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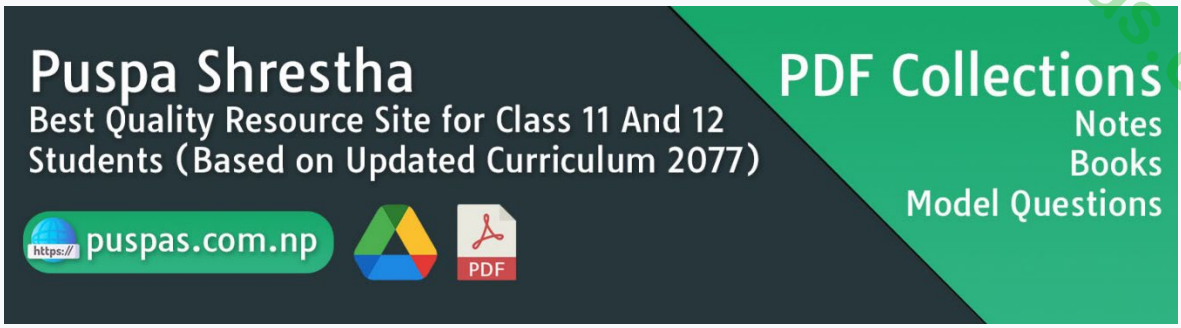

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MATHEMATICS (GRADE- XI)
NEW MODEL QUESTIONS - 2067

Time: 3hrs. **Full Marks: 100**
Pass Marks: 35

Candidates are required to give their answers in their own words as far as possible.
The figures in the margin indicate full marks.

Group 'A' [5 × 3 × 2 = 30]

Attempt ALL Questions:

1. (a) Write truth table for $p \wedge q \Rightarrow p \vee q$ hence draw a conclusion from the truth table.

SOLUTION

Truth table for $p \wedge q \Rightarrow p \vee q$

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \Rightarrow p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

From above truth table, we conclude that $p \wedge q \Rightarrow p \vee q$ is a tautology.

(b) If A = {1, 2, 3}, find the relation on A satisfying the condition $x + y < 4$. Is this relation a function? Give reason.

SOLUTION

Given
$A = \{1, 2, 3\}$
$A \times A = \{1, 2, 3,\} \times \{1, 2, 3\}$
$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
The relation on A satisfying the condition $x + y < 4$ is $\{(1, 1), (1, 2), (2, 1)\}$
This relation is not a function because the element 1 in the domain is associated with two elements 1 and 2 in the range set. A relation becomes a function only when each element in the domain has only one image.

(c) Test periodicity and symmetricity of the function y = cosx.

SOLUTION

Given
$y = \cos x$
Since $\cos(2\pi + x) = \cos x$, the function $y = \cos x$ is periodic with period 2π.
And, since $\cos(-x) = \cos x$; $y = \cos x$ is the symmetric about y – axis.

2. (a) Find the value of x for which $\sin x = \frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$ $(0 \le x \le 2\pi)$.

SOLUTION

$\sin x = \frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$

Since sine of an angle is positive and cosine of the same angle is negative, so the angle must lie in second quadrant.

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6}, \frac{7\pi}{6}$

So, the common solution of x lying between 0 and 2π is $\frac{5\pi}{6}$.

(b) Using mathematical induction method, prove that
$1 + 3 + 5 + \ldots + n$ terms $= n^2$.

SOLUTION

Let P(n) be the given statement. Then
P(n): $1 + 3 + 5 + \ldots + n$ terms $= n^2$
i.e. P(n): $1 + 3 + 5 + \ldots + (2n - 1) = n^2$
When $n = 1$:
L.H.S = 1
R.H.S = $1^2 = 1$
$\therefore$ L.H.S = R.H.S i.e. P(1) is true.
Let P(m) be true for $m \in N$. That is,
P(m): $1 + 3 + 5 + \ldots + (2m - 1) = m^2$ ... (i)
Now, we shall show that P(m + 1) is true when P(m) is true.
For this, we add $2(m + 1) - 1 = 2m + 1$ on both sides of (i), we get,
$(1 + 3 + 5 + \ldots + 2m - 1) + 2m + 1$
$= m^2 + 2m + 1$ (Using (i))
$= (m + 1)^2$
which shows that P(m + 1) is true whenever P(m) is true. Hence, by the principle of mathematical induction P(n) is true for all $n \in N$.

(c) Find the inverse of $\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$.

SOLUTION

Let,
$A = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$
$|A| = \begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} = 18 + 2 = 20 \neq 0$
So, A^{-1} exists.
A_{11} = Cofactor of 3 = 6
A_{12} = Cofactor of 2 = –(–1) = 1

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A_{13} = Cofactor of $-1 = -2$

A_{14} = Cofactor of $6 = 3$

Matrix of cofactors $= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}$

Adj. A = transpose of $\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix}$

$\therefore A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{20}\begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{10} & \frac{-1}{10} \\ \frac{1}{20} & \frac{3}{20} \end{pmatrix}$

3. (a) Solve the following system of linear equations by Cramer's rules, if possible: $5x - 3y = 9$; $10x - 6y = 16$.

SOLUTION

Given equations are

$5x - 3y = 9$

$10x - 6y = 16$

Coeff. of x	Coeff. of y	Constant
5	−3	9
10	−6	16

$D = \begin{vmatrix} 5 & -3 \\ 10 & -6 \end{vmatrix} = -30 + 30 = 0$

As D = 0, we can't solve the given equations by Cramer's rule.

(b) Find the value of the real numbers x and y if $(x + 2) + yi = (3 + i)(1 - 2i)$.

SOLUTION

Given $(x + 2) + yi = (3 + i)(1 - 2i)$

or, $(x + 2) + yi = 3 + i - 6i - 2i^2$

or, $(x + 2) + yi = 3 - 5i - 2(-1)$ $[i^2 = -1]$

or, $(x + 2) + yi = 5 - 5i$

Equating real and imaginary parts we have,

$x + 2 = 5 \Rightarrow x = 5 - 2 = 3$

and $y = -5$

$\therefore x = 3$

$y = -5$

(c) Determine the nature of the roots of the equation $2x^2 + 3x - 2 = 0$.

SOLUTION

Given equation is $2x^2 + 3x - 2 = 0$

Comparing this with $ax^2 + bx + c = 0$,

We get,

$a = 2, b = 3, c = -2$

Now, $b^2 - 4ac = (3)^2 - 4 \times 2 \times (-2) = 9 + 16 = 25 = (5)^2$

Since $b^2 - 4ac$ is positive as well as a perfect square, the roots of given equation are rational and unequal.

4. (a) Show that the points (1, 2) and (2, −3) lie on the opposite sides of the line $5x - 2y - 3 = 0$.

SOLUTION

The given equation of the line is $5x - 2y - 3 = 0$

For (1, 2): $5x - 2y - 3 = 5 \times 1 - 2 \times 2 - 3 = -2 < 0$.

For (2, −3): $5x - 2y - 3 = 5 \times 2 - 2 \times (-3) - 3 = 13 > 0$

Since $5x - 2y - 3$ has opposite signs for the points (1, 2) and (2, −3), So the points lie on the opposite sides of the line $5x - 2y - 3 = 0$.

(b) Find the equation of the circle concentric with $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5, 4).

SOLUTION

The equation of the circle concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ is

$x^2 + y^2 - 8x + 12y + k = 0$ ... (i)

If equation (i) passes through the point (5, 4) then

$5^2 + 4^2 - 8 \times 5 + 12 \times 4 + k = 0$

or, $k = -49$

Substituting the value of k in equation (i), we get

$x^2 + y^2 - 8x + 12y - 49 = 0$.

(c) Find the limit of $f(x) = \frac{x^2 - 4}{x - 2}$ as $x \to 2$. Is $f(x)$ continuous? If not, find the point of discontinuity.

SOLUTION

The limit of $f(x) = \frac{x^2 - 4}{x - 2}$ as $x \to 2$ is

$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 2} \frac{(x - 2)(x + 2)}{(x - 2)} = 2 + 2 = 4$

And $f(2) = \frac{2^2 - 4}{2 - 2} = \frac{0}{0}$

which is an indeterminate form.

So $f(2)$ doesn't exists.

Hence $f(x)$ is not continuous at $x = 2$

The function $f(x) = \frac{x^2 - 4}{x - 2}$ is continuous for all real values of x except where

$x - 2 = 0$ i.e. $x = 2$

So, $x = 2$ is the point of discontinuity of $f(x) = \frac{x^2 - 4}{x - 2}$

5. (a) Find the derivative of $\sec^2(\tan\sqrt{x})$.

SOLUTION

Let $y = \sec^2(\tan\sqrt{x})$

Differentiating both sides w.r.to 'x',

$\frac{dy}{dx} = \frac{d}{dx}\{\sec^2(\tan\sqrt{x})\}$

$= \frac{d}{d\{\sec(\tan\sqrt{x})\}^2} \cdot \frac{d\{\sec(\tan\sqrt{x})\}}{d(\tan\sqrt{x})} \cdot \frac{d(\tan\sqrt{x})}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$

$= 2\sec(\tan\sqrt{x})\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{\sqrt{x}}\sec^2(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2(\sqrt{x})$

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(b) For any curve $y = f(x)$, what do $f'(x) > 0$ and $f'(x) < 0$ represent?

SOLUTION

For any curve, $f'(x) > 0$ represents an increasing function and $f'(x) < 0$ represents a decreasing function.

(c) Evaluate $\int \frac{x}{(1-x^2)^{\frac{3}{2}}} dx$.

SOLUTION

Let $I = \int \frac{x\,dx}{(1-x^2)^{\frac{3}{2}}}$

Put $1 - x^2 = t$

or, $-2x dx = dt$

or, $x dx = -\frac{dt}{2}$

$$\therefore I = \int \frac{-\frac{dt}{2}}{t^{\frac{3}{2}}} = -\frac{1}{2}\int t^{-\frac{3}{2}} dt = -\frac{1}{2}\frac{t^{\frac{-1}{2}}}{\left(\frac{-1}{2}\right)} + C$$

$$= \frac{1}{\sqrt{t}} + C = \frac{1}{\sqrt{1-x^2}} + C$$

$$\therefore \int \frac{x}{(1-x^2)^{\frac{3}{2}}} dx = \frac{1}{\sqrt{1-x^2}} + C$$

Group 'B' [5 × 2 × 4 = 40]

6. (a) If A, B and C are any three non-empty sets, prove that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

SOLUTION

L.H.S = $A \cup (B \cap C)$

$= \{x : x \in A \text{ or } x \in B \cap C\}$

$= \{x : x \in A \text{ or } (x \in B \text{ and } x \in C)\}$

$= \{x : (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)\}$

$= \{x : x \in A \cup B \text{ and } x \in A \cup C\}$

$= (A \cup B) \cap (A \cup C)$ = R.H.S proved.

OR Define absolute value of a real number. If a be any positive real number and $x \in R$, prove that $|x| < a \Leftrightarrow -a < x < a$.

SOLUTION

Let x be any real number. The absolute value (or modulus or numerical value) of x is denoted by $|x|$ is a non negative real number defined by

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

For e.g. $|3| = 3$

$|-2| = -(-2) = 2$

For all $x \in R$, $x \leq |x|$

Given $|x| < a$

$\therefore x \leq |x| < a$

$\Rightarrow x < a$ ... (i)

Again, for all $x \in R$, $-x \leq |x|$

Given $|x| < a$

$\therefore -x \leq |x| < a$

$\Rightarrow -x < a$

$\Rightarrow x > -a$

$\Rightarrow -a < x$ ... (ii)

Combining (i) and (ii) $-a < x < a$.

Conversely, let $-a < x < a$

At first, $x < a$

If $x \geq 0$, then $|x| = x$

$\therefore |x| = x < a$ ... (iii)

Again, $-a < x$

$\Rightarrow a > -x$

$\Rightarrow -x < a$

But for $x < 0$, $|x| = -x$

$\therefore |x| = -x < a$ ... (iv)

Hence, from (iii) and (iv), for all $x \in R$, $|x| < a$.

(b) Sketch the graph of $y = x^2 - 6x + 9$ indicating its different characteristics.

SOLUTION

$y = x^2 - 6x + 9 = (x-3)^2$

This function represents parabola

The graph of given function has the following characteristics

(i) When $x = 0$, $y = 9 \neq 0$, so the curve does not pass through the origin but cuts y–axis at (0, 9).

(ii) The curve meets the x – axis at the points where $y = 0$

i.e. $x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$x = 3$

$\therefore$ The curve cut the x–axis at (3, 0)

(iii) Comparing $y = x^2 - 6x + 9$ with $ax^2 + bx + c = 0$,

$a = 1, b = -6, c = 9$

$x = \frac{-b}{2a} = \frac{6}{2 \times 1} = 3$

$y = 3^2 - 6 \times 3 + 9 = 0$

$\therefore$ The vertex of the parabola is (3, 0)

Also, $y = x^2 - 6x + 9$

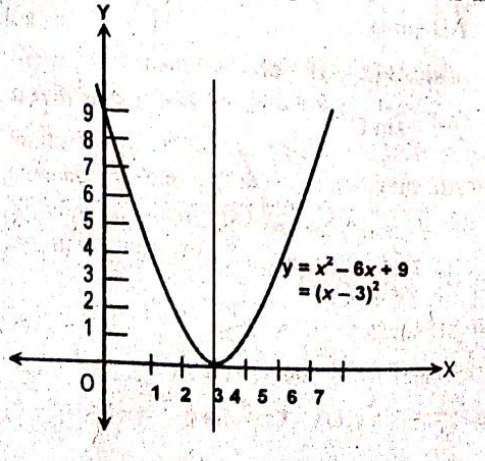
$= (x-3)^2$

So, the axis of the parabola is $x = 3$

(iv) Domain = $(-\infty, \infty)$

Range = $[0, \infty)$

The sketch of the graph of given function is as follows:



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7. (a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

SOLUTION

We have,

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

Given,

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

or, $\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

or, $\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \pi - \cos^{-1}z$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1}z)$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\cos(\cos^{-1}z)$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

or, $xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

Squaring on both sides, we get,

$(xy + z)^2 = (1-x^2)(1-y^2)$

or, $x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$

$\therefore x^2 + y^2 + z^2 + 2xyz = 1$

Hence proved.

OR State and prove sine law.

SOLUTION

Let ABC be a triangle. Since the sum of three angles of a triangle is 180°, at least one angle say B, must always be acute, then another angle, say C may be either acute, or obtuse, or a right angle as shown is the figure (1) or figure (2) or figure (3).

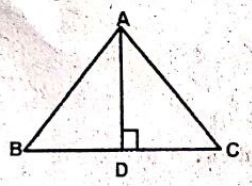

Fig. (1)

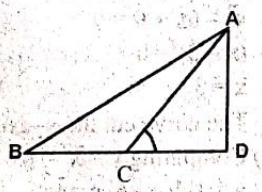

Fig. (2)

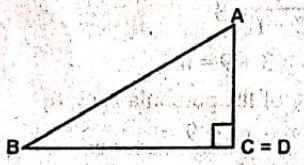

Fig. (3)

Draw AD perpendicular to BC,

In fig. (1), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B [$\because$ AB = c]

Also, $\frac{AD}{AC} = \sin C$

$\Rightarrow$ AD = AC sin C

$\Rightarrow$ AD = b sin C

In fig. (2), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B

Also, $\frac{AD}{AC} = \sin(180° - C) = \sin C$

$\Rightarrow$ AD = AC sin C

$\Rightarrow$ AD = b sin C

In fig. (3), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B

Also, $\frac{AD}{AC} = \sin 90°$

$\Rightarrow$ AD = AC sin 90°

$\Rightarrow$ AD = b sin C ($\because \angle C = 90°$)

In all the three cases,

AD = c sin B = b sin C

$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}$ ... (i)

Similarly, we can prove that $\frac{c}{\sin C} = \frac{a}{\sin A}$ ... (ii)

From (i) and (ii), we have

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Hence proved.

(b) Prove that:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$

SOLUTION

$$\text{L.H.S} = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} \quad R_1 \to R_1 + R_3$$

Taking (a + b + c) common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ b-a & c-b & a+b \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \\ -1(a-b) & -1(b-c) & a+b \end{vmatrix}$$

Taking common (a – b) from C_1 and (b – c) from C_2

$$= (a-b)(b-c)(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a+b & b+c & c^2 \\ -1 & -1 & a+b \end{vmatrix}$$

Expanding along R_1

$$= (a-b)(b-c)(a+b+c)\begin{vmatrix} a+b & b+c \\ -1 & -1 \end{vmatrix}$$

$$= (a-b)(b-c)(a+b+c)(-a-b+b+c)$$

$$= (b-c)(c-a)(a-b)(a+b+c)$$

= R.H.S. proved.

8. (a) Applying Inverse matrix method or row equivalent method, solve the system of linear equations:

$3x + y + 2z = -1$, $2x + 3y + z = 5$

$x + 2y - z = 8$

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SOLUTION

Inverse matrix method

Given equations: $3x + y + 2z = -1$

$2x + 3y + z = 5$

$x + 2y - z = 8$

The given equations in matrix form can be written as

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 8 \end{bmatrix}$$

which is of the form

$AX = B$

So, $X = A^{-1}B$ ... (i)

Now,

$$|A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$$

$= 3(-3-2) - 1(-2-1) + 2(4-3)$

$= -15 + 3 + 2 = -10 \neq 0$

So, A^{-1} exists

Now, for cofactors

A_{11} = Cofactor of $a_{11} = \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -3 - 2 = -5$

A_{12} = Cofactor of $a_{12} = -\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -(-2-1) = 3$

A_{13} = Cofactor of $a_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$

A_{21} = Cofactor of $a_{21} = -\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -(-1-4) = 5$

A_{22} = Cofactor of $a_{22} = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -3 - 2 = -5$

A_{23} = Cofactor of $a_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -(6-1) = -5$

A_{31} = Cofactor of $a_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = -5$

A_{32} = Cofactor of $a_{32} = -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -(3-4) = 1$

A_{33} = Cofactor of $a_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 9 - 2 = 7$

$$\text{Matrix of cofactors} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -5 & 3 & 1 \\ 5 & -5 & -5 \\ -5 & 1 & 7 \end{bmatrix}$$

$$\text{Adj. } A = \begin{bmatrix} -5 & 3 & 1 \\ 5 & -5 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} -5 & 5 & -5 \\ 3 & -5 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj. } A}{|A|} = -\frac{1}{10}\begin{bmatrix} -5 & 5 & -5 \\ 3 & -5 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

From (i)

$$X = -\frac{1}{10}\begin{bmatrix} -5 & 5 & -5 \\ 3 & -5 & 1 \\ 1 & -5 & 7 \end{bmatrix}\begin{bmatrix} -1 \\ 5 \\ 8 \end{bmatrix}$$

or, $X = -\frac{1}{10}\begin{bmatrix} 5 + 25 - 40 \\ -3 - 25 + 8 \\ -1 - 25 + 56 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{10}\begin{bmatrix} -10 \\ -20 \\ 30 \end{bmatrix}$

or, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

$\therefore x = 1; y = 2; z = -3$

OR

Row-Equivalent Matrix Method

The augmented matrix is

$$\begin{bmatrix} 3 & 1 & 2 & : & -1 \\ 2 & 3 & 1 & : & 5 \\ 1 & 2 & -1 & : & 8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 2 & 3 & 1 & : & 5 \\ 3 & 1 & 2 & : & -1 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 0 & -1 & 3 & : & -11 \\ 0 & -5 & 5 & : & -25 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 0 & 1 & -3 & : & 11 \\ 0 & -5 & 5 & : & -25 \end{bmatrix} R_2 \to (-1)R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 0 & 1 & -3 & : & 11 \\ 0 & 0 & -10 & : & 30 \end{bmatrix} R_3 \to R_3 + 5R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 0 & 1 & -3 & : & 11 \\ 0 & 0 & 1 & : & -3 \end{bmatrix} R_3 \to \left(-\frac{1}{10}\right) R_3$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & : & 8 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -3 \end{bmatrix} R_2 \to R_2 + 3R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & : & 5 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -3 \end{bmatrix} R_1 \to R_1 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -3 \end{bmatrix} R_1 \to R_1 - 2R_2$$

$\therefore x = 1; y = 2, z = -3$

(b) If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$.

SOLUTION

Let α be the common root of the given equations. Then α will satisfy both the equations.

$\alpha^2 + p\alpha + q = 0$ ... (i)

$\alpha^2 + q\alpha + p = 0$ ... (ii)

Solving (i) and (ii) by the method of cross–multiplication.

$$\begin{array}{ccccc} & \alpha^2 & & \alpha & & 1 \\ 1 & p & q & 1 & p & \\ 1 & q & p & 1 & q & \end{array}$$

$$\frac{\alpha^2}{p^2 - q^2} = \frac{\alpha}{q - p} = \frac{1}{q - p}$$

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From last two ratios, we have, $\alpha = 1$ ... (iii)

Again, from first two, we have $\alpha = \frac{p^2 - q^2}{q - p}$ ... (iv)

From (iii) and (iv), we get,

$\frac{p^2 - q^2}{q - p} = 1$

or, $p^2 - q^2 = q - p$

or, $(p + q)(p - q) + (p - q) = 0$

or, $(p - q)(p + q + 1) = 0$

Either $p - q = 0 \rightarrow p = q$

or, $p + q + 1 = 0$

Hence proved.

9. (a) Prove that the line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1 + m^2}$. Also show that $3x + 4y = 20$ is tangent to the circle $x^2 + y^2 = 16$.

SOLUTION

First part:

Let $y = mx + c$ ... (i) be a straight line and $x^2 + y^2 = a^2$ ... (ii) be a circle.

Solving (i) and (ii), we get

$x^2 + (mx + c)^2 = a^2$

or, $x^2 + m^2x^2 + 2mcx + c^2 - a^2 = 0$

or, $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ which is quadratic in x. The line (i) becomes tangent to the circle (ii) if two values of x obtained from this quadratic equation are equal i.e. the discriminant $B^2 - 4AC = 0$.

i.e. $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

or, $m^2c^2 - c^2 - m^2c^2 + a^2 + a^2m^2 = 0$

or, $c^2 = a^2(1 + m^2)$

$c = \pm a\sqrt{1 + m^2}$

$\therefore$ The line $y = mx + c$ will be tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1 + m^2}$

And the equation of the tangent line is

$y = mx \pm a\sqrt{1 + m^2}$

Last Part:

Given circle is $x^2 + y^2 = 16$ ... (i) comparing this with $x^2 + y^2 = a^2$,

we have $a^2 = 16$.

The equation of the line is $3x + 4y = 20$

or, $y = -\frac{3}{4}x + 5$ ... (ii)

Comparing this with $y = mx + c$, we have,

$m = -\frac{3}{4}$ and $c = 5$

The line (ii) will be a tangent to the circle (i) if

$c^2 = a^2(1 + m^2)$

$(5)^2 = 16\left\{1 + \left(-\frac{3}{4}\right)^2\right\}$

$25 = 16\left(\frac{16 + 9}{16}\right)$

$25 = 25$ (true)

Hence $3x + 4y = 20$ is a tangent to the circle $x^2 + y^2 = 16$ proved.

(b) Evaluate $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$.

SOLUTION

$$\lim_{x\to\theta} \frac{x\cot\theta - \theta\cot x}{x - \theta} \left(\frac{0}{0} \text{ form at } x = \theta\right)$$

$$= \lim_{x\to\theta} \frac{x\cot\theta - \theta\cot\theta + \theta\cot\theta - \theta\cot x}{x - \theta}$$

$$= \lim_{x\to\theta} \frac{(x - \theta)\cot\theta + \theta(\cot\theta - \cot x)}{x - \theta}$$

$$= \lim_{x\to\theta} \left\{\cot\theta + \frac{\theta}{x - \theta}(\cot\theta - \cot x)\right\}$$

$$= \cot\theta + \lim_{x\to\theta} \frac{\theta}{x - \theta}\left(\frac{\cos\theta}{\sin\theta} - \frac{\cos x}{\sin x}\right)$$

$$= \cot\theta + \lim_{x\to\theta} \frac{\theta}{x - \theta} \frac{\sin(x - \theta)}{\sin\theta \sin x}$$

$$= \cot\theta + \frac{\theta}{\sin\theta} \lim_{x\to\theta} \frac{\sin(x - \theta)}{(x - \theta)} \sin x$$

$$= \cot\theta + \frac{\theta}{\sin\theta} \lim_{x - \theta \to 0} \frac{\sin(x - \theta)}{(x - \theta)} \lim_{x\to\theta} \frac{1}{\sin x}$$

$$= \cot\theta + \frac{\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} = \cot\theta + \theta\,\text{cosec}^2\theta.$$

OR A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x - 3 & \text{for } x < 2 \\ 2 & \text{for } x = 2 \\ 3x - 5 & \text{for } x > 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$? If not, how can $f(x)$ be made continuous at $x = 2$.

SOLUTION

The left hand limit of $f(x)$ at $x = 2$ is

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x - 3)$$

$$= 2 \times 2 - 3 = 1$$

The right hand limit of $f(x)$ at $x = 2$ is.

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x - 5)$$

$$= 3 \times 2 - 5$$

$$= 1$$

Functional value at $x = 2$,

$f(2) = 2$

Here,

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \neq f(2)$$

So, $f(x)$ is not continuous at $x = 2$. It can be made continuous at $x = 2$ by redefining the function as follows:

$$f(x) = \begin{cases} 2x - 3 & \text{for } x < 2 \\ 1 & \text{for } x = 2 \\ 3x - 5 & \text{for } x > 2 \end{cases}$$

10. (a) Find the derivative of $\sqrt{3 - 2x}$ from first principles.

SOLUTION

Let $y = \sqrt{3 - 2x}$

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Also, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sqrt{3 - 2(x + \Delta x)}$

or, $\Delta y = \sqrt{3 - 2(x + \Delta x)} - y$

or, $\Delta y = \sqrt{3 - 2(x + \Delta x)} - \sqrt{3 - 2x}$

$$\Delta y = \frac{3 - 2(x + \Delta x) - 3 + 2x}{\sqrt{3 - 2(x + \Delta x)} + \sqrt{3 - 2x}}$$

$$\Delta y = \frac{-2\Delta x}{\sqrt{3 - 2(x + \Delta x)} + \sqrt{3 - 2x}}$$

Dividing both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-2}{\sqrt{3 - 2(x + \Delta x)} + \sqrt{3 - 2x}}$$

By definition of derivative,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-2}{\sqrt{3 - 2(x + \Delta x)} + \sqrt{3 - 2x}}$$

$$= \frac{-2}{\sqrt{3 - 2x} + \sqrt{3 - 2x}} = \frac{-2}{2\sqrt{3 - 2x}} = \frac{-1}{\sqrt{3 - 2x}}$$

(b) Find area between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

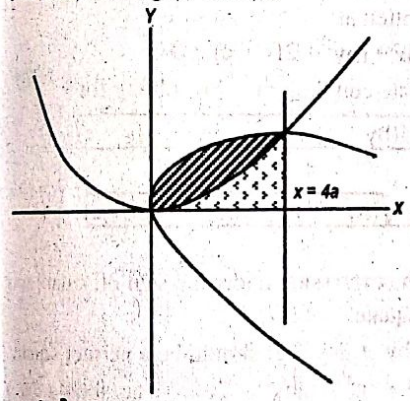
SOLUTION

The curve

$y^2 = 4ax$ ... (i)

$x^2 = 4ay$ ... (ii)

The curve (i) and (ii) represent parabola as shown in figure. The shaded region between parabolas is to be determined. For this, solving (i) and (ii)



$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$

$x^4 = 64a^3x$

i.e. $x(x^3 - 64a^3) = 0$

Either $x = 0$ or $x^3 = 64a^3$ i.e. $x = 4a$

The area bounded by parabola (i), x – axis with ordinates $x = 0, x = 4a$ is given by

$$A_1 = (\text{Shaded} + \text{cross}) = \int_0^{4a} y\,dx$$

$$= \int_0^{4a} \sqrt{4ax}\,dx = \sqrt{4a} \int_0^{4a} x^{1/2}dx = 2\sqrt{a}\left[\frac{x^{1/2+1}}{\frac{1}{2}+1}\right]_0^{4a}$$

$$= 2\sqrt{a} \times \frac{2}{3} \cdot [x^{3/2}]_0^{4a} = \frac{4\sqrt{a}}{3} \cdot [(4a)^{3/2} - 0]$$

$$= \frac{4\sqrt{a}}{3} \cdot 4^{3/2} \cdot a^{3/2} = \frac{4\sqrt{a}}{3} \cdot (2)^3 \cdot a^{3/2}$$

$$= \frac{32}{3} \cdot a^{1/2 + 3/2} = \frac{32}{3}a^2 \text{ sq. units}$$

$$A_2 = (\text{Crossed only}) = \int_0^{4a} y\,dx$$

$$= \int_0^{4a} \frac{x^2}{4a}dx = \frac{1}{4a}\int_0^{4a} x^2dx = \frac{1}{4a} \cdot \left[\frac{x^3}{3}\right]_0^{4a}$$

$$= \frac{1}{12a} \cdot [(4a)^3 - 0] = \frac{1}{12a} \cdot 64a^3 = \frac{16a^2}{3} \text{ sq. units}$$

Required area (shaded only) $= A_1 - A_2 = \frac{32}{3}a^2 - \frac{16a^2}{3}$

$$= \frac{16a^2}{3} \text{ sq.units}$$

Group 'C' [5 × 6 = 30]

11. Define domain and range of a function. Find the domain and range of $f(x) = \sqrt{21 - 4x - x^2}$.

SOLUTION

Let f is a function from a set A to B. Then the set A is known as the domain of the function. The domain of the function is denoted by dom (f). Thus,

$\text{dom}(f) = \{x : x \in A\}$

Range of the function: The set of values of $y = f(x) \in B$ for each $x \in A$ is known as the range of the function f. It is denoted by range (f). Thus,

Range (f) = $\{y : y \in B, y = f(x) \text{ for all } x \in A\}$

Next part:

$y = \sqrt{21 - 4x - x^2}$

$= \sqrt{25 - (x + 2)^2} = \sqrt{5^2 - (x + 2)^2}$

Thus the value of y is defined

if $5^2 - (x + 2)^2 \geq 0$

i.e. $5^2 \geq (x + 2)^2$

$\Rightarrow (x + 2)^2 \leq 5^2$

$\Rightarrow |x + 2|^2 \leq 5^2$

$\Rightarrow |x + 2| \leq 5$

$\Rightarrow -5 \leq x + 2 \leq 5$

$\Rightarrow -7 \leq x \leq 3$

$\therefore$ The domain of the function = $[-7, 3]$

Again,

$y^2 = 5^2 - (x + 2)^2$

or, $(x + 2)^2 = 5^2 - y^2$

$\Rightarrow (x + 2)^2 \geq 0$ for all $x \in R$

So, $5^2 - y^2 \geq 0$

$\Rightarrow y^2 \leq 5^2$

Since y is a positive square root, so $0 \leq y \leq 5$

$\therefore$ The range of f = $[0, 5]$

12. If AM, GM and HM be the arithmetic, geometric and harmonic mean between two unequal positive numbers, prove that.

(i) $GM^2 = (AM.HM)$ (ii) AM > GM > HM

SOLUTION

Let a and b be any two unequal positive numbers. Then,

$$A.M = \frac{a+b}{2}, \quad G.M = \sqrt{ab} \ \& \ H.M = \frac{2ab}{a+b}$$

(i) $A.M \times H.M = \frac{(a+b)}{2} \cdot \frac{2ab}{a+b} = ab = (\sqrt{ab})^2$

$= (G.M)^2$

(ii) $A.M - G.M = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{b} + (\sqrt{b})^2}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

which is always positive as it is a square of the difference between any two unequal positive numbers.

$\therefore$ $A.M - G.M > 0$

i.e. $A.M > G.M$ . . . (a)

From (i)

$A.M \times H.M = G.M \times G.M$

or, $\frac{A.M}{G.M} = \frac{G.M}{H.M}$

Since $A.M > G.M$, we must have $G.M > H.M$ . . . (b)

Hence from (a) & (b) $A.M > G.M > H.M$

13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to a line $x \cos\alpha + y \sin\alpha = p$. Also find the distance between the parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$.

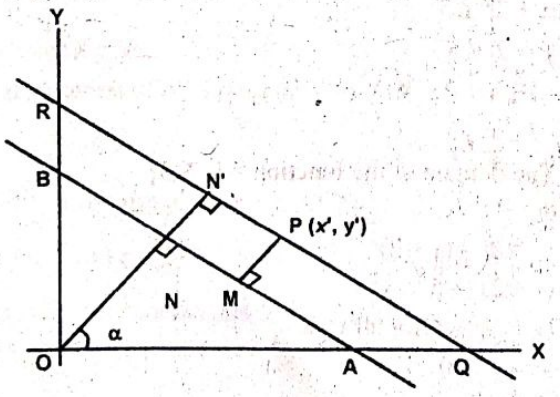
SOLUTION

Let AB be the line represented by the equation

$x \cos\alpha + y \sin\alpha = p$ ... (i)

So, $ON = p$ and $\angle AON = \alpha$

Let $P(x', y')$ be the point from which PM is drawn perpendicular to AB.



Draw a line QR through $P(x', y')$ parallel to the given line AB. Let the perpendicular from O to this line QR be ON' (= p'). Then

$PM = ON' - ON = p' - p$ ... (ii)

If the point P and the origin O lie on the same side of AB, then

$PM = ON - ON' = p - p' = -(p' - p)$ ... (iii)

$\therefore$ $PM = \pm(p' - p)$ ... (iv)

The proper sign is taken so as to make PM positive.

$\therefore$ Equation of QR is $x \cos\alpha + y \sin\alpha = p'$

Since, $P(x', y')$ lies on this line QR,

$x' \cos\alpha + y' \sin\alpha = p'$

$\therefore$ $PM = \pm(x' \cos\alpha + y' \sin\alpha - p)$

Which is the required length of the perpendicular drawn from (x', y') on the line $x \cos\alpha + y \sin\alpha = p$.

Next Part:

Any point on the $5x - 12y + 8 = 0$ is (8, 4).

Length of perpendicular from (8, 4) on the line $10x - 24y - 3 = 0$ is

$$\left|\frac{10 \times 8 - 24 \times 4 - 3}{\sqrt{10^2 + (-24)^2}}\right| = \left|\frac{80 - 96 - 3}{\sqrt{100 + 576}}\right|$$

$$= \left|\frac{-19}{\sqrt{676}}\right| = \frac{19}{26} \text{ units}$$

OR Find the condition that the general equation of second degree may represent a line pair. $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ represents a line pair, show that the lines are perpendicular.

SOLUTION

The general equation of 2nd degree in x and y is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

This can be written as

$ax^2 + 2(hy + g)x + (by^2 + 2fy + c) = 0$

which is quadrate equation in x, So solving for x,

$$x = \frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4 \cdot a(by^2 + 2fy + c)}}{2 \cdot a}$$

or, $$x = \frac{-(hy+g) \pm \sqrt{(hy+g)^2 - a(by^2 + 2fy + c)}}{a}$$

To get a line pair, the expression under the sign of square root should be a perfect square.

i.e. $(hy + g)^2 - a(by^2 + 2fy + c)$ should be a perfect square

i.e. $h^2y^2 + 2hy.g + g^2 - aby^2 - 2afy - ac$ should be perfect square.

i.e. $(h^2 - ab)y^2 + 2(hg - af)y + (g^2 - ac)$ should be a perfect square.

To be a perfect square, the discriminant should be zero

i.e. $4(hg - af)^2 - 4(h^2 - ab)(g^2 - ac) = 0$

i.e. $(hg - af)^2 - (h^2 - ab)(g^2 - ac) = 0$

i.e. $h^2g^2 - 2hg.af + a^2f^2 - (h^2g^2 - abg^2 - ach^2 + a^2bc) = 0$

i.e. $-a[abc + 2fgh - af^2 - bg^2 - ch^2] = 0$

Since $-a \neq 0$ as the coefficient of leading term.

$\therefore$ $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ is the required condition.

The condition can be written as

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

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Second part:

Given equation is

$3x^2 + 5xy - 3y^2 + 2x + 3y = 0$

Here, Coefficient of x^2 + coefficient of y^2

$= a + b = 3 + (-3) = 0$

So, the lines represented by given equations are perpendicular.

14. Define conjugate of a complex number. Find the square root of $\frac{2-36i}{2+3i}$.

SOLUTION

Let $Z = a + ib$ be a complex number then its conjugate is denoted by $\overline{Z}$ and is

$\overline{Z} = a - ib$.

For example, $z = 3 + 4i$ defined by

$\overline{Z} = 3 - 4i$

Next part:

Let $z = \frac{2-36i}{2+3i} = \frac{2-36i}{2+3i} \times \frac{2-3i}{2-3i} = \frac{4-72i-6i+108i^2}{(2)^2-(3i)^2}$

$= \frac{4-78i-108}{4+9} = \frac{-104-78i}{13} = -8-6i$

Let $x + iy$ be the square roots of $-8 - 6i$. Then,

$(x + iy)^2 = -8 - 6i$

$(x^2 - y^2) + i2xy = -8 - 6i$

Equating real and imaginary parts,

$x^2 - y^2 = -8$ ... (i)

$2xy = -6$ ... (ii)

Again, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$= (-8)^2 + (-6)^2 = 64 + 36 = (10)^2$

$\therefore x^2 + y^2 = 10$ ... (iii)

Solving (i) and (iii)

$x^2 = 1$ and $y^2 = 9$

$\therefore x = \pm 1$ and $y = \pm 3$

Since xy is negative (from (iii)) So the possible values of x and y are.

$x = 1, y = -3$ and $x = -1, y = 3$

$\therefore$ The required square roots of $\frac{2-36i}{2+3i}$ are $1 - 3i$ and $-1 + 3i$

i.e. $\pm(1 - 3i)$

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval $(-1, 2)$. Also examine whether the function is increasing or decreasing at $x = 0$.

SOLUTION

First part:

Given curve is $y = 4x^3 - 6x^2 - 9x + 1$ ... (i)

Differentiating both sides with respect to 'x', we get

$\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 6x^2 - 9x + 1)$

$\frac{dy}{dx} = 12x^2 - 12x - 9$ ... (ii)

Again, differentiating both sides with respect to 'x', we get

$\frac{d^2y}{dx^2} = 24x - 12$ ... (iii)

For maxima or minima $\frac{dy}{dx} = 0$

i.e. $12x^2 - 12x - 9 = 0$

or, $4x^2 - 4x - 3 = 0$

or, $4x^2 - 6x + 2x - 3 = 0$

or, $2x(2x - 3) + 1(2x - 3) = 0$

or, $(2x - 3)(2x + 1) = 0$

$\therefore x = \frac{3}{2}$ or $-\frac{1}{2}$

Clearly the values of x lie in the interval $(-1, 2)$

At $x = -\frac{1}{2}$

$\frac{d^2y}{dx^2} = 24\left(-\frac{1}{2}\right) - 12 = -24 < 0$ (Max)

$\therefore$ The given function is maximum at $x = -\frac{1}{2}$.

Maximum value is

$y_{max} = 4\left(-\frac{1}{2}\right)^3 - 6\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) + 1$

$= 4\left(-\frac{1}{8}\right) - 6\left(\frac{1}{4}\right) + \frac{9}{2} + 1 = -\frac{1}{2} - \frac{3}{2} + \frac{9}{2} + 1$

$= \frac{-1-3+9+2}{2} = \frac{7}{2}$

At $x = \frac{3}{2}$

$\frac{d^2y}{dx^2} = 24 \times \frac{3}{2} - 12 = 24 > 0$ (min)

So, the given function is minimum at $x = \frac{3}{2}$ and its minimum value is

$y_{min} = 4\left(\frac{3}{2}\right)^3 - 6\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 1$

$= 4 \times \frac{27}{8} - 6 \times \frac{9}{4} - \frac{27}{2} + 1 = \frac{27}{2} - \frac{27}{2} - \frac{27}{2} + 1$

$= -\frac{27}{2} + 1 = \frac{-27+2}{2} = \frac{-25}{2}$

Further, for the point of inflection,

$\frac{d^2y}{dx^2} = 0$

i.e. $24x - 12 = 0$

$\Rightarrow x = \frac{1}{2}$

At $x = \frac{1}{2}$

$\frac{d^3y}{dx^3} = 24 \neq 0$

$\therefore$ At $x = \frac{1}{2}$, there is a point of inflection.

Where, $y = 4 \times \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 - 9 \times \frac{1}{2} + 1$

$= 4 \times \frac{1}{8} - 6 \times \frac{1}{4} - \frac{9}{2} + 1$

$y = \frac{1}{2} - \frac{3}{2} - \frac{9}{2} + 1 = \frac{1-3-9+2}{2} = \frac{3-12}{2} = \frac{-9}{2}$

$\therefore$ Point of inflection is $\left(\frac{1}{2}, -\frac{9}{2}\right)$

Next part:

$f''(x) = 12x^2 - 12x - 9$

At $x = 0$,

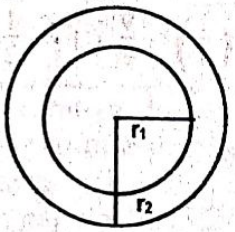
$f''(x) = 12 \times 0^2 - 12 \times 0 - 9 = -9 < 0$

So, the function f(x) is decreasing at $x = 0$.

OR Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 8cm/sec and that of the outer circle at the rate of 5 cm/sec. At a certain instant the radii of the inner and outer circles are respectively 24cm and 30cm. At what rate does the area between the two circles change?

SOLUTION

Let r_1 and r_2 be the radii of inner and outer circles in time t. Also let A_1 and A_2 be the areas of inner and outer circles respectively,



Then,

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

By question

$\frac{dr_1}{dt} = 8\text{cm/sec.}$

$\frac{dr_2}{dt} = 5\text{cm/sec}$

$\frac{dA_1}{dt} = \frac{d(\pi r_1^2)}{dt}$

$= \pi \frac{dr_1^2}{dr_1} \cdot \frac{dr_1}{dt}$

$= \pi 2r_1 \cdot 8$

$= 16\pi r_1$

$\frac{dA_2}{dt} = \frac{d}{dt}(\pi r_2^2)$

$= \frac{\pi dr_2^2}{dr_2} \cdot \frac{dr_2}{dt}$

$= \pi 2r_2 \cdot 5$

$= 10\pi r_2$

Let A be the area enclosed between two circles, then

$A = A_2 - A_1$

or, $\frac{dA}{dt} = \frac{d}{dt}(A_2 - A_1)$

$= \frac{dA_2}{dt} - \frac{dA_1}{dt}$

$= 10\pi r_2 - 16\pi r_1$

When $r_1 = 24\text{cm}$ and $r_2 = 30\text{cm}$

$\frac{dA}{dt} = 10\pi \times 30 - 16\pi \times 24$

$= 300\pi - 384\pi$

$= -84\pi$

This negative sign shows that the area between the circles is decreasing in time t at the rate 84 sq.cm/sec when $r_1 = 24\text{cm}$ and $r_2 = 30\text{cm}$.

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UNIT

SETS, REAL NUMBER SYSTEM AND LOGIC

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A. SETS

2 MARKS QUESTIONS

1. 2074 Set A Q.No. 1a

Prove that $A - \bar{B} = A \cap B$, where A and B are any two sets. [2]

SOLUTION

L.H.S. $= A - \bar{B}$

$= \{x: x \in A \text{ and } x \notin \bar{B}\}$

$= \{x: x \in A \text{ and } x \in B\} = \{x: x \in A \cap B\}$

$= A \cap B = R.H.S.$

2. 2073 Supp Q.No. 1a

If A and B are two non-empty sets, prove that: $\overline{A \cup B} = \bar{A} \cap \bar{B}$. [2]

SOLUTION

L.H.S. $= \{x: x \notin A \cup B\}$

$= \{x: x \notin A \text{ and } x \notin B\}$

$= \{x: x \in \bar{A} \text{ and } x \in \bar{B}\}$

$= \{x: x \in \bar{A} \cap \bar{B}\}$

$= \bar{A} \cap \bar{B} = R.H.S.$

3. 2072 Set E Q.No. 1a

If A and B are subsets of U, prove that $A - B = A \cap \bar{B}$. [2]

SOLUTION

L.H.S. $= A - B$

$= \{x: x \in A - B\}$

$= \{x: x \in A \text{ and } x \notin B\}$

$= \{x: x \in A \text{ and } x \in \bar{B}\}$

$= \{x: x \in A \cap \bar{B}\}$

$= A \cap \bar{B} = R.H.S.$

4. 2070 Old Q.No. 1a

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} denotes the universal set, P = {2, 3, 5, 7} and E = {2, 4, 6, 8} Find P^c and P – E. [2]

SOLUTION

Here, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$P = \{2, 3, 5, 7\}$

$E = \{2, 4, 6, 8\}$

$P^c = U - P$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9\}$

$P - E = \{2, 3, 5, 7\} - \{2, 4, 6, 8\}$

$= \{3, 5, 7\}$

5. 2069 [Set A] Old Q. No. 1a

Define power set. Write the power set of A = {x, y, z} [2]

SOLUTION

Power set: The set of all possible subsets of any set S is called the power set of S. It is denoted by 2^S or P(S).

Power of set A $(2^A) = \{\phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{z, x\}, \{x, y, z\}\}$

6. 2069 [Set B] Old Q. No. 1a

For non empty sets A and B if $A \cap B = \Phi$, prove that: $B \subset A$. [2]

SOLUTION

Let $x \in B$

$\Rightarrow x \notin A \ (\because A \cap B = \phi)$

$\Rightarrow x \notin \bar{A}$

$\therefore B \subset \bar{A}$

7. 2067 Q.No. 1a

Define power set. Write the power set of the set A ={a, b, c} [2]

SOLUTION

Power set: The set of all possible subsets of any set S is called the power set of S. It is denoted by 2^S or P(S).

Here $A = \{a, b, c\}$

The power set of A is

$2^A = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$.

8. 2066 Q.No. 1a

Given A = {1,2, 3} and B = {3, 4, 5, 6}, show that A – (A – B) = A∩B. [2]

SOLUTION

Given,

$A = \{1, 2, 3\}$

$B = \{3, 4, 5, 6\}$

$A - B = \{1,2,3\} - \{3, 4, 5, 6\} = \{1,2\}$

$A - (A - B) = \{1, 2, 3\} - \{1,2\} = \{3\}$ ... (i)

Again, $A \cap B = \{1,2,3\} \cap \{3, 4, 5, 6\} = \{3\}$ ... (ii)

From (i) and (ii)

$A - (A - B) = A \cap B$

9. 2064 Q.No. 1a

Find A∪B if: A = {x : x = 2n + 1, n ≤ 5, n∈N} and B = {x:x = 2n – 2, n ≤ 4, n∈N} [2]

SOLUTION

Here,

$A = \{x : x = 2n + 1, n \le 5, n \in N\}$

$B = \{x : x = 2n - 2, n \le 4, n \in N\}$

$\therefore A = \{2\times1+1, 2\times2+1, 2\times3+1, 2\times4+1, 2\times5+1\}$
$= \{3, 5, 7, 9, 11\}$

$B = \{2\times1-2, 2\times2-2, 2\times3-2, 2\times4-2\}$
$= \{0, 2, 4, 6\}$

So,

$A\cup B = \{3, 5, 7, 9, 11\} \cup \{0, 2, 4, 6\}$
$= \{0, 2, 3, 4, 5, 6, 7, 9, 11\}$

10. 2063 Q.No. 1a

Find A∩B if:

$A = \{x: x = 2n + 1, n \le 6, n \in N\}$

$B = \{x: x = 3n - 2, n \le 3, n \in N\}$ [2]

SOLUTION

Here,

$A = \{x: x = 2n + 1, n \le 6, n \in N\}$
$= \{3, 5, 7, 9, 11, 13\}$

$B = \{x: x = 3n - 2, n \le 3, n \in N\}$
$= \{1, 4, 7\}$

So,

$A\cap B = \{3, 5, 7, 9, 11, 13\} \cap \{1, 4, 7\} = \{7\}$

11. 2062 Q.No. 1a

If A = {1, 3, 5, 7} and B = {2, 3, 5}, find A∩B and A – B. Show them in Venn-diagram. [2]

SOLUTION

Here,

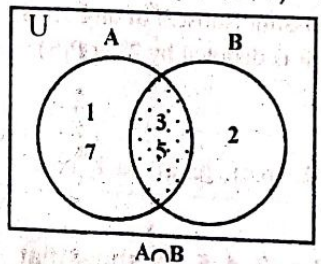
$A = \{1, 3, 5, 7\}$; $B = \{2, 3, 5\}$

Now,

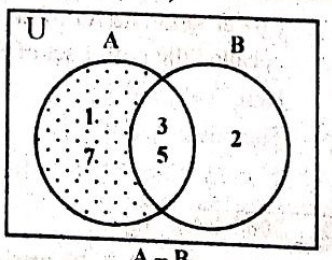
$A\cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5\} = \{3, 5\}$

and

$A - B = \{1, 3, 5, 7\} - \{2, 3, 5\} = \{1, 7\}$



A∩B



A – B

12. 2061 Q.No. 1a

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} be the universal set and M = {1, 3, 5, 7}, N = {2, 4, 6, 8}, then find $\overline{M\cup N}$ and $\overline{M\cap N}$. [2]

SOLUTION

Here,

$M\cup N = \{1, 3, 5, 7\} \cup \{2, 4, 6, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\therefore \overline{M\cup N} = U - (M\cup N)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 5, 6, 7, 8\}$
$= \{9\}$

And,

$M\cap N = \{1, 3, 5, 7\} \cap \{2, 4, 6, 8\} = \phi$

$\therefore \overline{M\cap N} = U - (M\cap N)$
$= \{1, 2, ..., 9\} - \phi$
$= \{1, 2, ..., 9\} = U$

13. 2060 Q.No. 1a

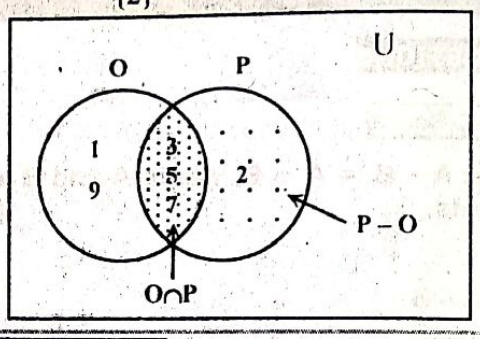
If O = {1, 3, 5, 7, 9} and P = {2, 3, 5, 7}, find O∩P and P – O with the help of venn-diagram. [2]

SOLUTION

Here,

$O\cap P = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$
$= \{3, 5, 7\}$

and, $P - O = \{2, 3, 5, 7\} - \{1, 3, 5, 7, 9\}$
$= \{2\}$



14. 2059 Q.No. 1a

If a = {a, e, i, o, u} and B = {a, b, c, d, e} are two given sets, find A∪B and A–B. [2]

SOLUTION

$A\cup B = \{a, e, i, o, u\} \cup \{a, b, c, d, e\}$
$= \{a, b, c, d, e, i, o, u\}$

and,

$A-B = \{a, e, i, o, u\} - \{a, b, c, d, e\}$
$= \{i, o, u\}$

15. 2058 Q.No. 1a

If U = {1, 2, 3, ..., 10}, A = {x: x ≥ 4}, B = {x: x < 8}, find A∩B and A – B. [2]

SOLUTION

Here,

$U = \{1, 2, 3, ..., 10\}$;

$A = \{4, 5, 6, ..., 10\}$,

$B = \{1, 2, 3, ..., 7\}$

Then,

$A\cap B = \{4, 5, 3, ..., 10\} \cap \{1, 2, 3, ..., 7\}$
$= \{4, 5, 6, 7\}$

And,

$A - B = \{4, 5, 6, ..., 10\} - \{1, 2, 3, ..., 7\}$
$= \{8, 9, 10\}$.

$\therefore A\cap B = \{4, 5, 6, 7\}$ and $A - B = \{8, 9, 10\}$

16. 2056 Q.No. 1a

If A = {a, e, i}, B = {e, u}, U = {a, e, i, o, u}, find $\overline{A\cup B}$ and $\overline{A\cap B}$. [2]

SOLUTION

Here, $A\cup B = \{a, e, i\} \cup \{e, u\} = \{a, e, i, u\}$

$\therefore \overline{A\cup B} = U - (A\cup B) = \{a, e, i, o, u\} - \{a, e, i, u\} = \{o\}$

Again, $A\cap B = \{a, e, i\} \cap \{e, u\} = \{e\}$

$\therefore \overline{A\cap B} = U - (A\cap B) = \{a, e, i, o, u\} - \{e\} = \{a, i, o, u\}$

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17. 2055 Old Model Q.No. 1a

If A = {a, e, i}, B = {e, u}, U = {a, e, i, o, u}, find $\overline{A \cup B}$ and $\overline{A} \cap \overline{B}$. [2]

➥ Please refer to 2056 Q.No. 1a

4 MARKS QUESTIONS

18. 2076 Set B Q.No. 6a

For any non-empty subsets A, B, C of universal set U, prove that:

$A - (B \cup C) = (A - B) \cap (A - C)$. [4]

SOLUTION

L.H.S. $= A - (B \cup C)$
$= \{x : x \in A \text{ and } x \notin B \cup C\}$
$= \{x : x \in A \text{ and } (x \notin B \text{ and } x \notin C)\}$
$= \{x : (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)\}$
$= \{x : x \in A - B \text{ and } x \in A - C\}$
$= \{x : x \in (A - B) \cap (A - C)\}$
$= (A - B) \cap (A - C) =$ R.H.S.

19. 2075 GIE Q.No. 6a

For non-empty sets A, B, C prove that:

$A - (B \cap C) = (A - B) \cup (A - C)$. [4]

SOLUTION

L.H.S. $= A - (B \cap C)$
$= \{x : x \in A \text{ and } x \notin (B \cap C)\}$
$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C\}$
$= \{x : (x \in A \text{ and } x \notin B\} \text{ or } (x \in A \text{ and } x \notin C\}$
$= \{x : (x \in A - B) \cup (x \in A - C)\}$
$= \{x : x \in (A - B) \cup (A - C)\}$
$= (A - B) \cup (A - C) =$ R.H.S.

20. 2075 Set A Q.No. 6a

Let A and B are subset of universal set U. Show that $A - (B \cap C) = (A - B) \cup (A - C)$. [4]

SOLUTION

L.H.S. $= A - (B \cap C)$
$= \{x : x \in A \text{ and } x \notin B \cap C\}$
$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C)\}$
$= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$
$= \{x : x \in A - B \text{ or } x \in A - C\}$
$= \{x : x \in (A - B) \cup (A - C)\}$
$= (A - B) \cup (A - C) =$ R.H.S

21. 2075 Set B Q.No. 6a

Define union of two sets. Also for any non-empty sets A, B, C, prove that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

SOLUTION

Union: Let A and B be two sets. Then the union of two sets A and B is the set of all elements that belong either to A or to B or to both. It is denoted by $A \cup B$.

Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection: Let A and B be two sets. Then the intersection of two sets A and B is the set of all elements that belong to both A and B. It is denoted by $A \cap B$.

Symbolically, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

For example, if $A = \{1, 2, 3, 4, 5\}$; $B = \{4, 5, 6\}$ then
$A \cup B = \{1, 2, 3, 4, 5\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$
And $A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 5, 6\} = \{4, 5\}$

Next part:

L.H.S. $= A \cup (B \cap C)$
$= \{x : x \in A \text{ or } x \in (B \cap C)\}$
$= \{x : x \in A \text{ or } (x \in B \text{ and } x \in C)\}$
$= \{x : (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)\}$
$= \{x : (x \in A \cup B) \text{ and } (x \in A \cup C)\}$
$= \{x : x \in (A \cup B) \text{ and } (A \cup C)\}$
$= (A \cup B) \cap (A \cup C) =$ R.H.S. Proved.

22. 2075 Set C Q.No. 6a

Let U be the universal set and let A and B be any subset of U. Prove that
$A \Delta B = (A \cup B) - (A \cap B)$. [4]

SOLUTION

L.H.S. $= A \Delta B$
$= (A - B) \cup (B - A)$
$= \{x : x \in (A - B) \cup (B - A)\}$
$= \{x : x \in A - B \text{ or } x \in B - A\}$
$= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)$
$= \{x : (x \in A \text{ or } x \in B) \text{ and } (x \notin A \text{ or } x \notin B)\}$
$= \{x : x \in A \cup B \text{ and } x \notin A \cap B\}$
$= \{x : x \in (A \cup B) - (A \cap B)\}$
$= (A \cup B) - (A \cap B) =$ R.H.S.

23. 2074 Supp Q.No. 6a

Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

➥ Please refer to 2075 Set B Q.No. 6a

24. 2074 Set B Q.No. 6a

State De Morgan's Laws of sets. Also for any two non empty subsets A and B of universal set U, prove that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$. [4]

SOLUTION

Statement of De-Morgan's laws: Let A and B be the subsets of the universal set U, then

i. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
ii. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

L.H.S. $= \overline{A \cup B}$
$= \{x : x \notin A \cup B\}$
$= \{x : x \notin A \text{ and } x \notin B\}$
$= \{x : x \in \overline{A} \text{ and } x \in \overline{B}\}$
$= \{x : x \in \overline{A} \cap \overline{B}\}$
$= \overline{A} \cap \overline{B} =$ R.HS.

25. 2073 Set C Q.No. 6a OR

Define complement of a set. If A and B are the subsets of the universal set U, prove that: $(A \cup B)' = A' \cap B'$. [4]

Complement of a set: Let U be the universal set and $A \subset U$. Then the complement of the set A, denoted by $\overline{A}$, A' or A^c, is defined as the set of all elements in the universal set U that do not belong to the set A.

Symbolically, $\overline{A} = \{x : x \in U \text{ but } x \notin A\} = U - A$

For example:

If $U = \{1, 2, 3, \ldots, 15\}$ and $A = \{1, 2, 3, 4, 5, 6\}$

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Then,

$\bar{A} = U - A = \{1, 2, 3, \ldots, 15\} - \{1, 2, 3, 4, 5, 6\}$
$= \{7, 8, 9, \ldots 15\}$

Next part: Please refer 2074 Set B Q.No. 6a

26. 2073 Set D Q.No. 6a

For non-empty sets A, B, C, prove that : $A - (B \cup C) = (A - B) \cap (A - C)$ [4]

➥ Please refer to 2076 Set B Q.No. 6a

27. 2072 Supp. Q.No. 6a

If A,B and C be any three non–empty sets, prove that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [4]

SOLUTION

L.H.S. $= A \cap (B \cup C)$
$= \{x : x \in A \cap (B \cup C)\}$
$= \{x : x \in A \text{ and } x \in B \cup C\}$
$= \{x : x \in A \text{ and } (x \in B \text{ or } x \in C)\}$
$= \{x : (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)\}$
$= \{x : x \in A \cap B \text{ or } x \in A \cap C\}$
$= \{x : x \in (A \cap B) \cup (A \cap C)\}$
$= (A \cap B) \cup (A \cap C) = \text{R.H.S.}$

28. 2072 Set C Q.No. 6a

If A and B are the non-empty subsets of the universal set U, prove that [4]

(i) $(A \cup B)^c = A^c \cap B^c$ (ii) $(A \cap B)^c = A^c \cup B^c$.

SOLUTION

i. $(A \cup B)^c = \{x : x \notin A \cup B\}$
$= \{x : x \notin A \text{ and } x \notin B\}$
$= \{x : x \in A^c \text{ and } x \in B^c\}$
$= \{x : x \in A^c \cap B^c\}$
$= A^c \cap B^c$. Hence proved.

ii. $(A \cap B)^c = \{x : x \in (A \cap B)^c\}$
$= \{x : x \notin A \cap B\}$
$= \{x : x \notin A \text{ or } x \notin B\}$
$= \{x : x \in A^c \text{ or } x \in B^c\}$
$= \{x : x \in A^c \cup B^c\}$
$= A^c \cup B^c$. Hence proved.

29. 2072 Set D Q.No. 6a

For any non-empty sets A, B and C, prove that $A - (B \cup C) = (A - B) - C$. [4]

SOLUTION

L.H.S. $= (A - B) - C$
$= \{x : x \in A - B \text{ and } x \notin C\}$
$= \{x : (x \in A \text{ and } x \notin B) \text{ and } x \notin C\}$
$= \{x : x \in A \text{ and } (x \notin B \text{ and } x \notin C)\}$
$= \{x : x \in A \text{ and } x \notin B \cup C\}$
$= \{x : x \in A - (B \cup C)\}$
$= A - (B \cup C)$

30. 2071 Supp. Q.No. 6a

Define the symmetric difference of two sets. Prove that: $A \Delta B = (A \cup B) - (A \cap B)$ [4]

SOLUTION

Symmetric difference: The symmetric difference of two sets A and B, denoted by $A\Delta B$, is defined by

$A\Delta B = (A - B) \cup (B - A)$

Next Part: Please refer to 2075 Set C Q.No. 6a

31. 2071 Old Q.No. 7a

If A and B are sub sets of a universal set U.

Prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]

➥ Please refer to 2076 Set B Q.No. 6a

32. 2071 Set C Q.No. 6a

For any three non-empty sets A, B, C, prove that: $(A - B) - C = A - (B \cup C)$. [4]

➥ Please refer 2072 Set D Q.No. 6a

33. 2071 Set D Q.No. 6a

Prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

➥ Please refer to 2075 Set B Q.No. 6a

34. 2070 Supp Q.No. 6a

If A and B are the subsets of the universal set U prove that:

i. $\overline{A \cup B} = \bar{A} \cap \bar{B}$ ii. $\overline{A \cap B} = \bar{A} \cup \bar{B}$ [4]

➥ Please refer 2072 Set C Q.No. 6a

35. 2070 Set C Q.No. 6 a

Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [4]

➥ **Definition:** Please refer to 2075 Set B Q.No. 6a

Next part: Please refer 2072 Supp. Q.No. 6a

36. 2070 Set D Q.No. 6 a

Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

➥ Please refer to 2075 Set B Q.No. 6a

37. 2070 Old Q.No. 7 a

For non-empty sets A, B, C, prove that $A - (B \cup C) = (A - B) \cap (A - C)$. [4]

➥ Please refer to 2076 Set B Q.No. 6a

38. 2069 Supp Q.No. 6 a

For any non-empty sets A, B, C define distributive laws. Also prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

SOLUTION

Distributive laws:

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Next part: Please refer to 2075 Set B Q.No. 6a

39. 2069 [Set A] Q. No. 6a

Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4]

➥ Please refer to 2075 Set B Q.No. 6a

40. 2069 [Set B] Q. No. 6a

Define De-Morgan's law. For any non-empty sets A, B, C, prove: $A - (B \cup C) = (A - B) \cap (A - C)$ [4]

SOLUTION

De Morgan's Law: If A and B are the subsets of the universal set U, then:

(i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (ii) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Next part: Please refer to 2076 Set B Q.No. 6a

41. 2069 [Set A] Old Q. No. 7a

For any non-empty sets A, B, C show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [4]

➥ Please refer 2072 Supp. Q.No. 6a

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42. 2068 Q.No. 6a

If A, B and C be any three non-empty sets, prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]

➥ Please refer to 2076 Set B Q.No. 6a

43. 2067 Q.No. 7a

In a group of students 30 study maths, 24 study physics, 22 study chemistry, 14 study maths only, 8 study physics only, 6 study maths and chemistry, 2 study maths and physics only and 8 study none. How many students are in the group? How many study chemistry only? How many study all three subjects? [4]

SOLUTION

Let U be the set of all students. Also let M, P and C denote the set of students who study maths, physics and chemistry respectively

Then, by question,

$n(M) = 30$ $n(P) = 24$

$n(C) = 22$ $n_0(M) = 14$

$n_0(P) = 8$

$n_0(M \cap C) = 6$ $n_0(M \cap P) = 2$

$n(\overline{M \cup P \cup C}) = 8$

$n(U) = ?$ $n_0(C) = ?$

$n(M \cap P \cap C) = ?$

We have,

$n(M) = n_0(M) + n_0(M \cap C) + n_0(M \cap P) + n(M \cap P \cap C)$

or, $30 = 14 + 6 + 2 + n(M \cap P \cap C)$

$\therefore n(M \cap P \cap C) = 30 - 22 = 8$

No. of students studying all three subjects $= n(M \cap P \cap C) = 8$

Again,

$n(P) = n_0(P) + n_0(M \cap P) + n_0(P \cap C) + n(M \cap P \cap C)$

or, $24 = 8 + 2 + n_0(P \cap C) + 8$

or, $24 - 18 = n_0(P \cap C)$

$n_0(P \cap C) = 6$

And,

$n(C) = n_0(C) + n_0(M \cap C) + n_0(P \cap C) + n(M \cap P \cap C)$

or, $22 = n_0(C) + 6 + 6 + 8$

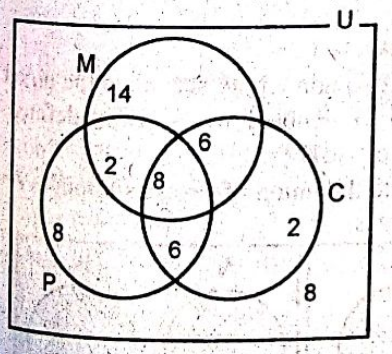
$\therefore n_0(C) = 22 - 20 = 2$

$\therefore$ No. of students studying chemistry only $= n_0(C) = 2$

Total students $= n(U)$

$= 8 + 6 + 2 + 6 + 14 + 2 + 8 + 8$

$= 54$



44. 2066 Q.No. 7a

Twenty three medals are awarded for folksongs, eight for Deuda songs and eleven for Maithili songs. If the total number of singers awarded is thirty two and only three singers received medals in all three types of songs, find how many singers received medals in exactly two of three types of songs. [4]

SOLUTION

Let F, D and M denote the set of singers who received medals for folk songs, Deuda and Maithili songs respectively.

Then,

$n(F) = 23$ $n(D) = 8$

$n(M) = 11$ $n(F \cup D \cup M) = 32$

$n(F \cap D \cap M) = 3$

$n_0(F \cap D) + n_0(D \cap M) + n_0(M \cap F) = ?$

Let

$n_0(F \cap D) = p$,

$n_0(D \cap M) = q$

and $n_0(M \cap F) = r$

Then $p + q + r = ?$

We have

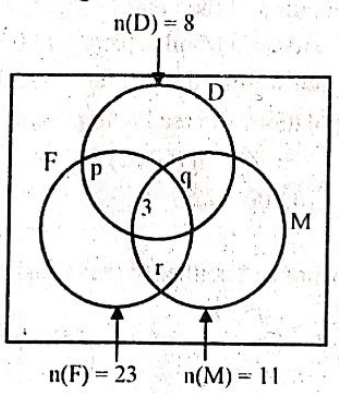
$n(F \cup D \cup M) = n(F) + n(D) + n(M) - n(F \cap D) - n(D \cap M) - n(M \cap F) + (F \cap D \cap M)$

or, $32 = 23 + 8 + 11 - (p + 3) - (q + 3) - (r + 3) + 3$

or, $32 = 45 - p - q - r - 9$

or, $p + q + r = 36 - 32 = 4$

$\therefore$ 4 singers received medals in exactly two of three types of songs.



45. 2065 Q.No. 7a

Out of group of 20 teachers in a school, 10 teach Maths, 9 teach Physics, 7 teach Chemistry, 4 teach Maths and Physics, but none teach both Maths and Chemistry: [4]

i. How many teach Physics and Chemistry?

ii. How many teach only Physics?

iii. How many teach only Chemistry?

SOLUTION

Let U be the set of all teachers, Also, let M, P and C denote the set of teachers who teachers Maths, Physics and Chemistry respectively. Then,

$n(U) = 20$ $n(M) = 10$

$n(P) = 9$ $n(C) = 7$

$n(M \cap P) = 4$ $n(M \cap C) = 0$

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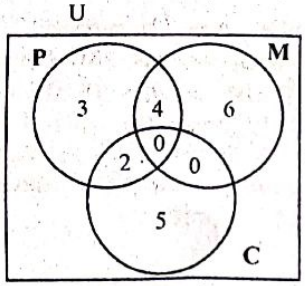
Then, $n(M \cap C \cap P) = 0$ [$\because (M \cap C) = 0$]



i. $n(U) = n(P) + n(M) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(P \cap M \cap C)$

or, $20 = 9 + 10 + 7 - 4 - 0 - n(P \cap C) + 0$

or, $20 = 22 - n(P \cap C)$

$\therefore n(P \cap C) = 22 - 20 = 2$

ii. $n(P) = n_0(P) + n(M \cap P) + n(P \cap C) - n(P \cap M \cap C)$

or, $9 = n_0(P) + 4 + 2 - 0$

or, $n_0(P) = 9 - 6$

or, $n_0(P) = 3$

$\therefore$ 3 teachers teach only Physics.

iii. $n(C) = n_0(C) + n(P \cap C) + n(M \cap C) - n(P \cap M \cap C)$

or, $7 = n_0(C) + 2 + 0 - 0$

or, $n_0(C) = 7 - 2 = 5$

$\therefore$ 5 teachers teach Chemistry only.

46. 2064 Q.No. 7a

A village has total population 25,000 out of which 13,000 read 'Gorkhapatra' and 10,500 read 'Kantipur' and 2500 read both paper. Find the percentage of population who read neither of these papers. [4]

SOLUTION

Let, $n(U)$ = Total population of the village = 25,000

$n(G)$ = No. of population who read Gorkhapatra = 13,000

$n(K)$ = No. of population who read Kantipur = 10,500

$n(G \cap K)$ = No. of population who read both papers.

Now, $n(G \cup K) = n(G) + n(K) - n(G \cap K)$

$= 13{,}000 + 10{,}500 - 2{,}500$

$= 21{,}000$

Now, the population who read neither of these papers

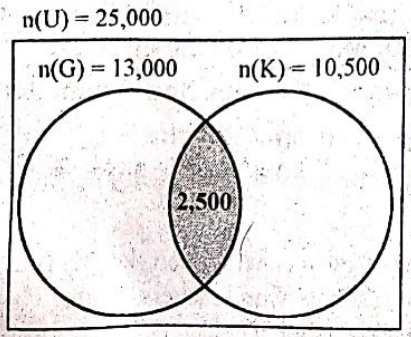
$= n(G \cup K) = n(U) - n(G \cup K)$

$= 25{,}000 - 21{,}000$

$= 4{,}000$

And its $\% = \frac{4{,}000}{25{,}000} \times 100\% = 16\%$

Hence, 16% people read neither of these papers.



47. 2063 Q.No. 7a

Of the number of three athletic teams, 21 are in the basketball team, 26 in hockey team and 29 in football team, 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. How many members are there in all? [4]

SOLUTION

Let B, H, F be the sets of members playing basketball, hockey and football respectively.

Then,

$n(B)$ = No. of members playing basketball = 21

$n(H)$ = No. of members playing hockey = 26

$n(F)$ = No. of members playing football = 29

$n(H \cap B)$ = No. of members playing hockey and basketball = 14

$n(H \cap F)$ = No. of members playing hockey and football = 15

$n(F \cap B)$ = No. of members playing football and basketball = 12

$n(B \cap H \cap F)$ = No. of members playing all three games = 8

$n(U) = n(B \cup H \cup F)$ = Total no. of members = ?

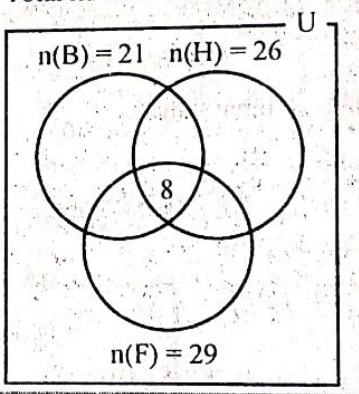
We know,

$n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

$= 21 + 26 + 29 - 14 - 15 - 12 + 8$

$= 84 - 41 = 43$

$\therefore$ Total number of members = 43



48. 2062 Q.No. 7a

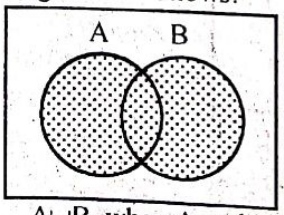
Define union and intersection of two sets. Illustrate them through Venn-diagrams. Let A, B, C be any non-empty subsets of U, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

SOLUTION

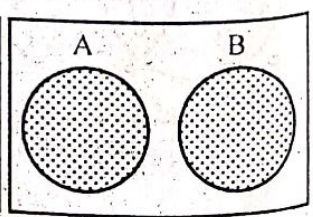
Union of two sets

Let A and B are two non empty sets, then the union of two sets A and B denoted by $A \cup B$ is defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Now, we illustrate the union of two sets through Venn-diagrams as follows:



$A \cup B$, when A and B are intersecting



$A \cup B$, when A and B are disjoint

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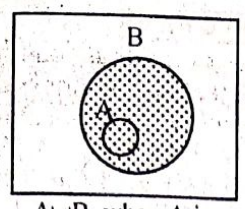
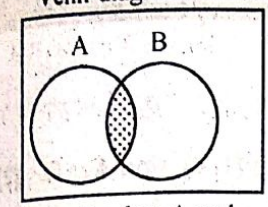


A∪B, when A is a proper subset of B

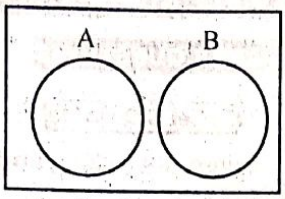
Intersection of two sets: The intersection of A and B denoted by A∩B is defined as

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

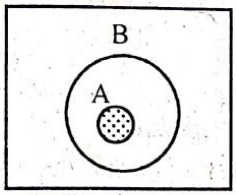
Now, we illustrate the intersection of two sets through Venn-diagrams as follows:



A∩B, when A and B are intersecting



A∩B, when A and B are disjoint



A∩B, when A is a proper subset of B

Last part: Please refer 2075 Set B Q.No. 6a

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49. 2061 Q.No. 7a

In a group of twenty eight teachers of a school, 15 teach English, 15 teach maths, 14 teach Nepali, 7 teach English and maths, 6 teach English and Nepali, 5 teach maths and Nepali. Find how many teach all three subjects, how many teach maths only and Nepali only. [4]

SOLUTION

Let E, M and N be the sets of teachers teaching English, Maths and Nepali respectively. Now,

$n(E \cup M \cup N)$ = no. of teachers of a school = 28

$n(E)$ = no. of teachers teaching English = 15

$n(M)$ = no. of teachers teaching Maths = 15

$n(N)$ = no. of teachers teaching Nepali = 14

$n(E \cap M)$ = no. of teachers teaching English and Maths = 7

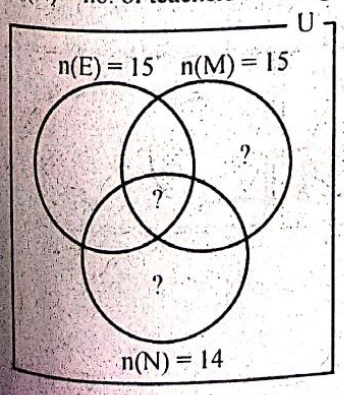
$n(E \cap N)$ = no. of teachers teaching English and Nepali = 6

$n(M \cap N)$ = no. of teachers teaching Maths and Nepali = 5

$n(E \cap M \cap N)$ = no. of teachers teaching all 3 subjects = ?

$n_0(M)$ = no. of teachers teaching Maths only = ?

$n_0(N)$ = no. of teachers teaching Nepali only = ?



We know that,

$n(E \cup M \cup N) = n(E) + n(M) + n(N) - n(E \cap M) - n(M \cap N) - n(E \cap N) + n(E \cap M \cap N)$

or, $28 = 15 + 15 + 14 - 7 - 5 - 6 + n(E \cap M \cap N)$

or, $n(E \cap M \cap N) = 28 - 26$

$\therefore n(E \cap M \cap N) = 2$

Again,

$n_0(M) = n(M) - n(E \cap M) - n(M \cap N) + n(E \cap M \cap N)$

$= 15 - 7 - 5 + 2$

$\therefore n_0(M) = 5$

and, $n_0(N) = n(N) - n(M \cap N) - n(E \cap N) + n(E \cap M \cap N)$

$= 14 - 5 - 6 + 2$

$n_0(N) = 5$

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50. 2060 Q.No. 7a

In a group of students 24 study Maths, 30 study Biology, 22 study Physics, 8 study Math only, 14 study Biology only, 6 study Biology and physics only and 2 study maths and biology only. Find:

i. How many study all three subjects

ii. How many students were in the group [4]

SOLUTION

Let M, B and P be the sets of students studying Mathematics, Biology and Physics respectively. Then,

$n(M)$ = no. of students studying Maths = 24

$n(B)$ = no. of students studying Biology = 30

$n(P)$ = no. of students studying Physics = 22

$n_0(M)$ = no. of students studying Maths only = 8

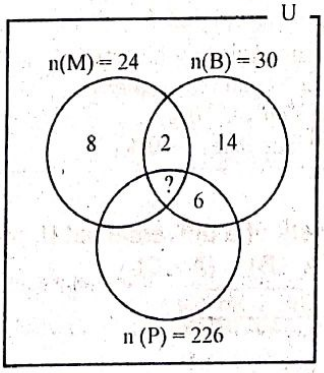
$n_0(B)$ = no. of students studying Biology only = 14

$n_0(B \cap P)$ = no. of students studying Biology and Physics only = 6

$n_0(M \cap B)$ = no. of students studying Maths and Biology only = 2

$n(M \cap B \cap P)$ = no. of students studying all three subjects = ?

$n(M \cup B \cup P)$ = no. of students in the group = ?



Now,

i. We know that

$n(B) = n_0(B) + n_0(B \cap P) + n_0(M \cap B) + n(M \cap B \cap P)$

or, $30 = 14 + 6 + 2 + n(M \cap B \cap P)$

$\therefore n(M \cap B \cap P) = 30 - 22 = 8$

ii. We have,

$n(M) = n_0(M) + n_0(M \cap B) + n_0(M \cap P) + n(M \cap B \cap P)$

or, $24 = 8 + 2 + n_0(M \cap P) + 8$

$\therefore n_0(M \cap P) = 24 - 18 = 6$

Again, we have,

$n(P) = n_0(P) + n_0(B \cap P) + n_0(M \cap P) + n(M \cap B \cap P)$

or, $22 = n_0(P) + 6 + 6 + 8$
or, $n_0(P) = 22 - 20 = 2$
Now,

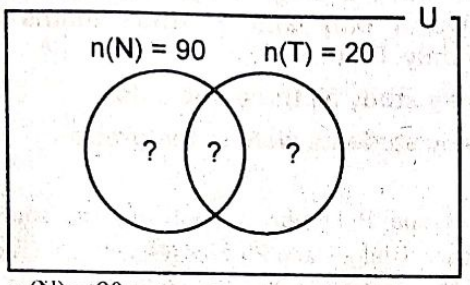
$$n(M \cup B \cup P) = n_0(M) + n_0(B) + n_0(P) + n_0(B \cap P) + n_0(M \cap B) + n_0(M \cap P) + n(M \cap B \cap P)$$
$$= 8 + 14 + 2 + 6 + 2 + 6 + 8 = 46$$

51. 2059 Q.No. 7a

In a certain village in Nepal, all the people speak Nepali or Tharu or both language. If 90% speak Nepali and 20% Tharu language, how many speak (i) Nepali only (ii) Tharu language only and (iii) both languages. [4]

SOLUTION

Let
n(N) denote the no. of people speaking Nepali language
n(T) denote the no. of people speaking Tharu language.
Let $n(N \cup T) = 100$.



$\therefore$ $n(N) = 90$
$n(T) = 20$
We have,
$n(N \cup T) = n(N) + n(T) - n(N \cap T)$
or, $100 = 90 + 20 - n(N \cap T)$
or, $100 = 110 - n(N \cap T)$
$\therefore$ $n(N \cap T) = 110 - 100 = 10$
$\therefore$ No. of people speaking both languages = 10%
Again, let
$n_0(N)$ = No. of people speaking Nepali only.
and $n_0(T)$ = No. of people speaking Tharu only.
Then, $n_0(N) = n(N) - n(N \cap T)$
$= 90 - 10 = 80$
and, $n_0(T) = n(T) - n(N \cap T)$
$= 20 - 10 = 10$
Thus, people speaking Nepali only = 80%
People speaking Tharu only = 10%
People speaking both languages = 10%.

52. 2058 Q.No. 7a

If A, B, C are subsets of a universal set U, prove that $A - (B \cap C) = (A - B) \cup (A - C)$ [4]

➥ Please refer to 2075 GIE Q.No. 6a

53. 2057 Q.No. 7a

Define the complement of a set. State and prove De-Morgan's laws. [4]

➥ **First Part:** Please refer 2073 Set C Q.No. 6a OR
Next Part: Please refer 2072 Set C Q.No. 6a

54. 2056 Q.No. 7a

Define the union and the intersection of two sets. If A, B and C are subsets of U; prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

➥ Please refer to 2075 Set B Q.No. 6a

55. 2055 Old Model Q.No. 7a

If A and B are subsets of universal set U prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4

SOLUTION

L.H.S. $= A \cup (B \cap C)$
$= \{x : x \in A \text{ or } x \in (B \cap C)\}$
$= \{x : x \in A \text{ or } (x \in B \text{ and } x \in C)\}$
$= \{x : (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)\}$
$= \{x : (x \in A \cup B) \text{ and } (x \in A \cup C)\}$
$= \{x : x \in (A \cup B) \text{ and } (A \cup C)\}$
$= (A \cup B) \cap (A \cup C)$ = R.H.S. Proved.

B. REAL NUMBERS

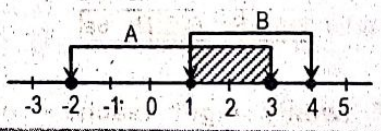
2 MARKS QUESTIONS

1. 2076 Set C Q.No. 1a

Given A = [– 2, 3] and B = [1, 4], find $A \cap B$. [2

SOLUTON

Given,
$A = [-2, 3]$
$B = [1, 4]$
Now,
$A \cap B = [-2, 3] \cap [1, 4]$
$= \{x : -2 \le x \le 3\} \cap \{x : 1 \le x \le 4\}$
$= \{x : 1 \le x \le 3\}$
$= [1, 3]$



2. 2075 Set B Q.No. 1a

Represent the solution set of x of $-2 \le 3x + 4 \le 1$ in interval form. [2

SOLUTION

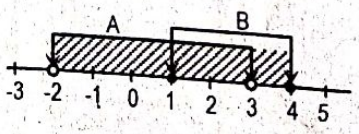
Given, $-2 \le 3x + 4 \le 10$
Subtracting 4 from each side, we get
$-2 - 4 \le 3x + 4 - 4 \le 10 - 4$
or, $-6 \le 3x \le 6$
Dividing each side by 3
$-2 \le x \le 2$
In interval form, this can be written as [-2, 2]

3. 2074 Set B Q.No. 1a

If A = [–2, 3) and B = [1, 4], compute $A \cup B$. [2

SOLUTION

Given,
$A = [-2, 3)$ and $B = [1, 4]$
Now, $A \cup B = [-2, 3) \cup [1, 4]$
$= \{x : -2 \le x < 3\} \cup \{x : 1 \le x \le 4\}$
$= \{x : -2 \le x \le 4\}$
$= [-2, 4]$



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4. 2073 Set C Q.No. 1a

Write $|x+2|<4$ without using modulus sign. [2]

SOLUTION

$|x+2|<4$

or, $-4<x+2<4$

or, $-4-2<x+2-2<4-2$

$\therefore -6<x<2$

5. 2071 Old Q.No. 1a

Let A = [–1, 3) and B = [2, 4], compute: A∪B and A – B. [2]

SOLUTION

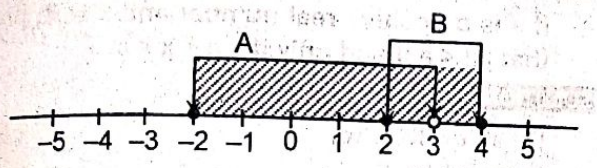
Here, A = [–1, 3) and B = [2, 4]

$A\cup B = [-1,3)\cup[2,4]$

$= \{x: -1\le x<3\}\cup\{x: 2\le x\le 4\}$

$= \{x: -1\le x\le 4\}$

$= [1,4]$



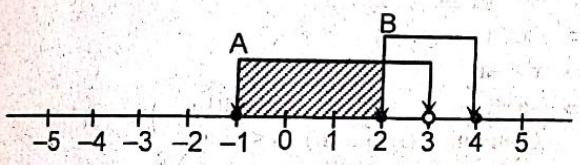
A∪B

$A-B = [-1,3)-[2,4]$

$= \{x: -1\le x<3\}-\{x: 2\le x\le 4\}$

$= \{x: -1\le x<2\}$

$= [-1,2)$



A∪B

6. 2071 Old Q.No. 1b

Rewrite, using absolute value sign for $|3x+2|\le 1$ [2]

SOLUTION

Here, $|3x+2|\le 1$

or, $-1\le 3x+2\le 1$ $\quad(\because |x|\le a \Leftrightarrow -a\le x\le a)$

or, $-1-2\le 3x+2-2\le 1-2$

or, $-3\le 3x\le -1$

or, $\frac{-3}{3}\le\frac{3x}{3}\le\frac{-1}{3}$

$\therefore -1\le x\le -\frac{1}{3}$

7. 2071 Set C Q.No. 1b

For any two real numbers x and y, prove that: $|x+y|\le|x|+|y|$. [2]

SOLUTION

$|x+y|^2 = (x+y)^2$

$= x^2+2xy+y^2$

$= |x|^2+|y|^2+2xy$

$\le |x|^2+|y|^2+2|x||y|$ $\quad[\because x\le|x|, y\le|y|]$

$= (|x|+|y|)^2$

$\therefore |x+y|\le|x|+|y|$

8. 2071 Set D Q.No. 1b

For any two real numbers x and y, prove that: $|x-y|\ge|x|-|y|$. [2]

SOLUTION

$|x| = |x-y+y|$

$\le |x-y|+|y|$ $\quad[\because |x+y|\le|x|+|y|$ triangle inequality$]$

or, $|x|-|y|\le|x-y|$

$\therefore |x-y|\ge|x|-|y|$

9. 2068 Old Q.No. 1a

Rewrite $|2x-1|\le 5$ without using absolute value sign. [2]

SOLUTION

$|2x-1|\le 5$

or, $-5\le 2x-1\le 5$ $\quad(\because |x|\le a \Leftrightarrow -a\le x\le a]$

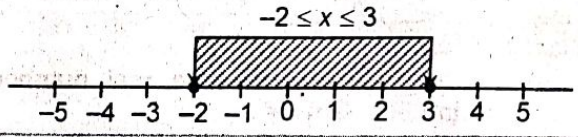
or, $-5+1\le 2x-1+1\le 5+1$

or, $-4\le 2x\le 6$

or, $\frac{-4}{2}\le\frac{2x}{2}\le\frac{6}{2}$

$\therefore -2\le x\le 3.$

The graph is as follows.



10. 2065 Q.No. 1a

Given A = [–2, 4) and B = (2, 5], compute A∪B and A∩B. [2]

SOLUTION

Here,

A = [–2, 4)

B = (2, 5]

Now,

$A\cup B = [-2,4)\cup(2,5]$

$= \{x: -2\le x<4\}\cup\{x: 2<x\le 5\}$

$= \{x: -2\le x\le 5\}$

$= [-2,5]$

$A\cap B = [-2,4)\cap(2,5]$

$= \{x: -2\le x<4\}\cup\{x: 2<x\le 5\}$

$= \{x: -2<x<4\}$

$= (-2,4)$

11. 2064 Q.No. 3c

Write $|x-7|<3$ without using the modules sign. [2]

SOLUTION

We have, $|x-7|<3$

$\Rightarrow -3<x-7<3$

$\Rightarrow -3+7<x-7+7<3+7$

$\Rightarrow 4<x<10$

12. 2063 Q.No. 3c

Write the following by using the modulus sign: $-1\le x\le 5$ [2]

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SOLUTION

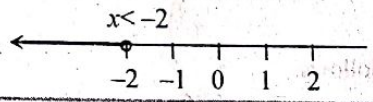
We have,
$-1 \le x \le 5$
or, $-1-2 \le x-2 \le 5-2$
or, $-3 \le x-2 \le 3$
$\Rightarrow |x-2| \le 3$ [$\because -a \le x \le a \Leftrightarrow |x| \le a$]

13. 2059 Q.No. 3c

Determine the solution set of $2x - 1 > 4x + 3$ [2]

SOLUTION

We have,
$2x - 1 > 4x + 3$
$\Rightarrow 2x - 1 + 1 > 4x + 3 + 1$
$\Rightarrow 2x > 4x + 4$
$\Rightarrow 2x - 4x > 4x - 4x + 4$
$\Rightarrow -2x > 4$
$\Rightarrow 2x < -4$
$\Rightarrow \frac{2x}{2} < -\frac{4}{2}$
$\Rightarrow x < -2$
$\therefore$ Solution set $= \{x: x < -2, x \in \Re\}$



14. 2057 Q.No. 1a

Prove that for any positive real number a, $|x| < a$ implies $-a < x < a$. [2]

SOLUTION

If x is negative i.e. $x < 0$, then $|x| = -x$
So, $|x| < a$ implies $-x < a$
or, $x > -a$ i.e. $-a < x$ ... (i)
Again, if x is positive i.e. $x > 0$, then $|x| = x$
So, $|x| < a$ implies $x < a$ ... (ii)
Combining (i) and (ii) we get $-a < x < a$. Proved

15. 2057 Q.No. 3c

Find the solution set of $4x + 3 \ge 2x - 1$. [2]

SOLUTION

We have, $4x + 3 \ge 2x - 1$
or, $4x - 2x \ge -1 - 3$
or, $2x \ge -4$
or, $\frac{2x}{2} \ge \frac{-4}{2}$
or, $x \ge -2$
$\therefore$ The required solution set $= \{x: x \ge -2\}$

16. 2056 Q.No. 1b

Rewrite, without using absolute value sign for $|3x + 2| \le 1$. [2]

SOLUTION

We have,
$|3x + 2| \le 1$
$\Rightarrow -1 \le 3x + 2 \le 1$ [$\because |x| \le a \Rightarrow -a \le x \le a$]
$\Rightarrow -1 - 2 \le 3x + 2 - 2 \le 1 - 2$
$\Rightarrow -3 \le 3x \le -1$
$\Rightarrow -\frac{3}{3} \le \frac{3x}{3} \le -\frac{1}{3}$
$\Rightarrow -1 \le x \le \frac{-1}{3}$

4 MARKS QUESTIONS

17. 2076 Set B Q.No. 6a OR

Define absolute value of a real number. Also prove that $|x| - |y| \le |x + y|$, $\forall x, y \in \mathbb{R}$. [4

SOLUTION

The absolute value or modulus or numerical value of any real number x is a non negative real number denoted by $|x|$, is defined as

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

$|x| = |x + y - y|$
$= |(x + y) + (-y)|$
$\le |x + y| + |-y|$, [Using triangle inequality]
$= |x + y| + |y|$ [$\because |-y| = |y|$]
$\therefore |x| - |y| \le |x + y|$.

18. 2076 Set C Q.No. 6a OR

If a is a positive real number and $x \in \mathbb{R}$, prove that $|x| < a$ if and only if $-a < x < a$. [4

SOLUTION

For all $x \in R$, $x \le |x|$
Given $|x| < a$
$\therefore x \le |x| < a$
$\Rightarrow x < a$ ... (i)
Again, for all $x \in R$, $-x \le |x|$
Given $|x| < a$
$\therefore -x \le |x| < a$
$\Rightarrow -x < a$
$\Rightarrow x > -a$
$\Rightarrow -a < x$ ... (ii)
Combining (i) and (ii) $-a < x < a$.
Conversely, let $-a < x < a$
At first, $x < a$
If $x \ge 0$, then $|x| = x$
$\therefore |x| = x < a$ ... (iii)
Again, $-a < x$
$\Rightarrow a > -x$
$\Rightarrow -x < a$
But for $x < 0$, $|x| = -x$
$\therefore |x| = -x < a$ ... (iv)
Hence, from (iii) and (iv), for all $x \in R$, $|x| < a$.

19. 2075 GIE Q.No. 6a OR

Define absolute value of a real number. If a is any positive real number prove that $|x| \le a$ and only if $-a \le x \le a$ for all $x \in R$. [

SOLUTION

Let x be any real number. The absolute value (or modulus or numerical value) of x is denoted by $|x|$ is non negative real number defined by

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$$

For e.g. $|3| = 3$
$|-2| = -(-2) = 2$
For all $x \in R$, $x \le |x|$
Given $|x| \le a$
$\therefore x \le |x| \le a$

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$\Rightarrow x \le a$ ... (i)

Again, for all $x \in R$, $-x \le |x|$

Given $|x| \le a$

$\therefore -x \le |x| \le a$

$\Rightarrow -x \le a$

$\Rightarrow x \ge -a$

$\Rightarrow -a \le x$ ... (ii)

Combining (i) and (ii) $-a \le x \le a$.

Conversely, let $-a \le x \le a$

At first, $x \le a$

If $x \ge 0$, then $|x| = x$

$\therefore |x| = x \le a$ ... (iii)

Again, $-a \le x$

$\Rightarrow a \ge -x$

$\Rightarrow -x \le a$

But for $x \le 0$, $|x| = -x$

$\therefore |x| = -x \le a$ ... (iv)

Hence, from (iii) and (iv), for all $x \in R$, $|x| \le a$.

20. 2075 Set A Q.No. 6a OR

Solve the inequality $\frac{x+2}{x^2-3x} > 0$. [4]

SOLUTION

The corresponding equation of given inequality is

$\frac{x+2}{x^2-3x} = 0$

or, $x + 2 = 0$

or, $x = -2$

Also, $\frac{x+2}{x^2-3x}$ exists for all $x \ne 0$ & $x \ne 3$.

These points divide the whole real line into subintervals $(-\infty, -2)$, $(-2, 0)$, $(0, 3)$ & $(3, \infty)$

Intervals	Value of $\frac{x+2}{x(x-3)}$
$(-\infty, -2)$	− ve
$(-2, 0)$	+ ve
$(0, 3)$	− ve
$(3, \infty)$	+ ve

From above table, the possible intervals are $(-2, 0)$ and $(3, \infty)$

The required solution is $x \in (-2, 0) \cup (3, \infty)$

21. 2075 Set C Q.No. 6a OR

Solve the inequality $x^2 + 7x + 10 < 0$. [4]

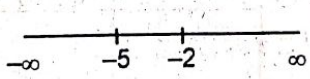
SOLUTION

The corresponding equation of given inequality is

$x^2 + 7x + 10 = 0$

or, $(x + 5)(x + 2) = 0$

$\therefore x = -5, -2$



These two points divide the real line into 3 sub-intervals $(-\infty, -5)$, $(-5, 2)$ and $(-2, \infty)$.

Now,

Intervals	Sign of		
	$(x + 5)$	$(x + 2)$	$(x + 5)(x + 2)$
$(-\infty, -5)$	−ve	− ve	+ ve
$(-5, -2)$	+ ve	− ve	− ve
$(-2, \infty)$	+ ve	+ ve	+ ve

$\therefore$ The possible interval is $(-5, -2)$

22. 2074 Supp Q.No. 6a OR

Solve the inequality $|2x - 1| \ge 3$ and represent the solution set in the real line. [4]

SOLUTION

If $(2x - 1) > 0$, then

$2x - 1 \ge 3$

or, $2x \ge 4$

or, $x \ge 2$

$\therefore x \in [2, \infty)$

Again, if $(2x - 1) < 0$, then

$-(2x - 1) \ge 3$

or, $2x - 1 \le -3$

or, $2x \le -2$

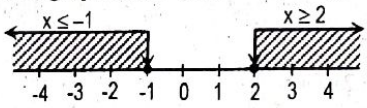
or, $x \le -1$

$\therefore x \in (-\infty, -1]$

The required solution of the given inequality is $\{x: x \le -1 \text{ or } x \ge 2\}$

i.e. $x \in (-\infty, -1] \cup [2, \infty)$

The graph of the given inequality is shown below:



23. 2074 Set A Q.No. 6a OR

Solve the inequality $|2x - 1| \ge 3$ and draw its graph. [4]

➥ Please refer to 2074 Supp Q.No. 6a OR

24. 2073 Supp Q.No. 6a OR

If A = (−1, 4) and B = [3, 5), find A∪B, A∩B, A−B and B−A. [4]

SOLUTION

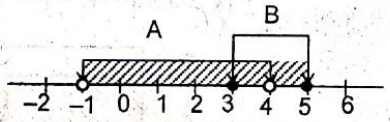
Here,

$A = (-1, 4)$ and $B = [3, 5]$

$A \cup B = (-1, 4) \cup [3, 5)$

$= \{x: -1 < x < 4\} \cup \{x: 3 \le x \le 5\}$

$= \{x: -1 < x \le 5\} = (-1, 5]$



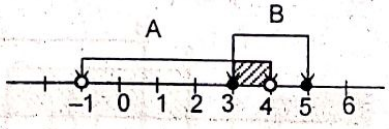
A∪B

$A \cap B = (-1, 4) \cap [3, 5]$

$= \{x: -1 < x < 4\} \cap \{x: 3 \le x \le 5\}$

$= \{x: 3 \le x < 4\}$

$= [3, 4)$



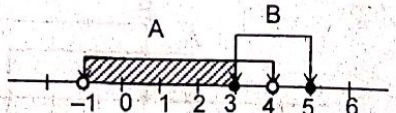
A∩B

$A - B = (-1, 4) - [3, 5]$.

$= \{x: -1 < x < 4\} - \{x: 3 \le x \le 5\}$

$= \{x: -1 < x < 3\}$

$= (-1, 3)$



A−B

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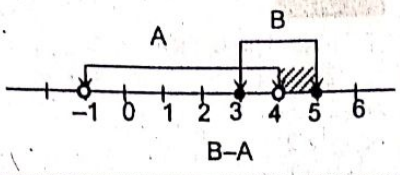
$B-A = [3, 5] - (-1, 4)$
$= \{x: 3 \le x \le 5\} - \{x: -1 < x < 4\}$
$= \{x: 4 \le x \le 5\}$
$= [4, 5]$



B–A

25. 2073 Set D Q.No. 6a OR

Define absolute value of a real number.

Prove that $|x + y| \le |x| + |y|$ for all $x, y \in \mathbb{R}$ [4]

SOLUTION

Let x be any real number. The absolute value (or modulus or numerical value) of x is denoted by $|x|$ is a non negative real number defined by

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$$

Next part

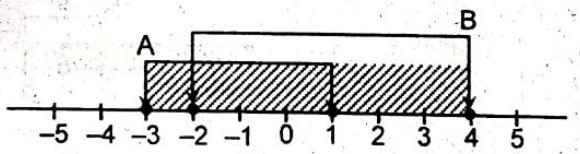
$|x + y|^2 = (x + y)^2$
$= x^2 + 2xy + y^2$
$= |x|^2 + |y|^2 + 2xy$
$\le |x|^2 + |y|^2 + 2|x|\,|y| \quad (\because x \le |x|,\ y \le |y|)$
$= (|x| + |y|)^2$
$\therefore |x + y| \le |x| + |y|$

26. 2072 Supp. Q.No. 6a OR

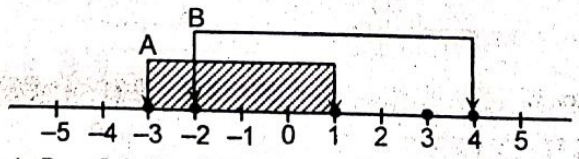
If A = [-3, 1] and B = [-2, 4], find A∪B, A∩B and A – B. Present them in the graph. [4]

SOLUTION

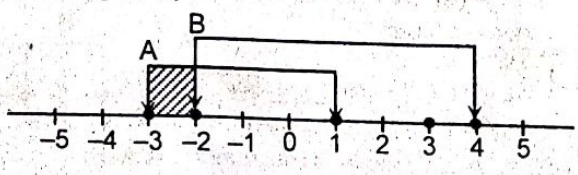
Here, $A = [-3, 1]$
$B = [-2, 4]$
$A \cup B = [-3, 1] \cup [-2, 4]$
$= \{x : -3 \le x \le 1\} \cup \{x : -2 \le x \le 4\}$
$= \{x : -3 \le x \le 4\} = [-3, 4]$



$A \cap B = [-3, 1] \cap [-2, 4]$
$= \{x : -3 \le x \le 1\} \cap \{x : -2 \le x \le 4\}$
$= \{x : -2 \le x \le 1\}$
$= [-2, 1]$



$A - B = [-3, 1] - [-2, 4]$
$= \{x : -3 \le x \le 1\} - \{x : -2 \le x \le 4\}$
$= \{x : -3 \le x < -2\}$
$= [-3, -2)$



27. 2072 Set C Q.No. 6b OR

Define absolute value of a real number. If $x \in \mathbb{R}$ and a is any positive real number, prove that $|x| < a \Rightarrow -a < x < a$ and conversely. [4]

➥ **Definition:** Please refer to 2076 Set B Q.No. 6a OR

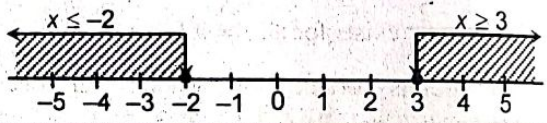
Next Part: Please refer to 2076 Set C Q.No. 6a OR

28. 2072 Set D Q.No. 6a OR

Solve the inequality $|2x - 1| \ge 5$ and represent the solution set in the real line. [4]

SOLUTION

If $(2x - 1) > 0$, then
$2x - 1 \ge 5$
or, $2x \ge 6$
or, $x \ge 3$
$\therefore x \in [3, \infty)$
Again, if $(2x - 1) < 0$, then
$-(2x - 1) \ge 5$
or, $2x - 1 \le -5$
or, $2x \le -4$
or, $x \le -2$
$\therefore x \in (-\infty, -2]$
The required solution of the given inequality is $\{x : x \le -2 \text{ or } x \ge 3\}$
i.e. $x \in (-\infty, -2] \cup [3, \infty)$
The graph of the given inequality is shown below.

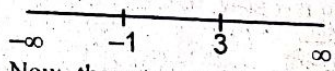


29. 2072 Set E Q.No. 6a OR

Solve: $x^2 - 2x - 3 \ge 0$. [4

SOLUTION

The corresponding equation of given inequality is
$x^2 - 2x - 3 = 0$
or, $x^2 - 3x + x - 3 = 0$
or, $x(x - 3) + 1(x - 3) = 0$
or, $(x + 1)(x - 3) = 0$
$\therefore x = -1, 3$



Now, these two points divide the real line into 3 sub intervals $(-\infty, -1)$, $(-1, 3)$ and $(3, \infty)$.

Intervals	Sign of		
	$(x + 1)$	$(x - 3)$	$(x + 1)(x - 3)$
$(-\infty, -1)$	–ve	– ve	+ ve
$(-1, 3)$	+ ve	– ve	– ve
$(3, \infty)$	+ ve	+ ve	+ ve

Also, at $x = -1$ and $x = 3$, $x^2 - 2x - 3 = 0$
$\therefore$ The possible intervals is $(-\infty, -1] \cup [3, \infty)$

30. 2071 Supp. Q.No. 6a OR

Solve the inequality: $x^2 + 7x + 10 < 0$. [4

➥ Please refer to 2075 Set C Q.No. 6a OR

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31. 2071 Set C Q.No. 6a Or

Define absolute value of a real number. Also, a is a positive real number and $x \in R$ then prove that: $|x| < a$ if and only if $-a < x < a$. [4]

➥ **Definition:** Please refer to 2076 Set B Q.No. 6a OR

Next Part: Please refer to 2076 Set C Q.No. 6a OR

32. 2071 Set D Q.No. 6a Or

Define absolute value of a real number. If a is a positive real number and $x \in R$, then prove that: $|x| < a$ if and only if $-a < x < a$. [4]

➥ **Definition:** Please refer to 2076 Set B Q.No. 6a OR

Next Part: Please refer to 2076 Set C Q.No. 6a OR

33. 2070 Supp Q.No. 6a Or

Solve the inequality: $|2x + 1| \geq 3$. Represent the solution in a real line. [4]

SOLUTION

Case I: If $2x + 1 > 0$, then

$|2x + 1| \geq 3$ gives

$(2x + 1) \geq 3$

or, $2x \geq 2$

$\therefore x \geq 1$

Case II: If $2x + 1 < 0$, then

$|2x + 1| \geq 3$ gives

or, $-(2x + 1) \geq 3$

or, $2x + 1 \leq -3$

or, $2x \leq -4$

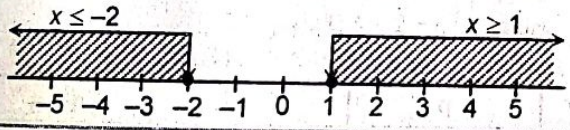
$\therefore x \leq -2$

Required solution set is

$\{x : x \leq -2 \text{ or } x \geq 1\}$

$= \{x : x \in (-\infty, -2] \cup [1, 0)\}$

The graph is shown below.



34. 2070 Set C Q.No. 6 a Or

Let A = [–3, 1] and B = [–2, 4]. Find A ∪ B, A ∩ B, A – B and B – A. [4]

SOLUTION

A ∪ B, A ∩ B, A – B:

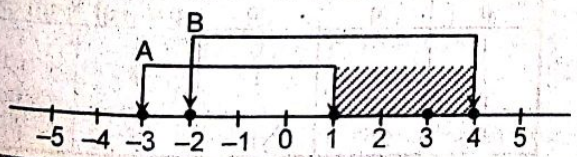
Please refer 2072 Supp. Q.No. 6a OR

B–A $= [-2, 4] - [-3, 1]$

$= \{x : -2 \leq x \leq 4\} - \{x : -3 \leq x \leq 1\}$

$= \{x : 1 < x \leq 4\}$

$= (1, 4]$



35. 2070 Set D Q.No. 6 a Or

Solve the inequality: $6 + 5x - x^2 \geq 0$. [4]

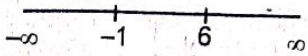
SOLUTION

The corresponding equation of given inequality is

$6 + 5x - x^2 = 0$

or, $(6 - x)(x + 1) = 0$

$\therefore x = -1, x = 6$



These two points divide the whole real line into 3 sub-intervals $(-\infty, -1)$, $(-1, 6)$ and $(6, \infty)$

Intervals	Sign of		
	$(6 - x)$	$(x + 1)$	$(6 - x)(x + 1)$
$(-\infty, -1)$	+ ve	− ve	− ve
$(-1, 6)$	+ ve	+ ve	+ ve
$(6, \infty)$	− ve	+ ve	− ve

Also, at $x = -1$, and $x = 6$, $6 + 5x - x^2 = 0$

The possible interval is $(-1, 6) \cup \{-1, 6\} = [-1, 6]$

36. 2069 Supp Q.No. 6 a Or

If a is a positive real number and $x \in \mathbb{R}$, prove that: $|x| < a$ if and only if $-a < x < a$. [4]

➥ Please refer to 2076 Set C Q.No. 6a OR

37. 2069 [Set A] Q. No. 6a OR

If $x \in R$ and a is any positive real number, prove that: $|x| < a \Rightarrow -a < x < a$ and conversely. [4]

➥ Please refer to 2076 Set C Q.No. 6a OR

38. 2069 [Set B] Q. No. 6a OR

Define absolute value of a real number. For any two real numbers x and y, prove that: $|x + y| \leq |x| + |y|$. [4]

➥ Please refer 2073 Set D Q.No. 6a OR

39. 2068 Q.No. 6a(Or)

Define absolute value of a real number. Rewrite the following relation without using absolute value sign $|2x - 1| \leq 5$. Also, draw the graph of the inequality. [4]

SOLUTION

Absolute value: Please refer to 2076 Set B Q.No. 6a OR

Next part: $|2x - 1| \leq 5$

or, $-5 \leq 2x - 1 \leq 5$ $\quad (\because |x| \leq a \Leftrightarrow -a \leq x \leq a]$

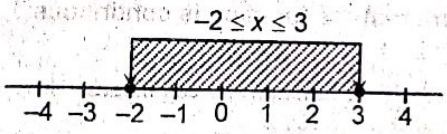
or, $-5 + 1 \leq 2x - 1 + 1 \leq 5 + 1$

or, $-4 \leq 2x \leq 6$

or, $\frac{-4}{2} \leq \frac{2x}{2} \leq \frac{6}{2}$

$\therefore -2 \leq x \leq 3$.

The graph is as follows.



C. LOGICS

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 1a

Find the truth value of the biconditional statement of p and q where p represents '2 is an even number' and q represents '4 is an even number'. [2]

SOLUTION

Given, p : 2 is an even number.

q : 4 is an even number.

Here, p and q are both true statements.

Now,

p	q	$p \Leftrightarrow q$
T	T	T

Thus, the truth value of $p \Leftrightarrow q$ is T.

2. 2075 GIE Q.No. 1a

For any statements p and q, prove that $p \vee (p \wedge q) \equiv p$ with the help of truth table. [2]

SOLUTION

Truth Table

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

From above table, we see that $p \vee (p \wedge q) = p$

3. 2075 Set C Q.No. 1a

For any statements p and q, prepare a truth table for $\sim(p \wedge (\sim q))$. [2]

SOLUTION

Truth Table for $\sim (p \wedge (\sim q))$

p	q	$\sim q$	$p \wedge (\sim q)$	$\sim (p \wedge (\sim q))$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

4. 2074 Supp Q.No. 1a

If p and q are two statements, construct a truth table for the compound statement $(\sim p) \vee (\sim q)$. [2]

SOLUTION

Truth table

p	q	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

5. 2075 Set A Q.No. 1a

Write the converse and contrapositive of the statement "For a function to be integrable it is sufficient that the function is continuous." [2]

SOLUTION

Converse: If a function is continuous, then it is differentiable.

Contrapositive: If a function is not continuous, then it is not differentiable.

6. 2073 Set D Q.No. 1a

For any statements p and q, prepare a truth table for $q \wedge \{\sim (p \vee q)\}$. [2]

SOLUTION

p	q	$p \vee q$	$\sim (p \vee q)$	$q \wedge (\sim (p \vee q))$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

7. 2072 Supp. Q.No. 1a

Define conjunction of a statement. Construc truth table form the compound stateme $\sim (p \wedge q)$.

SOLUTION

Two simple statements combined by the word 'and' an equivalent word) to form, a compound statement called the conjunction of the given statements.

Truth table for $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

8. 2072 Set C Q.No. 1a

Construct a truth table of $\sim (p \vee q) \wedge q$.

SOLUTION

Truth table for $\sim (p \vee q) \wedge q$

p	q	$p \vee q$	$\sim (p \vee q)$	$\sim (p \vee q) \wedge q$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

9. 2072 Set D Q.No. 1a

Find the truth value and the negation of th statement "3 + 2 = 6 or 6 is a multiple of 3".

SOLUTION

Given compound statement is

pvq: 3 + 2 = 6 or 6 is a multiple of 3.

Here, p : 3 + 2 = 6 (False)

q : 6 is a multiple of 3. (True)

p	q	$p \vee q$
F	T	T

Hence, the truth value of given statement is true i 'T'.

The negation of pvq is $\sim p \wedge \sim q$.

So, negation of given statement is $3 + 2 \neq 6$ and 6 is a multiple of 3.

10. 2071 Supp. Q.No. 1a

Construct a truth table for the stateme $(p \vee (\sim q)) \Rightarrow q$.

SOLUTION

p	q	$\sim q$	$p \vee (\sim q)$	$(\sim q)) \Rightarrow$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

11. 2071 Set C Q.No. 1a

Define a statement. If p is true, q is true and is true, find the truth value of $(p \vee q) \wedge (\sim q)$.

SOLUTION

Statement: An assertion expressed in words symbols, which is either true or false but not both at t same time, is called statement.

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Next Part:

Given p is true, q is true and r is true.

Now,

p	q	~q	p∨q	(p∨q) ∧ (~q)
T	T	F	T	F

∴ The truth value of (p∨q)∧ (~q) is false i.e. 'F'.

12. 2071 Set D Q.No. 1a

Construct truth table of ~(p∨q). [2]

SOLUTION

p	q	p∨q	~(p∨q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

13. 2070 Supp Q.No. 1a

Find the negation of the following statements

i. Light travels in a straight line.

ii. 3+2 = 5 or 6 is a multiple of 5. [2]

SOLUTION

i. Light does not travel in a straight line.

ii. $3 + 2 \neq 5$ and 6 is not a multiple of 5.

14. 2070 Set C Q.No. 1 a

Prepare a truth table for the compound statement p ∨ ~ (p ∧ q). What would you conclude from the truth table? [2]

SOLUTION

p	q	p∧q	~(p∧q)	p∨ ~ (p∧q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since the compound statement p∨ ~ (p∧q) has truth value T in all cases, it is a tautology.

15. 2070 Set D Q.No. 1 a

If p and q are any two statements, prove that: pvq ≡q v p. [2]

SOLUTION

p	q	p ∨ q	q ∧ p
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

From above truth table, $p \vee q = q \vee p$

16. 2069 Supp Q.No. 1 a

For any statement p show that p ∧ ~ p is a contradiction. [2]

OLUTION

p	~p	p ∧ ~p
T	F	F
F	T	F

Since the compound statement p ∧ ~p has truth value F in all cases, p ∧ ~p is a contradiction.

17. 2069 [Set A] Q. No. 1a

Define negation of a statement. Construct a truth table for the compound statement ~(p ∨ (~q)). [2]

SOLUTION

Negation: A statement which denies the given statement is known as negation of a given statement.

p	q	~q	p ∧ (~q)	~ (p ∨ (~q))
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

18. 2069 [Set B] Q. No. 1a

Write inverse and converse of the statement 'if 3 is an odd number then 6 is not an odd number'. [2]

SOLUTION

Given statement is

$p \Rightarrow q$: If 3 is an odd number then 6 is not an odd number.

Converse: $(q \Rightarrow p)$: If 6 is not an odd number then 3 is an odd number.

Inverse: $(\sim p \Rightarrow \sim q)$: If 3 is not an odd number then 6 is an odd number.

19. 2068 Q.No. 1a

Define disjunction of two statements. Prepare a truth table for the compound statement ~ (p v q). [2]

SOLUTION

Disjunction: Two simple statements combined by the word 'or' to form a compound statement is known as disjunction of the given statements.

p	q	p ∨ q	~(p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

4 MARKS QUESTIONS

20. 2076 Set C Q.No. 6a

Define conditional and bi-conditional statements. For any statements p and q, construct the truth table for (p v q) ⇔ (q v p). [4]

SOLUTON

Conditional statement: Two simple statements combined by "If ... then ..." to form a compound statement, is called the conditional of given statements. The symbol used for the conditional is ⇒.

Biconditional statement: Two simple statements combined by 'if and only if' abbreviated as iff to form a compound statement is called biconditional of the given statements. The symbol used for biconditional is ⇔.

Truth Table for $(p \vee q) \Leftrightarrow (q \wedge p)$

p	q	p ∨ q	q ∧ p	(p ∨ q) ⇔ (q ∧ p)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

21. 2075 Set B Q.No. 6a OR

Define connectives in logic. For the simple statements p, q, r, construct the truth table for the statement $(p \Leftrightarrow q) \vee (q \Leftrightarrow r)$. [4]

SOLUTION

Connectives: The compound statements are formed from the simple statements by using the word or phrase like 'or', 'and', 'if … then', 'if and only if', etc. These are known as logical connectives.

Truth Table $(p \Leftrightarrow q) \vee (q \Leftrightarrow r)$

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$(p \Leftrightarrow q) \vee (q \Leftrightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	T	T

22. 2074 Set A Q.No. 6a

Define conjunction of the statements. Prepare a truth table for the compound statement $(p \wedge q) \wedge \sim (p \vee q)$. Draw the conclusion about the statement from the truth table. [4]

SOLUTION

Conjunction:– Two simple statements combined by the word 'and' (or equivalent word) to form a compound statement is called the conjunction of the given statements.

Truth Table for $(p \wedge q) \wedge \sim (p \vee q)$

p	q	$p \wedge q$	$(p \vee q)$	$\sim (p \vee q)$	$(p \wedge q) \wedge \sim (p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since the compound statement $(p \wedge q) \wedge \sim (p \vee q)$ has truth value F in all cases, it is a contradiction.

23. 2074 Set B Q.No. 6a OR

Let p, q, r be any simple statements, construct truth table for the compound statement $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$. [4]

SOLUTION

Truth Table

p	q	r	$\sim q$	$p \Rightarrow \sim q$	$p \Rightarrow r$	$(p \Rightarrow \sim q) \wedge (p \Rightarrow q)$
T	T	T	F	F	T	F
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

24. 2073 Supp Q.No. 6a

If p and q are two statements, prove that $p \vee \sim(p \wedge q)$ is a tautology. [4]

SOLUTION

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since the compound statement $p \vee \sim (p \wedge q)$ has truth value T in all cases, it is a tautology.

25. 2073 Set C Q.No. 6a

Define conditional statement. Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$. What conclusion can be drawn about the statement from the truth table? [4]

SOLUTION

Conditional: Two simple statements combined by the word 'If … then' to form a compound statement is known as the conditional of the given statements.

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

From above truth table, $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology.

26. 2072 Set E Q.No. 6a

Let p, q and r be statements, show that the statement $(p \wedge q \Rightarrow r) \Leftrightarrow [p \Rightarrow (q \Rightarrow r)]$ is true. [4]

SOLUTION

p	q	r	$p \wedge q$	$p \wedge q \Rightarrow r$	$q \Rightarrow r$	$\lvert p \Rightarrow (q \Rightarrow r)\rvert$	$(p \wedge q \Rightarrow r) \Leftrightarrow \lvert p \Rightarrow (q \Rightarrow r)\rvert$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

From above truth table, $(p \wedge q \Rightarrow r) \Leftrightarrow [p \Rightarrow (q \Rightarrow r)]$ true.

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2 UNIT

RELATIONS, FUNCTIONS AND GRAPHS

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2 MARKS QUESTIONS

1. 2076 Set B Q.No. 1b

If $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 3$, find f^{-1}. [2]

SOLUTION

Let $y = f(x) = 2x - 3$

or, $y = f(x)$

or, $x = f^{-1}(y)$ ... (i)

Also,

$y = 2x - 3$

or, $y + 3 = 2x$

or, $x = \frac{y+3}{2}$ ... (ii)

From (i) and (ii), we have,

$f^{-1}(y) = \frac{y+3}{2}$

Since y is a dummy variable, replacing y by x, we get

$f^{-1}(x) = \frac{x+3}{2}$

2. 2076 Set C Q.No. 1b

Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 9x - 1$, $x \in \mathbb{R}$, examine whether f is one-one. [2]

SOLUTON

Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 9x - 1$.

Let $x_1, x_2 \in \mathbb{R}$ (domain).

Then, $f(x_1) = 9x_1 - 1$ and $f(x_2) = 9x_2 - 1$.

Now,

$f(x_1) = f(x_2)$

$\Rightarrow 9x_1 - 1 = 9x_2 - 1$

$\Rightarrow 9x_1 = 9x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

3. 2075 GIE Q.No. 1b

Find the domain & range for $f(x) = \frac{x}{x-1}$, $x \in IR$. [2]

SOLUTION

Let $y = \frac{x}{x-1}$

Clearly y is defined for all x in $\mathbb{R}$ except at $x = 1$.

So, domain $= \mathbb{R} - \{1\}$

Again,

$y = \frac{x}{x-1}$

or, $xy - y = x$

or, $x(y - 1) = y$

or, $x = \frac{y}{y-1}$

So, x is defined for all $y \in \mathbb{R}$ except at $y = 1$.

So, range $= \mathbb{R} - \{1\}$

4. 2075 Set A Q.No. 1b

Evaluate: $2 \log_5 \sqrt{5} + 3 \log_2 8$. [2]

SOLUTION

$2 \log_5 \sqrt{5} + 3 \log_2 8$

$= 2 \log_5 5^{1/2} + 3 \log_2 2^3$

$= 2 \times \frac{1}{2} \log_5 5 + 3 \times 3 \log_2 2$

$= \log_5 5 + 9 \log_2 2$

$= 1 + 9 \times 1$ [$\because \log_a a = 1$]

$= 10$

5. 2075 Set B Q.No. 1b

Let A = {1, 2, 3} and B = {3, 4, 5}. Show that $A \times B \neq B \times A$. [2]

SOLUTION

Here,

$A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$A \times B = \{1, 2, 3\} \times \{3, 4, 5\}$

$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3) (3, 4), (3, 5)\}$

$B \times A = \{3, 4, 5\} \times \{1, 2, 3\}$

$= \{(3, 1), (3, 2) (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$

$\therefore A \times B \neq B \times A$

6. 2075 Set C Q.No. 1b

Let A = {1, 2, 3}. Find the relation in A × A satisfying the condition x > y for all (x, y) ∈ A×A. Find the domain of the relation. [2]

SOLUTION

Here, $A = \{1, 2, 3\}$

Now,
$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$
$= \{(1, 1), (1, 2), (1, 3), (2, 1) (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
The relation in $A \times A$ satisfying the condition $x > y$ for all $(x, y) \in A \times A$ is $\{(2, 1), (3, 1) (3, 2)\}$
Domain of the relation $= \{2, 3\}$.

7. 2074 Supp Q.No. 1b

Let A = {0, 1, 2, 3, 4, 5, 6) and a function f : A → Q is defined by $f(x) = \frac{x}{2}$. Find the range of f. Q is the set of rational numbers. [2]

SOLUTION

Here, $f : A \to Q$ be defined by $f(x) = \frac{x}{2}$;
where $A = \{0, 1, 2, 3, 4, 5, 6\}$
when $x = 0$, $f(0) = \frac{0}{2} = 0$
when $x = 1$, $f(1) = \frac{1}{2}$
when $x = 2$, $f(2) = \frac{2}{2} = 1$
when $x = 3$, $f(3) = \frac{3}{2}$
when $x = 4$, $f(4) = \frac{4}{2} = 2$
when $x = 5$, $f(5) = \frac{5}{2}$
when $x = 6$, $f(6) = \frac{6}{2} = 3$
$\therefore$ Range of $f = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$

8. 2074 Set A Q.No. 1b

Let A = {a, b), B = {b, c} and C = {c, d). Find A ×(B∪C) and A×(B∩C). [2]

SOLUTION

Here,
$A = \{a, b\}, B = \{b, c\}, C = \{c, d\}$
$B \cup C = \{b, c\} \cup \{c, d\} = \{b, c, d\}$
$B \cap C = \{b, c\} \cap \{c, d\} = \{c\}$
Now, $A \times \{B \cup C\} = \{a, b\} \times \{b, c, d\}$
$= \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)\}$
and $A \times (B \cap C) = \{a, b\} \times \{c\}$
$= \{(a, c), (b, c)\}$

9. 2074 Set B Q.No. 1b

If A = {3, 4, 5, 6}, find the relation R in A×A, satisfying the condition x + y = 9; x∈A, y∈A. [2]

SOLUTION

Given,
$A = \{3, 4, 5, 6\}$
$A \times A = \{3,4, 5, 6\} \times \{3, 4, 5, 6\}$
$= \{(3, 3), (3, 4), (3, 5), (3, 6), (4, 3), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$
The relation in $A \times A$ satisfying the condition $x + y = 9$ is $\{(3, 6), (4, 5), (5, 4), (6, 3)\}$

10. 2073 Supp Q.No. 1b

Let A = {1, 2, 3, 4} and B = {1,3,5}. Find the relation R from set A to set B satisfying the condition y = 2x. [2]

SOLUTION

Here,
$A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$
$A \times B = \{1, 2, 3, 4\} \times \{1, 3, 5\}$
$= \{(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$
The relation R from set A to set B satisfying the condition $y = 2x$ is $\{\}$.

11. 2073 Set C Q.No. 1b

Find the domain and range of the relation R = {(1,1), (2, 2), (4, 4)}. What types of relation is this? [2]

SOLUTION

Given relation is $R = \{(1,1), (2, 2), (4, 4)\}$
Domain of $R = \{1, 2, 4\}$
Range of $R = \{1, 2, 4\}$
The given relation is reflexive since $(a, a) \in R$ for all a

12. 2073 Set D Q.No. 1b

Let A = {–1, 0, 2, 4, 6, 8} and f: A →R is defined by $f(x) = \frac{x}{x+2}$. Find range of f. [2]

SOLUTION

Here, $f(x) = \frac{x}{x+2}$
and $A = \{-1, 0, 2, 4, 6, 8\}$
when $x = -1$, $f(-1) = \frac{-1}{-1+2} = -1$
when $x = 0$, $f(0) = \frac{0}{0+2} = 0$
when $x = 2$, $f(2) = \frac{2}{2+2} = \frac{1}{2}$
when $x = 4$, $f(4) = \frac{4}{4+2} = \frac{2}{3}$
when $x = 6$, $f(6) = \frac{6}{6+2} = \frac{3}{4}$
when $x = 8$, $f(8) = \frac{8}{8+2} = \frac{4}{5}$
$\therefore$ Range of $f = \left\{-1, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\}$

13. 2072 Supp. Q.No. 1b

Let A = {a,b}, B= {b,c} and C = {c,d}, find (A×B)∪(A×C) and (A×B)∩(A×C). [2]

SOLUTION

Here, $A = \{a,b\}$, $B = \{b,c\}$ and $C = \{c,d\}$
$A \times B = \{(a, b), (a, c), (b, b), (b, c)$
$A \times C = \{(a, c), (a, d), (b, c), (b,d)\}$
$(A \times B) \cup (A \times C) = \{(a, b), (a, c), (b, b), (b, c), (a, d), (b, d)\}$
$(A \times B) \cap (A \times C) = \{(a, c), (b, c)\}$

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14. 2072 Set C Q.No. 1b

Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = 3x - 1$, find $(g_0f)(x)$ and $(f_0g)(x)$. [2]

SOLUTION

Here,

$f(x) = 2x + 1$

$g(x) = 3x - 1$

$gof(x) = g[f(x)] = g(2x + 1) = 3(2x + 1) - 1$

$= 6x + 3 - 1 = 6x + 2$

and $fog(x) = f[g(x)] = f(3x - 1) = 2(3x - 1) + 1$

$= 6x - 2 + 1 = 6x - 1$

15. 2072 Set D Q.No. 1b

If A = {1, 2, 3} and B = {1, 3, 5, 7}, find a relation from set A to set B determined by $x \le y$. [2]

SOLUTION

Here, A = {1, 2, 3} and B = {1, 3, 5, 7}

$A \times B = \{1, 2, 3\} \times \{1, 3, 5, 7\}$

$= \{(1, 1), (1\ 3)\ (1, 5), (1, 7), (2, 1), (2, 3), (2, 5), (2, 7), (3, 1), (3, 3), (3, 5), (3, 7)\}$

The relation from set A to set B determined by $x \le y$ is $\{(1, 1), (1\ 3)\ (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7)\}$

16. 2072 Set E Q.No. 1b

Find the value of $\log_a \sqrt{a\sqrt{a\sqrt{a}}}$. [2]

SOLUTION

$\log_a \sqrt{a\sqrt{a\sqrt{a}}}$

$= \log_a \sqrt{a\sqrt{a \cdot a^{1/2}}} = \log_a \sqrt{a\sqrt{a^{3/2}}}$

$= \log_a \sqrt{a \cdot a^{3/4}} = \log_a \sqrt{a^{7/4}}$

$= \log_a a^{7/8}$

$= \frac{7}{8} \log_a a \quad (\because \log a^m = m \log a)$

$= \frac{7}{8} \times 1 \quad (\because \log_a a = 1)$

$= \frac{7}{8}$

17. 2071 Supp. Q.No. 1b

Prove that $x^{\ln\left(\frac{y}{z}\right)} y^{\ln\left(\frac{z}{x}\right)} z^{\ln\left(\frac{x}{y}\right)} = 1$ [2]

SOLUTION

Let $a = x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln\left(\frac{z}{x}\right)} \cdot z^{\ln\left(\frac{x}{y}\right)}$

Taking 'ln' on both sides, we get

$\ln a = \ln \left\{ x^{\ln\left(\frac{y}{z}\right)} y^{\ln\left(\frac{z}{x}\right)} \cdot z^{\ln\left(\frac{x}{y}\right)} \right\}$

or, $\ln a = \ln\left(\frac{y}{z}\right) \ln x + \ln\left(\frac{z}{x}\right) \ln y + \ln\left(\frac{x}{y}\right) \ln z$

or, $\ln a = (\ln y - \ln z) \ln x + (\ln z - \ln x) \ln y + (\ln x - \ln y) \ln z$

or, $\ln a = \ln y \ln x - \ln z \ln x + \ln z \ln y - \ln x \ln y + \ln x \ln z - \ln y \ln z$

or, $\ln a = 0 = \ln 1$

or, $a = 1$

$\therefore x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln\left(\frac{z}{x}\right)} \cdot z^{\ln\left(\frac{x}{y}\right)} = 1$

18. 2070 Supp Q.No. 1b

If A = {1, 2, 3} and B = {a, b}, find A × B and B × A and show that A × B ≠ B × A. [2]

SOLUTION

Here, A = {1, 2, 3}

B = {a, b}

$A \times B = \{1, 2, 3\} \times \{a, b\}$

$= \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$B \times A = \{a, b\} \times \{1, 2, 3\}$

$= \{(a, 1), (a, 2), (a, 3), (b, 1) (b, 2), (b, 3)\}$

$\therefore A \times B \ne B \times A$

19. 2070 Old Q.No. 1 c

A function f is defined as $f(x) = \begin{cases} x^2, & x \le 0 \\ \frac{1}{x}, & x > 0 \end{cases}$, find f(0), f(+2) and $f(\sqrt{3})$. [2]

SOLUTION

$f(0) = 0^2 = 0$

$f(2) = \frac{1}{2}$

$f(\sqrt{3}) = \frac{1}{\sqrt{3}}$

20. 2070 Set C Q.No. 1b

Let A = {1, 2, 3, 4}. Find the relation on A satisfying the condition $x + y \le 4$. [2]

SOLUTION

Here, A = {1, 2, 3, 4}

$A \times A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1) (2, 2), (2, 3), (2, 4), (3, 1) (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

The relation on A satisfying the condition $x + y \le 4$ is $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

21. 2070 Set D Q.No. 1 b

Let A = {a, b}, B = {b, c} and C = {c, d}. Find: $A \times (B \cup C)$ and $A \times (B \cap C)$. [2]

SOLUTION

Here, A = {a, b}, B = {b, c} and C = {c,d}

$B \cup C = \{b, c\} \cup \{c, d\} = \{b, c, d\}$

$B \cap C = \{b, c\} \cap \{c, d\} = \{c\}$

$A \times (B \cup C) = \{a, b\} \times \{b, c, d\}$

$= \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)\}$

and $A \times (B \cap C) = \{a, b\} \times \{c\} = \{(a, c) (b, c)\}$

22. 2069 Supp Q.No. 1 b

Define inverse relation and give its example. [2]

SOLUTION

Inverse relation: A relation obtained by interchanging the first and second members of a relation R is known as inverse relation. It is denoted by R^{-1}.

For example, let A = {a, b}, B = {1, 2}

$A \times B = \{a, b\} \times \{1, 2\}$

$= \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

Let R = {(a, 1), (b, 2)}

Here R is a relation from A to B because $R \subset A \times B$.
Inverse relation $(R^{-1}) = \{(1, a), (2, b)\}$

23. 2069 [Set A] Q. No. 1b

Find the domain of the function $y = \sqrt{x-2}$. [2]

SOLUTION

Here, $y = \sqrt{x-2}$
For $x - 2 < 0$, y will be imaginary. So y will be defined only for $x - 2 \geq 0$

$\Rightarrow x \geq 2$

$\therefore$ Domain of the function $= [2, \infty)$

24. 2069 [Set B] Q. No. 1b

Find the domain, range and inverse of the relation R = {(1, 2), (2, 4), (3, 6), (4, 8)} [2]

SOLUTION

Given relation is $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$
Domain $= \{1, 2, 3, 4\}$
Range $= \{2, 4, 6, 8\}$
Inverse relation $= \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

25. 2069 [Set A] Old Q. No. 1b

Prove that:
$\log(1 + 2 + 3) = \log 1 + \log 2 + \log 3$ [2]

SOLUTION

L.H.S. $= \log(1 + 2 + 3)$
$= \log 6$
$= \log(1 \times 2 \times 3)$
$= \log 1 + \log 2 + \log 3$
$=$ R.H.S.

26. 2068 Q.No. 1b

Let A = {1, 2, 3, 4} and B = {1, 3, 5}. Find the relation R from Set A to Set B determined by the condition $x > y$. [2]

SOLUTION

Given, $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$
$A \times B = \{1, 2, 3, 4\} \times \{1, 3, 5\}$
$= \{(1, 1), (1, 3), (1, 5), (2, 1) (2, 3), (2 5), (3, 1) (3, 3), (3, 5), (4, 1) (4, 3,), (4, 5)\}$
The relation R from set A to set B determined by $x > y$ is $(2, 1), (3, 1), (4, 1), (4, 3)\}$

27. 2068 Old Q.No. 1b

Let A = {–1, 0, 2, 4, 6} and a function $f : A \to R$ is defined by $y = f(x) = \frac{x}{x+2}$. Find the range of f. [2]

SOLUTION

Here, $f(x) = \frac{x}{x+2}$
and $A = \{-1, 0, 2, 4, 6\}$

when $x = -1$, $f(-1) = \frac{-1}{-1+2} = -1$

when $x = 0$, $f(0) = \frac{0}{0+2} = 0$

when $x = 2$, $f(2) = \frac{2}{2+2} = \frac{1}{2}$

when $x = 4$, $f(4) = \frac{4}{4+2} = \frac{2}{3}$

when $x = 6$, $f(6) = \frac{6}{6+2} = \frac{3}{4}$

$\therefore$ Range of $f = \left\{-1, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}$

28. 2067 Q.No. 1b

If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that $xyz = 1$ [2]

SOLUTION

Given,
$\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$ (say)
Then, $\log x = k(y - z)$
$\log y = k(z - x)$
$\log z = k(x - y)$
On adding
$\log x + \log y + \log z = k(y - z) + k(z - x) + k(x - y)$
or, $\log(x\,y\,z) = ky - kz + kz - kx + kx - ky$
or, $\log(xyz) = 0$
or, $\log(xyz) = \log 1$ ($\because \log 1 = 0$)
$\therefore xyz = 1$

29. 2066 Q.No. 1b

Prove that $\log_a x^2 - 2\log_a \sqrt{x} = \log_a x$ [2]

SOLUTION

L.H.S. $= \log_a x^2 - 2\log_a \sqrt{x}$
$= \log_a x^2 - \log_a (\sqrt{x})^2$
$= \log_a x^2 - \log_a x$
$= \log_a \left(\frac{x^2}{x}\right)$
$= \log_a x =$ R.H.S.

30. 2065 Q.No. 1b

Let $f: R \to R$ be defined by $f(x) = 2x + 5$. Compute $f^{-1}(x)$. [2]

SOLUTION

Here,
$f(x) = 2x + 5$
i.e. $y = 2x + 5$
Now, interchanging the role of x & y, the value of y thus, obtained is $f^{-1}(x)$.
$x = 2y + 5$
or, $2y = x - 5$
or, $y = \frac{x-5}{2}$
$\therefore f^{-1}(x) = \frac{x-5}{2}$

31. 2064 Q.No. 1b

Let f, g be real valued functions defined as $f(x) = 4x + 7$, $x \in R$ and $g(x) = 5x - 2$, $x \in R$. Find fog (x) and gof (x). [2]

SOLUTION

Here, $f(x) = 4x + 7, x \in R$
$g(x) = 5x - 2, x \in R$
Now,
i. $fog(x) = f[g(x)] = f(5x - 2)$
$= 4(5x - 2) + 7 = 20x - 8 + 7 = 20x - 1$

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ii. $gof(x) = g[f(x)] = g(4x+7)$
$= 5(4x+7) - 2 = 20x + 35 - 2 = 20x + 33.$

32. 2063 Q.No. 1b

Let f, g be real valued function defined as $f(x) = x^2 + 5x + 7$ and $g(x) = 5x - 3$, $x \in R$
Find fog(x) and gof(x) [2]

SOLUTION

Given functions are
$f(x) = x^2 + 5x + 7$
$g(x) = 5x - 3$

Now,
$fog(x) = f\{g(x)\}$
$= f(5x-3)\}$
$= (5x-3)^2 + 5(5x-3) + 7$
$= 25x^2 - 30x + 9 + 25x - 15 + 7$
$= 25x^2 - 5x + 1$

and $gof(x) = g\{f(x)\}$
$= g(x^2 + 5x + 7)$
$= 5(x^2 + 5x + 7) - 3$
$= 5x^2 + 25x + 35 - 3$
$= 5x^2 + 25x + 32$

33. 2062 Q.No. 1b

Check whether the function f: [– 2, 3] → R given by $f(x) = x^3$ is one to one, onto or both. [2]

SOLUTION

The given function is $f(x) = x^3$
Let a, b ∈ [– 2, 3] such that $a \neq b$

$\Rightarrow a^3 \neq b^3$ [∵ Cubes of distinct real numbers are different]
$\Rightarrow f(a) \neq f(b)$
∴ f is one to one
Again,
f is onto if range of f = codomain of f
But range of f = $[f(-2), f(3)] = [(-2)^3, 3^3]$
= $[-8, 27] \neq$ Co–domain of f, because codomain of f is R
So, f is not onto.
Hence, f is one to one but not onto.

34. 2061 Q.No. 1b

Let h: R → R be defined by $h(x) = 3x - 6$, find a formula that defines h^{-1}. [2]

SOLUTION

Let $y = h(x) = 3x - 6$
Now, interchanging the role of x and y, the value of y thus obtained is $h^{-1}(x)$.

$\Rightarrow x = 3y - 6$
or, $3y = x + 6$
or, $y = \frac{x+6}{3}$
∴ $h^{-1}(x) = \frac{x+6}{3}$

35. 2060 Q.No. 1c

Define even and odd functions with examples. [2]

SOLUTION

A function $f(x)$ is said to be an even function if $f(-x) = f(x)$.
For example, $f(x) = x^2$ is an even function. For, $f(-x) = (-x)^2 = x^2 = f(x)$.
A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$.
For example $f(x) = x^3$ is an odd function. For, $f(-x) = (-x)^3 = -x^3 = -f(x)$

36. 2059 Q.No. 1c

Let f: R → R be defined by $y = f(x) = 2x - 3$, $x \in R$, find formula that defines f^{-1}. [2]

SOLUTION

Here, the given function is
$y = f(x) = 2x - 3$ ... (i)
Now, interchanging the role of x and y in (i), the value of y thus, obtained is $f^{-1}(x)$

$\Rightarrow x = 2y - 3$
or, $2y = x + 3$
or, $y = \frac{x+3}{2}$
∴ $f^{-1}(x) = \frac{x+3}{2}$

37. 2058 Q.No. 1b

Let f: A → R be given by $f(x) = 2|x| + 3$
Where A = {– 2, 0, 1, 2}, find the range of f. [2]

SOLUTION

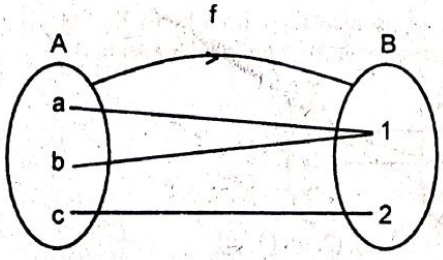
When $x = -2$ $f(-2) = 2|-2| + 3 = 2 \times 2 + 3 = 7$
When $x = 0$, $f(0) = 2|0| + 3 = 3$
When $x = 1$, $f(1) = 2|1| + 3 = 2 \times 1 + 3 = 5$
When $x = 2$, $f(2) = 2|2| + 3 = 2 \times 2 + 3 = 7$
∴ Range of f = {f(– 2), f(0), f(1), f(2)}
= {7, 3, 5, 7} = {3, 5, 7}

38. 2057 Q.No. 1b

When does a function f: A → B become an onto and one to one? [2]

SOLUTION

Onto function (Surjective function):
A function f from a set A to another set B i.e. f: A→ B is said to be an onto if every element of B is an image of at least one element of A i.e. if f(A) = B. i.e. if range of f = co-domain of f.
For example,



f: A → B is onto function

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One to one function (injective function):

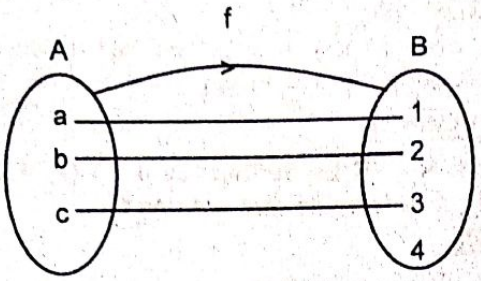
A function f from a set A to another set B i.e. f: A → B is said to be one to one (1 – 1) if distinct elements in A have distinct images in B.

In Symbol, for any $x_1, x_2 \in A$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Or, equivalently, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

For example,



f: A → B is one to one function

39. 2055 Old Q.No. 1b

When a function f: A → B becomes an onto function and one-to-one function? [2]

➥ Please refer to 2057 Q.No. 1b

6 MARKS QUESTIONS

40. 2076 Set B Q.No. 11

Define function. Distinguish relation and function with example. Find the domain and range of $f(x) = \sqrt{x^2 - 2x - 8}$, $x \in \mathbb{R}$. [6]

SOLUTON

Function: Let A and B be two non-empty sets. Then, a function f from a set A to set B is a relation that associates each elements in A with a unique element of B. It is denoted by f : A → B.

Difference between relation and function:

Let A and B be two non-empty sets. Then any subsets of Cartesian product A × B is called a relation from A to B. But, a function is a relation in which every element in A has unique image in B. Thus, every function is a relation but a relation may not be a function.

Example: Let A = {1, 2, 3} and B = {a, b, c}

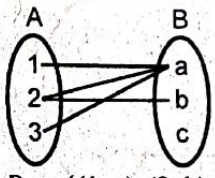
Then,

A × B = {1, 2, 3} × {a, b, c}
= {(1, a), (1, b), (1, c), (2, a) (2, b), (2, c), (3, a), (3, b), (3, c)}

i. R_1 = {(1, a), (2, a), (2, b), (3, a)}

Here, $R_1 \subseteq A \times B$

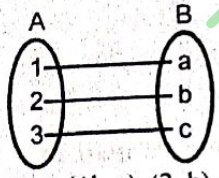
So, R_1 is a relation from A to B. But R_1 is not a function since 2 ∈ A has two images in B.



ii. R_2 = {(1, a), (2, b), (3, c)}

Here, $R_2 \subseteq A \times B$

So, R_2 is a relation from A to B. Also, every element in A has unique (only one) image in B, so it is a function.



iii. R_3 = {(1, a), (2, b), (c, 2)}

Here, $R_3 \not\subseteq A \times B$

So, R_3 is not a relation from A to B. Since R_3 is no relation, so it is not a function.

Last Part:

Let $y = f(x) = \sqrt{x^2 - 2x - 8}$

For $x^2 - 2x - 8 < 0$, y will be imaginary.

So, y will be defined only for

$x^2 - 2x - 8 \geq 0$

$\Rightarrow x^2 - 4x + 2x - 8 \geq 0$

$\Rightarrow x(x - 4) + 2(x - 4) \geq 0$

$\Rightarrow (x - 4)(x + 2) \geq 0$

The associated equation is $(x - 4)(x + 2) = 0$

$\therefore$ $x = -2, 4$

Intervals	Sign of		
	x + 2	x – 4	(x + 2)(x – 4)
(–∞, –2)	–	–	+
(–2, 4)	+	–	–
(4, ∞)	+	+	+

From above table, it is clear that the solution set $(-\infty, -2) \cup (4, \infty)$.

Also, at x = –2 and x = 4, $x^2 - 2x - 8 = 0$

Hence, required solution set is

$x \in (-\infty, -2] \cup [4, \infty)$.

$\therefore$ Domain = $(-\infty, -2] \cup [4, \infty)$.

For range

$y^2 = x^2 - 2x - 8$

$\Rightarrow y^2 = (x - 1)^2 - 9$

$\Rightarrow (x - 1)^2 = y^2 + 9$

Since, $(x - 1)^2 \geq 0$, so we have $y^2 + 9 \geq 0 \Rightarrow y^2 \geq -9$.

Since y is positive square root, so $y \in [0, \infty)$

$\therefore$ Range = $[0, \infty)$.

41. 2076 Set C Q.No. 11

Define domain and range of a function. Fi the domain and range of $f(x) = \sqrt{21 - 4x - x^2}$.

SOLUTION

Domain for a function: If f : A → B be a funct then the set A is called domain of f. It is denoted dom f. Thus,

dom (f) = {$x : x \in A$}

Range of a function: Let f : A → B be a functi Then, the set of values of $y = f(x) \in B$ for every $x \in$ is known as the range of the function f. It is denoted range (f). Thus,

Range (f) = {$y : y \in B, y = f(x)$ for all $x \in A$}

Next part:

$y = \sqrt{21 - 4x - x^2}$

$= \sqrt{25 - (x + 2)^2} = \sqrt{5^2 - (x + 2)^2}$

Thus the value of y is defined

if $5^2 - (x + 2)^2 \geq 0$

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i.e. $5^2 \geq (x+2)^2$

$\Rightarrow (x+2)^2 \leq 5^2$

$\Rightarrow |x+2|^2 \leq 5^2$

$\Rightarrow |x+2| \leq 5$

$\Rightarrow -5 \leq x+2 \leq 5$

$\Rightarrow -7 \leq x \leq 3$

$\therefore$ The domain of the function = $[-7, 3]$

Again,

$y^2 = 5^2 - (x+2)^2$

or, $(x+2)^2 = 5^2 - y^2$

$\Rightarrow (x+2)^2 \geq 0$ for all $x \in R$

So, $5^2 - y^2 \geq 0$

$\Rightarrow y^2 \leq 5^2$

$\Rightarrow |y|^2 \leq 5^2$

$\Rightarrow |y| \leq 5$

$\Rightarrow -5 \leq y \leq 5$

Since y is a positive square root, so $0 \leq y \leq 5$

$\therefore$ The range of f = $[0, 5]$

42. 2075 GIE Q.No. 11

Define one-one function and onto function. Let $f(x) = x^3 + 5$, $x \in R$. Find a formula that defines inverse function f^{-1}. [6]

First Part: Please refer to 2057 Q.No. 1b

Last part

Let $y = f(x) = x^3 + 5$

$\Rightarrow x = f^{-1}(y)$ ...(i)

and $y = x^3 + 5$

$\Rightarrow y - 5 = x^3$

$\Rightarrow x = \sqrt[3]{y-5}$ ...(ii)

From (i) and (ii), we have

$f^{-1}(y) = \sqrt[3]{y-5}$

Since y is a dummy variable, so replacing y by x, we get

$f^{-1}(x) = \sqrt[3]{x-5}$

43. 2075 Set A Q.No. 11

Define domain and range of a function. Find the domain and range of the function: [6]

a. $f(x) = 5 - (x+3)^2$

b. $f(x) = \frac{x}{|x|}$.

SOLUTION

First Part: Please refer to 2076 Set C Q.No. 11

Second part

a. Here, $f(x) = 5 - (x+3)^2$.

Let $y = f(x) = 5 - (x+3)^2$.

The given function is a polynomial of degree two in x.

So, y is defined for all $x \in \mathbb{R} = (-\infty, \infty)$.

Hence, domain of $f = \mathbb{R} = (-\infty, \infty)$.

Again, $y = 5 - (x+3)^2$

or, $(x+3)^2 = 5 - y$

Since $(x+3)^2 \geq 0$, so we have $5 - y \geq 0$

$\Rightarrow 5 \geq y$

$\Rightarrow y \leq 5$.

$\therefore$ Range = $(-\infty, 5]$.

b. Let $y = f(x) = \frac{x}{|x|}$

Here, y is not defined when $x = 0$.

Hence, domain of $f = \mathbb{R} - \{0\}$

Also, by definition of absolute value

$$|x| = \begin{cases} x \text{ if } & x \geq 0 \\ -x \text{ if } & x < 0. \end{cases}$$

So, for $x \geq 0$, $y = \frac{x}{|x|} = \frac{x}{x} = 1$

And, for $x < 0$, $y = \frac{x}{|x|} = \frac{x}{-x} = -1$.

$\therefore$ Range = $\{-1, 1\}$

44. 2075 Set B Q.No. 11

Define function, domain of a function, and range of a function. Show that $f(x) = 2x + 3$ is bijective ($f : R \to R$). Also find f^{-1} (2). [6]

SOLUTION

Function: A function f from a set A to set B is a rule which associates each elements of A with a unique element of B.

Domain of a function: If $f : A \to B$ be a function then the set A is called domain of f. It is denoted by dom f. Thus,

dom (f) = $\{x : x \in A\}$.

Range of a function: Let $f : A \to B$ be a function. Then, the set of values of $y = f(x) \in B$ for every $x \in A$ is known as the range of the function f. It is denoted by range (f). Thus,

range (f) = $\{y : y \in B, y = f(x)$ for all $x \in A\}$

f is one to one (injective)

Let $x_1, x_2 \in R$.

Then $f(x_1) = 2x_1 + 3$ and $f(x_2) = 2x_2 + 3$

Now, $f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is injective

f is onto (surjective)

Let $y \in R$. Then,

$y = f(x) = 2x + 3$

or, $2x = y - 3$

or, $x = \frac{y-3}{2} \in R$ for all $y \in R$

and $f\left(\frac{y-3}{2}\right) = 2\left(\frac{y-3}{2}\right) + 3 = y$

$\therefore$ y is the image of $\frac{y-3}{2}$

$\therefore$ f is onto (surjective).

Hence, f is bijective.

Let $y = f(x) = 2x + 3$

or, $y = f(x)$

$\Rightarrow x = f^{-1}(y)$ ... (i)

Also, $y = 2x + 3$

or, $y - 3 = 2x$

or, $\frac{y-3}{2} = x$ ... (ii)

From (i) and (ii), we get

$f^{-1}(y) = \frac{y-3}{2}$

Since y is dummy, replacing y by x.

$f^{-1}(x) = \frac{y-3}{2}$

When $x = 2$, $f^{-1}(2) = \frac{2-3}{2} = -\frac{1}{2}$

45. 2075 Set C Q.No. 11

Find the domain and range of the function $f(x) = \sqrt{2-x-x^2}$. [6]

SOLUTION

We have,

$$y = \sqrt{2-x-x^2}$$
$$= \sqrt{2-(x+x^2)}$$
$$= \sqrt{2-\left\{x^2+2\cdot x\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right\}}$$
$$= \sqrt{2-\left(x+\frac{1}{2}\right)^2+\frac{1}{4}}$$
$$= \sqrt{\frac{9}{4}-\left(x+\frac{1}{2}\right)^2}$$

Now, $y \geq 0$, for $\frac{9}{4}-\left(x+\frac{1}{2}\right)^2 \geq 0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 \leq \frac{9}{4}$

$\Rightarrow \left|x+\frac{1}{2}\right|^2 \leq \left(\frac{3}{2}\right)^2$

$\Rightarrow \left|x+\frac{1}{2}\right| \leq \frac{3}{2}$

$\Rightarrow -\frac{3}{2} \leq x+\frac{1}{2} \leq \frac{3}{2}$

$\Rightarrow -2 \leq x \leq 1$

Domain of the function is $[-2, 1]$.

Again, $\frac{9}{4}-\left(x+\frac{1}{2}\right)^2 = y^2$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 \leq \frac{9}{4}-y^2$

As $\left(x+\frac{1}{2}\right)^2 \geq 0$, for all $x \in \mathbb{R}$,

$\frac{9}{4}-y^2 \geq 0 \Rightarrow y^2 \leq \frac{9}{4}$.

Since y cannot be negative, $0 \leq y \leq \frac{3}{2}$. Hence range of $f = \left[0, \frac{3}{2}\right]$.

46. 2074 Supp Q.No. 11

Show that f: R→R defined by f(x) = cx + d where c (c ≠ 0) and d are real numbers is one to one and onto. Also find f^{-1}. [6]

SOLUTION

Here,

$f(x) = cx + d$ $(c \neq 0)$

One to one

Let $x_1, x_2 \in R$ (domain).

Then $f(x_1) = cx_1 + d$ and $f(x_2) = cx_2 + d$.

Now, $f(x_1) = f(x_2) \Rightarrow cx_1 + d = cx_2 + d$

$\Rightarrow cx_1 = cx_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one.

Onto

Let $y \in R$ (Range). Then,

$y = f(x) = cx + d$

or, $y - d = cx$

or, $x = \frac{y-d}{c} \in R$ (domain) for all $y \in R$ (Range).

Also, $f(x) = f\left(\frac{y-d}{c}\right) = c\left(\frac{y-d}{c}\right) + d = y$.

$\therefore$ f is onto.

To find f^{-1}

Let $y = f(x) = cx + d$.

or, $x = f^{-1}(y)$ ... (i)

Again, $y = cx + d$

or, $y - d = cx$

or, $x = \frac{y-d}{c}$ ... (ii)

From (i) and (ii), $f^{-1}(y) = \frac{y-d}{c}$

Since y is dummy variable, replacing y by x, we get,

$f^{-1}(x) = \frac{x-d}{c}$.

47. 2074 Set A Q.No. 11

Show that f : R → R defined by f(x) = cx + d where c(≠0) and d are real numbers, is one one and onto. Find $f^{-1}(x)$.

➥ Please refer to 2074 Supp Q.No. 11

48. 2074 Set B Q.No. 11

Let f: R → R and g: R → R be defined f(x) = x^3 + 2 and g(x) = 4x – 1. Find (fog) (x) and (gof) (x). Is (fog) (x) = (gof) (x)? Are (fog) and (gof) (x) one to one?

SOLUTION

Here,

$f(x) = x^3 + 2$

$g(x) = 4x - 1$

Now, $fog(x) = f[g(x)]$
$= f(4x-1)$
$= (4x-1)^3 + 2$

$gof(x) = g[f(x)] = g(x^3+2)$
$= 4(x^3+2) - 1$
$= 4x^3 + 8 - 1 = 4x^3 + 7$

Clearly $fog(x) \neq gof(x)$

For one to one of fog (x)

Let $x_1, x_2 \in R$. Then,

$fog(x_1) = (4x_1 - 1)^3 + 2$

and $fog(x_2) = (4x_2 - 1)^3 + 2$

Now, $fog(x_1) = fog(x_2) \Rightarrow (4x_1-1)^3 + 2 = (4x_2-1)^3 + 2$

$\Rightarrow (4x_1-1)^3 = (4x_2-1)^3$

$\Rightarrow 4x_1 - 1 = 4x_2 - 1$

$\Rightarrow 4x_1 = 4x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ fog(x) is one to one.

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Again, for one to one of gof(x)

Let $x_1, x_2 \in R$. Then, $gof(x_1) = 4x_1^3 + 7$ & $gof(x_2) = 4x_2^3 + 7$

Now, $gof(x_1) = gof(x_2) \Rightarrow 4x_1^3 + 7 = 4x_2^3 + 7$

$\Rightarrow 4x_1^3 = 4x_2^3$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ gof(x) is one to one.

49. 2073 Supp Q.No. 11

Define composite functions of two functions f and g. Let f: R → R and g: R → R be defined by $f(x) = 3x^2 - 4$ and $g(x) = 2x-5$, find (fog) (x) and (gof) (x). Are the functions (fog) (x) and (gof) (x) one to one? Give reasons. [6]

SOLUTION

Composite function: If f: A→B and g: B →C be any two functions then the composite of f and g, denoted by gof, is defined by gof(x)=g(f(x)) for all x∈A.

Next part:

$f(x) = 3x^2 - 4$

$g(x) = 2x - 5$

Now, $fog(x) = f(g(x)) = f(2x-5)$

$= 3(2x-5)^2 - 4$

$= 3(4x^2 - 20x + 25) - 4$

$= 12x^2 - 60x + 75 - 4$

$= 12x^2 - 60x + 71$

and $gof(x) = g(f(x)) = g(3x^2 - 4)$

$= 2(3x^2 - 4) - 5$

$= 6x^2 - 8 - 5$

$= 6x^2 - 13$

To check (fog) (x) for one to one.

Let $x_1, x_2 \in R$. Then

$fog(x_1) = 12x_1^2 - 60x_1 + 71$

and $fog(x_2) = 12x_2^2 - 60x_2 + 71$

Now, $(fog)(x_1) = (fog)(x_2)$

$\Rightarrow 12x_1^2 - 60x_1 + 71 = 12x_2^2 - 60x_2 + 71$

$\Rightarrow 12x_1^2 - 12x_2^2 - 60x_1 + 60x_2 = 0$

$\Rightarrow x_1^2 - x_2^2 - 5x_1 + 5x_2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) - 5(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 - 5) = 0$

Thus, $x_1 + x_2 - 5 = 0$

$x_1 + x_2 = 5$

Here, we can choose 2, 3 as two distinct element in domain as $2 + 3 = 5$ but

$(fog)(2) = 12 \times 2^2 - 60 \times 2 + 71 = -1$

$(fog)(3) = 12 \times 3^2 - 60 \times 3 + 71 = -1$

Since 2 and 3 are distinct elements in domain but their images is same. So, (fog) (x) is not one to one.

[Note: We can choose other distinct x_1 & x_2 so that $x_1 + x_2 = 5$]

Next for (gof) (x)

Let $x_1, x_2 \in R$. Then, $gof(x_1) = 6x_1^2 - 13$ and $gof(x_2) = 6x_2^2 - 13$.

Now, $(gof)(x_1) = (gof)(x_2) \Rightarrow 6x_1^2 - 13 = 6x_2^2 - 13$

$\Rightarrow 6x_1^2 = 6x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

Thus, $x_1 + x_2 = 0$

Here, we can take −1 & 1 as distinct elements of domain as $-1 + 1 = 0$ but,

$gof(-1) = 6 \times (-1)^2 - 13 = 6 - 13 = -7$

$gof(1) = 6 \times 1^2 - 13 = 6 - 13 = -7$

Since −1 and 1 are distinct elements in domain but their images are same. So gof is not one to one.

50. 2073 Set C Q.No. 11

Let the function $f(x) = x^3$ and $g(x) = \sin x$, $x \in IR$. Find f_og and g_of. Is $f_og = g_of$? Examine whether f is one to one and onto or not. [6]

SOLUTION

Here, $f(x) = x^3$

$g(x) = \sin x$

Now, $fog(x) = f(g(x))$

$= f(\sin x) = (\sin x)^3 = \sin^3 x$

and $gof(x) = g(f(x))$

$= g(x^3) = \sin x^3$

$\therefore fog(x) \neq gof(x)$

f is one to one

Let $x_1, x_2 \in R$. Then, $f(x_1) = x_1^3$ and $f(x_2) = x_2^3$

Now, $f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one.

f is onto

Let $y = f(x) = x^3$

or, $x = \sqrt[3]{y} \in R$, for all $y \in \Re$

Also, $f(x) = f(\sqrt[3]{y}) = (\sqrt[3]{y})^3 = y$

$\therefore$ f is onto.

51. 2073 Set D Q.No. 11

Define function. Also define one-one onto and one-one into function. Show that the function f: [1, 4]→ R defined by $f(x) = x^2$ is one-one but not onto. [6]

SOLUTION

Function: Please refer to 2075 Set B Q.No. 11

Second part: Please refer to 2057 Q.No. 1b

Third part: Here, $f: [1, 4] \to R$ be defined by $f(x) = x^2$

One-One

Let $x_1, x_2 \in [1, 4]$. Then, $f(x_1) = x_1^2$ and $f(x_2) = x_2^2$

Now, $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$, for all $x_1, x_2 \in [1, 4]$

$\therefore$ f is one-one.

Onto

Consider $100 \in R$, then,

$y = 100 = f(x) = x^2$

or, $100 = x^2$

$\therefore x = \pm 10 \notin [1, 4]$

So, f is not onto.

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Alternatively, for onto

$f(1) = 1^2 = 1$

and $f(4) = 4^2 = 16$

Hence, range of f = [1, 16] ≠ codomain of f.

So, f is onto.

52. 2072 Supp. Q.No. 11

Define one to one function and the onto function. Let a function $f: A \to B$ be defined by $f(x) = \frac{x+1}{2x-1}$ with A = {–1,0,1,2,3,4} and B = {–1, 0, $\frac{4}{5}$, $\frac{5}{7}$, 1, 2, 3}. Find the range of f. Is the function f one to one and onto both? [6]

SOLUTION

First part: Please refer to 2057 Q.No. 1b

Next part

Here, f : A → B is defined by $f(x) = \frac{x+1}{2x-1}$

Where, A = {–1, 0 , 1, 2, 3, 4}

and B = {–1, 0, 4/5. 5/7, 1, 2, 3,}

When $x = -1$, $f(-1) = \frac{-1+1}{2(-1)-1} = 0$

When $x = 0$, $f(0) = \frac{0+1}{2(0)-1} = -1$

When $x = 1$, $f(1) = \frac{1+1}{2(1)-1} = 2$

When $x = 2$, $f(2) = \frac{2+1}{2(2)-1} = 1$

When $x = 3$, $f(3) = \frac{3+1}{2(3)-1} = \frac{4}{5}$

When $x = 4$, $f(4) = \frac{4+1}{2(4)-1} = \frac{5}{7}$

Range of f = {–1, 0, 4/5, 5/7, 1, 2}

Since every distinct elements in A have distinct images in B, f is one to one.

And, f is not onto because 3 ∈ B doesn't have any pre-image in A.

53. 2072 Set C Q.No. 11

Let a function f : A → B be defined by $f(x) = \frac{x-1}{x+2}$ with A = {–1, 0, 1, 2, 3, 4} and B = $\left\{-2, 1, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$. Find the range of f. Is the function f one to one and onto both? If not, how can you make it one to one and onto both? [6]

SOLUTION

Here, the function f: A → B is defined by $f(x) = \frac{x-1}{x+2}$ and

A = {–1, 0, 1, 2, 3, 4} and B = $\left\{-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$

when $x = -1$, $f(-1) = \frac{-1-1}{-1+2} = -2$

when $x = 0$, $f(0) = \frac{0-1}{0+2} = -\frac{1}{2}$

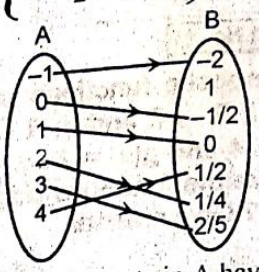
when $x = 1$, $f(1) = \frac{1-1}{1+2} = 0$

when $x = 2$, $f(2) = \frac{2-1}{2+2} = \frac{1}{4}$

when $x = 3$, $f(3) = \frac{3-1}{3+2} = \frac{2}{5}$

when $x = 4$, $f(4) = \frac{4-1}{4+2} = \frac{1}{2}$

Range of f = $\left\{-2, -\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{1}{2}\right\}$



Since the different elements in A have different image in B, so f is one to one.

Again, the element 1 in B has no pre-image in A, i f(A) ≠ B. So, f is not onto.

The function f can be made both one to one and on when 1 is removed from B.

i.e. $f: A \to B - \{1\}$ defined by $f(x) = \frac{x-1}{x+2}$ is both one one and onto.

54. 2072 Set D Q.No. 11

Define composite function. Let f : R → R b defined by f(x) = 2x and g: R → R be define by g(x) = x + 1. Find ($g_o f$) (x), ($f_o g$) (x), ($f_o f$) (and ($g_o g$) (x). Are the functions ($g_o f$) (x) an ($f_o g$) (x) one to one? [6

SOLUTION

Composite function: Please refer to 2073 Supp Q.No. 1

Next Part

f(x) = 2x

g(x) = x + 1

Now, (gof) (x) = g(f(x)) = g(2x) = 2x + 1

(fog) (x) = f(g(x) = f(x + 1) = 2(x + 1) = 2x + 2

(fof) (x) = f(f(x)) = f(2x) = 2(2x) = 4x

(gog) (x) = g(g(x)) = g(x + 1) = (x+1) + 1 = x + 2

For (gof) (x)

Let x_1 and $x_2 \in R$. Then (gof) $(x_1) = 2x_1 + 1$ and (go $(x_2) = 2x_2 + 1$

Now, (gof) (x_1) = (gof) (x_2)

$\Rightarrow$ $2x_1 + 1 = 2x_2 + 1$

$\Rightarrow$ $2x_1 = 2x_2$

$\Rightarrow$ $x_1 = x_2$

∴ gof is one to one.

Again, for (fog) (x)

(fog) $(x_1) = 2x_1 + 2$ and (fog) $(x_2) = 2x_2 + 2$

Now, (fog) (x_1) = (fog) (x_2)

$\Rightarrow$ $2x_1 + 2 = 2x_2 + 2$

$\Rightarrow$ $2x_1 = 2x_2$

$\Rightarrow$ $x_1 = x_2$

∴ fog is one to one.

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55. 2072 Set E Q.No. 11

What condition makes a function to have its inverse? Show that f : IR – {2} → IR – {0} given by $f(x) = \frac{1}{x-2}$ is bijective. Also, find f^{-1}. [6]

SOLUTION

If a function f: A → B is one to one and onto i.e. bijective then f^{-1} exists.

Next Part

Let $x_1, x_2 \in$ IR – {2}. Then,

$$f(x_1) = \frac{1}{x_1-2} \text{ and } f(x_2) = \frac{1}{x_2-2}$$

Suppose $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1-2} = \frac{1}{x_2-2}$

$\Rightarrow x_2 - 2 = x_1 - 2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one or injective.

Again, let $y \in$ IR – {0}. Then,

$$f(x) = y = \frac{1}{x-2}$$

or, $y = \frac{1}{x-2}$

or, $y(x-2) = 1$

or, $x - 2 = \frac{1}{y}$

or, $x = \frac{1}{y} + 2$

or, $x = \frac{1+2y}{y} \in$ IR – {2} as $y \neq 0$.

and $f\left(\frac{1+2y}{y}\right) = \frac{1}{\frac{1+2y}{y} - 2} = y$

$\therefore$ y is the image of $\frac{1+2y}{y}$

$\therefore$ f is onto (surjective).

Hence, f is bijective.

For f^{-1}

Let $y = f(x) = \frac{1}{x-2}$

$\Rightarrow x = f^{-1}(y)$ ...(i)

Again, $y = \frac{1}{x-2}$

$\Rightarrow x - 2 = \frac{1}{y}$

$\Rightarrow x = \frac{1}{y} + 2$

$\Rightarrow x = \frac{1+2y}{y}$ ...(ii)

From (i) and (ii),

$f^{-1}(y) = \frac{1+2y}{y}$

Since y is dummy variable, replacing y by x, we get

$f^{-1}(x) = \frac{2x+1}{x}$

56. 2071 Supp. Q.No. 11

Define bijective function. Prove that f: IR→IR defined by f(x) = 2x+3 is bijective. [6]

SOLUTION

Bijective function: A function f: A → B is said to be bijective if it is both one to one (injective) and onto (surjective).

f is one to one (injective)

Let $x_1, x_2 \in$ R. Then $f(x_1) = 2x_1 + 3$ and $f(x_2) = 2x_2 + 3$

Now, $f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is injective

f is onto (surjective)

Let $y \in$ R. Then,

$y = f(x) = 2x + 3$

or, $2x = y - 3$

or, $x = \frac{y-3}{2} \in$ R for all $y \in$ R

and $f\left(\frac{y-3}{2}\right) = 2\left(\frac{y-3}{2}\right) + 3 = y$

$\therefore$ y is the image of $\frac{y-3}{2}$

$\therefore$ f is onto (surjective).

Hence, f is bijective.

