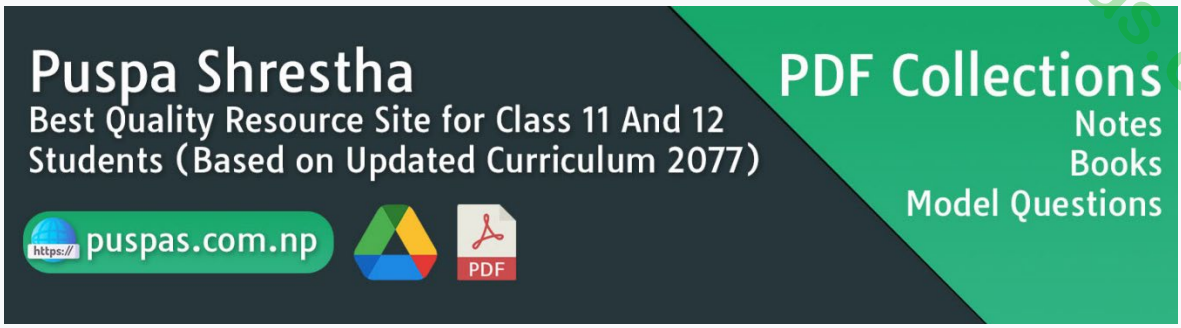
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SOLUTION

PHYSICS

Grade XI

With Model Solution

Dharm Bahadur Rokay

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Model Questions Solution - 2065

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Time: 3 Hours **Full Marks: 75**

General Instructions

i. This paper contains three groups of questions: Group A, Group B and group C
ii. Group A contains short answers questions, carrying 2 marks each.
iii. Group B contains long answer questions, carrying 4 marks each.
iv. Group C contains numerical problems, carrying 4 marks each except question 12 (last question) which carry 3 marks.
v. You may use the following values of physical constants wherever necessary:
$c = 3 \times 10^8$ m/s
$g = 10$ m/s^2
$k = 1.38 \times 10^{-23}$ J/K
Density of steel = 7800 kg/m^3
Sp. Heat capacity of water = 4200 Jkg^{-1} K^{-1}

Group A

1. Answer, in brief, any SIX questions **[2×6=12]**

a. What is the difference between accurate and precise measurement?

Accurate measurement means capable of providing a correct reading or measurement. In physical science, it means correct. A measurement is accurate if it correctly reflects the size of the thing being measured.

Precise measurement means exact, as in performance, execution, or amount. In physical science it means repeatable, reliable, getting the same measurement each time.

b. A ball having a momentum p hits a bat and its momentum becomes – p. What is the change in momentum of the ball?

From the question, we have
Initial momentum of the ball before hitting = P
Final momentum of the ball after hitting = – P
Thus, the change in momentum = Final momentum – initial momentum
$= -P - P = -2P$
Thus, the change in momentum after hitting by bat will be 2P (in magnitude).

c. State the conditions of equilibrium of a system of coplanar force.

The coplanar forces remain in equilibrium if the vector sum of all the forces is zero. i.e., $\Sigma \vec{F} = 0$ where $\vec{F}$s are coplanar forces. At this condition the total torque is also zero.

d. Does the acceleration of a body moving in a circular path remain constant?

No, the acceleration of a body moving in a circular path does not remain constant. Although, the magnitude of acceleration of the body moving in a circular path remains constant, its direction changes with time. Being acceleration a vector quantity, it has both magnitude and direction. Due to change in direction, the acceleration changes with time in a circular path but not remain constant.

e. Sand is thrown on the roads covered with snow. Why?

Sand is thrown on the roads covered with snow because it increases friction. When the sand is thrown in the icy road, it increases the coefficient of friction between the tyres of the vehicles and the road which prevents the sliding of vehicles.

f. Since the moon is constantly attracted towards the earth by the gravitational interaction, why doesn't it crash into the earth?

The moon is continuously and constantly attracted towards the earth by the gravitational interaction between the earth and moon. At the same time, the moon is also trying to move straight ahead because of its speed and the inertia of direction. So, its motion is tangential to the circular path. Due to this, the gravitational force and the motion are perpendicular to each other and the work done by this force is thus, $W = \vec{F}.\vec{S} = FS\cos\theta = 0$ being $\theta = 90°$, $\cos 90 = 0$. Therefore, the moon does not crash onto the earth.

g. Explain in terms of breaking stress why elephant has thicker legs as compared to human beings?

The breaking stress is defined as the ratio of the weight to the area of cross section.

i.e. Breaking stress = $\frac{\text{Breaking weight}}{\text{Area}}$

Breaking weight = Breaking stress × Area

This shows that the breaking weight is directly proportional to area. Thus, to balance the heavier weight of elephant, the area of the legs should be thicker. Hence, Elephant has thicker legs (to prevent from breaking) as compared to human beings.

2. Answer, in brief, any TWO questions: [2×2=4]

a. Why are two thin blankets warmer than a single blanket of double the thickness?

Since the rate of flow of heat from a body is given by

$\frac{Q}{t} = k A \frac{d\theta}{dt} \Rightarrow \frac{Q}{t} \propto k$, for constant A, dθ and dt.

A layer of air is trapped between two blankets, which is bad conductor of heat. In other words, the coefficient of thermal conductivity (k) of air is very small in comparison with that of blanket. It prevents the flow of heat from our body to the surrounding. Hence, two thinner blankets are warmer than single blanket of double thickness.

b. Distinguish between saturated and unsaturated vapor pressure.

Difference between saturated and unsaturated vapours are as follows:

Unsaturated Vapour	Saturated Vapour
a. If a space contains less than the maximum amount of vapour that it can hold at that temperature, the vapour in the space is unsaturated vapour.	a. If a space contains the maximum amount of vapour that it can hold at that temperature, vapour in the space is called saturated vapour.
b. Vapour in closed space but not in contact with liquid can be unsaturated.	b. Vapour in a closed space and in contact with liquid is saturated vapour.
c. Unsaturated vapour obeys gas laws approximately.	c. Saturated vapour doesn't obey gas laws.
d. Unsaturated vapour changes into saturated by decreasing volume or temperature.	d. Saturated vapour is independent of temperature.

c. Does the coefficient of linear expansion depend on length? Explain.

No, the coefficient of linear expansion of a material doesn't depend on length. The linear expansivity of a material is given as

$\alpha = \frac{\Delta l}{l(\theta_2 - \theta_1)}$

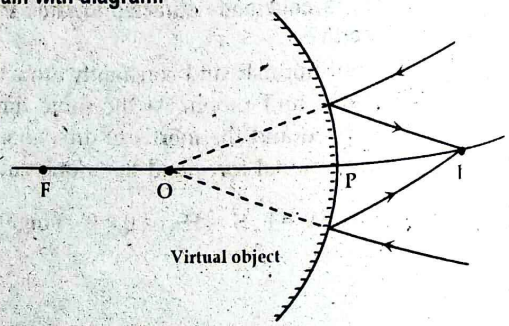
The ratio of $\frac{\Delta l}{l}$ is same in each case for a material so, the coefficient of linear expansion is same for all lengths of a material and does not depend on the length.

3. Answer, in brief, any ONE question: [2×1=2]

a. Can a convex mirror ever form a real image? If so, explain with diagram.

Yes, a convex mirror can form a real image if the object is virtual. That is, if a converging beam of light is incident on a convex mirror a real image is formed in front of it. In such case, the object distance is less than the focal length of the mirror and the object becomes virtual.

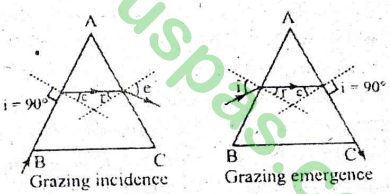
In figure 'I' be a real image of virtual object 'O'.



b. What are grazing incidence and grazing emergence?

For a prism, the deviation is maximum when angle of incidence is 90°. In this case, the ray has grazing incidence on the first refracting face of the prism. In grazing incidence, refracted angle on the first face of the prism is the critical angle C and the emergent ray makes an angle e (say) with the normal to the II face of the prism.

According to the principle of reversibility of light, it follows that if the ray of light is incident on the first face at an angle 'e', the angle of emergence on the other face will be 90°. This is what we call grazing emergence as shown in figure.



4. Answer, in brief, any ONE question: [2×1=2]

a. A man inside an insulated metallic cage does not receive shock when the cage is highly charged. Explain.

The potential at each point inside cage is same as that of the cage itself so that there is no potential difference between the man and the highly charged cage. Thus, the man does not receive any electric shock.

b. Is it possible to charge a capacitor to any potential?

No, we cannot charge a capacitor to any potential. It is limited by the dielectric strength of the medium between the two plates of the capacitor. If we apply the electric potential greater than its dielectric strength, its insulation will break and charges cannot be stored. So, we can charge a capacitor to a potential less than the dielectric strength of the insulator kept between the plates of capacitor.

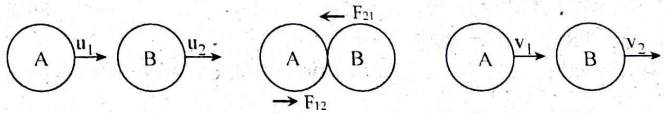
Group B

5. Answer any THREE questions: [4×3=12]

a. State the principle of conservation of linear momentum. How does Newton's third law of motion lead to the principle of conservation of linear momentum?

Statement: The principle of the conservation of linear momentum is stated as; "the total linear momentum of a system remains constant if no external forces act on the system." Or it states that "the total linear momentum of the system remains same before and after collision if there is no external force applied." For example, when a bullet is fired in the forward direction, there is recoil of the rifle in the backward direction so that the total linear momentum before and after firing is zero and hence, conserve the momentum.

Proof:



Let two bodies of masses m_1 and m_2 be moving in a straight line with respective initial velocities u_1 and u_2. If $u_1 > u_2$, they collide after some time. Let t be the time of collision, after that they move with respective final velocities v_1 and v_2 in the same straight line as shown in the figure.

During collision, let F_{12} be the force exerted by body m_1 on the body m_2 and F_{21} be the force exerted by body m_2 on the body m_1. According to Newton's third law of motion, the body m_2 also exerts force F_{21} in the opposite direction on the body m_1 such that,

$F_{12} = -F_{21}$ ... (i)

According to Newton's second law of motion,

$F_{12} = \frac{\text{change in momentum of body of mass } m_2}{t}$

$F_{12} = \frac{\text{final momentum of } m_2 - \text{initial momentum of } m_2}{t}$

$F_{12} = \frac{m_2 v_2 - m_2 u_2}{t}$

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$F_{12} \times t = m_2 v_2 - m_2 u_2$ ... (ii)

Similarly, $F_{21} \times t = m_1 v_1 - m_1 u_1$ ... (iii)

From equation (i), we have

$F_{12} \times t = -F_{21} \times t$ [Using (ii) and (iii)]

$m_2 v_2 - m_2 u_2 = -(m_1 v_1 - m_1 u_1)$ ... (iv)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

∴ Total momentum before collision = total momentum after collision

Hence, principle of conservation of linear momentum is verified.

b. Prove that the total mechanical energy in gravitational field is always conserved. Also explain conservative and non-conservative forces.

Principle of conservation of energy: It is stated as "energy is neither created nor destroyed but can be transformed from one form of energy into another form of energy." So, when a body falls freely under the action of gravity, the total mechanical energy is conserved.

Proof of conservation of mechanical energy: Consider a body of mass 'm' held at rest at height h (at point A) from the ground of the earth. If it is allowed to fall freely under the action of gravity, let B be the position of the body at any instant t and it finally reaches to ground at point C.

At point A, $u_A = u = 0$ m/s

Kinetic energy (K. E.) $= \frac{1}{2} mu^2 = 0$

Potential energy (P.E.) = mgh

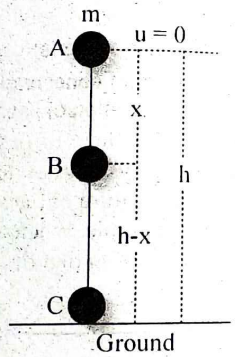
Total energy (E_A) = K.E. + P.E.

$E_A = 0 + mgh = mgh$ ... (i)

At point B:

Suppose the body falls from A to B through distance x from A, so height of point B is (h – x).

Let v_B be the velocity gained by the body falling from A to B.



Since $v_B^2 = u^2 + 2ax$

∴ $v_B^2 = 0 + 2gx$

⇒ $v_B^2 = 2gx$

Now, K.E. at point $B = \frac{1}{2} mv_B^2 = \frac{1}{2} m.2gx = mgx$

and P.E. at B = mg (h – x)

Total energy at B is E_B = K.E. + P.E.

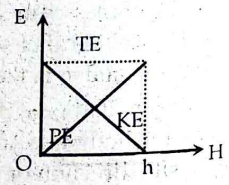
= mgx + mg (h – x)

= mgx + mgh – mgx

E_B = mgh ...(ii)

At point C:

Let v_C be the velocity at point C at which body covers distance h.



Since, $v_C^2 = v_A^2 + 2ah$

$v_C^2 = 0 + 2gh$ (a = g)

$v_C^2 = 2gh$

Kinetic energy at point (K.E.) $= \frac{1}{2} mv_C^2$

$= \frac{1}{2} m\, 2gh = mgh$

Potential energy (P.E.) = mg h = mg 0 = 0

Total energy at point C is E_C = mgh + 0

= mgh ...(iii)

From equation (i), (ii) and (iii), we see that

$E_A = E_B = E_C$ = mgh. Thus, when a body falls freely under the action of gravity, the kinetic energy and potential energy vary from point to point during motion but the total mechanical energy (K.E. + P.E.) remains constant at each point. So, the mechanical energy is conserved when the body falls under the action of gravity.

Conservative and Non-Conservative Forces:

A force is said to be conservative if the total work done by it on moving a particle round a closed path is zero. The work done by it in moving a particle from one point to another point depends only on the distance between these points but not the actual path followed. For example, gravitational force.

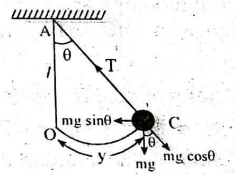
A force is said to be non-conservative if the work done by it on moving a particle round a closed path is not zero. The work done by non-conservative force depends upon the path followed and initial and final position also. For example, frictional force.

c. What is SHM? Show that the motion of a bob of simple pendulum is simple harmonic. Find its time period.

SHM: The to and fro motion in which acceleration is directly proportional to the displacement and directed towards mean position is called SHM.

Simple pendulum: A simple pendulum is a heavy mass suspended from a rigid support by a weightless, inextensible and flexible string which can vibrates about a fixed point. The point by which the string is suspended is called point of suspension.

Expression for the time period of simple pendulum



Let us consider mass of the simple pendulum 'm' suspended by a string of length l. If the mass is slightly displaced and left, it oscillates along the arc of a circle of central point A.

Let 'θ' be the angular displacement of bob and the new position of the bob is at point C as shown in figure.

The weight mg of the bob acting vertically downwards and T is the tension in the string acting towards A. The weight of the bob has two perpendicular components mg cosθ along the string which balance the tension and mg sin θ along the direction perpendicular to the tension.

It is clearly seen that the tension is balanced by mg cosθ whereas mg sinθ is unbalanced force called restoring force directed towards mean position.

Thus, the restoring force (F) = –mg sinθ ... (i)

[–ve sign indicates that the force is directed towards mean position]

Since,

F = ma ... (ii)

From equation (i) and (ii)

$a = -g \sin\theta$ ... (iii)

For small angle, $\sin\theta \simeq \theta$

From figure, $\theta = \frac{y}{l}$ where l is the length of the pendulum.

∴ $a = -g\frac{y}{l} \Rightarrow a \propto -y$ ... (iv)

This equation (iv) is in the form of simple harmonic motion as g/l is constant. So, motion of simple pendulum is simple harmonic motion.

Which is given by, $a = -\omega^2 y$ ... (v)

comparing equation (iv) and (v),

$\omega^2 = \frac{g}{l}$ [ω is angular velocity]

$\omega = \sqrt{\frac{g}{l}}$

Since, $\omega = \frac{2\pi}{T}$, T be the time period of oscillation.

∴ $T = \frac{2\pi}{\omega}$

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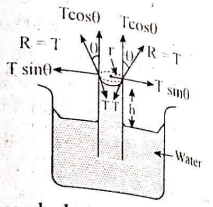
$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

Hence, time period of oscillation of simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$.

d. What is capillarity? Deduce an expression for the rise of a liquid in a capillary tube.

Capillarity: The phenomenon of rise or fall of liquid inside the capillary tube is called capillarity.

Let us consider a glass capillary tube of internal radius r and dipped in a liquid, such as water whose angle of contact is small, the liquid rises in the tube as shown in the figure: Due to surface tension, the liquid surface tries to contract with the glass tube. For this, liquid molecules pull the glass molecules with a force T per unit length tangentially at the point of contact of the liquid surface and glass. The glass molecules also exert equal and opposite reaction R = T, to the liquid molecules along the circle of contact. This reaction force (R = T) acting per unit length can be resolved into two components: T sinθ per unit length along horizontal which act radically outward along the circle of contact and cancel the effect of one another and T cosθ per unit length along vertical in upward direction which are responsible for the rise of liquid in the capillary tube. The liquid rises in the tube until total upward force due to surface tension becomes equal to the weight of the liquid in the tube above the outside level of liquid.



Here,

Total upward force = vertical component of T × circumference

$= T\cos\theta \times 2\pi r = 2\pi r\, T\cos\theta$ .... (i)

Volume of the liquid in the tube above the free surface of liquid is given by

V = Volume of cylinder of height h and radius r + Volume of cylinder of height r and radius r - Volume of hemisphere of radius r

$= \pi r^2 h + \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{1}{3}\pi r^3$

$V = \pi r^2\left(h + \frac{r}{3}\right)$

Weight of liquid column= Mass of liquid column × g = ρ × V × g = $\rho\,\pi r^2\left(h + \frac{r}{3}\right) g$

For equilibrium,

Total upward force = Weight of the liquid column

$T\cos\theta \times 2\pi r = \rho\,\pi r^2\left(h + \frac{r}{3}\right) g$

or, $T = \frac{\rho r\left(h + \frac{r}{3}\right) g}{2\cos\theta}$

When the tube is of a very fine bore, $\frac{r}{3}$ may be neglected compared to h, so surface tension T becomes

$T = \frac{\rho r h g}{2\cos\theta}$

$\therefore T = \frac{rh\rho g}{2\cos\theta}$

or, $h = \frac{2T\cos\theta}{r\rho g}$

This is required formula for the rise of liquid in a capillary tube.

6. Answer any TWO questions: [4×2=8]

a. Define specific latent heat of fusion of a solid. Develop an expression for determination of the latent heat of fusion of ice.

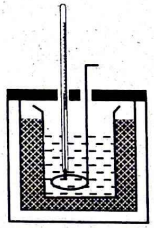
Specific Latent heat of fusion of a solid: The amount of heat required to change unit mass of solid to liquid at its melting point is called Specific latent heat of fusion of solid.

Latent heat of fusion of ice by the method of mixture:

Let us take a clean and dry calorimeter with stirrer and weighted.

The calorimeter is filled with water about one third and weighted again. The calorimeter is now placed into the wooden box and initial temperature is noted.

Small pieces of ice are taken and water in them is soaked with blotting paper and then dropped gently into the calorimeter containing water. The mixture is stirred gently until the temperature of mixture falls down and becomes constant. The final temperature of mixture is noted and weighted the mixture again.



Suppose,

Mass of calorimeter and stirrer = m_c

Mass of calorimeter stirrer and water = m_1

Mass of water taken = $m_1 - m_c = m_w$

Initial temperature of calorimeter and water= θ_1

Mass of calorimeter, stirrer, water and ice = m_2

Mass of ice taken = $m_2 - m_1 = m_i$

Final temperature of mixture = θ

Specific heat capacity of water= S_w

Specific heat capacity of calorimeter system = S_c

Latent heat of fusion of ice = L_i

Heat gained by the ice during melting = $m_i L_i$

Heat gained by the melted ice being heated from 0°C to the constant final temperature θ°C = $m_i S_w (\theta - 0) = m_i S_w \theta$

Thus, the total heat gained = $m_i L_i + m_i S_w \theta$

The heat lost by the calorimeter system and water

$= m_c S_c (\theta_1 - \theta) + m_w S_w (\theta_1 - \theta) = (m_c S_c + m_w S_w)(\theta_1 - \theta)$

From the principle of calorimetry,

Heat gained by ice = heat lost by calorimeter system and water

$m_i L_i + m_i S_w \theta = (m_c S_c + m_w S_w)(\theta_1 - \theta)$

$m_i L_i = (m_c S_c + m_w S_w)(\theta_1 - \theta) - m_i S_w \theta$

$L_i = \frac{(m_c S_c + m_w S_w)(\theta_1 - \theta) - m_i S_w \theta}{m_i}$

$\therefore L_i = \frac{(m_c S_c + m_w S_w)(\theta_1 - \theta)}{m_i} - S_w \theta$

This is the required expression for the latent heat of fusion of ice.

b. Define the coefficient of thermal conductivity of a substance. Describe Searle's method for determination of the thermal conductivity of a good conductor.

Thermal Conductivity: The thermal conductivity of material of a body is defined as the amount of heat that flows in one second across the opposite faces of a unit cube, whose opposite faces are kept at a temperature difference of 1K. The coefficient of thermal conductivity is represented by 'k'. Its Sl unit is $Wm^{-1}k^{-1}$.

Mathematically, $\frac{Q}{t} = k\frac{A(\theta_1 - \theta_2)}{x}$

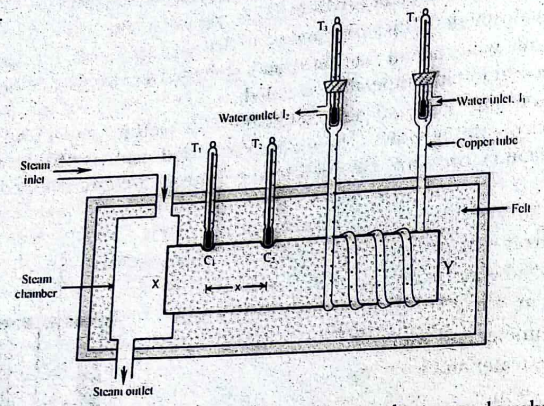
When $A = 1m^2$, $\theta_1 - \theta_2 = 1k$ and $x = 1$ m, then $k = \frac{Q}{t}$

Experimental determination of thermal conductivity (k) of solid bar (*Searle's method*)

The apparatus required for the determination of thermal conductivity is as shown in figure. XY is the

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metal bar whose thermal conductivity is to be determined. One end of bar is enclosed within steam chamber and a copper wire wound near the other end. Steam is passed through steam chamber and water flows through copper tube at constant rate. T_1 and T_2 are thermometers to measure the temperature of bar at a small distance x apart. Thermometer T_3 and T_4 measure the temperature of outgoing and incoming water. To ensure good thermal contact, small amount of mercury is placed into cavity.



As the end X of bar is heated by passing steam through steam chamber, so that heat is conducted from X to end Y and circulating water is warmed up. At steady state, the reading of all thermometers becomes constant. Heat lost from the surface of bar is neglected because the bar is lagged by some non-conducting material.

Let, θ_1, θ_2, θ_3 and θ_4 are the temperatures recorded by thermometers T_1, T_2, T_3 and T_4 respectively in steady state.

Area of cross section = A

Distance between cavities = x

Time for which the heat is considered = t

Mass of water that flows through the copper tube in time t = m

The amount of heat flows through metal bar in time t is,

$$Q = \frac{kA(\theta_1 - \theta_2)t}{x} \quad \ldots (i)$$

where k is the thermal conductivity of metal bar and $\theta_1 - \theta_2$ is temperature difference between two cavities.

The temperature of water flowing through the copper tube rises from θ_4 to θ_3. So the amount of heat absorbed by water in time t is

$$Q = mS(\theta_3 - \theta_4) \quad \ldots (ii)$$

From the principle of calorimetry,

Heat lost = heat gain

$$\frac{kA(\theta_1 - \theta_2)t}{x} = mS(\theta_3 - \theta_4)$$

$$k = \frac{mS(\theta_3 - \theta_4)x}{A(\theta_1 - \theta_2)t}$$

Which is the required expression of thermal conductivity of material bar. As all the quantities in above expression (right hand side) are known, then k can be experimentally measured.

What is a Carnot engine? Derive an expression for its efficiency.

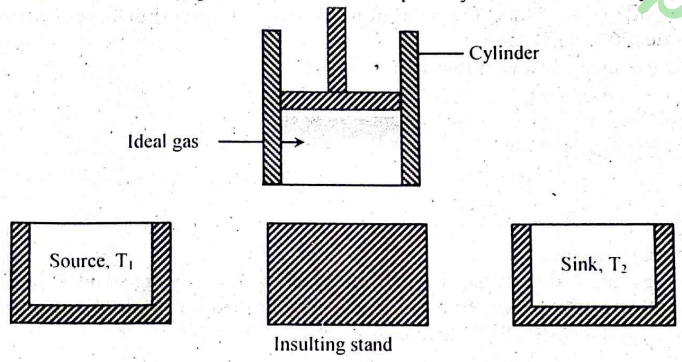
A Carnot engine is a theoretical engine, which is used to convert heat into mechanical work. It is free from all the defects of practical engines. Its efficiency is maximum and it is an ideal heat engine. A Carnot heat engine consists of,

Source: It has fixed high temperature (T_1) from which the heat engine can draw heat and has infinite thermal capacity.

Sink: It has fixed low temperature (T_2) to which any amount of heat can be rejected and has infinite thermal capacity.

c. **Working substance:** A cylinder with non-conducting sides and conducting bottom containing a perfect gas as the working substance.

The working substance (1 mole of ideal gas) is taken round a reversible cycle consisting of two adiabatic and two isothermal processes, so this complete cycle called *Carnot cycle*.



The Carnot cycle completes in four steps:

Step 1: *(Isothermal expansion)*:

The cylinder containing the working substance is placed on the source maintained at a higher temperature T_1. The cylinder absorbs Q_1 amount of heat from source through its conducting base and piston moves out of the cylinder. The expansion is isothermal, in which the gas expands from initial state A (P_1, V_1, T_1) to final state B (P_2, V_2, T_1) shown in PV indicator diagram.

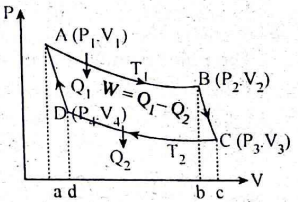
According to first law of thermodynamics, the workdone during this process,

$$W_1 = Q_1$$

$$W_1 = \int_{V_1}^{V_2} P\,dV \qquad [\because dU = 0]$$

$$W_1 = RT_1 \log_e \frac{V_2}{V_1} \quad \ldots (i)$$

= Area of ABbaA



P-V Diagram of Carnot Cycle

Step 2: *(Adiabatic expansion)*:

The cylinder is now removed from the source and placed on the insulating stand. The working substance is allowed to expand further. No heat enters or leaves the cylinder, i.e. adiabatic expansion in which ($T_2 < T_1$) gas expands from initial state B(P_2, V_2,T_1) to final state C(P_3,V_3,T_2) as shown in PV diagram. Let, W_2 be the workdone during adiabatic expansion. Then using first law of thermodynamics,

$$W_2 = \int_{V_2}^{V_3} PdV = \int_{V_2}^{V_3} \frac{dV}{V^\gamma} \cdot k \qquad [Q_2 = 0,\ PV^\gamma = \text{constant}]$$

$$= \int_{V_2}^{V_3} \frac{K}{V^\gamma}\,dV = K\int_{V_2}^{V_3} V^{-\gamma}\,dV = \frac{K}{1-\gamma}\left[\frac{1}{V_3^{\gamma-1}} - \frac{1}{V_2^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{P_3V_3^\gamma}{V_3^{\gamma-1}} - \frac{P_2V_2^\gamma}{V_2^{\gamma-1}}\right] \qquad [\because P_3V_3^\gamma = K = P_2V_2^\gamma]$$

$$W_2 = \frac{1}{1-\gamma}[P_3V_3 - P_2V_2] \qquad [\because PV = RT]$$

$$= \frac{1}{\gamma-1}[P_2V_2 - P_1V_1] = \frac{R(T_1 - T_2)}{\gamma - 1} \quad \ldots (ii)$$

= area of BCcbB

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Step 3: (Isothermal compression):
The cylinder is then removed from the stand and placed on the sink maintained at a lower temperature T_2. The working substance is compressed slowly until the state represented by D is reached. During this process, Q_2 amount of heat is rejected to sink at temperature T_2. This compression is isothermal. Thus, the workdone during compression from initial state C (P_3, V_3,T_2) to final state D(P_4, V_4,T_1) be

$W_3 = Q_3$ [U = 0, using 1st law of thermodynamics]

or, $Q_3 = W_3$

$$= -\int_{V_3}^{V_4} PdV = RT_2 \log_e \frac{V_3}{V_4} \quad \ldots (iii)$$

= Area of CDdcC

Step 4: (Adiabatic compression):
Finally the cylinder is removed from the sink and placed back on insulating stand. Pressure is increased from P_4 to P_1 and volume gets decreased from V_4 to V_1. The process is completely adiabatic compression and hence, reaches from state D (P_4, V_4,T_2) to initial state A(P_1, V_1,T_1).

$$\text{Workdone from D to A } (W_4) = -\int_{V_4}^{V_1} PdV = \int_{V_1}^{V_4} PdV = -\frac{R(T_1 - T_2)}{\gamma - 1} \quad \ldots (iv)$$

= Area of DAadD

The net workdone by the working substance in one complete cycle

= Area of ABbaA + Area of BCcbB + Area of CDdcC – Area of DAadD

= Area of ABCDA

$$= W_1 + W_2 - W_3 - W_4 = W_1 - W_3 + RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4} \quad \ldots (v)$$

The point B and C lie on same adiabatic, so

$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

Since, $\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \Rightarrow \frac{V_1}{V_4} = \frac{V_2}{V_3}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \ldots (vi)$$

$$\therefore W = R \log_e \frac{V_2}{V_1} [T_1 - T_2] \quad \text{[using (vi)]}$$

Now, Efficiency of heat engine is defined as the ratio of external work done by the engine to the amount of heat energy absorbed from the source.

$$\text{Efficiency } (\eta) = \frac{\text{External workdone (W)}}{\text{Heat absorbed from source}(Q_1)} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Therefore, Efficiency of Carnot engine (in percentage) is

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

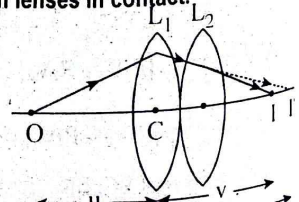
7. Answer any ONE question: (4 x 1 = 4)

a. Define power of a lens. Derive the formula for the effective power of two thin lenses in contact.

Power of Lens:
The ability of a lens to converge or diverge the light rays after refraction is called power of lens.

Mathematically, it is reciprocal to the focal length of lens. i.e., $P = \frac{1}{F}$,

where F = Focal length.



Its SI unit is Diopter when F is in meter.

Let us consider two thin convex lenses L_1 and L_2, placed in contact. f_1 and f_2 be the focal lengths of L_1 and L_2 respectively. There is a point object O placed on the principal axis at a distance 'u' from optical centre C. In the absence of lens L_2, I' be the image of O formed by lens L_1 at a distance v' from C shown in figure.

Using lens formula, we get

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \ldots (i)$$

When lens L_2 is placed in contact with L_1, I' acts as object for L_2 and real image I is formed at a distance v from C. Since lenses are thin, therefore point of contact can be taken as C. Now using lens formula for L_2, we get

$$\frac{1}{f_2} = \frac{1}{(-v')} + \frac{1}{v} \quad \ldots (ii)$$

Adding (i) and (ii) we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots (iii)$$

If we replaced the combined lens by a single lens having focal length F produced image I of object O so that

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \ldots (iv)$$

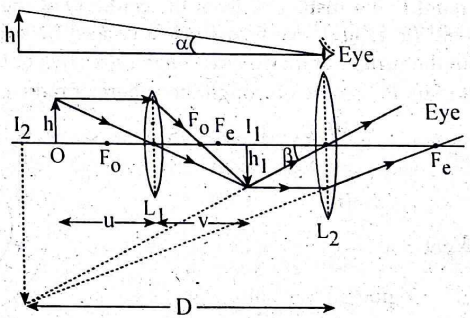
From eqn (iii) and (iv), we get $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$\therefore P = P_1 + P_2$, where $P = \frac{1}{F}$, $P_1 = \frac{1}{f_1}$ and $P_2 = \frac{1}{f_2}$

This is the required expression for the equivalent power of two thin convex lenses having powers P_1 and P_2 respectively. This shows that the combined power is greater than the individual power of lens.

b. Describe the construction and working of a compound microscope and hence, derive an expression for its magnifying power.

Compound microscope: A microscope which is a combination of two convex lenses of short focal length and has a larger magnification is called the compound microscope. A lens of short aperture and short focal length facing the object is called objective lens. And, another lens of short focal length and large aperture lying towards eye is called eye lens.



Principle: When a small object is placed just outside the focus of the objective lens, its real, inverted and magnified image is produced on the other side of the lens. The image produced by the objective lens acts as object for eye lens. The position of eye lens is so adjusted that the final image is formed at least distance (near point) of distinct vision from the eye.

Let us consider O be an object placed just outside the focus F_0 of the object lens. Its real image I_1 formed on the other side of the lens. This image I_1 acts as object for the eye lens. Finally, virtual and magnified image I_2 is formed on the same side at least distance of distinct vision as shown in figure.

Angular magnification: The angular magnification M, when the microscope is in normal use is given by the ratio of angle subtended at the eye by the final image to angle subtended at the eye by the object when both are at least distance of distinct vision (D).

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i.e. $M = \frac{\text{Angle subtended at the eye by the final image at D}}{\text{Angle subtended at the eye by the object at D}} = \frac{\beta}{\alpha} = \frac{h_2/D}{h/D} = \frac{h_2}{h} = \frac{h_2}{h_1} \times \frac{h_1}{h} = M_e \times M_o$

where, $M_e = \frac{h_2}{h_1}$ is the magnification of the eyepiece.

and $M_o = \frac{h_1}{h}$ is the magnification of the objective lens.

Here,

$M_e = \frac{h_2}{h_1} = -\left(\frac{D}{f_e} + 1\right)$

and $M_o = \frac{h_1}{h} = \frac{v}{u}$

where, u and v are the object and image distances for the objects.

Hence, $M = -\left(\frac{D}{f_e} + 1\right)\frac{v}{u}$

Since, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f_o} \Rightarrow \frac{v}{u} = \frac{v}{f_o} - 1$

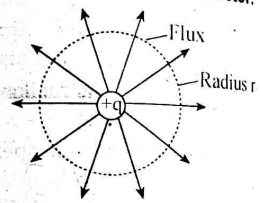
$\therefore M = \left(\frac{D}{f_e} + 1\right)\left(\frac{v}{f_o} - 1\right)$

Thus, for high angular magnification, here f_o and f_e should be small.

8. Answer any ONE question: (4 × 1 = 4)

a. State Gauss law. Apply it to obtain an expression for electric field outside the charged spherical conductor.

Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system)



i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$\therefore \phi = \frac{1}{\epsilon_0} \times q$

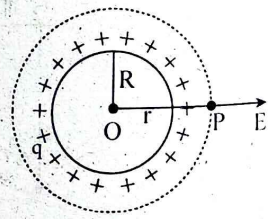
Electric field outside the charged sphere: Let us consider a charged sphere of radius R has charge q on its surface. There is a point P at a distance r from the centre O of the charged sphere, where we have to find the electric field (intensity). We now draw a concentric sphere of radius r so that the point P will lie outside from the surface of the sphere. Therefore, Area of this sphere = $4\pi r^2$.

If E be the field intensity at point P, then flux through the sphere of radius r is

$\phi = E A = E\,4\pi r^2$ ... (i)

From Gauss theorem, we have

$\phi = \frac{1}{-\epsilon_0} q$ ... (ii)



Using equation (i) & (ii), we get

$E\,4\pi r^2 = \frac{1}{\epsilon_0} q$

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

This is the electric field intensity at a point outside the charge sphere.

b. Find the equivalent capacitance of two capacitors when they are in (i) series and (ii) parallel.

Capacitance of capacitor: The ability of a conductor to store charge is called its capacitance. The rise in potential difference between conducting plates is directly proportional to the charge given to them.

i.e. $q \propto V \Rightarrow q = CV$

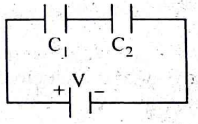
where, C is a constant of proportionality & is called the capacitance of the conductor. Thus, $C = \frac{q}{V}$

When V = 1 volt, then C = q

Thus, capacitance of a capacitor is numerically equal to the charge required to rise p.d. by 1 volt. The SI unit of capacitance is Farad (F).

A. Series Combination of Capacitor:

The capacitors are said to be joined in series, if the charge on each capacitor is same while the voltage across them is different. Suppose C_1 and C_2 are the capacitance of two capacitors which are connected to the terminals of a d.c. cell with p.d. V so that charge on each capacitor is q. The voltages across the capacitors are $V_1 = \frac{q}{C_1}$ and $V_2 = \frac{q}{C_2}$.



Thus, the total voltage across them becomes

$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2}$

$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$, where, $C = \frac{q}{V}$ is the total (or equivalent or net) capacitance of the series arrangement.

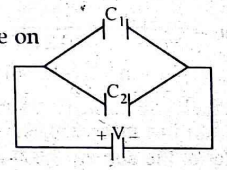
Thus, for series combination of capacitors, the reciprocal or equivalent capacitance is equal to the sum of the reciprocal of capacitance of individual capacitance and which is smaller than the smaller capacitance of individual capacitance.

B. Parallel Combination of Capacitors:

The capacitors are said to be joined in parallel, if each capacitor is directly connected to a source so that voltage across each capacitor remains same while the charge is different.

Suppose C_1 and C_2 are the capacitance of two capacitors which are joined in parallel across a source of emf V.

Suppose V is the p.d. between the plates of each capacitor, then charge on each capacitor is $q_1 = C_1V$ and $q_2 = C_2V$ respectively.



Thus, the total charge becomes

$q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V$

$\frac{q}{V} = C_1 + C_2$

Therefore, $C = C_1 + C_2$

where, $C = \frac{q}{V}$ is the resultant (or net or total or equivalent) capacitance of the parallel combination.

Hence, total capacitance of parallel combination is equal to the algebraic sum of the capacitance of individual capacitor.

Group C

Solve any THREE numerical questions: (4 × 3 = 12)

A ballet dancer spins with 2.4 rev/s with her arms outstretched, when the moment of inertia about the axis of rotation is I. With her arms folded, the moment of inertia about the same axis becomes 0.61. Calculate the new rate of spin.

olution

Given,

When arms outstretched,

Frequency (f_1) = 2.4 rev/sec

Moment of inertia (I_1) = I

When arms folded,

Frequency (f_2) = ?

Moment of inertia (I_2) = 0.6I

We have, from principle of conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

r, $I_1\,2\pi f_1 = I_2.\,2\pi f_2$

r, $I_1 f_1 = I_2 f_2$

r, $I.2.4 = 0.6\,I.\,f_2$

r, $f_2 = 4$ rev/sec

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b. A canoe has a velocity of 0.40 m/s southeast relative to the earth. The canoe is on the river that is flowing 0.50 m/s east relative to the Earth. Find the velocity (magnitude and direction) of the canoe relative to the river.

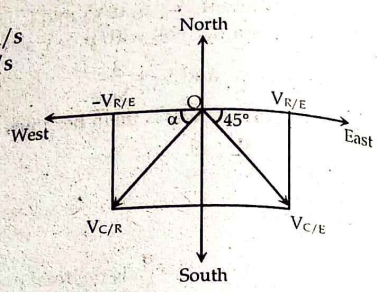
Solution

Given,

Velocity of canoe with respect to earth ($V_{C/E}$) = 0.4 m/s

Velocity of river with respect to earth ($V_{R/E}$) = 0.5 m/s

Velocity of canoe with respect to river ($V_{C/R}$) = ?



From figure,

$v_{C/E} = v_{R/E} + v_{C/R}$

$v_{C/R} = v_{C/E} - v_{R/E}$ .....(i)

The component of velocity $v_{C/R}$ are

$v_{C/R}(x) = v_{C/E}\cos 45° - v_{R/E}$

$= 0.4 \times \frac{1}{\sqrt{2}} - 0.5$

$= 0.2828 - 0.5 = 0.2117\ ms^{-1}$ (towards east)

and $v_{C/R}(y) = v_{C/E}\sin 45°$

$= 0.4 \times \frac{1}{\sqrt{2}} = 0.2828\ ms^{-1}$

Now, $v_{C/R} = \sqrt{(v_{C/R}(x))^2 + (v_{C/R}(y))^2}$

$= \sqrt{(-0.2117)^2 + (0.2828)^2} = 0.353\ ms^{-1}$

Again,

$\tan\alpha = \frac{v_{C/R}(y)}{v_{C/R}(x)} = \frac{0.2828}{-0.2117} = -1.336$

$\alpha = \tan^{-1}(-1.336) = 53.18°$, south east

Thus, the velocity of canoe relative to river be 0.353 ms^{-1} in a direction 53.18° south east.

c. The sun has mass 330,000 times that of the earth. For a person on the earth, the average distance to the center of the sun is 23,500 times the distance to the center of earth. In magnitude, what is the ratio of the sun's gravitational force on you to the earth's gravitational force on you?

Solution

Given,

Let mass and radius of earth are M_e and R_e respectively, then the mass of sun is 330000 M_e and distance between man and sun is 23500 R_e.

Let, m be the mass of man on the surface of earth then,

force on mass due to earth is

$F_e = \frac{GM_em}{R_e^2}$

and force on man due to sun it,

$F_s = \frac{GM_sm}{R_s^2}$

Now,

$\frac{F_e}{F_s} = \frac{M_e \times R_s^2}{M_s \times R_e^2} = \frac{M_e}{330000M_e} \times \frac{(23500)^2 R_e^2}{R_e^2} = \frac{(23500)^2}{330000} = 1673.5$

$\therefore \frac{F_s}{F_e} = \frac{1}{1673.5}$

d. Castor oil at 20°C has a coefficient of viscosity 2.42 N.s/m² and a density 940 kg/m³. Calculate the terminal velocity of a steel ball of radius 2.0 mm falling under gravity in the oil taking the density of steel as 7800 kg/m³.

Solution

Given,

Coefficient of viscosity (η) = 2.42 Nsm^{-2}

Density of oil (σ) = 940 kgm^{-3}

Density of steel (ρ) = 7800 kgm^{-3}

Radius of steel ball (r) = 2mm = 2×10^{-3} m

Terminal velocity (V) = ?

We have,

$V = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times (2 \times 10^{-3})^2 (7800 - 940) \times 10}{9 \times 2.42} = 0.025$ m/ sec

10. Solve any TWO numerical questions: (4 × 2 = 8)

a. A ball of copper of specific heat capacity 400 J kg^{-1} K^{-1} weighing 400 g is transferred from a furnace to 1 kg of water at 20°C. The temperature of water rises to 50°C. What is the original temperature of the ball?

Solution

Given,

Mass of copper ball (M_c) = 400 gm = 0.4 kg

Mass of water (M_w) = 1 kg

Temperature of water (θ_2) = 20°C

Find temperature of mixture (θ) = 50°C

Initial temperature of copper ball (θ_1) = ?

Specific heat capacity of copper (S_c) = 400 J/kg K

Specific heat capacity of water (S_w) = 4200J /kgK

Heat lost by ball (Q_1) = $m_c S_c (\theta_1 - \theta)$

$= 0.4 \times 400 (\theta_1 - 50)$

Heat gained by water (Q_2) = $M_wS_c (\theta - \theta_2) = 1 \times 4200 (50 - 20) = 4200 \times 30 = 126000$ J

From principle of colorimetry,

Heat loss = Heat gained

or, $Q_1 = Q_2$

or, $126000 = 0.4 \times 400 (\theta_1 - 50)$

or, $\theta_1 = 50 + 787.5$

or, $\theta_1 = 837.5°C$

b. An ideal gas initially at 4.00 atm and 350 K is permitted to expand adiabatically to 1.50 times its initial volume. Find the final pressure and temperature if the gas is (a) monatomic, (b) diatomic with $c_v = \frac{5}{2}R$.

Solution

Given,

Initial pressure (P_1) = 4 atm

Final pressure (P_2) = ?

Initial temperature (T_1) = 35k

Final temperature (T_2) = ?

Initial volume (V_1) = V

Final volume (V_2) = 1.5V

i. For mono-atomic gas $\gamma = 5/3 = 1.67$

We have

$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 4 \times \left(\frac{1}{1.5}\right)^{1.67} = 2.0$ atm

Also,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 350 \times \left(\frac{1}{1.5}\right)^{0.67} = 266.74$ K

i. For diatomic $\gamma = 7/5 = 1.4$

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 4 \times \left(\frac{V}{2V}\right)^{1.4} = 4 \times \left(\frac{1}{1.5}\right)^{1.4} = 2.27$ atm

Also,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 350 \times \left(\frac{1}{1.5}\right)^{0.4} = 297.6$K

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c. At what temperature will the average speed of oxygen molecule be sufficient so as to escape from the earth? Escape velocity from the earth is 11.2 km/s and mass of one oxygen molecule is 5.34×10^{-26} kg.

Solution

Given,

Escape velocity from earth $(\bar{c}) = 11.2$ Km/sec $= 11200$ m/sec

Mass of oxygen molecule $(m) = 5.34 \times 10^{-26}$ kg

Boltzmann constant $(K) = 1.38 \times 10^{-23}$ J/k

Temperature $(T) = ?$

We have,

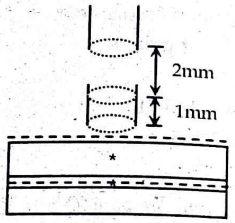
$$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT \qquad [\because \bar{c} = \sqrt{\bar{c}^2} \Rightarrow \bar{c}^2 = (\bar{c})^2]$$

or, $\frac{3}{2} \times 1.38 \times 10^{-28} \times T = \frac{1}{2} \times 5.34 \times 10^{-24} \times (11200)^2$

or, $T = 1.62 \times 10^5$ K

11. A microscope is focused on the upper surface of a glass plate. A second plate is then placed over the firs In order to focus on the bottom surface of the second plate, the microscope must be raised another 1mm. I order to focus on the upper surface, it must be raised another 2 mm. Find the index of refraction of th second plate.

Solution



Given,

Real length $= 2 + 1 = 3$ mm

Apparent depth $= 2$mm

Refractive index $(\mu) = ?$

We have,

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{3}{2} = 1.5$$

12. Two point charges + 1 μC and + 4 μC are placed at a distance of 0.12 m apart. Determining the point joinir the line between two charges where net force acting on the unit positive charge is zero?

Solution

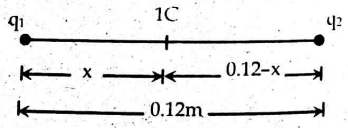
Given,

Charge on smaller point charge $(q_1) = 1\mu C = 1 \times 10^{-6}$ C

Charge on large point charge $(q_2) = 4\mu C = 4 \times 10^{-6}$C

Distance between charge $(r) = 0.12$ m

Let, a point unit charge is kept at distance x form q_1, then by question,



$F_1 = F_2$

$$\frac{1}{4\pi\epsilon_0}\frac{q_1.1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2.1}{(0.12-x)^2}$$

or, $\frac{q_1}{x^2} = \frac{q_2}{(0.12-x)^2}$

or, $\left(\frac{1}{x}\right)^2 = \left(\frac{2}{0.12-x}\right)^2$

or, $\frac{1}{x} = \frac{2}{0.12-x}$

or, $2x = 0.12 - x$

or, $3x = 0.12$

or, $x = 0.04$ m

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UNIT 1 Mechanics

Chapter 1: Physical Quantities

Short Answer Questions

1. 2076 Set B Q.No. 1a The energy of a photon is given by E = hf. Find the dimension and unit of Planck's constant h, where f is the frequency of radiation. [2]

Energy of photon, E = hf,

where, h = Planck's constant and f is frequency of photon

Dimension formula of f = $[T^{-1}]$

Dimension formula of E = $[ML^2T^{-2}]$

Now,

$$h = \frac{E}{f} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

$h = [ML^2T^{-1}]$

∴ Dimension of h are 1 in M, 2 in L and -1 in T.

Again,

$$h = \frac{E}{f} = \frac{\text{Joule}}{s^{-1}} = \text{Js}$$

So, unit of plank's constant is Js.

2. 2076 Set C Q.No. 1a Distinguish between precise and accurate measurements. [2]

Accurate measurement means capable of providing a correct reading or measurement. In physical science, it means correct. A measurement is accurate if it correctly reflects the size of the thing being measured.

Precise measurement means exact, as in performance, execution, or amount. In physical science it means repeatable, reliable, getting the same measurement each time.

3. 2075 Set A Q.No. 1a Differentiate between accuracy and precession of measurement. [2]

Please refer to 2076 Set C Q.No. 1a

4. 2075 Set B Q.No. 1g In one of the printed documents the unit of universal gravitational constant is given as $NmKg^{-2}$. Check its correctness from dimensional analysis. [2]

The unit of gravitational constant on printed document is $Nmkg^{-2}$. Here N is unit of force, m is unit of distance and kg is unit of mass. So, dimension of force = MLT^{-2}, dimension of distance = L & dimension of mass = M, then

$Nmkg^{-2} = MLT^{-2}.L.M^{-2}$

$= M^{-1}L^2T^{-2}$

Again,

$$F = G\frac{m_1m_2}{d^2}$$

or, $G = \frac{Fd^2}{m_1m_2}$

$$= \frac{MLT^{-2}.L^2}{M^2} = M^{-1}L^3T^{-2}$$

Thus given unit is not correct.

5. 2074 Supp. Q.No. 1a 2063 Q.No. 1 a Is dimensionally correct equation necessarily be a correct physical relation? What about dimensionally wrong equation? [2]

No, dimensionally correct equation may not be a correct physical relation. Consider the following relation, $v = u + 2at$

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Dimensional formula of v is $[LT^{-1}]$ and that of $(u + 2at)$ is also $[LT^{-1}]$. Hence, the above relation is dimensionally correct. But, it is established fact that the above relation isn't correct physical relation. Dimensionally wrong equation can't be correct physical relation. For example, $v = u + at^2$ is dimensionally wrong equation and it is also incorrect physical relation.

6. 2074 Set B Q.No. 1f | 2067 Q.No. 1 a What is the difference between accurate and precise measurements? [2]

Please refer to 2076 Set C Q.No. 1a

7. 2073 Supp Q.No. 1g A student writes an expression for the momentum (p) of a body of mass (m) with total energy (E) and considering the duration of time (t) as $p = \sqrt{2mE/t}$. Check its correctness on dimensional analysis. [2]

The given formula is

$$p = \sqrt{\frac{2mE}{t}},$$

where,

p = momentum, m = mass
E = Energy t = time

The dimensions of each terms are;

Dimension of momentum, $p = [MLT^{-1}]$
Dimension of mass, $m = [M]$
Dimension of time, $t = [T]$
Dimension of energy, $E = [ML^2T^{-2}]$

Then substituting the quantities in above relation, we get,

$$MLT^{-1} = \sqrt{\frac{M.ML^2T^{-2}}{T}} \quad [\because 2 \text{ is dimensionless constant}]$$

$$= \sqrt{M^2L^2T^{-1}} = MLT^{-1/2}$$

Dimension of LHS ≠ Dimension of RHS

So, the formula is dimensionally incorrect.

8. 2073 Set C Q.No. 1a Check dimensionally the correctness of the Stoke's formula, $F = 6\pi\eta rv$, where symbols have their usual meanings. [2]

The given Stoke's formula is

$$F = 6\pi\eta rv \quad \text{.... (i)}$$

The dimension of $F = [MLT^{-2}]$
The dimension of $6\pi = 0$ (dimensionless quantity)
The dimension of $\eta = [ML^{-1}T^{-1}]$
The dimension of $r = [L]$
The dimension of $v = [LT^{-1}]$

Substituting the value of dimensions in equation (i), we get,

$$[MLT^{-2}] = [ML^{-1}T^{-1}][L][LT^{-1}] = [M^{-1+1+1}L^{-1+1}T^{-1-1}] = [MLT^{-2}]$$

∴ Dimensions on left hand side are equal to the dimensions on right hand side. So the formula is dimensionally correct.

9. 2072 Supp Q.No. 1a Check the correctness of the formula, PV = RT. [2]

Given formula: PV = RT, where, P = pressure,
V = volume, R = gas constant and T = temperature.
The dimension of pressure, $P = [ML^{-1}T^{-2}]$
The dimension of volume, $V = [L^3]$
The dimension of gas constant, $R = [ML^2T^{-2}K^{-1}]$
The dimension of temperature, $T = [K]$

Now,

Dimensions of left hand side, PV = $[ML^{-1}T^{-2}L^3]$ = $[ML^2T^{-2}]$
Dimensions of right hand side, RT = $[ML^2T^{-2}K^{-1}K]$ = $[ML^2T^{-2}]$

∴ Dimensions on left hand side are equal to dimensions on right hand side of a formula PV = RT. Hence, the formula is dimensionally correct.

10. 2072 Set C Q.No. 1a The diameter of a steel rod is given as 56.47 ± 0.02 mm. What does it mean? [2]

The diameter of a steel rod is given as 56.47 ± 0.02 mm. This means that 56.47 mm is the actual diameter of the rod and ± 0.02 mm is least count of the instrument. It means the least count of the device that is used to measure the diameter of given steel rod is 0.02mm. The uncertainty number of device is '2'. The device is unable to measure the length beyond the two digits after decimal in mm scale.

11. 2072 Set D Q.No. 1a The length of rod is exactly 1 cm. An observer records the readings as 1.0 cm, 1.00 cm, and 1.000 cm, which is the most accurate measurement? [2]

If the length of rod is exactly 1 cm, when an observer records the readings as 1.0 cm, 1.00 cm and 1.000 cm, then the measurement which measures 1.000 cm is most accurate because it has more number of significant figure and the instrument which measure this data has less least count than the other. The instrument which measures length of rod 1.0 cm can measure one digit beyond the decimal, the instrument which measures length 1.00 cm can measure two digits after the decimal and the third instrument which measures lengths 1.000 cm can measure three digits after decimal. So, the third instrument can measure accurate.

12. 2071 Supp Q.No. 1a What do you mean by significant figure? [2]

The meaningful digits in a number are called significance figure. The number of digits in a number is called its number of significant figures. The number of significant figures in a measurement depends upon the least count of the measuring instrument. For example, the length of a scale is 2.17 cm when measured by vernier caliper and 2.176 cm when measured by micrometer screw gauze. Then significant figures are 3 and 4. The more significant figure gives more precious value of a measurement.

13. 2071 Set D Q.No. 1 d A student writes an expression of the force causing a body of mass (m) to move in a circular motion with a velocity (v) as $F = mv^2$ use the dimensional method to check its correctness. [2]

The given relation is $F = mv^2$
Where F is force, m is mass and v is speed of particle.
The dimension of force $F = [MLT^{-2}]$
The dimension of velocity $v = [LT^{-1}]$
The dimension of mass $m = [M]$
The dimension of left hand side $F = [MLT^{-2}]$
Dimension of right hand side $mv^2 = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

∴ Dimensions on left hand side are not equal to dimension on right hand side of equation. So, the formula is dimensionally wrong.

14. 2070 Supp (Set B) Q.No. 1 g Check the correctness of the relation $h = \frac{2T\cos\theta}{r\rho g}$, where symbols have usual meaning. [2]

Here, given formula is

$$h = \frac{2T\cos\theta}{r\rho g}$$

Here,

Dimension of h = [L]
Dimension of r = [L]
Dimension of $\rho = [ML^{-3}]$
Dimension of $g = [LT^{-2}]$
Dimension of T = $[MT^{-2}]$ and $2\cos\theta$ is dimensionless.
Then, dimension of left hand side, h = [L]

Dimension of right hand side, $\frac{2T\cos\theta}{r\rho g} = \frac{[MT^{-2}]}{[L][ML^{-3}][LT^{-2}]} = [M^{1-1}L^{-1+3-1}T^{-2+2}] = [L]$

∴ Dimensions on left hand side are equal to the dimensions to the right side of the formula. So the formula is dimensionally correct.

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15. 2070 Set C Q.No. 1 a Name any two physical quantities which have the same dimensions. Can a quantity have unit but no dimension? Explain.

Two physical quantities which have same dimensions are work and torque. Yes, a physical quantity which has unit but not dimension is plane angle. Its SI unit is radian but it has no dimension.

16. 2070 Set D Q.No. 1 b If $y = a + bt + ct^2$, where y is the distance and t is the time. What is the dimension and unit of c?

Given equation is,

$y = a + bt + ct^2$, where y is the distance and t is time, so according to principle of homogeneity, the dimensions and unit of c such that,

Unit of c × unit of t^2 = unit of y

$c \times sec^2 = m$

or, $c = m/sec^2$

∴ Unit of c is m/sec^2 and dimension of c are 0 in M, 1 in L and – 2 in T and dimension formula is $[M^0 L T^{-2}]$.

17. 2069 Supp Q.No. 1a Find the dimensions of Planck's constant 'h' from the given equation: $\lambda = \frac{h}{p}$; where λ is wavelength and p is the momentum of photon.

The formula for Planck's constant is given as $\lambda = \frac{h}{p}$ or $h = \lambda \times p$, where λ is wavelength and p is momentum. The dimension of $\lambda = [L]$ and dimension of $P = [M L T^{-1}]$. Then dimensions of h is,

$h = [L] [M L T^{-1}]$

$h = [M L^2 T^{-1}]$

∴ Dimensions of h are 1 in M, 2 in L and – 1 in T and dimension formula is $[M L^2 T^{-1}]$

18. 2069 (Set A) Old Q. No. 1a What are the limitations of dimensional analysis?

The limitations of dimensional analysis are given below.

a. It does not give any information about the dimensionless constant.
b. If a physical quantity depends on more than three factors having dimension, the formula cannot be derived.
c. We cannot derive the formulae containing trigonometric function, exponential functions logarithmic function etc.
d. It does not tell whether a physical quantity is a scalar or vector.
e. The dimensionally correct relation may not be physically correct.

19. 2069 (Set B) Q. No. 1a Check the correctness of the formula $t = 2\pi\sqrt{l/g}$ using dimensional analysis, t is the time period of simple pendulum, l is the length of simple pendulum and g is the acceleration due to gravity.

We have,

$t = 2\pi\sqrt{l/g}$

Dimension of $l = [L]$

Dimension of $g = [LT^{-2}]$

Then, in dimension form

$[T] = \text{constant}\sqrt{\frac{L}{LT^{-2}}}$

$[T] = [T]$

∴ Dimension of left hand side is equal to dimension of right hand side. So, the given formula is dimensionally correct.

20. 2069 Old (Set B) Q. No. 1a The force F is given in terms of time 't' and displacement 'x' by the equation $F = A \sin Bx + C \sin Dt$. What is the dimension of D/B?

Given formula is

$F = A \sin Bx + C \sin Dt$

Since, Bx and Dt are dimensionless quantities and x has dimension of length and t has dimension of second. So, dimension of B is $[L^{-1}]$ and dimension of D is $[T^{-1}]$.

Dimension of $\frac{D}{B} = \frac{T^{-1}}{L^{-1}} = [LT^{-1}]$

21. 2068 Old Q.No. 1 a Determine the dimensional formula for gravitational constant (G). [2]

We have,

$F = G\frac{m_1 m_2}{d^2}$

or, $G = \frac{Fd^2}{m_1 m_2}$ ... (ii)

where F = force, d = distance,

m_1 and m_2 = masses of two bodies.

Dimension of F = $[MLT^{-2}]$

Dimension of d = $[L]$

Dimension of m = $[M]$

∴ Dimension of $G = \frac{[MLT^{-2} \times L^2]}{[M \times M]} = [M^{-1} L^3 T^{-2}]$

22. 2067 Supp Q.No. 1 a Write dimensional formula of gravitational constant and latent heat. [2]

Gravitational constant: Please refer to 2068 Old Q.No. 1 a

The latent heat 'L' is defined as,

$L = \frac{Q}{m}$, where Q is amount of heat and m be the mass.

Dimensional formula of Q = $[ML^2T^{-2}]$

Dimensional formual of m = $[M]$

∴ Dimensional formula of $L = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$

23. 2066 Old Q. No. 1 a Is a dimensionally correct equation necessarily be a correct physical relation? Justify your answer. [2]

Please refer to 2074 Supp. Q.No. 1a

24. 2064 Q.No. 1 b Check the correctness of formula $t = 2\pi\sqrt{m/k}$ where t be the time period, m is the mass and k is the force per unit displacement. [2]

We have,

$t = 2\pi\sqrt{\frac{m}{k}}$

Since, 2π is constant

Dimension of m = $[M]$

Dimension of k = $[MT^{-2}]$

Dimension of t = $[T]$

Hence,

$t = \sqrt{\frac{M}{MT^{-2}}}$

$[T] = \sqrt{T^2}$

$[T] = [T]$

Hence, the given equation, $t = 2\pi\sqrt{\frac{m}{k}}$ is dimensionally correct.

25. 2062 Q.No. 1 b A student writes $\sqrt{\frac{R}{2GM}}$ for escape velocity. Check the correctness of the formula by using dimensional analysis.

The escape velocity, $v = \sqrt{\frac{R}{2GM}}$

The dimension of v = $[M^0 LT^{-1}]$

The dimension of G = $[M^{-1}L^3T^{-2}]$

The dimension of M = $[ML^0T^0]$ and

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The dimension R = [M^0LT^0]

In dimensional formula, this equation becomes

$$[LT^{-1}] = \sqrt{\frac{L}{M^{-1}L^3T^{-2} \times ML^0T^0}} = \sqrt{\frac{L}{L^3T^{-2}}} = \sqrt{\frac{1}{L^2T^{-2}}} = \sqrt{\frac{T^2}{L^2}}\,[LT^{-1}] = [TL^{-1}]$$

which is not true. Hence, given equation is dimensionally incorrect.

26. 2061 Q.No. 1 a Convert 10 ergs in joules.

Since erg and Joule are CGS and SI unit of workdone (or energy) respectively.

The dimensional formula of workdone is W = [ML^2T^{-2}] and compare with [$M^xL^yT^z$], we get

⇒ x = 1, y = 2 and z = -2

CGS system	SI system
n_1 = 10 ergs	n_2 = ?
M_1 = 1 gm	M_2 = 1 kg
L_1 = 1 cm	L_2 = 1 m
T_1 = 1 sec	T_2 = 1 sec

We have, $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{l_1}{l_2}\right]^y\left[\frac{T_1}{T_2}\right]^z = 10 \times \left[\frac{1\text{ gm}}{1\text{ kg}}\right]^1\left[\frac{1\text{ cm}}{1\text{ m}}\right]^2\left[\frac{1\text{ sec}}{1\text{ sec}}\right]^{-2}$

$= 10 \times \left[\frac{1\text{ gm}}{1000\text{ gm}}\right]^1\left[\frac{1\text{ cm}}{100\text{ cm}}\right]^2 [1]^{-2}$

$= 10 \times 10^{-3} \times 10^{-4} \times 1 = 10^{-6}$ J

i.e., 10 erg = 10^{-6}J

27. 2059 Q.No. 1 a Check the correctness of the formula $v^2 = u^2 + 2as$ using dimensional analysis.

The given formula is $v^2 = u^2 + 2as$, where v is final velocity, u is initial velocity, a is acceleration and s is distance.

Since, the dimension of v = [M^0LT^{-1}]

the dimension of u = [M^0LT^{-1}]

the dimension of a = [M^0LT^{-2}] and

the dimension of s = [M^0LT]

Hence, the dimensional formula of LHS = Dimension of v^2 = [$M^0LT^{-1}]^2$ = [$M^0L^2T^{-2}$]

Dimensional formula of RHS = Dimension of $u^2 + 2as$

= [$M^0LT^{-1}]^2$ + 2 [M^0LT^{-2}] [M^0LT^0] = 3[$M^0L^2T^{-2}$]

As 3 being dimensionless quantity, dimensions of LHS = dimensions of RHS.

Thus, the given formula is dimensionally correct.

28. 2057 Q.No. 1 a Taking force, length and time to be fundamental quantities, find the dimensional formula for the density.

Dimension of density (D):

Since, $D = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Force/Acceleration}}{\text{Volume}} = \frac{F}{a \times V}$

Dimension of F = [F]

Dimension of a = [LT^{-1}]

Dimension of V = [L^3] = $\frac{[F]}{[LT^{-2}]\,[L^3]}$ = [$FL^{-4}T^2$]

Hence, the dimensions of D are 1 in F, -4 in L and 2 in T i.e. [$FL^{-4}T^2$].

29. 2056 Q.No. 1 e Calculate the dimensional formula for Universal Gravitational Constant 'G'.

Please refer to 2068 Old Q.No. 1 a

30. 2055 Q.No. 1 e The escape velocity of a body is $V_g = \sqrt{\frac{2GM}{R}}$. Check the correctness of the formula using dimension.

The escape velocity of a body is, $v = \sqrt{\frac{2GM}{R}}$, where G is the universal gravitational constant, M is the mass and R is the radius of the earth.

Since, the dimension of v = [M^0LT^{-1}]

the dimension of G = [$M^{-1}L^3T^{-2}$]

the dimension of M = [ML^0T^0] and

the dimension R = [M^0LT^0]

So, LHS = dimension of v = [M^0LT^{-1}]

Dimension of $\sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\frac{M^{-1}L^3T^{-2}.\,M}{L}} = \sqrt{2}\sqrt{L^2T^{-2}} = \sqrt{2}\,[LT^{-1}]$

∴ $\sqrt{2}$ is dimension less quantity.

∴ LHS is demensinally equal to RHS hence, formula is correct dimensionally.

31. 2055 Q.No. 1 i The density of gold is 19.3 gm/cc. Express its value in SI unit.

Since gm/cc and kg/m^3 are CGS and SI unit of density respectively.

The dimensional formula of density is d = [$ML^{-3}T^0$] and compare with [$M^xL^yT^z$], we get

⇒ x = 1, y = -3 and z = 0

CGS system	SI system
n_1 = 19.3 g/cc	n_2 = ?
M_1 = 1 gm	M_2 = 1 kg
L_1 = 1 cm	L_2 = 1 m
T_1 = 1 sec.	T_2 = 1 sec

We have,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{l_1}{l_2}\right]^y\left[\frac{T_1}{T_2}\right]^z = 19.3 \times \left[\frac{1\text{ gm}}{1\text{ kg}}\right]^1\left[\frac{1\text{ cm}}{1\text{ m}}\right]^{-3}\left[\frac{1\text{ sec}}{1\text{ sec}}\right]$$

$$= 19.3 \times \left[\frac{1\text{ gm}}{1000\text{ gm}}\right]^1\left[\frac{1\text{ cm}}{100\text{ cm}}\right]^{-3}[1]$$

$= 19.3 \times 10^{-3} \times 10^6 \times 1 = 19300$

i.e. 19.3 gm/cc = 19300kg/m^3

32. 2054 Q.No. 1 a Obtain dimensions of specific heat capacity and gravitational constant.

i. Specific heat capacity of a substance is defined by the relation;

$Q = ms\Delta\theta$

or, $s = \frac{Q}{\Delta\theta m}$ ... (i)

where m = mass, Δθ = change in temperature,

Q = Heat energy supplied.

∴ Dimension of Q = [ML^2T^{-2}]

Dimension of m = [M]

Dimension of θ = [K]

Dimension of s = $\frac{[ML^2T^{-2}]}{[M\,K]}$ = [$M^0L^2T^{-2}K^{-1}$]

ii. Please refer to 2068 Old Q.No. 1 a

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1. **2076 Set C Q.No. 1b** **If $\vec{A} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{B} = 7\hat{i} + 5\hat{j} + \hat{k}$, find the angle between the vectors $\vec{A}$ and $\vec{B}$.** [2]

Given,

Vector $(\vec{A}) = 4\hat{i} - \hat{j} + 3\hat{k}$

Vector $(\vec{B}) = 7\hat{i} + 5\hat{j} + \hat{k}$

Angle $(\theta) = ?$

Now,

$\vec{A} \cdot \vec{B} = (4\hat{i} - \hat{j} + 3\hat{k}) \cdot (7\hat{i} + 5\hat{j} + \hat{k})$

$= (4 \times 7)(\hat{i} \cdot \hat{i}) - (1 \times 5)(\hat{j} \cdot \hat{j}) + (3 \times 1)(\hat{k} \cdot \hat{k})$

$= 28 - 5 + 3 = 26$

Again

$A = |\vec{A}| = \sqrt{4^2 + (-1)^2 + 3^2} = \sqrt{16 + 1 + 9} = \sqrt{26}$

$B = |\vec{B}| = \sqrt{7^2 + 5^2 + 1^2} = \sqrt{49 + 25 + 1} = \sqrt{75}$

Now,

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{A \cdot B} = \frac{26}{\sqrt{26} \times \sqrt{75}} = 0.589$$

or, $\theta = \cos^{-1}(0.589)$

$= 54°$

$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is 54°.

2. **2075 Set B Q.No. 1a** **$\vec{A}$ and $\vec{B}$ are two non-zero vectors. If $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$, what is the angle between $\vec{A}$ and $\vec{B}$?** [2]

If $\vec{A}$ and $\vec{B}$ are two vectors and θ is angle between them, then their scalar product is:

$\vec{A} \cdot \vec{B} = AB\cos\theta$ ... (i)

and the magnitude of their vector product is

$|\vec{A} \times \vec{B}| = AB\sin\theta$ ... (ii)

According to question, we have: $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$

or, $AB\cos\theta = AB\sin\theta$

$\Rightarrow \tan\theta = 1 = \tan 45°$

$\therefore \theta = 45°$

Thus, the angle between two vectors is 45°.

3. **2074 Supp. Q.No. 1b** **If $\vec{A} \cdot \vec{B} = 0$, what is the value of angle between $\vec{A}$ and $\vec{B}$?** [2]

The dot product between $\vec{A}$ and $\vec{B}$ is given by

$\vec{A} \cdot \vec{B} = AB\cos\theta$

where θ is angle between $\vec{A}$ and $\vec{B}$. Since $\vec{A} \cdot \vec{B} = 0$, then

$AB\cos\theta = 0$

or, $\cos\theta = 0$

or, $\theta = 90°$

So angle between $\vec{A}$ and $\vec{B}$ is 90°. i.e. $\vec{A}$ and $\vec{B}$ are perpendicular.

4. **2074 Set A Q.No. 1c** **Resultant of two equal forces may have the magnitude equal to one of the forces. At what angle between the two equal forces this is possible? Justify your answer.** [2]

Let θ be the angle between two equal forces $\vec{P}$ and $\vec{Q}$, each forces having magnitude P, so that resultant (sum) forces $\vec{R}$ has also magnitude P.

We know that,

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$P^2 = P^2 + P^2 + 2P^2\cos\theta$ [If P= Q = R]

$P^2 = 2P^2 + 2P^2\cos\theta$

$1 = 2 + 2\cos\theta$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

Hence, at inclination 120°, the sum of two equal forces equal to either of the force.

5. **2074 Set B Q.No. 1e** **Two vectors are given as = $\vec{V_1} = 2\hat{i}+3\hat{j} + 4\hat{k}$ and $\vec{V_2} = 3\hat{i} + 2\hat{j} - 4\hat{k}$. Which one of two is larger in magnitude? Justify your answer.** [2]

Two vectors are given as: $\vec{V_1} = 2\hat{i}+3\hat{j} + 4\hat{k}$ & $\vec{V_2} = 3\hat{i} - 2\hat{j} - 4\hat{k}$, then magnitudes of each vector are

$\vec{V_1} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$ units and

$\vec{V_2} = \sqrt{3^2 + (-2)^2 + (-4)^2} = \sqrt{29}$ units.

so, two vector are equal in magnitude.

6. **2073 Supp Q.No. 1e** **A vector is defined as $\vec{E} = 2\hat{i} + 3\hat{j} - 4\hat{k}$. What is the magnitude of the Y-component of $\vec{E}$?** [2]

Given vector is $\vec{E} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, comparing this with a vector in x, y and z components as,

$\vec{F} = a\hat{x} + b\hat{y} + c\hat{z}$, where a, b and c are magnitudes of x, y and z components. So, a = 2, b = 3 and c = – 4. Then

$|\vec{E}_Y| = \sqrt{b^2} = \sqrt{3^3} = 3$

$\therefore$ Y-component is 3.

7. **2073 Set D Q.No. 1a** **Two vectors have equal magnitudes and their resultant also has the same magnitude. What is the angle between the two vectors?** [2]

Let θ be the angle between two equal vectors $\vec{P}$ and $\vec{Q}$, each vectors having magnitude P, so that resultant (sum) vector $\vec{R}$ has also magnitude P.

We know that,

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$P^2 = P^2 + P^2 + 2P^2\cos\theta$ [If P= Q = R]

$P^2 = 2P^2 + 2P^2\cos\theta$

$1 = 2 + 2\cos\theta$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

Hence, at inclination 120°, the sum of two equal vector equcal to either of the vector.

8. **2072 Supp Q.No. 1e** **Can the sum of two equal vectors be equal to either vector?** [2]

Please refer to **2073 Set D Q.No. 1c**

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9. 2072 Set D Q.No. 1b **Given two vectors $\vec{A} = 4.00\,\hat{i} + 3.00\,\hat{j}$ and $\vec{B} = 5.00\,\hat{i} - 2.00\,\hat{j}$. Find the magnitude of each vector.** [2]

Given, $\vec{A} = 4.00\,\hat{i} + 3.00\,\hat{j}$ and $\vec{B} = 5.00\,\hat{i} - 2.00\,\hat{j}$

$|\vec{A}| = ?$

$|\vec{B}| = ?$

Now,

$|\vec{A}| = \sqrt{(\text{Coeff of } \hat{i})^2 + (\text{Coeff of } \hat{j})^2}$,

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= 5$ units

$|\vec{B}| = \sqrt{(\text{Coeff of } \hat{i})^2 + (\text{Coeff of } \hat{j})^2}$

$= \sqrt{5^2 + (-2)^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$ units

$\therefore$ Magnitude of $\vec{A}$ is 5 and $\vec{B}$ is $\sqrt{29}$

10. 2072 Set E Q.No. 1a **A vector $\vec{F} = \hat{i} + 2\,\hat{j} - 3\,\hat{k}$ is given. What is the magnitude of the Y-component of the vector?** [2]

Given vector $\vec{F} = \hat{i} + 2\,\hat{j} - 3\,\hat{k}$

Comparing with a vector with x, y and z- components as,

$\vec{F} = a\,\hat{x} + b\,\hat{y} + c\,\hat{z}$ where a, b and c are magnitude of x, y and z components so, $b = 2$

or, $|\vec{F}_y| = \sqrt{2^2} = 2$ units

$\therefore$ y-component is 2 units.

11. 2071 Supp Q.No. 1b **Is a physical quantity having magnitude and direction necessarily a vector quantity? Explain.** [2]

No, a physical quantity having magnitude & direction is not necessarily a vector quantity because for a physical quantity to be a vector quantity, it should have magnitude and direction and it must follow vector algebra like vector addition, subtraction and multiplication. For example, Electric current which has both magnitude and direction but it cannot be added vectorically so it is not a vector quantity.

12. 2071 Set C Q.No. 1b **A force (in Newton) expressed in vector notation as $\vec{F} = 2\,\hat{i} + \hat{j} - 3\,\hat{k}$ is applied on a body so that the displacement produced in meter is given by $\vec{D} = \hat{i} - 2\,\hat{j} - 3\,\hat{k}$. Express the result and nature of the work done.** [2]

Here, $\vec{F} = 2\,\hat{i} + \hat{j} - 3\,\hat{k}$ and $\vec{D} = \hat{i} - 2\,\hat{j} - 3\,\hat{k}$

Then, the work done is

$W = \vec{F} \cdot \vec{D} = (2\,\hat{i} + \hat{j} - 3\,\hat{k}) \cdot (\hat{i} - 2\,\hat{j} - 3\,\hat{k})$

$= 2.1 - 2.1 - 3 \times (-3) = 9$ J

The work done is scalar because it has only magnitude.

13. 2071 Set D Q.No. 1c **A force (in Newton) expressed in vector notation as $\vec{F} = 4\,\hat{i} + 7\,\hat{j} - 3\,\hat{k}$ is applied on a body and produces a displacement (in meter), $\vec{D} = 3\hat{i} - 2\hat{j} - 5\hat{k}$ in 4 seconds. Estimate the power.** [2]

Given,

Fore $(\vec{F}) = 4\,\hat{i} + 7\,\hat{j} - 3\,\hat{k}$

Displacement $(\vec{D}) = 3\hat{i} - 2\hat{j} - 5\hat{k}$

Time taken $(t) = 4$ sec

Work done $(W) = \vec{F} \cdot \vec{D} = (4\,\hat{i} + 7\,\hat{j} - 3\,\hat{k}) \cdot (3\hat{i} - 2\hat{j} - 5\hat{k}) = 12 - 14 + 15 = 13$

Power $(P) = \frac{W}{t} = \frac{13}{4} = 3.25$ Watt

14. 2070 Supp (Set A) Q.No. 1a **What does $\vec{A} \cdot \vec{A}$, the scalar product of a vector with itself gives? What about $\vec{A} \times \vec{A}$, the vector product of a vector with itself?** [2]

We have,

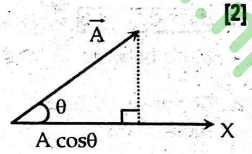
$\vec{A} \cdot \vec{A} = |\vec{A}| \cdot |\vec{A}| \cos\theta = A.A. \cos 0$ [$\theta = 0$ angle between $\vec{A}$ and $\vec{A}$] $= A^2$

Also, $\vec{A} \times \vec{A} = |\vec{A}| \times |\vec{A}| \sin\theta\,\hat{n}$ [$\hat{n}$ = unit vector] $= AA \sin 0\,\hat{n}$ [$\theta = 0$ angle between $\vec{A}$ and $\vec{A}$] $= 0$

$\therefore$ $\vec{A} \cdot \vec{A} = A^2$ i.e. the scalar product of a vector itself give the square of its magnitude and $\vec{A} \times \vec{A} = 0$, i.e. the vector product of a vector itself is zero.

15. 2070 Supp (Set B) Q.No. 1a **Can you find a vector quantity that has a magnitude of zero but components that are different from zero? Explain.** [2]

No, we can not find a vector quantity that has a magnitude of zero but components that are different from zero. Let $\vec{A}$ be a vector inclined at an angle θ with x-axis, and then component of $\vec{A}$ along x-axis is $A\cos\theta$. If $A = 0$, then, $A\cos\theta = 0$,



16. 2070 Set C Q.No. 1b **If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively, find $\hat{i} \cdot (\hat{j} \times \hat{k})$.** [2]

Let, $\hat{i}$, $\hat{j}$ and $\hat{k}$ be the three unit vector along x, y and z-axis respectively then,

$\hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot (|\hat{j}| \cdot |\hat{k}| \sin 90\,\hat{x})$

$= \hat{i} \cdot (1.1.1.\,\hat{x})$

$= \hat{i} \cdot \hat{x}$ [$\hat{x}$ has the direction of perpendicular to both $\hat{j}$ and $\hat{k}$ or plane mad by $\hat{j} \times \hat{k}$ which is $\hat{i}$]

$= \hat{i} \cdot \hat{i} = |\hat{i}| \cdot |\hat{i}| \cos 0 = 1.1.1 = 1$

17. 2070 Set D Q.No. 1a **The angle between two vectors $\vec{A}$ and $\vec{B}$ is θ. Find the magnitude and direction of $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$.** [2]

Let the angle between two vector $\vec{A}$ and $\vec{B}$ is θ, then, its cross product is $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta\,\hat{n}$

Magnitude of $\vec{A} \times \vec{B}$ is, $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

Direction of $\vec{A} \times \vec{B}$ is perpendicular to the direction of plane making by $\vec{A}$ and $\vec{B}$.

Again, the vector product is

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

It is scalar product, so, it has only magnitude but not direction.

18. 2069 Old (Set B) Q. No. 1b **If a vector has zero magnitude, is it meaning full to call it vector?** [2]

Yes, a vector which has zero magnitude is also a vector in case of two vectors travelling in opposite direction with equal magnitude. At this case, the resultant vector has zero magnitude still it is vector.

19. 2069 (Set A) Q. No. 1d **If $\vec{A}$ and $\vec{B}$ are non zero vectors, is it possible for $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$ both to be zero Explain.** [2]

No, it is not possible. If $\vec{A}$ and $\vec{B}$ are two non zero vectors, then their cross & dot products are

$\vec{A} \times \vec{B} = AB \sin\theta\,\hat{n}$

If $\theta = 0$, $\vec{A} \times \vec{B} = 0$

Similarly,

$\vec{A} \cdot \vec{B} = AB \cos\theta$

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If $\theta = 90°$, $\vec{A} \cdot \vec{B} = 0$

So, it is not possible for $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$ both to be zero simultaneously.

20. 2068 Q.No. 1 a 2060 Q.No. 1 a If the scalar product of two vectors in equal to the magnitude of their vector product, find the angle between them. [2]

Please refer to 2075 Set B Q.No. 1a

21. 2067 Supp Q.No. 1 b What should be the angle between the two vectors of same magnitude, so that their resultant is equal to either of them? [2]

Please refer to 2073 Set D Q.No. 1a

22. 2067 Q.No. 1 b $\vec{C}$ is the vector sum of $\vec{A}$ and $\vec{B}$ i.e. $\vec{C} = \vec{A} + \vec{B}$ for C = A + B to be true, What is the angle between $\vec{A}$ and $\vec{B}$? [2]

Suppose $\vec{A}$ and $\vec{B}$ are two vectors so that $\vec{A} + \vec{B} = \vec{C}$ and A + B = C. If θ be the angle between them, then

$\vec{C} = \vec{A} + \vec{B}$

$\Rightarrow C^2 = A^2 + B^2 + 2AB\cos\theta$ ... (i)

Also, C = A + B

$\Rightarrow C^2 = A^2 + B^2 + 2AB$ ... (ii)

Comparing Equation (i) and (ii), we get

$\cos\theta = 1 \quad \Rightarrow \theta = 0°$

Thus, the angle between two vectors $\vec{A}$ and $\vec{B}$ is 0°.

23. 2066 Old Q. No. 1 b 2063 Q.No. 1 b The magnitude of two vectors are 3 and 4, and their dot product is 6, what is the angle between them? [2]

If $\vec{A}$ =3 and $\vec{B}$ =4 are two vectors and θ is angle between them, then their scalar product is:

$\vec{A} \cdot \vec{B} = 12\cos\theta$ ... (i)

and the magnitude of their product is

$\vec{A} \cdot \vec{B} = 6$ ... (ii)

According to question, we have

$\vec{A} \cdot \vec{B} \Rightarrow 12\cos\theta = 6$

$\cos\theta = 1/2 = \cos 60° \Rightarrow \theta = 60°$

Thus, the angle between two vectors is 60°.

24. 2065 Q. No. 1 a If $\vec{A} \cdot \vec{B} = 0$, what is angle between $\vec{A}$ and $\vec{B}$. [2]

Please refer to 2074 Supp. Q.No. 1b

25. 2064 Q.No. 1a Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A} - \vec{B} = \vec{C}$ and A – B = C. Find the angle between them. [2]

Suppose $\vec{A}$ and $\vec{B}$ are two vectors so that $\vec{A} - \vec{B} = \vec{C}$ and A – B = C. If θ be the angle between them, then

$\vec{C} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

or, $(\vec{C})^2 = \{\vec{A} + (-\vec{B})\}^2$

or, $\vec{C}.\vec{C} = \vec{A}.\vec{A} + \vec{B}.\vec{B} - 2\vec{A}.\vec{B}$

$\Rightarrow C^2 = A^2 + B^2 - 2AB\cos\theta$ ... (i)

Also, C = A – B

$\Rightarrow C^2 = A^2 + B^2 - 2AB$ ... (ii)

Comparing Equation (i) and (ii), we get

$\cos\theta = 1 \quad \Rightarrow \theta = 0°$

Thus, the angle between two vectors $\vec{A}$ and $\vec{B}$ is 0°.

26. 2062 Q.No. 1 e What is the difference between scalar and vector products of two vectors? Explain. [2]

The difference between scalar and vector products of two vectors are as follows:

Scalar Product	Vector Product
1. The product of two vector which gives scalar quantity is called scalar product.	1. The product of two vector which gives vector quantity is called vector product.
2. The scalar product of two vectors is written as $\vec{A} \cdot \vec{B} = AB\cos\theta$	2. The vector product of two vectors is written as $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$.
3. It is denoted by dot operation.	3. It is denoted by cross operation.
4. It becomes zero if two vectors are perpendicular to each other.	4. It becomes zero if two vectors are parallel to each other.

27. 2061 Q.No. 1 c Can the sum of two equal vectors be equal to either of the vectors? Explain. [2]

Please refer to 2073 Set D Q.No. 1a

28. 2054 Q.No. 1 b If B is added to A, under what condition does the resultant vector have a magnitude equal to A + B? Under what conditions is the resultant vector equal to zero? [2]

Resultant vector of two vectors $\vec{A}$ and $\vec{B}$ is given by

$R^2 = A^2 + B^2 + 2AB\cos\theta$ ...(i)

Case–I

If R = A + B, then from (1), we get

or, $(A + B)^2 = A^2 + B^2 + 2AB\cos\theta$

or, $A^2 + 2AB + B^2 = A^2 + B^2 + 2AB\cos\theta$

or, $2AB = 2AB\cos\theta$

or, $\cos\theta = 1$

or, $\cos\theta = \cos 0°$

$\therefore \theta = 0°$

i.e. When two vectors $\vec{A}$ and $\vec{B}$ are in same directions ($\theta = 0°$), their resultant vector is (A + B).

Case – II

If $\theta = 180°$, from (i), we get

$R^2 = A^2 + B^2 + 2AB\cos 180°$

or, $R^2 = A^2 + B^2 - 2AB$

or, $R^2 = A - B$ or $B - A$

Thus, magnitude of resultant vector $\vec{R}$ of the two vectors $\vec{A}$ and $\vec{B}$ is zero only if (i) A = B and (ii) angle between them is $\theta = 180°$.

ng Answer Questions

2075 GIE Q.No. 5a 2069 Set A Q. No. 5a State the parallelogram law of vector addition. Derive expressions for the magnitude and direction of the resultant of two vectors inclined at an angle θ from each other. [4]

Parallelogram law of vector: It state that If two vectors acting simultaneously at a point are represented both in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, then the diagonal of the parallelogram passing through that point represents the resultant both in magnitude and direction.

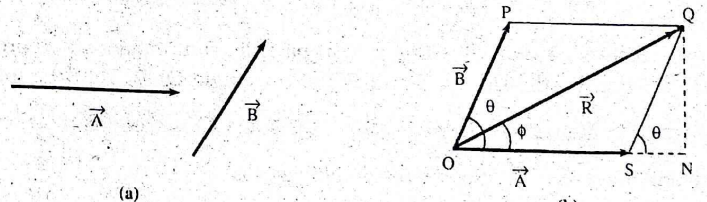


Fig. Parallelogram law of vectors.

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As shown in the fig., vectors $\vec{A}$ and $\vec{B}$ are represented by the sides of a parallelogram OPQS, and t
resultant $\vec{R}$ is represented by diagonal OQ such that $\vec{R} = \vec{A} + \vec{B}$.

Magnitude of $\vec{R}$: To calculate the magnitude of the resultant vector $\vec{R}$, let us draw a perpendicul
at N from Q when OS is produced. Let the angle between vectors $\vec{A}$ and $\vec{B}$ be θ.

In ΔONQ,

$OQ^2 = ON^2 + NQ^2$ ... (i)

or, $OQ^2 = (OS + SN)^2 + NQ^2$

$\sin\theta = \frac{NQ}{QS} = \frac{NQ}{B}$

or, $NQ = B \sin\theta$

and $\cos\theta = \frac{SN}{QS} = \frac{SN}{B}$

or, $SN = B\cos\theta$

From Equation (i)

$OQ^2 = (OS + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$

or, $R^2 = A^2 + 2AB\cos\theta + B^2$

or, $R = \sqrt{A^2 + 2AB\cos\theta + B^2}$

which is the magnitude of the resultant vector.

Direction of $\vec{R}$: Let the angle made by the resultant $\vec{R}$ with the vector $\vec{A}$ be ϕ.

In ΔONQ,

$\tan\phi = \frac{QN}{ON} = \frac{QN}{OS + SN}$

or, $\tan\phi = \frac{B\sin\theta}{A + B\cos\theta}$

or, $\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$

which is the direction of the resultant vector.

30. 2070 Supp (Set A) Q.No. 5 a State and explain the parallelogram law of vector addition.

Please refer to 2075 GIE Q.No. 5a

31. 2066 Q. No. 5 a State triangle law of vector addition. Obtain an expression for the resultant of two vector and Q inclined at angle θ.

Triangle law of vector: If two sides of a triangle completely represent two vectors both in magnitu and direction taken in same order, then the third side taken in opposite order represents the result of the two vectors both in magnitude and direction.

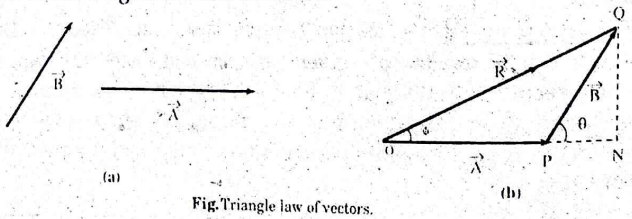


Fig. Triangle law of vectors.

Let there be two vectors $\vec{A}$ and $\vec{B}$ acting on a particle simultaneously represented both magnitudes and direction by the sides OP and PQ of a triangle OPQ. Let θ be the angle betw them. Then, according to the triangle law of vectors, the side OQ represents their resultant $\vec{R}$ wh makes an angle ϕ with the vector $\vec{A}$. So, the resultant vector is

$\vec{R} = \vec{A} + \vec{B}$

Magnitude of $\vec{R}$: Let us drop a perpendicular at N from Q where OP is produced to ON.

In ΔONQ,

$OQ^2 = ON^2 + NQ^2$

$OQ^2 = (OP + PN)^2 + NQ^2$ ... (i)

In ΔPNQ,

$\cos\theta = \frac{PN}{PQ} = \frac{PN}{B}$

or, $PN = B\cos\theta$

and $\sin\theta = \frac{QN}{PQ} = \frac{QN}{B}$

or, $QN = B\sin\theta$

From equation (i)

$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$

or, $R^2 = A^2 + 2AB\cos\theta + B^2$

or, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

which is the magnitude of the resultant vector.

Direction of $\vec{R}$: As the resultant $\vec{R}$ makes an angle ϕ with $\vec{A}$, then in ΔONQ,

$\tan\phi = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$

$\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$

This is required direction of the resultant vector.

2. 2055 Q.No. 2 OR State the parallelogram law of vector addition. Derive the magnitude and direction of the resultant vector.

Please refer to 2075 GIE Q.No. 5a

umerical Problems

1. 2076 Set B Q.No. 9a A disoriented physics professor drives 3.25 km north, then 4.75 km west and then 1.50km south. Find the magnitude and direction of the resultant displacement. [4]

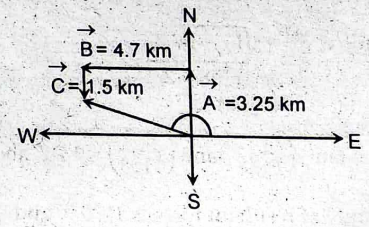
olution

Given,

Distance along north $|\vec{A}| = 3.25$ km

Distance along west $|\vec{B}| = 4.75$ km

Distance along south $|\vec{C}| = 1.5$ km



Now,

$A_x = |\vec{A}|\cos 90° = 0$, $A_y = |\vec{A}|\sin 90° = 3.25$ km

$B_x = |\vec{B}|\cos 180° = -4.75$ km, $B_y = |\vec{B}|\sin 180° = 0$

$C_x = |\vec{C}|\cos 270° = 0$, $C_y = |\vec{C}|\sin 270° = -1.50$ km

So, resultant component along x-axis

$R_x = A_x + B_x + C_x$

$= 0 - 4.75 + 0$

$= -4.75$ km

Similarly, resultant component along Y-axis.

$R_y = A_y + B_y + C_y$

$= 3.25 + 0 - 1.5$

$= 1.75$ km

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$\therefore$ Resultant, $R = \sqrt{R_x^2 + R_y^2}$
$= \sqrt{(-4.75)^2 + (1.75)^2}$
$= 5.06$ km

$\therefore$ Magnitude of resultant displacement is 5.06 km
Direction of resultant displacement

$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$= \tan^{-1}\left(\frac{1.75}{-4.75}\right)$

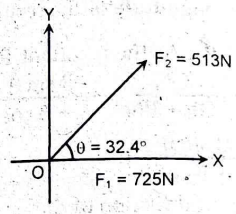
$= -20.2°$

– ve sign indicates that the resultant lies on second quadrant. So, $\beta = 20.2°$ north of west.

34. 2075 Set A Q.No. 9a A rocket fires two engines simultaneously. One produces a thrust of 725N directly forwa while the other gives a 513 N thrust at 32.4° above the forward direction. Find the magnitude and direction the resultant force that these engines exert on the rocket.

Solution

Given,
First force (F_1) = 725 N
Angle (θ) = 32.4°
Second force (F_2) = 513 N
Magnitude of resultant force (F) = ?
Direction of resultant force (β) = ?



Now, using method of components,

$F_{1X} = F_1 = 725$ N
$F_{2X} = F_2 \cos\theta$
$= 513 \times \cos 32.4$
$= 433$ N
$F_X = F_{1X} + F_{2X}$
$= 725 + 433$
$= 1158$ N

$F_{1Y} = 0$
$F_{2Y} = F_2 \sin\theta$
$= 513 \times \sin 32.4$
$= 275$ N
$F_Y = F_{1X} + F_{2X}$
$= 0 + 275$
$= 275$ N

Now
Total force, $F = \sqrt{F_X + F_Y}$
$= \sqrt{1158 + 275}$
$= 1190$ N

and, $\beta = \tan^{-1}\left(\frac{F_Y}{F_X}\right) = \tan^{-1}\left(\frac{275}{1158}\right) = 13.4°$ above F_1.

$\therefore$ Magnitude of resultant force is 1190 N and direction is 13.4°.

❑❑❑

Chapter 3: Kinematics

Short Answer Questions

1. 2076 Set B Q.No. 1b From a flying aeroplane a body should be dropped in advance to hit the target, why? [2]

When the bomb is dropped from aeroplane, it moves in parabolic path as shown in figure. So, to hit the target by body it should drop before reaching above the target as shown in figure.



2. 2075 GIE Q.No. 1b Is it possible to get a body accelerated even if it is moving with a uniform speed? Justify. [2]

Yes, it is possible to get a body accelerated even it is moving with uniform speed. In case of body revolving in a circular path with uniform speed, the magnitude of velocity (i.e. speed) in each and every point of its path is constant but the direction changes and directed towards centre of circle i.e. centripetal acceleration is acting on a body moving in a circular path at uniform speed.

3. 2075 GIE Q.No. 1f A ball is dropped gently from the top of a tower and another ball is thrown horizontally at the same time. Which ball will hit the ground first? [2]

The time of flight of a projectile when it is thrown from a tower of height h is $t = \sqrt{\frac{2h}{g}}$, where g is the acceleration due to gravity. It means the time of flight is independent to the initial velocity of projectile, as it is thrown from the tower. Hence, height being same, both balls hit the ground at the same time.

4. 2075 Set A Q.No. 1b Can an object with constant acceleration reverse its direction of travel? Explain. [2]

Yes, when a body moving in straight-line eastwards undergoes retardation, then its acceleration acts westwards. For example, suppose a bus is running eastwards in a straight road with velocity 4 m/s. Due to the application of brake, after 3 seconds, suppose its velocity be 1 m/s. Then its acceleration becomes $a = \frac{1-4}{3} = -1$ m/s². The negative sign indicates that acceleration is in the direction opposite to the direction of velocity. Thus, if the velocity is eastward, then constant acceleration is directed along the west in reverse direction.

5. 2074 Set A Q.No. 1e Velocities of two bodies A and B are given in vector notation as $\vec{V_A} = \hat{i}+2\hat{j} - 3\hat{k}$ and $\vec{V_B} = 3\hat{i} + 2\hat{j} - \hat{k}$ respectively. What will be the relative velocity of B with respect to A in this notation? [2]

Given,

If, $\vec{V_A} = \hat{i}+2\hat{j} - 3\hat{k}$ & $\vec{V_B} = 3\hat{i} + 2\hat{j} - \hat{k}$, then.
Relative velocity of B with respect to A is denoted by V_{BA} & give by

$\vec{V_{BA}} = V_B - V_A = (3\hat{i} + 2\hat{j} - \hat{k}) - (\hat{i}+2\hat{j} - 3\hat{k}) = 2\hat{i} + 2\hat{k}$

6. 2074 Set B Q.No. 1b Two stones P and Q of different masses m and 2m respectively are dropped simultaneously from the top of a tower and reach the ground with different energies. Which one is faster? [2]

The time of flight of a body projected from top of tower of height h is $t = \sqrt{\frac{2h}{g}}$, where g is acceleration due to gravity. It means time of flight is independent to the mass of projected body. So, the two different bodies P and Q of mass m and 2m respectively dropped simultaneously from the top of tower reach to the ground at the same time.

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7. **2073 Supp Q.No. 1a A ball is thrown with a velocity ... another ang... of projection achieve the same horizontal range? Justify your answer.**

The horizontal range (R) of projectile for an angle of projection θ is

$$R_1 = \frac{u^2 \sin 2\theta}{g}$$

If the angle of projection is 90 – θ, than horizontal range is

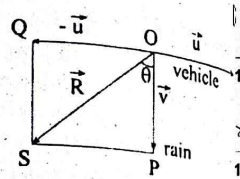
$$R_2 = \frac{u^2 \sin 2(90-\theta)}{g} = \frac{u^2 \sin(180-2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

$\therefore R_1 = R_2$

So, the ranges are same in two angles of projection, θ and 90° – θ.

8. **2072 Supp Q.No. 1b 2070 Set C Q.No. 1 c 2069 Old (Set B) Q. No. 1c Rain drops hitting the side windows of car in motion often leave diagonal streaks. Why?**

Suppose rain is falling vertically downward with a velocity $\vec{v}$ and a vehicle is moving in horizontal direction with velocity $\vec{u}$. To find the relative velocity of rain with respect to the vehicle, take $\overrightarrow{OP} = \vec{v}$ and $\overrightarrow{OQ} = -\vec{u}$.



Here $\overrightarrow{OQ}$ represents an equal and opposite velocity of the vehicle. The resultant $\vec{R}$ is represented magnitude and direction by vector $\overrightarrow{OS}$. Suppose $\vec{R}$ makes an angle θ with vertical. Form the figu... we have

$\text{Tan}\,\theta = u/v$, or $\theta = \text{Tan}^{-1}(u/v)$

Therefore, rain strikes the front wind screen making an angle θ with the vertical. This explains wh... front wind screen of an automobile moving in rain gets wet whereas the hind screen remains dry.

9. **2072 Set C Q.No. 1b If the displacement of a body is proportional to square of time, state whether the body moving with uniform velocity or uniform acceleration.**

Let, s be the displacement of a body which is proportional to the squre of time t, i.e.,

$s \propto t^2$

or, $s = kt^2$, where k = constant

$$\frac{ds}{dt} = 2kt$$

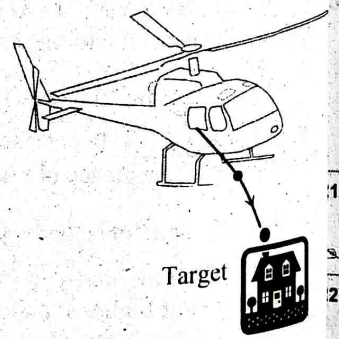
or, $$\frac{d^2s}{dt^2} = 2k$$

or, acceleration = constant

$\therefore$ The body is moving with constant acceleration if $s \propto t^2$.

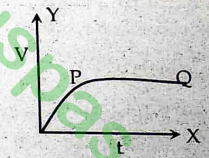
10. **2071 Set D Q.No. 1 b A bomb is to be dropped from a moving helicopter on a target on the ground. Explai... how it can hit the target.**

When the bomb is dropped from helicopter, it moves in parabolic path as shown in figure. So, to hit the target by bomb it should drop before reaching above the target as shown in figure.



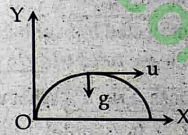
11. **2070 Supp (Set A) Q.No. 1 b Can you have a zero acceleration but non-zero velocity? Explain with help of a graph. [2]**

Yes, we have a zero acceleration and non zero velocity in a uniform motion. In the given v–t graph, the velocity through out the portion OP is increasing where acceleration is non-zero whereas throughout the region PQ, the velocity is constant or uniform and acceleration is zero.



12. **2070 Set D Q.No. 1 c A projectile moves in a parabolic path without air resistance. Is there any point at which its acceleration is perpendicular to the velocity? Explain. [2]**

When a projectile moves in a parabolic path without air resistance, at the maximum height, velocity and acceleration are perpendicular to each other as shown figure where velocity is horizontal and acceleration vertical.



13. **2069 (Set A) Q. No. 1c Can the direction of the velocity of a body be changed when its acceleration is constant? [2]**

Please refer to 2075 Set A Q.No. 1b

14. **2069 (Set B) Q. No. 1b Can an object with constant acceleration reverse its direction? Explain. [2]**

Please refer to 2075 Set A Q.No. 1b

15. **2069 Old (Set B) Q. No. 1d If the displacement of a body is proportional to the square of time, state the nature of motion of the body. [2]**

Please refer to 2072 Set C Q.No. 1b

16. **2068 Q.No. 1 b From a high tower, one ball is dropped from rest and the second ball is simultaneously projected horizontally. Neglecting air resistance, which ball will reach the ground earlier? [2]**

Please refer to 2075 GIE Q.No. 11

17. **2068 Old Q.No. 1 b Can a body have a constant speed but changing velocity? Explain with example. [2]**

Yes, when a body moving in a horizontal circular path with constant centripetal force (i.e., at constant centripetal acceleration), the magnitude of velocity (i.e., speed) in each point of tis path is constant, but the body chandes its direction of motion in each successive point of its path. In this case, the body can have constant speed but changing velocity.

18. **2066 Q. No. 1 a Can an object with constant acceleration reverses its direction? Explain. [2]**

Please refer to 2075 Set A Q.No. 1b

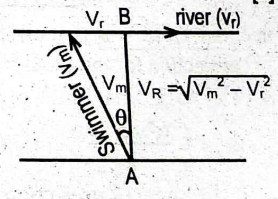
19. **2066 Old Q. No. 1 c If the displacement of a body is proportional to square of time, state the nature of motion of the body. [2]**

Please refer to 2072 Set C Q.No. 1b

20. **2065 Q. No. 1 b A swimmer wants to reach to a point just opposite on the other bank of the river. How should he swim and why? [2]**

Let us consider river is flowing with velocity v_r as shown in figure. A swimmer wants to reach to a point B just opposite on the other bank of the river from the point A. Then he has to move with the direction making an angle θ with the AB as shown in figure. From the figure,

$$\theta = \tan^{-1}\left(\frac{V_r}{V_R}\right)$$



21. **2062 Q.No. 1 c A ball is dropped gently from the top of a tower and another ball is thrown horizontally from the same point at the same time. Which ball will hit the ground earlier?**

Please refer to 2075 GIE Q.No. 11

22. **2061 Q.No. 1 b Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.**

Let a body is projected with velocity u makes an angle θ

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Horizontal range = maximum height

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}$$

$$\tan\theta = 4$$

$\therefore \theta = 75.96°$

Hence, required angle of projection is 75.96°.

23. 2059 Q.No. 1 b Two stones are projected simultaneously from a height. One falls freely while the other projected horizontally. Which one reaches the ground first? Explain.

Please refer to 2075 GIE Q.No. 1f

24. 2058 Q.No. 1 a What would be the effect on maximum range in doubling the initial velocity of a projectile?

The horizontal range of a projectile moving with initial velocity u making an angle θ with t horizontal is given by: $R = \frac{u^2 \sin 2\theta}{g}$

For maximum range, $\sin 2\theta = 1 \Rightarrow \theta = 45°$. Then $R_{max} = \frac{u^2}{g}$

Suppose R^1_{max} and R^2_{max} are the maximum horizontal ranges for initial velocity u and 2u respectivel

$$R^1_{max} = \frac{u^2}{g} \text{ and } R^2_{max} = \frac{(2u)^2}{g} = \frac{4u^2}{g}$$

$$\text{So, } \frac{R^1_{max}}{R^2_{max}} = \frac{u^2/g}{4u^2/g} = \frac{1}{4}$$

$\therefore R^2_{max} = 4 R^1_{max}$

It means that the maximum horizontal range is increased by four times when initial velocity is double

25. 2057 Q.No. 1 b A ball is dropped gently from the top of a tower and another ball is thrown horizontally at t same time. Which ball hit the ground earlier? Explain.

Please refer to 2075 GIE Q.No. 1f

26. 2056 Q.No. 1 f Can an object have an eastward velocity experiencing a westward acceleration? Give reason

Yes, when a body moving in straight-line eastwards undergoes retardation, then its acceleration a westwards. For example, suppose a bus is running eastwards in a straight road with velocity 4 m Due to the application of brake, after 3 seconds, suppose its velocity be 1 m/s. Then its accelerati becomes $a = \frac{1-4}{3} = -1 \text{ m/s}^2$. The negative sign indicates that acceleration is in the direction oppos to the direction of velocity. Thus, if the velocity is eastward, then acceleration is directed along t west.

27. 2053 Q.No. 1 b A car is moving on the road when the rain is falling vertically downwards. Why does the fr wind screen get wet?

Please refer to 2072 Supp Q.No. 1b

28. 2052 Q.No. 1 e A projectile fired at an angle of 18° has certain horizontal range. State another angle projection for the same horizontal range.

The horizontal range of a projectile for angle of projection θ is

$$R_1 = \frac{u^2 \sin 2\theta}{g}$$

If the angle of projection is 90° - θ, the horizontal range is

$$R_2 = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2 \sin (180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

$\therefore R_1 = R_2$

So, the ranges are same in two angles of projection, θ and 90° – θ. Here one angle is 18° and anoth angle should be 90° - 18° = 72°. Hence, the projectile must be fired with an angle 72° to achieve horizontal range of angle 18°.

29. 2051 Q.No. 1 i Give with an example a case where the velocity of an object is zero but its acceleration is not zero.

When an object is thrown up, then at the highest point, velocity is zero but acceleration due to gravity is g. Also at the extreme point of the simple pendulum or simple harmonic motion, the velocity is zero but not acceleration.

30. 2050 Q.No. 1 i Because of air resistance, two objects of unequal mass do not fall at precisely the same rate. If two bodies of identical shape but unequal mass be dropped simultaneously from the same height, which one reaches the ground first?

As the bodies have identical shape, they would displace equal volume of air. Hence, the buoyant force (B) on bodies due to the upthrust of air is same. Let m_1 and m_2 ($m_1 > m_2$) be masses of two bodies.

For first body, we have

$m_1 g - B = m_1 a_1$

or, $a_1 = g - \frac{B}{m_1}$ ... (i)

For second body, we have

$m_2 g - B = m_2 a_2$

or, $a_2 = g - \frac{B}{m_2}$ ... (ii)

$\because m_1 > m_2$, from (i) and (ii) we conclude that, $a_1 > a_2$.

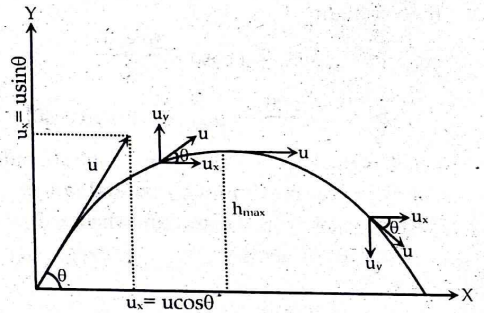
Hence, the body having a large mass falls more rapidly.

Long Answer Questions

31. 2076 Set C Q.No. 5a A projectile is fired at an angle of θ with the horizontal. Show that its trajectory is a parabola. Also, derive expressions for the maximum height attained and the time of flight. [4]

Let us consider a projectile having mass 'm' is projected with velocity u at an angle θ with horizontal. The projectile goes up with decreasing velocity, reaches at the highest point and falls back to level of projection as shown in figure.

If the air resistance is negligible, the horizontal component u_x remains constant throughout the motion whereas vertical component goes on decreasing and becomes zero at the highest point of its path then finally it increases downwards due to gravitational attraction.



Projectile motion

The horizontal distance travelled by projectile at an instant of time 't' is

$s = u_x.t \quad [\because s = ut + \frac{1}{2}at^2 \text{ and } a_x = 0]$

$x = u\cos\theta\, t$ ... (i)

The vertical distance travelled by the projectile at an instant of time 't' is

$s = ut - \frac{1}{2}gt^2$ [∵ g is -ve in upward motion.]

$y = u\sin\theta\, t - \frac{1}{2}gt^2$ ... (ii)

From equation, (i) $t = \frac{x}{u\cos\theta}$

$$y = u \sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \left(\frac{1}{2u^2}\frac{g}{\cos^2\theta}\right)x^2$$

$y = Ax + Bx^2$ ... (iii)

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where, $A = \tan\theta$ and $B = -\frac{1}{2}\frac{g}{\cos^2\theta\, u^2}$

Equation (iii) represents equation of parabola. Thus, the projectile follows parabolic trajectory.

a. Maximum height (h_{max}): The maximum vertical distance covered by the projectile during its time of flight is called maximum height. The vertical component of velocity (v_y) at maximum height is zero (i.e. $v_y = 0$).

We have,

$v^2 = u^2 + 2as$

or, $0 = (u\sin\theta)^2 + 2(-g)h_{max}$

$2g\,h_{max} = u^2\sin^2\theta$

$\Rightarrow h_{max} = \frac{u^2\sin^2\theta}{2g}$ ... (iv)

which is the required expression for maximum height.

b. Time of flight (T): It is the total time for which the projected projectile remains in air. At the end of flight, the projectile reaches to the ground, so vertical distance travelled by projectile is zero i.e., $h = 0$.

From equation (ii), we can write,

$0 = u\sin\theta\, T - \frac{1}{2}gT^2$

$\frac{1}{2}gT^2 = u\sin\theta T$

$T = \frac{2u\sin\theta}{g}$ ... (v)

This is the required expression for time of flight.

c. Horizontal range (R): The horizontal distance covered by the projected body during its time of flight is called Horizontal range. It is equal to product of horizontal component of velocity (u_x) and total time of flight.

i.e. $R = u_x \times T = u\cos\theta \times \frac{2u\sin\theta}{g}$

$R = u^2\frac{\sin 2\theta}{g}$ $[\sin 2\theta = 2\sin\theta\cos\theta]$ ... (vi)

which is the required expression for horizontal range.

Condition for maximum horizontal range:

The horizontal range is maximum when $\sin 2\theta = 1$ in the expression of R.

i.e. $\sin 2\theta = \sin 90°$

$\Rightarrow 2\theta = \sin 90°$

$\Rightarrow \theta = \frac{90}{2} = 45°$

Thus, the maximum horizontal range for velocity of projection (u) occours when it is projected in an angle of 45° and the maximum horizontal range is given by

$R_{max} = \frac{u^2}{g}$.

32. 2074 Set A Q.No. 5a A projectile is projected with a velocity V making an angle 0 with the horizontal. Derive relation for its trajectory. Also show the components of velocity at any point on the trajectory in a diagram. [4]

Please refer to 2076 Set C Q.No. 5a

33. 2073 Set D Q.No. 5a A projectile is fired at an angle θ with the horizontal. Show that its trajectory is parabola. Derive expressions for the maximum height attained and the time of its flight. [4]

Please refer to 2076 Set C Q.No. 5a

34. 2071 Supp Q.No. 5a Show that path followed by a projectile fired at an angle with horizontal is a parabola. Assuming that the point of projection and the landing point are at the same horizontal level, find the expression for horizontal range. [4]

35. 2069 Supp Q.No. 5a Show that the path followed by a projectile fired at an angle 'θ' with the horizontal is a parabola and derive general expression for the time of light and horizontal range. [4]

Please refer to 2076 Set C Q.No. 5a

36. 2067 Supp Q.No. 5 a A projectile is fired with a velocity v and making an angle θ with the horizontal. Derive expressions for the maximum height obtained, the time of flight and the horizontal range. [4]

Please refer to 2076 Set C Q.No. 5a

37. 2067 Q.No. 5 a Derive expressions for maximum height, range of projectile fired at angle 'θ' with horizontal. [3]

Please refer to 2076 Set C Q.No. 5a

38. 2064 Q.No. 3 a OR A stone is projected with a velocity in a direction making an angle θ with horizontal. Derive expression for: (i) maximum height (ii) time of flight (iii) horizontal range. Also find the condition for maximum horizontal range. [3+1]

Please refer to 2076 Set C Q.No. 5a

Numerical Problems

39. 2075 GIE Q.No. 9a A swimmer's speed along the river is 20 kmph and up stream is 8 kmph. Calculate the velocity of the stream and the swimmer's possible speed in still water. [4]

Solution

Given,

Speed of swimmer in down stream (V_R) = 20km/hr

Speed of swimmer in up stream (V'_R) = 8 kh/hr

Let, v_s = velocity of swimmer and v_r be the velocity of river with respect to still water. Then,

For down stream, the resultant velocity

$V_R = v_s + v_r$

or, $20 = v_s + v_r$ ... (i)

For upward stream, the resultant velocity is

$V'_R = v_s - v_r$

or, $8 = v_s - v_r$ ... (ii)

Adding (i) and (ii)

$28 = 2v_s$

or, $v_s = 14$km/hr

and from (i), $v_r = 6$ km/hr

40. 2075 Set B Q.No. 9d A body is projected upwards making an angle θ with the horizontal with a velocity of 300 ms^{-1}. Find the value of θ so that the horizontal range will be maximum. Hence find its range and time of flight. [4]

Solution

Given, Angle (θ) = ?

Velocity (v) = 300 m/sec.

We have,

Horizontal range $(R) = \frac{u^2\sin 2\theta}{g}$

For maximum, R

$\sin 2\theta = 1$

or, $2\theta = \sin^{-1}(1)$

or, $2\theta = 90°$

$\therefore \theta = 45°$

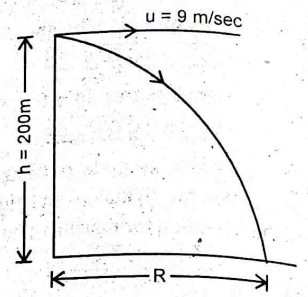
Again,

$R = \frac{u^2\sin 2\theta}{g} = \frac{300^2\sin 90°}{10} = 9000$ m

Time of flight, $T = \frac{2u\sin\theta}{g} = \frac{2\times 300\sin 45}{10} = 42.42$ sec.

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41. 2074 Supp. Q.No. 9a **A stone on the edge of a vertical cliff is kicked so that its initial velocity is 9 m/s horizontally. If the cliff is 200m high, calculate:**
i. time taken by stone to reach the ground.
ii. How far from the cliff the stone will hit the ground? [4]



Solution

Given,
Initial velocity (u) = 9m/sec
Height of cliff (h) = 200 m
Time to reach ground (t) = ?
Horizontal range (R) = ?

i. We have,

$h = u_y t + \frac{1}{2} g t^2 = 0 + \frac{1}{2} g t^2$ [$u_x = 9$ m/sec, $u_y = 0$]

or, $200 = \frac{1}{2} \times 10 \times t^2$

or, $t^2 = \frac{400}{10}$

or, $t = \sqrt{40} = 6.32$ sec.

ii. Horizontal distance covered $R = u_x \times t = 9 \times 6.32 = 56.92$ m

42. 2074 Set B Q.No. 9c 2073 Supp Q.No. 9a 2066 Q. No. 9 a **A projectile is fired from the ground with 500 ms^{-1} a 30° with the horizontal. Find the horizontal range, the greatest height and the time to reach the greates height.** [4]

Solution

Given,
Velocity of projectile (u) = 500 ms^{-1}
Angle with horizontal (θ) = 30°
Horizontal range (R) = ?
Greatest vertical height (H) = ?
Time to reach greatest height (t) = ?
We know,

$R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \sin 60°}{10} = 21650.64$ m

$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{(500)^2 (\sin 30°)^2}{2 \times 10} = 3125$ m

Time for greatest height, $t = \frac{u \sin\theta}{g} = \frac{500 \times \sin 30}{10} = 25$ sec

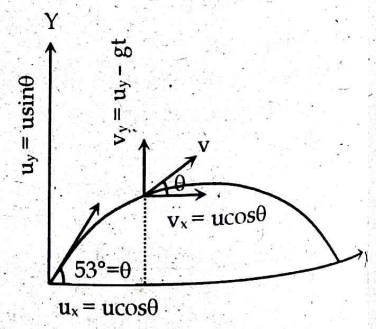
43. 2073 Set C Q.No. 9a **A batter hits a baseball so that it leaves the bat with an initial speed 37m/s at an angle o 53°. Find the position of the ball and the magnitude and direction of its velocity after 2 seconds. Treat th baseball as a projectile.** [4]

Solution

Given,
Initial velocity (u) = 37 m/sec
Angle of projection (θ) = 53°
Time taken (t) = 2 sec
Velocity (v) = ?
Position (x) = ?
Now after 2 sec

$v_x = u \cos\theta = 37 \times \cos 53° = 22.27$ m/sec

$v_y = u_y - gt = u \sin\theta - gt = 37 \times \sin 53° - 10 \times 2 = 9.55$ m/sec



i. For position,

$x = v_x.t = 22.27 \times 2 = 44.54$ m

$y = u_y t - \frac{1}{2} g t^2 = u \sin\theta.t - \frac{1}{2} \times 10 \times 2^2 = 37 \times \sin 53 \times -20 = 39.1$ m

∴ The position of projectile (x, y) = (44.54, 39.1)

ii. Velocity

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{22.27^2 + 9.55^2} = 24.23$ m/sec

$\tan\theta = \frac{v_y}{v_x}$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{9.55}{22.27}\right) = 23.21°$

∴ Magnitude of velocity is 24.23 m/sec and direction is 23.214°.

44. 2072 Supp Q.No. 9a **A base ball is thrown towards a player with an initial velocity 20ms^{-1} and 45° with the horizontal. At the moment, the ball is thrown the player is 50m from the thrower. At what speed and in what direction must he run to catch the ball at the same height at which it is released?** [4]

Solution

Given,
Velocity of ball (v) = 20 m/s
Direction of ball (θ) = 45°
Distance covered by ball is,

$R = \frac{v^2 \sin 2\theta}{g} = \frac{20^2 \times \sin^2 90°}{10} = 40$ m

Ad distance butween two players is 50 m, so
distance to be run = 50 – 40 = 10 m

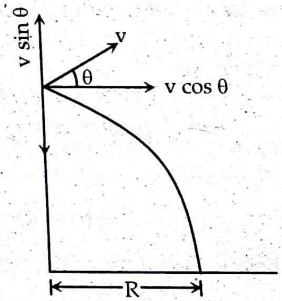
Time of flight $(T) = \frac{2v \sin\theta}{g} = \frac{2 \times 20 \times \sin 45°}{10} = \frac{4}{\sqrt{2}} = 2.83$ Sec

Speed of player $(V_d) = \frac{d}{t} = \frac{10}{2.83} = 3.5$ ms^{-1} toward the thrower.

45. 2072 Set D Q.No. 9a **An airplane is flying with a velocity of 90.0 m/s at an angle of 23.0° above the horizontal. When the plane is 114 m directly above a dog that is standing on level ground, a suitcase drops out of luggage compartment. How far from the dog will the suitcase land? You can ignore air resistance.** [4]

Solution

Given,
Velocity (v) = 90.0 m/s
Angle (θ) = 23.0°
Height of plane (h) = 114 m
Distance of suitcase from dog (R) = ?



$h = ut + \frac{1}{2} at^2$

or, $114 = -v \sin\theta t + \frac{1}{2} gt^2$

or, $114 = -90 \sin 23° t + \frac{1}{2} \times 10 \times t^2$

or, $114 = -35.16t + 5t^2$

or, $5t^2 - 35.16t - 114 = 0$

Solving this equation t, we get
t = 9.4, –2
But, time (t) cannot be negative, so t = 9.4 sec.
Now,

$R = v_x t = v \cos\theta \times t$
$= v \cos 23° \times t = 90 \times \cos 23° \times 9.4 = 778.7$ m from the dog

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46. 2071 Set C Q.No. 9 d **A bullet is fired with a velocity of 100 m/s from the ground at an angle of 60° with the horizontal. Calculate the horizontal range covered by the bullet. Also calculate the maximum height attained.** [4]

Solution

Given,
Velocity of projectile (v) = 100m/sec
Angle (θ) = 60° with horizontal
Horizontal range (R) = ?
Maximum height (H) = ?
We have

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{100^2 \times \sin 120^\circ}{10} = 866 \text{ m}$$

$$H = \frac{v^2 \sin^2\theta}{2g} = \frac{100^2 \times (\sin 60^\circ)^2}{2 \times 10} = 375 \text{ m}$$

47. 2070 Supp (Set A) Q.No. 9 a **A car travelling with a speed of 15m/s is braked and it slows down with uniform retardation. It covers a distance of 88 m as its velocity reduces to 7m/s. If the car continues to slow down with the same rate, after what further distance will it be brought to rest?** [4]

Solution

Given,

Case I
Initial velocity (u) = 15m/sec
Distance covered (s) = 88 m
Final velocity (v) = 7m/sec
Retardation (a) = ?
We have,
$v^2 = u^2 + 2as$

$$\text{or, } a = \frac{v^2 - u^2}{2.s} = \frac{7^2 - 15^2}{2 \times 88} = -1 \text{ m/sec}^2$$

Case II
Initial velocity (u) = 7 m/sec
Retardation (a) = – 1m/sec²
Final velocity (v) = 0m/sec
Distance covered (s) = ?
Again we have,
$v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 7^2}{2 \times (-1)} = 24.5 \text{ m}$$

48. 2070 Supp (Set B) Q.No. 9 a **A stone is projected horizontally with 20m/s from top of a tall building. Calculate its position and velocity after 3 sec neglecting the air resistance.** [4]

Solution

Given,
Initial velocity, u = 20m/sec
Time taken, t = 3 sec
Let, the stone be at point P after 3 sec.
Now,

$v_x = u = 20 \text{m/sec}$

$v_y = u_y + gt = 0 + 10 \times 3 = 30 \text{m/sec}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 30^2} = 36.06 \text{ m/s}$

Vertical distance, y is

$$y = u_y t + \frac{1}{2} gt^2 = 0 \times \frac{1}{2} \times 10 \times 3^2 = 45 \text{ m}$$

Horizontal distance x = u.t = 20 × 3 = 60 m
angle,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{30}{20}\right) = 56.3^\circ$$

So, the particle lies 45 m below the top of tower with an angle 56.3° with horizontal.

