{5}{3}\vec{j}$

10F. $\frac{10}{3}\vec{i} + \frac{5}{3}\vec{j}$ **10G.** $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$ **10H.** $10\vec{i} + 4\vec{j}$ **10I.** $4(-\vec{i} + \vec{j})$ **12C.** $\vec{a} + \vec{b}$ **12D.** $\vec{a} - \vec{b} + \vec{x}$

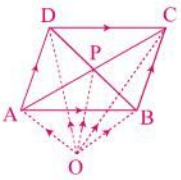
CHALLENGING EXERCISE 1

निम्नलिखित समस्याहरू पाठ्यक्रममा नभए तापनि केवल थप अभ्यासको लागि मात्र राखिएको हो ।

1. चित्रमा P, Q, R र S क्रमशः चतुर्भुज ABCD को भुजा CD, DA, AB र BC का मध्य बिन्दुहरू भए PR र QS आपसमा समद्विभाजित हुन्छन् भनी सिद्ध गर्नुहोस् ।
In the given figure, P, Q, R and S are the mid points of the sides CD, DA, AB and BC respectively of the quadrilateral ABCD. Prove that PR and QS bisect each other.

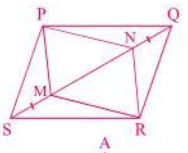


2. ABCD एउटा स. च. हो । विकर्णहरू बिन्दु G मा प्रतिच्छेदन भएका छन् । यदि O कुनै बिन्दु भए प्रमाणित गर्नुहोस्: $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4.\overrightarrow{OG}$
ABCD is a parallelogram and G is the point of intersection of its diagonals. If O is any point, prove that : $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4.\overrightarrow{OG}$ **[2060 R]**

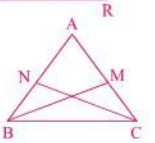


3. ABCD एउटा चर्तुभुज हो । सिद्ध गर्नुहोस् : $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{PQ}$ जसमा P र Q बिन्दुहरू क्रमशः AC र BD का मध्यबिन्दु हुन् ।
ABCD is a quadrilateral. Show that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{PQ}$ where P and Q are the midpoints of AC and BD respectively.

4. चित्रमा PQRS एउटा समानान्तर चतुर्भुज हो । विकर्ण SQ मा M र N दुई बिन्दुहरू छन् । यदि SM = NQ भए भेक्टर विधिबाट PMRN एउटा समानान्तर चतुर्भुज हो भनी सिद्ध गर्नुहोस् ।
In the given figure, PQRS is a parallelogram. M and N are two points on the diagonal SQ. If SM = NQ, prove by vector method that PMRN is a parallelogram.



5. सँगैको चित्रमा BM र CN त्रिभुज ACB को मध्यिकाहरू हुन् । यदि BM = CN भए त्रिभुज ACB समद्विबाहु त्रिभुज हो भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In the adjoining figure, BM and CN are the medians of the ΔACB. If BM = CN, prove by vector method that the ΔACB is an isosceles.



puspas.com.np

www.drmathnp.com

puspas.com.np

ASSIGNMENT ON VECTOR

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

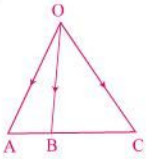
Attempt all the Questions.

Group 'A' $3 \times 1 = 3$

1. स्केलार गुणनफलको परिभाषा दिनुहोस् । (Define scalar product.)
2. भित्री विभाजन सम्बन्धी खण्ड सूत्र लेख्नुहोस् । (Write the section formula of internal division.)
3. यदि $\vec{a} \cdot \vec{b} = 0$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् । (If $\vec{a} \cdot \vec{b} = 0$, write the angle between $\vec{a}$ and $\vec{b}$.)

Group B $4 \times 2 = 8$

4. यदि $\overrightarrow{OA} = \begin{pmatrix} \sqrt{3} \\ 7 \end{pmatrix}$ र $\overrightarrow{OB} = \begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$ भए कोण AOB पत्ता लगाउनुहोस् ।
   If $\overrightarrow{OA} = \begin{pmatrix} \sqrt{3} \\ 7 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$ calculate the angle AOB.
5. यदि $\vec{a} = 2\vec{i} - p\vec{j}$ र $\vec{b} = \vec{i} - 3\vec{j}$ बीचको कोण 45° भए p को मान पत्ता लगाउनुहोस् ।
   If the angle between $\vec{a} = 2\vec{i} - p\vec{j}$ and $\vec{b} = \vec{i} - 3\vec{j}$ is 45° then what is the value of p ?
6. एउटा बिन्दु C ले रेखाखण्ड AB लाई भित्रपट्टीबाट 3 : 1 को अनुपातमा विभाजन गर्दछ । यदि A र B का स्थिति भेक्टरहरू क्रमशः $\vec{i} - 3\vec{j}$ र $2\vec{i} + 5\vec{j}$ छन् भने बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
   The point C divides the line segment AB internally in the ratio of 3 : 1. If the position vectors of A and B are $\vec{i} - 3\vec{j}$ and $2\vec{i} + 5\vec{j}$ respectively, find the position vector of the point C.
7. दिइएको चित्रमा $\overrightarrow{OA} = \vec{a}$ र $\overrightarrow{OB} = \vec{b}$ छन् । यदि $\overrightarrow{AC} = 3\overrightarrow{AB}$ भए $\overrightarrow{OC}$ पत्ता लगाउनुहोस् ।
   In the given figure, $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. If $\overrightarrow{AC} = 3\overrightarrow{AB}$, find $\overrightarrow{OC}$.



Group D $3 \times 5 = 15$

8. ΔABC मा AB र AC का मध्यबिन्दुहरू क्रमशः P र Q छन् । भेक्टर विधिबाट प्रमाणित गर्नुहोस् : PQ // BC र BC = 2PQ
   P and Q are the middle points of AB and AC respectively of the ΔABC. Prove by the vector method that: PQ // BC and BC = 2PQ.
9. एउटा चतुर्भुजको भुजाहरूका मध्यबिन्दुहरू क्रमशः जोड्दा एउटा समानान्तर चतुर्भुज बन्दछ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
   Prove that the lines joining the middle points of the sides of a quadrilateral taken in order is a parallelogram.
10. एउटा समबाहु चतुर्भुजको विकर्णहरू 90° हुने गरी प्रतिच्छेदन भएका हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
    Prove by the vector method that the diagonals of a rhombus intersect at 90°.

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

3. 90° **4.** 4.54 **5.** 1, – 4 **6.** $\frac{7}{4}\vec{i} + 3\vec{j}$ **7.** $3\vec{b} - 2\vec{a}$

www.drmathnp.com

puspas.com.np

Unit VII

स्थानान्तरण
Transformation



Felix Klein

During the 17th Century, René Descartes (1596-1650), a French mathematician and philosopher, laid the stepping stones for modern mathematics, specifically in analytical geometry, through the use of the Cartesian coordinate system. **Felix Klein** *(1849-1925), a German geometer, showed the importance of groups in geometry. This new idea allowed Klein to unify geometry. Klein's 1872 address in Erlanger, Germany proposed that the study of geometry be defined as the study of transformations that leave objects invariant (unchanged). This viewpoint is now known as the Erlanger Program.*

SEE SPECIFICATION GRID **1 Very Short, 1 Long & 1 Very Long Question**

Contents	Topics	Knowledge	Understanding	Application	Higher Ability	Total no. of Questions	Total Marks
		Each of 1 Mark	Each of 2 Marks	Each of 4 Marks	Each of 5 Marks		
स्थानान्तरण Transformation	संयुक्त स्थानान्तरण Combined Transformation विपरित स्थानान्तरण र विपरित वृत्त Inversion Transformation and Inversion Circle मेट्रिक्स स्थानान्तरण Matrix Transformation	1	-	1	1	3	10

SYLLABUS ISSUED BY CDC

7. **स्थानान्तरण (Transformation)** - 15 Periods

- परावर्तन, परिक्रमण, विस्थापन र विस्तारीकरणमध्ये कुनै दुईओटा स्थानान्तरणहरूको संयुक्त स्थानान्तरण, Combination of Transformation (Combination of any two at a time among reflection, rotation, translation and enlargement)
- विपरीत स्थानान्तरण र विपरीत वृत्त (Inversion transformation and inversion circle)
- मेट्रिक्सको प्रयोगबाट स्थानान्तरण (Transformation Using Matrix)

TRANSFORMATION

1. संयुक्त स्थानान्तरण
COMBINED TRANSFORMATION

DEFINITIONS

➢ **संयुक्त स्थानान्तरण (Combined Transformation)**

जब एउटा वस्तुलाई स्थानान्तरण गरिएको छ र यसको प्रतिबिम्बलाई पुनः नयाँ प्रतिबिम्ब पाउनको लागि स्थानान्तरण गरिएमा यस्तो स्थानान्तरणलाई संयुक्त स्थानान्तरण भनिन्छ ।

When an object has been transformed, its image can again be transformed to form a new image. Such transformation is called combined transformations.

FORMULAE AND KEY POINTS

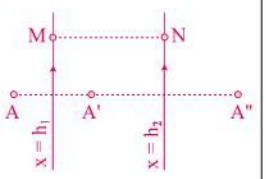
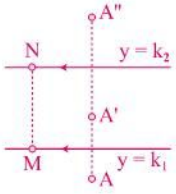
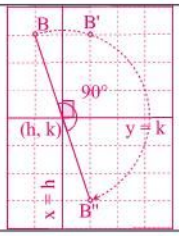
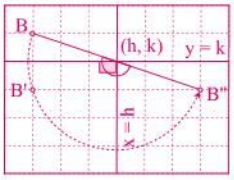
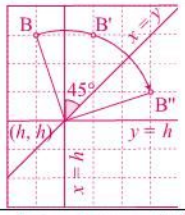
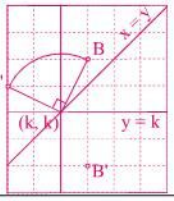
1. **परावर्तन (Reflection)**

परावर्तनको अक्ष (Axis of Reflection)		वस्तु (Object)		आकृति (Image)
(i)	X-axis or $y = 0$	$P(x, y)$	$\longrightarrow$	$P'(x, -y)$
(ii)	Y-axis or $x = 0$	$P(x, y)$	$\longrightarrow$	$P'(-x, y)$
(iii)	$y = x$ or $x = y$	$P(x, y)$	$\longrightarrow$	$P'(y, x)$
(iv)	$y = -x$ or $x = -y$	$P(x, y)$	$\longrightarrow$	$P'(-y, -x)$
(v)	$x = h$	$P(x, y)$	$\longrightarrow$	$P'(2h - x, y)$
(vi)	$y = k$	$P(x, y)$	$\longrightarrow$	$P'(x, 2k - y)$

www.drmathnp.com

puspas.com.np

संयुक्त परावर्तन (Combined Reflection)

S.N.	परावर्तनका अक्षहरू Axes of Reflections	एकल स्थानान्तरण Single transformation	चित्र Figure
	समानान्तर रेखाहरू (Parallel Lines)	विस्थापन (Translation)	
1.	पहिले $x = h_1$ र पछि $x = h_2$ मा परावर्तन गरिएमा If reflection $x = h_1$ is followed by reflection in $x = h_2$	विस्थापन भेक्टर $\begin{pmatrix} 2(h_2 - h_1) \\ 0 \end{pmatrix}$ वा $\begin{pmatrix} 2\overrightarrow{MN} \\ 0 \end{pmatrix}$ द्वारा हुने विस्थापन The translation by the translation vector $\begin{pmatrix} 2(h_2 - h_1) \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2\overrightarrow{MN} \\ 0 \end{pmatrix}$	
2.	पहिले $y = k_1$ र पछि $y = k_2$ मा परावर्तन गरिएमा (समानान्तर अक्षमा) If reflection $y = k_1$ is followed by reflection in $y = k_2$ (parallel axis)	विस्थापन भेक्टर $\begin{pmatrix} 0 \\ 2(k_2 - k_1) \end{pmatrix}$ वा $\begin{pmatrix} 0 \\ 2\overrightarrow{MN} \end{pmatrix}$ द्वारा हुने विस्थापन The translation by the translation vector $\begin{pmatrix} 0 \\ 2(k_2 - k_1) \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 2\overrightarrow{MN} \end{pmatrix}$	
	प्रतिच्छेदित रेखाहरू (Intersecting Lines)	परिक्रमण (Rotation)	
3.	पहिले $x = h$ र पछि $y = k$ मा परावर्तन गरिएमा If reflection $x = h$ is followed by reflection in $y = k$	चित्रअनुसार, बिन्दु (h, k) को वरिपरि हुने ऋणात्मक अर्ध परिक्रमण According to the figure, negative half turn rotation about the the point (h, k).	
4.	पहिले $y = k$ र पछि $x = h$ मा परावर्तन गरिएमा If reflection $y = k$ is followed by reflection in $x = h$	चित्रअनुसार, बिन्दु (h, k) को वरिपरि हुने धनात्मक अर्ध परिक्रमण According to the figure, positive half turn rotation about the point (h, k).	
5.	पहिले $x = h$ र पछि $x = y$ मा परावर्तन गरिएमा If reflection $x = h$ is followed by reflection in $x = y$	चित्रअनुसार, बिन्दु (h, h) को वरिपरि हुने ऋणात्मक चौथाइ परिक्रमण According to the figure, negative quarter turn rotation about the point (h, h).	
6.	पहिले $y = k$ र पछि $x = y$ मा परावर्तन गरिएमा If reflection $y = k$ is followed by reflection in $x = y$	चित्रअनुसार, बिन्दु (k, k) को वरिपरि हुने ऋणात्मक चौथाइ परिक्रमण Negative quarter turn rotation about the point (k, k).	

Note: परिक्रमणको दिशा वस्तुको स्थिति (चतुर्थांश) मा भर पर्छ ।
The direction of rotation is based on the position of the object.

www.drmathnp.com

puspas.com.np

2. परिक्रमण (Rotation)

	परिक्रमणको कोण Angle of rotation	परिक्रमणको केन्द्र Centre of rotation	वस्तु Object	प्रतिबिम्ब Image
(i)	$+90°$ or $-270°$	$(0, 0)$	$P(x, y)$	$\longrightarrow P'(-y, x)$
(ii)	$-90°$ or $+270°$	$(0, 0)$	$P(x, y)$	$\longrightarrow P'(y, -x)$
(iii)	$\pm 180°$	$(0, 0)$	$P(x, y)$	$\longrightarrow P'(-x, -y)$
(iv)	$+90°$ or $-270°$	(a, b)	$P(x, y)$	$\longrightarrow P'(-y + a + b, x - a + b)$
(v)	$-90°$ or $+270°$	(a, b)	$P(x, y)$	$\longrightarrow P'(y + a - b, -x + a + b)$
(vi)	$\pm 180°$	(a, b)	$P(x, y)$	$\longrightarrow P'(-x + 2a, -y + 2b)$

(iv) यदि R_1 र R_2 हरू दुई एउटै केन्द्रमा आधारीत परिक्रमणहरू भए संयुक्त परिक्रमण :
If R_1 and R_2 are two rotations about the same centre then combined rotation:
$(R_1 \circ R_2) = (R_2 \circ R_1) = R_1 + R_2$

अर्थात् ,

यदि $R_1[(0, 0), \theta_1]$ र $R_2 = [(0, 0), \theta_2]$ दुईओटा परिक्रमणहरू भए संयुक्त परिक्रमण:

$R_2 \circ R_1 = R_1 \circ R_2 = R[(0, 0), (\theta_1 + \theta_2)]$ हुन्छ ।

If $R_1[(0, 0), \theta_1]$ and $R_2 = [(0, 0), \theta_2]$ are two rotations then the combined rotation:

$R_2 \circ R_1 = R_1 \circ R_2 = R[(0, 0), (\theta_1 + \theta_2)]$

3. विस्थापन (Translation)

(i) यदि $T = \begin{pmatrix} a \\ b \end{pmatrix}$ विस्थापन भेक्टर र $P(x, y)$ वस्तु भए आकृति $P'(x + a, y + b)$ हुन्छ ।

If $T = \begin{pmatrix} a \\ b \end{pmatrix}$ is translation vector and $P(x, y)$ be an object then image is $P'(x + a, y + b)$.

(ii) यदि T_1 र T_2 हरू दुई विस्थापनहरू भए संयुक्त विस्थापन: $(T_1 \circ T_2) = (T_2 \circ T_1) = T_1 + T_2$

If T_1 & T_2 are 2 translations then combined translation: $(T_1 \circ T_2) = (T_2 \circ T_1) = T_1 + T_2$

4. विस्तारीकरण (Enlargement)

	विस्तारीकरणको केन्द्र Centre of enlargement	नापो Scale factor	वस्तु Object	आकृति Image
(i)	$(0, 0)$	k	(x, y)	$\longrightarrow (kx, ky)$
(ii)	(a, b)	k	(x, y)	$\longrightarrow [k(x - a) + a, k(y - b) + b]$

(iii) यदि $E_1 = [(a, b), k_1]$ र $E_2 = [(a, b), k_2]$ दुईओटा विस्तारीकरणहरू भए संयुक्त विस्तारीकरण :

$E_2 \circ E_1 = E_1 \circ E_2 = E[(a, b), k_1 \times k_2]$ हुन्छ ।

If $E_1 = [(a, b), k_1]$ and $E_2 = [(a, b), k_2]$ are two enlargements then the combined enlargement:

$E_2 \circ E_1 = E_1 \circ E_2 = E[(a, b), k_1 \times k_2]$

5. यदि F ले एउटा स्थानान्तरण र G ले अर्को स्थानान्तरण जनाउँछन् भने,

If F denotes a transformation and G denotes another transformation then,

(i) संयुक्त स्थानान्तरण (G∘F) (The combined transformation (G∘F)
= पहिले F द्वारा हुने स्थानान्तरण र पछि सोही प्रतिबिम्बलाई G द्वारा हुने स्थानान्तरण जनाउँछ ।
= First the transformation by F and then the transformation of the image by G

अर्थात् **(Or)**

G∘F = पहिले F र पछि G (G∘F = F is followed by G)

(ii) संयुक्त स्थानान्तरण (F∘G) = पहिले G द्वारा हुने स्थानान्तरण र पछि सोही प्रतिबिम्बलाई F द्वारा हुने स्थानान्तरण जनाउँछ ।
The combined transformation (F∘G)
= First the transformation by G and then the transformation of the first image by F

अर्थात् **(Or)**

F∘G = पहिले G र पछि F (F∘G = G is followed by F)

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

B. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2065 R, 2066 R, 2068 R', 2068 S, 2068 S', 2069 R, 2071 R, 2071 R', 2071 S	9
Model 2	2066 S	1
Model 4	2065 E, 2065 S, 2068 R, 2069 R', 2069 S, 2070 R', SEE 2073 S, SEE 2073 S'	7
Model 5	SEE 2075 R'	1
Model 6	2067 S, 2070 S', SEE 2074 R', SEE 2075 R'$_2$, SEE MODEL 2076	5
Model 7	2072 R, SEE 2074 R, SEE 2074 S, 2074 S', SEE 2075 R	5
Model 8	2065 R', 2066 R', 2067 R, 2072 S, 2073 R, SEE 2075 R_2	6
Model 9	-	-
Model 10	2067 R', 2070 R, 2070 S, 2072 R', 2073 R'	5

SEE EXERCISE 1

A. VERY SHORT QUESTIONS

Model 1 *Definitions and formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् । (Define the following terms.)

A. संयुक्त विस्थापन (Combined translation) **B.** संयुक्त परावर्तन (Combined reflection)

C. संयुक्त परीक्रमण (Combined rotation) **D.** संयुक्त विस्तारीकरण (Combined enlargement)

2A. स्थानान्तरणका चारओटा विधिहरू के के हुन् ? (What are the four methods of transformation?)

2B. संयुक्त स्थानान्तरण भन्नाले के बुझिन्छ ? (What do you meant by combined transformation?)

Model 2 *Problems based on combination of reflections in axes.*

3A. x-अक्षको परावर्तन पछि y-अक्षमा परावर्तन गर्दा हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् ।
State the single transformation when the reflection in X-axis is followed by the reflection in Y-axis.

Or, यदि r_1 ले रेखा x- अक्षमा हुने परावर्तन र r_2 ले y- अक्षमा हुने परावर्तनलाई जनाउँछन् भने $r_2 \circ r_1$ ले के जनाउँदछ ?
If r_1 represents the reflection on x-axis and r_2 represents the reflection on y-axis then what does $r_2 \circ r_1$ represent?

3B. $x = y$ मा परावर्तन गरेपछि $x = -y$ मा परावर्तन गर्दा हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् ।
State the single transformation when the reflection in $x = y$ is followed by reflection in $x = -y$.

4A. स्थानान्तरण r_1 ले एउटा बिन्दु A लाई A' मा र स्थानान्तरण r_2 ले A' लाई A" मा स्थानान्तरण गर्दछ भने $r_2 \circ r_1$ ले के गर्छ ?
Transformation r_1 transforms a point A to A' and r_2 transforms point A' to A". What does $r_2 \circ r_1$ do?

4B. यदि r_1 ले X-अक्षमा हुने परावर्तन तथा r_2 ले Y-अक्षमा हुने परावर्तनलाई जनाउँछन् भने $r_1 \circ r_2$ ले के जनाउँदछ ?
If r_1 represents the reflection on X-axis and r_2 represents the reflection on Y-axis then what does $r_1 \circ r_2$ represent?

4C. यदि r_1 ले $x = y$ रेखामा भएको परावर्तन र r_2 ले y-अक्षमा भएको परावर्तनलाई जनाउँछ भने $r_1 \circ r_2$ ले के जनाउँदछ ?If r_1 is the reflection about a line $x = y$ and r_2 is reflection about the y-axis, what does $r_1 \circ r_2$ represent?

5A. यदि F ले रेखा $y = x$ मा हुने परावर्तन र G ले रेखा $x = 0$ मा हुने परावर्तनलाई जनाउँछन् भने G∘F ले कुन स्थानान्तरणलाई प्रतिनिधित्व गर्छ ?
If F be the reflection on line $y = x$ and G be the reflection on line $x = 0$ then state what does G∘F represent ?

5B. बिन्दु A(x, y) लाई पहिला $y = x$ रेखामा परावर्तन गरी फेरि $y = 0$ मा परावर्तन गर्दा आउने प्रतिबिम्ब लेख्नुहोस् ।
Find the co-ordinates of the image of point A(x, y) after reflection about the line $y = x$, followed by the reflection about the line $y = 0$.

6A. बिन्दु (x, y) लाई पहिले रेखा $x = 0$ मा परावर्तन गरेपछि प्राप्त प्रतिबिम्बलाई $y = k$ मा परावर्तन गर्दा अन्तिम प्रतिबिम्ब पत्ता लगाउनुहोस् ।
Point (x, y) is reflected in the line $x = 0$ at first and then the image so formed is reflected in the line $y = k$. Find the final image.

6B. बिन्दु (x, y) लाई पहिले रेखा $x = 0$ मा परावर्तन गरेपछि प्राप्त प्रतिबिम्बलाई $y - m = 0$ मा परावर्तन गर्दा अन्तिम प्रतिबिम्ब पत्ता लगाउनुहोस् ।
Point (x, y) is reflected in the line $x = 0$ at first and then the image so formed is reflected in the line $y - m = 0$. Find the final image.

www.drmathnp.com

puspas.com.np

Model 3 *Combination of translations under the given translation vectors.*

7A. $F : (x, y) \to (2x, y)$ र $T : (x, y) \to (x, y - 1)$ दुईओटा स्थानान्तरण हुन् । उक्त स्थानान्तरणको संयुक्त स्थानान्तरण F∘T पत्ता लगाउनुहोस् ।

$F : (x, y) \to (2x, y)$ and $T : (x, y) \to (x, y - 1)$ are any two transformation. Find the combined transformation F∘T.

7B. यदि $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ र $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$ भए T_1T_2, पत्ता लगाउनुहोस् । (If $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ and $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$, find T_1T_2)

7C. यदि $T_1 = \begin{pmatrix} m \\ n \end{pmatrix}$ र $T_2 = \begin{pmatrix} a \\ b \end{pmatrix}$ दुई विस्थापन हुन भने को $T_1 \circ T_2$ निकाल्नुहोस् ।

If $T_1 = \begin{pmatrix} m \\ n \end{pmatrix}$ and $T_2 = \begin{pmatrix} a \\ b \end{pmatrix}$ are two translations, find the translation $T_1 \circ T_2$.

Model 4 *Problems based on combination of rotations.*

8A. एउटा परिक्रमण $R_1[(0, 0); x°]$ पछि $R_2[(0, 0); y°]$ गरिन्छ । संयुक्त परिक्रमण के हुन्छ ?

A rotation $R_1[(0, 0); x°]$ is followed by $R_2[(0, 0); y°]$. What is the combined rotation?

8B. एउटा परिक्रमण $R_1[(0, 0); 90°]$ पछि $R_2[(0, 0); 180°]$ गरिन्छ । संयुक्त परिक्रमण के हुन्छ ?

A rotation $R_1[(0, 0); 90°]$ is followed by $R_2[(0, 0); 180°]$. What is the combined rotation?

Model 5 *Problems based on combination of enlargements.*

9A. दुईओटा लगातार विस्तारीकरणहरू $E_1[O, K_1]$ र $E_2[O, K_2]$ द्वारा परिभाषित गरिएको भए संयुक्त विस्तारीकरण के होला ?

What will be the combined enlargement of two successive enlargements defined by $E_1[O, K_1]$ and $E_2[O, K_2]$?

9B. दुईओटा लगातार विस्तारीकरणहरू $E_1[(a, b), m]$ र $E_2[(a, b), n]$ द्वारा परिभाषित गरिएको भए यिनीहरूको एकल विस्तारीकरण पत्ता लगाउनुहोस् ।

What is the single enlargement of two successive enlargements defined by $E_1[(a, b), m]$ and $E_2[(a, b), n]$?

Model 6 *Problems based on combination of transformation under reflection is followed by rotation.*

10A. बिन्दु $P(a, b)$ लाई रेखा $y = x$ मा परावर्तन गरेपछि उद्गम बिन्दुको वरिपरि $-90°$ मा घुमाउँदा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

Find the image of point $P(a, b)$ after reflection on the line $y = x$ followed by the rotation through $-90°$ about the origin.

10B. बिन्दु (p, q) लाई रेखा $y = x$ मा परावर्तन गर्दा हुने प्रतिविम्बलाई उद्गमबिन्दु O बाट $+90°$ घुमाउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

Point (p, q) is reflected on the line $y = x$. The image so obtained is rotated about O through $+90°$. Find the co-ordinates of this image.

10C. बिन्दु (a, b) लाई सुरूमा रेखा $y = -x$ मा परावर्तन गरी आएको प्रतिविम्बलाई उद्गमबिन्दुबाट $+180°$ घुमाउँदा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

Find the co-ordinates of the image of the point (a, b), when it is first reflected on the line $y = -x$ and then the image so formed is rotated about the origin O through an angle of $+180°$.

10D. बिन्दु (a, b) लाई रेखा Y-अक्षमा परावर्तन गर्दा हुने प्रतिविम्बलाई उद्गमबिन्दु $O(0, 0)$ बाट $+90°$ घुमाउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

The image formed by reflecting the point (a, b) on the Y-axis is rotated about origin $O(0, 0)$ through $+90°$. Find the co-ordinates of this image.

10E. बिन्दु $A(x, y)$ लाई Y-अक्षले परावर्तन गर्दा बन्ने प्रतिविम्बलाई पुनः उद्गम बिन्दुबाट $-90°$ मा परिक्रमण गराउँदा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।

A point $A(x, y)$ is reflected on Y-axis and the image so obtained is rotated about origin through $-90°$. Find the coordinates of final image.

www.drmathnp.com

puspas.com.np

Model 7 *Problems based on combination of transformation under rotation is followed by reflection.*

11A. बिन्दु (a, b) लाई उद्गमबिन्दु O बाट $+90°$ घुमाउँदा बन्ने प्रतिविम्ब Y-अक्षमा परावर्तन गराउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Point (a, b) is rotated about the origin O through $+90°$ and the image so obtained is relfected on the Y-axis. Find the co-ordinates of the image.

11B. बिन्दु (x, y) लाई पहिले उद्गमबिन्दु 0 बाट $+90°$ मा घुमाउँदा हुने प्रतिविम्बलाई X-अक्षमा परावर्तन गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् र ती दुवै स्थानान्तरण जनाउने एउटै स्थानान्तरण पनि लेख्नुहोस् ।
Find the co-ordinates of the image of a point (x, y) when it is first rotated by $+90°$ about the origin O and then reflected on the X-axis. Also, write down the single transformation which represents both of these two transformations.

11C. बिन्दु (m, n) लाई पहिले उद्गमबिन्दु O बाट $+180°$ मा घुमाउँदा हुने प्रतिविम्बलाई Y- अक्षमा परावर्तन गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the image of a point (m, n) when it is first rotated by $+180°$ about the origin O and then reflected in the Y-axis.

11D. बिन्दु (a, b) लाई उद्गमबिन्दुबाट $-180°$ मा घुमाउँदा बन्ने प्रतिविम्बलाई $y = x$ रेखामा परावर्तन गराउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Point (a, b) is rotated about the origin through $-180°$ and the image so formed is reflected on the line $y = x$. Find the coordinates of the image so obtained.

11E. बिन्दु (m, n) लाई उद्गम बिन्दुबाट $-90°$ मा घुमाउँदा बन्ने प्रतिविम्बलाई $y = -x$ रेखामा परावर्तन गराउँदा बन्ने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Point (m, n) is rotated about the origin through $-90°$ and the image so obtained is reflected on the line $y = -x$. Find the co-ordinates of the image so formed.

12A. यदि r_1 ले X-अक्षद्वारा हुने परावर्तन र r_2 ले उद्गम बिन्दुबाट $+90°$ को परिक्रमण जनाउँछ भने बिन्दु $A(x, y)$ को संयुक्त स्थानान्तरण $r_1 \circ r_2$ द्वारा हुने प्रतिबिम्बित बिन्दु पत्ता लगाउनुहोस् ।
If r_1 is the reflection about the X-axis and r_2 is the rotation about $+90°$ through the origin, find the image of the point $A(x, y)$ under the combined transformation of $r_1 \circ r_2$.

12B. यदि R_1 ले x- अक्षद्वारा हुने परावर्तन र R_2 ले उद्गम बिन्दुको वरिपरि $-90°$ को परिक्रमण जनाउँछ भने बिन्दु $A(p, q)$ को संयुक्त स्थानान्तरण $R_1 \circ R_2$ द्वारा प्राप्त हुने प्रतिबिम्ब पत्ता लगाउनुहोस् ।
If R_1 is the reflection on x-axis and R_2 is the rotation through $-90°$ about the origin, find the image of a point $A(p, q)$ under the combined transformation of $R_1 \circ R_2$.

12C. यदि r_1 ले y-अक्षद्वारा हुने परावर्तन र r_2 ले उद्गम बिन्दुबाट $-90°$ को परिक्रमण जनाउँछ भने बिन्दु $P(x, y)$ को संयुक्त स्थानान्तरण $r_1 \circ r_2$ द्वारा हुने प्रतिबिम्बित बिन्दु पत्ता लगाउनुहोस् ।
If r_1 is the reflection about the y-axis and r_2 is the rotation through $-90°$ about the origin, find the image of point $P(x, y)$ under the combined transformation of $r_1 \circ r_2$.

Model 8 *Combination of transformations (reflection is followed by translation or vice-versa).*

13A. बिन्दु (a, b) लाई पहिले X-अक्षमा परावर्तन गरी $\begin{pmatrix} m \\ n \end{pmatrix}$ द्वारा स्थानान्तरण गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of image of point (a, b) when it is first reflected on the X-axis and then translated by $\begin{pmatrix} m \\ n \end{pmatrix}$.

13B. बिन्दु (x, y) लाई पहिले $\begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा स्थानान्तरण गरी Y-अक्षमा परावर्तन गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the image of the point (x, y) when it is first translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ and then reflected on the Y-axis.

13C. विस्थापन $\begin{pmatrix} a \\ b \end{pmatrix}$ पछि $x = y$ मा परावर्तन गरिएको भए बिन्दु $P(x, y)$ को प्रतिविम्बको निर्देशाङ्क लेख्नुहोस् ।
What is the co-ordinates of the image of a point $P(x, y)$ under the translation by $\begin{pmatrix} a \\ b \end{pmatrix}$ followed by the reflection in $x = y$?

www.drmathnp.com

puspas.com.np

Model 9 *Combination of transformations under rotation is followed by translation.*

14A. बिन्दु (x, y) लाई पहिले उद्गमबिन्दु O बाट $-90°$ घुमाउँदा हुने प्रतिविम्बलाई $\begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा विस्थापन गर्दा हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the image of a point (x, y) when it is first rotated about the origin O through $-90°$ and then translated by $\begin{pmatrix} a \\ b \end{pmatrix}$.

14B. बिन्दु (m, n) लाई पहिले उद्गम बिन्दु O को वरिपरि $+90°$ द्वारा परिक्रमण गराई $\begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा विस्थापन गर्दा हुने प्रतिबिम्बको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Find the co-ordinates of the image of a point (m, n), when it is first rotated about origin O through $+90°$ and then translated by $\begin{pmatrix} a \\ b \end{pmatrix}$.

Model 10 *Combination of transformations under reflection is followed by enlargement.*

15A. बिन्दु $P(x, y)$ लाई $x = h$ रेखामा परावर्तन गर्नुहोस् । प्राप्त प्रतिविम्बलाई $E[(0, 0), k]$ ले विस्तारीकरण गर्नुहोस् । अन्तिम प्रतिविम्बको निर्देशाङ्कहरू लेख्नुहोस् ।
Reflect a point $P(x, y)$ on the line $x = h$. Enlarge the image so obtained by $E[(0, 0), k]$. Write the coordinates of the final image.

15B. बिन्दु $A(x, y)$ लाई $y = k$ रेखामा परावर्तन गर्नुहोस् । प्राप्त प्रतिविम्बलाई $E[(0, 0), m]$ ले विस्तारीकरण गर्नुहोस् । अन्तिम प्रतिविम्बको निर्देशाङ्कहरू लेख्नुहोस् ।
Reflect a point $A(x, y)$ on the line $y = k$. Enlarge the image so obtained by $E[(0, 0), m]$. Write the coordinates of the final image.

Model 11 *Problems based on translation and enlargement.*

16A. बिन्दु $A(x, y)$ लाई भेक्टर $\begin{pmatrix} a \\ b \end{pmatrix}$ ले स्थानान्तरण गरी आउने प्रतिविम्बलाई $E[(0, 0), k]$ ले विस्तारीकरण गर्नुहोस् र अन्तिम प्रतिविम्बको निर्देशाङ्कहरू लेख्नुहोस् ।
The point $A(x, y)$ is translated by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ and enlarge the image so obtained by $E[(0, 0), k]$. Write the co-ordinates of the final image.

16B. उद्गम बिन्दु केन्द्र र नापो 2 अनुसार बिन्दु P (m, n) लाई विस्तारीकरण गर्दा प्रतिबिम्ब P' हुन्छ । फेरि P' लाई भेक्टर $\begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा P'' मा विस्थापन गरिएको छ भने P' र P'' को निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
Under the enlargement with centre origin and scale factor 2, the image of the point $P(m, n)$ is P'. Again P' is displaced by vector $\begin{pmatrix} a \\ b \end{pmatrix}$ to P''. Find the co-ordinates of P' and P''.

TRANSFORMATION

Model 12 *Concept Recall*

17A. यदि $T_1 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ र $T_2 = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ भए $T_2 \circ T_1$ भेक्टर पत्ता लगाउनुहोस् ।
If $T_1 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$, find the vector for $T_2 \circ T_1$.

17B. यदि $T_1 \circ T_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ भए सो संयुक्त स्थानतान्तरण अनुसार (4, 1) को प्रतिबिम्ब के हुन्छ ?
If $T_1 \circ T_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, what point will have image (4, 1) under the combined translation?

17C. एउटा विस्तारीकरण $E_1\left[(0, 0); \frac{3}{2}\right]$ पछि अर्को विस्तारीकरण $E_2[O, 2]$ छ । एकल स्थानान्तरण के होला ?
An enlargement $E_1\left[(0, 0); \frac{3}{2}\right]$ is followed by another enlargement $E_2[O, 2]$. What is the single transformation?

www.drmathnp.com

puspas.com.np

B. LONG QUESTIONS

Model 1 *Problems based on combination of reflections in intersecting axes.*

1A. X-अक्ष र Y-अक्षमा क्रमश: हुने संयुक्त परावर्तनहरूसँग समतुल्य हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् । सोही एकल स्थानान्तरण प्रयोग गरेर P(4, 3), Q(1, 1) र R(5, – 1) शीर्षबिन्दुहरू भएको ΔPQR को प्रतिविम्बको शीर्षबिन्दुहरूका निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै वस्तु र प्रतिविम्बलाई एउटै लेखाचित्रमा खिच्नुहोस् ।
State the single transformation equivalent to the combination of reflections on the X-axis and Y-axis respectively. Using this single transformation, find the coordinates of the vertices of the image of ΔPQR having vertices P(4, 3), Q(1, 1) and R(5, – 1). Also, draw the object and image on the same graph. **[2071 S]**

1B. X- अक्ष र Y- अक्षमा क्रमश: हुने संयुक्त परावर्तनहरूसँग समतुल्य हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् । सोही एकल स्थानान्तरण प्रयोग गरेर A(2, 3), B(3, – 4) र C(1, – 2) शीर्षबिन्दुहरू भएको ΔABC को प्रतिविम्बको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेख्नुहोस् । साथै वस्तु र प्रतिविम्बलाई एउटै लेखाचित्रमा खिच्नुहोस् ।
State the single transformation equivalent to the combination of reflections on the X-axis and Y-axis respectively. Using this single transformation, find the coordinates of the vertices of the image of ΔABC having vertices A(2, 3), B(3, – 4) and C(1, – 2). Also, draw the object and image on the same graph. **[2071 R]**

1C. शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको ΔABC लाई रेखा $y = x$ र X-अक्षमा क्रमश: परावर्तन गर्दा प्राप्त हुने प्रतिविम्बको निर्देशाङ्कहरू लेखी लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटा स्थानान्तरणले जनाउन सकिन्छ ? उल्लेख गर्नुहोस् ।
A ΔABC with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the line $y = x$ and the X-axis. Find the stating co-ordinates and graphically represent the image under these transformations. State also the single transformation given by the combinations of these transformations. **[2071 R']**

1D. शीर्षबिन्दुहरू A(6, 2), B(10, 2), C(11, 4) र D(5, 4) भएको चतुर्भुज ABCD लाई x- अक्षमा र त्यसपछि y-अक्षमा परावर्तन गराउदा संयुक्त स्थानान्तरण बन्ने प्रतिविम्बको निर्देशाङ्कहरू निकाल्नुहोस् । **[2068 R']**
Find the co-ordinates of the image of a quadrilateral ABCD with vertices A(6, 2), B(10, 2), C(11, 4) and D(5, 4) under the combined transformations of the reflection on x-axis then on the y-axis.

1E. लेखाचित्रमा A(5, 4), B(2, 2) र C(5, 2) शीर्षबिन्दुहरू भएको त्रिभुज ABC खिच्नुहोस् । ΔABC लाई X-अक्षमा र त्यसपछि प्राप्त प्रतिविम्बलाई रेखा $y = x$ मा लगातार परावर्तन गर्दा बन्ने प्रतिविम्बको निर्देशाङ्कहरू लेखाचित्रमा देखाउनुहोस् ।
On a graph paper, draw a triangle ABC having the vertices A(5, 4), B(2, 2) and C(5, 2). Find the image of ΔABC by stating co-ordinates and graphing them after successive reflection in X-axis followed by a reflection in the line $y = x$. **[2063 M]**

1F. लेखाचित्रमा P(1, 4), Q(4, 1) र R(7, 5) शीर्षबिन्दुहरू भएको त्रिभुज PQR खिच्नुहोस् । ΔPQR लाई Y-अक्षमा र त्यसपछि प्राप्त प्रतिविम्बलाई रेखा $x = y$ मा लगातार परावर्तन गर्दा बन्ने प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाई प्रतिबिम्बित त्रिभुजहरू लेखाचित्रमा देखाउनुहोस् ।
On a graph paper, draw a triangle PQR having the vertices P(1, 4), Q(4, 1) and R(7, 5). Find the image of ΔPQR by stating co-ordinates and graphing them after successive reflection in Y-axis followed by a reflection in the line $x = y$. **[2063 R]**

2A. शीर्षबिन्दुहरू A(1, 2), B (4, – 1) र C(2, 5) भएको त्रिभुजलाई रेखाहरू $x = 5$ र $y = -2$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिविम्बको निर्देशाङ्कहरू लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।
A triangle with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the lines $x = 5$ and $y = -2$. Find by stating coordinates and graphically represent the images under these transformations. State also the single transformation given by the combinations of these transformations. **[2065 M]**

2B. शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको त्रिभुजलाई रेखा $x = -1$ र $y = 2$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिविम्बको निर्देशाङ्कहरू लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् । **[2065 R, 2069 R]**
A triangle with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the line $x = -1$ and $y = 2$. Find the stating co-ordinates and represent the images graphically under these transformations. State also the single transformation given by the combinations of these transformations.

2C. शीर्षबिन्दुहरू A (1, 2), B (4, – 1) र C (2, 5) भएको त्रिभुजलाई रेखाहरू $x = 4$ र $y = -3$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिविम्बको निर्देशाङ्कहरू लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।
A triangle with vertices A (1, 2), B (4, – 1) and C (2, 5) is reflected successively in the lines $x = 4$ and $y = -3$. Find by stating co-ordinates and graphically represents the images under these transformations given by the combination of these transformations. **[2066 R]**

www.drmathnp.com

2D. शीर्ष बिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) भएको त्रिभुजलाई रेखाहरू $x = -1$ र $y = -2$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बहरूको निर्देशाङ्कहरू लेखी लेखाचित्रमा देखाउनुहोस् ।
A triangle with vertices A(1, 2), B(4, – 1) and C(2, 5) is reflected successively in the lines $x = -1$ and $y = -2$. Write the co-ordinates of the images and show on the same graph. **[2068 S]**

2E. शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, 4) भएको त्रिभुजलाई x-अक्ष र रेखा $y = 2$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बका निर्देशाङ्कहरू लेखेर लेखाचित्रमा व्यक्त गर्नुहोस् । **[2068 S']**
A triangle with vertices A(2, 3), B(4, 5) and C(1, 4) is reflected successively in the x-axis and $y = 2$. Find by stating co-ordinates and graphically represent the images under these transformations.

Model 2 Problems based on combination of rotations about origin.

3A. बिन्दुहरू H(4, – 2), R(2, 1) र T(5, 2) एउटा त्रिभुज HRT का तिनओटा शीर्षबिन्दुहरू हुन् । यी बिन्दुहरूलाई परिक्रमण [(0, 0), 180°] र उही दिशामा परिक्रमण [(0, 0), 90°] बाट हुने संयुक्त स्थानान्तरणले स्थानान्तरण गराउँदा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।
Points H(4, – 2), R(2, 1) and T(5, 2) are the vertices of a triangle ΔHRT. These points are transformed by a single transformation obtained by a rotation [(0, 0), 180°] and on the same direction a rotation [(0, 0), 90°]. Find the co-ordinates of the images of the points. Draw both the triangles on the same graph paper. **[2066 S]**

3B. बिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) एउटा त्रिभुज ABC का तिनओटा शीर्षबिन्दुहरू हुन् । यी बिन्दुहरूलाई परिक्रमण [(0, 0), 90°] र उही दिशामा परिक्रमण [(0, 0), 180°] बाट हुने संयुक्त स्थानान्तरणले स्थानान्तरण गराउँदा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।
Points A(1, 2), B(4, – 1) and C(2, 5) are the vertices of a triangle ΔABC. These points are transformed by a single transformation obtained by a rotation [(0, 0), 90°] and on the same direction a rotation [(0, 0), 180°]. Find the co-ordinates of the images of the points. Draw both the triangles on the same graph paper.

3C. यदि $R_1[(0, 0) ; 90°]$ र $R_2[(0, 0) ; 180°]$ परिक्रमणहरू भए $R_1 \circ R_2$ पत्ता लगाउनुहोस् । कुनै समकोणी त्रिभुजको निर्देशाङ्कहरू लेखाचित्रको माध्यमबाट लेखी $R_1 \circ R_2$ द्वारा उक्त त्रिभुजलाई स्थानान्तरण गर्नुहोस् । साथै प्राप्त प्रतिबिम्बलाई पनि सोही लेखाचित्रमा देखाउनुहोस् ।
If $R_1[(0, 0) ; 90°]$ and $R_2[(0, 0) ; 180°]$ are rotations then find $R_1 \circ R_2$. Write the co-ordinates of a right angled triangle using graph. Transform the triangle by $R_1 \circ R_2$. Also, represent the image in the same graph.

Model 3 Problems based on combination of enlargements when E_1 is followed by E_2.

4A. ΔABC का शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(6, 2) छन् । यदि $E_1 = [(0, 0), 2]$ र $E_2 = [(0, 2), 2]$ भए ΔABC लाई $E_2 \circ E_1$ संयुक्त विस्तारीकरण गरी प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुवै त्रिभुजलाई उही ग्राफमा खिच्नुहोस् ।
The vertices of ΔABC are A(2, 3), B(4, 5) and C(6, 2). If $E_1 = [(0, 0), 2]$ and $E_2 = [(0, 2), 2]$, then find the co-ordinates of image of ΔABC under enlargement $E_2 \circ E_1$. Draw both triangle on the same graph. **[2064 R']**

4B. ΔABC का शीर्षबिन्दुहरू A(1, 2), B(4, – 1) र C(2, 5) छन् । यदि $E_1 = [(0, 0), 2]$ र $E_2 = [(0, 0), 3]$ भए ΔABC लाई $E_2 \circ E_1$ संयुक्त विस्तारीकरण गरी प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुवै त्रिभुजलाई उही ग्राफमा खिच्नुहोस् ।
The vertices of ΔABC are A(1, 2), B(4, – 1) and C(2, 5). If $E_1 = [(0, 0), 2]$ and $E_2 = [(0, 0), 3]$, then find the co-ordinates of image of ΔABC under enlargement $E_2 \circ E_1$. Draw both triangle on the same graph.

4C. यदि E_1 [(1, 2), 4] र E_2 [(1, 2), $\frac{1}{2}$] विस्तारहरू भए $E_1 \circ E_2$ पत्ता लगाउनुहोस् । कुनै एउटा समकोणी त्रिभुजको निर्देशाङ्कहरू लेखाचित्रको माध्यमबाट लेखी $E_1 \circ E_2$ द्वारा उक्त त्रिभुजलाई स्थानान्तरण गर्नुहोस् र प्राप्त प्रतिबिम्बलाई पनि सोही लेखाचित्रमा देखाउनुहोस् ।
If E_1 [(1, 2), 4] and E_2 [(1, 2), $\frac{1}{2}$] are enlargements, find $E_1 \circ E_2$. Write the co-ordinates of a right-angled triangle in a graph. Transform the triangle by $E_1 \circ E_2$ and represent the image in same graph.

Model 4 Combination of transformation [translation and reflection.]

5A. शिर्षबिन्दुहरू X(1, 2), Y(– 1, – 2) र Z(5, 0) भएको त्रिभुज XYZ लाई भेक्टर $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई X-अक्षमा परावर्तन गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शिर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔXYZ र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
A triangle XYZ with vertices X(1, 2), Y(– 1, – 2) and Z(5, 0) is translated by a vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. The image so formed is reflected on the X-axis. Write the co-ordinates of the vertices of the images thus obtained and represent the ΔXYZ and its images on the same graph paper. **[SEE 2073 S]**

puspas.com.np

www.drmathnp.com

5B. शीर्षबिन्दुहरू P(2, 1), Q(4, 3) र R(6, – 1) भएको त्रिभुज PQR लाई X-अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई भेक्टर $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ ले विस्थापन गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle PQR with vertices P(2, 1), Q(4, 3) and R(6, – 1) is reflected on the X-axis. The image so formed is translated by the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$. Write the co-ordinates of the vertices of the images thus obtained and represent the ΔPQR and its images on the same graph paper. **[SEE 2073 S']**

5C. शीर्षबिन्दुहरू P(2, 4), Q(– 1, 2) र R(5, – 1) भएको ΔPQR लाई विस्थापन भेक्टर $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ले विस्थापन गर्नुहोस् र त्यसको प्रतिबिम्बलाई रेखा $x + y = 0$ मा परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर वस्तु र प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।

Translate the ΔPQR with vertices P(2, 4), Q(– 1, 2) and R(5, – 1) by translation vector $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and then reflect the image so obtained in the line $x + y = 0$. Write the co-ordinates of the vertices of the image and draw the object and images on the same graph paper. **[2070 R']**

5D. शीर्षबिन्दुहरू O(0, 0), A(2, 0), B(3, 2) र C(1, 2) भएको चतुर्भुज OABC लाई विस्थापन भेक्टर $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ ले विस्थापन गरी प्राप्त प्रतिबिम्बलाई $x = 3$ मा परावर्तन गर्नुहोस् । यसरी प्राप्त प्रतिबिम्बहरू र वस्तुलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Translate the quadrilateral OABC having vertices O(0, 0), A(2, 0), B(3, 2) and C(1, 2) by translation vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. Reflect the image so formed on the line $x = 3$. Represent the images and object in the same graph. **[2068 R]**

5E. O(0, 0), A(2, 0), B(3, 1), र C(1, 1) शीर्षबिन्दु भएको चित्र लेखाचित्रमा खिची यसलाई $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ द्वारा स्थानान्तरण गरी सोही लेखाचित्रमा O'A'B'C' खिच्नुहोस् । त्यसपछि चित्र O'A'B'C' लाई $x = 3$ रेखामा परावर्तन गरी त्यसै लेखाचित्रमा O"A"B"C" चित्र खिच्नुहोस् र O", A", B" र C" का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । **[2057 R]**

Draw a figure having the vertices O(0, 0), A(2, 0), B(3, 1) and C(1, 1) on a graph paper. It is translated by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ and plot the figure O'A'B'C' on the same graph paper then the figure O'A'B'C' is reflected on the line $x = 3$ to form the figure O"A"B"C" on the same graph. Determine the vertices of O", A", B" and C".

5F. त्रिभुज ABC का शीर्षबिन्दुहरू क्रमशः (– 2, – 4), (– 4, – 2) र (– 6, – 4) छन् । त्रिभुज ABC लाई $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ले विस्थापन गर्नुहोस् र प्राप्त प्रतिबिम्बलाई $x = 0$ मा परावर्तन गरी संयुक्त स्थानान्तरणका प्रतिबिम्बका शीर्षबिन्दुका निर्देशाङ्क लेख्नुहोस् र सबै त्रिभुजहरूलाई लेखाचित्रमा देखाउनुहोस् ।

The vertices of ΔABC are (– 2, – 4), (– 4, – 2) and (– 6, – 4) respectively. Translate ΔABC by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and reflect the image on the line $x = 0$. Find the co-ordinates of the vertices of the image under compound transformation and draw all the triangles in the graph. **[2065 E]**

5G. शीर्षबिन्दुहरू A(1, 0), B(2, 1) र C(3, – 1) भएको त्रिभुजलाई $T\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ मा विस्थापन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क लेखेर लेखाचित्रमा देखाउनुहोस् । उक्त प्रतिबिम्बलाई फेरि रेखा $x = 2$ मा परावर्तन गरी सोही लेखाचित्रमा देखाउनुहोस् ।

A triangle with vertices A(1, 0), B(2, 1) and C(3, – 1) is translated by $T\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and draw its image in graph. Again its image is reflected in the line $x = 2$ and draw its image in same graph. **[2065S ,2069S]**

5H. शीर्षबिन्दुहरू D(– 2, – 4), E(– 4, – 2) र F(– 6, – 4) भएको ΔDEF लाई $T = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ ले विस्थापन गर्नुहोस् र त्यसको प्रतिबिम्बलाई रेखा $y = x$ मा परावर्तन गर्दा प्राप्त प्रतिबिम्बहरूलाई एउटै लेखाचित्र खिच्नुहोस् ।

Translate the ΔDEF with vertices D(– 2, – 4), E(– 4, – 2) and F(– 6, – 4) by translation vector $T = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ and then reflect image so obtained in line $y = x$. Write down the co-ordinates of vertices and draw images on the same graph paper. **[2069 R']**

www.drmathnp.com

puspas.com.np

Model 5 *Combination of transformation [translation and rotation.]*

6A. शीर्षबिन्दुहरू P(2, 1), Q(2, 4) र R(5, 2) भएको ΔPQR लाई भेक्टर $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरि धनात्मक दिशातिर 90° मा परीक्रमण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको निर्देशाङ्कहरू लेखी ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

ΔPQR with vertices P(2, 1), Q(2, 4) and R(5, 2) is translated by the vector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and then the image so formed is rotated about the origin through 90° in positive direction. Write down the co-ordinates of the vertices of the images thus obtained and present ΔPQR and its images in the same graph paper. **[2075 R']**

6B. A (2, 3) , B (0, 3) र O(0, 0) भएको ΔAOB लाई $\overrightarrow{AB}$ द्वारा विस्थापन र O को वरिपरि + 90° परिक्रमण गरिएको छ । विस्थापनलाई T र परिक्रमणलाई R ले जनाइएको छ । संयुक्त स्थानान्तरण R∘T अनुसार प्रतिविम्ब पत्ता लगाउनुहोस् ।

ΔABO having A(2, 3), B(0, 3) and O(0, 0) is transformed by the translation vector $\overrightarrow{AB}$ and rotated through 90°(+) ve rotation about O. Translation is denoted by T and the rotation is denoted by R. Find the image under the combined transformation R∘T.

6C. A (2, 3), B (0, 3) र O(0, 0) भएको ΔAOB लाई $\overrightarrow{AB}$ द्वारा विस्थापन र O को वरिपरि + 90° परिक्रमण गरिएको छ । विस्थापनलाई T र परिक्रमणलाई R ले जनाइएको छ । संयुक्त स्थानान्तरण T∘R अनुसार प्रतिविम्ब पत्ता लगाउनुहोस् ।

ΔABO having A(2, 3), B(0, 3) and O(0, 0) is transformed by the translation vector $\overrightarrow{AB}$ and rotated through 90° (+) ve rotation about O. Translation is denoted by T and the rotation is denoted by R. Find the image under the combined transformation T∘R.

Model 6 *Combination of transformation [rotation and reflection.]*

7A. शीर्षबिन्दुहरू क्रमशः L(– 3, 3), M(– 6, 3) र N(– 3, 9) भएको त्रिभुज LMN लाई y - अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरी 180° मा परीक्रमण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी त्रिभुज LMN र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle LMN with vertices L(– 3, 3), M(– 6, 3) and N(– 3, 9) respectively is reflected in y-axis and then image formed is rotated about the origin through 180°. Write the co-ordinates of the vertices of the images thus obtained and present the triangle LMN and its images in the same graph paper. **[SEE 2075 R'$_2$]**

7B. शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, 4) भएको ΔABC लाइ x- अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A ΔABC with vertices A(2, 3), B(4, 5) and C(1, 4) is reflected in x-axis and then image so formed is rotated about the origin through 180°. Write the coordinates of the vertices of the images thus obtained and present the ΔABC and its images in the same graph paper. **[SEE 2074 R']**

7C. शीर्षबिन्दुहरू A(4, 2), B(1, 3) र C(3, 5) भएको ΔABC लाई घडीको दिशामा उद्गम बिन्दुको वरिपरि 90° मा परिक्रमण गरी प्राप्त प्रतिविम्बलाई Y-अक्षमा परावर्तन गरिएको छ । अब प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर वस्तु र प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा खिच्नुहोस् ।

ΔABC with vertices A(4, 2), B(1, 3) and C(3, 5) is rotated through 90° in the clockwise direction about the origin and the image so formed is reflected on the Y-axis. Now, write the co-ordinates of the vertices of the images and draw the object and images on the same graph. **[2070 S']**

7D. शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) भएको ΔABC लाई लेखाचित्रमा खिची यसलाई उद्गमबिन्दु O बाट – 90° परिक्रमण गरी सोही लेखाचित्रमा ΔA'B'C' खिच्नुहोस् । त्यसपछि ΔA'B'C' लाई रेखा $x - y = 0$ मा परावर्तन गरी त्यसै लेखाचित्रमा ΔA"B"C" खिच्नुहोस् । ΔA"B"C" को शीर्षबिन्दुहरूको निर्देशाङ्क लेख्नुहोस् । **[2062 K]**

Draw ΔABC having the vertices A(2, 0), B(3, 1) and C(1, 1) on a graph paper. It is rotated about the origin O through – 90° and present ΔA'B'C' on the same graph paper. Then the ΔA'B'C' is reflected on the line $x - y = 0$ and plot the ΔA"B"C" on the same graph paper. Write down the co-ordinates of vertices of the ΔA"B"C".

7E. शीर्षबिन्दुहरू A(2, 5), B(– 1, 3) र C(4, 1) भएको त्रिभुजलाई उद्गमबिन्दुको वरिपरि + 90° परिक्रमण गराउँदा प्राप्त प्रतिबिम्बलाई रेखा $x = 0$ मा परावर्तन गराइन्छ । यसरी प्राप्त प्रतिबिम्बका शीर्षबिन्दुका निर्देशाङ्कहरू पत्ता लगाउनुहोस् । दुवै प्रतिबिम्बीत त्रिभुजहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् र उक्त दुवै स्थानान्तरणलाई जनाउने एकल स्थानान्तरण पनि लेख्नुहोस् ।

A triangle having vertices A(2, 5), B(– 1, 3) and C(4, 1) is rotated about origin through + 90°. The image so obtained is reflected on the line $x = 0$. Find the vertices of image triangles. Show all the triangles in the same graph paper and also write the single transformation to represent these two transformations. **[2067 S]**

www.drmathnp.com

7F. यदि r_1 ले $x = 2$ रेखामा हुने परावर्तन र r_2 ले उद्गम बिन्दुबाट 180° मा हुने परिक्रमण जनाउँदछ भने $r_1 \circ r_2 (x, y)$ पत्ता लगाउनुहोस् । एउटा समकोण त्रिभुजको निर्देशाङ्कहरू लेखाचित्रको माध्यमबाट लेखी $r_1 \circ r_2$ अनुसार उक्त त्रिभुजलाई स्थानान्तरण गर्नुहोस् । साथै प्राप्त प्रतिबिम्बलाई पनि सोही लेखाचित्रमा देखाउनुहोस् ।

If r_1 is the reflection in the line $x = 2$ and r_2 is the rotation through 180° about the origin then find $r_1 \circ r_2$ (x, y). Write the co-ordinates of a right angled triangle using a graph paper. Transform the triangle by $r_1 \circ r_2$. Also, represent the image in same graph.

7G. चतुर्भुज ABCD का शीर्षबिन्दुका निर्देशाङ्कहरू A(1, 1), B(2, 3), C(4, 2) र D(3, − 2) छन् । उक्त चतुर्भुजलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गर्नुहोस् । प्रतिबिम्ब चतुर्भुजलाई $y = -x$ मा परावर्तन गर्नुहोस् । माथिका दुई स्थानान्तरणको संयुक्त स्थानान्तरणलाई जनाउने स्थानान्तरण लेख्नुहोस् । **[SEE MODEL 2076]**

The coordinates of vertices of a quadrilateral ABCD are A(1, 1), B(2, 3), C(4, 2) and D(3, − 2).Rotate this quadrilateral about origin through180°. Reflect this image of quadrilateral about $y = -x$. Write the name of transformation which denotes the combined transformation of above two transformations.

Model 7 Combination of transformation [translation and enlargement.]

8A. शिर्षबिन्दुहरू A(2, 3), B(2, 6) र C(3, 4) भएको त्रिभुज ABC लाई $T = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका दुवै प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle ABC with vertices A(2, 3), B(2, 6) and C(3, 4) is translated by $T = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and the image so obtained is enlarged by E[(0, 0), 2]. Write the co-ordinates of the vertices of the images so formed and represent ΔABC and its both images in the same graph. **[SEE 2075 R]**

8B. P(3, 4), Q(2, 1) र R(4, 2) शीर्षबिन्दुहरू भएको ΔPQR लाई $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

ΔPQR having the vertices P(3, 4), Q(2, 1) and R(4, 2) is translated by $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. The image so formed is enlarged by E[(0, 0), 2]. Writing the co-ordinates of the vertices of images thus obtained, represent the ΔPQR and its images in the same graph paper. **[SEE 2074 R]**

8C. ΔABC का शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) छन् । ΔABC लाई भेक्टर $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] द्वारा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्ष बिन्दुहरूका निर्देशाङ्कहरू लेखेर वस्तु र प्रतिबिम्बहरूलाई एउटै लेखचित्रमा प्रस्तुत गर्नुहोस् ।

The vertices of ΔABC are A(2, 0), B(3, 1) and C(1, 1). ΔABC is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$. The image so formed is enlarged by E[(0, 0), 2]. Write the coordinates of the vertices of images thus obtained and represent the object and the images on the same graph paper. **[2072 R]**

8D. ΔMNP का शीर्षबिन्दुहरू M(1, 1), N(3, 1) र P(2, 3) छन् । ΔMNP लाई विस्थापन भेक्टर $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ अनुसार विस्थापन गरी फेरि उद्गमबिन्दुलाई केन्द्र मानी नापो 2 लिएर विस्तारीकरण गर्नुहोस् ।

The vertices of ΔMNP are M(1, 1), N(3, 1) and P(2, 3). Find the image of ΔMNP under the translation by the translation vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Then enlarge by taking the scale factor 2 and centre at origin.

8E. शीर्षबिन्दुहरू A(3, 2), B(– 1, 0) र C(4, – 1) भएको त्रिभुज ABC लाई भेक्टर $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ले विस्थापन गरी बन्ने प्रतिबिम्बलाई केन्द्रबिन्दु (0, 0) र नापो 3 को आधारमा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

The image of the triangle ABC with vertices A(3, 2), B(– 1, 0) and C(4, – 1) formed by translating it by the vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is enlarged on the basis of the centre (0, 0) and scale factor 3. Write the coordinates of the vertices of the images thus obtained and present ΔABC and its images on the same graph paper. **[SEE 2074 S]**

www.drmathnp.com

puspas.com.np

8F. शीर्षबिन्दुहरू A(3, 3), B(1, 1) र C(5, 0) भएको त्रिभुज ABC लाई भेक्टर $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई केन्द्रबिन्दु (0, 0) र नापो 2 को आधारमा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A triangle ABC with vertices A(3, 3), B(1, 1) and C(5, 0) is translated by a vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$. The image so formed is enlarged with the centre (0, 0) and scale factor 2. Write the coordinates of the vertices of the images thus obtained and present ΔABC and its images on the same graph paper. **[SEE 2074 S']**

8G. यदि T ले $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ भेक्टरद्वारा विस्थापन र R ले $y = 3$ रेखामा परावर्तन जनाउँदछ भने T∘R पत्ता लगाउनुहोस् । कुनै एउटा समकोणी त्रिभुजको निर्देशाङ्कहरू लेखाचित्रको माध्यमबाट लेखी T∘R द्वारा उक्त त्रिभुजलाई स्थानान्तरण गर्नुहोस् । साथै प्राप्त प्रतिबिम्बलाई पनि सोही लेखाचित्रमा देखाउनुहोस् ।

If T denotes a translation by a vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and R dentoes the reflection in the line $y = 3$, find T∘R. Write the co-ordinates of a right angled triangle in a graph. Transform the triangle by T∘R and represent the image in same graph.

Model 8 *Combination of transformation [reflection and enlargement.]*

9A. E ले केन्द्रबिन्दु (3, 1) र नापो 2 भएको विस्तारीकरण र R ले $y = x$ मा हुने परावर्तन जनाउँदछन् । शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, – 2) भएको ΔABC को संयुक्त स्थानान्तरण E∘R द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै लेखाचित्रमा खिच्नुहोस् ।

E denotes the enlargement about the centre (3, 1) with a scale factor of 2 and R denotes a reflection on the line $y = x$. Find the image of ΔABC having the vertices A(2, 3), B(4, 5) and C(1, – 2) under the combined transformation E∘R. Draw both figures on the same graph paper. **[SEE 2075 R_2]**

9B. शीर्षबिन्दुहरू P(2, 1), Q(5, 3) र R(7, – 1) भएको त्रिभुज PQR लाई Y-अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] द्वारा विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Triangle PQR having vertices P(2, 1), Q(5, 3) and R(7, – 1) is reflected on the y-axis. The image so formed is enlarged by E[(0, 0), 2]. Write the co-ordinates of the vertices of the images thus obtained and present the ΔPQR and its images in the same graph paper. **[SEE 2073 R]**

9C. ΔPQR को शीर्षबिन्दुहरू P(1, 2), Q(2, 1) र R(4, 3) छन् । ΔPQR लाई $y = x$ मा परावर्तन गरेपछि E[(0, 0), 2] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । ΔPQR र यसको प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

P(1, 2), Q(2, 1) and R(4, 3) are the vertices of the triangle PQR. Find the coordinates of the images of the triangle PQR under the reflection on the line $y = x$ followed by the enlargement E[(0, 0), 2]. Present the ΔPQR and its images on the same graph. **[2072 S]**

9D. r_1 ले केन्द्रबिन्दु (– 3, – 4) बाट हुने नापो 3 भएको विस्तारीकरण जनाउँछ र r_2 ले $y = -x$ मा परावर्तन हुने स्थानान्तरण जनाउँछ । यदि शीर्षबिन्दुहरू A(– 4, 6), B(– 6, – 10) र C(12, – 8) भएको ΔABC लाई $r_1 \circ r_2$ द्वारा स्थानान्तरण गर्दा ΔABC को प्रतिबिम्ब ΔA'B'C' भए A', B' र C' को निर्देशाङ्क पत्ता लगाउनुहोस् ।

r_1 denotes enlargement about the centre (– 3, – 4) with a scale factor of 3 and r_2 denotes a reflection on the line $y = -x$. ΔABC having the vertices A(– 4, 6), B(– 6, – 10) and C(12, – 8) is mapped onto the ΔA'B'C' under the transformation $r_1 \circ r_2$. Find the co-ordinates of A', B' and C'. **[2063 R']**

9E. E ले केन्द्रबिन्दु (– 3, – 4) बाट हुने नापो 2 भएको विस्तारीकरण र R ले $y = 0$ मा हुने परावर्तन जनाउँछ । शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) भएको ΔABC लाई संयुक्त स्थानान्तरण E∘R द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै ग्राफमा खिच्नुहोस् ।

E denotes enlargement about the centre (– 3, – 4) with a scale factor of 2 and R denotes a reflection on the line $y = 0$. ΔABC having the vertices A(2, 0), B(3, 1) and C(1, 1) is mapped under the combined transformation E∘R. Find the image of ΔABC and draw two figure on the same graph paper. **[2065 R']**

9F. त्रिभुज ABC का शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) छन् । त्रिभुज ABC लाई रेखा $x = y$ मा परावर्तन गरेपछि E[(– 3, – 4), 2] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । त्रिभुज ABC र त्यसका प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

A(2, 0), B(3, 1) and C(1, 1) are the vertices of a triangle ABC. Find the co-ordinates of the vertices of the triangle ABC under the reflection on the line $x = y$ followed by the enlargement E[(– 3, – 4), 2]. Present the triangle ABC and its images on the same graph paper. **[2067 R]**

www.drmathnp.com

9G. त्रिभुज EFG का शीर्षबिन्दुहरू क्रमशः E(1, 0), F(2, 3) र G(1, 3) छन् । ΔEFG लाई रेखा $x = y$ मा परावर्तन गरेपछि E[(0, 0), 3] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बको निर्देशाङ्कहरू लेख्नुहोस् । वस्तु र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

The co-ordinates of the vertices of triangle EFG are E(1, 0), F(2, 3) and G(1, 3) respectively. Find the co-ordinates of vertices of images of ΔEFG under the reflection in the line $x = y$ followed by enlargement E[(0, 0), 3]. Represent object and its images in same graph. **[2066 R']**

9H. लेखाचित्रको माध्यमबाट एउटा समद्विबाहु त्रिभुजका शीर्षबिन्दुहरू लेख्नुहोस् । सो त्रिभुजलाई रेखा $x = y$ मा परावर्तन गरेपछि E[(0, 0) ; 2] द्वारा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बका निर्देशाङ्कहरू लेख्नुहोस् । वस्तु र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Using a graph paper, write the co-ordinates of an isosceles triangle. Find the co-ordinates of vertices of image of the triangle under the reflection in the line $x = y$ followed by enlargement E[(0, 0) ; 2]. Represent the object and image in the same graph.

Model 9 Combination of transformation [enlargement and reflection.]