57. 2071 Set C Q.No. 11

Define inverse function. If f:R→R be defined by $f(x) = x^2 - 3$, find $f^{-1}(x)$. Also determine whether $f_0f^{-1}(x) = f^{-1}{}_0f(x)$. [6]

SOLUTION

Inverse function: When a function f: A → B is bijective, then there exists a function f^{-1} : B → A called the inverse function of f.

Next Part

The f^{-1} exists if f is one to one and onto.

For one to one:

Consider –2, 2 ∈ ℝ (domain). Then,

$f(-2) = (-2)^2 - 3 = 4 - 3 = 1$

$f(2) = 2^2 - 3 = 4 - 3 = 1$

Thus, f(–2) = f(2) but $-2 \neq 2$.

So, f is not one to one.

Hence, f is not bijective and f^{-1} does not exist.

Since f^{-1} does not exist, so last part cannot be checked.

58. 2071 Set D Q.No. 11

Prove that the function f:ℜ–{3} →ℜ – {1} given by $f(x) = \frac{x}{x-3}$ is bijective. [6]

SOLUTION

Given the function f: R–{3} → R – {1} is defined by

$f(x) = \frac{x}{x-3}$.

f is injective (one to one)

Let $x_1, x_2 \in$ R –{3}.

Then, $f(x_1) = \frac{x_1}{x_1-3}$ and $f(x_2) = \frac{x_2}{x_2-3}$

Now, $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1-3} = \frac{x_2}{x_2-3}$

$\Rightarrow x_1x_2 - 3x_1 = x_1x_2 - 3x_2$

$\Rightarrow -3x_1 = -3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is injective (one to one)

f is surjective (onto)

Let $y \in R - \{1\}$. Then, $y = f(x) = \frac{x}{x-3}$

$\Rightarrow xy - 3y = x$

$\Rightarrow xy - x = 3y$

$\Rightarrow x(y-1) = 3y$

$\Rightarrow x = \frac{3y}{y-1} \in R - \{3\}$ as $y \neq 1$

and $f\left(\frac{3y}{y-1}\right) = \frac{\frac{3y}{y-1}}{\frac{3y}{y-1} - 3}$

$= \frac{\frac{3y}{y-1}}{\frac{3y - 3y + 3}{y-1}} = \frac{3y}{3} = y$

$\therefore$ y is the image of $\frac{3y}{y-1}$

$\therefore$ f is surjective (onto)

Hence, $f(x) = \frac{x}{x-3}$ is bijective.

59. 2070 Supp Q.No. 11

Define composite function of two functions. If $f: R \to R$ be defined by $f(x) = \frac{x-1}{x+1}$, $(x \neq -1)$ show that $(fof)(4) = -\frac{1}{4}$. [6]

SOLUTION

Composite function: Please refer to 2073 Supp Q.No. 11

Next part:

Here, $f(x) = \frac{x-1}{x+1}$

Now, $f_0f(x) = f[f(x)]$

$= f\left(\frac{x-1}{x+1}\right)$

$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$

$= \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}} = \frac{-2}{2x} = -\frac{1}{x}$

When $x = 4$, $f_0f(4) = -\frac{1}{4}$

60. 2070 Set C Q.No. 11

Let function $f: A \to B$ be defined by $f(x) = \frac{x+1}{2x-1}$. Find the range of f. Is the function f one to one and onto both? If not, how can the function be made one to one and onto both? [6]

SOLUTION

Here, $f(x) = \frac{x+1}{2x-1}$

Let $y = f(x) = \frac{x+1}{2x-1}$

or, $y = \frac{x+1}{2x-1}$

or, $2xy - y = x + 1$

or, $2xy - x = y + 1$

or, $x(2y-1) = y + 1$

or, $x = \frac{y+1}{2y-1}$ which is not defined for $2y - 1 = 0$

i.e. $y = \frac{1}{2}$

$\therefore$ Range of $f = R - \left\{\frac{1}{2}\right\}$

One to One

Let x_1 and x_2 be any two point of domain. Then,

$f(x_1) = \frac{x_1+1}{2x_1-1}$ and $f(x_2) = \frac{x_2+1}{2x_2-1}$

Now, $f(x_1) = f(x_2) \Rightarrow \frac{x_1+1}{2x_1-1} = \frac{x_2+1}{2x_2-1}$

$\Rightarrow (x_1+1)(2x_2-1) = (x_2+1)(2x_1-1)$

$\Rightarrow 2x_1x_2 - x_1 + 2x_2 - 1 = 2x_1x_2 - x_2 + 2x_1 -$

$\Rightarrow -x_1 + 2x_2 = -x_2 + 2x_1$

$\Rightarrow -3x_1 = -3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one

Onto

Here, $y = \frac{1}{2}$ is not in the range (image) set i.e. y

the co-domain has no pre image in the domain. S

not onto.

The function f can be made onto by removing

the co-domain,

Hence, f will be onto when $B = R - \left\{\frac{1}{2}\right\}$

61. 2070 Set D Q.No. 11

Define composite function of two functions g. Let $f: R \to R$ and $g: R \to R$ be two func defined by $f(x) = 2x^2 - 3$ and $g(x) = 3x$ Determine (fog) (x), (gof) (x) and (gog) ((fog) (x) = (gof) (x)? Are the function (fog) ((gog) (x) one to one?

SOLUTION

Composite function: Please refer to 2073 Supp Q.

Next part:

Given $f(x) = 2x^2 - 3$

$g(x) = 3x + 2$

Here, $(fog)(x) = f[g(x)]$

$= f(3x+2)$

$= 2(3x+2)^2 - 3$

$= 2(9x^2 + 12x + 4) - 3$

$= 18x^2 + 24x + 8 - 3$

$= 18x^2 + 24x + 5$

$(gof)(x) = g(f(x))$

$= g(2x^2 - 3)$

$= 3(2x^2 - 3) + 2$

$= 6x^2 - 9 + 2$

$= 6x^2 - 7$

$\therefore (fog)(x) \neq (gof)(x)$

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To check one – one for (fog) (x)

Let $(fog)(x_1) = (fog)(x_2)$, $x_1, x_2 \in R$

$18x_1^2 + 24x_1 + 5 = 18x_2^2 + 24x_2 + 5$

or, $18x_1^2 - 18x_2^2 + 24x_1 - 24x_2 = 0$

or, $3x_1^2 - 3x_2^2 + 4x_1 - 4x_2 = 0$

or, $3(x_1 - x_2)(x_1 + x_2) + 4(x_1 - x_2) = 0$

or, $(x_1 - x_2)(3x_1 + 3x_2 + 4) = 0$

Thus, $3x_1 + 3x_2 + 4 = 0$

or, $x_1 + x_2 = \frac{-4}{3}$

Here, $-\frac{4}{3}$, 0 are two different elements in domain but

$$(fog)\left(-\frac{4}{3}\right) = 18\left(-\frac{4}{3}\right)^2 + 24\left(-\frac{4}{3}\right) + 5$$

$$= 18\left(\frac{16}{9}\right) - \frac{96}{3} + 5$$

$$= 32 - 32 + 5 = 5$$

and $(fog)(0) = 18 \times 0^2 + 24 \times 0 + 5 = 5$

Since $-\frac{4}{3}$ and 0 are different elements in domain but their images is same. So $(fog)(x)$ is not one–one

Remaining part:

$g_0g(x) = g(g(x)) = g(3x + 2)$

$= 3(3x + 2) + 2$

$= 9x + 6 + 2$

$= 9x + 8$

Let $x_1, x_2 \in R$. Then,

$g_0g(x_1) = 9x_1 + 8$

and

$g_0g(x_2) = 9x_2 + 8$

Now,

$g_0g(x_1) = g_0g(x_2)$

$\Rightarrow 9x_1 + 8 = 9x_2 + 8$

$\Rightarrow 9x_1 = 9x_2$

$\Rightarrow x_1 = x_2$

$\therefore g_0g(x)$ is one to one.

62. 2069 Supp Q.No. 11

Define domain and range of a function. Find the domain and range of the function $y = x^2 - 4x + 3$ for $x \in R$ **[6]**

SOLUTION

First Part: Please refer to 2076 Set C Q.No. 11

Next Part

Here, $y = x^2 - 4x + 3$

The given function is a polynomial of degree 2 in x. So, y is defined for all $x \in R$. Hence, domain of $f = dom(f) = R = (-\infty, \infty)$

Again, $y = x^2 - 4x + 3$

or, $y - 3 = x^2 - 4x$

or, $y - 3 = x^2 - 4x + 4 - 4$

or, $y - 3 = (x - 2)^2 - 4$

or, $y + 1 = (x - 2)^2$

Since $(x - 2)^2 \geq 0$, so for all $x \in R$, $y + 1 \geq 0 \Rightarrow y \geq -1$

$\therefore$ range of $f = [-1, \infty)$

63. 2069 [Set A] Q. No. 11

Define one to one function and onto function.

Let a function $f: A \to B$ be defined by $f(x) = \frac{x^2}{6}$ with $A = \{-2, -1, 0, 1, 2\}$ and $B = \left\{0, \frac{1}{6}, \frac{2}{3}\right\}$ Find the range of f. Is the function f one to one and onto both? **[6]**

SOLUTION

Definition: Please refer to 2057 Q.No. 1b

Next Part

Here, $f(x) = \frac{x^2}{6}$

$A = \{-2, -1, 0, 1, 2\}$

$B = \left\{0, \frac{1}{6}, \frac{2}{3}\right\}$

when $x = -2$, $f(-2) = \frac{(-2)^2}{6} = \frac{4}{6} = \frac{2}{3}$

when $x = -1$, $f(-1) = \frac{(-1)^2}{6} = \frac{1}{6}$

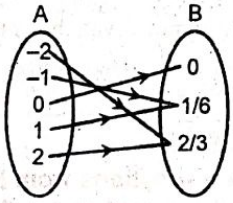
when $x = 0$, $f(0) = \frac{0^2}{6} = 0$

when $x = 1$, $f(1) = \frac{1^2}{6} = \frac{1}{6}$

when $x = 2$, $f(2) = \frac{2^2}{6} = \frac{2}{3}$

Range of $f = \left\{0, \frac{1}{6}, \frac{2}{3}\right\}$

Arrow diagram



Since range of f = B i.e. every elements in B has at least one pre–image in A, so f is onto.

Here, $-2 \neq 2$ but $f(-2) = \frac{2}{3} = f(2)$

So, f is not one to one.

64. 2069 [Set B] Q. No. 11

Define function. State the condition for a function to be bijective.

Given, $f(x) = x^3 + 5$, $x \in R$, find f^{-1}. **[6]**

SOLUTION

Function: Please refer to 2076 Set B Q.No. 11

Next part: A function f is bijective if it is both one to one (injective) and onto (surjective).

Last Part: Please refer to 2075 GIE Q.No. 11

65. 2068 Q.No. 11

Define the domain and the range of a function. Find the domain and the range of the function $f(x) = -x^2 + 4x - 3$. **[6]**

SOLUTION

First part: Please refer to 2075 Set A Q.No. 11

Next Part

Let $y = f(x) = -x^2 + 4x - 3$

The given function is a polynomial of degree 2 in x. So, y is defined for all $x \in R$.

So, domain of f = dom(f) = R = $(-\infty, \infty)$

Again, $y = -x^2 + 4x - 3$

or, $-y = x^2 - 4x + 3$

or, $-y = x^2 - 4x + 4 - 4 + 3$

or, $-y = (x-2)^2 - 1$

or, $1 - y = (x-2)^2$

Since $(x-2)^2 \geq 0$, so for all $x \in IR$,

$1 - y \geq 0$

$\Rightarrow 1 \geq y$

$\Rightarrow y \leq 1$

$\therefore$ Range of f = $(-\infty, 1]$

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

66. 2071 Old Q.No. 7b

Let $f: R \to R$ be defined by $y = f(x) = 2x - 3$, $x \in R$. Find a formula that defines the inverse function f^{-1}. [4]

SOLUTION

Let $y = f(x) = 2x - 3$

$\Rightarrow x = f^{-1}(y)$ ...(i)

And, $y = 2x - 3$

$\Rightarrow y + 3 = 2x$

$\Rightarrow x = \frac{y+3}{2}$ ...(ii)

From (i) and (ii), $f^{-1}(y) = \frac{y+3}{2}$

Since y is dummy variable, replacing y by x, we get $f^{-1}(x) = \frac{x+3}{2}$

67. 2070 Old Q.No. 7 b

If f and g are functions from R → R defined by $f(x) = x^2 - x + 1$ and $g(x) = 2x + 3$, find (fog)x and (gof)x. [4]

SOLUTION

Here, $f(x) = x^2 - x + 1$

$g(x) = 2x + 3$

Now, $(fog)(x) = f(g(x))$

$= f(2x+3)$

$= (2x+3)^2 - (2x+3) + 1$

$= 4x^2 + 12x + 9 - 2x - 3 + 1$

$= 4x^2 + 10x + 7$

And, $gof(x) = g(f(x))$

$= g(x^2 - x + 1)$

$= 2(x^2 - x + 1) + 3$

$= 2x^2 - 2x + 2 + 3$

$= 2x^2 - 2x + 5$

68. 2069 [Set A] Old Q. No. 7b

Let $f: R \to R$ and $g: R \to R$ be defined by $f(x) = 2x^2 - 3$, $g(x) = 3x + 2$. Determine the following composite functions: (i) (f 0 g) (x) (ii) (g 0 f) (x). [4]

SOLUTION

Given $f(x) = 2x^2 - 3$

$g(x) = 3x + 2$

Here, $(fog)(x) = f[g(x)]$

$= f(3x+2)$

$= 2(3x+2)^2 - 3$

$= 2(9x^2 + 12x + 4) - 3$

$= 18x^2 + 24x + 8 - 3$

$= 18x^2 + 24x + 5$

$(gof)(x) = g(f(x))$

$= g(2x^2 - 3)$

$= 3(2x^2 - 3) + 2$

$= 6x^2 - 9 + 2 = 6x^2 - 7$

69. 2069 [Set B] Old Q. No. 7a

If $A = \{x: x \in N \text{ and } x < 4\}$ and $B = \{x: x \in N$ a $x^2 - 9 = 0\}$, find A × B, B × A, A × A, B × B.

SOLUTION

Here, $A = \{x: x \in N \text{ and } x < 4\}$

$= \{1, 2, 3\}$

and $B = \{x: x \in N \text{ and } x^2 - 9 = 0\}$

$= \{3\}$

$A \times B = \{1, 2, 3\} \times \{3\} = \{(1, 3), (2, 3), (3, 3)\}$

$B \times A = \{3\} \times \{1, 2, 3\} = \{(3, 1), (3, 2), (3, 3)\}$

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, (3, 1), (3, 2), (3, 3)\}$

$B \times B = \{3\} \times \{3\} = \{(3, 3)\}$

70. 2069 [Set B] Old Q. No. 7b

If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that: $x^x y^y z^z = 1$

SOLUTION

Let $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

Then, $\log x = k(y-z)$, $\log y = k(z-x)$ and $\log z = k(x-y)$

Now,

$x \log x + y \log y + z \log z = x \cdot k(y-z) + y \cdot k(z-x) + z \cdot k(x-y)$

or, $\log x^x + \log y^y + \log z^z = kxy - kzx + kyz - kxy + kzx - kyz$

or, $\log(x^x \cdot y^y \cdot z^z) = 0 = \log 1$

$\therefore x^x \cdot y^y \cdot z^z = 1$

71. 2068 Old Q.No. 7b

Let R be the set of real numbers. Show that f functions f: R→R defined by $f(x) = 3x - 2$ for $x \in R$ is one to one and onto. Also find f^{-1}.

SOLUTION

Here, $f(x) = 3x - 2$

f is one to one

Let $x_1, x_2 \in R$ (domain). Then, $f(x_1) = 3x_1 - 2$ $f(x_2) = 3x_2 - 2$

Now, $f(x_1) = f(x_2) \Rightarrow 3x_1 - 2 = 3x_2 - 2$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one to one.

f is onto

Let $y \in R$ (domain). Then,

$f(x) = y = 3x - 2$

$\Rightarrow y + 2 = 3x$

$\Rightarrow x = \frac{y+2}{3} \in R$ for all $x \in R$.

and $f\left(\frac{y+2}{3}\right) = 3\left(\frac{y+2}{3}\right) - 2 = y + 2 - 2 = y$

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$\therefore$ y is the image of $\frac{y+2}{3}$

$\therefore$ f is onto.

Hence, f is one to one and onto.

For f^{-1}

Let $y = f(x) = 3x - 2$

$\Rightarrow x = f^{-1}(y)$ ...(i)

and $y = 3x - 2$

$\Rightarrow y + 2 = 3x$

$\Rightarrow x = \frac{y+2}{3}$ ...(ii)

From (i) and (ii), we have

$f^{-1}(y) = \frac{y+2}{3}$

Since y is a dummy variable, so replacing y by x, we have

$f^{-1}(x) = \frac{x+2}{3}$.

72. 2067 Q.No. 7b

For the function $f(x) = 2x^2 - 3$ and $g(x) = 3x + 2$, $x \in R$ examine whether fog and gof are one-one. [4]

SOLUTION

The given functions are

$f(x) = 2x^2 - 3$ ... (i)

$g(x) = 3x + 2$ ... (ii)

Now, $(fog)(x) = f[g(x)] = f(3x+2)$
$= 2(3x+2)^2 - 3$
$= 2(9x^2 + 12x + 4) - 3$
$= 18x^2 + 24x + 8 - 3$
$= 18x^2 + 24x + 5$

And, $(gof)(x) = g[f(x)] = g(2x^2 - 3) = 3(2x^2 - 3) + 2$
$= 6x^2 - 9 + 2$
$= 6x^2 - 7$

Now, let $a, b \in R$

Suppose $(fog)(a) = (fog)(b)$

Then $18a^2 + 24a + 5 = 18b^2 + 24b + 5$

$\Rightarrow 18a^2 - 18b^2 + 24a - 24b = 0$

$\Rightarrow 18(a^2 - b^2) + 24(a - b) = 0$

$\Rightarrow (a - b)[18(a + b) + 24] = 0$

$\Rightarrow$ Either $a - b = 0$ OR $18(a + b) + 24 = 0$

i.e. $a = b$ or, $(a + b) = \frac{-24}{18} = \frac{-4}{3}$

$\Rightarrow a \neq b$

Since for $(fog)(a) = (fog)(b) \Rightarrow a \neq b$, the function (fog) is not one to one.

Again, suppose

$(gof)(a) = (gof)(b)$

$\Rightarrow 6a^2 - 7 = 6b^2 - 7$

$\Rightarrow 6a^2 - 6b^2 = 0$

$\Rightarrow 6(a - b)(a + b) = 0$

$\Rightarrow$ Either $a - b = 0$ OR $a + b = 0$

i.e. $a = b$ or, $a = -b$

$\Rightarrow a \neq b$

Thus for $(gof)(a) = (gof)(b) \Rightarrow a \neq b$, the function (gof) is not one – one.

73. 2066 Q.No. 7b

Let R be the set of real numbers. Show that the function $f: R \to R$ such that $f(x) = 5x - 3$ for all $x \in R$ is one to one and onto. [4]

SOLUTION

Given $f: R \to R$ is defined by $f(x) = 5x - 3$ for all $x \in R$

f is one to one

Let $x_1, x_2 \in R$. Then $f(x_1) = 5x_1 - 3$, $f(x_2) = 5x_2 - 3$

Now, $f(x_1) = f(x_2)$

$\Rightarrow 5x_1 - 3 = 5x_2 - 3$

$\Rightarrow 5x_1 = 5x_2$

$\Rightarrow x_1 = x_2$

Hence f is one to one.

f is onto

Let y be any arbitrary element in R. Then,

$y = f(x) = 5x - 3$

$\Rightarrow 5x = y + 3$

$\Rightarrow x = \frac{y+3}{5}$ which is obviously a real number for each $y \in R$.

And, $f\left(\frac{y+3}{5}\right) = 5\left(\frac{y+3}{5}\right) - 3 = y$

This shows that y is the image of $\frac{y+3}{5}$

$\therefore$ f is onto.

Hence f is one to one and onto.

74. 2065 Q.No. 7b

If $f, g: R \to R$ defined by $f(x) = x^3 + 2$ and $g(x) = 4x - 1$, find $fog(x)$ and $gof(x)$, and show that the composite function is not commutative. [4]

SOLUTION

Given functions are

$f: R \to R$ defined by $f(x) = x^3 + 2$ ... (1)

$g: R \to R$ defined by $g(x) = 4x - 1$ ... (2)

Now,

i. $(fog)(x) = f[g(x)]$
$= f(4x - 1)$
$= (4x - 1)^3 + 2$
$= (4x)^3 - 3(4x)^2 .1 + 3(4x).1^3 - 1^3 + 2$
$= 64x^3 - 48x^2 + 12x - 1 + 2$
$= 64x^3 - 48x^2 + 12x + 1$

ii. $(gof)(x) = g[f(x)]$
$= g(x^3 + 2)$
$= 4(x^3 + 2) - 1$
$= 4x^3 + 8 - 1 = 4x^3 + 7$

From above, it is clear that $(fog)(x) \neq (gof)(x)$.

So, that the composite function is not commutative.

75. 2064 Q.No. 7b

Let Q be the set of all rational numbers. Show that the function: $f: Q \to Q$ such that $f(x) = 3x + 5$ for all $x \in Q$ is one to one and onto. Find f^{-1}. [4]

SOLUTION

The given function $f: Q \to Q$ is defined as $f(x) = 3x + 5$, where Q be the set of all rational numbers.

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i. f is one to one:

Let f (a) = f (b) then

$\Rightarrow 3a + 5 = 3b + 5$

$\Rightarrow 3a = 3b$

$\Rightarrow a = b$

Since for f(a) = f (b) $\Rightarrow$ a = b, f is one to one function.

ii. f is onto function:

Let y $\in$Q (where Q is the co-domain of f) be arbitrarily chosen.

Then, $y = 3x + 5$

or, $y - 5 = 3x$

$x = \frac{y-5}{3} \in Q$ for all $y \in Q$

Thus, for every y $\in$Q, there exists $\frac{y-5}{3} \in Q$ such that

$f\left(\frac{y-5}{3}\right) = 3\left(\frac{y-5}{3}\right) + 5 = y$

So, f is onto function.

iii. To find f^{-1}:

Since f is one to one and onto, f^{-1} exists

Here, $f(x) = 3x + 5$

If y is the image of x under f, then $y = 3x + 5$

Also since one to one and onto function has an inverse, $f^{-1}: Q \to Q$, exists and x will be the image of y under f^{-1}, i.e.

$x = f^{-1}(y)$ ... (i)

Solving for x in terms of y, we have

$y = 3x + 5$

or, $y - 5 = 3x$

or, $x = \frac{y-5}{3}$ ... (ii)

Then $f^{-1}(y) = \frac{y-5}{3}$

Here y is just a dummy variable and can be replaced by x. Thus, $f^{-1}(x) = \frac{x-5}{3}$

---

76. 2063 Q.No. 7b

Let R be the set of rational numbers. Show that the function defined by f: R→ R such that f(x) = 4x – 7, $x \in$ R is one-one and onto. Find a formula for f^{-1}? [4]

SOLUTION

Here f: R→ R is a function defined as f(x) = 4x– 7, where R is the set of rational number

i. f : R→ R is one to one function:

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 - 7 = 4x_2 - 7$

$\Rightarrow 4x_1 = 4x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f (x) is one to one function.

ii. f : R→ R is onto function:

Let, $y \in R$ be arbitrarily chosen such that

$y = 4x - 7$

or, $4x = y + 7$

$\Rightarrow x = \frac{y+7}{4}$

$\therefore$ For each, $y \in R$; there exists $x = \left(\frac{y+7}{4}\right) \in R$, that, $f(x) = y$.

So f (x) is onto function.

Since f: R → R is one-one and onto, f^{-1} exists.

Let $y = f(x)$

$\therefore x = f^{-1}(y)$ ... (i)

Now, $y = f(x) = 4x - 7$

or $y = 4x - 7$

or, $4x = y + 7$

or, $x = \frac{y+7}{4}$ ... (ii)

From (i) and (ii) we have, $f^{-1}(y) = \frac{y+7}{4}$.

Since y is dummy variable, y can be replaced by

$f^{-1}(x) = \frac{x+7}{4}$.

$\therefore$ The inverse function of f(x) is given by

$f^{-1}(x) = \frac{x+7}{4}$

---

77. 2062 Q.No. 7b

When does an inverse of a function exist? $R \to R$ be defined by $f(x) = x^3 + 5$, find formula that defines f^{-1}.

SOLUTION

The inverse of a function exists if it is one to one onto function.

Here, the given function is

$y = f(x) = x^3 + 5$ ... (1)

To find, $f^{-1}(x)$, we interchange the role of x and (1), and the value of y thus obtained is $f^{-1}(x)$.

i.e. $x = y^3 + 5$

or, $y^3 = x - 5$

or, $y = \sqrt[3]{x-5}$

$\therefore f^{-1}(x) = \sqrt[3]{x-5}$

---

78. 2061 Q.No. 7b

For the functions $f(x) = 2x^2 - 3$ and $g(x) = 3x$ where $x \in R$, determine (fog) (x), (gof) (x). Are (fog) and (gof) one–one?

➥ Please refer 2067 Q.No. 7b

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79. 2060 Q.No. 7b

Let f: R → R is defined by f(x) = 2x + 3 g: R → R is defined by $g(x) = x^2$, find (gof) and (fog) (x).

SOLUTION

The given functions are

$f(x) = 2x + 3$

and, $g(x) = x^2$

Now,

i. $(gof)(x) = g[f(x)]$

$= g(2x + 3)$

$= (2x + 3)^2$

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ii. $(fog)(x) = f[g(x)]$
$= f(x^2)$
$= 2(x^2) + 3$
$= 2x^2 + 3$

80. 2059 Q.No. 7b

Let, f: R→ R and g: R→R be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$, find f^{-1}, (gof) (x); (fog) (x). [4]

SOLUTION

We have,
$f(x) = x^2 + 1$ ... (i)
$g(x) = x^5$ ... (ii)
Now, f^{-1} exists, if f is bijective.
For one to one,
Consider $-1, 1 \in \mathbb{R}$ (domain). Then
$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$
and
$f(1) = 1^2 + 1 + 1 + 1 = 2$
Thus, $f(-1) = f(1)$, but $-1 \neq 1$.
So, f is not one to one. Hence, f^{-1} does not exist.
Again, $(gof)(x) = g[f(x)]$
$= g(x^2 + 1)$
$= (x^2 + 1)^5$
and, $(fog)(x) = f[g(x)]$
$= f(x^5)$
$= (x^5)^2 + 1$
$= x^{10} + 1$

81. 2058 Q.No. 7b

Check whether the function $f = [-2, 3] \to R$ given by $f(x) = x^3$ is one to one, onto or both. [4]

SOLUTION

The given function is $f(x) = x^3$
Let $a, b \in [-2, 3]$ such that $a \neq b$
$\Rightarrow a^3 \neq b^3$ [∵ Cubes of distinct real numbers are different]
$\Rightarrow f(a) \neq f(b)$
∴ f is one to one
Again,
f is onto if range of f = codomain of f
But range of f = $[f(-2), f(3)] = [(-2)^3, 3^3]$
= $[-8, 27] \neq$ Co–domain of f, because codomain of f is R
So, f is not onto.
Hence, f is one to one but not onto.

82. 2057 Q.No. 7b

A function f(x) is defined as follows:

$$f(x) = \begin{cases} = 4x - 2 & \text{for } x \geq 1 \\ = 2x & \text{for } x < 1 \end{cases}$$

, find f (2); f(1); f(0); $f(-1), \frac{f(h) - f(1)}{h}$ for $1 \leq h$ [4]

SOLUTION

We have,

$$f(x) = \begin{cases} 4x - 2 & \text{for } x \geq 1 \\ 2x & \text{for } x < 1 \end{cases}$$

Now,
i. **f (2)**
For $x = 2$; $f(x) = 4x - 2$
∴ $f(2) = 4 \times 2 - 2 = 8 - 2 = 6$
ii. **f(1)**
For $x = 1$, $f(x) = 4x - 2$
∴ $f(1) = 4 \times 1 - 2 = 4 - 2 = 2$
iii. **f(0)**
For $x = 0$, $f(x) = 2x$
∴ $f(0) = 2 \times 0 = 0$
iv **f (–1)**
For $x = -1$; $f(x) = 2x$
∴ $f(-1) = 2(-1) = -2$
v. $\frac{f(h) - f(1)}{h}$ **for $1 \leq h$ (i.e. $h \geq 1$)**
$= \frac{(4h - 2) - (4 \times 1 - 2)}{h} = \frac{4h - 2 - 2}{h} = \frac{4(h - 1)}{h}$

83. 2056 Q.No. 7b

Let f: R→ R and g: R→ R be defined by $f(x) = x^2 + 1$; $g(x) = x^5$ find f^{-1}; (gof) (x) and (fog) (x). [4]

➥ Please refer to 2059 Q.No. 7b

84. 2055 Old Model Q.No. 7b

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 4x - 2 & \text{for } x \geq 1 \\ 2x & \text{for } x < 1 \end{cases}$$

Find: (i) f(2) (ii) f(1) (iii) f (– 1) (iv) $\frac{f(h) - f(1)}{h}$ for $1 \leq h$ [4]

SOLUTION

We have,

$$f(x) = \begin{cases} 4x - 2 & \text{for } x \geq 1 \\ 2x & \text{for } x < 1 \end{cases}$$

i. $f(2) = 4 \times 2 - 2$ [∵ 2 belongs to the interval $x \geq 1$]
$= 6$
ii. $f(1) = 4 \times 1 - 2$ [∵ 1 belongs to the interval $x \geq 1$]
$= 2$
iii. $f(-1) = 2 \times (-1)$ [∵ –1 belongs to the interval $x < 1$]
$= -2$
iv. $f(h) = 4h - 2$ as $1 \leq h$
Now,
$\frac{f(h) - f(1)}{h} = \frac{(4h - 2) - 2}{h} = \frac{4h - 4}{h} = \frac{4(h - 1)}{h}$

3 UNIT

CURVE SKETCHING

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2 MARKS QUESTIONS

1. 2076 Set B Q.No. 1c

Test the even or odd nature and symmetricity of the function $y = 8x^2$. [2]

SOLUTION

Let $y = f(x) = 8x^2$

Now,

$f(-x) = 8(-x)^2$

$= 8x^2$

$= f(x)$.

Thus, f(x) is an even function.

Since, $f(-x) = f(x)$, so f(x) is symmetric about y-axis.

2. 2076 Set C Q.No. 1c

State the condition for which a function is symmetric with respect to origin. [2]

SOLUTON

A function $y = f(x)$ is said to be symmetric about the origin if $f(-x) = -f(x)$ for all x in the domain of f i.e. f(x) is an odd function.

3. 2075 GIE Q.No. 1c

Examine even and odd property of $f(x) = \sqrt{x^2 - 1}$. [2]

SOLUTION

Given, $f(x) = \sqrt{x^2 - 1}$

Now, $f(-x) = \sqrt{(-x)^2 - 1} = \sqrt{x^2 - 1} = f(x)$.

$\therefore$ $f(x)$ is an even function.

4. 2075 Set A Q.No. 1c

Test the function $f(x) = x^5 + x^3 + x$ for symmetry. [2]

SOLUTION

Here, $f(x) = x^5 + x^3 + x$

Now, $f(-x) = (-x)^5 + (-x)^3 + (-x)$

$= -x^5 - x^3 - x$

$= -(x^5 + x^3 + x)$

$= -f(x)$

$\therefore$ f(x) is symmetric about origin.

5. 2075 Set B Q.No. 1c

Examine the symmetricity of the function $f(x) = e^x + \frac{1}{e^x}$. [2]

SOLUTION

Given, $f(x) = e^x + \frac{1}{e^x}$

Now, $f(-x) = e^{-x} + \frac{1}{e^{-x}} = \frac{1}{e^x} + e^x = f(x)$

$\therefore$ $f(-x) = f(x)$

Hence the function is symmetric about y-axis.

6. 2075 Set C Q.No. 1c

Examine whether the function $f(x) = \frac{e^x -$ even or odd. Also examine for its symmetri

SOLUTION

Here, $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$f(-x) = \frac{e^{-x} - e^{-(-x)}}{e^{-x} + e^{-(-x)}} = \frac{e^{-x} - e^x}{e^{-x} + e^x} = -\frac{(e^x - e^{-x})}{e^x + e^{-x}} = -f(x)$

So, $f(x)$ is an odd function.

Also, $f(-x) = -f(x)$

So, $f(x)$ is symmetric about origin.

7. 2074 Supp Q.No. 1c

Test the periodicity of the function f(x) = s and find its period.

SOLUTION

Here,

$f(x) = \sin\pi x$

Period $= \frac{2\pi}{\pi} = 2$

Now,

$f(x + 2) = \sin\pi(x + 2) = \sin(2\pi + \pi x)$

$= \sin\pi x$ [$\because \sin(2\pi + \theta) = \sin\theta$]

$= f(x)$

$\therefore$ $f(x) = \sin \pi x$ is the periodic function, its period be

8. 2074 Set A Q.No. 1c

Test the periodicity of the functionf(x) = c and find its period.

SOLUTION

Given $f(x) = \cos\pi x$

Now,

$f(x + 2) = \cos\pi(x + 2)$

$= \cos(2\pi + \pi x)$

$= \cos\pi x = f(x)$

Hence f(x) is periodic and its period is 2.

9. 2074 Set B Q.No. 1c

Show that the function f (x) = 5 · decreasing for all $x \in \mathbb{R}$.

SOLUTION

Let $x_1, x_2 \in R$ and $x_1 > x_2$.

Then,

$x_1 > x_2 \Rightarrow -2x_1 < -2x_2$

$\Rightarrow 5 - 2x_1 < 5 - 2x_2$

$\Rightarrow f(x_1) < f(x_2)$

$\therefore$ $f(x)$ is decreasing function for all $x \in R$.

Alternative method

$f(x) = 5 - 2x$

$f'(x) = -2 < 0$ for all $x \in R$

So, $f(x)$ is decreasing for all $x \in R$.

10. 2073 Supp Q.No. 1c

Examine whether the function $f(x) = x^3 - x$ is even or odd. Also, test for symmetricity. [2]

SOLUTION

Given, $f(x) = x^3 - x$

Now,

$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x = -(x^3 - x)$

$= -f(x)$

So, $f(x)$ is an odd function.

Also, since $f(-x) = -f(x)$, $f(x)$ is symmetric about origin.

11. 2073 Set C Q.No. 1c

Examine the symmetry and even or odd nature of the function $y = x^3$. [2]

SOLUTION

Let $f(x) = y = x^3$

Now, $f(-x) = (-x)^3 = -x^3 = -f(x)$

So, $f(x)$ is an odd function.

Since $f(x)$ is an odd function, the curve is symmetric about origin.

12. 2073 Set D Q.No. 1c

Define even and odd function with suitable examples. [2]

SOLUTION

A function $f(x)$ is said to be even when $f(-x) = f(x)$ and odd when $f(-x) = -f(x)$

For example, $f(x) = x^2$ is an even function as

$f(-x) = (-x)^2 = x^2 = f(x)$

and $g(x) = \sin x$ is an odd function as

$g(-x) = \sin(-x) = -\sin x = -g(x)$

13. 2072 Supp. Q.No. 1c

Test the periodicity of the function $f(x) = \tan\frac{x}{4}$ and find its period. [2]

SOLUTION

Given, $f(x) = \tan\frac{x}{4}$

Here, $f(x + 4\pi) = \tan\left(\frac{x + 4\pi}{4}\right)$

$= \tan\left(\frac{x}{4} + \pi\right) = \tan\frac{x}{4} = f(x)$

$\therefore$ $f(x) = \tan\frac{x}{4}$ is a periodic function of period 4π.

14. 2072 Set C Q.No. 1c

Examine the even or odd nature and the symmetricity of the function $y = 10^x - 10^{-x}$. [2]

SOLUTION

Let $f(x) = 10^x - 10^{-x}$

Now, $f(-x) = 10^{-x} - 10^{-(-x)}$

$= 10^{-x} - 10^x$

$= -(10^x - 10^{-x}) = -f(x)$

$\therefore$ $f(x)$ is odd function.

Also, since $f(-x) = -f(x)$, $f(x)$ is symmetric about origin.

15. 2072 Set D Q.No. 1c

Find the periodicity of the function $y = \tan\frac{3x}{5}$ and find its period. [2]

SOLUTION

Let $f(x) = y = \tan\frac{3x}{5}$

For any integer n,

$f\left(x + \frac{5n\pi}{3}\right) = \tan\frac{3}{5}\left(x + \frac{5n\pi}{3}\right)$

$= \tan\left(n\pi + \frac{3x}{5}\right) = \tan\frac{3x}{5} = f(x)$

Hence, $f(x) = \tan\frac{3x}{5}$ is a periodic function.

Also, $f\left(x + \frac{5\pi}{3}\right) = \tan\frac{3}{5}\left(\frac{5\pi}{3} + x\right)$

$= \tan\left(\pi + \frac{3x}{5}\right) = \tan\frac{3x}{5} = f(x)$

$\therefore$ Period of given function $= \frac{5\pi}{3}$

16. 2072 Set E Q.No. 1c

Test for symmetry: $y = \sqrt{x^2 - 1}$ [2]

SOLUTION

Let $y = f(x) = \sqrt{x^2 - 1}$

Now, $f(-x) = \sqrt{(-x)^2 - 1} = \sqrt{x^2 - 1} = f(x)$

$\therefore$ $f(x)$ is symmetric about y-axis.

17. 2071 Supp. Q.No. 1c

Test for symmetry: $y = x^{-3} + x^{-1}$ [2]

SOLUTION

Let $y = f(x) = x^{-3} + x^{-1} = \frac{1}{x^3} + \frac{1}{x}$

Now, $f(-x) = \frac{1}{(-x)^3} + \frac{1}{(-x)}$

$= -\frac{1}{x^3} - \frac{1}{x} = -\left(\frac{1}{x^3} + \frac{1}{x}\right) = -f(x)$

$\therefore$ $f(x)$ is symmetric about the origin.

18. 2071 Set C Q.No. 1c

Determine whether the function $f(x) = \sqrt{x^2 - 1}$ is even or odd. Also test its symmetricity. [2]

SOLUTION

Given, $f(x) = \sqrt{x^2 - 1}$

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$f(-x) = \sqrt{(-x)^2 - 1} = \sqrt{x^2 - 1} = f(x)$

$\therefore$ $f(x)$ is an even function.

Also, $f(-x) = f(x)$

So, $f(x)$ is symmetrical about y-axis.

19. 2071 Set D Q.No. 1a

Define even and odd functions and give example of each. [2]

➥ Please refer to 2073 Set D Q.No. 1c

20. 2070 Supp Q.No. 1c

Test the periodicity of the function $f(x) = \tan \frac{x}{4}$ and find its period. [2]

➥ Please refer to 2072 Supp. Q.No. 1c

21. 2070 Set C Q.No. 1 c

Examine whether the function: $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd. Also examine for its symmetricity. [2]

➥ Please refer to 2075 Set C Q.No. 1c

22. 2070 Set D Q.No. 1 c

Test the even or odd nature and the symmetricity of the function $f(x) = x^4 + 3x^2 + 1$. [2]

SOLUTION

Here, $f(x) = x^4 + 3x^2 + 1$

Now, $f(-x) = (-x)^4 + 3(-x)^2 + 1$

$= x^4 + 3x^2 + 1 = f(x)$

$\therefore$ $f(x)$ is an even function.

Again, since $f(-x) = f(x)$, the function is symmetric about y-axis.

23. 2069 Supp Q.No. 1 c

Show that $y = x^3$ is an odd function and test its symmetricity. [2]

➥ Please refer 2073 Set C Q.No. 1c

24. 2069 [Set A] Q. No. 1c

Test the periodicity of the function $f(x) = \sin 2x$ and find its period. [2]

SOLUTION

Here, $f(x) = \sin 2x$

Now, $f(x + \pi) = \sin 2(x + \pi)$

$= \sin (2\pi + 2x)$

$= \sin 2x = f(x)$

$\therefore$ $f(x)$ is a periodic function of period 2π.

25. 2069 [Set B] Q. No. 1c

Examine the function $y = \cos x$ for symmetry and even or odd nature. [2]

SOLUTION

Here, $y = f(x) = \cos x$

Now, $f(-x) = \cos(-x) = \cos x = f(x)$

So, $f(x)$ is even function.

Since $f(-x) = f(x)$, so the function is symmetric about y-axis.

26. 2068 Q.No. 1c

Test the periodicity and the symmetricity of the function $y = \sin x$.

SOLUTION

Here, $y = f(x) = \sin x$

Now, $f(x + 2\pi) = \sin(x + 2\pi) = \sin x = f(x)$.

So, $y = \sin x$ is a periodic function of period 2π. Also $f(-x) = \sin(-x) = -\sin x = -f(x)$.

So, the given function is symmetric about origin.

4 MARKS QUESTIONS

27. 2076 Set B Q.No. 6b

Sketch the graph of $f(x) = (x - 1)^3$ indicating its characteristics.

SOLUTION

Here, $y = f(x) = (x - 1)^3$

The sketch of the graph of $y = f(x)$ can be obtained using transformation technique. The graph $y = (x - 1)^3$ is obtained by shifting the graph of $y = x^3$ horizontally by 1 unit right. First we sketch the graph of $y = x^3$. The characteristics of $y = f(x) = x^3$ are as follows:

i. Domain $= (-\infty, \infty)$

Range $= (-\infty, \infty)$

ii. When $x = 0$, $y = 0$. So the curve passes through the origin.

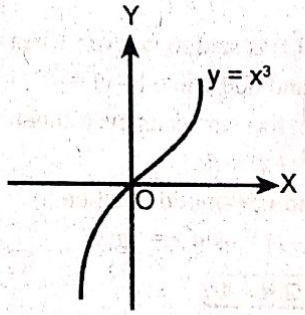
iii. Since $f(-x) = (-x)^3 = -x^3 = -f(x)$, it is an odd function and hence it is symmetric about origin.

iv. Since $x_1 < x_2 \Rightarrow x_1^3 < x_1^3$

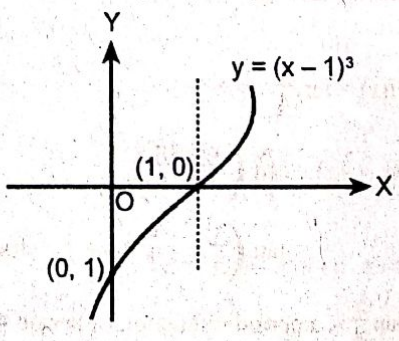
$\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in \mathbb{R}$

The graph is always increasing.

v. When $x > 0$ then $y > 0$ and when $x < 0$, then $y < 0$. This shows that the graph lies in first and third quadrant only. With the above characteristics the graph of $y = f(x) = x^3$ is as follows:



The graph of $y = f(x) = (x - 1)^3$ can be obtained by shifting the graph of $y = f(x) = x^3$ horizontally by 1 unit right. The sketch of $y = (x - 1)^3$ is



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28. 2076 Set C Q.No. 6b

Sketch the graph of $f(x) = x^2 - 3x$ indicating its characteristics. [4]

SOLUTON

Let $y = f(x) = x^2 - 3x$

The characteristics of given function are as follows:

i. Since $y = x^2 - 3x$ is in the form of $y = ax^2 + bx + c$, so it represents a parabola.
Here, $a = 1, b = -3, c = 0$.
Since, $a = 1 > 0$, the parabola turns upwards.

ii. Vertex:

$$x = -\frac{b}{2a} = -\frac{(-3)}{2 \times 1} = \frac{3}{2}$$

When, $x = \frac{3}{2}$,

$$y = \left(\frac{3}{2}\right)^2 - 3 \times \frac{3}{2} = \frac{9}{4} - \frac{9}{2} = \frac{9-18}{4} = -\frac{9}{4}$$

$\therefore$ Vertex $= \left(\frac{3}{2}, -\frac{9}{4}\right)$

iii. Domain $= (-\infty, \infty)$

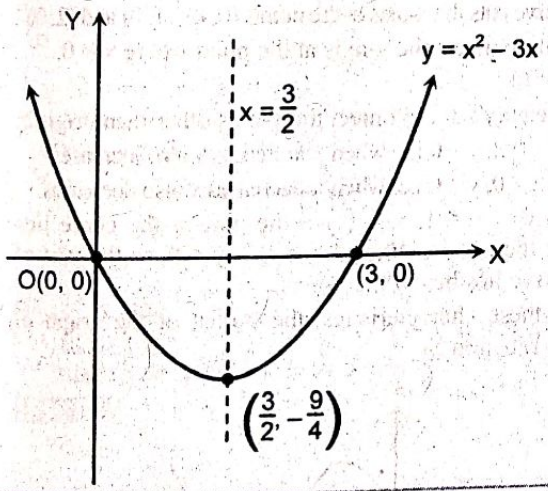
Range $= \left[-\frac{9}{4}, \infty\right)$

iv. Axis of the parabola: $x = \frac{3}{2}$ is the axis of the parabola.

v. When $x = 0$,
$y = 0^2 - 3 \times 0 = 0$
So the curve passes through the origin.

vi. When $y = 0$,
$0 = x^2 - 3x$
or, $x(x - 3) = 0$
$\therefore$ $x = 0, x = 3$.
So the curve cuts x–axis at (0, 0) and (3, 0).

With the above characteristics, the sketch of the graph of given function is as follows:



29. 2075 GIE Q.No. 6b

Sketch the curve of $f(x) = x^2 + 2x + 3$ mentioning its major characteristics. [4]

SOLUTION

Given curve is $y = f(x) = x^2 + 2x + 3$

The characteristics of above function are as follows:

i. The given curve $y = x^2 + 2x + 3$ is in the form $y = ax^2 + bx + c$, so it represents a parabola.
Here, $a = 1, b = 2, c = 3$
Also, $a = 1 > 0$, so the parabola turns upward.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-2}{2 \times 1} = -1$

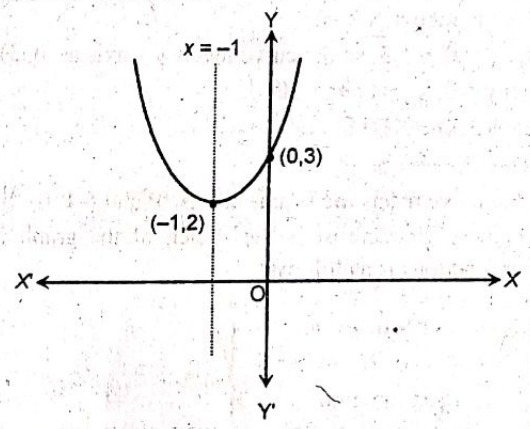
Using $x = -1$ in $y = x^2 + 2x + 3$, we get
$y = (-1)^2 + 2 \times (-1) + 3$
$= 1 - 2 + 3 = 2$

$\therefore$ Vertex $= (-1, 2)$

iii. Domain $= (-\infty, \infty)$
Range $= [2, \infty)$

iv. Axis of the parabola: The axis of the parabola is $x = -1$. So the parabola is symmetric about the line $x = -1$.

v. When $x = 0$, $y = 0^2 + 2 \times 0 + 3 = 3$. So, the parabola intersects y-axis at (0, 3).
When $y = 0$, $x^2 + 2x + 3 = 0$ which gives no real values of x.
With above characteristics, the sketch of the graph of given function is



30. 2075 Set A Q.No. 6b

Sketch the graph of $y = 3 \sin x$ using its different characteristics. [4]

SOLUTION

The characteristics of given functions are:

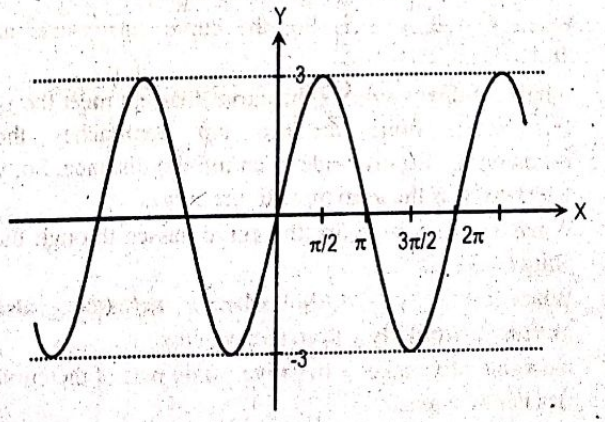
a. When $x = 0$, $y = 0$. So the curve passes through the origin.

b. Period $= 2\pi$

c. Domain $= (-\infty, \infty)$ and range $= [-3, 3]$

d. Since $y = 3 \sin x$ is odd, it is symmetric about origin.

e. The end point, mid point and quarter point of the base period 2π are $(0, 0), (\pi/2, 3), (\pi, 0), (3\pi/2, -3), (2\pi, 0)$
With these characteristics, the sketch of the graph of $y = 3 \sin x$ is as follows.



31. 2075 Set B Q.No. 6b

Stating main characteristics of the function $f(x) = x^2 + 4x + 3$, sketch the graph. [4]

SOLUTION

The characteristics of the function $y=f(x)=x^2+4x+3$ are as follows:

i. Comparing $y = x^2 + 4x + 3$ with $y = ax^2 + bx + c$, we have $a = 1, b = 4, c = 3$
Here $a = 1 > 0$, so the parabola turns upwards.

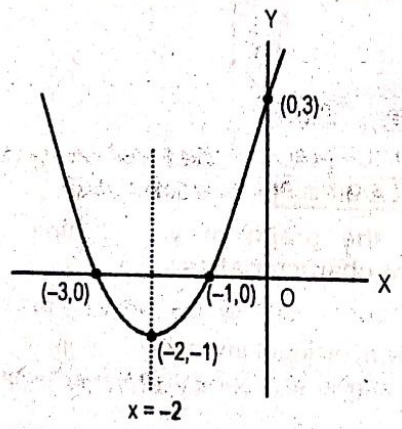
ii. Vertex $= \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$
$= \left(\frac{-4}{2.1}, \frac{4.1.3 - 4^2}{4.1}\right)$
$= (-2, -1)$

iii. Domain $= (-\infty, \infty)$
Range $= [-1, \infty)$

iv. Axis of parabola: $x = -2$

v. When $x = 0$, $y = 3$, so the curve meets y – axis at $(0, 3)$.

vi. When $y = 0$, $x^2 + 4x + 3 = 0$
or, $(x + 3)(x + 1) = 0$
$\therefore x = -3, -1$
So, the curve meets the x–axis at $(-3, 0)$ and $(-1, 0)$.
With these characteristics the sketch of the graph of given functions is as follows:



32. 2075 Set C Q.No. 6b

Sketch the graph of $y = \left(\frac{1}{3}\right)^x$ indicating its different characteristics. [4]

SOLUTION

First we sketch the graph of $y = 3^x$.
The characteristics of $y = 3^x$ are as follows:

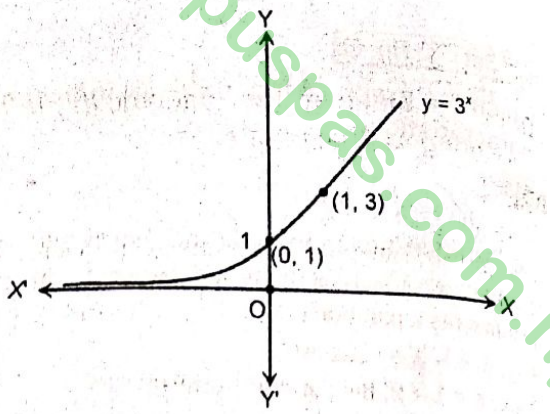
i. When $x = 0$, $y = 1$. So, the curve cuts y-axis at $(0, 1)$

ii. When $y = 0$, $x = -\infty$. So, the curve does not meet the x-axis at a finite distance but approaches the x-axis on its negative side at an infinite distance. So, $y = 0$ (x-axis) is the asymptote to the curve.

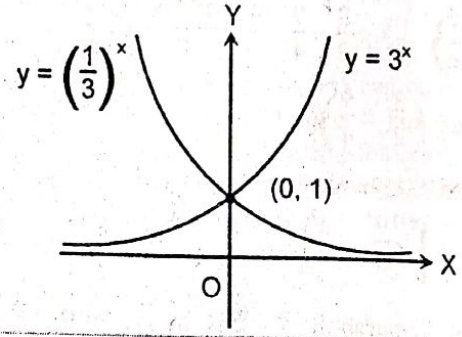
iii. When $x = 1$, $y = 3$. So, the curve passes through the point $(1, 3)$.

iv. When $x > 1$, $y > 3$. So, when x increases y also increases. Similarly y decreases when $x < 0$.

v. No value of x makes y negative, so no part of the curve lies below x-axis.



Now the graph of $y = f(x) = \left(\frac{1}{3}\right)^x$ is obtained reflecting the graph of $f(x) = 3^x$ about y-axis as sho below.



33. 2074 Supp Q.No. 6b

Draw the graph of the function $f(x) = x(x-1)(x-$ using its different characteristics.

SOLUTION

Here, $y = f(x) = x(x-1)(x-2)$
The characteristics of given function are as follows:

i. When $x = 0$, $y = 0$, so the curve passes through the origi

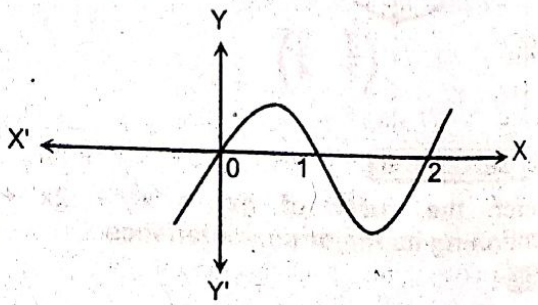
ii. The curve meets the x – axis at the point where $y = 0$
i.e. $x(x-1)(x-2) = 0$
$\therefore x = 0, 1, 2$
The curve cuts the x-axis at the points $(0, 0)$, $(1, 0)$ and $(2,$

iii. The curve meets the y-axis at the point where $x = 0$.
i.e. $y = 0$
So, the curve doesn't meet the y-axis other than origi

iv. When $x > 2$, $y > 0$ i.e. when x increases y also increases.

v. When $x < 0$, $y < 0$ i.e. when x decreases y also decreases

vi. When $0 < x < 1$, $y > 0$, so the part of the curve l above the x-axis. When $1 < x < 2$, $y < 0$, so the part the curve lies below the x-axis.
With these characteristics, the sketch of the graph given function is



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34. 2074 Set A Q.No. 6b

Draw the graph of the function $f(x) = x - x^2$ indicating its characteristics. [4]

SOLUTION

Here, $y = x - x^2$

The given function represents a parabola. It has following characteristics.

i. Comparing $y = x - x^2$ with $y = ax^2 + bx + c$, we get
$a = -1, b = 1, c = 0$
Since $a < 0$, the parabola turns downwards.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-1}{2 \times (-1)} = \frac{1}{2}$

When $x = \frac{1}{2}$, $y = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\therefore$ Vertex $= \left(\frac{1}{2}, \frac{1}{4}\right)$

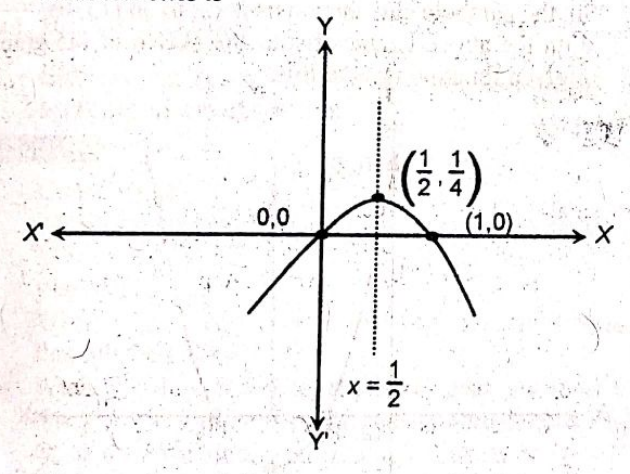
iii. Domain $= (-\infty, \infty)$

Range $= \left(-\infty, \frac{1}{4}\right]$

iv. The axis of the parabola is $x = \frac{1}{2}$

The parabola is symmetric about $x = \frac{1}{2}$.

The sketch of the graph of $y = x - x^2$ with above characteristics is



v. When $x = 0$, $y = 0$. So it passes through origin.
When $y = 0$, $x - x^2 = 0$
$\Rightarrow x(1 - x) = 0$
$\Rightarrow x = 0, 1$
i.e. the curve cuts the x-axis at the point (0, 0) and (1, 0)

35. 2074 Set B Q.No. 6b

Sketch the graph of the function $f(x) = x^2 - 4x + 3$ stating its main characteristics. [4]

SOLUTION

Given function is $y = x^2 - 4x + 3$

The characteristics of given function are:

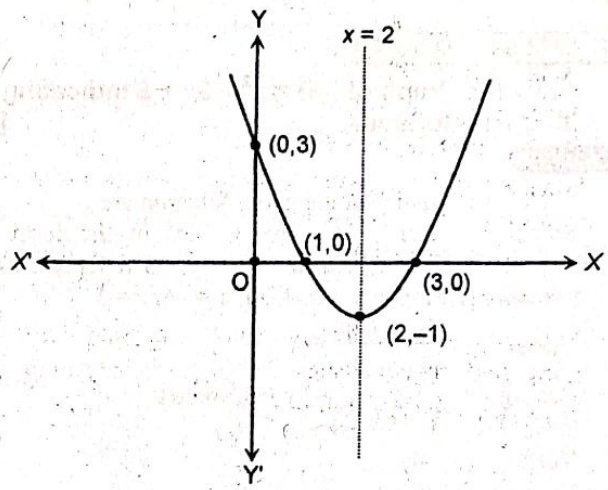
i. The given function $y = x^2 - 4x + 3$ is in the form $y = ax^2 + bx + c$, so it represents a parabola.
Here, $a = 1, b = -4, c = 3$
Since $a = 1 > 0$, the parabola turns upward.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-(-4)}{2 \times 1} = 2$
When $x = 2$, $y = 2^2 - 4 \times 2 + 3 = -1$
$\therefore$ Vertex $= (2, -1)$

iii. Domain $= (-\infty, \infty)$
Range $= [-1, \infty)$

iv. Axis of the parabola: The line $x = 2$ is the axis of the parabola.
The parabola is symmetric about $x = 2$.

v. When $x = 0$, $y = 0^2 - 4 \times 0 + 3 = 3$
So, the curve cuts y-axis at (0, 3)
When $y = 0$, $x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$\therefore$ $x = 1, 3$
So, the curve cuts x-axis at (1, 0) and (3, 0).
With the above characteristics, the sketch of the graph of given function is



36. 2073 Supp Q.No. 6b

Sketch the graph of the function $y = x^2 + 2x - 3$ using its different characteristics. [4]

SOLUTION

Let $y = f(x) = x^2 + 2x - 3$

The characteristics of the above function are as follows:

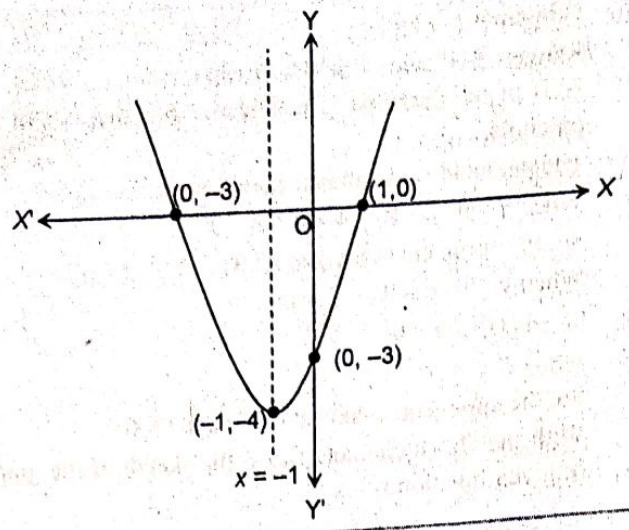
i. Comparing $y = x^2 + 2x - 3$ with $y = ax^2 + bx + c$, we get
$a = 1, b = 2$ and $c = -3$
Since $a = 1 > 0$, so the parabola turns upwards.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-2}{2 \times 1} = -1$
Putting $x = -1$ in $y = x^2 + 2x - 3$, we get
$y = (-1)^2 + 2(-1) - 3$
$= 1 - 2 - 3 = -4$
$\therefore$ Vertex $= (-1, -4)$

iii. Domain $= (-\infty, \infty)$
Range $= (-4, \infty)$

iv. Axis of the parabola: $x = -1$ is the axis of the parabola.

v. When $x = 0$, $y = -3$, so the parabola cuts y-axis at $(0, -3)$.
When $y = 0$, $x^2 + 2x - 3 = 0$
or, $(x + 3)(x - 1) = 0$
$\therefore$ $x = -3, 1$
So, the parabola cuts the x-axis at $(-3, 0)$ and $(1, 0)$ with the above characteristics, the sketch of the graph of given function is as follows:



37. 2073 Set C Q.No. 6b

Draw the graph of $f(x) = x^2 - 6x + 5$ indicating its characteristics. **[4]**

SOLUTION

The characteristics of the given function are

i. Since $y = f(x) = x^2 - 6x + 5$ is in the form of $y = ax^2 + bx + c$ with $a = 1 > 0$, so it represents a parabola turning upward. Also, $b = -6$, $c = 5$.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-(-6)}{2 \times 1} = 3$

Putting $x = 3$ in $y = x^2 - 6x + 5$, we get

$y = 3^2 - 6 \times 3 + 5 = -4$

∴ Vertex = (3, –4)

iii. Domain = $(-\infty, \infty)$

Range = $[-4, \infty)$

iv. Axis: The axis of the parabola is $x = 3$. So, the parabola is symmetric about the line $x = 3$.

v. When $x = 0$, $y = 5$, so the parabola cuts the y-axis at (0, 5).

Again, when $y = 0$,

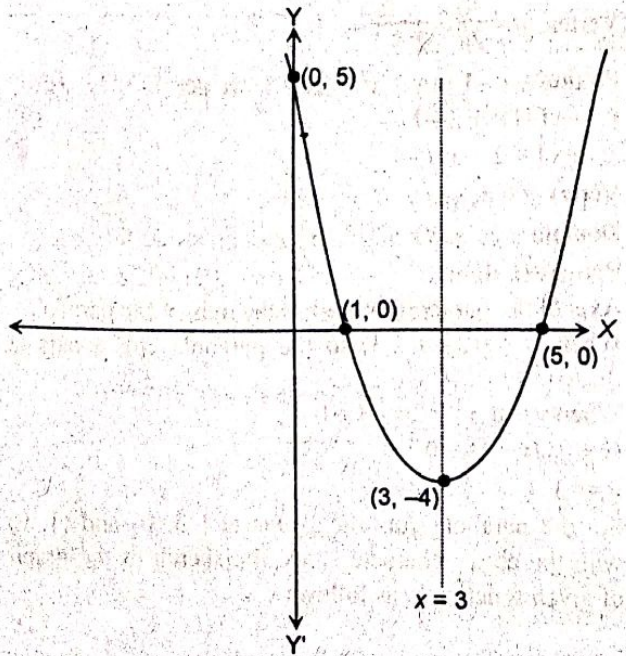
$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$

∴ $x = 1, 5$

So, the parabola cuts the x-axis at (1, 0) and (5, 0).

With the above characteristics the sketch of the graph of given function is as follows:



38. 2073 Set D Q.No. 6b

Stating the different characteristics sketch t curve of the function $f(x) = x^2 + 2x - 3$.

➥ Please refer to 2073 Supp Q.No. 6b

39. 2072 Supp.Q.No. 6b

Draw the graph of the function $y = x^2 - 5x + 6$ w its different characteristics.

SOLUTION

The characteristics of the given function are

i. Since $y = f(x) = x^2 - 5x + 6$ is in the form $y = ax^2 + bx + c$ with $a = 1 > 0$, it represents a parab turning upward. Also, $b = -5$, $c = 6$.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-(-5)}{2 \times 1} = 2.5$

Putting $x = 2.5$ in $y = x^2 - 5x + 6$, we get

$y = (2.5)^2 - 5 \times 2.5 + 6 = -0.25$

∴ Vertex = (2.5, –0.25)

iii. Domain = $(-\infty, \infty)$; Range = $[-0.25, \infty)$

iv. Axis: The axis of the parabola is $x = 2.5$. So, parabola is symmetric about the line $x = 2.5$.

v. When $x = 0$, $y = 6$, so the parabola cuts the y-axis at (0,

Again, when $y = 0$,

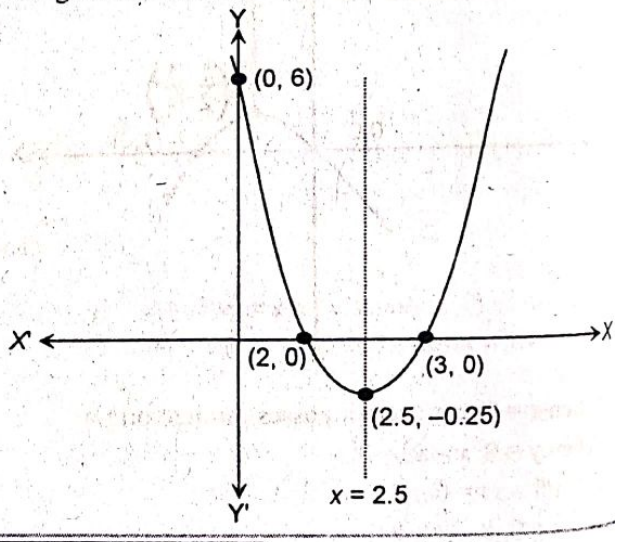
$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

∴ $x = 2, 3$

So, the parabola cuts the x-axis at (2, 0) and (3, 0).

With the above characteristics the sketch of the gra of given function is as follows:



40. 2072 Set C Q.No. 6b

Sketch the graph of $y = x - x^2$ indicating characteristics.

➥ Please refer to 2074 Set A Q.No. 6b

41. 2072 Set D Q.No. 6b

Sketch the graph of $y = x^2 - 2x - 3$ with different characteristics.

SOLUTION

Given $y = x^2 - 2x - 3$

The given function has following characteristics

i. Comparing it with $y = ax^2 + bx + c$, we get

$a = 1$, $b = -2$, $c = -3$

Since $a > 0$, it represents a parabola turning upward.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-(-2)}{2 \times 1} = 1$

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when $x = 1$, $y = 1^2 - 2 \times 1 - 3 = -4$

$\therefore$ Vertex = $(1, -4)$

iii. Domain = $(-\infty, \infty)$; Range = $[-4, \infty)$

iv. Axis of the parabola: The axis of the parabola is $x = 1$. So, the parabola is symmetric about the line $x = 1$.

v. When $x = 0$, $y = -3$. So, the parabola cuts y-axis at $(0, -3)$

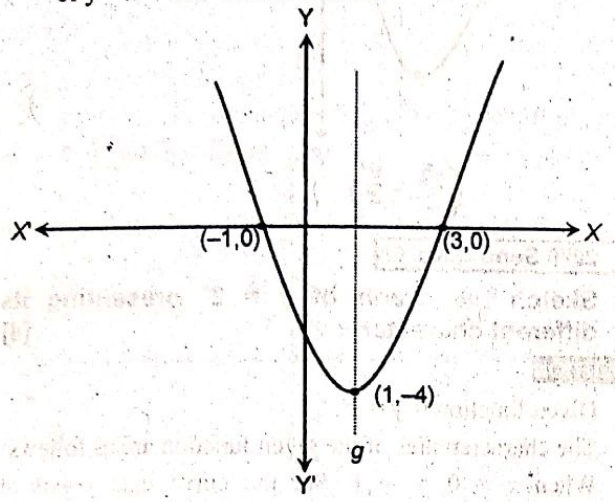
When $y = 0$, $x^2 - 2x - 3 = 0$

$\Rightarrow (x + 1)(x - 3) = 0$

$\Rightarrow x = -1, x = 3$

So, the curve cuts x-axis at $(-1, 0)$ and $(3, 0)$

With the above characteristics, the sketch of the graph of $y = x^2 - 2x - 3$ is as follows.



42. 2072 Set E Q.No. 6b

Sketch the graph of $y = 2^x + 1$ indicating its specific characteristics. [4]

SOLUTION

Given, $y = 2^x + 1 \Rightarrow y - 1 = 2^x$

The characteristics of the above function are as follows:

i. When $x = 0$, $y = 2^0 + 1 = 1 + 1 = 2$. So, the curve cuts y-axis at $(0, 2)$

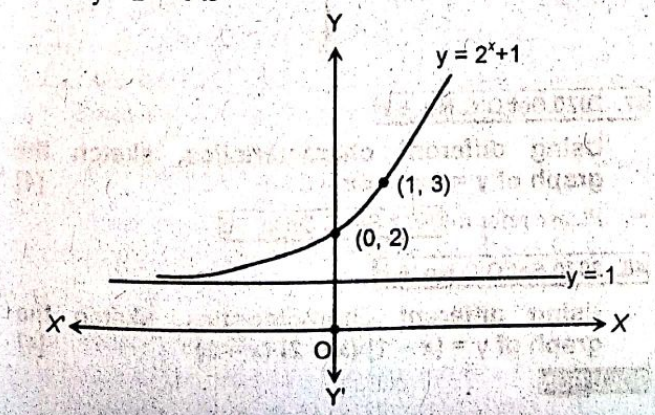
ii. When $x = 1$, $y = 2^1 + 1 = 3$. So, the curve passes through the points $(1, 3)$

iii. When $y = 1$, $x = -\infty$ i.e. the curve does not meet the line $y = 1$ at a finite distance but it approaches the line on its negative side at an infinite distance. So, $y = 1$ is the asymptote to the curve.

iv. When $x > 1$, $y > 3$, so when x increases y also increases. Similarly, y decreases to 1 but never becomes 1 when $x < 0$.

v. There is no value of x which makes y less than or equal to 1. So the curve lies above the line $y - 1 = 0$

With the above characteristics, the sketch of the graph of $y = 2^x + 1$ is



43. 2071 Supp. Q.No. 6b

Sketch the graph of the function $y = (x + 1)(x - 2)(x - 3)$ indicating its specific characteristics. [4]

SOLUTION

Given, $y = (x + 1)(x - 2)(x - 3)$

The given function has following characteristics.

i. When $x = 0$, $y = (1)(-2)(-3) = 6$. So, the curve cuts the y-axis at $(0, 6)$

ii. When $y = 0$, $(x + 1)(x - 2)(x - 3) = 0$

$\therefore x = -1, 2, 3$

Hence the curve cuts x-axis at $(-1, 0)$ $(2, 0)$ and $(3, 0)$

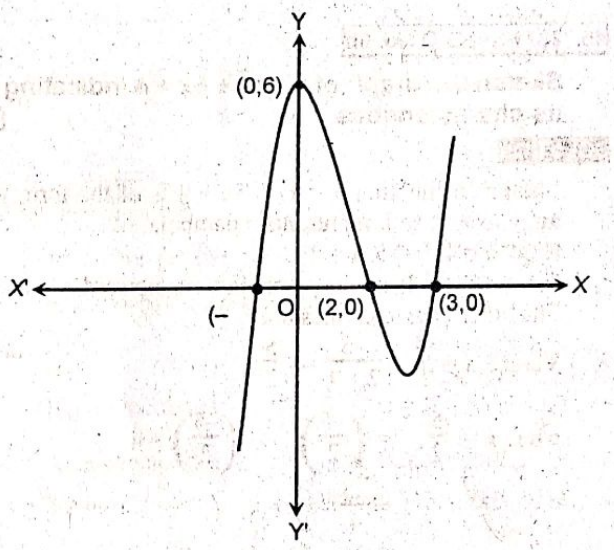
iii. When $x < -1$, $y < 0$ i.e. when x decreases y also decreases.

iv. When $-1 < x < 0$, y increases from 0 to 6 and $0 < x < 2$, y decreases from 6 to 0.

v. When $2 < x < 3$, $y < 0$. So, the curve lies below x-axis when $2 < x < 3$.

vi. When $x > 3$, then $y > 0$.

With the above characteristics the sketch of the given function is as follows.



44. 2071 Set C Q.No. 6b

Sketch the graph of $y = -x^2 + 4x - 3$ indicating its characteristics. [4]

SOLUTION

Given function is $y = -x^2 + 4x - 3$

The characteristics of given function are as follows:

i. The given function $y = -x^2 + 4x - 3$ is in the form of $y = ax^2 + bx + c$, it represents a parabola.

Here, $a = -1$, $b = 4$, $c = -3$

Since $a = -1 < 0$, the parabola turns downwards.

ii. Vertex: $x = -\frac{b}{2a} = -\frac{4}{2 \times (-1)} = 2$

When $x = 2$, $y = -2^2 + 4 \times 2 - 3$

$= -4 + 8 - 3 = 1$

$\therefore$ Vertex = $(2, 1)$

iii. Domain = $(-\infty, \infty)$

Range = $(-\infty, 1]$

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iv. Axis of the parabola: The axis of the parabola is $x = 2$. The parabola is symmetric about $x = 2$.

v. When $x = 0$, $y = -0^2 + 4 \times 0 - 3 = -3$. So, the curve cuts y-axis at $(0, -3)$

vi. When $y = 0$, $-x^2 + 4x - 3 = 0$

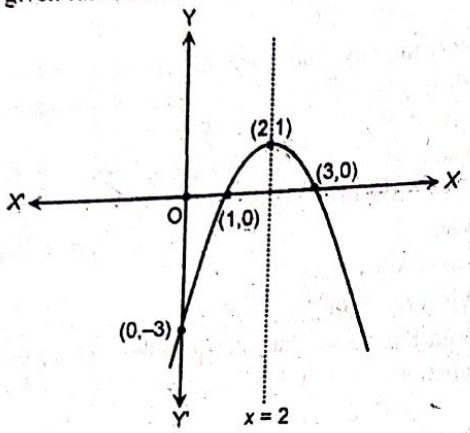
or, $x^2 - 4x + 3 = 0$

or, $(x - 1)(x - 3) = 0$

$\therefore$ $x = 1, 3$

So, the curve cuts x-axis at $(1, 0)$ and $(3, 0)$.

With the above characteristics the sketch of the graph of given function is



45. 2071 Set D Q.No. 6b

Sketch the graph of $y = x^2 + 5x + 4$ indicating its characteristics. [4]

SOLUTION

The given function $y = x^2 + 5x + 4$ is of the form $y = ax^2 + bx + c$, so it represents a parabola.

Here, $a = 1, b = 5, c = 4$

Since $a = 1 > 0$, so the parabola turns upward.

The other characteristics are:

i. Vertex: $x = \frac{-b}{2a} = \frac{-5}{2 \times 1} = -\frac{5}{2}$

when $x = -\frac{5}{2}$, $y = \left(\frac{-5}{2}\right)^2 + 5\left(\frac{-5}{2}\right) + 4$

$= \frac{25}{4} - \frac{25}{2} + 4$

$= \frac{25 - 50 + 16}{4} = \frac{-9}{4}$

Vertex $= \left(-\frac{5}{2}, -\frac{9}{4}\right)$

ii. Domain $= (-\infty, \infty)$

Range $= \left[-\frac{9}{4}, \infty\right)$

iii. Axis of the parabola: $x = -\frac{5}{2}$ is the axis of the parabola.

The parabola is symmetric about this line.

iv. When $x = 0$, $y = 4$. So the curve cuts the y-axis at $(0, 4)$

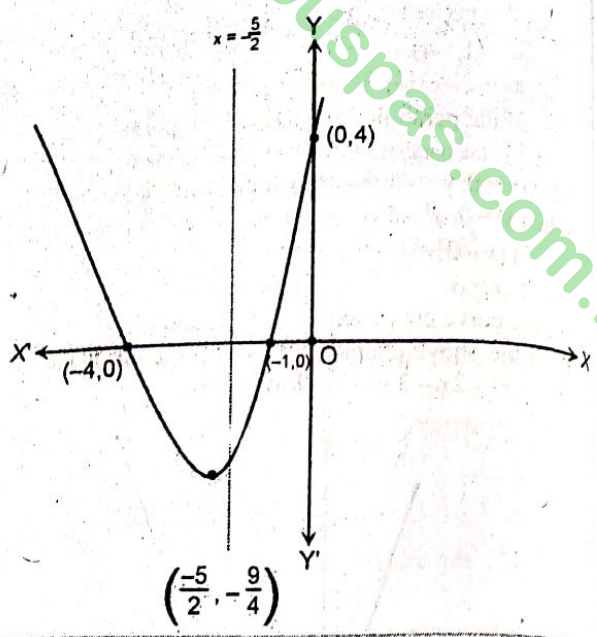
v. When $y = 0$, $x^2 + 5x + 4 = 0$

$\Rightarrow (x + 1)(x + 4) = 0$

$\Rightarrow x = -1, -4$

$\therefore$ So the parabola cuts the x-axis is at $(-1, 0)$ and $(-4, 0)$

With these characteristics the sketch of the graph of the parabola is as follows:



46. 2070 Supp Q.No. 6b

Sketch the graph of $y = 2^x$ presenting i different characters.

SOLUTION

Given function is $y = 2^x$.

The characteristics of the given function are as follow

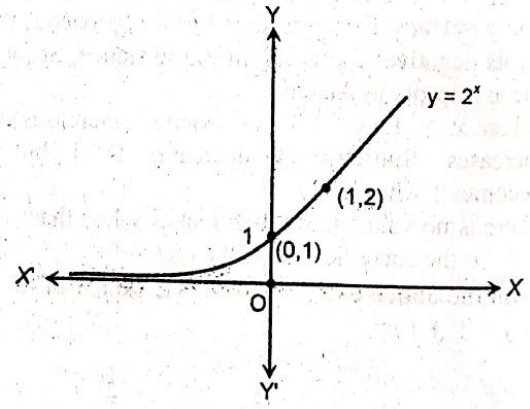
i. When $x = 0$, $y = 1$. So, the curve cuts y-axis (0, 1)

ii. When $y = 0$, $x = -\infty$. So, the curve does not meet t x-axis at a finite distance but approaches t x-axis on its negative side at an infinite distance. S $y = 0$ (x-axis) is the asymptote to the curve.

iii. When $x = 1$, $y = 2$. So, the curve passes through t point (1, 2)

iv. When $x > 1$, $y > 2$. So, when x increases y als increases. Similarly y decreases when $x < 0$.

v. No value of x makes y negative, so no part of the cur lies below x-axis.



47. 2070 Set C Q.No. 6 b

Using different characteristics, sketch th graph of $y = -x^2 + 4x - 3$. [4

➥ Please refer to 2071 Set C Q.No. 6b

48. 2070 Set D Q.No. 6 b

Using different characteristics, sketch th graph of $y = (x - 1)(x - 2)(x - 3)$ [4

SOLUTION

Given function is $y = (x - 1)(x - 2)(x - 3)$

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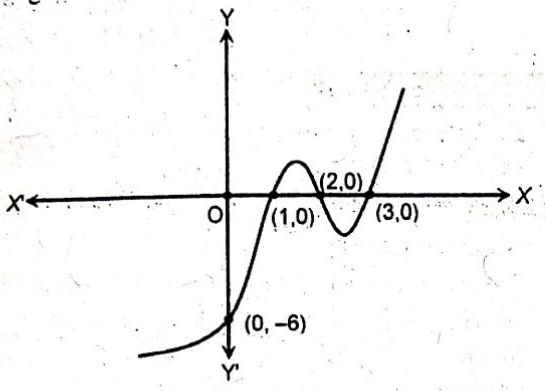
The characteristics of given function are:

i. When $x = 0$, $y = (-1)(-2)(-3) = -6$. So, the curve cuts y-axis at $(0, -6)$

ii. When $y = 0$, $(x-1)(x-2)(x-3) = 0 \Rightarrow x = 1, 2, 3$
So, the curve cuts x-axis at $(1, 0)$ $(2, 0)$ and $(3, 0)$

iii. When $x > 3$, $y > 0$. This shows that when x increases y also increases for $x > 3$.
Also when $x < 1$, $y < 0$. This shows that when x decreases, y also decreases.

iv. When $1 < x < 2$, $y > 0$. So the curve lies above x-axis when $1 < x < 2$.
Also, when $2 < x < 3$, $y < 0$. So the curve lies below the x-axis when $2 < x < 3$.
With the above characteristics, the sketch of the graph of given function is:



49. 2069 Supp Q.No. 6 b

Sketch the graph of $f(x) = x^2 + 2x + 3$ indicating its characteristics. [4]

➥ Please refer to 2075 GIE Q.No. 6b

50. 2069 [Set A] Q. No. 6b

Draw the graph of the function $y = x^2 - 4x + 3$ using its different characteristics. [4]

➥ Please refer to 2074 Set B Q.No. 6b

51. 2069 [Set B] Q. No. 6b

Sketch the graph of $f(x) = (x-4)^2 - 8$ indicating its characteristics. [4]

SOLUTION

Given, $y = f(x) = (x-4)^2 - 8$
$= x^2 - 8x + 16 - 8$
$= x^2 - 8x + 8$

The characteristics of given function are as follows:

i. The given curve $y = x^2 - 8x + 8$ is in the form of $y = ax^2 + bx + c$, so it represents a parabola.
Here, $a = 1$, $b = -8$, $c = 8$
Since $a = 1 > 0$, so the parabola turns upward.

ii. Vertex: $x = \frac{-b}{2a} = \frac{-(-8)}{2 \times 1} = 4$
When $x = 4$, $y = 4^2 - 8 \times 4 + 8 = -8$
$\therefore$ Vertex $= (4, -8)$

iii. Domain $= (-\infty, \infty)$
Range $= [-8, \infty)$

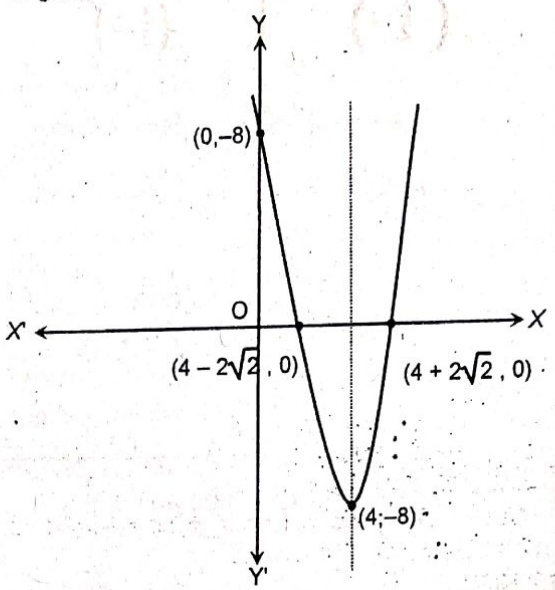
iv. Axis of the parabola: The line $x = 4$ is the axis of the parabola. The parabola is symmetric about $x = 4$.

v. When $x = 0$, $y = 0^2 - 8 \times 0 + 8 = 8$.
So, the parabola cuts y-axis at $x = 0$.
When $y = 0$, $x^2 - 8x + 8 = 0$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{32}}{2} = 4 \pm 2\sqrt{2}$$

$\therefore$ The curve cuts the x-axis at the points $(4 \pm 2\sqrt{2}, 0)$
With the above characteristics the sketch of the graph of given function is



52. 2068 Q.No. 6b

Draw the graph of $y = \cos x \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$ using its different characteristics. [4]

SOLUTION

Given function is $y = \cos x \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$

The characteristics of the above function are:

i. When $x = 0$, $y = 1$, so the curve cuts y-axis at $(0, 1)$

ii. When $y = 0$, $\cos x = 0$

$\therefore \cos x = \cos\left(\pm\frac{\pi}{2}\right)$

$\therefore x = \pm\frac{\pi}{2}$

Hence, the curve meets the x-axis at $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$

iii. Since $\cos x$ is an even function, so it is symmetric about y-axis for $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$

iv. For $\frac{-\pi}{2} \le x \le 0$, y increases and for $0 \le x \le \frac{\pi}{2}$, y decreases.

v. For all $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y \ge 0$, so the curve does not lie below x-axis.

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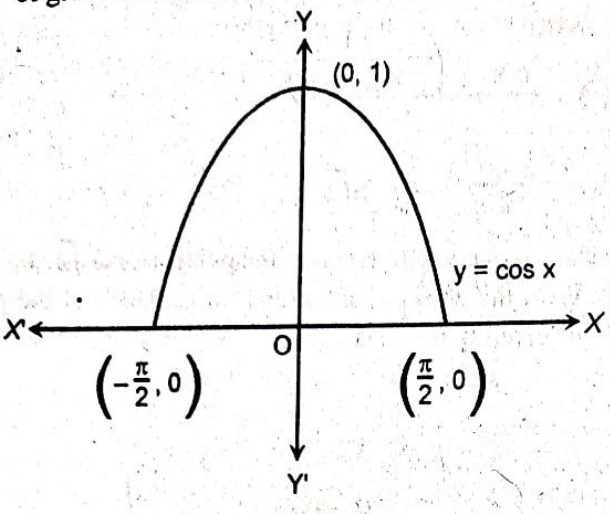
With the above characteristics, the sketch of the graph of given function is



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4 UNIT

TRIGONOMETRY

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A. INVERSE CIRCULAR FUNCTIONS

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 2a

Express $\tan^{-1} x$ in terms of inverse of sine function. [2]

SOLUTION

Let $\theta = \tan^{-1} x$

or, $x = \tan\theta$

We have,

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

or, $$\sin 2\theta = \frac{2x}{1+x^2}$$

or, $$2\theta = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

or, $$\theta = \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$\therefore$ $$\tan^{-1} x = \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

2. 2076 Set C Q.No. 2a

Prove that: $\sin(3\sin^{-1} x) = 3x - 4x^3$. [2]

SOLUTON

Let $\sin^{-1} x = \theta$

Then, $x = \sin\theta$

Now,

$\sin(3\sin^{-1} x) = \sin(3\theta)$

$= 3\sin\theta - 4\sin^3\theta$

$= 3x - 4x^3$

3. 2075 GIE Q.No. 2a

Prove that $\sin(2\sin^{-1} x) = 2x\sqrt{1-x^2}$. [2]

SOLUTION

Let $\sin^{-1} x = \theta$, then it implies

$\sin\theta = x$

Now, L.H.S. $= \sin(2\sin^{-1} x)$

$= \sin 2\theta$

$= 2\sin\theta.\cos\theta$

$= 2\sin\theta\sqrt{1-\sin^2\theta}$

$= 2x\sqrt{1-x^2}$ = R.H.S., proved

4. 2075 Set B Q.No. 2a

Show that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$. [2]

SOLUTION

Let $\sin^{-1} x = \theta$

Then $x = \sin\theta$

We have

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$= 3x - 4x^3$

or, $3\theta = \sin^{-1}(3x - 4x^3)$

$\therefore$ $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$

5. 2075 Set C Q.No. 2a

Prove that $\sin(2\sin^{-1} x) = 2x\sqrt{1-x^2}$. [2]

➥ Please refer to 2075 GIE Q.No. 2a

6. 2074 Supp Q.No. 2a

Prove that

$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$. [2]

SOLUTION

Let $\sin^{-1} x = A$ and $\sin^{-1} y = B$

Then, $x = \sin A$ and $y = \sin B$

$\therefore \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-x^2}$

and $\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-y^2}$

We have,

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

or, $\sin(A+B) = x.\sqrt{1-y^2} + y.\sqrt{1-x^2}$

or, $A + B = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$\therefore$ $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

7. 2074 Set A Q.No. 2a

Prove that $\sin(2\sin^{-1} x) = 2x\sqrt{1-x^2}$. [2]

➥ Please refer to 2075 GIE Q.No. 2a

8. 2074 Set B Q.No. 2a

If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$, prove that $x^2 + y^2 = 1$. [2]

SOLUTION

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$$

or, $$\cos^{-1} x = \frac{\pi}{2} - \cos^{-1} y$$

or, $\cos^{-1} x = \sin^{-1} y$

or, $\cos^{-1} x = \cos^{-1}\sqrt{1-y^2}$

or, $x = \sqrt{1-y^2}$

$\therefore$ $x^2 + y^2 = 1$

9. 2073 Set D Q.No. 2a

Prove: $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ [2]

SOLUTION

Let $\cos^{-1}x = \theta$

Then, $\cos\theta = x$

We know that,

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= 4x^3 - 3x$

or, $3\theta = \cos^{-1}(4x^3 - 3x)$

$\therefore 3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

10. 2072 Supp.Q.No. 2a

Prove that: $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$. [2]

SOLUTION

L.H.S. $= \tan^{-1}a - \tan^{-1}c$

$= \tan^{-1}a - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c$

$= \tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right)$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right]$

$=$ R.H.S.

11. 2072 Set C Q.No. 2a

Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$. [2]

➥ Please refer to 2075 GIE Q.No. 2a

12. 2071 Old Q.No. 2a

Prove that: $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$ [2]

SOLUTION

Let $\tan^{-1}\sqrt{x} = \theta$

$\Rightarrow \sqrt{x} = \tan\theta$

$\Rightarrow x = \tan^2\theta$

Now, We have

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

or, $\cos 2\theta = \frac{1-x}{1+x}$ $[\because \tan^2\theta = x]$

or, $2\theta = \cos^{-1}\left(\frac{1-x}{1+x}\right)$

or, $\theta = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$\therefore \tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

13. 2070 Set C Q.No. 2 a

Prove that: [2]

$\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$

➥ Please refer to 2072 Supp.Q.No. 2a

14. 2070 Set D Q.No. 2 a

Prove that:

$\cos(\sin^{-1}u + \cos^{-1}v) = v\sqrt{1-u^2} - u\sqrt{1-v^2}$

SOLUTION

Let $\sin^{-1}u = A$ and $\cos^{-1}v = B$

Then, $u = \sin A$ and $v = \cos B$

L.H.S. $= \cos(\sin^{-1}u + \cos^{-1}v)$

$= \cos(A + B)$

$= \cos A\cos B - \sin A\sin B$

$= \sqrt{1-\sin^2 A}\cos B - \sin A\sqrt{1-\cos^2 B}$

$= \sqrt{1-u^2}\cdot v - u\sqrt{1-v^2}$

$= v\sqrt{1-u^2} - u\sqrt{1-v^2}$ = R.H.S.

15. 2070 Old Q.No. 2 a

Express $\tan^{-1}x$ in terms of sine function.

➥ Please refer to 2076 Set B Q.No. 2a

16. 2069 [Set A] Old Q. No. 2a

Show that: $\tan^{-1}x = \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

SOLUTION

Let $x = \tan\theta$

R.H.S. $= \frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}$

$= \frac{1}{2}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\sin^{-1}(\sin 2\theta)$

$= \frac{1}{2}\times 2\theta$

$= \theta$

$= \tan^{-1}x$ $(\because x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$=$ L.H.S. proved.

17. 2069 [Set B] Old Q. No. 2a

Prove that: $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \frac{\pi}{2} - \frac{x}{2}$.

SOLUTION

L.H.S. $= \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$= \tan^{-1}\left(\frac{1+2\cos^2\frac{x}{2}-1}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$= \tan^{-1}\left(\cot\frac{x}{2}\right)$

$= \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right\}$

$= \frac{\pi}{2} - \frac{x}{2}$ = R.H.S.

18. 2068 Old Q.No. 2a

Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

➥ Please refer to 2071 Old Q.No. 2a

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19. 2067 Q.No. 2a

Show that $\tan^{-1} x = \frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}$ [2]

➥ Please refer 2069 [Set A] Old Q. No. 2a

20. 2066 Q.No. 2b

Prove that $\cos^{-1}(-x) = \pi - \cos^{-1} x$ [2]

SOLUTION

Let $\cos^{-1} x = \theta$

Then $x = \cos\theta$

So, $-x = -\cos\theta = \cos(\pi - \theta)$

or, $\cos^{-1}(-x) = \pi - \theta$

$\therefore$ $\cos^{-1}(-x) = \pi - \cos^{-1} x$. proved.

21. 2065 Q.No. 2a

Prove that: $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$. [2]

SOLUTION

Let $\sec^{-1}x = A$

$\therefore$ $x = \sec A$

Then, L.H.S. becomes

$A + \text{cosec}^{-1}(\sec A)$

$= A + \text{cosec}^{-1}\left[\text{cosec}\left(\frac{\pi}{2} - A\right)\right]$

$= A + \frac{\pi}{2} - A = \frac{\pi}{2}$

= R.H.S. Proved.

22. 2064 Q.No. 2a

Prove that:

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0$$

[2]

SOLUTION

$$\text{L.H.S.} = \tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right)$$

$$= (\tan^{-1}a - \tan^{-1}b) + (\tan^{-1}b - \tan^{-1}c) + (\tan^{-1}c - \tan^{-1}a)$$

$$= \tan^{-1}a - \tan^{-1}a + \tan^{-1}b - \tan^{-1}b + \tan^{-1}c - \tan^{-1}c$$

$$= 0$$

= R.H.S. Proved.

23. 2063 Q.No. 2a

Prove that: $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$ [2]

SOLUTION

$$\text{L.H.S.} = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$\left[\because \sin 2\theta = 2\sin\theta\cos\theta,\ \cos 2\theta = 2\cos^2\theta - 1\right]$$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2} \qquad [\because \tan^{-1}(\tan\theta) = \theta]$$

= R.H.S. proved.

24. 2062 Q.No. 2a

Solve: $\cos(\sin^{-1}x) = \frac{1}{2}$ [2]

SOLUTION

We have,

$\cos(\sin^{-1}x) = \frac{1}{2}$ ... (i)

Put $\sin^{-1}x = \theta$

$\therefore$ $x = \sin\theta$

Now, $\cos(\sin^{-1}x) = \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

$\therefore$ Equation (i) becomes, $\sqrt{1-x^2} = \frac{1}{2}$

or, $1 - x^2 = \frac{1}{4}$

or, $x^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ $x = \pm\frac{\sqrt{3}}{2}$

25. 2061 Q.No. 2a

Solve: $2\tan^{-1}x = \sin^{-1}\frac{2m}{1+m^2} + \sin^{-1}\frac{2n}{1+n^2}$ [2]

SOLUTION

Here,

$$2\tan^{-1}x = \sin^{-1}\frac{2m}{1+m^2} + \sin^{-1}\frac{2n}{1+n^2}$$

Let $m = \tan\theta$

$\therefore$ $\theta = \tan^{-1}m$

Then,

$$\sin^{-1}\frac{2m}{1+m^2} = \sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \sin^{-1}\sin 2\theta$$

$$= 2\theta$$

$$= 2\tan^{-1}m$$

$\therefore$ $\sin^{-1}\frac{2m}{1+m^2} = 2\tan^{-1}m$

Similarly, $\sin^{-1}\frac{2n}{1+n^2} = 2\tan^{-1}n$

Now, we have,

$$2\tan^{-1}x = \sin^{-1}\frac{2m}{1+m^2} + \sin^{-1}\frac{2n}{1+n^2}$$

or, $2\tan^{-1}x = 2\tan^{-1}m + 2\tan^{-1}n$

or, $2\tan^{-1}x = 2(\tan^{-1}m + \tan^{-1}n)$

or, $\tan^{-1}x = \tan^{-1}\left(\frac{m+n}{1-m\times n}\right)$

$\therefore$ $x = \frac{m+n}{1-mn}$

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26. 2060 Q.No. 2a

Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

SOLUTION

$$\text{L.H.S.} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{3+2}{6-1}\right) = \tan^{-1}\left(\frac{5}{5}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4} = \text{R.H.S. proved.}$$

27. 2059 Q.No. 2a

Show that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{6}{17}$ [2]

SOLUTION

$$\text{L.H.S.} = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-x.y}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{7+5}{35}}{\frac{35-1}{35}}\right) = \tan^{-1}\left(\frac{12}{34}\right)$$

$$= \tan^{-1}\frac{6}{17} = \text{R.H.S. Proved}$$

28. 2058 Q.No. 2a

Find the value of $\tan^{-1} 3 + \tan^{-1}\frac{1}{3}$ [2]

SOLUTION

We have,

$$\tan^{-1} 3 + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\left(\frac{3+\frac{1}{3}}{1-3\times\frac{1}{3}}\right)$$

$$\left[\because \tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)\right]$$

$$= \tan^{-1}\left(\frac{10}{0}\right) = \tan^{-1}(\infty) = \tan^{-1}\left(\tan\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}$$

29. 2057 Q.No. 2a

Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$ [2]

➥ Please refer to 2075 GIE Q.No. 2a

30. 2056 Q.No. 2a

Find the value of $\tan^{-1} 3 + \tan^{-1}\frac{1}{3}$. [2]

➥ Please refer to 2058 Q.No. 2a

31. 2055 Old Model Q.No. 2a

Show that: $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{6}{17}$ [2]

➥ Please refer to 2059 Q.No. 2a

4 MARKS QUESTIONS

32. 2075 Set A Q.No. 7a

If $-\frac{\pi}{2} < \tan^{-1}x + \tan^{-1}y + \tan^{-1}z < \frac{\pi}{2}$, then prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx}$. [4]

SOLUTION

$$\text{L.H.S.} = \tan^{-1}x + \tan^{-1}y + \tan^{-1}z$$

$$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$$

$$= \tan^{-1}\left\{\frac{\frac{x+y}{1-xy}+z}{1-\frac{x+y}{1-xy}.z}\right\}$$

$$= \tan^{-1}\left\{\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{1-xy-xz-yz}{1-xy}}\right\}$$

$$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \text{R.H.S.}$$

33. 2073 Supp Q.No. 7a

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$. [4]

SOLUTION

We have,

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$$

Given,

$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$$

or, $\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

or, $\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \pi - \cos^{-1}z$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1}z)$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\cos(\cos^{-1}z)$

or, $xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

or, $xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

Squaring on both sides, we get,

$$(xy+z)^2 = (1-x^2)(1-y^2)$$

or, $x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$

$\therefore\ x^2 + y^2 + z^2 + 2xyz = 1$

Hence proved.

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34. 2073 Set C Q.No. 7a OR

Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$ [4]

↪ Please refer to 2075 GIE Q.No. 2a

35. 2072 Set D Q.No. 7a

Prove that: [4]

$$\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi=2\left(\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right).$$

SOLUTION

L.H.S. $= \tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

$= (\tan^{-1}1 + \tan^{-1}2) + \tan^{-1}3$

$= \tan^{-1}\left(\frac{1+2}{1-1\times2}\right) + \tan^{-1}3$

$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \tan^{-1}\left(\frac{3}{-1}\right) + \tan^{-1}3$

$= \tan^{-1}(-3) + \tan^{-1}3$

$= \tan^{-1}\left[\frac{-3+3}{1-(-3)\times3}\right]$

$= \tan^{-1}\left(\frac{0}{10}\right)$

$= \tan^{-1}(0)$

$= \tan^{-1}(\tan\pi)$

$= \pi$

R.H.S. $= 2\left(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$

$= 2\left[\tan^{-1}\left(\frac{1+\frac{1}{2}}{1-1.\frac{1}{2}}\right) + \tan^{-1}\frac{1}{3}\right]$

$= 2\left[\tan^{-1}\left(\frac{3}{1}\right) + \tan^{-1}\frac{1}{3}\right]$

$= 2\left[\tan^{-1}3 + \tan^{-1}\frac{1}{3}\right]$

$= 2\left[\tan^{-1}\left(\frac{3+\frac{1}{3}}{1-3\times\frac{1}{3}}\right)\right]$

$= 2\left[\tan^{-1}\left(\frac{10}{0}\right)\right]$

$= 2\tan^{-1}(\infty)$

$= 2\tan^{-1}\left(\tan\frac{\pi}{2}\right)$

$= 2\times\frac{\pi}{2}$

$= \pi$

Hence, L.H.S. = R.H.S. proved.

36. 2072 Set E Q.No. 7a

Prove that $\cos(\sin^{-1}x + \cos^{-1}y) = y\sqrt{1-x^2} - x\sqrt{1-y^2}$. [4]

SOLUTION

Let $\sin^{-1}x = A$ and $\cos^{-1}y = B$

Then, $x = \sin A$ and $y = \cos B$

L.H.S. $= \cos(\sin^{-1}x + \cos^{-1}y)$

$= \cos(A + B)$

$= \cos A\cos B - \sin A\sin B$

$= \sqrt{1-\sin^2 A}\cos B - \sin A\sqrt{1-\cos^2 B}$

$= \sqrt{1-x^2}\cdot y - x\sqrt{1-y^2}$

$= y\sqrt{1-x^2} - x\sqrt{1-y^2}$

$=$ R.H.S.

37. 2071 Supp. Q.No. 7a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$,

prove that $xy+yz+zx = 1$. [4]

SOLUTION

Given, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

or, $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$

or, $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{2} - \tan^{-1}z$

or, $\frac{x+y}{1-xy} = \tan\left(\frac{\pi}{2} - \tan^{-1}z\right)$

or, $\frac{x+y}{1-xy} = \cot(\tan^{-1}z)$

or, $\frac{x+y}{1-xy} = \cot\left(\cot^{-1}\frac{1}{z}\right)$

or, $\frac{x+y}{1-xy} = \frac{1}{z}$

or, $xz + yz = 1 - xy$

$\therefore xy + yz + zx = 1$

38. 2071 Set C Q.No. 7a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi/2$,
show that: $xy + yz + zx = 1$. [4]

↪ Please refer to 2071 Supp. Q.No. 7a.

39. 2071 Set D Q.No. 7a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$
then show that: $x + y + z = xyz$. [4]

SOLUTION

Here,

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

or, $\tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

or, $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi - \tan^{-1}z$

or, $\frac{x+y}{1-xy} = \tan(\pi - \tan^{-1}z)$

or, $\frac{x+y}{1-xy} = -\tan(\tan^{-1}z)$ $[\because \tan(\pi-\theta) = -\tan\theta]$

or, $\frac{x+y}{1-xy} = -z$

or, $x + y = -z + xyz$

Hence, $x + y + z = xyz$.

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40. 2071 Old Q.No. 8a

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ show that: $x^2 + y^2 + z^2 + 2xzy = 1$. [4]

➥ Please refer to 2073 Supp Q.No. 7a

41. 2070 Supp Q.No. 7a

Prove that $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$. [4]

SOLUTION

Here, L.H.S. $= 4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5})$

$$= 4\left[\tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right] \quad \ldots (i)$$

Let $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$

$\therefore \sin\theta = \frac{1}{\sqrt{5}}$

Now,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{\frac{1}{\sqrt{5}}}{\sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}} = \frac{\frac{1}{\sqrt{5}}}{\sqrt{\frac{5-1}{5}}}$$

$$= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{4}} = \frac{1}{2}$$

or, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\therefore \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

Now,

L.H.S. $= 4\left\{\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right\}$

$$= 4\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)$$

$$\left[\because \tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)\right]$$

$$= 4\tan^{-1}\left(\frac{2+3}{6-1}\right) = 4\tan^{-1}\left(\frac{5}{5}\right)$$

$$= 4\tan^{-1}(1) = 4\tan^{-1}\left(\tan\frac{\pi}{4}\right) = 4\times\frac{\pi}{4} = \pi.$$

= R.H.S. proved.

42. 2070 Old Q.No. 8 a

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that: $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. [4]

SOLUTION

Let $\sin^{-1}x = A$, $\sin^{-1}y = B$, $\sin^{-1}z = c$

Then $x = \sin A$, $y = \sin B$, $z = \sin C$

L.H.S $= x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$

$= \sin A\sqrt{1-\sin^2 A} + \sin B\sqrt{1-\sin^2 B} + \sin c\sqrt{1-\sin^2 C}$

$= \sin A\cos A + \sin B\cos B + \sin C\cos C$

$$= \frac{2\sin A\cos A}{2} + \frac{2\sin B\cos B}{2} + \frac{2\sin C\cos C}{2}$$

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2}$$

$$= \frac{4\sin A\sin B\sin C}{2}$$

$= 2\sin A\sin B\sin C$

$= 2xyz$

= R.H.S proved

$\left(\because A + B + C = \pi\right.$
So, $\sin 2A + \sin 2B + \sin 2C$
$\left. = 4\sin A\sin B\sin C\right)$

43. 2069 Supp Q.No. 7 a

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$. [4]

➥ Please refer to 2073 Supp Q.No. 7a

44. 2069 [Set A] Q. No. 7a

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$ [4]

➥ Please refer to 2073 Supp Q.No. 7a

45. 2069 [Set B] Q. No. 7a

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ show that: $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. [4]

➥ Please refer to 2070 Old Q.No. 8 a

46. 2069 [Set A] Old Q. No. 8a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, prove that: $yz + zx + xy = 1$. [4]

➥ Please refer to 2071 Supp. Q.No. 7a

47. 2069 [Set B] Old Q. No. 8a

Prove that:

$$\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}.$$ [4]

SOLUTION

Let $\cos^{-1}x = \theta$ ...(i)

or, $x = \cos\theta$

Now,

$$2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\sin^{-1}\sqrt{\frac{1-\cos\theta}{2}}$$

$$= 2\sin^{-1}\sqrt{\frac{1-1+2\sin^2\frac{\theta}{2}}{2}}$$

$$= 2\sin^{-1}\left(\sin\frac{\theta}{2}\right)$$

$$= 2\cdot\frac{\theta}{2} = \theta \quad \ldots(ii)$$

and $2\cos^{-1}\sqrt{\frac{1+x}{2}} = 2\cos^{-1}\sqrt{\frac{1+\cos\theta}{2}}$

$$= 2\cos^{-1}\sqrt{\frac{1+2\cos^2\frac{\theta}{2}-1}{2}}$$

$$= 2\cos^{-1}\sqrt{\cos^2\frac{\theta}{2}}$$

$$= 2\cos^{-1}\left(\cos\frac{\theta}{2}\right)$$

$$= 2\cdot\frac{\theta}{2} = \theta \quad \ldots(iii)$$

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Combining (i), (ii) and (iii), we have

$$\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$$

48. 2068 Q.No. 7a

Prove that: $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi = 2(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$ [4]

➥ Please refer to 2072 Set D Q.No. 7a

49. 2068 Old Q.No. 8a

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$. [4]

➥ Please refer to 2073 Supp Q.No. 7a

50. 2067 Q.No. 8a

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$$ [4]

➥ Please refer to 2070 Old Q.No. 8 a

51. 2066 Q.No. 8a

If $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi$ show that $xy + yx + zx = 1$ [4]

SOLUTION

$$\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi$$

or, $\tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{y} + \tan^{-1}\frac{1}{z} = \pi$

or, $\tan^{-1}\left(\frac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{x}\cdot\frac{1}{y}}\right) = \pi - \tan^{-1}\frac{1}{z}$

or, $\frac{\frac{y+x}{xy}}{\frac{xy-1}{xy}} = \tan\left(\pi - \tan^{-1}\frac{1}{z}\right)$

or, $\frac{x+y}{xy-1} = -\tan\left(\tan^{-1}\frac{1}{z}\right)$

or, $\frac{x+y}{xy-1} = -\frac{1}{z}$

or, $zx + yz = -xy + 1$

$\therefore$ $xy + yz + zx = 1$ proved.

52. 2065 Q.No. 8a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$. [4]

SOLUTION

Given that

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

or, $\tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

or, $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi - \tan^{-1}z$

or, $\tan\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan(\pi - \tan^{-1}z)$

or, $\frac{x+y}{1-xy} = -\tan\tan^{-1}z$

or, $\frac{x+y}{1-xy} = -z$

or, $x + y = -z + xyz$

$\therefore$ $x + y + z = xyz$ Proved.

53. 2063 Q.No. 8a OR

Prove that $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = 2\left(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$ [4]

➥ Please refer to 2072 Set D Q.No. 7a

54. 2061 Q.No. 8a

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that $x + y + z = xyz$. [4]

➥ Please refer to 2071 Set D Q.No. 7a

55. 2060 Q.No. 8a

Prove that $\cot^{-1}(\tan 2x) - \cot^{-1}(\tan 3x) = x$ [4]

SOLUTION

L.H.S. $= \cot^{-1}(\tan 2x) - \cot^{-1}(\tan 3x)$

$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - 2x\right)\right] - \cot^{-1}\left[\cot\left(\frac{\pi}{2} - 3x\right)\right]$

$= \left(\frac{\pi}{2} - 2x\right) - \left(\frac{\pi}{2} - 3x\right)$ [$\because \cot^{-1}(\cot x) = x$]

$= \frac{\pi}{2} - 2x - \frac{\pi}{2} + 3x$

$= x$

= R.H.S. proved.

56. 2059 Q.No. 8a

Prove: $\tan(2\tan^{-1}x) = 2\tan(\tan^{-1}x + \tan^{-1}x^3)$. [4]

SOLUTION

Put $\tan^{-1}x = \theta$;

Then, $\tan\theta = x$

Now, L.H.S. $= \tan(2\tan^{-1}x)$

$= \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2x}{1-x^2}$

R.H.S. $= 2\tan(\tan^{-1}x + \tan^{-1}x^3)$

$= 2\tan\left[\tan^{-1}\left(\frac{x+x^3}{1-x.x^3}\right)\right]$

$= 2\tan\left[\tan^{-1}\frac{x(1+x^2)}{(1+x^2)(1-x^2)}\right]$

$= 2\tan\left[\tan^{-1}\frac{x}{1-x^2}\right]$

$= 2\left(\frac{x}{1-x^2}\right)$ [$\because \tan(\tan^{-1}x) = x$]

$\therefore$ $= \frac{2x}{1-x^2}$

$\therefore$ L.H.S. = R.H.S. Proved.

57. 2058 Q.No. 8a

Prove that $\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$ [4]

➥ Please refer 2070 Supp Q.No. 7a

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58. 2057 Q.No. 8a OR

Find the value of $\cos \tan^{-1} \sin \cot^{-1} x$. [4]

SOLUTION

Let $\cot^{-1} x = \theta$; $\therefore x = \cot\theta$

Now,

$$\sin \cot^{-1} x = \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \tan^{-1} \sin \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

Again, let $\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \beta$

So, $\tan\beta = \left(\frac{1}{\sqrt{1+x^2}}\right)$

Now,

$$\cos \tan^{-1} \sin \cot^{-1} x = \cos \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \cos\beta = \frac{1}{\sec\beta} = \frac{1}{\sqrt{1+\tan^2\beta}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{1}{\sqrt{1+x^2}}\right)^2}}$$

$$= \frac{1}{\sqrt{1+\frac{1}{1+x^2}}}$$

$$= \frac{1}{\sqrt{\frac{1+x^2+1}{1+x^2}}} = \sqrt{\frac{1+x^2}{2+x^2}}$$

59. 2056 Q.No. 8a

Solve: $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1} x$. [4]

SOLUTION

Let $a = \tan A$ and $b = \tan B$

So, $\frac{2a}{1+a^2} = \frac{2\tan A}{1+\tan^2 A} = \sin 2A$

And, $\frac{2b}{1+b^2} = \frac{2\tan B}{1+\tan^2 B} = \sin 2B$

Now, $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1} x$ becomes,

$\sin^{-1}(\sin 2A) + \sin^{-1}(\sin 2B) = 2\tan^{-1} x$

or, $2A + 2B = 2\tan^{-1} x$

or $A + B = \tan^{-1} x$

or, $x = \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$

$\therefore x = \frac{a+b}{1-ab}$

60. 2055 Old Model Q.No. 8a

Prove that $\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$ [4]

SOLUTION

Let, $\cot^{-1} 3 = A$

Then, $\cot A = 3$

$\therefore A = \tan^{-1}\frac{1}{3}$

$\Rightarrow \cot^{-1} 3 = \tan^{-1}\frac{1}{3}$

Again, let $\operatorname{cosec}^{-1}\sqrt{5} = B$

Then, $\operatorname{cosec} B = \sqrt{5}$

or, $\operatorname{cosec}^2 B = 5$

or, $1 + \cot^2 B = 5$

or, $\cot^2 B = 5 - 1$

or, $\cot^2 B = 4$

or, $\cot B = 2$

or, $\tan B = \frac{1}{2}$

$\Rightarrow B = \tan^{-1}\frac{1}{2}$

$\therefore \operatorname{cosec}^{-1}\sqrt{5} = \tan^{-1}\frac{1}{2}$

Now,

L.H.S. $= \cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5}$

$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)$

$= \tan^{-1}\left(\frac{5}{5}\right)$

$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi^c}{4}$ = R.H.S.

B. GENERAL VALUES

2 MARKS QUESTIONS

1. 2075 Set A Q.No. 2a

Solve the trigonometric equation. [2]

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

SOLUTION

Given,

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

or, $\tan\theta + \tan 2\theta = \sqrt{3} - \sqrt{3}\tan\theta\tan 2\theta$

or, $\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

or, $\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$

or, $\tan(\theta + 2\theta) = \sqrt{3}$

or, $\tan 3\theta = \tan\frac{\pi}{3}$

General value of $3\theta = n\pi + \frac{\pi}{3}$

or, $\theta = \frac{n\pi}{3} + \frac{\pi}{9}$

or, $\theta = \frac{3n\pi + \pi}{9}$

$\therefore \theta = (3n+1)\frac{\pi}{9}$

2. 2073 Supp Q.No. 2a

Solve: $7\sin^2x + 3\cos^2x = 4$. [2]

SOLUTION

Here, $7\sin^2 x + 3\cos^2 x = 4$

or, $7\sin^2 x + 3(1 - \sin^2 x) = 4$

or, $7\sin^2 x + 3 - 3\sin^2 x = 4$

or, $4\sin^2 x = 4 - 3$

or, $\sin^2 x = \frac{1}{4}$

or, $\sin x = \pm\frac{1}{2}$

or, $\sin x = \pm\sin\frac{\pi}{6}$

or, $\sin x = \sin\left(\pm\frac{\pi}{6}\right)$

$\therefore\ x = n\pi \pm \frac{\pi}{6}$

3. 2073 Set C Q.No. 2a

Solve : $\cot^2x + \text{cosec}^2x = 3\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$. [2]

SOLUTION

$\cot^2x + \text{cosec}^2x = 3$

or, $\cot^2x + 1 + \cot^2x = 3$

or, $2\cot^2x = 2$

or, $\cot^2x = 1$

or, $\cot x = \pm 1$

or, $\cot x = \cot\left(\pm\frac{\pi}{4}\right)$

or, $x = n\pi \pm \frac{\pi}{4}$

When $n = 0$, $x = \pm\frac{\pi}{4}$

When $n = 1$, $x = \pi \pm \frac{\pi}{4} = \frac{3\pi}{4}, \frac{5\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

When $n = -1$, $x = -\pi \pm \frac{\pi}{4} = \frac{-5\pi}{4}, \frac{-3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\therefore\ x = \pm\frac{\pi}{4}$

4. 2072 Set D Q.No. 2a

Solve $\tan ax = \cot bx$. [2]

SOLUTION

$\tan ax = \cot bx$

or, $\tan ax = \tan\left(\frac{\pi}{2} - bx\right)$

or, $ax = n\pi + \left(\frac{\pi}{2} - bx\right)$

or, $ax + bx = n\pi + \frac{\pi}{2}$

or, $(a + b)x = (2n + 1)\cdot\frac{\pi}{2}$

$\therefore\ x = \frac{(2n+1)\pi}{2(a+b)}$

5. 2072 Set E Q.No. 2a

Solve: $\sin 3x + \cos 3x = 0$ [2]

SOLUTION

Here, $\sin 3x + \cos 3x = 0$

or, $\sin 3x = -\cos 3x$

or, $\frac{\sin 3x}{\cos 3x} = -1$

or, $\tan 3x = -1 = \tan\frac{3\pi}{4}$

or, $3x = n\pi + \frac{3\pi}{4}$

or, $3x = \frac{4n\pi + 3\pi}{4}$

$\therefore\ x = (4n + 3)\frac{\pi}{12}$

6. 2071 Supp. Q.No. 2a

Solve: $\text{cosec}\theta = \cot\theta + \sqrt{3}$ [2]

SOLUTION

Given equation is $\text{cosec}\,\theta = \cot\theta + \sqrt{3}$

or, $\frac{1}{\sin\theta} = \frac{\cos\theta}{\sin\theta} + \sqrt{3}$

or, $\frac{1}{\sin\theta} = \frac{\cos\theta + \sqrt{3}\sin\theta}{\sin\theta}$

or, $\cos\theta + \sqrt{3}\sin\theta = 1$

Dividing both sides by $\sqrt{1^2 + (\sqrt{3})^2} = 2$, we get

$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \frac{1}{2}$

or, $\cos\theta\cos\frac{\pi}{3} + \sin\theta\sin\frac{\pi}{3} = \frac{1}{2}$

or, $\cos\left(\theta - \frac{\pi}{3}\right) = \cos\frac{\pi}{3}$

or, $\theta - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}$

or, $\theta = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{3}$

7. 2071 Set C Q.No. 2a

Solve for x: $\tan 2x = \tan x$ [2]

SOLUTION

Given function is $\tan 2x = \tan x$

or, $2x = n\pi + x$

$\therefore\ x = n\pi$

8. 2071 Set D Q.No. 2a

Solve: $\sqrt{3}\sin x - \cos x = \sqrt{3}$ *for* $0 \le x \le 2\pi$. [2]

SOLUTION

Given equation is $\sqrt{3}\sin x - \cos x = \sqrt{3}$

Dividing both sides by $\sqrt{(\sqrt{3})^2 + (-1)^2} = 2$, we get

$\sin x\cdot\frac{\sqrt{3}}{2} - \cos x\cdot\frac{1}{2} = \frac{\sqrt{3}}{2}$

or, $\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

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or, $\sin\left(x-\frac{\pi}{6}\right)=\sin\frac{\pi}{3}$

or, $\sin\left(x-\frac{\pi}{6}\right)=\sin\frac{\pi}{3}$

or, $x-\frac{\pi}{6}=n\pi+(-1)^n\frac{\pi}{3}$

or, $x=n\pi+\frac{\pi}{6}+(-1)^n\frac{\pi}{3}$

Putting $n=0, x=\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$

Putting $n=1, x=\pi+\frac{\pi}{6}-\frac{\pi}{3}=\frac{5\pi}{6}$

Putting $n=2, x=2\pi+\frac{\pi}{6}-\frac{\pi}{3}=\frac{5\pi}{2}>2\pi$

∴ Required solutions are $\frac{\pi}{2}$ and $\frac{5\pi}{6}$.

9. 2070 Supp Q.No. 2a

Solve: $\tan 2x - \cot x = 0$ [2]

SOLUTION

Given equation is $\tan 2x - \cot x = 0$

or, $\tan 2x = \cot x$

or, $\tan 2x = \tan\left(\frac{\pi}{2}-x\right)$

or, $2x = n\pi+\frac{\pi}{2}-x$

or, $3x=\frac{2n\pi+\pi}{2}$

∴ $x=(2n+1)\frac{\pi}{6}$

10. 2069 Supp Q.No. 2 a

Solve: $\cos 4x = \cos 2x \left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$ [2]

SOLUTION

Given, $\cos 4x = \cos 2x$

or, $\cos 4x - \cos 2x = 0$

or, $-2\sin\left(\frac{4x+2x}{2}\right)\cdot\sin\left(\frac{4x-2x}{2}\right)=0$

or, $\sin 3x \cdot \sin x = 0$

Either $\sin 3x = 0$ ...(i)

or, $\sin x = 0$ ...(ii)

From (i)

$\sin 3x = \sin n\pi$

$3x = n\pi$

$x=\frac{n\pi}{3}$

when $n=0, x=0$

when $n=1, x=\frac{\pi}{3}$

when $n=-1, x=\frac{-\pi}{3}$

when $n=2, x=\frac{2\pi}{3}>\frac{\pi}{2}$

when $n=-2, x=\frac{-2\pi}{3}<\frac{-\pi}{2}$

From (ii)

$\sin x = 0$

$x = n\pi$

when $n=0, x=0$

when $n=1, x=\pi>\frac{\pi}{2}$

when $n=-1, x=-\pi<\frac{-\pi}{2}$

∴ Required solutions are $\frac{-\pi}{3}, 0, \frac{\pi}{3}$

11. 2069 [Set A] Q. No. 2a

Solve: $\sin x - \cos x = \sqrt{2}$. [

SOLUTION

Given equation is $\sin x - \cos x = \sqrt{2}$

Dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$, we get

$\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x=\frac{\sqrt{2}}{\sqrt{2}}$

or, $\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}=1$

or, $\sin\left(x-\frac{\pi}{4}\right)=\sin\frac{\pi}{2}$

or, $x-\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{2}$

∴ $x=n\pi+(-1)^n\frac{\pi}{2}+\frac{\pi}{4}$

12. 2069 [Set B] Q. No. 2a

Solve: $\tan 2x = \tan x (-\pi \le x \le \pi)$. [2

SOLUTION

$\tan 2x = \tan x$

or, $2x = n\pi + x$

∴ $x = n\pi$

when $n=0, x=0$

when $n=1, x=\pi$

when $n=2, x=2\pi>\pi$

when $n=-1, x=-\pi$

when $n=-2, x=-2\pi<-\pi$

So the required solutions are $-\pi, 0, \pi$

13. 2068 Q.No. 2a

Solve: $\cot x + \tan x = 2 \quad (0 \le x \le \pi)$. [2

SOLUTION

Given equation is $\cot x + \tan x = 2$

or, $\frac{1}{\tan x}+\tan x=2$

or, $1+\tan^2 x = 2\tan x$

or, $\tan^2 x - 2\tan x + 1 = 0$

or, $(\tan x - 1)^2 = 0$

or, $\tan x = 1 = \tan\frac{\pi}{4}$

∴ $x = n\pi+\frac{\pi}{4}$

When $n=0, x=\frac{\pi}{4}$

When $n=1, x=\pi+\frac{\pi}{4}=\frac{5\pi}{4}>\pi$

∴ Required solution is $x=\frac{\pi}{4}$

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4 MARKS QUESTIONS

14. 2076 Set B Q.No. 7a

Solve for general values of θ: [4]

$\sin\theta + \sin 2\theta + \sin 3\theta = \cos\theta + \cos 2\theta + \cos 3\theta$.

SOLUTION

$\sin\theta + \sin 2\theta + \sin 3\theta = \cos\theta + \cos 2\theta + \cos 3\theta$

or, $2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) + \sin 2\theta = 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) + \cos 2\theta$

or, $2\sin 2\theta.\cos\theta + \sin 2\theta = 2\cos 2\theta.\cos\theta + \cos 2\theta$

or, $\sin 2\theta\,[2\cos\theta + 1] = \cos 2\theta\,[2\cos\theta + 1]$

or, $\sin 2\theta\,(2\cos\theta + 1) - \cos 2\theta\,(2\cos\theta + 1) = 0$

or, $(2\cos\theta + 1)(\sin 2\theta - \cos 2\theta) = 0$

Either, $2\cos\theta + 1 = 0$ ...(i)

or, $\sin 2\theta - \cos 2\theta = 0$ ...(ii)

From (i),

$\cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore\ \theta = 2n\pi \pm \frac{2\pi}{3} = (6n \pm 2)\frac{\pi}{3}$

From (ii)

$\sin 2\theta = \cos 2\theta$

or, $\frac{\sin 2\theta}{\cos 2\theta} = 1$

or, $\tan 2\theta = 1 = \tan\frac{\pi}{4}$

or, $2\theta = n\pi + \frac{\pi}{4}$

or, $2\theta = \frac{4n\pi + \pi}{4}$

$\therefore\ \theta = (4n + 1)\frac{\pi}{8}$

So, the required solutions are $(6n \pm 2)\frac{\pi}{3}$, $(4n + 1)\frac{\pi}{8}$

15. 2076 Set C Q.No. 7a

Solve for general values of x: $\tan^2 x = \sec x + 1$. [4]

SOLUTION

We have,

$\tan^2 x = \sec x + 1$

or, $\sec^2 x - 1 - \sec x - 1 = 0$

or, $\sec^2 x - \sec x - 2 = 0$

or, $\sec^2 x - 2\sec x + \sec x - 2 = 0$

or, $\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

or, $(\sec x - 2)(\sec x + 1) = 0$

Either $(\sec x - 2) = 0$

or, $\sec x = 2$ or, $\sec x + 1 = 0$

or, $\cos x = \frac{1}{2}$ or, $\sec x = -1$

or, $\cos x = \cos\frac{\pi}{3}$ or, $\cos x = -1$

or, $\cos x = \cos\pi$

or, $x = 2n\pi \pm \frac{\pi}{3}$ or, $x = 2n\pi + \pi$

Hence, the required values of x are $2n\pi \pm \frac{\pi}{3}$ or $2n\pi + \pi$, where $n \in Z$.

16. 2075 GIE Q.No. 7a

Solve for general values of x:

$\tan x - \cot x = \operatorname{cosec} x$. [4]

SOLUTION

Given equation is

$\tan x - \cot x = \operatorname{cosec} x$

or, $\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{1}{\sin x}$

or, $\frac{\sin^2 x - \cos^2 x}{\sin x\cos x} = \frac{1}{\sin x}$

or, $-(\cos^2 x - \sin^2 x) = \cos x$

or, $-\cos 2x = \cos x$

or, $\cos 2x + \cos x = 0$

or, $2\cos\frac{2x + x}{2}\cos\frac{2x - x}{2} = 0$

or, $\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

Either $\cos\frac{3x}{2} = 0$ ...(i)

or $\cos\frac{x}{2} = 0$ ...(ii)

From (i),

$\cos\frac{3x}{2} = 0$

or, $\frac{3x}{2} = (2n + 1)\frac{\pi}{2}$

$\therefore\ x = (2n + 1)\frac{\pi}{3}$

From (ii),

$\cos\frac{x}{2} = 0$

or, $\frac{x}{2} = (2n + 1)\frac{\pi}{2}$

$\therefore\ x = (2n + 1)\pi$

General values of x are $(2n + 1)\frac{\pi}{3}$ and $(2n + 1)\pi$.

17. 2075 Set B Q.No. 7a

Solve for general values of x :

$\tan x + \tan 2x = \tan 3x$. [4]

SOLUTION

$\tan x + \tan 2x = \tan 3x$

or, $\tan x + \tan 2x - \tan 3x = 0$

or, $\tan x + \tan 2x - \tan(x + 2x) = 0$

or, $\tan x + \tan 2x - \frac{\tan x + \tan 2x}{1 - \tan x.\tan 2x} = 0$

or, $(\tan x + \tan 2x)\left(1 - \frac{1}{1 - \tan x\ \tan 2x}\right) = 0$

Either, $\tan x + \tan 2x = 0$ ... (i)

or, $1 - \frac{1}{1 - \tan x\tan 2x} = 0$ ... (ii)

From (i)

$\tan x + \tan 2x = 0$

or, $\tan 2x = \tan(-x)$

or, $2x = n\pi + (-x)$

or, $3x = n\pi$

$\therefore x = \frac{n\pi}{3}$

From (ii)

$1 = \frac{1}{1 - \tan x \tan 2x}$

or, $1 - \tan x \tan 2x = 1$

or, $\tan x \tan 2x = 0$

Either, $\tan x = 0 \Rightarrow x = n\pi$

or, $\tan 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2}$

$\therefore x = n\pi, \frac{n\pi}{2}, \frac{n\pi}{3}$

18. 2075 Set C Q.No. 7a OR

Solve: $2\sin^2 x + \sin x = 1$. [4]

SOLUTION

$2\sin^2 x + \sin x = 1$

or, $2\sin^2 x + \sin x - 1 = 0$

or, $2\sin^2 x + 2\sin x - \sin x - 1 = 0$

or, $2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

or, $(\sin x + 1)(2\sin x - 1) = 0$

Either, $\sin x + 1 = 0$ ... (i)

or, $2\sin x - 1 = 0$ ... (ii)

From (i) $\sin x = -1$

or, $\sin x = -\sin\frac{\pi}{2}$

or, $\sin x = \sin\left(-\frac{\pi}{2}\right)$

$\therefore x = n\pi + (-1)^n \cdot \left(-\frac{\pi}{2}\right)$

From (ii), $2\sin x - 1 = 0$

or, $\sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n \cdot \frac{\pi}{6}$

Required general solutions are

$\therefore x = n\pi + (-1)^n \cdot \left(-\frac{\pi}{2}\right), n\pi + (-1)^n \cdot \frac{\pi}{6}$

19. 2074 Supp Q.No. 7a

Solve: $\sin\theta - \sqrt{3}\cos\theta = 2$ [4]

SOLUTION

Given, $\sin\theta - \sqrt{3}\cos\theta = 2$

Dividing each term by

$\sqrt{(\text{coeff. of } \sin\theta)^2 + (\text{coeff. of } \cos\theta)^2} = \sqrt{1^2 + (\sqrt{3})^2} = 2$,

we get

$\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta = \frac{2}{2}$

or, $\sin\theta\cos\frac{\pi}{3} - \cos\theta\sin\frac{\pi}{3} = 1$

or, $\sin\left(\theta - \frac{\pi}{3}\right) = 1$

or, $\theta - \frac{\pi}{3} = (4n+1)\frac{\pi}{2}$ [$\because \sin\theta = 1$, G.v. of $\theta = (4n+1)\frac{\pi}{2}$]

$\therefore \theta = (4n+1)\frac{\pi}{2} + \frac{\pi}{3}$

20. 2074 Set A Q.No. 7a OR

Solve: $\sec x \tan x = \sqrt{2}$. [4

SOLUTION

Given, $\sec x \cdot \tan x = \sqrt{2}$

or, $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sqrt{2}$

or, $\sin x = \sqrt{2}\cos^2 x$

or, $\sin x = \sqrt{2}(1 - \sin^2 x)$

or, $\sin x = \sqrt{2} - \sqrt{2}\sin^2 x$

or, $\sqrt{2}\sin^2 x + \sin x - \sqrt{2} = 0$

or, $\sqrt{2}\sin^2 x + 2\sin x - \sin x - \sqrt{2} = 0$

or, $\sqrt{2}\sin x(\sin x + \sqrt{2}) - 1(\sin x + \sqrt{2}) = 0$

or, $(\sin x + \sqrt{2})(\sqrt{2}\sin x - 1) = 0$

Either $\sin x + \sqrt{2} = 0$

$\Rightarrow \sin x = -\sqrt{2}$ impossible as $-1 \le \sin x \le 1$

or, $\sqrt{2}\sin x - 1 = 0$

or, $\sin x = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\therefore$ General value of x is $n\pi + (-1)^n\frac{\pi}{4}$

21. 2074 Set B Q.No. 7a

Solve for general values of x:
$\cos^3 x - \cos x \sin x - \sin^3 x = 1$. [4

SOLUTION

$\cos^3 x - \cos x \sin x - \sin^3 x = 1$

or, $(\cos^3 x - \sin^3 x) - \cos x \sin x - 1 = 0$

or, $(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x) - \cos x \sin x - 1 = 0$

or, $(\cos x - \sin x)(1 + \sin x \cos x) - 1(1 + \sin x \cos x) = 0$

or, $(1 + \sin x \cos x)(\cos x - \sin x - 1) = 0$

Since $1 + \sin x \cos x \neq 0$,

$\cos x - \sin x - 1 = 0$

$\cos x - \sin x = 1$

Dividing both sides by $\sqrt{2}$

$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

or, $\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

or, $\cos\left(x + \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

or, $x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$\therefore x = 2n\pi \pm \frac{\pi}{4} - \frac{\pi}{4}$

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22. 2073 Set D Q.No. 7a

Solve: $\tan x + \tan 2x + \tan 3x = 0$ [4]

SOLUTION

Given,

$(\tan x + \tan 2x) + \tan 3x = 0$

or, $(\tan x + \tan 2x) + \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$

or, $(\tan x + \tan 2x)\left\{1 + \frac{1}{1 - \tan x \tan 2x}\right\} = 0$

or, Either, $\tan x + \tan 2x = 0$ ...(i)

or, $1 + \frac{1}{1 - \tan x \tan 2x} = 0$ ...(ii)

From (i),

$2\tan 2x = -\tan x = \tan(-x)$

$2x = n\pi - x$

$3x = n\pi$

$x = \frac{n\pi}{3}$

From (ii)

$1 + \frac{1}{1 - \tan x \tan 2x} = 0$

or, $\tan x \tan 2x = 2$

or, $\tan x\left(\frac{2\tan x}{1 - \tan^2 x}\right) = 2$

or, $\tan^2 x = 1 - \tan^2 x$

or, $2\tan^2 x = 1$

or, $\tan^2 x = \frac{1}{2}$

or, $\tan x = \pm\frac{1}{\sqrt{2}}$

or, $\tan x = \tan\tan^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$

$x = n\pi + \tan^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$

Hence, the general solutions are

$x = \frac{n\pi}{3}$ and $x = n\pi + \tan^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$.

23. 2072 Supp. Q.No. 7a

Solve: $\sec x \tan x = \sqrt{2}$. [4]

➥ Please refer to 2074 Set A Q.No. 7a OR

24. 2072 Set C Q.No. 7a

Solve: $\sec x . \tan x = \sqrt{2}$ [4]

➥ Please refer to 2074 Set A Q.No. 7a OR

25. 2071 Old Q.No. 8a Or

Solve: $\sqrt{3}\sin x - \cos x = \sqrt{3}$ [4]

SOLUTION

Given equation is $\sqrt{3}\sin x - \cos x = \sqrt{3}$

Dividing both sides by $\sqrt{(\sqrt{3})^2 + (-1)^2} = 2$, we get

$\sin x \cdot \frac{\sqrt{3}}{2} - \cos x \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$

or, $\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

or, $\sin\left(x - \frac{\pi}{6}\right) = \sin\frac{\pi}{3}$

or, $\sin\left(x - \frac{\pi}{6}\right) = \sin\frac{\pi}{3}$

or, $x - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{3}$

$\therefore\ x = n\pi + \frac{\pi}{6} + (-1)^n\frac{\pi}{3}$

26. 2070 Set C Q.No. 7 a

Solve: $\sin x + \cos x = \sqrt{2}$ $(-2\pi \le x \le 2\pi)$. [4]

SOLUTION

Given equation is $\sin x + \cos x$

Dividing both sides by $\sqrt{1^2 + 1^2} = \sqrt{2}$, we get

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{2}}{\sqrt{2}}$

or, $\cos x \cos\frac{\pi}{4} + \sin x \cdot \sin\frac{\pi}{4} = 1$

or, $\cos\left(x - \frac{\pi}{4}\right) = \cos 0^\circ$

or, $x - \frac{\pi}{4} = 2n\pi$

or, $x = 2n\pi + \frac{\pi}{4}$

when $n = 0$, $x = 2 \times 0 \times \pi + \frac{\pi}{4} = \frac{\pi}{4}$

when $n = 1$, $x = 2 \times 1 \times \pi + \frac{\pi}{4} = \frac{9\pi}{4} > 2\pi$

when $n = -1$, $x = -2\pi + \frac{\pi}{4} = \frac{-7\pi}{4}$

when $n = -2$, $x = -4\pi + \frac{\pi}{4} = \frac{-15\pi}{4} < -2\pi$

Hence, the required values lying between -2π and 2π are $\frac{\pi}{4}$ and $\frac{-7\pi}{4}$.

27. 2070 Set D Q.No. 7 a

Solve: $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$. [4]

SOLUTION

Given, $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$

or, $4\sin^2\theta - 8\cos\theta + 1 = 0$

or, $4(1 - \cos^2\theta) - 8\cos\theta + 1 = 0$

or, $4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

or, $-4\cos^2\theta - 8\cos\theta + 5 = 0$

or, $4\cos^2\theta + 8\cos\theta - 5 = 0$

or, $(2\cos\theta + 5)(2\cos\theta - 1) = 0$

Either, $2\cos\theta + 5 = 0$

$\Rightarrow\ \cos\theta = \frac{-5}{2}$ which is not possible as $-1 \le \cos\theta \le 1$

or, $2\cos\theta - 1 = 0$

$\Rightarrow\ \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore\ \theta = 2n\pi \pm \frac{\pi}{3}$

28. 2070 Old Q.No. 8 a Or

Solve for general values of θ,

$\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$. [4]

➥ Please refer 2070 Set D Q.No. 7 a

29. 2069 [Set B] Old Q. No. 8a OR

Solve: $2\tan 3x \cos 2x + 1 = \tan 3x + 2\cos 2x$ [4]

SOLUTION

$2 \tan 3x \cos 2x + 1 = \tan 3x + 2 \cos 2x$

or, $2 \tan 3x \cos 2x + 1 - \tan 3x - 2 \cos 2x = 0$

or, $\tan 3x (2 \cos 2x - 1) - 1 (2 \cos 2x - 1) = 0$

or, $(2 \cos 2x - 1)(\tan 3x - 1) = 0$

Either, $2 \cos 2x - 1 = 0$

or, $\cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$

or, $2x = 2n\pi \pm \frac{\pi}{3}$

$\therefore \quad x = n\pi \pm \frac{\pi}{6}$

or, $\tan 3x - 1 = 0$

or, $\tan 3x = 1 = \tan \frac{\pi}{4}$

or, $3x = n\pi + \frac{\pi}{4} = (4n + 1)\frac{\pi}{4}$

$x = (4n + 1)\frac{\pi}{12}$

$\therefore \quad x = n\pi \pm \frac{\pi}{6}, (4n + 1)\frac{\pi}{12}$

30. 2069 [Set A] Old Q. No. 8a Or

Solve for general values of x:

$\cos x + \cos 3x + \cos 5x + \cos 7x = 0$. [4]

SOLUTION

$\cos x + \cos 3x + \cos 5x + \cos 7x = 0$

or, $(\cos 7x + \cos x) + (\cos 5x + \cos 3x) = 0$

or, $2\cos \frac{7x + x}{2} \cdot \cos \frac{7x - x}{2} + 2\cos \frac{5x + 3x}{2} \cdot \cos \frac{5x - 3x}{2} = 0$

or, $2 \cos 4x \cdot \cos 3x + 2 \cos 4x \cos x = 0$

or, $\cos 4x (\cos 3x + \cos x) = 0$

or, $\cos 4x \cdot 2 \cos \left(\frac{3x + x}{2}\right) \cdot \cos \left(\frac{3x - x}{2}\right) = 0$

or, $\cos 4x \cdot \cos 2x \cdot \cos x = 0$

Either $\cos 4x = 0$ ...(i)

or $\cos 2x = 0$ ...(ii)

or $\cos x = 0$ ...(iii)

From (i)

$\cos 4x = 0$

or, $\cos 4x = \cos \frac{\pi}{2}$

or, $4x = 2n\pi \pm \frac{\pi}{2}$

or, $4x = \frac{4n\pi \pm \pi}{2}$

$\therefore \quad x = (4n \pm 1)\frac{\pi}{8}$

From (ii)

$\cos 2x = 0 = \cos \frac{\pi}{2}$

or, $2x = 2n\pi \pm \frac{\pi}{2}$

or, $2x = \frac{4n\pi \pm \pi}{2}$

$\therefore \quad x = (4n \pm 1)\frac{\pi}{4}$

and, from (iii)

$\cos x = 0 = \cos \frac{\pi}{2}$

or, $x = 2n\pi \pm \frac{\pi}{2}$

$\therefore \quad x = (4n \pm 1)\frac{\pi}{2}$

Required solutions are

$(4n \pm 1)\frac{\pi}{8}, (4n \pm 1)\frac{\pi}{4}$ and $(4n \pm 1)\frac{\pi}{2}$

31. 2068 Old Q.No. 8a(Or)

Solve: $\cos \theta + \sqrt{3} \sin \theta = 2 \quad (-2\pi \le \theta \le 2\pi)$ [4]

SOLUTION

Given, $\cos \theta + \sqrt{3} \sin \theta = 2$

Dividing both sides by $\sqrt{1^2 + (\sqrt{3})^2} = 2$, we get

$\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta = \frac{2}{2}$

or, $\cos \theta \cos \frac{\pi}{3} + \sin \theta \sin \frac{\pi}{3} = 1$

or, $\cos \left(\theta - \frac{\pi}{3}\right) = \cos 0$

or, $\theta - \frac{\pi}{3} = 2n\pi \pm 0$

or, $\theta = 2n\pi + \frac{\pi}{3}$

when $n = 0, \theta = \frac{\pi}{3}$

when $n = 1, \theta = 2\pi + \frac{\pi}{3} = \frac{7\pi}{3} > 2\pi$

when $n = -1, \theta = -2\pi + \frac{\pi}{3} = \frac{-5\pi}{3}$

when $n = -2, \theta = -4\pi + \frac{\pi}{3} = \frac{-12\pi + \pi}{3} = \frac{-11\pi}{3} < -2\pi$

$\therefore$ Required solutions are $\theta = \frac{\pi}{3}$ and $\frac{-5\pi}{3}$

32. 2067 Q.No. 8a OR

Solve for general values of x:

$2 \sin^2 x + \sin^2 2x = 2$ [4]

SOLUTION

$2\sin^2 x + \sin^2 2x = 2$

or, $2 \sin^2 x + (2 \sin x \cos x)^2 = 2$

or, $2 \sin^2 x + 4 \sin^2 x \cos^2 x = 2$

or, $2 \sin^2 x + 4 \sin^2 x (1 - \sin^2 x) - 2 = 0$

or, $2 \sin^2 x + 4 \sin^2 x - 4 \sin^4 x - 2 = 0$

or, $4 \sin^4 x - 6 \sin^2 x + 2 = 0$

or, $4 \sin^4 x - 4 \sin^2 x - 2 \sin^2 x + 2 = 0$

or, $4 \sin^2 x (\sin^2 x - 1) - 2(2 \sin^2 x - 1) = 0$

or, $(4 \sin^2 x - 2)(\sin^2 x - 1) = 0$

Either $4 \sin^2 x - 2 = 0$ ... (i)

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OR, $\sin^2 x - 1 = 0$ ... (ii)

From (i), $4\sin^2 x - 2 = 0$

$\sin^2 x = \frac{2}{4} = \frac{1}{2}$

or, $\sin x = \pm\frac{1}{\sqrt{2}}$

or, $\sin x = \pm\sin\frac{\pi}{4} = \sin\left(\pm\frac{\pi}{4}\right)$

$\therefore x = n\pi + (-1)^n\left(\pm\frac{\pi}{4}\right)$

$= n\pi \pm \frac{\pi}{4} = (4n \pm 1)\frac{\pi}{4}$

From (ii),

$\sin^2 x - 1 = 0$

or, $\sin^2 x = 1$

or, $\sin x = \pm 1$

or, $\sin x = \pm\sin\frac{\pi}{2} = \sin\left(\pm\frac{\pi}{2}\right)$

$\therefore x = n\pi + (-1)^n\left(\pm\frac{\pi}{2}\right) = n\pi \pm \frac{\pi}{2}$

$= (2n \pm 1)\frac{\pi}{2}$

$\therefore$ The general solutions are

$x = (4n \pm 1)\frac{\pi}{4}, (2n \pm 1)\frac{\pi}{2}$

33. 2066 Q.No. 8a OR

Solve for general values of θ:

$\tan(\theta + \alpha).\tan(\theta - \alpha) = 1$ [4]

SOLUTION

$\tan(\theta + \alpha)\tan(\theta - \alpha) = 1$

or, $\left(\frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}\right)\left(\frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha}\right) = 1$

or, $\frac{\tan^2\theta - \tan^2\alpha}{1 - \tan^2\theta\tan^2\alpha} = 1$

or, $\tan^2\theta - \tan^2\alpha = 1 - \tan^2\theta\tan^2\alpha$

or, $\tan^2\theta\tan^2\alpha - \tan^2\alpha + \tan^2\theta - 1 = 0$

or, $\tan^2\alpha(\tan^2\theta - 1) + 1(\tan^2\theta - 1) = 0$

or, $(\tan^2\alpha + 1)(\tan^2\theta - 1) = 0$

or, $\tan^2\theta - 1 = 0$ $(\because \tan^2\alpha + 1 \neq 0)$

or, $\tan^2\theta = 1$

or, $\tan\theta = \pm 1$

or, $\tan\theta = \pm\tan\frac{\pi}{4} = \tan\left(\pm\frac{\pi}{4}\right)$

General value of $\theta = n\pi \pm \frac{\pi}{4} = (4n \pm 1)\frac{\pi}{4}$

34. 2065 Q.No. 8a OR

Solve: [4]

$\sin\theta + \sin 2\theta + \sin 3\theta = \cos\theta + \cos 2\theta + \cos 3\theta$.

Please refer to 2076 Set B Q.No. 7a

35. 2064 Q.No. 8a OR

If $\sin 2x = 3\sin 2y$, prove that:

$2\tan(x - y) = \tan(x + y)$ [4]

SOLUTION

Given that: $\sin 2x = 3\sin 2y$

or, $\frac{\sin 2x}{\sin 2y} = \frac{3}{1}$

or, $\frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{3+1}{3-1}$

[$\because$ By using componendo and dividendo]

or, $\frac{2\sin\left(\frac{2x+2y}{2}\right).\cos\left(\frac{2x-2y}{2}\right)}{2\cos\left(\frac{2x+2y}{2}\right).\sin\left(\frac{2x-2y}{2}\right)} = 2$

or, $\frac{\sin(x+y).\cos(x-y)}{\cos(x+y).\sin(x-y)} = 2$

or, $\frac{\sin(x+y)}{\cos(x+y)} = 2 \times \frac{\sin(x-y)}{\cos(x-y)}$

or, $\tan(x+y) = 2\tan(x-y)$

i.e. $2\tan(x-y) = \tan(x+y)$; Hence proved.

36. 2064 Q.No. 8a

Find the general value of x when

$\cos x + \sin x = \cos 2x + \sin 2x$ [4]

SOLUTION

We have,

$\cos x + \sin x = \cos 2x + \sin 2x$

Dividing both sides by $\sqrt{2}$

$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}\cos 2x + \frac{1}{\sqrt{2}}\sin 2x.$

or, $\cos\frac{\pi}{4}.\cos x + \sin\frac{\pi}{4}\sin x = \cos\frac{\pi}{4}\cos 2x + \sin\frac{\pi}{4}\sin 2x$

or, $\cos\left(x - \frac{\pi}{4}\right) = \cos\left(2x - \frac{\pi}{4}\right)$

$\therefore \left(2x - \frac{\pi}{4}\right) = 2n\pi \pm \left(x - \frac{\pi}{4}\right)$

or, $2x - \frac{\pi}{4} \pm \left(x - \frac{\pi}{4}\right) = 2n\pi$

Taking +ve sign

$2x - \frac{\pi}{4} + x - \frac{\pi}{4} = 2n\pi$

$3x - \frac{\pi}{2} = 2n\pi$

or, $3x = \left(2n\pi + \frac{\pi}{2}\right)$

or, $3x = \left(2n + \frac{1}{2}\right)\pi$

$\therefore x = \left(2n + \frac{1}{2}\right)\frac{\pi}{3} = \frac{(4n+1)}{2}.\frac{\pi}{3} = (4n+1)\frac{\pi}{6}$

Again, taking –ve sign

$2x - \frac{\pi}{4} - x + \frac{\pi}{4} = 2n\pi$

or, $x = 2n\pi$

Thus, the general values of x are $2n\pi$ or $(4n+1)\frac{\pi}{6}$.

37. 2063 Q.No. 8a

Find the general values of x, when $\sin 2x \tan x + 1 = \sin 2x + \tan x$. [4]

SOLUTION

We have,

$\sin 2x \tan x + 1 = \sin 2x + \tan x$

or, $\sin 2x \tan x + 1 - \sin 2x - \tan x = 0$

or, $\sin 2x \tan x - \sin 2x - \tan x + 1 = 0$

or, $\sin 2x (\tan x - 1) - 1 (\tan x - 1) = 0$

or, $(\tan x - 1)(\sin 2x - 1) = 0$

Either $\tan x - 1 = 0$ or, $\sin 2x - 1 = 0$

$\Rightarrow \tan x = 1$ $\Rightarrow \sin 2x = 1$

$\Rightarrow \tan x = \tan \frac{\pi}{4}$ $\Rightarrow \sin 2x = \sin \frac{\pi}{2}$

$\Rightarrow x = n\pi + \frac{\pi}{4}$ $\Rightarrow 2x = (4n+1)\frac{\pi}{2}$ $\left[\begin{array}{l}\sin\theta = 1 \\ \Rightarrow \theta = (4n+1)\frac{\pi}{2}\end{array}\right]$

$\therefore x = (4n+1)\frac{\pi}{4}$ $\therefore x = (4n+1)\frac{\pi}{4}$

Hence, the required general value of x is $(4n+1)\frac{\pi}{4}$.

38. 2062 Q.No. 8a

Solve: $\cos 3x + \cos 2x = \sin \frac{3x}{2} + \sin \frac{x}{2}$, $0 \le x \le \pi$. [4]

SOLUTION

We have,

$\cos 3x + \cos 2x = \sin \frac{3x}{2} + \sin \frac{x}{2}$

or, $2\cos\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right) = 2\sin\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right)$

or, $2\cos\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right) = 2\sin\left(\frac{4x}{4}\right)\cos\left(\frac{2x}{4}\right)$

or, $\cos\frac{5x}{2}\cos\frac{x}{2} - \sin x \cos\frac{x}{2} = 0$

or, $\cos\frac{x}{2}\left(\cos\frac{5x}{2} - \sin x\right) = 0$

Either, $\cos\frac{x}{2} = 0$ ... (1)

OR, $\cos\frac{5x}{2} - \sin x = 0$ ... (2)

From (1)

$\cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{x}{2} = \cos\frac{\pi}{2}$

or, $\frac{x}{2} = (2n+1)\frac{\pi}{2}$

$\therefore x = (2n+1)\pi$

For $n = 0$, $x = \pi \in [0, \pi]$

Again, from (2)

$\cos\frac{5x}{2} - \sin x = 0$

or, $\cos\frac{5x}{2} = \sin x$

or, $\cos\frac{5x}{2} = \cos\left(\frac{\pi}{2} - x\right)$

$\Rightarrow \frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x\right)$ ... (3)

Taking +ve sign in (3), we get

$\frac{5x}{2} = 2n\pi + \frac{\pi}{2} - x$

or, $\frac{5x}{2} + x = 2n\pi + \frac{\pi}{2}$

or, $\frac{7x}{2} = \frac{4n\pi + \pi}{2}$

or, $7x = 4n\pi + \pi$

$\therefore x = \frac{4n\pi + \pi}{7}$

For $n = 0$, $x = \frac{\pi}{7} \notin [0, \pi]$

For $n = 1$, $x = \frac{5\pi}{7} \in [0, \pi]$

Taking –ve sign in (3), we get

$\frac{5x}{2} = 2n\pi - \frac{\pi}{2} + x$

or, $\frac{5x}{2} - x = 2n\pi - \frac{\pi}{2}$

or, $\frac{3x}{2} = \frac{4n\pi - \pi}{2}$

or, $3x = 4n\pi - \pi$

$\therefore x = \frac{4n\pi - \pi}{3}$

For $n = 0$, $x = -\frac{\pi}{3} \notin [0, \pi]$

For $n = 1$, $x = \pi \in [0, \pi]$

Hence, the values of x in the interval $0 \le x \le \pi$ is $[0, \pi]$ are $\frac{\pi}{7}, \frac{5\pi}{7}$ and π.

39. 2062 Q.No. 8a OR

Solve: $\tan^2 x = \sec x + 1$ [4

➥ Please refer to 2076 Set C Q.No. 7a

40. 2061 Q.No. 8a OR

Solve for general values of x:

$\cos x + \cos 2x + \cos 3x = 0$ [4

SOLUTION

Here,

$\cos x + \cos 2x + \cos 3x = 0$

or, $(\cos 3x + \cos x) + \cos 2x = 0$

or, $2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \cos 2x = 0$

or, $2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right) + \cos 2x = 0$

or, $2\cos 2x \cos x + \cos 2x = 0$

or, $\cos 2x (2\cos x + 1) = 0$

Either $\cos 2x = 0$ Or $2\cos x + 1 = 0$

$\Rightarrow \cos 2x = \cos\frac{\pi}{2}$ $\Rightarrow 2\cos x = -1$

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or, $2x = (2n+1)\frac{\pi}{2}$ or, $\cos x = -\frac{1}{2}$

or, $x = (2n+1)\frac{\pi}{4}$ or, $\cos x = \cos\frac{2\pi}{3}$

or, $x = 2n\pi \pm \frac{2\pi}{3} = (6n \pm 2)\frac{\pi}{3}$

Hence, the general values of x are $(2n+1)\frac{\pi}{4}$ and $(6n \pm 2)\frac{\pi}{3}$.

41. 2060 Q.No. 8a OR

Solve for general values of x:

$7\sin^2 x + 3\cos^2 x = 4$ [4]

SOLUTION

Here, $7\sin^2 x + 3\cos^2 x = 4$

or, $7\sin^2 x + 3(1 - \sin^2 x) = 4$

or, $7\sin^2 x + 3 - 3\sin^2 x = 4$

or, $4\sin^2 x = 4 - 3$

or, $\sin^2 x = \frac{1}{4}$

or, $\sin x = \pm\frac{1}{2}$

or, $\sin x = \pm\sin\frac{\pi}{6}$

or, $\sin x = \sin\left(\pm\frac{\pi}{6}\right)$

$\therefore\ x = n\pi \pm \frac{\pi}{6}$

42. 2059 Q.No. 8a OR

Solve: $\cot x + \tan x = 2$ [4]

SOLUTION

We have,

$\cot x + \tan x = 2$

or, $\frac{1}{\tan x} + \tan x = 2$

or, $1 + \tan^2 x = 2\tan x$

or, $\tan^2 x - 2\tan x + 1 = 0$

or, $(\tan x - 1)^2 = 0$

or, $\tan x = 1$

or, $\tan x = \tan\frac{\pi}{4}$

$\therefore\ x = n\pi + \frac{\pi}{4}$, for any integer n.

43. 2058 Q.No. 8a OR

Solve: $\tan^2 x = \sec x + 1$ [4]

➥ Please refer to 2076 Set C Q.No. 7a

44. 2057 Q.No. 8a

Solve: $2\sin 3x - 2\sin x + 5\cos 2x = 0$ [4]

SOLUTION

We have

$2\sin 3x - 2\sin x + 5\cos 2x = 0$

or, $2(3\sin x - 4\sin^3 x) - 2\sin x + 5(1 - 2\sin^2 x) = 0$

or, $6\sin x - 8\sin^3 x - 2\sin x + 5 - 10\sin^2 x = 0$

or, $-8\sin^3 x - 10\sin^2 x + 4\sin x + 5 = 0$

or, $8\sin^3 x + 10\sin^2 x - 4\sin x - 5 = 0$

or, $2\sin^2 x(4\sin x + 5) - 1(4\sin x + 5) = 0$

or, $(4\sin x + 5)(2\sin^2 x - 1) = 0$

or, $(4\sin x + 5)\cos 2x = 0$

Either, $4\sin x + 5 = 0$

or, $\sin x = \frac{-5}{4} = -1.25$,

which is impossible as $-1 \le \sin x \le 1$.

So, the required solution is given by

$\cos 2x = 0$

or, $2x = (2n+1)\frac{\pi}{2}$

$\therefore\ x = (2n+1)\frac{\pi}{4}$, where $n \in Z$.

45. 2056 Q.No. 8a OR

Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$ [4]

SOLUTION

Here, $\sqrt{(\text{Coeff. of } \sin x)^2 + (\text{Coeff. of } \cos x)^2}$

$= \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

Dividing both sides by 2, we get

$\sin x \cdot \frac{1}{2} + \cos x \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2}$

or, $\sin x.\sin\frac{\pi}{6} + \cos x.\cos\frac{\pi}{6} = \frac{1}{\sqrt{2}}$

or, $\cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}$

or, $x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$ $[\because \cos x = \cos\alpha \Rightarrow x = 2n\pi \pm \alpha]$

$\therefore\ x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$

46. 2055 Old Model Q.No. 8a OR

Solve: $\sin x + \sqrt{3}\cos x = \sqrt{2}$ [4]

➥ Please refer 2056 Q.No. 8a OR

C. PROPERTIES OF TRIANGLE

2 MARKS QUESTIONS

1. 2071 Old Q.No. 1c

Prove that: $1 - \tan\frac{A}{2}\tan\frac{B}{2} = \frac{2c}{a+b+c}$. [2]

SOLUTION

L.H.S. $= 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$= 1 - \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

$= 1 - \sqrt{\frac{(s-b)(s-c)(s-c)(s-a)}{s(s-a)\cdot s(s-b)}}$

$= 1 - \frac{s-c}{s} = \frac{s-s+c}{s} = \frac{c}{\frac{a+b+c}{2}} = \frac{2c}{a+b+c}$

= R.H.S.

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2. 2069 [Set B] Old Q. No. 1c

In any triangle ABC, prove that: $a^2 + b^2 + c^2 - 2(bc\cos A + ca\cos B + ab\cos C) = 0$. [2]

SOLUTION

$$\text{L.H.S.} = a^2 + b^2 + c^2 - 2(bc\cos A + ca\cos B + ab\cos C)$$
$$= a^2 + b^2 + c^2 - 2bc \cdot \frac{b^2 + c^2 - a^2}{2bc} - 2ca \cdot \frac{c^2 + a^2 - b^2}{2ca} - 2ab \cdot \frac{a^2 + b^2 - c^2}{2ab}$$
$$= a^2 + b^2 + c^2 - (b^2 + c^2 - a^2) - (c^2 + a^2 - b^2) - (a^2 + b^2 - c^2)$$
$$= a^2 + b^2 + c^2 - b^2 - c^2 + a^2 - c^2 - a^2 + b^2 - a^2 - b^2 + c^2$$
$$= 0 = \text{R.H.S.}$$

3. 2067 Q.No. 1c

In any triangle if $\cos B = \frac{\sin A}{2\sin C}$, prove that the triangle is isosceles. [2]

SOLUTION

$$\cos B = \frac{\sin A}{2\sin C}$$

or, $$\frac{c^2 + a^2 - b^2}{2ac} = \frac{\frac{a}{2R}}{2\frac{b}{2R}}$$

or, $$\frac{c^2 + a^2 - b^2}{2ac} = \frac{a}{2c}$$

or, $c^2 + a^2 - b^2 = a^2$

or, $b^2 = c^2$

$\therefore$ $b = c$

Since the length of two sides is equal, the given triangle is isosceles.

4. 2064 Q.No. 1c

In a ΔABC, prove that $\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$ [2]

SOLUTION

$$\text{L.H.S.} = \frac{c - b\cos A}{b - c\cos A}$$
$$= \frac{2R\sin C - 2R\sin B\cos A}{2R\sin B - 2R\sin C\cos A}$$
$$= \frac{2R(\sin C - \sin B\cos A)}{2R(\sin B - \sin C\cos A)}$$
$$= \frac{\sin[180° - (A + B)] - \cos A.\sin B}{\sin[180° - (C + A)] - \cos A.\sin C}$$
$$= \frac{\sin(A + B) - \cos A\sin B}{\sin(C + A) - \cos A\sin C}$$
$$= \frac{\sin A\cos B + \cos A\sin B - \cos A\sin B}{\sin C\cos A + \cos C\sin A - \cos A\sin C}$$
$$= \frac{\sin A\cos B}{\cos C\sin A}$$
$$= \frac{\cos B}{\cos C} = \text{R.H.S. Proved.}$$

5. 2063 Q.No. 1c

In a ΔABC, prove that: [2]

$(b + c)\cos A + (c + a)\cos B + (a + b)\cos C = a + b + c$

SOLUTION

$$\text{L.H.S.} = (b + c)\cos A + (c + a)\cos B + (a + b)\cos C$$
$$= b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$$
$$= (b\cos C + c\cos B) + (c\cos A + a\cos C) + (a\cos B + b\cos A)$$
$$= a + b + c = \text{R.H.S. proved.}$$

6. 2061 Q.No. 1c

In any triangle ABC, if $\cos B = \frac{\sin A}{2\sin C}$, show that the triangle is isosceles. [2]

SOLUTION

Given that, in any ΔABC,

$$\cos B = \frac{\sin A}{2\sin C}$$

or, $2\cos B\sin C = \sin A$

or, $2\cos B\sin C = \sin[\pi - (B + C)]$ [$\because A+B+C=\pi$]

or, $2\cos B\sin C = \sin(B + C)$ [$\because \sin(\pi-\theta)=\sin\theta$]

or, $2\cos B\sin C = \sin B\cos C + \cos B\sin C$

or, $\sin B\cos C + \cos B\sin C - 2\cos B\sin C = 0$

or, $\sin B\cos C - \cos B\sin C = 0$

or, $\sin(B - C) = 0 = \sin 0°$

or, $B - C = 0$

or, $B = C$

$\Rightarrow$ ABC is an isosceles triangle. Proved.

7. 2060 Q.No. 1b

In any triangle ABC, prove that

$\sin A + \sin B + \sin C = \frac{s}{R}$. [2]

SOLUTION

We know from sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,$$ where R is the circum-radius.

$\Rightarrow$ $\sin A = \frac{a}{2R}$; $\sin B = \frac{b}{2R}$; $\sin C = \frac{c}{2R}$

Now, $$\text{L.H.S.} = \sin A + \sin B + \sin C$$
$$= \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$$
$$= \frac{a + b + c}{2R} = \frac{2s}{2R} = \frac{s}{R}$$
$$= \text{R.H.S.}$$

8. 2057 Q.No. 1c

Prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s - a)}{bc}}$. [2]

SOLUTION

In any triangle ABC, by cosine law, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or, $$2\cos^2\frac{A}{2} - 1 = \frac{b^2 + c^2 - a^2}{2bc}$$

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$$\text{or, } 2\cos^2\frac{A}{2} = \frac{b^2+c^2-a^2}{2bc} + 1$$

$$= \frac{b^2+c^2-a^2+2bc}{2bc}$$

$$= \frac{b^2+2bc+c^2-a^2}{2bc}$$

$$= \frac{(b+c)^2-(a)^2}{2bc}$$

$$= \frac{(b+c+a)(b+c-a)}{2bc}$$

$$= \frac{(a+b+c)(b+c-a)}{2bc}$$

$$= \frac{2s(b+c+a-2a)}{2bc} \quad (\because a+b+c=2s)$$

$$= \frac{2s(2s-2a)}{2bc}$$

$$\text{or, } 2\cos^2\frac{A}{2} = \frac{4s(s-a)}{2bc}$$

$$\text{or, } \cos^2\frac{A}{2} = \frac{s(s-a)}{bc}$$

$$\text{or, } \cos^2\frac{A}{2} = \frac{s(s-a)}{bc}$$

$$\therefore \cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} = \text{R.H.S., proved.}$$

4 MARKS QUESTIONS

9. 2076 Set C Q.No. 7a OR

If $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ are in any triangle ABC, prove that $\angle C = 60°$. What will be the relation between a, b, c if $\angle C = 90°$? [4]

SOLUTION

Given, $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$

or, $\frac{b+c+a+c}{(a+c)(b+c)}=\frac{3}{a+b+c}$

or, $(a+b+2c)(a+b+c) = 3(a+c)(b+c)$

or, $a^2+ab+ac+ab+b^2+bc+2ac+2bc+2c^2 = 3ab+3ac+3bc+3c^2$

or, $a^2+b^2-c^2 = ab$

Dividing both sides by 2ab, we get

$$\frac{a^2+b^2-c^2}{2ab} = \frac{ab}{2ab}$$

or, $\frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

or, $\cos C = \cos 60°$

$\therefore C = 60°$

10. 2075 Set A Q.No. 7a OR

Prove that: $\cos A + \cos B = \frac{2(a+b)}{c}\sin^2\frac{C}{2}$. [4]

SOLUTION

R.H.S

$$= \frac{2(a+b)}{c}\sin^2\frac{C}{2}$$

$$= \frac{2(2R\sin A + 2R\sin B)}{2R\sin C}\sin^2\frac{C}{2}$$

$$= \frac{4R(\sin A+\sin B)}{2R\sin C}\sin^2\frac{C}{2}$$

$$= \frac{2(\sin A+\sin B)}{\sin C}\sin^2\frac{C}{2}$$

$$= \frac{2(\sin A+\sin B)}{2\sin\frac{C}{2}\cos\frac{C}{2}}\sin^2\frac{C}{2}$$

$$= \frac{\sin A+\sin B}{\cos\frac{C}{2}}\sin\frac{C}{2}$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{\cos\frac{C}{2}}\sin\frac{C}{2}$$

$$= \frac{2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}\sin\frac{C}{2}$$

$$= 2\cos\left(\frac{A-B}{2}\right)\sin\frac{C}{2}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= \cos\left(\frac{A+B}{2}+\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}-\frac{A-B}{2}\right)$$

$$= \cos A + \cos B = \text{L.H.S.}$$

11. 2075 Set B Q.No. 7a OR

If 2cos A = sin B: sin C, show that the triangle is isosceles. [4]

SOLUTION

Given, $2\cos A = \sin B : \sin C$

or, $2\cos A = \frac{\sin B}{\sin C}$

or, $2\cos A\sin C = \sin B$

or, $2\cos A\sin C = \sin(A+C)$

or, $2\cos A\sin C = \sin A\cos C + \cos A\sin C$

$$\left[\because A+B+C=\pi \Rightarrow \sin(A+C)=\sin(\pi-B)=\sin B\right]$$

or, $2\cos A\sin C - \cos A\sin C = \sin A\cos C$

or, $\cos A\sin C = \sin A\cos C$

or, $\sin A\cos C - \cos A\sin C = 0$

or, $\sin(A-C) = 0$

or, $\sin(A-C) = \sin 0$

or, $A - C = 0$

$\therefore A = C.$

$\therefore$ Triangle ABC is an isosceles triangle.

12. 2075 Set C Q.No. 7a

State cosine law. Using cosine law, prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$. [4]

SOLUTION

First Part:

In any ΔABC, cosine law states that

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Second Part: Please refer to 2057 Q.No. 1c

13. 2074 Supp Q.No. 7a OR

Prove that: **[4]**

$$(b + c - a)\left(\cot\frac{1}{2}B + \cot\frac{1}{2}C\right) = 2a\cot\frac{1}{2}B.$$

SOLUTION

L.H.S.

$$= (b + c - a)\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$$

$$= (b + c - a)\left\{\sqrt{\frac{s(s-b)}{(s-c)(s-a)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}\right\}$$

$$= (b + c - a)\sqrt{\frac{s}{s-a}}\left(\sqrt{\frac{s-b}{s-c}} + \sqrt{\frac{s-c}{s-b}}\right)$$

$$= (b + c - a)\sqrt{\frac{s}{s-a}}\left(\frac{s-b+s-c}{\sqrt{(s-b)(s-c)}}\right)$$

$$= (b+c+a-2a)\sqrt{\frac{s}{(s-a)(s-b)(s-c)}}(2s-b-c)$$

$$= (2s-2a)\sqrt{\frac{s}{(s-a)(s-b)(s-c)}}(a+b+c-b-c)$$

$$= 2(s-a).\sqrt{\frac{s}{(s-a)(s-b)(s-c)}}.a$$

$$= 2a\sqrt{\frac{s.(s-a)^2}{(s-a)(s-b)(s-c)}}$$

$$= 2a\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$= 2a\cot\frac{A}{2} = \text{R.H.S.}$$

14. 2074 Set A Q.No. 7a

In any triangle ABC, prove that:

$$\tan\frac{1}{2}(B - C) = \frac{b-c}{b+c}.\cot\frac{A}{2}.$$ **[4]**

SOLUTION

$$\text{R.H.S.} = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$$

$$= \left(\frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}\right) \times \cot\frac{A}{2}$$

[∵ From sine law b= 2R sin B and c = 2R sin C]

$$= \left(\frac{\sin B - \sin C}{\sin B + \sin C}\right) \times \cot\frac{A}{2}$$

$$= \frac{2\cos\left(\frac{B+C}{2}\right).\sin\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)} \times \cot\frac{A}{2}$$

$$= \frac{\cos\left(\frac{\pi}{2}-\frac{A}{2}\right).\sin\left(\frac{B-C}{2}\right)}{\sin\left(\frac{\pi}{2}-\frac{A}{2}\right).\cos\left(\frac{B-C}{2}\right)} \times \cot\frac{A}{2}$$

$$\left[\because \text{in } \Delta ABC, A + B + C = \pi \Rightarrow \frac{B+C}{2} = \frac{\pi}{A} - \frac{A}{2}\right]$$

$$= \frac{\sin\frac{A}{2}.\sin\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}.\cos\left(\frac{B-C}{2}\right)} \times \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$= \tan\left(\frac{B-C}{2}\right)$$

$$= \tan \tfrac{1}{2}(B - C)$$

= L.H.S. Proved.