49. 2069 Supp Q.No. 9a **A swimmer's speed along the river (down stream) is 20 km/h and can swim up-stream at 8 km/h. Calculate the velocity of the stream and the swimmer's possible speed in still water.** [4]

Please refer to 2075 GIE Q.No. 9a

50. 2069 (Set A) Q. No. 9a **A man wishes to swim across a river 600m wide. If he can swim at the rate of 4km/h in still water and the river flows at 2km/h. Then in what direction must he swim to reach a point exactly opposite to the starting point and when will he reach it?** [4]

Solution

Given,
Width of river (d) = 600m
Speed of man (v_m) = 4km/h
Speed of river (v_r) = 2km/h
Time to cross river (t) = ?
Direction of swimmer (θ) = ?
The man should swim making angle θ as shown in figure, then

$$\sin\theta = \frac{v_r}{v_m} = \frac{2}{4} = \frac{1}{2}$$

$\theta = 30^\circ$
The time taken (t) is given as,

$$t = \frac{AC}{v_m \text{ along } AC} = \frac{d}{v_m \cos\theta} = \frac{600}{4 \times \cos 30^\circ} = 623.54 \text{ sec} = 10.4 \text{ min}$$

51. 2069 (Set B) Q. No. 9a **A base ball is thrown towards a player with an initial velocity 20ms⁻¹ and 45° with the horizontal. At the moment the ball is thrown, the player is 50m from the thrower. At what speed and in what direction must he run to catch the ball at the same height at which it was released?** [4]

Please refer to 2072 Supp Q.No. 9a

52. 2069 (Set B) Q. No. 9c **A body falls freely from the top of a tower and during the last second of its fall, it falls through 25m. Find the height of tower.** [4]

Solution

Given,
Height of tower (h) = ?
Distance covered in last sec (S_n) = 25m
We have,

$$S_n = u + \frac{a}{2}(2t - 1)$$

$$25 = 0 + \frac{10}{2}(2t - 1)$$

$\text{or, } 5 = 2t - 1$
$\text{or, } t = 3 \text{ sec}$
Also,

$$h = ut + \frac{1}{2} gt^2 = 0 \times 3 + \frac{1}{2} \times 10 \times 3^2 = 45 \text{m}$$

53. 2069 Old (Set B) Q. No. 2b **A projectile is launched with an initial velocity of 30ms⁻¹ at an angle of 60° above the horizontal. Calculate the magnitude and direction of its velocity 5s after launch.** [4]

Soluiton

Given,
Initial velocity (u) = 30m/sec
Angle of projection (θ) = 60°
Time taken (t) = 5 sec
Velcoity (v) = ?
Now, after 5 sec

$v_x = u\cos\theta = 30 \times \cos 60^\circ = 15 \text{ m/sec}$

$v_y = u_y - gt = u\sin\theta - gt = 30\sin 60 - 10 \times 5$
$= -24 \text{ m/sec}$

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$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 24^2} = 28.3 \text{ m/sec}$$

$$\tan\phi = \frac{v_y}{v_x}$$

or, $\phi = \tan^{-1}\frac{v_y}{v_x} \doteq -58°$

54. 2068 Old Q.No. 2 b A projectile is fired from the ground level with velocity 150m/s at 30° to the horizontal. Find its horizontal range. What will be the least speed with which it can be projected to achieve the same horizontal range?

Solution

Given,
Initial velocity (v) = 150m/sec
Angle (θ) = 30°
Horizontal range (R) = ?
Least speed for same range (u') = ?
We have,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{150^2 \sin 60°}{10} = 1948.55 \text{ m}$$

R becomes maximum for least speed

$$R = \frac{u'^2}{g}$$

$$1948.55 = \frac{u'^3}{10}$$

$$u' = 139.59 \text{ m/sec}$$

55. 2065 Q. No. 2 b A projectile is fired from the ground level with a velocity of 500ms^{-1} at 30° to horizon. Find the horizontal range, and greatest vertical height to which it rises. What is the least speed with which it could be projected in order to achieve the same horizontal range? [g = 10 N kg^{-1}] [4]

Soluiton

Given,
Velocity of projectile (u) = 500 ms^{-1}
Angle with horizontal (θ) = 30°
Horizontal range (R) = ?
Greatest vertical height (H) = ?
Least speed to achieve same horizontal range (v_{least}) = ?
We know,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \sin 60}{10} = 21650.64 \text{ m}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(500)^2 (\sin 30°)^2}{2 \times 10} = 3125 \text{ m}$$

Again, we know

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{or, } u^2 = \frac{Rg}{\sin 2\theta}$$

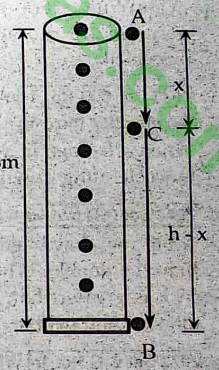
$$\text{or, } u = \sqrt{\frac{Rg}{\sin 2\theta}}$$

Here u will be least when sin 2θ will be maximum. The maximum value of sin 2θ = 1, and the least velocity to obtain the same range is

$$u_{least} = \sqrt{\frac{Rg}{1}} = \sqrt{Rg} = \sqrt{21651 \times 10} = 465.3 \text{ m/s}$$

56. 2063 Q.No. 2 b An object is dropped from the top of the tower of height 156.8 m and at the same time another object is thrown vertically upward with the velocity of 78.1 ms^{-1} from the foot of the tower, when and where the object meet?

Solution



Given,
Height (h) = 156.8m
Initial velocity (u) = 78.1 m/sec
Let, A be the position of top of the tower and B be the position of ground. Suppose they meet at point C.
For dropped object, distance moved

$$s = ut + \frac{1}{2}gt^2$$

$$\text{or, } x = 0 \times t + \frac{1}{2}gt^2$$

$$\text{or, } x = \frac{1}{2}gt^2$$

For projected object,

$$s = ut - \frac{1}{2}gt^2$$

$$\text{or, } h - x = 78.1\,t - \frac{1}{2}gt^2$$

$$\text{or, } h - \frac{1}{2}gt^2 = 78.1\,t - \frac{1}{2}gt^2$$

$$\text{or, } 156.8 = 78.1\,t$$

$$\therefore t = 2 \text{ s}$$

Putting t = 2 s in Eq. (i), we get

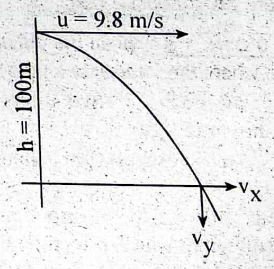
$$x = \frac{1}{2} 10 \times 2^2 = 20 \text{m}$$

Hence, they meet below 20 m from top of the tower after 2 s.

7. 2060 Q.No. 2 b A body is projected horizontally from the top of a tower 100 m high with a velocity of 9.8 ms^{-1}. Find the velocity with which it hits the ground.

olution

Given,
Horizontal velocity (u) = 9.8 ms^{-1}
Height from which body is projected (h) = 100 m
Final velocity (v) =?



$$\text{Since, } t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 100}{10}}$$

$$= \sqrt{20} = 4.47 \text{ sec}$$

$$v^2 = u^2 + (gt)^2$$

$$= (9.8)^2 + (10 \times 4.47)^2$$

$$v = 45.76 \text{ m/s}$$

If θ be the angle made by the body with the horizontal

$$\tan\theta = \frac{v_y}{v_x} = \frac{45.76}{9.8} = 4.56$$

$$\theta = 77.9°$$

❑❑❑

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Short Answer Questions

1. **2075 GIE Q.No. 1a Can a body be said to be at rest and in motion at the same time?**

Yes, a body can be regarded in a state of rest as well as in motion at the same time. No body rema in absolute rest or motion; they are comparative state of bodies. For example, passengers in a movi train are at rest with respect to each other but they are in motion with respect to the surroundi objects like trees, buildings.

2. **2073 Set C Q.No. 1b A woman in an elevator lets go of her briefcase but it does not fall to the floor. How the elevator moving?**

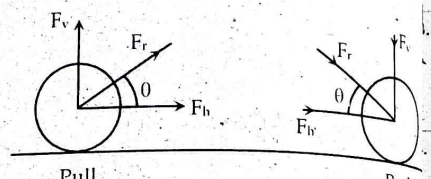
When a women in an elevator lets go of her briefcase but it does not fall to the floor, then the eleva is accelerating downwards with rate greater than 9.8 m/sec².

3. **2073 Set D Q.No. 1b It can hurt our foot more to kick a big rock than a small pebble. Why?**

A big rock has large mass than smaller pebble. Thus, the big rock has greater change in momentu than the pebble. When we kick to large rock, the change in momentum i.e. the force exerted by la rock is greater than smaller pebble. Due to this reason, it can hurt our foot more to kick a big r than a small pebble.

4. **2072 Set C Q.No. 1g Why is it easier to pull a body than to push it?**

When we pull a lawn mover, the vertical component of the applied pull acts opposite to the weight of the mover and it reduces its effective weight. On the other hand, when a lawn mover is pushed, the vertical component of the applied push adds to the weight of the mover and hence, its effective weight increases. As the effective weight is lesser, when the lawn mover is pulled, it is easier to pull the lawn mover than to push it.



5. **2072 Set D Q.No. 1g Why does a cricketer lower his hand while catching cricket ball?**

Rate of change of linear momentum= Force = $\frac{\text{Total change in momentum}}{\text{time}}$

The total change in momentum of the ball is constant. Hence, if time taken by a player in bringing ball to rest is more, rate of momentum change will be less. Thus, the player will have to apply sma force and his hands will also be safe from hitting.

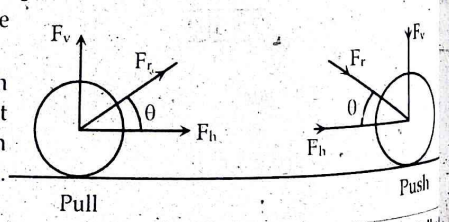
6. **2072 Set E Q.No. 1b Explain why a fielder lowers his hand while catching a cricket ball?**

Please refer to 2072 Set D Q.No. 1g

7. **2071 Set D Q.No. 1 a Explain the difference between a "push" and a "pull" in the case of a heavy roller o level road where the force acts on the roller making an angle with the horizontal road.**

The pull and push are two forces applied on the body as shown in figures.

In pulling, the vertical component of force is in opposite to weight and effective weight reduces. But in pushing, the vertical component of force is in direction of weight and effective weight increases. Hence, pull a roller is easier than to push.



8. **2070 Supp (Set B) Q.No. 1 e A firecracker at rest explodes, sending fragments off in all directions. Initially firecracker has zero momentum, but after the explosion the fragments flying off each other have quite a of momentum. Hasn't momentum been created? If not, explain why not?**

A firecracker at rest has zero momentum. When it explodes sends its fragments in all directions w high momentum. Here, each fragments moves in different direction such that the total momentum all fragments is zero (being momentum vector quantity). Therefore, the total momentum before after explosion be equal but there is no creation of momentum (principle of conservation of lin momentum).

2069 Supp Q.No. 1b It is easier to pull a heavy load than to push it. Why? [2]

Please refer to 2072 Set C Q.No. 1g

2069 Set A Q. No. 1f A man drops his briefcase in an elevator but it does not fall to the floor. What can be concluded about the situation? [2]

If a man drops his briefcase in an elevator but it does not fall to the floor, it conclude that the elvator is in free fall i.e. the elevator moves with the acceleration of 9.8 m/s² and the briefcase also falls with this acceleration. They fall with same acceleration and never meet with each other.

2069 Set B Q. No. 1c 2066 Q. No. 1b When a large heavy truck collides with a passenger car, the occupants of the car are more likely to be hurt than the truck driver. Why? [2]

A large heavy truck has large mass and momentum and hence change in momentum i.e. impulse for large truck is greater than the smaller car. Due to this, the large truck exerts more force on smaller car and hence, on collision between truck and car, the occupants of the car are more likely to be hurt than the truck driver.

2069 Set A Old Q. No. 1c A cricketer lowers his hands while catching balls. Why? [2]

Please refer to 2072 Set D Q.No. 1g

2069 Set A Old Q. No. 1d In rain a scooter may slip on the turning of a road. Why? [2]

In a rain, the coefficient of friction between the tyres of scooter and the road is very less. So, the frictional force (F = μR) is small and there is not source of necessary centripetal force for the scooter. Hence, the scooter in a rainy day may slip on the turning of a road.

2068 Old Q.No. 1 c It is easier to pull a lawn roller than to push it, why? [2]

Please refer to 2072 Set C Q.No. 1g

2067 Supp Q.No. 1 c Why does a heavy rifle not kick as strongly as a light rifle using the same cartridges? [2]

When a rifle recoils during firing of a bullet, the recoil velocity is given by $v_g = -\frac{m_b v_b}{m_g}$. This means the recoil velocity also depends on the mass of the rifle. If heavy rifle is used, mass would be higher which makes the recoil velocity less. If the rifle used has less mass, the recoil velocity will be high. So, there is precision problem in case of light weapons compared to heavier ones.

2065 Q. No. 1 d Why a man getting out of a moving bus must run in the same direction for a certain distance?

When a man jumps out of a moving bus his foot comes to rest whereas his upper part is still in motion due to inertia of motion. So, to prevent from falling, he has to run in the direction of the bus.

2063 Q.No. 1 d Explain why a coin placed on a cardboard covering a glass falls into the glass when the cardboard is pulled suddenly to one side?

When sudden jerk is given to cardboard, the cardboard comes into motion but coin at rest, remains at rest due to inertia of rest and gets separated from cardboard. Due to this reason, the coin get falls into glass.

2061 Q.No. 1 d Explain how Newton's first law of motion follows from the second law.

According to Newton's second law of motion, the external force applied on a body and acceleration produced by this application of force are related by $\vec{F} = m\vec{a}$. If no external force applied $\vec{F} = 0$ which implies $\vec{a} = 0$. We have, $\vec{v} = \vec{u} + \vec{a}t$ $\therefore \vec{v} = \vec{u}$

Since, $\vec{a} = 0$, this implies that if no external force acts on the body, then a body initially at rest ($\vec{u} = 0$) will remain at rest ($\vec{v} = 0$) or if it is moving with velocity ($\vec{u}$), it will keep on moving with the same velocity ($\vec{u} = \vec{v}$). This is the statement first law of motion. Hence, Newton's 1st law follows from Newton's second law.

2061 Q.No. 1 f Why is the kinetic friction less than the limiting friction?

Limiting friction occurs when the body just starts to move from rest. When a body is static or rest, the surfaces are well entrenched upon each other. So, overcoming them is difficult which makes the static

friction high. When the body is moving, the surfaces cannot entrench tightly or deeply overcoming them is also easy. This makes the kinetic friction less. Due to this reason, static friction limiting friction is always greater than the kinetic friction.

20. 2060 Q.No. 1 b Can a body be regarded in a state of rest as well as in motion at the same time? Give example.

Please refer to 2075 GIE Q.No. 1a

21. 2060 Q.No. 1 e Why do we slip on a rainy day?

The water on a muddy road provides a thin layer in between our feet and road. This layer br interlocking and decreases the friction. Hence, we slip on a muddy road on rainy days.

22. 2059 Q.No. 1 c Is friction a necessary evil? Explain.

Due to the friction, heating the machine loses a lot of energy. It also reduces the efficiency of machine. Friction damages the machine. In this sense friction is an evil.

However, our daily life would be impossible if there would no friction because,

a. It helps us to work and walk.
b. It stops vehicles by using brakes.
c. It helps in holding screws.
d. It makes possible to fasten nails on wall.
e. It helps in transmitting motion from one body to another.
f. It helps in shaping weapons like knives.

Thus, friction is more useful but not necessary evil.

23. 2056 Q.No. 1 b The leaves fall when a tree is shaken. Give reason.

Before shaking, both the branches and the leaves are at rest. When we shake the branches, branches come in to motion. But due to inertia of rest, the leaves tend to continue the state of rest a result, the leaves are separated from the branches and fall down.

24. 2056 Q.No. 1 c Why is it difficult to run fast on sand?

As we press the sandy ground in backward direction, the sand gets pushed away so that we get small reaction from the ground which makes us difficult to run fast.

25. 2054 Q.No. 1 c Explain why a cricketer moves his hands backwards while catching a ball?

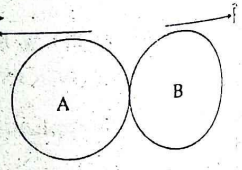
Please refer to 2072 Set D Q.No. 1g

26. 2053 Q.No. 1 c Give reasons why a man getting out of moving bus must run in the same direction certain distance.

Please refer to 2065 Q. No. 1 d

27. 2052 Q.No. 1 a If action and reaction are always equal and opposite, why don't they always cancel each leave no force for acceleration of the body?

According to Newton's third law of motion, the body B will exert a force $\vec{F}_2$ on the body A such that $\vec{F}_1 = -\vec{F}_2$



The negative sign shows that $\vec{F}_1$ and $\vec{F}_2$ act in opposite direction.

Since, action $\vec{F}_1$ and reaction $\vec{F}_2$ act on two different bodies (i.e. $\vec{F}_1$ acts on B and $\vec{F}_2$ acts on A), never cancel each other.

28. 2052 Q.No. 2 a 2050 Q.No. 2 a State Newton's second law of motion.

The rate of change of momentum of a body is directly proportional to applied force and takes pla the direction of applied force. According to the Newton's second law of motion, rate of chang momentum is proportional to the force (net external force).

i.e. $\frac{dP}{dt} \alpha F$

i.e. $F \alpha \frac{dP}{dt}$

$F = \frac{d}{dt}(mv) = m\frac{dv}{dt}$

[For constant mass and taking proportionality constant unity.]

$F = ma$, where 'a' is acceleration produced $= \frac{dv}{dt}$

By knowing the values of mass and acceleration, the force acting on the body can be calculated. Therefore, Newton's second law of motion defines the unit of force and it gives the quantitative measurement of force.

2051 Q.No. 1 iv When a balloon filled with air and its mouth downwards is released, it moves upwards, why?

According to Newton's third law of motion, action and reaction are equal and opposite. So when a balloon filled with air and it's mouth downward is released, air escapes out through the mouth downward with higher velocity (action) and as a result balloon moves upward with higher velocity (reaction).

2050 Q.No. 1 v The acceleration of a falling body is measured in elevator travelling at a constant speed of 9.8 m/s. What result is obtained?

As the body is falling freely and it is not in contact with the floor of the elevator, it has the usual acceleration of free-fall (9.8 ms^{-2}). If the body was on the floor, it would have not acceleration since the elevator is moving with a constant velocity.

2050 Q.No. 1 x Is momentum of a moving body a vector or a scalar?

Momentum of a body is defined as the product of velocity and mass of that body and it is completely related to the velocity. The velocity is a vector quantity and mass is the scalar quantity. The product of vector and scalar is a vector quantity so momentum of a body is a vector quantity.

ong Answer Questions

2076 Set B Q.No. 5b Define angle of friction and angle of repose. Establish a relation between them. [4]

Friction: The retarding force which comes into play when one body moves in contact with another is called friction or frictional force.

Cause of friction: According to classical theory, the surface of body contains large number of irregularities even if the surface is highly polished, when viewed by microscope. Thus, the friction arises due to interlocking of irregularities of two surfaces in contact.

According to modern theory, the force of friction is due to the force of adhesion or cohesion between the molecules on the two surfaces at the point of actual contact. But actual cause of friction is still unknown.

Anlge of friction: Angle between the resultant of limiting friction and normal reaction is called angle of friction.

Angle of Repose: Angle of repose is defined as the minimum angle of inclination of a plane structure with the horizontal such that a body kept on it just begins to slide down along the plane.

Let us consider an object of mass 'm' is kept on a plane surface AB and its inclination is slowly increased to θ such that the object placed on it starts to slide down. The weight of the body can be resolved into two components: mg cosθ and mg sinθ.

The component of the weight mg normal to the surface is mg cosθ which balances the normal reaction R and the component mg sinθ along the inclined surface acts downward along the surface while the frictional force acts upward along the surface. In equilibrium,

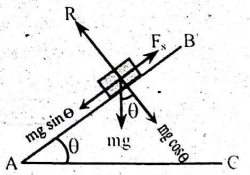
$mg \sin\theta = F_s$ ... (i)

$mg \cos\theta = R$ ... (ii)

Dividing Eqs. (i) by(ii) we get

$\frac{mg \sin\theta}{mg \cos\theta} = \frac{F_s}{R}$

$\tan\theta = \frac{F_s}{R}$



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As, $\frac{F_s}{R} = \mu$

$\therefore \tan\theta = \mu$

The tangent of the angle of repose is equal to the coefficient of friction between the surfaces.

$\tan\alpha = \mu$ where α is the angle of friction,

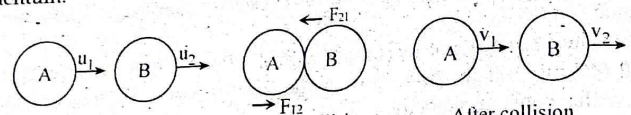
$\therefore \theta = \alpha$

i.e. angle of repose is equal to angle of friction.

33. 2075 Set A Q.No. 5a State and explain the principle of conservation of linear momentum. Hence derive it Newton's law of motion.

Statement: The principle of the conservation of linear momentum is stated as, "the total l momentum of a system remains constant if no external forces act on the system." Or, it states that total linear momentum of an system remains same before and after collision if there is no external applied." For example, when a bullet is fired in the forward direction, there is recoil of the rifle backward direction so that the total linear momentum before and after firing is zero. Hence, con the momentum.

Proof:



Let two bodies of masses m_1 and m_2 be moving in a straight line with respective initial velocities u_1 an If $u_1 > u_2$, they collide after some time. Let t be the time of collision, after that they move with respe final velocities v_1 and v_2 in the same straight line as shown in the figure.

During collision, let F_{12} be the force exerted by body m_1 on the body m_2 and F_{21} be the force exerte body m_2 on the body m_1. According to Newton's third law of motion, the body m_2 also exerts forc in the opposite direction on the body m_1 such that,

$F_{12} = -F_{21}$ ... (i)

According to Newton's second law of motion,

$$F_{12} = \frac{\text{Change in momentum of body of mass } m_2}{t}$$

$$F_{12} = \frac{\text{Final momentum of } m_2 - \text{initial momentum of } m_2}{t}$$

$$F_{12} = \frac{m_2v_2 - m_2u_2}{t}$$

$F_{12} \times t = m_2v_2 - m_2u_2$ ... (ii)

Similarly, $F_{21} \times t = m_1v_1 - m_1u_1$ ... (iii)

From Equation (i), we have

$F_{12} \times t = -F_{21} \times t$

$m_2v_2 - m_2u_2 = -(m_1v_1 - m_1u_1)$ [Using (ii) and (iii)]

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ... (iv)

$\therefore$ Total momentum before collision = total momentum after collision

Hence, principle of conservation of linear momentum is verified.

34. 2075 Set B Q.No. 5a Explain the difference between conservative and non-conservative forces. Also state prove law of conservation of linear momentum.

First part: A force is said to be conservative if the total work done by it on moving a particle round a closed is zero. The work done by it in moving a particle from one point to another point depends only on the dist between these points but not the actual path followed. For example; gravitational force.

A force is said to be non-conservative if the work done by it on moving a particle round a closed is not zero. The work done by non-conservative force depends upon the path followed and initial final position also. For example, frictional force.

Second part: Please refer to 2075 Set A Q.No. 5a

35. 2074 Supp. Q.No. 5a 2050 Q.No. 6 State the principle of conservation of linear momentum and show how it follows from Newton's second law of motion. [4]

Please refer to 2075 Set A Q.No. 5a

36. 2074 Set B Q.No. 5c State and explain the principle of conservation of linear momentum. [4]

Please refer to 2075 Set A Q.No. 5a

37. 2073 Set C Q.No. 5a Define angle of repose. Show that the angle of repose and the angle of friction are equal for the given pair of surfaces. [4]

Please refer to 2076 Set B Q.No. 5b

38. 2072 Set D Q.No. 5a State Newton's laws of motion. How does it lead to the principle of conservation of linear momentum? [4]

Newton's Laws of Motion

a. **First law of motion:** Everybody continues in its state of rest or of uniform motion in a straight line, unless external forces act on it. It is also called law of inertia.

b. **Second law of motion:** The rate of change of momentum of a body is directly proportional to impressed force and takes place in the direction of applied force.

c. **Third law of motion:** The action and reaction are always equal but opposite to the each others.

Second Part: Please refer to 2075 Set A Q.No. 5a

39. 2072 Set E Q.No. 5c What do you understand by friction and mention its cause. Obtain expression for the relation between angle of friction and angle of repose. [4]

Please refer to 2076 Set B Q.No. 5b

40. 2070 Supp (Set B) Q.No. 5 a What do you mean by cold weldings? Show that angle of repose and angle of friction are equal. [4]

Cold welding is the process of joining the two flat surfaces of solids without fusion than by any liquid or other process. This is due to the intermolecular force of attraction between two surfaces i.e. due to the adhesive force between two surface molecules. This is the modern theory of the friction.

Second Part: Please refer to 2076 Set B Q.No. 5b

41. 2069 (Set A) Q. No. 5c State the principle of conservation of linear momentum. How does the Newton's third law of motion lead to the principle of conservation of linear momentum? [4]

Please refer to 2075 Set A Q.No. 5a.

42. 2069 (Set A) Old Q. No. 3a State and prove the principle of conservation of linear momentum. [4]

Please refer to 2075 Set A Q.No. 5a

43. 2064 Q.No. 2 a State Newton's laws of motion. Show that Newton's first law of motion defines force and second law of motion defines the unit of force. [1+1+3]

Newton's Laws of Motion: Please refer to 2072 Set D Q.No. 5a

The Newton's first law constitutes two parts:

i. Every body continues in its state of rest, unless external forces act on it which is experienced in our daily life, because a body lying on a table continues to remain there in absence of an external force.

ii. Everybody continues in its state of uniform motion in a straight line, unless external forces act on it. This part of law may seem contrary to our common experience. For example, a rolling ball comes to rest after sometimes. The ball comes to the rest due to resistance of air and friction. The resistance of air and friction behaves as resistive forces. It gives the idea that to change the state of rest or uniform motion of a body something needed to act on it, which is called force. Thus, Newton's first law of motion gives the qualitative definition of force.

According to the Newton's second law of motion, rate of change in momentum is proportional to the force (net external force).

i.e. $\frac{dP}{dt} \propto F$ i.e. $F \propto \frac{dP}{dt}$

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt}$$

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F = ma, where 'a' is acceleration produced.
By knowing the values of mass and acceleration, the force acting on the body can be calculat
Therefore, Newton's second law of motion defines the unit of force and it gives the quantitat
measurement of force.

44. 2063 Q.No. 2 a What is the angle of repose? Show that when a body just begins to slide down on an inclin plane, the coefficient of friction is equal to the tangent of inclination of the plane.

Please refer to 2076 Set B Q.No. 5b

45. 2062 Q.No. 3 a What do you understand by friction? Explain its cause. Show that the coefficient of sl friction is equal to the angle of repose.

Please refer to 2076 Set B Q.No. 5b

46. 2060 Q.No. 2 a State the principle of conservation of linear momentum. Show that in collision between moving bodies in which no external forces act,

Please refer to 2075 Set A Q.No. 5a

47. 2055 Q.No. 5 State the laws of limiting friction. How would you measure the coefficient of friction betwe body and an inclined plane?

Laws of (limiting) friction:

a. The direction of the limiting friction is always opposite to that in which the motion tend take place.
b. The limiting friction acts tangentially to the two surfaces in contact.
c. The magnitude of the limiting friction is directly proportional to the normal react between the two surfaces.
d. The limiting friction depends upon the material and the nature of the surfaces in cont and their state of polish.
e. For any two given surfaces, the magnitude of the limiting friction is independent of shape or the area of the surfaces in contact so long as the normal reaction remains the sam

Coefficient of static friction: Let us consider an object of mass 'm' is kept on a plane surface AB its inclination is slowly increased to θ such that the object placed on it starts to slide down. T weight of the body can be resolved into two components: mg cosθ and mg sinθ.
The component of the weight mg normal to the surface is mgcosθ which balances the normal react R and the component mgsinθ along the inclined surface acts downward along the surface while frictional force acts upward along the surface. In equilibrium,

$mg \sin\theta = F_s$ ... (i)

$mg \cos\theta = R$ ... (ii)

Dividing Eqs. (i) by (ii), we get,

$$\frac{mg\sin\theta}{mg\cos\theta} = \frac{F_s}{R}$$

$$\tan\theta = \frac{F_s}{R}$$

As, $\frac{F_s}{R} = \mu$

$\therefore \tan\theta = \mu$

i.e., coefficient of friction is equal to tangent of angle of repose.
Thus, If 'θ' is known then coefficient of friction (μ) can be calculated.

48. 2052 Q.No. 2 b Show that the principle of conservation of linear momentum can be verified by us Newton's law.

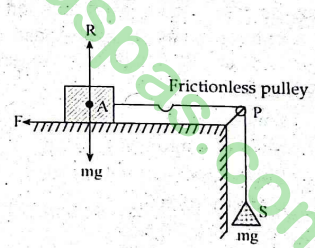
Please refer to 2075 Set A Q.No. 5a

49. 2051 Q.No. 6 What are the laws of friction? How are they experimentally verified?

Laws of (limiting) friction: Please refer to 2055 Q.No. 5

Experimental Verification

Consider a wooden block placed on a horizontal surface. It is attached to a string which passes over a frictionless pulley carrying a scale pan at the free end as shown in the figure. Add weights in the scale pan till wooden block just starts sliding. Clearly, frictional force opposes the motion which verifies first law. The frictional force F acts along the horizontal surface, thus, verifying second law. When the block just starts sliding, the total weight added in the scale pan along with the weight of the scale pan is equal to the limiting friction. The normal reaction R is equal to the weight of the block. It is found that the ratio of limiting friction is proportional to the normal reaction R. This verifies third law. If the wooden block is replaced by glass block of the same weight mg. It is found that the limiting friction is different in this case. This verifies fourth law.



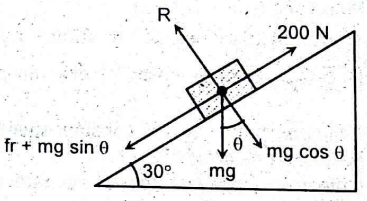
If the wooden block is placed on its side instead of on its base, the same force is found to be required to move the block as when it was placed on its base. This verifies fifth law.

Numerical Problems

50. 2076 Set C Q.No. 9a An iron block of mass 10 kg rests on a wooden plane inclined at 30° to the horizontal. It is found that the least force parallel to the plane which causes the block to slide up is 100N. Calculate the coefficient of friction between the two surfaces. [4]

Solution

Given,
Mass of iron block (m) = 10 kg
Inclination (θ) = 30°
Force (F) = 100 N
Coefficient of friction (μ) = ?
We have,
Frictional force (f_r) $= \mu R$
$= \mu mg\cos\theta$
$= \mu\ 10 \times 10 \times \cos 30°$
$= 86.6\ \mu$ [$\because R = mg\cos\theta$]

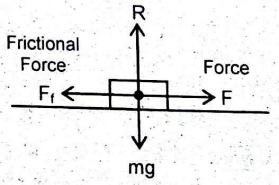


Again,
$F = fr + mg\sin\theta$
or, $100 = 86.6\ \mu + 10 \times 10 \times \sin 30°$
or, $100 = 86.6\ \mu + 50$
$\therefore \mu = \frac{100 - 50}{86.6} = 0.577$

51. 2074 Set A Q.No. 9a A box of mass 15 kg placed on horizontal floor is pulled by a horizontal force. What will be the work done by the force if the coefficient of sliding friction between the box and the surface of the floor is 0.3 and body moved at unit distance. [4]

Solution

Given,
Mass of box (m) = 15 kg
Work-done (W) = ?
Coefficient of sliding friction (μ) = 0.3
Distance traveled (d) = 1m
We know,
Force (F) = μR [$\because F = F_f = \mu R$]
= μmg [$\because$ Reaction, R = mg]
= 0.3 × 15 × 10
= 45 N



Now,
Work-done (W) = F.d = 45 × 1 = 45 J

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52. 2073 Set C Q.No. 9b **A little red wagon with mass 7 kg moves in a straight line on a frictionless horizontal surface. It has an initial speed of 4 m/s and then is pushed 3 m in the direction of the initial velocity by a force of 10N. Calculate the wagon's final speed and the acceleration produced by the force.**

Solution

Given,
Mass of wagon (m) = 7 kg
Initial speed (u) = 4 m/sec
Distance (s) = 3m
Force (F) = 10 N
Final speed (v) = ?
Acceleration (a) = ?
We have,
F = ma

or, $a = \frac{F}{m} = \frac{10}{7} = 1.43 \text{ m/sec}^2$

Again,
$v^2 = u^2 + 2as$
or, $v^2 = 4^2 + 2 \times 1.43 \times 3$
or, $v^2 = 24.58$
or, $v = 4.96$ m/sec
∴ Final speed = 4.96 m/sec, acceleration = 1.43 m/sec².

53. 2073 Set D Q.No. 9a **In a physics lab experiment, a 6 kg box is pushed across a flat table by a horizontal force F.**

i. If the box is moving at a constant speed of 0.35 m/s and the coefficient of kinetic friction is 0.12, what is the magnitude of F?

ii. If the box is speeding up with a constant acceleration of 0.18 m/s², what will be the magnitude of F?

Solution

Given,
mass of box (m) = 6 kg

i. Speed of box (u) = 0.35 m/sec
Coefficient of kinetic friction (μ_k) = 0.12
Frictional force (F) = ?
We have,

$\mu_K = \frac{F_K}{R}$

or, $F_K = \mu_K R = \mu_K mg$
Now,
$F = F_u - F_K$
or, $F_a = F + F_k$; for uniform motion
$= ma + F_k = 0 + \mu_K mg$
$= 0.12 \times 6 \times 10 = 7.2$ N

ii. When the box moves with acceleration 0.18 m/sec². Then,
$F_a = ma + \mu_K mg$
$= 6 \times 0.18 + 0.12 \times 6 \times 10 = 8.28$ N

54. 2072 Supp Q.No. 9c **A ball of mass 0.05Kg strikes a smooth wall normally four times in 2 seconds with velocity of 10ms⁻¹. Each time the ball rebounds with the same speed of 10ms⁻¹. Calculate the average force on the wall.**

Solution

Given,
Mass of a ball (m) = 0.05 kg
Initial velocity (u) = 10 m s⁻¹

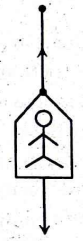
Final velocity (v) = –10m s⁻¹ (in opposite direction)
No. of striking (n) = 4 times.
Time taken (t) = 2 sec.
Average force on wall (F) = ?
We have,

$F = \frac{n(mv - mu)}{t} = \frac{4(10 \times 0.05 - (-10 \times 0.05))}{2} = \frac{4 \times (0.5 + 0.5)}{2} = 2$ N

55. 2072 Set C Q.No. 9a **A 550N physics student stands on a bathroom scale in an elevator. As the elevator starts moving the scale reads 450N. Draw free body diagram of the problem and find the magnitude and direction of the acceleration of the elevator.** **[4]**

Solution

Given,
Weight of the body (mg) = 550 N
Normal reaction (R) = 450 N
Acceleration (a) = ?
Here, the weight of the body is greater than the normal reaction, hence, the elevator is moving downward with acceleration 'a'. Then, for downward motion,

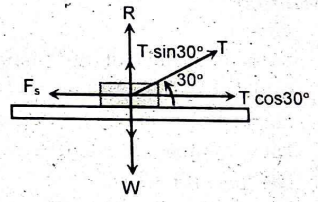


$F = mg - R$
or, $F = 550 - 450$
or, $F = 100$ N
Again,
$F = ma$
or, $100 = \frac{550}{10} \times a$
or, $a = \frac{100}{55} = 1.8$
∴ $a = 1.8$ m/sec² in downward motion.

56. 2071 Supp Q.No. 9a **Suppose you try to move a crate by tying a rope around it and pulling on the rope at angle of 30° above the horizontal. What is the tension required to keep the crate moving with constant velocity? Assume weight of the crate 'w' = 500 N and coefficient of dynamic friction $\mu_k = 0.40$.** **[4]**

Solution

Let W be the weight of crate, R be normal reaction, T be tension and f_s be the frictional force. Let $\theta = 30°$ be angle between T and horizontal. Then,



$W = R + T \sin 30°$
$\Rightarrow R = W - T \sin 30°$
In horizontal direction,
$ma = T\cos\theta - f_s$
For constant velocity, $a = 0$
$T\cos\theta = f_s$
or, $T\cos 30° = \mu_k R$ $[F_s = \mu_k R]$

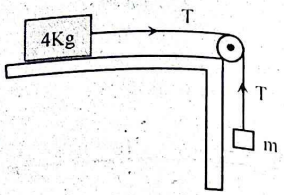
or, $T = \frac{\mu_k R}{\cos 30°} = \frac{0.4 \times \left(500 - T \cdot \frac{1}{2}\right)}{\cos 30°} = 187.61$ N

57. 2070 Set C Q.No. 9 a **A light rope is attached to a block with mass 4 kg that rests on a frictionless, horizontal surface. The horizontal rope passes over a frictionless pulley and a block with mass m is suspended from the other end. When the blocks are released, the tension in the rope is 10N. Draw free body diagrams and calculate the acceleration of either block and the mass m of the hanging block.** **[4]**

Solution

Let us consider a 4 kg mass is rest on horizontal frictionless surface attached by a light string which is passed over pully and suspended a mass m on it.
Now,
For 4 kg

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$T = 4 \times a$

or, $10N = 4\,a$

or, $a = 2.5\ m/sec^2$

For mass m

$mg - T = ma$

or, $m(g-a) = T$

or, $m(10 - 2.5) = 10$

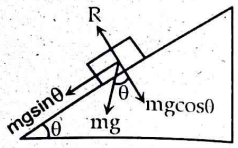
or, $m = \frac{10}{7.5} = 1.33\ kg$

58. 2070 Set D Q.No. 9 a **In a physics lab experiment, a 6kg box is pushed across a flat table by a horizontal force F. If the box is moving at a constant speed of 0.35m/s and the coefficient of kinetic friction is 0.12, find the magnitude of force F. What is the magnitude of force F if the box is moving with a constant acceleration 0.18 m/s²?**

Please refer to 2073 Set D Q.No. 9a

59. 2069 Supp Q.No. 9b **What would be the acceleration of a block sliding down an inclined plane that makes an angle of 45⁰ with the horizontal if the coefficient of sliding friction between two surfaces in 0.3?**

Solution



Given,

Angle of inclination (θ) = 45°

Coefficient of friction (μ) = 0.3

Downward acceleration (a) = ?

The downward force $= mg\sin\theta$

The frictional force $= \mu R = \mu\, mg\cos\theta$

Net downward force along the plane is,

$F = mg\sin\theta - \mu mg\cos\theta$

$= mg(\sin 45 - 0.3 \times \cos 45) = mg \times 0.495$

Then, acceleration a is given as,

$a = \frac{F}{m} = \frac{mg \times 0.495}{m} = 40 \times 0.495 = 4.95\ m/sec^2$

60. 2069 (Set A) Q. No. 9c **A cricket ball of mass 145 gm is moving with a velocity of 14m/s and is being hit by bat, so that the ball is turned back with a velocity of 22m/s. The force of blow acts on the ball for 0.015 sec. Find the average force exerted by the bat on the ball.**

Solution

Given,

Mass of cricket ball (m) = 145gm = 0.145 kg

Initial velocity of ball (u) = 14 m/sec

Final velocity of ball (v) = 22 m/sec

Time of impact (t) = 0.015 sec

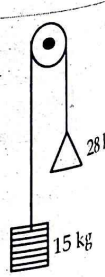
The force (F) = ?

$\therefore F = \frac{m(v-u)}{t} = \frac{0.145\{22 - (-14)\}}{0.015}$ [Being two acting in velocities are oppsite direction]

$= 348\ N$

61. 2068 Q.No. 9 a

A 15 kg load of bricks hangs from one end of a rope that passes over a small, frictionless pulley. A 28 kg counterweight is suspended from the other end of the rope as shown in figure. The system is released from rest. Using free body diagram method, find the magnitude of upward acceleration of the load and the tension in the rope while the load is moving. **[4]**



Solution

Let a be the acceleration & T be the tension in both cases, then,

For upward motion,

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$F = T - mg$

or, $ma = T - mg$

or, $T = m(a + g)$ ...(i)

For downward motion

$F' = Mg - T$

or, $Ma = Mg - T$

or, $T = M(g - a)$ ...(ii)

From equation (i) & (ii), we have

$m(a + g) = M(g - a)$

or, $15(a + 10) = 28(10 - a)$

or, $15a + 150 = 280 - 28a$

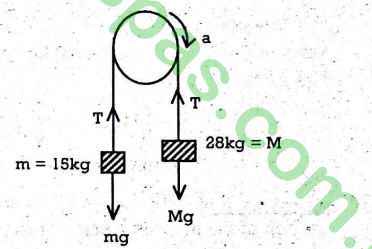
or, $43a = 130$

or, $a = \frac{130}{43} = 3.02 m/sec^2$

From equation (i)

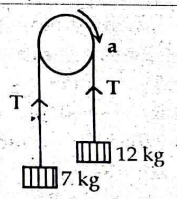
$T = m(a + g) = 15(3.02 + 10) = 195.35N$

$\therefore$ Acceleration is 3.02 m/sec² & tension is 195.35N.



62. 2067 Supp Q.No. 9 a

Two masses 7 kg and 12 kg are connected at the two ends of a light inextensible string that passes over a frictional pulley. Using free body diagram method, find the acceleration of masses and the tension in the siring, when the masses are released. **[4]**



Solution

Let, a be the acceleration for both case & T be tension of the string. For upward motion,

$F = T - mg$ (for small mass)

or, $ma = T - mg$

or, $T = m(a + g)$ ...(i)

For downward motion

$F = Mg - T$ (for big mass)

or, $Ma = Mg - T$

or, $T = M(g - a)$ ...(ii)

From equation (i) and (ii), we have

$m(a+g) = M(g - a)$

or, $7(a + 10) = 12(10 - a)$

or, $7a + 70 = 120 - 12a$

or, $70 - 120 = -12a - 7a$

or, $-50 = -19a$

or, $a = \frac{50}{19}$

$a = 2.63 m/sec^2$

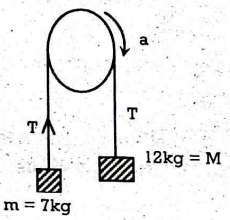
From equation (i),

$T = m(a + g)$

or, $T = 7(10 + 2.63)$

or, $T = 88.42N$

$\therefore$ Acceleration is 2.63m/sec² & tension is 88.42 N.



63. 2066 Old Q. No. 2 b **A vehicle having a mass of 500 kg is moving with a speed of 10 ms⁻¹. Sand is dropped into it at the rate of 10 kg/min. What force is needed to keep the vehicle moving with a uniform speed?** **[4]**

Solution

Given,

Mass of sand dropping per second $\left(\frac{m}{t}\right) = \frac{1}{6}$ kg s^{-1}

Initial speed (u) = 10 m s^{-1}
Mass of the vehicle (M) = 500 kg
The force needed,

$F = \frac{mv - mu}{t} = \frac{m}{t}(v - u) = \frac{1}{6}(10 - 0)$

F = 1.667 N in magnitude

64. 2059 Q.No. 2 b A lift moves (i) up and (ii) down with an acceleration of 2ms^{-2}. In each case, calculate the reaction of the floor on a man of mass 50 kg standing in the lift.

Solution

Given,
Acceleration (a) = 2 m s^{-2}
Mass of the man (m) = 50 kg
i. Reaction of the floor when lift moves up (R_1) = ?
ii. Reaction of the floor when lift moves down (R_2) = ?
Now,
i. When lift moves upward, normal reaction R_1 is given by

$R_1 - mg = ma$
or, $R_1 = ma + mg = m(a + g) = 50(2 + 10)$
$\therefore R_1 = 600$ N

ii. When the lift moves down,

$mg - R_2 = ma$
or, $mg - ma = R_2$
or, $m(g - a) = R_2$
or, $50(10 - 2) = R_2$
or, $400 = R_2$
$\therefore R_2 = 400$ N

65. 2057 Q.No. 2 b The mass of gas emitted from the rear of toy rocket is initially 0.2 kgs^{-1}. If the speed of the gas relative to the rocket is 40 ms^{-1}, and the mass of rocket is 4 kg, what is the initial acceleration of the rocket?

Solution

Given,

Mass of gas emitted per second $\left(\frac{m}{t}\right) = 0.2$ kg s^{-1}

Initial speed relative to the rocket (u) = 0
Final speed relative to the rocket (v) = 40 m s^{-1}
Mass of the rocket (M) = 4 kg
Acceleration of the rocket (a) = ?

The force with which gas emitted (F) $= \frac{mv - mu}{t} = \frac{m}{t}(v - u) = 0.2(40 - 0)$

F = 8 N

From Newton's third law of motion, the same 8 N force acts on the rocket that moves in opposite direction. So we have, force acting on the rocket = 8 N

or, $Ma = 8$
or, $4a = 8$
or, $a = 2$ m s^{-2}
$\therefore$ Initial acceleration of the rocket, $a = 2$ m s^{-2}.

❑❑❑

Chapter 5: Work and Energy

Short Answer Questions

1. 2076 Set B Q.No. 1c A light body and a heavy body have the same momentum. Which one will have greater kinetic energy? Explain. [2]

We know that momentum and K.E. of a body are related as,

$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$ [p = mv]

$\therefore K.E. = \frac{p^2}{2m}$

If a small body and a heavy body have same momentum, the K.E. of lighter body will be more than the heavier body, since $K.E. \propto \frac{1}{m}$ for same momentum.

2. 2076 Set C Q.No. 1c A body is moving with constant velocity v along a circular path of radius r. How much work is done by the centripetal force in revolving the body? [2]

A body is moving with constant velocity v along a circular path of radius r. The work done by the centripetal force is zero as the force and displacement of body at each point are perpendicular to each other. The work done is given as, $W = \vec{F} \cdot \vec{s} = Fs\cos\theta$. Since $\theta = 90°$, W = 0. So, no work is done by centripetal force.

3. 2075 Set A Q.No. 1g 2069 (Set A) Old Q. No. 1f 2066 Old Q. No. 1 f 2057 Q.No. 1 c Distinguish between conservative and non-conservative forces. [2]

A force is said to be conservative if the total work done by it on moving a particle round a closed path is zero. The work done by it in moving a particle from one point to another point depends only on the distance between these points but not the actual path followed. For example; gravitational force.
A force is said to be non-conservative if the work done by it on moving a particle round a closed path is not zero. The work done by non-conservative force depends upon the path followed and initial and final position also. For example, frictional force.

4. 2074 Supp. Q.No. 1c A stationary mass suddenly explodes into two fragments; one heavy and another light. Which one has greater kinetic energy and why? [2]

Let M be the mass of stationary mass, m_1 and m_2 be the masses of smaller and bigger fragments with velocity v_1 and v_2 respectively. Let U be the velocity of initial velocity which is zero (being stationary). Then from principles of conservation of momentum,

$m_1v_1 + m_2v_2 = MU$
or, $m_1v_1 + m_2v_2 = 0$ [U = 0]
or, $v_1 = -\frac{m_2v_2}{m_1}$
or, $v_1^2 = \frac{m_2^2v_2^2}{m_1^2}$

$\therefore$ K.E. smaller $= \frac{1}{2}m_1v_1^2$

$= \frac{1}{2}m_1\frac{m_2^2v_2^2}{m_1^2}$

$= \frac{1}{2}\frac{m_2^2v_2^2}{m_1}$

$= \frac{m_2}{m_1}\left(\frac{1}{2}m_2v_2^2\right)$

$= \frac{m_2}{m_1}$ K.E. bigger

Since, $m_2 > m_1$, K.E. $_{smaller}$ > K.E. $_{bigger}$
So, smaller fragment has greater energy due to greater velocity and momentum.

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5. 2073 Set D Q.No. 1c **A man carrying a bucket of water is walking on a level road with a uniform velocity. Does he do any work on the bucket while carrying it?**

The mechanical work done by a moving body is given by

$$W = F \cos\theta \quad \ldots (i)$$

where, F is applied force, S is displacement and θ is angle between force applied and displacement. When a man carrying a bucket of water is walking on a level road with a uniform velocity, the angle between force and displacement is 90°.

So, from equation (i) is

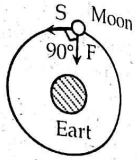
$$W = Fs \cos 90° = 0$$

$\therefore$ $w = 0$

i.e. No, work is done.

6. 2070 Supp (Set A) Q.No. 1 c **The moon is accelerating toward the earth. Why isn't it getting closer to us?**

The moon is revolving round the earth in a circular path where the centripetal force acting towards earth or centre is provided by the gravitational force. The moon thereby due to this pull moves fast enegough to avoid falling into the earth. It the speed is out of this balance then it would have moved away or fall into the earth. The work done by this force is zero as



$W = F.S.\cos 90° = 0$

So, no work is done by centripetal or gravitational force and hence, it does not get closer to earth.

7. 2070 Supp (Set B) Q.No. 1 b **Explain why the force of gravity due to the Earth does not pull the Moon in closer and closer on an inward spiral until it hit the Earth surface?**

Please refer to 2070 Supp (Set A) Q.No. 1 c

8. 2069 (Set A) Q. No. 1a **How does the kinetic energy of a body change if its momentum is halved?**

The kinetic energy of a particle of mass m moving velocity v is given by

$$K.E = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{P^2}{2m} \quad [\because P = mv = \text{momentum}]$$

If momentum is halved,

$$K.E' = \frac{\left(\frac{P}{2}\right)^2}{2m} = \frac{1}{4}\left(\frac{P^2}{2m}\right)$$

$$K.E' = \frac{1}{4}K.E$$

$\therefore$ Kinetic energy is decreased by 4 times when its momentum is halved.

9. 2068 Q.No. 1 d **What are elastic and inelastic collisions? Give examples of each.**

Elastic collision: The collision between two bodies in which both kinetic energy and momentum remain conserved, is called elastic collision. For example, collision between any two elemental particles can be taken as elastic collision.

i.e. Linear momentum conservation: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy conservation: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Here, m_1, m_2 are masses of bodies. u_1, u_2 and v_1, v_2 are initial and final velocities of two bodies respectively before and after elastic collision.

Inelastic collision: The collision between two bodies in which linear momentum is conversed but kinetic energy is called inelastic collision.

i.e. Linear momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

All the collisions occour in physical world are generally inelastic.

10. 2066 Old Q. No. 1 d **If you are given two objects one is heavier and another is lighter, both have the same kinetic energy. Which one has more linear momentum?** [2]

We know that, kinetic energy of a body of mass m and velocity v is

$$K.E. = \frac{1}{2}mv^2 = \frac{P^2}{2m} \Rightarrow P^2 = 2m \times K.E. \quad \text{Then, } P = \sqrt{2m \times K.E.}$$

This relation shows that the momentum of body is proportional to mass. Since both bodies have same kinetic energy, kinetic energy being constant in our case, thus, the lighter body has lesser momentum than the heavier body.

11. 2064 Q.No. 1 e **Two bodies of different masses are moving with the same kinetic energy of translation. Which one has more momentum?** [2]

Please refer to 2066 Old Q. No. 1 d

12. 2063 Q.No. 1 c **"The earth moving round the sun in an orbit is acted upon by a force, hence, the work must be done on the earth by this force" .Do you agree with this statement?**

No, when the earth rounds the sun, the centripetal force required is provided by the gravitational force between the earth and sun. The gravitational force is directed towards the sun and displacement of the earth is perpendicular to the force at every moment. Therefore, work done is given by $W = FS\cos\theta = FS\cos 90° = 0$. Hence, no work has to be done.

13. 2060 Q.No. 1 d **How does the K.E. of an object change if its momentum is doubled?**

Suppose a body of mass m is moving with velocity u. So its initial momentum (p) = mu and

Initial kinetic energy, $K.E. = \frac{1}{2}mu^2 = \frac{1}{2}\frac{m^2u^2}{m} = \frac{P^2}{2m}$.

When momentum of the body is doubled. Then its KE is

$$KE' = \frac{(2P)^2}{2m} = 4\left(\frac{P^2}{2m}\right) = 4 \text{ initial KE}$$

$\therefore$ New K.E. = 4 × initial K.E.

It means when linear momentum is doubled, its K.E. is increased by four times than previous K.E.

14. 2058 Q.No. 1 b **If a moving bullet striking a block of wood on a frictionless table embeds inside it, what happens to the KE of the bullet?**

If a moving bullet striking a block of wood on a frictionless table embeds inside it, the velocity of bullet decreases and hence, the kinetic energy of the bullet also decreases. This is because some of the kinetic energy is utilized in creating the hole where the bullet gets embedded as given by the relation $KE = 1/2mv^2 = F \times S$, where S is the distance travelled. In addition, some energy is spent in creating sound during collision and associtated heat.

15. 2055 Q.No. 1 a **Differentiate between conservation of kinetic energy and conservation of linear momentum.**

The total kinetic energy of an isolated system always remains same or total kinetic energy before collision is equal to total kinetic energy after collision is called conservation of kinetic energy. The conservation of K.E. equation is

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

The total linear momentum of an isolated system remains same or total linear momentum before collision is equal to total linear momentum after collision is called conservation of linear momentum. The conservation of linear momentum equation is

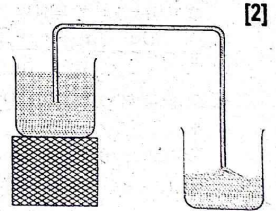
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

16. 2054 Q.No. 2 a **What is meant by elastic and non-elastic collision?** [2]

Please refer to 2068 Q.No. 1 d

17. 2050 Q.No. 1 ii **In a syphon, water is lifted above its original level during its flow from one container to another. Where does it get the needed potential energy from?** [2]

In a syphon, water is lifted above its original level during its flow from one container to another container as shown in figure. The potential energy needed in this case arises from the air pressure differences between two containers.



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Long Answer Questions

18. 2073 Supp Q.No. 5c What is the physical difference between elastic and inelastic collisions? Prove that colliding objects having same masses exchange their velocities in one dimensional elastic collision.

Collision: Collision is defined as the mutual interaction between two particles for relatively short interval of time as a result of which the energy and momentum of interacting particles change. There are two types of collision; elastic and inelastic collision.

Elastic collision: The collision between two bodies in which both kinetic energy and momentum remain conserved, is called elastic collision. In this collision, total mechanical energy remains conserved and forces involved are conservative in nature. For example, collision between any two elementary particles can be taken as elastic collision.

i.e. Linear momentum conservation: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy conservation: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

here, m_1, m_2 are masses of bodies. u_1, u_2 and v_1, v_2 are initial and final velocities of two bodies respectively before and after elastic collision.

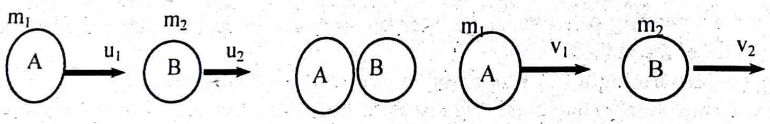
Inelastic collision: The collision between two bodies in which linear momentum is conversed but kinetic energy is called inelastic collision. In this collision, the forces involved may be conservative or non-conservative in nature.

i.e. Linear momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Mud thrown into wall is an example of inelastic collision.

One dimensional elastic collision: Let us consider two particles A and B of masses m_1 and m_2 moving with initial velocities u_1 and u_2 such that $u_1 > u_2$ in same path, collide with each other and after collision their velocities be v_1 and v_2 on same line as shown in figure. In elastic collision, both linear momentum and kinetic energy remain conserved.



Before collision During collision After collision

From the principle of conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or, $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ... (i)

Since the collision is elastic, the kinetic energy is also conserved i.e.

K.E. before collision = K.E. after collision

or, $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

or, $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

or, $m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$ ... (ii)

Dividing Eq. (ii) by Eq. (i), we get

$u_1 + v_1 = v_2 + u_2$

or, $u_1 - u_2 = v_2 - v_1$ ... (iii)

From equation (iii)

$v_2 = u_1 - u_2 + v_1$

Substituting the value of v_2 in equation (i)

$m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$

or, $v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$ ... (iv)

Again from equation (iii)

$v_1 = v_2 - u_1 + u_2$

Substituting the value of v_1 in equation (i) we get

$m_1(u_1 - v_2 + u_1 - u_2) = m_2(v_2 - u_2)$

or, $v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$ ... (v)

When $m_1 = m_2$ then from equation (iv) and (v), we get

$v_2 = u_1$ and $v_1 = u_2$

i.e. if two bodies of equal masses suffer elastic collision, then after the collision, they will interchange their velocity.

19. 2072 Supp Q.No. 5a 2069 (Set B) Q. No. 5a 2069 Old (Set B) Q. No. 2a 2057 Q.No. 2a What is the principle of conservation of energy? Show that total mechanical energy of a body is conserved when it moves under the action of gravitational field. [4]

Principle of conservation of energy: It is stated as "energy is neither created nor destroyed but can be transformed from one form of energy into another form of energy." So, when a body falls freely under the action of gravity, the total mechanical energy is conserved.

Proof of conservation of mechanical energy: Consider a body of mass 'm' held at rest at height h (at point A) from the ground of the earth. If it is allowed to fall freely under the action of gravity, let B be the position of the body at any instant t and it finally reaches to ground point C.

At point A, $u_A = u = 0$ m/s

Kinetic energy, (K. E.) $= \frac{1}{2}mu^2$

Potential energy (P.E.) = mgh

Total energy (E_A) = K.E. + P.E.

$E_A = 0 + mgh = mgh$ ... (i)

At point B:

Suppose the body falls from A to B through distance x from A, so height of point B is (h – x).

Let v_B be the velocity gained by the body falling from A to B.

Since, $v_B^2 = u^2 + 2ax$

$v_B^2 = 0 + 2gx$

$v_B^2 = 2gx$

Now, K.E. at point B $= \frac{1}{2}mv_B^2 = \frac{1}{2}m.2gx = mgx$

and P.E. at B = mg (h – x)

Total energy at B is E_B = K.E. + P.E.

$= mgx + mg(h - x)$

$= mgx + mgh - mgx$

$E_B = mgh$ ...(ii)

At point C:

Let v_C be the velocity at point C.

Since, $v_C^2 = v_A^2 + 2ah$

$v_C^2 = 0 + 2gh$ (a = g)

$v_C^2 = 2gh$

Kinetic energy at point C (K.E.) $= \frac{1}{2}mv_C^2$

$= \frac{1}{2}m\,2gh = mgh$

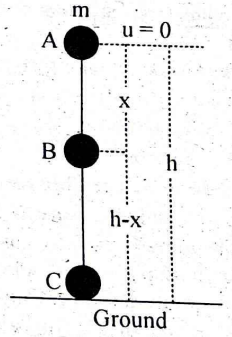
Potential energy (P.E.) = mg h = mg 0 = 0

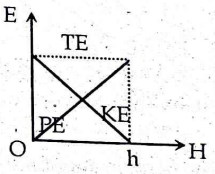
Total energy of point C is $E_C = mgh + 0$

$E_C = mgh$ ... (iii)

From equation (i), (ii) and (iii), we see that

$E_A = E_B = E_C = mgh$. Thus, when a body falls freely under the action of gravity, the kinetic energy and potential energy vary from point to point during motion but the total mechanical energy (K.E. + P.E.) remains constant at each point. So, the mechanical energy is conserved when the body falls under the action of gravity.





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20. **2072 Set C Q.No. 5b Write expressions for work done by a constant and a variable force. Show that the work done by the resultant force on a particle is equal to the change in kinetic energy of the particle.**

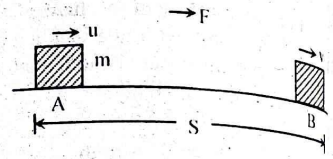
Work done by constant force: The work done by constant force is given by the product of force and distance travelled. i.e.

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

Work done by variable force: The work done by variable force is given by integration of product of force and small displacement. i.e.

$$W = \int \vec{F} \cdot d\vec{x}$$

Let us, consider a body of mass 'm' is moving from position A to B by applying a force F changing velocity from u to v travelling a distance s.



Then,

Work done by force is

$$W = F.s \quad \ldots (i)$$

From Newton's second law of motion,

$$F = ma \quad \ldots (ii)$$

Then, work done,

$$W = mas \quad \ldots (iii)$$

During the work done, the initial velocity 'u', changes to final velocity 'v'. Then,

$$v^2 = u^2 + 2as$$

or, $$as = \frac{v^2 - u^2}{2} \quad \ldots (iv)$$

From equation (iii) and (iv), we have

$$W = m\left(\frac{v^2 - u^2}{2}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \text{Final K.E.} - \text{Initial K.E.}$$

$\therefore$ Work done = Change in K.E., proved.

21. **2071 Supp Q.No. 5b What do you mean by variable force? Derive an expression for work done by a force.**

Work: Work is said to be done when the force acting on body is able to move it through a distance in the direction of force and it is defined as the product of force applied and displacement covered by the body i.e.

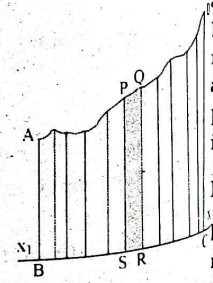
Work = F.s cos θ

Work done by variable force.

Let us consider a body is displaced from B to C by using variable force F. Consider the displacement from B to C is made up of large no. of infinitesimal displacement from x_1 to x_2. Such one such displacement is PQ. As, the displacement, PQ = dx is such displacement. Consider the force is constant in magnitude and as well as in direction.

Therefore, small amount of work done for small displacement PQ is

$$dw = F \times dx$$
$$= PS \times PQ$$
$$= \text{Area of strip PQRS}$$



Total amount of work done for displacement A to B,

$$W = \sum dw$$

As the displacement allowed to zero, then we can write,

$$W = \lim_{d_{DIS} \to 0} \Sigma F.dx = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \text{Area of strip PQRS}$$

$$W = \text{Area of ABCD}$$

i.e. work done by variable force is equal to area under force and the displacement curve.

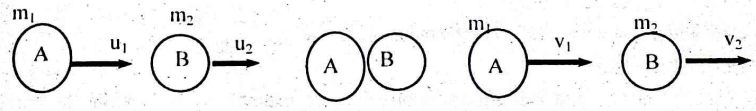
2071 Set C Q.No. 5 d What is elastic collision? Prove that the colliding objects exchange their velocities in one dimensional elastic collision. [4]

Please refer to 2073 Supp Q.No. 5c

2070 Set C Q.No. 5 a What is elastic collision? Show that in an elastic collision between two particles, the relative velocity of separation after collision is equal to the relative velocity of approach before collision. [4]

Elastic collision: The collision between two bodies in which both kinetic energy and momentum remain conserved, is called elastic collision. For example, collision between any two elementary particles can be taken as elastic collision.

Let us consider two particles A and B of masses m_1 and m_2 are moving with initial velocities u_1 and u_2 such that $u_1 > u_2$ in same path, collide with each other and let after collision their velocities be v_1 and v_2 on same line as shown in figure. In elastic collision, both linear momentum and kinetic energy remain conserved.



Before collision During collision After collision

From the principle of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \ldots (i)$$

Since the collision is elastic, the kinetic energy is also conserved i.e.

K.E. before collision = K.E. after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$
$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \quad \ldots (ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1 \ldots (iii)$$

This shows that the in an elastic collision between two particles, the relative velocity of separation after collision is equal to the relative velocity of approach before collision.

2070 Set D Q.No. 5 a 2067 Supp Q.No. 5b Define work. Derive an expression to calculate the work done by a variable force. [4]

Please refer to 2071 Supp Q.No. 5b

2068 Old Q.No. 2 a What do you mean by elastic and inelastic collisions? Write with one example of each. Show that the sum of kinetic energy and potential energy of a freely falling body remains constant at any instant. [2+3=5]

Collision: Collision is defined as the mutual interaction between two particles for relatively short interval of time as a result of which the energy and momentum of interacting particles change. There are two types of collision; elastic and inelastic collision.

Elastic collision: The collision between two bodies in which both kinetic energy and momentum remain conserved, is called elastic collision. In this collision, total mechanical energy remains constant and forces involved are conservative in nature. For example, collision between any two elementary particles can be taken as elastic collision.

i.e. Linear momentum conservation: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy conservation: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

here, m_1, m_2 are masses of bodies. u_1, u_2 and v_1, v_2 are initial and final velocities of two bodies respectively before and after elastic collision.

Inelastic collision: The collision between two bodies in which linear momentum is conversed but not kinetic energy is called inelastic collision. In this collision, the forces involved may be conservative or

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non-conservative in nature.

i.e. Linear momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Kinetic energy: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Mud thrown into wall is an example of inelastic collision.

Second part: Please refer to 2072 Supp Q.No. 5a

26. 2067 Q.No. 5 b Define work. Derive an expression to calculate work done by a variable force. [2]

Please refer to 2071 Supp Q.No. 5b

27. 2066 Q. No. 5 d What are conservative and non-conservative forces? Illustrate your answer by reference to the energy changes occurring in a body whilst falling freely under gravity. [6]

1st part: Please refer to 2075 Set A Q.No. 1g

2nd part: Please refer to 2072 Supp Q.No. 5a

28. 2065 Q. No. 2 a State and prove conservation of the mechanical energy. [1½]

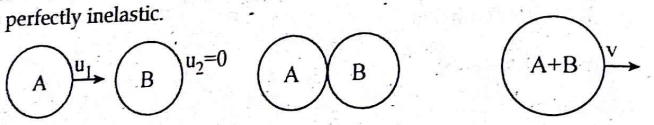
Please refer to 2072 Supp Q.No. 5a

29. 2059 Q.No. 3 a What are elastic and inelastic collision? Give an example of each. Write the energy and momentum equations for an inelastic collision. [2½]

First part: Please refer to 2068 Old Q.No. 2 a

Energy and momentum equation for inelastic collision:

Consider m_1 and m_2 are the masses of two bodies which are undergoing collision. Let u_1 and u_2 are initial velocities of two masses. After collision occour, both bodies move with a common velocity v and the collision is perfectly inelastic.



Before collision During collision After collision

By the principle of conservation of linear momentum,

$m_1u_1 + m_2u_2 = m_1v + m_2v = (m_1 + m_2)v$ ... (i)

$v = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$

Equation (i) represents the momentum equation.

If the second body is initially at rest i.e. $u_2 = 0$, then we have,

$m_1u_1 = (m_1 + m_2)v$

$\Rightarrow v = \frac{m_1}{m_1m_2}u_1$ ... (ii)

Now, total kinetic energy of particle before collision,

$E_1 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1u_1^2$ ... (iii) $[\because u_2 = 0]$

And total Kinetic energy after collision,

$E_2 = \frac{1}{2}(m_1 + m_2)v^2$

$= \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\right)^2u_1^2$ [Using equation (ii)]

$E_2 = \frac{1}{2}\frac{m_1^2}{m_1 + m_2}u_1^2$ ... (iv)

Dividing equation (iv) by equation (ii), we get

$\frac{E_2}{E_1} = \frac{\frac{1}{2}\frac{m_1^2}{m_1 + m_2}u_1^2}{\frac{1}{2}m_1u_1^2}$

$\frac{E_2}{E_1} = \frac{m_1}{m_1 + m_2}$ ... (v)

This is energy equation for inelastic collision.

$\Rightarrow E_2 < E_1$

This shows that kinetic energy before collision is greater than kinetic energy after collision in perfectly inelastic collision. This shows energy get loss during inelastic collision. This loss in energy is converted into heat and sound energy.

Numerical Problems

30. 2075 GIE Q.No. 9b A car of mass 1000 kg moves at a constant speed of 20m/s along a horizontal road. If a constant frictional force of 200N is acting between the car and the road. Calculate the power developed by the engine. [4]

Solution

Given,

Mass of car (m) = 1000kg

Speed of car (v) = 20 m/sec

Frictional force (F) = 200N

Power developed (P) = ?

Since speed of car is constant so, acceleration is zero, i.e., $a = 0m/sec^2$

Total force needed F = Fr + m.a

= 200 + 1000 × 0 = 200N.

$\therefore P = F \times v = 200 \times 20 = 4000W = 4\ KW$

31. 2075 Set B Q.No. 9c A water reservoir tank of capacity 250 m^3 is situated at a height of 20 m from the water level. What will be the power of an electric motor to be used to fill the tank in 3 hours? Efficiency of motor is 70%. [4]

Solution

Given,

Volume (V) = 250 m^3

Height (h) = 20m

time (t) = 3 hrs = 3 × 60 × 60 sec = 10800 sec.

Efficiency of meter (η) = 70%

Power of motor (P_{in}) = ?

Now,

Mass of water filled = Volume × density of water

= 250 × 1000

= 250000 kg

Work done to fill the tank, W = mgh

= 250000 × 10 × 20 = 5×10^7 J

Power (P_{out}) $= \frac{\text{Work done}}{\text{time}}$

$= \frac{5 \times 10^7}{10800}$

$= 4.63 \times 10^3$ Watt

Efficiency $= \frac{P_{out}}{P_{in}} \times 100\%$

or, $0.7 = \frac{4.63 \times 10^3}{P_{in}}$

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or, $P_{in} = \frac{4.63 \times 10^3}{0.7} = 6614$ Watt.

∴ Power of motor = 6614 Watt.

32. 2074 Set A Q.No. 9c A stationary mass explodes into two parts of mass 4 units and 40 units respectively. If the larger mass has an initial K.E. 10 J, what is the initial kinetic energy of the smaller mass? [4]

Solution

Given,

Mass of smaller part (m_1) = 4 kg

Mass of larger part (m_2) = 40 kg

Let v_1 and v_2 be the velocity of smaller and larger part, then, from the conservation of linear momentum,

$m_1v_1 + m_2v_2 = MV$ [Here, M = mass of body before explosion and V = Velocity of the stationary body before explosion = 0 m/s]

or, $m_1v_1 + m_2v_2 = 0$

or, $v_1 = -\frac{m_2}{m_1}.v_2$

Now,

K.E. of smaller mass $= \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\left(-\frac{m_2}{m_1}v_2\right)^2$

$= \frac{1}{2}\frac{m_2}{m_1} \times \left(\frac{1}{2}m_2v_2^2\right) \times 2$

$= \frac{m_2}{m_1} \times$ K.E. of larger part

$= \frac{40}{4} \times 10 = 100$ J

33. 2074 Set B Q.No. 9a An explosive of mass M placed at a point explodes into one-third and two-third parts. If the initial kinetic energy of the smaller part is 1000 J. What will be the initial K.E. of the bigger part? [4]

Solution

Given,

Mass of the body = M

Mass of smaller part (m_1) = $\frac{M}{3}$

Mass of bigger part (m_2) = $\frac{2M}{3}$

Let v_1 and v_2 be the velocity of smaller and bigger part, then, from the conservation of the linear momentum,

$m_1v_1 + m_2v_2 = 0$ [∵ Initial momentum (m.v) = 0 as v = 0]

or, $v_2 = -\frac{m_1}{m_2}v_1$

Now,

K.E. of bigger part $= \frac{1}{2}m_1v_2^2 = \frac{1}{2}m_2\left(-\frac{m_1}{m_2}v_1\right)^2$

$= \frac{1}{2}\frac{m_1}{m_2} \times \left(\frac{1}{2}m_1v_1^2\right) \times 2$

$= \frac{m_1}{m_2} \times$ K.E. of smaller part $= \frac{m/3}{2m/3} \times 1000 = \frac{1000}{2} = 500$ J

34. 2072 Set D Q.No. 9b You throw a 20 N rock vertically into the air from ground level. You observe that when it is 15 m above the ground, it is travelling at 25 m/s upward. Use the work-energy theorem to find (i) its speed as it left the ground and (ii) its maximum height. [4]

Solution

Given,

Weight of rock (W_F) = 20 N

Mass of rock (m) = 20/10 = 2 kg

Height (h) = 15 m

Velocity (v) = 25 m/sec

Initial speed (u) = ?

Maximum height (h_{max}) = ?

i. We have, from work energy theorem,

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

or, $F.h = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

or, $-20 \times 15 = \frac{1}{2} \times 2 \times 25^2 - \frac{1}{2} \times 2u^2$

or, $u^2 = 625 + 300$

or, $u^2 = 925$

or, $u = 30.41$ m/sec

ii. Again,

$v^2 = u^2 - 2g\,h_{max}$

$0 = 925 - 2 \times 10 \times h_{max}$ [For h_{max}, v = 0]

or, $h_{max} = \frac{925}{20} = 46.25$ m

35. 2071 Set C Q.No. 9 a A 650 KW power engine of a vehicle of mass 1.5×10^5 kg is rising on an inclined plane of inclination 1 in 100 with a constant speed of 60 km/hr. Find the frictional force between the wheels of the vehicle and the plane. [4]

Solution

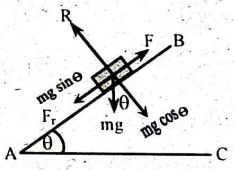
Given,

Power of engine (P) = 650 KW = 650000 W

Mass of vehicle (m) = 1.5×10^5 kg

Inclination $\sin\theta = \frac{1}{100}$

Speed of vehicle (v) = 60 km/hr $= \frac{60 \times 1000}{60 \times 60} = \frac{50}{3}$ m/sec



Frictional force (f_r) = ?

We have, total upward force

$F = f_r + mg\sin\theta$

or, $\frac{P}{v} = f_r + mg\sin\theta$ [∵ P = F.v]

or, $f_r = \frac{P}{v} - mg\sin\theta = \frac{650000 \times 3}{50} - 1.5 \times 10^5 \times 10 \times \frac{1}{100}$

∴ $f_r = 24000$ N

36. 2071 Set D Q.No. 9 b A stationary mass explodes into two parts of mass 4 kg and 40 kg. The initial kinetic energy of larger mass is 10 J. Find the velocity of the smaller mass. [4]

Solution

Given,

Smaller mass of (m_1) = 4 units

Larger mass (m_2) = 40 units

$K.E_{small}$ = ?

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$K.E_{large} = 10\ J$

We know that

$$\frac{K.E_{small}}{K.E_{large}} = \frac{m_2}{m_1}$$

or, $K.E_{small} = \frac{40}{4} \times 10 = 100J$

Again,

$$100 = \frac{1}{2} m_1 v_1^2$$

or, $v_1^2 = \frac{200}{4}$

$\therefore\ v_1 = 7.07\ m/sec$

37. 2070 Supp (Set A) Q.No. 9 b A block of weight 150N is pulled 20m along a horizontal surface at constant velocity. Calculate the work done by the pulling force if the coefficient of kinetic friction is 0.20 and the pulling force makes an angle of 60° with the vertical. [4]

Solution

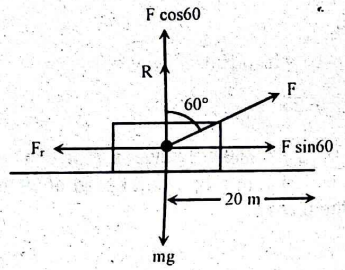
Given,

Weight of block (W) = mg = 150 N

The vertical and horizontal components of pulling force F are Fcos60° and Fsin60°. Then,

$R + F\cos60 = mg$

or, $R = mg - F\cos60 = 150 - \frac{F}{2}$



Also,

$$\mu = \frac{F\sin60}{R}$$

or, $R = \frac{F.\frac{\sqrt{3}}{2}}{0.2}$

or, $150 - \frac{F}{2} = \frac{\sqrt{3}}{0.4} F$

or, $F = 31.056\ N$

Then work done,

$$W = F\sin60 \times 20 = 31.056 \times \frac{\sqrt{3}}{2} \times 20 = 537.9\ J$$

38. 2070 Supp (Set B) Q.No. 9 b The constant force resisting the motion of a car of mass 1500 kg is equal to one fifteenth of its weight if, when travelling at 48 km/h, the car is brought to rest in a distance of 50 m by applying the brakes, find the additional retarding force due to the brakes (assumed constant) and heat developed in the brakes. [4]

Solution

Given,

Mass of car (m) = 1500 kg

Frictional force $(Fr) = \frac{1}{15} \times mg = \frac{1}{15} \times 1500 \times 10 = 1000\ N$

Initial velocity $(u) = 48km/hr = \frac{48 \times 1000}{60 \times 60} = \frac{40}{3}\ m/sec$

Distance covered (S) = 50 m

Final velocity (v) = 0

Additional force (f_a) = ?

Now,

$v^2 = u^2 + 2as$

$0^2 = \left(\frac{40}{3}\right)^2 + 2 \times a \times 50$

or, $a = -\frac{(40/3)^2}{2 \times 50} = -\frac{16}{9}\ m/sec^2$

The retarding force due to friction is

$fr = ma = -\frac{16}{9} \times 1500 = 2666.67\ N$

But frictional force (fr) = 1000 N

So, additional force (fa) = fr – Fr = 2666.67 – 1000 = 1666.67 N

Heat developed = work done

$\therefore\ W = fa \times S = 1666.67 \times 50 = 83333.5\ J$

39. 2069 (Set B) Q. No. 9b A 0.15 kg glider is moving to the right on a frictionless horizontal air track with a speed of 0.80ms^{-1}. It has a head on collision with a 0.300 kg glider that is moving to the left with a speed of 2.2 ms^{-1}. Find the final velocity (magnitude and direction) of each glider if the collision is elastic.

Solution

Given,

Mass of glider A (M_1) = 0.15 kg

Mass of glider B (M_2) = 0.3kg

Initial velocity of A (u_1) = 0.80 ms^{-1}

Initial velocity of (B) (u_2) = – 2.2m/s

Final velocity of A (v_1) = ?

Final velocity of B (v_2) = ?

We have,

$M_1 u_1 + M_2 u_2 = M_1 v_1 + M_2 v_2$

$0.15 \times 0.8 + 0.3 \times (-2.2) = 0.15 v_1 + 0.3 v_2$

or, $-0.54 = 0.15 v_1 + 0.3 v_2$ ... (i)

Again for elastic collision,

$u_1 + v_1 = u_2 + v_2$

or, $0.8 + v_1 = -2.2 + v_2$

or, $v_1 - v_2 = -3$ ... (ii)

Solving (i) & (ii), we get,

$v_2 = -0.2$ m/sec

& $v_1 = -3.2$ m/sec

40. 2069 (Set A) Old Q. No. 3b 2068 Q.No. 9 b A stationary mass explodes into two parts of mass 4 units and 40 units respectively. If the larger mass has an initial K.E. 100J, what is the initial K.E. of the smaller mass?

Solution

Given,

Smaller mass m_1 = 4 units

Larger mass m_2 = 40 units

K.E small =?

K.E larger = 100 J

We know that,

$$\frac{K.E_{smaller}}{K.E_{larger}} = \frac{m_2}{m_1}$$

$K.E_{smaller} = \frac{40}{4} \times 100 = 1000\ J$

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41. 2067 Q.No. 9 a **A typical car weighs about 1200 N. If the coefficient of rolling friction is $\mu_r = 0.015$. What horizontal force is needed to make the car move with constant speed of 72 km/h on a level road? Also calculate the power developed by the engine to maintain this speed.**

Solution

Given,
Weight of car (W) = 1200 N
Coefficient of friction (μ) = 0.015
Velocity (v) = 72 km/hr = 20 m/sec
Power developed by the engine (P) = ?
horizontal force (F) = ?
Now, we have,
F – f = ma
Since a = 0 Where F = horizontal force, f = frictional force
$\therefore$ F = f = μ_r mg = 0.015 × 1200 = 18 N
Also, Power developed by engine (P) = f × v = 18 × 20 = 360 watt.
Therefore, horizontal force needed = 18 N and
Power developed by engine = 360 watt

42. 2066 Q. No. 9 c **A car of mass 1000 kg moves at a constant speed of 25 m/s along a horizontal road where frictional force is 200 N. Calculate the power developed by the engine.** [4]

Solution

Given,
Mass of car (m) = 1000kg
Speed of car (v) = 25 m/sec
Frictional force (F) = 200N
Power developed (P) = ?
Since speed of car is constant. So, acceleration is zero, i.e., a = 0m/sec
Total force needed, F = Fr + m.a
= 200 + 1000 × 0 = 200N
$\therefore$ P = F × v = 200 × 25 = 5000W = 5 KW

43. 2061 Q.No. 2 b **A train of mass 2×10^5 kg moves at a constant speed of 72 kmh^{-1} up a straight inclined against a frictional force of 1.28×10^4N. The incline is such that the train rises vertically 1.0 m for every 100 m travelled along the incline. Calculate the necessary power developed by the train.**

Solution

Given,
Mass of train (M) = 2×10^5 kg
Speed of train (v) = 72 kmh^{-1} = $\frac{72 \times 1000}{60 \times 60}$ = 20m/sec
Frictional force (Fr) = 1.28×10^4N
Inclination = 1 in 100
Power developed (P) = ?
The total force needed
F = Fr + mg sinθ

$= 1.28 \times 10^4 + 2 \times 10^5 \times 10^5 \times \frac{1}{100}$ $\quad [\because \sin\theta = \frac{1}{100}]$

= 32800N
Power, P = F.v
= 32800 × 20
= 656000W
= 656 KW

44. 2058 Q.No. 2 b **A ball A of mass 0.1 kg moving with a velocity of 6ms^{-1} collides directly with a ball B of mass 0.2 kg at rest. Calculate their common velocity if both balls move off together. If ball A has rebounded with a velocity of 2ms^{-1} in the opposite direction after collission, what would be the new velocity of B?**

Solution

Given,
Mass of ball A (m_1) = 0.1kg
Mass of ball B (m_2) = 0.2 kg
Velocity of ball A(u_1) = 6m/sec
Velocity of ball B(u_2) = 0m/sec
Common velocity (v) = ?
We have, from principle of conservation of linear momentum
$m_1u_1 + m_2u_2 = m_1v + m_2v$

$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{0.1 \times 6 + 0.2 \times 0}{0.1 + 0.2} = \frac{0.6}{0.3} = 2m/sec$

If the ball A rebounds then,
v_1 = – 2m/sec
v_2 = ?
Again using conservation of momentum principle,
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
or, $0.1 \times 6 + 0.2 \times 0 = 0.1 \times (-2) + 0.2 \times v_2$
or, $0.6 + 0.2 = 0.2\, v_2$
or, $v_2 = \frac{0.8}{0.2} = 4m/sec$

45. 2058 Q.No. 3 b **A car of mass 1000 kg moves at a constant speed of 20 ms^{-1} along a horizontal road where the friction force is 200 N. Calculate the power developed by the engine.**

Please refer to 2075 GIE Q.No. 9b

46. 2056 Q.No. 2 b **A ball of mass 4 kg moving with a velocity 10 ms^{-1} collides with another body of mass 16 kg moving with 4 ms^{-1} from the opposite direction and then coalesces into a single body. Compute the loss of energy on impact.**

Solution

Given,
Mass of first ball (m_1) = 4kg
Initial velocity of first ball (u_1) = 10 ms^{-1}
Mass of second ball (m_2) = 16 kg
Initial velocity of second ball (u_2) = 4ms^{-1}
Loss of energy on impact (ΔE) = ?
Since, the balls are moving in opposite direction.
$m_1u_1 - m_2u_2 = (m_1 + m_2)\, v$, where v is the common velocity

$v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$

$= \frac{4 \times 10 - 16 \times 4}{4 + 16}$

= –1.2 m/s (i.e. in the direction of m_2)

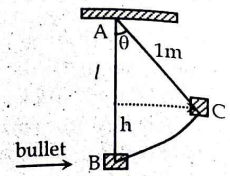
Kinetic energy before collision, $E_i = \frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} \times 4 \times 102 + \frac{1}{2} \times 16 \times u^2 = 328$ J

Kinetic energy after collision, $E_f = \frac{1}{2} (m_1 + m_2)\, u^2 = \frac{1}{2} (4 + 16) \times (1.2)^2 = 14.4$ J

$\Delta E = E_i - E_f = 328 - 14.4 = 313.6$ J
$\therefore$ Loss of energy is 313.6J

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47. **2053 Q.No. 3** **A bullet of mass 20 g travelling horizontally at 100 ms^{-1} embeds itself in the centre of a block of wood of mass 1 kg which is suspended by light vertical string 1 m in length. Calculate the maximum inclination of the string to the vertical.**



Solution

Given,

Mass of bullet (m_1) = 20 g = 20 × 10^{-3} kg

Initial velocity of bullet (u_1) = 100 m/s

Mass of wooden block (m_2) = 1 kg

Initial velocity of wooden block (u_2) = 0

Maximum inclination of the string to the vertical (θ) = ?

Let v be the velocity of the block and the bullet just after impact, then

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

$$= \frac{20 \times 10^{-3} \times 100 \times 1 \times 0}{(20 \times 10^{-3} + 1)}$$

$\therefore$ v = 1.96 m/s

Let θ be the maximum inclination that the string makes with the vertical and h is the vertical height through which the block rises.

For motion of the block, we have

u = 1.96 m/s

and v = 0

So, $v^2 = u^2 + 2gh$

$0^2 = (1.96)^2 + 2(-g) \times h$

$\therefore$ h = 0.19 m

From the figure, we have

$\cos\theta = \frac{l-h}{l}$, where l is the length of the string i.e. AB = AC = l

$$\therefore \cos\theta = \frac{1 - 0.19}{1}$$

$\therefore$ θ = 36°

Hence, maximum inclination of the string to the vertical is 36°.

48. **2051 Q.No. 5** **A bullet of mass 10g is fired from a gun of mass 1 kg. with a velocity of 100 ms^{-1}. Calculate the ratio of the kinetic energy of the bullet and the gun.**

Solution

Given,

Mass of bullet (m) = 10g = 0.01kg

Mass of gun (M) = 1 kg

Velocity of bullet (v) = 100m/sec

Ratio of K.E. = ?

From principle of conservation of linear momentum,

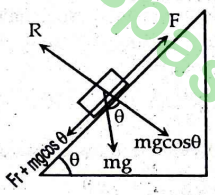
MV = mv [$\because$ V be velocity of gun]

$$V = \frac{m}{M}v = \frac{0.01}{1} \times 100 = 1\text{m/sec}$$

$$\therefore \frac{\text{K.E. of bullet}}{\text{K.E. of gun}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2} = \frac{0.01 \times 100^2}{1 \times 1} = 100:1$$

49. **2050 Q.No. 3 OR** **Find the power of an engine in kilowatts which pulls a train of mass 600 tonnes up an incline of 1 in 100 at the rate of 60 km/hr. The weight of the engine is 200 tonnes and the resistance due to friction is 50 Newton's per tonne.**

Solution



Given,

Mass of train = 600 tonnes

Mass of engine = 200 tonnes

Total mass (m) = (600 + 200) tonnes

= 800 × 10^3 kg

Inclination = 1 in 100

Speed of train (u) = 60km/hr = $\frac{60 \times 1000}{60 \times 60}$ = 16.67 m/sec

Total frictional force (Fr) = (50 Newton / tonnes × 800 tones) = 50 × 800 = 40000N

Power of engine (P) = ?

Then total force needed is

$F = Fr + mg\sin\theta$

$$= 40000 + 800 \times 10^3 \times \frac{1}{100} \times 10 \qquad (\because \sin\theta = \frac{1}{100})$$

= 120000 N

Now,

Power (P) = F.u = 120000 × 16.67 = 2000 KW

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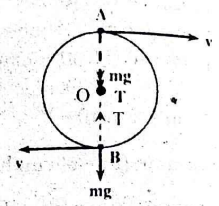
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Chapter 6: Circular Motion

Short Answer Questions

1. 2075 Set B Q.No. 1e A stone tied at one end of a string is whirled in a vertical circle. Where will the tension in the string be maximum and minimum? Show them in a diagram.

When a solid object of mass m is revolved in a vertical circle of radius r with uniform speed v, at highest point A;



$$T + mg = \frac{m v^2}{r}$$

where, T is tension on the string.

or, $T = \frac{mv^2}{r} - mg$ ... (i)

At lowest point B;

$$T - mg = \frac{m v^2}{r}$$

or, $T = \frac{mv^2}{r} + mg$ ... (ii)

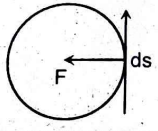
Thus, from equation (i) and (ii), it is clear that tension is maximum at lowest point and minimum at the highest point.

2. 2074 Supp. Q.No. 1d 2060 Q.No. 1 c What is the source of centripetal force to a satellite revolving round earth?

Gravitational attraction between the earth and the satellite is given as $F = \frac{GMm}{r^2}$ which is the source of centripetal force to a satellite revolving round the earth.

3. 2073 Set C Q.No. 1c If there is a net force acting on a particle in uniform circular motion, why does particle's speed not change?

If there is net force acting on a particle in uniform circular motion, the particle's speed does change because the force is perpendicular to the displacement at each & every point whose work done is zero as,



$$W = \vec{F} \cdot \vec{ds}$$
$$= F.ds \cos 90°$$
$$= 0.$$

4. 2073 Set D Q.No. 1e Why is it more difficult to move a stone tied with a longer string than a stone tied with shorter string?

When a stone of mass m is tying with a string and revolve it to a circle, the centrifugal force acting on it is,

$$F = \frac{mv^2}{r}$$, where v is velocity of stone.

or, $F = m\omega^2 r$ $[\because v = r\omega]$

When the length 'r' of string is more, then centrifugal force is more and more force is required to balance it or more centripetal force is required and hence, it is more difficult to revolve a stone by tying it to a longer string than by tying to a shorter string.

5. 2072 Set C Q.No. 1d Why is it more difficult to revolve a stone by tying it to a longer string than by tying to a shorter string?

Please refer to 2073 Set D Q.No. 1e

6. 2071 Supp Q.No. 1c In a washing machine, there is a spin dryer, which removes the water from cloth easily. What could be its working principle? Justify.

In a washing machine, there is a spin dryer which works on the principle of centrifugal force which removes the water particles outward from the clothes and the clothes get dried faster.

7. 2071 Set C Q.No. 1 f Explain the significance of the banking of a curved road. [2]

When a curved road is unbanked, force of friction between the tyres and the road provides the necessary centripetal force. Friction has to be increased suitably, which causes wear and tear of tyres. In the absence of necessary frictional force, the vehicle get skid. When the curved road is banked, horizontal component of normal reaction of ground provides the necessary centripetal force. The banking angle is given by

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$, where v is speed of vehicles

and r is radius of circular path. Thus to produce the source of centripetal force, road is banked.

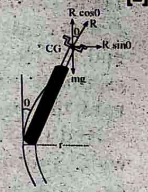
8. 2071 Set D Q.No. 1 e Explain the difference between a simple and conical pendulum. [2]

The differences between simple and conical pendulum are:

Simple pendulum	Conical pendulum
a. Simple pendulum moves in to and fro motion.	a. Conical pendulum moves in horizontal circle.
b. It obeys SHM principle.	b. It does not obey the SHM principle.
c. Acceleration is directed towards mean position.	c. Acceleration is towards the center of circle.
d. Time period is $t = 2\pi\sqrt{\frac{l}{g}}$.	d. Time period is $t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

9. 2070 Supp (Set B) Q.No. 1 f Why does a cyclist lean from vertical while turning on curved track? [2]

A cyclist inclines himself to the vertical while moving round a circular path to provide necessary centripetal force. The normal reaction of the road makes an angle with the vertical and the horizontal component of the reaction provides the necessary centripetal force to move the cycle in the circular path as shown in figure. The bending angle is



$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

10. 2067 Supp Q.No. 1 d Roads are banked at the turnings, why? [2]

Please refer to 2071 Set C Q.No. 1 f

11. 2067 Q.No. 1 c 2054 Q.No. 1 e Explain why a cyclist inclines himself to the vertical while moving round a circular path. [2]

Please refer to 2070 Supp (Set B) Q.No. 1 f

12. 2063 Q.No. 1 e 2056 Q.No. 1 a The positively charged nucleus of an atom attracts the electrons in the orbit. Why do electrons not collapse into the nucleus?

The necessary centripetal force for an electron to move in a circular path is provided by the attractive force between the electron and the nucleus. Since the velocity of electron and electrostatic forces are perpendicular to each other, the work done is zero and electron continues its circular motion and does not collapse into the nucleus.

13. 2062 Q.No. 1 a Why are curved roads banked? Explain.

Please refer to 2071 Set C Q.No. 1 f

14. 2059 Q.No. 1 d Why are roads banked on curved path?

Please refer to 2071 Set C Q.No. 1 f

15. 2055 Q.No. 1 g Why does a cyclist bend while going along curved road?

Please refer to 2070 Supp (Set B) Q.No. 1 f

16. 2053 Q.No. 2 a What is meant by angular velocity? [2]

The time rate of change of angular displacement is called angular velocity. It is denoted by ω and given by

$$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{d\theta}{dt}$$

Angular velocity is measured in radian per second (rad. s^{-1}).

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17. 2052 Q.No. 1 f **When a bus takes a turn, passengers are thrown away from the centre of the curved path. Why?**

When a bus is turning round a curved path, the passengers sitting in the car experience an outward force (centrifugal force). Since, they do not provide the necessary centripetal force and there is no source of ceritripetal force. Due to this reason passengers are thrown away from the centre of curved path, when a bus takes a turn.

18. 2052 Q.No. 1 g **A solid tied at the end of a string is revolved in vertical. At what point the tension in the string will be the greatest?**

Please refer to 2075 Set B Q.No. 1e

Long Answer Questions

19. 2076 Set C Q.No. 5b **What is meant by banking of a road? Discuss the motion of a vehicle round a banked circular track and obtain an expression for the banking angle in terms of speed of the vehicle.**

Banking of a road: The outer edges of roads are raised above than the inner edges at the curve to provide necessary centripetal force. This is called banking of road.

Centripetal force: the force acting on a body moving in a circular path towards its centre is called centripetal force. If a body of mass m moving in a circular path of radius r with constant speed v, the centripetal force acting on it is $F = \frac{mv^2}{r}$.

Motion of Car in a Banked Circular Road

Let us consider a car of mass m is moving in a circular banked road with banking angle θ and radius r with constant speed v as shown in figure. Generally, the frictional force between the tyres of the car and the road provides the necessary centripetal force on it but it cannot always provide such force especially if the road is icy or wet. In order not to depend on friction or to reduce wear on the tyres of road, the roads are banked. The effect of banking is to tilt the normal reaction toward the centre of the curved road. The normal reaction can be resolved into two components: R sinθ towards the centre which provides the necessary centripetal force and vertical component R cosθ which balances the weight of the car where θ is the banking angle or angle of inclination of the plane with the horizontal. We have for horizontal component,

$R \sin\theta = \frac{mv^2}{r}$ ...(i)

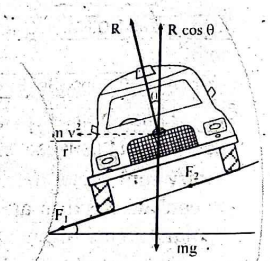
and vertical component,

$R \cos\theta = mg$ ...(ii)

Dividing equation (i) by (ii), we get,

$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$

or, $\tan\theta = \frac{v^2}{rg}$ ... (iii)



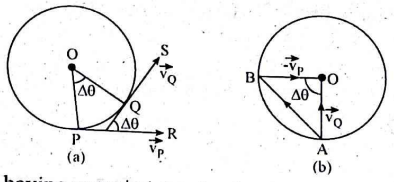
The above equation (iii) gives the angle of banking for a speed v and a curved track of horizontal radius r. So, the tangent of the banking angle is directly proportional to the square of the speed and inversely proportional to the radius of the path.

20. 2075 GIE Q.No. 5b 2069 Supp Q.No. 5b **Define centripetal force. Show that the acceleration of the body moving in a circular path of radius r with uniform speed v is v^2/r and draw a diagram to show the direction of the acceleration.**

Centriptal Force

If a body moves in a circular path with a constant angular velocity, has a constant linear speed along the circumference of circular path but the direction of velocity continuously change at every point of the circle and hence, it has acceleration which is directed towards the centre of the circle. However magnitude of the speed remains the same. This implies that if a body is moving with uniform speed in a circle, force is acting on the body at every point on the circle towards centre of the circle. This force that acts on body moving in circle towards the centre of the circle is called centripetal force. This force is required for the body to keep it in a circular path.

Expression for centripetal force:



Let us consider a body having mass 'm' moving in a circle of radius 'r' with uniform speed v. $\vec{v}_P$ and $\vec{v}_Q$ are the velocities at point P and Q in the direction of their tangents.

The change in velocity when body moves from point P to Q in interval Δt be $\Delta\vec{v} = \vec{v}_Q - \vec{v}_P$.

From figure (b), $\vec{AO}$ and $\vec{BO}$ represent $\vec{v}_Q$ and $\vec{v}_P$ in magnitude and direction. Then, the side $\vec{AB}$ of ΔAOB represents the change in velocity $\Delta\vec{v}$.

i.e. $\Delta\vec{v} = \vec{v}_Q + (-\vec{v}_P) = \vec{AB}$ ... (i)

Now, acceleration = Rate of change in velocity $= \frac{\Delta v}{\Delta t}$

i.e $a = \frac{\vec{AB}}{\Delta t}$ ... (ii)

If 'Δθ' be the angle subtended by OQ with radius vector OP in small time Δt and $|\vec{v}_P| = |\vec{v}_Q| = v$

From figure (b),

$\Delta\theta = \frac{AB}{OA} = \frac{\Delta v}{v}$

$AB = v\,\Delta\theta$ ... (iii)

Then, taking magnitude of equation (ii)

$a = \frac{AB}{\Delta t} = v\frac{\Delta\theta}{\Delta t}$ [using equation (iii)]

If $\Delta t \to 0$, $\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$, angular velocity

$\therefore a = v\omega = \frac{v^2}{r}$ ... (iv)

Since, $v = r\omega$ then $a = r\omega.\omega$

$\Rightarrow a = r\omega^2$ ... (v)

The centripetal force is given by $F = ma = \frac{mv^2}{r}$

$\Rightarrow F = m\omega^2 r$. This is required expression for centripetal forece.

21. 2075 Set A Q.No. 5b **What is a conical pendulum? Obtain an expression for its time period.** [4]

A conical pendulum is a weight (or bob) fixed on the end of a string (or rod) suspended from a pivot. Its construction is similar to an ordinary simple pendulum; however, instead of rocking back and forth, the bob of a conical pendulum moves at a constant speed in a horizontal circle with the string tracing out a cone.

Consider an object of mass m tied to a string of length *l* is moving in a horizontal circle of radius r as shown in figure. The tension T in the string has two components Tcosθ and Tsinθ. The component Tcosθ balances the weight of the object and Tsinθ provides the required centripetal force.

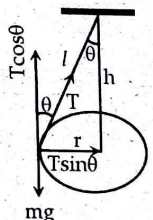
$T\cos\theta = mg$ ...(i)

$T\sin\theta = \frac{mv^2}{r}$ ... (ii)

Dividing (ii) by (i)

$\tan\theta = \frac{v^2}{rg}$

or, $v = \sqrt{rg\tan\theta}$



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Time period of conical pendulum: The time taken by the conical pendulum to complete one complete revolution around the horizontal circle is its time period. We have, $t = \frac{2\pi}{\omega}$

or, $t = \frac{2\pi r}{\omega r}$

or, $t = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}}$, $(v = \sqrt{r\,g\tan\theta})$

or, $t = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

From figure, $\sin\theta = \frac{r}{l}$

or, $r = l\sin\theta$

$\therefore t = 2\pi\sqrt{\frac{l\sin\theta}{g\cdot\frac{\sin\theta}{\cos\theta}}}$

$t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

which is expression for time period of conical pendulum.

22. 2075 Set B Q.No. 5d **What is the difference between the conceptual design of a conical pendulum with respect to that of a simple pendulum? Derive relations of time period and frequency of a conical pendulum.**

First Part: Please refer to 2071 Set D Q.No. 1e

Second Part: Please refer to 2075 Set A Q.No. 5b

23. 2074 Set A Q.No. 5b **What is a conical pendulum? Derive an expression for its time period.**

Please refer to 2075 Set A Q.No. 5b

24. 2073 Set D Q.No. 5b **Describe the motion of a conical pendulum and derive an expression for its time period.**

Please refer to 2075 Set A Q.No. 5b

25. 2072 Set C Q.No. 5a **Define centripetal and centrifugal forces. Derive an expression for the force acting on a body moving with uniform speed along a circular path.**

Centripetal force: The force acting on a body moving in a circular path towards its centre is called centripetal force.

Centrifugal forces: The outward force acting on a body moving in a circular path is called centrifugal force.

Second Part: Please refer to 2075 GIE Q.No. 5b

26. 2071 Set D Q.No. 5 a **What do you mean by the banking of a curved path? Derive an expression for the banking angle.**

Please refer to 2076 Set C Q.No. 5b

27. 2070 Supp (Set A) Q.No. 5 b **Define centripetal force. Show that the acceleration of a body moving in a circular path of radius 'r' with uniform speed v is v^2/r and is directed towards the centre of circular.**

Please refer to 2075 GIE Q.No. 5b

28. 2069 Old (Set B) Q. No. 3a **Define centripetal force. Calculate the force acting on a body moving with uniform speed along circular path.** [1+

Please refer to 2075 GIE Q.No. 5b

29. 2068 Q.No. 5 c **What is conical pendulum? Show that the period of oscillation of this pendulum is given by** $t = 2\pi\sqrt{\frac{1\cos\theta}{g}}$

Please refer to 2075 Set A Q.No. 5b

30. 2068 Old Q.No. 3 a **Define centripetal force. Discuss the motion of a car moving round in a circular banked track.** [1+3=4]

Please refer to 2076 Set C Q.No. 5b

31. 2066 Q. No. 5 c **What is conical pendulum? Show that the period of oscillation of this pendulum is given by** $t = 2\pi\sqrt{l\cos\theta / g}$ **where symbols have their usual meanings.** [4]

Please refer to 2075 Set A Q.No. 5b

32. 2066 Old Q. No. 3 a **Define the centripetal force. Derive an expression for the force acting on a body moving with uniform speed along a circular path.** [1+3]

Please refer to 2075 GIE Q.No. 5b

33. 2062 Q.No. 3 OR **What is centripetal force? Derive it in the case of motion of a bi-cycle on a curved road.**

Centripetal force: The force acting on a body moving in a circular path toward its centre is called centripetal force.

Motion of bicycle in a curved road: A cyclist to take a safe turn in curved road leans inward to provide necessary centripetal force.

Let us consider a cyclist is moving in a circular path of radius r with uniform speed v. Let θ be the angle of inclination of cycle with vertical.

Let m be the mass of the cyclist and cycle, then the total weight mg acts vertically downward from the centre of gravity. The reaction R offered by ground on the system has two components. They are (i) R cosθ, vertical component of R which balance the weight of the system and (ii) R sinθ, horizontal component of R which provides necessary centripetal force $\frac{mv^2}{r}$.

So, $mg = R\cos\theta$ ... (i)

[Vertical component]

and $\frac{mv^2}{r} = R\sin\theta$ ... (ii)

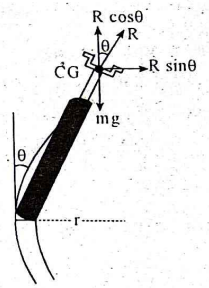
[Horizontal component]

Dividing equation (ii) by (i), we get

$\frac{mv^2/r}{mg} = \frac{R\sin\theta}{R\cos\theta}$

$\Rightarrow \tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ ... (iii)



Equation (ii) represents the centripetal force acting on cyclist and cycle and equation (iii) represents angle of inclination of bicycle with the vertical.

34. 2061 Q.No. 2 a **Explain what is meant by angular velocity. Show that the acceleration of a body moving in a circular path of radius r with uniform speed v is v^2/r.** [1+3]

Angular Velocity: The time rate of change of angular displacement is called angular velocity. It is denoted by ω and given by

$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{d\theta}{dt}$

Angular velocity is measured in radian per second (rad. s^{-1}).

Second part: Please refer to 2075 GIE Q.No. 5b

35. 2058 Q.No. 2 OR **Define centripetal acceleration. Derive an expression for it.** [1+3]

An acceleration acting on a body moving in a circle towards its centre is called centripteal acceleration.

Second part: Please refer to 2075 GIE Q.No. 5b

36. 2056 Q.No. 2 OR **Define centripetal force. Calculate the force acting on a body moving with a uniform speed along a circular path.**

Please refer to 2075 GIE Q.No. 5b

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37. 2053 Q.No. 2 b **Derive an expression for the force required to make a particle of mass m move in a circle of radius r with uniform angular velocity ω.**

Please refer to 2075 GIE Q.No. 5b

38. 2051 Q.No. 2 **Why a force is necessary to keep a body moving with uniform speed in a circular motion? Deduce its expression.**

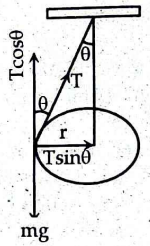
Please refer to 2075 GIE Q.No. 5b

Numerical Problems

39. 2076 Set B Q.No. 9b **A bob of mass 200 gram is whirled in a horizontal circle of radius 50cm by a string inclined at 30° to the vertical. Calculate the tension in the string and the speed of the bob in the horizontal circle.**

Solution

Given,
Mass of bob (m) = 200 gm = 0.2 kg
Radius of circle (r) = 50 cm = 0.5 m
Inclination (θ) = 30°
Tension on the string (T) = ?
Speed of bob (v) = ?
We have,



$T \sin\theta = \frac{mv^2}{r}$ ... (i)

$T \cos\theta = mg$ ... (ii)

$\tan\theta = \frac{v^2}{rg}$ ... (iii)

From equation (ii)

$T \cos\theta = mg$

or, $T \cos 30° = 0.2 \times 10$

or, $T = \frac{0.2 \times 10}{\cos 30}$

$= 2.3$ N

From equation (iii)

$\tan\theta = \frac{v^2}{rg}$

or, $v^2 = rg \tan\theta$

or, $v = \sqrt{rg \tan\theta}$

$= \sqrt{0.5 \times 10 \times \tan 30°}$

$= 1.7$ m/sec

40. 2073 Supp Q.No. 9d **An object of mass 0.5 kg is rotated in a horizontal circle by a string in 1 m long. The maximum tension in the string before it breaks is 50 N. What is the greatest number of revolutions per second of the object?**

Solution

Given,
Mass of the object (m) = 0.5 kg
Radius of horizontal circle (r) = 1 m
Tension in the string (T) = 50 N
Maximum number of revolutions (f) = ?
We know, for horizontal circle,

$T = m\omega^2 r = m(2\pi f)^2 . r$

or, $f^2 = \frac{T}{4\pi^2 . m . r}$

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or, $f = \frac{1}{2\pi}\sqrt{\frac{T}{m.r}}$

or, $f = \frac{1}{2\pi}\sqrt{\frac{50}{0.5 \times 1}}$

$\therefore f = 1.6$ rev/sec

41. 2072 Set D Q.No. 9c **A stone with mass 0.8 kg is attached to one end of a string 0.9 m long. The string will break if its tension exceeds 600 N. The stone is whirled in a horizontal circle, the other end of the string remains fixed. Find the maximum speed, the stone can attain without breaking the string.** [4]

Solution

Given,
Mass of stone (m) = 0.8 kg
Length of string (r) = 0.9 m
Maximum tension (T_{max}) = 600 N
Maximum speed (v_{max}) = ?
For maximum speed at which the string will not break,

$T_{max} = \frac{mv^2_{max}}{r}$

or, $600 = \frac{0.8}{0.9} v^2_{max}$

or, $v^2_{max} = 675$

or, $v_{max} = 25.98$ m/sec

42. 2072 Set E Q.No. 9a **An object of mass 4 kg is whirled round a vertical circle of radius 1 m with a constant speed of 3 ms^{-1}. Calculate the maximum tension in the string.** [4]

Solution

Given,
Mass of object (m) = 4 kg
Radius of circle (r) = 1 m
Speed of object (v) = 3 m/sec
Maximum tension (T_{max}) = ?
We have,

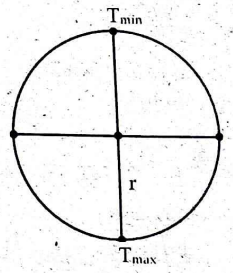
$T_{max} = \frac{mv^2}{r} + mg$

$= \frac{4 \times 3^3}{1} + 4 \times 10 = 76$ N

43. 2071 Set C Q.No. 9 b **A mass of 0.2 Kg is rotated by a string at a constant speed in a vertical circle of radius 1 m. If the minimum tension in the string is 3 N, calculate the magnitude of the speed and the maximum tension in the string.** [4]

Solution

Given,
Mass of stone (m) = 0.2 kg
Radius of circle (r) = 1m
Minimum Tension on string (T_{min}) = 3 N
Magnitude of speed (v) = ?
Maximum tension (T_{max}) = ?
We have,



$T_{max} = \frac{mv^2}{r} + mg$ ... (i)

$T_{min} = \frac{mv^2}{r} - mg$ ... (ii)

From equation (ii)

$$3 = \frac{0.2 \times v^2}{1} - 0.2 \times 10$$

$$\text{or, } 5 = 0.2\, v^2$$

$$\text{or, } v^2 = \frac{5}{0.2} = 25$$

$$\text{or, } v = 5 \text{ m/sec}$$

From equation (i)

$$T_{max} = \frac{0.2 \times 5^2}{1} + 0.2 \times 10$$

$$= 0.2 \times 25 + 2 = 7 \text{ N}$$

44. 2071 Set D Q.No. 9 c At what angle should a circular road be banked so that a car running at 50 Km/hr be s to go round the circular turn of 200 m radius?

Solution

Given,

Speed of car (v) = 50 km/hr = $\frac{50 \times 1000}{60 \times 60}$ m/sec

Radius of circle (r) = 200m

Banking angle (θ) = ?

We know,

$$\tan\theta = \frac{v^2}{rg}$$

$$\text{or, } \tan\theta = \frac{\left(\frac{50 \times 1000}{60 \times 60}\right)^2}{200 \times 10} = 0.0965$$

$$\text{or, } \theta = \tan^{-1}(0.0965) = 5.5°$$

45. 2070 Set C Q.No. 9 b A mass of 1 kg is attached to the lower end of a string 1 m long whose upper en fixed. The mass is made to rotate in a horizontal circle of radius 60 cm. If the circular speed of the mas constant, find the tension in the string and the period of motion.

Solution

Given,

Mass of object (m) = 1kg

Length of string (l) = 1 m

Radius of circle (r) = 60 cm = 0.6 m

Tension on the string (T) = ?

Period of motion (t) = ?

From figure,

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg} \text{ and}$$

$$\sin\theta = \frac{r}{l} = \frac{0.6}{1} = 0.6$$

$$\text{or, } \theta = \sin^{-1}(0.6) = 36.87°$$

So, from relation

$$T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta} = \frac{1 \times 10}{\cos 36.87} = 12.5 \text{ N}$$

Again, from relation

$$t = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{1 \times \cos 36.87}{10}} = 1.78 \text{ sec.}$$

46. 2070 Set D Q.No. 9 b A certain string breaks when a weight of 25N acts on it. A mass of 500 gram is attached to one end of the string of 1 m long and is rotated in a horizontal circle. Find the greatest number of revolutions per minute which can be made without breaking the string. [4]

Solution

Given,

Maximum tension (T_{max}) = 25 N

Mass of object (m) = 500 g = 0.5 kg

Radius of circle (r) = 1m

Frequency (f) = ?

We have,

$T_{max} = m\omega^2 r$ = centripetal force

$$\text{or, } T_{max} = m(2\pi f)^2 \times r$$

$$\text{or, } 25 = 0.5 \times 4\pi^2 \times f^2 \times 1$$

$$\text{or, } f^2 = \frac{25}{0.5 \times 4 \times \pi^2 \times 1}$$

$$\text{or, } f^2 = 1.2666 \text{ rev/sec.}$$

$$\therefore f = 1.125 \text{ rev/sec} = 1.125 \times 60 \text{ rpm} = 67.5 \text{ rev/min}$$

47. 2059 Q.No. 3 b An object of mass 8.0 kg is whirled round in a vertical circle of radius 2m with a constant speed of 6ms^{-1}. Calculate the maximum and the minimum tensions in the string.

Solution

Given,

Mass of object (m) = 8.0 kg

Radius (r) = 2m

Speed (v) = 6 ms^{-1}

Tension T is maximum when the mass is at lower end (i.e. at end B)

$$\therefore T_{max} = mg + \frac{mv^2}{r} = 8 \times 10 + \frac{8 \times 6^2}{2} = 224 \text{ N}$$

Tension T is minimum when the mass is at upper end (i.e. at end A)

$$\therefore T_{min} = \frac{mv^2}{r} - mg = \frac{8 \times 6^2}{2} - 8 \times 10 = 64\text{N}$$

48. 2052 Q.No. 3 A coin placed on a disc rotates with speed of 33⅓ rev. min^{-1} provided that the coin is not more than 10 cm from the axis. Calculate the coefficient of static friction between the coin and the disc.

Solution

Given,

Frequency of rotation (f) = $33\frac{1}{3}$ rev/min = $\frac{100}{3 \times 60}$ rev/sec

Radius (r) = 10cm = 0.1m

Coefficient of static friction (μ_s) = ?

When the coin just sustain on the disc, the frictional force is equal to the centripetal force, i.e.,

$$\frac{mv^2}{r} = f_s$$

$$\text{or, } m\omega^2 r = \mu_s R$$

$$\text{or, } \mu_s = \frac{m\omega^2 r}{mg} \quad [\because R = mg]$$

$$= \frac{(2\pi f)^2 r}{g} = \frac{4\pi^2\left(\frac{100}{3 \times 60}\right)^2 \cdot 0.1}{10} = 0.122$$

□□□

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Chapter 7: Gravitation

Short Answer Questions

1. 2076 Set B Q.No. 1d A person sitting in an artificial satellite of the earth feels weightlessness, but a person standing on moon has weight through the moon is a satellite of the earth. Why? [2]

An astronaut inside a small space ship orbiting around the earth does not experience any gravity. But an astronaut on the moon which is also orbiting around the earth does experience gravity because force of gravity is given as,

$$F = \frac{GM}{R^2},$$

The mass of spaceship is very small compared to the moon. Due to this region, the force of gravity of spaceship is negligible compared to that of moon.

2. 2076 Set C Q.No. 1d If the earth suddenly stops rotating about its axis, what would be the effect on 'g'? Would this effect be the same at all places? [2]

We know that, acceleration due to gravity by rotation of earth is

$$g' = g - g R\omega^2 \cos^2\theta \quad \ldots (i)$$

where θ is the latitude at the given place on the earth surface, R radius of earth and ω is angular speed of rotation of earth.

The value of g' decreases due to rotation of earth. If earth stops rotating ($\omega = 0$), it is clear from the equation (i) that $g' = g$. Thus, value of g' increases to g. This effect is maximum at equator and minimum at pole because at pole the value of g is not changed.

3. 2075 GIE Q.No. 1c Suppose the radius of the earth were to shrink by 2%, its mass remaining the same, would the acceleration due to gravity 'g' increase or decrease and by what percent? [2]

Solution

Let R be the radius of earth, when it is shrink by 2% then new radius R' is given as

$$R' = R - 2\% \text{ of } R$$

$$= R - \frac{2R}{100} = R - \frac{R}{50} = \frac{49}{50}R$$

The acceleration due to gravity of earth is,

$$g = \frac{GM}{R^2}, \text{ where } M = \text{mass of earth}$$

G = Universal gravitational constant

When R is shrink by 2%, then new g' is

$$g' = \frac{GM}{R'^2} = \frac{GM}{\left(\frac{49}{50}R\right)^2} = \frac{GM}{R^2} \times \left(\frac{50}{49}\right)^2$$

$$g' = g \times 1.041$$

$$\% \text{ change} = \frac{g' - g}{g} \times 100\% = 4.1\%$$

∴ The acceleration due to gravity is changed by 4.1%.

4. 2075 GIE Q.No. 1d An astronaut releases a spoon out of a satellite in the space. Will the spoon fall on the earth? [2]

No, the spoon will not fall on the earth when releasing from a satellite in the space. The gravitational force provided by earth is spent to produce centripetal force and no net force acting on it. So, it moves around the orbit of satellite with uniform speed.

5. 2075 Set A Q.No. 1f How does the weight of a body vary on going from earth to moon? Would its mass change? [2]

The weight of the body ($W = mg$) depends upon the acceleration due to gravity (g). The acceleration due to gravity is maximum at earth surface, decreases as going outer space, minimum at null point and less significant at moon surface. The weight of the body is also maximum at earth surface, minimum at null point and less significant when reaches to moon surface. The mass of body does not change wherever it has taken in moon or earth or space.

6. 2074 Set A Q.No. 1f Distinguish between gravitational potential and gravitational field strength. [2]

Differences between gravitational potential & field strength are given below.

Gravitational Potential	Gravitational field strength
1. It is defined as the amount of work done in bringing a body of unit mass from infinity to a point inside gravitational field.	1. It is defined as the force experienced by a body of unit mass inside the gravitational field.
2. It is given as $V_g = \frac{W}{m}$.	2. It is given as $E = \frac{F}{m}$
3. Its unit and dimensions are J/kg and $M^0L^2T^{-2}$.	3. Its unit and dimensions are N/kg or m/s² and $M^0L^1T^{-2}$
4. It is scalar quantity.	4. It is vector quantity.

7. 2074 Set B Q.No. 1g What is a black hole? Explain its characteristic nature. [2].

Black hole is the last stage of death of star. It has following characteristics;

- Its radius is less than critical radius.
- Its specific gravity is very high.
- Its escape velocity is greater than velocity of light.
- No light escape from its surface and hence, its surface is always black.

8. 2073 Set C Q.No. 1d An astronaut inside a small space ship orbiting around the earth does not experience any gravity. But an astronaut on the moon which is also orbiting around the earth does experience gravity. Why? [2]

Please refer to 2076 Set B Q.No. 1d

9. 2073 Set D Q.No. 1d If the earth suddenly stops rotating about its axis, what would be the effect on acceleration due to gravity at

i. the equator, ii. the poles? [2].

The acceleration due to gravity due to rotation of the earth is,

$$g' = g\left(1 - \frac{R\omega^2\cos^2\phi}{g}\right) \quad \ldots (i)$$

When earth stops to rotate, then, $\omega = \frac{2\pi}{T} = 0$

Then, $g' = g$.

∴ The acceleration due to gravity at equator increases and reaches to the value at pole and at pole no change in gravity.

10. 2072 Set C Q.No. 1e An astronaut in a space capsule orbiting the earth experiences weightlessness. Why? [2]

The centripetal force necessary to keep the space-ship in an orbit around the earth is provided by the gravitational force between the earth and the spaceship. Hence, the only force of gravity that an astronaut in an orbiting space-ship experiences is that which is due to the gravitational force exerted by the space ship. But the space-ship is small, this force is negligibly small and the astronaut doesn't experience any gravity. Hence, the astronaut experiences a feeling of weightlessness.

11. 2072 Set D Q.No. 1c At what condition, does a body become weightless at the equator? [2]

In order that a body placed at the equator become weightless, the effective acceleration due to gravity should be zero i.e. $g_r = 0$.

$$\text{or, } g - \omega^2 R = 0$$

$$\text{or, } g = \omega^2 R$$

$$\text{or, } \omega = \sqrt{\frac{g}{R}} = \frac{9.8}{6400000}$$

$$= 1.237 \times 10^{-3} \text{ rad/ sec.}$$

Again,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.237 \times 10^{-3}} = 5077.6 \text{ sec} = 1.41 \text{ hrs}$$

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So, when earth rotating in such a way that the time period of rotation is 1.41 hrs, bodies becom weightless at the equator.

12. 2072 Set E Q.No. 1c Write the unit and dimension of gravitational field with its definition.

The space around heavenly bodies to which its gravity can be experienced is called gravitation field. This is just area or region, so, it is measure is area or m^2 and dimension is L^2.

13. 2071 Set C Q.No. 1 e 2071 Set D Q.No. 1 g What is a black hole? Write down its characteristic features.

Please refer to 2074 Set B Q.No. 1g

14. 2070 Supp (Set A) Q.No. 1 e If heavier bodies are attracted more strongly by the earth why do they not fa faster than lighter ones? (neglect air resistance)

The acceleration due to gravity of earth is given by $g = \frac{GM}{R^2}$, where G is gravitational constant, M mass of earth and R is radius of earth. This shows that acceleration due to gravity depends upon t mass and radius of earth but not on the mass of falling object i.e. g is same for all falling object neglecting air resistance. Hence, heavier bodies even attracted more strongly by earth, they do not fa faster than lighter bodies.

15. 2070 Set C Q.No. 1 g What will happen to the value of acceleration due to gravity if the earth stops rotati about its axis?

Please refer to 2076 Set C Q.No. 1d

16. 2069 Supp Q.No. 1c Suppose the radius of the earth were to shrink by 2%, its mass remaining the sa calculate the percent change in the acceleration due to gravity 'g'.

Please refer to 2075 GIE Q.No. 1c

17. 2069 Set A Q. No. 1b If the sun some how collapsed to form a black-hole, what effect would this event h on the orbit of the earth?

There would be no effect on the orbit of he earth if the sun somehow collapsed fo fomr a black hol the sun collapsed into a black hole, it would have the same mass and but much smaller radius. gravitational force of attraction between earth and sun is given by $F = \frac{GM\,m}{r^2}$, where M be the m of the sun, m be the mass of the earth and r be the distance between the sun and earth. This sho that the gravitational attraction of the sun on the earth would not be affected. Hence, there would no effect on the orbit of the earth.

18. 2068 Q.No. 1 e If the force of gravity acts on all bodies is proportion to their masses, why does not a hea body fall faster than a light body?

Force of gravity acts on all bodies is proportional to their masses but the acceleration due to grav does not depend upon their masses and only depends upon the mass and radius of the earth $g = \frac{GM}{R^2}$. Hence, all the bodies fall at the same acceleration.

19. 2067 Q.No. 1 d 2066 Q. No. 1 c If the sun somehow collapsed to form a black hole, what effect would th event have on the orbit of the earth?

Please refer to 2069 Set A Q. No. 1b

20. 2064 Q.No. 1 f If earth suddenly stops rotating about its axis, what would be the effect on g?

Please refer to 2076 Set C Q.No. 1d

21. 2062 Q.No. 1 d What do you mean by geo-stationary satellite? Explain.

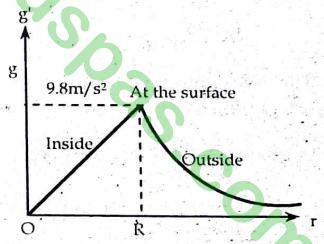
A satellite put in a circular orbit concentric and coplanar with the equatorial plane of the earth and a height, such that its period of revolution is just 24 hours is called geostationary satellite. T geostationary satellite moves around earth at distance about 36,000 km from the surface of earth.

22. 2058 Q.No. 1 d Explain why the moon has no atmosphere?