10A. मानौं A(1, – 1), B(5, 2) र C(3, 5) त्रिभुज ABC का शिर्षबिन्दुका निर्देशाङ्कहरू हुन् । उक्त त्रिभुजलाई लेखाचित्रमा प्रस्तुत गरी E_1[O(0, 0) ; 2] द्वारा बिस्तार गर्नुहोस् । पुनः विस्तारीत प्रतिबिम्बलाई Y-अक्षमा परावर्तन गरी लेखाचित्रमा प्रस्तुत गर्नुहोस् । त्रिभुज ABC र स्थानान्तरण पछिका प्रतिबिम्बहरूको प्रकृती लेख्नुहोस् ।

Let A(1, – 1), B(5, 2) and C(3, 5) are the coordinates of vertices of a triangle ABC. Plot this triangle in graph. Enlarge this triangle with E_1[O(0, 0) ; 2] and then reflect the enlarged image about Y-axis in that graph. Write the nature of object triangle and its images after transformations.

10B. R_1 ले केन्द्रबिन्दु (3, 4) बाट हुने नापो 2 भएको विस्तारीकरण जनाउँछ र R_2 ले रेखा $y = x$ मा परावर्तन हुने स्थानान्तरण जनाउँछ । यदि शीर्षबिन्दुहरू P(– 3, 5), Q(7, 4) र R(6, – 2) भएको ΔPQR लाई $R_2 \circ R_1$ द्वारा स्थानान्तरण गर्दा ΔPQR को प्रतिबिम्ब $\Delta P_1Q_1R_1$ भए P_1, Q_1 र R_1 को निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

R_1 denotes an enlargement about the center (3, 4) with a scale factor of 2 and R_2 denotes a reflection on the line $y = x$. ΔPQR with the vertices P(– 3, 5), Q(7, 4) and R(6, – 2) is mapped onto the $\Delta P_1Q_1R_1$ under the transformation $R_2 \circ R_1$. Find the coordinates of P_1, Q_1 and R_1. **[2064 R]**

10C. शीर्षबिन्दुहरू A(1, 0), B(0, 2) र C(2, 1) भएको ΔABC लाई लेखाचित्रमा खिच्नुहोस् । ΔABC लाई पहिले E(O, 2) द्वारा विस्तार गरी प्राप्त प्रतिबिम्बित ΔA'B'C' लाई Y-अक्षद्वारा परावर्तन गर्नुहोस् र सबै त्रिभुजहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Sketch the graph of ΔABC having vertices A(1, 0), B(0, 2) and C(2, 1) in a graph paper. Enlarge the ΔABC by E(O, 2) first and then reflect the image ΔA'B'C' in the Y-axis. Show all triangles on same graph. **[2064 S]**

Model 10 Combination of transformation [Rotation and enlargement.]

11A. त्रिभुज CAT का शीर्षबिन्दुहरू C(2, 5), A(– 1, 3) र T(4, 1) छन् । त्रिभुज CAT लाई उद्गम बिन्दुको वरिपरि 90° ले धनात्मक दिशामा परिक्रमण गरी फेरि E[(0, 0), 2] मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै वस्तु र प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

C(2, 5), A(– 1, 3) and T(4, 1) are the vertices of a triangle CAT. Find the coordinates of the vertices of the image of ΔCAT under the rotation of positive 90° about origin followed by enlargement E[(0, 0), 2]. Represent the object and images on the same graph paper. **[2072 R']**

11B. ΔABC का शीर्षबिन्दुहरू A(2, 5) , B(– 1, 3) र C(4, 1) छन् । ΔABC लाई उद्गम बिन्दुको वरिपरि ऋणात्मक चौथाई परिक्रमण गरिएको छ र प्राप्त प्रतिबिम्बलाई केन्द्र उद्गम बिन्दु र नापो 2 लिएर विस्तारीकरण गरिएको छ । यसरी ΔABC का प्रतिबिम्बहरूका शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाई वस्तु र प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा देखाउनुहोस् ।

The vertices of ΔABC are A(2, 5), B(– 1, 3) and C(4, 1). ΔABC is rotated through negative quarter turn about the origin and the image so obtained is enlarged by taking origin as the centre and scale factor 2. Hence, find the co-ordinates of the vertices of the images of ΔABC and show the object and images on the same graph. **[2070 S]**

11C. त्रिभुज KLM का शीर्षबिन्दुहरू K(2, 5), L(– 1, 3) र M(4, 1) छन् । ΔKLM लाई उद्गम बिन्दुको वरिपरि 90° ऋणात्मक परिक्रमण गरी फेरि E[(0, 0), 2] मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू निकाल्नुहोस् । साथै वस्तु र प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

K(2, 5), L(– 1, 3) and M(4, 1) are the vertices of a triangle KLM. Find the co-ordinates of the vertices of the images of ΔKLM under the rotation of negative 90° about the origin followed by the enlargement E[(0, 0), 2]. Present object and its images on the same graph paper. **[2070 R]**

11D. त्रिभुज PQR का शीर्षबिन्दुहरू P(2, 0), Q(– 1, 3) र R(2, 4) छन् । त्रिभुज PQR लाई बिन्दु (0, 0) को वरिपरी + 90° परिक्रमण गरी फेरी केन्द्र (2, 1) र नापो – 2 को आधारमा विस्तारीकरण गर्दा बन्ने प्रतिबिम्ब पत्ता लगाउनुहोस् ।

P(2, 0), Q(– 1, 3) and R(2, 4) are the vertices of Δ PQR. Find the image of ΔPQR under the rotation of + 90° about the point (0, 0) followed by the enlargement about the centre (2, 1) and scale factor – 2. **[2067R']**

puspas.com.np

www.drmathnp.com

11E. एउटा समानान्तर चतुर्भुज LOVE का शीर्षबिन्दुहरू L(2, 2), O(6, 2), V(7, 4) र E(3, 4) छन् । □LOVE लाई उद्गम बिन्दुको वरिपरि 90° ले धनात्मक दिशामा परिक्रमण गरी फेरि E[(0, 0), 3] मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै वस्तु प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
L(2, 2), O(6, 2), V(7, 4) and E(3, 4) are the vertices of a parallelogram LOVE. Find the co-ordinates of the vertices of the images of □ LOVE under the rotation of positive 90° about origin followed by enlargement E[(0, 0), 3]. Represent the object and images on the same graph paper. **[SEE 2073 R']**

Model 11 Combination of rotation (a, b) is followed by reflection, translation and enlargement.

12A. ΔABC का शीर्षबिन्दुहरू क्रमशः A(4, 6), B(2, 2) र C(8, 2) छन् । यसलाई बिन्दु P(1, 3) को वरिपरि 90° परिक्रमण गरी फेरी x-अक्षमा परावर्तन गरिएको छ । ΔABC का प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of ΔABC are A(4, 6), B(2, 2) and C(8, 2) respectively. It is rotated through 90° about a point P(1, 3) followed by reflection in x-axis. Find the co-ordinates of images of ΔABC.

12B. समानान्तर चतुर्भुज ABCD का शीर्षबिन्दुहरू क्रमशः A(2, 3), B(1, 1), C(4, 1) र D(5, 3) छन् । यसलाई बिन्दु P(3, 0) को वरिपरि 90° परिक्रमण गरी फेरी $x = y$ रेखामा परावर्तन गरिएको छ । समानान्तर चतुर्भुज ABCD का प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of a parallelogram ABCD are A(2, 3), B(1, 1), C(4, 1) and D(5, 3) respectively. It is rotated through 90° about a point P(3, 0) followed by reflection in $x = y$ line. Find the co-ordinates of images of parallelogram ABCD.

13A. ΔABC का शीर्षबिन्दुहरू क्रमशः A(4, 8), B(2, 4) र C(8, 4) छन् । यसलाई बिन्दु (4, – 2) को वरिपरि – 90° परिक्रमण गरी फेरी विस्थापन भेक्टर $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ द्वारा विस्थापन गरिएको छ । ΔABC का प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of ΔABC are A(4, 8), B(2, 4) and C(8, 4) respectively. It is rotated through – 90° about a point (4, – 2) followed by the translation $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the co-ordinates of images of ΔABC.

13B. समानान्तर चतुर्भुज PQRS का शीर्षबिन्दुहरू क्रमशः P(– 2, 4), Q(– 4, 2), R(– 1, 1) र S(1, 3) छन् । यसलाई बिन्दु D(– 1, – 1) को वरिपरि – 90° परिक्रमण गरी फेरी विस्थापन भेक्टर $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ द्वारा विस्थापन गरिएको छ । समानान्तर चतुर्भुज PQRS का प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of a parallelogram PQRS are P(– 2, 4), Q(– 4, 2), R(– 1, 1) and S(1, 3) respectively. It is rotated through – 90° about a point D(– 1, – 1) followed by the translation $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. Find the co-ordinates of images of parallelogram PQRS.

TRANSFORMATION

14A. ΔABC का शीर्षबिन्दुहरू क्रमशः A(2, 4), B(6, 6) र C(4, 0) छन् । यसलाई बिन्दु P(0, 2) को वरिपरि 180° द्वारा परिक्रमण गरी फेरी E[(0, 0); 2] द्वारा विस्तारीकरण गरिएको छ । ΔABC का प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of ΔABC are A(2, 4), B(6, 6) and C(4, 0) respectively. It is rotated through 180° about a point P(0, 2) followed by the enlargement E[(0, 0) ; 2]. Find the co-ordinates of images of ΔABC.

14B. चतुर्भुज PREM का शीर्षबिन्दुहरू क्रमशः P(1, 3), R(3, 3), E(2, 0) र M(– 3, 1) छन् । यसलाई बिन्दु (0, 1) को वरिपरि 180° परिक्रमण गरी फेरी उद्गम बिन्दु केन्द्र र नापो 2 भएको विस्तारीकरण अनुसार विस्तार गरिएको छ । चतुर्भुज PREM को प्रतिबिम्बका निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of a quadrilateral PREM are P(1, 3), R(3, 3), E(2, 0) and M(– 3, 1) respectively. It is rotated through 180° about a point (0, 1) followed by an enlargement of centre at origin and scale factor 2. Find the co-ordinates of images of quadrilateral PREM.

ANSWERS

A. VERY SHORT QUESTIONS

3A. $R_o[\pm 180°; (0, 0)]$ **3B.** Half turn rotation about the origin **4B.** $R_o[\pm 180°; (0, 0)]$ **4C.** $R_o[-90°; (0, 0)]$
5A. $R_o[+90°; (0, 0)]$ **5B.** $(y, -x)$ **6A.** $(-x, 2k-y)$ **6B.** $A(-x, 2m-y)$ **7A.** $(2x, y-1)$
7B. $(a+c, b+d)$ **7C.** $(a+m, b+n)$ **8A.** $R[(0, 0); x° + y°]$ **8B.** $R[(0, 0); 270°]$ **9A.** $E_2 E_1 [O, K_1K_2]$
9B. $E[(a, b); mn]$ **10A.** $(a, -b)$ **10B.** $(-p, q)$ **10C.** (b, a) **10D.** $(-b, -a)$
10E. (y, x) **11A.** (b, a) **11B.** $(-y, -x)$; reflection in $x = -y$ **11C.** $(m, -n)$
11D. $(-b, -a)$ **11E.** $(m, -n)$ **12A.** $(-y, -x)$ **12B.** (q, p) **12C.** $(-y, -x)$
13A. $(a+m, n-b)$ **13B.** $(-x-a, y+b)$ **13C.** $(y+b, x+a)$ **14A.** $(a+y, b-x)$ **14B.** $(a-n, b+m)$
15A. $(2hk - xk, yk)$ **15B.** $(xm, 2km - ym)$ **16A.** $(x+a)k, (y+b)k$ **16B.** $P'(2m, 2n)$ and $P''(2m+a, 2n+b)$
17A. $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **17B.** $(2, -2)$ **17C.** $E[(0, 0); 3]$

www.drmathnp.com

puspas.com.np

B. LONG QUESTIONS

1A. P'(– 4, – 3), Q'(– 1, – 1) and R'(– 5, 1) **1B.** Rotation [180°, O], A'(–2, –3), B'(–3, 4), C'(–1, 2)
1C. A'(2, 1), B'(– 1, 4), C'(5, 2) and A"(2, – 1), B"(– 1, – 4), C"(5, – 2), $R_0[(0, 0), -90°]$
1D. A'(– 6, – 2), B'(– 10, – 2), C'(– 11, – 4), D'(– 5, – 4) **1E.** A'(5, – 4), B'(2, – 2), C'(5, – 2), and A"(– 4, 5), B"(– 2, 2), C"(– 2, 5)
1F. P'(– 1, 4), Q'(– 4, 1) R'(–7, 5) and P"(4, – 1), Q"(1, – 4), R"(5, – 7)
2A. A'(9, 2), B'(6, –1), C'(8, 5); A"(9, –6), B"(6, –3), C"(8, –9) and Half turn rotation about (5, – 2)
2B. A'(– 3, 2), B'(– 6, – 1), C'(– 4, 5); A"(– 3, 2), B"(– 6, 5), C"(– 4, –1) and Half turn rotation about (– 1, 2)
2C. A' (7, 2), B'(– 6, – 1), C' (– 4, 5) ; A" (7, – 8), B" (4, – 5), C" (6, – 11); Rotation of 180° about (4, – 3)
2D. A'(– 3, 2), B'(– 6, – 1), C'(– 4, 5), A"(– 3, – 6), B"(– 6, – 3), C"(– 4, – 9)
2E. A'(2, – 3), B'(4, – 5), C'(1, – 4), A"(2, 7), B"(4, 9), C"(1, 8)
3A. H'(– 2, – 4), R'(1, – 2), T'(2, – 5) **3B.** A'(2, – 1), B'(– 1, – 4), C'(5, – 2) **4A.** A'(8, 10), B'(16, 18), C'(24, 6)
4B. A'(6, 12), B'(24, – 6), C'(12, 30) **5A.** X'(3, 3), Y'(1, – 1), Z'(7, 1) and X"(3, – 3), Y"(1, 1), Z"(7, – 1)
5B. P'(2, – 1), Q'(4, – 3), R'(6, 1), and P"(– 1,1), Q"(1,– 1), R"(3, 3)
5C. P'(0, 7), Q'(– 3, 5), M'(3, 2) and P"(–7, 0), Q"(– 5, 3), M"(– 2, – 3)
5D. O'(0, 2), A'(2, 2), B'(3, 4), C'(1, 4) and O"(6, 2), A"(4, 2), B"(3, 4), C"(5, 4)
5E. O'(0, 2), A'(2, 2), B'(3, 3), C'(1, 3) and O" (6, 2), A" (4, 2), B" (3, 3), C" (5, 3)
5F. A'(0, – 1), B'(– 2, 1), C'(– 4, – 1) and A"(0, – 1), B" (2, 1), C"(4, – 1)
5G. A'(2, 2), B'(3, 3), C'(4, 1) and A"(2, 2), B"(1, 3), C"(0, 1)
5H. D'(– 5, – 6), E'(– 7, – 4), F'(– 9, – 6) and D"(– 6, – 5), E"(– 4, – 7), F"(– 6, – 9)
6A. P'(3, – 1), Q'(3, 2), R'(6, 0) and P"(1, 3), Q"(– 2, 3), R"(0, 6)
6B. A'(0, 3), B'(– 2, 3), O'(– 2, 0) and A"(– 3, 0), B"(– 3, – 2), C"(0, – 2)
6C. A'(– 3, 2), B'(– 3, 0), O'(0, 0) and A"(– 5, 2), B"(– 5, 0), O"(0, 0)
7A. L'(3, 3), M'(6, 3), N'(3, 9) and L"(– 3, – 3), M"(– 6, – 3), N"(– 3, – 9)
7B. A'(2, – 3), B'(4, – 5), C'(1, – 4) and A"(– 2, 3), B"(– 4, 5), C"(– 1, 4)
7C. A'(2, – 4), B'(3, – 1), C'(5, – 3) and A"(– 2, – 4), B"(– 3, – 1), C"(– 5, – 3)
7D. A' (0, –2), B'(1, – 3), C'(1, – 1) and A" (2, 0), B"(–3, 1), C"(–1, 1)
7E. A'(–5, 2), B'(–3, – 1), C'(– 1, 4) and A"(5, 2), B"(3, – 1), C"(1, 4), Reflection in $x = -y$.
7G. A'(–1, – 1), B'(– 2, – 3), C'(– 4, – 2), D'(– 3, 2) and A"(1, 1), B"(3, 2), C"(2, 4), D"(– 2, 3), reflection in $y = x$ line
8A. A'(0, 6), B'(0, 9), C'(1, 7) and A"(0, 12), B"(0, 18), C"(2, 14)
8B. P'(1, 7), Q'(0, 4), R'(2, 5) and P"(2, 14), Q"(0, 8), R"(4, 10)
8C. A'(4, – 3), B'(5, – 2), C'(3, – 2) and A"(8, – 6), B"(10, – 4), C"(6, – 4)
8D. M'(3, 4), N'(5, 4), P'(4, 6) and M"(6, 8), N"(10, 8), P"(8, 12)
8E. A'(2, 4), B'(– 2, 2), C'(3, 1) and A" (6, 12), B" (– 6, 6), C" (9, 3)
8F. A'(1, 5), B'(– 1, 3), C'(3, 2) and A" (2, 10), B" (– 2, 6), C" (6, 4)
9A. A'(3, 2), B'(5, 4), C'(– 2, 1) and A"(3, 3), B"(7, 7), C"(– 7, 1)
9B. P'(– 2, 1), Q'(– 5, 3), R'(– 7, – 1), and P"(– 4, 2), Q'(– 10, 6), R"(– 14, – 2)
9C. P'(2, 1), Q'(1, 2), R'(3, 4) and P"(4, 2), Q"(2, 4), R"(6, 8)
9D. A'(– 12, 20), B'(36, 26), C'(30, – 28) **9E.** A'(7, 4), B'(9, 2), C'(5, 2)
9F. A'(0, 2), B'(1, 3), C'(1, 1) and A"(3, 8), B"(5, 10), C"(5, 6)
9G. E'(0, 1), F'(3, 2), G'(3, 1) and E"(0, 3), F"(9, 6), G"(9, 3)
10A. A'(2, – 2), B'(10, 4), C'(6, 10) and A"(– 2, – 2), B"(– 10, 4), C"(– 6, 10)
10B. P_1 (6, – 9), Q_1(4, 11), R_1 (– 8, 9) **10C.** A'(2, 0), B'(0, 4), C'(4, 2) & A"(2, 0), B"(0, 4), C"(– 4, 2)
11A. C'(– 5, 2), A'(– 3, – 1), T'(– 1, 4), and C"(– 10, 4), A"(– 6, – 2), T"(– 2, 8)
11B. A'(5, – 2), B'(3, 1), C'(1, – 4) and A"(10, – 4), B"(6, 2), C"(2, – 8)
11C. K'(– 5, 2), L'(– 3, – 1), M'(– 1, – 4) and K"(10, –4),L"(6, 2), M" (2, –8)
11D. P'(0, 2), Q'(– 3, – 1), R'(– 4, 2) and P"(6, – 1), Q"(12, 5), R"(14, – 1)
11E. L'(– 2, 2), O'(– 2, 6), V'(– 4, 7), E'(– 4, 3) and L"(– 6, 6), O"(– 6, 18), V"(– 12, 21), E"(– 12, 9)
12A. A'(– 2, 6), B'(2, 4), C'(2, 10) and A"(– 2, – 6), B"(2, – 4), C"(2, – 10)
12B. A'(0, – 1), B'(2, –2), C'(2, 1), D'(0, 2) and A"(– 1, 0), B"(– 2, 2), C"(1, 2), D"(2, 0)
13A. A'(14, – 2), B'(10, 0), C'(10, – 6) and A"(16, – 1), B"(12, 1), C"(12, – 5)
13B. P'(4, 0), Q'(2, 2), R'(1, – 1), S'(3, – 3) and P"(8, 3), Q"(6, 5), R"(5, 2), S"(7, 0)
14A. A'(–2, 0), B'(– 6, – 2), C'(– 4, 4) and A"(– 4, 0), B"(– 12, – 4), C"(– 8, 8)
14B. P'(–1, – 1), R'(– 3, – 1), E'(– 2, 2), M'(3, 1) and P"(– 2, – 2), R"(– 6, – 2), E"(– 4, 4), M"(6, 2)

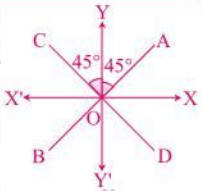
GENERAL EXERCISE 1

LONG QUESTIONS

Model 1 ***Problems based on combination of reflections in intersecting axes.***

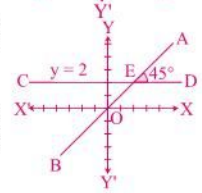
1A. बिन्दुहरू P(– 2, 1), Q(– 4, 2) र R(– 6, 4) भएको ΔPQR लाई रेखा CD मा परावर्तन पछि रेखा AB मा परावर्तन गराउँदा आउने प्रतिविम्बहरूलाई संयुक्त स्थानान्तरणबाट पत्ता लगाउनुहोस् जहाँ ∡COA = 90° छ । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा देखाउनुहोस् ।

ΔPQR of vertices P(– 2, 1), Q(– 4, 2) and R(– 6, 4) is reflected first on the line CD and then on the line AB. Find the vertices of image triangle ΔP'Q'R' under the combined transformation if ∡COA = 90°. Show both the triangles on the same graph paper.



1B. बिन्दुहरू P(2, 1), Q(4, 2) र R(6, 4) भएको ΔPQR लाई रेखा CD मा परावर्तन पछि रेखा AB मा परावर्तन गराउँदा आउने प्रतिविम्बहरूलाई संयुक्त स्थानान्तरणबाट पत्ता लगाउनुहोस् जहाँ ∡AED = 45° छ । दुईओटै त्रिभुजहरूलाई एउटै लेखाचित्रमा देखाउनुहोस् ।

ΔPQR of vertices P(2, 1), Q(4, 2) and R(6, 4) is reflected first on the line CD and then on the line AB. Find the vertices of image triangle ΔP'Q'R' under the combined transformation if ∡AED = 45°. Show both the triangles on the same graph paper.



www.drmathnp.com

puspas.com.np

2A. एउटा स. च. ABCD का शीर्षबिन्दुहरू A(3, 0), B(4, 2), C(2, 4) र D(1, 2) छन् । यसलाई रेखा $y = 1$ मा परावर्तन गरेपछि रेखा $x = 3$ मा परावर्तन गरिएको छ । अन्तिम प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
A parallelogram ABCD has the vertices A(3, 0), B(4, 2), C(2, 4) and D(1, 2). Find the image of the parallelogram ABCD under the reflection in the line $y = 1$ followed by reflection in the line $x = 3$. Also, find the single transformation to represent this combination of reflections.

2B. शीर्षबिन्दुहरू A(0, 1), B(5, 1), C(8, 5) र D(3, 5) भएको एउटा समबाहु चतुर्भुज पहिले $x = 4$ मा परावर्तन भएको छ र पछि $y = 3$ मा परावर्तन भएको छ । सो समबाहु चतुर्भुजको प्रतिविम्ब पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
A rhombus with vertices A(0, 1), B(5, 1), C(8, 5) and D(3, 5) is reflected in $x = 4$ and then in $y = 3$. Find the image of the rhombus. Also, find the single transformation to represent this combination of reflections.

3A. एउटा चतुर्भुज ABCD का शीर्षबिन्दुहरू A(0, 2), B(3, 1), C(4, 3) र D(2, 4) छन् । चतुर्भुज ABCD लाई पहिले $y = x$ मा परावर्तन गरेपछि X-अक्षमा परावर्तन गर्दा हुने प्रतिविम्ब पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
A quadrilateral ABCD has the vertices A(0, 2), B(3, 1), C(4, 3) and D(2, 4). Find the image of the quadrilateral ABCD under the reflection in $y = x$ followed by the reflection about X-axis. Also find the single transformation to represent this combination of reflections.

3B. एउटा त्रिभुज ABC का शीर्षबिन्दुहरू A(1, 2), B(4, 4) र C(3, 7) छन् । पहिले रेखा $y = 0$ मा परावर्तन गरी $\Delta A'B'C'$ भएको छ र फेरि त्रिभुज A'B'C' लाई रेखा $y = x$ मा परावर्तन गरी A"B"C" भएको छ भने $\Delta A'B'C'$ र $\Delta A"B"C"$ का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
The triangle whose vertices are A(1, 2), B(4, 4) and C(3, 7) is first reflected in the line $y = 0$ into $\Delta A'B'C'$ and then $\Delta A'B'C'$ is reflected in the line $y = x$ into $\Delta A"B"C"$. Write down the co-ordinates of $\Delta A'B'C'$ and $\Delta A"B"C"$. Also, find single transformation to represent this combination of reflections.

Model 2 *Problems based on combination of reflections in parallel axes.*

4A. बिन्दुहरू A(– 3, 3), B(– 6, 3) र C(– 3, 9) बाट बनेको त्रिभुज पहिले Y-अक्षमा परावर्तन गरी A', B', C' भएको छ र त्यसपछि रेखा $x = 3$ मा पुनः परावर्तन गरी A", B", C" भएको छ । सबै प्रतिविम्बका बिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
A triangle formed by the points A(– 3, 3), B(– 6, 3) and C(– 3, 9) is reflected in the Y-axis. The $\Delta A'B'C'$ is reflected in the line $x = 3$ to give a second image $\Delta A"B"C"$. Find the co-ordinates of all the image points. Also, find single transformation to represent this combination of reflections.

4B. A(4, 8), B(6, 6), C(3, 6) र D(4, 4) शीर्षबिन्दु भएको चित्र लेखाचित्रमा खिची यसलाई $y = 4$ मा परावर्तन गरी सोही लेखाचित्रमा A'B'C'D' खिच्नुहोस् । त्यसपछि चित्र A'B'C'D' लाई $y = 0$ मा परावर्तन गरी त्यसै लेखाचित्रमा A"B"C"D" खिँच्नुहोस् र A", B", C" र D" का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै यो संयुक्त परावर्तनलाई जनाउने एकल स्थानान्तरण पनि पत्ता लगाउनुहोस् ।
Draw a figure having the vertices A(4, 8), B(6, 6), C(3, 6) and D(4, 4) on the graph. It is reflected on the line $y = 4$ then draw A'B'C'D'. Then after reflect it on the line $y = 0$ and draw A"B"C"D" on the same graph and find the coordinates of A", B", C" and D". Also, find single transformation to represent this combination of reflections.

Model 3 *Problems based on finding the image under the combination of rotations about the origin.*

5. चतुर्भुज ABCD का शीर्षबिन्दुहरूका निर्देशाङ्कहरू A(– 4, 4), B(– 2, 4),C(– 2, 1) र D(– 4, 1) छन् । उद्गमबिन्दुलाई परिक्रमणको केन्द्र मानेर तलका संयुक्त परिक्रमण अनुसार प्रतिविम्ब पत्ता लगाउनुहोस्:
The co-ordinates of vertices of quadrilateral ABCD are A(– 4, 4), B(– 2, 4), C(– 2, 1) and D(– 4, 1). Find the image of ABCD under following combined rotation with origin as the centre of rotation:

A. पहिले 80° र त्यसपछि 190° (80° is followed by 190°) **B.** पहिले 100° र त्यसपछि 80° (100° is followed by 80°)

6A. A (2, 5), B (6, 4) र C (3, 7) बिन्दुहरू ΔABC का शीर्षहरू हुन् । ΔABC लाई केन्द्र मा 99° र 81° मा परिक्रमण गरी संयुक्त परिक्रमणबाट प्राप्त प्रतिविम्बको चित्र बनाउनुहोस् र प्रतिविम्बका निर्देशाङ्कहरू पनि लेख्नुहोस् ।
Points A(2, 5), B(6, 4) and C(3, 7) are the vertices of ΔABC. Draw the figure of image of ΔABC under the combined rotation of 99° followed by 81° by taking (0, 0) as centre of rotation and write the co-ordinates of the image.

6B. एउटा समानान्तर चतुर्भुज ABCD का शीर्षबिन्दुहरू A(3, 0), B(4, 2), C(2, 4) र D(1, 2) छन् । यसलाई उद्गमबिन्दुको वरिपरि 180° परिक्रमण गरेपछि उद्गमबिन्दुको वरिपरि 90° परिक्रमण गरिएको छ । अन्तिम प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् । समानान्तर चतुर्भुज ABCD र यसको प्रतिविम्बलाई लेखाचित्रमा देखाउनुहोस् ।
A parallelogram ABCD has the vertices A(3, 0), B(4, 2), C(2, 4) and D(1, 2). Find the image of the parallelogram ABCD under the rotation of 180° about the origin followed by rotation of 90° about the origin. Present the parallelogram ABCD and its image on the same graph.

www.drmathnp.com

puspas.com.np

Model 4 *Problems based on finding the image under the combination of translation.*

7A. चतुर्भुज ABCD लाई लेखाचित्रमा उतार्नुहोस् जसका शीर्षहरू A(8, 8), B(12, 12), C(8, 12) र D(0, 12) छन् । यदि $T_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ र $T_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ भए संयुक्त स्थानान्तरण $T_1\ T_2$ अनुसार प्रतिविम्ब चतुर्भुज A'B'C'D' पत्ता लगाउनुहोस् ।

In the graph paper, draw quadrilateral ABCD with vertices A(8, 8), B(12, 12), C(8, 12) and D(0, 12). Find image quadrilateral A'B'C'D' under the transformation T_1T_2 if $T_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

7B. एउटा चतुर्भुज PQRS का शीर्षबिन्दुहरू P(1, 2), Q(2, 3), R(5, 2) र S(6, 4) छन् । सो चतुर्भुजलाई पहिले $T_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ले र पछि $T_2 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ ले बिस्थापन गर्नुहोस् ।

The vertices of a quadrilateral PQRS are P(1, 2), Q(2, 3), R(5, 2) and S(6, 4). Translate the quadrilateral under $T_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ followed by $T_2 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

8A. यदि A(2, 3) , B (4, 5) र C(6, 2) तिनओटा बिन्दुहरू भए बिस्थापन भेक्टर $\overrightarrow{AB}$ पछि $\overrightarrow{BC}$ द्वारा बिस्थापन गरी ΔABC को अन्तिम प्रतिविम्ब पत्ता लगाउनुहोस् ।

If A(2, 3), B(4, 5) and C(6, 2) are three points, find the final image of ΔABC under the translation vector $\overrightarrow{AB}$ followed by $\overrightarrow{BC}$.

8B. एउटा त्रिभुज ABC का शीर्षबिन्दुहरू A(1, 5), B(4, 8) र C(7, 3) छन् । यदि $T_1 = \overrightarrow{AB}$ र $T_2 = \overrightarrow{AC}$ भए $T_1\ T_2$ (Δ ABC) पत्ता लगाउनुहोस् ।

The vertices of a triangle ABC are A(1, 5), B(4, 8) and C(7, 3). If $T_1 = \overrightarrow{AB}$ and $T_2 = \overrightarrow{AC}$ then find $T_1\ T_2$ (ΔABC).

8C. A(– 2, 2), B (2, 2) र C(2, 8) शीर्षबिन्दुहरू भएको ΔABC लाई स्थानान्तरण भेक्टर $T_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ र $T_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ द्वारा स्थानान्तरण गरिएको छ । $T_1\ T_2$ पछि आउने ΔABC को प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

ΔABC with vertices A(– 2, 2), B (2, 2) and C(2, 8) is translated by the translation vectors $T_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the the co-ordinates of image ΔABC after $T_1\ T_2$.

Model 5 *On finding a translation vector when next T.V., object & final image are given.*

9A. A(– 2, 2), B(2, 2) र C(2, 8) शीर्षबिन्दुहरू भएको ΔABC लाई स्थानान्तरण भेक्टर $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ र $T_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ द्वारा स्थानान्तरण गरिएको छ । यदि अन्तिम प्रतिविम्बको निर्देशाङ्कहरू A' (1, 5), B'(3, 5) र C'(5, 11) छन् भने स्थानान्तरण भेक्टर T_1 पत्ता लगाउनुहोस् ।

A triangle ABC having vertices A(– 2, 2), B(2, 2) and C(2, 8) is translated by the translating vectors $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ and $T_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. If the co-ordinates of the final image are A' (1, 5), B' (3, 5) and C' (5, 11) after $T_1\ T_2$, find the translating vector T_1.

9B. A(– 2, 2), B(2, 2) र C(2, 8) शीर्षबिन्दुहरू भएको ΔABC लाई स्थानान्तरण भेक्टर $T_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ र $T_2 = \begin{pmatrix} a \\ b \end{pmatrix}$ द्वारा स्थानान्तरण गरिएको छ । यदि अन्तिम प्रतिविम्बको निर्देशाङ्कहरू A' (1, 5), B' (3, 5) र C' (5, 11) छन् भने स्थानान्तरण भेक्टर T_2 पत्ता लगाउनुहोस् ।

ΔABC having vertices A(– 2, 2), B(2, 2) and C(2, 8) is translated by the translating vectors $T_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $T_2 = \begin{pmatrix} a \\ b \end{pmatrix}$. If the co-ordinates of the final image are A'(1, 5), B'(3, 5) and C'(5, 11) after $T_1\ T_2$, find the translating vector T_2 .

www.drmathnp.com

puspas.com.np

Model 6 *Problems based on finding image under the combination of enlargements.*

10A. ΔPQR का शीर्षहरू P(1, 2), Q (3, 2) र R (3, 4) छन् । $E_1[O, 2]$ र $E_2\left[O, 1\frac{1}{2}\right]$ अनुसार ΔPQR लाई विस्तार गरी संयुक्त विस्तारीकरण अनुसार हुने अन्तिम प्रतिविम्ब पत्ता लगाउनुहोस् ।

P(1, 2), Q (3, 2) and R (3, 4) are the vertices of ΔPQR. Enlarge ΔPQR with respect to $E_1[O, 2]$ and $E_2\left[O, 1\frac{1}{2}\right]$. Find the final image of ΔPQR under combined enlargements.

10B. P(6, 1), Q(6, 5) र R(8, 5) भएको ΔPQR लाई लेखाचित्रमा खिच्नुहोस् । केन्द्रबिन्दु (2, 3) र नापो 1/2 द्वारा ΔPQR लाई विस्तारीकरण गरेपछि E_1 [(2, 3); 4] द्वारा विस्तारीकरण गर्दा ΔP'Q'R' भए P',Q' र R' को निर्देशाङ्क पत्ता लगाई ΔP'Q'R' लाई सोही लेखाचित्रमा देखाउनुहोस् ।

On a graph paper, draw the ΔPQR, whose vertices are P(6, 1), Q(6, 5) and R(8, 5). If the ΔPQR maps to ΔP'Q'R' by an enlargement about the center (2, 3) with scale factor of 1/2 ; followed by E_1 [(2, 3); 4], find the co-ordinates of P', Q' and R'. Show this image ΔP'Q'R' on the same graph paper.

10C. शीर्षबिन्दुहरू A(0, 1), B(4, 1), C(3, 3) र D(1, 3) भएको एउटा समलम्ब चतुर्भुज पहिले $E_1[O; 2]$ द्वारा विस्तारीकरण भएको छ र पछि $E_2[O; 3/2]$ द्वारा विस्तारीकरण भएको छ । सो समलम्ब चतुर्भुजको संयुक्त परावर्तनअनुसार प्रतिविम्ब पत्ता लगाउनुहोस् । सो समलम्ब चतुर्भुज र यसको प्रतिविम्बलाई लेखाचित्रमा देखाउनुहोस् ।

A trapezium ABCD with vertices A(0, 1), B(4, 1), C(3, 3) and D(1, 3) is enlarged by $E_1[O; 2]$ and then by $E_2[O; 3/2]$. Find the image of the trapezium under the combined transformation. Show the trapezium and its image on the graph.

Model 7 *Problems based on combination of reflections and rotation or vice-versa.*

11A. A (3, 0) र B (2, 1) भएको ΔOAB लाई + 90° मा परिक्रमण गरी रेखा $x = y$ मा प्रतिविम्बलाई परावर्तन गर्नुहोस् र ΔAOB को अन्तिम प्रतिविम्बको निर्देशाङ्क लेख्नुहोस् । साथै, यी संयुक्त स्थानान्तरणलाई जनाउने एकल स्थानान्तरण लेख्नुहोस् ।

Rotate ΔOAB, where A (3, 0) and B (2, 1) are given, through + 90° about O and then reflect the image in the line $x = y$. Find the final image of ΔOAB and also find the single transformation which is equivalent to the combination of these transformations.

11B. ΔPQR मा P(1, 6), Q(2, 8) र R(7, 2) छन् । ΔPQR लाई $y = 6$ मा परावर्तन गरी उद्गमबिन्दुको वरिपरि 90° परिक्रमण गर्दा बन्ने अन्तिम प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

The vertices of a ΔPQR are P(1, 6), Q(2, 8) and R(7, 2). ΔPQR is reflected in $y = 6$ and then rotated about the origin by 90°. Find the co-ordinates of final image.

Model 8 *Problems based on combination of translation and reflection or vice-versa.*

12A. A(1, 1), B(4, 4), C(4, 8) र D(1, 5) एउटा समबाहु चतुर्भुजका शीर्षबिन्दुहरू हुन् । यसलाई $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ द्वारा विस्थापन गरेपछि रेखा $y = 8$ मा परावर्तन गरी प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

A(1, 1), B(4, 4), C(4, 8) and D(1, 5) are the vertices of a parallelogram. Translate it by $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ and then reflect in the line $y = 8$. Determine the coordinates of the image.

12B. ΔABC का शीर्षबिन्दुरू A(1, 2), B(3, 2) र C(1, 4) छन् । यो पहिले X- अक्षमा परावर्तन भएको छ र पछि विस्थापन भेक्टर $\overrightarrow{A'B'}$ द्वारा विस्थापित भए अन्तिम प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

A(1, 2), B(3, 2) and C (1, 4) are the vertices of ΔABC. It is reflected in X-axis and then translated by translation vector $\overrightarrow{A'B'}$. Find the co-ordinates of the final image.

12C. ΔABC का शीर्षबिन्दुरू A(4, 3), B(7, 4) र C(2, 8) छन् । यो पहिले Y-अक्षमा परावर्तन भएको छ र पछि विस्थापन भेक्टर $\overrightarrow{AC}$ द्वारा विस्थापित भए अन्तिम प्रतिविम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

A(4, 3), B(7, 4) and C (2, 8) are the vertices of ΔABC. ΔABC is reflected in Y-axis and then translated by translation vector $\overrightarrow{AC}$. Find the co-ordinates of the final image.

www.drmathnp.com

puspas.com.np

Model 9 *Problems based on combination of enlargement and reflection or vice-versa.*

13A. O(0, 0), B (0, 1), C(1, 1) र D (1, 0) निर्देशाङ्कहरू भएको ☐OBCD को प्रतिविम्ब संयुक्त स्थानान्तरणद्वारा पत्ता लगाउनुहोस्, जहाँ पहिले X-अक्षमा हुने परावर्तन र पछि नापो 2 लिएर उद्गमबिन्दुलाई केन्द्र मानी विस्तारीकरण गरिएको छ ।
Find the image of ☐OBCD having co-ordinates O(0, 0), B(0, 1), C(1, 1) and D(1, 0) under combined transformation where a reflection in X-axis is followed by an enlargement of scale factor 2 with origin as the centre of enlargement .

13B. ΔABC का शीर्षबिन्दुहरू A(2, 1), B(4, 5) र C(– 1, 4) छन् । ΔABC लाई उद्गमबिन्दुको वरिपरि नापो 3 लिएर विस्तारीकरण गरी $x = y$ रेखामा परावर्तन गर्नुहोस् ।
The vertices of ΔABC are A(2, 1), B(4, 5) and C(– 1, 4). Enlarge ΔABC taking centre as origin and scale factor 3 then reflect in the line $x = y$.

Model 10 *Problems based on combination of rotation and enlargement or vice-versa.*

14A. ΔMNP का शीर्षबिन्दुहरू M(1, 1), N(3, 1) र P(2, 3) छन् । ΔMNP को प्रतिविम्ब उद्गमबिन्दुको वरिपरि 90° धनात्मक परिक्रमणअनुसार पत्ता लगाउनुहोस् र सोही प्रतिविम्बलाई फेरि उद्गमबिन्दुलाई केन्द्र मानी नापो 2 लिएर विस्तारीकरण गर्नुहोस् ।
The vertices of ΔMNP are M(1, 1), N(3, 1) and P(2, 3). Find the image of ΔMNP under the rotation through 90° in positive direction about the origin. Then enlarge the image so obtained by taking the scale factor 2 and centre as origin.

14B. ΔPQR का शीर्षबिन्दुहरू P(1, – 2), Q(3, 4) र R(– 4, 0) छन् । यदि विस्तारको केन्द्र (2, 2) र नापो 2 भए ΔPQR लाई विस्तारीकरण गरी R[O, + 90°] द्वारा परिक्रमण गर्नुहोस् ।
The vertices of ΔPQR are P(1, – 2), Q(3, 4) and R(– 4, 0). If the centre of enlargement is (2, 2) and scale factor 2, then enlarge ΔPQR and rotate by R[O, + 90°].

14C. लेखाचित्रको माध्यमबाट एउटा समद्विबाहु त्रिभुज PQR का शीर्षबिन्दुहरू लेख्नुहोस् । यदि विस्तारको केन्द्र (2, 2) र नापो 2 भए सो त्रिभुजलाई PQR लाई विस्तारीकरण गरी R[O, + 90°] द्वारा परिक्रमण गर्नुहोस् ।
Using a graph paper write the co-ordinates of an isosceles triangle PQR. If the centre of enlargement is (2, 2) and scale factor 2, then enlarge ΔPQR and rotate by R[O, + 90°].

14D. लेखाचित्रको माध्यमबाट एउटा समकोण त्रिभुज MNP का शीर्षबिन्दुहरू लेख्नुहोस् । ΔMNP को प्रतिविम्ब उद्गमबिन्दुको वरिपरि 90° धनात्मक परिक्रमणअनुसार पत्ता लगाउनुहोस् र सोही प्रतिविम्बलाई फेरि उद्गमबिन्दुलाई केन्द्र मानी नापो 2 लिएर विस्तारीकरण गर्नुहोस् ।
Using a graph paper write the co-ordinates of a right angled triangle MNP. Find the image of ΔMNP under the rotation through 90° in positive direction about the origin. Then enlarge the image so obtained by taking the scale factor 2 and centre as origin.

ANSWERS

LONG QUESTIONS

1A. P'(2, – 1) Q'(4, – 2) R' (6, – 4) **1B.** P'(3, 2) Q'(2, 4) R' (0, 6)

2A. A'(3, 2), B'(4, 0), C'(2, – 2), D'(1, 0) and A"(3, 2), B"(2, 0), C"(4, – 2), D"(5, 0); Half turn rotation about (3, 1)

2B. A'(8, 1), B'(3, 1), C'(0, 5), D'(5, 5) and A"(8, 5), B"(3, 5), C"(0, 1), D"(5, 1) ; Half turn rotation about (4, 3)

3A. A'(2, 0), B'(1, 3), C'(3, 4), D'(4, 2) and A"(2, 0), B"(1, – 3), C"(3, – 4), D"(4, – 2); – 90° rotation about (0, 0)

3B. A'(1,– 2), B'(4, – 4), C'(3, – 7) and A"(– 2, 1), B"(– 4, 4), C"(– 7, 3); + 90°, rotation about (0, 0)

4A. A'(3, 3), B'(6, 3), C'(3, 9) and A"(3, 3), B"(0, 3), C"(3, 9); A translation by the translation vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

4B. A'(4, 0), B'(6, 2), C'(3, 2), D'(4, 4) and A"(4, 0), B"(6, – 2), C"(3, – 2), D"(4, – 4); A translation by the TV $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$

5A. A'(4, 4) B'(4, 2), C'(1, 2), D'(1, 4) **5B.** A'(4, – 4), B'(2, – 4), C'(2, – 1) D'(4, – 1) **6A.** A'(– 2, – 5), B'(– 6, – 4), C'(– 3, – 7)

6B. A"(0, – 3), B"(2, – 4), C"(4, – 2), D"(2, – 1) **7A.** A'(13, 7), B' (17, 11), C'(13,11), D' (5, 11)

7B. P'(7,10), Q' (8, 11) R' (11, 10), S' (12, 12) **8A.** A' (6, 2), B'(8, 4), C'(10, 1) **8B.** A' (10, 6), B' (13, 9), C' (16, 4)

8C. A' (1, 5), B' (3, 5), C' (5, 11) **9A.** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **9B.** $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **10A.** P'(3, 6) Q'(9, 6), R'(9, 12)

10B. P'(10, – 1), Q'(10, 7), R'(14, 7) **10C.** A'(0, 3), B'(12, 3), C'(9, 9), D'(3, 9)

11A. O' (0, 0), A'(0, 3), B'(– 1, 2) and O"(0, 0), A"(3, 0), B"(2, – 1)

11B. P'(1, 6), Q'(2, 4), R'(7, 10) and P"(– 6, 1), B"(– 4, 2), R"(–10, 7)

12A. A'(6, 8), B'(9, 11), C'(9, 15), D'(6, 12) and A"(6, 8), B"(9, 5), C"(9, 1), D"(6, 4)

12B. A'(1, – 2), B'(3, – 2), C'(1, – 4) and A"(3, –2), B"(5, – 2), C"(3, – 4)

12C. A'(– 4, 3), B'(– 7, 4), C'(– 2, 8) and A"(– 6, 8), B"(–9, 9), C"(– 4, 13)

13A. O"(0, 0), B"(0, – 2), C"(2, – 2), D"(2, 0) **13B.** A'(3, 6), B'(15, 12), C'(12, – 3)

14A. M'(– 1, 1), N'(– 1, 3), P'(– 3, 2) and M"(– 2, 2), N"(– 2, 6), P"(– 6, 4) **14B.** P"(6, 0), Q"(– 6, 4), R"(2, – 10)

14C. Show to your teacher. **14D.** Show to your teacher.

www.drmathnp.com

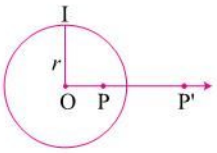
2. विपरीत स्थानान्तरण र विपरीत वृत्त
INVERSION TRANSFORMATION AND INVERSION CIRCLE

2.1. विपरीत स्थानान्तरण (Inversion Transformation)

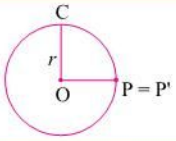
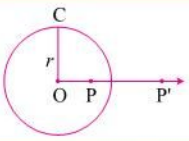
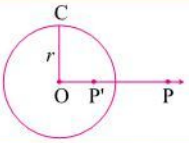
कुनै वृत्तमा आधारीत स्थानान्तरणलाई विपरीत स्थानान्तरण भनिन्छ ।
A transformation with respect to a circle is called inversion transformation or simply inversion.

2.2 उत्क्रम वृत्त वा विपरीत वृत्त र उत्क्रम बिन्दु (Inversion circle or circle inversion and inversion point)

केन्द्रबिन्दु O र अर्धव्यास r भएको वृत्तको केन्द्रबिन्दु हुँदै गएको रेखामा केन्द्रबिन्दु O बाहेकको कुनै बिन्दु P को प्रतिविम्ब OP × OP' = r^2 हुनेगरी P' हुन्छ भने सो वृत्तलाई उत्क्रम वृत्त र P' लाई उत्क्रम बिन्दु भनिन्छ ।
In a circle with centre O and radius r, if P be a point other than origin then the inverse of P with respect to the circle is a point P' on line OP such that OP × OP' = r^2. The image P' is the inversion point of P and the circle is inversion circle.



2.3 उत्क्रम बिन्दु र उत्क्रम वृत्तका विभिन्न अवस्थाहरू (Different conditions of inversion point and inversion circle)

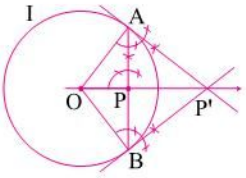
वृत्तको परिधिमा बिन्दु हुँदा Point is at the circumference	वृत्त भित्र बिन्दु हुँदा Point is inside the circle	वृत्त बाहिर बिन्दु हुँदा Point is outside the circle
		
वस्तु र प्रतिविम्ब एउटै हुन्छ । Object and image are at the same position.	वस्तु र प्रतिविम्ब भित्र र बाहिर हुन्छन् । Object and image are inside and outside the circle.	वस्तु र प्रतिविम्ब बाहिर र भित्र हुन्छन् । Object and image are outside and inside the circle.
OP = OP' = r	**OP < r & OP' > r**	**OP > r & OP' < r**

Note: (i) (P')' = P (ii) P' को उत्क्रम बिन्दु P हुन्छ । (Inversion point of P' is P.)

2.4. उत्क्रम बिन्दुको ज्यामितीय व्याख्या (Geometrical Interpretation of Inverse Point)

(i) उत्क्रम वृत्त I को भित्र बिन्दु P हुँदा (Point P is being inside the inversion circle I)

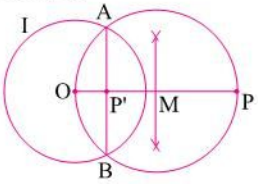
P को उत्क्रम बिन्दुको रचनाका चरणहरू :
Steps of construction of inversion point of P:



1. OP जोडेर लम्ब्याऔं । (Join OP and produce.)
2. OP को बिन्दु P मा लम्ब खिचेर लम्ब्याऔं ।
   Construct perpendicular to OP at P and produce.
3. लम्ब रेखाले परिधिका बिन्दुहरू A र B मा भेटेको छ ।
   The perpendicular line meets the points A and B of the circumference.
4. OA को बिन्दु A र OB को बिन्दु B मा लम्बहरू खिचौं ।
   Construct the perpendiculars to OA at A and OB at B.
5. प्रतिच्छेदन बिन्दु P' लाई P को उत्क्रम बिन्दु भनिन्छ ।
   The point of intersection P' is the inversion point of P.

(ii) उत्क्रम वृत्त I को बाहिर बिन्दु P हुँदा (Point P is being outside the inversion circle I):

P को उत्क्रम बिन्दुको रचनाका चरणहरू:
Steps of construction of inversion point of P:

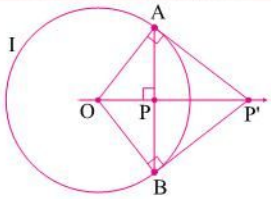
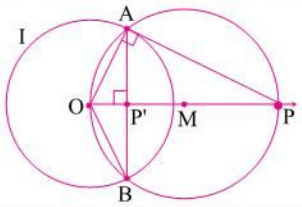


1. OP जोडौं । (Join OP.)
2. OP को मध्यबिन्दु M पत्ता लगाऔं । (Find the midpoint M of OP.)
3. MP लाई अर्धव्यास र M लाई केन्द्र मानेर वृत्त खिचौं ।
   Construct the circle with radius MP and M as a centre.
4. सो वृत्तले वृत्त I को A र B मा काटेको छ । (The circle will intersect circle I at two points, A and B.)
5. OP र AB को प्रतिच्छेदन बिन्दु P' नै P को उत्क्रम बिन्दु हुन्छ ।
   The point of intersection P' of OP and AB is the inversion point of P.

puspas.com.np

www.drmathnp.com

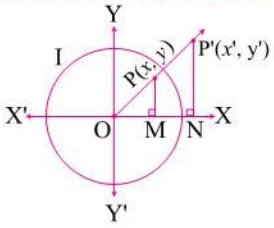
puspas.com.np

वृत्त I को भित्र P हुँदाको प्रमाण Proof of P inside circle I		वृत्त I को बाहिर P हुँदाको प्रमाण Proof of P outside circle I	
			
Statements	**Reasons**	**Statements**	**Reasons**
1. In ΔAOP and $\Delta AOP'$ (i) $\measuredangle OPA = \measuredangle OAP'$ (ii) $\measuredangle AOP = \measuredangle AOP'$	 (i) By construction (ii) Common angles	1. In $\Delta AOP'$ and ΔAOP (i) $\measuredangle OP'A = \measuredangle OAP$ (ii) $\measuredangle AOP' = \measuredangle AOP$	 (i) Both are right angles. (ii) Both are common angles.
2. $\Delta AOP \sim \Delta AOP'$	By AA similarity	2. $\Delta AOP' \sim \Delta AOP$	By AA similarity
3. $\frac{OA}{OP'} = \frac{AP}{AP'} = \frac{OP}{OA}$	The corresponding sides of similar ΔS.	3. $\frac{AO}{OP} = \frac{AP'}{AP} = \frac{OP'}{OA}$	Corresponding sides of similar triangles.
4. $\frac{OA}{OP'} = \frac{OP}{OA}$ $\therefore$ $OP \times OP' = OA^2$ i.e. $OP \times OP' = r^2$	From the statement (3)	4. $\frac{AO}{OP} = \frac{OP'}{OA}$ $\therefore$ $OP \times OP' = OA^2$ i.e. $OP \times OP' = r^2$	From the statement (3)

2.5. दिइएको बिन्दुको उत्क्रम बिन्दु पत्ता लगाउने तरिका । (Method of finding the inversion point of given point.)

Case - I

When the centre of inversion circle is origin (0, 0).



In the figure, P'(x', y') is the inverse of P(x, y) with respect to the inversion circle I having equation $x^2 + y^2 = r^2$.

Let O be the origin and r be the radius of the circle.

Then, $OP \times OP' = r^2$

Draw PM $\perp$ OX and P'N $\perp$ OX.

Then, OM = x, ON = x', PM = y and P'N = y'

Using Pythagorean theorem we get,

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} \text{ and } OP' = \sqrt{(x'-0)^2 + (y'-0)^2} = \sqrt{x'^2 + y'^2}$$

We have, $\Delta P'NO$ and ΔPMO are similar by AA test of similarity of triangles.

Then we know that, $\frac{ON}{OM} = \frac{P'N}{PM} = \frac{OP'}{OP}$

or, $\frac{x'}{x} = \frac{y'}{y} = \frac{OP'}{OP} \times \frac{OP}{OP}$

or, $\frac{x'}{x} = \frac{y'}{y} = \frac{r^2}{OP^2}$ [$\because OP \times OP' = r^2$]

or, $\frac{x'}{x} = \frac{y'}{y} = \frac{r^2}{x^2 + y^2}$

Taking first and third ratios then,	Taking second and third ratios then,
$\frac{x'}{x} = \frac{r^2}{x^2 + y^2}$	$\frac{y'}{y} = \frac{r^2}{x^2 + y^2}$
$\therefore$ $x' = \frac{r^2 x}{x^2 + y^2}$	$\therefore$ $y' = \frac{r^2 y}{x^2 + y^2}$

Thus, the co-ordinates of the inverse P'(x', y') of P(x, y) with respect to the circle I is

$$(x', y') = \left(\frac{r^2 x}{x^2 + y^2}, \frac{r^2 y}{x^2 + y^2}\right)$$

Note:

(i) If $x^2 + y^2 = 0$ for the centre of inversion and x' and y' do not have real values then the inverse of the centre of inversion is a point at infinity.

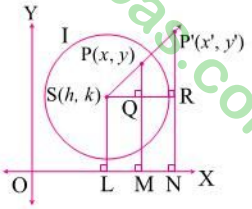
(ii) For any point on the inversion circle $x^2 + y^2 = r^2$, the inverse point P'(x' , y') of P(x, y) is given by $x' = x$ and $y' = y$. i.e. P'(x', y') = P'(x, y)

(iii) The inverse of the inverse point is given by $x = \frac{r^2 x'}{x'^2 + y'^2}$ and $y = \frac{r^2 y'}{x'^2 + y'^2}$

www.drmathnp.com

puspas.com.np

Case - II



When the centre of inversion is other than origin i.e. (h, k).

In the figure, $P'(x', y')$ is the inverse of a point $P(x, y)$ with respect to the circle I whose equation is $(x-h)^2 + (y-k)^2 = r^2$. The centre of the circle is (h, k) and the radius of the circle is r.

Draw SL ⊥ OX, P'N ⊥ OX, PM ⊥ OX, SR ⊥ P'N.

Then, OL = h, OM = x, ON = x', SL = k, PM = y, P'N = y'

We have, SP = $\sqrt{(x-h)^2 + (y-k)^2}$ and SP' = $\sqrt{(x'-h)^2 + (y'-k)^2}$

Now, ΔSRP' and ΔSQP are similar by AA similarity.

Then, we have, $\frac{SR}{SQ} = \frac{P'R}{PQ} = \frac{SP'}{SP}$

or, $\frac{LN}{LM} = \frac{P'N - NR}{PM - QM} = \frac{SP' \times SP}{SP^2}$

or, $\frac{ON - OL}{OM - OL} = \frac{P'N - SL}{PM - SL} = \frac{SP' \times SP}{SP^2}$

or, $\frac{x'-h}{x-h} = \frac{y'-k}{y-k} = \frac{r^2}{(x-h)^2 + (y-k)^2}$ $[\because SP' \times SP = r^2]$

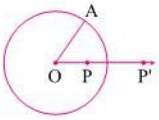
Taking first and third ratios then,	Similarly taking second and third ratios then,
$\frac{x'-h}{x-h} = \frac{r^2}{(x-h)^2 + (y-k)^2}$	$\frac{y'-k}{y-k} = \frac{r^2}{(x-h)^2 + (y-k)^2}$
or, $x' - h = \frac{r^2(x-h)}{(x-h)^2 + (y-k)^2}$	or, $y' - k = \frac{r^2(y-k)}{(x-h)^2 + (y-k)^2}$
$\therefore$ $x' = h + \frac{r^2(x-h)}{(x-h)^2 + (y-k)^2}$	$\therefore$ $y' = k + \frac{r^2(x-k)}{(x-h)^2 + (y-k)^2}$

So, the coordinates of the inverse point $P'(x', y')$ of the point $P(x, y)$ with respect to the circle I is;

$$\left(h + \frac{r^2(x-h)}{(x-h)^2 + (y-k)^2}, k + \frac{r^2(y-k)}{(x-h)^2 + (y-k)^2}\right)$$

SOLVED EXAMPLES

Example 1: चित्रमा OA = 6 cm र OP = 4 cm छन् । PP' को नाप कति होला ?
In the figure, OA = 6 cm and OP = 4 cm. What will be the measure of PP'?



Solution: Here, OA = 6 cm, OP = 4 cm

By the definition, OP × OP' = r^2

or, 4 × OP' = 6^2

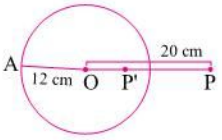
$\therefore$ OP' = 9 cm

Now, PP' = OP' − OP = 9 cm − 4 cm = 5 cm.

Thus, the measure of PP' is 5 cm.

- *The inverse of a line through the centre of inversion is the same line.*
- *The inverses of triangle, quadrilateral and other polygons not through the centre of inverse with respect to circle are also triangle, quadrilateral and other polygons respectively.*

Example 2: चित्रमा O वृत्तको केन्द्रबिन्दु हो, OA = 12 cm र OP = 20 cm छन् । यदि P को उत्क्रम बिन्दु P' भए PP' को नाप पत्ता लगाउनुहोस् ।
In the figure, O is the centre of circle, OA = 12 cm and OP = 20 cm. If P' is the inversion image of P then find the measure of PP'.



Solution: Here, radius (OA) = 12 cm and OP = 20 cm, PP' = ?

We know that, OP × OP' = r^2

or, 20 cm × OP' = $(12 \text{ cm})^2$

or, 20 cm × OP' = 144 cm^2

$\therefore$ OP' = 7.2 cm

Now, PP' = OP − OP' = 20 cm − 7.2 cm

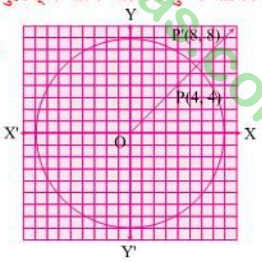
$\therefore$ PP' = 12.8 cm

Thus, the measure of PP' is 12.8 cm.

www.drmathnp.com

Example 3: वृत्त $x^2 + y^2 = 64$ को आधारमा बिन्दु (4, 4) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै सो बिन्दु र सोको उत्क्रम बिन्दुलाई एउटै लेखाचित्रमा पनि देखाउनुहोस् ।

Find the inverse of the point (4, 4) with respect to the circle $x^2 + y^2 = 64$. Also, show the inversion point in the same graph.



Solution:

Here, given equation of circle is $x^2 + y^2 = 64$

or, $x^2 + y^2 = 8^2$

Comparing it with $x^2 + y^2 = r^2$ then, centre = (0, 0) and radius $(r) = 8$

Also, given point; (4, 4) = (x, y)

We know that,

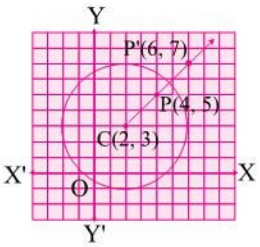
$$P(x, y) \xrightarrow{Inversion\ [(0,0);\ r]} P\left[\frac{r^2x}{x^2+y^2}, \frac{r^2y}{x^2+y^2}\right]$$

$$P(4, 4) \xrightarrow{Inversion\ [(0,0);\ 8]} P\left[\frac{8^2\times 4}{4^2+4^2}, \frac{8^2\times 4}{4^2+4^2}\right]$$

Thus, the inverse of given point (4, 4) is (8, 8)

Example 4: दिइएको बिन्दु (4, 5) को वृत्त $x^2 + y^2 - 4x - 6y - 3 = 0$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै सोको उत्क्रम बिन्दुलाई लेखाचित्रमा देखाउनुहोस् ।

Find the inversion point of the given point (4, 5) with respect to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$. Also show the inversion point in the graph.



Solution:

Here, the given equation of circle is; $x^2 + y^2 - 4x - 6y - 3 = 0$

or, $x^2 - 4x + y^2 - 6y - 3 = 0$

or, $x^2 - 2 \,.\, x \,.\, 2 + 2^2 + y^2 - 2 \,.\, y \,.\, 3 + 3^2 = 3 + 2^2 + 3^2$

or, $(x - 2)^2 + (y - 3)^2 = 16$

$\therefore\ (x - 2)^2 + (y - 3)^2 = 4^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ then,

centre $(h, k) = (2, 3)$ and radius $(r) = 4$

Given point P(4, 5) = P(x, y)

Let, the point of inverse be P' (x', y')

We have, $x' = \dfrac{r^2(x-h)}{(x-h)^2+(y-k)^2} + h$ and $y' = \dfrac{r^2(y-k)}{(x-h)^2+(y-k)^2} + k$

or, $x' = \dfrac{4^2(4-2)}{(4-2)^2+(5-3)^2} + 2$ and $y' = \dfrac{4^2(5-3)}{(4-2)^2+(5-3)^2} + 3$

or, $x' = \dfrac{16\times 2}{4+4} + 2$ and $y' = \dfrac{16\times 2}{4+4} + 3$

or, $x' = 4 + 2$ and $y' = 4 + 3$

$\therefore\ x' = 6$ and $y' = 7$

Thus, the inversion point is (6, 7).

Now, the graphical representation is shown alongside.

Example 5: यदि केन्द्रबिन्दु O भएको वृत्तको आधारमा बिन्दु A(3, 4) को उत्क्रम बिन्दु A' (12, 16) छ भने सो वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।

If the inversion of a point A(3, 4) with respect to a circle having centre as origin is A'(12, 16), find the radius of the circle.

Solution:

Here, centre of inversion = (0, 0)

Given point = A(3, 4) = A(x, y)

Inversion point = A'(12, 16) = A'(x', y')

Radius of the circle (r) = ?

We know that, $x' = \dfrac{r^2x}{x^2+y^2}$ and $y' = \dfrac{r^2y}{x^2+y^2}$

or, $12 = \dfrac{r^2\times 3}{3^2+4^2}$ or, $16 = \dfrac{r^2\times 4}{3^2+4^2}$

or, $12 = \dfrac{r^2\times 3}{25}$ or, $16 = \dfrac{r^2\times 4}{25}$

or, $\dfrac{12\times 25}{3} = r^2$ or, $\dfrac{16\times 25}{4} = r^2$

or, $r^2 = 100$ or, $100 = r^2$

$\therefore\ r = 10$ $\therefore\ r = 10$

Thus, the radius of the inversion circle is 10 units.

pustaks.com.np

www.drmathnp.com

puspas.com.np

Example 6: बिन्दुहरू P(4, 5) र Q(4, 3) जोड्ने रेखाखण्डको केन्द्र C(2, 3) र अर्धव्यास 4 एकाइ भएको वृत्तको आधारमा उत्क्रम रेखाखण्ड पत्ता लगाउनुहोस् । सो उत्क्रमलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Find the inverse line segment joining the point P(4, 5) and Q(4, 3) with respect to a circle with centre C(2, 3) and radius 4 units. Show the inversion on the same graph.

Solution: Here, in the circle, centre C(2, 3) = C(h, k)
Radius (r) = 4 units
Let, the inversion point of P(4, 5) or P(x, y) is P'(x', y')

$$\text{We know that, } P'(x', y') = P'\left(\frac{r^2(x-h)}{(x-h)^2+(y-k)^2}+h, \frac{r^2(y-k)}{(x-h)^2+(y-k)^2}+k\right)$$

$$= P'\left(\frac{4^2(4-2)}{(4-2)^2+(5-3)^2}+2, \frac{4^2(5-3)}{(4-2)^2+(5-3)^2}+3\right)$$

$$= P'\left(\frac{16\times 2}{4+4}+2, \frac{16\times 2}{4+4}+3\right)$$

$$= P'(4+2, 4+3)$$

$\therefore$ P'(x', y') = P'(6, 7)
Let, the inversion point of Q(4, 3) or Q(x, y) is Q'(x', y')

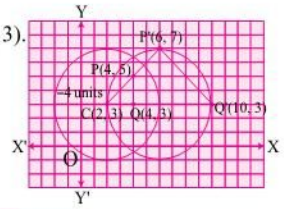
$$\text{We know that, } Q'(x', y') = Q'\left(\frac{r^2(x-h)}{(x-h)^2+(y-k)^2}+h, \frac{r^2(y-k)}{(x-h)^2+(y-k)^2}+k\right)$$

$$= Q'\left(\frac{4^2(4-2)}{(4-2)^2+(3-3)^2}+2, \frac{4^2(3-3)}{(4-2)^2+(3-3)^2}+3\right)$$

$$= Q'\left(\frac{16\times 2}{4+0}+2, \frac{16\times 0}{4+0}+3\right)$$

$$= Q'(8+2, 0+3) = Q'(10, 3)$$

Thus, the inversion line segment has the points P'(6, 7) and Q'(10, 3).
Now, representation on the graph;



SEE / SLC REVIEW MESSAGE

A. VERY SHORT QUESTIONS

Model	SEE Examination History (2065 to 2075)	n(Q)
Model 1	SEE MODEL 2076	1

SEE EXERCISE 2

TRANSFORMATION

A. VERY SHORT QUESTIONS

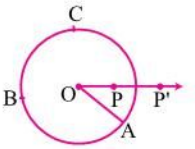
Model 1 *Definitions or formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms) :

A. विपरीत स्थानान्तरण (Inversion transformation) **B.** विपरीत वृत्त (Inversion circle)

C. उत्क्रम बिन्दु (Inversion point)

2. सँगैको चित्रबाट तलका पदहरूको नाम लेख्नुहोस् ।
On the basis of adjoining figure, name the following terms.



A. विपरीत वृत्त (Inversion circle)

B. बिन्दु P को उत्क्रम बिन्दु (Inverse of a point P)

C. बिन्दु P' को उत्क्रम बिन्दु (Inverse of a point P')

D. विपरीत वृत्तको अर्धव्यास (Radius of inversion circle)

E. OP, OP' र OA को सम्बन्ध (Relation between OP, OP' and OA.)

F. OP.OP' र r को सम्बन्ध (Relation between OP.OP] and r) **[SEE MODEL 2076]**



3A. यदि उत्क्रम वृत्तको बाहिर बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if point P is outside the circle of inversion?

www.drmathnp.com

puspas.com.np

3B. यदि उत्क्रम वृत्तको भित्र बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if point P is inside the circle of inversion?

3C. यदि उत्क्रम वृत्तको परिधिमा बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if point P is on the circumference of circle of inversion?

3D. यदि उत्क्रम वृत्तको केन्द्रबिन्दुमा बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if point P lies at the centre of the circle of inversion?

4A. उत्क्रम वृत्तको केन्द्रबाट जाने रेखाको विपरीत के हुन्छ ?
What is the inverse of a line through the centre of inversion circle?

4B. उत्क्रम वृत्तको बाहिर पट्टीबाट जाने रेखाको विपरीत के हुन्छ ?
What is the inverse of a line through the outside of the circle?

5A. केन्द्रबिन्दु उद्गम बिन्दुमा पर्ने वृत्तमा $P(x, y)$ को उत्क्रम बिन्दु $P'(x', y')$ भए $P'(x', y')$ निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to find $P'(x', y')$ if $P'(x', y')$ is the inversion point of $P(x, y)$ in a circle with centre at origin.

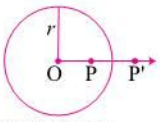
5B. केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' भएको वृत्तमा $P(x, y)$ को उत्क्रम बिन्दु $P'(x', y')$ भए x' निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate x' if $P'(x', y')$ is the inversion point of $P(x, y)$ of a circle whose centre is (h, k) and radius 'r'.