15. 2074 Set B Q.No. 7a OR

State and prove cosine law of trigonometry. **[4]**

SOLUTION

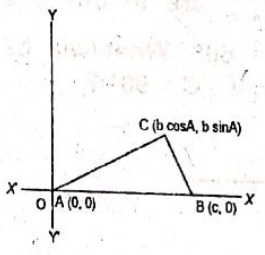
Statement of cosine law:

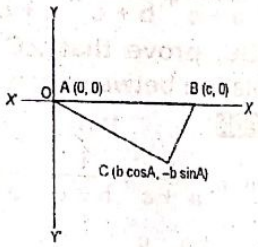
In any triangle ABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$





We place the ΔABC in the standard positive with the vertex A at the origin and the side AB along the positive x-axis. Then, the coordinates of three vertices A, B and C are clearly (0, 0), (c, 0) and (bcosA, ±bsinA) respectively. The positive sign is to be taken if the vertex C is above the x-axis and the negative sign if it is below the x-axis.

Now, using the distance formula, we have

$$BC^2 = (b\cos A - c)^2 + (\pm b\sin A - 0)^2$$

or, $a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$

or, $a^2 = b^2(\sin^2 A + \cos^2 A) - 2bc\cos A + c^2$

or, $a^2 = b^2 - 2bc\cos A + c^2$

or, $2bc\cos A = b^2 + c^2 - a^2$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Similarly, we can show

$$\cos B = \frac{c^2 + a^2 - b^2}{2bc}$$

and $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

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16. 2073 Supp Q.No. 7a OR

Prove, in any triangle that

$$\sin\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$ **[4]**

SOLUTION

cosine law: In any ΔABC,

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{c^2+a^2-b^2}{2ca}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Next Part

From cosine law, we have

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

or, $2bc\cos A = b^2+c^2-a^2$

or, $-2bc\cos A = a^2-b^2-c^2$

Adding $2bc$ on both sides, we get

$2bc - 2bc\cos A = a^2-(b^2+c^2)+2bc$

or, $2bc(1-\cos A) = a^2-(b^2-2bc+c^2)$

or, $2bc(1-\cos A) = a^2-(b-c)^2$

or, $2bc\left(1-1+2\sin^2\frac{A}{2}\right) = (a-b+c)(a+b-c)$

or, $2bc\cdot 2\sin^2\frac{A}{2} = (2s-2b)(2s-2c)$ $(\because a+b+c=2s)$

or, $bc\sin^2\frac{A}{2} = (s-b)(s-c)$

or, $\sin^2\frac{A}{2} = \frac{(s-b)(s-c)}{bc}$

$\therefore \sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, the positive sign is chosen as $\frac{A}{2}$ is an acute angle.

17. 2073 Set C Q.No. 7a

State and prove Sine law in any triangle. **[4]**

SOLUTION

Let ABC be a triangle. Since the sum of three angles of a triangle is 180°, at least one angle say B, must always be acute, then another angle, say C may be either acute, or obtuse, or a right angle as shown is the figure (1) or figure (2) or figure (3).

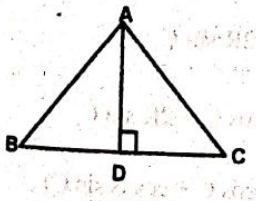

Fig. (1)

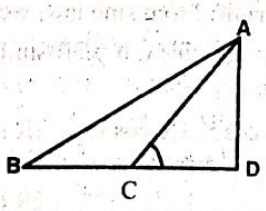

Fig. (2)

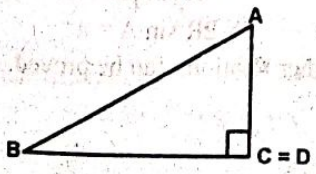

Fig. (3)

Draw AD perpendicular to BC,

In fig. (1), $\frac{AD}{AB} = \sin B$

$\Rightarrow AD = AB\sin B$

$\Rightarrow AD = c\sin B$ $[\because AB = c]$

Also, $\frac{AD}{AC} = \sin C$

$\Rightarrow AD = AC\sin C$

$\Rightarrow AD = b\sin C$

In fig. (2), $\frac{AD}{AB} = \sin B$

$\Rightarrow AD = AB\sin B$

$\Rightarrow AD = c\sin B$

Also, $\frac{AD}{AC} = \sin(180°-C) = \sin C$

$\Rightarrow AD = AC\sin C$

$\Rightarrow AD = b\sin C$

In fig. (3), $\frac{AD}{AB} = \sin B$

$\Rightarrow AD = AB\sin B$

$\Rightarrow AD = c\sin B$

Also, $\frac{AD}{AC} = 1 = \sin 90°$

$\Rightarrow AD = AC\sin 90°$

$\Rightarrow AD = b\sin C$ $(\because \angle C = 90°)$

In all the three cases,

$AD = c\sin B = b\sin C$

$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}$ ... (i)

Similarly, we can prove that $\frac{c}{\sin C} = \frac{a}{\sin A}$ ... (ii)

From (i) and (ii), we have

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Hence proved.

18. 2073 Set D Q.No. 7a OR

In any triangle ABC, if $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, prove that the triangle is either isosceles or right angled. **[4]**

SOLUTION

Here, $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$

or, $\cos A\sin C + 2\sin C\cos C = \cos A\sin B + 2\sin B\cos B$

or, $\cos A\sin C + \sin 2C = \cos A\sin B + \sin 2B$

or, $\cos A(\sin B - \sin C) + \sin 2B - \sin 2C = 0$

or, $\cos A\cdot 2\cos\frac{B+C}{2}\sin\frac{B-C}{2} + 2\cos(B+C)\cdot\sin(B-C) = 0$

or, $\cos A\cdot 2\cos\frac{B+C}{2}\sin\frac{B-C}{2} - 2\cos A\sin(B-C) = 0$

$[\because \cos(B+C) = -\cos A]$

or, $2\cos A\left\{\cos\frac{B+C}{2}\sin\frac{B-C}{2} - 2\sin\frac{B-C}{2}\cdot\cos\frac{B-C}{2}\right\} = 0$

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or, $2\cos A \sin\frac{(B-C)}{2}\left\{\cos\frac{B+C}{2} - 2\cos\frac{B-C}{2}\right\} = 0$

Since $\cos\frac{B+C}{2} - 2\cos\frac{B-C}{2} \neq 0$ as $\frac{1}{2}(B-C) < \frac{1}{2}(B+C)$

Hence,

either $\cos A = 0 \Rightarrow \cos A = \cos 90° \Rightarrow A = 90°$

or, $\sin\frac{B-C}{2} = 0 \Rightarrow \sin\frac{B-C}{2} = \sin 0 \Rightarrow B = C$

Hence, the triangle is either right angled or isosceles.

19. 2072 Supp. Q.No. 7a OR

If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that $C = 45°$ or $135°$ [4]

SOLUTION

We have,

$a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$

or, $(a^2)^2 + (b^2)^2 + (c^2)^2 = 2c^2(a^2 + b^2)$.

or, $(a^2 + b^2)^2 - 2a^2b^2 + (c^2)^2 = 2(a^2 + b^2)c^2$

$[\because a^2 + b^2 = (a+b)^2 - 2ab]$

or, $(a^2 + b^2)^2 - 2(a^2 + b^2).c^2 + (c^2)^2 = 2a^2b^2$

or, $(a^2 + b^2 - c^2)^2 = 2a^2b^2$

or, $\left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2 = \frac{2}{4}$

or, $\cos^2 C = \frac{1}{2}$ $\left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}\right]$

or, $\cos C = \pm\frac{1}{\sqrt{2}}$

or, $\cos C = \cos 45°$ or $\cos 135°$

Hence, $\angle C = 45°$ or $135°$ Proved.

20. 2072 Set C Q.No. 7a OR

In any triangle ABC, prove that $\tan\frac{1}{2}(B-C) = \frac{b-c}{b+c}\cot\frac{A}{2}$ [4]

➥ Please refer to 2074 Set A Q.No. 7a

21. 2072 Set D Q.No. 7a OR

If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60°$. [4]

➥ Please refer to 2076 Set C Q.No. 7a OR

22. 2072 Set E Q.No. 7a OR

If $b - a = mc$, prove that $\cot\frac{B-A}{2} = \frac{1 + m\cos B}{m\sin B}$. [4]

SOLUTION

Given, $b - a = mc$

$\therefore m = \frac{b-a}{c}$

R.H.S. $= \frac{1 + m\cos B}{m\sin B}$

$= \frac{1 + \left(\frac{b-a}{c}\right)\cos B}{\left(\frac{b-a}{c}\right)\sin B}$

$= \frac{c + (b-a)\cos B}{(b-a)\sin B}$

$= \frac{2R\sin C + (2R\sin B - 2R\sin A)\cos B}{(2R\sin B - 2R\sin A)\sin B}$

$= \frac{\sin C + (\sin B - \sin A)\cos B}{(\sin B - \sin A)\sin B}$

$= \frac{\sin(A+B) + (\sin B - \sin A)\cos B}{(\sin B - \sin A)\sin B}$

$= \frac{\sin A\cos B + \cos A\sin B + \sin B\cos B - \sin A\cos B}{(\sin B - \sin A)\sin B}$

$= \frac{\sin B(\cos A + \cos B)}{\sin B(\sin B - \sin A)}$

$= \frac{\cos A + \cos B}{\sin B - \sin A}$

$= \frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{B-A}{2}}$

$= \frac{\cos\left(\frac{B-A}{2}\right)}{\sin\left(\frac{B-A}{2}\right)}$ $[\because \cos(-\theta) = \cos\theta]$

$= \cot\frac{B-A}{2}$ = L.H.S.

23. 2071 Supp. Q.No. 7a OR

If $b - a = mc$, prove that: $\cot\frac{B-A}{2} = \frac{1 + m\cos B}{m\sin B}$ [4]

➥ Please refer 2072 Set E Q.No. 7a OR

24. 2071 Set C Q.No. 7a Or

State sine law. Use this law to prove the projection law. [4]

SOLUTION

Sine law: In any ΔABC,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Projection law

In any ΔABC,

$b\cos C + c\cos B = a$

$c\cos A + a\cos C = b$

$a\cos B + b\cos A = c$

Proof: From sine law, we have

$a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$

Now,

$b\cos C + c\cos B = 2R\sin B\cos C + 2R\sin C\cos B$

$= 2R(\sin B\cos C + \cos B\sin C)$

$= 2R\sin(B + C)$

$= 2R\sin A = a$

Similarly, other relations can be proved.

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25. 2071 Set D Q.No. 7a OR

In any triangle ABC, if $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, then prove that the triangle is either isosceles of right angled. [4]

➥ Please refer 2073 Set D Q.No. 7a OR

26. 2070 Supp Q.No. 7a Or

State cosine law. Using consine law, prove that: $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ [4]

➥ Please refer to 2073 Supp Q.No. 7a OR

27. 2070 Set C Q.No. 7 a Or

State sine law. Using sine law, prove that: $\tan\frac{1}{2}(C-A) = \frac{c-a}{c+a}\cdot\cot\frac{B}{2}$ [4]

SOLUTION

Sine law: In any ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Next Part

$$c - a = 2R\sin C - 2R\sin A$$
$$= 2R(\sin C - \sin A)$$
$$= 2R\cdot 2\cos\frac{C+A}{2}\cdot\sin\frac{C-A}{2}$$
$$= 4R\cos\frac{C+A}{2}\sin\frac{C-A}{2}$$

and $c + a = 2R(\sin C + \sin A)$
$$= 2R\cdot 2\sin\frac{C+A}{2}\cos\frac{C-A}{2}$$
$$= 4R\sin\frac{C+A}{2}\cos\frac{C-A}{2}$$

Hence, $\frac{c-a}{c+a} = \frac{4R\cos\frac{C+A}{2}\sin\frac{C-A}{2}}{4R\sin\frac{C+A}{2}\cos\frac{C-A}{2}}$

or, $\frac{c-a}{c+a} = \cot\left(\frac{C+A}{2}\right)\tan\left(\frac{C-A}{2}\right)$

or, $\tan\left(\frac{C-A}{2}\right) = \frac{1}{\cot\left(\frac{C+A}{2}\right)}\cdot\frac{c-a}{c+a}$

$$= \frac{c-a}{c+a}\tan\left(\frac{C+A}{2}\right)$$
$$= \frac{c-a}{c+a}\tan\left(90^\circ - \frac{B}{2}\right)$$

$\left[\because A+B+C = 180^\circ \Rightarrow \frac{C+A}{2} = 90^\circ - \frac{B}{2}\right]$

$$= \frac{c-a}{c+a}\cot\frac{B}{2}$$

28. 2070 Set D Q.No. 7 a Or

If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that: $C = 45^\circ$ or 135°. [4]

➥ Please refer to 2072 Supp. Q.No. 7a OR

29. 2070 Old Q.No. 8 b

In any triangle ABC, prove that: [4]

$a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0.$

SOLUTION

L.H.S.
$$= a\sin(B-C) + b\sin(C-A) + c\sin(A-B)$$
$$= 2R\sin A\sin(B-C) + 2R\sin B\sin(C-A) + 2R\sin C\sin(A-B)$$
$$= 2R\sin(B+C)\sin(B-C) + 2R\sin(C+A)\sin(C-A) + 2R\sin(A+B)\sin(A-B)$$
$$= 2R(\sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B)$$

[$\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$]

$$= 2R\times 0$$
$$= 0 = \text{R.H.S.}$$

30. 2069 Supp.Q.No. 7 a OR

In any triangle ABC, prove that: [4]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$.

➥ Please refer to 2074 Set A Q.No. 7a

31. 2069 [Set A] Q. No. 7a OR

State sine law.

Prove that: $\tan\frac{1}{2}(B-C) = \frac{b-c}{b+c}\cot\frac{A}{2}$. [4]

➥ Please refer to 2074 Set A Q.No. 7a

32. 2069 [Set A] Old Q. No. 8b

In any triangle ABC, prove that: [4]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$.

➥ Please refer to 2074 Set A Q.No. 7a

33. 2068 Q.No. 7a Or

State cosine Law, using cosine Law, prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ [4]

➥ Please refer 2057 Q.No. 1c

34. 2068 Old Q.No. 8b

If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that: $C = 45^\circ$ or 135° [4]

➥ Please refer to 2072 Supp. Q.No. 7a OR

35. 2067 Q.No. 8b OR

In any triangle ABC if $8R^2 = a^2 + b^2 + c^2$, prove that the triangle is right angled. [4]

SOLUTION

$$8R^2 = a^2 + b^2 + c^2$$
or, $8R^2 = (2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2$
or, $8R^2 = 4R^2\sin^2 A + 4R^2\sin^2 B + 4R^2\sin^2 C$
or, $2 = \sin^2 A + \sin^2 B + \sin^2 C$
or, $2 = \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + 1 - \cos^2 C$
or, $2 = \frac{1}{2} + \frac{1}{2} + 1 - \frac{1}{2}(\cos 2A + \cos 2B) - \cos^2 C$
or, $\frac{1}{2}(\cos 2A + \cos 2B) + \cos^2 C = 0$

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or, $\frac{1}{2}\left[2\cos\frac{2A+2B}{2}\cdot\cos\frac{2A-2B}{2}\right]+\cos^2C=0$

or, $\cos(A+B)\cos(A-B)+\cos^2C=0$

or, $-\cos C\cos(A-B)+\cos^2C=0$

or, $\cos C[\cos C-\cos(A-B)]=0$

or, $\cos C[-\cos(A+B)-\cos(A-B)]=0$

or, $\cos C[\cos(A+B)+\cos(A-B)]=0$

or, $\cos C(\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B)=0$

or, $2\cos A\cos B\cos C=0$

Either $\cos A=0$ or $\cos B=0$ or $\cos C=0$

i.e. either $A=90°$ or $B=90°$ or $C=90°$

Hence, the triangle must be right angled triangle.

36. 2066 Q.No. 8b OR

In a ΔABC, if (sin A + sin B + sin C) (sin A + sin B − sin C) = 3 sin A sin B then prove ∠C = 60°. [4]

SOLUTION

$(\sin A+\sin B+\sin C)(\sin A+\sin B-\sin C)=3\sin A\sin B$

or, $\left(\frac{a}{2R}+\frac{b}{2R}+\frac{c}{2R}\right)\left(\frac{a}{2R}+\frac{b}{2R}-\frac{c}{2R}\right)=3\cdot\frac{a}{2R}\cdot\frac{b}{2R}$

or, $\left(\frac{a+b+c}{2R}\right)\left(\frac{a+b-c}{2R}\right)=\frac{3ab}{(2R)^2}$

or, $(a+b+c)(a+b-c)=3ab$

or, $(a+b)^2-c^2=3ab$

or, $a^2+2ab+b^2-c^2=3ab$

or, $a^2+b^2-c^2=ab$

or, $\frac{a^2+b^2-c^2}{ab}=1$

or, $\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$

or, $\cos C=\frac{1}{2}$

or, $\cos C=\cos 60°$

∴ $C=60°$. Hence proved.

37. 2065 Q.No. 8b

In any triangle, state and prove cosine law. [4]

➥ Please refer to 2074 Set B Q.No. 7a OR

38. 2064 Q.No. 8b

In any triangle, prove that:

a cos A + b cos B + c cos c = 4 R sin A sin B sin C [4]

SOLUTION

L.H.S. = a cos A + b cos B + c cos C

= 2R sin A. cos A + 2 R sin B. cos B + 2R sin C. cos C

= R (sin 2A + sin 2B + sin 2C)

$=R\left[2\sin\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right)+\sin 2C\right]$

= R [2 sin (180° − C). cos (A − B) + sin 2C]

[∵ A + B + C = 180°]

= R [2 sin C. cos (A − B) + 2 sin C cos C]

= 2R sin C [cos (A − B) + cos C]

= 2 R sin C [cos (A − B) + cos {180° − (A + B)}]

= 2 R sin C [cos (A − B) − cos (A + B)]

= 2 R sin C × 2 sin A sin B

= 4 R sin A. sin B. sin C.

= R.H.S. Proved.

39. 2062 Q.No. 8b

In any ΔABC, prove that: $\frac{\cos B-\cos C}{\cos A+1}=\frac{c-b}{a}$ [4]

SOLUTION

L.H.S. $=\frac{\cos B-\cos C}{\cos A+1}$

$=\frac{\frac{c^2+a^2-b^2}{2ca}-\frac{a^2+b^2-c^2}{2ab}}{\frac{b^2+c^2-a^2}{2bc}+1}$ [∵ By using cosine law]

$=\frac{\frac{b(c^2+a^2-b^2)-c(a^2+b^2-c^2)}{2abc}}{\frac{b^2+c^2-a^2+2bc}{2bc}}$

$=\frac{bc^2+a^2b-b^3-ca^2-b^2c+c^3}{b^2+c^2-a^2+2bc}\times\frac{1}{a}$

$=\frac{bc^2-b^2c+a^2b-ca^2+c^3-b^3}{(b^2+c^2-a^2+2bc)a}$

$=\frac{bc(c-b)-a^2(c-b)+(c-b)(c^2+bc+b^2)}{a(b^2+c^2-a^2+2bc)}$

$=\frac{(c-b)(bc-a^2+c^2+bc+b^2)}{a(b^2+c^2-a^2+2bc)}$

$=\frac{(c-b)(b^2+c^2-a^2+2bc)}{a(b^2+c^2-a^2+2bc)}$

$=\frac{(c-b)}{a}$ = R.H.S. proved.

40. 2061 Q.No. 8b

In any triangle ABC, prove that

$\frac{a^2\sin(B-C)}{\sin B+\sin C}+\frac{b^2\sin(C-A)}{\sin C+\sin A}+\frac{c^2\sin(A-B)}{\sin A+\sin B}=0$ [4]

SOLUTION

We know by sine law, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$

⇒ a = 2R sin A, b = 2R sin B and c = 2R sin C

Now, $\frac{a^2\sin(B-C)}{\sin B+\sin C}=\frac{a\,a\sin(B-C)}{\sin B+\sin C}$

$=\frac{a\,2R\sin A\sin(B-C)}{\sin B+\sin C}$ [∵ a = 2R sinA]

$=\frac{a\,2R\sin(B+C)\sin(B-C)}{\sin B+\sin C}$ [∵ sinA=sin[π-(B+C)]

$=\frac{2R\,a(\sin^2B-\sin^2C)}{\sin B+\sin C}$

$=\frac{2R\,a(\sin B+\sin C)(\sin B-\sin C)}{(\sin B+\sin C)}$

= 2Ra (sin B − sin C)

= a (2R sin B − 2R sin C)

= a (b − c)

= ab − ca

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$\therefore \frac{a^2 \sin(B - C)}{\sin B + \sin C} = ab - ca$ ... (i)

Similarly,

$\frac{b^2 \sin(C - A)}{\sin C + \sin A} = bc - ab$ ... (ii)

and $\frac{c^2 \sin(A - B)}{\sin A + \sin B} = ca - bc$ ... (iii)

Now,

$$\text{L.H.S.} = \frac{a^2 \sin(B - C)}{\sin B + \sin C} + \frac{b^2 \sin(C - A)}{\sin C + \sin A} + \frac{c^2 \sin(A - B)}{\sin A + \sin B}$$
$$= (ab - ca) + (bc - ab) + (ca - bc)$$
$$= ab - ca + bc - ab + ca - bc$$
$$= 0 = \text{R.H.S. proved.}$$

41. 2060 Q.No. 8b

Prove that in any triangle ABC,

$$\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0 \quad [4]$$

SOLUTION

By sine law, we know

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R;$$ where R is circum radius.

$$\Rightarrow \sin A = \frac{a}{2R}, \sin B = \frac{b}{2R}, \sin C = \frac{c}{2R}$$

And, by cosine law, we know

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Now,

L.H.S.

$$= \frac{b^2 - c^2}{a^2} 2 \sin A \cos A + \frac{c^2 - a^2}{b^2} 2 \sin B \cos B + \frac{a^2 - b^2}{c^2} \times 2 \sin C \cos C$$

$$= \left(\frac{b^2 - c^2}{a^2}\right) \times 2 \times \frac{a}{2R} \times \left(\frac{b^2 + c^2 - a^2}{2bc}\right) + \left(\frac{c^2 - a^2}{b^2}\right) \times 2 \times \frac{b}{2R} \times \left(\frac{c^2 + a^2 - b^2}{2ca}\right) + \left(\frac{a^2 - b^2}{c^2}\right) + 2 \times \frac{c}{2R} \times \left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$= \frac{(b^2 - c^2)(b^2 + c^2 - a^2)}{2abc\,R} + \frac{(c^2 - a^2)(c^2 + a^2 - b^2)}{2abc\,R} + \frac{(a^2 - b^2)(a^2 + b^2 - c^2)}{2abc\,R}$$

$$= \frac{(b^4 + b^2c^2 - a^2b^2 - b^2c^2 - c^4 + c^2a^2) + (c^4 + c^2a^2 - b^2c^2 - c^2a^2 - a^4 + a^2b^2) + (a^4 + a^2b^2 - c^2a^2 - a^2b^2 - b^4 + b^2c^2)}{2abcR}$$

$$= \frac{0}{2abc\,R}$$
$$= 0$$
$$= \text{R.H.S.}$$

42. 2059 Q.No. 8b

In any triangle ABC prove that:

$$\tan \tfrac{1}{2}(B - C) = \left(\frac{b - c}{b + c}\right) \cdot \cot \frac{A}{2}. \quad [4]$$

➥ Please refer to 2074 Set A Q.No. 7a

43. 2058 Q.No. 8b

In any ΔABC prove that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [4]

SOLUTION

Let ABC be a triangle. since the sum of three angles of a triangle is 180°, at least one angle say B, must always be acute, then another angle, say C may be either acute, or obtuse, or a right angle as shown is the figure (1) or figure (2) or figure (3).

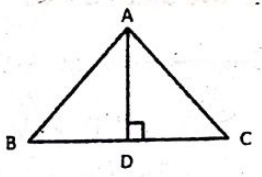


Fig. (1)

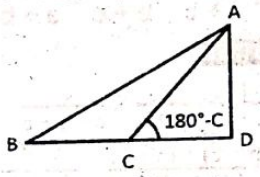


Fig. (2)

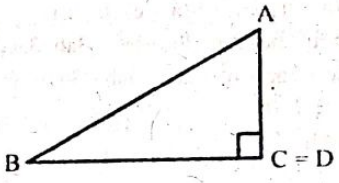


Fig. (3)

Draw AD perpendicular to BC,

In fig. (1), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B [$\because$ AB = c]

Also, $\frac{AD}{AC} = \sin C$

$\Rightarrow$ AD = AC sin C

$\Rightarrow$ AD = b sin C

In fig. (2), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B

Also, $\frac{AD}{AC} = \sin(180° - C) = \sin C$

$\Rightarrow$ AD = AC sin C

$\Rightarrow$ AD = b sin C

In fig. (3), $\frac{AD}{AB} = \sin B$

$\Rightarrow$ AD = AB sin B

$\Rightarrow$ AD = c sin B

Also, $\frac{AD}{AC} = 1 = \sin 90°$

$\Rightarrow$ AD = AC sin 90°

$\Rightarrow$ AD = b sin C ($\because \angle C = 90°$)

In all the three cases,

$AD = c \sin B = b \sin C$

$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}$ ... (i)

Similarly, we can prove that $\frac{c}{\sin C} = \frac{a}{\sin A}$ ... (ii)

From (i) and (ii), we have

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

44. 2056 Q.No. 8b

If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that $\angle C = 45°$ or $135°$. [4]

➥ Please refer to 2072 Supp. Q.No. 7a OR

45. 2055 Old Model Q.No. 8b

If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, prove that $\angle C = 60°$ [4]

SOLUTION

$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

or, $\frac{(b+c)+(a+c)}{(a+c)(b+c)} = \frac{3}{(a+b+c)}$

or, $(a+b+2c)(a+b+c) = 3(a+c)(b+c)$

or, $a^2+ab+ac+ab+b^2+bc+2ac+2bc+2c^2 = 3ab+3ac+3bc+3c^2$

or, $a^2 + 2ab + 3bc + 3ac + b^2 + 2c^2 - 3ab - 3ac - 3bc - 3c^2 = 0$

or, $a^2 - ab + b^2 - c^2 = 0$

or, $a^2 + b^2 - c^2 = ab$

or, $\frac{a^2+b^2-c^2}{ab} = 1$

or, $\frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

or, $\cos C = \frac{1}{2} = \cos 60°$

or, $C = 60°$

$\therefore \angle C = 60°$ Proved.

D. SOLUTION OF TRIANGLE

2 MARKS QUESTIONS

1. 2070 Old Q.No. 1 b

In any triangle ABC, if $a = 3$, $b = 3\sqrt{3}$, $A = 30°$, find B. [2]

SOLUTION

In ΔABC, $a = 3$, $b = 3\sqrt{3}$, $A = 30°$, $B = ?$

From sine law, we have

$\frac{a}{\sin A} = \frac{b}{\sin B}$

or, $\frac{3}{\sin 30°} = \frac{3\sqrt{3}}{\sin B}$

or, $\frac{3}{1/2} = \frac{3\sqrt{3}}{\sin B}$

or, $6 = \frac{3\sqrt{3}}{\sin B}$

or, $\sin B = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

or, $\sin B = \sin 60°$ or $\sin 120°$

$\therefore B = 60°$ or $120°$

4 MARKS QUESTIONS

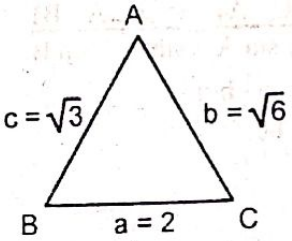
2. 2076 Set B Q.No. 7a OR

If $a = 2$, $b = \sqrt{6}$ and $c = \sqrt{3} - 1$, find the angles of the triangle ABC. [4]

SOLUTION

Here,

$b = \sqrt{6}$, $a = 2$ and $c = \sqrt{3} - 1$



By cosine law, we know that

$\cos C = \frac{a^2+b^2-c^2}{2ba}$

$= \frac{4+6-(\sqrt{3}-1)^2}{2\times\sqrt{6}\times 2}$

$= \frac{10-(3-2\sqrt{3}+1)}{4\sqrt{6}}$

$= \frac{10-4+2\sqrt{3}}{4\sqrt{6}}$

$= \frac{6+2\sqrt{3}}{4\sqrt{6}}$

$= \frac{2\times 3+2\sqrt{3}}{4\sqrt{6}}$

$= \frac{2\sqrt{3}(\sqrt{3}+1)}{4\sqrt{3}\times\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15°$

$\therefore C = 15°$

Again,

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$= \frac{4+(\sqrt{3}-1)^2-6}{2\times 2\times(\sqrt{3}-1)}$

$= \frac{-2+3-2\sqrt{3}+1}{4(\sqrt{3}-1)}$

$= \frac{2-2\sqrt{3}}{4(\sqrt{3}-1)}$

$= \frac{2(1-\sqrt{3})}{4(\sqrt{3}-1)}$

$= \frac{-2(\sqrt{3}-1)}{4(\sqrt{3}-1)}$

$= -\frac{1}{2} = \cos 120°$

$\therefore B = 120°$

Also, we know that the sum of angles of triangle $= 180°$

i.e. $A + B + C = 180°$

or, $A = 180° - B - C$

$= 180° - 120° - 15°$

$= 45°$

Hence, $A = 45°$, $B = 120°$ and $C = 15°$

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3. 2075 GIE Q.No. 7a OR

Given a = 2, b = $\sqrt{6}$, c = $\sqrt{3}$ –1, solve the triangle ABC. [4]

➥ Please refer to 2076 Set B Q.No. 7a OR

4. 2071 Old Q.No. 8b Or

In a triangle ABC, C = 30°; b = $\sqrt{3}$ and a = 1. Find the other angles and the sides. [4]

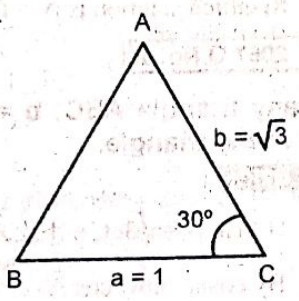
SOLUTION

In ΔABC, C = 30°; b = $\sqrt{3}$ and a = 1

we have, from cosine law,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

or, $\cos 30° = \frac{1^2 + (\sqrt{3})^2 - c^2}{2 \cdot 1 \cdot \sqrt{3}}$



or, $\frac{\sqrt{3}}{2} = \frac{4 - c^2}{2\sqrt{3}}$

or, $4 - c^2 = 3$

or, $c^2 = 1$

∴ $c = 1$

Again, by sine law

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

or, $\frac{1}{\sin A} = \frac{1}{\sin 30°}$

∴ $A = 30°$

since, $A + B + C = 180$

$B = 180 - A - C$

$= 180 - 30 - 30 = 120°$

∴ $c = 1$, $A = 30°$, $B = 120°$

5. 2069 [Set B] Q. No. 7a OR

If a = 2, b = 1 + $\sqrt{3}$, C = 60°, solve the triangle ABC. [4]

SOLUTION

In ΔABC, a = 2, b = 1 + $\sqrt{3}$, C = 60°

From cosine law, we have

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

or, $\cos 60° = \frac{2^2 + (1 + \sqrt{3})^2 - c^2}{2 \cdot 2 \cdot (1 + \sqrt{3})}$

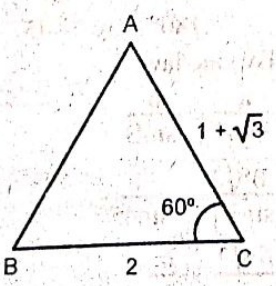
or, $\frac{1}{2} = \frac{4 + 1 + 2\sqrt{3} + 3 - c^2}{4(1 + \sqrt{3})}$

or, $2 + 2\sqrt{3} = 8 + 2\sqrt{3} - c^2$

or, $c^2 = 6$

∴ $c = \sqrt{6}$

Again, from sine law



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

or, $\frac{2}{\sin A} = \frac{\sqrt{6}}{\sin 60°}$

or, $\frac{2}{\sin A} = \frac{\sqrt{6}}{\frac{\sqrt{3}}{2}}$

or, $\frac{2}{\sin A} = \sqrt{2}$

or, $\sin A = \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \sin 45°$

∴ $A = 45°$

Also, $A + B + C = 180°$

or, $45 + B + 60 = 180$

or, $B = 180 - 105 = 75°$

∴ Required solution is

$c = \sqrt{6}$, $A = 45°$, $B = 75°$

6. 2069 [Set A] Old Q. No. 8b OR

In any triangle ABC, a = 3, b = $3\sqrt{3}$, and A = 30°. Solve the triangle. [4]

SOLUTION

In ΔABC, a = 3, b = $3\sqrt{3}$ and A = 30°

From cosine law, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or, $\cos 30° = \frac{(3\sqrt{3})^2 + c^2 - 3^2}{2 \cdot 3\sqrt{3} \cdot c}$

or, $\frac{\sqrt{3}}{2} = \frac{27 + c^2 - 9}{6\sqrt{3}\, c}$

or, $9c = 18 + c^2$

or, $c^2 - 9c + 18 = 0$

or, $(c - 3)^2 = 0$

∴ $c = 3$

Again, using sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

or, $\frac{3}{\sin 30°} = \frac{3\sqrt{3}}{\sin B}$

or, $\frac{3}{1/2} = \frac{3\sqrt{3}}{\sin B}$

or, $\sin B = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

or, $\sin B = \sin 60°$ or $\sin 120°$

∴ $B = 60°$ or $120°$.

When B = 60

$A + B + C = 180$

$30 + 60 + C = 180$

⇒ $C = 90°$

When B = 120

$A + B + C = 180$

$30 + 120 + C = 180$

⇒ $C = 30°$

Required solutions are

$c = 3$, $B = 60°$, $C = 90°$

or, $c = 3$, $B = 120°$, $C = 30°$

7. 2069 [Set B] Old Q. No. 8b OR

Two sides of a triangle are $\sqrt{3}$ + 1 and $\sqrt{3}$ – 1 and the included angle is 60°, solve the triangle. [4]

SOLUTION

In ΔABC, let a = $\sqrt{3}$ + 1, b = $\sqrt{3}$ – 1 and C = 60°

From cosine law, we have

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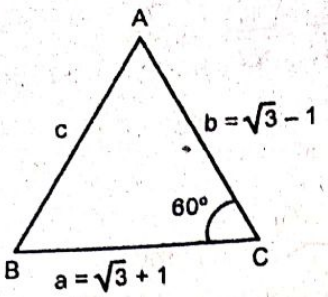


$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

or, $\cos 60° = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - c^2}{2(\sqrt{3}+1)(\sqrt{3}-1)}$

or, $\frac{1}{2} = \frac{3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1 - c^2}{2(3-1)}$

or, $\frac{1}{2} = \frac{8 - c^2}{4}$

or, $2 = 8 - c^2$

or, $c^2 = 6$

$\therefore$ $c = \sqrt{6}$

Again, from sine law,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

or, $\frac{\sqrt{3}+1}{\sin A} = \frac{\sqrt{6}}{\sin 60°}$

or, $\frac{\sqrt{3}+1}{\sin A} = \frac{\sqrt{6}}{\frac{\sqrt{3}}{2}}$

or, $\frac{\sqrt{3}+1}{\sin A} = \sqrt{2}$

or, $\sin A = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 15°$

$\therefore$ $A = 15°$

Also, $A + B + C = 180°$

or, $15 + B + 60 = 180$

$\Rightarrow$ $B = 180 - 75 = 105°$

Required solution is $c = \sqrt{6}$, $A = 15°$, $B = 105°$

8. 2068 Old Q.No. 8b(Or)

If C = 30°, B = 45° and c = 6 √2, solve the triangle. [4]

SOLUTION

In ΔABC, $C = 30°$, $B = 45°$ and $c = 6\sqrt{2}$

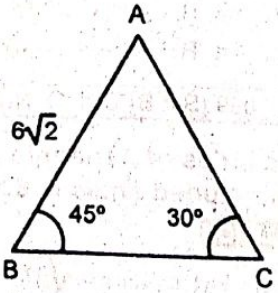
We have,

$A + B + C = 180°$

or, $A + 45 + 30 = 180$

$\therefore$ $A = 180 - 75 = 105°$

Again, from sine law,



$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

or, $\frac{b}{\sin 45°} = \frac{6\sqrt{2}}{\sin 30°}$

or, $\frac{b}{\frac{1}{\sqrt{2}}} = \frac{6\sqrt{2}}{\frac{1}{2}}$

or, $b = 6\sqrt{2} \times 2 \times \frac{1}{\sqrt{2}} = 12$

Again,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

or, $\frac{a}{\sin 105°} = \frac{12}{\sin 45°}$

or, $a = \frac{12}{\sin 45°} \times \sin 105°$

$= \frac{12}{\frac{1}{\sqrt{2}}} \times \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= 6(\sqrt{3}+1)$

$\therefore$ Required solution is $A = 105°$, $b = 12$, $a = 6(\sqrt{3}+1)$

9. 2067 Q.No. 8b

In any triangle ABC, b = √3, c = 1 and A = 30°, solve the triangle. [4]

SOLUTION

Given, $b = \sqrt{3}$, $C = 1$, $\angle A = 30°$

By cosine law, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

or, $\cos 30° = \frac{(\sqrt{3})^2 + 1^2 - a^2}{2\sqrt{3} \times 1}$

or, $\frac{\sqrt{3}}{2} = \frac{3 + 1 - a^2}{2\sqrt{3}}$

or, $3 = 4 - a^2$

or, $a^2 = 4 - 3$

or, $a = 1$

Since, $a = 1$, ΔABC is an isosceles triangles

$\therefore$ $\angle C = \angle A = 30°$

Finally, we know that

$\angle A + \angle B + \angle C = 180°$

or, $\angle B = 180° - \angle A - \angle C$

$= 180° - 30° - 30° = 120°$

Hence, $a = 1$, $b = \sqrt{3}$, $\angle C = 30°$ and $\angle B = 120°$

10. 2066 Q.No. 8b

If A = 30°, B = 45°, a = 6√2, solve the triangle ABC. [4]

SOLUTION

$A = 30°$, $B = 45°$, $a = 6\sqrt{2}$

We know that

$A + B + C = 180°$

So, $C = 180° - A - B$

$= 180° - 30° - 45° = 105°$

By sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

or, $\frac{6\sqrt{2}}{\sin 30°} = \frac{b}{\sin 45°}$

or, $\frac{6\sqrt{2}}{\frac{1}{2}} = \frac{b}{\frac{1}{\sqrt{2}}}$

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$\therefore\ b = \frac{12\sqrt{2}}{\sqrt{2}} = 12$

Also, $\frac{a}{\sin A} = \frac{c}{\sin C}$

or, $\frac{6\sqrt{2}}{\sin 30°} = \frac{c}{\sin 105°}$

or, $\frac{6\sqrt{2}}{\frac{1}{2}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

or, $c = \frac{6\sqrt{2}(\sqrt{3}+1)}{\sqrt{2}} = 6(\sqrt{3}+1)$

$\therefore\ C = 105°,\ b = 12,\ c = 6(\sqrt{3}+1)$

11. 2065 Q.No. 8b OR

In any triangle ABC, $b = \sqrt{3}$, $c = 1$ and $\angle A = 30°$, solve the triangle. [4]

➥ Please refer to 2067 Q.No. 8b

12. 2064 Q.No. 8b OR

If the angles of a triangle are to one another as 1:2:3, prove that the corresponding sides are $1:\sqrt{3}:2$. [4]

SOLUTION

In the Δ ABC, angles are in the ratio 1:2:3, so let $\angle A = K$, $\angle B = 2K$ and $\angle C = 3K$.

$\therefore\ A + B + C = K + 2K + 3K = 6K = 180°$

or, $K = 30°$

$\therefore\ A = 30°,\ B = 60°,\ C = 90°$

From sine law, we have

$a : b : c = \sin A : \sin B : \sin C$

$= \sin 30° : \sin 60° : \sin 90°$

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$

$= \frac{1}{2} \times 2 : \frac{\sqrt{3}}{2} \times 2 : 1 \times 2$

$= 1 : \sqrt{3} : 2$

Hence, the corresponding sides are in the ratio $1:\sqrt{3}:2$. Proved

13. 2062 Q.No. 8b OR

In any ΔABC, $a = 2$, $b = \sqrt{6}$ and $c = \sqrt{3} - 1$, find $\angle B$. [4]

SOLUTION

Here,

$a = 2,\ b = \sqrt{6}$ and $c = \sqrt{3} - 1$

Now, $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

$= \frac{(\sqrt{3}-1)^2 + 2^2 - (\sqrt{6})^2}{2(\sqrt{3}-1) \times 2}$

$= \frac{3 - 2\sqrt{3} + 1 + 4 - 6}{4(\sqrt{3}-1)}$

$= \frac{2 - 2\sqrt{3}}{4(\sqrt{3}-1)} = \frac{-2(\sqrt{3}-1)}{4(\sqrt{3}-1)}$

or, $\cos B = -\frac{1}{2}$

or, $\cos B = \cos 120°$

$\therefore\ \angle B = 120$

14. 2061 Q.No. 8b OR

In any triangle ABC, if $A = 30°$ and $B = 90°$, find a:b:c. [4]

SOLUTION

We have,

In ΔABC, $\angle A = 30°$ and $\angle B = 90°$

$\therefore\ \angle C = 180° - (\angle A + \angle B)$

$= 180° - 30° - 90° = 60°$

Now, by sine law, we have,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\therefore\ a = 2R \sin A,\ b = 2R \sin B,\ c = 2R \sin C$

So, $a:b:c = 2R \sin A : 2R \sin B : 2R \sin C$

$= \sin A : \sin B : \sin C$

$= \sin 30° : \sin 90° : \sin 60°$

$\therefore\ a:b:c = \frac{1}{2} : 1 : \frac{\sqrt{3}}{2} = 1 : 2 : \sqrt{3}$

15. 2060 Q.No. 8b OR

In any triangle ABC, $b = \sqrt{3}$, $c = 1$ and $A = 30°$, solve the triangle. [4]

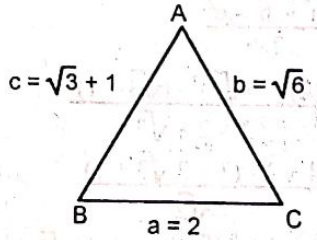
➥ Please refer to 2067 Q.No. 8b

16. 2059 Q.No. 8b OR

Solve the triangle, if $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3} + 1$. [4]

SOLUTION

Given, $a = 2,\ b = \sqrt{6},\ c = \sqrt{3} + 1$



By cosine law, we have

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{2^2 + (\sqrt{6})^2 - (\sqrt{3}+1)^2}{2 \times 2 \times \sqrt{6}}$

$= \frac{4 + 6 - (3 + 2\sqrt{3} + 1)}{4\sqrt{6}}$

$= \frac{10 - 4 - 2\sqrt{3}}{4\sqrt{6}} = \frac{6 - 2\sqrt{3}}{4\sqrt{6}}$

$= \frac{2(3 - \sqrt{3})}{4\sqrt{6}} = \frac{3 - \sqrt{3}}{2\sqrt{6}}$

$= \frac{\sqrt{3}(\sqrt{3} - 1)}{2\sqrt{6}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

or, $\cos C = \cos 75°$

$\therefore\ C = 75°$

Again,

$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

$= \frac{(\sqrt{3}+1)^2 + 2^2 - (\sqrt{6})^2}{2(\sqrt{3}+1).2}$

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$$= \frac{3 + 2\sqrt{3} + 1 + 4 - 6}{4(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{3}+2}{4(\sqrt{3}+1)} = \frac{2(\sqrt{3}+1)}{4(\sqrt{3}+1)}$$

or, $\cos B = \frac{1}{2}$

$\cos B = \cos 60°$

$\therefore$ $B = 60°$

since, $A + B + C = 180$

$A = 180 - C - B$

$= 180° - 75° - 60°$

$= 180° - 135° = 45°$

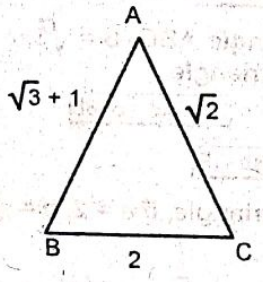
$\therefore$ $A = 45°$, $B = 60°$, $C = 75°$

17. 2058 Q.No. 8b OR

If a= 2, b = $\sqrt{2}$, c = $\sqrt{3}$ + 1, solve the triangle. [4]

SOLUTION

Given, $a = 2$; $b = \sqrt{2}$; $c = \sqrt{3} + 1$



By cosine law,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(2)^2 + (\sqrt{2})^2 - (\sqrt{3}+1)^2}{2 \times 2 \times \sqrt{2}}$$

$$= \frac{4 + 2 - (3 + 2\sqrt{3} + 1)}{4\sqrt{2}}$$

$$= \frac{6 - 3 - 2\sqrt{3} - 1}{4\sqrt{2}} = \frac{2 - 2\sqrt{3}}{4\sqrt{2}}$$

$$= \frac{2(1 - \sqrt{3})}{4\sqrt{2}} = \frac{1 - \sqrt{3}}{2\sqrt{2}} = \cos 105°$$

$\therefore$ $C = 105°$

Again,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(\sqrt{2})^2 + (\sqrt{3}+1)^2 - 2^2}{2 \times \sqrt{2} \times (\sqrt{3}+1)}$$

$$= \frac{2 + 3 + 2\sqrt{3} + 1 - 4}{2\sqrt{2}(\sqrt{3}+1)}$$

$$= \frac{2 + 2\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}$$

$$= \frac{2(\sqrt{3}+1)}{2\sqrt{2}(\sqrt{3}+1)} = \frac{1}{\sqrt{2}} = \cos 45°$$

$\therefore$ $A = 45°$

Since $A + B + C = 180°$

$B = 180° - A - C$

$= 180° - 45° - 105° = 30°$

$\therefore$ $A = 45°$, $B = 30°$, $C = 105°$

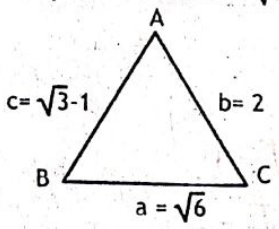
18. 2057 Q.No. 8b OR

Solve the triangle if $a = \sqrt{6}$, $b = 2$ and $c = \sqrt{3}$. [4]

SOLUTION

Here,

$a = \sqrt{6}$, $b = 2$ and $c = \sqrt{3} - 1$



By cosine law, we know that

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{6 + 4 - (\sqrt{3}-1)^2}{2 \times \sqrt{6} \times 2}$$

$$= \frac{10 - (3 - 2\sqrt{3} + 1)}{4\sqrt{6}}$$

$$= \frac{10 - 4 + 2\sqrt{3}}{4\sqrt{6}}$$

$$= \frac{6 + 2\sqrt{3}}{4\sqrt{6}}$$

$$= \frac{2 \times 3 + 2\sqrt{3}}{4\sqrt{6}}$$

$$= \frac{2\sqrt{3}(\sqrt{3}+1)}{4\sqrt{3} \times \sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15°$$

$\therefore$ $C = 15°$

Again,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{4 + (\sqrt{3}-1)^2 - 6}{2 \times 2 \times (\sqrt{3}-1)}$$

$$= \frac{-2 + 3 - 2\sqrt{3} + 1}{4(\sqrt{3}-1)}$$

$$= \frac{2 - 2\sqrt{3}}{4(\sqrt{3}-1)}$$

$$= \frac{2(1 - \sqrt{3})}{4(\sqrt{3}-1)}$$

$$= \frac{-2(\sqrt{3}-1)}{4(\sqrt{3}-1)} = -\frac{1}{2} = \cos 120°$$

$\therefore$ $A = 120°$

Also, we know that the sum of angles of triangle = 180°

i.e. $A + B + C = 180°$

or, $B = 180° - A - C = 180° - 120° - 15° = 45°$

Hence, $A = 120°$, $B = 45°$ and $C = 15°$.

19. 2056 Q.No. 8b OR

If three sides of a triangle are in the rati 2:$\sqrt{6}$:$\sqrt{3}$ + 1, find the angles. [

SOLUTION

Given that a: b: c = 2: $\sqrt{6}$: $\sqrt{3}$ + 1

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Now, from sine law

$\sin A : \sin B : \sin C = a : b : c$

$= 2 : \sqrt{6} : (\sqrt{3}+1)$

$= \frac{2}{2\sqrt{2}} : \frac{\sqrt{6}}{2\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+1}{2\sqrt{2}}$

(Dividing by $2\sqrt{2}$)

$= \sin 45° : \sin 60° : \sin 75°$

Thus, $A = 45°$, $B = 60°$, $C = 75°$.

Note: Supposing $a = 2k$, $b = \sqrt{6}\,k$ and $c = (\sqrt{3}+1)k$ and applying cosine law, we can also get the required angles.

20. 2055 Old Model Q.No. 8b OR

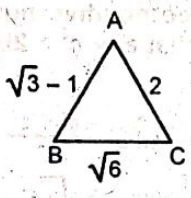
If $a = \sqrt{6}$, $b = 2$, $c = \sqrt{3} - 1$, solve the triangle [4]

SOLUTION

Here,

$a = \sqrt{6}$, $b = 2$, $c = \sqrt{3} - 1$

From cosine law, we have



$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2^2 + (\sqrt{3}-1)^2 - (\sqrt{6})^2}{2 \times 2(\sqrt{3}-1)}$

$= \frac{4 + 3 - 2\sqrt{3} + 1 - 6}{4(\sqrt{3}-1)}$

$= \frac{2 - 2\sqrt{3}}{4(\sqrt{3}-1)} = \frac{-2(\sqrt{3}-1)}{4(\sqrt{3}-1)} = -\frac{1}{2} = \cos 120°$

$\therefore A = 120°$

Again, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{(\sqrt{6})^2 + 2^2 - (\sqrt{3}-1)^2}{2\sqrt{6} \times 2}$

$= \frac{6 + 4 - (3 - 2\sqrt{3} + 1)}{4\sqrt{6}}$

$= \frac{6 + 2\sqrt{3}}{4\sqrt{6}} = \frac{2\sqrt{3}(\sqrt{3}+1)}{2\sqrt{3} \times 2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\cos C = \cos 15°$

$\therefore C = 15°$

Also, we have,

$A + B + C = 180°$

or, $B = 180° - (A + C)$

$= 180° - (120° + 15°) = 45°$

Hence, $A = 120°$, $B = 45°$ and $C = 15°$.

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5 UNIT

SEQUENCE AND SERIES AND MATHEMATICAL INDUCTION

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A. SEQUENCE AND SERIES

2 MARKS QUESTIONS

1. 2071 Supp. Q.No. 2b

If a, b, c are in A.P. and x, y, z are in G.P, show that: $x^{b-c} y^{c-a} z^{a-b} = 1$ [2]

SOLUTION

If a, b, c are in A.P, then $b - a = c - b = d$ (say)

Then, $a - b = -d$, $b - c = -d$ and $c - a = 2d$.

And, if x, y, z are in G.P, then $y^2 = xz$

L.H.S. $= x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$

$= x^{-d} \cdot y^{2d} \cdot z^{-d}$

$= x^{-d} \cdot (y^2)^d \cdot z^{-d}$

$= x^{-d} \cdot (xz)^d \cdot z^{-d}$

$= x^{-d} \cdot x^d \cdot z^d \cdot z^{-d}$

$= x^{-d+d} \cdot z^{d-d} = x^0 z^0 = 1 =$ R.H.S.

2. 2068 Q.No. 1a

If A.M., G.M. and H.M. be the arithmetic mean, geometric mean and harmonic mean between two unequal positive numbers, prove that: AM × HM = $(GM)^2$ [2]

SOLUTION

Let a and b be any two unequal positive numbers. Then,

$$A.M. = \frac{a+b}{2},\ G.M = \sqrt{ab},\ H.M = \frac{2ab}{a+b}$$

Now, $A.M \times H.M = \frac{a+b}{2} \times \frac{2ab}{a+b}$

$= ab = (\sqrt{ab})^2 = (G.M)^2$

3. 2067 Q.No. 1a

Prove that a^2, b^2, c^2 are in A.P., if b + c, c+ a, a + b are in H. P. [2]

SOLUTION

Here, b + c, c + a, a + b will be in H.P if

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

i.e. if $\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

or. $\frac{b+c-c-a}{(b+c)(c+a)} = \frac{c+a-a-b}{(a+b)(c+a)}$

or. $\frac{b-a}{c+b} = \frac{c-b}{b+a}$

or, $b^2 - a^2 = c^2 - b^2$

i.e. if a^2, b^2, c^2 are in A.P, which is given. Hence proved.

4. 2066 Q.No. 1 a

If a, b, c are three positive numbers in harmonic sequence show that $a^2 + c^2 > 2b^2$. [2]

SOLUTION

A.M between a^2 and $c^2 = \frac{a^2+c^2}{2}$

G.M between a^2 and $c^2 = \sqrt{a^2c^2} = ac$

Then, $\frac{a^2+c^2}{2} > ac$ ... (i) [$\because$ A.M > G.M.]

G.M between a and c $= \sqrt{ac}$

H.M between a and c = b (given)

Then, $\sqrt{ac} > b$ [$\because$ G.M. > H.M]

$\therefore$ $ac > b^2$ ... (ii)

From (i) and (ii)

$\frac{a^2+c^2}{2} > b^2$

$a^2 + c^2 > 2b^2$ proved.

5. 2066 C Q.No. 1 a

If G be the geometric mean between two distinct positive numbers a and b, show that $\frac{1}{G-a} + \frac{1}{G-b} = \frac{1}{G}$. [2]

SOLUTION

Since G is G.M between two distinct positive numbers a and b so, $G = \sqrt{ab}$

L.H.S $= \frac{1}{G-a} + \frac{1}{G-b}$

$= \frac{1}{\sqrt{ab}-a} + \frac{1}{\sqrt{ab}-b}$

$= \frac{\sqrt{ab}+a}{(\sqrt{ab})^2 - a^2} + \frac{\sqrt{ab}+b}{(\sqrt{ab})^2 - b^2}$

$= \frac{\sqrt{ab}+a}{ab-a^2} + \frac{\sqrt{ab}+b}{ab-b^2}$

$= \frac{\sqrt{ab}+b}{b(a-b)} - \frac{\sqrt{ab}+a}{a(a-b)}$

$$= \frac{a\sqrt{ab} + ab - b\sqrt{ab} - ab}{ab(a-b)}$$

$$= \frac{(a-b)\sqrt{ab}}{ab(a-b)} = \frac{1}{\sqrt{ab}} = \frac{1}{G} = \text{R.H.S proved.}$$

6. 2065 Q.No 1 a

If a, b, c are in G.P. Prove that $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P. [2]

SOLUTION

If a, b, c are in G.P then

$b^2 = ac$ ... (i)

$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

if $\frac{1}{2b} - \frac{1}{a+b} = \frac{1}{b+c} - \frac{1}{2b}$

or, $\frac{a+b-2b}{2b(a+b)} = \frac{2b-b-c}{2b(b+c)}$

or, $\frac{a-b}{a+b} = \frac{b-c}{b+c}$

or, $(a-b)(b+c) = (b-c)(a+b)$

or, $ab + ca - b^2 - bc = ab + b^2 - ca - bc$

or, $2ca = 2b^2$

$b^2 = ca$ which is true by (i)

Hence $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

7. 2064 Q.No. 1 a

If A.M. and G.M. be the arithmetic mean and geometric mean between two unequal positive numbers, prove that A.M. > G.M. [2]

SOLUTION

Let a and b be any two unequal positive numbers. Then,

$A.M = \frac{a+b}{2}$, $G.M = \sqrt{ab}$ & $H.M = \frac{2ab}{a+b}$

i. $A.M \times H.M = \frac{(a+b)}{2} \cdot \frac{2ab}{a+b}$

$= ab = (\sqrt{ab})^2$

$= (G.M)^2$

ii. $A.M - G.M = \frac{a+b}{2} - \sqrt{ab}$

$= \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{b} + (\sqrt{b})^2}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

which is always positive as it is a square of the difference between any two unequal positive numbers.

$\therefore$ $A.M - G.M > 0$

i.e. $A.M > G.M$ ... (a)

From (i)

$A.M \times H.M = G.M \times G.M$

or, $\frac{A.M}{G.M} = \frac{G.M}{H.M}$

Since A.M > G.M. we must have G.M > H.M ... (b)

Hence from (a) & (b) A.M > G.M > H.M

8. 2063 Q.No. 1 a

If a, b, c be in A.P.; b, c, d in G.P. and c, d, e in H.P, prove that a, c, e are in G.P. [2]

SOLUTION

a, b, c be in A.P.

$b = \frac{a+c}{2}$ ... (i)

b, c, d be in G.P

$c^2 = bd$ ... (ii)

c, d, e in H.P

$d = \frac{2ce}{c+e}$ ... (iii)

Now, from (ii)

$c^2 = bd$

or, $c^2 = \frac{a+c}{2} \cdot \frac{2ce}{c+e}$

$c = \frac{c(a+c)}{c+e}$

or, $c^2 + ce = ae + ce$

$\therefore$ $c^2 = ae$

i.e. a, c, e are in G.P. proved.

9. 2062 Q.No. 1 a

If H be the harmonic mean between a and b, prove that $\frac{1}{H-a} + \frac{1}{H-b} = \frac{1}{a} + \frac{1}{b}$ [2]

Solution

Since H be harmonic mean between a & b, $H = \frac{2ab}{a+b}$

L.H.S. $= \frac{1}{H-a} + \frac{1}{H-b}$

$= \frac{1}{\frac{2ab}{a+b} - a} + \frac{1}{\frac{2ab}{a+b} - b}$

$= \frac{1}{\frac{2ab - a^2 - ab}{a+b}} + \frac{1}{\frac{2ab - ab - b^2}{a+b}}$

$= \frac{a+b}{ab - a^2} + \frac{a+b}{ab - b^2}$

$= \frac{a+b}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right)$

$= \frac{a+b}{b-a}\left(\frac{b-a}{ab}\right) = \frac{a+b}{ab}$

$= \frac{1}{a} + \frac{1}{b}$ = R.H.S. proved.

10. 2061 Q.No. 1 a

If A be the A.M. and H the harmonic mean between two quantities a and b, show that: $\frac{a-A}{a-H} \times \frac{b-A}{b-H} = \frac{A}{H}$.

SOLUTION

$A = \frac{a+b}{2}$, $H = \frac{2ab}{a+b}$

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L.H.S. $= \frac{a-A}{a-H} \cdot \frac{b-A}{b-H}$

$= \frac{a-\frac{a+b}{2}}{a-\frac{2ab}{a+b}} \times \frac{b-\frac{a+b}{2}}{b-\frac{2ab}{a+b}}$

$= \frac{(a-b)(a+b)}{2(a^2-ab)} \times \frac{(b-a)(a+b)}{2(b^2-ab)}$

$= \frac{(a-b)(a+b)}{2a(a-b)} \times \frac{(b-a)(a+b)}{2b(b-a)}$

$= \frac{a+b}{2a} \cdot \frac{a+b}{2b}$

$= \frac{\frac{a+b}{2}}{\frac{2ab}{a+b}} = \frac{A}{H}$

= R.H.S Proved.

11. 2060 Q.No. 1 a

If $a^x = b^y = c^z$ and a, b, c are in G.P. then x, y, z are in H.P. [2]

SOLUTION

Since $a^x = b^y = c^z$

So, $a^x = b^y \Rightarrow a = b^{y/x}$

and $c^z = b^y \Rightarrow c = b^{y/z}$

If a, b, c are in G.P., then

$b^2 = ac$

or, $b^2 = b^{y/x} \cdot b^{y/z}$

or, $b^2 = b^{y/x + y/z}$

or, $2 = \frac{y}{x} + \frac{y}{z}$

or, $2 = \frac{yz + xy}{xz}$

or, $2xz = y(x + z)$

$\therefore y = \frac{2xz}{x+z}$

Hence x, y, z are in H.P.

12. 2059 Q.No. 1 a

If p is A.M. between q and r, q is G.M. between r and p then prove that r will be H.M. between p and q. [2]

SOLUTION

Since p is A.M between q and r, then

$p = \frac{q+r}{2}$ ... (i)

Similarly, since q is G.M between r and p, then

$q = \sqrt{rp}$

$q^2 = rp$ ... (ii)

From (i) $2p = q + r$

or, $2pq = q^2 + qr$ (Multiplying both sides by q)

or, $2pq = rp + qr$ [using (ii)]

or, $2pq = r(p + q)$

$\therefore r = \frac{2pq}{p+q}$

$\therefore$ r is the H.M between p and q.

13. 2058 Q.No. 1 a

Find the sum:

1.3 + 2.4 + 3.5 + ... to n terms.

SOLUTION

The nth term of 1. 2. 3, . . . is n

The nth term of 3. 4. 5, . . . is (n + 2)

Hence the nth term of the given series is n(n + 2)

i.e. $t_n = n^2 + 2n$

Let S_n be the sum to n terms of the given series. then

$S_n = \Sigma t_n = \Sigma(n^2 + 2n)$

$= \Sigma n^2 + 2\Sigma n = \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$

$= n(n+1)\left\{\left(\frac{2n+1}{6} + 1\right)\right\}$

$= n(n+1)\left(\frac{2n+1+6}{6}\right)$

$= \frac{n(n+1)(2n+7)}{6}$

14. 2057 Q.No. 1 a

Sum to infinity:

$1 + 3x + 5x^2 + 7x^3 + \ldots (-1 < x < 1)$. [2]

SOLUTION

Let $S_x = 1 + 3x + 5x^2 + 7x^3 + \ldots$

Then $xS_x = x + 3x^2 + 5x^3 + \ldots$

By subtraction, we get

$(1-x)S_x = 1 + 2x + 2x^2 + 2x^3 + \ldots$

or, $(1-x)S_x = 1 + (2x + 2x^2 + 2x^3 + \ldots)$

or, $(1-x)S_x = 1 + \frac{2x}{1-x}$ $\left[\because S_x = \frac{a}{1-r},\ |r| < 1\right]$

or, $(1-x)S_x = \frac{1-x+2x}{1-x}$

or, $(1-x)S_x = \frac{1+x}{1-x}$

$\therefore S_x = \frac{1+x}{(1-x)^2}$.

6 MARKS QUESTIONS

15. 2076 Set B Q.No. 12

If A,G and H are A.M., G.M. and H.M. respectively between any two unequal positive numbers then prove that [6]

i. A > G > H and

ii. $G^2 = A \times H$

SOLUTON

Let a and b be any two unequal positive numbers Then,

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$ & $H = \frac{2ab}{a+b}$

i. $A \times H = \frac{(a+b)}{2} \cdot \frac{2ab}{a+b}$

$= ab = (\sqrt{ab})^2$

$= G^2$

ii. $A - G = \frac{a+b}{2} - \sqrt{ab}$

$= \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{b} + (\sqrt{b})^2}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

which is always positive as it is a square of the difference between any two unequal positive numbers.

$\therefore$ $A - G > 0$

i.e. $A > G$ ... (a)

From (i)

$A \times H = G \times G$

or, $\frac{A}{G} = \frac{G}{H}$

Since $A > G$, we must have

$G > H$ ... (b)

Hence from (a) & (b) $A > G > H$

16. 2076 Set C Q.No. 12

If p, q, r be in A. P., q, r, x be in G. P. and r, x, y in H.P., prove that p, r, y are in G.P. [6]

SOLUTION

If p, q, r be in A.P. then $q = \frac{p+r}{2}$ ...(i)

If q, r, x be in G.P. then $r^2 = qx$ ...(ii)

If r, x, y be in H.P. then $x = \frac{2ry}{r+y}$ ...(iii)

Putting the value of q and x from (i) and (iii) in (ii)

$r^2 = \frac{p+r}{2} \cdot \frac{2ry}{r+y}$

or, $r^2 = (p+r) \cdot \frac{ry}{r+y}$

or, $r = (p+r) \cdot \frac{y}{r+y}$

or, $r(r+y) = (p+r)y$

or, $r^2 + ry = py + ry$

or, $r^2 = py$

This shows that p, r, y are in G.P.

17. 2075 GIE Q.No. 12

If A, G, H are A.M., G.M. and H.M. between any two unequal positive numbers, prove that (i) $A > G > H$ and (ii) $G^2 = A \times H$. [6]

➥ Please refer to 2076 Set B Q.No. 12

18. 2075 Set A Q.No. 12

If a, b, c are in H.P., prove that $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. [6]

SOLUTION

If a, b, c are in H.P then

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Multiplying each term by $(a+b+c)$

$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.

Subtracting 2 from each term.

$\frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ are in A.P

or, $\frac{a+b+c-2a}{a}, \frac{a+b+c-2b}{b}, \frac{a+b+c-2c}{c}$ are in A.P

or, $\frac{a+b+c-2a}{a}, \frac{a+b+c-2b}{b}, \frac{a+b+c-2c}{c}$ are in A.P

$\therefore$ $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P

19. 2075 Set B Q.No. 12

Sum to n terms the series:

$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$ [6]

SOLUTION

Let S_n be the sum to n term of the given series.

Then,

$S_n = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots + \frac{3n-5}{5^{n-2}} + \frac{3n-2}{5^{n-1}}$

$\frac{1}{5}S_n = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots\ldots\ldots + \frac{3n-5}{5^{n-1}} + \frac{3n-2}{5^n}$

Subtracting, we get

$\frac{4}{5}S_n = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \ldots + \frac{3}{5^{n-1}} - \frac{3n-2}{5^n}$

$= 1 + 3\left\{\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots + \frac{1}{5^{n-1}}\right\} - \frac{3n-2}{5^n}$

$= 1 + 3\frac{\frac{1}{5}\left\{1 - \left(\frac{1}{5}\right)^{n-1}\right\}}{1 - \frac{1}{5}} - \frac{3n-2}{5^n}$

$= 1 + \frac{3}{4} - \frac{3}{4} \cdot \frac{1}{5^{n-1}} - \frac{3n-2}{5^n}$

$= 1 + \frac{3}{4}\left\{1 - \frac{1}{5^{n-1}}\right\} - \frac{3n-2}{5^n}$

$= \frac{7}{4} - \frac{3}{4} \cdot \frac{1}{5^{n-1}} - \frac{3n-2}{5^n}$

$= \frac{7}{4} - \left\{\frac{15 + 12n - 8}{4.5^n}\right\}$

$\frac{4}{5}S_n = \frac{7}{4} - \frac{12n+7}{4.5^n}$

$S_n = \frac{35}{16} - \frac{5(12n+7)}{16.5^n}$

$\therefore$ $S_n = \frac{35}{15} - \frac{12n+7}{16.5^{n-1}}$

20. 2075 Set C Q.No. 12

Sum to n terms to the series. [6]

$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$

➥ Please refer to 2075 Set B Q.No. 12

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21. 2074 Supp Q.No. 12

Find the nth term and the sum of the n terms of the series: 3 + 7 + 13 +21 + 31 + ... [6]

SOLUTION

Given series is

$3 + 7 + 13 + 21 + 31 + \ldots$

$= (1^2 + 2) + (2^2 + 3) + (3^2 + 4) + (4^2 + 5) + \ldots$

The nth term of $1^2, 2^2, 3^2, 4^2, 5^2 \ldots$ is n^2

& the nth term of 2, 3, 4, 5, 6 ... is

$2 + (n-1) \cdot 1$ [$\because t_n = a + (n-1) \cdot l$]

$= 2 + n - 1$

$= n + 1$

The nth term of given series $(t_n) = n^2 + n + 1$

We have,

$$S_n = \Sigma t_n$$

$$= \Sigma(n^2 + n + 1)$$

$$= \Sigma n^2 + \Sigma n + \Sigma 1$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$$= n\left\{\frac{(n+1)(2n+1)}{6} + \frac{n+1}{2} + 1\right\}$$

$$= n\left\{\frac{(n+1)(2n+1) + 3(n+1) + 6}{6}\right\}$$

$$= n\left(\frac{2n^2 + 2n + n + 1 + 3n + 3 + 6}{6}\right)$$

$$= \frac{n(2n^2 + 6n + 10)}{6} = \frac{n}{3}(n^2 + 3n + 5)$$

22. 2074 Set A Q.No. 12

The sum of three numbers in A. P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G.P. Find the numbers. [6]

SOLUTION

Let three numbers in A.P. be a – d, a and a + d. By question,

$(a - d) + a + (a + d) = 36$

or, $3a = 36$

$\therefore a = 12$

Again, by question,

a – d + 1, a + 4 and a + d + 43 are in G.P.

i.e. 12 – d + 1, 12 + 4 and 12 + d + 43 are in G.P.

i.e. 13 – d, 16 and 55 + d are in G.P.

$\Rightarrow \frac{16}{13 - d} = \frac{55 + d}{16}$ (by definition of G.P.)

or, $256 = (55 + d)(13 - d)$

or, $256 = 715 - 55d + 13d - d^2$

or, $d^2 + 42d - 459 = 0$

or, $d^2 + 51d - 9d - 459 = 0$

or, $d(d + 51) - 9(d + 51) = 0$

or, $(d + 51)(d - 9) = 0$

$\therefore d = -51, 9$

When d = –51, a = 12

a – d = 12 + 51 = 63, a = 12, a + d = 12 – 51 = –39

When d = 9, a = 12

a – d = 12 – 9 = 3, a = 12, a + d = 9 + 12 = 21.

Required three numbers are 63, 12, –39 or 3, 12, 21

23. 2074 Set B Q.No. 12

Sum to n-terms of the series:

$$\frac{2}{5} + \frac{6}{5^2} + \frac{10}{5^3} + \frac{14}{5^4} + \ldots\ldots, \ldots\ldots\ldots, \ldots\ldots$$ [6]

SOLUTION

Let, $S_n = \frac{2}{5} + \frac{6}{5^2} + \frac{10}{5^3} + \frac{14}{5^4} + \ldots\ldots + \frac{4n-6}{5^{n-1}} + \frac{4n-2}{5^n}$

$$\frac{1}{5}S_n = \frac{2}{5^2} + \frac{6}{5^3} + \frac{10}{5^4} + \ldots\ldots\ldots\ldots + \frac{4n-6}{5^n} + \frac{4n-2}{5^{n+1}}$$

Subtracting, we get,

$$\left(1 - \frac{1}{5}\right)S_n = \frac{2}{5} + \frac{4}{5^2} + \frac{4}{5^3} + \frac{4}{5^4} + \ldots + \frac{4}{5^n} - \frac{4n-2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{2}{5} + \frac{\frac{4}{5^2}\left\{1 - \left(\frac{1}{5}\right)^{n-1}\right\}}{1 - \frac{1}{5}} - \frac{4n-2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{2}{5} + \frac{\frac{4}{5^2}\left(1 - \frac{1}{5^{n-1}}\right)}{\frac{4}{5}} - \frac{4n-2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{2}{5} + \frac{1}{5}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{4n-2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{2}{5} + \frac{1}{5} - \frac{1}{5^n} - \frac{4n-2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{3}{5} - \left(\frac{1}{5^n} + \frac{4n-2}{5^{n+1}}\right)$$

$$\text{or, } \frac{4}{5}S_n = \frac{3}{5} - \frac{5 + 4n - 2}{5^{n+1}}$$

$$\text{or, } \frac{4}{5}S_n = \frac{3}{5} - \frac{4n+3}{5^{n+1}}$$

$$\text{or, } S_n = \frac{5}{4}\left(\frac{3}{5} - \frac{4n+3}{5^{n+1}}\right)$$

$$\therefore S_n = \frac{3}{4} - \frac{5(4n+3)}{4.5^{n+1}} = \frac{3}{4} - \frac{4n+3}{4.5^n}$$

24. 2073 Supp Q.No. 12

Find the sum of n terms of the series [6]

$1^2 . 1 + 2^2 . 3 + 3^2 . 5 + \ldots$

SOLUTION

The nth term of $1^2, 2^2, 3^2, \ldots$ is n^2

and the nth term of 1, 3, 5, ... is 2n – 1

So, the nth term of the given series $(t_n) = n^2(2n - 1)$

$= 2n^3 - n^2$

Let S_n be the sum to n terms of the given series then

$$S_n = \Sigma t_n = 2\Sigma n^3 - \Sigma n^2$$

$$= \frac{2n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) - \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left(\frac{3n^2 + 3n - 2n - 1}{3}\right)$$

$$= \frac{n(n+1)(3n^2 + n - 1)}{2}$$

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25. 2073 Set C Q.No. 12

Find the general term and then find the sum of first n terms of the series $n + 2(n-1) + 3(n-2) + \ldots\ldots$ [6]

SOLUTION

Let t_r be the general term and S_n be the sum of first n terms.

Then, t_r = (r^{th} term of 1,2,3, …) × (r^{th} term of n, n − 1, n − 2, …)

$= \{(1 + (r-1)\cdot 1\} \times \{n + (r-1)\cdot(-1)\}$

$[\because t_n = a + (n-1)d]$

$= r(n - r + 1)$

We have,

$$S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n} r(n-r+1)$$

$$= \sum_{r=1}^{n} \{r(n+1) - r^2\} = \sum_{r=1}^{n} r(n+1) - \sum_{r=1}^{n} r^2$$

$$= (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$$

$$= (n+1)\cdot\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left\{(n+1) - \frac{2n+1}{3}\right\}$$

$$= \frac{n(n+1)}{2}\cdot\frac{(3n+3-2n-1)}{3}$$

$$= \frac{n(n+1)(n+2)}{6}$$

26. 2073 Set D Q.No. 12

Sum to n terms of the series: $3.2 + 5.2^2 + 7.2^3 + \ldots$ [6]

SOLUTION

Let, $S_n = 3\cdot2 + 5\cdot2^2 + 7\cdot2^3 + \ldots + (2n-1)\cdot2^{n-1} + (2n+1)\cdot2^n$ … (i)

Multiplying equation (i) on both sides by 2, we get.

$2S_n = 3\cdot2^2 + 5\cdot2^3 + 7\cdot2^4 + \ldots + (2n-1)2^n + (2n+1)2^{n+1}$ … (ii)

Subtracting equation (ii) from equation (i), we get

$S_n = 3\cdot2 + 5\cdot2^2 + 7\cdot2^3 + \ldots + (2n-1)\cdot2^{n-1} + 2(n+1)2^n$

$2S_n = \quad 3\cdot2^2 + 5\cdot2^3 + \ldots\ldots\ldots\ldots + (2n-1)\cdot2^n + (2n+1)2^{n+1}$

$-\quad -\quad -\quad\quad -\quad -$

$-S_n = 3\cdot2 + 2\cdot2^2 + 2\cdot2^3 + \ldots\ldots\ldots\ldots + 2\cdot2^n - (2n+1)2^{n+1}$

or, $-S_n = 3\cdot2 + 2(2^2 + 2^3 + \ldots + 2^n) - (2n+1)2^{n+1}$

or, $-S_n = 3\cdot2 + 2\cdot2^2\frac{(2^{n-1}-1)}{2-1} - (2n+1)2^{n+1}$

or, $-S_n = 6 + 8(2^{n-1} - 1) - (2n+1)2^{n+1}$

or, $-S_n = 6 + 8\cdot2^{n-1} - 8 - 2n\cdot2^{n+1} - 2^{n+1}$

or, $-S_n = -2 + 2^3\cdot2^{n-1} - n\cdot2^1\cdot2^{n+1} - 2^{n+1}$

or, $-S_n = -2 + 2^{n+2} - n\cdot2^{n+2} - 2^{n+1}$

or, $-S_n = -2 - (n-1)2^{n+2} - 2^{n+1}$

$\therefore S_n = 2 + 2^{n+1} + (n-1)2^{n+2}$

27. 2072 Supp. Q.No. 12

The sum of three numbers in A.P. is 36. When the number are increased by 1,4,43 respectively, the resulting numbers are in G.P. Find the numbers. [6]

➥ Please refer to 2074 Set A Q.No. 12

28. 2072 Set C Q.No. 12

The sum of three numbers in A.P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G.P. Find the numbers. [6]

➥ Please refer to 2074 Set A Q.No. 12

29. 2072 Set D Q.No. 13

Prove that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations: [6]

(a) $(G.M.)^2 = A.M. \times H.M.$ (b) A.M. > G.M. > H.M.

If A.M. between two numbers = 25 and H.M. = 16, find G.M.

SOLUTION

Let a and b be any two unequal positive numbers. Then,

$A.M = \frac{a+b}{2}$, $G.M = \sqrt{ab}$ & $H.M = \frac{2ab}{a+b}$

i. $A.M \times H.M = \frac{(a+b)}{2}\cdot\frac{2ab}{a+b}$

$= ab = (\sqrt{ab})^2$

$= (G.M)^2$

ii. $A.M - G.M = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a})^2 - 2\sqrt{a}\cdot\sqrt{b} + (\sqrt{b})^2}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

which is always positive as it is a square of the difference between any two unequal positive numbers.

$\therefore$ A.M − G.M > 0

i.e. A.M > G.M. … (a)

From (i)

$A.M \times H.M = G.M \times G.M$

or, $\frac{A.M}{G.M} = \frac{G.M}{H.M}$

Since A.M > G.M, we must have G.M > H.M … (b)

Hence from (a) & (b) A.M > G.M > H.M

Next part:

A.M. = 25, H.M. = 16, G.M. = ?

We have, $(G.M.)^2 = A.M. \times H.M.$

$(G.M.)^2 = 25 \times 16$

or, $(G.M.)^2 = 400 = (20)^2$

$\therefore$ G.M. = 20

30. 2072 Set E Q.No. 12

Sum to n terms of the series $1.1^2 + 4.2^2 + 7.3^2 + \ldots$ [6]

SOLUTION

Here,

n^{th} term (t_n) = (n^{th} term of 1, 4, 7, …) × (n^{th} term of 1^2, 2^2, 3^2, …)

$= (3n-2) \times n^2$

$= 3n^3 - 2n^2$

Sum to n terms $(S_n) = \Sigma t_n$

$= \Sigma(3n^3 - 2n^2)$

$= 3\Sigma n^3 - 2\Sigma n^2$

$= 3\left[\frac{n(n+1)}{2}\right]^2 - 2\cdot\frac{n(n+1)(2n+1)}{6}$

$= n(n+1)\left\{\frac{3n(n+1)}{4} - \frac{2n+1}{3}\right\}$

$= n(n+1)\left\{\frac{9n(n+1) - 4(2n+1)}{12}\right\}$

$= n(n+1)\frac{(9n^2+9n-8n-4)}{12}$

$= \frac{n(n+1)(9n^2+n-4)}{12}$

31. 2071 Set C Q.No. 12

Sum to n terms of the series: $\frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \ldots$ [6]

SOLUTION

Let $S_n = \frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \ldots + (-1)^n\frac{4n-6}{5^{n-1}} + (-1)^{n+1}\frac{4n-2}{5^n}$

$\frac{1}{5}S_n = \frac{2}{5^2} - \frac{6}{5^3} + \frac{10}{5^4} + \ldots\ldots\ldots + (-1)^n\frac{4n-6}{5^n} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

On adding, we get,

$\left(1+\frac{1}{5}\right)S_n = \frac{2}{5} - \frac{4}{5^2} + \frac{4}{5^3} - \frac{4}{5^4} + \ldots\ldots + (-1)^n\frac{4}{5^n} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2}\left\{1 - \frac{1}{5} + \frac{1}{5^2} - \ldots + (n-1)\text{ terms}\right\} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2}\cdot\frac{\left\{1-\left(-\frac{1}{5}\right)^{n-1}\right\}}{1-\left(-\frac{1}{5}\right)} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{4}{5^2}\cdot\frac{\left\{1-(-1)^{n-1}\cdot\frac{1}{5^{n-1}}\right\}}{\frac{6}{5}} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{2}{15}\left\{1 - \frac{(-1)^{n-1}}{5^{n-1}}\right\} + (-1)^{n+1}\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{2}{5} - \frac{2}{15} + \frac{2}{3}\cdot\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}\cdot\frac{4n-2}{5^{n+1}}$

or, $\frac{6}{5}S_n = \frac{4}{15} + \frac{2}{3}\cdot\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}\cdot\frac{4n-2}{5^{n+1}}$

or, $S_n = \frac{5}{6}\left(\frac{4}{15} + \frac{2}{3}\cdot\frac{(-1)^{n-1}}{5^n} + (-1)^{n+1}\cdot\frac{4n-2}{5^{n+1}}\right)$

$= \frac{2}{9} + \frac{1}{9}\cdot\frac{(-1)^{n-1}}{5^{n-1}} + \frac{(-1)^{n+1}(4n-2)}{6\cdot 5^n}$

$= \frac{2}{9} + \frac{(-1)^{n-1}}{9\cdot 5^{n-1}} + \frac{(-1)^{n+1}(4n-2)}{6\cdot 5^n}$

32. 2071 Set D Q.No. 12

Sum to n terms of the series:
$1.n + 2.(n-1) + 3.(n-2) + \ldots$

➥ Please refer to 2073 Set C Q.No. 12

33. 2070 Supp Q.No. 12

Find the general term and also the sum of n terms of the series:

$1.n + 2.(n-1) + 3.(n-2) + \ldots\ldots$ [6]

➥ Please refer to 2073 Set C Q.No. 12

34. 2070 Set C Q.No. 12

Show that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations. [6]

a. $(G.M.)^2 = A.M. \times H.M.$

b. $A.M. > G.M. > H.M.$

➥ Please refer to 2072 Set D Q.No. 13

35. 2070 Set D Q.No. 12

Sum to infinity the following series.

$1 - 5a + 9a^2 - 13a^3 + \ldots$ to ∞ $(-1 < a < 1)$. [6]

SOLUTION

Let $S_\infty = 1 - 5a + 9a^2 - 13a^3 + \ldots$

$aS_\infty = \quad a - 5a^2 + 9a^3 - \ldots$

$(1+a)S_\infty = 1 - 4a + 4a^2 - 4a^3 + \ldots$

or, $(1+a)S_\infty = 1 - 4a(1 - a + a^2 \ldots)$

or, $(1+a)S_\infty = 1 - 4a\cdot\frac{1}{1-(-a)}$ $\left[\because S_\infty = \frac{a}{1-r}\right]$

or, $(1+a)S_\infty = 1 - \frac{4a}{1+a}$

or, $(1+a)S_\infty = \frac{1+a-4a}{1+a}$

or, $(1+a)S_\infty = \frac{1-3a}{1+a}$

$\therefore S_\infty = \frac{1-3a}{(1+a)^2}$

36. 2069 Supp Q.No. 12

Define geometric mean. Find the sum of the first n terms of the series.

$1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \ldots$ [6]

SOLUTION

Geometric mean: If a and b are both positive numbers then $G = \sqrt{ab}$ is called the geometric mean between a and b. Again, if a and b are both negative numbers then $G = -\sqrt{ab}$ is called the geometric mean between a and b.

Next Part

$S_n = 1 + \frac{3}{2} + \frac{7}{14} + \frac{15}{8} + \ldots$ to n terms

$= 1 + \left(\frac{4-1}{2}\right) + \left(\frac{8-1}{4}\right) + \left(\frac{16-1}{8}\right) + \ldots$ n terms

$= 1 + \left(2 - \frac{1}{2}\right) + \left(2 - \frac{1}{4}\right) + \left(2 - \frac{1}{8}\right) + \ldots$ n terms

$= 1 + \{2 + 2 + 2 + \ldots$ to $(n-1)$ terms$\}$

$- \left\{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \text{ to } (n-1)\text{ terms}\right\}$

$= 1 + 2\cdot(n-1) + \frac{\frac{1}{2}\left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}}{1 - \frac{1}{2}}$

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$= 1 + 2(n-1) + 1 - \frac{1}{2^{n-1}}$

$= 2 + 2n - 2 - \frac{1}{2^{n-1}} = 2n - \frac{1}{2^{n-1}}$

37. 2069 [Set A] Q. No. 12

Find the sum of n terms of the series $3.1^2 + 4.2^2 + 5.3^2 + \ldots$. [6]

SOLUTION

The nth term of 3, 4, 5 ...is $n + 2$

And, the nth term of $1^2, 2^2, 3^2, \ldots$ is n^2

Hence, the nth term of given series $(t_n) = (n+2) \cdot n^2$
$= n^3 + 2n^2$

Sum of n terms $(S_n) = \Sigma t_n$

$= \Sigma (n^3 + 2n^2)$

$= \Sigma n^3 + 2\Sigma n^2$

$= \left[\frac{n(n+1)}{2}\right]^2 + 2.\frac{n(n+1)(2n+1)}{6}$

$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{3}$

$= n(n+1)\left\{\frac{n(n+1)}{4} + \frac{2n+1}{3}\right\}$

$= n(n+1)\frac{(3n^2+3n+8n+4)}{12}$

$= \frac{n(n+1)(3n^2+11n+4)}{12}$

38. 2069 [Set B] Q. No. 12

Find the nth term and then the sum of the first n-terms of the series: [6]

$1^2.2 + 2^2.3 + 3^2.4 + \ldots$.

SOLUTION

The nth term of $1^2, 2^2, 3^2, \ldots$ is n^2

And, the nth term of 2, 3, 4 ... is $(n + 1)$

Hence, the nth term of given series $(t_n) = n^2(n+1)$

Sum to n terms $(S_n) = \Sigma t_n$

$= \Sigma n^2 (n+1)$

$= \Sigma n^3 + \Sigma n^2$

$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)}{2}\left\{\frac{n(n+1)}{2} + \frac{2n+1}{3}\right\}$

$= \frac{n(n+1)}{2}\left\{\frac{3n^2+3n+4n+2}{6}\right\}$

$= \frac{n(n+1)(3n^2+7n+2)}{12}$

$= \frac{n(n+1)(3n^2+6n+n+2)}{12}$

$= \frac{n(n+1)\{3n(n+2)+1(n+2)\}}{12}$

$= \frac{n(n+1)(n+2)(3n+1)}{12}$

39. 2068 Q.No. 12

Prove that:

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$ **[6]**

SOLUTION

Let $S_n = 1^2 + 2^2 + 3^3 + \ldots + n^2$

We know that for all values of x

$x^3 - (x-1)^3 = 3x^2 - 3x + 1$

Putting $x = 1, 2, 3, \ldots n$, we get

$1^3 - 0^3 = 3.1^2 - 3.1 + 1$

$2^3 - 1^3 = 3.2^2 - 3.2 + 1$

$3^3 - 2^3 = 3.3^2 - 3.3 + 1$

. . . . . .

$n^3 - (n-1)^3 = 3.n^2 - 3.n + 1$

Adding, we get,

$n^3 - 0^3 = 3(1^2 + 2^2 + 3^2 + \ldots + n^2) - 3(1 + 2 + 3 + \ldots + n) + \underbrace{(1+1+\ldots+1)}_{n \text{ terms}}$

or, $n^3 = 3S_n - \frac{3n(n+1)}{2} + n$

or, $3S_n = n^3 - n + \frac{3n(n+1)}{2}$

or, $3S_n = n(n^2-1) + \frac{3n(n+1)}{2}$

or, $3S_n = n(n+1)(n-1) + \frac{3n(n+1)}{2}$

or, $3S_n = n(n+1)\left\{(n-1) + \frac{3}{2}\right\}$

or, $3S_n = n(n+1)\left(\frac{2n-2+3}{2}\right)$

$\therefore S_n = \frac{n(n+1)(2n+1)}{6}$

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

40. 2067 Q.No. 7a

The sides of a square are each 16 cm. A second square is drawn by joining the mid points of the sides, successively. In the second square the process is repeated to drawing the third square. If this process is continued indefinitely, find the sum of the areas of all the squares. [4]

SOLUTION

Let BCD be the square whose each side is 16 cm.

$\therefore$ Area of ABCD $= (16)^2 = 256$ cm^2

Again, let MNOP be the square obtained by joining the mid points M, N, O, P of sides AB, BC, CD and DA of a square ABCD.

Then, $MN = \sqrt{(MB)^2 + (BN)^2} = \sqrt{8^2 + 8^2} = 8\sqrt{2}$ cm.

The area of the square MNOP $= (8\sqrt{2})^2 = 128$ cm^2

Also, WXYZ be the square obtained by joining the midpoints W, X, Y, Z of the sides MN, NO, OP and PQ of a square MNOP.

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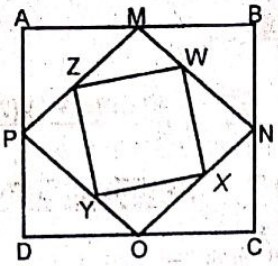

Then, $WX = \sqrt{(WN)^2 + (NX)^2}$

$= \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = 8$

Area of square $WXYZ = 8^2 = 64$ cm^2 and so on.

The series of the areas of all the squares is $256 + 128 + 64 + \ldots$

Here, $a = 256$, $r = \frac{128}{256} = \frac{1}{2}$, $S_\infty = ?$

We have,

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{256}{1-\frac{1}{2}} = \frac{256}{1/2}$$

$$= 256 \times 2 = 512 \text{ cm}^2$$

41. 2066 Q.No. 7 a

Sum to n terms of the series:

$$1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots$$ **[4]**

SOLUTION

Let S_n be the sum to n terms of given series

Then,

$$S_n = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots + \frac{n-1}{2^{n-2}} + \frac{n}{2^{n-1}}$$

$$\frac{1}{2}S_n = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n-1}{2^{n-1}} + \frac{n}{2^n}$$

By subtraction, we get,

$$\frac{1}{2}S_n = \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^{n-1}}\right) - \frac{n}{2^n}$$

or, $\frac{1}{2}S_n = \frac{1\left[1 - \left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} - \frac{n}{2^n}$ $\left[\because S_n = \frac{a(1-r^n)}{1-r} \text{ if } r < 1\right]$

or, $\frac{1}{2}S_n = \frac{1-\frac{1}{2^n}}{\frac{1}{2}} - \frac{n}{2^n}$

or, $\frac{1}{2}S_n = \left(\frac{2^n-1}{2^n}\right)\frac{2}{1} - \frac{n}{2^n}$

or, $\frac{1}{2}S_n = \frac{2.2^n - 2 - n}{2^n}$

or, $\frac{1}{2}S_n = 2 - \frac{2}{2^n} - \frac{n}{2^n}$

or, $S_n = 4 - \frac{2^2}{2^n} - \frac{2n}{2^n}$

$\therefore \quad S_n = 4 - \frac{1}{2^{n-2}} - \frac{n}{2^{n-1}}$

42. 2066 C Q.No. 7 a

Sum of n terms of series: $1 + \frac{4}{5} + \frac{7}{5^2} + \ldots\ldots$ **[4]**

➥ Please refer to 2075 Set B Q.No. 12

43. 2065 Q.No 7 a

Sum to infinity: $\frac{a}{x} + \frac{b}{x^2} + \frac{a}{x^3} + \frac{b}{x^4} + \ldots$, when $|x| > 1$. **[4]**

SOLUTION

Let $S_\infty = \frac{a}{x} + \frac{b}{x^2} + \frac{a}{x^3} + \frac{b}{x^4} + \ldots$

$$= \left(\frac{a}{x} + \frac{b}{x^2}\right) + \left(\frac{a}{x^3} + \frac{b}{x^4}\right) + \ldots$$

$$= \frac{ax+b}{x^2} + \frac{ax+b}{x^4} + \ldots$$

$$= \frac{\frac{ax+b}{x^2}}{1-\frac{1}{x^2}} \quad \left(\because S_\infty = \frac{a}{1-r}, |r| < 1\right)$$

$$= \frac{ax+b}{x^2-1}$$

44. 2064 Q.No. 7 a

Sum to n terms of the series:

$1^2 . 1 + 2^2 . 3 + 3^2 . 5 + \ldots\ldots$ **[4]**

➥ Please refer to 2073 Supp Q.No. 12

45. 2063 Q.No. 7 a

Prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

➥ Please refer to 2068 Q.No. 12

46. 2062 Q.No. 7 a

If A.M., G.M. and H.M. are the arithmetic mean, geometric mean and harmonic mean between two unequal positive numbers a and b, prove that: A.M. > G.M. > H.M. **[4]**

SOLUTION

Let a and b be any two unequal positive numbers. Then,

$$A.M = \frac{a+b}{2}, \quad G.M = \sqrt{ab} \ \& \ H.M = \frac{2ab}{a+b}$$

i. $A.M \times H.M = \frac{(a+b)}{2} \cdot \frac{2ab}{a+b}$

$= ab = (\sqrt{ab})^2 = (G.M)^2$

ii. $A.M - G.M = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{b} + (\sqrt{b})^2}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

which is always positive as it is a square of the difference between any two unequal positive numbers.

$\therefore \quad A.M - G.M > 0$

i.e. $A.M > G.M$. . . (a)

From (i)

$A.M \times H.M = G.M \times G.M$

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or, $\frac{A.M}{G.M} = \frac{G.M}{H.M}$

Since A.M > G.M, we must have G.M > H.M ... (b)

Hence from (a) & (b) A.M > G.M > H.M.

47. 2061 Q.No. 7 a

Sum to n term the following series: [4]

$$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$$

➥ Please refer 2066 C Q.No. 7 a

48. 2060 Q.No. 7 a

Prove that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the relation A.M. > G.M. > H.M. [4]

➥ Please refer to 2062 Q.No. 7 a

49. 2059 Q.No. 7 a

Prove that:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$ [4]

➥ Please refer to 2063 Q.No. 7 a

50. 2058 Q.No. 7 a

If a, b, c are in H.P., prove that: 2a – b, b, 2c – b are in G.P. [4]

SOLUTION

If a, b, c are in H.P, then

$$b = \frac{2ac}{a+c} \quad \ldots (i)$$

Now, (2a – b) (2c – b)

$$= \left(2a - \frac{2ac}{a+c}\right)\left(2c - \frac{2ac}{a+c}\right) \text{ [Using (i)]}$$

$$= \left(\frac{2a^2 + 2ac - 2ac}{a+c}\right)\left(\frac{2ac + 2c^2 - 2ac}{a+c}\right)$$

$$= \frac{2a^2}{a+c} \cdot \frac{2c^2}{a+c} = \frac{4a^2c^2}{(a+c)^2}$$

$$= \left(\frac{2ac}{a+c}\right)^2 = b^2$$

$\therefore$ (2a – b) (2c – b) = b^2

This shows that (2a – b), b, (2c – b) are in G.P.

51. 2057 Q.No. 7 a

If a, b, c are in H.P. prove that a(b + c), b(c + a), c(a + b) are in A.P. [4]

SOLUTION

If a, b, c are in H.P then

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{1}{a} - \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b} - \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right), \frac{1}{c} - \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ are in A.P.

$\Rightarrow -\left(\frac{1}{b}+\frac{1}{c}\right), -\left(\frac{1}{a}+\frac{1}{c}\right), -\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

$\Rightarrow \frac{1}{b}+\frac{1}{c}, \frac{1}{a}+\frac{1}{c}, \frac{1}{a}+\frac{1}{b}$ are in A.P.

$\Rightarrow \frac{b+c}{bc}, \frac{c+a}{ca}, \frac{a+b}{ab}$ are in A.P

Multiplying each term by abc, we get

a(b + c), b(c + a), c(a + b) are in A.P.

B. MATHEMATICAL INDUCTION

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 2b

Prove by mathematical induction:

$1 + 3 + 5 + \ldots + (2n - 1) = n^2$. [2]

SOLUTION

Let P(n) be the given statement. Then

P(n): $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

When n = 1:

L.H.S = 1

R.H.S = $1^2 = 1$

$\therefore$ L.H.S = R.H.S i.e. P(1) is true.

Let P(m) be true for $m \in N$. That is,

P(m): $1 + 3 + 5 + \ldots + (2m - 1) = m^2$ ... (i)

Now, we shall show that P(m + 1) is true when P(m) is true.

For this, we add 2(m + 1) – 1 = 2m + 1 on both sides of (i), we get,

$(1 + 3 + 5 + \ldots + 2m - 1) + 2m + 1 = m^2 + 2m + 1$

(Using (i))

$= (m + 1)^2$

which shows that P(m + 1) is true whenever P(m) is true. Hence, by the principle of mathematical induction P(n) is true for all $n \in N$.

2. 2076 Set C Q.No. 2b

For the statement P_n: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$, write P_1, P_k and P_{k+1}. [2]

SOLUTON

Given statement is

P_n: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

When n = 1,

P_1: $1 = \frac{1(1+1)}{2}$;

When n = k,

P_k: $1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$,

When n = k + 1,

P_{k+1}: $1+2+3+ \ldots +k+ (k+1) = \frac{(k+1)\{(k+1)+1\}}{2}$

3. 2075 GIE Q.No. 2b

Using the principle of mathematical induction, prove that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$. [2]

➥ Please refer to 2076 Set B Q.No. 2b

4. 2075 Set A Q.No. 2b

Use mathematical induction to prove that for any positive integer n, $(n+3)^2 > 2n+7$. [2]

SOLUTION

Let P(n): $(n+3)^2 > 2n+7$

When n = 1, P(1): $(1+3)^2 > 2\times 1+7$

$16 > 9$ which is true.

So, P(1) is true

Suppose that P(k) is true for some $k \in N$.

Then, P(k): $(k+3)^2 > 2k+7$ ... (1)

We show P(k + 1) is true.

Now,

$\{(k+1)+3\}^2$

$= \{(k+3)+1\}^2$

$= (k+3)^2 + 2\cdot(k+3)\cdot 1 + 1^2 > (2k+7) + 2\cdot(k+3) + 1$. [Using (i)]

$= 2k+7+2k+6+1$

$= 2k+2+7+2k+5$

$= 2(k+1)+7+(2k+5) > 2(k+1)+7$ [$\because k \in N, 2k+5 > 0$]

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

5. 2075 Set B Q.No. 2b

Use mathematical induction to prove that the sum of first n natural numbers is $\frac{n(n+1)}{2}$. [2]

SOLUTION

Let P(n) be the given statement. Then,

P(n) : $1+2+3+\ldots+n = \frac{n(n+1)}{2}$

When n = 1, L.H.S. = 1 and R.H.S. $= \frac{1(1+1)}{2} = 1$

$\therefore$ L.H.S. = R.H.S i.e. P(1) is true.

Assume that P(k) is true for some $k \in N$. Then,

P(k) : $1+2+3+\ldots+k = \frac{k(k+1)}{2}$ ...(i)

We show that P(k + 1) is true.

Adding (k + 1) on both sides of (i), we get

$1+2+3+\ldots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$

$= (k+1)\left\{\frac{k}{2}+1\right\}$

$= (k+1)\frac{(k+2)}{2}$

$= \frac{(k+1)\{(k+1)+1\}}{2}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

6. 2075 Set C Q.No. 2b

Using the principle of mathematical induction, prove that for any positive integer, $4^n - 1$ is divisible by 3. [2]

SOLUTION

Let P(n) be the given statement. Then, P(n) : $4^n - 1$ is divisible by 3.

When n = 1, $P(1) = 4^1 - 1 = 3$ which is divisible by 3.

i.e. P(1) is true.

Let us assume that P(k) is true for some $k \in N$. Then,

P(k): $4^k - 1$ is divisible by 3. ...(i)

We show that P(k + 1) is true.

Now, P(k + 1) : $4^{k+1} - 1$

$= 4^k \cdot 4 - 4 + 4 - 1$

$= 4(4^k - 1) + 3$

which is divisible by 3 as first part is divisible by 3 by (i) and the second term is multiple of 3.

This shows that P(k + 1) is true whenever p(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

7. 2074 Supp Q.No. 2b

Using the principle of mathematical induction, prove that $1+3+5+7+\ldots+(2n-1) = n^2$. [2]

➥ Please refer to 2076 Set B Q.No. 2b

8. 2074 Set A Q.No. 2b

Using the principle of mathematical induction, prove that: $2+2^2+2^3+\ldots+2^n = 2(2^n-1)$. [2]

SOLUTION

Let P(n) be the given statement. Then,

P(n) : $2+2^2+2^3+\ldots+2^n = 2(2^n-1)$

When n = 1,

L.H.S = 2 & R.H.S. $= 2(2^1-1) = 2$

$\therefore$ L.H.S = R.H.S

Hence P(1) is true.

Suppose P(k) is true for some $k \in N$. Then,

$2+2^2+2^3+\ldots+2^k = 2(2^k-1)$ ... (i)

We show that P(k + 1) is true.

Adding 2^{k+1} on both sides of (i), we get,

$2+2^2+2^3+\ldots+2^k+2^{k+1}$

$= 2(2^k-1)+2^{k+1} = 2.2^k - 2 + 2^{k+1}$

$= 2^{k+1}+2^{k+1}-2 = 2.2^{k+1}-2 = 2(2^{k+1}-1)$

which shows that P(k + 1) is true whenever P(k) is true. Hence by principle of mathematical induction P(n) is true for all $n \in N$.

9. 2074 Set B Q.No. 2b

Prove by mathematical induction for all $n \in N$ $2+4+6+\ldots+2n = n(n+1)$. [2]

SOLUTION

Let P(n) be the given statement.

Then, P(n) : $2+4+6+\ldots+2n = n(n+1)$

When n = 1, L.H.S. = 2, R.H.S. = 1(1 + 1) = 2

$\therefore$ L.H.S. = R.H.S. i.e. P(1) is true.

Let us assume that P(k) is true for some $k \in N$. Then,

P(k) : $2+4+6+\ldots+2k = k(k+1)$ ...(i)

We show that P(k + 1) is true.

Adding (2k + 2) on both sides of (i), we get

$2+4+6+\ldots+2k+(2k+2) = k(k+1)+(2k+2)$

$= k(k+1)+2(k+1)$

$= (k+1)(k+2)$

$= (k+1)\{(k+1)+1\}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

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10. 2073 Supp Q.No. 2b

Using the principle of mathematical induction, prove that $3^{2n} - 1$ is divisible by 8. [2]

SOLUTION

Let P(n) be the given statement. Then,

P(n): $3^{2n} - 1$ is divisible by 8.

When n = 1, P(1): $3^{2\cdot1} - 1 = 9 - 1 = 8$ which is divisible by 8.

i.e. P(1) is true.

Assume that P(k) is true for some $k \in N$. Then,

P(k): $3^{2k} - 1$ is divisible by 8. ... (i)

We show P(k + 1) is true.

Now, P(k + 1) : $3^{2(k+1)} - 1$

$= 3^{2k+2} - 1 = 3^{2k} \cdot 3^2 - 1$

$= 3^{2k} \cdot 3^2 - 3^2 + 3^2 - 1$

$= 3^2(3^{2k} - 1) + 8$

which is divisible by 8 as first part is divisible by 8 by (i)

This shows that P(k + 1) is true whenever P(k) is true. Then by principle of mathematical induction P(n) is true for all $n \in N$.

11. 2073 Set C Q.No. 2b

Using the principle of Mathematical induction prove that 1 + 3 + 5 + ... to n terms = n^2. [2]

Please refer to 2076 Set B Q.No. 2b

12. 2073 Set D Q.No. 2b

Using the principle of mathematical induction, prove that: $1^3 + 2^3 + 3^3 + \ldots n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$. [2]

SOLUTION

Let P(n) be the given statement.

Then, P(n) : $1^3 + 2^3 + 3^3 + \ldots + n^3$

$= \left\{\frac{n(n+1)}{2}\right\}^2$

When n = 1 L.H.S. = $1^3 = 1$ and R.H.S.

$= \left\{\frac{1(1+1)}{2}\right\}^2 = 1$

∴ L.H.S. = R.H.S. i.e. P(1) is true.

Let us assume that P(k) is true for some $k \in N$. Then,

$1^3 + 2^3 + 3^3 + \ldots + k^3 = \left\{\frac{k(k+1)}{2}\right\}^2$ ...(i)

We show that P(k + 1) is true.

Adding $(k + 1)^3$ on both sides of (i), we get

$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3$

$= \left\{\frac{k(k+1)}{2}\right\}^2 + (k+1)^3$

$= (k+1)^2\left(\frac{k^2}{4} + k + 1\right)$

$= (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$

$= \frac{(k+1)^2(k+2)^2}{4}$

$= \left[\frac{(k+1)\{(k+1)+1\}}{2}\right]^2$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

13. 2072 Supp. Q.No. 2b

Prove by the principle of mathematical induction that: $1+3+5+ \ldots + 2n-1) = n^2$ [2]

Please refer to 2076 Set B Q.No. 2b

14. 2072 Set C Q.No. 2b

Prove by the principle of mathematical induction that: [2]

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

SOLUTION

Let P(n) be the given statement. Then,

P(n): $\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

When n = 1, L.H.S. = $\frac{1}{1\cdot2} = \frac{1}{2}$, R.H.S. = $\frac{1}{1+1} = \frac{1}{2}$

∴ L.H.S. = R.H.S i.e. P(1) is true.

Let P(k) be true for $k \in N$. Then,

$\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$ ...(i)

We show that P(k + 1) is true.

Adding $\frac{1}{(k+1)(k+2)}$ on both sides of (i), we get

$\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \ldots \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$

$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{(k+1)(k+1+1)}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

15. 2072 Set D Q.No. 2b

Prove by the principle of mathematical induction that [2]

$$2 + 5 + 8 + \ldots + (3n-1) = \frac{n(3n+1)}{2}.$$

SOLUTION

Let P(n) be the given statement. Then,

P(n) : $2 + 5 + 8 + \ldots + (3n-1) = \frac{n(3n+1)}{2}$

When n = 1, L.H.S. = 2, R.H.S. = $\frac{1(3\times1+1)}{2} = 2$

∴ L.H.S. = R.H.S. i.e. P(1) is true.

Let P(k) be true for some $k \in N$.

That is P(k) : $2 + 5 + 8 + \ldots + (3k-1) = \frac{k(3k+1)}{2}$ ...(i)

We show that P(k + 1) is true.

Adding (3k + 2) on both sides of (i), we get

$2+5+8+ \ldots + (3k-1) + (3k+2)$

$= \frac{k(3k+1)}{2} + (3k+2) = \frac{3k^2+k+6k+4}{2}$

$= \frac{3k^2+7k+4}{2} = \frac{3k^2+3k+4k+4}{2}$

$= \frac{3k(k+1)+4(k+1)}{2} = \frac{(k+1)(3k+4)}{2}$

$= \frac{(k+1)(3k+3+1)}{2} = \frac{(k+1)\{3(k+1)+1\}}{2}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all n ∈ N.

16. 2072 Set E Q.No. 2b

Use mathematical induction to prove that for any positive integer n, $9^n - 1$ is divisible by 4.

SOLUTION

Let P(n) be the given statement. Then, P(n) : $9^n - 1$ is divisible by 4.

When n = 1, P(1) = $9^1 - 1 = 8$ which is divisible by 4.

i.e. P(1) is true.

Let P(k) be true for some k ∈ N.

Then, P(k) : $9^k - 1$ is divisible by 4 ...(i)

We show P k + 1) is true

Now, P(k + 1) : $9^{k+1} - 1$

$= 9^k \cdot 9 - 1$

$= 9^k \cdot 9 - 9 + 9 - 1$

$= 9(9^k - 1) + 8$

which is divisible by 4 as first part is divisible by 4 by (i) and the second is multiple of 4.

This shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all n ∈ N.

17. 2071 Set C Q.No. 2b

Prove by mathematical induction:

1 + 3 + 5 + ... + n terms = n^2 [2]

➥ Please refer to 2076 Set B Q.No. 2b

18. 2071 Set D Q.No. 2b

Prove by the principle of mathematical induction that: $4^n - 1$ is divisible by 3. [2]

➥ Please refer to 2075 Set C Q.No. 2b

19. 2070 Supp Q.No. 2b

Using the principle of mathematical induction, prove that: 2 + 4 + 6 + 8 + ... + 2n = n(n + 1) [2]

➥ Please refer to 2074 Set B Q.No. 2b

20. 2070 Set C Q.No. 2 b

Using principle of mathematical induction, Prove that: 1 + 3 + 5 + 7 + ... + (2n–1) = n^2. [2]

➥ Please refer to 2076 Set B Q.No. 2b

21. 2070 Set D Q.No. 2 b

Using principle of mathematical induction, prove that: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \frac{1}{2^n} = 1 - \frac{1}{2^n}$. [2]

SOLUTION

Let P(n) be the given statement.

Then, P(n): $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

When n = 1, L.H.S. = $\frac{1}{2}$ and R.H.S. = $1 - \frac{1}{2^1} = \frac{1}{2}$

∴ L.H.S. = R.H.S. i.e. P(1) is true.

Let us suppose that P(k) is true for some k ∈ N. Then,

p(k) : $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$ ...(i)

We show that P(k + 1) is true.

Adding $\frac{1}{2^{k+1}}$ on both sides of (i), we get,

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 1 - \left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right)$

$= 1 - \frac{2-1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all n ∈ N.

22. 2069 Supp Q.No. 2 b

Prove by the principle of mathematical induction:

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$ [2]

➥ Please refer to 2073 Set D Q.No. 2b

23. 2069 [Set A] Q. No. 2b

Use the principle of mathematical induction:

2 +4 + 6 + ... + 2n = n(n + 1). [2]

➥ Please refer to 2074 Set B Q.No. 2b

24. 2069 [Set B] Q. No. 2b

Prove by the principle of mathematical induction: 2 + 4 + 6 ... + 2n = n(n + 1). [2]

➥ Please refer to 2074 Set B Q.No. 2b

25. 2068 Q.No. 2b

Using the principle of mathematical induction, prove that: $1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$. [2]

➥ Please refer to 2075 Set B Q.No. 2b

6 MARKS QUESTIONS

26. 2071 Supp. Q.No. 12

State the principle of mathematical induction. Using it prove that: [6]

$1.3 + 2.4 + 3.5 + \ldots + n(n+2) = \frac{1}{6} n(n+1)(2n+7)$.

SOLUTION

Principle of mathematical induction: The principle of mathematical induction states that if P(n) be the statement and if

(i) P(1) is true

(ii) P(k + 1) is true whenever P(k) is true then P(n) is true for all n ∈ N.

Next part:

Let P(n) be the given statement. Then,

P(n) : $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots + n(n+2)$

$= \frac{1}{6} n(n+1)(2n+7)$

When n = 1, L.H.S. = $1 \cdot 3 = 3$,

R.H.S. = $\frac{1}{6} \cdot 1 (1+1)(2 \cdot 1 + 7) = 3$

∴ L.H.S. = R.H.S. i.e. P(1) is true.

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Let P(k): $1\cdot3 + 2\cdot4 + 3\cdot5 + \ldots k\cdot(k+2)$

$= \frac{1}{6}k(k+1)(2k+7)$ ...(i)

We show that P(k + 1) is true

Adding (k + 1) (k + 3) on both sides of (i), we get

$1\cdot3+2\cdot4+3\cdot5+\ldots+k\cdot(k+2)+(k+1)\cdot(k+3)$

$= \frac{1}{6}k(k+1)(2k+7)+(k+1)(k+3)$

$= (k+1)\left\{\frac{1}{6}k(2k+7)+(k+3)\right\}$

$= (k+1)\frac{(2k^2+7k+6k+18)}{6}$

$= \frac{(k+1)(2k^2+13k+18)}{6}$

$= \frac{(k+1)(2k^2+4k+9k+18)}{6}$

$= \frac{(k+1)\{2k(k+2)+9(k+2)\}}{6}$

$= \frac{(k+1)(k+2)(2k+9)}{6}$

$= \frac{(k+1)(k+1+1)(2k+2+7)}{6}$

$= \frac{(k+1)\{(k+1)+1\}\{2(k+1)+7\}}{6}$

which shows that P(k + 1) is true whenever P(k) is true. Hence, by principle of mathematical induction P(n) is true for all $n \in N$.

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6 UNIT

MATRICES AND DETERMINANTS

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A. MATRICES

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 2c

Find the adjoint of the matrix $\begin{pmatrix} 2 & 5 \\ 3 & -7 \end{pmatrix}$. [2]

SOLUTION

Let $A = \begin{pmatrix} 2 & 5 \\ 3 & -7 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

The cofactors are

A_{11} = cofactors of $a_{11} = (-1)^{1+1}(-7) = -7$

A_{12} = cofactors of $a_{12} = (-1)^{1+2}(3) = -3$

A_{21} = cofactors of $a_{21} = (-1)^{2+1}(5) = -5$

A_{22} = cofactors of $a_{22} = (-1)^{2+2}(2) = 2$

Matrix of cofactors $= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} -7 & -3 \\ -5 & 2 \end{pmatrix}$

Adjoint of $A = \begin{pmatrix} -7 & -3 \\ -5 & 2 \end{pmatrix}^T = \begin{pmatrix} -7 & -5 \\ -3 & 2 \end{pmatrix}$

2. 2076 Set C Q.No. 2c

If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, show that $(A^T)^T = A$. [2]

SOLUTON

Given $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

$A^T = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}^T = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$

$(A^T)^T = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}^T = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = A$

$\therefore (A^T)^T = A$

3. 2075 GIE Q.No. 2c

$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$, verify that $(AB)^T = B^T.A^T$. [2]

SOLUTION

$AB = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2+0 & -1-2 \\ 6+0 & -3-1 \end{pmatrix}$

$= \begin{pmatrix} 2 & -3 \\ 6 & -4 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} 2 & -3 \\ 6 & -4 \end{pmatrix}^T$

$= \begin{pmatrix} 2 & 6 \\ -3 & -4 \end{pmatrix}$ ...(i)

Again,

$B^TA^T = \begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}^T \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}^T$

$= \begin{pmatrix} 2 & 0 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2+0 & 6+0 \\ -1-2 & -3-1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 6 \\ -3 & -4 \end{pmatrix}$ ...(ii)

From (i) and (ii),

$(AB)^T = B^TA^T$. Hence verified.

4. 2075 Set A Q.No. 2c

Define skew-symmetric matrix. For any square matrix A, show that A – A' is skew-symmetric. [2]

SOLUTION

Skew-symmetric matrix: A square matrix $A = (a_{ij})$ is said to be skew - symmetric matrix if $a_{ij} = -a_{ji}$ for all i & j.

In other words, a square matrix A is said to be skew-symmetric if $A' = -A$.

Next part:

$(A - A')' = A' - (A')'$

$= A' - A$

$= -(A - A')$

$\therefore (A - A')' = -(A - A')$

This shows that A – A' is skew symmetric matrix.

5. 2075 Set B Q.No. 2c

Find the adjoint of the matrix $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$. [2]

SOLUTION

Let $A = \begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$

The cofactors are

$A_{11} = 2$ $\quad A_{21} = -3$

$A_{12} = -4$ $\quad A_{22} = 5$

Matrix of cofactors $= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -3 & 5 \end{pmatrix}$

Adj. $A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \end{pmatrix}^T = \begin{pmatrix} 2 & -3 \\ -4 & 5 \end{pmatrix}$

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6. 2075 Set C Q.No. 2c

If $A = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$ be a symmetric matrix, find the value of x. [2]

SOLUTION

If $A = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$ be a symmetric matrix then $A^T = A$.

i.e. $\begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}^T = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$

or, $\begin{pmatrix} 4 & 2x-1 \\ x+3 & -1 \end{pmatrix} = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$

Comparing corresponding elements, we get

$2x - 1 = x + 3$

or, $2x - x = 3 + 1$

$\therefore x = 4$.

7. 2074 Supp Q.No. 2c

If $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$, find AA^T. [2]

SOLUTION

Here,

$A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$

$A^T = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}^T = \begin{pmatrix} 7 & 6 \\ -3 & 2 \end{pmatrix}$

Now,

$AA^T = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 7 & 6 \\ -3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 7\times7+(-3)\times(-3) & 7\times6+(-3)\times2 \\ 6\times7+2\times(-3) & 6\times6+2\times2 \end{pmatrix}$

$= \begin{pmatrix} 49+9 & 42-6 \\ 42-6 & 36+4 \end{pmatrix}$

$= \begin{pmatrix} 58 & 36 \\ 36 & 40 \end{pmatrix}$

8. 2074 Set A Q.No. 2c

If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T. [2]

SOLUTION

Here,

$A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$

$\therefore A^T = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$

Now, $A \cdot A^T = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$

$= \begin{pmatrix} 4+1 & 2-2 \\ 2-2 & 1+4 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

9. 2074 Set B Q.No. 2c

Find the adjoint of the matrix $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$. [2]

SOLUTION

Let $A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

The Cofactor are

A_{11} = Cofactor of $a_{11} = (-1)^{1+1}(-1) = -1$

A_{12} = Cofactor of $a_{12} = (-1)^{1+2}(4) = -4$

A_{21} = Cofactor of $a_{21} = (-1)^{2+1}(1) = -1$

A_{22} = Cofactor of $a_{22} = (-1)^{2+2}(2) = 2$

Matrix of cofactors $= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ -1 & 2 \end{pmatrix}$

Adjoint of $A = \begin{pmatrix} -1 & -4 \\ -1 & 2 \end{pmatrix}^T = \begin{pmatrix} -1 & -1 \\ -4 & 2 \end{pmatrix}$

10. 2073 Supp Q.No. 2c

If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, find B^TA^T. [2]

SOLUTION

$B^TA^T = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}^T \cdot \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}^T$

$= \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 \\ -5 & 6 \end{pmatrix}$

$= \begin{pmatrix} 8+5 & 6-6 \\ 12+10 & 9-12 \end{pmatrix} = \begin{pmatrix} 13 & 0 \\ 22 & -3 \end{pmatrix}$

11. 2073 Set C Q.No. 2c

If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, verify that $(A + B)^1 = A^1 + B^1$. [2]

SOLUTION

$A + B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix}$

$(A + B)' = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix}' = \begin{pmatrix} 3 & 9 \\ 6 & 12 \end{pmatrix}$

And, $A' + B' = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}' + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}'$

$= \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 6 & 12 \end{pmatrix}$

$\therefore (A + B)' = A' + B'$

12. 2073 Set D Q.No. 2c

If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, show that $A^T \cdot A = I$. [2]

SOLUTION

Here, $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$A^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$A^T \cdot A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

13. 2072 Supp. Q.No. 2c

If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T. [2]

➥ Please refer to 2074 Set A Q.No. 2c

14. 2072 Set C Q.No. 2c

If $A = \begin{pmatrix} 3 & -5 \\ 4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$, find $(AB)^T$. [2]

SOLUTION

$$AB = \begin{pmatrix} 3 & -5 \\ 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 0-5 & 9+10 \\ 0+6 & 12-12 \end{pmatrix} = \begin{pmatrix} -5 & 19 \\ 6 & 0 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} -5 & 19 \\ 6 & 0 \end{pmatrix}^T = \begin{pmatrix} -5 & 6 \\ 19 & 0 \end{pmatrix}$$

15. 2072 Set D Q.No. 2c

If $A = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$, find $(A^T)^T$. [2]

SOLUTION

Here, $A = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$

$$A^T = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$$

16. 2072 Set E Q.No. 2c

If $A = \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}$ and $f(x) = x^2 - 5x + 3$, find f(A). [2]

SOLUTION

Here, $A = \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}$

$f(x) = x^2 - 5x + 3$

$f(A) = A^2 - 5A + 3I$

$$= \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} - 5\begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-12 & 3+6 \\ -4-8 & -12+4 \end{pmatrix} - \begin{pmatrix} 5 & 15 \\ -20 & 10 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -11 & 9 \\ -12 & -8 \end{pmatrix} - \begin{pmatrix} 5 & 15 \\ -20 & 10 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -11-5+3 & 9-15+0 \\ -12+20+0 & -8-10+3 \end{pmatrix}$$

$$= \begin{pmatrix} -13 & -6 \\ 8 & -15 \end{pmatrix}$$

17. 2071 Supp. Q.No. 2c

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, find $(AB)^T$. [2]

SOLUTION

We have,

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

Now, $AB = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 3+3 & 2+12 \\ 6+4 & 4+16 \end{pmatrix} = \begin{pmatrix} 6 & 14 \\ 10 & 20 \end{pmatrix}$$

$\therefore$ The transpose of $AB = \begin{pmatrix} 6 & 14 \\ 10 & 20 \end{pmatrix}^T = \begin{pmatrix} 6 & 10 \\ 14 & 20 \end{pmatrix}$

18. 2071 Set C Q.No. 2c

Define singular matrix. Test whether the matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & -1 \\ -1 & 3 & -2 \end{pmatrix}$ is singular or not. [2]

SOLUTION

Singular Matrix: A square matrix A is to be singular if $|A| = 0$.

Here, $|A| = \begin{vmatrix} 3 & 1 & 0 \\ -2 & 1 & -1 \\ -1 & 3 & -2 \end{vmatrix}$

$$= 3\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix}$$

[Expanding along R_1]

$$= 3(-2+3) - 1(4-1)$$

$$= 3 - 3 = 0$$

$\therefore$ A is a singular matrix.

19. 2071 Set D Q.No. 2c

If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, show that $A^{-1} = \frac{1}{19}A$ [2]

SOLUTION

Here, $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$$

So, A^{-1} exists.

The cofactors are

$A_{11} = -2$, $A_{12} = -5$, $A_{21} = -3$, $A_{22} = 2$

$$\text{Adj. } A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^T = \begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}^T$$

$$= \begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}$$

We have,

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A$$

$$= \frac{1}{-19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} = \frac{1}{19}A$$

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20. 2070 Supp Q.No. 2c

Define symmetric matrix.

If $A = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix}$ find $A + A^T$. [2]

SOLUTION

Symmetric matrix: A square matrix $A = \{a_{ij}\}$ is said to be symmetric matix if $a_{ij} = a_{ji}$ for all i and j.

For example: $A = \begin{bmatrix} 2 & 3 & -5 \\ 3 & 4 & 2 \\ -5 & 2 & 1 \end{bmatrix}$

Second part

$$A + A^T = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 2 & 5 \\ 4 & 3 & 2 \\ 3 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 6 & 8 \\ 6 & 6 & 6 \\ 8 & 6 & 12 \end{pmatrix}$$

21. 2070 Set C Q.No. 2 c

If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T. [2]

➥ Please refer to 2074 Set A Q.No. 2c

22. 2070 Set D Q.No. 2 c

If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, find $B^T A^T$. [2]

➥ Please refer to 2073 Supp Q.No. 2c

23. 2070 Old Q.No. 3 a

Given $A = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ show that $(A + B)^T = A^T + B^T$. [2]

SOLUTON

$$A + B = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 5 \end{pmatrix}$$

$$(A+B)^T = \begin{pmatrix} 3 & 4 \\ 5 & 5 \end{pmatrix}^T = \begin{pmatrix} 3 & 5 \\ 4 & 5 \end{pmatrix} \quad \ldots(i)$$

$$\text{And, } A^T + B^T = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}^T + \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 4 & 5 \end{pmatrix} \ldots(ii)$$

From (i) and (ii), $(A + B)^T = A^T + B^T$

24. 2069 Supp Q.No. 2c

Show that if $A = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$ then $(A')' = A$. [2]

SOLUTION

Here, $A = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$

$$A' = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$$

$$(A')' = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix} = A$$

25. 2069 [Set A] Q. No. 2c

If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$ find $(AB)^T$. [2]

SOLUTION

Here, $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$

$$AB = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 8+5 & 12+10 \\ 6-6 & 9-12 \end{pmatrix} = \begin{pmatrix} 13 & 22 \\ 0 & -3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 13 & 0 \\ 22 & -3 \end{pmatrix}$$

26. 2069 [Set B] Q. No. 2c

For the given matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$, show that $(A+B)^1 = A^1 + B^1$. [2]

SOLUTION

$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$

$$A + B = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$(A+B)' = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}' = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\text{Again, } A'+B' = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}' + \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}'$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$\therefore (A+B)' = A' + B'$

27. 2069 [Set A] Old Q. No. 3a

If $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ show that: $A^2 - 2A + I = 0$ where I is the unit matrix and 0 is the null matrix. [2]

SOLUTION

$$\text{L.H.S.} = A^2 - 2A + I$$

$$= A \cdot A - 2A + I$$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2+1 & 2-2+0 \\ 0-0+0 & 1-2+1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$= 0 = \text{R.H.S.}$

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28. 2069 [Set B] Old Q. No. 4b

If $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$, show that: $AB \neq BA$. [2]

SOLUTION

$$AB = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 1+1 \\ 2-4 & 2+1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -2 & 3 \end{pmatrix}$$

$$\text{and } BA = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+2 & -1-1 \\ 4-2 & -4+1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -3 \end{pmatrix}$$

$\therefore AB \neq BA$

29. 2068 Q.No. 2c

Find the inverse of the matrix $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$ [2]

SOLUTION

Here, $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 7 & -3 \\ 6 & 2 \end{vmatrix} = 14 + 18 = 32 \neq 0$$

So, A^{-1} exists.

The cofactors are

$A_{11} = 2, A_{12} = -6, A_{21} = -(-3) = 3, A_{22} = 7$

$$\text{Adj. } A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & -6 \\ 3 & 7 \end{pmatrix}^T = \begin{pmatrix} 2 & 3 \\ -6 & 7 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj.} A = \frac{1}{32} \begin{pmatrix} 2 & 3 \\ -6 & 7 \end{pmatrix}$$

30. 2068 Old Q.No. 3a

If $A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$, find the matrix $AB - 3I$ where I is the unit matrix of order 2. [2]

SOLUTION

$AB - 3I$

$$= \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix} - 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-6+3 & 3-2+6 \\ -2+6+1 & -3+2+2 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 7 \\ 5 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1-3 & 7-0 \\ 5-0 & 1-3 \end{pmatrix} = \begin{pmatrix} -4 & 7 \\ 5 & -2 \end{pmatrix}$$

31. 2067 Q.No. 3a

Construct (3 × 3) matrix with the elements given by $a_{ij} = 2i + j$ [2]

SOLUTION

Let the 3×3 matrix is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given $a_{ij} = 2i + j$

$\therefore a_{11} = 2 \times 1 + 1 = 3$ $\quad a_{12} = 2 \times 1 + 2 = 4$

$a_{13} = 2 \times 1 + 3 = 5$ $\quad a_{21} = 2 \times 2 + 1 = 5$

$a_{22} = 2 \times 2 + 2 = 6$ $\quad a_{23} = 2 \times 2 + 3 = 7$

$a_{31} = 2 \times 3 + 1 = 7$ $\quad a_{32} = 2 \times 3 + 2 = 8$

$a_{33} = 2 \times 3 + 3 = 9$

$\therefore$ The required 3×3 matrix with the elements given by $a_{ij} = 2i + j$ is $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix}_{3 \times 3}$.

32. 2066 Q.No. 3a

If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then determine the matrix A. [2]

SOLUTION

$$A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad \ldots \text{(i)}$$

$$A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \quad \ldots \text{(ii)}$$

Multiplying (i) by 2 and adding it with (ii)

$$2A + 2B + A - 2B = 2 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\text{or, } 3A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\text{or, } 3A = \begin{bmatrix} 2-1 & 0+1 \\ 2+0 & 2-1 \end{bmatrix}$$

$$\text{or, } 3A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\therefore A = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

33. 2065 Q.No. 3a

Define symmetric and skew symmetric matrix with examples. [2]

SOLUTION

Symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be symmetric matix if $a_{ij} = a_{ji}$ for all i and j.

For example: $A = \begin{bmatrix} 2 & 3 & -5 \\ 3 & 4 & 2 \\ -5 & 2 & 1 \end{bmatrix}$

Skew-symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be skew-symmetric if $a_{ij} = -a_{ji}$ for all non-diagonal elements and the diagonal elements are zero.

For example: $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

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34. 2064 Q.No. 4b

If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$; find A^{-1}. [2]

SOLUTION

We have,

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

To find: A^{-1}

Now, $|A| = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 3 - 4 = -1 \neq 0$. So A^{-1} exists.

Let A_{ij} denote the cofactor of the element a_{ij}. So,

$A_{11} = (-1)^2 |1| = 1$

$A_{12} = (-1)^3 |2| = -2$

$A_{21} = (-1)^3 |2| = -2$

$A_{22} = (-1)^4 |3| = 3$

$\therefore$ Matrix of cofactors of

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

$$\text{And, Adj. } A = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}$ (Adj. A)

$$= \frac{1}{-1} \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

35. 2063 Q.No. 3a

Define a triangular matrix. How do you distinguish between upper and lower triangular matrices? [2]

SOLUTION

A square matrix whose all the elements below (or above) the leading diagonal are all zero is called a triangular matrix.

A square matrix whose all the elements below the leading diagonal are zero is called an upper triangular matrix and if all the elements above the leading diagonal are zero is called a lower triangular matrix.

For example,

$A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & 4 & 1 \\ 0 & 0 & 8 \end{bmatrix}$ which is an upper triangular matrix.

$B = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ -1 & 2 & 7 \end{bmatrix}$ which is a lower triangular matrix.

36. 2062 Q.No. 3a

If $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, show that $(AB)^T = B^T A^T$ [2]

SOLUTION

Here,

$$AB = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -4+0 & 2+0 \\ -2+9 & 1+6 \end{pmatrix}$$

or, $AB = \begin{pmatrix} -4 & 2 \\ 7 & 7 \end{pmatrix}$

$\therefore (AB)^T = \begin{pmatrix} -4 & 7 \\ 2 & 7 \end{pmatrix}$

Again, we have

$B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$

$\therefore B^T = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$

and $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

$\therefore A^T = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$

$\therefore B^T A^T = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} -4+0 & -2+9 \\ 2+0 & 1+6 \end{pmatrix} = \begin{pmatrix} -4 & 7 \\ 2 & 7 \end{pmatrix}$

$\therefore (AB)^T = B^T A^T$ Proved.

37. 2061 Q.No. 3a

Construct a 3 × 3 matrix whose elements are $a_{ij} = 2i + j$ [2]

SOLUTION

Let the required 3×3 matrix $= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

We have, $a_{ij} = 2i + j$

Now,

$a_{11} = 2 \times 1 + 1 = 3$ $\quad a_{21} = 2 \times 2 + 1 = 5$

$a_{31} = 2 \times 3 + 1 = 7$ $\quad a_{12} = 2 \times 1 + 2 = 4$

$a_{22} = 2 \times 2 + 2 = 6$ $\quad a_{32} = 2 \times 3 + 2 = 8$

$a_{13} = 2 \times 1 + 3 = 5$ $\quad a_{23} = 2 \times 2 + 3 = 7$

$a_{33} = 2 \times 3 + 3 = 9$

$\therefore$ The required 3×3 matrix $= \begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}$

38. 2060 Q.No. 3a

$A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, Show that $(AB)^T = B^T A^T$. [2]

➥ Please refer to **2062 Q.No. 3a**

39. 2059 Q.No. 3a

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$, find $(AB)^T$. [2]

Here, $AB = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$

$= \begin{pmatrix} 5+18 & 7+24 \\ 10+24 & 14+32 \end{pmatrix} = \begin{pmatrix} 23 & 31 \\ 34 & 46 \end{pmatrix}$

$\therefore (AB)^T = \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$

40. 2058 Q.No. 3a

Let $A = \begin{pmatrix} 2 & 1 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix}$, find $(AB)^T$ [2]

SOLUTION

We have $A = \begin{pmatrix} 2 & 1 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix}$

Now $(AB) = \begin{pmatrix} 2 & 1 \\ 0 & 7 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2+3 & 10+2 \\ 0+21 & 0+14 \end{pmatrix}$

$= \begin{pmatrix} 5 & 12 \\ 21 & 14 \end{pmatrix}$

$\therefore (AB)^T = \begin{pmatrix} 5 & 21 \\ 12 & 14 \end{pmatrix}$

41. 2057 Q.No. 3a

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ find the transpose of AB. [2]

➥ Please refer to 2071 Supp. Q.No. 2c

42. 2056 Q.No. 3a

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$, then prove that AB≠BA. [2]

SOLUTION

Here,

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5+18 & 7+24 \\ 10+24 & 14+32 \end{pmatrix} = \begin{pmatrix} 23 & 31 \\ 34 & 46 \end{pmatrix}$$

Again,

$$BA = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} \times \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5+14 & 15+28 \\ 6+16 & 18+32 \end{pmatrix} = \begin{pmatrix} 19 & 43 \\ 22 & 50 \end{pmatrix}$$

Hence, AB ≠ BA, Proved.

43. 2055 Old Model Q.No. 3a

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$, find $(AB)^T$. [2]

➥ Please refer to 2059 Q.No. 3a

4 MARKS QUESTIONS

44. 2058 Q.No. 10a

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, show that $A^2 - 2A - 5I = O$ [4]

SOLUTION

Here, $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$

Now, $A^2 = A \cdot A$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6 & 2+2 \\ 3+3 & 6+1 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix}$$

We know that

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

L.H.S. $= A^2 - 2A - 5I$

$$= \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 6 & 2 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 7-2-5 & 4-4-0 \\ 6-6-0 & 7-2-5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$= O$

L.H.S. = R.H.S. proved.

B. DETERMINANTS

2 MARKS QUESTIONS

1. 2071 Old Q.No. 3a

Without expanding the determinant prove that

$$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ [2]

SOLUTION

L.H.S. $= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \frac{1}{xyz} \begin{vmatrix} xyz & x & x^2 \\ xyz & y & y^2 \\ xyz & z & z^2 \end{vmatrix}$

Taking commons x, y, z common from R_1 R_2, R_3 respectively

$$= \frac{xyz}{xyz} \begin{vmatrix} yz & 1 & x \\ zx & 1 & y \\ xy & 1 & z \end{vmatrix}$$

$$= - \begin{vmatrix} x & 1 & yz \\ y & 1 & zx \\ z & 1 & xy \end{vmatrix} \quad [\because C_1 \leftrightarrow C_3]$$

$$= (-)(-) \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} \quad [\because C_1 \leftrightarrow C_2]$$

$$= \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

= R.H.S. proved.

4 MARKS QUESTIONS

2. 2076 Set B Q.No. 7b

Prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$$ [4]

SOLUTION

L. H.S. $= \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

$$= \frac{1}{abc} \begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix}$$

Taking common a from R_1, b from R_2 and c from R_3

$$= \frac{1}{abc} \cdot abc \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

Taking common $1 + a^2 + b^2 + c^2$ from C_1.

$$(1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1-1 & b^2-(b^2+1) & c^2-c^2 \\ 1 & b^2+1 & c^2 \end{vmatrix}$$

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$= (1+a^2+b^2+c^2)\begin{vmatrix} 0 & -1 & 0 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$

Expanding along R_1

$(1+a^2+b^2+c^2) \cdot 1 \begin{vmatrix} 1 & c^2 \\ 1 & c^2+1 \end{vmatrix}$

$= (1+a^2+b^2+c^2)(c^2+1-c^2)$

$= 1+a^2+b^2+c^2$

$=$ R.H.S

3. 2076 Set C Q.No. 7b

Show that: [4]

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

SOLUTION

Here,

$$\text{L.H.S.} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

Taking (a+b+c) common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= (a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ 2b-b+c+a & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ a+b+c & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from C_1

$$= (a+b+c)(a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)^2\begin{vmatrix} 0 & 0 & 1 \\ 1 & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix} \quad C_2 \to C_2 - C_3$$

Taking $(a+b+c)$ common from C_2

$$= (a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

Expanding along R_1

$$= (a+b+c)^3 \cdot 1 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}$$

$= (a+b+c)^3 (1-0)$

$= (a+b+c)^3$

$=$ R.H.S. proved.

4. 2075 GIE Q.No. 7b

Evaluate: $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$. [4]

SOLUTION

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 - R_3$

$$= \begin{vmatrix} b^2+c^2+b^2-c^2 & a^2+c^2+a^2-c^2 & a^2+b^2-a^2-b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 2b^2 & 2a^2 & 0 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Expanding along R_1

$$2b^2\begin{vmatrix} c^2+a^2 & b^2 \\ c^2 & a^2+b^2 \end{vmatrix} - 2a^2\begin{vmatrix} b^2 & b^2 \\ c^2 & a^2+b^2 \end{vmatrix}$$

$= 2b^2\{(c^2+a^2)(a^2+b^2) - b^2c^2\} - 2a^2\{b^2(a^2+b^2) - b^2c^2\}$

$= 2b^2\{a^2c^2 + b^2c^2 + a^4 + a^2b^2 - b^2c^2\} - 2a^2\{a^2b^2 + b^4 - b^2c^2\}$

$= 2a^2b^2c^2 + 2a^4b^2 + 2a^2b^4 - 2a^4b^2 - 2a^2b^4 + 2a^2b^2c^2$

$= 4a^2b^2c^2$

5. 2075 Set A Q.No. 7b

What is a determinant? Use properties of determinant prove that: [4]

$$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x-a)(x-b)(x+a+b).$$

SOLUTION

The arrangement of numbers in square form and enclosed by | | bracket is called a determinant. It has a unique value.

$$\text{L.H.S} = \begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} x-a & a-x & b-b \\ a-a & x-b & b-x \\ a & b & x \end{vmatrix}$$

$$= \begin{vmatrix} x-a & -(x-a) & 0 \\ 0 & x-b & -(x-b) \\ a & b & x \end{vmatrix}$$

Taking $(x-a)$ common from R_1 and $(x-b)$ from R_2.

$$= (x-a)(x-b)\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a & b & x \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$

$$= (x-a)(x-b)\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ a+b & b & x \end{vmatrix}$$

Expanding along R_1

$$(x-a)(x-b) \cdot 1 \begin{vmatrix} 1 & -1 \\ a+b & x \end{vmatrix}$$

$= (x-a)(x-b)(x+a+b)$

$=$ R.H.S.

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6. 2075 Set B Q.No. 7b

Prove that:

$$\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix} = 1 + x + y + z. \quad [4]$$

SOLUTION

$$\text{L.H.S} = \begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+x+y+z & y & z \\ 1+x+y+z & 1+y & z \\ 1+x+y+z & y & 1+z \end{vmatrix}$$

Taking common $(1 + x + y + z)$ from C_1

$$= (1+x+y+z) \begin{vmatrix} 1 & y & z \\ 1 & 1+y & z \\ 1 & y & 1+z \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$= (1+x+y+z) \begin{vmatrix} 1-1 & y-(1+y) & z-z \\ 1-1 & 1+y-y & z-(1+z) \\ 1 & y & 1+z \end{vmatrix}$$

$$= (1+x+y+z) \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & y & 1+z \end{vmatrix}$$

Expanding along C_1

$$(1+x+y+z) \cdot 1 \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix}$$

$$= (1+x+y+z)(1-0)$$

$$= 1 + x + y + z = \text{R.H.S}$$

7. 2075 Set C Q.No. 7b

Prove that: [4]

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2.$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix}$$

Applying $C_1 \to xC_1$, $C_2 \to yC_2$, $C_3 \to zC_3$

$$= \frac{1}{xyz} \begin{vmatrix} x(x^2+1) & xy^2 & xz^2 \\ x^2y & y(y^2+1) & yz^2 \\ x^2z & y^2z & z(z^2+1) \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \frac{1}{xyz} \begin{vmatrix} x(x^2+1+y^2+z^2) & xy^2 & xz^2 \\ y(x^2+y^2+1+z^2) & y(y^2+1) & yz^2 \\ z(x^2+y^2+z^2+1) & y^2z & z(z^2+1) \end{vmatrix}$$

Taking $1 + x^2 + y^2 + z^2$ common from C_1

$$= \frac{1}{xyz}(1+x^2+y^2+z^2) \begin{vmatrix} x & xy^2 & xz^2 \\ y & y(y^2+1) & yz^2 \\ z & y^2z & z(z^2+1) \end{vmatrix}$$

Taking x, y, z common from, R_1, R_2 and R_3 respectively

$$= \frac{1}{xyz}(1+x^2+y^2+z^2)(x.y.z) \begin{vmatrix} 1 & y^2 & z^2 \\ 1 & y^2+1 & z^2 \\ 1 & y^2 & z^2+1 \end{vmatrix}$$

$$= (1+x^2+y^2+z^2) \begin{vmatrix} 0 & -1 & 0 \\ 1 & y^2+1 & z^2 \\ -1 & y^2 & z^2+1 \end{vmatrix} \quad R_1 \to R_1 - R_2$$

Expanding along R_1

$$= (1+x^2+y^2+z^2) \begin{vmatrix} -1 & z^2 \\ 1 & z^2+1 \end{vmatrix}$$

$$= (1+x^2+y^2+z^2)(z^2+1-z^2)$$

$$= 1 + x^2 + y^2 + z^2$$

$$= \text{R.H.S. Proved.}$$

8. 2074 Supp Q.No. 7b

Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & xz & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy) \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$$

Applying $C_1 \to xC_1$, $C_2 \to yC_2$, $C_3 \to zC_3$

$$= \frac{1}{xyz} \begin{vmatrix} x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \\ xyz & xyz & xyz \end{vmatrix}$$

Taking common xyz from R_3

$$= \frac{1}{xyz} \cdot xyz \begin{vmatrix} x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$\begin{vmatrix} x^2-y^2 & y^2-z^2 & z^2 \\ x^3-y^3 & y^3-z^3 & z^3 \\ 1-1 & 1-1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (x-y)(x+y) & (y-z)(y+z) & z^2 \\ (x-y)(x^2+xy+y^2) & (y-z)(y^2+yz+z^2) & z^3 \\ 0 & 0 & 1 \end{vmatrix}$$

Taking $(x-y)$ common from C_1 and $(y-z)$ common from C_2

$$(x-y)(y-z) \begin{vmatrix} x+y & y+z & z^2 \\ x^2+xy+y^2 & y^2+yz+z^2 & z^3 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3

$$(x-y)(y-z) \cdot 1 \cdot \begin{vmatrix} x+y & y+z \\ x^2+xy+y^2 & y^2+yz+z^2 \end{vmatrix}$$

$$= (x-y)(y-z)\{(x+y)(y^2+yz+z^2) - (y+z)(x^2+xy+y^2)\}$$

$$= (x-y)(y-z)(xy^2+xyz+xz^2+y^3+y^2z+yz^2-x^2y-xyz-y^2z-x^2z-xyz-y^2z)$$

$$= (x-y)(y-z)(yz^2-x^2y+xz^2-x^2z)$$

$$= (x-y)(y-z)\{y(z^2-x^2)+xz(z-x)\}$$

$$= (x-y)(y-z)\{y(z-x)(z+x)+xz(z-x)\}$$

$$= (x-y)(y-z)(z-x)\{y(z+x)+zx\}$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$= \text{R.H.S.}$$

9. 2074 Set A Q.No. 7b

Show that:

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-z)(z-x)(x-y).$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

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$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} \quad [R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1]$$

Expanding along C_1

$$= 1 \begin{vmatrix} y-x & y^2-x^2 \\ z-x & z^2-x^2 \end{vmatrix}$$

$$= \begin{vmatrix} -(x-y) & -(x^2-y^2) \\ (z-x) & (z^2-x^2) \end{vmatrix}$$

$$= -(x-y)(z^2-x^2) + (x^2-y^2)(z-x)$$

$$= (x+y)(x-y)(z-x) - (x-y)(z-x)(z+x)$$

$$= (x-y)(z-x)(x+y-z-x)$$

$$= (x-y)(y-z)(z-x) = \text{R.H.S.}$$

$\therefore$ L.H.S. = R.H.S. Proved.

10. 2074 Set B Q.No. 7b

Prove that: $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$. [4]

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$$

Applying $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$

$$= \frac{1}{abc} \begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix}$$

Taking abc common from C_1

$$= \frac{1}{abc} \cdot abc \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \text{R.H.S. proved.}$$

11. 2073 Supp Q.No. 7b

Prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$. [4]

SOLUTION

$$\text{L.H.S} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 1-1 & 1-1 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

Taking common $(a-b)$ form C_1 and $(b-c)$ form C_2

$$(a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$$

Expanding along R_1

$$(a-b)(b-c) \cdot 1 \begin{vmatrix} 1 & 1 \\ -c & -a \end{vmatrix}$$

$$= (a-b)(b-c)(-a+c)$$

$$= (a-b)(b-c)(c-a)$$

$$= \text{R.H.S.}$$

12. 2073 Set C Q.No. 7b

Without expanding show that

$$\begin{vmatrix} b & c & b+c \\ c & a & c+a \\ a & b & a+b \end{vmatrix} = 0.$$ [4]

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} b & c & b+c \\ c & a & c+a \\ a & b & a+b \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$

$$= \begin{vmatrix} b+c & c & b+c \\ c+a & a & c+a \\ a+b & b & a+b \end{vmatrix}$$

$$= 0 \quad (\because C_1 = C_3)$$

$$= \text{R.H.S.}$$

13. 2073 Set D Q.No. 7b

Prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$ [4]

SOLUTION

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

Taking common $(a-b)$ and $(b-c)$ from C_1 and C_2

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

Expanding along R_1

$$= (a-b)(b-c) \times 1 \begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(b^2+bc+c^2-a^2-ab-b^2)$$

$$= (a-b)(b-c)(bc-ab+c^2-a^2)$$

$$= (a-b)(b-c)[b(c-a)+(c^2-a^2)]$$

$$= (a-b)(b-c)(c-a)(b+c+a)$$

$$= (a-b)(b-c)(c-a)(a+b+c) = \text{Proved}$$

14. 2072 Supp. Q.No. 7b

Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$$. [4]

➥ Please refer to 2074 Supp Q.No. 7b

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15. 2072 Set C Q.No. 7b

Show that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy) \quad [4]$$

➥ Please refer to 2074 Supp Q.No. 7b

16. 2072 Set D Q.No. 7b

Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c). \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$$

Applying $R_3 \to R_3 + R_1$

$$= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a+b+c & a+b+c & a+b+c \end{vmatrix}$$

Taking common $(a+b+c)$ from R_3

$$(a+b+c)\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ 1-1 & 1-1 & 1 \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} a-b & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \\ 0 & 0 & 1 \end{vmatrix}$$

Taking $(a-b)$ common from C_1 and $(b-c)$ from C_2,

$$(a-b)(b-c)(a+b+c)\begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3

$$(a-b)(b-c)(a+b+c)\cdot 1\begin{vmatrix} 1 & 1 \\ a+b & b+c \end{vmatrix}$$

$= (a-b)(b-c)(a+b+c)(b+c-a-b)$

$= (b-c)(c-a)(a-b)(a+b+c) = \text{R.H.S.}$

17. 2072 Set E Q.No. 7b

Prove that:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca) \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$$

Taking abc common from C_3

$$= \frac{1}{abc}\cdot abc\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2-b^2 & a^3-b^3 & 1-1 \\ b^2-c^2 & b^3-c^3 & 1-1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a-b)(a+b) & (a-b)(a^2+ab+b^2) & 0 \\ (b-c)(b+c) & (b-c)(b^2+bc+c^2) & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Taking common $(a-b)$ from R_1 and $(b-c)$ from R_2

$$(a-b)(b-c)\begin{vmatrix} a+b & a^2+ab+b^2 & 0 \\ b+c & b^2+bc+c^2 & 0 \\ c^2 & c^3 & 1 \end{vmatrix}$$

Expanding along C_3

$$(a-b)(b-c)\cdot 1\begin{vmatrix} a+b & a^2+ab+b^2 \\ b+c & b^2+bc+c^2 \end{vmatrix}$$

$= (a-b)(b-c)\{(a+b)(b^2+bc+c^2) - (b+c)(a^2+ab+b^2)\}$

$= (a-b)(b-c)(ab^2+abc+ac^2+b^3+b^2c+bc^2-a^2b-ab^2-b^3-a^2c-abc-b^2c)$

$= (a-b)(b-c)(bc^2 - a^2b + ac^2 - a^2c)$

$= (a-b)(b-c)\{b(c^2-a^2) + ac(c-a)\}$

$= (a-b)(b-c)\{b(c-a)(c+a) + ac(c-a)\}$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$

$= \text{R.H.S.}$

18. 2071 Supp. Q.No. 7b

Show that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right) \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix}$$

$$= xyz\begin{vmatrix} \frac{a+x}{x} & \frac{b}{y} & \frac{c}{z} \\ \frac{a}{x} & \frac{b+y}{y} & \frac{c}{z} \\ \frac{a}{x} & \frac{b}{y} & \frac{c+z}{z} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= xyz\begin{vmatrix} 1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z} & \frac{b}{y} & \frac{c}{z} \\ 1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z} & 1+\frac{b}{y} & \frac{c}{z} \\ 1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z} & \frac{b}{y} & 1+\frac{c}{z} \end{vmatrix}$$

Taking common $\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)$ from C_1

$$= xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\begin{vmatrix} 1 & \frac{b}{y} & \frac{c}{z} \\ 1 & 1+\frac{b}{y} & \frac{c}{z} \\ 1 & \frac{b}{y} & 1+\frac{c}{z} \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$= xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1+\frac{b}{y} & \frac{c}{z} \\ 1 & \frac{b}{y} & 1+\frac{c}{z} \end{vmatrix}$$

Expanding along R_1

$$= xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\cdot 1\begin{vmatrix} 1 & \frac{c}{z} \\ 1 & 1+\frac{c}{z} \end{vmatrix}$$

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$$= xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(1+\frac{c}{z}-\frac{c}{z}\right)$$

$$= xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right) = \text{R.H.S.}$$

19. 2071 Set C Q.No. 7b

Without expanding the determinant show that:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} a^2 & bc & a \\ b^2 & ca & b \\ c^2 & ab & c \end{vmatrix} \quad [4]$$

SOLUTION

$$\text{R.H.S.} = \begin{vmatrix} a^2 & bc & a \\ b^2 & ca & b \\ c^2 & ab & c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^3 & abc & a^2 \\ b^3 & abc & b^2 \\ c^3 & abc & c^2 \end{vmatrix}$$

Taking abc common from C_2

$$\frac{1}{abc}\cdot abc\begin{vmatrix} a^3 & 1 & a^2 \\ b^3 & 1 & b^2 \\ c^3 & 1 & c^2 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & a^3 & a^2 \\ 1 & b^3 & b^2 \\ 1 & c^3 & c^2 \end{vmatrix} \quad (C_1 \leftrightarrow C_2)$$

$$= (-)(-)\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad (C_2 \leftrightarrow C_3)$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \text{L.H.S.}$$

20. 2071 Set D Q.No. 7b

Without expanding the determinant show that:

$$\begin{vmatrix} y+z & y & z \\ z+x & z & x \\ x+y & x & y \end{vmatrix} = 0 \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} y+z & y & z \\ z+x & z & x \\ x+y & x & y \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= \begin{vmatrix} y+z-y & y & z \\ z+x-z & z & x \\ x+y-x & x & y \end{vmatrix}$$

$$= \begin{vmatrix} z & y & z \\ x & z & x \\ y & x & y \end{vmatrix}$$

$= 0 \quad (\because C_1 = C_3)$

$= \text{R.H.S.}$

21. 2071 Old Q.No. 10a

Show that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ [4]

➥ Please refer to **2076 Set C Q.No. 7b**

22. 2070 Supp Q.No. 7b

Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 \quad [4]$$

➥ Please refer to **2076 Set C Q.No. 7b**

23. 2070 Set C Q.No. 7 b

Prove that:

$$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2. \quad [4]$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking common a from C_1, b from C_2 and c from C_3

$$= abc\begin{vmatrix} a & c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2 - R_3$

$$= abc\begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Taking $-2b$ common from R_1

$$= abc(-2b)\begin{vmatrix} 1 & 1 & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$

$$= -2ab^2c\begin{vmatrix} 1 & 0 & 0 \\ a+b & -a & a \\ b & c & c \end{vmatrix}$$

Expanding along R_1

$$= -2ab^2c \cdot 1\begin{vmatrix} -a & a \\ c & c \end{vmatrix} = -2ab^2c(-ac-ac)$$

$$= -2ab^2c(-2ac) = 4a^2b^2c^2$$

$$= \text{R.H.S.}$$

24. 2070 Set D Q.No. 7 b

Prove that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right) \quad [4]$$

➥ Please refer to **2071 Supp. Q.No. 7b**

25. 2070 Old Q.No. 10 a

Using properties of determinant, show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 \quad [4]$$

➥ Please refer to **2076 Set C Q.No. 7b**

26. 2069 Supp Q.No. 7 b

Without expanding, show that:

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad [4]$$

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SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & a+b+c-a \\ 1 & ca & a+b+c-b \\ 1 & ab & a+b+c-c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & a+b+c \\ 1 & ca & c+a+b \\ 1 & ab & a+b+c \end{vmatrix} - \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} - \frac{1}{abc} \begin{vmatrix} a & abc & a^2 \\ b & abc & b^2 \\ c & abc & c^2 \end{vmatrix}$$

[$\because$ Taking $(a + b + c)$ common from C_3 of 1st determinant and multiplying R_1 by a, R_2 by b and R_3 by c in 2nd determinant]

$$= (a+b+c) \times 0 - \frac{1}{abc} \times abc \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

[$\because C_1 = C_3$ of 1st determinant and taking abc common from C_2 of 2nd determinant]

$$= - \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad [\because C_1 \leftrightarrow C_2]$$

Hence, proved.

27. 2069 [Set A] Q. No. 7b

Show that: $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$

$= (b-c)(c-a)(a-b)(a+b+c)$. [4]

➥ Please refer to 2072 Set D Q.No. 7b

28. 2069 [Set B] Q. No. 7b

Without expanding show that:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0.$$ [4]

SOLUTION

We have, $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

$$= \begin{vmatrix} 1 & bc & ab+ca \\ 1 & ca & bc+ab \\ 1 & ab & ca+bc \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2$

$$= \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & bc+ab+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$

Taking $(ab + bc + ca)$ common from C_3

$$= (ab+bc+ca) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$$= (ab+bc+ca) \times 0 \quad [\because C_1 = C_3]$$

$$= 0$$

29. 2069 [Set A] Old Q. No. 10a

Show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$ [4]

➥ Please refer to 2076 Set C Q.No. 7b

30. 2069 [Set B] Old Q. No. 10a

Prove that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$$ [4]

➥ Please refer to 2071 Supp. Q.No. 7b

31. 2068 Q.No. 7b

Show that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$ [4]

➥ Please refer to 2074 Supp Q.No. 7b

32. 2068 Old Q.No. 10a

Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(xy+yz+zx)$$ [4]

➥ Please refer to 2074 Supp Q.No. 7b

33. 2067 Q.No. 10a

Prove that:

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$$ [4]

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

Applying $C_3 \to C_3 - xC_1 - yC_2$

$$= \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ ax+by & bx+cy & -(ax^2+2bxy+cy^2) \end{vmatrix}$$

Expanding along C_3

$$= -(ax^2+2bxy+cy^2) \begin{vmatrix} a & b \\ b & c \end{vmatrix}$$

$$= -(ax^2+2bxy+cy^2)(ac-b^2)$$

$$= (b^2-ac)(ax^2+2bxy+cy^2)$$

$$= \text{R.H.S. Proved}$$

34. 2066 Q.No. 10a

If a, b, c are non zero and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0,$

then show that abc = 1. [4]

SOLUTION

$$\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$$

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$$\text{or, } \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\text{or, } abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = 0$$

$$\text{or, } abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\text{or, } (abc-1) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\text{or, } (abc-1) \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = 0 \quad \begin{bmatrix} R_1 \to R_1-R_2 \\ R_2 \to R_2-R_3 \end{bmatrix}$$

$$\text{or, } (abc-1) \begin{vmatrix} 0 & a-b & (a+b)(a-b) \\ 0 & b-c & (b+c)(b-c) \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\text{or, } (abc-1)(a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\text{or, } (abc-1)(a-b)(b-c) \cdot 1 \begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix} = 0$$ (Expanding along C_1)

or, $(abc-1)(a-b)(b-c)(b+c-a-b) = 0$

or, $(abc-1)(a-b)(b-c)(c-a) = 0$

Since a, b, c are non-zero and not equal,
so $(a-b) \neq 0, (b-c) \neq 0, (c-a) \neq 0$.

Then, we must have

$abc - 1 = 0$

$\Rightarrow abc = 1$. Hence proved.

35. 2065 Q.No. 10a

Show that:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2 \quad [4]$$

Please refer to 2075 Set C Q.No. 7b

36. 2064 Q.No. 10a

Prove (without expanding): [4]

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Please refer to 2069 Supp Q.No. 7 b

37. 2063 Q.No. 10a

Prove (without expanding) [4]

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Please refer to 2074 Set B Q.No. 7b

38. 2062 Q.No. 10a

Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ [4]

Please refer to 2076 Set C Q.No. 7b

39. 2061 Q.No. 10a

Without expanding show that

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ [4]

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & a+b+c-a \\ 1 & ca & a+b+c-b \\ 1 & ab & a+b+c-c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & bc & a+b+c \\ 1 & ca & a+b+c \\ 1 & ab & a+b+c \end{vmatrix} - \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

Taking $(a + b + c)$ common from C_3 of first determinant, we get

$$= (a+b+c) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} - \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

$$= (a+b+c) \times 0 - \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

[$\because C_1 = C_3$ in the first determinant]

$$= - \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

Multiplying C_1 by a, C_2 by b and C_3 by c, we get

$$= -\frac{1}{abc} \begin{vmatrix} a & abc & a^2 \\ b & abc & b^2 \\ c & abc & c^2 \end{vmatrix}$$

$$= -\frac{1}{abc}(abc) \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

[Taking abc common from C_2]

$$= -(-) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad [\because C_1 \leftrightarrow C_2]$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

= R.H.S. proved.

40. 2060 Q.No. 10a

Use properties of determinant to show that [4]

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \frac{1}{xyz} \begin{vmatrix} xyz & x & x^2 \\ xyz & y & y^2 \\ xyz & z & z^2 \end{vmatrix}$$

Taking commons x, y, z common from R_1 R_2, R_3 respectively

$$= \frac{xyz}{xyz}\begin{vmatrix} yz & 1 & x \\ zx & 1 & y \\ xy & 1 & z \end{vmatrix}$$

$$= -\begin{vmatrix} x & 1 & yz \\ y & 1 & zx \\ z & 1 & xy \end{vmatrix} \qquad [\because C_1 \leftrightarrow C_3]$$

$$= (-)(-)\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} \qquad [\because C_1 \leftrightarrow C_2]$$

$$= \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

= R.H.S. proved.

41. 2059 Q.No. 10 a

Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ **[4]**

SOLUTION

We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

Taking common (a - b) and (b - c) from C_1 and C_2

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

Expanding along R_1

$$= (a-b)(b-c) \times 1 \begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(b^2+bc+c^2-a^2-ab-b^2)$$

$$= (a-b)(b-c)(bc-ab+c^2-a^2)$$

$$= (a-b)(b-c)[b(c-a)+(c^2-a^2)]$$

$$= (a-b)(b-c)(c-a)(b+c+a)$$

$$= (a-b)(b-c)(c-a)(a+b+c)$$

42. 2057 Q.No. 10a

Find the value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ **[4]**

➥ Please refer to 2069 [Set B] Q. No. 7b

43. 2056 Q.No. 10a

Evaluate: $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ **[4]**

SOLUTION

We have, $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 1+x-1 & 1-1 & 1 \\ 1-1-y & 1+y-1 & 1 \\ 1-1 & 1-1-z & 1+z \end{vmatrix}$$

$$= \begin{vmatrix} x & 0 & 1 \\ -y & y & 1 \\ 0 & -z & 1+z \end{vmatrix}$$

$$= x\begin{vmatrix} y & 1 \\ -z & 1+z \end{vmatrix} - 0\begin{vmatrix} -y & 1 \\ 0 & 1+z \end{vmatrix} + 1\begin{vmatrix} -y & y \\ 0 & -z \end{vmatrix}$$

[Expanding along R_1]

$$= x(y+yz+z) - 0 + 1(yz-0)$$

$$= xy + xyz + zx + yz$$

$$= xyz + xy + yz + zx$$

$$= xyz\left(1+\frac{1}{z}+\frac{1}{x}+\frac{1}{y}\right) = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

44. 2055 Old Model Q.No. 10a

Prove that $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = 0$ **[4]**

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$

$$= \begin{vmatrix} a+b+c & 1 & b+c \\ a+b+c & 1 & c+a \\ a+b+c & 1 & a+b \end{vmatrix}$$

Taking (a + b + c) common from C_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$$= (a+b+c) \times 0 \qquad [\because C_1 = C_2]$$

$$= 0$$

= R.H.S., Proved.

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7

UNIT

SYSTEM OF LINEAR EQUATIONS

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A. ROW EQUIVALENT MATRIX METHOD

2 MARKS QUESTIONS

1. 2069 Set B Old Q.No. 3a

Solve by row equivalent method:

$4x + 3y = 7$, $5x - 3y = 8$ [2]

SOLUTION

The augmented matrix is

$$\begin{bmatrix} 4 & 3 & : & 7 \\ 5 & -3 & : & 8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3/4 & : & 7/4 \\ 5 & -3 & : & 8 \end{bmatrix} R_1 \to \frac{1}{4}R_1$$

$$\sim \begin{bmatrix} 1 & 3/4 & : & 7/4 \\ 0 & -27/4 & : & -3/4 \end{bmatrix} R_2 \to R_2 - 5R_1$$

$$\sim \begin{bmatrix} 1 & 3/4 & : & 7/4 \\ 0 & 1 & : & 1/9 \end{bmatrix} R_2 \to \frac{-4}{27} R_2$$

$$\sim \begin{bmatrix} 1 & 0 & : & 5/3 \\ 0 & 1 & : & 1/9 \end{bmatrix} R_1 \to R_1 - \frac{3}{4}R_2$$

2. 2064 Q.No. 3a

Solve by row–equivalent method:

$3x - 2y = 8$; $5x + 3y = 7$ [2]

SOLUTION

The given equation of straight lines are

$3x - 2y = 8$

$5x + 3y = 7$

The augmented matrix form is

$$\begin{bmatrix} 3 & -2 & :8 \\ 5 & 3 & :7 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\sim \begin{bmatrix} 1 & -\frac{2}{3} & \frac{8}{3} \\ 5 & 3 & :7 \end{bmatrix}$$

Applying $R_2 \to R_2 - 5R_1$

$$\sim \begin{bmatrix} 1 & -\frac{2}{3} & \frac{8}{3} \\ 0 & \frac{19}{3} & \frac{-19}{3} \end{bmatrix}$$

Applying $R_2 \to \left(\frac{3}{19}\right) R_2$

$$\sim \begin{bmatrix} 1 & -\frac{2}{3} & \frac{8}{3} \\ 0 & 1 & :-1 \end{bmatrix}$$

Applying $R_1 \to R_1 + \frac{2}{3}R_2$

$$\sim \begin{bmatrix} 1 & 0 & :2 \\ 0 & 1 & :-1 \end{bmatrix}$$

$\therefore \quad x = 2; y = -1$

4 MARKS QUESTIONS

3. 2075 Set A Q.No. 8a

What are elementary row operations? Use it to solve: [4]

$x + y + z = 6$, $x - y + z = 2$, $x + y - z = 0$.

SOLUTION

The elementary row operations are

i. interchange any two rows

ii. Addition of one row to another.

iii. Multiplication of any row by a non-zero real number.

iv. Multiplication of any row by a non-zero number and addition of the resulting row to another.

Row equivalent matrix method

The augmented matrix is:

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 1 & : & 2 \\ 1 & 1 & -1 & : & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 0 & : & -4 \\ 0 & 0 & -2 & : & -6 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & -2 & : & -6 \end{bmatrix} R_2 \to \frac{-1}{2}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & -2 & : & -6 \end{bmatrix} R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to \frac{-1}{2}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_1 \to R_1 - R_3$$

$\therefore \quad x = 1, y = 2$ and $= 3$

4. 2072 Set E Q.No. 8a

Solve the following system of equations by the row–equivalent matrix method: [4]

$x + y + z = -2$, $2x + 8y + 5z = 5$, $x + 2y - z = 2$.