Due to the low value of acceleration due to gravity, the escape velocity on the moon is quite sm about 2 km/s. The root mean square velocity of different gases is greater than the escape velocity the moon. As a result, all the gases molecules escaped out from moon with the passage of tim Hence, the moon has no atmosphere.

23. 2055 Q.No. 1 b How does 'g' at a point vary with the distance from the centre of the earth? Where is the highest value of g? Explain.

The variation of acceleration due to gravity at depth d inside the surface and at a height h above of the earth of radius R is given by $g' = g\left(1-\frac{d}{R}\right)$ and $g' = \frac{R^2}{g(R+h)^2}$, where g be the acceleration due to gravity at the surface of earth. These two formulae show that the acceleration due to gravity is zero at the centre of earth, linearly increasing from centre to surface of the earth, maximum at the surface and decreasing exponentially outside the surface of earth as shown in figure.



24. 2054 Q.No. 1 d Why an astronaut in a space capsule orbiting the earth experiences a feeling of weightlessness?

Please refer to 2072 Set C Q.No. 1e

25. 2053 Q.No. 1 d The weight of a body is less inside the earth than on the surface. Explain.

The value of acceleration due to gravity at a depth d from the earth's surface is given by

$g' = g\left(1-\frac{d}{R}\right)$ ..... (i) where g is acceleration due to gravity at earth's surface, R is the radius of the earth.

Multiplying equation (i) by m. we get

$mg' = mg\left(1-\frac{d}{R}\right)$

or, $mg' = mg - mg\left(\frac{d}{R}\right)$

$\therefore mg' < mg \qquad \left[\because \left(\frac{d}{R}\right) \text{ is positive quantity}\right]$

Weight of a body of mass m on surface > weight of the body at depth d below at the earth's surface. Hence, the weight of a body is less inside the earth than on the surface.

26. 2052 Q.No. 1 d Obtain an expression for gravitational potential energy and establish its dimension.

Gravitational potential energy of an object at a point in the gravitational field is defined as the amount of work done to bring an object from infinity to that point without gravitational acceleration. The gravitational potential energy at any point in space at a distance r is given by

$U = -\frac{GM\,m}{r}$

Since it is energy, so, its dimension is same as energy i.e. $[ML^2T^{-2}]$.

27. 2051 Q.No. 1 v What will happen to the value of 'g' if the earth stops rotating?

Please refer to 2076 Set C Q.No. 1d

Long Answer Questions

28. 2076 Set B Q.No. 5a What is escape velocity? Derive an expression for it on the surface of the earth.

Escape velocity: The minimum velocity with which a body must be thrown upwards in order that may just escape the gravitational pull of the planet is called escape velocity. If a body is thrown b velocity equal to escape velocity, it never comes back to earth.

Expression for escape velocity: Let M and R be the mass and radius of the earth. Let 'm' be the ma of the body thrown vertically upwards with escape velocity 'v' from the surface of earth. Let the bo be at a distance 'x' from the centre of earth at an instant then the force of attraction between these tw masses is

$F = \frac{GmM}{x^2}$

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If we displace this mass through small displacement dx, small work done dW is given by,

$dW = F \times dx$

$= \frac{GmM}{x^2} \times dx$

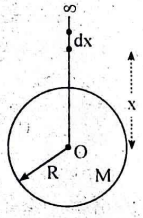
The total amount of workdone in taking the object from the surface to infinity is calculated by integrating this equation from limit x = R to x = ∞, we get

$$W_{R\infty} = \int_R^\infty \frac{GmM}{x^2} \times dx = GmM \int_R^\infty x^{-2}\, dx = GmM\left[\frac{x^{-1}}{-1}\right]_R^\infty = -GmM\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$W_{R\infty} = \frac{GmM}{R} \quad \ldots \text{(i)}$$

The kinetic energy by which the body is projected so that it just leaves the gravitational pull of the earth is given by,

$$K.E. = \frac{1}{2} mv_e^2 \quad \ldots \text{(ii)},$$



where v_e is the escape velocity of earth.

Equating equation (i) & (ii), we get

$$\text{i.e. } \frac{1}{2} mv_e^2 = \frac{GmM}{R} = m\frac{GM}{R} \quad [\because g = \frac{GM}{R^2}]$$

$$\Rightarrow v_e^2 = \frac{2gR^2}{R} = 2gR$$

$$\therefore v_e = \sqrt{2gR}$$

This is the required expression of the escape velocity.

Taking, g = 9.8 ms^{-2} and R = 6400 km, we get the escape velocity for the earth is 11.2 km/sec.

29. 2075 GIE Q.No. 5c What is escape velocity? Show that the velocity of escape of a body is $\sqrt{2}$ times the orbital velocity?

Please refer to 2076 Set B Q.No. 5a

30. 2074 Set B Q.No. 5a Explain the meaning of escape velocity based on the concept of gravitational potential. Hence derive expression for escape velocity of a body thrown from the surface of the earth.

Please refer to 2076 Set B Q.No. 5a

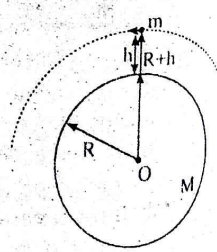
31. 2073 Set C Q.No. 5b What is escape velocity? Derive an expression for the escape velocity on the surface of the earth.

Please refer to 2076 Set B Q.No. 5a

32. 2072 Set D Q.No. 5b What is a geostationary satellite? Derive an expression for the time period of the satellite revolving around the earth?

Geostationary satellite: The artificial satellite that always stays over the same place on the earth while the earth rotates about its axis is called geostationary satellite. The orbit of the geostationary satellite is called parking orbit. The height of the parking orbit (h) from the surface of earth is nearly 36,000 km.

Orbital velocity and time period of the satellite: A heavenly body that revolves round the earth in a circular orbit is called satellite of earth. The orbital velocity of a satellite is the velocity required to be given to the satellite in order put it in an orbit around the earth.



Let M and R be the mass and radius of the earth respectively. In addition, m is the mass of satellite moving in the orbit of radius r = R + h, where h is the height of the satellite above the earth's surface.

In order to keep the satellite moving in the orbit of constant radius r,

Gravitational force = Centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

where v is the velocity of satellite in the orbit.

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} \quad [\because g = \frac{GM}{R^2}]$$

$$\therefore v = R\sqrt{\frac{g}{R+h}}$$

This is the required expression for orbital velocity.

Time period: The time period T of a satellite is the total time taken by the satellite to complete one revolution around the earth is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{gR^2}} = 2\pi\sqrt{\frac{r^3}{gR^2}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\therefore T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

This is the required expression for time period of satellite revolving around the earth.

33. 2072 Set E Q.No. 5a Derive an expression for the time period and height of artificial satellite from the surface of the earth. [4]

Please refer to 2072 Set D Q.No. 5b

34. 2071 Supp Q.No. 5c Obtain an expression for variation of 'g' with rotation of earth. [4]

Variation of g due to rotation of the earth:

Let us consider an object of mass m at a point P on the earth surface at latitude of θ. When the earth is not rotating, its weight, W = mg acts along the radius of the earth toward the centre.

When the earth rotates with angular velocity ω on its axis, the object at P will also rotate about the centre C of a circle of radius r where r = R cos θ and R is the radius of the earth. So the object will experience centrifugal force,

$F_c = m\omega^2 r = m\omega^2 R\cos\theta$

and the object is under the action of two forces centrifugal force and its weight.

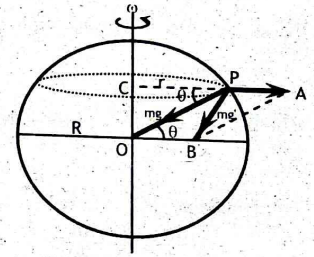


Figure: Variation of g' due to the rotation of the earth.

The resultant of two forces is shown by PB in Fig. which is the apparent weight mg' of the object.

By using parallelogram law of vector addition,

$PB^2 = PO^2 + PA^2 + 2PO \times PA\cos(180 - \theta)$

or, $m^2g'^2 = (mg)^2 + (m R\omega^2\cos\theta)^2 + 2 \times mg \times m R\omega^2\cos\theta \times (-\cos\theta)$

or, $g' = g\left[1 + \frac{R^2\omega^4\cos^2\theta}{g^2} - \frac{2R\omega^2\cos^2\theta}{g}\right]^{1/2}$

Since $\frac{R^2\omega^2}{g^2}$ is a small quantity, the terms containing the factor $\frac{R^2\omega^4}{g^2}$ can be neglected.

Hence, $g' = g\left[1 - \frac{2R\omega^2\cos^2\theta}{g}\right]^{1/2}$

Expanding the above equation by using binomial theorem

$g' = g\left[1 - \frac{1}{2} \times \frac{2R\omega^2\cos^2\theta}{g} + \ldots\right]$

Higher powers of $\frac{R\omega^2\cos^2\theta}{g}$ can be neglected and

$g' = g\left[1 - \frac{R\omega^2\cos^2\theta}{g}\right]$

$\therefore g' = g - gR\omega^2\cos^2\theta$

This is required expression for the variation of acceleration due to gravity due to rotation of earth. The equation shows that due to the rotation of the earth, the acceleration due to gravity decreases.

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35. 2071 Set C Q.No. 5 c **Explain the concept of geostationary satellite. Find an expression for the total energy of the moon revolving around the earth.** [4]

Geostationary Satellite: The artificial satellite that always stays over the same place of the earth while the earth rotates about its axis is called geostationary satellite. This is possible because the time period of rotation of earth equals to the time period of revolution of geostationary satellite.

A satellite revolving round the earth has both the kinetic energy and potential energy. Suppose a satellite of mass m is at a height of h from the earth's surface of mass M and radius R. The radius of the orbit of the satellite is R + h. The gravitation force between the earth and the satellite is

$$F_g = \frac{GMm}{(R+h)^2}$$

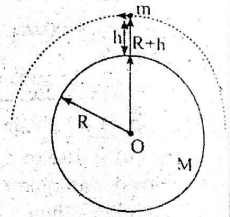
which provides the required centripetal force to the satellite. If v_0 is the orbital velocity of the satellite then centripetal force is

$$F_c = \frac{m v_0^2}{(R+h)}$$

As $F_g = F_c$,

$$\frac{GMm}{(R+h)^2} = \frac{m v_0^2}{(R+h)}$$

or, $$v_0^2 = \frac{GM}{R+h}$$



$$\therefore \text{Kinetic energy} = \frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{GM}{(R+h)}$$

$$\text{K.E.} = \frac{GMm}{2(R+h)}$$

The potential energy of the satellite at the height h above the earth's surface is

$$\text{P.E.} = -\frac{GMm}{(R+h)}$$

$$\therefore \text{Total energy} = \text{P.E.} + \text{K.E.}$$

$$E = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$E = -\frac{GMm}{2(R+h)}$$

The negative sign in above equation means that the satellite is bounded to the earth i.e. energy has to supply from outside to free a satellite from its orbit.

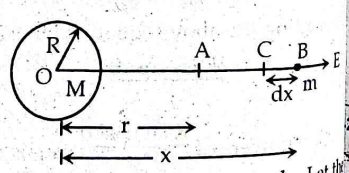
36. 2070 Supp (Set A) Q.No. 5 c **What is escape velocity? Show that the escape velocity of a body is $\sqrt{2R}$ Where symbols have usual meanings?**

Please refer to 2076 Set B Q.No. 5a

37. 2070 Supp (Set B) Q.No. 5 b **Define gravitational potential energy and derive its expression.**

Gravitational potential energy: Gravitational potential energy of an object at a point in the gravitational field is defined as the amount of work done to bring an object from infinity to that point without gravitational acceleration.

Let earth be a perfect sphere of radius R and mass M. The mass of the earth can be supposed to be concentrated at its centre O. Let us calculate the gravitational potential energy of the body of mass m at point A in the gravitational field, where OA = r as shown in Fig.



Take two points B and C on the line of OE produced onwards such that OB = x and BC = dx. Let the body is placed at point B.

Then, the gravitational force of attraction on the body at point B will be

$$F = \frac{GMm}{x^2}$$

When the body is moved through a very small displacement BC = dx, then the small amount of work has to be done which is given by

$$dW = F\,dx = \frac{GMm}{x^2}dx \quad \ldots (i)$$

Total work done in bringing the body from infinity to point A is calculated by intergrating equation (i) from ∞ to r, as,

$$W = \int_{\infty}^{r}\frac{GMm}{x^2}dx = GMm\int_{\infty}^{r}x^{-2}dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$\therefore W = -\frac{GMm}{r} \quad \ldots (ii)$$

Since, this work done is stored in the body as its gravitational potential energy U, the gravitational potential energy of the body at point A in the gravitational field will be

$$U = -\frac{GMm}{r} \quad \ldots (iii)$$

Since it is energy, so, its dimension is same as energy i.e. $[ML^2T^{-2}]$.

38. 2070 Set C Q.No. 5 b **Define escape velocity. Find an expression for the escape velocity from the surface of the earth.** [4]

Please refer to 2076 Set B Q.No. 5a

39. 2070 Set D Q.No. 5 b **What is a gravitational potential energy? Obtain an expression for the gravitational potential energy of a body at a distance r from the centre of the earth.** [4]

Please refer to 2070 Supp (Set B) Q.No. 5 b

40. 2069 (Set A) Q. No. 5b **What is geo-stationary satellite? Obtain an expression for the total energy of a satellite orbitting round the earth?** [4]

Please refer to 2071 Set C Q.No. 5 c

41. 2069 (Set B) Q. No. 5c **What do you mean by the terms black hole and event horizon? Obtain an expression for the schwarzschild radius.** [4]

Black Holes and Event Horizon: When the hydrogen gas at the centre of star which acts as the fuel of the star to produce the light and heat energy is finished, there is no outward force. Consequently, the radius of the star decreases and density becomes high remaining same mass. The escape velocity of any particle on the star depends on the mass and radius of the star as $v = \sqrt{\frac{2GM}{R}}$. If the radius decreases the escape velocity increases. At a certain radius, the escape velocity becomes equal to the velocity of light. This radius is called critical radius or Schwarzschild radius R_S. This is given as

$$c = \sqrt{\frac{2GM}{R_S}}$$

or, $$R_S = \frac{2GM}{c^2}$$

When the radius of star becomes less than the Schwarzschild radius R_s, the escape velocity becomes more than the velocity of light and nothing even light can escape from the surface of star. The star becomes dark or black and called black hole which has very large density and gravity. The surface of sphere with radius R_S surrounding a black hole is called event horizon.

42. 2069 (Set A) Old Q. No. 2a or **What is escape velocity? Derive an expression for it.** [1+4]

Please refer to 2076 Set B Q.No. 5a

43. 2069 Old (Set B) Q. No. 3a OR **What do you mean by parking orbit? Derive an expression for the orbital velocity and hence, find time period of the satellite revolving around the earth.** [1+3]

Please refer to 2072 Set D Q.No. 5b

44. 2068 Q.No. 5 a **What is escape velocity? Derive its expression on the surface of the earth.** [4]

Please refer to 2076 Set B Q.No. 5a

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45. 2068 Old Q.No. 3 a or Derive an expression for the variation of acceleration due to gravity of the earth with the altitude and explain its meaning.

Acceleration due to gravity (g): It is the accleration with which a body falls freely toward the centre of earth due to its gravity. It is denoted by 'g' and its average value on earth's surface is 9.8 m/s². It varies with altitude.

Variation of 'g' with altitude above earth surface: Let us consider g and g' are the acceleration due to gravity on the surface of earth and at the height 'h' above the surface of earth respectively. Then,

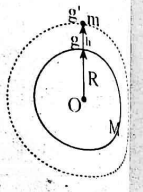
Acceleration due to gravity at the surface of earth,

$$g = \frac{GM}{R^2} \quad \ldots (i)$$

where M is mass of earth and R be the radius of earth.

Similarly, acceleration due to gravity at the height 'h' from the surface of earth is given by,

$$g' = \frac{GM}{(R+h)^2} \quad \ldots (ii)$$



Dividing equation (ii) by (i), we get

$$\frac{g'}{g} = \frac{GM/(R+h)^2}{GM/R^2} = \frac{R^2}{(R+h)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

$$\frac{g'}{g} = \left[1 - \frac{2h}{R} + \ldots\right] \quad \left[\because \left(1-\frac{x}{R}\right)^{-2} = 1 - \frac{2x}{R} + \ldots, \text{ binomial expansion}\right]$$

$$\therefore g' = g\left(1 - 2\frac{h}{R}\right)$$

Thus, value of acceleration due to gravity decreases with increase in height above the earth surface. If h = 0 then g' = g.

46. 2067 Q.No. 5 c What is a satellite? Obtain an expression of the total energy of a satellite orbiting round the earth.

Satellite: Satellite is a heavenly body which revolves round the earth in its own orbit. There are two types of satellites: artificial and natural satellite.

Second Part: Please refer to 2071 Set C Q.No. 5 c

47. 2066 Old Q. No. 2 a What is escape velocity? Prove that escape velocity of a body from earth's surface is $\sqrt{2gR}$ where g is the acceleration due to gravity on the surface of the earth and R is the radius of the earth.

Please refer to 2076 Set B Q.No. 5a

48. 2065 Q. No. 3 a OR What do you mean by parking orbit? Derive an expression for the orbital velocity hence, find time period of the satellite revolving around the earth.

Please refer to 2072 Set D Q.No. 5b

49. 2064 Q.No. 3 a How does acceleration due to gravity vary with distance from the centre of earth above and below its centre?

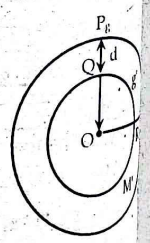
Acceleration due to gravity (g): It is the accleration with which a body falls freely toward the centre of earth due to its gravity. It is denoted by 'g' and its average value on earth's surface is 9.8 m/s². It varies with altitude.

Variation of acceleration due to gravity below the earth surface:

Let g and g' be the values of acceleration due to gravity on the surface of earth and at a depth d below the surface of earth.

The acceleration due to gravity at the surface of earth,

$$g = \frac{GM}{R^2} \quad \ldots (i)$$



where M is mass of earth and R be the radius of earth. At a depth 'd' below the earth's surface, acceleration due to gravity is given by

$$g' = \frac{GM'}{(R-d)^2} \quad \ldots (ii)$$

where M' is mass of earth having radius (R – d). Taking earth to be perfectly spherical, then

$$M = \frac{4}{3}\pi R^3\rho \quad \ldots (iii)$$

where ρ is density of earth.

and $M' = \frac{4}{3}\pi (R-d)^3\rho \quad \ldots (iv)$

Using equation (iii) and (iv) in equation (i) and (ii)

$$g = \frac{G}{R^2}\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi GR\rho \quad \ldots (v)$$

and $g' = \frac{G}{(R-d)^2}\frac{4}{3}\pi (R-d)^3\rho$

$$= \frac{4}{3}\pi G(R-d)\rho \quad \ldots (vi)$$

Dividing equation (vi) by (v) we get

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R\rho} = \frac{R-d}{R} = 1 - \frac{d}{R}$$

$$\Rightarrow g' = g\left(1 - \frac{d}{R}\right)$$

which is the required expression for the variation of acceleration due gravity with depth. The above expression shows that the value of acceleration due to gravity decreases with increasing the value of depth and zero at the centre of earth i.e. at d = R.

Variation of acceleration due to gravity above the earth surface:

Please refer to 2068 Old Q.No. 3 a or

50. 2063 Q.No. 3 a Suppose that a strong man can throw a stone so that it will never return to the surface of earth. How much work does he have to do in throwing up the stone of mass 'm' and find an expression for its minimum velocity?

Please refer to 2076 Set B Q.No. 5a

51. 2061 Q.No. 3 a Explain what is meant by the universal gravitational constant. Discuss the variation of acceleration due to gravity with altitude and derive an expression for its value at height h above the surface of the earth. [1+4]

Universal gravitational constant, G is defined as the force of attraction between two bodies each of unit mass unity and placed at a unit distance apart. Its value is 6.67×10^{-11} N m² kg⁻².

Please refer to 2068 Old Q.No. 3 a or

52. 2060 Q.No. 3 a Explain what is meant by the universal gravitational constant. Discuss the variation of acceleration due to gravity with depth and derive an expression for its value at depth d below the surface of the earth. [1+3]

Universal gravitational constant, G is defined as the force of attraction between two bodies each of unit mass and placed at a unit distance apart. Its value is 6.67×10^{-11} N m² kg⁻².

Please refer to 2064 Q.No. 3 a

53. 2059 Q.No. 2 a Define escape velocity of a body on a planet. Derive an expression for it. [1+3]

Please refer to 2076 Set B Q.No. 5a

54. 2058 Q.No. 3 a What is acceleration due to gravity? Explain how it varies with the altitude. [1+3]

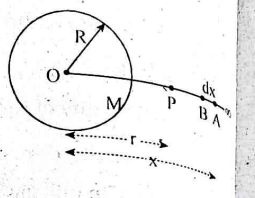
Please refer to 2068 Old Q.No. 3 a or

55. 2057 Q.No. 3 a What is gravitational potential? Derive the relation for gravitational potential at a point due to a point mass. [1+4]

Gravitational potential (V): The gravitational potential of a heavenly body like earth at a point inside the gravitational field is defined as the amount of workdone in taking the body of unit mass from infinity to that point without gravitational accelaration.

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Let M and R be the mass and radius of the earth. To find the gravitational potential (V) at a point P, at a distance 'r' from the centre of earth, we consider a body of unit mass at the point A, at distance x from O as shown in figure below.



The gravitational force of attraction experience by a unit mass at a distance 'x' from centre of earth is

$$F = \frac{GM.1}{x^2} \quad \ldots (i)$$

If dx be the small distance (AB in fig) through which unit mass is moved towards O, the small workdone (dW) is given by

$$dW = \text{Force} \times \text{displacement}$$

$$= \frac{GM}{x^2}\,dx \quad \ldots (ii)$$

The total workdone in taking the mass form infinity to the point 'P' is the calculated by integrating equation (ii) from limit x=∞ to x=r, we get

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} \frac{GM}{x^2}\,dx = GM \int_{\infty}^{r} x^{-2}\,dx = GM\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r} = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = -\frac{GM}{r} \quad \ldots (iii)$$

This work done is equal to gravitation P.E. at point A.

Thus, the gravitational potential at a distance 'r' from centre of earth is

$$V = -\frac{GM}{r}$$

which is required gravitational potential at a point P.

56. 2056 Q.No. 2 a What is a satellite? Calculate the orbital velocity of an artificial satellite.

Satellite: Satellite is heavenly body which revolves round the earth in its own orbit. There are two types of satellites: artificial and natural satellite.

Orbital velocity and time period of the satellite:

A heavenly body that revolves round the earth in a circular orbit is called satellite of earth. The orbital velocity of a satellite is the velocity required to be given to the satellite in order to put it in orbit around the earth.

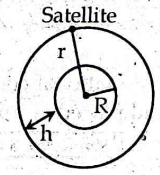
Let M and R be the mass and radius of the earth respectively. In addition, m is the mass of satellite moving in the orbit of radius r = R + h, where h is the height of the satellite above the earth's surface.

In order to keep the satellite moving in the orbit of constant radius r,

Gravitational force = Centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

where v is the velocity of satellite in the orbit.



$$v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} \quad [\because g = \frac{GM}{R^2}]$$

$$\therefore v = R\sqrt{\frac{g}{R+h}}$$

This is the required expression for orbital velocity.

57. 2053 Q.No. 4 Discuss the variation of acceleration due to gravity below the earth's surface.

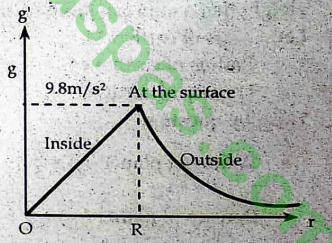
Please refer to 2064 Q.No. 3 a

58. 2050 Q.No. 6 OR Assuming the earth to be perfectly spherical, give sketch graphs to show how:

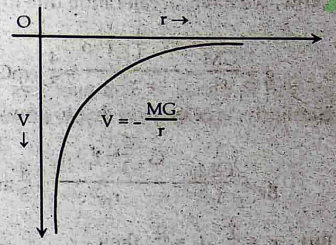
a. The acceleration due to gravity, and

b. The gravitational potential due to earth's mass vary with distance from the surface of the earth to point external to it.

The variation of acceleration due to gravity at depth d inside the surface and at a height h above of the earth of radius R is given by $g' = g\left(1 - \frac{d}{R}\right)$ and $g' = g\frac{R^2}{(R+h)^2}$, where g be the acceleration due to gravity at the surface of earth. These two formulae show that the acceleration due to gravity is zero at the centre of earth, linearly increasing from centre to surface of the earth, maximum at the surface and decreasing exponentially outside the surface of earth as shown in figure.



The gravitational potential at a point at distance r from centre of earth is given by $V = -\frac{MG}{r}$, where M is mass of earth, G is universal gravitational constant and r = R + h. Where R be the radius of earth and h is the distance from surface of earth. The variation of potential with distance from earth is shown in figure. The potential is decreasing as we move from surface to outwards and tends to zero at $r \to \infty$, or infinity.



Numerical Problems

59. 2076 Set C Q.No. 9b A remote sensing satellite of the earth revolves in a circular orbit at a height of 250 km above the earth's surface. What is the orbital speed and period of revolution of the satellite? [4]

Solution

Given,

Radius of earth (R) = 6.38×10^6 m

Height of satellite (h) = 250 km = 250×10^3 m

Orbital speed (v_0) = ?

Time period (T) = ?

We know,

Centripetal force = Gravitation force

$$\text{or, } \frac{mv_0^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$\text{or, } v_0 = \sqrt{\frac{GM}{(R+h)}} \quad \left[g = \frac{GM}{R^2},\ GM = gR^2\right]$$

$$\text{or, } v_0 = \sqrt{\frac{gR^2}{(R+h)}}$$

$$= \sqrt{\frac{10 \times (6.38 \times 10^6)^2}{(6.38 \times 10^6 + 250 \times 10^3)}}$$

$$= 7835.5 \text{ m/sec.}$$

Again, Time of revolution:

$$T = \frac{2\pi(R+h)}{v_0}$$

$$= \frac{2\pi(6.38 \times 10^6 + 250 \times 10^3)}{7835.5}$$

$$= 5314 \text{ sec}$$

60. 2075 Set A Q.No. 9b A man can jump 1.5 m on earth. Calculate the approximate height he might be able to jump on a planet whose density is one quarter of the earth and where radius is one third that of the earth. [4]

Solution

Given,

Height that a man jumps on earth (h_e) = 1.5 m

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Height that a man can jump on planet (h_P) = ?

Let, Mass of earth = M

Radius of earth = r

Density of earth = ρ

Then, by question

Radius of planet (r_P) = $\frac{1}{3}r$

Density of planet (ρ_P) = $\frac{\rho}{4}$

Now, we have, acceleration due to gravity on earth,

$$g = \frac{GM}{r^2} = \frac{G \cdot \frac{4}{3}\pi r^3 \rho}{r^2} = \frac{4}{3}\pi r\rho G \quad \ldots (i)$$

Similarly, acceleration due to gravity on planet

$$g_P = \frac{GM_P}{r_P^2} = \frac{G \cdot \frac{4}{3}\pi r_P^3 \rho_P}{r_P^2} = \frac{4}{3}\pi r_P \rho_P G$$

But $r_P = \frac{r}{3}$, $\rho_P = \frac{\rho}{4}$, then,

$$g_P = \frac{1}{3}\pi \frac{r}{3} \cdot \frac{\rho}{4}$$

$$g_P = \frac{1}{9}\pi r\rho G \quad \ldots (ii)$$

Dividing equation (ii) by (i), we get

$$\frac{g_P}{g} = \frac{\frac{1}{9}}{\frac{4}{3}} = \frac{1}{9} \times \frac{3}{4} = \frac{1}{12}$$

or, $g_P = \frac{g}{12}$ ... (iii)

By question, to jump the height h and h_p, the work done be same, i.e.

Work done on earth = Work done on planet

or, $m.g_e.h_e = m.g_P.h_P$

or, $m.g_e.h_e = m \cdot \frac{g_e}{12} \cdot h_P$

or, $h_P = 12 \times h_e$

$= 12 \times 1.5$

$= 18$ m

∴ The man can jump 18 m on the planet.

61. 2075 Set B Q.No. 9a **An artificial satellite revolves round the earth in 2.5 hours in a circular orbit. Find the height of the satellite above the earth assuming earth as a sphere of radius 6370 km.**

Solution

Given,

Time (T) = 2.5 hours = $2.5 \times 60 \times 60 = 9000$ sec.

Height of satellite (h) = ?

Radius of earth (R) = 6370 km = 6370000 m = 6.37×10^6 m

We have

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

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or, $\frac{T^2}{4\pi^2} = \frac{(R+h)^3}{gR^2}$

or, $\frac{9000^2}{4 \times 3.14^2} = \frac{(R+h)^3}{10 \times (6.37 \times 10^6)^2}$

or, $(R+h)^3 = \frac{9000^2 \times 10 \times (6.37 \times 10^6)^2}{4 \times 3.14^2}$

or, $(R+h)^3 = 8.334 \times 10^{20}$

or, $R + h = (8.334 \times 10^{20})^{1/3}$

or, $R + h = 9.4 \times 10^6$ m

or, $h = 9.4 \times 10^6 - 6.37 \times 10^6$

or, $h = 3.04 \times 10^6$ m

∴ $h = 3040$ km

62. 2074 Supp. Q.No. 9b **Obtain the value of g from the motion of moon assuming that its period of rotation round the earth is 27 days 8 hours and the radius of its orbit is 60.1 times the radius of the earth.** [4]

Solution

Given,

Period of rotation (T) = 27 days and 8 hours

$= 27 \times 24 \times 60 \times 60 + 8 \times 60 \times 60$

$= 2361600$ sec

Radius of earth (R) = 6.36×10^6 m

Radius of orbit (r) = $6.36 \times 10^6 \times 60.1$

Acceleration due to gravity (g) = ?

We have,

$$T = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$$

or, $T^2 = \frac{4\pi^2}{R^2}\frac{r^3}{g}$

or, $g = \frac{4\pi^2 \times r^3}{T^2 R^2}$

$= \frac{4\pi^2 \times (6.36 \times 10^6 \times 60.1)^3}{2361600^2 (6.36 \times 10^6)^2}$

$= 9.76$ m/sec^2

63. 2074 Set A Q.No. 9d **The period of moon revolving under the gravitational force of the earth is 27.3 days. Find the distance of the moon from the centre of the earth if the mass of earth is 5.97×10^{24} kg.**

Solution

Given,

Time period of moon (T) = 27.3 days = 2.359×10^6 sec

Mass of earth (M) = 5.97×10^{24} kg

Distance of moon from centre of the earth (d) = ?

We know, fro the moon to be in circular orbit,

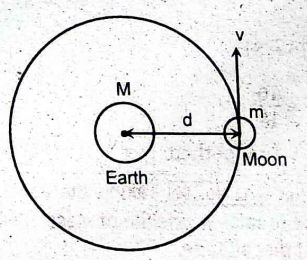
Gravitational force = Centripetal force

i.e., $\frac{GMn}{d^2} = \frac{mv^2}{d}$; m = mass of moon

or, $v = \sqrt{\frac{GM}{d}}$

Also,

$$T = \frac{2\pi d}{v} = \frac{2\pi d}{\sqrt{\frac{GM}{d}}}$$



$$\text{or, } \frac{GM}{d} = \left(\frac{2\pi d}{T}\right)^2$$

$$\text{or, } d^3 = \frac{GMT^2}{4\pi^2} = \frac{6.7 \times 10^{-11} \times 5.97 \times 10^{24} \times (2.359 \times 10^6)^2}{4\pi^2} = 5.64 \times 10^{25}$$

$$\therefore d = 3.83 \times 10^8 \text{m}$$

64. 2073 Supp Q.No. 9b Mass of earth is nearly 81 times heavier than the moon. Its diameter is about 4 times larger than that of the moon. Estimate the value of acceleration of gravity on the surface of the moon.

Solution

Let M_e, M_o and M be the mass of earth, moon and any object respectively.

From earth,

Gravitational force (F) on the object is,

$$F = \frac{GM_em}{R_e^2}$$; G = Universal gravitational constant, R_e = Radius of the earth

$$\text{or, } mg = \frac{GM_e.m}{R_e^2} \quad \left[\because F = mg \text{ is weight of the object, } \& \ g = \text{accleration due to gravity on earth's surface.}\right]$$

$$\text{or, } gR_e^2 = GM_em \quad \ldots \text{(i)}$$

For moon,

Gravitational force (F') on the object is,

$$F' = \frac{GM_mm}{R_m^2}$$;

$$\text{or, } mg' = \frac{GM_m.m}{R_m^2} \quad \left[\because g' = \text{accleration due to gravity on moon's surface, } R_m = \text{Radius of the moon}\right]$$

$$\text{or, } g'R_m^2 = GM_mm \quad \ldots \text{(ii)}$$

Now, dividing equation (i) and (ii), we get

$$\frac{gR_e^2}{g'R_m^2} = \frac{M_e}{M_m}$$

$$\text{or, } \frac{g}{g'}\left(\frac{R_e}{R_m}\right)^2 = \frac{M_e}{M_m}$$

$$\text{or, } \frac{g}{g'}\left(\frac{D_e/2}{D_m/2}\right)^2 = \frac{M_e}{M_m} \quad \left[\because \text{Radius, } R = \frac{\text{Diameter (D)}}{2}\right]$$

$$\text{or, } \frac{g}{g'}\left(\frac{D_e}{D_m}\right)^2 = \frac{M_e}{M_m}$$

$$\text{or, } \frac{g}{g'}\left(\frac{4D_m}{D_m}\right)^2 = \frac{81M_m}{M_m} \quad \left[\because \text{Given, diameter of earth, } D_e = 4 \times \text{diameters of moon } (D_m),\ M_e = 81 \times M_m\right]$$

$$\text{or, } \frac{g}{g'} \times 16 = 81$$

$$\text{or, } g' = \frac{16}{18}g$$

If $g = 9.8\ m/s^2$ then, $g' = 1.94\ m/s^2$

65. 2073 Set D Q.No. 9b Taking the earth to be uniform sphere of radius 6400km, calculate the total energy needed to raise a satellite of mass 1000 kg to a height of 600 km above the ground and to set it into circular orbit at that altitude.

Solution

Given,

Radius of earth (R) = 6400 km = 6400000 m

Mass of satellite (m) = 1000 kg

Height of satellite (h) = 600 km = 600000 m

Total energy (E) = ?

Energy needed = Increase in P.E. + K.E. at Orbit

$$= -\frac{GMm}{r} - \left(\frac{-GMm}{R}\right) + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + \frac{GMm}{R} + \frac{1}{2}m\,GM,R+n$$

$$= GMm\left[\frac{1}{R} - \frac{1}{2(R+h)}\right] = gR^2m\left[\frac{1}{R} - \frac{1}{2(R+h)}\right] = gm\left[R - \frac{R^2}{2(R+h)}\right]$$

$$= 2000 \times 10\left[6400000 - \frac{6400000^2}{2(6400000 + 600000)}\right]$$

$$= 3.47 \times 10^{10}\ \text{J}$$

66. 2072 Set C Q.No. 9b Taking the earth to be uniform sphere of radius 6400 km and the value of g at the surface to be 10 ms^{-2}, calculate the total energy needed to raise a satellite of mass 2000 kg to a height of 800 km above the ground and to set it into circular orbit at that altitude. [4]

Solution

Given,

Radius of earth (R) = 6400km = 6400000m

Mass of satellite (m) = 2000kg

Height of satellite (h) = 800 km = 800000m

Total energy needed (E) = ?

Energy required = Increase in P.E. + K.E at orbit

= P.E. at orbit – P.E surface + K.E at orbit

$$= -\frac{GMm}{r} - \left(\frac{-GMm}{R}\right) + \frac{1}{2}mv^2 = \frac{-GMm}{R+h} + \frac{GMm}{R} + \frac{1}{2}m\frac{GM}{R+h}$$

$$= GMm\left[\frac{1}{R} - \frac{1}{2(R+h)}\right] = gR^2m\left[\frac{1}{R} - \frac{1}{2(R+h)}\right] = gm\left[R - \frac{R^2}{2(R+h)}\right]$$

$$= 2000 \times 10\left[6400000 - \frac{(6400000)^2}{2(6400000 + 80000)}\right]$$

$$= 7.12 \times 10^{10}\ \text{J}$$

67. 2071 Set D Q.No. 9 d An earth satellite moves in a circular orbit with a speed of 6.2 kms^{-1}. Find the time of one revolution and its centripetal acceleration. [4]

Solution

Given,

Orbital velocity of earth satellite (v_0) = 6.2 kms^{-1} = 6200 m/sec

Radius of earth (R) = 6400000 m

Time period (T) = ?

Radial acceleration (a) = ?

We have,

$$v_0 = R\sqrt{\frac{g}{R+h}}$$

$$\text{or, } 6200 = 6400000\sqrt{\frac{10}{6400000 + h}}$$

$$\text{or, } 9.385 \times 10^{-7} = \sqrt{\frac{10}{6400000 + h}}$$

$$\text{or, } 6400000 + h = \frac{10}{9.385 \times 10^{-7}}$$

$$\text{or, } h = 10655301 - 6400000$$

$$h = 4255301\ \text{m}$$

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Now,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$= 2\pi\sqrt{\frac{(4255301 + 6400000)^3}{10 \times 6400000}} = 2\pi\sqrt{\frac{295350}{66}}$$

$$= 2\pi \times 1718.57 = 10792.65 \text{ sec} = 2.99 \text{ hrs}$$

Now, radial acceleration

$$a = \frac{v^2}{r+R} = \frac{(6200)^2}{10655301} = 3.6 \text{ m/sec}^2$$

68. 2069 Supp Q.No. 9d Calculate the points along a line joining the centres of earth and moon where there is no gravitational force. [4]

Solution

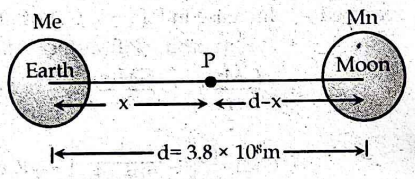
Given,

Mass of earth (Me) = 6×10^{24} kg

Mass of moon (M_m) = 7.2×10^{22} kg

Distance between centres of earth and moon (d) = 3.8×10^8 m

Let P be a point at which gravitational force between earth and moon is zero i.e. equal and opposite consider unit, mass is kept at P, them



$$\frac{G.M_e.1}{x^2} = \frac{G.M_m.1}{(d-x)^2}$$

$$\frac{6 \times 10^{24}}{x^2} = \frac{7.2 \times 10^{22}}{(d-x)^2}$$

or, $\frac{600}{x^2} = \frac{7.2}{(d-x)^2}$

or, $\left(\frac{24.5}{x}\right)^2 = \left(\frac{2.68}{d-x}\right)^2$

or, $\frac{24.5}{x} = \frac{2.68}{d-x}$

or, $24.5d - 24.5x = 2.68x$

or, $24.5d = 27.18x$

or, $x = \frac{24.5 \times 3.8 \times 10^8}{27.18} = 3.42 \times 10^8$ m

3.42×10^8 m from earth, gravitational force is zero.

69. 2067 Supp Q.No. 9 b Assuming the earth to be uniform sphere of radius 6400 km, calculate the total energy needed to raise a satellite of mass 2000 kg to a height of 800 km above the surface of the earth and to set it into circular orbit at that altitude. [4]

Please refer to 2072 Set C Q.No. 9b

70. 2066 Q. No. 9 d What is the period of revolution of a satellite of mass m that orbits the earth in a circular path of radius 7880 km about 1500 km above the surface of the earth? [4]

Solution

Given,

Radius of orbit (r) = 7880 km

Height (h) = 1500 km = 1500000m

Period of revolution (T) = ?

Radius of earth (R) = r – h

= 7880 – 1500 = 6380 km = 6380000 m

We have;

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{(6380000 + 1500000)^3}{10 \times (63800000)^2}} = 6888.88 \text{ sec} = 1.91 \text{ hrs}$$

71. 2054 Q.No. 3 Calculate the period of revolution of a satellite revolving at a distance of 20 km above the surface of the earth, (Radius of the earth = 6400 km. acceleration due to gravity = 10 m/s²).

Solution

Given,

Height from earth (h) = 20km = 20000m

Radius of earth (R) = 6400km = 6400000m

Acceleration due to gravity (g) = 10m/sec²

Time period (T) = ?

We have;

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$= 2\pi\sqrt{\frac{(6400000 + 20000)^3}{10 \times (6400000)^2}} = 5050.12 \text{ sec}$$

72. 2051 Q.No. 3 A 200 kg satellite is lifted to an orbit of 2.2×10^4km radius. If the radius and mass of the earth are 6.37×10^6 m and 5.98×10^{24} kg respectively, how much additional potential energy is required to lift the satellite?

Solution

Given,

Mass of satellite (m) = 200 kg

Radius of orbit (r) = 2.2×10^4 km = 2.2×10^7 m

Radius of earth (R) = 6.37×10^6 m

Mass of earth (M) = 5.98×10^{24} kg

Additional potential energy = ?

We know,

Additional P.E. = P.E. on the obit – P.E. at the earth's surface

$$= -\frac{GMm}{r} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R} - \frac{GMm}{r} = GMm\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$= 6.7 \times 10^{-11} \times 5.98 \times 10^{24} \times 200\left[\frac{1}{6.37 \times 10^6} - \frac{1}{2.2 \times 10^7}\right]$$

$$= 4.47 \times 10^7 \text{ J}$$

∴ Required additional P.E. is 4.47×10^7 J.

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Chapter 8: Equilibrium

Short Answer Questions

1. **2076 Set B Q.No. 1e The passengers in a small boat are not allowed to stand, why?**

The centre of gravity (CG) of a body should be as low as possible to be stable. If one stands on a small boat, the CG of boat raised up which makes it less stable. Due to this reason, the turning of boat more possible and passengers become unsafe. Due to this reason, the passengers in a small boat are not allowed to stand.

2. **2076 Set C Q.No. 1e A table lamp has a heavy and large base, why?**

A body is in stable equilibrium when it is C.G. is in the lowest position as well as the base of the body should be large. If the table lamp has heavy base, its C.G. lies in lower position. Again if the base table lamp is large, the vertical line drawn from displaced lamp lies within base and the lamp attain its initial position without failing on the table. So, the table lamp has a heavy and large base.

3. **2075 Set A Q.No. 1c 2051 Q.No. 1 vi Why is a cow more stable than a man?**

Any body is in stable equilibrium when its C.G. is in the lowest position as well as the base of body should be large. The base of the cow is larger than the base of the man being four footed animal. The vertical line drawn from the C.G. passes through the base if the cow is slightly displaced position but not in man. Hence, the cow is more stable than a man.

4. **2074 Supp. Q.No. 1e Why a wrench of longer arm is preferred more?**

We know, torque = force × perpendicular distance from axis of rotation. A smaller force can produce large torque (i.e. turning effect) if the wrench has longer arm. If the wrench has shorter arm, we have to apply large for to produce small torque. Therefore, we prefer a wrench of longer arm as compared to that of shorter one.

5. **2074 Set B Q.No. 1d One end of a solid rod of length L (in m) is fixed at one end. Can the magnitude direction of torque be estimated if a force $\vec{F}$ (in N) act at a point l (in m) from the fixed end making an angle 150^0 with the horizontal? Explain.**

Yes, the magnitude and direction of torque can be estimated of force $\vec{F}$ (in N) act at a point l(in m) a solid rod of length L(in m) from the fixed end making an angle 150° with the horizontal. The torque is defined as

$$\vec{\tau} = \vec{l} \times \vec{F}$$

or, $|\vec{\tau}| = \tau = lF\sin\theta\,|\hat{n}|$

If $\theta = 150°$, then

$$\tau = lF\sin 150° = \frac{1}{2}\,l.F \text{ in magnitude}$$

The direction of τ is found by using right hand rule.

6. **2073 Set C Q.No. 1e 2073 Set D Q.No. 1f 2072 Set C Q.No. 1c 2070 Supp (Set A) Q.No. 1 d 2070 Set C Q.No. 1 Does the centre of gravity of a solid body always lie within the material of the body? Explain.**

No, the centre of gravity (C.G.) may or may not be within the body. In other words, it is possible that there may be no matter at the C.G. For example, C.G. of a circular ring, sphere, football, etc., lies at their geometrical centre at which there is no matter.

7. **2072 Supp Q.No. 1c Does the centre of mass and centre of gravity lie at the same point? Explain with example.**

The centre of gravity of a body is defined as the point where the whole weight of the body can be supposed to act.

The centre of mass of a body is the point where the whole mass of the body can be supposed to be concentrated in which an applied force produces linear motion and not rotation.

For ordinary bodies, the C.G and C.M. coincide but, for a body of such large dimensions that the value of g is different for its different parts, the C.M. may not coincide with C.G. At a gravity free space, C.G. makes no sense but C.M. has still a meaning.

8. **2072 Set D Q.No. 1d A handle or a knob is fixed at the free end of the door. Explain why? [2]**

The moment of force is given by $\tau = Fr\sin\theta$, where r is perpendicular distance between point of applied force and axis of rotation. When the knob of door is made at free end, r increases and hence, small applied force also can produce large turning effect of force and easy to open door. So, a handle or a knob is fixed at the free end of door.

9. **2071 Supp Q.No. 1d What is the difference between centre of mass and centre of gravity of a system? [2]**

Please refer to 2072 Supp Q.No. 1c

10. **2071 Set C Q.No. 1 g Mention the conditions of stable equilibrium. [2]**

The conditions for a body in stable equilibrium are:

i. The C.G. of the body should lie as low as possible.
ii. The base of the body should be as large as possible.
iii. C.G. should lie within the base of the body on displaced position.

11. **2070 Set D Q.No. 1 e When four-legged animals walk, they always have three of their legs on the ground at any instant. Explain why? [2]**

A body, to be stable should have large base area and vertical line drawn from centre of gravity should lie within base. When the three legs of an animal remains on ground while walking, its base area increases and becomes stabe. Due to stability, it does not fall down on the ground easily. So, four legged animal always have three of their legs on the ground at any instant while walking.

12. **2069 Supp Q.No. 1g A man carrying a load on his back leans forward. Why should he do so? [2]**

When a man carrying a load on his back leans forward, the vertical line through the centre of gravity passes through base. If he carries the load straight upon his back, the centre of gravity shifted backward and the vertical line through the center of gravity falls outside the base (backward) and the man will feel difficulty in carrying the load due to the moment acting on the man by the load which makes his/her unstable. Hence, a man carrying a load on his back leans forward to be more stable.

13. **2069 (Set A) Old Q. No. 1b A coolie carrying a load on his back leans forward. Why? [2]**

Please refer to 2069 Supp Q.No. 1g

14. **2069 (Set B) Q. No. 1d Does the centre of gravity of a solid always lie within the material of the body? If not, give a counter example. [2]**

Please refer to 2073 Set C Q.No. 1e

15. **2068 Q.No. 1 c 2053 Q.No. 1e Explain why a man carrying a load on his back leans forward. [2]**

Please refer to 2069 Supp Q.No. 1g

16. **2068 Old Q.No. 1 d A man carrying a bucket full of water on his right hand always leans towards his left hand. Why? [2]**

When a man carrying bucket of water in one side leans opposite side, the vertical line through the centre of gravity passes through base. If he carries the bucket straight upon his hand the vertical line through the center of gravity falls outside the base and the man will feel difficulty in carrying the load due to the moment acting on the man by the load. Hence, man carrying a bucket of water on his hand always leans to the opposite side.

17. **2067 Q.No. 1 e During pregnancy, women often develop back pains from leaning backward while walking. Why do they have to walk this way? [2]**

For normal woman, centre of gravity lies at the middle of her body. But the stomach of a pregnant woman bolges out from her body and so the centre of gravity is shifted in front than normal, due to which she will have a tendency to fall in forward direction. But, if she leans back, the centre of gravity will be shifted backward towards the normal position, the vertical line draw from CG lies within base and she will become more stable. This is why for more stability, a pregnant woman leans backward while walking.

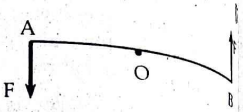
18. **2065 Q. No. 1 c Why a wrench of longer arm is preferred in comparison to a wrench of short arm? [2]**

Please refer to 2074 Supp. Q.No. 1e

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19. 2064 Q.No. 1 d Define the terms: Couple and moment of couple.

Two equal and unlike parallel forces acting on a rigid body at different points and different lines of action constitute a couple. Turning effect of such forces is torque or moment of couple. Mathematically, it is the product of one of the force and perpendicular distance between them.



Couple = F × A B

20. 2058 Q.No. 1 e A man carrying a bucket of water on his hand always leans to the opposite side. Explain why

Please refer to 2068 Old Q.No. 1 d

21. 2055 Q.No. 1 f Why is a horse more stable than a man?

Please refer to 2075 Set A Q.No. 1c

22. 2054 Q.No. 1 f Can a body be in equilibrium if it is in it motion? Explain.

Yes, the body can be in equilibrium if is in motion also. From Newton's second law of motion, $F = \frac{}{}$ If, $F = 0$, so $0 = ma \Rightarrow a = 0 \Rightarrow v =$ constant. This means the body is either at rest or in a unifo linear motion. Hence, the body is in equilibrium if the external force on the body is zero.Thus, whe body is in translational equilibrium, it is either at rest or it is moving with constant velocity.

23. 2053 Q.No. 1 g What is meant by the moment of a couple?

Please refer to 2064 Q.No. 1 d

24. 2052 Q.No. 1 b State principle of moment and give an example.

If a body is in equilibrium under the action of number of forces, at different points of the body, the algebraic sum of the moment of all the forces about any point is zero i.e. clock wise moment m be equal to anticlockwise moment, which is called principle of moment.

25. 2051 Q.No. 1 ii What is the difference between centre of gravity and centre of mass of a system?

Please refer to 2072 Supp Q.No. 1c

26. 2050 Q.No. 1 vi Is it possible for a solid body to have no matter at it centre of gravity?

Please refer to 2073 Set C Q.No. 1e

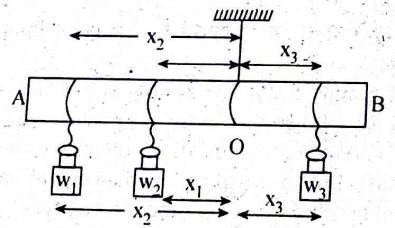
Long Answer Questions

27. 2059 Q.No. 3 a OR State principle of moments. Explain how you verify the principle of moment in y laboratory. [4+

Principle of moment: *If a body is in equilibrium under the action of number of forces, at differ points of the body, then the algebraic sum of the moments of all the forces about any point is zero clock wise moment must be equal to anticlockwise moment, which is called principle of moment.*

Verification of principle of moment

Let us take a 1 meter wooden scale suspends it with thread on stand. Adjust this scale so it can be equilibrium i.e. can stay horizontally. The position of thread on the scale on this state is the positi of centre of gravity. Suspend three different weights on scale by the help of thread on both sides the scale i.e. along OA and OB.



As shown in the figure, w_1, w_2 and w_3 are placed at distances x_1, x_2 and x_3 respectively from the cent of gravity. Since, O represents centre of gravity the weights w_1 and w_2 produce anticlockwi moments whereas w_3 produces clockwise moment about centre of gravity. According to principle moment, the sum of clockwise moment is equal to sum of anticlockwise moment in equilibriu condition.

i.e. $w_3 x_3 = w_1 x_1 + w_2 x_2$

which proofs the principle of moments. The experiment can perform for different sets of weights.

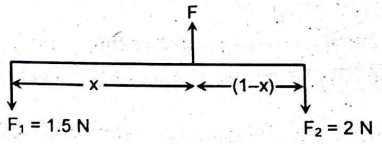
Numerical Problems

28. 2075 Set B Q.No. 9b Two forces of 1.5 N and 2.0 N act vertically at the two ends of a metre scale. Where and in what direction should a force be applied so that the scale remains horizontally stable. [4]

Solution

Let F_1 (= 1.5N) be the force acting vertically at one end and F_2 (= 2N) be another force acting vertically at second end. Let F be a force acting vertically upward direction at distance x from F_1 and 1 – x from F_2. When the meter scale is in horizontal equilibrium,

Clockwise moment = Anticlockwise moment



or, $F_1 \times x = F_2 \times (1 - x)$

or, $1.5 \times x = 2 \times (1 - x)$

or, $1.5x = 2 - 2x$

or, $3.5x = 2$

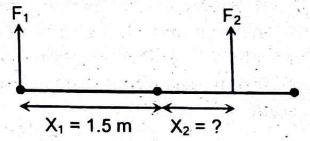
$\therefore x = \frac{2}{3.5} = 0.57$

$\therefore 1 - x = 1 - 0.57 = 0.43$

The third force is acting at a distance of 0.43 m from 2N force or 0.57 m from 1.5 N force.

29. 2072 Supp Q.No. 9b Two people are carrying a uniform wooden board that is 3m long and weighs 160N. If one person applies an upward force equal to 60N at one end, at what point does the other person lift? [4]

Solution



Given,

Length of wooden board (l) = 3m

Weight of wooden board (W) = 160 N

Upward force to one end (F_1) = 60 N

Distance of F_1 from centre (X_1) = 1.5 m

Upward force to other point (F_2) = 160 – 60

= 100 N

Distance of F_2 from centre (X_2) = ?

We have,

$F_1 \times X_1 = F_2 \times X_2$

$60 \times 1.5 = 100 \times X_2$

$X_2 = 0.9$ m from center

30. 2050 Q.No. 4 A roller whose diameter is 1.0 m weights 360 N. What horizontal force is necessary to pull the roller over a brick 0.1m high when the force is applied at the centre? [4]

Solution

Given,

Diameter of roller (d) = 1.0m

Weight of roller (W) = 360 N

Height of brick = 0.1 M

Let F be the horizontal force applied to pull the roller over brick.

Taking the equilibrium at P,

We have,

$F \times BP = W \times AP$

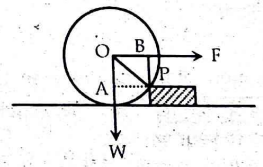
or, $F \times (0.5 - 0.1) = W \times \sqrt{OP^2 - OA^2}$

or, $F \times 0.4 = W \times \sqrt{0.5^2 - 0.4^2}$

or, $F \times 0.4 = 360 \times 0.3$

or, $F = \frac{360 \times 0.3}{0.4}$

$F = 270$ N



□□□

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Short Answer Questions

1. **2076 Set B Q.No. 1f, 2073 Set C Q.No. 1f If the ice on the polar caps of the earth melts, how will it affect the duration of the day? Explain.**

If the ice on the polar caps of the earth melts, the water (changing from ice to water), spreads on the surface of earth towards the equator. Due to this reason, the moment of inertia increases, to conserve angular momentum, the angular speed of earth decreases.

i.e. $I\omega$ = constant

or, $\frac{I2\pi}{T}$ = constant

or, $T \propto I$,

As moment of inertia increases, the time period increases and hence duration of day increased.

2. **2076 Set C Q.No. 1f Can you distinguish a raw egg and a hardboiled egg by spinning each one on the table? Explain.**

Boiled egg acts just like a rigid body while rotating, but it is not the case in a raw egg because of liquid matter present in it. In case of raw egg, the liquid matter rises to go away from centre, thereby increasing its moment of inertia. As moment of inertia is more, the raw egg will spin with less angular velocity provided the same torque applied in the both cases. Thus, hard boiled egg will spin faster than raw egg.

3. **2075 Set A Q.No. 1e Experienced cooks can tell whether an egg is raw or hard boiled by rolling it down a slope. How is this possible? What are they looking for?**

Experienced cooks can tell whether an egg is raw or hard boiled by rolling it down a slope. A raw egg has liquid inside it and hard boil has solid. When raw egg rolls, the liquid tries to force out towards the outer walls which increases moment of inertia $I = \Sigma mr^2$ of the egg. But solid i.e. boiled egg has less moment of inertia. From principle of conservation of linear momentum,

$I\omega$ = constant

or, $I\frac{2\pi}{T}$ = constant

or, $I \propto T$

If, I_r & I_h are M.I. of raw and hard boiled egg while T_r & T_h are corresponding spining the time period.

Since, $I_h < I_r$

$T_h < T_r$

The hard boiled egg move faster than the raw egg.

4. **2075 Set B Q.No. 1d Both "the work done by a force" and "the torque produced by a force" are the product of force and the position vector. How can one make the difference between the two? Explain.**

Work done = Force × Distance and Torque = Force × perpendicular distance i.e. both work done and torque both are defined as the product of force and position vector or distance. However work done is dot product and torque is vector product i.e. work done is scalar quantity and torque is vector quantity. This is the main difference between work done and torque.

5. **2074 Supp. Q.No. 1f If earth shrinks, how will be the duration of a day affected?**

If earth shrinks, the moments of inertia ($I = \Sigma mr^2$) decreases due to decreasing radius of the particles from axis of rotation. From conservation of angular momentum,

$I\omega$ = constant.

or, $I\frac{2\pi}{T}$ = constant

or, $T \propto I$,

As the moment of inertia decrease, time duration of rotation decreases and hence day will be shorter.

2074 Set A Q.No. 1g Does the angular momentum of a body moving in a circular path change? Give explanation to your answer.

No, the angular momentum of a body does not change moving in a circular path because the angular momentum always remains conserved it the external torque on it is zero. i.e. $L = I\omega$ remain constant where I is moment of inertia and ω is angular speed.

7. **2072 Supp Q.No. 1g, 2069 (Set B) Q. No. 1e The cap of a bottle can be easily opened with the help of two fingers than with one finger. Why? [2]**

When we apply the force by two fingers to open the cap of a bottle, a moment of couple i.e. torque is produced which helps to open it easily. When we apply the force by a single finger, it can not produce torque or turning effect or produce less turning effect and hence, be difficult to open cap of bottle.

8. **2071 Supp Q.No. 1e The work done by a force is the product of force and distance. The torque due to the force is the product of force and distance. Does this mean that torque and work are equivalent? Explain. [2]**

No, work (W) is scalar quantity and torque (τ) is a vector quantity as,

$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$, where s is linear distance.

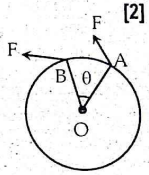
$\tau = \vec{F} \times \vec{d} = Fd\sin\theta$, where d is the perpendicular distance between the point of applied force and axis of rotation.

9. **2070 Supp (Set A) Q.No. 1 f A ballet dancer stretches her hands when she wants to come to rest. Why? [2]**

When a ballet dancer stretches her arms, the moment of inertia (I) of the dancer about axis of rotation increased. Since, no external torque is applied, total angular momentum L($I\omega$ = constant) remains constant. Due to increase in I, ω (angular velocity) decreases and the dancer comes to rest gradually. So, the ballet dancer stretches her arms when she wants to come to rest.

10. **2070 Supp (Set B) Q.No. 1 d A flywheel rotates with constant angular velocity. Does a point on its rim have a tangential acceleration? A radial acceleration? Are these accelerations constant in magnitude? In direction? In each case, give the reasoning behind your answer. [2]**

When a flywheel rotates with constant angular velocity, then it has constant linear velocity on the rim as $v = \omega r$. The magnitude of linear velocity is constant but direction changes at each point. So, there is tangential acceleration. There is also radial acceleration as, $a = v^2/r = \omega^2 r$. The direction of tangential as well as radial acceleration changes at each and every point. The direction of radial acceleration is directed towards the radius.



11. **2070 Set D Q.No. 1 g, 2067 Supp Q.No. 1e, 2051 Q.No. 1iii A fan with blades takes longer time to come to rest than without blades. Why? [2]**

Inertia of a body depends upon its mass and hence in the quantity of matter contained in the body. Heavier bodies have more inertia than lighter bodies. Thus, inertia of motion is larger for a fan with blades than without blades. That's why; the fan with blades takes longer time to come to rest than without the blades.

12. **2069 Old (Set B) Q. No. 1e A solid sphere and a hollow cylinder of same mass and same size is rolling down on an inclined plane from rest. Which one reaches the bottom first? Why? [2]**

The solid sphere reaches to the ground at first than the hollow cylinder because hollow cylinder has more moment of inertia than the hollow sphere.

The moment of inertia of solid sphere of radius R is $\frac{2}{5}MR^2$ and that of hollow cylinder is MR^2. From principle of conservation of angular momentum,

$I\omega$ = Constant (K)

or, $I.\frac{2\pi}{T}$ = Constant

or, $I \propto T$

Since $I_H > I_S$

$T_H > T_s$

$\therefore$ Solid sphere reaches first.

13. **2068 Old Q.No. 1 e A ballet dancer can increase or decrease her spining rate by using the principle of conservation of angular momentum, how? [2]**

When the ballet dancer (or dancing girl in a rotating turn table) stretches her arms, the moment of inertia

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(I) of the dancer about axis of rotation increases. Since no external torque is applied, total angular momentum L = Iω remains constant. For this, angular velocity correspondingly decreases. Thus, spinning rate decreases. When she lowers her hands, the moment of inertia decreases and hence spinning rate (ω) gets increased automatically to conserve the angular momentum.

14. 2058 Q.No. 1 c A ballet dancer stretches her arms to reduce her motion. Explain.

Please refer to 2070 Supp (Set A) Q.No. 11

15. 2057 Q.No. 1 e A dancer girl is rotating over a turntable with her arms outstretched. If she lowers her arms, how does this affect her motion?

Please refer to 2068 Old Q.No. 1 e

16. 2056 Q.No. 1 d Explain why spokes are fitted in the cycle wheel.

The spokes of cycle wheel increases its mass at rim and hence increases its moment of inertia. The greater the moment of inertia, the more is the opposition to any change in uniform rotational motion. As a result, cycle runs smoothly and steadily. If the cycle wheels had no spokes, the cycle would be driven jerky and hence unsafe.

17. 2055 Q.No. 1 c What is the counterpart of the mass and force in rotational motion?

In linear motion, we have

force = mass × linear acceleration ... (i)

In rotational motion, we have

torque = (moment of inertia) × angular acceleration

or, $\tau = I\alpha$ ... (ii)

Hence, the counterparts of mass and force in rotational motion are moment of inertia and torque respectively because they play the same role in rotational motion as mass and acceleration play in linear motion.

18. 2054 Q.No. 1 h Suppose that only two external forces act on a rigid body and the two forces are equal in magnitude but opposite in direction. Under what conditions will the body rotate?

Suppose that only two external forces act on a rigid body and the two forces are equal in magnitude but opposite in direction, if these two are acting at different line of actions, the body will rotate.

19. 2053 Q.No. 1 f If the earth is struck by meteorites, the earth will slow down slightly. Why?

The earth rotates about an axis passing throughout its geographic north and south poles with a period of one day. If it is struck by meteorites, effective mass of earth has increased, the M.I. of earth is increased. So, by principle of conservation of angular momentum (i.e., Iω = constant), ω decreases and earth will slow down slightly.

20. 2050 Q.No. 1 iii Why is it easier to hold down a 10 kg body in your hand at your side than to hold it with your arm extended horizontally?

When a 10 kg body is held in our hand with our arm extended horizontally, the force (weight of the mass) and the distance of the force from the rotation axis are at right angle. This produces maximum torque. But when the body is held at our side, the force always acts through the rotation axis, so there is a very small turning effect about the shoulder joint. Hence, it is more convenient to hold down a 10 kg mass in our hand at our side than to hold with our arm extended horizontally.

Long Answer Questions

21. 2076 Set B Q.No. 5c Define moment of inertia. Derive an expression for the moment of inertia of thin rod about an axis through its centre and perpendicular to its length.

Moment of inertia: Moment of inertia of a rigid body about a particular axis is defined as the sum of the products of masses of all the particles contains on that rigid body and square of their respective distances from the axis of rotation. It is represented by I. Mathematically;

$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$

$I = \frac{1}{2}(\Sigma_i m_i r_i^2)$

It is a scalar quantity and it has the unit of kgm². It is also called rotational inertia.

The moment of inertia of a thin and uniform rod about an axis passing through the centre and perpendicular to its length.

Let us consider a thin uniform rod of length l and mass M rotating about an axis YY' as shown as figure. We have to find out the moment of inertia of this uniform rod about an axis passing through its centre and perpendicular to its length. For this, take a small portion of rod at a distance x from O. It has length dx and mass dm.

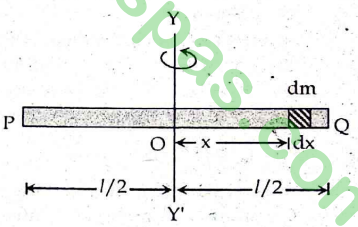


Fig: M.I of a thin uniform rod.

The mass per unit length of rod is $\frac{M}{l}$ and hence, mass of dx portion is $dm = \frac{M}{l}dx$.

The moment of inertia of rod about an axis YY' is

$$I = \int_{-l/2}^{l/2} x^2 dm = \int_{-l/2}^{l/2} x^2 \frac{M}{l} dx = \frac{M}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{M}{3l}\left[\frac{l^3}{8} + \frac{l^3}{8}\right] = \frac{M}{3l}\left[\frac{l^3}{4}\right] = \frac{Ml^2}{12}$$

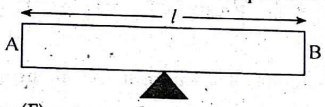
$\therefore I = \frac{1}{12}Ml^2$

This is required expression for moment of inertia of a uniform rod about an axis passing through its centre and perpendicular to its length.

22. 2075 Set A Q.No. 5c Show that in rotational motion, power is the product of torque and angular velocity. [4]

Couple: Two equal and opposite parallel forces, not acting along a single point, which tends to rotate a body, is called couple.

Moment of couple (τ): The product of one of the force and shortest distance between two forces which always have turning effect, is called moment of couple. It is also called torque (τ).



Torque (τ) = force (F) × perpendicular distance (r)

Expression for the workdone by couple

Let us consider a wheel of radius 'r' rotates through an angle 'θ' due to a couple of force acting tangentially to the rim as shown in figure.

The torque acting on the wheel is given by,

$\tau = F \times AB$

$\tau = F \times 2r$ ... (i)

(where AB = 2r, diameter)

The workdone by the couple of force F is given by

$W = F \times AA' + F \times B'B'$

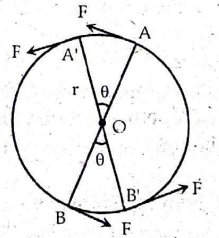
$= F \times r\theta + F \times r\theta$ [BB' = A'A' = rθ]

$= 2(F \times r\theta)$

$= (F \times 2r)\theta$ ... (ii)

$= \tau\theta$ [using equation (ii)]

$\therefore W = \tau\theta$



Power P is defined as the rate of work done,

Then,

$P = \frac{dW}{d\tau} = \frac{d}{d\tau}(\tau\theta)$

$= \tau\frac{d\theta}{dt} = \tau\omega$

Thus in rotational motion power is the product of torque and angular velocity.

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23. 2075 Set B Q.No. 5c What is the physical meaning of moment of inertia of a rigid body? Also derive an expression in the case of a thin and uniform rod about an axis passing through one end and perpendicular to its length.

Moment of inertia: Moment of inertia of a rigid body about a particular axis is defined as the sum of the products of masses of all the particles contains on that rigid body and square of their respective distances from the axis of rotation. It is represented by I. Mathematically;

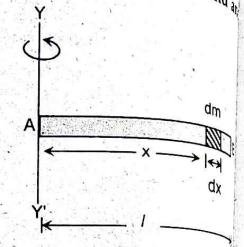
$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$

$I = \frac{1}{2}(\Sigma_i m_i r_i^2)$

It is a scalar quantity and it has the unit of kgm^2. It is also called rotational inertia.

Calculation of moment of inertia of uniform thin rod about an axis passing through one end and perpendicular to its length:

Consider a thin uniform rod AB of mass M and length *l*. Suppose the rod be rotating about an axis YY' passing through its one end and perpendicular to its length. To find the moment of inertia of this rod about the axis YY', consider a small element of length dx whose mass is dm at a distance x from one end A. Its moment of inertia is x^2 dm.



The moment of inertia of the rod about the axis YY' is

$$I = \int_0^l x^2 dm$$

The mass per unit length of the bar $= \frac{M}{l}$ and mass of the element of length dx $= \left(\frac{M}{l}\right) dx$

or, $dm = \frac{M}{l} dx$

Therefore, $I = \int_0^l \frac{M}{l} x^2 dx = \frac{M}{l}\int_0^l x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{M\,l^2}{3}$

This is required moment of inertia of a uniform rod about an axis passig through one end perpendicular to its length.

24. 2074 Supp. Q.No. 5b What is meant by moment of inertia? How is it related with the rotational kinetic energy of a body?

Moment of inertia: Moment of inertia of a rigid body about a particular axis is defined as the sum of products of masses of all the particles contains on that rigid body and square of their respective distances from the axis of rotation. It is represented by 'I'. Mathematically;

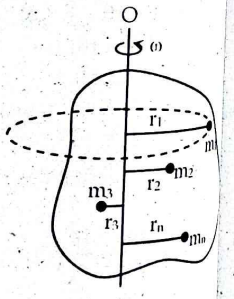
$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$

$I = \frac{1}{2}(\Sigma_i m_i r_i^2)$

It is a scalar quantity and it has the unit of kgm^2. It is also called rotational inertia.

Expression of rotational kinetic energy

Let particles of the body of masses $m_1, m_2, m_3, \ldots, m_n$ are situated at distances $r_1, r_2, r_3, \ldots r_n$ respectively from an axis of rotation O of a rigid body as shown in fig. Let the body be rotated with uniform angular velocity ω about this axis. Although, each particle within the body has the same angular velocity ω but their linear velocity will be different. Let $v_1, v_2, v_3, \ldots, v_n$ be the linear velocities of the particles of masses $m_1, m_2, m_3, \ldots, m_n$ respectively. Then, $v_1 = \omega r_1$, $v_2 = \omega r_2$, $\ldots$, $v_n = \omega r_n$



A rigid body rotating about an axis

Now, the kinetic energy of the particle having mass m_1 is

$= \frac{1}{2} m_1 v_1^2$ [v_1 is liner velocity of m_1]

$E_1 = \frac{1}{2} m_1\omega^2 r_1^2$ [$\because v = r\omega$]

Similarly, kinetic energy of the particle having mass m_2 is

$E_2 = \frac{1}{2} m^2 v_2^2 = \frac{1}{2} m_2 r_2^2 \omega^2$ and so on.

$\therefore$ Total rotational kinetic energy of the body is given by

$E = E_1 + E_2 + \ldots + E_n$

$= \frac{1}{2} m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 + \ldots + \frac{1}{2} m_n r_n^2 \omega_n^2$

$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2)\,\omega^2$

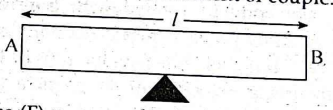
$= \frac{1}{2}(\Sigma_i m_i r_i^2)\,\omega^2$

$E = \frac{1}{2} I\omega^2$

This is the required expression of rotational kinetic energy in terms of moment of inertia.

25. 2074 Set A Q.No. 5c Explain the concept of torque and angular acceleration in the case of a rigid body. Derive a relation between them. [4]

Torque: The product of one of the force and shortest distance between two forces which always have turning effect is called torque (τ). It is also called moment of couple.



Torque (τ) = force (F) × perpendicular distance (r)

Angular Acceleration: The time rate of change of angular velocity is called angular acceleration. It is denote by α and given by $\alpha = \frac{d\omega}{dt}$. It is measured in radian per sec^2 (rad sec^{-2}).

Relation between Torque and Moment of Inertia

Let consider a rigid body rotating about an axis O under the action of a constant torque τ. Suppose the body consists of n particles of masses $m_1, m_2, m_3 \ldots m_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ from the axis of rotation respectively. The torque will produce a constant angular acceleration α in each particle. Since the particle of mass m_1 follows a circular path of radius r_1, the magnitude of the linear acceleration of this particle is

$a_1 = r_1\alpha$

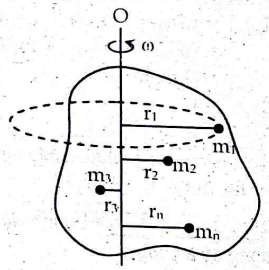
The net external force acting on this particle is

$F_1 = m_1a_1 = m_1(r_1\alpha) = m_1r_1\alpha$

The magnitude of torque acting on this particle due to this force

$\tau_1 = F_1r_1$

$= (m_1 r_1 \alpha) r_1 = m_1 r_1^2 \alpha$



A rigid body rotating about an axis

Similarly, the magnitude of torque on the particles of masses $m_2, m_3, \ldots$ are $m_2r_2^2\alpha$, $m_3r_3^2\alpha \ldots$ respectively.

Then the total torque is the sum of the individual torques acting on the constituent particles, so,

$\tau = m_1r_1^2\alpha + m_2r_2^2\alpha + m_3r_3^2\alpha + \ldots$

$= (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots)\,\alpha$

$= (\Sigma m_i r_i^2)\,\alpha$

$\therefore \tau = I\alpha$

where, $I = \Sigma m_i r_i^2$ is the moment of inertia of the body about the axis Y. this is required relationship between touque and angular acceleration.

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26. 2074 Set B Q.No. 5d Define moment of inertia. How is it related with angular momentum of a body rotating about an axis of rotation? [4]

Moment of inertia: Moment of inertia of a rigid body about a particular axis is defined as the sum of the products of masses of all the particles contains on that rigid body and square of their respective distances from the axis of rotation. It is represented by I. Mathematically;

$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$

$I = \frac{1}{2}(\Sigma_i m_i r_i^2)$

It is a scalar quantity and it has the unit of kgm^2. It is also called rotational inertia.

Angular Momentum: Angular momentum is defined as the product of linear momentum and distance between the object and axis of rotation. It is represented by L and given by L = linear momentum × perpendicular distance from the axis of rotation.

or, $L = pr$

$\therefore L = mvr$ ... (i)

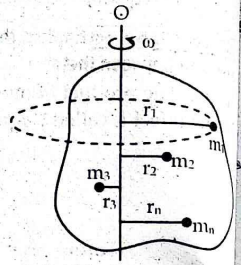
Since $v = \omega r$, where ω is the angular velocity of the object, the angular momentum can be written as

$L = m(\omega r)r$

or, $L = mr^2\omega$ .... (ii)

Relation between Angular Momentum and Moment of Inertia

Let particles of the body of masses $m_1, m_2, m_3, \ldots, m_n$ are situated at distances $r_1, r_2, r_3, \ldots r_n$ respectively from an axis of rotation O of a rigid body as shown in fig. Let the body be rotated with uniform angular velocity ω about this axis. Although, each particle within the body has the same angular velocity ω but their linear velocity will be different. Let $v_1, v_2, v_3, \ldots, v_n$ be the linear velocities of the particles of masses $m_1, m_2, m_3, \ldots, m_n$ respectively. Then, $v_1 = \omega r_1$, $v_2 = \omega r_2$, ..., $v_n = \omega r_n$. The magnitude of the angular momentum of particle of mass m_1 about the axis of rotation = linear momentum × $r_1 = m_1 v_1 r_1 = m_1 r_1^2 \omega$



A rigid body rotating about an axis

Similarly, the magnitude of angular momentum of particles of masses $m_2, m_3, \ldots$ are $m_2 r_2^2 \omega$, $m_3 r_3^2 \omega, \ldots$ respectively. Now, the total angular momentum, L of the body about the axis O is,

$L = m_1r_1^2\omega + m_2r_2^2\omega + m_3r_3^2\omega + \ldots$

$= (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots)\omega$

$= (\Sigma m_i r_i^2)\omega$

$= I\omega$

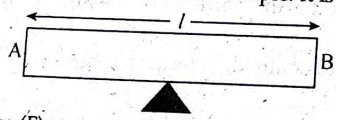
$\therefore L = I\omega$

This is required relationship between angular momentum and moment of inertia. Here, $\Sigma m_i r_i^2 = I$ is moment of inertia of the body.

27. 2073 Supp Q.No. 5b Define torque and couple in rotational motion. Also derive an expression for the work done by a couple. [4]

Couple: Two equal and opposite parallel forces, not acting along a single point, which tends to rotate a body, is called couple.

Moment of couple (τ): The product of one of the force and shortest distance between two forces which always have turning effect, is called moment of couple. It is also called torque (τ).



Torque (τ) = force (F) × perpendicular distance (r)

Expression for the workdone by couple

Let us consider a wheel of radius 'r' rotates through an angle 'θ' due to a couple of force acting tangentially to the rim as shown in figure.

The torque acting on the wheel is given by,

$\tau = F \times AB$

$\tau = F \times 2r$ ... (i)

(where AB = 2r, diameter)

The workdone by the couple of force F is given by

$W = F \times AA' + F \times B'B'$

$= F \times r\theta + F \times r\theta$ [BB' = A'A' = rθ]

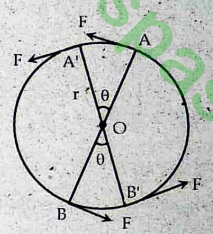
$= 2(F \times r\theta)$ ... (ii)

$= (F \times 2r)\theta$

$= \tau\theta$ [using equation (ii)]

$W = \tau\theta$

which is the required expression for workdone by couple.



8. 2073 Set C Q.No. 5c Define moment of inertia. Obtain an expression for the moment of inertia of a thin and uniform rod about an axis passing through the centre and perpendicular to its length. [4]

Please refer to 2076 Set B Q.No. 5c

9. 2072 Supp Q.No. 5d What is radius of gyration? Show that the acceleration of a body rolling down an inclined plane is $a = \frac{MgSin\theta}{(M + \frac{I}{R^2})}$, where θ is the angle of inclination of the plane through which a body rolls down, M is the mass of the body, I is the moment of inertia and R is the radius. [4]

Radius of gyration is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated such that the moment of inertia remains same. It is denoted by K and given by

$I = MK^2$

where I = Moment of inertia & M = mass of body. The radius of gyration is also defined as,

$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}}$

Radius of gyration of a body about a given axis may also be defined as the root mean square distances of the various particles of the body from the axis of rotation.

Acceleration of a Rolling body on an Inclined plane.

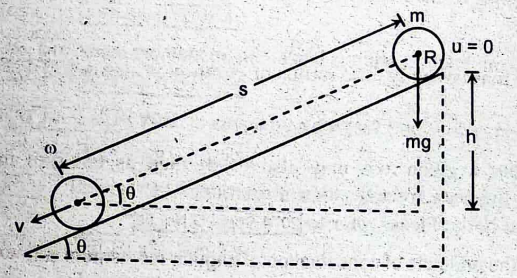


Fig: A rolling body on inclined plane

Let us consider a circular symmetrical body like ball, ring, disc etc. of mass m, radius R is rolling down on an inclined plane of inclination θ with horizontal as shown in figure.

Let v be the velocity along the plane covering distance s and below at height. Then, loss of potential energy is given as,

Loss of P.E = mgh ... (i)

Along this rolling, the body gain the K.E which is equal to the sum of rotational K. E & translational K. E. as,

$K.E. = K.E_t + K.E_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}v^2\left(m + \frac{I}{R^2}\right)$

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From principle of conservation of energy;

Loss in P.E = gain in K. E.

$$mgh = \frac{1}{2} v^2 \left(m + \frac{I}{R^2}\right)$$

or, $$mgs\sin\theta = \frac{1}{2} v^2 \left(m + \frac{I}{R^2}\right) \qquad [\because \sin\theta = \frac{h}{s}]$$

or, $$v^2 = \frac{2mgs\sin\theta}{m + \frac{I}{R^2}}$$

Again,

$$v^2 = u^2 + 2as \qquad [\because u = 0]$$

$$v^2 = 2as$$

or, $$2as = \frac{2mgs\sin\theta}{m + \frac{I}{R^2}}$$

$$a = \frac{mg\sin\theta}{m + \frac{I}{R^2}}$$

which is required acceleration of a rolling body along the inclined plane.

30. 2072 Set D Q.No. 5c Define moment of inertia and radius of gyration. Derive an expression for the ki energy of rotation of a rigid body.

Moment of inertia: Moment of inertia of a rigid body about a particular axis is defined as the sum product of masses of all the particles contains on that rigid body and square of their respective dista from the axis of rotation. It is represented by I. Mathematically;

$$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$$

$$I = \frac{1}{2}(\Sigma_i\, m_ir_i^2)$$

It is a scalar quantity and it has the unit of kgm^2.

Radius of gyration is defined as the distance from the axis of rotation to a point where the total m of the body is supposed to be concentrated such that the moment of inertia remains same. denoted by K and given by

$$I = MK^2$$

where I = Moment of inertia & M = mass of body. The radius of gyration is also defined as,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}}$$

∴ Radius of gyration of a body about a given axis may also be defined as the root mean sq distances of the various particles of the body from the axis of rotation.

Kinetic energy of rotation of a rigid body: Please refer to 2074 Supp. Q.No. 5b

31. 2072 Set E Q.No. 5d State and explain the principle of conservation of angular momentum with example.

Principle of Conservation of Angular Momentum:

Principle of conservation of angular momentum states that the total angular momentum of a syst remains conserved if the external torque applied on it is zero.

i.e. $I\omega = L = \text{Constant}$

where I is moment of inertia, ω is angular speed and L is angular momentum.

Explanation:

The time rate of change of angular momentum is called torque, then

$$\tau = \frac{dL}{dt}$$

According to statement, applied torque is zero is $\tau = 0$,

or, $$\frac{dL}{dt}$$

Integrating we get

L = constant

or, $L = I\omega$ = constant

or, $I_1\omega_1 = I_2\omega_2$

This is principle of conservation of angular momentum.

Examples:

i. A ballet dancer can increase or decrease the spinning rate during her performance using principle of conservation of angular momentum.

ii. A diver uses this principle in the swimming pool.

iii. Motion of planet revolving in elliptical orbit round the sum.

32. 2071 Set C Q.No. 5 a Define torque and couple in rotational motion. Derive an expression for the work done by the couple. [4]

Please refer to 2073 Supp Q.No. 5b

33. 2071 Set D Q.No. 5 d What do you mean by a rigid body? Obtain an expression for the moment of inertia of a thin and uniform rod about an axis passing through the centre and perpendicular to its length. [4]

Rigid Body: A rigid body is defined as a sold body in which particles are arranged compactly so that the interparticle distances are very small and fixed. These distances are not disturbed by any external force. So, the rigid bodies have fixed shape and size. They undergo translational and rotational motion.

Second Part: Please refer to 2076 Set B Q.No. 5c

34. 2070 Set C Q.No. 5 c Derive a relation between torque applied and angular acceleration produced in a rigid body and hence, define moment of inertia. [4]

Please refer to 2074 Set A Q.No. 5c

35. 2070 Set D Q.No. 5 d Derive a relation between angular momentum and moment of inertia of a rigid body and hence, define moment of inertia. [4]

Please refer to 2074 Set B Q.No. 5d

36. 2069 Supp Q.No. 5d Define moment of inertia. Obtain an expression for the moment of inertia of a thin rod about an axis passing through one end and perpendicular to its length. [4]

Please refer to 2075 Set B Q.No. 5c

37. 2068 Q.No. 5 b Define moment of inertia and angular momentum. Establish a relation between them.

Please refer to 2074 Set B Q.No. 5d

38. 2066 Old Q. No. 3 a OR What is meant by a couple? Derive an expression for the work done by a couple. [1+3]

Please refer to 2073 Supp Q.No. 5b

39. 2062 Q.No. 2 a Define the terms: couple and moment of a couple. Derive an expression for the work done by a couple. [1+1+3]

Please refer to 2073 Supp Q.No. 5b

40. 2060 Q.No. 3 OR Explain the meaning of the term 'moment of inertia'. Show that the quantity 1/2 IW² is the Kinetic energy of rotation of a rigid body rotating about an axis with angular velocity w. [1+3]

Please refer to 2074 Supp. Q.No. 5b

41. 2057 Q.No. 3 OR Define moment of inertia. How is it related with rotational kinetic energy of a body? [5]

Please refer to 2074 Supp. Q.No. 5b

42. 2051 Q.No. 4 OR What is meant by moment of inertia? How is it related with the rotational kinetic energy of a body?

Please refer to 2074 Supp. Q.No. 5b

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Numerical Problems

43. 2076 Set C Q.No. 9c A wheel starts from rest and accelerates with constant angular acceleration to an angular velocity of 15 revolutions per second in 10 seconds. Calculate the angular acceleration and angle which the wheel has rotated at the end of 2 second. [4]

Solution

Given,
Initial angular velocity (ω_0) = 0 rad/ sec.
Final frequency (f) = 15 rps
Time taken (t) = 10 sec.
Angular acceleration (α) = ?
Angular displacement (θ) = ?
Time (t_1) = 2 sec.
Now,

Final angular velocity (ω) $= 2\pi f$
$= 2\pi \times 15$
$= 94.2$ rad/ sec.

We have,

$\omega = \omega_0 + \alpha t$

or, $94.2 = 0 + \alpha \times 10$

$\therefore \alpha = \frac{94.2}{10} = 9.42$ rad/sec^2

Angular displacement (θ) $= \omega \times t$
$= 94.2 \times 2$
$= 188.4$ radian

44. 2075 GIE Q.No. 9c A ballet dancer spins with 2.4 rev/s with her arms outstretched when the moment of inertia about the axis of rotation is I. With her arms folded, the moment of inertia about same axis becomes 0.6I. Calculate the new rate of spin. [4]

Solution

Given,
When arms outstretched,
Frequency (f_1) = 2.4 rev/sec
Moment of inertia (I_1) = I
When arms folded,
Frequency (f_2) = ?
We have, from principle of conservation of angular momentum,

$I_1 \omega_1 = I_2 \omega_2$

or, $I_1 2\pi f_1 = I_2 . 2\pi f_2$

or, $I_1 f_1 = I_2 f_2$

or, $I.2.4 = 0.6\, I. f_2$

or, $f_2 = 4$ rev/sec

45. 2073 Set D Q.No. 9c An electric fan is turned off, and its angular velocity decreases uniformly from 500 rev/min to 200 rev/min in 4 seconds. Find the angular acceleration and the number of revolutions made by the motor in the 4 sec interval. [4]

Solution

Given,

Initial frequency (f_1) = 500 rev/min = $\frac{500}{60}$ rev/ sec

Final frequency (f_2) = 200rev/min = $\frac{200}{60}$ rev/sec

Time (t) = 4.00 sec

Angular acceleration (α) = ?

Frequency (f) = ?

Time to come to rest (t') = ?

$\therefore \alpha = \frac{\omega - \omega_0}{t} = \frac{2\pi f_2 - 2\pi f_1}{t} = \frac{2\pi}{4}\left(\frac{200}{60} - \frac{500}{60}\right) = -7.85$ rad/sec^2

Now,

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{500}{60} \times 2\pi \times 4 + \frac{1}{2} \times (-7.85) \times 4^2 = 146.64$ radians.

$\therefore n = \frac{\theta}{2\pi}$

$= \frac{146.64}{2\pi} = 23.3$

Let, t' be the extra time to come rest then,

$\alpha = \frac{\omega - \omega_0}{t'}$

$-7.85 = \frac{0 - 2\pi f_2}{t'}$

$t' = \frac{2\pi \times 200}{7.85 \times 60} = 2.67$sec

46. 2072 Set C Q.No. 9d 2070 Supp (Set A) Q.No. 9 d 2061 Q.No. 3 b 2054 Q.No. 5 A constant torque of 500 Nm turns a wheel which has a moment of inertia 20 kgm^2 about its center. Find the angular velocity gained in 2 second and the kinetic energy gained. [4]

Solution

Given,
Torque (τ) = 500 N m
Moment of inertia (I) = 20 kg m^2
Time (t) = 2 sec
Angular velocity gained after 2 sec (ω) = ?
Kinetic energy gained (K.E.) = ?
Initial angular velocity (ω_0) = 0
We know that,

$\omega = \omega_0 + \alpha t$

or, $\omega = 0 + \frac{\tau}{I} t$

or, $\omega = 0 + \frac{500}{20} 2 \quad \left[\because \alpha = \frac{\tau}{I}\right]$

$\therefore \omega = 50$ rad sec^{-1}

K.E. gained, K.E. $= \frac{1}{2} I \omega^2 = \frac{1}{2} \times 20 \times (50)^2 = 25{,}000$ J

Hence, the required angular velocity gained in 2 sec. is 50 rad sec^{-1} and the required kinetic energy gained is 25,000 J.

47. 2072 Set E Q.No. 9b A disc of radius 1 m and mass 5 kg is rolling along a horizontal plane. Its moment of inertia about its centre is 2.5 kg m^2. If its velocity along the plane is 2 ms^{-1}, find its angular velocity and the total energy. [4]

Solution

Given,
Radius of disc (r) = 1 m
Mass of disc (m) = 5 kg
Moment of Inertia (I) = 2.5 kg m^2
Velocity (v) = 2 m/sec
Angular velocity (ω) = ?

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Total energy (T.E.) = ?

We have,

$v = \omega r$

or, $\omega = \frac{v}{r} = \frac{2}{1} = 2$ rad/sec

Total Energy = Linear energy + Rotational energy

$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 2.5 \times 2^2 = 10 + 5 = 15$ Joules

48. 2071 Supp Q.No. 9b A constant torque of 500Nm turns a wheel about its center. The moment of inertia about this axis is 100 kgm². Find the angular velocity gained in 4 seconds and kinetic energy gained after revolutions.

Solution

Given,

Torque (τ) = 500 Nm

M.I. (I) = 100 Kgm²

Time (t) = 4 sec.

No. of revolutions = 20 rev

Angular velocity (ω) = ?

Kinetic energy (K.E.) = ?

Now,

$\omega = \omega_0 + \alpha.t = 0 + \frac{\tau}{I}.t = \frac{500}{100} \times 4 = 20$ rad/sec

Let θ be total angular distance covered the $\theta = 2\pi r = 2\pi \times 20 = 40\pi$ radian

Now,

$\omega^2 = \omega_0^2 + 2\alpha\theta = 2 \times \frac{\tau}{I} \times 40\pi = 2\frac{500}{100} \times 40\pi = 4\pi$

$\omega = \sqrt{400\pi}$

Again,

K.E. $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 100 \times 400\pi = 62831.85$ J.

49. 2070 Supp (Set B) Q.No. 9 c A computer disk drive is turned on starting from the rest and has constant angular acceleration, (a) how long did it take to make the first complete rotation, and (b) what is its angular acceleration? Given that the disk took 0.750 sec for the drive to make its second complete revolution.

Solution

Given,

Initial angular velocity (ω_0) = 0 rad/sec

Time taken for first complete rotation (t_1) = ?

Time for second complete rotation (t_2) = t_1 + 0.75

Angular acceleration (α) = ?

For first rotation,

$\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$

or, $2\pi = 0 + \frac{1}{2}\alpha t_1^2$

or, $4\pi = \alpha t_1^2$ ... (i)

For second rotation,

$\theta_1 = \omega_0 t_2 + \frac{1}{2}\alpha t_2^2$

or, $4\pi = 0 + \frac{1}{2}\alpha (0.75 + t_1)^2$

or, $4\pi = \frac{1}{2}\alpha (0.75 + t_1)^2$ ... (ii)

Substrating equation (i) from (ii) we get

$0 = \frac{1}{2}\alpha (0.75 + t_1)^2 - \alpha t_1^2$

or, $(0.75 + t_1)^2 = 2t_1^2$

or, $0.75^2 + 2 \times 0.75 \times t_1 + t_1^2 = 2t_1^2$

or, $0.5625 + 1.5\, t_1 + t_1^2 = 2t_1^2$

or, $0.5625 + 1.5\, t_1 - t_1^2 = 0$

or, $t_1^2 - 1.5t_1 - 0.5625 = 0$

Solving this quarditic equation, we get

$t_1 = 1.81$ sec

Substituting the value of t_1 is equation (i), we get

$4\pi = \alpha (1.81)^2$

or, $\alpha = 3.83$ rad/sec²

∴ Time to complete first rotation is 1.81 sec and angular acceleration is 3.83 rad/sec².

50. 2069 Supp Q.No. 9c A constant torque of 200 Nm turns a wheel about its centre. The moment of Inertia about the axis in 100 kg m². Find the angular velocity gained in 4 seconds and the kinetic energy gained after 10 revolutions. [4]

Solution

Given,

Torque (τ) = 200 N m

Moment of inertia (I) = 100 kg m²

Time (t) = 4 sec

Angular velocity (ω) = ?

i. Using the relation

$\tau = I\alpha = I \times \frac{\omega}{t}$

or, $\omega = \frac{\tau t}{I} = \frac{200 \times 4}{100} = 8$ rad sec⁻¹

ii. Number of revolutions (n) = 10

Kinetic energy gained (K.E) = ?

If θ is the total angular distance covered, then

$\theta = 2\pi \times n = 2\pi \times 10 = 20\,\pi$ radians

Now, Using the relation

$\omega^2 = \omega_0^2 + 2\alpha\,\theta = 0 + 2 \times 2 \times 20\pi$

or, $\omega = \sqrt{251.33}$ [$\because \alpha = \frac{\tau}{I} = \frac{200}{100} = 2$ rad s⁻²]

$= 15.85$ rad s⁻¹

∴ K.E. gained after 10 rev. $(E_2) = \frac{1}{2} I\omega^2 = \frac{1}{2} 100 \times (15.85)^2 = 12561$ J

Hence, required angular velocity gained in 4 sec is 8 rads s⁻¹ and the required kinetic energy gained after 10 revolutions is 12561 J.

51. 2069 Set A Q. No. 9b A ballet dancer spins with 2.4 rev/sec with her arms outstretched when the moment of inertia about the axis of rotation is I. With her arms folded, the moment of inertia about the same axis becomes 0.6I. Calculate the new rate of spin. [4]

Please refer to 2075 GIE Q.No. 9c

52. 2069 (Set A) Old Q. No. 2b 2055 Q.No. 3 A constant torque of 200N turns a wheel about its center. The moment of inertia of it about the axis is 100 kg m². Find the K.E. gained after 20 revolutions when it starts from rest. [4]

Solution

Given,

Torque (τ) = 200 N m

Moment of inertia (I) = 100 kg m²

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Time (t) = 4 sec
Angular velocity (ω) = ?

i. Using the relation

$\tau = I\alpha = I \times \frac{\omega}{t}$

or, $\omega = \frac{\tau t}{I} = \frac{200 \times 4}{100} = 8$ rad sec^{-1}

ii. Number of revolutions (n) = 20
Kinetic energy gained (K.E) = ?
If θ is the total angular distance covered, then

$\theta = 2\pi \times n$
$= 2\pi \times 20 = 40\,\pi$ radians

Now, Using the relation

$\omega^2 = \omega_0^2 + 2\alpha\theta = 0 + 2 \times 2 \times 40\pi$

or, $\omega = \sqrt{502.72}$ $[\because \alpha = \frac{\tau}{I} = \frac{200}{100} = 2$ rad s$^{-2}]$
$= 22.42$ rad s^{-1}

∴ K.E. gained after 20 rev. $(E_2) = \frac{1}{2} I\omega^2 = \frac{1}{2} 100 \times (22.42)^2 = 25132.82$ J $= 25133$ J

Hence, required angular velocity gained in 4 sec is 8 rads s^{-1} and the required kinetic energy ga after 20 revolutions is 25133 J.

53. 2067 Supp Q.No. 9 c A ballet dancer spins about a vertical axis at 1 revolution per second with her stretched. With her arms folded, her moment of inertia about the axis decreases by 40%, calculate th rate of revolution.

Solution

Given,
When Arm stretched,
Moment of inertia $(I_1) = I$
Frequency $(f_1) = 1$ rev/sec
We Arm folded,
Moment of inertia $(I_2) = (100 - 40)$ % of $I = 60\%$ of $I = 0.6\,I$
Frequency $(f_2) = ?$
From principle of conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$
$I_1 2\pi f_1 = I_2 2\pi f_2$
$I_1 f_1 = I_2 f_2$
$I.1 = 0.6\,I\,f_2$
$f_2 = 1.67$ rev/ sec

54. 2067 Q.No. 9 b An electric fan is turned off, and its angular velocity decreases uniformly from 500 rev/m 200 rev/min in 4.00s. (a) Find the angular acceleration and the number of revolutions made by the mot 4.00s interval. (b) How many more seconds are required for the fan to come to rest if the ang acceleration remains constant?

Please refer to 2073 Set D Q.No. 9c

55. 2066 Q. No. 9 b A disc of moment of inertia 5×10^{-4} kg m^2 is rotating freely about the axis through its cent 40 rpm. Calculate the new revolution per minute if some wax of mass 0.02 kg is dropped gently on to disc 0.08 m from the axis.

Solution

Given,
Moment of inertia $(I_1) = 5 \times 10^{-4}$ kg m^2
Frequency $(f_1) = 40$ rpm
Mass added $(m) = 0.02$ kg
Radius of mass $(r) = 0.08$m

New frequency $(f_2) = ?$
New moment of inertia $I_2 = I_1 + mr^2$
or, $I_2 = 5 \times 10^{-4} + 0.02 \times 0.08^2 = 6.28 \times 10^{-4}$ kgm^2
From principle of conservation of angular momentum,
or, $I_1w_1 = I_2w_2$
or, $I_1 2\pi f_1 = I_2 2\pi f_2$
or, $I_1 f_1 = I_2 f_2$
or, $f_2 = \frac{I_1 f_1}{I_2} = \frac{5 \times 10^{-4} \times 40}{6.28 \times 10^{-4}} = 32$ rpm

∴ New frequency is 32 rpm.

56. 2063 Q.No. 3 b A constant torque of 200 Nm turns a wheel about its centre. The moment of inertia about this axis is 100 kg m^2. Find the kinetic energy gained after 20 revolutions.

Please refer to 2069 (Set A) Old Q. No. 2b

57. 2052 Q.No. 3 OR Speed of a body spinning about an axis increase from rest to 100 rev min^{-1} in 5 sec., if a constant torque of 20 Nm is applied. The external torque is then removed and the body comes to rest in 100 sec due to friction. Calculate the frictional torque.

Solution

Given,
Case I:
Initial angular velocity $(\omega_o) = 0$ rad/sec
Time taken (t) = 5 s

Final angular velocity $(\omega) = 2\pi \times 100$ rad min^{-1} $= 2\pi \times \frac{100}{60}$ rad sec^{-1}

Torque applied $(\tau) = 20$ Nm
Frictional torque $(\tau') = ?$
If I is the moment of inertia of a body about a give axis, then

$\tau = I\alpha$

$\alpha = \frac{\tau}{I}$ ... (i)

where, α is the angular acceleration.
Again,

$\omega = \omega_o + \alpha t$

$2\pi \times \frac{100}{60} = 0 + \frac{20}{I} \times 5$ ($\because$ using equation(i))

∴ $I = \frac{30}{\pi}$ ... (ii)

Case II
Time taken (t) = 100 sec
Final angular velocity $(\omega) = 0$

Initial angular velocity $(\omega_o) = 2\pi \times \frac{100}{60}$ rad sec^{-1}

Let α' be the angular acceleration, then

$\omega = \omega_0 + \alpha' t$

$0 = 2\pi \frac{100}{60} + \alpha' \times 100$

∴ $\alpha' = \frac{-2\pi}{60} = \frac{2\pi}{60}$ (in magnitude)

Thus, the magnitude of frictional torque becomes,

Frictional torque $= I\alpha' = \frac{30}{\pi} \times \frac{2\pi}{60} = 1$ Nm

Thus, the frictional torque (τ') is 1 Nm.

□□□

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Short Answer Questions

1. 2075 GIE Q.No. 1g Why do spring balances show wrong readings after long use?

When a spring balance has been used for a long time, elastic fatigue is developed. So, the spring of such a balance takes longer time to recover its original position and it shows wrong readings.

2. 2074 Set A Q.No. 1d 2073 Supp Q.No. 1c Will the Young's modulus of elasticity change if the load hanging on it is doubled? Why?

The Young's modulus of elasticity of a body is given by

$$Y = \frac{F/A}{e/l} = \frac{Fl}{Ae}$$ where, F/A = stress, e/l = strain

When the load hanging on the material is doubled, the Young's modulus of elasticity remains constant because the increase in load is balanced by the extension produced. The Young's modulus of elasticity of a material is constant quantity. It depends on ther nature of material, but not on the stress is applied to it. Therefore, Young's modulus of elasticity remains same although the load hanging on it is changed.

3. 2072 Set D Q.No. 1e What is elastic limit and breaking stress?

Elastic limit: The maximum value of the deforming force which a body can experience and still regains its original size and shape once the force has been removed is called elastic limit.

Breaking stress: Beyond the yield point of the body, if the deforming force is increased, it suddenly breaks. This point of loading is called the breaking point of the body and the stress which appears in the body at the breaking point is known as breaking stress.

4. 2072 Set E Q.No. 1e Explain the physical meaning of Poisson's ratio.

Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain. Hence, it gives the decrease in lateral size per unit increase in length of a wire within elastic limit. i.e. Lateral train / longitudinal strain.

Lateral strain = σ Longitudinal strain.

$$\sigma = \frac{\text{Lateral strain } (\beta)}{\text{Longitudinal strain } (\alpha)} = \frac{\beta}{\alpha}$$

5. 2071 Supp Q.No. 1f 2057 Q.No. 1 a Why are bridges declared unsafe after a long use?

Due to the regular loading and unloading of traffic after long use, bridges are elastically fatigued. Therefore, the bridges will lose gradually their elastic strength. The amount of strains in the bridge for a given stress will become large and they will collapse ultimately at any time. That's why, bridges are declared unsafe after long use.

6. 2070 Supp (Set A) Q.No. 1 g What happens in the Young's modulus of elasticity of a material when the load hanging on it is doubled?

Please refer to 2074 Set A Q.No. 1d

7. 2069 Supp Q.No. 1f Steel bridges are declared unsafe after a few decades of use. Why?

Please refer to 2071 Supp Q.No. 1f

8. 2069 (Set B) Q. No. 1f Compare the mechanical properties of a steel cable, made by twisting many thin wires together, with those of solid steel rod of the same diameter.

The mechanical strength of the steel cable, made by twisting many thin wires together is greater than the cable of solid steel rod of the same diameter. It means the breaking stress of a material with twinsting is greater than without twisting. Hence a twisting steel cable can hold the greater load than as solid rod of the same diameter.

9. 2069 Old (Set B) Q. No. 1f 2066 Q. No. 1f Explain in terms of breaking stress why elephant has thicker legs as compared to human beings?

The breaking stress is defined as the ratio of maximum breaking load to which the wire is subjected to the original cuoss sectional area.

$$\text{Breaking stress} = \frac{\text{Breaking weight}}{\text{Area}}$$

Breaking weight = Breaking stress × Area

This shows that the breaking weight is directly proportional to area. Thus, to balance the heavier weight of elephant, the area of the legs should be thicker. Hence, Elephant has thicker legs (to prevent from breaking) as compared to human beings.

10. 2065 Q.No. 1 a How will you justify that stone is more rigid than iron? [2]

We know that for a good rigid body, change in dimensions will be negligible for a given stress. Stone shows negligible strain for a given stress than that by iron. Therefore, stone is more rigid than iron.

11. 2064 Q.No. 2 a Two wires A and B have equal lengths and are made of same material. If the diameter of wire A is twice that of wire B, which wire has the greater extension for a given load?

Let l be the length and Y be the Young's modulus of elasticity of a wire, then its extension e is obtained as,

$$Y = \frac{F/A}{e/l} = \frac{Fl}{eA}$$

$$\text{or, } e = \frac{Fl}{YA}$$

$$\text{or, } e \propto \frac{1}{d^2}, \text{ for same Y, } l \text{ and F,} \quad [\because A = \frac{\pi d^2}{4}]$$

$$\text{or, } e_A \propto \frac{1}{d_A^2} \quad \ldots \text{(i)}$$

$$\text{and } e_B \propto \frac{1}{d_B^2} \quad \ldots \text{(ii)}$$

Dividing equation (i) by (ii), we get

$$\frac{e_A}{e_B} = \frac{d_B^2}{d_A^2}$$

Since, $d_A = 2d_B$

$$\frac{e_A}{e_B} = \frac{d_B^2}{4d_B^2} = \frac{1}{4}$$

$$\text{or, } e_B = 4e_A$$

∴ Wire B extends four times than A.

12. 2063 Q.No. 1d 2059 Q.No. 1 a Why are rubbers used as vibration absorber? [2]

The rubber can be compressed or stretched easily because of its coiled form of molecules. The area of elastic hysteresis loop of rubber is large. It takes longer time during compressing and stretching. Being so, it can store large amount of elastic potential energy and finally converts into heat energy. Thus, a rubber sheet absorbs vibration kinetic energy and releases it slowly to heat energy. Hence, the rubber sheets are used as vibration absorber.

13. 2062 Q.No. 1 a Differentiate between elasticity and plasticity. [2]

The main two differences between *elasticity and plasticity* are:

a. The property of the body, which regains its original configuration after removal of deforming force, is called elasticity. For example: the property of the body like quartz, phosphor and bronze is perfectly elastic in nature. The property of the body, which does not regain its original configuration after removal of deforming force, is called plasticity. For example, the property of the body like paraffin, wax is perfectly plasticity.

b. For elastic body, even a large deforming force brings about a small change in configuration that may or may not be noticed. For plastic body, even a small deforming force brings about a large change in configuration.

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14. 2061 Q.No. 1 a Explain why steel is said to be more elastic than rubber.

The Young's modulus for a solid is given by:

$$Y = \frac{F/A}{e/L} = \frac{FL}{Ae}$$

For rubber and steel, we have,

$$Y_r = \frac{F\ L}{A\ e_r} \text{ and } Y_s = \frac{F\ L}{A\ e_s} \Rightarrow \frac{Y_r}{Y_s} = \frac{e_s}{e_r}$$

When both samples are stretched by same force, the extension produced in the rubber will be much greater than that in steel.

i.e. $e_r > e_s$

Hence, $Y_s > Y_r$.

Being greater value of Young's modulus, steel is more elastic than rubber.

15. 2058 Q.No. 1 a Explain which one is more elastic-rubber or steel.

Please refer to 2061 Q.No. 1 a

Long Answer Questions

16. 2076 Set C Q.No. 5c Define elastic limit and Young's modulus of elasticity. How can you determine Young's modulus for a material in the laboratory? Explain.

Elastic limit: The extension produced in an elastic body is directly proportional to the producing it but this proportionality holds the force within a certain limit of deformation after the law of proportionality does not hold. This limit is known as elastic limit.

Young's modulus of elasticity (Y): It is defined as the ratio of normal stress to longitudinal strain

i.e. $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

The SI unit of Young's modulus of elasticity is Nm^{-2}.

Determination of Young's modulus (Y): The magnitude of Y for a material in the form of wire be determined using the apparatus as shown in figure. A and B are two long thin wires of material suspended from a rigid support. The wire A is kept taut by weight L called dead attached to its lower end which carries a scale M. V is vernier caliper carried by wire B.

The initial reading on the scale with no load is noted. Now, the wire B is loaded in equal steps kg and corresponding readings on the vernier are noted. The loads, W, are gradually remov steps of 0.5 kg and the vernier readings are again noted. If these readings are nearly the same readings when the weights were added, this shows that elastic limit was not exceeded.

The magnitude of Young's modulus can be calculated by plotting a graph of the force F (= mg) in n against the average extension e in meter. A straight line passing through the origin is obtained as sho figure:

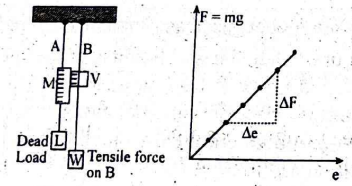


Fig: Determination of Y

We have,

$$Y = \frac{F}{e} \times \frac{l}{A}$$

From the graph, the value of $\frac{F}{e}$ can be calculated which is equal to the gradient $\frac{\Delta F}{\Delta e}$ of the str line.

Thus, knowing the value of $\frac{F}{e}$, the original length l and the cross-section A of the wire, the Yo modulus of elasticity can be calculated.

17. 2075 GIE Q.No. 5d 2071 Supp Q.No. 5d 2069 (Set A) Q. No. 5d 2067 Q.No. 5 d Derive an expression for the energy stored in a stretched string. Define the term energy density of a body under strain. [4]

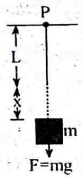
Energy Stored in a stretched wire: When a deforming force is applied to a wire, it is extended against restoring force set up in the wire due to its elasticity. Thus, the applied force works against the restoring force. This work done by deforming force is stored as potential energy in the wire.

To calculate energy stored in a stretched wire, consider a wire of length, L, suspended from point P. A be the area of cross section of the wire. A mass m, is attached at its lower end, therefore, mg is the deforming force, F applied to the wire. Let x be extension produced in the wire as in figure. If Y be the Young's modulus of elasticity of material of the wire, then

$$F = \frac{YAx}{L} \quad \ldots (i)$$

If dx is the further small extension, then small work done, dW is given by

$$dW = F.dx = \frac{YAx}{L}\,dx \quad \ldots (ii)$$



The total work done by deforming force F in extending the wire from 0 to l is

$$W = \int_0^l \frac{YA}{L} x\,dx = \frac{AY}{L} \cdot \frac{l^2}{2} \quad \ldots (iii)$$

This is the elastic potential energy stored in the wire. This may be written as,

$$W = \frac{1}{2}\left(\frac{YA}{L} \times l\right) \times l \Rightarrow W = \frac{1}{2} \times F \times l$$

$$W = \frac{1}{2} \times \text{maximum stretching force} \times \text{extension}$$

$$W = \text{average force} \times \text{extension}$$

Equation (iii) also can be written as,

$$W = \frac{1}{2} \times \frac{Yl}{LA} \times \frac{l}{L} \times AL$$

$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume of the wire.}$$

Thus, the energy per unit volume of wire is called energy density and is given by

$$U = \frac{W}{V} = \frac{1}{2} \times \text{stress} \times \text{strain.}$$

18. 2075 Set B Q.No. 5b Explain Hooke's law with necessary figure and derive and expression for the energy density in a stretched wire. [4]

Hooke's Law: It states that within elastic limit, the stress is proportional to strain. i.e., stress ∝ strain

$$\frac{\text{stress}}{\text{strain}} = \text{constant (E)}$$

where E is called modulus of elasticity or coefficient of elasticity of the elastic body.

Since, strain has no dimension and then dimension of modulus of elasticity is same as that of stress.

Experimental verification of Hooke's law:

Consider a spring of length L suspended from a rigid support as in figure (a). The spring carries a pan at its lower end so that it can be loaded. A pointer P slides over a scale, fitted parallel to the spring, so that the position of the spring can be noted.

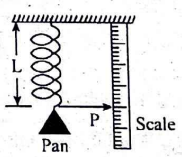


Figure (a)

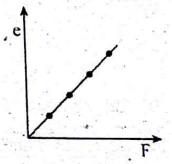


Figure (b)

The pan is loaded in steps of 0.5 kg, and corresponding pointer readings are noted. The difference between each reading with first reading give the extension produced in the spring by different weights (or load).

A graph between load 'F' and extension produced 'e' is drawn as shown in figure (b), which a straight line is passing through origin. This indicates that,

$F \propto e$

$F = ke \quad \ldots (i)$

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Dividing both sides of equation (i) by A and also multiplying and dividing the R.H.S. by L, we get

$$\frac{F}{A} = \left(\frac{kL}{A}\right)\frac{e}{L}$$

Since, cross-sectional area A and L are constants, $\frac{kL}{A}$ is also constant.

$\frac{F}{A} \propto \frac{e}{L}$ i.e. stress α strain.

This verifies Hooke's law.

Second part: Please refer to 2075 GIE Q.No. 5d

19. 2074 Supp. Q.No. 5c What does the term Strain refer to? Find an expression for energy stored in a stretched wire.

Strain: The strain of body is defined as the ratio of change in configuration (i.e. shape, length or volume) to the original configuration of the body when deforming force is applied on it, i.e.

$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$; it has no unit and dimension.

Second Part: Please refer to 2075 GIE Q.No. 5d

20. 2073 Supp Q.No. 5a Derive an expression for the energy stored in a stretched wire. Also define Poisson's ratio.

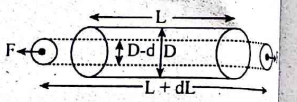
First Part: Please refer to 2075 GIE Q.No. 5d

Poisson's ratio: The ratio of the lateral strain to the longitudinal strain due to given stress, within the elastic limit is called Poisson's ratio. It is denoted by σ and given by

$\sigma = \frac{\text{lateral strain } (\beta)}{\text{tangential strain } (\alpha)}$

This ratio is found to be constant for the given material.

Consider a wire of length L, and diameter D. It is stretched with force F, therefore (L + dL) is its extended length i.e. dL is extension of the wire. (D – d) be the new diameter of extended wire, i.e. d is contraction in diameter as shown in figure.



∴ Tangential (or linear) strain, $(\alpha) = \frac{dL}{L}$ and Lateral strain, $\beta = \frac{d}{D}$

Thus, $\sigma = \frac{\beta}{\alpha} = \frac{d}{D}\cdot\frac{L}{dL}$

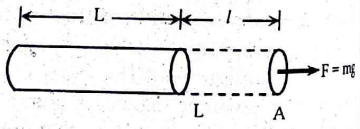
21. 2073 Set D Q.No. 5d Define Young's modulus of elasticity. Deduce an expression for the energy stored in stretched wire.

Young's Modulus of Elasticity, Y

It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit. Thus,

$\text{Young's modulus, } Y = \frac{\text{normal stress}}{\text{longtidinal strain}}$

$= \frac{F/A}{l/L} = \frac{FL}{Al}$ ... (i)



Energy stored in a Stretched Wire: Please refer to 2075 GIE Q.No. 5d

22. 2071 Set C Q.No. 5 b What do you understand by Poisson's ratio? Derive an expression for energy stored in stretched wire.

Poisson's ratio: Please refer to 2073 Supp Q.No. 5a

Second Part: Please refer to 2075 GIE Q.No. 5d

23. 2071 Set D Q.No. 5 b Define Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio.

Types of Moduli of elasticity

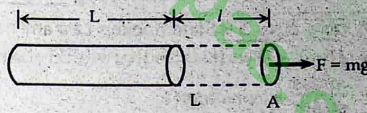
There are three types of Moduli of elasticity depending upon the types of strees and strain, described below:

i. Young's Modulus of Elasticity, Y

It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit. Thus,

$\text{Young's modulus, } Y = \frac{\text{normal stress}}{\text{longtidinal strain}}$

$= \frac{F/A}{l/L} = \frac{FL}{Al}$ ... (i)



ii. Bulk Modulus of Elasticity, B

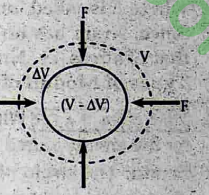
It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit. Thus,

$B = \frac{\text{normal stress}}{\text{volumetric strain}} = \frac{F/A}{-v/V}$

If P is the increase in pressure applied on the spherical body then,

$F/A = P$

$\therefore B = -\frac{PV}{v}$



iii. Shear Modulus or Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain, within the elastic limit. Thus

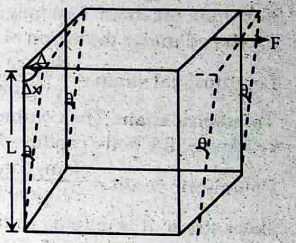
$\eta = \frac{\text{tangential stress}}{\text{shearing strain}} = \frac{F/A}{\theta}$

From figure, $\tan\theta = \frac{\Delta x}{L}$

Since θ is very small, so $\tan\theta \approx \theta$

$\therefore \theta = \frac{\Delta x}{L}$

Hence, $\eta = \frac{F/A}{\Delta x/L} = \frac{FL}{A\,\Delta x}$



Poisson's ratio: Please refer to 2073 Supp Q.No. 5a

24. 2069 (Set B) Q. No. 5b What are the proportional limit and the elastic limit? Derive an expression for the energy stored in a stretched wire? [4]

Proportional limit and the elastic limit: In a stress strain graph, proportional limit is a point at which the stress is directly proportional to the strain and the elastic limit is the point beyond the proportional limit at which the wire regains its original length but Hooke's law is not followed is called elastic limit.

Second part: Please refer to 2075 GIE Q.No. 5d

25. 2069 (Set A) Old Q.No. 3a What do you mean by elastic limit? Derive an expression for the energy stored in stretched wire. [3+1]

Elastic limit: The extension produced in an elastic body is directly proportional to the force producing it but this proportionality holds the force within a certain limit of deformation after which the law of proportionality does not hold. This limit is known as elastic limit.

Second part: Please refer to 2075 GIE Q.No. 5d

26. 2068 Q.No. 5 d Define stress and strain. Derive an expression for energy stored in a stretched wire. [4]

Stress: The restoring force per unit area of the body is called stress. It is also defined as the deforming force per unit area of the body. That is,

$\text{Stress} = \frac{\text{external force or deforming force}}{\text{area of the body}} = \frac{F}{A}$

In SI, the unit of stress is Nm^{-2} and dimensional formula of stress is $[ML^{-1}T^{-2}]$.

Strain: The strain of the body is defined as the ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body. i.e.

$\text{strain} = \frac{\text{change in configuration}}{\text{original configuration}}$

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Since, strain is a ratio of two similar quantities and has no unit and dimensions.

Second part: Please refer to 2075 GIE Q.No. 5d

27. 2066 Q.No. 3 a State Hooke's Law and describe different types of modulus of elasticity. [1+

Hook's law: It states that within elastic limit, the stress is proportional to strain. i.e. stress ∝ strain

$$\frac{\text{stress}}{\text{strain}} = \text{constant (E)}$$

where E is called modulus of elasticity or coefficient of elasticity.

Since, strain has no dimension and then the unit and dimension of modulus of elasticity is same that of stress. i.e. Nm^{-2} and $[ML^{-1}T^{-2}]$

Types of Moduli of elasticity: Please refer to 2071 Set D Q.No. 5 b

28. 2062 Q.No. 3 a OR What does the term strain refer to? What are different types of strain? Find an expressio for the energy stored in a wire.

Strain: The strain of the body is defined as the ratio of the change in configuration (i.e. shape, leng or volume) to the original configuration of the body. i.e.

$$\text{Strain} = \frac{\text{change in configuration}}{\text{original configuration}}$$

Since, strain is a ratio of two similar quantities and has no unit and dimensions. The types of strain are

i. **Longitudinal strain:** The longitudinal strain is defined as the change in length per unit original leng produced under the action of the deforming force.

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}}$$

ii. **Volumetric strain:** The volumetric strain in a body is defined as the change in volume per unit origi volume of the body produced under the action of the deforming force.

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

iii. **Shear strain:** It is defined as the angle, θ through which the face of the body originally perpendicu to the fixed face is turned when it is under the shearing stress.

Second Part: Please refer to 2075 GIE Q.No. 5d

29. 2061 Q.No. 3 a Why is energy stored in a wire? Prove that the elastic energy stored per unit volume i stretched wire is equal to ½ × stress × strain. [5

Please refer to 2075 GIE Q.No. 5d

30. 2060 Q.No. 3 a Define elastic limit and Young's modulus of elasticity. Explain how would you determ Young's modulus for a material in the form of a wire. [5

Please refer to 2076 Set C Q.No. 5c

31. 2058 Q.No. 3 a State Hooke's law. How would you verify it experimentally?

Please refer to 2075 Set B Q.No. 5b

32. 2057 Q.No. 3 a OR What is Poisson's ratio? Derive an expression for the energy stored in a stretched wire?

Poisson's ratio: Please refer to 2073 Supp Q.No. 5a

Second Part: Please refer to 2075 GIE Q.No. 5d

Numerical Problems

33. 2076 Set B Q.No. 9c A copper wire and a steel wire of the same cross sectional area and of length 1m and respectively are connected end to end. A force is applied, which stretches their combined length by 1c Find how much each wire is elongated.

Solution

Given,

Length of copper wire (l_c) = 1m

Length of steel wire (l_s) = 2m

Total extension (e) = 1 cm = 10^{-2} m

Young's modules for copper (Y_c) = 1.2×10^{11} Nm^{-2}

Young's modules for steel (Y_s) = 2×10^{11} Nm^{-2}

Extension for copper wire (e_c) = ?

Extension for steel wire (e_s) = ?

Here, given

$e = e_c + e_s$

i.e. $e_c + e_s = 10^{-2}$m ...(i)

Now for copper wire

$$Y_c = \frac{\text{Stress}}{\text{Strain}}$$

or, $\text{Stress} = Y_C \times \text{Strain} = Y_C \times \frac{e_C}{l_C}$

Similarly for steel wire,

$\text{Stress} = Y_S \times \frac{e_s}{l_s}$

Since cross sectional area and force on both wire be same, then stress (i.e. force per unit area) is equal for both wire. Then,

$Y_C \times \frac{e_C}{l_C} = Y_S \times \frac{e_S}{l_S}$

or, $\frac{1.2 \times 10^{11} \times e_C}{1} = \frac{2 \times 10^{11} \times e_S}{2}$

or, $e_c \times 1.2 = e_s$

or, $e_s = 1.2\, e_C$ ...(ii)

From equation (i) and (ii), we have

$e_C + 1.2\, e_C = 10^{-2}$

or, $2.2 e_C = 10^{-2}$

or, $e_C = 0.0045$ m

and

$e_s = 1.2 \times e_C = 1.2 \times 0.0045 = 0.0055$

$\therefore$ $e_C = 4.5 \times 10^{-3}$ m and $e_S = 5.5 \times 10^{-3}$ m

34. 2075 Set A Q.No. 9c The rubber cord of a catapult has a cross-sectional area 1.0mm² and total unstretched length 10 cm. It is stretched to12 cm and then released to project a missile of mass 5.0g. Calculate the velocity of projection. [4]

Solution

Given,

Area of cross section (A) - 10mm² = 1×10^{-4} m²

Length of wire (l) = 10 cm = 0.1m

Stretched length (L) = 12 cm = 0.12 m

Elongation (e) = L – l = 0.2 – 0.1 = 0.02 m

Mass (m) = 5g = 5×10^{-3} kg

Young's modulus of elasticity (Y) = 5×10^8 N/m²

Velocity of projection (v) = ?