5C. केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' भएको वृत्तमा $P(x, y)$ को उत्क्रम बिन्दु $P'(x', y')$ भए y' निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate y' if $P'(x', y')$ is the inversion point of $P(x, y)$ of a circle whose centre is (h, k) and radius 'r'.

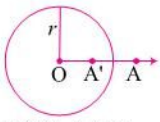
Model 2 *Concept Recall*

6A. सँगैको चित्रमा P को विपरीत P' छ । यदि $OP \times OP' = 25\ cm^2$ भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
In the adjoining figure, P' is the inverse of P. If $OP \times OP' = 25\ cm^2$ then find the radius of the inversion circle.

6B. सँगैको चित्रमा P को विपरीत P' छ । यदि $OP \times OP' = 49\ cm^2$ भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
In the adjoining figure, P' is the inverse of P. If $OP \times OP' = 49\ cm^2$ then find the radius of the inversion circle.



Q. 6(a)& 6(b)



Q. 7(a) & 7(b)

7A. उत्क्रम वृत्तको चित्रमा A को विपरीत A' छ । यदि $r = 8$ cm भए $OA \times OA'$ को नाप पत्ता लगाउनुहोस् ।
In the figure of inversion circle, A' is the inverse of A. If $r = 8$ cm then find the measure of $OA \times OA'$,

7B. उत्क्रम वृत्तको चित्रमा A को विपरीत A' छ । यदि $r = 9$ cm भए $OA \times OA'$ को नाप पत्ता लगाउनुहोस् ।
In the figure of inversion circle, A' is the inverse of A. If $r = 9$ cm then find the measure of $OA \times OA'$,

8A. केन्द्रबिन्दु O र अर्धव्यास r भएको वृत्तमा P को उत्क्रम बिन्दु P' छ । यदि $OP = 5$ cm र $OP' = 20$ cm भए r को मान पत्ता लगाउनुहोस् ।
In a circle with centre O and radius r, P' is the inversion point of P. If $OP = 5$ cm and $OP' = 20$ cm, find the value of r.

8B. केन्द्रबिन्दु O र अर्धव्यास r भएको वृत्तमा P को उत्क्रम बिन्दु P' छ । यदि $OP = 2$ cm र $OP' = 8$ cm भए r को मान पत्ता लगाउनुहोस् ।
In a circle with centre O and radius r, P' is the inversion point of P. If $OP = 2$ cm and $OP' = 8$ cm, find the value of r.

9A. केन्द्रबिन्दु 'O' भएको वृत्तमा अर्धव्यास 'r' छ । P को उत्क्रम बिन्दु P' छ । यदि $r = 4$ cm र $OP' = 2$ cm भए OP को नाप पत्ता लगाउनुहोस् ।
In a circle with centre 'O' and radius 'r', the image of P is P'. If $r = 4$ cm and $OP' = 2$ cm, find the measure of OP.

9B. केन्द्रबिन्दु 'O' भएको वृत्तमा अर्धव्यास 'r' छ । P को उत्क्रम बिन्दु P' छ । यदि $r = 3$ cm र $OP' = 1$ cm भए OP को नाप पत्ता लगाउनुहोस् ।
In a circle with centre 'O' and radius 'r', the image of P is P'. If $r = 3$ cm and $OP' = 1$ cm, find the measure of OP.

B. LONG QUESTIONS

Model 1 *Problems based on the formula $OP \times OP' = r^2$.*

1A. एउटा वृत्तको उद्गम बिन्दु O बाट तीनओटा बिन्दुहरू A, B र C क्रमशः 2, 3 र 4 एकाइ दूरीमा छन् । यदि वृत्तको अर्धव्यास 6 एकाइ भए A, B र C का उत्क्रम बिन्दुहरूको वृत्तको केन्द्रबाट दूरी पत्ता लगाउनुहोस् ।
Three points A, B and C are at distances 2, 3 and 4 units respectively from the centre O of a circle. If the radius of the circle is 6 units then find the distance of inversion points of A, B and C from the centre of the circle.

www.drmathnp.com

1B. एउटा वृत्तको उद्‌गम बिन्दु O बाट तीनओटा बिन्दुहरू A, B र C क्रमशः 4, 10 र 20 एकाइ दूरीमा छन् । यदि वृत्तको अर्धव्यास 10 एकाइ भए A, B र C का उत्क्रम बिन्दुहरूको वृत्तको केन्द्रबाट दूरी पत्ता लगाउनुहोस् ।
Three points A, B and C are at distances 4, 10 and 20 units respectively from the centre O of a circle. If the radius of the circle is 10 units then find the distance of inversion points of A, B and C from the centre of the circle.

1C. एउटा वृत्तको उद्‌गम बिन्दु O बाट तीनओटा बिन्दुहरू A, B र C क्रमशः 2, 4 र 16 एकाइ दूरीमा छन् । यदि वृत्तको अर्धव्यास 8 एकाइ भए A, B र C का उत्क्रम बिन्दुहरूको वृत्तको केन्द्रबाट दूरी पत्ता लगाउनुहोस् ।
Three points A, B and C are at distances 2, 4, and 16 units respectively from the centre O of a circle. If the radius of the circle is 8 units then find the distance of inversion points of A, B and C from the centre of the circle.

1D. एउटा वृत्तको उद्‌गम बिन्दु O बाट तीनओटा बिन्दुहरू A, B र C क्रमशः 6, 12 र 18 एकाइ दूरीमा छन् । यदि वृत्तको अर्धव्यास 24 एकाइ भए A, B र C का उत्क्रम बिन्दुहरूको वृत्तको केन्द्रबाट दूरी पत्ता लगाउनुहोस् ।
Three points A, B and C are at distances 6, 12 and 18 units respectively from the centre O of a circle. If the radius of the circle is 24 units then find the distance of inversion points of A, B and C from the centre of the circle.

Model 2 *Problems based on inversion point when inversion centre is at the origin.*

2A. वृत्त $x^2 + y^2 = 20$ को आधारमा बिन्दुहरू P(2, 1) र Q(3, 1) का उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the inversion points of P(2, 1) and Q(3, 1) with respect to the circle $x^2 + y^2 = 20$.

2B. वृत्त $x^2 + y^2 = 50$ को आधारमा बिन्दुहरू A(6, 8) र B(3, 4) का उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the inversion points of A(6, 8) and B(3, 4) with respect to the circle $x^2 + y^2 = 50$.

2C. वृत्त $x^2 + y^2 = 100$ को आधारमा बिन्दुहरू A(1, 2) र B(7, 1) का उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the inversion points of A(1, 2) and B(7, 1) with respect to the circle $x^2 + y^2 = 100$.

2D. वृत्त $x^2 + y^2 = 60$ को आधारमा बिन्दुहरू P(2, 4) र Q(1, 3) का उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the inversion points of P(2, 4) and Q(1, 3) with respect to the circle $x^2 + y^2 = 60$.

3A. वृत्त $x^2 + y^2 = 9$ को आधारमा बिन्दु A(3, 0) को उत्क्रम बिन्दु पत्ता लगाई एउटै लेखाचित्रमा उत्क्रम वृत्त, वस्तु र प्रतिबिम्ब देखाउनुहोस् ।
Find the inversion point of a point A(3, 0) with respect to the circle $x^2 + y^2 = 9$ and show the inversion circle, object and image in the same graph.

3B. वृत्त $x^2 + y^2 = 16$ को आधारमा बिन्दु P(2, 0) को उत्क्रम बिन्दु पत्ता लगाई एउटै लेखाचित्रमा उत्क्रम वृत्त, वस्तु र प्रतिबिम्ब देखाउनुहोस् ।
Find the inversion point of a point P(2, 0) with respect to the circle $x^2 + y^2 = 16$ and show the inversion circle, object and image in the same graph.

3C. वृत्त $x^2 + y^2 = 16$ को आधारमा बिन्दु A(1, 0) को उत्क्रम बिन्दु पत्ता लगाई एउटै लेखाचित्रमा उत्क्रम वृत्त, वस्तु र प्रतिबिम्ब देखाउनुहोस् ।
Find the inversion point of a point A(1, 0) with respect to the circle $x^2 + y^2 = 16$ and show the inversion circle, object and image in the same graph.

3D. वृत्त $x^2 + y^2 = 4$ को आधारमा बिन्दु A(1, 0) को उत्क्रम बिन्दु पत्ता लगाई एउटै लेखाचित्रमा उत्क्रम वृत्त, वस्तु र प्रतिबिम्ब देखाउनुहोस् ।
Find the inversion point of a point A(1, 0) with respect to the circle $x^2 + y^2 = 4$ and show the inversion circle, object and image in the same graph.

3E. वृत्त $x^2 + y^2 = 64$ को आधारमा बिन्दु D(0, 4) को उत्क्रम बिन्दु पत्ता लगाई एउटै लेखाचित्रमा उत्क्रम वृत्त, वस्तु र प्रतिबिम्ब देखाउनुहोस् ।
Find the inversion point of a point D(0, 4) with respect to the circle $x^2 + y^2 = 64$ and show the inversion circle, object and image in the same graph.

Model 3 *Problems based on inversion point when inversion centre is not at the origin.*

4A. बिन्दु A(3, 4) को वृत्त $(x-2)^2 + (y-3)^2 = 24$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inversion point of A(3, 4) with respect to the circle $(x-2)^2 + (y-3)^2 = 24$.

4B. बिन्दु B(5, 6) को वृत्त $(x-4)^2 + (y-2)^2 = 68$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inversion point of B(5, 6) with respect to the circle $(x-4)^2 + (y-2)^2 = 68$.

4C. बिन्दु P(2, 2) को वृत्त $(x-2)^2 + (y-1)^2 = 3^2$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inversion point of P(2, 2) with respect to the circle $(x-2)^2 + (y-1)^2 = 3^2$.

4D. बिन्दु B(3, 2) को वृत्त $(x-2)^2 + (y-3)^2 = 4$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
Find the inversion point of B(3, 2) with respect to the circle $(x-2)^2 + (y-3)^2 = 4$.

puspas.com.np

www.drmathnp.com

5A. एउटा उत्क्रम वृत्तको व्यास 8 एकाइ र केन्द्र (0, 1) छ । सो वृत्तको आधारमा बिन्दु C(– 1, 2) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The diameter of an inversion circle is 8 units and centre (0, 1). Find the inversion point of C(– 1, 2) with respect to the circle.

5B. एउटा उत्क्रम वृत्तको व्यास 10 एकाइ र केन्द्र (1, 2) छ । सो वृत्तको आधारमा बिन्दु P(– 3, – 1) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The diameter of an inversion circle is 10 units and centre (1, 2). Find the inversion point of P(– 3, – 1) with respect to the circle.

6A. एउटा उत्क्रम वृत्तको समीकरण $x^2 + y^2 - 8x - 6y - 2 = 0$ छ । सो वृत्तको आधारमा बिन्दु D(1, 3) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The equation of an inversion circle is $x^2 + y^2 - 8x - 6y - 2 = 0$. Find the inversion point of D(1, 3) with respect to the circle.

6B. एउटा उत्क्रम वृत्तको समीकरण $x^2 + y^2 - 4x - 6y - 23 = 0$ छ । सो वृत्तको आधारमा बिन्दु A(3, 4) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The equation of an inversion circle is $x^2 + y^2 - 4x - 6y - 23 = 0$. Find the inversion point of A(3, 4) with respect to the circle.

6C. एउटा उत्क्रम वृत्तको समीकरण $x^2 + y^2 + 6x - 8y + 8 = 0$ छ । सो वृत्तको आधारमा बिन्दु B(1, 3) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The equation of an inversion circle is $x^2 + y^2 + 6x - 8y + 8 = 0$. Find the inversion point of B(1, 3) with respect to the circle.

6D. एउटा उत्क्रम वृत्तको समीकरण $x^2 + y^2 - 4x - 6y = 7$ छ । सो वृत्तको आधारमा बिन्दु C(4, 5) को उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।
The equation of an inversion circle is $x^2 + y^2 - 4x - 6y = 7$. Find the inversion point of C(4, 5) with respect to the circle.

7A. दिइएको बिन्दु A(4, 5) को वृत्त $x^2 + y^2 - 4x - 6y - 3 = 0$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै उत्क्रम वृत्त, वस्तु र प्रतिबिम्बलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Find the inversion point of the given point A(4, 5) with respect to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$. Also, show the inversion circle, object and image in the graph.

7B. दिइएको बिन्दु A(0, 4) को वृत्त $x^2 + y^2 - 2x - 6y = 6$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै उत्क्रम वृत्त, वस्तु र प्रतिबिम्बलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Find the inversion point of the given point A(0, 4) with respect to the circle $x^2 + y^2 - 2x - 6y = 6$. Also, show the inversion circle, object and image in the graph.

Model 4 ***Problems based on line inversion with respect to the centre is not at the origin.***

8A. बिन्दुहरू P(12, 15) र Q(12, 9) जोड्ने रेखाखण्डको केन्द्र (6, 9) र अर्धव्यास 12 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
Find the inverse of line segment joining the points P(12, 15) and Q(12, 9) with respect to a circle with centre C(6, 9) and radius 12 units. Show the inversion transformation in the graph.

8B. बिन्दुहरू A(8, 10) र B(8, 6) जोड्ने रेखाखण्डको केन्द्र (4, 6) र अर्धव्यास 8 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
Find the inverse of line segment joining the points A(8, 10) and B(8, 6) with respect to a circle with centre C(4, 6) and radius 8 units. Show the inversion transformation in the graph.

8C. बिन्दुहरू P(6, 4) र Q(7, 2) जोड्ने रेखाखण्डको केन्द्र (2, 2) र अर्धव्यास 10 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
Find the inverse of line segment joining the points P(6, 4) and Q(7, 2) with respect to a circle with centre R(2, 2) and radius 10 units. Show the inversion transformation in the graph.

TRANSFORMATION

ANSWERS

A. VERY SHORT QUESTIONS

2A. ACB **2B.** P' **2C.** P **2D.** OA **2E.** $OP \times OP' = OA^2$ **2F.** $OP.OP' = r^2$

5A. $x' = \frac{r^2 x}{x^2 + y^2}$, $y' = \frac{r^2 y}{x^2 + y^2}$ **5B.** $x' = \frac{r^2 (x - h)}{(x - h)^2 + (y - k)^2} + h$ **5C.** $y' = \frac{r^2 (y - k)}{(x - h)^2 + (y - k)^2} + k$ **6A.** 5 cm

6B. 7 cm **7A.** 64 cm^2 **7B.** 81 cm^2 **8A.** 10 cm **8B.** 4 cm **9A.** 8 cm **9B.** 9 cm

B. LONG QUESTIONS

1A. 18 units, 12 units, 9 units **1B.** 25 units, 10 units, 5 units **1C.** 32 units, 16 units, 4 units

1D. 96 units, 48 units, 32 units **2A.** P'(8, 4), Q'(6, 2) **2B.** A'(3, 4), B'(6, 8) **2C.** A'(20, 40), B'(14, 2)

2D. P'(6, 12), Q'(6, 18) **3A.** A'(3, 0) **3B.** P'(8, 0) **3C.** A'(16, 0) **3D.** A'(4, 0) **3E.** D'(0, 16)

4A. A'(14, 15) **4B.** B'(8, 18) **4C.** P'(2, 10) **4D.** B'(4, 1) **5A.** C'(– 8, 9) **5B.** P'(– 3, – 1)

6A. D' (– 5, 3) **6B.** A' (20, 21) **6C.** B' (1, 3) **6D.** C' (7, 8) **7A.** A'(6, 7) **7B.** A'(– 7, 11)

8A. P'(18, 21), Q'(30, 9) **8B.** A'(12, 14), B'(20, 6) **8C.** P'(22, 12), Q'(22, 2)

www.drmathnp.com

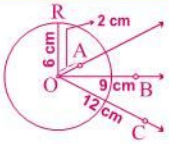
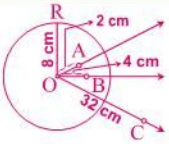
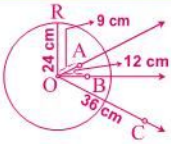
puspas.com.np

GENERAL EXERCISE 2

LONG QUESTIONS

Model 1 *Problems based on the formula $OP \times OP' = r^2$.*

1. तल दिइएका चित्रहरूबाट A, B र C का उत्क्रम बिन्दुहरूको उद्गमबिन्दुबाट दूरी पत्ता लगाउनुहोस् ।
From the figures given below, find the distance of inversion points of A, B and C from the origin.

A.	B.	C.
		

Model 2 *Problems based on finding radius/diameter.*

2A. यदि बिन्दु A(2, 0) को उत्क्रम वृत्त $x^2 + y^2 = r^2$ अनुसार उत्क्रम बिन्दु A'(8, 0) भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the inversion point of A(2, 0) with respect to the inversion circle $x^2 + y^2 = r^2$ is A'(8, 0), find the radius of the inversion circle.

2B. यदि बिन्दु P(1, 0) को उत्क्रम वृत्त $x^2 + y^2 = r^2$ अनुसार उत्क्रम बिन्दु P'(16, 0) भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the inversion point of P(1, 0) with respect to the inversion circle $x^2 + y^2 = r^2$ is P'(16, 0), find the radius of the inversion circle.

2C. केन्द्र उद्गम बिन्दु भएको एउटा वृत्तमा बिन्दु A(2, 0) को उत्क्रम बिन्दु A'(18, 0) भए सो वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the inverse of a point A(2, 0) with respect to a circle having centre at origin is A'(18, 0) then find the radius of the circle.

2D. केन्द्र उद्गम बिन्दु भएको एउटा वृत्तमा बिन्दु P(3, 0) को उत्क्रम बिन्दु P'(12, 0) भए सो वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the inverse of a point P(3, 0) with respect to a circle having centre at origin is P'(12, 0) then find the radius of the circle.

2E. केन्द्र उद्गम बिन्दु भएको एउटा वृत्तमा बिन्दु Q(4, 8) को उत्क्रम बिन्दु Q'(5, 10) भए सो वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
If the inverse of a point Q(4, 8) with respect to a circle having centre at origin is Q'(5, 10) then find the radius of the circle.

3A. यदि एउटा बिन्दु P(2, 2) को केन्द्र C(1, 2) भएको वृत्तको आधारमा उत्क्रम बिन्दु P'(17, 2) भए सो वृत्तको व्यास पत्ता लगाउनुहोस् ।
If the inverse of a point P(2, 2) with respect to a circle having centre at C(1, 2) is P'(17, 2) then find the diameter of the circle.

3B. यदि एउटा बिन्दु A(4, 4) को केन्द्र C(2, 4) भएको वृत्तको आधारमा उत्क्रम बिन्दु A'(34, 4) भए सो वृत्तको व्यास पत्ता लगाउनुहोस् ।
If the inverse of a point A(4, 4) with respect to a circle having centre at C(2, 4) is A'(34, 4) then find the diameter of the circle.

3C. यदि एउटा बिन्दु Q(8, 12) को केन्द्र C(4, 6) भएको वृत्तको आधारमा उत्क्रम बिन्दु $Q'\left(\frac{152}{13}, \frac{228}{13}\right)$ भए सो वृत्तको व्यास पत्ता लगाउनुहोस् ।

If the inverse of a point Q(8, 12) with respect to a circle having centre at C(4, 6) is $Q'\left(\frac{152}{13}, \frac{228}{13}\right)$ then find the diameter of the circle.

3D. यदि उद्गम बिन्दु केन्द्र हुने वृत्तको आधारमा एउटा बिन्दु A(3, 4) को उत्क्रम बिन्दु A'(12, 16) भए सो वृत्तको व्यास पत्ता लगाउनुहोस् । साथै बिन्दु B(6, 8) को उत्क्रम बिन्दु पनि सोही वृत्त अनुसार पत्ता लगाउनुहोस् ।
If the inversion of a point A(3, 4) with respect to a circle having centre as origin is A'(12, 16), find the diameter of the circle. Also find the inversion point of B(6, 8) with respect to the circle.

www.drmathnp.com

puspas.com.np

Model 3 *Finding object in inversion transformation.*

4A. वृत्त $x^2 + y^2 = 20$ को आधारमा बिन्दुहरू P र Q का उत्क्रम बिन्दुहरू क्रमश : P'(2, 1) र Q'(3, 1) छन् भने P र Q का निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The inversion points of P and Q with respect to the circle $x^2 + y^2 = 20$ are P'(2, 1) and Q'(3, 1) respectively. Find the coordinates of P and Q.

4B. वृत्त $x^2 + y^2 = 50$ को आधारमा बिन्दुहरू A र B का उत्क्रम बिन्दुहरू क्रमश : A'(6, 8) र B'(3, 4) छन् भने A र B का निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The inversion points of A and B with respect to the circle $x^2 + y^2 = 50$ are A'(6, 8) and B'(3, 4) respectively. Find the coordinates of A and B.

5A. वृत्त $x^2 + y^2 = 100$ को आधारमा बिन्दुहरू A(m, n) र B(p, q) का उत्क्रम बिन्दुहरू क्रमश : A'(1, 2) र B'(7, 1) भए m, n, p र q का मानहरू पत्ता लगाउनुहोस् ।
The inversion points of A(m, n) and B(p, q) with respect to the circle $x^2 + y^2 = 100$ are A'(1, 2) and B'(7, 1) respectively. Find the values of m, n, p and q.

5B. वृत्त $x^2 + y^2 = 60$ को आधारमा बिन्दुहरू P(a, b) र Q(c, d) का उत्क्रम बिन्दुहरू क्रमश : P'(2, 4) र Q'(1, 3) भए a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।
The inversion points of P(a, b) and Q(c, d) with respect to the circle $x^2 + y^2 = 60$ are P'(2, 4) and Q'(1, 3) respectively. Find the values of a, b, c and d.

6A. बिन्दु A(m, n) को वृत्त $(x - 2)^2 + (y - 3)^2 = 24$ को आधारमा उत्क्रम बिन्दु A'(3, 4) भए m र n का मानहरू पत्ता लगाउनुहोस् ।
The inversion point of A(m, n) with respect to the circle $(x - 2)^2 + (y - 3)^2 = 24$ is A'(3, 4). Find the values of m and n.

6B. बिन्दु P(a, b) को वृत्त $(x - 4)^2 + (y - 2)^2 = 68$ को आधारमा उत्क्रम बिन्दु P'(5, 6) भए a र b का मानहरू पत्ता लगाउनुहोस् ।
The inversion point of P(a, b) with respect to the circle $(x - 4)^2 + (y - 2)^2 = 68$ is P' (5, 6). Find the values of a and b.

7A. बिन्दुहरू A र B जोड्ने रेखाखण्डको केन्द्र C(6, 9) र अर्धव्यास 12 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू क्रमश : A'(12, 15) र B'(12, 9) भए A र B का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखाचित्रमा देखाउनुहोस् ।
The inverse points of line segment joining the points A and B with respect to a circle with centre C(6, 9) and radius 12 units are A'(12, 15) and B'(12, 9) respectively. Find the coordinates of A and B. Show the inversion transformation in the graph.

7B. बिन्दुहरू A र B जोड्ने रेखाखण्डको केन्द्र C(4, 6) र अर्धव्यास 8 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू क्रमश : A'(8, 10) र B'(8, 6) भए A र B का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखाचित्रमा देखाउनुहोस् ।
The inverse points of line segment joining the points A and B with respect to a circle with centre C(4, 6) and radius 8 units are A'(8, 10) and B'(8, 6) respectively. Find the coordinates of A and B. Show the inversion transformation in the graph.

7C. बिन्दुहरू P र Q जोड्ने रेखाखण्डको केन्द्र R(2, 2) र अर्धव्यास 10 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू क्रमश : P'(6, 4) र Q'(7, 2) भए P र Q का निर्देशाङ्कहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
The inverse points of line segment joining the points P and Q with respect to a circle with centre R(2, 2) and radius 10 units are P'(6, 4) and Q'(7, 2) respectively. Find the coordinates of P and Q. Show the inversion transformation in the graph.

ANSWERS

1A. OA' = 18 cm, OB' = 4 cm, OC' = 3 cm
1B. OA' = 32 cm, OB' = 16 cm, OC' = 2 cm
1C. OA' = 64 cm, OB' = 48 cm, OC' = 16 cm
2A. 4 units **2B.** 4 units **2C.** 6 units **2D.** 6 units
2E. 10 units **3A.** 8 units **3B.** 16 units **3C.** 20 units **3D.** 20 cm, B'(6, 8)
4A. P(8, 4), Q(6, 2) **4B.** A(3, 4), B(6, 8) **5A.** $m = 20, n = 40, p = 14, q = 2$ **5B.** $a = 6, b = 12, c = 6, d = 18$
6A. $m = 14, n = 15$ **6B.** $a = 8, b = 18$ **7A.** A(18, 21), B(30, 9) **7B.** A(12, 14), B(20, 6)
7C. P(22, 12), Q(22, 2)

www.drmathnp.com

3. मेट्रिक्स स्थानान्तरण MATRIX TRANSFORMATION

DEFINITIONS

➢ **मेट्रिक्स स्थानान्तरण (Matrix transformation)**

यदि स्थानान्तरण मेट्रिक्स T र P मेट्रिक्सको रूपमा रहेको वस्तु भए आकृति P' = TP हुन्छ ।

यदि T_1 र T_2 हरू दुई स्थानान्तरण मेट्रिक्सहरू भए संयुक्त स्थानान्तरण मेट्रिक्स $T_2 \times T_1$ हुन्छ ।

If T be the transformation matrix and P be the object in matrix form then image P' = TP.

If T_1 & T_2 are two transformation matrices, the combined transformation matrix = $T_2 \times T_1$.

FORMULAE AND KEY POINTS

(i)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	X-अक्षमा परावर्तन Reflection in x-axis	$P(x, y) \to P'(x, -y)$
(ii)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Y-अक्षमा परावर्तन Reflection in y-axis	$P(x, y) \to P'(-x, y)$
(iii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$y = x$ मा परावर्तन Reflection in $y = x$	$P(x, y) \to P'(y, x)$
(iv)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$y = -x$ मा परावर्तन Reflection in $y = -x$	$P(x, y) \to P'(-y, -x)$
(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	R_o [O, + 90°]	$P(x, y) \to P'(-y, x)$
(vi)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	R_o [O, − 90°]	$P(x, y) \to P'(y, -x)$
(vii)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	R_o [O, ± 180°]	$P(x, y) \to P'(-x, -y)$
(viii)	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	E[O, k]	$P(x, y) \to P'(kx, ky)$
(ix)	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x-a \\ y-b \end{pmatrix}+\begin{pmatrix} a \\ b \end{pmatrix}$	E[(a, b), k]	$P(x, y) \to P'[k(x-a)+a, k(y-b)+b]$

स्थानान्तरण मेट्रिक्स पत्ता लगाउने तरिका:

जस्तै: X-अक्षमा परावर्तन हुँदा वस्तु = (x, y) भए प्रतिबिम्ब = $(x, -y)$ हुन्छ ।

अब, प्रतिबिम्बका x-सदस्य र y-सदस्यलाई समीकरणको रूपमा व्यक्त गर्दा,

$x = 1 \times x + 0 \times y \to 1 \times x + 0 \times y = x$

$-y = 0 \times x + (-1) \times y \to 0 \times x + (-1) \times y = -y$

यसलाई मेट्रिक्सको रूपमा लेख्दा,

$$\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

प्रतिबिम्ब = स्थानान्तरण मेट्रिक्स वस्तु

Method of finding transformation matrix:

If an object (x, y) is reflected on X -axis then the image will be $(x, -y)$.

Now, expressing the x-component and y-component of this image in matrix form then.

$x = 1 \times x + 0 \times y \to 1 \times x + 0 \times y = x$

$-y = 0 \times x + (-1) \times y \to 0 \times x + (-1) \times y = -y$

Writing this equations in matrix form then,

$$\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Image = Transformation Matrix Object

TRANSFORMATION

puspas.com.np

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

B. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 R, 2066 R', 2067 S, 2068 S, 2068 S', 2069 R', 2071 R, 2071 S	8
Model 2	2069 R	1
Model 3	SEE 2075 R, SEE 2075 R_2	1
Model 4	2070 R, 2072 R', SEE 2075 R'_2	3
Model 5	2065 E, R', S, M, 2066 R, S, 2067 R, R', 2068 R, R', 2069 S, 2070 R', S, S', 2071 R', 2072 R, S, 2073 R, R', SEE 2073 S, SEE 2073 S', SEE 2074 R', SEE 2074 S, S', SEE 2075 R', SEE MODEL 2076	15
Model 6	SEE 2074 R	1

SEE EXERCISE 3

A. VERY SHORT QUESTIONS

Model 1 *Definitions and formulae Recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
A. एकाइ वर्ग (Unit square) **B.** एकात्मक स्थानान्तरण (Identity transformation)
C. मेट्रिक्स स्थानान्तरण (Matrix transformation)

2A. X-अक्षको परावर्तनसँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write down the matrix associated with reflection in X-axis.

2B. उद्गम बिन्दुको वरिपरी हुने $+90°$ सँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write down the matrix associated with the rotation through $+90°$ about the origin.

2C. एकात्मक स्थानान्तरणसँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write down the matrix associated with identity transformation.

2D. नापो k र केन्द्र $(0, 0)$ भएको विस्तारीकरणसँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write the matrix to represent an enlargement of scale factor k with centre $(0, 0)$.

Model 2 *Identifying the transformation represented by given matrix (reflection) .*

3. दिइएको मेट्रिक्सले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् ।
Find the transformation represented by the given matrix.

A. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **B.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **D.** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Model 3 *Identifying the transformation represented by given matrix (rotation) .*

4. दिइएको मेट्रिक्सले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् ।
Find the transformation represented by the given matrix.

A. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **B.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **C.** $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Model 4 *Problems based on matrix transformation on enlargement.*

5. दिइएको मेट्रिक्सले प्रतिनिधित्व गर्ने स्थानान्तरण पत्ता लगाउनुहोस् ।
Find the transformation represented by the given matrix.

A. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **B.** $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ **C.** $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ **D.** $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

Model 5 *Problems based on identifying the transformation matrix.*

6A. बिन्दु $A(x, y)$ लाई बिन्दु $A(-x, y)$ मा स्थानान्तरण गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2×2 matrix which transforms $A(x, y)$ to $A(-x, y)$.

6B. कुनै एउटा 2×2 स्थानान्तरण मेट्रिक्सद्वारा (x, y) यदि (y, x) मा स्थानान्तरण हुन्छ भने उक्त स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
If a point (x, y) is transformed into (y, x) by a 2×2 transformation matrix, then find the matrix. [2067R']

www.drmathnp.com

6C. कुनै एउटा 2×2 स्थानान्तरण मेट्रिक्सद्वारा (a, b) यदि $(b, -a)$ मा स्थानान्तरण हुन्छ भने उक्त स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
If a point (a, b) is transformed into $(b, -a)$ by a 2×2 transformation matrix, find the matrix. **[2068 R']**

6D. रेखा $y = x$ मा हुने परावर्तनलाई जनाउने स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the transformation matrix which represents the reflection on the line $y = x$. **[2070 R']**

6E. Y-अक्षमा हुने परावर्तनलाई जनाउने स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the transformation matrix which represents the reflection on Y-axis. **[2072 R]**

6F. रेखा $y = -x$ मा हुने परावर्तनलाई जनाउने स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the transformation matrix which represents the reflection on the line $y = -x$. **[SEE 2074 S, 2072 S]**

6G. उद्गम बिन्दु वरिपरि $+270°$ मा परिक्रमण जनाउने 2×2 स्थानान्तरण मेट्रिक्स लेख्नुहोस् ।
Write down the 2×2 transformation matrix which represents the rotation about the origin through $+270°$.

Model 6 ***Problems based on 2×1 transformation matrix.***

7A. एउटा बिन्दु (a, b) लाई बिन्दु $(a + 2, b - 3)$ मा स्थानान्तरण गर्ने एउटा 2×1 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×1 matrix which transforms a point (a, b) into the point $(a + 2, b - 3)$.

7B. एउटा बिन्दु (a, b) लाई बिन्दु $(a + 4, b - 5)$ मा स्थानान्तरण गर्ने एउटा 2×1 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×1 matrix which transforms a point (a, b) into the point $(a + 4, b - 5)$.

7C. एउटा बिन्दु (p, q) लाई बिन्दु $(p + 3, q - 2)$ मा स्थानान्तरण गर्ने एउटा 2×1 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×1 matrix which transforms a point (p, q) into the point $(p + 3, q - 2)$.

7D. बिन्दु $(-3, 2)$ लाई 2×1 स्थानान्तरण मेट्रिक्स $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ ले स्थानान्तरण गर्नुहोस् ।
Transform a point $(-3, 2)$ by a 2×1 transformation matrix $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

7E. बिन्दु $(4, 5)$ लाई 2×1 स्थानान्तरण मेट्रिक्स $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ले स्थानान्तरण गर्नुहोस् ।
Transform a point $(4, 5)$ by a 2×1 transformation matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

B. LONG QUESTIONS

Model 1 ***Problems based on finding image using transformation matrix and object.***

TRANSFORMATION

1A. त्रिभुज ΔPQR का शीर्षबिन्दुहरू $P(2, 3)$, $Q(-5, 4)$ र $R(3, 2)$ छन् । त्यस त्रिभुजलाई मेट्रिक्स $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ ले स्थानान्तरण गराई आउने प्रतिविम्बको शीर्ष बिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।
ΔPQR is a triangle with vertices $P(2, 3)$, $Q(-5, 4)$ and $R(3, 2)$. Transform the given triangle by a matrix $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ and find the coordinates of the vertices of its image. **[2068 S]**

1B. $A(1, -1)$, $B(3, 2)$ र $C(0, 2)$ भएको ΔABC लेखाचित्रमा खिच्नुहोस् । सो ΔABC लाई मेट्रिक्स $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ ले स्थानान्तरण गर्दा हुन आउने प्रतिविम्ब सोही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
Draw a ΔABC where vertices are $A(1, -1)$, $B(3, 2)$ and $C(0, 2)$ on a graph paper. Draw on the same graph, the image of the ΔABC under the transformation given by the matrix $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$. **[2057 S]**

1C. शीर्षबिन्दुहरू $A(3, 1)$, $B(-2, -1)$ र $C(4, 2)$ भएको त्रिभुजलाई स्थानान्तरण मेट्रिक्स $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ द्वारा स्थानान्तरण गराउँदा बन्ने प्रतिविम्ब $\Delta A'B'C'$ का शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । **[2066 R']**
Find the co-ordinates of the vertices of image $\Delta A'B'C'$ which is transformed by the transformation of ΔABC having vertices $A(3, 1)$, $B(-2, -1)$ and $C(4, 2)$ under the transformation by a matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

puspas.com.np

www.drmathnp.com

puspas.com.np

2A. समानान्तर चतुर्भुज QRST का शीर्षबिन्दुहरू Q(– 1, 1), R(– 2, – 1), S(2, – 1) र T(3, 1) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरी हुने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

The parallelogram QRST has the vertices Q(– 1, 1), R(– 2, – 1), S(2, – 1) and T(3, 1). Transform the given parallelogram under the matrix $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ and find the coordinates of vertices of its image. **[2071 S]**

2B. वर्ग WXYZ का शीर्षबिन्दुहरू W(0, 3), X(1, 1), Y(3, 2) र Z(2, 4) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरी हुने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

The square WXYZ has the vertices W(0, 3), X(1, 1), Y(3, 2) and Z(2, 4). Transform the given square WXYZ under the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ and find the coordinates of the vertices of its image. **[2071 R]**

2C. चतुर्भुज WXYZ का शीर्षबिन्दुहरू W(3, 2), X(3, – 1), Y(2, – 1) र Z(4, 3) छन् । यसलाई मेट्रिक्स $\begin{pmatrix} 4 & 3 \\ 2 & 0 \end{pmatrix}$ द्वारा स्थानान्तरण गरी हुने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

The quadrilateral WXYZ has the vertices W(3, 2), X(3, – 1), Y(2, – 1) and Z(4, 3). Transform the given quadrilateral under the matrix $\begin{pmatrix} 4 & 3 \\ 2 & 0 \end{pmatrix}$ and find the vertices of its image. **[2069 R']**

2D. शीर्षबिन्दुहरू A(2, 1), B(4, 1), C(4, 3) र D(2, 3) भएको वर्ग ABCD लाई मेट्रिक्स $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ ले स्थानान्तरण गर्दा आउने प्रतिविम्बहरू पत्ता लगाउनुहोस् ।

Find the co-ordinates of the vertices of image which is transformed by the transformation of square ABCD having vertices A(2, 1), B(4, 1), C(4, 3) and D(2, 3) under the transformation by a matrix $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. **[2068 S']**

3A. एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ मेट्रिक्सले स्थानान्तरण गराई आउने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्क लेख्नुहोस् ।

Transform a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ under the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and write the co-ordinates of the vertices of the images. **[2065 R]**

3B. एकाइ वर्गलाई मेट्रिक्स $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ ले स्थानान्तरण गर्दा प्राप्त हुने प्रतिविम्बको निर्देशाङ्क पत्ता लगाउनुहोस् । **[2067 S]**

Find the co-ordinates of the vertices of the image of the unit square transformed by the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$.

3C. एकाइ वर्गलाई $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ मेट्रिक्सले स्थानान्तरण गराई आउने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्कहरू लेख्नुहोस् ।

Transform a unit square by the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and write the coordinates of the image.

TRANSFORMATION

Model 2 ***Problems based on finding image by calculating and using the transformation matrix.***

4A. शीर्षबिन्दुहरू M(0, 0), N(1, 0), O(1, 1) र P(0, 1) भएको एकाइ वर्ग MNOP लाई $y = -x$ अक्षमा स्थानान्तरण गर्ने मेट्रिक्सद्वारा स्थानान्तरण गर्दा हुने प्रतिविम्ब चतुर्भुज M'N'O'P' का शीर्षबिन्दुहरू लेख्नुहोस् ।

A unit square MNOP having vertices M(0, 0), N(1, 0), O(1, 1) and P(0, 1) is transformed under the matrix transformation through $y = -x$ and write the vertices of the images quadrilateral M'N'O'P' so formed. **[2069 R]**

4B. शीर्षबिन्दुहरू P(– 3, 2), Q(– 1, 4) र R(2, 0) भएको ΔPQR लाई X-अक्षमा परावर्तन हुने प्रतिविम्ब मेट्रिक्स विधि प्रयोग गरी यसका आकृतिको प्रतिविम्बहरू पत्ता लगाउनुहोस् ।

P(– 3, 2), Q(– 1, 4) and R(2, 0) are the vertices of ΔPQR. Find the vertices of the image triangle under the reflection on X-axis by using matrix method. **[2064 R']**

4C. यदि मेट्रिक्स M_1 ले X- अक्षमा हुने परावर्तन र मेट्रिक्स M_2 ले उद्गम बिन्दुको वरिपरि 180° मा हुने परिक्रमणलाई जनाउँछन् भने M_1M_2 ले एकाइ वर्गलाई स्थानान्तरण गर्दा बन्ने चतुर्भुजको शीर्ष बिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

If M_1 is the matrix represented by reflection on x-axis and M_2 is the matrix represented by rotation through 180° about origin then transform unit square by M_1M_2 and write the co-ordinates of vertices of image quadrilateral.

www.drmathnp.com

4D. यदि मेट्रिक्स M_1 ले Y- अक्षमा हुने परावर्तन र मेट्रिक्स M_2 ले उद्गम बिन्दुको वरिपरि $+90°$ मा हुने परिक्रमणलाई जनाउँछन् भने M_1M_2 ले एकाइ वर्गलाई स्थानान्तरण गर्दा बन्ने चतुर्भुजको शीर्ष बिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।
If M_1 is the matrix represented by reflection on y-axis and M_2 is the matrix represented by rotation through $+90°$ about origin then transform unit square by M_1M_2 and write the co-ordinates of vertices of image quadrilateral.

Model 3 *On finding transformation matrix when object & image are given (line segment).*

5A. बिन्दुहरू A(4, 1) र B(7, 5) जोड्ने रेखाखण्ड AB लाई स्थानान्तरण गर्दा A'(– 4, 1) र B'(– 7, 5) बिन्दुहरू जोड्ने रेखाखण्ड A'B' बन्दछ । यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
A line segment AB joining the points A(4, 1) and B(7, 5) is transformed to the line segment A'B' joining the points A'(– 4, 1) and B'(– 7, 5). Find the 2×2 matrix that represents this transformation. **[SEE 2075 R, 2075 R_2]**

5B. बिन्दु P(5, 1) र Q(8, 6) भएको रेखा PQ लाई स्थानान्तरण गर्दा PQ को प्रतिबिम्ब P_1(– 5, 1) र Q_1(– 8, 6) भएको रेखा P_1Q_1 बन्दछ भने कुन एउटै स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2×2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।
Line PQ having P(5, 1) and Q(8, 6) maps onto the line P_1Q_1 having P_1(– 5, 1) and Q_1(– 8, 6) so that the image P_1Q_1 of PQ is formed. Which is the single transformation for this mapping ? Also find 2×2 matrix that represents the transformation. **[2059 R]**

5C. बिन्दु P(2, 8) र Q(7, 1) भएको रेखा PQ लाई स्थानान्तरण गर्दा PQ को प्रतिबिम्ब P_1(– 8, – 2) र Q_1(– 1, – 7) भएको रेखा P_1Q_1 बन्दछ भने कुन एउटै स्थानान्तरणले गर्दा सो प्रतिबिम्ब बन्दछ ? यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2×2 मेट्रिक्स पनि पत्ता लगाउनुहोस् ।
Line PQ with P(2, 8) and Q(7, 1) maps onto the line P_1Q_1 having P_1(– 8, – 2) and Q_1(– 1, – 7) so that the image P_1Q_1 of PQ is formed. Which is the single transformation for this mapping? Also find 2×2 matrix which represents this transformation. **[2061 R]**

Model 4 *On finding transformation matrix when object & image are given (triangle).*

6. दिइएका शीर्षबिन्दुहरू भएको Δ लाई सँगैको प्रतिबिम्ब Δ मा स्थानान्तरण गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×2 matrix which transforms a Δ with the image Δ having given vertices.

	वस्तुका शीर्षबिन्दुहरू (Vertices of object)	प्रतिविम्बका शीर्षबिन्दुहरू (Vertices of image)	
A.	ΔPQR ; P(1, 2), Q(4, 1) & R(2, 5)	ΔP'Q'R'; P'(5, 2), Q'(6, 1) & R'(12, 5)	**[2075 R'$_2$]**
B.	ΔPQR ; P(4, 3), Q(6, 4) & R(8, 1)	ΔP'Q'R'; P'(– 3, – 4), Q'(– 4, – 6) & R'(– 1 , – 8)	**[2072 R', 2060 CP]**
C.	ΔABC; A(3, 6), B(5, – 3) & C(– 4, 2)	ΔA'B'C'; A'(– 3, – 6), B'(– 5, 3) & C'(4, – 2)	**[2070 R]**
D.	ΔPQR; P(3, 6), Q(4, 2) & R(2, 2)	$\Delta P_1Q_1R_1$; P_1(– 6, 3), Q_1(– 2, 4), & R_1(– 2, 2)	**[2058 R]**
E.	ΔABC; A(2, 7), B(2, 9) & C(6, 7)	ΔA'B'C'; A'(7, 2), B'(9, 2) & C'(7, 6)	**[2058 S]**
F.	ΔPQR; P(1, 2), Q(5, 2) & R(1, 5)	ΔP'Q'R'; P'(1,– 2), Q'(5,– 2), & R'(1,– 5)	**[2059 S]**
G.	ΔPQR; P(– 2, 4), Q(6, 2) & R(1, – 1)	$\Delta P_1 Q_1 R_1$; P_1(4, 2), Q_1 (2, – 6) & R_1 (– 1,– 1)	**[2062 R]**
H.	ΔPQR; P(– 3, – 4), Q(– 4, – 6) & R(–1, – 8)	$\Delta P_1Q_1R_1$; P_1(4, 3), Q_1 (6, 4) & R_1(8, 1)	**[2062 S]**
I.	ΔPQR; P(– 3, 5), Q(7, 4) & R(6, – 2)	$\Delta P_1Q_1R_1$; P_1(5, 3), Q_1(4, – 7) & R_1(– 2, – 6)	**[2063 R']**

Model 5 *On finding transformation matrix when object & image are given (quadrilateral).*

7A. एउटा 2×2 मेट्रिक्स पत्ता लगाउनुहोस् जसले शीर्षबिन्दुहरू A(2, 3), B(4, 3), C(4, 5) र D(2, 5) भएको एउटा वर्ग ABCD लाई शीर्षबिन्दुहरू A'(3, 2), B'(3, 4), C'(5, 4) र D'(5, 2) भएको वर्ग A'B'C'D' मा स्थानान्तरण गर्दछ ।
Find the 2×2 transformation matrix which transforms a square ABCD with vertices A(2, 3), B(4, 3), C(4, 5) and D(2, 5) into a square A'B'C'D' with vertices A'(3, 2), B'(3, 4), C'(5, 4) and D'(5, 2). **[2070 S]**

7B. A(0, 0), B(0, 1), C(1, 1), D(1, 0) भएको वर्ग ABCD लाई 2×2 मेट्रिक्सद्वारा स्थानान्तरण गर्दा A'(0, 0), B'(0, – 2), C'(2, – 2) र D'(2, 0) भएको वर्ग A' B' C' D' हुन्छ भने सो 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
A square ABCD whose vertices are A(0, 0), B(0, 1), C(1, 1), D(1, 0) is mapped onto the square A' B' C' D' by a 2×2 matrix so that the vertices of the square A' B' C' D' are A'(0, 0), B'(0, –2), C'(2, – 2) and D'(2, 0). Find the 2×2 matrix. **[2060 R]**

puspas.com.np

www.drmathnp.com

8. दिइएका शीर्षबिन्दुहरू भएको वर्गलाई सँगैको प्रतिबिम्ब समानान्तर चतुर्भुजमा स्थानान्तरण गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×2 matrix which transforms the square with the image parallelogram having given vertices.

	वर्गका शीर्षबिन्दुहरू (Vertices of square)	समानान्तर चतुर्भुजका शीर्षबिन्दुहरू (Vertices of a parallelogram)	
A.	A(2, 0), B(5, 1), C(4, 4) & D(1, 3)	A'(2, 2), B'(7, 3), C'(12, – 4) & D'(7, – 5)	[2072 R, 2066 R]
B.	A(0, 3), B(1, 1), C(3, 2) & D(2, 4)	A'(6, – 6), B(3, – 1), C'(7, – 1) & D'(10, – 6)	[2057 R, 60 S, 2064 S, 2065 R']
C.	A (0, 0), B (1, 0), C (1, 1) & D (0,1)	A'(0, 0), B'(3, 0), C'(4, 1) & D'(1, 1)	[2057S, 2060S']
D.	P(0, 3), Q(1, 1), R(3, 2) & S(2, 4)	P'(3, 0), Q'(1, – 1), R'(2, – 3) & S'(4, – 2)	[2061 S, 2069 S]
E.	A(0, 0), B(1, 0), C(1, 1) & D(0, 1)	A'(0, 0), B'(2, 0), C'(2, 1) & D'(0, 1)	[2063 M]
F.	O(0, 0), A(2, 0), B(2, 2) & C(0, 2)	A'(4, 0), B'(10, 4) & C'(6, 4).	[2063 S]
G.	P(0, 0), Q(2, 0), R(2, 1) & S(0, 1)	P'(0, 0), Q'(2, 4), R'(2, 5) & S'(0, 1)	[2066 S]
H.	P(– 2, 0), Q(– 5, 1), R(– 4, 4) & S(– 1, 3)	P'(– 2, – 2), Q'(– 3, – 7), R'(4, – 12) & S'(5, – 7)	[2070 R']

9. एकाइ वर्ग वा $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई तलको समानान्तर चतुर्भुजमा स्थानान्तरण गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2×2 transformation matrix which transforms a unit square or $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ to the following parallelogram.

A. $\begin{bmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ [SEE 2075 R', 2068 R', 2065 M, 2073 S] **B.** $\begin{bmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{bmatrix}$ [2068 R]

C. $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ [2063 R, 2070 S', 2072 S, 2073 S', 2074 R']

D. $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ [SEE MODEL 2076, 2074 S', 2065 E] **E.** $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ [2067 R']

F. $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & -1 & -2 & -1 \end{pmatrix}$ [2071 R']

G. $\begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ [SEE 2074 S, 2067 R, SEE 2073 R]

10A. मेट्रिक्स $\begin{pmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ लाई समानान्तर चतुर्भुज $\begin{pmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ मा स्थानान्तरण गर्ने एउटा 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find a 2×2 transformation matrix in which a matrix $\begin{pmatrix} 2 & 4 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ is transformed into a parallelogram $\begin{pmatrix} -2 & -4 & -4 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$. [2065 S]

10B. मेट्रिक्स $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ले आयत $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई आयत $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ मा स्थानान्तरण गर्दछ भने सो मेट्रिक्स $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ को मान निकाल्नुहोस् । कुन स्थानान्तरले गर्दा सो प्रतिबिम्ब बन्दछ ?

The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ maps the rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ onto the rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$, find the value of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Which is the transformation for this mapping? [2062 K]

10C. $\begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$ लाई $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।

Find 2×2 matrix, which transforms $\begin{bmatrix} 0 & 2 & 3 & 2 \\ 0 & 3 & 5 & 1 \end{bmatrix}$ into $\begin{bmatrix} 0 & 3 & 5 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$. [SEE 2073 R']

www.drmathnp.com

puspas.com.np

Model 6 *Problems based on finding values of a, b, c and d when object, TM and image are given.*

11A. यदि मेट्रिक्स $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ ले एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$ मा स्थानान्तरण गर्दछ भने a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।

If the matrix $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ transforms a unit square to the parallelogram $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$, find the values of a, b, c and d. **[SEE 2074 R]**

11B. एकाइ वर्ग $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ लाई $\begin{bmatrix} 3 & b \\ c & 1 \end{bmatrix}$ मेट्रिक्सले स्थानान्तरण गर्दा आउने प्रतिबिम्ब $\begin{bmatrix} 0 & a & 5 & 2 \\ 0 & 1 & 2 & d \end{bmatrix}$ भए a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।

A unit square $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ is transformed by the matrix $\begin{bmatrix} 3 & b \\ c & 1 \end{bmatrix}$ to get $\begin{bmatrix} 0 & a & 5 & 2 \\ 0 & 1 & 2 & d \end{bmatrix}$. Find the values of a, b, c and d.

11C. एकाइ वर्ग $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ लाई $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ मेट्रिक्सले स्थानान्तरण गर्दा आउने प्रतिबिम्ब $\begin{bmatrix} 0 & 3 & d & 1 \\ 0 & c & 3 & 2 \end{bmatrix}$ भए a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।

A unit square $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ is transformed by the matrix $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ to get $\begin{bmatrix} 0 & 3 & d & 1 \\ 0 & c & 3 & 2 \end{bmatrix}$. Find the values of a, b, c and d.

11D. वर्ग $\begin{bmatrix} 0 & b & 3 & 2 \\ a & 1 & c & d \end{bmatrix}$ लाई $\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$ मेट्रिक्सले स्थानान्तरण गर्दा आउने प्रतिबिम्ब समानान्तर चतुर्भुज $\begin{bmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{bmatrix}$ भए a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।

A square $\begin{bmatrix} 0 & b & 3 & 2 \\ a & 1 & c & d \end{bmatrix}$ is transformed by the matrix $\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$ to get a parallelogram $\begin{bmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{bmatrix}$. Find the values of a, b, c and d.

11E. यदि मेट्रिक्स $\begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}$ ले एकाइ वर्ग लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$ मा स्थानान्तरण गर्दछ भने a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।

If a matrix $\begin{pmatrix} a+1 & 2 \\ b-1 & 2 \end{pmatrix}$ transform a unit square to the parallelogram $\begin{pmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{pmatrix}$, find the values of a, b, c and d.

Model 7 *Problems based on matrix transformation and other transformations.*

12A. त्रिभुज ΔPQR का शीर्षबिन्दुहरू P(2, 3), Q(– 5, 4) र R(3, 2) छन् । त्यस त्रिभुजलाई मेट्रिक्स $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ ले स्थानान्तरण गराई रेखा $x = 2$ मा परावर्तन गर्दा आउने प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

ΔPQR is a triangle with vertices P(2, 3), Q(– 5, 4) and R(3, 2). Transform the given triangle by a matrix $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ and reflect in the line $x = 2$ and find the coordinates of the vertices of its image.

12B. A (1, – 1), B (3, 2) र C (0, 2) भएको ΔABC लेखाचित्रमा खिच्नुहोस् । सो ΔABC लाई मेट्रिक्स $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ ले स्थानान्तरण गरेपछि रेखा $y = 2$ मा परावर्तन गर्दा हुन आउने प्रतिबिम्ब सोही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।

Draw a ΔABC where vertices are A (1, – 1), B (3, 2) and C (0, 2) on a graph paper. Draw on the same graph, the image of the ΔABC under the transformation given by the matrix $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ followed by reflection in $y = 2$.

www.drmathnp.com

12C. शीर्षबिन्दुहरू A(3, 1), B(– 2, – 1) र C(4, 2) भएको त्रिभुजलाई स्थानान्तरण मेट्रिक्स $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ द्वारा स्थानान्तरण गरेपछि उद्गम बिन्दुको वरिपरि 90° परिक्रमण गराउँदा बन्ने प्रतिबिम्ब ΔA'B'C' का शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

Find the co-ordinates of the vertices of image ΔA'B'C' which is transformed by the transformation of ΔABC having vertices A(3, 1), B(– 2, – 1) and C(4, 2) under the transformation by a matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ followed by the rotation of 90° about the origin.

13A. समानान्तर चतुर्भुज QRST का शीर्षबिन्दुहरू Q(– 1, 1), R(– 2, – 1), S(2, – 1) र T(3, 1) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरेपछि उद्गम बिन्दुको वरिपरि 180° परिक्रमण गर्दा हुने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

The parallelogram QRST has the vertices Q(– 1, 1), R(– 2, – 1), S(2, – 1) and T(3, 1). Transform the given parallelogram under the matrix $\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$ followed by rotation of 180° about the origin. Find the coordinates of vertices of its image.

13B. वर्ग WXYZ का शीर्षबिन्दुहरू W(0, 3), X(1, 1), Y(3, 2) र Z(2, 4) छन् । यसलाई मेट्रिक्स $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ द्वारा स्थानान्तरण गरेपछि रेखा $y = x$ मा परावर्तन गर्दा हुने प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

The square WXYZ has the vertices W(0, 3), X(1, 1), Y(3, 2) and Z(2, 4). Transform the given square WXYZ under the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ followed by the reflection in the line $x = y$. Find the coordinates of the vertices of its image.

13C. चतुर्भुज WXYZ का शीर्षबिन्दुहरू W(3, 2), X(3, – 1), Y(2, – 1) र Z(4, 3) छन् । यसलाई मेट्रिक्स $\begin{pmatrix} 4 & 3 \\ 2 & 0 \end{pmatrix}$ द्वारा स्थानान्तरण गरेपछि विस्थापन भेक्टर $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ द्वारा हुने विस्थापन अनुसार प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

The quadrilateral WXYZ has the vertices W(3, 2), X(3, – 1), Y(2, – 1) and Z(4, 3). Transform the given quadrilateral under the matrix $\begin{pmatrix} 4 & 3 \\ 2 & 0 \end{pmatrix}$ followed by the translation of translation vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the vertices of its image.

13D. शीर्षबिन्दुहरू A(2, 1), B(4, 1), C(4, 3) र D(2, 3) भएको वर्ग ABCD लाई मेट्रिक्स $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ ले स्थानान्तरण गरेपछि विस्थापन भेक्टर $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ द्वारा हुने विस्थापन अनुसार आउने प्रतिबिम्बहरू पत्ता लगाउनुहोस् ।

Find the co-ordinates of the vertices of image which is transformed by the transformation of square ABCD having vertices A(2, 1), B(4, 1), C(4, 3) and D(2, 3) under the transformation by a matrix $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ followed by the translation of translation vector $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

14A. एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ मेट्रिक्सले स्थानान्तरण गरेपछि x-अक्षमा परावर्तन गराउँदा आउने प्रतिबिम्बको शीर्षबिन्दुहरूको निर्देशाङ्क लेख्नुहोस् ।

Transform a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ under the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ followed by the reflection in x-axis. Write the co-ordinates of the vertices of the images.

14B. एकाइ वर्गलाई मेट्रिक्स $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ ले स्थानान्तरण गरेपछि विस्तारीकरण E[(0, 0) ; 2] द्वारा विस्तार गराउँदा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

Find the co-ordinates of the vertices of the image of the unit square transformed by the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ followed by the enlargement E[(0, 0) ; 2].

puspas.com.np

www.drmathnp.com

14C. एकाइ वर्गलाई $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ मेट्रिक्सले स्थानान्तरण गरेपछि विस्तारीकरण E[(0, 0) ; 3] द्वारा विस्तार गर्दा आउने प्रतिविम्बको शीर्षबिन्दुहरूको निर्देशाङ्कहरू लेख्नुहोस् ।

Transform a unit square by the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ followed by the enlargement E[(0, 0) ; 3].Write the coordinates of the image.

15A. एउटा बिन्दु A (3, 2) लाई $y = 3$ रेखामा परावर्तन गरी मेट्रिक्स $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ द्वारा स्थानान्तरण गर्नुहोस् । उक्त प्रतिबिम्बको वृत्त $x^2 + y^2 = 200$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।

A point A(3, 2) is reflected in the line $y = 3$ and then transformed by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Find the inversion point of the image with respect to the circle $x^2 + y^2 = 200$.

15B. एउटा बिन्दु A (2, 3) लाई $x = 3$ रेखामा परावर्तन गरी मेट्रिक्स $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ द्वारा स्थानान्तरण गर्नुहोस् । उक्त प्रतिबिम्बको वृत्त $x^2 + y^2 = 75$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् ।

A point A(2, 3) is reflected in the line $x = 3$ and then transformed by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. Find the inversion point of the image with respect to the circle $x^2 + y^2 = 75$.

ANSWERS

A. VERY SHORT QUESTIONS

2A. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **2B.** $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **2C.** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **2D.** $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ **3A.** Reflection in X-axis

3B. Reflection in y-axis **3C.** Reflection in $y = -x$ **3D.** Reflection in $y = x$ **4A.** Rotation by [O, + 90°] or, [– 270°, (0, 0)]

4B. – 90° Rotation about the origin **4C.** R_o[± 180°; (0, 0)] **4D.** R_o[360° ; (0, 0)] **5A.** Identity transformation **5B.** E [O, 3]

5C. E[O, 2] **5D.** $E\left[O, \frac{1}{2}\right]$; Reduction by s.f. $\frac{1}{2}$ with centre at origin **6A.** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **6B.** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

6C. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **6D.** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **6E.** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **6F.** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ **6G.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **7A.** $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

7B. $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ **7C.** $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **7D.** (1, 7) **7E.** (6, 8)

B. LONG QUESTIONS

1A. P'(11, 7), Q'(7, – 6) & R'(9, 8) **1B.** A' (3, 3), B' (– 1, 4), C' (– 4, – 2) **1C.** A'(6, 2), B'(– 4, – 2), C'(8, 4)

2A. Q'(– 1, – 2), R'(– 11, – 4), S'(5, 4) and T'(15, 6) **2B.** W'(– 3, 0), X'(– 1, – 1), Y'(– 2, – 3), Z'(– 4, – 2)

2C. W'(18, 6), X'(9, 6), Y'(5, 4), Z'(25, 8) **2D.** A'(4, 7), B'(6, 11), C'(10, 17), D'(8, 13)

3A. $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ **3B.** $\begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ **3C.** (0, 0), (3, 1), (5, 2), (2, 1) **4A.** M'(0, 0), N'(0, – 1), O'(– 1, – 1), P'(– 1, 0)

4B. P'(– 3, – 2), Q'(– 1, – 4),R'(2, 0) **4C.** (0, 0), (– 1, 0), (– 1, 1), (0, 1) **4D.** (0, 0), (0, – 1), (– 1, – 1), (– 1, 0)

5A. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **5B.** Reflection in Y-axis, $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **5C.** Refl^n in $x = -y$, $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ **6A.** $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

6B. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ **6C.** $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ **6D.** $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **6E.** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **6F.** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **6G.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

6H. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ **6I.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **7A.** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **7B.** $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ **8A.** $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ **8B.** $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$

8C. $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ **8D.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ **8E.** $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ **8F.** $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ **8G.** $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ **8H.** $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$

9A. $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ **9B.** $\begin{pmatrix} 6 & 2 \\ 2 & 4 \end{pmatrix}$ **9C.** $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ **9D.** $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ **9E.** $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ **9F.** $\begin{pmatrix} 3 & 2 \\ -1 & -1 \end{pmatrix}$

9G. $\begin{pmatrix} 4 & 2 \\ 1 & 2 \end{pmatrix}$ **10A.** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **10B.** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Reflection in X-axis **10C.** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

11A. $a = 4, b = 1, c = 6, d = 2$ **11B.** $a = 3, b = 2, c = 1, d = 1$ **11C.** $a = 3, b = 2, c = 1, d = 4$ **11D.** $a = 3, b = 1, c = 2, d = 4$

11E. 3, 2, 6, 2 **12A.** P'(11, 7), Q'(7, – 6), R'(9, 8) and P"(– 7, 7), Q"(– 3, – 6), R"(– 5, 8)

12B. A' (3, 3), B' (– 1, 4), C' (– 4, – 2) and A"(3, 1), B"(– 1, 0), C"(– 4, 6)

12C. A'(6, 2), B'(– 4, – 2), C'(8, 4) and A"(– 2, 6), B"(2, – 4), C"(– 4, 8)

13A. Q'(– 1, – 2), R'(– 11, – 4), S'(5, 4) and T'(15, 6) and Q"(1, 2), R"(11, 4), S"(– 5, – 4), T"(– 15, – 6)

puspas.com.np

www.drmathnp.com

13B. W'(– 3, 0), X'(– 1, – 1), Y'(– 2, – 3), Z'(– 4, – 2) and W"(0, – 3), X"(– 1 , – 1), Y"(– 3, – 2), Z"(– 2, – 4)
13C. W'(18, 6), X'(9, 6), Y'(5, 4), Z'(25, 8) and W"(20, 9), X"(11, 9), Y"(7, 7), Z"(27, 11)
13D. A'(4, 7), B'(6, 11), C'(10, 17), D'(8, 13) and A"(5, 11), B"(7, 15), C"(11, 21), D"(9, 17)
14A. $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ and (0, 0), (3, – 1), (5, – 2), (2, – 1) **14B.** $\begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{pmatrix}$ and (0, 0), (8, 2), (12, 6), (4, 4)
14C. (0, 0), (3, 1), (5, 2), (2, 1) and (0, 0), (9, 3), (15, 6), (6, 3) **15 A.** A'"(12, 16) **15 B.** P'"(4, 3)

GENERAL EXERCISE 3

LONG QUESTIONS

Model 1 *On finding transformation matrix when image of unit square is given.*

एकाइ वर्गलाई तलका प्रत्येकमा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स M पत्ता लगाउनुहोस्:
Find the 2 × 2 matrix M which transforms a unit square into each of the following:

1A. $\begin{pmatrix} 0 & 3 & 3 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ **1B.** $\begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ **1C.** $\begin{pmatrix} 0 & 3 & 3 & 0 \\ 0 & 0 & -3 & -3 \end{pmatrix}$

Model 2 *Problems based on finding image of polygon using given transformation matrix.*

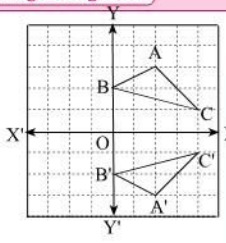
2A. एउटा पञ्चभुज ABCDE का शीर्षबिन्दुहरू A(2, 3), B(3, 1), C(5, 1), D(6, 3) र E(4, 5) छन् । स्थानान्तरण मेट्रिक्स $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ को प्रयोग गरी सो पञ्चभुजको प्रतिविम्बका निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
Ther vertices of a pentagon ABCDE are A(2, 3), B(3, 1), C(5, 1), D(6, 3) and E(4, 5). Transform the pentagon by the transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ and find the co-ordinates of image.

2B. एउटा आयत ABCD का शीर्षबिन्दुहरू A(– 3, 3), B(2, 3), C(2, 5) र D(– 3, 5) छन् । स्थानान्तरण मेट्रिक्स $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ ले विस्तार गरेपछिको ABCD को प्रतिविम्ब पत्ता लगाई लेखाचित्रमा देखाउनुहोस् ।
The vertices of a rectangle ABCD are A(– 3, 3), B(2, 3), C(2, 5) and D(– 3, 5). Find the image of ABCD under the transformation matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and show in graph.

Model 3 *Problems based on finding 2 × 2 transformation matrix when object and image are given.*

3A. बिन्दु (1, 2) लाई (4, 4) र बिन्दु (3, – 2) लाई (4, – 4) मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2 × 2 matrix of transformation which maps the point (1, 2) to (4, 4) and the point (3, – 2) to (4, – 4).

3B. चित्रमा ΔABC लाई 2 × 2 को मेट्रिक्सद्वारा ΔA'B'C' मा स्थानान्तरण गरिएको छ । सो स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
In the figure, ΔABC is transformed by 2 × 2 matrix to ΔA'B'C'. Find the transformation matrix.



3C. एउटा 2 × 2 स्थानान्तरण मेट्रिक्सले बिन्दु A (2, 3) लाई A'(6, 2) मा र बिन्दु B (– 1, 4) लाई B' (8, – 1) मा स्थानान्तरण गर्दछ भने उक्त स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
A 2 × 2 transformation matrix maps A(2, 3) to A'(6, 2) and B(– 1, 4) to B'(8, – 1). Find the 2 × 2 transformation matrix representing the mapping.

3D. यदि मेट्रिक्स $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ले बिन्दुहरू (2, 0), (3, 2) र (0, 4) लाई बिन्दुहरू (0, – 2), (– 2, – 3) र (– 4, 0) मा स्थानान्तरण गर्दछ भने मेट्रिक्स $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ पत्ता लगाउनुहोस् । साथै, उक्त मेट्रिक्सले प्रतिनिधित्व गर्ने स्थानान्तरणको प्रकार पनि उल्लेख गर्नुहोस् ।
If the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ transforms the points (2, 0), (3, 2) and (0, 4) onto (0, – 2), (– 2, – 3) and (– 4, 0) respectively, find the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Also, mention the type of transformation represented by this matrix.

puspas.com.np

www.drmathnp.com

puspas.com.np

Model 4 *Problems based on finding final image under the given combination of matrix transformation.*

4A. त्रिभुज $\begin{pmatrix} 2 & 6 & 6 \\ 1 & 1 & 5 \end{pmatrix}$ लाई स्थानान्तरण $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ पछि $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ द्वारा संयुक्त स्थानान्तरित गरिएको छ भने अन्तिम प्रतिविम्ब पत्ता लगाउनुहोस् ।
Triangle $\begin{pmatrix} 2 & 6 & 6 \\ 1 & 1 & 5 \end{pmatrix}$ is transformed under the combined transformation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ followed by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Find the final image.

4B. एउटा त्रिभुज ABC का निर्देशाङ्कहरू A(2, 3), B(4, 5) र C(6, 2) छन् । ΔABC लाई $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ पछि $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ले संयुक्त स्थानान्तरण गर्दा प्राप्त हुने प्रतिविम्बका निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
The vertices of a ΔABC are A(2, 3), B(4, 5) and C(6, 2). Find the co-ordinates of the image of ΔABC under the combined transformation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ followed by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

4C. यदि $X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ र $Y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ भए मेट्रिक्स XY अनुसार A(1, 1), B(0, 1) र C(4, 4) का प्रतिविम्बहरू पत्ता लगाउनुहोस् ।
If $X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $Y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ then find the images of A(1, 1), B(0, 1) and C(4, 4) under the matrix XY.

Model 5 *On finding image by using the 2 × 2 matrix equivalent to given transformation.*

5A. एउटा विस्तारीकरणको केन्द्र O र नापो 3 छ । यो विस्तारीकरण जनाउने स्थानान्तरण मेट्रिक्स लेखेर शीर्षहरू P(2, 1), Q(1, 2) र R (2, 2) भएको ΔPQR लाई सो मेट्रिक्सको प्रयोग गरी प्रतिविम्ब पत्ता लगाउनुहोस् ।
An enlargement O has scale factor 3. Write down the matrix to represent this enlargement and hence use it to find the image of ΔPQR having vertices P(2, 1), Q(1, 2) and R (2, 2).

5B. एउटा विस्तारीकरणको केन्द्र O र नापो 2 छ । यो विस्तारीकरण जनाउने स्थानान्तरण मेट्रिक्स लेखेर शीर्षहरू A(1, 4), B(2, 6) र C(5, 3) भएको ΔABC लाई सो मेट्रिक्सको प्रयोग गरी प्रतिविम्ब पत्ता लगाउनुहोस् ।
An enlargement with centre at origin has scale factor 2. Write down the matrix to represent this enlargement and hence use it to find the image of ΔABC having vertices A(1, 4), B(2, 6) and C(5, 3).