SOLUTION

The augmented matrix is:

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & -2 \\ 2 & 8 & 5 & 5 \\ 1 & 2 & -1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & -2 \\ 0 & 6 & 3 & 9 \\ 0 & 1 & -2 & 4 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & -2 \\ 0 & 1 & 1/2 & 3/2 \\ 0 & 1 & -2 & 4 \end{array}\right] R_2 \to \frac{1}{6} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1/2 & -7/2 \\ 0 & 1 & 1/2 & 3/2 \\ 0 & 0 & -5/2 & 5/2 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1/2 & -7/2 \\ 0 & 1 & 1/2 & 3/2 \\ 0 & 0 & 1 & -1 \end{array}\right] R_3 \to \frac{-2}{5} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - \frac{1}{2} R_3 \\ R_2 \to R_2 - \frac{1}{2} R_3 \end{array}$$

$\therefore \quad x = -3, y = 2, z = -1$

5. 2071 Old Q.No. 10b Or

Solve by row–equivalent matrix method [4]

$x - y + 2z = 0$; $x - 2y + 3z = -1$; $2x - 2y + z = -3$

SOLUTION

The augmented matrix is:

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right] R_2 \to (-1) R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right] R_1 \to R_1 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_3 \to -\frac{1}{3} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 + R_3 \end{array}$$

$\therefore \quad x = 0, y = 2$ and $z = 1$

6. 2070 Old Q.No. 10 b

Solve by row equivalent matrix method

$9y - 5x = 3$, $x + z = 1$, $2y + z = 2$. [4]

SOLUTION

The augmented matrix is:

$$\left[\begin{array}{ccc:c} -5 & 9 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -9/5 & 0 & -3/5 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \end{array}\right] R_1 \to \frac{-1}{5} R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -9/5 & 0 & -3/5 \\ 0 & 9/5 & 1 & 8/5 \\ 0 & 2 & 1 & 2 \end{array}\right] R_2 \to R_2 - R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -9/5 & 0 & -3/5 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 2 & 1 & 2 \end{array}\right] R_2 \to \frac{5}{9} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 0 & -1/9 & 2/9 \end{array}\right] \begin{array}{l} R_1 \to R_1 + \frac{9}{5} R_2 \\ R_3 \to R_3 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5/9 & 8/9 \\ 0 & 0 & 1 & -2 \end{array}\right] R_3 \to -9R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - \frac{5}{9} R_3 \end{array}$$

$\therefore \quad x = 3, y = 2$ and $z = -2$

7. 206? Q.No. 10b

Solve by row–equivalent method: [4]

$9y - 5x = 3$; $x + z = 1$ and $z + 2y = 2$

SOLUTION

The augmented matrix of the given equations is

$$\left[\begin{array}{ccc:c} -5 & 9 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -\frac{9}{5} & 0 & -\frac{3}{5} \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \end{array}\right] R_1 \to -\frac{1}{5} \times R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -\frac{9}{5} & 0 & -\frac{3}{5} \\ 0 & \frac{9}{5} & 1 & \frac{8}{5} \\ 0 & 2 & 1 & 2 \end{array}\right] R_2 \to R_2 - R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -\frac{9}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & \frac{5}{9} & \frac{8}{9} \\ 0 & 2 & 1 & 2 \end{array}\right] R_2 \to \frac{5}{9} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & \frac{5}{9} & \frac{8}{9} \\ 0 & 0 & \frac{-1}{9} & \frac{2}{9} \end{array}\right] \begin{array}{l} R_1 \to R_1 + \frac{9}{5} R_2 \\ R_3 \to R_3 - 2R_2 \end{array}$$

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$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & \frac{5}{9} & : & \frac{8}{9} \\ 0 & 0 & 1 & : & -2 \end{bmatrix} R_3 \to -9R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - \frac{5}{9}R_3 \end{matrix}$$

$\therefore x = 3, y = 2, z = -2$

8. 2061 Q.No. 10b OR

Solve by Row–equivalent matrix method:

$x - y - z = -2$, $x + 4z = 4$ and $y - 2z = 1$ **[4]**

SOLUTION

The augmented matrix of the given system of linear equations is

$$\begin{bmatrix} 1 & -1 & -1 & : & -2 \\ 1 & 0 & 4 & : & 4 \\ 0 & 1 & -2 & : & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & : & -2 \\ 0 & 1 & 5 & : & 6 \\ 0 & 1 & -2 & : & 1 \end{bmatrix} R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 4 & : & 4 \\ 0 & 1 & 5 & : & 6 \\ 0 & 0 & -7 & : & -5 \end{bmatrix} \begin{matrix} R_1 \to R_1 + R_2 \\ R_3 \to R_3 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 4 & : & 4 \\ 0 & 1 & 5 & : & 6 \\ 0 & 0 & 1 & : & 5/7 \end{bmatrix} R_3 \to (-1/7)R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 8/7 \\ 0 & 1 & 0 & : & 17/7 \\ 0 & 0 & 1 & : & 5/7 \end{bmatrix} \begin{matrix} R_1 \to R_1 - 4R_3 \\ R_2 \to R_2 - 5R_3 \end{matrix}$$

$\therefore x = \frac{8}{7}, y = \frac{17}{7}, z = \frac{5}{7}$

9. 2060 Q.No. 10b

Solve by row equivalent matrix method:

$x - 2y - 3z = -1$, $2x + y + z = 6$,

$x + 3y - 2z = 13$ **[4]**

SOLUTION

Given equations are

$x - 2y - 3z = -1$, $2x + y + z = 6$, $x + 3y - 2z = 13$

The augmented matrix of the given system of equation is

$$\begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 2 & 1 & 1 & : & 6 \\ 1 & 3 & -2 & : & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 0 & 5 & 7 & : & 8 \\ 0 & 5 & 1 & : & 14 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 0 & 1 & 7/5 & : & 8/5 \\ 0 & 5 & 1 & : & 14 \end{bmatrix} R_2 \to (1/5)R_2$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 0 & 1 & 7/5 & : & 8/5 \\ 0 & 0 & -6 & : & 6 \end{bmatrix} R_3 \to R_3 - 5R_2$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 0 & 1 & \frac{7}{5} & : & \frac{8}{5} \\ 0 & 0 & 1 & : & -1 \end{bmatrix} R_3 \to (-1/6)R_3$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & : & -1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & -1 \end{bmatrix} R_2 \to R_2 - 7/5\, R_3$$

$$\sim \begin{bmatrix} 1 & -2 & 0 & : & -4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & -1 \end{bmatrix} R_1 \to R_1 + 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & -1 \end{bmatrix} R_1 \to R_1 + 2R_2$$

$\therefore x = 2;\ y = 3;\ z = -1$

10. 2059 Q.No. 10b

Solve by row equivalent matrix method **[4]**

$x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$

SOLUTION

The augmented matrix of the given system of linear equations can be written as

$$\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 2 & 1 & : & 2 \\ 5 & -9 & 0 & : & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 2 & 1 & : & 2 \\ 0 & -9 & -5 & : & -8 \end{bmatrix} R_3 \to R_3 - 5R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & \frac{1}{2} & : & 1 \\ 0 & -9 & -5 & : & -8 \end{bmatrix} R_2 \to \left(\frac{1}{2}\right)R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & \frac{1}{2} & : & 1 \\ 0 & 0 & -\frac{1}{2} & : & 1 \end{bmatrix} R_3 \to R_3 + 9R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & \frac{1}{2} & : & 1 \\ 0 & 0 & 1 & : & -2 \end{bmatrix} R_3 \to (-2)R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - \frac{1}{2}R_3 \end{matrix}$$

$\therefore x = 3, y = 2, z = -2$

11. 2058 Q.No. 10b

Solve, by row equivalent matrix method,

$x + y + z = 1$, $x + 2y + 3z = 4$, $x + 3y + 7z = 13$ **[4]**

SOLUTION

The augmented matrix of the given system of linear equations is

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 3 & : & 4 \\ 1 & 3 & 7 & : & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 2 & : & 3 \\ 0 & 2 & 6 & : & 12 \end{bmatrix} \text{Applying } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & : & -2 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 2 & : & 6 \end{bmatrix} \text{Applying } \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

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$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 3 \end{array}\right] \text{ Applying } R_3 \to \frac{1}{2}R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right] \text{ Applying } \begin{matrix} R_1 \to R_1 + R_3 \\ R_2 \to R_2 - 2R_3 \end{matrix}$$

$\therefore x = 1, y = -3, z = 3$

12. 2057 Q.No. 10b

Solve by using row equivalent matrix method: $x - 2y + 2z = 0$; $x - 2y + 3z = -1$; $2x - 2y + z = -3$ [4]

SOLUTION

The augmented matrix of the given system of linear equations is

$$\left[\begin{array}{ccc:c} 1 & -2 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right]$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & -2 & 2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 2 & -3 & -3 \end{array}\right]$$

Applying $R_2 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & -2 & 2 & 0 \\ 0 & 2 & -3 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Applying $R_2 \to \frac{1}{2}R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & -2 & 2 & 0 \\ 0 & 1 & \frac{-3}{2} & \frac{-3}{2} \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Applying $R_1 \to R_1 - 2R_3$ and $R_2 \to R_2 + \frac{3}{2}R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & -2 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Applying $R_1 \to R_1 + 2R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$\therefore x = -4, y = -3, z = -1$

13. 2056 Q.No. 10b

Solve by row equivalent matrix method:

$x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$ [4]

SOLUTION

The augmented matrix of the given system of linear equations is

$$\left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 5 & -9 & 0 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & -9 & -5 & -8 \end{array}\right] R_3 \to R_3 - 5R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & -9 & -5 & -8 \end{array}\right] R_2 \to \frac{1}{2}R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & -\frac{1}{2} & 1 \end{array}\right] R_3 \to R_3 + 9R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] R_3 \to (-2)R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] R_1 \to R_1 - R_3;\ R_2 \to R_2 - \frac{1}{2}R_3$$

Hence, $x = 3$; $y = 2$, $z = -2$

14. 2055 Old Model Q.No. 10b

Solve, by row equivalent matrix method. [4]

$x - y + 2z = 0$; $x - 2y + 3z = -1$; $2x - 2y + z = -3$

SOLUTION

The augmented matrix of given system of linear equations is

$$\left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right] \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right] R_2 \to (-1)R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right] R_1 \to R_1 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_3 \to \left(-\frac{1}{3}\right)R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 + R_3 \end{matrix}$$

Hence, $x = 0$; $y = 2$; $z = 1$.

B. INVERSE MATRIX METHOD

2 MARKS QUESTIONS

1. 2073 Supp Q.No. 3a

Using inverse matrix method, solve the following system of equations:

$2x + 5y = 17$, $5x - 2y = -1$. [2]

SOLUTION

Writing the given equation in matrix form

$$\begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ -1 \end{bmatrix}$$

which is in the form of $AX = B$

$\Rightarrow X = A^{-1}B$ ... (i)

Now, $|A| = \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix} = -4 - 25 = -29 \neq 0.$

So, A^{-1} exists.

The cofactors are

$A_{11} = (-1)^{1+1}(-2) = -2$

$A_{12} = (-1)^{1+2}(5) = -5$

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$A_{21} = (-1)^{2+1}(5) = -5$

$A_{22} = (-1)^{2+2}(2) = 2$

Adj. A = transpose of matrix of cofactors.

$$= \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ Adj } A$$

$$= \frac{1}{-29} \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix}$$

From (i)

$$X = A^{-1}B = \frac{1}{-29} \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 17 \\ -1 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29} \begin{bmatrix} -34+5 \\ -85-2 \end{bmatrix}$$

$$= -\frac{1}{29} \begin{bmatrix} -29 \\ -87 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1, y = 3$

2. 2071 Set C Q.No. 3a

Solve by Inverse matrix method: $2x + y = 7$, $x + 3y = 11$ [2]

SOLUTION

Writing the given equation in matrix form:

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

which is in the form of $AX = B$

$\therefore$ $X = A^{-1}B$

Now,

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5 \neq 0$$

$\therefore$ A^{-1} exists

The co–factors are

$A_{11} = (-1)^{1+1} \cdot (3) = 3$

$A_{12} = (-1)^{1+2} \cdot (1) = -1$

$A_{21} = (-1)^{2+1} \cdot (1) = -1$

$A_{22} = (-1)^{2+2} \cdot (2) = 2$

$$\text{Adj. } A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{Adj} \cdot A$$

$$= \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

We have,

$X = A^{-1}B$

$$X = \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 21-11 \\ -7+22 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 2$ and $y = 3$

3. 2071 Set D Q.No. 3a

Solve by Inverse matrix method: $5x - 3y = 9$; $2x + 5y = 16$ [2]

SOLUTION

Writing the given equations in matrix form:

$$\begin{bmatrix} 5 & -3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$$

which is in the form of $AX = B$

$X = A^{-1}B$ ...(i)

For A^{-1}

$$|A| = \begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix} = 25 + 6 = 31 \neq 0$$

So, A^{-1} exists.

The cofactors are

$A_{11} = (-1)^{1+1}(5) = 5,$

$A_{12} = (-1)^{1+2}(2) = -2$

$A_{21} = (-1)^{2+1}(-3) = 3,$

$A_{22} = (-1)^{2+2}(-3) = 5$

$$\text{Adj. } A = \begin{bmatrix} 5 & -3 \\ 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 5 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A = \frac{1}{31} \begin{bmatrix} 5 & 3 \\ -2 & 5 \end{bmatrix}$$

From (i), we get

$$X = \frac{1}{31} \begin{bmatrix} 5 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 9 \\ 16 \end{bmatrix}$$

or, $$X = \frac{1}{31} \begin{bmatrix} 45+48 \\ -18+80 \end{bmatrix}$$

or, $$X = \frac{1}{31} \begin{bmatrix} 93 \\ 62 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 3, y = 2$

4. 2069 [Set A] Old Q. No. 3b

Solve by inverse matrix method: $x = y = 5$ and $x - y = 3$. [2]

SOLUTION

Writing the given equations in matrix form:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

which is in the form of $AX = B$

$\Rightarrow$ $X = A^{-1}B$ ...(i)

For A^{-1}

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 \neq 0$$

So, A^{-1} exists.

The cofactors are

$A_{11} = (-1)^{1+1}(-1) = -1$

$A_{12} = (-1)^{1+2}(1) = -1$

$A_{21} = (-1)^{2+1}(1) = -1$

$A_{22} = (-1)^{2+2}(1) = 1$

$$\text{Adj. } A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

We have,

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj.} A = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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From (i), $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 5+3 \\ 5-3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$\therefore\ x = 4, y = 1$

5. 2067 Q.No. 3b

Solve by matrix inversion method:

$x + y = 3$ and $x - y = 1$ **[2]**

SOLUTION

Given equations are

$x + y = 3$

$x - y = 1$

Writing these equation in matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

which is in form of $AX = B$

$\therefore\ X = A^{-1}B$ ... (i)

Now, $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = -2 \neq 0.$$

So, inverse of A exists.

We have, $A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

$\because A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

From (i)

$$X = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3+1 \\ 3-1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore\ x = 2; y = 1$

6. 2063 Q.No. 4b

Solve by inverse matrix method: **[2]**

$3x + 2y = 5, 7x + 5y = 12$

SOLUTION

The given equations are

$3x + 2y = 5$

$7x + 5y = 12$

The given equations in matrix form can be written as

$AX = B$, where

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

The solution is given by

$X = A^{-1}B$ ... (i)

Now, $|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 15 - 14$

$= 1 \neq 0$. So A^{-1} exists.

Again,

$$(\text{Adj. } A) = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$\left[\because A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \text{Adj. } A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\right]$$

And,

$$A^{-1} = \frac{1}{|A|}(\text{Adj. } A) = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

|Note: If $|A| = 1$, then $(\text{Adj. } A) = A^{-1}$|

Hence, $X = A^{-1}B$ becomes

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 25-24 \\ -35+36 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore\ x = 1, y = 1$

7. 2062 Q.No. 3b

Solve: $x = 2y + 3$ and $3x - 5y = 8$ by inverse matrix method. [2]

SOLUTION

The given equations are

$x - 2y = 3$ ... (1)

$3x - 5y = 8$ ... (2)

The matrix form of above linear equations is $AX = B$

Where,

$$A = \begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix};\ X = \begin{pmatrix} x \\ y \end{pmatrix};\ B = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

The solution is given by $X = A^{-1}B$ ... (3)

Now,

$$|A| = \begin{vmatrix} 1 & -2 \\ 3 & -5 \end{vmatrix} = -5 + 6 = 1 \neq 0.$$ So A^{-1} exists.

$$\text{Adj. } A = \begin{pmatrix} -5 & 2 \\ -3 & 1 \end{pmatrix}$$

$$\left[\because A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \text{Adj. } A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\right]$$

And, $A^{-1} = \frac{1}{|A|} \times (\text{Adj. } A) = \frac{1}{1}\begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}$$

Hence, $X = A^{-1}B$ becomes

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} -15+16 \\ -9+8 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\therefore\ x = 1, y = -1$

8. 2061 Q.No. 3b

Solve by matrix inversion method, $x + y =$ [2]

and $x - y = 0$

SOLUTION

Given equations are

$x + y = 2$

$x - y = 0$

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The matrix form of the above system of equations is $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

The solution is given by $X = A^{-1} B$ ... (i)

Now, $|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$

$= -1 - 1 = -2 \neq 0$

So, A^{-1} exists.

The cofactors of elements of A are

$A_{11} = (-1)^{1+1} |-1| = -1$

$A_{12} = (-1)^{1+2} |1| = -1$

$A_{21} = (-1)^{2+1} |1| = -1$

$A_{22} = (-1)^{2+2} |1| = 1$

$$\text{Cofactor matrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

We know that,

$$\text{Adj. A} = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}$ (Adj. A)

From (1), we have

$X = A^{-1} B$

or, $$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -2+0 \\ -2+0 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1; y = 1$

4 MARKS QUESTIONS

9. 2076 Set B Q.No. 8a

Solve by inverse matrix or row equivalent method: [4]

$x - y = 0$, $2x - y + 4z = 18$, $-3x + z + 2 = 0$.

SOLUTION

The given equations are:

$x - y + 0 \cdot z = 0$

$2x - y + 4z = 18$

$-3x + 0 \cdot y + z = -2$

Row Equivalent Matrix Method

The augmented matrix is

$$\begin{bmatrix} 1 & -1 & 0 & : & 0 \\ 2 & -1 & 4 & : & 18 \\ -3 & 0 & 1 & : & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 0 & : & 0 \\ 0 & 1 & 4 & : & 18 \\ 0 & -3 & 1 & : & -2 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 4 & : & 18 \\ 0 & 1 & 4 & : & 18 \\ 0 & 0 & 13 & : & 52 \end{bmatrix} \begin{matrix} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 4 & : & 18 \\ 0 & 1 & 4 & : & 18 \\ 0 & 0 & 1 & : & 4 \end{bmatrix} R_3 \to \frac{1}{13} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 4 \end{bmatrix} \begin{matrix} R_1 \to R_1 - 4R_3 \\ R_2 \to R_2 - 4R_3 \end{matrix}$$

$\therefore x = 2, y = 2, z = 4$

Inverse Matrix Method

Writing the given equation in matrix form:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 4 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ -2 \end{bmatrix}$$

which is in the form of $AX = B$

$\therefore X = A^{-1}B$ ...(i)

Now,

$$|A| = \begin{vmatrix} 1 & -1 & 0 \\ 2 & -1 & 4 \\ -3 & 0 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix}$$

$= 1(-1) + 1(2+12)$

$= -1 + 14 = 13 \neq 0$

$\therefore A^{-1}$ exists.

The cofactors are:

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix} = 1(-1) = -1$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = -1(2 + 12) = -14$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ -3 & 0 \end{vmatrix} = 1(-3) = -3$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1(-1) = 1$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = 1(1) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ -3 & 0 \end{vmatrix} = -1(-3) = 3$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 0 \\ -1 & 4 \end{vmatrix} = 1(-4) = -4$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix} = -1(4) = -4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1(-1 + 2) = 1(1) = 1$$

$$\text{Matrix of cofactors} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -14 & -3 \\ 1 & 1 & 3 \\ -4 & -4 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 1 & -4 \\ -14 & 1 & -4 \\ -3 & 3 & 1 \end{bmatrix}$$

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Now, $A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A$

$$= \frac{1}{13}\begin{bmatrix} -1 & 1 & -4 \\ -14 & 1 & -4 \\ -3 & 3 & 1 \end{bmatrix}$$

From (i)

$$X = \frac{1}{13}\begin{bmatrix} -1 & 1 & -4 \\ -14 & 1 & -4 \\ -3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 18 \\ -2 \end{bmatrix}$$

or, $$X = \frac{1}{13}\begin{bmatrix} 0+18+8 \\ 0+18+8 \\ 0+54-2 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 26 \\ 26 \\ 52 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$\therefore$ $x = 2$, $y = 2$ and $z = 4$.

10. 2076 Set C Q.No. 8a

Solve by row-equivalent or inverse matrix method: [4]

$x - y = 1$, $z + x = -6$, $x + y - 2z = 3$

SOLUTION

Row equivalent Matrix Method

The augmented matrix is

$$\begin{bmatrix} 1 & -1 & 0 & : & 1 \\ 1 & 0 & 1 & : & -6 \\ 1 & 1 & -2 & : & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 0 & : & 1 \\ 0 & 1 & 1 & : & -7 \\ 0 & 2 & -2 & : & 2 \end{bmatrix} R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & -6 \\ 0 & 1 & 1 & : & -7 \\ 0 & 0 & -4 & : & 16 \end{bmatrix} R_1 \to R_1 + R_2,\ R_3 \to R_3 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & -6 \\ 0 & 1 & 1 & : & -7 \\ 0 & 0 & 1 & : & -4 \end{bmatrix} R_3 \to \left(\frac{1}{-4}\right) R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & -2 \\ 0 & 1 & 0 & : & -3 \\ 0 & 0 & 1 & : & -4 \end{bmatrix} R_2 \to R_2 - R_3,\ R_1 \to R_1 - R_3$$

$\therefore$ $x = -2$, $y = -3$, $z = -4$

Inverse Matrix Method

Writing the given system of equations in matrix form

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 3 \end{bmatrix}$$

which is in the form of $AX = B \Rightarrow X = A^{-1}B$... (i)

Now, for A^{-1}

$$|A| = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 1(0-1) + 1(-2-1)$$

$$= -1 - 3 = -4 \neq 0.$$

So, A^{-1} exists.

The cofactors are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 0 - 1 = -1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} = -(2-0) = -2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = -2 - 0 = -2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1 - 0 = -1$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -(1-0) = -1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 0 + 1 = 1$$

$$\text{Adj. } A = \begin{bmatrix} -1 & 3 & 1 \\ -2 & -2 & -2 \\ -1 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -2 & -1 \\ 3 & -2 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A = \frac{1}{-4}\begin{bmatrix} -1 & -2 & -1 \\ 3 & -2 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

From (i)

$$X = \frac{1}{-4}\begin{bmatrix} -1 & -2 & -1 \\ 3 & -2 & -1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -6 \\ 3 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -1+12-3 \\ 3+12-3 \\ 1+12+3 \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} 8 \\ 12 \\ 16 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}$$

$\therefore$ $x = -2$, $y = -3$, $z = -4$.

11. 2075 GIE Q.No. 8a

Using row equivalent method or inverse matri method solve: [4

$2x - 3y + z = 0$, $x + y + z = 7$, $3x + 2y - z = 3$.

SOLUTION

Row Equivalent Method

The augmented matrix is

$$\begin{bmatrix} 2 & -3 & 1 & : & 0 \\ 1 & 1 & 1 & : & 7 \\ 3 & 2 & -1 & : & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 2 & -3 & 1 & : & 0 \\ 3 & 2 & -1 & : & 3 \end{bmatrix} R_2 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 0 & -5 & -1 & : & -14 \\ 0 & -1 & -4 & : & -18 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 0 & 1 & \frac{1}{5} & : & \frac{14}{5} \\ 0 & -1 & -4 & : & -18 \end{bmatrix} R_2 \to \left(-\frac{1}{5}\right) R_2$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{4}{5} & : & \frac{21}{5} \\ 0 & 1 & \frac{1}{5} & : & \frac{14}{5} \\ 0 & 0 & -\frac{19}{5} & : & -\frac{76}{5} \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + R_2 \end{matrix}$$

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$$\left[\begin{array}{ccc:c} 1 & 0 & \frac{4}{5} & \frac{21}{5} \\ 0 & 1 & \frac{1}{5} & \frac{14}{5} \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 \to \left(-\frac{5}{19}\right) R_3$$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right] \begin{array}{l} R_1 \to R_1 - \frac{4}{5}R_3 \\ R_2 \to R_2 - \frac{1}{5}R_3 \end{array}$$

$\therefore$ $x = 1, y = 2, z = 4$

Inverse Matrix Method

Writing the given equations in matrix from

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 3 \end{bmatrix}$$

which is in the form of

$AX = B$

$X = A^{-1}B$ ...(i)

Now,

$$|A| = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} - (-3)\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

$= 2(-1-2) + 3(-1-3) + 1(2-3)$

$= -6 - 12 - 1 = -19 \neq 0$

$\therefore$ A^{-1} exists.

The cofactors are

$$A_{11} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1-2 = -3$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -(-1-3) = 4$$

$$A_{13} = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = 2-3 = -1$$

$$A_{21} = -\begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix} = -(3-2) = -1$$

$$A_{22} = \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -2-3 = -5$$

$$A_{23} = -\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = -(4+9) = -13$$

$$A_{31} = \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} = -3-1 = -4$$

$$A_{32} = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -(2-1) = -1$$

$$A_{33} = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2+3 = 5$$

Matrix of cofactor $= \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

$$= \begin{pmatrix} -3 & 4 & -1 \\ -1 & -5 & -13 \\ -4 & -1 & 5 \end{pmatrix}$$

Adj. A $= \begin{pmatrix} -3 & 4 & -1 \\ -1 & -5 & -13 \\ -4 & -1 & 5 \end{pmatrix}^T$

$$= \begin{pmatrix} -3 & -1 & -4 \\ 4 & -5 & -1 \\ -1 & -13 & 5 \end{pmatrix}$$

$A^{-1} = \frac{1}{|A|}$. Adj. A

$$= -\frac{1}{19}\begin{pmatrix} -3 & -1 & -4 \\ 4 & -5 & -1 \\ -1 & -13 & 5 \end{pmatrix}$$

$$= \frac{1}{19}\begin{pmatrix} 3 & 1 & 4 \\ -4 & 5 & 1 \\ 1 & 13 & -5 \end{pmatrix}$$

Then, from (i),

$$X = \frac{1}{19}\begin{pmatrix} 3 & 1 & 4 \\ -4 & 5 & 1 \\ 1 & 13 & -5 \end{pmatrix}\begin{pmatrix} 0 \\ 7 \\ 3 \end{pmatrix}$$

or, $$X = \frac{1}{19}\begin{pmatrix} 0+7+12 \\ 0+35+3 \\ 0+91-15 \end{pmatrix}$$

or, $$X = \frac{1}{19}\begin{pmatrix} 19 \\ 38 \\ 76 \end{pmatrix}$$

or, $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$\therefore$ $x = 1, y = 2, z = 4$

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12. 2075 Set B Q.No. 8a

Solve by row-equivalent or inverse matrix method: [4]

$x + z = 1$, $z + 2y = 2$, $5x - 9y = -3$.

Row Equivalent Matrix Method:

Please refer to 2056 Q.No. 10b [From 7 A]

Inverse Matrix Method:

Given equations are

$-5x + 9y + 0.z = 3$

$x + 0.y + z = 1$,

$0.x + 2y + z = 2$,

Writing the given equation in matrix method

$$\begin{bmatrix} -5 & 9 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

which is in the form of $AX = B$

$\therefore$ $X = A^{-1}B$

Now,

$$|A| = \begin{bmatrix} -5 & 9 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} = -5\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 9\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$= -5(-2) - 9(1)$

$= 10 - 9 = 1 \neq 0$

$\therefore$ A^{-1} exists.

The cofactors are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1(1) = -1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1(2) = 2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 9 & 0 \\ 2 & 1 \end{vmatrix} = -1(9) = -9$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix} = 1(-5) = -5$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} -5 & 9 \\ 0 & 2 \end{vmatrix} = -1(-10) = 10$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 9 & 0 \\ 0 & 1 \end{vmatrix} = 1(9) = 9$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} -5 & 0 \\ 1 & 1 \end{vmatrix} = -1(-5) = 5$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} -5 & 9 \\ 1 & 0 \end{vmatrix} = 1(-9) = -9$$

$$\text{Adj. } A = \begin{bmatrix} -2 & -1 & 2 \\ -9 & -5 & 10 \\ 9 & 5 & -9 \end{bmatrix}^T = \begin{bmatrix} -2 & -9 & 9 \\ -1 & -5 & 5 \\ 2 & 10 & -9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj. } A = \frac{1}{1}\begin{bmatrix} -2 & -9 & 9 \\ -1 & -5 & 5 \\ 2 & 10 & -9 \end{bmatrix}$$

We have

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -2 & -9 & 9 \\ -1 & -5 & 5 \\ 2 & 10 & -9 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 - 9 + 18 \\ -3 - 5 + 10 \\ 6 + 10 - 18 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

$\therefore$ $x = 3$, $y = 2$ and $z = -2$

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13. 2075 Set C Q.No. 8a

Using row-equivalent method or inverse matrix method, solve the following system of equations. [4]

$x + z = 1$, $z + 2y = 2$, $5x - 9y = -3$.

➥ **Raw Equivalent Matrix Method:**

Please refer to 2056 Q.No. 10b [From 7 A]

Inverse Matrix Method:

Please refer to 2075 Set B Q.No. 8a

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14. 2074 Supp Q.No. 8a

Using row equivalent matrix method or inverse matrix method solve the following system of equation: [4]

$x + 4y + z = 18$, $3x + 3y - 2z = 2$, $-4y + z = -7$

SOLUTION

Row–equivalent matrix method

The augmented matrix is

$$\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 3 & 3 & -2 & : & 2 \\ 0 & -4 & 1 & : & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & -9 & -5 & : & -52 \\ 0 & -4 & 1 & : & -7 \end{bmatrix} \quad R_2 \to R_2 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & -4 & 1 & : & -7 \end{bmatrix} \quad R_2 \to \left(-\frac{1}{9}\right)R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -11/9 & : & -46/9 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & 0 & 29/9 & : & -145/9 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to R_3 + 4R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -11/9 & : & -46/9 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \quad R_3 \to \frac{9}{29}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 + \frac{11}{9}R_3 \\ R_2 \to R_2 - \frac{5}{9}R_3 \end{matrix}$$

$\therefore$ $x = 1$, $y = 3$ and $z = 5$

Inverse Matrix Method

Writing the given equation in matrix method

$$\begin{bmatrix} 1 & 4 & 1 \\ 3 & 3 & -2 \\ 0 & -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 2 \\ 7 \end{bmatrix}$$

Which is in the form of $AX = B$

$\therefore$ $X = A^{-1}B$

Now,

$$|A| = \begin{vmatrix} 1 & 4 & 1 \\ 3 & 3 & -2 \\ 0 & -4 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix} - 4\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & -3 \\ 0 & -4 \end{vmatrix}$$

$$= 1(3 - 8) - 4(3) + 1(-12)$$

$$= -5 - 12 - 12 = -29 \neq 0$$

$\therefore$ A^{-1} exists.

The cofactors are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix} = 3 - 8 = -5$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} = -1(3) = -3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -3 \\ 0 & -4 \end{vmatrix} = 1(-12) = -12$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 1 \\ -4 & 1 \end{vmatrix} = -1(4 + 4) = -8$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 4 \\ 0 & -4 \end{vmatrix} = -1(-4) = 4$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 4 & 1 \\ 3 & -2 \end{vmatrix} = (-8 - 3) = -11$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -1(-2 - 3) = 5$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 4 \\ 3 & 3 \end{vmatrix} = (3 - 12) = -9$$

$$\text{Adj. } A = \begin{bmatrix} -5 & -8 & -12 \\ -8 & 1 & 4 \\ -11 & 5 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -11 \\ -8 & 1 & 5 \\ -12 & 4 & -9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot \text{Adj.}A = \frac{1}{-29}\begin{bmatrix} -5 & -8 & -11 \\ -8 & 1 & 5 \\ -12 & 4 & -9 \end{bmatrix}$$

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We have,

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -5 & -8 & -11 \\ -3 & 1 & 5 \\ -12 & 4 & -9 \end{bmatrix}\begin{bmatrix} 18 \\ 2 \\ -7 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -90-16+77 \\ -54+2-35 \\ -216+8+63 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -29 \\ -87 \\ -145 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$\therefore x = 1, y = 3$ and $z = 5$

15. 2074 Set A Q.No. 8a

Using row-equivalent method or inverse matrix method, solve the following system of equations. [4]

$x + y + z = 1, x + 2y + 3z = 4, x + 3y + 7z = 13.$

SOLUTION

Row Equivalent Matrix Method

Please refer 2058 Q.No. 10b [From 7 A]

Inverse Matrix Method

Writing the given equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 13 \end{bmatrix}$$

which is in the form of $AX = B$

$\Rightarrow X = A^{-1}B$ ...(i)

For A^{-1}

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 7 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$= 1(14-9) - 1(7-3) + 1(3-2)$

$= 5 - 4 + 1 = 2 \neq 0$

So, A^{-1} exists.

The cofactors are

$$A_{11} = \begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} = 14 - 9 = 5$$

$$A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = -(7-3) = -4$$

$$A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$A_{21} = -\begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = -(7-3) = -4$$

$$A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 7 \end{vmatrix} = 7 - 1 = 6$$

$$A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$$

$$\text{Adj. } A = \begin{bmatrix} 5 & -4 & 1 \\ -4 & 6 & -2 \\ 1 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -4 & 1 \\ -4 & 6 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj. } A = \frac{1}{2}\begin{bmatrix} 5 & -4 & 1 \\ -4 & 6 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

Now, from (i),

$X = A^{-1}B$

$$\text{or, } X = \frac{1}{2}\begin{bmatrix} 5 & -4 & 1 \\ -4 & 6 & -2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 13 \end{bmatrix}$$

$$\text{or, } X = \frac{1}{2}\begin{bmatrix} 5-16+13 \\ -4+24-26 \\ 1-8+13 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ -6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$

$\therefore x = 1, y = -3, z = 3$

16. 2074 Set B Q.No. 8a

Solve by row-equivalent or inverse matrix method: $x - y = 1, z + x = -6, x + y - 2z = 3.$ [4]

➥ Please refer to 2076 Set C Q.No. 8a

17. 2073 Set C Q.No. 8a

Using row equivalent or inverse matrix method, solve the following system of equations

$x + z = 1; 2y + z = 2; 5x - 9y + 3 = 0$ [4]

SOLUTION

Row equivalent matrix method

Please refer 2056 Q.No. 10 b [From 7A]

Inverse matrix method

Please refer to 2075 Set B Q.No. 8a

18. 2073 Set D Q.No. 8a

Using inverse matrix method or row equivalent method, solve:

$x + 2y + 3z = 4, 2x + y = 4, 3y - 2z = 0.$ [4]

SOLUTION

Inverse matrix method

Writing the given equations in matrix form,

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$$

which is in the form of $AX = B$

$\Rightarrow X = A^{-1}B$ ...(i)

Now, for A^{-1}

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$= 1(-2-0) - 2(-4-9) = -2 + 26 = 24 \neq 0$

The cofactors are

$A_{11} = \begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix} = -2$

$A_{12} = -\begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = 4$

$A_{13} = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = 13$

$A_{22} = \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} = -2$

$A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = -3$

$A_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$

$A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = 5$

$A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

$$\text{Adj. } A = \begin{pmatrix} -2 & 4 & 6 \\ 13 & -2 & -3 \\ -3 & 5 & -3 \end{pmatrix}^T = \begin{pmatrix} -2 & 13 & -3 \\ 4 & -2 & 5 \\ 6 & -3 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{24}\begin{pmatrix} -2 & 13 & -3 \\ 4 & -2 & 5 \\ 6 & -3 & -3 \end{pmatrix}$$

From (i)

$$X = A^{-1}B = \frac{1}{24}\begin{pmatrix} -2 & 13 & -3 \\ 4 & -2 & 5 \\ 6 & -3 & -3 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \frac{1}{24}\begin{pmatrix} -8+52-0 \\ 16-8+0 \\ 24-12-0 \end{pmatrix} = \frac{1}{24}\begin{pmatrix} 44 \\ 8 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11/6 \\ 1/3 \\ 1/2 \end{pmatrix}$$

$\therefore$ $x = 11/6$, $y = 1/3$ and $z = 1/2$

Row equivalent matrix method

The augmented matrix is

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 4 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -4 \\ 0 & 3 & -2 & 0 \end{array}\right] R_2 \to R_2 - 2R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4/3 \\ 0 & 3 & -2 & 0 \end{array}\right] R_2 \to \frac{1}{(-3)} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -1 & 4/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & -8 & -4 \end{array}\right] \begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - 3R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -1 & 4/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & 1 & 1/2 \end{array}\right] R_3 \to \frac{1}{(-8)} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 11/6 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_3 \\ R_1 \to R_1 + R_3 \end{matrix}$$

$\therefore$ $x = \frac{11}{6}$, $y = \frac{1}{3}$ and $z = \frac{1}{2}$

19. 2072 Supp. Q.No. 8a

Using row equivalent matrix method or the inverse matrix method, solve the following system of equations: [4]

$9y - 5x = 3$, $x + z = 1$, $z + 2y = 2$

SOLUTION

Row equivalent matrix method

Please refer 2070 Old Q.No. 10b [from 7A]

Inverse matrix method

Please refer to 2075 Set B Q.No. 8a

20. 2072 Set C Q.No. 8a

Using row equivalent matrix or inverse matrix method, solve the following equations: [4]

$x + 4y + z = 18$, $3x + 3y - 2z = 2$, $-4y + z = -7$

Please refer to 2074 Supp Q.No. 8a

21. 2072 Set D Q.No. 8a

Using row equivalent matrix method or inverse matrix method, solve the following equations:

$x + 2y - 3z = 9$, $2x - y + 2z = -8$, $3x - y - 4z = 3$. [4]

SOLUTION

Row equivalent matrix method

The augmented matrix is:

$$\left[\begin{array}{ccc:c} 1 & 2 & -3 & 9 \\ 2 & -1 & 2 & -8 \\ 3 & -1 & -4 & 3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & 9 \\ 0 & -5 & 8 & -26 \\ 0 & -7 & 5 & -24 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & 9 \\ 0 & 1 & -8/5 & -26/5 \\ 0 & -7 & 5 & -24 \end{array}\right] R_2 \to \frac{-1}{5} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1/5 & -7/5 \\ 0 & 1 & -8/5 & 26/5 \\ 0 & 0 & -31/5 & 62/5 \end{array}\right] \begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 7R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 1/5 & -7/5 \\ 0 & 1 & -8/5 & 26/5 \\ 0 & 0 & 1 & -2 \end{array}\right] R_3 \to \frac{-5}{31} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{matrix} R_1 \to R_1 - \frac{1}{5} R_3 \\ R_2 \to R_2 + \frac{8}{5} R_3 \end{matrix}$$

$\therefore$ $x = -1$, $y = 2$ and $z = -2$

Inverse matrix method

Writing the given equation in matrix form:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & -1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -8 \\ 3 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & -1 & -4 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 2 \\ -1 & -4 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} - 3\begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix}$$

$$= 1(4+2) - 2(-8-6) - 3(-2+3)$$

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$= 1(6) - 2(-14) - 3(1)$

$= 6 + 28 - 3 = 31 \neq 0$

$\therefore A^{-1}$ exits

The cofactors are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 2 \\ -1 & -4 \end{vmatrix} = 1\cdot(4+2) = 6$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 2 \\ 3 & -4 \end{vmatrix} = -1(-8-6) = 14$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix} = 1(-2+3) = 1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -3 \\ -1 & -4 \end{vmatrix} = -1(-8-3) = 11$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = 1(-4+9) = 5$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -1(-1-6) = 7$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = 1(4-3) = 1$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -1(2+6) = -8$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1(1-4) = -5$$

Again,

$$\text{Adj. } A = \begin{bmatrix} 6 & 14 & 1 \\ 11 & 5 & 7 \\ 1 & -8 & -5 \end{bmatrix}^T = \begin{bmatrix} 6 & 11 & 1 \\ 14 & 5 & -8 \\ 1 & 7 & -5 \end{bmatrix}$$

So, we have

$X = A^{-1}B$

$$\text{or, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{31}\begin{pmatrix} 6 & 11 & 1 \\ 14 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}\begin{pmatrix} 9 \\ -8 \\ 3 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{31}\begin{pmatrix} 54-88+3 \\ 126-40-24 \\ 9-56-15 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{31}\begin{pmatrix} -31 \\ 62 \\ -62 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$\therefore x = -1$, $y = 2$ and $z = -2$

22. 2071 Supp. Q.No. 8a

Solve by row equivalent or inverse matrix method: [4]

$x + y + z = 6$, $x - y + z = 2$, $x + y - z = 0$.

SOLUTION

Row equivalent matrix method

Please refer to 2075 Set A Q.No. 8a [From 7 A]

Inverse matrix method

Writing the given equation in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$$

which is in the form of $AX = B$

$X = A^{-1}B$

Now,

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}$$

$= 1(1-1) - 1(-1-1) + 1(1+1)$

$= 0 + 2 + 2 = 4 \neq 0$

$\therefore A^{-1}$ exists

The cofactors are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1(1-1) = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1(1+1) = 2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1-1) = -2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -1(0) = 0$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1(1+1) = 2$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -1(0) = 0$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1-1) = -2$$

$$\text{Adj. } A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|}\cdot \text{Adj. } A = \frac{1}{4}\begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

We have,

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 0 & 2 & 2 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 0+4+0 \\ 12-4+0 \\ 12+0-0 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1$, $y = 2$ and $z = 3$

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23. 2071 Old Q.No. 10b

Solve by inverse matrix method: $x - y = 2$; $2x + 3y = 9$. [4]

SOLUTION

Writing the given equation in matrix method

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

Which is in the form of $AX = B$

$\therefore X = A^{-1}B$

Now,

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$$

$\therefore A^{-1}$ exists.

The cofactors are:

$A_{11} = (-1)^{1+1} \cdot (3) = 3$

$A_{12} = (-1)^{1+2} \cdot (2) = -2$

$A_{21} = (-1)^{2+1} \cdot (-1) = 1$

$A_{22} = (-1)^{2+2} \cdot (1) = 1$

$$\text{Adj. } A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj.} A = \frac{1}{5} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

We have,

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 6+9 \\ -4+9 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\therefore x = 3$ and $y = 1$

24. 2070 Supp Q.No. 8a

Applying row equivalent matrix method or the inverse matrix method, solve the following equations. $x + 4y + 3z = 6$, $3x + 9y = 18$, $-5x - 6y + 2z = -5$. [4]

SOLUTION

Row Equivalent Matrix Method

The augmented matrix is

$$\left[\begin{array}{ccc:c} 1 & 4 & 3 & 6 \\ 3 & 9 & 0 & 18 \\ -5 & -6 & 2 & -5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 3 & 6 \\ 0 & -3 & -9 & 0 \\ 0 & 14 & 17 & 25 \end{array}\right] \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + 5R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 3 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 14 & 17 & 25 \end{array}\right] R_2 \to \frac{1}{(-3)} R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -9 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & -25 & 25 \end{array}\right] \begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to R_3 - 14R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -9 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] R_3 \to \frac{1}{(-25)} R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{matrix} R_1 \to R_1 + 9R_3 \\ R_2 \to R_2 - 3R_3 \end{matrix}$$

$\therefore x = -3, y = 3, z = -1$

Inverse matrix method

Writing the given equation in matrix form:

$$\begin{bmatrix} 1 & 4 & 3 \\ 3 & 9 & 0 \\ -5 & -6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 18 \\ -5 \end{bmatrix}$$

which is in the form of $AX = B$

$X = A^{-1}B$

Now,

$$|A| = \begin{vmatrix} 1 & 4 & 3 \\ 3 & 9 & 0 \\ -5 & -6 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 9 & 0 \\ -6 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 0 \\ -5 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & 9 \\ -5 & -6 \end{vmatrix}$$

$$= 1(18) - 4(6) + 3(-18 + 45)$$

$$= 18 - 24 + 3(27)$$

$$= -6 + 81 = 75 \neq 0$$

$\therefore A^{-1}$ exist

The co–factors are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 9 & 0 \\ -6 & 2 \end{vmatrix} = 1(18) = 18$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 0 \\ -9 & 2 \end{vmatrix} = -1(6) = -6$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 9 \\ -5 & -6 \end{vmatrix} = 1(-18 + 45) = 27$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 3 \\ -6 & -2 \end{vmatrix} = -1(8 + 18) = -26$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ -5 & -6 \end{vmatrix} = 2 + 15 = 17$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 4 \\ -5 & -6 \end{vmatrix} = -1(-6 + 20) = -14$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 4 & 3 \\ 9 & 0 \end{vmatrix} = 1(-27) = -27$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} = -1(-9) = 9$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 4 \\ 3 & 9 \end{vmatrix} = 1(9 - 12) = -3$$

Again,

$$\text{Adj. } A = \begin{bmatrix} 18 & -6 & 27 \\ -26 & 17 & -14 \\ -27 & 9 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 18 & -26 & -27 \\ -6 & 17 & 9 \\ 27 & -14 & 9 \\ 27 & -14 & -3 \end{bmatrix}$$

Now, we have

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{75} \begin{bmatrix} 18 & -26 & -27 \\ -6 & 17 & 9 \\ 27 & -14 & 9 \\ 27 & -14 & -3 \end{bmatrix} \begin{bmatrix} 6 \\ 18 \\ -5 \end{bmatrix}$$

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$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{75}\begin{bmatrix} 108 - 468 + 135 \\ -36 + 306 - 45 \\ 162 - 252 + 15 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{75}\begin{bmatrix} -225 \\ 225 \\ -75 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{75}\begin{bmatrix} -3 \\ 3 \\ -1 \end{bmatrix}$$

$\therefore x = -3, y = 3$ and $z = -1$

25. 2070 Set C Q.No. 8 a

Using row equivalent matrix method or inverse matrix method, solve the following equations. $x + 4y + z = 18$, $3x + 3y - 2z = 2$, $-4y + z = -7$ [4]

➥ Please refer to 2074 Supp Q.No. 8a

26. 2070 Set D Q.No. 8 a

Using row equivalent matrix method or inverse matrix method, solve the following equations. $9y - 5x = 3$, $x + z = 1$, $z + 2y = 2$. [4]

SOLUTION

Row equivalent matrix method

Please refer 2056 Q.No. 10 b [From 7A]

Inverse matrix method

Please refer to 2075 Set B Q.No. 8a

27. 2070 Old Q.No. 10 b Or

Using matrix inversion method solve: $x + 4y = 3$, $3x - 2y + 5 = 0$. [4]

SOLUTION

Given equations are $x + 4y = 3$, $3x - 2y = -5$

Writing the given equations in matrix form

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

which is in the form of $AX = B$

$\Rightarrow X = A^{-1}B$ ...(i)

For A^{-1}

$$|A| = \begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix} = 2 - 12 = -14 \neq 0$$

So, A^{-1} exists.

The cofactors are

$A_{11} = (-1)^{1+1}(-2) = -2$

$A_{12} = (-1)^{1+2}(3) = -3$

$A_{21} = (-1)^{2+1}(4) = -4$

$A_{22} = (-1)^{2+2}(1) = 1$

$$\therefore \text{Adj. } A = \begin{bmatrix} -2 & -3 \\ -4 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & -4 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot$$

$$\text{Adj. } A = \frac{1}{-14}\begin{bmatrix} -2 & -4 \\ -3 & 1 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$$

From (i), we get

$$X = \frac{1}{14}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 6 - 20 \\ 9 + 5 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} -14 \\ 14 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\therefore x = -1, y = 1$

28. 2069 Supp Q.No. 8 a

Using row equivalent or inverse matrix method, solve the following system of equations: $2x - y + 4z + 3 = 0$, $x - 4z = 5$, $6x - y + 2z = 10$. [4]

SOLUTIONS

Row Equivalent Matrix Method

The augmented matrix is

$$\begin{bmatrix} 2 & -1 & 4 & : & -3 \\ 1 & 0 & -4 & : & 5 \\ 6 & -1 & 2 & : & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 2 & -1 & 4 & : & -3 \\ 6 & -1 & 2 & : & 10 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & -1 & 12 & : & -13 \\ 0 & -1 & 26 & : & -20 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 6R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & -1 & 26 & : & -20 \end{bmatrix} \quad R_2 \to (-1)R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & -1 & 26 & : & -20 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & 0 & 1 & : & -1/2 \end{bmatrix} \quad R_3 \to \frac{1}{14}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 7 \\ 0 & 0 & 1 & : & -1/2 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 + 4R_3 \\ R_2 \to R_2 + 12R_3 \end{matrix}$$

$\therefore x = 3, y = 7, z = -\frac{1}{2}$

Inverse Matrix Method

Writing the given equations in matrix form

$$\begin{bmatrix} 2 & -1 & 4 \\ 1 & 0 & -4 \\ 6 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} -3 \\ 5 \\ 10 \end{bmatrix}$$

Which is in the form of $AX = B$

$\Rightarrow X = A^{-1}B$ ...(i)

For A^{-1}

$$|A| = \begin{vmatrix} 2 & -1 & 4 \\ 1 & 0 & -4 \\ 6 & -1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -4 \\ 6 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 0 \\ 6 & -1 \end{vmatrix}$$

$= 2(0 - 4) + 1(2 + 24) + 4(-1 - 0)$

$= -8 + 26 - 4 = 14 \neq 0$

So, A^{-1} exists.

The cofactors are

$$A_{11} = \begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} = 0 - 4 = -4$$

$$A_{12} = -\begin{vmatrix} 1 & -4 \\ 6 & 2 \end{vmatrix} = -(2 + 24) = -26$$

$$A_{13} = \begin{vmatrix} 1 & 0 \\ 6 & -1 \end{vmatrix} = -1 - 0 = -1$$

$$A_{21} = -\begin{vmatrix} -1 & 4 \\ -1 & 2 \end{vmatrix} = -(-2 + 4) = -2$$

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$A_{22} = \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} = 4 - 24 = -20$

$A_{23} = -\begin{vmatrix} 2 & -1 \\ 6 & -1 \end{vmatrix} = -(-2 + 6) = -4$

$A_{31} = \begin{vmatrix} -1 & 4 \\ 0 & -4 \end{vmatrix} = 4 - 0 = 4$

$A_{32} = -\begin{vmatrix} 2 & 4 \\ 1 & -4 \end{vmatrix} = -(-8 - 4) = 12$

$A_{33} = \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 0 + 1 = 1$

$$\text{Adj. } A = \begin{bmatrix} -4 & -26 & -1 \\ -2 & -20 & -4 \\ 4 & 12 & 1 \end{bmatrix}^T = \begin{bmatrix} -4 & -2 & 4 \\ -26 & -20 & 12 \\ -1 & 4 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A = \frac{1}{14}\begin{bmatrix} -4 & -2 & 4 \\ -26 & -20 & 12 \\ -1 & 4 & 1 \end{bmatrix}$$

Now, from (i), we get

$$X = A^{-1}B = \frac{1}{14}\begin{bmatrix} -4 & -2 & 4 \\ -26 & -20 & 12 \\ -1 & 4 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 5 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 12 - 10 + 40 \\ 78 - 100 + 120 \\ 3 - 20 + 10 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 42 \\ 98 \\ -7 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1/2 \end{bmatrix}$$

$\therefore \quad x = 3, y = 7, z = -\frac{1}{2}$

29. 2069 [Set A] Q. No. 8a

Using row equivalent matrix method or inverse matrix method, solve the following equations: $x - 2y - z = -7$; $2x + y + z = 0$; $3x - 5y + 8z = 13$ [4]

SOLUTION

Row Equivalent Matrix Method

The augmented matrix is

$$\begin{bmatrix} 1 & -2 & -1 & : & -7 \\ 2 & 1 & 1 & : & 0 \\ 3 & -5 & 8 & : & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -1 & : & -7 \\ 0 & 5 & 3 & : & 14 \\ 0 & 1 & 11 & : & 34 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -1 & : & -7 \\ 0 & 1 & 3/5 & : & 14/5 \\ 0 & 1 & 11 & : & 34 \end{bmatrix} R_2 \to \frac{1}{5}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1/5 & : & -7/5 \\ 0 & 1 & 3/5 & : & 14/5 \\ 0 & 0 & 52/5 & : & 156/5 \end{bmatrix} \begin{matrix} R_1 \to R_1 + 2R_2 \\ R_3 \to R_3 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1/5 & : & -7/5 \\ 0 & 1 & 3/5 & : & 14/5 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to \frac{5}{52}R_3$$

$$\text{or, } \begin{bmatrix} 1 & 0 & 0 & : & -2 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \begin{matrix} R_1 \to R_1 - \frac{1}{5}R_3 \\ R_2 \to R_2 - \frac{3}{5}R_3 \end{matrix}$$

$\therefore \quad x = -2, y = 1, z = 3$

Inverse Matrix Method

writing the given equations in matrix form,

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & 1 & 1 \\ 3 & -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 0 \\ 13 \end{bmatrix}$$

which is in the form of $AX = B$

$\Rightarrow \quad X = A^{-1}B \quad \ldots(i)$

For A^{-1}

$$|A| = \begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 1 \\ 3 & -5 & 8 \end{vmatrix}$$

$= 1\begin{vmatrix} 1 & 1 \\ -5 & 8 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ 3 & 8 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}$

$= 1(8 + 5) + 2(16 - 3) - 1(-10 - 3)$

$= 13 + 26 + 13 = 52 \neq 0$

So, A^{-1} exists.

The cofactors are

$A_{11} = \begin{vmatrix} 1 & 1 \\ -5 & 8 \end{vmatrix} = 8 + 5 = 13$

$A_{12} = -\begin{vmatrix} 2 & 1 \\ 3 & 8 \end{vmatrix} = -(16 - 3) = -13$

$A_{13} = \begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix} = -10 - 3 = -13$

$A_{21} = -\begin{vmatrix} -2 & -1 \\ -5 & 8 \end{vmatrix} = -(-16 - 5) = 23$

$A_{22} = \begin{vmatrix} 1 & -1 \\ 3 & 8 \end{vmatrix} = 8 + 3 = 11$

$A_{23} = -\begin{vmatrix} 1 & -2 \\ 3 & -5 \end{vmatrix} = -(5 + 6) = -1$

$A_{31} = \begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = -2 + 1 = -1$

$A_{32} = -\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -(1 + 2) = -3$

$A_{33} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 + 4 = 5$

$$\text{Adj. } A = \begin{bmatrix} 13 & -13 & -13 \\ 23 & 11 & -1 \\ -1 & -3 & 5 \end{bmatrix}^T = \begin{bmatrix} 13 & 23 & -1 \\ -13 & 11 & -3 \\ -13 & -1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj. } A = \frac{1}{52}\begin{bmatrix} 13 & 23 & -1 \\ -13 & 11 & -3 \\ -13 & -1 & 5 \end{bmatrix}$$

Now, from (i), we get

$$\text{or, } X = A^{-1}B = \frac{1}{52}\begin{bmatrix} 13 & 23 & -1 \\ -13 & 11 & -3 \\ -13 & -1 & 5 \end{bmatrix}\begin{bmatrix} -7 \\ 0 \\ 13 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{52}\begin{bmatrix} -91 + 0 - 13 \\ 91 + 0 - 39 \\ 91 + 0 + 65 \end{bmatrix} = \frac{1}{52}\begin{bmatrix} -104 \\ 52 \\ 156 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore \quad x = -2, y = 1, z = 3$

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30. 2069 Set B Q. No. 8a

Using row equivalent or inverse matrix method, solve the following system of equations: $x - y = 0$, $2x - y + 4z = 18$, $-3x + z + 2 = 0$. [4]

➥ Please refer to 2076 Set B Q.No. 8a

31. 2068 Q.No. 8a

Applying row equivalent matrix method or inverse matrix method, solve the following system of equations: $x + y + z = 1$; $x + 2y + 3z = 4$; $x + 3y + 7z = 13$ [4]

➥ Please refer to 2074 Set A Q.No. 8a

32. 2068 Old Q.No. 10b

Solve the following system of equations by row equivalent matrix method or inverse matrix method: $x - y - z = -2$; $5x + 10z = 20$; $10y - 20z = 10$ [4]

SOLUTION

Row–equivalent matrix method

The augmented matrix is:

$$\left[\begin{array}{ccc:c} 1 & -1 & -1 & -2 \\ 5 & 0 & 10 & 20 \\ 0 & 10 & -20 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & -1 & -2 \\ 0 & 5 & 15 & 30 \\ 0 & 10 & -20 & 10 \end{array}\right] \quad R_2 \to R_2 - 5R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -1 & -1 & -2 \\ 0 & 1 & 3 & 6 \\ 0 & 10 & -20 & 10 \end{array}\right] \quad R_2 \to \frac{1}{5}R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -50 & -50 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + R_2 \\ R_3 \to R_3 - 10R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_3 \to \left(-\frac{1}{50}\right) R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_3 \\ R_2 \to R_2 - 3R_3 \end{array}$$

$\therefore x = 2$, $y = 3$ and $z = 1$.

Inverse Matrix Method

Writing the given equations in matrix form

$$\begin{bmatrix} 1 & -1 & -1 \\ 5 & 0 & 10 \\ 0 & 10 & -20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 20 \\ 10 \end{bmatrix}$$

which is in the form of

$AX = B$

$\Rightarrow X = A^{-1}B$

Now, for A^{-1}

$$|A| = \begin{vmatrix} 1 & -1 & -1 \\ 5 & 0 & 10 \\ 0 & 10 & -20 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & 10 \\ 10 & -20 \end{vmatrix} - 5\begin{vmatrix} -1 & -1 \\ 10 & -20 \end{vmatrix}$$

[Expanding along C_1]

$$= 1(0 - 100) - 5(20 + 10)$$

$$= -100 - 150 = -250 \neq 0$$

So, A^{-1} exists.

The cofactors are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 10 \\ 10 & -20 \end{vmatrix} = 0 - 100 = -100$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 10 \\ 10 & -20 \end{vmatrix} = -(-100 - 0) = 100$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 0 \\ 0 & 10 \end{vmatrix} = 50 - 0 = 50$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & -1 \\ 0 & -20 \end{vmatrix} = -(20 + 10) = -30$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 0 & -20 \end{vmatrix} = -20 - 0 = -20$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 0 & 10 \end{vmatrix} = -(10 - 0) = -10$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & -1 \\ 0 & 10 \end{vmatrix} = -10 - 0 = -10$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 5 & 10 \end{vmatrix} = -(10 + 5) = -15$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} -1 & -1 \\ 5 & 0 \end{vmatrix} = 0 + 5 = 5$$

$$\text{Matrix of cofactors} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -100 & 100 & 50 \\ -30 & -20 & -10 \\ -10 & -15 & 5 \end{bmatrix}$$

$$\text{Adj. } A = \begin{bmatrix} -100 & 100 & 50 \\ -30 & -20 & -10 \\ -10 & -15 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -100 & -30 & -10 \\ 100 & -20 & -15 \\ 50 & -10 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ Adj. } A = \frac{1}{-250}\begin{bmatrix} -100 & -30 & -10 \\ 100 & -20 & -15 \\ 50 & -10 & 5 \end{bmatrix}$$

Now, from (i)

$X = A^{-1}B$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-250}\begin{bmatrix} -100 & -30 & -10 \\ 100 & -20 & -15 \\ 50 & -10 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ 20 \\ 10 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-250}\begin{bmatrix} 200 - 600 - 100 \\ -200 - 400 - 150 \\ -100 - 200 + 50 \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-250}\begin{bmatrix} -500 \\ -750 \\ -250 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$\therefore x = 2$, $y = 3$ and $z = 1$.

33. 2066 Q.No. 10b

Solve by row equivalent or inverse matrix method: $x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$ [4]

SOLUTION

Row equivalent matrix method

Please refer 2056 Q.No. 10 b [From 7A]

Inverse matrix method

Please refer to 2075 Set B Q.No. 8a

34. 2065 Q.No. 10b

Solve by matrix method: [4]

$x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1$

SOLUTION

Given linear equations are

$x + y + z = 6$ ... (i)

$x - y + z = 2$ ... (ii)

$2x + y - z = 1$ ... (iii)

The matrix equation of above system of linear equations is $AX = B$ where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

The solution is given by $X = A^{-1} \cdot B$

Now,

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$= 1(1-1) - 1(-1-2) + 1(1+2)$

$= 0 + 3 + 3 = 6 \neq 0$. A^{-1} exists.

Let A_{ij} denote the cofactor of a_{ij} then,

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = +(1-1) = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -(-1-2) = 3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = +(1+2) = 3$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = -(-1-1) = 2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = +(-1-2) = -3$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = +(1+1) = 2$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -(1-1) = 0$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = +(-1-1) = -2$$

$\therefore$ Matrix of cofactors of $A = \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

Again, $(\text{Adj. } A) = \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}^T$

$$= \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \times \text{Adj. } A$

Hence, $X = A^{-1} \cdot B$ becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{|A|} \times (\text{Adj. } A) \cdot B$$

$$= \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 4+2 \\ 18-6 \\ 18+2-2 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$; $y = 2$ and $z = 3$

35. 2060 Q.No. 10b OR

Solve by matrix inversion method:

$3x + 5y = 7$, $5x + 9y = 7$

SOLUTION

Given equations are

$3x + 5y = 7$

$5x + 9y = 7$

The matrix form of the above system of linear equati is

$AX = B$, where

$$A = \begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$$

The solution is given by $X = A^{-1} B$ ... (i)

Now, $|A| = \begin{vmatrix} 3 & 5 \\ 5 & 9 \end{vmatrix} = 27 - 25 = 2 \neq 0$

So, A^{-1} exists.

The co–factor elements of A are

$A_{11} = (-1)^{1+1}|9| = 9$

$A_{12} = (-1)^{1+2}|5| = -5$

$A_{21} = (-1)^{2+1}|5| = -5$

$A_{22} = (-1)^{2+2}|3| = 3$

$\therefore$ Matrix of cofactors of $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$$= \begin{bmatrix} 9 & -5 \\ -5 & 3 \end{bmatrix}$$

Now, $\text{Adj. } A = \begin{bmatrix} 9 & -5 \\ -5 & 3 \end{bmatrix}^T = \begin{bmatrix} 9 & -5 \\ -5 & 3 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{|A|}(\text{Adj. } A) = \frac{1}{2}\begin{bmatrix} 9 & -5 \\ -5 & 3 \end{bmatrix}$

Hence, from (1)

$X = A^{-1} B$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 9 & -5 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 7 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 63-35 \\ -35+21 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 28 \\ -14 \end{bmatrix} = \begin{bmatrix} 14 \\ -7 \end{bmatrix}$$

$\therefore$ $x = 14$, $y = -7$

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36. 2059 Q.No. 10b OR

Solve by using inverse matrix method [4]

$2x + 4y = 7$

$8x - 6y = -5$

SOLUTION

The given linear equations are

$2x + 4y = 7$

$8x - 6y = -5$

The matrix form of above system of linear equations is $AX = B$, where

$$A = \begin{bmatrix} 2 & 4 \\ 8 & -6 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

The solution is given by $X = A^{-1}B$

Now, $|A| = \begin{vmatrix} 2 & 4 \\ 8 & -6 \end{vmatrix}$

$= (-12 - 32) = -44 \neq 0$. So A^{-1} exists.

The cofactor elements of matrix A are as follows:

$A_{11} = (-1)^{1+1}|-6| = -6$

$A_{12} = (-1)^{1+2}|8| = -8$

$A_{21} = (-1)^{2+1}|4| = -4$

$A_{22} = (-1)^{2+2}|2| = 2$

$\therefore$ Matrix of cofactors of $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$= \begin{bmatrix} -6 & -8 \\ -4 & 2 \end{bmatrix}$

Now, (Adj. A) $= \begin{bmatrix} -6 & -8 \\ -4 & 2 \end{bmatrix}^T = \begin{bmatrix} -6 & -4 \\ -8 & 2 \end{bmatrix}$

We know that,

$A^{-1} = \frac{1}{|A|}$ (Adj. A)

Hence, $X = A^{-1}B$ becomes

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-44}\begin{bmatrix} -6 & -4 \\ -8 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

$$= \frac{-1}{44}\begin{bmatrix} -42 + 20 \\ -56 - 10 \end{bmatrix}$$

$$= \frac{-1}{44}\begin{bmatrix} -22 \\ -66 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 3/2 \end{bmatrix}$$

Hence, $x = \frac{1}{2}$; $y = \frac{3}{2}$

37. 2058 Q.No. 10b OR

Solve by inverse matrix method [4]

$2x + 5y = 7$, $5x + 2y = -3$

SOLUTION

The matrix form of the above system of linear equations, is $AX = B$, where,

$$A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

The solution is given by $X = A^{-1}B$

Now, $|A| = \begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix} = 4 - 25 = -21 \neq 0$

So A^{-1} exists

The cofactor elements of matrix A are as follows:

$A_{11} = (-1)^{1+1}|2| = 2$

$A_{12} = (-1)^{1+2}|5| = -5$

$A_{21} = (-1)^{2+1}|5| = -5$

$A_{22} = (-1)^{2+2}|2| = 2$

$\therefore$ Matrix of cofactors of $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$

$= \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}$

Now, Adj. $A = \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}^T = \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}$

We know that,

$A^{-1} = \frac{1}{|A|} \times$ (Adj. A)

Hence $X = A^{-1}B$ becomes

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-21} \times \begin{pmatrix} 2 & -5 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$= \frac{-1}{21}\begin{pmatrix} 14 + 15 \\ -35 - 6 \end{pmatrix}$$

$$= \frac{-1}{21}\begin{pmatrix} 29 \\ -41 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-29}{21} \\ \frac{41}{21} \end{pmatrix}$$

$\therefore$ $x = \frac{-29}{21}$, $y = \frac{41}{21}$

38. 2057 Q.No. 10b OR

Solve by inverse matrix method:

$-2x + 4y = 3$; $3x - 7y = 1$ [4]

SOLUTION

The given linear equations are

$-2x + 4y = 3$

$3x - 7y = 1$

The matrix form of the above systems of linear equations is $AX = B$ where

$$A = \begin{pmatrix} -2 & 4 \\ 3 & -7 \end{pmatrix}; X = \begin{pmatrix} x \\ y \end{pmatrix}; B = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

The solution is given by $X = A^{-1}B$ ... (i)

Here, $|A| = \begin{vmatrix} -2 & 4 \\ 3 & -7 \end{vmatrix}$

$= 14 - 12 = 2 \neq 0$. So, A^{-1} exists

The cofactor elements of A are as follows:

$A_{11} = (-1)^{1+1}|-7| = -7$

$A_{12} = (-1)^{1+2}|3| = -3$

$A_{21} = (-1)^{2+1}|4| = -4$

$A_{22} = (-1)^{2+2}|-2| = -2$

$\therefore$ Matrix of cofactors of $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$

$= \begin{pmatrix} -7 & -3 \\ -4 & -2 \end{pmatrix}$

Now, (Adj. A) $= \begin{pmatrix} -7 & -3 \\ -4 & -2 \end{pmatrix}^T = \begin{pmatrix} -7 & -4 \\ -3 & -2 \end{pmatrix}$

We know that,

$A^{-1} = \frac{1}{|A|}$ (Adj. A) $= \frac{1}{2}\begin{pmatrix} -7 & -4 \\ -3 & -2 \end{pmatrix}$

$\therefore$ $X = A^{-1}B$ becomes

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -7 & -4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -21-4 \\ -9-2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -25 \\ -11 \end{pmatrix} = \begin{pmatrix} -25/2 \\ -11/2 \end{pmatrix}$$

$\therefore x = \frac{-25}{2}$ and $y = \frac{-11}{2}$

39. 2056 Q.No. 10b OR

Solve by using inverse matrix method:

$x - y = 2$; $2x + 3y = 9$ **[4]**

SOLUTION

The given linear equations are:

$x - y = 2$ ... (i)

$2x + 3y = 9$ ... (ii)

The matrix form of the given system of linear equations is $AX = B$ where,

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

The solution is given by $X = A^{-1}B$ ... (iii)

Now, $|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$.

So, A^{-1} exists.

The cofactor elements of matrix A are

$A_{11} = (-1)^{1+1}|3| = 3$

$A_{12} = (-1)^{1+2}|2| = -2$

$A_{21} = (-1)^{2+1}|-1| = -(-1) = 1$

$A_{22} = (-1)^{2+2}|1| = 1$

$\therefore$ Matrix of cofactors of $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$$= \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$$

Again, $(\text{Adj. } A) = (\text{Cof. } A)^T = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(\text{Adj. } A)$

Hence, from equation (iii)

$$X = A^{-1}B$$

$$= \frac{1}{5} \times \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$= \frac{1}{5} \times \begin{bmatrix} 6+9 \\ -4+9 \end{bmatrix}$$

$$= \frac{1}{5} \times \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

or, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\therefore x = 3; y = 1$

40. 2055 Old Model Q.No. 10b OR

Solve by using inverse matrix method:

$-2x + 4y = 3$, $3x + 7y = 1$. **[4]**

SOLUTION

In matrix form the given equations can be written as

$$\begin{pmatrix} -2 & 4 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

which is in the form of $AX = B$, where,

$$A = \begin{pmatrix} -2 & 4 \\ 3 & 7 \end{pmatrix}; X = \begin{pmatrix} x \\ y \end{pmatrix}; B = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

The solution is given by

$X = A^{-1}B$ ... (i)

Now, $|A| = \begin{vmatrix} -2 & 4 \\ 3 & 7 \end{vmatrix} = -14 - 12 = -26 \neq 0$.

So, A^{-1} exists

The cofactor elements of A are

$A_{11} = (-1)^{1+1}|7| = 7$

$A_{12} = (-1)^{1+2}|3| = -3$

$A_{21} = (-1)^{2+1}|4| = -4$

$A_{22} = (-1)^{2+2}|-2| = -2$

$\therefore$ Matrix of cofactors of $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$

$$= \begin{pmatrix} 7 & -3 \\ -4 & -2 \end{pmatrix}$$

$$\text{Now, (Adj. } A) = \begin{pmatrix} 7 & -3 \\ -4 & -2 \end{pmatrix}^T = \begin{pmatrix} 7 & -4 \\ -3 & -2 \end{pmatrix}$$

We know that,

$$A^{-1} = \frac{1}{|A|}(\text{Adj. } A)$$

$$= -\frac{1}{26}\begin{pmatrix} 7 & -4 \\ -3 & -2 \end{pmatrix}$$

Hence, $X = A^{-1}B$ becomes

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{26}\begin{pmatrix} 7 & -4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= -\frac{1}{26}\begin{pmatrix} 7 \times 3 - 4 \times 1 \\ -3 \times 3 - 2 \times 1 \end{pmatrix}$$

$$= -\frac{1}{26}\begin{pmatrix} 21-4 \\ -9-2 \end{pmatrix}$$

$$= -\frac{1}{26}\begin{pmatrix} 17 \\ -11 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-17}{26} \\ \frac{11}{26} \end{pmatrix}$$

$\therefore x = \frac{-17}{26}$; $y = \frac{11}{26}$

C. CRAMER'S RULE

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 3a

Using Cramer's rule, solve: $3x + 2y + 9 = 0$, $2x - 3y + 6 = 0$. **[2]**

SOLUTION

Given equations are

$3x + 2y = -9$

$2x - 3y = -6$

Coeff. of x	Coeff. of y	Constant term
3	2	-9
2	-3	-6

$$D = \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -9 - 4 = -13$$

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$$D_1 = \begin{vmatrix} -9 & 2 \\ -6 & -3 \end{vmatrix} = 27 + 12 = 39$$

$$D_2 = \begin{vmatrix} 3 & -9 \\ 2 & -6 \end{vmatrix} = -18 + 18 = 0$$

Using Cramer's rule, we have

$$x = \frac{D_1}{D} = \frac{39}{-13} = -3$$

$$y = \frac{D_2}{D} = \frac{0}{-13} = 0$$

2. 2076 Set C Q.No. 3a

Solve by Cramer's rule:

$2x + y = 7$, $x + 3y = 11$. **[2]**

SOLUTION

Coeff. of x	Coeff. of y	Constant term
2	1	7
1	3	11

Now,

$$D = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5 \neq 0$$

$$D_1 = \begin{vmatrix} 7 & 1 \\ 11 & 3 \end{vmatrix} = 21 - 11 = 10$$

$$D_2 = \begin{vmatrix} 2 & 7 \\ 1 & 11 \end{vmatrix} = 22 - 7 = 15$$

Using Cramer's Rule, we have,

$$x = \frac{D_1}{D} = \frac{10}{5} = 2$$

$$y = \frac{D_2}{D} = \frac{15}{5} = 3$$

3. 2075 GIE Q. No. 3a

Solve by Cramer's rule: $x + y = 5$, $6x - 4y = 0$. **[2]**

SOLUTION

Coeff. of x	Coeff. of y	Constant term
1	1	5
6	−4	0

$$D = \begin{vmatrix} 1 & 1 \\ 6 & -4 \end{vmatrix} = -4 - 6 = -10 \neq 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 0 & -4 \end{vmatrix} = -20 - 0 = -20$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} = 0 - 30 = -30$$

Using Cramer's Rule, we have,

$$x = \frac{D_1}{D} = \frac{-20}{-10} = 2$$

$$y = \frac{D_2}{D} = \frac{-30}{-10} = 3$$

4. 2075 Set A Q.No. 3a

Determine consistency and independence of the system of linear equations. **[2]**

$3x - 2y = 3$, $-4y + 6x = 9$.

SOLUTION

Given lines are

$3x - 2y = 3$ ... (i)

$6x - 4y = 9$ ... (ii)

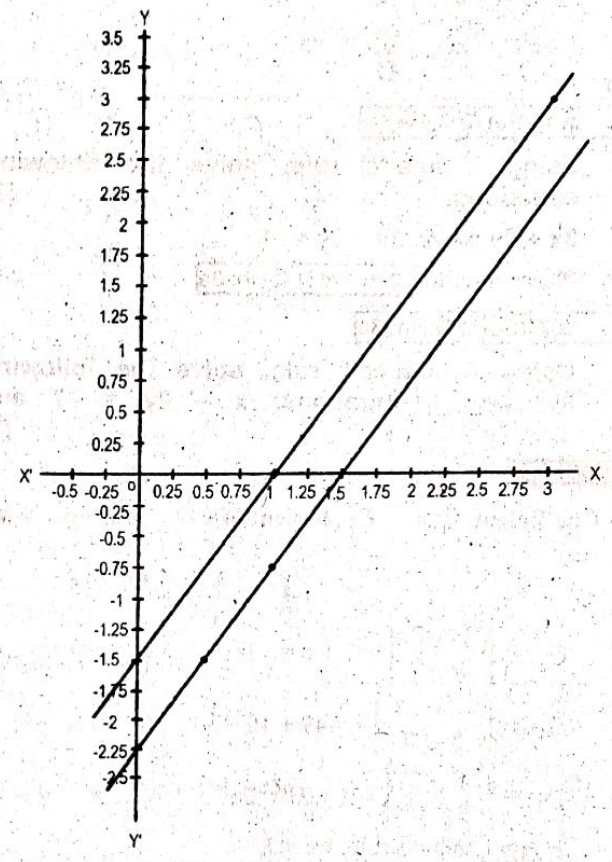
From (i) $3x - 2y = 3$

x	1	3
y	0	3

From (ii)

x	0.5	1
y	-1.5	-0.75

Now,



From above graph, we see that the lines are parallel.

Hence the system is inconsistent and independent.

Alternative method

Here, $a_1 = 3$, $b_1 = -2$, $c_1 = 3$, $a_2 = 6$, $b_2 = -4$, $c_2 = 9$

Now, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{3}{9} = \frac{1}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

This shows that the lines are parallel (but not coincident). So the system of equations is inconsistent and independent.

5. 2075 Set B Q.No. 3a

Solve by Cramer's rule:

$3x + 4y = 2$, $5x - 7y = 24$. **[2]**

SOLUTION

Coeff. of x	Coeff. of y	Constant term
3	4	2
5	−7	24

$$D = \begin{vmatrix} 3 & 4 \\ 5 & -7 \end{vmatrix} = 3 \times (-7) - 4 \times 5 = -41$$

$$D_1 = \begin{vmatrix} 2 & 4 \\ 24 & -7 \end{vmatrix} = 2 \times (-7) - 4 \times 24 = -110$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 5 & 24 \end{vmatrix} = 3 \times 24 - 2 \times 5 = 62$$

Using cramer's Rule, we have

$$x = \frac{D_1}{D} = \frac{-110}{-41} = \frac{110}{41}$$

$$y = \frac{D_2}{D} = \frac{62}{-41} = -\frac{62}{41}$$

6. 2075 Set C Q.No. 3a

Using Cramer's rule, solve the following equations. [2]

$3x + 2y = -9$, $2x - 3y = -6$.

Please refer to 2076 Set B Q.No. 3a

7. 2074 Supp Q.No. 3a

Using Crammer's rule, solve the following system of equations: $x - 2y = -7$ and $3x + 7y = 5$. [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant form
1	−2	−7
3	7	5

$$D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix} = 7 + 6 = 13$$

$$D_1 = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix} = -49 + 10 = -39$$

$$D_2 = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix} = 5 + 21 = 26$$

Using Cramer's rule, we get

$$x = \frac{D_1}{D} = \frac{-39}{13} = -3$$

$$y = \frac{D_2}{D} = \frac{26}{13} = 2$$

$\therefore x = -3$ and $y = 2$

8. 2074 Set A Q.No. 3a

Using Cramer's rule, solve the following equations: [2]

$2x + 5y = 17$, $5x - 2y = -1$.

SOLUTION

Coeff. of x	Coeff. of y	Constant term
2	5	17
5	−2	−1

$$D = \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix} = -4 - 25 = -29$$

$$D_1 = \begin{vmatrix} 17 & 5 \\ -1 & -2 \end{vmatrix} = -34 + 5 = -29$$

$$D_2 = \begin{vmatrix} 2 & 17 \\ 5 & -1 \end{vmatrix} = -2 - 85 = -87$$

Using Cramer's rule, we have

$$x = \frac{D1}{D} = \frac{-29}{-29} = 1$$

$$y = \frac{D_2}{D} = \frac{-87}{-29} = 3$$

$\therefore x = 1, y = 3$

9. 2074 Set B Q.No. 3a

Solve by Cramer's rule: $x + y = 5$ and $6x - 4y = 0$. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constant term
1	1	5
6	−4	0

$$D = \begin{vmatrix} 1 & 1 \\ 6 & -4 \end{vmatrix} = -4 - 6 = -10$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 0 & -4 \end{vmatrix} = -20 - 0 = -20$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} = 0 - 30 = -30$$

Using Cramer's rule, we have, $x = \frac{D_1}{D} = \frac{-20}{-10} = 2$

$$y = \frac{D_2}{D} = \frac{-30}{-10} = 3$$

$\therefore x = 2, y = 3$

10. 2073 Set C Q.No. 3a

Use Cramer's rule to solve $3x + 2y = 8$ and $4x + y = 9$. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constant term
3	2	8
4	1	9

$$D = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3 - 8 = -5$$

$$D_1 = \begin{vmatrix} 8 & 2 \\ 9 & 1 \end{vmatrix} = 8 - 18 = -10$$

$$D_2 = \begin{vmatrix} 3 & 8 \\ 4 & 9 \end{vmatrix} = 27 - 32 = -5$$

Using Cramer's Rule, we have,

$$x = \frac{D_1}{D} = \frac{-10}{-5} = 2$$

$$y = \frac{D_2}{D} = \frac{-5}{-5} = 1$$

Hence, the solution is (2, 1).

11. 2073 Set D Q.No. 3a

Solve by Cramer's rule: $2x + 3y = 5$ and $x + 7y = 8$. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constant term
2	3	5
1	7	8

$$D = \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix} = 14 - 3 = 11$$

$$D_1 = \begin{vmatrix} 5 & 3 \\ 8 & 7 \end{vmatrix} = 35 - 24 = 11$$

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$D_2 = \begin{vmatrix} 2 & 5 \\ 1 & 8 \end{vmatrix} = 16 - 5 = 11$

By Cramer's rule, we have,

$x = \frac{D_1}{D} = \frac{11}{11} = 1$

$y = \frac{D_2}{D} = \frac{11}{11} = 1$

$\therefore x = 1$ and $y = 1$

12. 2072 Supp. Q.No. 3a

Using Cramer's rule, solve the following equations: $3x + 4y = -2$, $5x - 7y = 24$ [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant term
3	4	−2
5	−7	24

$D = \begin{vmatrix} 3 & 4 \\ 5 & -7 \end{vmatrix} = -21 - 20 = -41$

$D_1 = \begin{vmatrix} -2 & 4 \\ 24 & -7 \end{vmatrix} = 14 - 96 = -82$

$D_2 = \begin{vmatrix} 3 & -2 \\ 5 & 24 \end{vmatrix} = 72 + 10 = 82$

By Cramer's rule, we have

$x = \frac{D_1}{D} = \frac{-82}{-41} = 2$

$y = \frac{D_2}{D} = \frac{82}{-41} = -2$

$\therefore x = 2$ and $y = -2$

13. 2072 Set C Q.No. 3a

Solve by Cramer's rule, the following equations: $3x + 4y = -2$; $5x - 7y = 24$ [2]

Please refer to 2072 Supp. Q.No. 3a

14. 2072 Set D Q.No. 3a

Using Cramer's rule, solve: [2]

$5x - 3y = 20$; $2x + 5y = 8$

SOLUTION

Coefficient of x	Coefficient of y	Constant form
5	−3	20
2	5	8

$D = \begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix} = 25 + 6 = 31$

$D_1 = \begin{vmatrix} 20 & -3 \\ 8 & 5 \end{vmatrix} = 100 + 24 = 124$

$D_2 = \begin{vmatrix} 5 & 20 \\ 2 & 8 \end{vmatrix} = 40 - 40 = 0$

By Cramer's rule,

$x = \frac{D_1}{D} = \frac{124}{31} = 4$

$y = \frac{D_2}{D} = \frac{0}{31} = 0$

$x = 4$ and $y = 0$

15. 2072 Set E Q.No. 3a

Solve by Cramer's rule: $4x + 3y + 4 = 0$, $6x + 5y + 7 = 0$. [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant term
4	3	−4
6	5	−7

$D = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} = 20 - 18 = 2$

$D_1 = \begin{vmatrix} -4 & 3 \\ -7 & 5 \end{vmatrix} -20 + 21 = 1$

$D_2 = \begin{vmatrix} 4 & -4 \\ 6 & -7 \end{vmatrix} = -28 + 24 = -4$

By Cramer's rule,

$x = \frac{D_1}{D} = \frac{1}{2}$

$y = \frac{D_2}{D} = \frac{-4}{2} = -2$

$\therefore x = \frac{1}{2}$ and $y = -2$

16. 2071 Supp. Q.No. 3a

Using Cramer's rule, solve:

$\frac{4}{x} + \frac{6}{y} = 0, \frac{3}{x} - \frac{4}{y} = \frac{-17}{6}$ [2]

SOLUTION

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

Then, $4a + 6b = 0$ and $3a - 4b = \frac{-17}{6}$

Coefficient of a	Coefficient of b	Constant term
4	6	0
3	−4	−17/6

$D = \begin{vmatrix} 4 & 6 \\ 3 & -4 \end{vmatrix} = -16 - 18 = -34$

$D_1 = \begin{vmatrix} 0 & 6 \\ -17/6 & -4 \end{vmatrix} = \frac{17}{6} \cdot 6 = 17$

$D_2 = \begin{vmatrix} 4 & 3 \\ 0 & -17/6 \end{vmatrix} = 4 \cdot \frac{-17}{6} = \frac{-34}{3}$

Using Cramer's rule, we have

$a = \frac{D_1}{D} = \frac{17}{-34} = \frac{-1}{2}$

$b = \frac{D_2}{D} = \frac{-34}{3 \cdot 34} = \frac{1}{3}$

Now, $\frac{1}{x} = a = \frac{-1}{2}$

or, $x = -2$

Again, $\frac{1}{y} = b = \frac{1}{3}$

or, $y = 3$

$\therefore x = -2, y = 3$

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17. 2071 Old Q.No. 3b

Solve by using Cramer's Rule $x - y = 2$; $2x + 3y = 9$ [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant form
1	−1	2
2	3	9

Now,

$D = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$

$D_1 = \begin{vmatrix} 2 & -1 \\ 9 & 3 \end{vmatrix} = 6 + 9 = 15$

$D_2 = \begin{vmatrix} 1 & 2 \\ 2 & 9 \end{vmatrix} = 9 - 4 = 5$

By Cramer's rule

$x = \frac{D_1}{D} = \frac{15}{5} = 3$

$y = \frac{D_2}{D} = \frac{5}{5} = 1$

$\therefore$ $x = 3$ and $y = 1$

18. 2070 Supp Q.No. 3a

Using Cramer's rule, solve the following equations: $5x - 2y = -2$, $2x + 5y = -24$ [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant form
5	−2	−2
2	5	−24

$D = \begin{vmatrix} 5 & -2 \\ 2 & 5 \end{vmatrix} = 25 + 4 = 29$

$D_1 = \begin{vmatrix} -2 & -2 \\ -24 & 5 \end{vmatrix} = -10 - 48 = -58$

$D_2 = \begin{vmatrix} 5 & -2 \\ 2 & -24 \end{vmatrix} = -120 + 4 = -116$

By Cramer's rule,

$x = \frac{D_1}{D} = \frac{-58}{29} = -2$

$Y = \frac{D_2}{D} = \frac{-116}{29} = -4$

$\therefore$ $x = -2$ and $y = -4$

19. 2070 Set C Q.No. 3 a

Using Cramer's rule, solve the following equations: $3x - 2y = 8$, $5x + 3y = 7$. [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant form
3	−2	8
5	3	7

$D = \begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix} = 9 + 10 = 19$

$D_1 = \begin{vmatrix} 8 & -2 \\ 7 & 3 \end{vmatrix} = 24 + 14 = 38$

$D_2 = \begin{vmatrix} 3 & 8 \\ 5 & 7 \end{vmatrix} = 21 - 40 = -19$

By Cramer's rule,

$x = \frac{D_1}{D} = \frac{38}{19} = 2$

$Y = \frac{D_2}{D} = \frac{-19}{19} = -1$

$\therefore$ $x = 2$ and $y = -1$

20. 2070 Set D Q.No. 3 a

Applying Cramer's rule, solve the following equations:

$3x + \frac{4}{y} = 10, -2x + \frac{3}{y} = -1.$ [2]

SOLUTION

Let $\frac{1}{y} = a$. Then, given equations can bee written as

$3x + 4a = 10$

and $-2x + 3a = -1$

Now,

Coeff. of x	Coeff. of a	Constant term
3	4	10
−2	3	−1

$D = \begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix} = 9 + 8 = 17$

$D_1 = \begin{vmatrix} 10 & 4 \\ -1 & 3 \end{vmatrix} = 30 + 4 = 34$

$D_2 = \begin{vmatrix} 3 & 10 \\ -2 & -1 \end{vmatrix} = -3 + 20 = 17$

Using Cramer's rule, we have

$x = \frac{D_1}{D} = \frac{34}{17} = 2$

and $a = \frac{D_2}{D} = \frac{17}{17} = 1$

$\frac{1}{y} = 1$

$\therefore$ $x = 2, y = 1$

21. 2070 Old Q.No. 3 b

Solve by Cramer's rule $x + y = 3$, $x - y = 1$. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constant term
1	1	3
1	−1	1

$D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$

$D_1 = \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -3 - 1 = -4$

$D_2 = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 1 - 3 = -2$

Using Cramer's rule, we have

$x = \frac{D_1}{D} = \frac{-4}{-2} = 2$

$y = \frac{D_2}{D} = \frac{-2}{-2} = 1$

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22. 2069 Supp Q.No. 3a

Use Cramer's rule to solve the system of equations $2x+3y=2$, $3x-2y=5$. [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant form
2	3	2
3	−2	5

$$D = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4 - 9 = -13$$

$$D_1 = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$$

$$D_2 = \begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} = 10 - 6 = 4$$

By Cramer's rule,

$$x = \frac{D_1}{D} = \frac{-19}{-13} = \frac{19}{13}$$

$$y = \frac{D_2}{D} = \frac{4}{-13} = -\frac{4}{13}$$

$\therefore x = \frac{19}{13}$ and $y = \frac{4}{13}$

23. 2069 [Set A] Q. No. 3a

Using Cramer's rule, solve the following equations: [2]

$x - 2y = -7$; $3x + 7y = 5$

➥ Please refer to 2074 Supp Q.No. 3a

24. 2069 [Set B] Q. No. 3a

Solve by Cramer's rule: $3x + 2y + 9 = 0$, $2x - 3y + 6 = 0$. [2]

➥ Please refer to 2076 Set B Q.No. 3a

25. 2068 Old Q.No. 3b

Using Cramer's rule, solve the following system: $3x + 4y = -2$; $5x - 7y = 24$ [2]

SOLUTION

Coeff. of x	Coeff. of y	Constant term
3	4	−2
5	−7	24

$$D = \begin{vmatrix} 3 & 4 \\ 5 & -7 \end{vmatrix} = -21 - 20 = -41$$

$$D_1 = \begin{vmatrix} -2 & 4 \\ 24 & -7 \end{vmatrix} = 14 - 96 = -82$$

$$D_2 = \begin{vmatrix} 3 & -2 \\ 5 & -24 \end{vmatrix} = 72 + 10 = 82$$

Using Cramer's rule, we have

$$x = \frac{D_1}{D} = \frac{-82}{-41} = 2$$

$$y = \frac{D_2}{D} = \frac{82}{-41} = -2$$

$\therefore x = 2$ and $y = -2$.

26. 2068 Q.No. 3a

Using Cramer's rule, solve the system of equations: [2]

$2x + 5y = 17$; $5x - 2y = -1$

➥ Please refer to 2074 Set A Q.No. 3a

27. 2066 Q.No. 3b

Solve by Cramer's rule: $-x + y = 9$, $x - 3y = 5$. [2]

SOLUTION

Coefficient of x	Coefficient of y	Constant
−1	1	9
1	−3	5

$$D = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$$

$$D_1 = \begin{vmatrix} 9 & 5 \\ 1 & -3 \end{vmatrix} = -27 - 5 = -32$$

$$D_2 = \begin{vmatrix} -1 & 1 \\ 9 & 5 \end{vmatrix} = -5 - 9 = -14$$

By Cramer's rule,

$$x = \frac{D_1}{D} = \frac{-32}{2} = -16$$

$$y = \frac{D_2}{D} = \frac{-14}{2} = -7$$

$\therefore x = -16$

$y = -7$

28. 2065 Q.No. 3b

Solve by Crammer's rule: $3x + 2y = 8$; $4x + y = 9$ [2]

➥ Please refer to 2073 Set C Q.No. 3a

29. 2060 Q.No. 3b

Give reason why simultaneous equation $x + 2y = 5$ and $3x + 6y = 12$ are not solvable by Cramer's rule. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constants
1	2	5
3	6	12

Now, $D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$

Since $D = 0$, the given system of linear equation are not solvable by Cramer's rule.

Moreover, $D_1 = \begin{vmatrix} 5 & 2 \\ 12 & 6 \end{vmatrix} = 30 - 24 = 6 \neq 0$

Since $D = 0$ and $D_1 \neq 0$, given system of linear equations is inconsistent.

30. 2059 Q.No. 3b

Solve by Cramer's rule $2x - y = 5$; $x - 2y = 1$ [2]

SOLUTION

The given linear equations are

$2x - y = 5$

$x - 2y = 1$

Coeff. of x	Coeff. of y	Constants
2	−1	5
1	−2	1

Now, $D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = -4 + 1 = -3$

$$D_1 = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = -10 + 1 = -9$$

$$D_2 = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2 - 5 = -3$$

By Cramer's rule, we get

$$x = \frac{D_1}{D} = \frac{-9}{-3} = 3$$

$$y = \frac{D_2}{D} = \frac{-3}{-3} = 1$$

Hence, $x = 3, y = 1$.

31. 2057 Q.No. 3b

Solve by Cramer's rule: $2x - y = 5$; $x - 2y = 1$. [2]

SOLUTION

Coeff. of x	Coeff. of y	Constants
2	−1	5
1	−2	1

Now, $D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = -4 + 1 = -3$

$$D_1 = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = -10 + 1 = -9$$

$$D_2 = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2 - 5 = -3$$

Since $D = -3 \neq 0$, there is unique solution. So by Cramer's rule

$$x = \frac{D_1}{D} = \frac{-9}{-3} = 3$$

and $y = \frac{D_2}{D} = \frac{-3}{-3} = 1$

$\therefore\ x = 3, y = 1$.

32. 2058 Q.No. 3b

Solve by Cramer's rule: [2]

$x - 2y = -7$, $3x + 7y = 5$

SOLUTION

Coeff. of x	Coeff. of y	Constants
1	−2	−7
3	7	5

$$D = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix} = 7 + 6 = 13$$

$$D_1 = \begin{vmatrix} -7 & -2 \\ 5 & 7 \end{vmatrix} = -49 + 10 = -39$$

$$D_2 = \begin{vmatrix} 1 & -7 \\ 3 & 5 \end{vmatrix} = 5 + 21 = 26$$

Using Cramer's rule, we get

$$x = \frac{D_1}{D} = \frac{-39}{13} = -3;$$

$$y = \frac{D_2}{D} = \frac{26}{13} = 2$$

33. 2056 Q.No. 3b

Solve by Cramer's rule: $-x + y = 9$, $x - 3y = 5$ [2]

SOLUTION

Coefficient of x	Coefficient of y	Constants
−1	1	9
1	−3	5

Now, $D = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$

$$D_1 = \begin{vmatrix} 9 & 1 \\ 5 & -3 \end{vmatrix} = -27 - 5 = -32$$

$$D_2 = \begin{vmatrix} -1 & 9 \\ 1 & 5 \end{vmatrix} = -5 - 9 = -14$$

Using Cramer's rule, we have

$$x = \frac{D_1}{D} = \frac{-32}{2} = -16$$

$$y = \frac{D_2}{D} = \frac{-14}{2} = -7$$

Hence, $x = -16$; $y = -7$

and $\frac{1}{y} = b = \frac{1}{3}$

$\therefore\ y = 3$.

34. 2055 Old Model Q.No. 3b

Solve by Cramer's rule:

$-x + y = 9$, $x - 3y = 5$ [2]

➥ Please refer to 2056 Q.No. 3b

4 MARKS QUESTIONS

35. 2073 Supp Q.No. 8a

Using row equivalent matrix method or the Cramer's rule, solve the following system of equations: [4]

$x - 2y - z = -7$, $2x + y + z = 0$, $3x - 5y + 8z = 13$.

SOLUTION

Row Equivalent Matrix Method

Please refer to 2069 Set A Q.No. 8a Chapter 7B

Cramer's Rule

Coeff. of x	Coeff. of y	Coeff. of z	Constant term
1	−2	−1	−7
2	1	1	0
3	−5	8	13

Now, $D = \begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 1 \\ 3 & -5 & 8 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & 1 \\ -5 & 8 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ 3 & 8 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}$

$= 1(8 + 5) + 2(16 - 3) - 1(-10 - 3)$

$= 13 + 26 + 13 = 52$

$D_1 = \begin{vmatrix} -7 & -2 & -1 \\ 0 & 1 & 1 \\ 13 & -5 & 8 \end{vmatrix}$

$= -7\begin{vmatrix} 1 & 1 \\ -5 & 8 \end{vmatrix} - (-2)\begin{vmatrix} 0 & 1 \\ 13 & 8 \end{vmatrix} + (-1)\begin{vmatrix} 0 & 1 \\ 13 & -5 \end{vmatrix}$

$= -7(8 + 5) + 2(0 - 13) - 1(0 - 13)$

$= -91 - 26 + 13 = -104$

$D_2 = \begin{vmatrix} 1 & -7 & -1 \\ 2 & 0 & 1 \\ 3 & 13 & 8 \end{vmatrix}$

$= 1\begin{vmatrix} 0 & 1 \\ 13 & 8 \end{vmatrix} - (-7)\begin{vmatrix} 2 & 1 \\ 3 & 8 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 0 \\ 3 & 13 \end{vmatrix}$

$= 1(0 - 13) + 7(16 - 3) - 1(26 - 0)$

$= -13 + 91 - 26$

$= 52$

$D_3 = \begin{vmatrix} 1 & -2 & -7 \\ 2 & 1 & 0 \\ 3 & -5 & 13 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & 0 \\ -5 & 13 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 0 \\ 3 & 13 \end{vmatrix} + (-7)\begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}$

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$= 1(13-0) + 2(26-0) - 7(-10-3)$
$= 13 + 52 + 91$
$= 156$

By cramer's rule,

$x = \frac{D_1}{D} = \frac{-104}{52} = -2$

$y = \frac{D_2}{D} = \frac{52}{52} = 1$

$z = \frac{D_3}{D} = \frac{156}{52} = 3.$

36. 2071 Set D Q.No. 8a

Using Cramer's rule or row–equivalent matrix method solve the following system:

$x + y + z = 6$, $x - y + z = 2$, $5x - 9y = -3$. **[4]**

SOLUTION

Cramer's rule

Coeff. of x	Coeff. of y	Coeff. of z	Constant term
1	1	1	6
1	−1	1	2
5	−9	0	−3

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 5 & -9 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ -9 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 5 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 5 & -9 \end{vmatrix}$$

$= 1(0+9) - 1(0-5) + 1(-9+5)$
$= 9 + 5 - 4 = 10$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ -3 & -9 & 0 \end{vmatrix}$$

$$= 6\begin{vmatrix} -1 & 1 \\ -9 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -3 & -9 \end{vmatrix}$$

$= 6(0+9) - 1(0+3) + 1(-18-3)$
$= 54 - 3 - 21 = 30$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 5 & -3 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix} - 6\begin{vmatrix} 1 & 1 \\ 5 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 5 & -3 \end{vmatrix}$$

$= 1(0+3) - 6(0-5) + 1(-3-10)$
$= 3 + 30 - 13 = 20$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 5 & -9 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 2 \\ -9 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 5 & -3 \end{vmatrix} + 6\begin{vmatrix} 1 & -1 \\ 5 & -9 \end{vmatrix}$$

$= 1(3+18) - 1(-3-10) + 6(-6+5)$
$= 21 + 13 - 24 = 10$

Using Cramer's rule, we have

$x = \frac{D_1}{D} = \frac{30}{10} = 3$

$y = \frac{D_2}{D} = \frac{20}{10} = 2$

$z = \frac{D_3}{D} = \frac{10}{10} = 1$

$\therefore x = 3$, $y = 2$ and $z = 1$

Row equivalent matrix method

The augmented are:

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 1 & : & 2 \\ 5 & -9 & 0 & : & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 0 & : & -4 \\ 0 & -14 & -5 & : & -33 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 0 & : & 2 \\ 0 & -14 & -5 & : & -33 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & -5 & : & -15 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 14R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to \frac{1}{-5} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_1 \to R_1 - R_3$$

$\therefore$ $x = 1$, $y = 2$ and $z = 3$

37. 2071 Set C Q.No. 8a

Using Cramer's rule or row–equivalent method solve the following system:

$x + y + z = 6$, $2x + 3y + 5z = 23$,

$7x + 5y - 2z = 11$. **[4]**

SOLUTION

Cramer's rule

Coeff. of x	Coeff. of y	Coeff. of z	Constant form
1	1	1	6
2	3	5	23
7	5	−2	11

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 7 & 5 & -2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 7 & 5 \end{vmatrix}$$

$= 1(-6-25) - 1(-4-35) + 1(10-21)$
$= -31 + 39 - 11 = -3$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 23 & 3 & 5 \\ 11 & 5 & -2 \end{vmatrix}$$

$$= 6\begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 23 & 5 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 23 & 3 \\ 1 & 5 \end{vmatrix}$$

$= 6(-6-25) - 1(-46-5) + 1(115-33)$
$= 6(-31) - 1(-101) + 1(82)$
$= -186 + 101 + 82 = -3$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 23 & 5 \\ 7 & 11 & -2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 23 & 5 \\ 11 & -2 \end{vmatrix} - 6\begin{vmatrix} 2 & 5 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 23 \\ 7 & 11 \end{vmatrix}$$

$= (-46-55) - 6(-4-35) + 1(22-161)$
$= -101 + 234 - 139 = -6$

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$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 23 \\ 7 & 5 & 11 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 23 \\ 5 & 11 \end{vmatrix} - 1\begin{vmatrix} 2 & 23 \\ 7 & 11 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 7 & 5 \end{vmatrix}$$

$$= 1(33 - 115) - (22 - 161) + 6(10 - 21)$$

$$= -82 + 139 - 66 = -9$$

By Cramer's rule,

$$x = \frac{D_1}{D} = \frac{-3}{-3} = 1$$

$$y = \frac{D_2}{D} = \frac{-6}{-3} = 2$$

$$z = \frac{D_3}{D} = \frac{-9}{-3} = 3$$

$\therefore$ $x = 1$, $y = 2$ and $z = 3$

Row equivalent matrix method

The augmented are:

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 2 & 3 & 5 & : & 23 \\ 7 & 5 & -2 & : & 11 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 3 & : & 11 \\ 0 & -2 & -9 & : & -31 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & : & -5 \\ 0 & 1 & 3 & : & 11 \\ 0 & 0 & -3 & : & -9 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 2R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & : & -5 \\ 0 & 1 & 3 & : & 11 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to \left(-\frac{1}{3}\right) R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \begin{matrix} R_1 \to R_1 + 2R_3 \\ R_2 \to R_2 - 3R_3 \end{matrix}$$

$\therefore$ $x = 1$, $y = 2$ and $z = 3$

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38. 2069 [Set A] Old Q. No. 10b

Solve by Row–equivalent or Cramer's rule. [4]

$2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$

SOLUTION

Row Equivalent Matrix Method

The augmented matrix is:

$$\begin{bmatrix} 2 & -1 & 3 & : & 9 \\ 1 & 1 & 1 & : & 6 \\ 1 & -1 & 1 & : & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 2 & -1 & 3 & : & 9 \\ 1 & -1 & 1 & : & 2 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -3 & 1 & : & -3 \\ 0 & -2 & 0 & : & -4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & -1/3 & : & 1 \\ 0 & -2 & 0 & : & -4 \end{bmatrix} R_2 \to \left(-\frac{1}{3}\right) R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 4/3 & : & 5 \\ 0 & 1 & -1/3 & : & 1 \\ 0 & 0 & -2/3 & : & -2 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 2R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 4/3 & : & 5 \\ 0 & 1 & -1/3 & : & 1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to \left(-\frac{3}{2}\right) R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \begin{matrix} R_1 \to R_1 - 4/3R_3 \\ R_2 \to R_2 + 1/3R_3 \end{matrix}$$

$\therefore$ $x = 1$, $y = 2$ and $z = 3$

Cramer's Rule

Coeff. of x	Coeff. of y	Coeff. of z	Constant term
2	−1	3	9
1	1	1	6
1	−1	1	2

Here, $D = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$$= 2\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 2(1+1) + 1(1-1) + 3(-1-1) = 4 - 6 = -2$$

$$D_1 = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= 9\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 6 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= 9(1+1) + 1(6-2) + 3(-6-2)$$

$$= 18 + 4 - 24 = -2$$

$$D_2 = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 6 & 1 \\ 2 & 1 \end{vmatrix} - 9\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix}$$

$$= 2(6-2) - 9(1-1) + 3(2-6)$$

$$= 8 - 0 - 12 = -4$$

$$D_3 = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 6 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix} + 9\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 2(2+6) + 1(2-6) + 9(-1-1)$$

$$= 16 - 4 - 18 = -6$$

$\therefore$ By Cramer's Rule

$$x = \frac{D_1}{D} = \frac{-2}{-2} = 1$$

$$y = \frac{D_2}{D} = \frac{-4}{-2} = 2$$

$$z = \frac{D_3}{D} = \frac{-6}{-2} = 3$$

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39. 2069 [Set B] Old Q. No. 10b

Solve the following system of equation by inve matrix method Or Cramer's rule.

$2x - y + 4z = -3$, $x - 4z = 5$, and $6x - y + 2z = 10$

SOLUTION

Inverse Matrix Method

Please refer to 2069 Supp. Q. No. 8a [Chapter 7B]

Cramer's Rule

Coeff. of x	Coeff. of y	Coeff. of z	Constant term
2	−1	4	−3
1	0	−4	5
6	−1	2	10

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$$D = \begin{vmatrix} 2 & -1 & 4 \\ 1 & 0 & -4 \\ 6 & -1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -4 \\ -6 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 0 \\ 6 & -1 \end{vmatrix}$$

$$= 2(0-4) + 1(2+24) + 4(-1-0)$$

$$= 4(-1-0)$$

$$= -8 + 26 - 4 = 14$$

$$D_1 = \begin{vmatrix} -3 & -1 & 4 \\ 5 & 0 & -4 \\ 10 & -1 & 2 \end{vmatrix}$$

$$= -3\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 5 & -4 \\ 10 & 2 \end{vmatrix} + 4\begin{vmatrix} 5 & 0 \\ 10 & -1 \end{vmatrix}$$

$$= -3(0-4) + 1(10+40) + 4(-5-0)$$

$$= 12 + 50 - 20 = 42$$

$$D_2 = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 5 & -4 \\ 6 & 10 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 5 & -4 \\ 10 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -4 \\ 6 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ 6 & 10 \end{vmatrix}$$

$$= 2(10+40) + 3(2+24) + 4(10-30)$$

$$= 100 + 78 - 80 = 98$$

$$D_3 = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 0 & 5 \\ 6 & -1 & 10 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 5 \\ -1 & 10 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 5 \\ 6 & 10 \end{vmatrix} + (-3)\begin{vmatrix} 1 & 0 \\ 6 & -1 \end{vmatrix}$$

$$= 2(0+5) + 1(10-30) - 3(-1-0)$$

$$= 10 - 20 + 3 = -7$$

Using Cramer's rule, we have

$$x = \frac{D_1}{D} = \frac{42}{14} = 3$$

$$y = \frac{D_2}{D} = \frac{98}{14} = 7$$

$$z = \frac{D_3}{D} = \frac{-7}{14} = -\frac{1}{2}$$

40. 2067 Q.No. 10b

Solve by Cramer's rule or Row–equivalent method: [4]

$x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + y - z = 0$

SOLUTION

Cramer's rule

Coefficient of x	Coefficient of y	Coefficient of z	Constant
1	1	1	9
2	5	7	52
2	1	−1	0

Now, $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix}$

$$= 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$$= 1(-5-7) - 1(-2-14) + 1(2-10)$$

$$= -12 + 16 - 8 = -4 \neq 0$$

$$D_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$$= 9(-5-7) - 1(-52-0) + 1(52-0)$$

$$= -108 + 52 + 52$$

$$= -4$$

$$D_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$$= 1(-52-0) - 9(-2-14) + 1(0-104)$$

$$= -52 + 144 - 104$$

$$= -12$$

$$D_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$$= 1(0-52) - 1(0-104) + 9(2-10)$$

$$= -52 + 104 - 72$$

$$= -20$$

By Cramer's rule,

$$x = \frac{D_1}{D} = \frac{-4}{-4} = 1$$

$$y = \frac{D_2}{D} = \frac{-12}{-4} = 3$$

$$z = \frac{D_3}{D} = \frac{-20}{-4} = 5$$

$\therefore$ $x = 1;\ y = 3;\ z = 5$ Ans.

Row equivalent matrix method

The augmented matrix is

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 5 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & 18 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & \frac{5}{3} & \frac{34}{3} \\ 0 & -1 & -3 & 18 \end{array}\right] R_2 \to \frac{1}{3}R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & \frac{5}{3} & \frac{34}{3} \\ 0 & 0 & \frac{-4}{3} & \frac{-20}{3} \end{array}\right] R_3 \to R_3 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & \frac{5}{3} & \frac{34}{3} \\ 0 & 0 & 1 & 5 \end{array}\right] R_3 \to \left(\frac{-3}{4}\right)R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right] R_2 \to R_2 - \frac{5}{3}R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & :4 \\ 0 & 1 & 0 & :3 \\ 0 & 0 & 1 & :5 \end{bmatrix} R_1 \to R_1 - R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & :1 \\ 0 & 1 & 0 & :3 \\ 0 & 0 & 1 & :5 \end{bmatrix} R_1 \to R_1 - R_2$$

Hence $x = 1, y = 3, z = 5$

41. 2064 Q.No. 10b

Solve the following system of equation by inverse matrix method or Cramer's rule: $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$. [4]

SOLUTION

Inverse Matrix Method:

The matrix form of given system of linear equations is $AX = B$ where

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

The solution of the matrix equation $AX = B$ is given by $X = A^{-1}B$.

Now,

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 2(1+1) + 1(1-1) + 3(-1-1)$$

$$= 4 - 6 = -2 \neq 0.$$ So, A^{-1} exists.

Let A_{ij} be the cofactor element of a_{ij}, So

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = +(1+1) = 2$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -(1-1) = 0$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = +(-1-1) = -2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 3 \\ -1 & 1 \end{vmatrix} = -(-1+3) = -2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = +(2-3) = -1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = +(-1-3) = -4$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = +(2+1) = 3$$

$$\therefore \text{Matrix of cofactors of } A = \begin{bmatrix} 2 & 0 & -2 \\ -2 & -1 & 1 \\ -4 & 1 & 3 \end{bmatrix}$$

$$\text{Adj. } A = \begin{bmatrix} 2 & 0 & -2 \\ -2 & -1 & 1 \\ -4 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{Adj. } A) = \frac{1}{-2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{-2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 18-12-8 \\ 0-6+2 \\ -18+6+6 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence $x = 1, y = 2, z = 3$

Cramer's Rule:

Please refer to 2069 [Set A] Old Q. No. 10b

42. 2063 Q.No. 10b

Solve by Cramer's rule or by row equivalent method. [4]

$x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1$

SOLUTION

Cramer's rule

Coeff. of x	Coeff. of y	Coeff. of z	Constants
1	1	1	6
1	−1	1	2
2	1	−1	1

Now,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= 1(1-1) - 1(-1-1) + 2(1+1)$$

$$= 0 - 1 \times (-2) + 2 \times 2$$

$$= 2 + 4$$

$$= 6$$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 6\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= 6(1-1) - 2(-1-1) + 1(1+1)$$

$$= 0 - 2(-2) + 1(2)$$

$$= 6$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 6 & 1 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 6 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= 1(-2-1) - 1(-6-1) + 2(6-2)$$

$$= 1(-3) - 1(-7) + 2(4)$$

$$= -3 + 7 + 8$$

$$= 12$$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 6 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 6 \\ -1 & 2 \end{vmatrix}$$

$$= 1(-1-2) - 1(1-6) + 2(2+6)$$

$$= -3 + 5 + 16$$

$$= 18$$

By using Cramer's rule, we get

$$x = \frac{D_1}{D} = \frac{6}{6} = 1$$

$$y = \frac{D_2}{D} = \frac{12}{6} = 2$$

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$z = \frac{D_3}{D} = \frac{18}{6} = 3$

$\therefore x = 1, y = 2, z = 3$

Row equivalent matrix method

Given equations are

$x + y + z = 6$

$x - y + z = 2$

$2x + y - z = 1$

The augmented matrix of the given system of equations is

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 1 & : & 2 \\ 2 & 1 & -1 & : & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 0 & : & -4 \\ 0 & -1 & -3 & : & -11 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 0 & : & 2 \\ 0 & -1 & -3 & : & -11 \end{bmatrix} R_2 \to -\frac{1}{2} \times R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & -3 & : & -9 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_3 \to -\frac{1}{3} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} R_1 \to R_1 - R_3$$

$\therefore x = 1, y = 2, z = 3$

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43. 2062 Q.No. 10b OR

Solve the equations by Cramer's rule

$9y - 5x = 3$, $x + z = 1$ and $z + 2y = 2$ **[4]**

SOLUTION

Coeff. of x	Coeff. of y	Coeff. of z	Constants
−5	9	0	3
1	0	1	1
0	2	1	2

Here,

$$D = \begin{vmatrix} -5 & 9 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= -5 \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 9 & 0 \\ 2 & 1 \end{vmatrix} + 0 \begin{vmatrix} 9 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= -5(0 - 2) - 1(9 - 0) + 0(9 - 0)$$

$$= -5(-2) - 1(9) + 0$$

$$= 10 - 9 = 1$$

$$D_1 = \begin{vmatrix} 3 & 9 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 9 & 0 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 9 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 3(0 - 2) - 1(9 - 0) + 2(9 - 0)$$

$$= 3(-2) - 1(9) + 2(9)$$

$$= -6 - 9 + 18$$

$$= 3$$

$$D_2 = \begin{vmatrix} -5 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= -5 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} + 0 \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= -5(1 - 2) - 1(3 - 0) + 0(3 - 0)$$

$$= -5(-1) - 1(3) + 0(3)$$

$$= 5 - 3 + 0$$

$$= 2$$

$$D_3 = \begin{vmatrix} -5 & 9 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{vmatrix}$$

$$= -5 \begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} - 9 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$$

$$= -5(0 - 2) - 9(2 - 0) + 3(2 - 0)$$

$$= -5(-2) - 9(2) + 3(2)$$

$$= 10 - 18 + 6$$

$$= -2$$

Now, by using Cramer's Rule we get,

$x = \frac{D_1}{D} = \frac{3}{1} = 3$

$y = \frac{D_2}{D} = \frac{2}{1} = 2$

$z = \frac{D_3}{D} = -\frac{2}{1} = -2$

Hence, $x = 3, y = 2, z = -2$

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44. 2061 Q.No. 10b

Solve by Cramer's rule:

$x + 2y + 3z = 6$, $2x + 4y + z = 7$

and $3x + 2y + 9z = 14$ **[4]**

SOLUTION

Coeff. of x	Coeff. of y	Coeff. of z	Constants
1	2	3	6
2	4	1	7
3	2	9	14

Now, $D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 9 \end{vmatrix}$

$$= 1 \begin{vmatrix} 4 & 1 \\ 2 & 9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 \\ 2 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$= 1(36 - 2) - 2(18 - 6) + 3(2 - 12)$$

$$= 34 - 24 - 30 = -20$$

$$D_1 = \begin{vmatrix} 6 & 2 & 3 \\ 7 & 4 & 1 \\ 14 & 2 & 9 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 4 & 1 \\ 2 & 9 \end{vmatrix} - 7 \begin{vmatrix} 2 & 3 \\ 2 & 9 \end{vmatrix} + 14 \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$= 6(36 - 2) - 7(18 - 6) + 14(2 - 12)$$

$$= 6 \times 34 - 7 \times 12 + 14 \times (-10)$$

$$= 204 - 84 - 140 = -20$$

$$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 2 & 7 & 1 \\ 3 & 14 & 9 \end{vmatrix}$$

$$= 1\begin{vmatrix} 7 & 1 \\ 14 & 9 \end{vmatrix} - 2\begin{vmatrix} 6 & 3 \\ 14 & 9 \end{vmatrix} + 3\begin{vmatrix} 6 & 3 \\ 7 & 1 \end{vmatrix}$$

$$= 1(63 - 14) - 2(54 - 42) + 3(6 - 21)$$

$$= 49 - 24 - 45 = -20$$

and $$D_3 = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 7 \\ 3 & 2 & 14 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & 7 \\ 2 & 14 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 2 & 14 \end{vmatrix} + 3\begin{vmatrix} 2 & 6 \\ 4 & 7 \end{vmatrix}$$

$$= 1(56 - 14) - 2(28 - 12) + 3(14 - 24)$$

$$= 42 - 32 - 30 = -20$$

By using Cramer's rule, we have

$$x = \frac{D_1}{D} = \frac{-20}{-20} = 1$$

$$y = \frac{D_2}{D} = \frac{-20}{-20} = 1$$

$$z = \frac{D_3}{D} = \frac{-20}{-20} = 1$$

Hence, $x = 1, y = 1, z = 1$.

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8 UNIT

COMPLEX NUMBER

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2 MARKS QUESTIONS

1. 2076 Set B Q.No. 3b

Express $\sqrt{3} + i$ in polar form. **[2]**

SOLUTION

Given complex number is $\sqrt{3} + i$

Here, $x = \sqrt{3}$, $y = 1$

We have, $r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + 1^2} = 2$

$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \theta = 30°$

In polar form, given complex number is

$r(\cos\theta + i\sin\theta) = 2(\cos 30° + i \sin 30°)$

2. 2076 Set C Q.No. 3b

Find the conjugate of (3 + 5i) (7 – 5i). **[2]**

SOLUTON

Let $z = (3 + 5i)(7 - 5i)$

Then,

$z = (3 + 5i)(7 - 5i)$

$= 21 - 15i + 35i - 25i^2$

$= 21 + 20i + 25$

$= 46 + 20i$

Conjugate of $z = \bar{z}$

$= \overline{46 + 20i}$

$= 46 - 20i$

3. 2075 GIE Q.No. 3b

Express $\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$ in a + ib form. **[2]**

SOLUTION

$\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$

$= \frac{2 - 5i}{1 - 4i} \times \frac{1 + 4i}{1 + 4i}$

$= \frac{2 - 5i + 8i - 20i^2}{1^2 - (4i)^2}$

$= \frac{2 + 3i - 20i^2}{1 - 16i^2}$

$= \frac{2 + 3i + 20}{1 + 16}$ $[\because i^2 = -1]$

$= \frac{22 + 3i}{17}$

$= \frac{22}{17} + \frac{3}{17}i$

which is in the form of a + ib, where

$a = \frac{22}{17}$ and $b = \frac{3}{17}$.

4. 2075 Set A Q.No. 3b

Express the complex number $(1 - i)^9 \left(1 - \frac{1}{i^3}\right)^9$ in the standard form a + bi. **[2]**

SOLUTION

$(1 - i)^9 \left(1 - \frac{1}{i^3}\right)^9$

$= (1 - i)^9 \left(1 - \frac{1}{i^2 \cdot i}\right)^9$

$= (1 - i)^9 \left(1 - \frac{1}{-1 \cdot i}\right)^9$ $[\because i^2 = -1]$

$= (1 - i)^9 \left(1 + \frac{1}{i}\right)^9$

$= (1 - i)^9 \left(\frac{i + 1}{i}\right)^9$

$= \frac{\{(1 - i).(1 + i)\}^9}{i^9}$

$= \frac{(1^2 - i^2)^9}{i^8.i}$

$= \frac{\{1 - (-1)\}^9}{(i^2)^4.i}$

$= \frac{2^9}{i} \times \frac{i}{i} = \frac{2^9 i}{-1} = -512i = 0 + (-512)i$

which is in the form of a + ib where a = 0, b = –512.

5. 2075 Set B Q.No. 3b

Find the conjugate of the complex number $\frac{5 - 7i}{5 + 8i}$. **[2]**

SOLUTION

Let $z = \frac{5 - 7i}{5 + 8i}$

or, $z = \frac{5 - 7i}{5 + 8i} \times \frac{5 - 8i}{5 - 8i}$

$$\text{or, } z = \frac{25 - 40i - 35i + 56i^2}{25 - 64i^2}$$

$$\text{or, } z = \frac{25 - 75i - 56}{25 + 64}$$

$$\text{or, } z = \frac{-31 - 75i}{89}$$

$$\text{or, } z = \frac{-31}{89} - \frac{75}{89}i$$

$$\therefore \bar{z} = \frac{-31}{89} + \frac{75}{89}i$$

6. 2075 Set C Q.No. 3b

If $\sqrt{x - iy} = a - ib$ prove that $\sqrt{x + iy} = a + ib$. [2]

SOLUTION

Given, $\sqrt{x - iy} = a - ib$

Squaring both sides, we get

$x - iy = (a - ib)^2$

or, $x - iy = a^2 - 2.a.ib + (ib)^2$

or, $x - iy = a^2 - 2abi - b^2$

or, $x - iy = a^2 - b^2 - 2abi$

Equating real and imaginary parts, we get,

$x = a^2 - b^2$, $y = 2ab$.

Now, L.H.S $= \sqrt{x + iy}$

$= \sqrt{a^2 - b^2 + i.\,2ab}$

$= \sqrt{a^2 + 2.a.ib + (ib)^2}$

$= \sqrt{(a + ib)^2}$

$= a + ib$ = R.H.S

7. 2074 Supp Q.No. 3b

If ω is a complex cube root of unity, show that: [2]

$(1 - \omega + \omega^2)^4 (1 + \omega - \omega^2)^4 = 256$

SOLUTION

$(1 - \omega + \omega^2)^4 (1 + \omega - \omega^2)^4$

$= (1 + \omega^2 - \omega)^4 (1 + \omega - \omega^2)^4$

$= (-\omega - \omega)^4 (-\omega^2 - \omega^2)^4$ $\quad (\because 1 + \omega + \omega^2 = 0)$

$= (-2\omega)^4 (-2\omega^2)^4$

$= 16\omega^4 \times 16\omega^8$

$= 256\,\omega^{12}$

$= 256 \times 1 = 256$

8. 2074 Set A Q.No. 3b

Find the real numbers x and y if $(x - 1)i + (y + 1) = (1 + i)(4 - 3i)$. [2]

SOLUTION

Given,

$(x - 1)i + (y + 1) = (1 + i)(4 - 3i)$

or, $(x - 1)i + (y + 1) = 4 - 3i + 4i - 3i^2$

or, $(x - 1)i + (y + 1) = 4 - 3i + 4i + 3$

or, $(x - 1)i + (y + 1) = 7 + i$

Equating real & imaginary parts, we get,

$y + 1 = 7$ & $x - 1 = 1$

or, $y = 6$ & $x = 2$

$\therefore x = 2$ & $y = 6$.

9. 2074 Set B Q.No. 3b

Find the multiplicative inverse of the complex number $3 + 4i$. [2]

SOLUTION

The multiplicative inverse (reciprocal) of the complex number $3 + 4i$ is $\frac{1}{3 + 4i}$

$$= \frac{1}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} = \frac{3 - 4i}{3^2 - (4i)^2}$$

$$= \frac{3 - 4i}{25} = \frac{3}{25} - \frac{4}{25}i$$

10. 2073 Supp Q.No. 3b

Express the complex number $\sqrt{\frac{1+i}{1-i}}$ into a+ib form. [2]

SOLUTION

$$\sqrt{\frac{1+i}{1-i}} = \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}$$

$$= \frac{1+i}{\sqrt{1-i^2}} = \frac{1+i}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

which is in the form of $a + ib$, where $a = \frac{1}{\sqrt{2}}$ and $b = \frac{1}{\sqrt{2}}$

11. 2073 Set C Q.No. 3b

Find the conjugate of $1 + i + i^2 + i^3 + i^4$. [2]

SOLUTION

Let $z = 1 + i + i^2 + i^3 + i^4$

$= 1 + i + i^2 + i^2 \cdot i + (i^2)^2$

$= 1 + i + (-1) + (-1) \cdot i + (-1)^2$ $[\because i^2 = -1]$

$= 1 + i - 1 - i + 1$

$\therefore \bar{z} = 1$

12. 2073 Set D Q.No. 3b

Find the modulus of $(3 + 4i)(4 + i)$. [2]

SOLUTION

Let $z = (3 + 4i)(4 + i)$

$|z| = |(3 + 4i)(4 + i)|$

$= |3 + 4i|\,|4 + i|$

$= \sqrt{3^2 + 4^2}\sqrt{4^2 + 1^2} = 5\sqrt{17}$

13. 2072 Supp. Q.No. 3b

If $x - iy = \frac{3 - 2i}{3 + 2i}$, prove that $x^2 + y^2 = 1$. [2]

SOLUTION

Given, $x - iy = \frac{3 - 2i}{3 + 2i}$

Taking modulus on both sides, we get,

$$|x - iy| = \left|\frac{3 - 2i}{3 + 2i}\right|$$

$$\text{or, } \sqrt{x^2 + (-y)^2} = \frac{\sqrt{3^2 + (-2)^2}}{\sqrt{3^2 + 2^2}}$$

$$\text{or, } \sqrt{x^2 + y^2} = 1$$

Squaring both sides, we get.

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14. 2072 Set C Q.No. 3b

Express the complex number: $i-\sqrt{3}$ into polar form. [2]

SOLUTION

Given complex number is $i-\sqrt{3} = -\sqrt{3} + 1 \cdot i$

$\therefore x = -\sqrt{3}, y = 1$

We have, $r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2+1^2} = 2$

$\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = \tan 150^\circ \quad (\because x<0, y>0)$

$\therefore \theta = 150^\circ$

$\therefore$ In polar form given complex number is

$r(\cos\theta + i\sin\theta) = 2(\cos 150^\circ + i\sin 150^\circ)$

15. 2072 Set D Q.No. 3b

If $x = a + b$, $y = a\omega + b\omega^2$ and $z = a\omega^2 + b\omega$, show that $x + y + z = 0$. [2]

SOLUTION

Here, $x = a + b$

$y = a\omega + b\omega^2$

$z = a\omega^2 + b\omega$

L.H.S. $= x + y + z$

$= (a+b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega)$

$= (a + a\omega + a\omega^2) + (b + b\omega^2 + b\omega)$

$= a(1 + \omega + \omega^2) + b(1 + \omega^2 + \omega)$

$= a \times 0 + b \times 0 \quad (\because 1 + \omega + \omega^2 = 0)$

$= 0 =$ R.H.S.

16. 2072 Set E Q.No. 3b

If z and w are complex numbers, show that: $|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$ [2]

SOLUTION

L.H.S. $= |z+w|^2 + |z-w|^2$

$= (z+w)(\overline{z+w}) + (z-w)(\overline{z-w}) \quad (\because |z|^2 = z\bar{z})$

$= (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w})$

$= z\bar{z} + z\bar{w} + \bar{z}w + w\bar{w} + z\bar{z} - z\bar{w} - \bar{z}w + w\bar{w}$

$= 2z\bar{z} + 2w\bar{w}$

$= 2|z|^2 + 2|w|^2$

$= 2(|z|^2 + |w|^2) =$ R.H.S

17. 2071 Supp. Q.No. 3b

Express the complex number $1 - \sqrt{3}\,i$ into polar form. [2]

SOLUTION

Given complex number is $1 - \sqrt{3}\,i$

$\therefore x = 1, y = -\sqrt{3}$

We have, $r = \sqrt{x^2+y^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3} = \tan 300^\circ \quad (\because x>0, y<0)$

$\therefore \theta = 300^\circ$

In polar form, given complex number can be written as

$r(\cos\theta + i\sin\theta) = 2(\cos 300^\circ + i\sin 300^\circ)$

18. 2071 Set C Q.No. 3b

Prove that the modulus of a complex number and its conjugate are equal. [2]

SOLUTION

Let z be a complex number. We have to prove that $|z| = |\bar{z}|$

Let $z = x + iy$

Then $\bar{z} = x - iy$

Now, $|z| = \sqrt{x^2+y^2}$

and $|\bar{z}| = \sqrt{x^2+(-y)^2} = \sqrt{x^2+y^2}$

$\therefore |z| = |\bar{z}|$

19. 2071 Set D Q.No. 3b

For any complex number Z, prove that: $Z\bar{Z} = |Z|^2$ [2]

SOLUTION

Let $Z = a + ib$

$\bar{Z} = a - ib$

$|Z| = \sqrt{a^2+b^2}$

L.H.S. $= Z\bar{Z}$

$= (a+ib) + (a-ib)$

$= a^2 - i^2 b^2$

$= a^2 + b^2$

$= (\sqrt{a^2+b^2})^2$

$= |Z|^2 =$ R.H.S.

20. 2071 Old Q.No. 4a

Express $\frac{i}{i+1}$ in the polar form. [2]

SOLUTION

Here,

$\frac{i}{i+1} = \frac{i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{i-i^2}{1-i^2}$

$= \frac{i+1}{1+1} = \frac{1}{2} + \frac{1}{2}\cdot i$

$\therefore x = \frac{1}{2}, y = \frac{1}{2}$

We have,

$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{y}{x} = \frac{1/2}{1/2} = 1 = \tan 45^\circ$

In polar form, given complex number can be written as:

$r(\cos\theta + i\sin\theta) = \frac{1}{\sqrt{2}}(\cos 45^\circ + i\sin 45^\circ)$

21. 2070 Supp Q.No. 3b

Express the complex number $\sqrt{\frac{1+i}{1-i}}$ in the form of a + ib. [2]

➥ Please refer to 2073 Supp Q.No. 3b

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22. 2070 Set C Q.No. 3 b

If $\alpha = \frac{1}{2}(-1+\sqrt{-3})$, $\beta = \frac{1}{2}(-1-\sqrt{-3})$, show that: $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$. [2]

SOLUTION

Given, $\alpha = \frac{1}{2}(-1+\sqrt{-3})$, $\beta = \frac{1}{2}(-1-\sqrt{-3})$

Here, α and β are imaginary cube roots of unitary, so if $\alpha = \omega$ and $\beta = \omega^2$.

L.H.S. $= \alpha^4 + \alpha^2\beta^2 + \beta^4$
$= \omega^4 + \omega^2(\omega^2)^2 + (\omega^2)^4$
$= \omega^3 \cdot \omega + \omega^6 + \omega^8$
$= 1 \cdot \omega + (\omega^3)^2 + \omega^6 \cdot \omega^2$
$= \omega + (1)^2 + (\omega^3)^2 \cdot \omega^2$
$= \omega + 1 + (1)^2 \cdot \omega^2$
$= \omega + 1 + \omega^2 = 0 = $ R.H.S.

23. 2070 Set D Q.No. 3 b

Find the values of x and y if $(x+2) + yi = (3+i)(1-2i)$. [2]

SOLUTION

Here, $(x+2) + yi = (3+i)(1-2i)$
or, $(x+2) + yi = 3 + i - 6i - 2i^2$
or, $(x+2) + yi = 3 - 5i + 2$
or, $(x+2) + yi = 5 - 5i$

Equating the real and imaginary parts, we get
$x + 2 = 5$ and $y = -5$
$\therefore$ $x = 3$ and $y = -5$

24. 2070 Old Q.No. 4 a

Express the complex number in Cartesian form whose modulus is 6 and amplitude is 60°. [2]

SOLUTION

Given, $|z| = 6$ and $\theta = 60°$
We know that $r = |z|$
$\therefore$ $r = 6$
Required complex number is $r(\cos\theta + i\sin\theta)$
$= 6(\cos 60° + i \sin 60°)$
$= 6\left(\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}\right) = 3 + 3\sqrt{3}\,i$

25. 2069 Supp Q.No. 3 b

Express $1 + i\sqrt{3}$ in polar form. [2]

SOLUTION

Given complex number is $1 + i\sqrt{3}$
$\therefore$ $x = 1$, $y = \sqrt{3}$
We have, $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2} = 2$
$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1} = \tan 60°$
$\therefore$ $\theta = 60°$
In polar form, given complex number is $2(\cos 60° + i \sin 60°)$

26. 2069 [Set A] Q. No. 3b

If ω be a complex cube root of unity, find the value of: $(1 - \omega + \omega^2)^4 (1 + \omega - \omega^2)^4$ [2]

➥ Please refer to **2074 Supp Q.No. 3b**

27. 2069 [Set B] Q. No. 3b

Express $\sqrt{3} + i$ in polar form. [2]

➥ Please refer to **2076 Set B Q.No. 3b**

28. 2069 [Set A] Old Q. No. 4a

Express the complex number $\frac{1}{1-i}$ in the form $a + ib$. [2]

SOLUTION

$\frac{1}{1-i} \times \frac{1+i}{1+i}$
$= \frac{1+i}{1-i^2} = \frac{1+i}{2} = \frac{1}{2} + \frac{1}{2} \cdot i$

Which is in the form of $a + ib$, where $a = \frac{1}{2}$ and $b = \frac{1}{2}$.

29. 2069 [Set B] Old Q. No. 3b

Find the values of the real number x and y if $(x+2) + yi = (3+i)(1-2i)$. [2]

➥ Please refer to **2070 Set D Q.No. 3 b**

30. 2068 Q.No. 3b

Find the cube roots of unity. [2]

SOLUTION

Let Z be the cube roots of unity. Then,
$z^3 = 1$
or, $z^3 - 1 = 0$
or, $(z-1)(z^2 + z + 1) = 0$
Either, $z - 1 = 0 \Rightarrow z = 1$
or, $z^2 + z + 1 = 0$
or, $z = \frac{-1 \pm \sqrt{3}i}{2}$
$\therefore$ $z = 1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

31. 2068 Old Q.No. 4a

Find the real numbers x and y if: $x + iy = (2-3i)(3-2i)$ [2]

SOLUTION

Given,
$x + iy = (2-3i)(3-2i)$
or, $x + iy = 6 - 4i - 9i + 6i^2$
or, $x + iy = 6 - 13i - 6$
or, $x + iy = 0 + (-13)i$

Equating real and imaginary parts, we have
$x = 0$, $y = -13$

32. 2067 Q.No. 4a

Express the complex number $\frac{i}{1-i}$ in the polar form. [2]

SOLUTION

Given complex number is
$\frac{i}{1-i} = \frac{i}{1-i} \times \frac{1+i}{1+i}$

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$= \frac{i + i^2}{1^2 - i^2}$

$= \frac{i - 1}{1 + 1}$ $(\because i^2 = -1)$

$= \frac{i - 1}{2} = -\frac{1}{2} + \frac{1}{2}i$

Let $\frac{i}{1-i} = -\frac{1}{2} + \frac{1}{2}i = r(\cos\theta + i\sin\theta)$

Then, $r\cos\theta = -\frac{1}{2}$, $r\sin\theta = \frac{1}{2}$

Squaring and adding

$r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\therefore r = \frac{1}{\sqrt{2}}$

And hence $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = 135°$

Thus, $\frac{i}{1-i} = r(\cos\theta + i\sin\theta)$

$= \frac{1}{\sqrt{2}}(\cos 135° + i \sin 135°)$

33. 2066 Q.No. 4a

Prove that:

$(2 + \omega)(2 + \omega^2)(2 - \omega^2)(2 - \omega^4) = 21$ **[2]**

SOLUTION

L.H.S. $= (2 + \omega)(2 + \omega^2)(2 - \omega^2)(2 - \omega^4)$

$= (2 + \omega)(4 - \omega^4)(2 - \omega^3 . \omega)$

$= (2 + \omega)(4 - \omega^3 . \omega)(2 - \omega^3 . \omega)$

$= (2 + \omega)(4 - \omega)(2 - \omega)$ $[\because \omega^3 = 1]$

$= (4 - \omega^2)(4 - \omega)$

$= 16 - 4\omega - 4\omega^2 + \omega^3$

$= 16 - 4(\omega + \omega^2) + 1$

$= 16 - 4(-1) + 1$

$[\because 1 + \omega + \omega^2 = 0, \ \therefore \omega + \omega^2 = -1]$

$= 21$

= R.H.S. proved.

34. 2065 Q.No. 4a

If α, β are the complex cube roots of unity then show that: $\alpha^4 + \beta^4 + \frac{1}{\alpha\beta} = 0$ **[2]**

SOLUTION

If $\alpha = \frac{-1 + \sqrt{3}i}{2} = \omega$ then $\beta = \frac{-1 - \sqrt{3}i}{2} = \omega^2$

Now,

L.H.S. $= \alpha^4 + \beta^4 + \frac{1}{\alpha\beta}$

$= \omega^4 + (\omega^2)^4 + \frac{1}{\omega \cdot \omega^2}$

$= \omega^4 + \omega^8 + \frac{1}{\omega^3}$

$= \omega^4 + \omega^8 + 1$ $[\because \omega^3 = 1]$

$= \omega^3 \cdot \omega + \omega^3 \cdot \omega^3 \cdot \omega^2 + 1$

$= 1 \cdot \omega + 1 \cdot 1 \cdot \omega^2 + 1$

$= \omega + \omega^2 + 1$

$= -1 + 1$ $[\because 1 + \omega + \omega^2 = 0]$

= 0 = R.H.S. proved.

35. 2064 Q.No. 3b

If 1, ω, ω^2 be the cube roots of unity, prove that $(1 + \omega^2)^3 - (1 + \omega)^3 = 0$ **[2]**

SOLUTION

Here, 1, ω, ω^2 are the cube roots of unity. so, we must have $1 + \omega + \omega^2 = 0$ and $w^3 = 1$

Now,

L.H.S. $= (1 + \omega^2)^3 - (1 + \omega)^3$

$= (-\omega)^3 - (-\omega^2)^3$

$= -\omega^3 - (-\omega^6) = \omega^6 - \omega^3$

$= (\omega^3)^2 - \omega^3 = 1^2 - 1$

$= 1 - 1 = 0$ = R.H.S. Proved.

36. 2063 Q.No. 3b

Find the square roots of 7 + 24i. **[2]**

SOLUTION

Let $x + iy$ be the square roots of 7 + 24i. Then,

$7 + 24i = (x + iy)^2$

$= x^2 + 2ixy + (iy)^2$

$= x^2 - y^2 + 2ixy$ $[\because i^2 = -1]$

Equating the real and imaginary parts, we get

$x^2 - y^2 = 7$ ... (i)

$2xy = 24$ ... (ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2 = 7^2 + (2xy)^2$

$= 49 + 24^2 = 49 + 576$

$= 625 = (25)^2$

$\therefore x^2 + y^2 = 25$ ... (iii)

Adding equations (i) and (iii), we get

$x^2 - y^2 = 7$

$x^2 + y^2 = 25$

$2x^2 = 32$

or, $x^2 = 16$

$\therefore x = \pm 4$

Putting the value of x^2 in (iii), we get

$16 + y^2 = 25$

$y^2 = 9$

$\therefore y = \pm 3$

From equation (ii), $xy = 12$, which is positive

$\therefore$ x and y should be of the same sign.

So if $x = 4$;

$y = 3$ and if $x = -4$; $y = -3$.

$\therefore x + iy = 4 + 3i$ and $-4 - 3i = -(4 + 3i)$

Hence, the required roots are $\pm(4 + 3i)$

37. 2062 Q.No. 4a

Simplify: $\left[3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{16}$ **[2]**

SOLUTION

$\left[3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{16}$

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$$= 3^{16}\left[\cos\left(16\times\frac{\pi}{4}\right) + i\sin\left(16\times\frac{\pi}{4}\right)\right]$$

[$\because$ By De-Moivre's theorem]

$= 3^{16}(\cos 4\pi + i\sin 4\pi)$

$= 3^{16}(1 + i\times 0) = 3^{16}$

38. 2061 Q.No. 4a

Find the conjugate of the complex number $\frac{3+4i}{3-4i}$. [2]

SOLUTION

Let $Z = \frac{3+4i}{3-4i}$

Now, $Z = \frac{3+4i}{3-4i}\times\frac{(3+4i)}{(3+4i)}$

$= \frac{(3+4i)^2}{3^2-(4i)^2}$

$= \frac{9+24i+16i^2}{9-16i^2}$

$= \frac{9+24i-16}{9+16}$ ($\because i^2 = -1$)

$= \frac{-7+24i}{25}$

or, $Z = \frac{-7}{25}+\frac{24}{25}i$

$\therefore$ The conjugate of $Z = \bar{Z} = \frac{-7}{25}-\frac{24}{25}i$

39. 2060 Q.No. 4a

Express the complex number, $-\sqrt{2}+i\sqrt{2}$ in polar form. [2]

SOLUTION

Let $-\sqrt{2}+i\sqrt{2} = r(\cos\theta + i\sin\theta)$ ... (1)

Equating the real and imaginary parts, we get

$r\cos\theta = -\sqrt{2}$ ... (2)

$r\sin\theta = \sqrt{2}$ ... (3)

Now, $(r\cos\theta)^2 + (r\sin\theta)^2 = (-\sqrt{2})^2 + (\sqrt{2})^2$

or, $r^2(\cos^2\theta + \sin^2\theta) = 2+2$

or, $r^2 = 4$

$\therefore r = 2$

And, $\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$ [Dividing (3) by (2)]

or, $\tan\theta = -1 = \tan 135°$

$\therefore \theta = 135°$

Substituting the value of r and θ in equation (1) we get

$-\sqrt{2}+i\sqrt{2} = 2(\cos 135° + i\sin 135°)$

This is the required polar form.

40. 2059 Q.No. 4a

Express in the polar form for $z = 2 + 2i$ [2]

SOLUTION

Let $2+2i = r(\cos\theta + i\sin\theta)$ ... (i)

Now, equating the real and imaginary parts, we get

$r\cos\theta = 2$ ... (ii)

$r\sin\theta = 2$ ... (iii)

Now, $r^2\cos^2\theta + r^2\sin^2\theta = 2^2 + 2^2$

or, $r^2(\cos^2\theta + \sin^2\theta) = 4+4 = 8$

or, $r^2 = 8$

$\therefore r = \sqrt{8} = 2\sqrt{2}$

Again,

$\frac{r\sin\theta}{r\cos\theta} = \frac{2}{2}$ [Dividing (iii) by (ii)]

or, $\tan\theta = 1 = \tan 45°$

$\therefore \theta = 45°$

Hence, from equation (i), the required polar form o

$z = 2+2i$ is $z = 2\sqrt{2}(\cos 45° + i\sin 45°)$.

41. 2058 Q.No. 4a

Express the complex number $(2, 2\sqrt{3})$ in th polar form. [2

SOLUTION

Here, $(2, 2\sqrt{3}) = 2 + i2\sqrt{3}$

Now, let $2 + i2\sqrt{3} = r(\cos\theta + i\sin\theta)$ ... (1)

Equating the real and imaginary parts, we get,

$r\cos\theta = 2$ ... (2)

$r\sin\theta = 2\sqrt{3}$ ... (3)

Now, $r^2(\cos^2\theta + \sin^2\theta) = 2^2 + (2\sqrt{3})^2 = 4+12 = 16$

or, $r^2\times 1 = 16$

$\therefore r = 4$

Again, dividing equation (3) by equation (2) we get,

$\frac{r\sin\theta}{r\cos\theta} = \frac{2\sqrt{3}}{2}$

i.e. $\tan\theta = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$

Hence, from equation (1), the required polar form $(2 + i2\sqrt{3})$ is $4(\cos 60° + i\sin 60°)$

42. 2057 Q.No. 4b

If $Z_1 = (3, 2)$; $Z_2 = (5, 3)$, compute $Z_1 Z_2$ a $\overline{Z_1 + Z_2}$. [

SOLUTION

We have,

$Z_1 = (3, 2)$; $Z_2 = (5, 3)$ which can be written as

$Z_1 = 3 + 2i$ and $Z_2 = 5 + 3i$

Now,

$Z_1 Z_2 = (3+2i)(5+3i)$

$= 15 + 9i + 10i + 6i^2$

$= 15 + 19i - 6$ [$\because i^2 = -1$]

$= 9 + 19i$

$\therefore Z_1 Z_2 = (9, 19)$

And, $Z_1 + Z_2 = (3+2i) + (5+3i)$

$= 3 + 2i + 5 + 3i = 8 + 5i$

Now, $\overline{Z_1 + Z_2} = \overline{8+5i} = 8 - 5i$

$\therefore \overline{Z_1 + Z_2} = (8, -5)$

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43. 2056 Q.No. 4a

If $z = 3 + 4i$ and $w = 2 + i$, find $|zw|$ and $\left|\frac{z}{w}\right|$ [2]

SOLUTION

We have,

$z = 3 + 4i$ and $w = 2 + i$

Now, $|zw| = |z|\,|w|$

$= |3 + 4i|\,|2 + i|$

$= \sqrt{3^2 + 4^2} \times \sqrt{2^2 + 1^2}$

$= \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$

And $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{\sqrt{3^2 + 4^2}}{\sqrt{2^2 + 1^2}} = \frac{\sqrt{25}}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$.

44. 2055 Old Model Q.No. 4a

If $z_1 = (a, b)$ and $z_2 = (c, d)$; compute $z_1 z_2$ and $\overline{z_1 + z_2}$ [2]

SOLUTION

Here,

$z_1 = (a, b) = a + ib$

$z_2 = (c, d) = c + id$

Now,

$z_1 \cdot z_2 = (a + ib).(c + id)$

$= ac + iad + ibc + i^2 bd$

$= (ac - bd) + i(ad + bc)$ $[\because i^2 = -1]$

$= (ac - bd, ad + bc)$

Again,

$\overline{z_1 + z_2} = \overline{(a + ib) + (c + id)}$

$= \overline{a + c + i(b + d)}$

$= a + c - i(b + d)$

$= a + c + i(-b - d)$

$= (a + c, -b - d)$

6 MARKS QUESTIONS

45. 2076 Set B Q.No. 14

Define conjugate of a complex number. Using De Moivre's theorem, find the square roots of $-2 + 2\sqrt{3}\,i$. [6]

SOLUTON

Conjugate of a complex number: If $z = a + ib$ be a complex number then the conjugate of z, denoted by $\bar{z}$, is defined by $\bar{z} = \overline{a + ib} = a - ib$.

For example, if $z = 3 + 4i$ then $\bar{z} = 3 - 4i$.

Next Part:

Let $z = -2 + 2\sqrt{3}\,i = x + iy$

Then, $x = -2$, $y = 2\sqrt{3}$.

Now,

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-2)^2 + (2\sqrt{3})^2}$

$= \sqrt{16}$

$= 4$

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$= \tan 120°$ [$\because$ z lies in second quadrant]

$\therefore \theta = 120°$

In polar form,

$z = 4(\cos 120° + i \sin 120°)$

$= 4\{\cos(120 + 360k) + i \sin(120 + 360k)\}$

Now,

$\sqrt{z} = z^{1/2}$

$= [4\{\cos(120 + 360k) + i \sin(120 + 360k)\}]^{1/2}$

$= 4^{1/2}\left\{\cos\left(\frac{120 + 360k}{2}\right) + i\left(\frac{120 + 360k}{2}\right)\right\}$

[Using De moivre's Theorem] where k = 0, 1

$= 2\{\cos\{60 + 180k) + i \sin(60 + 180k)\}$

When k = 0,

$z^{1/2} = 2(\cos 60° + i \sin 60°)$

$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 1 + \sqrt{3}\,i$

When k = 1,

$z^{1/2} = 2\{\cos(60 + 180) + i \sin(60 + 180)\}$

$= 2(-\cos 60 - i \sin 60)$

$= 2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$= -1 - \sqrt{3}\,i$

$= -(1 + \sqrt{3}\,i)$

$\therefore$ Required square roots are $\pm(1 + \sqrt{3}\,i)$

46. 2076 Set C Q.No. 14

State De Moivre's theorem and apply it to find fourth roots of $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$. [6]

SOLUTION

De Moivre's theorem: If n is any positive integer, $[r(\cos\theta + i \sin\theta)]^n = r^n(\cos n\theta + i \sin n\theta)$.

Next Part:

Let $z^4 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

Here, $x = -\frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$

We have, $r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

and, $\tan\theta = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3} = \tan 120$

[$\because$ the number lies in 2nd quadrant]

$\therefore \theta = 120°$

Thus, $z^4 = 1(\cos 120 + i \sin 120)$

or, $z^4 = \cos(120 + 360k) + i \sin(120 + 360\,k)$

or, $z = [\cos(120 + 360k) + i \sin(120 + 360k)]^{1/4}$

$= \cos\left(\frac{120 + 360k}{4}\right) + i \sin\left(\frac{120 + 360k}{4}\right)$

[Using De Moivre's Theorem]
where k = 0, 1, 2, 3, ...

$= \cos(30 + 90k) + i\sin(30 + 90k)$

When k = 0,

$z = \cos 30 + i\sin 30$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}i$

When k = 1,

$z = \cos 120 + i \sin 120$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i = -\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

When k = 2,

$z = \cos(30+180) + i \sin(30+180) - \cos 30 - i \sin 30$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

When k = 3,

$z = \cos(30 + 270) + i \sin(30 + 270)$

$= \sin 30 - i \cos 30$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}i$

Hence, the required 4th roots are

$\pm\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right), \pm\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

47. 2075 GIE Q.No. 14

Define modulus of a complex number. If z and w are two complex numbers, prove that: $|z-w| \le |z| + |w|$ and $|z + w| \ge |z| - |w|$. [6]

SOLUTION

Modulus of a complex number: If $z = x + iy$ is a complex number, then the absolute value or modulus of z is a non-negative real number, denoted by $|z|$, and is defined by $|z| = \sqrt{x^2 + y^2}$.

Let z and w be two complex numbers.

Now,

$|z - w| = |z + (-w)|$

$\le |z| + |w|$ $[\because |z + w| \le |z| + |w|]$

$\therefore |z - w| \le |z| + |w|$

And,

$|z| = |z + w - w|$

$= |(z + w) + (-w)|$

$\le |z + w| + |-w|$ $[\because |z + w| \le |z| + |w|]$

$= |z + w| + |w|$

or, $|z| - |w| \le |z + w|$

$\therefore |z + w| \ge |z| - |w|$

48. 2075 Set A Q.No. 13

State De Moivre's theorem. Find the cube roots of the complex number -27i using this theorem. [6]

SOLUTION

Statement of De-Moivre's Theorem:

If n is any positive integer, then

$[r(\cos\theta + i \sin\theta)]^n = r^n(\cos n\theta + i \sin n\theta)$

Next part:

Let z be the cube roots of $-27i$.

Then, $z^3 = -27i$.

Here, $x = 0, y = -27$

$r = \sqrt{0^2 + (-27)^2} = 27$

and $\tan\theta = \frac{y}{x} = \frac{-27}{0} = \tan 270^\circ$

$\therefore \theta = 270^\circ$

Thus,

$z^3 = 27(\cos 270^\circ + i \sin 270^\circ)$

$= 27[\cos(270 + 360k) + i \sin(270 + 360k)]$ [This is general polar form.]

$z = [27\{\cos(270 + 360k) + i \sin(270 + 360k)\}]^{1/3}$

$= 3\left\{\cos\left(\frac{270 + 360k}{3}\right) + i \sin\left(\frac{270 + 360k}{3}\right)\right\}$

(Using De Moivre's Theorem) where k = 0, 1, 2

When k = 0,

$z = 3(\cos 90 + i \sin 90)$

$= 3(0 + i.1) = 3i$

When k = 1,

$z = 3\left(\cos\frac{630}{3} + i \sin\frac{630}{3}\right)$

$= 3(\cos 210 + i \sin 210)$

$= 3\left(\frac{-\sqrt{3}}{2} - i.\frac{1}{2}\right)$

$= \frac{-3\sqrt{3}}{2} - \frac{3}{2}i$

When k = 2,

$z = 3\left(\cos\frac{990}{3} + i \sin\frac{990}{3}\right)$

$= 3(\cos 330 + i \sin 330^\circ)$

$= 3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= \frac{3\sqrt{3}}{2} - \frac{3}{2}i.$

49. 2075 Set B Q.No. 14

State De-Moivre's theorem for a positive inc n. Using De-Moivre's theorem, find the fou roots of $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$.

Please refer to 2076 Set C Q.No. 14

50. 2075 Set C Q.No. 14

Define conjugate of a complex number. W its geometrical meaning.

Also, find the square root of $\frac{8 - 6i}{1 + i}$

SOLUTION

Let $z = a + ib$ be a complex number. The conjuga z, denoted by $\bar{z}$, is defined by $\bar{z} = a - ib$.

Geometrically, the complex conjugate of a com number is the reflection of the complex number a the x–axis (real axis).

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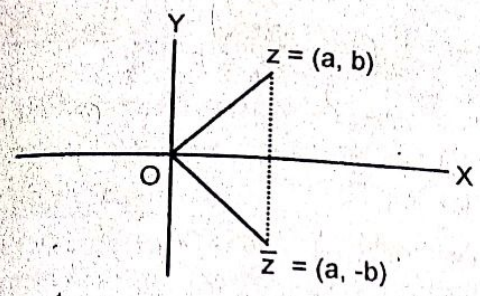

Next part

$$\frac{8-6i}{1+i} = \frac{8-6i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{8-6i-8i+6i^2}{i-i^2} = \frac{8-14i-6}{1-(-1)} = \frac{2-14i}{2}$$

$$= 1-7i$$

Let $x + iy$ be the square roots of $1 - 7i$

Then, $(x+iy)^2 = 1-7i$

or, $x^2 + 2.x.iy + (iy)^2 = 1-7i$

or, $x^2 + 2xyi + i^2y^2 = 1-7i$

or, $x^2 - y^2 + 2xyi = 1-7i$

Comparing real and imaginary parts, we get,

$x^2 - y^2 = 1$ ... (i)

$2xy = -7$

$xy = \frac{-7}{2}$ ... (ii)

We have

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= (1)^2 + (-7)^2$

$= 50 = (5\sqrt{2})^2$

or, $x^2 + y^2 = 5\sqrt{2}$ ... (iii)

Adding (i) and (iii)

$x^2 - y^2 = 1$

$x^2 + y^2 = 5\sqrt{2}$

$2x^2 = 5\sqrt{2} + 1$

or, $x^2 = \frac{5\sqrt{2}+1}{2}$

or, $x = \pm\sqrt{\frac{5\sqrt{2}+1}{2}}$

Substituting the value of x^2 in (iii)

$\frac{5\sqrt{2}+1}{2} + y^2 = 5\sqrt{2}$

or, $y^2 = 5\sqrt{2} - \frac{5\sqrt{2}+1}{2}$

$= \frac{10\sqrt{2}-5\sqrt{2}-1}{2} = \frac{5\sqrt{2}-1}{2}$

Since from (ii) xy is negative, so x and y are of opposite signs.

$\therefore x = \sqrt{\frac{5\sqrt{2}+1}{2}}$ and $y = -\sqrt{\frac{5\sqrt{2}-1}{2}}$

or, $x = -\sqrt{\frac{5\sqrt{2}+1}{2}}$ and $y = \sqrt{\frac{5\sqrt{2}-1}{2}}$

Hence the square roots are

$\sqrt{\frac{5\sqrt{2}+1}{2}} - i\sqrt{\frac{5\sqrt{2}-1}{2}}$

and $-\sqrt{\frac{5\sqrt{2}+1}{2}} + i\sqrt{\frac{5\sqrt{2}-1}{2}}$

i.e. $\pm\left(\sqrt{\frac{5\sqrt{2}+1}{2}} - i\sqrt{\frac{5\sqrt{2}-1}{2}}\right)$.

51. 2074 Supp Q.No. 14

Define Modulus of a complex number. If z and w are two complex numbers, prove that: $|z+w|^2 = |z|^2 + |w|^2 + 2\,Re\,(z\bar{w})$. [6]

SOLUTION

If $z = x + iy$ be a given complex number then modulus of z, denoted by $|z|$, is defined by

$|z| = \sqrt{x^2+y^2}$.

Next Part:

Let $z = a + ib$ and $w = c + id$

$\therefore |z| = \sqrt{a^2+b^2}$, $|w| = \sqrt{c^2+d^2}$

Now, $z\bar{w} = (a+ib)(c-id)$

$= ac - i\,ad + i\,bc - i^2\,bd$

$= ac + i(bc-ad) + bd$

$= (ac+bd) + i(bc-ad)$

$Re\,(z\bar{w}) = ac + bd$

Also, $z+w = (a+ib) + (c+id)$

$= (a+c) + i(b+d)$

L.H.S. $= |z+w|^2 = (a+c)^2 + (b+d)^2$

R.H.S. $= |z|^2 + |w|^2 + 2Re\,(z\bar{w})$

$= a^2 + b^2 + c^2 + d^2 + 2(ac+bd)$

$= a^2 + 2ac + c^2 + b^2 + 2bd + d^2$

$= (a+c)^2 + (b+d)^2$

$\therefore$ L.H.S. = R.H.S. proved

Alternative method:

L.H.S. $= |z+w|^2$

$= (z+w)(\overline{z+w})$ $[\because |z|^2 = z\bar{z}]$

$= (z+w)(\bar{z}+\bar{w})$ $\left[\because \overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2\right]$

$= z\bar{z} + w\bar{w} + z\bar{w} + \bar{z}w$

$= |z|^2 + |w|^2 + z\bar{w} + \overline{z\bar{w}}$ $\left[\because z\bar{z} = |z|^2 \text{ and } \bar{\bar{w}} = w\right]$

$= |z|^2 + |w|^2 + 2Re\,(z\bar{w})$ $[\because z + \bar{z} = 2Re(z)]$

= R.H.S.

52. 2074 Set A Q.No. 14

State De Moivre's theorem. Using De Moivre's theorem, find the cube roots of unity. If $\omega = \frac{-1+\sqrt{3}\,i}{2}$ be complex cube root of unity, show that $\omega^2 = \frac{-1-\sqrt{3}\,i}{2}$. [6]

SOLUTION

Statement of the De-Moivre's Theorem: For any position integer n,

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

In fact the theorem is true not only for a positive integer but also for any real number n.

Second part:

Suppose, $z_k = (1)^{1/3} = (1+i.0)^{1/3} = (\cos 0° + i\sin 0°)^{1/3}$

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or, $z_k = [\cos(0° + k.360°) + i\sin(0° + k.360°)]^{1/3}$, k = 0, 1, 2.

By using De-Moivre's theorem

$$z_k = \left[\cos\left(\frac{0° + k.360°}{3}\right) + i\sin\left(\frac{0° + k.360°}{3}\right)\right]$$

or, $z_k = \cos(k \times 120°) + i\sin(k \times 120°)$

Putting k = 0, 1, 2 the required cube roots of unity are as follows:

If k = 0, then $z_0 = \cos 0° + i\sin 0° = 1 + i \times 0 = 1$

If k = I, then $z_1 = \cos 120° + i\sin 120°$

$$= -\frac{1}{2} + i \times \frac{\sqrt{3}}{2} = \frac{-1 + i\sqrt{3}}{2}$$

If k = 2, then

$z_2 = \cos 240° + i\sin 240°$

$= \cos(180° + 60°) + i\sin(180° + 60°)$

$= -\cos 60° - i\sin 60°$

$$= -\frac{1}{2} - i\frac{\sqrt{3}}{2} = \frac{-1 - i\sqrt{3}}{2}$$

∴ The three cube roots of unity are 1, $\frac{-1 + i\sqrt{3}}{2}$ and $\frac{-1 - i\sqrt{3}}{2}$.

Last Part:

Given $\omega = \frac{-1 + \sqrt{3}\,i}{2}$

Then, $\omega^2 = \left(\frac{-1 + \sqrt{3}\,i}{2}\right)^2$

$$= \frac{(-1)^2 + 2.(-1).\sqrt{3}\,i + (\sqrt{3}\,i)^2}{4}$$

$$= \frac{1 - 2\sqrt{3}\,i + 3i^2}{4}$$

$$= \frac{1 - 2\sqrt{3}\,i - 3}{4} = \frac{-2 - 2\sqrt{3}\,i}{4} = \frac{-1 - \sqrt{3}i}{2}$$

---

53. 2074 Set B Q.No. 14

Define De Moivre's theorem for a positive index n. Using De Moivre's theorem find the square roots of $-1 + \sqrt{3}\,i$. [6]

SOLUTION

Statement of De Moivre's Theorem: If n is any positive integer, then

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Next Part:

Let $z = -1 + \sqrt{3}i$

Here, $x = -1$, $y = \sqrt{3}$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3} = \tan 120°$$

$\theta = 120°$

In polar form

$z = 2(\cos 120° + i\sin 120)$

$= 2\{\cos(120 + 360k) + i\sin(120 + 360k)\}$

$\sqrt{z} = z^{1/2}$

$= [2\{\cos(120 + 360k) + i\sin(120 + 360k)\}]^{1/2}$

$$= \sqrt{2}\left\{\cos\left(\frac{120 + 360k}{2}\right) + i\sin\left(\frac{120 + 360k}{2}\right)\right\}$$

(Using De Moivre's theorem), where, k = 0,1

$= \sqrt{2}\{\cos(60 + 180k) + i\sin(60 + 180k)\}$

When k = 0,

$\sqrt{z} = \sqrt{2}(\cos 60 + i\sin 60)$

$$= \sqrt{2}\left(\frac{1}{2} + i.\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{2}\,\frac{1}{2}(1 + \sqrt{3}i) = \frac{1}{\sqrt{2}}(1 + \sqrt{3}i)$$

When k = 1,

$\sqrt{z} = \sqrt{2}\{\cos(60 + 180) + i\sin(60 + 180)\}$

$= \sqrt{2}(-\cos 60 - i\sin 60)$

$$= \sqrt{2}\left(\frac{-1}{2} - i.\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{\sqrt{2}}(1 + \sqrt{3}i)$$

∴ Hence the required square roots are $\pm\frac{1}{\sqrt{2}}(1 + \sqrt{3}\,i)$

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54. 2073 Supp Q.No. 14

If z and w are two complex numbers, prove that $|z+w| \le |z| + |w|$. [6]

SOLUTION

$|z + w|^2 = (z + w)\overline{(z + w)}$ $[\because |z|^2 = z\bar{z}]$

$= (z + w)(\bar{z} + \bar{w})$ $[\because \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2]$

$= z\bar{z} + w\bar{w} + z\bar{w} + \bar{z}w$

$= |z|^2 + |w|^2 + z\bar{w} + \overline{z\bar{w}}$ $[\because z\bar{z} = |z|^2$ and $\bar{\bar{w}} = w]$

$= |z|^2 + |w|^2 + 2\mathrm{Re}(z\bar{w})$ $[\because z + \bar{z} = 2\mathrm{Re}(z)]$

$\le |z|^2 + |w|^2 + 2|z\bar{w}|$

$= |z|^2 + |w|^2 + 2|z|\,|\bar{w}|$ $[\because w = |\bar{w}|]$

$= |z|^2 + |w|^2 + 2|z|\,|w|$

$= (|z| + |w|)^2$

∴ $|z + w| \le |z| + |w|$.

Alternative method

Let $z = a + ib$ and $w = c + id$. Then,

$z + w = (a + ib) + (c + id) = (a + c) + i(b + d)$

Also,

$|z| = \sqrt{a^2 + b^2}$,

$|w| = \sqrt{c^2 + d^2}$

$|z + w| = \sqrt{(a + c)^2 + (b + d)^2}$.

Now, $|z| + |w| \ge |z + w|$ will be true if

$$\sqrt{a^2 + b^2} + \sqrt{c^2 + d^2} \ge \sqrt{(a + c)^2 + (b + d)^2}$$

Squaring both sides,

i.e., $a^2 + b^2 + 2\sqrt{a^2 + b^2}\,\sqrt{c^2 + d^2} + c^2 + d^2 \ge a^2 + 2ac + c^2 + b^2 + 2bd + d^2$

i.e., $2\sqrt{a^2 + b^2}\,\sqrt{c^2 + d^2} \ge 2(ac + bd)$

i.e., $\sqrt{a^2 + b^2}\,\sqrt{c^2 + d^2} \ge ac + bd$

i.e., $(a^2 + b^2)(c^2 + d^2) \ge (ac + bd)^2$

i.e., $a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \ge a^2c^2 + 2abcd + b^2d^2$

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i.e., $a^2d^2 - 2abcd + b^2c^2 \geq 0$

i.e., $(ad - bc)^2 \geq 0$ which is true for all real numbers a, b, c and d.

Hence, $|z| + |w| \geq |z + w|$.

55. 2073 Supp Q.No. 14 OR

State De-moivre's theorem. Using De-moivre's theorem find the cube roots of unity. Also establish the properties of the cube roots of unity. [6]

SOLUTION

First & Second Part: Please refer to 2074 Set A Q.No. 14

Properties

1. Each imaginary cube root of unity is the square roots of thee other:

For, $\left(\frac{-1+\sqrt{3}\,i}{2}\right)^2 = \frac{1-2\sqrt{3}\,i+3i^2}{4}$

$= \frac{1-2\sqrt{3}\,i-3}{4} = \frac{-1-\sqrt{3}\,i}{2}$

and $\left(\frac{-1-\sqrt{3}\,i}{2}\right)^2 = \left(\frac{1+\sqrt{3}\,i}{2}\right)^2$

$= \frac{1+2\sqrt{3}\,i+3i^2}{4}$

$= \frac{1+2\sqrt{3}\,i-3}{4}$

$= \frac{-2+2\sqrt{3}\,i}{4} = \frac{-1+\sqrt{3}\,i}{2}$

ii. The product of two imaginary cube roots of unity is 1.

For, $\left(\frac{-1+\sqrt{3}\,i}{2}\right)\cdot\left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$= \frac{(-1)^2-(\sqrt{3}\,i)^2}{4} = \frac{1-3i^2}{4} = \frac{4}{4} = 1$

iii. The sum of three cube roots of unity is zero.

For, $1 + \left(\frac{-1+\sqrt{3}\,i}{2}\right) + \left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$= 1 - \frac{1}{2} + \frac{\sqrt{3}\,i}{2} - \frac{1}{2} - \frac{\sqrt{3}\,i}{2} = 1 - 1 = 0$

56. 2073 Set C Q.No. 14

State De Moivre's theorem for any positive index n. Using De Moivre's theorem find the square roots of $4 + 4\sqrt{3}\,i$. [6]

SOLUTION

De Moivre's theorem: If n is any positive index, then $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Next part

Let $z = 4 + 4\sqrt{3}\,i$

Here, $x = 4$, $y = 4\sqrt{3}$

We have, $r = \sqrt{x^2+y^2} = \sqrt{4^2 + (4\sqrt{3})^2} = 8$

$\tan\theta = \frac{y}{x} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$

In polar form,

$z = r(\cos\theta + i\sin\theta)$

$= 8(\cos 60° + i\sin 60°)$

$= 8\{\cos(60 + 360k) + i\sin(60 + 360k)\}$ in general polar form

$\therefore \sqrt{z} = z^{1/2}$

$= 8\{\cos(60 + 360k) + i\sin(60 + 360k)\}]^{1/2}$

$= 8^{1/2}\left\{\cos\left(\frac{60+360k}{2}\right) + i\sin\left(\frac{60+360k}{2}\right)\right\}$

[Using De Moivre's theorem] where, $k = 0, 1$.

$= 2\sqrt{2}\{\cos(30 + 180k) + i\sin(30 + 180k)$

When $k = 0$,

$\sqrt{z} = 2\sqrt{2}\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right)$

$= \sqrt{6} + i\sqrt{2}$

When $k = 1$

$\sqrt{z} = 2\sqrt{z}\{\cos(30 + 180) + i\sin(30 + 180)\}$

$= 2\sqrt{z}(-\cos 30° - i\sin 30°)$

$= 2\sqrt{z}\left(-\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right)$

$= -(\sqrt{6} + i\sqrt{2})$

Hence, the required square roots are $\pm(\sqrt{6} + \sqrt{2}\,i)$

57. 2073 Set D Q.No. 14

State De Moivre's theorem. Using De Moivre's theorem find the cube roots of unity. Also show that sum of the three cube roots of unity is zero. [6]

SOLUTION

First part: Please refer to 2074 Set A Q.No. 14

Last part:

$1 + \left(\frac{-1+\sqrt{3}i}{2}\right) + \left(\frac{-1-\sqrt{3}i}{2}\right)$

$= 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i = 1 - 1 = 0$

58. 2072 Supp. Q.No. 14

If z and w be two complex numbers, prove that: $|z + w| \leq |z| + |w|$. [6]

➥ Please refer to 2073 Supp Q.No. 14

59. 2072 Supp. Q.No. 14 OR

Define conjugate of a complex number. Find the square roots of the complex number $\frac{2-36i}{2+3i}$. [6]

SOLUTION

Conjugate of a complex number:

Let $z = x + iy$ be a complex number. Then its conjugate, denoted by $\bar{z}$, is defined by $\bar{z} = x - iy$.

Next part:

Here,

$\frac{2-36i}{2+3i} = \frac{2-36i}{2+3i} \times \frac{2-3i}{2-3i}$

$= \frac{4-6i-72i-108i^2}{2^2-9i^2}$

$= \frac{4-78i-108}{4+9} = \frac{-104-78i}{13}$

$= -8 - 6i$

Let $x + iy$ be the square roots of $-8 - 6i$. Then,

$(x + iy)^2 = -8 - 6i$

or, $x^2 + 2xyi + i^2y^2 = -8 - 6i$

or, $x^2 - y^2 + 2xy\,i = -8 - 6i$

Equating real and imaginary parts, we get

$x^2 - y^2 = -8$ ...(i)

and $2xy = -6$

$xy = -3$ ...(ii)

We have, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (-8)^2 + 4(-3)^2$

$= 100$

$= (10)^2$

$\therefore \quad x^2 + y^2 = 10$ ...(iii)

Adding (i) and (ii), we get,

$x^2 - y^2 = -8$

$x^2 + y^2 = 10$

$2x^2 = 2$

$x^2 = 1$

$\therefore \quad x = \pm 1$

Putting the value of x^2 in (i), we get

$1 - y^2 = -8$

$y^2 = 9$

$y = \pm 3$

From (ii) $xy = -3$ which is negative, so x and y have opposite sign,

$\therefore \quad x = 1, y = -3$ and $x = -1, y = 3$

Required square roots are

$1 - 3i$ and $-1 + 3i = -(1 - 3i)$

i.e. $\pm(1 - 3i)$

60. 2072 Set C Q.No. 14

If z and w are two complex numbers, prove that: $|z + w|^2 = |z|^2 + |w|^2 + 2 \text{ Re } (z\bar{w})$. [6]

➥ Please refer to 2074 Supp Q.No. 14

61. 2072 Set D Q.No. 14

State De Moivre's theorem. Using De Moivre's theorem solve $Z^6 = 1$. [6]

SOLUTION

De Moivre's theorem: If n is any positive integer, then

$[r(\cos\theta + i \sin\theta)]^n = r^n (\cos n\theta + i \sin n\theta)$

Next part

$z^6 = 1 = 1 (\cos 0° + i \sin 0°)$

or, $z = [1 (\cos 0° + i \sin 0°)]^{1/6}$

$= [1\{\cos(0° + k\,360°) + i \sin(0° + k\,360°)\}]^{1/6}$

$= (1)^{1/6}\left\{\cos\left(\frac{0° + k\,360°}{6}\right) + i \sin\left(\frac{0° + k\,360°}{6}\right)\right\}$

$\left[\because \text{By De-Moivre's theorem}\right]$ where, $k = 0, 1, 2, 3, 4, 5$

$\therefore \quad z = \cos\left(\frac{k\,360°}{6}\right) + i \sin\left(\frac{k\,360°}{6}\right)$ ... (i)

Now, putting $k = 0, 1, 2, 3, 4, 5$ in (1), we get

$z_0 = \cos\left(\frac{0 \times 360°}{6}\right) + i \sin\left(\frac{0 \times 360°}{6}\right)$

$= \cos 0° + i \sin 0°$

$= 1 + i \times 0$

$= 1$

$z_1 = \cos\left(\frac{1 \times 360°}{6}\right) + i \sin\left(\frac{1 \times 360°}{6}\right)$

$= \cos 60° + i \sin 60°$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$z_2 = \cos\left(\frac{2 \times 360°}{6}\right) + i \sin\left(\frac{2 \times 360°}{6}\right)$

$= \cos 120° + i \sin 120°$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$z_3 = \cos\left(\frac{3 \times 360°}{6}\right) + i \sin\left(\frac{3 \times 360°}{6}\right)$

$= \cos 180° + i \sin 180°$

$= -1 + i \times 0$

$= -1$

$z_4 = \cos\left(\frac{4 \times 360°}{6}\right) + i \sin\left(\frac{4 \times 360°}{6}\right)$

$= \cos 240° + i \sin 240°$

$= \cos(180° + 60°) + i \sin(180° + 60°)$

$= -\cos 60° + i(-\sin 60°)$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

$z_5 = \cos\left(\frac{5 \times 360°}{6}\right) + i \sin\left(\frac{5 \times 360°}{6}\right)$

$= \cos 300° + i \sin 300°$

$= \cos(360° - 60°) + i \sin(360° - 60°)$

$= \cos 60° + i(-\sin 60°)$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} i$

62. 2072 Set E Q.No. 13

State and prove De Moivre's theorem. Is the theorem true for integers? Justify your answer. [6]

SOLUTION

Statement of De Moivre's theorem:

If n is any positive integer, then

$[r(\cos\theta + i \sin\theta)]^n = r^n (\cos n\theta + i \sin n\theta)$

Proof: Let n be positive integer. We prove the theorem by induction method.