Now, we have

$$Y = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{Fl}{A.e.}$$

or, $F = \frac{Y.A.e}{l} = \frac{5 \times 10^8 \times 10^{-6} \times 0.02}{0.01} = 100$ N

Now,

Energy stored, $E = \frac{1}{2} F.e. = \frac{1}{2} \times 100 \times 0.02 = 1$ J

Let v be the velocity of projection, then,

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$$\frac{1}{2}mv^2 = E$$

or, $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1}{5 \times 10^{-3}}} = 20 \text{ m/sec}$

35. 2074 Set B Q.No. 9b **A wire of length 2.5 m and area of cross-section 1×10^{-6} m² has a mass of 15 kg hanged on it. What is the extension produced? How much is the energy stored in the extended wire if Young's modulus of wire is 2×10^{11} Nm^{-2}.**

Solution

Given,
Length of wire, $l = 2.5$ m
Area of cross-section, $A = 1 \times 10^{-6}$ m²
Mass hanged, $m = 15$ kg
Young's modulus, $Y = 2 \times 10^{11}$ Nm^{-2}
Elongation produced (e) = ?
Energy stored (E) = ?
We know,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{l/e} = \frac{Fl}{Ae}$$

or, $e = \frac{F.l}{AY} = \frac{mg.l}{AY}$ [$\because F = mg$]

$$= \frac{15 \times 10 \times 2.5}{1 \times 10^{-6} \times 2 \times 10^{11}} = 1.875 \times 10^{-3} \text{ m}$$

And

Energy stored (E) $= \frac{1}{2} \times F \times e = \frac{1}{2} \times 150 \times 1.875 \times 10^{-3}$ [$\because F = mg = 15 \times 10 = 150$N]

$= 1.41 \times 10^{-1}$ J

36. 2073 Set C Q.No. 9d **A uniform steel wire of density 8000 kg m^{-3} weight 20 gm and is 2.5 m long. It lengthens by 1 mm when stretched by a force of 80 N. Calculate the value of the Young's modulus of steel and energy stored in the wire.**

Solution

Given,
Density of steel (ρ) = 8000 kgm^{-3}
Mass of steel (m) = 20 gm = 0.020 kg
Length of wire (l) = 2.5 m
Elongation (e) = 1mm = 1×10^{-3}m
Force (F) = 80 N
Young's modulus of Elasticity (Y) = ?
Energy stored (E) = ?
We have,

$$\rho = \frac{m}{V}$$

$$\rho = \frac{m}{A.c.}$$

$$A = \frac{m}{\rho.c.}$$

Again,

$$Y = \frac{F/A}{e/l} = \frac{F.l}{A.e} = \frac{F.l}{e.\frac{m}{\rho.l}} = \frac{f.l^2\rho}{e.m} = \frac{80 \times 2.5^2 \times 8000}{1 \times 10^{-3} \times 0.02} = 2 \times 10^{11} \text{ N/m}^2$$

$$E = \frac{1}{2}F.e. = \frac{1}{2} \times 80 \times 1 \times 10^{-3} = 40 \times 10^{-3} \text{ J}$$

37. 2072 Supp Q.No. 9d **What force is required to stretch a steel wire of cross-sectional area 1cm² to double its length?** **[4]**

Solution

Given,
Force (F) = ?
Area of cross section (A) = 1cm² = 10^{-4} m²
Young' modulus for seel (Y) = 2×10^{11} N/m²
Let length of wire (l) = x m.
After extension its length (L) becomes double
i.e. 2x, so extension (e) = $\Delta l = L - l = 2x - x = x$ m.
Now, We have,

$$Y = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{F}{A}$$

or, $F = Y \times A = 2 \times 10^{11} \times 10^{-4} = 2 \times 10^7$ N

38. 2072 Set C Q.No. 9c **Calculate the work done in stretching a steel wire 100 cm in length and of cross-sectional area 0.030 cm² when a load of 100N is slowly applied before the elastic limit is reached.** **[4]**

Solution

Given,
Length of wire of steel (l) = 1000 cm = 1m.
Cross-sectional area (A) = 0.03 cm² = 0.03×10^{-4} m²
Load applied (F) = 100 N
Young's modulus of elasticity (y) = 2×10^{11} N/m²
Work done (W) = ?
Now,

$$Y = \frac{F/A}{e/l}$$ [$\because$ e = extension]

or, $Y = \frac{F \times l}{e \times A}$

or, $e = \frac{F \times l}{A \times Y} = \frac{100 \times 1}{0.03 \times 10^{-4} \times 2 \times 10^{11}} = 1.667 \times 10^{-4}$ m

Now, work done to stretch wire by e is

$W = \frac{1}{2}F.e. = \frac{1}{2} \times 100 \times 1.667 \times 10^{-4} = 8.33 \times 10^{-3}$ J

39. 2070 Supp (Set A) Q.No. 9 c **A uniform steel wire of density 7800 kgm^{-3} weights 16gm and is 250 cm long. It lengthens by 1.2 mm when stretched by a force of 80 N. Calculate the Young's modulus and the energy stored in the wire.** **[4]**

Solution

Given,
Density of steel (ρ) = 7800 kg m^{-3}
Mass of wire (m) = 16gm = 0.016 kg
Length of wire (l) = 250cm = 2.5 m
Elongation (e) = 1.2mm = 1.2×10^{-3} m
Load (F) = mg = $8 \times 10 = 80$ N
Yours' modulus of elasticity (Y) = ?
Energy stored (E) = ?
We have,

$$Y = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{F.l}{e.A} = \frac{F.l}{e.\frac{m}{l.\rho}} = \frac{F.l^2\rho}{e.m} = \frac{80 \times 2.5^2 \times 7800}{1.2 \times 10^{-3} \times 0.016} = 2.0 \times 10^{11} \text{N/m}^2$$

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$\therefore$ E $= \frac{1}{2}$.F.e $= \frac{1}{2}$.8 . $1.2 \times 10^{-3} = 4.8 \times 10^{-2}$ J

40. 2070 Supp (Set B) Q.No. 9 d **A steel cable with cross-sectional area 3 cm² has an elastic limit of 2.40×10^8 Pa. Find the maximum upward acceleration that can be given a 1200 kg elevator supported by the cable if the stress is not exceed one third of the elastic limit.**

Solution

Given,
Area of cross section of cable (A) = 3 cm² = 3×10^{-4} m²
Elastic limit = 2.4×10^8 Pa

Stress $= \frac{1}{3} \times 2.4 \times 10^8$ Pa

Mass (m) = 1200 kg
Upward acceleration (a) = ?
We have,
Stress = F/A

or, F = Stress × A $= \frac{1}{3} \times 2.4 \times 10^8 \times 3 \times 10^{-4} = 2.4 \times 10^4$ N

a $= F/m = \frac{2.4 \times 10^4}{1200}$

$\therefore$ a $= 20$ m/sec²

41. 2070 Set C Q.No. 9 c **A vertical brass rod of circular section is loaded by placing a 5 kg weight on top of it. If its length is 50 cm and radius of cross section is 1 cm, find the contraction of rod and the energy stored in it.**

Solution

Given,
Mass of the brass (m) = 5 kg
Length of rod (l) = 50 cm = 0.5 m
Radius of brass rod (r) = 1 cm = 0.01m
Contraction of rod (e) = ?
Energy stored (W) = ?
Young's modulus of brass (Y) = 3.5×10^{10} N/m²
Now,
Load on brass F = mg = 5 × 10 = 50N
We have, for contraction of rod,

$Y = \frac{F/A}{e/l} = \frac{Fl}{Ae}$

$e = \frac{Fl}{Y \pi r^2} = \frac{50 \times 0.5}{3.5 \times 10^{10} \times \pi \times (0.01)^2} = 2.27 \times 10^{-6}$ m

Energy stored $= \frac{1}{2}$ F.e $= \frac{1}{2} \times 50 \times 2.27 \times 10^{-6} = 5.7 \times 10^{-5}$ J

42. 2070 Set D Q.No. 9 c **Calculate the work done is stretching a steel wire 100 cm in length end of cross sectional area 0.03cm² when a load of 100N is slowly applied without the elastic limit being reached.**

Please refer to 2072 Set C Q.No. 9c

43. 2069 (Set B) Q. No. 9d **How much force is required to punch a hole 1 cm in diameter in a steel sheet 5mm thick whose shearing strength is 2.76×10^8 Nm⁻².**

Solution

Given,
Diameter of hole (d) = 1m = 0.01m
Thickness of sheet (t) = 5mm = 5×10^{-3}m
Shear in strength (η) = 2.76×10^8 Nm⁻²
Force required (F) = ?
We have,

Shear strength $= \frac{F}{A}$

$2.76 \times 10^8 = \frac{F}{\pi.d.t}$

[Length of circumference $l = 2\pi r$
$\therefore$ $A = l \times t = 2\pi r \times t = \pi d.t$]

F $= 2.76 \times 10^8 \times \pi \times 0.01 \times 5 \times 10^{-3} = 43353.97$ N

44. 2068 Old Q.No. 3 b **Find the work done in stretching a wire of cross sectional area 10^{-2} cm² and 2m long through 0.1 mm, if Y for the material of wire is 2×10^{11} Nm⁻². [3]**

Solution

Given,
Work done (W) = ?
Cross sectional area (A) = 10^{-2} cm² = 10^{-6}m²
Length of wire (l) = 2m
Elongation of wire (e) = 0.1mm = 0.1×10^{-3} m
Young's modulus (Y) = 2×10^{11} N/m²
We have;

$Y = \frac{F/A}{e/l} = \frac{F.l}{e\,A}$

or, F $= \frac{Y.e.\,A}{l}$

$\therefore$ W $= \frac{1}{2}$F.e. $= \frac{1}{2}\frac{Y.e^2\,A}{l} = \frac{1}{2} \times \frac{2 \times 10^{11}.(0.1 \times 10^{-3})^2.10^{-6}}{2} = 5 \times 10^{-4}$ J

45. 2067 Supp Q.No. 9 d **A uniform steel wire of density 7800 kgm⁻³ weighs 16 gram and is 250 cm. It lengthens by 1.2 mm when a load of 8 kg is applied. Calculate the value Young's modulus for the steel and the energy stored in the wire.**

Please refer to 2070 Supp (Set A) Q.No. 9 c

46. 2064 Q.No. 3 b **A steel wire of density 8000 kg m⁻³ weighs 24 gm and is 250 cm. long. It lengthens by 1.2 mm when stretched by a force of 80N. Calculate the Young's modulus for the steel and the energy stored in the wire.**

Solution

Given,
Density of steel (ρ) = 8000 kg m⁻³
Mass of wire (m) = 24gm = 0.024 kg
Length of wire (l) = 250cm = 2.5 m
Elongation (e) = 1.2mm = 1.2×10^{-3} m
Load (F) = 80 N
Young's modulus of elasticity (Y) = ?
Energy stored (E) = ?
We have,

$Y = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{F.l}{e.A} = \frac{F.l}{e.\frac{m}{l.\rho}} = \frac{F.l^2\rho}{e.m}$ $\quad [\because \rho = \frac{m}{v} = \frac{m}{A \times c} \Rightarrow A = \frac{m}{\rho \times c}]$

$= \frac{80 \times 2.5^2 \times 8000}{1.2 \times 10^{-3} \times 0.024} = 1.4 \times 10^{11}$ N/m²

$\therefore$ E $= \frac{1}{2}$.F.e $= \frac{1}{2} \times 80 \times 1.2 \times 10^{-3} = 4.8 \times 10^{-2}$ J

□□□

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Short Answer Questions

1. 2075 Set B Q.No. 1f The position of a certain object in S.H.M. is given as x = 0.05 cos (290t + 2.5); where x is in meter and t is in sec. What are the amplitude, period and initial phase angle for this motion?

The given equation of S.H.M is

$x = 0.05 \cos(290t + 2.5)$ ... (i)

Comparing equation (i), with

$x = r\cos(\omega t + \phi)$

We get,

Amplitude (r) = 0.05 m

$\omega = 290$

or, $T = \frac{2\pi}{\omega}$

or, $T = \frac{2 \times 3.14}{290}$

i.e. Period (T) = 0.022 sec.

Phase angle, ϕ = 2.5 rad

2. 2074 Set B Q.No. 1c If the length of second's pendulum is increased further by 200 percent, will it lose or gain time?

The time period of pendulum is $t = 2\pi\sqrt{\frac{l}{g}}$, when the length l of the second pendulum increased by 200%, then time period of the pendulum also increased. So, the pendulum will gain time and become late.

3. 2073 Supp Q.No. 1b 2072 Set E Q.No. 1d What do you understand by a second's pendulum? If it is taken to moon, will it gain or lose time? Why?

The simple pendulum whose time period is 2 seconds is called second pendulum. When it is taken to moon, the acceleration due to gravity is less in moon, the time period will be more and hence, the second pendulum will loose time.

4. 2072 Supp Q.No. 1f A simple harmonic motion is represented, in usual notation, by y =a sin (wt +ø). Find its acceleration.

Given displacement of SHM is

$y = a\sin(\omega t + \phi)$

or, $\frac{dy}{dt} = a.\omega.\cos(\omega t + \phi)$

or, $\frac{d^2y}{dt^2} = a\omega^2\{-\sin(\omega t + \phi)\}$

or, acceleration $= \frac{d^2y}{dt^2} = -\omega^2 y$ $[\because y = a\sin(\omega t + \phi)]$

$\therefore a = -\omega^2 y$

This is required acceleration.

5. 2071 Set C Q.No. 1 a How does the frequency of vibration of a simple pendulum related with its length? Hence, estimate the frequency of a second's pendulum.

The time period of vibration of simple pendulum is given as

$t = 2\pi\sqrt{\frac{l}{g}}$ ... (i)

where l is length of pendulum and g is acceleration due to gravity. Since,

frequency, $f = \frac{1}{t} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

for second pendulum, t = 2 sec, then

$2 = 2\pi\sqrt{\frac{l}{g}}$

or, $\frac{1}{\pi} = \sqrt{\frac{l}{g}}$

or, $l = \frac{g}{\pi^2}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{g}{\frac{g}{\pi^2}}} = \frac{1}{2}$

Hence, frequency of vibration of second's pendulum is $\frac{1}{2}$ Hz.

6. 2071 Set C Q.No. 1 d In usual notation, a simple harmonic motion is given as y = a sin (ωt – ϕ). Find its acceleration. [2]

Given displacement of SHM is

$y = a\sin(\omega t - \phi)$

or, $\frac{dy}{dt} = a.\omega.\cos(\omega t - \phi)$

or, $\frac{d^2y}{dt^2} = a\omega^2\{-\sin(\omega t - \phi)\}$

or, acceleration $= \frac{d^2y}{dt^2} = -\omega^2 y$ $[\because y = a\sin(\omega t - \phi)]$

$\therefore a = -\omega^2 y$

This is required acceleration.

7. 2071 Set D Q.No. 1 f A SHM is represented as y = a cos (ωt + ϕ) in usual notation. Find its acceleration. [2]

Given displacement of SHM is

$y = a\cos(\omega t + \phi)$

or, $\frac{dy}{dt} = -a.\omega.\sin(\omega t + \phi)$

or, $\frac{d^2y}{dt^2} = -a\omega^2\{\cos(\omega t + \phi)\}$

or, acceleration $= \frac{d^2y}{dt^2} = -\omega^2 y$ $[\because y = a\cos(\omega t + \phi)]$

$\therefore a = -\omega^2 y$

This is required acceleration.

8. 2069 Supp Q.No. 1e A pendulum clock is taken to moon, will it gain or lose the time? Why? [2]

The time period of pendulum clock is $t = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of pendulum. When it is taken to moon, acceleration due to gravity decreases and the time period will be more and hence, the pendulum will loose time.

9. 2068 Q.No. 1 g Explain why soldiers are ordered to break steps while crossing a bridge. [2]

Soldiers are ordered to break their steps while crossing the bridge because when they march in steps, the frequency of marching coincides with the natural frequency of the bridge. In this case, resonance will occur in the bridge and begins to oscilate with maximum amplitude which may cause the destruction of the bridge. To avoid such a destruction of the bridge, soldiers are ordered to break their steps while crossing it.

10. 2067 Supp Q.No. 1 f Why are bells made of metal and not of wood? Explain. [2]

There is high damping of sound waves in wood but very low damping in metals. If the damping is high, sound can travel for short distance but if the damping is less, sound can travel for long distances for longer duration and sound can be heard at greater distances. Hence, bells are made of

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metals but not of wood.

11. 2066 Q. No. 1 d A body is moving in a circular path with constant speed. Is this motion a simple harmonic motion? Why?

No, if a body is moving in a circular path with constant speed, it does not simple harmonic motion because, for a motion to be simple harmonic, the acceleration should be proportional to displacement and always directed towards the mean posotion. The body moving in circular path with constant speed does not satisfy these conditions. So, its motion is not simple harmonic motion.

12. 2066 Old Q. No. 1 e A pendulum clock is in an elevator that descends at constant velocity. Does it keep correct time? If the some clock is in an elevator in free fall, does it keep correct time?

The time period of a pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. In an elevator that descends constant velocity, the acceleration due to gravity does not altered and hence, it gives the correct time. When the elevator in free fall, the effective value of acceleration due to gravity g is zero and hence time period is infinite. Such a pendulum will be at rest and do not oscillate when displaced from its mean positions. So, it cannot keep time, no possibility of correct time.

13. 2065 Q. No. 1 e A pendulum clock is taken to moon. Will it gain or lose time?

Please refer to 2069 Supp Q.No. 1e

14. 2059 Q.No. 1 e What are the drawbacks of simple pendulum?

The drawbacks of simple pendulum are:

a. The string used cannot be weightless, perfectly inextensible and flexible.
b. No heavy point mass can be realized in practice.
c. The point by which the string is supported is not perfectly rigid.
d. If angular velocity is large, the formula of time period is not valid. The motion of the simple pendulum can also become circular rather than linear.
e. te time period cannot be measured in space, where $g = 0$.

i.e., $T = 2\pi\sqrt{\frac{l}{g}}$, for $g = 0$, $T = \infty$

15. 2057 Q.No. 1 d On what factors does the period of a simple pendulum depend?

The time period of simple pendulum is given by: $T = 2\pi\sqrt{\frac{l}{g}}$. Thus, the period of simple pendulum depends upon

a. The effective length of simple pendulum ($T \propto \sqrt{l}$).
b. The acceleration due to gravity act on the bob ($T \propto \sqrt{\frac{1}{g}}$).

16. 2052 Q.No. 1 c If length of a simple pendulum increased by 4 times its original length, will its time period change? If yes, by how much?

Time period of a simple pendulum of length l is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots (i)$$

Let T_1 be the time - period of the pendulum when its length becomes $4l$.

$$\therefore T_1 = 2\pi\sqrt{\frac{4l}{g}} \quad \ldots (ii)$$

Dividing equation (i) by (ii), we get

$$\frac{T}{T_1} = 2\pi\sqrt{\frac{l}{g}} \times \frac{1}{2\pi}\sqrt{\frac{g}{4l}}$$

or, $\frac{T}{T_1} = \frac{1}{\sqrt{4}} \Rightarrow T_1 = 2T$

Hence, new time-period (T_1) will be double if length is increased by four times.

17. 2050 Q.No. 1 iv If a pendulum clock is taken to a mountain top, does it gain or lose time?

The time period of a pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. At the mountain top, g is less than that at the sea level, hence, the time peroid will be more. So, such pendulum will be late and lose time.

Long Answer Questions

18. 2076 Set B Q.No. 5d What is simple harmonic motion? Calculate the total energy of a particle executing simple harmonic motion. [4]

Simple harmonic motion (SHM): The motion of the body is said to be simple harmonic if its acceleration is always,

a. directed towards a fixed point (mean position)
b. directly proportional to distance from fixed point.

It is a to and fro motion of an object about a mean position.

Energy is SHM: A particle excuting SHM has both kinetic energy and potential energy and their sum is total energy.

Potential Energy

Suppose a particle of mass m is executing S.H.M. with amplitude r and angular velocity ω. If y is the displacement, then acceleration

$a = -\omega^2 y$

Restoring force $F = m a$

$= -m\omega^2 y = -k y$

where $m\omega^2 = k$, a constant. If the particle is displaced further by a small displacement dy against the force, then work done

$dW = -Fdy = -(-ky)\,dy = ky\,dy$

$\therefore$ Total work done to displace the particle from mean position to the position of displacement y is,

$$W = \int_0^y F\,dy = \int_0^y k\,y\,dy = k\left[\frac{y^2}{2}\right]_0^y = \frac{1}{2}k y^2$$

$$\therefore W = \frac{1}{2}k y^2$$

The work done on the particle will remain in the form of potential energy. Thus,

$$E_p = \frac{1}{2}k y^2$$

$$= \frac{1}{2}m\omega^2 y^2 \quad \ldots (i)$$

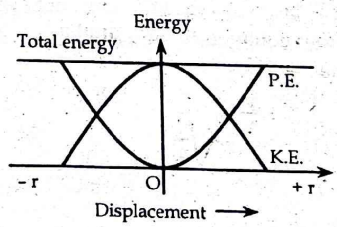
Kinetic Energy

Kinetic energy of the particle with velocity v is given by,

$$E_k = \frac{1}{2}m v^2$$

$$= \frac{1}{2}m\left(\omega\sqrt{r^2 - y^2}\right)^2$$

$$= \frac{1}{2}m\omega^2(r^2 - y^2) \quad \ldots (ii)$$



Total energy of the particle at any point is

$$E = E_p + E_k = \frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\omega^2(r^2 - y^2) = \frac{1}{2}m\omega^2 r^2$$

$$\therefore E = \frac{1}{2}m\omega^2 r^2 \quad \ldots (iii)$$

As m, ω and r are constants, the total energy remains constant for a particle executing S.H.M.

Case (i):

When the particle is at the mean position, $y = 0$ and

$$P.E. = \frac{1}{2}m\omega^2(0)^2 = 0$$

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and K.E. $= \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 (0)^2$

$= \frac{1}{2} m \omega^2 r^2 = T.E$

Case (ii):
When the particle is at the extreme position, $y = r$

P.E. $= \frac{1}{2} m \omega^2 r^2 = T.E$

and K.E. $= \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 r^2 = 0.$

The variation of kinetic energy and P.E. potential energy as a function of position in SHM is shown in figure.

19. 2075 Set A Q.No. 5d What is simple harmonic oscillator? Obtain an expression for its total energy, and time period.

Please refer to 2076 Set B Q.No. 5d [4]

20. 2073 Supp Q.No. 5d Show that the small oscillations of a mass loaded spring suspended vertically are simple harmonic. Deduce expression for its time period. [4]

Simple harmonic motion (SHM): The motion of the body is said to be simple harmonic if its acceleration is always,

a. directed towards a fixed point (mean position)
b. directly proportional to distance from fixed point.

It is a to and fro motion of an object about a mean position.

SHM produce by loaded mass at the free end of a spring:

Let us consider a spring S suspend from a fixed point A as shown in figure. When the mass m is loaded to the lower end of the spring, it is extended through a distance 'l'. If 'k' is the force constant of spring, then at equilibrium position,

Weight = restoring force

i.e. $mg = kl$

$\Rightarrow k = \frac{mg}{l}$ = force per unit extension ... (i)

Now, if the mass is pulled down through a distance x, then the restoring force is given by

$F = mg - k(l + x)$

$= mg - kl - kx$

$= mg - mg - kx$ [using equation (i)]

$F = -kx$ ... (ii)

If 'a' be the acceleration produced then, $F = ma$ ...(iii)

From equation (ii) and (iii),

$ma = -kx$

$a = -\frac{k}{m} x$

$a = -\omega^2 x$ $\left[\omega^2 = \frac{k}{m}\right]$ ... (iv)

i.e. $a \propto -x$ ... (v)

From equation (iv) and (v) we can say that, the motion of the loaded spring is simple harmonic. The negative sign indicate that the acceleration is directed towards to mean position and directly proportional to the displacement.

The time period of oscillation is given by

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

or, $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$T = 2\pi \sqrt{\frac{m}{k}}$

which is the required expression for the time period.

21. 2073 Set D Q.No. 5c Define simple harmonic motion. Deduce a relation for total energy of a simple harmonic oscillator. [4]

Please refer to 2076 Set B Q.No. 5d

22. 2072 Supp Q.No. 5c 2069 (Set B) Q. No. 5d What is a simple pendulum? Show that motion of the bob of a simple pendulum is simple harmonic. Obtain an expression for its frequency. [4]

Simple pendulum: A simple pendulum is a heavy mass suspended from a rigid support by a weightless, inextensible and flexible string which can vibrates about a fixed point. The point by which the string is suspended is called point of suspension.

Expression for the time period of simple pendulum

Let us consider a mass of the simple pendulum m suspended by a string of length l. If the mass is slightly displaced and left, it oscillates along the arc of a circle of central point A. Let θ be the angular displacement of bob and the new position of the bob is at point C as shown in figure.

The weight mg of the bob acting vertically downwards and T is the tension in the string. The weight of the bob has two perpendicular components mg cosθ along the string which balance the tension and mg sin θ along the direction perpendicular to the tension.

It is clearly seen that the tension is balanced by mg cosθ whereas mg sinθ is unbalanced force called restoring force directed towards mean position.

Thus, the restoring force (F) = $-mg \sin\theta$ ... (i)

[-ve sign indicates that the force is directed towards mean position]

Since,

$F = ma$ ... (ii)

From equation (i) and (ii)

$a = -g \sin\theta$ ... (iii)

For small angle, $\sin\theta \simeq \theta$

From figure, $\theta = \frac{y}{l}$ where l is the length of the pendulum and y be the displacement i.e., arc length.

$\therefore a = -g \frac{y}{l} \Rightarrow a \propto -y$ ... (iv)

This equation (iv) is in the form of simple harmonic motion. So, motion of smple pendulum is simple harmonic motion. Here, the acceleration is directly proportional to the displacement y and -ve sign indicates that the acceleration is opposite to displacement or directed towards mean position.

Which is given by, $a = -\omega^2 y$ ... (v)

comparing equation (iv) and (v),

$\omega^2 = \frac{g}{l}$ [ω is angular velocity]

$\omega = \sqrt{\frac{g}{l}}$

Since, $\omega = \frac{2\pi}{T}$, T be the time period of oscillation.

$\therefore T = \frac{2\pi}{\omega}$

$\Rightarrow T = 2\pi \sqrt{\frac{l}{g}}$

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Hence, time period of oscillation of simple

23. 2072 Set C Q.No. 5c What are characteristics of simple harmonic motion? Show that motion of vertical spring system is simple harmonic and hence, derive formula for its time period.

Please refer to 2073 Supp Q.No. 5d

24. 2070 Supp (Set B) Q.No. 5 c Find an expression for the energy of particle in S.H.M. and show that the system obeys the law of conservation of energy.

Please refer to 2070 Set B Q.No. 5d

25. 2069 (Set A) Old Q. No. 2a Define Simple harmonic motion and obtain an expression of time period of simple pendulum.

Simple harmonic motion (SHM): The motion of the body is said to be simple harmonic if its acceleration is always,

a. directed towards a fixed point (mean position)
b. directly proportional to distance from fixed point.

It is a to and fro motion of an object about a mean position.

Second Part: Please refer to 2072 Supp Q.No. 5c

26. 2067 Supp Q.No. 5 c Define SHM. Derive a relation for total energy of a simple harmonic oscillator.

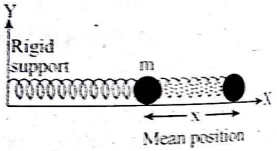
Please refer to 2076 Set B Q.No. 5d

27. 2065 Q. No. 3 a What is meant by simple harmonic motion? Show that the bob of a simple pendulum move with simple harmonic motion and finds its time period.

Please refer to 2072 Supp Q.No. 5c

28. 2063 Q.No. 3 a OR Obtain an expression for the time period of a mass 'm' attached with a spring placed horizontally on a smooth table.

Let a small body of mass m be placed on a smooth (frictionless) horizontal surface as shown in the figure. One end of a spring having negligible mass is attached to a rigid support and other end is attached to the mass m. The mass m is pulled along the horizontal surface and then it is released. The mass m begins to oscillate about its mean position. Let x be the position of the mass m at any instant. The restoring force acting on the mass m is given by Hooke's law as $F = -kx$ where k is the spring constant or force constant of the spring.



According to Newton's second law of motion, $F = ma$

i.e. $-kx = ma$

$$a = \frac{-k}{m}x = \omega^2 x,$$

where $\omega^2 = \frac{k}{m}$ (in magnitude)

This is the equation of simple harmonic motion ($a = -\omega^2 x$). Hence, the motion of the mass attached to the spring in the horizontal place is SHM. We have

$$\omega^2 = \frac{k}{m}$$ where ω is the angular velocity in SHM.

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

This is the expression for time-period of oscillation.

29. 2061 Q.No. 3 a OR Define simple harmonic motion. Show that the oscillation of mass suspended from a spring is simple harmonic.

Please refer to 2073 Supp Q.No. 5d

30. 2058 Q.No. 2 a Show that the motion of a simple pendulum is a simple harmonic motion. Derive its time period. [3+1=4]

Please refer to 2072 Supp Q.No. 5c

31. 2056 Q.No. 3 a Define simple harmonic motion. Show that the vertical oscillations of a mass suspended by a light helical spring are simple harmonic. [1+4]

Please refer to 2073 Supp Q.No. 5d

32. 2055 Q.No. 4 What is simple harmonic motion? Derive the relation between the acceleration and displacement of the practicle executing S.H.M. [1+3]

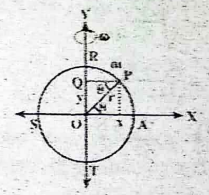
Simple harmonic motion (SHM): The motion of the body is said to be simple harmonic if its acceleration is always,

a. directed towards a fixed point (mean position)
b. directly proportional to distance from fixed point.

It is a to and fro motion of an object about a mean position.

Relation between acceleration and displacement of particle executing SHM:

Consider a particle having mass 'm' moving with uniform circular motion with an angular velocity ω in a circle of radius 'r' and centre at 'O' shown in figure. Let P be the position of the particle at time 't'. Draw PQ perpendicular to OY, then Q is foot of perpendicular drawn from point P on y-axis. As the particle moves from point P to R, Q moves from point O to point R along the vertical diameter. When the particle moves from R to S, Q comes back from point R to point O. As the particle moves from point S to point T, Q moves from point O to T and when the particle moves from point T to point A, Q comes back from point T to point O. If the particle continuously goes round the circle, the projection Q goes on vibrating to and fro about the centre O of the circle. Thus, the foot of perpendicular has to and fro motion along the diameter, is simple harmonic motion (SHM).



Now from figure, in ΔOQP, we have

$$\sin\theta = \frac{OQ}{OP}$$

$OQ = OP\sin\theta$

i.e. $y = r\sin\theta$ ... (i)

where y is displacement of point Q from mean position.

$y = r\sin\omega t$ [$\because \theta = \omega t$]

$v = r\omega\cos\omega t$ ... (ii)

Further,

$v = \omega\sqrt{r^2\cos^2\omega t}$

$= \omega\sqrt{r^2(1-\sin^2\omega t)}$

$= \omega\sqrt{r^2 - r^2\sin^2\omega t}$

$v = \omega\sqrt{r^2 - y^2}$ ... (iii) [$\because$ using equations (i)]

Now, acceleration is the rate of change in velocity.

i.e. $a = \frac{dv}{dt} = \frac{d}{dt}(r\omega\cos\omega t) = r\omega(-\omega\sin\omega t) = -\omega^2 r\sin\omega t$

$= -\omega^2 y$ ... (iv) [$\because y = r\sin\omega t = r\sin\theta$]

Equation (iv) is the equation relating the acceleration of a particle executing simple harmonic motion and displacement.

33. 2054 Q.No. 4 What is a simple pendulum? Show that motion of the bob of a simple pendulum is simple harmonic. Obtain an expression for its time period. [4]

Please refer to 2072 Supp Q.No. 5c

34. 2052 Q.No. 4 Define simple harmonic motion. Show that system of a spring hung from a support with a mass hanging at free end move simple harmonically. [4]

Please refer to 2073 Supp Q.No. 5d

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Numerical Problems

35. 2076 Set C Q.No. 9d The velocity of a particle executing simple harmonic motion is 16cms^{-1} at a distance of cm from the mean position and 8 cms^{-1} at a distance of 12 cm from the mean position. Calculate the amplitude of the motion.

Solution

Given,

1st case

Velocity (v_1) = 16 cms^{-1}

Distance (y_1) = 8 cm

2nd case

Velocity (v_2) = 8 cms^{-1}

Distance (y_2) = 12 cm

Amplitude (A) = ?

We know,

$v = \omega\sqrt{A^2 - y^2}$

Now

$v_1 = \omega\sqrt{A^2 - y_1^2}$

$v_2 = \omega\sqrt{A^2 - y_2^2}$

Dividing V_1 and V_2 we get

$$\frac{v_1}{v_2} = \frac{\omega\sqrt{A^2 - y_1^2}}{\omega\sqrt{A^2 - y_2^2}}$$

$$\text{or, } \frac{v_1}{v_2} = \sqrt{\frac{A^2 - y_1^2}{A^2 - y_2^2}}$$

$$\text{or, } \frac{v_1^2}{v_2^2} = \frac{A^2 - y_1^2}{A^2 - y_2^2}$$

$$\text{or, } \left(\frac{16}{8}\right)^2 = \frac{A^2 - 8^2}{A^2 - 12^2}$$

$$\text{or, } 4 = \frac{A^2 - 64}{A^2 - 144}$$

$$\text{or, } 4A^2 - 576 = A^2 - 64$$

$$\text{or, } 4A^2 - A^2 = 576 - 64$$

$$\text{or, } 3A^2 = 512$$

$$\text{or, } A^2 = 170.66$$

$$\text{or, } A = \sqrt{170.66}$$

$$\therefore A = 13.06 \text{ cm}$$

36. 2074 Supp. Q.No. 9c 2062 Q.No. 2 b A simple pendulum 4m long swings with an amplitude of 0.2m. Compute the velocity of the pendulum at its lowest point and its acceleration at extreme ends.

Solution

Given,

Length of pendulum (l) = 4m

Amplitude (r) = 0.2m

Velocity at lowest point (V_{max}) = ?

Acceleration at extreme point (a_{max}) = ?

We have

Time period of pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\frac{4}{10} = 4 \text{ sec}$$

$$\therefore V_{max} = \omega r = \frac{2\pi}{T} \cdot r = \frac{2\pi}{4} \cdot 0.2 = 0.32 \text{ m/s}$$

$$a_{max} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 \cdot r = \left(\frac{2\pi}{T}\right)^2 \cdot 0.2 = 0.5 \text{m/s}^2$$

37. 2074 Set A Q.No. 9b A body of mass 0.1 kg is undergoing simple harmonic motion of amplitude 1 m and period 0.2 second. If the oscillation is produced by a spring what will be the maximum value of the force and the force constant of the spring? [4]

Solution

Given,

Mass of the body (m) = 0.1 kg

Amplitude (r) = 1 m

Time period (T) = 0.25

Maximum force (F_{max}) = ?

Force constant (k) = ?

We know,

$$F_{max} = ma_{max}$$

$$= m.\omega^2 r \qquad [\because a = \omega^2 r]$$

$$= m.\left(\frac{2\pi}{T}\right)^2 r \qquad \left[\because \omega = 2\pi f \text{ and } f = \frac{1}{T}\right]$$

$$= \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 \times 0.1 \times 1}{0.2^2} = 98.596 \text{ N}$$

Also,

$$\omega^2 = \frac{k}{m}$$

$$\text{or, } k = \left(\frac{2\pi}{T}\right)^2 \times m = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 0.1}{0.2^2} = 98.596 \text{ N/m}$$

38. 2073 Set C Q.No. 9c A body of mass 200 gm is executing simple harmonic motion with amplitude of 20 mm. The maximum force which acts upon it is 0.8N. Calculate its maximum velocity and its period of oscillation. [4]

Solution

Given,

Mass of body (m) = 200 gm = 200 × 10^{-3} kg

Amplitude (r) = 20mm = 20 × 10^{-3} m

Maximum force (F) = 0.8 N

Maximum velocity (V_{max}) = ?

Period of oscillation (T) = ?

We know, that

$$F_{max} = \frac{mv^2_{max}}{r}$$

$$\text{or, } 0.8 = \frac{200 \times 10^{-3} \times v^2_{max}}{20 \times 10^{-3}}$$

$$\text{or, } \frac{0.8 \times 20}{200} = v^2_{max}$$

$$\text{or, } v_{max} = 0.28 \text{ m/sec}$$

Again,

$$v_{max} = \omega r$$

$$\text{or, } v = \frac{2\pi}{T} \cdot r$$

$$\text{or, } T = \frac{2\pi r}{v} = \frac{2\pi \times 20 \times 10^{-3}}{0.28} = 0.45 \text{ sec}$$

$\therefore$ Maximum velocity = 0.28 m/sec

Time period = 0.45 sec.

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39. 2072 Set E Q.No. 9c **A body of mass 2 kg is suspended from a spring of negligible mass and is found to stretch the spring 0.1 m. What is its force constant and the time period?**

Solution

Given,
Mass of a body (m) = 2 kg
Extension (l) = 0.1 m
Force constant (k) = ?
Time period (t) = ?
We have,

$F = kl$

or, $mg = kl$

or, $k = \frac{mg}{l} = \frac{2 \times 10}{0.1}$

$= 200$ N/m

Again,

$t = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times 0.1 = 0.628$ sec

40. 2071 Supp Q.No. 9c **A glider with mass m = 2.00 kg sits on a frictionless horizontal air track, connected to a spring with force constant k = 5.00 N/m. You pull the glider, stretching the spring 0.100 m and then release it with no initial velocity. The glider begins to move back toward its equilibrium position (x = 0). What is its velocity when x = 0.080m?**

Solution

Given,
Mass of glider (m) = 2.00 kg
Force constant (k) = 5.00 N/m
Max. amplitude (r) = .100 m
Amplitude at instant is (y) = 0.08m
Velocity at instant (v) = ?
Now,

$v = \omega\sqrt{r^2 - y^2}$

or, $v = \sqrt{\frac{k}{m}}\sqrt{r^2 - y^2}$ $\quad \left[\because \omega^2 = \frac{k}{m}\right]$

$= \sqrt{\frac{5}{2}} \times \sqrt{0.1^2 - 0.08^2} = 0.1$ m/sec

41. 2070 Set C Q.No. 9 d **A simple pendulum 5 m long swings with an amplitude 25 cm. Find the velocity of the pendulum at its lowest point and the acceleration at the end of its path.**

Solution

Given,
Length of pendulum (l) = 5m
Amplitude of vibration (r) = 25 cm = 0.25m
Velocity at lowest point (v_{max}) = ?
Acceleration at end (a_{max}) = ?
We have,

$v_{max} = \omega r = \frac{2\pi}{T} \cdot r = \frac{2\pi \cdot r}{2\pi\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} \cdot r = \sqrt{\frac{10}{5}} \times 0.25 = 0.35$ m/sec

$a_{max} = \omega^2 r = \frac{g}{l} \cdot r = \frac{10}{5} \times 0.25 = 2 \times 0.25 = 0.5$ m/sec^2

42. 2070 Set D Q.No. 9 d **A body is vibrating with simple harmonic motion of amplitude 15 cm and frequency 4 Hz. Calculate the maximum value of acceleration and velocity.** [4]

Solution

Given,
Applitude of vibration (r) = 15cm = 0.15 m
Frequency of vibration (f) = 4 Hz
Maximum value of acceleration (a_{max}) = ?
Maximum value of velocity (V_{max}) = ?
We have,

$V_{max} = \omega r = (2\pi f) \times r = (2\pi \times 4) \times 0.15 = 2\pi \times 4 \times 0.15 = 3.77$ m/sec

$a_{max} = \omega^2 r = (2\pi f)^2 \cdot r = (2\pi \times 4)^2 \times 0.15 = 94.75$ m/sec^2

43. 2069 Old (Set B) Q. No. 3b **A small mass of 0.2 kg is attached to one end of helical spring and produces an extension of 15mm. The mass is now set into vertical oscillation of amplitude 10 mm. What is:**
i. the period of oscillation?
ii. the maximum kinetic energy of the mass?
iii. the potential energy of the spring when the mass is 5mm below the centre of oscillation? ($g = 9.8 ms^{-2}$)

Solution

Given,
Mass of object (m) = 0.2 kg
Extension (e) = 15 mm = 15×10^{-3}m
Amplitude (r) = 10mm = 10×10^{-3}m
Period (T) = ?
$K.E_{max}$ = ?
P.E. below 5 mm = ?
Now,

i. Period (T) $= 2\pi\sqrt{\frac{e}{g}} = 2\pi\sqrt{\frac{15 \times 10^{-3}}{10}} = 0.243$ sec

ii. $K.E_{max} = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2} \times 0.2 \times \left(\frac{2\pi}{0.243}\right)^2 \times (10 \times 10^{-3})^2 = 6.68 \times 10^{-3}$J

iii. P.E. below 5 mm of Oscillation,

P.E. $= \frac{1}{2}m\omega^2 y^2 = \frac{1}{2} \times 0.2 \times \left(\frac{2\pi}{0.243}\right)^2 \times (5 \times 10^{-3})^2 = 1.67 \times 10^{-3}$J

44. 2068 Q.No. 9 c **A simple pendulum has a period of 4.2 second. When the pendulum is shortened by 1m, the period is 3.7 second. From these measurements, calculate the acceleration of free fall and the original length of the pendulum.** [4]

Solution

Given,
Time period (T_1) = 4.2 sec.
Length shortened = 1m
Time period (T_2) = 3.7 sec
Length (l) = ?
Acceleration of free ball (g) = ?
We have,

$T_1 = 2\pi\sqrt{\frac{l}{g}}$

and $T_2 = 2\pi\sqrt{\frac{l-1}{g}}$

$\frac{T_2}{T_1} = \sqrt{\frac{l-1}{l}}$

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$$\text{or, } \left(\frac{T_2}{T_1}\right)^2 = \frac{l-1}{l}$$

$$\text{or, } \left(\frac{3.7}{4.2}\right)^2 = 1 - \frac{1}{l}$$

$$\text{or, } \frac{1}{l} = 1 - 0.776$$

$$\text{or, } \frac{1}{l} = 0.224$$

$$l = 4.5 \text{ m}$$

Again,

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{or, } 4.2 = 2\pi\sqrt{\frac{4.5}{g}}$$

$$\text{or, } 4.2^2 = 4\pi^2\frac{4.5}{g}$$

$$\text{or, } g = \frac{4\pi^2 \times 4.5}{4.2^2}$$

$$g = 10 \text{m/sec}^2$$

45. 2067 Q.No. 9 c Calculate the period of oscillation of a simple pendulum of length 1.8m with a bob of ma kg. If the bob of this pendulum is pulled aside a horizontal distance of 20 cm and released. What will values of (i) the K.E. and (ii) the velocity of the bob at the lowest point of the swing?

Solution

Given,

Length of pendulum (l) = 1.8 m

Mass of bob (m) = 2.2 kg

Amplitude (r) = 20cm = 0.2 m

Time period (T) = ?

Velocity (V_{max}) = ?

$K.E_{max.}$ = ?

We have

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.8}{10}} = 2.66\text{sec}$$

$$V_{max} = \omega r = \frac{2\pi}{T}.r = \frac{2\pi}{2.66}.0.2 = 0.47 \text{ ms}^{-1}$$

$$KE_{max} = \frac{1}{2}m\omega^2r^2 = \frac{1}{2} \times 2.2 \times \left(\frac{2\pi}{2.66}\right)^2 0.2^2 = 0.243\text{J}$$

46. 2064 Q.No. 2 b The displacement y of a mass vibrating with simple harmonic motion is giv y = 20 sin 10 πt. Where y is in millimeter and t is in second. What is:
(i) amplitude (ii) the period (iii) the velocity at t = 0

Solution

Given,

The displacement equation is, $y = 20 \sin 10\pi t$

Since, General displacement equation is, $y = r \sin \omega t$

Comparing these two equations, we have

i. Amplitude, (r) =20 mm = 20×10^{-3} m

ii. Time period, (T) = ?

We have,

$\omega = 10\pi$

$$\frac{2\pi}{T} = 10\pi$$

$$\therefore T = \frac{2}{10} = 0.2 \text{ sec}$$

iii. Velocity (at t = 0), v = ?

$$y = 20 \sin 10\pi t$$

$$\text{or, } \frac{dy}{dt} = 20 \cos 10\pi t \times 10\pi$$

$$\text{or, } v = 20 \cos 0 \times 10\pi \quad \left(\because v = \frac{dy}{dt}\right) \text{ for } t = 0$$

$$\text{or, } v = 20 \times 10\pi$$

$$\therefore v = 628 \text{ mm/s} = 0.628 \text{ m/sec}$$

47. 2060 Q.No. 3 b A particle of mass 0.3 kg vibrates with a period of 2 seconds. If its amplitude is 0.5m, what is its maximum kinetic energy?

Solution

Given,

Mass (m) = 0.3 kg

Time period (T) = 2 second

Amplitude (r) = 0.5 m

Maximum K.E. = ?

We have

$$K.E_{max} = \frac{1}{2}m\omega^2r^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2.r^2 = \frac{1}{2}.0.3\left(\frac{2\pi}{2}\right)^2.0.5^2 = 0.37\text{J}$$

48. 2053 Q.No. 5 A small mass rests on a horizontal platform which vibrates in simple harmonic motion with a period of 0.25s. Find the maximum amplitude of the motion which will allow the mass to remain in contact with the platform throughout the motion.

Solution

Given,

Time period (T) = 0.25 sec

Maximum amplitude (r) = l

When the mass remain in contact with platform vibrating vertically, acceleration must be equal to acceleration due to gravity.

i.e. $a = g = \omega^2 r$

$$\text{or, } g = \left(\frac{2\pi}{T}\right)^2.r$$

$$\text{or, } 10 = \left(\frac{2\pi}{0.25}\right)^2.r$$

$$\text{or, } r = 0.0158 \text{ m}$$

49. 2050 Q.No. 5 A simple pendulum 4m long swings with an amplitude of 0.2 m. [4]
(a) Compute the velocity of the pendulum at its lowest point.
(b) Compute its acceleration at the end of its path.

Solution

Given,

Length of pendulum (l) = 4m

Amplitude (a) = 0.2m

Angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{4}} = 1.56 \text{ rad sec}^{-1} \left(\because T = 2\pi\sqrt{\frac{l}{g}}\right)$

Velocity at its lowest position is, $v = \omega r = 1.56 \times 0.2 = 0.31 \text{ ms}^{-1}$

Acceleration at the end of its path is, $a = \omega^2 r = (1.56)^2 \times 0.2 = 0.49 \text{ ms}^{-2}$

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Chapter 12: Fluid Mechanics
[Fluid Statics, Surface Tension and Viscosity]

A. Fluid Statics

Short Answer Questions

1. **2075 GIE Q.No. 1e Why do clouds seem to be floating in the sky?** [2]

Clouds are tiny particles of water. The upthrust force acting on the clouds and viscous force on it balances its weight. As a result, the cloud particles gain terminal velocity and move with a constant speed in the sky. So, they seem to be floating in the sky.

2. **2075 Set B Q.No. 1c Which one gives the feeling of heaviness in the case of a kilogram of cotton or a kilogram of lead? Why?** [2]

A kilogram of lead gives the feeling of heaviness than a kilogram of cotton because the density of lead is greater than density of cotton. The one kilogram of cotton occupies more space than lead of same mass i.e. volume of one kilogram of cotton is more than lead.

3. **2074 Supp. Q.No. 1g 2063 Q.No. 1f A body floats in a liquid contained in a beaker. The whole system falls under gravity. What is the value of upthrust on the body due to liquid?** [2]

A body can float in water if the weight of body is equal to upthrust acting on it. But when the whole system falls under gravity, a = g and we have

Weight of the body – upthrust = ma

or, mg – upthrust = ma

or, upthrust = mg – ma

or, upthrust = mg - mg =0

∴ upthrust = 0, hence, uptrrust on the body falling freely under gravity is zero.

4. **2074 Set A Q.No. 1b Why is the bottom of a ship made heavy? Explain.** [2]

When the bottom of a ship is heavy, the centre of gravity (C.G.) of the ship lies below the centre of buoyancy (C.B) and hence, meta-centre (C.M.) of the displaced liquid and the floating ship comes in stable condition even its equilibrium position is disturbed. So, this makes the stability of the ship.

5. **2070 Supp (Set B) Q.No. 1 c The purity of the gold can be tested by weighing it in air and water. How?** [2]

The purity of the gold can be tested by determining its specific gravity. Although the mass and volume of a substance varies, the specific gravity remains constant at certain temperature. It is calculated as,

For gold, specific gravity = $\frac{\text{Weight of gold in air}}{\text{weight of gold in water}}$ x specific gravity of water at t°C

This specific gravity is compared to the standard value of specific gravity of pure gold (19.3), we can test the purity of gold.

6. **2070 Set C Q.No. 1 f In still air, a helium filled balloon rises up to a certain height and then stops rising. Why?** [2]

The atmospheric pressure is greater near the surface of earth and gradually decreases to the height. Since, the density of helium gas is lower than that of air near the ground. The weight of the ballon will be smaller than the upthrust acting on it and the balloon rises to a certain height when upthrust is equal to its weight where the density of air and helium is same. Atfter this height, the ballon stops to rise because weight of ballon becomes more than uptrust.

7. **2069 (Set A) Q. No. 1e In hot air ballooning, a large balloon is filled with air heated by a gas burner at the bottom. Why must the air be heated?** [2]

The density of hot air is less than the cold air and hence, the ballon becomes light which helps to move upward. Therefore, the air is heated to let the balloon up in the sky.

8. **2069 Set A Q. No. 1g An ice cube floats in a glass of water. When the ice melts, will the water level in the glass rise, fall or remain unchanged? Explain.** [2]

No, The water level will remain same if piece of ice melts completely in a container of water. Let V' be the volume of ice cube inside water and V, the whole volume of ice. For floating body,

weight of ice = upthrust

$m_i \times g = V' \times \sigma_w g$

or, $V \times \rho_i = V' \sigma_w$

or, $V' = V \times \frac{\rho_i}{\sigma_w}$

When whole ice melts, volume of water obtained = $\frac{\text{mass of water}}{\text{density of water}} = \frac{\text{mass of ice}}{\text{density of water}} = \frac{V \times \rho_i}{\sigma_w}$

Since, the volume of water obtained from melted ice is equal to the volume of ice inside water level, so water level remains the same.

9. **2069 (Set A) Old Q. No. 1e 2051 Q.No. 1 vii Why is the bottom of a ship made heavy?** [2]

Please refer to 2074 Set A Q.No. 1b

10. **2068 Old Q.No. 1 f What will happen to the level of water in a container when a submerged piece of ice melts completely?** [2]

Please refer to 2069 Set A Q. No. 1g

11. **2065 Q. No. 1 f Distinguish between density and specific gravity.** [2]

The differences between density and specific gravity are:

Density	Specific Gravity
a. The quantity of matter (mass) per unit volume is called density of a substance.	a. The ratio of density of any substance to the density of water at 4°C is called specific gravity of that substance.
b. It is absolute value.	b. It is the relative density.
c. It's unit is kg/m³ in SI and gm/ cm³ in CGS.	c. It is unit less quantity.

12. **2064 Q.No. 1 c If a cork is released from the bottom of a jar filled with water, what will happen and why?** [2]

The cork comes up when it is released from the bottom of a jar filled with water. It is because of upthrust acting on it is greater than its wight and it floats in water. A body can float in water if the weight of body is equal to upthrust acting on it.

13. **2062 Q.No. 1 f How will you make difference between density and specific gravity of a body?**

Please refer to 2065 Q. No. 1 f

14. **2061 Q.No. 1 e A cork is floating in water. What is the apparent weight of the cork?**

A body can float in water if the weight of body is equal to upthrust acting on it. The centre of gravity of the body and the centre of buoyancy of displaced water lie in the same line. Thus, the cork becomes weightless.

15. **2060 Q.No. 1 f A helium filled balloon rises to a certain height and then halts. Why?**

Please refer to 2070 Set C Q.No. 1 f

16. **2059 Q.No. 1 f Why is it easier to lift a body in a liquid than in air?**

The density of a liquid is greater than that of air. Since the upthrust depends on the density, the upthrust acting on the body is greater in water than in air and hence it is easier to lift the body in water.

17. **2058 Q.No. 1 f A piece of ice is floating in water. Will the water level rise if the ice melts completely? Explain.**

Please refer to 2069 Set A Q. No. 1g

18. **2057 Q.No. 1 f What are centre of buoyancy and metacentre?**

Centre of buoyancy: The position of the centre of gravity of the displaced fluid is called centre of buoyancy. The buoyant force or upthrust acts vertically upward from the centre of buoyancy.

Metacenter: When a body floating in a liquid is in equilibrium, the centre of gravity of the body and centre of buoyancy both lie in vertical line and form a couple. When a body get slightly tilt, both CG and CB change their positions. The point where the vertical line passing through new centre of buoyancy in the displaced position of the fluid meets the original vertical line passing through the centre of gravity of floating body is known as metacenter.

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19. 2055 Q.No. 1 d State the laws of floatation.

For a body floating freely in a liquid, the following laws, called laws of flotation, must be followed.

i. The weight of the body is equal to the weight of the displaced liquid.

ii. The centre of gravity of the body and the centre of gravity of the displaced liquid are in same vertical line.

20. 2054 Q.No. 1 i Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?

We know that,

Upthrust = Weight of water (fluid) displaced

or, U = mass of water displaced (m) × g

or, U = volume of water displaced (V) × density of water (ρ) × g $\left(\because \rho = \frac{m}{V}\right)$

or, U = Vρg ... (i)

If lead and iron objects have equal volume, they displace equal volume of water. Hence, the equation (i) indicates that buoyant force on the lead object is same as that on the iron object of same volume whatever be their densities.

21. 2053 Q.No. 1 h Does a ship sink more in river water or in sea water? Explain.

Since density of sea water is greater than that of fresh water due to presence of impurity like salt, so weight of displaced water for same volume is greater for sea water than river water. Therefore, lesser volume of water is needed to float the same boat in sea than in river.

22. 2052 Q.No. 1 i Steel balls sink in water but they don't sink in mercury. Why?

The density of steel (7800 kgm^{-3}) is greater than density of water (1000 kgm^{-3}) but smaller than density of mercury (13600 kgm^{-3}). When steel balls are dipped in water, they sink because upthrust due to water is less than the weight of the steel balls. But steel balls don't sink in mercury because upthrust due to mercury is higher than the weight of the steel balls.

23. 2050 Q.No. 1 vii Why does ice float in water?

For a body to float, the density of body should less than the density of liquid. Here, the density of ice is less than the density of water. Due to this reason, the weight of water displaced by ice is greater than its weight. So, ice floats in water.

24. 2050 Q.No. 1 ix A rigid, lighter than air balloon filled with Helium can not continue to rise indefinitely. Why?

Please refer to 2070 Set C Q.No. 1f

Numerical Problems

25. 2074 Set B Q.No. 9d An iceberg having volume 2.06 litre floats in sea water of density 1.03 gm /cm^{-3} with a portion of 224 CC above the surface. Calculate the density of ice. [4]

Solution

Given,

Volume of iceberg = 2.06 litre = 2.06 × 10^{-3} m^3

Density of sea water = 1.03 g/cm^3 = 1030 kgm^{-3}

Volume of iceberg above sea water = 224 cc = 224 × 10^{-6} m^3

Density of ice, ρ = ?

From principle of flotation, we have

weight of iceberg = weight of the displaced water

mass of iceberg × g = mass of displaced water × g

or, density of ice × volume of ice = density of sea water × volume of displaced water

or, ρ × 2.06 × 10^{-3} = 1030 × (2.06 × 10^{-3} – 0.224 × 10^{-3})

or, ρ × 2.06 × 10^{-3} = 1030 × (2.06 – 0.224) 10^{-3}

or, $\rho = \frac{1030 \times 1.836}{2.06}$

∴ ρ = 918 kgm^{-3}

26. 2073 Supp. Q.No. 11 2066 Q. No. 1 e A boy can lift a maximum load of 250 N of water. How many litres of mercury of density 13600 kgm^{-3} he can lift in an identical vessel? [2]

Solution

Given,

The maximum load (F) = 250N

density of mercury (ρ) = 13600 kg m^{-3}

volume of mercury (V) = ?

We have,

F = mg = V. ρ.g

250 = 13600. V . 10

V = 1.838 × 10^{-3} m^3 = 1.838 litres

Hence, 1.838 litres of mercury a boy can lift.

27. 2069 Old Q. No. 3 b A geologist finds that a moon rock whose mass is 7.2 kg has an apparent mass 5.88 kg when submerged in water. What is the density of the rock? [1+3]

Solution

Given,

Mass of rock (m_r) = 7.2 kg

Mass of rock in water (m_{rw}) = 5.88 kg

Density of rock = ?

We have,

Weight of rock = Apparent wt + loss wt

Weight of rock = Apparent wt + wt of water displaced

or, 7.2 × 10 = 5.88 × 10 + weight of water displaced

or, 7.2 = 58.8 + mass of water displaced × 10

or, 13.2 = $\frac{\text{mass of rock}}{\text{density rock}} \times 1000 \times 10$

or, density of rock = $\frac{7.2 \times 1000 \times 10}{13.2}$

density of rock = 5454.54 kg/m^3

28. 2066 Q. No. 3 b A string supports a solid iron object of mass 200gm totally immersed in a liquid of specific gravity 0.9. Calculate the tension in the string it the density of iron is 8000 kg m^{-3}. [3]

Solution

Given,

Mass of iron (m) = 200 gm = 0.2 kg

Density of iron (ρ) = 8000 kg/m^3

Density of liquid (σ) = 900 kg/m^3

Tension in the string (T) = ?

We know,

T = weight – upthrust

= mg – weight of displaced liquid

= 0.2 × 10 – mass of displaced liquid × g

= 2 – density of liquid × volume of displaced liquid × g

= 2 – 6 × volume of iron × g

= $2 - 900 \times \frac{0.2}{8000} \times 10$

= 2 – 0.225 = 1.775

∴ Tension in the string, T = 1.775 N

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29. 2064 Q.No. 3 b **An alloy of mass 588 gm and volume 1000 c.c. is made of iron of density 8.0 gm/cc and aluminum of density 2.7 gm/cc. Calculate the proportion by (i) volume (ii) by mass of the constituents of the alloy.**

Solution

Given,

Mass of alloy (m) = 588 gm

Volume of alloy (V) = 1000 c.c.

Density of iron (ρ_i) = 8.0 gm/cc

Density of aluminium (ρ_{al}) = 2.7 gm/cc.

Proportion of volume ($V_i : V_a$) = ?

Proportion by mass ($m_i : m_a$) = ?

Let V_i be the volume of iron, then volume of aluminium, $V_a = 100 - V_i$

Mass of iron, $m_i = V_i \times 8 = 8V_i g$

Mass of aluminium = $(100 - V_i) \times 2.7$ g

$\therefore$ $m_i + m_a = 588$

or, $8V_i + (100 - V_i) \times 2.7 = 588$

$8V_i + 270 - 2.7V_i = 588$

$5.3\ V_i = 588 - 270$

or, $V_i = \frac{318}{5.3} = 60$ cc

$V_a = 100 - 60 = 40$cc

$\therefore$ $V_i : V_a = 60 : 40 = 3 : 2$

$m_i = 8V_i = 8 \times 60 = 480$ g

$m_a = 40 \times 2.7 = 108$g

$\therefore$ $m_i : m_a = 480 : 108 = 40 : 9$

30. 2062 Q.No. 3 b **A string supports a solid iron object of mass 200gm totally immersed in a liquid of density 900km^{-3}. The density of iron is 8000 kgm^{-3}. Calculate the tension in the string.**

Please refer to 2065 Q. No. 3 b

31. 2057 Q.No. 3 b **A string supports a solid iron of mass 200 gm totally immersed in a liquid of density 900 kg m^{-3}. Calculate the tension in the string if the density of iron is 8000 kg m^{-3}.**

Please refer to 2065 Q. No. 3 b

32. 2056 Q.No. 3 b **An iceberg having a volume of 2060 cc floats in sea-water of density 1030 kg m^{-3} with a portion of 224 cc above the surface. Calculate the density of ice.**

Please refer 2074 Set B Q.No. 9d

33. 2055 Q.No. 6 OR **A piece of gold-aluminium alloy weights 100 gm in air and 80 g in water. What is the weight of the gold in the alloy if the relative density of gold is 19.3 and that of aluminium is 2.5?**

Solution

Given,

Weight of alloy in air = 100 g

Weight of alloy in water = 80 g

Relative density of gold = 19.3

Relative density of aluminium = 2.5

Weight of gold in alloy = ?

Loss in weight of alloy = 100 – 80 = 20 g

Let, mass of gold in alloy = x

$\therefore$ mass of aluminium in alloy = 100 – x

Weight of aluminium in alloy = (100 – x) g

Now,

volume of alloy = volume of displaced water = $\frac{\text{mass of displaced water}}{\text{density of water}}$

volume of alloy = $\frac{20}{1} = 20$ cm^3

Then, volume of gold in alloy = $\frac{x}{19.3}$

and, volume of aluminium in alloy = $\frac{100 - x}{2.5}$

Since, the sum of volume of gold and aluminium in alloy is equal to the volume of alloy, so

$\frac{x}{19.3} + \frac{100 - x}{2.5} = 20$

or, $0.052\ x + 40 - 0.4\ x = 20$

or, $-0.348\ x = 20 - 40$

or, $x = 57.5$g

$\therefore$ mass of gold in alloy = 57.5 g.

34. 2054 Q.No. 5 OR **A string supports a solid iron object of mass 180g totally immersed in a liquid of density 800 kg/m^3. The density of iron is 8000 kg/m^3. Calculate the tension in the string.**

Solution

Given,

Mass of iron (m_i) = 180g = 0.180 kg

Density of liquid (ρ_l) = 800 kgm^{-3}

Density of iron (ρ_i) = 8000 kgm^{-3}

Tension in the string (T) = ?

Tension in the string is equal to the apparent weight of body i.e. weight of solid iron in liquid.

$\therefore$ Apparent weight = weight in air – loss weight

or, $T = m_i g - m_l g$

$= m_i g - \frac{m_i}{\rho_i} \cdot \rho_l g = 0.18 \times 10 - 0.18 \times 10 \times \frac{800}{8000} = 1.62$ N

35. 2052 Q.No. 5 **A 25 cm thick block of ice floating on fresh water can support an 80 kg man standing on it, what is the smallest area of the ice block? (sp gr. of ice = 0.917).**

Solution

Given,

Thickness of ice (t) = 25cm = 0.25m

Weight of mass (m_m) = 80kg

Smallest area of ice (A) = ?

Specific gravity of ice = 0.917

$\therefore$ Density of ice = 0.917×1000

$= 917$ kg m^{-3}

For floating, weight of (ice + man) = weight of water displaced

or, $m_i g + m_m g = m_w . vg$

or, $V_i \times \rho_i + m_m = V_i \cdot \rho_w$ [$\because$ for just floating body]

or, $V_i \times (\rho_w - \rho_i) = m_m$

$\therefore$ $A = \frac{m_m}{t(\rho_w - \rho_i)} = \frac{80}{0.25\ (1000 - 917)} = 3.85 m^2$

36. 2051 Q.No. 3 OR **The density of ice is 971 km m^{-3}, and the approximate density of seawater in which an iceberg floats is 1025 kg. m^{-3}. What fraction of the iceberg, is beneath the water surface?**

Solution

Given,

Density of ice (ρ_i) = 971 kgm^{-3}

Density of sea water (ρ_w) = 1025 kg m^{-3}

Fraction of ice beneath the water surface = ?

For just floating body ice,

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Weight of ice = weight of water displaced

$V_i \cdot \rho_i \cdot g = V_w \cdot \rho_w \cdot g$

or, $\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w}$ [∵ Volume of water displaced is equal to volume of ice beneath the surface of water.]

$= \frac{971}{1025} = 0.95$

B. Surface Tension

Short Answer Questions

1. 2076 Set B Q.No. 1g **Why does hot soup taste better than cold soup?** [2]

The whole taste of soup is based on how well soup (hot or cold) spreads over the taste buds of the tongue. When temperature is low, the surface tension of water is high, so water molecules are held back, which prevents them to reach to the taste buds. This makes the taste incomplete. However, increase of temperature reduces the surface tension. This makes the water (soup) spread better, increasing their access to the taste buds. So, the taste is good and complete. Due to this reason, hot soup is tastier than a cold one.

2. 2076 Set C Q.No. 1g **A tiny liquid drop is spherical but a larger drop has oval shape. Why?** [2]

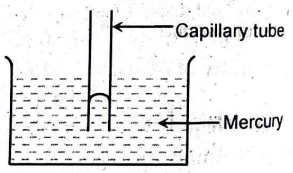
The shape of a liquid drop depends upon the surface tension and the force of gravity acting on it. The surface tension depends upon the surface area of the drop whereas the force of gravity depends upon the volume of the drop. Therefore, the ratio of surface tension to the force of gravity is inversely proportional to the radius of drop (r). When radius of drop is small, the surface tension dominates the force of gravity and the drop is nearly spherical. But, when radius of drop is large, force of gravity dominates the surface tension and hence, the bigger drops of mercury become oval in shape.

3. 2075 Set A Q.No. 1d 2070 Set D Q.No. 1 f 2066 Q.No. 1 a **Why is soap solution a better cleaning agent than ordinary water?** [2]

The surface tension of soap solution is less than the ordinary water due to which the water reaches to every region of clothes and cleans them. If the surface tension of water is high, the water does not enter deep inside every part of clothes and cannot remove the dust particle. Due to less surface tension soup solution is a better cleansing agent than ordinary water.

4. 2075 Set B Q.No. 1b **How does the shape of the surface of mercury look like in a capillary tube dipped in it? Explain in a figure with proper justification.** [2]

When a capillary tube dipped in liquid of mercury, the liquid is depressed in the tube below the level of mercury in the vessel as shown in figure. The shaped of liquid mercury inside the tube is convex meniscus. This is because the cohesive force between mercury molecules is greater than the adhesive force between mercury molecule and the molecules of the tube. As a result the angle of constant becomes greater than 90° and the liquid dippressed and takes convex meniscus inside the tube.



5. 2074 Set A Q.No. 1a **Explain why liquid drops are spherical in shape?** [2]

Please refer to 2076 Set C Q.No. 1g

6. 2074 Set B Q.No. 1a **Why are liquid drops spherical in shape? Explain.** [2]

Please refer to 2076 Set C Q.No. 1g

7. 2073 Supp Q.No. 1d 2072 Set E Q.No. 1f **It is observed that the surface of a liquid in a capillary tube dipped in it is either convex or concave. What may be the reason? Explain.** [2]

It is observed that the surface of a liquid in a capillary tube dipped in it is either convex or concave. The reason behind this is cohesive force of liquid molecules and adhesive force between liquid molecule and molecules of tube. If cohesive force between liquid molecules is more, the surface is convex and if the adhesive force between liquid and solid molecules is more, then the surface of liquid is concave.

8. 2073 Set D Q.No. 1g **Hot soup gives better taste than cold one, why?** [2]

Please refer to 2076 Set B Q.No. 1g

9. 2072 Supp Q.No. 1d **On what factors does the surface tension of a liquid depend? Explain.** [2]

Surface tension of a liquid depends on its temperature and impurity mixed on it like soap, detergent. Moreover, the surface tension is different for different liquid i.e. it depends upon nature of a liquid.

10. 2072 Set C Q.No. 1f **Why the small liquid drops are spherical while large drops are flat?** [2]

Please refer to 2076 Set C Q.No. 1g

11. 2070 Set C Q.No. 1 e **Soap bubbles are almost perfect spheres. Why?** [2]

The soap bubbles are lighter such that the surface tension dominaties its weight which makes soap bubbles almost perfect spheres.

12. 2069 Supp Q.No. 1d **Antiseptics used in cuts and wounds of human flesh have low surface tension, Why?**

If the antiseptics used for cut and wounds in human flesh have low surface tension, they reach to each & every parts of wounds and make relief from pain as well as cure fast.

13. 2069 (Set A) Old Q.No. 1a **How do the leaves of tree get water from ground?** [2]

The leaves of tree get water from ground by capillary action. The stem of the body of tree consists of small bores like capilary tube. Since the roots are deep in conact with the water resource below the ground, water travels through the channels of bores according to the relation $h = \frac{2T\cos\theta}{r\rho g}$. Since the pores are very fines, r is very small. so, they can rise to leaves.

14. 2069 (Set A) Old Q.No. 2a **What are cohesive and adhesive forces?** [2]

Cohesive force: The force of attraction between the molecules of the same substance is called force of cohesion or cohesive force. It is maximum in solids, lesser in liquids and the least in gases.

Force of adhesion: The force of attraction between the molecules of the different substances is called the force of adhesion or adhesive force. Force of adhesion is different for different substances. For example, gum has greater adhesive force for a solid surface than water.

15. 2069 (Set B) Q. No. 1g **Why the antiseptics used for cuts and wounds in human flesh have low surface tension?** [2]

Please refer to 2069 Supp Q.No. 1d

16. 2068 Q.No. 1 f **What will happen if a capillary tube of insufficient height is dipped in water?**

If a capillary tube of insufficient height is dipped in water, the curvature of meniscus increases and water does not overflow. The height climbed by a liquid and radius of curvature of the meniscus at any instant is given by, $hr = \frac{2T}{\rho g}$, where T is the surface tension of a liquid and ρ be the density of liquid. This relation shows that radius of curvature of the meniscus depends on the height available for it to expand. Less the height available, more is the radius of curvature of the meniscus. When h = 0, as in the case of tube of insufficient length, r becomes infinite, i.e. the meniscus becomes flat.

17. 2068 Old Q.No. 1 a **Why are small liquid drops always spherical in shape?**

Please refer to 2076 Set C Q.No. 1g

18. 2067 Q.No. 1 f **The tip of the nib of a pen is split. Why?**

The function of a pen is to ensure a uniform flow of ink to the tip of pen for proper writing. The major component of ink is water which wets other substance with its high adhesive capacity and surface tension. When a split is made, it resembles a capillary tube. Since, the split has a small separation, it resembles small r. So, liquid (ink) slides along the tube away from the mass to other locations according to the equation $h = \frac{2T\cos\theta}{\rho g r}$. This will carry the ink to enough extent, i.e. up to the tip (larger h due to small r).

19. 2063 Q.No. 1 a **Small particles of camphor dance on the surface of water, why?**

The surface of water acts as a stretched elastic membrane due to surface tension. When a small piece

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of camphor is placed on the surface, it will lower the surface tension at the point of contact. A net unbalanced force arises there by pulling the camphor across the water. This process continues until the camphor dissolves completely. Due to the pull of unbalanced surface tension, the piece of camphor appears to be dancing on the surface of water.

20. 2063 Q.No. 2 a **Hot soup is tastier than a cold one, why?**

Please refer to 2076 Set B Q.No. 1g

21. 2062 Q.No. 2 a **We use towels to dry our body after taking a shower, why?**

Towel consists of a large number of pores, which acts as the capillary tubes. When the towel is pressed against a wet body, the water is absorbed from the wet body into the fibers of the towel by the action of capillarity. That's why; towels are used to dry our body after taking shower.

22. 2061 Q.No. 2 a **Hairs of a brush spread out when it is dipped in water and cling together as soon as it is taken out of water. Explain.**

In the water, intermolecular force is balanced, so that no force is applied to minimize the surface area of a body kept inside it. Hence, hairs of a brush spread out when it is dipped into water. When the brush is taken out from the water, some of it still remains in the water. The surface of the water tends to minimixe the surface area of itseld. This ultimately minimizes the surface area of brush, hence the hair clin together.

23. 2060 Q.No. 1 a **Why small drops of mercury are spherical and bigger drops oval in shape?**

Please refer to 2076 Set C Q.No. 1g

24. 2059 Q.No. 2 a **Why does mercury inside the capillary tube made of glass depress when dipped in a reservoir of mercury?**

Since the height of liquid column on dipping the capillary tube is

$$h = \frac{2T\cos\theta}{\rho g r}$$

For mercury inside the glass tube, the value of angle of contact is greater than 90°.

For, $\theta > 90°$, $\cos\theta$ = '– ve

$$\therefore h = -\frac{2T\cos\theta}{\rho g r}$$ (-ve sign indicates the depression of liquid inside the tube)

Moreover, the cohesive force between the molecules of mercury is greater than the adhesive force between the mercury and the capillary tube. Therefore, the mercury cannot stick to the wall of the capillary tube and the contact surface of mercury will curve downward. As a result, the mercury inside the capillary tube made of glass depress when dipped in a reservoir of mercury.

25. 2058 Q.No. 2 a **Why mercury does not wet the glass tube?**

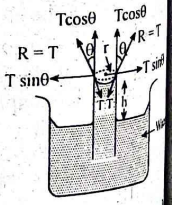
The angle of contact for mercury in glass tube is greater than 90°. Also, the force of cohesion between the molecules of mercury is large in comparison to the force of adhesion between the glass and mercury molecules. As a result, the mercury does not wet the glass tube.

Long Answer Questions

26. 2073 Set C Q.No. 5d **What is capillarity? Deduce a relation for the height of liquid column rise in a capillary tube when one end is dipped in the liquid.**

Capillarity: The process of rise or fall of liquid inside the capilarity tube is called capillarity.

Let us consider a glass capillary tube of internal radius r and dipped in a liquid, such as water whose angle of contact is small, the liquid rises in the tube at height h as shown in the figure. Due to surface tension, the liquid surface tries to contract with the glass tube. For this, liquid molecules pull the glass molecules with a force T per unit length tangentially at the point of contact of the liquid surface and glass. The glass molecules also exert equal and opposite reaction R = T, to the liquid molecules along the circle of contact. This reaction force (R = T) acting per unit length can be resolved into two components; T sinθ per unit length along horizontal which act radically outward along the circle of contact and cancel the effect of one another and T cosθ per unit length along vertical in upward



direction which are responsible for the rise of liquid in the capillary tube. The liquid rises in the tube until total upward force due to surface tension becomes equal to the weight of the liquid in the tube above the outside level of liquid.

Here,

Total upward force = vertical component of T × circumference

$= T\cos\theta \times 2\pi r = 2\pi r\, T\cos\theta$ .... (i)

Volume of the liquid in the tube above the free surface of liquid is given by

V = Volume of cylinder of height h and radius r + Volume of cylinder of height r and radius r – Volume of hemisphere of radius r

$$= \pi r^2 h + \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{1}{3}\pi r^3$$

$$V = \pi r^2\left(h + \frac{r}{3}\right)$$

Weight of liquid column= Mass of liquid column × g = ρ × V × g = $\rho\pi r^2\left(h + \frac{r}{3}\right) g$

For equilibrium,

Total upward force = Weight of the liquid column

$$T\cos\theta \times 2\pi r = \rho\pi r^2\left(h + \frac{r}{3}\right) g$$

$$\text{or, } T = \frac{\rho r\left(h + \frac{r}{3}\right) g}{2\cos\theta}$$

When the tube is of a very fine bore, $\frac{r}{3}$ may be neglected compared to h, so surface tension T becomes

$$T = \frac{\rho r h g}{2\cos\theta}$$

$$\therefore T = \frac{rh\rho g}{2\cos\theta}$$

$$\text{or, } h = \frac{2T\cos\theta}{r\rho g}$$

This is required formula for the rise of liquid in a capillary tube.

27. 2072 Set D Q.No. 5d **Define surface tension and angle of contact. Deduce an expression for rise of a liquid in a capillary tube.** [4]

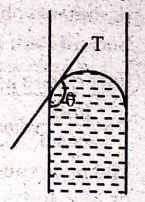
Surface tension: The property of liquid by virtue of which its surface acts as an elastic membrane covering the liquid is called surface tension. Mathematically, the surface tension of a liquid is defined as the amount of force acting on unit length on either side of an imaginary line drawn on the liquid surface at rest.

If F is the force acting on a line of length L, then the surface tension is given by,

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} \Rightarrow T = \frac{F}{L}$$

Its SI unit is Nm^{-1} and its dimensional form formula is $[M L^0 T^{-2}]$

Angle of contact: Angle of contact is defined as the angle made by tangent drawn with the convex surface inside the liquid at the point of contact of liquid with glass. In figure θ is the angle of contact when the liquid is convex when viewed from above.



Second Part: Please refer to 2073 Set C Q.No. 5d

28. 2068 Old Q.No. 3 a Or **Explain, with necessary theory, the capillary tube method for the measurement of surface tension of a liquid.** [4]

Please refer to 2073 Set C Q.No. 5d

29. 2067 Supp Q.No. 5 d **What is capillarity? Deduce an expression for the rise of a liquid in a capillary tube.** [4]

Please refer to 2073 Set C Q.No. 5d

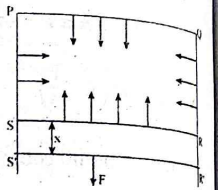
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30. 2065 Q.No. 3 a OR Prove that the surface energy and surface tension are numerically same. Explain the concept of the angle of contact, with necessary figure, when the surface of the liquid is convex if viewed from the above. [3+1]

Surface tension: Please refer to 2072 Set D Q.No. 5d

Surface energy: The free surface of a liquid at rest has a tendency to contract and occupy minimum surface area. If the surface area of the liquid is to be increased, some work must be done. This work is done increasing per unit area is stored in the form of potential energy at the surface of liquid. This energy is called *surface energy*. Moreover, the surface tension of liquid is numerically equals to its surface energy. Mathematically, surface energy $= \frac{\text{Workdone}}{\text{Area}}$.

Relation between Surface Energy and Surface Tension



In order to find the relation between surface energy of a liquid and its surface energy, we consider a rectangular frame of wire PQRS in which the wire SR is movable. If we dip the frame in soap solution, a thin film is formed which pulls the wire SR inward due to surface tension. The force on SR due to the surface tension is given by

$$F = T \times 2l$$

where, T is the surface tension of the film and l is the length of the wire SR. The length is taken twice because the soap film has two free surfaces on two sides of the wire.

If the film is stretched outward by a small distance x to the position S'R' against the force due to surface tension T, then work done is given by

Work done = force × distance = F × x

$= T \times 2l \times x$

$2l\,x$ is the total increase in area of the soap film on both side.

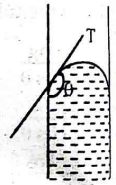
∴ From the definition of surface energy

$$\text{Surface energy } (\sigma) = \frac{\text{Work done in increasing surface area}}{\text{increase in surface area}} = \frac{T \times 2lx}{2lx} = T$$

$\therefore \sigma = T$

Hence, surface tension t is numerically equal to the surface energy σ.

Angle of contact: Angle of contact is defined as the angle made by tangent drawn with the convex surface inside the liquid at the point of contact of liquid with glass. In figure θ is the angle of contact when the liquid is convex when viewed from above.



31. 2064 Q.No. 3 a Define surface tension and surface energy of a liquid and derive a relation between them. [2+2]

Please refer to 2065 Q.No. 3 a OR

32. 2062 Q.No. 3 a What causes liquid fall or rise in a capillary tube? Derive an expression for the rise or fall of liquid in a capillary tube.

The force of cohesion and adhesion between the molecules determine the rise or fall of liquid inside the capillary tube. If the force of adhesion is dominant, the liquid rises inside the tube while the dominant cohesive force cause the fall of liquid inside the capillary tube.

Second Part: Please refer to 2073 Set C Q.No. 5d

33. 2061 Q.No. 3 a OR 2059 Q.No. 3 a OR What causes the surface of a liquid as if it is under tension? Show that $T = \frac{h\rho g r}{2\cos\theta}$ where the symbols have their usual meaning. [4]

The main cause of the surface of a liquid as it is under tension is its intermolecular force of attraction or the force of cohesion. The molecules inside the surface of liquid are attracted by all the neighbouring molecules and the resultant force is zero. But the molecule lying on the surface of the liquid is only attracted inward and hence, the surface of liquid acts as under tension.

Second Part: Please refer to 2073 Set C Q.No. 5d

34. 2057 Q.No. 3 a Deduce an expression for the rise of a liquid in a capillary tube.

Please refer to 2073 Set C Q.No. 5d

Numerical Problems

35. 2074 Supp. Q.No. 9d Angle of contact of mercury with glass is 135°. A narrow tube of glass having diameter 2mm is dipped in a beaker containing mercury. By what height does the mercury go down in the tube relative to the level of mercury outside? [4]

Solution

Given,

Angle of contact (θ) = 135°

Diameter of tube (d) = 2mm

$\therefore$ Radius of tube (r) $= \frac{d}{2} = \frac{2}{2} = 1\text{mm} = 1 \times 10^{-3}\text{m}$

Height of mercury go down (h) = ?

Density of mercury (ρ) = 13600 kgm^{-3}

Surface tension of mercury (T) = 0.547 Nm^{-1}

We have,

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$= \frac{2 \times 0.547 \times \cos 135°}{1 \times 10^{-3} \times 13600 \times 10}$$

$$= -5.7 \times 10^{-3}\text{m}$$

$$= -5.7\text{mm}$$

-ve sign indicates that height of mercury column inside tube decreases or mercury go down.

36. 2072 Set E Q.No. 9d 2058 Q.No. 3 b A capillary tube of 0.4 mm diameter is placed vertically inside a liquid of density 800 kgm^{-3}, surface tension 5 × 10^{-2}Nm^{-1} and angle of contact 30°. Calculate the height to which liquid rises in the capillary tube. [4]

Solution

Given,

Diameter of capillary tube (d) = 0.4 mm

Radius of capillary tube (r) = d/2 = 0.2 mm = 0.2 × 10^{-3} m.

Density of a liquid (ρ) = 800 kg m^{-3}

Surface tension (T) = 5.0 × 10^{-2} Nm^{-1}

Angle of contact (θ) =30°

Height of the liquid (h) = ?

We have,

$$h = \frac{2T\cos\theta}{\rho g r} = \frac{2 \times (5 \times 10^{-2}) \times \cos 30°}{800 \times 9.8 \times (0.2 \times 10^{-3})} = 0.055\text{ m}$$

37. 2071 Supp Q.No. 9d A rectangular plate of dimensions 6cm by 4 cm and thickness 2 mm is placed with its largest face flat on the surface of water. Calculate the downward force on the plate due to surface tension assuming zero angle of contact. What is the downward force if the plate is placed vertical so that it's longest side just touches the water? [Surface tension of water =7.0× 10^{-2} Nm^{-1}] [4]

Solution

Given,

Length of the plate (l) = 6 cm = 0.06 m

Breath of the plate (b) = 4 cm = 0.04 m

Thickness of the plate (t) = 2 mm = 2 × 10^{-3} m

Angle of contact (θ) = 0°

Surface tension of water (T) = 7 × 10^{-2} N m^{-1}

When the plate is placed with its largest face flat on the surface of water, then,

Total length of contact (L) $= 2(l + b) = 2(0.06 + 0.04) = 0.2$ m

Downward force due to surface tension = T × L = 7 × 10^{-2} × 0.2 = 1.4 × 10^{-2} N

If the plate is vertical, then total length of contact, L = 2 (l + d)= 2(0.06 + 0.02) = 0.124 cm

So downward force due to surface tension = T × L = 7 × 10^{-2} × 0.124 = 8.7 × 10^{-2} N

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38. 2071 Set C Q.No. 9 c **Find the work done required to break up a drop of water of radius 5×10^{-3} m into eight drops of water, assuming isothermal condition. [Surface tension of water $=72 \times 10^{-3}$ N/m]** [4]

Solution

Given,

Radius of large drop (R) $= 5 \times 10^{-3}$ m

No. of drops (n) = 8

Work done (w) = ?

Surface Tension of water (T) $= 72 \times 10^{-3}$ Nm^{-1}

Since mass of water drop remains constant

$$\left(\frac{4}{3}\pi R^3\right)\rho = 8\left(\frac{4}{3}\pi r^3\right)\rho$$

or, $R^3 = 8r^3$

or, $R = 2r$

or, $r = \frac{R}{2} = \frac{5 \times 10^{-3}}{2} = 2.5 \times 10^{-3}$ m

Area of large drop $= 4\pi R^2 = 4\pi(5 \times 10^{-3})^2 = 3.14 \times 10^{-4}$ m^2

Area of smaller drops $= 8 \times 4\pi r^2 = 8 \times 4\pi \times (2.5 \times 10^{-3})^2 = 6.28 \times 10^{-4}$ m^2

Increase in area $= (6.28 - 3.14) \times 10^{-4}$ m^2 $= 3.14 \times 10^{-4}$ m^3

Work done = Increase in area $\times$ T $= 3.14 \times 10^{-4} \times 72 \times 10^{-3} = 2.26 \times 10^{-5}$ J

39. 2071 Set D Q.No. 9 a **A capillary tube of 0.3 m diameter is placed vertically inside a liquid of density 800 Kgm^{-3}, surface tension 5×10^{-4} Nm^{-1} and angle of contact 30°. Calculate the height to which the liquid rises in the capillary tube.** [4]

Solution

Given,

Radius of capillary tube (r) $= \frac{0.3}{2} = 0.15$ m

Density of liquid (ρ) = 800 kg /m^3

Surface tension (T) $= 5 \times 10^{-4}$ N/m

Angle of constant (θ) = 30°

Height of liquid rises (h) = ?

We have

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 5 \times 10^{-4} \times \cos 30°}{0.15 \times 800 \times 10} = 7.2 \times 10^{-7}\ m$$

40. 2069 (Set A) Old Q.No. 3b **A rectangular plate of dimensions 6cm by 4 cm and thickness 2mm is placed with its largest face flat on the surface of water. Calculate the downward force on the plate due to the surface tension assuming zero angle of contact.**
(Surface tension of water = 7×10^{-2}Nm^{-1}) [3]

Please refer to 2071 Supp Q.No. 9d

41. 2066 Q.No. 3 b **Calculate the work done in breaking a drop of water of 2mm diameter into million droplets of same size. The surface tension of water is 72×10^{-3}Nm^{-1}.** [3]

Solution

Given,

Radius of water drop (R) = 1mm = 1×10^{-3}m

No. of drops (n) = 10^6

Surface tension (T) $= 72 \times 10^{-3}$ Nm^{-1}

Work done(W) = ?

Let, r be the radius of small drop, since mass of water drop contanst

$$\left(\frac{4}{3}\pi R^3\right)\rho = 10^6\left(\frac{4}{3}\pi r^3\right)\rho$$

or, $R^3 = 10^6 r^3$

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or, $R = 100r$

or, $r = \frac{R}{100} = \frac{10^{-3}}{100} = 1 \times 10^{-5}$ m

Area of large drop $= 4\pi R^2$
$= 4\pi (1 \times 10^{-3})^2$
$= 1.25 \times 10^{-5}$m^2

Area of smaller drops $= 10^6 \times 4\pi r^2$
$= 10^6 \times 4\pi \times (1 \times 10^{-5})^2$
$= 1.25 \times 10^{-3}$m^2

Increase in area $= 1.25 \times 10^{-3} - 1.25 \times 10^{-5}$
$= 1.244 \times 10^{-3}$ m^2

∴ Work done (W) = Increase in area $\times$ T
$= 1.244 \times 10^{-3} \times 72 \times 10^{-3}$
$= 8.96 \times 10^{-5}$J

42. 2060 Q.No. 3 b **A rectangular plate of dimensions 6cm. by 4 cm. and thickness 2mm. is placed vertical so that its largest side flat on the surface of the water. Calculate the downward force on the plate due to surface tension. (Surface tension of water = 7.0×10^{-2} Nm^{-1})** [3]

Please refer to 2071 Supp Q.No. 9d

C. Viscosity

Short Answer Questions

1. 2073 Set C Q.No. 1g **Small air bubbles rise slowly while big bubbles rise rapidly through the liquid. Why?** [2]

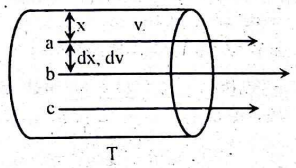
The terminal velocity of a body through a fluid is given by, $v = \frac{2r^2(\rho - \sigma)g}{9\eta}$. This shows that if the radius increases, the terminal velocity also increases. If the air bubbles are large, they have larger radius, so the terminal velocity will be more. However, if the radius is small, as in the case of a small air bubble, the terminal velocity will be less. Hence, small air bubbles rise slowly while big bubbles rise rapidly through the liquid.

2. 2072 Set D Q.No. 1f **Why is a suction effect experienced by a person standing close to the platform at a station when a fast train passes?** [2]

When a fast moving train passes to a person standing nearby its track, the air between the person and the train moves with a high speed. According to the Bernoulli formula, $P + \frac{1}{2}\rho v^2$ = constant, the air pressure becomes low near him/her compared to the pressure at the other regions including behind the person. So the air moves from back side of the person to front side pushing him/her on the way.

3. 2072 Set E Q.No. 1g **Explain with a diagram, the meaning of velocity gradient in the case of liquid flowing in a tube.** [2]

Let us consider a tube T in which a liquid is flowing as shown in figure. Let, a, b and c be three layers of liquid in the tube. A layer a is at a distance x from the surface of tube moving at a velocity v. The layer b is at a distance x +dx from the surface of tube and moving with a velocity of v +dv. Hence, the velocity increases on increasing distance. The change in velocity per unit change in distance is velocity gradient.



i.e. Velocity gradient $= \frac{dv}{dx}$

4. 2071 Supp Q.No. 1g **Airports at high elevations have longer runways for take offs and landings than the airports at sea level, why?** [2]

From Bernoulli's Theorem, $P + \frac{1}{2}\rho v^2$ = constant, where P = pressure, ρ density and v = velocity. Airport at high elevation has low air pressure than the airport at low elevation and the velocity of

airplane is greater in high elevation airport than low elevation airport. Hence, the airports at high elevations have longer runways for take offs and landings than the airports at sea level.

5. 2071 Set C Q.No. 1 c Express the dimension of the velocity gradient from the definition of the coefficient of viscosity.

The velocity gradient $\left(\frac{dv}{dx}\right)$ in terms of coefficient of viscosity (η) is given by, [2]

$F = \eta A \frac{dv}{dx}$ [in magnitude]

where A is area of surface of contact and F is viscous force, Now,

$$\frac{dv}{dx} = \frac{F}{\eta A}$$

Here,
Dimension of F = $[MLT^{-2}]$
Dimension of η = $[ML^{-1}T^{-1}]$
Dimension of A = $[L^2]$
Then,

$$\frac{dv}{dx} = \frac{[MLT^{-2}}{[ML^{-1}T^{-1}]\,[L^2]} = [T^{-1}]$$

Hence, dimension of $\frac{dv}{dx}$ is $[T^{-1}]$

6. 2070 Set D Q.No. 1 d When a smooth-flowing stream of water comes out of a faucet, it narrows as it falls. Explain.

When a smooth-flowing stream of water comes out of a faucet, it narrows as it falls. This happens due to the law of continuity of liquid flow, which states that for an incompressible fluid, the product of cross-section and velocity of fluid should be constant i.e. av = constant. When the water falls down, its velocity goes on increasing due to gravity and area decreases hence, becomes narrow. [2]

7. 2069 (Set A) Old Q.No. 1b Define coefficient of viscosity and poise.

The coefficient of viscosity is defined as [2]

or, $F = -\eta A \frac{dv}{dx}$ ... (i)

where A is area of layer of fluid, $\frac{dv}{dx}$ is the velocity gradient and η is a constant of proportionality and is called coefficient of viscosity. The negative sign shows that the direction of viscous force, F is opposite to the direction of motion of the liquid.

If A = 1 and $\frac{dv}{dx} = 1$

Then, from Eq. (i), we have

$\eta = -F$

Hence, coefficient of viscosity of a liquid is defined as the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid. Poise is the cgs unit of coefficient of viscosity & Nsm^{-2} or deca-poise in SI.

8. 2069 (Set A) Old Q.No. 2b State Bernoulli's theorem.

It states that for the streamline flow of incompressible non-viscous fluid, the sum of the pressure energy, kinetic energy and potential energy per unit mass is always constant. Thus, $\frac{P}{\rho} + \frac{1}{2}v^2 + gh =$ constant. This is Bernoull's equation. [2]

9. 2068 Old Q.No. 2 a Why does a flag flutter on strong wind?

Fluttering of flags is due to the pressure difference created by the movement of air over and below the flag. In general, velocity of air is high at greater heights and less, close to the ground. Then according to the Bernoulli's, theorem, $P + \frac{1}{2}\rho v^2$ = constant, higher velocity leads to low pressure. This gives low pressure at height and high below, creating a resultant upward force and pressure. So the flag is lifted from its drifted position and start to flutter.

10. 2067 Supp Q.No. 1 g Why do bigger air bubbles rise faster than the smaller ones in boiling water? [2]

Please refer to 2073 Set C Q.No. 1g

11. 2067 Q.No. 1 g Explain why a suction effect is experienced by a person standing close to the platform at a station when a fast train passes. [2]

Please refer to 2072 Set D Q.No. 1f

12. 2066 Q. No. 1 g When a smooth-flowing stream of water comes out of a faucet, it narrows as it falls. Explain why this happens. [2]

Please refer to 2070 Set D Q.No. 1 d

13. 2066 Q.No. 2 a Why are light roofs blown off during cyclones or strong wind storms?

During windstorm, the velocity of wind suddenly decreases when enters into the houses, but its velocity remains high above the roof. According to Bernoulli's principle, $(P + \frac{1}{2}\rho v^2 =$ constant) the pressure on the top of region is lower than that at bottom region. The inside pressure becomes very much larger than the outside pressure of the roof because the inside air has lower kinetic energy than that of outside air of the roof. This difference of pressure results in an increment of upthrust. As a result, the roof is lifted up and blown off by the wind.

14. 2064 Q.No. 1 a Define viscosity. Does it depend on temperature? [2]

The property of fluid by virtue of which opposes the relative motion of different layers of fluid is called viscosity. It is measured in terms of coefficient of viscosity (η) as given by Newton's formula: $F = -\eta A \frac{dv}{dx}$

Its SI unit is $Ns\,m^{-2}$ and its dimension is $[ML^{-1}\,T^{-1}]$. The viscosity of a liquid varies inversely with its temperature.

15. 2060 Q.No. 2 a During certain windstorm, light roofs are blown off. Why?

Please refer to 2066 Q.No. 2 a

16. 2057 Q.No. 2 a Machine parts are jamed in cold day. Why?

The viscosity of a liquid varies inversely with its temperature. As the temperature decreases in winter, the coefficient of viscosity of the lubricant increases. Due to the increase in viscosity of lubricant, the machine parts are jammed in winter.

Long Answer Questions

17. 2076 Set C Q.No. 5d What is terminal velocity? Derive an expression for the terminal velocity of a small spherical body falling through a viscous fluid. [4]

Terminal velocity: If a small object is falling through a viscous liquid, it accelerates at first due to gravity but after some time its velocity reaches a steady value, known as *terminal velocity*. In this condition, the sum of upthrust (U) and viscous force (F) acting upward is equal to the weight (W) of the object. Thus, resultant force on the object is zero so it moves with a uniform velocity (i.e. terminal velocity).
i.e. For terminal velocity, Upthrust (U) + Viscous force (F) = weight (W)
That's why; a sphere falling through a viscous medium acquires a terminal velocity.

Expression for Terminal Velocity: Consider a sphere of radius (r) and density (ρ) falling under gravity in a fluid of density (σ). The forces on the sphere are upthrust (U), and viscous force (F) acting in upward direction and weight (W) acting in downward direction, as shown in figure. When the sphere reaches a terminal velocity (v),

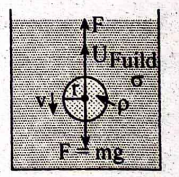
$F + U = W$ ... (i)

We have,

$F = 6\pi\eta r v$ (From Stokes' law)

U = weight of fluid displaced (From Archimedes principle)

$= \frac{4}{3}\pi r^3 \sigma g$ and $W = \frac{4}{3}\pi r^3 \rho g$ [$\because$ weight = volume × density × g]



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Therefore, from equation (i) we have,

$$6\pi\eta rv + \frac{4}{3}\pi r^3\sigma g = \frac{4}{3}\pi r^3\rho g$$

or, $6\pi\eta rv = \frac{4}{3}\pi r^3(\rho - \sigma)g$

$\therefore v = \frac{2r^2(\rho - \sigma)g}{9\eta}$

This is the required expression for terminal velocity.

18. 2074 Supp. Q.No. 5d 2058 Q.No. 3 a OR Using dimensional consideration, deduce Poiseulle's formula for the rate of flow of a liquid through a tube.

Poiseulle derived an expression for the volume of a liquid steadily flowing per second through a tube of uniform bore. This formula can be derived by the method of dimensions. [4]

Consider a liquid flowing steadily though a pipe AB of length l and radius r. It is found that the volume of liquid flowing per second through the pipe depends on

a. the coefficient of viscosity (η)
b. the radius (r) of the pipe, and
c. the pressure gradient $\frac{P}{l}$ where, P is pressure difference between the ends of the pipe of length (l).

Let, $V \propto \eta^x$, $V \propto r^y$ and $V \propto \left(\frac{P}{l}\right)^z$

Where, indices x, y and z are the numbers to be determined.

$\therefore V \propto \eta^x r^y \left(\frac{P}{l}\right)^z$

$V = k\eta^x r^y \left(\frac{P}{l}\right)^z$ ... (i)

where, k is proportionality constant.

We have,

The dimensions of V= $[L^3T^{-1}]$

The dimension of η= $[ML^{-1}T^{-1}]$

The dimension of r= [L]

The dimension of $\left(\frac{P}{l}\right)$= $[ML^{-2}T^{-2}]$

Thus, from equation (i), it follows that, $[L^3T^{-1}] = [ML^{-1}T^{-1}]^x [L]^y [ML^{-2}T^{-2}]^z$

$[L^3T^{-1}] = [M^{x+z}L^{-x+y-2z}T^{-x-2z}]$

Equating the powers of M, L & T on both sides, we get

$x + z = 0$

$-x+y-2z = 3$

$x + 2z = 1$

Solving these equations, we get,

$x = -1$, $y = 4$ and $z = 1$

Substituting values of x, y & z in equation (i), we obtain,

$$V = K\eta^{-1}r^4\left(\frac{P}{l}\right)^1$$

$$\Rightarrow V = K\frac{Pr^4}{\eta l}$$

Value of K cannot be determined by the method of dimension. Mathematical investigation shows that, $K = \frac{\pi}{8}$.

$\therefore V = \frac{\pi Pr^4}{8\eta l}$

This is Poiseulle's equation.

19. 2074 Set A Q.No. 5d Describe Stoke's method to find the coefficient of viscosity of liquid in the laboratory with necessary theory. [4]

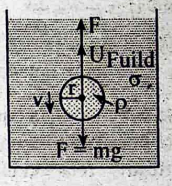
If a small object is falling through a viscous liquid, it accelerates at first due to gravity but after some time its velocity reaches a steady value, known as *terminal velocity*. In this condition, the sum of upthrust (U) and viscous force (F) acting upward is equal to the weight (W) of the object. Thus, resultant force on the object is zero so it moves with a uniform velocity (i.e. terminal velocity).

i.e. For terminal velocity, Upthrust (U) + Viscous force (F) = weight (W)

That's why; a sphere falling through a viscous medium acquires a terminal velocity.

Consider a sphere of radius (r) and density (ρ) falling under gravity in a fluid of density (σ). The forces on the sphere are upthrust (U), and viscous force (F) acting in upward direction and weight (W) acting in downward direction, as shown in figure. When the sphere reaches a terminal velocity (v),

$F + U = W$ ... (i)



We have,

$F = 6\pi\eta r v$ (From Stokes' law)

U = weight of fluid displaced (From Archimedes principle)

$= \frac{4}{3}\pi r^3\sigma g$ and $W = \frac{4}{3}\pi r^3\rho g$ [$\because$ weight = volume × density × g]

Therefore, from equation (i) we have,

$$6\pi\eta rv + \frac{4}{3}\pi r^3\sigma g = \frac{4}{3}\pi r^3\rho g$$

or, $6\pi\eta rv = \frac{4}{3}\pi r^3(\rho - \sigma)g$

$\therefore \eta = \frac{2r^2(\rho - \sigma)g}{9v}$

This is the required expression for determination of coefficient of viscosity.

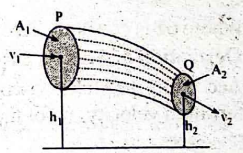
20. 2074 Set B Q.No. 5b 2064 Q.No. 3 a OR State and prove Bernoulli's principle. [4]

Bernoull's Principle: It states that for the streamline flow of incompressible non-viscous fluid, the sum of the pressure energy, kinetic energy and potential energy per unit mass is always constant.

Thus, $\frac{P}{\rho} + \frac{1}{2}v^2 + g\,h$ = constant. This is Bernoull's equation

Proof: Consider section PQ of a pipe. A_1 and A_2 are cross-sections at P and Q respectively. h_1 and h_2 are heights at P and Q from reference level. P_1 and P_2 be the fluid pressure at P and Q, v_1 and v_2 be the velocities at respective point; as shown in figure.

According to the principle of continuity, velocity at any section of the tube is inversely proportional to the area of cross-section at that point. Thus, velocity v_1, at greater cross-section A_1 is less than velocity, v_2 at smaller cross-section A_2. Hence, the liquid is accelerated when moving form P to Q.



Now, Force at P = P_1A_1

Distance traveled, at P, in 1 sec = v_1

$\therefore$ Work done W_1 on the liquid entering at P in 1 sec is,

W_1 = Force × distance traveled in 1 sec = $P_1A_1v_1$.

Similarly, work done, W_2, by the liquid, leaving at Q, in 1 sec is,

$W_2 = P_2A_2v_2$

Net work done by the fluid is,

$W = W_1 - W_2 = P_1A_1v_1 - P_2A_2v_2 = P_1V - P_2V$

$W = (P_1 - P_2)\frac{m}{\rho}$ ... (i)

Where, $V = A_1v_1 = A_2v_2 = \frac{m}{\rho}$; is the volume of the fluid that flows through the tube in 1 second, m is mass of the fluid flowing through the tube in one second.

This work done W, should be equal to the sum of change in kinetic energy and potential energy of the mass.

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Here, increase in kinetic energy $= \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

Decrease in potential energy $= mg(h_1 - h_2)$

Net gain in the energy $= \frac{1}{2}m\left(v_2^2 - v_1^2\right) - mg(h_1 - h_2)$ ... (ii)

From equations (i) and (ii), we have,

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}m\left(v_2^2 - v_1^2\right) - mg(h_1 - h_2)$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g}$$

$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant} \quad \text{... (iii)}$$

This is Bernoulli's equation for streamline flow of liquid.

The Bernoull's equation is valid only for steady, non-viscous and incompressible fluid because continuity relation is used for it.

21. 2072 Supp Q.No. 5b What is terminal velocity? Describe Stoke's method to determine the coefficient of viscosity of a liquid.

Please refer to 2074 Set A Q.No. 5d

22. 2072 Set C Q.No. 5d State and prove Bernoulli's theorem for the steady flow of an incompressible and non-viscous flow.

Please refer to 2074 Set B Q.No. 5b

23. 2072 Set E Q.No. 5b State and prove Bernoulli's theorem in flowing liquid.

Please refer to 2074 Set B Q.No. 5b

24. 2071 Set D Q.No. 5 c Describe the experimental method to find the coefficient of viscosity of a liquid by Stoke's method.

Please refer to 2074 Set A Q.No. 5d

25. 2070 Supp (Set A) Q.No. 5 d State Stoke's law and deduce it from dimensional analysis. Define coefficient of viscosity of a liquid.

Stoke's law of viscous force states that, viscous force is directly proportional to coefficient of viscosity of medium (η), radius of spherical body (r) and terminal velocity of body in viscous medium (v).

i.e. $F \propto \eta r v$

or, $F = 6\pi\eta r v$ ... (i)

where 6π is constant. This equation (i) is Stoke's law.

Derivation of Stoke's Law by Dimensional Method

According to Stoke, viscous force, F depends upon (i) coefficient of viscosity, η of the medium, (ii) terminal velocity, (v) of the body, and (iii) radius, r of the spherical body. This can be written as

$F \propto \eta^a v^b r^c$

or, $F = k\eta^a v^b r^c$ ... (ii)

where k is dimensionless constant and a, b, c are constant to be determined.

In dimensional form, the Eq. (ii) can be written as

$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [LT^{-1}]^b [L]^c$

or, $[MLT^{-2}] = [M^a L^{-a+b-c} T^{-a-b}]$

Applying the principle of homogeneity of dimensions, we get

$a = 1$ ... (iii)

$-a + b + c = 1$ ... (iv)

and $-a - b = -2$ ... (v)

Solving these three equations, we get

$a = 1, b = 1, c = 1$

Substituting the values of a, b and c in equation (ii), we get

$F = k\eta r v$

For small spheres, the value of k is found to be 6π and the viscous force is

$F = 6\pi\eta r v$

This is known as Stoke's law.

Coefficient of viscocity: Coefficient of viscosity of a liquid is defined as the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid.

26. 2070 Supp (Set B) Q.No. 5 d State and prove Bernoulli's principle for a liquid flow. [4]

Please refer to 2074 Set B Q.No. 5b

27. 2070 Set C Q.No. 5 d State and prove Bernoulli's theorem for the flow of non-viscous fluids. [4]

Please refer to 2074 Set B Q.No. 5b

28. 2070 Set D Q.No. 5 c Define terminal velocity and hence, describe the method of determining the coefficient of viscosity of a liquid using Stoke's law. [4]

Please refer to 2074 Set A Q.No. 5d

29. 2069 Supp Q.No. 5c 2066 Q. No. 5 b Derive an expression for terminal velocity of small spherical ball of radius 'a' dropped gently in a viscous liquid of density 'ρ' and the coefficient of viscosity 'η'. [4]

Please refer to 2076 Set C Q.No. 5d

30. 2069 (Set A) Old Q.No. 3aOR What is terminal velocity? Using Stoke's formula, find the expression for the terminal velocity. What is the acceleration of the body inside the liquid before it attains terminal velocity?

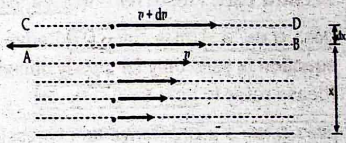
Please refer to 2076 Set A Q.No. 5d

31. 2068 Old Q.No. 3 a Derive Newton's formula for viscous force Define coefficient of viscosity. Calculate its dimensions. [2+1+1]

Viscosity: The property of a fluid (liquid or gas) by virtue of which an internal friction comes in to play between the layers of fluid when the fluid is in motion and opposes the relative motion of its different layers is called viscous force or simply viscosity.

Newton's Law of Viscosity, Coefficient of Viscosity

Consider a liquid flowing steadily over a fixed solid horizontal surface as shown in figure. The layer in contact with the fixed surface is at rest while the velocity of other layers increases uniformly upwards.



Consider two layers AB and CD moving with velocities v and $v + dv$ at a distance x and $(x + dx)$ respectively from the fixed solid surface.

The change of velocity divided by the distance, dv/dx, in a direction perpendicular to the velocity is called the velocity gradient.

According to Newton, the viscous force F depends upon the following factors:

i. It is directly proportional to area, A of the layers in contact i.e.

$F \propto A$

ii. It is directly proportional to the velocity gradient between the layers i.e.

$F \propto \frac{dv}{dx}$

Combining these two factors, we have

$F \propto A\frac{dv}{dx}$

or, $F = -\eta A\frac{dv}{dx}$ ... (i)

where η is a constant of proportionality and is called coefficient of viscosity. Its value depends upon

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the nature of the liquid. The negative sign shows that the direction of viscous force, F is opposite to the direction of motion of the liquid.

If $A = 1$ and $\frac{dv}{dx} = 1$

Then, from Eq. (i), we have

$\eta = -F$

Hence, coefficient of viscosity of a liquid is defined as the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid.

Dimensional Formula for η

The coefficient of viscosity of a fluid is

$$\eta = \frac{F}{A\frac{dv}{dx}}$$

The dimension of force, $F = [ML\,T^{-2}]$, dimension of area, $A = [L^2]$ and dimension of $dv/dx = [LT^{-1}/L]$

Then, The dimensional of coefficient of viscosity is

$$\eta = \frac{[M\,L\,T^{-2}]}{[L^2]\left[\frac{L\,T^{-1}}{L}\right]} = [M\,L^{-1}\,T^{-1}]$$

The coefficient of viscosity is also expressed in $kg\ m^{-1}s^{-1}$ in SI-units.

32. 2066 Q.No. 3 a OR **What is viscosity? Obtain an expression of viscous force from Newton's laws for viscosity.** [1+3]

Please refer to 2068 Old Q.No. 3 a

33. 2065 Q.No. 3 a **How is the coefficient of viscosity related with the velocity gradient of the following liquid? Use dimensional method to obtain Poiseuille's formula for the flow of fluid.** [1+3]

Relationship between η and $\frac{dv}{dx}$: The coefficient of viscosity (η) of a liquid is related with the velocity gradient $\left(\frac{dv}{dx}\right)$ as $F = -\eta A\frac{dv}{dx}$ where, A is area of liquid flowing and F is viscous force acting between the layer of flowing liquid.

Second Part: Please refer to 2074 Supp. Q.No. 5d

34. 2063 Q.No. 3 a 2059 Q.No. 3 a **Derive Bernoulli's equation from the work energy theorem. Explain why the equation is valid only for steady non-viscous and incompressible fluid?**

Please refer to 2074 Set B Q.No. 5b

35. 2063 Q.No. 3 a OR **Derive an expression for terminal velocity of a small spherical ball dropped gently in a viscous liquid.**

Please refer to 2076 Set C Q.No. 5d

36. 2060 Q.No. 3 a OR **Explain why a sphere falling through a viscous medium acquires a terminal velocity. Derive an expression for it.**

Please refer to 2076 Set C Q.No. 5d

Numerical Problems

37. 2076 Set B Q.No. 9d **Eight spherical rain drops of the same mass and radius are falling down with a terminal speed 5 cms^{-1}. If they coalesce to form one big drop, what will be its terminal speed?** [4]

Solution

Given,

Velocity of small rain drop (v) = 5 cm/sec = 0.05 m/sec

Velocity of large rain drop (V) = ?

Let r be the radius of small rain drop,

Then viscous force = $6\pi\eta rv$ and

Weight of drop $= \frac{4}{3}\pi r^3\rho g$

$\therefore 6\pi\eta rv = \frac{4}{3}\pi r^3\rho g$ ...(i)

Since volume of eight small volume is equal to the volume of a large drop. If R be the radius of large drop, then

$\frac{4}{3}\pi R^3 = 8.\frac{4}{3}\pi r^3$

$R^3 = 8r^3$

$R = 8^{1/3}r = 2^{3/3}\,r = 2r$

Viscous force for large drop $= 6\pi\eta RV$

Weight of large drop $= \frac{4}{3}\pi R^3\rho g$

$6\pi\eta RV = \frac{4}{3}\pi R^3\rho g$ ... (ii)

Dividing (ii) by (i), we get

$\frac{RV}{rv} = \left(\frac{R}{r}\right)^3$

or, $\frac{V}{v} = \left(\frac{R}{r}\right)^2 = \left(\frac{2r}{r}\right)^2$

or, $\frac{V}{v} = 4$

or, $V = 4v = 4 \times 0.05 = 0.2$ m/sec

38. 2075 GIE Q.No. 9d 2069 (Set A) Q. No. 9d **Castor oil at 20°C has a coefficient of viscosity 2.42 Ns/m^2 and density 940kg/m^3. Calculate the terminal velocity of a steel ball of radius 2.00 mm falling under gravity in the oil.** [4]

Solution

Given,

Coefficient of viscosity (η) = 2.42 Nsm^{-2}

Density of oil (σ) = 940 kgm^{-3}

Density of steel (ρ) = 7800 kgm^{-3}

Radius of steel ball (r) = 2mm = 2×10^{-3} m

Terminal velocity (v) = ?

We have,

$$v = \frac{2r^2(\rho - \sigma)\,g}{9\eta} = \frac{2\times(2\times10^{-3})^2(7800 - 940)\times10}{9\times2.42} = 0.025\ \text{m/sec}$$

39. 2075 Set A Q.No. 9d **Calculate mass of an aeroplane with the wings of area 55m^2 flying horizontally. The velocity of air above and below the wings is 155 m/s and 140 m/s respectively.** [4]

Solution

Given,

Mass of aeroplane (m) = ?

Area of wings (A) = 55 m^2

Velocity of air above wings (v_1) = 155 m/sec

Velocity of air below wings (v_2) = 140 m/sec

Density of air (ρ) = 1.29 Kg/m^3

Now,

From Bernoulli's principle

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or, $P_1 - P_2 = \frac{1}{2}\rho(v_1^2 - v_2^2)$

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$= \frac{1}{2} \times 1.29 (155^2 - 140^2)$

$= 2854 \text{ N/m}^2$

Maximum lifting force = Pressure difference × Area

$= (P_1 - P_2) \times A$

$= 2854 \times 55$

$= 1.57 \times 10^5 \text{ N}$

Mass of aeroplane, $M = \frac{F}{g}$

$= \frac{1.57 \times 10^5}{10}$

$= 1.57 \times 10^4 \text{ Kg}$

40. 2073 Supp Q.No. 9c 2062 Q.No. 3 b **Calculate the magnitude and direction of the terminal velocity of an air bubble of radius 1 mm passing through an oil of viscosity 0.2 Nsm^{-2} and specific gravity 0.9 if the density of air is 1.29 Kgm^{-3}.** [4]

Solution

Given,

Radius of air bubble (r) = 1mm = 1×10^{-3} m

Viscosity of oil (η) = 0.20 Nm^{-2}s

Density of air (ρ) = 1.29 kg/m^3

Density of oil (σ) = 0.9×1000 = 900 kg/m^3

Terminal velocity (v) = ?

We have,

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times (1 \times 10^{-3})^2 (1.29 - 900) \times 10}{9 \times 0.20} = -0.0099 \text{m/sec}$$

The negative sign indicates that the bubble moves in upward direction against the force of gravity.

41. 2073 Set D Q.No. 9d **Eight spherical raindrops of equal size are falling vertically through air with a terminal velocity of 0.15 m/s. What would be the terminal velocity, if they coalesce to form a big drop?** [4]

Solution

Given,

Velocity of small rain drop (v) = 0.15m/sec

Let r be the radius of small rain drop,

Then viscous force = $6\pi\eta rv$ and

Weight of drop = $\frac{4}{3}\pi r^3 \rho g$

$\therefore 6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g$ ...(i)

Since volume of three small volume is equal to the volume of a large drop. If R be the radius of large drop, then

$\frac{4}{3}\pi R^3 = 8.\frac{4}{3}\pi r^3$

$R^3 = 8r^3$

$R = 8^{1/3} r = 2^{3/3} r = 2r$

Viscous force for large drop = $6\pi\eta RV$

Weight of large drop = $\frac{4}{3}\pi R^3 \rho g$

$6\pi\eta RV = \frac{4}{3}\pi R^3 \rho g$ ... (ii)

Dividing (ii) by (i), we get

$\frac{RV}{rv} = \left(\frac{R}{r}\right)^3$

$\text{or, } \frac{V}{v} = \left(\frac{R}{r}\right)^2 = \left(\frac{2R}{r}\right)^2$

$\text{or, } \frac{V}{v} = 4$

$\text{or, } V = 4v = 4 \times 0.15 = 0.6 \text{ m/sec}$

42. 2072 Set D Q.No. 9d 2057 Q.No. 3 b **Castor oil at 20°C has a coefficient of viscosity 2.42 Ns/m^2 and density 940 kg/m^3. Calculate the terminal velocity of steel ball of radius 2.00 mm falling under gravity in the oil, taking density of steel as 7800 kg/m^3.** [4]

Please refer to 2075 GIE Q.No. 9d

43. 2068 Q.No. 9 d **Water flows steadily through a horizontal pipe of non-uniform cross section. If the pressure of water is 4×10^4 Nm^{-2} at a point where the velocity of flow is 2ms^{-1} and cross section is 0.02m^2, what is the pressure at a point where cross section reduces to 0.01m^2?** [4]

Solution

Given,

For one end of pipe,

Pressure (p_1) = 4×10^4 Nm^{-2}

Velocity (v_1) = 2ms^{-1}

Area (a_1) = 0.02m^2

For another end of pipe,

Pressure (p_2) = ?

Area (a_2) = 0.01m^2

Velocity (v_2) = ?

We know from continuity equation

$a_1v_1 = a_2v_2$

$v_2 = \frac{2 \times 0.02}{0.01} = 4 \text{m/sec}$

Now, from Bernooll's equation,

$\therefore p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$ [$\because \rho = 1000 \text{ kg/m}^2$]

$\text{or, } 4 \times 10^4 + \frac{1}{2} \times 1000 \times 2^2 = p_2 + \frac{1}{2} \times 1000 \times 4^2$

$\text{or, } 42000 = p_2 + 8000$

$\text{or, } p_2 = 42000 - 8000 = 34000 \text{ Nm}^{-2} = 3.4 \times 10^4 \text{ Nm}^{-2}$

44. 2067 Q.No. 9 d **Three spherical raindrops of equal size are falling vertically through air with a terminal velocity of 0.150 m/s. What would be the terminal velocity if these three drops were to coalesce to form a larger spherical drop?** [4]

Solution

Given,

Velocity of small rain drop (v) = 0.15m/sec

Let r be the radius of small rain drop,

Then viscous force = $6\pi\eta rv$ and

Weight of drop = $\frac{4}{3}\pi r^3 \rho g$

$\therefore 6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g$ ...(i)

Since volume of three small volume is equal to the volume of a large drop. If R be the radius of large drop, then

$\frac{4}{3}\pi R^3 = 3.\frac{4}{3}\pi r^3$

$R^3 = 3r^3$

$R = 3^{1/3} r^3$

Viscous force for large drop = $6\pi\eta RV$

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Weight of large drop $= \frac{4}{3}\pi R^3\rho g$

$6\pi\eta RV = \frac{4}{3}\pi R^3\rho g$ ... (ii)

Dividing (ii) by (i), we get

$\frac{RV}{rv} = \frac{R^3}{r^3}$

or, $\frac{V}{v} = \left(\frac{R}{r}\right)^2 = \left(\frac{3^{1/3}.r}{r}\right)^2$

or, $V = v.\ 3^{2/3} = 0.15 \times 3^{2/3} = 0.31$ m/sec

45. 2065 Q.No. 3 b Two drops of same liquid of same radius are falling through air with steady velocity of 2 ms^{-1}. If the two drops coalesce, what would be the terminal velocity?

Solution

Given,

Velocity of liquid drop (v) = 2.0 m/sec

Let r be the radius of liquid drop.

We know,

Viscous force $= 6\pi\eta rv$

Weight of liquid drop $= \frac{4}{3}\pi r^3\rho g$

$\therefore\ 6\pi\eta rv = \frac{4}{3}\pi r^3\rho g$ ...(i)

Since volume of two small drop and volume of single large drop becomes equal. Let R be the radius of large drop when two small drops coalesce.

$\therefore\ \frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$

or, $R^3 = 2r^3$

or, $R = 2^{1/3}r$

Hence, for larger drop

$6\pi\eta RV = \frac{4}{3}\pi R^3\rho g$ ...(ii) [V be terminal velocity of larger drop]

Dividing (ii) by (i), we get

$\frac{RV}{rv} = \frac{R^3}{r^3}$

or, $\frac{V}{v} = \left(\frac{R}{r}\right)^2$

or, $V = v\left(\frac{2^{1/3}r}{r}\right)^2 = v.\ 2^{2/3} = 2\ .\ 2^{2/3} = 3.17$ m/sec

46. 2061 Q.No. 3 b What is the terminal velocity of a glass ball falling through a tall jar containing glycerene? The densities of the glass ball and glycerene are 2.6 g/cc and 1.32 g/cc respectively and the viscosity of the glycerene is 0.85 poise and radius of the glass ball is 2mm.

Solution

Given,

Density of glass (ρ) = 2.6 g/cc

Density of glycerene (σ) = 1.32 g/cc

Viscosity of glycence (η) = 0.85 pise

Radius of ball (r) = 2mm = 0.2 cm

Terminal velocity (v) = ?

We know,

$v = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times 0.2^2\ (2.6 - 1.32) \times 980}{9 \times 0.85} = 13.1$ cm/s = 0.13 m/sec

47. 2059 Q.No. 3 b What is the terminal velocity of a steel ball falling through a tall jar containing glycerine? The densities of the steel ball and glycerine are 8.5 g/cc and 1.32 g/cc respectively and the viscosity of the glycerine is 0.85 poise and radius of the steel ball is 2 mm.

Solution

Given,

Density of glass (ρ) = 8.5 g/cc

Density of glycerene (σ) = 1.32 g/cc

Viscosity of glycence (η) = 0.85 poise

Radius of ball (r) = 2mm = 0.2 cm

Terminal velocity (v) = ?

We know,

$v = \frac{2r^2(\rho - \sigma)g}{9\eta} = \frac{2 \times 0.2^2\ (8.5 - 1.32) \times 980}{9 \times 0.85} = 73.5$ cm/s

□□□

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UNIT 2

Heat and Thermodynamics

Chapter 1: Heat and Temperature

Short Answer Questions

1. **2074 Supp. Q.No. 2a** **A student claimed that thermometers are useless because a thermometer always registers its own temperature. How would you respond?**

When a thermometer is used to measure temperature of a body then there will be exchange of heat until the body and thermometer attain a common temperature. The student's claim is true that the thermometer registers its own temperature but the temperature registered is also the temperature of the body. Hence, the registered temperature is the temperature of thermometer as well as body.

2. **2074 Set B Q.No. 2a** **Express 98°F into Kelvin scale.**

we have,

$$\frac{F-32}{180} = \frac{K-273}{100}$$

Given F = 98°F, then

$$\text{or, } \frac{98-32}{180} = \frac{K-273}{100}$$

$$\text{or, } \frac{98-32}{18} = \frac{K-273}{10}$$

or, 980 - 320 18K - 273 x 18

or, 660 = 18K - 4914

or, K = 310K

3. **2073 Supp Q.No. 2a** **Two metallic rod of the same material but of different length are heated. Smaller rod has circular area of cross-section but larger rod has rectangular cross-section. Will their linear expansivity be the same or different? Give justification of your answer.**

The coefficient of linear expansivity of a material is given as

$$\alpha = \frac{\Delta l}{l(\theta_2 - \theta_1)}; \quad \theta_2 - \theta_1 = \text{Change in temperature}$$

The ratio $\frac{\Delta l}{l}$ is same in each case for a material. Hence, a smaller rod has circular area of cross section and a bigger rod of the rectangular cross-section have same linear expansivity.

4. **2073 Set D Q.No. 2a** **Why is it sometimes possible to loosen caps on screw top bottles by dipping the cap briefly in hot water?**

When the cap of a screw top bottle is dipped briefly in hot water, there will be unequal expansion of the metal cap and the glass neck in which it is fitted. Since the expansivity of metal is greater than the expansivity of glass, so cap expands before the heat reaches the bottle and hence, the metal cap will be loosened.

5. **2072 Set E Q.No. 2a** **Explain why the possibility of "water taps burst" rises in severe winter.**

In cold weather, when water trapped in a pipe cools further from 4° C to 0° C, it freezes. The water on cooling from 4° C to 0° C, expands considerably due to anomalous expansion of water. But the size of the pipes contract due to the fall in temperature. The expansion in volume of water in the ice gives outward pressure in the pipe due to which the pipe bursts.

6. **2071 Supp Q.No. 2a** **A square brass plate has a large circular hole cut in its centre. If the plate is heated, it will expand. Will the diameter of the hole expand or contract? Explain your answer.** [2]

When a metallic block with a hole on it is heated, its all the dimensions are expanded including hole. A useful analogy to thermal expansion is dimension of the object undergoes the same fractional change. Thus, increase in temperature will expand the hole, not reduce it.

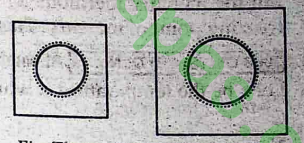


Fig: Thermal expansion of a plate with a hole.