6A. शीर्षबिन्दुहरू P(– 1, 1), Q(– 6, 1), R(– 6, 5) र S(– 1, 5) भएको वर्गलाई रेखा $y = -x$ मा परावर्तन गरिएको छ । वर्ग PQRS को प्रतिविम्बको निर्देशाङ्कहरू 2 × 2 मेट्रिक्स प्रयोग गरी पत्ता लगाउनुहोस् ।
A square having vertices P(– 1, 1), Q(– 6,1), R(– 6,5) and S(– 1, 5) is reflected in line $y = -x$. Find the co-ordinates of image of square PQRS by using 2 × 2 matrix.

6B. P(0, 3), Q(– 6, 1), R(– 6, 5) र S(– 1, 5) शीर्षबिन्दु भएको चतुर्भुज PQRS लाई रेखा $y = x$ मा परावर्तन गरिएको छ । चतुर्भुज PQRS को प्रतिविम्बको निर्देशाङ्कहरू 2 × 2 मेट्रिक्स प्रयोग गरी पत्ता लगाउनुहोस् ।
A quadrilateral having vertices P(0, 3), Q(– 6, 1), R(– 6, 5) and S (– 1, 5) is reflected in the line $y = x$. Find the co-ordinates of image of quad. PQRS by using 2 × 2 matrix.

7A. A(3, 2), B(1, 4) र C(– 1, 1) त्रिभुज ABC को शीर्षबिन्दुहरू हुन् । ΔABC लाई उद्गमबिन्दुको वरिपरी 90° कोणमा परिक्रमण गरिएको छ भने 2 × 2 मेट्रिक्सको प्रयोग गरी प्रतिविम्ब ΔA'B'C' को निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
ΔABC has vertices at A(3, 2), B(1, 4) and C(– 1, 1). Find the co-ordinates of image ΔA'B'C' when ΔABC is rotated about the origin by an angle of 90° using 2 × 2 matrix.

7B. D(3, 2), R(1, 4) र S(– 1, 1) त्रिभुज DRS का शीर्षबिन्दुहरू हुन् । जब ΔDRS लाई उद्गमबिन्दुको वरिपरि 180° कोणमा परिक्रमण गरिएको छ भने 2 × 2 मेट्रिक्सको प्रयोग गरी प्रतिविम्ब ΔD'R'S' को निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।
ΔDRS has vertices D(3, 2), R(1, 4) and S(– 1, 1). Using 2 × 2 matrix, find the co-ordinates of image ΔD'R'S' when ΔDRS is rotated about the origin by an angle of 180°.

Model 6 *On proving the single transformation equivalent to combined transformation using matrix.*

8A. रेखा $y = x$ मा परावर्तन गरी Y-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरणलाई उद्गमबिन्दुमा 90° को परिक्रमणद्वारा देखाउन सकिन्छ भनी मेट्रिक्स विधिबाट पुष्टि गर्नुहोस् ।
By using matrix method, show that a reflection in the line $y = x$ followed by a reflection in Y-axis is equivalent to the rotation through 90° about the origin.

8B. रेखा $y = x$ मा परावर्तन गरी उद्गमबिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण Y-अक्षमा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।
Show by using matrix method that a reflection about the line $y = x$ followed by rotation about the origin through 90° (+) ve is equivalent to the reflection about Y-axis.

www.drmathnp.com

8C. X-अक्षमा परावर्तन गरी उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the reflection about X-axis followed by rotation about the origin through + 90° is equivalent to reflection about the line $y = x$.

8D. Y-अक्षमा परावर्तन गरी उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = -x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the reflection about Y-axis followed by rotation about the origin through + 90° is equivalent to reflection about the line $y = -x$.

9A. उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गरी X-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = -x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the rotation about the origin through + 90° followed by the reflection about the line X-axis is equivalent to the reflection about the line $y = -x$.

9B. उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गरी Y-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the rotation about the origin through + 90° followed by the reflection about the line Y-axis is equivalent to the reflection about the line $y = x$.

9C. उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गरी $y = x$ रेखामा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा X-अक्षमा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the rotation about origin through + 90° followed by the reflection in the line $y = x$ is equivalent to the reflection in X-axis.

9D. X-अक्षमा परावर्तन गरी उद्गम बिन्दुको वरिपरि – 90° मा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = -x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the reflection in X-axis followed by rotation through – 90° about the origin is equivalent to the reflection about the line $y = -x$.

10A. Y-अक्षमा परावर्तन गरी उद्गम बिन्दुको वरिपरि – 90° मा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the reflection in Y-axis followed by rotation through – 90° about the origin is equivalent to the reflection about the line $y = x$.

10B. रेखा $y = x$ मा परावर्तन गरी उद्गम बिन्दुको वरिपरि – 90° मा परिक्रमण गर्दा हुने संयुक्त स्थानान्तरण रेखा X-अक्षमा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the reflection about the line $y = x$ followed by the rotation through – 90° about the origin is equivalent to the reflection about X-axis.

10C. उद्गम बिन्दुको वरिपरि – 90° मा परिक्रमण गरी X-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the rotation about the origin through – 90° followed by reflection in X-axis is equivalent to the reflection about the line $y = x$.

10D. उद्गम बिन्दुको वरिपरि – 90° परिक्रमण गरी Y-अक्षमा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा $y = -x$ मा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the rotation about origin through – 90° followed by reflection in Y-axis is equivalent to the reflection about the line $y = -x$.

10E. उद्गम बिन्दुको वरिपरि – 90° परिक्रमण गरी $y = -x$ मा परावर्तन गर्दा हुने संयुक्त स्थानान्तरण रेखा X-अक्षमा हुने परावर्तनसँग समतुल्य हुन्छ भनी मेट्रिक्स विधिबाट प्रमाणित गर्नुहोस् ।
Prove by the matrix method that the rotation about origin through – 90° followed by reflection in $y = -x$ is equivalent to the reflection about the line X-axis.

ANSWERS

LONG QUESTIONS

1A. $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ **1B.** $\begin{pmatrix} 4 & 2 \\ 0 & 2 \end{pmatrix}$ **1C.** $\begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix}$ **2A.** A'(– 3, – 2), B'(– 1, – 3), C'(– 1, – 5), D'(– 3, – 6), E'(– 5, – 4)

2B. A' (– 9, 9), B' (6, 9), C' (6, 15), D' (– 9, 15) **3A.** $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ **3B.** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **3C.** $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

3D. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ Reflection in the line $y = -x$. **4A.** (2, – 1), (6, – 1),(6, – 5) **4B.** A' (– 2, 3), B'(– 4, 5), C'(– 6, 2)

4C. A'(– 1, – 1), B'(– 4, – 1), C'(– 4, – 4) **5A.** P'(6, 3), Q'(3, 6), R'(6, 6) **5B.** A'(2, 8), B'(4, 12), C'(10, 6)

6A. P'(– 1, 1), Q'(– 1, 6),R'(– 5, 6) S'(– 5, 1) **6B.** P'(3, 0) , Q' (1, – 6), R'(5, – 6), S'(5,– 1)

7A. A'(– 2, 3), B'(– 4, 1), C'(– 1,– 1) **7B.** D'(– 3, – 2), R'(– 1, – 4), S' (1, – 1)

puspas.com.np

www.drmathnp.com

puspas.com.np

CHALLENGING EXERCISE 1

1. दुईओटा विस्थापन भेक्टरहरू $T_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ र $T_2 = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$ छन् । यदि ΔABC लाई $T_1 T_2$ ले विस्थपन गर्दा A'(2, 5), B'(6, 7) र C'(8, 3) भएको त्रिभुज A' B' C' बन्दछ भने ΔABC का शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

Two translation vectors are $T_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $T_2 = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$ If ΔABC is translated by $T_1 T_2$ then the image $\Delta A'B'C'$ having vertices A'(2, 5), B'(6, 7) and C'(8, 3) is formed. Find the co-ordinates of the vertices of ΔABC.

2. दुईओटा विस्थापन भेक्टरहरू $T_1 = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ र $T_2 = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ छन् । यदि ΔABC लाई $T_1 T_2$ ले विस्थपन गर्दा A'(9, 10), B'(12, 14) र C' (15, 6) भएको त्रिभुज A'B'C' बन्दछ भने ΔABC का शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

Two translation vectors are $T_1 = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ and $T_2 = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ If ΔABC is translated by $T_1 T_2$ then the image $\Delta A'B'C'$ having vertices A'(9, 10), B' (12, 14) and C' (15, 6) is formed. Find the co-ordinates of the vertices of ΔABC.

3. ΔABC का शीर्षबिन्दुहरू A(2, 2), B (1, – 1) र C (3, 0) छन् । यदि $E_1 = [(0, 2), 2]$ र $E_2 = [(0, – 2), – 1/2]$ भए यसलाई $E_2 E_1$ ले विस्तार गर्दा बन्ने अन्तिम प्रतिविम्बका निर्देशाङ्कहरू पत्ता लगाई सो संयुक्त विस्तारीकरणको केन्द्र र नापो पनि पत्ता लगाउनुहोस् ।

The vertices of ΔABC are A(2, 2), B (1, – 1) and C (3, 0). If $E_1 = [(0, 2), 2]$ and $E_2 = [(0, – 2), – 1/2]$ then find the final image of ΔABC when it is enlarged by $E_2 E_1$ and also find the centre and scale factor of the combined enlargement.

4. R ले रेखा $x = 2$ मा हुने परावर्तन र S ले उद्गमबिन्दुको वरिपरि 90° परिक्रमण जनाउँदछ । यदि संयुक्त स्थानान्तरण R∘S अनुसार ΔABC का अन्तिम प्रतिविम्ब A'(7, 2), B' (9, 4) र C'(6, 6),भए ΔABC का शीर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

R = reflection in the line $x = 2$ and S = rotation of 90° about the origin. If the image of ΔABC under the combined transformation R∘S are A'(7, 2), B' (9, 4) and C'(6, 6) then, find the coordinates of vertices of ΔABC.

5. R ले रेखा $y = 5$ मा हुने परावर्तन र T ले भेक्टर $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ले हुने विस्थापन जनाउँदछ । यदि संयुक्त स्थानान्तरण T∘R अनुसार ΔABC को प्रतिविम्बका शीर्षबिन्दुहरू A'(4, 10), B'(6, 8) र C'(8,11) भए ΔABC का शीर्षबिन्दुहरू पत्ता लगाउनुहोस् ।

R denotes the reflection in the line $y = 5$ and T is the translation by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. If the co-ordinates of vertices of image of ΔABC under T∘R are A'(4, 10), B'(6, 8) and C'(8, 11), find the co-ordinates of vertices of ΔABC.

6. ΔABC लाई भेक्टर $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ ले विस्थापन गरी उद्गमबिन्दुको वरिपरि 90° परिक्रमण गर्दा अन्तिम प्रतिविम्बका निर्देशाङ्कहरू A"(– 8, 6), B"(– 10, 8) र C"(– 7, 10) भए ΔABC का शीर्षबिन्दुहरू पत्ता लगाउनुहोस् ।

ΔABC is translated by the vector $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and then rotated by 90° about the origin so that the co-ordinates of final images are A"(– 8, 6), B"(– 10, 8) and C"(– 7, 10). Find the co-ordinates of vertices of ΔABC.

7. ΔABC लाई $x = 2$ रेखामा परावर्तन गरी नापो 2 र उद्गमबिन्दुलाई केन्द्र मानी विस्तारीकरण गर्दा अन्तिम प्रतिविम्बका निर्देशाङ्कहरू A"(4, 6), B"(0, 10) र C"(– 4, 4) भए ΔABC का शीर्षबिन्दुहरू पत्ता लगाउनुहोस् ।

ΔABC is reflected in the line $x = 2$ and then enlarged by the scale factor 2 and the centre is at origin so that the co-ordinates of final image are A"(4, 6), B"(0, 10) and C"(– 4, 4). Find the co-ordinates of vertices of ΔABC.

8. ΔABC लाई उद्गमबिन्दुको वरिपरि 90° द्वारा परिक्रमण गरी नापो 2 र केन्द्र (0, 0) भएको विस्तारीकरणद्वारा विस्तार गर्दा अन्तिम प्रतिविम्बका निर्देशाङ्कहरू A"(– 6, 4), B"(– 10, 8) र C"(– 4, 12) भए ΔABC शीर्षबिन्दुहरूका निर्देशाङ्कहरू पत्ता लगाउनुहोस् ।

ΔABC is rotated about the origin by 90° then enlarged by the enlargement of scale factor 2 and centre is at the origin so that the co-ordinates of final image of ΔABC are A"(– 6, 4), B"(– 10, 8) and C"(– 4, 12). Find the co-ordinates of vertices of ΔABC .

9. ΔABC का शीर्षबिन्दुहरू A(4, 0), B(4, 2) र C(2, 4) छन् । ΔABC का शीर्षबिन्दुहरूको वृत्त $x^2 + y^2 = 20$ को आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । साथै ΔABC र विपरीत $\Delta A'B'C'$ लाई एउटै लेखाचित्रमा पनि देखाउनुहोस् ।

The vertices of ΔABC are A(4, 0), B(4, 2) and C(2, 4). Find the inversion points of vertices of ΔABC with respect to circle $x^2 + y^2 = 20$. Also, show ΔABC and inversion $\Delta A'B'C'$ in the same graph.

10. ΔABC का शीर्षबिन्दुहरू A(2, 6), B(2, 4) र C(9, 3) छन् । ΔABC का शीर्षबिन्दुहरूको वृत्त $x^2 + y^2 = 60$ को आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । साथै ΔABC र विपरीत $\Delta A'B'C'$ लाई एउटै लेखाचित्रमा पनि देखाउनुहोस् ।

The vertices of ΔABC are A(2, 6), B(2, 4) and C(9, 3). Find the inversion points of vertices of ΔABC with respect to circle $x^2 + y^2 = 60$. Also, show ΔABC and inversion $\Delta A'B'C'$ in the same graph.

www.drmathnp.com

puspas.com.np

ANSWERS

1. A (4, 7), B (8, 9), C (10, 5)
2. A (7, 7), B (10, 11), C (13, 3)
3. Centre = (0, –1), s.f.= – 1
4. A(2, 3), B(4, 5), C(6, 2)
5. A(2, 3), B(4, 5), C(6, 2)
6. A(2, 3), B(4, 5), C(6, 2)
7. A(2, 3), B(4, 5), C(6, 2)
8. A(2, 3), B(4, 5), C(6, 2)
9. A'(5, 0), B'(4, 2), C'(2, 4)
10. A'(3, 9), B'(6, 12), C'(6, 2)

ASSIGNMENT ON TRANSFORMATION

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' 2 × 1 = 2

1. एउटा उत्क्रम वृत्तमा, P को उत्क्रम बिन्दु P' हो । यदि OP × OP' = 81 cm² भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
In an inversion circle, P' is the inversion point of P. If OP × OP' = 81 cm², then find the radius of the inversion circle.

2. कुन स्थानान्तरण मेट्रिक्स रेखा $x = y$ मा हुने परावर्तनसँग सम्बन्धित छ ?
Which transformation matrix is associated with reflection in the line $x = y$?

Group C 2 × 4 = 8

3. ΔPQR का शीर्षबिन्दुहरू P(– 3, 2), Q(– 1, 4) र R(2, 0) छन् । मेट्रिक्स विधि प्रयोग गरी X-अक्षमा हुने परावर्तनको प्रतिबिम्ब त्रिभुजका शीर्षबिन्दुहरू पत्ता लगाउनुहोस् ।
P(– 3, 2), Q(– 1, 4) and R(2, 0) are the vertices of ΔPQR. Find the vertices of the image triangle under the reflection on X-axis by using matrix method.

4. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 2 & 5 & 3 \\ 0 & 4 & 6 & 2 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2 × 2 transformation matrix which transforms a unit square to a parallelogram $\begin{bmatrix} 0 & 2 & 5 & 3 \\ 0 & 4 & 6 & 2 \end{bmatrix}$.

Group D 3 × 5 = 15

5. शीर्षबिन्दुहरू A(1, 0), B(0, 2) र C(2, 1) भएको ΔABC लाई लेखाचित्रमा खिच्नुहोस् । ΔABC लाई E[O, 2] द्वारा विस्तार गर्नुहोस् र त्यसपछि प्रतिबिम्ब ΔA'B'C' लाई उद्गम बिन्दुको वरिपरि 90° धनात्मक दिशामा परिक्रमण गर्नुहोस् । सबै त्रिभुजहरूलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Sketch the graph of ΔABC having vertices A(1, 0), B(0, 2) and C(2, 1) in a graph paper. Enlarge the ΔABC by E[O, 2] first and then rotate the image ΔA'B'C' by 90° +ve about the origin. Show all triangles in same graph.

6. केन्द्र (2, 3) र अर्धव्यास 4 एकाइ भएको वृत्तको आधारमा बिन्दुहरू P(4, 5) र Q(4, 3) जोड्ने रेखाखण्डको विपरीत पत्ता लगाउनुहोस् । सो विपरीत स्थानान्तरणलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Find the inverse line segment joining the points P(4, 5) and Q(4, 3) with respect to a circle with centre C(2, 3) and radius 4 units. Show the inversion transformation on the same graph.

7. शीर्षबिन्दुहरू P(3, 6), Q(4, 2) र R(2, 2) भएको ΔPQR लाई शीर्षबिन्दुहरू $P_1(– 6, 3)$, $Q_1(– 2, 4)$ र $R_1(– 2, 2)$ भएको $\Delta P_1Q_1R_1$ मा प्रतिचित्रण गरिएको छ । यो प्रतिचित्रणलाई कुन एकल स्थानान्तरणले जनाउँदछ ? साथै यो स्थानान्तरण जनाउने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
ΔPQR whose vertices are P(3, 6), Q(4, 2) and R(2, 2) maps onto the $\Delta P_1Q_1R_1$ with vertices $P_1(– 6, 3)$, $Q_1(– 2, 4)$ and $R_1(– 2, 2)$. Which is the single transformation for this mapping ? Also find the 2 × 2 matrix that represents this transformation.

TRANSFORMATION

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C⁺	B	B⁺	A	A⁺

ANSWERS

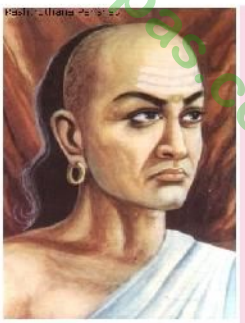
1. 9 cm
2. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
3. P'(– 3, – 2), Q'(– 1, – 4), R'(2, 0)
4. $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$
5. A' (2, 0), B'(0, 4), C'(4, 2) and A"(0, 2), B"(– 4, 0), C"(– 2, 4)
6. P'(6, 7), Q'(10, 3)
7. Rotation about (0, 0) in + ve 90°, $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

www.drmathnp.com

puspas.com.np

Unit VIII

तथ्याङ्कशास्त्र
Statistics



KAUTHILYA

'Statistics' is derived from the Latin word 'status' which means a political state. This suggests that statistics is as old as human civilization. In the year 3050 B.C., perhaps the first census was held in Egypt. In India also, about 2000 years ago, we had an efficient system of collecting administrative statistics, particularly, during the regime of Chandra Gupta Maurya (324-300 B.C.). The system of collecting data related to births and deaths is mentioned in **Kautilya**'s *Arthshastra (around 300 B.C.). A detailed account of administrative surveys conducted during akbar's regime is given in Ain-I-Akbari written by Abul Fazl.*

SEE SPECIFICATION GRID **1 Short Question and 2 Long Questions**

Contents	Topics	Knowledge	Understanding	Application	Higher Ability	Total no. of Questions	Total Marks
		Each of 1 Mark	Each of 2 Marks	Each of 4 Marks	Each of 5 Marks		
तथ्याङ्क शास्त्र (Statistics)	चतुर्थांशीय विचलन Quartile Deviation मध्यक भिन्नता Mean Deviation स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क Standard Deviation and Coefficient of Variation	-	1	2	-	3	10

SYLLABUS ISSUED BY CDC

8. तथ्याङ्क शास्त्र (Statistics) **- 12 Periods**

विचरणशीलता (Dispersion)

- चतुर्थांशीय विचलन र यसको गुणाङ्क (अविछिन्न श्रेणी)
  Quartile deviation and its coefficient (Continuous series)
- मध्यक भिन्नता (मध्यक र मध्यिकाबाट) र यसको गुणाङ्क (अविछिन्न श्रेणी)
  Mean deviation (from mean and median) and its coefficient (Continuous series)
- स्तरीय भिन्नता र यसको गुणाङ्क र विश्लेषण (अविछिन्न श्रेणी)
  Standard deviation and its coefficient and analysis (Continuous series)
- विचरणशीलताको गुणाङ्क (Coefficient of variation) (अविछिन्न श्रेणी)
  Coefficients of variation and its coefficient (Continuous series)

1. चतुर्थांशीय विचलन र यसको गुणाङ्क
QUARTILE DEVIATION AND ITS COEFFICIENT

DEFINITIONS

1. विचरणशीलताको मापन (Measure of Dispersion)

तथ्याङ्कमा मानहरू सामान्यतया छरिएको (scatterness), विविधता (variability), विचलन (deviation), उतार चढाव, कति टाढा, कति ठूला वा साना छन् र एक आपसमा कति सम्बन्धित छन् भनी हेर्नको लागि गरिने मापनलाई विचरणशीलताको मापन भनिन्छ । तथ्याङ्कमा सबै मान एउटै भए त्यसको विचरणशीलता हुँदैन ।

A measure of dispersion is the method of finding information regarding the amount of variability or spread or deviation or scatterness present in the data. If all the values of a data are identical, there is no dispersion.

- यदि तथ्याङ्कका पदहरू आपसमा नजिक भए विचरणशीलताको मान सानो हुन्छ ।
  The amount of dispersion in different values may be small, if the values are close together.

www.drmathnp.com
puspas.com.np

विचरणशीलताको मापन गर्ने प्रमुख विधिहरू यस प्रकारका छन् । (The major methods of measure of dispersion are as follows.)

(a) विस्तार (range)
(b) चतुर्थांशीय विचलन (Quartile deviation)
(c) मध्यक भिन्नता (Mean deviation)
(d) स्तरीय भिन्नता (Standard deviation)

2. चतुर्थांशीय भिन्नता (Quartile deviation)

तेस्रो चतुर्थांश (Q_3) र पहिलो चतुर्थांश (Q_1) बीचको अन्तरलाई चतुर्थांशको विस्तार भनिन्छ । चतुर्थांशको विस्तारको आधालाई अर्धचतुर्थांशीय विस्तार अथवा चतुर्थांशीय भिन्नता भनिन्छ ।

The difference between the first quartile (Q_1) and the third quartile (Q_3) is known as inter-quartile range. The half of the inter-quartile range is called **Semi-interquartile Range** or **Quartile Deviation.**

3. चतुर्थांशीय भिन्नताको गुणाङ्क (Coefficient of Quartile Deviation)

चतुर्थांशीय भिन्नता निरपेक्ष मान भएकाले यसलाई अरू तथ्याङ्कका समूहसँग तुलना गर्नका लागि चतुर्थांशीय भिन्नताको गुणाङ्क निकालिन्छ । चतुर्थांशीय भिन्नताको गुणाङ्क $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$ हुन्छ ।

As the quartile deviation is an absolute value, we need to find the coefficient of quartile deviation to compare this with other data. Coefficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

4. चतुर्थांशीय भिन्नताका गुणहरू र दोषहरू (Merits and Demerits of Quartile Deviation)

गुणहरू (Merits)	दोषहरू (Demerits)
1. यो बुझ्नको लागि सामान्य र गणना गर्न सजिलो हुन्छ । QD is simple to understand and easy to calculate.	1. सबै मानहरूमा QD आधारित हुँदैन । QD is not based on all observations.
2. अधिकतम मानले यसलाई असर गर्दैन किनकी माथिल्लो 25% र तल्लो 25% छोडिन्छ । It is not affected by extreme terms because 25% of upper and 25% of lower terms are left out.	2. मानहरूको घटबढले तथ्याङ्कको गणनामा असर गर्दछ । It is too much affected by the fluctuations of samples in the calculation.
3. सुरूको र अन्तिमका खुल्ला वर्गान्तरहरू भएको तथ्याङ्कको पनि QD गणना गर्न सकिन्छ । QD can be calculated even for open-end interval.	3. तथ्याङ्कमा पदका मानहरूको क्रम मिलाउनुपर्छ । Arrangement of data are necessary.
4. 6QD = 5MD = 4SD प्रयोग गरी MD र SD पत्ता लगाउन सकिन्छ । MD and SD can be calculated using 6QD = 5MD = 4SD	4. यदि मानहरू अनियमित भए परीणाम नराम्रो सँग बिग्रिन्छ । If the values are irregular then result is affected badly.

5. सुधार गुणाङ्क (Correction Factor)

यदि दिइएको तथ्याङ्क विच्छिन्न छ भने यसलाई सुधार गुणाङ्कद्वारा निरन्तर बनाउन सकिन्छ ।

If the given data is discontinuous, it can be made continuous by correction factor.

सुधार गुणाङ्क $(C_F) = \frac{\text{दोस्रो वर्गान्तरको तल्लो सीमा} - \text{पहिलो वर्गान्तरको माथिल्लो सीमा}}{2}$ हुन्छ ।

Correction Factor $(C_F) = \frac{\text{Lower limit of } 2^{nd} \text{ class} - \text{Upper limit of first class}}{2}$

दिइएको तथ्याङ्कलाई निरन्तर बनाउन $L - C_F$ र $U + C_F$ गरिन्छ ।

To make the given data continuous; $L - C_F$ and $U + C_F$ should be done.

6. विच्छिन्न तथ्याङ्कको उदाहरण (Example of discontinuous data)

Example:

Marks	1 - 19	21 - 39	41 - 59	61 - 79
Frequency	6	7	8	5

To make it continuous, Correction factor $(C_F) = \frac{21 - 19}{2} = 1$

Now, make $L - C_F$ and $U + C_F$ then,

Marks	0 - 20	20 - 40	40 - 60	60 - 80
Frequency	6	7	8	5

This is now continuous data.

www.drmathnp.com

puspas.com.np

FORMULAE AND KEY POINTS

तथ्याङ्क Data	पहिलो चतुर्थांश First Quartile (Q_1)	दोस्रो चतुर्थांश वा मध्यिका 2nd Quartile or Median (Q_2)	तेस्रो चतुर्थांश 3rd Quartile (Q_3)
अविच्छिन्न श्रेणी (Continuous Series) x: 10-20, 20-30, 30-40, 50-60 f: 2, 3, 5, 2	Q_1 class $= \left(\frac{N}{4}\right)^{th}$ class $Q_1 = L + \frac{i}{f}\left(\frac{N}{4} - c.f\right)$	Q_2 class $= \left(\frac{N}{2}\right)^{th}$ class $Q_2 = L + \frac{i}{f}\left(\frac{N}{2} - c.f\right)$	Q_3 class $= \left(\frac{3N}{4}\right)^{th}$ class $Q_3 = L + \frac{i}{f}\left(\frac{3N}{4} - c.f\right)$

चतुर्थांशीय विचलन (Quartile Deviation) $= \frac{Q_3 - Q_1}{2}$,

चतुर्थांशीय विचलनको गुणाङ्क (Coefficient of Quartile Deviation) $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

SOLVED EXAMPLE

Example 1 : तलको तथ्याङ्कबाट चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the quartile deviation and its coefficient of the following data.

Marks obtained	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	5	15	10	8	6	2

Solution: Here, from the given data;

Marks	f	c.f.
20 - 30	5	5
30 - 40	15	20
40 - 50	10	30
50 - 60	8	38
60 - 70	6	44
70 - 80	2	46
	N = 46	

Q_1 lies at $\left(\frac{N}{4}\right)^{th}$ class $= \frac{46}{4}^{th}$ class $= 11.5^{th}$ class $= (30 - 40)$

$\therefore \quad Q_1 = L + \frac{i}{f}\left(\frac{N}{4} - c.f.\right) = 30 + \frac{10}{15}(11.5 - 5) = 34.34$

Q_3 lies at $\frac{3N}{4}^{th}$ class $= \frac{3 \times 46}{4}^{th}$ class $= 34.5^{th}$ class $= (50 - 60)$

$\therefore \quad Q_3 = L + \frac{i}{f}\left(\frac{3N}{4} - c.f.\right) = 50 + \frac{10}{8}(34.5 - 30) = 55.63$

We know that, Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{55.63 - 34.34}{2} = \frac{21.29}{2} = 10.65$

Now, the coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{55.63 - 34.34}{55.63 + 34.34}$ $\therefore$ QD = 0.23

Thus, the QD is 0.23.

SEE EXERCISE 1

STATISTICS

A. SHORT QUESTIONS

Model 1 *Definitions and concept recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् (Define the following terms):
   - **A.** पहिलो चतुर्थांश (First quartile)
   - **B.** दोस्रो चतुर्थांश (Second quartile)
   - **C.** तेस्रो चतुर्थांश (Third quartile)
   - **D.** चतुर्थांशीय विचलन (Quartile deviation)
   - **E.** चतुर्थांशीय विचलनको गुणाङ्क (Coefficient of quartile deviation)
   - **F.** विचरणशीलताको मापन (Measure of dispersion)

www.drmathnp.com
puspas.com.np

2A. विचरणशीलताको मापनका विधिहरू के-के हुन् ? (What are the methods of measure of dispersion?)

2B. पहिलो चतुर्थांश र तेस्रो चतुर्थांशको भिन्नता लेख्नुहोस् । (Write the difference between first quartile and third quartile.)

Model 2 Problems based on first quartile and third quartile.

3A. एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको स्थान $15^{\text{औं}}$ पद छ । बारम्बारताहरूको योगफल र तेस्रो चतुर्थांशको स्थान पत्ता लगाउनुहोस् ।
In a continuous data, the position of first quartile is 15^{th} term. Find the sum of frequencies of the data and the position of the third quartile.

3B. एउटा निरन्तर श्रेणीमा दोस्रो चतुर्थांशको स्थान $20^{\text{औं}}$ पद छ । बारम्बारताहरूको योगफल र पहिलो चतुर्थांशको स्थान पत्ता लगाउनुहोस् ।
In a continuous data, the position of the second quartile is 20^{th} item. Find the sum of frequencies of the data and the position of the first quartile.

3C. एउटा निरन्तर श्रेणीमा तेस्रो चतुर्थांशको स्थान $81^{\text{औं}}$ पद छ । बारम्बारताहरूको योगफल र दोस्रो चतुर्थांशको स्थान पत्ता लगाउनुहोस् ।
In a continuous data, the position of the third quartile is 81^{st} item. Find the sum of frequencies of the data and the position of the second quartile.

Model 3 Problems based on finding Q. D.

4A. एउटा तथ्याङ्कको पहिलो चतुर्थांश 35 र तेस्रो चतुर्थांश 75 छ । चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, first quartile is 35 and the third quartile is 75. Find the quartile deviation and its coefficient.

4B. एउटा तथ्याङ्कको पहिलो चतुर्थांश 35 र तेस्रो चतुर्थांश 75 छ । चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, first quartile is 35 and the third quartile is 75. Find the quartile deviation and its coefficient.

4C. एउटा तथ्याङ्कको पहिलो चतुर्थांश 45 र तेस्रो चतुर्थांश 55 छ । चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, the first quartile is 45 and the third quartile is 55. Find the quartile deviation and its coefficient.

5A. एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको दुई गुणा तेस्रो चतुर्थांश छ । यदि सो तथ्याङ्कको मध्यिकाको मान 45 भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In a continuous series, third quartile is two times of first quartile. If the median of the data is 45, find the quartile deviation.

5B. एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको तीन गुणा तेस्रो चतुर्थांश छ । यदि सो तथ्याङ्कको मध्यिकाको मान 20 भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In a continuous series, third quartile is three times of first quartile. If the median of the data is 20, find the quartile deviation.

Model 4 Problems based on Q.D and its coefficient.

6A. एउटा तथ्याङ्कको चतुर्थांशीय विचलन र पहिलो चतुर्थांश क्रमशः 20 र 35 छन् । सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, quartile deviation and the first quartile are 20 and 35 respectively. Find the third quartile and the coefficient of quartile deviation.

6B. एउटा तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विचलन क्रमशः 17.5 र 20 छन् । सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, the first quartile and the quartile deviation are 17.5 and 20 respectively. Find the third quartile and the coefficient of quartile deviation.

7A. एउटा निरन्तर तथ्याङ्कमा चतुर्थांशीय विस्तार 20 र $Q_1 = 10$ छन् । चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
The interquartile range of a continuous data is 20 and $Q_1 = 10$. What is the value of quartile deviation and its coefficient?

7B. एउटा निरन्तर तथ्याङ्कमा चतुर्थांशीय विस्तार 30 र $Q_3 = 40$ छन् । चतुर्थांशीय विचलन र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
The interquartile range of a continuous data is 30 and $Q_3 = 40$. What is the value of quartile deviation and its coefficient?

8A. एउटा तथ्याङ्कमा Q_3 र Q_1 को अन्तर 20 छ र तिनीहरूको योग 40 छ । QD र QD को गुणाङ्क पत्ता लगाउनुहोस् ।
The difference of Q_3 and Q_1 in a data is 20 and their sum is 40. Find QD and the coefficient of QD.

8B. एउटा तथ्याङ्कमा Q_3 र Q_1 को अन्तर 30 छ र तिनीहरूको योग 60 छ । QD र QD को गुणाङ्क पत्ता लगाउनुहोस् ।
The difference of Q_3 and Q_1 in a data is 30 and their sum is 60. Find QD and the coefficient of QD.

Model 5 Problems based on finding quartiles when Q.D. & its coefficient are given.

9A. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमशः 15 र $\frac{3}{7}$ छन् । पहिलो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data, the quartile deviation and its coefficient are 15 and $\frac{3}{7}$ respectively. Find the first quartile.

www.drmathnp.com

puspas.com.np

9B. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमशः 14 र $\frac{7}{22}$ छन् । पहिलो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data, the quartile deviation and its coefficient are 14 and $\frac{7}{22}$ respectively. Find the first quartile.

10A. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमशः 15 र $\frac{3}{7}$ छन् । तेस्रो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data, the quartile deviation and its coefficient are 15 and $\frac{3}{7}$ respectively. Find the third quartile.

10B. एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमशः 14 र $\frac{7}{22}$ छन् । तेस्रो चतुर्थांश पत्ता लगाउनुहोस् ।

In a data the quartile deviation and its coefficient are 14 and $\frac{7}{22}$ respectively. Find the third quartile.

Model 6 ***On finding coefficient of Q.D. when Q_1 and Q_3 & Q.D. are given.***

11A. एउटा तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय भिन्नता क्रमशः 58 र 28 छन् । चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

The third quartile and inter-quartile range of a data are 58 and 28 respectively. Find the coefficient of the quartile deviation.

11B. एउटा तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार क्रमशः 13 र 2 छन् । चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

The first quartile and the inter-quartile range of a data are 13 and 2 respectively. Find the coefficient of the quartile deviation.

Model 7 ***Problems based on finding Q_1/Q_3 & inter quartile range.***

12A. एउटा तथ्याङ्कको तेस्रो चतुर्थांश 15 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{1}{14}$ भए सो तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The third quartile of a data is 15. If the coefficient of quartile deviation is $\frac{1}{4}$, find the first quartile and the inter-quartile range of the data.

12B. एउटा तथ्याङ्कको तेस्रो चतुर्थांश 58 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{7}{22}$ भए सो तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The third quartile of a data is 58. If the coefficient of quartile deviation is $\frac{7}{22}$, find the first quartile and the inter-quartile range of the data.

13A. एउटा तथ्याङ्कको पहिलो चतुर्थांश 13 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{1}{14}$ भए सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The first quartile of a data is 13. If the coefficient of quartile deviation is $\frac{1}{14}$, find the third quartile and the inter-quartile range of the data.

13B. एउटा तथ्याङ्कको पहिलो चतुर्थांश 30 छ । यदि यो तथ्याङ्कको चतुर्थांशीय विचलनको गुणाङ्क $\frac{7}{22}$ भए सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।

The first quartile of a data is 30. If the coefficient of quartile deviation is $\frac{7}{22}$, find the third quartile and the inter-quartile range of the data.

14A. कुनै श्रेणीको पहिलो चतुर्थांशीय मान x र चतुर्थांशीय विचलन (x) भए त्यसको तेस्रो चतुर्थांशीय मान तथा चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

In a data, value of first quartile is 'x' and quartile deviation is also (x). Find the third quartile and the coefficient of quartile deviation.

14B. कुनै श्रेणीको पहिलो चतुर्थांशीय मान $2a$ र चतुर्थांशीय विचलन (a) भए त्यसको तेस्रो चतुर्थांशीय मान तथा चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

In a data, value of first quartile is '$2a$' and quartile deviation is (a). Find the third quartile and the coefficient of quartile deviation.

www.drmathnp.com
puspas.com.np

B. LONG QUESTIONS

Model 1 *Problems based on quartile deviation.*

1A. दिइएको तथ्याङ्कमा $Q_1 = 25$ भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In the given data $Q_1 = 25$, find the quartile deviation.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	3	4	6	4	3

1B. दिइएको तथ्याङ्कमा $Q_1 = 9.6$ भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In the given data $Q_1 = 9.6$, find the quartile deviation.

CI	4 - 8	8 - 12	12 - 16	16 - 20	20 - 24
f	3	5	4	5	3

1C. दिइएको तथ्याङ्कमा $Q_3 = 165$ भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In the given data $Q_3 = 165$, find the quartile deviation.

CI	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250
f	7	20	15	10	8

1D. दिइएको तथ्याङ्कमा $Q_3 = 37.5$ भए चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
In the given data $Q_3 = 37.5$, find the quartile deviation.

CI	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
f	4	6	20	10	7	3

2. तलका तथ्याङ्कहरूबाट चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
Find the quartile deviation from the following data.

A.

CI	3 - 6	6 - 9	9 - 12	12 - 15
f	1	6	9	4

B.

Variable	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	10	16	9

C.

Marks	0 - 5	10 - 15	5 - 10	15 - 20	20 - 25
No. of students	4	10	8	6	4

D.

Marks	30 - 35	25 - 30	20 - 25	40 - 45	35 - 40
No. of students	5	10	12	7	6

Model 2 *Problems based on quartile deviation and its coefficient.*

3. तलका तथ्याङ्कहरूबाट चतुर्थांशीय विचलन र सो को गुणाङ्क पत्ता लगाउनुहोस् ।
From the following data, find the quartile deviation and its coefficient.

A.

Class interval	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
Frequency	4	12	16	6	2

B.

Age	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20
No. of students	14	18	23	18	7

C.

Age (in years)	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of workers	2	6	22	13	7

D.

Classes	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	3	4	6	10	3

E.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of students	4	3	7	22	7	2

F.

Weight (kg)	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22	22 - 24
No. of students	41	53	67	72	47	32

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on finding quartile deviation of the following data.*

4. तलका तथ्याङ्कहरूको चतुर्थांशीय विचलन पत्ता लगाउनुहोस् । (Find the quartile deviation of the following data.)

A.

Marks	$0 \le x < 10$	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$
No. of students	4	6	8	2

B.

Marks	$5 \le x < 15$	$15 \le x < 25$	$25 \le x < 35$	$35 \le x < 45$
No. of students	3	5	4	2

C.

Class	$20 \le x < 30$	$40 \le x < 50$	$60 \le x < 70$	$30 \le x < 40$	$50 \le x < 60$
Frequency	5	4	6	3	2

Model 4 *Problems based on quartile deviation (less than and more than).*

5. तलका तथ्याङ्कहरूको चतुर्थांशीय विचलन पत्ता लगाउनुहोस् । (Find the quartile deviation of the following data.)

A.

Class	$20 \le x < 30$	< 40	< 50	< 60	< 70
Frequency	5	8	12	14	20

B.

Class	< 5	< 10	< 15	< 20	< 25	< 30
Frequency	3	6	10	15	20	24

C.

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	3	7	13	23	26

D.

Class interval	More than 25	More than 20	More than 15	More than 10	More than 5
Frequency	2	8	24	36	40

E.

Time (in sec)	Above 0	Above 10	Above 20	Above 30	Above 40
No. of calls	20	18	13	6	1

6. तलका तथ्याङ्कबाट अर्ध चतुर्थांशीय विस्तार र चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the interquartile range and coefficient of quartile deviation from the following data:

A.

Weight (Kg)	10 - 20	10 - 30	10 - 40	10 - 50	10 - 60	10 - 70
No. of men	12	31	36	46	55	61

B.

Ages (in years)	10 - 12	10 - 14	10 - 16	10 - 18	10 - 20
No. of students	14	32	55	73	80

Model 5 *Problems based on mid value and frequency.*

7. तलका तथ्याङ्कबाट चतुर्थांशीय विचलन पत्ता लगाउनुहोस् । (Find the quartile deviation from the following data.)

A.

मध्यमान (Mid value)	25	35	45	55	65	75
बारम्बारता (Frequency)	3	4	6	9	4	4

B.

मध्यमान (Mid value)	25	35	45	55	65
बारम्बारता (Frequency)	5	3	4	2	6

C.

मध्यमान (Mid value)	2.5	7.5	12.5	17.5	22.5	27.5
बारम्बारता (Frequency)	3	3	4	5	5	4

ANSWERS

A. SHORT QUESTIONS

3A. 60, 45th term **3B.** 40, 10th term **3C.** 108, 54th term **4A.** 20, 0.36 **4B.** 20, 0.36 **4C.** 5, 0.1 **5A.** 15
5B. 10 **6A.** 75, 0.36 **6B.** 57.5, 0.53 **7A.** 10, 0.5 **7B.** 15, 0.6 **8A.** 10, 0.5 **8B.** 15, 0.5
9A. 20 **9B.** 30 **10A.** 50 **10B.** 58 **11A.** 0.32 **11B.** $\frac{1}{14}$ **12A.** 13 and 2
12B. 30 and 28 **13A.** 15, 2 **13B.** 58, 28 **14A.** $3x$, 0.5 **14B.** $4a$, 0.33

B. LONG QUESTIONS

1A. 5 **1B.** 4.4 **1C.** 47.5 **1D.** 8.125 **2A.** 1.83 **2B.** 7.1875
2C. 4.58 **2D.** 6.67 **3A.** 3.44, 0.216 **3B.** 1.945, 0.133 **3C.** 6.86 yrs and 0.18
3D. 17.75 and 0.32 **3E.** 6.45 and 0.198 **3F.** 2.32 kg and 0.13 **4A.** 7.29 **4B.** 7.625
4C. 15.83 **5A.** 15.83 **5B.** 6.5 **5C.** 17.75 **5D.** 3.43 **5E.** 8
6A. 28.04, 0.39 **6B.** 3.89, 0.133 **7A.** 10.21 **7B.** 15.83 **7C.** 6.5

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 1

Model 1 Problems based on making frequency distribution table.

1. तलको तथ्याङ्कको पहिलो वर्गान्तर 0 - 10 लिएर बारम्बारता तालिका बनाई चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
Taking 0 - 10 as a first class interval, make a frequency table and find the quartile deviation.

A. 2, 11, 21, 31, 41, 8, 15, 22, 35, 40, 16, 24, 25, 26, 28, 38

B. 8, 11, 21, 31, 41, 7, 12, 22, 32, 42, 9, 13, 23, 33, 45, 15, 24, 25, 27, 35

2A. 10 - 20 को पहिलो वर्गान्तर लिएर बारम्बारता तालिका बनाई चतुर्थांशीय विचलन पत्ता लगाउनुहोस् ।
Taking 10 - 20 as a first class interval, make a frequency table and find the quartile deviation.
48, 50, 34, 29, 56, 40, 14, 62, 28, 70, 22, 30, 38, 74, 13, 47, 20, 53, 64, 34, 75, 66, 60, 21, 45, 57, 15, 41

2B. 40 जना परीक्षार्थीहरूले अङ्ग्रेजीमा प्राप्त गरेका अङ्कहरू तल दिइएका छन् । उचित वर्गान्तर लिएर एउटा बारम्बारता तालिका बनाई चतुर्थांशीय विचलन पत्ता लगाउनुहोस् :
The following are the marks obtained by 40 examinees in English. Construct a frequency table by taking a suitable class interval and calculate the quartiles deviation.
30, 90, 54, 57, 64, 58, 78, 28, 44, 69, 83, 88, 17, 70, 93, 33, 59, 55, 20, 46, 23, 91, 51, 63, 18, 53, 38, 69, 27, 65, 61, 73, 85, 41, 87, 95, 67, 75, 15, 40

Model 2 Problems based on finding Q.D. and its coefficient in continuous data.

3. चतुर्थांशीय विचलन (अर्ध चतुर्थांशीय विस्तार) र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find Q.D. (semi inter-quartile range) and its coefficient :

A.

Weight (kg)	10-20	20-30	30-40	40-50	50-60	60-70
No. of men	12	19	5	10	9	6

B.

Mark obtained	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	15	10	8	6	2

C.

Marks	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	10	5	3	5	2	2

D.

C.I.	20-30	30-40	40-50	50-60	60-70
f	8	16	4	4	3

E.

Height	60-62	62-64	64-66	66-68	68-70	70-72
No. of students	4	6	8	12	7	2

F.

Age (in yrs)	0-5	5-10	10-15	15-20	20-25	25-30
Number	23	22	17	13	13	12

Model 3 Problems based on finding Q_1 and Q_3 in continuous data.

4. Q_1 र Q_3 पत्ता लगाउनुहोस् (Find Q_1 and Q_3):

A.

Marks	< 5	< 10	< 15	< 20	< 25	< 30
C.F.	2	4	7	13	27	40

B.

Marks	$5 \le x < 15$	$15 \le x < 25$	$25 \le x < 35$	$35 \le x < 45$
No. of students	3	5	4	2

Model 4 Problems based on correction factor.

5. तलका तथ्याङ्कहरूको चतुर्थांशीय विचलन पत्ता लगाउनुहोस् (Find the quartile deviation of the following data):

A.

Class	1 - 4	6 - 9	11 - 14	16 - 19	21 - 24
Frequency	4	8	10	6	4

B.

Class	41 - 49	51 - 59	61 - 69	71 - 79	81 - 89
Frequency	5	12	20	10	3

ANSWERS

1A. 8.33 **1B.** 10 **2A.** 16 **2B.** 17.5 **3A.** 14.02, 0.39 **3B.** 10.64, 0.236
3C. 28.87, 0.44 **3D.** 7.39, 0.195 **3E.** 1.0225, 0.015 **3F.** 7.275, 0.57 **4A.** 17.5, 26.15 **4B.** 16, 31.25
5A. 4.58 **5B.** 7.125

www.drmathnp.com

puspas.com.np

2. मध्यक भिन्नता र यसको गुणाङ्क
MEAN DEVIATION AND ITS COEFFICIENT

1. मध्यक भिन्नता (Mean Deviation)

कुनै सङ्ख्यात्मक तथ्याङ्कमा यसको प्रत्येक मानको मध्यक वा मध्यिका वा रीतबाट आउने भिन्नताहरूको मध्यकलाई मध्यक भिन्नता भनिन्छ ।

अर्को शब्दमा, मध्यक र मध्यिका जस्ता केन्द्रीय प्रवृतिका नापहरूबाट प्रत्येक पदको अन्तरको निरपेक्ष मानको औसतलाई मध्यक भिन्नता भनिन्छ । मध्यक भिन्नतालाई औसत भिन्नता पनि भनिन्छ ।

The mean of the deviation of the item from the mean or median or mode of a numerical data is known as mean deviation.

In other words, mean deviation is the average of the absolute deviation taken from central value (mean, median or mode). Mean deviation is also called average deviation.

2. मध्यक भिन्नताको गुणाङ्क (Coefficient of Mean Deviation)

मध्यक भिन्नतामा आधारित विचरणशीलताको तुलनात्मक मापन नै मध्यक भिन्नताको गुणाङ्क हो ।

The relative measure of dispersion based on mean deviation is called coefficient of mean deviation.

3. मध्यक भिन्नताका गुणहरू र दोषहरू (Merits and demerits of Mean Deviation)

गुणहरू (Merits)	दोषहरू (Demerits)
1. यो बुझ्नको लागि सामान्य र गणना गर्न सजिलो हुन्छ । It is simple to understand and easy to compute.	1. यदि औषत भिन्न या दशमलवमा आय गणना गर्न गाह्रो हुन्छ । If average is in fraction or in decimal then it is difficult to calculate.
2. यो तथ्याङ्कको हरेक मानसँग आधारित हुन्छ । It is based on each and every data.	2. विचलनको गणना गर्दा चिन्हलाई ख्याल गरिदैंन । Signs are ignored while taking the deviation.
3. यो मध्यिकाबाट गणना गरियो भने अधिकतम मानले कम असर गर्छ । If calculated from median, it is less effected by extreme term.	3. यसले धेरै वास्तविक नतिजा दिदैंन । This method will not give us very accurate result.

FORMULAE AND KEY POINTS

अविच्छिन्न श्रेणीमा मध्यक भिन्नता (Mean Deviation in Continuous Data)

1. मध्यकबाट मध्यक भिन्नता (Mean deviation from mean) : $\text{M.D.} = \frac{\Sigma f|m - \bar{X}|}{N}$ where m = mid value
2. मध्यिकाबाट मध्यक भिन्नता (Mean deviation from median) : $\text{M.D.} = \frac{\Sigma f|m - \text{Med}|}{N}$
3. MD को गुणाङ्क (Coefficient of MD) $= \frac{\text{MD}}{\text{मध्यक (Mean)}}$ OR $\frac{\text{MD}}{\text{मध्यिका (Median)}}$

SEE / SLC REVIEW MESSAGE

LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2072 R, SEE 2074 R, SEE 2074 S, SEE 2074 S	4
Model 2	2065 R', 2066 R, 2066 S, SEE MODEL 2076	4

SEE EXERCISE 2

A. SHORT QUESTIONS

Model 1 *Definitions and formulae recall.*

1. तलका पदहरूको परिभाषा दिनुहोस् र सो निकाल्ने सूत्र पनि लेख्नुहोस् ।
Define the following terms and also write the formula to calculate them.

A. मध्यक भिन्नता (Mean deviation) **B.** मध्यिका भिन्नता (Median deviation)

C. MD को गुणाङ्क (Coefficient of MD)

2A. मध्यक भिन्नताको एक गुण र एक दोष लेख्नुहोस् । (Write a merit and a demerit of mean deviation.)

2B. मध्यक भिन्नताको गणना गर्दा कुन कुन केन्द्रिय प्रवृत्तीका मानहरू प्रयोग गर्न सकिन्छ ? कुन विधि बढी उपयुक्त होला ? कारण सहित लेख्नुहोस् ।
Which values of central tendency can be used to calculate mean deviation? Which method is considered as the best? Write with reason.

www.drmathnp.com

puspas.com.np

Model 2 *Problems based on finding M.D. and its coefficient from mean.*

3A. एउटा निरन्तर श्रेणीमा $\sum fm = 120$, N = 10 र $\sum f|\ m - \bar{x}|\ = 34$ छ । सो तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
A continuous series has $\sum fm = 120$, N = 10 and $\sum f|\ m - \bar{x}|\ = 34$. Find mean deviation and its coefficient from mean.

3B. एउटा निरन्तर श्रेणीमा $\sum fm = 540$, N = 5 र $\sum f|\ m - \bar{x}|\ = 52$ छ । सो तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
A continuous series has $\sum fm = 540$, N = 5 and $\sum f|\ m - \bar{x}|\ = 52$. Find mean deviation and its coefficient from mean.

3C. एउटा निरन्तर श्रेणीमा $\sum fm = 441$, N = 9 र $\sum f|\ m - \bar{x}|\ = 44$ छ । सो तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
A continuous series has $\sum fm = 441$, N = 9 and $\sum f|\ m - \bar{x}|\ = 44$. Find mean deviation and its coefficient from mean.

3D. एउटा निरन्तर श्रेणीमा $\sum fm = 192$, N = 6 र $\sum f|\ m - \bar{x}|\ = 32$ छ । सो तथ्याङ्कको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
A continuous series has $\sum fm = 192$, N = 6 and $\sum f|\ m - \bar{x}|\ = 32$. Find mean deviation and its coefficient from mean.

4A. एउटा निरन्तर श्रेणीमा $\sum fm = 2400$, N = 100 र $\sum f(m - \bar{x}) = 448$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\sum fm = 2400$, N = 100 and $\sum f(m - \bar{x}) = 448$ then find mean deviation from mean and its coefficient.

4B. एउटा निरन्तर श्रेणीमा $\sum fm = 1950$, N = 50 र $\sum f(m - \bar{x}) = 546$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\sum fm = 1950$, N = 50 and $\sum f(m - \bar{x}) = 546$ then find mean deviation from mean and its coefficient.

4C. एउटा निरन्तर श्रेणीमा $\sum fm = 1000$, N = 50 र $\sum f(m - \bar{x}) = 208$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\sum fm = 1000$, N = 50 and $\sum f(m - \bar{x}) = 208$ then find mean deviation from mean and its coefficient.

4D. एउटा निरन्तर श्रेणीमा $\sum fm = 1200$, N = 100 र $\sum f(m - \bar{x}) = 224$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\sum fm = 1200$, N = 100 and $\sum f(m - \bar{x}) = 224$ then find mean deviation from mean and its coefficient.

Model 3 *Problems based on finding M.D. and its coefficient from median.*

5A. एउटा निरन्तर श्रेणीमा मध्यिका 50, N = 8 र $\sum f(m - md) = 59$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 50, N = 8 and $\sum f(m - md) = 59$. Find the mean deviation from median and its coefficient.

5B. एउटा निरन्तर श्रेणीमा मध्यिका 50, N = 9 र $\sum f(m - md) = 43$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 50, N = 9 and $\sum f(m - md) = 43$. Find the mean deviation from median and its coefficient.

5C. एउटा निरन्तर श्रेणीमा मध्यिका 45, N = 7 र $\sum f(m - md) = 105$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 45, N = 7 and $\sum f(m - md) = 105$. Find the mean deviation from median and its coefficient.

5D. एउटा निरन्तर श्रेणीमा मध्यिका 125, N = 5 र $\sum f(m - md) = 65$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 125, N = 5 and $\sum f(m - md) = 65$. Find the mean deviation from median and its coefficient.

www.drmathnp.com

puspas.com.np

6A. एउटा निरन्तर श्रेणीमा मध्यिका 12, N = 100 र $\sum f(m - md) = 224$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 12, N = 100 and $\sum f(m - md) = 224$. Find mean deviation from median and its coefficient.

6B. एउटा निरन्तर श्रेणीमा मध्यिका 20, N = 50 र $\sum f(m - md) = 208$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 20, N = 50 and $\sum f(m - md) = 208$. Find mean deviation from median and its coefficient.

6C. एउटा निरन्तर श्रेणीमा मध्यिका 40, N = 50 र $\sum f(m - md) = 530$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 40, N = 50 and $\sum f(m - md) = 530$. Find mean deviation from median and its coefficient.

6D. एउटा निरन्तर श्रेणीमा मध्यिका 24, N = 100 र $\sum f(m - md) = 448$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, median = 24, N = 100 and $\sum f(m - md) = 448$. Find mean deviation from median and its coefficient.

B. LONG QUESTIONS

Model 1 **Problems based on finding mean deviation of continuous series (from median).**

1. तलको तथ्याङ्कअनुसार मध्यिकाबाट मध्यक भिन्नता निकाल्नुहोस् ।
Find the mean deviation from the median of the following data:

A.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

[SEE 2074 R]

B.

Marks obtained	10-20	20-30	30-40	40-50	50-60
No. of students	5	4	5	4	2

[2072 R]

C.

Age in year	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of People	5	7	8	6	4

[2062R]

D.

Marks	5-15	15-25	25-35	35-45	45-55
No of Students	3	5	4	5	3

E.

X	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	4	5	6

[SEE 2074 S]

F.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	7	5	10	12	6

[SEE 2074 S']

Model 2 **Problems based on finding mean deviation of continuous series (from mean).**

2A. तल दिइएको तथ्याङ्कअनुसार मध्यकबाट मध्यक भिन्नता निकाल्नुहोस् :
Compute the mean deviation from mean of the data given below:

Age (in yrs)	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
No. of boys	12	8	10	6	4

[2060CP]

2B. औसत भिन्नता पत्ता लगाउनुहोस् (Find the average deviation):

CI	5 - 15	5 - 25	5 - 35	5 - 45	5 - 55
f	2	5	11	16	20

[2065 R']

3. तलको आँकडाको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :
From the data given, find the mean deviation from the mean and its coefficient :

A.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	2	3	6	5	4

[2066 R]

B.

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	2	3	6	5	4

[SEE MODEL 2076]

C.

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

[2066 S]

www.drmathnp.com

puspas.com.np

Model 3 *Mean deviation of the data (less than & more than).*

4. तलका तथ्याङ्कहरूको मध्यकबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation of the following data from mean.

A.

Class	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90
Frequency	3	64	196	349	489	540	542

B.

Class	Less than 20	Less than 30	Less than 40	Less than 50	Less than 10
Frequency	6	10	15	20	3

C.

Class	100 - 150	100 -200	100 - 250	100 - 300	100 - 350
Frequency	1	3	6	10	15

D.

Class	2 - 4	2 - 6	2 - 8	2 - 10
Frequency	4	9	11	12

E.

Class	0 - 50	Above 10	Above 20	Above 30	40 - 50
Frequency	20	18	15	10	4

5. तलका तथ्याङ्कहरूको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation of the following data from median.

A.

Height	above 0 cm	above 8 cm	above 16 cm	above 24 cm	above 32 cm	above 40 cm
No. of plants	50	42	35	30	18	6

B.

Class interval	More than 0	More than 10	More than 20	More than 30	More than 40
Frequency	50	45	37	22	6

C.

Class interval	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	5	6	7	10	12

D.

Age	Less than 50	Less than 40	Less than 30	Less than 20	Less than 10
No. of people	20	18	16	7	5

Model 4 *Problems based on frequency table.*

6A. 0 - 4 को एउटा वर्गान्तर लिएर बारम्बारता तालिका बनाउनुहोस् र मध्यक भिन्नता पत्ता लगाउनुहोस् :
Construct a frequency distribution table taking 0 - 4 as a class interval and find mean deviation: 1, 3, 2, 3, 4, 5, 6, 7, 5, 7, 10, 9, 15

6B. पहिलो वर्गान्तर 2 - 4 बनाई तलको तथ्याङ्कको मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्:
Taking 2 - 4 as a first class interval, find the mean deviation of the following data and its coefficient:
4, 2, 3, 9, 7, 6, 4, 2, 3, 5, 4, 5

6C. वर्गान्तर 10 बनाई तलको तथ्याङ्कको मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्:
Taking 10 as a class interval size, find the mean deviation of the following data and its coefficient:
59, 71, 45, 44, 35, 21, 29, 42, 37, 49, 58, 69, 55, 39, 79, 50, 65, 52, 60, 64

ANSWERS

A. SHORT QUESTIONS

3A. 3.4, 0.283 **3B.** 10.4, 0.096 **3C.** 4.89, 0.099 **3D.** 5.33, 0.17 **4A.** 4.48, 0.187 **4B.** 10.92, 0.28
4C. 4.16, 0.208 **4D.** 2.24, 0.187 **5A.** 7.375, 0.147 **5B.** 4.78, 0.095 **5C.** 15, 0.33 **5D.** 13, 0.104
6A. 2.24, 0.187 **6B.** 4.16, 0.208 **6C.** 10.6, 0.265 **6D.** 4.48, 0. 187

B. LONG QUESTIONS

1A. 9.56 **1B.** 11. 3 **1C.** 10.58 **1D.** 11 **1E.** 11.3 **1F.** 11.05
2A. 4.6 **2B.** 10.3 **3A.** 10.3, 0.37 (approx.) **3B.** 10.3, 0.3678 **3C.** 9·44 & 0.35
4A. 9.50 **4B.** 12 **4C.** 53.33 **4D.** 1.33 **4E.** 10.5 **5A.** 10.854
5B. 9.56 **5C.** 30 **5D.** 9.50 **6A.** 2.46 **6B.** 1.33, 0.266 **6C.** 12.3, 0.23

www.drmathnp.com

puspas.com.np

GENERAL EXERCISE 2

LONG QUESTIONS

Model 1 *Problems based on finding mean deviation of continuous series (from median).*

1. मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् (Find mean deviation from median):

A.

C.I	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	6	5	4

B.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

C.

C.I	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	6	8	11	14	8	3

D.

Height	95-105	105-115	115-125	125-135	135-145	145-155
No. of students	6	8	11	14	8	3

E.

Class	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Frequency	4	6	10	20	10	6	4

Model 2 *Problems based on finding mean deviation of continuous series (from mean).*

2. दिइएको तथ्याङ्कहरूबाट मध्यक भिन्नता या औसत भिन्नता पत्ता लगाउनुहोस् :
Find the M.D. or average deviation from the data given:

A.

C.I.	0-10	10-20	20-30	30-40	40-50	50-60
f	2	4	6	4	1	4

B.

Marks	0-10	10- 20	20-30	30-40	40-50
frequency	2	3	6	5	4

C.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	18	24	20	19	5

D.

Weight	20-25	25-30	30-35	35-40	40-45	45-50
No. of men	7	3	6	4	8	2

E.

Marks	$5 \le x < 10$	$10 \le x < 15$	$15 \le x < 20$	$20 \le x < 25$	$25 \le x < 30$
No. of students	7	4	5	6	3

Model 3 *Problems based on finding mean deviation and its coefficient of continuous series.*

3. तलका प्रत्येक तथ्याङ्कबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्:
From each of the following data, find the mean deviation and its coefficient.

A.

Class-Interval	0-10	0-20	0-30	0-40	0-50
Frequency	2	5	11	16	20

www.drmathnp.com

B.

Class-Interval	2-4	2-6	2-8	2-10
Frequency	4	9	11	12

C.

Class-Interval	1-3	1-5	1-7	1-9
Frequency	3	8	9	10

4. मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् (Find mean deviation and its coefficient) :

A.

CI	0-10	10-20	20-30	30-40	40-50	50-60	60-70
c.f.	4	10	20	40	50	56	60

B.

Class Interval	0-10	10-20	20-30	30-40	40-50
c.f.	5	13	28	44	50

C.

x	0-4	4-8	4-12	0-16	16-20	0-24
f	5	7	17	37	7	50

D.

Class Interval	2-4	2-6	6-8	0-10
Frequency	3	7	2	10

E.

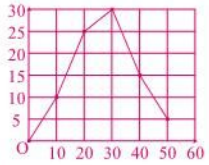
Class Interval	2-8	12-18	22-28	32-38	42-48
Frequency	5	8	15	16	6

F.

Marks	11-19	21-29	31-39	41-49	51-59	61-69
No. of students	6	8	11	14	8	3

5. सँगैको बारम्बारता वक्रले जनाउने जानकारीको मध्यक भिन्नता पत्ता लगाउनुहोस् ।

Find the mean deviation of the information shown by adjoining frequency curve.



Model 4 *Problems based on correction factor.*

6. तलका तथ्याङ्कहरूको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।

Find the mean deviation of the following data from median.

A.

Class	2-8	12-18	22-28	32-38	42-48
Frequency	5	8	15	16	6

B.

Class	22 - 28	32 - 38	42 - 48	52 - 58	62 - 68	72 - 78
Frequency	3	5	5	9	4	4

STATISTICS

ANSWERS

1A. 10.3 **1B.** 9.56 **1C.** 11.8 **1D.** 11.8 **1E.** 113.33 **2A.** 13.06
2B. 10.3 **2C.** 10.80 **2D.** 7.27 **2E.** 6.144 **3A.** 10.3, 0.37 **3B.** 1.33, 0.266
3C. 1.2, 0.3 **4A.** 11.33, 0.324 **4B.** 9.56, 0.34 **4C.** 4.83, 0.38 **4D.** 1.48, 0.28 **4E.** 9.44, 0.349
4F. (From Median) 11.8, 0.295 **5.** 8.65 **6A.** 9.56 **6B** 12.37

puspas.com.np

www.drmathnp.com

puspas.com.np

3. स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क
STANDARD DEVIATION AND THE COEFFICIENT OF VARIATION

1. स्तरीय भिन्नता (Standard Deviation)

मध्यक (mean) बाट लिइएको भिन्नताको वर्गको औसतको वर्गमूल नै स्तरीय भिन्नता हो । यसलाई σ द्वारा जनाइन्छ ।
The square root of the average of squares of the deviation taken from the mean is called the **standard deviation**. It is denoted by σ.

2. स्तरीय भिन्नताको गुणाङ्क (Coefficient of standard deviation)

स्तरीय भिन्नतामा आधारित विचरणशीलताको तुलनात्मक मापन नै स्तरीय भिन्नताको गुणाङ्क हो ।
The relative measure of dispersion based on the standard deviation is known as the coefficient of standard deviation.

3. विचरणशीलता (Variance)

स्तरीय भिन्नताको वर्गलाई नै विचरणशीलता (variance) भनिन्छ । यसलाई σ^2 द्वारा जनाइन्छ ।
The square of standard deviation is known as variance. It is denoted by σ^2.

4. विचरणशीलताको गुणाङ्क (Coefficient of Variation)

स्तरीय भिन्नताको गुणाङ्कलाई 100 ले गुणन गर्दा प्राप्त हुने नतिजालाई विचरणशीलताको गुणाङ्क भनिन्छ ।
If the coefficient of standard deviation is expressed as percentage then it is known as coefficient of variation.

5. स्तरीय भिन्नताका गुणहरू र दोषहरू (Merits and demerits of Standard Deviation)

गुणहरू (Merits)	दोषहरू (Demerits)
1. यो सबै मानहरूमा आधारित हुन्छ । It is based on all the observations.	1. अन्य मापनहरूसँग तुलना गर्दा यो गणना गर्न कठीन हुन्छ । As compared to other measures, it is difficult to compute.
2. यो विचरणशीलताको सर्वोत्कृष्ट मापन हो । It is the best measure of dispersion.	2. सुरूको र अन्तिम खुल्ला वर्गान्तरहरू भएको श्रेणीमा यसको गणना गर्न सकिदैंन । It can not be calculated for open end classes.
3. स्तरीय भिन्नता कठीन रूपमा परिभाषीत गरिएको छ र यसको मान सधैं निश्चित आउँछ । Standard deviation is rigidly defined and its value is always definite.	3. यसले अधिकतम मानलाई धेरै भार दिन्छ । It gives more weightage to extreme values.

FORMULAE AND KEY POINTS

अविच्छिन्न श्रेणीमा स्तरीय भिन्नता (Standard Deviation in Continuous Data)

प्रत्यक्ष बिधि Direct method	बास्तविक मध्यक बिधि Actual mean method	अनुमानित मध्यक बिधि Assumed mean method	पद विचलन बिधि Step deviation method
$\sigma = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma f(m-\bar{X})^2}{N}}$	$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$

मध्यक $(\bar{X}) = \frac{\Sigma fm}{N} = a + \frac{\Sigma fd}{N} = a + \frac{\Sigma fd'}{N} \times h, \quad d = m - a, \quad d' = \frac{m-a}{c}$

Where, $\bar{X}$ = मध्यक (Mean), $d = X -$ अनुमानित मध्यक (assumed mean)
N = बारम्बारता (frequency), m = मध्यमान (mid value), SD = σ

SD को गुणाङ्क (Coefficient of SD) $= \frac{\sigma}{\bar{X}}$ विचरणशीलताको गुणाङ्क (Coefficient of Variation) $= \frac{\sigma}{\bar{X}} \times 100\%$

SHORT CLIPS

1. विचरणशीलताको गुणाङ्क जति सानो छ, त्यतिनै बढी तथ्याङ्क एकरूपता वा स्थिरता भएको मानिन्छ ।
The smaller value of the coefficient of variation indicate the data is consistency or uniform.
2. विचरणशीलताको गुणाङ्क जति ठूलो छ, त्यतिनै बढी तथ्याङ्क एकरूपता नभएको वा स्थिरता नभएको मानिन्छ ।
The greater value of the coefficient of variation indicate the data is not consistency or uniform.
3. विभिन्न तथ्याङ्कहरू बिचको तुलनात्मक अध्ययन गर्दा विचरणशीलताको गुणाङ्क (Coefficient of variation) सानो भएको राम्रो मानिन्छ ।
In the comparative study of various data, the data with the smaller value of coefficient of variation is considered as good or uniform.
4. विचरणशीलताको अधिकतम गुणाङ्क 1 वा 100% हुन्छ ।
The maximum value of coefficient of variation is 1 or 100%.

www.drmathnp.com

puspas.com.np

SEE / SLC REVIEW MESSAGE

A. SHORT QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 5	SEE MODEL 2076	1

B. LONG QUESTIONS

Models	SEE / SLC Examination History (2065 to 2075)	n(Q)
Model 1	2065 M, 2065 S, 2067 S, 2068 R, 2069 S, 2070 R, 2070 S, 2071 R, 2071 S, 2072 R', 2072 S, 2073 R, S, S', SEE 2074 R', SEE 2075 R'	13
Model 2	2069 R', 2071 R'	2
Model 3	SEE 2075 R, SEE 2075 R_2, SEE MODEL 2076	3

SEE EXERCISE 3

A. SHORT QUESTIONS

Model 1 *Problems based on Definitions & Formulae.*

1. तलका पदहरूको परिभाषा दिनुहोस् र सो निकाल्ने एउटा सूत्र पनि लेख्नुहोस् ।
Define the following terms and also write a formula to calculate each.