When $n = 1$,

$[r(\cos\theta + i\sin\theta)]^1 = r(\cos\theta + i\sin\theta) = r^1(\cos 1\cdot\theta + i\sin 1\cdot\theta)$

Thus the theorem is true for $n = 1$.

Let us suppose that the theorem is true for $n = k$.

Then

$[r(\cos\theta + i \sin\theta)]^k = r^k (\cos k\theta + i \sin k\theta)$ ...(i)

We prove that the theorem is true for $n = k + 1$.

Multiplying both sides of (i) by $r(\cos\theta + i \sin\theta)$, we get,

$[r(\cos\theta + i\sin\theta)]^k \cdot r(\cos\theta + i\sin\theta) = r^k(\cos k\theta + i\sin k\theta) \cdot r(\cos\theta + i\sin\theta)$

$\therefore \quad [r(\cos\theta + i \sin\theta)]^{k+1}$

$= r^{k+1}\{\cos(k\theta + \theta) + i \sin(k\theta + \theta)\}$

$= r^{k+1}\{\cos(k + 1)\theta + i \sin(k + 1)\theta\}$

This shows that the theorem is true for $n = k + 1$, whenever it is true for $n = k$. Then, by principle of mathematical induction, the theorem is true for all positive integer n.

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Last part

When $n = -k, k > 0$

$$[r(\cos\theta + i\sin\theta)]^{-k}$$

$$= \frac{1}{[r(\cos\theta + i\sin\theta)]^{k}}$$

$$= \frac{1}{r^{k}(\cos k\theta + i\sin k\theta)}$$

$$= \frac{r^{-k}}{\cos k\theta + i\sin k\theta} \cdot \frac{\cos k\theta - i\sin k\theta}{\cos k\theta - i\sin k\theta}$$

$$= \frac{r^{-k}(\cos k\theta - i\sin k\theta)}{\cos^2 k\theta - i^2\sin^2 k\theta}$$

$$= \frac{r^{-k}\{\cos(-k\theta) + i\sin(-k\theta)\}}{\cos^2 k\theta + \sin^2 k\theta}$$

$$= r^{-k}\{\cos(-k\theta) + i\sin(-k\theta)\}$$

$$= r^{-k}\{\cos(-k)\theta + i\sin(-k)\theta\}$$

This shows that the theorem is true for $n = -k, k > 0$.

Hence, the theorem is true for any integer.

63. 2071 Supp. Q.No. 13

State De Moiver's theorem. Apply it to compute 4th roots of -1. [6]

SOLUTION

Statement of De Moivre's theorem:

If n is any positive integer, then

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Next part:

Let $z^4 = -1 = -1 + 0\cdot i$

Here, $x = -1, y = 0$

We have, $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 0^2} = 1$

$\tan\theta = \frac{y}{x} = \frac{0}{-1} = 0 = \tan 180^\circ$

$\therefore \theta = 180^\circ$

In polar form

$z^4 = r(\cos\theta + i\sin\theta)$

$= 1(\cos 180^\circ + i\sin 180^\circ)$

$= \cos(180 + 360k) + i\sin(180 + 360k)$

$z = \{\cos(180 + 360k) + i\sin(180 + 360k)\}^{1/4}$

$$= \cos\left(\frac{180 + 360k}{4}\right) + i\sin\left(\frac{180 + 360k}{4}\right)$$

(using De Moivre's theorem) where, $k = 0, 1, 2, 3$

When $k = 0$, $z = \cos 45^\circ + i\sin 45^\circ$

$$= \frac{1}{\sqrt{2}} + i\cdot\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1 + i)$$

When $k = 1$, $z = \cos 135^\circ + i\sin 135^\circ$

$$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i = -\frac{1}{\sqrt{2}}(1 - i)$$

When $k = 2$, $z = \cos 225^\circ + i\sin 225^\circ$

$$= -\frac{1}{\sqrt{2}} - i\cdot\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}(1 + i)$$

When $k = 3$, $z = \cos 315^\circ + i\sin 315^\circ$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i = \frac{1}{\sqrt{2}}(1 - i)$$

Hence the required roots are $\pm\frac{1}{\sqrt{2}}(1 + i)$ and $\pm\frac{1}{\sqrt{2}}(1 - i)$

64. 2071 Set C Q.No. 14

State De Moivre's theorem. Using De Moivre's theorem find the square roots of $4 + 4\sqrt{3}\,i$. [6]

➥ Please refer to 2073 Set C Q.No. 14

65. 2071 Set D Q.No. 14

State De Moivre's theorem. Using De Moivre's theorem find the square roots of $2 + 2\sqrt{3}\,i$. [6]

SOLUTION

Statement of De Moivre's theorem:

If n is any positive integer, then

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Next part

Let $z = 2 + 2\sqrt{3}\,i$

Here, $x = 2, y = 2\sqrt{3}$

We have, $r = \sqrt{x^2 + y^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{2} = \sqrt{3} = \tan 60^\circ$

$\therefore \theta = 60^\circ$

In polar form

$z = r(\cos\theta + i\sin\theta)$

$= 4(\cos 60^\circ + i\sin 60^\circ)$

$= 4\{\cos(60^\circ + 360k) + i\sin(60^\circ + 360^\circ k)\}$

Now, $\sqrt{z} = z^{1/2}$

$= [4\{\cos(60 + 360k) + i\sin(60 + 360k)\}]^{1/2}$

$$= 4^{1/2}\left\{\cos\left(\frac{60 + 360k}{2}\right) + i\sin\left(\frac{60 + 360k}{2}\right)\right\}$$ (Using De Moivre's theorem) where, $k = 0, 1$

$= 2\{\cos(30 + 180k) + i\sin(30 + 180k)\}$

When $k = 0$, $\sqrt{z} = 2(\cos 30^\circ + i\sin 30^\circ)$

$$= 2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \sqrt{3} + i$$

When $k = 1$,

$\sqrt{z} = 2\{\cos(30 + 180) + i\sin(30 + 180)\}$

$= 2(-\cos 30 - i\sin 30^\circ)$

$$= 2\left(\frac{-\sqrt{3}}{2} - i\cdot\frac{1}{2}\right) = -\sqrt{3} - i = -(\sqrt{3} + i)$$

$\therefore$ Required square roots are $\pm(\sqrt{3} + i)$.

66. 2070 Supp Q.No. 14

Find the square roots of the complex number $\frac{2 - 36i}{2 + 3i}$. [6]

➥ Please refer to 2072 Supp. Q.No. 14 OR

67. 2070 Set C Q.No. 14

Find the square root of the complex number $-5 + 12i$. [6]

SOLUTION

Let $x + iy$ be the square roots of $-5 + 12i$. Then,

or, $x^2 + 2xyi + i^2y^2 = -5 + 12i$

or, $x^2 - y^2 + 2xyi = -5 + 12i$

Equating real and imaginary parts, we get

$x^2 - y^2 = -5$ ...(i)

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$2xy = 12$

$xy = 6$ ...(ii)

We have,

$(x^2 + y^2)^2 = (x^2 - y^2) + 4x^2y^2$

$= (-5)^2 + 4(6)^2$

$= 169$

$= (13)^2$

$\therefore x^2 + y^2 = 13$ ...(iii)

Adding (i) and (iii), we get

$x^2 - y^2 = -5$

$x^2 + y^2 = 13$

$2x^2 = 8$

$x^2 = 4$

$\therefore x = \pm 2$

Substituting the value of x^2 in (i), we get,

$4 - y^2 = -5$

or, $y^2 = 9$

$\therefore y = \pm 3$

Since from (ii) $xy = 6$ which is positive, so x and y have same sign.

$\therefore x = 2, y = 3$ and $x = -2, y = -3$

The required square roots are

$2 + 3i$ and $-2 - 3i = -(2 + 3i)$

i.e. $\pm(2 + 3i)$

68. 2070 Set D Q.No. 14

Define absolute value of a complex number. If z and w are two complex numbers, prove that: $|z + w| \leq |z| + |w|$ [6]

➥ **Conjugate of a complex number:**

Please refer to 2072 Supp. Q.No. 14 OR

Next part: Please refer to 2073 Supp Q.No. 14

69. 2070 Set D Q.No. 14 Or

Find the cube roots of unity. Also, establish the properties of cube roots of unit [6]

➥ Please refer to 2073 Supp Q.No. 14 OR

70. 2069 Supp Q.No. 14

Define conjugate of a complex number and write any four of its properties. Also prove $|z_1| + |z_2| \geq |z_1 + z_2|$ where z_1 and z_2 are two complex numbers. [6]

➥ **Conjugate of a complex number:**

Let $z = x + iy$ be a complex number. Then its conjugate, denoted by $\bar{z}$, is defined by $\bar{z} = x - iy$.

Any four properties of conjugates: Let z and w be the two complex numbers. Then,

i. $\text{Re}(z) = \frac{1}{2}(z + \bar{z})$

ii. $\text{Im}(z) = \frac{1}{2i}(z - \bar{z})$

iii. $\overline{z + w} = \bar{z} + \bar{w}$

iv. $\overline{zw} = \bar{z}\,\bar{w}$

Next part: Please refer to 2073 Supp Q.No. 14

71. 2069 [Set A] Q. No. 14

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, prove that: $z_1z_2 = r_1r_2\{\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\}$ and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\}$. [6]

SOLUTION

$z_1z_2 = r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2)$

$= r_1r_2(\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2)$

$= r_1r_2\{\cos\theta_1\cos\theta_2 + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2) - \sin\theta_1\sin\theta_2\}$

$= r_1r_2\{(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\}$

$= r_1r_2\{\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\}$

$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} \times \frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

$= \frac{r_1(\cos\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - i^2\sin\theta_1\sin\theta_2)}{r_2\{\cos^2\theta_2 - i^2\sin^2\theta_2}$

$= \frac{r_1\{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\}}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}$

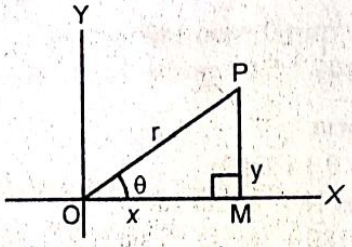
$= \frac{r_1}{r_2}\{\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\}$

72. 2069 [Set A] Q. No. 14 OR

Define complex number. Express a comple number into polar form. State De-Moivre's theorem. Using De'Moivre's theorem, find th cube roots of unity. [6

SOLUTION

A number which is in the form $z = x + iy = (x, y$ where x and y are real numbers and $i = \sqrt{-1}$ (iota), i called a complex number.



Polar form of a complex number:

Let $z = x + iy$ be a complex number in Cartesian form This number can be represented by a point P in th complex plane as shown in the figure. From figure, OM $= x$ and $PM = y$.

Let $OP = r$ and $\angle XOP = Q$.

Then, from right angled ΔPOM,

$\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{x}$

$x = r\cos\theta$ and $y = r\sin\theta$

Hence, $z = x + iy$

$= r\cos\theta + i \cdot r\sin\theta$

$= r(\cos\theta + i\sin\theta)$, where $r = \sqrt{x^2 + y^2}$,

$\theta = \tan^{-1}\left(\frac{y}{x}\right), z \neq 0$

$\therefore$ Polar form of a complex number is $r(\cos\theta + i\sin\theta)$

Next part: Please refer to 2074 Set A Q.No. 14

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73. 2069 [Set B] Q. No. 14

State De Moivre's theorem for any positive index n. Using De Moivre's theorem find the square roots of $4 + 4\sqrt{3}\,i$. [6]

➥ Please refer to 2073 Set C Q.No. 14

74. 2068 Q.No. 14

State De-Moivre's theorem. Using De Moivre's theorem, find the square roots of $-2 - 2\sqrt{3}\,i$. [6]

SOLUTION

De-Moivre's theorem: If n is any positive integer then $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$.

Let $z = -2 - 2\sqrt{3}\,i$

$\therefore x = -2$ and $y = -2\sqrt{3}$

We know that $r = \sqrt{x^2 + y^2}$

$= \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4$

and $\tan\theta = \frac{y}{x} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

$= \tan 240° \left(\because \begin{matrix} x < 0 \\ y < 0 \end{matrix}\right)$

In polar form

$z = r(\cos\theta + i\sin\theta)$

$= 4(\cos 240° + i\sin 240°)$

$= 4\{\cos(240 + 360k) + i\sin(240 + 360k)\}$

$\therefore \sqrt{z} = z^{1/2}$ $[4\{\cos(240+360k)+i\sin(240+360k)\}]^{1/2}$

$= 4^{1/2}\left\{\cos\left(\frac{240+360k}{2}\right) + i\sin\left(\frac{240+360k}{2}\right)\right\}$

[Using De Moivre's theorem] where, k = 0, 1

When k = 0,

$\sqrt{z} = 2(\cos 120° + i\sin 120°)$

$= 2\left(-\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}$

When k = 1,

$\sqrt{z} = 2(\cos 300° + i\sin 300°)$

$= 2\left(\frac{1}{2} - i\cdot\frac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3} = -(-1 + i\sqrt{3})$

Required square roots are $\pm(-1 + i\sqrt{3})$

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

75. 2070 Old Q.No. 11 a

Find the square root of $\frac{5 + 12i}{3 - 4i}$. [4]

SOLUTION

Given complex number is

$\frac{5+12i}{3-4i} = \frac{5+12\,i}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{15 + 20i + 36i + 48i^2}{9 - 16i^2}$

$= \frac{-33 + 56i}{25}$ $[\because i^2 = -1]$

$= \frac{-33}{25} + \frac{56}{25}\,i$

Let $x + iy$ be the square roots of $\frac{-33}{25} + \frac{56}{25}\,i$

Then, $(x + iy)^2 = -\frac{33}{25} + \frac{56}{25}\,i$

or, $x^2 + 2xyi + i^2y^2 = \frac{-33}{25} + \frac{56}{25}\,i$

or, $x^2 - y^2 + i2xy = -\frac{33}{25} + \frac{56}{25}\,i$

Equating real and imaginary parts, we get

$x^2 - y^2 = \frac{-33}{25}$ … (i)

$2xy = \frac{56}{25}$ … (ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= \left(\frac{-33}{25}\right)^2 + \left(\frac{56}{25}\right)^2$

$= \frac{1089}{625} + \frac{3136}{625} = \frac{4225}{625} = \left(\frac{65}{25}\right)^2$

$\therefore x^2 + y^2 = \frac{65}{25}$ … (iii)

Adding equations (i) and (iii), we get

$2x^2 = \frac{32}{25}$

or, $x^2 = \frac{16}{25}$

$\therefore x = \pm\frac{4}{5}$

From equation (ii), when $x = \frac{4}{5}$, $y = \frac{7}{5}$

and when $x = \frac{-4}{5}$, $y = \frac{-7}{5}$

Hence, the square roots are

$\frac{4}{5} + \frac{7}{5}\,i$ and $\frac{-4}{5} - \frac{7}{5}\,i$

$= -\left(\frac{4}{5} + \frac{7}{5}\,i\right)$ i.e. $\pm\left(\frac{4}{5} + \frac{7}{5}\,i\right)$

76. 2071 Old Q.No. 11a

State and prove De Moivre's theorem for any positive integer. [4]

➥ Please refer to 2072 Set E Q.No. 13

77. 2069 [Set A] Old Q. No. 11a

State De Moivre's theorem hence determines the cube roots of unity. [4]

➥ Please refer to 2074 Set A Q.No. 14

78. 2068 Old Q.No. 11a

Find the cube roots of unity. If ω be an imaginary cube root of unity, prove that: $1 + \omega + \omega^2 = 0$ [4]

SOLUTION

Let z be the cube of roots of unity. Then,

$z^3 = 1$

or, $z^3 - 1 = 0$

or, $(z - 1)(z^2 + z + 1) = 0$

Either, $z - 1 = 0 \Rightarrow z = 1$

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or, $z^2 + z + 1 = 0$

or, $z = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2}$

So, the three cubes roots of unity are, 1, $\frac{-1+\sqrt{3}\,i}{2}$ and $\frac{-1-\sqrt{3}\,i}{2}$.

Next part:

We know that, if $\frac{-1+\sqrt{3}\,i}{2} = \omega$

then $\frac{-1-\sqrt{3}\,i}{2} = \omega^2$

$\therefore 1 + \omega + \omega^2 = 1 + \left(\frac{-1+\sqrt{3}\,i}{2}\right) + \left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$= 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$= 1 - 1 = 0$. Proved

79. 2067 Q.No. 11a

Find the square root of $\frac{5+12i}{3-4i}$ [4]

➥ Please refer to 2070 Old Q.No. 11 a

80. 2066 Q.No. 11a

If $\sqrt{x+iy} = a + ib$, prove that $\sqrt{x-iy} = a - ib$ [4]

SOLUTION

Given, $\sqrt{x+iy} = a + ib$

Squaring on both sides,

$x + iy = a^2 + 2abi + i^2b^2$

or, $x + iy = a^2 - b^2 + 2abi$ $(\because i^2 = -1)$

Comparing real and imaginary part,

$x = a^2 - b^2$, $y = 2ab$

To prove $\sqrt{x-iy} = a - ib$

L.H.S. $= \sqrt{x-iy}$

$= \sqrt{a^2 - b^2 - i.2ab}$

$= \sqrt{a^2 - 2a.(ib) + (ib)^2}$

$= \sqrt{(a-ib)^2}$

$= a - ib$

= R.H.S Proved.

81. 2065 Q.No. 11a

Find the square roots of (–7 + 24i). [4]

SOLUTION

Let $x + iy$ be the square roots of $-7 + 24i$. Then,

$\therefore -7 + 24i = (x+iy)^2 = x^2 - y^2 + 2ixy$

Equating real and imaging parts, we get

$x^2 - y^2 = -7$ ... (i)

and $2xy = 24$

or, $xy = 12$ ... (ii)

Now, $x^2 + y^2 = \sqrt{(x^2-y)^2 + 4x^2y^2}$

$[\because a + b = \sqrt{(a-b)^2 + 4ab}]$

or, $x^2 + y^2 = \sqrt{(-7)^2 + 4\times12^2}$

or, $x^2 + y^2 = \sqrt{49 + 576}$

or, $x^2 + y^2 = \sqrt{625}$

or, $x^2 + y^2 = 25$ ... (iii)

$[\because x^2 + y^2$ cannot be negative]

Adding equations (ii) and (iii) we get,

$x^2 - y^2 = -7$

$x^2 + y^2 = 25$

$2x^2 = 18$

$x^2 = 9$

$\therefore x = \pm 3$

Again substituting the value of x in equation (iii), we get

$9 + y^2 = 25$

or, $y^2 = 16$

$\therefore y = \pm 4$

Since $xy = 12$, x and y should have same sign

$\therefore$ Either $x = 3$; $y = 4$ or $x = -3$; $y = -4$

Hence, the square roots are

$x + iy$ i.e. $3 + 4i$ and $-3 - 4i$

i.e. $\pm(3 + 4i)$

82. 2064 Q.No. 12a

Solve: $z^6 = 1$ [4

➥ Please refer to 2072 Set D Q.No. 14

83. 2063 Q.No. 12b

Find the cube roots of unity and discuss thei properties. [4

➥ **First part**: Please refer to 2074 Set A Q.No. 14

Second part: Properties of cubes roots of unity are a follows:

i. Each imaginary cube root of unity is the square of th other.

ii. The product of the two imaginary cube roots of unity i equal to 1 i.e. $\omega.\omega^2 = 1$ i.e $\omega^3 = 1$

iii. The sum of the three cube roots of unity is zero i.e. 1 $\omega + \omega^2 = 0$

84. 2062 Q.No. 11a

Find the cube roots of unity. Write thei properties. [4

➥ **First part**: Please refer to 2068 Old Q.No. 11a

Second part: Please refer to 2063 Q.No. 12b

85. 2061 Q.No. 11a

State De-Moivre's theorem hence solve $z^6 = 1$. [4

SOLUTION

Statement of De-Moivre's theorem: For any positiv integer n,

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Next part: Please refer to 2064 Q.No. 12a

86. 2060 Q.No. 11a

Using De-Moivres theorem, find the fourt roots of unity. [4

SOLUTION

Let $z_k^4 = 1 = 1 + 0 \cdot i = \cos 0° + i\sin 0°$

So, the polar form of $z_k^4 = 1\ (\cos 0° + i\sin 0°)$

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$\therefore z_k = [1\ (\cos 0° + i \sin 0°)]^{1/4}$

$= [1\ \{(\cos 0° + k \times 360°) + i\ (\sin 0° + k \times 360°)\}]^{1/4}$

By using De Moivre's theorem, we get

$$z_k = 1^{1/4}\left[\cos\left(\frac{0° + k\,360°}{4}\right) + i \sin\left(\frac{0° + k\,360°}{4}\right)\right]$$

or, $z_k = [\cos k\,90° + i \sin k\,90°]$

Now, for the fourth roots, putting k = 0, 1, 2, 3, the required four roots are

$z_0 = [\cos (0 \times 90°) + i \sin (0 \times 90°)]$
$= \cos 0° + i \sin 0°$
$= 1 + i \times 0$
$= 1$

$z_1 = [\cos (1 \times 90°) + i \sin (1 \times 90°)]$
$= \cos 90° + i \sin 90°$
$= 0 + i \times 1$
$= i$

$z_2 = [\cos (2 \times 90°) + i \sin (2 \times 90°)]$
$= \cos 180° + i \sin 180°$
$= -1 + i \times 0$
$= -1$

$z_3 = [\cos (3 \times 90°) + i \sin (3 \times 90°)]$
$= \cos 270° + i \sin 270°$
$= \cos (180° + 90°) + i \sin (180° + 90°)$
$= -\cos 90° + i (-\sin 90°)$
$= -0 - i \times 1$
$= -i$

Thus, the required fourth roots of unity are 1, i, −1, −i.

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87. 2059 Q.No. 11a

Find the square roots of z = 7 − 24 i [4]

SOLUTION

Let $x + iy$ be the square roots of $7 - 24i$. Then,

Squaring both sides, we get

$7 - 24i = (x + iy)^2 = x^2 + 2x\,(iy) + (iy)^2$
$= (x^2 - y^2) + i\,(2xy)$

Equating the real and imaginary parts,

$(x^2 - y^2) = 7$ ... (i)

and $2xy = 24$ ... (ii)

Then, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$
$= 7^2 + (2xy)^2$
$= 7^2 + 24^2$
$= 49 + 576$
$= 625$
$= (25)^2$

$\therefore x^2 + y^2 = 25$ ... (iii)

Adding equation (ii) and equation (iii) we get

$x^2 - y^2 = 7$
$x^2 + y^2 = 25$
$2x^2 = 32$

or, $x^2 = 16$

$\therefore x = \pm 4$

Subtracting (ii) from (iv) we get

$x^2 + y^2 = 25$
$x^2 - y^2 = 7$
$- \quad + \quad -$
$2y^2 = 18$

or, $y^2 = 9$

$\therefore y = \pm 3$

Since from equation (ii) xy is negative, so x and y have opposite sign.

So, $x = 4$, $y = -3$ and $x = -4$, $y = 3$

Hence, the required square roots are

$4 - 3i$ and $-4 + 3i = -(4 - 3i)$

i.e. $\pm (4 - 3i)$

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88. 2058 Q.No. 11 a

State De-Moivre's theorem. Use it to find the cube root of 1. [4]

➥ Please refer to 2074 Set A Q.No. 14

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89. 2057 Q.No. 11b

State De-Moivre's Theorem. Use it to find the value of $(1 + i)^{20}$ [4]

SOLUTION

Statement of De-Moivre's Theorem:

For any positive integer n,

$[r (\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$

Let $1 + i = r (\cos \theta + i \sin \theta)$ ... (i)

Equating the real and imaginary parts, we get

$r \cos \theta = 1$ ... (ii)

$r \sin \theta = 1$ ... (iii)

Squaring (ii) and (iii) and adding, we get

$(r \cos \theta)^2 + (r \sin \theta)^2 = 1^2 + 1^2$

or, $r^2 (\cos^2 \theta + \sin^2 \theta) = 2$

or, $r^2 = 2$

$\therefore r = \sqrt{2}$

Dividing (iii) by (ii), we get

$$\frac{r \sin \theta}{r \cos \theta} = \frac{1}{1}$$

or, $\tan \theta = 1 = \tan 45°$

$\therefore \theta = 45°$

Putting the value of r and θ in (i), we get

$(1 + i) = \sqrt{2}\ (\cos 45° + i \sin 45°)$

Now, $(1 + i)^{20} = [\sqrt{2}\ (\cos 45° + i \sin 45°)]^{20}$
$= (\sqrt{2})^{20} [\cos (20 \times 45°) + i \sin (20 \times 45°)]$
[Using De-Moivre's theorem]
$= (2^{\frac{1}{2} \times 20}) (\cos 900° + i \sin 900°)$
$= 2^{10} (-1 + i \times 0)$
$= -2^{10}$

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90. 2056 Q.No. 11a

State De-Moivre's theorem. Use it to find the cube roots of unity. [4]

➥ Please refer to 2074 Set A Q.No. 14

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91. 2055 Old Model Q.No. 11a

Using De Moivre's theorem, find the value of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^7$ **[4]**

SOLUTION

Firstly, we reduce the given complex number into polar form. For this,

Let $\frac{1}{2}+i\frac{\sqrt{3}}{2}=r(\cos\theta+i\sin\theta)$ ... (i)

Equating real and imaginary part, we get,

$r\cos\theta=\frac{1}{2}$ ... (ii)

$r\sin\theta=\frac{\sqrt{3}}{2}$ ... (iii)

Now, $r=\sqrt{(r\cos\theta)^2+(r\sin\theta)^2}$

$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{1}=1$

Also, dividing equation (iii) by equation (ii), we get

$\frac{r\sin\theta}{r\cos\theta}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

or, $\tan\theta=\sqrt{3}=\tan 60°$

Hence, from equation (i) $\frac{1}{2}+i\frac{\sqrt{3}}{2}=1(\cos 60°+i\sin 60°)$

$\therefore \left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^7=(\cos 60°+i\sin 60°)^7$

$=(\cos 7\times 60°+i\sin 7\times 60°)$

[Using De-Moivre's theorem]

$=(\cos 420°+i\sin 420°)$

$=[\cos(4\times 90°+60°)+i\sin(4\times 90°+60°)]$

$=(\cos 60°+i\sin 60°)$

$=\frac{1}{2}+i\frac{\sqrt{3}}{2}$

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9 UNIT

POLYNOMIAL EQUATIONS

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2 MARKS QUESTIONS

1. 2076 Set B Q.No. 3c

If one root of the equation $ax^2 + bx + c = 0$ be twice the other, show that $2b^2 = 9ac$. [2]

SOLUTION

Let α be a root of the equation $ax^2 + bx + c = 0$.
By question, the other root $= 2\alpha$.

Now, $\alpha + 2\alpha = -\frac{b}{a}$

or, $3\alpha = -\frac{b}{a}$

or, $\alpha = -\frac{b}{3a}$

Again, $\alpha \cdot 2\alpha = \frac{c}{a}$

or, $2\alpha^2 = \frac{c}{a}$

or, $2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$

or, $\frac{2b^2}{9a^2} = \frac{c}{a}$

$\therefore 2b^2 = 9ac$

2. 2076 Set C Q.No. 3c

Find the value of k so that the equation $9x^2 + kx + 1 = 0$ has equal roots. [2]

SOLUTION

Given equation is $9x^2 + kx + 1 = 0$ ...(i)

Comparing (i) with $ax^2 + bx + c = 0$, we get

$a = 9, b = k, c = 1$

For equal roots, we have $b^2 - 4ac = 0$

or, $k^2 - 4 \cdot 9 \cdot 1 = 0$

or, $k^2 = 36$

or, $k^2 = (\pm 6)^2$

$\therefore k = \pm 6$

3. 2075 GIE Q.No. 3c

For what values of k the equation $9x^2 + kx + 1 = 0$ has equal roots? [2]

Please refer to **2076 Set C Q.No. 3c**

4. 2075 Set A Q.No. 3c

Find the value of k if the roots of the equation $3kx^2 = 4(kx - 1)$ are real and equal. [2]

SOLUTION

Given equation is

$3kx^2 = 4(kx - 1)$

or, $3kx^2 - 4kx + 4 = 0$ ... (i)

Comparing (i) with $ax^2 + bx + c = 0$, we get,

$a = 3k, b = -4k, c = 4$

For real and equal roots, we have

$b^2 - 4ac = 0$

or, $(-4k)^2 - 4 \cdot 3k \cdot 4 = 0$

or, $16k^2 - 48k = 0$

or, $k^2 - 3k = 0$

or, $k(k - 3) = 0$

$\therefore k = 0, 3$

Here, $k = 0$ is not possible. Therefore $k = 3$.

5. 2075 Set B Q.No. 3c

If one root of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ be four times the other, show that $4b^2 = 25ac$. [2]

SOLUTION

Let one root of the equation $ax^2 + bx + c = 0$ be α.
Then, by question other root $= 4\alpha$.
Now,

$\alpha + 4\alpha = \frac{-b}{a}$

or, $5\alpha = \frac{-b}{a}$

$\therefore \alpha = \frac{-b}{5a}$ ... (i)

And $\alpha \cdot 4\alpha = \frac{c}{a}$

or, $4\alpha^2 = \frac{c}{a}$

or, $4\left(\frac{-b}{5a}\right)^2 = \frac{c}{a}$ [using (i)]

or, $\frac{4b^2}{25a^2} = \frac{c}{a}$

$\therefore 4b^2 = 25ac$

6. 2075 Set C Q.No. 3c

If $a + b + c = 0$, solve the equation $ax^2 + bx + c = 0$. [2]

SOLUTION

Given $a + b + c = 0$

or, $a = -b - c$

Given equation is $ax^2 + bx + c = 0$

or, $(-b-c)x^2 + bx + c = 0$. $[\because a + b + c = 0]$

or, $-(b + c)x^2 + bx + c = 0$

or, $(b + c)x^2 - bx - c = 0$

$$\text{or, } x = \frac{-(-b) \pm \sqrt{(-b)^2 - 4.(b+c).(-c)}}{2.(b+c)}$$

$$= \frac{b \pm \sqrt{b^2 + 4c(b+c)}}{2(b+c)}$$

$$= \frac{b \pm \sqrt{b^2 + 4bc + 4c^2}}{2(b+c)}$$

$$= \frac{b \pm \sqrt{(b+2c)^2}}{2(b+c)}$$

$$= \frac{b \pm (b+2c)}{2(b+c)}$$

Taking +ve sign, we get

$$x = \frac{b+b+2c}{2(b+c)} = \frac{2b+2c}{2(b+c)} = \frac{2(b+c)}{2(b+c)} = 1$$

Taking –ve sign,

$$x = \frac{b-(b+2c)}{2(b+c)} = \frac{b-b-2c}{2(b+c)} = \frac{-2c}{2(b+c)}$$

$$= \frac{-c}{b+c} = \frac{-c}{-a} \quad [\because a + b + c = 0 \Rightarrow b + c = -a]$$

$$= \frac{c}{a}$$

$$\therefore x = 1, \frac{c}{a}$$

7. 2074 Supp Q.No. 3c

If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k. [2]

SOLUTION

Given equation is $x^2 + 2(k + 2)x + 9k = 0$ ...(i)

Comparing equation (i) with $ax^2 + bx + c = 0$, we get,
$a = 1, b = 2(k + 2), c = 9k$

If the equation (i) has equal roots, then

$b^2 - 4ac = 0$

or, $\{2(k + 2)\}^2 - 4 \times 1 \times 9k = 0$

or, $4(k^2 + 4k + 4) - 36k = 0$

or, $k^2 + 4k + 4 - 9k = 0$

or, $k^2 - 5k + 4 = 0$

or, $(k - 1)(k - 4) = 0$

$\therefore k = 1, 4$

8. 2074 Set A Q.No. 3c

Find the value of k so that the equation $3x^2 + 7x + 6 - k = 0$ has one root equal to zero. [2]

SOLUTION

Given equation is $3x^2 + 7x + 6 - k = 0$ ... (i)

Comparing equation (i) with $ax^2+bx+c= 0$, we get,
$a = 3, b = 7, c = 6 - k$

For one root equal to zero, we have $c = 0$

or, $6 - k = 0$

$\therefore k = 6$.

9. 2074 Set B Q.No. 3c

If one root of the equation $x^2 - px + q = 0$ be twice the other, show that $2p^2 = 9q$. [2]

SOLUTION

Let α be one root of the equation
$x^2 - px + q = 0$. ... (i),

Then, by question, the other root $= 2\alpha$

Now, $\alpha + 2\alpha = \frac{-(-p)}{1} = p$

$3\alpha = p$

$\alpha = \frac{p}{3}$ ...(ii)

And, $\alpha.2\alpha = \frac{q}{1}$

or, $2\alpha^2 = q$

or, $2\left(\frac{p}{3}\right)^2 = q$ [Using (ii)]

$\therefore 2p^2 = 9q$

10. 2073 Supp Q.No. 3c

Find the value of k so that the equation $3x^2 + kx - 2 = 0$ has roots whose sum is equal to 6. [2]

SOLUTION

Given, equation is $3x^2 + kx - 2 = 0$ ... (i)

Comparing the equation (i) $ax^2+bx+c = 0$, we get,
$a = 3, b = k, c = -2$.

We have, sum of the roots $= \frac{-b}{a}$

or, $6 = \frac{-k}{3}$

or, $18 = -k$

$\therefore k = -18$.

11. 2073 Set C Q.No. 3c

Find the value of P so that equation $5x^2 - Px + 16 = 0$ has equal roots. [2]

SOLUTION

Comparing given equation with $ax^2 + bx + c = 0$, we get

$a = 5, b = -P, c = 16$

For equal roots, we have,

$b^2 - 4ac = 0$

$(-P)^2 - 4 \cdot 5 \cdot 16 = 0$

or, $P^2 - 320 = 0$

or, $P^2 = 320$

or, $P^2 = (\pm 8\sqrt{5})^2$

$P = \pm 8\sqrt{5}$

12. 2073 Set D Q.No. 3c

Find a quadratic equation whose roots are twice the roots of $4x^2 + 8x - 5 = 0$. [2]

SOLUTION

Let α and β be the roots of the equation
$4x^2 + 8x - 5 = 0$. Then,

$\alpha + \beta = \frac{-8}{4} = -2$ and $\alpha\beta = \frac{-5}{4}$.

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By question, the roots of required equations are 2α and 2β.

Now, $2\alpha + 2\beta = 2(\alpha+\beta) = 2(-2) = -4$

and $2\alpha \cdot 2\beta = 4\alpha\beta = 4\left(\frac{-5}{4}\right) = -5$.

$\therefore$ The required equation is

$x^2 - (-4)x + (-5) = 0$

$\therefore x^2 + 4x - 5 = 0$

13. 2072 Supp.Q.No. 3c

If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k. [2]

➥ Please refer to 2074 Supp Q.No. 3c

14. 2072 Set C Q.No. 3c

Find the value of k so that the equation $3x^2 + 7x + 6 - k = 0$ has one root equal to zero. [2]

SOLUTION

Given equation is $3x^2 + 7x + 6 - k = 0$ ...(i)

Since every root satisfies its equation, so put $x = 0$ in (i), we get

$3 \times 0^2 + 7 \times 0 + 6 - k = 0$

$\therefore k = 6$

15. 2072 Set D Q.No. 3c

If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 3:4, prove that $12b^2 = 49ac$. [2]

SOLUTION

Let 3α and 4α be the roots of the equation $ax^2 + bx + c = 0$. Then,

$3\alpha + 4\alpha = -\frac{b}{a}$

or, $7\alpha = -\frac{b}{a}$

$\therefore \alpha = -\frac{b}{7a}$ ...(i)

And, $3\alpha \cdot 4\alpha = \frac{c}{a}$

or, $12\alpha^2 = \frac{c}{a}$

or, $12\left(-\frac{b}{7a}\right)^2 = \frac{c}{a}$ [using ...(i)]

or, $\frac{12b^2}{49a^2} = \frac{c}{a}$

$\therefore 12b^2 = 49ac$

16. 2072 Set E Q.No. 3c

If the sum of the roots of the equation $(m+1)x^2 + (2m+3)x + (3m+4) = 0$, is zero find the product of its roots. [2]

SOLUTION

Given equation is

$(m+1)x^2 + (2m+3)x + (3m+4) = 0$

By given, sum of the roots = 0

i.e. $\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = 0$

or, $\frac{-(2m+3)}{m+1} = 0$

$\therefore m = \frac{-3}{2}$

Now, product of the roots $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$= \frac{3m+4}{m+1}$

$= \frac{3 \times \left(\frac{-3}{2}\right) + 4}{\frac{-3}{2} + 1}$

$= \frac{-9+8}{-3+2} = \frac{-1}{-1} = 1$

17. 2071 Supp. Q.No. 3c

If the difference of roots of the equation $x^2+px+q = 0$ is unity, show that: $p^2 = 1+4q$. [2]

SOLUTION

Given equation is $x^2+px+q = 0$

Let one root be α. By question, another root is $\alpha+1$

Now, $\alpha + (\alpha+1) = \frac{-p}{1}$

$2\alpha + 1 = -p$

$\alpha = \frac{-p-1}{2}$ ...(i)

And, $\alpha \cdot (\alpha+1) = \frac{q}{1}$

or, $\alpha^2 + \alpha = q$

or, $\left(\frac{-p-1}{2}\right)^2 + \left(\frac{-p-1}{2}\right) = q$

or, $\frac{p^2+2p+1}{4} - \frac{p+1}{2} = q$

or, $\frac{p^2+2p+1-2p-2}{4} = q$

or, $p^2 - 1 = 4q$

$\therefore p^2 = 1 + 4q$

18. 2071 Set C Q.No. 3c

Find a quadratic equation whose roots are twice the roots of: $4x^2 + 8x - 5 = 0$. [2]

➥ Please refer to 2073 Set D Q.No. 3c

19. 2071 Set D Q.No. 3c

For what values of k the equation $9x^2 + kx + 1 = 0$ has equal roots? [2]

➥ Please refer to 2076 Set C Q.No. 3c

20. 2071 Old Q.No. 4b

If α and β are the roots of $px^2 + qx + q = 0$ prove that: $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$ [2]

SOLUTION

If α and β are the roots of the equation $px^2 + qx + q = 0$, then

$\alpha + \beta = \frac{-q}{p}$ and $\alpha\beta = \frac{q}{p}$

L.H.S. $= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}}$

$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\frac{q}{p}}$

$$= \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{q}{p}}$$

$$= \frac{\frac{-q}{p}}{\sqrt{\frac{q}{p}}} + \sqrt{\frac{q}{p}} = -\sqrt{\frac{q}{p}} + \sqrt{\frac{q}{p}}$$

$= 0$ = R.H.S.

21. 2070 Supp Q.No. 3c

Determine the nature of the roots of the equation $4x^2 + 8x - 5 = 0$. [2]

SOLUTION

Given equation is $4x^2 + 8x - 5 = 0$ ...(i)
Comparing equation (i) with
$ax^2 + bx + c = 0$, we get
$a = 4, b = 8, c = -5$
Now, $b^2 - 4ac = 8^2 - 4 \times 4 \times (-5)$
$= 64 + 80$
$= 144$ which is a perfect square.

So, the roots of the given equations are rational, real and unequal.

22. 2070 Set C Q.No. 3 c

If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k. [2]

➥ Please refer to 2074 Supp Q.No. 3c

23. 2070 Set D Q.No. 3 c

Find the quadratic equation whose one root is $2 + \sqrt{3}$. [2]

SOLUTION

We know that the surd roots of a quadratic equation occurs in conjugate pairs. Given root $= 2 + \sqrt{3}$. Hence, another root $= 2 - \sqrt{3}$.
The required quadratic equation is
$x^2 -$ (sum of the roots)x + product of the roots $= 0$
or, $x^2 - (2 + \sqrt{3} + 2 - \sqrt{3})x + (2 + \sqrt{3})(2 - \sqrt{3}) = 0$
or, $x^2 - 4x + (4 - 3) = 0$
$\therefore\ x^2 - 4x + 1 = 0.$

24. 2070 Old Q.No. 4 b

If the sum of roots is $-p$ and product of roots is q, find the quadratic equation. [2]

SOLUTION

Sum of the roots $= -p$
Product of the roots $= q$
The quadratic equation is
$x^2 -$ (sum of the roots)x + product of the roots $= 0$
or, $x^2 - (-p)x + q = 0$
$\therefore\ x^2 + px + q = 0$

25. 2069 Supp Q.No. 3 c

Prove that the roots of $x^2 - 3x + 2 = 0$ are rational. [2]

SOLUTION

Given equation is
$x^2 - 3x + 2 = 0$ ...(i)
Comparing equation (i) with $ax^2 + bx + c = 0$, we get
$a = 1, b = -3, c = 2$
Now, $b^2 - 4ac = (-3)^2 - 4 \times 1 \times 2$
$= 9 - 8$
$= 1$ which is a perfect square.
Hence, the roots of given equation are rational.

26. 2069 [Set A] Q. No. 3c

For what values of p will the equation $5x^2 - px + 45 = 0$ have equal roots? [2]

SOLUTION

Given equation is $5x^2 - px + 45 = 0$
Comparing this equation with the quadratic equation $ax^2 + bx + c = 0$, we get, $a = 5$;
$b = -p$; $c = 45$
We know that, the given equation will have equal roots if $b^2 - 4ac = 0$.
i.e. $(-p)^2 - 4 \times 5 \times 45 = 0$
or, $p^2 = 900$
or, $p^2 = (\pm 30)^2$
$\therefore\ p = \pm 30.$

27. 2069 [Set B] Q. No. 3c

If one root of the equation $ax^2 + bx + c = 0$ be twice the other show that: $2b^2 = 9ac$. [2]

➥ Please refer to 2076 Set B Q.No. 3c

28. 2068 Q.No. 3c

Form a quadratic equation whose roots are -5 and 4. [2]

SOLUTION

Given roots are -5 and 4.
Sum of the roots $= -5 + 4 = -1$
Product of the roots $= (-5) \cdot 4 = -20$
Required quadratic equation is
$x^2 - (-1)x + (-20) = 0$
$\therefore\ x^2 + x - 20 = 0$

29. 2060 Q.No. 4b

If the roots of the quadratic equation are $p + q$ and $p - q$, find the quadratic equation. [2]

SOLUTION

We know,
The quadratic equation is
$x^2 -$ (Sum of the roots) x + (Product of the roots) $= 0$
or, $x^2 - [(p + q) + (p - q)]x + (p + q) \cdot (p - q) = 0$
or, $x^2 - 2px + (p^2 - q^2) = 0$
This is the required quadratic equation.

30. 2057 Q.No. 4a

Form the quadratic equation whose one root is $3 + 4i$. [2]

SOLUTION

It is given that one root of a quadratic equation is $3 + 4i$ which is complex. So, the other root must be $3 - 4i$.
Now, let $\alpha = 3 + 4i$ and $\beta = 3 - 4i$
Then, $\alpha + \beta = (3 + 4i) + (3 - 4i) = 6$
And, $\alpha\beta = (3 + 4i)(3 - 4i)$
$= 3^2 - (4i)^2 = 9 + 16 = 25$ [$\because i^2 = -1$]

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So, the required quadratic equation is
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
or, $x^2 - 6x + 25 = 0$
$\therefore$ $x^2 - 6x + 25 = 0$ is the required quadratic equation.

31. 2056 Q.No. 4b

For what value of p will the equation $5x^2 - px + 45 = 0$ have equal roots? [2]

➥ Please refer to 2069 [Set A] Q. No. 3c

4 MARKS QUESTIONS

32. 2076 Set B Q.No. 8b

Find the equation whose roots are reciprocal of the roots of $x^2 - x + 1 = 0$. [4]

SOLUTION

Let α and β be the roots of $x^2 - x + 1 = 0$;

Here, $\alpha + \beta = \left(-\frac{b}{a}\right) = \frac{-(-1)}{1} = 1$

$\alpha \cdot \beta = \frac{c}{a} = \frac{1}{1} = 1$

By question, the roots of the required equations are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha \times \beta} = \frac{1}{1} = 1$

$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha.\beta} = \frac{1}{1} = 1$

Now, the required quadratic equation is
x^2 – (sum of the roots) x + product of the roots = 0

or, $x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha}\frac{1}{\beta}\right) = 0$
or, $x^2 - 1x + 1 = 0$
$\therefore$ $x^2 - x + 1 = 0$

33. 2076 Set C Q.No. 8b

Find the equation whose roots are reciprocal of the roots of $x^2 - 3x + 2 = 0$. [4]

SOLUTION

Let α and β be the roots of the equation $x^2 - 3x + 2 = 0$.
Then,

$\alpha + \beta = -\frac{(-3)}{1} = 3$

$\alpha\beta = \frac{2}{1} = 2$

By question, the roots of the required equation are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{3}{2}$

$\frac{1}{\alpha}\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{2}$

The required quadratic equation is

$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha} \cdot \frac{1}{\beta} = 0$

or, $x^2 - \frac{3}{2}x + \frac{1}{2} = 0$

$\therefore$ $2x^2 - 3x + 1 = 0$

34. 2075 GIE Q.No. 8b

If α and β are the roots of the equation $ax^2 - bx + b = 0$, prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{b}{a}}$. [4]

SOLUTION

If α and β are the roots of the equation $ax^2 - bx + b = 0$, then

$\alpha + \beta = \frac{b}{a}$ and $\alpha\beta = \frac{b}{a}$

L.H.S. $= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$

$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}}$

$= \frac{\frac{b}{a}}{\sqrt{\frac{b}{a}}} = \frac{\sqrt{\frac{b}{a}} \times \sqrt{\frac{b}{a}}}{\sqrt{\frac{b}{a}}}$

$= \sqrt{\frac{b}{a}}$

= R.H.S

35. 2075 Set A Q.No. 8b

If the sum of the roots of the equation $px^2 + qx + r = 0$ are equal to the sum of their squares, show that $2pr = pq + q^2$. [4]

SOLUTION

Let α and β be the roots of the equation $px^2 + qx + r = 0$.
Then,

$\alpha + \beta = \frac{-q}{p}$

$\alpha\beta = \frac{r}{p}$

By question, $\alpha + \beta = \alpha^2 + \beta^2$
or, $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

or, $\frac{-q}{p} = \left(\frac{-q}{p}\right)^2 - 2.\frac{r}{p}$

or, $\frac{-q}{p} = \frac{q^2}{p^2} - \frac{2r}{p}$

or, $\frac{-q}{p} = \frac{q^2 - 2pr}{p^2}$

or, $-q = \frac{q^2 - 2pr}{p}$

or, $-pq = q^2 - 2pr$
$\therefore$ $2pr = pq + q^2$

36. 2075 Set A Q.No. 8b OR

The sum of roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. Prove that product of the roots is $-\frac{1}{2}(a^2 + b^2)$. [4]

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SOLUTION

Given equation is

$$\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$$

or, $\frac{x+b+x+a}{(x+a)(x+b)}=\frac{1}{c}$

or, $c(2x+a+b)=(x+a)(x+b)$

or, $2cx+(a+b)c=x^2+bx+ax+ab$

or, $0=x^2+(a+b-2c)x+ab-c(a+b)$

Let α and β be the roots of this equation.

Now, $\alpha+\beta=-\left(\frac{a+b-2c}{1}\right)$

or, $\alpha+\beta=2c-a-b$ ... (i)

And, $\alpha\beta=ab-c(a+b)$

or, $\alpha\beta=ab-c(a+b)$ ... (ii)

By question, $\alpha+\beta=0$

or, $2c-a-b=0$

or, $c=\frac{a+b}{2}$

Substituting the value of c in (ii), we get,

$$\alpha\beta=ab-\left(\frac{a+b}{2}\right).(a+b)$$

$$=ab-\frac{1}{2}(a+b)^2$$

$$=\frac{2ab-a^2-2ab-b^2}{2}$$

$$=\frac{-a^2-b^2}{2}=-\frac{1}{2}(a^2+b^2)$$

37. 2075 Set B Q.No. 8b

If the equation $x^2+px+q=0$ and $x^2+qx+p=0$ have a common root, prove that either $p=q$ or $p+q+1=0$. [4]

SOLUTION

Given equations are

$x^2+px+q=0$ ...(i)

and $x^2+qx+p=0$ ...(ii)

Let α be the common root. Then,

$\alpha^2+p\alpha+q=0$

$\alpha^2+q\alpha+p=0$

Solving these equations by cross multiplication method

$$\frac{\alpha^2}{p^2-q^2}=\frac{\alpha}{q-p}=\frac{1}{q-p}$$

Taking first two ratios,

$$\frac{\alpha^2}{p^2-q^2}=\frac{\alpha}{q-p}$$

$$\therefore\ \alpha=\frac{p^2-q^2}{q-p} \quad ...(iii)$$

Again, taking last two ratios,

$$\frac{\alpha}{q-p}=\frac{1}{q-p}$$

$$\alpha=\frac{q-p}{q-p} \quad ...(iv)$$

From (iii) and (iv), we get

$$\frac{p^2-q^2}{q-p}=\frac{q-p}{q-p}$$

or, $(p^2-q^2)(q-p)=(q-p)^2$

or, $(p^2-q^2)(q-p)-(q-p)^2=0$

or, $(p+q)(p-q)(q-p)-(q-p)^2=0$

or, $-(p+q)(q-p)(q-p)-(q-p)^2=0$

or, $-(p+q)(q-p)^2-(q-p)^2=0$

or, $(p+q)(q-p)^2+(q-p)^2=0$

or, $(q-p)^2(p+q+1)=0$

Either, $(q-p)^2=0\Rightarrow p=q$

or, $p+q+1=0$

38. 2075 Set C Q.No. 8b

Show that the roots of the equation $(a^2-bc)x^2+2(b^2-ca)x+(c^2-ab)=0$ will be equal, if either $b=0$ or $a^3+b^3+c^3-3abc=0$.

SOLUTION

The given quadratic eq^n. is

$(a^2-bc)x^2+2(b^2-ca)x+(c^2-ab)=0$; where

$A=(a^2-bc)$, $B=2(b^2-ca)$; $C=(c^2-ab)$

We know that a quadratic equation has equal roots, if its discriminant i.e. $B^2-4AC=0$

i.e. if $[2(b^2-ca)]^2-4(a^2-bc)(c^2-ab)=0$

or, $4(b^4-2ab^2c+c^2a^2)-4(c^2a^2-a^3b-bc^3+ab^2c)=0$

or, $4(b^4-2ab^2c+c^2a^2-c^2a^2+a^3b+bc^3-ab^2c)=0$

or, $b(b^3-2abc+a^3+c^3-abc)=0$

or, $b(a^3+b^3+c^3-3abc)=0$

Either $b=0$,

or, $a^3+b^3+c^3-3abc=0$. Hence proved.

39. 2074 Supp Q.No. 8b

If the equations $x^2+px+q=0$ and $x^2+qx+p=0$ have a common root, prove that $p=q$ or $p+q+1=0$. [4]

➥ Please refer to 2075 Set B Q.No. 8b

40. 2074 Set A Q.No. 8b

If one root of the equation is the square of the other, prove that $b^3+a^2c+ac^2=3abc$. [4]

SOLUTION

The given quadratic equation is

$ax^2+bx+c=0$ ... (i)

It is given that, one root of equation (i) is square of the other. So let α and α^2 be the roots of equation (i).

Then, sum of the roots $=\alpha+\alpha^2=-\frac{b}{a}$ ... (ii)

And, product of the roots $=\alpha\cdot\alpha^2=\frac{c}{a}$

i.e. $\alpha^3=\frac{c}{a}$ ... (iii)

Now, cubing both sides of (ii), we get

$$(\alpha+\alpha^2)^3=\left(\frac{-b}{a}\right)^3$$

or, $\alpha^3+3\alpha.\alpha^2(\alpha+\alpha^2)+(2^2)^3=\frac{-b^3}{a^3}$

or, $\frac{c}{a}+3\frac{c}{a}\left(-\frac{b}{a}\right)+\left(\frac{c}{a}\right)^2=\frac{-b^3}{a^3}$ [$\because$ Using (ii) and (iii)]

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$$\text{or, } \frac{c}{a}+\frac{c^2}{a^2}-\frac{3bc}{a^2}=\frac{-b^3}{a^3}$$

$$\text{or, } \frac{ac+c^2-3bc}{a^2}=\frac{-b^3}{a^3}$$

$$\text{or, } ac+c^2-3bc=\frac{-b^3}{a}$$

$$\text{or, } a^2c+ac^2-3abc=-b^3$$

$\therefore b^3+a^2c+ac^2=3abc$. Hence proved.

41. 2074 Set B Q.No. 8b

If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$. [4]

Please refer to 2075 Set B Q.No. 8b

42. 2073 Supp Q.No. 8b

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (x^2 + d^2) = 0$ are equal, prove that $\frac{a}{c} = \frac{b}{d}$. [4]

SOLUTION

Given equation is

$(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$ ... (i)

Comparing (i) with $Ax^2 + Bx + C = 0$, we get

$A = a^2 + b^2$, $B = -2(ac + bd)$, $C = c^2 + d^2$

If the roots of the equation (i) are equal,

then $B^2 - 4AC = 0$

$$\text{or, } \{-2(ac+bd)\}^2-4(a^2+b^2)(c^2+d^2)=0$$

$$\text{or, } (ac+bd)^2-(a^2+b^2)(c^2+d^2)=0$$

$$\text{or, } a^2c^2+2abcd+b^2d^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2=0$$

$$\text{or, } a^2d^2-2abcd+b^2c^2=0$$

$$\text{or, } (ad-bc)^2=0$$

$$\text{or, } ad-bc=0$$

$$\text{or, } ad=bc$$

$$\therefore \frac{a}{b}=\frac{c}{d}$$

43. 2073 Set C Q.No. 8b

Form the equation whose roots are reciprocal of the roots of $x^2 - x + 1 = 0$. [4]

SOLUTION

Let α and β be the roots of the equation $x^2 - x + 1 = 0$. Then,

$$\alpha+\beta=-\frac{(-1)}{1}=1$$

$$\alpha\beta=\frac{1}{1}=1$$

By question, the roots of the required equation are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

$$\text{Now, } \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{1}{1}=1$$

$$\text{And, } \frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{1}=1$$

So, the required equation is,

$$x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\frac{1}{\alpha}\cdot\frac{1}{\beta}=0$$

$$x^2-x+1=0$$

44. 2073 Set D Q.No. 8b

If one of the roots of the equation $ax^2 + bx + c = 0$ is thrice the other, show that $3b^2 = 16ac$. [4]

SOLUTION

Let one root of $ax^2 + bx + c = 0$ be α. Then by question other root is 3α.

Now, sum of two roots $= \frac{-b}{a}$

$$\alpha+3\alpha=\frac{-b}{a}$$

$$\text{or, } 4\alpha=\frac{-b}{a}$$

$$\text{or, } \alpha=\frac{-b}{4a} \quad \text{... (i)}$$

Again, product of two roots $= \frac{c}{a}$

$$\alpha\cdot 3\alpha=\frac{c}{a}$$

$$\text{or, } 3\alpha^2=\frac{c}{a}$$

$$\text{or, } 3\left(\frac{-b}{4a}\right)^2=\frac{c}{a} \quad \text{[using (i)]}$$

$$\text{or, } \frac{3b^2}{16a^2}=\frac{c}{a}$$

$\therefore 3b^2 = 16ac$. Hence proved.

45. 2072 Supp. Q.No. 8b

Find the condition that the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2 + b^1x + c^1 = 0$ may have one root common. [4]

SOLUTION

Given equations are

$a_1x^2+b_1x+c_1=0$ ... (i)

$a_2x^2+b_2x+c_2=0$ ... (ii)

Condition of one root common:

Let, α be the common root of equation (i) and (ii)

Then

$a_1\alpha^2+b_1\alpha+c_1=0$

and $a_2\alpha^2+b_2\alpha+c_2=0$

Solving these equations by cross multiplication, we get

$$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

Taking first two ratios, we get

$$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{c_1a_2-c_2a_1}$$

$$\Rightarrow \alpha=\frac{b_1c_2-b_2c_1}{c_1a_2-c_2a_1} \quad \text{... (iii)}$$

Taking last two ratios, we get

$$\frac{\alpha}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \alpha=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1} \quad \text{... (iv)}$$

From equations (iii) and (iv), we can write

$$\frac{b_1c_2-b_2c_1}{c_1a_2-c_2a_1}=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

$$\text{or, } (a_1b_2-a_2b_1)(b_1c_2-b_2c_1)=(c_1a_2-c_2a_1)^2$$

This is the required condition that the equations (i) and (ii) have one root common.

Condition for two roots common:

Let α and β be the two roots common of equations (i) and (ii)

α and β are roots of equation (i)

$\Rightarrow \alpha + \beta = -\frac{b_1}{a_1}$ and $\alpha \cdot \beta = \frac{c_1}{a_1}$ ... (v)

Also, α and β are roots of equations (ii)

$\Rightarrow \alpha + \beta = -\frac{b_2}{a_2}$ and $\alpha \cdot \beta = \frac{c_2}{a_2}$ ... (vi)

Now from equations (v) and (vi)

$-\frac{b_1}{a_1} = -\frac{b_2}{a_2}$ and $\frac{c_1}{a_1} = \frac{c_2}{a_2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$ and $\frac{a_1}{a_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

This is the required condition that the equations (i) and (ii) may have two roots common.

46. 2072 Set C Q.No. 8b

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$. [4]

➥ Please refer to 2073 Supp Q.No. 8b

47. 2072 Set D Q.No. 8b

If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

➥ Please refer to 2074 Set A Q.No. 8b

48. 2072 Set E Q.No. 8b

If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ are equal, show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$. [4]

SOLUTION

Given equation is

$p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ ...(i)

Comparing (i) with $ax^2 + bx + c = 0$, we get

$a = p(q - r)$, $b = q(r - p)$, $c = r(p - q)$

If the roots of (i) are equal, then

$b^2 - 4ac = 0$

or, $\{q(r - p)\}^2 - 4 \cdot p(q - r) \cdot r(p - q) = 0$

or, $q^2(r - p)^2 - 4pr(q - r)(p - q) = 0$

or, $q^2(r^2 - 2rp + p^2) - 4pr(pq - q^2 - pr + qr) = 0$

or, $q^2r^2 - 2pq^2r + p^2q^2 - 4p^2qr + 4pq^2r + 4p^2r^2 - 4pqr^2 = 0$

or, $q^2r^2 + 2pq^2r + p^2q^2 - 4pqr(r + p) + 4p^2r^2 = 0$

or, $q^2(r^2 + 2pr + p^2) - 4pqr(r + p) + 4p^2r^2 = 0$

or, $\{q(r + p)\}^2 - 2q(r + p) \cdot 2pr + (2pr)^2 = 0$

or, $\{q(r + p) - 2pr\}^2 = 0$

or, $q(r + p) - 2pr = 0$

or, $qr + pq = 2pr$

Dividing both sides by pqr, we get $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$

49. 2071 Supp. Q.No. 8b

If a, b, c are rational and $a+b+c = 0$, show that the roots of: $(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$ are rational. [4]

SOLUTION

Given equation is

$(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$ ...(i)

Comparing equation (i) with $Ax^2 + Bx + C = 0$, we get,

$A = b + c - a$, $B = c + a - b$, $C = a + b - c$

Also, given $a + b + c = 0$

Now,

$B^2 - 4AC = (c+a-b)^2 - 4(b+c-a)(a+b-c)$

$= (-b - b)^2 - 4(-a - a)(-c - c)$ [$\because a + b + c = 0$]

$= 4b^2 - 16ac$

$= 4(b^2 - 4ac)$

$= 4\{(-a - c)^2 - 4ac\}$

$= 4(a^2 + 2ac + c^2 - 4ac)$

$= 4(a^2 - 2ac + c^2)$

$= 4(a - c)^2$

$= \{2(a - c)\}^2$

which is a perfect square.

Hence, the roots of given equations are rational.

50. 2071 Set C Q.No. 8b

If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, show that $p^2m = l^2q$. [4]

SOLUTION

Given equations are

$x^2 + px + q = 0$ ...(i)

and $x^2 + lx + m = 0$ ...(ii)

Let α and β be the roots of the equation (i), then

$\alpha + \beta = -p$ ...(iii)

$\alpha\beta = q$ ...(iv)

Again, if the roots of equation (ii) are in the same ratio as those of equation (i), the roots of (ii) will be of the form $k\alpha$ and $k\beta$.

Now, $k\alpha + k\beta = \frac{-l}{1}$

or, $k(\alpha + \beta) = -l$

or, $k(-p) = -l$ [$\because$ using (iii)]

$\therefore k = \frac{l}{p}$ ...(v)

Again, $k\alpha \cdot k\beta = \frac{m}{1}$

or, $k^2(\alpha\beta) = m$

or, $k^2 \cdot q = m$ [$\because$ using (iv)]

or, $k^2 = \frac{m}{q}$ ...(vi)

Eliminating k from (v) and (vi)

$\left(\frac{l}{p}\right)^2 = \frac{m}{q}$

$\therefore p^2m = l^2q$

51. 2071 Set D Q.No. 8b

If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a root in common, show that: either $p = q$ or $p + q + 1 = 0$. [4]

➥ Please refer to 2075 Set B Q.No. 8b

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52. 2071 Old Q.No. 11b

Determine the value of p for which one root of the equation $x^2 + px + 1 = 0$ is the square of the other. [4]

SOLUTION

Let α be a root of the equation $x^2 + px + 1 = 0$. By question, another root is α^2.

Now, $\alpha + \alpha^2 = \frac{-p}{1}$

$\alpha + \alpha^2 = -p$ ...(i)

and $\alpha \cdot \alpha^2 = \frac{1}{1}$

$\alpha^3 = 1$ ...(ii)

Cubing (i) on both sides, we get

$(\alpha + \alpha^2)^3 = (-p)^3$

$\alpha^3 + 3\alpha \cdot \alpha^2 (\alpha + \alpha^2) + (\alpha^2)^3 = -p^3$

or, $1 + 3 \cdot 1 (-p) + 1 = -p^3$

or, $p^3 - 3p + 2 = 0$

or, $p^3 - p^2 + p^2 - p - 2p + 2 = 0$

or, $p^2 (p-1) + p(p-1) - 2(p-1) = 0$

or, $(p-1)(p^2 + p - 2) = 0$

or, $(p-1)(p-1)(p+2) = 0$

or, $(p-1)^2 (p+2) = 0$

Either $(p-1)^2 = 0 \Rightarrow p = 1$

or, $p + 2 = 0 \Rightarrow p = -2$

$\therefore p = 1, -2$

53. 2070 Supp Q.No. 8b

If α and β be the roots of the equation $px^2 + qx + q = 0$, prove that:

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$$ [4]

SOLUTION

If α and β are the roots of the equation $px^2 + qx + q = 0$, then

$\alpha + \beta = \frac{-q}{p}$ and $\alpha\beta = \frac{q}{p}$

L.H.S. $= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}}$

$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\frac{q}{p}}$

$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{q}{p}}$

$= \frac{\frac{-q}{p}}{\sqrt{\frac{q}{p}}} + \sqrt{\frac{q}{p}}$

$= -\sqrt{\frac{q}{p}} + \sqrt{\frac{q}{p}}$

$= 0 =$ R.H.S.

54. 2070 Set C Q.No. 8 b

Find the condition under which the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2 + b^1x + c^1 = 0$ may have one root common. [4]

Please refer to 2072 Supp. Q.No. 8b

55. 2070 Set D Q.No. 8 b

If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

Please refer to 2074 Set A Q.No. 8b

56. 2070 Old Q.No. 11 b

Find α, β be the roots of $ax^2 + bx + b = 0$, show that: $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = 0$. [4]

SOLUTION

If α and β are the roots of the equation $ax^2 + bx + b = 0$, then

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{b}{a}$

L.H.S. $= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}}$

$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\frac{b}{a}}$

$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}} = \frac{\frac{-b}{a}}{\sqrt{\frac{b}{a}}} + \sqrt{\frac{b}{a}}$

$= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}}$

$= 0 =$ R.H.S

57. 2069 Supp Q.No. 8 b

Determine the values of p for which the equations $3x^2 + 4px + 2 = 0$ and $2x^2 + 3x - 2 = 0$ may have a common root. [4]

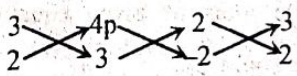
SOLUTION

Given equations are

$3x^2 + 4px + 2 = 0$

and $2x^2 + 3x - 2 = 0$

Writing the coefficient in order and replacing the first one, we have



Using the condition of one root common, we have

$(3 \cdot 3 - 2 \cdot 4p)\{4p \cdot (-2) - 3 \cdot 2\} = \{2 \cdot 2 - 3 \cdot (-2)\}^2$

or, $(9 - 8p)(-8p - 6) = (4 + 6)^2$

or, $2(9 - 8p)(-4p - 3) = 100$

or, $(9 - 8p)(-4p - 3) = 50$

or, $-36p - 27 + 32p^2 + 24p = 50$

or, $32p^2 - 12p - 77 = 0$

or, $p = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 32 \cdot (-77)}}{2 \cdot 32}$

$= \frac{12 \pm \sqrt{10000}}{64}$

$= \frac{12 \pm 100}{64}$

$= \frac{7}{4}, -\frac{11}{8}$

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58. 2069 [Set A] Q. No. 8b

Prove that a quadratic equation cannot have more than two roots. [4]

SOLUTION

Given quadratic equation is

$ax^2 + bx + c = 0 \ (a \neq 0)$ ... (1)

If possible, suppose x_1, x_2 and x_3 be the three different roots of the equation (i). So x_1, x_2 and x_3 must satisfy the equation (i). Thus,

$ax_1^2 + bx_1 + c = 0$ ... (2)

$ax_2^2 + bx_2 + c = 0$ ... (3)

$ax_3^2 + bx_3 + c = 0$ ... (4)

Subtracting equation (3) from equation (2), we get

$ax_1^2 - ax_2^2 + bx_1 - bx_2 = 0$

or, $a(x_1^2 - x_2^2) + b(x_1 - x_2) = 0$

or, $(x_1 - x_2)[a(x_1 + x_2) + b] = 0$

Since, $x_1 \neq x_2 \Rightarrow x_1 - x_2 \neq 0$

So, $a(x_1 + x_2) + b = 0$ ... (5)

Similarly, from equations (3) and (4) we get

$a(x_2 + x_3) + b = 0$ ... (6)

Now, subtracting equation (6) from (5), we get

$a(x_1 + x_2) - a(x_2 + x_3) = 0$

or, $a(x_1 + x_2 - x_2 - x_3) = 0$

or, $a(x_1 - x_3) = 0$

which is impossible, since by hypothesis $a \neq 0$ and by our assumption, $x_1 \neq x_3$, i.e. $x_1 - x_3 \neq 0$.

Hence, we arrive to the conclusion that the quadratic equation cannot have more than two roots.

59. 2069 [Set B] Q. No. 8b

Form the equation whose roots are the reciprocals of the roots of $ax^2 + bx + c = 0$. [4]

SOLUTION

Let α and β be the roots of the equation $ax^2 + bx + c = 0$. Then,

$\alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$

Since the roots of the required equation are the reciprocals of the roots of given equation, so the required roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c}$

and $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$

Required equation is

$x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0$

$\therefore \ cx^2 + bx + a = 0$

60. 2069 [Set A] Old Q. No. 11b

If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 3:4, prove that: $12b^2 = 49ac$. [4]

SOLUTION

Let 3α and 4α be the roots of the equation $ax^2 + bx + c = 0$. Then,

$3\alpha + 4\alpha = -\frac{b}{a}$

or, $7\alpha = -\frac{b}{a}$

$\therefore \ \alpha = -\frac{b}{7a}$ ...(i)

And, $3\alpha \cdot 4\alpha = \frac{c}{a}$

or, $12\alpha^2 = \frac{c}{a}$

or, $12\left(-\frac{b}{7a}\right)^2 = \frac{c}{a}$ [using ...(i)]

or, $\frac{12b^2}{49a^2} = \frac{c}{a}$

$\therefore \ 12b^2 = 49ac$

61. 2069 [Set B] Old Q. No. 11b

Find the condition that the roots of the quadratic equation $ax^2 + cx + c = 0$ may be in the ratio m : n. [4]

SOLUTION

If α and β be the roots of the equation $ax^2 + cx + c = 0$, then

$\alpha + \beta = -\frac{c}{a}$ and $\alpha\beta = \frac{c}{a}$

By given, $\alpha : \beta = m : n$

We know that $\frac{\alpha + \beta}{\sqrt{\alpha\beta}} = \frac{\frac{-c}{a}}{\sqrt{\frac{c}{a}}}$

or, $\frac{\alpha}{\sqrt{\alpha\beta}} + \frac{\beta}{\sqrt{\alpha\beta}} = -\sqrt{\frac{c}{a}}$

or, $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = -\sqrt{\frac{c}{a}}$

$\therefore \ \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} + \sqrt{\frac{c}{a}} = 0$

which is the required condition.

62. 2068 Q.No. 8b

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then show that: $\frac{a}{b} = \frac{c}{d}$ [4]

➥ Please refer to 2073 Supp Q.No. 8b

63. 2068 Old Q.No. 11b

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that: $\frac{a}{b} = \frac{c}{d}$ [4]

➥ Please refer to 2073 Supp Q.No. 8b

64. 2067 Q.No. 11b

If one root of the equation $ax^2 + bx + c = 0$ is triple of the other, show that $3b^2 = 16ac$. [4]

➥ Please refer to 2073 Set D Q.No. 8b

65. 2066 Q.No. 11b

If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

➥ Please refer to 2074 Set A Q.No. 8b

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66. 2065 Q.No. 11b

Find the equation whose roots are reciprocal to the roots of $x^2 - x + 1 = 0$. [4]

↳ Please refer to 2076 Set B Q.No. 8b

67. 2064 Q.No. 11b

Under what conditions are the roots of the quadratic equation $ax^2 + bx + c = 0$: (i) real and unequal (ii) imaginary. [4]

SOLUTION

If the discriminant $b^2 - 4ac > 0$ then the roots of the quadratic equation $ax^2 + bx + c = 0$ are real and unequal.

In particular, if $b^2 - 4ac > 0$ as well as a perfect square, the roots are rational and unequal, and if $b^2 - 4ac > 0$ but not a prefect square, the roots are irrational and unequal.

If the discriminant $b^2 - 4ac < 0$, then the roots are imaginary and unequal.

68. 2063 Q.No. 11b

Under what conditions will quadratic equation $ax^2 + bx + c = 0$ has

i. **one root the reciprocal of the other.**

ii. **roots equal in magnitude but opposite in sign.** [4]

SOLUTION

i. Let α and $\frac{1}{\alpha}$ be the roots of the quadratic equation $ax^2 + bx + c = 0$, then we have $\alpha\left(\frac{1}{\alpha}\right) = \frac{c}{a}$

$[\because$ product of the roots $= \frac{c}{a}]$

or, $1 = \frac{c}{a}$

$\Rightarrow a = c$

Which is the required condition for the equation to have one root, the reciprocal of the other.

ii. Let α and $-\alpha$ be the roots of the quadratic equation $ax^2 + bx + c = 0$, then we have

$\alpha + (-\alpha) = \frac{-b}{a}$ $\quad\left[\because \text{Sum of the roots} = -\frac{b}{a}\right]$

or, $\alpha - \alpha = -\frac{b}{a}$

or, $0 = -\frac{b}{a}$

$\Rightarrow b = 0$ which is the required condition for the quadratic equation to have roots equal in magnitude but opposite in sign.

69. 2062 Q.No. 11b

The quadratic equations: $ax^2 + bx + c = 0$ can not have more than two roots. Prove it. [4]

↳ Please refer to 2069 [Set A] Q. No. 8b

70. 2061 Q.No. 11b

If one root of the equation $lx^2 + mx + n = 0$ be four times the other, show that $4m^2 = 25ln$. [4]

SOLUTION

Let α and 4α be the roots of the given quadratic equation $lx^2 + mx + n = 0$ ... (1)

Then, sum of the roots $= \alpha + 4\alpha = \frac{-m}{l}$

or, $5\alpha = = \frac{-m}{l}$

or, $\alpha = \frac{-m}{5l}$ ... (2)

And, product of the roots $= \alpha \cdot 4\alpha = \frac{n}{l}$

or, $4\alpha^2 = \frac{n}{l}$ ... (3)

Now, using equation (2) in equation (3) we get

$4\left(\frac{-m}{5l}\right)^2 = \frac{n}{l}$

or, $\frac{4m^2}{25l^2} = \frac{n}{l}$

or, $\frac{4m^2}{25l} = n$

$\therefore 4m^2 = 25ln$.

Hence, proved.

71. 2060 Q.No. 11b

If the roots of $lx^2 + mx + n = 0$ be in the ratio 3:4, show that $12m^2 = 49\,ln$. [4]

SOLUTION

Let 3α and 4α be the roots of the given quadratic equation $lx^2 + mx + n = 0$

Then, sum of the roots $= 3\alpha + 4\alpha = \frac{-b}{a} = \frac{-m}{l}$

or, $7\alpha = \frac{-m}{l}$

$\therefore \alpha = \frac{-m}{7l}$ ... (2)

Again, product of roots $= 3\alpha \cdot 4\alpha = \frac{c}{a} = \frac{n}{l}$

or, $12\,\alpha^2 = \frac{n}{l}$

or, $\alpha^2 = \frac{n}{12\,l}$ ... (3)

Now, using equation (2) in equation (3) we get

$\left(\frac{-m}{7l}\right)^2 = \frac{n}{12l}$

or, $\frac{m^2}{49l^2} = \frac{n}{12\,l}$

or, $12\,m^2 = \frac{49\,l^2\,n}{l}$

$\therefore 12\,m^2 = 49\,ln$. Hence proved.

72. 2059 Q.No. 11b

Show that the roots of the equation $(a^2 - bc)\,x^2 + 2(b^2 - ca)\,x + (c^2 - ab) = 0$ will be equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$ [4]

↳ Please refer to 2075 Set C Q.No. 8b

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73. 2058 Q.No. 11b

If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

➥ Please refer to 2074 Set A Q.No. 8b

74. 2057 Q.No. 11a

Find the condition for two given quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ may have one root common and both roots common. [4]

➥ Please refer to 2072 Supp. Q.No. 8b

75. 2056 Q.No. 11b

If the roots of the equation $lx^2 + nx + n = 0$ be in the ratio p : q, find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$. [4]

SOLUTION

Given equation $lx^2 + nx + n = 0$ ... (i)

Let, α and β be the roots of the equation (i), then

$\alpha + \beta = -\frac{n}{l}$ ... (ii)

and $\alpha \cdot \beta = \frac{n}{l}$

$\Rightarrow \sqrt{\alpha \cdot \beta} = \sqrt{\frac{n}{l}}$ ... (iii)

By the question, $\alpha : \beta = p : q$

or, $\frac{\alpha}{\beta} = \frac{p}{q}$ ... (iv)

Now, dividing equation (ii) by equation (iii), we get,

$$\frac{\alpha + \beta}{\sqrt{\alpha.\beta}} = \frac{-\frac{n}{l}}{\sqrt{\frac{n}{l}}}$$

$$\Rightarrow \frac{\alpha}{\sqrt{\alpha.\beta}} + \frac{\beta}{\sqrt{\alpha.\beta}} = -\frac{n}{l} \times \sqrt{\frac{l}{n}} = -\sqrt{\frac{n}{l}}$$

$$\Rightarrow \frac{\alpha}{\sqrt{\alpha}.\sqrt{\beta}} + \frac{\beta}{\sqrt{\alpha}.\sqrt{\beta}} = -\sqrt{\frac{n}{l}}$$

$$\Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = -\sqrt{\frac{n}{l}}$$

$$\therefore \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = -\sqrt{\frac{n}{l}} \quad \left[\because \text{From equation (iv) } \frac{\alpha}{\beta} = \frac{p}{q}\right]$$

76. 2055 Old Model Q.No. 11b

Find the condition for two quadratic equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ may have one root common. [4]

SOLUTION

Given quadratic equations are

$ax^2 + bx + c = 0$ ... (i)

$a_1x^2 + b_1x + c_1 = 0$ ... (ii)

Let α be a root common to both the equations. Then,

$a\alpha^2 + b\alpha + c = 0$ ... (iii)

$a_1\alpha^2 + b_1\alpha + c_1 = 0$ ... (iv)

By the rule of cross-multiplication, we get

$$\frac{\alpha^2}{bc_1 - b_1c} = \frac{\alpha}{a_1c - ac_1} = \frac{1}{ab_1 - a_1b}$$

Taking first two ratios

$\Rightarrow \alpha = \frac{bc_1 - b_1c}{a_1c - ac_1}$

Again, taking last two ratios

$\alpha = \frac{a_1c - ac_1}{ab_1 - a_1b}$

Combining the two values of α, we get

$$\frac{bc_1 - b_1c}{a_1c - ac_1} = \frac{a_1c - ac_1}{ab_1 - a_1b}$$

$\therefore (ab_1 - a_1b)(bc_1 - b_1c) = (ca_1 - c_1a)^2$

This is the required condition.

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10 UNIT

CO-ORDINATE GEOMETRY

A. EQUATION OF STRAIGHT LINES

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 4a

Find the equation of the straight line that has y – intercept '2' and is parallel to $4x - 8y + 3 = 0$. [2]

SOLUTION

The straight line having y-intercept 2 means it passes through the point (0, 2).

Equation of any line parallel to $4x - 8y + 3 = 0$ is

$4x - 8y + k = 0$ ... (i)

If the equation (i) passes through the point (0, 2) then

$4 \times 0 - 8 \times 2 + k = 0$

$\therefore k = 16.$

Putting the value of k in (i), we get,

$4x - 8y + 16 = 0.$

2. 2076 Set C Q.No. 4a

Find the equation of a straight line which passes through the point (4, – 5) and is perpendicular to x + y = 5. [2]

SOLUTON

Equation of any line perpendicular to $x + y = 5$ is

$x - y + k = 0$ ...(i)

If equation (i) passes through the point (4, –5) then

$4 - (-5) + k = 0$

or, $k = -9$

Substituting the value of k in equation (i), we get,

$x - y - 9 = 0$

which is the required equation of straight line.

3. 2075 GIE Q.No. 4a

Find the equation of the line passing through the point (2, –1) and is perpendicular to $7x - 5y + 9 = 0$. [2]

SOLUTION

Equation of any line perpendicular to $7x - 5y + 9 = 0$ is

$5x + 7y + k = 0$ ...(i)

Since equation (i) passes through the point (2, –1), so

$7 \times 2 - 5 \times (-1) + k = 0$ ...(i)

or, $14 + 5 + k = 0$

$\therefore k = -19$

Putting the value of k in (i), we get,

$5x + 7y - 19 = 0$, which is the required equation of line.

4. 2075 Set A Q.No. 4a

Two lines passing through the point (2, 3) make an angle of 45°. If the slope of one of the line is 2, find the slope of the other. [2]

SOLUTION

Slope of one line $(m_1) = 2$

Let slope of another line $(m_2) = m$.

Here, $\theta = 45°$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

or, $$\tan 45° = \pm\frac{2 - m}{1 + 2m}$$

Taking positive sign, we get,

$$1 = \frac{2 - m}{1 + 2m}$$

or, $1 + 2m = 2 - m$

or, $3m = 1 \Rightarrow m = \frac{1}{3}$

Again, taking negative sign, we get,

$$1 = -\frac{2 - m}{1 + 2m}$$

or, $1 + 2m = -2 + m$

or, $m = -3$

$\therefore m = -3$ or $\frac{1}{3}$

5. 2075 Set B Q.No. 4a

Find the equations of the straight line passing through the point of intersection of the lines x – y = 0 and x + y = 0 and parallel to the line 2x + 3y = 5. [2]

SOLUTION

Given equations are

$x - y = 0$ ... (i)

& $x + y = 0$ ... (ii)

Adding (i) & (ii), we get

$x - y = 0$

$\underline{x + y = 0}$

$2x = 0$

$x = 0$

Putting the value of x in (ii) we get,

$0 + y = 0$

$y = 0$

∴ Point in intersection of line (i) & line (ii) is (0, 0)
Equation of any line parallel to $2x + 3y = 5$ is
$2x + 3y + k = 0$ ... (iii)
If equation (iii) passes through the point (0, 0) then $2 \times 0 + 3 \times 0 + k = 0$
⇒ $k = 0$
Putting the value of k in (iii), we get
$2x + 3y = 0$ which is the required equation of straight line.

6. 2075 Set C Q.No. 4a

Find the equation of a straight line which is parallel to the lines $2x + 3y = 5$ and making x-intercept 3 on the x-axis. [2]

SOLUTION

Equation of any line parallel to $2x + 3y = 5$ is
$2x + 3y + k = 0$ ... (i)
Since line (i) makes X–intercept 3, so it passes through the point (3, 0). Thus
$2 \times 3 + 3 \times 0 + k = 0$
or, $k = -6$
Putting the value of k in (i), we get,
$2x + 3y - 6 = 0$
∴ $2x + 3y = 6$

7. 2074 Supp Q.No. 4a

Find the equation of a straight line through the middle point of the line segment joining (2, – 4) and (2, 4) and parallel to the line $3x - 2y = 4$. [2]

SOLUTION

Mid point of line segment connecting (2, –4) and (2, 4)
$= \left(\frac{2+2}{2}, \frac{-4+4}{2}\right) = (2, 0)$
Equation of line parallel to $3x - 2y = 4$ is
$3x - 2y + k = 0$ ...(i)
If the line (i) passes through (2, 0) then,
$3 \times 2 - 2 \times 0 + k = 0$
∴ $k = -6$
Substituting the value of k in equation (i), we have
$3x - 2y - 6 = 0$
∴ $3x - 2y = 6$, which is the required equation of straight line.

8. 2074 Set A Q.No. 4a

Find the equation of a straight line through the mid-point of the line segment connecting (2, – 4) and (2, 4) and parallel to the line $3x - 2y = 4$. [2]

➥ Please refer to 2074 Supp Q.No. 4a

9. 2074 Set B Q.No. 4a

Find the equation of the straight line which is perpendicular to $3x - y + 4 = 0$ and passes through the point (2, 3). [2]

SOLUTION

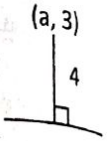
Equation of any line perpendicular to
$3x - y + 4 = 0$ is $x + 3y + k = 0$ ... (i)
If equation (i) passes through the point (2, 3) then
$2 + 3 \times 3 + k = 0$
$k = -11$
Substituting the value of k in (i), we get,
$x + 3y - 11 = 0$
∴ $x + 3y = 11$
which is the required equation of line.

10. 2073 Supp Q.No. 4a

The length of the perpendicular from the Poi (a, 3) on the line $3x + 4y + 5 = 0$ is 4. Find th value of a.

SOLUTION

Given equation of line is
$3x + 4y + 5 = 0$ ... (i)
$3x + 4y + 5 = 0$
By given, length of Perpendicular drawn form the po (a, 3) on the line (i) = 4
i.e. $\pm\left(\frac{3 \times a + 4 \times 3 + 5}{\sqrt{3^2 + 4^2}}\right) = 4$



or, $\pm\left(\frac{3 \times a + 4 \times 3 + 5}{5}\right) = 4$
or, $\pm(3a + 17) = 20$
or, $3a + 17 = \pm 20$
Taking +ve sign,
$3a + 17 = 20$
or, $3a = 20 - 17$
or, $3a = 3$
∴ $a = 1$
Again, taking –ve sign
$3a + 17 = -20$
or, $3a = -20 - 17$
or, $3a = -37$
∴ $a = \frac{-37}{3}$.
Hence, $a = 1, \frac{-37}{3}$.

11. 2073 Set C Q.No. 4a

Determine the value of k so that the li $kx + 3y + 10 = 0$ will be perpendicular to t line $3x - 2y = 5$.

SOLUTION

Given lines are:
$kx + 3y + 10 = 0$ ...(i)
and $3x - 2y = 5$ ...(ii)
Slope of line (i) $(m_1) = \frac{-k}{3}$
Slope of line (ii) $(m_2) = \frac{-3}{-2} = \frac{3}{2}$
Line (i) and line (ii) will be perpendicular to each o if
$m_1 \cdot m_2 = 1$
or, $\left(\frac{-k}{3}\right) \cdot \frac{3}{2} = -1$
∴ $k = 2$

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12. 2073 Set D Q.No. 4a

Prove that the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}-\frac{y}{a}=1$ are perpendicular to each other. [2]

SOLUTION

Given lines are

$\frac{x}{a}+\frac{y}{b}=1$ ...(i)

and $\frac{x}{b}-\frac{y}{a}=1$ ...(ii)

Slope of equation (i) $(m_1)=\frac{-\text{Coeff. of } x}{\text{Coeff. of } y}=\frac{-\frac{1}{a}}{\frac{1}{b}}=-\frac{b}{a}$ and

slope of equation (ii) $(m_2)=\frac{-\frac{1}{b}}{\frac{1}{a}}=\frac{a}{b}$

Now, $m_1 \cdot m_2=-\frac{b}{a}\times\frac{a}{b}=-1$

Hence, the lines (i) and (ii) are perpendicular to each other.

13. 2072 Supp.Q.No. 4a

Find the equation of the line through (4,1) which is perpendicular to the line $x-2y-4=0$. [2]

SOLUTION

Equation of line perpendicular to $x-2y-4=0$ is

$2x-y+k=0$ ...(i)

If equation (i) passes through the point (4, 1) then,

$2\times4+1+k=0$

$\therefore k=-9$

Substituting the value of k in (i), we get

$2x+y-9=0$

14. 2072 Set C Q.No. 4a

Find the equation of a straight line through the centroid of the triangle with vertices at (3, –4), (–2, 1) and (5, 0) and perpendicular to the line $x-3y=4$. [2]

SOLUTION

Centroid of the triangle $=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$=\left(\frac{3-2+5}{3}, \frac{-4+1+0}{3}\right)$

$=(2,-1)$

Equation of any line perpendicular to $x-3y=4$ is

$3x+y+k=0$ ...(i)

Since (i) passes through the point (2, –1), so

$3\times2-1+k=0$

$\therefore k=-5$

Putting the values of k in (i),

$3x+y-5=0$

$\therefore 3x+y=5$, which is the required equation of straight line.

15. 2072 Set D Q.No. 4a

If p is the length of the perpendicular dropped from the origin on the line $\frac{x}{a}+\frac{y}{b}=1$, prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$. [2]

SOLUTION

We have, the equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

or, $\frac{x}{a}+\frac{y}{b}-1=0$ ... (i)

Since, p = The length of perpendicular from the origin to the line (i)

i.e. $p=\left|\frac{Ax'+By'+C}{\sqrt{A^2+B^2}}\right|=\left|\frac{\frac{1}{a}\cdot0+\frac{1}{b}\cdot0-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}\right|$

$p=\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$

Squaring both sides, we get

$p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$

or, $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ Proved

16. 2072 Set E Q.No. 4a

Find the equation of the straight line passing through the point (x_0, y_0) and parallel to the line $ax+by+c=0$. [2]

SOLUTION

Equation of any line parallel to $ax+by+c=0$ is

$ax+by+k=0$ ...(i)

If equation (i) passes through the point (x_0, y_0) then

$ax_0+by_0+k=0$

or, $k=-ax_0-by_0$

Substituting the value of k in (1), we get

$ax+by-ax_0-by_0=0$

$\therefore ax+by=ax_0+by_0$ which is the required equation.

17. 2071 Supp. Q.No. 4a

Find the equation of the straight line passing through the point (x_0, y_0) and perpendicular to the straight line $ax+by+c=0$. [2]

SOLUTION

Equation of any line perpendicular to

$ax+by+c=0$ is

$bx-ay+k=0$ ...(i)

Since (i) passes through the point (x_0, y_0), so

$bx_0-ay_0+k=0$

or, $k=ay_0-bx_0$

Substituting the value of k in (i), we get,

$bx-ay+ay_0-bx_0=0$

18. 2071 Set C Q.No. 4a

Find the equation of the line through (4, 2) which is parallel to $x - 2y - 4 = 0$ [2]

SOLUTION

Equation of the line parallel to $x - 2y - 4 = 0$ is

$x - 2y + k = 0$ ...(i)

If (i) passes through the point (4, 2), then

$4 - 2 \times 2 + k = 0$

$\therefore \quad k = 0$

Substituting the value of k in (i), we get

$x - 2y = 0$

19. 2071 Set D Q.No. 4a

Find the obtuse angle between the lines $x - 3y = 6$ and $y = 2x + 5$. [2]

SOLUTION

Slope of the line $x - 3y = 6$ is $(m_1) = \frac{1}{3}$

Slope of the line $y = 2x + 5$ is $(m_2) = 2$

If θ be the angle between the given lines, then

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm\frac{\frac{1}{3} - 2}{1 + \frac{1}{3}\cdot 2} = \pm\frac{\frac{1-6}{3}}{\frac{3+2}{3}} = \pm(-1) = \pm 1$$

For obtuse angle $\tan\theta$ must be negative, so taking $-$ve sign, we have

$\tan\theta = -1 = \tan 135°$

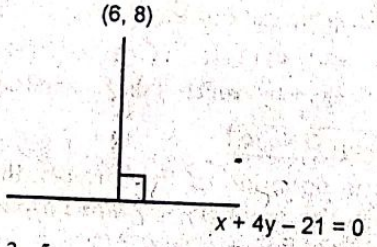
$\therefore \quad \theta = 135°$

20. 2071 Old Q.No. 2b

Find the length of the perpendicular from (6, 8) to the line through (1, 5) and (9, 3). [2]

SOLUTION

The equation of the line joining the points (1, 5) and (9, 3) is



$$y - 5 = \frac{3-5}{9-1}(x-1)$$

or, $y - 5 = \frac{-2}{8}(x-1)$

or, $y - 5 = \frac{-1}{4}(x-1)$

or, $4y - 20 = -x + 1$

or, $x + 4y - 21 = 0$ ...(i)

Now, the length of perpendicular from (6, 8) upon the line (i) is

$$\left|\frac{6 + 4\times 8 - 21}{\sqrt{1^2 + 4^2}}\right| = \frac{17}{\sqrt{17}} = \sqrt{17} \text{ units}$$

21. 2070 Supp Q.No. 4a

If p is the length of the perpendicular dropped from the (a, b) on the line $\frac{x}{a} + \frac{y}{b} = 1$, prove that: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$. [2]

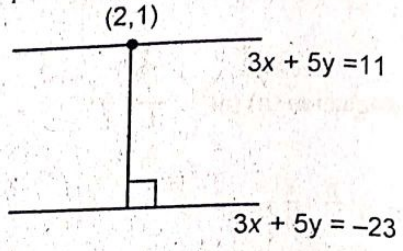
➥ Please refer to 2072 Set D Q.No. 4a

22. 2070 Set C Q.No. 4 a

Find the distance between the two parallel lines $3x + 5y = 11$ and $3x + 5y = -23$. [2]

SOLUTION

Given parallel lines are



$3x + 5y = 11$ ...(i)

$3x + 5y = -23$ ...(ii)

Put $x = 2$ in equation (i), we get

$6 + 5y = 11$

$\Rightarrow \quad y = 1$

$\therefore$ A point in line (i) is (2, 1)

Distance between given parallel lines = length of perpendicular from (2, 1) on the line (ii)

$$= \left|\frac{3\times 2 + 5\times 1 + 23}{\sqrt{3^2 + 5^2}}\right| = \frac{34}{\sqrt{34}} = \sqrt{34} \text{ units}$$

23. 2070 Set D Q.No. 4 a

Find the equation of the line passing through the middle point of the line segment connecting (2,–4) and (2, 4) and parallel to the line $3x - 2y = 4$. [2]

➥ Please refer to 2074 Supp Q.No. 4a

24. 2070 Old Q.No. 2 b

Find the equation of the line which passes through origin and is perpendicular to $4x + 3y - 12 = 0$. [2]

SOLUTION

Equation of line perpendicular to $4x + 3y - 12 = 0$ is

$3x - 4y + k = 0$ ...(i)

If equation (i) passes through (0, 0) then,

$3\times 0 - 4\times 0 + k = 0$

$\therefore \quad k = 0$

Putting the value of k in (i), we get

$3x - 4y = 0$, which is the requited equation of line.

25. 2069 Supp Q.No. 4 a

Find the equation to the straight line which passes through the point (4, –5) and is perpendicular to $3x + 4y + 5 = 0$. [2]

SOLUTION

The equation of line perpendicular to

$3x + 4y + 5 = 0$ is

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$4x - 3y + k = 0$ ...(i)

If equation (i) passes through the point (4, –5) then

$4 \times 4 - 3 \times (-5) + k = 0$

or, $k = -31$

Substituting the value of k in equation (i), we get

$4x - 3y - 31 = 0$

$4x - 3y = 31$, which is the requited equation of line.

26. 2069 [Set A] Q. No. 4a

Find the equation of a line through (5, 4) and perpendicular to the line $4x - 3y = 10$ [2]

SOLUTION

Equation of line perpendicular to the line

$4x - 3y = 10$ is

$3x + 4y + k = 0$ ...(i)

If equation (i) passes through the point (5, 4), then

$3 \times 5 + 4 \times 4 + k = 0$

$\therefore$ $k = -31$

Putting the value of k in equation (i), we get

$3x + 4y - 31 = 0$, which is the required equation of straight line.

27. 2069 [Set B] Q. No. 4a

Find the equation to the straight line that has y-intercept 3 and is parallel to the straight line $8x - 4y + 9 = 0$. [2]

SOLUTION

The straight line having y-intercept 3 means it passes through the point (0, 3)

Equation of any line parallel to $8x - 4y + 9 = 0$ is

$8x - 4y + k = 0$ ...(i)

If equation (i) passes through the point (0, 3), then

$8 \times 0 - 4 \times 3 + k = 0$

$\therefore$ $k = 12$

Putting the value of k in (i), we get

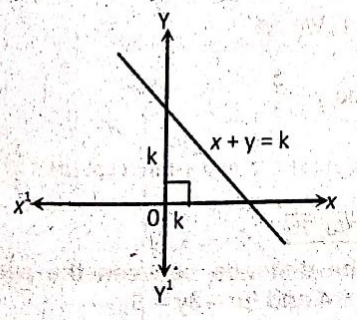
$8x - 4y + 12 = 0$, which is the required equation of line.

28. 2069 [Set A] Old Q. No. 2b

Find the value of k so that the line whose equation is $x + y = k$ will form a triangle of area 32 square units with coordinate axes. [2]

SOLUTION

Given equation is $x + y = k$



or, $\frac{x}{k} + \frac{y}{k} = 1$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$, we get

x – intercept (a) = k

y – intercept (b) = k

We know that,

Area of triangle $= \frac{1}{2}$ (base) × (height)

or, $32 = \frac{1}{2}$ (x – intercept) × (y – intercept)

or, $32 = \frac{1}{2} k.k$

or, $k^2 = 64$

$\therefore$ $k = \pm 8$.

29. 2069 [Set B] Old Q. No. 2b

If a line with equation $5x + 6y = 2k$ together with the coordinate axes form a triangle of area 135 sq units, find the value of k. [2]

SOLUTION

Given equation of line is

$5x + 6y = 2k$

or, $\frac{x}{2k/5} + \frac{y}{2k/6} = 1$

Where, x-intercept = 2k/5

y-intercept = 2k/6

The are of triangle $= \frac{1}{2}$ (x-intercept) (y-intercept)

or, $135 = \frac{1}{2} \cdot \frac{2k}{5} \cdot \frac{2k}{6}$

or, $135 = \frac{k^2}{15}$

or, $k^2 = 2025$

or, $k^2 = (\pm 45)^2$

$\therefore$ $k = \pm 45$

30. 2068 Q.No. 4a

Find the equation of the line parallel to the line $5x + 4y = 9$ and making an intercept–5 on the x-axis. [2]

SOLUTION

The line making an intercept –5 on the x-axis means it passes through the point (–5, 0)

Equation of any line parallel to $5x + 4y = 9$ is

$5x + 4y + k = 0$ ...(i)

If equation (i) passes through the point (–5, 0), then

$5 \times (-5) + 4 \times 0 + k = 0$

$\therefore$ $k = 25$

Putting the value of k in equation (i), we get

$5x + 4y + 25 = 0$

31. 2068 Old Q.No. 2b

Find the value of 'a' for which the lines $3x + y - 2 = 0$, $ax + 2y - 3 = 0$ and $2x - y - 3 = 0$ may be concurrent. [2]

SOLUTION

If the lines $3x + y - 2 = 0$, $ax + 2y - 3 = 0$ and $2x - y - 3 = 0$ are concurrent then

$$\begin{vmatrix} 3 & 1 & -2 \\ a & 2 & -3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

or, $3\begin{vmatrix} 2 & -3 \\ -1 & -3 \end{vmatrix} - 1\begin{vmatrix} a & -3 \\ 2 & -3 \end{vmatrix} + (-2)\begin{vmatrix} a & 2 \\ 2 & -1 \end{vmatrix} = 0$

or, $3(-6 - 3) - 1(-3a + 6) - 2(-a - 4) = 0$

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or, $-27 + 3a - 6 + 2a + 8 = 0$
or, $5a = 25$
$\therefore a = 5$

32. 2067 Q.No. 2b

Find the value of k so that the line whose equation is $x + y = k$ will form a triangle with the coordinate axes whose area is 32 sq. units. [2]

➥ Please refer to 2069 [Set A] Old Q. No. 2b

33. 2066 Q.No. 2a

Examine whether the points (0, 11), (2, 3) and (3, –1) are collinear or not. [2]

SOLUTION

The equation of the line joining $(x_1, y_1) = (0, 11)$ and $(x_2, y_2) = (2, 3)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $y - 11 = \frac{3 - 11}{2 - 0}(x - 0)$

or, $y - 11 = \frac{-8}{2}x$

or, $y - 11 = -4x$

or, $4x + y - 11 = 0$ ... (i)

The given points will be collinear if the third (remaining) point (3, –1) satisfies equation (i). For, put $x = 3, y = -1$ in equation (i)

$4 \times 3 + (-1) - 11 = 0$

or, $12 - 12 = 0$

$0 = 0$ (true)

So the given points are collinear.

34. 2065 Q.No. 2b

Find the equation of the line through the intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$, and cutting off equal intercepts from the axes. [2]

SOLUTION

The equation of two lines are

$3x - 4y + 1 = 0$ ...(i)
$5x + y - 1 = 0$ ...(ii)

Now, multiplying equation (ii) by 4 and adding them, we get

$3x - 4y + 1 = 0$
$20x + 4y - 4 = 0$
$23x - 3 = 0$

or, $x = \frac{3}{23}$

Putting the value of x in equation (i), we get,

$3 \times \frac{3}{23} - 4y + 1 = 0$ (i)

or, $4y = \frac{9}{23} + 1$

or, $4y = \frac{9 + 23}{23}$

or, $4y = \frac{32}{23}$

or, $y = \frac{32}{23 \times 4}$

or, $y = \frac{8}{23}$

The point of interaction is $\left(\frac{3}{23}, \frac{8}{23}\right)$

Let the equation of straight line which cut off equal intercept in the axes be

$\frac{x}{k} + \frac{y}{k} = 1$

or, $x + y = k$ (iii)

Since this line passes through the point $\left(\frac{3}{23}, \frac{8}{23}\right)$

We have,

$\frac{3}{23} + \frac{8}{23} = k$

$k = \frac{11}{23}$

Putting the value of k in equation (iii), the required equation of the straight line is

$x + y = \frac{11}{23}$

$23x + 23y = 11$

This is the required equation of straight line.

35. 2064 Q.No. 2b

If p be the perpendicular distance of the origin from a line whose intercepts on the axes are a and b, prove that: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$. [2]

➥ Please refer to 2072 Set D Q.No. 4a

36. 2063 Q.No. 2b

Find the intercepts on the axes made by the line $2x + 3y = 5$ [2]

SOLUTION

Given line is $2x + 3y = 5$

or, $\frac{2x}{5} + \frac{3y}{5} = 1$

or, $\frac{x}{5/2} + \frac{y}{5/3} = 1$

Comparing this equation with the equation $\frac{x}{a} + \frac{y}{b} = 1$, We get

$a = \frac{5}{2}$ and $b = \frac{5}{3}$

$\therefore$ x-intercept $(a) = \frac{5}{2}$ and y-intercept $(b) = \frac{5}{3}$

37. 2062 Q.No. 2b

Find the distance between the parallel lines $y = 2x + 4$ and $6x - 3y = 5$ [2]

SOLUTION

The given parallel lines are

$2x - y + 4 = 0$ ... (1)
$6x - 3y - 5 = 0$ ... (2)

Let (h, k) be any point on the line (1), then

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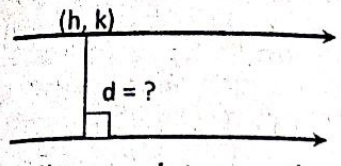
$2h - k + 4 = 0$

or, $2h - k = -4$ ... (3)



Now, distance between the lines (1) and (2) = Perpendicular distance of line (2) drawn from (h, k) of line (1)

$= \pm \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$= \pm \frac{6h - 3k - 5}{\sqrt{6^2 + (-3)^2}}$

$= \pm \frac{3(2h - k) - 5}{\sqrt{36 + 9}}$

$= \pm \frac{3(-4) - 5}{\sqrt{45}}$ [$\because$ using (3)]

$= \pm \frac{(-12 - 5)}{3\sqrt{5}} = \pm \frac{(-17)}{3\sqrt{5}}$

$= -\frac{(-17)}{3\sqrt{5}}$ [$\because$ Distance is always non-negative]

$= \frac{17}{3\sqrt{5}}$ units

38. 2061 Q.No. 2b

Find the straight lines which have slope –1 and form a triangle of area 8 square units with coordinate axes. [2]

SOLUTION

Given, slope $m = -1$.

We know that the equation of a straight line with slope m and y–intercept c is

$y = mx + c$ ... (i)

Since, $m = -1$, we have

$y = (-1)x + c$

or, $y = -x + c$ ... (ii)

If we put $x = 0$ in (ii), we get

$y = 0 + c$

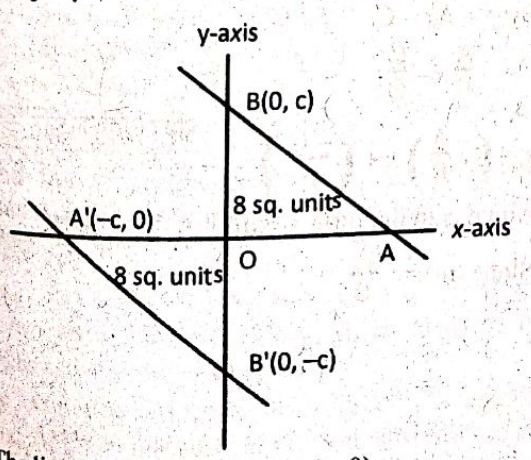
or, $y = c$

$\therefore$ The line (ii) meets y-axis at B(0, c).

If we put $y = 0$ in (ii), we get

$0 = -x + c$

or, $x = c$



$\therefore$ The line (ii) meets x-axis at A(c, 0)

Now, area of $\Delta OAB = \frac{1}{2} OA \times OB$

$= \frac{1}{2} \times c \times c$ [$\because OA = c, OB = c$]

$\Delta OAB = \frac{1}{2} c^2$

or, $8 = \frac{1}{2} c^2$

or, $c^2 = 16$

$\therefore$ $c = \sqrt{16} = \pm 4$

Substituting the value of c in (ii), we get

$y = -x \pm 4$

$\therefore$ $x + y \pm 4 = 0$ is the required straight lines.

39. 2060 Q.No. 2b

Find the equation of the straight line whose slope is $\frac{1}{3}$ and passes through the intersection of lines $y = x$ and $y = -x$. [2]

SOLUTION

The given lines are

$y = x$

i.e. $x - y = 0$ ... (1)

And, $y = -x$

i.e. $x + y = 0$ ... (2)

Adding equations (1) and (2) we get

$x - y = 0$

$x + y = 0$

$2x = 0$

$\therefore$ $x = 0$

Substituting the value of x in equation (i) we get

$y = 0$

$\therefore$ The point of intersection of the two lines is (0, 0).

We know that the equation of a line passing through a point (x_1, y_1) with slope m is

$y - y_1 = m(x - x_1)$.

$\therefore$ The equation of line passing through the point (0, 0) with slope $\frac{1}{3}$ is $y - 0 = \frac{1}{3}(x - 0)$

or, $3y = x$

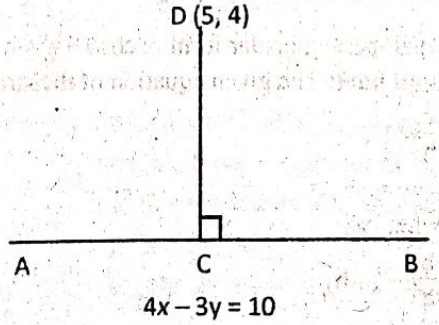
$\therefore$ $x - 3y = 0$, which is the required equation of straight line.

40. 2059 Q.No. 2b

Find the equation of the line through (5, 4) and perpendicular to the line $4x - 3y = 10$. [2]

SOLUTION:

The equation of any line perpendicular to the line $4x - 3y = 10$ is



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i.e. $-3x-4y+k=0$ ... (i)

Since it passes through the point (5, 4), so

$-3\times5-4\times4+k=0$

or, $k=31$

Substituting the value of k in equation (i), we get,

$-3x-4y+31=0$

$\therefore$ $3x+4y-31=0$, which is the required equation.

41. 2058 Q.No. 2b

Write the condition of which the straight lines given by $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ will be parallel and perpendicular. [2]

SOLUTION

The equation of the straight lines are

$A_1x + B_1y + C_1 = 0$ ... (i)

$A_2x + B_2y + C_2 = 0$ ... (ii)

Slope of line (i) $= m_1 = -\left(\frac{\text{Coeff. of } x}{\text{Coeff. of y}}\right)$

$= -\left(\frac{A_1}{B_1}\right)$

Slope of line (ii) $= m_2 = -\left(\frac{A_2}{B_2}\right)$

Lines (i) and (ii) will be parallel if $m_1 = m_2$

i.e. $-\left(\frac{A_1}{B_2}\right) = -\left(\frac{A_2}{B_2}\right)$

i.e. $\frac{A_1}{B_1} = \frac{A_2}{B_2}$

$\therefore$ $A_1B_2 = A_2B_1$

Also, the two lines (i) and (ii) will be perpendicular if

$m_1 \times m_2 = -1$

i.e. $-\left(\frac{A_1}{B_2}\right)\times-\left(\frac{A_2}{B_2}\right) = -1$

or, $A_1A_2 = -B_1B_2$

or, $A_1A_2 + B_1B_2 = 0$

42. 2057 Q.No. 2b

What are the standard forms of equation of a straight line? Find the slope of the line $\frac{x}{a}-\frac{y}{b}=1$ [2]

SOLUTION

First part: The three Standard forms of a straight line are as follows:

i. Slope intercept form: $y = mx + c$

ii. Double intercepts form: $\frac{x}{a}+\frac{y}{b}=1$

iii. Normal/ perpendicular form: $x\cos\alpha + y\sin\alpha = p$

Second part: The given equation of the straight line is

$\frac{x}{a}-\frac{y}{b}=1$

or, $bx - ay = ab$

or, $ay = bx - ab$

or, $y = \frac{b}{a}x + (-b)$

This is the equation of straight line in the slope intercept form $y = mx + c$

Where $m = \frac{b}{a}$

$\therefore$ Slope $(m) = \frac{b}{a}$

43. 2056 Q.No. 2b

Find the acute angle between the lines $x - 3y - 6 = 0$ and $y = 2x + 5$.

SOLUTION

The given lines are

$x - 3y - 6 = 0$ ... (i)

and $y = 2x + 5$

or, $2x - y + 5 = 0$ ... (ii)

Slope of line (i) $= m_1 = \frac{-\text{Coeff. of } x}{\text{Coeff. of y}} = \frac{-1}{-3} = \frac{1}{3}$

Slope of line (ii) $= m_2 = -\frac{2}{-1} = 2$

If θ be the angle between lines (i) and (ii), then $\tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1.m_2}\right)$

$= \pm\left(\frac{\frac{1}{3}-2}{1+\frac{1}{3}\times2}\right) = \pm\left(\frac{1-6}{3+2}\right) = \pm\left(\frac{-5}{5}\right) = \pm(-1)$

For acute angle, $\tan\theta > 0$, so take (–ve) sign.

i.e. $\tan\theta = -(-1) = 1$

or, $\tan\theta = \tan 45°$

$\therefore$ $\theta = 45°$

44. 2055 Old Model Q.No. 2b

What are the three standard forms of equation of a straight line? Also find the slope $Ax + By + C = 0$

SOLUTION

The three standard forms of a straight line are as follows:

i. Slope intercept form: $y = mx + c$

ii. Double intercepts form: $\frac{x}{a}+\frac{y}{b}=1$

iii. Normal/ perpendicular form: $x\cos\alpha + y\sin\alpha = p$

Now, given equation is $Ax + By + C = 0$

or, $By = -Ax - C$

or, $y = -\frac{A}{B}x - \frac{C}{B}$

i.e. $y = \left(-\frac{A}{B}\right)x + \left(\frac{-C}{B}\right)$

which is in the form $y = mx + c$

$\therefore$ Slope $(m) = -\frac{A}{B}$

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6 MARKS QUESTIONS

45. 2076 Set B Q.No. 13

Find the equations of the bisectors of the angles between the lines 4x – 3y + 1 = 0 and 12x – 5y + 7 = 0. Also prove that the bisectors are at right angle to each other, and identify the bisector which contains the origin. [6]

SOLUTION

First Part:

Given lines are

$4x - 3y + 1 = 0$

and $12x - 5y + 17 = 0$

The equations of bisectors are

$$\frac{4x-3y+1}{\sqrt{4^2+(-3)^2}} = \pm\frac{12x-5y+7}{\sqrt{12^2+(-5)^2}}$$

or, $\frac{4x-3y+1}{5} = \pm\frac{12x-5y+7}{13}$

Taking +ve sign,

$$\frac{4x-3y+1}{5} = \frac{12x-5y+7}{13}$$

or, $60x - 25y + 35 = 52x - 39y + 13$

or, $8x + 14y + 22 = 0$

$\therefore 4x + 7y + 11 = 0$ ...(i)

Again, taking –ve sign

$$\frac{4x-3y+1}{5} = -\left(\frac{12x-5y+7}{13}\right)$$

or, $52x - 39y + 13 = -60x + 25y - 35$

or, $112x - 64y + 48 = 0$

$\therefore 7x - 4y + 3 = 0$

Slope of bisector (i) $(m_1) = \frac{-4}{7}$

Slope of bisector (ii) $(m_2) = \frac{7}{4}$

Here, $m_1 m_2 = \frac{-4}{7} \times \frac{7}{4} = -1$

Hence, the bisectors are at right angles to each other.

Last Part

The bisector of the angle between the line containing the origin is

$$\frac{4x-3y+1}{\sqrt{4^2+(-3)^2}} = \frac{12x-5y+7}{\sqrt{12^2+(-5)^2}}$$

or, $\frac{4x-3y+1}{5} = \frac{12x-5y+7}{13}$

or, $52x - 39y + 13 = 60x - 25y + 35$

or, $8x + 14y + 22 = 0$

$\therefore 4x + 7y + 11 = 0$

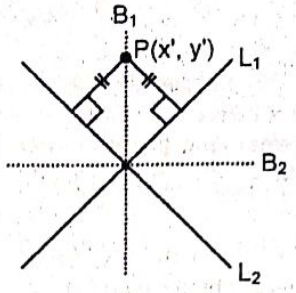
46. 2076 Set C Q.No. 13

Find the bisectors of the angles between the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$. [6]

SOLUTION

Let L_1: $A_1x + B_1y + C_1 = 0$ and L_2: $A_2x + B_2y + C_2 = 0$ be two given lines. Let B_1 and B_2 be bisectors of the angles between the lines L_1 and L_2.

We know that any point on a bisector is equidistant from the given lines. Let P(x', y') be any point in any of the bisectors.



Distance of L_1 from P(x', y') $= \pm\frac{A_1x' + B_1y' + C_1}{\sqrt{A_1^2 + B_1^2}}$

Distance of L_2 from P(x', y') $= \pm\frac{A_2x' + B_2y' + C_2}{\sqrt{A_2^2 + B_2^2}}$

Since P is equidistant from L_1 and L_2, we have,

$$\pm\frac{A_1x' + B_1y' + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm\frac{A_2x' + B_2y' + C_2}{\sqrt{A_2^2 + B_2^2}}$$

or, $\frac{A_1x' + B_1y' + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm\frac{A_2x' + B_2y' + C_2}{\sqrt{A_2^2 + B_2^2}}$

These relations are true for any point P(x', y') on the bisectors. Hence the locus of (x', y') are

$\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm\frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}}$ which are the required equations of bisectors.

47. 2075 GIE Q.No. 13

If p and p^1 be the lengths of perpendiculars from the origin upon the straight line whose equations are x sec θ + y cosec θ = a and x cos θ – y sin θ = a cos 2θ respectively, prove that $4p^2 + p_1^2 = a^2$. [6]

SOLUTION

Given equations are

$x \sec\theta + y \operatorname{cosec}\theta = a$

or, $\frac{x}{\cos\theta} + \frac{y}{\sin\theta} = a$

or, $x \sin\theta + y \cos\theta = a \sin\theta \cos\theta$

or, $x \sin\theta + y \cos\theta - a \sin\theta \cos\theta = 0$ ...(i)

and $x \cos\theta - y \sin\theta = a \cos 2\theta$

or, $x \cos\theta - y \sin\theta - a \cos 2\theta = 0$ ...(ii)

By given,

p = length of perpendicular from (0, 0) upon the line (i)

$$= \pm\left(\frac{0 \cdot \sin\theta + 0 \cdot \cos\theta - a \sin\theta \cos\theta}{\sqrt{\sin^2\theta + \cos^2\theta}}\right)$$

$= \pm(a \sin\theta \cos\theta)$

Again, p' = length of perpendicular from (0, 0) on the line (ii)

$$= \pm\left(\frac{0 \cdot \cos\theta - 0 \cdot \sin\theta - a \cos 2\theta}{\sqrt{\cos^2\theta + (-\sin\theta)^2}}\right)$$

$= \pm a \cos 2\theta$

Now, $4p^2 + p'^2 = 4\{\pm(a \sin\theta \cos\theta)\}^2 + (\pm a \cos 2\theta)^2$

$= 4a^2 \sin^2\theta \cos^2\theta + a^2 \cos^2 2\theta$

$= a^2 (2 \sin\theta \cos\theta)^2 + a^2 \cos^2 2\theta$

$= a^2 (\sin 2\theta)^2 + a^2 \cos^2 2\theta$

$= a^2 (\sin^2 2\theta + \cos^2 2\theta) = a^2 \times 1 = a^2$

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48. 2075 Set A Q.No. 14

Derive the length of the perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also find the distance between the parallel lines $y = mx + c$ and $y = mx + d$. [6]

SOLUTION

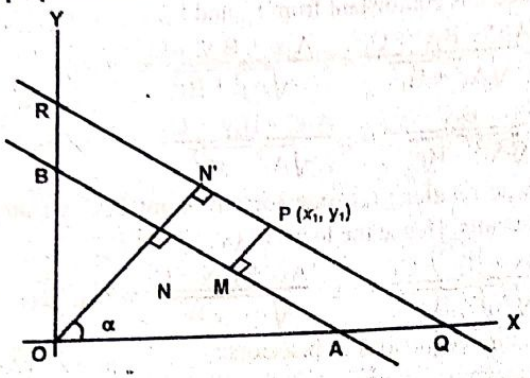
First part:

Let AB be the line represented by the equation

$x\cos\alpha + y\sin\alpha = p$ ... (i)

So, ON = p and $\angle AON = \alpha$

Let P (x_1, y_1) be the point from which PM is drawn perpendicular to AB.



Draw a line QR through P (x_1, y_1) parallel to the given line AB. Let the perpendicular from O to this line QR be ON' $(= p_1)$. Then

$PM = ON' - ON = p_1 - p$ ... (ii)

If the point P and the origin O lie on the same side of AB, then

$PM = ON - ON' = p - p_1 = -(p_1 - p)$ ... (iii)

$\therefore PM = \pm(p_1 - p)$ ... (iv)

The proper sign is taken so as to make PM positive.

$\therefore$ Equation of QR is $x\cos\alpha + y\sin\alpha = p_1$

Since, P (x_1, y_1) lies on this line QR,

$x_1\cos\alpha + y_1\sin\alpha = p_1$

$\therefore PM = \pm(x_1\cos\alpha + y_1\sin\alpha - p)$

Which is the required length of the perpendicular drawn from (x_1, y_1) on the line $x\cos\alpha + y\sin\alpha = p$.

Second part:

Given parallel lines are

$y = mx + c$

or, $mx - y + c = 0$ ... (i)

and $y = mx + d$

or, $mx - y + d = 0$ ... (ii)

When $x = 0$ from (i) $y = c$

So, a point in line (i) is (o, c)

Now,

Distance between line (i) and line (ii) = length of perpendicular from (0, c) to the line (ii)

$$= \pm\frac{m\cdot 0 - c + d}{\sqrt{m^2 + (-1)^2}}$$

$$= \pm\frac{d - c}{\sqrt{m^2 + 1}}$$

49. 2075 Set B Q.No. 13

If p and p' be the length of the perpendicul from the origin upon the straight lines wh equations are $x\sec\theta + y\operatorname{cosec}\theta = a$ $x\cos\theta - y\sin\theta = a\cos2\theta$, prove that $4p^2 + p'^2 =$

➥ Please refer to 2075 GIE Q.No. 13

50. 2075 Set C Q.No. 13

Find the incentre of the triangle formed by lines $x - y + 1 = 0$, $x - 7y + 7 = 0$ $x + y - 3 = 0$.

SOLUTION

Given lines are

$x - y + 1 = 0$ ... (1)

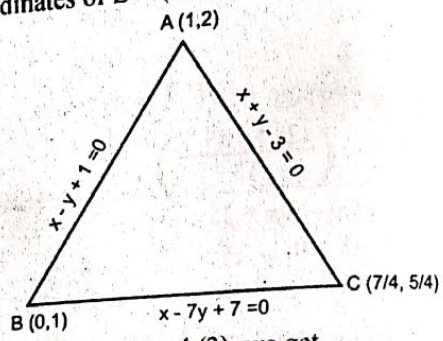
$x - 7y + 7 = 0$ ... (2)

$x + y - 3 = 0$ ... (3)

Let equation (1), equation (2) and equation represent sides AB, BC & CA of ΔABC respectivel

Solving (1) and (2), we get $x = 0$, $y = 1$

$\therefore$ Coordinates of B = (0, 1)



Again, solving (2) and (3), we get

$x = \frac{7}{4}$ and $y = \frac{5}{4}$.

Coordinates of $C = \left(\frac{7}{4}, \frac{5}{4}\right)$.

Again solving (1) and (2), we get

$x = 1$, $y = 2$

Coordinates of A = (1, 2)

Now,

$$a = BC = \sqrt{\left(\frac{7}{4} - 0\right)^2 + \left(\frac{5}{4} - 1\right)^2} = \frac{5\sqrt{2}}{4}$$

$$b = CA = \sqrt{\left(1 - \frac{7}{4}\right)^2 + \left(2 - \frac{5}{4}\right)^2} = \frac{3\sqrt{2}}{4}$$

$$c = AB = \sqrt{(0-1)^2 + (1-2)^2} = \sqrt{2}$$

Here, $(x_1, y_1) = (1, 2)$

$(x_2, y_2) = (0, 1)$

$(x_3, y_3) = \left(\frac{7}{4}, \frac{5}{4}\right)$

We know,

$$\text{Incentre } (x, y) = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

$$= \left(\frac{\frac{5\sqrt{2}}{4}\times 1 + \frac{3\sqrt{2}}{4}\times 0 + \sqrt{2}\times\frac{7}{4}}{\frac{5\sqrt{2}}{4} + \frac{3\sqrt{2}}{4} + \sqrt{2}}, \frac{\frac{5\sqrt{2}}{4}\times 2 + \frac{3\sqrt{2}}{4}\times 1 + \sqrt{2}\times\frac{5}{4}}{\frac{5\sqrt{2}}{4} + \frac{3\sqrt{2}}{4} + \sqrt{2}}\right)$$

$$= \left(\frac{\frac{5\sqrt{2}+7\sqrt{2}}{4}}{\frac{5\sqrt{2}+3\sqrt{2}+4\sqrt{2}}{4}}, \frac{\frac{10\sqrt{2}+3\sqrt{2}+5\sqrt{2}}{4}}{\frac{5\sqrt{2}+3\sqrt{2}+4\sqrt{2}}{4}}\right)$$

$$= \left(\frac{12\sqrt{2}}{12\sqrt{2}}, \frac{18\sqrt{2}}{12\sqrt{2}}\right) = \left(1, \frac{3}{2}\right)$$

51. 2074 Supp Q.No. 13

If p_1 and p_2 are the lengths of the perpendiculars drawn from the points $(\cos\theta, \sin\theta)$ and $(-\sec\theta, \operatorname{cosec}\theta)$ on the line $x\sec\theta + y\operatorname{cosec}\theta = 0$, prove that. $\frac{4}{p_1^2} - p_2^2 = 4$. [6]

SOLUTION

The equation of the line is

$x\sec\theta + y\operatorname{cosec}\theta = 0$ ...(i)

By question,

P_1 = Length of the perpendicular form $(\cos\theta, \sin\theta)$ on the line (i)

$$= \pm\frac{\cos\theta\sec\theta + \sin\theta\operatorname{cosec}\theta}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$= \pm\frac{2}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}} = \pm\frac{2}{\sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}}}$$

$$= \pm 2\sin\theta\cos\theta = \pm\sin 2\theta$$

Again, P_2 = Length of the perpendicular form $(-\sec\theta, \operatorname{cosec}\theta)$ on the line (i)

$$= \pm\frac{-\sec^2\theta + \operatorname{cosec}^2\theta}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$= \pm\left\{\frac{\left(\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}\right\}$$

$$= \pm\left(\frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\theta}}{\sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}}}\right)$$

$$= \pm\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \pm\frac{2\cos 2\theta}{\sin 2\theta} = \pm 2\cot 2\theta$$

Now,

$$\frac{4}{P_1^2} - P_2^2 = \frac{4}{\sin^2 2\theta} - 4\cot^2 2\theta$$

$$= 4(\operatorname{cosec}^2 2\theta - \cot^2 2\theta) = 4$$

52. 2074 Set A Q.No. 13

Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. Also prove that the bisectors are at right angles to each other. [6]

Please refer to 2076 Set B Q.No. 13

53. 2074 Set B Q.No. 13

Find the equations of the bisectors of the angles between the lines $3x + 4y + 2 = 0$ and $12x - 5y + 3 = 0$. Also show that bisectors are at right angle. [6]

SOLUTION

Given equations of lines are

$3x + 4y + 2 = 0$

and $12x - 5y + 3 = 0$

The equations of bisectors are

$$\frac{3x + 4y + 2}{\sqrt{3^2 + 4^2}} = \pm\frac{12x - 5y + 3}{\sqrt{12^2 + (-5)^2}}$$

or, $$\frac{3x + 4y + 2}{5} = \pm\frac{12x - 5y + 3}{13}$$

or, $39x + 52y + 26 = \pm(60x - 25y + 15)$

Taking positive sign, we get,

$39x + 52y + 26 = 60x - 25y + 15$

$\therefore$ $21x - 77y - 11 = 0$ ... (i)

Again, taking negative sign,

$39x + 52y + 26 = -60x + 25y - 15$

or, $99x + 27y + 41 = 0$ ... (ii)

Slope of equation (i) $(m_1) = \frac{21}{77}$

Slope of equation (ii) $(m_2) = \frac{-99}{27}$

Now, $m_1m_2 = \frac{21}{77} \times \left(\frac{-99}{27}\right) = -1$

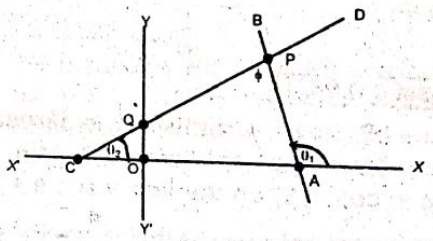
So, the bisectors are at right angles to each other.

54. 2073 Supp Q.No. 13

Find the angle between the lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the condition under which the two lines are (i) parallel, (ii) perpendicular. [6]

SOLUTION

Let, $y = m_1x + c_1$ and $y = m_2x + c_2$ be the equations of the two lines in slope intercept form. Let the lines be APB and CQD meeting the x axis at the points A and C respectively.



Let, $\angle PAX = \theta_1$ and $\angle PCX = \theta_2$

$\therefore$ Slope of line AB $(m_1) = \tan\theta_1$

and slope of line CD $(m_2) = \tan\theta_2$

If ϕ be the angle between the lines AB and CD then

$\theta_1 = \phi + \theta_2$

[$\because$ In a triangle Exterior Angle = Sum of two opposite interior angles]

or, $\phi = \theta_1 - \theta_2$

$\therefore$ $\tan\phi = \tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1.\tan\theta_2}$

or, $\tan\phi = \left(\frac{m_1 - m_2}{1 + m_1.m_2}\right)$ ... (i)

The other angle between the lines is $(\pi - \phi)$

So, for this angle, we have

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$$\tan(\pi-\phi)=\frac{m_1-m_2}{1+m_1.m_2}$$

or, $-\tan\phi=\frac{m_1-m_2}{1+m_1.m_2}$

or, $\tan\phi=-\left(\frac{m_1-m_2}{1+m_1.m_2}\right)$ ... (ii)

Combining (i) and (ii) we get,

$$\tan\phi=\pm\left(\frac{m_1-m_2}{1+m_1.m_2}\right)$$

$\therefore \phi = \tan^{-1}\left(\pm\frac{m_1-m_2}{1+m_1.m_2}\right)$

This is the required angle between the two given lines.

Second Part:

i. Condition of parallelism:

The two lines will be parallel to each other if $\phi = 0$. Then

$$\tan 0=\frac{m_1-m_2}{1+m_1m_2}$$

or, $0=\frac{m_1-m_2}{1+m_1m_2}$

or, $0=m_1-m_2$

$\therefore m_1=m_2$.

ii. Condition of perpendicularity:

The two lines will be perpendicular to each other if $\phi = 90°$.

Then,

$$\tan 90=\frac{m_1-m_2}{1+m_1m_2}$$

or, $\frac{1}{0}=\frac{m_1-m_2}{1+m_1m_2}$

or, $1+m_1m_2=0$

$\therefore m_1m_2=-1$

55. 2073 Set C Q.No. 13

If P_1 and P_2 be the lengths of the perpendicular drawn from the points(cos θ, sin θ) and (–sec θ, cosec θ) on the line x sec θ + y cosec θ = 0 respectively, prove that: $\frac{4}{P_1^2}-P_2^2=4$. [6]

➥ Please refer to 2074 Supp Q.No. 13

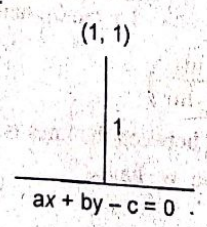
56. 2073 Set D Q.No. 13

If the length of the perpendicular from the point (1,1) to the line $ax + by - c = 0$ is 1, show that $\frac{1}{a}+\frac{1}{b}-\frac{1}{c}=\frac{2}{2ab}$. [6]

SOLUTION

Given line is $ax + by - c = 0$ ...(i)

By question, length of perpendicular from (1, 1) upon the line (i) is 1:



i.e. $\pm\frac{a\cdot1+b\cdot1-c}{\sqrt{a^2+b^2}}=1$

or, $\pm(a+b-c)=\sqrt{a^2+b^2}$

Squaring both sides, we get,

$(a+b-c)^2=a^2+b^2$

or, $a^2+b^2+c^2+2ab-2bc-2ca=a^2+b^2$

or, $c^2+2bc+2ca-2ab$

or, Dividing both sides by 2abc

or, $\frac{c^2}{2abc}=\frac{2bc}{2abc}+\frac{2ca}{2abc}-\frac{2ab}{2abc}$

$\therefore \frac{c}{2ab}=\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$

57. 2072 Supp. Q.No. 13

Find the equations of the bisectors of the angles between the lines, $4x-3y+1=0$ and $12x-5y+7=0$ and prove that the two bisectors are at right angles to each other. [6]

➥ Please refer to 2076 Set B Q.No. 13

58. 2072 Set C Q.No. 13

Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$, and prove that the bisectors are at right angles to each other. Also identify the bisector of the angle between the lines containing the origin. [6]

➥ Please refer to 2076 Set B Q.No. 13

59. 2072 Set D Q.No. 12

Find the length of the perpendicular drawn from the point (x', y') on the line whose equation is $ax + by + c = 0$. [6]

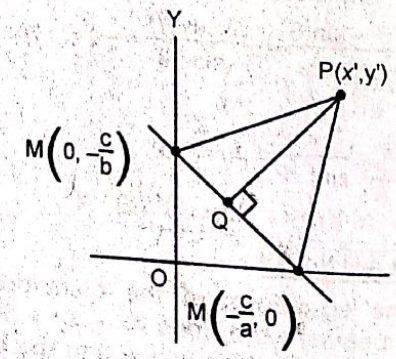
SOLUTION

Given equation of line is

$ax + by + c = 0$

It can be written in double intercept form as

$$\frac{x}{\left(\frac{-c}{a}\right)}+\frac{y}{\left(\frac{-c}{b}\right)}=1$$



Let the given line meets x-axis at $M\left(-\frac{c}{a}, 0\right)$ and y-axis at $L\left(0, -\frac{c}{b}\right)$.

Let PQ be the perpendicular from P(x', y') on LM. Join LP and MP.

For area of ΔLMP, we have

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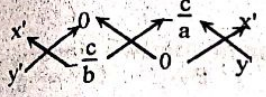


$\therefore$ Area of $\Delta LMP = \frac{1}{2}\left|-\frac{cx'}{b}-0+0-\frac{c^2}{ab}-\frac{cy'}{a}-0\right|$

$= \frac{1}{2}\left|\frac{cx'}{b}+\frac{c^2}{ab}+\frac{cy'}{a}\right|$

$= \frac{1}{2}\left|\frac{c}{ab}(ax'+by'+c)\right|$

$= \frac{1}{2}\left|\frac{c}{ab}\right| |ax'+by'+c|$ ...(i)

Also, area of $\Delta LMP = \frac{1}{2}$ LM · PQ

$= \frac{1}{2}\sqrt{\left(0+\frac{c}{a}\right)^2+\left(-\frac{c}{b}-0\right)^2}\cdot PQ$

$= \frac{1}{2}\sqrt{\frac{c^2(a^2+b^2)}{a^2b^2}}\cdot PQ$

$= \frac{1}{2}\left|\frac{c}{ab}\right|\cdot\sqrt{a^2+b^2}\cdot PQ$ ...(ii)

From (i) and (ii),

$\frac{1}{2}\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}\cdot PQ = \frac{1}{2}\left|\frac{c}{ab}\right| |ax'+by'+c|$

or, $\sqrt{a^2+b^2}\cdot PQ = |ax'+by'+c|$

$\therefore PQ = \left|\frac{ax'+by'+c}{\sqrt{a^2+b^2}}\right|$

Required length of perpendicular from (x', y') on the line $ax+by+c=0$ is

$$\left|\frac{ax'+by'+c}{\sqrt{a^2+b^2}}\right|$$

60. 2072 Set E Q.No. 14

Derive the formula for the length of the perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$. [6]

SOLUTION

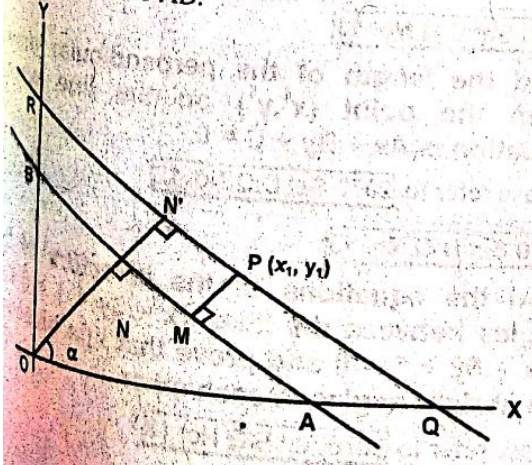
First part:

Let AB be the line represented by the equation

$x\cos\alpha + y\sin\alpha = p$ ... (i)

So, ON = p and $\angle AON = \alpha$

Let P (x_1, y_1) be the point from which PM is drawn perpendicular to AB.



Draw a line QR through P (x_1, y_1) parallel to the given line AB. Let the perpendicular from O to this line QR be ON' (= p_1). Then

$PM = ON' - ON = p_1 - p$ ... (ii)

If the point P and the origin O lie on the same side of AB, then

$PM = ON - ON' = p - p_1 = -(p_1 - p)$ ... (iii)

$\therefore$ $PM = \pm(p_1 - p)$ ... (iv)

The proper sign is taken so as to make PM positive.

$\therefore$ Equation of QR is $x\cos\alpha + y\sin\alpha = p_1$

Since, P (x_1, y_1) lies on this line QR,

$x_1\cos\alpha + y_1\sin\alpha = p_1$

$\therefore$ $PM = \pm(x_1\cos\alpha + y_1\sin\alpha - p)$

Which is the required length of the perpendicular drawn from (x_1, y_1) on the line $x\cos\alpha + y\sin\alpha = p$.

Next Part:

Any point on the $5x - 12y + 8 = 0$ is (8, 4).

Length of perpendicular from (8, 4) on the line $10x - 24y - 3 = 0$ is

$$\left|\frac{10\times 8 - 24\times 4 - 3}{\sqrt{10^2+(-24)^2}}\right| = \left|\frac{80-96-3}{\sqrt{100+576}}\right|$$

$$= \left|\frac{-19}{\sqrt{676}}\right| = \frac{19}{26} \text{ units}$$

61. 2071 Supp. Q.No. 14

Derive the formula for length of the perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between the parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$. [6]

➥ Please refer to 2072 Set E Q.No. 14

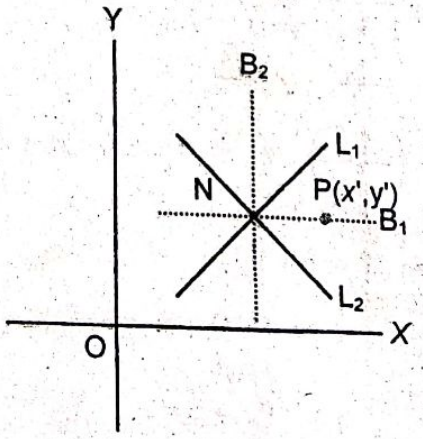
62. 2071 Set C Q.No. 13 Or

Find the bisectors of the angles between the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$. Also determine the condition that the bisector of the angle in which the origin lies. [6]

SOLUTION

Let $L_1 : l_1x + m_1y + n_1 = 0$ and $L_2 : l_2x + m_2y + n_2 = 0$ be two given lines meeting at N. Let B_1 and B_2 be the lines bisecting the angles between the lines L_1 and L_2. We have to find the equations of bisectors B_1 and B_2.

We know that any point on the bisector is equidistant from the two lines. Let P(x', y') be any point in any one of the bisectors.



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Distance of L_1 from P

$$= \pm\frac{l_1x' + m_1y' + n_1}{\sqrt{l_1^2 + m_1^2}}$$

Distance of L_2 from P

$$= \pm\frac{l_2x' + m_2y' + n_2}{\sqrt{l_2^2 + m_2^2}}$$

$$\therefore \pm\frac{l_1x' + m_1y' + n_1}{\sqrt{l_1^2 + m_1^2}} = \pm\frac{l_2x' + m_2y' + n_2}{\sqrt{l_2^2 + m_2^2}}$$

$$\text{or, } \frac{l_1x' + m_1y' + n_1}{\sqrt{l_1^2 + m_1^2}} = \pm\frac{l_2x' + m_2y' + n_2}{\sqrt{l_2^2 + m_2^2}}$$

Since these relations are true for any point (x', y') on the bisectors, so the equations of the bisectors are

$$\frac{l_1x + m_1y + n_1}{\sqrt{l_1^2 + m_1^2}} = \pm\frac{l_2x + m_2y + n_2}{\sqrt{l_2^2 + m_2^2}}$$

Bisectors of the angle containing origin

Let the equations $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ of two lines be written so that n_1 and n_2 are both positive. Hence, the coordinates of the origin when substituted in the equations make both equations positive. Let Q be any point on the bisector containing the origin then the coordinates of Q make $l_1x + m_1y + n_1$ positive because both the points O and Q lie on the same side of the line L_1. Similarly, $l_2x + m_2y + n_2$ must also be positive. Hence, the equation of the bisector of an angle which contains the origin is

$$\frac{l_1x + m_1y + n_1}{\sqrt{l_1^2 + m_1^2}} = +\frac{l_2x + m_2y + n_2}{\sqrt{l_2^2 + m_2^2}}$$

63. 2071 Set D Q.No. 13

Find the angle between the liens $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the condition of parallelism and perpendicularity of the lines. Hence find the angle between the lines $y = (2-\sqrt{3})x + 5$ and $y = (2+\sqrt{3})x - 7$. [6]

SOLUTION

First part: Please refer to 2073 Supp Q.No. 13

Last part

Given lines are

$y = (2-\sqrt{3})x + 5$ ...(i)

and $y = (2+\sqrt{3})x + 7$ ...(ii)

Slope of (i) $(m_1) = 2-\sqrt{3}$

Slope of (ii) $(m_2) = 2+\sqrt{3}$

If θ be the angle between the given lines then,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$\text{or, } \tan\theta = \pm\frac{2-\sqrt{3}-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}$$

$$\text{or, } \tan\theta = \pm\frac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3}$$

$$\text{or, } \tan\theta = \pm\left(\frac{-2\sqrt{3}}{2}\right)$$

$$\text{or, } \tan\theta = \pm(-\sqrt{3}) = \pm\sqrt{3}$$

$$\text{or, } \tan\theta = \tan 60^\circ \text{ or } \tan 120^\circ$$

$$\therefore \theta = 60^\circ \text{ or } 120^\circ$$

64. 2070 Supp Q.No. 13

Find the angle between the two lines whose equations are $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$.

Also, find the condition for two lines to be (i) parallel (ii) perpendicular to each other. [6]

SOLUTION

Given lines are

$A_1x + B_1y + C_1 = 0$ ...(i)

and $A_2x + B_2y + C_2 = 0$ ...(ii)

$$\text{Slope of line (i) } (m_1) = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-A_1}{B_1}$$

$$\text{Slope of line (ii) } (m_2) = \frac{-A_2}{B_2}$$

If θ be the angle between the given lines, then

$$\tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$$

$$= \pm\left\{\frac{-\frac{A_1}{B_1} + \frac{A_2}{B_2}}{1 + \left(-\frac{A_1}{B_1}\right)\left(-\frac{A_2}{B_2}\right)}\right\}$$

$$= \pm\left(\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right)$$

$$\therefore \theta = \tan^{-1}\left(\pm\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right)$$

i. Parallel lines

The two lines will be parallel if $\theta = 0^\circ$

$$\text{i.e. } \tan 0 = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

$$\text{or, } 0 = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

$$\text{or, } A_1B_2 - A_2B_1 = 0$$

$$\text{or, } A_1B_2 = A_2B_1$$

$$\text{i.e. } \frac{A_1}{A_2} = \frac{B_1}{B_2}$$

ii. Perpendicular lines

The lines will be perpendicular to each other if $\theta = 90^\circ$

$$\text{i.e. } \tan 90^\circ = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

$$\text{or, } \frac{1}{\cot 90^\circ} = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

$$\text{or, } \frac{1}{0} = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

$$\therefore A_1A_2 + B_1B_2 = 0$$

65. 2070 Set C Q.No. 13

Find the length of the perpendicular drawn from the point (x^1, y^1) on the line whose equation is $Ax + By + C = 0$. [6]

➥ Please refer to 2072 Set D Q.No. 12

66. 2070 Set D Q.No. 13

Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$ and prove that line bisectors are at right angles to each other. [6]

➥ Please refer to 2076 Set B Q.No. 13

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67. 2069 Supp Q.No. 13

Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. And prove that the bisectors are at right angles to each other. [6]

➥ Please refer to 2076 Set B Q.No. 13

68. 2069 [Set A] Q. No. 13

If P and P' be the lengths of the perpendiculars from origin upon the straight lines whose equations are $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ prove that: $4p^2 + p'^2 = a^2$. [6]

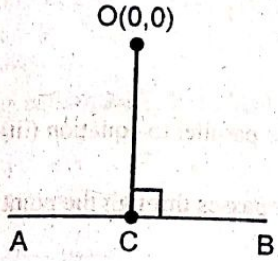
➥ Please refer to 2075 GIE Q.No. 13

69. 2069 [Set B] Q. No. 13

Prove that the perpendicular from the origin upon the straight line joining the points $(c\cos\alpha, c\sin\alpha)$ and $(c\cos\beta, c\sin\beta)$ bisects the distance between them. [6]

SOLUTION

Let A$(c\cos\alpha, c\sin\alpha)$ and B$(c\cos\beta, c\sin\beta)$ be two given points.



$$\text{Slope of AB} = \frac{c\sin\beta - c\sin\alpha}{c\cos\beta - c\cos\alpha} = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}$$

Slope of line perpendicular to AB

$$= -\frac{1}{\text{slope of AB}} = -\frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}$$

Equation of line perpendicular to AB is

$$y - 0 = -\frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}(x - 0)$$

or, $(\cos\beta - \cos\alpha)x + (\sin\beta - \sin\alpha)y = 0$ ...(i)

Let C be the mid point of AC. Then, coordinates of C

$$= \left(\frac{c\cos\alpha + c\cos\beta}{2}, \frac{c\sin\alpha + c\sin\beta}{2}\right)$$

Put $x = \frac{c}{2}(\cos\alpha + \cos\beta)$ and $y = \frac{c}{2}(\sin\alpha + \sin\beta)$ in equation (i), we have

$$(\cos\beta - \cos\alpha)\cdot\frac{c}{2}(\cos\alpha + \cos\beta) + (\sin\beta - \sin\alpha)\cdot\frac{c}{2}(\sin\alpha + \sin\beta) = 0$$

or, $\cos^2\beta - \cos^2\alpha + \sin^2\beta - \sin^2\alpha = 0$

or, $(\sin^2\beta + \cos^2\beta) - (\sin^2\alpha + \cos^2\alpha) = 0$

or, $1 - 1 = 0$

or, $0 = 0$ (true)

Hence, OC is perpendicular bisector of AB.

70. 2068 Q.No. 13

Find the angle between two straight lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the conditions under which the two straight lines will be (i) parallel (ii) perpendicular. [6]

➥ Please refer to 2073 Supp Q.No. 13

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

71. 2071 Old Q.No. 9a

Find the length of the perpendicular from the point (x^1, y^1) on a straight line $Ax + By + C = 0$. [4]

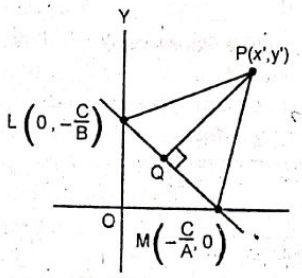
SOLUTION

Given equation of line is

$Ax + By + C = 0$

It can be written in double intercept form as

$$\frac{x}{\left(\frac{-C}{A}\right)} + \frac{y}{\left(\frac{-C}{B}\right)} = 1$$



Let the given line meets x-axis at $M\left(-\frac{C}{A}, 0\right)$ and y-axis at $L\left(0, -\frac{C}{B}\right)$.

Let PQ be the perpendicular from P(x', y') on LM. Join LP and MP.

For area of ΔLMP, we have

$$\begin{matrix} x' & & 0 & & -\frac{C}{A} & & x' \\ & \times & & \times & & \times & \\ y' & & -\frac{C}{B} & & 0 & & y' \end{matrix}$$

$$\therefore \text{Area of } \Delta LMP = \frac{1}{2}\left|-\frac{Cx'}{B} - 0 + 0 - \frac{C^2}{AB} - \frac{Cy'}{A} - 0\right|$$

$$= \frac{1}{2}\left|\frac{Cx'}{B} + \frac{C^2}{AB} + \frac{Cy'}{A}\right|$$

$$= \frac{1}{2}\left|\frac{C}{AB}(Ax' + By' + C)\right|$$

$$= \frac{1}{2}\left|\frac{C}{AB}\right||Ax' + By' + C| \quad ...(i)$$

Also, area of $\Delta LMP = \frac{1}{2}LM \cdot PQ$

$$= \frac{1}{2}\sqrt{\left(0 + \frac{C}{A}\right)^2 + \left(-\frac{C}{B} - 0\right)^2}\cdot PQ$$

$$= \frac{1}{2}\sqrt{\frac{C^2(A^2 + B^2)}{A^2B^2}}\cdot PQ$$

$$= \frac{1}{2}\left|\frac{C}{AB}\right|\cdot\sqrt{A^2 + B^2}\cdot PQ \quad ...(ii)$$

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From (i) and (ii),

$$\frac{1}{2}\left|\frac{C}{AB}\right|\sqrt{A^2+B^2}\cdot PQ=\frac{1}{2}\left|\frac{C}{AB}\right||Ax'+By'+C|$$

or, $\sqrt{A^2+B^2}\cdot PQ=|Ax'+By'+C|$

$\therefore$ $PQ=\left|\frac{Ax'+By'+C}{\sqrt{A^2+B^2}}\right|$

Required length of perpendicular from (x', y') on the line $Ax+By+C=0$ is

$$\left|\frac{Ax'+By'+C}{\sqrt{A^2+B^2}}\right|$$

72. 2070 Old Q.No. 9 a

Find the equation to the straight line which passes through the intersection of the lines $3x-4y+1=0$ and $5x+y-1=0$ and cuts off equal intercepts from the axes. [4]

SOLUTION

Given lines are

$3x-4y+1=0$ ...(i)

$5x+y-1=0$ ...(ii)

Multiplying equation (ii) by 4 and adding with equation (i), we get

$$\begin{array}{r} 3x-4y+1=0 \\ 20x+4y-4=0 \\ \hline 23x-3=0 \end{array}$$

or, $x=\frac{3}{23}$

Putting the value of x in (i), we get

$\frac{9}{23}-4y=1=0$

or, $\frac{32}{23}=4y$

or, $y=\frac{8}{23}$

$\therefore$ Point of intersection of given lines $=\left(\frac{3}{23},\frac{8}{23}\right)$

since the required line cuts off equal intercepts from the axes, so let x-intercept = y-intercept = a.

The equation of line in double intercept form is

$\frac{x}{a}+\frac{y}{a}=1$

or, $x+y=a$ ...(iii)

If equation (iii) passes through the point $\left(\frac{3}{23},\frac{8}{23}\right)$, then

$\frac{3}{23}+\frac{8}{23}=a$

or, $a=\frac{11}{23}$

Substituting the value of a in equation (iii), we get

$x+y=\frac{11}{23}$

$\therefore$ $23x+23y=11$, which is the required equation of straight line.

73. 2069 [Set A] Old Q. No. 9a

Find the equation of the straight line which passes through the point of intersection of $x+y=18$ and $3x-2y+1=0$, and is parallel to the straight line joining the points (3, 4) and (5, 6). [4]

SOLUTION

Given equations are

$x+y=18$ ...(i)

$3x-2y+1=0$ ...(ii)

Multiplying equation (i) by 2 and adding

$2x+2y=36$

$\underline{3x-2y=-1}$

$5x=35$

$\therefore$ $x=7$

Putting the value of x in (i) we get,

$7+y=18$

$y=11$

$\therefore$ Point of intersection of (i) and (ii) = (7, 11)

Equation of line joining the points (3, 4) and (5, 6) is

$y-4=\frac{6-4}{5-3}(x-3)$

or, $y-4=x-3$

or, $x-y+1=0$ ...(iii)

Equation of line parallel to equation (iii) is

$x-y+k=0$ ...(iv)

If equation (iv) passes through the point (7, 11), then

$7-11+k=0$

or, $k=4$

Putting the value of k in equation (iv), we get

$x-y+4=0$, which is the required equation of line.

74. 2069 [Set B] Old Q. No. 9a

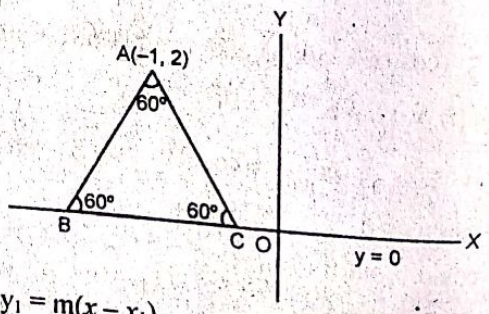
Find the equation of the sides of an equilateral triangle whose vertex is (–1, 2) and base is $y=0$. [4]

SOLUTION

Let ΔABC be an equilateral triangle whose vertex is at A(–1,2) and base is $y=0$.

Slope of AC = $\tan 120^\circ=-\sqrt{3}$

Equation of AC is



$y-y_1=m(x-x_1)$

or, $y-2=-\sqrt{3}(x+1)$

or, $y-2=-\sqrt{3}x-\sqrt{3}$

or, $\sqrt{3}x+y+\sqrt{3}-2=0$

Again, the slope of AB is $\tan 60^\circ=\sqrt{3}$

The equation of AB is

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$y - y_1 = m(x - x_1)$

or, $y - 2 = \sqrt{3}(x + 1)$

or, $y - 2 = \sqrt{3}x + \sqrt{3}$

$\sqrt{3}x - y + \sqrt{3} + 2 = 0$

∴ Required equations are

$\sqrt{3}x + y + \sqrt{3} - 2 = 0$ and

$\sqrt{3}x - y + \sqrt{3} + 2 = 0$

75. 2068 Old Q.No. 9a

Find the angle between two straight lines whose equations are $y = m_1 x + c_1$ and $y = m_2x + c_2$. Also find the conditions for the two lines to be (i) parallel (ii) perpendicular. [4]

Please refer to 2073 Supp Q.No. 13

76. 2067 Q.No. 9a

Find the equation to the straight line which makes equal intercepts on the axes and passes through the point of intersection of the lines. $2x - 3y + 1 = 0$ and $x + 2y - 2 = 0$. [4]

SOLUTION

The equations of the given lines are

$2x - 3y + 1 = 0$ ... (i)

$x + 2y - 2 = 0$ ... (ii)

Multiplying equation (ii) by 2 and subtracting it from equation (i)

$$\begin{array}{rrr} 2x - 3y + 1 & = & 0 \\ 2x + 4y - 4 & = & 0 \\ - \quad - \quad + & & + \\ \hline -7y + 5 & = & 0 \end{array}$$

$y = \frac{5}{7}$

Substituting the value of y in equation (i)

$2x - 3 \times \frac{5}{7} + 1 = 0$

or, $2x = \frac{15}{7} - 1$

or, $2x = \frac{15 - 7}{7}$

or, $x = \frac{8}{2 \times 7} = \frac{4}{7}$

So, the point of intersection of equation (i) and equation (ii) is $\left(\frac{4}{7}, \frac{5}{7}\right)$

The equation of straight line that makes equal intercepts on axes is

$\frac{x}{a} + \frac{y}{a} = 1$

or, $x + y = a$ ... (iii)

since equation (iii) passes through the point of intersection of equation (i) and equation (ii) i.e. $\left(\frac{4}{7}, \frac{5}{7}\right)$.

So, $\frac{4}{7} + \frac{5}{7} = a$

$a = \frac{9}{7}$

Substituting the value of a in equation (iii), we get

$x + y = \frac{9}{7}$

∴ $7x + 7y = 9$ is the required equation of the line.

77. 2066 Q.No. 9a

Determine the value of m for which the straight lines $y = x + 1$, $y = 2(x + 1)$ and $y = mx + 3$ are concurrent. [4]

SOLUTION

Given lines are

$y = x + 1$ ... (i)

$y = 2(x + 1)$ ... (ii)

$y = mx + 3$ ... (iii)

Now, solving equation (i) and equation (ii),

$x + 1 = 2(x + 1)$

or, $x + 1 = 2x + 2$

or, $1 - 2 = 2x - x$

∴ $x = -1$

Substituting the value of x in equation (i),

$y = -1 + 1 = 0$

So the point of intersection of equation (i) and equation (ii) is (–1, 0).

Since the given three lines are concurrent, so they must intersect at a point. For, that the point of intersection of line (i) and line (ii) must lie in line (iii). So put $x = -1$ and $y = 0$ in (iii), we get

$0 = m(-1) + 3$

or, $0 = -m + 3$

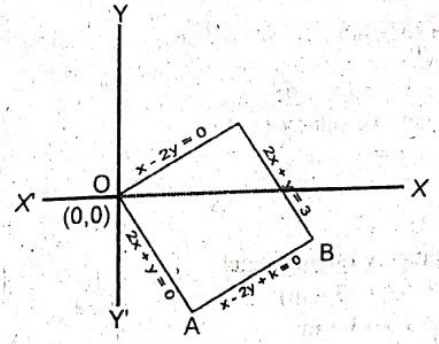
∴ $m = 3$ Ans.

78. 2065 Q.No. 9a

The origin is a corner of a square and two of its sides are given by $2x + y = 0$ and $2x + y = 3$. Find the equations of the other two sides. [4]

SOLUTION

Let OABC be a square as shown in the figure.



Here, equation of OA is $2x + y = 0$

equation of BC is $2x + y = 3$

equation of OC = ?

equation of AB = ?

Since OC is perpendicular to OA, so equation of OC is of the form

$x - 2y + k = 0$ ... (1)

Since equation (1) passes through the point (0,0), so

$0 - 2 \times 0 + k = 0$

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$\therefore$ $k = 0$

Putting the value of k in equation (i), we get

$x - 2y = 0$

Again, AB is parallel to OC, so equation of AB is of the form

$2x + y + k = 0$ ... (2)

Now,

OC = Length of perpendicular from (0, 0) on BC

$= \left|\frac{2\times 0 + 0 - 3}{\sqrt{2^2+1^2}}\right| = \frac{3}{\sqrt{5}}$

Since OABC is a square, so OC = OA

or, $\frac{3}{\sqrt{5}}$ = Perpendicular length from origin on the line (2)

or, $\frac{3}{\sqrt{5}} = \pm\frac{2\times 0 + 0 + k}{\sqrt{2^2+1^2}}$

$\therefore$ $k = \pm 3$

$\therefore$ Equations of AB are $x - 2y \pm 3 = 0$.

79. 2064 Q.No. 9b

Find the equation of the straight lines which passes through the point (2, 3) and are inclined at 45° to the straight line $x + 3y + 4 = 0$. [4]

SOLUTION

Let m be the slope of the required line. Since it passes through the point (2, 3), its equation is

$y - 3 = m(x - 2)$ ... (i)

Slope of the given line $x + 3y + 4 = 0$ is $-\frac{1}{3}$

Hence, $\tan 45° = \pm\frac{m - \left(\frac{-1}{3}\right)}{1 + m\times\left(\frac{-1}{3}\right)}$

or, $1 = \pm\left(\frac{3m+1}{3-m}\right)$

or, $\frac{3m+1}{3-m} = \pm 1$

or, $3m + 1 = \pm(3 - m)$

Taking +ve sign, we get

$3m + 1 = 3 - m$

or, $4m = 2$

$\therefore$ $m = ½$

Taking – ve sign, we get

$3m + 1 = -(3 - m)$

or, $3m + 1 = -3 + m$

or, $2m = -4$

$\therefore$ $m = -2$

Thus m = ½ or –2

Substituting these value of m in (i), the required equations of the lines are

$y - 3 = ½(x - 2)$ and $y - 3 = -2(x - 2)$

or, $2y - 6 = x - 2$ and $y - 3 = -2x + 4$

or, $x - 2y + 4 = 0$ and $2x + y - 7 = 0$

80. 2063 Q.No. 9a

Prove that the equation of the straight line which passes through the point $(a\cos^3\theta, a\sin^3\theta)$ and is perpendicular to the straight line $x\sec\theta + y\,\text{cosec}\,\theta = a$ is $x\cos\theta - y\sin\theta = a\cos 2\theta$. [4]

SOLUTION

The given equation of the line is

$x\sec\theta + y\,\text{cosec}\,\theta = a$ ... (i)

The equation of the line perpendicular to the line (i) is

$x\,\text{cosec}\,\theta - y\sec\theta + k = 0$ ... (ii)

The line (ii) passes through the point $(a\cos^3\theta, a\sin^3\theta)$, then we have

$a\cos^3\theta\,\text{cosec}\,\theta - a\sin^3\theta\sec\theta + k = 0$

or, $a\cos^3\theta\times\frac{1}{\sin\theta} - a\sin^3\theta\times\frac{1}{\cos\theta} + k = 0$

or, $\frac{a(\cos^4\theta - \sin^4\theta)}{\sin\theta\cos\theta} + k = 0$

or, $\frac{a(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta} + k = 0$

or, $\frac{a\cos 2\theta\times 1}{\sin\theta\cos\theta} + k = 0$ $\left[\because \begin{matrix}\cos^2\theta - \sin^2\theta = \cos 2\theta \\ \cos^2\theta + \sin^2\theta = 1\end{matrix}\right]$

$\therefore$ $k = \frac{-a\cos 2\theta}{\sin\theta\cos\theta}$

Substituting the value of k in (ii), we get

$x\,\text{cosec}\,\theta - y\sec\theta - \frac{a\cos 2\theta}{\sin\theta\cos\theta} = 0$

or, $x\cdot\frac{1}{\sin\theta} - y\cdot\frac{1}{\cos\theta} = \frac{a\cos 2\theta}{\sin\theta\cos\theta}$

or, $\frac{x\cos\theta - y\sin\theta}{\sin\theta\cos\theta} = \frac{x\cos 2\theta}{\sin\theta\cos\theta}$

$\therefore$ $x\cos\theta - y\sin\theta = a\cos 2\theta$. Hence proved.

81. 2063 Q.No. 9a OR

Find the equations of the bisectors of the angles between the straight lines $3x - 4y + 3 = 0$ and $12x - 5y - 1 = 0$. [4]

SOLUTION

Given lines are

$3x - 4y + 3 = 0$ ... (i)

$12x - 5y - 1 = 0$ ... (ii)

The equations of the bisectors of the angles between the lines (i) and (ii) are

> The equations of the bisectors of the angles between the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are
> $\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm\frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}}$

$\frac{3x - 4y + 3}{\sqrt{3^2 + (-4)^2}} = \pm\frac{(12x - 5y - 1)}{\sqrt{12^2 + (-5)^2}}$

or, $\frac{3x - 4y + 3}{\sqrt{9 + 16}} = \pm\frac{(12x - 5y - 1)}{\sqrt{144 + 25}}$

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$$\text{or, } \frac{3x-4y+3}{\sqrt{25}} = \pm\frac{(12x-5y-1)}{\sqrt{169}}$$

$$\text{or, } \frac{3x-4y+3}{5} = \pm\frac{(12x-5y-1)}{13}$$

$$\text{or, } 39x-52y+39 = \pm(60x-25y-5)$$

Now, taking positive sign,

$$39x-52y+39 = 60x-25y-5$$

$$\text{or, } 60x-25y-5-39x+52y-39=0$$

$$\text{or, } 21x+27y-44=0$$

Again, taking negative sign

$$39x-52y+39 = -(60x-25-5)$$

$$\text{or, } 39x-52y+39+60x-25y-5=0$$

$$\text{or, } 99x-77y+34=0$$

Hence, the equations of the bisectors are $21x + 27y - 44 = 0$ and $99x - 77y + 34 = 0$

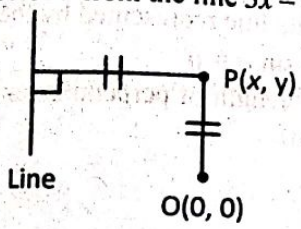
82. 2062 Q.No. 9a

Find the equation of the locus of a point P which is equidistant from $3x - 4y + 2 = 0$ and the origin. **[4]**

SOLUTION

Let (x, y) be the coordinates of a point P.

Now, distance of P from the line $3x - 4y + 2 = 0$ is



$$\pm\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}} = \pm\frac{3x-4y+2}{\sqrt{3^2+(-4)^2}} = \pm\frac{3x-4y+2}{5}$$

Again, distance of P from the origin O

$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(x-0)^2+(y-0)^2}$$

$$=\sqrt{x^2+y^2}$$

Now, according to the question

$$\sqrt{x^2+y^2} = \pm\frac{3x-4y+2}{5}$$

Squaring both sides, we get

$$x^2+y^2 = \left(\pm\frac{3x-4y+2}{5}\right)^2$$

$$\text{or, } x^2+y^2 = \frac{(3x-4y+2)^2}{25}$$

$$\text{or, } 25x^2+25y^2 = (3x)^2+(-4y)^2+2^2+2(3x)(-4y)+2(-4y)(2)+2(2)(3x)$$

$$\text{or, } 25x^2+25y^2 = 9x^2+16y^2+4-24xy-16y+12x$$

$$\text{or, } (25x^2-9x^2)+(25y^2-16y^2)+24xy-12x+16y-4=0$$

$$\text{or, } 16x^2+9y^2+24xy-12x+16y-4=0$$

This is the required equation of locus of a point P(x, y).

83. 2061 Q.No. 9a

Find the equation to the straight line which passes through the intersection of the straight lines $3x - 4y + 1 = 0$ and $5x + y = 1$ and cuts off equal intercepts from the axes. **[4]**

SOLUTION

The equation of two lines are

$$3x-4y+1=0 \quad \ldots (1)$$

and $5x + y - 1 = 0$ ... (2)

Multiplying equation (2) by 4 and adding with equation (1), we get

$$3x-4y+1=0$$

$$\underline{20x+4y-4=0}$$

$$23x-3=0$$

$$\text{or, } 23x=3$$

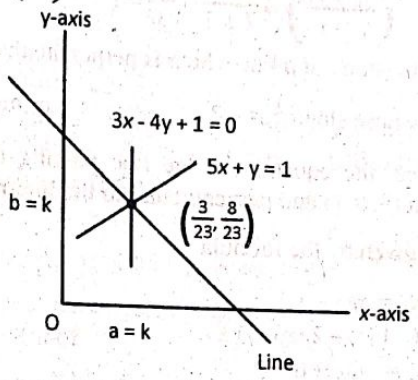
$$\therefore x=\frac{3}{23}$$

Putting $x=\frac{3}{23}$ in equation (1), we get

$$3\left(\frac{3}{23}\right)-4y+1=0$$

$$\text{or, } 4y=\frac{9}{23}+1$$

$$\text{or, } y=\frac{1}{4}\left(\frac{32}{23}\right)=\frac{8}{23}$$



$\therefore$ The point of intersection of the lines (1) and (2) is $\left(\frac{3}{23}, \frac{8}{23}\right)$.

Let the equation of straight line which cuts of equal intercepts to the axes be

$$\frac{x}{k}+\frac{y}{k}=1$$

$$\text{or, } x+y=k \quad \ldots (3)$$

Since this line passes through the point $\left(\frac{3}{23}, \frac{8}{23}\right)$

We have,

$$\frac{3}{23}+\frac{8}{23}=k$$

$$\text{or, } k=\frac{11}{23}$$

Putting the value of k in (3), the required equation of straight line is

$$x+y=\frac{11}{23}$$

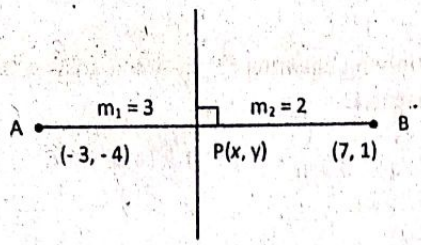
i.e. $23x + 23y = 11$

84. 2060 Q.No. 9a

Find the equation of the line through the point that divides the join of the points (– 3, – 4) and (7, 1) in the ratio 3:2 and is perpendicular to the join. **[4]**

SOLUTION

Let P(x, y) be the point which divides the line joining the points A(– 3, – 4) and B(7, 1) in the ratio 3:2. Then by section formula,



$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3 \times 7 + 2 \times -3}{3 + 2}, \frac{3 \times 1 + 2 \times -4}{3 + 2}\right)$$

$$= \left(\frac{21 - 6}{5}, \frac{3 - 8}{5}\right) = \left(\frac{15}{5}, \frac{-5}{5}\right)$$

$= (3, -1)$

Again, slope of line AB is

$$(m) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \frac{1 + 4}{7 + 3} = \frac{5}{10} = \frac{1}{2}$$

Again, slope of a line which is perpendicular to the line AB whose slope $\frac{1}{2}$ is -2. [$\because m_1m_2 = -1$]

Hence, the equation of the line passing through the point $(3, -1)$ and perpendicular to the line whose slope $\frac{1}{2}$ is given by the formula

$y - y_1 = m(x - x_1)$

or, $y - (-1) = -2(x - 3)$

or, $y + 1 = -2x + 6$

$\therefore$ $2x + y - 5 = 0$, which is the required equation of the straight line.

85. 2059 Q.No. 9a

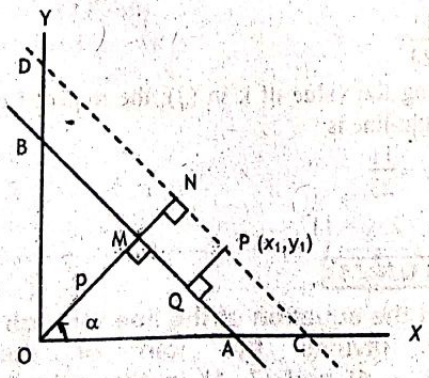
Find the length of the perpendicular from the point (x_1, y_1) on a straight line $x \cos \alpha + y \sin \alpha = p$. [4]

SOLUTION

Let AB be the line represented by the equation

$x \cos \alpha + y \sin \alpha = p$ ... (i)

Now, OM = length of perpendicular drawn from origin on the line (i)



$\therefore$ OM = p

And, $\angle AOM = \alpha$

Let, P (x_1, y) be the given point from which length of perpendicular is to be measured.

Let the perpendicular PQ be drawn from the point P (x_1, y_1) on the line (i).

Also, draw a line CD passing through the point P(x_1, y_1) which is parallel to AB. Let OM be produced to meet CD at N and Let ON = p'

Then, PQ = NM = ON – OM = p' – p ... (ii)

If P(x_1, y_1) and the origin O lie on the same side of AB then,

PQ = OM – ON = p – p' = –(p' – p) ... (iii)

$\therefore$ The equation of CD is

$x \cos \alpha + y \sin \alpha = p'$ ... (iv)

Since, the line CD passes through the point P(x_1, y_1), equation (iv) canm be written as

$x_1 \cos \alpha + y_1 \sin \alpha = p'$ ... (v)

From (ii) and (iii)

$PQ = \pm (p' - p)$

or, $PQ = \pm (x_1 \cos \alpha + y_1 \sin \alpha - p)$

[$\because$ using the relation (iv)]

This is the required length of the perpendicular drawn from the point (x_1, y_1) on the line $x \cos \alpha + y \sin \alpha = p$.