7. **2071 Set C Q.No. 2 b** **Explain the significance of anomalous expansion of water with an example observed in nature.** [2]

Most of the liquids expand on being heated but water behaves in a peculiar manner. Water in the range of 0°C to 4°C contracts with increasing temperature. Its coefficient of volume expansion is negative in this range. Above this temperature, water expands. Hence, water has maximum density at 0°C. Such behaviour of water is called anomalous expansion. The anomalous expansion of water has important effect on plant and animal life in lakes when the surface temperature is below 4°C. The water near the surface is less dense than water below it and so, it remains there colder than the water at the bottom. As the surface freezes, the ice floats on the surface while water at 4°C remains at bottom where aquatic life remains alive. Hence, aquatic plants and animals can serve in very cold places due to property of anomalous expansion of water.

8. **2071 Set D Q.No. 2 a** **Define the coefficient of cubical expansion of a solid and hence, write an expression for the variation of its density with temperature.** [2]

Coefficient of cubical expansion of solid is defined as the change in volume per unit original volume per unit rise in temperature.

i.e. $\gamma = \frac{\Delta V}{V_0 \Delta\theta}$

The variation of density with temperature is

$$\rho_2 = \frac{\rho_1}{1 + \gamma\Delta\theta}$$

ρ_2 = density at θ_2

ρ_1 = density at θ_1

$\Delta\theta = \theta_2 - \theta_1$

9. **2070 Supp (Set A) Q.No. 2 b** **Forzen water pipes often burst. Will a alcohol thermometer break if the temperature drops below the freezing point of alcohol?** [2]

Forzen water pipes often burst due to anomalous expansion i.e., expansion on cooling below 4°C. At 0°C the water has greater volume than at 4°C but pipe contract on cooling. so, water pipe burst on cooling. However, the alcohol thermometer will not break if the temperature drops below freezing point of alcohol because the volume of alcohol decreases (or alcohol contract) on cooling to its freezing point.

10. **2070 Set C Q.No. 2 a** **A hole is drilled in a flat metal sheet. What happens to the diameter of the hole as the metal sheet is heated to higher temperature?** [2]

Please refer to 2071 Supp Q.No. 2a

11. **2070 Set D Q.No. 2 a** **Does the cubical expansivity of a liquid depend on its original volume? Explain.** [2]

No, the cubical expansivity of a liquid does not depend upon original volume of liquid.

The cubical expansitivity of a liquid is given as, $\gamma = \frac{\Delta V}{V(\theta_2 - \theta_1)}$

The ratio of $\frac{\Delta V}{V}$ is same in each case for a liquid so, the coefficient of cubical expansion is same for all volume of a liquid and does not depend on the original volume.

12. **2069 (Set A) Q. No. 2a** **Frozen water pipes often burst, will a mercury thermometer break if the temperature of the thermometer is brought below the freezing point of mercury?** [2]

In cold weather, when water trapped in a pipe cools further from 4° C to 0° C, it freezes. The water on cooling from 4° C to 0° C expands considerably due to anomalous expansion of water. But the size of

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the pipes contract due to the fall in temperature of water in the ice. This creates a vacuum in the pipe. The outward pressure in the pipe due to which the pipe bursts.

But the mercury thermometer will not break if the temperature of thermometer is brought below the freezing point of mercury. This is because, the thermometer as well as mercury contract on freezing.

13. **2069 Old (Set B) Q. No. 4a At what temperature will the Celsius scale reading double the Fahrenheit reading?**

Let x be the temperature at which celsius scale doubles the Fahrenheit reading, then,

$C = 2x$.

Now,

$$\frac{C}{100} = \frac{F-32}{180}$$

or, $\frac{2x}{100} = \frac{x-32}{180}$

or, $\frac{x}{50} = \frac{x-32}{180}$

or, $180x = 50x - 1600$

or, $(180 - 50)x = -1600$

or, $130x = -1600$

$x = -12.3°F$

∴ At $-12.3°F$, the celsius scale reading doubles the Fahrenheit reading.

14. **2068 Q.No. 2 a Define absolute temperature.**

Absolute temperature: The temperature in Kelvin scale is absolute temperature. It's value is $(273 + \theta)°C$, where θ can take any values.

Absolute zero temperature: The temperature at which the pressure or volume of any gas reduces to zero is called absolute zero temperature. It is the lowest temperature that can exist in universe at which the matter is highly condensed and no collision between gas molecules occurs. Its value is 0 K or $-273°C$.

15. **2068 Old Q.No. 4 a At what temperature, do the Fahrenheit-thermometer and Celsius-thermometer show the same reading?**

Let y be the required value on both scales. So

$C = F = x$

We know,

$$\frac{C-0}{100} = \frac{F-32}{180}$$

or, $\frac{x}{100} = \frac{x-32}{180}$

or, $\frac{x}{5} = \frac{x-32}{9}$

or, $5x - 160 = 9x$

or, $-160 = 4x$

∴ $x = -40$

Thus, required value is $-40°C$ or $-40°F$.

16. **2067 Supp Q.No. 2 a Does the coefficient of linear expansion depend on length? Justify your answer.**

No, the coefficient of linear expansion of a material doesn't depend on length. The linear expansitivity of a material is given as

$$\alpha = \frac{\Delta l}{l(\theta_2 - \theta_1)}$$

The ratio $\frac{\Delta l}{l}$ is same in each case for a material. So, the coefficient of linear expansion is same for all lengths of a material and does not depend on the length.

17. **2066 Q. No. 2 b When a metallic block with hole in it is heated, why does not the material around the hole expand into the hole and make it smaller?** [2]

Please refer to 2071 Supp Q.No. 2a

18. **2065 Q. No. 4 a Why are glass windows possible to be cracked in very cold region?** [2]

In the very cold regions, the temperature of the air outside the room will be very low as compared to the temperature inside the room. There is a large difference of temperature inside and outside the room because glass is a bad conductor of heat. Due to this reason, there is an unequal expansion of the outer and inner sides of glass and hence, the glass windows cracks.

19. **2064 Q.No. 4 a 2050 Q.No. 7i Two bodies made of the same material have the same external dimension and appearance, but one is solid and the other is hollow. When they are heated, is the overall volume expansion the same or different?** [2]

For the same volume, the expansion in two bodies of the same material depends only on the temperature. If two identical bodies, one solid and other hollow, are given the same amount of heat, the rise in temperature in the hollow body is greater than that of solid body because of large mass of the solid body. As a result, from the relation, $V_2 = V_1(1 + \gamma \Delta\theta)$, the hollow body will have greater expansion though the same heat is given to the two bodies due to greater $\Delta\theta$.

20. **2063 Q.No. 4 a Why does a thick glass tumbler crack when boiling water is poured on it?** [2]

Glass is bad conductor of heat. As hot water is poured in it, the inner part of glass expands more quickly than outer part. Therefore, there is unequal expansion between inner and outer layers, which causes crack.

21. **2062 Q.No. 4 a Why a column of mercury in a thermometer first descends slightly and then rises when placed in hot water? Explain.** [2]

When a mercury thermometer is placed in hot water, the bulb at first gets heat from outside and expands and hence, the column of mercury in thermometer at first descends slightly. Later on, heat is passed to the mercury and it expands. We know that, the expansivity of Hg is greater than that of the material of the bulb and hence, the level of mercury in the thermometer first descends slightly and then rises when placed in hot water.

22. **2061 Q.No. 4 b At what point of thermometric scale does kelvin scale reading coincide with Fahrenheit scale reading?** [2]

Solution

Let x be the required value on both scales. So

$K = F = x$

We know,

$$\frac{K-273}{100} = \frac{F-32}{180}$$

or, $\frac{x-273}{100} = \frac{x-32}{180}$

or, $\frac{x-273}{5} = \frac{x-32}{9}$

∴ $x = 574.25$ K

Thus, required value is 574.25 K or 574.25°F.

23. **2059 Q.No. 4 a What are the differences between heat and temperature?** [2]

The differences between heat and temperature are given below:

Heat	Temperature
1. Heat is a form of energy.	1. Temperature is a degree of hotness or coldness of a body.
2. SI-unit of heat is joule and CGS- unit is calorie.	2. Its SI-unit is Kelvin.
3. It is a cause.	3. It is an effect.
4. Two bodies can be in thermal equilibrium without having same amount of heat.	4. Two bodies cannot be in thermal equilibrium if they are at different temperatures.
5. It is a measure of total kinetic energy of all molecules of a body.	5. It measures average kinetic energy of all molecules of the body.

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24. 2058 Q.No. 4 a Fishes stay alive in frozen pond in winter. Explain.

Fishes stay alive in frozen pond in winter because of anomalous expansion of water. At 4°C, water has lowest volume and highest density which remains inside the pond and the ice which has the temperature below 4°C lies on the surface of pond because it has lower density. This makes the fishes alive inside the frozen pond in winter.

25. 2058 Q.No. 4 c Why mercury is used in thermometer?

Mercury is commonly used as a thermometric substance due to following reasons:

i. It's boiling point is 357° C and freezing point - 39° C. Therefore, it can be used over a wide range of temperature.
ii. Its expansion is nearly uniform over the ordinary range of temperatures.
iii. It can be easily seen through the glass because it is opaque and shining in colour.
iv. It does not wet the glass.
v. It has low specific heat, so it doesn't take much heat from the body whose temperature is to be measured.

26. 2055 Q.No. 7 d Does the coefficient of linear expansion depend on length? Explain.

Please refer to 2067 Supp Q.No. 2 a

27. 2055 Q.No. 7 e Why is mercury used commonly as a thermometric substance? Give two reasons.

Please refer to 2058 Q.No. 4 c

28. 2054 Q.No. 7 b Explain why a column of mercury in thermometer first descends slightly and then rises when placed in hot water?

Please refer to 2062 Q.No. 4 a

29. 2052 Q.No. 7 c Water level initially falls in a vessel when it is heated. Why?

When a vessel containing water is heated, the vessel receives heat earlier than water. As a result, at first expansion of the vessel takes place and hence, initially the water level falls in a vessel.

30. 2052 Q.No. 7 d Why is it sometimes possible to loosen caps on screw top bottles by dipping the cap briefly in hot water?

Please refer to 2073 Set D Q.No. 2a

31. 2052 Q.No. 7 e Why do frozen water pipe bursts?

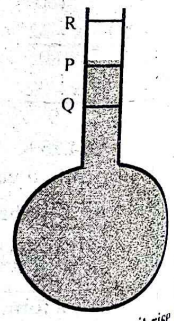
Please refer to 2072 Set E Q.No. 2a

Long Answer Questions

32. 2076 Set C Q.No. 6a Define the coefficients of real expansion and apparent expansion and hence derive the relation between them. [4]

Real and Apparent Expansions of Liquid

Let us consider a glass flask of long stem with uniform small bore and it is filled with liquid up to level P. When the vessel is heated gently, at first the vessel expands so that the liquid level lowers to a point Q and after then, the liquid expands so that it's level rises to a point R above P as shown in figure. Here, actually, PQ is the expansion of the vessel and QR is the real expansion of liquid. The difference in real expansion of the liquid V_{QR} and expansion of the vessel V_{PQ} is called the apparent expansion of the liquid. In figure,



$V_{QR} = V_{PR} + V_{PQ}$ ... (i)

∴ Real expansion of liquid= apparent expansion of liquid + expansion of the vessel

Coefficient of Real Expansion

It is defined as the ratio of the real increase in volume of the liquid to the original volume per unit rise in temperature. It is denoted by γ_r and is given by,

$$\gamma_r = \frac{\text{real increase in volume}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_r}{V \times \Delta\theta}$$

Coefficient of Apparent Expansion

It is defined as the ratio of the apparent increase in volume of the liquid to the original volume per unit rise in temperature. It is denoted by γ_a and is given by,

$$\gamma_a = \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_a}{V \times \Delta\theta}$$

Coefficient of Volume Expansion of Vessel

It is defined as the ratio of increase in volume of the vessel to the original volume per unit rise in temperature. It is denoted by γ_g and is given by,

$$\gamma_g = \frac{\text{increase in volume of the vessel}}{\text{original volume} \times \text{rise in temp.}} = \frac{\Delta V_v}{V \times \Delta\theta}$$

The unit of all these coefficients is K^{-1} or $°C^{-1}$.

Relation between Coefficient of Real and Apparent Expansion

Let us consider a glass vessel of volume V filled with some liquid at temperature θ_1° C. When the liquid is heated to θ_2° C, it expands. The real increase in volume of the liquid,

$$\Delta V_r = \gamma_r V \Delta\theta = \gamma_r V (\theta_2 - \theta_1)$$

Similarly, apparent increase in volume of the liquid,

$$\Delta V_a = \gamma_a V \Delta\theta = \gamma_a V (\theta_2 - \theta_1)$$

and the increase in volume of the vessel,

$$\Delta V_g = \gamma_g V \Delta\theta = \gamma_g V (\theta_2 - \theta_1)$$

$\therefore \Delta V_r = \Delta V_a + \Delta V_g$ [From equation (i)]

or, $\gamma_r V \Delta\theta = \gamma_a V \Delta\theta + \gamma_g V \Delta\theta$

or, $\gamma_r = \gamma_a + \gamma_g$

This equation gives the relation between coefficient of real and apparent expansion.

33. 2075 GIE Q.No. 6a Define linear and cubical expansivity of solids. Establish a relationship between the coefficients of linear and cubical expansions. [4]

Linear expansivity (α): The increase in length per unit original length per unit rise in temperature is called linear expansivity or coefficient of linear expansion. It is denoted by α and is given by

$$\alpha = \frac{\text{change in length}}{\text{original length x rise in temperature}}$$

$$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)} \Rightarrow \alpha = \frac{\Delta l}{l_1 \Delta\theta}$$

where Δl is the change in length, l_1 is original length and $\Delta\theta$ is the change in temperature. The unit of α is K^{-1}.

Cubical expansivity (γ): The increase in volume per unit original volume per unit rise in temperature is called cubical expansivity or coefficient of cubical expansion. It is denoted by γ and is given by

$$\gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{rise in temperature}}$$

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} \Rightarrow \gamma = \frac{\Delta V}{V_1 \Delta\theta}$$

where ΔV is the change in volume, V_1 is original volume and Δθ is the change in temperature. The unit of γ is K^{-1}.

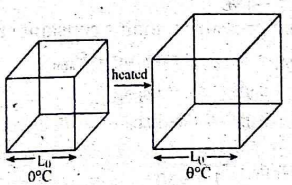
Relation between α and γ: Let us consider a cube whose sides having length L_0 at temperature 0°C and volume is V_0. If the cube is heated, then the side of cube becomes L_θ and volume becomes V_θ. If γ be the coefficient of cubical expansion, then

$V_\theta = V_0 (1 + \gamma\theta)$

Since, $V_0 = L_0^3$ and $V_\theta = L_\theta^3$

Then, $L_\theta^3 = L_0^3 (1 + \gamma\theta)$ ... (i)

But $L_\theta = L_0 (1 + \alpha\theta)$, ... (ii)



where α is coefficient of linear expansion.

Using (ii) in (i), we get

$L_0^3 (1 + \alpha\theta)^3 = L_0^3 (1 + \gamma\theta)$

$1 + 3\alpha\theta + 3\alpha^2\theta^2 + (\alpha\theta)^3 = 1 + \gamma\theta$

Since, α is small quantity, the terms containing higher powers of α can be neglected. Then

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$3\alpha\theta = \gamma\theta \Rightarrow \gamma = 3\alpha$

i.e., coefficient of cubical expansion is three times greater than the coefficient of linear expansion.

34. 2075 Set B Q.No. 6b **Define real and apparent expansivities of a liquid? Derive the relation between them.** [4]

Please refer to 2076 Set C Q.No. 6a

35. 2074 Supp. Q.No. 6a **Describe how the cubical expansivity of a liquid can be determined by the use of balanced columns.** [4]

Lest us consider a u-shaped glass tube ABCD filled with a liquid whose real expansivity γ should be determined. Initially the liquid is at equal height as shown in figure. One arm of tube is surrounded with ice-water jacket and another arm is surrounded with steam jacket. The temperature of each jacket is noted by thermometer fitted on them. To prevent the transformation of heat, a wet cloth is kept at middle. At different heights, the pressure at the base is same.

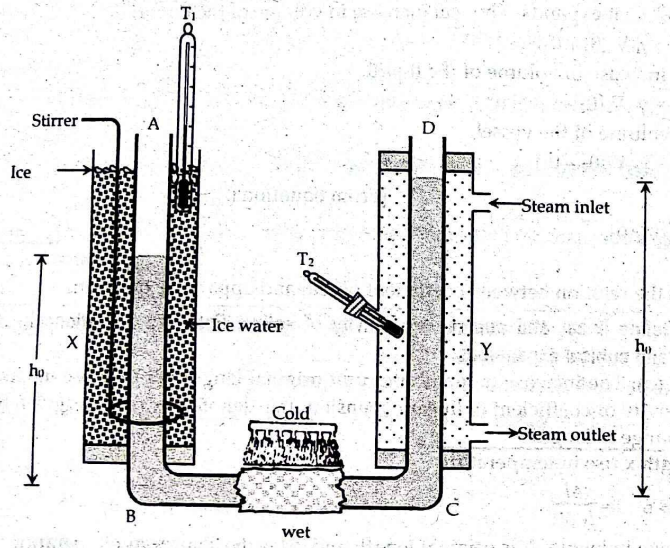


Fig. Apparatus for real expansivity of liquid.

For cooler arm

h_0 = height of liquid

ρ_0 = density of liquid

So, total pressure at the bottom of cooler arm is,

$P_B = h_0\rho_0 g + P_{atm}$

where, P_{atm} = atmospheric pressure on liquid surface

Also for hotter arm

h_θ = height of liquid

ρ_θ = density of liquid at θ°C

Therefore, total pressure at the bottom of hotter arm,

$P_C = h_\theta\rho_\theta g + P_{atm}$

Since pressure remains constant at both points, (i.e. $P_B = P_C$)

$h_0\rho_0 g + P_{atm} = h_\theta\rho_\theta g + P_{atm}$

$h_0\rho_0 g = h_\theta\rho_\theta g$

$h_0\rho_0 = h_\theta\rho_\theta$

Also, $\rho_\theta = \dfrac{\rho_0}{1+\gamma\Delta\theta}$

where, γ is absolute expansivity of liquid.

So,

$h_0\rho_0 = h_\theta \dfrac{\rho_0}{(1+\gamma\Delta\theta)}$

or, $h_0 = \dfrac{h_\theta}{(1+\gamma\Delta\theta)}$

or, $h_0(1+\gamma\Delta\theta) = h_\theta$

or, $h_0 + h_0\gamma\Delta\theta = h_\theta$

or, $h_0\gamma\Delta\theta = h_\theta - h_0$

$\gamma = \dfrac{h_\theta - h_0}{h_0\,\Delta\theta}$

This is the expression of determining absolute expansivity of liquid.

36. 2073 Set C Q.No. 6a **Define coefficients of real and apparent expansion of a liquid, and establish a relation between them.** [4]

Please refer to 2076 Set C Q.No. 6a

37. 2072 Supp Q.No. 6a **Define the coefficients of linear an cubical expansion of solid and establish their relation.** [4]

Please refer to 2075 GIE Q.No. 6a

38. 2072 Set C Q.No. 6a **Distinguish between real and apparent expansion of liquid. Describe with mathematical detail, a method to determine real expansivity of a liquid.** [4]

First part: Please refer to 2076 Set C Q.No. 6a

Second part: Please refer to 2074 Supp. Q.No. 6a

39. 2071 Set C Q.No. 6 c **Does cubical expansivity depend upon the initial volume of a solid? Write the unit of this expansivity. Also derive its relation with superficial expansivity.** [4]

No, the cubical expansivity of a solid does not depend upon initial volume of solid.

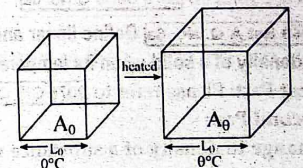
The cubical expansitivity of a solid is given as

$\gamma = \dfrac{\Delta V}{V(\theta_2 - \theta_1)}$

The ratio of $\dfrac{\Delta V}{V}$ is same in each case for a solid so, the coefficient of cubical expansion is same for all volume of a solid and does not depend on the volume. The unit of this expansivity is per kelvin (K^{-1}).

Relation between β and γ: Let us consider a cube whose sides having length L_0 and area of each face A_0 at temperature 0°C and volume is V_0. If the cube is heated, then the side of cube becomes L_θ, area A_θ and volume V_θ. If γ be the coefficient of cubical expansion then,

$V_\theta = V_0(1+\gamma\theta)$ ... (i)



Again,

$V_\theta = A_\theta \times L_\theta$

or, $V_\theta = A_0(1+\beta\Delta\theta) \times L_0(1+\alpha\Delta\theta)$

or, $V_\theta = A_0L_0(1 + \alpha\Delta\theta + \beta\Delta\theta + \alpha\beta\Delta\theta^2)$

or, $V_\theta = V_0[1 + (\alpha+\beta)\Delta\theta]$ ... (ii)

[$\because$ α and β are small quantities and hence, there product is very small which is neglected]

Comparing equation (i) and (ii)

$\gamma = \alpha + \beta$ [$\because \beta = 2\alpha$]

or, $\gamma = \dfrac{\beta}{2} + \beta = \dfrac{3\beta}{2}$

or, $\dfrac{\gamma}{3} = \dfrac{\beta}{2}$

This is the required relation between γ and β.

40. 2070 Supp (Set B) Q.No. 6 a **What are meant by real and apparent expansions of a liquid? Show that sum of coefficient of real expansion of a liquid is sum of coefficient of apparent expansion of the liquid and coefficient of cubical expansion of the vessel.** [4]

Please refer to 2076 Set C Q.No. 6a

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41. 2070 Set C Q.No. 6 a Describe a method to determine the linear expansivity of a solid. Can the cubical expansivity be derived from this value? [4]

Determination of Coefficient of Linear Expansion of a Solid by Pullinger's Apparatus

The experimental method to determine the linear expensivity of a solid consist of uniform metal (of brass or steel) rod AB whose coefficient of linear expansion is to be determined, a hollow metal tube with steam inlet and outlet, a thermometer, a spherometer, a galvanometer and heating system which are arranged as shown in figure.

Let, l_1 be the initial length of metal rod at room temperature say θ_1. When it is placed inside the metallic tube and heated its length increases. When central leg of the spherometer is lowered to rod, the spherometer reading is R_1. Steam is passed inside the tube continuously until the thermometer records a constant temperature θ_2. After the steady state is reached, the spherometer screw is rotated down so that it comes in contact with the end A. Let this reading of the spherometer be R_2.

We have,

Initial length of the rod = l_1

Initial temperature = θ_1°C

Initial spherometer reading = R_1

Final steady temperature = θ_2°C

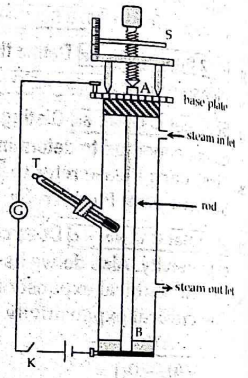
Final spherometer reading = R_2

$\therefore$ Increase in length = $R_2 - R_1$

The coefficient of linear expansion α of the material of the rod is given by

$$\alpha = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}}$$

$$= \frac{R_2 - R_1}{l_1 (\theta_2 - \theta_1)}$$



In this way, the coefficient of linear expansion of the material of the rod can be determined. The cubical expansivity of the solid can be determined by multiplying the linear expansivity by 3. If γ be the cubical expansivity of that material, then $\gamma = 3\alpha$.

42. 2070 Set D Q.No. 6 a Define the coefficient of real and apparent expansions of a liquid and derive a relation between them. [4]

Please refer to 2076 Set C Q.No. 6a

43. 2069 Set A Q. No. 6a Define linear and cubical expansivities of solids. Derive an expression for the variation in density of a solid when its temperature is raised from θ_1° C to θ_2°C. [4]

First Part: Please refer to 2075 GIE Q.No. 6a

Second Part:

Change in density of a substance with the change in temperature

When the temperature of a substance is increased, its volume increases. As the density is inversely proportional to the volume, the density of the substance changed due to change in volume.

Let m and V_1 be the mass and volume of a substance at a temperature θ_1. The density of the substance is given by

$$\rho_1 = \frac{m}{V_1} \quad \ldots (i)$$

Suppose, the substance be heated so that its volume increases to V_2 and temperature θ_2. The density of the substance at this temperature is

$$\rho_2 = \frac{m}{V_2} \quad \ldots (ii)$$

But, the volume V_2 at θ_2 is given by

$$V_2 = V_1 (1 + \gamma \Delta\theta) \quad \ldots (iii)$$

where γ is the coefficient of cubical expansion of the substance. So, the density is now,

$$\rho_2 = \frac{m}{V_2} = \frac{m}{V_1 [1 + \gamma \Delta\theta]}$$

$$\text{or, } \rho_2 = \frac{\rho_1}{1 + \gamma \Delta\theta}$$

$\therefore \rho_1 > \rho_2$ since $(1 + \gamma\Delta\theta) > 1$

Thus, the density decreases as the temperature increases.

44. 2069 (Set A) Old Q. No. 5a Define coefficient of linear expansion. Obtain the relation between α and γ. [5]

Please refer to 2075 GIE Q.No. 6a

45. 2066 Q. No. 6 a Define linear and cubical expansions of solid, and establish a relation between their coefficients. [4]

Please refer to 2075 GIE Q.No. 6a

46. 2060 Q.No. 5 a Define linear, superficial, and cubical expansivities. Show that $\beta = 2\alpha$ where α and β are linear and superficial expansivities. [2+3]

Linear expansivity (α): The increase in length per unit original length per unit degree rise in temperature is called linear expansivity or coefficient of linear expansion. It is denoted by α and is given by

$$\alpha = \frac{\text{change in length}}{\text{original length} \times \text{rise in temperature}} \Rightarrow \alpha = \frac{l_2 - l_1}{l_1 (\theta_2 - \theta_1)} \Rightarrow \alpha = \frac{\Delta l}{l_1 \Delta\theta}$$

where Δl is the change in length, l_1 is original length and $\Delta\theta$ is the change in temperature. The unit of α is K^{-1}.

Superficial expansivity (β): The increase in area per unit original area per unit rise in temperature is called superficial expansivity or coefficient of superficial expansion. It is denoted by β and is given by

$$\beta = \frac{\text{change in area}}{\text{original area} \times \text{rise in termperature}} \Rightarrow \beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)} \Rightarrow \beta = \frac{\Delta A}{A_1 \Delta\theta}$$

where ΔA is change in area A_1 is original area and $\Delta\theta$ is rise in temperature. The unit of β is K^{-1}.

Cubical expansivity (γ): The increase in volume per unit original volume per unit rise in temperature is called cubical expansivity or coefficient of cubical expansion. It is denoted by γ and is given by

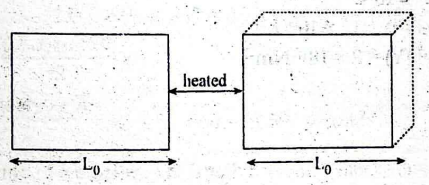
$$\gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{rise in temperature}} \Rightarrow \gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} \Rightarrow \gamma = \frac{\Delta V}{V_1 \Delta\theta}$$

where ΔV is the change in volume, V_1 is original volume and $\Delta\theta$ is the change in temperature. The unit of γ is K^{-1}.

Derivation of $\beta = 2\alpha$: Let us consider, a square sheet of side L_0 and L_θ at temperatures 0°C and θ°C, then

$$L_\theta = L_0 (1 + \alpha\theta) \quad \ldots (i)$$

where α is linear expansivity of material sheet. S_0 and S_θ are the area of the sheet at 0°C and at θ°C respectively.



Now, $S_\theta = S_0 (1 + \beta\theta)$ where, β is superficial expansivity.

or, $L_\theta^2 = L_0^2 (1 + \beta\theta)$

$L_\theta^2 (1 + \alpha\theta)^2 = L_0^2 (1 + \beta\theta)$

$(1 + \alpha\theta)^2 = (1 + \beta\theta)$

$1 + 2\alpha\theta + + (\alpha\theta)^2 = 1 + \beta\theta$

[$\because \alpha$ is small quantity and α^2 is still smaller so $\alpha^2\theta^2$ can be neglected]

Then, $1 + 2\alpha\theta = 1 + \beta\theta \Rightarrow 2\alpha\theta = \beta\theta$

$\therefore \beta = 2\alpha$

i.e. coefficient of superficial expansion is twice its linear expansivity.

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47. 2059 Q.No. 5 a **Why do substances expand on heating? Show that $\alpha = \frac{\gamma}{3}$ where α and γ are the coefficient of linear, and cubic... expansion of a substance.** [1+4]

When a substance is heated, its molecules get thermal energy. These molecules vibrate and push far each other. As a result, the substance expanded on heating.

2nd part: Please refer to 2075 GIE Q.No. 6a

48. 2056 Q.No. 5 a **Define linear and cubical expansivities. Derive a relation between them.** [1+1+3]

Please refer to 2075 GIE Q.No. 6a

49. 2051 Q.No. 8 **Obtain an expression for the change in density of a gas due to the thermal expansion.**

Please refer to 2069 Set A Q. No. 6a

Numerical Problems

50. 2076 Set B Q.No. 10a 2054 Q.No. 10 b **A glass flask of volume 400cm³ is just filled with mercury at 0°C. How much mercury will overflow when the temperature of the system rises to 80°C. The coefficient of cubical expansion of glass is 1.2×10^{-5} °C^{-1} and that of mercury is 1.8×10^{-5} °C^{-1}.** [4]

Solution

Given,

Volume of glass flask at 0°C (V_1) = 400 cm³

Initial temperature (θ_1) = 0°C

Final temperature (θ_2) = 80°C

Coefficient of cubical expansion of glass (γ_g) = 1.2×10^{-5} C^{-1}

Coefficient of cubical expansion of mercury (γ_m) = 1.8×10^{-5}C^{-1}

Amount of mercury overflows = ?

Let, V_2 be the volume of glass at 80°C, then

$V_2 = V_1 \{1 + \gamma_g(\theta_2 - \theta_1)\} = 400 \{1 + 1.2 \times 10^{-5} (80 - 0)\} = 400 (1.00096) = 400.384$ cm³

Let, V_2' be the volume of mercury at 80°C,

$V_2' = V_1 \{1 + \gamma_m (\theta_2 - \theta_1)\} = 400 \{1 + 1.8 \times 10^{-5} \times 80\} = 400.576$ cm³

Amount of mercury overflow = $V_2' - V_2 = 400.576 - 400.384 = 0.192$ cm³ $= 1.92 \times 10^{-7}$m³

51. 2075 GIE Q.No. 10c **A Steel wire having length 8m and diameter 4mm is fixed between two rigid support. Calculate increase in tension on a wire when temperature falls by 10°C. Where Young's modulus of wire: 2×10^{11}N/m², linear expansivity of steel = 1.2×10^{-5}K^{-1}.**

Solution

Given,

Length of wire (l) = 8m

Diameter of wire (d) = 4mm = 4×10^{-3} m

Charge in temperature (θ) = 10°C

Linear expansivity of steel (α) = 12×10^{-6} K^{-1}

Young's modulus for steel (Y) = 2×10^{11} Nm^{-2}

We have,

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\,\Delta l}$$

$$\text{or, } F = \frac{YA\Delta l}{l} = Y\frac{\pi d^2}{4}\Delta\theta.\,\alpha \qquad [\because \Delta l = l \propto \Delta\theta]$$

$$= 2 \times 10^{11} \times \frac{\pi (4 \times 10^{-3})^2}{4} \times 10 \times 12 \times 10^{-6}$$

$$F = 301.6N$$

The required increase in tension is 301.6 N.

52. 2075 Set A Q.No. 10a **An iron rod of length 100m at 10°C is used to measure a distance of 2km on a day when the temperature is 40°C. Calculate the error in measuring the distance.** [4]

Solution

Given,

Temperature of iron (θ_1) = 10°C

Length of iron rod (l_{10}) = 100 m

Temperature of day (θ_2) = 40°C

Linear expensivity of iron (α) = 1.2×10^{-5} °C^{-1}

Distance to be measured = 2000 m

Error in measuring distance = ?

Now, we have,

$l_{40} = l_{10}[1 + \alpha(\theta_2 - \theta_1)]$

$= 100 (1 + 1.2 \times 10^{-5} \times 30)$

$= 100.036$ m

Then, the rod measures the distance of 2000 m at 40°C = 2000.72 m. Then error in measuring distance is 2000.72 - 2000 = 0.72 m.

53. 2074 Set A Q.No. 10a **A seconds pendulum made of brass keeps correct time at 10°C. How many seconds it will lose or gain per day when the temperature of its surrounding rises to 35°C?** [4]

Solution

Given,

The time period of simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad ; \left[\begin{matrix} l = \text{length of pendulum} \\ g = \text{acceleration due to gravity} \end{matrix}\right]$$

At 35°C,

$$T_{35} = 2\pi\sqrt{\frac{l_{35}}{g}} \quad \ldots \text{(i)}$$

$$\& \; T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \ldots \text{(ii)}$$

Dividing equation (i) and (ii), we get

$$\frac{T_{35}}{T_{10}} = \sqrt{\frac{l_{35}}{l_{10}}} = \sqrt{\frac{l_{10}[1 + \alpha(\theta_2 - \theta_1)]}{l_{10}}} \qquad [\because l_2 = l_1 (1 + \alpha\,\Delta\theta)]$$

$$\text{or, } \frac{T_{35}}{T_{10}} = \sqrt{1 + \alpha\,\Delta\theta} \qquad \left[\begin{matrix} \because \Delta\theta = \theta_2 - \theta_1 = 35 - 10 = 25°C \\ \& \; \alpha_{brass} = 2 \times 10^{-5}\ k^{-1} \end{matrix}\right]$$

$$\text{or, } \frac{T_{35}}{T_{10}} = \sqrt{1 + 2 \times 10^{-5} \times 25} = 1.000249969$$

Since, the pendulum keeps the correct time at 10°C, then

T_{10} = 2 sec (for second pendulum)

So, $T_{35} = 2 \times 1.000249969$ sec = 2.000499938 sec

The pendulum clock lose time (2.000499938 - 2) = 0.000499938 sec in 2 s.

$\therefore$ In 1 second, the clock lose $\frac{0.000499938}{2}$ sec

$\therefore$ In 1 day, the clock lose $\frac{0.000499938}{2} \times 24 \times 60 \times 60 = 21.597$ seconds.

54. 2074 Set B Q.No. 10b **Two ends of a steel wire of length 8 m and 2 mm radius are fixed to two rigid supports. Calculate the increase in tension in the wire when temperature falls by 10°C.** [4]

Solution

Given,

Length of wire (l) = 8m

Diameter of wire (d) = 2 mm = 2×10^{-3} m

Charge in temperature (θ) = 10°C

Linear expansivity of steel (α) = 12×10^{-6} K^{-1}

Young's modulus for steel (Y) = 2×10^{11} Nm^{-2}

We have,

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$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\,\Delta l}$$

or, $F = \frac{YA\Delta l}{l} = Y\frac{\pi d^2}{4}\Delta\theta.\,\alpha \qquad [\because \Delta l = l \propto \Delta\theta]$

$$= 2 \times 10^{11} \times \frac{\pi\,(2 \times 10^{-3})^2}{4} \times 10 \times 12 \times 10^{-6}$$

$F = 75.36$ N

The required increase in tension is 75.36 N.

55. 2073 Set D Q.No. 10a **A glass flask with volume 200 cm³ is filled to the brim with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C?** [4]

Solution

Given,

Volume of glass flask at 20 °C (V_1) = 200 cm³

Initial temperature (θ_1) = 20°C

Final temperature (θ_2) = 100°C

Coefficient of cubical expansion of glass (γ_g) = 1.2×10^{-5} K⁻¹

Coefficient of cubical expansion of mercury (γ_m) = 18×10^{-5} K⁻¹

Amount of mercury overflow = ?

Let V_2 be the volume of glass at 100°C, then,

$V_2 = V_1[1 + \gamma(\theta_2 - \theta_1)]$

$= 200\,[1 + 1.2 \times 10^{-5}\,(100 - 20)] = 200.192$ cm³

Let V_2' be the volume of mercury at 100°C, then,

$V_2' = V_1[1 + \gamma_m(\theta_2 - \theta_1)]$

$= 200\,[1 + 18 \times 10^{-5}\,(100 - 20)] = 202.88$ cm³

Amount of mercury overflow at 100°C = $V_2' - V_1 = 202.88 - 200.192 = 2.688$ cm³

56. 2072 Set D Q.No. 10a **The marking on an aluminium ruler and a brass ruler are perfectly aligned at 0°C. How far apart will the 20.0 cm marks be on the two rulers at 100°C, if precise alignment of the left hand ends of the rulers is maintained? Coefficient of linear expansion of aluminium and brass are 2.4×10^{-5} K⁻¹ and 2.0×10^{-5} K⁻¹ respectively.** [4]

Solution

Given,

For aluminum rod	For brass rod
Initial length (l_1^a) = 20 cm	(l_1^b) = 20 cm
Initial temperature (θ_1) = 0° C	(θ_1) = 0° C
Final temperature (θ_2) = 100°C	(θ_2) = 100°C
Linear expansivity (α_a) = 2.4×10^{-5}K⁻¹	(k_b) = 2×10^{-5} K⁻¹
Final length (l_2^a) = ?	(l_2^b) = ?

We have,

$l_2^a = l_1^a\,(1 + \alpha_a\,\Delta\theta) = 20\,(1 + 2.4 \times 10^{-5} \times 100) = 20.048$ cm

$l_2^b = l_1^b\,(1 + \alpha_b\,\Delta\theta) = 20\,(1 + 2 \times 10^{-5} \times 10) = 20.04$ cm

Now,

$l_2^a - l_2^b = 20.48 - 20.04 = 0.008$ cm

∴ Right side of aluminum rod is 0.008 cm far from right side of brass rod.

57. 2072 Set E Q.No. 10a **A brass pendulum clock keeps correct time at 15°C. How many seconds per day it will lose or gain at 0°C?** [4]

Solution

Given,

Initial temperature (θ_1) = 15°C

Final temperature (θ_2) = 0°C

Linear expansivity (α) = 2×10^{-5} K⁻¹

Gain in time per day = ?

Let T_{15} and T_0 be the time period at 15°C and 0°C, then,

$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} = 2\pi\sqrt{\frac{l_0(1 + \alpha\Delta\theta)}{g}}$$

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$$

Now,

$$\frac{T_{15}}{T_0} = \sqrt{\frac{1 + \alpha\Delta\theta}{1}} = \sqrt{1 + 2 \times 10^{-5} \times 15} = 1.00015$$

Now,

$$\frac{T_{15} - T_0}{T_0} = 0.00015.$$

This is the gain in time in 1 sec.

Gain in time per day = $0.0015 \times 24 \times 60 \times 60 = 12.96$ sec

58. 2071 Supp Q.No. 10a **Using the following data, determine the temperature at which wood will just sink in benzene.** [4]

Density of benzene at 0°c = 9.0×10^2 kgm⁻³. Density of wood at 0°C = 8.8×10^2 kg/m³.

Cubical expansivity of benzene = 1.2×10^{-3}K⁻¹,

Cubical expansivity of wood = 1.5×10^{-4} K⁻¹

Solution

Given,

Density of benzene at 0°C($\rho_b°$) = 9.0×10^2 Kgm⁻³

Density of wood at 0°C($\rho_w°$) = 8.8×10^2 Kgm⁻³

Cubical explanation of benzene (γ_b) = 1.2×10^{-3} K⁻¹

Cubical explanation of wood (γ_w) = 1.5×10^{-4} K⁻¹

Temperature at which wood sink in benzene (θ) = ?

Let ρ_b^θ and ρ_w^θ be the densities of benzene and wood at θ°C. Then,

Now,

$$\rho_b^\theta = \frac{\rho_b°}{1 + \gamma_b\,.\,\theta} = \frac{9.0 \times 10^2}{1 + 1.2 \times 10^{-3} \times \theta}$$

$$\rho_w^\theta = \frac{\rho_w°}{1 + \gamma_w\,.\,\theta} = \frac{8.8 \times 10^2}{1 + 1.5 \times 10^{-4} \times \theta}$$

Wood will just sink in benzene, when $\rho_b^\theta = \rho_w^\theta$. So,

$$\frac{9.0 \times 10^2}{1 + 1.2 \times 10^{-3} \times \theta} = \frac{8.8 \times 10^2}{1 + 1.5 \times 10^{-4} \times \theta}$$

or, $\theta = 21.7$ °C

59. 2070 Supp (Set A) Q.No. 10 b 2069 (Set A) Q. No. 10c **A glass vessel of volume 50cm³ is filled with mercury and is heated from 20°C to 60°C. What volume of mercury will overflow?** [4]

Solution

Given,

Volume of glass (Vg) = 50cm³

Temperature change ($\Delta\theta$) = 60° – 20° = 40°C

Linear expansivity of glass (α_g) = 1.8×10^{-6}K⁻¹

Volume expansivity of mercury (γ_m) = 1.8×10^{-4}K⁻¹

Volume of mercury overflow = ?

After heating,

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$= V(\gamma_m - 3\alpha_g) \times \Delta\theta \quad [\because \gamma = 3\alpha]$

$= 50\ (1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}) \times 40 \quad = 0.35\ cm^3$

60. **2069 (Set B) Q. No. 10a** **The pendulum of a clock is made of brass. If the clock keeps correct time at 15°C, how many seconds per day will it lose at 20°C?** [4]

Solution

Given,

Initial temperature $(\theta_1) = 15°C$

Final temperature $(\theta_2) = 20°C$

Linear expansivity $(\alpha) = 0.000018°C^{-1}$

Loss in time per day = ?

Let, T_{15} and T_{20} be the time periods at 15°C and 20°C. Then,

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} \quad \ldots(i)$$

$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} \quad \ldots(ii)$$

Divide (i) by (ii)

$$\frac{T_{20}}{T_{15}} = \sqrt{\frac{l_{20}}{l_{15}}} = \sqrt{\frac{l_{15}(1+\alpha\Delta\theta)}{l_{15}}}$$

$$\frac{T_{20}}{T_{15}} = \sqrt{1+\alpha\Delta\theta} = \sqrt{1+0.000018\times 5} = 1.000045$$

$$\frac{T_{20}-T_{15}}{T_{15}} = 0.000045$$

This is the lose in time per sec.

$\therefore$ Lose in time per day $= 0.000045 \times 60 \times 60 \times 24 = 4$ sec.

61. **2069 Old (Set B) Q. No. 5b** **The length of an iron rod is measures by a brass scale. When both of them are at 10°C, the measured length is 50cm. What is the length of the rod at 40°C when measured by the brass scale at 40°C.? (α for brass = $24\times10^{-6}°C^{-1}$, α for iron = $16\times10^{-6}°C^{-1}$)** [4]

Solution

Given,

Length of brass at 10°C $(l_{10}) = 50$ cm

Length of bass at 40°C (l_{40}) = ?

Initial temperature $(\theta_1) = 10°C$

Final temperature $(\theta_2) = 40°C$

Linear expansivity of brass $(\alpha_i) = 24 \times 10^{-6}\ °C^{-1}$

Linear expansivity of iron $(\alpha_i) = 16 \times 10^{-6}°C^{-1}$

Now,

Length of iron rod at 40°C $= 50\ [1 + 16 \times 10^{-6}\ (40 - 10)] = 50.024$ cm

Length of one cm division of the brass scale at 40°C $= 1[1 + 24 \times 10^{-6}\ (40 - 10)] = 1.00072$ cm

$\therefore$ Length of the rod as measured by the brass scale at 40°C $= \dfrac{50.024}{1.0072} = 49.988$ cm.

62. **2068 Q.No. 10 a** **A steel wire 8m long and 4mm. in diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls by 10°C.** [4]

Please refer to 2075 GIE Q.No. 10c

63. **2068 Old Q.No. 5 b** **A brass rod of length 0.40 m and steel rod of length 0.60 m, both are initially at 0°C are heated to 75°C. If the increase in lengths is the same for both the rods, calculate the linear expansivity of brass. The linear expansivity of steel is $12\times10^{-6}\ °C^{-1}$.** [4]

Solution

Given,

Length of bass rod at 0°C $(l_o^b) = 0.40$m

Length of steel rod at 0°C $(l_o^s) = 0.60$m

Initial temperature $(\theta_1) = 0°C$

Final temperature $(\theta_2) = 75°C$

Linear expansivity of steel $(\alpha_s) = 12 \times 10^{-6}\ °C^{-1}$

Linear expansivity of brass (α_b) = ?

Let, l_2^b and l_2^s be the lengths of brass and steel at θ_2 °C then,

$l_2^b = l_o^b(1 + \alpha_b\ \Delta\theta)$

or, $l_2^b - l_o^b = \alpha_b l_o^b\ \Delta\theta$

Similarly,

$l_2^s - l_o^s = \alpha_s\ l_o^s\ \Delta\theta$

By question

$l_2^b - l_o^b = l_2^s - l_o^s$

or, $\alpha_b l_o^b\ \Delta\theta = \alpha_s l_o^s\ \Delta\theta$

$$\alpha_b = \frac{\alpha_s \cdot l_o^s}{l_o^b} = \frac{12\times10^{-6}\times 0.60}{0.40} = 18\times10^{-6}°C^{-1}$$

64. **2067 Q.No. 10 a** **A copper vessel with a volume of exactly $100m^3$ at a temperature of 15°C is filled with glycerin. If the temperature rises to 25°C, how much glycerin will spill out?** [4]

Solution

Given,

Volume of copper vessel at 15°C $(V_1) = 100m^3$

Initial temperature $(\theta_1) = 15°C$

Final temperature $(\theta_2) = 25°C$

Linear expansivity of copper $(\alpha_c) = 16.7\times 10^{-6}\ °C^{-1}$

Cubical expansivity of glycerene $(\gamma_m) = 5.3 \times 10^{-4}\ °C^{-1}$

Amount of glycerene spilt out = ?

Let, V_2 be the volume of copper at 25°C

$V_2 = V_1\ (1 + \gamma_c\Delta\theta) = 100\ (1 + 3 \times 16.7 \times 10^{-6} \times 10) = 100.05\ m^3$

Let, V_2' be the volume of mercury at 25°C, they

$V_2' = V_1(1 + \gamma_m\ \Delta\theta)$

$= 100\ (1 + 5.3 \times 10^{-4} \times 10) \quad [\because \gamma = 3\alpha]$

$= 100.53\ m^3$

Volume of mercury spilt,

$= V_2' - V_2 = (100.53 - 100.05)m^3 = 0.48\ m^3$

65. **2067 Supp Q.No. 10 a** **A glass flask of volume 500 cm3 is just filled with mercury at 0°c. How much mercury overflows when the temperature of the system is raised to 80°c.** [4]

Solution

Given,

Volume of glass at 0°C $(V_1) = 500\ cm^3$

Initial temperature $(\theta_1) = 0°C$

Final temperature $(\theta_2) = 80°C$

Cubical expansivity of glass $(\gamma_g) = 1.2 \times 10^{-5}\ °C^{-1}$

Cubical expansivity of mercury $(\gamma_m) = 1.8 \times 10^{-5}\ °C^{-1}$

Volume of mercury overflow at 80°C = ?

Let, V_2 be the volume of glass at 80°C, then

$V_2 = V_1\ (1 + \gamma_g\ \Delta\theta) = 500\ (1 + 1.2 \times 10^{-5} \times 80) = 500.48\ cm^3$

Let, V_2' be the volume of mercury at 80°C,

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$V_2' = V_1 (1 + \gamma_m \Delta\theta) = 500 (1 + 1.8 \times 10^{-5} \times 80) = 500.72 \text{ cm}^2$

Volume of mercury overflows = $V_2' - V_2 = 500.72 - 500.48 = 0.24 \text{ cm}^3$

66. 2066 Old Q. No. 5 b **A clock which has a brass pendulum beats seconds correctly when the temperature of the room is 30°C. How many seconds will it gain or lose per day when the temperature of the room falls to 10°C? (α for brass = 0.000018°C^{-1}).**

Solution

Given,

Initial temperature (θ_1) = 30°C

Final temperature (θ_2) = 10°C

Linear expansivity (α) = 0.000018°C^{-1}

Loss in time per day = ?

Let, T_{30} and T_{10} be the time periods at 30°C and 10°C.

We know, time period of pendulum is, $T = 2\pi\frac{l}{g}$.

So,

$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} \quad \ldots(i)$$

and,

$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \ldots(ii)$$

Divide (i) by (ii)

$$\frac{T_{30}}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}} = \sqrt{\frac{l_{10}(1 + \alpha\Delta\theta)}{l_{10}}} \quad [\because l_2 = l_1 (1 + \propto \Delta\theta]$$

$$\frac{T_{30}}{T_{10}} = \sqrt{1 + \alpha\Delta\theta} = \sqrt{1 + 0.000018 \times 20} = 1.00018$$

Now,

$$\frac{T_{30} - T_{10}}{T_{10}} = 0.00018$$

This is the gain in time per sec.

$\therefore$ Gain in time per day = $0.00018 \times 60 \times 60 \times 24 = 16$ sec.

67. 2063 Q.No. 3 b **A copper wire of diameter 0.5 mm. is stretched between two points at 25°C. Calculate the increase in tension in the wire if the temperature falls to 0°C. (Young's modulus for copper = 1.2 × 10^{11} Nm^{-2} linear expansivity for copper = 18 × 10^{-6} K^{-1})**

Solution

Given,

Diameter of copper wire (d) = 0.5mm

Radius of copper wire (r) = $\frac{d}{2}$ = 0.25 mm = 0.25×10^{-3} m

Initial temperature (θ_1) = 25°C

Final temperature (θ_2) = 0°C

Young's modules of copper (Y) = 1.2×10^{11} Nm^{-2}

Linear expansivity of copper (α) = 18×10^{-6} K^{-1}

Increase in tension (T) = ?

We know,

$$Y = \frac{T/A}{\Delta l/l} \quad [\because \alpha = \frac{\Delta l}{l\Delta\theta} \text{ or, } \Delta l = \alpha . l\Delta\theta]$$

$$\text{or, } Y = \frac{T}{A} \cdot \frac{l}{\Delta l}$$

$$\text{or, } T = YA . \frac{\Delta l}{l} = YA . \frac{\alpha l \Delta\theta}{l} = YA\,\alpha\Delta\theta = 1.2 \times 10^{11} \times \pi (0.25 \times 10^{-3})^2 \times 18 \times 10^{-6} \times 25$$

$$= 10.6 \text{ N}$$

68. 2065 Q.No. 9 OR **The density of silver at 0°C is 10310 kgm^{-3} and the coefficient of linear expansion is 0.000019/°C. Calculate its density at 100°C.**

Solution

Given,

Initial temperature (θ_1) = 0°C

Final temperature (θ_2) = 100°C

Initial density of silver at 0°C (ρ_1) = 10310 kg/m^3

Final density of silver at 100°C (ρ_2) = ?

Coefficient of liner expansion of steel (α) = 0.000019/°C

We have,

$$\rho_2 = \frac{\rho_1}{1 + \gamma(\theta_2 - \theta_1)} \quad [\because \gamma = 3\alpha]$$

$$= \frac{10310}{1 + 3 \times 0.000019\,(100 - 0)} = 10251.566 \text{ kg/m}^3$$

69. 2053 Q.No. 8 **An aluminum rod when measured with a steel scale, both being at 25°C appears to be 1 m long. If the scale is correct at 0°C, what will be the length of the rod at 0°C? (Linear expansivity of aluminum 26 × 10^{-6} K^{-1} and of steel = 12 × 10^{-6} K^{-1}).**

Solution

Given,

Linear expansivity of aluminium (α_{al}) = 26×10^{-6} K^{-1},

Linear expansivity of steel (α_s) = 12×10^{-6} K^{-1}

At 0° C, the steel scale is correct (l_0) = 1 m

At 25° C, length of the steel scale (l_{25}) = ?

From the linear expansion (l_{25}) = $l_0 (1 + \alpha\,\Delta\theta)$

$= 1[1 + \alpha\,(25 - 0)] = 1[1 + 12 \times 10^{-6}\,(25 - 0)] = 1.0003$ m

$\therefore$ True length l_{25} of steel scale at 25° C = 100.03 cm

$\therefore$ True length of aluminium rod at 25° C, $l'_{25} = l_{25} = 1.0003$ m

Suppose l_0 be the length of the aluminium rod at 0° C.

$\therefore l'_{25} = l'_0 [1 + \alpha_{al} . (25 - 0)]$

or, $1.0003 = l_0 [1 + 26 \times 10^{-6} \times 25]$

or, $l_0 = \frac{1000.03}{1.00065} = 0.9996$ m

$\therefore$ The length of the aluminimum rod at 0° C = 0.9996 m

□□□

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Short Answer Questions

1. 2076 Set B Q.No. 2a **The latent heat of vaporization is much larger than the latent heat of fusion of substance. Why?**

When a body is converted from liquid to vapour state, much more energy is required than the energy needed to convert same body or substance from solid to liquid state. It is due to the reason that to convert a material liquid to vapour, the molecules are to be separated out by large distance whereas to convert from solid to liquid, the molecules are to be separated by relatively smaller distance. Hence, latent heat of vapourization of a materials is greater than that of latent heat of fusion. [2]

2. 2075 Set A Q.No. 2c **Why does steam at 100°C give more severe burn than water at the same temperature?**

The amount of heat contained in steam at 100°C is 540 cal/gm (latent heat of vaporization) more than that of water at 100°C. So, when we touch the steam, more heat flows to our body than from water both being at 100°C. Hence, steam at 100°C produce more severe burn than water at the same temperature. [2]

3. 2073 Set C Q.No. 2a **Groundnuts are fried along with sand, why?**

The specific heat capacity of sand is low i.e. its temperature rapidly increases with small increase in heat. When the groundnut fried along with sand, the groundnut gets fried fastly with lower heat supplied due to greater temperature. [2]

4. 2072 Supp Q.No. 2a **Why is the groundnut fried along with sand?**

Please refer to 2073 Set C Q.No. 2a [2]

5. 2072 Set E Q.No. 2c **Water remains cool in earthen pots than in metal pots in summer. Explain why?**

The earthen pot consists of a large number of small pores. Through these pores, water comes out and is evaporated. For the evaporation, heat is absorbed from the water inside the pot. Since, the water inside the pot has lost heat, it remains cold in the earthen pot during the summer. [2]

6. 2069 Supp Q.No. 2a **A medical officer prescribes to put wet cloths on the forehead of a person suffering from high fever, why?**

Water has large specific heat capacity. When a wet cloth is kept on the forehead of a person, it absorbs large amount of heat from the person's forehead without rising its temperature and temperature of the high fever person falls down quickly. Thus, the person feels relief on loosing heat. Therefore, during high fever, a wet cloth is kept on the forehead of a person. [2]

7. 2069 Supp Q.No. 2c **Why does the temperature of ice fall when some salt is added to it?**

When salt is added to the ice, the ice provides latent heat to the salt to melt. Since, the ice loses heat its temperature falls. [2]

8. 2069 (Set A) Q. No. 2b **If you add heat to an object, do you necessarily increases its temperature? Justify your.**

No, the temperature of the object is not necessarily increased if the object is heated. As an example, the ice gets melted when it is heated at zero degree without gaining its temperature i.e. at latent heat of fusion of ice and similarly at the latent heat of vaporization. [2]

9. 2069 (Set A) Old Q. No. 4a **Water in earthen pot remains cold in summer. Explain, why?**

Please refer to 2072 Set E Q.No. 2c [2]

10. 2069 (Set B) Q. No. 2a **Why do you feel cool in the mouth when you eat halls?**

Halls needs certain heat energy to melt when it is kept into the mouth, it takes some energy. This means our body losses some heat energy to melt the halls. Since mouth losses heat, we feel cold. [2]

11. 2069 Old (Set B) Q. No. 4b **Why does a food cook faster in pressure cooker than open vessel with boiling water?**

When the amount of water vapor and hence, water vapor pressure above the surface of water level increases, the boiling point of the water also increases. Pressure cooker is a closed vessel. When water is heated in it, pressure of water vapor above the surface of water increases, so boiling point of water rises(since a liquid boils when its vapour pressure becomes equal to the pressure above its surface) above 100°C. It means, the same mass of the water contains more amount of heat. As a result, the food is cooked faster in a pressure cooker. [2]

12. 2069 Old (Set B) Q. No. 4c **Explain why a person feels cooling when he pours a little spirit on his hand.**

Perfume or spirit is a volatile substance; their molecules evaporate at a higher rate even at normal temperature. When they are sprayed over a skin surface, they take the body heat and evaporate faster. During evaporation, the body heat is also taken away by the molecules. So, that portion feels colder compared to other parts. [2]

13. 2067 Q.No. 2 a **During a high fever, a wet cloth is kept on the forehead of a person. Why?**

Please refer to 2069 Supp Q.No. 2a [2]

14. 2066 Old Q. No. 4 a **How is it that ice cream appears cooler to the mouth than ice at 0°C?**

The ice cream contains impurities than pure ice; as a result the melting point of ice cream is lower than the melting point of ice. So, ice cream appears cooler to the mouth than ice at 0°C. [2]

15. 2066 Old Q. No. 4 c **If you wet your hand and pickup an ice tray that is below 0°C, your hand may stick to it. This does not happen to wood. Why?**

If we wet our hands and pickup on ice tray which is below 0°C, our hands may stick to it because the water of our hand freezes. The ice particles get diffused to each other and stick to each other. If we touch wood, the water and wood do not stick because their molecules do not diffuse with each other. [2]

16. 2061 Q.No. 4 c 2053 Q.No. 7 c **Explain why water remains cool in earthen pot in summer.**

Please refer to 2072 Set E Q.No. 2c [2]

17. 2060 Q.No. 4 c **Why do we feel cold when we spray perfume on our body?**

Please refer to 2069 Old (Set B) Q. No. 4c [2]

18. 2056 Q.No. 4 a **When you come out of swimming pool, you feel cold. Why?**

When we come out of swimming pool, our body is wet with water. After some time, this water is evaporated from our body taking the heat of the body reducing the temperature of the body as a result we feel cool. [2]

19. 2056 Q.No. 4 b **Why does steam at 100°C causes severe burns than hot water at 100°C?**

Please refer to 2075 Set A Q.No. 2c [2]

20. 2054 Q.No. 7 c **Why can you get a more severe burn from steam at 100°C than from water at 100°C?**

Please refer to 2075 Set A Q.No. 2c [2]

21. 2053 Q.No. 7 b **Why is spark produced when two stones are stricken against one another?**

When two stones are stricken against each other, work is done on the expense of muscular energy used this work done is converted into heat energy due to friction producing spark. [2]

22. 2051 Q.No. 7 d **During high fever, a wet cloth is kept on the forehead of a person. Why?**

Please refer to 2069 Supp Q.No. 2a [2]

23. 2050 Q.No. 7 b **Why does food cook faster in a pressure cooker than in an open pot?**

Please refer to 2069 Old (Set B) Q. No. 4b

Long Answer Questions

24. 2076 Set B Q.No. 6a **State Newton's law of cooling. Use it to find the specific heat capacity of a unknown liquid.** [4]

Newton's law of cooling: It is stated as, "the rate of heat lost (rate of cooling) is proportional to the excess of temperature over the surrounding and is independent to the nature of the liquid."

The rate of cooling depends on the exposed area of the heat loosing surface and its nature. Higher the temperature difference between the body and surroundings, more heat lost by the body per unit time.

Suppose a liquid at a temperature θ°C loses a small amount of heat dQ in small time dt, let θ_o°C be the temperature of the surrounding. Then, according to Newton's law of cooling, we can write

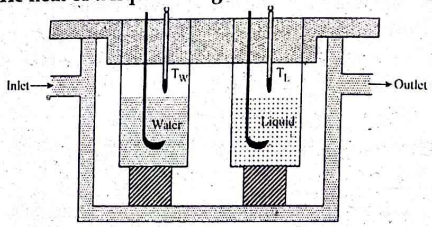
Rate of heat lost ∝ temperature difference between liquid and surrounding.

i.e. $-\frac{dQ}{dt} \propto (\theta - \theta_o) \Rightarrow -\frac{dQ}{dt} = K(\theta - \theta_o)$

where K is the proportionality constant and its value depends upon the nature of liquid and the surface area exposed to atmosphere. The negative sign indicates that as the time passes, the liquid gives out the heat.

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Expression for the specific heat of a liquid using Newton's law of cooling:



Let us consider m_1 and m_2 be the masses of two identical calorimeters made of same material of specific heat capacity S_C. Suppose, first calorimeter contains water having mass m_w and second calorimeter contains same volume of liquid (whose specific heat is to be determined) having mass m_L. Suppose t_1 and t_2 are the time in which water and liquid cools through temperature θ_1 to θ_2.

Now,

Heat lost by first calorimeter to cool from θ_1 to $\theta_2 = m_1 S_C(\theta_1 - \theta_2)$

Heat lost by water in calorimeter to cool from θ_1 to $\theta_2 = m_w S_w(\theta_1 - \theta_2)$

Total heat lost by calorimeter and water in cooling through θ_1 to θ_2 is

$= m_1 S_C (\theta_1 - \theta_2) + m_w S_w (\theta_1 - \theta_2) = (m_1 S_C + m_w S_w)(\theta_1 - \theta_2)$

Rate of cooling of water $= \dfrac{(m_1 S_C + m_w S_w)(\theta_1 - \theta_2)}{t_1}$ ... (i)

Similarly, if S_L is the specific heat of liquid, then

rate of cooling of liquid $= \dfrac{(m_2 S_C + m_L S_L)(\theta_1 - \theta_2)}{t_2}$ ... (ii)

As the liquid and water have been cooled under identical conditions, then their rate of cooling must be same accordance with Newton's law of cooling.

i.e. $\dfrac{(m_1 S_C + m_w S_w)(\theta_1 - \theta_2)}{t_1} = \dfrac{(m_2 S_C + m_L S_L)(\theta_1 - \theta_2)}{t_2}$

$m_2 S_C + m_L S_L = \dfrac{(m_1 S_C + m_w S_w) t_2}{t_1}$

$\therefore S_L = \dfrac{(m_1 S_C + m_w S_w) t_2}{m_L t_1} - \dfrac{m_2 S_C}{m_L}$

The above expression is used to find the specific heat of liquid by method of mixture.

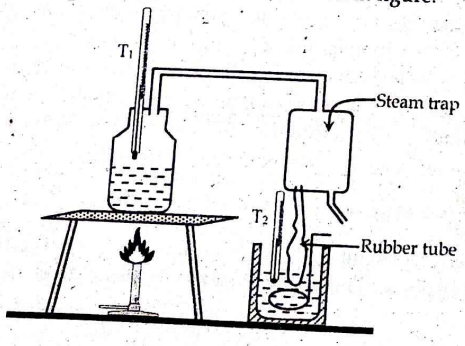
25. 2075 GIE Q.No. 6c 2066 Old Q. No. 5 a OR Describe the method to determine latent heat of vaporization of water in laboratory. [4]

Specific latent heat of vaporization of a liquid (L_V): The amount of heat required to convert unit mass of a substance from liquid state to vapour state at its boiling point is called specific latent heat of vaporization of the liquid. It is represented by L_V and is given by

$L_V = \dfrac{\text{amount of heat } (Q)}{\text{mass } (m)} = \dfrac{Q}{m}$

The latent heat of vaporization of water is 2.26×10^6 J/kg or 540 cal/ gm.

Measurement of specific latent heat of vaporization (L_V): The experimental arrangement of the determination of latent heat of vaporization is as shown in figure.



A calorimeter and stirrer is weighted after cleaning, fill half of calorimeter by water and then weighted it again and temperature of system, θ_1 is noted. Steam is prepared by boiling water at boiler so that the thermometer shows fixed temperature θ_2 of steam. The steam is then passed in the calorimeter containing water and stir by stirrer. When the temperature of the mixture is raised by about 5°C to 10°C, the temperature of mixture and whole mass of the system is taken.

Calculation:

Mass of calorimeter and stirrer = m_c

Specific heat of material of calorimeter = S_c

Initial temperature of calorimeter and water= θ_1

Mass of water taken = m_w

Temperature of steam = θ_2

Final temperature of mixture = θ

Mass of steam added = m_s

Latent heat of vaporization of water = L_v

Specific heat capacity of water = S_w

Now,

Heat lost by steam on condensation = $m_s L_v$

Heat lost by condensed steam (water) on cooling from θ_2 to $\theta = m_s(\theta_2 - \theta) S_w$

$\therefore$ Total heat lost by steam $= m_s L_v + m_s(\theta_2 - \theta) S_w$

And heat gained by water and calorimeter

$= m_c S_c (\theta - \theta_1) + m_w S_w (\theta - \theta_1)$

$= (m_c S_c + m_w S_w)(\theta - \theta_1)$

From the principle of calorimetry, we can write

heat lost by condensed steam = heat gained by calorimeter and water

$m_s L_v + m_w S_w (\theta_2 - \theta) = (m_c S_c + m_w S_w)(\theta - \theta_1)$

$m_s L_v = (m_c S_c + m_w S_w)(\theta - \theta_1) - m_w S_w (\theta_2 - \theta)$

$\therefore L_v = \dfrac{(m_c S_c + m_w S_w)(\theta - \theta_1)}{m_s} - S_w(\theta_2 - \theta)$

This is the required expression for the latent heat of vaporization.

26. 2075 Set A Q.No. 6a What is meant by heat capacity? Describe the method of mixture to determine specific heat capacity of a solid. [4]

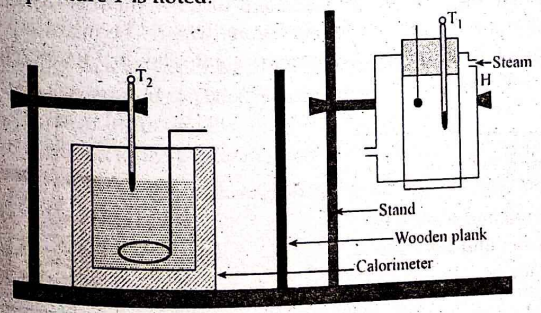
Heat capacity: The amount of heat energy required to increase the temperature of a substance by 1°C is called heat capacity.

Specific heat capacity: The amount of heat energy required to increase the temperature of unit mass of a substance by 1°C is called its specific heat capacity. It is denoted by S and given by

$S = \dfrac{\text{Amount of heat } (Q)}{\text{Mass } (m) \times \text{Temp. diff } (\Delta\theta)} = \dfrac{Q}{m\,\Delta\theta}$

Its unit is J/kg°C.

Determination of specific heat capacity of a solid by the method of mixture: Let us consider a solid of mass m_s and its specific heat capacity is S_s. The solid is heated to the temperature T_2 and dropped in a calorimeter with water at initial temperature T_1. Suppose m_c and S_c are the mass and specific heat capacity of calorimeter respectively and m_w is the mass of water taken in calorimeter and S_w specific heat capacity of water. When the solid is dropped into water, the temperature of water increased and temperature of solid decreased. By stiring water, it can reach to a constant temperature. The final constant temperature T is noted.



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Now, Heat lost by the body when its temperature fall from T_2 to $T = m_s S_s (T_2 - T)$

Heat gained by calorimeter and water system

$= m_c S_c (T - T_1) + m_w S_w (T - T_1)$

$= (m_c S_c + m_w S_w) (T - T_1)$

Now, according to principle of calorimetry, heat lost by hot body = heat gain by calorimeter and water system.

$m_s S_s (T_2 - T) = (m_c S_c + m_w S_w) (T - T_1)$

$\therefore S_s = \frac{(m_c S_c + m_w S_w)(T - T_1)}{m_s (T_2 - T)}$

Thus, the specific heat capacity of solid can be calculated by using above expression.

27. 2074 Supp. Q.No. 6b **State Newton's law of cooling. Use this law to determine the specific heat capacity of a liquid.** [4]

Please refer to 2076 Set B Q.No. 6a

28. 2074 Set A Q.No. 6a **State Newton's Law of cooling. Describe how is this law applied to determine the specific heat capacity of a liquid.** [4]

Please refer to 2076 Set B Q.No. 6a

29. 2074 Set B Q.No. 6c **Describe experimental method, with necessary theory, to determine the latent heat of vaporization of water.** [4]

Please refer to 2075 GIE Q.No. 6c

30. 2073 Supp Q.No. 6a **Describe, in brief, the laboratory method of mixture to determine the specific heat capacity of a solid.** [4]

Please refer to 2075 Set A Q.No. 6a

31. 2073 Set D Q.No. 6c **State and explain Newton's law of cooling. Describe with mathematical detail a method for the determination of specific heat capacity of a liquid.** [4]

Please refer to 2076 Set B Q.No. 6a

32. 2072 Supp Q.No. 6b **What is specific heat capacity of a substance? Describe the method of mixture to measure the specific heat capacity of solid.** [4]

Please refer to 2075 Set A Q.No. 6a

33. 2072 Set D Q.No. 6a **State and explain Newton's law of cooling. Describe with mathematical detail a method for the measurement of specific heat capacity of a liquid.** [4]

Please refer to 2076 Set B Q.No. 6a

34. 2072 Set E Q.No. 6b **What is the difference between the "specific heat capacity" and "latent heat"? Describe in brief, the laboratory experiment for the determination of latent heat of fusion of ice.** [4]

Specific heat capacity: This is amount of heat required to increase the temperature of a body of unit mass by unit degree of temperature.

Latent heat: This is amount of heat energy required to change the substance from solid state to liquid state.

Latent heat of fusion of ice by the method of mixture:

Let us take a clean and dry calorimeter with stirrer and weighted it. The calorimeter is filled with water about one third and weighted again. The calorimeter is now placed into the wooden box and initial temperature is noted. Small pieces of ice are taken and water in them is soaked with blotting paper and then dropped gently into the calorimeter containing water. The mixture is stirred gently until the temperature of mixture falls down and becomes constant. The final temperature of mixture is noted and weighted the mixture again.

Suppose,

Mass of calorimeter and stirrer = m_c

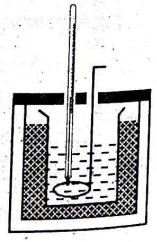
Mass of calorimeter stirrer and water = m_l

Mass of water taken = $m_l - m_c = m_w$

Initial temperature of calorimeter and water= θ_l

Mass of calorimeter, stirrer, water and ice = m_2

Mass of ice taken = $m_2 - m_l = m_i$



Final temperature of mixture = θ

Specific heat capacity of water= S_w

Specific heat capacity of calorimeter system = S_c

Latent heat of fusion of ice = L_i

Heat gained by the ice during melting = $m_i L_i$

Heat gained by the melted ice being heated from 0°C to the constant final temperature θ°C = $m_i S_w (\theta - 0) = m_i S_w \theta$

Thus, the total heat gained = $m_i L_i + m_i S_w \theta$

The heat lost by the calorimeter system and water

$= m_c S_c (\theta_l - \theta) + m_w S_w (\theta_l - \theta) = (m_c S_c + m_w S_w)(\theta_l - \theta)$

From the principle of calorimetry,

Heat gained by ice = heat lost by calorimeter system and water

$m_i L_i + m_i S_w \theta = (m_c S_c + m_w S_w)(\theta_l - \theta)$

$m_i L_i = (m_c S_c + m_w S_w)(\theta_l - \theta) - m_i S_w \theta$

$L_i = \frac{(m_c S_c + m_w S_w)(\theta_l - \theta) - m_i S_w \theta}{m_i}$

$\therefore L_i = \frac{(m_c S_c + m_w S_w)(\theta_l - \theta)}{m_i} - S_w \theta$

This is the required expression for the latent heat of fusion of ice.

35. 2071 Supp Q.No. 6a **Define specific latent heat of fusion of ice. Develop an expression for the determination of specific latent heat of fusion of ice.** [4]

Specific Latent heat of fusion: The amount of heat required to change unit mass of ice to water at its meting point or 0°C is called Specific latent heat of fusion of ice. The latent heat of fusion of ice is 80 cal/g.

Latent heat of fusion of ice by the method of mixture: Please refer to 2072 Set E Q.No. 6b

36. 2071 Set D Q.No. 6 a **Describe in brief the method of mixture to determine the specific heat of a solid in the laboratory?** [4]

Please refer to 2075 Set A Q.No. 6a

37. 2069 Supp Q.No. 6b **Describe the experiment that is applied to determine the specific heat capacity of a solid by the method of mixture.** [4]

Please refer to 2075 Set A Q.No. 6a

38. 2069 (Set B) Q. No. 6a, 2062 Q.No. 5 a **State and explain Newton's law of cooling and derive the expression for the specific heat of the liquid.** [4]

Please refer to 2076 Set B Q.No. 6a

39. 2069 Old (Set B) Q. No. 5a **Why the temperature is constant during change of state of the substance? Develop an expression for the determination of the latent heat of vaporization.** [1+4]

The temperature is constant during change of state of substance because heat is used to change the state of substance without raising its temperature.

Second Part: Please refer to 2075 GIE Q.No. 6c

40. 2068 Q.No. 6 a **Define latent heat of fusion of ice. Describe the method for the measurement of it in the laboratory.** [4]

Specific Latent heat of fusion: Please refer to 2071 Supp Q.No. 6a

Latent heat of fusion of ice by the method of mixture: Please refer to 2072 Set E Q.No. 6b

41. 2068 Old Q.No. 5 a **Define latent heat of fusion of ice. Explain the method of determining the latent heat of fussion of ice by the method of mixture in laboratory.** [1+4]

Specific Latent heat of fusion: Please refer to 2071 Supp Q.No. 6a

Latent heat of fusion of ice by the method of mixture: Please refer to 2072 Set E Q.No. 6b

42. 2067 Supp Q.No. 6 a **Explain with necessary theory how to determine the specific heat capacity of a liquid by the method of cooling.**

Please refer to 2076 Set B Q.No. 6a

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43. 2067 Q.No. 6 a 2060 Q.No. 5 a OR **Define specific heat of substance. Describe the method of mixture to determine the specific heat of a solid.** [4]

Please refer to 2075 Set A Q.No. 6a

44. 2065 Q. No. 5 a **State and explain Newton's law of cooling. Determine the specific heat capacity of liquid by method of cooling.** [1+4]

Please refer to 2076 Set B Q.No. 6a

45. 2064 Q.No. 5 a **Define specific heat capacity and heat capacity. Describe the method of mixture to determine the specific heat capacity of a liquid.** [1+4]

Please refer to 2075 Set A Q.No. 6a

46. 2063 Q.No. 5 a **State and explain Newton's Law of cooling.**

Please refer to 2076 Set B Q.No. 6a

47. 2058 Q.No. 5 a **Explain how you determine the specific heat of a solid by the method of mixture.**

Please refer to 2075 Set A Q.No. 6a

48. 2055 Q.No. 8 **What is specific latent heat of vapourisation of a liquid? Develop an expression for the determination of the latent heat of vapourisation.**

Please refer to 2075 GIE Q.No. 6c

Numerical Problems

49. 2076 Set B Q.No. 10b **A copper calorimeter of mass 300 gm contains 500 gm of water at temperature 15°C. A 560 gm block of aluminium at temperature 100°C is dropped in the calorimeter and the temperature is observed to increase to 22.5°C. Find the specific heat capacity of aluminium.** [4]

Solution

Given,

Specific heat capacity of copper (S_c) = 400 J kg^{-1} °C^{-1}

Specific heat capacity of water (S_w) = 4200J kg^{-1}°C^{-1}

Specific heat capacity of aluminium (S_{al}) = ?

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 500 gm = 0.5 kg

Mass of alluminium (m_{al}) = 560 gm = 0.56 kg

Temperature of water and calorimeter (θ_2) = 15°C

Temperature of alluminium (θ_1) = 100°C

Final temperature (θ) = 22.5°C

Now, by using principle of calorimetry which states as

Heat lost by alluminium = Heat gained by colarimetry

$m_{al} \times S_{al} \times (\theta_1 - \theta) = (m_c \times S_c + m_w \times S_w) \times (\theta - \theta_2)$

$0.56 \times S_{al} \times (100 - 22.5) = (0.3 \times 400 + 4200 \times 0.5)(22.5 - 15)$

or, S_{al} = 383.64 J kg^{-1} °C^{-1}

50. 2076 Set C Q.No. 10a **A substance takes 3 minutes in cooling from 50° C to 45° C and takes 5 minutes in cooling 45° C to 40° C. What is the temperature of the surroundings?** [4]

Solution

Given,

t_1 = 3 min,

t_2 = 5 min

Average temperature between 50°C and 45°C = $\frac{50+45}{2}$ = 47.5° C

Average temperature between 45°C and 40°C = $\frac{40+45}{2}$ = 42.5° C

Let θ° C be the temperature of surrounding,

From Newton's law of cooling,

$\left(\frac{dQ}{dt}\right)_1 = K(47.5 - \theta)$

or, $ms\left(\frac{d\theta}{dt}\right)_1 = K(47.5 - \theta)$ ...(i)

Similarly,

$ms\left(\frac{d\theta}{dt}\right)_2 = K(42.5 - \theta)$ ...(ii)

Again,

$\left(\frac{d\theta}{dt}\right)_1 = \frac{50-45}{3}$

and

$\left(\frac{d\theta}{dt}\right)_2 = \frac{45-40}{5}$

Dividing (i) and (ii) we get

$\frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{(47.5 - \theta)}{(42.5 - \theta)}$

or, $\frac{5}{3} = \frac{(47.5 - \theta)}{(42.5 - \theta)}$

or, $212.5 - 5\theta = 142.5 - 3\theta$

or, $70 = 2\theta$

$\therefore \theta$ = 35°C

51. 2075 GIE Q.No. 10a **How much heat is required to convert 10kg of ice at -10°C into steam at 100°C ? (SP. heat of ice = 2100Jkg^{-1} K^{-1}, latent heat of fusion of ice = 3.36×10^{5}Jkg^{-1}, latent heat of vaporization = 2.268 × 10^{6}Jkg^{-1})**

Solution

Given,

Mass of ice (m_i) = 10 kg

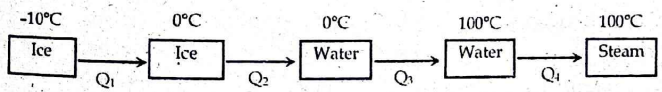
Specific heat capacity of ice (S_i) = 2100 J kg^{-1}°C^{-1}

Specific latent heat of ice (L_i) = 336000 J kg^{-1}

Specific heat capacity of water (S_w) = 4200 J kg^{-1}°C^{-1}

Latent heat of vapourization (L_v) = 2.26× 10^{6}J/kg

Amount of heat (Q) = ?



Amount of heat required to convert ice from -10°C to ice at 0°C is

$Q_1 = m_i S_i (0 - (-10))$

$= 10 \times 2100 \times 10 = 210000$ J

Amount of heat required to convert ice from 0°C to water at 0°C

$Q_2 = m_i L_i = 10 \times 3.36\ 10^5$

$= 3360000$ J

Amount of heat required to convert water at 0°C to water at 100°C

$Q_3 = m_w S_w (100 - 0)$

$= 10 \times 4200 \times 100 = 4200000$ J

Amount of heat required to convert water at 100°C to vapour at 100°C.

$Q_4 = m_w L_v$

$= 10 \times 2.26 \times 10^6 = 22600000$ J

Total heat required to convert 10 kg of ice from -10°C to vapour at 100°C is

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 210000 + 3360000 + 4200000 + 22600000$

$= 30370000 = 3.037 \times 10^7$ J

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52. 2075 Set B Q.No. 10a **A mixture of 500g water and 100 g ice at 0°C is kept in a copper calorimeter of mass 200 g. How much steam from the boiler be passed to the mixture so that the temperature of the mixture reaches 40°C?** [4]

Solution

Given,

Mass of water (m_w) = 500 g = 0.5 kg

Mass of ice (m_i) = 100 g = 0.1 kg

Mass of calorimeter (m_c) = 200 g = 0.2 kg

Mass of steam (m_s) = ?

Final temperature (θ) = 40°

Specific heat capacity of water (s_w) = 4200 J/kg K

Specific heat capacity of copper (s_c) = 400 J/kg K

Latent heat of fusion of ice (L_i) = 3.36×10^5 J/kg

Latent heat of vaporization of water (L_v) = 2.26×10^6 J/kg

Now,

Heat gained by 0°C ice to melto to 0°C water $= m_i L_i$

$= 0.1 \times 3.36 \times 10^5 = 3.36 \times 10^4$J

Heat gained by water from 0°C to water of 40°C $= m_w s_w(40 - 0) + m_i s_i(40 - 0) + m_c s_c(40 - 0)$

$= 0.5 \times 4200 \times 40 + 0.1 \times 4200 \times 40 + 0.2 \times 400 \times 40$

$= 1.04 \times 10^5$J

Total heat gained $= 1.04 \times 10^5 + 3.36 \times 10^4$

$= 1.37 \times 10^5$J

Again,

Heat loss by 100°C steam to 100°C water- $= m_s L_V$

Heat loss by 100°C water to 40°C water $= m_s s_w (100 - 40)$

Total heat loss $= m_s L_v + m_s s_w (100 - 40)$

$= m_s(L_v + 60 s_w)$

$= m_s(2.26 \times 10^6 + 60 \times 4200)$

$= m_s \times 2.512 \times 10^6$J

From principle of calorimeter,

Total heat gained = Total heat loss

1.37×10^5J $= m_s \times 2.512 \times 10^6$J

or, $m_s = \frac{1.37 \times 10^5}{2.512 \times 10^6 \text{J}} = 0.05478$ kg = 54.78 gm

∴ Mass of steam passed = 54.78 gm

53. 2073 Supp Q.No. 10c 2072 Supp Q.No. 10b 2069 (Set B) Q. No. 10b **What is the result of mixing 100 g of ice at 0°C into 100 g of water at 20°C in an iron vessel of mass 100 g?** [4]

Solution

Given,

Mass of ice (m_i) = 100gm at 0°C

Mass of water (m_w) = 100gm at 20°C

Mass of iron (m_v) = 100gm at 20°C

Specific heat of iron (S_v) = 0.1 cal g^{-1} °C^{-1}

Heat gain by ice to melt (Q_1) = mi Li

= 100 × 80 = 8000 cal

Heat lost by water + vessel (Q_2) = $(m_w S_w + m_v . S_v) \times \Delta\theta$

= (100 × 1 + 100 × 0.1) × 20 = 2200 cal.

Here, $Q_1 > Q_2$. So, total ice does not melt. So, mass of ice melt,

$mi = \frac{2200}{80} = 27.5$ gm

So, mixture contains 72.5 gm of ice & 127.5 gm of water at 0°C.

54. 2073 Set C Q.No. 10a **In an experiment on the specific heat of a metal, a 200 gm block of metal at 150°C is dropped in a copper calorimeter of mass 270 gm containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Calculate the specific heat of the metal. [S_c = 390 J/Kg°C, S_w = 4200 J/kg °C]** [4]

Solution

Given,

Mass of metal (m_m) = 200 gm = 0.2 kg

Mass of calorimeter (m_c) = 270 gm = 0.27 kg

Mass of water (m_w) = 1 × 150 gm = 0.15 kg [$\because m = \rho \times V = 1 \times 150 = 150$ gm]

Temperature of metal (θ_1) = 150° C

Temperature of water + calorimeter (θ_2) = 27° C

Final temperature (θ) = 40° C

Specific heat capacity of copper (S_c) = 390 J/kg °C

Specific heat capacity of water (S_w) = 4200 J/kg °C

Specific heat capacity of metal (S_m) = ?

We have, by principle of calorimetry

Heat lost by metal block = Heat gained by calorimeter and water

$m_m S_m(\theta_1 - \theta) = (m_c S_c + m_w S_w)(\theta - \theta_2)$

or, $0.2 \times S_m(150 - 40) = (0.27 \times 390 + 0.15 \times 4{,}200)(40 - 27)$

or, $S_m = \frac{9558.9}{22} = 434.5$ J/Kg°C

Specific heat capacity of metal (S_m) is 434.5 J/Kg °C.

55. 2072 Set C Q.No. 10a **A copper pot with mass 0.5 kg contains 0.170 kg of water at a temperature of 20°C. A 0.250 kg block of iron at 85°C is dropped into the pot. Find the final temperature assuming no heat loss to the surroundings.** [4]

Solution

Given,

Mass of copper pot (m_c) = 0.5 kg

Mass of water (m_w) = 0.170 kg

Temperature of water (θ_1) = 20° C

Mass of iron (m_i) = 0.250 kg

Temperature of iron (θ_2) = 85° C

Specific heat capacity of copper (S_c) = 390 J kg^{-1} K^{-1}

Specific heat capacity of water (S_w) = 4190 J kg^{-1} K^{-1}

Specific heat capacity of iron (S_i) = 470 J kg^{-1} k^{-1}

Temperature of mixture (θ) = ?

Now,

Heat gained by water + copper pot (Q_1) = $(m_c s_c + m_w s_w)(\theta - c)$

Heat lost by iron (Q_2) = $m_i s_i (\theta_2 - \theta)$

According to principle of calorimeter,

Heat gained = Heat lost

$Q_1 = Q_2$

or, $(m_c S_c + m_q S_w)(\theta - \theta_1) = m_i s_i(\theta_2 - \theta)$

or, $(0.5 \times 390 + 0.17 \times 4190)(\theta - 20) = 0.25 \times 470 (85 - \theta)$

or, $907.3\theta - 18146 = 9987.5 - 117.5\theta$

or, $\theta = \frac{28133.5}{1024.8} = 27.45°$ C

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56. 2071 Set C Q.No. 10 a **Find the result of mixing 0.8 kg of ice at –10°C with 0.8 kg of water at 80°C.**

Solution

Given,

Mass of ice (mi) = 0.8 kg

Mass of water (m_w) = 0.8 kg

Temperature of ice (θ_i) = – 10°C

Temperature of water (θ_w) = 80°C

Specific heat capacity of water (S_w) = 4200 Jkg^{-1} $°C^{-1}$

Specific heat capacity of ice (Si) = 2100 J kg^{-1} $°C^{-1}$

Specific latent heat of ice (Li) = 336 × 10^3 J /kg

Result of mixing (θ_f) = ?

Total amount of heat required to melt ice at

0°C (Q_1) = miSi(θ_i – 0) + miLi = 0.8 × 2100 × 10 + 0.8 × 336 × 10^3 = 285600 J

Total amount of heat lost in cooling from 80°C

to 0°C (Q_2) = M_wS_w (80 – 0) = 0.8 × 4200 × 80 = 268800 J

Since $Q_1 > Q_2$ total ice does not melt. Then amount of ice melted,

$$m_i = \frac{Q_2}{Li + Si \times 10} = \frac{268800}{2100 \times 10 + 336000} = 0.753 \text{ kg}$$

So, the result of mixing consists of 0.753 + 0.8 = 1.553 kg of water and 0.047 kg of ice at 0°C.

57. 2071 Set D Q.No. 10 c **From what height should a block of ice be dropped in order that it may melt completely?** [4]

Solution

Given,

Mass of ice = m_i

Height to be dropped for completely melt = h

Specific latent heat of ice = Li = 336000 J kg^{-1}

To be completely melt, the P.E. lost by the ice block must be equal to latent heat of ice of that block.

i.e. $m_igh = m_iL_i$

$$\text{or, } h = \frac{Li}{g} = \frac{336000}{10} = 33600 \text{ m}$$

∴ To be completely melt, a block of ice should dropped from 33600m height.

58. 2070 Supp (Set A) Q.No. 10 a **How much heat is required to convert 5 kg of ice at –10°C into steam at 100°C?** [4]

Solution

Given,

Mass of ice (m_i) = 5 kg

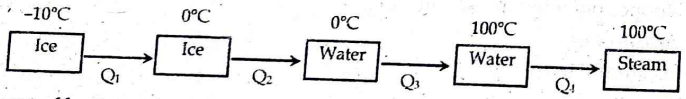
Specific heat capacity of ice (S_i) = 2100 J $kg^{-1}°C^{-1}$

Specific latent heat of ice (L_i) = 336000 J kg^{-1}

Specific heat capacity of water (S_w) = 4200 J $kg^{-1}°C^{-1}$

Latent heat of vapourization (L_v) = 2.26× 10^6J/kg

Amount of heat (Q) = ?



Amount of heat required to convert ice from –10°C to 0°C is

Q_1 = m_iS_i (0 – (–10)) = 5 × 2100 × 10 = 105000J

Amount of heat required to convert ice from 0°C to water at 0°C

Q_2 = m_iL_i = 5 × 336000 = 1680000 J

Amount of heat required to convert water at 0°C to water at 100°C

Q_3 = m_wS_w (100 – 0) = 5 × 4200 × 100 = 2100000 J

Amount of heat required to convert water at 100°C to steam at 100°C.

Q_4 = m_wL_V = 5 × 2.26 × 10^6 = 11.30 × 10^6 J

Total heat required to convert 5 kg of ice from –10°C to steam at 100°C is

Q = $Q_1 + Q_2 + Q_3 + Q_4$ = 105000 + 1680000 + 2100000 + 11300000 = 1.52 × 10^7 J

59. 2070 Supp (Set B) Q.No. 10 a **A copper calorimeter of mass 300 g contains 500 g of water at a temperature of 15°C. A 560g block of aluminium at temperature of 100°C is dropped in the water of the calorimeter and the temperature is observed to increase 22.5°C. Find the specific heat capacity of aluminium, neglecting the heat lost to the surroundings.** [4]

Solution

Given,

Specific heat capacity of copper (Sc) = 400 J kg^{-1} $°C^{-1}$

Specific heat capacity of water (S_w) = 4200J $kg^{-1}°C^{-1}$

Specific heat capacity of aluminium (S_{al}) = ?

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 500 gm = 0.5 kg

Mass of alluminium (m_{al}) = 560 gm = 0.56 kg

Temperature of water and calorimeter (θ_2) = 15°C

Temperature of alluminium (θ_1) = 100°C

Final temperature (θ) = 25°C

Now, by using principle of calorimetry which states as

Heat lost = Heat gained

$m_{al} \times S_{al} \times (\theta_1 - \theta) = (m_c \times S_c + m_w \times S_w) \times (\theta - \theta_2)$

$0.56 \times S_{al} \times (100 - 25) = (0.3 \times 400 + 4200 \times 0.5)(25 - 15)$

or, S_{al} = 528. 57 J $kg^{-1}°C^{-1}$

60. 2070 Set C Q.No. 10 a **A ball of copper weighing 400 gram is transferred from a furnace to a copper calorimeter of mass 300 gram and containing 1 kg of water at 20°C. The temperature of water rises to 50°C. What is the original temperature of the ball?** [4]

Solution

Given,

Mass of copper ball (m_b) = 400 gm = 0.4 kg

Mass of calorimeter (m_c) = 300 gm = 0.3 kg

Mass of water (m_w) = 1 kg

Initial temperature of water and calorimeter (θ_2) = 20°C

Final temperature (θ) = 50°C

Specific heat capacity of copper ball (S_c) = 400J/kg°C

Specific heat capacity of water (S_w) = 4200J/kg°C

Temperature of ball (θ_1) = ?

From principle of calorimetry

Heat loss = Heat gain

$m_bS_c(\theta_1 - \theta) = (m_cS_c + m_wS_w)(\theta - \theta_2)$

or, $0.4 \times 400(\theta_1 - 50) = (0.3 \times 400 + 1 \times 4200)(50 - 20)$

or, $160(\theta_1 - 50) = 129600$

$$\text{or, } \theta_1 = \frac{129600 + 8000}{160} = 860°C$$

∴ Temperature of ball is 860°C

61. 2070 Set D Q.No. 10 a **A copper calorimeter of mass 300 gram contains 500 gram of water at 15°C. A 560 gram of aluminium ball at temperature of 100°C is dropped in the calorimeter and the temperature is increased to 25°C. Find the specific heat capacity of aluminium.** [4]

Please refer to 2070 Supp (Set B) Q.No. 10 a

62. 2069 (Set A) Q. No. 10b **10 gm of steam at 100°C is passed into a mixture of 100 gm of water and 10 gm of ice at 0°C. Find the resulting temperature of the mixture.** [4]

Solution

Given,

Mass of steam (m_s) = 10gm

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Temp of steam (θ_s) = 100°C
Mass of water (m_w) = 100 gm
Temp of water & ice (θ_i) = 0°C
Mass of ice (m_i) = 10 gm
Temperature of mixture (θ) = ?
Latent heat of ice (L_i) = 80 cal/gm
Latent heat of steam (L_s) = 540 cal/gm
Heat loss by steam = $m_s L_s + m_s S_w (100-\theta)$
Heat gained by water & ice = $m_i L_i + (m_i + m_w) S_w. (\theta-0)$
By principle of calorimetly,
Heat loss = Heat gain,

or, $m_s L_s + m_s S_w (100-\theta) = m_i L_i + (m_i + m_w) S_w \theta$
$10 \times 540 + 10 \times 1 (100-\theta) = 10 \times 80 + (10 + 100). 1. \theta$
or, $5400 + 1000 - 10\theta = 800 + 110\theta$
or, $5600 = 120\theta$
or, $\theta = 46.66°C$.

63. 2066 Q. No. 10 b What is the result of mixing 10gm of ice at 0°C into 15gm of water at 20°C in a vessel of mass 100 gm and specific heat 0.09.

Solution

Given,
Mass of ice (m_i) = 10gm
Temperature of ice (θ_i) = 0°C
Latent heat of ice (L_i) = 80 cal
Mass of water (m_w) = 15 gm
Temperature of water + vessel (θ_w) = 20°C
Mass of vessel (m_v) = 100gm
Specific heat of vessel (s_v) = 0.09cal/gm°C
Result of mixing (θ) = ?
Heat gained by ice in melting
$Q_1 = m_i L_i = 10 \times 80 = 800$ cal
Heat lost by water + vessel from 20°C to 0°C
$Q_2 = (m_w S_w + m_v s_v)(20 - 0) = (15 \times 1 + 100 \times 0.09)\, 20 = (15 + 9) \times 20 = 24 \times 20 = 480$ cal
Here, $Q_1 > Q_2$, so, total ice does not melt.

Amount of ice melt = $\frac{480}{80} = 6$ gm

So, mixture contains (15 + 6) = 21 gm of water and (10 – 6)= 4 gm of ice at 0°C.

64. 2061 Q.No. 5 b What is the result of mixing 100g of ice at 0°C and 100g of water at 100°C. Latent heat of fusion of ice = 336×10^3 Jkg^{-1}, specific heat of water = 4200 Jkg^{-1} k^{-1}.

Solution

Given,
Mass of ice (m_i) = 100gm = 0.1 kg
Temperature of ice (θ_1) = 0°C
Temperature of water (θ_2) = 100°C
Mass of water (m_w) = 100°C
Latent heat of ice (L_i) = 336×10^3 J /kg
Specific heat of water (S_w) = 4200 Jkg^{-1}°C^{-1}
Amount of heat gained by ice in melting
$Q_1 = m_i L_i = 0.1 \times 336000 = 33600$ J
Total heat lost by water from 100°C to 0°C is
$Q_2 = m_w S_w (\theta_2 - \theta_1) = 0.1 \times 4200 (100 - 0) = 42000$ J

Here,
$Q_2 > Q_1$ and all ice melts.
So, the ice gains heat in two steps to melt at 0°C and to reach at a temperature θ say.
Total heat gain by ice,
$Q_1 = m_i L_i + m_i S_w (\theta - 0)$
$= 33600 + 0.1 \times 4200\, \theta = 33600 + 420\, \theta$
Heat lost by water
$Q_w = m_w S_w (\theta_2 - \theta)$
$= 0.1 \times 4200 (100 - \theta) = 42000 - 420\, \theta$
By principle of calorimeter,
Heat gained = heat lost
or, $33600 + 420\, \theta = 42000 - 420\, \theta$
or, $840\, \theta = 42000 - 33600$
$\theta = \frac{8400}{840} = 10°C$
∴ Temperature of mixture is 10°C

65. 2059 Q.No. 5 b 2051 Q.No. 11 How much heat is needed to change 10 g of ice at - 10°C to steam at 100°C. (Specific heat capacity of ice = 0.5 cal g^{-1} °C^{-1}, Latent heat of fusion of ice = 80 cal g^{-1}, Latent heat of vaporization = 540 cal g^{-1}). [4]

Solution

Given,
Mass of ice (m_i) = 10g
Specific heat of ice (S_i) = 0.5 cal/g °C
Amount of heat required to change ice at – 10°C to ice at 0°C is
$Q_1 = m_i s_i d\theta = 10 \times 0.5 \times [0 - (-10)] = 10 \times 0.5 \times 10 = 50$ cals.
Amount of heat required to change ice at 0°C to water at 0°C is
$Q_2 = m_i L_i = 10 \times 80$ [$\because L_i = 80$ cal g^{-1}]
$= 800$ cals
Amount of heat required to change water at 0°C to water at 100°C is
$Q_3 = m_i S_w d\theta = 10 \times 1 \times (100 - 0)$
$= 1000$ cals. [$\because S_w = 1$ cal g^{-1} °C^{-1}]
Amount of heat required to change water at 100° C to steam at 100° C is
$Q_4 = m_i \times L_s$ [$\because L_s = 540$ cal g^{-1}]
$= 10 \times 540 = 5400$ cal
Total heat gained
$Q = Q_1 + Q_2 + Q_3 + Q_4$
$= 50 + 800 + 1000 + 5400$
$= 7250$ cal
$= 7250 \times 4.2$
$= 30450$ J

66. 2057 Q.No. 5 b From what height a block of ice be dropped in order that it may completely melt. It is assumed that 20% of energy of fall is retained by ice. [L = 80 cal./g].

Solution

Given,
Latent heat of ice (L) = 80 cal/g
Height (h) = ?
Suppose h be the height from where a block of ice be dropped and m be the mass of the ice block. Then the loss in potential energy by the ice-block when it reaches to the ground is mgh. From question,
Energy retained by the ice-block = 20% of mg h

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Specific latent heat of ice (L) = 80 cal/g $= \frac{80 \times 4.2}{10^{-3}}$ J kg^{-1}

Amount of heat required to melt the ice block is given by

$E_2 = mL = m \times \frac{80 \times 4.2}{10^{-3}}$

$= m \times 336000$ ... (ii)

As the ice reaches the ground, it melts completely. So,

$E_1 = E_2$

$\frac{20}{100} \times mg\,h = m \times 336000$

or, $\frac{20}{100} \times 10\,h = 336000$

or, $h = \frac{336000 \times 100}{200} = 168000$ m

∴ Height of ice block from where it is dropped, h = 168000 m.

67. 2052 Q.No. 10 b 50 gm. of ice at - 6°C is dropped into water at 0°C. How many grams of water freeze? (Given SP heat capacity of ice = 2000 J. Kg^{-1} °C^{-1}).

Solution

Given,

Mass of ice (m_i) = 50 g =50 × 10^{-3} kg

SP heat capacity of ice(S_i)= 2000 J Kg^{-1} °C^{-1}

Amount of heat gained by ice at – 6°C to ice at 0°C is given by

$Q_1 = mS_{ice}\,d\theta$

$= 50 \times 10^{-3} \times 2000 \times [0 - (-6)]$

$= 600$ J ... (i)

If m is the mass of water frozen, then the amount of heat lost by water at 0° C to ice at 0° C is given by

$Q_2 = m\,L_w = m \times (336 \times 10^3)$ ... (ii)

By the principle of calorimetry,

Heat lost = Heat gained

$m \times 336 \times 10^3 = 600$

or, $m = 0.001785 = 1.785 \times 10^{-3}$ kg

∴ $m = 1.785$ g

68. 2050 Q.No. 8 Evaporation or perspiration is an important mechanism for temperature control of warm blooded animals. What mass of water must evaporate from the surface of an 80 kg human body to cool it by 1°C? The specific heat capacity of the human body is approximately 0.1 cal gm^{-1} °C and the latent heat of vaporization of water at the body temperature is 577 cal. gm^{-1}?

Solution

Given,

Mass of human (m) = 80kg

Change in temperature ($\Delta\theta$) = 1°C

Specific heat capacity of human (S_n) = 0.1 calgm^{-1} °C^{-1} = 420 J/Kg°C

Latent heat of vaporization of water at human body temperature

(L_n) = 577 cal/gm = 2423400 J/kg

Heat gained by water that evaporate is,

$m.L_n = 2423400 \times m$

Heat loss by human = $m \times S_n \times \Delta\theta = 80 \times 420 \times 1 = 33600$ J

But, heat lost = heat gained

$m \times 2423400 = 33600$

$m = 0.01386$ Kg

❑❑❑

Chapter 3: Thermal Properties of Matter

Short Answer Questions

1. 2075 Set B Q.No. 2a How can Kelvin scale, in principle, be designed by the experiment based on ideal gas law? Explain. [2]

Ans The ideal gas equation is given as

$PV = nRT$ ... (i)

when, T = 0°C, then PV = 0 which doesn't give any meaning (or becomes meaning less). So, the temperature scale in 0°C is converted to Kelvin scale i.e. 0°C = 0 + 273K = 273K which gives the physical meaning i.e. PV ∝ T. So, in ideal gas equation, temperature in Celsius scale is converted into Kelvin by adding 273 to it.

2. 2074 Set A Q.No. 2c When a car is driven some distance, the air pressure in the tyre increases. Why? [2]

Ans When a car is driven some distance, the tyre gets heated due to friction. When tyres get heated, the temperate of gas inside the tyre increased. Since the pressure of gas is the function of temperature.

i.e. $P = \frac{1}{3}\rho c^2$ $[\because \bar{c} \propto \sqrt{T}]$

Due to this reason the pressure inside the tyre increases on increasing temperature when the car is drive some distances.

3. 2073 Supp Q.No. 2c What physical concept is provided by universal gas constant? Write its unit. [2]

Ans The Ideal gas equation is given by PV = nRT, P is pressure, V is the volume, T is the temperature and constant R in equation of state is called universal gas constant. It is defined as the amount of work done per mole per unit temperature.

The unit of R is J mol^{-1} K^{-1} and value of R = 8.318 J/mol K.

4. 2072 Set C Q.No. 2c Explain on the basis of kinetic theory that the pressure of a gas increases with increase of temperature. [2]

Ans According to kinetic theory of gas, the pressure exerted by gas moving with average r m.s. speed $\bar{c}$ is given by

$P = \frac{1}{3}\rho\,\bar{c}^2$, ρ = density of gas

or, $P = \frac{1}{3}\frac{M}{V}\bar{c}^2$ At constant volume, $P \propto \bar{c}^2$. But $\bar{c}^2 \propto T$

∴ $P \propto T$

Hence, the pressure of gas increases on increasing temperature of gas.

5. 2072 Set D Q.No. 2c On reducing the volume of a gas at constant temperature, the pressure of a gas increase. Why? [2]

Ans On reducing the volume of gas at constant temperature, the number of molecules per unit volume increase. Therefore, more number of molecules collide with the walls per second and hence, large momentum is transferred to the walls per second. The K.E. also increases, as a result pressure increases.

6. 2072 Set E Q.No. 2b What does "the kinetic interpretation of temperature" signify? Explain. [2]

Ans According to Kinetic theory of gas, the Kinetic energy if gas molecule is given by

$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}KT$, where $\bar{v}$ is the root mean square velocity, T is temperature of gas and K is Boltzman constant. This shows that K.E. of gas depends upon the temperature.

7. 2071 Set C Q.No. 2 a What is the difference between a real and an ideal gas? Explain on the basis of the hypothesis of the kinetic theory. [2]

Ans Ideal gas is only hypothetical condition but the real gas is found in real life. These two gases have following differences on the basis of kinetic theory of gas:

i. The size of ideal gas molecules is negligible but the size of real gas has significant size.

ii. There is no force of attraction or repulsion between molecules of ideal gas but there is force of attraction or repulsion between real gas molecules.

iii. The collision of ideal gas molecular is perfectly elastic but not in case of real gas.

Question Bank Solutions

8. **2070 Supp (Set B) Q.No. 2 a At a fixed temperature, the volume of a vessel is compressed to half. How will the rms speed of the gas in it change?**

The rms speed of gas is given by

$$C_{rms} = \sqrt{\frac{3PV}{M}}$$

If temperature remains constant and volume is compressed to half, then pressured will increase such that PV = RT = constant at constant temperature. So, there is no change in rms speed of a gas on changing volume at constant temperature.

9. **2069 (Set A) Old Q. No. 4b Do you expect the gas in cooking gas cylinder to obey the ideal gas equation?**

No, the gas in cooking gas cylinder does not obey the ideal gas equation. To obey the ideal gas equation, the temperature of gas should be high and pressure should be low. But the gas in a cooking gas cylinder has high pressure and low temperature. So, it does not obey the ideal gas equation.

10. **2068 Old Q.No. 4 b What are the characteristics of a gas to be an ideal?**

According to kinetic theory of gas, a gas should have the characteristics to be an ideal,

a. The size of molecules should be negligibly small in comparison to distance between them.
b. The molecules of gas should alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules should be negligible.
d. The gas molecules should be a perfect elastic sphere.
e. The temperature of gas should be high and pressure should be low.

11. **2065 Q. No. 4 c**

Which has more molecules: A kilogram of hydrogen or a kilogram of oxygen?

The number of mole of a substance is defined as

$$n = \frac{\text{Mass of substance}}{\text{Molecular mass of substance}}$$

$$n = \frac{m}{M}$$

or, $n \propto \frac{1}{M}$; for a given mass = 1kg

Since $M_O > M_H$, $n_O < n_H$; i.e. 1 kg of Hydrogen gas has more atoms than 1 kg of oxygen.

12. **2064 Q.No. 4 c Absolute zero temperature is not zero energy temperature. Explain.**

According to kinetic theory of gases, the kinetic energy of the gas molecules is $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}KT$, where $\bar{v}$ is root means square velocity of the molecules of gas and K is Boltzmann constant. At absolute zero temperature, T = 0K, kinetic energy of gas is zero. However, all the molecules remain at rest and there exist potential energy. The total energy is the sum of kinetic and potential energy. At 0K the total energy is non zero due to its potential energy. So, absolute zero temperature is not zero energy temperature.

13. **2063 Q.No. 4 c In the kinetic theory of gases, why do we not take into account the changes in gravitational potential energy of the molecule?**

We do not take into account the change in gravitational potential energy in the kinetic theory of gases because the gas molecules are very small in size and its gravitational potential energy($U = -\frac{GMm}{r}$) is negligible compared to the kinetic energy ($E_k = \frac{1}{2}m\bar{v}^2$).

14. **2062 Q.No. 4 c Write the unit of the universal gas constant and give its physical meaning.**

Please refer to 2073 Supp Q.No. 2c

15. **2060 Q.No. 4 b Under what conditions do the real gases obey more strictly the gas equation PV = RT?**

The equation of state of ideal gas is given by: PV = RT, where R = 8.314J/mole K, is called universal gas constant. The real gases obey more strictly the gas equation PV = RT only (a) at high temperature and (b) at low pressure.

16. **2057 Q.No. 4 a Why does the cycle tube burst sometimes in summer? [2]**

The atmospheric temperature is high during summer. The temperature of air inside cycle tyre gets increased, which results to increase in kinetic energy of air molecules in the tube. This increase in kinetic energy increases the pressure in the tube-but the volume of tyre remains same. While riding the cycle in summer days, the pressure of gas increases tremendously due to the rise in temperature and so the tube burst suddenly.

17. **2057 Q.No. 4 b At absolute zero temperature, why the kinetic energy is zero? [2]**

At absoute zero, the gas does not produce pressure on the wall. from ideal gas equation,

PV = nRT

For, T = 0, P = 0

It means that molecules remains at rest, i.e. speed of molecules becomes zero. Since, the speed is zero; kinetic energy ($\frac{1}{2}m\bar{v}^2$) of gas molecules is also zero. Moreover, from kinetic theory of gases, the kinetic energy of the gas molecules is $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}KT$, where $\bar{v}$ is root mean square velocity of the molecules of gas and K is Boltzmann constant. At absolute zero temperature, T = 0K, kinetic energy of gas is zero.

18. **2055 Q.No. 7 a Why do you consider an ideal gas while formulating the pressure in the light of kinetic theory of gases? [2]**

While formulating the pressure exerted by a gas using the kinetic theory of gas, we consider an ideal gas which has the following properties:

i. The collisions between molecules of a gas are perfectly elastic. That is, there is no loss of kinetic energy during collisions.
ii. There is no molecular attraction or repulsion. That is, there is no potential energy between the molecules.

19. **2053 Q.No. 7 d Molecules of different gases have equal average kinetic energies, provided their temperature is the same. Do these molecules have equal velocities also?**

No, the gas molecules have not equal velocities. The average K.E. per molecule is given by

$$\frac{1}{2}mc^2_{rms} = \frac{3}{2}kT \quad \ldots (i)$$

If two gases have the same average kinetic energy at the same temperature, then

$$\frac{1}{2}m_1c_1^2 = \frac{1}{2}m_2c_2^2 = \frac{3}{2}kT \quad \ldots (ii)$$

where c_1 is the rms speed of a gas having mass of each molecule m_1 and c_2 is the rms speed of another gas having mass of each molecule m_2. Since, m_1 and m_2 are different for different gases, the equation (ii) indicates that c_1 and c_2 are also different.

20. **2053 Q.No. 10 a Outline the essential features of the kinetic theory of gases. [2]**

The kinetic theory of gas explains the macroscopic properties of gases like pressure, volume, temperature in terms of microscopic properties like molecular mass, diameter, velocity etc. This kinetic theory of gas based on following assumptions:

a. The size of molecules is negligibly small in comparison to distance between them.
b. The molecules of gas are alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules is negligible.
d. The motion of gas molecules is random. They have velocity in all directions.
e. The gas molecules are like perfect elastic sphere.
f. The gas molecules move in straight line between two collisions.
g. The collision between gas molecules is perfectly elastic.

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21. 2050 Q.No. 7 e Which has more atoms: a kilogram of hydrogen or a kilogram of iron?

The number of mole of a substance is defined as

$$n = \frac{\text{mass of substence}}{\text{Molecular mass of substance}} = \frac{m}{M}$$

or, $n \propto \frac{1}{M}$; for a given mass = 1kg

Since $M_i > M_H$, $n_H > n_i$; i.e. 1 kg of Hydrogen gas has more atoms than 1 kg of iron.

Long Answer Questions

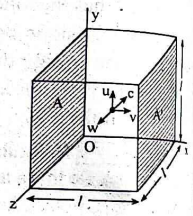
22. 2076 Set B Q.No. 6b 2076 Set C Q.No. 6b 2074 Set A Q.No. 6b Derive an expression for the pressure exerted by an ideal gas on the basis of Kinetic theory. [4]

The **kinetic theory of gas** explains the macroscopic properties of gases like pressure, volume, temperature in terms of microscopic properties like molecular mass, diameter, velocity etc. This kinetic theory of gas based on following assumptions:

a. The size of molecules is negligibly small in comparison to distance between them.
b. The molecules of gas are alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules is negligible.
d. The motion of gas molecules is random. They have velocity in all directions.
e. The gas molecules are like perfect elastic sphere.
f. The gas molecules move in straight line between two collisions.
g. The collision between gas molecules is perfectly elastic.

Pressure exerted by gas;

Let us consider a cubical box of side l contains N number of molecules of gas. Each molecules are identical and having mass m. Suppose c is the velocity of gas molecule at any time, then u, v and w are the component of velocity along OX, OY and OZ directions respectively. Then,



$c^2 = u^2 + v^2 + w^2$ ... (i)

Let us consider molecule striking against face A with momentum mu along OX and the molecule rebounds with same momentum as the collision is perfectly elastic. Hence, the momentum of molecule before collision is mu and after collision is –mu.

Change in momentum during collision = mu – (–mu) = 2mu

The time taken by the molecule to move from face A to A' and back to A is given by $\Delta t = \frac{2l}{u}$

Change in momentum per second $= \frac{\Delta p}{\Delta t} = 2mu\frac{u}{2l} = \frac{mu^2}{l}$

i.e. Force on the face A $= \frac{mu^2}{l}$

Pressure on the face A $= \frac{\text{Force}}{\text{area}} = \frac{mu^2}{l^3}$ ... (ii)

Equation (ii) gives the pressure on the face A due to single molecule whose x-component velocity is u. Since, the velocity of each molecules are different and their components in the direction OX also different. Let $u_1, u_2 \ldots u_N$ be the components of velocity of all molecules along OX direction. Then, total pressure on the face A is.

$$P = \frac{mu_1^2}{l^3} + \frac{mu_2^2}{l^3} + \ldots + \frac{mu_N^2}{l^3}$$

$$= \frac{m}{l^3}(u_1^2 + u_2^2 + \ldots + u_N^2) \quad \ldots \text{(iii)}$$

If $\bar{u}^2$ denoted mean velocity of $u_1^2, u_2^2 \ldots u_N^2$

$$\bar{u}^2 = \frac{u_1^2 + u_2^2 + \ldots + u_n^2}{N}$$

$u_1^2 + u_2^2 + \ldots u_N^2 = N\bar{u}^2$

Using equation (iv) in (iii), we get ... (iv)

$$P = \frac{m}{l^3} N\bar{u}^2 \quad \ldots \text{(v)}$$

Since all N molecules have random motion with varying speed in all direction, then

$$\bar{u}^2 = \bar{v}^2 = \bar{w}^2 = \frac{1}{3}\bar{c}^2 \quad \ldots \text{(vi) [Using equation (i)]}$$

where $\bar{c}^2$ is mean square velocity of gas.

Using equation (vi) in (v), we get

$$P = \frac{m}{l^3} N\left(\frac{1}{3}\bar{c}^2\right)$$

Since mass of gas (M) = mN and l^3 = V (Volume of the cube).

Hence, $P = \frac{M}{NV} N\left(\frac{1}{3}\bar{c}^2\right) = \frac{1}{3}\frac{M}{V}\bar{c}^2 = \frac{M}{V}\frac{1}{3}\bar{c}^2$

$$P = \frac{1}{3}\rho\bar{c}^2 \quad \ldots \text{(vii)}$$

Where $\rho = \frac{M}{V}$ is the density of gas. Equation (vii) gives the pressure exerted by gas on the wall of cube.

23. 2075 Set A Q.No. 6b Starting from basic postulate for kinetic theory of gas, derive an expression for the pressure exerted by the gas contained in a box. [4]

Please refer to 2076 Set B Q.No. 6b

24. 2074 Set B Q.No. 6b Deduce an expression for pressure exerted by a ideal gas from kinetic theory? [4]

Please refer to 2076 Set B Q.No. 6b

25. 2073 Supp Q.No. 6b State Boyle's and Charle's laws. Describe how these laws are combined to derive ideal gas law. [4]

Boyle's law: The volume (V) of a given mass of a gas is inversely proportional to its pressure (P) provided the temperature remains constant called Boyle's law. Mathematically,

$V \propto \frac{1}{P}$ ... (i) (at constant temperature)

$\therefore PV = \text{constant}$

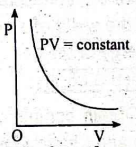
According to kinetic theory of gases $PV = \frac{1}{3}Mc^2$ where M is the mass of gas and $\bar{c}$ is the root mean square velocity. Since, mean square velocity ∝ temperature, i.e. $\bar{c}^2 \propto T$.

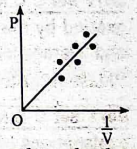
Thus, $PV \propto T$

For constant T, $PV = constant$.

Which is Boyle's law.

It is applicable only at high temperature and low pressure.





Charle's law: If pressure is kept constant, the volume (V) of gas is directly proportional to absolute temperature (T).

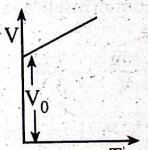
i.e. $V \propto T$, ... (ii) at constant P

Now, according to kinetic theory of gases; $PV = \frac{1}{3}Mc^2$

Where M = mass of gas

$\bar{c}$ = root mean square velocity

$V = \frac{1}{3}\left(\frac{M}{P}\right)\bar{c}^2$.



Since, we know $\bar{c}^2 \propto T$ and $\frac{M}{P}$ is constant. Then, $V \propto T$, which is Charles' law.

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Ideal gas equation (Equation of state)

For a given mass of gas, the three thermodynamic variables pressure (P), volume (V) and temperature (T) change simultaneously. The equation which relates these three variables pressure (P), volume (V) and temperature (T) is called equation of state.

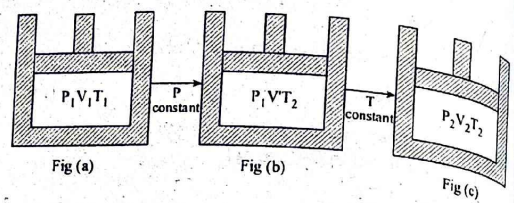


Fig (a) Fig (b) Fig (c)

As shown in above figure, the volume of gas is V_1 at pressure P_1 and absolute temperature T_1. If temperature of gas is increased from T_1 to T_2 at constant pressure (P_1), the volume of gas becomes V' as shown in figure (b). Applying Charle's law, we get

$$\frac{V_1}{T_1} = \frac{V'}{T_2} \Rightarrow V' = \frac{T_2}{T_1} V_1 \quad \ldots (i)$$

Now, the gas is compressed at constant temperature T_2 so that the new volume of gas is V_2 and pressure P_2. Applying Boyle's law, we get

$$P_1V' = P_2V_2$$

Using equation (i), we get

$$P_1V_1\frac{T_2}{T_1} = P_2V_2 \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \ldots (ii)$$

In general, we can write $\frac{PV}{T} = \text{constant}$ ... (iii)

The constant appears in equation (iii) does not depends on nature of gas taken and amount of gas taken. If one molecule of gas occupies a volume V at pressure P and at temperature (absolute) T, then

$PV = RT$, Where R is universal gas constant.

For n-mole of gas, $PV = nRT$

which is the equation of state of an ideal gas.

26. **2073 Set C Q.No. 6b** **Using the postulates of kinetic theory of gases, deduce an expression for the pressure exerted by an ideal gas on the walls of a container.** [4]

Please refer to 2076 Set B Q.No. 6b

27. **2073 Set D Q.No. 6a** **State and explain Boyle's law and Charle's law. Use these laws to derive the ideal gas equation for n-moles of a gas.** [4]

Please refer to 2073 Supp Q.No. 6b

28. **2072 Set C Q.No. 6c** **Use the concepts of the kinetic theory of gases to derive an expression for pressure exerted by the gas on the walls of a container. Extend your result to establish a relation between pressure and average kinetic energy of the gas.** [4]

Please refer to 2076 Set B Q.No. 6b

29. **2071 Set C Q.No. 6 a** **Obtain combined equation of state of an ideal gas on the basis of statements and explanations of Boyle's and Charle's laws.** [4]

Please refer to 2073 Supp Q.No. 6b

30. **2071 Set D Q.No. 6 b** **Derive ideal gas equation on the basis of kinetic theory of a gas.** [4]

Derivation of Boyle's law: According to kinetic theory of gases,

We have,

$$P = \frac{1}{3}\frac{mn}{V}\bar{c}^2 = \frac{1}{3}\frac{M}{V}\bar{c}^2 \quad \ldots (i)\ [\because M = m \times n]$$

$$PV = \frac{1}{3}M\bar{c}^2.$$

$$\bar{c}^2 = \frac{3PV}{M} = \frac{3RT}{M} \quad [\because PV = RT, \text{for one mole of gas}]$$

For a given gas, M is constant and R is molar gas constant then

$\bar{c}^2 \propto T$ ... (ii)

Using equation (ii) in (i), we get

$$P \propto \frac{T}{V} \quad \ldots (iii)$$

$$\Rightarrow PV \propto T$$

i.e. $P \propto \frac{1}{V}$, for constant temperature.

For constant temperature, PV = constant, which is Boyle's law.

Derivation of Charles law

If pressure is constant, then

$$P = \frac{1}{3}\rho\bar{c}^2$$

$$\Rightarrow P = \frac{1}{3}\frac{M}{V}\bar{c}^2$$

$$V = \frac{1}{3P}M\bar{c}^2 \quad \ldots (iv)$$

For a given mass of gas, M is constant.

$$V \propto \bar{c}^2 \quad \ldots (v)$$

Using equation (ii) on (v), we get

$V \propto T$

Which is Charle's constant pressure law.

Similarly, if volume is taken constant and using equation (ii) on (iv), we get

$P \propto T$

This is Charle's constant volume law.

Derivation of combined gas law,

According to kinetic theory of gases,

We have

$$P = \frac{1}{3}\frac{mn}{V}\bar{c}^2 = \frac{1}{3}\frac{M}{V}\bar{c}^2 \quad \ldots (vi)\ [\because M = m \times n]$$

$$PV = \frac{1}{3}M\bar{c}^2$$

or, $\bar{c}^2 = \frac{3PV}{M} = \frac{3RT}{M}$ [$\because PV = RT$, for one mole of gas]

For a given gas, M is constant and R is molar gas constant then

$\bar{c}^2 \propto T$ ... (vii)

Using equation (ii) in (i),

$\Rightarrow PV \propto T$ ... (viii)

$\therefore PV = RT$, where R is Universal Gas constant. This is the combined gas law.

31. **2070 Supp (Set B) Q.No. 6 b** **What do you mean by ideal gas? Derive Pv = mrT, where symbols have usual meanings.** [4]

Ideal gas: An ideal gas is hypothetical gas which has following characteristics:

a. The size of molecules should be negligibly small in comparison to distance between them,
b. The molecules of gas should alike in size and mass but differ from the molecules of other gas.
c. The force of attraction between molecules should be negligible.
d. The gas molecules should be a perfect elastic sphere.
e. The temperature of gas should be high and pressure should be low.

Second Part: Please refer to 2073 Supp Q.No. 6b

32. **2070 Set C Q.No. 6 b** **What do you mean by an ideal gas? Derive ideal gas equation for n mole of gas.** [4]

Please refer to 2073 Supp Q.No. 6b

33. **2070 Set D Q.No. 6 b** **What is perfect gas? Prove that the average kinetic energy of a gas molecule is directly proportional to the absolute temperature of the gas.** [4]

Please refer to 2076 Set B Q.No. 6b

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34. 2069 Supp Q.No. 6a **Using the postulates of kinetic theory of gases, derive an expression for the pressure exerted by an ideal gas.**
Please refer to 2076 Set B Q.No. 6b [4]

35. 2069 (Set B) Q. No. 6b **Use the kinetic theory of gases to derive an expression for the pressure exerted by a gas on the walls of its container.**
Please refer to 2076 Set B Q.No. 6b [4]

36. 2069 (Set A) Old Q. No. 5a OR **State Charle's law of gases. Derive the ideal gas equation PV = nrt where symbols have usual meaning.**
Please refer to 2073 Supp Q.No. 6b [1+4]

37. 2068 Q.No. 6 b **On the basis of kinetic theory of gases, deduce the relation, $P = \frac{1}{3}\rho\bar{c}^2$ where the symbols have their usual meanings.**
Please refer to 2076 Set B Q.No. 6b [4]

38. 2067 Supp Q.No. 6 b **State Boyle's law and Charle's law. Use them to derive the ideal gas equation PV = nRT.**
Please refer to 2073 Supp Q.No. 6b

39. 2066 Q. No. 6 b **Prove that the pressure exerted by a gas on the wall of a container is 2/3 times the kinetic energy per unit volume of the gas.**
Please refer to 2076 Set B Q.No. 6b [4]

40. 2066 Old Q. No. 5 a **Outline essential features of the kinetic theory of gases. Obtain an equation of state on the basis of kinetic theory.**
1st part: Please refer to 2076 Set B Q.No. 6b [3+2]
2nd part: Please refer to 2071 Set D Q.No. 6b

41. 2064 Q.No. 5 a OR **Define volume coefficient and pressure coefficient. How pressure coefficient and volume coefficient are related?** [1+4]

Volume coefficient of gas (γ_v): The volume coefficient of a gas is defined as the change in volume of gas per unit volume at 0°C per degree Celsius (Kelvin) change in temperature, provided the pressure remains constant. It is denoted by γ_v.