A. स्तरीय भिन्नता (Standard deviation) **B.** स्तरीय भिन्नताको गुणाङ्क (Coefficient of standard deviation)

C. विचरणशीलताको गुणाङ्क (Coefficient of variation)

2A. स्तरीय भिन्नताको एक गुण र एक दोष लेख्नुहोस् । (Write a merit and a demerit of standard deviation.)

2B. कुन कुन विधिबाट स्तरीय भिन्नता निकाल्न सकिन्छ ? सूत्रहरू पनि लेख्नुहोस् ।
By which methods, the standard deviation can be calculated? Write formulae also.

2C. स्तरीय भिन्नताको गुणाङ्क र विचरणशीलताको गुणाङ्क बिच फरक लेख्नुहोस् ।
Write the difference between the coefficient of standard deviation and coefficient of variation.

Model 2 *Problems based on SD by direct method.*

3A. एउटा निरन्तर श्रेणीमा $\Sigma fm = 1775$, $\Sigma fm^2 = 639375$ र N = 5 भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fm = 1775$, $\Sigma fm^2 = 639375$ and N = 5 then find standard deviation and its coefficient.

3B. एउटा निरन्तर श्रेणीमा $\Sigma fm = 126$, $\Sigma fm^2 = 2380$ र N = 7 भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fm = 126$, $\Sigma fm^2 = 2380$ and N = 7 then find standard deviation and its coefficient.

4A. एउटा निरन्तर श्रेणीमा N = 47, $\Sigma fm = 770$ र $\Sigma fm^2 = 16450$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, N = 47, $\Sigma fm = 770$ and $\Sigma fm^2 = 16450$ then find the standard deviation and its coefficient.

4B. एउटा निरन्तर श्रेणीमा N = 48, $\Sigma fm = 576$ र $\Sigma fm^2 = 6960$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, N = 48, $\Sigma fm = 576$ and $\Sigma fm^2 = 6960$ then find the standard deviation and its coefficient.

Model 3 *Problems based on SD by assumed mean method.*

5A. एउटा निरन्तर श्रेणीमा $\Sigma fd = -14$, $\Sigma fd^2 = 460$, $d = m - A$, A = 46 र N = 7 भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fd = -14$, $\Sigma fd^2 = 460$, $d = m - A$, A = 46 and N = 7 then find standard deviation and its coefficient.

5B. एउटा निरन्तर श्रेणीमा $\Sigma fd = -6$, $\Sigma fd^2 = 238$, $d = m - A$, A = 20 र N = 6 भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fd = -6$, $\Sigma fd^2 = 238$, $d = m - A$, A = 20 and N = 6 then find standard deviation and its coefficient.

6A. एउटा निरन्तर श्रेणीमा $\Sigma fd = 0$, $\Sigma fd^2 = 848$, N = 100, $d = m - A$ र A = 12 भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fd = 0$, $\Sigma fd^2 = 848$, N = 100, $d = m - A$ and A = 12 then find the standard deviation and its coefficient.

6B. एउटा निरन्तर श्रेणीमा $\Sigma fd = -20$, $\Sigma fd^2 = 5400$, N = 35, $d = m - A$ र A = 40 भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fd = -20$, $\Sigma fd^2 = 5400$, N = 35, $d = m - A$ and A = 40 then find the standard deviation and its coefficient.

Model 4 *Problems based on SD by actual mean method.*

7A. एउटा निरन्तर श्रेणीमा $\bar{x} = 7$, $\Sigma fd = 35$, $\Sigma fd^2 = 10$ र $d = m - \bar{x}$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\bar{x} = 7$, $\Sigma fd = 35$, $\Sigma fd^2 = 10$ and $d = m - \bar{x}$ then find standard deviation and its coefficient.

www.drmathnp.com

7B. एउटा निरन्तर श्रेणीमा $\bar{x} = 18, \Sigma fd^2 = 112, \Sigma fd = 126$ र $d = m - \bar{x}$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\bar{x} = 18, \Sigma fd^2 = 112, \Sigma fd = 126$ and $d = m - \bar{x}$ then find standard deviation and its coefficient.

8A. एउटा निरन्तर श्रेणीमा $N = 30, \Sigma fm = 870$ र $\Sigma f(m-\bar{x})^2 = 1070$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $N = 30, \Sigma fm = 870$ and $\Sigma f(m-\bar{x})^2 = 1070$ then find the standard deviation and its coefficient.

8B. एउटा निरन्तर श्रेणीमा $N = 48, \Sigma fm = 576$ र $\Sigma f(m-\bar{x})^2 = 48$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $N = 48, \Sigma fm = 576$ and $\Sigma f(m-\bar{x})^2 = 48$ then find the standard deviation and its coefficient.

9A. एउटा निरन्तर श्रेणीमा $\Sigma fm = 114, \bar{x} = 19$ र $\Sigma f(m-\bar{x})^2 = 232$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fm = 114, \bar{x} = 19$ and $\Sigma f(m-\bar{x})^2 = 232$ then find standard deviation and its coefficient.

9B. एउटा निरन्तर श्रेणीमा $\Sigma fm = 308, \bar{x} = 44$ र $\Sigma f(m-\bar{x})^2 = 432$ भए स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $\Sigma fm = 308, \bar{x} = 44$ and $\Sigma f(m-\bar{x})^2 = 432$ then find standard deviation and its coefficient.

Model 5 Problems based on finding variance and the coefficient of variation .

10A. कुनै तथ्याङ्कको स्तरीय भिन्नता 0.25 भए विचरणशीलता कति हुन्छ ? पत्ता लगाउनुहोस् ।
If the standard deviation of a set of data is 0.25, find its variance. **[SEE MODEL 2076]**

10B. कुनै तथ्याङ्कको स्तरीय भिन्नता 5.5 र मध्यक 20 भए विचरणशीलता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
If the standard deviation of a data is 5.5 and the mean is 20, find its variance and the coefficient of variation.

10C. एउटा निरन्तर श्रेणीमा $N = 10, \Sigma fm = 40, \Sigma fm^2 = 192$ र $\bar{x} = 4$ भए सो श्रेणीको विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $N = 10, \Sigma fm = 40, \Sigma fm^2 = 192$ and $\bar{x} = 4$ then find the coefficient of variation.

10D. एउटा निरन्तर श्रेणीमा $N = 20, \Sigma fm = 570, \Sigma fm^2 = 19100$ र $\bar{x} = 28.50$ भए श्रेणीको विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series, $N = 20, \Sigma fm = 570, \Sigma fm^2 = 19100$ and $\bar{x} = 28.50$ then find the coefficient of variation.

Model 6 Problems based on SD by step deviation method.

11A. एउटा अविच्छिन्न श्रेणीमा $N = 28, \Sigma fd' = -2, \Sigma fd^2 = 36$ र $10d' = m - a$ भए स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
In a continuous data, $N = 28, \Sigma fd' = -2, \Sigma fd^2 = 36$ and $10d' = m - a$ then find the standard deviation.

11B. एउटा निरन्तर श्रेणीमा $N = 50, \Sigma fd' = -20, \Sigma fd'^2 = 92$ र $h = 10$ भए सो श्रेणीको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
In a continuous data, $N = 50, \Sigma fd' = -20, \Sigma fd'^2 = 92$ and $h = 10$ then find the standard deviation of the data.

11C. एउटा निरन्तर श्रेणीमा $N = 75, \Sigma fd'^2 = 127, h = 10$ र स्तरीय भिन्नता 12.5 भए $\Sigma fd'$ पत्ता लगाउनुहोस् ।
In a continuous data, $N = 75, \Sigma fd'^2 = 127, h = 10$ and standard deviation = 12.5 then find $\Sigma fd'$.

11D. एउटा निरन्तर श्रेणीमा $N = 20, \Sigma fd' = -4, h = 10$ र स्तरीय भिन्नता 16 भए $\Sigma fd'^2$ को मान पत्ता लगाउनुहोस् ।
In a continuous data, $N = 20, \Sigma fd' = -4, h = 10$ and standard deviation 16 then find $\Sigma fd'^2$.

Model 7 Miscellaneous Problems.

12. दिइएको जानकारीबाट S.D. र यसको गुणाङ्क पत्ता लगाउनुहोस् (From the given information, find S.D. and its coefficient):

A. $N = 7, \Sigma fd = 28, \Sigma fd^2 = 224, d = m - A$ & $A = 10$ **B.** $N = 100, \Sigma fd = 0, \Sigma fd^2 = 848, d = m - A$ & $A = 6.92$

C. $N = 48, \Sigma fm = 576, \Sigma fm^2 = 6960$ **D.** $N = 100, \Sigma fm = 3100, \Sigma fm^2 = 112900$

B. LONG QUESTIONS

Model 1 Problems based on finding standard deviation of continuous series.

1. तलको आँकडाबाट स्तरीय भिन्नता निकाल्नुहोस् (Find the standard deviation of the following data):

A.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

[SEE 2075 R', SEE 2073 S']

B.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	6	5	4	3

[2072 S]

C.

Marks obtained	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	5	4

[2072 R']

D.

Class interval	25-35	35-45	45-55	55-65	65-75
Frequency	5	4	6	7	3

[2071 S]

E.

Class interval	5-15	15-25	25-35	35-45	45-55
Frequency	7	3	6	4	5

[2071 R]

F.

Class interval	0-4	4-8	8-12	12-16	16-20	20-24
Frequency	7	7	10	15	7	6

[2070 S]

puspas.com.np

www.drmathnp.com

puspas.com.np

G.

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	5	3	2

[2068 R]

H.

CI	0-10	10-20	20-30	30-40	40-50
f	8	10	12	8	2

[2067 S]

I.

Daily Sales	0-10	10-20	20-30	30-40	40-50
Numbers of Shops	2	9	10	7	1

[2059 R, SEE 2073S, SEE 2074 R']

J.

Weight (in kg.)	10-20	20-30	30-40	40-50	50-60	60-70
No. of boys	2	5	6	3	2	2

[2058 S]

K.

Marks Secured	0-20	20-40	40-60	60-80	80-100
No of students	2	8	16	10	4

[2058 R]

L.

Marks secured	10-20	20-30	30-40	40-50	50-60
No. of students	8	12	15	9	6

[2057 R]

M.

Daily sales	10-20	20-30	30-40	40-50	50-60
No. of shops	4	10	12	8	6

[2061 R,69S. SEE 2073 R]

N.

Marks	10-20	20-30	30-40	40-50	50-60
Number of students	3	5	8	12	2

[2061 S]

O.

Height in cm	0-8	8-16	16-24	24-32	32-40
Number of Plants	6	7	10	8	9

[2062R]

P.

Marks Obtained	0-10	10-20	20-30	30-40	40-50
No. of Students	16	12	18	3	1

[2060CP]

Q.

x	0-10	10-20	20-30	30-40	40-50
f	5	4	4	6	1

[2065 M]

R.

Marks obtained)	0-10	10-20	20-30	30-40	40-50
No. of students	1	4	7	3	5

[2065 S]

S.

Class interval	0-4	4-8	8-12	12-16	16-20
Frequency	15	12	10	8	5

[2070 R]

Model 2 *Problems based on finding standard deviation and its coefficient of continuous series.*

2A. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र त्यसको गुणाङ्क पत्ता लगाउनुहोस्:
Find the standard deviation and its coefficient from the given data: [2069 R']

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	15	25	35	45

2B. तलको तथ्याङ्कबाट स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and its coefficient from the following data. [2071 R']

Marks Obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	12	20	40	12	8

3. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र त्यसको गुणाङ्क पत्ता लगाउनुहोस्:
Find the standard deviation and its coefficient from the given data:

A.

Age (yrs.)	10-15	15-20	20-25	25-30	30-35	35-40
No. of men	16	23	28	18	10	5

B.

Daily wage (Rs)	100-125	125-150	150-175	175-200	200-225
No. of workers	75	57	81	19	12

C.

Class interval	0-4	4-8	8-12	12-16	16-20	20-24
Frequency	7	7	10	15	7	6

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on finding the coefficient of variation.*

4. तल दिइएको तथ्याङ्कबाट विचरणशीलताको गुणाङ्क गणना गर्नुहोस् ।
Calculate the coefficient of variation from the data given below.

A.

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

[SEE 2075 R]

B.

Class-interval	0-20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	2	3	4	5	6

[SEE 2075 R_2]

C.

Marks obtained (x)	30-40	40-50	50-60	60-70	70-80	80-90
No. of students (f)	4	8	10	16	6	6

D.

Expenditure (Rs)	60-63	63-66	66-69	69-72	72-75
Civil Servants	5	18	42	27	8

5. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and coefficient of variation from given data.

A.

Age	0-4	4-8	8-12	12-16	16-20	20-24
No. of students	7	7	10	15	7	6

[SEE MODEL 2076]

B.

Age	60-65	65-70	70-75	75-80	80-85
No. of people	7	6	6	4	3

C.

Class	0-10	10-20	20-30	30-40	40-50	50-60
No. of people	4	6	10	20	6	4

ANSWERS

A. SHORT QUESTIONS

3A. 43.01, 0.12 **3B.** 4, 0.22 **4A.** 9.03, 0.55 **4B.** 1, 0.083 **5A.** 7.8, 0.17 **5B.** 6.22, 0.327
6A. 2.91 and 0.24 **6B.** 12.41 and 0.31 **7A.** 1.41 and 0.2014 **7B.** 4, 0.22
8A. 5.97 and 0.206 **8B.** 1, 0.083 **9A.** 6.22, 0.33 **9B.** 7.8, 0.18 **10A.** 0.0625
10B. 30.25, 27.50% **10C.** 44.72% **10D.** 41.93% **11A.** 11.32 **11B.** 12.96
11C. 27.12 **11D.** 52 **12A.** 4, 0.28 **12B.** 2.91, 0.42 **12C.** 1, 0.08 **12D.** 12.96, 0.42

B. LONG QUESTIONS

1A. 11.48 **1B.** 12.25 **1C.** 12.29 **1D.** 13.11 **1E.** 14.78 **1F.** 6.05 **1G.** 12.35
1H. 11.52 **1I.** 9.73 **1J.** 14.35 **1K.** 20.27 **1L.** 12. 33 **1M.** 12.03 **1N.** 10.98
1O. 10.86 **1P.** 10.25 **1Q.** 12.68 **1R.** 11.94 **1S.** 5.32 **2A.** 11.66, 0.35
2B. 12.96, 0.42 **3A.** 6.89, 0.30 **3B.** 28.35, 0.194 **3C.** 6.05, 0.50 **4A.** 42.55% **4B.** 44.1%
4C. 23.14% **4D.** 4.29% **5A.** 6.05, 50.42% **5B.** 6.765, 20.38% **5C.** 12.96, 41%

GENERAL EXERCISE 3

LONG QUESTIONS

Model 1 *Problems based on SD by direct method.*

1. तलका तथ्याङ्कहरूको प्रत्यक्ष विधिबाट स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
Find the standard deviation of the following data by direct method.

A.

Class interval	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency	2	6	8	12	2

B.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	4	6	10	20	6	1

Model 2 *Problems based on SD by actual mean method.*

2. वास्तविक मध्यकको प्रयोग गरी स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation using actual mean):

A.

X	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
f	4	6	10	20	6	4

B.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	8	12	20	40	12	8

www.drmathnp.com

puspas.com.np

Model 3 *Problems based on SD by assumed mean method.*

3. कल्पित मध्यकको प्रयोग गरी स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation using assumed mean):

A.

Wage (in Rs.)	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
No. of workers	5	18	42	27	8

B.

Class interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	2	3	6	5	4

C.

Weight	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of students	3	7	10	5	3

Model 4 *Problems based on SD by step deviation method.*

4. तलका तथ्याङ्कहरूको पद विचलन विधिबाट स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and its coefficient of the following data by step deviation method.

A.

Class	1 - 3	3 - 5	5 - 7	7 - 9
Frequency	3	5	1	1

B.

Class	0 - 4	4 - 8	8 - 12	12 - 16
Frequency	4	8	2	1

C.

Marks	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	4	6	10	17	11	9	3

Model 5 *Problems based on combined class interval.*

5. स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation):

A.

X	0 - 10	0 - 20	0 - 30	0 - 40	0 - 50
f	2	5	10	13	15

B.

X	0 - 10	0 - 20	0 - 30	0 - 40	0 - 50
f	5	13	28	44	50

C.

Marks	20 - 30	20 - 40	30 - 50	40 - 60	60 - 70
Frequency	4	10	14	12	8

D.

Marks obtained	0 - 10	0 - 20	20 - 30	10 - 40	30 - 50
No. of students	5	9	4	14	7

Model 6 *Problems based on discontinuous data.*

6. तलका श्रेणीहरूको स्तरीय भिन्नता र सो को गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and its coefficient of the following data.

A.

Class interval	25-29	30-34	35-39	40-44	45-49	50-54
Frequency	2	3	4	7	2	1

B.

Class	32-38	42-48	52-58	62-68	72-78
Frequency	2	3	6	5	4

C.

Weight	12-18	22-28	32-38	42-48	52-58
No. of students	3	7	10	5	3

Model 7 *Problems based on constructing frequency table.*

7. अविच्छिन्न बारम्बारता तालिका बनाई स्तरीय भिन्नता पत्ता लगाउनुहोस्:
Construct a continuous frequency table and find the standard deviation:

A. 1, 3, 2, 3, 4, 5, 6, 7, 6, 7, 5, 7, 10, 9, 15, (one class interval is 4-8)

B. 2, 21, 11, 4, 5, 3, 24, 26, 22, 5, 6, 26, 9, 39, 41, 34, 35, 14, 20, 45 (First interval is 0-10)

www.drmathnp.com

8. 0-10 को एउटा वर्गान्तर लिएर अविच्छिन्न बारम्बारता तालिका बनाई स्तरीय भिन्नता पत्ता लगाउनुहोस्:
Prepare a continuous frequency table and find the standard deviation taking 0-10 as a class interval:

A. 2, 21, 5, 4, 11, 3, 26, 24, 22, 5, 6, 26, 9, 39, 41, 34, 35, 14, 20, 45

B. 7, 13, 34, 25, 20, 3, 27, 19, 40, 21, 36, 48, 35, 12, 24

9. 5 को वर्गान्तर तालिका बनाइ स्तरीय भिन्नता, यसको गुणाङ्क र विचरणशीलताको गुणाङ्क समेत पत्ता लगाउनुहोस् ।
Taking class interval of size 5, find the standard deviation, coefficient of standard deviation and also the coefficient of variation.
10, 11, 18, 20, 18, 18, 17, 16, 14, 12, 10, 22, 24, 28, 23, 14, 16, 20, 23, 26, 28, 29, 9, 4, 8, 12, 5, 9, 8, 10,

Model 8 *Problems based on finding variance and the coefficient of variation in continuous series.*

10. तलका तथ्याङ्कहरूबाट विचरणशीलता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस्:
From the following data, find the variance and the coefficient of variation:

A.

Class-interval	1-3	3-5	5-7	7-9
Frequency	3	5	1	1

B.

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	6	8	5	3

C.

Class-interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	4	6	10	20	6	4

11. तलका तथ्याङ्कहरूबाट विचरणशीलता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस्:
From the following data, find the variance and the coefficient of variation:

A.

Wage (in Rs.)	60-62	63-65	66-68	69-71	72 - 74
No. of workers	5	18	42	27	8

B.

Marks	5-15	25-35	45-55	65-75	85-95
Frequency	2	8	16	10	4

C.

Marks obtained	0-10	0-20	0-30	0-40	0-50
No. of students	1	5	12	15	20

Model 9 *Problems based on mid value and frequencies.*

12. स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation):

A.

mid value	10	20	30	40	50
f	8	12	15	9	6

B.

mid value	40	45	50	55	60
Frequency	2	2	5	3	2

13. तलको तथ्याङ्कहरूबाट स्तरीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्:
From the following data, find the standard deviation and its coefficient:

A.

mid-age	10	20	30	40	50	60
No. of men	1	5	10	12	8	4

B.

mid-marks	5	15	25	35	45	55
Frequency	4	6	10	20	6	4

14. वास्तविक मध्यकको प्रयोग गरी स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation using actual mean.):

A.

mid value	10	20	30	40	50	60
f	2	10	20	24	16	8

B.

mid value	10	20	30	40	50
f	3	8	7	7	5

puspas.com.np

www.drmathnp.com

puspas.com.np

ANSWERS

LONG QUESTIONS

1A. 10.46 **1B.** 11. 81 **2A.** 12.96 **2B.** 12.96 **3A.** 2.92 **3B.** 12.29
3C. 11.32 **4A.** 1.79, 0.4475 **4B.** 3.27, 0.545 **4C.** 15.48, 0.278 **5A.** 12.1 **5B.** 11.48
5C. 13.63 **5D.** 12.68 **6A.** 6.53, 0.168 **6B.** 12.29, 0.21 **6C.** 11.32, 0.33 **7A.** 3.27
7B. 13.58 **8A.** 13.58 **8B.** 12.11 **9.** 6.98, 0.43, 43.47% **10A.** 3.2, 44.75%
10B. 139.84, 48.09% **10C.** 168, 41.80% **11A.** 8.52, 4.33% **11B.** 410.87, 38.24% **11C.** 142.75, 0.42%
12A. 12.33 **12B.** 6.11 **13A.** 12.42, 0.324 **13B.** 12.96, 0.42 **14A.** 12.42 **14B.** 12.47

CHALLENGING EXERCISE ON STATISTICS

1. तपाईंहरूलाई मध्यमान र बारम्बारता दिइएको छ । वर्गान्तर, चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :
You are given mid value and frequency. Find class interval, quartile deviation and its coefficient :

Age (yrs.)	10	30	50	70	90
No. of students	12	20	25	18	5

2. तपाईंहरूलाई मध्यमान र बारम्बारता दिइएको छ । वर्गान्तर, चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :
You are given mid value and frequency. Find class interval, quartile deviation and its coefficient :

Marks	5	15	25	35	45	55	65	75	85
No. of students	11	18	25	28	30	33	22	15	22

3. दिइएको तथ्याङ्कको चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find the quartile deviation and its coefficient of the following data:

Marks	20-29	30-39	40-49	50-59	60-69	70-79
No. of students	306	182	144	96	42	34

4. चतुर्थांशहरूमा आधारित विचरणशीलताको मापनहरूको प्रयोग गरी वितरणहरू A र B मध्य कुन धेरै फैलिएका छन्, निकाल्नुहोस् ।
Using measures of dispersion based on quartiles, compare the following distributions A and B and state which distribution is more variable.

X	Distribution	
	A	B
5 - 15	2	2
15 - 25	5	4
25 - 35	10	9
35 - 45	3	5

मध्यक भिन्नता पत्ता लगाउनुहोस् (Find the mean deviation):

5.

C. I.	0 - 50	0 - 40	0 - 30	0 - 20	0 - 10
f	50	40	35	15	5

6.

C. I.	1 - 9	11 - 19	21 - 29	31 - 39	41 - 49	51 - 59
f	2	4	6	4	1	4

दिइएको तथ्याङ्कहरूको स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation of given data):

7.

C. I.	5 - 55	5 - 45	5 - 35	5 - 25	5 - 15
f	30	28	16	8	2

8.

C. I.	33 - 35	36 - 38	39 - 41	42 - 44	45 - 47
f	17	19	23	21	27

9.

Wages up to	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

www.drmathnp.com

10. यदि दिइएको तालिकाको मध्यिका 24 भए मध्यक भिन्नता कति होला ?
If the median of following data is 24, what is the mean deviation ?

C. I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	9	21	x	15	10

11. एउटा निरन्तर तथ्याङ्कको स्तरीय भिन्नता 15 छ । यदि विचरणशीलताको गुणाङ्क 75% भए स्तरीय भिन्नताको गुणाङ्क र मध्यक पत्ता लगाउनुहोस् ।
The standard deviation of a continuous data is 15. If the coefficient of variation of the data is 75%, find the coefficient of standard deviation and the mean.

12. एउटा निरन्तर तथ्याङ्कको स्तरीय भिन्नता 18 छ । यदि विचरणशीलताको गुणाङ्क 90% भए स्तरीय भिन्नताको गुणाङ्क र मध्यक पत्ता लगाउनुहोस् ।
The standard deviation of a continuous data is 18. If the coefficient of variation is 90% then find the coefficient of standard deviation and mean.

13. एउटा निरन्तर श्रेणीमा 200 जना विद्यार्थीहरूको प्राप्ताङ्कको मध्यक र स्तरीय भिन्नता क्रमशः 40 र 15 छन् । सबै प्राप्ताङ्कहरूको वर्गको योगफल पत्ता लगाउनुहोस् ।
In a continuous data, the mean and the standard deviation of the marks obtained by 200 students are 40 and 15 respectively. Find the sum of the square of all the marks.

Hint: Here, $N = 200$, $\overline{X} = 40$ and $SD = 15$

Now, $\overline{X} = \frac{\sum fm}{N}$ or, $40 = \frac{\sum fm}{200}$ $\therefore \sum fm = 8000$

Again, $SD = \sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$ or, $15^2 = \frac{\sum fm^2}{200} - \left(\frac{8000}{200}\right)^2$

Simplify it then $\sum fm^2 = 365000$.

14. एउटा निरन्तर श्रेणीमा 20 जना विद्यार्थीहरूको प्राप्ताङ्कको मध्यक र स्तरीय भिन्नता क्रमशः 10 र 2 छन् । सबै प्राप्ताङ्कहरूको वर्गको योगफल पत्ता लगाउनुहोस् ।
In a continuous data, the mean and the standard deviation of the marks obtained by 20 students are 10 and 2 respectively. Find the sum of the square of all the marks.

15. एउटा निरन्तर श्रेणीमा 20 जना विद्यार्थीहरूको प्राप्ताङ्कको मध्यक र स्तरीय भिन्नता क्रमशः 43.5 र 16 छन् । सबै प्राप्ताङ्कहरूको वर्गको योगफल पत्ता लगाउनुहोस् ।
In a continuous data, the mean and the standard deviation of the marks obtained by 20 students are 43.5 and 16 respectively. Find the sum of the square of all the marks.

16. एउटा फ्याक्ट्रीमा 2 किसिमका बल्वहरू A र B छन् । तिनीहरूको टिकाउ अवधिको बारेमा परीक्षण गर्दा निम्न परिणाम प्राप्त भयो । कुन बल्वको टिकाउ अवधि एकनास हुन्छ ?
A factory has two types of bulbs A and B. In an experiment relating to their life, the following results were obtained. Which types of bulbs has more uniform life?

Life in years	No. of bulbs	
	A	B
0-4	5	8
4-8	8	6
8-12	13	11
12-16	4	5

STATISTICS

ANSWERS

1. 0-20, 20-40, 40-60, 60-80, 80-100, Q.D. = 17.67 and coefficient = 0.39
2. 0-10, 10-20, 20-30, 30-40, 40-50,……, 80-90 Q.D. = 17.42 and coefficient = 0.37

3.	10.71,0.287	**4.**	B	**5.**	9	**6.**	13.09	**7.**	10. 46
8.	4. 23	**9.**	25. 8	**10.**	9. 5	**11.**	0.75, 20	**12.**	0.90, 20
14.	2080	**15.**	42965	**16.**	A				

puspas.com.np

www.drmathnp.com

puspas.com.np

ASSIGNMENT ON STATISTICS

परीक्षार्थीले दिइएको निर्देशनका आधारमा आफ्नै मौलिक शैलीमा सिर्जनात्मक उत्तर दिने गर्नु पर्दछ ।

Time : 40 minutes **Full Marks : 25**

Attempt all the Questions.

Group 'A' $3 \times 2 = 6$

1. एउटा तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार क्रमशः 13 र 2 छन् । चतुर्थांशीय भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
   The first quartile and the inter-quartile range of a data are 13 and 2 respectively. Find the coefficient of the quartile deviation.

2. एउटा निरन्तर श्रेणीमा $\overline{x} = 37, \sum f|d| = 544$ र $\sum f = 40$ छन् । मध्यकबाट मध्यक भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
   In a continuous series, $\overline{x} = 37, \sum f|d| = 544$ and $\sum f = 40$. Find its coefficient of mean deviation from mean.

3. एउटा अविच्छिन्न श्रेणीमा $\sum f = 70, \sum fm = 1660$ र $\sum fm^2 = 42500$ छन् । विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
   In a continuous data, $\sum f = 70, \sum fm = 1660$ and $\sum fm^2 = 42500$. Find coefficient of variation.

Group C $5 \times 4 = 20$

4. तलको तथ्याङ्कबाट चतुर्थांशीय भिन्नता पत्ता लगाउनुहोस् (Find the quartile deviation from the following data):

Daily sales	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of shops	2	9	10	7	1

5. दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् :
   Find the mean deviation from median of the given data:

Mid value	5	10	15	20	25
f	4	3	9	7	2

6. मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् (Find the mean deviation from mean and its coefficient):

Age (in yrs.)	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
No. of boys	12	8	10	6	4

7. स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् (Find the standard deviation and coefficient of variation):

C.I.	2 - 4	2 - 6	6 - 8	2 - 10
f	3	7	2	10

8. तलको तथ्याङ्कबाट स्तरीय भिन्नता पत्ता लगाउनुहोस् (Find the standard deviation from the following data):

C.I.	10	20	30	40	50
f	8	12	15	9	6

How well did you do?

0	1-3	3-6	6-9	9-12	12-15	15-18	18-21	21-25
N	E	D	C	C^+	B	B^+	A	A^+

ANSWERS

1. $\frac{1}{14}$ **2.** 0.37 **3.** 28.22% **4.** 7.62 **5.** 4.472 **6.** 4.6, 0.56

7. 1.89, 36.28% **8.** 12.33

www.drmathnp.com

pusnas.com.np

SEE NEW MODEL QUESTION 2076
2076 (2020) ISSUED BY CDC (SET 1)

पाठ्यक्रम विकास केन्द्र, सानोठिमी भक्तपुर

नमूना प्रश्न पत्र - २०७६

विषय : ऐच्छिक (अतिरिक्त) गणित पूर्णांक : १००
विषय कोड : २४१ समय : ३ घण्टा

GROUP A 10 × 1 = 10

1. (a) त्रिकोणमीतीय फलनको परिभाषा लेख्नुहोस् ।
Define trigonometric function.

(b) दुई सङ्ख्याहरू a र b बिचको अङ्कगणितीय मध्यक के हुन्छ ?
What is the arithmetic mean between two numbers a and b?

2. (a) सङ्ख्या रेखामा अविच्छिन्नता हुने सङ्ख्याहरूको समूह लेख्नुहोस् ।
Write a set of numbers which is continuous in number line.

(b) यदि मेट्रिक्स $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ भए $|A|$ को मान कति हुन्छ ?
If matrix $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, what is the value of $|A|$?

3. (a) यदि दुई सिधा रेखाहरूका झुकावहरू क्रमशः m_1 र m_2 छन् र तिनीहरू बिचको कोण θ भए $\tan\theta$ को मान पत्ता लगाउने सुत्र लेख्नुहोस् ।
If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for $\tan\theta$.

(b) एउटा सोलीलाई समतलीय सतहले आधारसँग समानान्तर हुनेगरि प्रतिच्छेदन गर्दा कस्तो आकृति बन्दछ ?
Which geometric figure will be formed if a plane intersects a cone parallel to its base?

4. (a) $\sin 2A$ लाई $\tan A$ को रूपमा व्यक्त गर्नुहोस् ।
Express $\sin 2A$ in terms of $\tan A$.

(b) उन्नतांश कोणको परिभाषा लेख्नुहोस् ।
Define angle of elevation.

5. (a) यदि $\vec{a}$ र $\vec{b}$ बिचको कोण θ भए $\vec{a}$ र $\vec{b}$ को स्केलर गुणनफल के हुन्छ ?
What is the scalar product of two vectors $\vec{a}$ and $\vec{b}$ if the angle between them is θ ?

(b) एउटा इन्भर्सन स्थानान्तरणमा P को प्रतिबिम्ब P' र केन्द्र O भएको इन्भर्सन वृत्तको अर्धव्यास r भए OP.OP' र r को सम्बन्ध लेख्नुहोस् ।
In an inversion transformation if P' is image of P and r is radius of inversion circle with centre O, write the relation of OP.OP' and r.

GROUP B 13 × 2 = 26

6. (a) यदि $f(x) = 4x + 5$ भए $f^{-1}(x)$ को मान पत्ता लगाउनुहोस् ।
Find $f^{-1}(x)$ if $f(x) = 4x + 5$.

(b) यदि $g(x) = 2x - 1$ र $f(x) = 4x$, भए $g_o f(x)$ को मान पत्ता लगाउनुहोस् ।
If $g(x) = 2x - 1$ and $f(x) = 4x$, find the value of $g_o f(x)$.

(c) $f(x) = x^2 - 1$ र $f(x) = 3$ प्रतिच्छेदित हुने बिन्दुहरू पत्ता लगाउनुहोस् ।
What will be the points of intersection of the curve $f(x) = x^2 - 1$ and $f(x) = 3$?

7. (a) यदि $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ भए $|A|$ पत्ता लगाई A^{-1} परिभाषित हुन्छ, हुँदैन लेख्नुहोस् ।
If $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find $|A|$ and write A^{-1} is defined or not.

(b) क्रामरको नियम अनुसार $ax + by = c$ र $px + qy = r$ मा D_1 र D_2 को मान पत्ता लगाउनुहोस् ।
According to Cramer's rule, find the values of D_1 and D_2 for $ax + by = c$ and $px + qy = r$.

8. (a) समीकरणहरू $3x + 4y + 5 = 0$ र $6x + 8y + 7 = 0$ का झुकाव पत्ता लगाई ती रेखाहरूको सम्बन्ध लेख्नुहोस् ।
Find the slopes of two straight lines $3x + 4y + 5 = 0$ and $6x + 8y + 7 = 0$ and write the relationship between them.

(b) जोडा रेखाहरू $3x + 2y = 0$ र $2x - 3y = 0$ लाई जनाउने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation for the pair of lines represented by $3x + 2y = 0$ and $2x - 3y = 0$.

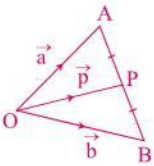
9. (a) $\sin 6A \cdot \cos 4A$ लाई sine वा cosine को योग वा अन्तरमा रूपान्तरण गर्नुहोस् ।
Convert $\sin 6A \cdot \cos 4A$ into sum or difference of sine or cosine.

(b) $\frac{\sin A}{1 + \cos A}$ लाई tangent अनुपातको अपवर्तकको रूपमा व्यक्त गर्नुहोस् ।
Express $\frac{\sin A}{1 + \cos A}$ in terms of sub-multiple angle of tangent.

(c) यदि $2\sin 2\theta = \sqrt{3}$ भए θ को मान पत्ता लगाउनुहोस् । $(0° \le \theta \le 180°)$
If $2\sin 2\theta = \sqrt{3}$, find the value of θ. $(0° \le \theta \le 180°)$

10. (a) यदि $|\vec{a}| = 2$, $|\vec{b}| = 12$ र $\vec{a}.\vec{b} = 12$ भए भेक्टरहरू $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।
Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 12$ and $\vec{a}.\vec{b} = 12$.

(b) दिइएको चित्रमा $\overrightarrow{AP}$ पत्ता लगाई $\vec{p}$ लाई $\vec{a}$ र $\vec{b}$ को रूपमा व्यक्त गर्नुहोस् ।
From the given figure find $\overrightarrow{AP}$ and express $\vec{p}$ in terms of $\vec{a}$ and $\vec{b}$.



MODEL QUESTIONS

www.drmathnp.com

pushas.com.np

(c) कुनै तथ्याङ्ककको स्तरीय भिन्नता 0.25 भए विचरणशीलता कति हुन्छ ? पत्ता लगाउनुहोस् ।
If the standard deviation of a set of data is 0.25, find its variance.

GROUP C 11 × 4 = 44

11. हल गर्नुहोस् (Solve) : $x^3 - 3x^2 - 4x + 12 = 0$

12. सर्तहरू $x - 2y \le 1, x + y \le 4, x \ge 0, y \ge 0$ को आधारमा $P = 5x + 4y$ को अधिकतम र न्यूनतम मान पत्ता लगाउनुहोस् ।
Optimize $P = 5x + 4y$ under the given constraints: $x - 2y \le 1, x + y \le 4, x \ge 0, y \ge 0$

13. वास्तविक फलन $f(x) = 2x + 3$ को लागि
(a) $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ र $f(3)$ का मानहरू पत्ता लगाउनुहोस् ।
(b) के यो फलन $x = 3$ मा अविच्छिन्न हुन्छ ?
For a real valued function $f(x) = 2x + 3$
(a) Find the values of $f(2.95)$, $f(2.99)$, $f(3.01)$, $f(3.05)$ and $f(3)$.
(b) Is this function continuous at $x = 3$?

14. म्याट्रिक्स विधिको प्रयोग गरी तल दिइएका समीकरणहरू हल गर्नुहोस् : $3x + 5y = 11, 2x - 3y = 1$
By using matrix method, solve the following system of equations: $3x + 5y = 11, 2x - 3y = 1$

15. उद्गम बिन्दुबाट जाने र $2x^2 - 5xy + 2y^2 = 0$ ले प्रतिनिधित्व गर्ने सरल रेखाहरूसँग लम्ब हुने सरल रेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$.

16. मान पत्ता लगाउनुहोस् (Find the value of):
$\sin 20° \cdot \sin 30° \cdot \sin 40° \cdot \sin 80°$

17. यदि $A + B + C = \pi$ भए प्रमाणित गर्नुहोस् :
If $A + B + C = \pi$, prove that:
$\sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A \cos B \sin C$

18 एउटा धरहराको ठीक अगाडि जमिनको सतहमा रहेको एक बिन्दुबाट धरहराको माथि ठड्याइएको 6 m अग्लो ध्वजदण्डको टुप्पो र फेदका उन्नतांश कोणहरू क्रमशः 60° र 45° पाईयो । धरहराको उचाइ र धरहराको फेदबाट सो बिन्दुसम्मको दूरी पत्ता लगाउनुहोस् ।
From a point at the ground level infront of a tower, the angle of elevations of the top and bottom of flagstaff 6 m high situated at the top of a tower are observed 60° and 45° respectively. Find the height of the tower and the distance between the base of the tower and point of observation.

19. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने 2×2 म्याट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2×2 matrix which transforms a unit square to a parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$.

20. दिइएको तथ्याङ्ककको मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the mean deviation from mean and its coefficient from given data.

प्राप्ताङ्क Marks obtained	0-10	10-20	20-30	30-40	40-50
विद्यार्थी सङ्ख्या No. of students	2	3	6	5	4

21. दिइएको तथ्याङ्कबाट स्तरीय भिन्नता र विचरणशीलताको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the standard deviation and coefficient of variation from given data.

उमेर(Age)	0-4	4-8	8-12	12-16	16-20	20-24
विद्यार्थी सङ्ख्या No. of students	7	7	10	15	7	6

GROUP D 4 × 5 = 20

22. एकजना ठेकेदारले कुनै काम निश्चित समय भन्दा ढिलो गरे वापत पहिलो दिनको रु. 200, दोस्रो दिनको रु 250 र तेस्रो दिनको रु 300 गर्दै प्रत्येक पछिल्लो दिनमा पहिलो दिनको भन्दा रु 50 बढि जरिवाना तिर्नुपर्छ । यदि उक्त ठेकेदारले कुनै काम 30 दिन ढिला गरेमा जम्मा कति जरिवाना तिर्नुपर्ला ?
A contractor on construction job specifies a penalty for delay of completion beyond a certain date as: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day and so on. The penalty for each succeding day being Rs 50 more than that of the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

23. एउटा पाङ्ग्राको परिधिमा रहेका तीनओटा बिन्दुहरू (5, 7) , (– 1, 7) र (5, – 1) एउटा निश्चित बिन्दुबाट बराबर दुरीमा पर्दछन् । उक्त निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । साथै उक्त तीन बिन्दुहरू पर्ने गरी बिन्दुपथ पत्ता लगाउनुहोस् ।
On a wheel, there are three points (5, 7) , (– 1, 7) and (5, – 1) located such that the distance from a fixed point to these points is always equal. Find the coordinate of the fixed point and then derive the equation representing the locus that contains all three points.

24. भेक्टर विधिको प्रयोग गरी कुनैपनि चतुर्भुजको भुजाहरूका मध्यबिन्दुहरू क्रमशः जोड्दै जाँदा बन्ने चतुर्भुज समानान्तर चतुर्भुज हुन्छ, भनी प्रमाणित गर्नुहोस् ।
By using vector method, prove that the quadrilateral formed by joining the midpoints of adjacent sides of a quadrilateral is a parallelogram.

25. चतुर्भुज ABCD का शीर्षबिन्दुका निर्देशाङ्कहरू A(1, 1), B(2, 3), C(4, 2) र D(3, – 2) छन् । उक्त चतुर्भुजलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गर्नुहोस् । प्रतिबिम्ब चतुर्भुजलाई $y = -x$ मा परावर्तन गर्नुहोस् । माथिका दुई स्थानान्तरणको संयुक्त स्थानान्तरणलाई जनाउने स्थानान्तरण लेख्नुहोस् ।
The coordinates of vertices of a quadrilateral ABCD are A(1, 1), B(2, 3), C(4, 2) and D(3, – 2).Rotate this quadrilateral about origin through180°. Reflect this image of quadrilateral about $y = -x$. Write the name of transformation which denotes the combined transformation of above two transformations.

www.drmathnp.com

माध्यमिक शिक्षा उत्तिर्ण परीक्षा
उत्तर कुञ्जिका - २०७६

उत्तरपुस्तिका परीक्षण कुञ्जिकाउत्तरपुस्तिका परीक्षणको निम्ति परीक्षकलाई सामान्य मार्गदर्शन हो । परीक्षकले उत्तरको शुद्धता, स्तरीयता, मौलिकता आदि हेरी आवश्यकताअनुसार विवेक प्रयोग गरी स्तरअनुसार सही, स्पष्ट, उपयुक्त, मापनीय र स्तरीय मूल्याङ्कन गर्नुपर्ने छ । सम्भव भएसम्म कुञ्जिकाले निर्देश गरेको परिधि र सीमाभित्र रही मूल्याङ्कन गर्नुपर्ने छ ।

Note: Give the relevant mark(s) for other correct method.

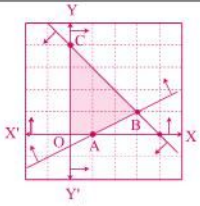
GROUP A 10 × 1 = 10

1. (a)	The function with trigonometric ratios as the independent variables.....	1
(b)	$\frac{a+b}{2}$	1
2. (a)	Set of real numbers	1
(b)	$ps - qr$.....	1
3. (a)	$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1.m_2}$.....	1
(b)	circle.....	1
4. (a)	$\frac{2\tan A}{1+\tan^2 A}$	1
(b)	Correct definition of angle of elevation.....	1
5. (a)	$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$.....	1
(b)	OP.OP' = r^2.....	1

GROUP B 13 × 2 = 26

6. (a)	Let, $y = 4x + 5$	
	(i) $x = 4y + 5$	1
	(ii) $f^{-1}(x) = \frac{x-5}{4}$	1
(b)	(i) $g_o f(x) = g(4x)$	1
	(ii) $g(4x) = 2(4x) - 1 = 8x - 1$	1
(c)	(i) $x^2 - 1 = 3$	
	$x = \pm 2$.....	1
	(ii) ∴ The points are (– 2, 3) and (2, 3)	1
7. (a)	(i) $\lvert A\rvert = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} = 2 + 3 = 5$	1
	(ii) $\lvert A\rvert \neq 0$, so A^{-1} exists or defines.....	1
(b)	(i) $D_1 = \begin{vmatrix} c & b \\ r & q \end{vmatrix} = cq - br$.....	1
	(ii) $D_2 = \begin{vmatrix} a & c \\ p & r \end{vmatrix} = ar - pc$.....	1
8. (a)	(i) $m_1 = -\frac{3}{4}$, $m_2 = -\frac{3}{4}$	1
	(ii) So they are parallel since both slope are equal.	1
(b)	(i) $(3x + 2y)(2x - 3y) = 0$	1
	(ii) $6x^2 - 5xy - 6y^2 = 0$.....	1
9. (a)	(i) $= \frac{1}{2}[2\sin 6A \cdot \cos 4A]$.....	1
	(ii) $= \frac{1}{2}[\sin(6A + 4A) + \sin(6A - 4A)]$	
	$= \frac{1}{2}[\sin 10A + \sin 2A]$	1
(b)	(i) $\frac{\sin A}{1 + \cos A} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}}$	1
	(ii) $= \tan\frac{A}{2}$	1
(c)	(i) $2\theta = 60°, 120°$.....	1
	(ii) ∴ $\theta = 30°, 60°$	1
10. (a)	(i) $\cos\theta = \frac{12}{2 \times 12}$	1
	(ii) ∴ $\theta = 60°$.....	1
(b)	(i) $\overrightarrow{AC} = \vec{p} - \vec{a}$	1
	(ii) $\vec{p} = \frac{\vec{a} + \vec{b}}{2}$	1
(c)	Standard deviation(σ) = 0.25, Variance = σ^2..	1
	Variance = $(0.25)^2 = 0.0625$	1

GROUP C 11 × 4 = 44

11.	Let $f(x) = x^3 - 3x^2 - 4x + 12$	
	(i) $f(2) = 0$, ∴ $x - 2$ is a factor of $f(x)$	1
	(ii) $(x - 2)(x^2 - x - 6) = 0$.....	1
	(iii) $(x - 2)(x + 2)(x - 3) = 0$.....	1
	(iv) ∴ $x = -2, 2, 3$	1
12.		
	(i) For correct graph	1+1
	(ii) Finding values of at (0, 0), (1, 0), (3, 1) and (0, 4).....	1
	(iii) Maximum value of P is 19 at (3, 1) And minimum value is 0 at (0, 0)	1
13.	(i) $f(2.95) = 8.9$, $f(2.99) = 8.98$, $f(3.01) = 9.02$, $f(3.05) = 9.1$, $f(3) = 9$	1+1
	(ii) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$.....	1
	(iii) ∴ $f(x)$ is continuous at $x = 3$.....	1
14.	(i) $\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$	1
	(ii) $A^{-1} = \frac{1}{-19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}$	1
	(iii) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ 1 \end{pmatrix}$.....	1
	(iv) ∴ $x = 2$ and $y = 1$.....	1
15.	(i) Finding separate equations $x - 2y = 0$ and $2x - y = 0$..... Separate equations perpendicular to above equations and passing through origin are	1
	(ii) $2x + y = 0$ and $x + 2y = 0$.....	1
	(iii) Equation of pair of lines is $(2x + y)(x + 2y) = 0$.....	1
	(iv) ∴ $2x^2 + 5xy + 2y^2 = 0$ is the required equation of pair of line	1
16.	(i) $= \frac{1}{4} \cdot \sin 20° \cdot (2\sin 80° \cdot \sin 40°)$	
	$= \frac{1}{4}.\sin 20°.[\cos(80° - 40°) - \cos(80° + 40°)]$	1

MODEL QUESTIONS

puspas.com.np

www.drmathnp.com

(ii) $= \frac{1}{4} . \sin 20°. \cos 40° + \frac{1}{8} . \sin 20°$ 1

(iii) $= \frac{1}{8}[\sin(40°+20°) - \sin(40°-20°)] + \frac{1}{8}\sin 20°$ 1

(iv) $= \frac{\sqrt{3}}{16}$ 1

17. (i) $\sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1}{2}(\cos 2B - \cos 2A) + \sin^2 C$ 1

(ii) $= \sin C . \sin(A - B) + \sin^2 C$ 1

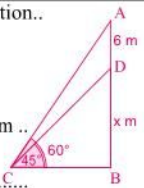
(iii) $= \sin C . [\sin(A - B) + \sin(A + B)]$ 1

(iv) $= 2 \sin A \cos B \sin C$ 1

18. (i) Correct figure with description..

Let, BD = x m,

$\tan 45° = \frac{x}{CB}$, CB = x m 1



(ii) $\tan 60° = \frac{x+6}{CB}$, $CB = \frac{x+6}{\sqrt{3}}$ m .. 1

(iii) $x = \frac{x+6}{\sqrt{3}}$, x = 8.19 m 1

(iv) ∴ Height = 8.19 m and distance = 8.19 m 1

19. (i) let, the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ 1

(ii) $\begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ 1

(iii) $a = 3$, $b = 1$, $c = 0$ and $d = 1$ 1

(iv) ∴ The matrix is $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ 1

20.

Marks	f	x	fx	$\lvert D\rvert = \lvert x - \bar{x}\rvert$	$f\lvert D\rvert$
0-10	2	5	10	23	46
10-20	3	15	45	13	39
20-30	6	25	150	3	18
30-40	5	35	175	7	35
40-50	4	45	180	17	68
Total	N = 20		*560*		*206*

(i) For correct table 1

(ii) $\bar{x} = \frac{560}{20} = 28$ 1

(iii) M.D. $= \frac{206}{20} = 10.3$ 1

(iv) Coefficient of M.D $= \frac{10.3}{28} = 0.3678$ 1

21.

Age	f	x	$d = x - 10$	fd	fd^2
0-4	7	2	− 8	− 56	448
4-8	7	6	− 4	− 28	112
8-12	10	10	0	0	0
12-16	15	14	4	60	240
16-20	7	18	8	56	448
20-27	6	22	12	72	864
Total	**N = 52**			**104**	**2112**

(i) For correct table 1

Let assumed mean (A) = 10

(ii) $\bar{x} = 10 + \frac{104}{52} = 12$ 1

(iii) $\sigma = \sqrt{\frac{2112}{52} - \left(\frac{104}{52}\right)^2} = 6.05$ 1

(iv) Coefficient of variation $= \frac{6.05}{12} \times 10\%$

$= 50.42\%$ 1

GROUP D 4 × 5 = 20

22. (i) Writing the series 200, 250, 300, ... 1

(ii) Writing $a = 200$, $d = 50$ and $n = 30$ 1

(iii) Formula for $S_n = \frac{n}{2}[2a + (n-1)d]$ 1

(iv) $\frac{30}{2}[2 \times 200 + 29 \times 50]$ 1

(v) $15[400 + 1450] = 27750$ 1

23. suppose the fixed point is P(x, y)

The three points are A(5, 7), B(−1, 7) and C(5, − 1)

(i) PA = PB and PA = PC 1

(ii) finding $x = 2$ from PA= PB..... 1

(iii) finding $y = 3$ from PA = PC 1

(iv) Finding Distance PA = 5 by

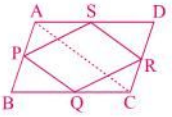
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

The locus of a point (X, Y) with center (2, 3) and radius 5 as

$\sqrt{(X - 2)^2 + (Y - 3)^2} = 5$

(iv) $x^2 + y^2 - 4x - 6y - 12 = 0$ 1

24. (i) For correct figure. 1



(ii) $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$ 1

(iii) $\overrightarrow{PQ} = K\overrightarrow{AC}$ where $k = \frac{1}{2}$ ∴ $\overrightarrow{PQ} \parallel \overrightarrow{AC}$ 1

(iv) $\overrightarrow{SR} = \frac{1}{2}\overrightarrow{AC}$ and $\overrightarrow{SR} \parallel \overrightarrow{AC}$..... 1

(v) ∴ PQRS is a parallelogram 1

25 (i) Rotating four points about the origin through 180° by formula

Object		Image
(x, y)	*Rotation [180°; (0, 0)]* →	$(-x, -y)$
A(1, 1)	*Rotation [180°; (0, 0)]* →	A'(− 1, − 1)
B(2, 3)	*Rotation [180°; (0, 0)]* →	B'(− 2, − 3)
C(4, 2)	*Rotation [180°; (0, 0)]* →	C'(− 4, − 2)
D(3, − 2)	*Rotation [180°; (0, 0)]* →	D'(− 3, 2)

(Rotation of each point 0.5 marks) 1 + 1

Object		Image
P(x, y)	*Reflection in y = − x* →	P'(− y, − x)
A'(− 1, − 1)	*Reflection in y = x* →	A"(1, 1)
B'(− 2, − 3)	*Reflection in y = x* →	B"(3, 2)
C'(− 4, − 2)	*Reflection in y = x* →	C"(2, 4)
D'(− 3, 2)	*Reflection in y = x* →	D"(− 2, 3)

(ii) (Reflection of each point 0.5 marks) 1 + 1

(iii)Writing the combined transformation = reflection about y = x line..... 1

pusmas.com.np

www.drmathnp.com

puspas.com.np

SEE MODEL QUESTION 2076 SET 2
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पूर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) $y = \sin x$ को y-खण्ड कति हुन्छ ?
What is the y-intercept of $y = \sin x$?

   (b) यदि एउटा बहुपदीय $f(x)$ लाई $(x - a)$ ले भाग गरिएको छ भने शेष कति हुन्छ ?
If a polynomial $f(x)$ is divided by $(x - a)$, what will be its remainder?

2. (a) फलनको निरन्तरताको परिभाषा दिनुहोस् ।
Define Continuity of a function.

   (b) यदि एउटा मेट्रिक्स A = [4] भए A को डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।
If a matrix A = [4], find the determinant of A.

3. (a) सोलीको परिभाषा दिनुहोस् । (Define cone.)

   (b) केन्द्र (0, 0) र अर्धव्यास 'r' भएको वृत्तको समीकरण के हुन्छ ?
What is the equation of circle whose centre is (0, 0) and radius 'r'?

4. (a) 2 cos A cos B लाई cosine को योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् ।
Express 2 cos A cos B in the sum or difference of cosine.

   (b) यदि sec A = cosec A भए A को न्यून कोणीय मान कति हुन्छ ?
If sec A = cosec A, what is the acute value of A?

5. (a) भेक्टरको स्केलर गुणनफलमा लम्ब हुने अवस्था लेख्नुहोस् ।
State the condition of perpendicularity in scalar product of two vectors.

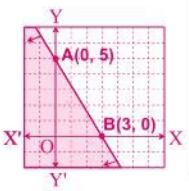
   (b) यदि उत्क्रम वृत्तको बाहिर बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if P is outside the circle of inversion?

GROUP B [13 × 2 = 26]

6. (a) यदि $x^3 - kx^2 - 13x + 10$ लाई $(x + 2)$ ले भाग गर्दा शेष 4 हुन्छ भने k को मान शेषसाध्य प्रयोग गरेर पत्ता लगाउनुहोस् ।
If $x^3 - kx^2 - 13x + 10$ is divided by $(x + 2)$, the remainder is 4. Find the value of k using the remainder theorem.

   (b) यदि श्रेणी 80 + 70 + 60 + ... को n औं पद शून्य भए n को मान निकाल्नुहोस् ।
If the n^{th} term of the series 80 + 70 + 60 + ... is zero, find the value of n.

   (c) दिइएको चित्रमा छाया पारेको भाग जनाउने असमानता पत्ता लगाउनुहोस्:
Find the inequality that represents the shaded region in the given figure:



7. (a) यदि मेट्रिक्स $A = \begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix}$ भए A^{-1} पत्ता लगाउनुहोस् ।
If matrix $A = \begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix}$ then find A^{-1}.

   (b) यदि मेट्रिक्स $P = \begin{bmatrix} 2x & -6 \\ 3 & 4 \end{bmatrix}$ र P को डिटरमिन्यान्ट 10 भए x को मान पत्ता लगाउनुहोस् ।
If matrix $P = \begin{bmatrix} 2x & -6 \\ 3 & 4 \end{bmatrix}$ and determinant of P is 10 then find the value of x.

8. (a) $x^2 - 3xy + 2y^2 = 0$ ले दिने जोडा रेखाको छुट्टाछुट्टै समीकरण पत्ता लगाउनुहोस् ।
Find the separate equations to line given by the equation $x^2 - 3xy + 2y^2 = 0$.

   (b) केन्द्रबिन्दु (3, 0) र अर्धव्यास 5 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle having the center (3, 0) and radius 5 units.

9. (a) प्रमाणित गर्नुहोस् (Prove that):
$$\frac{\sin\frac{A}{2} + \sin A}{1 + \cos\frac{A}{2} + \cos A} = \tan\frac{A}{2}$$

   (b) प्रमाणित गर्नुहोस् (Prove that):
$\sin 80° - \cos 110° = \sqrt{3} \sin 50°$

   (c) यदि $\sin\theta - \cos\theta = 0$ भए θ को न्युनकोणी मान पत्ता लगाउनुहोस् ।
If $\sin\theta - \cos\theta = 0$, find the acute value of θ.

10. (a) यदि $\vec{a} = 3\vec{i} + m\vec{j}$ र $\vec{b} = 6\vec{i} - 2\vec{j}$ एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।
If $\vec{a} = 3\vec{i} + m\vec{j}$ and $\vec{b} = 6\vec{i} - 2\vec{j}$ are perpendicular to each other then find the value of m.

   (b) बिन्दुहरू A र P का स्थिति भेक्टरहरू क्रमशः $3\vec{i} + 5\vec{j}$ र $\vec{i} + 4\vec{j}$ छन् । यदि AB को मध्यबिन्दु P भए B को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
The position vectors of the points A and P are $3\vec{i} + 5\vec{j}$ and $\vec{i} + 4\vec{j}$ respectively. If P is the middle point of AB then find the position vector of B.

   (c) एउटा तथ्याङ्कको पहिलो चतुर्थांश 35 र तेस्रो चतुर्थांश 75 छ । चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, first quartile is 35 and the third quartile is 75. Find the quartile deviation and its coefficient.

GROUP C [11 × 4 = 44]

11. यदि एउटा गुणोत्तर श्रेणीको दोस्रो पद 48 र पाँचौ पद 6 छ भने प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the second term of a geometric series is 48 and its fifth term is 6 then find the sum of the first 6 terms.

www.drmathnp.com

12. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically):
$x^2 - x - 2 = 0$

13. यदि $f(x) = 4x + 5$ एउटा वास्तविक मान भएको फलन भए

(a) $x = 1.9$ र 1.99 मा $f(x)$ का मानहरू के के हुन्छन् ?

(b) $x = 2.1$ र 2.01 मा $f(x)$ का मानहरू के के हुन्छन् ?

(c) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति-कति हुन्छन् ?

(d) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? पत्ता लगाउनुहोस् ।

If $f(x) = 4x + 5$ is a real valued function,

(a) What are the values of $f(x)$ at $x = 1.9$ and 1.99?

(b) What are the values of $f(x)$ at $x = 2.1$ and 2.01?

(c) What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?

(d) Is the function continuous at $x = 2$?

14. क्रामरको नियमबाट हल गर्नुहोस् :
Solve by the Cramer's rule:

$$\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

15. समीकरण $3x^2 + xy - 10y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग समानान्तर हुने र बिन्दु $(1, 0)$ भएर जाने एक जोडा रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a pair of lines passing through the point $(1, 0)$ and parallel to the lines represented by the equation $3x^2 + xy - 10y^2 = 0$.

16. प्रमाणित गर्नुहोस् (Prove that) : $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

17. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् :
If $A + B + C = \pi^c$, prove that:
$2 \sin^2 A + \cos 2B + \cos 2C - 1 = 4 \cos A \cdot \cos B \cdot \cos C + 1$

18. 20 मिटर अग्लो भवनबाट एउटा स्तम्भको टुप्पो र फेद हेर्दा अवनति कोणहरू क्रमश : 45° र 60° पाइयो भने स्तम्भको उचाइ र भवन र स्तम्भबिचको दूरी पत्ता लगाउनुहोस् ।
From the top of a building of 20 m high, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole and the distance between the building and the pole.

19. एकाइ वर्ग $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ लाई समानान्तर चतुर्भुज $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ मा स्थानान्तरण गर्ने एउटा 2×2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2×2 transformation matrix in which a unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ is transformed into a parallelogram $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

20. तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation from median of the data given below:

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

21. तल दिइएको तथ्याङ्कको आधारमा स्तरीय भिन्नता निकाल्नुहोस् :
Calculate the standard deviation form the following data:

x	0-10	10-20	20-30	30-40	40-50
f	5	4	4	6	1

GROUP D [4 × 5 = 20]

22. यदि $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ र $g \circ f(4) = -8$ भए $f^{-1}(-2)$ र b का मानहरू निकाल्नुहोस् । $g(x)$ कस्तो किसिमको फलन हो ?
If $f(x) = \frac{x}{2-x}$, $g(x) = bx - 2$ and $g \circ f(4) = -8$, find the values of $f^{-1}(-2)$ and b. What type of function is $g(x)$?

23. रेखा $x - y + 1 = 0$ सँग 45° को कोण बनाउने र बिन्दु $(-1, 2)$ भएर जाने सिधा रेखाहरूको समीकरण पत्ता लगाउनुहोस् । साथै, ती रेखाहरूको प्रतिच्छेदन बिन्दु पनि पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point $(-1, 2)$ and making an angle of 45° with the line $x - y + 1 = 0$. Also, find the point of intersection of the lines.

24. समानान्तर चतुर्भुजका विकर्णहरू आपसमा समद्विभाजन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals of a parallelogram bisect to each other.

25. लेखाचित्रमा कुनै समकोण त्रिभुज ABC लिनुहोस् । ΔABC लाई पहिले उद्गम बिन्दुको वरिपरि $+90^\circ$ मा परिक्रमण गरेपछि केन्द्र O र नापो 2 भएको विस्तारीकरणद्वारा स्थानान्तरण गरी प्रतिबिम्बका निर्देशाङ्कहरू पत्ता लगाउनुहोस् र तिनीहरू सबैलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
Take a right angled ΔABC in a graph. Rotate the Δ ABC through $+90^\circ$ about the centre at origin and then enlarge with centre at origin and the scale factor 2. Find the images and show them on the same graph paper.

ANSWERS

1. (a) 0 (b) $f(a)$ **2.** (b) 4
3. (b) $x^2 + y^2 = r^2$ **4.** (a) $\cos(A + B) + \cos(A - B)$
(b) 45° **5.** (a) Scalar product = 0
(b) Inside the circle **6.** (a) 6 (b) 9
(c) $5x + 3y \le 15$ **7.** (a) $\begin{pmatrix} 5 & 3 \\ -3 & -2 \end{pmatrix}$ (b) -1
8. (a) $x - y = 0$ and $x - 2y = 0$ (b) $x^2 + y^2 - 6x - 16 = 0$
9. (c) 45° **10.** (a) 9 (b) $\vec{i} + 3\vec{j}$ (c) 20, 0.36
11. 189 **12.** 2, – 1 **13.** (a) 12.6, 12.96
(b) 13.4, 13.04 (c) 13, 13 (d) Yes **14.** 2, 1
15. $3x^2 + xy - 10y^2 - 6x - y + 3 = 0$ **18.** 11.55 m
19. $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ **20.** 9.56 **21.** 12.69
22. 4, 3, linear function **23.** $y = 2$ and $x = -1$ and $(2, -1)$

pustas.com.np

www.drmathnp.com

puspas.com.np

SEE MODEL QUESTION 2076 SET 3
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) $y = \cos x$ को y-खण्ड कति हुन्छ ?
What is the y-intercept of $y = \cos x$?

(b) शेष कति होला जब एउटा बहुपदीय $x^3 - 1$ लाई $(x - 2)$ ले भाग गरिएको छ ?
What is the remainder when a polynomial $x^3 - 1$ is divided by $(x - 2)$?

2. (a) फलनको वक्रको बिच्छिन्नताको परिभाषा दिनुहोस् ।
Define discontinuity of a curve of a function.

(b) यदि $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ भए $|A|$ पत्ता लगाउनुहोस् ।
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find $|A|$.

3. (a) शाङ्कि भागको परिभाषा दिनुहोस् ।
Define conic section.

(b) केन्द्र (a, b) र अर्धव्यास 'r' भएको वृत्तको समीकरण लेख्नुहोस् ।
State the equation of circle whose centre is (a, b) and radius 'r'.

4. (a) 2 sin α sin β लाई cosine को योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् ।
Express 2 sin α sin β in the sum or difference of cosine.

(b) यदि cot X = tan X भए X को न्यून कोणीय मान कति हुन्छ ?
If cot X = tan X, what is the acute value of X?

5. (a) यदि $\vec{a} = k\vec{b}$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुन्छ ? जहाँ k एउटा स्केलर परिमाण हो ।
If $\vec{a} = k\vec{b}$, what is the angle between $\vec{a}$ and $\vec{b}$ where k is a scalar quantity?

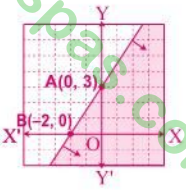
(b) यदि उत्क्रम वृत्तको भित्र बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if P is inside the circle of inversion?

GROUP B [13 × 2 = 26]

6. (a) यदि $f(x) = x^3 + kx^2 - 4x + 12$ को एउटा गुणनखण्ड $x + 2$ भए k को मान पत्ता लगाउनुहोस् ।
If $(x + 2)$ is a factor of $f(x) = x^3 + kx^2 - 4x + 12$ then find the value of k.

(b) यदि 25, m + 1, 35 एउटा समानान्तरीय अनुक्रम भए m को मान पत्ता लगाउनुहोस् ।
If 25, m + 1, 35 is an arithmetic sequence then find the value of m.

(c) दिइएको चित्रमा छाया पारेको भाग जनाउने असमानता पत्ता लगाउनुहोस्:
Find the inequality that represents the shaded region in the given figure:



7. (a) मेट्रिक्स $\begin{bmatrix} m & 2 \\ 7 & 3 \end{bmatrix}$ को विपरीत मेट्रिक्स $\begin{bmatrix} 3 & -2 \\ -7 & m \end{bmatrix}$ भए m को मान निकाल्नुहोस् ।
If the inverse of the matrix $\begin{bmatrix} m & 2 \\ 7 & 3 \end{bmatrix}$ is the matrix $\begin{bmatrix} 3 & -2 \\ -7 & m \end{bmatrix}$, find the value of m.

(b) यदि $\begin{vmatrix} 2p & 7 \\ -5 & 9 \end{vmatrix} = -1$ भए p को मान निकाल्नुहोस् ।
If $\begin{vmatrix} 2p & 7 \\ -5 & 9 \end{vmatrix} = -1$, find the value of p.

8. (a) यदि समीकरण $9x^2 + 12xy + (k + 1)y^2 = 0$ ले जनाउने रेखाहरू सम्पाती छन् भने k को मान निकाल्नुहोस् ।
If the lines represented by the equation $9x^2 + 12xy + (k + 1)y^2 = 0$ are coincident, find the value of k.

(b) एउटा व्यासको छेउ छेउका बिन्दुहरू (2, 3) र (4, 0) भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having (2, 3) and (4, 0) as the end points of a diameter.

9. (a) यदि $\sin\frac{\theta}{2} = \frac{1}{2}\left(a + \frac{1}{a}\right)$ भए प्रमाणित गर्नुहोस् :
If $\sin\frac{\theta}{2} = \frac{1}{2}\left(a + \frac{1}{a}\right)$, prove that:
$\cos\theta = \frac{-1}{2}\left(a^2 + \frac{1}{a^2}\right)$

(b) प्रमाणित गर्नुहोस् (Prove that):
$\frac{\sin A + \sin B}{\cos A + \cos B} = \tan\frac{A + B}{2}$

(c) हल गर्नुहोस् (Solve): $2\sin\theta - \sqrt{2} = 0 [0° \le \theta \le 90°]$

10. (a) यदि $|\vec{a}| = 3$, $|\vec{b}| = 5$ र $\vec{a}.\vec{b} = \frac{15}{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = \frac{15}{2}$ then find the angle between $\vec{a}$ and $\vec{b}$.

(b) यदि ΔABC का शीर्षबिन्दुहरू A, B र C का स्थिति भेक्टरहरू क्रमशः $(3\vec{i} + 4\vec{j})$, $(4\vec{i} + 5\vec{j})$ र $(5\vec{i} + 6\vec{j})$ छन् भने सो त्रिभुजको गुरुत्वकेन्द्र G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
If the position vectors of the vertices A, B, and C of ΔABC are $(3\vec{i} + 4\vec{j})$, $(4\vec{i} + 5\vec{j})$ and $(5\vec{i} + 6\vec{j})$ respectivley, find the position vector of the centroid G of the triangle.

www.drmathnp.com

(c) एउटा तथ्याङ्कको चतुर्थांशीय भिन्नता र पहिलो चतुर्थांश क्रमशः 20 र 35 छन् । सो तथ्याङ्कको तेस्रो चतुर्थांश र चतुर्थांशीय भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, quartile deviation and the first quartile are 20 and 35 respectively. Find the third quartile and the coefficient of quartile deviation.

GROUP C [11 × 4 = 44]

11. दुई सङ्ख्याहरूको समानान्तरीय मध्यमा 25 र तिनीहरूको गुणोत्तर मध्यमा 20 भए ती दुई सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The arithmetic mean of two numbers is 25 and their geometric mean is 20. Find the numbers.