86. 2058 Q.No. 9a

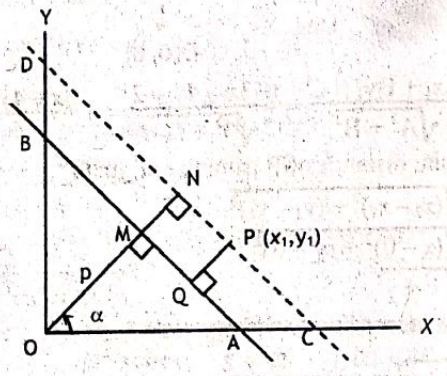
Find the length of the perpendicular from the point (h, k) on a straight line $x \cos \alpha + y \sin \alpha = p$. [4]

SOLUTION

Let AB be the line represented by the equation

$x \cos \alpha + y \sin \alpha = p$ ... (i)

Now, OM = length of perpendicular drawn from origin on the line (i)



$\therefore$ OM = p

And, $\angle AOM = \alpha$

Let, P (h, k) be the given point from which length of perpendicular is to be measured.

Let the perpendicular PQ be drawn from the point P (h, k) on the line (i).

Also, draw a line CD passing through the point P(h, k) which is parallel to AB. Let OM be produced to meet CD at N and Let ON = p'

Then, PQ = NM = ON – OM = p' – p ... (ii)

If P(h, k) and the origin O lie on the same side of AB then,

PQ = OM – ON = p – p' = –(p' – p) ... (iii)

$\therefore$ The equation of CD is

$x \cos \alpha + y \sin \alpha = p'$ ... (iv)

Since, the line CD passes through the point P(h, k), equation (iv) canm be written as

$h \cos \alpha + k \sin \alpha = p'$ ... (v)

From (ii) and (iii)

$PQ = \pm (p' - p)$

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or, $PQ = \pm (h \cos\alpha + k \sin\alpha - p)$ [$\because$ using the relation (iv)]
This is the required length of the perpendicular drawn from the point (h, k) on the line $x \cos\alpha + y \sin\alpha = p$.

87. 2057 Q.No. 9a

Find the length of the perpendicular from the point (x_1, y_1) on a straight line $x \cos\alpha + y \sin\alpha = p$. [4]

Please refer to 2059 Q.No. 9a

88. 2056 Q.No. 9a

Find the angles between two lines given by $y = m_1 x + c_1$ and $y = m_2 x + c_2$. Also state the condition for them to be perpendicular and parallel. [4]

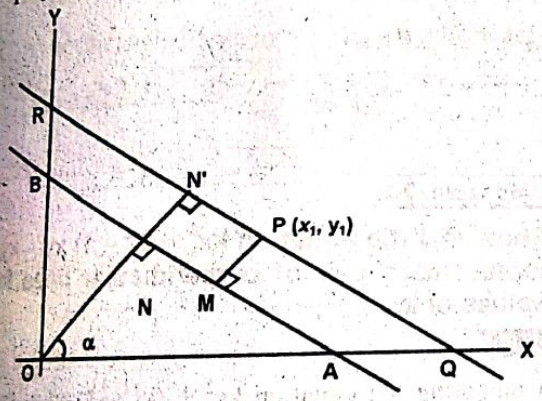
Please refer to 2073 Supp Q.No. 13

89. 2055 Old Model Q.No. 9a

Find the length of the perpendicular from the point (x_1, y_1) on a straight line $x \cos\alpha + y \sin\alpha = p$. [4]

SOLUTION

Let AB be the line represented by the equation
$x \cos\alpha + y \sin\alpha = p$ ... (i)
So, ON = p and $\angle AON = \alpha$
Let $P(x_1, y_1)$ be the point from which PM is drawn perpendicular to AB.



Draw a line QR through $P(x_1, y_1)$ parallel to the given line AB. Let the perpendicular from O to this line QR be ON' (= p_1). Then
$PM = ON' - ON = p_1 - p$ ... (ii)
If the point P and the origin O lie on the same side of AB, then
$PM = ON - ON' = p - p_1 = -(p_1 - p)$ ... (iii)
$\therefore PM = \pm (p_1 - p)$ ... (iv)
The proper sign is taken so as to make PM positive.
Equation of QR is $x \cos\alpha + y \sin\alpha = p_1$
Since, $P(x_1, y_1)$ lies on this line QR,
$x_1 \cos\alpha + y_1 \sin\alpha = p_1$
$\therefore PM = \pm (x_1 \cos\alpha + y_1 \sin\alpha - p)$
Which is the required length of the perpendicular drawn from (x_1, y_1) on the line $x \cos\alpha + y \sin\alpha = p$.

B. PAIR OF STRAIGHT LINES

2 MARKS QUESTIONS

1. 2071 Old Q.No. 2c

Find the equation of the bisectors of the angles between the lines represented by $2x^2 - 6xy - y^2 = 0$. [2]

SOLUTION

Comparing $2x^2 - 6xy - y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get
$a = 2, h = -3, b = -1$
The equation of bisectors are:
$h(x^2 - y^2) = (a - b)xy$
or, $-3(x^2 - y^2) = (2 + 1)xy$
or, $-3x^2 + 3y^2 = 3xy$
$\therefore x^2 + xy - y^2 = 0$

2. 2070 Old Q.No. 2 c

Write the conditions of perpendicularity and parallelism of the lines represented by $lx^2 + 2hxy + ny^2 = 0$. [2]

SOLUTION

Given equation is $lx^2 + 2hxy + ny^2 = 0$
The condition of perpendicularity is
coefficient of x^2 + coefficient of $y^2 = 0$
i.e. $l + n = 0$
Again, the condition of parallelism is
$h^2 = ab$ for $ax^2 + 2hxy + by^2 = 0$
Hence, the condition for given equation is $h^2 = ln$

3. 2069 [Set A] Old Q. No. 2c

Find the angle between the pair of lines given by $2x^2 + 7xy + 3y^2 = 0$. [2]

SOLUTION

Given equation is $2x^2 + 7xy + 3y^2 = 0$
Comparing this equation with
$ax^2 + 2hxy + by^2 = 0$, we get
$a = 2, h = \frac{7}{2}, b = 3$
If θ be the angle between the lines then,

$$\tan\theta = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm \frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2 \cdot 3}}{2 + 3}$$

$$= \pm \left(\frac{2 \times \frac{5}{2}}{5}\right)$$

$$= \pm 1$$

$$= \tan 45^\circ \text{ or } \tan 135^\circ$$

$\therefore \theta = 45^\circ$ or 135°

4. 2069 [Set B] Old Q. No. 2c

Find the value of k so that: $x^2 + kxy + 2y^2 + 3x + 5y + 2 = 0$ may represent a line pair. [2]

SOLUTION

Given equation is $x^2 + kxy + 2y^2 + 3x + 5y + 2 = 0$ ...(i)
Comparing (i) with $ax^2+2hxy+by^2+2gx+2fy+c=0$, we get

$a = 1, h = \frac{k}{2}, b = 2, g = \frac{3}{2}, f = \frac{5}{2}, c = 2$

The condition for the general equation of second degree to represent two straight line is
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

or, $1\times2\times2+2\times\frac{5}{2}\times\frac{3}{2}\times\frac{k}{2}-1\times\left(\frac{5}{2}\right)^2-2\times\left(\frac{3}{2}\right)^2-2\times\left(\frac{k}{2}\right)^2=0$

or, $4+\frac{15k}{4}-\frac{25}{4}-\frac{9}{2}-\frac{k^2}{2}=0$

or, $16 + 15k - 25 - 18 - 2k^2 = 0$
or, $2k^2 - 15k + 27 = 0$
or, $2k^2 - 6k - 9k + 27 = 0$
or, $2k(k-3) - 9(k-3) = 0$
or, $(k-3)(2k-9) = 0$
$\therefore k = 3, \frac{9}{2}$

5. 2068 Old Q.No. 2c

Find the equations of the bisectors of the angles between the lines represented by $3x^2 - 15xy + 2y^2 = 0$. [2]

SOLUTION

Given equation is
$3x^2 - 15xy + 2y^2 = 0$ ...(i)
Comparing (i) with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 3, h = -\frac{15}{2}, b = 2$

The equations of bisectors are
$h(x^2 - y^2) = (a - b)xy$

or, $-\frac{15}{2}(x^2 - y^2) = (3 - 2)xy$

or, $-\frac{15}{2}(x^2 - y^2) = xy$

or, $-15(x^2 - y^2) = 2xy$
or, $-15x^2 + 15y^2 = 2xy$
$\therefore 15x^2 + 2xy - 15y^2 = 0$

6. 2067 Q.No. 2c

Find the angle between the lines given by $x^2 - 2xy\cot\theta - y^2 = 0$ [2]

SOLUTION

Given equation is $x^2 - 2xy\cot\theta - y^2 = 0$ ... (i)
Comparing (i) with $ax^2 + 2hxy + by^2 = 0$, we get
$a = 1, h = -\cot\theta, b = -1$
Let ϕ be the angle between the lines represented by (i) then

$\tan\phi = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

or, $\tan\phi = \pm\frac{2\sqrt{(-\cot\theta)^2 - 1\times(-1)}}{1 + (-1)}$

$= \pm\frac{2\sqrt{\cot^2\theta + 1}}{1 - 1}$

$= \infty = \tan\frac{\pi}{2}$

$\therefore \phi = \frac{\pi}{2}$

7. 2066 Q.No. 2c

For what value of k, the equation $2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$ represents a line pair? [2]

SOLUTION

Given equation is, $2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$
Comparing this with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get,

$a = 2, h = \frac{7}{2}, b = 3, g = -2, f = \frac{-7}{2}, c = k$

The condition is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

or, $2\times3\times k + 2\times\left(\frac{-7}{2}\right)\times(-2)\times\frac{7}{2} - 2\left(\frac{-7}{2}\right)^2 - 3(-2)^2 - k\left(\frac{7}{2}\right)^2 = 0$

or, $6k + 49 - \frac{49}{2} - 12 - \frac{49k}{4} = 0$

or, $\frac{24k + 196 - 98 - 48 - 49k}{4} = 0$

or, $-25k + 50 = 0$

or, $k = \frac{50}{25} = 2$

$\therefore k = 2$

8. 2065 Q.No. 2c

Show that the equation $kx^2 + (k^2 - 1)xy - ky^2 = 0$ represents a pair of perpendicular lines for all values of k. [2]

SOLUTION

Comparing the equation $kx^2 + (k^2 - 1)xy - ky^2 = 0$ with equation $ax^2 + 2hxy + by^2 = 0$, we get
$a = k; b = -k$
The condition of perpendicularity is
$a + b = 0$
or, $k - k = 0$
$0 = 0$ (true)
So the line pairs are perpendicular for all the values of k.

9. 2064 Q.No. 2c

Find the equation of the two lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$ [2]

SOLUTION

The given equation of pair of straight lines is
$2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$
It may be written as
$y^2 + (3x + 2)y + (2x^2 + 5x - 3) = 0$
Solving for y, we get,

$y = \frac{-(3x + 2) \pm \sqrt{(3x + 2)^2 - 4.1.(2x^2 + 5x - 3)}}{2 \times 1}$

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$$\text{or, } y = \frac{-(3x+2) \pm \sqrt{9x^2+12x+4-8x^2-20x+12}}{2}$$

$$\text{or, } y = \frac{-(3x+2) \pm \sqrt{x^2-8x+16}}{2}$$

$$= \frac{-(3x+2) \pm \sqrt{(x-4)^2}}{2}$$

$$= \frac{-(3x+2) \pm (x-4)}{2}$$

$$\text{or, } y = \frac{-(3x+2)+(x-4)}{2} \text{ and } y = \frac{-(3x+2)-(x-4)}{2}$$

$$\text{or, } y = \frac{-2x-6}{2} \text{ and } y = \frac{-4x+2}{2}$$

$$\text{or, } y = \frac{2(-x-3)}{2} \text{ and } y = \frac{2(-2x+1)}{2}$$

or, $y = -x-3$ and $y = -2x+1$

or, $x+y+3=0$ and $2x+y-1=0$

These are the required equations of the lines.

10. 2063 Q.No. 2c

Find the angle between the lines represented by $2x^2 + 7xy + 3y^2 = 0$ [2]

SOLUTION

Comparing the equation $2x^2 + 7xy + 3y^2 = 0$ with the equation $ax^2 + 2hxy + by^2 = 0$, we get

$a = 2, h = \frac{7}{2}, b = 3$

If θ be the angle between the given pair of lines, then

$$\tan\theta = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

$$= \frac{\pm 2\sqrt{(7/2)^2 - 2\times 3}}{2+3}$$

$$= \frac{\pm 2\sqrt{\frac{49}{4}-6}}{5} = \frac{\pm 2\sqrt{\frac{49-24}{4}}}{5}$$

$$= \pm\left(\frac{2\times\frac{5}{2}}{5}\right) = \pm 2\times\frac{5}{2}\times\frac{1}{5}$$

or, $\tan\theta = \pm 1$

Taking positive sign,

$\tan\theta = 1 = \tan 45°$

$\therefore \theta = 45°$

Taking negative sign

$\tan\theta = -1 = \tan 135°$

$\therefore \theta = 135°$

Hence $\theta = 45°$ or $135°$

11. 2062 Q.No. 2c

Find the value of k so that $2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$ may represent a pair of lines. [2]

SOLUTION

The given equation is

$2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$

We compare it with the general equation of second degree

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 2, h = \frac{7}{2}, b = 3, g = -2, f = \frac{-7}{2}$ and $c = k$

The condition for the general equation of second degree to represent the pair of straight lines is

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\text{or, } 2\cdot3\cdot k + 2\left(\frac{-7}{2}\right)(-2)\times\frac{7}{2} - 2\left(\frac{-7}{2}\right)^2 - 3(-2)^2 - k\left(\frac{7}{2}\right)^2 = 0$$

$$\text{or, } 6k + 49 - \frac{49}{2} - 12 - \frac{49k}{4} = 0$$

$$\text{or, } 6k - \frac{49k}{4} = -49 + \frac{49}{2} + 12$$

$$\text{or, } \frac{24k-49k}{4} = \frac{-98+49+24}{2}$$

$$\text{or, } \frac{-25k}{2} = -25$$

$$\text{or, } \frac{k}{2} = 1$$

$\therefore k = 2$

12. 2061 Q.No. 2c

Find the angle between the pair of lines [2]

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$

SOLUTION

The given equation of pair of lines is

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ ...(i)

Comparing equation (i) with the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 1, h = 3, b = 9, g = 2, f = 6$ and $c = -5$

If θ be the angle between the lines represented by (i), then

$$\tan\theta = \frac{\pm 2\sqrt{h^2-ab}}{a+b}$$

$$= \frac{\pm 2\sqrt{3^2-1\times 9}}{1+9}$$

$$= \frac{\pm 2\sqrt{9-9}}{10} = 0$$

or, $\tan\theta = 0$

or, $\tan\theta = \tan 0°$

$\therefore \theta = 0°$

13. 2060 Q.No. 2c

Verify whether the second degree equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents a pair of straight lines or not. [2]

SOLUTION

We have,

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ ... (i)

Comparing this equation with the general equation of second degree

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 1, h = 3, b = 9, g = 2, f = 6, c = -5$

We know that the equation (i) represents the pair of straight lines, if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Now,

$abc + 2fgh - af^2 - bg^2 - ch^2$

$= (1 \times 9 \times -5) + (2 \times 6 \times 2 \times 3) - (1 \times 6^2) - (9 \times 2^2) - [(-5) \times 3^2]$

$= -45 + 72 - 36 - 36 + 45$

$= 72 - 72 = 0$

Hence, the given equation represents a pair of straight lines.

14. 2059 Q.No. 2c

Find the angle between the line pair $2x^2 + 7xy + 3y^2 = 0$. [2]

SOLUTION

The given equation is $2x^2 + 7xy + 3y^2 = 0$ ... (i)

Comparing the equation (i) with the equation $ax^2 + 2hxy + by^2 = 0$, we get

$a = 2, h = \frac{7}{2}, b = 3$

If θ be the angle between the given pair of lines, then

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm\frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2 \times 3}}{2 + 3}$$

$$= \pm\frac{2\sqrt{\frac{49}{4} - 6}}{5} = \pm\frac{2\sqrt{\frac{49 - 24}{4}}}{5} = \pm\frac{2 \times \frac{5}{2}}{5}$$

$$= \pm\frac{5}{5}$$

or, $\tan\theta = \pm 1$

$\Rightarrow \theta = \tan^{-1}(\pm 1) = 45°, 135°$

15. 2058 Q.No. 2c

Find the angle between the line pair given by $x^2 - 2xy\cot\theta - y^2 = 0$ [2]

SOLUTION

Comparing the given equation $x^2 - 2xy\cot\theta - y^2 = 0$, with the equation $ax^2 + 2hxy + by^2 = 0$, we get $a = 1, h = -\cot\theta, b = -1$.

If α be the angle between the lines represented by the lines pair $x^2 - 2xy\cot\theta - y^2 = 0$, then

$$\tan\alpha = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm\frac{2\sqrt{\cot^2\theta + 1}}{1 - 1} = \infty = \tan\frac{\pi}{2}$$

$\therefore \alpha = \frac{\pi}{2}$

16. 2057 Q.No. 2c

Determine the line represents by the equation $x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$ [2]

SOLUTION

The given equation is

$x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$

or, $y^2 + 2(x - 1)y + (x^2 - 2x - 15) = 0$ ... (i)

Equation (i) is quadratic in y, which gives two values of y, so solving (i) for y we get,

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or, $$y = \frac{-2(x-1) \pm \sqrt{\{2(x-1)\}^2 - 4 \times 1 \times (x^2 - 2x - 15)}}{2 \times 1}$$

$$= \frac{-2x + 2 \pm \sqrt{4x^2 - 8x + 4 - 4x^2 + 8x + 60}}{2}$$

$$= \frac{-2x + 2 \pm \sqrt{64}}{2} = \frac{-2x + 2 \pm 8}{2} = \frac{2(-x + 1 \pm 4)}{2}$$

$= -x + 1 \pm 4$

$\therefore y = -x + 1 \pm 4$

Taking positive sign, we get

$y = -x + 1 + 4$

or, $y = -x + 5$

or, $x + y - 5 = 0$

Taking negative sign, we get

$y = -x + 1 - 4$

or, $y = -x - 3$

or, $x + y + 3 = 0$

Hence, the required two straight lines are

$x + y - 5 = 0$ and $x + y + 3 = 0$

17. 2056 Q.No. 2c

Write the condition for which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a line pair. [2]

SOLUTION

The condition that general equation of second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a line pair is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

18. 2055 Old Model Q.No. 2c

Write the condition for which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a line pair. [2]

➥ Please refer to **2056 Q.No. 2c**

6 MARKS QUESTIONS

19. 2076 Set B Q.No. 13 OR

Prove that the bisectors of the angles between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$ [6]

SOLUTION

Let the two straight lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ be $y = m_1x$ and $y = m_2x$.

Then, $m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

The equations of the bisectors of the angle between the lines $y - m_1x = 0$ and $y - m_2x = 0$ are

$$\frac{y - m_1x}{\sqrt{1 + m_1^2}} = \pm\frac{y - m_2x}{\sqrt{1 + m_2^2}}$$

The combined equation is

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$$\left(\frac{y-m_1x}{\sqrt{1+m_1^2}}+\frac{y-m_2x}{\sqrt{1+m_2^2}}\right)\left(\frac{y-m_1x}{\sqrt{1+m_1^2}}-\frac{y-m_2x}{\sqrt{1+m_2^2}}\right)=0$$

or, $\left(\frac{y-m_1x}{\sqrt{1+m_1^2}}\right)^2-\left(\frac{y-m_2x}{\sqrt{1+m_2^2}}\right)^2=0$

or, $(1+m_2^2)(y^2-2m_1xy+m_1^2x^2)-(1+m_1^2)(y^2-2m_2xy+m_2^2x^2)=0$

or, $x^2(m_1^2-m_2^2)-2xy(m_1-m_2)(1-m_1m_2)-y^2(m_1^2-m_2^2)=0$

or, $(x^2-y^2)(m_1^2-m_2^2)=2xy(m_1-m_2)(1-m_1m_2)$

or, $x^2-y^2=2xy\left(\frac{1-m_1m_2}{m_1+m_2}\right)$

or, $x^2-y^2=2xy\left(\frac{1-\frac{a}{b}}{-\frac{2h}{b}}\right)$

or, $x^2-y^2=2xy\left(\frac{b-a}{-2h}\right)$

or, $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$ which is the required equation.

20. 2076 Set C Q.No. 13 OR

Find the angle between a pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. Also discuss the condition of parallelism and perpendicularity of the lines. [6]

SOLUTION

Then the equation $ax^2 + 2hxy + by^2 = 0$ ...(i) can be written as

$$\frac{a}{b}x^2+\frac{2h}{b}xy+y^2=0 \quad \text{...(ii)}$$

If the lines represented by (i) are $y = m_1x$ and $y = m_2x$ then their combined equation is given by

$$m_1m_2x^2-(m_1+m_2)xy+y^2=0 \quad \text{...(iii)}$$

since (ii) and (iii) represent the same pair of lines,

$$m_1+m_2=\frac{-2h}{b} \text{ and } m_1m_2=\frac{a}{b}$$

If θ be the angle between the lines, then

$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}$$

$$=\pm\frac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{1+m_1m_2}$$

$$=\pm\frac{\sqrt{\left(-\frac{2h}{b}\right)^2-4\frac{a}{b}}}{1+\frac{a}{b}}$$

$$=\pm\frac{2\sqrt{h^2-ab}}{a+b}$$

$$\theta=\tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$$

Condition for coincident lines

Two lines will be coincident if $\theta = 0$. Then,

$$\tan 0^\circ=\frac{2\sqrt{h^2-ab}}{a+b}$$

or, $0=h^2-ab$

$h^2 = ab$

Condition of perpendicularity

Two lines will be perpendicular to each other if $\theta = 90^\circ$

$$\tan 90^\circ=\frac{2\sqrt{h^2-ab}}{a+b}$$

or, $\cot 90^\circ=\frac{a+b}{2\sqrt{h^2-ab}}$

or, $0=\frac{a+b}{2\sqrt{h^2-ab}}$

or, $a + b = 0$

i.e. coefficient of x^2 + coefficient of $y^2 = 0$

21. 2075 GIE Q.No. 13 OR

Find the bisectors of angles between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$. [6]

➥ Please refer to 2076 Set B Q.No. 13 OR

22. 2075 Set A Q.No. 14 OR

Prove that the straight lines joining the origin to the intersection of the straight line $kx + hy = 2hk$ and the curve $(x - h)^2 + (y - k)^2 = a^2$ are perpendicular if $h^2 + k^2 = a^2$. [6]

SOLUTION

The given equation of the circle is

$(x-h)^2+(y-k)^2=c^2$

or, $x^2-2hx+h^2+y^2-2ky+k^2-c^2=0$

or, $x^2+y^2-2(hx+ky)+h^2+k^2-c^2=0$ ... (i)

The equation of the given line is

$kx + hy = 2hk$

or, $\frac{kx+hy}{2hk}=1$ ... (ii)

Making equation (i) homogeneous with the help of equation (ii), we get

$$x^2+y^2-2(hx+ky)\left(\frac{kx+hy}{2hk}\right)+(h^2+k^2-c^2)\left(\frac{kx+hy}{2hk}\right)^2=0$$

or, $x^2+y^2-\frac{(hx+ky)(kx+hy)}{hk}+(h^2+k^2-c^2)\frac{(k^2x^2+2khxy+h^2y^2)}{4h^2k^2}=0$

or, $4h^2k^2(x^2+y^2)-4hk(hx+ky)(kx+hy)+(h^2+k^2-c^2)(k^2x^2+2khxy+h^2y^2)=0$

or, $4h^2k^2(x^2+y^2)-4hk(hkx^2+h^2xy+k^2xy+hky^2)+(h^2+k^2-c^2)(k^2x^2+2khxy+h^2y^2)=0$

or, $\{4h^2k^2-4h^2k^2+k^2(h^2+k^2-c^2)\}x^2+4h^2k^2-4h^2k^2+\{(h^2+k^2-c^2)h^2\}y^2-4h^3kxy-4hk^3xy+2hk(h^2+k^2-c^2)xy=0$

or, $k^2(h^2+k^2-c^2)x^2+h^2(h^2+k^2-c^2)y^2+hk(h^2+k^2-c^2)-4h^3k-4hk^3)xy=0$

This is the equation of the lines joining the origin and the points of intersection of the given curve and the line. These straight lines are perpendicular to each other if coeff. of x^2 + coeff. of $y^2 = 0$

$k^2(h^2+k^2-c^2)+h^2(h^2+k^2-c^2)=0$

or, $(h^2+k^2-c^2)(h^2+k^2)=0$.

$\therefore\ h^2+k^2=c^2$

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23. 2075 Set B Q.No. 13 OR

Find the bisectors of angles between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$. [6]

➥ Please refer to 2076 Set B Q.No. 13 OR

24. 2075 Set C Q.No. 13 OR

Show that the two straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with x-axis angles such that the difference of their tangent is 2. [6]

SOLUTION

The equation of the straight line is

$x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ ... (i)

Let $y = m_1x$ and $y = m_2x$ be the two lines represented by the equation (i)

so that, $m_1 + m_2 = \frac{-2h}{b} = \frac{2\tan\theta}{\sin^2\theta}$

$$= \frac{2\sin\theta}{\cos\theta.\sin^2\theta} = \frac{2}{\sin\theta.\cos\theta}$$

$$\therefore m_1 \cdot m_2 = \frac{a}{b} = \frac{\tan^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{\frac{\sin^2\theta}{\cos^2\theta} + \cos^2\theta}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^4\theta}{\sin^2\theta.\cos^2\theta}$$

Now, the difference of the tangents of the angles made with the x-axis by the lines $y = m_1x$ and $y = m_2x$ is

$$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$$

$$= \left(\frac{2}{\sin\theta.\cos\theta}\right)^2 - \frac{4(\sin^2\theta + \cos^4\theta)}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{4}{\sin^2\theta.\cos^2\theta} - \frac{4(\sin^2\theta + \cos^4\theta)}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{4(1 - \sin^2\theta - \cos^4\theta)}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{4(\cos^2\theta - \cos^4\theta)}{\sin^2\theta.\cos^2\theta}$$

$$= \frac{4\cos^2\theta(1 - \cos^2\theta)}{\sin^2\theta.\cos^2\theta} = \frac{4\cos^2\theta\sin^2\theta}{\sin^2\theta.\cos^2\theta}$$

$(m_1 - m_2)^2 = 4$

or, $m_1 - m_2 = \sqrt{4}$

$\therefore m_1 - m_2 = 2$ Proved.

25. 2074 Supp Q.No. 13 OR

Prove that the equation $ax^2 + 2hxy + by^2 = 0$ always represent a pair of straight lines through the origin. Are they always real? [6]

SOLUTION

The homogeneous equation of second degree in x and y is $ax^2 + 2hxy + by^2 = 0$ ...(i)

Case I

If $b \neq 0$, then dividing both sides of (i) by b

$$y^2 + \frac{2h}{b}xy + \frac{a}{b}x^2 = 0$$

or, $\left(\frac{y}{x}\right)^2 + \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$

This equation is quadratic in $\frac{y}{x}$

Hence, $\frac{y}{x}$ will have two values say m_1 and m_2.

Then, $\frac{y}{x} = m_1$ and $\frac{y}{x} = m_2$

or, $y = m_1x$ and $y = m_2x$

which are the equations of lines through origin.

Case II

If $b = 0$, then equation (i) becomes

$ax^2 + 2hxy = 0$

$x(ax + 2hy) = 0$

$\therefore x = 0$ and $ax + 2hy = 0$

which are the lines through origin.

Hence, in all cases the homogeneous equation will represent two straight lines through origin.

No they are not always real. A pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ are

i. real and distinct if $h^2 - ab > 0$
ii. coincident if $h^2 - ab = 0$
iii. imaginary if $h^2 - ab < 0$.

26. 2074 Set A Q.No. 13 OR

Find the angle between the two lines represented by $ax^2 + 2hxy + by^2 = 0$. Also find the condition under which the two lines are (a) perpendicular (b) coincident. [6]

➥ Please refer to 2076 Set C Q.No. 13 OR

27. 2074 Set B Q.No. 13 OR

Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of parallelism and perpendicularity of the lines. [6]

➥ Please refer to 2076 Set C Q.No. 13 OR

28. 2073 Supp Q.No. 13 OR

Find the equations of the two straight lines represented by the equation $2x^2+3xy+y^2+5x+2y-3=0$. Find the point of intersection and also the angle between the lines. [6]

SOLUTION

Given equation is

$2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$

or, $2x^2 + (3y + 5)x + y^2 + 2y - 3 = 0$

or, $x = \frac{-(3y+5) \pm \sqrt{(3y+5)^2 - 4 \cdot 2 \cdot (y^2 + 2y - 3)}}{2 \cdot 2}$

or, $x = \frac{-(3y+5) \pm \sqrt{9y^2 + 30y + 25 - 8y^2 - 16y + 24}}{4}$

or, $x = \frac{-(3y+5) \pm \sqrt{y^2 + 14y + 49}}{4}$

or, $x = \frac{-(3y+5) \pm \sqrt{(y+7)^2}}{4}$

or, $x = \frac{-(3y+5) \pm (y+7)}{4}$

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Taking +ve sign,

$$x = \frac{-(3y+5)+(y+7)}{4}$$

or, $4x = -3y - 5 + y + 7$

or, $4x + 2y = 2$

or, $2x + y - 1 = 0$ ...(i)

Again, taking –ve sign,

$$x = \frac{-(3y+5)-(y+7)}{4}$$

or, $4x = -(3y+5) - (y+7)$

or, $4x = -3y - 5 - y - 7$

or, $4x + 4y + 12 = 0$

or, $x + y + 3 = 0$ ...(ii)

For point of intersection, we solve equation (i) and equation (ii)

Subtracting equation (ii) from equation (i)

$2x + y - 1 = 0$

$x + y + 3 = 0$

$- \quad - \quad -$

$x - 4 = 0$

$x = 4$

Putting the value of x in equation (ii), we get

$4 + y + 3 = 0$

$\therefore y = -7$

$\therefore$ Required point of intersection is $(4, -7)$

Again, slope of (i) $(m_1) = \frac{-2}{1} = -2$

Slope of (ii) $(m_2) = -\frac{1}{1} = -1$

If θ be the angle between (i) and (ii), then

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm\frac{-2-(-1)}{1+(-2)(-1)}$$

$$= \pm\left(-\frac{1}{3}\right)$$

$$= \pm\frac{1}{3}$$

$$\therefore \theta = \tan^{-1}\left(\pm\frac{1}{3}\right)$$

29. 2073 Set C Q.No. 13 OR

If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove that $pq = -1$. [6]

SOLUTION

Here, the given equations are

$x^2 - 2pxy - y^2 = 0$ ... (i)

and, $x^2 - 2qxy - y^2 = 0$

or, $(x^2 - y^2) = 2qxy$ ... (ii)

Now, the equation of bisectors of the angles between the lines (i) is

$h(x^2 - y^2) = (a - b)xy$

or, $-p(x^2 - y^2) = (1 + 1)xy$

or, $-p(x^2 - y^2) = 2xy$ ...(iii)

But according to the question, pair of the lines in (ii) also represents the equation of the bisectors of the angle between the lines represented by (i).

So, equations (ii) and (iii) represent the same. Thus the coefficient of like terms of equation (ii) and (iii) must be proportional.

i.e. $\frac{-p}{1} = \frac{2}{2q}$

or, $-pq = 1$

$\therefore pq = -1$

30. 2073 Set D Q.No. 13 OR

Find the angle between a pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. Also discuss the condition of parallelism and perpendicularity of the lines. [6]

➥ Please refer to 2076 Set C Q.No. 13 OR

31. 2072 Supp. Q.No. 13 OR

Find the condition under which the general equation of second degree represents a line pair. [6]

SOLUTION

Given equation is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \ (a \neq 0)$

or, $ax^2 + 2x(hy + g) + by^2 + 2fy + c = 0$

or, $x = \frac{-2(hy+g) \pm \sqrt{\{2(hy+g)\}^2 - 4a(by^2 + 2fy + c)}}{2 \cdot a}$

or, $x = \frac{-(hy+g) \pm \sqrt{h^2y^2 + 2ghy + g^2 - aby^2 - 2afy - ac}}{a}$

or, $x = \frac{-(hy+g) \pm \sqrt{(h^2 - ab)y^2 + (2gh - 2af)y + g^2 - ac}}{a}$

These equations will be linear only if $(h^2 - ab)y^2 + (2gh - 2af)y + (g^2 - ac)$ be a perfect square.

For this, discriminant of this equation = 0

$\{2(gh - af)\}^2 - 4(h^2 - ab)(g^2 - ac) = 0$

or, $(gh - af)^2 - (h^2 - ab)(g^2 - ac) = 0$

or, $g^2h^2 - 2afgh + a^2f^2 - h^2g^2 + h^2ac + abg^2 - a^2bc = 0$

or, $a(abc + 2fgh - af^2 - bg^2 - ch^2) = 0$

Since $a \neq 0$, the required conditions is

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

32. 2072 Set C Q.No. 13 OR

Deduce the condition that the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight lines. [6]

➥ Please refer to 2072 Supp. Q.No. 13 OR

33. 2072 Set D Q.No. 12 OR

Find the equation of the two lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$. Find their point of intersection and also the angle between them. [6]

➥ Please refer to 2073 Supp Q.No. 13 OR

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34. 2072 Set E Q.No. 14 OR

Show that two straight lines through the origin, which make an angle of 45° with the line $px + qy + r = 0$ are given by $(p^2 - q^2)(x^2 - y^2) + 4pqxy = 0$ [6]

SOLUTION

Let $y = mx$ ...(i) be the line through origin and making an angle of 45° with the line

$px + qy + r = 0$ ...(ii)

Slope of line (i) $(m_1) = m$

Slope of line (ii) $(m_2) = \frac{-p}{q}$

We have,

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\text{or, } \tan 45° = \frac{m - \left(-\frac{p}{q}\right)}{1 + m\cdot\left(-\frac{p}{q}\right)}$$

$$\text{or, } 1 = \pm\frac{m + \frac{p}{q}}{1 - \frac{mp}{q}}$$

$$\text{or, } 1 = \pm\frac{mq + p}{q - mp}$$

Squaring both sides, we get

$(q - mp)^2 = (mq + p)^2$ ...(iii)

From (i) $m = \frac{y}{x}$. Using this value of m in (iii)

$$\left(q - \frac{y}{x}\cdot p\right)^2 = \left(\frac{y}{x}\cdot q + p\right)^2$$

or, $(qx - yp)^2 = (yq + px)^2$

or, $q^2x^2 - 2pqxy + y^2p^2 = y^2q^2 + 2xypq + p^2x^2$

or, $0 = p^2x^2 - q^2x^2 + 4pqxy - y^2p^2 + y^2q^2$

or, $(p^2 - q^2)x^2 + 4pqxy - (p^2 - q^2)y^2 = 0$

$\therefore$ $(p^2 - q^2)(x^2 - y^2) + 4pqxy = 0$ which is the required equation.

35. 2071 Supp. Q.No. 14 OR

If θ be the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ then prove that: $\tan\theta = \pm 2\frac{\sqrt{h^2 - ab}}{a + b}$ when (i) $b \neq 0$ (ii) $b = 0$. [6]

SOLUTION

i. Let $b \neq 0$. Then the equation $ax^2 + 2hxy + by^2 = 0$ ...(1) can be written as

$$\frac{a}{b}x^2 + \frac{2h}{b}xy + y^2 = 0 \quad \text{...(2)}$$

If the lines represented by (1) are $y = m_1x$ and $y = m_2x$ then their combined equation is given by

$m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0$ ...(3)

Since (2) and (3) represent the same pair of lines,

$m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

If θ be the angle between the lines, then

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \pm\frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2}$$

$$= \pm\frac{\sqrt{\left(-\frac{2h}{b}\right)^2 - 4\frac{a}{b}}}{1 + \frac{a}{b}}$$

$$= \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\therefore \quad \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

ii. When $b = 0$, then

$$\tan\theta = \pm\frac{2\sqrt{h^2 - 0}}{a + 0} = \pm\frac{2h}{a}$$

36. 2071 Set C Q.No. 13

Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of parallelism and perpendicularity of the lines. Find the angle between the lines represented by $x^2 + 9xy + 14y^2 = 0$ [6]

SOLUTION

First Part: Please refer to **2076 Set C Q.No. 13 OR**

Last part

Comparing $x^2+9xy+14y^2 = 0$ with $ax^2+2hxy+by^2 = 0$, we get

$a = 1, h = \frac{9}{2}, b = 14$

If θ be the angle between the lines, then

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \pm\frac{2\sqrt{\left(\frac{9}{2}\right)^2 - 1\cdot 14}}{1 + 14}$$

$$= \pm\frac{2\sqrt{\frac{81 - 56}{4}}}{15} = \pm\frac{2\sqrt{\frac{25}{4}}}{15} = \pm\frac{2\times\frac{5}{2}}{15}$$

$$= \pm\frac{1}{3}$$

$$\therefore \quad \theta = \tan^{-1}\left(\pm\frac{1}{3}\right)$$

37. 2071 Set D Q.No. 13 Or

Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of coincidence and perpendicularity of the lines. Find the angles between the pair of lines represented by $7x^2 + 8xy + y^2 = 0$. [6]

SOLUTION

First Part: Please refer to **2076 Set C Q.No. 13 OR**

Last part

Comparing $7x^2 + 8xy + y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 7, h = 4, b = 1$

If θ be the angle between the lines then

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$$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b}$$

$$\text{or, } \tan\theta = \pm\frac{2\sqrt{4^2-7\cdot1}}{7+1}$$

$$= \pm\frac{2\times3}{8} = \pm\frac{3}{4}$$

$$\therefore \theta = \tan^{-1}\left(\pm\frac{3}{4}\right)$$

38. 2070 Supp Q.No. 13 Or

Find the equations of two lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$. Find their point of intersection. [6]

SOLUTION

Given equation is

$2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$

or, $2x^2 + (3y+5)x + y^2 + 2y - 3 = 0$

or, $x = \frac{-(3y+5) \pm \sqrt{(3y+5)^2 - 4\cdot2\cdot(y^2+2y-3)}}{2\cdot2}$

or, $x = \frac{-(3y+5) \pm \sqrt{9y^2+30y+25-8y^2-16y+24}}{4}$

or, $x = \frac{-(3y+5) \pm \sqrt{y^2+14y+49}}{4}$

or, $x = \frac{-(3y+5) \pm \sqrt{(y+7)^2}}{4}$

or, $x = \frac{-(3y+5) \pm (y+7)}{4}$

Taking +ve sign,

$x = \frac{-(3y+5)+(y+7)}{4}$

or, $4x = -3y - 5 + y + 7$

or, $4x + 2y = 2$

or, $2x + y - 1 = 0$ ...(i)

Again, taking –ve sign,

$x = \frac{-(3y+5)-(y+7)}{4}$

or, $4x = -(3y+5) - (y+7)$

or, $4x = -3y - 5 - y - 7$

or, $4x + 4y + 12 = 0$

or, $x + y + 3 = 0$ ...(ii)

For point of intersection, we solve equation (i) and equation (ii)

Subtracting equation (ii) from equation (i)

$2x + y - 1 = 0$

$x + y + 3 = 0$

$- \quad - \quad -$

$x - 4 = 0$

$x = 4$

Putting the value of x in equation (ii), we get

$4 + y + 3 = 0$

$\therefore y = -7$

$\therefore$ Required point of intersection is $(4, -7)$

39. 2070 Set C Q.No. 13 Or

Find the equation to the pair of lines joining the origin to the intersection of the straight line $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1+m^2)$. [6]

Please refer to 2068 Old Q.No. 9b

40. 2070 Set D Q.No. 13 Or

Find the condition under which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of lines. [6]

Please refer to 2072 Supp. Q.No. 13 OR

41. 2069 Supp Q.No. 13 Or

Show that the straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with x-axis angles such that the difference of their tangents is 2. [6]

Please refer to 2075 Set C Q.No. 13 OR

42. 2069 [Set A] Q. No. 13 OR

Show that the homogeneous equation of degree two always represents a pair of straight line passing through the origin. Also, find the angle between them. [6]

SOLUTION

The homogeneous equation of second degree in x and y is $ax^2 + 2hxy + by^2 = 0$ ...(i)

Case I

If $b \neq 0$, then dividing both sides of (i) by b

$y^2 + \frac{2h}{b}xy + \frac{a}{b}x^2 = 0$

or, $\left(\frac{y}{x}\right)^2 + \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$

This equation is quadratic in $\frac{y}{x}$

Hence, $\frac{y}{x}$ will have two values say m_1 and m_2.

Then, $\frac{y}{x} = m_1$ and $\frac{y}{x} = m_2$

or, $y = m_1x$ and $y = m_2x$

which are the equations of lines through origin.

Case II

If $b = 0$, then equation (i) becomes

$ax^2 + 2hxy = 0$

$x(ax + 2hy) = 0$

$\therefore x = 0$ and $ax + 2hy = 0$

Which are the lines through origin.

Hence, in all cases the homogeneous equation will represent two straight lines through origin.

Second part:

Then the equation $ax^2 + 2hxy + by^2 = 0$ ...(i) can be written as

$\frac{a}{b}x^2 + \frac{2h}{b}xy + y^2 = 0$ $(b \neq 0)$ ...(ii)

If the lines represented by (i) are $y = m_1x$ and $y = m_2x$ then their combined equation is given by

$m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0$ ...(iii)

since (ii) and (iii) represent the same pair of lines,

$m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

If θ be the angle between the lines, then

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$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm\frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$= \pm\frac{\sqrt{\left(-\frac{2h}{b}\right)^2 - 4\frac{a}{b}}}{1 + \frac{a}{b}} = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\therefore \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

43. 2069 [Set B] Q. No. 13 OR

Prove that the bisectors of the angles between the pair of straight lines $ax^2 - 2hxy + by^2 = 0$ is given by $\frac{x^2 - y^2}{xy} = \frac{a - b}{h}$. [6]

SOLUTION

Let the two straight lines represented by the equation $ax^2 - 2hxy + by^2 = 0$ be $y = m_1x$ and $y = m_2x$.

Then, $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

The equations of the bisectors of the angle between the lines $y - m_1x = 0$ and $y - m_2x = 0$ are

$$\frac{y - m_1x}{\sqrt{1 + m_1^2}} = \pm\frac{y - m_2x}{\sqrt{1 + m_2^2}}$$

The combined equation is

$$\left(\frac{y - m_1x}{\sqrt{1 + m_1^2}} + \frac{y - m_2x}{\sqrt{1 + m_2^2}}\right)\left(\frac{y - m_1x}{\sqrt{1 + m_1^2}} - \frac{y - m_2x}{\sqrt{1 + m_2^2}}\right) = 0$$

or, $\left(\frac{y - m_1x}{\sqrt{1 + m_1^2}}\right)^2 - \left(\frac{y - m_2x}{\sqrt{1 + m_2^2}}\right)^2 = 0$

or, $(1 + m_2^2)(y^2 - 2m_1xy + m_1^2x^2) - (1 + m_1^2)(y^2 - 2m_2xy + m_2^2x^2) = 0$

or, $x^2(m_1^2 - m_2^2) - 2xy(m_1 - m_2)(1 - m_1m_2) - y^2(m_1^2 - m_2^2) = 0$

or, $(x^2 - y^2)(m_1^2 - m_2^2) = 2xy(m_1 - m_2)(1 - m_1m_2)$

or, $x^2 - y^2 = 2xy\left(\frac{1 - m_1m_2}{m_1 + m_2}\right)$

or, $x^2 - y^2 = 2xy\left(\frac{1 - \frac{a}{b}}{\frac{2h}{b}}\right)$

or, $x^2 - y^2 = 2xy\left(\frac{b - a}{2h}\right)$

or, $\frac{x^2 - y^2}{b - a} = \frac{xy}{h}$

$\therefore h(x^2 - y^2) = (b - a)xy$ which is the required equation.

44. 2068 Q.No. 13(Or)

Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ [6]

SOLUTION

The equation of the curve is $x^2 + y^2 = c^2$ ... (i)

The equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

or, $bx + ay = ab$

or, $\left(\frac{bx + ay}{ab}\right) = 1$ ... (ii)

Using equation (ii) in equation (i) in order to make the equation (i) homogeneous, we get

$$x^2 + y^2 = c^2\left(\frac{bx + ay}{ab}\right)^2$$

or, $x^2 + y^2 = c^2\left(\frac{b^2x^2 + 2abxy + a^2y^2}{a^2b^2}\right)$

or, $a^2b^2x^2 + a^2b^2y^2 = b^2c^2x^2 + 2abc^2xy + a^2c^2y^2$

or, $(a^2b^2 - b^2c^2)x^2 - 2abc^2xy + (a^2b^2 - a^2c^2)y^2 = 0$ ... (iii)

Here, equation (iii) represents the lines joining the origin and the point of intersection of the given curve (i) and the line (ii).

We know that these straight lines are at right angle to each other if

Coeff. of x^2 + Coeff. of $y^2 = 0$

i.e. $(a^2b^2 - b^2c^2) + (a^2b^2 - a^2c^2) = 0$

or, $2a^2b^2 = b^2c^2 + a^2c^2$

or, $2 = \frac{b^2c^2 + a^2c^2}{a^2b^2}$

or, $\frac{c^2}{a^2} + \frac{c^2}{b^2} = 2$

$\therefore \frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. Hence proved.

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

45. 2071 Old Q.No. 9b

Prove that the straight lines joining the origin to the point of intersection of the line $bx + ay - ab = 0$ and the curve $x^2+y^2=c^2$ are right angels if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

➥ Please refer to 2068 Q.No. 13(Or)

46. 2070 Old Q.No. 9 b

Show that the straight lines joining the origin to the points of intersection of the line $kx + hy = 2hk$ with the curve $(x - h)^2 + (y - k)^2 = c^2$ are at right angles if $h^2 + k^2 = c^2$. [4]

➥ Please refer to 2075 Set A Q.No. 14 OR

47. 2069 [Set A] Old Q. No. 9b

Find the equation of two straight lines joining the origin to the points of intersection of the straight line $x - y = 2$ and the curve $5x^2 + 12xy - 8y^2 + 8x - 4y + 12 = 0$. 4 [4]

SOLUTION

Given curve is

$5x^2 + 12xy - 8y^2 + 8x - 4y + 12 = 0$ ...(i)

And, equation of straight line is

$x - y = 2$

or, $\frac{x - y}{2} = 1$ ...(ii)

Making equation (i) homogeneous with the help of (ii), we have

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$$5x^2+12xy-8y^2+8x\left(\frac{x-y}{2}\right)-4y\left(\frac{x-y}{2}\right)+12\left(\frac{x-y}{2}\right)^2=0$$

or, $5x^2+12xy-8y^2+4x(x-y)-2y(x-y)+3(x^2-2xy+y^2)=0$

or, $5x^2+12xy-8y^2+4x^2-4xy-2xy+2y^2+3x^2-6xy+3y^2=0$

or, $12x^2-3y^2=0$

or, $4x^2-y^2=0$ which is the required equation.

48. 2069 [Set B] Old Q. No. 9b

Show that: the two straight lines $x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$ make with the x-axis angles such that the difference between their tangents is 2. [4]

Please refer to 2075 Set C Q.No. 13 OR

49. 2068 Old Q.No. 9b

Find the equation to the pair of straight lines joining the origin to the intersection of the straight lines $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1 + m^2)$. [4]

SOLUTION

The given equation of the line is $y = mx + c$ which can be written as

$$\frac{y-mx}{c}=1 \quad \ldots (i)$$

The equation of the curve is

$x^2+y^2=a^2$

i.e. $x^2+y^2-a^2=0$ ... (ii)

Now, making the equation (ii) homogeneous with the help of equation (i), we get

$$x^2+y^2-a^2\left(\frac{y-mx}{c}\right)^2=0$$

or, $x^2+y^2-a^2\left(\frac{y^2-2ymx+m^2x^2}{c^2}\right)=0$

or, $x^2+y^2-\frac{a^2(y^2-2ymx+m^2x^2)}{c^2}=0$

or, $x^2c^2+y^2c^2-a^2y^2+2a^2ymx-a^2m^2x^2=0$

or, $(c^2-a^2m^2)x^2+(c^2-a^2)y^2+2a^2ymx=0$ ... (iii)

which is the equation of the pair of the lines joining the origin to the points of intersection of the line (i) and the curve (ii).

The straight lines represented by (iii) are at right angles to each other if

coeff. of x^2 + coeff. of $y^2 = 0$

or, $(c^2-a^2m^2)+(c^2-a^2)=0$

or, $c^2-a^2m^2+c^2-a^2=0$

or, $c^2+c^2=a^2+a^2m^2$

or, $2c^2=a^2(1+m^2)$ Proved.

2067 Q.No. 9b

Find the condition so that the straight lines joining the origin to the points of intersection of the line $kx + hy = 2hk$ with the circle $(x-h)^2+(y-k)^2=c^2$ are at right angle. [4]

Please refer to 2070 Old Q.No. 9 b

51. 2066 Q.No. 9b

Find the equation of the lines which are right angles to the lines represented by $ax^2 + 2hxy + by^2 = 0$. [4]

SOLUTION

Let $y = m_1x$ and $y = m_2x$ be the separate equations of the given line pair.

$ax^2 + 2hxy + by^2 = 0$. So $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

Equations of the lines through origin and perpendicular to

$y = m_1x$ and $y = m_2x$ are

$y = -\frac{1}{m_1}x$ and $y = -\frac{1}{m_2}x$ respectively.

or, $m_1y + x = 0$ and $m_2y + x = 0$

The combined equation is

$(m_1y + x)(m_2y + x) = 0$

or, $m_1m_2y^2 + (m_1 + m_2)xy + x^2 = 0$

or, $\frac{a}{b}y^2 - \frac{2h}{b}xy + x^2 = 0$

or, $ay^2 - 2hxy + bx^2 = 0$

$\therefore bx^2 - 2hxy + ay^2 = 0$

52. 2065 Q.No. 9b

Show that pair of lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with the axis of x angle such that the difference of their tangent is 2. [4]

Please refer to 2069 [Set B] Old Q. No. 9b

53. 2064 Q.No. 9a

Find the single equation of the lines through the origin and perpendicular to the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$. [4]

Please refer to 2066 Q.No. 9b

54. 2063 Q.No. 9b

Find the equations of the two lines represented by $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$. Prove that the two lines are parallel. [4]

SOLUTION

The given general equation of second degree is

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ ... (i)

The equation (i) can be written as

$x^2 + (6y + 4)x + 9y^2 + 12y - 5 = 0$,

It is quadratic in x.

Solving for x, we get

$$x = \frac{-(6y+4)\pm\sqrt{(6y+4)^2-4\times1\times(9y^2+12y-5)}}{2\times1}$$

$$= \frac{-(6y+4)\pm\sqrt{36y^2+48y+16-36y^2-48y+20}}{2}$$

$$= \frac{-(6y+4)\pm\sqrt{36}}{2}$$

$$= \frac{-(6y+4)\pm6}{2}$$

$$= -(3y+2)\pm3$$

$$x = -(3y+2)\pm3$$

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Taking positive sign, we get

$x = -(3y+2)+3$

or, $x = -3y-2+3$

or, $x+3y-1=0$ ... (ii)

Again, taking negative sign, we get

$x = -(3y+2)-3$

or, $x = -3y-2-3$

or, $x+3y+5=0$ ... (iii)

Hence, $x+3y-1=0$ and $x+3y+5=0$ are the required two lines.

Next, slope of line (ii) is $m_1 = -\frac{1}{3}$

slope of line (iii) is $m_2 = -\frac{1}{3}$

As, the slopes of two lines are equal, i.e. $m_1 = m_2$. Hence, two lines are parallel.

55. 2062 Q.No. 9b

Determine the two straight lines represented by: $6x^2 - xy - 12y^2 - 8x + 29y - 14 = 0$ [4]

SOLUTION

We have,

$6x^2 - xy - 12y^2 - 8x + 29y - 14 = 0$

or, $6x^2 - xy - 8x - 12y^2 + 29y - 14 = 0$

or, $6x^2 - (y+8)x - (12y^2 - 29y - 14) = 0$

It is quadratic in x, where $a = 6$, $b = -(y+8)$; $c = -(12y^2 - 29y - 14)$

We know,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

or, $$x = \frac{(y+8) \pm \sqrt{\{-(y+8)\}^2 - 4\times6\times-(12y^2-29y-14)}}{2\times6}$$

$$= \frac{(y+8) \pm \sqrt{y^2+16y+64+24(12y^2-29y-14)}}{12}$$

$$= \frac{(y+8) \pm \sqrt{y^2+16y+64+288y^2-696y-336}}{12}$$

$$= \frac{(y+8) \pm \sqrt{289y^2-680y+400}}{12}$$

$$= \frac{(y+8) \pm \sqrt{(17y)^2-2\times17y\times20+20^2}}{12}$$

$$= \frac{(y+8) \pm \sqrt{(17y-20)^2}}{12}$$

or, $$x = \frac{(y+8) \pm (17y-20)}{12}$$

or, $12x = (y+8) \pm (17y-20)$

Now, taking positive sign, we get

$12x = y+8+17y-20$

or, $12x - 18y + 12 = 0$

$\therefore 2x - 3y + 2 = 0$

Again, taking negative sign, we get

$12x = (y+8) - (17y-20)$

or, $12x + 16y - 28 = 0$

$\therefore 3x + 4y - 7 = 0$

Hence, the required straight lines are $2x - 3y + 2 = 0$ and $3x + 4y - 7 = 0$

56. 2061 Q.No. 9b

For what values of c, the lines which join the origin to the point of intersection of the line $x - y + c = 0$ and the curve $x^2 + y^2 + 4x - 6y - 36 = 0$ may be at right angle. [4]

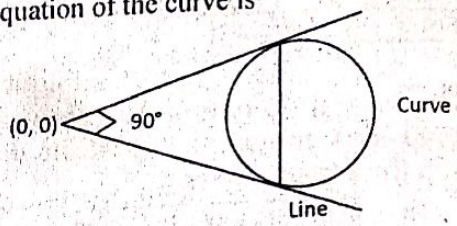
SOLUTION

The equation of the given line is $x - y + c = 0$

or, $y - x = c$

or, $\frac{y-x}{c} = 1$ ... (1)

The equation of the curve is



$x^2 + y^2 + 4x - 6y - 36 = 0$ ... (2)

Making equation (2) homogeneous with the help of (1), we get

$$x^2 + y^2 + 4x\left(\frac{y-x}{c}\right) - 6y\left(\frac{y-x}{c}\right) - 36\left(\frac{y-x}{c}\right)^2 = 0$$

or, $$x^2 + y^2 + 4\left(\frac{xy-x^2}{c}\right) - 6\left(\frac{y^2-xy}{c}\right) - 36\left(\frac{y^2-2xy+x^2}{c^2}\right) = 0$$

or, $c^2x^2 + c^2y^2 + 4cxy - 4cx^2 - 6cy^2 + 6cxy - 36y^2 + 72xy - 36x^2 = 0$

or, $(c^2-4c-36)x^2 + (10c+72)xy + (c^2-6c-36)y^2 = 0$ ... (3)

Equation (3) is the equation of the lines joining the origin to the points of intersection of the given line (1) and the curve (2).

The lines given by the equation (3) are at right angles to each other if (coeff. of x^2) + (coeff. of y^2) = 0

i.e. if $(c^2-4c-36) + (c^2-6c-36) = 0$

or, $2c^2 - 10c - 72 = 0$

or, $c^2 - 5c - 36 = 0$

or, $c^2 - 9c + 4c - 36 = 0$

or, $c(c-9) + 4(c-9) = 0$

or, $(c-9)(c+4) = 0$

$\therefore c = 9$ or -4

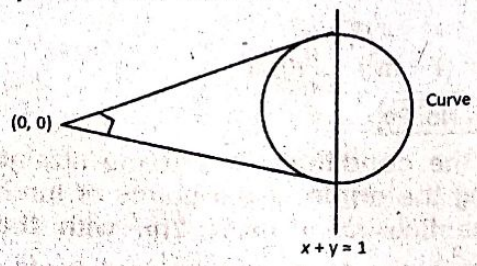
57. 2060 Q.No. 9b

Show that the lines joining the points of intersection of the line $x + y = 1$ with the curve $4x^2 + 4y^2 + 4x - 2y - 5 = 0$ with the origin are at right angles to each other. [4]

SOLUTION

The equation of the line is $x + y = 1$ ... (1)

The equation of the curve is



$4x^2 + 4y^2 + 4x - 2y - 5 = 0$

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which can be written as

$4x^2 + 4y^2 + 4x \times 1 - 2y \times 1 - 5 \times 1^2 = 0$ ... (2)

The equation of pair of lines joining the origin to the points of intersections of (1) and (2) can be obtained by making the homogeneous equation to (2) with the help of (1),

i.e. $4x^2 + 4y^2 + 4x(x + y) - 2y(x + y) - 5(x + y)^2 = 0$

or, $4x^2 + 4y^2 + 4x^2 + 4xy - 2xy - 2y^2 - 5x^2 - 10xy - 5y^2 = 0$

or, $3x^2 - 3y^2 - 8xy = 0$ ... (3)

Now, coefficient of x^2 + coefficient of $y^2 = 3 + (-3) = 0$

∴ Equation (3) represents the line pair perpendicular to each other.

58. 2059 Q.No. 9b

Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

Please refer to 2068 Q.No. 13(Or)

59. 2058 Q.No. 9b

Prove that the pair of straight lines joining the origin to the points of intersection of the line $y = mx + c$ and the curve $x^2 + y^2 = a^2$ are at right angles if $2c^2 = a^2(1 + m^2)$. [4]

Please refer to 2068 Old Q.No. 9b

60. 2057 Q.No. 9b

If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove $pq = -1$. [4]

Please refer to 2073 Set C Q.No. 13 OR

61. 2056 Q.No. 9b

Prove that the straight lines joining the origin to the point of intersections of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

Please refer to 2068 Q.No. 13(Or)

62. 2055 Old Model Q.No. 9b

Prove that the straight lines joining the origin to the point of intersections of the line $\frac{x}{a} + \frac{y}{b} = 1$, and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

Please refer to 2068 Q.No. 13(Or)

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11 UNIT

CIRCLE

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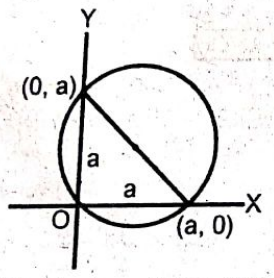
2 MARKS QUESTIONS

1. 2076 Set B Q.No. 4b

Find the equation of the circle which passes through origin and makes equal intercepts on both axes. [2]

SOLUTION

Let the equal intercept on the axes be a. Then the circle has (a, 0) and (0, a) as the end points of diameter since $\angle XOY = 90°$.



That is, $(x_1, y_1) = (a, 0)$ and $(x_2, y_2) = (0, a)$.

The equation of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - a)(x - 0) + (y - 0)(y - a) = 0$

or, $x^2 - ax + y^2 - ay = 0$

$\therefore\ x^2 + y^2 - ax - ay = 0.$

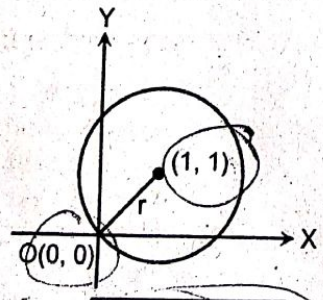
2. 2076 Set C Q.No. 4b

Find the equation of the circle which passes through origin and whose center is at (1, 1). [2]

SOLUTON

Centre $(h, k) = (1, 1)$

Since the circle passes through the origin, so radius is the distance between (0, 0) and (1, 1). So,



$r = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$

Equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$

or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$\therefore\ x^2 + y^2 - 2x - 2y = 0.$

3. 2075 GIE Q.No. 4b

Find the equation of the circle whose center is at (–3, –4) and touches the x-axis. [2]

SOLUTION

Centre $(h, k) = (-3, -4)$

Since the circle touches x-axis,

so radius $r = |k| = |-4| = 4$

Equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x + 3)^2 + (y + 4)^2 = 4^2$

or, $x^2 + 6x + 9 + y^2 + 8y + 16 = 16$

$\therefore\ x^2 + y^2 + 6x + 8y + 9 = 0.$

4. 2075 Set A Q.No. 4b

If one end of a diameter of circle $x^2 + y^2 - 2x + 6y - 15 = 0$ is (4, 1), find the coordinate of the other end. [2]

SOLUTION

Given equation of circle is

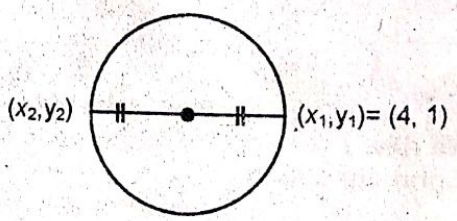
$x^2 + y^2 - 2x + 6y - 15 = 0$ ...(i)

Comparing (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$g = -1, f = 3, c = -15$

Centre $= (-g, -f) = (1, -3)$

Given, one end of diameter $(x_1, y_1) = (4, 1)$, other end $(x_2, y_2) = ?$



We know that centre is the mid point of the diameter. So, using mid point formula, we have

$x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$

or, $1 = \frac{4 + x_2}{2}$ and $-3 = \frac{1 + y_2}{2}$

$\therefore\ x_2 = -2$ and $y_2 = -7$

Required coordinates of other end = (–2, –7)

5. 2075 Set B Q.No. 4b

Find the equation of the circle which touches both axes and has radius 4 units. [2]

SOLUTION

Let (h, k) be the centre and r be the radius of the circle.

Given radius (r) = 4

Since the circle touches both the axes, r = h = k.

∴ centre (h, k) = (4, 4) & radius (r) = 4

Equation of circle is

$(x-h)^2+(y-k)^2=r^2$

$(x-4)^2+(y-4)^2=4^2$

or, $x^2-8x+16+y^2-8y+16=16$

or, $x^2+y^2-8x-8y+16=0$.

∴

6. 2075 Set C Q.No. 4b

Find the equation of the circle which touches both the axes and has its centre on the line x – 2y = 3. [2]

SOLUTION

Let (h, k) be the centre and 'r' be the radius of the circle.

Since the circle touches both axes, h = k = r.

The centre (h, h) lies in x – 2y = 3. So,

h – 2h = 3

or, –h = 3

∴ h = –3

∴ Centre (h, k) = (h, h) = (–3, –3)

Radius (r) = |h| = |–3| = 3.

Equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x+3)^2+(y+3)^2=3^2$

or, $x^2+6x+9+y^2+6y+9=9$

∴ $x^2+y^2+6x+6y+9=0$

7. 2074 Supp Q.No. 4b

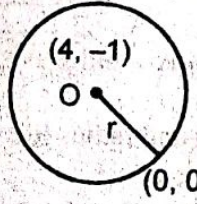
Find the equation of a circle with centre at (4, -1) and passing through the origin. [2]

SOLUTION

Given, centre of the circle (h, k) = (4, –1)

Since the circle passes through the origin, so

Radius (r) = distance between (0, 0) and (4, –1) $=\sqrt{(4-0)^2+(-1-0)^2}=\sqrt{17}$



Now, the equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x-4)^2+(y+1)^2=(\sqrt{17})^2$

or, $x^2-8x+16+y^2+2y+1=17$

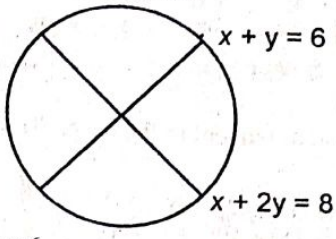
∴ $x^2+y^2-8x+2y=0$

8. 2074 Set A Q.No. 4b

Find the equation of a circle having radius 10 units and equations of any two diameters are x + 2y = 8 and x + y = 6. [2]

SOLUTION

Equations of diameters are



$x+y=6$ ...(i)

$x+2y=8$ ...(ii)

Subtracting (ii) from (i),

$x + y = 6$

$x + 2y = 8$

$-y = -2$

∴ $y = 2$

Putting the value of y in (i), we get

$x+2=6 \Rightarrow x=4$

∴ The point of intersection of the diameters is (4, 2). Since, the diameter intersect at the centre, so (4, 2) is the centre of the circle i.e. (h, k) = (4, 2)

Given radius (r) = 10

The equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x-4)^2+(y-2)^2=10^2$

or, $x^2-8x+16+y^2-4y+4=100$

∴ $x^2+y^2-8x-4y-80=0$

9. 2074 Set B Q.No. 4b

Find the equation of the circle which touches the x-axis at (4, 0) and passes through the point (1, 1). [2]

SOLUTION

Let (h, k) be the centre & r be the radius of the circle.

Since the circle touches X–axis, k = r.

Equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x-h)^2+(y-k)^2=k^2$ ... (i)

Since equation (i) passes through the points (4, 0) & (1, 1), then

$(4-h)^2+k^2=k^2$

or, $(4-h)^2=0 \Rightarrow h=4$.

And $(1-h)^2+(1-k)^2=k^2$

or, $(1-4)^2+1-2k+k^2=k^2$

or, $9+1-2k=0$

$k=\frac{10}{2}=5$

Hence the equation of circle is

$(x-4)^2+(y-5)^2=5^2$

or, $x^2-8x+16+y^2-10y+25=25$

∴ $x^2+y^2-8x-10y+16=0$.

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10. 2073 Supp Q.No. 4b

Find the equation of the tangent to the circle $x^2+y^2-2x-4y+3=0$ at (2, 3). [2]

SOLUTION

Given equation of circle is

$x^2+y^2-2x-4y+3=0$ ... (i)

Comparing (i) with $x^2+y^2+2gx+2fy+c=0$, we get,

$2g=-2, 2f=-4, c=3$

$g=-1, f=-2$.

The equation of tangent to the circle (i) at

$(x_1, y_1)=(2, 3)$ is

$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

or, $x.2+y.3+(-1)(x+2)+(-2)(y+3)+3=0$

or, $2x+3y-x-2-2y-6+3=0$

$\therefore x+y=5$

11. 2073 Set C Q.No. 4b

Find the equation to the circle which touches the coordinate axes at (a, 0) and (0, a). [2]

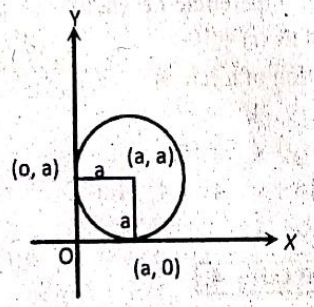
SOLUTION

Since the circle touches the co-ordinates axes at (a, 0) and (0, a)

So, centre (h, k) = (a, a)

Radius (r) = a

The required equation of the circle is



$(x-h)^2+(y-k)^2=r^2$

or, $(x-a)^2+(y-a)^2=a^2$

or, $x^2-2ax+a^2+y^2-2ay+a^2=a^2$

$\therefore x^2+y^2-2ax-2ay+a^2=0$

12. 2073 Set D Q.No. 4b

Find the equation of the circle touching x–axis whose center is at (4,3). [2]

SOLUTION

Centre (h, k) = (4, 3)

Since the circle touches x-axis, so radius

(r) = k = 3

The equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x-4)^2+(y-3)^2=3^2$

or, $x^2-8x+16+y^2-9y+9=9$

$\therefore x^2+y^2-8x-9y+16=0$

13. 2072 Supp. Q.No. 4b

Find the equation of the circle whose two of the diameters are $x+y=6$ and $x+2y=8$ and radius 10. [2]

➥ Please refer to 2074 Set A Q.No. 4b

14. 2072 Set C Q.No. 4b

Find the equation of the circle with centre at (p, q) and radius $\sqrt{p^2+q^2}$. [2]

SOLUTION

Here, centre (h, k) = (p, q)

radius (r) $=\sqrt{p^2+q^2}$

Equation of circle is

$(x-h)^2+(y-k)^2=r^2$

or, $(x-p)^2+(y-q)^2=(\sqrt{p^2+q^2})^2$

or, $x^2-2px+p^2+y^2-2qx+q^2=p^2+q^2$

$\therefore x^2+y^2-2px-2qx=0$

15. 2072 Set D Q.No. 4b

Find the equation of the circle with two of the diameters are $x+y=6$ and $x+2y=8$ and radius 10. [2]

➥ Please refer to 2074 Set A Q.No. 4b

16. 2072 Set E Q.No. 4b

Find the equation of a circle concentric with the circle $x^2+y^2+2x+6y-39=0$ and passing through (7, –7). [2]

SOLUTION

The equation of any circle concentric with

$x^2+y^2+2x+6y-39=0$ is

$x^2+y^2+2x+6y+k=0$ ...(i)

Since (i) passes through the point (7, –7), so

$7^2+(-7)^2+2\times7+6\times(-7)+k=0$

or, $49+49+14-42+k=0$

or, $k=-70$

Substituting the value of k in (i), we get

$x^2+y^2+2x+6y-70=0$ which is the required equation of circle.

17. 2071 Supp. Q.No. 4b

If one end of the diameter of circle $x^2+y^2-2x+6y-15=0$ is (4,1), find the coordinates of the other end. [2]

➥ Please refer to 2075 Set A Q.No. 4b

18. 2071 Set C Q.No. 4b

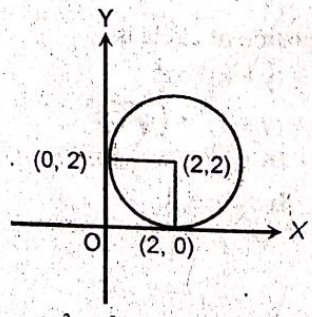
Find the equation of the circle which touches the axes at (2, 0) and (0, 2). [2]

SOLUTION

The circle touches x-axis and y-axis at (2, 0) and (0, 2) respectively. So,

centre (h, k) = (2, 2) and radius (r) = 2

The equation of circle is



$(x-h)^2+(y-k)^2=r^2$

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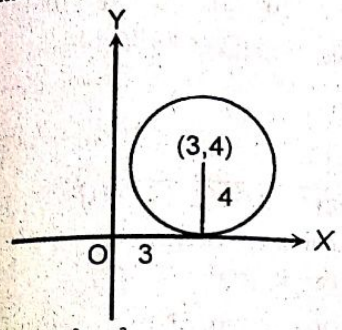
$(x-2)^2+(y-2)^2=2^2$
or, $x^2-4x+4+y^2-4y+4=4$
or, $x^2+y^2-4x-4y+4=0$

19. 2071 Set D Q.No. 4b

Find the equation of the circle whose center is at (3, 4) and touches the x–axis. [2]

SOLUTION

Centre $(h, k) = (3, 4)$
Since the circle touches the x-axis, radius $(r) = k = 4$
The equation of circle is



$(x-h)^2+(y-k)^2=r^2$
or, $(x-3)^2+(y-4)^2=4^2$
or, $x^2-6x+9+y^2-8y+16=16$
$\therefore x^2+y^2-6x-8y+9=0$

20. 2070 Supp Q.No. 4b

Prove that the line $5x + 12y + 78 = 0$ is tangent to the circle $x^2 + y^2 = 36$. [2]

SOLUTION

Given circle is
$x^2+y^2=36$ ...(i)
$\therefore$ Centre = (0, 0)
radius = 6
Given line is
$5x+12y+78=0$ ...(ii)
Now, the length of perpendicular from centre (0, 0) upon the given line

$= \left|\frac{5\times0+12\times0+78}{\sqrt{5^2+12^2}}\right| = \left|\frac{78}{13}\right| = 6$ which is equal to radius of the circle.

Hence, the given line is a tangent to the given circle.

21. 2070 Set C Q.No. 4 b

Find the equation of the circle whose two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [2]

Please refer to 2074 Set A Q.No. 4b

22. 2070 Set D Q.No. 4 b

Find the centre and the radius of the circle $x^2 + y^2 + 4x - 6y + 4 = 0$. [2]

SOLUTION

Given equation of circle is
$x^2+y^2+4x-6y+4=0$ ...(i)
Comparing equation (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = 4, 2f = -6, c = 4$
$\therefore$ $g = 2, f = -3$
Centre $= (-g, -f) = (-2, 3)$
Radius $= \sqrt{g^2+f^2-c} = \sqrt{2^2+(-3)^2-4}$
$= \sqrt{4+9-4} = 3$

23. 2069 Supp Q.No. 4 b

Find the equation to the circle which touches the x-axis and has centre at (3, 4). [2]

Please refer to 2071 Set D Q.No. 4b

24. 2069 [Set A] Q. No. 4b

Find the equation of the circle concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5, 4). [2]

SOLUTION

The equation of the circle concentric with
$x^2 + y^2 - 8x + 12y + 15 = 0$ is
$x^2 + y^2 - 8x + 12y + k = 0$ ... (i)
Since (i) passes through (5, 4),
so $5^2 + 4^2 - 8 \times 5 + 12 \times 4 + k = 0$
$\therefore$ $k = -49$
Substituting the value of k in (i)
$x^2 + y^2 - 8x + 12y - 49 = 0$
Which is the required equation of circle.

25. 2069 [Set B] Q. No. 4b

Find the equation to the circle which has the points (0, –1) and (2, 3) as ends of a diameter. [2]

SOLUTION

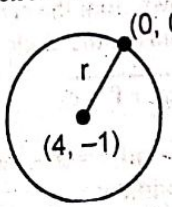
Given end points of a diameter are
$(x_1, y_1) = (0, -1)$
and $(x_2, y_2) = (2, 3)$
The equation of circle is
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
or, $(x-0)(x-2)+(y+1)(y-3)=0$
or, $x^2-2x+y^2+y-3y-3=0$
$\therefore$ $x^2+y^2-2x-2y-3=0$

26. 2068 Q.No. 4b

Find the equation of the circle with centre at (4, –1) and passing through the origin. [2]

SOLUTION

Centre of the circle $(h, k) = (4, -1)$
Since the circle passes through the point (0, 0),
radius (r) = distance between (0, 0) and (4,–1).
$= \sqrt{(4-0)^2+(-1-0)^2} = \sqrt{17}$
The equation of circle is



$(x-h)^2+(y-k)^2=r^2$
or, $(x-4)^2+(y+1)^2=(\sqrt{17})^2$
or, $x^2-8x+16+y^2+2y+1=17$
$\therefore$ $x^2+y^2-8x+2y=0$

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27. 2068 Q.No. 1c

Find the equation of a circle with centre at (4, 5) and $3x - 4y + 5 = 0$ as the line tangent to the circle. [2]

SOLUTION

Centre of the circle (h, k) = (4, 5)

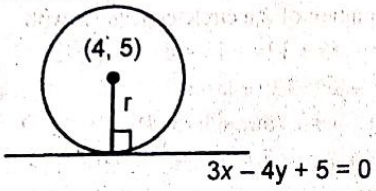
Equation of tangent is $3x - 4y + 5 = 0$

Radius (r) = length of perpendicular drawn from centre upon the tangent line

$$= \left|\frac{3\times4 - 4\times5 + 5}{\sqrt{3^2 + (-4)^2}}\right|$$

$$= \left|\frac{12 - 20 + 5}{5}\right| = \frac{3}{5}$$

Equation of circle is



$(x-h)^2 + (y-k)^2 = r^2$

or, $(x-4)^2 + (y-5)^2 = \left(\frac{3}{5}\right)^2$

or, $x^2 - 8x + 16 + y^2 - 10y + 25 = \frac{9}{25}$

or, $x^2 + y^2 - 8x - 10y + 41 = \frac{9}{25}$

or, $25x^2 + 25y^2 - 200x - 250y + 1025 = 9$

$\therefore$ $25x^2 + 25y^2 - 200x - 250y + 1016 = 0$

28. 2067 Q.No. 1c

Find the equation of the circle which is concentric to the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passes through (5, 4). [2]

➥ Please refer to 2069 [Set A] Q. No. 4b

29. 2066 C Q.No. 1 c

Write the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [2]

SOLUTION

Given equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

Centre = (–g, –f)

Radius = $\sqrt{g^2 + f^2 - c}$

The line $lx + my + n = 0$ ... (ii)

will be tangent to the circle (i) if the length of the perpendicular drawn from the centre (–g, –f) to line (ii) is equal to the radius of the circle.

i.e. $\pm\frac{n - lg - mf}{\sqrt{l^2 + m^2}} = \sqrt{g^2 + f^2 - c}$

Squaring, $(n - lg - mf)^2 = (l^2 + m^2)(g^2 + f^2 - c)$ which is the required condition.

30. 2066 Q.No. 1 c

Find the equation of the circle which touches the coordinate axes at (a, 0) and (0, a). [2]

➥ Please refer to 2073 Set C Q.No. 4b

31. 2065 Q.No 1 c

Find the equation of the circle which has its centre on the line y = 2 and passes through the points (2, 0), (4, 0) [2]

SOLUTION

Let the centre of the circle be (h, k)

Since (h,k) lies on the line $y = 2$

So, $k = 2$

Also, the circle passes through the points (2, 0) and (4, 0) so

$(h-2)^2 + (k-0)^2 = (h-4)^2 + (k-0)^2$

or, $h^2 - 4h + 4 + k^2 = h^2 - 8h + 16 + k^2$

or, $8h - 4h = 16 - 4$

$\therefore$ $h = 3$

$\therefore$ Centre (h, k) = (3, 2)

Radius = $\sqrt{(3-2)^2 + (2-0)^2} = \sqrt{5}$

$\therefore$ The equation of the circle is

$(x-h)^2 + (y-k)^2 = r^2$

or, $(x-3)^2 + (y-2)^2 = (\sqrt{5})^2$

or, $x^2 - 6x + 9 + y^2 - 4y + 4 = 5$

$\therefore$ $x^2 + y^2 - 6x - 4y + 8 = 0$

32. 2064 Q.No. 1 c

Find the equation of a circle whose two of the diameters are $x + y = 6$ and $x + 2y = 4$ and radius 10. [2]

SOLUTION

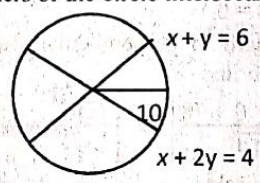
The equation of diameters are

$x + y = 6$ ... (i)

$x + 2y = 4$ ... (ii)

Solving (i) and (ii), we get, $x = 8$, $y = -2$

The diameters of the circle intersects at centre.



So, centre (h, k) = (8, –2)

Radius (r) = 10

The equation of the circle is

$(x-h)^2 + (y-k)^2 = r^2$

or, $(x-8)^2 + (y+2)^2 = 10^2$

or, $x^2 - 16x + 64 + y^2 + 4y + 4 = 100$

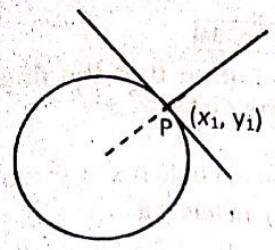
$\therefore$ $x^2 + y^2 - 16x + 4y - 32 = 0$

33. 2063 Q.No. 1 c

Find the equation of the normal to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1) [2]

SOLUTION

The equation of the tangent to the circle $x^2 + y^2 = a^2$ at $P(x_1, y_1)$ is $xx_1 + yy_1 = a^2$



So, the slope of the tangent $= \frac{-x_1}{y_1}$

Since the normal at P is perpendicular to the tangent at P. So, the slope of the normal is $\frac{y_1}{x_1}$ $[\because m_1 \times m_2 = -1]$

Also, the normal passes through the point $P(x_1, y_1)$.
Hence its equation is

$$y - y_1 = \frac{y_1}{x_1}(x - x_1)$$

or, $x_1 y - x_1 y_1 = xy_1 - x_1 y_1$

or, $x_1 y = y_1 x$

$\therefore$ $x_1 y - y_1 x = 0$

34. 2062 Q.No. 1 c

Find the equation of a circle with centre at (6, 5) and touching the line 3x–2y+2=0. **[2]**

SOLUTION

Centre $(h, k) = (6, 5)$

Equation of the tangent line is $3x - 2y + 2 = 0$

Radius of the circle is equal to the perpendicular distance from the centre to the tangent line.

i.e. $r = \pm\frac{3.6 - 2.5 + 2}{\sqrt{3^2 + (-2)^2}}$

$\therefore$ $r = \frac{10}{\sqrt{13}}$

The equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 6)^2 + (y - 5)^2 = \left(\frac{10}{\sqrt{13}}\right)^2$

or, $x^2 - 12x + 36 + y^2 - 10y + 25 = \frac{100}{13}$

or, $x^2 + y^2 - 12x - 10y + 61 = \frac{100}{13}$

$\therefore$ $13x^2 + 13y^2 - 156x - 130y + 693 = 0$.

35. 2061 Q.No. 1 c

Find the value of k so that the straight line $4x + 3y + k = 0$ may touch the circle $x^2 + y^2 - 4x + 10y + 4 = 0$ **[2]**

SOLUTION

Equation of the circle is

$x^2 + y^2 - 4x + 10y + 4 = 0$ ... (i)

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$

We get, $g = -2$, $f = 5$, $c = 4$

Centre $(-g, -f) = (2, -5)$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{4 + 25 - 4} = 5$

The line $4x + 3y + k = 0$ ..... (ii) will touch the circle (i) if the length of the perpendicular from centre of the circle (2, –5) to line (ii) is equal to the radius of the circle.

i.e. $5 = \pm\frac{4.2 + 3.(-5) + k}{\sqrt{3^2 + 4^2}}$

or, $5 = \pm\left(\frac{-7 + k}{5}\right)$

or, $25 = \pm(-7 + k)$

Taking +ve sign, $k = 32$

Taking –ve sign, $k = -18$

$\therefore$ $k = 38, -18$

36. 2060 Q.No. 1 c

Find the centre and radius of the circle $2x^2 + 2y^2 - 12x + 4y = 1$. **[2]**

SOLUTION

Given equation of the circle is

$2x^2 + 2y^2 - 12x + 4y - 1 = 0$

or, $x^2 + y^2 - 6x + 2y - \frac{1}{2} = 0$

Comparing this with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -6$, $2f = 2$, $c = -\frac{1}{2}$

$\therefore$ $g = -3$, $f = 1$

Centre $= (-g, -f) = (3, -1)$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{9 + 1 + \frac{1}{2}}$

$= \sqrt{\frac{21}{2} \times \frac{2}{2}}$

$= \frac{\sqrt{41}}{2}$ units

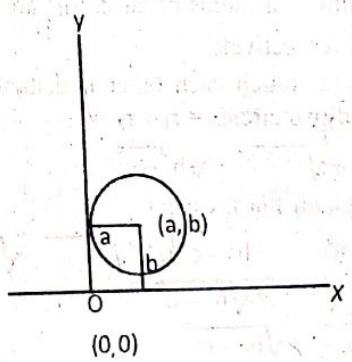
37. 2059 Q.No. 1 c

Find the equation of the circle which has centre at (a, b) and touches the y–axis. **[2]**

SOLUTION

Centre of the circle $(h, k) = (a, b)$

Since the circle touches y – axis



So, radius $(r) = a$

Equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - a)^2 + (y - b)^2 = a^2$

or, $x^2 - 2ax + a^2 + y^2 - 2by + b^2 = a^2$

$\therefore$ $x^2 + y^2 - 2ax - 2by + b^2 = 0$

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38. 2058 Q.No. 1 c

Determine the equation of the circle if the ends of a diameter be at (1, 2) and (3, 5). [2]

SOLUTION

The equation of the circle with end points of a diameter $(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (3, 5)$ is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x - 1)(x - 3) + (y - 2)(y - 5) = 0$

or, $x^2 - x - 3x + 3 + y^2 - 2y - 5y + 10 = 0$

$\therefore x^2 + y^2 - 4x - 6y + 13 = 0$

39. 2057 Q.No. 1 c

Find the value of k so that the length of the tangent from (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 1. [2]

SOLUTION

Length of the tangent from (5, 4) to the given circle $x^2 + y^2 + 2ky = 0$ is 1

i.e. $\sqrt{5^2 + 4^2 + 2.k.4} = 1$

or, $\sqrt{41 + 8k} = 1$

Squiring on both sides, we get,

$41 + 8k = 1$

or, $8k = 1 - 41$

or, $8k = -40$

$\therefore k = -5$

4 MARKS QUESTIONS

40. 2076 Set B Q.No. 9a

Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^{2+} + y^2 + 2by + c^2 = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

SOLUTION

The equations of circles are

$x^2 + y^2 + 2ax + c^2 = 0$ ... (i)

$x^2 + y^2 + 2by + c^2 = 0$ ... (ii)

Centre and radius of circle (i) are $(-a, 0)$ and $r_1 = \sqrt{a^2 - c^2}$ respectively.

And centre and radius of circle (ii) are $(0, -b)$ and $r_2 = \sqrt{b^2 - c^2}$ respectively.

Two circles touch each other if distance between the centres of two circles $= r_1 \pm r_2$

i.e. $\sqrt{a^2 + b^2} = \sqrt{a^2 - c^2} \pm \sqrt{b^2 - c^2}$

Squaring both sides, we get

or, $a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{a^2 - c^2}.\sqrt{b^2 - c^2}$

or, $2c^2 = \pm 2\sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

or, $c^2 = \pm \sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

Again, squaring, $c^4 = (a^2 - c^2)(b^2 - c^2)$

or, $c^4 = a^2b^2 - a^2c^2 - b^2c^2 + c^4$

or, $b^2c^2 + a^2c^2 = a^2b^2$

Dividing both sides by $a^2b^2c^2$, we get

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

41. 2076 Set C Q.No. 9a

Find the condition that the line $px + qy = r$ is tangent to the circle $x^2 + y^2 = a^2$. [4]

SOLUTON

Given equation of circle is $x^2 + y^2 = a^2$

Centre = (0, 0), radius = a

Given line is

$px + qy = r$

or, $px + qy - r = 0$ ...(i)

The line (i) will be a tangent to the given circle length of perpendicular from centre of the given circl to line (i) is equal to the radius of the circle.

i.e. $\pm\left(\frac{p \cdot 0 + q \cdot 0 - r}{\sqrt{p^2 + q^2}}\right) = a$

or, $\pm(-r) = a\sqrt{p^2 + q^2}$

Squaring on both sides, we get,

$r^2 = a^2(p^2 + q^2)$

$\therefore a^2(p^2 + q^2) = r^2$ which is the required condition.

42. 2075 GIE Q.No. 9a

Find the equation of the circle having th chord $x - y - 1 = 0$ of the circl $2x^2 + 2y^2 - 2x - 6y - 25 = 0$ as diameter. [4

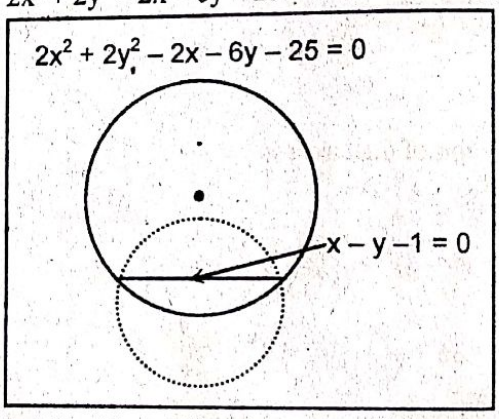
SOLUTION

Given chord is $x - y - 1 = 0$

or, $x = y + 1$ ...(i)

And, equation of circle is

$2x^2 + 2y^2 - 2x - 6y - 25 = 0$ ...(ii)



Solving (i) and (ii)

$2(y + 1)^2 + 2y^2 - 2(y + 1) - 6y - 25 = 0$

or, $2(y^2 + 2y + 1) + 2y^2 - 2y - 2 - 6y - 25 = 0$

or, $2y^2 + 4y + 4 + 2y^2 - 2y - 2 - 6y - 25 = 0$

or, $4y^2 - 4y - 25 = 0$

$\therefore y = 3, -2$ (approximately)

From (i),

when $y = 3$, $x = 4$

when $y = -2$, $x = -1$

End points of diameter are

$(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (4, 3)$

The equation of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x + 1)(x - 4) + (y + 2)(y - 3) = 0$

or, $x^2 - 4x + x - 4 + y^2 - 3y + 2y - 6 = 0$

$\therefore x^2 + y^2 - 3x - y - 10 = 0$

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43. 2075 Set A Q.No. 9a

Find the value of k if the length of the tangent from the point (1, 2) to the circle $x^2+y^2+x+y-4=0$ and $3x^2+3y^2-x-y-k=0$ are in the ratio 4:3. [4]

SOLUTION

Given circles are

$x^2+y^2+x+y-4=0$ ... (i)

and $3x^2+3y^2-x-y-k=0$

or, $x^2+y^2-\frac{x}{3}-\frac{y}{3}-\frac{k}{3}=0$ ... (ii)

Now, length of tangent from (1, 2) to the circle (i) is

$= \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

$= \sqrt{1^2+2^2+1+2-4}$

$= 2$

Again, length of tangent from (1, 2) to the circle (ii) is

$= \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

$= \sqrt{1^1+2^2-\frac{1}{3}-\frac{2}{3}-\frac{k}{3}}$

$= \sqrt{4-\frac{k}{3}}$

By question, $\frac{2}{\sqrt{4-\frac{k}{3}}}=\frac{4}{3}$

Squaring both sides, we get

or, $\frac{4}{4-\frac{k}{3}}=\frac{16}{9}$

or, $\frac{4}{\frac{12-k}{3}}=\frac{16}{9}$

or, $\frac{12}{12-k}=\frac{16}{9}$

or, $\frac{3}{12-k}=\frac{4}{9}$

or, $27=48-4k$

or, $4k=48-27$

or, $4k=21$

$\therefore k=\frac{21}{4}$

44. 2075 Set B Q.No. 9a

Find the condition that the line $lx + my + n = 0$ should be tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]

SOLUTION

Given equation of circle is

$x^2+y^2+2gx+2fy+c=0$ ... (i)

Centre = $(-g, -f)$

Radius = $\sqrt{g^2+f^2-c}$

The line $lx+my+n=0$ ... (ii)

will be tangent to the circle (i) if the length of the perpendicular drawn from the centre $(-g, -f)$ to line (ii) is equal to the radius of the circle.

i.e. $\pm\frac{n-lg-mf}{\sqrt{l^2+m^2}}=\sqrt{g^2+f^2-c}$

Squaring, $(n - lg - mf)^2 = (l^2 + m^2)(g^2 + f^2 - c)$ which is the required condition.

45. 2075 Set C Q.No. 9a

Find the equations of the tangent to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$. [4]

SOLUTION

Given equation of circle is

$x^2+y^2-6x+4y-12=0$ ...(i)

Comparing (i) with $x^2+y^2+2gx+2fy+c=0$, we get,

$2g=-6$, $2f=4$, $c=-12$, $g=-3$, $f=2$

$\therefore$ Centre = $(-g, -f) = (3, -2)$

Radius = $\sqrt{g^2+f^2-c}=\sqrt{9+4+12}=5$

Equation of any line parallel to $4x + 3y + 5 = 0$ is,

$4x+3y+k=0$. ...(ii)

This line will be the tangent to the circle (i), if

Radius = length of perpendicular from centre of circle (i) upon the line (ii)

i.e. $5=\pm\frac{4\cdot3+3(-2)+k}{\sqrt{4^2+3^2}}$

or. $5=\pm\left(\frac{6+k}{5}\right)$

or, $25=\pm(6+k)$

Taking positive sign,

$25=6+k$

$\therefore k=19$

Putting the value of k in (ii), we get,

$4x+3y+19=0$

Again, taking negative sign,

$25=-(6+k)$

or, $-25=6+k$

$\therefore k=-31$

Putting the value of k in (ii), we get,

$4x+3y-31=0$

Required equations are:

$4x+3y+19=0$

and $4x+3y-31=0$

46. 2074 Supp Q.No. 9a

Prove that the straight line $y = x + a\sqrt{2}$ touches the circle $x^2 + y^2 = a^2$. Also find the point of contact. [4]

SOLUTION

Given line is $y=x+a\sqrt{2}$ ... (i)

& circle is $x^2+y^2=a^2$ ... (ii)

Solving (i) and (ii), we get

$x^2+(x+a\sqrt{2})^2=a^2$

or, $x^2+x^2+2\sqrt{2}ax+2a^2=a^2$

or, $2x^2+2\sqrt{2}ax+a^2=0$

or, $x^2+\sqrt{2}ax+\frac{a^2}{2}=0$

or, $x^2+2.x.\frac{a}{\sqrt{2}}+\left(\frac{a}{\sqrt{2}}\right)^2=0$

or, $\left(x+\frac{a}{\sqrt{2}}\right)^2=0$

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This gives two equal values of x. Then corresponding values of y are also equal.

Hence the given line meets the given circle in two coincident points. So, given line touches the given circle. Here, $x = -\frac{a}{\sqrt{2}}$.

When $x = -\frac{a}{\sqrt{2}}$, from (i), $y = -\frac{a}{\sqrt{2}} + a\sqrt{2}$

$$= \frac{-a+2a}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

The point of contact $= \left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

47. 2074 Set A Q.No. 9a

Find the equations of the tangent and normal to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at (2, 3). [4]

SOLUTION

Given equation of circle is

$x^2 + y^2 - 2x - 4y + 3 = 0$ ... (i)

Comparing (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get,

$2g = -2, 2f = -4, c = 3$

$\therefore$ $g = -1, f = -2, c = 3$

The equation of tangent to the circle

(i) at $(x_1, y_1) = (2, 3)$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

or, $x.2 + y.3 + (-1)(x + 2) + (-2)(y + 3) + 3 = 0$

or, $2x + 3y - x - 2 - 2y - 6 + 3 = 0$

$\therefore$ $x + y = 5$

Slope of tangent $= -1$

$\therefore$ Slope of normal $= -\frac{1}{\text{slope of tangent}} = \frac{-1}{(-1)} = 1$

The equation of normal at

$(x_1, y_1) = (2, 3)$ is $y - 3 = 1(x - 2)$

or, $y - 3 = 1(x - 2)$

or, $y - 3 = x - 2$

or, $0 = x - y - 2 + 3$

$\therefore$ $x - y + 1 = 0$

48. 2074 Set B Q.No. 9a

Find the conditions for the circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ to touch (i) externally (ii) internally. [4]

SOLUTION

Given circles are

$x^2 + y^2 = a^2$ ... (i)

$\therefore$ centre $= (0, 0)$, radius $= a$

& $(x - c)^2 + y^2 = b^2$ ... (ii)

$\therefore$ Centre $= (c, 0)$ & radius $= b$.

i. The circles (i) & (ii) touch externally if distance between two centres is equal to sum of radii.

i.e. $\sqrt{(c-0)^2 + (0-0)^2} = a + b$

i.e. $c = a + b$.

ii. The circles touches internally if distance betwee centres $= a \sim b$

$c = a \sim b$.

49. 2073 Supp Q.No. 9a

Find the equation of a circle passing through the points (1, 2), (3, 1) and (–3, –1). [4

SOLUTION

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

If the circle (i) passes through the points (1, 2), (3, 1) and (–3, –1), then

$1^2 + 2^2 + 2g.1 + 2f.2 + c = 0$

or, $2g + 4f + c = -5$ ... (ii)

Similarly, $3^2 + 1^2 + 2g.3 + 2f.1 + c = 0$

or, $6g + 2f + c = -10$ ... (iii)

And, $(-3)^2 + (-1)^2 + 2g.(-3) + 2.f(-1) + c = 0$

or, $-6g - 2f + c = -10$ ... (iv)

Adding (iii) and (iv)

$$\begin{array}{rl} 6g + 2f + c & = -10 \\ -6g - 2f + c & = -10 \\ \hline 2c & = -20 \\ c & = -10 \end{array}$$

Putting the value of c in (ii) and (iii), we get,

$2g + 4f - 10 = -5$

$2g + 4f = 10 - 5$

$2g + 4f = 5$ ... (v)

And, $6g + 2f - 10 = -10$

$6g + 2f = 0$ ... (vi)

Multiplying equation (v) by 3 and subtracting (vi) from equation (v)

$$\begin{array}{r} 6g + 12f = 15 \\ 6g + 2f = 0 \\ - \quad - \\ \hline 10f = 15 \end{array}$$

$f = \frac{15}{10} = \frac{3}{2}$

Putting the value of f in (vi),

$6g + 2 \times \frac{3}{2} = 0$

or, $6g + 3 = 0$

or, $6g = -3$

$g = \frac{-3}{6} = \frac{-1}{2}$

Now, putting the value of f, g & c in (i) we get,

$x^2 + y^2 + 2\left(-\frac{1}{2}\right)x + 2.\frac{3}{2}.y + (-10) = 0$

$x^2 + y^2 - x + 3y - 10 = 0$

50. 2073 Set C Q.No. 9a

For what value of 'a', the line $x + 3y = a$ touches the circle $x^2 + y^2 - 3x - 3y + 2 = 0$. Find also the point of contact. [4]

SOLUTION

Given equation of circle is

$x^2 + y^2 - 3x - 3y + 2 = 0$ ... (i)

Comparing (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get,

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$g = -\frac{3}{2}, f = -\frac{3}{2}, c = 2$

$\therefore$ Centre $= (-g, -f) = \left(\frac{3}{2}, \frac{3}{2}\right)$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2 - 2} = \sqrt{\frac{5}{2}}$

The line $x + 3y = a$ ...(ii) will touch the circle (i) if,

$\sqrt{\frac{5}{2}} = \pm \frac{\frac{3}{2} + 3 \cdot \frac{3}{2} - a}{\sqrt{1^2 + 3^2}}$

or, $\sqrt{\frac{5}{2}} \cdot \sqrt{10} = \pm (6 - a)$

or, $5 = \pm (6 - a)$

Taking positive sign,

$5 = 6 - a$

$a = 1$

Again, taking negative sign,

$5 = -6 + a$

$a = 11$

$\therefore$ $a = 1, 11$

When $a = 1$, equation (ii) becomes

$x + 3y = 1$ ...(iii)

and when $a = 11$, equation (ii) becomes

$x + 3y = 11$ ...(iv)

For point of contact, solving (i) and (iii)

$(1 - 3y)^2 + y^2 - 3(1 - 3y) - 3y + 2 = 0$

or, $1 - 6y + 9y^2 + y^2 - 3 + 9y - 3y + 2 = 0$

or, $10y^2 = 0$

$\therefore$ $y = 0$

When $y = 0$, from (iii), $x = 1$

Required point is (1, 0)

Again, solving (i) and (ii)

$(11 - 3y)^2 + y^2 - 3(11 - 3y) - 3y + 2 = 0$

$121 - 66y + 9y^2 + y^2 - 33 + 9y - 3y + 2 = 0$

or, $10y^2 - 60y + 90 = 0$

or, $y^2 - 6y + 9 = 0$

or, $(y - 3)^2 = 0$

$\therefore$ $y = 3$

When $y = 3$, from (iv)

$x + 3 \times 3 = 11$

$\therefore$ $x = 2$

$\therefore$ Required points = (2, 3)

Hence, $a = 1, 11$ and required points are (1, 0) and (2, 3).

51. 2073 Set D Q.No. 9a

Find the condition that the line $lx + my + n = 0$ should be tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]

Please refer to 2066 C Q.No. 1 c

52. 2072 Supp. Q.No. 9a

Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$. [4]

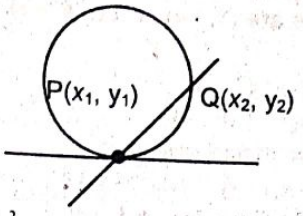
Please refer to 2075 Set C Q.No. 9a

53. 2072 Set C Q.No. 9a

Find the equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1). [4]

SOLUTION

Let $P(x_1, y_1)$ be the point on the circle $x^2 + y^2 = a^2$. Let us take another point $Q(x_2, y_2)$ on the circle which is near to P. Then,



$x_1^2 + y_1^2 = a^2$ ...(i)

and $x_2^2 + y_2^2 = a^2$ ...(ii)

Subtracting, we have

$x_2^2 - x_1^2 + y_2^2 - y_1^2 = 0$

or, $y_2^2 - y_1^2 = x_1^2 - x_2^2$

or, $(y_2 - y_1)(y_2 + y_1) = (x_1 - x_2)(x_1 + x_2)$

or, $(y_2 - y_1)(y_2 + y_1) = -(x_2 - x_1)(x_1 + x_2)$

or, $\frac{y_2 - y_1}{x_2 - x_1} = -\frac{(x_1 + x_2)}{y_1 + y_2}$ ...(iii)

The equation of PQ is

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y - y_1 = -\frac{x_1 + x_2}{y_1 + y_2}(x - x_1)$ [using (iii)]

By definition, the secant PQ becomes tangent at P when $Q \to P$ i.e. when $x_2 \to x_1$ and $y_2 \to y_1$

Hence, the equation of tangent at P is

$y - y_1 = -\frac{x_1}{y_1}(x - x_1)$

or, $yy_1 - y_1^2 = -xx_1 + x_1^2$

or, $xx_1 + yy_1 = x_1^2 + y_1^2$

$\therefore$ $xx_1 + yy_1 = a^2$ [using (i)]

54. 2072 Set D Q.No. 9a

Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ will be tangent to the circle. $(x - a)^2 + (y - b)^2 = r^2$ if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{r^2}$ [4]

SOLUTION

Equation of given line is $\frac{x}{a} + \frac{y}{b} - 1 = 0$ ...(i)

and the equation of given circle is

$(x - a)^2 + (y - b)^2 = r^2$ ...(ii)

Centre = (a, b) and radius = r

The line (i) will be a tangent to the circle (ii) if the length of perpendicular from centre (a, b) upon the line (i) is equal to radius.

i.e. if $\pm \frac{\frac{a}{a} + \frac{b}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = r$

or, $\pm \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = r$

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or, $\frac{1}{r^2}=\frac{1}{a^2}+\frac{1}{b^2}$

$\therefore \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{r^2}$

55. 2072 Set E Q.No. 9a

Find the equations of the circle and tangent to the circle perpendicular to the diameter whose end points are (3, –4) and (– 4, –3). [4]

SOLUTION

Here, $(x_1, y_1) = (3, -4)$

$(x_2, y_2) = (-4, -3)$

The equation of the circle is

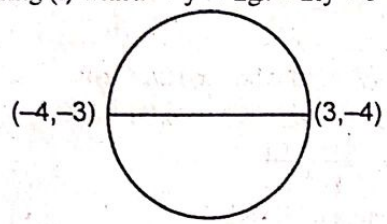
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

or, $(x-3)(x+4) + (y+4)(y+3) = 0$

or, $x^2 - 3x + 4x - 12 + y^2 + 4y + 3y + 12 = 0$

or, $x^2 + y^2 + x + 7y = 0$ ...(i)

Comparing (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get



$g = \frac{1}{2}, f = \frac{7}{2}, c = 0$

Centre $= (-g, -f) = \left(-\frac{1}{2}, -\frac{7}{2}\right)$

Radius $(r) = \sqrt{g^2 + f^2 - c}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{7}{2}\right)^2 - 0}$

$= \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$

Equation of diameter joining the points $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (-4, -3)$ is

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y + 4 = \frac{-3+4}{-4-3}(x-3)$

or, $y + 4 = \frac{1}{-7}(x-3)$

or, $-7y - 28 = x - 3$

or, $x + 7y + 28 - 3 = 0$

$\therefore x + 7y + 25 = 0$

Equation of any line perpendicular to this line is

$7x - y + k = 0$ ...(ii)

This line will be tangent to the circle (i) if length of perpendicular from centre of the circle upon line (ii) is equal to radius.

i.e. $\frac{5}{\sqrt{2}} = \pm\left\{\frac{7\left(-\frac{1}{2}\right) - \left(-\frac{7}{2}\right) + k}{\sqrt{7^2 + (-1)^2}}\right\}$

or, $\frac{5}{\sqrt{2}} = \pm\left(\frac{\frac{-7}{2} + \frac{7}{2} + k}{\sqrt{50}}\right)$

or, $\frac{5}{\sqrt{2}} = \pm\frac{k}{5\sqrt{2}}$

or, $k = \pm 25$

$\therefore$ The required equations of tangents are $x + 7y \pm 25 = 0$.

56. 2071 Supp. Q.No. 9a

Find the value of k if the line $4x - 3y + k = 0$ is tangent to the circle $x^2 + y^2 - 8x + 12y + 3 = 0$. [4]

SOLUTION

Given equation of line is

$4x - 3y + k = 0$ ...(i)

and equation of circle is

$x^2 + y^2 - 8x + 12y + 3 = 0$ ...(ii)

Comparing equation (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -8, 2f = 12, c = 3$

$\therefore g = -4, f = 6$

Centre $= (-g, -f) = (4, -6)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + 6^2 - 3} = 7$

If given line be a tangent to the given circle, then the perpendicular distance from the centre on the line should be equal to the radius.

i.e. $7 = \pm\left\{\frac{4(4) - 3(-6) + k}{\sqrt{4^2 + (-3)^2}}\right\}$

or, $7 = \pm\left(\frac{34 + k}{5}\right)$

Taking +ve sign,

$7 = \frac{34 + k}{5}$

or, $35 = 34 + k$

$\therefore k = 1$

Again, taking –ve sign,

$7 = -\left(\frac{34 + k}{5}\right)$

or, $35 = -(34 + k)$

or, $k = -35 - 34 = -69$

$\therefore k = 1$ or -69

57. 2071 Set C Q.No. 9a

Prove that the tangents to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$. [4]

SOLUTION

Equation of circle is $x^2 + y^2 = 5$ ... (i)

The equation of tangent to the circle (i) at (1, –2) is

$x(1) + y(-2) = 5$

$x - 2y = 5$ ... (ii)

The line (ii) will touch the circle

$x^2 + y^2 - 8x + 6y + 20 = 0$ ... (iii)

if radius of circle (iii) is equal to the perpendicular distance from centre of circle (iii) to the line (ii).

Comparing (iii) with $x^2 + y^2 + 2yx + 2fy + c = 0$, we get

$g = -4, f = 3, c = 20$

$\therefore$ Centre $= (-g, -f) = (4, -3)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{16 + 9 - 20} = \sqrt{5}$

Now, perpendicular distance from (4, –3) to line (ii) is

$$\left|\frac{4-2(-3)-5}{\sqrt{(1)^2+(-2)^2}}\right|$$

$$= \left|\frac{4+6-5}{\sqrt{5}}\right| = \left|\frac{5}{\sqrt{5}}\right| = \sqrt{5}$$

Hence the radius of the circle (iii) is equal to the perpendicular distance from centre of the circle (iii) to the line (ii). So the tangent to the circle (i) also touches the circle (iii)

Now, to find the point of contact we have to solve equation (ii) and (iii). For putting the value of x from (ii) in equation (iii)

$(2y+5)^2 + y^2 - 8(2y+5) + 6y + 20 = 0$

or, $4y^2 + 20y + 25 + y^2 - 16y - 40 + 6y + 20 = 0$

or, $5y^2 + 10y + 5 = 0$

or, $y^2 + 2y + 1 = 0$

or, $(y+1)^2 = 0$

or, $y + 1 = 0$

$y = -1$

Substituting the value of y in equation (ii)

$x - 2(-1) = 5$

$x + 2 = 5$

$x = 5 - 2 = 3$

∴ Hence the point of contact is $(3, -1)$

58. 2071 Set D Q.No. 9a

Prove that the two circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

➥ Please refer to 2076 Set B Q.No. 9a

59. 2070 Supp Q.No. 9a

Find the condition that the line $y = mx + c$ may be tangent to the circle $x^2 + y^2 = a^2$. Also, find the equation of the tangent in the slope form. [4]

SOLUTION

Let $y = mx + c$ . . . (i)

be a straight line and $x^2 + y^2 = a^2$ . . . (ii) be a circle.

Solving (i) and (ii), we get

$x^2 + (mx + c)^2 = a^2$

or, $x^2 + m^2x^2 + 2mcx + c^2 - a^2 = 0$

or, $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ which is quadratic in x.

The line (i) becomes tangent to the circle (ii) if two values of x obtained from this quadratic equation are equal

i.e. the discriminant $B^2 - 4AC = 0$.

i.e. $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

or, $m^2c^2 - c^2 - m^2c^2 + a^2 + a^2m^2 = 0$

or, $c^2 = a^2(1 + m^2)$

$c = \pm a\sqrt{1 + m^2}$

∴ The line $y = mx + c$ will be tangent to the circle

$x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1 + m^2}$

And the equation of the tangent line is

$y = mx \pm a\sqrt{1 + m^2}$

60. 2070 Set C Q.No. 9 a

Find the equations of the tangent and normal to the circle $x^2 + y^2 - 3x + 10y - 5 = 0$ at the point $(4, -11)$. [4]

SOLUTION

Given equation of circle is

$x^2 + y^2 - 3x + 10y - 5 = 0$ ...(i)

We have to find the equation of tangent and normal to the circle (i) at $(4, -11)$. But the point $(4, -11)$ does not lie in (i) as

$4^2 + (-11)^2 - 3 \times 4 + 10 \times (-11) - 5 = 0$

$16 + 121 - 12 - 110 - 5 = 0$

i.e. $10 = 0$ which is not true.

61. 2070 Set D Q.No. 9 a

Find the value of k so that the line $4x + 3y + k = 0$ may touch the circle $x^2 + y^2 - 4x + 10y + 4 = 0$. [4]

➥ Please refer to 2061 Q.No. 1c

62. 2069 Supp Q.No. 9 a

Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

➥ Please refer to 2076 Set B Q.No. 9a

63. 2069 [Set A] Q. No. 9a

Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ which are perpendicular to $3x - 4y = 14$ [4]

SOLUTION

Given equation of circle is

$x^2 + y^2 - 2x - 4y - 4 = 0$ ...(i)

Comparing (i) with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = -2, 2f = -4, c = -4$

∴ $g = -1, f = -2$

Centre $= (-g, -f) = (1, 2)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-1)^2 + (-2)^2 + 4} = 3$

Equation of any line perpendicular to $3x - 4y = 14$ is

$4x + 3y + k = 0$ ...(ii)

The line (ii) will be tangent to the circle (i) if length of perpendicular drawn from centre upon line (ii) is equal to radius.

i.e. $\pm\left(\frac{4 \cdot 1 + 3 \cdot 2 + k}{\sqrt{4^2 + 3^2}}\right) = 3$

or, $\pm\left(\frac{10 + k}{5}\right) = 3$

Taking +ve sign, we get

$10 + k = 15$

∴ $k = 5$

Putting the value of k in (ii), we get

$4x + 3y + 5 = 0$

Again, taking –ve sign, we get

$-\left(\frac{10 + k}{5}\right) = 3$

or, $10 + k = -15$

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$\therefore$ $k = -25$

$\therefore$ Putting the value of k in (ii), we get

$4x + 3y - 25 = 0$

$\therefore$ Required equations of tangents are

$4x + 3y + 5 = 0$ and $4x + 3y - 25 = 0$

64. 2069 [Set B] Q. No. 9a

Show that the circles $x^2 + y^2 - 6x - 6y + 10 = 0$ and $x^2 + y^2 = 2$ touch each other at (1, 1). [4]

SOLUTION

Given equations of circles are

$x^2 + y^2 - 6x - 6y + 10 = 0$ ...(i)

and $x^2 + y^2 = 2$ ...(ii)

Circle (i) and (ii) touches each other at (1, 1) if this point satisfies both the equations of circle.

Put $x = 1$, $y = 1$ in equation (i), we get

$1^2 + 1^2 - 6 \times 1 - 6 \times 1 + 10 = 0$

i.e. $0 = 0$ (true)

Again, put $x = 1$, $y = 1$ in equation (ii), we get

$1^2 + 1^2 = 2$

i.e. $2 = 2$ (true)

Hence, both circle touches at the point (1, 1)

65. 2068 Q.No. 9a

Show that the tangents to the circle $x^2 + y^2 = 100$ at the points (6, 8) and (8, –6) are perpendicular to each other. [4]

SOLUTION

Given equation of circle is $x^2 + y^2 = 100$.

The equation of tangent to the given circle at $(x_1, y_1) = (6, 8)$ is

$x \cdot x_1 + y \cdot y_1 = a^2$

i.e. $x \cdot 6 + y \cdot 8 = 100$

or, $3x + 4y = 50$ ...(i)

Slope of equation (i) $(m_1) = \frac{-3}{4}$

Again, equation of tangent to the given circle at (8, –6) is

$x \cdot 8 + y \cdot (-6) = 100$

or, $4x - 3y = 100$ ...(ii)

Slope of equation (ii) $(m_2) = \frac{4}{3}$

Here, $m_1 \cdot m_2 = \left(-\frac{3}{4}\right) \cdot \frac{4}{3} = -1$

So, the tangents are at perpendicular to each other.

66. 2068 Q.No. 9a

Find the equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1). [4]

➥ Please refer to 2072 Set C Q.No. 9a

67. 2067 Q.No. 9a

Prove that the tangent to the circle $x^2 + y^2 = 5$ at the point (1, – 2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ and find the point of contact. [4]

➥ Please refer to 2071 Set C Q.No. 9a

68. 2066 Q.No. 9 a

Find the equation of the line through (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ [4]

SOLUTION

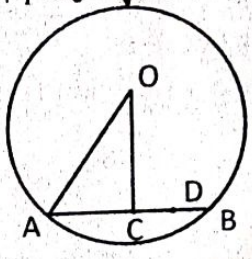
The equation of circle is $x^2 + y^2 - 6x + 4y - 3 = 0$

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$

We get, $g = -3$, $f = 2$, $c = -3$

$\therefore$ Centre $= (-g, -f) = (3, -2)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{9 + 4 + 3} = 4$



Let AB be the chord of the circle, O be the centre of the circle and $OC \perp AB$

Length of $AB = 4\sqrt{3}$

$\therefore$ $AC = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

Now, from right angled ΔAOC,

$OC = \sqrt{OA^2 - AC^2} = \sqrt{4^2 - (2\sqrt{3})^2} = \sqrt{4} = 2$

Let D (1, –1) be a point on the chord AB.

The equation of the chord AB through the point D (1, –1) is

$y - (-1) = m(x - 1)$

$m(x - 1) - y - 1 = 0$ ... (i)

Length of perpendicular from (3, –2) to the line (i) is 2

So, $\pm \frac{m(3 - 1) + 2 - 1}{\sqrt{m^2 + 1}} = 2$

or, $\pm \frac{2m + 1}{\sqrt{m^2 + 1}} = 2$

Squaring $\left(\pm \frac{2m + 1}{\sqrt{m^2 + 1}}\right)^2 = 4$

or, $(2m + 1)^2 = 4(m^2 + 1)$

or, $4m^2 + 4m + 1 = 4m^2 + 4$

or, $4m = 3$

$\therefore$ $m = \frac{3}{4}$

Substituting the value of m in equation (i),

$\frac{3}{4}(x - 1) - y - 1 = 0$

$3x - 3 - 4y - 4 = 0$

$\therefore$ $3x - 4y - 7 = 0$ which is the required equation.

69. 2066 C Q.No. 9 a

A circle touches the parallel lines $3x - 4y = 7$ and $3x - 4y + 43 = 0$ and has its centre on the line $2x - 3y + 13 = 0$. Find its equation. [4]

SOLUTION

Since diameter of the circle = distance between parallel tangent lines

Any point on $3x - 4y = 7$ is $(1, -1)$

Length of the perpendicular from (1, –1) to the line $3x - 4y + 43 = 0$ is

$\left|\frac{3.1 - 4(-1) + 41}{\sqrt{3^2 + (-4)^2}}\right| = \left|\frac{50}{\sqrt{25}}\right| = \left|\frac{50}{5}\right| = 10$

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∴ Radius of the circle $= \frac{10}{2} = 5$

Let (h, k) be the centre of the circle then

$2h - 3k + 13 = 0$ ... (i)

Also, length of the perpendicular from the centre (h, k) on the line $3x - 4y + 43 = 0$ is equal to the radius of the circle.

i.e. $\frac{3h - 4k + 43}{\sqrt{3^2 + (-4)^2}} = 5$

or, $3h - 4k + 18 = 0$ ... (ii)

Solving (i) and (ii), we get $h = -2$, $k = 3$

∴ The equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

or, $(x + 2)^2 + (y - 3)^2 = 5^2$

or, $x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

∴ $x^2 + y^2 + 4x - 6y - 12 = 0$

70. 2065 Q.No 9 a

Find the equation of the line through the point (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$. [4]

➥ Please refer to 2066 Q.No. 9 a

71. 2064 Q.No. 9 a

Find the equation of the circle whose centre is at the point (h, k) and which passes through the origin. Prove that the equation of the tangent to the circle at the origin is $hx + ky = 0$. [4]

SOLUTION

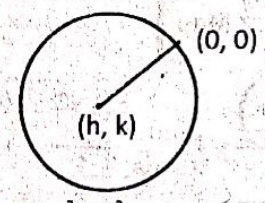
Centre of the circle is (h, k)

Since the circle passes through the origin.

So radius = distance between (h, k) and (0, 0)

$= \sqrt{h^2 + k^2}$

The equation of the circle is



$(x - h)^2 + (y - k)^2 = r^2$

or, $(x - h)^2 + (y - k)^2 = h^2 + k^2$

or, $x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = h^2 + k^2$

∴ $x^2 + y^2 - 2hx - 2ky = 0$ ... (i)

We know that the equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

So, the equation of tangent to the circle (i) at (0, 0) is

$x \cdot 0 + y \cdot 0 + (-h)(x + 0) + (-k)(y + 0) = 0$

or, $-hx - ky = 0$

∴ $hx + ky = 0$ proved

72. 2063 Q.No. 9 a

Find the value of k so that the straight line $4x + 3y + k = 0$ may touch the circle $x^2 + y^2 - 4x + 10y + 4 = 0$ [4]

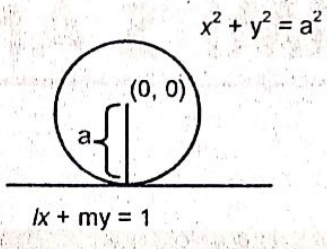
SOLUTION

Please refer to 2061 Q.No. 1 c

73. 2062 Q.No. 9 a

If the line $lx + my = 1$ touches the circle $x^2 + y^2 = a^2$ prove that the point (l, m) lies on a circle whose radius is $\frac{1}{a}$. [4]

SOLUTION



Equation of the circle is $x^2 + y^2 = a^2$

Centre = (0, 0)

Radius = a

If the line $lx + my = 1$ touches the given circle, then the radius of the circle is equal to the perpendicular distance from centre to the line $lx + my = 1$.

i.e. $a = \pm \frac{lx0 + m \times 0 - 1}{\sqrt{l^2 + m^2}}$

Squaring, $a^2(l^2 + m^2) = 1$

∴ $l^2 + m^2 = \frac{1}{a^2}$

This shows that (l, m) lies on the circle whose radius is $\frac{1}{a}$.

74. 2061 Q.No. 9 a

A circle touches the parallel lines $3x - 4y = 7$ and $3x - 4y + 43 = 0$ and has its centre on the line $2x - 3y + 13 = 0$. Find its equation. [4]

➥ Please refer to 2066 C Q.No. 9 a

75. 2060 Q.No. 9 a

Find the equation of the line through the point (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$. [4]

➥ Please refer to 2066 Q.No. 9 a

76. 2059 Q.No. 9 a

Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

➥ Please refer to 2076 Set B Q.No. 9a

77. 2058 Q.No. 9 a

Obtain the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]

➥ Please refer to 2075 Set B Q.No. 9a

78. 2057 Q.No. 9 a

Obtain the condition for the straight line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$. [4]

➥ Please refer to 2070 Supp Q.No. 9a

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12 UNIT

LIMITS AND CONTINUITY

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A. LIMIT

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 4c

Evaluate: $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$. [2]

SOLUTION

Here, $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$

$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{mx}{2}\right)}{2\sin^2\left(\frac{nx}{2}\right)} = \lim_{x \to 0} \left(\frac{\sin\frac{mx}{2}}{\sin\frac{nx}{2}}\right)^2$

$= \left[\frac{\lim\limits_{\frac{mx}{2} \to 0}\left(\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right) \times \frac{m}{2}}{\lim\limits_{\frac{nx}{2} \to 0}\left(\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right) \times \frac{n}{2}}\right]^2$

$= \left(\frac{1 \times \frac{m}{2}}{1 \times \frac{n}{2}}\right)^2 \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$

$= \frac{m^2}{n^2}$

2: 2076 Set C Q.No. 4c

Evaluate: $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 + 3x - 10}$. [2]

SOLUTON

$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 + 3x - 10} \left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 2} \frac{x^2 - 2x - 2x + 4}{x^2 + 5x - 2x - 10}$

$= \lim_{x \to 2} \frac{x(x-2) - 2(x-2)}{x(x+5) - 2(x+5)}$

$= \lim_{x \to 2} \frac{(x-2)(x-2)}{(x-2)(x+5)}$

$= \lim_{x \to 2} \frac{x-2}{x+5}$

$= \frac{2-2}{2+5} = \frac{0}{7} = 0$

3. 2075 GIE Q.No. 4c

Evaluate: $\lim_{x \to \infty} (\sqrt{x+a} - \sqrt{x})$. [2]

SOLUTION

Here,

$\lim_{x \to \infty} (\sqrt{x+a} - \sqrt{x})$ $(\infty - \infty \text{ form})$

$= \lim_{x \to \infty} (\sqrt{x+a} - \sqrt{x}) \times \left(\frac{\sqrt{x+a} + \sqrt{x}}{\sqrt{x+a} + \sqrt{x}}\right)$

$= \lim_{x \to \infty} \left(\frac{x + a - x}{\sqrt{x+a} + \sqrt{x}}\right) = \lim_{x \to \infty} \frac{a}{\sqrt{x+a} + \sqrt{x}}$

$= \frac{a}{\infty + \infty} = \frac{a}{\infty} = 0$

4. 2075 Set B Q.No. 4c

Evaluate: $\lim_{x \to \infty} \frac{3x^2 - 4}{4x^2}$. [2]

SOLUTION

$= \lim_{x \to \infty} \frac{3x^2 - 4}{4x^2} \left[\frac{\infty}{\infty} \text{ form}\right]$

$= \lim_{x \to \infty} \frac{x^2\left(3 - \frac{4}{x^2}\right)}{4x^2} = \lim_{x \to \infty} \frac{3 - \frac{4}{x^2}}{4} = \frac{3-0}{4} = \frac{3}{4}$

5. 2075 Set C Q.No. 4c

Evaluate: $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$. [2]

SOLUTION

$\lim_{x \to 0} \frac{1 - \cos x}{x^2} \left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2} = \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2$

$= 2 \times 1^2 = 2$

6. 2074 Supp Q.No. 4c

Evaluate: $\lim_{x \to a} \frac{\tan(x-a)}{2x - 2a}$ [2]

SOLUTION

$\lim_{x \to a} \frac{\tan(x-a)}{2x - 2a}$ $\left[\frac{0}{0} \text{ form}\right]$

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$$= \lim_{x \to a} \frac{\tan(x-a)}{2(x-a)}$$

$$= \frac{1}{2} \cdot \lim_{(x-a) \to 0} \frac{\tan(x-a)}{(x-a)}$$

$$= \frac{1}{2} \times 1 \quad \left[\because \lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= \frac{1}{2}$$

7. 2074 Set A Q.No. 4c

Evaluate: $\lim_{x \to p} \frac{x^2 - p^2}{\tan(x-p)}$. [2]

SOLUTION

$$\lim_{x \to p} \frac{x^2 - p^2}{\tan(x-p)} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to p} \frac{(x+p)(x-p)}{\tan(x-p)}$$

$$= \lim_{x \to p} \left\{\frac{(x+p)}{\frac{\tan(x-p)}{(x-p)}}\right\} = \frac{p+p}{1} = 2p$$

8. 2074 Set B Q.No. 4c

Evaluate: $\lim_{x \to \infty} (\sqrt{2+x} - \sqrt{x})$. [2]

SOLUTION

$$\lim_{x \to \infty} \sqrt{2+x} - \sqrt{x} \qquad (\infty - \infty \text{ form})$$

$$= \lim_{x \to \infty} \frac{\sqrt{2+x} - \sqrt{x}}{1} \times \frac{\sqrt{2+x} + \sqrt{x}}{\sqrt{2+x} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{2+x-x}{\sqrt{2+x} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{2+x} + \sqrt{x}} = \frac{2}{\infty} = 0.$$

9. 2073 Supp Q.No. 4c

Evaluate: $\lim_{x \to 64} \frac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4}$. [2]

SOLUTION

$$\lim_{x \to 64} \frac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4} \qquad \left[\frac{0}{0} \text{ from}\right]$$

$$= \lim_{x \to 64} \frac{x^{1/6} - 2}{x^{1/3} - 4}$$

$$= \lim_{x \to 64} \frac{x^{1/6} - 2}{(x^{1/6})^2 - 2^2}$$

$$= \lim_{x \to 64} \frac{x^{1/6} - 2}{(x^{1/6} - 2)(x^{1/6} + 2)}$$

$$= \lim_{x \to 64} \frac{1}{x^{1/6} + 2}$$

$$= \frac{1}{(64)^{1/6} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

10. 2073 Set D Q.No. 4c

Evaluate: $\lim_{x \to 1} \frac{\sqrt{2x} - \sqrt{3-x}}{x-1}$. [2]

SOLUTION

$$\lim_{x \to 1} \frac{\sqrt{2x} - \sqrt{3-x}}{x-1} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to 1} \frac{\sqrt{2x} - \sqrt{3-x}}{x-1} \times \frac{\sqrt{2x} + \sqrt{3-x}}{\sqrt{2x} + \sqrt{3-x}}$$

$$= \lim_{x \to 1} \frac{2x - 3 + x}{(x-1)(\sqrt{2x} + \sqrt{3-x})}$$

$$= \lim_{x \to 1} \frac{3(x-1)}{(x-1)(\sqrt{2x} + \sqrt{3-x})}$$

$$= \frac{3}{\sqrt{2 \cdot 1} + \sqrt{3-1}} = \frac{3}{2\sqrt{2}}$$

11. 2072 Supp.Q.No. 4c

Evaluate: $\lim_{x \to \pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$ [2]

SOLUTION

$$\lim_{x \to \pi/4} \frac{\sec^2 x - 2}{\tan x - 1} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to \pi/4} \frac{1 + \tan^2 x - 2}{\tan x - 1} = \lim_{x \to \pi/4} \frac{\tan^2 x - 1}{\tan x - 1}$$

$$= \lim_{x \to \pi/4} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$$

$$= \lim_{x \to \pi/4} (\tan x + 1)$$

$$= \tan\frac{\pi}{4} + 1 = 1 + 1 = 2$$

12. 2072 Set C Q.No. 4c

Evaluate: $\lim_{x \to \infty} (\sqrt{x} - \sqrt{x-3})$ [2]

SOLUTION

Here,

$$\lim_{x \to \infty} (\sqrt{x} - \sqrt{x-3}) \qquad (\infty - \infty \text{ form})$$

$$= \lim_{x \to \infty} (\sqrt{x} - \sqrt{x-3}) \times \frac{(\sqrt{x} + \sqrt{x-3})}{(\sqrt{x} + \sqrt{x-3})}$$

$$= \lim_{x \to \infty} \frac{x - x + 3}{\sqrt{x} + \sqrt{x-3}}$$

$$= \lim_{x \to \infty} \frac{3}{\sqrt{x} + \sqrt{x-3}} = \frac{3}{\infty + \infty} = \frac{3}{\infty} = 0$$

13. 2072 Set D Q.No. 4c

Evaluate: $\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$. [2]

SOLUTION

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to 4} \frac{(x-4)(x^2 + 4x + 16)}{(x-4)(x+4)}$$

$$= \lim_{x \to 4} \frac{x^2 + 4x + 16}{x+4} = \frac{4^2 + 4 \times 4 + 16}{4+4} = \frac{48}{8} = 6$$

14. 2072 Set E Q.No. 4c

Evaluate: $\lim_{x \to \infty} \frac{\sin x}{x}$. [2]

SOLUTION

$\lim_{x \to \infty} \frac{\sin x}{x}$

Put $y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$

Then, as $x \to \infty$, $y \to 0$

$\lim_{y \to 0} \frac{\sin \frac{1}{y}}{\frac{1}{y}} = \lim_{y \to 0} y \sin \frac{1}{y} = 0$

15. 2071 Supp. Q.No. 4c

Evaluate: $\lim_{x \to \infty} (\sqrt{x+1} - \sqrt{x})$. [2]

SOLUTION

$\lim_{x \to \infty} (\sqrt{x+1} - \sqrt{x})$ ($\infty - \infty$ form)

$= \lim_{x \to \infty} \frac{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{(\sqrt{x+1} + \sqrt{x})}$

$= \lim_{x \to \infty} \frac{x+1-x}{\sqrt{x+1} + \sqrt{x}}$

$= \lim_{x \to \infty} \frac{1}{\sqrt{x+1} + \sqrt{x}} = \frac{1}{\infty} = 0$

16. 2071 Set C Q.No. 4c

Evaluate $\lim_{x \to \infty} (\sqrt{x+a} - \sqrt{x})$ [2]

➥ Please refer to 2075 GIE Q.No. 4c

17. 2071 Set D Q.No. 4c

Evaluate: $\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1}$ [2]

SOLUTION

$\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1}$. $\left[\frac{\infty}{\infty} \text{ form}\right]$

Dividing both the numerator and denominator by x

$\lim_{x \to \infty} \left\{ \frac{\frac{\sqrt{x^2+1}}{x}}{\frac{x+1}{x}} \right\}$

$= \lim_{x \to \infty} \left\{ \frac{\sqrt{\frac{x^2+1}{x^2}}}{\frac{x}{x} + \frac{1}{x}} \right\}$

$= \lim_{x \to \infty} \frac{\sqrt{1 + \frac{1}{x^2}}}{1 + \frac{1}{x}}$

$= \frac{\sqrt{1 + \frac{1}{\infty^2}}}{1 + \frac{1}{\infty}} = \frac{\sqrt{1+0}}{1+0} = 1$

18. 2071 Old Q.No. 5a

Evaluate $\lim_{x \to p} \frac{x^2 - p^2}{\tan(x-p)}$ [2]

➥ Please refer to 2074 Set A Q.No. 4c

19. 2071 Old Q.No. 6a

Find the limit of the function

$f(x) = \begin{cases} 2x^2 + 1 & \text{for } x \le 1 \\ 4x + 1 & \text{for } x > 2 \end{cases}$ at $x = 2$ if it exists. [2]

SOLUTION

L.H.L. at $x = 2$ is $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x^2 + 1)$

$= 2 \times 2^2 + 1 = 9$

R.H.L. at $x = 2$ is $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (4x + 1)$

$= 4 \times 2 + 1 = 9$

Here, L.H.L. = R.H.L. = 9

$\therefore \lim_{x \to 2} f(x)$ exists and $\lim_{x \to 2} f(x) = 9$

20. 2070 Supp Q.No. 4c

Evaluate : $\lim_{x \to \infty} \sqrt{(x-a)} - \sqrt{(x-b)}$. [2]

SOLUTION

$\lim_{x \to \infty} \sqrt{(x-a)} - \sqrt{(x-b)}$ [$\infty - \infty$ form]

$= \lim_{x \to \infty} \left\{ \frac{\sqrt{x-a} - \sqrt{x-b}}{1} \times \frac{\sqrt{x-a} + \sqrt{x-b}}{\sqrt{x-a} + \sqrt{x-b}} \right\}$

$= \lim_{x \to \infty} \frac{x-a-x+b}{\sqrt{x-a} + \sqrt{x-b}}$

$= \lim_{x \to \infty} \frac{b-a}{\sqrt{x-a} + \sqrt{x-b}} = \frac{b-a}{\infty} = 0$

21. 2070 Set C Q.No. 4 c

Evaluate: $\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$ [2]

➥ Please refer to 2072 Set D Q.No. 4c

22. 2070 Set D Q.No. 4 c

Evaluate : $\lim_{x \to \pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$ [2]

➥ Please refer to 2072 Supp.Q.No. 4c

23. 2070 Old Q.No. 5 a

Evaluate : $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$. [2]

➥ Please refer to 2075 Set C Q.No. 4c

24. 2069 Supp Q.No. 4 c

Find the limit of the function

$f(x) = \begin{cases} 3x^2 - 1 & \text{when } x < 2 \\ 4x + 3 & \text{when } x \ge 2 \end{cases}$ at $x = 2$. [2]

SOLUTION

L.H.L. at $x = 2$ is $\lim_{x \to 2^-} f(x)$

$= \lim_{x \to 2^-} (3x^2 - 1) = 3 \times 2^2 - 1 = 11$

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R.H.L. at $x = 2$ is $\lim_{x \to 2^+} f(x)$

$= \lim_{x \to 2^+} (4x + 3) = 4 \times 2 + 3 = 11$

Here, L.H.L. = R.H.L.

So, $\lim_{x \to 2} f(x)$ exists and $\lim_{x \to 2} f(x) = 11$

25. 2069 [Set A] Q. No. 4c

Evaluate: $\lim_{x \to \infty} (\sqrt{x} - \sqrt{x-3})$ [2]

➥ Please refer to 2072 Set C Q.No. 4c

26. 2069 [Set B] Q. No. 4c

Evaluate: $\lim_{x \to 0} \frac{1 - \cos px}{1 - \cos qx}$ [2]

SOLUTION

Here, $\lim_{x \to 0} \frac{1 - \cos px}{1 - \cos qx}$

$= \lim_{x \to 0} \frac{2 \sin^2 \left(\frac{px}{2}\right)}{2 \sin^2 \left(\frac{qx}{2}\right)}$

$= \lim_{x \to 0} \left(\frac{\sin \frac{px}{2}}{\sin \frac{qx}{2}} \right)^2$

$= \left[\frac{\lim_{\frac{px}{2} \to 0} \left(\frac{\sin \frac{px}{2}}{\frac{px}{2}} \right) \times \frac{p}{2}}{\lim_{\frac{qx}{2} \to 0} \left(\frac{\sin \frac{qx}{2}}{\frac{qx}{2}} \right) \times \frac{q}{2}} \right]^2$

$= \left(\frac{1 \times \frac{p}{2}}{1 \times \frac{q}{2}} \right)^2 \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$

$= \frac{p^2}{q^2}$

27. 2069 [Set A] Old Q. No. 5a

Evaluate: $\lim_{x \to a} \frac{\sqrt{3x} - \sqrt{2x + a}}{2(x - a)}$ [2]

SOLUTION

$\lim_{x \to a} \frac{\sqrt{3x} - \sqrt{2x + a}}{2(x - a)}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to a} \frac{\sqrt{3x} - \sqrt{2x + a}}{2(x - a)} \times \frac{\sqrt{3x} + \sqrt{2x + a}}{\sqrt{3x} + \sqrt{2x + a}}$

$= \lim_{x \to a} \frac{3x - (2x + a)}{2(x - a)(\sqrt{3x} + \sqrt{2x + a})}$

$= \lim_{x \to a} \frac{3x - 2x - a}{2(x - a)(\sqrt{3x} + \sqrt{2x + a})}$

$= \lim_{x \to a} \frac{x - a}{2(x - a)(\sqrt{3x} + \sqrt{2x + a})}$

$= \lim_{x \to a} \frac{1}{2(\sqrt{3x} + \sqrt{2x + a})}$

$= \frac{1}{2(\sqrt{3a} + \sqrt{2a + a})} = \frac{1}{2 \times 2\sqrt{3a}} = \frac{1}{4\sqrt{3a}}$

28. 2069 [Set B] Old Q. No. 5c

Evaluate: $\lim_{x \to a} (a - x) \tan \frac{\pi x}{2a}$ [2]

SOLUTION

Put $x - a = h$ as $x \to a$, $h \to 0$

$\therefore \lim_{h \to 0} (-h) \tan \frac{\pi}{2a} (a + h)$

$= -\lim_{h \to 0} h \tan \left(\frac{\pi a}{2a} + \frac{\pi h}{2a} \right)$

$= -\lim_{h \to 0} h \tan \left(\frac{\pi}{2} + \frac{\pi h}{2a} \right)$

$= \lim_{h \to 0} h \cot \frac{\pi h}{2a} = \lim_{h \to 0} h \frac{\cos \frac{\pi h}{2a}}{\sin \frac{\pi h}{2a}}$

$= \lim_{h \to 0} \left\{ \frac{\cos \frac{\pi h}{2a}}{\sin \frac{\pi h}{2a}} \right\} = \frac{2a}{\pi}$

29. 2068 Q.No. 4c

Evaluate: $\lim_{x \to a} \frac{\sin (x - a)}{x^2 - a^2}$ [2]

SOLUTION

$\lim_{x \to a} \frac{\sin (x - a)}{x^2 - a^2}$

$= \lim_{x \to a} \frac{\sin (x - a)}{(x + a)(x - a)}$

$= \lim_{x \to a} \frac{1}{(x + a)} \times \lim_{x - a \to 0} \frac{\sin (x - a)}{x - a}$

$= \frac{1}{a + a} = \frac{1}{2a}$

30. 2068 Old Q.No. 5a

Evaluate: $\lim_{x \to a} \frac{\sin(x - a)}{x^2 - a^2}$ [2]

➥ Please refer to 2068 Q.No. 4c

31. 2067 Q.No. 5a

Evaluate: $\text{Lt}_{x \to \infty} (x - \sqrt{x^2 + x})$ [2]

SOLUTION

$\lim_{x \to \infty} (x - \sqrt{x^2 + x})$ $(\infty - \infty \text{ form})$

$= \lim_{x \to \infty} x - \sqrt{x^2 + x} \times \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}}$

$= \lim_{x \to \infty} \frac{x^2 - x^2 - x}{x + \sqrt{x^2 + x}}$

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$$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}}$$

Dividing both numerator and denominator by x

$$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}}$$

$$= \frac{-1}{1+\sqrt{1+\frac{1}{\infty}}} = \frac{-1}{1+\sqrt{1+0}} = \frac{-1}{1+1} = -\frac{1}{2}$$

32. 2066 Q.No. 5a

Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x}$ [2]

SOLUTION

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x} \times \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

$$= \lim_{x\to 0} \frac{1+x^2-1+x^2}{x(\sqrt{1+x^2}+\sqrt{1-x^2})}$$

$$= \lim_{x\to 0} \frac{2x^2}{x(\sqrt{1+x^2}+\sqrt{1-x^2})}$$

$$= \lim_{x\to 0} \frac{2x}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

$$= \frac{2\times 0}{\sqrt{1+0}+\sqrt{1-0}} = \frac{0}{2} = 0$$

33. 2066 Q.No. 6a

Determine the limit of, [2]

$$f(x) \begin{cases} = 2-x^2 & \text{for } x \le 2 \\ = x-4 & \text{for } x > 2 \end{cases} \text{ at } x = 2, \text{ if the exists.}$$

SOLUTION

Left hand limit at $x = 2$ is

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2-x^2)$$
$$= 2-2^2$$
$$= -2$$

Right hand limit at $x = 2$ is

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-4)$$
$$= 2-4$$
$$= -2$$

Here, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -2$

So, the limit of $f(x)$ at $x = 2$ exists and

$$\lim_{x\to 2} f(x) = -2$$

34. 2065 Q.No. 5a

Evaluate: $\lim_{x\to a} \frac{\sin(x-a)}{x^2-a^2}$ [2]

➥ Please refer to **2068 Q.No. 4c**

35. 2064 Q.No. 5c

Evaluate $\lim_{x\to 0} \frac{\sin x^\circ}{x}$ [2]

SOLUTION

$$\lim_{x\to 0} \frac{\sin x^\circ}{x}$$

$$= \lim_{x\to 0} \frac{\sin\frac{\pi x}{180}}{x} \quad \left[\because 1^\circ = \frac{\pi^c}{180}\right]$$

$$= \lim_{x\to 0} \frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$$

$$= \lim_{x\to 0} \left(\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}\right) \times \frac{\pi}{180} = \frac{\pi}{180} \lim_{\frac{\pi x}{180}\to 0} \left(\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}\right)$$

$$= \frac{\pi}{180} \cdot 1 = \frac{\pi}{180}$$

36. Q.No. 6c

Find the limit of the function for $f(x) = x + 2$ when $x \ge 0$ and $f(x) = 4x + 2$ when $x < 0$ at $x = 0$ [2]

SOLUTION

The given function is

$$f(x) = \begin{cases} x+2 & \text{when } x \ge 0 \\ 4x+2 & \text{when } x < 0 \end{cases}$$

L.H.L. of $f(x)$ at $x = 0$ is,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (4x+2) = 4\times 0 + 2 = 2$$

And R.H.L. of $f(x)$ at $x = 0$ is

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+2) = 0+2 = 2$$

Since, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 2$, the limit exists.

And, $\lim_{x\to 0} f(x) = 2$.

37. 2063 Q.No. 5c

Show that: $\lim_{x\to 2} \frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}} = 8$ [2]

SOLUTION

$$\lim_{x\to 2} \frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 2} \frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}} \times \frac{(\sqrt{3x-2}+\sqrt{x+2})}{(\sqrt{3x-2}+\sqrt{x+2})}$$

$$= \lim_{x\to 2} \frac{(x^2-4)(\sqrt{3x-2}+\sqrt{x+2})}{(3x-2-x-2)}$$

$$= \lim_{x\to 2} \frac{(x^2-4)(\sqrt{3x-2}+\sqrt{x+2})}{(2x-4)}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)(\sqrt{3x-2}+\sqrt{x+2})}{2(x-2)}$$

$$= \lim_{x\to 2} \frac{(x+2)(\sqrt{3x-2}+\sqrt{x+2})}{2}$$

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$= \frac{(2+2)(\sqrt{3\times2-2}+\sqrt{2+2})}{2}$

$= \frac{4(\sqrt{4}+\sqrt{4})}{2}$

$= 2(2+2) = 8$

38. 2062 Q.No. 5a

Evaluate: $\lim_{x\to a} \frac{x^{2/3}-a^{2/3}}{x-a}$ [2]

SOLUTION

$\lim_{x\to a} \frac{x^{2/3}-a^{2/3}}{x-a}$

$= \frac{2}{3}a^{\frac{2}{3}-1}$ $\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$

$= \frac{2}{3}(a^{-1/3}) = \frac{2}{3a^{1/3}}$

39. 2061 Q.No. 5a

Evaluate: $\lim_{x\to 1} \frac{\sqrt{2x}-\sqrt{3-x^2}}{x-1}$ [2]

SOLUTION

Here,

$\lim_{x\to 1} \frac{\sqrt{2x}-\sqrt{3-x^2}}{x-1}$ $\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to 1} \frac{\sqrt{2x}-\sqrt{3-x^2}}{x-1} \times \left(\frac{\sqrt{2x}+\sqrt{3-x^2}}{\sqrt{2x}+\sqrt{3-x^2}}\right)$

$= \lim_{x\to 1} \frac{2x-3+x^2}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x\to 1} \frac{x^2+2x-3}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x\to 1} \frac{x^2+3x-x-3}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x\to 1} \frac{x(x+3)-1(x+3)}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x\to 1} \frac{(x+3)(x-1)}{(x-1)(\sqrt{2x}+\sqrt{3-x^2})}$

$= \lim_{x\to 1} \frac{x+3}{\sqrt{2x}+\sqrt{3-x^2}}$

$= \frac{1+3}{\sqrt{2\times1}+\sqrt{3-1^2}} = \frac{4}{\sqrt{2}+\sqrt{2}}$

$= \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

40. 2060 Q.No. 5a

Does the limit of the function $f(x) = \begin{cases} x \text{ when } x>0 \\ -x \text{ when } x<0 \end{cases}$ **exist at $x = 0$? Justify your answer.** [2]

SOLUTION

The given function is

$f(x) = \begin{cases} x \text{ when } x>0 \\ -x \text{ when } x<0 \end{cases}$

L.H.L. of $f(x)$ at $x = 0$ is

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0$

R.H.L. of $f(x)$ at $x = 0$ is

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$

since L.H.L. = R.H.L. at $x = 0$, the limit exists.

$\therefore \lim_{x\to 0} f(x) = 0$

41. 2059 Q.No. 5a

Evaluate: $\lim_{x\to 0} \frac{1-\cos px}{1-\cos qx}$ [2]

➥ Please refer to 2069 [Set B] Q. No. 4c

42. 2059 Q.No. 6a

Does the limit of the function $f(x) = \begin{cases} 2x+1 \text{ for } x\geq 1 \\ 4x^2-1 \text{ for } x<1 \end{cases}$ **at $x = 1$ exist?** [2]

SOLUTION

Here, $f(x) = \begin{cases} 2x+1 \text{ for } x\geq 1 \\ 4x^2-1 \text{ for } x<1 \end{cases}$

Now, L.H.L. of $f(x)$ at $x = 1$ is

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (4x^2-1)$

$= 4\times 1^2 - 1 = 3$

And, R.H.L. of $f(x)$ at $x = 1$ is

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x+1)$

$= 2\times 1 + 1 = 3$

Since, L.H.L. = R.H.L. at $x = 1$, the limit of $f(x)$ at $x = 1$ exists and $\lim_{x\to 1} f(x) = 3$

43. 2058 Q.No. 5a

Evaluate: $\lim_{x\to\infty} (\sqrt{x+a}-\sqrt{x})$ [2]

➥ Please refer to 2075 GIE Q.No. 4c

44. 2057 Q.No. 5a

Evaluate: $\lim_{x\to 1} \frac{x^2+3x-4}{x-1}$ [2]

SOLUTION

$\lim_{x\to 1} \frac{x^2+3x-4}{x-1}$ $\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to 1} \frac{x^2+4x-x-4}{x-1}$

$= \lim_{x\to 1} \frac{x(x+4)-1(x+4)}{x-1}$

$= \lim_{x\to 1} \frac{(x-1)(x+4)}{(x-1)}$

$= \lim_{x\to 1} (x+4)$

$= 1+4 = 5$

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45. 2057 Q.No. 6a

Find the limit of the function

$$f(x) = \begin{matrix} x^2 + 2, x \le 5 \\ 3x + 12, x > 5 \end{matrix} \Big\} \text{ at } x = 5 \text{ if exists.} \quad [2]$$

SOLUTION

We have, $f(x) = \begin{cases} x^2 + 2 & , x \le 5 \\ 3x + 12, & x > 5 \end{cases}$

Now, left hand limit of $f(x)$ at $x = 5$ is

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (x^2 + 2) = 5^2 + 2 = 27$$

And, right hand limit of $f(x)$ at $x = 5$ is

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (3x + 12) = 3 \times 5 + 12 = 27$$

Here, $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 27$

Hence, the limit of the function $f(x)$ at $x = 5$ exists and $\lim_{x \to 5} f(x) = 27$.

46. 2056 Q.No. 5a

Evaluate: $\lim_{x \to \infty} (\sqrt{x} - \sqrt{x-3})$ [2]

➥ Please refer to 2072 Set C Q.No. 4c

47. 2056 Q.No. 6a

Determine the limit of

$$f(x) = \begin{matrix} 2 - x^2 \text{ for } x < 2 \\ x - 4 \text{ for } x > 2 \end{matrix} \Big\} \text{ at } x=2 \text{ if it exists.} \quad [2]$$

SOLUTION

We have,

$$f(x) = \begin{cases} 2 - x^2 & \text{for } x < 2 \\ x - 4 & \text{for } x > 2 \end{cases}$$

Now, L.H.L. of $f(x)$ at $x = 2$ is $\lim_{x \to 2^-} f(x)$

$$= \lim_{x \to 2^-} (2 - x^2)$$
$$= 2 - 2^2$$
$$= 2 - 4 = -2.$$

And, R.H.L. of $f(x)$ at $x = 2$ is $\lim_{x \to 2^+} f(x)$

$$= \lim_{x \to 2^+} (x - 4)$$
$$= 2 - 4 = -2$$

Since L.H.L. = R.H.L. at $x = 2$, so $\lim_{x \to 2} f(x)$ exists.

$\therefore \lim_{x \to 2} f(x) = -2.$

48. 2055 Old Model Q.No. 5a

Evaluate: $\lim_{x \to a} \frac{x^5 - a^5}{x^4 - a^4}$ [2]

SOLUTION

$$\lim_{x \to a} \frac{x^5 - a^5}{x^4 - a^4} = \lim_{x \to a} \left(\frac{\frac{x^5 - a^5}{x - a}}{\frac{x^4 - a^4}{x - a}} \right) = \frac{\lim_{x \to a} \frac{x^5 - a^5}{x - a}}{\lim_{x \to a} \frac{x^4 - a^4}{x - a}}$$

$$= \frac{5a^{5-1}}{4a^{4-1}} \quad \left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= \frac{5a^4}{4a^3} = \frac{5a}{4}$$

4 MARKS QUESTIONS

49. 2076 Set B Q.No. 9b

Evaluate: $\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$. [4]

SOLUTION

We have,

$$\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}. \quad \left[\frac{0}{0} \text{ form}\right]$$

Put $x = \theta + h$. Then $h \to 0$ as $x \to \theta$

Now,

$$\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$$
$$= \lim_{h \to 0} \frac{(\theta + h)\cos\theta - \theta\cos(\theta + h)}{h}$$
$$= \lim_{h \to 0} \frac{\cos\theta + h\cos\theta - \theta\cos(\theta + h)}{h}$$
$$= \lim_{h \to 0} \left[\frac{\theta\cos\theta - \theta\cos(\theta + h)}{h} + \frac{h\cos\theta}{h} \right]$$
$$= \lim_{h \to 0} \left[\frac{\theta\{\cos\theta - \cos(\theta + h)\}}{h} + \cos\theta \right]$$
$$= \lim_{h \to 0} \left[\frac{\theta\left(2\sin\frac{\theta + \theta + h}{2} \cdot \sin\frac{\theta + h - \theta}{2}\right)}{h} + \cos\theta \right]$$
$$= \lim_{h \to 0} \left[\frac{\theta\left(2\sin\frac{2\theta + h}{2} \cdot \sin\frac{h}{2}\right)}{h} + \cos\theta \right]$$
$$= 2\theta \lim_{h \to 0} \sin\left(\frac{2\theta + h}{2}\right) \times \frac{\sin\frac{h}{2}}{h} + \cos\theta$$
$$= 2\theta \lim_{h \to 0} \sin\left(\frac{2\theta + h}{2}\right) \times \lim_{\frac{h}{2} \to 0} \left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) \times \frac{1}{2} + \cos\theta$$
$$= 2\theta \sin\left(\frac{2\theta + 0}{2}\right) \times \frac{1}{2} + \cos\theta \quad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$$
$$= \theta \sin\left(\frac{2\theta}{2}\right) + \cos\theta = \theta\sin\theta + \cos\theta$$

Alternative method

$$\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta} \left[\frac{0}{0} \text{ form}\right]$$
$$= \lim_{x \to \theta} \left[\frac{x\cos\theta - \theta\cos\theta + \theta\cos\theta - \theta\cos x}{x - \theta} \right]$$
$$= \lim_{x \to \theta} \left[\frac{(x - \theta)\cos\theta + \theta(\cos\theta - \cos x)}{x - \theta} \right]$$
$$= \lim_{x \to \theta} \left[\frac{(x - \theta)\cos\theta}{x - \theta} + \frac{\theta(\cos\theta - \cos x)}{x - \theta} \right]$$
$$= \lim_{x \to \theta} \left[\cos\theta + \frac{\theta}{(x - \theta)} \left\{ 2\sin\left(\frac{\theta + x}{2}\right) \cdot \sin\left(\frac{x - \theta}{2}\right) \right\} \right]$$
$$= \cos\theta + \lim_{x \to \theta} \left\{ \theta \cdot 2\sin\left(\frac{\theta + x}{2}\right) \cdot \frac{\sin\left(\frac{x - \theta}{2}\right)}{\left(\frac{x - \theta}{2}\right)^2} \right\}$$

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$= \cos\theta + \theta \cdot 2\sin\left(\frac{\theta+\theta}{2}\right) \cdot 1 \cdot \frac{1}{2}$

$= \cos\theta + \theta \sin\theta$

50. 2076 Set C Q.No. 9b

Evaluate: $\lim_{x \to y} \frac{x \sin y - y \sin x}{x - y}$. **[4]**

SOLUTION

$\lim_{x\to y} \frac{x \sin y - y \sin x}{x-y}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x\to y} \frac{x \sin y - y \sin y + y \sin y - y \sin x}{x-y}$

$= \lim_{x\to y} \left\{\frac{(x-y)\sin y}{(x-y)} - \frac{y}{x-y}(\sin x - \sin y)\right\}$

$= \lim_{x\to y} \left\{\sin y - \frac{y}{x-y} \cdot 2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)\right\}$

$= \lim_{x\to y} \left\{\sin y - 2y\cos\left(\frac{x+y}{2}\right) \cdot \frac{\sin\left(\frac{x-y}{2}\right)}{\frac{(x-y)}{2} \times 2}\right\}$

$= \sin y - y\cos\left(\frac{y+y}{2}\right) \times 1$

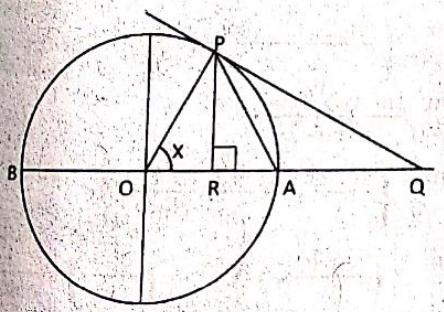
$= \sin y - y \cos y$

51. 2075 GIE Q.No. 9b

Prove geometrically: $\lim_{x\to 0} \frac{\sin x}{x} = 1$. **[4]**

SOLUTION

Let APB be a circle of radius r and AP be an arc which makes an angle xc at the center O. Draw a tangent PQ at the point P of the circle which meets BA produced at Q. Joint PA and draw PR perpendicular to BA. Then, from the figure.



Area of $\Delta OPA \le$ Area of sector OPA $\le$ Area of ΔOPQ ...(i)

Now, Area of $\Delta OPA = \frac{1}{2}$ base × height

$= \frac{1}{2} OA \times PR$

$= \frac{1}{2} \times r \times r \sin x = \frac{1}{2} r^2 \sin x$.

$\left[\because \text{In } \Delta OPR, \sin X = \frac{p}{h} = \frac{PR}{OP} = \frac{PR}{r} \therefore PR = r \sin X\right]$

Area of sector OPA $= \frac{1}{2} \times$ Arc length × Radius

$= \frac{1}{2} \times AP \times r$

$= \frac{1}{2} \times (r.x) \times r \quad [\because x = \frac{l}{r} \Rightarrow l = rx]$

$= \frac{1}{2} r^2 x$

And, Area of $\Delta OPQ = \frac{1}{2}$ base × height

$= \frac{1}{2} OQ \times PR$

$= \frac{1}{2} \times \frac{r}{\cos x} \times r \sin x = \frac{1}{2} r^2 \tan x$

$\left[\because \text{In } \Delta OPQ; \cos x = \frac{OP}{OQ} = \frac{r}{OQ}\right.$

$\left.\therefore OQ = \frac{r}{\cos x}\right]$

Hence, from relation (i)

$\frac{1}{2} r^2 \sin x \le \frac{1}{2} r^2 x \le \frac{1}{2} r^2 \tan x$

or, $\sin x \le x \le \tan x$

or, $1 \le \frac{x}{\sin x} \le \frac{1}{\cos x}$ [$\because$ dividing by sin x]

Taking reciprocal, we get

$1 \ge \frac{\sin x}{x} \ge \cos x$

Taking $\lim_{x\to 0}$, we get

or, $\lim_{x\to 0} 1 \ge \lim_{x\to 0} \frac{\sin x}{x} \ge \lim_{x\to 0} \cos x$

or, $1 \ge \lim_{x\to 0} \frac{\sin x}{x} \ge 1$

$\Rightarrow \lim_{x\to 0} \frac{\sin x}{x} = 1$, Proved.

52. 2075 Set A Q.No. 9b

Evaluate: $\lim_{h\to 0} \frac{\cos^2(x+h) - \cos^2 x}{h}$. **[4]**

SOLUTION

$\lim_{h\to 0} \frac{\cos^2(x+h) - \cos^2 x}{h}$ $\left[\frac{0}{0} \text{ from}\right]$

$= \lim_{h\to 0} \frac{\{\cos(x+h) + \cos x\}\{\cos(x+h) - \cos x\}}{h}$

$= \lim_{h\to 0} \frac{\{\cos(x+h) + \cos x\}\{-2 \cdot \sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\}}{h}$

$= \lim_{h\to 0} \{\cos(x+h) + \cos x\} \cdot \left\{-2\sin\left(x + \frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2} \cdot 2}\right\}$

$= (\cos x + \cos x)(-\sin x \cdot 1)$

$= -2 \cos x \sin x$

$= -\sin 2x$.

53. 2075 Set B Q.No. 9b

Evaluate: $\lim_{x\to y} \frac{x \sin y - y \sin x}{x - y}$ **[4]**

➥ Please refer to 2076 Set C Q.No. 9b

54. 2075 Set C Q.No. 9b

Evaluate: $\lim_{x\to 0} \frac{x\cos\theta - \theta\cos x}{x - \theta}$. **[4]**

➥ Please refer to 2076 Set B Q.No. 9b

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55. 2074 Supp Q.No. 9b

Evaluate: $\lim_{x\to a}\frac{\sqrt{2x}-\sqrt{3x-a}}{\sqrt{x}-\sqrt{a}}$. [4]

SOLUTION

$$\lim_{x\to a}\frac{\sqrt{2x}-\sqrt{3x-a}}{\sqrt{x}-\sqrt{a}}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to a}\frac{\sqrt{2x}-\sqrt{3x-a}}{\sqrt{x}-\sqrt{a}}\times\frac{\sqrt{2x}+\sqrt{3x-a}}{\sqrt{2x}+\sqrt{3x-a}}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$$

$$=\lim_{x\to a}\frac{\{2x-(3x-a)\}(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$$

$$=\lim_{x\to a}\frac{(2x-3x+a)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$$

$$=\lim_{x\to a}\frac{(-x+a)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$$

$$=\lim_{x\to a}\frac{-(x-a)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3x-a})}$$

$$=-\frac{(\sqrt{a}+\sqrt{a})}{(\sqrt{2a}+\sqrt{3a-a})}=\frac{-2\sqrt{a}}{2\sqrt{2a}}=-\frac{1}{\sqrt{2}}$$

56. 2074 Set A Q.No. 9b

Evaluate: $\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{6-x^2}}{x-2}$. [4]

SOLUTION

$$\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{6-x^2}}{x-2}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{6-x^2}}{x-2}\times\frac{\sqrt{x}+\sqrt{6-x^2}}{\sqrt{x}+\sqrt{6-x^2}}$$

$$=\lim_{x\to 2}\frac{x-6+x^2}{(x-2)(\sqrt{x}+\sqrt{6-x^2})}$$

$$=\lim_{x\to 2}\frac{x^2+x-6}{(x-2)(\sqrt{x}+\sqrt{6-x^2})}$$

$$=\lim_{x\to 2}\frac{x^2+3x-2x-6}{(x-2)(\sqrt{x}+\sqrt{6-x^2})}$$

$$=\lim_{x\to 2}\frac{x(x+3)-2(x+3)}{(x-2)(\sqrt{x}+\sqrt{6-x^2})}$$

$$=\lim_{x\to 2}\frac{(x+3)(x-2)}{(x-2)(\sqrt{x}+\sqrt{6-x^2})}$$

$$=\lim_{x\to 2}\frac{(x+3)}{\sqrt{x}+\sqrt{6-x^2}}$$

$$=\frac{2+3}{\sqrt{2}+\sqrt{6-2^2}}=\frac{5}{\sqrt{2}+\sqrt{2}}=\frac{5}{2\sqrt{2}}$$

57. 2074 Set B Q.No. 9b

Evaluate: $\lim_{y\to x}\frac{y\sec y-x\sec x}{y-x}$ [4]

SOLUTION

$$\lim_{y\to x}\frac{y\sec y-x\sec x}{y-x}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{y\to x}\left[\frac{y\sec y-x\sec y+x\sec y-x\sec x}{y-x}\right]$$

$$=\lim_{y\to x}\left[\frac{(y-x)\sec y+x(\sec y-\sec x)}{y-x}\right]$$

$$=\lim_{y\to x}\left[\frac{(y-x)\sec y}{y-x}+\frac{x}{y-x}(\sec y-\sec x)\right]$$

$$=\lim_{y\to x}\left[\sec y+\frac{x}{y-x}\left(\frac{1}{\cos y}-\frac{1}{\cos x}\right)\right]$$

$$=\lim_{y\to x}\left[\sec y+\frac{x}{y-x}\frac{(\cos x-\cos y)}{\cos x\cos y}\right]$$

$$=\sec x+\lim_{y\to x}\frac{x}{y-x}\left\{\frac{2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)}{\cos x\cos y}\right\}$$

$$=\sec x+\lim_{y\to x}\left\{\frac{x}{\cos x\cos y}\cdot 2\sin\left(\frac{x+y}{2}\right)\cdot\frac{\sin\left(\frac{y-x}{2}\right)}{\left(\frac{y-x}{2}\right).2}\right\}$$

$$=\sec x+\frac{x}{\cos x.\cos x}\cdot\sin\left(\frac{x+x}{2}\right).1$$

$$=\sec x+x\tan x\sec x$$

$$=\sec x(1+x\tan x)$$

58. 2073 Supp Q.No. 9b

Evaluate: $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}$ [4]

SOLUTION

$$\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to 0}\left\{\frac{\frac{\sin 2x}{\cos 2x}-\sin 2x}{x^3}\right\}$$

$$=\lim_{x\to 0}\left\{\frac{\sin 2x-\sin 2x\cos 2x}{x^3\cos 2x}\right\}$$

$$=\lim_{x\to 0}\left\{\frac{\sin 2x(1-\cos 2x)}{x^3\cos 2x}\right\}$$

$$=\lim_{x\to 0}\frac{\sin 2x\,.\,2\sin^2 x}{x^3\cos 2x}$$

$$=\lim_{x\to 0}\left\{\frac{\sin 2x}{x}\cdot\frac{2\sin^2 x}{x^2}\cdot\frac{1}{\cos 2x}\right\}$$

$$=\lim_{x\to 0}\left\{\left(\frac{\sin 2x}{2x}\times 2\right)\times 2\times\left(\frac{\sin x}{x}\right)^2\times\frac{1}{\cos 2x}\right\}$$

$$=1\times 2\times 2\times 1^2\times\frac{1}{1}=4$$

59. 2073 Set C Q.No. 9b

Evaluate: $\lim_{x\to\theta}\frac{x\cos\theta-\theta\cos x}{x-\theta}$. [4]

➥ Please refer to 2076 Set B Q.No. 9b

60. 2073 Set D Q.No. 9b

Evaluate: $\lim_{x\to y}\frac{x\cot y-y\cot x}{x-y}$. [4]

SOLUTION

$$\lim_{x\to y}\frac{x\cot y-y\cot x}{x-y}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to y}\frac{x\cot y-y\cot y+y\cot y-y\cot x}{x-y}$$

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$$= \lim_{x\to y}\left\{\frac{(x-y)\cot y}{x-y}+\frac{y(\cot y-\cot x)}{x-y}\right\}$$

$$= \lim_{x\to y}\left\{\cot y+\frac{y}{x-y}\left(\frac{\cos y}{\sin y}-\frac{\cos x}{\sin x}\right)\right\}$$

$$= \lim_{x\to y}\left\{\cot y+\frac{y}{x-y}\left(\frac{\sin x\cos y-\cos x\sin y}{\sin x\sin y}\right)\right\}$$

$$= \lim_{x\to y}\left\{\cot y+y\cdot\frac{\sin(x-y)}{x-y}\cdot\frac{1}{\sin x\sin y}\right\}$$

$$= \cot y+y\cdot 1\cdot\frac{1}{\sin y\sin y}$$

$$= \cot y+y\,\mathrm{cosec}^2 y$$

61. 2072 Supp. Q.No. 9b

Evaluate: $\lim_{x\to a}\dfrac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}$ **[4]**

SOLUTION

$$\lim_{x\to a}\frac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to a}\frac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}\times\frac{\sqrt{3a-x}+\sqrt{x+a}}{\sqrt{3a-x}+\sqrt{x+a}}$$

$$= \lim_{x\to a}\frac{3a-x-x-a}{4(x-a)(\sqrt{3a-x}+\sqrt{x+a})}$$

$$= \lim_{x\to a}\frac{2a-2x}{4(x-a)(\sqrt{3a-x}+\sqrt{x+a})}$$

$$= \lim_{x\to a}\frac{-2(x-a)}{4(x-a)(\sqrt{3a-x}+\sqrt{x+a})}$$

$$= \lim_{x\to a}\frac{-1}{2(\sqrt{3a-x}+\sqrt{x+a})}$$

$$= \frac{-1}{2(\sqrt{3a-a}+\sqrt{a+a})}=\frac{-1}{2\cdot 2\sqrt{2a}}=\frac{-1}{4\sqrt{2a}}$$

62. 2072 Set C Q.No. 9b

Evaluate: $\lim_{x\to\theta}\dfrac{x\cos\theta-\theta\cos x}{x-\theta}$ **[4]**

➥ Please refer to 2076 Set B Q.No. 9b

63. 2072 Set D Q.No. 9b

Evaluate: $\lim_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$ **[4]**

Here, $\lim_{x\to 0}\dfrac{\tan x-\sin x}{x^3}\quad\left[\frac{0}{0}\text{ form}\right]$

$$= \lim_{x\to 0}\frac{\left(\frac{\sin x}{\cos x}-\sin x\right)}{x^3}$$

$$= \lim_{x\to 0}\frac{(\sin x-\sin x\cos x)}{x^3\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^3\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x\,2\sin^2 x/2}{x^3\cos x}$$

$$= \lim_{x\to 0}2\,\frac{\sin x}{x}\cdot\frac{\sin^2 x/2}{x^2}\cdot\frac{1}{\cos x}$$

$$= \lim_{x\to 0}\frac{2\sin x}{x}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}\frac{1}{\cos x}$$

$$= 2\times 1\times 1^2\times\frac{1}{4}\times\frac{1}{1}\quad\left[\because\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$= \frac{1}{2}$$

64. 2072 Set E Q.No. 9b

Evaluate: $\lim_{x\to 1}\dfrac{\sqrt{x+2}-\sqrt{4-x}}{x^2-1}$ **[4]**

SOLUTION

$$\lim_{x\to 1}\frac{\sqrt{x+2}-\sqrt{4-x}}{x^2-1}\quad\left[\frac{0}{0}\text{ form}\right]$$

$$= \lim_{x\to 1}\left\{\frac{\sqrt{x+2}-\sqrt{4-x}}{x^2-1}\times\frac{\sqrt{x+2}+\sqrt{4-x}}{\sqrt{x+2}+\sqrt{4-x}}\right\}$$

$$= \lim_{x\to 1}\frac{x+2-4+x}{(x^2-1)(\sqrt{x+2}+\sqrt{4-x})}$$

$$= \lim_{x\to 1}\frac{2(x-1)}{(x+1)(x-1)(\sqrt{x+2}+\sqrt{4-x})}$$

$$= \lim_{x\to 1}\frac{2}{(x+1)(\sqrt{x+2}+\sqrt{4-x})}$$

$$= \frac{2}{(1+1)(\sqrt{3}+\sqrt{3})}=\frac{2}{2\cdot 2\sqrt{3}}=\frac{1}{2\sqrt{3}}$$

65. 2071 Supp. Q.No. 9b

Evaluate: $\lim_{x\to y}\dfrac{x\sin y-y\sin x}{x-y}$ **[4]**

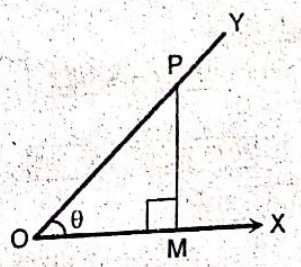
➥ Please refer to 2076 Set C Q.No. 9b

66. 2071 Set C Q.No. 9b

Prove geometrically: $\lim_{\theta\to 0}\sin\theta=0$ **[4]**

SOLUTION

Let OX be the initial line and $\angle XOY=\theta$. Let us take any point P on the line OY. Draw PM perpendicular to OX.