If V_o and V_θ are the volume of the given mass of gas at 0°C and at θ°C. If γ_V is the volume coefficient of gas and θ be the increase in temperature, then,

$V_\theta = V_0 + \gamma_v V_0\theta = V_0(1 + \gamma_v\theta)$

$V_\theta - V_0 = V_0\gamma_V\theta$

$\therefore \gamma_V = \frac{V_\theta - V_o}{V_0\theta}$ ... (i)

Experimentally, it is found that $\gamma_V = \frac{1}{273}$ °C^{-1}. It means that at constant pressure, the volume of a certain mass of gas increases or decreases by $\frac{1}{273}$ of its volume at 0 °C for each degree rise or fall in temperature.

Pressure coefficient of gas (γ_P): The pressure coefficient of a gas at constant volume is defined as the change in pressure per unit pressure at 0°C per degree Celsius change in temperature. It is denoted by γ_P.

If P_0 and P_θ are pressure of given mass of gas at temperature 0°C and θ°C and γ_P is the pressure coefficient of gas on raising temperature from 0°C to θ°C, then

$P_\theta = P_0(1 + \gamma_P\theta)$

$P_\theta - P_0 = P_0\gamma_P\theta$

$\gamma_P = \frac{P_\theta - P_0}{P_0\theta}$ ... (ii)

Experimentally, it is found that $\gamma_P = \frac{1}{273}$ °C^{-1}. It means that at constant volume, the pressure of a certain mass of gas increases or decreases by $\frac{1}{273}$ of its pressure at 0 °C for each degree rise or fall in temperature.

Relation between γ_V and γ_P: Hence, from above discussion, it is clear that the pressure coefficient γ_P and volume coefficient γ_V are same for all gases.

i.e. $\gamma_V = \gamma_P = \frac{1}{273}$ °C^{-1}

42. 2062 Q.No. 5 a OR **State Boyle's and Charle's law and hence, obtain the relationship for the combined gas law.** [2+3]
Please refer to 2073 Supp Q.No. 6b

43. 2061 Q.No. 5 a OR **State Boyle's law and derive ideal gas equation.** [1+4]
Please refer to 2073 Supp Q.No. 6b

44. 2059 Q.No. 5 a OR **Using postulates of kinetic theory of a gas, derive an expression for the pressure exerted by the gas on the wall of a box.**
Please refer to 2076 Set B Q.No. 6b

45. 2057 Q.No. 5 a **State Charle's law and derive ideal gas equation.** [1+4]
Please refer to 2073 Supp Q.No. 6b

46. 2054 Q.No. 9 **Use the kinetic theory of gases to derive an expression for the pressure exerted by a gas on the walls of its container.** [4]
Please refer to 2076 Set B Q.No. 6b

47. 2052 Q.No. 9 **State Boyle's and Charle's law. Show how can be combined to given equation of state of an ideal gas.**
Please refer to 2073 Supp Q.No. 6b

48. 2051 Q.No. 10 **Starting from the pressure relation $P = \frac{mn\,\bar{c}^2}{3V}$ in kinetic theory of gases, derive Boyle's law and Charle's law.**
Please refer to 2071 Set D Q.No. 6 b

49. 2050 Q.No. 10 OR **Show that, on the basis of the simple kinetic theory of gases, the pressure P of an ideal gas of density ρ is given by P = 1/3 ρ $\bar{c}^2$, where $\bar{c}^2$ is the mean square speed of the molecules. Explain the assumptions you have made in deriving this formula.**
Please refer to 2076 Set B Q.No. 6b

Numerical Problems

50. 2075 Set B Q.No. 10b 2072 Supp Q.No. 10a 2067 Q.No. 10 b 2065 Q. No. 5 b **Air at 273 K and 1.01×10^5 N/m^2 pressure contains 2.70×10^{25} molecules per cubic meter. How many molecules per cubic meter will there be at a place where the temperature is 223 K and the pressure is 1.33×10^4 N/m^2?** [4]

Solution

Given,

At temperature, 273K,

Temperature (T_1) = 273 K

Pressure (P_1) = 1.01×10^5 N/m^2

No. of molecules (n_1) = 2.7×10^{25} molecules/m^3

At temperature, 223 K

Temperature (T_2) = 223 K

Pressure (P_2) = 1.33×10^4 N/m^2

No. of molecules (n_2) = ?

We know,

$\frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1}$ $[V_1 = Vv_2 = 1m^3]$

or, $n_2 = \frac{P_2}{P_1}\cdot\frac{n_1 T_1}{T_2} = \frac{1.33\times10^4}{1.01\times10^5} \times \frac{2.7\times10^{25}\times 273}{223} = 4.35\times10^{24}$ molecules/m^3.

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51. 2074 Supp. Q.No. 10a The correct inflation of a tyre at 20°C is 2 kg/cm². After driving several hours, the driver checks the tyres. If the tyre's temperature is 50°C, what should be the pressure reading? [4]

Solution

Given,

Initial condition,

Inflation (m/A) = 2 kg/cm²

$= 2 \times 100 \times 100$ kgm^{-2}

$= 2 \times 10^4$ kgm^{-2}

Final condition,

Pressure (P_2) = ?

Temperature (T_2) = 50°C = 50 + 273 = 323 K

Pressure (P_1) $= \frac{mg}{A} = 2 \times 10^4 \times 10 = 2 \times 10^5$ Nm^{-2}

Temperature (T_1) = 20°C = 20 + 273 = 293 K

We have,

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $[\because V_1 = V_2]$

or, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

or, $\frac{2 \times 10^5}{293} = \frac{P_2}{323}$

or, $P_2 = \frac{323}{293} \times 2 \times 10^5 = 2.2 \times 10^5$ Nm^{-2}

52. 2073 Supp Q.No. 10a The root mean square (rms) speed of a gas molecule is 600 ms^{-1} at 500°C. Calculate the rms speed of the gas at 100°C. [4]

Solution

Given,

c_{rms} of the molecule at 500°C (c_1) = 600 m/s

Temperature (T_1) = 500°C = 773 k

c_{rms} of the molecule at 100°C (c_2) = ?

Temperature (T_2) = 100°C = 373 k

We know,

$c_{rms} = \sqrt{\frac{3kT}{m}}$

$\Rightarrow c_{rms} \propto \sqrt{T}$

So, we can write

$\frac{c_1}{c_2} = \sqrt{\frac{T_1}{T_2}}$

or, $c_2 = c_1 \times \sqrt{\frac{T_2}{T_1}} = 600 \times \sqrt{\frac{373}{773}} = 416.79$ m/s

53. 2072 Set D Q.No. 10b A cylindrical tank has a tight fitting piston that allows the volume of the tank to be changed. The tank originally contains 0.110 m³ of air at a pressure of 3.4 atm. The piston is slowly pulled out until the volume of the gas is increased to 0.390 m³. If the temperature remains constant what is the final value of the pressure? [4]

Solution

Given,

Initial volume (V_1) = 0.110 m³

Initial pressure (P_1) = 3.4 atm

Final volume (V_2) = 0.39 m³

Final pressure (P_2) = ?

Since, the temperature of the system remain constant, the process is isothermal. So, the isothermal equation is

$P_1V_1 = P_2V_2$

or, $P_2 = \frac{P_1V_1}{2} = \frac{0.11 \times 3.4}{0.39} = 0.95$ atm

54. 2072 Set E Q.No. 10c At a pressure of 700 mm of Hg, the root mean square speed of the molecules of a gas is 400 ms^{-1}. What is its density? [4]

Solution

Given,

Pressure (P) = 700 mm of Hg = $700 \times 10^{-3} \times 13600 \times 10 = 95200$ N/m²

rms Velocity ($\bar{c}$) = 400 m sec^{-1}

Density of gas (ρ) = ?

We have,

$P = \frac{1}{3} \rho \bar{c}^2$ $[\because c_{rms} = \sqrt{\bar{c}^2} \Rightarrow c_{rms}^2 = \bar{c}^2]$

or, $\rho = \frac{3P}{\bar{c}^2} = \frac{3 \times 95200}{400^2} = 1.785$ kg m^{-3}

55. 2071 Supp Q.No. 10b What is average translational K.E. of a molecule of an ideal gas at a temperature of 27°C? What is the total random translational kinetic energy of the molecules in 1 mole of this gas? What is the root-mean-square speed of oxygen molecules at this temperature? [4]
[Relative molecular mass of oxygen = 32; Universal gas constant = 8.31 Jmol^{-1}K^{-1}]

Solution

Given,

Average K.E. of one mole of an ideal gas is

$\frac{1}{2} M\bar{c}^2 = \frac{3}{2} RT$; [M = mole mass and T = 27° = 300K]

or, $K.E_1 = \frac{3}{2} \times 8.31 \times 300$

$= 3739.5$ J/mole

Average K.E. of a molecule of an ideal gas is,

$\frac{1}{2} m\bar{c}^2 = \frac{3}{2} \frac{R}{N} T$; [m = mass of one molecule]

or, $K.E_2 = \frac{3}{2} \times \frac{8.31}{6.023 \times 10^{23}} \times 300 = 1.24 \times 10^{-20}$ J

Again,

$c_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 1.67 \times 10^{-27} \times 6 \times 10^{23}}} = 484.4$ m/sec

56. 2070 Supp (Set A) Q.No. 10 c At what temperature will the average speed of oxygen molecule be sufficient so as to escape from the earth? [Escape velocity from the earth is 11.2 km/s and mass of one oxygen molecule is 53.4×10^{-24} g] [4]

Solution

Given,

Escape velocity of earth (Cs) = 11.2 km/sec = 11200 m/sec

Molecular mass of oxygen (M) = 53.4×10^{-24} g = 53.4×10^{-27} kg

Temperature (T) = ?

Gas constant (R) = 8.31 J mol^{-1} K^{-1}

We have

$Cs = \sqrt{\frac{3RT}{Nm}}$

or, $T = \frac{Cs^2 \times Nm}{3R} = \frac{(11200)^2 \times 53.4 \times 10^{-27}}{3 \times 8.31} \times 6.023 \times 10^{23} = 161833.29$ K

57. 2069 (Set A) Q. No. 10a Air at a temperature of 273 K and a pressure of 1.01×10^5 N/m² pressure contains 2.7×10^{25} molecules per cubic meter. How many molecules per cubic meter will be there at a place where the temperature is 223K and the pressure is 1.33×10^4 N/m². [4]

Please refer to 2075 Set B Q.No. 10b

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58. 2067 Supp Q.No. 10 b **Taking the density of Nitrogen at STP to 1.251 kgm-3, find the root mean square velocity of Nitrogen molecules at 127°c.**

Solution

Given,

Density of nitrogen at STP (ρ_1) = 1.251 Kg/m³

Molecular wt. of nitrogen (M) = 28×10^{-3} Kg

Root mean square speed at 127°C (c) = ?

Now,

Volume of 1 molecule of nitrogen gas at STP (V_1) = $M/\rho_1 = \frac{28 \times 10^{-3}}{1.251} = 0.022 m^3$

Pressure of nitrogen gas at STP (P_1) = 760 mm of Hg

$= 760 \times 10^{-3} \times 13600 \times 9.8 = 1.01 \times 10^{-5}$ N/m²

Volume of 1 mole of gas at STP (V_1) = 22.5×10^{-3}m

Temperature of nitrogen at STP (T_1) = 273 K

Now, gas constant

$R = \frac{P_1V_1}{T_1} = 8.304$ J mol^{-1} K^{-1}

Temperature of gas (T) = 127°C = (127 + 273) K

$c = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.304 \times 400}{28 \times 10^{-3}}} = 596.6$ m/sec

59. 2063 Q.No. 5 b **A cylinder of gas has a mass of 10 kg and pressure of 8 atmosphere at 27°C. When some gas is used in a cold room at -3°C, the gas remaining in the cylinder at this temperature has a pressure of 6.4 atmospheres. Calculate the mass of gas used.**

Solution

Given,

Mass of gas (M_1) = 10kg

Temperature of gas (T_1) = 27°C = 300K

Pressure of the gas (P_1) = 8 atm = $8 \times 1.01 \times 10^5$ N/m²

Again when some gas is used at -3°C

Let remaining mass of the gas be m_2

Temperature of gas (T_2) = – 3°C = 270 K

Pressure of gas (P_2) = 6.4 atm = $6.4 \times 1.01 \times 10^5$ N/m²

Let, V be the volume of the cylinder and r be the gas constant per unit mass.

Then,

$r = \frac{P_1V}{m_1T_1}$ ... (i)

and $r = \frac{P_2V}{m_2T_2}$ ... (ii)

Equating equation (i) and (ii), we get

$\frac{P_1V}{m_1T_1} = \frac{P_2V}{m_2T_2}$

or, $m_2 = \frac{P_2T_1m_1}{P_1T_2} = \frac{6.4 \times 1.01 \times 10^5 \times 10 \times 300}{8 \times 1.01 \times 10^5 \times 270} = 8.9$ kg

Mass of gas used = $m_1 - m_2 = 10 - 8.9 = 1.1$ kg

60. 2060 Q.No. 5 b **Helium gas occupies a volume of 0.04m³ at a pressure of 2×10^5 Nm^{-2} and temperature 300 K. Calculate the mass of the helium and root mean square speed of its molecules. (Relative molecular mass of helium = 4, molar gas constant = 8.3 J mol^{-1} K^{-1})**

Solution

Given;

Volume (V) = 0.04m³

Pressure (P) = 2×10^5 Nm^{-2}

Temperature (T) = 300K

Relative molecular mass of helium = 4

Molar mass (M) = 4g = 4×10^{-3} kg

Molar gas constant (R) = 8.3 J mol^{-1} K^{-1}

From ideal gas equation,

$PV = nRT$

$PV = \frac{m}{M}RT$ ($\because n = \frac{m}{M}$, m is the mass of the gas)

$m = \frac{PMV}{RT}$

$m = \frac{2 \times 10^5 \times 0.04 \times 4 \times 10^{-3}}{8.3 \times 300}$

$\therefore m = 1.29 \times 10^{-2}$ kg

We know,

$P = \frac{1}{3}\rho\bar{c}^2$

$\bar{c}^2 = \frac{3P}{\rho}$

$c_{rms} = \sqrt{\frac{3P}{m/V}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3 \times 2 \times 10^5 \times 0.04}{1.29 \times 10^{-2}}}$

$\therefore c_{rms} = 1363.98$ m/s

61. 2058 Q.No. 5 b **Find the rms speed of nitrogen at NTP. Density of N_2 = 1.29kg/m³ at NTP.**

Solution

Given,

At NTP, we have

Pressure (P) = 1.01×10^5 Nm^{-2}

Temperature (T) = 273 K

Density (ρ) = 1.29 kgm^{-3}

rms speed (c_{rms}) = ?

We know,

$P = \frac{1}{3}\rho\bar{c}^2_{rms}$

or, $1.01 \times 10^5 = \frac{1}{3} \times 1.29 \times \bar{c}^2_{rms}$

or, $\bar{c}^2_{rms} = 234883.72$

$\therefore c_{rms} = 484.65$ ms^{-1}

62. 2056 Q.No. 5 b **Two glass bulbs of equal volume are joined by a narrow tube and are filled with a gas at s.t.p. When one bulb is kept in melting ice and the other is placed in a hot bath, the new pressure is 877.6 mm of Hg. Calculate the temperature of the bath.**

Solution

Given,

Initial pressure of the gas at s.t.p. (P_1) = 760 mm Hg.

Final pressure of the gas in both bulbs (P_2) = 877.6 mm Hg.

Suppose V be the volume of each bulb and θ be the temperature of hot bath,

Mass of gas at s.t.p. in each bulb = $\frac{P_1V}{r \times 273}$

Total mass of gas at s.t.p. in both the bulbs = $\frac{2P_1V}{r \times 273}$ ... (i)

Final mass of gas in melting ice = $\frac{P_2V}{r \times 273}$ ... (ii)

Final mass of gas in hot bath = $\frac{P_2V}{r(273 + \theta)}$ ... (iii)

We know, total mass of the substance remains constant. So,

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Initial total mass of gas at s.t.p in both the bulbs = final total mass of the gas in both bulbs

$$\text{or, } \frac{2P_1V}{r \times 273} = \frac{P_2V}{r \times 273} + \frac{P_2V}{r(273+\theta)}$$

$$\text{or, } \frac{2P_1}{273} = \frac{P_2}{273} + \frac{P_2}{(273+\theta)}$$

$$\text{or, } \frac{2P_1}{273} - \frac{P_2}{273} = \frac{P_2}{(273+\theta)}$$

$$\text{or, } (273+\theta) = \frac{P_2}{\left(\frac{2P_1}{273} - \frac{P_2}{273}\right)} = \frac{P_2}{2P_1 - P_2} \times 273$$

$$\theta = \frac{P_2}{(2p_1 - p_2)} \times 273 - 273 = \frac{877.6}{2 \times 760 - 877.6} \times 273 - 273 = \frac{239584.8}{642.4} - 273 = 373 - 273$$

$\therefore \theta = 100°$ C

Temperature of hot bath is 100° C.

63. 2052 Q.No. 8 Two bulbs of equal volume are joined by a narrow tube and are filled with gas at STP. When one bulb is kept in melting ice and the other in boiling water, calculate the new pressure of the gas.

Solution

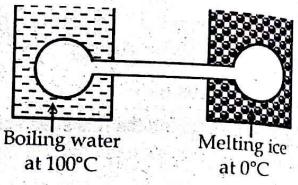
Given,

Standard pressure (P) = 760 mm Hg

Standard temperature (T) = 273 K

Let V be the volume of each bulb. Then

Mass of gas in each bulb (m) $= \frac{PV}{rT} = \frac{760V}{r \times 273}$



$\therefore$ Total mass of gas in both bulbs = $2m = 2 \times \frac{760V}{r \times 273}$ ... (i)

When one bulb is kept in melting ice and the other in boiling water, in this case, let P' be the new pressure. Then,

Mass of gas in the bulb which is kept in melting ice,

$m' = \frac{P'V}{r \times 273}$; where 273K is the temperature of the bulb which is kept in melting ice.

and Mass of gas in the bulb which is kept in boiling water, $m'' = \frac{P'V}{r \times 373}$ where 373K is the temperature of the bulb which is kept in boiling water.

$\therefore$ Total mass of gas is both bulbs = $m' + m''$

$$= \frac{P'V}{r \times 273} + \frac{P'V}{r \times 373} \quad \text{... (ii)}$$

In both the cases, the total mass of gas is constant in both bulbs. Therefore, from eqn. (i) and (ii), we have

$$2 \times \frac{760\,V}{r \times 273} = \frac{P'V}{r \times 273} + \frac{P'V}{r \times 373} = \frac{P'V}{r}\left(\frac{1}{273} + \frac{1}{373}\right)$$

$$\text{or, } \frac{2 \times 760}{273} = P'\left(\frac{373 + 273}{273 \times 373}\right)$$

$\therefore$ P' = 877.65 mm of Hg which is the required new pressure of the gas.

❑❑❑

Chapter 4: Hygrometry

Short Answer Questions

1. 2076 Set B Q.No. 2b 2071 Supp Q.No. 2b Why does it take longer time to dry wet clothes in rainy summer than in sunny winter? [2]

Ans: The evaporation depends upon the humidity of air. The process of drying clothes depends upon the evaporation process and hence, humidity of air. In rainy summer, there is more humidity in air than the sunny winter. So, clothes dry faster in sunny winter than rainy summer.

2. 2076 Set C Q.No. 2a 2073 Set C Q.No. 2b Dews are formed in the clear night but not in the cloudy night. Why? [2]

Ans: In clear nights, the atmosphere looses heat by radiation and cools below the dew point temperature. When the temperature falls below the dew point temperature, dews are formed. However, in the cloudy night, cloud acts as blankets. As a result, the earth surface as well as the atmosphere cannot loose heat and can't get cooled up to dew point. Hence, dews are formed in clear nights but not in cloudy night.

3. 2075 Set A Q.No. 2a Explain why we can see our breath in winter but not in summer? [2]

Ans: The exhaled gases consist of water vapour with other wastes. The winter temperature is very less than the summer. When the gases come out from mouth during breath, they are exposed to lower temperature in winter. The vapour condenses to water. So, white trace is seen more in winter. In summer, less vapour condenses to water and the white trace is not seen during breath.

4. 2075 Set B Q.No. 2c Point out the differences between saturated and unsaturated vapour pressures with examples. [2]

Ans: Difference between saturated and unsaturated vapours are as follows:

Unsaturated Vapour	Saturated Vapour
a. If a space contains less than the maximum amount of vapour that it can hold at that temperature, the vapour in the space is unsaturated vapour.	a. If a space contains the maximum amount of vapour that it can hold at that temperature, vapour in the space is called saturated vapour.
b. Vapour in closed space but not in contact with liquid can be unsaturated.	b. Vapour in a closed space and in contact with liquid is saturated vapour.
c. Unsaturated vapour obeys gas laws approximately.	c. Saturated vapour doesn't obey gas laws.
d. Unsaturated vapour changes into saturated by decreasing volume or temperature.	d. Saturated vapour is independent of temperature.

5. 2074 Supp. Q.No. 2b Why do we feel uncomfortable during very high relative humidity of air? [2]

Ans: We feel uncomfortable during very high relative humidity of air because at high relative humidity evaporation of water from our body is very slow.

6. 2074 Set A Q.No. 2b 2068 Q.No. 2 b Explain why dews are formed in clear night but not in cloudy night. [2]

Ans: Please refer to 2076 Set C Q.No. 2a

7. 2074 Set B Q.No. 2b Point out the differences between saturated and unsaturated vapour pressures. [2]

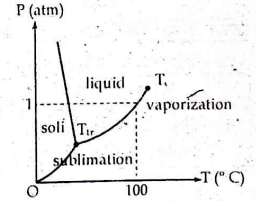
Ans: Please refer to 2075 Set B Q.No. 2c

8. 2073 Set D Q.No. 2b Dews are formed in the early morning hours, why? [2]

Ans: During the night, the surface of earth loses heat by radiation and gets cooled faster than the atmosphere. Beside this, the layers of the air, which are close to earth surface, are colder than the layers farther from earth surface. When the temperature of earth surface falls below dew point (i.e., saturated condition of atmosphere) at the early morning hours, the water vapour present in atmosphere condense in the form of tiny water drops on the leaves or grass. These tiny water drops are called dews.

9. 2072 Set C Q.No. 2a What is triple point? Show that there exist a single value of triple point. [2]

Ans: Triple point is the point in phase (P-T) diagram which represents a particular pressure and temperature at which the solid, liquid and vapour states of the substance can co-exist. In the diagram, the point T_{tr} is called the triple point. This point is taken as standard point in modern thermometry. For example, the values of pressure and temperature corresponding to triple point for water are 0.0061×10^5Pa and 273.16 K.



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10. 2072 Set D Q.No. 2a What is the difference between saturated and unsaturated vapour?

Please refer to 2075 Set B Q.No. 2c

11. 2071 Set D Q.No. 2 b Why are dews formed in the clear nights but not in the cloudy ones? Explain.

Please refer to 2076 Set C Q.No. 2a

12. 2070 Supp (Set B) Q.No. 2 b A weather report said relative humidity of 80%. Explain its meaning

Relative humidity is defined as the ratio of mass of water vapour present in a given volume of air to the mass of water vapour required to saturate the same volume at same temperature. i.e.,

$$\text{R.H.} = \frac{\text{Mass of water vapour present in a given volume of air at certain temperature}}{\text{Mass of water vapour required saturting the same volume of air at same temperature}}$$

So, the relative humidity of 80% means that the mass of water vapour present in air is 80 parts of mass of water vapour required to saturate the volume of air on that day or time.

13. 2070 Set D Q.No. 2 c 2067 Supp Q.No. 2 b Define triple point and write its significance.

Please refer to 2072 Set C Q.No. 2a

14. 2069 (Set A) Old Q. No. 4c What is meant by triple point?

Please refer to 2072 Set C Q.No. 2a

15. 2068 Old Q.No. 4 c What do you mean by triple point of water?

Please refer to 2072 Set C Q.No. 2a

16. 2067 Q.No. 2 b Why is the triple point, instead of the melting point or boiling point of water, taken as standard fixed point in modern thermometry?

The triple point of water is unique i.e. it occurs at a particular pressure and temperature. its values are 0.0061×10^5Pa and 273.16 K. On the other hand, melting point of ice and boiling point of water change due to pressure or due to the presence of impurities in water. Hence, melting point of ice and boiling point of water cannot be taken as standard fixed points and triple point of water is chosen as standard fixed point in modern thermometry.

17. 2066 Old Q. No. 4 b What is relative humidity? Why is its value more near the sea shore?

Relative humidity is defined as the ratio of mass of water vapour present in a given volume of air to the mass of water vapour required to saturate the same volume at same temperature. i.e.,

$$\text{R.H.} = \frac{\text{Mass of water vapour present in a given volume of air at certain temperature}}{\text{Mass of water vapour required to saturtate the same volume of air at same temperature}}$$

The amount of water vapour present in the atmosphere near the sea water is comparatively more. Because of this, the value of relative humidity is more near sea water.

18. 2064, Q.No. 4(b) Why are you more uncomfortable on a hot day when the humidity is high than when it is cooled?

In a dry hot day, the atmosphere contains less amount of water vapour (i.e. R.H. is low). If so, evaporation of water from our body to the atmosphere can take place easily. As a result, our body can regulate heat and water properly and we feel easy. However, in a humid day at the same temperature, atmosphere contains more amount water vapour (i.e. R.H. is more). Due to this, evaporation of water from our body to the atmosphere cannot take place properly. As a result, our body cannot regulate heat and water properly and we feel uncomfortable. Hence, a humid day is more uncomfortable than a dry hot day at the same temperature.

19. 2063 Q.No. 4 b What is relative humidity? Why is its value more near sea water?

Please refer to 2066 Old Q. No. 4 b

20. 2062 Q.No. 4 b Explain the difference between saturated and unsaturated vapour pressure.

Please refer to 2075 Set B Q.No. 2c

21. 2061 Q.No. 4 a Distinguish between a gas and vapour.

The difference between gas and vapour are as follows:

Gas	Vapour
a. The intermolecular forces between gas molecules are negligible.	a. The intermolecular force is more significant.
b. Gases cannot be liquefied whatever pressure applied above its critical temperature.	b. Vapour can easily liquefied by applying pressure.
c. In gases, there is no hydrogen bonding present.	c. In water vapour, hydrogen bonding is present.

22. 2059 Q.No. 4 b How are dews formed? [2]

Please refer to 2073 Set D Q.No. 2b

23. 2058 Q.No. 4 b What are saturated and unsaturated vapours? [2]

Please refer to 2075 Set B Q.No. 2c

24. 2057 Q.No. 4 c 2051 Q.No. 7 e Why are dews formed in the early morning hours? [2]

Please refer to 2073 Set D Q.No. 2b

25. 2056 Q.No. 4 c What do you mean by Triple Point? [2]

Please refer to 2072 Set C Q.No. 2a

26. 2055 Q.No. 7 b Distinguish between saturated and unsaturated vapour pressure. [2]

Please refer to 2075 Set B Q.No. 2c

27. 2054 Q.No. 7 a Why is the triple point of water chosen as a standard fixed point in modern thermometry? [2]

Please refer to 2067 Q.No. 2 b

28. 2054 Q.No. 10 a Compare the properties of saturated and unsaturated vapours. [2]

Please refer to 2075 Set B Q.No. 2c

29. 2053 Q.No. 7 f Define Triple point. [2]

Please refer to 2072 Set C Q.No. 2a

30. 2052 Q.No. 7 f Define dew point. [2]

It is defined as the temperature at which the water vapour present in certain volume of air become just sufficient to saturate it. At this temperature, the vapour pressure is equal to the saturated vapour pressure. Therefore,

$$\text{R.H.} = \frac{\text{Saturated vapour pressure at dew point}}{\text{Saturated vapour pressure at room temperature}}$$

Below the due point temperature, the water vapour condenses in the tiny water drop. These drops are called dews.

Numerical Problems

31. 2052 Q.No. 8 OR At certain day the air temperature in a room is 17.7°C and the dew point 5.3°C. Find the relative humidity (SVP at 5°, 6°, 17° and 18°C are 0.654 cm, 0.705 cm, 1.442 cm, 1.546 cm respectively).

Solution

Given,

Room temperature = 17.7°C, Dew point = 5.3°C

SVP at 17°C = 1.442 cm

SVP at 18°C = 1.546 cm

Relative humidity = ?

Differences of vapour pressure for 1°C = 1.546 – 1.442 = 0.104 cm

Difference of vapour pressure for 0.7°C = 0.7 × 0.104 = 0.0728 cm

SVP at 5°C = 0.654 cm

SVP at 6°C = 0.705 cm

Difference of vapour pressure for 1°C = 0.705 – 0.654 = 0.051 cm

Difference of vapour pressure for 0.3°C = 0.3 × 0.051 = 0.0153 cm

SVP at 5.3°C = 0.654 + 0.0153 = 0.6693 cm

SVP at 17.7°C = 1.442 + 0.0728 = 1.5148 cm

$$\therefore \text{Relative humidity} = \frac{\text{SVP at dew point}}{\text{SVP at room temperature}} \times 100\% = \frac{0.6693}{1.5148} \times 100\% = 44.18\%$$

□□□

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Chapter 5: Transfer of Heat

Short Answer Questions

1. **2076 Set C Q.No. 2b Cooking utensils are blackened at the bottom and polished on the upper surface. Explain, why?** [2]

The black bodies are good absorber of heat and polished shining bodies are good heat reflectors. Cooking utensils are blackened at the bottom because it absorbs heat while cooking and upper surface is polished to reflect heat to food, i.e., to avoid loss of heat. So, cooking utensils are blackened at the bottom and polished on the upper surface.

2. **2075 GIE Q.No. 2b How can water be boiled in a thin paper cup?** [2]

The boiling of water requires 100°C whereas the burning of paper requires its ignition temperature which is greater than 100°C. When water is heated in a paper cup, heat goes to the paper as well as water, so the temperature of water as well as the cup rise to 100°C first. Then water starts to boil. At that time, the ignition temperature of paper is still not arrived, so it does not burn. As the process of boiling goes on, a condition is reached when water finishes off. Then only all the heat is directed to paper, raising its temperature heavily and ultimately burning it.

3. **2075 GIE Q.No. 2c 2069 (Set A) Q. No. 2c Why are the polar regions much cooler than the equatorial regions despite the fact that the polar regions are periodically tilted towards the sun?** [2]

The heat received by the surface of earth is given by $E = \frac{I}{r^2}\ \text{Cos}\ \theta$, where , θ is the angle between incident ray and normal drawn on the surface, r be the distance between the sun and earth, I be the intensity of heat radiation. The heat rays of sun fall obliquely at the Polar Regions and straight at the equator. Due to this reason, the polar regions of the earth are much cooler than equatorial regions even they are tilted toward the sun periodically.

4. **2075 Set B Q.No. 2b What is the physical meaning of emissivity? Write its unit.** [2]

Emissivity of a body is the ratio of the emissive power of the body to the emissive power of the perfectly black body at the same temperature, i.e.

$$\text{Emissivity} = \frac{\text{Emissive power of a body}}{\text{Emissive power of perfectly black body}}$$

It is the ratio of same quantity. So, it has no unit. Its value ranges 0 to 1. Emissivity = 0 indicate the perfectly shining and 1 means the body is perfectly black body. Physically it gives the closeness of the body to the perfectly black body.

5. **2072 Supp Q.No. 2b Air is a bad conductor of heat. Why do you feel cool without cloth in your body?** [2]

Air is bad conductor of heat, when we wear cloth; it stops the transformation of air from outside to inside and vice versa. i.e., the air trapped inside the cloth does not conduct heat to outside. If we do not wear cloth, there is no layer of air trapped around our body. Then the heat conducts from body to surroundings and we feel cool. So, even air is bad conductor of heat, we feel cool without cloth in our body.

6. **2072 Supp Q.No. 2c 2069 (Set B) Q. No. 2c 2052 Q.No. 7a Although aluminium is good conductor of heat, how can aluminium foil with shining surface can be used to keep food hot for a long time?** [2]

The reflection of heat radiation depends up on the shining properties of the material. Aluminium foil is too much shiny, and reflects most of the heat radiations falling on it. The reflected heat from the aluminium surface again goes to the food materials. So loss of heat from the food is prevented and the food would remain warm for a long time. The shiny surface reflects heat radiations falling on it, thus, prevents the heat from food being lost and keeps the food warm from a long time.

7. **2070 Set C Q.No. 2 b Birds often swell their feathers in winter. Why?** [2]

Since the rate of flow of heat from a body is given by

$$\frac{Q}{t} = k\,A\,\frac{d\theta}{dt} \Rightarrow \frac{Q}{t} \propto k,$$ for constant A, $d\theta$ and dx.

A layer of air is trapped between the feathers, which is bad conductor of heat and low value of thermal conductivity (K). It prevents the flow of heat from bird's body to the surrounding. Hence, birds often swell their feathers in winter.

8. **2070 Set D Q.No. 2 b Animals curl into a ball, when they feel very cold. Why?** [2]

The amount of heat radiated by a body is directly proportional to surface area i.e., $P = A\sigma T^4$, where, A be the area of surface of the body. If the surface area is reduced, the heat radiated is also reduced. Since, spherical body has minimum surface area for a given volume, to reduce the heat radiation from their bodies by reducing surface area, the animals curl into ball in winter.

9. **2066 Q. No. 2 a 2060 Q.No. 4 a Why are two thin blankets warmer than a single blanket of double the thickness ?** [2]

Since, the rate of flow of heat from a body is given by

$$\frac{Q}{t} = k\,A\,\frac{d\theta}{dt} \Rightarrow \frac{Q}{t} \propto k,$$ for constant A, $d\theta$ and dx.

A layer of air is trapped between two blankets, which is bad conductor of heat. In other words, the coefficient of thermal conductivity (k) of air is very small in comparison with that of blanket. It prevents the flow of heat from our body to the surrounding. Hence, two thinner blankets are warmer than single blanket of double thickness.

10. **2059 Q.No. 4 c During the winter, the animals curl into a ball. Explain why?** [2]

Please refer to 2070 Set D Q.No. 2 b

11. **2055 Q.No. 7 c Why are good absorbers always good emitters?** [2]

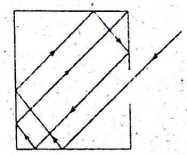
Temperature is the effect of heat that means when a body absorbs heat its temperature rises. From Stefan's law of radiation, amount of heat emitted per unit time per unit surface area is directly proportional to the fourth power of absolute temperature,

i.e. $E \propto T^4$.

It means hotter body can emit greater amount of heat than relatively cold body. Hence, good absorbers are always good emitters.

12. **2054 Q.No. 7 f What is a black body? How is it realized in practice?** [2]

A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it. A perfectly blackbody can't be realized in practice. The nearest approach to a perfectly blackbody is a surface coated with lamp black, platinum black, such a surface absorbs 96% to 98% of the incident radiation.



If a small hole is made in the lid of a closed empty tin, the hole looks almost black although the shining tin is a good reflector because the light which enters through it is reflected many times round the walls of the tin, before it passes through the hole again as shown in the figure. At each reflection, absorption occurs.

13. **2053 Q.No. 7 a Hot water pipes used in the room are painted black. Why?** [2]

When hot water flows through a pipe, the pipe also absorbs some heat. This absorbed heat is lost by the pipe in the form of radiation. We know that, good absorbers are good radiators also. So, the black painted pipe used in a room radiates more heat than the pipe painted with other colors and hence, keeps the room warm.

14. **2052 Q.No. 7 b Metal knob of door is colder than wooden parts at the same temperature. Why?** [2]

Metal is good conductor of heat whereas wood is bad conductor of heat. When we touch the metal knob of door, it conducts heat from our body consequently there is loss of heat from our body and we feel cold. However, when wood is touched, wood does not accept heat from the body, because it is not a conductor. So, the body heat remains preserved and we don not feel cool.

15. **2051 Q.No. 7 a How can water be boiled in a paper cup?** [2]

Please refer to 2075 GIE Q.No. 2b

Long Answer Questions

16. **2075 Set B Q.No. 6a On what factors the thermal conductivity of a solid depends? Describe, Searle's experimental method to determine the thermal conductivity of a solid.** [4]

Thermal Conductivity: The thermal conductivity of material of a body is defined as the amount of heat that flows in one second across the opposite faces of a unit cube, whose opposite faces are kept at a temperature difference of 1K. The coefficient of thermal conductivity is represented by k. Its SI unit

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is $Wm^{-1} k^{-1}$.

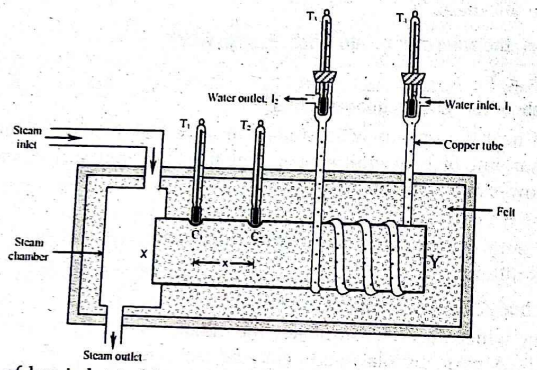
Mathematically, $\frac{Q}{t} = k \frac{A(\theta_1-\theta_2)}{x}$

When $A = 1m^2$, $\theta_1-\theta_2 = 1K$ and $x = 1$ m, then $k = \frac{Q}{t}$

Thermal conductivity depends upon nature of material.

Experimental determination of thermal conductivity (k) of solid bar(*Searle's method*)

The apparatus required for the determination of thermal conductivity of a metal bar is as shown in figure. XY is the metal bar whose thermal conductivity is to be determined. One end of bar is enclosed within steam chamber and a copper wire wound near the other end steam is passed through steam chamber and water flows through copper tube at constant rate. T_1 and T_2 are thermometers to measure the temperature of bar at a small distance x apart. Thermometer T_3 and T_4 measure the temperature of outgoing and incoming water. To ensure good thermal contact, small amount of mercury is placed into cavity.



As the end X of bar is heated by passing steam through steam chamber, so that heat is conducted from X to end Y and circulating water is warmed up. At steady state, the reading of all thermometers becomes constant. Heat lost from the surface of bar is neglected because the bar is lagged by some non-conducting material.

Let, θ_1, θ_2, θ_3 and θ_4 are the temperatures recorded by thermometers T_1, T_2, T_3 and T_4 respectively in steady state.

Area of cross section = A

Distance between cavities = x

Time for which the heat is considered = t

Mass of water that flows through the copper tube in time t = m

The amount of heat flows through metal bar in time t is,

$$Q = \frac{kA(\theta_1-\theta_2)t}{x} \quad \ldots (i)$$

where k is the thermal conductivity of metal bar and $\theta_1 - \theta_2$ is temperature difference between two cavities.

The temperature of water flowing through the copper tube rises from θ_4 to θ_3. So the amount of heat absorbed by water in time t is

$$Q = mS(\theta_3 - \theta_4) \quad \ldots (ii)$$

From the principle of calorimetry,

Heat lost = heat gain

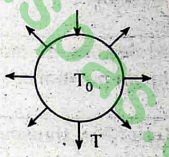
$$\frac{kA(\theta_1-\theta_2)t}{x} = mS(\theta_3-\theta_4)$$

$$k = \frac{mS(\theta_3-\theta_4)x}{A(\theta_1-\theta_2)t}$$

Which is the required expression of thermal conductivity of a metal bar. As all the quantities in above expression (right hand side) are known, then k can be experimentally measured.

17. 2072 Set D Q.No. 6c 2057 Q.No. 5 a OR **State and explain Stefan's law of black body radiation. Can a perfect black body be realized in practice?** [4]

Ans **Black body:** A perfect black body is one which absorbs heat radiation of all the wavelengths, which fall on it. It does not reflect or transmit radiations. Thus, black body is a good absorber.



Stefan's law of black body radiation: It is stated as, "the rate of the radiant energy per unit area from the surface of perfect blackbody is directly proportional to fourth power of absolute temperature."

i.e. $E \propto T^4 \Rightarrow E = \sigma T^4$ ... (i)

Where σ is Stafen's constant. The value of σ is 5.67×10^{-8} J s^{-1} $m^{-2}K^{-4}$. If the body isn't perfectly black and it has emissivity 'e' then Stafen's law becomes.

$E = e\sigma T^4$ ... (ii) where $0 < e < 1$

For perfect black body, e = 1. If A be the surface area of the blackbody, then the radiant power is given by

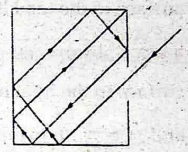
$P = A\sigma eT^4$ ... (iii)

Now, suppose the black body at absolute temperature (T) is kept in an enclosure at absolute temperature T_0, then the net rate of lose or gain of radiant energy per unit area is given by

$E = \sigma e (T - T_0)^4$... (iv) Where $T > T_0$

If A be the surface area of the body then the net radiant energy transfer per second by the body is given by

$P = A\sigma e (T - T_0)^4$ where $T > T_0$.



Black body: A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it. A perfectly blackbody can't be realized in practice. The nearest approach to a perfectly blackbody is a surface coated with lamp black, platinum black, such a surface absorbs 96% to 98% of the incident radiation.

If a small hole is made in the lid of a closed empty tin, the hole looks almost black although the shining tin is a good reflector because the light which enters through it is reflected many times round the walls of the tin, before it passes through the hole again as shown in the figure. At each reflection, absorption occurs.

18. 2072 Set E Q.No. 6c **State and explain Stefan's law of black body radiation.** [4]

Ans Please refer to 2072 Set D Q.No. 6c

19. 2071 Supp Q.No. 6b **State and explain Stefan's black body radiation. Explain how a black body can be realized in practice.** [4]

Ans Please refer to 2072 Set D Q.No. 6c

20. 2071 Set C Q.No. 6 b **Define the coefficient of thermal conductivity. Describe, with necessary theory, an experiment for its determination in the laboratory.** [4]

Ans Please refer to 2075 Set B Q.No. 6a

21. 2070 Supp (Set A) Q.No. 6 b **Define coefficient of thermal conductivity. Describe Searl's method of determining thermal conductivity.** [4]

Ans Please refer to 2075 Set B Q.No. 6a

22. 2069 Old (Set B) Q. No. 5a OR **Define thermal conductivity. Describe Searle's method of determination of thermal conductivity of good conductor.** [1+4]

Ans Please refer to 2075 Set B Q.No. 6a

23. 2068 Old Q.No. 5 a or **Discuss the methods of heat transmission. Define reflection, transmission and absorption coefficients of heat radiation and relate them.** [3+2=5]

Ans The methods of heat transmission are discussed below:

Conduction	Convection	Radiation
Material medium is essential.	Material medium is also essential.	Material medium is not essential.
Transfer of heat can be in any direction.	Heat is transmitted only vertically upward.	Transfer of heat takes place in all directions in straight line.

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Transfer of heat takes place at slow rate.	This process is slower than radiation and faster than conduction.	Transfer of heat is fastest (at a velocity of 3×10^8 ms^{-1})
Molecules do not leave their mean positions.	Molecules move from one place to another place.	Electromagnetic waves move from one place to another place.

Reflection, transmission and absorption coefficients of heat radiation: When heat radiation falls on a surface, a part of it is absorbed, a part of it is reflected and a part is transmitted. Suppose Q is the heat radiation incident on a surface, R is the reflected, A is the absorbed, and T is the transmitted part of heat through the surface. Since total energy reflected, absorbed and transmitted per unit area per second is equal to the energy falling on the surface, then

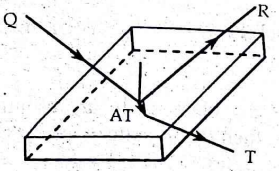
$R + A + T = Q$

Dividing both sides by Q, we get

$$\frac{R}{Q}+\frac{A}{Q}+\frac{T}{Q}=1$$

or, $r + a + t = 1$ ... (i)

where,



$r = \frac{R}{Q}$, reflectance or reflection coefficient and defined as the ratio of heat energy reflected from the surface to the total heat energy incident on it at the same time.

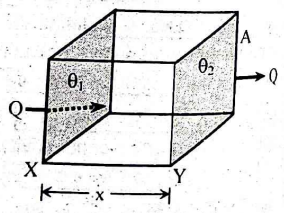
$a = \frac{A}{Q}$, absorptance or absorption coefficient and defined as the ratio of heat energy absorbed by the surface to the total heat energy incident on it in same time.

and $t = \frac{T}{Q}$, transmittance or transmission coefficient of the surface and defined as the ratio of heat energy transmitted by the surface to the total heat energy incident on it in same time.

The reflectance r, absorptance a and transmittance t depend on the nature of the surface of the body and on the wavelength of incident radiation but not on the nature of the material of the body.

24. 2067 Q.No. 6 b What do you mean by thermal conductivity of a substance? Deduce an expression for the thermal conductivity of a good conductor in steady state. [4]

Consider a section XY of a long metal bar, whose side length be x and area of cross-section A. When it's opposite faces are maintained at temperatures θ_1 and θ_2 such that $\theta_1 > \theta_2$, heat flows from left to right as shown in Fig. Experimentally, it is found that under steady state condition, the quantity of heat Q flowing through the section of the bar is



i. directly proportional to the cross-section area A of the face.
$Q \propto A$ ... (i)

ii. directly proportional to the temperature difference $(\theta_1 - \theta_2)$ between the two faces.
$Q \propto (\theta_1 - \theta_2)$ ... (ii)

iii. directly proportional to time t for which heat flows.
$Q \propto t$ ... (iii)

iv. inversely proportional to the perpendicular distance x between hot and cold faces.
$$Q \propto \frac{1}{x} \quad \text{... (iv)}$$

Combining Eqs. (i), (ii), (iii) and (iv), we get

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$\therefore Q = \frac{kA(\theta_1 - \theta_2)t}{x} \quad \text{... (v)}$$

where k is a constant called coefficient of thermal conduction or simply thermal conductivity. It's value depends upon material of the bar.

Again,

$$k = \frac{Qx}{A(\theta_1 - \theta_2)t}$$

If $A = 1\ m^2$, $\theta_1 - \theta_2 = 1°C$ or 1 K, $t = 1$ sec and $x = 1$ m, then $Q = k$.

Thus, thermal conductivity is defined as the amount of heat flowing per second across the opposite faces of a cube of side 1m maintained at a unit temperature difference. It's unit is W m^{-1} K^{-1} in SI-units and dimensional formula is $[M\,L\,T^{-3}\,K^{-1}]$.

25. 2065 Q. No. 5 a Or Define thermal conductivity. Write its units and dimensions. Describe Searle's method of determination of thermal conductivity of good conductor. [1+1+3]

Please refer to 2075 Set B Q.No. 6a

26. 2063 Q.No. 5 a OR Describe Searle's method of determination of thermal conductivity of a good conductor.

Please refer to 2075 Set B Q.No. 6a

27. 2061 Q.No. 5 a Define thermal conductivity. Describe with the necessary theory on experiment to determine the thermal conductivity of a metal bar. [1+4]

Please refer to 2075 Set B Q.No. 6a

28. 2058 Q.No. 5 a OR What is a black body? Derive Stefan's law of black body radiation. [5]

Please refer to 2072 Set D Q.No. 6c

29. 2056 Q.No. 5 OR What do you mean by perfectly black body. State and explain Stefan's law of black body radiation.

Please refer to 2072 Set D Q.No. 6c

30. 2051 Q.No. 9 Define thermal conductivity. Derive an expression for thermal conductivity of a good conductor in steady state.

Please refer to 2067 Q.No. 6 b

31. 2050 Q.No. 9 What is radiation and how does this mode of heat transfer differ from conduction and convection?

It is the process in which heat is transmitted from one place to another directly without intervening medium. Heat radiation comes to the earth from the sun through this process. Heat radiations can pass through the vacuum also. Their properties are similar to light radiations. Heat radiations also form a part of the electro magnetic spectrum.

Difference between Conduction, Convection and Radiation are:

There are following differences between conduction, convection and radiation.

Conduction	Convection	Radiation
Material medium is essential.	Material medium is also essential.	Material medium is not essential.
Transfer of heat can be in any direction.	Heat is transmitted only vertically upward.	Transfer of heat takes place in all directions in straight line.
Transfer of heat takes place at slow rate.	This process is slower than radiation and faster than conduction.	Transfer of heat is fastest (at a velocity of 3×10^8 ms^{-1})
Molecules do not leave their mean positions.	Molecules move from one place to another place.	Electromagnetic waves move from one place to another place.

Numerical Problems

32. 2076 Set C Q.No. 10b A bar 0.2m in length and 2.5 cm^2 in cross section is ideally lagged. One end is maintained at 100° C and the other end is maintained at 0° C by immersing in melting ice. Calculate the mass of ice melt in one hour. Thermal conductivity of the material of the bar is 4×10^{-2} Wm^{-1} K^{-1}. [4]

Solution

Given,

Length of bar (l) = 0.2 m

Cross sectional area (A) = 2.5×10^{-4} m^2

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Higher temperature (θ_1) = 100°C = 373k
Lower temperature (θ_2) = 0°C = 273k
Thermal conductivity of bar (K) = 4×10^{-2} $Wm^{-1} K^{-1}$
Latent heat of fusion of ice (L_i) = 3.36×10^5 J Kg^{-1}
Time taken (t) = 1 hour = 60 × 60 = 3600 sec
Amount of ice melt (m) = ?
We have,
Rate of heat lost,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \qquad [\because Q = mL_i]$$

or, $\frac{m(3.36 \times 10^5)}{3600} = \frac{4 \times 10^{-2} \times 2.5 \times 10^{-4} (373 - 273)}{0.2}$

or, $\frac{m(3.36 \times 10^5)}{3600} = 5 \times 10^{-3}$

$\therefore m = \frac{3600 \times 5 \times 10^{-3}}{3600 \times 10^5}$

$= 5.36 \times 10^{-5}$ kg

33. 2075 Set A Q.No. 10c The Sun is a black body of surface temperature about 6000K. If Sun's radius is 7×10^8m, calculate the energy per second radiated from its surface. The earth is about 1.5×10^{11} m from the Sun. Assuming all the radiation from the Sun falls on the surface of sphere of this radius, estimate the energy per second per meter² received by the earth. [4]

Solution

Given,
Radius of sun (r) = 7×10^8 m
Temperature of sun (T) = 6000 K
Power (P) = ?
Stefan's constant (σ) = 5.7×10^{-8} $wm^{-2} K^{-4}$
We know,

$P = \sigma AT^4$
$= \sigma 4\pi r^2 T^4$
$= 5.7 \times 10^{-8} \times 4\pi \times (7 \times 10^8)^2 \times 6000^4$
$= 4.55 \times 10^{26}$ watt

Again, Energy per second per m² received by earth

$= \frac{P}{4\pi R^2} = \frac{4.55 \times 10^{26}}{4\pi (1.5 \times 10^{11})^2}$ [R = Distance both earth and sun = 1.5×10^{11}m]

$= 1609$ Wm^{-2}

34. 2074 Supp. Q.No. 10b A sphere of radius 2.00 cm with a black surface is cooled and then suspended in a large evacuated enclosure with black walls maintained at 27°C. If the rate of change of thermal energy of sphere is 1.85JS⁻¹ when its temperature is –73°C, calculate the value of Stefan's Constant. [4]

Solution

Given,
Radius of sphere = 2.00 cm = 2×10^{-2}m
Temperature of sphere (T) = 27°C = 300 K
Power (P) = 1.85 JS^{-1}
Temperature of enclosure (T_0) = –73°C = 200 K
Stefan's constant (σ) = ?
We have,

$P = \sigma A (T^4 - T_0^4)$
or, $1.85 = \sigma \times 4 \times (2 \times 10^{-2})^2 (300^4 - 200^4)$
$\therefore \sigma = 5.66 \times 10^{-8}$ $Wm^{-2} K^{-4}$

35. 2074 Set A Q.No. 10b Estimate the power loss through unit area from a perfectly black body at 327°C to the surrounding environment at 27°C. [4]

Solution

Given,
Area (A) = 1m²
Temperature of black body (T_1) = 327°C = 600 K
Temperature of surrounding (T_0) = 27°C = 300 K
Power loss (P) = ?
We know, from Stefan's law,

$P = e\sigma A (T_1^4 - T_0^4)$; [$\sigma = 5.67 \times 10^{-8}$ $Wm^{-2} K^{-4}$ is Stefan's constant and e = 1 is emissivity of perfectly black body]

or, $P = 1 \times 5.67 \times 10^{-8} \times 1 \times (600^4 - 300^4)$
$\therefore P = 6889.05$ W

36. 2074 Set B Q.No. 10c A spherical blackbody of radius 5 cm has its temperature 127°C and its emissivity is 0.6. Calculate its radiant power. [4]

Solution

Given,
Radius of spherical black-body (R) = 5 cm = 5×10^{-2} m
Temperature of black body (T_1) = 127°C = 400 K
Emissivity (e) = 0.6
Radiant power (P) = ?
We know, from Stefan's law,

$P = \sigma Ae T^4$; [$\sigma = 5.67 \times 10^{-8}$ $Wm^{-2} k^{-4}$ is Stefan's constant]
$= 5.67 \times 10^{-8} \times 4\pi \times (5 \times 10^{-2})^2 \times 0.6 \times 400^4$ [$\because A = 4\pi R^2$]
$= 27.34$ W

37. 2073 Supp Q.No. 10b Estimate the radiant power loss from a human body at a temperature 38.5°C to the environment at 0°C if the surface area of the body is 1.5 m² and its emissivity is 0.6. [4]

Solution

Given,
Temperature of human body (T_1) = 38.5° C = 311.5 K
Temperature of environment (T_0) = 0° C = 273 K
Surface area of the body (A) = 1.5 m²
Emissivity (e) = 0.6
Radiant power loss (P) = ?
We know, from stefan's law,

$P = \sigma Ae (T_1^4 - T_0^4)$; σ = Stefan's constant = 5.67×10^{-8} $Wm^{-2} k^{-4}$
or, $P = 5.67 \times 10^{-8} \times 1.5 \times 0.6 \times (311.5^4 - 273^4)$
$\therefore P = 197$ W

38. 2073 Set C Q.No. 10b A pot with a steel bottom 8.5 mm thick rest on a hot stove. The area of the bottom of the pot is 0.15m². The water inside the pot is at 100°C and 390 gm of water is evaporated every 3 minute. Find the temperature of lower surface of the pot which is in contact with the stove. [k = 50.2 W/mK, $L_v = 2256 \times 10^3$ J/kg] [4]

Solution

Given,
Thickness of steep pot (l) = 8.5 mm = 8.5×10^{-3} m
Area of bottom of pot (A) = 0.15 m²
Temperature of water (θ_1) = 100°C
Temperature of bottom of pot (θ_2) = ?
Time period (t) = 3 min = 3 × 60 = 180 sec
Mass of water (m) = 390 gm = 0.39 kg

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Thermal conductivity of steel (k) = 50.2 W/mK
Latent heat of vaporization of water (L_v) = 2256×10^3 J/Kg
We have,

$$Q = \frac{kA(\theta_2 - \theta_1)\,t}{l}$$

or, $mL_v = \frac{kA(\theta_2 - \theta_1)\,t}{l}$

or, $0.39 \times 2256 \times 10^3 = \frac{50.2 \times 0.15\,(\theta_2 - 100) \times 180}{8.5 \times 10^{-3}}$

or, $\theta_2 - 100 = 5.52$

or, $\theta_2 = 105.52°C$

39. **2073 Set D Q.No. 10b** **A rod 1.3 m long consists of a 0.8 m length of aluminium joined end to end to a 0.5 m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece is maintained at 20°C. No heat is lost through the sides of the rod. At a steady state, what is the temperature at the point where the two metals are joined? [K_{al} = 205 W/mK, K_b = 110 w/mk]** [5]

Solution

Given,
Length of aluminum (l_1) = 0.8 m
Length of brass (l_2) = 0.5 m
Temperature of aluminum (θ_1) = 150 °C
Temperature of brass (θ_2) = 20°C
K_1 = 205 $Wm^{-1}K^{-1}$
K_2 = 109 $Wm^{-1}K^{-1}$
Temperature at joined (θ) = ?
We have at steady state,

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_1$$

$$\frac{k_1 A(150 - \theta)}{0.8} = \frac{k_2 A(\theta - 20)}{0.5}$$

or, $\frac{205 \times 150 - 205\,\theta}{0.8} = \frac{109 \times \theta - 109 \times 20}{0.5}$

or, $\theta = \frac{17119}{189.7} = 90.2°C$

40. **2072 Set C Q.No. 10c** **A rod 1.300 m long consists of a 0.800 m length of aluminium joined end to end to a 0.500 m length of brass. The free end of the aluminium section is maintained at 150°C and the free end of the brass piece is maintained at 20°C. No heat is lost through the side of the rod. At steady state, what is the temperature of the point when two metals are joined.**

Please refer to 2073 Set D Q.No. 10b

41. **2071 Set D Q.No. 10 b** **The element of an electric fire with an output of 1.5 KW is a cylinder of 0.3 m long and 0.04 m in radius. Calculate its temperature if it behaves as a black body.** [4]

Solution

Given,
Power of electric fire (P) = 1.5 KW = 1500 W
Length of cylinder (l) = 0.3 m
Radius of cylinder (r) = 0.04 m
Stefan's constant (σ) = 5.67×10^{-8} W m^{-2} k^{-4}
Temperature of cylinder (T) = ?
We have

$P = \sigma AT^4$

or, $1500 = 5.67 \times 10^{-8} \times 2\pi \times r \times l \times T^4$

or, $1500 = 5.67 \times 10^{-8} \times 2\pi \times 0.3 \times 0.04 \times T^4$

or, $T^4 = \frac{1500}{5.67 \times 10^{-8} \times 2\pi \times 0.3 \times 0.04} = 3.51 \times 10^{11}$

or, $T = (3.51 \times 10^{11})^{1/4} = 769.64$ k

42. **2070 Supp (Set B) Q.No. 10 b** **A bar 0.2 m in length and of cross-sectional area $2.5 \times 10^{-4} m^2$ is ideally lagged. One end is maintained at 373 K while the other is maintained at 273 K by immersing in melting ice Calculate the rate at which the ice melts owing to the flow of heat along the bar.** [4]

Solution

Given,
Length of bar (l) = 0.2 m
Cross sectional area (A) = 2.5×10^{-4} m^2
Higher temperature (θ_1) = 373 k
Lower temperature (θ_2) = 273 k
Thermal conductivity of bar (k) = 4.0×10^2 Wm^{-1} k^{-1}
Latent heat of ice (Li) = 336000 J/kg
Now,
The amount of heat gained by ice

$$Q = \frac{KA(\theta_2 - \theta_1)\,t}{l}$$

Then rate of heat lost

$$\frac{Q}{t} = \frac{KA(\theta_2 - \theta_1)}{l} = \frac{mL}{t}$$

or, $\frac{m}{t} = \frac{KA(\theta_2 - \theta_1)}{L\,l} = \frac{4.0 \times 10^2 \times 2.5 \times 10^{-4} \times 100}{336000 \times 0.2} = 1.5 \times 10^{-4}$ kg/sec

43. **2070 Set C Q.No. 10 b** **A slab of stone of area 0.36 m^2 and thickness 10 cm is exposed on the lower surface to steam at 100°C . A block of ice at 0°C rests on the upper surface of the slab. In one hour, 4.8 kg of ice is melted. Calculate the thermal conductivity of stone.** [4]

Solution

Given,
Area of stone (A) = 0.36 m^2
Thickness of stone (x) = 10 cm = 0.1 m
Temperature difference ($d\theta$) = $\theta_2 - \theta_1$ = 100 –0 = 100°C
Mass of ice melted (m) = 4.8 kg
Time (t) = 1hr = 3600 sec
Thermal conductivity of stone (k) = ?
We have,

$$\frac{Q}{t} = \frac{kA(\theta_2 - \theta_1)}{x}$$

$$\frac{mL}{t} = \frac{kAd\theta}{x}$$

or, $k = \frac{m.L.x}{t.A.d\theta} = \frac{4.8 \times 336000 \times 0.1}{3600 \times 0.36 \times 100} = 1.24$ $Wm^{-1}K^{-1}$

44. **2070 Set D Q.No. 10 b** **A metal rod of length 20cm and cross sectional area 3.14 cm^2 is covered with non-conducting substance. One of its end is maintained at 100°C, while the other end is put in ice at 0°C. It is found that 25 gram of ice melts in 5 minutes. Calculate the thermal conductivity of the metal.** [4]

Solution

Given,
Length of rod (l) = 20 cm = 0.2 m
Cross-sectional area (A) = 3.14 cm^2 = 3.14×10^{-4} m^2
Latent heat of ice (L) = 336000 J/kg
Mass of ice melted (m) = 25g = 0.025 kg

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Temperature difference, $(\theta_2 - \theta_1) = 100 - 0 = 100°c$
Thermal conductivity of metal (w) = ?
We have,

$$\frac{Q}{t} = \frac{KA(\theta_2 - \theta_1)}{l}$$

or, $$\frac{mL}{t} = \frac{KA(\theta_2 - \theta_1)}{l}$$

or, $$K = \frac{m \times L \times l}{t \times A \times (\theta_2 - \theta_1)} = \frac{0.025 \times 336000 \times 0.2}{5 \times 60 \times 3.14 \times 10^{-4} \times 100} = 178.3 \text{ Wm}^{-1} \text{K}^{-1}$$

45. 2069 Supp Q.No. 10c 2062 Q.No. 5 b Estimate the rate of heat loss through a glass window of area 2m² and thickness 4mm when the temperature of the room is 300K and temperature outside is 5°C.

Solution

Given,
Area of the glass window(A) = 2 m²
Thickness of the glass window (x) = 4mm = 4×10^{-3} m
Temperature inside the room (θ_1) = 300 K
Temperature outside the room(θ_2) = 5° C
= 5 + 273 = 278 K
If k is the coefficient of thermal conductivity of glass, then from the relation,

$$\frac{Q}{t} = \frac{kA(\theta_1 - \theta_2)}{x} \qquad [k = 1.2 \text{ Wm}^{-1}\text{K}^{-1}]$$

or, $$\frac{Q}{t} = \frac{1.2 \times 2 \times (300 - 278)}{4 \times 10^{-3}}$$

$$\therefore \frac{Q}{t} = 13200 \text{ W}$$

∴ The rate of heat loss through a glass window, $\frac{Q}{t} = 13200$ W

46. 2069 (Set A) Old Q. No. 5b The sun is a black body of surface temperature about 6000 K. If the sun's radius is 7×10^8m, calculate the energy per second radiated from its surface. (stefan's constant = 5.7×10^{-8} Wm^{-2}K^{-4})

Solution

Given,
Radius of the sun (r) = 7×10^8 m
Temperature of sun (T) = 6000 K
Power (p) = ?
Stefan's constant (σ) = 5.7×10^{-8} Wm^{-2}K^{-4}
We know,

$$P = \sigma AT^4 = \sigma 4\pi r^2 T^4$$
$$= 5.7 \times 10^{-8} \times 4\pi \times (7 \times 10^8)^2 \times (6000)^4$$
$$= 4.55 \times 10^{26} \text{ Watt}$$

47. 2068 Q.No. 10 b An ice box is made of wood 1.75 cm. thick lined inside with cork 2 cm. thick. If the temperature of inner surface of the cork is steady at 0°C and that of the outer surface of the wood is steady at 12°C; what is the temperature of the interface? The thermal conductivity of wood is five times that of cork.

Solution

Given,

Wood	Cork
1.75 cm.	2 cm.
$\theta_w = 12°C$	$\theta_2 = 0°C$

Let θ be the temperature of interface,
For wood.

$$\left(\frac{Q}{t}\right)_w = \frac{K_W A(12 - \theta)}{1.75 \times 10^{-2}}$$

For cork,

$$\left(\frac{Q}{t}\right)_C = \frac{K_C A(\theta - 0)}{2 \times 10^{-2}}$$

For steady's state,

$$\left(\frac{Q}{t}\right)_w = \left(\frac{Q}{t}\right)_C$$

or, $$\frac{K_W A(12 - \theta)}{1.75 \times 10^{-2}} = \frac{K_C A(\theta - 0)}{2 \times 10^{-2}}$$

or, $$\frac{5K_C A(12 - \theta)}{1.75} = \frac{K_C \theta}{2}$$

or, $10(12 - \theta) = 1.75\theta$
or, $120 = (1.75 + 10)\theta$
or, $120 = 11.75\theta$
∴ $\theta = 10.2°C$

48. 2066 Q. No. 10 c A man, the surface area of whose skin is 2m² is sitting in a room where the air temperature is 20°C. If his skin temperature is 37°C, find the rate at which his body loses heat. The emissivity of his skin is 0.97. [4]

Solution

Given,
Surface area of skin (A) = 2m²
Room temperature (T_o) = 20°C = 293 K
Skin temperature (T) = 37°C = 310 K
Emissivity of skin (e) = 0.97
Rate of heat loss (E) = ?
We have,

$$E = \sigma A.e.(T^4 - T_0^4) = 5.67 \times 10^{-8} \times 2 \times 0.97 [(310)^4 - (293)^4] = 205.2 \text{ W}$$

49. 2064 Q.No. 5 b Assuming that the thermal insulation provided by a woolen glove is equivalent to a layer of quiescent air 3 mm thick, determine the heat loss per minute from a man's hand, surface area 200 cm² on a winter day when the atmospheric air temperature is -3°C. The skin temperature is to be taken as 35°C and thermal conductivity of air as 24×10^{-3} Wm^{-1} K^{-1}. [4]

Solution

Given,
Thickness of slab (x) = 3 mm = 3×10^{-3} m
Area (A) = 200 cm² = 200×10^{-4} m²
Temperature of skin (θ_1) = 35° C = 308 K
Temperature of atmosphere (θ_2) = - 3°C = 270 K
Time (t) = 60 sec,
Tharmal conductivity (k) = 24×10^{-3} w m^{-1} K^{-1}
Heat loss per minute, Q = ?
We know,

$$Q = \frac{kA(\theta_1 - \theta_2)t}{x} = \frac{24 \times 10^{-3} \times 200 \times 10^{-4} (308 - 270)60}{3 \times 10^{-3}} \text{ J}$$

$$\therefore Q = 364.8 \text{J}$$

50. 2055 Q.No. 9 Estimate the rate of heat loss through a glass window of area 2m² and thickness 3mm when the temperature of the room is 20°C and that of air outside is 5°C.

Solution

Given,
Area(A) = 2m²

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Thickness of the glass (x) = 3 mm = 3×10^{-3} m
Tharmal conductivity (k) = 1.2 $Wm^{-1} K^{-1}$
Temperature inside the room (θ_1) = 20° C,
Temperature outside the room (θ_2) = 5° C

Rate of heat loss $\left(\frac{Q}{t}\right)$ = ?

We know,

$$\frac{Q}{t} = \frac{kA(\theta_1 - \theta_2)}{x} = \frac{1.2 \times 2(20-5)}{3 \times 10^{-3}}$$

$$\therefore \frac{Q}{t} = 12 \times 10^3 \text{ W} = 12 \text{ kW}$$

51. 2054 Q.No. 8 OR What is the ratio of the energy per second radiated by the filament of a lamp at 2500 K to that radiated at 2000 K, assuming the filament is a black body radiator?

Solution

Given,
Temperature of filament lamp (T_1) = 2500 K
Temperature of heat radiated (T_2) = 2000 K
Power Fadiated by Lamp at 2500 K (P_1) / Power Radiated by Lamp at 2000 K (P_2) = ?

The energy radiated per second by a black body of surface area is

$P = \sigma AT^4$

where σ is Stefan's constant and T is absolute temperature of black body.

If P_1 and P_2 be the energy radiated per second at 2500 K and 200 K respectively, then

$$\frac{P_1}{P_2} = \frac{\sigma AT_1^4}{\sigma AT_2^4}$$

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{2500}{2000}\right)^4 = 2.44$$

Hence, the required ratio of power is 2.44:1

52. 2053 Q.No. 10 b Estimate the rate at which ice would melt in a wooden box 2.5 cm. thick of inside measurement 100 cm. × 60 cm. × 40 cm. assuming that the external temperature is 35°C and thermal conductivity of wood is 0.168 $Wm^{-1} k^{-1}$.

Solution

Given,
Inside measurement of a wooden box is 100 cm × 60 cm × 4 cm.
∴ Total surface area of the walls of the wooden box,

$A = 2[100 \times 60 + 60 \times 40 + 40 \times 100]$
$= 24800 \text{ cm}^2 = 2.48 \text{ m}^2$

Thickness of wooden box (x) = 2.5 cm = 2.5×10^{-2} m
External temperature (θ_1) = 35° C
Internal temperature (θ_2) = 0° C
The rate of heat flow in to the box is given by

The flow of heat in to the box in time dt is, $dQ = \frac{kA(\theta_1 - \theta_2)\,dt}{x}$

or, $\frac{dQ}{dt} = \frac{0.168 \times 2.48 \times (35-0)}{2.5 \times 10^{-2}} = 583.29 \text{ W}$

Inside the box, the rate of heat gained by ice is 583.29 W
If L is the latent heat of fusion of ice.

Then rate of melting ice $= \frac{dQ/dt}{L} = \frac{538.29}{336 \times 10^3} = 1.6 \times 10^{-3} \text{kg s}^{-1}$

□□□

Chapter 6: First Law of Thermodynamics

Short Answer Questions

1. 2076 Set B Q.No. 2c The internal energy of a compressed gas is less than that of a rarefied gas at the same temperature. Why? [2]

A gas system consists of molecular motion and molecular interaction. The molecular motion possesses kinetic energy (E_K) and the interaction (or molecular configuration) possesses potential energy (E_P). The sum of kinetic and potential energy is total internal energy of gas. i.e.

$E = E_K + E_P$

At same temperature, E_K remains same and E_P varies with intermolecular distance. At high pressure, the volume is less i.e. the intermolecular distance is less and the mutual attraction between molecules increases and hence intermolecular potential energy decreases. But in case of rare fraction, the gas molecules separate apart and hence their intermolecular potential energy increases. Thus, internal energy of compressed gas is less than that of a rarefied gas.

2. 2076 Set C Q.No. 2c Why does the temperature of a gas drop during an adiabatic expansion? [2]

According to first law of thermodynamics, the amount of heat given to the gas is equal to the sum of the change in internal energy of gas and work done by the gas i.e. dQ = dU + dW. During adiabatic expansion, dQ = 0, i.e., dU = -dW, i.e., internal energy of the gas is used to do external work. So, internal energy decreases. The internal energy of the system is function of temperature, the decrease in internal energy means decrease in temperature.

3. 2075 GIE Q.No. 2a Why does the temperature of a gas undergoing adiabatic expansion decrease? [2]

Please refer to 2076 Set C Q.No. 2c

4. 2075 Set A Q.No. 2b Why does a gas have two specific heat capacities? [2]

From first law of thermodynamics,

$dQ = dU + dW$ ..... (i)
$dQ = dU + pdV$ .....(ii)

At constant volume, dQ = dU, so entire supplied heat is used up in raising temperature (internal energy). At constant pressure, heat supplied does an external work in addition to rise in internal energy. So, in making specific heat at constant pressure is high for same rise in temperature as achieved by the process at constant volume. The relation is given by $C_p - C_v = R$, where C_p, C_v = Molar heat capacities at constant pressure and volume and R = Universal gas constant. When a gas is heated, its pressure and volume increase effectively and simultaneously with the rise in temperature. Therefore, specific heat of a gas can be defined either at constant pressure or at constant volume and hence, gas has two specific heat capacities.

5. 2074 Set B Q.No. 2c Explain the significance of the first law of thermodynamics. Hence write its expression in terms of the change in entropy of the system. [2]

The first law of thermodynamics explains the conservation of energy in thermodynamic system. If some energy dQ is supplied to a system, then some of it is used to increase internal energy dU and some energy is used to do external work dW, then from first law of thermodynamics, dQ = dU + dW.

Entropy of the system is defined as disorderness as $dS = \frac{dQ}{T}$ i.e. dQ = TdS. So, dU + dW = TdS which represents first law of thermodynamics in terms of entropy.

6. 2073 Set D Q.No. 2c Compare the internal energy of a compressed gas with that of the rarefied gas at the same temperature. [2]

Please refer to 2076 Set B Q.No. 2c

7. 2072 Set C Q.No. 2b Why specific heat capacity of gas at constant pressure is greater than its specific heat capacity at constant volume? [2]

C_p and C_v are molar heat capacities at constant pressure and at constant volume respectively. According to first law of thermodynamics, the amount of heat given to the gas is equal to the sum of the change in internal energy of gas and work done by the gas i.e. dQ = dU + dW. If one mole of gas is heated at constant volume, work done is zero and hence, supplied heat is utilized in raising the temperature of gas i.e. internal energy. However, if one mole of gas is heated at constant pressure, the gas expands and hence, performs some work. That is, a part of supplied heat is utilized to raise the temperature and a part to perform work. Thus, greater amount of heat is required to raise the temperature of one mole of gas by 1°C at constant pressure than at constant volume. Mathematically, $C_p = C_v + R$, Since R>0, $C_p > C_v$. Therefore, C_p is greater than C_v.

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8. 2072 Set D Q.No. 2b **When a gas expands adiabatically, it does work on its surroundings. But, if there is heat input to the gas where does the energy come from?**

According to first law of thermodynamics, the heat equation is, dQ = dU + dW, dQ is heat given in or out, dU is change in internal energy and dW is external work done. During adiabatic expansion dQ = 0, dU = -dW, the external work done is performed by losing of internal energy of gas by lowering temperature.

9. 2071 Supp Q.No. 2c **Is C_p always greater than C_v?**

Please refer to 2072 Set C Q.No. 2b

10. 2070 Supp (Set A) Q.No. 2 a **Explain, why the temperature of gas drops in an adiabatic process.** [2]

Please refer to 2076 Set C Q.No. 2c

11. 2070 Supp (Set B) Q.No. 2 c **What are the limitations of first law of thermodynamics?**

The first law of thermodynamics has the following limitations.

i. First law of thermodynamics does not indicate the direction of heat transfer. Heat always flows from the body at higher temperature to the body at lower temperature. But, the first law does not indicate as to why the heat can not flow from a cold body to a hot one.

ii. When a bullet strikes a target, kinetic energy of the bullet is converted into heat energy. But, the first law does not indicate as to why heat energy developed in the target can not be converted back into mechanical energy of the bullet enabling it to fly back.

iii. The first law of thermodynamics does not give to what extent the mechanical energy is obtained from the heat energy.

It has been found that no heat engine can convert all the heat energy received from the source into mechanical energy with out rejecting a part to the surrounding continuously. The first law of thermodynamics is silent about this.

12. 2070 Set C Q.No. 2 c **A gas has two specific heats. Which one is greater and why?** [2]

Please refer to 2072 Set C Q.No. 2b

13. 2069 (Set B) Q. No. 2b **When we blow on the block of our hand with our mouth wide open, we feel warm. But if we partially close out mouth to form an 'O' and then blow on our hand, our breath feels cool. Why?** [2]

When we blow on the block of our hand with our mouth wide open, the air gets adiabatic compression and we feel warm. But if we partially close our mouth to form an 'O' and blow on our hand, air gets adiabatic expansion and we feel cool.

14. 2069 (Set A) Old Q.No. 1c **Can heat be considered as a form of potential energy?** [2]

Yes, heat can be considered in the form of potential energy. When a body is heated, its molecules become far from each other and molecular separation increases. This separation is against intermolecular force of attraction which is stored in the form of potential energy.

15. 2069 Supp Q.No. 2b **Air escaping from a tiny hole of a tube is felt cool, why?** [2]

When air escapes from a tiny hole at a tube, it undergoes adiabatic expansion. From first law of thermodynamics, we have dQ = dU + dW.

during adiabatic expansion, dQ = 0 which gives dU = - dW. This shows that the internal energy of gas decreases and temperature falls. Hence, Air escaping from a tiny hole of a tube is felt cool.

16. 2068 Q.No. 2 c 2065 Q. No. 4 b 2050 Q.No. 7 iv **Air escaping from an air hose at a gas station always feels cold. Why?** [2]

When air escapes from an air-hose at a gas station, it undergoes adiabatic expansion. From first law of thermodynamics, we have dQ = dU + dW.

during adiabatic expansion, dQ = 0 which gives dU = - dW. This shows that the internal energy of gas decreases and temperature falls. Hence, Air escaping from an air-hose at a gas station always feels cold.

17. 2068 Old Q.No. 1 b **Explain why temperature of gas drops in adiabatic expansion.**

Please refer to 2076 Set C Q.No. 2c

18. 2068 Old Q.No. 2b **Milk is poured into a cup of tea and is mixed with spoon. Is this an example of reversible process? Explain.** [2]

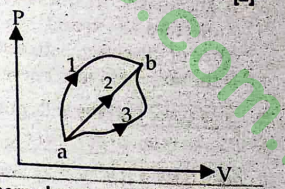
When milk is poured into a cup of tea and is mixed with spoon. This process is not reversible process because the reverse process is not possible. We cannot separate them again.

19. 2067 Supp Q.No. 2 c **Why is the molar heat capacity of a gas at constant pressure always greater than at constant volume?** [2]

Please refer to 2072 Set C Q.No. 2b

20. 2067 Q.No. 2 c **A system is taken from state 'a' to state 'b' along the three paths shown in adjacent figure. Along which path is the work done by the system greatest and the least? Give reason.** [2]

The work done by the system depends upon the area under the curve in PV-diagram. In a given figure, the area under curve 1 is greatest and hence, work done is also greatest while the area under curve 3 is least and hence, work done is also least.



21. 2066 Q.No. 1 b **What happens to the internal energy of a gas during (i) isothermal expansion (ii) adiabatic expansion?**

According to first law of thermodynamics, the amount of heat given to the gas is equal to the sum of the change in internal energy of gas and work done by the gas i.e. dQ = dU + dW. During isothermal process, temperature remains constant. Hence, no change in internal energy takes place because internal energy is the function of temperature. The applied heat is utilized only to perform external work. During adiabatic expansion, dQ = 0, i.e., dU = -dW, i.e., internal energy of the gas is used to do external work. So, internal energy decreases.

22. 2066 Q. No. 2 c **Explain why the temperature of a gas drops in an adiabatic process?** [2]

Please refer to 2076 Set C Q.No. 2c

23. 2065 Q.No. 1 b **What is the difference between isobaric and isochoric processes? Explain.** [2]

The differences between the isobaric and isochoric processes are given below:

Isobaric processes	Isochoric processes
1. This is the thermodynamic system carried at constant pressure i.e. dP = 0.	1. This is the thermodynamic process carried at constant volume i.e. dV = 0.
2. The work done in this process is zero i.e. dW = PdV = 0.	2. The work done in this process is not zero i.e. dW = $P(V_2-V_1)$.
3. In this system, dQ = dU.	3. In this process, dQ = dU + dW.

24. 2064 Q.No. 1 b **Explain why Cp is greater than Cv.**

Please refer to 2072 Set C Q.No. 2b

25. 2064 Q.No. 2 b **Is it possible to increase temperature of a body without giving heat to it? Explain.**

Yes, it is possible to increase temperature of a body without giving heat to it. From first law of thermodynamics, we have dQ = dU + dW.

According to question, dQ = 0 gives dU = -dW = -PdV.

It means the temperature of a body can be increased by decreasing its volume adiabatically.

26. 2062 Q.No. 1 b **Is internal energy of an ideal gas, the sum of kinetic energy and potential energy at a temperature greater than absolute zero? Explain.**

Ideal gas molecules have no intermolecular force and hence, no internal potential energy even if temperature is greater than absolute zero. However at absolute zero, kinetic energy of ideal gas is also zero. But, above absolute zero temperature, kinetic energy is taken into account. Hence, an ideal gas has only kinetic energy but not potential energy at a temperature greater than absolute zero temperature.

27. 2061 Q.No. 1 b **A cylinder filled with a gas is being carried inside a fast moving train, what change will be there in the internal energy of the gas?**

As the cylinder is kept within the train and the motion of the cylinder is zero w.r.t. train, vibration of gas molecules filled in the cylinder is not possible. Due to this, temperature of gas does not change. As it is known that internal energy of gas is function of temperature, there will be no change in internal energy.

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28. 2060 Q.No. 1 b What do you mean by internal energy of a gas?

A gas containing in a container has its intermolecular force of attraction i.e. it has potential energy. Also, the gas molecules of gas are in random motion i.e. it has kinetic energy. According to the first law of thermodynamics, the total amount of heat supplied to the gas is utilized to increase its temperature and rest to do the work against the external pressure. The increase in the temperature of a gas is due to it has to do the work against the intermolecular force of attraction between gas molecules to expand, when heated. The energy required to do the work against the intermolecular force of attraction between gas molecules is termed as internal energy of a gas. Hence, Internal energy is the sum of kinetic energy and potential energy of a substance.

29. 2059 Q.No. 1 b Why is C_p greater than C_v?

Please refer to 2072 Set C Q.No. 2b

30. 2059 Q.No. 2 b Why does internal energy remain constant in an isothermal system?

According to first law of thermodynamics, the amount of heat given to the gas is equal to the sum of the change in internal energy of gas and work done by the gas i.e. $dQ = dU + dW$. During isothermal process, temperature remains constant. Hence, there is no change in internal energy because it is the function of temperature. The applied heat is utilized only to perform external work.

31. 2058 Q.No. 1 b What is meant by adiabatic expansion? Explain.

The thermodynamic process in which the exchange of heat does not occur during expansion i.e. $dQ = 0$, this expansion is called adiabatic expansion. In this expansion, the wall of the container is perfectly non-conducting with surrounding and the system is so fast that there is no time of exchange of heat.

32. 2057 Q.No. 2 b 2053 Q.No. 7e Why has a gas two values of molar heat capacities?

Please refer to 2075 Set A Q.No. 2b

33. 2054 Q.No. 7 d Distinguish between an isothermal change and an adiabatic change. [2]

The difference between the isothermal and adiabatic change are given below:

Isothermal changes	Adiabatic changes
1. The thermodynamic process in which temperature of a system remains constant is known as isothermal process.	1. The thermodynamic process in which exchange of heat of a system with its surrounding is zero is known as adiabatic process.
2. The system is thermally conducting with the surrounding in this process.	2. The system is thermally insulated with the surrounding in this process.
3. It is slow process.	3. It is fast process.
4. In this, change in internal energy remains same i.e. $dU = 0$.	4. In this, change in internal energy changes i.e. $dU \neq 0$.
5. The equation of isothermal change is PV = constant.	5. The equation of adiabatic change is PV^γ = constant.

34. 2052 Q.No. 10 a Differentiate between isothermal and adiabatic change. [2]

Please refer to 2054 Q.No. 7 d

Long Answer Questions

35. 2076 Set C Q.No. 6c State and explain first law of thermodynamics. Derive the relation, $C_p - C_v = R$, where the symbols have their usual meanings. [4]

First Law of Thermodynamics

The first law of thermodynamics is stated as, "The amount of heat supplied to a system is equal to the sum of the increase in internal energy of the system and external work done by the system."

Mathematically,

$dQ = dU + dW$

It is based on the principle of conservation of energy and signifies that heat can only be produced at the expenditure of mechanical energy.

Relation between two specific heat capacities of a gas

C_p and C_v are molar specific heat capacities at constant pressure and constant volume and R is the universal gas constant. To derive relation between them consider one mole of an ideal gas taken in a cylinder fitted with frictionless piston. Let the gas is heated at a constant volume so that its temperature increases by dT and no work is done. The amount of heat is used to increase only the internal energy. i.e.

$dQ = dU = 1 \times C_v dT$ ... (i) $[\because Q = m \times s \times d\theta]$

where C_v is the molar specific heat capacity at constant volume.

Again, when the gas is heated at constant pressure, so that temperature increases by dT. Let dQ be the heat supplied then,

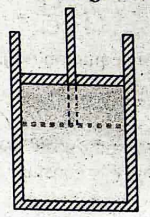
$dQ = 1 \times C_p \times dT = C_p\, dT$ ... (ii)

where C_p is the molar specific heat at constant pressure

Then from first law of thermodynamics, we can write

$dQ = dU + PdV$ ... (iii)

where dU is the increase in internal energy, P is the constant pressure at which the gas is heated and dV is increase in volume.



Using equation (i) and (ii) in (iii), we get,

$C_p dT = C_v dT + PdV$ ... (iv)

For 1 mole gas, ideal gas equation is $PV = RT$.

Differentiating above equation keeping pressure constant,

$\frac{d(PV)}{dT} = \frac{d(RT)}{dT}$

$PdV = RdT$ ... (v)

Substituting the value of PdV from equation (v) to equation (iv), we get,

$C_p dT = C_v dT + RdT$

$C_p dT = (C_v + R)\, dT$

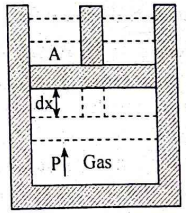
$C_p - C_v = R$

which is required reaction between Cp and Cv.

36. 2075 Set A Q.No. 6c What do you mean by adiabatic process? Deduce the equation PV^γ = const. where the symbols have their usual meaning. [4]

Adiabatic process: The thermodynamic process in which heat neither enters nor leaves the system but pressure, volume and temperature may change is called adiabatic process. This process is held quickly.

Consider one mole of an ideal gas contained in a cylinder having perfectly non-conducting walls. Let the gas be compressed adiabatically so that the piston of cross-section area A moves inwards through a distance dx.



So, work done during adiabatic compression is

$dW = P \times A \times dx = PdV$ ...(i)

Let dT be the rise in temperature during the compression.

Since, $dU = C_v dT$ ... (ii)

where C_v is molar specific heat capacity at constant volume.

According to first law of thermodynamics,

$dQ = dU + dW$

$dQ = C_v dT + PdV$ ... (iii)

For adiabatic process, $dQ = 0$, then equation (iii) becomes

$C_v dT + PdV = 0$ ... (iv)

For an ideal gas, equation of state is

$PV = RT$ ... (v)

Partially differentiating equation (v), we get,

$PdV + VdP = RdT$

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$$dT = \frac{PdV + VdP}{R} \quad \ldots (vi)$$

Putting the value of dT form equation (vi) in equation (iv), we have

$$C_v\frac{PdV + VdP}{R} + PdV = 0$$

$$C_vPdV + C_vVdP + R\,PdV = 0$$

$$C_vPdV + C_vVdP + (C_p - C_v)\,PdV = 0 \qquad [\because C_p - C_v = R]$$

$$C_vPdV + C_vVdP + C_p\,PdV - C_v\,PdV = 0$$

$$C_vVdP + C_p\,PdV = 0$$

Dividing both sides by C_vPV, we get

$$\frac{C_v\,VdP}{C_v\,PV} + \frac{C_pPdV}{C_v\,PV} = 0$$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \quad \ldots (vii)$$

$$[\because \frac{CP}{C_v} = \gamma, \text{ is the ratio of two specific heat capacities}]$$

Integrating equation (vii), we get

$$\int\frac{dP}{P} + \gamma\int\frac{dV}{V} = 0$$

$$\log_eP + \gamma\log_eV = \text{constant}$$

$$\log_eP + \log_eV^\gamma = \text{constant}$$

$$\log_e PV^\gamma = \text{constant}$$

$$\Rightarrow PV^\gamma = \text{constant}$$

This is equation of adiabatic process.

37. 2074 Supp. Q.No. 6c Why does a gas have two molar heat capacities? Show that for an ideal gas; $c_p - c_v = R$, where symbols have their usual meanings.

First Part: Please refer to 2075 Set A Q.No. 2b [4]

Second Part: Please refer to 2076 Set C Q.No. 6c

38. 2074 Set A Q.No. 6c Why does a gas have two molar heat capacities? Derive relation between them for an ideal gas.

First Part: Please refer to 2075 Set A Q.No. 2b [4]

Second Part: Please refer to 2076 Set C Q.No. 6c

39. 2074 Set B Q.No. 6a What is the difference between isothermal and adiabatic process. Prove that PV^γ is constant in an adiabatic process. r is the ratio of C_p and C_v. [4]

First Part: Please refer to 2054 Q.No. 7 d

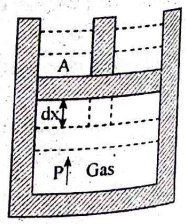
Second Part: Please refer to 2075 Set A Q.No. 6c

40. 2073 Supp Q.No. 6c What is adiabatic process? Derive an expression for the work done in this process. [4]

Adiabatic process: The thermodynamic process in which heat neither enters nor leaves the system but pressure, volume and temperature may change is called adiabatic process. This process is held quickly. In this process, dQ = 0.

Work done during adiabatic process: Let us consider one mole of gas is expanding adiabatically from volume V_1 to volume V_2 as shown in indicator diagram. The work done for an increase in volume dV is dW = PdV. The total work done when gas expand from initial state A (P_1,V_1) to final state B(P_2,V_2) is given by

$$W = \int_{V_1}^{V_2} PdV \quad \ldots (i)$$



We know,

$PV^\gamma = \text{constant} = K$ [Adiabatic gas equation]

$$P = \frac{K}{V^\gamma} \quad \ldots (ii)$$

Using equation (ii) to (i)

$$W = \int_{V_1}^{V_2}\frac{K}{V^\gamma}dV = K\int_{V_1}^{V_2}V^{-\gamma}\,dV$$

$$= \frac{K}{-\gamma+1}[V^{-\gamma+1}]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}\left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right]$$

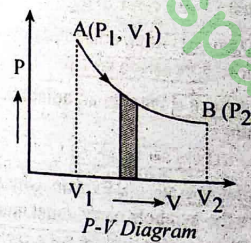
$$= \frac{1}{1-\gamma}\left[\frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}}\right] = \frac{1}{1-\gamma}\left[\frac{P_2V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1V_1^\gamma}{V_1^{\gamma-1}}\right] \quad [\because P_1V_1^\gamma = K = P_2V_2^\gamma]$$

$$W = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1]$$

In terms of temperature,

$$W = \frac{R}{1-\gamma}(T_2 - T_1)[\text{Since } P_2V_2 = RT_2 \text{ and } P_1V_1 = RT_1]$$

Which is required workdone in adiabatic process.



P-V Diagram

41. 2073 Set C Q.No. 6c What is an isothermal process? Derive the relation, PV^γ = constant, where symbols have their usual meanings. [4]

Isothermal Process: The process in which pressure and volume of a system change without any change in its temperature is called an isothermal process. In such a process, there is a free exchange of heat between the system and its surroundings. During isothermal process: the walls of the cylinder are perfectly conducting so free exchange of heat between the gas and its surroundings makes temperature constant in the isothermal process and the expansion and compression of the system is slow. E.g., the boiling of water and melting of ice. The equation of state of isothermal process is PV = constant.

Second Part: Please refer to 2075 Set A Q.No. 6c

42. 2072 Set D Q.No. 6b Explain the term, Thermodynamic process. Obtain an expression for the work done by a gas during adiabatic expansion. [4]

Thermodynamic Processes: The thermodynamic process takes place when any change in thermodynamic system occurs. The change is due to the change in thermodynamic parameters which are pressure, temperature and volume of the system. The important thermodynamic processes are isothermal process, adiabatic process, isochoric process, isobaric process, reversible process and irreversible process.

Second Part: Please refer to 2073 Supp Q.No. 6c

43. 2072 Set E Q.No. 6a What do you understand by isothermal and adiabatic processes? Derive PV^γ = constant, where symbols have their usual meaning. [4]

Isothermal Process: The process in which pressure and volume of a system change without any change in its temperature is called an isothermal process. In such a process, there is a free exchange of heat between the system and its surroundings. During isothermal process: the walls of the cylinder are perfectly conducting so free exchange of heat between the gas and its surroundings makes temperature constant in the isothermal process and the expansion and compression of the system is slow. E.g., the boiling of water and melting of ice. The equation of state of isothermal process is PV = constant.

Adiabatic Process: A thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and its surrounding is called adiabatic process.

In this process, the heat flow can be prevented by surrounding the system with thermally insulating material or by carrying out the process so quickly that there is no time for appreciable heat flow. eg., sudden bursting of tube of bicycle tyre. The equation of state of adiabatic process is PV^γ=constant,

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where γ is the ratio of two specific heat capacity of gas.

Second Part: Please refer to 2075 Set A Q.No. 6c

44. **2071 Supp Q.No. 6c For an adiabatic process show that PV^{γ} = constant.**

Please refer to 2075 Set A Q.No. 6c

45. **2071 Set D Q.No. 6 c What is an adiabatic process? Prove the relation PV^{γ} = constant, where notations carry usual meanings.** [4]

Please refer to 2075 Set A Q.No. 6c

46. **2070 Supp (Set A) Q.No. 6 a Explain, why do the gases have two specific heat capacities. Also, show that C_p-Cv=R. where the symbols carry usual meaning.** [4]

First Part: Please refer to 2075 Set A Q.No. 2b

Second Part: Please refer to 2076 Set C Q.No. 6c

47. **2070 Set C Q.No. 6 c What is thermodynamic process? Describe different thermodynamic processes.** [4]

Thermodynamic Processes:

The thermodynamic process takes place when any change in thermodynamic system occurs. The change is due to the change in thermodynamic parameters which are pressure, temperature and volume of the system. The important thermodynamic processes are:

i. **Isothermal Process:** The process in which pressure and volume of a system change without any change in its temperature is called an isothermal process. In such a process, there is a free exchange of heat between the system and its surroundings. During isothermal process: the walls of the cylinder are perfectly conducting so free exchange of heat between the gas and its surroundings makes temperature constant in the isothermal process and the expansion and compression of the system is slow. E.g.,the boiling of water and melting of ice. The equation of state of isothermal process is PV = constant.

ii. **Adiabatic Process:** A thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and its surrounding is called adiabatic process.

In this process, the heat flow can be prevented by surrounding the system with thermally insulating material or by carrying out the process so quickly that there is no time for appreciable heat flow. eg., sudden bursting of tube of bicycle tyre. The equation of state of adiabatic process is PV^{γ} =constant, where γ is the ratio of two specific heat capacity of gas.

iii. **Isochoric Process:** The thermodynamic process in which the volume remains constant is called an isochoric process. In this process, the working substance is kept in a non expanding chamber. Since there is no change in volume, the work done in such a process is zero. Therefore, from the first law of thermodynamics, we have

$dQ = dU + dW = dU + 0$

or, $dQ = dU$

Thus, whole of the heat supplied is used to increase the internal energy of the system.

iv. **Isobaric Process:** The thermodynamic process in which the pressure remains constant is called isobaric process. This system is carried out in an expanding chamber in which the pressure is kept constant. So, the temperature and the volume of the system change. The amount of heat dQ given to the working substance is partly used in increasing the temperature of the working substance by dT and partly used in doing external work.

v. **Reversible Process:** The process which can be retraced in the reverse order, so that it passes exactly through the same state as in the direct process is called reversible process. Example of reversible process is an infinitesimally slow expansion and compression of an ideal gas at a constant pressure.

vi. **Irreversible Process:** The processes which cannot be retraced in the opposite order by reversing the controlling factors are known as irreversible processes. Almost all processes in nature are irreversible process. Examples, rusting of iron, dissolving of soap in water, decay of matter, etc.

48. **2070 Set D Q.No. 6 c Derive an expression for the work done during the adiabatic expansion of an ideal gas. Does the internal energy of the system change during adiabatic expansion?** [4]

First Part: Please refer to 2073 Supp Q.No. 6c

Second Part: Please refer to 2076 Set C Q.No. 2c

49. **2069 (Set A) Q. No. 6b Define adiabatic process in thermodynamics. Show that: PV^{γ} = constant, where symbols have their usual meanings.** [4]

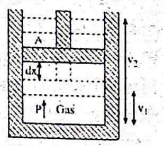
Please refer to 2075 Set A Q.No. 6c

50. **2069 (Set A) Old Q.No. 4a What do you mean by isothermal process? Obtain an expression for the work done by an ideal gas in the isothermal process.** [1+3]

Isothermal Process: The process in which pressure and volume of a system change without any change in its temperature is called an isothermal process. In such a process, there is a free exchange of heat between the system and its surroundings. During isothermal process: the walls of the cylinder are perfectly conducting so free exchange of heat between the gas and its surroundings makes temperature constant in the isothermal process and the expansion and compression of the system is slow. E.g., the boiling of water and melting of ice. The equation of state of isothermal process is PV = constant.

Work done during isothermal expansion: Let us consider one mole of an ideal gas contained in a cylinder having perfectly conducting walls. When piston move through infinitesimally small distance dx, then small work done dw is given by

$$dW = F \times dx = \frac{F}{A} \times A \times dx = PA\,dx$$



where A is the area of cross section of the piston.

$dW = PdV \qquad [\because dV = A\,dx]$

When gas is allowed to expand from initial state A (P_1, V_1, T_1) to final state B (P_2, V_2, T_2), the amount of work done is given as

$$W = \int_{V_1}^{V_2} dW = \int_{V_1}^{V_2} PdV \qquad \ldots (i)$$

For one mole of an ideal gas, we have $PV = RT \Rightarrow P = \frac{RT}{V}$

For isothermal process, temperature remains constant. Then equation (i) can be written as,

$$W = \int_{V_1}^{V_2} \frac{RT}{V} dV = RT \int_{V_1}^{V_2} \frac{dV}{V} = RT\,[\log_e V_2 - \log_e V_1]$$

$$\therefore W = RT \log_e \frac{V_2}{V_1}$$

i.e. $W = 2.3026\ RT \log_{10} \frac{V_2}{V_1}$

In terms of pressure change, work done during isothermal process is given by

$$W = RT \log_e \frac{P_1}{P_2} = 2.3026\ RT \log_{10} \frac{P_1}{P_2}$$

51. **2069 (Set A) Old Q.No. 4a OR Define two specific heat capacity of the gas. Show that for an ideal gas Cp – Cv = R.** [1+3]

Two specific heat capacity of the gas

i. **Specific heat capacity at constant volume (C_V):** It is defined as the amount of heat required to raise the temperature of unit mass of a gas though 1°C at constant volume. Its unit is J kg^{-1} K^{-1}. It is given as-

$$C_V = \left(\frac{dQ}{dt}\right)_V.$$

ii. **Specific heat capacity at constant pressure (C_P):** It is defined as the amount of heat required to raise the temperature of unit mass of a gas though 1°C at constant pressure. Its unit is J kg^{-1} K^{-1}. It is given as

$$C_P = \left(\frac{dQ}{dt}\right)_P.$$

Second Part: Please refer to 2076 Set C Q.No. 6c

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52. 2068 Q.No. 6 c Why does a gas have two molar heat capacities? Show that for an ideal gas: $C_p - C_v = R$. Where R is the molar gas constant. [4]

First part: Please refer to 2075 Set A Q.No. 2b
Second part: Please refer to 2076 Set C Q.No. 6c

53. 2068 Old Q.No. 4 a Show that for an ideal gas Cp - Cv = R. [4]

Please refer to 2076 Set C Q.No. 6c

54. 2066 Q.No. 4 a What is adiabatic process? Derive the relation $PV^\gamma = K$, where symbols have their usual meaning. [1+3]

Please refer to 2075 Set A Q.No. 6c

55. 2065 Q.No. 4 a OR Discuss the limitations of the first law of thermodynamics. Also derive an expression for equation of ideal gas during adiabatic process relating its temperature with pressure. [1+3]

Limitations of the First Law of Thermodynamics: Please refer to 2070 Supp (Set B) Q.No. 2 c
Please refer to 2075 Set A Q.No. 6c

56. 2064 Q.No. 4 a State and explain first law of thermodynamics and use it to derive the relation, PV^γ = constant, where the symbols have their usual meanings. [1+3]

First law of thermodynamics: Please refer to 2076 Set C Q.No. 6c
Second Part: Please refer to 2075 Set A Q.No. 6c

57. 2063 Q.No. 4 a OR What is adiabatic process? Derive an expression for work done in adiabatic system.

Please refer to 2073 Supp Q.No. 6c

58. 2062 Q.No. 4 a OR Explain why C_P is greater than C_V? Derive an expression relating C_P, C_V and R.

1st part: Please refer to 2072 Set C Q.No. 2b
2nd part: Please refer to 2076 Set C Q.No. 6c

59. 2061 Q.No. 4 a One mole of ideal gas undergoes an isothermal expansion from pressure V_1 to V_2 at constant temperature T. Find the expression for work done in this process.

Please refer to 2069 (Set A) Old Q.No. 4a

60. 2060 Q.No. 4 a What are reversible and irreversible processes? Derive an expression for the work done by an ideal gas during isothermal expansion. [1+3]

Please refer to 2069 (Set A) Old Q.No. 4a

61. 2057 Q.No. 4 a Derive the relation $C_p - C_v = R$, where the symbols have their usual meaning.

Please refer to 2076 Set C Q.No. 6c

62. 2053 Q.No. 9 Derive a relation between the two molar capacities of a gas. [4]

Please refer to 2076 Set C Q.No. 6c

Numerical Problems

63. 2076 Set B Q.No. 10c A certain volume of dry air at NTP is allowed to expand five times of its original volume under adiabatic condition. Calculate the final pressure and temperature. [4]

Solution

Given,

Initial temperature (T_1) = 27°C = 373 K (at NTP)

Final temperature (T_2) = ?

Initial pressure (P_1) = 760 mm of Hg = 1.01×10^5 N/m²

Final pressure (P_2) = ?

Initial volume (V_1) = V, let

Final volume (V_2) = 5V

Ratio of molar capacity (γ) = 1.4

We have for adiabatic expansion,

$P_1V_1^\gamma = P_2V_2^\gamma$

or, $1.01 \times 10^5 \times (V)^\gamma = P_2(5V)^\gamma$

$$\text{or, } P_2 = \frac{1.01 \times 10^5 \times V^\gamma}{5^\gamma \times V^\gamma}$$

$$= \frac{1.01 \times 10^5}{5^{1.4}}$$

$$= 1.06 \times 10^4 \text{ N/m}^2$$

Again,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\text{or, } T_2 = T_1 \frac{V^{\gamma-1}}{5^{\gamma-1}V^{\gamma-1}}$$

$$= \frac{373}{5^{1.4-1}}$$

$$= 196 \text{ K}$$

64. 2075 GIE Q.No. 10b A certain volume of dry air at NTP is allowed to expand four times of its original volume under (i) isothermal conditions (ii) adiabatic conditions. Calculate the final pressure and temperature in each case. (γ=1.4).

Solution

Given,

Initial temperature (T_1) = 273 K

Initial pressure (P_1) = 1.01×10^5 Nm⁻²

Initial volume (V_1) = V (let)

γ = 1.4

Final pressure (P_2) = ?

Final temperature (T_2) = ?

i. When air expanded isothermally; to 4 times its volume i.e.

$V_2 = 4V$

Then,

$P_2V_2 = P_1V_1$ [Isothermal condition]

$$\text{or, } P_2 = \frac{P_1V_1}{V_2} = \frac{1.01 \times 10^5 \times V}{4V}$$

$$= 25250 \text{ Nm}^{-2}$$

Again,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\text{or, } T_2 = \frac{V_2 \times T_1}{V_1}$$

$$= \frac{4V \times 273}{V} = 273 \times 4 = 1092 \text{ K}$$

ii. When air is expanded adiabatically to 4 times its volume, then by adiabatic equation,

$P_1V_1^\gamma = P_2V_2^\gamma$

$$\text{or, } P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma$$

$$= 1.01 \times 10^5 \left(\frac{1}{4}\right)^{1.4}$$

$$= 1.01 \times 10^5 \times 0.1436$$

$$= 14502.3 \text{ N/m}^{-2}$$

Again,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\text{or, } T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

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$$= 273\left(\frac{V}{4V}\right)^{1.4-1}$$

$$= 273 \times \left(\frac{1}{4}\right)^{0.4}$$

$$= 157 \text{ K}$$

65. 2075 Set B Q.No. 10c **A gas in a cylinder initially at a temperature of 10°C and one atmospheric pressure is to be compressed adiabatically to 1/8 of its volume. Find the final temperature. (Take $\gamma = 1.4$)** [4]

Solution

Given,

Initial temperature (T_1) = 10°C = 273 + 10 = 283 K

Initial pressure (P_1) = 1 Atmospheric pressure = 1.01×10^5 N/m²

Final temperature (T_2) = ?

$\gamma = 1.4$

Initial volume (V_1) = V

Final volume (V_2) = $\frac{V}{8}$

We have,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or, $283\ V^{\gamma-1} = T_2\left(\frac{V}{8}\right)^{\gamma-1}$

or, $283\ V^{1.4-1} = T_2\left(\frac{V}{8}\right)^{1.4-1}$

$\therefore\ T_2 = 283 \times 8^{1.4-1} = 650$ K

$\therefore$ Final temperature = 650 K

66. 2073 Set D Q.No. 10c **A gas in a cylinder is initially at a temperature of 17°C and pressure 1.01×10^5 Nm^{-2}. If it is compressed adiabatically to one-eighth of its original volume, what would be the final temperature and pressure of the gas?** [4]

Solution

Given,

Initial temperature (T_1) = 17°C = 273 + 17 = 290 K

Initial pressure (P_1) = 1.01×10^5 N/m²

Final pressure (P_2) = ?

Final temperature (T_2) = ?

γ = 1.40

Initial volume (V_1) = V

Final volume (V_2) = V/8

We have,

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$1.01 \times 10^5 \times V^{\gamma} = P_2 \times \left(\frac{V}{8}\right)^{\gamma}$

or, $P_2 = 1.01 \times 10^5 \times 8^{1.4}$

$= 18.56 \times 10^5$ N/m²

Again

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or, $290 \times V^{\gamma-1} = T_2\left(\frac{V}{8}\right)^{\gamma-1}$

or, $T_2 = 290 \times 8^{1.4-1} = 666.24$K

67. 2072 Supp Q.No. 10c **For hydrogen, the molar heat capacities at constant volume and constant pressure are 20.5 Jmol^{-1}K^{-1} and 28.8 Jmol^{-1}K^{-1}. Calculate (i) the heat needed to raise the temperature of 8g of hydrogen from 10°C to 15°C at constant pressure, (ii) the increase in internal energy of the gas. [molar mass of H_2 = 2gm]** [4]

Solution

Given,

Molar heat capacity at constant volume (C_v) = 20.5 J mol^{-1}K^{-1}

Molar heat capacity at constant pressure (C_p) = 28.8 J mol^{-1}K^{-1}

Mass of hydrogen (m) = 8 gm

Increase in temperature = (15 –10) = 5K

a. Heat required to raise 8 gm of hydrogen from 10° to 15 °C at constant pressure,

$dQ = \frac{m}{M}C_p\,dT = \frac{8}{2} \times 28.8 \times 5 = 576$ Joule

b. Increase in internal energy $dU = \frac{m}{M}C_vdT = \frac{8}{2} \times 20.5\ (15 - 10) = 410$ Joule.

68. 2072 Set C Q.No. 10b **Five moles of an ideal gas are kept at constant temperature of 53°C while the pressure of the gas is increased from 1.00 atm to 3.00 atm. Calculate work done by the gas.** [4]

Solution

Given,

No. of males (n) = 5

Temperature (T) = 53°C = 273 + 53 = 326 k

Initial pressure (P_1) = 1.00 atm = 1.0×10^5 N/m²

Final pressure (P_2) = 3.00 atm = 3.0×10^5 N/m²

Work done (W) = ?

We know work done in isothermal pressure,

$$W = nRT \ln\left(\frac{V_2}{V_1}\right) \quad \left[\begin{matrix} P_1V_1 = P_2V_2 \\ \frac{P_1}{P_2} = \frac{V_2}{V_1} \end{matrix}\right]$$

$$\therefore\ W = nRT \ln\left(\frac{P_1}{P_2}\right) = 5 \times 8.31 \times 326 \ln\left(\frac{1}{3}\right) = -14881\ \text{J}$$

The negative sign indicates that work is done to the gas.

69. 2071 Set C Q.No. 10 b **Air is compressed adiabatically to half its volume. Calculate the change in its temperature.** [4]

Solution

Given,

Let, initial volume (V_1) = V

Final volume (V_2) = $\frac{V}{2}$

Initial temperature = T_1

Final temperature = T_2

The change in temperature = ?

γ for air (γ_a) = 1.4

We have

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1V^{\gamma-1} = T_2\left(\frac{V}{2}\right)^{\gamma-1}$

or, $T_1V^{1.4-1} = \frac{T_2V^{1.4-1}}{2^{1.4-1}}$

or, $T_1 \times 2^{0.4} = T_2$

or, $\frac{T_2}{T_1} = 2^{0.4} = 1.32$

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or, $\frac{T_2}{T_1} - 1 = 1.32 - 1$ (Subtracting 1(one) on both sides)

or, $\frac{T_2 - T_1}{T_1} = 0.32$

or, $\left(\frac{T_2 - T_1}{T_1}\right)\% = 32\%$

Hence, temperature d is changed by 32%.

70. **2070 Supp (Set B) Q.No. 10 c** **A monoatomic ideal gas that is initially at a pressure of 1.50×10^5 Pa and has a volume of $0.08 m^3$ compressed adiabatically to a volume of $0.04 m^3$. (a) What is the final pressure? (b) How much work is done by the gas? (c) What is the ratio of the final temperature of the gas to its initial temperature?** [4]

Solution

Given,

Initial pressure (P_1) = 1.50×10^5 Pa

Initial volume (V_1) = $0.08 m^3$

Final volume (V_2) = $0.04 m^3$

a. Final pressure (P_2) = ?

b. Work done (W) = ?

c. Ratio of final to initial temperature = ?

$\gamma = \frac{5}{3}$ for monoatomic gas,

The adiabatic equation is

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

or, $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1.5 \times 10^5 \left(\frac{0.08}{0.04}\right)^{5/3} = 1.5 \times 10^5 \times (2)^{5/3} = 476220$ Pa

Work done,

$W = \frac{1}{\gamma - 1}(P_1V_1 - P_2V_2) = \frac{1}{\frac{5}{3} - 1}(1.5 \times 10^5 \times 0.08 - 476220 \times 0.04)$

$= \frac{3}{2}(12000 - 19048.8) = -10573$ J

('-ve' sign indicates that work is done on the gas).

Again,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{0.08}{0.04}\right)^{5/3-1} = 1.58$

∴ Ratio of final to initial temperature is 1.58.

71. **2069 Supp Q.No. 10a** **If the ratio of specific heat capacities of a gas is 1.4 and its density at S.T.P. is 0.09 kgm^3, calculate the values of specific heat capacities at constant pressure and at constant volume.** [4]

Solution

Given,

Ratio of specific heat capacities (γ) = 1.4

Density of gas at STP (ρ) = 0.09 kg m^{-3}

Specific heat capacity of gas at constant pressure (r) = ?

Specific heat capacity of gas at constant volume (C_v) = ?

We have ideal gas equation,

$PV = mrT$

or, $r = \frac{PV}{mT} = \frac{P}{\frac{m}{V}T} = \frac{P}{\rho T}$

$= \frac{1.01 \times 10^5}{0.09 \times 273}$ [At STP, $P = 1.01 \times 10^5$ N/m^2, $T = 273$ K]

$= 4110.7$ J kg^{-1} K^{-1}

Now,

$\frac{C_p}{C_v} = 1.40$

or, $C_p = 1.4\ C_v$

But, $C_p - C_V = r$

or, $1.4C_v - C_V = r$

or, $0.4 \times C_v = 4110.7$

$C_v = 10276.76 = 1.03 \times 10^4$ J kg^{-1} K^{-1}

$C_p = 1.4\ C_v = 1.4 \times 1.03 \times 10^4 = 1.44 \times 10^4$ J kg^{-1} K^{-1}

72. **2067 Supp Q.No. 10 c** **Gas in a cylinder initially at a temperature of 17°C and pressure of 1.01×10^5 Nm^{-2} is to be compressed adiabatically to one-eight of its volume. Find the temperature and pressure of the gas.** [4]

Please refer to 2073 Set D Q.No. 10c

73. **2067 Q.No. 10 c** **A liter of air, initially at 20°C and at 760 mm of Hg pressure, is heated at constant pressure until its volume is doubled. Find (i) the temperature, (ii) external work done by the air in expanding, and (iii) the quantity of heat supplied. Specific heat capacity at constant volume = 714 J/kg K.** [4]

Solution

Given,

Initial volume (V_1) = 10^{-3} m^{-3} = 1 liter.

Initial temperature (T_1) = 20 + 273 = 293K

Initial pressure (P_1) = 760mm Hg = 1.01×10^5 Nm^{-2}

Final volume (V_2) = 2×10^{-3} m^{-3}

Final temperature (T_2) = ?

Since at constant pressure,

$\frac{V_2}{T_2} = \frac{V_1}{T_1}$

or, $T_2 = \frac{V_2 \times T_1}{V_1}$

or, $T_2 = \frac{10^{-3} \times 293 \times 2}{10^{-3}}$

∴ $T_2 = 586$ K

The external work done (dw) = $P_1(V_2 - V_1) = 1.01 \times 10^5 (2 \times 10^{-3} - 10^{-3}) = 101.3$J

Mass of air = $1.293 \times 0.93 \times 10^{-3}$ $\left[\because \frac{V_0}{T_0} = \frac{V_1}{T_1} \Rightarrow V_0 = 0.93 \times 10^{-3}\ m^3\right.$, $\left.\text{Also, } (\rho_{NTP})_{air} = 1.293 \times kg.m^3\right]$

$= 1.202 \times 10^{-3}$ kg

Change in internal energy = mc_vdt

$= 1.202 \times 10^{-3} \times 714 \times 293$

$= 251.5$J

∴ Heat supplied = du + dw = 251.5 + 101.3J = 352.8J

74. **2066 Q.No. 4 b** **A litre of air initially at 20°C and at 760mm of Hg pressure is heated at constant pressure until its volume is doubled. Find the final temperature and the external work done by the gas in expanding.** [3]

Please refer to 2067 Q.No. 10 c

75. **2066 Q. No. 10 a** **An ideal gas initially at 4 atmosphere and 300 K is permitted to expand adiabatically twice its initial volume. Find the final pressure and temperature if the gas is (i) mono-atomic and (ii) diatomic with $C_V = 5/2$ R** [4]

Solution

Given,

Initial pressure (P_1) = 4 atm

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Final pressure (P_2) = ?
Initial temperature (T_1) = 300K
Final temperature (T_2) = ?
Initial volume (V_1) = V
Final volume (V_2) = 2V

i. For mono-atomic gas, $\gamma = 5/3 = 1.67$
We have
$P_1V_1^{\gamma} = P_2V_2^{\gamma}$
$P_2 = 4/2^{1.67} = 1.257$ atm
Also,
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times \left(\frac{1}{2}\right)^{0.67} = 188.55K$

ii. For diatomic $\gamma = 7/5 = 1.4$
$P_1V_1^{\gamma} = P_2V_2^{\gamma}$
$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 4 \times \left(\frac{V}{2V}\right)^{1.4} = 4 \times \left(\frac{1}{2}\right)^{1.4} = 1.52$ atm
Also,
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times \left(\frac{1}{2}\right)^{0.4} = 227.36K$

76. 2065 Q.No. 4 b **The density of an ideal gas is 1.6 kgm^{-3} at 27°C and 10^5Nm^{-2} pressure. Its specific heat capacity at constant volume is 312 Jkg^{-1}K^{-1}. Find the ratio of the specific heat at constant pressure to that at constant volume.**

Solution
Given,
Density of ideal gas (ρ) = 1.6 kg m^{-3}
Temperature (T) = 27° C = 27 + 273 = 300 K
Pressure (P) = 1×10^5 N m^{-2}
Specific heat capacity at constant volume (c_v) = 312 J kg^{-1} K^{-1}
c_p = ?
Since, we know that
$PV = mrT$
$P = \frac{m}{V}rT = \rho rT$
$\therefore r = \frac{P}{\rho T} = \frac{1.00 \times 10^5}{1.6 \times 300} = 208.3$ J kg^{-1} K^{-1}
Since,
$c_p = c_v + r = 312 + 208.3 = 520.3$ J kg^{-1} K^{-1}
$\therefore \gamma = \frac{c_p}{c_v} = \frac{520.3}{312} = 1.67$

77. 2064 Q.No. 4 b 2061 Q.No. 4 b **The density of a gas is 1.775 Kg m^{-3} at 27°C and 10^5 Nm^{-2} pressure. If the specific heat capacity at constant pressure is 846 Jkg^{-1} k^{-1}. Find the ratio of specific heat capacity at constant pressure to that at constant volume.**

Solution
Given,
Density of gas (ρ) = 1.775 kg/m^3
Temperature of gas (T) = 27°C = 27 + 273 = 300K
Pressure (P) = 10^5 N/m^2
Specific heat capacity at constant pressure (Cp) = 846 Jkg^{-1}/K^{-1}
γ = ?
We have
$PV = nRT$
or, $PV = mrT$
or, $r = \frac{PV}{mT} = \frac{P}{\rho T} = \frac{10^5}{1.775 \times 300} = 187.8$ JKg^{-1} K^{-1}
Again
$C_p - C_v = r$
or, $846 - C_v = 187.8$
or, $C_v = 846 - 187.8 = 658.2$ J/Kg.K
Also,
$\gamma = \frac{C_p}{C_v} = \frac{846}{658.2} = 1.29$

78. 2062 Q.No. 4 b **An ideal gas in slowly compressed at constant temperature of 50°C to one half of its original volume. In this process, 80 cal of heat was given. How much work was done and what was the change in the internal energy of the gas? Assume one mole of an ideal gas.**

Solution
Given,
Constant temperature (T) = 50° C
= 50 + 273 = 323 K
Initial volume (V_1) = V (say)
∴ Final volume (V_2) = $\frac{V}{2}$
Heat given (Q) = 80 cal
= 80 × 4.2 J = 336 J [∵ 4.2 J = 1 cal]
Number of mole of gas (n) = 1
∴ Work done is given by
$W = RT \log_e \frac{V_2}{V_1} = 8.3 \times 323 \log_e \frac{V}{2V} = -807.03$ J
Change in internal energy = 336 + 807.03 = 1143.03 J

79. 2060 Q.No. 4 b **16g of oxygen having volume 0.02m^3 at a temperature of 27°C and pressure of 2×10^5 Nm^{-2} is heated at constant pressure until its volume increases to 0.03m^3. Calculate the external work done and increase in internal energy of the gas if its Molar heat capacity at constant volume is 0.8 J mol^{-1} K^{-1} and Molar mass of oxygen is 32.**

Solution
Given,
Initial volume (V_1) = 0.02 m^3
Final volume (V_2) = 0.03 m^3
Initial temperature (T_1) = 27°C = 300 K
Initial pressure (P_1) = 2×10^5 Nm^{-2} = final pressure (∵ Pressure is constant)
External workdone (W) = $P\Delta V = 2 \times 10^5 (0.03 - 0.02) = 2 \times 10^3$J
Again, Molar heat capacity at constant volume C_v = 0.8 Jmol^{-1}K^{-1}
Molar mass of oxygen = 32 g
Mass of oxygen taken = 16 g
No. of moles (n) = $\frac{16}{32} = 0.5$
Increase in internal energy, ΔU = ?
At constant pressure,
$\frac{V_1}{T_1} = \frac{V_2}{T_2}$
$T_2 = \frac{T_1V_2}{V_1} = \frac{300 \times 0.03}{0.02} = 450$ K

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Hence, increase in internal energy is 60 J.

80. 2059 Q.No. 4 b **A gasoline engine takes in air at 25°C and one atmospheric pressure and compresses adiabatically to one-tenth of its original volume. Find the final temperature and pressure. ($\gamma = 1.4$).**

Solution

Given,

Initial temperature (T_1) = 25°C = 298 K

Final temperature (T_2) = ?

Initial pressure (P_1) = 1 atm = $1.01 \times 10^5 N/m^2$

Final pressure (P_2) = ?

Initial volume (V_1) = V, let

Final volume (V_2) = V/10

$\gamma = 1.4$

We have, for adiabatic change,

$P_1V_1^\gamma = P_2V_2^\gamma$

$1.01 \times 10^5 \times V^\gamma = P_2\left(\frac{V}{10}\right)^\gamma$

$P_2 = 1.01 \times 10^5 \times 10^{1.4} = 25.37 \times 10^5 N/m^2 = 25.12$ atm

Again,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 \times \left(\frac{V}{10}\right)^{\gamma-1} = V^\gamma \times 298$

$T_2 = 10^{1.4-1} \times 298 = 748.54$ K

81. 2058 Q.No. 4 b **A mass of air occupying initially a volume 2×10^{-3} m^{-3} at a pressure of 760 mm of mercury and a temperature of 20°C is expanded adiabatically and reversibly to twice its volume, and then compressed isothermally and reversibly to a volume of $3\times10^{-3}m^3$. Find the final pressure assuming the ratio of the specific heat capacities of air to be 1.4.**

Solution

Given,

For adiabatic expansion,

Initial volume (V_1) = $2 \times 10^{-3}m^3$

Final volume (V_2) = $2\times2\times10^{-3}m^3$

Initial pressure (P_1) = 760 mm of Hg

Initial temperature (T_1) = 20°C = 293 K

We have,

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 760\left(\frac{2 \times 10^{-3}}{2 \times 2 \times 10^{-3}}\right)^\gamma = 760 \times (2)^{-1.4}$

$\therefore P_2 = 287.98$ mm of Hg

For isothermal compression,

Let P_3 be the final pressure

Pressure (P_2) = 287.98 mm of Hg

Volume (V_3) = 3×10^{-3} m^3

We have,

$P_2V_2 = P_3V_3$

$P_3 = \frac{P_2V_2}{V_3} = \frac{287.98 \times 2 \times 2 \times 10^{-3}}{3 \times 10^{-3}}$

$\therefore P_3 = 383.98$ mm of Hg

Hence, the final pressure is 383.98 mm of Hg.

82. 2057 Q.No. 4 b **Gas in a cylinder, initially at a temperature of 10°C and pressure of 1.01×10^5 Nm^{-2} is to be compressed adiabatically to one eight of its volume. Find final pressure and temperature. (Ratio of molar heat capacities = 1.40)**

Solution

Given,

Initial temperature (T_1) = 10°C = 273 + 10 = 283 K

Initial pressure (P_1) = 1.01×10^5 N/m^2

Final pressure (P_2) = ?

Final temperature (T_2) = ?