12. वर्ग समीकरण $x^2 - 3x - 4 = 0$ लाई लेखा चित्रद्वारा हल गर्नुहोस् ।
Solve graphically the quadratic equation $x^2 - 3x - 4 = 0$.

13. यदि $f(x) = 3x + 2$ एउटा वास्तविक मान भएको फलन भए
(a) $x = 1.9$ र 1.99 मा $f(x)$ का मानहरू के के हुन्छन् ?
(b) $x = 2.1$ र 2.01 मा $f(x)$ का मानहरू के के हुन्छन् ?
(c) $\lim_{x \to 2} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति-कति हुन्छन् ?
(d) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 2$ is a real valued function,
(a) What are the values of $f(x)$ at $x = 1.9$ and 1.99?
(b) What are the values of $f(x)$ at $x = 2.1$ and 2.01?
(c) What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?
(d) Is the function continuous at $x = 2$?

14. मेट्रिक्स विधिबाट हल गर्नुहोस् ।
Solve by matrix method : $\frac{x+y}{5} = 1 = y - x$

15. यदि $x^2 + 2xy\text{cosec}\,\alpha + y^2 = 0$ ले प्रतिनिधित्व गर्ने एक जोडा सरल रेखाबिचको न्यूनकोण θ भए प्रमाणित गर्नुहोस् : $\theta = 90° - \alpha$.
If the acute angle between the pair of straight lines represented by $x^2 + 2xy\text{cosec}\alpha + y^2 = 0$ is θ then prove that: $\theta = 90° - \alpha$.

16. प्रमाणित गर्नुहोस् (Prove that):
cosec 2A + cosec 4A = cot A – cot 4A

17. यदि A + B + C = 90° भए प्रमाणित गर्नुहोस् :
If A + B + C = 90° then prove that:
2sin Acos A + sin 2B + sin 2C = 4cos A cos B cos C

18. 24 मिटर अग्लो एउटा स्तम्भको टुप्पोबाट एउटा खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा अवनति कोणहरू क्रमशः 45° र 60° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।
From the top of a tower 24 meter high, the angles of depression of the top and the foot of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

19. यदि मेट्रिक्स $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ ले एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$ मा स्थानान्तरण गर्दछ भने a, b, c र d का मानहरू पत्ता लगाउनुहोस् ।
If the matrix $\begin{bmatrix} a & 2 \\ b & 2 \end{bmatrix}$ transforms a unit square to the parallelogram $\begin{bmatrix} 0 & 4 & c & 2 \\ 0 & 1 & 3 & d \end{bmatrix}$, find the values of a, b, c and d.

20. तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नताको गणना गर्नुहोस् :
Calculate the mean deviation of the data given below from median:

Marks obtained	10-20	20-30	30-40	40-50	50-60
No. of students	5	4	5	4	2

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
Find the standard deviation of the data given below:

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	2	9	10	7	1

GROUP D [4 × 5 = 20]

22. यदि $f(x) = 3x - 2$ र $f \circ g(x) = 6x - 2$ भए $g \circ f(x) = 8$ हुँदा x को मान पत्ता लगाउनुहोस् । फलनहरू $f(x)$ र $g(x)$ लेखाचित्रमा कुन बिन्दुमा काटिन्छन् ?
If $f(x) = 3x - 2$ and $f \circ g(x) = 6x - 2$ then find the value of x such that $g \circ f(x) = 8$. In which point $f(x)$ and $g(x)$ intersect in the graph.

23. एउटा त्रिभुजका शीर्षबिन्दुहरू A(– 2, 1), B(2, 2) र C(6, – 2) छन् । A बाट BC मा खिचिएको उचाइ AD र मध्यिका AP का समीकरणहरू पत्ता लगाउनुहोस् ।
The vertices of a triangle are A(– 2, 1), B(2, 2) and C(6, – 2). Find the equation of the altitude AD and median AP drawn from A to BC.

24. त्रिभुजमा गुरुत्वकेन्द्र भनेको के हो ? ΔPQR मा मध्यरेखा QS र गुरुत्वकेन्द्र G छन् । यदि O उद्गम बिन्दु हो भने, प्रमाणित गर्नुहोस् :
What is the centroid of a triangle? In ΔPQR, QS is a median and G is the centroid. If O is the origin then prove that:
$$\overrightarrow{OG} = \frac{1}{3}\left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$$

25. शीर्षबिन्दुहरू A (3, 5), B(7, 2) र C(5, 8) भएको एउटा त्रिभुजलाई पहिले $x = -3$ मा परावर्तन गरी त्यसपछि प्राप्त प्रतिबिम्बलाई बिन्दु (0, 0) को वरिपरि घडीको दिशामा 90° परिक्रमण गरिएको छ । यसरी बन्ने प्रतिबिम्बका शीर्षबिन्दुहरू पत्ता लगाउनुहोस् र साथै ΔABC र यसको प्रतिबिम्बलाई लेखाचित्रमा खिच्नुहोस् ।
A triangle with vertices A (3, 5), B(7, 2) and C(5, 8) is reflected about the line $x = -3$ and then the image formed is rotated about the point (0, 0) through 90° in clockwise direction. Find the vertices of the images so formed and also draw ΔABC and its images on graph.

ANSWERS

1. (a) 1 (b) 7 **2.** (b) $ad - bc$
3. (b) $(x-a)^2 + (y-b)^2 = r^2$
4. (a) $\cos(\alpha - \beta) - \cos(\alpha + \beta)$ (b) 45°
5. (a) 0° or 180° (b) outside the circle **6.** (a) – 3 (b) 29 (c) $2y - 3x \le 6$ **7.** (a) 15 (b) –2
8. (a) 3 (b) $x^2 + y^2 - 6x - 6y + 8 = 0$ **9.** (c) 45°
10. (a) 60° (b) $4\vec{i} + 5\vec{j}$ (c) 75, 0.36
11. 10 and 40 or 40 and 10 **12.** 4, – 1 **13.** (a) 7.7, 7.97 (b) 8.3, 8.03 (c) 8, 8 (d) Yes **14.** 2, 3 **18.** 10.15 m
19. $a = 4$, $b = 1$, $c = 6$, $d = 2$ **20.** 11.3 **21.** 9.76
22. 2 and (2, 4) **23.** $x - y + 3 = 0$ and $x + 6y = 4$
25. A'(– 9, 5), B'(– 13, 2), C'(– 11, 8), A"(5, 9), B"(2, 13), C"(8, 11)

pustas.com.np

www.drmathnp.com

pusmas.com.np

SEE MODEL QUESTION 2076 SET 4
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पूर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1+1) = 10]

1. (a) समीकरण $y = ax^2 + bx + c$, $a \neq 0$ भएको पाराबोलाको शीर्षको x- निर्देशाङ्क के हुन्छ ?
What is the x- coordinate of vertex of parabola whose equation is $y = ax^2 + bx + c$, $a \neq 0$?

(b) असमानता $4x + 3y \geq 12$ मा बिन्दु (4, 5) पर्छ कि पर्दैन परीक्षण गर्नुहोस् ।
Test whether the point (4, 5) lies in the solution set of the inequality $4x + 3y \geq 12$ or not.

2. (a) "$-3 \leq x \leq 3$ मान्य हुने गरी सबै सङ्ख्याहरू x को समूह" का लागि अन्तराल सङ्केत (Interval notation) मा लेख्नुहोस् ।
Write the interval notation for "the set of all numbers x such that $-3 \leq x \leq 3$".

(b) मान पत्ता लगाउनुहोस् (Evaluate): $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

3. (a) पाराबोलाको परिभाषा दिनुहोस् । (Define parabola.)

(b) वृत्तको व्यास रूपको समीकरण लेख्नुहोस् ।
State the diameter form of equation of circle.

4. (a) $2 \sin x \cos y$ लाई sine को योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् ।
Express $2 \sin x \cos y$ in the sum or difference of sine.

(b) यदि $\sin\theta = \frac{\sqrt{3}}{2}$, $0° \leq \theta \leq 180°$ भए θ को मान पत्ता लगाउनुहोस् ।
If $\sin\theta = \frac{\sqrt{3}}{2}$, $0° \leq \theta \leq 180°$, find the value of θ.

5. (a) स्केलर गुणनफल ($\vec{a} \cdot \vec{b}$) पत्ता लगाउने सूत्र लेख्नुहोस् । जहाँ $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ छन् ।
Write down the formula to calculate scalar product ($\vec{a} \cdot \vec{b}$), where $\vec{a} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

(b) यदि उत्क्रम वृत्तको परिधिमा बिन्दु P पर्छ भने P को उत्क्रम बिन्दु कता पर्छ ?
Where is the inverse of P if P is on the circumference of circle of inversion?

GROUP B [13 × 2 = 26]

6. (a) यदि $h(x) = 2x + 3$ र $g(x) = 3x - 2$ भए $g \circ h(5)$ को मान पत्ता लगाउनुहोस् ।
If $h(x) = 2x + 3$ and $g(x) = 3x - 2$, find the value of $g \circ h(5)$.

(b) श्रेणी $3 + 6 + 12 + \ldots\ldots$ को कुनचाहिँ पद 192 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series $3 + 6 + 12 + \ldots\ldots$ is 192 ? Find it.

(c) श्रेणी $9 + 3 + 1 + \ldots + \frac{1}{27}$ को योगफल निकाल्नुहोस् ।
Find the sum of the series $9 + 3 + 1 + \ldots + \frac{1}{27}$.

7. (a) यदि मेट्रिक्स A को विपरित मेट्रिक्स $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ भए मेट्रिक्स A पत्ता लगाउनुहोस् ।
If an inverse matrix of a matrix A is $A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, find the matrix A.

(b) यदि मेट्रिक्स $M = \begin{bmatrix} -5 & -8 \\ p & 12 \end{bmatrix}$ को डिटरमिन्यान्ट 20 भए, p को मान पत्ता लगाउनुहोस् ।
If the determinant of matrix $M = \begin{bmatrix} -5 & -8 \\ p & 12 \end{bmatrix}$ is 20, find the value of p.

8. (a) एक जोडा सरल रेखाहरू $x = 3y$ र $3x = y$ लाई प्रतिनिधित्व गर्ने एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation which represents a pair of straight lines $x = 3y$ and $3x = y$.

(b) यदि वृत्त $x^2 + y^2 - 4x - 6y - k = 0$ को अर्धव्यास 4 एकाइ भए k को मान पत्ता लगाउनुहोस् ।
If the radius of the circle $x^2 + y^2 - 4x - 6y - k = 0$ is 4 units, find the value of k.

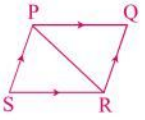
9. (a) यदि $\sin\frac{\theta}{3} = \frac{3}{5}$ भए $\sin\theta$ को मान निकाल्नुहोस् ।
If $\sin\frac{\theta}{3} = \frac{3}{5}$, find the value of $\sin\theta$.

(b) प्रमाणित गर्नुहोस् (Prove that):
$2 \sin 7\theta \cdot \cos 2\theta = \sin 9\theta + \sin 5\theta$

(c) हल गर्नुहोस् (Solve):
$\cos^2\theta - \cos\theta + \frac{1}{4} = 0$ $(0° \leq \theta \leq 90°)$

10. (a) यदि $10\vec{i} - 7\vec{j}$ र $a\vec{i} + 10\vec{j}$ आपसमा लम्ब भए a को मान पत्ता लगाउनुहोस् ।
If $10\vec{i} - 7\vec{j}$ and $a\vec{i} + 10\vec{j}$ are perpendicular to each other then find the value of a.

(b) दिइएको समानान्तर चतुर्भुजमा $\overrightarrow{SR} = 4\vec{i} - 2\vec{j}$ र $\overrightarrow{PR} = 6\vec{i} + 5\vec{j}$ भए $\overrightarrow{QR}$ पत्ता लगाउनुहोस् ।



In the given parallelogram $\overrightarrow{SR} = 4\vec{i} - 2\vec{j}$ and $\overrightarrow{PR} = 6\vec{i} + 5\vec{j}$, find $\overrightarrow{QR}$.

(c) एउटा तथ्याङ्कमा चतुर्थांशीय भिन्नता र यसको गुणाङ्क क्रमशः 15 र 3/7 छन् । तेस्रो चतुर्थांश पत्ता लगाउनुहोस् ।
In a data, the quartile deviation and its coefficient are 15 and $\frac{3}{7}$ respectively. Find the third quartile.

www.drmathnp.com

GROUP C [11 × 4 = 44]

11. यदि $f(x) = \frac{x}{2x-3}$ र $f(x) = f^{-1}$ भए x को मान निकाल्नुहोस् ।

If $f(x) = \frac{x}{2x-3}$ and $f(x) = f^{-1}(x)$, find the value of x.

12. P = $5x + 3y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् :

Find the maximum value of $P = 5x + 3y$ under the following constraints
$2x + y \leq 20$, $2x + 3y \leq 24$, $x \geq 0$ and $y \geq 0$

13. यदि $f(x) = 4x + 7$ एउटा वास्तविक मान भएको फलन भए

(a) $x = 1.9$ र 1.99 मा $f(x)$ का मानहरू के के हुन्छन् ?

(b) $x = 2.1$ र 2.01 मा $f(x)$ का मानहरू के के हुन्छन् ?

(c) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन्?

(d) के $x = 2$ मा $f(x)$ निरन्तर हुन्छ ? पत्ता लगाउनुहोस् ।

If $f(x) = 4x + 7$ is a real valued function,

(a) What are the values of $f(x)$ at $x = 1.9$ and 1.99?

(b) What are the values of $f(x)$ at $x = 2.1$ and 2.01?

(c) What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?

(d) Is the function continuous at $x = 2$?

14. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$4x - \frac{9}{y} + 11 = 0$, $\frac{6}{y} - 3x = 8$

15. समीकरण $6x^2 - xy - y^2 = 0$ ले प्रतिनिधित्व गर्ने बेग्लाबेग्लै रेखाहरूको समीकरण पत्ता लगाउनुहोस् । ती रेखाहरूबिचको कोण पनि निकाल्नुहोस् ।

Find the separate equations of the pair of lines represented by the equation $6x^2 - xy - y^2 = 0$. Also find the angle between them.

16. प्रमाणित गर्नुहोस् (Prove that):
$\sin^2 A - \cos^2 A.\cos 2B = \sin^2 B - \cos^2 B.\cos 2A$

17. यदि P + Q + R = 180° भए प्रमाणित गर्नुहोस् :

If P + Q + R = 180° then prove that:
$2 \sin \frac{P}{2} \cos \frac{P}{2} + \sin Q + \sin R = 4\cos \frac{P}{2} . \cos \frac{Q}{2} . \cos \frac{R}{2}$

18. कुनै एउटा बिन्दुबाट एक स्तम्भको टुप्पोको उन्नतांश कोण 45° अवलोकन गरियो र सो बिन्दुबाट 80 मिटर टाढा जाँदा सो कोण 30° पाइयो भने स्तम्भको उचाइ पत्ता लगाउनुहोस् ।

The angles of elevation of the top of a tower from a point was observed to be 45° and on walking 80 m away from that point, it was found to be 30°. Find the height of the tower.

19. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 3 & 4 & 2 \\ 0 & 1 & 3 & 2 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।

Find the 2 × 2 transformation matrix which transforms the unit square into a parallelogram $\begin{bmatrix} 0 & 3 & 4 & 2 \\ 0 & 1 & 3 & 2 \end{bmatrix}$.

20. तलको तथ्याङ्कबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस्:

From the following data, find the mean deviation and its coefficient.

Class-Interval	2-4	2-6	2-8	2-10
Frequency	4	9	11	12

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।

Find the standard deviation of the data given below:

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	5	3	2

GROUP D [4 × 5 = 20]

22. अभिव्यञ्जक $x^2 + bx + c$ लाई $(x + 1)$ ले भाग गर्दा शेष 1 रहन्छ र जब यसलाई $x - 2$ ले भाग गरिन्छ तब शेष 13 रहन्छ भने b र c का मानहरू पत्ता लगाउनुहोस् ।

The expression $x^2 + bx + c$ leaves the remainder 1 when it is divided by $(x + 1)$ and when it is divided by $(x - 2)$, the remainder is 13. Find the values of b and c.

23. यदि रेखा $my + x = cm$ रेखाहरू $x - 23 = -4y$ र $7x = 3y + 6$ को प्रतिच्छेदन बिन्दु भएर जान्छ र $5x - 4y = 6$ सँग समानान्तर हुन्छ भने m र c का मानहरू पत्ता लगाउनुहोस् ।

If the line $my + x = cm$ passes through the point of intersection of the lines $x - 23 = -4y$ and $7x = 3y + 6$ and parallel to the line $5x - 4y = 6$, then find the values of m and c.

24. यदि PQRS एउटा आयत भए भेक्टर विधिबाट प्रमाणित गर्नुहोस् :

If PQRS is a rectangle then prove by vector method:
PR = QS.

25. शीर्षबिन्दुहरू A(1, 2), B(4, − 1) र C(2, 5) भएको त्रिभुजलाई रेखाहरू $x = 5$ र $y = -2$ मा लगातार परावर्तन गर्दा प्राप्त हुने प्रतिबिम्बको निर्देशाङ्क लेखेर लेखाचित्रमा प्रस्तुत गर्नुहोस् । साथै यसरी हुने संयुक्त स्थानान्तरणलाई कुन एउटै स्थानान्तरणले जनाउन सकिन्छ, उल्लेख गर्नुहोस् ।

A triangle with vertices A(1, 2), B(4, − 1) and C(2, 5) is reflected successively in the lines $x = 5$ and $y = -2$. Find the stating coordinates and graphically represent the images under these transformations. State also the single transformation given by the combinations of these transformations.

ANSWERS

1. **(a)** $-\frac{b}{2a}$ **(b)** Yes, it lies. **2.** **(a)** $[-3, 3]$ **(b)** -2 **3.** **(b)** $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
4. **(a)** $\sin(A + B) + \sin(A - B)$ **(b)** 60°, 120°
5. **(a)** $\vec{a} . \vec{b} = a_1a_2 + b_1b_2$ **(b)** At the same point P
6. **(a)** 37 **(b)** 7^{th} term **(c)** $\frac{364}{27}$
7. **(a)** $\begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$ **(b)** 10 **8.** **(a)** $3x^2 - 10xy + 3y^2 = 0$
(b) $k = 3$ **9.** **(a)** $\frac{117}{125}$ **(c)** 60° **10.** **(a)** 7
(b) $\vec{i} + 7\vec{j}$ **(c)** 50 **11.** $x = 0, 2$
12. 51 at (9, 2) **13.** **(a)** 14.6, 14.96 **(b)** 15.4, 15.04
(c) 15, 15 **(d)** Yes **14.** $-2, 3$
15. $2x - y = 0$ and $3x + y = 0$, θ = 45° or 135° **18.** 109.28 m
19. $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ **20.** 133, 0.266 **21.** 12.36 **22.** $b = c = 3$
23. $\frac{4}{5}, -1$ **25.** A'(9, 2), B'(6, − 1), C'(8, 5), A"(9, − 6), B"(6, − 3), C"(8, − 9); Half turn rotation about (5, − 2)

pustas.com.np

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 5
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पूर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) सीमा रेखाको समीकरण $2x + 3y = 6$ र हल समूहमा बिन्दु (0, 0) भएको असमानता पत्ता लगाउनुहोस् ।
Find the inequality whose boundary line is $2x + 3y = 6$ and the solution set contains (0, 0).

(b) समीकरण $ax^2 + bx + c = 0$ भएको पाराबोलाको सममिति रेखाको समीकरण के हुन्छ ?
What is the equation of line of symmetry of the parabola whose equation is $ax^2 + bx + c = 0$?

2. (a) "$-4 < x < 4$ मान्य हुने गरी सबै सङ्ख्याहरू x को समूह" का लागि अन्तराल सङ्केत (Interval notation) मा लेख्नुहोस् ।
Write the interval notation for "the set of all numbers x such that $-4 < x < 4$ ".

(b) एकाइ मेट्रिक्सको डिटरमिन्यान्टको मान कति हुन्छ ?
What is the value of determinant of an identity matrix ?

3. (a) दीर्घ वृत्तको परिभाषा दिनुहोस् । (Define ellipse.)

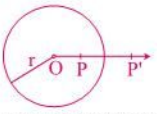
(b) $y = m_1x + c_1$ र $y = m_2x + c_2$ रेखाहरू बीचको कोण निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

4. (a) $2 \cos\theta \sin\theta$ लाई sine को योग वा अन्तरको रूपमा व्यक्त गर्नुहोस् ।
Express $2 \cos\theta \sin\theta$ in the sum or difference of sine.

(b) यदि $\cos A = \frac{\sqrt{3}}{2}, 0° \le A \le 180°$ भए A को मान पत्ता लगाउनुहोस् ।
If $\cos A = \frac{\sqrt{3}}{2}, 0° \le A \le 180°$, find the value of A.

5. (a) यदि X - अक्षतिरको एकाइ भेक्टर $\vec{i}$ भए $\vec{i}^2$ पत्ता लगाउनुहोस् ।
If $\vec{i}$ is unit vector along X-axis, what is the value of $\vec{i}^2$?

(b) सँगैको चित्रमा P को विपरीत P' छ । यदि $OP \times OP' = 25\ cm^2$ भए उत्क्रम वृत्तको अर्धव्यास पत्ता लगाउनुहोस् ।
In the adjoining figure, P' is the inverse of P. If $OP \times OP' = 25\ cm^2$ then find the radius of the inversion circle.



GROUP B [13 × 2 = 26]

6. (a) यदि $f(x) = 3x - 2$ र $f \circ g(x) = 6x - 2$ भए $g(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 3x - 2$ and $f \circ g(x) = 6x - 2$, find $g(x)$.

(b) श्रेणी $5 + 9 + 13 + \ldots$ को कुनचाहिँ पद 85 हुन्छ ? पत्ता लगाउनुहोस् ।
Which term of the series $5 + 9 + 13 + \ldots$ is 85? Find it.

(c) यदि गुणोत्तर श्रेणीको तेस्रो पद 3 छ भने यसका प्रथम पाँच पदहरूको गुणनफल निकाल्नुहोस् ।
If the third term of a geometric series is 3, find the product of its first five terms.

7. (a) यदि $E = \begin{bmatrix} -3 & 4 \\ 2 & -2a \end{bmatrix}$ र $|E| = 22$ भए a को मान पत्ता लगाउनुहोस् ।
If $E = \begin{bmatrix} -3 & 4 \\ 2 & -2a \end{bmatrix}$ and $|E| = 22$, find the value of a.

(b) यदि मेट्रिक्स $\begin{pmatrix} 2x & 7 \\ 5 & 9 \end{pmatrix}$ को विपरीत मेट्रिक्स $\begin{pmatrix} 9 & y \\ -5 & 4 \end{pmatrix}$ भए x र y का मानहरू निकाल्नुहोस् ।
If the inverse of the matrix $\begin{pmatrix} 2x & 7 \\ 5 & 9 \end{pmatrix}$ is the matrix $\begin{pmatrix} 9 & y \\ -5 & 4 \end{pmatrix}$, calculate the values of x and y.

8. (a) झुकावहरू क्रमशः $-\sqrt{3}$ र $\sqrt{3}$ भएका रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines having slopes $-\sqrt{3}$ and $\sqrt{3}$ respectively.

(b) यदि वृत्त $x^2 + y^2 - ax - by - 12 = 0$ को केन्द्र (2, 3) भए a र b का मानहरू निकाल्नुहोस् ।
If the centre of the circle $x^2 + y^2 - ax - by - 12 = 0$ is (2, 3), find the values of a and b.

9. (a) यदि $\tan\frac{A}{2} = \frac{3}{4}$ भए $\sin A$ को मान निकाल्नुहोस् ।
If $\tan\frac{A}{2} = \frac{3}{4}$ then find the value of $\sin A$.

(b) प्रमाणित गर्नुहोस् (Prove that):
$2\cos 70°. \cos 20° = \cos 50°$

(c) हल गर्नुहोस् (Solve) :
$\sin^2\theta - \sin\theta + \frac{1}{4} = 0 \ [0° \le \theta \le 90°]$

10. (a) यदि $\vec{a} = 4\vec{i} + 2\vec{j}$ र $\vec{b} = -\vec{i} + 2\vec{j}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = 4\vec{i} + 2\vec{j}$ and $\vec{b} = -\vec{i} + 2\vec{j}$, then find the angle between $\vec{a}$ and $\vec{b}$.

(b) A र B का स्थिति भेक्टरहरू क्रमशः $9\vec{i} + 7\vec{j}$ र $\vec{i} - 3\vec{j}$ छन् । यदि AB को मध्यबिन्दु M भए M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
The position vectors of A and B are $9\vec{i} + 7\vec{j}$ and $\vec{i} - 3\vec{j}$ respectively. If M is the mid-point of AB, find the position vector of M.

pusnas.com.np

www.drmathnp.com

(c) एउटा तथ्याङ्कको तेस्रो चतुर्थांश 15 छ । यदि यो तथ्याङ्कको चतुर्थांशीय भिन्नताको गुणाङ्क $\frac{1}{14}$ भए सो तथ्याङ्कको पहिलो चतुर्थांश र चतुर्थांशीय विस्तार पत्ता लगाउनुहोस् ।
The third quartile of a data is 15. If the coefficient of quartile deviation is $\frac{1}{14}$, find the first quartile and the inter-quartile range of the data.

GROUP C [11 × 4 = 44]

11. दुई फलनहरू $f(x) = \frac{2x+5}{8}$ र $g(x) = 3x - 4$ छन् । यदि $(f \circ g)^{-1}(x)$ एउटा एकात्मक फलन हो भने x को मान पत्ता लगाउनुहोस् ।
Two functions are $f(x) = \frac{2x+5}{8}$ and $g(x)$ $3x - 4$. If $(f \circ g)^{-1}(x)$ is an identity function, find the value of x.

12. $P = 3x + 2y$ को निम्नलिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।
Find the maximum value of $P = 3x + 2y$ in the following conditions.
$x + y \geq 0, x - y \leq 0, y \leq 2$ and $x \geq -1$

13. यदि $f(x) = \begin{cases} x+2, & 1 \leq x \leq 2 \\ 4x-4, & x > 2 \end{cases}$ भए
(a) $x = 1.99$ मा $f(x)$ को मान पत्ता लगाउनुहोस् ।
(b) $x = 2.01$ मा $f(x)$ को मान पत्ता लगाउनुहोस् ।
(c) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू के के हुन् ?
(d) के फलन $x = 2$ मा निरन्तर छ ?
If $f(x) = \begin{cases} x+2, & 1 \leq x \leq 2 \\ 4x-4, & x > 2 \end{cases}$ then,
(a) For $x = 1.99$, find the value of $f(x)$.
(b) For $x = 2.01$, find the value of $f(x)$.
(c) What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?
(d) Is the function continuous at $x = 2$?

14. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$2x - 5y = 1, 7x + 3y = 24$

15. केन्द्रबिन्दु (1, 2) भएको र रेखाहरू $x + 2y = 3$ र $3x + y = 4$ को प्रतिच्छेदन बिन्दु भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle having centre (1, 2) and passing through the point of intersection of the lines $x + 2y = 3$ and $3x + y = 4$.

16. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sec 4\theta - 1}{\sec 2\theta - 1} = \tan 4\theta . \cot \theta$

17. यदि $P + Q + R = 180°$ भए प्रमाणित गर्नुहोस्:
If $P + Q + R = 180°$, then prove that:
$\cos(Q + R - P) + \cos(R + P - Q) + \cos(P + Q - R)$
$= \sin^2 P + 4\cos P . \cos Q . \cos R + \cos^2 P$

18. एउटा चउरमा रहेको कुनै स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 60° भएको समयमा भन्दा सूर्यको उचाइ 45° भएको समयमा 45 मि बढी लामो हुन्छ भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।
The shadow of a tower on the ground is found to be 45 m longer when the sun's altitude is 45° than when it is 60°. What will be the height of the tower? Find it.

19. शीर्षबिन्दुहरू A(2, 0), B(5, 1), C(4, 4) र D(1, 3) भएको एउटा वर्ग ABCD लाई 2 × 2 मेट्रिक्सले समानान्तर चतुर्भुज A'B'C'D' मा स्थानान्तरण गर्दा समानान्तर चतुर्भुजका शीर्षबिन्दुहरू A'(2, 2), B'(7, 3), C'(12, – 4) र D'(7, – 5) हुन्छन् भने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
A square ABCD with vertices A(2, 0), B(5, 1), C(4, 4) and D(1, 3) is mapped onto a parallelogram A'B'C'D' by a 2 × 2 matrix so that the vertices of the parallelogram are A'(2, 2), B'(7, 3), C'(12, – 4) and D'(7, – 5). Find the 2 × 2 transformation matrix.

20. 0-4 को एउटा वर्गान्तर लिएर बारम्बारता तालिका बनाउनुहोस् र मध्यक भिन्नता पत्ता लगाउनुहोस् :
Construct a frequency distribution table taking 0 - 4 as a class interval and find mean deviation:
1, 3, 2, 3, 4, 5, 6, 7, 5, 7, 10, 9, 15

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस्:
Find the standard deviation of the data given below:

Class Interval	10-20	20-30	30-40	40-50	50-60
Frequency	4	10	12	8	6

GROUP D [4 × 5 = 20]

22. यदि $p(x) = 6x^3 + ax^2 - 10x - 3$ को एउटा गुणनखण्ड $x + 1$ भए अन्य गुणनखण्डहरू पत्ता लगाउनुहोस् ।
If $p(x) = 6x^3 + ax^2 - 10x - 3$ has a factor $x + 1$, find the other factors.

23. समीकरणहरू $(3b^2 - a^2)x^2 - 8ab\,xy + (3a^2 - b^2)y^2 = 0$ ले जनाउने रेखाहरूबिचको न्यूनकोण 60° हुन्छ भनी प्रमाणित गर्नुहोस् ।
Prove that the acute angle between the straight lines represented by the equations $(3b^2 - a^2)x^2 - 8ab\,xy + (3a^2 - b^2)y^2 = 0$ is 60°.

24. समद्विबाहु त्रिभुजको आधारको मध्यबिन्दु र शीर्षबिन्दु जोड्ने रेखा सो त्रिभुजको आधारमा लम्ब हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the line joining the mid-point of the base and the vertex of an isosceles triangle is perpendicular to the base.

25. ΔABC का शीर्षबिन्दुहरू A(2, – 1), B(– 4, 2) र C(– 3, – 2) छन् । भेक्टर $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ ले विस्थापन गरेपछि $x = 2$ मा परावर्तन गराउँदा हुने ΔABC को प्रतिबिम्ब पत्ता लगाउनुहोस् ।
A(2, – 1), B(– 4, 2) and C(– 3, – 2) are the vertices of ΔABC. Find the image of ΔABC under the translation by the vector $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ followed by reflection on $x = 2$. Also, plot all the triangles on the same graph paper.

ANSWERS

1. (a) $2x + 3y \leq 6$ (b) $x = -\frac{b}{2a}$ **2.** (a) (– 4, 4) (b) 1
3. (b) $\theta = \tan^{-1}\left(\pm\frac{m_1 - m_2}{-1 + m_1 m_2}\right)$
4. (a) $\sin(A + B) - \sin(A - B)$ (b) 30° **5.** (a) 1
(b) 5 cm **6.** (a) $2x$ (b) 21st term (c) 243 **7.** (a) 5
(b) $x = 2, y = -7$ **8.** (a) 60° (b) $a = 4, b = 6$
9. (a) $\frac{24}{25}$ (c) 30° **10.** (a) 90° (b) $5\vec{i} + 2\vec{j}$
(c) 13 and 2 **11.** $-\frac{3}{2}$ **12.** 10 at (2, 2)
13. (a) 3.99 (b) 4.04 (c) 4, 4 (d) Yes **14.** 3, 1
15. $x^2 + y^2 - 2x - 4y + 4 = 0$ **18.** 106.47 m
19. $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ **20.** 2.46 **21.** 12.03
22. $(3x + 1)(2x - 3)$ **25.** A"(4, – 2), B"(10, 1), C"(9, – 3)

www.drmathnp.com

puspas.com.np

SEE MODEL QUESTION 2076 SET 6
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) यदि $f : A \to B$ र $g : B \to C$ दुई फलनहरू हुन् भने संयुक्त फलन $A \to C$ लाई के ले जनाउँदछ ?
If $f : A \to B$ and $g : B \to C$ be the two functions then what will denote the composite function from $A \to C$?

(b) यदि बहुपदीय $p(x)$ को एउटा गुणनखण्ड $(x - m)$ भए $p(m)$ को मान कति होला ?
If $(x - m)$ is a factor of polynomial $p(x)$, what is the value of $p(m)$?

2. (a) $\{x : -14 \le x < -11\}$ का लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् ।
Write the interval notation for $\{x : -14 \le x < -11\}$.

(b) यदि $\begin{vmatrix} 1 & 0 \\ 2 & x \end{vmatrix} = 0$ भए x को मान पत्ता लगाउनुहोस् ।
If $\begin{vmatrix} 1 & 0 \\ 2 & x \end{vmatrix} = 0$, find the value of x.

3. (a) अतिपरावलयको परिभाषा दिनुहोस् । (Define hyperbola.)

(b) कस्तो अवस्थामा दुई सीधा रेखाहरू लम्ब हुन्छन् ?
What is the condition under which two straight lines will be perpendicular?

4. (a) $\sin\alpha + \sin\beta$ लाई sine वा cosine को गुणनफलको रूपमा बदल्नुहोस् ।
Convert $\sin\alpha + \sin\beta$ in terms of product of sine or cosine.

(b) यदि $\cos\theta = \frac{1}{2}$, $0° \le \theta \le 180°$ भए θ को मान पत्ता लगाउनुहोस् ।
If $\cos\theta = \frac{1}{2}$, $0° \le \theta \le 180°$, find the value of θ.

5. (a) खण्ड सूत्र $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$ मा m र n ले के के जनाउँदछन् ?
What do m and n represent in the section formula $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$?

(b) X - अक्षको परावर्तनसँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write down the matrix associated with reflection in X-axis.

GROUP B [13 × 2 = 26]

6 (a) यदि बहुपदीय $x^3 + 6x^2 + kx + 10$ लाई $(x + 2)$ ले भाग गर्दा शेष 4 हुन्छ भने k को मान शेष साध्य प्रयोग गरेर पत्ता लगाउनुहोस् ।
If the polynomial $x^3 + 6x^2 + kx + 10$ is divided by $(x + 2)$, the remainder is 4. Find the value of k using the remainder theorem.

(b) यदि $\frac{1}{3}$, p, q, 9 गुणोत्तरीय अनुक्रममा भए p र q का मानहरू पत्ता लगाउनुहोस् ।
If $\frac{1}{3}$, p, q, 9 are in the geometric sequence, find the values of p and q.

(c) दिइएको असमानता $4x + 5y \ge -20$ लाई लेखाचित्रमा देखाउनुहोस् ।
Show the given inequality $4x + 5y \ge -20$ on a graph.

7. (a) यदि $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ र $B = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ भए $|AB|$ को मान पत्ता लगाउनुहोस् ।
If $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, find the value of $|AB|$.

(b) यदि $A = \begin{bmatrix} 1 & -2 \\ 0 & x \end{bmatrix}$ को विपरीत मेट्रिक्स $B = \begin{bmatrix} 1 & 4 \\ y & 2 \end{bmatrix}$ भए x र y का मानहरू पत्ता लगाउनुहोस् ।
If the inverse of matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & x \end{bmatrix}$ is the matrix $B = \begin{bmatrix} 1 & 4 \\ y & 2 \end{bmatrix}$, determine the values of x and y.

8. (a) यदि रेखाहरू $2x - 8y + 6 = 0$ र $px - 12y - 4 = 0$ एक आपसमा समानान्तर भए, p को मान पत्ता लगाउनुहोस् ।
If the lines $2x - 8y + 6 = 0$ and $px - 12y - 4 = 0$ are parallel to each other, find the value of p.

(b) समीकरण $x^2 + y^2 - mx + 6y - 12 = 0$ भएको वृत्तको केन्द्रबिन्दु $(-2, -3)$ भए m को मान पत्ता लगाउनुहोस् ।
Find the value of m when the centre of a circle with equation $x^2 + y^2 - mx + 6y - 12 = 0$ is $(-2, -3)$.

9. (a) प्रमाणित गर्नुहोस् (Prove that):
$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$

(b) प्रमाणित गर्नुहोस् (Prove that):
$\frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°} = \cot 35°$

(c) हल गर्नुहोस् (Solve): $4 - 3\sec^2\theta = 0$ $(0° \le \theta \le 90°)$

10. (a) यदि $\vec{p}.\vec{q} = 18\sqrt{3}$, $|\vec{p}| = 6$ र $|\vec{q}| = 6$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{p}.\vec{q} = 18\sqrt{3}$, $|\vec{p}| = 6$ and $|\vec{q}| = 6$, find the angle between $\vec{p}$ and $\vec{q}$.

(b) रेखाखण्ड EF को मध्यबिन्दुको स्थिति भेक्टर $4\vec{i} + 7\vec{j}$ र बिन्दु E को स्थिति भेक्टर $-3\vec{i} - 4\vec{j}$ छन् भने बिन्दु F को स्थिति भेक्टर पत्ता लगाउनुहोस् ।

www.drmathnp.com

pushpas.com.np

If the position vector of the midpoint of the line segment EF is $4\vec{i} + 7\vec{j}$ and the position vector of the point E is $-3\vec{i} - 4\vec{j}$, find the position vector of the point F.

(c) एउटा निरन्तर श्रेणीमा मध्यिका 50, N = 8 र $\sum f(m - md) = 59$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series median 50, N = 8 and $\sum f(m - md) = 59$. Find the mean deviation from median and its coefficient.

GROUP C [11 × 4 = 44]

11. यदि $f(x) = 2x + 5$, $g(x) = \frac{3x-1}{2}$ र $fg^{-1}(x) = f(x)$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x + 5$, $g(x) = \frac{3x-1}{2}$ and $fg^{-1}(x) = f(x)$ then find the value of x.

12. लेखाचित्रद्वारा हल गर्नुहोस् (Solve graphically):
$x^2 - x - 2 = 0$

13. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3, & x > 0 \end{cases}$ को $x = 0$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of $f(x) = \begin{cases} 3-x, & x \le 0 \\ 3, & x > 0 \end{cases}$ at $x = 0$ by calculating left hand limit, right hand limit and functional values.

14. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\frac{4}{x} + \frac{3}{y} = 1, \frac{3}{x} - \frac{2}{y} = \frac{1}{24}$

15. बिन्दुहरू (3, 2) र (5, 4) भएर जाने र केन्द्र $3x - 2y - 1 = 0$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the circle which passes through the points (3, 2) and (5, 4) and its centre lies on the line $3x - 2y - 1 = 0$.

16. यदि $A + B + C = 180°$ भए प्रमाणित गर्नुहोस्:
If $A + B + C = 180°$ then prove that:
$\cos(B + C - A) + \cos(C + A - B) + \cos(A + B - C) = \sec^2 A + 4\cos A . \cos B . \cos C - \tan^2 A$

17. हल गर्नुहोस् (Solve):
$2\sqrt{3}\cos^2\theta = \sin\theta \ (0° \le \theta \le 360°)$

18. समतल सतहमा ठड्याइएको एउटा स्तम्भको छायाको लम्बाइ सूर्यको उचाइ 45° भएको समयमा भन्दा सूर्यको उचाइ 30° भएको समयमा 60 फिट बढी लामो पाइयो भने सो स्तम्भको उचाइ कति होला ? पत्ता लगाउनुहोस् ।
The shadow of a tower standing on a plain is found to be 60 feet longer when the sun's altitude is 30° than when it is 45°. What will be the height of the tower ? Find it.

19. E ले केन्द्रबिन्दु (– 3, – 4) बाट हुने नापो 2 भएको विस्तारीकरण र R ले $y = 0$ मा परावर्तन हुने जनाउँछ । शीर्षबिन्दुहरू A(2, 0), B(3, 1) र C(1, 1) भएको ΔABC लाई संयुक्त स्थानान्तरण EOR द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै ग्राफमा खिच्नुहोस् ।
E denotes enlargement about the centre (– 3, – 4) with a scale factor of 2 and R denotes a reflection on the line $y = 0$. ΔABC having the vertices A(2, 0), B(3, 1) and C(1, 1) is mapped under the combined transformation EOR. Find the image of ΔABC and draw two figure on the same graph paper.

20. चतुर्थांशीय भिन्नता (अर्ध चतुर्थांशीय विस्तार) र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find Q.D. (semi inter-quartile range) and its coefficient:

Marks	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	10	5	3	5	2	2

21. तल दिइएको आँकडाबाट स्तरीय भिन्नता पत्ता लगाउनुहोस् :
Calculate the standard deviation of the data given below:

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	6	5	4	3

GROUP D [4 × 5 = 20]

22. एउटा मानिसले रु. 975 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 5 ले कम छ । पहिलो किस्ता वापतको रकम रु. 100 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 975 in monthly installments, each installment being less than the former by Rs 5. The amount of first installment is Rs 100. In how many installments will the entire amount be paid ? Give reason.

23. रेखाहरू $x^2 - xy - 2y^2 = 0$ सँग लम्ब र उद्गम बिन्दुबाट जाने रेखाहरूको एकल समीकरण निकाल्नुहोस् ।
Find the single equation of a pair of straight lines passing through the origin and perpendicular to the line pair represented by $x^2 - xy - 2y^2 = 0$.

24. समबाहु चतुर्भुज PQRS मा PQ = QR = RS = SP भए भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् : PR ⊥ QS
In a rhombus PQRS, PQ = QR = RS = SP. Prove by vector method: PR ⊥ QS.

25. दिइएको बिन्दु (4, 5) को वृत्त $x^2 + y^2 - 4x - 6y - 3 = 0$ को आधारमा उत्क्रम बिन्दु पत्ता लगाउनुहोस् । साथै लेखाचित्रमा पनि देखाउनुहोस् ।
Find the inversion point of the given point (4, 5) with respect to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$. Also, show in the graph.

ANSWERS

1. (a) $g°f$ (b) 0 **2.** (a) $[-14, -11)$ (b) 0
3. (b) $m_1 \times m_2 = -1$ **4.** (a) $2\sin\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right)$
(b) 60° **5.** (a) Ratios of the sections
(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **6.** (a) 11 (b) 1, 3
7. (a) – 2 (b) $x = \frac{1}{2}, y = 0$ **8.** (a) 3 (b) – 4
9. (c) 30° **10.** (a) 30° (b) $11\vec{i} + 18\vec{j}$
(c) 7.38, 0.15 **11.** 1 **12.** 2, – 1 **13.** Continuous
14. 8, 6 **15.** $x^2 + y^2 - 6x - 8y + 21 = 0$
17. 60°, 120° **18.** 81.96 m
19. A'(2, 0), B'(3, – 1), C'(1, – 1), A"(7, 4), B"(9, 2), C"(5, 2)
20. 28.87, 0.44 **21.** 12.24 **22.** 15
23. $2x^2 + xy + y^2 = 0$ **25.** (6, 7)

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 7
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) कस्तो अवस्थामा कुनै फलनको विपरीत फलन सम्भावित हुन्छ ?
Under what condition the inverse function of a function is possible?

(b) गुणनखण्ड साध्य उल्लेख गर्नुहोस् ।
State the Factor Theorem.

2. (a) $\{x : 6 < x \leq 20\}$ का लागि अन्तराल सङ्केत (Interval notation) लेख्नुहोस् ।
Write the interval notation for $\{x : 6 < x \leq 20\}$.

(b) कस्तो अवस्थामा एउटा मेट्रिक्सको विपरीत सम्भव हुन्छ ?
Under what condition, the inverse of a matrix is possible?

3. (a) यदि प्रतिच्छेदित सतह समकोणी सोलीको आधार सँग समानान्तर भए कुन शाङ्कि क भाग बन्दछ ?
If the intersection plane is parallel to the base of cone then what conic does it form?

(b) एउटा रेखा ℓ_1 को झुकाव $\frac{3}{4}$ छ । रेखा ℓ_1 सँग समानान्तर हुने रेखाको झुकाव पत्ता लगाउनुहोस् ।
The slope of a line (ℓ_1) is $\frac{3}{4}$. Find the slope of another line parallel to the line (ℓ_1).

4. (a) अपवर्त्य कोण भनेको के हो ?
What is multiple angle?

(b) यदि $A + B + C = \pi^c$ भए $\cos(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\cos(A + B)$ in terms of angle C.

5. (a) भित्री विभाजन सम्बन्धी खण्ड सूत्र लेख्नुहोस् ।
Write the section formula of internal division.

(b) एकात्मक स्थानान्तरण सँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
Write down the matrix associated with identity transformation.

GROUP B [13 × 2 = 26]

6. (a) यदि $x^3 - mx^2 - 4x + m + 12$ को एउटा गुणनखण्ड $(x - m)$ भए m को मान निकाल्नुहोस् ।
If $(x - m)$ is a factor of $x^3 - mx^2 - 4x + m + 12$, calculate the value of m.

(b) पहिलो पद 2 र तेस्रो पद 242 भएको गुणोत्तर अनुक्रमको समान अनुपात पत्ता लगाउनुहोस् ।
Find the common ratio of a geometric sequence, whose first term is 2 and the third term is 242.

(c) $(2x - 1) \geq \frac{1}{2}(x + 13)$ लाई लेखाचित्रमा खिच्नुहोस् र यसका मानहरू जनाउने भाग छायाँ पारी प्रस्तुत गर्नुहोस् ।
Draw $(2x - 1) \geq \frac{1}{2}(x + 13)$ on a graph and shade the region which represents its solution.

7. (a) यदि मेट्रिक्स $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ र मेट्रिक्स $B = \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$ भए मेट्रिक्स AB को डिटरमिनान्ट पत्ता लगाउनुहोस् ।
If matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 2 & -3 \end{bmatrix}$, find the determinant of AB.

(b) यदि मेट्रिक्स $\begin{pmatrix} m & 4 \\ -1 & 3 \end{pmatrix}$ र मेट्रिक्स $\begin{pmatrix} 3 & -4 \\ 1 & m \end{pmatrix}$ एक अर्काका विपरीत मेट्रिक्स भए m को मान पत्ता लगाउनुहोस् ।
If matrix $\begin{pmatrix} m & 4 \\ -1 & 3 \end{pmatrix}$ and matrix $\begin{pmatrix} 3 & -4 \\ 1 & m \end{pmatrix}$ are inverse to each other, find the value of m.

8. (a) यदि दुईओटा सरल रेखाहरू $px + qy + r = 0$ र $\ell x - my + n = 0$ एक आपसमा लम्ब भए, $p\ell = qm$ हुन्छ भनी देखाउनुहोस् ।
If two straight lines $px + qy + r = 0$ and $\ell x - my + n = 0$ are perpendicular to each other, show that $p\ell = qm$.

(b) केन्द्र $(1, -2)$ र व्यास $2\sqrt{5}$ एकाइ भएको एउटा वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of a circle having centre $(1, -2)$ and the diameter $2\sqrt{5}$ units.

9. (a) प्रमाणित गर्नुहोस् (Prove that):
$$\frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} = \tan\frac{A}{2}$$

(b) प्रमाणित गर्नुहोस् (Prove that):
$$\frac{\cos A - \cos 5A}{\sin 5A - \sin A} = \tan 3A$$

(c) हल गर्नुहोस् (Solve): $1 - \tan^2\theta = -2 \; (0° \leq \theta \leq 90°)$

10. (a) यदि $\vec{p} + \vec{q} + \vec{r} = 0$, $|\vec{p}| = 6$, $|\vec{q}| = 7$ र $|\vec{r}| = \sqrt{127}$ भए $\vec{p}$ र $\vec{q}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{p} + \vec{q} + \vec{r} = 0$, $|\vec{p}| = 6$, $|\vec{q}| = 7$ and $|\vec{r}| = \sqrt{127}$, find the angle between $\vec{p}$ and $\vec{q}$.

(b) यदि M र N का स्थिति भेक्टरहरू क्रमशः $9\vec{i} + 3\vec{j}$ र $3\vec{i} + 5\vec{j}$ छन् भने $\overrightarrow{MP} = \overrightarrow{PN}$ हुने गरी एउटा बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
If the position vectors of M and N are $9\vec{i} + 3\vec{j}$ and $3\vec{i} + 5\vec{j}$ respectively, find the position vector of a point P such that $\overrightarrow{MP} = \overrightarrow{PN}$.

puspas.com.np

www.drmathnp.com

(c) एउटा निरन्तर श्रेणीमा $\sum fm = 2400$, N = 100 र $\sum f(m - \bar{x}) = 448$ भए मध्यकबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।

In a continuous series $\sum fm = 2400$, N = 100 and $\sum f(m - \bar{x}) = 448$ then find mean deviation from mean and its coefficient.

GROUP C [11 × 4 = 44]

11. यदि $f(x) = \frac{x+3}{2}$ र $g(x) = 2x - 3$ भए $f^{-1} \circ f(x) = f \circ g(x)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

If $f(x) = \frac{x+3}{2}$ and $g(x) = 2x - 3$, prove that $f^{-1} \circ f(x) = f \circ g(x)$.

12. वर्ग समीकरण $x^2 - 3x + 2 = 0$ लाई लेखा चित्रद्वारा हल गर्नुहोस् ।
Solve graphically the quadratic equation $x^2 - 3x + 2 = 0$.

13. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 4x - 1, & x < 1 \\ 7x, & x \geq 1 \end{cases}$ को $x = 1$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of $f(x) = \begin{cases} 4x - 1, & x < 1 \\ 7x, & x \geq 1 \end{cases}$ at $x = 1$ by calculating left hand limit, right hand limit and functional values.

14. क्रामरको नियमबाट हल गर्नुहोस् (Solve by the Cramer's rule): $2x = 3(6 - y)$, $3x = 2y + 1$

15. वृत्त $2x^2 + 2y^2 + 4x - 2y + 1 = 0$ सँग समकेन्द्रीत हुने र बिन्दु (4, – 2) भएर जाने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle concentric with the circle $2x^2 + 2y^2 + 4x - 2y + 1 = 0$ and passing through the point (4, – 2).

16. यदि A + B + C = 180° भए प्रमाणित गर्नुहोस्
If A + B + C = 180°, Prove that:
$2 \sin \frac{B}{2} \cos \frac{B}{2} + \sin C = 4 \sin \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

17. हल गर्नुहोस् (Solve): $[0° \leq \theta \leq 360°]$
$\sqrt{3} \sin \theta + \cos \theta = \sqrt{2}$

18. एउटा धरहराको टुप्पोलाई यसको फेदबाट 27 m र 75 m दूरीमा एकैतिरबाट अवलोकन गर्दा उन्नतांशकोणहरू समपूरक पाइयो भने उक्त धरहराको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of a tower observed from 27 m and 75 m away from its foot on the same side are found to be complementary. Find the height of the tower.

19. P(3, 4), Q(2, 1) र R(4, 2) शीर्षबिन्दुहरू भएको ΔPQR लाई $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखेर ΔPQR र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
ΔPQR having the vertices P(3, 4), Q(2, 1) and R(4, 2) is translated by $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. The image so formed is enlarged by E[(0, 0), 2]. Writing the co-ordinates of the vertices of images thus obtained, represent ΔPQR and its images in the same graph paper.

20. चतुर्थांशीय भिन्नता (अर्ध चतुर्थांशीय विस्तार) र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find Q.D. (semi inter-quartile range) and its coefficient :

C.I.	20-30	30-40	40-50	50-60	60-70
f.	8	16	4	4	3

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
Find the standard deviation of the data give below.

Class interval	25-35	35-45	45-55	55-65	65-75
Frequency	5	4	6	7	3

GROUP D [4 × 5 = 20]

22. एउटा मानिसले रु. 45750 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 50 ले बढी छ । पहिलो किस्ता वापतको रकम रु. 800 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।
A person pays a loan of Rs 45750 in monthly installments, each installment being more than the former by Rs 50. The amount of first installment is Rs 800. In how many installments will the entire amount be paid ? Give reason.

23. समीकरण $6x^2 + xy - y^2 - 24x + 8y = 0$ ले प्रतिनिधित्त्व गरेका दुई फरक रेखाका समीकरणहरू र तिनीहरू बिचको कोण पनि निकाल्नुहोस् ।
Find two separate equations and the angle between them as represented by the second degree equation: $6x^2 + xy - y^2 - 24x + 8y = 0$

24. यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ र रेखाखण्ड AB लाई बाहिरपट्टीबाट बिन्दु M ले $m : n$ को अनुपातमा विभाजन गर्दछ भने, प्रमाणित गर्नुहोस्:
If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and the point M divides the line segment AB externally in the ratio of $m : n$ then prove that : $\overrightarrow{OM} = \frac{m\vec{b} - n\vec{a}}{m - n}$

25. यदि उद्गम बिन्दु केन्द्र हुने वृत्तको आधारमा एउटा बिन्दु A(3, 4) को उत्क्रम बिन्दु A'(12, 16) भए सो वृत्तको व्यास पत्ता लगाउनुहोस् । साथै बिन्दु B(6, 8) को उत्क्रम बिन्दु पनि सोही वृत्त अनुसार पत्ता लगाउनुहोस् ।
If the inversion of a point A(3, 4) with respect to a circle having centre as origin is A'(12, 16), find the diameter of the circle. Also find the inversion point of B(6, 8) with respect to the circle.

ANSWERS

1. (a) one to one onto **2.** (a) (6, 20] (b) Determinant ≠ 0 **3.** (a) Circle (b) $-\frac{4}{3}$
4. (b) – cos C **5.** (a) $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m + n}$ (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **6.** (a) 4 (b) ±11 **7.** (a) 40 (b) – 1 **8.** (b) $x^2 + y^2 - 2x + 4y = 0$ **9.** (c) 60°
10. (a) 60° (b) $6\vec{i} + 4\vec{j}$ (c) 4.48, 0.187
12. 2, 1 **13.** Discontinuous **14.** 3, 4
15. $x^2 + y^2 + 2x - y - 30 = 0$ **17.** 15° or 105° **18.** 45 m
19. P'(1, 7), Q'(0, 4), R'(2, 5), P''(2, 14), Q''(0, 8), R''(4, 10)
20. 7.58, 0.199 **21.** 13.11 **22.** 30
23. $3x - y = 0$ and $2x + y = 8$; 45° **25.** 20 units, (6, 8)

pustas.com.np

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 8
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) यदि $f = \{(2, -4)\}$, भए f^{-1} कति हुन्छ ?
If $f = \{(2, -4)\}$, what is f^{-1}?

   (b) शेष साध्य उल्लेख गर्नुहोस् ।
State Remainder Theorem.

2. (a) बायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् ।
Write down the notational representation of left hand limit.

   (b) A, B र X तीनओटा मेट्रिक्सहरू छन् । यदि AX = B र $|A| \neq 0$ भए X को मान पत्ता लगाउनुहोस् ।
A, B and X are three matrices. If AX = B and $|A| \neq 0$, what is the value of X?

3. (a) यदि प्रतिच्छेदित सतह सोलीको जेनेरेटर सँग समानान्तर भए कुन शाङ्कुिक भाग बन्दछ ?
If the intersection plane is parallel to the generator of cone then what conic does it form?

   (b) कस्तो अवस्थामा जोडा सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ लम्ब हुन्छन् ?
Under what condition, the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will be perpendicular?

4. (a) cos 2A लाई cos A को रूपमा लेख्नुहोस् ।
Convert cos 2A in terms of cos A.

   (b) यदि $A + B + C = \pi^c$ भए sin (B + A) लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express sin (B + A) in terms of angle C.

5. (a) बाहिरी विभाजन सम्बन्धी खण्ड सूत्र उल्लेख गर्नुहोस् ।
State the section formula of external division.

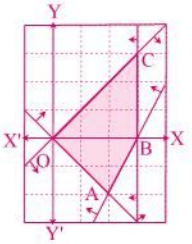
   (b) नापो k र केन्द्र (0, 0) भएको विस्तारीकरण सँग सम्बन्धित मेट्रिक्स लेख्नुहोस् ।
What is the matrix to represent an enlargement of scale factor k with centre (0, 0).

GROUP B [13 × 2 = 26]

6. (a) यदि $f(x) = 3x$, $g(x) = x + 2$ र $f \circ g(x) = 18$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = 3x$, $g(x) = x + 2$ and $f \circ g(x) = 18$, find the value of x.

   (b) गुणोत्तर अनुक्रम 2, 4, 8, 16 ..... को पन्ध्रौं पद पत्ता लगाउनुहोस् ।
Find the fifteen (15^{th}) term of a geometric sequence 2, 4, 8, 16, ...

   (c) रङ्ग्याइएको बहुभुजीय क्षेत्रको आधारमा $Z = 2x + 3y$ को अधिकतम र न्यूनतम मानहरू पत्ता लगाउनुहोस्:
Find the maximum and minimum values of $Z = 2x + 3y$ over the given shaded polygonal region.



7. (a) यदि $P = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ भए P^{-1} पत्ता लगाउनुहोस् ।
If $P = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, find P^{-1}.

   (b) यदि मेट्रिक्स $P = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$ र मेट्रिक्स $Q = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ भए मेट्रिक्स PQ को डिटरमिनान्ट पत्ता लगाउनुहोस् ।
If matrix $P = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$ and matrix $Q = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$, find the determinant of the matrix PQ.

8. (a) यदि बिन्दुहरू $(4, -p)$ र $(-2, 6)$ भएर जाने रेखा समीकरण $2y + 3x = -3$ भएको रेखासँग समानान्तर छ भने p को मान पत्ता लगाउनुहोस् ।
If a line passing through the points $(4, -p)$ and $(-2, 6)$ is parallel to the line having an equation $2y + 3x = -3$, find the value of p.

   (b) यदि समीकरण $mx^2 - 5xy - 6y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरू एक आपसमा लम्ब भए m को मान पत्ता लगाउनुहोस् ।
If the pair of lines represented by an equation $mx^2 - 5xy - 6y^2 = 0$ are perpendicular to each other, find the value of m.

9. (a) प्रमाणित गर्नुहोस् (Prove that):
$\text{cosec}\,\theta - \cot\theta = \tan\frac{\theta}{2}$

   (b) प्रमाणित गर्नुहोस् (Prove that):
$\frac{\cos 40° - \sin 30°}{\sin 60° - \cos 50°} = \tan 50°$

   (c) हल गर्नुहोस् (Solve): $\sin x - \sin 2x = 0$ $[0° \leq x \leq \frac{\pi^c}{2}]$

10. (a) एकाइ भेक्टर $\vec{i}$ र $\vec{a} = \sqrt{3}\,\vec{i} + \vec{j}$ बिचको कोण निकाल्नुहोस् ।
Find the angle between unit vector $\vec{i}$ and $\vec{a} = \sqrt{3}\vec{i} + \vec{j}$.

   (b) ΔABC को भुजा BC को मध्यबिन्दु D छ । यदि बिन्दुहरू A र D का स्थिति भेक्टर क्रमश: $3\vec{i} + 5\vec{j}$ र $3\vec{i} - 4\vec{j}$ भए ΔABC को भार केन्द्र (G) को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
D is the mid-point of the side BC of ΔABC. If the position vectors of the points A and D are $3\vec{i} + 5\vec{j}$ and $3\vec{i} - 4\vec{j}$ respectively, find the position vector of the centroid (G) of ΔABC.

   (c) एउटा निरन्तर श्रेणीमा मध्यिका 12, N = 100 र $\sum f(m - md.) = 224$ भए मध्यिकाबाट मध्यक भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series median 12, N = 100 and $\sum f(m - md.) = 224$. Find mean deviation from mean and its coefficient.

puspas.com.np

www.drmathnp.com

GROUP C [11 × 4 = 44]

11. हल गर्नुहोस् (Solve): $3x^3 = 7x^2 - 4$

12. यदि a र b को बिचमा पाँचओटा गुणोत्तर मध्यमाहरू छन् । यदि दोस्रो र अन्तिम मध्यमा क्रमशः 2 र 16 छन् भने a र b का मानहरू पत्ता लगाउनुहोस् ।
There are five geometric means between a and b. If second and last mean are 2 and 16 respectively, find the values of a and b.

13. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 5x+2, & x \le 2 \\ 6x, & x > 2 \end{cases}$ को $x = 2$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of $f(x) = \begin{cases} 5x+2, & x \le 2 \\ 6x, & x > 2 \end{cases}$ at $x = 2$ by calculating left hand limit, right hand limit and functional values.

14. क्रामरको नियमबाट हल गर्नुहोस् (Solve by the Cramer's rule):
$2x + 3y = 12,\ 3x + 4y = 17$

15. शीर्षबिन्दुहरू P(3, 3), Q(– 2, – 6) र R(5, – 3) भएको ΔPQR को भारकेन्द्र भएर जाने र QR रेखासँग समानान्तर हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line passing through the centroid of ΔPQR with vertices P(3, 3), Q(– 2, – 6) and R(5, – 3) and parallel to the line QR.

16. प्रमाणित गर्नुहोस् (Prove that):
$2\tan 2\theta + 4\cot 4\theta = \cot\theta - \tan\theta$

17. हल गर्नुहोस् (Solve):
$1 + \sqrt{3}\tan\alpha - \sec\alpha = 0\ (0° \le \alpha \le 360°)$

18. एउटा ठाडो खम्बा AB लाई बिन्दु C ले AC : CB = 2 : 1 हुने गरी विभाजन गरिएको छ । जमिनको कुनै एउटा बिन्दुबाट C को उन्नतांश कोण 30° छ भने सोही बिन्दुबाट A को उन्नतांश कोण कति हुन्छ ? पत्ता लगाउनुहोस् जहाँ B खम्बाको फेद हो ।
A vertical pole AB is divided by a point C such that AC : CB = 2 : 1. The angle of elevation of C from a point on the ground is 30°. What is the angle of elevation of A from the same point? Find it where B is the foot of the pole.

19. शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, 4) भएको ΔABC लाई x- अक्षमा परावर्तन गर्दा बन्ने प्रतिबिम्बलाई उद्गम बिन्दुको वरिपरि 180° मा परिक्रमण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूको शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
A ΔABC with vertices A(2, 3), B(4, 5) and C(1, 4) is reflected in x-axis and then image so formed is rotated about the origin through 180°. Write the coordinates of the vertices of the images thus obtained and present the ΔABC and its images in the same graph paper.

20. चतुर्थांशीय भिन्नता (अर्ध चतुर्थांशीय विस्तार) र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find Q.D. (semi inter-quartile range) and its coefficient :

Height	60-62	62-64	64-66	66-68	68-70	70-72
No. of students	4	6	8	12	7	2

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् :
Find the standard deviation of the data given below:

Class interval	5-15	15-25	25-35	35-45	45-55
Frequency	7	3	6	4	5

GROUP D [4 × 5 = 20]

22. लेखाचित्रविधि र प्रतिस्थापन विधिद्वारा हल गर्नुहोस् । कुन विधिबाट हल गर्न तपाईंलाई सजिलो भयो कारण दिनुहोस् ।
Solve by graphical method & method of substitution. Which method did you feel easier? Give reason.
$y = x^2 - x - 3$ and $x = y$

23. यदि (1, 4) भएर जाने कुनै वृत्तको दुई व्यासहरूका समीकरणहरू $x - y = 1$ र $2x + 3y = 7$ भए सो वृत्तको समीकरण निकाल्नुहोस् ।
Find the equation of the circle which passes through the point (1, 4) and whose equations of two diameters are $x - y = 1$ and $2x + 3y = 7$.

24. त्रिभुज PQR मा ∡QPR = 90° र QR को मध्यबिन्दु S छ भने SP = SQ = SR हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In a triangle PQR, ∡QPR = 90° and S is a mid-point of QR, prove by the vector method that : SP = SQ = SR.

25. बिन्दुहरू P(4, 5) र Q(4, 3) जोड्ने रेखाखण्डको केन्द्र (2, 3) र अर्धव्यास 4 भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
Find the inverse of line segment joining the points P(4, 5) and Q(4, 3) with respect to a circle with centre C(2, 3) and radius 4 units. Show the inversion transformation in the graph.

ANSWERS

1. (a) $f^{-1} = \{(-4, 2)\}$ **2.** (a) $\lim_{x \to a^-} f(x)$ (b) $X = A^{-1}B$ **3.** (a) Ellipse (b) $a + b = 0$
4. (a) $2\cos^2 A - 1$ (b) sin C
5. (a) $\overrightarrow{OP} = \frac{m\overrightarrow{OB} - n\overrightarrow{OA}}{m-n}$ (b) $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$
6. (a) 4 (b) 32768 (c) 15, – 2
7. (a) $\frac{1}{6}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ (b) – 77 **8.** (a) 3
(b) 6 **9.** (c) 0°, 60° **10.** (a) 30° (b) $3\vec{i} - \vec{j}$
(c) 2.24, 0.184 **11.** 1, 2, $-\frac{2}{3}$ **12.** $\frac{1}{2}$, 32
13. Continuous **14.** (3, 2) **15.** $3x - 7y - 20 = 0$
17. 0°, 120°, or 360° **18.** 60°
19. A'(2, – 3), B'(4, – 5), C'(1, – 4), A"(– 2, 3), B"(– 4, 5), C"(– 1, 4)
20. 1.0225, 0.015 **21.** 14.78 **22.** (3, 3), (–1, –1)
23. $x^2 + y^2 - 4x - 2y - 5 = 0$ **25.** P'(6, 7), Q'(10, 3)

pustas.com.np

www.drmathnp.com

pusmas.com.np

SEE MODEL QUESTION 2076 SET 9
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् **(Answer all the questions):**

GROUP A [5 × (1 + 1) = 10]

1. **(a)** $f \circ g(x)$ को विस्तार क्षेत्र f र g मध्ये कुन फलनको विस्तार क्षेत्रसँग बराबर हुन्छ ?
Which range of the functions between f and g is equal to the range of the functions $f \circ g(x)$?

(b) यदि $f(x)$ को डिग्री '$4m$' र $g(x)$ को डिग्री 'm' भए $f(x) \div g(x)$ को डिग्री कति होला ?
If the degree of $f(x)$ is '$4m$' and $g(x)$ is 'm' what is the degree of $f(x) \div g(x)$?)

2. **(a)** दायाँ पक्षबाट सीमान्त मानलाई साङ्केतिक रूपमा लेख्नुहोस् ।
Write down the notational representation of right hand limit.

(b) A, B र X तीनओटा मेट्रिक्सहरू छन् । यदि $XA = B$ र $|A| \neq 0$ भए x को मान पत्ता लगाउनुहोस् ।
A, B and X are three matrices. If $XA = B$ and $|A| \neq 0$, what is the value of X?

3. **(a)** यदि प्रतिच्छेदित सतह सोलीको अक्षसँग समानान्तर भए कुन शाङ्कि क भाग बन्दछ ?
If the intersection plane is parallel to the axis of cone then what conic does it form?

(b) सीधा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ सम्पाती हुने अवस्था उल्लेख गर्नुहोस् ।
State the condition under which the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will be parallel.

4. **(a)** $\sin 3A$ र $\sin A$ बीचको सम्बन्ध लेख्नुहोस् ।
What is the relation between $\sin 3A$ and $\sin A$?

(b) यदि $A + B + C = \pi^c$ भए $\tan(A + B)$ लाई कोण C को रूपमा व्यक्त गर्नुहोस् ।
If $A + B + C = \pi^c$, express $\tan(A + B)$ in terms of angle C.

5. **(a)** $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ हुनेगरी ΔABC एउटा त्रिभुज छ, भने बिन्दु G को विशिष्ट नाम उल्लेख गर्नुहोस् । जहाँ O उद्गम बिन्दु हो ।
ABC is a triangle such that $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ and O be the origin, write the special name of the point G.

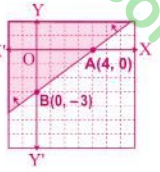
(b) यदि $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ र $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$ भए $T_1 \circ T_2$ जनाउने भेक्टर के हुन्छ ?
If $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ and $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$, what is the vector representing $T_1 \circ T_2$?

GROUP B [13 × 2 = 26]

6. **(a)** यदि $f : x \to 3x + b$ र $ff(2) = 12$ भए b को मान पत्ता लगाउनुहोस् ।
If $f : x \to 3x + b$ & $ff(2) = 12$, find the value of b.

(b) यदि $a,\ a - 2,\ a + 1$ गुणोत्तरीय अनुक्रममा भए a को मान पत्ता लगाउनुहोस् ।
If $a,\ a - 2,\ a + 1$ are in geometric sequence, find the value of a.

(c) दिइएको ग्राफमा छाया पारिएको भागले प्रतिनिधित्व गरेको सम्बन्धित असमानता लेख्नुहोस् ।
Write down the corresponding inequality represented by the shaded region in the adjoining graph.



7. **(a)** यदि $P = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$ भए P को विपरीत मेट्रिक्स पत्ता लगाउनुहोस् ।
If $P = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$, find the inverse matrix of P.