Then, from right angled triangle OPM,

$$\sin\theta=\frac{PM}{OP}$$

When θ is small, PM will be small and P will be close to M. When θ is small enough PM will be small enough and P will be very close to M. This implies that as $\theta\to 0$, $PM\to 0$

Therefore $\lim_{\theta\to 0}\sin\theta=\lim_{\theta\to 0}\frac{PM}{OP}=0$

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67. 2071 Set D Q.No. 9b

Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$. [4]

SOLUTION

$$\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to \theta} \left\{\frac{x\cot\theta - \theta\cot\theta + \theta\cot\theta - \theta\cot x}{x - \theta}\right\}$$

$$= \lim_{x \to \theta} \left\{\frac{(x-\theta)\cot\theta + \theta(\cot\theta - \cot x)}{x - \theta}\right\}$$

$$= \lim_{x \to \theta} \left\{\frac{(x-\theta)\cot\theta}{(x-\theta)} + \frac{\theta}{(x-\theta)}\left(\frac{\cos\theta}{\sin\theta} - \frac{\cos x}{\sin x}\right)\right\}$$

$$= \lim_{x \to \theta} \left\{\cot\theta + \frac{\theta}{x-\theta}\left(\frac{\sin x\cos\theta - \cos x\sin\theta}{\sin\theta\sin x}\right)\right\}$$

$$= \lim_{x \to \theta} \left\{\cot\theta + \theta \cdot \frac{\sin(x-\theta)}{(x-\theta)} \cdot \frac{1}{\sin\theta\sin x}\right\}$$

$$= \cot\theta + \theta \cdot 1 \cdot \frac{1}{\sin\theta \cdot \sin\theta}$$

$$= \cot\theta + \theta\,\text{cosec}^2\theta$$

68. 2071 Old Q.No. 12b Or

Evaluate: $\lim_{x \to 1} \frac{1 + \cos\pi x}{\tan^2 \pi x}$ [4]

SOLUTION

$$\lim_{x \to 1} \frac{1 + \cos\pi x}{\tan^2 \pi x} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to 1} \frac{1 + \cos\pi x}{\tan^2 \pi x} \times \frac{1 - \cos\pi x}{1 - \cos\pi x}$$

$$= \lim_{x \to 1} \frac{1 - \cos^2\pi x}{\tan^2 \pi x (1 - \cos\pi x)}$$

$$= \lim_{x \to 1} \frac{\sin^2 \pi x}{\frac{\sin^2 \pi x}{\cos^2 \pi x}(1 - \cos\pi x)}$$

$$= \lim_{x \to 1} \frac{\cos^2 \pi x}{1 - \cos\pi x}$$

$$= \frac{\cos^2\pi}{1 - \cos\pi} = \frac{(-1)^2}{1 - (-1)} = \frac{1}{2}$$

69. 2070 Supp Q.No. 9b

Evaluate: $\lim_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$ [4]

➥ Please refer to 2073 Supp Q.No. 9b

70. 2070 Set C Q.No. 9 b

Evaluate : $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$. [4]

➥ Please refer to 2071 Set D Q.No. 9b

71. 2070 Set D Q.No. 9 b

Evaluate : $\lim_{x \to \infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a})$. [4]

SOLUTION

Here,

$$\lim_{x \to \infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a}) \ (\infty - \infty \text{ form})$$

$$= \lim_{x \to \infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a}) \times \frac{(\sqrt{x} + \sqrt{x-a})}{(\sqrt{x} + \sqrt{x-a})}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}(x - x + a)}{(\sqrt{x} + \sqrt{x-a})}$$

$$= \lim_{x \to \infty} \frac{a\sqrt{x}}{(\sqrt{x} + \sqrt{x-a})} \left(\frac{\infty}{\infty} \text{ form}\right)$$

Dividing both numerator and denominator by $\sqrt{x}$

$$= \lim_{x \to \infty} \left(\frac{\frac{a\sqrt{x}}{\sqrt{x}}}{\frac{\sqrt{x}}{\sqrt{x}} + \frac{\sqrt{x-a}}{\sqrt{x}}}\right)$$

$$= \lim_{x \to \infty} \left(\frac{a}{1 + \sqrt{\frac{x-a}{x}}}\right) = \lim_{x \to \infty} \frac{1}{1 + \sqrt{1 - \frac{a}{x}}}$$

$$= \frac{a}{1 + \sqrt{1-0}} = \frac{a}{2}$$

72. 2070 Old Q.No. 12 b

Evaluate: $\lim_{x \to y} \frac{x\cos y - y\cos x}{x - y}$ [4]

SOLUTION

$$\lim_{x \to y} \frac{x\cos y - y\cos x}{x - y} \quad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to y} \frac{x\cos y - y\cos y + y\cos y - y\cos x}{x - y}$$

$$= \lim_{x \to y} \left\{\frac{(x-y)\cos y}{x-y} + \frac{y}{x-y}(\cos y - \cos x)\right\}$$

$$= \lim_{x \to y} \left\{\cos y + \frac{y}{x-y} 2\sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)\right\}$$

$$= \lim_{x \to y} \left\{\cos y + 2y\sin\left(\frac{x+y}{2}\right) \cdot \frac{\sin\left(\frac{x-y}{2}\right)}{\frac{x-y}{2} \times 2}\right\}$$

$$= \cos y + y\sin y$$

73. 2069 Supp Q.No. 9 b

Prove geometrically that: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$. [4]

➥ Please refer to 2075 GIE Q.No. 9b

74. 2069 [Set A] Q. No. 9b

Evaluate: $\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$ [4]

➥ Please refer to 2076 Set B Q.No. 9b

75. 2069 [Set B] Q. No. 9b

Evaluate: $\lim_{x \to 2} \frac{x - \sqrt{8 - x^2}}{\sqrt{x^2 + 12} - 4}$. [4]

SOLUTION

$$\lim_{x \to 2} \frac{x - \sqrt{8 - x^2}}{\sqrt{x^2 + 12} - 4} \quad \left[\frac{0}{0} \text{ form}\right]$$

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$$= \lim_{x\to 2}\left\{\frac{x-\sqrt{8-x^2}}{\sqrt{x^2+12}-4}\times\frac{x+\sqrt{8-x^2}}{x+\sqrt{8-x^2}}\times\frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}\right\}$$

$$= \lim_{x\to 2}\left[\frac{\{(x)^2-(\sqrt{8-x^2})^2\}(\sqrt{x^2+12}+4)}{\{(\sqrt{x^2+12})^2-4^2\}(x+\sqrt{8-x^2})}\right]$$

$$= \lim_{x\to 2}\frac{(x^2-8+x^2)(\sqrt{x^2+12}+4)}{(x^2+12-16)(x+\sqrt{8-x^2})}$$

$$= \lim_{x\to 2}\frac{2(x^2-4)(\sqrt{x^2+12}+4)}{(x^2-4)(x+\sqrt{8-x^2})}$$

$$= \frac{2(\sqrt{2^2+12}+4)}{2+\sqrt{8-2^2}} = \frac{2(4+4)}{2+2} = 4$$

76. 2069 [Set A] Old Q. No. 12b

Evaluate: $\lim_{x\to y}\frac{x\cos y - y\cos x}{x-y}$. **[4]**

➥ Please refer to 2070 Old Q.No. 12 b

77. 2069 [Set B] Old Q. No. 13b

Evaluate: $\lim_{x\to 1}\frac{x-\sqrt{2-x^2}}{2x-\sqrt{2+2x^2}}$ **[4]**

SOLUTION

Here,

$$\lim_{x\to 1}\frac{x-\sqrt{2-x^2}}{2x-\sqrt{2+2x^2}} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{(x-\sqrt{2-x^2})}{(2x-\sqrt{2+2x^2})}\times\frac{(2x+\sqrt{2+2x^2})}{(2x+\sqrt{2+2x^2})}\times\frac{(x+\sqrt{2-x^2})}{(x+\sqrt{2-x^2})}$$

$$= \lim_{x\to 1}\frac{(x^2-2+x^2)(2x+\sqrt{2+2x^2})}{(4x^2-2-2x^2)(x+\sqrt{2-x^2})}$$

$$= \lim_{x\to 1}\frac{(2x^2-2)(2x+\sqrt{2+2x^2})}{(2x^2-2)(x+\sqrt{2-x^2})}$$

$$= \lim_{x\to 1}\frac{2x+\sqrt{2+2x^2}}{x+\sqrt{2-x^2}}$$

$$= \frac{2\times 1+\sqrt{2+2\times 1^2}}{1+\sqrt{2-1^2}}$$

$$= \frac{2+\sqrt{2+2}}{1+\sqrt{1}} = \frac{2+2}{1+1} = \frac{4}{2} = 2 \text{ Proved.}$$

78. 2068 Q.No. 9b

Evaluate: $\lim_{x\to a}\left(\frac{\sqrt{3x}-\sqrt{2x+a}}{2(x-a)}\right)$ **[4]**

➥ Please refer to 2069 Set 'A' Old Q. No. 5a

79. 2068 Old Q.No. 12b

Evaluate: $\lim_{x\to a}\frac{\sqrt{3a-x}-\sqrt{x+a}}{4(x-a)}$ **[4]**

➥ Please refer to 2072 Supp. Q.No. 9b

80. 2067 Q.No. 12b

Evaluate: $\lim_{x\to 0}\frac{(a+x)\sec(a+x)-a\sec a}{x}$ **[4]**

SOLUTION

$$\lim_{x\to 0}\frac{(a+x)\sec(a+x)-a\sec a}{x} \quad \left(\frac{0}{0}\text{ from}\right)$$

$$= \lim_{x\to 0}\frac{a\sec(a+x)+x\sec(a+x)-a\sec a}{x}$$

$$= \lim_{x\to 0}\left[\sec(a+x)+\frac{a\{\sec(a+x)-\sec a\}}{x}\right]$$

$$= \sec a + \lim_{x\to 0}\frac{a\left\{\frac{1}{\cos(a+x)}-\frac{1}{\cos a}\right\}}{x}$$

$$= \sec a + \lim_{x\to 0}\frac{a\{\cos a-\cos(a+x)\}}{x\cos a\cos(a+x)}$$

$$= \sec a + \lim_{x\to 0}\frac{2a\sin\left(\frac{a+x+a}{2}\right)\sin\left(\frac{a+x-a}{2}\right)}{x\cos a\cos(a+x)}$$

$$= \sec a + \lim_{x\to 0}\frac{2a\sin\left(\frac{2a+x}{2}\right)\sin\frac{x}{2}}{\cos a\cos(a+x)\frac{x}{2}\times 2}$$

$$= \sec a + \frac{a.\sin a}{\cos a.\cos a}$$

$$= \sec a + a\sin a\sec^2 a$$

81. 2066 Q.No. 12b

Evaluate: $\lim_{x\to\theta}\frac{x\sin\theta-\theta\sin x}{x-\theta}$ **[4]**

SOLUTION

$$\lim_{x\to\theta}\frac{x\sin\theta-\theta\sin x}{x-\theta} \quad \left[\frac{0}{0}\text{ form}\right]$$

Put $x-\theta = h$ then $x = \theta + h$ so that when $x\to\theta$, $h\to 0$

Now, $\lim_{x\to\theta}\frac{x\sin\theta-\theta\sin x}{x-\theta}$

$$= \lim_{h\to 0}\frac{(\theta+h)\sin\theta-\theta\sin(\theta+h)}{h}$$

$$= \lim_{h\to 0}\frac{\theta\sin\theta+h\sin\theta-\theta\sin(\theta+h)}{h}$$

$$= \lim_{h\to 0}\frac{h\sin\theta+\theta\{\sin\theta-\sin(\theta+h)\}}{h}$$

$$= \lim_{h\to 0}\left[\sin\theta+\frac{\theta}{h}2\cos\left(\frac{\theta+\theta+h}{2}\right)\sin\left(\frac{\theta-\theta-h}{2}\right)\right]$$

$$= \lim_{h\to 0}\left[\sin\theta+\frac{2\theta}{h}\cos\left(\frac{2\theta+h}{2}\right)\sin\left(-\frac{h}{2}\right)\right]$$

$$= \lim_{h\to 0}\left[\sin\theta-\frac{2\theta}{h}\cos\left(\theta+\frac{h}{2}\right)\sin\frac{h}{2}\right]$$

$$= \lim_{h\to 0}\sin\theta - \lim_{h\to 0}\frac{2\theta}{h}\cos\left(\theta+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2}}\times\frac{h}{2}$$

$$= \sin\theta-\theta\cos\theta.1$$

$$= \sin\theta-\theta\cos\theta$$

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82. 2065 Q.No. 12b

Evaluate: $\lim_{x \to \pi/2} \frac{\tan x + \cot x}{\tan x - \cot x}$ [4]

SOLUTION

$$\lim_{x \to \pi/2} \frac{\tan x + \cot x}{\tan x - \cot x}$$

$$= \lim_{x \to \pi/2} \left(\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}} \right)$$

$$= \lim_{x \to \pi/2} \left(\frac{\frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}}{\frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}} \right)$$

$$= \lim_{x \to \pi/2} \frac{1}{\sin^2 x - \cos^2 x}$$

$$= \lim_{x \to \pi/2} \frac{-1}{\cos^2 x - \sin^2 x} = \lim_{x \to \pi/2} \frac{-1}{\cos 2x}$$

$$= \frac{-1}{\cos\left(2 \times \frac{\pi}{2}\right)} = \frac{-1}{\cos \pi} = \frac{-1}{-1} = 1$$

83. 2064 Q.No. 13b

Evaluate: $\lim_{x \to y} \frac{\tan x - \tan y}{x - y}$ [4]

SOLUTION

Here, $\lim_{x \to y} \frac{\tan x - \tan y}{x - y}$.

$$= \lim_{x \to y} \frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{x - y}$$

$$= \lim_{x \to y} \frac{\sin x \cdot \cos y - \cos x \cdot \sin y}{(x - y) \cos x \cdot \cos y}$$

$$= \lim_{x \to y} \frac{\sin (x - y)}{(x - y) \cos x \cdot \cos y}$$

$$= \lim_{(x-y) \to 0} \frac{\sin (x - y)}{(x - y)} \times \lim_{x \to y} \frac{1}{\cos x \cdot \cos y}$$

$$= 1 \times \frac{1}{\cos y \cdot \cos y} = \frac{1}{\cos^2 y} = \sec^2 y$$

84. 2063 Q.No. 13b OR

Evaluate: $\lim_{x \to \theta} \frac{x \cos \theta - \theta \cos x}{x - \theta}$ [4]

➥ Please refer to 2076 Set B Q.No. 9b

85. 2062 Q.No. 12b

Prove that: $\lim_{x \to 1} \frac{x - \sqrt{2 - x^2}}{2x - \sqrt{2 + 2x^2}} = 2$ [4]

➥ Please refer to 2069 [Set B] Old Q. No. 13b

86. 2062 Q.No. 12b OR

A function is defined as

$$f(x) = \begin{cases} 3x^2 + 2 \text{ if } x < 1 \\ 2x + 3 \text{ if } x \geq 1 \end{cases}, \text{ find } \lim_{x \to 1} f(x).$$ [4]

SOLUTION

The left hand limit of $f(x)$ at $x = 1$ is

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x^2 + 2)$$

$$= 3 \times 1^2 + 2 = 5$$

The right hand limit of $f(x)$ at $x = 1$ is

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x^2 + 3)$$

$$= 2 \times 1 + 3 = 5$$

Here, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

So, the limit exists and $\lim_{x \to 1} f(x) = 5$

87. 2061 Q.No. 12b

Evaluate: $\lim_{x \to y} \frac{\tan x - \tan y}{x - y}$ [4]

SOLUTION

Here, $\lim_{x \to y} \frac{\tan x - \tan y}{x - y}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to y} \frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{x - y}$$

$$= \lim_{x \to y} \frac{\sin x \cos y - \cos x \sin y}{(x - y) \cos x \cos y}$$

$$= \lim_{x \to y} \frac{\sin (x - y)}{(x - y)} \frac{1}{\cos x \cos y}$$

$$= \lim_{x - y \to 0} \frac{\sin (x - y)}{x - y} \lim_{x \to y} \frac{1}{\cos x \cos y}$$

$$= 1 \frac{1}{\cos y \cos y} \left(\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right)$$

$$= \sec^2 y$$

88. 2060 Q.No. 12b

Evaluate: $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ [4]

➥ Please refer to 2072 Set D Q.No. 9b

SOLUTION

89. 2059 Q.No. 12b

Evaluate: $\lim_{x \to \infty} \sqrt{x} (\sqrt{x} - \sqrt{x - a})$ [4]

➥ Please refer to 2070 Set D Q.No. 9 b

90. 2058 Q.No. 12b OR

Prove geometrically $\lim_{x \to 0} \frac{\sin x}{x} = 1$ [4]

➥ Please refer to 2075 GIE Q.No. 9b

91. 2057 Q.No. 12b

Evaluate: $\lim_{x \to y} \frac{\sin x - \sin y}{(x - y)}$ [4]

SOLUTION

$$\lim_{x \to y} \frac{\sin x - \sin y}{(x - y)}$$

$$= \lim_{x \to y} \frac{2 \cos\left(\frac{x + y}{2}\right) \cdot \sin\left(\frac{x - y}{2}\right)}{(x - y)}$$

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$$= 2\lim_{x\to y}\cos\left(\frac{x+y}{2}\right)\lim_{\left(\frac{x-y}{2}\right)\to 0}\left\{\frac{\sin\left(\frac{x-y}{2}\right)}{\left(\frac{x-y}{2}\right)}\right\}\times\frac{1}{2}$$

$$= 2\cos\left(\frac{y+y}{2}\right)\times 1\times\frac{1}{2}$$

$$= \cos\left(\frac{2y}{2}\right) = \cos y$$

92. 2056 Q.No. 12b

Prove geometrically $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ [4]

➥ Please refer to 2075 GIE Q.No. 9b

93. 2055 Old Model Q.No. 12b OR

Prove geometrically $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ [4]

➥ Please refer to 2075 GIE Q.No. 9b

B. CONTINUITY

2 MARKS QUESTIONS

1. 2075 Set A Q.No. 4c

Find the value of k for which the function

$$f(x) = \begin{cases} \frac{x^2-2x}{x-2} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$$

is continuous at the point x = 3. [2]

SOLUTION

When $x = 3$, $f(3) = k$.

Also, $\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{x^2-2x}{x-2}$

$= \lim_{x\to 3}\frac{x(x-2)}{x-2} = \lim_{x\to 3}(x) = 3$

Since $f(x)$ is continuous at $x = 3$,

$\lim_{x\to 3} f(x) = f(3)$

or, $3 = k$

$\therefore k = 3$

2. 2073 Set C Q.No. 4c

Find the limit of $f(x)\ \frac{x^2-4}{x-2}$ as $x\to 2$. If $f(x)$ continuous? If not, find the point of discontinuity. [2]

SOLUTION

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)}$$

$$= \lim_{x\to 2}(x+2) = 2+2 = 4$$

Functional value $= f(2) = \frac{2^2-4}{2-2} = \frac{0}{0}$ which is an indeterminate form.

Hence, $f(x)$ is not continuous at $x = 2$.

Point of discontinuity is at where $x - 2 = 0$

i.e. $x = 2$

3. 2070 Old Q.No. 6 a

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} x^2 & \text{when } x > 1 \\ 2 & \text{when } x = 1 \\ x & \text{when } x < 1 \end{cases}$$

. Is the function continuous at $x = 2$? Justify. [2]

SOLUTION

L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x) = 1$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x^2) = 1^2 = 1$

Functional value $= f(1) = 2$

Here, $\lim_{x\to 1^-} f(x) = 0$ $f(x) \neq f(1)$

So, $f(x)$ is discontinuous at $x = 1$

4. 2069 [Set A] Old Q. No. 6a

Show that the function $f(x) = \begin{cases} x+1 & x\neq 1 \\ 0 & x=1 \end{cases}$ is not continuous at $x = 1$. [2]

SOLUTION

Here, $f(1) = 0$

and $\lim_{x\to 1} f(x) = \lim_{x\to 1}(x+1) = 1+1 = 2$

$\therefore f(1) \neq \lim_{x\to 1} f(x)$

So, $f(x)$ is not continuous at $x = 1$.

5. 2068 Old Q.No. 6a

Examine the continuity of the function $f(x) = \frac{x^2-16}{x-4}$ at $x = 4$. [2]

SOLUTION

Here, $f(x) = \frac{x^2-16}{x-4}$

Now, $f(4) = \frac{4^2-16}{4-4} = \frac{0}{0}$, which is an indeterminate form.

So, $f(4)$ doesn't exist.

Hence, $f(x)$ is not continuous at $x = 4$.

6. 2067 Q.No. 6a

Test the continuity of $f(x) = x + 2$ when $x \neq 2$, $f(x) = 4$ when $x = 2$ at $x = 2$. [2]

SOLUTION

Left hand limit at $x = 2$ is

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x+2) = 2+2 = 4$

Right hand limit at $x = 2$ is

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+2) = 2+2 = 4$

Here, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 4$

$\therefore \lim_{x\to 2} f(x) = 4$

Functional value at $x = 2$ is

$f(2) = 4$

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$\therefore \lim_{x \to 2} f(x) = f(2)$

So, the given function $f(x)$ is continuous at $x = 2$.

7. 2065 Q.No. 6a

Discuss the continuity of the function $\frac{x^2-9}{x-3}$ and point out the discontinuity if exists. [2]

SOLUTION

Given function is $f(x) = \frac{x^2-9}{x-3}$

Since for $x = 3$; $f(x) = \frac{9-9}{3-3} = \frac{0}{0}$ which is indeterminate form. So, the given function is discontinuous at $x = 3$.

Except at $x = 3$; $f(x)$ is well define every where, as well as limiting value also exists and equals to the functional value. So the given function is continuous everywhere except at $x = 3$.

8. 2064 Q.No. 6c

Why the function $f(x) = \sin\frac{1}{x}$ is not continuous at $x = 0$? [2]

SOLUTION

The given function is $f(x) = \sin\frac{1}{x}$

Now, L.H.L. at $x = 0$

$= \lim_{x \to 0^-} \sin\left(\frac{1}{x}\right)$

$= \lim_{h \to 0} \sin\left(\frac{1}{0-h}\right)$ where $h > 0$

$= \lim_{h \to 0} \sin\left(\frac{-1}{h}\right)$

$= \lim_{h \to 0} -\sin\left(\frac{1}{h}\right)$

$= -\lim_{h \to 0} \sin\left(\frac{1}{h}\right)$

= Any value between -1 and 0

R.H.L. at $x = 0$

$= \lim_{x \to 0^+} \sin\left(\frac{1}{x}\right)$

$= \lim_{h \to 0} \sin\left(\frac{1}{0+h}\right)$ where $h > 0$

$= \lim_{h \to 0} \sin\left(\frac{1}{h}\right)$

= Any value between 0 and 1.

Thus, L.H.L. $\neq$ R.H.L.

$\therefore \lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ doesn't exist

Hence, the given function is discontinuous at $x = 0$.

9. 2062 Q.No. 6a

Is the function $f(x) = \frac{1}{1-x}$ continuous at $x = 1$? [2]

SOLUTION

The given function is $f(x) = \frac{1}{1-x}$

Now, $f(1) = \frac{1}{1-1} = \frac{1}{0} = \infty$

$\therefore$ $f(1)$ is undefined

i.e. $f(1)$ does not exist.

So, the given function is not continuous at $x = 1$.

10. 2061 Q.No. 6a

Show that the function $f(x) = \begin{cases} x+2 & \text{for } x \neq 2 \\ 0 & \text{for } x = 2 \end{cases}$, is not continuous at $x = 2$. [2]

SOLUTION

The given function is

$f(x) = \begin{cases} x+2 & \text{for } x \neq 2 \\ 0 & \text{for } x = 2 \end{cases}$

When $x = 2$, $f(x) = 0$

$\therefore$ $f(2) = 0$

L.H.L. of $f(x)$ at $x = 2$ is

$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} (x+2) = 2+2 = 4$

Since, $f(2) \neq \lim_{x \to 2-} f(x)$, the given function is not continuous at the point $x = 2$.

11. 2060 Q.No. 6a

A function is defined as

$$f(x) = \begin{cases} x^2-1 & \text{when } x < 1 \\ x^2+1 & \text{when } x \geq 1 \end{cases}$$

Examine whether the function is continuous or not at $x = 1$. [2]

SOLUTION

The given function is

$f(x) = \begin{cases} x^2-1 & \text{when } x < 1 \\ x^2+1 & \text{when } x \geq 1 \end{cases}$

When $x = 1$, $f(x) = x^2 + 1$

$\therefore$ $f(1) = 1^2 + 1 = 2$

L.H.L. of $f(x)$ at $x = 1$ is

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - 1 = 1^2 - 1 = 0$

Since $f(1) \neq \lim_{x \to 1^-} f(x)$, the given function is discontinuous at $x = 1$.

12. 2058 Q.No. 6a

Is the function $f(x) = \frac{x^2-9}{x-3}$ continuous at $x = 3$? Justify your answer. [2]

SOLUTION

The given function is $f(x) = \frac{x^2-9}{x-3}$

Now, $f(3) = \frac{3^2-9}{3-3} = \frac{0}{0}$, which is indeterminate form. So $f(x)$ is undefined at $x = 3$. Hence $f(x)$ is discontinuous at $x = 3$.

13. 2055 Old Model Q.No. 6a

Is the function $f(x) = \frac{x^2-9}{x-3}$ continuous at $x = 3$? [2]

SOLUTION

Here, the given function is $f(x) = \frac{x^2-9}{x-3}$

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Now, $f(3) = \frac{3^2-9}{3-3} = \frac{0}{0}$, which is indeterminate form.

So, $f(x)$ is undefined at $x = 3$. Hence, the given function is discontinuous at $x = 3$

4 MARKS QUESTIONS

14. 2076 Set B Q.No. 9b OR

Given $f(x) = \begin{cases} 3-x, & 0 \le x < \frac{3}{2} \\ -3+x, & x \ge \frac{3}{2} \end{cases}$

Test f(x) for continuity at $x = \frac{3}{2}$. If f(x) is not continuous at $x = \frac{3}{2}$, how would you make it continuous? [4]

SOLUTION

$$\text{L.H.L.} = \lim_{x \to 3/2^-} f(x) = \lim_{x \to 3/2^-} (3-x)$$
$$= 3 - \frac{3}{2} = \frac{3}{2}$$
$$\text{R.H.L.} = \lim_{x \to 3/2^+} f(x) = \lim_{x \to 3/2^+} (-3+x)$$
$$= -3 + \frac{3}{2} = -\frac{3}{2}$$

Here, $\lim_{x \to 3/2^-} f(x) \neq \lim_{x \to 3/2^+} f(x)$

So, $f(x)$ has ordinary or jump discontinuity.

Hence, $f(x)$ has irremovable discontinuity.

15. 2076 Set C Q.No. 9b OR

Given $f(x) = \begin{cases} \frac{1}{2}+x, & 0 < x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{3}{2}-x, & \frac{1}{2} < x < 1 \end{cases}$

Show that f(x) has removable discontinuity at $x = \frac{1}{2}$. Define f(x) so as to make f(x) continuous at $x = \frac{1}{2}$. [4]

SOLUTON

Left hand limit at $x = \frac{1}{2}$ is

$$\lim_{x \to 1/2^-} f(x) = \lim_{x \to 1/2^-} \left(\frac{1}{2}+x\right) = \frac{1}{2}+\frac{1}{2} = 1$$

Right hand limit at $x = \frac{1}{2}$ is

$$\lim_{x \to 1/2^+} f(x) = \lim_{x \to 1/2^+} \left(\frac{3}{2}-x\right) = \frac{3}{2}-\frac{1}{2} = 1$$

Thus, $\lim_{x \to 1/2^-} f(x) = \lim_{x \to 1/2^+} f(x) = 1$

$\lim_{x \to 1/2} f(x) = 1$

Again, functional value $= f\left(\frac{1}{2}\right) = \frac{1}{2}$

Here, $\lim_{x \to 1/2} f(x) \neq f\left(\frac{1}{2}\right)$.

So, f(x) has removable discontinuity at $x = \frac{1}{2}$.

The function f(x) can be made continuous by redefining f(x) as follows:

$$f(x) = \begin{cases} \frac{1}{2}+x, & 0 < x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2}-x, & \frac{1}{2} < x < 1 \end{cases}$$

16. 2075 GIE Q.No. 9b OR

A function f(x) is defined as

$$f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$$

Show that f(x) is continuous at x = 1 and x = 2. [4]

SOLUTION

$$\text{Here, } f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$$

At x = 1

$$\text{L.H.L} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1^2 = 1$$
$$\text{R.H.L} = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x = 1$$

Functional value (F.V) = f(1) = 1

$\therefore$ L.H.L = R.H.L = F.V.

So, f(x) is continuous at x = 1.

At x = 2

$$\text{L.H.L} = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x = 2$$
$$\text{R.H.L} = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{1}{4}x^3 = \frac{1}{4} \times 2^3 = 2$$
$$\text{F.V} = f(2) = \frac{1}{4} \times 2^3 = 2$$

$\therefore$ L.H.L = R.H.L = F.V.

So, f(x) is continuous at x = 2.

17. 2075 Set B Q.No. 9b OR

A function is defined as follows: [4]

$$f(x) = \begin{cases} 3+2x \text{ for } -\frac{3}{2} \le x < 0 \\ 3-2x \text{ for } 0 \le x < \frac{3}{2} \\ -3-2x \text{ for } x \ge \frac{3}{2} \end{cases}$$

Show that f(x) is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$.

SOLUTION

$$\text{Here, } f(x) = \begin{cases} 3+2x & \text{for } \frac{-3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

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At $x = 0$

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3 + 2x)$$
$$= 3 + 2 \times 0 = 3$$

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3 - 2x)$$
$$= 3 - 2 \times 0 = 3$$

Functional value $= f(0) = 3 - 2 \times 0 = 3$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

Hence, $f(x)$ is continuous at $x = 0$.

At $x = \frac{3}{2}$

$$\text{L.H.L.} = \lim_{x \to 3/2^-} f(x)$$
$$= \lim_{x \to 3/2^-} (3 - 2x) = 3 - 2 \times \frac{3}{2} = 0$$

$$\text{R.H.L.} = \lim_{x \to 3/2^+} f(x)$$
$$= \lim_{x \to 3/2^+} (-3 - 2x) = -3 - 2 \times \frac{3}{2} = -6$$

$\therefore \lim_{x \to 3/2^-} f(x) \neq \lim_{x \to 3/2^+} f(x)$

Hence, $f(x)$ is discontinuous at $x = \frac{3}{2}$

18. 2075 Set C Q.No. 9b OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x + 1 \text{ for } x < 1 \\ 2 \text{ for } x = 1 \\ 3x \text{ for } x > 1 \end{cases}$$

Is the function continuous at x = 1? If not, can it be made continuous at x = 1 ? **[4]**

SOLUTION

$$\text{L.H.L.} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x + 1)$$
$$= 2 \times 1 + 1 = 3$$

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x)$$
$$= 3 \times 1 = 3$$

Functional value $f(1) = 2$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$

So, $f(x)$ is not continuous at $x = 1$. The function $f(x)$ can be made continuous at $x = 1$ by redefining as follows:

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

19. 2074 Supp Q.No. 9b OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x^2 - 2x - 3} & \text{for } x \neq 3 \\ \dfrac{5}{3} & \text{for } x = 3 \end{cases}$$

Prove that f(x) is discontinuous at x = 3. Can the definition of f(x) be modified so as to make it continuous at x = 3? If so, how? **[4]**

SOLUTION

L.H.L. at $x = 3$ is $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2 - x - 6}{x^2 - 2x - 3}$

$$= \lim_{x \to 3^-} \frac{(x - 3)(x + 2)}{(x - 3)(x + 1)}$$
$$= \lim_{x \to 3^-} \frac{x + 2}{x + 1} = \frac{3 + 2}{3 + 1} = \frac{5}{4}$$

L.H.L. at $x = 3$ is $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x^2 - x - 6}{x^2 - 2x - 3}$

$$= \lim_{x \to 3^+} \frac{(x + 3)(x + 2)}{(x - 3)(x + 1)}$$
$$= \lim_{x \to 3^+} \frac{x + 2}{x + 1} = \frac{3 + 2}{3 + 1} = \frac{5}{4}$$

L.H.L. = R.H.L. $= \frac{5}{4}$

So, limit of $f(x)$ as $x \to 3$ exists and $\lim_{x \to 3} f(x) = \frac{5}{4}$

Functional value at $x = 3$ is $f(3) = \frac{5}{3}$

Here, $\lim_{x \to 3} \neq f(3)$

So, $f(x)$ is discontinuous at $x = 3$.

The function $f(x)$ can be made continuous at $x = 3$ by redefining as

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x^2 - 2x - 3}, & x \neq 3 \\ \dfrac{5}{4}, & x = 3 \end{cases}$$

20. 2074 Set A Q.No. 9b OR

A function f(x) is defined as follows: **[4]**

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < 1 \\ 4 & \text{for } x = 1 \\ 6x - 1 & \text{for } x > 1 \end{cases}$$

Is the function continuous at x = 1? If not, how can you make it continuous?

SOLUTION

$$\text{L.H.L.} = \lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1^-} (2x + 3) = 2 \times 1 + 3 = 5$$

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x)$$
$$= \lim_{x \to 1^+} (6x - 1) = 6 \times 1 - 1 = 5$$

Functional value $f(1) = 4$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$

So, $f(x)$ is discontinuous at $x = 1$. The function $f(x)$ can be made continuous at $x = 1$ by redefining as follows:

$$f(x) = \begin{cases} 2x + 3 & \text{for} \quad x < 1 \\ 5 & \text{for} \quad x = 1 \\ 6x - 1 & \text{for} \quad x > 1 \end{cases}$$

21. 2074 Set B Q.No. 9b OR

A function f(x) is defined in (0, 3) as follows: **[4]**

$$f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \leq x < 2 \\ \dfrac{1}{4}x^3 & \text{when } 2 \leq x < 3 \end{cases}$$

Show that f(x) is continuous at x = 1 and x = 2.

➥ Please refer to **2075 GIE Q.No. 9b OR**

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22. 2073 Supp 9b OR

A function f(x) is defined in (0, 3) in the following way:

$$f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$$

Examine the continuity of the function f(x) at x = 1 and x = 2. [4]

➥ Please refer to 2075 GIE Q.No. 9b OR

23. 2073 Set C Q.No. 9b OR

A function f(x) is defined by

$$f(x) = \begin{cases} 3 + x & \text{when } -\frac{3}{2} \le x < 0 \\ 3 - x & \text{when } 0 \le x < \frac{3}{2} \\ -3 + x & \text{when } x \ge \frac{3}{2} \end{cases}$$ [4]

Test the continuity of f(x) at $x = 0$ and $x = \frac{3}{2}$.

SOLUTION

At $x = 0$,

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3 + x)$$
$$= 3 + 0 = 3$$

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3 - x)$$
$$= 3 - 0 = 3$$

Functional value = f(0) = 3 − 0 = 3

L.H.L. = R.H.L. = F.V.

So, f(x) is continuous at $x = 0$.

At $x = \frac{3}{2}$,

$$\text{L.H.L.} = \lim_{x \to 3/2^-} f(x)$$
$$= \lim_{x \to 3/2^-} (3 - x) = 3 - \frac{3}{2} = \frac{3}{2}$$

$$\text{R.H.L.} = \lim_{x \to 3/2^+} f(x)$$
$$= \lim_{x \to 3/2^+} (-3 + x) = -3 + \frac{3}{2} = -\frac{3}{2}$$

L.H.L. ≠ R.H.L.

So, f(x) is discontinuous at $x = \frac{3}{2}$.

24. 2073 Set D Q.No. 9b OR

A function f(x) is defined by

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ 1 & \text{when } x = \frac{1}{2} \\ 1 - x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

Examine whether f(x) is continuous at $x = \frac{1}{2}$. If not, define f(x) so as to make it continuous. [4]

SOLUTION

Given function is

$$f(x) = \begin{cases} x, & \text{when } 0 \le x < \frac{1}{2} \\ 1, & \text{when } x = \frac{1}{2} \\ 1 - x, & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

$$\text{L.H.L.} = \lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} (x) = \frac{1}{2}$$

$$\text{R.H.L.} = \lim_{x \to \frac{1}{2}^+} f(x) = \lim_{x \to \frac{1}{2}^+} (x - 1) = 1 - \frac{1}{2} = \frac{1}{2}$$

At $x = \frac{1}{2}$, $f\left(\frac{1}{2}\right) = 1$.

Here, L.H.L. = R.H.L. ≠ Functional value.

So, f(x) is not continuous at $x = \frac{1}{2}$.

The function f(x) can be made continuous by redefining as follows:

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{when } x = \frac{1}{2} \\ 1 - x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

25. 2072 Supp. Q.No. 9b OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3 + 2x & \text{for } -3/2 \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < 3/2 \\ -3 - 2x & \text{for } x \ge 3/2 \end{cases}$$

Show that the function f(x) is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$. [4]

➥ Please refer to 2075 Set B Q.No. 9b OR

26. 2072 Set C Q.No. 9b OR

A function f(x) is defined as:

$$f(x) = \begin{cases} \frac{x^2 - x - 6}{x^2 - 2x - 3}, & x \ne 3 \\ \frac{5}{3}, & x = 3 \end{cases}$$ [4]

Prove that f(x) is discontinuous at x = 3. Can the definition of f(x) for x = 3 be modified so as to make it continuous there?

➥ Please refer to 2074 Supp Q.No. 9b OR

27. 2072 Set D Q.No. 9b OR

A function f(x) is defined as follows: [4]

$$f(x) = \begin{cases} x^2 + 2 & \text{for } x < 5 \\ 20 & \text{for } x = 5 \\ 3x + 12 & \text{for } x > 5 \end{cases}$$

Show that f(x) has removable discontinuity at x = 5. Is it possible to make the above function continuous at x = 5? If yes, how can it be made?

SOLUTION

$$\text{L.H.L. at } x = 5 \text{ is } \lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (x^2 + 2)$$
$$= 5^2 + 2 = 27$$

R.H.L. at $x = 5$ is $\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (3x + 12)$
$= 3 \times 5 + 12 = 27$

$\therefore$ L.H.L. = R.H.L. = 27

$\therefore \lim_{x \to 5} f(x) = 27$

Functional value at $x = 5$ is $f(5) = 20$

Here, $\lim_{x \to 5} f(x) \neq f(5)$

So, $f(x)$ has removable discontinuity at $x = 5$.

Yes, the function $f(x)$ can be made continuous at $x = 5$ by redefining as follows

$$f(x) = \begin{cases} x^2 + 2 & \text{for } x < 5 \\ 27 & \text{for } x = 5 \\ 3x + 12 & \text{for } x > 5 \end{cases}$$

28. 2072 Set E Q.No. 9b OR

A function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2 - 5 & \text{for } x < 4 \\ 8 & \text{for } x = 4 \\ 2x + 3 & \text{for } x > 4 \end{cases}$$

Show that the function is discontinuous at $x = 4$. Is it possible to make it continuous at $x = 4$. If possible, how? [4]

SOLUTION

L.H.L. at $x = 4$ is $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (x^2 - 5)$
$= 4^2 - 5 = 11$

R.H.L. at $x = 4$ is $\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (2x + 3)$
$= 2 \times 4 + 3 = 11$

L.H.L. = R.H.L. = 11

$\therefore \lim_{x \to 4} f(x) = 11$

Functional value at $x = 4$ is $f(4) = 8$

Here, $\lim_{x \to 4} f(x) \neq f(4)$

Hence, $f(x)$ is discontinuous at $x = 4$.

Yes, $f(x)$ can be made continuous at $x = 4$ by redefining as follows

$$f(x) = \begin{cases} x^2 - 5 & \text{for } x < 4 \\ 11 & \text{for } x + 4 \\ 2x + 3 & \text{for } x < 4 \end{cases}$$

29. 2071 Supp. Q.No. 9b OR

A function $f(x)$ is defined as: [4]

$$f(x) = \begin{cases} 5x^2 + 3 & \text{for } x > 1 \\ 9 & \text{for } x = 1 \\ 6x + 2 & \text{for } x < 1 \end{cases}$$

Show that $f(x)$ has a removable discontinuity at $x = 1$.

SOLUTION

L.H.L. at $x = 1$ is $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (6x + 2)$
$= 6 \times 1 + 2 = 8$

R.H.L. at $x = 1$ is $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (5x^2 + 3)$
$= 5 \times 1^2 + 3 = 8$

L.H.L. = R.H.L. = 8

$\therefore \lim_{x \to 1} f(x) = 8$

Here, $\lim_{x \to 1} f(x) \neq f(1)$

Hence $f(x)$ has a removable discontinuity at $x = 1$.

30. 2071 Set C Q.No. 9bOr

Show that the function

$$f(x) = \begin{cases} x, & \text{when } 0 \leq x < 1/2 \\ 1, & \text{when } x = 1/2 \\ 1 - x, & \text{when } 1/2 < x < 1 \end{cases}$$ **is**

discontinuous at $x = \frac{1}{2}$,

Also, redefine $f(x)$ so as to $f(x)$ be continuous at $x = \frac{1}{2}$. [4]

➥ Please refer to 2073 Set D Q.No. 9b OR

31. 2071 Set D Q.No. 9b OR

Show that the function

$$f(x) = \begin{cases} \frac{1}{2-x}, & \text{when } 0 < x < 1/2 \\ \frac{1}{2}, & \text{when } x = 1/2 \\ \frac{3}{2-x}, & \text{when } 1/2 < x < 1 \end{cases}$$

is discontinuous at $x = 1/2$. Also, redefine $f(x)$ so as to $f(x)$ be continuous at $x = 1/2$. [4]

SOLUTION

L.H.L. $= \lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} \frac{1}{2-x}$

$= \frac{1}{2 - \frac{1}{2}} = \frac{1}{\frac{4-1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

R.H.L. $= \lim_{x \to \frac{1}{2}^+} f(x) = \lim_{x \to \frac{1}{2}^+} \frac{3}{2-x}$

$= \frac{3}{2 - \frac{1}{2}} = \frac{3}{\frac{3}{2}} = 3 \times \frac{2}{3} = 2$

L.H.L. $\neq$ R.H.L.

So, $f(x)$ has jump discontinuity at $x = \frac{1}{2}$. So, $f(x)$ cannot be redefined to make it continuous at $x = \frac{1}{2}$.

32. 2071 Old Q.No. 12b

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3 + 2x & \text{for } 3/2 \leq x < 0 \\ 3 - 2x & \text{for } 0 \leq x < 3/2 \\ -3 - 2x & \text{for } x \geq 3/2 \end{cases}$$

Show that: $f(x)$ is continuous at $x = 0$ and discontinuous at $x = \frac{3}{2}$. [4]

➥ Please refer to 2075 Set B Q.No. 9b OR

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33. 2070 Supp Q.No. 9b Or

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Is the function continuous at $x = 1$? If not, why? How can the function be made continuous at $x = 1$? [4]

Please refer to 2075 Set C Q.No. 9b OR

34. 2070 Set C Q.No. 9 b Or

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3+2x & \text{for } \frac{-3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

Show that $f(x)$ is continuous at $x = 0$ but discontinuous at $x = \frac{3}{2}$. [4]

Please refer to 2075 Set B Q.No. 9b OR

35. 2070 Set D Q.No. 9 b Or

A function $f(x)$ is defined below:

$$f(x) = \begin{cases} kx+3 & \text{for } x \ge 2 \\ 3x-1 & \text{for } x < 2 \end{cases}$$

Find the value of k so that $f(x)$ is continuous at $x = 2$. [4]

SOLUTION

Here, $f(x) = \begin{cases} kx+3 & \text{for } x \ge 2 \\ 3x-1 & \text{for } x < 2 \end{cases}$

$$\text{L.H.L.} = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x-1)$$
$$= 3 \times 2 - 1 = 5$$

$$\text{R.H.L.} = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (kx+3)$$
$$= k \cdot 2 + 3 = 2k+3$$

Functional value $= f(2) = k \cdot 2 + 3 = 2k+3$

Since $f(x)$ is continuous at $x = 2$, so

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

i.e. $5 = 2k+3$

or, $2k = 2$

$\therefore k = 1$

36. 2070 Old Q.No. 12 b Or

Investigate, the following function for continuity

$$f(x) = \begin{cases} 1+x & \text{when } x > 0 \\ 2 & \text{when } x = 0 \\ -1-x & \text{when } x < 0 \end{cases} \text{ at } x = 0.$$ [4]

SOLUTION

$$\text{L.H.L.} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x) = 1$$

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2) = 1^2 = 1$$

Functional value $= f(1) = 2$

Here, $\lim_{x \to 1^-} f(x) = 0$ $f(x) \ne f(1)$

So, $f(x)$ is discontinuous at $x = 1$

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-1-x)$$
$$= -1 - 0 = -1$$

$$\text{R.H.L.} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (1+x) = 1+0 = 1$$

Here, $\lim_{x \to 0^-} f(x) \ne \lim_{x \to 0^+} f(x)$

So, $f(x)$ is discontinuous at $x = 0$.

37. 2069 Supp Q.No. 9 b Or

Given $f(x) = \begin{cases} 3-x & \text{when } 0 \le x < 3/2 \\ -3+x & \text{when } x \ge 3/2 \end{cases}$

Is $f(x)$ continuous at $x = 3/2$? If not, how would you make it continuous? [4]

Please refer to 2076 Set B Q.No. 9b OR

38. 2069 [Set A] Q. No. 9b OR

Define continuity of a function at a point. A function is defined as follows:

$$f(x) = \begin{cases} \frac{2x^2-18}{x-3} & \text{for } x \ne 3 \\ k & \text{for } x = 3 \end{cases}$$

Find the value of k so that $f(x)$ is continuous at $x = 3$. [4]

SOLUTION

Continuity: A function $f(x)$ is said to be continuous at $x = a$, if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Second part:

Here, $f(3) = k$

And, $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x^2-18}{x-3}$

$$= \lim_{x \to 3} \frac{2(x^2-9)}{x-3}$$
$$= \lim_{x \to 3} \frac{2(x-3)(x+3)}{x-3}$$
$$= 2(3+3)$$
$$= 12$$

If $f(x)$ is continuous at $x = 3$, then

$$\lim_{x \to 3} f(x) = f(3)$$

i.e. $12 = k$

$\therefore k = 12$

39. 2069 [Set B] Q. No. 9b OR

Show that the function

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ 1 & \text{when } x = \frac{1}{2} \\ 1-x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

is discontinuous at $x = \frac{1}{2}$. Also, write how it could be made continuous? [4]

Please refer to 2073 Set D Q.No. 9b OR

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40. 2069 [Set A] Old Q. No. 12b OR

Define continuity of a function at a point.

Given $f(x) = \begin{cases} 2-x^2 & \text{for } x<2 \\ 3 & \text{for } x=2 \\ x-4 & \text{for } x>2 \end{cases}$

Test the continuity of the functions at $x = 2$. [4]

SOLUTION

Continuity: A function $f(x)$ is said to be continuous at $x = a$, if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Next part:

$$\text{L.H.L.} = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2-x^2) = 2-2^2 = -2$$

$$\text{R.H.L.} = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x-4) = 2-4 = -2$$

Functional value $= f(2) = 3$

$$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \neq f(2)$$

Hence, $f(x)$ is discontinuous at $x = 2$.

41. 2069 [Set B] Old Q. No. 13b OR

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x+3 & \text{for } x<1 \\ 4 & \text{for } x=1 \\ 6x-1 & \text{for } x>1 \end{cases}$$

is the function continuous at $x = 1$? If not, how can you make it continuous? [4]

➥ Please refer to 2074 Set A Q.No. 9b OR

42. 2068 Q.No. 9b(Or)

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x<1 \\ 2 & \text{for } x=1 \\ 3x & \text{for } x>1 \end{cases}$$

Is the function continuous at $x = 1$? If not, can it be made continuous at $x = 1$? [4]

➥ Please refer to 2075 Set C Q.No. 9b OR

43. 2068 Old Q.No. 12b (Or)

A functions $f(x)$ is defined by as follows: $f(x) =$

$$\begin{cases} 2x+1 & \text{for } x<1 \\ 2 & \text{for } x=1 \\ 3x & \text{for } x>1 \end{cases}$$

Examine whether $f(x)$ is continuous at $x = 1$. If not, can it be made continuous at $x = 1$? [4]

➥ Please refer to 2075 Set C Q.No. 9b OR

44. 2067 Q.No. 12b OR

Discuss the continuity of the function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

at $x = 0$. [4]

SOLUTION

We know that for all real x, $-1 \le \sin\frac{1}{x} \le 1$.

If $x > 0$, we have $-x \le x \sin\frac{1}{x} \le x$.

Taking $\lim_{x \to 0^+}$ on each side, we get

$$0 \le \lim_{x \to 0^+} x \sin\frac{1}{x} \le 0.$$

Hence, by the squeeze theorem,

$$\lim_{x \to 0^+} x \sin\frac{1}{x} = 0$$

Similarly, if $x < 0$, we have $-x \ge x \sin\frac{1}{x} \ge x$.

Taking $\lim_{x \to 0^-}$ on each side, we get

$$0 \ge \lim_{x \to 0^-} x \sin\frac{1}{x} \ge 0.$$

Hence, by the squeeze theorem, we get

$$\lim_{x \to 0^-} x \sin\frac{1}{x} = 0.$$

Also, $f(0) = 0$.

Since left hand limit = right hand limit = functional value,

i.e., $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0) = 0$,

$\therefore$ $f(x)$ is continuous at $x = 0$.

45. 2066 Q.No. 12b Or

A function is defined as follows:

$$f(x) = \begin{cases} -x & \text{when } x<0 \\ x & \text{when } 0<x<1 \\ 2-x & \text{when } x \ge 1 \end{cases}$$

show that it is continuous at $x = 0$ and $x = 1$. [4]

SOLUTION

Left hand limit at $x = 0$ is

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$$

Right hand limit at $x = 0$ is

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x) = 0$$

And $f(0) = 0$

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

Hence $f(x)$ is continuous at $x = 0$

Next,

Left hand limit at $x = 1$,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x) = 1$$

Right hand limit at $x = 1$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} f(2-x) = 2-1 = 1$$

and $f(1) = 2-1 = 1$

$$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

Hence, $f(x)$ is continuous at $x = 1$

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46. 2065 Q.No. 12b OR

Test the continuity of the function:

$$f(x) = \begin{cases} x, \text{ when } 0 \le x < \frac{1}{2} \\ 1, \text{ when } x = \frac{1}{2} \\ 1 - x, \text{ when } \frac{1}{2} < x < 1 \end{cases} \quad \text{at } x = \frac{1}{2} \qquad [4]$$

➥ Please refer to 2073 Set D Q.No. 9b OR

47. 2064 Q.No. 13b OR

Show that the function $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & (x \neq 0) \\ 1, & (x = 0) \end{cases}$ is discontinuous at $x = 0$. Redefine the function in such a way that it can becomes continuous at $x = 0$. [4]

SOLUTION

The given function is, $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & (x \neq 0) \\ 1, & (x = 0) \end{cases}$

Now,

L.H.L. at $x = 0 = \lim_{x \to 0^-} f(x)$

$$= \lim_{x \to 0^-} \frac{\sin^2 ax}{x^2}$$

$$= \lim_{x \to 0^-} \left(\frac{\sin ax}{ax^2}\right)^2 \times a^2$$

$$= \lim_{x \to 0^-} \left(\frac{\sin^2 ax}{ax}\right)^2 \times a^2$$

$$= 1^2 \times a^2 = a^2$$

R.H.L. at $x = 0 = \lim_{x \to 0^+} f(x)$

$$= \lim_{x \to 0^+} \frac{\sin^2 ax}{x^2}$$

$$= \lim_{x \to 0^+} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$$

$$= \lim_{ax \to 0^+} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$$

$$= 1^2 \times a^2 = a^2$$

At $x = 0$; $f(x) = 1$

$\therefore f(0) = 1$

Since, L.H.L. = R.H.L. $\neq f(x)$ at $x = 0$,

$f(x)$ is discontinuous at $x = 0$

This type of discontinuity is known as removable discontinuity.

If we redefine the function as follows:

$$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & (x \neq 0) \\ a^2, & (x = 0) \end{cases}$$

Now, it becomes continuous at $x = 0$

48. 2063 Q.No. 13b

Let a function $f(x)$ be defined by [4]

$$f(x) \begin{cases} 2 - x^2 & (x < 2) \\ 3 & (x = 2) \\ x - 4 & (x > 2) \end{cases}$$

Verify that the limit of the function exists at $x = 2$. Is the function continuous at $x = 2$? State how can you make it continuous.

SOLUTION

Here, given function is $f(x) = \begin{cases} 2 - x^2 & (x < 2) \\ 3 & (x = 2) \\ x - 4 & (x > 2) \end{cases}$

L.H.L. of $f(x)$ at $x = 2$ is

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2 - x^2) = 2 - 2^2 = 2 - 4 = -2$

R.H.L. of $f(x)$ as $x = 2$ is

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x - 4) = 2 - 4 = -2$

Since, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$, limit of the function $f(x)$ exists at $x = 2$

$\therefore \lim_{x \to 2} f(x) = -2$

Further, testing the continuity of $f(x)$ at $x = 2$

When $x = 2$, $f(x) = 3$

$\therefore f(2) = 3$

From above,

$\lim_{x \to 2} f(x) = -2$

Since, $f(2) \neq \lim_{x \to 2} f(x)$ the function $f(x)$ is not continuous at $x = 2$. It can be made continuous at $x = 2$ by redefining the function as follows.

$$f(x) = \begin{cases} 2 - x^2 & (x < 2) \\ -2 & (x = 2) \\ x - 4 & (x > 2) \end{cases}$$

49. 2061 Q.No. 12b OR

Discuss the continuity of the function

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{where } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases} \qquad [4]$$

➥ Please refer to 2067 Q.No. 12b OR

50. 2060 Q.No. 12b OR

Discuss the continuity of the function:

$f(x) = |x|$ at $x = 0$ [4]

SOLUTION

Given function is $f(x) = |x|$

We know, $|x| = \begin{cases} -x & \text{when } x < 0 \\ 0 & \text{when } x = 0 \\ x & \text{when } x > 0 \end{cases}$

When $x = 0$, $f(x) = 0$

$\therefore f(0) = 0$

L.H.L. of $f(x)$ at $x = 0$ is

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$

R.H.L. of $f(x)$ at $x = 0$ is

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$

Since, $f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$, the given function is continuous at $x = 0$.

51. 2059 Q.No. 12b OR

When does a function $f(x)$ become continuous at $x = a$? Test the continuity of

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \text{ at } x = 1 \\ 3x & \text{for } x > 1 \end{cases} \qquad [4]$$

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SOLUTION

We say a function $y = f(x)$ is continuous at a point $x = a$ if $\lim_{x \to a} f(x) = f(a)$

i.e. the limiting value of $f(x)$ is equal to the functional value at the point $x = a$.

L.H.L. of $f(x)$ at $x = 1$

$= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-} (2x + 1) = (2 \times 1 + 1) = 3$

R.H.L. of $f(x)$ at $x = 1$

$= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} (3x) = 3 \times 1 = 3$

When, $x = 1$, $f(1) = 2 \times 1 = 2$

Since, L.H.L. = R.H.L. $\neq f(1)$, the given function $f(x)$ is discontinuous at $x = 1$.

52. 2058 Q.No. 12b

Discuss the continuity of the function, [4]

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases} \text{ at } x = 1$$

SOLUTION

The given function is

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

$f(x)$ at $x = 1$

When $x = 1$, $f(x) = 2x$

$\therefore f(1) = 2 \times 1 = 2$

L.H.L. of $f(x)$ at $x = 1$

$= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x + 1)$

$= 2 \times 1 + 1 = 3$

Since $f(1) \neq \lim_{x \to 1-} f(x)$ i.e. functional value $\neq$ left hand limit at $x = 1$, the given function is discontinuous at $x = 1$.

53. 2057 Q.No. 12b OR

When does a function $f(x)$ becomes continuous at $x = a$? Discuss the continuity of: [4]

$$f(x) = \begin{cases} = 2x + 1 & \text{for } x < 1 \\ = 2x & \text{for } x = 1 \\ = 3x & \text{for } x > 1 \end{cases} \text{ at } x = 1$$

➥ Please refer to **2059 Q.No. 12b OR**

54. 2056 Q.No. 12b OR

When does a function $f(x)$ become continuous at $x = a$? Is the function $f(x)$ defined by

$$f(x) = \begin{cases} = 3 + 2x & -3/2 \leq x < 0 \\ = 3 - 2x & 0 \leq 3/2 \\ = -3 - 2x & x \geq 3/2 \end{cases} \text{ continuous at } x = \frac{3}{2} \text{ [4]}$$

SOLUTION

A function $f(x)$ becomes continuous at a point $x = a$ if $f(a) = \lim_{x \to a} f(x)$

i.e. When functional value = limiting value of $f(x)$ at $x = a$.

or, Equivalently, $f(x)$ becomes continuous at $x = a$ if

$f(a) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

The given function is

$$f(x) = \begin{cases} 3 + 2x & \text{for } -3/2 \leq x < 0 \\ 3 - 2x & \text{for } 0 \leq x < 3/2 \\ -3 - 2x & \text{for } x \geq 3/2 \end{cases}$$

For $x = \frac{3}{2}$; $f(x) = -3 - 2x$

So, $f\left(\frac{3}{2}\right) = -3 - 2 \times \frac{3}{2} = -6$

L.H.L. of $f(x)$ at $x = \frac{3}{2}$ is

i.e. $\lim_{x \to \frac{3}{2}-} f(x) = \lim_{x \to \frac{3}{2}-} (3 - 2x)$

$= 3 - 2 \times \frac{3}{2}$

$= 0$

Since, $f\left(\frac{3}{2}\right) \neq$ L.H.L, the given function is not continuous at $x = \frac{3}{2}$.

55. 2055 Old Model Q.No. 12b

A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 3 + 2x & \text{for } -3/2 \leq x < 0 \\ 3 - 2x & \text{for } 0 \leq x < 3/2 \\ -3 - 2x & \text{for } x \geq 3/2 \end{cases}$$

Is the function continuous at $x = \frac{3}{2}$? [4]

➥ Please refer to **2056 Q.No. 12b OR**

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13 UNIT

THE DERIVATIVES

A. DERIVATIVES OF ALGEBRAIC FUNCTIONS

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 5a

Find $\frac{dy}{dx}$ when $x = 2at$ and $y = 2at^2$. [2]

SOLUTION

Here, $x = 2at$

Differentiating both sides w.r. to t,

$$\frac{dx}{dt} = \frac{d}{dt}(2at) = 2a$$

Again, $y = 2at^2$

Differentiating both sides w.r. to t,

$$\frac{dy}{dx} = \frac{d}{dt}(2at^2) = 4at$$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at}{2a} = 2t$$

2. 2075 GIE Q.No. 5a

Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$. [2]

SOLUTION

Given, $x^3 + y^3 - 3axy = 0$

or, $x^3 + y^3 = 3axy$

Differentiating both sides with respect to 'x',

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3axy)$$

or, $\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 3a\frac{d}{dx}(xy)$

or, $3x^2 + \frac{d(y^3)}{dy}\cdot\frac{dy}{dx} = 3a\left\{x\frac{dy}{dx} + y\frac{d(x)}{dx}\right\}$

or, $3x^2 + 3y^2\frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y\right)$

or, $x^2 + y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$

or, $y^2\frac{dy}{dx} - ax\frac{dy}{dx} = ay - x^2$

or, $(y^2 - ax)\frac{dy}{dx} = ay - x^2$

$\therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$

3. 2074 Supp Q.No. 5a

Find $\frac{dy}{dx}$, when $y = \frac{1}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}}$. [2]

SOLUTION

Here,

$$y = \frac{1}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}}$$

$$= \frac{1}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} \times \frac{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}$$

$$= \frac{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{x^2 + a^2 - (x^2 - a^2)}$$

$$= \frac{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{x^2 + a^2 - x^2 + a^2}$$

$$= \frac{1}{2a^2}\left(\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}\right)$$

Differentiating both sides w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{2a^2}\left\{(x^2 + a^2)^{\frac{1}{2}} + (x^2 - a^2)^{\frac{1}{2}}\right\}\right]$$

$$= \frac{1}{2a^2}\left\{\frac{(x^2 + a^2)^{\frac{1}{2}}}{d(x^2 + a^2)}\cdot\frac{d(x^2 + a^2)}{dx} + \frac{(x^2 - a^2)^{\frac{1}{2}}}{d(x^2 - a^2)}\cdot\frac{d(x^2 - a^2)}{dx}\right\}$$

$$= \frac{1}{2a^2}\left\{\frac{1}{2}(x^2 + a^2)^{\frac{1}{2}-1}\cdot 2x + \frac{1}{2}(x^2 - a^2)^{\frac{1}{2}-1}\cdot 2x\right\}$$

$$= \frac{x}{2a^2}\left\{(x^2 + a^2)^{-\frac{1}{2}} + (x^2 - a^2)^{-\frac{1}{2}}\right\}$$

$$= \frac{x}{2a^2}\left(\frac{1}{\sqrt{x^2 + a^2}} + \frac{1}{\sqrt{x^2 - a^2}}\right)$$

4. 2073 Set C Q.No. 5a

Find $\frac{dy}{dx}$ when $y = (x+1)(x+2)(x+3)$. [2]

SOLUTION

Here, $y = (x+1)(x+2)(x+3)$

$$= (x^2 + 3x + 2)(x + 3)$$

$$= x^3 + 6x^2 + 11x + 6$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 6x^2 + 11x + 6)$$

$$= 3x^2 + 12x + 11$$

5. 2073 Set D Q.No. 5a

Find $\frac{dy}{dx}$ when $x = at$, $y = \frac{a}{t}$. [2]

SOLUTION

Here, $x = at$

$\frac{dx}{dt} = \frac{d}{dt}(at) = a$

and $y = \frac{a}{t} = at^{-1}$

$\frac{dy}{dt} = \frac{d}{dt}(at^{-1}) = a \cdot (-1) \cdot t^{-2} = -\frac{a}{t^2}$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-a/t^2}{a} = -\frac{1}{t^2}$

6. 2070 Supp Q.No. 5a

Find $\frac{dy}{dx}$ when $y = \frac{1}{\sqrt[3]{3x^2 - 4x - 1}}$ [2]

SOLUTION

Here, $y = \frac{1}{\sqrt[3]{3x^2 - 4x - 1}} = (3x^2 - 4x - 1)^{-1/3}$

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 4x - 1)^{-1/3}$$

$$= \frac{d(3x^2 - 4x - 1)^{-1/3}}{d(3x^2 - 4x - 1)} \cdot \frac{d(3x^2 - 4x - 1)}{dx}$$

$$= -\frac{1}{3}(3x^2 - 4x - 1)^{-1/3 - 1} \cdot (6x - 4)$$

$$= -\frac{1}{3}(3x^2 - 4x - 1)^{-4/3} \cdot 2(3x - 2)$$

$$= -\frac{2(3x - 2)}{3(3x^2 - 4x - 1)^{4/3}} = \frac{-2(3x - 2)}{3(3x^2 - 4x - 1)^{4/3}}$$

7. 2070 Set C Q.No. 5 a

Find $\frac{dy}{dx}$ when $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. [2]

SOLUTION

Here, $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

$\frac{dx}{dt} = \frac{d}{dt}(t + t^{-1}) = 1 + (-1)\, t^{-1-1}$

$= 1 - \frac{1}{t^2} = \frac{t^2 - 1}{t^2}$

Again, $y = t - \frac{1}{t} = t - t^{-1}$

$\frac{dy}{dt} = \frac{d}{dt}(t - t^{-1}) = 1 - (-1)\, t^{-1-1}$

$= 1 + \frac{1}{t^2} = \frac{t^2 + 1}{t^2}$

Now,

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{t^2 + 1}{t^2}}{\frac{t^2 - 1}{t^2}} = \frac{t^2 + 1}{t^2 - 1}$

8. 2069 Supp Q.No. 5 a

Find the $\frac{dy}{dx}$; when $y = \frac{1}{\sqrt{x+a} + \sqrt{x-a}}$ [2]

SOLUTION

Here, $y = \frac{1}{\sqrt{x+a} + \sqrt{x-a}}$

$= \frac{1}{\sqrt{x+a} + \sqrt{x-a}} \times \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}}$

$= \frac{\sqrt{x+a} - \sqrt{x-a}}{x + a - x + a}$

$= \frac{1}{2a}\{(x+a)^{1/2} - (x-a)^{1/2}\}$

Differentiating both sides with respect to 'x', we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{2a}\{(x+a)^{1/2} - (x-a)^{1/2}\}\right]$$

$$= \frac{1}{2a}\left\{\frac{d(x+a)^{1/2}}{d(x+a)} \cdot \frac{d(x+a)}{dx} - \frac{d(x-a)^{1/2}}{d(x-a)} \cdot \frac{d(x-a)}{dx}\right\}$$

$$= \frac{1}{2a}\left\{\frac{1}{2}(x+a)^{-1/2} \cdot 1 - \frac{1}{2}(x-a)^{-1/2} \cdot 1\right\}$$

$$= \frac{1}{4a}\left(\frac{1}{\sqrt{x+a}} - \frac{1}{\sqrt{x-a}}\right)$$

9. 2069 [Set A] Q. No. 5a

Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3axy = 0$ [2]

➥ Please refer to 2075 GIE Q.No. 5a

10. 2068 Q.No. 5a

Find the derivative of $\frac{1}{x - \sqrt{a^2 + x^2}}$ [2]

SOLUTION

Let $y = \frac{1}{x - \sqrt{a^2 + x^2}} \times \frac{x + \sqrt{a^2 + x^2}}{x + \sqrt{a^2 + x^2}}$

$= \frac{x + \sqrt{a^2 + x^2}}{x^2 - a^2 - x^2}$

$= -\frac{1}{a^2}(x + \sqrt{a^2 + x^2})$

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}\left\{-\frac{1}{a^2}(x + \sqrt{a^2 + x^2})\right\}$$

$$= -\frac{1}{a^2}\left\{\frac{d(x)}{dx} + \frac{d(a^2 + x^2)^{1/2}}{d(a^2 + x^2)} \cdot \frac{d(a^2 + x^2)}{dx}\right\}$$

$$= -\frac{1}{a^2}\left\{1 + \frac{1}{2}(a^2 + x^2)^{-1/2} \cdot 2x\right\}$$

$$= -\frac{1}{a^2}\left\{1 + \frac{x}{(a^2 + x^2)^{1/2}}\right\}$$

$$= -\frac{1}{a^2}\left(1 + \frac{x}{\sqrt{a^2 + x^2}}\right)$$

11. 2066 Q.No. 5c

Find $\frac{dy}{dx}$ if $ax^2 + 2hxy + by^2 = 1$ [2]

SOLUTION

$ax^2 + 2hxy + by^2 = 1$

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Differentiating both sides w.r. to 'x'

$$\frac{d}{dx}(ax^2 + 2hxy + by^2) = \frac{d}{dx}(1)$$

or, $a\frac{d}{dx}(x^2) + 2h\frac{d}{dx}(xy) + b\frac{d}{dx}(y^2) = 0$

or, $2ax + 2h\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + b\frac{dy^2}{dy}\cdot\frac{dy}{dx} = 0$

or, $2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} = 0$

or, $(2ax + 2hy) + (2hx + 2by)\frac{dy}{dx} = 0$

or, $\frac{dy}{dx} = \frac{-(2ax + 2hy)}{2hx + 2by}$

$\therefore \frac{dy}{dx} = -\frac{(ax + hy)}{hx + by}$

12. 2055 Old Model Q.No. 5c

Find dy/dx of $x = a\sin t$, $y = a\cos t$ [2]

SOLUTION

Here, $x = a\sin t$

$\therefore \frac{dx}{dt} = \frac{d}{dt}(a\sin t) = a\cos t$

Also, $y = a\cos t$

$\therefore \frac{dy}{dt} = \frac{d}{dt}(a\cos t) = -a\sin t$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-a\sin t}{a\cos t}$

$\therefore \frac{dy}{dx} = -\tan t$

4 MARKS QUESTIONS

13. 2076 Set B Q.No. 10a

Find from first principles, the derivative of $(bx - a)^n$. [4]

SOLUTION

Let $y = (bx - a)^n$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \{b(x + \Delta x) - a\}^n$

or $\Delta y = (bx + b\Delta x - a)^n - y$

or, $\Delta y = (bx + b\Delta x - a)^n - (bx - a)^n$

Dividing both sides by Δx,

$$\frac{\Delta y}{\Delta x} = \frac{(bx + b\Delta x - a)^n - (bx - a)^n}{\Delta x}$$

Taking limit $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{(bx + b\Delta x - a)^n - (bx - a)^n}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left\{\frac{(bx + b\Delta x - a)^n - (bx - a)^n}{b\Delta x}\cdot b\right\}$$

$$= \lim_{(bx+b\Delta x-a)\to(bx-a)}\left\{\frac{(bx + b\Delta x - a)^n - (bx - a)^n}{(bx + b\Delta x - a) - (bx - a)}\cdot b\right\}$$

$$= n(bx - a)^{n-1}\cdot b \left[\because \lim_{x \to 0}\frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= nb(bx - a)^{n-1}$$

14. 2076 Set C Q.No. 10a

Find from definition, the derivative of $(ax + b)^n$. [4]

SOLUTON

Let $y = (ax + b)^n$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \{a(x + \Delta x) + b\}^n$

or, $\Delta y = (ax + a\Delta x + b)^n - y$

or, $\Delta y = (ax + a\Delta x + b)^n - (ax + b)^n$

Dividing both sides by Δx and taking limit $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{(ax + a\Delta x + b)^n - (ax + b)^n}{\Delta x}$$

or, $\frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{(ax + a\Delta x + b)^n - (ax + b)^n}{a\Delta x}\cdot a$

$$= \lim_{(ax+a\Delta x+b)\to(ax+b)}\left\{\frac{(ax + a\Delta x + b)^n - (ax + b)^n}{(ax + a\Delta x + b) - (ax + b)}\cdot a\right\}$$

$$= n(ax + b)^{n-1}\cdot a \left[\because \lim_{x \to a}\frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= na(ax + b)^{n-1}$$

15. 2075 GIE Q.No. 10a

Find from definition, the derivative of $\sqrt{3x - 2}$. [4]

SOLUTION

Let $y = \sqrt{3x - 2}$

Again, let Δx and Δy be the small increments in x and y respectively.

Then,

$y + \Delta y = \sqrt{3(x + \Delta x) - 2}$

or, $\Delta y = \sqrt{3x + 3\Delta x - 2} - y$

or, $\Delta y = \frac{\sqrt{3x + 3\Delta x - 2} - \sqrt{3x - 2}}{1}\times\frac{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$

or, $\Delta y = \frac{3x + 3\Delta x - 2 - (3x - 2)}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$

or, $\Delta y = \frac{3x + 3\Delta x - 2 - 3x + 2}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$

or, $\Delta y = \frac{3\Delta x}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$

or, $\frac{\Delta y}{\Delta x} = \frac{3}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$

Taking limit $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{3}{\sqrt{3x + 3\Delta x - 2} + \sqrt{3x - 2}}$$

$$\frac{dy}{dx} = \frac{3}{\sqrt{3x + 0 - 2} + \sqrt{3x - 2}}$$

$$= \frac{3}{2\sqrt{3x - 2}}$$

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16. 2075 Set C Q.No. 10a

Find, from first principles, the derivatives of $\frac{3x+5}{\sqrt{x}}$. [4]

SOLUTION

Let $y = \frac{3x+5}{\sqrt{x}} = \frac{3x}{\sqrt{x}} + \frac{5}{\sqrt{x}} = 3\sqrt{x} + \frac{5}{\sqrt{x}}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = 3\sqrt{x+\Delta x} + \frac{5}{\sqrt{x+\Delta x}}$

or, $\Delta y = 3\sqrt{x+\Delta x} + \frac{5}{\sqrt{x+\Delta x}} - y$

or, $\Delta y = 3\sqrt{x+\Delta x} + \frac{5}{\sqrt{x+\Delta x}} - 3\sqrt{x} - \frac{5}{\sqrt{x}}$

or, $\Delta y = 3(\sqrt{x+\Delta x} - \sqrt{x}) + 5\left(\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}\right)$

or, $\Delta y = 3\frac{\sqrt{x+\Delta x} - \sqrt{x}}{1} \times \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} + 5\left(\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}\right)$

or, $\Delta y = \frac{3(x+\Delta x - x)}{\sqrt{x+\Delta x} + \sqrt{x}} + \frac{5(\sqrt{x} - \sqrt{x+\Delta x})}{\sqrt{x}\sqrt{x+\Delta x}} \times \frac{\sqrt{x} + \sqrt{x+\Delta x}}{\sqrt{x} + \sqrt{x+\Delta x}}$

or, $\Delta y = \frac{3\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}} + \frac{5(x - x - \Delta x)}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})}$

or, $\Delta y = \frac{3\Delta x}{\sqrt{x+\Delta x} + \sqrt{x}} - \frac{5\Delta x}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})}$

or, $\Delta y = \Delta x \left\{\frac{3}{\sqrt{x+\Delta x} + \sqrt{x}} - \frac{5}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})}\right\}$

or, $\frac{\Delta y}{\Delta x} = \frac{3}{\sqrt{x+\Delta x} + \sqrt{x}} - \frac{5}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})}$

Taking limit $\Delta x \to 0$ on both sides, we get,

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left\{\frac{3}{\sqrt{x+\Delta x} + \sqrt{x}} - \frac{5}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})}\right\}$

$\frac{dy}{dx} = \frac{3}{\sqrt{x} + \sqrt{x}} - \frac{5}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})}$

$= \frac{3}{2\sqrt{x}} - \frac{5}{2x\sqrt{x}} = \frac{3x-5}{2x.\sqrt{x}} = \frac{3x-5}{2x^{3/2}}$

17. 2074 Supp Q.No. 10a

Find from first principles the derivative of $\sqrt{2x+3}$. [4]

SOLUTION

Let $y = \sqrt{2x+3}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sqrt{2(x+\Delta x) + 3}$

or, $\Delta y = \sqrt{2x + 2\Delta x + 3} - y$

or, $\Delta y = \sqrt{2x + 2\Delta x + 3} - \sqrt{2x+3}$

or, $\Delta y = \frac{\sqrt{2x+2\Delta x+3} - \sqrt{2x+3}}{1} \times \frac{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

or, $\Delta y = \frac{2x + 2\Delta x + 3 - 2x - 3}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

or, $\frac{\Delta y}{\Delta x} = \frac{2}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

Taking limit $\Delta x \to 0$ on both sides, we get

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{2}{\sqrt{2x+2\Delta x+3} + \sqrt{2x+3}}$

$\frac{dy}{dx} = \frac{2}{\sqrt{2x+3} + \sqrt{2x+3}} = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}$

18. 2074 Set A Q.No. 10a

Find from first principles, the derivative of $\sqrt{2-3x}$. [4]

SOLUTION

Let $y = \sqrt{2-3x}$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sqrt{2 - 3(x+\Delta x)}$

or, $\Delta y = \sqrt{2 - 3x - 3\Delta x} - y$

or, $\Delta y = \frac{\sqrt{2-3x-3\Delta x} - \sqrt{2-3x}}{1} \cdot \frac{\sqrt{2-3x-3\Delta x} + \sqrt{2-3x}}{\sqrt{2-3x-3\Delta x} + \sqrt{2-3x}}$

or, $\Delta y = \frac{2 - 3x - 3\Delta x - 2 + 3x}{\sqrt{2-3x-3\Delta x} + \sqrt{2-3x}}$

or, $\frac{\Delta y}{\Delta x} = \frac{-3}{\sqrt{2-3x-3\Delta x} + \sqrt{2-3x}}$

Taking limit $\Delta x \to 0$ on both sides, we get

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-3}{\sqrt{2-3x-3\Delta x} + \sqrt{2-3x}}$

$\frac{dy}{dx} = \frac{-3}{\sqrt{2-3x} + \sqrt{2-3x}} = \frac{-3}{2\sqrt{2-3x}}$

19. 2074 Set B Q.No. 10a

Find the principles the derivative of $\frac{1}{\sqrt{x-1}}$ [4

SOLUTION

Let $y = \frac{1}{\sqrt{x-1}}$

Also let Δx and Δy be the small increments in x and y respectively.

Then,

$y + \Delta y = \frac{1}{\sqrt{x+\Delta x-1}}$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x-1}} - y$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x-1}} - \frac{1}{\sqrt{x-1}}$

or, $\Delta y = \frac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\sqrt{x+\Delta x-1}\sqrt{x-1}}$

or, $\Delta y = \frac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\sqrt{x+\Delta x-1}\sqrt{x-1}} \times \frac{\sqrt{x-1} + \sqrt{x+\Delta x-1}}{\sqrt{x-1} + \sqrt{x+\Delta x-1}}$

or, $\Delta y = \frac{x - 1 - x - \Delta x + 1}{\sqrt{x+\Delta x-1}\sqrt{x-1}(\sqrt{x-1} + \sqrt{x+\Delta x-1})}$

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$$\text{or, } \Delta y = \frac{-\Delta x}{\sqrt{x+\Delta x-1}\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x+\Delta x-1}\right)}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{-1}{\sqrt{x+\Delta x-1}\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x+\Delta x-1}\right)}$$

Taking limit $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\left\{\frac{-1}{\sqrt{x+\Delta x-1}\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x+\Delta x-1}\right)}\right\}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x-1}\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x-1}\right)}$$

$$= \frac{-1}{(x-1).\,2\sqrt{x-1}} = \frac{-1}{2(x-1)^{3/2}}$$

20. 2073 Set D Q.No. 10a

Find from definition the derivative of $\sqrt{1+x}$. [4]

SOLUTION

Let $y = \sqrt{1+x}$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta y = \sqrt{1+x+\Delta x}$$

$$\text{or, } \Delta y = \sqrt{1+x+\Delta x} - y$$

$$\text{or, } \Delta y = \sqrt{1+x+\Delta x} - \sqrt{1+x}$$

$$\text{or, } \Delta y = \frac{\sqrt{1+x+\Delta x}-\sqrt{1+x}}{1} \times \frac{\sqrt{1+x+\Delta x}+\sqrt{1+x}}{\sqrt{1+x+\Delta x}+\sqrt{1+x}}$$

$$\text{or, } \Delta y = \frac{1+x+\Delta x-1-x}{\sqrt{1+x+\Delta x}+\sqrt{1+x}}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{1}{\sqrt{1+x+\Delta x}+\sqrt{1+x}}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{1}{\sqrt{1+x+\Delta x}+\sqrt{1+x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1+x}+\sqrt{1+x}} = \frac{1}{2\sqrt{1+x}}$$

21. 2072 Supp. Q.No. 10a

Find from first principles, the derivative of $\sqrt{2-3x}$. [4]

Please refer to 2074 Set A Q.No. 10a

22. 2072 Set E Q.No. 10a

Find the derivative of $\sqrt[5]{1+x}$ using the first principles. [4]

SOLUTION

Let $y = \sqrt[5]{1+x} = (1+x)^{1/5}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta y = (1+x+\Delta x)^{1/5}$$

$$\Delta y = (1+x+\Delta x)^{1/5} - y$$

$$\Delta y = (1+x+\Delta x)^{1/5} - (1+x)^{1/5}$$

Dividing both sides by Δx, we get,

$$\frac{\Delta y}{\Delta x} = \frac{(1+x+\Delta x)^{1/5} - (1+x)^{1/5}}{\Delta x}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{(1+x+\Delta x)^{1/5} - (1+x)^{1/5}}{(1+x+\Delta x)-(1+x)}$$

$$\frac{dy}{dx} = \frac{1}{5}(1+x)^{1/5-1} \qquad \left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{1}{5}(1+x)^{-4/5} = \frac{1}{5(1+x)^{4/5}}$$

23. 2071 Supp. Q.No. 10a

Find the derivative of $\sqrt{1+x^2}$ from the first principles. [4]

SOLUTION

Let $y = \sqrt{1+x^2}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta y = \sqrt{1+(x+\Delta x)^2}$$

$$\text{or, } \Delta y = \sqrt{1+(x+\Delta x)^2} - y$$

$$\text{or, } \Delta y = \frac{\sqrt{1+(x+\Delta x)^2}-\sqrt{1+x^2}}{1} \times \frac{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

$$\text{or, } \Delta y = \frac{1+(x+\Delta x)^2-(1+x^2)}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

$$\text{or, } \Delta y = \frac{1+x^2+2x\cdot\Delta x+(\Delta x)^2-1-x^2}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

$$\text{or, } \Delta y = \frac{\Delta x\,(2x+\Delta x)}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{2x+\Delta x}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{2x+\Delta x}{\sqrt{1+(x+\Delta x)^2}+\sqrt{1+x^2}}$$

$$\frac{dy}{dx} = \frac{2x}{\sqrt{1+x^2}+\sqrt{1+x^2}} = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$$

24. 2071 Set C Q.No. 10a

Find, from definition, the derivative of $\sqrt{\frac{1}{1-x}}$. [4]

SOLUTION

Let $y = \sqrt{\frac{1}{1-x}} = \frac{1}{\sqrt{1-x}}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta x = \frac{1}{\sqrt{1-(x+\Delta x)}}$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{1-(x+\Delta x)}} - y$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{1-x-\Delta x}} - \frac{1}{\sqrt{1-x}}$$

$$\text{or, } \Delta y = \frac{\sqrt{1-x}-\sqrt{1-x-\Delta x}}{\sqrt{1-x-\Delta x}\sqrt{1-x}} \times \frac{\sqrt{1-x}+\sqrt{1-x-\Delta x}}{\sqrt{1-x}+\sqrt{1-x-\Delta x}}$$

$$\text{or, } \Delta y = \frac{1-x-1+x+\Delta x}{\sqrt{1-x}\sqrt{1-x-\Delta x}\left(\sqrt{1-x}+\sqrt{1-x-\Delta x}\right)}$$

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or, $\frac{\Delta y}{\Delta x} = \frac{1}{\sqrt{1-x}\sqrt{1-x-\Delta x}\,(\sqrt{1-x}+\sqrt{1-x-\Delta x})}$

Taking *lim*it $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\left\{\frac{1}{\sqrt{1-x}\sqrt{1-x-\Delta x}\,(\sqrt{1-x}+\sqrt{1-x-\Delta x})}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}\sqrt{1-x}\,(\sqrt{1-x}+\sqrt{1-x})}$$

$$= \frac{1}{(1-x)\cdot 2\sqrt{1-x}} = \frac{1}{2(1-x)^{3/2}}$$

25. 2071 Set D Q.No. 10a

Find from first principles the derivative of $\sqrt{1/x}$. [4]

SOLUTION

Let $y = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta y = \frac{1}{\sqrt{x+\Delta x}}.$$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x}} - y$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}$

or, $\Delta y = \frac{\sqrt{x}-\sqrt{x+\Delta x}}{\sqrt{x+\Delta x}\sqrt{x}} \times \frac{\sqrt{x}+\sqrt{x+\Delta x}}{\sqrt{x}+\sqrt{x+\Delta x}}$

or, $\Delta y = \frac{x-x-\Delta x}{\sqrt{x+\Delta x}\sqrt{x}\,(\sqrt{x}+\sqrt{x+\Delta x})}$

or, $\frac{\Delta y}{\Delta x} = \frac{-1}{\sqrt{x+\Delta x}\sqrt{x}\,(\sqrt{x}+\sqrt{x+\Delta x})}$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\left\{\frac{-1}{\sqrt{x+\Delta x}\sqrt{x}\,(\sqrt{x}+\sqrt{x+\Delta x})}\right\}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x}\cdot\sqrt{x}\,(\sqrt{x}+\sqrt{x})} = \frac{-1}{x\cdot 2\sqrt{x}} = \frac{-1}{2x^{3/2}}$$

26. 2070 Set C Q.No. 10 a

Find from first principles the derivative of $\sqrt{2-3x}$. [4]

➥ Please refer to 2074 Set A Q.No. 10a

27. 2070 Set D Q.No. 10 a

Find from first principles the derivative of $\sqrt{1+x}$. [4]

➥ Please refer to 2073 Set D Q.No. 10a

28. 2069 [Set A] Q. No. 10a

Find from first principles the derivative of $\sqrt{2x+3}$. [4]

➥ Please refer to 2074 Supp Q.No. 10a

29. 2069 [Set B] Q. No. 10a

Find from first principles the derivative of $f(x) = \frac{1}{\sqrt{x+a}}$ [4]

SOLUTION

Here, $f(x) = y = \frac{1}{\sqrt{x+a}}$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$$y + \Delta y = \frac{1}{\sqrt{x+\Delta x+a}}$$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x+a}} - y$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x+a}} - \frac{1}{\sqrt{x+a}}$

or, $\Delta y = \frac{\sqrt{x+a}-\sqrt{x+\Delta x+a}}{\sqrt{x+\Delta x+a}\sqrt{x+a}} \times \frac{\sqrt{x+a}+\sqrt{x+\Delta x+a}}{\sqrt{x+a}+\sqrt{x+\Delta x+a}}$

or, $\Delta y = \frac{x+a-x-\Delta x-a}{\sqrt{x+\Delta x+a}\sqrt{x+a}\,(\sqrt{x+a}+\sqrt{x+\Delta x+a})}$

or, $\frac{\Delta y}{\Delta x} = \frac{-1}{\sqrt{x+\Delta x+a}\sqrt{x+a}\,(\sqrt{x+a}+\sqrt{x+\Delta x+a})}$

Taking *lim*it $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\left\{\frac{-1}{\sqrt{x+\Delta x+a}\sqrt{x+a}\,(\sqrt{x+a}+\sqrt{x+\Delta x+a})}\right\}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x+a}\sqrt{x+a}\,(\sqrt{x+a}+\sqrt{x+a})}$$

$$= \frac{-1}{(x+a)\cdot 2\sqrt{x+a}} = \frac{-1}{2(x+a)^{3/2}}$$

30. 2068 Old Q.No. 13a

Find from first principle, the derivative of $\sqrt{2x+3}$. [4]

➥ Please refer to 2074 Supp Q.No. 10a

31. 2067 Q.No. 13a

Find from definition, the derivative of $\frac{1}{\sqrt{x}}$. [4]

SOLUTION

Let, $y = \frac{1}{\sqrt{x}}$.

Again let, Δx and Δy be the small increments of x & y respectively, then

$$y + \Delta y = \frac{1}{\sqrt{x+\Delta x}}$$

or, $\Delta y = \frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}$

or $\Delta y = \frac{\sqrt{x}-\sqrt{x+\Delta x}}{\sqrt{x}\,(\sqrt{x+\Delta x})} \times \frac{\sqrt{x}+\sqrt{x+\Delta x}}{\sqrt{x}+(\sqrt{x+\Delta x})}$

or, $\Delta y = \frac{x-x-\Delta x}{\sqrt{x}\,(\sqrt{x+\Delta x})\,(\sqrt{x}+\sqrt{x+\Delta x})}$

or, $\frac{\Delta y}{\Delta x} = \frac{-\Delta x}{\Delta x\,(\sqrt{x}\,(\sqrt{x+\Delta x})\,(\sqrt{x}+\sqrt{x+\Delta x}))}$

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$$\text{or, } \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x}(\sqrt{x+\Delta x})(\sqrt{x}+\sqrt{x+\Delta x})}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{x}(\sqrt{x+0})(\sqrt{x}+\sqrt{x+0})}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{2x^{3/2}}$$

32. 2063 Q.No. 12a OR

Find from definition, the derivative of $\frac{1}{\sqrt{x+2}}$ [4]

SOLUTION

Let $y = \frac{1}{\sqrt{x+2}}$

Let Δx and Δy be the small increments in x and y respectively, then

$$y + \Delta y = \frac{1}{\sqrt{x+\Delta x+2}}$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{x+\Delta x+2}} - y$$

$$\Delta y = \frac{1}{\sqrt{x+\Delta x+2}} - \frac{1}{\sqrt{x+2}}$$

$$\text{or, } \Delta y = \frac{\sqrt{x+2}-\sqrt{x+\Delta x+2}}{\sqrt{x+\Delta x+2}\sqrt{x+2}}$$

$$= \frac{\sqrt{x+2}-\sqrt{x+\Delta x+2}}{\sqrt{x+\Delta x+2}\sqrt{x+2}} \times \frac{\sqrt{x+2}+\sqrt{x+\Delta x+2}}{\sqrt{x+2}+\sqrt{x+\Delta x+2}}$$

$$= \frac{x+2-x-\Delta x-2}{\sqrt{x+\Delta x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+\Delta x+2})}$$

$$\Delta y = \frac{-\Delta x}{\sqrt{x+\Delta x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+\Delta x+2})}$$

$$\frac{\Delta y}{\Delta x} = \frac{-1}{\sqrt{x+\Delta x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+\Delta x+2})}$$

By the definition, we have

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x+\Delta x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+\Delta x+2})}$$

$$= \frac{-1}{\sqrt{x+0+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+0+2})}$$

$$= \frac{-1}{\sqrt{x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+2})}$$

$$= \frac{-1}{(x+2)2\sqrt{x+2}}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{2(x+2)^{3/2}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x+2}}\right) = \frac{-1}{2(x+2)^{3/2}}$$

3. 2061 Q.No. 13a

Find $\frac{dy}{dx}$ from first principle when $y = x + \sqrt{x}$. [4]

OLUTION

Here, $y = x + \sqrt{x}$

Let Δx and Δy be the small increments in x and y respectively, then

$$y + \Delta y = (x + \Delta x) + \sqrt{x + \Delta x}$$

$$\text{or, } \Delta y = (x + \Delta x) + \sqrt{x + \Delta x} - y$$

$$\text{or, } \Delta y = x + \Delta x + \sqrt{x + \Delta x} - x - \sqrt{x}$$

$$\therefore \Delta y = \Delta x + \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \Delta x + (\sqrt{x + \Delta x} - \sqrt{x}) \times \left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}\right)$$

$$= \Delta x + \frac{(x + \Delta x - x)}{(\sqrt{x + \Delta x} + \sqrt{x})}$$

$$= \Delta x + \frac{\Delta x}{(\sqrt{x + \Delta x} + \sqrt{x})}$$

$$= \Delta x \left(1 + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}\right)$$

$$\therefore \frac{\Delta y}{\Delta x} = 1 + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

By definition, we have

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(1 + \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}\right)$$

$$= 1 + \frac{1}{\sqrt{x + 0} + \sqrt{x}}$$

$$= 1 + \frac{1}{\sqrt{x} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}$$

34. 2059 Q.No. 13a

Find from the first principles the derivative of $\frac{1}{\sqrt{3x+4}}$. [4]

SOLUTION

Let $y = \frac{1}{\sqrt{3x+4}}$

Let Δx and Δy be the small increments in x and y respectively, then

$$y + \Delta y = \frac{1}{\sqrt{3(x + \Delta x) + 4}}$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{3(x + \Delta x) + 4}} - \frac{1}{\sqrt{3x+4}}$$

$$\Delta y = \frac{\sqrt{3x+4} - \sqrt{3x + 3\Delta x + 4}}{\sqrt{3x + 3\Delta x + 4}\sqrt{3x+4}}$$

$$= \frac{(\sqrt{3x+4} - \sqrt{3x + 3\Delta x + 4})}{\sqrt{3x + 3\Delta x + 4}\sqrt{3x+4}} \times \left(\frac{\sqrt{3x+4} - \sqrt{3x + 3\Delta x + 4}}{\sqrt{3x + 3\Delta x + 4}\sqrt{3x+4}}\right)$$

$$= \frac{3x + 4 - 3x - 3\Delta x - 4}{\sqrt{3x + 3\Delta x + 4}\sqrt{3x+4}} \times \frac{1}{(\sqrt{3x+4} + \sqrt{3x + 3\Delta x + 4})}$$

$$= \frac{-3\Delta x}{\sqrt{3x+3\Delta x+4}\sqrt{3x+4}\,(\sqrt{3x+4}+\sqrt{3x+3\Delta x+4})}$$

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{-3}{\sqrt{3x+3\Delta x+4}\sqrt{3x+4}\,(\sqrt{3x+4}+\sqrt{3x+3\Delta x+4})}$$

We know from definition

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3}{\sqrt{3x+3\Delta x+4}\sqrt{3x+4}\,(\sqrt{3x+4}+\sqrt{3x+3\Delta x+4})}$$

$$= \frac{-3}{\sqrt{3x+0+4}\sqrt{3x+4}\,(\sqrt{3x+4}+\sqrt{3x+0+4})}$$

$$= \frac{-3}{\sqrt{3x+4}\sqrt{3x+4}\,(\sqrt{3x+4}+\sqrt{3x+4})}$$

$$= \frac{-3}{(3x+4)\,2\,(\sqrt{3x+4})} = \frac{-3}{2(3x+4)^{3/2}}$$

$$\therefore \frac{d}{dx}\left(\frac{1}{\sqrt{3x+4}}\right) = \frac{-3}{2\,(3x+4)^{3/2}}$$

35. 2056 Q.No. 13a

Find, from the first principle the derivative of: $y = \frac{1}{\sqrt{ax+b}}$ [4]

SOLUTION

We have, $y = \frac{1}{\sqrt{ax+b}}$

Let Δx and Δy be the small increments in x and y respectively, then

$$y + \Delta y = \frac{1}{\sqrt{a\,(x+\Delta x)+b}}$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{a\,(x+\Delta x)+b}} - y$$

$$\text{or, } \Delta y = \frac{1}{\sqrt{ax+a\Delta x+b}} - \frac{1}{\sqrt{ax+b}}$$

$$= \frac{\sqrt{ax+b}-\sqrt{ax+a\Delta x+b}}{\sqrt{ax+a\Delta x+b}\sqrt{ax+b}}$$

$$= \frac{\sqrt{ax+b}-\sqrt{ax+a\Delta x+b}}{\sqrt{ax+a\Delta x+b}\sqrt{ax+b}} \times \frac{\sqrt{ax+b}+\sqrt{ax+a\Delta x+b}}{\sqrt{ax+b}+\sqrt{ax+a\Delta x+b}}$$

$$= \frac{ax+b-ax-a\Delta x-b}{\sqrt{ax+a\Delta x+b}.\sqrt{ax+b}\,(\sqrt{ax+b}+\sqrt{ax+a\Delta x+b})}$$

$$= \frac{-a\Delta x}{\sqrt{ax+a\Delta x+b}.\sqrt{ax+b}\,(\sqrt{ax+b}+\sqrt{ax+a\Delta x+b})}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-a}{\sqrt{ax+a\Delta x+b}.\sqrt{ax+b}\,(\sqrt{ax+b}+\sqrt{ax+a\Delta x+b})}$$

By definition, we have

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-a}{\sqrt{ax+a\Delta x+b}.\sqrt{ax+b}\,(\sqrt{ax+b}+\sqrt{ax+a\Delta x+b})}$$

$$= \frac{-a}{\sqrt{ax+b}.\sqrt{ax+b}\,(\sqrt{ax+b}+\sqrt{ax+b})}$$

$$= \frac{-a}{(ax+b).2\sqrt{ax+b}}$$

$$\therefore \frac{dy}{dx} = \frac{-a}{2\,(ax+b)^{3/2}}$$

36. 2055 Old Model Q.No. 13a

Find, from the first principles, the derivative of: $\frac{1}{\sqrt{3x+4}}$ [4]

➥ Please refer to 2059 Q.No. 13a

B. DERIVATIVES OF TRIGONOMETRIC AND INVERSE CIRCULAR FUNCTIONS

2 MARKS QUESTIONS

1. 2076 Set C Q.No. 5a

Find $\frac{dy}{dx}$ when $x^2y^2 = \sin(x+y)$. [2]

SOLUTON

Given, $x^2y^2 = \sin(x+y)$

Differentiating both sides w.r. to x

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}\{\sin(x+y)\}$$

$$\text{or, } x^2\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x^2) = \frac{d\{\sin(x+y)\}}{d(x+y)} \cdot \frac{d}{dx}(x+y)$$

$$\text{or, } x^2\frac{d}{dy}(y^2)\frac{dy}{dx} + y^2 \cdot 2x = \cos(x+y)\left\{\frac{d}{dx}(x) + \frac{d}{dx}(y)\right\}$$

$$\text{or, } x^2 \cdot 2y\frac{dy}{dx} + 2xy^2 = \cos(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$\text{or, } 2x^2y\frac{dy}{dx} - \cos(x+y)\frac{dy}{dx} = \cos(x+y) - 2xy^2$$

$$\text{or, } \{2x^2y - \cos(x+y)\}\frac{dy}{dx} = \cos(x+y) - 2xy^2$$

$$\therefore \frac{dy}{dx} = \frac{\cos(x+y) - 2xy^2}{2x^2y - \cos(x+y)}$$

2. 2075 Set B Q.No. 5a

Find $\frac{dy}{dx}$, when x = tant and y = sint cost. [2]

SOLUTION

Here, $x = \tan t$

$$\frac{dx}{dt} = \frac{d}{dt}(\tan t) = \sec^2 t$$

And, $y = \sin t \cos t = \frac{1}{2}(2 \sin t \cos t) = \frac{1}{2}\sin 2t$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{1}{2}\sin 2t\right) = \frac{1}{2} \cdot 2\cos 2t = \cos 2t$$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos 2t}{\sec^2 t} = \cos 2t \cdot \cos^2 t$

3. 2074 Set A Q.No. 5a

Find $\frac{dy}{dx}$ when x = 2a tanθ, y = a secθ. [2]

SOLUTION

Here,

$x = 2a\tan\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(2a\tan\theta) = 2a\frac{d}{d\theta}(\tan\theta) = 2a.\sec^2\theta$$

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Again,
$y = a\sec\theta$
$\frac{dy}{d\theta} = \frac{d}{d\theta}(a\sec\theta) = a\frac{d}{d\theta}(\sec\theta) = a.\sec\theta.\tan\theta$
Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sec\theta\tan\theta}{2a\sec^2\theta}$
$= \frac{\tan\theta}{2\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{2.\frac{1}{\cos\theta}} = \frac{1}{2}\sin\theta$

4. 2074 Set B Q.No. 5a

Find $\frac{dy}{dx}$, when x = a (1 - cost) and y = a(t + sin t). [2]

SOLUTION

Here, $y = a(t + \sin t)$
$\frac{dy}{dt} = a\left\{\frac{d(t)}{dt} + \frac{d(\sin t)}{dt}\right\} = a(1 + \cos t)$
And, $x = a(1 - \cos t)$
$\frac{dx}{dt} = a\left\{\frac{d(1)}{dt} - \frac{d}{dt}(\cos t)\right\} = a(0 + \sin t) = a\sin t$
Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a(1 + \cos t)}{a \cdot \sin t}$
$= \frac{2\cos^2 t/2}{2\sin t/2\cos t/2} = \cot t/2$

5. 2073 Supp Q.No. 5a

Find $\frac{dy}{dx}$ when

x = a(cost + tsint), y = a(sint – tcost). [2]

SOLUTION

Here,
$x = a(\cos t + t\sin t)$
$\frac{dx}{dt} = \frac{d}{dt}\{a(\cos t + t\sin t)\}$
$= a\{\frac{d}{dt}(\cos t) + \frac{d}{dt}(t\sin t)\}$
$= a\{-\sin t + t\frac{d}{dt}(\sin t) + \sin t\frac{d}{dt}(t)\}$
$= a(-\sin t + t.\cos t + \sin t.1)$
$= a(t\cos t) = at\cos t$
And, $y = a(\sin t - t\cos t)$
$\frac{dy}{dt} = \frac{d}{dt}\{a(\sin t - t\cos t)\}$
$= a\{\frac{d}{dt}(\sin t) - \frac{d}{dt}(t\cos t)\}$
$= a\left[\cos t - \left\{t\frac{d}{dt}\cos t + \cos t\frac{d}{dt}(t)\right\}\right]$
$= a[\cos t - \{t.(-\sin t) + \cos t.1\}]$
$= a(\cos t + t\sin t - \cos t)$
$= at\sin t$
Now,
$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at\sin t}{at\cos t} = \tan t$

6. 2072 Supp. Q.No. 5a

Find $\frac{dy}{dx}$ If $x^2 + y^2 = \sin xy$ [2]

SOLUTION

Given, $x^2 + y^2 = \sin xy$
Differentiating both sides with respect to x, we get.
$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(\sin xy)$
or, $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d(\sin xy)}{d(xy)} \cdot \frac{d(xy)}{dx}$
or, $2x + \frac{d(y^2)}{dy} \cdot \frac{dy}{dx} = \cos xy \cdot \left\{x\frac{d(dy)}{dx} + y\frac{d}{dx}(x)\right\}$
or, $2x + 2y\frac{dy}{dx} = \cos xy\left(x\frac{dy}{dx} + y\right)$
or, $2x + 2y\frac{dy}{dx} = \cos xy\frac{dy}{dx} + y\cos xy$
or, $2x - y\cos xy = x\cos xy\frac{dy}{dx} - 2y\frac{dy}{dx}$
or, $(x\cos xy - 2y)\frac{dy}{dx} = 2x - y\cos xy$
$\therefore \frac{dy}{dx} = \frac{2x - y\cos xy}{x\cos xy - 2y}$

7. 2072 Set D Q.No. 5a

Find $\frac{dy}{dx}$ when $y = \sec^2(\tan\sqrt{x})$. [2]

SOLUTION

Here, $y = \sec^2(\tan\sqrt{x})$
Differentiating both sides with respect to 'x', we get
$\frac{dy}{dx} = \frac{d}{dx}\{\sec^2(\tan\sqrt{x})\}$
$= \frac{d\{\sec(\tan\sqrt{x})\}^2}{d\{\sec(\tan\sqrt{x})\}} \cdot \frac{d\{\sec(\tan\sqrt{x})\}}{d(\tan\sqrt{x})} \cdot \frac{d(\tan\sqrt{x})}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$
$= 2\sec(\tan\sqrt{x}) \cdot \sec(\tan\sqrt{x})\tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$
$= \frac{1}{\sqrt{x}}\sec^2(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x}$

8. 2071 Set C Q.No. 5a

Find $\frac{dy}{dx}$ when $x + y = \sin(x + y)$. [2]

SOLUTION

Here, $x + y = \sin(x + y)$
Differentiating both sides with respect to 'x', we get
$\frac{d}{dx}(x + y) = \frac{d}{dx}\{\sin(x + y)\}$
or, $\frac{d}{dx}(x) + \frac{d}{dx}(y) = \frac{d\{\sin(x + y)\}}{d(x + y)} \cdot \frac{d(x + y)}{dx}$
or, $1 + \frac{dy}{dx} = \cos(x + y)\left\{\frac{d(x)}{dx} + \frac{d(y)}{dx}\right\}$
or, $1 + \frac{dy}{dx} = \cos(x + y)\left(1 + \frac{dy}{dx}\right)$
or, $1 + \frac{dy}{dx} = \cos(x + y) + \cos(x + y)\frac{dy}{dx}$
or, $1 - \cos(x + y) = -\frac{dy}{dx} + \cos(x + y)\frac{dy}{dx}$

or, $\{\cos(x+y)-1\}\frac{dy}{dx} = 1-\cos(x+y)$

or, $\frac{dy}{dx} = \frac{-\{\cos(x+y)-1\}}{\cos(x+y)-1} = -1$

9. 2071 Set D Q.No. 5a

Find the derivative of tanx with respect to cotx [2]

SOLUTION

Derivative of $\tan x$ with respect to $\cot x$ is

$$\frac{d(\tan x)}{d(\cot x)} = \frac{\frac{d(\tan x)}{dx}}{\frac{d(\cot x)}{dx}} = \frac{\sec^2 x}{-\operatorname{cosec}^2 x} = -\frac{\sin^2 x}{\cos^2 x} = -\tan^2 x$$

10. 2070 Set D Q.No. 5 a

Find $\frac{dy}{dx}$ when $x - y = \tan xy$. [2]

SOLUTION

Here, $x - y = \tan xy$

Differentiating both sides with respect to 'x', we get

$$\frac{d}{dx}(x-y) = \frac{d}{dx}(\tan xy)$$

or, $\frac{d(x)}{dx} - \frac{d(y)}{dx} = \frac{d(\tan xy)}{d(xy)} \cdot \frac{d(xy)}{dx}$

or, $1 - \frac{dy}{dx} = \sec^2(xy)\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right)$

or, $1 - \frac{dy}{dx} = \sec^2(xy)\left(x\frac{dy}{dx} + y\right)$

or, $1 - \frac{dy}{dx} = x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy)$

or, $1 - y\sec^2(xy) = x\sec^2(xy)\frac{dy}{dx} + \frac{dy}{dx}$

or, $1 - y\sec^2(xy) = \frac{dy}{dx}\{x\sec^2(xy) + 1\}$

$\therefore \frac{dy}{dx} = \frac{1 - y\sec^2(xy)}{x\sec^2(xy) + 1}$

11. 2070 Old Q.No. 5 c

Find $\frac{dx}{dy}$ when $y = \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)$. [2]

SOLUTION

Let $y = \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)$

$= \tan^{-1}\left(\frac{2\sin x\cos x}{1+2\cos^2 x - 1}\right)$

$= \tan^{-1}\left(\frac{\sin x}{\cos x}\right) = \tan^{-1}(\tan x) = x$

Differentiating both sides w.r. to 'x' $\frac{dy}{dx} = \frac{dx}{dx} = 1$

12. 2069 [Set A] Old Q. No. 5c

Find $\frac{dy}{dx}$, when $x = a(t + \sin t)$, $y = a(1 - \cos t)$. [2]

SOLUTION

Here, $x = a(t + \sin t)$

$$\frac{dx}{dt} = a\left\{\frac{d(t)}{dt} + \frac{d(\sin t)}{dt}\right\} = a(1 + \cos t)$$

And, $y = a(1 - \cos t)$

$$\frac{dy}{dt} = a\left\{\frac{d(1)}{dt} - \frac{d}{dt}(\cos t)\right\} = a(0 + \sin t) = a\sin t$$

Now,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{a\cdot\sin t}{a(1+\cos t)} = \frac{2\sin t/2\cos t/2}{2\cos^2 t/2} = \tan t/2$$

13. 2069 [Set B] Q. No. 5a

Find $\frac{dy}{dx}$ when $y = \frac{1}{\sec x - \tan x}$. [2]

SOLUTION

$$y = \frac{1}{\sec x - \tan x} \times \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} = \sec x + \tan x$$

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{d}{dx}(\sec x) + \frac{d}{dx}(\tan x)$$

$$= \sec x\tan x + \sec^2 x$$

$$= \sec x(\tan x + \sec x)$$

14. 2069 [Set B] Old Q. No. 5b

Find $\frac{dy}{dx}$ of $y = 2\theta - \tan\theta$ and $x = \tan\theta$ [2]

SOLUTION

Here, $y = 2\theta - \tan\theta$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(2\theta - \tan\theta) = 2 - \sec^2\theta$$

Again, $x = \tan\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\tan\theta) = \sec^2\theta$$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2 - \sec^2\theta}{\sec^2\theta}$

$$= \frac{2}{\sec^2\theta} - 1 = 2\cos^2\theta - 1 = \cos 2\theta$$

15. 2068 Old Q.No. 5b

Find $\frac{dy}{dx}$ when $x = a\cos^2\theta$ and $y = b\sin^2\theta$. [2]

SOLUTION

Here, $x = a\cos^2\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos^2\theta)$$

$$= a\frac{d(\cos\theta)^2}{d(\cos\theta)} \cdot \frac{d(\cos\theta)}{d\theta}$$

$$= a\cdot 2\cos\theta\cdot(-\sin\theta)$$

$$= a(-\sin 2\theta) = -a\sin 2\theta$$

and $y = b\sin^2\theta$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\sin^2\theta) = b\frac{d(\sin\theta)^2}{d(\sin\theta)} \cdot \frac{d(\sin\theta)}{d\theta}$$

$$= b\cdot 2\sin\theta\cdot\cos\theta = b\sin 2\theta$$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\sin 2\theta}{-a\sin 2\theta} = \frac{-b}{a}$

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16. 2067 Q.No. 5c

Find the derivative of $\tan^{-1}\frac{\sin 2x}{1+\cos 2x}$ [2]

➥ Please refer to 2070 Old Q.No. 5 c

17. 2065 Q.No. 5c

Find $\frac{dy}{dx}$, when $y = \sin\theta$ and $\theta = 5x^2 - 6x + 2$. [2]

SOLUTION

$y = \sin\theta$ (i)

$\theta = 5x^2 - 6x + 2$ (ii)

Now, differentiating equation (i) w.r.t θ and differentiating equation (ii) w.r.t. x, we get

$\frac{dy}{dv} = \frac{d}{d\theta}\sin\theta = \cos\theta = \cos(5x^2 - 6x + 2)$

[∵ using (ii)]

And $\frac{dv}{dx} = \frac{d}{dx}(5x^2 - 6x + 2) = 10x - 6$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \cos(5x^2 - 6x + 2) \times (10x - 6)$

$= (10x - 6)\cos(5x^2 - 6x + 2)$.

Alternatively,

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{d}{d\theta}\sin\theta \times \frac{d}{dx}(5x^2 - 6x + 2)$

$= \cos\theta \times (10x - 6)$

$= (10x - 6)\cos\theta$)

18. 2064 Q.No. 5b

Find $\frac{dy}{dx}$ where $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ [2]

SOLUTION

We have, $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Suppose, $x = \tan\theta$ ... (i)

$\therefore y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

or, $y = \tan^{-1}(\tan 2\theta)$

$\therefore y = 2\theta$ ... (ii)

From (i)

$x = \tan\theta$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}(\tan\theta) = \sec^2\theta$

and from (ii) $y = 2\theta$

$\therefore \frac{dy}{d\theta} = \frac{d}{d\theta}(2\theta) = 2$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2}{\sec^2\theta} = \frac{2}{1+\tan^2\theta} = \frac{2}{1+x^2}$

Alternatively,

We have,

$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ... (i)

Suppose, $x = \tan\theta$ ... (ii)

Now, (i) becomes

$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \tan^{-1}(\tan 2\theta)$

or, $y = 2\theta$

or, $y = 2\tan^{-1}x$ [∵ from (ii)]

Differentiating both sides with respect to x,

$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x)$

$= 2\frac{d}{dx}(\tan^{-1}x) = 2\left(\frac{1}{1+x^2}\right)$

Hence, $\frac{dy}{dx} = \frac{2}{1+x^2}$

19. 2063 Q.No. 5b

Find $\frac{dy}{dx}$ when $y = \sin^{-1}(3x - 4x^3)$ [2]

SOLUTION

We have, $y = \sin^{-1}(3x - 4x^3)$

Put, $x = \sin\theta$ ... (i)

Then, $y = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

or, $y = \sin^{-1}(\sin 3\theta)$

$\therefore y = 3\theta$ ... (ii)

From (i),

$\frac{dx}{d\theta} = \frac{d}{d\theta}(\sin\theta) = \cos\theta$

From (ii),

$\frac{dy}{d\theta} = \frac{d}{d\theta}(3\theta) = 3$

Now, by Chain Rule,

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3}{\cos\theta} = \frac{3}{\sqrt{1-\sin^2\theta}} = \frac{3}{\sqrt{1-x^2}}$ [∵ Using (i)]

$\therefore \frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$

20. 2062 Q.No. 5c

Find $\frac{dy}{dx}$ if $y = \tan^{-1}\frac{2x}{1-x^2}$ [2]

SOLUTION

We have,

$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put, $x = \tan\theta$ ... (i)

So, $\frac{2x}{1-x^2} = \frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

$\therefore y = \tan^{-1}(\tan 2\theta) = 2\theta$ ... (ii)

Now, differentiating (i) w.r.to θ, we get

$\frac{dx}{d\theta} = \frac{d}{d\theta}(\tan\theta) = \sec^2\theta$

And, differentiating (ii) w.r. to θ, we get

$\frac{dy}{d\theta} = \frac{d}{d\theta}(2\theta) = 2$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2}{\sec^2\theta}$

$= \frac{2}{1+\tan^2\theta} = \frac{2}{1+x^2}$ [∵ Using (i)]

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21. 2061 Q.No. 5c

Differentiate sin x with respect to tan x [2]

SOLUTION

Let $y = \sin x$ ... (i)

and $u = \tan x$ ... (ii)

Then, we have to find $\frac{d(\sin x)}{d(\tan x)}$ i.e. $\frac{dy}{du}$

Now, from (i)

$y = \sin x$

$\frac{dy}{dx} = \frac{d}{dx}\sin x = \cos x$

and from (ii) $\frac{du}{dx} = \frac{d}{dx}\tan x = \sec^2 x$

$\therefore \frac{dy}{du} = \frac{dy/dx}{du/dx} = \frac{\cos x}{\sec^2 x} = \cos^3 x$

Hence, $\frac{d \sin x}{d \tan x} = \cos^3 x$

Alternatively,

$$\frac{d(\sin x)}{d(\tan x)} = \frac{\frac{d(\sin x)}{dx}}{\frac{d(\tan x)}{dx}} = \frac{\cos x}{\sec^2 x}$$

$= \cos x . \cos^2 x = \cos^3 x$

22. 2060 Q.No. 5c

Find $\frac{dy}{dx}$ when $x = 2a \tan\theta$ and $y = a \sec^2\theta$. [2]

SOLUTION

We have,

$x = 2a \tan\theta$ ... (1)

$y = a \sec^2\theta$ ... (2)

From (1),

$\frac{dx}{d\theta} = \frac{d}{d\theta}(2a \tan\theta) = 2a \sec^2\theta$

From (2),

$\frac{dy}{d\theta} = \frac{d}{d\theta}(a \sec^2\theta)$

$= a\frac{d}{d\theta}(\sec^2\theta)$

$= a \times \frac{d(\sec^2\theta)}{d(\sec\theta)} \times \frac{d(\sec\theta)}{d\theta}$

$= a \times 2 \sec\theta \times \sec\theta \tan\theta$

$= 2a \sec^2\theta \tan\theta$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2a \sec^2\theta \tan\theta}{2a \sec^2\theta} = \tan\theta$

$\therefore \frac{dy}{dx} = \tan\theta$

23. 2059 Q.No. 5c

Find $\frac{dy}{dx}$ of $x = a \sin t$, $y = a \cos t$. [2]

SOLUTION

We have,

$x = a \sin t$ ... (i)

$y = a \cos t$ ... (ii)

From (i), we get

$\frac{dx}{dt} = \frac{d}{dt}(a \sin t) = a \cos t$

From (ii), we get

$\frac{dy}{dt} = \frac{d}{dt}(a \cos t) = -a \sin t$

Now by Chain Rule,

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-a \sin t}{a \cos t} = -\tan t$

24. 2058 Q.No. 5c

Find $\frac{dy}{dx}$ if $x = a \sin t$, $y = a \cos t$ [2]

➥ Please refer to 2059 Q.No. 5c

4 MARKS QUESTIONS

25. 2075 Set A Q.No. 10a

Find, from the first principles, the differential coefficient of the function $\sin^2(3x - 5)$. [4]

SOLUTION

Let $y = \sin^2(3x - 5)$

Also let Δx and Δy be the small increments of x and y respectively. Then,

$y + \Delta y = \sin^2\{3(x + \Delta x) - 5\}$

or, $\Delta y = \sin^2(3x - 3\Delta x - 5) - y$

or, $\Delta y = \sin^2(3x + 3\Delta x - 5) - \sin^2(3x - 5)$

or, $\Delta y = \{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\}\{\sin(3x + 3\Delta x - 5) - \sin(3x - 5)\}$

or, $\Delta y = \{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\}.2 \cos\left(\frac{3x + 3\Delta x - 5 + 3x - 5}{2}\right).\sin\left(\frac{3x + 3\Delta x - 5 - 3x + 5}{2}\right)$

or, $\Delta y = \{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\} \cdot 2 \cos\left(3x - 5 + \frac{3\Delta x}{2}\right).\sin\left(\frac{3\Delta x}{2}\right)$

Dividing both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{\{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\}.2 \cos\left(3x - 5 + \frac{3\Delta x}{2}\right).\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Taking limit $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left[\frac{\{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\}.2 \cos\left(3x - 5 + \frac{3\Delta x}{2}\right).\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}\right]$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left[\{\sin(3x + 3\Delta x - 5) + \sin(3x - 5)\} \cdot 2 \cos\left(3x - 5 + \frac{3\Delta x}{2}\right)\frac{\sin\left(\frac{3\Delta x}{2}\right)}{\frac{3\Delta x}{2}} \times \frac{3}{2}\right]$$

$= \{\sin(3x - 5) + \sin(3x - 5)\}.3 \cos(3x - 5).1$

$= 2 \sin(3x - 5).3 \cos(3x - 5)$

$= 3\{2 \sin(3x - 5) \cos(3x - 5)\}$

$= 3 \sin 2(3x - 5)$

26. 2075 Set B Q.No. 10a

Find, from the first principles the derivative of $\cos^2 x$. [4]

SOLUTION

Let $y = \cos^2 x$

Then $y + \Delta y = \cos^2(x + \Delta x)$

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where Δx and Δy are the corresponding increments in x and y respectively

Now, $\Delta y = \cos^2(x+\Delta x) - \cos^2 x$

or, $\Delta y = [\cos(x+\Delta x) - \cos x][\cos(x+\Delta x) + \cos x]$

or, $\Delta y = 2\sin\left(\frac{x+\Delta x+x}{2}\right)\cdot\sin\left(\frac{x-x-\Delta x}{2}\right) \times 2\cos\left(\frac{x+\Delta x+x}{2}\right)\cos\left(\frac{x+\Delta x-x}{2}\right)$

$= 4\sin\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{-\Delta x}{2}\right)\times\cos\left(\frac{2x+\Delta x}{2}\right)\cos\left(\frac{\Delta x}{2}\right)$

$= -4\sin\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)\times\cos\left(\frac{2x+\Delta x}{2}\right)\cos\left(\frac{\Delta x}{2}\right)$

$= -2\sin\left(\frac{2x+\Delta x}{2}\right)\cos\left(\frac{2x+\Delta x}{2}\right)\times 2\sin\left(\frac{\Delta x}{2}\right)\cos\left(\frac{\Delta x}{2}\right)$

$= -\sin 2\left(\frac{2x+\Delta x}{2}\right)\times\sin 2\left(\frac{\Delta x}{2}\right)$

$= -\sin(2x+\Delta x)\sin\Delta x$

Dividing both sides by Δx, we get

$\frac{\Delta y}{\Delta x} = \frac{-\sin(2x+\Delta x).\sin\Delta x}{\Delta x}$

$\therefore \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{-\sin(2x+\Delta x).\sin\Delta x}{\Delta x}$

or, $\frac{dy}{dx} = -\lim_{\Delta x\to 0}\sin(2x+\Delta x)\times\left(\frac{\sin\Delta x}{\Delta x}\right)$

or, $\frac{dy}{dx} = -\sin 2x \times 1$

$\therefore \frac{d}{dx}(\cos^2 x) = -\sin 2x$

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27. 2073 Supp Q.No. 10a

Find from first principles, the derivative of $\sqrt{\sin 2x}$. [4]

SOLUTION

Let $y = \sqrt{\sin 2x}$

Let, Δx be the small increments in x and Δy be the corresponding increments in y,

then

$y + \Delta y = \sqrt{\sin 2(x+\Delta x)} = \sqrt{\sin(2x+2\Delta x)}$

or, $\Delta y = \sqrt{\sin(2x+2\Delta x)} - y$

$\therefore \Delta y = \sqrt{\sin(2x+2\Delta x)} - \sqrt{\sin 2x}$

or, $\Delta y = \left(\sqrt{\sin(2x+2\Delta x)} - \sqrt{\sin 2x}\right)\times\frac{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}$

$= \frac{\sin(2x+2\Delta x) - \sin 2x}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}$

$= \frac{2\cos\left(\frac{2x+2\Delta x+2x}{2}\right)\cdot\sin\left(\frac{2x+2\Delta x-2x}{2}\right)}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}$

$= \frac{2\cos\left(\frac{4x+2\Delta x}{2}\right)\cdot\sin\Delta x}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}$

$= \frac{2\cos(2x+\Delta x)\cdot\sin\Delta x}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}$

$\therefore \frac{\Delta y}{\Delta x} = \frac{2\cos(2x+\Delta x)}{\left(\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}\right)}\times\left(\frac{\sin\Delta x}{\Delta x}\right)$

$= 2\cos(2x+\Delta x)\times\frac{1}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}\times\left(\frac{\sin\Delta x}{\Delta x}\right)$

By the definition, we know that

$\frac{dy}{dx} = \lim_{\Delta x\to 0}\left(\frac{\Delta y}{\Delta x}\right)$

$= \lim_{\Delta x\to 0} 2\cos(2x+\Delta x)\times \frac{1}{\sqrt{\sin(2x+2\Delta x)} + \sqrt{\sin 2x}}\times\left(\frac{\sin\Delta x}{\Delta x}\right)$

$= 2\cos 2x \times \frac{1}{\sqrt{\sin 2x} + \sqrt{\sin 2x}}\times 1$

$= \frac{2\cos 2x}{2\sqrt{\sin 2x}} = \frac{\cos 2x}{\sqrt{\sin 2x}}$

$\therefore \frac{d}{dx}\left(\sqrt{\sin 2x}\right) = \frac{\cos 2x}{\sqrt{\sin 2x}}$

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28. 2073 Set C Q.No. 10a

Find from first principles the derivative of $f(x) = \sqrt{\sin 2x}$. [4]

➥ Please refer to 2073 Supp Q.No. 10a

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29. 2072 Set C Q.No. 10a

Find from the first principles, the derivative of $\tan(3x-4)$. [4]

SOLUTION

Let $y = \tan(3x-4)$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \tan\{3(x+\Delta x) - 4\}$

or, $\Delta y = \tan(3x+3\Delta x-4) - y$

or, $\Delta y = \tan(3x+3\Delta x-4) - \tan(3x-4)$

or, $\Delta y = \frac{\sin(3x+3\Delta x-4)}{\cos(3x+3\Delta x-4)} - \frac{\sin(3x-4)}{\cos(3x-4)}$

or, $\Delta y = \frac{\sin(3x+3\Delta x-4)\cos(3x-4) - \cos(3x+3\Delta x-4)\sin(3x-4)}{\cos(3x+3\Delta x-4)\cos(3x-4)}$

or, $\Delta y = \frac{\sin(3x+3\Delta x-4-3x+4)}{\cos(3x+3\Delta x-4)\cos(3x-4)}$

or, $\Delta y = \frac{\sin(3\Delta x)}{\cos(3x+3\Delta x-4)\cos(3x-4)}$

Dividing both sides by Δx, we get

$\frac{\Delta y}{\Delta x} = \frac{\sin(3\Delta x)}{\Delta x\cos(3x+3\Delta x-4)\cos(3x-4)}$

Taking *lim*it $\Delta x\to 0$ on both sides, we get

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\left\{\frac{\sin 3\Delta x}{3\Delta x}\cdot 3\cdot\frac{1}{\cos(3x+3\Delta x-4)\cos(3x-4)}\right\}$

$\frac{dy}{dx} = 1\cdot 3\cdot\frac{1}{\cos(3x-4)\cdot\cos(3x-4)}$

$= 3\sec^2(3x-4)$

---

30. 2072 Set D Q.No. 10a

Find from the first principles, the derivation of $\sin x$. [4]

SOLUTION

Let $y = \sin x$

Also, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sin(x + \Delta x)$

or, $\Delta y = \sin(x + \Delta x) - y$

or, $\Delta y = \sin(x + \Delta x) - \sin x$

Dividing both sides by Δx, we get,

$$\frac{\Delta y}{\Delta x} = \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$$

$$= \frac{2\cos\left(\frac{x + \Delta x + x}{2}\right)\sin\left(\frac{x + \Delta x - x}{2}\right)}{\Delta x}$$

$$= \frac{2\cos\left(x + \frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}}{\Delta x}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left\{2\cos\left(x + \frac{\Delta x}{2}\right)\cdot\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}\times 2}\right\}$$

$$\frac{dy}{dx} = \cos(x + 0)\cdot 1 = \cos x$$

31. 2070 Supp Q.No. 10a

Find from first principles, the derivative of cos2x. [4]

SOLUTION

Let $y = \cos 2x$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \cos 2(x + \Delta x)$

or, $\Delta y = \cos 2(x + \Delta x) - \cos 2x$

or, $\Delta y = -2\sin\left(\frac{2x + 2\Delta x + 2x}{2}\right)\cdot\sin\left(\frac{2x + 2\Delta x - 2x}{2}\right)$

or, $\Delta y = -2\sin(2x + \Delta x)\cdot\sin\Delta x$

Dividing both sides by Δx and taking *lim*it $\Delta x \to 0$, we get

$$\lim_{\Delta \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left\{-2\sin(2x + \Delta x)\cdot\frac{\sin\Delta x}{\Delta x}\right\}$$

$$\frac{dy}{dx} = -2\sin 2x\cdot 1 = -2\sin 2x$$

32. 2070 Old Q.No. 13 a Or

Find $\frac{dy}{dx}$ if $xy = \tan(x^2 + y^2)$. [4]

SOLUTION

Here, $xy = \tan(x^2 + y^2)$

Differentiating both sides with respect to 'x'

$$\frac{d}{dx}(xy) = \frac{d}{dx}\{\tan(x^2 + y^2)\}$$

or, $x\frac{dy}{dx} + y\frac{dx}{dx} = \frac{d\{\tan(x^2 + y^2)\}}{d(x^2 + y^2)}\cdot\frac{d(x^2 + y^2)}{dx}$

or, $x\frac{dy}{dx} + y = \sec^2(x^2 + y^2)\cdot\left\{\frac{d(x)^2}{dx} + \frac{d(y^2)}{dy}\cdot\frac{dy}{dx}\right\}$

or, $x\frac{dy}{dx} + y = \sec^2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right)$

or, $x\frac{dy}{dx} + y = 2x\sec^2(x^2 + y^2) + 2y\sec^2(x^2 + y^2)\cdot\frac{dy}{dx}$

or, $y - 2x\sec^2(x^2 + y^2) = 2y\sec^2(x^2 + y^2)\frac{dy}{dx} - x\frac{dy}{dx}$

or, $\{2y\sec^2(x^2 + y^2) - x\}\frac{dy}{dx} = y - 2x\sec^2(x^2 + y^2)$

$\therefore \frac{dy}{dx} = \frac{y - 2x\sec^2(x^2 + y^2)}{2y\sec^2(x^2 + y^2) - x}$

33. 2069 Supp Q.No. 10 a

Find from first principles the derivative of sec x. [4]

SOLUTION

Let $y = \sec x$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sec(x + \Delta x)$

or, $\Delta y = \sec(x + \Delta x) - y$

or, $\Delta y = \sec(x + \Delta x) - \sec x$

or, $\Delta y = \frac{1}{\cos(x + \Delta x)} - \frac{1}{\cos x}$

or, $\Delta y = \frac{\cos x - \cos(x + \Delta x)}{\cos(x + \Delta x)\cos x}$

or, $\Delta y = \frac{2\sin\left(\frac{x + \Delta x + x}{2}\right)\cdot\sin\left(\frac{x + \Delta x - x}{2}\right)}{\cos(x + \Delta x)\cos x}$

or, $\Delta y = \frac{2\sin\left(x + \frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}}{\cos(x + \Delta x)\cos x}$

Dividing both sides by Δx and taking *lim*it $\Delta x \to 0$ on both sides, we get,

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left\{\frac{2\sin\left(x + \frac{\Delta x}{2}\right)}{\cos(x + \Delta x)\cos x}\cdot\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}\cdot 2}\right\}$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos x\cdot\cos x} = \tan x\cdot\sec x$$

34. 2069 [Set A] Old Q. No. 13a

Find $\frac{dy}{dx}$ of $y = \sin 2x$ from first principles. [4]

SOLUTION

Let, $y = \sin 2x$

Let, Δx and Δy be the small increments in x and y respectively. Then

$y + \Delta y = \sin 2(x + \Delta x) = \sin(2x + 2\Delta x)$

or, $\Delta y = \sin 2(x + \Delta x) - y$

or, $\Delta y = \sin(2x + 2\Delta x) - \sin 2x$

or, $\Delta y = 2\cos\left(\frac{2x + 2\Delta x + 2x}{2}\right)\sin\left(\frac{2x + 2\Delta x - 2x}{2}\right)$

$= \cos(2x + \Delta x)\sin\Delta x$

Dividing both sides by Δx, we get

$\frac{\Delta y}{\Delta x} = 2\cos(2x + \Delta x)\left(\frac{\sin\Delta x}{\Delta x}\right)$

Now, from the definition

$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right)$

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$$= \lim_{\Delta x \to 0} 2\cos(2x + \Delta x)\left(\frac{\sin \Delta x}{\Delta x}\right)$$
$$= 2\cos(2x + 0) \times 1$$
$$= 2\cos 2x$$

$$\therefore \frac{d}{dx}(\sin 2x) = 2\cos 2x$$

35. 2069 [Set B] Old Q. No. 12b OR

Find from definition the derivative of $\sin^2 3x$. [4]

SOLUTION

Let $y = \sin^2 3x$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \sin^2 3(x + \Delta x)$

or, $\Delta y = \sin^2(3x + 3\Delta x) - \sin^2 3x$

or, $\Delta y = \{\sin(3x+3\Delta x) + \sin 3x\}\{\sin(3x+3\Delta x) - \sin 3x\}$

Dividing both sides by Δx, we get

$$\frac{\Delta y}{\Delta x} = \frac{\{\sin(3x+3\Delta x) + \sin 3x\}\, 2\cos\left(\frac{3x+3\Delta x+3x}{2}\right)\sin\left(\frac{3x+3\Delta x-3x}{2}\right)}{\Delta x}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left[\{\sin(3x+3\Delta x) + \sin 3x\} \cdot 2\cos\left(3x + \frac{3\Delta x}{2}\right) \cdot \frac{\sin\frac{3\Delta x}{2}}{\frac{3\Delta x}{2}} \cdot \frac{2}{3}\right]$$

$$\frac{dy}{dx} = (\sin 3x + \sin 3x) \cdot 3\cos 3x \cdot 1$$
$$= 2\sin 3x \cdot 3\cos 3x$$
$$= 3(2\sin 3x \cos 3x)$$
$$= 3\sin(2 \cdot 3x) = 3\sin 6x$$

36. 2068 Q.No. 10a

Find from first principle, the derivative of $\sin 4x$.[4]

SOLUTION

Let $y = \sin 4x$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y + \sin 4(x + \Delta x)$

or, $\Delta y = \sin(4x + 4\Delta x) - \sin 4x$

Dividing both sides by Δx, we get,

$$\text{or, } \frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{4x+4\Delta x+4x}{2}\right) \cdot \sin\left(\frac{4x+4\Delta x-4x}{2}\right)}{\Delta x}$$

$$= \frac{2\cos(4x+2\Delta x)\sin(2\Delta x)}{\Delta x}$$

Taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left\{2\cos(4x+2\Delta x) \cdot \frac{\sin 2\Delta x}{2\Delta x} \cdot 2\right\}$$

$$\frac{dy}{dx} = 2\cos 4x \cdot 1 \cdot 2 = 4\cos 4x$$

37. 2066 Q.No. 13a

Find from first principles the derivatives of *sin* 2x.[4]

SOLUTION

Let $y = \sin 2x$

Let Δx and Δy be the small increments in x and y respectively, then

$y + \Delta y = \sin 2(x + \Delta x)$

or, $\Delta y = \sin 2(x + \Delta x) - y$

or, $\Delta y = \sin 2(x + \Delta x) - \sin 2x$

$$\text{or, } \Delta y = 2\cos\left(\frac{2x + 2\Delta x + 2x}{2}\right) \cdot \sin\left(\frac{2x + 2\Delta x - 2x}{2}\right)$$

$$\text{or, } \Delta y = 2\cos\left(\frac{4x + 2\Delta x}{2}\right)\sin(\Delta x)$$

Dividing both sides by Δx

$$\frac{\Delta y}{\Delta x} = 2\cos(2x + \Delta x)\frac{\sin(\Delta x)}{\Delta x}$$

By definition of derivative,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 2\cos(2x + \Delta x)\frac{\sin(\Delta x)}{\Delta x}$$
$$= 2\cos 2x$$

38. 2065 Q.No. 13a

Find from first principles, the derivative of $\sqrt{\sin 2x}$. [4]

➥ Please refer to 2073 Supp Q.No. 10a

39. 2064 Q.No. 12b OR

Find from definition the derivative of $\cos^2 x$. [4]

➥ Please refer to 2075 Set B Q.No. 10a

40. 2062 Q.No. 13a

Find from definition, the derivative of $\sqrt{\tan x}$.

SOLUTION

We have $y = \sqrt{\tan x}$

Let Δx and Δy be the small increments in x and y respectively, then

$\Delta y = \sqrt{\tan(x + \Delta x)}$

$\Delta y = \sqrt{\tan(x + \Delta x)} - y$

or, $\Delta y = \sqrt{\tan(x + \Delta x)} - \sqrt{\tan x}$

$$= \left(\sqrt{\tan(x+\Delta x)} - \sqrt{\tan x}\right) \times \frac{\left(\sqrt{\tan(x+\Delta x)} + \sqrt{\tan x}\right)}{\left(\sqrt{\tan(x+\Delta x)} + \sqrt{\tan x}\right)}$$

$$= \frac{\tan(x + \Delta x) - \tan x}{\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}}$$

$$= \frac{\frac{\sin(x+\Delta x)}{\cos(x+\Delta x)} - \frac{\sin x}{\cos x}}{\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}}$$

$$= \frac{\sin(x + \Delta x)\cos x - \cos(x + \Delta x)\sin x}{\cos(x + \Delta x)\cos x\left(\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}\right)}$$

$$= \frac{\sin(x + \Delta x - x)}{\cos(x + \Delta x)\cos x\left(\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}\right)}$$

$$\Delta y = \frac{\sin \Delta x}{\cos(x + \Delta x)\cos x\left(\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}\right)}$$

Dividing both sides by Δx, we get

$$\frac{\Delta y}{\Delta x} = \frac{\sin \Delta x}{\Delta x}\frac{1}{\cos(x + \Delta x)\cos x\left(\sqrt{\tan(x + \Delta x)} + \sqrt{\tan x}\right)}$$

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We know, from definition

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \times \frac{1}{\cos(x+\Delta x)\cos x\left(\sqrt{\tan(x+\Delta x)}+\sqrt{\tan x}\right)}$$

$$= 1 \times \frac{1}{\cos(x+0)\cos x\left(\sqrt{\tan(x+0)}+\sqrt{\tan x}\right)}$$

$$= \frac{1}{\cos x \cos x\left(\sqrt{\tan x}+\sqrt{\tan x}\right)}$$

$$= \frac{1}{\cos^2 x \cdot 2\sqrt{\tan x}} = \frac{1}{\cos^2 x} \cdot \frac{1}{2\sqrt{\tan x}}$$

$$\therefore \frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$$

41. 2060 Q.No. 13a

Find $\frac{dy}{dx}$ from first principles of $y = \sqrt{\tan x}$ [4]

➥ Please refer to 2062 Q.No. 13a

42. 2058 Q.No. 13a

Find from definition, the derivatives of sin 2x. [4]

➥ Please refer to 2069 [Set A] Old Q. No. 13a

43. 2057 Q.No. 13a

Find from the first principle the derivative of $y = \sqrt{\sin 2x}$ [4]

➥ Please refer to 2073 Supp Q.No. 10a

C. DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

2 MARKS QUESTIONS

1. 2075 Set A Q.No. 5a

Find the derivative of $\ln(\ln\sqrt{x})$. [2]

SOLUTION

Let $y = \ln(\ln\sqrt{x})$

Differentiating both sides w.r.to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\{\ln(\ln\sqrt{x})\}$$

$$= \frac{d\{\ln(\ln\sqrt{x})\}}{d(\ln\sqrt{x})} \cdot \frac{d(\ln\sqrt{x})}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$$

$$= \frac{1}{\ln\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{\ln\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x \ln\sqrt{x}}$$

2. 2075 Set C Q.No. 5a

Find the derivative of $e^{\sqrt{\cos x}}$. [2]

SOLUTION

Differentiating both sides w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sqrt{\cos x}}\right)$$

$$= \frac{d\left(e^{\sqrt{\cos x}}\right)}{d\left(\sqrt{\cos x}\right)} \cdot \frac{d\left(\sqrt{\cos x}\right)}{d(\cos x)} \cdot \frac{d(\cos x)}{dx}$$

$$= e^{\sqrt{\cos x}} \cdot \frac{1}{2}(\cos x)^{\frac{1}{2}-1} \cdot (-\sin x)$$

$$= \frac{-\sin x\, e^{\sqrt{\cos x}}}{2\sqrt{\cos x}}$$

3. 2072 Set C Q.No. 5a

If $y = e^{\sin(\log x)}$, find $\frac{dy}{dx}$. [2]

SOLUTION

Here, $y = e^{\sin(\log x)}$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{\sin(\log x)}\right]$$

$$= \frac{d\left[e^{\sin(\log x)}\right]}{d[\sin(\log x)]} \times \frac{d[\sin(\log x)]}{d(\log x)} \times \frac{d(\log x)}{dx}$$

$$= e^{\sin(\log x)} \cdot \cos(\log x) \cdot \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{x} \cdot \cos(\log x) \cdot e^{\sin(\log x)}$$

4. 2072 Set E Q.No. 5a

Find the differential coefficient of $\ln(\sec x + \tan x)$. [2]

SOLUTION

Let $y = \ln(\sec x + \tan x)$

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}\{\ln(\sec x + \tan x)\}$$

$$= \frac{d\{\ln(\sec x + \tan x)\}}{d(\sec x + \tan x)} \cdot \frac{d(\sec x + \tan x)}{dx}$$

$$= \frac{1}{(\sec x + \tan x)} \cdot (\sec x \tan x + \sec^2 x)$$

$$= \frac{1}{(\sec x + \tan x)} \cdot \sec x(\tan x + \sec x)$$

$$= \sec x$$

5. 2071 Supp. Q.No. 5a

Find the derivative of $e^{\sqrt{\sin x}}$ [2]

SOLUTION

Let $y = e^{\sqrt{\sin x}}$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sqrt{\sin x}}\right)$$

$$= \frac{d\left(e^{\sqrt{\sin x}}\right)}{d\left(e^{\sqrt{\sin x}}\right)} \cdot \frac{d(\sin x)^{1/2}}{d(\sin x)} \cdot \frac{d(\sin x)}{dx}$$

$$= e^{\sqrt{\sin x}} \cdot \frac{1}{2}(\sin x)^{1/2-1} \cdot \cos x$$

$$= \frac{1}{2} e^{\sqrt{\sin x}} \cdot \frac{1}{\sqrt{\sin x}} \cdot \cos x$$

$$= \frac{\cos x}{2\sqrt{\sin x}} \cdot e^{\sqrt{\sin x}}$$

6. 2071 Old Q.No. 5c

Find $\frac{dy}{dx}$ of $y = e^{\sin(\log x)}$. [2]

➥ Please refer to 2072 Set C Q.No. 5a

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7. 2057 Q.No. 5c

Find $\frac{dy}{dx}$ of $y = e^{\sin(\log x)}$ [2]

Please refer to 2072 Set C Q.No. 5a

8. 2056 Q.No. 5c

Find $\frac{dy}{dx}$ of $y = e^{5x} \cdot \sin(\log x)$ [2]

SOLUTION

We have,

$y = e^{5x} \cdot \sin(\log x)$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\{e^{5x} \cdot \sin(\log x)\}$$

$$= e^{5x} \times \frac{d}{dx}\sin(\log x) + \sin(\log x) \cdot \frac{d}{dx}(e^{5x})$$

$$= e^{5x} \times \frac{d\sin(\log x)}{d(\log x)} \times \frac{d(\log x)}{dx} + \sin(\log x) \times \frac{d(e^{5x})}{d(5x)} \times \frac{d(5x)}{dx}$$

$$= e^{5x} \times \cos(\log x) \times \frac{1}{x} + \sin(\log x) \times e^{5x} \times 5$$

$$= \frac{1}{x} \cdot e^{5x}\cos(\log x) + 5e^{5x}\sin(\log x)$$

$$\therefore \frac{dy}{dx} = e^{5x}\left[\frac{1}{x}\cos(\log x) + 5\sin(\log x)\right]$$

4 MARKS QUESTIONS

9. 2071 Old Q.No. 13a

Find from definition, the derivative of $\log_e x$. [4]

SOLUTION

Let $y = \log_e x$

Again, let Δx and Δy be the small increments in x and y respectively. Then,

$y + \Delta y = \log_e(x + \Delta x)$

or, $\Delta y = \log_e(x + \Delta x) - y$

or, $\Delta y = \log_e(x + \Delta x) - \log_e x$

or, $\Delta y = \log_e\left(\frac{x + \Delta x}{x}\right)$

or, $\Delta y = \log_e\left(1 + \frac{\Delta x}{x}\right)$

Dividing both sides by Δx and taking *lim*it $\Delta x \to 0$ on both sides, we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{\log_e\left(1 + \frac{\Delta x}{x}\right)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\log_e\left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x} \times x}$$

$$= \lim_{\Delta x \to 0}\left\{\frac{\log_e\left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}} \cdot \frac{1}{x}\right\} = 1 \times \frac{1}{x} = \frac{1}{x}$$

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14 UNIT

APPLICATIONS OF DERIVATIVES

A. MAXIMA AND MINIMA

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 5c

Determine the interval in which the function $f(x) = \frac{1}{2}x^2 - x$ is increasing or decreasing. [2]

SOLUTION

Given, $f(x) = \frac{1}{2}x^2 - x$

$f'(x) = \frac{1}{2} \cdot 2x - 1 = x - 1$

For stationary point, we have,

$f'(x) = 0$

or, $x - 1 = 0$

$\therefore x = 1$

The point $x = 1$ divides the real line into two subintervals $(-\infty, 1)$ and $(1, \infty)$.

Now,

Intervals	Sing of $f'(x)$	Nature of the function
$(-\infty, 1)$	−ve	decreasing
$(1, \infty)$	+ve	increasing

$\therefore$ $f(x)$ is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.

2. 2076 Set C Q.No. 5c

State what do $f'(x) > 0$ and $f'(x) < 0$ represent for any curve $y = f(x)$. [2]

SOLUTION

For any curve, $f'(x) > 0$ represents an increasing function and $f'(x) < 0$ represents a decreasing function.

3. 2075 GIE Q.No. 5c

Test whether the function $f(x) = 2x^2 - 4x + 3$ is increasing or decreasing on the interval (1, 4]. [2]

SOLUTION

Given, $f(x) = 2x^2 - 4x + 3$

Differentiating both sides w.r. to x

$f'(x) = 4x - 4$

For stationary point, we have $f'(x) = 0$

i.e. $4x - 4 = 0$

$\therefore x = 1$

For $x > 1$, $f'(x) = 4x - 4 = 4(x - 1) > 0$

So, $f(x)$ is increasing for $x > 1$ i.e. $(1, \infty)$.

Clearly $(1, 4] \subset (1, \infty)$.

So, $f(x)$ is increasing on the interval $(1, 4]$.

4. 2075 Set A Q.No. 5b

Find the minimum value of the functio $2x^2 + 4x + 7$. [2

SOLUTION

Let $y = 2x^2 + 4x + 7$

$\frac{dy}{dx} = 4x + 4$

$\frac{d^2y}{dx^2} = 4$

For max. or min values, we have $\frac{dy}{dx} = 0$

or, $4x + 4 = 0$

$\therefore x = -1$

When $x = -1$, $\frac{d^2y}{dx^2} = 4 > 0$

So, y has min. value at $x = -1$.

Min. value $= y_{min}$

$= 2(-1)^2 + 4(-1) + 7$

$= 2 - 4 + 7$

$= 5$

5. 2075 Set B Q.No. 5c

Examine the increasing or decreasing natur of the function $y = x - \frac{1}{x}$. [2

SOLUTION

Given,

$y = x - \frac{1}{x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(x - \frac{1}{x}\right)$

$= \frac{d}{dx}(x - x^{-1})$

$= \frac{d}{dx}(x) - \frac{d}{dx}(x^{-1})$

$= 1 + \frac{1}{x}$ which is always positive except $x = 0$.

$\therefore$ $f(x)$ is increasing for all $x \in \mathbb{R}$ except $x = 0$.

6. 2074 Supp Q.No. 5c

Show that the function $f(x) = 2x^3 - 24x + 15$ is increasing at $x = 3$ and decreasing at $x = \frac{3}{2}$. [2]

SOLUTION

Here, $f(x) = 2x^3 - 24x + 15$

$\therefore f'(x) = 6x^2 - 24$

At $x = 3$, $f'(3) = 6 \times 3^2 - 24 = 30 > 0$

So, $f(x)$ is increasing at $x = 3$

At $x = \frac{3}{2}$, $f'\left(\frac{3}{2}\right) = 6 \times \left(\frac{3}{2}\right)^2 - 24$

$= \frac{27}{2} - 24 = \frac{27 - 48}{2} = -\frac{21}{2} < 0$

$\therefore$ $f(x)$ is decreasing at $x = \frac{3}{2}$

7. 2074 Set A Q.No. 5c

Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [2]

SOLUTION

Here, $f(x) = 15x^2 - 14x + 1$

$f'(x) = 30x - 14$

At $x = \frac{2}{5}$, $f'\left(\frac{2}{5}\right) = 30 \times \frac{2}{5} - 14 = -2 > 0$

$\therefore$ $f(x)$ is decreasing at $x = \frac{2}{5}$

At $x = \frac{5}{2}$, $f'\left(\frac{5}{2}\right) = 30 \times \frac{5}{2} - 14 = 61 > 0$

So, $f(x)$ is increasing at $x = \frac{5}{2}$.

8. 2074 Set B Q.No. 5c

Find the intervals in which the function $f(x) = 5x^3 - 135x + 22$ is increasing or decreasing. [2]

SOLUTION

Here, $f(x) = 5x^3 - 135x + 22$

$f'(x) = 15x^2 - 135$

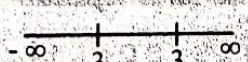
For stationary points, $f'(x) = 0$

or, $15x^2 - 135 = 0$

or, $x^2 - 9 = 0$

or, $(x + 3)(x - 3) = 0$

$\therefore x = -3, 3$



Interval	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -3)$	+ ve	increasing
$(-3, 3)$	− ve	decreasing
$(3, \infty)$	+ ve	increasing

$f(x)$ is increasing on $(-\infty, -3) \cup (3, \infty)$ and decreasing on $(-3, 3)$

9. 2073 Set C Q.No. 5c

Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [2]

SOLUTION

Given, $f(x) = 3x^2 - 6x + 5$

$f'(x) = 6x - 6$



For stationary points, $f'(x) = 0$

$6x - 6 = 0$

$x = 1$

Interval	Sign of $f'(x)$	Remarks
$(-\infty, 1)$	−ve	decreasing
$(1, \infty)$	+ve	increasing

$\therefore$ $f(x)$ is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.

10. 2073 Set D Q.No. 5c

Show that $f(x) = x - \frac{1}{x}$, is increasing for all $x \in R$ except at $x = 0$. [2]

SOLUTION

Here, $f(x) = x - \frac{1}{x} = x - x^{-1}$

$f'(x) = \frac{d}{dx}(x - x^{-1})$

$= 1 - (-1)x^{-1-1}$

$= 1 + \frac{1}{x^2}$, which is always positive for any real x except $x = 0$.

$\therefore$ $f(x)$ is increasing for all $x \in R$ except at $x = 0$

11. 2072 Set C Q.No. 5c

If $f(x) = 2x^3 - 6x^2 + 5$, determine where does the graph of the function concave upward. [2]

SOLUTION

Here, $f(x) = 2x^3 - 6x^2 + 5$

$f'(x) = 6x^2 - 12x$

$f''(x) = 12x - 12$

For point of inflection $f''(x) = 0$

$12x - 12 = 0$



$\therefore x = 1$

When $x > 1$, $f''(x) = 12x - 12 > 0$

So, the graph of $f(x)$ is concave upward for $x > 1$

$\therefore$ $f(x)$ is concave upward in $(1, \infty)$.

12. 2072 Set D Q.No. 5c

Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [2]

➥ Please refer to **2074 Set A Q.No. 5c**

13. 2071 Supp. Q.No. 5b

Show that the function $y = x^3 - 3x^2 + 6x + 3$ has neither maximum nor minimum values. [2]

SOLUTION

Here, $y = x^3 - 3x^2 + 6x + 3$

$\frac{dy}{dx} = 3x^2 - 6x + 6$

$= 3(x^2 - 2x + 2)$

$= 3\{(x - 1)^2 + 1\} \neq 0$ for all real values of x.

Hence, y has neither maximum nor minimum.

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14. 2071 Set C Q.No. 5c

Find the intervals in which $f(x) = x^2 - 2x + 10$ is increasing or decreasing. [2]

SOLUTION

Here, $f(x) = x^2 - 2x + 10$

$f'(x) = 2x - 2$

For stationary points, we have

$f'(x) = 0$

$2x - 2 = 0$

$\therefore x = 1$



Intervals	Sign of $f'(x)$	Remarks
$(-\infty, 1)$	− ve	decreasing
$(1, \infty)$	+ ve	increasing

$\therefore$ $f(x)$ is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.

15. 2071 Set D Q.No. 5c

Find the minimum of $f(x) = 3x^2 - 6x + 4$. [2]

SOLUTION

Here, $f(x) = 3x^2 - 6x + 4$

$f'(x) = 6x - 6$

$f''(x) = 6$

For minimum,

$f'(x) = 0$

or, $6x - 6 = 0$

or, $x = 1$

When $x = 1$; $f''(1) = 6 > 0$

So, $f(x)$ has minimum value at $x = 1$.

Minimum value $= f(1)$

$= 3 \cdot 1^2 - 6 \cdot 1 + 4$

$= 1$

16. 2070 Supp Q.No. 5c

Examine whether the function $f(x) = 2x^3 - 24x + 15$ is increasing or decreasing at $x = 3$ and $x = \frac{3}{2}$. [2]

➥ Please refer to 2074 Supp Q.No. 5c

17. 2070 Set C Q.No. 5 c

Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [2]

SOLUTION

Given $f(x) = 3x^2 - 6x + 5$

$f'(x) = 6x - 6$

For stationary points, we have

$f'(x) = 0$

or, $6x - 6 = 0$



$\therefore$ $x = 1$

Intervals	Sign of $f'(x)$	Remarks
$(-\infty, 1)$	− ve	decreasing
$(1, \infty)$	+ ve	increasing

$\therefore$ $f(x)$ is decreasing on the interval $(-\infty, 1)$ and increasing on the interval $(1, \infty)$.

18. 2069 Supp Q.No. 5 c

Find the interval in which the function $f(x) = x^2 - 5x + 6$ is increasing or decreasing. [2]

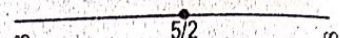
SOLUTION

Here, $f(x) = x^2 - 5x + 6$

$f'(x) = 2x - 5$

For stationary points, $f'(x) = 0$

or, $2x - 5 = 0$



$\therefore$ $x = \frac{5}{2}$

Intervals	Sign of $f'(x)$	Remarks
$(-\infty, 5/2)$	− ve	decreasing
$(5/2, \infty)$	+ ve	increasing

$\therefore$ $f(x)$ is increasing on the interval $\left(\frac{5}{2}, \infty\right)$ and decreasing on the interval $\left(-\infty, \frac{5}{2}\right)$.

19. 2069 [Set B] Q. No. 5c

For any curve $y = f(x)$, what do $f'(x) > 0$ and $f'(x) < 0$ represent? [2]

➥ Please refer to 2076 Set C Q.No. 5c

20. 2068 Q.No. 5c

Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = 2/5$ and $x = 5/2$. [2]

SOLUTION

Here, $f(x) = 15x^2 - 14x + 1$

$f'(x) = 30x - 14$

At $x = \frac{2}{5}$, $f'\left(\frac{2}{5}\right) = 30 \times \frac{2}{5} - 14 = -2 < 0$

$\therefore$ $f(x)$ is decreasing at $x = \frac{2}{5}$

At $x = \frac{5}{2}$, $f'\left(\frac{5}{2}\right) = 30 \times \frac{5}{2} - 14 = 61 > 0$

$\therefore$ $f(x)$ is increasing at $x = \frac{5}{2}$

6 MARKS QUESTIONS

21. 2076 Set B Q.No. 15

Write the criteria for the function $y = f(x)$ to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection. [6]

SOLUTION

First part: The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

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Next Part:

Here, $f(x) = 2x^3 - 15x^2 + 36x + 10$

$f'(x) = 6x^2 - 30x + 36$

$f''(x) = 12x - 30$

For stationary points, we have $f'(x) = 0$

or, $6x^2 - 30x + 36 = 0$

or, $x^2 - 5x + 6 = 0$

or, $(x-2)(x-3) = 0$

$\therefore x = 2, 3$

At $x = 2$

$f''(2) = 12 \times 2 - 30 = -6 < 0$

So, $f(x)$ has maximum value at $x = 2$

Maximum value $= f(2)$

$= 2\times2^3 - 15\times2^2 + 36 \times 2 + 10 = 38$

At $x = 3$

$f''(3) = 12 \times 3 - 30 = 6 > 0$

So, $f(x)$ has minimum value at $x = 3$

Minimum value $= f(3)$

$= 2\times 3^3 - 15\times3^2 + 36\times3 + 10 = 37$

For point of inflection, $f''(x) = 0$

or, $12x - 30 = 0$

$\therefore x = \frac{5}{2}$

22. 2076 Set C Q.No. 15

Find the absolute maxima and absolute minima of the function $f(x) = x^3 - 3x^2 + 5$ on $[-2, 2]$. Also find the point of inflection, if any. [6]

SOLUTION

Here, $f(x) = x^3 - 3x^2 + 5$. on $[-2, 2]$

Differentiating both sides w.r.to x

$\frac{d}{dx}[f(x)] = \frac{d}{dx}(x^3 - 3x^2 + 5)$

$f'(x) = 3x^2 - 6x$

Again, differentiating both sides w.r.to x,

$f''(x) = 6x - 6$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$

or, $3x(x-2) = 0$

$\therefore x = 0, 2.$

When $x = -2$, $f(-2) = (-2)^3 - 3(-2)^2 + 5$

$= -8 - 12 + 5$

$= -15.$

When $x = 0$, $f(0) = 0^3 - 3\times0^2 + 5$

$= 5$

When $x = 2$, $f(2) = 2^3 - 3.2^2 + 5$

$= 8 - 12 + 5$

$= 1$

$\therefore$ Absolute max. value $= 5$ at $x = 0$.

Absolute min. value $= -15$ at $x = -2$.

For point of inflection, we have

$f''(x) = 0$

$6x - 6 = 0$

or, $x = 1$.

$\therefore$ Point of inflection is at $x = 1$.

23. 2075 GIE Q.No. 15

Write the criteria for the function $y = f(x)$ to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection, if any. [6]

➥ Please refer to 2076 Set B Q.No. 15

24. 2075 GIE Q.No. 15 OR

A window is in the form of a rectangle surmounted by a semicircle. If the total perimeter is 9m, find the radius of the semicircle for the greatest window's area. [6]

SOLUTION

Let $2x$ be the length and y be the breadth of the rectangle of the window which is surmounted by a semi–circle as shown in the figure.

Total perimeter = 9m

or, $y + 2x + y + \pi x = 9$

or, $2y = 9 - 2x - \pi x$

or, $y = \frac{9 - 2x - \pi x}{2}$ ...(i)

Area of the window (A)

= area of rectangle + area of semi–circle

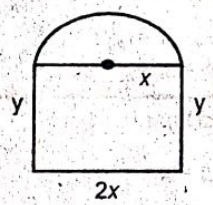
$= 2x \cdot y + \frac{1}{2} \cdot \pi \cdot x^2$

$= \frac{\pi x^2}{2} + 2x\left(\frac{9 - 2x - \pi x}{2}\right)$ [using (i)]

$= \frac{\pi x^2}{2} + 9x - 2x^2 - \pi x^2 = 9x - 2x^2 - \frac{\pi x^2}{2}$

$\frac{dA}{dx} = 9 - 4x - \pi x$

$\frac{d^2A}{dx^2} = -4 - \pi$



For maximum area, $\frac{dA}{dx} = 0$

or, $9 - 4x - \pi x = 0$

or, $(4 + \pi)x = 9$

$\therefore x = \frac{9}{4 + \pi}$

When $x = \frac{9}{4+\pi}$, $\frac{d^2A}{dx^2} = -4 - \pi < 0$

So, area is maximum when $x = \frac{9}{4+\pi}$

Hence, for the greatest window area, the radius of semi-circle $= \frac{9}{4+\pi}$ m.

25. 2075 Set A Q.No. 15

Determine the intervals in which the function $f(x) = x^4 - 4x^3$ is concave upwards or downwards. Also, find the point of inflection, if they exist. [6]

SOLUTION

Given,

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

Setting $f'(x) = 0$, we get

$12x^2 - 24x = 0$

or, $12x(x-2) = 0$

$\therefore x = 0, 2.$

Now,

Intervals	Sign of f"(x)	Concavity
$(-\infty, 0)$	+ ve	Concave upward
$(0, 2)$	− ve	Concave downward
$(2, \infty)$	+ ve	Concave upward

$\therefore$ f(x) is concave upward on $(-\infty, 0) \cup (2, \infty)$ and concave downward on $(0, 2)$.

26. 2075 Set B Q.No. 15

Find the absolute maxima and absolute minima of the function $f(x) = x^3 - 3x^2 + 5$ on $[-2, 2]$. Also find the point of inflection, if any. [6]

➥ Please refer to 2076 Set C Q.No. 15

27. 2075 Set C Q.No. 15

List the criteria for a function y = f(x), to have the local maximum and local minimum at a point. Prove that $f(x) = x + \frac{1}{x}$ whose maximum value is less than its minimum value. [6]

SOLUTION

First part: The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Next part:

$f(x) = x + \frac{1}{x}$

$f'(x) = 1 - \frac{1}{x^2}$

$f''(x) = \frac{2}{x^3}$

For max. or min values, $f'(x) = 0$

or, $1 - \frac{1}{x^2} = 0$

or, $1 = \frac{1}{x^2}$

or, $x^2 = 1$

or, $x^2 = (\pm 1)^2$

$\therefore x = \pm 1$

When $x = 1$, $f''(x) = \frac{2}{1^3} = 2 > 0$

So, f(x) has minimum value at $x = 1$.

Min. value $= 1 + \frac{1}{1} = 1 + 1 = 2$

When $x = -1$, $f''(x) = \frac{2}{(-1)^3} = -2 < 0$.

So, f(x) has maximum value at $x = -1$.

Max. value $= (-1) + \frac{1}{(-1)} = -1 - 1 = -2$.

$\therefore$ Max. value is less than min. value.

28. 2074 Supp Q.No. 15

Determine where the graph is concave upwards and where it is concave downwards of the function $f(x) = x^4 - 8x^3 + 18x^2 - 24$ [6]

SOLUTION

Given function is $f(x) = x^4 - 8x^3 + 18x^2 - 24$

Diff. both sides with respect to 'x', we get

$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^4 - 8x^3 + 18x^2 - 24)$.

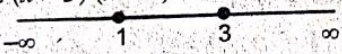
$\therefore f'(x) = 4x^3 - 24x^2 + 36x$

And $f''(x) = 12x^2 - 48x + 36$

$= 12(x^2 - 4x + 3)$

$= 12(x^2 - 3x - x + 3)$

$= 12(x-3)(x-1)$



For the points of inflection, $f''(x) = 0$

$12(x-3)(x-1) = 0$

$\therefore$ Required points of inflection are $x = 1$ and $x = 3$.

Intervals	Sign of f''(x)	Remarks
$(-\infty, 1)$	+ ve	Concave upwards
$(1, 3)$	− ve	Concave downwards
$(3, \infty)$	+ ve	Concave upwards

$\therefore$ $f(x)$ is concave upwards on $(-\infty, 1) \cup (3, \infty)$ and downwards on $(1, 3)$.

29. 2074 Set A Q.No. 15

List the criteria for a function y = f(x), to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 4x^3 - 15x^2 + 12x + 7$. [6]

SOLUTION

First part:

The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Second part

$f(x) = 4x^3 - 15x^2 + 12x + 7$

$f'(x) = 12x^2 - 30x + 12$

$f''(x) = 24x - 30$

For stationary points, we have $f'(x) = 0$

or, $12x^2 - 30x + 12 = 0$

or, $2x^2 - 5x + 2 = 0$

or, $(2x-1)(x-2) = 0$

$\therefore x = \frac{1}{2}, 2$

At $x = \frac{1}{2}$

$f''\left(\frac{1}{2}\right) = 24 \times \left(\frac{1}{2}\right) - 30 = -18 < 0$

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$\therefore$ $f(x)$ has maximum value at $x = \frac{1}{2}$

Maximum value $= f\left(\frac{1}{2}\right)$

$= 4 \times \left(\frac{1}{2}\right)^3 - 15 \times \left(\frac{1}{2}\right)^2 + 12 \times \frac{1}{2} + 7$

$= \frac{1}{2} - \frac{15}{4} + 6 + 7$

$= \frac{2 - 15 + 52}{4} = \frac{39}{4}$

At $x = 2$

$f''(2) = 24 \times 2 - 30 = 18 > 0$

So, $f(x)$ has minimum value at $x = 2$

Minimum value $= f(2)$

$= 4 \times 2^3 - 15 \times 2^2 + 12 \times 2 + 7$

$= 32 - 60 + 24 + 7 = 3$

For point of inflection

$f''(x) = 0$

or, $24x - 30 = 0$

$\therefore$ $x = \frac{5}{4}$

30. 2074 Set B Q.No. 15

Write the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection, if any. [6]

➥ Please refer to 2076 Set B Q.No. 15

31. 2073 Supp Q.No. 15

List the criteria at which the function y = f(x) will have local maximum and local minimum values. Find the maximum and the minimum values of the function $y = 4x^3 - 6x^2 - 9x + 1$ on the interval $-1 < x < 2$. Also find the point of inflection. [6]

SOLUTION

First Part: The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Next Part:

Given curve is $y = 4x^3 - 6x^2 - 9x + 1$ ...(i)

Differentiating both sides with respect to 'x', we get

$\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 6x^2 - 9x + 1)$

$\frac{dy}{dx} = 12x^2 - 12x - 9$ ... (ii)

Again, differentiating both sides with respect to 'x', we get

$\frac{d^2y}{dx^2} = 24x - 12$ ... (iii)

For maxima or minima $\frac{dy}{dx} = 0$

i.e. $12x^2 - 12x - 9 = 0$

or, $4x^2 - 4x - 3 = 0$

or, $4x^2 - 6x + 2x - 3 = 0$

or, $2x(2x - 3) + 1(2x - 3) = 0$

or, $(2x - 3)(2x + 1) = 0$

$\therefore$ $x = \frac{3}{2}$ or $-\frac{1}{2}$

Clearly the values of x lie in the interval $(-1, 2)$

At $x = -\frac{1}{2}$

$\frac{d^2y}{dx^2} = 24\left(-\frac{1}{2}\right) - 12 = -24 < 0$

$\therefore$ The given function is maximum at $x = -\frac{1}{2}$.

Maximum value is

$y_{max} = 4\left(-\frac{1}{2}\right)^3 - 6\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) + 1$

$= 4\left(-\frac{1}{8}\right) - 6\left(\frac{1}{4}\right) + \frac{9}{2} + 1$

$= -\frac{1}{2} - \frac{3}{2} + \frac{9}{2} + 1$

$= \frac{-1 - 3 + 9 + 2}{2} = \frac{7}{2}$

At $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = 24 \times \frac{3}{2} - 12 = 24 > 0$

So, the given function is minimum at $x = \frac{3}{2}$ and its minimum value is

$y_{min} = 4\left(\frac{3}{2}\right)^3 - 6\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 1$

$= 4 \times \frac{27}{8} - 6 \times \frac{9}{4} - \frac{27}{2} + 1$

$= \frac{27}{2} - \frac{27}{2} - \frac{27}{2} + 1$

$= -\frac{27}{2} + 1 = \frac{-27 + 2}{2} = \frac{-25}{2}$

Further, for the point of inflection,

$\frac{d^2y}{dx^2} = 0$

i.e. $24x - 12 = 0$

$\Rightarrow$ $x = \frac{1}{2}$ which is the point of inflection.

32. 2073 Set C Q.No. 15

Determine the maximum and minimum value of $f(x) = 2x^3 - 9x^2 + 12x - 4$. Also find the point of inflection, if any. [6]

SOLUTION

Here, $f(x) = 2x^3 - 9x^2 + 12x - 4$

$f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18$

For maxima or minima, we have

$f'(x) = 0$

or, $6x^2 - 18x + 12 = 0$

or, $x^2 - 3x + 2 = 0$

or, $(x - 1)(x - 2) = 0$

$\therefore$ $x = 1, 2$

When $x = 1$, $f''(x) = 12 \times 1 - 18 = -6 < 0$

So, f(x) has maximum value at $x = 1$.
Maximum value $= 2 \times 1^3 - 9 \times 1^2 + 12 \times 1 - 4 = 1$
When $x = 2$, $f''(x) = 12 \times 2 - 18 = 6 > 0$
So, f(x) has minimum value at $x = 2$.
Minimum value $= 2 \times 2^3 - 9 \times 2^2 + 12 \times 2 - 4 = 0$
For point of inflection,
$f''(x) = 0$
or, $12x - 18 = 0$
$x = \frac{18}{12} = \frac{3}{2}$

33. 2073 Set D Q.No. 15

Define the absolute maxima and absolute minima of the function. Find the absolute maxima and absolute minima of $f(x) = x^3 - 3x^2 + 5$ on $[-2,2]$. Also find the point of inflection, if any. [6]

SOLUTION

A function f defined on an interval [a, b] is said to have absolute (global) maximum value at $x = x_0$ if $f(x) \le f(x_0)$ for all $x \in [a, b]$.
Similarly, a function f defined on an interval [a, b] is said to have absolute (global) minimum value at $x = x_0$ if $f(x) \ge f(x_0)$ for all $x \in [a, b]$.
Here, $f(x) = x^3 - 3x^2 + 5$
$f'(x) = 3x^2 - 6x$
$f''(x) = 6x - 6$
Now, $f'(x) = 0$ gives
$3x^2 - 6x = 0$
or, $x^2 - 2x = 0$
or, $x(x - 2) = 0$
$\therefore x = 0, 2 \in [-2, 2]$
When $x = -2$, $f(-2) = (-2)^3 - 3(-2)^2 + 5 = -15$
When $x = 0$, $f(0) = 0^3 - 3 \times 0^2 + 5 = 5$
When $x = 2$, $f(2) = 2^3 - 3 \times 2^2 + 5 = 1$
$\therefore$ Absolute maximum value $= 5$ at $x = 0$
Absolute minimum value $= -15$ at $x = 2$
Point of inflection
$f''(x) = 0$
or, $6x - 6 = 0$
$\therefore x = 1$

34. 2072 Supp. Q.No. 15

List the criteria for the function $y = f(x)$ to have the maximum and minimum value at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 5$. Also, find the point of inflection. [6]

SOLUTION

First part: The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Second Part:
Here, $f(x) = 2x^3 - 15x^2 + 36x + 5$
$f'(x) = 6x^2 - 30x + 36$
$f''(x) = 12x - 30$
For stationary points, we have $f'(x) = 0$
or, $6x^2 - 30x + 36 = 0$
or, $x^2 - 5x + 6 = 0$
or, $(x - 2)(x - 3) = 0$
$\therefore x = 2, 3$
At $x = 2$
$f''(2) = 12 \times 2 - 30 = -6 < 0$
So, f(x) has maximum value at $x = 2$
Maximum value $= f(2)$
$= 2 \times 2^3 - 15 \times 2^2 + 36 \times 2 + 5 = 33$
At $x = 3$
$f''(3) = 12 \times 3 - 30 = 6 > 0$
So, f(x) has minimum value at $x = 3$
Minimum value $= f(3)$
$= 2 \times 3^3 - 15 \times 3^2 + 36 \times 3 + 5 = 32$
For point of inflection, $f''(x) = 0$
or, $12x - 30 = 0$
$\therefore x = \frac{5}{2}$

35. 2072 Set C Q.No. 15

If $y = f(x)$ represents a certain curve, what do $f^1(x) > 0$, $f^1(x) < 0$ and $f^1(x) = 0$ at a point represent?