$\gamma = 1.40$

Initial volume (V_1) = V

Final volume (V_2) = V/8

We have,

$P_1V_1^\gamma = P_2V_2^\gamma$

$1.01 \times 10^5 \times V^\gamma = P_2 \times \left(\frac{V}{8}\right)^\gamma$

or, $P_2 = 1.01 \times 10^5 \times 8^{1.4} = 18.56 \times 10^5$ N/m^2

Again

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or, $283 \times V^{\gamma-1} = T_2\left(\frac{V}{8}\right)^{\gamma-1}$

or, $T_2 = 283 \times 8^{1.4-1} = 650.16$K

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Short Answer Questions

1. 2074 Supp. Q.No. 2c **Why does a refrigerator consume more power in summer than in winter to cool the same quantity of food by same degree?**

A refrigerator is a heat engine which extracts heat from a cold body and rejects to hot body by doing work in working substance. Higher the temperature difference between hot and cold body, more energy is required to be done by the refrigerator. In summer temperature difference is higher than winter. So, more work is to be done by the refrigerator in summer than winter. Hence, a refrigerator consumes more power in summer than in winter to cool the same quantity of food by same degree of temperature.

2. 2074 Set A Q.No. 2a **Why not an ideal heat engine is cent percent efficient?**

The efficiency of an engine never be 100%. The ideal heat engine called Carnot engine, which works between two temperature limits T_1 and T_2, where T_1 is the temperature of source and T_2 is the temperature of sink. The efficiency of engine is given by

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

For Carnot engine,

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

For efficiency to be 100%, the low temperature reservoir (sink) should be at absolute zero temperature but it is impossible to have cold reservoir at absolute zero. Hence, the efficiency of engine is always less than 100%.

3. 2073 Supp Q.No. 2b **An engine cannot have 100% efficiency. Why and which basic expression of thermodynamic law suggest this?**

Please refer to 2074 Set A Q.No. 2a

4. 2073 Set C Q.No. 2c **Explain the significance of second law of thermodynamics.**

The second law of thermodynamics simply ensure the direction of heat flow, i.e. heat always flows from a body at higher temperature to the body at lower temperature and heat never flows itself from a body at lower temperature to higher temperature. The second law of thermodynamics, according to:

a. **Kelvin-Planck Statement:** It is impossible to get continuous supply of work from a body, which can transfer heat with a single source of heat.
b. **Clausius Statement:** It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance.

5. 2071 Set C Q.No. 2 c **Entropy is defined in the second law of thermodynamics. Can the first law be expressed in terms of entropy? How?**

Entropy is a measure of the disorder of a system. The change in entropy, dS, when a system changes from one equilibrium state to another is defined as,

$$dS = \frac{dQ}{T}$$

or, $dQ = TdS$

Here, dQ is amount of heat added or subtracted to change entropy, T is absolute temperature. If heat dQ is added to a system, its desordreness increases while decreases disorderness on decreasing heat. According to first law of thermodynamics,

$dQ = dU + dW$

To increase dQ, some work has to be done on the system. So, entropy can also be defined in terms of first law of thermodynamics.

6. 2071 Set D Q.No. 2 c **Explain the significant difference between the first and the second laws of thermodynamics.** [2]

The first law of thermodynamics asserts that heat and work are only different forms of energy and mutually inter-convertible. It does not tell us how and to what extent one can be converted into the other. It only tells us the energy is conserved. The second law of thermodynamics goes a step further and imposes additional restrictions on the first law by asserting that when heat is converted into work, only a partial conversion is possible, even under the best conditions and that any such conversion must require a hot body and a cold body. To summarize, the first law denies the possibility of destroying or creating energy including heat energy and the second law denies the possibility of using heat from a single body for the purpose of converting it into work.

7. 2070 Supp (Set A) Q.No. 2 c **Spark plug is not necessary in a diesel engine, why?** [2]

In petrol engine, the petrol is mixed with air in ratio 2 : 98 and the mixtures are compressed adiabatically. The temperature of compressed mixture rises to about 200°C that is not enough temperature for ignition of petrol. It is, therefore spark plug is needed to ignite the mixture. However, in diesel engine before mixing diesel with air, it is compressed adiabatically so that its temperature rises about 700°C then mixed with air. This temperature is enough to ignite the mixture. So, no spark plug is needed in diesel engine.

8. 2066 Q.No. 2 b **Is it possible to construct a heat engine that creates no thermal pollution?**

No, we cannot construct a heat engine that creates no thermal pollution. The heat engine works between two temperature ranges. Let, T_1 and T_2 be the temperatures of source and sink and its efficiency is,

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\ \%$$

For, no thermal pollution, T_2 should be zero. But according to Kelvin's statement, no engine can work in a single source at temperature T_1.

9. 2065 Q.No. 2 b **Write down the statements of second law of thermodynamics.** [2]

The second law of thermodynamics simply ensure the direction of heat flow, i.e. heat always flows from a body at higher temperature to the body at lower temperature and heat never flows itself from a body at lower temperature to higher temperature. The second law of thermodynamics, according to:

a. **Kelvin-Planck Statement:** It is impossible to get continuous supply of work from a body, which can transfer heat with a single source of heat.
b. **Clausius Statement:** It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance.

10. 2063 Q.No. 2 h **Why do diesel engines need no spark plugs?** [2]

Please refer to 2070 Supp (Set A) Q.No. 2 c

11. 2062 Q.No. 2 h **Petrol engine is less efficient than diesel engine. Explain why?** [2]

The efficiency of heat engine in terms of compression ratio is given as, $\eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}$, where ρ is called compression ratio $\left(\rho = \frac{V_2}{V_1}\right)$, where V_2 = volume of gas after compression and V_1 = volume of gas before compression, γ is called specific heat ratio, i.e., $\gamma = \frac{C_p}{C_v}$. Since the compression coefficient of diesel engine $\left(\rho = \frac{V_2}{V_1}\right)$ is greater than that of petrol engine. Hence, the efficiency of diesel engine is greater than petrol engine.

12. 2061 Q.No. 2b 2054 Q.No. 7e **State second law of thermodynamics.** [2]

Please refer to 2065 Q.No. 2 b

13. 2060 Q.No. 2 b **Can the thermal efficiency of an engine ever be 100%? Give reason.** [2]

Please refer to 2074 Set A Q.No. 2a

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14. 2058 Q.No. 2 b **Can a room be cooled by leaving the doors of an electric refrigerator open in a closed room?**

No, a room can not be cooled by leaving the door of an electric refrigerator open. A refrigerator is a heat engine that operates in the reverse direction i.e. it extracts heat from a cold body and transfers it to a room which is at the higher temperature. Hence, the temperature of room will not be cooled down but the room gets heated slightly.

15. 2057 Q.No. 1 b **Distinguish between petrol and diesel engine.** [5]

The differences between a petrol engine and a diesel engine are as follows:

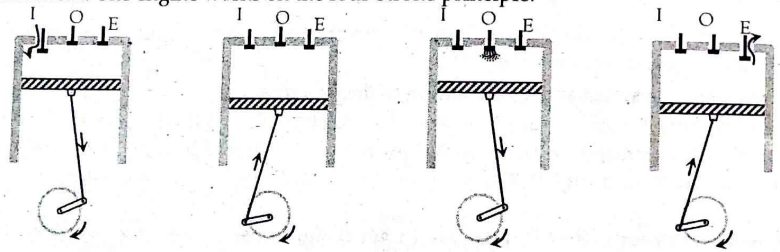
Petrol Engine	Diesel Engine
a. It uses petrol as fuel.	a. It uses diesel as fuel.
b. It has spark plug.	b. It does not have spark plug.
c. As compared to diesel engine, it has low efficiency.	c. As compared to petrol engine, its efficiency is high.
d. It is lighter than diesel engine and hence, it is used for vehicles like cars, airplanes, scooter, etc.	d. Diesel engine is a very heavy engine and hence, it can not be used in vehicle such as cars, scooters etc.

Long Answer Questions

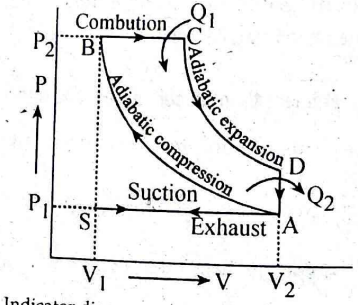
16. 2076 Set B Q.No. 6c **Describe the working principle of diesel engine with the help of PV diagram.** [4]

A diesel engine is an internal combustion heat engine, which uses diesel as fuel. It has higher efficiency than that of petrol engine. It does not have spark plug.

The cylinder of diesel engine is provided with three valves namely air inlet valve I, oil supply valve O and the exhaust valve E. These valves are opened and closed at proper moments by suitable mechanism. The engine works on the four-stroke principle.



(i) (ii) (iii) (iv)



Indicator diagram

The P-V diagram of diesel engine is as shown in figure.

a. **Suction stroke:** Air valve I is opened and pure air at atmospheric pressure is drawn into the cylinder by the outward motion of the piston as shown in Fig (i). It is represented by SA in P-V diagram.

b. **Compression stroke:** All the valves I and E are closed, as the piston start moving inwards as in Fig (ii). Air is compressed adiabatically to about one-seventeenth of its initial volume by the inward motion of the piston. In this process, the temperature is raised to about 1000° C. This is represented by AB in P-V diagram.

c. **Working stroke:** The oil valve O is opened and diesel oil is forced in under pressure as shown in Fig (iii). At high temperature, the oil burns spontaneously. The supply of oil is so regulated that during combustion, as the piston moves forward, the pressure remains constant. In this process, the burning fuel supplies heat to the air at constant pressure. At a certain stage when the temperature has reached the maximum value, about 2000 K, the supply of oil is cut off. This is the stroke in which useful work is done. During this process, Q_1 amount of heat is absorbed as shown by BC in PV diagram.

d. **Exhaust stroke:** As the piston moves inwards, exhaust valve E is opened and the useless gas mixture is forced out and expands adiabatically in the cylinder as shown in Fig.(iv). In P-V diagram at point C, the supply of fuel is cut off and the remaining part of working stroke is represented by adiabatic expansion CD. At D, the exhaust valve is opened so that the pressure drops to A, the atmospheric pressure. During the process DA, Q_2 amount of heat is rejected to the surroundings. The portion AS represents the exhaust stroke in which useless gas mixture is forced out. At E outlet valve O closes and inlet valve I is open. Then the cycle again starts.

Efficiency of a Diesel Engine

Let Q_1 be the amount of heat absorbed by the working substance at a constant pressure during combustion BC. Let Q_2 be the amount of heat rejected at constant volume during release of heat DA. Then the efficiency is given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}$$

where, $\rho = V_2/V_1$ is called compression ratio.

Merits and Demerits of Diesel Engine

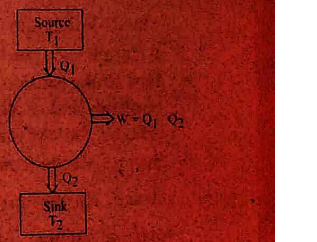
A diesel engine has higher efficiency than that of a petrol engine. As the diesel is less inflammable than petrol, it has less risk of firing. Its reliability is greater than that of petrol engine. Diesel oil is cheaper than petrol so the running expense is less than petrol engine.

Diesel engine is however, heavier than petrol engine, so it could not be used in light vehicles like aeroplanes, scooters etc.

17. 2075 GIE Q.No. 6b **State and explain second law of thermodynamics. Define the efficiency of a heat engine and explain qualitatively why the efficiency of such an engine is always less than 100%?** [4]

Second law of thermodynamics: Please refer to 2065 Q.No. 2 b

Heat engine: It is a device used for converting heat energy into mechanical energy. There are two types of heat engines such as external combustion engine (for example, steam engine) and internal combustion engine (for example, petrol engine). In external combustion heat engine, fuel is burnt outside of working portion of engine whereas in internal combustion heat engine fuel is burnt within the working portion of engine.



A heat engine consists of three basic parts. They are

a. The source of heat (hot body)
b. The working substance and
c. The sink of heat (cold body).

The working substance is taken along the cycle of operations, during which it absorb Q_1 amount of heat from the source at temperature T_1, converts a part of absorbed heat into useful work (W) and rejects the remaining heat Q_2 to the sink which is at lower temperature T_2.

After undergoing chain of operations, the working substance comes back to its original state without disturbing its internal energy.

The mechanical work obtained in each cycle, $W = Q_1 - Q_2$

The efficiency of heat engine (η) is defined as the ratio of the net external work done by the engine to the amount of heat absorbed from the heat source.

$$\text{i.e., } \eta = \frac{\text{external work obtained}}{\text{heat energy absorbed from the source}} = \frac{W}{Q1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

In terms of percentage, $\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$

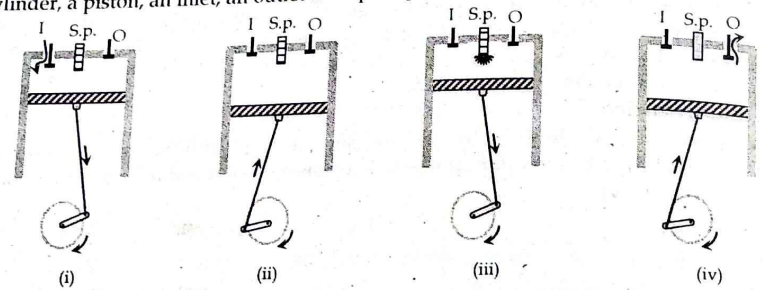
For ideal heat engine, $\eta = 100\%$, for which $Q_2 = 0$, but according to second law of thermodynamics, no

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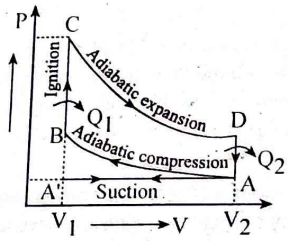
heat engine can be designed so that whatever heat it will absorb from the source, will be completely converted into mechanical work. Thus, heat engine do not have 100% efficiency. Moreover, the efficiency of heat engine can be expressed in terms of temperature of source (T_1) and that of sink (T_2) as, $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$.

18. 2075 Set B Q.No. 6c **Describe the working principle of petrol engine with the help of its PV diagram.** [4]

The internal combustion heat engine which uses petrol as a fuel is called petrol engine. It consists of a cylinder, a piston, an inlet, an outlet and spark plug as shown in figure.



(i) (ii) (iii) (iv)



Indicator diagram

(v)

The P –V diagram of petrol engine is as shown in figure.

It completes in four strokes. They are:

a. **First stroke (suction stroke):** It is represented by A'A. As the flywheel rotates, the piston attached to it moves outwards and creates a partial vacuum inside the cylinder as shown in fig (i). Due to low pressure inside cylinder, the inlet valve opens and air petrol mixture is sucked into cylinder.

b. **Second stroke (compression stroke):** In this stroke, both the inlet valve I and the outlet valve O are closed. The mixture undergoes adiabatic compression and its volume is reduced to about $\frac{1}{5}^{th}$ of the original volume. In P –V diagram it is represented by line AB. The mixture undergoes adiabatic compression from A to B so that temperature of system rises up to 600°C. After compression, spark plug produces spark and ignition takes place. It is represented by BC on the indicator diagram. This stroke is shown in Fig(ii).

c. **Third stroke (working stroke):** Due to the ignition of the mixture, the piston is pushed out with great force i.e. adiabatic expansion takes place as shown in figure (iii). During adiabatic expansion, the engine performs some external work. It is represented by CD line in indicator diagram.

d. **Fourth stroke (Exhaust stroke):** The outlet valve opens due to high pressure and allows the gases to escape out until the pressure falls to the atmospheric pressure as shown in figure (iv). Heat is rejected to the surrounding practically at a constant volume. It is represented by curve DA in indicator diagram. Finally, the burnt gases are forced out of the cylinder with the inward movement of the piston. The cycle is completed and ready to start another cycle. It is represented by the curve AA' in indicator diagram.

The efficiency of petrol engine is given by,

$$\eta = 1 - \frac{Q_2}{Q_1}$$

But $\frac{Q_1}{Q_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{\rho}\right)^{\gamma-1}$ [Where $\rho = \frac{V_2}{V_1}$ is adiabatic compression ratio]

$\therefore \eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}$.

19. 2073 Set D Q.No. 6b **What is efficiency of a heat engine? Describe the working of a petrol engine with the help of its P–V diagram.**

First Part: Please refer to 2075 GIE Q.No. 6b (Second part) [4]

Second Part: Please refer to 2075 Set B Q.No. 6c

20. 2072 Supp Q.No. 6c 2070 Supp (Set B) Q.No. 6c 2069 (Set A) Q. No. 6c 2067 Q.No. 6c 2066 Q. No. 6c **Describe the working of a petrol engine with the help of its P-V diagram.**

Please refer to 2075 Set B Q.No. 6c [4]

21. 2072 Set C Q.No. 6b **Describe the working of diesel engine with a P-V diagram. What are its merits and demerits?**

Please refer to 2076 Set B Q.No. 6c [4]

22. 2070 Supp (Set A) Q.No. 6 c **Discuss the working principle of a diesel engine with the help of p-v diagram.** [4]

Please refer to 2076 Set B Q.No. 6c

23. 2069 Supp Q.No. 6c 2067 Supp Q.No. 6 c **Describe the working of a diesel engine with the help of P-V diagram.**

Please refer to 2076 Set B Q.No. 6c [4]

24. 2069 Set B Q. No. 6c **Describe the working of carnot's engine with the help of its P-V diagram.** [4]

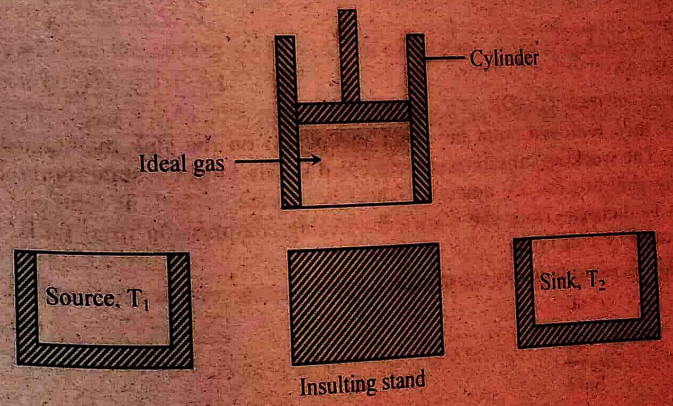
A Carnot engine is a theoretical engine, which is used to convert heat into mechanical work. It is free from all the defects of practical engines. Its efficiency is maximum and it is an ideal heat engine. A Carnot heat engine consists of,

a. **Source:** It has fixed high temperature (T_1) from which the heat engine can draw heat and has infinite thermal capacity.

b. **Sink:** It has fixed low temperature (T_2) to which any amount of heat can be rejected and has infinite thermal capacity.

c. **Working substance:** A cylinder with non-conducting sides and conducting bottom containing a perfect gas as the working substance.

The working substance (1 mole of ideal gas) is taken round a reversible cycle consisting of two adiabatic and two isothermal processes, so this complete cycle called *Carnot cycle*.



The Carnot cycle completes in four steps:

Step 1: (Isothermal expansion A→B): -

The cylinder containing the working substance is placed on the source maintained at a higher temperature T_1. The cylinder absorbs Q_1 amount of heat from source through its conducting base and piston moves out of the

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cylinder. The expansion is isothermal, in which the gas expands from initial state A (P_1, V_1, T_1) to final state B (P_2, V_2, T_1) shown in PV indicator diagram.

According to first law of thermodynamics, the work done during this process,

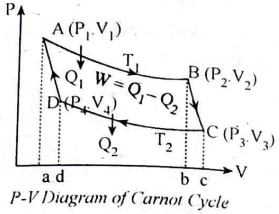
$W_1 = Q_1$

Since,

$dU = 0$

So,

$$W_1 = \int_{V_1}^{V_2} P\,dV$$



P-V Diagram of Carnot Cycle

$$W_1 = RT_1 \log_e \frac{V_2}{V_1} \quad \ldots (i)$$

= Area of ABbaA

Step 2: *(Adiabatic expansion B→C):*

The cylinder is now removed from the source and placed on the insulating stand. The working substance is allowed to expand further. No heat enters or leaves the cylinder, i.e. adiabatic expansion in which ($T_2 < T_1$) gas expands from initial state B(P_2, V_2, T_1) to final state C(P_3, V_3, T_2) as shown in PV diagram. Let, W_2 be the work done during adiabatic expansion. Then using first law of thermodynamics,

$$W_2 = \int_{V_2}^{V_3} P\,dV = \int_{V_2}^{V_3} \frac{dV}{V^\gamma} \cdot k \qquad [Q_2 = 0,\ PV^\gamma = \text{constant}]$$

$$= \int_{V_2}^{V_3} \frac{K}{V^\gamma}\,dV = K\int_{V_2}^{V_3} V^{-\gamma}\,dV = \frac{K}{1-\gamma}\left[\frac{1}{V_3^{\gamma-1}} - \frac{1}{V_2^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{P_3V_3^\gamma}{V_3^{\gamma-1}} - \frac{P_2V_2^\gamma}{V_2^{\gamma-1}}\right] \qquad [\because P_3V_3^\gamma = K = P_2V_2^\gamma]$$

$$W_2 = \frac{1}{1-\gamma}[P_3V_3 - P_2V_2] \qquad [\because PV = RT]$$

$$= \frac{1}{\gamma-1}[P_2V_2 - P_3V_3]$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} \quad \ldots (ii)$$

= area of BCcbB

Step 3: *(Isothermal compression C→D):*

The cylinder is then removed from the stand and placed on the sink maintained at a lower temperature T_2. The working substance is compressed slowly until the state represented by D is reached. During this process, Q_2 amount of heat is rejected to sink at temperature T_2. This compression is isothermal. Thus, the work done during compression from initial state C (P_3, V_3, T_2) to final state D(P_4, V_4, T_2) be

$W_3 = Q_3$ [U = 0, using 1st law of thermodynamics]

or, $Q_3 = W_3$

$$= -\int_{V_3}^{V_4} P\,dV = RT_2 \log_e \frac{V_3}{V_4} \quad \ldots (iii)$$

= Area of CDdcC

Step 4: *(Adiabatic compression D→A):*

Finally the cylinder is removed from the sink and placed back on insulating stand. Pressure is increased from P_4 to P_1 and volume gets decreased from V_4 to V_1. The process is completely adiabatic compression

and hence, reaches from state D (P_4, V_4, T_2) to initial state A(P_1, V_1, T_1).

Work done from D to A (W_4)

$$= -\int_{V_4}^{V_1} P\,dV = \int_{V_1}^{V_4} P\,dV = -\frac{R(T_1 - T_2)}{\gamma - 1} \quad \ldots (iv)$$

= Area of DAadD

The net work done:

Let W be the net work done by the working substance in one complete cycle, then

= Area of ABbaA + Area of BCcbB + Area of CDdcC + Area of DAadD

= Area of ABCDA

$$= W_1 + W_2 - W_3 - W_4 = W_1 - W_3 + RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4} \quad \ldots (v)$$

The point B and C lie on same adiabatic, so

$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

Since, $\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \Rightarrow \frac{V_1}{V_4} = \frac{V_2}{V_3}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \ldots (vi)$$

$$\therefore W = R \log_e \frac{V_2}{V_1}[T_1 - T_2] \qquad [\text{using (vi)}]$$

Now, Efficiency of heat engine is defined as the ratio of external work done by the engine to the amount of heat energy absorbed from the source.

$$\text{Efficiency } (\eta) = \frac{\text{External workdone (W)}}{\text{Heat absorbed from source}(Q_1)} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Therefore, Efficiency of Carnot engine (in percentage) is

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

25. **2068 Old Q.No. 4 a OR What is a heat engine? Calculate the thermal efficiency of a heat engine.** [1+3]
   Please refer to 2075 GIE Q.No. 6b

26. **2066 Q.No. 4 a OR Draw and explain the P–V diagram of petrol engine.** [1+3]
   Please refer to 2075 Set B Q.No. 6c

27. **2065 Q.No. 4 a Explain the working of a diesel engine on P-V diagram and write down its merits and demerits.** [3+1]
   Please refer to 2076 Set B Q.No. 6c

28. **2064 Q.No. 4 a OR 2060 Q.No. 4 a OR 2059 Q.No. 4 a OR Explain the working mechanism of a Petrol engine with the help of a PV diagram.**
   Please refer to 2075 Set B Q.No. 6c

29. **2063 Q.No. 4 a State and explain the second law of thermodynamics.**
   Please refer to 2065 Q.No. 2 b

30. **2062 Q.No. 4 a Explain the working mechanism of a diesel engine with the help of a PV diagram.**
   Please refer to 2076 Set B Q.No. 6c

31. **2061 Q.No. 4 a OR Explain the working of a Carnot engine with the help of a P-v diagram.**
   Please refer to 2069 Set B Q. No. 6c

32. **2059 Q.No. 4 a What is the basis difference between the first law and the second law of thermodynamics? Explain on the basis of Carnot's engine that no heat engine have efficiency of unity? (Derivation not required).** [2+2]
   Difference between first and second law of thermodynamics: Please refer to 2071 Set D Q.No. 2 c

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According to Carnot engine, the efficiency of Carnot engine is given by

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

where T_2 be the temperature of sink and T_1 be the temperature of source. Thus, the efficiency only depends upon the temperature of source and sink but not on nature of working substance. Efficiency will be higher for higher temperature of source and lower temperature of sink. Complete conversion of heat into work ($\eta = 100\%$) is possible only when $T_2 = 0$ or $T_1 = \infty$. In other words, the engine works between absolute zero and a higher temperature. Since, absolute zero is unattainable η is less than unity or less than 100%.

33. 2058 Q.No. 4 a What do you mean by a heat engine? How do you define its efficiency?

Please refer to 2075 GIE Q.No. 6b

34. 2058 Q.No. 4 a OR Explain the working of a diesel engine with the help of P-V diagram.

Please refer to 2076 Set B Q.No. 6c

35. 2057 Q.No. 4 a OR Draw the P-V diagram for petrol engine and explain its working on the basis of the diagram.

Please refer to 2075 Set B Q.No. 6c

Numerical Problems

36. 2076 Set C Q.No. 10c The efficiency of a Carnot cycle is 15%. If on reducing the temperature of sink by 65°, the efficiency becomes double, find the temperature of source and sink. [4]

Solution

Given,

The efficiency of engine (η) = 15%

Reduced efficiency (η') = 30%

Let T_1 be the temperature of source and T_2 be the temperature of sink, then

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

$$\text{or, } 15\ \% = \frac{T_1 - T_2}{T_1} \times 100\% \quad \ldots \text{(i)}$$

Again,

$$\eta' = \left(1 - \frac{T_2 - 65}{T_1}\right) \times 100\%$$

$$\text{or, } 30\ \% = \frac{T_1 - (T_2 - 65)}{T_1} \times 100\ \% \quad \ldots \text{(ii)}$$

Divide (ii) by (i), we get

$$\frac{30}{15} = \frac{T_1 - (T_2 - 65)}{T_1 - T_2}$$

$$2T_1 - 2T_2 = T_1 - T_2 + 65$$

$$\text{or, } 2T_1 - 2T_2 - T_1 + T_2 = 65$$

$$\text{or, } T_1 - T_2 = 65$$

$$\text{or, } T_1 = T_2 + 65, \text{ then from (ii)}$$

$$30\% = \frac{T_1 - T_2 + 65}{T_2 + 65} \times 100\%$$

$$\text{or, } 30\% = \frac{T_2 + 65 - T_2 + 65}{T_2 + 65} \times 100\%$$

$$\text{or, } 0.3\,T_2 + 19.5 = 130$$

$$\text{or, } 0.3T_2 = 110.5$$

$$T_2 = 368.3 \text{ K}$$

$$T_1 = 60 + 368.3 = 428.3 \text{ K}$$

37. 2075 Set A Q.No. 10b A Carnot engine has 40% efficiency with a sink at 10°C. By how many degrees should the temperature of the source be increased in order to raise the efficiency to 65%? [4]

Solution

Given,

Initial efficiency (η_1) = 40%

Temperature of sink (T_2) = 10°C = 273 + 10 = 283 K

Final efficiency (η') = 65%

Temperature of source to be increases (ΔT) = ?

Now, we have,

$$\eta_1 = 1 - \frac{T_2}{T_1}$$

$$\text{or, } 0.40 = 1 - \frac{283}{T_1}$$

$$\text{or, } \frac{283}{T_1} = 1 - 0.40$$

$$\text{or, } T_1 = \frac{283}{0.6} = 472 \text{ K}$$

Again,

$$\eta' = 1 - \frac{T_2}{T_1} \qquad [\because T_2 = T_1']$$

$$0.65 = 1 - \frac{283}{T_1'}$$

$$\text{or, } \frac{283}{T_1'} = 1 - 0.65$$

$$\text{or, } T_1' = \frac{283}{0.35} = 808.57 \text{ K}$$

Temperature of source to be raised, $\Delta T = T_1' - T_1$

$= 808.57 - 472$

$= 336.57 \text{ K} = 63.57\ ^\circ\text{C}$

38. 2074 Supp. Q.No. 10c 2071 Supp Q.No. 10c 2068 Q.No. 10 c A Carnot engine has 50% efficiency with a sink at 9°C. By how many degrees should temperature of source be increased in order to raise the efficiency to 70%? [4]

Solution

Given,

First case

Efficiency (η) = 50%

Temp. of cold reservoir (T_2) = 9°C = 282K

Temp of source (T_1) = ?

We have,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\text{or, } 0.50 = 1 - \frac{282}{T_1}$$

$$\text{or, } T_1 = 564 \text{ K}$$

Second case

Efficiency (η') = 70%

Temp. of source (T_1') = ?

Temp. of resourvoir (T_2) = 9°C = 282K

Also,

$$\eta' = 1 - \frac{T_2}{T_1'}$$

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$T_1' = \frac{282}{0.3} = 940$ K

Temp. to be raised
$= T_1' - T_1 = 940 - 564 = 376$ K

39. **2074 Set A Q.No. 10c** **An ideal heat engine operates between two reservoirs at two temperatures. In order to achieve 30% efficiency when the temperature of the sink is 50°C, what should be the temperature of the source?** [4]

Solution

Given,
Efficiency of engine (η) = 30%
Temperature of sink (T_2)= 50°C = 323 K
Temperature of source (T_1) = ?
We know,

$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

or, $30\% = \left(1 - \frac{323}{T_1}\right) \times 100\%$

or, $0.3 = 1 - \frac{323}{T_1}$

or, $\frac{323}{T_1} = 1 - 0.3$

or, $T_1 = \frac{323}{0.7}$

$\therefore T_1 = 461.43$ k

40. **2074 Set B Q.No. 10a** **The source reservoir of a carnot engine is at a temperature of 400 K and takes 400 J of heat and rejects 20 J of heat to the sink reservoir in each cycle. What is the efficiency of engine and the temperature of the sink?** [4]

Solution

Given,
Temperature of source, $T_1 = 400$ K
Heat absorbed, $Q_1 = 400$ J
Heat rejected, $Q_2 = 20$ J
Temperature of sink, T_2 = ?
Efficiency of engine, η = ?
We know,

$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\% = \left(1 - \frac{20}{400}\right) \times 100\% = 95\%$

Also,

$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

or, $\frac{95}{100} = 1 - \frac{T_2}{400}$

or, $\frac{T_2}{400} = 1 - \frac{95}{100} = 0.05$

$\therefore T_2 = 20$ k

41. **2073 Set C Q.No. 10c** **A Carnot's engine has 25% efficiency with a sink at 9°c. By how many degrees should the temperature of the source be increased in order to raise the efficiency to 70%?** [4]

Solution

Given,
First case
Efficiency (η) = 25%
Temp. of cold reservoir (T_2) = 9°C = 282K
Temp of source (T_1) = ?
We have,

$\eta = 1 - \frac{T_2}{T_1}$

or, $0.25 = 1 - \frac{282}{T_1}$

or, $T_1 = 376$ K

Second case
Efficiency (η') = 70%
Temp. of source (T_1') = ?
Temp. of resourvoir (T_2) = 9°C = 282K
Also,

$\eta' = 1 - \frac{T_2}{T_1'}$

$0.70 = 1 - \frac{282}{T_1'}$

$T_1' = \frac{282}{0.3} = 940$ K

Temp. to be raised
$= T_1' - T_1 = 940 - 376 = 564$ K

42. **2072 Set D Q.No. 10c** **A diesel engine performs 2200 J of mechanical work and discards 4300 J of heat each cycle. (i) How much heat must be supplied to the engine in each cycle? (ii) What is the thermal efficiency of the engine?** [4]

Solution

Given,
Work done (W) = 2200 J
Discards of heat (Q_2) = 4300 J
Heat supplied (Q_1) = ?
Thermal efficiency (η) = ?
Now we have
$W = Q_1 - Q_2$
or, $2200 = Q_1 - 4300$
or, $Q_1 = 6500$ J
Again,

$\eta = \frac{W}{Q_1} \times 100\%$

$= \frac{2200}{6500} \times 100\% = 33.85\%$

43. **2072 Set E Q.No. 10b** **What will be the thermal efficiency of an engine if it takes 8 KJ heat from the source and rejects 6 KJ to the sink in one cycle?** [4]

Solution

Given,
Input energy (Q_1) = 8 KJ = 8000 J
Rejection of heat (Q_2) = 6 KJ = 6000 J
Work done $W = Q_1 - Q_2 = 8000 - 6000 = 2000$ J

Question Bank Solutions

Thermal efficiency (η) = ?

We have,

$$\eta = \frac{W}{Q_1} \times 100\% = \frac{2000}{8000} \times 100\% = 25\%$$

44. 2071 Set C Q.No. 10 c **A Carnot engine works between the source and the sink with efficiency 40%. How much temperature of the sink be lowered keeping the source temperature constant at 27°C so that its efficiency increases by 10%?** [4]

Solution

Given,

Case I

Efficiency (η_1) = 40%

Temperature of source (T_1) = 27° C = 300 K

Temperature of sink (T_2) = ?

Case II

Efficiency (η_2) = (40 + 10)% = 50%

Temperature of source (T_1') = T

Temperature of sink (T_2') = ?

Now,

$$\eta_1 = 1 - \frac{T_2}{T_1}$$

$$0.4 = 1 - \frac{T_2}{300}$$

or, $\frac{T_2}{300} = 0.6$

or, $T_1 = 0.6 \times 300 = 180$ K

Again,

$$\eta_2 = 1 - \frac{T_2'}{T_1'}$$

$$0.5 = 1 - \frac{T_2'}{300}$$

or, $\frac{T_2'}{300} = 1 - 0.5 = 0.50$

or, $T_2' = 150$ K

∴ Temperature of sink lowered = 180 – 150 = 30 K

45. 2071 Set D Q.No. 10 a **A Carnot engine works between the source and the sink with 25% efficiency. How much temperature of the sink be raised keeping the sink at the same constant temperature at 27°C so that the efficiency is increased by 15°C?** [4]

Solution

Given,

First case

Efficiency (η) = 25%

Temp. of cold reservoir (T_2) = 27°C = 273+27 = 300K

Temp of source (T_1) = ?

We have,

$$\eta = 1 - \frac{T_2}{T_1}$$

or, $0.25 = 1 - \frac{300}{T_1}$

or, $T_1 = 400$ K

Second case

Efficiency (η') = 25 + 15 = 40%

Temp. of source (T_1') = ?

Temp. of resourvoir (T_2) = 27°C = 300K

Also,

$$\eta' = 1 - \frac{T_2}{T_1'}$$

$$0.40 = 1 - \frac{300}{T_1'}$$

$$T_1' = \frac{300}{0.6} = 500 \text{ K}$$

Temp. to be raised = $T_1' - T_1$

= 500 – 400 = 100 K

46. 2070 Set C Q.No. 10 c **The efficiency of a Carnot cycle is 15%. If on reducing the temperature of the sink by 65°C, the efficiency becomes 30%, find the initial and final temperatures between which the cycle is working.** [4]

» Please refer to 2076 Set C Q.No. 10c

47. 2070 Set D Q.No. 10 c **A petrol engine consumes 25kg of petrol per hour. The calorific value of petrol is 11.4×10^6 cal/kg. The power of the engine is 99.75KW. Calculate the efficiency of the engine.** [4]

Solution

Given,

Efficiency of engine (η) = ?

Calorific value of petrol = 11.4×10^6 cal/kg

= $11.4 \times 10^6 \times 4.2$ J/kg

Engine consumes = 25 kg/hr

Now, input power = $\frac{25 \times 11.4 \times 10^6 \times 4.2}{60 \times 60}$ = 332500 watt

Output power = 99.75 KW = 99750 W

Now,

Efficiency (η) = $\frac{\text{Power output}}{\text{Power input}} \times 100\% = \frac{99750}{332500} \times 100\% = 30\%$

48. 2069 Supp Q.No. 10b **A Carnot Engine works between 800°C and 400° C. If it is possible either to increase the source temperature by 50°C or to decrease the sink temperature by 50°C, which of these actions will be causing more increase in the efficiency? Justify your answer.** [4]

Solution

Given,

Temperature of source (T_1) = 800°C

Temperature of sink (T_2) = 400°C

Now,

Efficiency (η) = $\left(1 - \frac{T_2}{T_1}\right) \times 100\% = \left(1 - \frac{400 + 273}{800 + 273}\right) \times 100\% = 37.28\%$

When temperature of source is increased by 50°C then new efficiency is,

$\eta' = \left(1 - \frac{T_2}{T_1'}\right) \times 100\% = \left(1 - \frac{400 + 273}{850 + 273}\right) \times 100\% = 40.1\%$

Again, the efficiency of engine when the temperature of sink is decreased by 50°C is,

$\eta'' = \left(1 - \frac{T_2'}{T_1}\right) \times 100\% = 1 - \frac{400 - 50 + 273}{800 + 273} = 42\%$

∴ To decrease the temperature of sink is more efficient.

49. 2069 (Set B) Q. No. 10c **A petrol engine consumes 10 kg of petrol in one hour. The calorific value of petrol is 11.4×10^3 cal/gm. The power of the engine is 20 kwatts. Calculate the efficiency of the engine.** [4]

Solution

Given,

Power of engine (P) = 20 KW = 20000 W

Efficiency of heat engine = ?

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Calorific value of petrol = 11.4×10^3 k cal/gm = 11.4×10^6 cal kg^{-1}

Petrol consumed per second or input power

$= \frac{10}{3600} \times 11.4 \times 10^6$ cals^{-1} $= \frac{10}{3600} \times 11.4 \times 10^6 \times 4.2$ Js^{-1} = 133000 W

% efficiency of engine $= \frac{\text{output power}}{\text{input power}} \times 100\% = \frac{20000}{133000} \times 100\% = 15\ \%$

Hence, the efficiency of heat engine is 15%.

50. 2068 Old Q.No. 4 b **A carnot engine whose low temperature reservoir is at 27°C has an efficiency of 25%. In order to increase the efficiency to 50%, how much the temperature of the high temperature reservoir be increased, if the temperature of the low temperature reservoir remains constant.**

Solution

Given,

First case

Efficiency (η) = 25%

Temp. of cold reservoir (T_2) = 27°C = 273+27 = 300K

Temp of source (T_1) = ?

We have,

$\eta = 1 - \frac{T_2}{T_1}$

or, $0.25 = 1 - \frac{300}{T_1}$

or, $T_1 = 400$ K

Second case

Efficiency (η') = 50%

Temp. of source (T_1') = ?

Temp. of resourvoir (T_2) = 27°C = 300K

Also,

$\eta' = 1 - \frac{T_2}{T_1'}$

$0.50 = 1 - \frac{300}{T_1'}$

$T_1' = \frac{300}{0.5} = 600$ K

Temp. to be raised

$= T_1' - T_1$

$= 600 - 400 = 200$ K

51. 2063 Q.No. 4 b **A petrol engine consumes 5 kg of petrol per hour. If the power of engine is 20 K W and the calorific value of petrol is 11×10^3 k cal per kg. Calculate the efficiency of the engine.**

Solution

Given,

Power of engine (P) = 20 KW = 20000 W

Efficiency of heat engine = ?

Calorific value of petrol = 11×10^3 k cal kg^{-1} = 11×10^6 cal kg^{-1}

Petrol consumed per second or input power

$= \frac{5}{3600} \times 11 \times 10^6$ cals^{-1} $= \frac{5}{3600} \times 11 \times 10^6 \times 4.2$ Js^{-1} = 64166.67 W

% efficiency of engine $= \frac{\text{output power}}{\text{input power}} \times 100\% = \frac{20000}{64166.67} \times 100\% = 31.2\ \%$

Hence, the efficiency of heat engine is 31.2 %

□□□

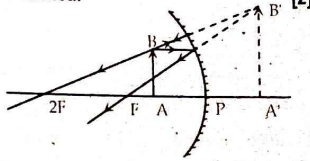
UNIT 3 Geometrical Optics

Chapter 1: Reflection at Curved Mirrors

Short Answer Questions

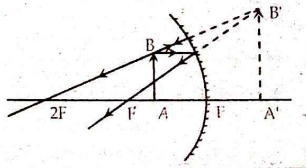
1. 2074 Set B Q.No. 3a **Use graphical method to locate image of an erect object placed before a concave mirror between its pole and the focus. Hence write down the nature of image so formed.** [2]

When an object AB lies between principal focus and pole of the mirror, it's image A'B' will be formed behind the mirror which is virtual, erect and magnified.



2. 2073 Set C Q.No. 3a **A concave mirror is often used as an aid for applying cosmetics to the face, why?** [2]

For shaving or make-up purpose, I suggest concave mirror. When a concave mirror is used for this purpose, our face or object lies between focus and optical centre, so that highly enlarged, virtual, erect image is formed behind the mirror show in diagram.



3. 2073 Set D Q.No. 3a **Convex mirrors are used as rear view mirrors in cars, why?** [2]

Convex mirrors are used as rear view mirrors in automobiles to see the traffic at the backside. They provide wider angle of view, although the images are diminished. They make virtual, erect and diminished image of wide range.

4. 2072 Supp Q.No. 3b **Does focal length change when the curve mirror is dipped in liquid? Why?** [2]

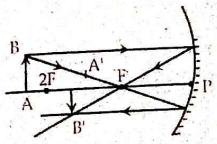
No, the focal length of a spherical mirror does not change when it is immersed in water because the focal length of mirror is independent to the medium in which it is kept. As $\left(\frac{1}{f} = \frac{1}{u} + \frac{1}{v}\right)$, it is independent of refractive index of medium. So, the focal length of mirror does not change when it is kept in water.

5. 2071 Set C Q.No. 3 b **Spherical mirror may behave as a plane mirror as a special case. Explain.** [2]

For spherical mirror, the focal length and radius of curvature are related as, $f = \frac{R}{2}$. For $f = \infty$, $R = \infty$, which is case of plane mirror. Hence, spherical mirror may behave as a plane mirror as special case when its radius of curvature becomes infinity.

6. 2070 Supp (Set B) Q.No. 3 a **A spherical mirror is cut in half horizontally. Will an image be formed by the botom half of the mirror? How?** [2]

When we cut a shperical mirror into half horizontally, the radius of curvature is not changed. The focal length thus remains the same and the image formed will be that of the usual case. The only difference that happens is that the intensity of the images will be reduced as half of the part of the mirror is removed.



7. 2070 Set D Q.No. 3 b **Distinguish between real and virtual images.** [2]

The differences between real image and virtual image are as follows:

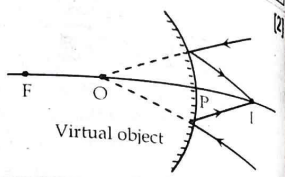
Real image	Virtual image
a. An image is said to be real, if it is formed by the actual intersection of reflected or refracted rays.	a. An image is said to be virtual if it is formed by the virtual intersection of reflected or refracted ray.

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b. They can be received in screen.	b. They cannot be received in screen.
c. Real images are inverted with respect to the object.	c. Virtual images are erect with respect to object.
d. Image formed by concave mirror is generally real.	d. Image formed by convex mirror is generally virtual.

8. 2069 (Set A) Q. No. 3a **Can a convex mirror ever form real image? Justify your answer.** [2]

Yes, a convex mirror can form a real image if the object is virtual. That is, if a converging beam is incident on a convex mirror, a real image is formed in front of it. In such case, the object distance is less than the focal length of the mirror.



9. 2069 (Set A) Old Q. No. 6a **Can a convex mirror ever produce a real image?** [2]

Please refer to 2069 (Set A) Q. No. 3a

10. 2068 Old Q.No. 6 b **What types of mirror will you suggest for shaving or make-up purpose and why?** [2]

Please refer to 2073 Set C Q.No. 3a

11. 2065 Q. No. 6 a **A spherical mirror is immersed in water. Will its focal length change?** [2]

Please refer to 2072 Supp Q.No. 3b

12. 2059 Q.No. 6 b **Trace the position of an image formed by a concave mirror when real object is placed at a distance less than its focal length.** [2]

Please refer to 2074 Set B Q.No. 3a

13. 2056 Q.No. 6 a **Why are convex mirrors used in cars for rear view?** [2]

Please refer to 2073 Set D Q.No. 3a

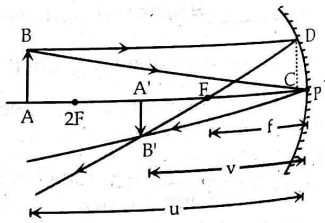
Long Answer Questions

14. 2076 Set C Q.No. 7a **Derive mirror formula for a concave mirror and state the sign convention used.** [4]

Mirror formula: The relationship between focal length of mirror with object distance and image distance is called mirror formula.

Consider a concave mirror of focal length f which has to relate with object distance u and image distance v. Let AB be an object placed normally on the principal axis of the mirror. The rays of light from the object AB, after reflection from concave mirror meet at point B'. So A'B' is the real image of the object AB. If the aperture of the mirror is small, the point P i.e pole of the mirror coincides with point C. Since Δ's ABC and A'B'C are similar, so their corresponding sides are proportional.

$$\frac{AB}{A'B'} = \frac{CA}{CA'} \quad \text{... (i)}$$



Similarly Δ's CDF and A'B'F are similar.

$$\therefore \frac{CD}{A'B'} = \frac{CF}{FA'}$$

But CD = AB

$$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'} \quad \text{... (ii)}$$

From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

$$\text{or, } \frac{CA}{CA'} = \frac{CF}{CA' - CF}$$

$$\text{or, } \frac{u}{v} = \frac{f}{v - f},$$

$$\text{or, } uv - uf = vf$$

$$\text{or, } uv = uf + fv$$

where, CA = u is object distance taken +ve for real object

CA' = v is image distance taken +ve for real image

CF = f is focal length taken +ve for concave mirror

Dividing both sides by uv

$$\frac{uv}{uv} = \frac{uf}{uv} + \frac{vf}{uv}$$

$$\text{or, } 1 = \frac{f}{v} + \frac{f}{u}$$

$$\text{or, } 1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$ which is mirror formula.

The nature of image is inverted, real and different size according to position of object.

15. 2074 Set A Q.No. 7b **Derive mirror formula in the case of concave mirror. Also discuss the nature of the image formed due to object placed at different positions.** [4]

Please refer to 2076 Set C Q.No. 7a

16. 2073 Supp Q.No. 7a **Derive mirror formula in the case of concave mirror. Also discuss the nature of images.** [4]

Please refer to 2076 Set C Q.No. 7a

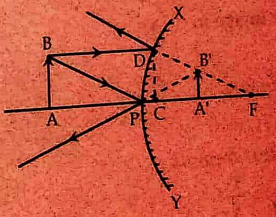
17. 2071 Set C Q.No. 7 b **How will you make difference between real and virtual images? Obtain an expression for the relation between object distance, image distance and the focal length in the case of a convex mirror.** [4]

The differences between real image and virtual image are as follows:

Real image	Virtual image
a. An image is said to be real, if it is formed by the actual intersection of reflected or refracted rays.	a. An image is said to be virtual if it is formed by the virtual intersection of reflected or refracted ray.
b. They can be received in screen.	b. They cannot be received in screen.
c. Real images are inverted with respect to the object.	c. Virtual images are erect with respect to object.
d. Image formed by concave mirror is generally real.	d. Image formed by convex mirror is generally virtual.

Mirror formula for convex mirror: Let us consider a convex mirror XY of focal length f which has to relate with object distance u and image distance v. Let AB be an object placed normally on the principal axis of the mirror. The rays of light from the object AB after reflection from convex mirror appear to meet at point B'. So A'B' is the virtual image of the object AB. If the aperture of the mirror is small, the point P i.e pole of the mirror coincides with point C. Since Δ's ABC and A'B'C are similar so their corresponding sides are proportional.

$$\therefore \frac{AB}{A'B'} = \frac{CA}{CA'} \quad \text{... (i)}$$



Similarly Δ's CDF and A'B'F are similar.

$$\therefore \frac{CD}{A'B'} = \frac{CF}{FA'}$$

But CD = AB

$$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'} \quad \text{... (ii)}$$

From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

$$\text{or, } \frac{CA}{CA'} = \frac{CF}{CF + A'C}$$

$$\text{or, } \frac{u}{-v} = \frac{-f}{-f + v},$$

$$\text{or, } uv - uf = vf$$

$$\text{or, } uv = uf + fv$$

where, CA = u is object distance

CA' = -v is image distance for virtual image

CF = -f is focal length for convex mirror.

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Dividing both sides by uv

$$\frac{uv}{uf} = \frac{uf}{uv} + \frac{vf}{uv}$$

$$\text{or, } 1 = \frac{f}{v} + \frac{f}{u}$$

$$\text{or, } 1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$ which is mirror formula.

From ΔABC and $A'B'C$ the magnification is defined as

$$m = \frac{A'B'}{AB} = \frac{CA'}{CA} = \frac{-v}{u}$$

$$\therefore m = \frac{-v}{u}$$

18. 2069 (Set B) Q. No. 7a 2061 Q.No. 7a **Point out the difference between real image and virtual image. Obtain a relation connecting the object distance, image distance and focal length of concave mirror.** [4]

First Part: Please refer to 2071 Set C Q.No. 7 b

Second Part: Please refer to 2076 Set C Q.No. 7a

19. 2064 Q.No. 7 a **What do you mean by principal focus of a convex mirror? Prove mirror formula for a convex mirror and also show that m = – v/u for the same mirror.** [1+3+1]

Principal Focus: When a narrow beam of light, parallel to the principal axis and close to it is incident on curved mirror, after reflection appears to meet at a point, this point is called principal focus. It is usually denoted by F.

Second Part: Please refer to 2071 Set C Q.No. 7 b

Numerical Problems

20. 2071 Set D Q.No. 11 **At what position an object be placed in front of a concave mirror of radius of curvature 0.4 m so that an erect image of magnification 3 be produced?** [4]

Solution

Given,
Radius of curvature (R) = 0.4 m
Magnification (m) = 3
Object distance (u) = ?
Now,

$$\text{Focal length } (f) = \frac{R}{2} = \frac{0.4}{2} = 0.2 \text{ m}$$

For erect image m = – ve
So,

$$m = \frac{-v}{u} = 3$$

$$\text{or, } v = -3u$$

From mirror formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{0.2} = \frac{1}{u} - \frac{1}{3u}$$

$$\text{or, } 5 = \frac{3-1}{3u}$$

$$\text{or, } 5 = \frac{2}{3u}$$

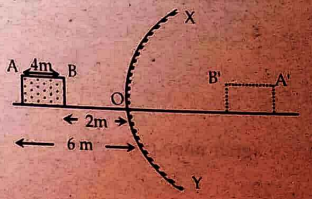
$$\text{or, } u = \frac{2}{3 \times 5}$$

$$\therefore u = 0.133 \text{ m}$$

21. 2066 Old Q. No. 7 b **A pole 4 m long is laid along the principal axis of a convex mirror of focal length 1m. The end of the pole nearer the mirror is 2 m from it. Find the length of the image of the pole.** [4]

Solution

Given,
Length of pole AB = 4m
Focal length of mirror (f) = – 1m
Length of image A'B' = ?
Let B be nearer point from the mirror which is 2m from the pole of mirror, then far point A is 2+4 = 6m form the pole of mirror.
Now, for nearer end,
u = 2m, f = – 1m, v = ?



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{-1} = \frac{1}{2} + \frac{1}{v}$$

$$\text{or, } \frac{1}{v} = -1 - \frac{1}{2} = -\frac{3}{2} \text{ m}$$

$$\text{or, } v = -\frac{2}{3} \text{ m}$$

For fat end,
u' = 6m, v' = ?, f = – 1m

$$\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$$

$$\text{or, } \frac{1}{-1} = \frac{1}{6} + \frac{1}{v'}$$

$$\text{or, } \frac{1}{v'} = -1 - \frac{1}{6} = -\frac{7}{6} \text{ m}$$

$$\text{or, } v' = -\frac{6}{7} \text{ m}$$

$$\therefore \text{The length of image } A'B' = \left(\frac{6}{7} - \frac{2}{3}\right) = 0.19\text{m}$$

22. 2060 Q.No. 7 b **An erect image, three times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?**

Solution

Given,
Magnification (m) = 3
Radius of curvature (r) = 36 cm

$$\text{Focal length } (f) = \frac{r}{2} = \frac{36}{2} = 18 \text{ cm}$$

Object distance (u) = ?
We have,

$$m = \frac{-v}{u} = 3$$

$$v = -3u$$

$$\text{or, } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{18} = \frac{1}{u} - \frac{1}{3u}$$

$$\text{or, } \frac{1}{18} = \frac{3-1}{3u} = \frac{2}{3u}$$

$$\text{or, } 3u = 36$$

$$\text{or, } u = 12 \text{ cm}$$

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23. 2057 Q.No. 7 b **An object 10 cm high is placed in front of a convex mirror of focal length 20 cm and the object is 30 cm from the mirror. Find the height of the image.**

Solution

Given,
Height of object (O) = 10 cm
Height of image (I) = ?
Focal length of convex mirror (f) = –20 cm
Object distance (u) = 30 cm
If v is the image distance, then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-20} = \frac{1}{30} + \frac{1}{v} \quad \Rightarrow v = -12 \text{ cm}$$

Since, magnification (m) $= \frac{I}{O} = \frac{v}{u}$

$$\therefore \frac{I}{10} = \frac{12}{30}$$

$$\Rightarrow I = 4 \text{ cm} = 0.04 \text{ m}$$

Thus, height of image is 0.04 m.

24. 2055 Q.No. 15 b **Calculate the focal length of a concave mirror when an object placed at a distance of 40 cm makes image equal to the size of the object.**

Solution

Given,
Focal length (f) = ?
Object distance (u) = 40 cm
We have,

$$m = \frac{v}{u} = \frac{I}{0}$$

But, I = 0, m = 1

$$m = \frac{v}{u} = 1$$

or, $v = u = 40$ cm

So, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{40} + \frac{1}{40}$$

or, $f = 20$cm

25. 2053 Q.No. 13 **A metre scale is placed along the axis of a convex mirror of focal length 25 cm, its nearer end being at a distance of 50 cm. Calculate the size of the image formed.**

Solution

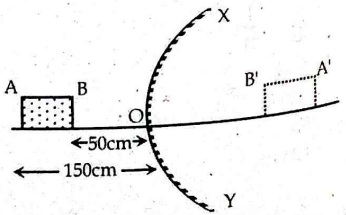
Given,
Focal length of the convex mirror (f) = – 25cm
In the given figure, nearer end B of a meter scale AB is at a distance of 50 cm from the pole (O) of the convex mirror and far end A of the scale is at a distance of 150 cm form the pole O.
For the near point B of scale, u = 50 cm
Using mirror formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$-\frac{1}{25} = \frac{1}{50} + \frac{1}{v}$$

where v is the image distance for B

$v = -16.67$ cm



Again, for far end A,
$u' = 150$ cm
Using mirror formula, we get

$$\frac{1}{f'} = \frac{1}{u'} + \frac{1}{v'}$$

$$-\frac{1}{25} = \frac{1}{150} + \frac{1}{v'}$$

$\therefore v' = -21.43$ cm

So, size of image (A'B') = (21.43 – 16.67) cm = 4.67 cm.

26. 2050 Q.No. 15 **A convex mirror with a radius of curvature 30 cm forms a real image 20 cm from its pole. Explain how it is possible and find whether the image is erect or inverted.**

Solution

Given,
Radius of curvature of a convex mirror (r) = –30 cm
Image distance (v) = 20 cm
Object distance (u) = ?
Focal length of convex mirror (f) $= \frac{r}{2} = \frac{-3}{2} = -15$ cm

We know, $\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{-15} - \frac{1}{20} = \frac{3-4}{60} = \frac{-7}{60}$

$\therefore u = -\frac{60}{7} = -8.57$ cm

The negative sign shows that the object is virtual. Since u and v have opposite signs, the image is erect with respect to the object.

□□□

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Chapter 2: Refraction at Plane Surfaces

Short Answer Questions

1. **2076 Set C Q.No. 3a Why does the bottom of a swimming pool always appear shallower than it actually is? [2]**

This is due to refraction (bending) of light while passing from water to air. In such case, object lies in denser medium and is viewed from rarer medium. So when light passes from denser to rarer medium, it bends away from normal so that real position is shifted to apparent position. Apparent depth is always less than real depth and hence, a clear pool of water always appears to be shallower than it is actually.

2. **2070 Supp (Set A) Q.No. 3 a Why does the sun look slightly oval when it is near the horison? [2]**

This is due to refraction of light when passes one medium to another. The density of the air in the atmosphere is greater near the surface of earth than above it. Thus, the magnitude of refraction increases with decreasing altitude. Due to this, the rays from the lower edge of the sun are bent more than the rays from the upper edge when situated at horizon. Hence, vertical diameter of the sun is shortened more than the horizontal diameter. That is why, the sun appears flattened at the time of sunset and sunrise because of refraction of light.

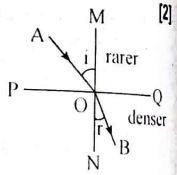
3. **2069 (Set A) Old Q. No. 6b A stick partially dipped in water seems to be bent. Why? [2]**

This is due to refraction of light. When a stick is partially dipped in water, due to refraction of light, it seems to be bent. When light rays pass from denser medium to rarer medium, then rays bent away from the normal. When a stick is placed in water partially, the light rays from different portions of the dipped part of the stick undergo bending away from the normal to reach the eyes. So, it seems to be bending.

4. **2062 Q.No. 6 b Can total internal reflection be achieved if object originates in rarer medium? Explain with a diagram to justify your answer. [2]**

No, total internal reflection can not be achieved if object originates in rarer medium. For total internal reflection, the light ray must pass from denser to rarer media and angle of incidence in denser medium must be greater than critical angle so that angle of refraction in rarer medium is 90°.

If light passes from rarer to denser medim, the light ray bend towards normal and can't be achieved total internal reflection.



5. **2061 Q.No. 6 b A ray of light in air strikes a glass surface. Is there a range of angles for which total internal reflection takes place? [2]**

No, there is not a range of angles for which total internal reflection takes place when a ray of light in air strikes a glass surface. It is because for total internal reflection, light should travel from denser medium to rarer medium. But here, air is rarer medium and glass is denser medium. So, this phenomenon does not happen in this case.

6. **2058 Q.No. 6 c Why does diamond sparkle? [2]**

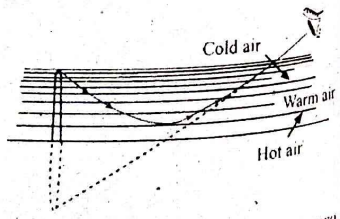
Diamond shines because of total internal reflection of light. The refractive index of diamond is 2.4 and hence, its critical angle is small, 24.6°. The surfaces of the diamond are cut such that light in it undergoes multiple total internal reflections. The light coming out of some faces makes brilliancy of the diamond.

7. **2057 Q.No. 6 b Why does the sun look a little oval when it is at the horizon? [2]**

Please refer to 2070 Supp (Set A) Q.No. 3 a

8. **2054 Q.No. 11 b Why do in summer, roads often appear to be covered with water when seen from a distance? Explain. [2]**

This is due to total internal reflection of light. In summer, during day time, the layers of air in contact with the road become very hot and upper layer of air remains cold, so the densities and refractive indices of these layers are lower than those of the cooler layers higher up. The rays of light from a distance object bend more and more as they pass through denser (cold air) to rarer (hot air) medium. When the beam falls on a layer at an angle greater than the critical angle for it, it gets totally reflected. These reflected rays then travel up and undergo a series of refractions, but in a direction opposite to those in the first case. These rays finally reach observer's eyes, which see an inverted image of the object, as though reflected from a pool of water.



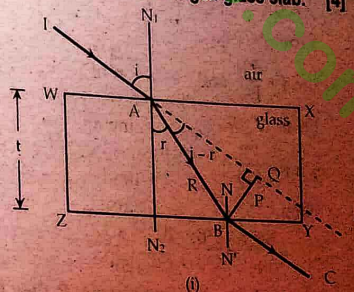
9. **2054 Q.No. 11 b Why does a clear pool of water always appear to be shallower than it actually is? [2]**

Please refer to 2076 Set C Q.No. 3a

Long Answer Questions

10. **2076 Set B Q.No. 7a Define lateral shift and derive an expression for it due to a parallel edged glass slab. [4]**

The perpendicular distance between the direction of incident ray and emergent ray of light is called the lateral shift. It is denoted by P.



(i)

Let us consider a glass slab WXYZ of thickness t. When a ray of light IA is incident on the surface of the slab at A, it is refracted along AB and at last emerges out along BC as shown in fig. N_1N_2 and NN' are normals drawn at A and B respectively. BQ is a perpendicular distance between direction of incident ray and emergent ray which is called as BQ is lateral shift.

From ΔABQ, we have

$$\sin(i-r) = \frac{BQ}{AB}$$

or, $BQ = AB \sin(i-r)$ ... (i)

Again from ΔABN_2, we have

$$\cos r = \frac{AN_2}{AB} = \frac{t}{AB} \quad [\because t = AN_2 = \text{thickness of slab}]$$

or, $AB = \frac{t}{\cos r}$

So, Eq. (i) becomes

$$BQ = \frac{t}{\cos r}\sin(i-r)$$

$$\therefore BQ = \frac{t\sin(i-r)}{\cos r}$$

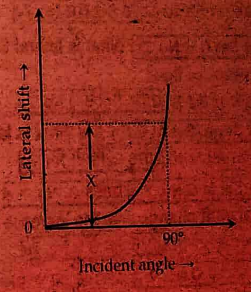
$$\therefore P = \frac{t\sin(i-r)}{\cos r}$$

when, $i = 90°$

$$BQ = P = \frac{t\sin(90°-r)}{\cos r}$$

$$= \frac{t\cos r}{\cos r}$$

$$\therefore P = t$$



(ii)

It means that, when incident angle increases, the lateral shift increases and at incident angle of 90° on the surface of glass slab, lateral shift produced by it is equal to the thickness of the slab as shown in fig(ii). From fig(ii), we see that the value of lateral shift slightly increases on increasing incident angle and become hightest on incident angle of 90°.

11. **2074 Supp. Q.No. 7a What causes the bending of light when it passes from one medium to another medium? [4]**

Deduce the relation, $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

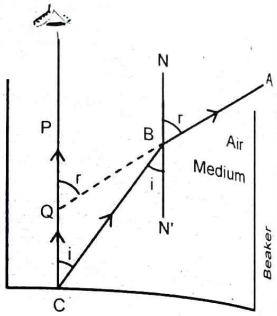
When light passes from one medium to another medium, its wavelength and speed changes. Due to change in speed, it deviates from its original path and hence bends at boundary of two media. Thus difference in speed of light in different medium is the cause of bending of light while passes from one medium to another medium.

Let us consider a coin C is placed in a breaker containing a medium of refractive index, μ. Let CB be the ray of light passing from denser medium and bends to BA while passing from denser medium to air medium. Let NN' be the normal making incident angle i and refracted angle r. Draw AB to Q such

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that the coin is seen at Q from the eye. Then, PC = real depth and PQ = apparent depth.
The refractive index of medium in this case is given as

$${}_a\mu_m = \frac{\sin r}{\sin i}$$ [here light is passing denser to rarer medium]



From figure,
$\angle CBN' = \angle QCB = \angle i$
$\angle NBA = \angle PQB = \angle r$
From ΔCPB,

$\text{Sin } i = \frac{PB}{BC}$ and

From ΔQPB,

$\text{Sin } r = \frac{PB}{BQ}$

Then,

$${}_a\mu_m = \frac{\text{Sir } r}{\sin i} = \frac{PB}{QB} \times \frac{BC}{PB} = \frac{BC}{BQ}$$

If point B is very close to point P, then BC ≈ PC, BQ ≈ PQ

$$\therefore {}_a\mu_m = \frac{PC}{PQ} = \frac{\text{Real depth}}{\text{Apparent depth}} \qquad \left[\begin{array}{l}\because \text{PC = Real depth} \\ \text{PQ = Apparent depth}\end{array}\right]$$

Hence proved.

12. 2073 Set D Q.No. 7a **Define lateral shift. Derive an expression for it. Show in a graph the variation of lateral shift with the angle of incidence.** [4]
Please refer to 2076 Set B Q.No. 7a

13. 2072 Set C Q.No. 7b **What is lateral Shift? Deduce lateral shift in terms of thickness of a slab and the angle of incidence of light.** [4]
Please refer to 2076 Set B Q.No. 7a

14. 2070 Supp (Set A) Q.No. 7 a **What is lateral shift? Derive an expression for it due to a rectangular glass slab.** [4]
Please refer to 2076 Set B Q.No. 7a

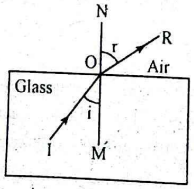
15. 2070 Set D Q.No. 7 a 2065 Q. No. 7 a **What is lateral shift? Derive an expression for its value. How does the lateral shift change with the increase in the angle of incidence?** [4]
Please refer to 2076 Set B Q.No. 7a

16. 2067 Supp Q.No. 7 a **Derive an expression for lateral shift. Show in a graph how does the lateral shift vary with the increase in angle of incidence '?** [4]
Please refer to 2076 Set B Q.No. 7a

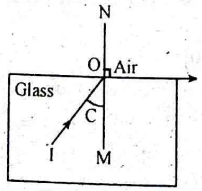
17. 2067 Q.No. 7 a **What is lateral shift? Derive an expression for it due to a parallel edged glass slab.** [4]
Please refer to 2076 Set B Q.No. 7a

18. 2056 Q.No. 7 a **What do you mean by critical angle and total internal reflection? Derive a relation between critical angle and refractive index.** [1+1+3]

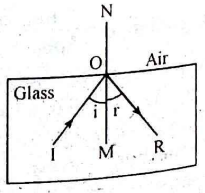
Critical angle: When light passes from denser medium to rarer medium, the refracted ray bends away from normal. If angle of incidence in denser medium is increased gradually, angle of refraction also goes on increasing and at certain incident angle; the refracted angle is 90°. The angle of incidence in denser medium for which angle of refraction is 90° in rarer medium is called critical angle. It is denoted by 'C'.



i = angle of incidence
r = angle of refraction
i < c



C = critical angle
i = c



i > c

Total internal reflection: When the angle of incidence in denser medium is made slightly greater than critical angle C, the light is reflected in the same denser medium obeying the laws of reflection and losed the laws of refraction. This is known as total internal reflection.
Total internal reflection is shown in figure for which angle of incidence is greater than critical angle.
Relation between μ and C: Let 'μ' be the refractive index of denser medium with respect to rarer medium. If C be the angle of incidence C (critical angle), r is angle of refraction in rarer medium. Then refractive index is defined as the ratio of the sine of angle of refraction to the sine of angle of incidence.

i.e. $\mu = \frac{\sin r}{\sin i}$

But for total internal reflection, i = C and r = 90°

$\mu = \frac{\sin 90°}{\sin C} \Rightarrow \mu = \frac{1}{\sin C}$

For glass air system, $\mu = 1.5$ and $C = \sin^{-1}(\frac{1}{1.5}) = 41.5°$

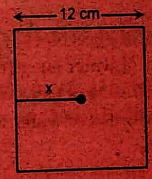
i.e. angle of incidence as critical angle is 41.5° for glass air system.

19. 2052 Q.No. 15 a **Define critical angle and total internal reflection.** [3]
Please refer to 2056 Q.No. 7 a

Numerical Problems

20. 2075 Set A Q.No. 11 **A transparent cube of 12 cm edge contains a small air bubble. Its apparent depth when viewed through one face of the cube is 6 cm and when viewed through the opposite face is 2 cm. What is the actual distance of the bubble from the first face?** [4]

Solution
Given,
Edge of cube = 12 cm
Let the actual distance of the bubble from the first face be x cm than,



$\mu = \frac{x}{6}$ ... (i)

For second face,
The actual distance of air bubble = 12 – x, then

$\mu = \frac{12 - x}{2}$ ... (ii)

Equating equation (i) and (ii), we get,

$\frac{x}{6} = \frac{12 - x}{2}$

or, $2x = 72 - 6x$
or, $8x = 72$
$\therefore x = 9$ cm

21. 2075 Set B Q.No. 11 **Calculate the critical angles of (i) glass-water and (ii) water-air interfaces if object lies in the denser medium.** [4]

Solution
Given,
Refractive index of glass $({}_a\mu_g) = 1.5$
Refractive index of water $({}_a\mu_w) = 1.33$
Critical angle of glass water interface = ?
Critical angle of water-air interface = ?
i. We have,
Here, the objective is in denser medium, then critical angle C_{gw} is given as

$$\sin C_{gw} = \frac{1}{{}_w\mu_g} = \frac{1}{{}_w\mu_a \times {}_a\mu_g} = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{1.33}{1.5} = 0.8867$$

or, $C_{gw} = \sin^{-1}(0.8867)$

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$\therefore C_{gw} = 62.46°$

ii. For water-air interface, the objective is in denser medium means in water. So critical angle for water, C_w is

$$\sin C_w = \frac{1}{{}_w\mu_g}$$

or, $\sin C_w = \frac{1}{1.33} = 0.752$

$\therefore C_w = \sin^{-1}(0.752) = 48.76°$

22. **2072 Set D Q.No. 11** **An optical fiber with refractive index 1.72 is surrounded by a glass coating having refractive index 1.50. Find the critical angle for total internal reflection at the fiber glass interface.** [4]

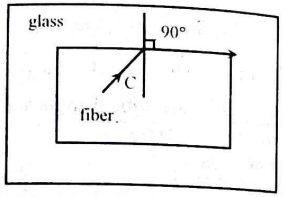
Solution

Given,

R.I. of optical fiber $({}_a\mu_0) = 1.72$

R.I. of glass $({}_a\mu_g) = 1.50$

Critical angle (C) = ?



We have,

$$\text{Sin } C = \frac{1}{{}_g\mu_0} = \frac{1}{{}_g\mu_a \cdot {}_a\mu_0} = \frac{{}_a\mu_g}{{}_a\mu_0}$$

$$= \frac{1.5}{1.72} \quad [{}_g\mu_0 = {}_g\mu_a \cdot {}_a\mu_0 \text{ and } {}_g\mu_a = \frac{1}{{}_a\mu_g}]$$

or, $C = \text{Sin}^{-1}(0.872)$

$= 60.7°$

23. **2071 Set C Q.No. 11** **What is the apparent position of an object below a rectangular glass slab of refractive index 1.45 and the thickness 0.06 m if a layer of water 0.05 m thick is on the top of the glass slab?** [4]

Solution

Given,

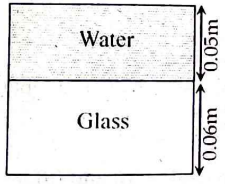
Thickness of glass block $(t_1) = 0.06$ m

Thickness of water layer $(t_2) = 0.05$ m

Refractive index of glass w.r.t. air $(\mu_g) = 1.45$

Refractive index of water w.r.t. air $(\mu_w) = 1.33$

We know,



$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

For glass, $d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 0.06\left(1 - \frac{1}{1.45}\right) = 0.0186$ m

For water $d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 0.05\left(1 - \frac{1}{1.33}\right) = 0.0124$ m

$\therefore$ Total displacement (d) = $d_1 + d_2 = 0.0186 + 0.0124 = 0.031$ m

Hence, the apparent position of the object is 0.031 m above the bottom and $0.11 - 0.031 = 0.0789$ m below the top.

24. **2068 Old Q.No. 7 b** **What is the apparent position of an object below a rectangular block of glass 8 cm thick if a layer of water 6 cm thick is on the top of the glass? (Take: $_a\mu_g = 1.50$ and $_a\mu_w = 1.33$)**

Solution

Given,

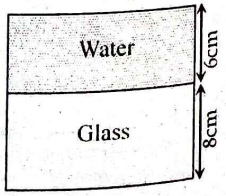
Thickness of glass block $(t_1) = 8$ cm

Thickness of water layer $(t_2) = 6$ cm

Refractive index of glass w.r.t. air $(\mu_g) = 1.5$

Refractive index of water w.r.t. air $(\mu_w) = 1.33$

We know,



$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

For glass, $d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 8\left(1 - \frac{1}{1.5}\right) = 2.6$ cm

For water $d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 6\left(1 - \frac{1}{1.33}\right) = 1.488$ cm

$\therefore$ Total displacement (d) = $d_1 + d_2 = 2.6 + 1.488 = 4.088$ cm

Hence, the apparent position of the object is 4.088 cm above the bottom and $14 - 4.088 = 9.912$ cm below the top.

25. **2067 Q.No. 11** **What is the apparent position of an object below a rectangular glass slab 6 cm thick if a layer of water 4 cm thick is on the top of the glass slab?** [4]

Solution

Given,

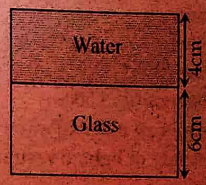
Thickness of glass block $(t_1) = 6$ cm

Thickness of water layer $(t_2) = 4$ cm

Refractive index of glass w.r.t. air $(\mu_g) = 1.5$

Refractive index of water w.r.t. air $(\mu_w) = 1.33$

We know,



$$\text{Displacement (d)} = t\left(1 - \frac{1}{\mu}\right)$$

For glass, $d_1 = t_1\left(1 - \frac{1}{\mu_g}\right) = 6\left(1 - \frac{1}{1.5}\right) = 2$ cm

For water $d_2 = t_2\left(1 - \frac{1}{\mu_w}\right) = 4\left(1 - \frac{1}{1.33}\right) = 0.99$ cm

$\therefore$ Total displacement (d) = $d_1 + d_2 = 2 + 0.99 = 2.99$ cm

Hence, the apparent position of the object is 2.99 cm above the bottom and $10 - 2.99 = 7.01$ cm below the top.

26. **2063 Q.No. 7 b** **Light from a luminous point on the lower face of a rectangular glass slab, 2 cm thick, strikes the upper face and the totally reflected rays outline a circle of 3.2 cm radius on the lower face. What is refractive index of the glass?**

Solution

Let O be the object lying at a lower face of a rectangular glass slab EFGH.

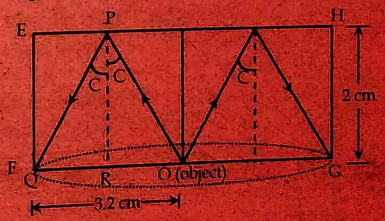
Now,

Thickness of glass slab, PR = 2 cm

Radius of circle, OQ = 3.2 cm

Refractive index of glass, μ = ?

From figure,



$$QR = \frac{OQ}{2} = \frac{3.2}{2} = 1.6 \text{ cm}$$

$\tan C = \frac{QR}{PR}$, where C is critical angle

$$C = \tan^{-1}\left(\frac{1.6}{2}\right) = 38.66°$$

So, $\mu = \frac{1}{\sin C} = \frac{1}{\sin 38.66°} = 1.6$

27. **2069 Q.No. 7 b** **A microscope is focused on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4 cm, and it is found necessary to raise the microscope through a vertical distance of 1.25 cm to bring the scratch again into focus. Find the refractive index of turpentine.** [4]

Solution

Given,

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Real depth = 4 cm

Since microscope is raised through a vertical distance of 1.25 cm to focus the scratch again,

$\therefore$ Apparent depth = 4 – 1.25 = 2.75 cm

We have,

$$\text{Refractive index } (\mu) = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{4}{2.75} = 1.45$$

Hence, refractive index of turpentine is 1.45.

28. 2058 Q.No. 7 b **What is the apparent position of an object below a rectangular block of glass 6 cm thick a layer of water 4 cm thick is on the top of the glass? (Refractive index of glass = 3/2 and that of water = 4/3**

Please refer to 2067 Q.No. 11

29. 2054 Q.No. 13 OR **How long will the light take in travelling a distance of 500 m in water? Refractive index water is 1.33 and velocity of light in vacuum is 3×10^8 m/s.**

Solution

Given,

Time (t) = ?

Distance (d) = 500m

Refractive index of water (μ_w) = 1.33

Velocity of light in vacuum (c) = 3×10^8 m/sec

We have,

$$\mu = \frac{c}{v}$$

$$\text{or, } v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \text{m/sec}$$

$$\text{Now, } t = \frac{d}{V} = \frac{500}{2.26 \times 10^8} = 2.2 \times 10^{-6} \text{ sec.}$$

□□□

Chapter 3: Refraction through Prisms

Short Answer Questions

1. 2076 Set C Q.No. 3b **Under what conditions does a prism produce the angle of minimum deviation? Explain. [2]**

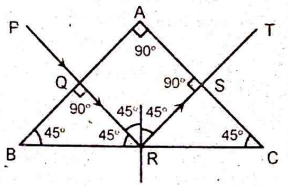
The minimum deviation (δ_{min}) of light occurs when the ray passes symmetrically through the prism. In this case, the angle of emergence is equal to the angle of incidence and angle of refracton at two faces are also equal. The refracted ray through the prism is also parallel to the base of the prism.

2. 2075 Set A Q.No. 3a **What are the conditions for minimum deviation of a ray of light passing through a prism? [2]**

Please refer to 2076 Set C Q.No. 3b

3. 2075 Set B Q.No. 3a **A ray of light originated in air falls perpendicularly on the shorter face of a right angled isosceles (90° – 45° – 45°) triangular glass prism of refractive index 1.5. Find the angle of deviation in this case in a diagram. [2]**

Let ABC is an isosceles right angled prism such that AB and AC are equal. A ray of light PQ incidents on face AB perpendicularly such that it does not derivate and strikes face BC at R with incident angle 45°, but $\mu = 1.5$ i.e. critical angle (C) becomes 42° $\left(\therefore \mu = \frac{1}{\sin C}\right)$.



Here incident angle becomes greater than critical angle and total internal reflection takes place. As a result light ray emerges through path RST. Here derivation angle becomes 90°.

4. 2074 Supp. Q.No. 3b **A glass prism is immersed in water. What happens to the value of angle of minimum deviation? [2]**

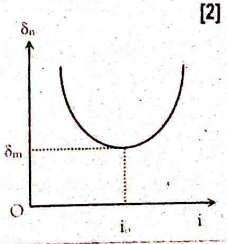
The refractive index (μ) of a glass prism with angle (A) and minimum deviation δ_m are related by a formula,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

When the glass prism is immersed in water, the refractive index decreases and hence angle of minimum deviation also decease being direct relationship.

5. 2074 Set A Q.No. 3b **What do you understand by minimum deviation in the case of refraction through a prism? Plot a graph between angles of incidence and the deviation produced. [2]**

When the angle of incidence increases, the deviation angle at first decreases, reaches to a minimum value and again increases. The minimum value of deviation angle is called minimum deviation angle. It is the condition at which the deviation angle is minimum and starts to increase on increasing incident angle beyond this. At minimum deviation condition, angle of incidence and angle of emergence are equal, the refracting angles at both faces are equal and the refracted light ray inside the prism is parallel to base of prism.



6. 2066 Old Q. No. 6 c **State the factors on which the deviation produced by a prism depend. [2]**

The deviation produced by the prism depends upon the angle of prism, refractive index of a medium, angle of incidence, color of light and the medium around the prism.

7. 2060 Q.No. 6 c **What are the advantages of total reflecting prism over plane mirror? [2]**

The advantages of total reflecting prism over plane mirror are:

a. When light is reflected from total reflecting prism, whole of the light is totally internally reflected. However, from plane mirror, only about 90% of incident light is reflected. Therefore, intensity of reflected light is more in prism than that of the light reflected from plane mirror.
b. No multiple reflections take place in total reflecting prism as in plane mirror.
c. The image is not laterally inverted in case of total reflecting prism as in plane mirror.

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d. Use of total reflecting prism reduces the size of optical instrument than in using plane mirror.

e. It does not require silvering which is one of the advantages.

8. 2051 Q.No. 12 e **Under what condition does a prism produce the angle of minimum deviation?**

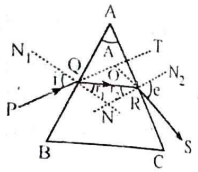
Please refer to 2076 Set C Q.No. 3b [2]

Long Answer Questions

9. 2074 Set B Q.No. 7a **Explain with necessary theory of measuring the refractive index of a prism by minimum deviation method.**

Prism is a triangular shaped refracting medium. It has two refracting faces, a base and a refracting angle. Let us consider a prism ABC, AB and AC are its two refracting faces with angle of prism A as shown in figure. [4]

Let us consider light ray PQ incident on the face AB of prism with an angle i then refracted along QR and finally emerge out with emergent angle from the face AC of the prism. r_1 and r_2 are the angle of refractions for the prism. The angle of deviation δ of light caused by the prism is the angle between the direction of incident ray and emergent ray.



From figure, the angle of deviation $\delta = \angle TOS$

and $\angle TOS = \angle OQR + \angle ORQ$

$= (\angle OQN - \angle RQN) + (\angle ORN - \angle QRN)$

$= (i - r_1) + (e - r_2)$

i.e. $\delta = i + e - (r_1 + r_2)$ ... (i)

Again, from quadrilateral AQNR, $\angle AQN = 90°$ and $\angle ARN = 90°$

$\therefore \angle A + \angle QNR = 180°$ ... (ii)

Also, from ΔQRN, $r_1 + r_2 + \angle QNR = 180°$ ... (iii)

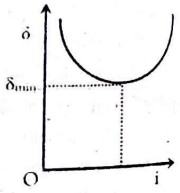
From (ii) and (iii), we get

$r_1 + r_2 = A$ ... (iv)

Using (iv) in (i), we get

$\delta = i + e - A$ ... (v)

Experimentally, it is found that if the angle of incidence is gradually increased from zero, the deviation δ goes on decreasing to a certain minimum value δ_{min} for a certain angle of incidence and then again the deviation angle increases as shown in figure.



In minimum deviation position ($\delta = \delta_{min}$), i = e and $r_1 = r_2$.

Thus, from (v), we get

$\delta_{min} = i + i - A = 2i - A$

$i = \frac{A + \delta_{min}}{2}$ ... (vi)

Similarly, $r_1 = r_2 = r$, equation (iv) gives

$r + r = A \Rightarrow 2r = A$

$\Rightarrow r = \frac{A}{2}$ ... (vii)

If μ is the refractive index of material defined as the ratio of the sine of angle of incidence to sine of angle of reflection, then

$\mu = \frac{\sin i}{\sin r}$

$\therefore \mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}}$ [using (vi) and (vii)]

This is the required expression of refractive index relating the angle of prism and angle of minimum deviation.

10. 2072 Set D Q.No. 7a **Discuss the phenomenon of refraction through a prism. Derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation.** [4]

Please refer to 2074 Set B Q.No. 7a

11. 2072 Set E Q.No. 7a **Derive a relation between the refractive index, angle of minimum deviation and the angle of prism. Also discuss the nature of the change in deviation with respect to the angle of incidence on one of the refracting surface of the prism.**

Please refer to 2074 Set B Q.No. 7a [4]

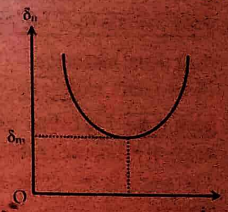
12. 2071 Set C Q.No. 7 a **Explain why a ray of light in air deviates while entering into a medium. Derive an expression for the refractive index of a glass prism if the ray suffers a condition of minimum deviation.** [4]

First Part: When a ray of light passes through a medium, its speed change due to change in wavelength and it deviates.

Second Part: Please refer to 2074 Set B Q.No. 7a

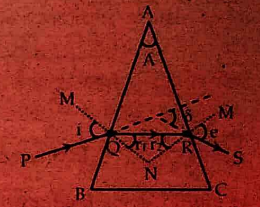
13. 2070 Supp (Set B) Q.No. 7 a **What do you mean by minimum deviation of light when passing through a prism? Show that the deviation produced by a small angle prism is independence of angle of incidence.** [4]

Minimum Deviation: When the angle of incidence increases, the deviation angle at first decreases, reaches to a minimum value and again increases. The minimum deviation is the condition at which the deviation angle is minimum and starts to increase on increasing incident angle beyond it. At minimum deviation condition, angle of incidence and angle of emergence are equal, the refracting angles at both faces are equal and the refracted light ray inside the prism is parallel to base of prism.



Let us consider ABC is the prism having small angle A. PQ is the incident light ray making angle of incidence i, QR is the refracted ray in the prism, RS is the emergent ray.

r_1, r_2, δ and e are angle of refraction at faces AB and AC, angle of deviation and emergent angle respectively. μ be the refractive index of material of prism.



From Snell's law, $\mu = \frac{\sin i}{\sin r_1} \approx \frac{i}{r_1}$ [for refraction at face AB]

[sin i ~ i, for small i]

$\therefore i = \mu r_1$ ... (i) [if i is small r also small]

Similarly, for refraction at face AC, using Snell's law

$\mu = \frac{\sin e}{\sin r_2} \approx \frac{e}{r_2}$

$\Rightarrow e = \mu r_2$ ... (ii)

For prism, we have

$i + e = A + \delta$

$\mu r_1 + \mu r_2 = A + \delta$ [using (i) and (ii)]

$\delta = \mu (r_1 + r_2) - A$

$= \mu A - A$ [$\because A = r_1 + r_2$]

$\therefore \delta = A (\mu - 1)$

This shows that deviation produced by a small angled prism is independent of angle of incidence provided it is small.

14. 2070 Set C Q.No. 7 a **What is prism? Show that the deviation produced by a small angle prism is independent of the angle of incidence, provided the angle of incidence is small.** [4]

First Part: A prism is a refracting surface with its two refracting surfaces inclined at an angle to each other. In figure AB and AC are two refracting surfaces. A is the angle of prism ABC.

The phenomenon of bending of light, when it passes from one medium to another medium is called refraction. When white light is incident on one face of prism, it is dispersed into its constituent seven colours from another side of prism. This is due to the peculier geometrical shape of prism. The refractive index of prism can be calculated by using Cauchy formula;

$\mu = A + \frac{B}{\lambda^2}$

where λ is the wavelength of light incident on prism.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

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15. 2069 (Set B) Q. No. 7b Show that $\mu = \frac{\sin\left(\frac{A+\delta m}{n}\right)}{\sin A/2}$ for a prism where the notations carry their usual meanings. [4]

Please refer to 2074 Set B Q.No. 7a

16. 2069 (Set A) Old Q. No. 7a What is the deviation of light? Show that deviation produced by a small angled prism is independent of the angle of incidence. [1+4]

Deviation of light is the bending of light from its original direction and angle between the direction of incident ray and emergent ray of light is called deviation angle.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

17. 2069 Old (Set B) Q. No. 7a What is prism? Show that $\mu = \frac{\sin\left(\frac{A+\delta m}{n}\right)}{\sin A/2}$ for a prism where the notations carry their usual meanings. [1+4]

Please refer to 2074 Set B Q.No. 7a

18. 2068 Q.No. 7 a Define angle of prism. Prove that the deviation produced by a small angle prism for small angle of incidence is independent of angle of incidence. [4]

The angle between refracting faces of a prism is called angle of prism.

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

19. 2068 Old Q.No. 7 a or What is the cause of dispersion of light? Prove that the deviation produced by a small angle prism for small angle of incidence is independent of angle of incidence. [2+3]

Please refer to 2070 Supp Set B Q.No. 7a

20. 2066 Q. No. 7 b What is minimum deviation for prism? Prove that the refractive index of the prism is $\mu = \frac{\text{Sin}\left(\frac{A+Dm}{2}\right)}{\text{Sin}\,(A/2)}$. Each symbol has its usual meaning. [4]

Minimum deviation for prism: Please refer to 2070 Supp (Set B) Q.No. 7 a

Second Part: Please refer to 2074 Set B Q.No. 7a

21. 2063 Q.No. 7 a Derive an expression connecting refractive index of the material prism with minimum deviation.

Please refer to 2074 Set B Q.No. 7a

22. 2060 Q.No. 7 a Discuss the phenomenon of refraction through prism and show that the deviation of incident ray produced by a small angled prism for small angle of incidence is independent of the angle of prism. [2+3]

First Part: Please refer to 2070 Set C Q.No. 7 a

Second part: Please refer to 2070 Supp (Set B) Q.No. 7 a

23. 2059 Q.No. 7 a Show that $\frac{\sin\frac{(A+D_m)}{2}}{\sin\left(\frac{A}{2}\right)}$ for a prism where the notations carry usual meaning. [5]

Please refer to 2074 Set B Q.No. 7a

24. 2057 Q.No. 7 a Or Derive an expression connecting the refractive index of the material of the prism with the angle of minimum deviation.

Please refer to 2074 Set B Q.No. 7a

25. 2054 Q.No. 12 Derive an expression for the refractive index of the material of the prism in terms of the refracting angle and the angle of minimum deviation. [5]

Please refer to 2074 Set B Q.No. 7a

26. 2050 Q.No. 12 Derive the formula connecting the refractive index of a prism and the angle of minimum deviation, if the angle of the prism is A. [5]

Please refer to 2074 Set B Q.No. 7a

Numerical Problems

27. 2073 Set C Q.No. 11 A ray of light is refracted through a prism of angle 60°. Find the angle of incidence so that the emergent ray just grazes in the second face. Refractive index of the material of the prism is 1.45. [4]

Solution

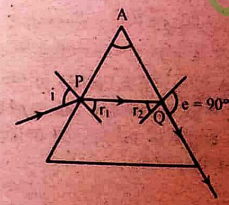
Given,

Angle of prism (A) = 60°

Incidence Angle (i) = ?

Emergent Angle (e) = 90°

Refractive index (μ) = 1.45



We have, at Q,

$_g\mu_a = \frac{\text{Sin}\,r_2}{\text{sine}}$

or, $\text{sin}\,r_2 = \frac{\sin 90°}{a\mu_g} = \frac{1}{1.45} = 0.689$

$\therefore r_2 = 43.6°$

Again,

$A = r_1 + r_2 = r_1 = A - r_2 = 60 - 43.6 = 16.4°$

Now, at P,

$\mu = \frac{\text{Sin}\,i}{\sin r_1}$

or, $\sin I = 1.45 \times \sin 16.4 = 0.41$

$\therefore i = 24.2°$

28. 2072 Supp Q.No. 11 A narrow beam of light is incident normally on one face of an equilateral prism and finally emerges from the prism. The prism is now surrounded by water. What is the angle between the directions of the emergent beam in the two cases? [$\mu_g = 1.45$, $\mu_w = 1.33$] [4]

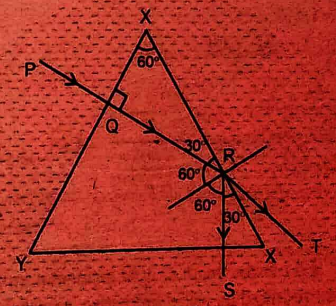
Solution

Given,

Refractive index of glass (μ_g) = 1.45

Refrative index of water (μ_w) = 1.33

Angle between emergence beam = ?



First case, when the prism is in air,

$\sin C = \frac{1}{\mu_g}$

or, $C = \sin^{-1}\left(\frac{1}{\mu_g}\right)$

$= \sin^{-1}\left(\frac{1}{1.45}\right)$

$= 43.6°$

i.e. the light ray suffers total internal relfection.

Second case, when the prism is surrounded by water,

$\sin C = \frac{1}{{}_w\mu_g}$

or, $C = \sin^{-1}\left(\frac{\mu_w}{\mu_g}\right)$

$= \sin^{-1}\left(\frac{1.33}{1.45}\right)$

$= 66.5°$

Since $i < C$, the ray refracted from second face.

Then,

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$$_{g}\mu_{w} = \frac{\sin i}{\sin e}$$

$$\text{or, } \frac{\mu_w}{\mu_g} = \frac{\sin 60}{\sin e}$$

$$\text{or, } e = 70.71°C$$

Thus, the angle between two beams ($\angle$ SRT)

$$= (90° - 60°) + (90° - 70.71°)$$

$$= 49.29°$$

29. **2071 Supp Q.No. 11** **A certain prism is found to produce a minimum deviation of 51°, while it produces a deviation of 62°48' for two values of angle of incidence, namely 40°6' and 82°42' respectively. Determine the refractive angle of prism, the angle of incidence at minimum deviation and the refractive index of the material of the prism.** [4]

Solution

Given,

Angle of minimum deviation (ρ_m) = 51°

Angle of deviation (δ) = 62°48'

Angle of incidence at face 1, (i_1) = 40°6'

Angle of incidence at face 2, (i_2) = 82°42'

Refracting angle of prism (A) = ?

Angle of incidence at minimum deviation (i) = ?

R.I. of prism (μ) = ?

We know,

$$\delta + A = i_1 + i_2$$

$$62° 48' + A = 40°6' + 82°42'$$

or, $A = 60°$

At minimum deviation,

$i_2 = i_2 = i$ and $\delta = \delta_{min}$, so

$$\delta_m + A = i + i = 2i$$

$$\text{or, } i = \frac{\delta_m + A}{2} = \frac{51° + 60°}{2} = 55.5°$$

$$\mu = \frac{\text{Sin}\left(\frac{A + \delta_m}{2}\right)}{\text{Sin}\left(\frac{A}{2}\right)} = \frac{\text{Sin}\left(\frac{60 + 51}{2}\right)}{\text{Sin}\left(\frac{60}{2}\right)} = 1.65$$

$\therefore \mu = 1.65$

30. **2064 Q.No. 7 b** **A narrow beam of light incident normally on one face of an equilateral prism (refractive index 1.45) being surrounded by water (refractive index 1.44). At what angle the ray of light emerges out?** [4]

Solution

Given,

Refractive index of glass ($_a\mu_g$) = 1.45

Refractive index of water ($_a\mu_w$) = 1.33

Angle of emergent (e) = ?

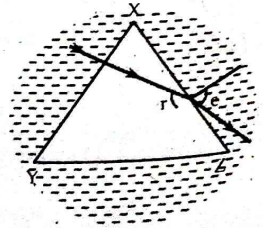
When light falls normally on one face, say XY of prism XYZ, it passes without bending to the face XZ. At face XZ angle of incident is 60° and angle of emergence is e which is shown in Fig.

From Shell's law, we have

$$_g\mu_w = \frac{\sin i}{\sin e}$$

$$\text{or, } {}_g\mu_a \times {}_a\mu_w = \frac{\sin i}{\sin e}$$

$$\text{or, } \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{\sin i}{\sin e}$$



$$\text{or, } \sin e = \frac{{}_a\mu_g}{{}_a\mu_w} \sin i = \frac{2.45}{.33} \times \sin 60°$$

$$\text{or, } \sin e = \frac{1.45}{1.33} \times \frac{\sqrt{3}}{2} = 0.9442$$

$$\text{or, } e = \sin^{-1}(0.9442) = 70.8°$$

Required angle of emergent is 70.8°.

31. **2062 Q.No. 7 B** **2055 Q.No. 13** **A glass prism of angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. Find the angle of minimum deviation for parallel beam of light passing through the prism.**

Solution

Given,

Angle of prism (A) = 72°

Refractive index of glass (η_g) = 1.66

Refractive index of a liquid (η_l) = 1.33

Angle of minimum deviation (δ_{min}) = ?

If $_l\eta_g$ is the refractive index of the material of the glass prism w.r.t a liquid, then

$$_l\eta_g = \frac{\sin\frac{A + \delta_{min}}{2}}{\sin\frac{A}{2}}$$

$$\frac{\eta_g}{\eta_l} = \frac{\sin\frac{72° + \delta_{min}}{2}}{\sin\frac{72°}{2}}$$

$$\frac{1.66}{1.33} = \frac{\sin\frac{72° + \delta_{min}}{2}}{\sin 36°}$$

$$\sin\frac{72° + \delta_{min}}{2} = \frac{1.66}{1.33} \sin 36°$$

$$\frac{72° + \delta_{min}}{2} = \sin^{-1}\left(\frac{1.66}{1.33} \sin 36°\right)$$

$\therefore \delta_{min} = 22.38°$

Hence, required angle of minimum deviation is 22.38°.

32. **2051 Q.No. 16 OR** **A glass prism of angle A and refractive index 1.5 produces the angle of minimum derivation equal to 40°. Calculate the value of angle of prism.**

Solution

Given,

Refractive index (μ) = 1.5

Angle of minimum deviation (δ_m) = 40°

Angle of prism (A) = ?

We know,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\text{or, } 1.5 = \frac{\sin\left(\frac{A + 40°}{2}\right)}{\sin\frac{A}{2}}$$

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$$\text{or, } 1.5 = \frac{\sin\left(\frac{A}{2}+20°\right)}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2}\cdot\cos 20° + \cos\frac{A}{2}\cdot\sin 20°}{\sin\frac{A}{2}}$$

$$= \cos 20° + \cot\frac{A}{2}\cdot\sin 20°$$

$$\text{or, } \cot\frac{A}{2} = \frac{1.5-\cos 20°}{\sin 20°} = \frac{1.5-0.94}{0.342}$$

$$\cot\frac{A}{2} = 1.638$$

$$\text{or, } \tan\frac{A}{2} = \frac{1}{1.63} = 0.61$$

$$\text{or, } \frac{A}{2} = \tan^{-1}(0.61) = 31.4°$$

$$A = 62.8°$$

Hence, required angle of prism is 62.8°.

□□□

Short Answer Questions

1. 2074 Set B Q.No. 3b | 2069 Old (Set B) Q. No. 6c | 2067 Q.No. 3 a A convex lens is immersed in water. Will its focal length change? Explain. [2]

Ans: Yes, the focal length of a lens changes when it is placed in water. From lens maker's formula, focal length of lens in air is

$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \quad \ldots (i)$$

where ${}^a\mu_g$ is refractive index of glass w.r.t. air, f_a is focal length of lens in air, r_1 and r_2 are radii of curvature of two surfaces of the lens.

When the lens in immersed in water, then

$$\frac{1}{f_w} = ({}_w\mu_g - 1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \quad \ldots (ii)$$

where ${}_w\mu_g$ is refractive index of glass w.r.t. water and f_w is focal length of lens in water.

Dividing eqn. (i) by (ii)

$$\frac{f_w}{f_a} = \left(\frac{{}^a\mu_g - 1}{{}_w\mu_g - 1}\right) = \frac{{}^a\mu_g - 1}{\left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)} \qquad \left[\because {}_w\mu_g = {}_w\mu_a \cdot {}^a\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w}\right]$$

$$= \frac{1.5-1}{\frac{1.5}{1.33}-1} = \frac{0.5}{1.1278-1} = \frac{0.5}{0.1278}$$

$$\frac{f_w}{f_a} = 3.9$$

$$\text{or, } f_w = 3.9\, f_a$$

It means f_w is greater than f_a. Hence, focal length of a lens increases in immersing in water.

2. 2073 Set C Q.No. 3b Sun glasses have curved surfaces but their power is zero, why? [2]

Ans: Sun glasses have curved faces or surfaces, but their power is zero because they have two curved surfaces, one of them is convex and another is concave having the same radii of curvature. So, $R_1 = -R_2 = R$, then from lens maker's formula, we know that,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$$

or, P = 0 i.e. Power of sun glasses zero.

3. 2073 Set D Q.No. 3b A lens is immersed in water. Is there any change in its focal length? Explain. [2]

Ans: Please refer to 2074 Set B Q.No. 3b

4. 2070 Supp (Set B) Q.No. 3 b A converging lens is dipped in a liquid having refractive index greater than the lens. Does its power change? Explain. [2]

Ans: A converging lens of power P_1 is kept in a liquid of refractive index μ_l, then the power of lens be P_2. Then we have

$$P_1 = ({}_a\mu_g - 1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$P_2 = ({}_l\mu_g - 1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{P_2}{P_1} = \frac{{}_l\mu_g - 1}{{}_a\mu_g - 1}$$

$$\text{or, } \frac{P_2}{P_1} = \frac{\frac{{}_a\mu_g}{{}_a\mu_l} - 1}{{}_a\mu_g - 1}$$

Here, ${}_a\mu_l > {}_a\mu_g$, so, the power decreases and becomes negative.

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5. 2070 Supp (Set A) Q.No. 3 b **If a lens made of glass is immersed into water, what will happen to its power? Explain.**

The focal length (f_1) of a lens in air is given by: $\frac{1}{f_1} = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ [2]

where ${}_a\mu_g$ is the refractive index of the material of the lens in air, r_1 and r_2 are radii of curvature of two surfaces of the lens.

The power of lens is given by: $P = \frac{1}{\text{focal length (m)}}$. So, power of lens when placed in air is given by:

$$P_1 = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots \text{(i)},$$

Similarly power (P_2) of the lens when it is immersed in water is given by:

$$P_2 = ({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots \text{(ii)}$$

where, ${}_w\mu_g$ is the refractive index of the material of prism w.r.t. to water.
Dividing equation (ii) by equation (i), we get.

$$\frac{P_2}{P_1} = \frac{({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}{({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

$$\frac{P_2}{P_1} = \frac{({}_w\mu_g - 1)}{({}_a\mu_g - 1)} = \frac{\left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right)}{{}_a\mu_g - 1}$$

$$\frac{P_2}{P_1} = \frac{\left(\frac{1.5}{1.33} - 1\right)}{1.5 - 1} = \frac{0.17}{1.33 \times 0.5}$$

$\Rightarrow P_2 = 0.26 \times P_1 \Rightarrow P_2 < P_1$

That means power of lens is decreased when immersed in water.

6. 2069 Supp Q.No. 3a **A convex lens is immersed in water, will its focal length be changed?** [2]

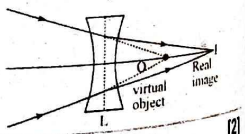
Please refer to 2074 Set B Q.No. 3b

7. 2064 Q.No. 6 c **If a converging lens and a diverging lens having the same focal length be in contact, how will the combination of lenses behave?** [2]

Let f_1 and f_2 be the focal lengths of convex and concave lens respectively, then their combined focal length is given as $\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$, where the focal length of concave lens is taken as negatve. By question, if $f_1 = f_2$, then $F = \infty$. Hence, the combined focal length of the combination is infinity and acts as a plane glass slab.

8. 2061 Q.No. 6 c **Draw the ray diagram showing the formation of real image by a concave lens.** [2]

When converging beam of light is incident on a concave lens, the rays of light are diverged and meet at a point on the principal axis as shown in figure below. In this case, object is virtual and the image of the object is real.



9. 2060 Q.No. 6 b **A lens made of glass is immersed into water. Will its power increase or decrease?** [2]

Please refer to 2070 Supp (Set A) Q.No. 3 b

10. 2059 Q.No. 6 c **Does the focal length of a lens change when immersed in water? Will it increase or decrease?** [2]

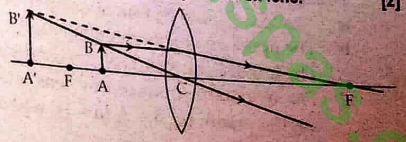
Please refer to 2074 Set B Q.No. 3b

11. 2055 Q.No. 11 a **Can a concave lens from a real image? Give the condition.** [2]

Please refer to 2061 Q.No. 6 c

12. 2053 Q.No. 11 a **Draw the ray diagram showing the formation of a virtual image by a convex lens.** [2]

When an object is placed between focus and optical centre, its image is formed on the same side which is virtual, erect and enlarged as shown in figure. AB is an object placed between optical centre and focus whose image is A'B' which is virtual, erect and magnified formed on the same side.

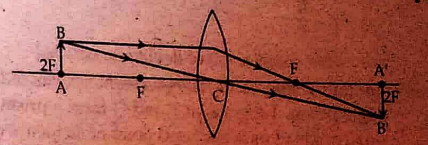


13. 2052 Q.No. 11 b **Does the focal length of a lens change if it is immersed in water? Will it increase or decrease?** [2]

Please refer to 2074 Set B Q.No. 3b

14. 2051 Q.No. 12 a **At what distance from a convex lens should an object be placed on the axis, so as to form a real image of the same size? Illustrate your answer with a ray diagram.** [2]

An object placed at twice of focal length of a convex lens on its axis, it forms a real image of the same size which is real and inverted in nature as shown in figure. In figure, AB is the object place at 2F and A' B' is the image formed at 2F on other side.



15. 2051 Q.No. 12 c **Under what conditions does a concave lens form a real image?** [2]

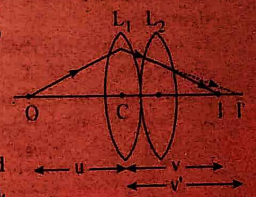
Please refer to 2061 Q.No. 6 c

Long Answer Questions

16. 2076 Set C Q.No. 7b **Derive an expression for the combined focal length of two thin lenses in contact.** [4]

Let us consider two thin convex lenses L_1 and L_2, placed in contact, f_1 and f_2 be their respective focal lengths.
There is a point object O placed on the principal axis at a distance u from optical centre C. In the absence of lens L_2, I' be the image of O formed by lens L_1, at a distance v' from C as shown in figure.
Using lens formula, we get

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \ldots \text{(i)}$$



When lens L_2 is placed in contact with L_1, I' acts as object for L_2 and real image I is formed at a distance v from C. Since lenses are thin, therefore point of contact can be taken as C. Now using lens formula for L_2, we get

$$\frac{1}{f_2} = \frac{1}{(-v')} + \frac{1}{v} \quad \ldots \text{(ii)}$$

Adding (i) and (ii) we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots \text{(iii)}$$

If we replaced the combined lens by a single lens having focal length F produced image I of object O so that

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \ldots \text{(iv)}$$

From eqn (iii) and (iv), we get $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

This is the required expression for the equivalent focal length of two thin convex lenses having focal length f_1 and f_2 respectively.

17. 2075 GIE Q.No. 7a 2070 Set C Q.No. 7 b 2058 Q.No. 7 a 2057 Q.No. 7 a 2056 Q.No. 7 a OR **Derive lens maker's formula.** [4]

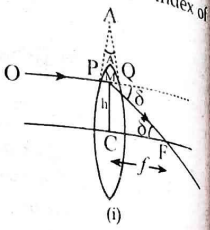
Lens Maker formula: Lens Maker formula gives the relation between focal length, radii of curvature of surfaces and refractive index (μ) of its material.

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Postulates (assumptions):

a. The lens should be thin in comparision to the radii of curvature of the lens surface.
b. The aperture of lens shold be small.
c. The incident rays and refracted rays make small angle with the principal axis and
d. The deviation produced by lens should be similar to the deviation produced by a small angled prism. i.e. $\delta = A(\mu - 1)$, where A is the angle of lens and μ is the refractive index of the lens.

Derivation: Let us consider a ray OP incident on a convex lens parallel to the principal axis at a height h shown in fig (i). Let f be the focal length of the lens. After refraction through the lens, the emergent ray passes through the focus F. If δ is the angle of deviation, then.



(i)

$$\tan\delta = \frac{h}{f} \quad \text{(in } \Delta MCF\text{)}$$

$$\delta = \frac{h}{f} \qquad \ldots (i)$$

[For small angle of deviation, $\tan\delta \approx \delta$]

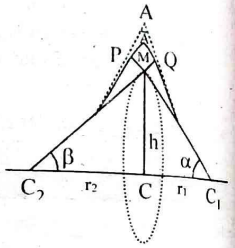
For a thin lens, the deviation can be considered as deviation by a small angle prism. Tangents at P and Q form a prism of small angle A. For a small angle of incidence and point of incidence close to C,

We can write, $\delta = A(\mu - 1)$ ... (ii)

From (i) and (ii), we get

$$\frac{h}{f} = A(\mu - 1)$$

$$\frac{1}{f} = (\mu - 1)\frac{A}{h} \qquad \ldots (iii)$$



(ii)

The normal at P and Q pass through the centres of curvature C_1 and C_2 of lens surfaces as shown in figure (ii).

From the figure (ii) $\angle PMC_2 = \angle QMC_1 = A$

Also, $\angle QMC_1 = \angle MC_2C + \angle MC_1C = \alpha + \beta$

[Since α and β are small angles made by PC_1 and PC_2 with principal axis.]

$$A = \alpha + \beta \approx \tan\alpha + \tan\beta$$

$$= \frac{h}{r_1} + \frac{h}{r_2}$$ [r_1 and r_2 be radii of curvatures which are positive for convex lens.]

$$\therefore \frac{A}{h} = \frac{1}{r_1} + \frac{1}{r_2} \quad \ldots (iv)$$

Using (iv) in (iii), we get

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

which is the required lens Maker formula.

Sign conventions:

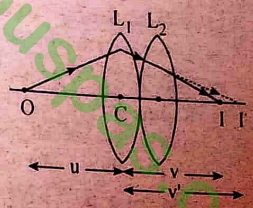
a. Radius of curvature and focal length for convex lens is taken positive and
b. Radius of curvature and focal length for concave lens is taken negative.
c. Image distance and object distance for real image and object are taken as positive.
d. Image distance and object distance for virtual image and object are taken as negative.

18. 2075 Set A Q.No. 7a 2071 Supp Q.No. 7a Define power of a lens. Derive the formula for the effective power of two thin lenses in contact. [4]

Power of lens: Power of lens is its ability to converge or diverge the light rays after passing through it. Mathematically power of lens is defined as the reciprocal of the focal length. i.e.

$P = \frac{1}{f}$, when f is in meter, P is in Diopter.

Let us consider two thin convex lenses L_1 and L_2, placed in contact. f_1 and f_2 be the focal lengths of L_1 and L_2 respectively. There is a point object O placed on the principal axis at a distance u from optical centre C. In the absence of lens L_2, I' be the image of O formed by lens L_1 at a distance v' from C shown in figure.



Using lens formula, we get

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \qquad \ldots (i)$$

When lens L_2 is placed in contact with L_1, I' acts as object for L_2 and real image I is formed at a distance v from C. Since lenses are thin therefore point of contact can be taken as C. Now using lens formula for L_2, we get

$$\frac{1}{f_2} = \frac{1}{(-v')} + \frac{1}{v} \qquad \ldots (ii)$$

Adding (i) and (ii) we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \ldots (iii)$$

If we replaced the combined lens by a single lens having focal length F produced image I of object O so that

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \qquad \ldots (iv)$$

From eqⁿ (iii) and (iv), we get $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

This is the required expression for the equivalent focal length of two thin convex lenses having focal length f_1 and f_2 respectively. The power of the combined lens is then given as,

$$P = \frac{1}{F}$$

$$P = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\therefore P = P_1 + P_2$

Hence, the equivalent power increases on joining two thin lenses.

19. 2075 Set B Q.No. 7b Derive expression for the focal length in Lens maker's formula. [4]

Please refer to 2075 GIE Q.No. 7a

20. 2072 Supp Q.No. 7b 2052 Q.No. 12 Derive lens maker's formula and state the sign convention for radius of curvature. [4]

Please refer to 2075 GIE Q.No. 7a

21. 2072 Set C Q.No. 7a Define principal focus and the power of lens. Derive the formula for the focal length of two thin lenses in contact. [4]

Principal focus: It is the point on principal axis at which all rays parallel to principal axis pass through it after refraction. It is denoted by F. In a concave lens, the rays diverge away from the lens, which appear to come from the focus. By symmetry, every lens has two principal foci, one on each side of the lens. The focus of a convex lens is real and that of the concave lens is virtual.

Power of lens: Power of lens is its ability to converge or diverge the light rays after passing through it. Mathematically power of lens is defined as the reciprocal of the focal length. i.e.

$P = \frac{1}{f}$, when f is in meter, P is in Diopter.

Second Part: Please refer to 2076 Set C Q.No. 7b

22. 2071 Set D Q.No. 7b Derive lens Maker's formula and write the unit of the power of a lens. [4]

Please refer to 2075 GIE Q.No. 7a

Unit of power of lens is diopter (D) or m^{-1}.

23. 2069 Supp Q.No. 7a Define an expression for combined focal length when two lenses are combined coaxially. [4]

Please refer to 2076 Set C Q.No. 7b

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24. 2069 (Set A) Q. No. 7b **Derive an expression for lens maker's formula.**

Please refer to 2075 GIE Q.No. 7a [4]

25. 2068 Old Q.No. 7 a **Define radius of curvature of a lens. Derive Lens maker's formula.** [1+4]

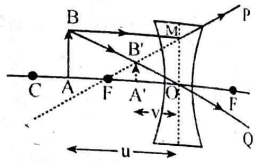
Radius of curvature: The distance between centre of curvature and optical centre of lens or radius of sphere whose small portions form the lens is called radius of curvature of a lens.

2nd part: Please refer to 2075 GIE Q.No. 7a

26. 2066 Old Q. No. 7 a OR **Derive the formula relating the object distance, the image distance and the focal length for a concave lens.** [5]

Lens formula for diverging (concave) lens:

Let us consider an object AB is placed between principal focus F and centre of curvature C as shown in figure. A ray BM parallel to principal axis appears to pass through F after refraction through the lens at M. Another ray BO, originating from B passes directly through optical centre O, which appears to meet at B' so that A'B' became virtual image of object AB formed in between pricnipal focus and optical centre O.



In figure, AO = u, object distance
A'O = -v, image distance and
FO = -f, focal length of lens.

As ΔABO and ΔA'B'O are similar so that

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad \ldots (i)$$

Again, ΔA'B'F and ΔMOF are similar.

So that, $\frac{A'B'}{MO} = \frac{FA'}{FO}$

Since MO = AB,

$$\frac{A'B'}{AB} = \frac{FA'}{FO} \quad \ldots (ii)$$

From (i) and (ii), we get

$$\frac{A'O}{AO} = \frac{FA'}{FO} \Rightarrow \frac{A'O}{AO} = \frac{FO - A'O}{FO}$$

$$\frac{-v}{u} = \frac{-f-(-v)}{-f}$$

[For concave lens, virtual distances are taken negative]

$$\frac{-v}{u} = \frac{-f+v}{-f}$$

$$vf = -uf + uv$$

$$vf + uf = uv$$

Dividing both sides by uvf, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

which is the required lens formula relating object distance.

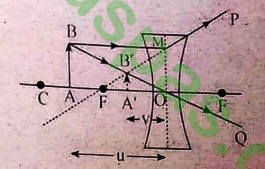
27. 2065 Q. No. 7 a OR **Define principal focus. Derive the lens formula for the concave lens and show that m = -v/u for the same lens.** [1+4]

Principal focus: It is the point on principal axis at which all rays parallel to principal axis pass through it after refraction. It is denoted by F. In a concave lens, the rays diverge away from the lens, which appear to come from the focus. By symmetry, every lens has two principal foci, one on each side of the lens. The focus of a convex lens is real and that of the concave lens is virtual.

Lens formula for diverging (concave) lens:

Let us consider an object AB is placed between principal focus F and centre of curvature C as shown in figure (ii). A ray BM parallel to principal axis appears to pass through F after refraction the lens at M. Another ray BO, originating from B passes directly through optical centre O, which appears to meet at B' so that A'B' became virtual image A'B' of object AB formed in between pricnipal focus and optical centre O.

In figure, AO = u, object distance
A'O = -v, image distance and
FO = -f, focal length of lens.



As ΔABO and ΔA'B'O are similar so that

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad \ldots (i)$$

Again, ΔA'B'F and ΔMOF are similar.

So that $\frac{A'B'}{MO} = \frac{FA'}{FO}$

Since MO = AB,

$$\frac{A'B'}{AB} = \frac{FA'}{FO} \quad \ldots (ii)$$

From (i) and (ii), we get

$$\frac{A'O}{AO} = \frac{FA'}{FO} \Rightarrow \frac{A'O}{AO} = \frac{FO - A'O}{FO}$$

$$\frac{-v}{u} = \frac{-f-(-v)}{-f}$$

[For concave lens, virtual distances are taken negative]

$$\frac{-v}{u} = \frac{-f+v}{-f}$$

$$vf = -uf + uv$$

$$vf + uf = uv$$

Dividing both sides by uvf, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

which is the required lens formula relating object distance.

Magnification: The ratio of height of image to object is called magnification. From above figure

$$m = \frac{A'B'}{AB} = \frac{-v}{u}$$

28. 2064 Q.No. 7 a OR **What do you mean by conjugate focii? Derive lens maker's formula and state sign convention for the radius of curvature.** [1+4]

Conjugate foci: Two points, one on each side on the principal axis lies in two sides of lens, where rays of parallel beam of light also parallel to the principal axis actually converge is called conjugate foci.

Please refer to 2075 GIE Q.No. 7a

29. 2063 Q.No. 7 a OR **Derive lens maker formula for a convex lens.**

Please refer to 2075 GIE Q.No. 7a

30. 2062 Q.No. 7 a **Draw ray diagram to locate the image of a point object placed between the centre of curvature and the principal focus of a diverging lens. Derive lens formula for the diverging lens.** [2+3]

First Part: Figure drawn shows that image formed by the diverging lens when point object is placed between centre of curvature and principal focus.



Second Part: Please refer to 2066 Old Q. No. 7 a OR

31. 2059 Q.No. 7 a OR **Derive the formula relating object distance, image distance and the focal length for a convex lens.**

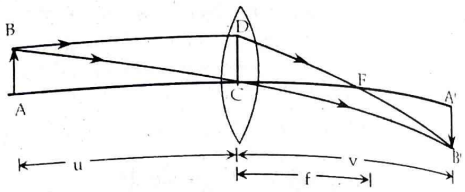
Consider a convex lens of focal length f. Let AB be an object placed normally on the principal axis of the lens. The rays of light from the object AB after refracting through the convex lens meets at point B'. So A'B' is the real image of the object AB.

Since Δs ABC and A'B'C are similar so their corresponding sides are proportional.

$$\frac{AB}{A'B'} = \frac{CA}{CA'} \quad \ldots (i)$$

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Similarly Δ's CDF and A'B'F are similar. So,



$$\frac{CD}{A'B'} = \frac{CF}{FA'}$$

But CD = AB

$$\therefore \frac{AB}{A'B'} = \frac{CF}{FA'} \quad \text{... (ii)}$$

From Eqs. (i) & (ii), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

$$\text{or, } \frac{CA}{CA'} = \frac{CF}{CA' - CF}$$

$$\text{or, } \frac{u}{v} = \frac{f}{v - f},$$

where CA = u is object distance

CA' = v is image distance

CF = f is focal length.

$$\text{or, } uv - uf = vf$$

$$\text{or, } uv = uf + fv$$

Dividing both sides by uv

$$\frac{uv}{uv} = \frac{uf}{uv} + \frac{vf}{uv}$$

$$\text{or, } 1 = \frac{f}{v} + \frac{f}{u}$$

$$\text{or, } 1 = f\left(\frac{1}{v} + \frac{1}{u}\right)$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}, \text{ which is lens formula.}$$

32. **2055 Q.No. 12 Derive the combined focal length of two thin lenses in contact.** [4]

Please refer to 2076 Set C Q.No. 7b

33. **2051 Q.No. 13 Derive an expression for the equivalent focal length of two thin convex lenses in contact.** [4]

Please refer to 2076 Set C Q.No. 7b

34. **2050 Q.No. 14 Describe a method, with a suitable diagram to determine the focal length of a convex lens.**

Please refer to 2059 Q.No. 7 a OR

Numerical Problems

35. **2076 Set B Q.No. 11 A convex lens of focal length 24 cm and of refractive index 1.5 is totally immersed in water of refractive index 1.33. Find its focal length in water.** [4]

Solution

Given,

Focal length of a convex lens (f) = 24 cm

Refractive index of glass ($_a\mu_g$) = 1.5

Refractive index of water ($_a\mu_w$) = 1.33

Focal length of the lens in water (f') = ?

When the lens is in air, then

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{24} = (1.5 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{24} = 0.5\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\text{or, } \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{12} \quad \text{... (i)}$$

When the lens is in water,

$$\frac{1}{f'} = ({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= ({}_w\mu_a \times {}_a\mu_g - 1)\left(\frac{1}{12}\right) \quad \left[\because \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{12} \text{ from eq}^n. \text{ (i)}\right]$$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right) \times \frac{1}{12} = \left(\frac{1.5}{1.33} - 1\right) \times \frac{1}{12} = 0.0106$$

$$\therefore f' = 93.88 \text{ cm}$$

36. **2074 Supp. Q.No. 11 The image of an object is obtained on a screen with the help of a converging lens. The distance of the object is 40 cm from the screen and size of the image is 9 times the size of the object. Calculate distance of the screen from the lens and focal length of the lens.** [4]

Solution

Given,

Let u be the object distance and v be the image distance from the lens. Then from given condition,

$$u + v = 40 \text{ cm}$$

Again, magnification, $m = \frac{v}{u} = 9$

$$\text{or, } v = 9u$$

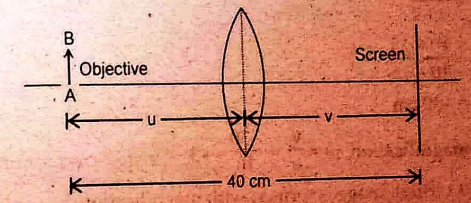
Then,

$$u + 9u = 40$$

$$\text{or, } 10u = 40$$

$$\therefore u = 4$$

and $v = 4 \times 9 = 36$ cm



Again,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{f} = \frac{1}{4} + \frac{1}{36}$$

$$\text{or, } \frac{1}{f} = \frac{9+1}{36}$$

$$\therefore f = \frac{36}{10} = 3.6 \text{ cm}$$

$\therefore$ f = 3.6 cm and v = 36 cm

37. **2074 Set A Q.No. 11 An object is placed at a distance 30 cm from a thin lens of power 4 Dioptre. Discuss the nature of image formed by the lens.** [4]

Solution

Given,

Power of lens (P) = 4D

Object distance (u) = 30 cm

Image distance (v) = ?

We know,

$$\text{Focal length (f)} = \frac{1}{P} = \frac{1}{4} = 0.25\text{m} = 25 \text{ cm}$$

Also, from lens formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{or, } v = \frac{uf}{u - f}$$

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or, $v = \frac{30 \times 25}{30 - 25}$

$\therefore v = 150$ cm

Also, magnification, $m = \frac{v}{u} = \frac{1}{0} \Rightarrow \frac{1}{0} = \frac{150}{30} = 5$

Thus, a real image with magnification 5 is formed.

38. **2074 Set B Q.No. 11** **An erect object is placed at a distance of 10 cm before a convex lens of focal length 15 cm. When will the image be formed? Discuss the nature of image.** [4]

Solution

Given,
Object distance, u = 10 cm
Focal length, f = 15 cm
Image distance, v = ?
We know, from lens formula,

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

or, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

or, $v = \frac{uf}{u - f}$

or, $v = \frac{10 \times 15}{10 - 15}$

$\therefore v = -30$ cm

Also, magnification, $m = \frac{v}{u} = \frac{-30}{10} = -3$

Thus, a virtual image is formed with magnification 3.