(b) यदि $M = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ र $N = \begin{pmatrix} 0 & -2 \\ 5 & -1 \end{pmatrix}$ भए $|M^TN|$ को मान पत्ता लगाउनुहोस् ।
If $M = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 0 & -2 \\ 5 & -1 \end{pmatrix}$ then find the value of $|M^TN|$.

8. **(a)** समीकरणहरू $y + \sqrt{3}x + 4 = 0$ र $x - \sqrt{3}y = 5$ एक आपसमा लम्ब हुन्छन् भनी देखाउनुहोस् ।
Show that the lines $y + \sqrt{3}x + 4 = 0$ and $x - \sqrt{3}y = 5$ are perpendicular to each other.

(b) समीकरण $3x^2 + 7xy + 2y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको कोण पत्ता लगाउनुहोस् ।
Find the angle between the pair of lines represented by equation $3x^2 + 7xy + 2y^2 = 0$.

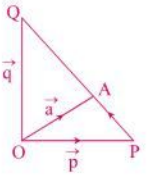
9. **(a)** प्रमाणित गर्नुहोस् (Prove that) :
$\cot\frac{A}{2} - \tan\frac{A}{2} = 2\cot A$

(b) प्रमाणित गर्नुहोस् (Prove that) :
$\frac{\cos 20° - \sin 20°}{\cos 20° + \sin 20°} = \tan 25°$

(c) हल गर्नुहोस् (Solve) : $\sin 2\theta + \cos\theta = 0\ [0° \le \theta \le 90°]$

10. **(a)** यदि $\vec{a} + 2\vec{b}$ र $5\vec{a} - 4\vec{b}$ आपसमा लम्ब तथा $\vec{a}$ र $\vec{b}$ एकाइ भेक्टरहरू भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other and $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$.

(b) चित्रमा, यदि $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$ छ भने प्रमाणित गर्नुहोस्:
In the figure, if $\overrightarrow{PA} = \frac{1}{4}\overrightarrow{PQ}$, prove that :
$\vec{a} = \frac{1}{4}(3\vec{p} + \vec{q})$



(c) एउटा निरन्तर श्रेणीमा $\Sigma fd = 0$, $\Sigma fd^2 = 848$,

www.drmathnp.com

pustas.com.np

N = 100, $d = m - A$ र A = 12 भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series $\Sigma fd = 0$, $\Sigma fd^2 = 848$, N = 100, $d = m - A$ and A = 12 then find the standard deviation and its coefficient.

GROUP C [11 × 4 = 44]

11. हल गर्नुहोस् (Solve): $6x^3 + x^2 - 19x + 6 = 0$

12. एउटा गुणोत्तर श्रेणीको दोस्रो पद र पाचौं पद क्रमशः 15 र 405 भए उक्त श्रेणीको प्रथम 6 पदहरूको योगफल पत्ता लगाउनुहोस् ।
If the second term and fifth term of a geometric series are 15 and 405 respectively, find the sum of the series from first to 6 terms.

13. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} x^2+1, & x \le 3 \\ 2x+4, & x > 3 \end{cases}$ को $x = 3$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of $f(x) = \begin{cases} x^2+1, & x \le 3 \\ 2x+4, & x > 3 \end{cases}$ at $x = 3$ by calculating left hand limit, right hand limit and functional values.

14. क्रामरको नियमबाट हल गर्नुहोस् :
Solve by the Cramer's rule:
$3x + 4y = 2, 2x + 3y = x + 4y + 3$

15. बिन्दुहरू (4, – 5) र (– 8, 9) जोडने रेखाखण्डको लम्बार्धकको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the perpendicular bisector of the line segment joining the points (4, – 5) and (– 8, 9).

16. प्रमाणित गर्नुहोस् (Prove that):
$\cos^3\theta.\cos 3\theta + \sin^3\theta.\sin 3\theta = \cos^3 2\theta$

17. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस्
If $A + B + C = \pi^c$, Prove that:
$\frac{\cos A}{\sin B.\sin C} + \frac{\cos B}{\sin C.\sin A} + \frac{\cos C}{\sin A.\sin B} = \text{cosec } 30°$

18. 21 m अग्लो पहाडको टुप्पोबाट एउटा धरहराको टुप्पो र फेद अवलोकन गर्दा अवनति कोणहरू क्रमशः 45° र 60° पाइयो भने धरहराको उचाइ पत्ता लगाउनुहोस् ।
From the top of mountain 21 meter high, the angles of depression of the top and the bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower.

19. बिन्दुहरू A(8, 10) र B(8, 6) जोड्ने रेखाखण्डको केन्द्र (4, 6) र अर्धव्यास 8 एकाइ भएको वृत्तको आधारमा उत्क्रम बिन्दुहरू पत्ता लगाउनुहोस् । सो उत्क्रम स्थानान्तरणलाई लेखा चित्रमा देखाउनुहोस् ।
Find the inverse of line segment joining the points A(8, 10) and B(8, 6) with respect to a circle with centre C(4, 6) and radius 8 units. Show the inversion transformation in the graph.

20. दिइएको तथ्याङ्कको चतुर्थांशीय भिन्नता र यसको गुणाङ्क पत्ता लगाउनुहोस् :
Find the quartile deviation and its coefficient of the following data:

Marks	20-29	30-39	40-49	50-59	60-69	70-79
No. of students	306	182	144	96	42	34

21. निम्न प्राप्ताङ्कको मध्यकबाट मध्यक भिन्नता पत्ता लगाउनुहोस् । यसको गुणाङ्क पनि पत्ता लगाउनुहोस् ।
Find the mean deviation from mean of the following data. Also find its coefficient :

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

GROUP D [4 × 5 = 20]

22. वर्ग समीकरण $y = x^2 + 2$ ले दिने पाराबोला र बिन्दुहरू (– 2, 6) र (2, 6) भएर जाने रेखाको प्रतिच्छेदन बिन्दुहरू पत्ता लगाउनुहोस् ।
Find the point of intersections of the parabola of a quadratic equation $y = x^2 + 2$ and the line passing through the points (– 2, 6) and (2, 6).

23. तीन जना विद्यार्थीहरू एउटा खेल मैदानमा वृत्ताकार आकृति बन्ने गरी परिधिमा बसेका छन् । उनीहरूको बसेको ठाउँको निर्देशाङ्कहरू (1, 2), (3, 4) र (3, 2) छन् । तिनीहरूबाट बराबर दूरीमा पर्ने निश्चित बिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् । उक्त वृत्ताकार पथको बिन्दु पथको समीकरण पत्ता लगाउनुहोस् ।
Three students are sitting in a playground to make a circular path. They are sitting on the circumference, the co-ordinates of their position on the circle are (1, 2), (3, 4) and (3, 2). Find the co-ordinates of the point equidistant from their position. Also, find the equation of the locus.

24. वृत्तार्धको कोण एक समकोण हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that the angle formed in a semi circle is a right angle.

25. ΔPQR का शीर्षबिन्दुहरू P(– 3, 2), Q(– 1, 4) र R(2, 0) छन् । x-अक्षमा हुने परावर्तन अनुसार मेट्रिक्स विधि प्रयोग गरी सो त्रिभुजको प्रतिबिम्ब पत्ता लगाउनुहोस् ।
P(– 3, 2), Q(– 1, 4) and R(2, 0) are the vertices of ΔPQR. Find the vertices of the image triangle under the reflection on x-axis by using matrix method.

ANSWERS

1. (a) $f(x)$ (b) $3m$ **2.** (a) $\lim_{x \to a^+} f(x)$ (b) $X = BA^{-1}$
3. (a) Hyperbola (b) $h^2 = ab$
4. (a) $3\sin A - 4\sin^3 A$ (b) $-\tan C$
5. (a) G(Centroid) (b) $\begin{pmatrix} a+c \\ b+d \end{pmatrix}$ **6.** (a) $-\frac{3}{2}$
(b) $\frac{4}{5}$ (c) $3x - 4y \le 12$ **7.** (a) $\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$
(b) 20 **8.** (b) 45°, 135° **9.** (c) 90° **10.** (a) 60°
(c) 2.91 and 0.24 **11.** $-1, \frac{3}{2}, \frac{1}{3}$ **12.** 1820
13. Continuous **14.** 2, –1 **15.** $7y = 6x + 26$
18. 8.88 m **19.** A'(12, 14), B'(20, 6) **20.** 10.71,0.287
21. 9.44, 0.3496 **22.** (– 2, 6), (2, 6)
23. $x^2 + y^2 - 4x - 6y + 11 = 0$
25. P'(– 3, – 2), Q'(– 1, – 4), R'(2, 0)

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 10
2076 (2020)

दिइएका निर्देशनका आधारमा आफ्नो शैलीमा सिर्जनात्मक उत्तर दिनुहोस् ।

समय : ३ घण्टा पूर्णाङ्क – १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) यदि भागफल $q(x)$, शेष $r(x)$, र भाजक $d(x)$ भए बहुपदीय $f(x)$ लाई $q(x)$, $r(x)$ र $d(x)$ को रूपमा लेख्नुहोस् ।
If the quotient, $q(x)$, remainder, $r(x)$, and divisor $d(x)$ then write the polynomial $f(x)$ in terms of $q(x)$, $r(x)$ and $d(x)$.

(b) पहिलो पद 'a' र समान अन्तर 'd' भएको AS को n औं समानान्तरीय मध्यमा के हुन्छ ?
What is the n^{th} arithmetic mean of an AS whose first term is 'a' and common difference is 'd'?

2. (a) $\lim_{x \to a^-} f(x)$ लाई वाक्यमा लेख्नुहोस् ।
Write in $\lim_{x \to a^-} f(x)$ in sentence.

(b) क्रामरको नियम प्रयोग गरी कस्ता समीकरणहरू हल गर्न सकिन्छ ?
What type of equations can be solved using Cramer's rule?

3. (a) एउटा सतहले लम्ब हुने गरी र सोलीको शीर्ष बिन्दु बाट नजाने गरी प्रतिच्छेदित गर्दछ । यसरी बन्ने शाङ्कि भागको नाम लेख्नुहोस् ।
A plane perpendicularly intersects a cone not passing through the vertex. Name the section so formed.

(b) जोडा रेखाहरू $ax^2 + 2hxy + by^2 = 0$ बीचको कोण पत्ता लगाउने सूत्र लेख्नुहोस् ।
Write the formula to find angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$.

4. (a) sin 2P + sin 2Q लाई गुणनको रूपमा व्यक्त गर्नुहोस् ।
Express sin 2P + sin 2Q into product form.

(b) यदि $\tan\theta = \frac{1}{\sqrt{3}}$, $0° \le \theta \le 180°$ भए θ को मान पत्ता लगाउनुहोस् ।
If $\tan\theta = \frac{1}{\sqrt{3}}$, $0° \le \theta \le 180°$, find the value of θ.

5. (a) यदि O उद्गम बिन्दु र $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ भए AB को मध्यबिन्दु M को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
If O be the origin and $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$ then find the position vector of M which is the mid point of AB.

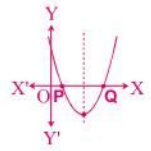
(b) X-अक्षको परावर्तन पछि Y-अक्षमा परावर्तन गर्दा हुने एकल स्थानान्तरण उल्लेख गर्नुहोस् ।
State the single transformation when the reflection in X-axis is followed by the reflection in Y-axis.

GROUP B [13 × 2 = 26]

6. (a) यदि $f(x) = 3x + 2$ र $g(x) = 2x - 1$ भए, $g \circ f(x)$ पत्ता लगाउनुहोस् ।
If $f(x) = 3x + 2$ and $g(x) = 2x - 1$, find $g \circ f(x)$.

(b) श्रेणी 5 + 9 + 13 + ... + 77 मा कतिओटा पदहरू छन्, पत्ता लगाउनुहोस् ।
How many terms are there in the series 5 + 9 + 13 + ... + 77 ?

(c) दिइएको लेखाचित्रबाट आवश्यक समीकरण लेख्नुहोस् र तोकिएका दुई बिन्दुहरू P र P' का निर्देशाङ्कहरू समेत उल्लेख गर्नुहोस् ।
Write the required equation from the given graph and determine the co-ordinates of two points P and P'.



7. (a) m को मान पत्ता लगाउनुहोस् (Find the value of m):
$A = \begin{pmatrix} 1 & 2 \\ 3 & m \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $|AB| = 24$

(b) तल दिइएका मेट्रिक्सहरू एकअर्काका विपरीत मेट्रिक्सहरू भए m को मान पत्ता लगाउनुहोस्:
If the matrices given below are inverse to each other, find the value of m:
$\begin{pmatrix} m & 1 \\ 5 & 2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ -5 & m \end{pmatrix}$

8. (a) यदि सीधा रेखाहरू $2x + 3y + 6 = 0$ र $ax - 5y + 20 = 0$ एक आपसमा लम्ब छन् भने a को मान पत्ता लगाउनुहोस् ।
If the straight lines $2x + 3y + 6 = 0$ and $ax - 5y + 20 = 0$ are perpendicular to each other, find the value of a.

(b) समीकरण $x^2 - 4xy + y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the pair of lines represented by equation $x^2 - 4xy + y^2 = 0$.

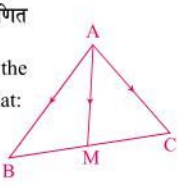
9. (a) प्रमाणित गर्नुहोस् (Prove that) :
$\frac{1 - 2\sin A.\cos A}{2} = \sin^2\left(\frac{\pi^c}{4} - A\right)$

(b) प्रमाणित गर्नुहोस् (Prove that) :
$\cos 20° + \cos 140° + \cos 100° = 0$

(c) हल गर्नुहोस् (Solve) : $3\sec\theta = 4\cos\theta$ $[0° \le \theta \le 90°]$

10. (a) x को मान कति हुँदा भेक्टरहरू $3\vec{i} - 2\vec{j}$ र $x\vec{i} + 3\vec{j}$ आपसमा लम्ब हुन्छन् ?
For what value of x are the two vectors $3\vec{i} - 2\vec{j}$ and $x\vec{i} + 3\vec{j}$ perpendicular to each other ?

(b) दिइएको चित्रमा, ΔABC को मध्यिका AM छ । प्रमाणित गर्नुहोस् :
In the given figure, AM is the median of ΔABC. Prove that:
$\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$



puspas.com.np

www.drmathnp.com

(c) एउटा निरन्तर श्रेणीमा $N = 30$, $\Sigma fm = 870$ र $\Sigma f(m - \bar{x})^2 = 1070$ भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series $N = 30$, $\Sigma fm = 870$ and $\Sigma f(m - \bar{x})^2 = 1070$ then find the standard deviation and its coefficient.

GROUP C [11 × 4 = 44]

11. हल गर्नुहोस् (Solve): $2x^3 + 6 - 3x^2 - 11x = 0$

12. एउटा गुणोत्तर श्रेणीको तेस्रो पद र सातौँ पद क्रमशः 20 र 320 भए उक्त श्रेणीको प्रथम 9 पदहरूको योगफल पत्ता लगाउनुहोस् ।
Find the sum of the first 9 terms of a geometric series whose third term and seventh term are 20 and 320 respectively.

13. फलन $f(x) = \begin{cases} 2x-1, & x<2 \\ 3, & x=2 \\ x+1, & x>2 \end{cases}$ बिन्दु $x = 2$ मा निरन्तर छ भनी प्रमाणित गर्नुहोस् ।
Prove that the function $f(x) = \begin{cases} 2x-1, & x<2 \\ 3, & x=2 \\ x+1, & x>2 \end{cases}$ is continuous at a point $x = 2$.

14. क्रामरको नियमबाट हल गर्नुहोस् (Solve by the Cramer's rule):
$y + 2x = 4$, $3y - 2x + 4 = 8$

15. बिन्दु $(-6, 2)$ भएर जाने र बिन्दुहरू $(-3, 2)$ र $(5, -3)$ जोड्ने रेखामा लम्ब हुने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight line passing through the point $(-6, 2)$ and perpendicular to the line joining the points $(-3, 2)$ and $(5, -3)$.

16. प्रमाणित गर्नुहोस् (Prove that) :
$\cos^3 A.\cos 3A + \sin^3 A.\sin 3A = \cos^3 2A$

17. यदि $A + B + C = 180°$ भए, प्रमाणित गर्नुहोस्
If $A + B + C = 180°$, then prove that :
$(\cos A + \cos B)(\cos A - \cos B) - \cos^2 C = 2\cos A \sin B \sin C - 1$

18. 20 m अग्लो एउटा घरको छत र फेदबाट एउटा स्तम्भको टुप्पोको उन्नतांश कोणहरू क्रमशः 45° र 60° छन् भने उक्त स्तम्भको उचाइ पत्ता लगाउनुहोस् ।
From the roof and basement of a house 20 m high, the angles of elevation of the top of a tower are 45° and 60° respectively. Find the height of the tower.

19. ΔABC का शिर्षबिन्दुहरू A(2, 5) , B(– 1, 3) र C(4, 1) छन् । ΔABC लाई उद्गम बिन्दुको वरिपरि ऋणात्मक चौथाइ परिक्रमण गरिएको छ र प्राप्त प्रतिबिम्बलाई केन्द्र उद्गम बिन्दु र नापो 2 लिएर विस्तारीकरण गरिएको छ । यसरी ΔABC का प्रतिबिम्बहरूका शिर्षबिन्दुहरूको निर्देशाङ्कहरू पत्ता लगाई वस्तु र प्रतिबिम्बहरूलाई एउटै लेखाचित्रमा देखाउनुहोस् ।
The vertices of ΔABC are A(2, 5), B(– 1, 3) and C(4, 1). ΔABC is rotated through negative quarter turn about the origin and the image so obtained is enlarged by taking origin as the centre and scale factor 2. Hence, find the co-ordinates of the vertices of the images of ΔABC and show the object and images on the same graph.

20. तलको तथ्याङ्कबाट चतुर्थांशीय भिन्नता पत्ता लगाउनुहोस् ।
Find the quartile deviation of the following data.

Marks obtained	20-30	30-40	40-50	50-60	60-70	70-80
No of students	5	15	10	8	6	2

21. मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस्:
Find mean deviation from median:

C.I	0-10	10-20	20-30	30-40	40-50
f	2	3	6	5	4

GROUP D [4 × 5 = 20]

22. शर्तहरू $3x + 5y \le 24$, $0, \le y \le 4$ र $0 \le x \le 7$ को आधारमा उद्देश्य फलन $P(x, y) = 16x - 2y$ को अधिकतम र न्यूनतम मानहरू पत्ता लगाउनुहोस् :
Find the maximum and minimum values of the objective function $P(x, y) = 16x - 2y$ under the constraints:
$3x + 5y \le 24$, $0, \le y \le 4$ and $0 \le x \le 7$

23. बिन्दुहरू (4, 1) र (6, 5) भएर जाने र केन्द्रबिन्दु रेखा $4x + y = 16$ मा पर्ने वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of a circle passing through the points (4, 1) and (6, 5) and having its centre on the line $4x + y = 16$.

24. समानान्तर चतुर्भुजका विकर्णहरू परस्पर समद्विभाजन हुन्छन् भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove by vector method that the diagonals of a parallelogram bisect to each other.

25. P (0, 3), Q (1, 1), R (3, 2) र S (2, 4) भएको वर्ग PQRS लाई 2 × 2 मेट्रिक्सद्वारा परिवर्तन गर्दा P'(3, 0), Q'(1, – 1), R'(2, – 3) र S'(4, – 2) भएको सामानान्तर चतुर्भुज P'Q'R'S' हुन्छ भने सो 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् । सो मेट्रिक्सले कुन स्थानान्तरणलाई जनाउँदछ ?
A square PQRS with the vertices P(0, 3), Q(1, 1), R(3, 2) and S(2, 4) is mapped onto the parallelogram P'Q'R'S' by 2 × 2 matrix so that the vertices of the parallelogram are P'(3, 0), Q'(1, – 1), R'(2, – 3) and S'(4, – 2). Find the 2 × 2 matrix. Which transformation does the matrix denote?

ANSWERS

1. (a) $f(x) = d(x) \times q(x) + r(x)$ (b) $a + nd$
2. (b) Simultaneous **3.** (a) Hyperbola
(b) $\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$
4. (a) $2\sin(P + Q)\cos(P - Q)$ (b) 30°
5. (a) $4\vec{i} + 6\vec{j}$ (b) Half turn rotation about the origin
6. (a) $6x + 3$ (b) 19 (c) (3, 6) & (– 3, 6); $y = \frac{2}{3}x^2$
7. (a) 30 (b) 3 **8.** (a) $a = \frac{15}{2}$ (b) 60°
9. (c) 30° **10.** (a) $x = 2$ (c) 5.97 and 0.206
11. $-2, \frac{1}{2}, 3$ **12.** 2560 **13.** Continuous **14.** (1, 2)
15. $8x - 5y + 58 = 0$ **18.** 47.32 m
19. A'(5, – 2), B'(3, 1), C'(1, – 4) & A"(10, – 4), B"(6, 2), C"(2, – 8) **20.** 10.65 **21.** 10.3
22. 112 at (7, 0) → Maximum, – 8 at (0, 4) → Minimum
23. $x^2 + y^2 - 6x - 8y + 15 = 0$ **25.** $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} R_o[-90°;(0, 0)]$

www.drmathnp.com

pusmas.com.np

SEE MODEL QUESTION 2076 SET 11
2076 (2020)

समय : ३ घण्टा पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) जब एउटा बहुपदीय $q(x)$ लाई शेष $q(-a) = 0$ हुने गरी अर्को बहुपदीयले भाग गरिन्छ भने एउटा गुणनखण्ड के होला ?
When a polynomial $q(x)$ is divided by another polynomial such that remainder $q(-a) = 0$. What will be its one of the factor?

(b) यदि 'a' र 'b' दुई धनात्मक सङ्ख्याहरू भए AM र GM बीचको सम्बन्ध लेख्नुहोस् ।
If 'a' and 'b' are the two positive numbers, write the relation between their AM and GM.

2. (a) $\lim_{x \to a^+} f(x)$ लाई वाक्यमा लेख्नुहोस् ।
Write $\lim_{x \to a^+} f(x)$ in sentence.

(b) क्रामरको नियम अनुसार D, D_x र D_y दिइएको बेला x को मान निकाल्ने सूत्र के हुन्छ ?
What should be the formula to calculate x using Cramer's rule when D, D_x and D_y are given?

3. (a) एउटा सतहले समकोणी सोलीलाई शीर्ष बिन्दुबाट नजाने गरी र सोलीको अक्ष सँग सो सतहले बनाएको कोण भन्दा अर्धशीर्षकोण (Semi vertical angle) α सानो हुने गरी प्रतिच्छेदन गर्दछ । यसरी बन्ने शाङ्कि भागको नाम लेख्नुहोस् ।
A plane cuts the right cone not passing its vertex and semi vertical angle a is less than angle made by the plane to the axis of cone. Name the section so formed.

(b) जोडा रेखाहरू $x + a = 0$ र $x - a = 0$ ले जनाउने एकल समीकरण के हुन्छ ?
What is the single equation to represent the pair of lines $x + a = 0$ and $x - a = 0$?

4. (a) $\cos x - \cos y$ लाई sine को गुणनफलको रूपमा बदल्नुहोस् ।
Convert $\cos x - \cos y$ in terms of product of sine.

(b) यदि $\sin\theta - \frac{\sqrt{3}}{2} = 0,\ 0° \le \theta \le 180°$ भए θ को मान पत्ता लगाउनुहोस् ।
If $\sin\theta - \frac{\sqrt{3}}{2} = 0,\ 0° \le \theta \le 180°$, find the value of θ.

5. (a) यदि एउटा त्रिभुजको शीर्षबिन्दुहरूका स्थिति भेक्टरहरूको योगफल $6\vec{i} + 9\vec{j}$ भए यसको गुरूत्व केन्द्रको स्थिति भेक्टर पत्ता लगाउनुहोस् ।
If the sum of position vectors of vertices of a triangle is $6\vec{i} + 9\vec{j}$ then find the position vector of its centroid.

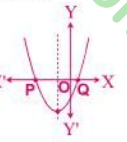
(b) दुई ओटा लगातार विस्तारीकरणहरू $E_1[O, K_1]$ र $E_2[O, K_2]$ द्वारा परिभाषित गरिएको भए संयुक्त विस्तारीकरण के होला ?
What will be the combined enlargement of two successive enlargements defined by $E_1[O, K_1]$ and $E_2[O, K_2]$?

GROUP B [13 × 2 = 26]

6. (a) यदि $f^{-1}(x) = \frac{x-1}{3}$ भए $f(x)$ पत्ता लगाउनुहोस् ।
If $f^{-1}(x) = \frac{x-1}{3}$ then find $f(x)$.

(b) दुई सङ्ख्याहरू 5 : 3 को अनुपातमा छन् । यदि तिनीहरूको अङ्कगणितीय मध्यमा 40 भए उक्त सङ्ख्याहरू पत्ता लगाउनुहोस् ।
Two numbers are in the ratio of 5 : 3. If their arithmetic mean is 40, find the numbers.

(c) सँगैको चित्रमा $y = x^2 + 4x - 5$ को लेखाचित्र देखाइएको छ । यसले X-अक्षको बिन्दुहरू P र Q मा काटेको छ । P र Q का निर्देशाङ्कहरू के-के हुन् ? यी निर्देशाङ्कको X-निर्देशाङ्कहरूले के जनाउँदछन् ?



Adjoining figure represents the sketch of the graph $y = x^2 + 4x - 5$. It cuts the X-axis at P and Q. What are the co-ordinates of P and Q? What do these X-co-ordinates denote?

7. (a) मेट्रिक्सहरू $A = \begin{bmatrix} 5 & 9 \\ 6 & 11 \end{bmatrix}$ र $B = \begin{bmatrix} 11 & -9 \\ -6 & 5 \end{bmatrix}$ विपरीत (प्रतिलोमी) छन् भनी सिद्ध गर्नुहोस् ।
Prove that $A = \begin{bmatrix} 5 & 9 \\ 6 & 11 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & -9 \\ -6 & 5 \end{bmatrix}$ are inverse matrices.

(b) प्रमाणित गर्नुहोस् (Prove that) :
$$\begin{vmatrix} b+c & a \\ c+a & c \end{vmatrix} = \begin{vmatrix} c+a & a-b \\ c & c-a \end{vmatrix}$$

8. (a) सरल रेखा $3x - my = 19$ ले सरल रेखा $3x + 5y = 7$ सँग 45° को कोण बनाउँछ भने m को मान निकाल्नुहोस् ।
Calculate the value of m if the line $3x - my = 19$ makes an angle of 45° with the line $3x + 5y = 7$.

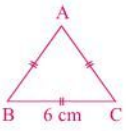
(b) समीकरण $x^2 - 2xy \operatorname{cosec}\alpha + y^2 = 0$ ले जनाउने एक जोडा सरल रेखाहरूबिचको कोण पत्ता लगाउनुहोस् ।
Find the angle between a pair of straight lines represented by an equation $x^2 - 2xy \operatorname{cosec}\alpha + y^2 = 0$.

9. (a) यदि $\cos\frac{\alpha}{3} = \frac{1}{2}$ भए $\sin\alpha$ को मान निकाल्नुहोस् ।
If $\cos\frac{\alpha}{3} = \frac{1}{2}$, find the value of $\sin\alpha$.

(b) प्रमाणित गर्नुहोस् (Prove that) :
$\frac{1}{2}(\cos 2\theta - \cos 8\theta) = \sin 5\theta . \sin 3\theta$

(c) हल गर्नुहोस् (Solve): $\sqrt{3}\tan\theta - 3 = 0\ [0° \le \theta \le \pi^c]$

10. (a) दिइएको ΔABC मा AB = BC = CA = 6 से.मि. छन् भने $\vec{AB}.\vec{AC}$ को मान निकाल्नुहोस् ।
In the given ΔABC, AB = BC = CA = 6 cm. Find the value of $\vec{AB}.\vec{AC}$.



(b) बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $5\vec{i} + 3\vec{j}$ र $2\vec{i} - 3\vec{j}$ छन् भने AB लाई 2 : 1 को अनुपातमा विभाजन गर्ने बिन्दु P को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
The position vectors of A and B are $5\vec{i} + 3\vec{j}$ and $2\vec{i} - 3\vec{j}$ respectively. Find the position vector of the point P which divides AB in the ratio 2 : 1.

www.drmathnp.com

pustas.com.np

(c) एउटा निरन्तर श्रेणीमा N = 47, Σfm = 770, Σfx^2 = 16450 र $\overline{x}$ = 16.38 भए सो श्रेणीको स्तरीय भिन्नता र सोको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous series N = 47, Σfm = 770, Σfx^2 = 16450 and $\overline{x}$ = 16.38 then find the standard deviation and its coefficient.

GROUP C [11 × 4 = 44]

11. हल गर्नुहोस् (Solve): $x^3 = 7x^2 - 36$

12. 4 र 24 का बिचमा n ओटा समानान्तरीय मध्यमाहरू छन् । यदि तेस्रो मध्यमा र अन्तिम मध्यमाको अनुपात 4 : 5 भए उक्त श्रेणीमा जम्मा कतिओटा पदहरू छन् ? पत्ता लगाउनुहोस् ।
There are n arithmetic means between 4 and 24. If the ratio of third mean to the last mean is 4 : 5 then find the number of terms in the series.

13. फलन $f(x) = \frac{x+1}{4x+4}$ लाई $x = 4$ मा निरन्तरता छ कि विच्छिन्नता छ; परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of the function $f(x) = \frac{x+1}{4x+4}$ at $x = 4$.

14. क्रामरको नियमबाट हल गर्नुहोस् (Solve by the Cramer's rule):
$\frac{4}{x} + \frac{5}{y} = 43, \frac{7}{x} + \frac{3}{y} = 58$

15. रेखा $3x - 2y = 5$ सँग समानान्तर भई बिन्दुहरू (– 4, 2) र (2, 4) जोड्ने रेखाखण्डको मध्य बिन्दु भएर जाने रेखाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the line parallel to the line $3x - 2y = 5$ and passing through the midpoint of the line segment joining the points (– 4, 2) and (2, 4).

16. यदि $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{\sin B} + \frac{1}{\cos B}$ भए प्रमाणित गर्नुहोस् :
If $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{\sin B} + \frac{1}{\cos B}$, prove that :
$\cot\left(\frac{A+B}{2}\right) = \tan A . \tan B$

17. यदि $\alpha + \beta + \theta = \pi^c$ भए प्रमाणित गर्नुहोस्:
If $\alpha + \beta + \theta = \pi^c$, prove that:
$\sin(\alpha + \beta - \theta) + \sin(\beta + \theta - \alpha) + \sin(\theta + \alpha - \beta) = 4 \sin\alpha . \sin\beta . \sin\theta$

18. समतल स्थानमा भएका कुनै दुईओटा बिन्दुहरूबाट एउटा टावरको टुप्पोमा हेर्दा बन्ने उन्नतांश कोणहरू समपूरक पाइयो । यदि ती बिन्दुहरू उक्त टावरको फेदबाट 20 मिटर र 45 मिटरको दूरीमा छन् भने, सो टावरको उचाइ पत्ता लगाउनुहोस् ।
The angles of elevation of the top of a tower observed from two points on the same plane are found to be complementary. If the points are at a distance of 20 m and 45 m from the foot of the tower, find the height of the tower.

19. यदि उद्गम बिन्दु केन्द्र हुने वृत्तको आधारमा एउटा बिन्दु A(3, 4) को उत्क्रम बिन्दु A'(12, 16) भए सो वृत्तको व्यास पत्ता लगाउनुहोस् । साथै बिन्दु B(6, 8) को उत्क्रम बिन्दु पनि सोही वृत्त अनुसार पत्ता लगाउनुहोस् ।
If the inversion of a point A(3, 4) with respect to a circle having centre as origin is A'(12, 16), find the diameter of the circle. Also find the inversion point of B(6, 8) with respect to the circle.

20. तलको तथ्याङ्कबाट चतुर्थांशीय विस्तार र चतुर्थांशीय भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
Find the interquartile range and coefficient of quartile deviation from the following data:

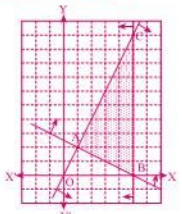
Weight (Kg)	10 - 20	10 -30	10 - 40	10 - 50	10 - 60	10 - 70
No. of workers	12	31	36	46	55	61

21. तलको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Calculate the mean deviation from the median on the basis of the following data.

Marks	10 - 20	20 -30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	6	8	11	14	8	3

GROUP D [4 × 5 = 20]

22. ΔABC भित्र छाया पारिएको भागले तीनओटा असमानताहरू जनाउँदछ । ती असमानताहरू लेख्नुहोस् । साथै तीनओटै असमानताहरूलाई सन्तुलन गर्ने मानहरूबाट $3x + 5y$ को अधिकतम मान पनि पत्ता लगाउनुहोस् ।



The shaded region inside the ΔABC is represented by three inequalities. Write down the inequalities. Also calculate the maximum value of $3x + 5y$ from the values which satisfy all three inequalities.

23. एउटा वृत्ताकार पोखरीको किनारामा बिन्दुहरू (2, – 2), (6, 6) र (5, 7) पर्दछन् । सो पोखरीको किनारामा पर्ने सबै बिन्दुहरूको बिन्दुपथ पत्ता लगाउनुहोस् ।
The points at the edge of a circular pond are (2, – 2), (6, 6) and (5, 7). Find the equation of locus of all points of the circular edge.

24. चतुर्भुज ABCD मा AD, AB, BC र CD का मध्यबिन्दुहरू क्रमशः M, N, R र S भए MNRS समानान्तर चतुर्भुज हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In a quadrilateral ABCD if M, N, R and S are the midpoints of the sides AD, AB, BC and CD respectively. Prove vectorically that MNRS is a parallelogram.

25. एउटा एकाइ वर्ग OABC लाई शीर्षबिन्दुहरू O'(0, 0), A'(3, 0), B'(4, 1) र C'(1, 1) भएको समानान्तर चतुर्भुज O'A'B'C' मा स्थानान्तरण गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
Find a 2 × 2 matrix which transforms a unit square OABC to a parallelogram O'A'B'C' with vertices O'(0, 0), A'(3, 0), B'(4, 1) and C'(1, 1).

ANSWERS

1. (a) $(x - a)$ (b) AM ≥ GM **2.** (b) $\frac{D_x}{D}$
3. (a) Ellipse (b) $x^2 - a^2 = 0$ **4.** (a) $2\sin\frac{x+y}{2} . \sin\frac{y-x}{2}$
(b) 60°, 120° **5.** (a) $2\vec{i} + 3\vec{j}$
(b) $E_2 E_1$ [O, K_1K_2] **6.** (a) $3x + 1$ (b) 50, 30
(c) Q(1, 0), P(– 5, 0) **8.** (a) 12
(b) $\tan^{-1}(\pm\cot\alpha)$ **9.** (a) 0 (c) 60°
10. (a) 18 (b) $\vec{p} = 3\vec{i} - \vec{j}$ (c) 9.03, 0.55
11. – 2, 3, 6 **12.** 6 **13.** Continuous **14.** $\frac{1}{7}, \frac{1}{3}$
15. $3x - 2y + 9 = 0$ **18.** 30 m **19.** 10 units, (6, 8)
20. 28.04, 0.39 **21.** 11.8
22. $x + 2y \geq 5, y \leq 2x, x \leq 5$; 65 at (5, 10)
23. $x^2 + y^2 - 4x - 6y - 12 = 0$ **25.** $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 12
2076 (2020)

समय : ३ घण्टा | पुर्णाङ्क - १००

सबै प्रश्नहरूको उत्तर दिनुहोस् (Answer all the questions):

GROUP A [5 × (1 + 1) = 10]

1. (a) सूत्र $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ मा 'n' ले के जनाउँदछ ? जहाँ सङ्केतहरूका आफ्नै प्रचलित अर्थ छन् ।
What does 'n' represent in the formula $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$? where the symbols have their own usual meanings.

(b) बहुपदीयको शेष साध्य उल्लेख गर्नुहोस् ।
State the remainder theorem of polynomial.

2. (a) बायाँबाट लिइने सीमान्त मानलाई साङ्केतिक प्रस्तुतीमा लेख्नुहोस् ।
Write the symbolic notation of left hand limit.

(b) $\begin{vmatrix} m & i \\ n & a \end{vmatrix}$ को डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।
Find the determinant of $\begin{vmatrix} m & i \\ n & a \end{vmatrix}$.

3. (a) एउटा समकोणी सोलीको आधारसँग प्रतिच्छेदन गर्ने समतल सतह समानान्तर भए यसबाट कुन शाङ्कि भाग बन्दछ ?
If the intersection plane is parallel to the base of cone, what conic does it form?

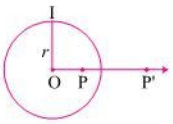
(b) $ax^2 + 2hxy + by^2 = 0$ ले जनाउने रेखाहरू बीचको कोण पत्ता लगाउने सूत्र के हुन्छ ?
What is the formula of angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$?

4. (a) sin 2A लाई tan A को रूपमा व्यक्त गर्नुहोस् ।
Express sin 2A in terms of tan A.

(b) θ को कुन न्यूनकोणीय मानले त्रिकोणमितीय समीकरण $\tan\theta - \sqrt{3} = 0$ लाई सन्तुलन गर्दछ ?
What acute value of θ satisfies the trigonometric equation $\tan\theta - \sqrt{3} = 0$?

5. (a) एउटै आरम्भिक बिन्दु भएका $\vec{a}$ र $\vec{b}$ बीचको स्केलार गुणनफललाई परिभाषीत गर्नुहोस् ।
Define the scalar product between $\vec{a}$ and $\vec{b}$ having same initial point.

(b) यदि सँगैको उत्क्रम वृत्तको अर्धव्यास 8 cm भए OP × OP' पत्ता लगाउनुहोस् ।
If the radius of the adjoining inversion circle is 8 cm then find OP × OP'.



GROUP B [13 × 2 = 26]

6. (a) यदि $f(x + 2) = 2x + 3$ भए $f^{-1}(4)$ पत्ता लगाउनुहोस् ।
If $f(x + 2) = 2x + 3$ then find $f^{-1}(4)$.

(b) $x^3 - 6x^2 + 11x - 8$ मा कति जोड्दा एउटा गुणनखण्ड $(x - 3)$ भएको बहुपदीय बनाउन सकिन्छ ?
What must be added to $x^3 - 6x^2 + 11x - 8$ to make it a polynomial having a factor $(x - 3)$?

(c) एउटा वर्ग समीकरणको पाराबोला Y - अक्षसँग सममिति भएको छ । यदि सो पाराबोला (0, 0) र (2, 4) भएर जान्छ भने समीकरण पत्ता लगाउनुहोस् ।
The parabola of a quadratic equation is symmetric with Y-axis. If its passes through (0, 0) and (2, 4), find the equation.

7. (a) यदि $A = \begin{pmatrix} p & 3 \\ 2 & 1 \end{pmatrix}$ र $B = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ भए p को मान कति हुँदा AB एउटा एकल मेट्रिक्स हुन्छ ?
If $A = \begin{pmatrix} p & 3 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$, for what value of p, AB is a singular matrix?

(b) समीकरणहरू $2x + 3y = 12$ र $x + y = 5$ मा D को मान -1 छ । क्रामरको नियम प्रयोग गरी x को मान पत्ता लगाउनुहोस् ।
In the equations $2x + 3y = 12$ and $x + y = 5$, the value of D = -1. Find the value of x using Cramer's rule.

8. (a) जोडा रेखाहरू $\frac{x}{y} = 2$ र $\frac{y}{x} = 2$ बाट दुई डिग्रीको वर्ग समघातीय समीकरण पत्ता लगाउनुहोस् ।
Find the homogeneous equation of second degree from the pair of lines $\frac{x}{y} = 2$ and $\frac{y}{x} = 2$.

(b) वृत्त $(x - 2)^2 + (y - 3)^2 = 49$ को केन्द्रबिन्दु र अर्धव्यास पत्ता लगाउनुहोस् ।
Find the centre and radius of circle $(x - 2)^2 + (y - 3)^2 = 49$.

9. (a) तालिका वा क्याल्कुलेटरको प्रयोग नगरी $(\sin 75° + \sin 15°)$ को मान पत्ता लगाउनुहोस् ।
Find the value of $(\sin 75° + \sin 15°)$ without using table or calculator.

(b) प्रमाणित गर्नुहोस् (Prove that):
$\frac{1 + \cos 2\theta + \sin 2\theta}{1 - \cos 2\theta + \sin 2\theta} = \cot\theta$

(c) न्यूनकोणको लागि हल गर्नुहोस्:
Solve for acute angle:
$2\sin^2\theta - 3\sin\theta + 1 = 0$

10. (a) यदि $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 6$, $|\vec{b}| = 10$ र $|\vec{c}| = 14$ भए $\vec{a}$ र $\vec{b}$ बीचको कोण पत्ता लगाउनुहोस् ।
If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 6$, $|\vec{b}| = 10$ and $|\vec{c}| = 14$, find the angle between $\vec{a}$ and $\vec{b}$.

(b) बिन्दुहरू A र B का स्थिति भेक्टरहरू क्रमशः $(8\vec{i} + 6\vec{j})$ र $(3\vec{i} + \vec{j})$ छन् । AB लाई 2 : 3 को अनुपातमा भित्रपट्टीबाट विभाजन गर्ने बिन्दु C को स्थिति भेक्टर पत्ता लगाउनुहोस् ।
The position vectors of the points A and B are $(8\vec{i} + 6\vec{j})$ and $(3\vec{i} + \vec{j})$ respectively. Find the position vector of the point C, which divides AB in the ratio of 2 : 3 internally.

(c) एउटा निरन्तर श्रेणीमा पहिलो चतुर्थांशको मान चतुर्थांशीय भिन्नताको दुई गुणा छ । चतुर्थांशीय भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
In a continuous data, the value of the first quartile is two times of the value of quartile deviation. Find the coefficient of the quartile deviation.

puspas.com.np

www.drmathnp.com

pusras.com.np

GROUP C [11 × 4 = 44]

11. दुईओटा फलनहरू $f(x)=\frac{x}{4}+\frac{5}{8}$ र $f\circ g(x)=\frac{3x}{4}-\frac{3}{8}$ छन् ।
यदि $(g\circ f)^{-1}(x)$ एउटा एकात्मक फलन भए x को मान पत्ता लगाउनुहोस् ।
Two functions are $f(x)=\frac{x}{4}+\frac{5}{8}$ and $f\circ g(x)=\frac{3x}{4}-\frac{3}{8}$.
If $(g\circ f)^{-1}(x)$ is an identity function, find the value of x.

12. एउटा AP का तीनओटा पदहरूको योगफल 21 छ । यदि दोस्रो सङ्ख्या 1 ले घटाइयो र तेस्रो सङ्ख्या 1 ले बढाइयो भने एउटा GP बन्दछ । ती सङ्ख्याहरू पत्ता लगाउनुहोस् ।
The sum of three numbers in an AP is 21. If the second number is reduced by 1 and the third term is increased by 1, we obtain the GP. Find the numbers.

13. फलन $h(x)=\frac{4x^2-25}{2x-5}$ को निरन्तरता वा विच्छिन्नता $x=2.5$ मा पहिचान गर्नुहोस् ।
Identify whether the function $h(x)=\frac{4x^2-25}{2x-5}$ has continuity or discontinuity at $x=2.5$.

14. मेट्रिक्स विधिबाट हल गर्नुहोस् :
Solve by the matrix method:
$$\frac{3x+5y}{24}=\frac{5x-2y}{9}=\begin{vmatrix}3&2\\4&3\end{vmatrix}$$

15. P(3, 6), Q(10, 7) र R(7, 3) एउटा वृत्तको परिधिका बिन्दुहरू हुन् । यदि $\angle PRQ = 90°$ भए वृत्तको समीकरण र अर्धव्यास RC को समीकरण पत्ता लगाउनुहोस् ।
P(3, 6), Q(10, 7) and R(7, 3) are the points at the circumference of a circle. If $\angle PRQ = 90°$ then find the equation of circle and the equation of radius (RC).

16. प्रमाणित गर्नुहोस् (Prove that):
$$\sin^6\frac{\pi^c}{8}+\sin^6\frac{3\pi^c}{8}+\sin^6\frac{5\pi^c}{8}+\sin^6\frac{7\pi^c}{8}=\frac{5}{4}$$

17. यदि $A+B+C=\sin\pi^c$ भए प्रमाणित गर्नुहोस् :
If $A+B+C=\sin\pi^c$ then prove that :
$$2\cos^2\frac{A}{2}+\cos B+\cos C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

18. एउटा धरहराको उचाइ यसमाथि रहेको ध्वजदण्डको उचाइको आधा छ । यदि जमिनको कुनै बिन्दुबाट धरहराको टुप्पाको उन्नतांश कोण 30° भए ध्वजदण्डको टुप्पाको उन्नतांशकोण पत्ता लगाउनुहोस् ।
The height of a tower is half the height of the flagstaff at its top. The angle of elevation of the top of the tower as seen from any point on the ground is 30°. Find the angle of elevation of the top of the flagstaff from the same point.

19. एउटा 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् जसले शीर्षबिन्दुहरू A(2, 3), B(4, 3), C(4, 5) र D(2, 5) भएको एउटा वर्ग ABCD लाई शीर्षबिन्दुहरू A'(3, 2), B'(3, 4), C'(5, 4) र D'(5, 2) भएको वर्ग A'B'C'D' मा स्थानान्तरण गर्दछ ।
Find the 2 × 2 transformation matrix which transforms a square ABCD with vertices A(2, 3), B(4, 3), C(4, 5) and D(2, 5) into a square A'B'C'D' with vertices A'(3, 2), B'(3, 4), C'(5, 4) and D'(5, 2).

20. तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation from median of the data given below:

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

21. तलको तथ्याङ्कबाट स्तरीय भिन्नता निकाल्नुहोस् ।
Calculate the standard deviation from the following data.

x	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	4	4	6	1

GROUP D [4 × 5 = 20]

22. तलका शर्तहरूको आधारमा $z = 2x - 3y$ को न्यूनतम मान पत्ता लगाउनुहोस् ।
Find the minimum value of $z = 2x - 3y$ under the following constraints.
$x + y \geq 0, x - y \leq 0, x \geq -1$ and $y \leq 2$

23. रेखा $2x - y + 7 = 0$ सँग 45° को कोण बनाउने र बिन्दु (4, 5) भएर जाने रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the straight lines passing through the point (4, 5) and making an angle of 45° with the line $2x - y + 7 = 0$.

24. समबाहु चतुर्भुजको विकर्णहरू समकोण हुने गरी प्रतिच्छेदन हुन्छन् भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
Prove by vector method that, the diagonals of rhombus intersect to each other at a right angle.

25. त्रिभुज CAT का शीर्षबिन्दुहरू C(2, 5), A(– 1, 3) र T(4, 1) छन् । त्रिभुज CAT लाई उद्गम बिन्दुको वरिपरि 90° ले धनात्मक दिशामा परिक्रमण गरी फेरि E[(0, 0), 2] मा विस्तारीकरण गर्दा बन्ने प्रतिबिम्बहरूको निर्देशाङ्कहरू पत्ता लगाउनुहोस् । साथै वस्तु र प्रतिबिम्बहरूलाई उही लेखाचित्रमा प्रस्तुत गर्नुहोस् ।
C(2, 5), A(– 1, 3) and T(4, 1) are the vertices of a triangle CAT. Find the coordinates of the vertices of the image of ΔCAT under the rotation of positive 90° about origin followed by enlargement E[(0, 0), 2]. Represent the object and images on the same graph paper.

ANSWERS

1. (a) No. of means (b) $ma - ni$ **2.** (a) $\lim_{x\to a} f(x)$ **3.** (a) Circle
4. (a) $\frac{2\tan A}{1+\tan^2 A}$ (b) 60° **5.** (b) 64 cm²
6. (a) $\frac{5}{2}$ (b) 2 (c) $y = x^2$ **7.** (a) 6
(b) 3 **8.** (a) $2x^2 - 5xy + 2y^2 = 0$
(b) (2, 3); 7 **9.** (a) $\sqrt{\frac{3}{2}}$ (c) 30° **10.** (a) 60°
(b) $6\vec{i}+4\vec{j}$ (c) 0.33 **11.** $\frac{13}{2}$
12. 3, 7, 11 or 12, 7, 2 **13.** discontinuity at $x = 2.5$
14. 3, 3 **15.** $x^2 + y^2 - 13x - 13y + 72 = 0; \frac{5}{\sqrt{2}}$
18. 60° **19.** $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ **20.** 9.56
21. 12.68 **22.** – 8 at (– 1, 2)
23. $x - 3y + 11 = 0, 3x + y = 17$
25. C'(– 5, 2), A'(– 3, – 1), T'(– 1, 4), C''(– 10, 4), A''(– 6, – 2), T''(– 2, 8)

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 13
2076 (2020)

विषय : ऐच्छिक (अतिरिक्त) गणित **पूर्णाङ्क : १००**
विषय कोड : ३३१ **समय : ३ घण्टा**

GROUP A 10 × 1 = 10

1. (a) घनघातीय फलनको परिभाषा लेख्नुहोस् ।
Write the definition of a cubic function.

(b) दुईओटा धनात्मक सङ्ख्याहरू m र n बिचको गुणोत्तर मध्यमा कति हुन्छ ?
What is the geometric mean between two positive numbers m and n?

2. (a) सङ्ख्या रेखामा प्राकृतिक सङ्ख्याहरूको समूह र वास्तविक सङ्ख्याहरूको समूह मध्ये कुन समूहमा निरन्तरता हुन्छ ?
In which set of numbers, do you find continuity between set of natural numbers and set of real numbers in number line?

(b) मेट्रिक्स $\begin{pmatrix} l & o \\ v & e \end{pmatrix}$ को डिटरमिनान्ट कति हुन्छ ?
What is the determinant of the matrix $\begin{pmatrix} l & o \\ v & e \end{pmatrix}$?

3. (a) यदि झुकाव a र b भएका दुईओटा सिधा रेखाहरूबीचको कोण θ भए $\tan\theta$ कति हुन्छ ?
If θ be the angle between two straight lines having slope a and b, what is $\tan\theta$?

(b) दीर्घ वृत्तको परिभाषा दिनुहोस् । (Define ellipse.)

4. (a) $\cos 2A$ लाई $\cos A$ को रूपमा व्यक्त गर्नुहोस् ।
Express $\cos 2A$ in terms of $\cos A$.

(b) यदि $2\cos\theta = \sqrt{3}$ भए $0 \le \theta \le 90°$ मा θ को मान कति हुन्छ ?
If $2\cos\theta = \sqrt{3}$, what is the value of θ for $0 \le \theta \le 90°$?

5. (a) यदि $\vec{p} = (a, b)$ र $\vec{q} = (b, a)$ भए $\vec{p} \cdot \vec{q}$ को मान कति हुन्छ ?
If $\vec{p} = (a, b)$ and $\vec{q} = (b, a)$, what is the value of $\vec{p} \cdot \vec{q}$?

(b) दिइएको उत्क्रम वृत्त एकाइ वृत्त भए OP × OP' को नाप कति होला ?
Given inversion circle is a unit circle. What is the measure of OP × OP'?



GROUP B 13 × 2 = 26

6. (a) शेष साध्यको कथन लेख्नुहोस् । उक्त साध्य प्रयोग गरी $x^6 - 64$ को गुणनखण्ड $(x-2)$ हो , होइन यकिन गर्नुहोस् ।
State remainder theorem. Use it and determine whether $(x-2)$ is a factor of $x^6 - 64$ or not?

(b) पाराबोलाको मानक समीकरण लेखी $y = x^2 - 4x + 5$ को शीर्षबिन्दुको निर्देशाङ्क पत्ता लगाउनुहोस् ।
Write the standard equation of a parabola and find the vertex of $y = x^2 - 4x + 5$.

(c) $f : R \to R$ का लागि $f(x) = 3x - 2$ भए $f^{-1}(4)$ पत्ता लगाउनुहोस् ।
For, $f : R \to R$, $f(x) = 3x - 2$, find $f^{-1}(4)$.

7. (a) यदि $A = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix}$ भए A को विपरित मेट्रिक्स A^{-1} पत्ता लगाउनुहोस् ।
If $A = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix}$, find the inverse matrix A^{-1} of A.

(b) समीकरणहरू $x + 2y = 7$ र $2x - y = 4$ मा क्रामरको नियम प्रयोग गर्दा $x = \frac{D_1}{D} = \frac{D_1}{(-5)}$ र $y = \frac{D_2}{D} = \frac{D_2}{(-5)}$ भए D_1 र D_2 का मानहरू पत्ता लगाउनुहोस् ।
In Cramer's rule for solving equations $x + 2y = 7$ and $2x - y = 4$, $x = \frac{D_1}{D} = \frac{D_1}{(-5)}$ and $y = \frac{D_2}{D} = \frac{D_2}{(-5)}$. Find the values of D_1 and D_2.

8. (a) जोडा समीकरणहरू $mx = ny$ र $nx = -my$ लाई प्रतिनिधित्व गर्ने एउटै समीकरण लेख्नुहोस् ।
Write the single equation representing the pair of equations $mx = ny$ and $nx = -my$.

(b) केन्द्र (4, 5) र व्यास 6 एकाइ भएको वृत्तको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of circle having centre (4, 5) and diameter 6 units.

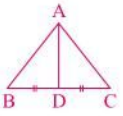
9. (a) क्याल्कुलेटर वा त्रिकोणमितीय तालिकाको प्रयोग विना $\cos 15°$ को मान पत्ता लगाउनुहोस् ।
Find the value of $\cos 15°$ without using calculator or trigonometric table.

(b) प्रमाणित गर्नुहोस् (Prove that) :
$\frac{\sin 10x - \sin 6x}{\cos 10x + \cos 6x} = \tan 2x$

(c) हल गर्नुहोस् (Solve) : $2\cos\alpha - 1 = 0$ $(0 \le \alpha \le 360°)$

10. (a) यदि $\vec{a} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ र $\vec{b} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ भए $\vec{a}$ र $\vec{b}$ बीच बन्ने कोण पत्ता लगाउनुहोस् ।
If $\vec{a} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$, find the angle between $\vec{a}$ and $\vec{b}$.

(b) चित्रमा BD = DC भए $\overrightarrow{AC} = 2\overrightarrow{AD} + \overrightarrow{BA}$ हुन्छ भनी देखाउनुहोस् ।
In the figure, BD = DC, prove that: $\overrightarrow{AC} = 2\overrightarrow{AD} + \overrightarrow{BA}$.



(c) एउटा तथ्याङ्कको पहिलो चतुर्थांश 70 र तेस्रो चतुर्थांश पहिलो चतुर्थांशको दोब्बर भन्दा 25 ले धेरै छ । सो तथ्याङ्ककको चतुर्थांशीय भिन्नताको गुणाङ्क पत्ता लगाउनुहोस् ।
The first quartile of a data is 70 and third quartile is 25 more than the double of the first quartile. Find the coefficient of the quartile deviation of the data.

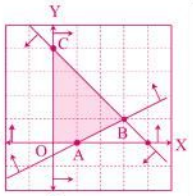
GROUP C 11 × 4 = 44

11. दुई फलनहरू $f(x) = \frac{2x+5}{8}$ र $g(x) = 3x - 4$ छन् । यदि $(f \circ g)^{-1}(x)$ एउटा एकात्मक फलन हो भने x को मान पत्ता लगाउनुहोस् ।
Two functions are $f(x) = \frac{2x+5}{8}$ and $g(x)$ $3x - 4$. If $(f \circ g)^{-1}(x)$ is an identity function, find the value of x.

puspas.com.np

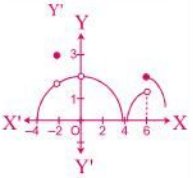
www.drmathnp.com

12. दिइएको लेखाचित्रबाट उद्देश्य फलन $z(x, y) = 3x + 2y$ का शर्तहरू र अधिकतम मान पत्ता लगाउनुहोस् ।



From the given graph, find the constraints and maximum value of the objective function $z(x, y) = 3x + 2y$

13. चित्रमा एउटा आनुपातिक फलनको लेखाचित्र दिइएको छ । उक्त लेखाचित्र कुनै एउटा कम्पनिको नाफा/ नोक्सानसँग सम्बन्धित छ । उक्त वक्र कुन-कुन बिन्दुमा विच्छिन्न छ र किन ? कारण लेख्नुहोस् ।



In the figure, the graph of a rational function is shown. The graph represent the profit/loss of a company. At which points does the function is discontinuous and why ? Give reason.

14. दिइएका जोडा समीकरणहरूको मेट्रिक्स विधि प्रयोग गरी हल गर्नुहोस् : $2x - 3y = 7, 4y - 3x + 10 = 0$

Solve the given pair of equations using matrix method : $2x - 3y = 7, 4y - 3x + 10 = 0$

15. समीकरण $3x^2 + xy - 10y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूसँग समानान्तर हुने र बिन्दु (1, 0) भएर जाने एक जोडी रेखाहरूको समीकरण पत्ता लगाउनुहोस् ।

Find the equation of a pair of lines passing through the point (1, 0) and parallel to the lines represented by the equation $3x^2 + xy - 10y^2 = 0$.

16. प्रमाणित गर्नुहोस् (Prove that):

$$2\sin^2\left(\frac{\pi^c}{4} - A\right) = \frac{(1 - \tan A)^2}{1 + \tan^2 A}$$

17. यदि $A + B + C = 180°$ भए प्रमाणित गर्नुहोस् :

If $A + B + C = 180°$, prove that:

$$2\sin\frac{A}{2}\cos\frac{A}{2} + \sin B - \sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$

18. 24 मिटर अग्लो एउटा स्तम्भको टुप्पोबाट एउटा खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा अवनति कोणहरू क्रमश: 45° र 60° पाइयो भने खम्बाको उचाइ पत्ता लगाउनुहोस् ।

From the top of a tower 24 meter high, the angles of depression of the top and the foot of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

19. यदि मेट्रिक्स M_1 ले X- अक्षमा हुने परावर्तन र मेट्रिक्स M_2 ले उद्गम बिन्दुको वरिपरि 180° मा हुने परिक्रमणलाई जनाउँछन् भने M_1M_2 ले एकाइ वर्गलाई स्थानान्तरण गर्दा बन्ने चतुर्भुजको शीर्षबिन्दुहरूको निर्देशाङ्क पत्ता लगाउनुहोस् ।

If M_1 is the matrix represented by reflection on X-axis and M_2 is the matrix represented by rotation through 180° about origin then transform unit square by M_1M_2 and write the co-ordinates of vertices of image quadrilateral.

20. तलको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।

Find the mean deviation from median of the data given :

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	8	15	16	6

21. दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस्:

Find the standard deviation of the given data:

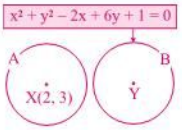
Class Interval	10-20	20-30	30-40	40-50	50-60
Frequency	4	10	12	8	6

GROUP D 4 × 5 = 20

22. एउटा मानिसले रु. 3900 ऋण मासिक किस्तामा तिर्ने विचार गरेछ । प्रत्येक किस्ता ठीक अघिल्लो किस्ता भन्दा रु. 20 ले कम छ । पहिलो किस्ता वापतको रकम रु. 400 छ । जम्मा रकम कति ओटा किस्तामा तिर्न सकिन्छ ? कारण दिनुहोस् ।

A person pays a loan of Rs 3900 in monthly installments, each installment being less than the former by Rs 20. The amount of first installment is Rs 400. In how many installments will the entire amount be paid ? Give reason.

23. दिइएको चित्रमा बराबर वृत्तहरू A र B का केन्द्रबिन्दुहरू क्रमश: X र Y छन् । यदि X को निर्देशाङ्क (2, 3) र वृत्त B को समीकरण $x^2 + y^2 - 2x + 6y + 1 = 0$ छ भने वृत्त A को समीकरण निकाल्नुहोस् । साथै रेखा XY को समीकरण पनि पत्ता लगाउनुहोस ।



In the given figure, the centres of two equal circles A and B are X and Y respectively. If the coordinates of X are (2, 3) and the equation of circle B is $x^2 + y^2 - 2x + 6y + 1 = 0$ then find the equation of the circle A. Also, find the equation of line XY.

24. त्रिभुजमा गुरुत्वकेन्द्रको परिभाषा दिनुहोस् । ΔPQR मा मध्यरेखा QS र गुरुत्वकेन्द्र G छन् । यदि O उद्गम बिन्दु हो भने, प्रमाणित गर्नुहोस् : $3\overrightarrow{OG} = \left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$

Define centroid of a triangle. In ΔPQR, QS is a median and G is the centroid. If O is the origin then prove that: $3\overrightarrow{OG} = \left(\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}\right)$

25. यदि E_1 [(1, 2), 4] र E_2 [(1, 2), $\frac{1}{2}$] विस्तारहरू भए $E_1 \circ E_2$ पत्ता लगाउनुहोस् । कुनै एउटा समकोणी त्रिभुजको निर्देशाङ्कहरू लेखाचित्रको माध्यमबाट लेखी $E_1 \circ E_2$ द्वारा उक्त त्रिभुजलाई स्थानान्तरण गर्नुहोस् र प्राप्त प्रतिबिम्बलाई पनि सोही लेखाचित्रमा देखाउनुहोस् ।

If E_1 [(1, 2), 4] and E_2 [(1, 2), $\frac{1}{2}$] are enlargements, find $E_1 \circ E_2$. Write the co-ordinates of a right-angled triangle in a graph. Transform the triangle by $E_1 \circ E_2$ and represent the image in same graph.

ANSWERS

1. (b) $\sqrt{mn}$ **2.** (a) Set of Real numbers

2. (b) *le* – vo **3.** (a) $\tan\theta = \pm\left(\frac{a-b}{1+ab}\right)$

4. (a) $2\cos^2 A - 1$ (b) $\theta = 30°$ **5.** (a) 2ab

(b) $(OP) \times (OP') = 1$ **6.** (a) Yes (b) (2, 1)

(c) 2 **7.** (a) $\begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}$ (b) – 15, – 10

8. (a) $mnx^2 + (m^2 - n^2)xy - mny^2 = 0$

(b) $x^2 + y^2 - 8x - 10y + 32 = 0$ **9.** (a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(c) 60°, 300° **10.** (a) 45° (c) 0.404

11. $-\frac{3}{2}$ **12.** $2y \geq x - 1, x + y \leq 4, x \geq 0, y \geq 0$, 11 at (3, 1)

13. $x = -2, x = 0, x = 4, x = 6$ **14.** $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

15. $3x^2 + xy - 10y^2 - 6x - y + 3 = 0$ **18.** 10.15 *m*

19. (0, 0), (– 1, 0), (– 1, 1), (0, 1) **20.** 9.56 **21.** 12.03

22. 15 installments **23.** $x^2 + y^2 - 4x - 6y + 4 = 0, 6x - y = 9$

pustakalaya.com.np

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 14
2076 (2020)

विषय : ऐच्छिक (अतिरिक्त) गणित | पूर्णाङ्क : १००
विषय कोड : ३३१ | समय : ३ घण्टा

GROUP A 10 × 1 = 10

1. (a) $y = \tan x$ को y-खण्ड कति हुन्छ ?
What is the y-intercept of $y = \tan x$?

(b) पहिलो पद 'a' र अन्तिम पद 'b' भएको समानान्तरीय अनुक्रमको n ओटा पदहरूको योगफल कति हुन्छ ?
What is the sum of n terms of arithmetic sequence whose first term is 'a' and last term is 'b'?

2. (a) अन्तराल (interval) $[a, b]$ को अर्थ के हुन्छ ?
What is the meaning of the interval $[a, b]$?

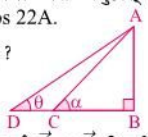
(b) एउटा मेट्रिक्समा मुख्य विकर्णका सदस्यहरूको गुणनफल m र सहायक विकर्णका सदस्यहरूको गुणनफल n भए सो मेट्रिक्सको डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।
In a matrix, the product of elements of leading diagonal is m and the product of elements of secondary diagonal is n. Find the determinant of the matrix.

3. (a) यदि दुई रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ आपसमा समानान्तर भए यिनीहरूको झुकावको सम्बन्ध लेख्नुहोस् ।
If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel to each other, write the relation between their slopes.

(b) यदि प्रतिच्छेदित सतह सोलीको जेनेरेटरसँग समानान्तर भए कुन शाङ्कि क भाग बन्दछ ?
If the intersection plane is parallel to the generator of cone then what conic does it form?

4. (a) cos 44A लाई cos 22A को रूपमा व्यक्त गर्नुहोस् ।
Express cos 44A interms of cos 22A.

(b) चित्रमा α र θ मध्य कुन ठूलो हुन्छ ?
In the figure, which is greater between α and θ?



5. (a) $\vec{a} \cdot \vec{b}$ को अधिकतम मान प्राप्त गर्न $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुनुपर्छ ?
What should be the angle between $\vec{a}$ and $\vec{b}$ to get the maximum value of $\vec{a} \cdot \vec{b}$?

(b) केन्द्रबिन्दु उद्गम बिन्दुमा पर्ने वृत्तमा $P(x, y)$ को उत्क्रम बिन्दु $P'(x', y')$ भए $P'(x', y')$ निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to find $P'(x', y')$ if $P'(x', y')$ is the inversion point of $P(x, y)$ in a circle with centre at origin.

GROUP B 13 × 2 = 26

6. (a) यदि $f(x) = \frac{2x-3}{5}$ भए $f^{-1}\left(\frac{1}{5}\right)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = \frac{2x-3}{5}$, find the value of $f^{-1}\left(\frac{1}{5}\right)$.

(b) यदि $f = \{(1, 3), (2, 4), (3, 5)\}$ र $g = \{(3, 1), (4, 2), (5, 3)\}$ भए $(f \circ g)$ लाई मिलान चित्रमा देखाउनुहोस् । $(f \circ g)$ का क्रमजोडाहरू लेख्नुहोस् ।
If $f = \{(1, 3), (2, 4), (3, 5)\}$ and $g = \{(3, 1), (4, 2), (5, 3)\}$, then show $(f \circ g)$ in arrow-diagram. Write the ordered pairs of $(f \circ g)$.

(c) उद्गमबिन्दुमा शीर्ष भएको एउटा पाराबोला बिन्दु (4, 32) भएर जान्छ भने सो पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the parabola which passes through the point (4, 32) and vertex lies at the origin.

7. (a) यदि मेट्रिक्स $A = \begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ र A को डिटरमिन्यान्ट 4 भए x को मान पत्ता लगाउनुहोस् ।
If the matrix $A = \begin{bmatrix} 2x & 6 \\ 4 & 1 \end{bmatrix}$ and determinant of A is 4 then, find the value of x.

(b) समीकरणहरू $mx + ny = 2$ र $nx + my = 2$ मा क्रामरको नियम अनुसार $\frac{D}{D_y}$ पत्ता लगाउनुहोस् ।
According to the Cramer's rule, find $\frac{D}{D_y}$ in the equations $mx + ny = 2$ and $nx + my = 2$.

8. (a) रेखाहरू MN र PQ का झुकावहरू क्रमशः 2 र – 3 छन् भने ती रेखाहरूबीचको अधिककोण पत्ता लगाउनुहोस् ।
The slopes of the lines MN and PQ are 2 and – 3 respectively. Find the obtuse angle between the lines.

(b) कुनै वृत्तको एउटा व्यासका दुई छेउका बिन्दुहरू (0, 6) र (8, 0) छन् भने सो वृत्तको समीकरण पत्ता लगाउनुहोस् ।
The end points of a diameter of a circle are (0, 6) and (8, 0). Find the equation of the circle.

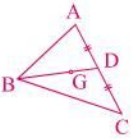
9. (a) यदि $\sin\frac{A}{3} = \frac{1}{2}\left(a + \frac{1}{a}\right)$ भए $\sin A = -\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
If $\sin\frac{A}{3} = \frac{1}{2}\left(a + \frac{1}{a}\right)$, prove that: $\sin A = -\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

(b) प्रमाणित गर्नुहोस् (Prove that):
$\frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A} = \tan 5A$

(c) हल गर्नुहोस् (Solve): $2\cos\theta = \sec\theta\ [0° \le \theta \le 90°]$

10. (a) यदि $\vec{p} \cdot \vec{q} = 6$, $|\vec{q}| = 2\sqrt{3}$ एकाइ र $\vec{p}$ र $\vec{q}$ बिचको कोण 30° भए $\vec{p}$ को लम्बाइ पत्ता लगाउनुहोस् ।
If $\vec{p} \cdot \vec{q} = 6$, $|\vec{q}| = 2\sqrt{3}$ unit and the angle between $\vec{p}$ and $\vec{q}$ is 30°, find the length of $\vec{p}$.

(b) दिइएको चित्रमा AD = DC र बिन्दु G त्रिभुज ABC को गुरुत्वकेन्द्र हो । यदि बिन्दुहरू B र D का स्थिति भेक्टरहरू क्रमश : $3\vec{i} + 7\vec{j}$ र $3\vec{i} - 2\vec{j}$ छन् भने G को स्थिति भेक्टर पत्ता लगाउनुहोस् ।



In the given figure, AD = DC and the point G is the centroid of the triangle ABC. If the position vectors of the points B and D are $3\vec{i} + 7\vec{j}$ and $3\vec{i} - 2\vec{j}$ respectively, find the position vector of G.