Find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing. Also, find the point of inflection. [6]

SOLUTION

a. If $f'(x) > 0$ at a certain point, then f(x) is increasing in the neighbourhood of the point.
b. If $f'(x) < 0$ at a certain point, then f(x) is decreasing in the neighbourhood of that point.
c. If $f'(x) = 0$ at a certain point then f(x) remains stationary (constant) at that point.

Next part
Here, $f(x) = 2x^3 - 15x^2 + 36x + 1$
$f'(x) = 6x^2 - 30x + 36$
$f''(x) = 12x - 30$
For stationary points, we have $f'(x) = 0$
or, $6x^2 - 30x + 36 = 0$
or, $x^2 - 5x + 6 = 0$
or, $(x - 2)(x - 3) = 0$
$\therefore x = 2, 3$

Intervals	Sign of $f'(x)$	Remarks
$(-\infty, 2)$	+ ve	increasing
$(2, 3)$	− ve	decreasing
$(3, \infty)$	+ ve	increasing

$\therefore$ f(x) is increasing on $(-\infty, 2) \cup (3, \infty)$ and decreasing on $(2, 3)$.
For point of inflection, $f''(x) = 0$
or, $12x - 30 = 0$
or, $12x = 30$
$\therefore x = \frac{5}{2}$

36. 2072 Set D Q.No. 15

List the criteria for the function $y = f(x)$ to have the local maxima and local minima at the point $x = a$. Find the local maxima and local minima of the function $f(x) = 2x^3 - 9x^2 - 24x + 3$. Also find the point of inflection. [6]

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SOLUTION

The condition for the function $y = f(x)$ to have the local maximum and local minimum at the point $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Next part:

Here, $f(x) = 2x^3 - 9x^2 - 24x + 3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

For stationary points, $f'(x) = 0$

or, $6x^2 - 18x - 24 = 0$

or, $x^2 - 3x - 4 = 0$

or, $(x + 1)(x - 4) = 0$

$\therefore x = -1, 4$

At $x = -1$

$f''(-1) = 12(-1) - 18 = -30 < 0$

So, $f(x)$ has maximum at $x = -1$

Maximum value $= f(-1)$

$= 2(-1)^3 - 9(-1)^2 - 24(-1) + 3$

$= 16$

At $x = 4$

$f''(4) = 12 \times 4 - 18 = 30 > 0$

So, $f(x)$ has minimum at $x = 4$

Minimum value $= f(4)$

$= 2 \times 4^3 - 9 \times 4^2 - 24 \times 4 + 3$

$= -109$

Last Part:

For point of inflection,

$f''(x) = 0$

$12x - 18 = 0$

or, $12x = 18$

$\therefore x = \frac{18}{12} = \frac{3}{2}$

37. 2072 Set E Q.No. 15

List the criteria for a function $y = f(x)$ to have local maximum and local minimum at a point. Find the local maxima and local minima of the function $f(x) = 12x^5 - 45x^4 + 4x^3 + 6$. [6]

SOLUTION

First part

The condition for the function $y = f(x)$ to have the local maximum and local minimum at the poin t $x = a$ are as follows

Conditions	Maxima	Minima
First order derivative	$\frac{dy}{dx} = f'(x) = 0$	$\frac{dy}{dx} = f'(x) = 0$
Second order derivative	$\frac{d^2y}{dx^2} = f''(x) < 0$	$\frac{d^2y}{dx^2} = f''(x) > 0$

Next part

$f(x) = 12x^5 - 45x^4 + 40x^3 + 6$

$f'(x) = 60x^4 - 180x^3 + 120x^2$

$f''(x) = 240x^3 - 540x^2 + 240x$

For stationary points, we have

$f'(x) = 0$

or, $60x^4 - 180x^3 + 120x^2 = 0$

or, $60x^2(x^2 - 3x + 2) = 0$

or, $x^2(x - 1)(x - 2) = 0$

$\therefore x = 0, 1, 2$

At $x = 0$

$f''(0) = 240 \times 0 - 540 \times 0 + 240 \times 0 = 0$

and $f'''(x) = 720x^2 - 1080x + 240$

$f'''(0) = 0 - 0 + 240 = 240 \neq 0$

Hence, $f(x)$ has neither maximum nor minimum value at $x = 0$.

At $x = 1$

$f''(1) = 240 \times 1^3 - 540 \times 1^2 + 240 \times 1 = -60 < 0$

$\therefore$ $f(x)$ has a maximum value at $x = 1$

Max. value $= f(1)$

$= 12 \times 1^5 - 45 \times 1^4 + 40 \times 1^3 + 6 = 13$

At $x = 2$

$f''(2) = 240 \times 2^3 - 540 \times 2^2 + 240 \times 2 = 1680 > 0$

$\therefore$ $f(x)$ has a minimum value at $x = 2$

Minimum value is

$f(2) = 12 \times 2^5 - 45 \times 2^4 + 40 \times 2^3 + 6 = -10$

38. 2071 Supp. Q.No. 15

Find the local maxima and minima and points of inflection: $f(x) = 2x^3 - 15x^2 + 36x + 5$. [6]

SOLUTION

Here, $f(x) = 2x^3 - 15x^2 + 36x + 5$

$f'(x) = 6x^2 - 30x + 36$

$f''(x) = 12x - 30$

For stationary points, we have $f'(x) = 0$

or, $6x^2 - 30x + 36 = 0$

or, $x^2 - 5x + 6 = 0$

or, $(x - 2)(x - 3) = 0$

$\therefore x = 2, 3$

At $x = 2$

$f''(2) = 12 \times 2 - 30 = -6 < 0$

So, $f(x)$ has maximum value at $x = 2$

Maximum value $= f(2)$

$= 2 \times 2^3 - 15 \times 2^2 + 36 \times 2 + 5 = 33$

At $x = 3$

$f''(3) = 12 \times 3 - 30 = 6 > 0$

So, $f(x)$ has minimum value at $x = 3$

Minimum value $= f(3)$

$= 2 \times 3^3 - 15 \times 3^2 + 36 \times 3 + 5 = 32$

For point of inflection, $f''(x) = 0$

or, $12x - 30 = 0$

$\therefore x = \frac{5}{2}$

39. 2071 Set C Q.No. 15

A window is in the form of a rectangle surmounted by a semi circle. If the total perimeter is 9m, find the radius of the semicircle for the greatest window area. [6]

➥ Please refer to 2075 GIE Q.No. 15 OR

40. 2071 Set D Q.No. 15

Find the absolute maxima and absolute minima of the function.

$f(x) = x^3 - 3x^2 + 5$ on [–2, 2]. Also find the point of inflection of f(x), if any. [6]

➥ Please refer to 2073 Set D Q.No. 15

41. 2070 Supp Q.No. 15

Find the local maxima and local minima of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1, 2). Also find the point of inflection. [6]

➥ Please refer to 2073 Supp Q.No. 15

42. 2070 Set C Q.No. 15

List the criteria for the function y = f(x) to have the local maxima and local minima at a point? Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. [6]

➥ Please refer to 2074 Set A Q.No. 15

43. 2070 Set D Q.No. 15

Write criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 9x^2 - 24x + 3$. Also find the point of inflection. [6]

➥ Please refer to 2072 Set D Q.No. 15

44. 2069 Supp Q.No. 15

Determine the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1,2). Also find the point of inflection. [6]

➥ Please refer to 2073 Supp Q.No. 15

45. 2069 [Set A] Q. No. 15

What are the criteria for a function y = f(x) to have the local maxima and local minima at a point? Find the local maxima and local minima of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1, 2). Also find the point of inflection. [6]

➥ Please refer to 2073 Supp Q.No. 15

46. 2069 [Set B] Q. No. 15

Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x - 2$. Also, find the point of inflection, if any. [6]

SOLUTION

Here, $f(x) = x^3 - 6x^2 + 9x - 2$

$f'(x) = 3x^2 - 12x + 9$

$f''(x) = 6x - 12$

For stationary points, we have $f'(x) = 0$

or, $3x^2 - 12x + 9 = 0$

or, $x^2 - 4x + 3 = 0$

or, $(x - 1)(x - 3) = 0$

$\therefore x = 1, 3$

At $x = 1$

$f''(1) = 6 \times 1 - 12 = -6 < 0$

So, $f(x)$ has maximum value at $x = 1$

Maximum value $= f(1) = 1^3 - 6 \times 1^2 + 9 \times 1 - 2 = 2$

At $x = 3$

$f''(3) = 6 \times 3 - 12 = 6 > 0$

So, $f(x)$ has minimum value at $x = 3$

Minimum value $= f(3) = 3^3 - 6 \times 3^2 + 9 \times 3 - 2$

$= 27 - 54 + 27 - 2 = -2$

For point of inflection $f''(x) = 0$

$6x - 12 = 0$

$\therefore x = 2$

47. 2068 Q.No. 15

List the criteria for the function y = f(x) to have local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. [6]

➥ Please refer to 2074 Set A Q.No. 15

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

48. 2071 Old Q.No. 13a Or

Determine where the graph is concave upwards and where it is concave downwards. Also find the point of inflection $f(x) = (x^2 - 1)(x^2 - 5)$. [4]

SOLUTION

Here, $f(x) = (x^2 - 1)(x^2 - 5)$

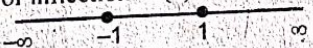
$= x^4 - 5x^2 - x^2 + 5$

$= x^4 - 6x + 5$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

For points of inflection $f''(x) = 0$



or, $12x^2 - 12 = 0$

or, $x^2 - 1 = 0$, $\therefore x = \pm 1$

Intervals	Sign of $f''(x)$	Remarks
$(-\infty, -1)$	+ ve	concave upwards
$(-1, 1)$	– ve	concave downwards
$(1, \infty)$	+ ve	concave upwards

$\therefore$ The graph of $f(x)$ is concave upwards on the interval $(-\infty, -1) \cup (1, \infty)$ and concave downwards on the interval $(-1, 1)$

49. 2070 Old Q.No. 13 a

Examine the maxima and minima of the function $4x^3 - 6x^2 + 3x$. Also find the point of inflexion, if any. [4]

SOLUTION

Let $y = 4x^3 - 6x^2 + 3x$

$\frac{dy}{dx} = 12x^2 - 12x + 3$

$\frac{d^2y}{dx^2} = 24x - 12$

For maxima or minima, $\frac{dy}{dx} = 0$

or, $12x^2 - 12x + 3 = 0$

or, $4x^2 - 4x + 1 = 0$

or, $(2x - 1)^2 = 0$

$\therefore x = \frac{1}{2}$

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When $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = 24 \times \frac{1}{2} - 12 = 0$

$\frac{d^3y}{dx^3} = 24 \neq 0$

So, y has neither maximum value nor minimum value.

For point of inflection, $f''(x) = 0$

or, $24x - 12 = 0$

$\therefore x = \frac{1}{2}$

50. 2069 [Set A] Old Q. No. 13a OR

Show that: $x^5 - 5x^4 + 5x^3 - 1$ is maximum when $x = 1$ and minimum when $x = 3$. [4]

SOLUTION

Let $y = x^5 - 5x^4 + 5x^3 - 1$

$\frac{dy}{dx} = 5x^4 - 20x^3 + 15x^2$

$\frac{d^2y}{dx^2} = 20x^3 - 60x^2 + 30x$

When $x = 1$,

$\frac{d^2y}{dx^2} = 20 \times 1^3 - 60 \times 1^2 + 30 \times 1$

$= -10 < 0$

$\therefore f(x)$ has maximum when $x = 1$.

Again, when $x = 3$,

$\frac{d^2y}{dx^2} = 20 \times 3^3 - 60 \times 3^2 + 30 \times 3$

$= 540 - 540 + 90 = 90 > 0$

$\therefore f(x)$ has minimum value when $x = 3$.

51. 2068 Old Q.No. 13a(Or)

Find the maximum and the minimum value of the function:

$f(x) = 2x^3 - 9x^2 - 24x + 3$. [4]

SOLUTION

Here, $f(x) = 2x^3 - 9x^2 - 24x + 3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

For stationary points, $f'(x) = 0$

or, $6x^2 - 18x - 24 = 0$

or, $x^2 - 3x - 4 = 0$

or, $(x + 1)(x - 4) = 0$

$\therefore x = -1, 4$

At $x = -1$

$f''(-1) = 12(-1) - 18 = -30 < 0$

So, $f(x)$ has maximum at $x = -1$

Maximum value $= f(-1)$

$= 2(-1)^3 - 9(-1)^2 - 24(-1) + 3$

$= 16$

At $x = 4$

$f''(4) = 12 \times 4 - 18 = 30 > 0$

So, $f(x)$ has minimum at $x = 4$

Minimum value $= f(4)$

$= 2 \times 4^3 - 9 \times 4^2 - 24 \times 4 + 3$

$= -109$

52. 2067 Q.No. 13a OR

A man wishes to fence a rectangular garden with 256 meter fencing material. Find the maximum area he can enclose. [4]

SOLUTION

Let x and y be the length and the breadth of a rectangular garden respectively.

Perimeter = 256

i.e. $2(x + y) = 256$

or, $x + y = 128$

or, $y = 128 - x$ . . . (i)

Let A be the area of rectangular garden. Then

$A = xy$

or, $A = x(128 - x)$ (Using (i))

or, $A = 128x - x^2$

$\frac{dA}{dx} = 128 - 2x$

$\frac{d^2A}{dx^2} = -2$

For maxima or minima,

$\frac{dA}{dx} = 0$

or, $128 - 2x = 0$

or, $x = \frac{128}{2} = 64$

When $x = 64$,

$\frac{d^2A}{dx^2} = -2 < 0$. Hence A is maximum when $x = 64$.

From (i)

$y = 128 - 64 = 64$

Maximum area $= x \times y = 64 \times 64 = 4096\ m^2$.

53. 2066 Q.No. 13a OR

Using derivatives, find two numbers whose sum is 10 and sum of whose squares is minimum. [4]

SOLUTION

Let x and y be two numbers such that $x + y = 10$

$\therefore y = 10 - x$ . . . (i)

Again, let $S = x^2 + y^2$

or, $S = x^2 + (10 - x)^2$ (using (i))

or, $S = x^2 + 100 - 20x + x^2$

or, $S = 2x^2 - 20x + 100$

$\therefore \frac{dS}{dx} = 4x - 20$

and $\frac{d^2S}{dx^2} = 4$

For maxima or minima, $\frac{ds}{dx} = 0$

or, $4x - 20 = 0$

or, $x = \frac{20}{4} = 5$

When $x = 5$, $\frac{d^2S}{dx^2} = 4 > 0$

So, when $x = 5$, S is minimum.

So, from (i)

$y = 10 - x = 10 - 5 = 5$

$\therefore$ Hence the required two numbers are 5 and 5.

54. 2065 Q.No. 13a OR

Show that the rectangle of largest possible area for a given perimeter is a square. [4]

SOLUTION

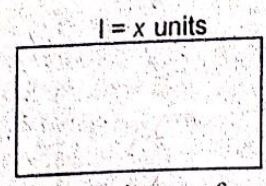
Let length and breadth of a rectangle be x units and y units respectively.

$\therefore$ Perimeter of rectangle $= 2(l + b)$

or, $P = 2(x + y)$.

or, $x + y = \frac{P}{2}$

or, $y = \frac{P}{2} - x$ ... (i)



where, P is the perimeter of rectangle which is constant

Now, area of the rectangle (A) = lb

or, $A = xy$

or, $A = x\left(\frac{P}{2} - x\right)$

or, $A = \frac{Px}{2} - x^2$ (ii)

$\frac{dA}{dx} = \frac{d}{dx}\left(\frac{Px}{2} - x^2\right) = \left(\frac{P}{2} - 2x\right)$

Also, $\frac{d^2A}{dx^2} = -2$.

For Maxima, $\frac{dA}{dx} = 0$

i.e. $\frac{P}{2} - 2x = 0$

or, $2x = \frac{P}{2}$

$\therefore x = \frac{P}{4}$

When $x = \frac{P}{4}$, $\frac{d^2A}{dx^2} = -2 < 0$

Thus, when $x = \frac{P}{4}$, the area of rectangle is largest (i.e. Maxima)

Substituting the value of x in equation (i), we get

$y = \frac{P}{2} - \frac{P}{4} = \frac{2P - P}{4} = \frac{P}{4}$.

$\therefore x = y = \frac{P}{4}$

Hence, we arrive to the conclusion that, the rectangle of largest possible area for a given perimeter is a square.

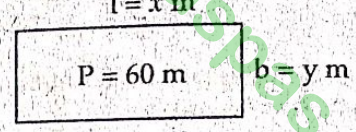
55. 2064 Q.No. 12b

Find the maximum area of a rectangular plot of land, which can be enclosed by a rope of length 60 metres. [4]

SOLUTION

Suppose x m. and y m. be the length and breadth respectively of a rectangular plot of land.

It is given that, the land should be enclosed by a rope of length 60 meters.



$\therefore$ Perimeter of the land = 60 m

i.e. $2(x + y) = 60$

or, $y = 30 - x$ ... (i)

To find: The maximum area of the land.

For this, Area of the land (A) = $l \times b = x \cdot y$

or, $A = x(30 - x)$ [$\because$ using i]

$\therefore A = 30x - x^2$ ... (ii)

Now, $\frac{dA}{dx} = \frac{d}{dx}(30x - x^2) = 30 - 2x$ [$\because$ using (i)]

For maxima or minima, $\frac{dA}{dx} = 0$

i.e. $30 - 2x = 0$

or, $30 = 2x$

$\therefore x = 15$m

And, $\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{dA}{dx}\right) = \frac{d}{dx}(30 - 2x) = 2 < 0$

Hence, the area of rectangular plot of land is maximum when the length

i.e. $x = 15$m

$\therefore$ The maximum area of the land from equation (ii) is

$A = 30 \times 15 - 15^2 = 450 - 225 = 225\text{m}^2$.

56. 2063 Q.No. 12a

Calculate the maximum and minimum values of $x^3 - 3x^2 - 9x + 27$. [4]

SOLUTION

Let $f(x) = x^3 - 3x^2 - 9x + 27$ ... (i)

Differentiating both sides w.r. to 'x', we get

$\frac{d}{dx}f(x) = \frac{d}{dx}(x^3 - 3x^2 - 9x + 27)$

i.e. $f'(x) = 3x^2 - 6x - 9$ ... (ii)

Again, differentiating both sides w.r. to 'x' we get

$f''(x) = 6x - 6$ ... (iii)

For the maximum or minimum values of $f(x)$,

$f'(x) = 0$

$\therefore 3x^2 - 6x - 9 = 0$

or, $3x^2 - 9x + 3x - 9 = 0$

or, $3x(x - 3) + 3(x - 3) = 0$

or, $(x - 3)(3x + 3) = 0$

$\therefore x = 3, -1$

When $x = 3$, $f''(x) = 6 \times 3 - 6 = 12 > 0$.

So $f(x)$ has a minimum value at $x = 3$ and the minimum value of $f(x)$ is

$f(3) = 3^3 - 3(3)^2 - 9 \times 3 + 27$

$= 27 - 27 - 27 + 27 = 0$

And when $x = -1$, $f''(x) = 6 \times (-1) - 6 = -12 < 0$.

So $f(x)$ has a maximum value at $x = -1$ and the maximum value of $f(x)$ is

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 27$

$= -1 - 3 + 9 + 27$

$= 32$

Thus, the maximum value of $f(x) = 32$ at $x = -1$,

And, the minimum value of $f(x) = 0$ at $x = 3$

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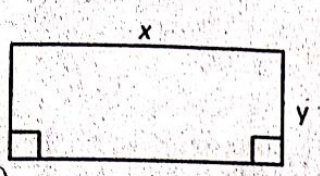
57. 2062 Q.No. 13a OR

Show that the rectangle of largest possible area for a given perimeter is a square. [4]

SOLUTION

Let x and y be the length and breadth, A be the area and P be the given perimeter of the given rectangle. Then



$\therefore P = 2(x + y)$

or, $\frac{P}{2} = x + y$

or, $y = \left(\frac{P}{2} - x\right)$ ... (i)

Now, $A = xy$

or, $A = x\left(\frac{P}{2} - x\right)$ [$\because$ Using (i)]

or, $A = \frac{Px}{2} - x^2$ ... (ii)

Diff. both sides w.r. to x, we get

$\frac{dA}{dx} = \frac{d}{dx}\left(\frac{Px}{2} - x^2\right) = \frac{P}{2} - 2x$

Again, diff. w.r. to x, we get

$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{P}{2} - 2x\right) = 0 - 2 = -2$

For maximia or minima, we must have $\frac{dA}{dx} = 0$

i.e. $\frac{P}{2} - 2x = 0$

or, $2x = \frac{P}{2}$

$\therefore x = \frac{P}{4}$

When $x = \frac{P}{4}$, $\frac{d^2A}{dx^2} = -2 < 0$

So, the area will be largest (maximum) when $x = \frac{P}{4}$

And, when $x = \frac{P}{4}$, from (i)

$y = \frac{P}{2} - x = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$

$\therefore x = y = \frac{P}{4}$

This shows that for the largest area of rectangle, length and breadth must be equal. Hence, the rectangle of largest possible area of the given perimeter is a square.

58. 2061 Q.No. 13a OR

A man wishes to fence a rectangular garden with 256 m fencing material. Find the maximum area he can enclose. [4]

SOLUTION

Let x and y be the length and breadth of the rectangular garden. Then,

$2(x + y) = 256$

or, $x + y = 128$

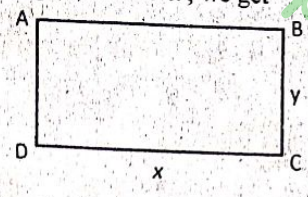
or, $y = 128 - x$ ... (i)

Let A be the area of the rectangular garden, then

$A = xy = x(128 - x)$

$\therefore A = 128x - x^2$ ... (ii)

Differentiating (ii) w.r. to 'x', we get



$\frac{dA}{dx} = \frac{d}{dx}(128x - x^2)$

$\therefore \frac{dA}{dx} = 128 - 2x$ ... (iii)

Again, differentiating (iii) w.r. to 'x', we get

$\frac{d^2A}{dx^2} = -2$ ... (iv)

For maximum area of the rectangular garden, we must have,

$\frac{dA}{dx} = 0$

i.e. $128 - 2x = 0$

$\Rightarrow x = 64$

When $x = 64$, $\frac{d^2A}{dx^2} = -2 < 0$

$\therefore$ The area of the garden is maximum when $x = 64$

Substituting the value of x in (i), we get

$y = 128 - 64 = 64$

Hence, the maximum area of the garden

$A = xy = 64 \times 64 = 4096\ m^2$

59. 2060 Q.No. 13a OR

Find the maximum and minimum value of the function $x^3 - 3x^2 + 6x + 5$, if exist. Also find the point of inflection. [4]

SOLUTION

Let $y = x^3 - 3x^2 + 6x + 5$ ... (i)

Diff. both sides w.r. to 'x', we get

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 6x + 5)$

or, $\frac{dy}{dx} = 3x^2 - 6x + 6$ ... (ii)

Again, diff. b.s. w.r. to 'x' we get

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(3x^2 - 6x + 6)$

or, $\frac{d^2y}{dx^2} = 6x - 6$ ... (iii)

And, $\frac{d^3y}{dx^3} = 6$ ... (iv)

For maxima or minima, $\frac{dy}{dx} = 0$

i.e. $3x^2 - 6x + 6 = 0$

or, $3(x^2 - 2x + 2) = 0$

or, $x^2 - 2x + 2 = 0$

or, $(x - 1)^2 + 1 = 0$, since $(x - 1)^2$ is positive, so it is not possible to find any real value of x.

So, given function has neither maxima nor minima.

For the point of inflection,

$\frac{d^2y}{dx^2} = 0$

or, $6x - 6 = 0$

or, $6x = 6$

$\therefore x = 1$

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60. 2059 Q.No. 13a OR

Show that the rectangle of largest possible area for a given perimeter, is a square. [4]

➥ Please refer to 2062 Q.No. 13a OR

61. 2058 Q.No. 13a OR

Show that the rectangle of largest possible area for a given perimeter is a square. [4]

➥ Please refer to 2062 Q.No. 13a OR

62. 2057 Q.No. 13a OR

Find the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$. Also find the point of inflection. [4]

SOLUTION

Given curve is $y = 4x^3 - 6x^2 - 9x + 1$ ... (i)

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 6x^2 - 9x + 1)$$

$$\frac{dy}{dx} = 12x^2 - 12x - 9 \quad \text{... (ii)}$$

Again, differentiating both sides with respect to 'x', we get

$$\frac{d^2y}{dx^2} = 24x - 12 \quad \text{... (iii)}$$

For maxima or minima $\frac{dy}{dx} = 0$

i.e. $12x^2 - 12x - 9 = 0$

or, $4x^2 - 4x - 3 = 0$

or, $4x^2 - 6x + 2x - 3 = 0$

or, $2x(2x-3) + 1(2x-3) = 0$

or, $(2x-3)(2x+1) = 0$

$\therefore x = \frac{3}{2}$ or $-\frac{1}{2}$

At $x = -\frac{1}{2}$

$$\frac{d^2y}{dx^2} = 24\left(-\frac{1}{2}\right) - 12 = -24 < 0$$

$\therefore$ The given function is maximum at $x = -\frac{1}{2}$.

Maximum value is

$$y_{max} = 4\left(-\frac{1}{2}\right)^3 - 6\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) + 1$$

$$= 4\left(-\frac{1}{8}\right) - 6\left(\frac{1}{4}\right) + \frac{9}{2} + 1$$

$$= -\frac{1}{2} - \frac{3}{2} + \frac{9}{2} + 1 = \frac{-1-3+9+2}{2} = \frac{7}{2}$$

At $x = \frac{3}{2}$

$$\frac{d^2y}{dx^2} = 24 \times \frac{3}{2} - 12 = 24 > 0$$

So, the given function is minimum at $x = \frac{3}{2}$ and its minimum value is

$$y_{min} = 4\left(\frac{3}{2}\right)^3 - 6\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 1$$

$$= 4 \times \frac{27}{8} - 6 \times \frac{9}{4} - \frac{27}{2} + 1$$

$$= \frac{27}{2} - \frac{27}{2} - \frac{27}{2} + 1 = -\frac{27}{2} + 1 = \frac{-27+2}{2} = \frac{-25}{2}$$

For the point of inflection,

$\frac{d^2y}{dx^2} = 0$

i.e. $24x - 12 = 0$

$\therefore x = \frac{1}{2}$

63. 2056 Q.No. 13a OR

Determine where the graph is concave upwards or concave downwards for $f(x) = x^4 - 8x^3 + 18x^2 - 24$. Also find the point of inflection. [4]

SOLUTION

Given function is $f(x) = x^4 - 8x^3 + 18x^2 - 24$

Diff. both sides with respect to 'x', we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^4 - 8x^3 + 18x^2 - 24).$$

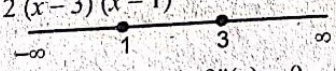
$\therefore f'(x) = 4x^3 - 24x^2 + 36x$

And $f''(x) = 12x^2 - 48x + 36$

$= 12(x^2 - 4x + 3)$

$= 12(x^2 - 3x - x + 3)$

$= 12(x-3)(x-1)$



For the points of inflection, $f''(x) = 0$

$12(x-3)(x-1) = 0$

$\therefore$ Required points of inflection are $x = 1$ and $x = 3$.

Intervals	Sign of $f''(x)$	Remarks
$(-\infty, 1)$	+ ve	Concave upwards
$(1, 3)$	− ve	Concave downwards
$(3, \infty)$	+ ve	Concave upwards

$\therefore$ $f(x)$ is concave upwards on $(-\infty, 1) \cup (3, \infty)$ and downwards on $(1, 3)$.

For point of inflection, $f''(x) = 0$

or, $12(x-3)(x-1) = 0$

or, $(x-3)(x-1) = 0$,

$\therefore x = 1, 3$

64. 2055 Old Model Q.No. 13a OR

Find the maximum and minimum values of the curve $y = 4x^3 - 6x^2 - 9x + 1$ on the interval $(-1, 2)$. Also find the point of inflection. [4]

SOLUTION

Given curve is $y = 4x^3 - 6x^2 - 9x + 1$ ... (i)

Differentiating both sides with respect to 'x', we get

$$\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 6x^2 - 9x + 1)$$

$$\frac{dy}{dx} = 12x^2 - 12x - 9 \quad \text{... (ii)}$$

Again, differentiating both sides with respect to 'x', we get

$$\frac{d^2y}{dx^2} = 24x - 12 \quad \text{... (iii)}$$

For maxima or minima $\frac{dy}{dx} = 0$

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i.e. $12x^2 - 12x - 9 = 0$
or, $4x^2 - 4x - 3 = 0$
or, $4x^2 - 6x + 2x - 3 = 0$
or, $2x(2x-3) + 1(2x-3) = 0$
or, $(2x-3)(2x+1) = 0$
$\therefore x = \frac{3}{2}$ or $-\frac{1}{2}$

Clearly the values of x lie in the interval $(-1, 2)$

At $x = -\frac{1}{2}$

$\frac{d^2y}{dx^2} = 24\left(-\frac{1}{2}\right) - 12 = -24 < 0$

$\therefore$ The given function is maximum at $x = -\frac{1}{2}$.

Maximum value is

$y_{max} = 4\left(-\frac{1}{2}\right)^3 - 6\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) + 1$

$= 4\left(-\frac{1}{8}\right) - 6\left(\frac{1}{4}\right) + \frac{9}{2} + 1$

$= -\frac{1}{2} - \frac{3}{2} + \frac{9}{2} + 1 = \frac{-1-3+9+2}{2} = \frac{7}{2}$

At $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = 24 \times \frac{3}{2} - 12 = 24 > 0$

So, the given function is minimum at $x = \frac{3}{2}$ and its minimum value is

$y_{min} = 4\left(\frac{3}{2}\right)^3 - 6\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 1$

$= 4 \times \frac{27}{8} - 6 \times \frac{9}{4} - \frac{27}{2} + 1$

$= \frac{27}{2} - \frac{27}{2} - \frac{27}{2} + 1$

$= -\frac{27}{2} + 1 = \frac{-27+2}{2} = \frac{-25}{2}$

Further, for the point of inflection,

$\frac{d^2y}{dx^2} = 0$

i.e. $24x - 12 = 0$

$\Rightarrow x = \frac{1}{2}$ which is the point of inflection.

B. RATE MEASURE

2 MARKS QUESTIONS

1. 2075 Set C Q.No. 5c

Find the rate of change of the volume of a cylinder of radius r and height h will respect to a change in the radius. [2]

SOLUTION

Let V be the volume of cylinder.

We have, $V = \pi r^2 h$.

Now,

$\frac{dV}{dr} = \frac{d}{dr}(\pi r^2 h)$

$= \pi h \frac{d}{dr}(r^2)$ [$\because$ h is constant]

$= 2\pi rh$

Therefore, the rate of change of V with respect to a change in radius is $2\pi rh$.

2. 2073 Supp Q.No. 5c

A stone thrown into a pond produces circular ripples which expand from the point of impact. If the radius of the ripple increases at the rate of 3.5 cm/sec, how fast is the area growing when the radius is 15cm? ($\pi = \frac{22}{7}$) [2]

SOLUTION

Let r be the radius and A be the area of a circular ripple in time t. By given,

$\frac{dr}{dt} = 3.5$ cm/sec

We have, $A = \pi r^2$

$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

When r = 15 cm, then

$\therefore \frac{dA}{dt} = \pi \times 2 \times 15 \times 3.5 = \frac{22}{7} \times 30 \times 3.5 = 330$ cm^2/sec

3. 2072 Supp. Q.No. 5c

A stone thrown into a pond produces a circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5cm/sec, how fast is the area growing when the radius is 15cm? ($\pi = \frac{22}{7}$) [2]

➥ Please refer to 2073 Supp Q.No. 5c

4. 2072 Set E Q.No. 5b

Find the rate of change of volume of a cylinder of radius r and height h with respect to a change in radius. [2]

➥ Please refer to 2075 Set C Q.No. 5c

5. 2070 Set D Q.No. 5 c

A stone thrown into a pond produces circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5 cm/sec, find how fast is the area growing when the radius is 15 cm. (π = 22/7) [2]

➥ Please refer to 2073 Supp Q.No. 5c

6. 2069 [Set A] Q. No. 5c

The side of a square sheet is increasing at the rate of 5cm/min. At what rate is the area increasing when the side is 12cm. long? [2]

SOLUTION

Let x be the length of side and A be the area of the square at time t. By question,

$\frac{dx}{dt} = 5$cm/min

When $x = 12$ cm, $\frac{dA}{dt} = ?$

We have, $A = x^2$

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

When $x = 12$ cm, $\frac{dA}{dt} = 2 \times 12 \times 5 = 120$ cm^2/min

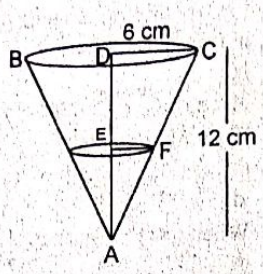
$\therefore \frac{dA}{dt} = 120$ cm^2/min

6 MARKS QUESTIONS

7. 2076 Set B Q.No. 15 OR

Water flows into an inverted conical tank at the rate of 42cm³/sec. When the depth of the water is 8cm, how fast is the level rising? Assume that the height of the tank is 12cm and the radius of the top is 6cm. [6]

SOLUTION



Let ABC be the conical water tank into which water is flowing. At a certain time t, let h be the height AE of the water and EF be the radius of water surface.

Since,

$\Delta ACD \sim \Delta AFE$, so

$\frac{AE}{AD} = \frac{EF}{DC}$

or, $\frac{AE}{12} = \frac{r}{6}$

$\therefore r = \frac{h}{2}$

By question, $\frac{dv}{dt} = 42\ cm^3/sec$

When $h = 8$ cm, $\frac{dh}{dt} = ?$

Let V be the volume of water in the tank.

We have,

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt} = \frac{\pi h^2}{4} \cdot \frac{dh}{dt}$

When $h = 8$ cm

$42 = \pi \times \frac{8^2}{4} \times \frac{dh}{dt}$

or, $\frac{42 \times 4}{64 \times \pi} = \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{21}{8\pi}$ cm/sec

8. 2076 Set C Q.No. 15 OR

The volume of a spherical balloon is increasing at the rate of 25cm³/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

SOLUTION

Let V be the volume, r be the radius and A be the area of a spherical balloon at time t.

By given,

$\frac{dv}{dt} = 25\ cm^3/sec$

When $r = 5$ cm, $\frac{dA}{dt} = ?$

We have, $V = \frac{4}{3}\pi r^3$

or, $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

or, $25 = 4\pi r^2 \frac{dr}{dt}$

or, $\frac{dr}{dt} = \frac{25}{4\pi r^2}$ ...(i)

Again, we have

$A = 4\pi r^2$

$\frac{dA}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt}$

$= 4\pi \cdot 2r \cdot \frac{25}{4\pi r^2}$ ...[using (i)]

$= \frac{50}{r}$

When $r = 5$ cm, $\frac{dA}{dt} = \frac{50}{5} = 10\ cm^2/sec$

Hence, the surface is increasing at the rate of 10 cm^2/sec.

9. 2075 Set A Q.No. 15 OR

A man 150 cm tall, walks away from a source of light situated at the top of a pole 5 m high at the rate of 0.7 m/s. Find the rate at which [6]

a. his shadow is lengthening.

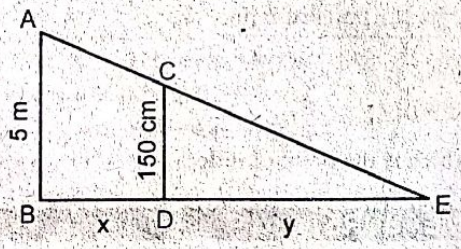
b. the tip of his shadow is moving, when he is 2m away from the pole.

SOLUTION

a. Let AB be a pole and CD be the man. Let BD = x and DE = y.

Given,

$\frac{dx}{dt} = 0.7$ m/s, AB = 5m, CD = 150 cm.



Since,

$\Delta ABE \sim \Delta CDE$,

$\frac{AB}{CD} = \frac{BE}{DE}$

or, $\frac{500\ cm}{150\ cm} = \frac{x+y}{y}$

or, $\frac{10}{3} = \frac{x+y}{y}$

or, $10y = 3x + 3y$

or, $3x = 7y$

or, $\frac{d}{dt}(7y) = \frac{d}{dt}(3x)$

or, $7\frac{dy}{dt} = 3\frac{dx}{dt}$

$\therefore \frac{dy}{dt} = \frac{3}{7} \times 0.7 = 0.3$ m/sec

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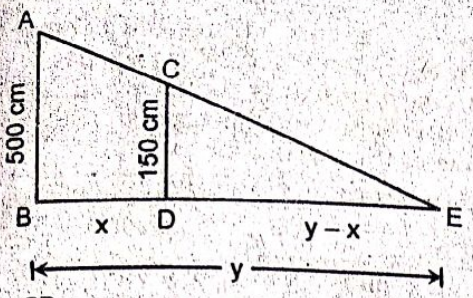
b. By similarity of ΔABE and ΔCDE,



$$\frac{DE}{BE} = \frac{CD}{AB}$$

or, $\frac{y-x}{y} = \frac{150}{500}$

or, $\frac{y-x}{y} = \frac{3}{10}$

or, $10y - 10x = 3y$

or, $7y = 10x$

Diff. both sides w.r.t. 't'

$$7\frac{dy}{dt} = 10 \cdot \frac{dx}{dt}$$

$$\Rightarrow 7\frac{dy}{dx} = 10 \times 0.7$$

$$\Rightarrow \frac{dy}{dx} = 1$$

When, $x = 2m$, $\frac{dy}{dx} = 1$

Hence, required rate at which the tip of his shadow is moving = 1 m/sec.

10. 2075 Set B Q.No. 15 OR

The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

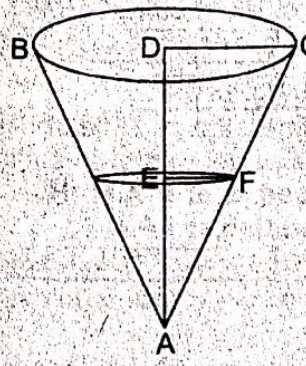
Please refer to 2076 Set C Q.No. 15 OR

11. 2075 Set C Q.No. 15 OR

Water flows into an inverted conical tank at the rate of 24cm^3/min. When the depth of water is 9 cm, how fast is the level rising? Assume that the height of the tank is 15 cm and the radius at the top is 5 cm. [6]

SOLUTION

Let ABC be the conical water tank into which water is flowing at the rate of 24 cm^3/min. At a certain time t, let h be the height AE of the water and r be the radius EF of the water surface. From the figure, we see that ΔACD and ΔAEF are similar.



So, $\frac{AE}{AD} = \frac{EF}{CD}$

or, $\frac{h}{15} = \frac{r}{5}$

$\therefore r = \frac{h}{3}$

Let V be the volume of water in the tank in time t.

We have,

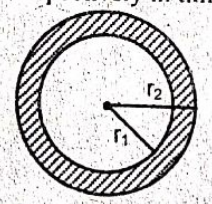
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 . h = \frac{1}{27}\pi h^3$$

12. 2074 Supp Q.No. 15 OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 10 cm/sec and that of the outer circle at the rate of 7 cm/sec. At a certain time, the radii of the inner and the outer circles are respectively 24 cm and 30 cm. At that time, is the area between the circles increasing or decreasing? [6]

SOLUTION

Let r_1 and r_2 be the radius of inner and outer circle respectively in time t.



By given,

$\frac{dr_1}{dt} = 10$ cm/sec

and $\frac{dr_2}{dt} = 7$ cm/sec

If A be the area between the outer and inner circles at time t, then

A = area of outer circle – area of inner circle

$= \pi r_2^2 - \pi r_1^2$

$$\frac{dA}{dt} = 2\pi r_2 \frac{dr_2}{dt} - 2\pi r_1 \frac{dr_1}{dt}$$

When, $r_1 = 24$ cm and $r_2 = 30$ cm

$$\frac{dA}{dt} = 2\pi \times 30 \times 7 - 2\pi \times 24 \times 10$$

$= -60\pi$ cm^2/sec

This shows that the area between the circles is decreasing at the rate of 60π cm^2/sec.

13. 2074 Set A Q.No. 15 OR

A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75 cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm. [6]

SOLUTION

Let r be the radius and V be the volume of the spherical ball of salt in time t. By given,

$\frac{dV}{dt} = 0.75$ cm^3/min

We have,

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$$V = \frac{4}{3}\pi r^3$$

or, $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

or, $0.75 = 4\pi r^2 \frac{dr}{dt}$

or, $\frac{3}{4} = 4\pi r^2 \frac{dr}{dt}$

or, $\frac{dr}{dt} = \frac{3}{16\pi r^2}$

When r = 6 cm, $\frac{dr}{dt} = \frac{3}{16\pi \times 6^2} = \frac{1}{192\pi}$ cm/min

14. 2074 Set B Q.No. 15 OR

The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of the i) perimeter of the square and ii) area of square when the side of the square is 12 cm. [6]

SOLUTION

Let x be the length of side, A be the area and P be the perimeter of the square in time t. By given

$\frac{dx}{dt} = 0.2$ cm/sec

i. We know

$P = 4x$

$\frac{dP}{dt} = 4 \cdot \frac{dx}{dt}$

When $x = 12$ cm, $\frac{dP}{dt} = 4 \times 0.2 = 0.8$ cm/sec

∴ The perimeter is increasing at the rate of 0.8 cm/sec.

ii. We have

$A = x^2$

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

When $x = 12$ cm, $\frac{dA}{dt} = 2 \times 12 \times 0.2 = 4.8$ cm^2/sec.

∴ The area is increasing at the rate of 4.8 cm^2/sec.

15. 2073 Set C Q.No. 15 OR

If the radius of a circle increases at a uniform rate, prove that its area will increase at a rate which varies as its radius. [6]

SOLUTION

Let r be the radius and A be the area of the circle at time t. By question,

$\frac{dr}{dt} = k$, where k is a constant

We have,

$A = \pi r^2$

$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = 2\pi r \frac{dr}{dt}$

$= 2\pi r \cdot k = (2\pi k)\, r$

$=$ (constant) r

∴ $\frac{dA}{dt} \propto r$.

Hence, the area will increase at a rate which varies as its radius.

16. 2073 Set D Q.No. 15 OR

Water flows into an inverted conical tank at the rate of 42cm^3/sec. When the depth of water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

➥ Please refer to 2076 Set B Q.No. 15 OR

17. 2072 Set E Q.No. 15 OR

A ship sailing east at 15 km/hr passed a point A at 11 AM. A second ship sailing south at 10 km/hr passed A at 9 AM. How fast was the ships separating at 1 PM? [6]

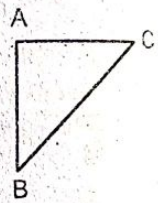
SOLUTION

Let A be the position of the port. Again, let B and C be the position of the ships at 1 PM. Then, distance travelled by the first ship in 2 hrs (11 AM to 1 PM)

AB = 15 × 2 = 30 km

And, distance travelled by second ship in 4 hrs (9 AM to 1 PM)

AC = 10 × 4 = 40 km



Now, BC $= \sqrt{AB^2 + AC^2}$

$= \sqrt{30^2 + 40^2} = 50$

Let BC = x km in time t.

From figure,

$x^2 = AB^2 + AC^2$

$2x\frac{dx}{dt} = 2AB\frac{d(AB)}{dt} + 2AC\frac{d(AC)}{dt}$

When $x = 50$,

$2 \times 50 \times \frac{dx}{dt} = 2 \times 30 \times 15 + 2 \times 40 \times 10$

or, $100\frac{dx}{dt} = 900 + 800$

or, $\frac{dx}{dt} = \frac{1700}{100} = 17$ km/hr

∴ The ships were separating at the rate of 17 km/hr at 1 PM.

18. 2072 Set C Q.No. 15 OR

A spherical ball of salt dissolving in water decreases its value at the rate of 0.75 cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm. [6]

➥ Please refer to 2074 Set A Q.No. 15 OR

19. 2072 Set D Q.No. 15 OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate 10 cm/sec and that of the outer circle at the rate of 7 cm/sec. At a certain instant, the radii of the inner and the outer circles are respectively 24 cm and 30 cm. At that time, is the area between the circles increasing or decreasing? How fast? [6]

➥ Please refer to 2074 Supp Q.No. 15 OR

20. 2071 Supp. Q.No. 15 OR

A ship sailing east at 15 km/hr passed a point A at 11 A.M. A second ship sailing south at 10 km/hr passed A at 9 A.M. How fast were the ships separating at 1 P.M.? [6]

➥ Please refer to 2072 Set E Q.No. 15 OR

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21. 2071 Set C Q.No. 15 Or

Water flows into an inverted conical tank at the rate of 42 cm^3/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

➥ Please refer to 2076 Set B Q.No. 15 OR

22. 2071 Set D Q.No. 15 Or

The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of the (i) perimeter of the square, and (ii) the area of the square, when the side of square is 12 cm. [6]

➥ Please refer to 2074 Set B Q.No. 15 OR

23. 2070 Supp Q.No. 15 Or

Water flows into an inverted conical tank at the rate of 42cm^3/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

➥ Please refer to 2076 Set B Q.No. 15 OR

24. 2070 Set C Q.No. 15 Or

A spherical ball of salt is dissolving in water in such a way that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at the constant rate. [6]

SOLUTION

Let r be the radius, V be the volume and be S be the surface area of the spherical ball of salt at time t. By question,

$$\frac{dV}{dt} \propto S$$

or, $$\frac{dV}{dt} = -ks \quad \ldots(i)$$

where –k is a constant.

We have, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

Then (i) can be written as

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k \cdot 4\pi r^2$$

or, $$\frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = -k \cdot 4\pi r^2$$

or, $\frac{dr}{dt} = -k$ which is a constant.

∴ The radius is decreasing at a constant rate.

25. 2069 Supp Q.No. 15 OR

If the volume of an expanding cube is increasing at the rate of 5cm^3/min, how fast is its surface area increasing when the surface area is 24 sq cm? [6]

SOLUTION

Let V be the volume, x be the length of side and A be the surface area of an expanding cube at time t. By question,

$$\frac{dV}{dt} = 5 \text{ cm}^3/\text{min}$$

When $A = 24$ cm^2, $\frac{dA}{dt} = ?$

We have,

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

or, $$5 = 3x^2 \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{5}{3x^2} \quad \ldots(i)$$

Again, we have

$$A = 6x^2$$

$$\frac{dA}{dt} = 6 \cdot 2x \cdot \frac{dx}{dt}$$

$$= 12x \cdot \frac{5}{3x^2} \text{ [using (i)]}$$

$$= \frac{20}{x}$$

When $A = 24$

i.e. $6x^2 = 24$

$\Rightarrow x = 2, \frac{dA}{dt} = \frac{20}{2} = 10$

Hence, the surface area is increasing at the rate of 10 cm^2/min.

26. 2069 [Set B] Q. No. 15 Or

The volume of a spherical balloon is increasing at the rate of 25 cubic cm/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

➥ Please refer to 2076 Set C Q.No. 15 OR

27. 2068 Q.No. 15(Or)

A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6cm. [6]

➥ Please refer to 2074 Set A Q.No. 15 OR

4 MARKS QUESTIONS [OLD SYLLABUS QUESTIONS]

28. 2068 Q.No. 10b Or

A spherical ball of salt is dissolving in water in such a manner that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate. [4]

SOLUTION

Let r be the radius, V be the volume and be S be the surface area of the spherical ball of salt at time t. By question,

$$\frac{dV}{dt} \propto S$$

or, $$\frac{dV}{dt} = -ks \quad \ldots(i)$$

where –k is a constant.

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We have, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

Then (i) can be written as

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k \cdot 4\pi r^2$$

or, $\frac{4}{3}\pi \cdot 3r^2\frac{dr}{dt} = -k \cdot 4\pi r^2$

or, $\frac{dr}{dt} = -k$ which is a constant.

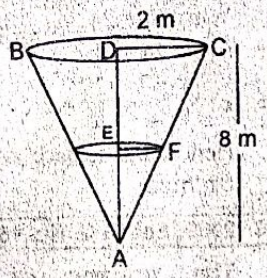
∴ The radius is decreasing at a constant rate.

29. 2067 Q.No. 10b OR

Water flows into an inverted conical vessel at the rate of 24 m^3/m. When the depth of water depth of water is 4 m, how fast is the level rising, assuming that the height of the vessel is 8 m and the radius at the top is 2 m? [4]

SOLUTION

Let ABC be the conical tank into which water is flowing. Let h be the height AE of the water and r be the radius EF of the water surface at time t. Now,



ΔACD and ΔAFE are similar.

So, $\frac{AE}{AD} = \frac{EF}{DC}$

or, $\frac{h}{8} = \frac{r}{2}$

⇒ $r = \frac{h}{4}$

Let V be the volume of water in the tank. By given, $\frac{dV}{dt} = 24 m^3/m$. Then,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 \cdot h = \frac{\pi h^3}{48}$$

$$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2\frac{dh}{dt} = \frac{\pi h^2}{16} \cdot \frac{dh}{dt}$$

When h = 4m, $24 = \frac{\pi \times 4^2}{16} \times \frac{dh}{dt}$

∴ $\frac{dh}{dt} = \frac{24}{\pi}$

Hence, the level is rising at the rate of $\frac{24}{\pi}$ m/min.

30. 2066 Q.No. 10 b OR

If the area of a circle increases at a uniform rate prove that the rate of increase of the perimeter varies inversely as the radius. [4]

SOLUTION

Let r be the radius, A be the area and P be the perimeter of the circle at time t. We have,

$A = \pi r^2$ and $P = 2\pi r$

By question, $\frac{dA}{dt} = k$ where k is a constant.

Now, $P^2 = (2\pi r)^2 = 4\pi^2 r^2 = 4\pi(\pi r^2) = 4\pi A$

$$\Rightarrow 2P\frac{dP}{dt} = 4\pi\frac{dA}{dt}$$

$$\Rightarrow \frac{dP}{dt} = \frac{4\pi}{2P} \cdot \frac{dA}{dt}$$

$$= \frac{4\pi}{2 \cdot 2\pi r} \cdot \frac{dA}{dt} = \frac{1}{r} \cdot k$$

$$\Rightarrow \frac{dP}{dt} \propto \frac{1}{r}$$

Hence, the rate of increase of the perimeter varies inversely as the radius.

31. 2066 C Q.No. 10 b OR

If the volume of an expanding cube is increasing at the rate of 4ft^3/min, how fast is its surface area increasing when the surface area is 24 sq.ft.? [4]

SOLUTION

Let x be the length of side, V be the volume and S be the surface area at time t.

By question, $\frac{dV}{dt} = 4ft^3/min$

When S = 24 sq.ft, $\frac{dS}{dt} = ?$

We have, $V = x^3$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

or, $4 = 3x^2\frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = \frac{4}{3x^2} \quad \ldots(i)$$

Again, we have, $S = 6x^2$

$$\frac{dS}{dt} = 6 \cdot 2x \cdot \frac{dx}{dt}$$

$$= 12x \times \frac{4}{3x^2} \text{ [using (i)]}$$

$$= \frac{16}{x}$$

When S = 24 i.e. $6x^2 = 24 \Rightarrow x = 2$, $\frac{dS}{dt} = \frac{16}{2} = 8$ ft/min

Hence, the surface area is increasing at the rate of 8 ft^2/min.

32. 2065 Q.No 10 b OR

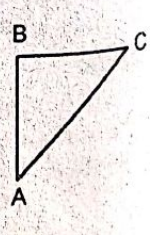
An aeroplane is flying horizontally at a height of $\frac{2}{3}$ mile with a velocity of 15m.p.h. Find the rate at which it is receding from a fixed point on the ground which it passed over two minutes ago. [4]

SOLUTION

Let A be the fixed point on the ground. Let B be the position of aeroplane flying at a height of $\frac{2}{3}$ mile from A i.e. AB = $\frac{2}{3}$ mile.

Let B be the position of aeroplane at time t. Distance travelled in 2 minutes

$= 15 \times \frac{2}{60} = \frac{1}{2}$ mile



Now, $AC^2 = AB^2 + BC^2$

or, $AC^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^2$

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$$= \frac{4}{9} + \frac{1}{4} = \frac{25}{36}$$

$$\therefore AC = \frac{5}{6}$$

At any time t, let BC = x and AC = y. Then,

$$y^2 = \left(\frac{2}{3}\right)^2 + x^2$$

$$\frac{dy^2}{dt} = 0 + \frac{dx^2}{dt}$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$y\frac{dy}{dt} = x\frac{dx}{dt}$$

When, $x = BC = \frac{1}{2}$, $\frac{dx}{dt} = 15$ mph and $y = \frac{5}{6}$, we have

$$\frac{5}{6}\cdot\frac{dy}{dt} = \frac{1}{2} \times 15$$

$$\therefore \frac{dy}{dt} = 9 \text{ mph}$$

33. 2064 Q.No. 10 b OR

A stone thrown into a pond produces circular ripples which expand from the point of impact. If the radius of the ripple increases at the rate of 25 cm/sec, how fast is the (i) perimeter (ii) area increasing when the radius is 80 cm? [4]

SOLUTION

Let r be the radius, P be the perimeter and A be the area of the circular ripples at time t.

By given, $\frac{dr}{dt} = 25$ cm/sec

i. We have, $P = 2\pi r$

$$\frac{dP}{dt} = 2\pi\frac{dr}{dt} = 2\pi \times 25 = 50\pi$$

Hence, perimeter is increasing at the rate of 50π cm/sec.

ii. $A = \pi r^2$

$$\frac{dA}{dt} = \pi.2r.\frac{dr}{dt}$$

When r = 80 cm,

$$\frac{dA}{dt} = 2\pi \times 80 \times 25 = 4000\,\pi \text{ cm}^2/\text{sec}$$

$\therefore$ Area is increasing at the rate of $4000\,\pi$ cm^2/sec.

34. 2063 Q.No. 10 b OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 6 cm/sec and that of the outer circle at the rate of 2.5 cm/sec. At a certain time the radius of the inner and the outer circles are respectively 20 cm and 32 cm. At that time how fast is the area between the circle increasing or decreasing? [4]

SOLUTION

Let r_1 be the radius of inner circle and r_2 be the radius of the outer circles at time t. By question,

$\frac{dr_1}{dt} = 6$ cm/sec and

$\frac{dr_2}{dt} = 2.5$ cm/sec

Let A be the area between the circles at time t. Then,

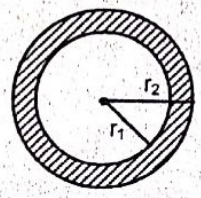
A = area of outer circle – area of inner circle

$$= \pi r_2^2 - \pi r_1^2$$

$$\frac{dA}{dt} = \pi \cdot 2r_2 \cdot \frac{dr_2}{dt} - \pi \cdot 2r_1 \cdot \frac{dr_1}{dt}$$

or, $\frac{dA}{dt} = 2\pi r_2 \times 2.5 - 2\pi r_1 \times 6$

$$= 5\pi r_2 - 12\pi r_1$$



When $r_1 = 20$ cm and $r_2 = 32$ cm

$$\frac{dA}{dt} = 5\pi \times 32 - 12\pi \times 20$$

$$= 160\pi - 240\pi$$

$$= -80\pi$$

Hence, the area between the circles is decreasing at the rate of 80π cm^2/sec.

35. 2062 Q.No. 10 b OR

A circular plate of metal expands by heat so that its radius increases at the rate of 0.25 cm/sec. Find the rate at which the surface is increasing when the radius is 7cm. [4]

SOLUTION

Let r be the radius and S be the surface area of circular plate. By question,

$\frac{dr}{dt} = 0.25$ cm/sec

When r = 7 cm, $\frac{dS}{dt} = ?$

We have,

$S = \pi r^2$

$$\frac{dS}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

$$= \pi \times 2r \times 0.25$$

$$= 0.5\,\pi r$$

When r = 7, $\frac{dS}{dt} = 0.5\pi \times 7 = 3.5\pi$ cm^2/sec

36. 2061 Q.No. 10 b OR

If the volume of an expanding cube is increasing at the rate of 4 ft^3/min, how fast is its surface area increasing when the surface area is 24 sq. ft.? [4]

➥ Please refer to 2066 C Q.No. 10 b OR

37. 2060 Q.No. 10 b OR

A spherical balloon is inflated at the rate of 10 cubic cm/sec. At what rate is the radius increasing when the radius is 6 cm. [4]

SOLUTION

Let r be the radius and V be the volume of spherical balloon at time t. By question,

$\frac{dV}{dt} = 10$ cm^3/sec

When r = 6 cm, $\frac{dr}{dt} = ?$

We have, $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

When r = 6 cm

$10 = \frac{4}{3}\pi \times 3 \times 6^2 \times \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{5}{72\pi}$ cm/sec

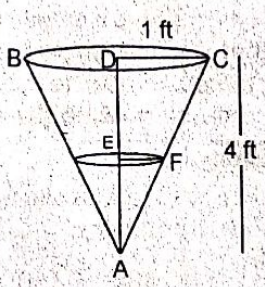
The radius is increasing at the rate of $\frac{5}{72\pi}$ cm/sec

38. 2059 Q.No. 10 b OR

Water flows into an inverted conical tank at the rate of 27ft³/min. When the depth of water is 2ft, how fast is the level rising? Assume that the height of the tank is 4ft and the radius of the top is 1 ft. [4]

SOLUTION

Let ABC be the conical tank into which water is flowing at the rate of 27 ft³/min. Let h be the height AE of the water surface and r be the radius EF of the water surface at time t. Now ΔACD and ΔAFE are similar. So,



$\frac{AE}{AD} = \frac{EF}{DC}$

or, $\frac{h}{4} = \frac{r}{1}$

$\therefore r = \frac{h}{4}$

Let V be the volume of water in the tank. We have,

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h = \frac{1}{48}\pi h^3$

$\therefore \frac{dV}{dt} = \frac{1}{48}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$

$= \frac{\pi}{16} \cdot h^2 \cdot \frac{dh}{dt}$

When h = 2ft,

$27 = \frac{\pi}{16} \times 2^2 \times \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{108}{\pi}$ ft/min.

39. 2058 Q.No. 10 b OR

A spherical ballon is inflated at the rate of 10 cubic cm/sec. At what rate is the radius increasing when the radius is 10 cm. [4]

SOLUTION

let r be the radius and V be the volume of the spherical balloon at time t. Then,

$\frac{dV}{dt} = 10$ cm³/sec

When r = 10 cm, $\frac{dr}{dt} = ?$

We have, $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3} \cdot \pi \cdot 3r^2 \cdot \frac{dr}{dt}$

or, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

When r = 10 cm.

$10 = 4\pi \times 10^2 \times \frac{dr}{dt}$

or, $\frac{1}{40\pi} = \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{1}{40\pi}$ cm/sec.

40. 2057 Q.No. 10 b OR

A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x–coordinate is increasing at the rate of 4 ft/sec. Find the rate at which the distance of the point from the origin is increasing when the point is at (2, 4). [4]

SOLUTION

Let P(x,y) be a point moving along the curve $y = 2x^3 - 3x^2$. By given, $\frac{dx}{dt} = 4$ft/sec.

Again, $\frac{dy}{dt} = \frac{d}{dt}(2x^3 - 3x^2)$

$= (6x^2 - 6x)\frac{dx}{dt}$

$= (6x^2 - 6x) \times 4$

$= 24x(x - 1)$

When (x, y) = (2,4), $\frac{dy}{dt} = 24 \times 2(2 - 1) = 48$

Let S be the distance between the origin O(0,0) and the point P(x,y). Then,

$S = \sqrt{x^2 + y^2}$

$\frac{dS}{dt} = \frac{d}{dt}(x^2 + y^2)^{1/2}$

$= \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$

$= \frac{1}{\sqrt{x^2 + y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$

When (x,y) = (2,4)

$\frac{dS}{dt} = \frac{1}{\sqrt{2^2 + 4^2}}(2 \times 4 + 4 \times 48)$

$= \frac{1}{\sqrt{20}}(200)$

$= \frac{1}{2\sqrt{5}} \times 200$

$\therefore \frac{dS}{dt} = 20\sqrt{5}$ ft/sec.

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15 UNIT

ANTI–DERIVATIVES AND ITS APPLICATIONS

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A. INDEFINITE INTEGRALS

2 MARKS QUESTIONS

1. 2076 Set B Q.No. 5b

Evaluate: $\int \sin 2x.\ \sin 3x.\ dx.$ [2]

SOLUTION

$$\int \sin 2x.\ \sin 3x.\ dx$$

$$= \frac{1}{2}\int 2\sin 3x \sin 2x\ dx$$

$$= \frac{1}{2}\int \{\cos(3x-2x) - \cos(3x+2x)\}dx$$

$$= \frac{1}{2}\int (\cos x - \cos 5x)dx$$

$$= \frac{1}{2}\left[\int \cos x\ dx - \int \cos 5x\ dx\right]$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right] + c$$

$$= \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + c$$

2. 2076 Set C Q.No. 5b

Evaluate: $\int \log x\ dx.$ [2]

SOLUTION

Here, $I = \int \log x\ dx = \int 1.\ \log x\ dx$

$$= \log x \int 1dx - \int \left[\frac{d}{dx}(\log x).\int 1.dx\right] dx$$

$$= x\log x - \int \frac{1}{x}.x\ dx = x\log x - \int 1.\ dx$$

$$= x\log x - x + C$$

3. 2075 GIE Q.No. 5b

Evaluate: $\int \frac{\log x}{x} dx.$ [2]

SOLUTION

Let $I = \int \frac{\log x}{x} dx$

Put $y = \log x$

$dy = \frac{1}{x} dx$

Then,

$$I = \int y\ dy$$

$$= \frac{y^2}{2} + C$$

$$= \frac{1}{2}(\log x)^2 + C$$

4. 2075 Set A Q.No. 5c

Evaluate: $\int \frac{x+2}{x^2-x-6} dx.$ [2]

SOLUTION

$$\int \frac{x+2}{x^2-x-6} dx$$

$$= \int \frac{x+2}{x^2-3x+2x-6} dx$$

$$= \int \frac{x+2}{x(x-3)+2(x-3)} dx$$

$$= \int \frac{x+2}{(x+2)(x-3)} dx$$

$$= \int \frac{1}{x-3} dx$$

$$= \frac{1}{1}.\log(x-3) + C \quad \left[\because \int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b) + C\right]$$

$$= \log(x-3) + C$$

5. 2075 Set B Q.No. 5b

Evaluate: $\int \frac{x^2+1}{x+1} dx.$ [2]

SOLUTION

$$\int \frac{x^2+1}{x+1} dx$$

$$= \int \frac{x^2-1+1+1}{x+1}\,dx$$

$$= \int \left(\frac{x^2-1}{x+1}+\frac{2}{x+1}\right)dx$$

$$= \int \left\{\frac{(x-1)(x+1)}{x+1}+\frac{2}{x+1}\right\}dx$$

$$= \int \left(x-1+\frac{2}{x+1}\right)dx$$

$$= \int x\,dx - \int dx + 2\int \frac{1}{x+1}dx$$

$$= \frac{x^2}{2} - x + 2\log(x+1) + C.$$

6. 2076 Set C Q.No. 5b

Evaluate: $\int \frac{x+3}{x-3}\,dx$. [2]

SOLUTION

$$\int \frac{x+3}{x-3}\,dx$$

$$= \int \frac{x-3+3+3}{x-3}\,dx$$

$$= \int \left(\frac{x-3}{x-3}+\frac{6}{x-3}\right)dx$$

$$= \int dx + 6\int \frac{1}{x-3}\,dx$$

$$= x + 6.\frac{1}{1}.\log(x-3) + C$$

$$\left[\because \int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)+c\right]$$

$$= x + 6\log(x-3) + C.$$

7. 2074 Supp Q.No. 5b

Evaluate: $\int xe^x dx$ [2]

SOLUTION

$$\int xe^x dx = x\int e^x dx - \int \left\{\frac{d}{dx}(x)\int e^x dx\right\}dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

8. 2074 Set A Q.No. 5b

Evaluate: $\int (3\sin x - 4)^2 \cos x\,dx$ [2]

SOLUTION

Let $I = \int (3\sin x - 4)^2 \cos x\,dx$

Put $y = 3\sin x - 4$

$\frac{dy}{dx} = 3\cos x$

$dy = 3\cos x\,dx$

$\frac{dy}{3} = \cos x\,dx$

Then,

$$I = \int y^2.\frac{dy}{3}$$

$$= \frac{1}{3}\int y^2 dy$$

$$= \frac{1}{3}.\frac{y^3}{3} + c$$

$$= \frac{y^3}{9} + c$$

$$= \frac{1}{9}(3\sin x - 4)^3 + c$$

9. 2074 Set B Q.No. 5b

Evaluate: $\int x\log x\,dx$. [2]

SOLUTION

$$\int x\log x\,dx$$

$$= \int \log x.\,x\,dx$$

$$= \log x\int x\,dx - \int \left\{\frac{d}{dx}(\log x)\right\}\int x\,dx\,dx$$

$$= \log x.\frac{x^2}{2} - \int \left(\frac{1}{x}.\frac{x^2}{2}\right)dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}.\frac{x^2}{2} + c$$

$$= \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

10. 2073 Supp Q.No. 5b

Evaluate: $\int \left(1+\frac{1}{x^2}\right)e^{x-\frac{1}{x}}\,dx$. [2]

SOLUTION

Let $I = \int \left(1+\frac{1}{x^2}\right)e^{x-\frac{1}{x}}\,dx$.

Put $y = x - \frac{1}{x}$

Then, $\frac{dy}{dx} = 1 + \frac{1}{x^2}$

$$dy = \left(1+\frac{1}{x^2}\right)dx$$

$$\therefore I = \int e^y dy = e^y + c$$

$$= e^{x-\frac{1}{x}} + c$$

11. 2073 Set C Q.No. 5b

Evaluate: $\int \frac{x\,dx}{(1-x^2)^{3/2}}$. [2]

SOLUTION

Let $I = \int \frac{x\,dx}{(1-x^2)^{3/2}}$

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Put $y = 1 - x^2$

$dy = -2x\,dx$

$\frac{dy}{-2} = x\,dx$

Now, $I = \int \frac{-dy/2}{y^{3/2}} = -\frac{1}{2}\int y^{-3/2}\,dy$

$= -\frac{1}{2}\frac{y^{-3/2+1}}{\left(\frac{-3}{2}+1\right)} + C = -\frac{1}{2}\frac{y^{-1/2}}{\left(\frac{-1}{2}\right)} + C$

$= \frac{1}{y^{1/2}} + C = \frac{1}{\sqrt{y}} + C = \frac{1}{\sqrt{1-x^2}} + C$

12. 2073 Set D Q.No. 5b

Evaluate: $\int \frac{dx}{x \log x}$. [2]

SOLUTION

Let $I = \int \frac{dx}{x \log x}$

Put $y = \log x$

$dy = \frac{1}{x}dx$

Then, $I = \int \frac{1}{y}dy$

$= \log y + C$

$= \log(\log x) + C$

13. 2072 Supp.Q.No. 5b

Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$. [2]

SOLUTION

$\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$

$= \int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}} \times \frac{\sqrt{2x+1} + \sqrt{2x-3}}{\sqrt{2x+1} + \sqrt{2x-3}}$

$= \int \frac{(\sqrt{2x+1} + \sqrt{2x-3})\,dx}{2x+1-2x+3}$

$= \frac{1}{4}\int \{(2x+1)^{1/2} + (2x-3)^{1/2}\}\,dx$

$= \frac{1}{4}\left\{\frac{(2x+1)^{3/2}}{\frac{3}{2}\cdot 2} + \frac{(2x-3)^{3/2}}{\frac{3}{2}\cdot 2}\right\} + C$

$= \frac{1}{12}\{(2x+1)^{3/2} + (2x-3)^{3/2}\} + C$

14. 2072 Set C Q.No. 5b

Evaluate: $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}\,dx$ [2]

SOLUTION

Let $I = \int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}\,dx$

Put $y = x + \frac{1}{x}$

Then, $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$dy = \left(1 - \frac{1}{x^2}\right)dx$

Now, $I = \int e^y\,dy = e^y + C = e^{x+\frac{1}{x}} + C$

15. 2072 Set D Q.No. 5b

Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$ [2]

➥ Please refer to 2072 Supp.Q.No. 5b

16. 2072 Set E Q.No. 5c

Evaluate: $\int e^{bx}(e^{ax} - e^{-ax})\,dx$. [2]

SOLUTION

$\int e^{bx}(e^{ax} - e^{-ax})\,dx$

$= \int (e^{bx} \cdot e^{ax} - e^{bx} \cdot e^{-ax})\,dx$

$= \int \{e^{(a+b)x} - e^{(b-a)x}\}\,dx$

$= \int e^{(a+b)x}\,dx - \int e^{(b-a)x}\,dx$

$= \frac{e^{(a+b)x}}{a+b} - \frac{e^{(b-a)x}}{b-a} + C \quad \left[\because \int e^{ax}\,dx = \frac{e^{ax}}{a} + C\right]$

$= \frac{e^{(a+b)x}}{a+b} + \frac{e^{(b-a)x}}{a-b} + C$

17. 2071 Supp. Q.No. 5c

Evaluate: $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$. [2]

SOLUTION

Let $I = \int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$.

Put $y = \sqrt{x}$

Then, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore 2dy = \frac{1}{\sqrt{x}}dx$

Then, $I = \int \sec^2 y \cdot 2dy$

$= 2\int \sec^2 y\,dy$

$= 2\tan y + C = 2\tan\sqrt{x} + C$

18. 2071 Set C Q.No. 5b

Evaluate: $\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx.$ [2]

SOLUTION

Let $I = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$

Put $y = \cos x + \sin x$

Then, $\frac{dy}{dx} = -\sin x + \cos x$

$dy = (\cos x - \sin x)\,dx$

$\therefore I = \int \frac{(\cos x - \sin x)dx}{\cos x + \sin x} = \int \frac{dy}{y}$

$= \log y + C$

$= \log(\cos x + \sin x) + C$

19. 2071 Set D Q.No. 5b

Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx.$ [2]

SOLUTION

Let $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Put $y = \sqrt{x}$

Then, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

or, $2dy = \frac{1}{\sqrt{x}}\,dx$

$\therefore I = \int \sin y \cdot 2dy$

$= 2\int \sin y\,dy$

$= 2(-\cos y) + C = -2\cos\sqrt{x} + C$

20. 2071 Old Q.No. 6c

Integrate: $\int \log x\,dx$ [2]

➥ Please refer to 2076 Set C Q.No. 5b

21. 2070 Supp Q.No. 5b

Evaluate: $\int (2x-1)e^{2x}\,dx.$ [2]

SOLUTION

$\int (2x-1)e^{2x}\,dx$

$= (2x-1)\int e^{2x}\,dx - \int \left[\frac{d}{dx}(2x-1)\int e^{2x}\,dx\right]dx$

$= (2x-1)\cdot\frac{e^{2x}}{2} - \int 2\cdot\frac{e^{2x}}{2}\,dx$

$= \frac{e^{2x}}{2}(2x-1) - \frac{e^{2x}}{2} + C$

$= \frac{e^{2x}}{2}(2x-1-1) + C$

$= \frac{e^{2x}}{2}\cdot 2(x-1) + C$

$= (x-1)e^{2x} + C$

22. 2070 Set C Q.No. 5 b

Evaluate: $\int \frac{1}{\sqrt{2x+1} - \sqrt{2x-3}}\,dx.$ [2]

➥ Please refer to 2072 Supp.Q.No. 5b

23. 2070 Set D Q.No. 5 c

Evaluate : $\int \frac{1}{x}\cdot\sin(\log x)\,dx.$ [2]

SOLUTION

Here, $I = \int \frac{\sin(\log x)}{x}\,dx$

Suppose, $\log x = y$

Differentiating with respect to x, we get.

$\frac{d}{dx}(\log x) = \frac{dy}{dx}$

Or, $\frac{1}{x} = \frac{dy}{dx}$

Or, $\frac{1}{x}\,dx = dy$

$\therefore I = \int \sin y\,dy$

$= -\cos y + C$

24. 2070 Old Q.No. 6 b

Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}.$ [2]

SOLUTION

Let $I = \int \frac{dx}{\sin^2 x \cdot \cos^2 x}$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx$

$= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}\right)dx$

$= \int (\sec^2 x + \operatorname{cosec}^2 x)\,dx$

$= \int \sec^2 x\,dx + \int \operatorname{cosec}^2 x\,dx$

$= \tan x - \cot x + C$

25. 2069 Supp Q.No. 5 b

Evaluate: $\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)dx.$ [2]

SOLUTION

Let $I = \int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)dx$

$= \int \left(x^2 - \frac{1}{x^2}\right)dx$

$= \int x^2\,dx - \int x^{-2}\,dx$

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$= \frac{x^3}{3} - \frac{x^{-2+1}}{-2+1} + C$

$= \frac{x^3}{3} - \frac{x^{-1}}{-1} + C$

$= \frac{x^3}{3} + x^{-1} + C$

$= \frac{x^3}{3} + \frac{1}{x} + C$

26. 2069 Set A Q.No, 5b

Evaluate: $\int \cot x \, (\log \sin x)^3 \, dx$ **[2]**

SOLUTION

Let $I = \int \cot x \, (log \sin x)^3 \, dx$

Put $y = log \sin x$

Then, $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x$

$\therefore dy = \cot x \, dx$

Now, $I = \int y^3 \, dy = \frac{y^4}{4} + C = \frac{1}{4} (log \sin x)^4 + C$

27. 2069 [Set B] Q. No. 5b

Evaluate: $\int \frac{dx}{\sin^2 x . \cos^2 x}$ **[2]**

➥ Please refer to 2070 Old Q.No. 6 b

28. 2069 [Set A] Old Q. No. 6b

Evaluate: $\int \frac{dx}{1 + \sin x}$ **[2]**

SOLUTION

$$\int \frac{dx}{1+\sin x} = \int \frac{(1-\sin x)\,dx}{(1+\sin x)(1-\sin x)}$$

$$= \int \frac{(1-\sin x)dx}{1\sin^2 x}$$

$$= \int \left(\frac{1-\sin x}{\cos^2 x}\right) dx$$

$$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int (\sec^2 x - \sec x \tan x)\, dx$$

$$= \int \sec^2 x \, dx \int \sec x \tan x \, dx$$

$$= \tan x - \sec x + C.$$

29. 2069 [Set B] Old Q. No. 6b

Evaluate: $\int \cot x \, dx.$ **[2]**

SOLUTION

Let $I = \int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$

Put $y = \sin x$

Then, $\frac{dy}{dx} = \cos x$

$\therefore dy = \cos x \, dx$

Now, $I = \int \frac{1}{y} dy$

$= log \, y + C = log (\sin x) + C$

30. 2068 Q.No. 5b

Evaluate: $\int \left(1 - \frac{1}{x^2}\right) e^{x + \frac{1}{x}} dx$ **[2]**

➥ Please refer to 2072 Set C Q.No. 5b

31. 2068 Old Q.No. 6b

Evaluate: $\int (a\sin x - b)^3 \cos x \, dx.$ **[2]**

SOLUTION

Let $I = \int (a\sin x - b)^3 \cos x \, dx$

Put $y = a \sin x - b$

Then, $\frac{dy}{dx} = a \cos x$

$\frac{dy}{a} = \cos x \, dx$

$\therefore I = \int y^3 . \frac{dy}{a}$

$= \frac{1}{a} . \frac{y^4}{4} + C = \frac{1}{4a} (a \sin x - b)^4 + C$

32. 2067 Q.No. 6b

Evaluate: $\int \frac{dx}{1 + \sin x}$ **[2**

➥ Please refer to 2069 [Set A] Old Q. No. 6b

33. 2066 Q.No. 6b

Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$ **[2**

➥ Please refer to 2072 Supp.Q.No. 5b

34. 2065 Q.No. 6b

Evaluate: $\int log \, x \, dx$ **[2**

➥ Please refer to 2076 Set C Q.No. 5b

35. 2064 Q.No. 6b

Integrate: $\int cosec \, x \, dx$ **[**

SOLUTION

We have, $I = \int cosec \, x \, dx$

$$= \int \frac{cosec \, x \, (cosec \, x - \cot x)}{(cosec \, x - \cot x)} dx$$

$$= \int \frac{cosec^2 x - cosec \, x \cot x}{(cosec \, x - \cot x)} dx$$

$$= \int \frac{\frac{d}{dx}(cosec \, x - \cot x)}{(cosec \, x - \cot x)} dx$$

$= log \, (cosec \, x - \cot x) + C$

$$\left[\because \int \frac{f'(x)}{f(x)} dx = \log f(x) + C\right]$$

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36. 2063 Q.No. 6b

Integrate: $\int \sec x\, dx$ [2]

SOLUTION

Here,

$I = \int \sec x\, dx$

$= \int \frac{\sec x.(\sec x + \tan x)}{(\sec x + \tan x)} dx$

$= \int \frac{\sec^2 x + \sec x.\tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = y$

Differentiating with respect to 'x', we get

$\frac{d}{dx}(\sec x + \tan x) = \frac{dy}{dx}$

or, $\sec^2 x + \sec x.\tan x = \frac{dy}{dx}$

$\therefore (\sec^2 x + \sec x.\tan x)\, dx = dy$

$\therefore I = \int \frac{dy}{y}$

$= \log y + c$

$= \log(\sec x + \tan x) + c$

37. 2062 Q.No. 6b

Evaluate: $\int x \sin x\, dx$ [2]

SOLUTION

Here, $I = \int x \sin x\, dx$

Integrating by parts, we get

$I = x\int x \sin x\, dx - \int\left[\frac{d}{dx}(x)\int \sin x\, dx\right] dx$

$= x(-\cos x) - \int 1 \times (-\cos x)\, dx$

$= -x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x + C$

38. 2061 Q.No. 6b

Evaluate: $\int \frac{1}{x}\cos(\log x)\, dx$ [2]

SOLUTION

Here, $I = \int \frac{1}{x}\cos(\log x)\, dx$

Put $\log x = y$

Diff. w.r. to x, we get

$\frac{d}{dx}(\log x) = \frac{dy}{dx}$

or, $\frac{1}{x}\, dx = dy$

$\therefore I = \int \cos y\, dy$

$= \sin y + C$

$I = \sin(\log x) + C$

39. 2060 Q.No. 6b

Evaluate: $\int \sin^2 2x\, dx$ [2]

SOLUTION

Here, $I = \int \sin^2 2x\, dx$

$= \frac{1}{2}\int 2\sin^2 2x\, dx$

$= \frac{1}{2}\int [1 - \cos 2(2x)]\, dx$

$= \frac{1}{2}\int (1 - \cos 4x)\, dx$

$= \frac{1}{2}\left[\int dx - \int \cos 4x\, dx\right]$

$= \frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + C$

$\therefore I = \frac{x}{2} - \frac{\sin 4x}{8} + C$

40. 2059 Q.No. 6b

Integrate $\int \sec x\, dx$ [2]

➥ Please refer to **2063 Q.No. 6b**

41. 2058 Q.No. 6c

Integrate: $\int \log x\, dx$ [2]

➥ Please refer to **2076 Set C Q.No. 5b**

42. 2057 Q.No. 6b

Integrate: $\int x \sin x\, dx$. [2]

SOLUTION

Here, $I = \int x \sin x\, dx$

Consider x as the first and $\sin x$ as the second functions. Integrating by parts, we get

$I = x\int \sin x\, dx - \int\left[\frac{d}{dx}(x).\int \sin x\, dx\right] dx$

$= x(-\cos x) - \int 1 \times (-\cos x)\, dx$

$= -x\cos x + \int \cos x\, dx = -x\cos x + \sin x + C$

$\therefore \int x \sin x\, dx = -x\cos x + \sin x + C$

43. 2056 Q.No. 6c

Integrate: $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$ [2]

SOLUTION

Here, $I = \int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

$= \int \sqrt{x}\, dx - \int \frac{1}{\sqrt{x}}\, dx$

$= \int x^{1/2}\, dx - \int x^{-1/2}\, dx$

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$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2x^{3/2}}{3} - 2x^{1/2} + C$$

$$= \frac{2x\sqrt{x}}{3} - 2\sqrt{x} + C$$

$$= \frac{2\sqrt{x}}{3}(x-3) + C$$

44. 2055 Old Model Q.No. 6c

Integrate: $\int \sec x\, dx$ [2]

SOLUTION

Here, $I = \int \sec x\, dx$

$$= \int \frac{\sec x.(\sec x + \tan x)}{(\sec x + \tan x)} dx$$

$$= \int \frac{\sec^2 x + \sec x.\tan x}{\sec x + \tan x} dx$$

Put $\sec x + \tan x = y$

Differentiating with respect to 'x', we get

$$\frac{d}{dx}(\sec x + \tan x) = \frac{dy}{dx}$$

or, $\sec^2 x + \sec x.\tan x = \frac{dy}{dx}$

$\therefore (\sec^2 x + \sec x.\tan x)\, dx = dy$

$\therefore I = \int \frac{dy}{y}$

$= \log y + c$

$= \log(\sec x + \tan x) + c$

4 MARKS QUESTIONS

45. 2075 Set A Q.No. 10b

Evaluate: $\int (x^2+1)e^x\, dx$. [4]

SOLUTION

$\int (x^2+1)e^x\, dx$.

$= (x^2+1)\int e^x\, dx - \int \left\{\frac{d}{dx}(x^2+1)\int e^x dx\right\} dx$

$= (x^2+1).e^x - \int 2x.e^x\, dx$

$= e^x(x^2+1) - 2\left[x\int e^x dx - \int\{\frac{d(x)}{dx}\int e^x dx\}dx\right]$

$= e^x(x^2+1) - 2[x.e^x - \int 1.e^x\, dx]$

$= e^x(x^2+1) - 2(xe^x - e^x) + C$

$= e^x(x^2+1) - 2xe^x + 2e^x + C$

$= e^x(x^2+1-2x+2) + C$

$= e^x(x^2-2x+3) + C$

46. 2072 Set E Q.No. 10b

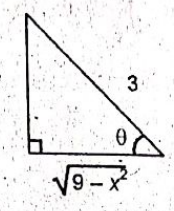
Evaluate: $\int \frac{1}{x^2\sqrt{9-x^2}} dx$ [4]

SOLUTION

Put $x = 3\sin\theta$

Then, $dx = 3\cos\theta\, d\theta$

$\therefore I = \int \frac{1}{x^2\sqrt{9-x^2}} = \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9-9\sin^2\theta}}$



$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta \cdot 3\cos\theta}$

$= \frac{1}{9}\int \operatorname{cosec}^2\theta\, d\theta$

$= -\frac{1}{9}\cot\theta + C$

$= -\frac{1}{9}\cdot\frac{\sqrt{9-x^2}}{x} + C$

$= \frac{-\sqrt{9-x^2}}{9x} + C$

47. 2071 Supp. Q.No. 10b

Evaluate: $\int \sin^2 ax\, dx$. [4]

SOLUTION

Let $I = \int \sin^2 ax\, dx$

$= \int \frac{1-\cos 2\cdot ax}{2} dx \quad \left[\because \sin^2 x = \frac{1-\cos 2x}{2}\right]$

$= \frac{1}{2}\{\int dx - \int \cos 2ax\, dx\}$

$= \frac{1}{2}\left(x - \frac{\sin 2ax}{2a}\right) + C = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C$

48. 2071 Old Q.No. 13b

Integrate: $\int \sec^3 x dx$ [4]

SOLUTION

Here, $I = \int \sec^3 x\, dx = \int \sec x \sec^2 x\, dx$

Integrating by parts, we get

$$I = \sec x.\int \sec^2 x\, dx - \int\left[\frac{d}{dx}\sec x.\int \sec^2 x\, dx\right] dx$$

$= \sec x\tan x - \int \sec x\tan x\tan x\, dx$

$= \sec x\tan x - \int \sec x\tan^2 x\, dx$

$= \sec x\tan x - \int \sec x(\sec^2 x - 1)\, dx$

$[\because \tan^2 x = \sec^2 x - 1]$

$= \sec x\tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

or, $I = \sec x.\tan x - I + \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} dx$

$[\because I = \int \sec^3 x\, dx]$

or, $I + I = \sec x.\tan x + \int \frac{(\sec x\tan x\ \sec^2 x)}{(\sec x + \tan x)} dx$

or, $2I = \sec x.\tan x + \log(\sec x + \tan x) + C$

$\left[\because \int \frac{f'(x)}{f(x)} dx = \log f(x)\right]$

$\therefore I = \frac{1}{2}\sec x\tan x + \frac{1}{2}\log(\sec x + \tan x) + C$

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49. 2069 [Set A] Old Q. No. 13b

Integrate: $\int \cot x(\log \sin x)^3\, dx$. [4]

➥ Please refer to 2069 Set 'A', Q.No. 5b

50. 2068 Old Q.No. 13b

Evaluate: $\int e^{ax} \cos bx\ dx$ [4]

SOLUTION

Here,

$I = \int e^{ax} \cos bx\, dx$ ... (i)

$= \int \cos bx \cdot e^{ax}\, dx$

$\therefore I = \cos bx \int e^{ax} dx - \int \left[\frac{d}{dx}(\cos bx) \int e^{ax} dx\right] dx$

$= \cos bx \times \frac{e^{ax}}{a} - \int -\sin bx \times b \cdot \frac{e^{ax}}{a} dx$

$= \frac{e^{ax}}{a} \cos bx + \frac{b}{a} \int e^{ax} \sin bx\, dx$

$= \frac{e^{ax}}{a} \cos bx + \frac{b}{a}\left\{\sin bx \int e^{ax} dx - \int \left[\frac{d}{dx} \sin bx \int e^{ax} dx\right] dx\right\}$

$= \frac{e^{ax}}{a} \cos bx + \frac{b}{a}\left\{\sin bx \frac{e^{ax}}{a} - \int \cos bx \times b \times \frac{e^{ax}}{a} dx\right\}$

$= \frac{e^{ax}}{a} \cos bx + \frac{b}{a}\left\{\frac{e^{ax}}{a} \sin bx - \frac{b}{a} \int e^{ax} \cos bx\, dx\right\}$

or, $I = \frac{e^{ax}}{a} \cos bx + \frac{b}{a^2} e^{ax} \sin bx - \frac{b^2}{a^2} I + C_1$ [$\because$ using i]

or, $I + \frac{b^2}{a^2} I = \frac{e^{ax}}{a} \cos bx + \frac{b}{a^2} e^{ax} \sin bx + C_1$

or, $I\left(\frac{a^2 + b^2}{a^2}\right) = \frac{e^{ax}}{a} \cos bx + \frac{b}{a^2} e^{ax} \sin bx + C_1$

$\therefore I = \left(\frac{a^2}{a^2 + b^2}\right)\left[\frac{e^{ax}}{a} \cos bx + \frac{be^{ax}}{a^2} \sin bx\right] + C$

$= \left(\frac{a^2}{a^2 + b^2}\right) \cdot \frac{e^{ax}}{a^2}(a \cos bx + b \sin bx) + C$

$= \left(\frac{e^{ax}}{a^2 + b^2}\right)(a \cos bx + b \sin bx) + C$

51. 2067 Q.No. 13 b

Evaluate: $\int \sec^3 x\, dx$ [4]

➥ Please refer to 2071 Old Q.No. 13b

52. 2065 Q.No. 13b

Evaluate: $\int e^{ax} \cos bx\, dx$ [4]

➥ Please refer to 2068 Old Q.No. 13b

53. 2064 Q.No. 13a

Evaluate: $\int \frac{\sin(\log x)}{x} dx$ [4]

SOLUTION

Here, $I = \int \frac{\sin(\log x)}{x} dx$

Suppose, $\log x = y$

Differentiating with respect to x, we get.

$\frac{d}{dx}(\log x) = \frac{dy}{dx}$

or, $\frac{1}{x} = \frac{dy}{dx}$

or, $\frac{1}{x} dx = dy$

$\therefore I = \int \sin y\, dy$

$= -\cos y + C$

54. 2061 Q.No. 13b

Integrate: $\int \sec^3 x\, dx$ [4]

➥ Please refer to 2071 Old Q.No. 13b

55. 2060 Q.No. 13b

Evaluate: $\int x \sin^2 x\, dx$ [4]

SOLUTION

Here,

$I = \int x \sin^2 x\, dx$

$= \int x\left(\frac{1 - \cos 2x}{2}\right) dx$

$= \frac{1}{2} \int (x - x \cos 2x)\, dx$

$= \frac{1}{2} \int x\, dx - \frac{1}{2} \int x \cos 2x\, dx$

$= \frac{1}{2} \times \frac{x^2}{2} - \frac{1}{2}\left\{x \int \cos 2x\, dx - \int \left[\frac{d}{dx} x \int \cos 2x\, dx\right] dx\right\}$

$= \frac{x^2}{4} - \frac{1}{2}\left(x \cdot \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} dx\right)$

$= \frac{x^2}{4} - \frac{1}{4} x \sin 2x + \frac{1}{4} \int \sin 2x\, dx$

$= \frac{x^2}{4} - \frac{x}{4} \sin 2x + \frac{1}{4} \times \frac{-\cos 2x}{2} + C$

$= \frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + C$

56. 2059 Q.No. 13b

Evaluate: $\int e^x \cos x\, dx$ [4]

SOLUTION

Here, $I = \int e^x \cos x\, dx$

Integrating by parts, we get

$I = \cos x \int e^x dx - \int \left[\frac{d}{dx}(\cos x) \int e^x dx\right] dx$

$= e^x \cos x - \int (-\sin x)\, e^x dx$

$= e^x \cos x + \int \sin x\, e^x dx$

$= e^x \cos x + \left\{\sin x \int e^x dx - \int \left[\frac{d}{dx}(\sin x) \int e^x dx\right] dx\right\}$

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$= e^x \cos x + e^x \sin x - \int \cos x\, e^x\, dx$

$= e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$

or, $I = e^x \cos x + e^x \sin x - I \quad \left[\because I = \int e^x \cos x\, dx\right]$

or, $2I = e^x \cos x + e^x \sin x + C$

$\therefore I = \frac{e^x}{2}(\cos x + \sin x) + C$

57. 2058 Q.No. 13b

Evaluate: $\int x^2\, e^{ax}\, dx$ **[4]**

SOLUTION

Here, $I = \int x^2 \cdot e^{ax}\, dx$

Take x^2 as the 1st function and e^{ax} as the 2nd function.

$= x^2 \int e^{ax}\, dx - \int \left[\frac{d}{dx}(x^2) \int e^{ax}\, dx\right] dx$

$= x^2 \times \frac{e^{ax}}{a} - \int 2x \times \frac{e^{ax}}{a}\, dx$

$= \frac{x^2 e^{ax}}{a} - \frac{2}{a}\int x\, e^{ax}\, dx$

$= \frac{x^2 e^{ax}}{a} - \frac{2}{a}\left\{x \int e^{ax}\, dx - \int \left[\frac{d}{dx}(x) \int e^{ax}\, dx\right] dx\right\}$

$= \frac{x^2 e^{ax}}{a} - \frac{2x}{a} \times \frac{e^{ax}}{a} + \frac{2}{a}\int 1 \times \frac{e^{ax}}{a}\, dx$

$= \frac{x^2 e^{ax}}{a} - \frac{2x e^{ax}}{a^2} + \frac{2}{a^2} \times \frac{e^{ax}}{a} + C$

$= \frac{e^{ax}}{a}\left[x^2 - \frac{2x}{a} + \frac{2}{a^2}\right] + C$

58. 2057 Q.No. 13b

Integrate $\int x^2 \sin x\, dx$ **[4]**

SOLUTION

Here, $I = \int x^2 \sin x\, dx$

$= x^2 \int \sin x\, dx - \int \left[\frac{dx^2}{dx} - \int \sin x\, dx\right] dx$

$= x^2(-\cos x) - \int 2x.(-\cos x)\, dx$

$= -x^2 \cos x + 2\int x \cos x\, dx$

$= -x^2 \cos x + 2\left\{x \int \cos x\, dx - \int \left[\frac{dx}{dx}\int \cos x\right] dx\right\}$

$= -x^2 \cos x + 2\left\{x \sin x - \int 1.\sin x\, dx\right\}$

$= -x^2 \cos x + 2x \sin x - 2\int \sin x\, dx$

$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$

$\therefore I = -x^2 \cos x + 2x \sin x + 2\cos x + C$

59. 2056 Q.No. 13b

Evaluate: $\int \frac{dx}{\sqrt{a^2 + x^2}}$ **[4]**

SOLUTION

Let $I = \int \frac{dx}{\sqrt{a^2 + x^2}}$

Put $x = a \tan\theta$

Then, $dx = a \sec^2\theta\, d\theta$ and $a^2 + x^2 = a^2 + a^2 \tan^2\theta = a^2(1 + \tan^2\theta) = a^2 \sec^2\theta$

Now, $I = \int \frac{dx}{\sqrt{a^2 + x^2}}$

$= \int \frac{a \sec^2\theta\, d\theta}{\sqrt{a^2 \sec^2\theta}} = \int \frac{a \sec^2\theta\, d\theta}{a \sec\theta}$

$= \int \sec\theta\, d\theta = \int \frac{\sec\theta(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}\, d\theta$

Again, put $\sec\theta + \tan\theta = y$

Then, $dy = (\sec\theta \tan\theta + \sec^2\theta)\, d\theta$

$= \sec\theta(\sec\theta + \tan\theta)\, d\theta$

Now, I becomes

$I = \int \frac{dy}{y} = \log y + C$

$= \log(\sec\theta + \tan\theta) + C \quad [\because y = \sec\theta + \tan\theta]$

$= \log\left(\sqrt{1 + \tan^2\theta} + \tan\theta\right) + C$

$= \log\left[\sqrt{1 + \left(\frac{x}{a}\right)^2} + \frac{x}{a}\right] + C \quad [\because x = a\tan\theta]$

$= \log\left[\sqrt{\frac{a^2 + x^2}{a^2}} + \frac{x}{a}\right] + C$

$= \log\left(\frac{\sqrt{x^2 + a^2} + x}{a}\right) + C$

$= \log(\sqrt{x^2 + a^2} + x) - \log a + C$

$\left[\because \log\left(\frac{x}{y}\right) = \log x - \log y\right]$

$= \log(\sqrt{x^2 + a^2} + x) + C'$

where, $C' = C - \log a$

60. 2055 Old Model Q.No. 13b

Evaluate: $\int x^2 \sin x\, dx$ **[4]**

SOLUTION

Here, $I = \int x^2 \sin x\, dx$

$= x^2 \int \sin x\, dx - \int \left[\frac{dx^2}{dx} - \int \sin x\, dx\right] dx$

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$= x^2(-\cos x) - \int 2x.(-\cos x)\,dx$

$= -x^2\cos x + 2\int x\cos x\,dx$

$= -x^2\cos x + 2\left\{x\int\cos x\,dx - \int\left[\frac{dx}{dx}\int\cos x\right]dx\right\}$

$= -x^2\cos x + 2\left\{x\sin x - \int 1.\sin x\,dx\right\}$

$= -x^2\cos x + 2x\sin x - 2\int\sin x\,dx$

$= -x^2\cos x + 2x\sin x - 2(-\cos x) + C$

$\therefore I = -x^2\cos x + 2x\sin x + 2\cos x + C$

B. DEFINITE INTEGRALS

2 MARKS QUESTIONS

1. 2059 Q.No. 6c

Evaluate $\int_1^2 \frac{\sin(\log x)}{x}\,dx$ [2]

SOLUTION

Here, $I = \int_1^2 \frac{\sin(\log x)}{x}\,dx$

Put $\log x = y$

Differentiating both sides with respect to x, we get

$\frac{1}{x} = \frac{dy}{dx}$

or, $\left(\frac{1}{x}\right)dx = dy$

when $x = 1$, $y = \log 1 = 0$

when $x = 2$, $y = \log 2$

$\therefore I = \int_1^{\log 2} \sin y\,dy = [-\cos y]_0^{\log 2} = [\cos y]_{\log 2}^0$

$= \cos 0^c - \cos(\log 2)$

$= 1 - \cos(\log 2)$

2. 2057 Q.No. 6c

Find the value of $\int_{-\pi/3}^{\pi/3} \cos t\,dt$. [2]

SOLUTION

Here, $I = \int_{-\pi/3}^{\pi/3} \cos t\,dt$

$= [\sin t]_{-\pi/3}^{\pi/3} = \sin\frac{\pi}{3} - \sin\left(-\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} + \sin\frac{\pi}{3}$ $[\because \sin(-\theta) = -\sin\theta]$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

4 MARKS QUESTIONS

3. 2070 Old Q.No. 13 b

Evaluate: $\int_0^{\pi/4} \frac{\cos x}{1+\sin x}\,dx$. [4]

SOLUTION

$\int_0^{\pi/4} \frac{\cos x}{1+\sin x}$

Indefinite integral $(I) = \int \frac{\cos x}{1+\sin x}\,dx$

Put $y = 1 + \sin x$

$dy = \cos x\,dx$

$\therefore I = \int \frac{1}{y}\,dy = \log y = \log(1+\sin x)$

Definite integral $= \int_0^{\pi/4} \frac{\cos x}{1+\sin x}\,dx$

$= [\log(1+\sin x)]_0^{\pi/4}$

$= \log\left(1+\sin\frac{\pi}{4}\right) - \log(1+\sin 0)$

$= \log\left(1+\frac{1}{\sqrt{2}}\right)$

4. 2069 [Set B] Old Q. No. 13 a

Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ [4]

SOLUTION

$\int_0^{\pi/2} \frac{dx}{1+\sin x}$

$= \int_0^{\pi/2} \frac{dx}{1-\cos\left(\frac{\pi}{2}+x\right)}$ $\left[\because \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta\right]$

$= \int_0^{\pi/2} \frac{dx}{2\sin^2\frac{1}{2}\left(\frac{\pi}{2}+x\right)}$ $\left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

$= \frac{1}{2}\int_0^{\pi/2} \operatorname{cosec}^2\frac{1}{2}\left(\frac{\pi}{2}+x\right)$

$= \frac{1}{2}\left[\frac{-\cot\frac{1}{2}\left(\frac{\pi}{2}+x\right)}{\frac{1}{2}}\right]_0^{\pi/2}$

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$$= \left[-\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]_0^{\pi/2}$$

$$= -\left[\cot\left(\frac{\pi}{4}+\frac{\pi}{4}\right) - \cot\frac{\pi}{4}\right]$$

$$= -\cot\frac{\pi}{2} + \cot\frac{\pi}{4}$$

$$= -0 + 1 = 1$$

5. 2066 Q.No. 13b

Evaluate: $\int_1^3 \frac{x\,dx}{1+x^2}$ **[4]**

SOLUTION

$$\int_1^3 \frac{x\,dx}{1+x^2} = \frac{1}{2}\int_1^3 \frac{2x\,dx}{1+x^2}$$

$$= \frac{1}{2}[\log(1+x^2)]_1^3$$

$$= \frac{1}{2}[\log(1+3^2) - \log(1+1^2)]$$

$$= \frac{1}{2}[\log 10 - \log 2]$$

$$= \frac{1}{2}\left[\log\left(\frac{10}{2}\right)\right]$$

$$= \frac{1}{2}\log 5$$

$$= \log(5)^{1/2}$$

$$= \log\sqrt{5}$$

6. 2063 Q.No. 13a

Evaluate: $\int_0^2 \frac{x\,dx}{\sqrt{x^2+4}}$ **[4]**

SOLUTION

We have, $I = \int_0^2 \frac{x\,dx}{\sqrt{x^2+4}}$

Put, $x^2 + 4 = y$

Then, $2x = \frac{dy}{dx}$

$\therefore x\,dx = \frac{1}{2}dy$

When $x = 0$, $y = 0^2 + 4 = 4$

When $x = 2$, $y = 2^2 + 4 = 8$

Now,

$$\int_0^2 \frac{x\,dx}{\sqrt{x^2+4}} = \int_4^8 \frac{1}{2\sqrt{y}}dy$$

$$= \frac{1}{2}\int_4^8 \frac{dy}{\sqrt{y}} = \frac{1}{2}\int_4^8 y^{-1/2}\,dy$$

$$= \frac{1}{2}\left[\frac{y^{-1/2+1}}{-\frac{1}{2}+1}\right]_4^8 = \frac{1}{2}\left[\frac{y^{-1/2}}{\frac{1}{2}}\right]_4^8$$

$$= \frac{1}{2}\times 2\left[y^{1/2}\right]_4^8$$

$$= 8^{1/2} - 4^{1/2}$$

$$= \sqrt{8} - \sqrt{4}$$

$$= 2\sqrt{2} - 2$$

$$= 2(\sqrt{2} - 1)$$

Hence, $\int_0^2 \frac{x\,dx}{\sqrt{x^2+4}} = 2(\sqrt{2}-1)$

7. 2062 Q.No. 13b OR

Evaluate: $\int_0^{-1} \frac{dx}{\sqrt{4-x^2}}$ **[4]**

SOLUTION

We have,

$$I = \int_0^{-1} \frac{dx}{\sqrt{4-x^2}} \quad \text{... (i)}$$

Put, $x = 2\sin\theta$

Diff. w.r. to θ, we get

$\frac{dx}{d\theta} = 2\cos\theta$

or, $dx = 2\cos\theta\,d\theta$

Now, $\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta$

When $x = 0$,

$2\sin\theta = 0$

or, $\sin\theta = 0$

or, $\sin\theta = \sin 0$

$\therefore \theta = 0$

When $x = -1$,

$2\sin\theta = -1$

or, $\sin\theta = \frac{-1}{2}$

or, $\sin\theta = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

$\therefore \theta = \frac{-\pi}{6}$

$\therefore$ From (i)

$$I = \int_0^{-\pi/6} \frac{2\cos\theta\,d\theta}{2\cos\theta}$$

$$= \int_0^{-\pi/6} 1.\,d\theta$$

$$= [\theta]_0^{-\pi/6} = \frac{-\pi}{6}$$

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C. APPLICATION OF INTEGRATION

2 MARKS QUESTIONS

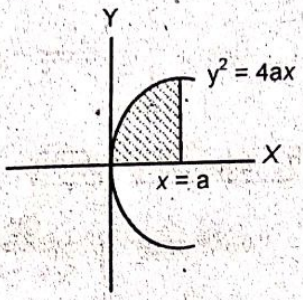
1. 2071 Old Q.No. 6b

Find the area bounded by the curve $y^2 = 4ax$ the *x-axis* and the ordinates $x = 0$ and $x = a$. [2]

SOLUTION

Given curve is $y^2 = 4ax$ ...(i)

The area bounded by curve (i), the *x-axis* and the ordinates of $x = 0$ and $x = a$ is



$$\int_0^a y\,dx = \int_0^a \sqrt{4ax}\,dx$$

$$= 2\sqrt{a}\int_0^a x^{1/2}\,dx$$

$$= 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$$

$$= 2\sqrt{a}\cdot\frac{2}{3}[x^{3/2}]_0^a$$

$$= \frac{4}{3}\sqrt{a}\cdot a^{3/2} = \frac{4a^2}{3}$$

2. 2070 Old Q.No. 6 c

Find the area under the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 1$. [2]

SOLUTION

The area bounded by the curve $y = 2\sqrt{x}$ and between $x = 0$ and $x = 1$ is

$$\int_0^1 y\,dx = 2\int_0^1 2x^{1/2}\,dx$$

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^1 = \frac{4}{3}[x^{3/2}]_0^1$$

$$= \frac{4}{3}[1^2 - 0]$$

$$= \frac{4}{3}\text{ square units.}$$

3. 2069 [Set A] Old Q. No. 6c

Find the area bounded by curve $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$. [2]

SOLUTION

Given curve is $y = \cos x$ ...(i)

The area bounded by the curve (i), $x = 0$ and $x = \frac{\pi}{2}$ is

$$\int_0^{\pi/2} y\,dx = \int_0^{\pi/2}\cos x\,dx = [\sin x]_0^{\pi/2}$$

$$= \sin\frac{\pi}{2} - \sin 0$$

$$= 1 - 0 = 1\text{ sq. unit.}$$

4. 2069 [Set B] Old Q. No. 6a

Find the area enclosed by $y = 3x$, the *x-axis* and ordinates $x = 0$, $x = 4$. [2]

SOLUTION

The area enclosed by $y = 3x$, the *x-axis* and the ordinates $x = 0$, $x = 4$ is

$$\int_0^4 y\,dx = \int_0^4 3x\,dx$$

$$= \left[\frac{3x^2}{2}\right]_0^4$$

$$= \frac{3}{2}\cdot 4^2 - 0 = 24\text{ sq. units}$$

5. 2068 Old Q.No. 6c

Find the area bounded by the curve $y = 3x^2 - 2$, x –axis and the two ordinates $x = 1$ and $x = 4$. [2]

SOLUTION

The area bounded by the curve $y = 3x^2 - 2$, *x-axis* and the two ordinates $x = 1$ and $x = 4$ is

$$\int_1^4 y\,dx = \int_1^4 (3x^2 - 2)\,dx = \left[\frac{3x^3}{3} - 2x\right]_1^4$$

$$= [x^3 - 2x]_1^4$$

$$= (4^3 - 2\times 4) - (1^3 - 2\times 1)$$

$$= 56 + 1$$

$$= 57\text{ sq. units.}$$

6. 2067 Q.No. 6c

Find the area under the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 1$. [2]

➥ Please refer to 2070 Old Q.No. 6 c

7. 2066 Q.No. 6c

Find the area bounded by the curve $y = \sin x$, $x = 0$, $x = \pi$. [2]

SOLUTION

Given, $y = \sin x$

$$\text{Required area} = \int_0^\pi \sin x\,dx$$

$$= [-\cos x]_0^\pi$$

$$= (-\cos\pi + \cos 0)$$

$$= -(-1) + 1$$

$$= 2\text{ sq. units.}$$

8. 2065 Q.No. 6c

Find the area of the region bounden by the curve $y = e^x$, the x –axis and the ordinates at $x = 1$; $x = 2$. [2]

SOLUTION

Here, we have to find the area bounded by the curve $y = e^x$, the x –axis and the ordinates at $x = 1$; $x = 2$.

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$$\therefore \text{The required area} = \int_1^2 e^x\,dx$$
$$= [e^x]_1^2$$
$$= (e^2 - e^1)$$
$$= (e^2 - e)$$

9. 2064 Q.No. 6a

Find the are bounded by the *x-axis* and the following curve and ordinates

$y = log\ x$, $x = 1$, $x = e$ [2]

SOLUTION

The given curve is $y = log\ x$

Now, the area of the curve bounded by the *x-axis* and the ordinates at $x = 1$ and $x = e$ is given by

$$\text{Area} = \int_1^e log\ x\,dx$$
$$= [x\ log\ x - x]_1^e$$
$$= (e\ log\ e - e) - (log\ 1 - 1)$$
$$= (e \times 1 - e) - (0 - 1)$$
$$= e - e + 1$$
$$= 1 \text{ sq. units}$$

10. 2063 Q.No. 6a

Find the area bounded by the *x-axis* and the following curve and ordinates. [2]

$xy = 8$, $x = 3$, $x = 8$

SOLUTION

The given curve is

$$xy = 8$$

or, $y = \frac{8}{x}$ ... (i)

The ordinates are at $x = 3$ and at $x = 8$

Now, the area of the region bounded by the curve (i), *x-axis* and the ordinates at $x = 3$ and $x = 8$ is given by

$$\int_a^b f(x)\,dx = \int_3^8 y\,dx$$
$$= \int_3^8 \frac{8}{x}\,dx$$
$$= 8\int_3^8 \frac{1}{x}\,dx$$
$$= 8\left[log\ x\right]_3^8$$
$$= 8(log\ 8 - log\ 3) = 8log\left(\frac{8}{3}\right) \text{ sq units.}$$

11. 2062 Q.No. 6c

Find the area bounded by the *x-axis* and the curve $y = log\ (1 + x)$ and ordinates $x = 0$ and $x = 1$. [2]

SOLUTION

The given curve is

$$y = log\ (1 + x) \quad \text{... (i)}$$

The area of the region bounded by the curve (i), *x-axis* and the ordinates $x = 0$ and $x = 1$ is

$$\int_a^b y\,dx = \int_0^1 log\ (1 + x)\,dx = \left[I\right]_0^1$$

Where, $I = \int \log\ (1 + x)\,dx$

$$= \int \log\ (1 + x) \times 1\,dx$$

Integrating by parts, we get

$$I = log\ (1 + x)\int 1\,dx - \int\left[\frac{d}{dx}\log\ (1 + x)\int 1\,dx\right]dx$$
$$= x\ log\ (1 + x) - \int \frac{1}{(1 + x)} \times x\,dx$$
$$= x\ log\ (1 + x) - \int \frac{x}{1 + x}\,dx$$
$$= x\ log\ (1 + x) - \int \frac{(1 + x) - 1}{1 + x}\,dx$$
$$= x\ log\ (1 + x) - \int \frac{1 + x}{1 + x}dx + \int \frac{dx}{1 + x}$$
$$= x\ log\ (1 + x) - \int 1\,dx + \log\ (1 + x)$$
$$= x\ log\ (1 + x) - x + log\ (1 + x)$$

or, $I = (1 + x)\ log\ (1 + x) - x$

$$\therefore \int_0^1 y\,dx = \left[I\right]_0^1$$
$$= \left[(1 + x)\ log\ (1 + x) - x\right]_0^1$$
$$= [(1 + 1)\ log\ (1 + 1) - 1] - [(1 + 0)\ log\ (1 + 0) - 0]$$
$$= (2\ log\ 2 - 1) - 0 \qquad [\because log\ 1 = 0]$$
$$= log\ 2^2 - 1$$
$$= log\ 4 - 1$$

12. 2061 Q.No. 6c

Find the area bounded by curves $y = 3x^2$, $x = 1$ and $x = 3$. [2]

SOLUTION

The region bounded by the curves $y = 3x^2$, *x-axis*, $x = 1$ and $x = 3$ is

$$\int_1^3 y\,dx = \int_1^3 3x^2\,dx$$

$$= \left[\frac{3x^3}{3}\right]_1^3$$

$$= [x^3]_1^3$$

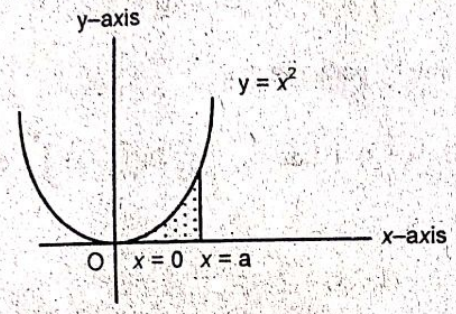
$$= 3^3 - 1^3 = 27 - 1 = 26 \text{ sq. units}$$

13. 2060 Q.No. 6c

Find the area under the curve $y = x^2$ bounded by *x-axis* and between the ordinates $x = 0$ and $x = a$. [2]

SOLUTION

The area under the curve $y = x^2$ bounded by *x-axis* and between the ordinates $x = 0$ and $x = a$ is given by:



$$\text{Area} = \int_0^a y\,dx$$

$$= \int_0^a x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^a$$

$$= \frac{a^3}{3} - 0 = \frac{a^3}{3} \text{ square units.}$$

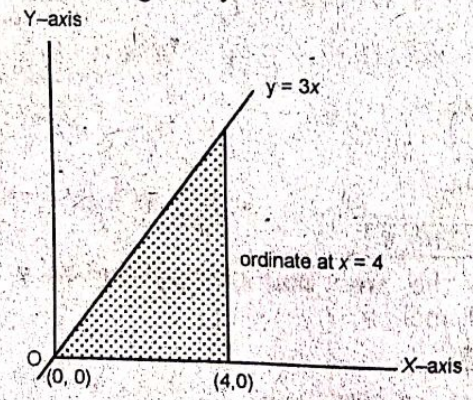
14. 2058 Q.No. 6b

Find the area enclosed by the curve $y = 3x$, the *x-axis* and ordinates at $x = 0$ and $x = 4$. [2]

SOLUTION

The given curve is $y = 3x$

∴ Required area is given by



$$\text{Area} = \int_0^4 y\,dx = \int_0^4 3x\,dx$$

$$= \left[\frac{3x^2}{2}\right]_0^4 = \left(\frac{3 \times 4^2}{2} - 0\right)$$

$$= 24 \text{ sq. unit}$$

15. 2056 Q.No. 6b

Find the area bounded by the axis of *x* and the curve $y = x^3$ and the ordinates at $x = 2$ and $x = 4$. [2]

SOLUTION

Here, the required region bounded by x– axis and the curve $y = 4x^3$ and the ordinates at $x = 2$ and $x = 4$.

The area of the region PQRS

$$\text{Area} = \int_2^4 y\,dx = \int_2^4 4x^3\,dx = \left[\frac{4x^4}{4}\right]_2^4$$

$$= 4^4 - 2^4 = 256 - 16 = 240 \text{ sq. units.}$$

16. 2055 Old Model Q.No. 6b

Find the area bounded by the axis of *x* and the curve $y = 4x^3$ and the ordinates at $x = 2$ and $x = 4$. [2]

➥ Please refer to 2056 Q.No. 6b

4 MARKS QUESTIONS

17. 2076 Set B Q.No. 10b

Find the area of the circle $x^2 + y^2 = 36$, using method of integration. [4]

SOLUTION

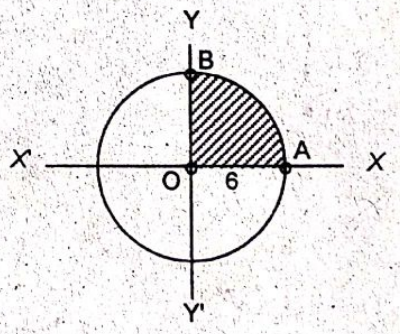
Equation of given circle is $x^2 + y^2 = 36$

∴ $y = \sqrt{36 - x^2}$ ... (i)

The centre of circle (i) is at (0, 0) and radius OA = 6 units

Since, circle (i) is symmetrical about both the axes, *x-axis* and y-axis divide it into four equal parts.

OAB is the portion of the circle lying in the first quadrant, which is bounded by the axes and the curve (i).



$$\text{Now, Area of OAB} = \int_0^6 y\,dx = \int_0^6 \sqrt{36 - x^2}\,dx$$

Put $x = 6 \sin\theta$, then $dx = 6\cos\theta\,d\theta$

When $x = 0$, then $0 = 6\sin\theta$

or, $\sin\theta = 0 = \sin 0 \Rightarrow \theta = 0$

When $x = 6$, then $6 = 6\sin\theta \Rightarrow \sin\theta = 1$

or, $\sin\theta = \sin \pi/2 \Rightarrow \theta = \pi/2$

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Now, Area of OAB $= \int_0^{\pi/2} \sqrt{36 - 36\sin^2\theta} \times 6\cos\theta \, d\theta$

$= \int_0^{\pi/2} 6\sqrt{36\cos^2\theta} \times \cos\theta \, d\theta$

$= 36 \int_0^{\pi/2} \cos\theta \times \cos\theta \, d\theta$

$= \frac{36}{2} \int_0^{\pi/2} 2\cos^2\theta \, d\theta$

$= 18 \int_0^{\pi/2} (1 + \cos 2\theta) \, d\theta$

$= 18 \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= 18 \left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0 + 0)\right]$

$= 18\left(\frac{\pi}{2} + \frac{0}{2}\right) = 9\pi$

If A be the area of circle, then

A = 4 × Area of OAB = 4 × 9π = 36 π sq. units

Hence, the area of given circle = 36 π sq. units.

18. 2076 Set C Q.No. 10b

Using method of integration, find the area under the curve $4x^2 + 9y^2 = 36$. [4]

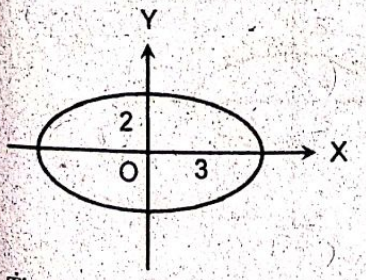
SOLUTON

Given curve is $4x^2 + 9y^2 = 36$

Dividing both sides by 36, we get

$\frac{x^2}{9} + \frac{y^2}{4} = 1$.

The curve is symmetrical about x–axis and y–axis. So, to find the area of the whole ellipse, we first find the area of the portion lying in the first quadrant and then multiply it by 4.



The area of the ellipse (A)

= 4(Area of the portion lying in first quadrant)

$= 4\int_0^3 y \, dx$

$= 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2} \, dx$

$= \frac{8}{3}\int_0^3 \sqrt{9 - x^2} \, dx$

Put $x = 3\sin\theta$.

Then, $dx = 3\cos\theta \, d\theta$

When, $x = 0, \theta = 0$

When, $x = 3, \theta = \frac{\pi}{2}$

Hence,

$A = \frac{8}{3}\int_0^{\pi/2} \sqrt{9 - 9\sin^2\theta} \cdot 3\cos\theta \, d\theta$

$= \frac{8}{3}\int_0^{\pi/2} 3\cos\theta \cdot 3\cos\theta \, d\theta$

$= 24\int_0^{\pi/2} \cos^2\theta \, d\theta$

$= 24\int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} \, d\theta$

$= 12\int_0^{\pi/2} (1 + \cos 2\theta) \, d\theta$

$= 12\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= 12\left[\left(\frac{\pi}{2} + 0\right) - (0 + 0)\right]$

$= 6\pi$ sq. units

19. 2075 GIE Q.No. 10b

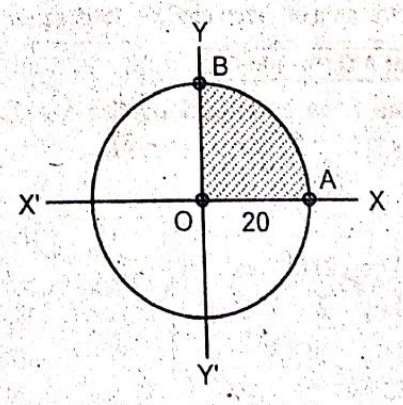
Find the area under the curve $x^2 + y^2 = 400$, using method of integration. [4]

SOLUTION

Equation of given circle is $x^2 + y^2 = 400$

$\therefore y = \sqrt{400 - x^2}$ ... (i)

The centre of circle (i) is at (0, 0) and radius OA = 20 units



Since, circle (i) is symmetrical about both the axes, x-axis and y-axis divide it into four equal parts.

OAB is the portion of the circle lying in the first quadrant, which is bounded by the axes and the curve (i).

Now, Area of OAB $= \int_0^{20} y\,dx = \int_0^{20} \sqrt{400 - x^2}\,dx$

Put $x = 20 \sin\theta$, then $dx = 20\cos\theta\,d\theta$

When $x = 0$, then $0 = 20\sin\theta$

or, $\sin\theta = 0 = \sin 0 \Rightarrow \theta = 0$

When $x = 20$, then $20 = 20\sin\theta \Rightarrow \sin\theta = 1$

or, $\sin\theta = \sin \pi/2 \Rightarrow \theta = \pi/2$

Now,

$$\text{Area of OAB} = \int_0^{\pi/2} \sqrt{400 - 400\sin^2\theta} \times 20\cos\theta\,d\theta$$

$$= \int_0^{\pi/2} 20\sqrt{400\cos^2\theta} \times \cos\theta\,d\theta$$

$$= 400\int_0^{\pi/2} \cos\theta \times \cos\theta\,d\theta$$

$$= \frac{400}{2}\int_0^{\pi/2} 2\cos^2\theta\,d\theta$$

$$= 200\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$$

$$= 200\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 200\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0 + 0)\right]$$

$$= 200\left(\frac{\pi}{2} + \frac{0}{2}\right)$$

$$= 200 \times \frac{\pi}{2} = 100\pi$$

If A be the area of circle, then

A $= 4 \times$ Area of OAB

$= 4 \times 100\pi = 400\pi$ sq. units

Hence, the area of given circle $= 400\pi$ sq. units.

20. 2075 Set A Q.No. 10b OR

Find the area bounded by the x-axis and the curve y = (x + 1) (x – 2) (x – 3). [4]

SOLUTION

Here,

y $= (x + 1)(x - 2)(x - 3)$

$= (x + 1)(x^2 - 2x - 3x + 6)$

$= (x + 1)(x^2 - 5x + 6)$

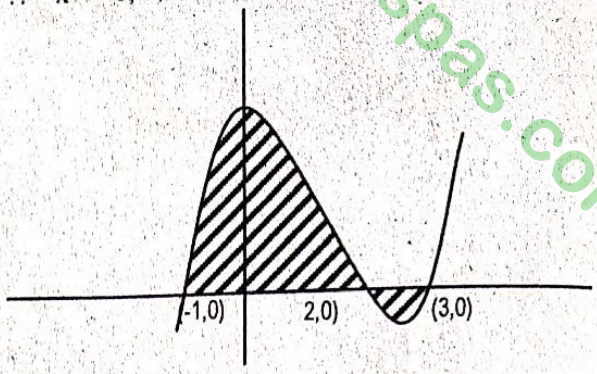
$= x^3 - 5x^2 + 6x + x^2 - 5x + 6$

$= x^3 - 4x^2 + x + 6$

At x–axis, y = 0

or, $(x + 1)(x - 2)(x - 3) = 0$

$\therefore$ $x = -1, 2, 3$.



Required area

$$(A) = \left|\int_{-1}^{2} y\,dx\right| + \left|\int_{2}^{3} y\,dx\right|$$

$$= \left|\int_{-1}^{2} (x^3 - 4x^2 + x + 6)\,dx\right| + \left|\int_{-1}^{2} (x^3 - 4x^2 + x + 6)\,dx\right|$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_{-1}^{2} + \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_{2}^{3}$$

$$= \left|\left(\frac{2^4}{4} - \frac{4\cdot 2^3}{3} + \frac{2^2}{2} + 6\cdot 2\right) - \left\{\frac{(-1)^4}{4} - 4\cdot\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 6(-1)\right\}\right|$$

$$+ \left|\left(\frac{3^4}{4} - \frac{4.3^3}{3} + \frac{3^2}{2} + 6.3\right) - \left(\frac{2^4}{4} - \frac{4.2^3}{3} + \frac{2^2}{2} + 6.2\right)\right|$$

$$= \left|\frac{45}{4}\right| + \left|-\frac{7}{12}\right| = \frac{45}{4} + \frac{7}{12} = \frac{142}{12} = \frac{71}{6}$$

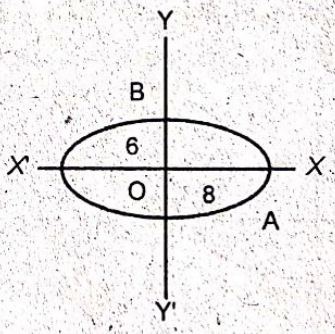
21. 2075 Set B Q.No. 10b

Find the area of the curve enclosed by $\frac{x^2}{64} + \frac{y^2}{36} = 1$, using the method of integration. [4]

SOLUTION

Given curve is $\frac{x^2}{64} + \frac{y^2}{36} = 1$

The curve is symmetrical about *x-axis* and y-axis. So, to find the area of the whole ellipse, we first find the area of the portion lying in the first quadrant and then multiply it by 4. The area of the portion lying in the first quadrant is bounded by the curve, *x-axis* and the ordinates $x = 0$ and $x = 8$. So its area is



$$A = \int_0^8 y\,dx$$

$$= \int_0^8 \frac{6}{8}\sqrt{64 - x^2}\,dx \quad \ldots(i)$$

Put $x = 8\sin\theta$

Then, $dx = 8\cos\theta\,d\theta$

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$\sqrt{64-x^2} = \sqrt{64-64\sin^2\theta} = 8\cos\theta$

When $x = 0$, $\theta = 0$

When $x = 8$, $\theta = \frac{\pi}{2}$

Then, from (i)

$$A = \frac{6}{8}\int_0^{\pi/2} 8\cos\theta \cdot 8\cos\theta\, d\theta$$

$$= 8 \times 6 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 48 \int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{48}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 24\left[\frac{\pi}{2} + \frac{\sin\pi}{2} - 0\right]$$

$$= 24 \times \frac{\pi}{2}$$

$$= 12\pi$$

∴ The whole area of the ellipse

= 4(area of the portion lying in the first quadrant)

$= 4 \times 12\pi = 48\pi$ sq. units

22. 2075 Set C Q.No. 10b

Find the area of the circle $x^2 + y^2 = a^2$. [4]

SOLUTION

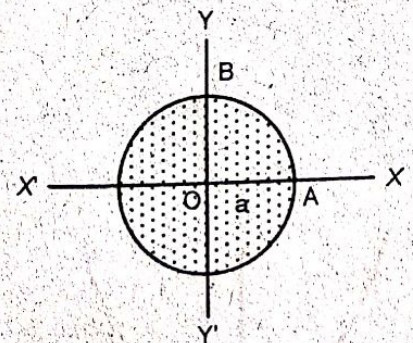
The equation of circle is

$x^2 + y^2 = a^2$

or, $y = \sqrt{a^2 - x^2}$ ... (i)

The centre of the given circle is at origin (0, 0) and its radius is a. since the given circle is symmetrical about axes, *x-axis* and y-axis divide it into four equal parts as shown in the figure.

OAB is the portion of the circle lying in the first quadrant which is bounded by the given curve and the axes.



At A, $y = 0 \Rightarrow x^2 = a^2$

$x = a$

At O, $x = y = 0$

∴ Area of OAB $= \int_0^a y\, dx$

Hence,

Area of the circle $= 4 \times$ Area of OAB

$$= 4\int_0^a y\, dx = 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

Put $x = a\sin\theta$, then $dx = a\cos\theta\, d\theta$

When $x = 0$, $\sin\theta = 0$

or, $\sin\theta = \sin 0$

∴ $\theta = 0$

When $x = a$, $a\sin\theta = a$

or, $\sin\theta = 1$

or, $\sin\theta = \sin\frac{\pi}{2}$

∴ $\theta = \frac{\pi}{2}$

∴ Area of the circle $= 4\int_0^{\pi/2} \sqrt{a^2 - a^2\sin^2\theta} \times a\cos\theta\, d\theta$

$$= 4\int_0^{\pi/2} a\cos\theta \times a\cos\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{4a^2}{2}\int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= 2a^2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2a^2\left[\left(\frac{\pi}{2} + \frac{\sin 2\times\pi/2}{2}\right) - \left(0 + \frac{\sin 2\times 0}{2}\right)\right]$$

$$= 2a^2\left[\frac{\pi}{2} + \frac{\sin\pi}{2} - \frac{\sin 0}{2}\right]$$

$$= 2a^2\left[\frac{\pi}{2} + \frac{0}{2} - \frac{0}{2}\right]$$

$= \pi a^2$ sq. units

23. 2074 Supp Q.No. 10b

Using integration, find the area of the circle $x^2 + y^2 = a^2$. [4]

➥ Please refer to 2075 Set C Q.No. 10b

24. 2074 Set A Q.No. 10b

Find the area of the region between the curve $y^2 = 4x$ and the line $x = y$. [4]

SOLUTION

The given curves are

$y^2 = 4x$ ...(i)

and $y = x$ ...(ii)

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Solving (i) and (ii), we get

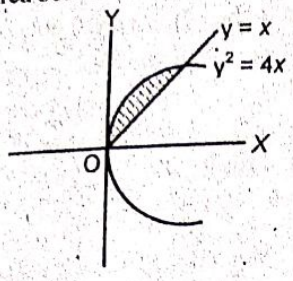
$x^2 = 4x$

or, $x^2 - 4x = 0$

or, $x(x-4) = 0$

$\therefore x = 0, x = 4$

Required area between the curves



$= \int_0^4 (y_2 - y_1)\,dx$ where $y_2 = \sqrt{4x}$ (upper curve) and $y_1 = x$ (lower curve)

$= \int_0^4 (\sqrt{4x} - x)\,dx$

$= \int_0^4 (2x^{1/2} - x)\,dx$

$= \left[2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^4 = \left[\frac{4}{3}x^{3/2} - \frac{x^2}{2}\right]_0^4$

$= \frac{4}{3} \cdot 4^{3/2} - \frac{4^2}{2} = \frac{32}{3} - 8 = \frac{8}{3}$ sq. units.

25. 2074 Set B Q.No. 10b

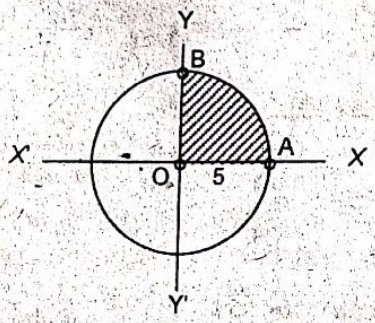
Using integration, find the area of the curve enclosed by $x^2 + y^2 = 25$. [4]

SOLUTION

Equation of given circle is $x^2 + y^2 = 25$

$\therefore y = \sqrt{25 - x^2}$ ... (i)

The centre of circle (i) is at (0, 0) & radius OA = 5 units



Since, circle (i) is symmetrical about both the axes, x-axis and y-axis divide it into four equal parts.

OAB is the portion of the circle lying in the first quadrant, which is bounded by the axes and the curve (i).

Now, Area of OAB $= \int_0^5 y\,dx = \int_0^5 \sqrt{25 - x^2}\,dx$

Put $x = 5\sin\theta$, then $dx = 5\cos\theta\,d\theta$

When $x = 0$, then $0 = 5\sin\theta$

or, $\sin\theta = 0 = \sin 0 \Rightarrow \theta = 0$

When $x = 5$, then $5 = 5\sin\theta \Rightarrow \sin\theta = 1$

or, $\sin\theta = \sin\pi/2 \Rightarrow \theta = \pi/2$

Now, Area of OAB $= \int_0^{\pi/2} \sqrt{25 - 25\sin^2\theta} \times 5\cos\theta\,d\theta$

$= \int_0^{\pi/2} 5\sqrt{25\cos^2\theta} \times \cos\theta\,d\theta$

$= 25\int_0^{\pi/2} \cos\theta \times \cos\theta\,d\theta$

$= \frac{25}{2}\int_0^{\pi/2} 2\cos^2\theta\,d\theta$

$= \frac{25}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$

$= \frac{25}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= \frac{25}{2}\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0 + 0)\right]$

$= \frac{25}{2}\left(\frac{\pi}{2} + \frac{0}{2}\right)$

$= \frac{25\pi}{4}$

If A be the area of circle, then

A $= 4 \times$ Area of OAB $= 4 \times \frac{25\pi}{4} = 25\pi$ sq. units

Hence, the area of given circle $= 25\pi$ sq. units.

26. 2073 Supp 10b

Find the area bounded by the axis of coordinates, the curve $x^2 = 4a(y - 2a)$ and the ordinate of the point (h, k). [4]

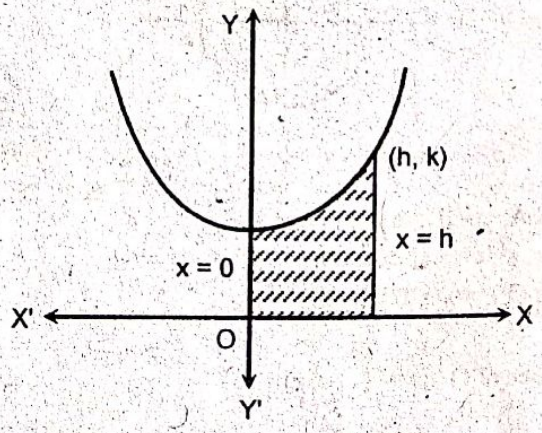
SOLUTION

Given curve is

$x^2 = 4a(y - a)$

or, $\frac{x^2}{4a} = y - 2a$

or, $y = \frac{x^2}{4a} + 2a$ ... (i)



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Thus, we have to find the area bounded by the curve (i), x-axis, and the ordinates $x = 0$ and $x = h$.

$$\text{Required area} = \int_0^h y\,dx$$

$$= \int_0^h \left(\frac{x^2}{4a} + 2a\right) dx$$

$$= \left[\frac{x^3}{4a \cdot 3} + 2ax\right]_0^h$$

$$= \left(\frac{h^3}{12a} + 2ah\right) - 0$$

$$= \frac{h^3 + 24a^2h}{12a}$$

$$= \frac{h(h^2 + 24a^2)}{12a}$$

$$= \frac{h}{12a}(h^2 + 24a^2) \text{ sq. units}$$

27. 2073 Set C Q.No. 10b

Using integration, find the area of the circle $x^2 + y^2 = 36$. [4]

➥ Please refer to 2076 Set B Q.No. 10b

28. 2073 Set D Q.No. 10b

Find the area enclosed by ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, using method of integration. [4]

SOLUTION

The given curve is

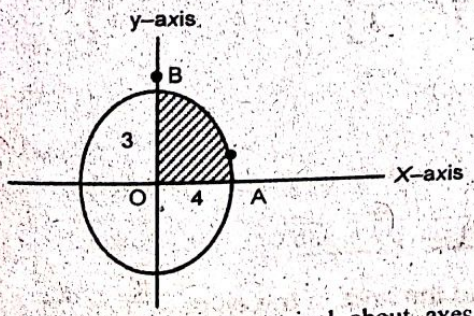
$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \quad \ldots (1)$$

or, $\frac{y^2}{9} = 1 - \frac{x^2}{16}$

or, $\frac{y^2}{9} = \frac{16 - x^2}{16}$

or, $y^2 = \frac{9}{16}(16 - x^2)$

$\therefore y = \frac{3}{4}\sqrt{16 - x^2} \quad \ldots (2)$



The given curve is symmetrical about axes. So the coordinate axes divide the area enclosed by the given curve in four equal parts as shown in the figure.

So, area of ellipse = 4 (Area of the portion OAB shaded on the figure lying in 1st quadrant).

Here, OA = 4, OB = 3

$\therefore$ Area of the portion OAB $= \int_0^4 y\,dx$

$$= \int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx$$

[$\because$ From eq" (2)]

$$= \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\,dx \quad \ldots (3)$$

Put $x = 4 \sin\theta$

Differentiating both sides with respect to θ, we get,

$\therefore \frac{dx}{d\theta} = 4\cos\theta$

or, $dx = 4\cos\theta\,d\theta$

And, $\sqrt{16 - x^2} = \sqrt{16 - 16\sin^2\theta} = \sqrt{16\cos^2\theta} = 4\cos\theta$

When $x = 0$,

$4\sin\theta = 0$

or, $\sin\theta = 0$

or, $\sin\theta = \sin 0$

$\therefore \theta = 0$

When $x = 4$,

$4\sin\theta = 4$

or, $\sin\theta = 1$

or, $\sin\theta = \sin\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$

So form (3) Area of portion OAB

$$= \frac{3}{4}\int_0^{\pi/2} 4\cos\theta \times 4\cos\theta\,d\theta$$

$$= 12\int_0^{\pi/2} \cos^2\theta\,d\theta = 12\int_0^{\pi/2} \left(\frac{1 + \cos 2\theta}{2}\right) d\theta$$

$$= 6\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$$

$$= 6\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 6\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$

$$= 6\left[\left(\frac{\pi}{2} + \frac{0}{2}\right) - 0\right] = 6 \times \frac{\pi}{2}$$

$= 3\pi$ sq. units

Hence, the area of ellipse $= 4 \times 3\pi = 12\pi$ sq. units.

29. 2072 Supp. Q.No. 10b

Find the area of the circle $x^2 + y^2 = a^2$ (Use the method of calculus). [4]

➥ Please refer to 2075 Set C Q.No. 10b

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30. 2072 Set C Q.No. 10b

Find the area of the region between the curve $y^2 = 4x$ and the line $y = x$. [4]

➥ Please refer to 2074 Set A Q.No. 10b

31. 2072 Set D Q.No. 10b

Using integration, find the area of the circle $x^2 + y^2 = 16$.. [4]

SOLUTION

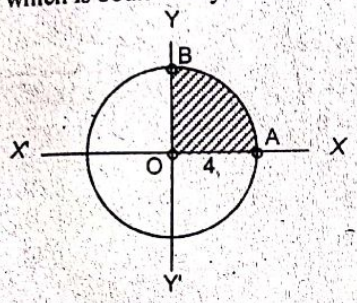
Equation of given circle is $x^2 + y^2 = 16$

$\therefore y = \sqrt{16 - x^2}$ ...(i)

The centre of circle (i) is at (0, 0) and radius OA = 4 units

Since, circle (i) is symmetrical about both the axes, *x-axis* and y-axis divide it into four equal parts.

OAB is the portion of the circle lying in the first quadrant, which is bounded by the axes and the curve (i).



Now, Area of OAB $= \int_0^4 y\,dx = \int_0^4 \sqrt{16 - x^2}\,dx$

Put $x = 4\sin\theta$, then $dx = 4\cos\theta\,d\theta$

When $x = 0$, then $0 = 4\sin\theta$

or, $\sin\theta = 0 = \sin 0 \Rightarrow \theta = 0$

When $x = 4$, then $4 = 4\sin\theta \Rightarrow \sin\theta = 1$

or, $\sin\theta = \sin\pi/2 \Rightarrow \theta = \pi/2$

Now, Area of OAB $= \int_0^{\pi/2} \sqrt{16 - 16\sin^2\theta} \times 4\cos\theta d\theta$

$= \int_0^{\pi/2} 4\sqrt{16\cos^2\theta} \times \cos\theta\,d\theta$

$= 16\int_0^{\pi/2} \cos\theta \times \cos\theta\,d\theta$

$= \frac{16}{2}\int_0^{\pi/2} 2\cos^2\theta\,d\theta$

$= 8\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$

$= 8\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= 8\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0 + 0)\right]$

$= 8\left(\frac{\pi}{2} + \frac{0}{2}\right) = 4\pi$

If A be the area of circle, then

A $= 4 \times$ Area of OAB $= 4 \times 4\pi = 16\pi$ sq. units

Hence, the area of given circle $= 16\pi$ sq. units.

32. 2072 Set E Q.No. 10b OR

Find the area of the region between the curves $y = x^2$ and $x = y^2$. [4]

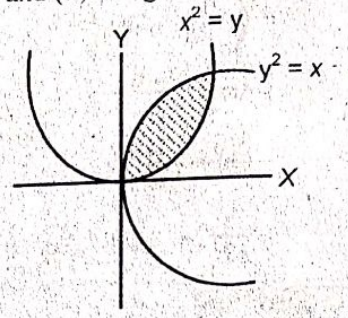
SOLUTION

Given curves are:

$x^2 = y$ ...(i)

and $y^2 = x$ ...(ii)

Solving (i) and (ii). we get



$(x^2)^2 = x$

or, $x^4 - x = 0$

or, $x(x^3 - 1) = 0$

$\therefore x = 0, 1$

Required area between the curves

$= \int_0^1 (y_1 - y_2)\,dx$ where $y_1 = \sqrt{x}$ and $y_2 = x^2$

$= \int_0^1 (\sqrt{x} - x^2)\,dx$

$= \int_0^1 (x^{1/2} - x^2)\,dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$

$= \frac{2}{3}\cdot(1)^{3/2} - \frac{1^3}{3}$

$= \frac{1}{3}$ sq. unit.

33. 2071 Old Q.No. 13b Or

Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [4]

SOLUTION

Given curve is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The curve is symmetrical about *x-axis* and y-axis. So, to find the area of the whole ellipse, we first find the area of the portion lying in the first quadrant and then multiply it by 4. The area of the portion lying in the

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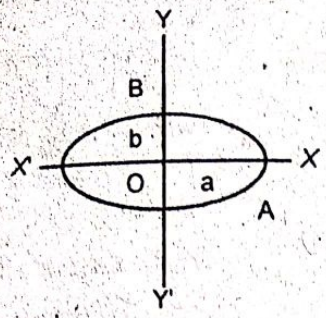
first quadrant is bounded by the curve, *x-axis* and the ordinates $x = 0$ and $x = a$. So its area is



$$A = \int_0^a y\,dx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx \quad \ldots(i)$$

Put $x = a \sin\theta$

Then, $dx = a\cos\theta\,d\theta$

$\therefore \sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2\theta} = a\cos\theta$

When $x = 0$, $\theta = 0$

When $x = a$, $\theta = \frac{\pi}{2}$

Then, from (i)

$$A = \frac{b}{a}\int_0^{\pi/2} a\cos\theta \cdot a\cos\theta\,d\theta$$

$$= ab\int_0^{\pi/2} \cos^2\theta\,d\theta$$

$$= ab\int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{ab}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{ab}{2}\left[\frac{\pi}{2} + \frac{\sin\pi}{2} - 0\right]$$

$$= \frac{ab\pi}{4}$$

$\therefore$ The whole area of the ellipse

= 4(area of the portion lying in the first quadrant)

$= 4 \cdot \frac{ab\pi}{4} = \pi ab$ sq. units

34. 2071 Set C Q.No. 10b

Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **[4]**

Please refer to 2071 Old Q.No. 13b Or

35. 2071 Set D Q.No. 10b

Using integration, find the area under the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ **[4]**

SOLUTION

The given curve is

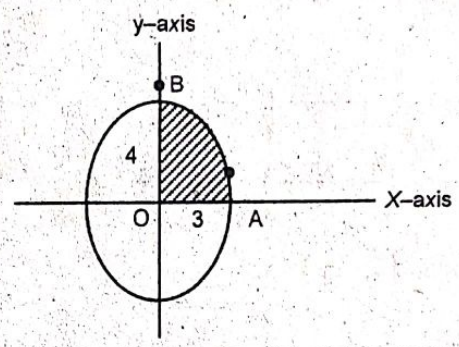
$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \quad \ldots.(1)$$

or, $\frac{y^2}{16} = 1 - \frac{x^2}{9}$

or, $\frac{y^2}{16} = \frac{9 - x^2}{9}$

or, $y^2 = \frac{16}{9}(9 - x^2)$

$\therefore y = \frac{4}{3}\sqrt{9 - x^2} \quad \ldots(2)$



The given curve is symmetrical about axes. So the coordinate axes divide the area enclosed by the given curve in four equal parts as shown in the figure.

So, area of ellipse = 4 (Area of the portion OAB shaded on the figure lying in 1st quadrant).

Here, OA = 3, OB = 4

$\therefore$ Area of the portion OAB $= \int_0^3 y\,dx$

$$= \int_0^3 \frac{4}{3}\sqrt{9 - x^2}\,dx$$

[$\because$ From eq^n (2)]

$$= \frac{4}{3}\int_0^3 \sqrt{9 - x^2}\,dx \quad \ldots(3)$$

Put $x = 3\sin\theta$

Differentiating both sides with respect to θ, we get,

$\therefore \frac{dx}{d\theta} = 3\cos\theta$

or, $dx = 3\cos\theta\,d\theta$

And, $\sqrt{9 - x^2} = \sqrt{9 - 9\sin^2\theta} = \sqrt{9\cos^2\theta} = 3\cos\theta$

When $x = 0$,

$3\sin\theta = 0$

or, $\sin\theta = 0$

or, $\sin\theta = \sin 0^c$

$\therefore \theta = 0$

When $x = 3$,

$3\sin\theta = 3$

or, $\sin\theta = 1$

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or, $\sin\theta = \sin\frac{\pi^c}{2}$

$\therefore \theta = \frac{\pi^c}{2}$

So form (3) Area of portion

$$OAB = \frac{4}{3}\int_0^{\pi/2} 3\cos\theta \times 3\cos\theta\, d\theta$$

$$= 12\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 12\int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= 6\int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= 6\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 6\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$

$$= 6\left[\left(\frac{\pi}{2} + \frac{0}{2}\right) - 0\right]$$

$$= 6 \times \frac{\pi}{2}$$

$= 3\pi$ sq. units

Hence, the area of ellipse $= 4 \times 3\pi = 12\pi$ sq. units.

36. 2070 Supp Q.No. 10b

Find the region between the curves $y^2 = 4ax$ and $x^2 = 4ay$. [4]

SOLUTION

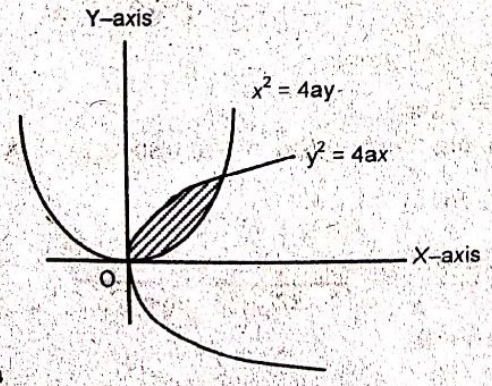
The two curves are

$y^2 = 4ax$ ... (1)

$x^2 = 4ay$ ... (2)

From (2) putting $y = \frac{x^2}{4a}$ in (1) we get,

$$\left(\frac{x}{4a}\right)^2 = 4ax$$



or, $\frac{x^4}{16a^2} = 4ax$

or, $x^4 = 64a^3x$

or, $x^4 - 64a^3x = 0$

or, $x(x^3 - 64a^3) = 0$

Either, $x = 0$ OR, $x^3 - 64a^3 = 0$

$\Rightarrow x^3 = 64a^3 = (4a)^3$

$\therefore x = 4a$

$\therefore$ The required area

$$= \int_0^{4a} (y_2 - y_1)\, dx \quad \text{where, } y_1 = \frac{x^2}{4a} \text{ and } y_2 = \sqrt{4ax}$$

$$= \int_0^{4a} y_2\, dx - \int_0^{4a} y_1\, dx$$

$$= \int_0^{4a} 2\sqrt{a}\sqrt{x}\, dx - \int_0^{4a} \frac{x^2}{4a}\, dx$$

$$= 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^{4a} - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3}[(4a)^{3/2} - 0] - \frac{1}{12a}[(4a)^3 - 0]$$

$$= \frac{4\sqrt{a}}{3} \times a^{3/2} \times (4)^{3/2} - \frac{1}{12a} \times a^3 \times 4^3$$

$$= \frac{32a^2}{3} - \frac{64a^2}{12}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3} \text{ sq. units}$$

37. 2070 Old Q.No. 13 b Or

Using method of integration, find the area of the portion of the circle $x^2 + y^2 = r^2$ in the first quadrant. [4]

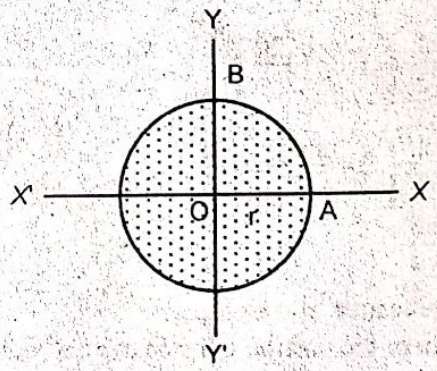
SOLUTION

The equation of circle is

$x^2 + y^2 = r^2$

or, $y = \sqrt{a^2 - r^2}$ ... (i)

The centre of the given circle is at origin (0, 0) and its radius is r. Here, OAB is the portion of the circle lying in the first quadrant which is bounded by the given curve and the axes.



At A, $y = 0 \Rightarrow x^2 = r^2$

$\therefore x = r$

At O, $x = y = 0$

$$\therefore \text{Area of OAB} = \int_0^r y\, dx$$

$$= \int_0^r \sqrt{a^2 - x^2}\, dx$$

Put $x = r\sin\theta$, then $dx = r\cos\theta\, d\theta$

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When $x = 0$, $\sin\theta = 0$

or, $\sin\theta = \sin 0$

$\theta = 0$.

When $x = r$, $r\sin\theta = r$

or, $\sin\theta = 1$

or, $\sin\theta = \sin\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$.

$\therefore$ Area of the circle in first quadrant

$$= \int_0^{\pi/2} \sqrt{r^2 - r^2\sin^2\theta} \times r\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} r\cos\theta \times r\cos\theta\, d\theta$$

$$= r^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= a^2 \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{r^2}{2} \int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= \frac{r^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{r^2}{2}\left[\left(\frac{\pi}{2} + \frac{\sin 2\times\pi/2}{2}\right) - \left(0 + \frac{\sin 2\times 0}{2}\right)\right]$$

$$= \frac{r^2}{2}\left[\frac{\pi}{2} + \frac{\sin\pi}{2} - \frac{\sin 0}{2}\right]$$

$$= \frac{r^2}{2}\left[\frac{\pi}{2} + \frac{0}{2} - \frac{0}{2}\right]$$

$$= \frac{\pi r^2}{4} \text{ sq. units}$$

38. 2070 Set C Q.No. 10 b

Find the area bounded by y-axis, the curve $x^2 = 4a(y - 2a)$ and $y = 6a$. [4]

SOLUTION

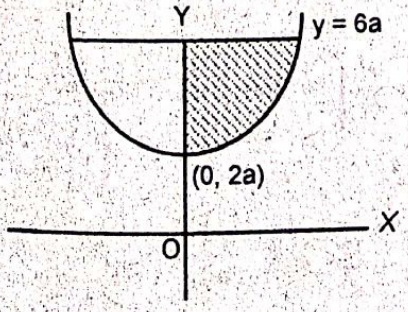
The given curve is $x^2 = 4a(y - 2a)$ ...(i)

At y-axis, $x = 0$.

Then, from (i),

$0 = 4a(y - 2a)$

$\Rightarrow y = 2a$



Also, given line is $y = 6a$ ...(ii)

$\therefore$ Hence, the required area bounded by y-axis, the curve (i) and (ii) is

$$\int_{2a}^{6a} x\, dy = \int_{2a}^{6a} \sqrt{4a(y-2a)}\, dy$$

$$= 2\sqrt{a}\int_{2a}^{6a} (y-2a)^{1/2}\, dy$$

$$= 2\sqrt{a}\left[\frac{(y-2a)^{3/2}}{3/2\cdot 1}\right]_{2a}^{6a}$$

$$= \frac{4\sqrt{a}}{3}[(y-2a)^{3/2}]_{2a}^{6a}$$

$$= \frac{4\sqrt{a}}{3}[(6a-2a)^{3/2} - (2a-2a)^{3/2}]$$

$$= \frac{4\sqrt{a}}{3}(4a)^{3/2} = \frac{4a^{1/2}}{3}\cdot 8a^{3/2}$$

$$= \frac{32a^2}{3} \text{ sq. units}$$

39. 2070 Set D Q.No. 10 b

Find the area enclosed by axis of x and the curve $y = 3x - 5x^2$. [4]

SOLUTION

Given curve is $y = 3x - 5x^2$ ...(i)

This curve meets the *x-axis* at the point, where $y = 0$

$\therefore 3x - 5x^2 = 0$

or, $x(3 - 5x) = 0$

$\therefore x = 0, x = \frac{3}{5}$

The required area between the *x-axis* and the given curve is

$$\int_0^{3/5} y\, dx$$

$$= \int_0^{3/5} (3x - 5x^2)\, dx$$

$$= \left[\frac{3x^2}{2} - \frac{5x^3}{3}\right]_0^{3/5}$$

$$= \left[\frac{3}{2}\cdot\left(\frac{3}{5}\right)^2 - \frac{5}{3}\cdot\left(\frac{3}{5}\right)^3\right] - \left[\frac{3}{2}\cdot 0^2 - \frac{5}{3}\cdot 0^3\right]$$

$$= \frac{3}{2}\cdot\frac{9}{25} - \frac{5}{3}\cdot\frac{27}{125}$$

$$= \frac{27}{50} - \frac{9}{25} = \frac{9}{50} \text{ sq. units}$$

40. 2069 Supp Q.No. 10 b

Find the area of the region bounded by the curves. $y = \sqrt{x}$ and $y = x$. [4]

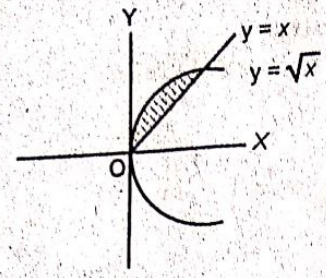
SOLUTION

Given curves are

$y = \sqrt{x}$ ...(i)

and $y = x$ ...(ii)

Solving (i) and (ii), we get



$x = \sqrt{x}$

or, $x^2 - x = 0$

or, $x(x-1) = 0$

$\therefore$ $x = 0, 1$

Required area between the curves

$= \int_0^1 (y_1 - y_2)\,dx$ where $y_1 = \sqrt{x}$ and $y_2 = x$

$= \int_0^1 (\sqrt{x} - x)\,dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$

$= \left(\frac{2}{3}\cdot 1^{3/2} - \frac{1^2}{2}\right) - 0 = \frac{1}{6}$ sq. unit

41. 2069 [Set A] Q. No. 10b

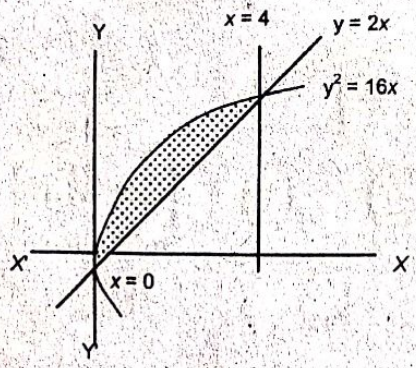
Find the area of the region between the curves $y^2 = 16x$ and the line $y = 2x$. [4]

SOLUTION

The given equations of the curve and the line are respectively,

$y^2 = 16x$ ... (i)

$y = 2x$ ... (ii)



Firstly, we find the point of intersections of the curve (i) and line (ii). For this using equation (ii) in equation (i), we get

$(2x)^2 = 16x$

or, $4x^2 - 16x = 0$

or, $4x(x-4) = 0$

or, $x = 0; x = 4$

When $x = 0$; $y = 0$

When $x = 4$; $y = 8$

So, the curve (i) and line (ii) intersect at the points (0, 0) and (4, 8)

$\therefore$ Required area

$= \int_0^4 (y_2 - y_1)\,dx$, where $y_2 = \sqrt{16x} = 4\sqrt{x}$ and $y_1 = 2x$

$= \int_0^4 (4\sqrt{x} - 2x)\,dx$

$= \left[\frac{4x^{3/2}}{3/2} - \frac{2x^2}{2}\right]_0^4$

$= \frac{8}{3}\left[x^{3/2}\right]_0^4 - \left[x^2\right]_0^4$

$= \frac{8}{3}\cdot\left[4^{3/2} - 0\right] - (4^2 - 0)$

$= \frac{8}{3}(2^3) - 16 = \frac{8}{3}\times 8 - 16$

$= \frac{64 - 48}{3} = \frac{16}{3}$ sq. units

42. 2069 [Set B] Q. No. 10b

Find the area of the region bounded by the curves $x^2 = 4y$ and $x = y$. [4]

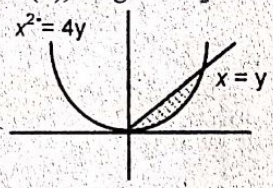
SOLUTION

The given curves are

$x^2 = 4y$ ...(i)

$x = y$ ...(ii)

Solving (i) and (ii), we get



$x^2 = 4x$

or, $x^2 - 4x = 0$

or, $x(x-4) = 0$

$\therefore$ $x = 0, 4$

Required area bounded by the curves

$= \int_0^4 (y_1 - y_2)\,dx$ where, $y_1 = x$, and, $y_2 = \frac{x^2}{4}$

$= \int_0^4 \left(x - \frac{x^2}{4}\right)dx$

$= \left[\frac{x^2}{2} - \frac{1}{4}\cdot\frac{x^3}{3}\right]_0^4$

$= \frac{4^2}{2} - \frac{1}{4}\cdot\frac{4^3}{3}$

$= \frac{96 - 64}{12} = \frac{32}{12} = \frac{8}{3}$ sq. units.

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43. 2069 [Set A] Old Q. No. 13b OR

Find the area of the circle $x^2 + y^2 = 16$ using integration. [4]

Please refer to 2072 Set D Q.No. 10b

44. 2069 [Set B] Old Q. No. 13 a OR

Find the area bounded by y-axis, the curve $x^2 = 4a(y - 2a)$ and $y = 6a$. [4]

Please refer to 2070 Set C Q.No. 10 b

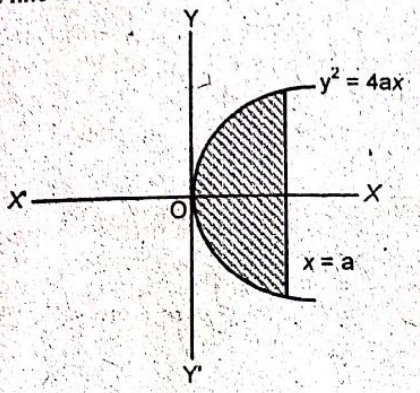
45. 2068 Q.No. 10b

Find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$. [4]

SOLUTION

The given curve is $y^2 = 4ax$ ...(i)

and the line is $x = a$ ...(ii)



The curve (i) is symmetrical about *x-axis*. So, to find the whole area bounded by (i) and (ii), we first find the area lying in first quadrant and multiply it by 2.

Required area $= 2\int_0^a y\,dx$

$= 2\int_0^a \sqrt{4ax}\,dx = 2 \cdot 2\sqrt{a}\int_0^a x^{1/2}\,dx$

$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$

$= 4\sqrt{a} \cdot \frac{2}{3}[a^{3/2} - 0^{3/2}]$

$= \frac{8a^2}{3}$ sq. units

46. 2068 Old Q.No. 13b(Or)

Find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$. [4]

Please refer to 2068 Q.No. 10b

47. 2067 Q.No. 13b OR

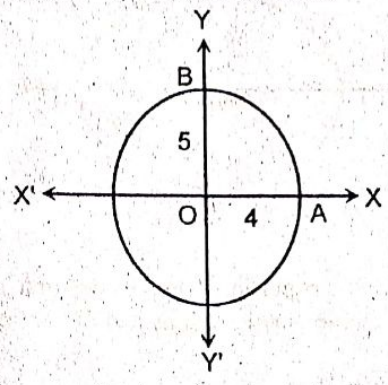
Find the area under the curves $\frac{x^2}{16} + \frac{y^2}{25} = 1$ using method of integration. [4]

SOLUTION

The given curve is $\frac{x^2}{16} + \frac{y^2}{25} = 1$

which is symmetrical about both x– and y– axis. So to find the area of the whole ellipse, we first find the area of the portion lying in the first quadrant and then multiply it by 4.

Here $0A = 4$, $0B = 5$, The area of the portion lying in the first quadrant is bounded by the curve, x – axis and the ordinates $x = 0$, $x = 4$. So its area is



$A = \int_0^4 y\,dx = \int_0^4 \frac{5}{4}\sqrt{16 - x^2}\,dx$

$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = \frac{5}{4}\sqrt{16 - x^2}\right)$

Put $x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

When $x = 0$, $\theta = 0$, and when $x = 4$, $\theta = \frac{\pi}{2}$

$\therefore A = \int_0^{\frac{\pi}{2}} \frac{5}{4}\sqrt{16 - 16\sin^2\theta} \cdot 4\cos\theta\,d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{5}{4} \cdot 4\cos\theta \cdot 4\cos\theta\,d\theta = 20\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$

$= 20\int_0^{\frac{\pi}{2}} \left(\frac{1 + \cos 2\theta}{2}\right) d\theta$

$= 10\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\,d\theta$

$= 10\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = 10 \times \frac{\pi}{2} = 5\pi$

$\therefore$ The whole area $= 4 \times 5\pi = 20\pi$ square units.

48. 2066 Q.No. 13b OR

Find the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ [4]

SOLUTION

The curve is symmetrical about both the x– and y– axis. So to find the area of the whole ellipse first we find the area of the portion lying in the first quadrant and then

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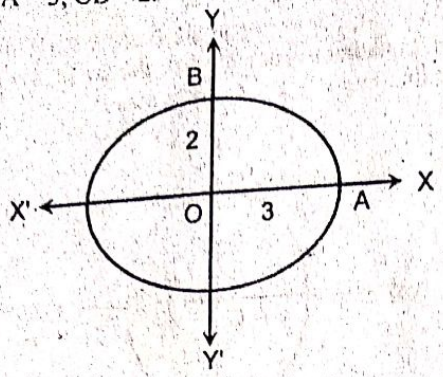
multiply it by 4. Here, the given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$,

so OA = 3, OB = 2.



The area of the portion lying in the first quadrant is bounded by the curve x – axis and the ordinates $x = 0$, $x = 3$.

So its area is

$$A = \int_0^3 y\,dx = \int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx$$

$\because \frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow y=\frac{2}{3}\sqrt{9-x^2}$

Put $x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\therefore \sqrt{9-x^2}=\sqrt{9-\sin^2\theta}= = 3\cos\theta$

When $x = 0$, $\theta = 0$

When $x = 3$, $\theta=\frac{\pi}{2}$

$$\therefore A = \int_0^{\frac{\pi}{2}} \frac{2}{3}.3\cos\theta.3\cos\theta\,d\theta$$

$$= 6\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = 6\int_0^{\frac{\pi}{2}} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= 3\left(\theta+\frac{\sin 2\theta}{2}\right)_0^{\frac{\pi}{2}}$$

$$= 3.\frac{\pi}{2}=\frac{3\pi}{2}$$

The whole area $= 4A$

$$= 4\times\frac{3\pi}{2} = 6\pi \text{ square units.}$$

49. 2065 Q.No. 13b OR

Using method of integration, find the area under the curve $x^2 + y^2 = a^2$. [4]

➥ Please refer to 2075 Set C Q.No. 10b

50. 2064 Q.No. 13a OR

Find the area of the circle $x^2 + y^2 = 9$ using method of integration [4]

SOLUTION

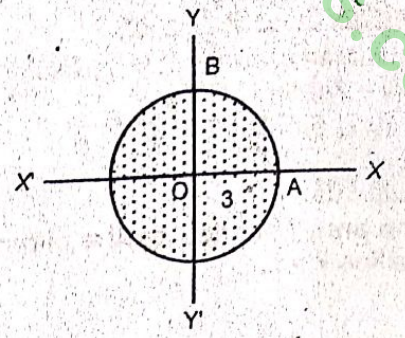
The equation of circle is

$x^2 + y^2 = 9$

or, $y = \sqrt{9-x^2}$ ... (i)

The centre of the given circle is at origin (0, 0) and its radius is 3. since the given circle is symmetrical about axes, x-axis and y-axis divide it into four equal parts as shown in the figure.

OAB is the portion of the circle lying in the first quadrant which is bounded by the given curve and the axes.



At A, $y = 0 \Rightarrow x^2 = 9$

$\therefore x = 3$

At O, $x = y = 0$

$$\therefore \text{Area of OAB} = \int_0^3 y\,dx$$

Hence,

Area of the circle $= 4 \times$ Area of OAB

$$= 4\int_0^3 y\,dx$$

$$= 4\int_0^3 \sqrt{9-x^2}\,dx$$

Put $x = 3\sin\theta$, then $dx = 3\cos\theta\,d\theta$

When $x = 0$, $\sin\theta = 0$

or, $\sin\theta = \sin 0$

$\therefore \theta = 0$

When $x = 3$, $3\sin\theta = 3$

or, $\sin\theta = 1$

or, $\sin\theta = \sin\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$

$\therefore$ Area of the circle.

$$= 4\int_0^{\pi/2} \sqrt{9-9\sin^2\theta}\times 3\cos\theta\,d\theta$$

$$= 4\int_0^{\pi/2} 3\cos\theta\times 3\cos\theta\,d\theta$$

$$= 4\times 9\int_0^{\pi/2} \cos^2\theta\,d\theta = 36\int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= 36\times\frac{1}{2}\int_0^{\pi/2} (1+\cos 2\theta)\,d\theta$$

$$= 18\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

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$$= 18\left[\left(\frac{\pi}{2} + \frac{\sin 2 \times \pi/2}{2}\right) - \left(0 + \frac{\sin 2 \times 0}{2}\right)\right]$$

$$= 18\left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \frac{\sin 0}{2}\right] = 18\left[\frac{\pi}{2} + \frac{0}{2} - \frac{0}{2}\right]$$

$= 9\pi$ sq. units

51. 2063 Q.No. 13a OR

Find the area of the circle $x^2 + y^2 = 25$, using method of integration. [4]

↪ Please refer to 2074 Set B Q.No. 10b

52. 2062 Q.No. 13b

Find the area of the ellipse: $\frac{x^2}{9} + \frac{y^2}{16} = 1$ [4]

↪ Please refer to 2071 Set D Q.No. 10b

53. 2061 Q.No. 13b OR

Using integration find the area of the circle $x^2 + y^2 = a^2$. [4]

↪ Please refer to 2075 Set C Q.No. 10b

54. 2060 Q.No. 13b OR

Find using method of integration the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$. [4]

SOLUTION

The two curves are

$y^2 = 4x$ ... (1)

$x^2 = 4y$ ... (2)

From (2) Putting $y = \frac{x^2}{4}$ in (1), we get

$$\left(\frac{x^2}{4}\right)^2 = 4x$$

or, $\frac{x^4}{16} = 4x$

or, $x^4 - 64x = 0$

or, $x(x^3 - 64) = 0$

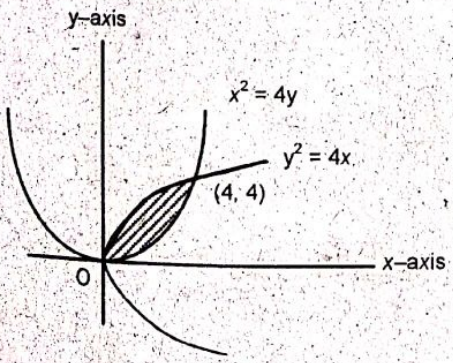
Either $x = 0$

or, $x^3 - 64 = 0$

$\Rightarrow x^3 = 64$

$\Rightarrow x = 4$

$\therefore x = 0, 4$



$\therefore$ The required area

$$= \int_0^4 (y_2 - y_1)\, dx \quad \text{where, } y_1 = \frac{x^2}{4},\ y_2 = \sqrt{4x}$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$$

$$= \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{4 \times 3}\right]_0^4$$

$$= \frac{4}{3}\left[x^{3/2}\right]_0^4 - \frac{1}{12}\left[x^3\right]_0^4$$

$$= \frac{4}{3} \times (4^{3/2} - 0) - \frac{1}{12}(4^3 - 0)$$

$$= \frac{4}{3} \times 2^{2 \times 3/2} - \frac{1}{12} \times 64$$

$$= \frac{4 \times 8}{3} - \frac{16}{3}$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$= \frac{32 - 16}{3}$$

$$= \frac{16}{3} \text{ sq. units}$$

55. 2059 Q.No. 13b OR

Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$ [4]

↪ Please refer to 2069 [Set A] Q. No. 10b

56. 2058 Q.No. 13b OR

Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$. [4]

↪ Please refer to 2070 Supp Q.No. 10b

57. 2057 Q.No. 13b OR

Find the area of the circle $x^2 + y^2 = 25$ [4]

↪ Please refer to 2074 Set B Q.No. 10b

58. 2056 Q.No. 13b OR

Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [4]

↪ Please refer to 2069 [Set A] Q. No. 10b

59. 2055 Old Model Q.No. 13b OR

Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [4]

↪ Please refer to 2069 [Set A] Q. No. 10b

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