(c) कुनै श्रेणीको पहिलो चतुर्थांशीय मान x र चतुर्थांशीय विचलन (x) भए त्यसको तेस्रो चतुर्थांशीय मान तथा चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।
In a data, value of first quartile is 'x' and quartile deviation is also (x). Find the third quartile and the coefficient of quartile deviation.

www.drmathnp.com

GROUP C 11 × 4 = 44

11. हल गर्नुहोस् (Solve): $2x^3 - 9x^2 + 7x + 6 = 0$

12. $Z = 10x + 12y$ को निम्न लिखित अवस्थामा अधिकतम मान निकाल्नुहोस् ।
Find the maximum value of $Z = 10x + 12y$ under the following constraints: $x + y \le 6$, $x - y \le 4$, $x \ge 0$ and $y \ge 0$

13. यदि $f(x) = \begin{cases} x+2, & 1 \le x \le 2 \\ 4x-4, & x > 2 \end{cases}$ भए,
If $f(x) = \begin{cases} x+2, & 1 \le x \le 2 \\ 4x-4, & x > 2 \end{cases}$ then,
(i) $x = 1.9999$ मा $f(x)$ को मान पत्ता लगाउनुहोस् ।
For $x = 1.9999$, find the value of $f(x)$.
(ii) $x = 2.0001$ मा $f(x)$ को मान पत्ता लगाउनुहोस् ।
For $x = 2.0001$, find the value of $f(x)$.
(iii) $\lim_{x \to 2^-} f(x)$ र $\lim_{x \to 2^+} f(x)$ का मानहरू कति कति हुन्छन् ?
What are the values of $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?
(iv) के फलन $x = 2$ मा निरन्तर छ ?
Is the function continuous at $x = 2$?

14. मेट्रिक्स विधिबाट हल गर्नुहोस् ।
Solve by matrix method: $\frac{3}{y} = \frac{2}{x} + 2, \frac{6}{x} - \frac{2}{y} = 1$

15. यदि समीकरण $2x^2 + kxy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको कोण 45° भए k को धनात्मक मान पत्ता लगाई रेखाहरूका बेग्लाबेग्लै समीकरणहरू पनि पत्ता लगाउनुहोस् ।
If an angle between the pair of lines represented by the equation $2x^2 + kxy + 3y^2 = 0$ is 45°, then find the positive value of k and also find the separate equations of the lines.

16. क्याल्कुलेटर अथवा मान तालिकाको प्रयोग नगरीकन मान पत्ता लगाउनुहोस् ।
Without using the calculator or table, find the value of: sin 100° . sin 120° . sin 140° . sin 160°

17. P, Q र R एउटा ΔPQR का कोणहरू हुन् भने प्रमाणित गर्नुहोस् ।
If P, Q and R are the angles of a ΔPQR, prove that:
$2\left(\sin\frac{Q}{2} + \sin\frac{R}{2}\right)\left(\sin\frac{Q}{2} - \sin\frac{R}{2}\right) - \cos P = 1 - 4\cos\frac{P}{2}.\cos\frac{Q}{2}.\sin\frac{R}{2}$

18. एउटा घरको छत र फेदबाट एउटा रुखको टुप्पोको अवनति कोण र उन्नतांश कोणहरू क्रमशः 60° र 30° छन् । यदि रुखको उचाइ 15 फिट भए घरको उचाइ पत्ता लगाउनुहोस् ।
From the roof and foot of a house, the angles of depression and elevation of the top of a tree are 60° and 30° respectively. If the height of the tree is 15 ft, find the height of the house.

19. बिन्दुहरू A(4, 1) र B(7, 5) जोड्ने रेखाखण्ड AB लाई स्थानान्तरण गर्दा A'(– 4, 1) र B'(– 7, 5) बिन्दुहरू जोड्ने रेखाखण्ड A'B' बन्दछ । यस स्थानान्तरणलाई प्रतिनिधित्व गर्ने 2 × 2 मेट्रिक्स पत्ता लगाउनुहोस् ।
A line segment AB joining the points A(4, 1) and B(7, 5) is transformed to the line segment A'B' joining the points A'(– 4, 1) and B'(– 7, 5). Find the 2 × 2 matrix that represents this transformation.

20. तलको तथ्याङ्ककको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation from median of the data below:

X	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	4	5	6

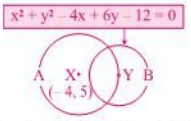
21. तलको तथ्याङ्कबाट विचरणशीलताको गुणाङ्क गणना गर्नुहोस् ।
Calculate the coefficient of variation from the data below.

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

GROUP D 4 × 5 = 20

22. एउटा गुणोत्तर श्रेणीको पहिला पाँच पदहरूको गुणनफल 243 छ । यदि उक्त गुणोत्तर श्रेणीको तेस्रो पद एउटा समानान्तर श्रेणीको दशौं पदसँग बराबर भए उक्त समानान्तर श्रेणीको पहिला 19 पदहरूको योगफल पत्ता लगाउनुहोस् ।
The product of the first five terms of a geometric series is 243. If the third term of the geometric series is equal to the tenth term of an arithmetic series, find the sum of the first 19 terms of the arithmetic series.

23. दिइएको चित्रमा केन्द्रबिन्दु X भएको वृत्त A वृत्त B को केन्द्रबिन्दु Y भएर गएको छ ।



यदि वृत्त B को समीकरण $x^2 + y^2 - 4x + 6y - 12 = 0$ र X को निर्देशाङ्कहरू(– 4, 5) छन् भने वृत्त A को समीकरण पत्ता लगाउनुहोस् ।
In the given figure, the circle A with centre X passes through the centre Y of the circle B. If the equation of circle B is $x^2 + y^2 - 4x + 6y - 12 = 0$ and the co-ordinates of X are (– 4, 5), then find the equation of the circle A.

24. व्यास AB भएको एउटा अर्धवृत्तको परिधिकोण ACB एक समकोण हुन्छ भनी भेक्टर विधिद्वारा प्रमाणित गर्नुहोस् ।
Prove by vector method that the circumference angle ACB of a semi-circle with a diameter AB is a right-angle.

25. शिर्षबिन्दुहरू A(2, 3), B(2, 6) र C(3, 4) भएको त्रिभुज ABC लाई $T = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ ले विस्थापन गर्दा बन्ने प्रतिबिम्बलाई E[(0, 0), 2] ले विस्तारीकरण गरिएको छ । यसरी प्राप्त प्रतिबिम्बहरूका शीर्षबिन्दुहरूका निर्देशाङ्कहरू लेखी ΔABC र यसका दुवै प्रतिबिम्बहरू एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस् । साथै, प्रतिबिम्बका प्रकृतिहरूको बारेमा लेख्नुहोस् ।
A triangle ABC with vertices A(2, 3), B(2, 6) and C(3, 4) is translated by $T = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and the image so obtained is enlarged by E[(0, 0), 2]. Write the co-ordinates of the vertices of the images so formed and represent ΔABC and its both images in the same graph. Also, write the nature of images.

ANSWERS

1. (a) 0 (b) $\frac{n}{2}(a+b)$ **2.** (b) $m - n$
3. (a) $m_1 = m_2$ (b) Parabola
4. (a) $2\cos^2 22A - 1$ (b) α **5.** (a) 0°
(b) $x' = \frac{r^2 x}{x^2+y^2}$, $y' = \frac{r^2 y}{x^2+y^2}$ **6.** (a) 2
(b) {(3, 3), (4, 4), (5, 5)} (c) $y = 2x^2$ **7.** (a) 14
(b) $\frac{m+n}{2}$ **8.** (a) 135° (b) $x^2 + y^2 - 8x - 6y = 0$
9. (c) 45° **10.** (a) 2 (b) $3\vec{i} + \vec{j}$ (c) 3x, 0.5
11. $\frac{-1}{2}$, 2, 3 **12.** 72 at (0, 6)
13. (i) 3.9999 (ii) 4.0004 (iii) 4, 4 (iv) Yes
14. $x = 2, y = 1$ **15.** $k = 7$, $2x + y = 0$ and $x + 3y = 0$
16. $\frac{3}{16}$ **18.** 60 ft **19.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
20. 11.3 **21.** 42.55% **22.** 57
23. $x^2 + y^2 + 8x - 10y - 59 = 0$ **25.** A"(0, 12), B"(0, 18), C"(2, 14)

pustakalaya.com.np

www.drmathnp.com

SEE MODEL QUESTION 2076 SET 15
2076 (2020)

विषय : ऐच्छिक (अतिरिक्त) गणित **पूर्णाङ्क : १००**
विषय कोड : ३३१ **समय : ३ घण्टा**

GROUP A 10 × 1 = 10

1. (a) $y = \cos x$ को न्यूनतम मान कति हुन्छ ?
What is the minimum value of $y = \cos x$?

(b) पहिलो पद 'a' र समान अन्तर 'd' भएको AS को n औं समानान्तरीय मध्यमा के हुन्छ ?
What is the n^{th} arithmetic mean of an AS whose first term is 'a' and common difference is 'd'?

2. (a) अन्तराल (interval) (a, b) को अर्थ के हुन्छ ?
What do you meant by the interval (a, b)?

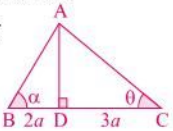
(b) एउटा मेट्रिक्समा मुख्य विकर्णका सदस्यहरूको गुणनफल 50 र सहायक विकर्णका सदस्यहरूको गुणनफल 40 भए सो मेट्रिक्सको डिटरमिन्यान्ट पत्ता लगाउनुहोस् ।
In a matrix, the product of elements of leading diagonal is 50 and the product of elements of secondary diagonal is 40. Find the determinant of the matrix.

3. (a) यदि दुई रेखाहरू $y = m_1x + c_1$ र $y = m_2x + c_2$ आपसमा लम्ब भए यिनीहरूको झुकावको सम्बन्ध लेख्नुहोस् ।
If the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular to each other, write the relation between their slopes.

(b) एउटा सतहले सोलीको अक्षसँग लम्ब हुने गरी र सोलीको शीर्षबिन्दुबाट नजाने गरी प्रतिच्छेदित गर्दछ । यसरी बन्ने शाङ्कि भागको नाम लेख्नुहोस् ।
A plane perpendicularly intersects a cone not passing through the vertex. Name the section so formed.

4. (a) cos 16A लाई sin 8A को रूपमा व्यक्त गर्नुहोस् ।
Express cos 16A interms of sin 8A.

(b) चित्रमा α र θ मध्य कुन ठूलो हुन्छ ?
In the figure, which is greater between α and θ?



5. (a) $\vec{a} \cdot \vec{b}$ को न्यूनतम मान प्राप्त गर्न $\vec{a}$ र $\vec{b}$ बीचको कोण कति हुनुपर्छ ?
What should be the angle between $\vec{a}$ and $\vec{b}$ to get the minimum value of $\vec{a} \cdot \vec{b}$?

(b) केन्द्रबिन्दु (h, k) र अर्धव्यास 'r' भएको वृत्तमा $P(x, y)$ को उत्क्रम बिन्दु $P'(x', y')$ भए x' निकाल्ने सूत्र लेख्नुहोस् ।
Write the formula to calculate x' if $P'(x', y')$ is the inversion point of $P(x, y)$ of a circle whose centre is (h, k) and radius 'r'.

GROUP B 13 × 2 = 26

6. (a) यदि $f(x) = 2x - 1$ भए $ff(-1)$ को मान पत्ता लगाउनुहोस् ।
If $f(x) = 2x - 1$ then find the value of $ff(-1)$.

(b) यदि $f(x) = \frac{3x+4}{5}$ एउटा एक एक सम्पूर्ण फलन हो र $f^{-1}(x) = 12$ भए x को मान पत्ता लगाउनुहोस् ।
If $f(x) = \frac{3x+4}{5}$ is one to one onto function and $f^{-1}(x) = 12$ then find the value of x.

(c) उद्गमबिन्दुमा शीर्ष भएको एउटा पाराबोला बिन्दु (2, 8) भएर जान्छ भने सो पाराबोलाको समीकरण पत्ता लगाउनुहोस् ।
Find the equation of the parabola which passes through the point (2, 8) and vertex lies at the origin.

7. (a) यदि $M = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, $N = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ र $|MN| = 10$ भए P को मान पत्ता लगाउनुहोस् ।
If $M = \begin{bmatrix} 2 & 1 \\ 3 & P \end{bmatrix}$, $N = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$ and $|MN| = 10$, find the value of P.

(b) समीकरणहरू $ax + by = 1$ र $bx + ay = 1$ मा क्रामरको नियम अनुसार $\frac{D_x}{D}$ पत्ता लगाउनुहोस् ।
According to the Cramer's rule, find $\frac{D_x}{D}$ in the equations $ax + by = 1$ and $bx + ay = 1$.

8. (a) समीकरणहरू $2x - y + 1 = 0$ र $x + my - 7 = 0$ भएका दुई रेखाहरू आपसमा लम्ब छन् भने m को मान पत्ता लगाउनुहोस् ।
The two straight lines having equations $2x - y + 1 = 0$ and $x + my - 7 = 0$ are perpendicular to each other. Find the value of m.

(b) समीकरण $2x^2 - 7xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस् ।
Find the acute angle between the lines represented by the equation $2x^2 - 7xy + 3y^2 = 0$.

9. (a) प्रमाणित गर्नुहोस् (Prove that):
$\sin^4 B + \cos^4 B = 1 - \frac{1}{2}\sin^2 2B$

(b) यदि $\cos\frac{B}{3} = \frac{1}{2}\left(m + \frac{1}{m}\right)$ भए प्रमाणित गर्नुहोस् :
$\cos B = \frac{1}{2}\left(m^3 + \frac{1}{m^3}\right)$
If $\cos\frac{B}{3} = \frac{1}{2}\left(m + \frac{1}{m}\right)$ then, prove that:
$\cos B = \frac{1}{2}\left(m^3 + \frac{1}{m^3}\right)$

(c) हल गर्नुहोस् (Solve): $\text{cosec } A = 2\sin A \ [0° \le A \le 90°]$

pusmas.com.np

www.drmathnp.com

10. (a) यदि $|\vec{a}| = 10$, $|\vec{b}| = 3$ र $\vec{a} \cdot \vec{b} = 15\sqrt{2}$ भए $\vec{a}$ र $\vec{b}$ बिचको कोण पत्ता लगाउनुहोस् ।

If $|\vec{a}| = 10$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 15\sqrt{2}$ then find the angle between $\vec{a}$ and $\vec{b}$.

(b) यदि $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ र M ले BA लाई भित्रपट्टिबाट 3 : 2 को अनुपातमा विभाजन गर्दछ भने प्रमाणित गर्नुहोस् : $\vec{m} = \frac{1}{5}(3\vec{a} + 2\vec{b})$

If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OM} = \vec{m}$ and M divides BA internally in the ratio of 3 : 2 then, prove that: $\vec{m} = \frac{1}{5}(3\vec{a} + 2\vec{b})$

(c) कुनै श्रेणीको पहिलो चतुर्थांशीय मान $2a$ र चतुर्थांशीय विचलन (a) भए त्यसको तेस्रो चतुर्थांशीय मान तथा चतुर्थांशीय विचलनको गुणाङ्क पत्ता लगाउनुहोस् ।

In a data, value of first quartile is '$2a$' and quartile deviation is (a). Find the third quartile and the coefficient of quartile deviation.

GROUP C 11 × 4 = 44

11. गुणन साध्य प्रयोग गरेर हल गर्नुहोस् : $2x^3 + 13x^2 = 36$
Solve by using factor theorem: $2x^3 + 13x^2 = 36$

12. उद्देश्य फलन $P = 3x + 2y$ को निम्न लिखित अवस्थाहरूमा अधिकतम मान निकाल्नुहोस् ।
Maximize the objective function $P = 3x + 2y$ under the following constraints.
$x + y \geq 6$, $x - y \geq 4$, $x \geq 0$ and $y \geq 0$

13. बायाँ तर्फको सीमान्त मान, दायाँ तर्फको सीमान्त मान र फलनको मान पत्ता लगाई फलन $f(x) = \begin{cases} 3-x, & x \leq 0 \\ 3, & x > 0 \end{cases}$ को $x = 0$ मा निरन्तरता वा विच्छिन्नता परीक्षण गर्नुहोस् ।
Examine the continuity or discontinuity of $f(x) = \begin{cases} 3-x, & x \leq 0 \\ 3, & x > 0 \end{cases}$ at $x = 0$ by calculating left hand limit, right hand limit and functional value.

14. मेट्रिक्स विधिबाट हल गर्नुहोस् (Solve by matrix method):
$\frac{3}{y} + \frac{4}{x} = 1$, $\frac{3}{x} - \frac{2}{y} = \frac{1}{24}$

15. समीकरण $x^2 + 3xy - 4y^2 = 0$ ले प्रतिनिधित्व गर्ने एकजोडा सरलरेखाहरूसँग समानान्तर हुने र बिन्दु (0, 1) भएर जाने एकजोडा सरलरेखाहरूको एकल समीकरण पत्ता लगाउनुहोस् ।
Find the single equation of a pair of straight lines passing through the point (0, 1) and parallel to the pair of straight lines represented by the equation $x^2 + 3xy - 4y^2 = 0$.

16. प्रमाणित गर्नुहोस् (Prove that):
$8 \sin 20° \cdot \sin 40° \cdot \sin 80° = \sqrt{3}$

17. यदि $A + B + C = \pi^C$ भए प्रमाणित गर्नुहोस् :
If $A + B + C = \pi^C$ then prove that:
$\sin A \cos A + \sin B \cos B + \sin C \cos C = 2 \sin A \sin B \sin C$

18. 16 मिटर अग्लो एउटा भवनको छतबाट ठीक अगाडि रहेको एउटा बत्तीको खम्बाको टुप्पो र फेदलाई अवलोकन गर्दा उन्नतांश कोण र अवनति कोण क्रमशः 60° र 30° पाइयो भने सो खम्बाको उचाइ पत्ता लगाउनुहोस् ।
From the roof of a building 16 meter high, the angles of elevation and depression of the top and the foot of an electric pole are observed to be 60° and 30° respectively. Find the height of the pole.

19. एकाइ वर्गलाई समानान्तर चतुर्भुज $\begin{bmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ मा स्थानान्तरण गर्ने 2 × 2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस् ।
Find the 2 × 2 transformation matrix which transforms the unit square into a parallelogram $\begin{bmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.

20. तल दिइएको तथ्याङ्कको मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस् ।
Find the mean deviation from median of the data given below:

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	7	5	10	12	6

21. तल दिइएको तथ्याङ्कको स्तरीय भिन्नता पत्ता लगाउनुहोस् ।
Find the standard deviation of the data given below.

Class-interval	0 - 10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

GROUP D 4 × 5 = 20

22. सुधिरले रु. 4368 वार्षिक 6 ओटा किस्ताहरूमा, हरेक किस्ता अघिल्लो किस्ताको तेब्बर हुने गरी तिर्ने शर्तमा ऋणमा लियो । पहिलो किस्ता र अन्तिम किस्ता पत्ता लगाउनुहोस् ।
Sudhir borrows Rs 4368 which he promises to pay in 6 annual installments, each installment being treble of the preceding one. Find the first and the last installment.

23. एउटा वृत्ताकार डिजाइनको समीकरण $2x^2 + 2y^2 - 20x - 28y + 98 = 0$ छ । सो डिजाइनको केन्द्रबिन्दु, व्यास र क्षेत्रफल पत्ता लगाउनुहोस् ।
Equation of a circular design is $2x^2 + 2y^2 - 20x - 28y + 98 = 0$. Find the centre, diameter and area of the design.



24. चित्रमा ΔOAB कस्तो किसिमको त्रिभुज हो ? यदि ΔAOB को मध्यिका OC भए $OC \perp AB$ हुन्छ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस् ।
In the figure, what type of triangle is ΔAOB? If OC is the median of ΔAOB then prove by the vector method that $OC \perp AB$.



25. E ले केन्द्रबिन्दु (3, 1) र नापो 2 भएको विस्तारीकरण र R ले $y = x$ मा हुने परावर्तन जनाउँदछन् । शीर्षबिन्दुहरू A(2, 3), B(4, 5) र C(1, – 2) भएको ΔABC को संयुक्त स्थानान्तरण E∘R द्वारा हुने प्रतिबिम्ब पत्ता लगाउनुहोस् । दुवै चित्र एउटै लेखाचित्रमा खिच्नुहोस् । साथै, प्रतिबिम्ब त्रिभुजहरूको प्रकृति पनि लेख्नुहोस् ।
E denotes the enlargement about the centre (3, 1) with a scale factor of 2 and R denotes a reflection on the line $y = x$. Find the image of ΔABC having the vertices A(2, 3), B(4, 5) and C(1, – 2) under the combined transformation E∘R. Draw both figures on the same graph paper. Also, write the nature of image triangles.

pustak.com.np

www.drmathnp.com

puspas.com.np

ANSWERS

1. (a) -1 (b) $a+nd$ **2.** (b) 10 **3.** (a) $m_1 \times m_2 = -1$ (b) Circle **4.** (a) $1-2\sin^2 8A$ (b) α **5.** (a) 180° (b) $x' = \frac{r^2(x-h)}{(x-h)^2+(y-k)^2} + h$ **6.** (a) -7 (b) 8 (c) $y = 2x^2$ **7.** (a) 4 (b) $\frac{1}{a+b}$ **8.** (a) 2 (b) 45° **9.** (c) 45°

10. (a) 45° (c) $4a$, 0.33 **11.** $-2, -6, \frac{3}{2}$ **12.** $P_{max} = 18$ at (6, 0) **13.** Continuous **14.** $x = 8, y = 6$

15. $x^2 + 3xy - 4y^2 - 3x + 8y - 4 = 0$ **18.** 64 m **19.** $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ **20.** 11.05 **21.** 11.48

22. Rs 12, Rs 2916 **23.** (5, 7), 10 units, 25π sq. units **25.** A"(3, 3), B"(7, 7), C"(– 7, 1), Similar and Congruent

HINTS FOR SOME EXTRA QUESTIONS OF TRIGONOMETRY

1. प्रमाणित गर्नुहोस् (Prove that): $(\sec 2\theta + 1)(\sec 4\theta + 1)(\sec 8\theta + 1) = \tan 8\theta . \cot \theta$

☞ LHS $= (\sec 2\theta + 1)(\sec 4\theta + 1)(\sec 8\theta + 1)$

$= \left(\frac{1}{\cos 2\theta} + 1\right)\left(\frac{1}{\cos 4\theta} + 1\right)\left(\frac{1}{\cos 8\theta} + 1\right) = \frac{1+\cos 2\theta}{\cos 2\theta} \times \frac{1+\cos 4\theta}{\cos 4\theta} \times \frac{1+\cos 8\theta}{\cos 8\theta}$

$= \frac{2\cos^2\theta}{\cos 2\theta} \times \frac{2\cos^2 2\theta}{\cos 4\theta} \times \frac{2\cos^2 4\theta}{\cos 8\theta} = \frac{8}{\cos 8\theta}(\cos^2\theta \cos 2\theta \cos 4\theta)$

$= \frac{\cos\theta}{\sin\theta} \times \frac{4}{\cos 8\theta} \times (2\sin\theta . \cos\theta)\cos 2\theta . \cos 4\theta$

$= \cot\theta \times \frac{2}{\cos 8\theta} \times \sin 2\theta \cos 2\theta . \cos 4\theta = \cot\theta \times \frac{2}{\cos 8\theta} \times (2\sin 2\theta \cos 2\theta)\cos 4\theta$

$= \cot\theta \times \frac{2}{\cos 8\theta} \times \sin 4\theta \cos 4\theta = \cot\theta \times \frac{\sin 8\theta}{\cos 8\theta}$

$= \cot\theta . \tan 8\theta = \tan 8\theta \cot\theta = \text{RHS}$

2. प्रमाणित गर्नुहोस् (Prove that): $\cot\left(\frac{A}{2} + 45°\right) - \tan\left(\frac{A}{2} - 45°\right) = \frac{2\cos A}{1+\sin A}$

☞ Here, LHS $= \cot\left(\frac{A}{2} + 45°\right) - \tan\left(\frac{A}{2} - 45°\right) = \frac{\cos\left(\frac{A}{2}+45°\right)}{\sin\left(\frac{A}{2}+45°\right)} - \frac{\sin\left(\frac{A}{2}-45°\right)}{\cos\left(\frac{A}{2}-45°\right)}$

$= \frac{\cos\left(\frac{A}{2}+45°\right)\cos\left(\frac{A}{2}-45°\right) - \sin\left(\frac{A}{2}+45°\right)\sin\left(\frac{A}{2}-45°\right)}{\sin\left(\frac{A}{2}+45°\right)\cos\left(\frac{A}{2}-45°\right)}$

$= \frac{\cos\left(\frac{A}{2}+45°+\frac{A}{2}-45°\right)}{\left(\sin\frac{A}{2}\cos 45° + \cos\frac{A}{2}\sin 45°\right)\left(\cos\frac{A}{2}\cos 45° + \sin\frac{A}{2}\sin 45°\right)}$

$= \frac{\cos A}{\left(\sin\frac{A}{2}\times\frac{1}{\sqrt{2}} + \cos\frac{A}{2}\times\frac{1}{\sqrt{2}}\right)\left(\cos\frac{A}{2}\times\frac{1}{\sqrt{2}} + \sin\frac{A}{2}\times\frac{1}{\sqrt{2}}\right)}$

$= \frac{\cos A}{\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)} = \frac{\cos A}{\frac{1}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)^2}$

$= \frac{2\cos A}{\sin^2\frac{A}{2} + \cos^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{2\cos A}{1+\sin A} = \text{RHS}$ Proved

www.drmathnp.com

puspas.com.np

3. प्रमाणित गर्नुहोस् (Prove that): $2\cos x - \cos 3x - \cos 5x = 16\sin^2 x \cdot \cos^3 x$

☞ Here, LHS $= 2\cos x - \cos 3x - \cos 5x = 2\cos x - (\cos 3x + \cos 5x)$

$= 2\cos x - 2\cos\frac{3x+5x}{2}\cos\frac{3x-5x}{2} = 2\cos x - 2\cos 4x \cos x$

$= 2\cos x(1 - \cos 4x) = 2\cos x.\, 2\sin^2 2x = 4\cos x(\sin 2x)^2$

$= 4\cos x.\,(2\sin x \cos x)^2 = 4\cos x.\, 4\sin^2 x \cos^2 x$

$= 16\cos^3 x \sin^2 x = \text{LHS}$

4. प्रमाणित गर्नुहोस् (Prove that): $\sin 6°.\sin 42°.\sin 66°.\sin 78° = \frac{1}{16}$

☞ Here, LHS $= \sin 6°.\sin 42°.\sin 66°.\sin 78° = \sin 6°\cos(90° - 48°)\cos(90° - 24°)\cos(90° - 12°)$

$= \sin 6°.\cos 48°.\cos 24°.\cos 12° = \cos 12°.\cos 24°.\cos 48°.\cos 84°$

$= \frac{1}{2\sin 12°}\sin 24°.\cos 24°.\cos 48°.\sin 6° = \frac{1}{4\sin 12°}(2\sin 24°\cos 24°)\cos 48°\sin 6°$

$= \frac{1}{8\sin 12°}(2\sin 48°\cos 48°)\sin 6° = \frac{1}{8\sin 12°}\sin 96°\sin 6°$

$= \frac{1}{16\sin 12°}2\sin(90° + 6°)\sin 6° = \frac{1}{16\sin 12°}2\cos 6°\sin 6°$

$= \frac{\sin 12°}{16\sin 12°} = \frac{1}{16} = \text{RHS}$

5. प्रमाणित गर्नुहोस् (Prove that): $\cos 12°.\cos 24°.\cos 48°.\cos 84° = \frac{1}{16}$

☞ Here, LHS $= \cos 12°.\cos 24°.\cos 48°.\cos 84°$

$= \frac{1}{2\sin 12°}\sin 24°.\cos 24°.\cos 48°.\cos 84° = \frac{1}{4\sin 12°}(2\sin 24°\cos 24°)\cos 48°\cos 84°$

$= \frac{1}{8\sin 12°}(2\sin 48°\cos 48°)\cos 84° = \frac{1}{8\sin 12°}\sin 96°\cos 84°$

$= \frac{1}{16\sin 12°}2\sin(90° + 6°)\cos(90° - 6°) = \frac{1}{16\sin 12°}2\cos 6°\sin 6°$

$= \frac{\sin 12°}{16\sin 12°} = \frac{1}{16} = \text{RHS}$

6. प्रमाणित गर्नुहोस् (Prove that): $\cos^3\theta.\cos 3\theta + \sin^3\theta.\sin 3\theta = \cos^3 2\theta$.

☞ Here, LHS $= \cos^3\theta.\cos 3\theta + \sin^3\theta.\sin 3\theta$

$= \frac{1}{4}(3\cos\theta + \cos 3\theta)\cos 3\theta + \frac{1}{4}(3\sin\theta - \sin 3\theta)\sin 3\theta$

$= \frac{1}{4}[3\cos\theta\cos 3\theta + \cos^2 3\theta + 3\sin\theta\sin 3\theta - \sin^2 3\theta]$

$= \frac{1}{4}[3(\cos\theta\cos 3\theta + \sin\theta\sin 3\theta) + \cos^2 3\theta - \sin^2 3\theta]$

$= \frac{1}{4}[3\cos(\theta - 3\theta) + \cos 2\times 3\theta] = \frac{1}{4}[3\cos 2\theta + \cos 6\theta]$

$= \frac{1}{4}[3\cos 2\theta + \cos 3(2\theta)] = \frac{1}{4}[3\cos 2\theta + 4\cos^3 2\theta - 3\cos 2\theta]$

$= \frac{1}{4}.4\cos^3 2\theta = \cos^3 2\theta = \text{RHS}$

7. प्रमाणित गर्नुहोस् (Prove that): $\cos^3\theta.\sin^2\theta = \frac{1}{16}(2\cos\theta - \cos 3\theta - \cos 5\theta)$

☞ Here, RHS $= \frac{1}{16}(2\cos\theta - \cos\theta\, 3\theta - \cos 5\theta) = \frac{1}{16}\{2\cos\theta - (\cos 3\theta + \cos 5\theta)\}$

$= \frac{1}{16}\left\{2\cos\theta - 2\cos\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)\right\}$

$= \frac{1}{16}(2\cos\theta - 2\cos 4\theta\cos\theta) = \frac{1}{16}\times 2\cos\theta(1 - \cos 4\theta)$

$= \frac{1}{8}\cos\theta.\, 2\sin^2 2\theta = \frac{1}{4}\cos\theta(\sin 2\theta)^2$

$= \frac{1}{4}\cos\theta.\,(2\sin\theta\cos\theta)^2 = \frac{1}{4}\cos\theta.\, 4\sin^2\theta\cos^2\theta$

$= \cos^3\theta\sin^2\theta = \text{LHS}$

www.drmathnp.com

puspas.com.np

8. एउटा कोण θ लाई α र β दुई भागमा विभाजन गरिएको छ । यदि $\tan\alpha : \tan\beta = x : y$ भए, $\sin(\alpha-\beta)(x+y) = (x-y)\sin\theta$ हुन्छ भनी प्रमाणित गर्नुहोस् ।
An angle θ is divided into two parts α and β. If $\tan\alpha : \tan\beta = x : y$, prove that: $\sin(\alpha-\beta)(x+y) = (x-y)\sin\theta$

☞ Here, $\alpha+\beta=\theta$ and $\frac{\tan\alpha}{\tan\beta}=\frac{x}{y}$

Now, $\frac{x-y}{x+y} = \frac{\frac{x-y}{y}}{\frac{x+y}{y}} = \frac{\frac{x}{y}-1}{\frac{x}{y}+1} = \frac{\frac{\tan\alpha}{\tan\beta}-1}{\frac{\tan\alpha}{\tan\beta}+1} = \frac{\tan\alpha-\tan\beta}{\tan\beta}{\frac{\tan\alpha+\tan\beta}{\tan\beta}}$

$= \frac{\tan\alpha-\tan\beta}{\tan\alpha+\tan\beta} = \frac{\frac{\sin\alpha}{\cos\alpha}-\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}} = \frac{\sin\alpha\cos\beta-\sin\beta\cos\alpha}{\cos\alpha\cos\beta}{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$

$\therefore \frac{x-y}{x+y} = \frac{\sin(\alpha-\beta)}{\sin\theta}$ Thus, $\sin(\alpha-\beta)(x+y) = (x-y)\sin\theta$

9. प्रमाणित गर्नुहोस् (Prove that): $16\cos^3 A.\sin^2 A = 2\cos A - \cos 3A - \cos 5A$

☞ Here, RHS $= 2\cos A - \cos 3A - \cos 5A = 2\cos A - (\cos 3A + \cos 5A)$

$= 2\cos A - 2\cos\frac{3A+5A}{2}\cos\frac{3A-5A}{2} = 2\cos A - 2\cos 4A\cos A$

$= 2\cos A(1-\cos 4A) = 2\cos A.2\sin^2 2A = 4\cos A(\sin 2A)^2$

$= 4\cos A.(2\sin A\cos A)^2 = 4\cos A.4\sin^2 A\cos^2 A$

$= 16\cos^3 A\sin^2 A$ = LHS

10. प्रमाणित गर्नुहोस् (Prove that): $16\cos^2 A.\sin^3 A = 2\sin A + \sin 3A - \sin 5A$

☞ Here, RHS $= 2\sin A + \sin 3A - \sin 5A = 2\sin A + (\sin 3A - \sin 5A)$

$= 2\sin A + 2\cos\frac{3A+5A}{2}\sin\frac{3A-5A}{2} = 2\sin A + 2\cos 4A\sin(-A)$

$= 2\sin A(1-\cos 4A) = 2\sin A.2\sin^2 2A = 4\sin A(\sin 2A)^2$

$= 4\sin A.(2\sin A\cos A)^2 = 4\sin A.4\sin^2 A\cos^2 A$

$= 16\sin^3 A\cos^2 A$ = LHS

11. प्रमाणित गर्नुहोस् (Prove that): $\frac{\sec 12A-1}{\sec 6A-1} = \frac{\tan 12A}{\tan 3A}$

☞ Here, LHS $= \frac{\sec 12A-1}{\sec 6A-1}$

$= \frac{\frac{1}{\cos 12A}-1}{\frac{1}{\cos 6A}-1}$

$= \frac{1-\cos 12A}{\cos 12A}\times\frac{\cos 6A}{1-\cos 6A}$

$= \frac{2\sin^2 6A\cos 6A}{\cos 12A.2\sin^2 3A}$

$= \frac{2\sin 6A\cos 6A}{\cos 12A}.\frac{2\sin 3A\cos 3A}{2\sin^2 3A}$

$= \frac{\sin 12A}{\cos 12A}.\frac{\cos 3A}{\sin 3A}$

$= \tan 12A.\cot 3A$

$= \frac{\tan 12A}{\tan 3A}$ = RHS

12. प्रमाणित गर्नुहोस् (Prove that): $\operatorname{cosec} 4A - \cot 8A = \tan 2A + \operatorname{cosec} 8A$

☞ Here, LHS $= \operatorname{cosec} 4A - \cot 8A = \frac{1}{\sin 4A}-\frac{\cos 8A}{\sin 8A}$

$= \frac{1}{\sin 4A}-\frac{\cos 8A}{2\sin 4A\cos 4A} = \frac{2\cos 4A-\cos 8A}{2\sin 4A\cos 4A}$

$= \frac{2\cos 4A-[2\cos^2 4A-1]}{2\sin 4A\cos 4A} = \frac{2\cos 4A-2\cos^2 4A+1}{2\sin 4A\cos 4A}$

$= \frac{2\cos 4A(1-\cos 4A)+1}{2\sin 4A\cos 4A} = \frac{2\cos 4A.2\sin^2 2A+1}{2\sin 4A\cos 4A}$

$= \frac{4\cos 4A\sin^2 2A}{2\sin 4A\cos 4A}+\frac{1}{2\sin 4A\cos 4A} = \frac{2\sin^2 2A}{2\sin 2A\cos 2A}+\frac{1}{\sin 8A}$

$= \frac{\sin 2A}{\cos 2A}+\frac{1}{\sin 8A}$ = $\tan 2A + \operatorname{cosec} 8A$ = RHS

www.drmathnp.com

puspas.com.np

13. प्रमाणित गर्नुहोस् (Prove that): $2\sin x(\cos x + \cos 3x + \cos 5x) = \sin 6x$

☞ Here, LHS $= 2\sin x[\cos x + \cos 3x + \cos 5x]$

$= 2\sin x\left[\cos x + 2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)\right]$

$= 2\sin x[\cos x + 2\cos 4x\cos x] = 2\sin x.\cos x(1 + 2\cos 4x)$

$= \sin 2x(1 + 2\cos 4x) = \sin 2x + 2\sin 2x\cos 4x$

$= \sin 2x + \sin(2x + 4x) + \sin(2x - 4x)$

$= \sin 2x + \sin 6x - \sin 2x$

$= \sin 6x = \text{RHS}$ *Proved.*

14. प्रमाणित गर्नुहोस् (Prove that): $\operatorname{cosec}\frac{\alpha}{4} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha = \cot\frac{\alpha}{8} - \cot\alpha$

☞ Here, RHS $= \cot\frac{\alpha}{8} - \cot\alpha = \cot\frac{\alpha}{8} - \frac{\cos\alpha}{\sin\alpha}$

$= \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{2} - 1}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} + \frac{1}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$= \cot\frac{\alpha}{8} - \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} + \frac{1}{\sin\alpha} = \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{4} - 1}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}} + \operatorname{cosec}\alpha$

$= \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{4}}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}} + \frac{1}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}} + \operatorname{cosec}\alpha$

$= \cot\frac{\alpha}{8} - \frac{\cos\frac{\alpha}{4}}{\sin\frac{\alpha}{4}} + \frac{1}{\sin\frac{\alpha}{2}} + \operatorname{cosec}\alpha = \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{8} - 1}{2\sin\frac{\alpha}{8}\cos\frac{\alpha}{8}} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha$

$= \cot\frac{\alpha}{8} - \frac{2\cos^2\frac{\alpha}{8}}{2\sin\frac{\alpha}{8}\cos\frac{\alpha}{8}} + \frac{1}{2\sin\frac{\alpha}{8}\cos\frac{\alpha}{8}} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha$

$= \cot\frac{\alpha}{8} - \frac{\cos\frac{\alpha}{8}}{\sin\frac{\alpha}{8}} + \frac{1}{\sin\frac{\alpha}{4}} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha$

$= \cot\frac{\alpha}{8} - \cot\frac{\alpha}{8} + \operatorname{cosec}\frac{\alpha}{4} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha = \operatorname{cosec}\frac{\alpha}{4} + \operatorname{cosec}\frac{\alpha}{2} + \operatorname{cosec}\alpha = \text{LHS}$

15. प्रमाणित गर्नुहोस् (Prove that): $16\sin 10°.\sin 30°.\sin 50°.\sin 70° = 1$

☞ Here, LHS $= 16\sin 10°.\sin 30°.\sin 50°.\sin 70°$

$= 8\sin 10°\sin 50°\sin 70° = 4\sin 10°[2\sin 50°\sin 70°]$

$= 4\sin 10°[\cos(50° - 70°) - \cos(50° + 70°)]$

$= 4\sin 10°\left[\cos 20° + \frac{1}{2}\right] = 4\sin 10°\cos 20° + 2\sin 10°$

$= 2[2\sin 10°\cos 20°] + 2\sin 10° = 2[\sin(10° + 20°) + \sin(10° - 20°)] + 2\sin 10°$

$= 2(\sin 30° - \sin 10°) + 2\sin 10° = 2 \times \frac{1}{2} - 2\sin 10° + 2\sin 10° = 1 = \text{RHS}$

16. यदि $\cos A + \sin B = p$ र $\sin A + \cos B = q$ **भए** $\operatorname{cosec}(A + B) = \frac{2}{p^2 + q^2 - 2}$ **हुन्छ भनी प्रमाणित गर्नुहोस् ।**

If $\cos A + \sin B = p$ and $\sin A + \cos B = q$, then prove that: $\operatorname{cosec}(A + B) = \frac{2}{p^2 + q^2 - 2}$

☞ Here, Squaring and adding the given results then, $(\cos A + \sin B)^2 + (\sin A + \cos B)^2 = p^2 + q^2$

or, $\cos^2 A + 2\cos A\sin B + \sin^2 B + \sin^2 A + 2\sin A\cos B + \cos^2 B = p^2 + q^2$

or, $2 + 2(\cos A\sin B + \sin A\cos B) = p^2 + q^2$ or, $2\sin(A + B) = p^2 + q^2 - 2$

or, $\sin(A + B) = \frac{p^2 + q^2 - 2}{2}$ or, $\frac{1}{\sin(A + B)} = \frac{2}{p^2 + q^2 - 2}$

$\therefore \operatorname{cosec}(A + B) = \frac{2}{p^2 + q^2 - 2}$

www.drmathnp.com

puspas.com.np

17. प्रमाणित गर्नुहोस् (Prove that): $\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}=\frac{1}{16}$

☞ Here, LHS $= \cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15} = \cos 24°\cos 48°\cos 96°\cos 192°$

$= \frac{1}{2\sin 24°}(2\cos 24°\sin 24°)\cos 48°\cos 96°\cos 192°$

$= \frac{1}{4\sin 24°}.2\sin 48°\cos 48°\cos 96°\cos 192°$

$= \frac{1}{8\sin 24°}2\sin 96°\cos 96°\cos 192° = \frac{1}{16\sin 24°}.2\sin 192°\cos 192°$

$= \frac{1}{16\sin 24°}.\sin 384° = \frac{\sin(360°+24°)}{16\sin 24°} = \frac{\sin 24°}{16\sin 24°} = \frac{1}{16} = \text{RHS}$

18. प्रमाणित गर्नुहोस् (Prove that): $\cos^6\beta - \sin^6\beta = \frac{1}{16}(15\cos 2\beta + \cos 6\beta)$

☞ Here, LHS $= \cos^6\beta - \sin^6\beta = (\cos^2\beta)^3 - (\sin^2\beta)^3$

$= (\cos^2\beta - \sin^2\beta)^3 + 3\cos^2\beta\sin^2\beta(\cos^2\beta - \sin^2\beta)$

$= \cos^3 2\beta + \frac{3}{4}(2\cos\beta\sin\beta)^2.\cos 2\beta = \cos^3 2\beta + \frac{3}{4}\sin^2 2\beta.\cos 2\beta$

$= \cos^3 2\beta + \frac{3}{8}(1-\cos 4\beta)\cos 2\beta = \cos 2\beta\left[\cos^2 2\beta + \frac{3}{8}(1-\cos 4\beta)\right]$

$= \cos 2\beta\left[\frac{1}{2}2\cos^2 2\beta + \frac{3}{8} - \frac{3}{8}\cos 4\beta\right] = \cos 2\beta\left[\frac{1}{2}(1+\cos 4\beta) + \frac{3}{8} - \frac{3}{8}\cos 4\beta\right]$

$= \cos 2\beta\left[\frac{1}{2} + \frac{3}{8} + \frac{1}{8}\cos 4\beta\right] = \frac{7}{8}\cos 2\beta + \frac{1}{8}\cos 2\beta\cos 4\beta$

$= \frac{7}{8}\cos 2\beta + \frac{1}{16}2\cos 2\beta\cos 4\beta = \frac{7}{8}\cos 2\beta + \frac{1}{16}(\cos 6\beta + \cos 2\beta)$

$= \frac{7}{8}\cos 2\beta + \frac{1}{16}\cos 2\beta + \frac{1}{16}\cos 6\beta = \frac{15}{16}\cos 2\beta + \frac{1}{16}\cos 6\beta$

$= \frac{1}{16}(15\cos 2\beta + \cos 6\beta) = \text{RHS}$

19. सिद्ध गर्नुहोस् (Prove that): $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

☞ Here, LHS $= \cos(3A+2A) = \cos 3A.\cos 2A - \sin 3A.\sin 2A$

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)2\sin A.\cos A$

$= 8\cos^5 A - 6\cos^3 A - 4\cos^3 A + 3\cos A - 2\cos A(3\sin^2 A - 4\sin^4 A)$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 2\cos A[(3(1-\cos^2 A) - 4(1-\cos^2 A)^2]$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 2\cos A(3 - 3\cos^2 A - 4 + 8\cos^2 A - 4\cos^4 A)$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 2\cos A(-1 + 5\cos^2 A - 4\cos^4 A)$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A + 2\cos A - 10\cos^3 A + 8\cos^5 A$

$= 16\cos^5 A - 20\cos^3 A + 5\cos A = \text{RHS}$

20. प्रमाणित गर्नुहोस् (Prove that): $\tan\left(\frac{\pi^c}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi^c}{4}-\frac{\theta}{2}\right) = 2\sec\theta$

☞ Here, Let $\frac{\theta}{2}$ = A then $\tan(45°+A) + \tan(45°-A)$

$= \frac{1+\tan A}{1-\tan A} + \frac{1-\tan A}{1+\tan A}$

$= \frac{1+2\tan A + \tan^2 A + 1 - 2\tan A + \tan^2 A}{1-\tan^2 A} = \frac{2(1+\tan^2 A)}{1-\tan^2 A}$

$= 2\frac{1}{\cos 2A}$

$= 2\sec 2A$

$= 2\sec\theta = \text{RHS}$

21. प्रमाणित गर्नुहोस् (Prove that): $\frac{\cos^2\beta - \sin^2\alpha}{\sin\alpha.\cos\alpha + \sin\beta.\cos\beta} = \cot(\alpha+\beta)$

☞ Here, LHS $= \frac{\cos^2\beta - \sin^2\alpha}{\sin\alpha\cos\alpha + \sin\beta\cos\beta} = \frac{2\cos^2\beta - 2\sin^2\alpha}{2\sin\alpha\cos\alpha + 2\sin\beta\cos\beta}$

$= \frac{1+\cos 2\beta - (1-\cos 2\alpha)}{\sin 2\alpha + \sin 2\beta} = \frac{1+\cos 2\beta - 1 + \cos 2\alpha}{\sin 2\alpha + \sin 2\beta}$

$= \frac{\cos 2\alpha + \cos 2\beta}{\sin 2\alpha + \sin 2\beta} = \frac{2\cos\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)}{2\sin\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)}$

$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \cot(\alpha+\beta) = \text{RHS}$

www.drmathnp.com

puspas.com.np

22. प्रमाणित गर्नुहोस् (Prove that): $\cos^2\phi + \cos^2\left(\phi + \frac{\pi}{3}\right) + \cos^2\left(\phi - \frac{\pi}{3}\right) = \frac{3}{2}$

☞ Here, LHS $= \cos^2\phi + \cos^2(\phi + 60°) + \cos^2(\phi - 60°)$

$= \cos^2\phi + \frac{2\cos^2(\phi + 60°) + 2\cos^2(\phi - 60°)}{2}$

$= \cos^2\phi + \frac{1 + \cos(2\phi + 120°) + 1 + \cos(2\phi - 120°)}{2}$

$= \cos^2\phi + \frac{2}{2} + \frac{2}{2}\cos\frac{2\phi + 120° + 2\phi - 120°}{2}.\cos\frac{2\phi + 120° - 2\phi + 120}{2}$

$= 1 + \cos^2\phi + \cos 2\phi.\left(-\frac{1}{2}\right)$ $= 1 + \cos^2\phi - \frac{1}{2}(2\cos^2\phi - 1)$

$= 1 + \cos^2\phi - \cos^2\phi + \frac{1}{2}$ $= \frac{3}{2} = \text{RHS}$

23. प्रमाणित गर्नुहोस् (Prove that): $\frac{\cos 7A + \cos 3A - \cos 5A - \cos A}{\sin 7A - \sin 5A - \sin 3A + \sin A} = \cot 2A$

☞ Here, LHS $= \frac{\cos 7A + \cos 3A - \cos 5A - \cos A}{\sin 7A - \sin 5A - \sin 3A + \sin A}$ $= \frac{(\cos 7A - \cos A) + (\cos 3A - \cos 5A)}{(\sin 7A + \sin A) - (\sin 5A + \sin 3A)}$

$= \frac{2\sin\frac{7A + A}{2}.\sin\frac{A - 7A}{2} + 2\sin\frac{3A + 5A}{2}\sin\frac{5A - 3A}{2}}{2\sin\frac{7A + A}{2}.\cos\frac{7A - A}{2} - 2\sin\frac{5A + 3A}{2}.\cos\frac{5A - 3A}{2}}$

$= \frac{2\sin 4A.\sin(-3A) + 2\sin 4A.\sin A}{2\sin 4A.\cos 3A - 2\sin 4A.\cos A} = \frac{2\sin 4A(-\sin 3A + \sin A)}{2\sin 4A(\cos 3A - \cos A)}$

$= \frac{\sin A - \sin 3A}{\cos 3A - \cos A} = \frac{2\cos.\frac{A + 3A}{2}.\sin\frac{A - 3A}{2}}{2\sin\frac{3A + A}{2}.\sin\frac{A - 3A}{2}} = \frac{\cos 2A}{\sin 2A} = \cot 2A = \text{RHS}$

24. प्रमाणित गर्नुहोस् (Prove that): $\cos^2A.\sin^4A = \frac{1}{32}(\cos 6A - 2\cos 4A - \cos 2A + 2)$

☞ Here, LHS $= \cos^2A.\sin^4A$

$= \frac{1}{8}[4\sin^2A\cos^2A]\,2\sin^2A$ $= \frac{1}{16}2\sin^2 2A\,[1 - \cos 2A)$

$= \frac{1}{16}[2\sin^2 2A - 2\sin^2 2A\cos 2A)$ $= \frac{1}{16}[1 - \cos 4A - 2\sin 2A\cos 2A\sin 2A)$

$= \frac{1}{16}[1 - \cos 4A - \sin 4A\sin 2A)$ $= \frac{1}{32}[2 - 2\cos 4A - 2\sin 4A\sin 2A)$

$= \frac{1}{32}[2 - 2\cos 4A - \{\cos(4A - 2A) - \cos(4A + 2A)\}]$

$= \frac{1}{32}[2 - 2\cos 4A - \cos 2A + \cos 6A] = \text{RHS}$

25. प्रमाणित गर्नुहोस् (Prove that): $16\cos^3 A = 16\cos^5 A + 2\cos A - \cos 3A - \cos 5A$.

☞ Here, RHS $= 16\cos^5 A + 2\cos A - \cos 3A - \cos 5A$

$= 16\cos^5 A + 2\cos A - 2\cos\left(\frac{3A + 5A}{2}\right)\cos\left(\frac{3A - 5A}{2}\right)$

$= 16\cos^5 A + 2\cos A - 2\cos 4A\cos A$

$= 2\cos A[8\cos^4 A + 1 - \cos 4A]$

$= 2\cos A[8\cos^4 A + \sin^2 2A]$

$= 2\cos A[8\cos^4 A + 2 \times 4\sin^2 A\cos^2 A]$

$= 2\cos A \times 8\cos^2 A[\cos^2 A + \sin^2 A]$

$= 16\cos^3 A = \text{LHS}$

26. प्रमाणित गर्नुहोस् (Prove that): $\frac{1}{\sin 2\theta} + \frac{\cos 4\theta}{\sin 4\theta} = \cot\theta - \text{cosec}\,4\theta$

☞ Here, LHS $= \frac{1}{\sin 2\theta} + \frac{\cos 4\theta}{\sin 4\theta}$

$= \frac{1}{\sin 2\theta} + \frac{\cos 4\theta}{2\sin 2\theta\cos 2\theta}$

$= \frac{2\cos 2\theta + \cos 4A}{2\sin 2\theta\cos 2\theta}$

$= \frac{2\cos 2\theta + 2\cos^2 2\theta - 1}{2\sin 2\theta\cos 2\theta}$

$= \frac{2\cos 2\theta(1 + \cos 2\theta) - 1}{2\sin 2\theta\cos 2\theta}$

$= \frac{2\cos 2\theta(1 + \cos 2\theta)}{2\sin 2\theta\cos 2\theta} - \frac{1}{2\sin 2\theta.\cos 2\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta.\cos\theta} - \frac{1}{\sin 4\theta}$

$= \cot\theta - \text{cosec}\,4\theta = \text{RHS}$

www.drmathnp.com

puspas.com.np

27. प्रमाणित गर्नुहोस् (Prove that): $\tan(45°+\theta) = \frac{\cos 2\theta}{1-\sin 2\theta}$

☞ Here, LHS $= \tan(45°+\theta) = \frac{\sin(45°+\theta)}{\cos(45°+\theta)}$

$= \frac{\sin 45°\cos\theta + \cos 45°\sin\theta}{\cos 45°\cos\theta - \sin 45°\sin\theta} = \frac{\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}{\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta} = \frac{\frac{1}{\sqrt{2}}(\cos\theta+\sin\theta)}{\frac{1}{\sqrt{2}}(\cos\theta-\sin\theta)}$

$= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} \times \frac{\cos\theta-\sin\theta}{\cos\theta-\sin\theta} = \frac{\cos^2\theta - \sin^2\theta}{(\cos\theta-\sin\theta)^2}$

$= \frac{\cos 2\theta}{\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta} = \frac{\cos 2\theta}{1-\sin 2\theta} = \text{RHS}$

28. प्रमाणित गर्नुहोस् (Prove that): $\text{cosec } 50° + \cot 100° = \cot 25° - \text{cosec } 100°$

☞ Here, LHS $= \text{cosec } 50° + \cot 100°$

$= \frac{1}{\sin 50°} + \frac{\cos 100°}{\sin 100°}$

$= \frac{1}{\sin 50°} + \frac{\cos 100°}{2\sin 50°\cos 50°} = \frac{2\cos 50° + \cos 100°}{2\sin 50°\cos 50°}$

$= \frac{2\cos 50° + 2\cos^2 50° - 1}{2\sin 50°\cos 50°} = \frac{2\cos 50°(1+\cos 50°) - 1}{2\sin 50°\cos 50°}$

$= \frac{2\cos 50°(1+\cos 50°)}{2\sin 50°\cos 50°} - \frac{1}{2\sin 50°.\cos 50°}$

$= \frac{2\cos 50° 2\cos^2 25°}{4\sin 25°\cos 25°\cos 50°} - \frac{1}{2\sin 50°.\cos 50°}$

$= \frac{\cos 25°}{\sin 25°} - \frac{1}{\sin 100°}$

$= \cot 25° - \text{cosec } 100° = \text{RHS}$

29. प्रमाणित गर्नुहोस् (Prove that): $4\sin^3\theta\cos 3\theta + 4\cos^3\theta.\sin 3\theta = 3\sin 4\theta$

☞ Here, LHS $= 4\sin^3\theta.\cos 3\theta + 4\cos^3\theta.\sin 3\theta$

$= (3\sin\theta - \sin 3\theta)\cos 3\theta + (3\cos\theta + \cos 3\theta)\sin 3\theta$

$= 3\sin\theta\cos 3\theta - \sin 3\theta\cos 3\theta + 3\cos\theta\sin 3\theta + \cos 3\theta\sin 3\theta$

$= 3\sin\theta\cos 3\theta + 3\cos\theta\sin 3\theta$

$= 3\sin(\theta+3\theta) = 3\sin 4\theta = \text{RHS}$

30. प्रमाणित गर्नुहोस् (Prove that): $\cos^3 A + \cos^3(120°+A) + \cos^3(240°+A) = \frac{3}{4}\cos 3A$

☞ Here, LHS $= \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos(360°+3A) + 3\cos(120°+A)\} + \frac{1}{4}\{\cos(720°+3A) + 3\cos(240°+A)\}$

$= \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos 3A + 3\cos(120°+A)\} + \frac{1}{4}\{\cos 3A + 3\cos(240°+A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A + \cos(120°+A)\ \cos(240°A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}[\cos A + \cos\{180° - (60°-A)\} + \cos\{180° + (60°+A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A - \cos(60°-A) - \cos(60°+A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A - \cos(60°-A) + \cos(60°+A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A - \cos(60°-A) + \cos(60°+A)\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}\left\{\cos A - \left(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A + \frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right)\right\}$

$= \frac{3}{4}\cos 3A + \frac{3}{4}(\cos A - \cos A) = \frac{3}{4}\cos 3A = \text{RHS}$

31. प्रमाणित गर्नुहोस् (Prove that): $\sqrt{\frac{1+\sin A}{1-\sin A}} = \tan\left(\frac{\pi}{4} + \frac{A}{2}\right)$

☞ RHS $= \frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}} = \frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}} = \sqrt{\frac{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)^2}{\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)^2}} = \sqrt{\frac{1+2\sin\frac{A}{2}\cos\frac{A}{2}}{1-2\sin\frac{A}{2}\cos\frac{A}{2}}} = \sqrt{\frac{1+\sin A}{1-\sin A}} = \text{LHS}$

www.drmathnp.com

puspas.com.np

32. प्रमाणित गर्नुहोस् (Prove that): $\tan A + \tan(60^\circ + A) + \tan(60^\circ - A) = 3\tan 3A$

☞ Here, LHS $= \tan A + \tan(60° + A) - \tan(60° - A) = \tan A + \dfrac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \dfrac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$

$= \tan A + \dfrac{8\tan A}{1 - 3\tan^2 A} = \dfrac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$

$= \dfrac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A} = 3\tan 3A = \text{RHS}$

33. प्रमाणित गर्नुहोस् (Prove that) : $\sin\theta.\sin(120° - \theta).\sin(120° + \theta) = \dfrac{1}{4}\sin 3\theta$.

☞ Here, LHS $= \sin\theta.\sin(120° - \theta).\sin(120° + \theta) = \dfrac{1}{2}\sin\theta\,[2\sin(120° - \theta)\sin(120° + \theta)]$

$= \dfrac{1}{2}\sin\theta\,[\cos(120° - \theta - 120° - \theta) - \cos(120° - \theta + 120° + \theta)]$

$= \dfrac{1}{2}\sin\theta\,[\cos 2\theta - \cos 240°] = \dfrac{1}{2}\sin\theta\cos 2\theta - \dfrac{1}{2}\sin\theta\cos(180° + 60°)$

$= \dfrac{1}{4}[2\sin\theta\cos 2\theta] + \dfrac{1}{2}\sin\theta\cos 60° = \dfrac{1}{4}[\sin(\theta + 2\theta) + \sin(\theta - 2\theta)] + \dfrac{1}{2}\times\dfrac{1}{2}\sin\theta$

$= \dfrac{1}{4}\sin 3\theta - \dfrac{1}{4}\sin\theta + \dfrac{1}{4}\sin\theta = \dfrac{1}{4}\sin 3\theta = \text{RHS}$

34. यदि $A + B = C$ भए प्रमाणित गर्नुहोस् (If $A + B = C$, then prove that): $\cos^2 A + \cos^2 B + \cos^2 C = 1 + 2\cos A\cos B\cos C$

☞ Here, LHS $= \cos^2 A + \cos^2 B + \cos^2 C$

$= \dfrac{1 + \cos 2A}{2} + \dfrac{1 + \cos 2B}{2} + \cos^2 C = 1 + \dfrac{1}{2}[\cos 2A + \cos 2B] + \cos^2 C$

$= 1 + \dfrac{1}{2}\times 2\cos(A + B)\cos(A - B) + \cos^2 C = 1 + \cos C\cos(A - B) + \cos^2 C$

$= 1 + \cos C\,[\cos(A - B) + \cos C] = 1 + \cos C\,[\cos(A - B) + \cos(A + B)]$

$= 1 + \cos C\,2\cos A\cos B = 1 + 2\cos A\cos B\cos C = \text{RHS}$

35. यदि $A + B + C = \pi^c$ भए, दिइएको सम्बन्ध स्थापित गर्नुहोस् : (If $A + B + C = \pi^c$, establish the given relation): $\sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C) = 4\sin A\sin B\sin C$

OR $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

☞ Here, $A + B + C = \pi^c$ Then, $A + B = \pi^c - C$

Taking sin and cos on both the sides then, $\sin(A + B) = \sin(\pi^c - C) = \sin C$

$\cos(A + B) = \cos(\pi^c - C) = -\cos C$

LHS $= \sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C)$

$= \sin(180° - 2A) + \sin(180° - 2B) + \sin(180° - 2C)$

$= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\dfrac{2A + 2B}{2}\right)\cos\left(\dfrac{2A - 2B}{2}\right) + 2\sin C\cos C$

$= 2\sin(A + B)\cos(A - B) + 2\sin C\cos C$

$= 2\sin C\cos(A - B) + 2\sin C\cos C = 2\sin C[\cos(A - B) + \cos C]$

$= 2\sin C[\cos(A - B) - \cos(A + B)] = 2\sin C\,2\sin A\sin B$

$= 4\sin A\sin B\sin C = \text{RHS}$

36. यदि $A + B + C = \pi^c$ भए प्रमाणित गर्नुहोस् (If $A + B + C = \pi^c$ then prove that) :

$\sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B) = 4\sin\left(\dfrac{B - C}{2}\right)\sin\left(\dfrac{C - A}{2}\right)\sin\left(\dfrac{A - B}{2}\right)$

☞ Here, $A + B + C = \pi^c$

$B + 2C = \pi^c - (A - C)$, | $C + 2A = \pi^c - (B - A)$ | and $A + 2B = \pi^c - (C - B)$

LHS $= \sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B) = \sin(A - C) + \sin(B - A) + \sin(C - B)$

$= 2\sin\left(\dfrac{A - C + B - A}{2}\right)\cos\left(\dfrac{A - C - B + A}{2}\right) + 2\sin\left(\dfrac{C - B}{2}\right)\cos\left(\dfrac{C - B}{2}\right)$

$= 2\sin\left(\dfrac{B - C}{2}\right)\cos\left(\dfrac{2A - B - C}{2}\right) - 2\sin\left(\dfrac{B - C}{2}\right)\cos\left(\dfrac{B - C}{2}\right)$

$= 2\sin\left(\dfrac{B - C}{2}\right)\left[\cos\left(\dfrac{2A - B - C}{2}\right) - \cos\left(\dfrac{B - C}{2}\right)\right]$

$= 2\sin\dfrac{B - C}{2}\left[2\sin\left(\dfrac{A - C}{2}\right)\sin\left(\dfrac{B - A}{2}\right)\right] = 4\sin\left(\dfrac{B - C}{2}\right)\sin\left(\dfrac{C - A}{2}\right)\sin\left(\dfrac{A - B}{2}\right) = \text{RHS}$

www.drmathnp.com

puspas.com.np

37. यदि $A+B+C=\pi^c$ भए (If $A+B+C=\pi^c$ then prove that): $\frac{\sin A+\sin B-\sin C}{\sin A-\sin B+\sin C}=\cot\frac{C}{2}\tan\frac{B}{2}$

☞ Here, $A+B+C=\pi^c$ or, $A+B=\pi^c-C$ or, $\frac{A+B}{2}=90°-\frac{C}{2}$

or, $\sin\left(\frac{A+B}{2}\right)=\sin\left(90°-\frac{C}{2}\right)=\cos\frac{C}{2}$ or, $\cos\left(\frac{A+B}{2}\right)=\cos\left(90°-\frac{C}{2}\right)=\sin\frac{C}{2}$

$$\text{LHS}=\frac{\sin A+\sin B-\sin C}{\sin A-\sin B+\sin C}=\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)-\sin C}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)+\sin C}$$

$$=\frac{2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}}=\frac{2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\frac{C}{2}\right]}{2\sin\frac{C}{2}\left[\sin\left(\frac{A-B}{2}\right)+\cos\frac{C}{2}\right]}$$

$$=\cot\frac{C}{2}\frac{\left[\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right]}{\left[\sin\left(\frac{A-B}{2}\right)+\sin\left(\frac{A+B}{2}\right)\right]}=\cot\frac{C}{2}\frac{2\sin\frac{A}{2}\sin\frac{B}{2}}{2\sin\frac{A}{2}\cos\frac{B}{2}}=\cot\frac{C}{2}\tan\frac{B}{2}=\text{RHS}$$

38. यदि $A+B+C=\pi^c$ भए (If $A+B+C=\pi^c$ then prove that): $\sin 3A+\sin 3B+\sin 3C=-4\cos\frac{3A}{2}\cos\frac{3B}{2}\cos\frac{3C}{2}$

☞ Here, $A+B+C=\pi^c$ or, $A+B=\pi^c-C$ or, $\frac{3(A+B)}{2}=270°-\frac{3C}{2}$

or, $\sin\frac{3(A+B)}{2}=\sin\left(270°-\frac{3C}{2}\right)=-\cos\frac{3C}{2}$ or, $\cos\frac{3(A+B)}{2}=\cos\left(270°-\frac{3C}{2}\right)=-\sin\frac{3C}{2}$

$$\text{LHS}=\sin 3A+\sin 3B+\sin 3C=2\sin\frac{3(A+B)}{2}\cos\frac{3(A-B)}{2}+\sin 3C$$

$$=-2\cos\frac{3C}{2}\cos\frac{3(A-B)}{2}+2\sin\frac{3C}{2}\cos\frac{3C}{2}=-2\cos\frac{3C}{2}\left[\cos\frac{3(A-B)}{2}-\sin\frac{3C}{2}\right]$$

$$=-2\cos\frac{3C}{2}\left[\cos\frac{3(A-B)}{2}+\cos\frac{3(A+B)}{2}\right]$$

$$=-2\cos\frac{3C}{2}\,2\cos\left[\frac{3(A-B)+3(A+B)}{4}\right]\cos\left[\frac{3(A-B)-3(A+B)}{4}\right]$$

$$=-2\cos\frac{3C}{2}\,2\cos\frac{3A}{2}\cos\frac{3B}{2}=-4\cos\frac{3A}{2}\cos\frac{3B}{2}\cos\frac{3C}{2}=\text{RHS}$$

39. यदि $A+B+C=\pi^c$ भए (If $A+B+C=\pi^c$ then prove that): $\frac{\cot B+\cot C}{\tan B+\tan C}+\frac{\cot C+\cot A}{\tan C+\tan A}+\frac{\cot A+\cot B}{\tan A+\tan B}=1$

☞ Here, $A+B+C=\pi^c$ or, $\cot(A+B)=\cot(\pi^c-C)$

or, $\frac{\cot A\cot B-1}{\cot A+\cot B}=-\cot C$ or, $\cot A\cot B-1=-\cot C\cot A-\cot B\cot C$

or, $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$ or, $\frac{1}{\tan A\tan B}+\frac{1}{\tan B\tan C}+\frac{1}{\tan C\tan A}=1$

or, $\frac{\frac{1}{\tan A}+\frac{1}{\tan B}}{\tan A+\tan B}+\frac{\frac{1}{\tan B}+\frac{1}{\tan C}}{\tan B+\tan C}+\frac{\frac{1}{\tan C}+\frac{1}{\tan A}}{\tan C+\tan A}=1$ $\therefore$ $\frac{\cot A+\cot B}{\tan A+\tan B}+\frac{\cot B+\cot C}{\tan B+\tan C}+\frac{\cot C+\cot A}{\tan C+\tan A}=1$

40. यदि $A+B+C=\pi^c$ भए (If $A+B+C=\pi^c$ then prove that):

$\frac{\tan A}{\tan B}+\frac{\tan B}{\tan C}+\frac{\tan C}{\tan A}+\frac{\tan B}{\tan A}+\frac{\tan C}{\tan B}+\frac{\tan A}{\tan C}=\cos A\sec B\sec C+\cos B\sec C\sec A+\cos C\sec A\sec B$

☞ Here, $A+B+C=\pi^c$

$A+B=\pi^c-C$, $\sin(A+B)=\sin(\pi^c-C)=\sin C$ | $B+C=\pi^c-A$, $\sin(B+C)=\sin(\pi^c-A)=\sin A$ | $C+A=\pi^c-B$, $\sin(C+A)=\sin(\pi^c-B)=\sin B$

$$\text{LHS}=\frac{\tan A}{\tan B}+\frac{\tan B}{\tan C}+\frac{\tan C}{\tan A}+\frac{\tan B}{\tan A}+\frac{\tan C}{\tan B}+\frac{\tan A}{\tan C}=\frac{\frac{\sin A}{\cos A}+\frac{\sin C}{\cos C}}{\frac{\sin B}{\cos B}}+\frac{\frac{\sin B}{\cos B}+\frac{\sin A}{\cos A}}{\frac{\sin C}{\cos C}}+\frac{\frac{\sin C}{\cos C}+\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}}$$

$$=\frac{\sin(A+C)\cos B}{\sin B\cos A\cos C}+\frac{\sin(A+B)\cos C}{\sin C\cos A\cos B}+\frac{\sin(B+C)\cos A}{\sin A\cos B\cos C}$$

$$=\frac{\sin B\cos B}{\sin B\cos A\cos C}+\frac{\sin C\cos C}{\sin C\cos A\cos B}+\frac{\sin A\cos A}{\sin A\cos B\cos C}$$

$$=\cos B\sec A\sec C+\cos C\sec A\sec B+\cos A\sec B\sec C=\text{RHS}$$

♣ Best of Luck ♣