39. **2073 Supp Q.No. 11** **A convex lens of focal length 24 cm is totally immersed in water. Find its focal length in water if the refractive indices of glass and water are 1.5 and 1.33 respectively.** [4]

Please refer to 2076 Set B Q.No. 11

40. **2072 Set E Q.No. 11** **An equiconvex lens has a power of 4 dioptre. What will be the radius of curvatures of each surface if the lens is made of glass of refractive index 1.5?** [4]

Solution

Given,
Power of lens (P) = 4 dioptre
Radius of curvature (R) = ?
Refractive index (μ) = 1.5
We have, lens maker's formula,

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

For equiconvex lens,

$R_1 = R_2 = R$

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$

or, $P = (\mu - 1)\frac{2}{R}$

or, $R = (\mu - 1)\frac{2}{P} = (1.5 - 1) \times \frac{2}{4} = \frac{0.5 \times 2}{4} = \frac{1}{4}$

$= 0.25$ m

41. **2069 (Set A) Q. No. 11** **An object to the left of a lens is imaged by the lens on a screen 30cm to the right of the lens. When the lens is moved 4cm to the right, the screen must be moved 4cm to the left to refocus the image. Determine the focal length of the lens.** [4]

Solution

In first case,
Object distance (u) = x
Image distance (v) = 30 cm
Then, focal length, f is

$\frac{1}{f} = \frac{1}{x} + \frac{1}{30}$

In second case,
Object distance (u^1) = x + 4
Image distance (v^1) = 30 – 4 – 4 = 22cm
Focal length f is,

$\frac{1}{f} = \frac{1}{x+4} + \frac{1}{22}$

$\therefore \frac{1}{x} + \frac{1}{30} = \frac{1}{x+4} + \frac{1}{22}$

or, $\frac{1}{x} - \frac{1}{x+4} = \frac{1}{22} - \frac{1}{30}$

or, $\frac{4}{x(x+4)} = 0.012$

or, $\frac{x^2 + 4x}{4} = 82.5$

or, $x^2 + 4x - 330 = 0$

or, $x = 16.27$ cm or – 20.3 cm. But 'x' cannot be negative.

Now,

$\frac{1}{f} = \frac{1}{16.27} + \frac{1}{30}$

or, $f = 10.55$cm

42. **2068 Q.No. 11** **A thin equiconvex lens of glass of refractive index 1.5 whose surfaces have a radius of curvature of 24cm is placed on a horizontal plane mirror. When the space between the lens and mirror is filled with a liquid, a pin held 40 cm vertically above the lens is found to coincide with its own image. Calculate the refractive index of the liquid.** [4]

Solution

Given,
R.I of lens (μ_1) = 1.5
Radius of curvature (r) = 24 cm
Now, focal length f is

$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{r} + \frac{1}{r}\right) = (1.5 - 1)\left(\frac{1}{24} + \frac{1}{24}\right) = 0.04167$

$f_1 = 24$ cm
Combined focal length = 40 cm

Now, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{40} - \frac{1}{24} = \frac{1}{f_2}$

$f_2 = -60$ cm
For liquid,
$r_1 = -24$ cm $\quad r_2 = \infty$
$f_2 = -60$ cm $\quad \mu_2 = ?$
Again,

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$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{-60} = (\mu_2 - 1)\left(\frac{1}{-24} + \frac{1}{\infty}\right)$$

or, $\frac{24}{60} = \mu_2 - 1$

or, $\mu_2 = 1.4$

43. 2067 Supp Q.No. 11 **A convex lens of focal length 24 cm is totally immersed in water. Find its focal length in water.** [4]

Please refer to 2076 Set B Q.No. 11

44. 2066 Q. No. 11 **The real image obtained by a lens of power 5D is three times the length of the object. Calculate the object and image distances.** [4]

Solution

Given,

Power of lens (P) = 5D

Focal length (f) $= \frac{1}{P} = \frac{1}{5} = 0.2$ m

Magnification (m) = 3

Object distance (u) = ?

Image distance (v) = ?

We have,

$m = \frac{v}{u} = 3$

$\therefore v = 3u$

Again,

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{0.2} = \frac{1}{u} + \frac{1}{3u}$

$5 = \frac{4}{3u}$

or, $u = 0.267$ m $= 26.7$cm and $v = 3u = 80.1$ cm.

45. 2061 Q.No. 7 b **The radii of curvature of the faces of a thin converging meniscus lens of glass of refractive index 1.5 are 15 cm and 30 cm. What is the focal length of the lens when it is completely immersed in water of refractive index 4/3?**

Solution

Given,

Radii of curvature of a thin converging meniscus are given by

$r_1 = 15$ cm $= 0.15$ m and $r_2 = 30$ cm $= 0.3$ m

R.I. of glass (μ_g) = 1.5

R.I. of lass (μ_w) = 1.33

For converging meniscus, its focal length should be positive. So, in the lens Maker formula: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, we should take r_1 positive and r_2 = negative i.e., $r_1 = 0.15$ m and $r_2 = -0.3$ m.

Then,

$\therefore \frac{1}{f_1} = (\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ ... (i) [In air]

and $\frac{1}{f_2} = ({}_w\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ ... (ii) [In water]

where μ_g is the refrective index of glass w.r.t. air, ${}_w\mu_g$ is the refrective index of glass w.r.t. water. f_1 and f_2 are focal length of the lens in air and in water respectively. Dividing (i) and (ii) we get,

$$\frac{f_2}{f_1} = \frac{\mu_g - 1}{{}_w\mu_g - 1} = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_w} - 1} \quad \left(\because {}_w\mu_g = \frac{\mu_g}{\mu_w}\right)$$

$$\frac{f_2}{f_1} = \frac{1.5 - 1}{\frac{1.5}{1.33} - 1} = 3.91 \quad \text{... (iii)}$$

From (i), we have

$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{0.15} - \frac{1}{0.3}\right]$

$f_1 = 0.6$ m

From (iii), we get

$f_2 = 3.91 \times 0.6$

$f_2 = 2.34$ m

Hence, required focal length is 2.346 m.

46. 2054 Q.No. 13 **A convex lens of focal length 24 cm. (refractive index = 1.5) is totally immersed in water (refractive index = 1.33). Find its focal length in water.**

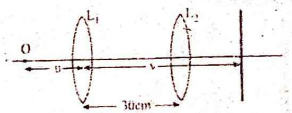
Please refer to 2076 Set B Q.No. 11

47. 2053 Q.No. 13 OR **A luminous object and a screen are placed on an optimal bench and a converging lens is placed between them to produce a sharp image of the object on the screen. The linear magnification of the image is found to be 2.5. The lens is now moved 30 cm. nearer the screen and a sharp image again formed. Calculate the focal length of the lens.**

Solution

First case:

If u and v are the object and image distance respectively, then



$m = \frac{v}{u}$

$2.5 = \frac{v}{u}$

$v = 2.5\,u$ ... (i)

Since, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where f is the focal length of the lens.

$\frac{1}{f} = \frac{1}{2.5u} + \frac{1}{u}$ [using (i)]

$\therefore \frac{1}{f} = \frac{1.4}{u}$ ... (ii)

Second case :

When the lens is moved towards the screen from the position L_1 to the position L_2 such that $L_1L_2 = 30$ cm.

new object distance (u') = u + 30

new image distance (v') = v − 30 = 2.5u − 30 [using (i)]

$\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$

$\therefore \frac{1}{f} = \frac{1}{u + 30} + \frac{1}{2.5\,u - 30}$ ... (iii)

Solving equation (ii) and (iii),

We get, u = 20 cm

From equation (ii) $\frac{1}{f} = \frac{1.4}{20} \Rightarrow f = 14.28$ cm $= 0.143$ m

Hence, focal length of the lens is 14.3 cm.

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48. 2052 Q.No. 13 A converging meniscus of glass (μ = 1.5) having radius of curvature 4cm and 8 cm is put on a horizontal surface facing upward. If it is filed with water, what will be the focal length of the combination?

Solution

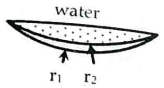
Given,

Refractive index of glass $({}_a\mu_g) = 1.5$

Radius of curvature of one surface $(r_1) = 4$ cm

Radius of curvature of next surface $(r_2) = 8$ cm

Combined focal length (F) = ?



For converging meniscus,

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or, $\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{4} - \frac{1}{8}\right) = 0.5 \times \frac{1}{8}$

or, $f_1 = 16$ cm

For water lens

$$\frac{1}{f_2} = ({}_a\mu_w - 1)\left(\frac{1}{r_1'} + \frac{1}{r_2'}\right)$$

or, $\frac{1}{f_2} = (1.33 - 1)\left(\frac{1}{8} + \frac{1}{\infty}\right)$ [$\because r_1' = r_2 = 8$ cm, same with meniscus and $r_2' = \infty$, for plane surface]

or, $\frac{1}{f_2} = 0.33 \times \frac{1}{8}$

or, $f_2 = 24.24$ cm

Again, we know

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or, $\frac{1}{F} = \frac{1}{16} + \frac{1}{24.24}$

$\therefore F = 9.64$ cm

Hence, combined focal length is 9.64 m.

49. 2051 Q.No. 14 A converging lense of 5 cm focal length used as a simple magnifier, produces a virtual image 25 cm from the eye. How far from the lens should the object be place? What is the magnification?

Solution

Given,

Focal length of converging lens (f) = 5 cm

Image distance (v) = – 25 cm

Object distance (u) = ?

Magnification (m) = ?

We know,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or $\frac{1}{5} = \frac{1}{u} - \frac{1}{25}$

or, $\frac{1}{u} = \frac{1}{5} + \frac{1}{25}$

$\therefore u = 4.17$ cm

Again, we know

$$m = \frac{v}{u} = \frac{-25}{4.17}$$

$\therefore m = -6$

QQQ

Chapter 5: Dispersion

Short Answer Questions

1. 2076 Set B Q.No. 3b 2072 Set D Q.No. 3b 2064 Q.No. 6 b 2052 Q.No. 11 a When white light is dispersed by a prism, red light appears at the top of the spectrum where as violet at the bottom. Why? [2]

When white light is passed through a prism, it splits up into its seven-constituent colours because of their different wavelengths in the prism (denser medium). From Cauchy dispersion relation, we have

$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. The wavelength of red colour is the highest among the constituent colours of white light. So its refractive index is the lowest. Similarly, wavelength of violet colour being the smallest among the constituent colours of white light, its refractive index is the highest. Moreover, the deviation produced by a small angled prism is given by, $\delta = A(\mu - 1)$

As $\mu_r < \mu_v$, then $\delta_r < \delta_v$. It means red colour is least deviated and the violet colour is most deviated. Hence, red light appears at the top of the spectrum whereas the violet at the bottom.

2. 2075 GIE Q.No. 3b Why does the sky look blue in a day? [2]

Sky is seen blue due to scattering of light by air molecules. According to law of Rayleigh, the intensity of scattered light (I) depends with the wavelength (λ) as, $I \propto \frac{1}{\lambda^4}$

As the wavelength of blue color being nearly half of the wavelength of red color, the intensity of scattering of blue color is sixteen times greater than that of red color of light. As a result, the blue color predominates other colors and the sky appears blue.

3. 2075 Set A Q.No. 3b Why does the sky appear blue? [2]

Please refer to 2075 GIE Q.No. 3b

4. 2074 Set A Q.No. 3a How do the colours of light related with the refractive index of a lens? Explain. [2]

From the Cauchy dispersion relation, the refractive index (μ) of a medium is given by

$$\mu = A + \frac{B}{\lambda^2} + \ldots.$$

where A and B are constant and λ is wavelength of light. Since the different colors of light have different wavelengths & hence different refractive index. The red light has longer wavelength then the blue color of light. So, the refractive index of lens for red light is less then the blue color of light.

5. 2073 Supp Q.No. 3b Define dispersive power of a lens and write expression for it in terms of the refractive indices of lens with respect to different colours used. [2]

Dispersive power of a material for any two colours is defined as the ratio of the angular dispersion for these two colours to the deviation suffered by mean light (i.e. yellow light). It is denoted by ω.

So, for violet and red light, the dispersive power is

$$\omega = \frac{\delta_v - \delta_r}{\delta} = \frac{A(\mu_v - 1) - A(\mu_r - 1)}{A(\mu - 1)} \Rightarrow \omega = \frac{\mu_v - \mu_r}{\mu - 1},$$

where μ_v, μ_r and μ are the refractive index of the material for violet, red and mean light respectively.

6. 2072 Supp Q.No. 3a How does the refractive index relate with the wavelength of light? [2]

From the Cauchy dispersion relation, the refractive index (μ) of a medium is given by

$$\mu = A + \frac{B}{\lambda^2} + \ldots.$$

where A and B are constant and λ is wavelength of light. Thus, refractive index relates inversely with wavelength of light.

7. 2072 Set C Q.No. 3b State the necessary conditions for the production of a pure spectrum. [2]

The necessary conditions for the production of pure spectrum are,

i. Instead of point source, a narrow slit should be used.

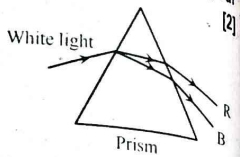
ii. A convex lens is required to make the beam parallel.

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iii. The prism should be kept at minimum deviation position.
iv. Another convex lens should be kept in the path of emergent light.

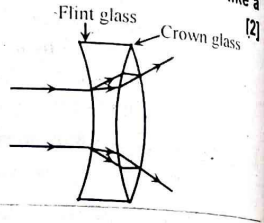
8. 2072 Set E Q.No. 3a **Which one of blue and red light will be deviated more by a prism? Give reason for your answer.** [2]

When a white light passes through a prism, red light deviates less than the blue light. This is because the red light has more velocity, longest wavelength and hence, least refractive index but blue light has less velocity less wavelength and more refractive index. Since, $\delta = A(\mu - 1)$, blue light deviates more than red light.



9. 2072 Set E Q.No. 3b **Can a concave and a convex lens be combined so that the combination behaves like a diverging lens? Justify your answer.** [2]

Yes, a concave and a convex lens can be combined so that the combination behaves like a diverging lens. When a diverging lens of flint glass (with more R.I.) and a convex lens of crown glass (with less R.I.) are combined the combination becomes diverging lens.



10. 2071 Supp Q.No. 3b 2066 Q. No. 3 a **What is the cause of dispersion of light?** [2]

When white light is passed through a prism, it splits up into its seven-constituent colours because of their different wavelength in the prism (denser medium). From Cauchy dispersion relation, we have

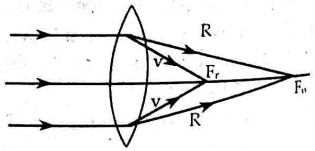
$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. The wavelength of red colour is the highest among the constituent colours of white light. So its refractive index is the lowest. Similarly wavelength of violet colour being the smallest among the constituent colours of white light, its refractive index is the highest. Moreover, the deviation produced by a small angled prism is given by, $\delta = A(\mu - 1)$.

As $\mu_r < \mu_v$ then $\delta_r < \delta_v$. It means red colour is least deviated and the violet colour is most deviated. Hence, red light appears at the top of the spectrum whereas the violet at the bottom. Hence, the cause of dispersion of light is the difference in wavelength of light in medium.

11. 2071 Set D Q.No. 3 a **Draw an outline sketch to show the chromatic aberration in the case of a lens. How can achromatism be achieved?** [2]

When a white light beam incident on a lens parallel to the principal axis, it splits into its constituent colour. The focal length for various colours light is different depending up on the refractive index of the material. This defect of lens, which is unable to focus the light of different colours to a single point, is called chromatic aberration of lens. Achromatism is obtained by combining convex lens of crown glass and concave lens of flint glass.



12. 2070 Set C Q.No. 3 b **A spherical mirror cannot give rise to chromatic aberration. Why?** [2]

Chromatic aberration occurs when light passes through a denser medium like lens, i.e. in refraction phenomena, because of different focal lengths of lens for different colours (wavelengths). But focal length of mirror $\left(\frac{1}{f} = \frac{1}{u} + \frac{1}{v}\right)$ is independent of refractive index of incident light i.e. all the colours are reflected by same angle. Hence, such defects cannot be seen in a mirror.

13. 2069 Supp Q.No. 3b **Why does the sun appears red during the sun-rise?** [2]

We know that the intesity of scattering of light is inversely proportional to the forth power of wavelength of light $\left(I \propto \frac{1}{\lambda^4}\right)$. The red light has longest wavelength and hence, get least scattered

than the other light ray. At sun-rise and sun-set the sun is far and oblique from us than the noon. The red light get least scattered and reach to us but other light scattered and hence, sun looks red during sun-rise and sun-set.

14. 2069 (Set A) Old Q. No. 6c **What is the cause of dispersion of a white ray of light?** [2]

Please refer to 2071 Supp Q.No. 3b

15. 2069 Old (Set B) Q. No. 6b **Does the refractive index depend upon the wavelength of light?** [2]

Yes, the R.I. depends upon the wavelength of light. From Cauchy dispersion relation, we have

$$\mu = A + \frac{B}{\lambda^2} + \ldots$$

This shows that refractive index of a prism for a particular colour varies inversely with the wavelength of the colour. Hence, the refractive index of light depends upon the wavelength of light.

16. 2068 Q.No. 3 b **What is chromatic aberration? Explain why a mirror cannot give rise to chromatic aberration.** [2]

Please refer to 2070 Set C Q.No. 3 b

17. 2068 Old Q.No. 6 c 2065 Q. No. 6 c **What do you mean by chromotic aberration?** [2]

Please refer to 2071 Set D Q.No. 3 a

18. 2067 Supp Q.No. 3 b **Distinguish between chromatic aberration and spherical aberration.** [2]

Chromatic abberration	Spherical abberration
1. This is inability of a lens to focus all colours of light at a single point is called the chromatic aberration	1. This is inability of a lens to focus all the light rays (paraxial and marginal) at a single point is called spherical aberration.
2. Chormatic aberration occurs due to deviation at different angle by different color of light.	2. Spherical aperration occurs due to deviation at different angle by marginal and paraxial rays.
3. This defect can be removed by achromatic combination of lenses.	3. This defect can be removed by using slit or stop.
4. This is color defect.	4. This is not color defect.

19. 2067 Q.No. 3 b **The sun looks red at sun-rise and sun-set. Why?** [2]

Please refer to 2069 Supp Q.No. 3b

20. 2066 Q. No. 3 b **Why is sky blue? Explain.**

Please refer to 2075 GIE Q.No. 3b

21. 2066 Old Q. No. 6 b **If a plane glass slab is placed on letter of different colours, then violet coloured letters appear more and more raised up. Why?** [2]

If a plane glass slab is placed on letter of different colors, then violet colored letters appear more and more raised up means the apparent displacement is more which is given by the formula, $d = t\left(1 - \frac{1}{\mu}\right)$ where t is apparent depth and μ is the refractive index of glass. The violet light has shortest wavelength than the other visible light, hence its refractive index is more and the displacement is more. Hence, the violet colored letter appears to rise more than other.

22. 2063 Q.No. 6 c 2057 Q.No. 6 c 2054 Q.No. 11 c **Explained why can a mirror not give rise to chromatic aberration?** [2]

Please refer to 2070 Set C Q.No. 3 b

23. 2062 Q.No. 6 c **Explain the meaning of achromatism in a lens.** [2]

The aberration or defect produced due to the dispersion of light is called chromatic aberration. Due to this, the image of a white object is colored and blurred. Using a combination of a crown glass convex lens with a flint glass concave lens can reduce chromatic aberration. This combination is called achromatism. The method of reducing chromatic aberration is called achromatization.

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24. 2056 Q.No. 6 c What do you mean by dispersive power? [2]

Please refer to 2073 Supp Q.No. 3b

25. 2055 Q.No. 11 b Why are a number of dark lines seen in the spectrum of light from the sun? [2]

When the light from sun comes to earth, the white light of sun gets dispersed and spectrum is formed. In the spectrum of sun light, some of the colors of light are absorbed as a result dark lines are seen.

26. 2055 Q.No. 11 c Why there is no dispersion of monochromatic light? [2]

When white light is passed through a lens (or a prism), it is splitted into its seven constituent different colours. This is because different colours (wavelengths) have different speed in the medium (lens or prism) and angle of refraction is different for different colour. But, when monochromatic light (light of particular wavelength) is passed through a lens (or a prism), it doesn't get splitted and hence, no dispersion.

27. 2053 Q.No. 11 b How is dispersive power related to refractive index of the material? [2]

Please refer to 2073 Supp Q.No. 3b

Long Answer Questions

28. 2073 Set C Q.No. 7a What is chromatic aberration in lens? Deduce the condition for achromatism in two thin lenses in contact. [4]

When a beam of white light incident on a lens parallel to the principal axis, it splits into its constituent colours. The focal length for various colours of light is different depending upon the refractive index of the material. This defect of lens, which is unable to bring the light of different colours to focus at a single point, is called chromatic aberration of lens.

Let us consider two thin lenses L_1 and L_2 (convex and concave) placed in contact. Let μ_v, μ and μ_r be the refractive indices of the material of lens L_1 for violet, mean and red light and f_v, f and f_r be the focal lengths for violet, mean and red lights. Similarly, μ_v', μ', μ_r' and f_v', f' and f_r' be the refractive indices and focal lengths for violet, mean and red light respectively for lens L_2.

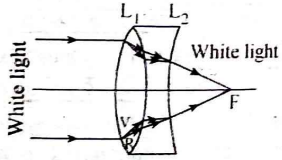
For lens L_1, the focal length for mean light,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots (i)$$

where r_1 and r_2 be the radii of curvature of two surfaces of the lens L_1.

Focal length of lens L_1 for the violet light

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \ldots (ii)$$



From equation (i), we get

$$\frac{1}{r_1} + \frac{1}{r_2} = f(\mu - 1) \quad \ldots (iii)$$

$$\therefore \frac{1}{f_v} = \frac{\mu_v - 1}{(\mu - 1)f} \quad \ldots (iv)$$

Similarly, for red light

$$\frac{1}{f_r} = \frac{\mu_r - 1}{(\mu - 1)f} \quad \ldots (v)$$

For lens L_2, Focal length for violet light is

$$\frac{1}{f_v'} = \frac{\mu_v' - 1}{(\mu' - 1)f'} \quad \ldots (vi)$$

Focal length for red light is

$$\frac{1}{f_r'} = \frac{\mu_r' - 1}{(\mu' - 1)f'} \quad \ldots (vii)$$

If F_v and F_r represents the focal length of the combinations of two lens for violet and red light respectively, then

$$\frac{1}{F_v} = \frac{1}{f_v} + \frac{1}{f_v'} = \frac{\mu_v - 1}{(\mu - 1)f} + \frac{\mu_v' - 1}{(\mu' - 1)f'} \quad \ldots (viii)$$ [Using equation (iv) and (vi)

and $$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} = \frac{\mu_r - 1}{(\mu - 1)f} + \frac{\mu_r' - 1}{(\mu' - 1)f'} \quad \ldots (ix)$$

[Using equation (v) and (vii)]

When two lenses are combined scuh that the combination is free from chromatic abberation, then the violet and red rays will come to focus at a single point. i.e.

$F_v = F_r$

$$\frac{1}{F_v} = \frac{1}{F_r}$$

$$\frac{\mu_v - 1}{(\mu - 1)f} + \frac{\mu_v' - 1}{(\mu' - 1)f'} = \frac{\mu_r - 1}{(\mu - 1)f} + \frac{\mu_r' - 1}{(\mu' - 1)f'}$$

$$\frac{\mu_v - 1}{(\mu - 1)f} - \frac{\mu_r - 1}{(\mu - 1)f} + \frac{\mu_v' - 1}{(\mu' - 1)f'} - \frac{\mu_r' - 1}{(\mu' - 1)f'} = 0$$

$$\frac{\mu_v - \mu_r}{(\mu - 1)f} + \frac{\mu_v' - \mu_r'}{(\mu' - 1)f'} = 0$$

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0 \quad \Rightarrow \frac{\omega}{f} = -\frac{\omega'}{f'} \quad \ldots (x)$$

where $\omega = \frac{\mu_v - \mu_r}{(\mu - 1)}$ is dispersive power of crown glass lens L_1 and $\omega' = \frac{\mu_v' - \mu_r'}{\mu' - 1}$ is dispersive power of flint glass lens L_2.

The relation given by the equation (x) is the required expression for achromatism in two lenses in contact.

29. 2070 Supp (Set B) Q.No. 7 b What is chromatic aberration in a lens? How will you combine two lenses of different materials so that there is no chromatic aberration. [4]

Please refer to 2073 Set C Q.No. 7a

30. 2070 Set D Q.No. 7 b What is chromatic aberration? Show that for a lens, the chromatic aberration is the product of dispersive power and focal length of mean light. [4]

Please refer to 2073 Set C Q.No. 7a

31. 2069 (Set A) Old Q. No. 7a OR What is chromatic aberration? Show that the chromatic aberration is the product of dispersive power and mean focal length of a lens. [4]

Please refer to 2073 Set C Q.No. 7a

32. 2069 Old (Set B) Q. No. 7a OR 2060 Q.No. 7 a OR What is chromatic aberration? Show that two lenses form an achromatic doublet if the ratio of their focal lengths is numerically equal to the ratio of the corresponding dispersive powers of their materials. [1+4]

Please refer to 2073 Set C Q.No. 7a

33. 2066 Old Q. No. 7 a What do you mean by chromatic aberration? Show for a lens, the chromatic aberration is the product of dispersive power and, its mean focal length. [1+4]

Please refer to 2073 Set C Q.No. 7a

34. 2058 Q.No. 7 a OR Derive the condition for achromatism in two lenses in contact. [5]

Please refer to 2073 Set C Q.No. 7a

35. 2055 Q.No. 14 What is Chromatic abberation in lenses? Derive the condition for achromatic lenses.

Please refer to 2073 Set C Q.No. 7a

Numerical Problems

36. 2073 Set D Q.No. 11 The dispersive powers for crown and flint glass are 0.013 and 0.026 respectively. Calculate the focal lengths of the lenses which form an achromatic doublet of focal length 80 cm, when placed in contact. [4]

Solution

Given,

Dispersive power of crown glass (ω_1) = 0.013

Dispersive power of flint glass (ω_2) = 0.026

Combined mean focal length (F) = 80 cm

Focal length of crown glass (f_1) = ?

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Focal length of flint glass (f_2) = ?

We have,

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

or, $\frac{f_1}{f_2} = -\frac{0.013}{0.026} = -\frac{1}{2}$

or, $f_1 = -\frac{1}{2} f_2$

or, $f_2 = -2f_1$ ... (i)

Again,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

or, $\frac{1}{f_1} + \frac{1}{2f_1} = \frac{1}{80}$

or, $\frac{1}{2f_1} = \frac{1}{80}$

or, $2f_1 = 80$ cm

$\therefore f_1 = 40$ cm

$f_2 = -2f_1 = -2 \times 40 = -80$ cm

37. 2069 (Set B) Q. No. 11 An achromatic converging lens of mean focal length 40cm is made by combining two lenses of different materials. If the dispersive powers of the two lenses are in the ratio 1:3, find the focal lengths of each lens. [4]

Solution

Given,

Mean focal length (F) = 40 cm

Ratio of dispersive powes $\left(\frac{\omega_1}{\omega_2}\right) = \frac{1}{3}$

Focal lengths of each lens f_1 and f_2 = ?

Let the focal length of the lenses be f_1 and f_2, then

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

or, $\frac{f_1}{f_2} = -\frac{1}{3}$

or, $f_2 = -3f_1$

But $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$

or, $\frac{1}{f_1} - \frac{1}{3f_1} = \frac{1}{40} \quad = \quad \frac{3-1}{3f_1} = \frac{1}{40}$

or, $f_1 = \frac{40 \times 2}{3} = \frac{80}{3} = 26\frac{2}{3}$ cm

and $f_2 = -3f_1 = -3 \times \frac{80}{3} = -80$cm

38. 2069 (Set A) Old Q.No. 6b An achromatic lens of focal length +30cm is to be constructed by combining a crown glass lens and flint glass lens. What must be the focal lengths of the component lenses if the dispersive powers of crown glass and flint glass are 0.018 and 0.027 respectively? [3]

Solution

Given,

Mean focal lenth (F) = 30cm

Dispersive power of crown glass (ω_1) = 0.018

Dispersive power of flint glass (ω_2) = 0.027

Focal length of crown glass (f_1) = ?

Focal length of flint glass (f_2) = ?

We have

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = \frac{0.018}{0.027}$$

or, $\frac{f_1}{f_2} = -\frac{2}{3}$

or, $f_1 = -\frac{2}{3} f_2$

Again,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or, $\frac{1}{F} = \frac{-3}{2f_2} + \frac{1}{f_2}$

or, $\frac{1}{30} = \frac{-3}{2f_2} + \frac{1}{f_2}$

or, $\frac{1}{30} = \frac{-3+2}{2f_2}$

or, $f_2 = -15$ cm

or, $f_1 = -\frac{2}{3} \times (-15) = 10$ cm

$\therefore f_1 = 10$ cm and $f_2 = -15$ cm ▫▫▫

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Chapter 6: Optical Instruments

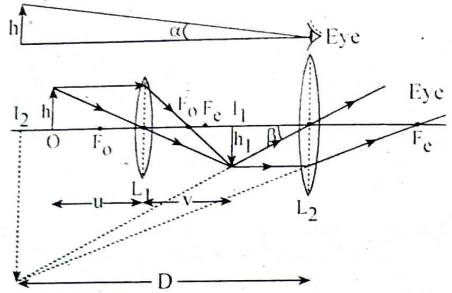
Short Answer Questions

1. 2075 GIE Q.No. 3a **How is it possible to block the huge sun just with a small coin placed right in front of your eye?** [2]

The visual angle subtended by the sun at eye is very small. So, it looks like a small disc of small diameter. If a coin of such diameter is placed in front of eye, it can block the huge sun.

2. 2073 Supp Q.No. 3a **Two convex lenses of focal length f_1 and f_2 are provided. How will you arrange them as eye piece and object lens to design a compound microscope? Explain with an outline diagram.** [2]

A microscope which is a combination of two convex lenses of short focal length and has a larger magnification is called the compound microscope. A lens of short aperture and short focal length facing the object is called objective lens. And, another lens of short focal length and large aperture lying towards eye is called eye lens.



3. 2071 Set C Q.No. 3 a **What do you mean by accommodation of an eye?** [2]

The ability of the eye to bring objects lying at various distances to focus on the retina is called accommodation of eye. For this, the image distance is fixed for all objects of different distances. This is possible if the focal length of eye changes for different distant objects. The ciliary muscles, by contracting cause the lens to bulge out and reduces to focal length of eye lens bringing nearby objects to focus on the retina. When ciliary muscle gets relaxed, the edges of the lens are pulled out, the focal leneth increases, bringing distant objects to focus on the retiana.

4. 2069 (Set A) Q. No. 3b **A large object when taken away from you eye appears smaller. Why?** [2]

When a large object is taken away from our eye, the angle made by the object at unaided eye i.e., visual angle decreases and the object appears smaller.

5. 2069 (Set B) Q. No. 3b **Why do far objects appear to be smaller than the nearer ones?** [2]

Please refer to 2069 (Set A) Q. No. 3b

6. 2069 (Set A) Old Q.No. 1f **Define power of accommodation and persistance of vision.** [2]

Power of Accommodation of eye: Please refer to 2071 Set C Q No. 3 a

Persistence of vision: Persistence of vision is the propety of an eye by virtue of which the impression of the image persists upon retina upto $1/10^{th}$ of a second after the object has been removed.

7. 2066 Supp Q.No. 1d **Define accommodation of eye and its near point.** [2]

Power of Accommodation of eye: Please refer to 2071 Set C Q.No. 3 a

Near point: The nearest distance from our eye that can be seen by our normal nacked eye. This distance is 25 cm for normal human eye.

8. 2055 Q.No. 11 c **Why has nature provided us with two eyes? Explain.** [2]

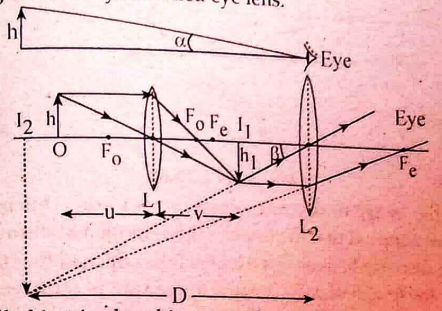
The benefits of two eyes for human being are:

a. Two eyes give wide field of view than one eye.
b. Two eyes give a better judgment of distance of the object.
c. The left eye sees more of the right sides of the object and the right eye sees more of the left sides of the objects. The result is that two images of an object overlap and give impression of solidity of the object.

Long Answer Questions

9. 2076 Set B Q.No. 7b **Explain the construction and working of a compound microscope with final image at near point. Also, derive an expression for its magnifying power.** [4]

Compound microscope: A microscope which is a combination of two convex lenses of short focal length and has a larger magnification is called the compound microscope. A lens of short aperture and short focal length facing the object is called objective lens. And, another lens of short focal length and large aperture lying towards eye is called eye lens.



Principle: When a small object is placed just outside the focus of the object lens, its real, inverted and magnified image is produced on the other side of the lens. The image produced by the object lens acts as object for eye lens. The position of eye lens is so adjusted that the final image is formed at least distance (near point) of distinct vision from the eye.

Let us consider O be an object placed just outside the focus F_0 of the object lens. Its real image I_1 is formed on the other side of the lens. This image I_1 acts as object for the eye lens. Finally, virtual and magnified image I_2 is formed on the same side at least distance of distinct vision as shown in figure.

Angular magnification: The angular magnification M, when the microscope is in normal use is given by the ratio of angle subtended at the eye by the final image to angle subtended at the eye by the object when both are at least distance of distinct vision (D).

i.e. $$M = \frac{\text{Angle subtended at the eye by the final image at D}}{\text{Angle subtended at the eye by the object at D}} = \frac{\beta}{\alpha} = \frac{h_2/D}{h/D} = \frac{h_2}{h} = \frac{h_2}{h_1} \times \frac{h_1}{h} = M_e \times M_o$$

where, $M_e = \frac{h_2}{h_1}$ is the magnification of the eyepiece.

and $M_o = \frac{h_1}{h}$ is the magnification of the objective lens.

Here,

$$M_e = \frac{h_2}{h_1} = -\left(\frac{D}{f_e} + 1\right)$$

and $$M_o = \frac{h_1}{h} = \frac{v}{u}$$

where u and v are the object and image distances for the objects.

Hence, $$M = -\left(\frac{D}{f_e} + 1\right)\frac{v}{u}$$

Since, $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_o} \Rightarrow \frac{v}{u} = \frac{v}{f_o} - 1$$

$$\therefore M = \left(\frac{D}{f_e} + 1\right)\left(\frac{v}{f_o} - 1\right)$$

Thus, for high angular magnification, here f_o and f_e should be small.

10. 2075 GIE Q.No. 7b **Describe the construction and working of a refraction type astronomical telescope obtain its angular magnification and length of telescope, when the final image is formed at infinity.** [4]

Working of Astronomical Telescope in Normal Adjustment:

Telescope: Telescope is an optical instrument to view distant objects like heavenly bodies. It consists

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of two converging lenses of long focal length placed coaxially in a tube. The lens of large aperture and long focal length which is facing towards the object is called objective lens and the lens of short aperture facing towards eye is called eye lens. The distance between them is adjusted by sliding the eyepiece to view the image at required position.

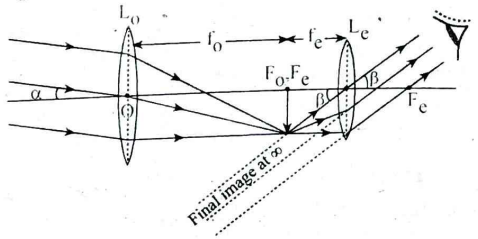


Fig. Astronomical telescope

The objective lens forms the image of distance object on its focus. This image acts as the object for eyepiece. The eyepiece is so adjusted that the final image is formed at desired place.

At normal adjustment, the distance between two lenses is so adjusted that the final image is formed at infinity. At this condition, the length of tube is equal to the sum of focal lengths of two lenses (i.e. $L = f_o + f_e$).

Magnifying power: Magnifying power (M) of telescope is defined as the ratio of angle sustended by final image at eye (β) to angle subtended by object at unaided eye (α). i.e. $M = \frac{\beta}{\alpha}$.

The angular magnification for astronomical telescope in normal adjustment is

$$M = \frac{\text{Angle subtended by the image at the eye through telescope}}{\text{Angle subtended at the unaided eye by a distant object}} = \frac{\beta}{\alpha} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$$

This is required magnifying power of astronomical telescope forming final image at infinity.

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11. 2075 Set A Q.No. 7b **Describe the construction and working of a compound microscope producing final image at infinity and hence derive an expression for its magnifying power.** [4]

Compound microscope: A microscope which is a combination of two convex lenses of short focal length and has a larger magnification is called the compound microscope. A lens of short aperture and short focal length facing the object is called objective lens. And, another lens of short focal length and large aperture lying towards eye is called eye lens.

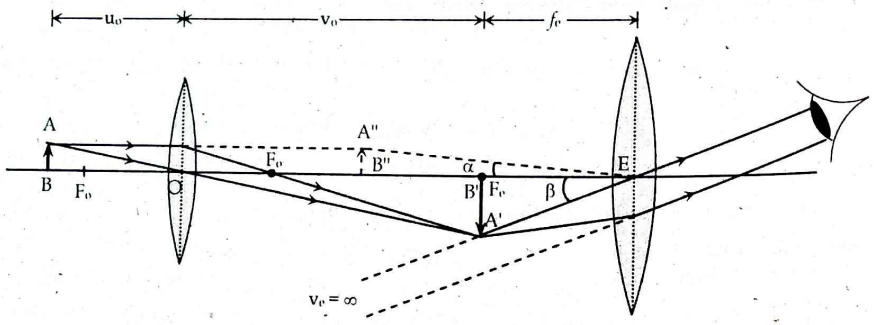


Fig. Compound microscope for adjustment at infinity

Principle: When a small object is placed just outside the focus of the objective lens, its real, inverted and magnified image is produced on the other side of the lens. The image produced by the objective lens acts as object for eye lens. The position of eye lens is so adjusted that the final image is formed at infinite at which the image of objective lies at focus of eye lens.

Let us consider AB be an object placed just outside the focus F_0 of the objective lens. Its real inverted and magnified image A'B' is formed on the other side of the objective lens. This image A'B' acts as object for the eye lens and lies at focus of it whose final image is formed at infinity as shown in figure.

Magnifying Power: Magnifying power (M) of microscope is defined as the ratio of angle sustended by final image at eye (β) to angle sutenced by object at unaided eye (α). i.e. $M = \frac{\beta}{\alpha}$.

The angle subtended at unaided eye by the object, $\alpha = \frac{A"B"}{D}$,

and the angle subtended by the final image at the eyepiece, $\beta = \frac{A'B'}{f_e}$

Thus, Magnifying power,

$$M = \frac{\beta}{\alpha}$$
$$= \frac{A'B'}{f_e} \times \frac{D}{A"B"}$$
$$= \frac{D}{f_e} \times \frac{A'B'}{A"B"}$$
$$= \frac{D}{f_e} \times \frac{A'B'}{AB} \qquad [A"B" = AB]$$
$$= \frac{D}{f_e} \times \frac{v_o}{u_o}$$

$$\therefore M = \frac{v_o}{u_o} \times \frac{D}{f_e} \qquad \left[\frac{v_o}{u_o} = m = \frac{A'B'}{AB} \text{height of image/height of object is the magnification of objective}\right]$$

This is required expression of magnifying power of compound microscope when final image is formed at infinity. The length of the microscope tube = $v_o + f_e$.

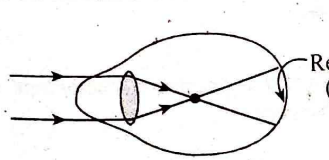
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12. 2075 Set B Q.No. 7a **What are the defects of vision known to you? How are myopic and hypermetropic defects removed? Explain with necessary theory.** [4]

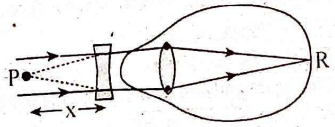
When the lens of the eye cannot change its focal length according to the distance of object, then eye becomes unable to see the object placed at any point with the range of vision. This is called the defects of vision and the eye is called defective eye. So the lens of defected eye focuses the image of the object either in front of the retina or beyond the retina. According to this theory, there are two kinds of common defects of vision, namely: Shortsightedness (myopia) and Long-sightedness (hypermetropia).

Short-sightedness (myopia)

It is that defect of vision in which a person can clearly see the objects lying near it but cannot see distinctly objects lying beyond a certain distance. For this defect, the far point is much near than infinity.



Short sight



Correction for short sight

Causes of short-sightedness

This defect arises due to two reasons:

a. Elongation of the eye ball than the normal size and

b. Shortening of focal length of the lens, when the eye is fully relaxed.

Remedy: This defect can be removed by placing a concave lens of suitable focal length close to the eye as shown in figure. The parallel rays of light from infinity after refraction through concave lens appear to come from the far point P.

If x is the distance of far point from the eye, then for the concave lens placed before the eye

u = ∞

v = −x (virtual image)

Let f be the focal length of the required concave lens, then using the lens formula, me have

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$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{\infty} + \frac{1}{-x} = -\frac{1}{x}$$

or, $f = -x$

Thus, myopic eye is cured against the defect by using a concave lens of focal length equal to the distance of the far point from the eye.

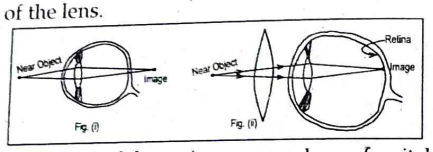
Long sightnedness (hypermetropia)

The defect of an eye in which it can see the distant objects clearly but not the objects nearer to the eye is called long sightedness. The near point of such eye lies farther away from the least distance of distinct vision. This defect, also called hypermetropia. The rays from an object at normal near point (i.e. 25 cm from the eye) are brought to focus at a point behind the retina as shown in Fig (i)

This defect arises due to two reasons:

a. Shortening of the eye ball than the normal size and

b. Elongation of focal length of the lens.



Remedy: This defect can be remedied by using convex lens of suitable focal length. The light rays coming from an object lying near to the eye converges by convex lens and further converges by eye lens and image formed at retina as shown in fig.(ii).

Let x be the distance of near point from the eye. For the convex lens, object distance u = D and image distance v = – x. The focal length of lens is given by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{D} - \frac{1}{x}$$

Since x > D, f is positive. So, a converging lens is needed to correct this defect.

13. **2074 Supp. Q.No. 7b** **What is compound microscope? Outline the ray diagram and find the angular magnification of compound microscope.** [4]

Please refer to 2076 Set B Q.No. 7b

14. **2074 Set A Q.No. 7a** **What do you understand by accommodation and adaptation of an eye? How are the defects related to myopic and hyper-metropic defects corrected?** [4]

First Part: Please refer to 2069 (Set A) Old Q.No. 1f

Second Part: Please refer to 2075 Set B Q.No. 7a

15. **2074 Set B Q.No. 7b** **Describe the construction and working of an astronomical telescope. Also obtain its angular magnification when the final image is formed at infinity.** [4]

Please refer to 2075 GIE Q.No. 7b

16. **2073 Supp Q.No. 7b** **Sketch an outline diagram for designing a compound microscope and deduce expression for its magnifying power, when the image is at near point.** [4]

Please refer to 2076 Set B Q.No. 7b

17. **2073 Set C Q.No. 7b** **Describe the structure and working of an astronomical telescope at normal adjustment with a suitable ray diagram. Also, calculate its magnifying power.** [4]

Please refer to 2075 GIE Q.No. 7b

18. **2073 Set D Q.No. 7b** **Draw a labelled diagram and describe the working of a compound microscope with final image at near point. Also, obtain an expression for its magnifying power.** [4]

Please refer to 2076 Set B Q.No. 7b

19. **2072 Supp Q.No. 7a** **Draw a labeled diagram to illustrate the action of a compound microscope with final image at the near point of normal eye. Also, derive an expression for its magnifying power.** [4]

Please refer to 2076 Set B Q.No. 7b

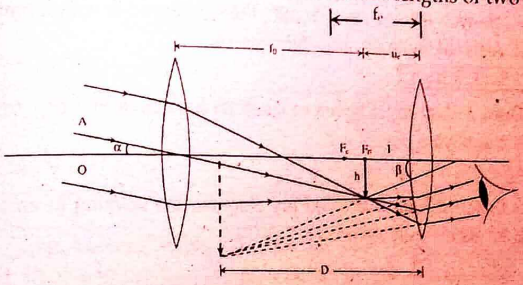
20. **2072 Set D Q.No. 7b** **With a ray diagram, explain the working of a compound microscope. Obtain an expression for its magnifying power, when the image is at near point.** [4]

Please refer to 2076 Set B Q.No. 7b

21. **2072 Set E Q.No. 7b** **Sketch a diagram of an astronomical telescope and derive an expression for its magnifying power when the final image is formed at near point.** [4]

Telescope: Telescope is an optical instrument to view distant objects like heavenly bodies. It consists of two converging lenses of long focal length placed coaxially in a tube. The lens of large aperture and long focal length which is facing towards the object is called objective lens and the lens of short aperture facing towards eye is called eye lens. The distance between them is adjusted by sliding the eyepiece to view the image at required position.

The objective lens forms the image of distance objects on its focus. This image acts as the object for eyepiece. The eyepiece is so adjusted that the final image is formed at desired place. For final image at near point, the length of telecope is greater than the sum of focal lengths of two lenses.



Magnifying power: Magnifying power (M) of telescope is defined as the ratio of angle sustended by final image at eye (β) to angle sutenced by object at unaided eye (α). i.e. $M = \frac{\beta}{\alpha}$.

The angle subtended at unaided eye by the object, $\alpha = \frac{h}{f_0}$

and the angle subtended by the final image at the eyepiece, $\beta = \frac{h}{u_e}$

where u_e is the distance of object at F_0 from eyepiece.

Angular magnification, $M = \frac{\beta}{\alpha} = \frac{(h/u_e)}{(h/f_0)} = \frac{f_0}{u_e}$

To express u_e in terms of D and f_e, we have from lens formula,

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ and $f = f_e$, $v = -D$.

$\therefore \frac{1}{f_e} = \frac{1}{u_e} - \frac{1}{D}$

or, $\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$

or, $\frac{1}{u_e} = \left(1 + \frac{f_e}{D}\right)\frac{1}{f_e}$

$\therefore M = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

This is required magnifying power of astronomical telescope forming final image at nearer point.

22. **2071 Supp Q.No. 7b** **2067 Q.No. 7 b** **Describe the construction and working of a refraction type astronomical telescope. Obtain its angular magnification when final image is formed at infinity.** [4]

Please refer to 2075 GIE Q.No. 7b

23. **2071 Set D Q.No. 7 a** **Sketch an outline diagram of an astronomical telescope and derive expression for its angular magnification when the final image is formed at the least distant of distinct vision.** [4]

Please refer to 2072 Set E Q.No. 7b

24. **2070 Supp (Set A) Q.No. 7 b** **With the help of a ray diagram, derive an expression for angular magnifications of astronomical telescope, when the final image is formed at the least distiance of distint vision.** [4]

Please refer to 2072 Set E Q.No. 7b

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25. 2069 Supp Q.No. 7b Describe the working of a refracting type astronomical telescope with the help of labelled diagram. Obtain its angular magnification when the final image is formed at infinity. [4]

Please refer to 2075 GIE Q.No. 7b

26. 2069 (Set A) Q. No. 7a What is long sightedness? Discuss the possible causes of long sightedeness. Explain, how this deflect can be remedied. [4]

Please refer to 2075 Set B Q.No. 7a (2nd part)

27. 2069 (Set A) Old Q.No. 6a OR With the help of a labelled diagram, explain the principle and working of a compound microscope. Derive an expression for the magnifying power. [1+3]

Please refer to 2076 Set B Q.No. 7b

28. 2068 Q.No. 7 b Describe with labeled ray diagram, the working of an astronomical telescope when image is formed at near point. Also calculate its magnifying power. [4]

Please refer to 2072 Set E Q.No. 7b

29. 2068 Old Q.No. 6 a Draw a clear ray diagram to illustrate the action of a compound microscope. Also derive an expression for its magnifying power. [2+2]

Please refer to 2076 Set B Q.No. 7b

30. 2067 Supp Q.No. 7 b Describe with a labelled ray diagram, the working of an astronomical telescope Also, derive an expression for its magnifying power. [4]

Please refer to 2075 GIE Q.No. 7b

31. 2066 Q. No. 7 a Describe the construction and working of a compound microscope and hence, derive an expression for its magnifying power. [4]

Please refer to 2076 Set B Q.No. 7b

32. 2065 Q.No. 6 a Draw an outline diagram of a compound microscope and derive expression for the magnification. [1+3]

Please refer to 2076 Set B Q.No. 7b

33. 2053 Q.No. 12 What are defects of vision? How are they remedied?

Please refer to 2075 Set B Q.No. 7a

Numerical Problems

34. 2076 Set C Q.No. 11 A compound microscope has lenses of focal length 1 cm and 3 cm. An object is placed 1.2 cm from objective lens. If a virtual image is formed 25 cm from the eye, calculate the separation of the lenses. [4]

Solution

Given,

For the objective,

Focal length (f) = 1 cm

Object distance (u_0) = 1.2 cm

Image distance $(v_0) = \frac{u_0 f_0}{u_0 - f_0} = \frac{1 \times 1.2}{1.2 - 1} = 6$ cm $\quad \left[\frac{1}{f_0} = \frac{1}{u_0} + \frac{1}{v_0}\right]$

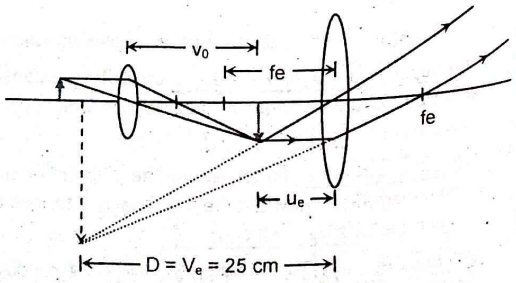
For the eyepiece:

Focal length (f_e) = 3 cm

Image distance (v_e) = -25 cm [Virtual image]

Object distance $(u_e) = \frac{v_e f_e}{v_e - f_e} = \frac{-25 \times 3}{-25 - 3} = 2.7$ cm

Separation of lens: $v_0 + u_e = 6 + 2.7 = 8.7$ cm



35. 2075 GIE Q.No. 11 A person who can see distinctly from a distance of one meter wants to read a book placed at a distance of 0.25m. What kind of lens would be required? Calculate its power. [4]

Solution

Given,

The book is kept at a distance 0.25 m.

So, u = -25 m

Its image is formed at a distance of 1 m.

So, V = -1m

Now,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$= \frac{1}{-1} - \frac{1}{-0.25} = 3$

or, $f = \frac{1}{3}$ m

Since, if is positive, convex lens is required power of lens (P) $= \frac{1}{f} = +3$D

36. 2072 Set C Q.No. 11 The eyepiece of a refracting telescope has a focal length of 90 cm. The distance between objective and eyepiece is 1.8 m and the final image is at infinity. What is the angular magnification of the telescope? [4]

Solution

Given,

Focal length of ice-piece (f_e) = 90 cm = 0.9 m

Length of telescope (l) = 1.8 m

Angular magnification (m) = ?

We have,

$l = f_0 + f_e$ [f_0 = focal length of objective]

$1.8 = f_0 + 0.9$

or, $f_0 = 1.8 - 0.9 = 0.9$ m

$m = \frac{f_0}{f_e} = \frac{0.9}{0.9} = 1$

$\therefore m = 1$

37. 2070 Supp (Set A) Q.No. 11 In order to correct his near point to 25 cm a man is prescribed with spectacles with conversing lenses of 50 cm focal length and to correct his far point to infinity, he is given diverging lenses of 200 cm focal length. Calculate his far point and near point when not wearing the spectacles. [4]

Solution

Given,

Focal length of convex lens (f) = 50 cm

Focal length of concave lens (f) = - 200 cm

To correct his near point,

u = 25 cm, f = 50cm, v = ?

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u - f}{uf}$

$v = \frac{uf}{u - f} = \frac{25 \times 50}{25 - 50} = -50$cm

$\therefore$ His near point is 50 cm

To correct his far point,

$u = \infty$, f = - 200 cm , v = ?

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$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $v = -200$ cm

$\therefore$ His far point is 200 cm.

38. 2070 Supp (Set B) Q.No. 11 **The focal length of eyepiece and objective lens of a certain microscope is 2.5 cm and 16 mm respectively. The distance between objective and eye piece is 22.1 cm. The final image formed by eyepiece is at infinity. Treat all lenses as thin. (a) What should be the distance of the object from the objective lens? (b) What is the linear magnification produced by the objective?** [4]

Solution

Given,

Focal length of eyepiece (f_e) = 2.5 cm

Focal length of objective (f_0) = 16 mm = 1.6cm

Distance between two lenses, (d) = 22.1 cm

We have

$d = f_e + v_0$

or, $22.1 = 2.5 + v_0$

$v_0 = 22.1 - 2.5 = 19.6$ cm

From lens formula,

$$\frac{1}{f_0} = \frac{1}{u_0} + \frac{1}{v_0}$$

$$\frac{1}{1.6} = \frac{1}{u_0} + \frac{1}{19.6}$$

$$\frac{1}{u_0} = \frac{1}{1.6} - \frac{1}{19.6}$$

or, $u_0 = 1.74$ cm

$\therefore$ Object is kept at 1.74 cm from objective lens.

Linear magnification $= \frac{D}{f_e}\left(\frac{v_0}{f_0} - 1\right) = \frac{25}{2}\left(\frac{19.6}{1.6} - 1\right) = 140.63$

39. 2070 Set C Q.No. 11 **A compound microscope has lenses of focal lengths 1 cm and 3 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed is 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.** [4]

Solution

Given,

Focal length of objective (f_0) = 1 cm

Focal length of eye piece (f_e) = 3 cm

Object distance (u_0) = 1.2 cm

Separation between lenses = ?

Magnifying power (P) = ?

For objective lens,

Image distance $(v) = \frac{uf_0}{u - f_0} = \frac{1 \times 12}{1.2 - 1} = 6$ cm

For the eyepiece,

$f_e = 3$cm, $v' = -25$ cm

Object distance $u' = \frac{v'f_e}{v' - f_e} = \frac{-25 \times 3}{-25 - 3} = 2.7$ cm

The separation of the lenses = $v + u'$

$= 6 + 2.7 = 8.7$ cm

Magnification $(P) = -\left(\frac{D}{f_e} + 1\right)\frac{v}{u} = -\left(\frac{25}{3} + 1\right)\frac{6}{12} = -46.7$

$\therefore P = 46.7$

Hence, magnification = 46.7

40. 2070 Set D Q.No. [illegible] **An objective of focal length 1 m and an eye piece of focal length 2 cm. A real image of the sun, 10 cm in diameter, is formed on a screen 24cm from the eye piece. What angle does the sun subtend at the objective?** [4]

Solution

Given,

Image distance (v_e) = 24m

Focal length of eyepiece (f_e) = 2 cm

Focal length of objective (f_0) = 1 m = 100cm

Height of image (I) = 10 cm

Angle subtended at objective by sun (α) = ?

Now, for eye piece,

$$u_e = \frac{v_e f_e}{v_e - f_e} = \frac{24 \times 2}{24 - 2} = \frac{24}{11} \text{ cm}$$

also,

$$\frac{v_e}{u_e} = \frac{I}{O} \text{ (in magnitude)}$$

$$\text{or, } O = \frac{u_e}{v_e} \times I$$

$$= \frac{24}{11 \times 24} \times 10 = \frac{10}{11} \text{ cm}$$

Now,

$$\tan\alpha = \frac{O}{f_0} = \frac{10}{11 \times 100} = \frac{1}{110}$$

or, $\alpha = \tan\alpha = \frac{1}{110} = 0.009$ radian

41. 2069 Supp Q.No. 11 **An optician prescribes converging lenses of focal length 50cm to correct the near point to 25 cm and diverging lenses of focal length 200cm to correct the far point to infinity to a person. Calculate his far point and near point without using the spectacles.** [4]

Please refer to 2070 Supp (Set A) Q.No. 11

42. 2068 Old Q.No. 6 b **Where is the near point of an eye for which a spectacle lens of power +2D is prescribed?** [3]

Solution

Given,

Power of lens (P) = 2D

Object distance (u) = 25 cm

Near point (v) = ?

Now,

$F = \frac{1}{P} = \frac{1}{2} = 0.5$ m

For the spectacles lens,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\text{or, } \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1-2}{50} = -\frac{1}{50}$$

or, $v = -50$ cm

$\therefore$ Near point is 50 cm.

43. 2066 Q.No. 6 b **A telescope is made of an object glass of focal length 20 cm and an eye-piece of 5cm, both converging lenses. Find the magnifying power (i) When the eye is focused to receive parallel rays, and (ii) when the eye sees the image situated at near point which can be taken as 25 cm.**

Solution

Given,

Focal length of objective (F_0) = 20cm

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Focal length of eyepiece (F_e) = 5 cm

Magnifying power (M) = ?

i. For normal adjustment

$$M = \frac{F_o}{F_e} = \frac{20}{5} = 4$$

ii. For least distance of distinct vision

$$M = \left(1 + \frac{F_e}{D}\right)\frac{F_o}{F_e} = \left(1 + \frac{5}{25}\right)\frac{20}{5} = 4.8$$

44. 2050 Q.No. 15 OR **An elderly man cannot clearly see objects nearer than 200 cm. What spectacles will he need to reduce this distance to 25 cm? If his eyes can focus rays to points not less than 150 cm. behind them, calculate his range of vision while using spectacles.**

Solution

Given,

Object distance (u) = 25cm

Image distance (v) = – 200cm

Range of vision = ?

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{1}{f} = \frac{1}{25} - \frac{1}{200}$

or, $\frac{1}{f} = \frac{7}{200}$

or, $f = 28.6$ cm

He should use the converging lens of focal length 28.6cm. Let x be the object distance for v = -150cm, then

$$\frac{1}{x} + \frac{1}{150} = \frac{1}{f} = \frac{7}{200}$$

or, $\frac{1}{x} = \frac{7}{200} - \frac{1}{150}$

$$= \frac{21 - 4}{600}$$

$$x = \frac{600}{17} = 35.3 \text{ cm}$$

∴ Range of vision = 25cm to 35.3cm

□□□

Chapter 7: Photometry

Short Answer Questions

1. 2076 Set B Q.No. 3a 2070 Set C Q.No. 3 a 2068 Q.No. 3 a **The sun is less bright in morning and evening as compared to that at noon although its distance from the observer is almost the same. Why?** [2]

The illuminance of a surface is the luminous flux falling normally on unit area of the surface. It is denoted by E. From Lambert cosine law, if I be the luminous intensity of source, then illuminance of the surface at a distance r is given by: $E = \frac{I}{r^2}\cos\theta$,

where θ is the angle between incident ray and normal drawn on the surface. At the evening and morning the sun makes angle of 90 degree with the normal of the earth surface but at the day time the sun is just above the observer i.e parallel to the normal of surface and hence, the sun is less bright in morning and in evening as compared to that at noon although its distance form the observer is almost the same.

2. 2075 Set B Q.No. 3b **What do lumen and lux signify? Explain.** [2]

One lumen is the luminous flux (ϕ) emitted per unit solid angle by a uniform point source of 1 candle power.

Illuminance of a $1m^2$ surface is said to be 1 lux if it is held at a distance of 1 m from the source of luminous intensity 1 candela normally to the direction of propagation of light.

3. 2074 Supp. Q.No. 3a 2064 Q.No. 6 a 2051 Q.No. 12 d **Why does the illuminance of a surface decrease as it is moved away from the light source?** [2]

In photometry, inverse square law states that illuminance (E) of a surface due to a point source of light varies inversely as the square of the distance (r) between the source and the surface i.e. $E \propto \frac{1}{r^2}$.

That means, as r increases, E decreases and vice-versa. Hence, illuminnace (E) decreases with the increase in distance r (i.e. on moving the surface away from the light source).

4. 2072 Set C Q.No. 3a **What is illuminance? Does the direction of incidence of light affect illuminance of surface? Explain.** [2]

The illuminance of a surface is the luminous flux falling normally on unit area of the surface. It is denoted by E. From Lambert cosine law, if I be the luminous intensity of source, then illuminance of the surface at a distance r is given by: $E = \frac{I}{r^2}\cos\theta$,

where θ is the angle between incident ray and normal drawn on the surface.

Thus, the illuminance (intensity of illumination) of a surface depends on following factors:

a. Luminous intensity (I) of the source.
b. Distance of the surface from the source (r).
c. Direction along which light is incident on the surface (θ).

5. 2072 Set D Q.No. 3a **Define luminous intensity of a source. What is its unit?** [2]

The luminous intensity of a luminous source (energy emitted by such a source stimulates the sensation of vision) is defined as luminous flux per unit solid angle. It is denoted by I and given by $I = \frac{\phi}{4\pi}$ its unit is lum/sr called candela (cd).

6. 2071 Supp Q.No. 3a **Explain why illumination of a surface decreases as it is moved away from the light sources.** [2]

Please refer to 2074 Supp. Q.No. 3a

7. 2071 Set D Q.No. 3 b **What do you understand by 1 lumen and 1 phot? What do they signify?** [2]

1 lumen is defined as the amount of luminous flux emitted per unit solid angle by a point source of one candle power.

1 phot is defined as the amount of light energy falling normally on a square centimeter spherical surface of radius one centimeter when a source of one candle power is kept at the centre of sphere.

One phot=one lumen/square centimeter.

They are importance because they are the unit of measurement of luminous flux and illumination in the photometry.

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8. 2070 Set D Q.No. 3 a **What is illuminance? Name the factors upon which it depends.** [2]

Please refer to 2072 Set C Q.No. 3a

9. 2069 (Set B) Q. No. 3a **The sun is less bright in the morning and in the evening as compared to the noon although its distance from the observer is almost the same. Explain.** [2]

Please refer to 2076 Set B Q.No. 3a

10. 2068 Old Q.No. 6 a **Distinguish between illuminance and luminance.** [2]

The difference between illuminance and luminance are given below:

Illuminance	Luminance
a. The amount of light energy falling on a surface or a body is called illuminance.	a. The amount of light energy reflected by the surface of a body is called luminance
b. It is luminous flux falling per unit area of surface.	b. It is luminous flux reflected by unit area of surface.
c. It does not depend upon the roughness of surface.	c. It depends upon the roughness of surface
d. It is same for white and black surface.	d. It is greater for white surface and less for dark surface.

11. 2067 Supp Q.No. 3 a **Define illuminance. Name the factors upon which it depends.** [2]

Please refer to 2072 Set C Q.No. 3a

12. 2063 Q.No. 6 a **Why illumination of a surface decreases as it is moved away from the light sources?** [2]

Please refer to 2074 Supp. Q.No. 3a

13. 2062 Q.No. 6 a **What is the difference between lumen and lux?** [2]

Please refer to 2075 Set B Q.No. 3b

14. 2061 Q.No. 6 a **Define luminous efficiency of source of light and give its SI unit.** [2]

Luminous efficiency of a light source is the ratio of luminous flux given out by the source to the input power fed to the source.

i.e. Luminous efficiency $(\eta) = \frac{\text{luminous flux}}{\text{power fed}} = \frac{\phi}{P}$

The SI unit of luminous efficiency lumen/watt

15. 2060 Q.No. 6 a **What are the factors on which the illuminance of a surface depends?** [2]

Please refer to 2072 Set C Q.No. 3a

16. 2059 Q.No. 6 a **What is luminous flux? State its unit.** [2]

The luminous energy emitted by a source per second is called luminous flux. It is denoted by ϕ. The unit of luminous flux is lumen. A lumen is unit of energy per second. From experiments, it has been found that 621 lumen of green light (λ = 5550A°) is equal to 1 watt. It should be cleared that the luminous flux depends on (i) radiant flux and (ii) the wavelength contained in the source of emission.

17. 2058 Q.No. 6 b **Define luminous flux. State its unit of measurement.** [2]

Please refer to 2059 Q.No. 6 a

18. 2054 Q.No. 11 a **What is the meaning of Luminous intensity?** [2]

Please refer to 2072 Set D Q.No. 3a

Numerical Problems

19. 2069 (Set A) Old Q. No. 7b 2069 Old (Set B) Q. No. 7b **A lamp radiating uniformly with a luminous intensity of 200 cd is hung 3 m above the centre of a room of dimensions 8m by 8m. Calculate the illuminance at a point in a corner at floor level.** [4]

Solution

Given,

Luminous intensity (I) = 200 cd.

Side of room = 8 m

Distance of bulb from centre of room = 3m

Illumnace at corner (E) = ?

Let, ABCD is a square and O be its centre.

Now, $DB = \sqrt{8^2 + 8^2} = \sqrt{128} = 8\sqrt{2}$ m

and $OB = \frac{DB}{2} = 4\sqrt{2}$ m

Again, $LB = \sqrt{3^2 + (4\sqrt{2})^2} = \sqrt{9 + 32} = \sqrt{41}$ m

$\cos\theta = \frac{3}{\sqrt{41}}$

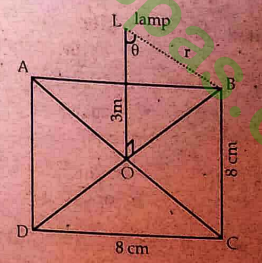
The illuminance at corner B is

$E = \frac{I}{r^2}\cos\theta = \frac{I}{LB^2}\cos\theta$

$= \frac{200 \times \frac{3}{\sqrt{41}}}{(\sqrt{41})^2}$

$= \frac{200 \times 3}{41 \times \sqrt{41}}$

$= \frac{600}{262.5} = 2.3$ lux



20. 2065 Q. No. 7 b **When a light falls normally on earth, illuminance of 1.57×10^5 lumen/m² is produced on earth. The distance between earth and sun is 1.5×10^8 km. Calculate the luminous intensity and luminous flux of the sun.** [4]

Solution

Given,

Illuminance (E) = 1.57×10^5 lumen/m²

Distance (r) = 1.5×10^8 km = 1.5×10^{11}m

luminous flux (ϕ) = ?

Since the luminous intensity of the sun is

$E = \frac{I}{r^2}$

$I = E.\ r^2 = 1.57 \times 10^5 \times (1.5 \times 10^{11})^2 = 3.5 \times 10^{27}$ candela

Also, the luminous flux of the sun is

$I = \frac{\phi}{4\pi}$

$\therefore \phi = 4\pi \times I = 4\pi \times 3.5 \times 10^{27} = 4.44 \times 10^{28}$ lumen.

Thus, the luminous intensity of the sun is 3.5×10^{27} candela and its luminous flux is 4.44×10^{28} lumen.

21. 2056 Q.No. 7 b **If two identical lamps are 1m a part, where should be a screen be placed between them so that the intensity on one side of the screen is four times the intensity on the other side.**

Solution

Given,

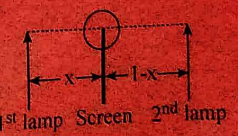
Distance between two lamps = 1 m

According to the question, intensity of other side of the screen due to second lamp is $I_2 = 4I$.

We have,

$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$

$\therefore \frac{I}{4I} = \frac{x^2}{(1-x)^2}$



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Where, $r_1 = x$ is the distance of first lamp from the screen.

$4x^2 = 1 - 2x + x^2$

$3x^2 + 2x - 1 = 0$

It is in the form $ax^2 + bx + c = 0$

Whose solution is $= x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 3(-1)}}{2 \times 3}$

$x = \frac{-2 \pm \sqrt{16}}{6} = -1\text{m or } 0.33\text{ m}$

$\therefore$ $x = -1$ m is not possible, so $x = 0.33$ m.

❑❑❑

UNIT 4

Electrostatics

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Chapter 1: Electrostatics

Short Answer Questions

1. **2076 Set B Q.No. 4a** **Bits of paper are attracted to an electrified comb, even though they have no net charge. How is this possible?** [2]

Ans When a comb runs through hair, it gets charged due to friction with hair. This charged comb when brought close to paper, it induces opposite charge on paper pieces and attracts to paper even though they no net charge. If the hair is wet, instead of dry, the friction between hair and comb is highly reduced and hence, comb is less charged. Thus, it cannot attract the paper.

2. **2076 Set B Q.No. 4b** **Can two electric lines of force intersect each other? Explain.** [2]

Ans We know that, the tangent drawn at any point on electric lines of force gives the direction of electric field intensity at that point. If two lines of force intersect at a point, then it demands two different directions of electric field intensity at the point of intersection. But, it is impossible. Hence, two lines of force never intersect to each other.

3. **2076 Set C Q.No. 4a** **What do you mean by quantization of charge? Explain.** [2]

Ans The charges on any charged body are integral multiple of basic unit of charge. This basic unit of charge is taken as the charge on an electron or a proton which is equal to $\pm 1.6 \times 10^{-19}$ C. Therefore the charge on any body is, $Q = \pm n e$ where e is the charge on an electron and n is any integer. This property of charge is called the quantization of electric charge. This means the charges on the body are not in fraction number i.e. they are in whole number.

4. **2075 GIE Q.No. 4a** **We often experience the sparks during the winter nights when removing the cloths from our body. Why?** [2]

Ans In winter season, our body is dry but not wet. When we remove our clothes from out body, the charges are produced due to friction between the body and clothes. As a result, some sparks are produced.

5. **2075 Set A Q.No. 4a** **Why metallic chain is used in the tanker transporting highly inflammable liquid?** [2]

Ans The vehicles get charged due to the friction during motion. The body of vehicles also gets charged due to friction of air. If the accumulation of charge is enough, it causes sparking and the inflammable material may catch fire and explosion may happen. When an iron or metallic chain is suspended from the vehicle, it drags charges from body of vehicle to ground. Hence, the iron chain prevent from explosion.

6. **2075 Set A Q.No. 4b** **Is it possible that electric potential at a point is zero but not electric field?** [2]

Ans No, the electric potential is zero at a point; the electric field intensity is not zero at that point. For example, the point which lies exactly at the middle or the points on the plane which is perpendicular bisector of the line joining two equal but opposite charges, the potential due to the charges will be equal but opposite. Hence, the total potential becomes zero but both of the charges produce the electric field in the same direction which makes electric field non-zero.

7. **2075 Set B Q.No. 4b** **How can a body be charged with positive electricity by the method of induction? Explain.** [2]

Ans **Charging a body positively by induction**

Let us consider a negatively charged glass rod taken to near an uncharged body AB which is fitted on the insulated stand. This body AB can be charged positively by induction in the following figure:

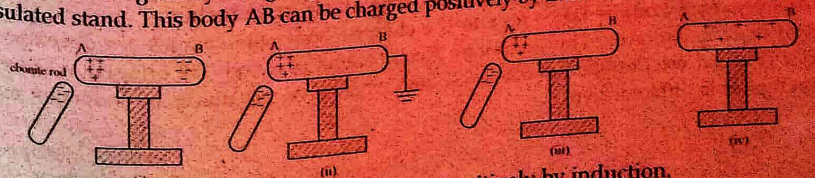


Figure: Charging a body positively by induction.

8. 2074 Supp. Q.No. 4a **What is the magnitude of an electric field which will balance the weight of an electron on the surface of earth?** [2]

Let E be the value of electric field strength of an electron which will just balance its weight on the surface of earth, then,

Electric force = Weight of electron

or, $eE = W$

or, $eE = m_e g$

or $E = \frac{m_e g}{e}$

Since, mass of electron, $m_e = 9.1 \times 10^{-31}$ kg

Charge of an electron, $e = 1.6 \times 10^{-19}$C

Then,

$$E = \frac{9.1 \times 10^{-31} \times 10}{1.6\ 10^{-19}}$$

$$= 5.68 \times 10^{-11}\ N/C$$

9. 2074 Set A Q.No. 4a **Can two electric lines of force ever intersect? Explain.** [2]

Please refer to 2076 Set B Q.No. 4b

10. 2074 Set B Q.No. 4a **Different charges $q_1, q_2, q_3, \ldots$ are placed at $r_1, r_2, r_3, \ldots$ respectively in an electrostatic field. Write expressions for the total electric potential and intensity at R in proper notations. $r_1, r_2, r_3, \ldots$ and R refer to the distances from origin of the field (or space).** [2]

Let different charges $q_1, q_2, q_3, \ldots, q_n$ are placed at distance $r_1, r_2, r_3, \ldots, r_n$ respectively in an electrostatic field. The potential at a point at a distance R is the sum of potentials due to each charge i.e.,

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

The total electric field intensity is the vector sum of the electric field intensity due to each charge

$$\vec{E} = \vec{E_1} + \vec{E_2} + \ldots + \vec{E_n} = \frac{1}{4\pi\epsilon_0}\sum_{i,j} \frac{a_i}{r_{ij}^2}\hat{r}_{ij}$$

11. 2074 Set B Q.No. 4b **Define and write the unit of potential gradient in an electrostatic field.** [2]

The rate of change of electric potential with respect to distance is called electric potential gradiant. It is given as

$$\text{Potential gradiant} = \frac{dV}{dr}$$

Its unit is Volt/meter(V/m) or Newton /Columb (N/C).

12. 2073 Supp Q.No. 4a **Explain how a pith ball can be electrified with positive or negative charge.** [2]

Electrostatic Induction for Positive Charge: Please refer to 2075 Set B Q.No. 4b

Electrostatic Induction for Negative Charge

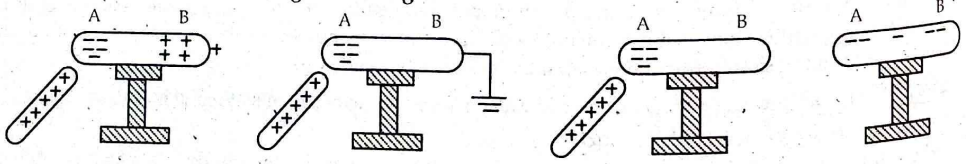


Figure: Charging a body negatively by induction.

When an uncharged body is taken near to the charged body, the charges are developed on the uncharged body temporarily. This temporary electrification of a body by bringing a charged body close to it is called electrostatic induction. A body is charged negatively by induction method as shown in figures in different steps.

13. 2073 Set C Q.No. 4a **What is potential gradient? How is it related with the electric field intensity?** [2]

The rate of change of electric potential with respect to distance is called electric potential gradient. It is given as;

$$\text{Potential gradient} = \frac{dV}{dr}$$

It is related to electric field intensity as,

$$E = \frac{-dV}{dr}$$

14. 2073 Set D Q.No. 4a **Can a charged body attract an uncharged body? Explain.** [2]

Yes, a charged body can attract an uncharged body. When the charged body is taken to near an uncharged body, the uncharged body gets charged by induction method. The induced charges nearer to the end of charge body are unlike while far end are like charges. According to the nature of unlike charges, they attract with each other.

15. 2072 Supp Q.No. 4a 2072 Set E Q.No. 4a 2065 Q. No. 8 b **A man inside an insulated metallic cage does not receive shock when the cage is highly charged, why?** [2]

The potential at each point inside cage is same as that of the cage itself so that there is no potential difference between the man and the highly charged cage. Thus, the man does not receive any electric shock.

16. 2072 Supp Q.No. 4b **How do you charge a body with positive electricity by induction? Explain.** [2]

Please refer to 2075 Set B Q.No. 4b

17. 2072 Set C Q.No. 4a **Why does bodies get electrified when they are rubbed together for a while?** [2]

When two bodies rubbed together for a while, one body loses valence electrons while other body gains electrons. The body which loses electrons is positively charged because there will deficit of electrons and the other body which gains electrons is negatively charged because there will be excess of electrons (negative charge). In this way, transformation of electrons from one body to another body electrified both two bodies on rubbing.

18. 2072 Set D Q.No. 4a **What is electrostatic shielding?** [2]

When a hollow metallic conductor is charged, all the charges will be distributed on its outer surface. Inside the conductor, there will be no electric field. Hence, a hollow conductor acts as a shield due to which there will be zero electric field inside it is known as electrostatic shielding.

19. 2072 Set D Q.No. 4b **Two charged conductors are touched mutually and then separated. What will be the charge on them?** [2]

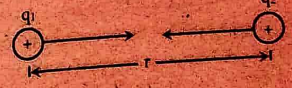
When two charged conductor are touched mutually and then separated, the opposite charge will be produced due to electrostatic induction.

20. 2071 Supp Q.No. 4a **If the electric potential is zero at a point, is the electric field intensity also zero at that point? Explain.** [2]

Please refer to 2075 Set A Q.No. 4b

21. 2071 Set C Q.No. 4 a **Two positive charges separated by a distance create an electrostatic field. Can (i) electric potential, and (ii) electric field intensity be zero at a point between the charges? Justify your answer.** [2]

Let us consider two charges q_1 and q_2 are separated by some distance r. Then the force at a point between two charges is zero due to equal and opposite force. So, the electric field intensity at that point is also zero as $E = \frac{F}{q}$.



However,

$$E = -\frac{dV}{dr}$$

or, $\frac{dV}{dr} = 0$ [$\because E = 0$]

or, $dV = 0$ on integrating

or, $V = \text{constant}$

$\therefore$ (i) Electric field is zero (ii) potential is constant

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22. 2071 Set D Q.No. 4 b How can a neutral body be changed with negative electricity by induction? Explain. [2]

Please refer to 2073 Supp Q.No. 4a

23. 2070 Supp (Set A) Q.No. 4 a Two electric lines of force never intersect each other, why? [2]

Please refer to 2076 Set B Q.No. 4b

24. 2070 Supp (Set B) Q.No. 4 a The tyres of airplane are made from a special rubber which is resonably good conductor of electricity. What would be the reason behind it? [2]

During the landing there is a friction between the tyres of the aircraft and the runway. This develops charge on the tyres. If the accumulation of charge becomes high, it may lead to sparkling and fire. If the tyres are made slightly conducting, the charge will not accumulate on it and leak to ground. So, the tyres of airplane are made from a rubber which is reasonably good conductor of electricity.

25. 2070 Set C Q.No. 4 b Can a charged body attract an uncharged body? Explain. [2]

Please refer to 2073 Set D Q.No. 4a

26. 2070 Set D Q.No. 4 a More charge can be stored on a metal if it is highly polished that when its surface is rough. Explain. [2]

If the surface of a charged metal sphere is rough, it contains many minute projections through which charges start to leak. This is because the electric field intensity is very high at such projections which cause to occur discharge of gases near it. This can be minimized if the surface of the charged metal sphere is polished. That's why, more charge can be stored on a metal sphere if it is highly polished than when its surface is rough.

27. 2069 Supp Q.No. 4a Why are the four footed animals posed to more threat during close by lightning strike than the two footed humans? [2]

The four feet of the four footed animal remain in contact with ground while two feet of the human are attached with earth. Due to this reason, during lightening, the conductivity of four footed animal is more than the two footed human. So, the four footed animals posed to more threat during close by lightning strike than the two footed humans.

28. 2069 (Set A) Q. No. 4a Why is it dangerous to take shelter under a tall tree during lightening? [2]

It is dangerous to take shelter under a tall tree during lighting because the green tree is good conductor of electricity and there is much more possibility of the lightening through the tree.

29. 2069 (Set A) Q. No. 4b Can two electric lines of force ever intersect each other? Explain. [2]

Please refer to 2076 Set B Q.No. 4b

30. 2069 (Set B) Q. No. 4a The vehicles carrying inflammable fluid drag a chain along the ground. Why? [2]

Please refer to 2075 Set A Q.No. 4a

31. 2069 Set B Q. No. 4b Sharp projections are avoided in machines. Why? [2]

According to action of point, the surface charge density will be maximum at the point, if there is a sharp edge in electrostatic machine. ($\sigma = \frac{q}{A} \Rightarrow \sigma \propto \frac{1}{A}$, for constant charge). Due to this, when air molecule come in contact with such edges, the molecules get similar charge from them and get repulsion. Other fresh air molecule comes in contact and gets some charge and move away. The process is continuous so that the conductor leaks the charge continuously and an electric wind is formed, if it has point edge. Therefore, such pointed edges are avoided in electrical machines.

32. 2069 Old (Set B) Q. No. 8b Two lines of electric field do not intersect with each other. Why? [2]

Please refer to 2076 Set B Q.No. 4b

33. 2068 Q.No. 4 a What do you mean by quantization of charge? [2]

Please refer to 2076 Set C Q.No. 4a

34. 2068 Old Q.No. 8 b What do you mean by charging by conduction and charging by induction? [2]

The process of making charged body from uncharged body by any method is called charging. We can charge a body by method of conduction or induction. In charging by conduction, we can charge a conductor by supplying or touching a charge body to it. While in induction method, a charge body is taken near an uncharged body without touching it and an uncharged body becomes charged. In a body when is charged by conduction has same type of charge but in induction opposite type of charge is developed.

35. 2067 Supp Q.No. 4 a More charge can be stored on a metal if it is highly polished than when its surface is rough, why? [2]

Please refer to 2070 Set D Q.No. 4 a

36. 2067 Supp Q.No. 4 b Define one electron volt. [2]

The amount of kinetic energy gained by an electron when accelerating through a potential difference of 1 volt is called one electron volt or 1 eV. i.e. 1 eV = 1 electronic charge × 1 volt = 1.6×10^{-19} C × 1 J/C.

$1 eV = 1.6 \times 10^{-19}$ J. Therefore, one electron volt energy is numerically equal to 1.6×10^{-19} J.

37. 2067 Q.No. 4 a Some of the free electrons in a good conductor (such as a piece of copper) move at speed of 10^6 m/s or faster. Why don't these electrons fly out of the conductor completely? [2]

Metal (or good conductor) have free electrons at its surface, these electrons are strongly attracted towards the positively charged protons of the same metal by the strong electrostatic force of attraction by the formula,

$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{x^2}$ where q_1 and q_2 are the charges of protons and electrons and x be the distance between the protons and electrons. Due to this strong force of attraction, some of the free electrons in a good conductor (such as a piece of copper) move at speeds of 10^6 m/s or faster but do not fly out of the conductor completely.

38. 2066 Q. No. 4 a If the electric field is zero throughout a certain region of space, is the potential also zero in the region or not? Explain. [2]

The electric field intensity and potential are related as

$E = -\frac{dV}{dr}$, If E = 0, V = constant.

Thus, the electric potential may be non-zero, if the electric field is zero throughout a certain region like equipotentail surface where its value is non-zero remains constant.

39. 2066 Q.No. 8 c A man inside an insulated metallic cage does not receive shock when the cage is highly charged. Explain.

Please refer to 2072 Supp Q.No. 4a

40. 2064 Q.No. 8 b Prove $1 Vm^{-1} = 1 NC^{-1}$. [2]

Electric field intensity between two parallel plates is given by $E = \frac{V}{d}$, where V is potential difference between the plates and d is separation of the plates.

If V = 1 volt and d = 1 m, then

E = 1 V/m. ... (i)

Again in terms of force F experienced by a charge q, the electric field intensity is given by, $E = \frac{F}{q}$.

If F = 1 N for q = 1 C, then

E = 1 N/C. ... (ii)

From (i) and (ii), we have

1 V/m = 1 N/C. Hence, proved

41. 2064 Q.No. 8 c A comb run through one's dry hair attracts bits of paper. Why?

Please refer to 2076 Set B Q.No. 4a

42. 2063 Q.No. 8 b Why do sharp edges are strictly avoided in an electrical machine?

Please refer to 2069 Set B Q. No. 4b

43. 2063 Q.No. 8 c What is meant by relative permittivity? What is its minimum value?

Dielectric constant (relative permittivity) of a medium is defined as the ratio of the permittivity of that medium to the permittivity of free space (or vacuum). It is denoted by ε_r and given by

$\varepsilon_r = \frac{\text{permittivity of medium}}{\text{permittivity of free space}} \Rightarrow \varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$

It is also called as dielectric constant (k). Thus, the dielectric constant of a medium is the relative

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permittivity of medium with respect to the vacuum. The minimum value of dielectric constant or relative permittivity is 1 for vacuum or air and greater than 1 for other medium.

44. 2062 Q.No. 8 a Explain the phenomenon of action point in a charged conical sphere. [2]

According to action of point, the surface charge density will be maximum at the point, ($\sigma = \frac{q}{A} \Rightarrow \sigma \propto \frac{1}{A}$, for constant charge). A conical charged conductor has maximum charged density at conical part than the other part of the surface. As a result, the voltage at the conical part of the conductor is maximum. When air molecules come in contact with such edges, the molecules get similar charge from them and get repulsion. Other fresh air molecule comes in contact and gets some charge and move away. The process is continuous so that the conductor leaks the charge continuously from the conical part.

The charged sphere has no such a points and does not loose charge continuously and remain for a long time.

45. 2061 Q.No. 8 a Why can more charge be placed on a metal if it is highly polished than when its surface is rough? [2]

Please refer to 2070 Set D Q.No. 4 a

46. 2060 Q.No. 8 a A charged conical conductor loses its charge earlier than a similarly charged sphere. Why? [2]

Please refer to 2062 Q.No. 8 a

47. 2059 Q.No. 8 a Why pointed ends are not kept in the electrostatics machine? [2]

Please refer to 2069 Set B Q. No. 4b

48. 2058 Q.No. 8 a Why are sharp edges or points avoided in electrical machines? [2]

Please refer to 2069 Set B Q. No. 4b

49. 2058 Q.No. 8 b What is an eV? [2]

Please refer to 2067 Supp Q.No. 4 b

50. 2057 Q.No. 8 a A man inside an insulated metallic cage does not receive a shock when the cage is highly charged. Explain. [2]

Please refer to 2072 Supp Q.No. 4a

51. 2056 Q.No. 8 b What do you mean by one electron volt? [2]

Please refer to 2067 Supp Q.No. 4 b

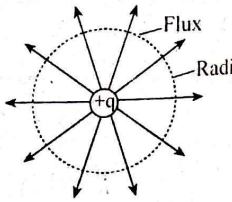
52. 2056 Q.No. 8 c No two lines of force in an electric field ever intersect each other. Why? [2]

Please refer to 2076 Set B Q.No. 4b

Long Answer Questions

53. 2076 Set B Q.No. 8a State and explain Gauss's law in electrostatics and use it to determine the electric field intensity due to a line charge. [4]

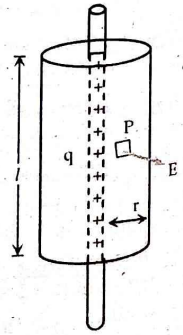
Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).



i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$\therefore \quad \phi = \frac{1}{\epsilon_0} \times q$

Suppose a very long positive line charge having linear charge density is λ. The field due to this charge will have cylindrical symmetry because all the points at certain distance around the line are identical to view it. To find the electric field intensity E at a point P at a normal distance r from the conductor, let us draw a cylindrical Gaussian surface is of length l and radius r so that the point P lies at its curved surface as shown in figure.



Here, the charge enclosed by the Gaussian surface is

$q = \lambda l$

From Gauss theorem, electric flux passing through the closed surface is

$\phi = \frac{1}{\epsilon_0} q = \frac{1}{\epsilon_0} \lambda l$ ...(i)

Also, the surface area of closed surface is

$A = 2\pi r l$ ... (ii)

So, the electric flux through the curved surface is

$\phi = E.A = E.\ 2\pi r l$

From equations (i) and (ii), we get

$E.\ 2\pi r l = \frac{1}{\epsilon_0} \lambda l$

$\therefore \quad E = \frac{\lambda}{2\pi\epsilon_0 r}$ ... (iii)

Hence, the electric field intensity due to a long linear charge distribution depends only on the linear charge density but not on the total charge and length.

54. 2076 Set C Q.No. 8a State and explain Gauss's theorem and use it to find the electric field intensity due to infinite plane sheet of charge. [4]

Gauss theorem: It states that the total electric flux passing through a closed surface enclosing a charge is equal to $\frac{1}{\epsilon_0}$ times the magnitude of net charge enclosed by the closed surface. If a surface encloses a charge a inside it, the electric flux passing through it is $\phi = \frac{q}{\epsilon_0}$ where ϵ_0 is the permittivity of vacuum. The closed surface enclosing the charge may be of any shape and such surfaces are called Gaussian surface.

Electric Field of an Infinite Plane Sheet of Charge:

Let us consider an infinite plane sheet with a uniform surface charge density σ. We have to find out the electric field intensity at a point P near the sheet. For this, consider a small area A surrounding P with its plane parallel to the sheet and construct a cylindrical Gaussian surface of base area A whose walls are perpendicular to the sheet, extending by an equal distance on both sides of the sheet. By symmetry, the electric field intensity E has the same magnitude on both sides of the surface and is directed normally away from the charged sheet. The flux crossing the side walls of the cylinder is zero because E is parallel to the walls.

Total electric flux passing through the two end flat faces,

$\phi = 2 \times E\,A$

Net charge enclosed by the Gaussian surface,

$q = \sigma \times A$

By Gauss's theorem, we have

$\phi = \frac{q}{\epsilon_0}$

or, $2 \times E\,A = \frac{1}{\epsilon_0} \times \sigma\,A$

or, $E = \frac{\sigma}{2\epsilon_0}$

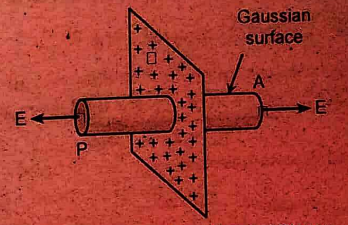
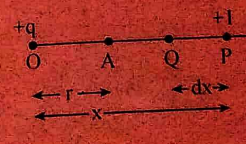


Fig: Infinite Plane Sheet of Charge

Thus, for points near the sheet, the electric field intensity is independent of the distance from the sheet.

55. 2075 GIE Q.No. 8a Define electric potential and derive an expression for it due to a point charge at a distance 'd' from it. [4]

Electric Potential (V): The electric potential at a point inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from infinity to that point.



If $W_{\infty A}$ is the amount of work done in moving unit positive charge from infinity to that point A, then, it is equal to the electric potential at that point A.

i.e., $V = W_{\infty A}$

Suppose a unit positive charge is at any point P at a distance x a point charge + q situated at O so that OP = x.

Since the force experienced by unit positive charge is equal to the electric intensity at that point, we

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have

$$F = \frac{1}{4\pi\epsilon_o}\frac{q}{x^2}$$

The work done in moving a unit positive charge from P to Q through small distance dx is

$$dW = F \times (-dx) = -\frac{1}{4\pi\epsilon_o}\frac{q}{x^2}dx$$

where, -ve sign indicate the work is done against electrostatic force.

Therefore, the total work done in moving a unit positive charge from infinity to a point A is

$$W_{\infty A} = \int_{\infty}^{r} dW = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}dx = -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r} = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

Hence, the electric potential at point A is $V = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$.

56. 2075 Set A Q.No. 8a Define electric field intensity. Write down its units. Obtain an expression for electric field due to an electric dipole at a point equidistant from each charge. [4]

Electric Field Intensity: The amount of electric force (F) experienced by a unit positive charge kept at a point inside the electric field is called electric field intensity. It is denoted by E and given by

$$E = \frac{F}{q}$$

The unit of E is NC^{-1}, Since F is in Newton & q is in Coulomb.

Electric Field Intensity due to a Dipole:

Let us consider a dipole of charges +q and -q, separated by a distance r. We have to find out the electric field intensity at a point R at equidistance r from P and Q, i.e. to charges. The electric field intensity at a point R due to a charge +q situated at Point P is given as

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$, directed point U along RU.

Similarly, electric field intensity at a point R due to a charge -q at Q is given by E_2 in both magnitude and direction as,

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$, directed along RS.

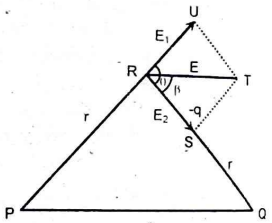


Fig: E due to a dipole

The resultant intensity E in both magnitude and direction is given by diagonal RT of a parallelogram RSTU. The resultant electric field is given by

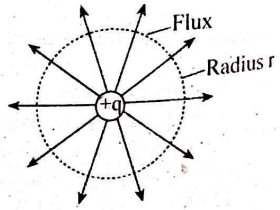
$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

and direction is given as

$$\beta = \tan^{-1}\left(\frac{E_1\sin\theta}{E_2 + E_1\cos\theta}\right)$$

57. 2075 Set B Q.No. 8b State Gauss law of electrostatics and use it to find the electric field intensity due to a plane charged conductor. [4]

Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).

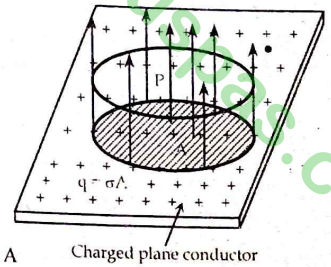


i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$$\therefore \quad \phi = \frac{1}{\epsilon_0} \times q$$

Electric field intensity due to a plane charged conductor,

Consider a plane conductor with the uniform surface charge density, σ. To find the electric field intensity at a point P out-side the conductor, draw a cylinder of cross-sectional area A whose top surface lies at P and the bottom surface lies in the plane of the conductor as shown in figure. Then by symmetry, the electric field intensity at every point on the cross sectional area A should be same. So, the flux passing normally through the surface area A is given by $\phi = E \times A$



Charged plane conductor

The net charge Q enclosed by the Gaussian surface is $q = \sigma \times A$

From Gauss's theorem, the total flux passing through the cylinder is

$$\phi = \frac{q}{\epsilon_0}$$

or, $E \times A = \frac{\sigma A}{\epsilon_0}$

or, $E = \frac{\sigma}{\epsilon_0}$

This is required electric field intensity due to plane charged conductor.

58. 2074 Supp. Q.No. 8a Define electric potential and derive an expression for it due to point charge. [4]

Please refer to 2075 GIE Q.No. 8a

59. 2074 Set A Q.No. 8a, 2066 Q. No. 8 a State and explain Gauss law of electrostatics. Apply it to obtain an expression for electric field of a linearly charged body. [4]

Please refer to 2076 Set B Q.No. 8a

60. 2074 Set A Q.No. 8b Are electric potential and electric field vector quantities? Justify with their definitions. Also obtain an expression for electric potential at a point due to a point charge. [4]

Electric potential: The electric potential at a point inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from infinity to that point.

Electric field: The electric field is defined as the region around the charge where electric force can be experienced. The electric field intensity E at a point inside the field is defined as the force experienced by unit positive charge when placed at that point.

They are not vector quantities because they do not obey the laws of vector algebra.

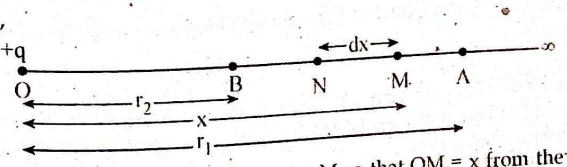
Second Part: Please refer to 2075 GIE Q.No. 8a

61. 2074 Set B Q.No. 8a Use Coulomb's law to define potential difference between two points near a static charge. Also derive expression for it. [4]

Electric potential difference: The electric potential difference between two points inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from one point to another point against the field.

If W_{BA} is the amount of work done in moving unit positive charge from point B to point A. Then, it is equal to the potential difference between two points A and B.

i.e. $V_{AB} = W_{BA}$,



Suppose a unit positive charge is at any point M so that OM = x from the point charge +q at O. We have to find electric potential difference between two points A and B.

Since the force experienced by unit positive charge is equal to the electric intensity at that point, then,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}$$,

The work done in moving a unit positive charge from M to N through small displacement dx is

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$dW = F \times (-dx) = -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}dx$

Negative sign indicates that E and dx are in opposite directions.

Therefore, the total workdone in moving a unit positive charge from point B to point A is

$$W_{BA} = \int dW = \int_{r_1}^{r_2} -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}dx = \int_{r_1}^{r_2} -\frac{q}{4\pi\epsilon_0}x^{-2}dx = -\frac{q}{4\pi\epsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_{r_1}^{r_2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x}\right]_{r_1}^{r_2}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = V_{AB}$$

Hence, the potential difference between two points A and B is

$$V_{AB} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

62. 2074 Set B Q.No. 8b State and explain Gauss law of electrostatics with one of its applications. [4]

Please refer to 2076 Set B Q.No. 8a

63. 2073 Supp Q.No. 8b Define electric field intensity and potential gradient and establish a relation between them. [4]

Electric field intensity: The amount of electric force experienced by a unit positive charge kept at a point inside the electric field is called electric field intensity. It is denoted as E and given as

$$E = \frac{F}{q}$$

Electric potential gradient: The rate of change of electric potential with respect to distance is called electric potential gradient. That is electric potential gradient $= \frac{dv}{dt}$.

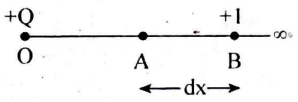
Relation between the electric potential gradient and the electric field strength at a point

Let A and B are two points on a line of force due to a point charge + Q, as shown in figure. These points are so close that electric field intensity (E) is constant between them.

Work done in taking a unit charge from B to A = Force × displacement.

$= -Edx$ ... (i)

$\left(\because E = \frac{F}{q'}$ for unit charge $q' = 1 \therefore E = F\right)$



Negative sign shows that force on the unit charge and displacement are in opposite directions.

If the potential difference between A and B is dV, (A is at higher potential), then work done in taking a unit charge from B to A = dV. ... (ii)

From (i) and (ii), we get

$dV = -Edx \Rightarrow E = -\frac{dV}{dx}$ ...(iii)

Thus, electric field strength is equal to the negative of the potential gradient. The direction of E is always in the direction of decrease of potential.

64. 2073 Set C Q.No. 8a What is electrostatic induction? Explain with necessary diagrams, a method of charging a body positively by induction. [4]

Electrostatic Induction

When an uncharged body is taken near to the charged body, the charges are developed on the uncharged body temporarily. This temporary electrification of a body by bringing a charged body close to it is called electrostatic induction.

a. Charging a body positively by induction:

Let us consider a negatively charged glass rod taken to near an uncharged body AB which is fitted on the insulated stand. This body AB can be charged positively by induction in the following ways:

Step I: If a negatively charged glass rod is brought near a body AB, the end A of the body acquires bound negative charge and the far end B acquires free negative charges due to induction as shown in Fig. (i).

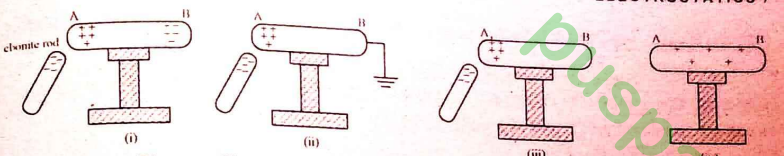


Figure: Charging a body positively by induction.

Step II: The body, AB is earthed with the help of the metal wire as shown in Fig.(ii). In this case, the free negative charge flows to the earth. However the positive charges being the bound charges do not move in to the earth.

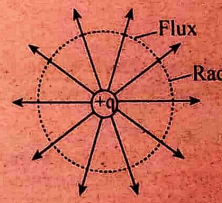
Step III: The earthing is removed keeping the glass rod still in its position. The bound positive charge on end B remains there as shown in Fig.(iii).

Step IV: Finally, the positively charged glass rod is removed away from the body, AB. The bound negative charge spreads over whole surface of the body, AB as shown in Fig. (iv). Therefore, the body, AB becomes positively charged by induction.

65. 2073 Set C Q.No. 8b State and explain Gauss's theorem and use it to find the electric field due to a charged sphere at a point [4]

i. outside the sphere and ii. inside the sphere.

Gauss's Theorem: The total electric flux passing through any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the closed surface in (SI system).



i.e., Electric flux $= \frac{1}{\epsilon_0} \times$ net charge enclosed

$\therefore \phi = \frac{1}{\epsilon_0} \times q$

Electric field due to a solid charged sphere: Let us consider a charged sphere of radius R has charge q on its surface.

There is a point P at a distance r from the centre O of the charged sphere, where we have to find the electric field (intensity).

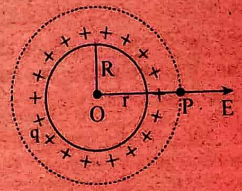
We now draw a concentric sphere of radius r so that the point P will lie outside from the surface of the sphere. Therefore, Area of this sphere $= 4\pi r^2$.

If E be the field intensity at point P, then flux through the sphere of radius r is

$\phi = E A = E 4\pi r^2$ ... (i)

From Gauss theorem, we have

$\phi = \frac{1}{\epsilon_0} q$ ... (ii)



Using equation (i) & (ii), we get

$E \times 4\pi r^2 = \frac{1}{\epsilon_0} q$

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

Thus, the electric field outside the charged sphere is same as whole of the charge were concentrated at its centre.

When the point p lies at the surface of the charged sphere (R=r). Electric field intensity at this surface is

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$

When the point p lies inside the sphere, then $r < R$.

Since, no charge lies inside the conductor, so $q = 0$.

Therefore, $E = \frac{1}{4\pi\epsilon_0}\frac{0}{r^2} = 0$

66. 2073 Set D Q.No. 8a State and explain Gauss's theorem and use it to find the electric field intensity due to a line charge. [4]

Please refer to 2076 Set B Q.No. 8a

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67. 2073 Set D Q.No. 8b **Define potential gradient and electric field intensity. Establish a relation between them.** [4]

Please refer to 2073 Supp. Q.No. 8b

68. 2072 Supp Q.No. 8a **State Gauss's theorem in electrostatics. Use this theorem to calculate the electric field due to a solid charged sphere at a point outside it.** [4]

Please refer to 2073 Set C Q.No. 8b

69. 2072 Set C Q.No. 8b **Define potential difference and potential gradient. Obtain an expression for the potential difference between two points in electric field due to a point charge.** [4]

Please refer to 2074 Set B Q.No. 8a

70. 2072 Set D Q.No. 8a **What is electric flux? State and explain Gauss law in electrostatics. Use it to find electric field intensity due to infinite plane sheet of charge.** [4]

Please refer to 2076 Set C Q.No. 8a

71. 2072 Set E Q.No. 8a **Define electric field intensity and dielectric constant of a medium. Also derive a relation between the electric field with the potential gradient.** [4]

Electric field intensity: The amount of electric force experienced by a unit positive charge kept at a point inside the electric field is called electric field intensity. It is denoted as E and given as

$$E = \frac{F}{q}$$

Dielectric constant: The dielectric constant or relative permittivity of a medium is defined as the ratio of permittivity of free space to permittivity of a medium.

i.e. $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

Second part: Please refer to 2073 Supp Q.No. 8b

72. 2071 Supp Q.No. 8a **State Gauss theorem in electrostatics. Use it to calculate electric field near an infinite plane sheet of charge.** [4]

Please refer to 2076 Set C Q.No. 8a

73. 2071 Set C Q.No. 8 a **State and explain Gauss law. Use it to find the electric field due to a charged conducting sphere (i) outside, and (ii) inside.** [4]

Please refer to 2073 Set C Q.No. 8b

74. 2071 Set D Q.No. 8 a **What is the physical difference between electric potential are electric field intensity? Derive an expression for the potential differences between two points in the electrostatic field.** [4]

Please refer to 2074 Set B Q.No. 8a

75. 2071 Set D Q.No. 8 b **State and explain Gauss law of electrostatics and apply it to find an electric field intensity on the surface and inside the surface of a charged sphere.** [4]

Please refer to 2073 Set C Q.No. 8b

76. 2070 Supp (Set A) Q.No. 8 a **Define electric potential. Derive an expression for the electric potential due to a point charge at a distance 'r' from it.** [4]

Please refer to 2075 GIE Q.No. 8a

77. 2070 Supp (Set B) Q.No. 8 a **Define potential difference and potential gradient. Derive a relation between the potential gradient and electric field intensity.** [4]

Electric potential difference: The electric potential difference between two points inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from one point to another point against the field.

Second Part: Please refer to 2073 Supp Q.No. 8b

78. 2070 Set C Q.No. 8 a **Define electric field intensity and potential gradient. Establish a relation between them.**

Please refer to 2073 Supp Q.No. 8b

79. 2070 Set C Q.No. 8 b **State and explain Gauss's law in electrostatics. Use this law to obtain electric field intensity due to a charged sphere i) outside it ii) inside it.**

Please refer to 2073 Set C Q.No. 8b

80. 2070 Set D Q.No. 8 a **What is electrostatic induction? How can you charge a body positively by induction?** [4]

Please refer to 2073 Set C Q.No. 8a

81. 2070 Set D Q.No. 8 b **State and explain Gauss's law in electrostatics. Use it to find the electric field intensity due a line charge.** [4]

Please refer to 2076 Set B Q.No. 8a

82. 2069 Supp Q.No. 8a **Define electric field intensity and electric potential. Deduce an expression for the potential difference between two points in an electric field.** [4]

Electric field intensity: The amount of electric force experienced by a unit positive charge kept at a point inside the electric field is called electric field intensity. It is denoted as E and given as

$$E = \frac{F}{q}$$

Electric Potential (V): The electric potential at a point inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from infinity to that point.

Second Part: Please refer to 2074 Set B Q.No. 8a

83. 2069 (Set A) Q. No. 8a **State Gauss Theorem. Use this theorem to find the electric field intensity due to a plane charged conductor.** [4]

Please refer to 2075 Set B Q.No. 8b

84. 2069 (Set B) Q. No. 8a 2069 Old (Set B) Q. No. 9a 2066 Q.No. 9 a 2063 Q.No. 9 a OR **Define electric potential. Derive an expression for the potential due to a point charge at any point in space.** [4]

Please refer to 2075 GIE Q.No. 8a

85. 2069 (Set A) Old Q. No. 9b **State Gauss law. Use it to find the electric field due to an infinite plane conductor.** [4]

Please refer to 2075 Set B Q.No. 8b

86. 2069 Old (Set B) Q. No. 9a OR 2058 Q.No. 9 a **State Gauss's theorem in electrostatics. Use this theorem to calculate the electric field due to a solid charged sphere at a point inside it.** [1+4]

Gauss's Theorem: Please refer to 2076 Set B Q.No. 8a

Electric field intensity inside a solid charged sphere: Let us consider a charged sphere whose electric field intensity at a point P inside the surface has to be determined. For this, draw a sphere of radius r, such that r < R and concentric to the charged sphere as shown in figure. Thus, the surface of this sphere is the Gaussian surface. Since there is no charge inside the charged sphere, the Gaussian surface does not enclosed any charge. So, the charge inside the Gaussian surface, is q = 0. If E be the electric field intensity at point P, the flux passing through the surface is

$\phi = E \times A$

But from Gauss's theorem, the flux passing through the sphere is

$\phi = \frac{q}{\epsilon_0}$

where, q is the charge inside the Gaussian surface, which is equal to zero.

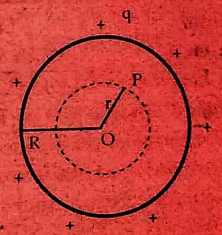
Then, we have

$E \times A = \frac{q}{\epsilon_0}$

or, $E \times 4\pi r^2 = \frac{0}{\epsilon_0}$

or, $E = 0$

Thus, the electric field intensity is zero everywhere inside the charged sphere.



87. 2068 Q.No. 8 a **State and explain Gauss's theorem in electrostatics. Use this theorem to find the electric field intensity due to a plane charged conductor.** [4]

Please refer to 2075 Set B Q.No. 8b

88. 2068 Q.No. 8 b **Define potential gradient. Derive a relation between electric field intensity and potential gradient.** [4]

Please refer to 2073 Supp. Q.No. 8b

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89. 2067 Supp Q.No. 8 a **State and explain Gauss's theorem in electrostatics. Use it to obtain an expression for electric field intensity due to a linearly charged body.** [4]

Please refer to 2076 Set B Q.No. 8a

90. 2067 Q.No. 8 a **Define electric potential and derive an expression for it due to a point charge.** [4]

Please refer to 2075 GIE Q.No. 8a

91. 2066 Q. No. 8 b **What is electric potential? Write its unit. Obtain an expression for electric potential at a point near an electrostatic charge.** [4]

Please refer to 2075 GIE Q.No. 8a

92. 2065 Q. No. 9 a **Define electric potential Derive an expression for the potential difference formula and hence, obtain the potential at point due to a point charge.** [1+3+1]

Please refer to 2074 Set B Q.No. 8a

93. 2062 Q.No. 9 a **Define electric potential and intensity at a point due to a charge. Obtain an expression for the potential difference between two points r_1 and r_2 from charge +q.** [5]

Please refer to 2074 Set B Q.No. 8a

94. 2061 Q.No. 9 a **Define potential and electric field at a point in an electrostatic field. Derive a relation between the electric potential gradient and the electric field strength at a point.** [5]

Electric potential: The electric potential at a point inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from infinity to that point.

Electric field: The electric field is defined as the region around the charge where electric force can be experienced. The electric field intensity E at a point inside the field is defined as the force experienced by unit positive charge when placed at that point.

Second Part: Please refer to 2073 Supp Q.No. 8b

95. 2060 Q.No. 9 a **State and explain Gauss's theorem in electrostatics and use it to find the electric field intensity due to a hollow charged spherical conductor.** [5]

Please refer to 2073 Set C Q.No. 8b

96. 2057 Q.No. 9 a **What is an electric potential? Derive the formula for the potential at a point due to a point charge.** [1+4]

Please refer to 2075 GIE Q.No. 8a

Numerical Problems

97. 2076 Set B Q.No. 12 **Three equal charges 1.8×10^{-6} C each are located at the corners of an equilateral triangle ABC of side 10cm. Calculate the electric potential due to these charges at the mid point of AB.** [3]

Solution

Given,

Charges (q) = 1.8×10^{-6} C

Side of triangle (r) = 10 cm = 0.1m

Electric potential (V) = ?

Permittivity of free space (ε_0) = 8.85×10^{-12} $C^2 N^{-1} m^{-2}$

Let, ABC be an equilateral triangle of side 0.1 m. Let D be the mid point on AB such that AD = BD = 0.05 m.

Three charges of charge q are located at three corners of triangle ABC.

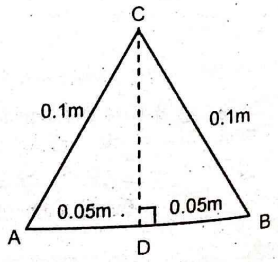
Now,

Potential at D due to the charge of A, $V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AD}$

Potential at D due to the charge of B, $V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BD}$

Potential at D due to the charge of C, $V_C = \frac{1}{4\pi\varepsilon_0}\frac{q}{CD}$

$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{AC^2 - AD^2}}$



Total potential at D, $V = V_A + V_B + V_C$

$= \frac{1}{4\pi\varepsilon_0}\frac{q}{AD} + \frac{1}{4\pi\varepsilon_0}\frac{q}{BD} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{AC^2 - AD^2}}$

$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{AD} + \frac{1}{BD} + \frac{1}{\sqrt{AC^2 - AD^2}}\right)$

$= \frac{1.8 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12}}\left(\frac{1}{0.05} + \frac{1}{0.05} + \frac{1}{\sqrt{0.1^2 - 0.05^2}}\right)$

$= 9 \times 10^9 \times 1.8 \times 10^{-6} \times 51.547 = 8.35 \times 10^5$ V/m

98. 2075 GIE Q.No. 12 2069 Supp Q.No. 12 **Calculate the value of two equal charges if they repel each other with a force of 0.1N when situated 50 cm apart in vacuum. What would be the distance between them if they are placed in an insulating medium of dielectric constant 10?** [3]

Solution

Given,

Force (f) = 0.1N

Distance between two charges (r) = 50 cm = 0.5m

Value of charge (q) = ?

We have, in air,

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ [$\because q = q_1 = q_2$]

$0.1 = \frac{9 \times 10^9 \times q^2}{(0.5)^2}$

or, $q^2 = \frac{0.1 \times (0.5)^2}{9 \times 10^9}$

or, $q^2 = 2.778 \times 10^{-12}$

or, $q = 1.667 \times 10^{-6}$C

$\therefore q = 1.667\ \mu$C

When they are kept in medium of dielectric constant 10,

$F = \frac{1}{4\pi \epsilon . \epsilon_0} \times \frac{q^2}{r^2}$

$0.1 = \frac{9 \times 10^9 \times (1.667 \times 10^{-6})^2}{10\, r^2}$

$r = \sqrt{\frac{9 \times 10^9 \times (1.667 \times 10^{-6})^2}{10 \times 0.1}} = 0.16$ m

99. 2075 Set A Q.No. 12 **Two large parallel metal plates carry opposite charges. They are separated by 10 cm and p.d. of 500 volts is applied on them. (a) what is the magnitude of electric field strength between them? Compute the work done by this field on a charge of 2×10^{-9} C as it moves from higher to the lower potential.** [3]

Solution

Given,

Distance between plate (d) = 10 cm = 0.1 m

Potential difference between plate (V) = 500 V

Electric field strength (E) = ?

Work done (W) = ?

Charge (q) = 2×10^{-9}C

Now, we have,

$E = \frac{V}{d} = \frac{500}{0.1} = 5000$ V/m

Again, Work done, W = q . V

$= 2 \times 10^{-9} \times 500 = 10^{-6}$ J

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100. 2075 Set B Q.No. 12 **Two large parallel metal plates carry opposite charges. They are separated by 0.20 m and the p.d between them is 500 V. What is the magnitude of electric field, if it is uniform, in the region between them?** [3]

Solution

Given,
Distance (d) = 0.2 m
Potential difference (d) = 500 V
Electric field intensity (E) = ?
We have,

$$E = \frac{V}{d} = \frac{500}{0.2} = 2500 \text{ V/m}$$

101. 2072 Set C Q.No. 12 2070 Supp (Set A) Q.No. 12 **What distance must an electron move in a uniform potential gradient 200V cm^{-1} in order to gain kinetic energy 3.2×10^{-18} J?** [3]

Solution

Given,
Potential gradient (E) = 200 V/cm = 20000 V/m
K.E. gained (K.E.) = 3.2×10^{-18} J
Distance moved (d) = ?
We have

$$eV = \frac{1}{2} mv^2 = \text{K.E.}$$

$$eV = 3.2 \times 10^{-18}$$

or, e.d.E. = 3.2×10^{-18} $\quad [\because E = \frac{V}{d}]$

$$\text{or, } d = \frac{3.2 \times 10^{-18}}{e \times E} = \frac{3.2 \times 10^{-18}}{1.6 \times 10^{-19} \times 20000} = 1 \times 10^{-3} \text{ m}$$

102. 2072 Set D Q.No. 12 **An electron of mass 9.1×10^{-31} kg and charge 1.6×10^{-19} C is situated in a uniform electric field of intensity 1.2×10^4 Vm^{-1}. Find the time it takes to travel 1 cm from rest.** [3]

Solution

Given,
Mass of electron (m_e) = 9.1×10^{-31} kg
Charge of electron (e) = 1.6×10^{-19} C
Electric field intensity (E) = 1.2×10^4 V/m
Distance covered (y) = 1 cm = 10^{-2} m
Time taken (t) = ?
We have,

$$F = eE$$

$$\text{or, } ma = eE$$

$$\text{or, } a = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 1.2 \times 10^4}{9.1 \times 10^{-31}} = 2.1 \times 10^{15} \text{ m/sec}^2$$

Again,

$$y = ut + \frac{1}{2} at^2 \quad [\because u = 0 \text{ m/sec}]$$

$$10^{-2} = 0 + \frac{1}{2} \times 2.1 \times 10^{15} t^2$$

$$\text{or, } t^2 = 9.5 \times 10^{-18}$$

$$t = 3.1 \times 10^{-9} \text{ sec}$$

103. 2072 Set E Q.No. 12 **A solid sphere of radius 1 cm is carrying a charge of 2C. Find the electric field intensity at the centre, on its surface and at a point 2 cm from the centre of the charged sphere.** [3]

Solution

Given,
Radius of sphere (R) = 1 cm = 10^{-2} m
Charge on sphere (q) = 2C
Electric field intensity (E) = ?

Now,

i. At the centre

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = 0, \text{ due to } q = 0 \text{ at centre}$$

ii. At the surface of sphere,

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} \times \frac{2}{(10^{-2})^2} = \frac{9 \times 10^9 \times 2}{(10^{-2})^2}$$

$$= 1.8 \times 10^{14} \text{ N/C}$$

iii. At 2m from the centre

$$E_3 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = 9 \times 10^9 \times \frac{2}{(2 \times 10^{-2})^2} = 4.5 \times 10^{13} \text{ N/C}$$

104. 2071 Supp Q.No. 12 **An alpha particle is a nucleus of doubly ionized helium. It has mass of 6.68×10^{-27} kg and charge of 3.2×10^{-19}C. Compare the force of electrostatic repulsion between the two alpha particles with the force of gravitational attraction between them.** [3]
[Universal gravitational constant = 6.67×10^{-11} Nm^{2}kg^{-2}; Permittivity of free space = 8.85×10^{-12}Fm^{-1}]

Solution

Given,
Mass of helium (m) = 6.68×10^{-27} kg
Charge of helium (q) = 3.2×10^{-19} C
(F_e/ Fe) = ?

Now, $F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$

$$F_g = G\frac{m^2}{r^2}$$

Now, $\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G} \frac{q^2}{m^2} = \frac{1}{4\pi \times 8.85 \times 10^{-12} \times 6.67 \times 10^{-11}} \times \frac{(3.2 \times 10^{-19})^2}{(6.68 \times 10^{-27})^2} = 3.1 \times 10^{35}$

105. 2070 Supp (Set B) Q.No. 12 **An electron of charge 1.6×10^{-19}C is situated in a uniform electric field of intensity 120,000 V/m. Find the force on it and its acceleration.** [3]

Solution

Given,
Charge of an electron (e) = 1.6×10^{-19}C
Electric field intensity (E) = 120,000 V/m
Force (F) = ?
Acceleration (a) = ?
Mass of electron (m_e) = 9.1×10^{-31} kg
We have

$$F = eE = 1.6 \times 10^{-19} \times 120000 = 1.92 \times 10^{-14} \text{ N}$$

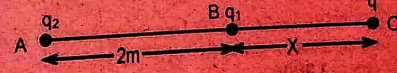
$$a = F/me = \frac{1.92 \times 10^{-14}}{9.1 \times 10^{-31}} = 2.1 \times 10^{16} \text{ m/sec}^2$$

106. 2069 (Set A) Q. No. 12 **Two charges $+1 \times 10^{-6}$ C and -4×10^{-6}C are separated by a distance of 2m. Determine the position of the null point.** [3]

Solution

Given,
Charge (q_1) = 1×10^{-6} C
Charge (q_2) = -4×10^{-6} C
Distance (d) = 2m
Null distance = ?
From figure,
Force on q due to q_1 is

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{x^2} \text{ along BC}$$



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Force on q due to q_2 is

$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+2)^2}$ along CA

By question, $F_1 = F_2$ (for null point)

or, $\frac{q_2}{x^2} = \frac{q_1}{(x+2)^2}$

or, $\frac{10^{-6}}{x^2} = \frac{4\times10^{-6}}{(x+2)^2}$

or, $\left(\frac{1}{x}\right)^2 = \left(\frac{2}{x+2}\right)^2$

$\therefore$ x = 2m from charge q_1 and 4 m from charge q_2.

107. 2068 Old Q.No. 9 b **Two horizontal parallel plates, 20mm apart, have a potential difference of 1000V between them the upper plate being at a positive potential. A negatively-charged oil drop, mass 4.8×10^{-15} Kg, is situated between the plates. Calculate the number of electrons on the drop if it is stationary in the air, neglecting the density of air. (take g = 10 ms^{-2} and e = 1.6×10^{-19}C)** [4]

Solution

Given,

Distance (d) = 20mm = 20×10^{-3} m

Potential difference (V) = 1000V

Mass of oil drop (m) = 4.8×10^{-15} kg

No. of electrons (n) = ?

We have,

For stationary charge,

W = F_e

mg = q.E

or, $q.\frac{V}{d} = mg \quad \left[\because E = \frac{V}{d}\right]$

or, $q = \frac{mgd}{V} = \frac{4.8\times10^{-15}\times10\times20\times10^{-3}}{1000} = 9.6\times10^{-19}$C

Again from quantization of charge q = ne

$n = \frac{q}{e}$

$= \frac{9.6\times10^{-19}}{1.6\times10^{-19}} = 6$

108. 2067 Q.No. 12 **A hollow spherical conductor of radius 12 cm is charged to 6×10^{-6}C. Find the electric field strength at the surface of sphere, inside the sphere at 8cm and at distance 15 cm from the sphere.** [3]

Solution

Given,

Radius of charged sphere (R) = 12cm

Charge (q) = 6×10^{-6}C

Electric field intensity (E) = ?

i. On the surface

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} = \frac{9\times10^9\times6\times10^{-6}}{(0.12)^2} = 3.75\times10^6$ N/C

ii. Inside the sphere

E = 0 because charge enclosed by Gaussian surface is zero.

iii. Outside the sphere at distance 15 cm

r = 12 + 15 = 27 cm = 0.27m

$E = \frac{1}{4\pi E_o}\frac{q}{r^2}$

$= \frac{9\times10^9\times6\times10^{-6}}{(0.27)^2} = 7.41\times10^5$ N/C

109. 2066 Q.No. 9 b **Two horizontal parallel plates each of area 500 cm^2 are mounted 2mm apart in vacuum. The lower plate is earthed and the upper one is given a positive charge of 0.05 μC. Find the electric field intensity and the potential difference between the plates.** [4]

Solution

Given,

Area of plates (A) = $500cm^2 = 500 \times 10^{-4}$ m^2

Distance (d) = 2mm = 2×10^{-3} m

Charge (q) = 0.05 μc = 0.05×10^{-6} C

Electric field intensity (E) = ?

Electric potential (V) = ?

We have

$E = \frac{\sigma}{\epsilon_o} = \frac{q}{A\epsilon_o}$

$= \frac{0.05\times10^{-6}}{500\times8.85\times10^{-12}\times10^{-4}}$

$= 1.13\times10^5$ V/ m

and

$V = E \times d$

$= 1.13\times10^5\times2\times10^{-3}$

$= 226$ V

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Chapter 2: Capacitance and Dielectrics

Short Answer Questions

1. 2076 Set C Q.No. 4b **Two plates of a charged capacitor are brought closer. What happens to its capacitance and energy stored?** [2]

The capacitance of a parallel plate capacitor is given by $C = \frac{\epsilon A}{d}$, where ϵ is permittivity of a medium kept inside the plates. A be area of plate and d is the distance between the plates of capacitor. If two plates of a charged capacitor are brought closer to each other, d will be decreased. As a result capacitance, C increased. Again energy stored in a charged capacitor is given as

$u = \frac{1}{2}CV^2$, as C increases, u also increases.

2. 2075 GIE Q.No. 4b 2070 Supp (Set A) Q.No. 4 b **Two identical capacitors are connected in series. Is the resulting capacitance greater or less than that of each individual capacitor?** [2]

Let two identical capacitors of capacitance C of each are connected in series, then the net capacitance is $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$ or $C' = \frac{C}{2}$. So, net capacitance is less than the individual capacitance. If they are connected in parallel, then their resultant capacitance is C" = C + C = 2C. So, net capacitance is greater than individual capacitance.

3. 2075 Set B Q.No. 4a **What is the physical significance of relative permittivity of a material placed between two plates of a capacitor?** [2]

The capacitance of parallel plate capacitor is given by

$C_0 = \frac{\epsilon_0 A}{d}$

Where A is the area of cross-section, d is distance between plates and ϵ_0 is permittivity of free space. If a dielectric is inserted between the plate, then capacitance is

$C_m = \frac{\epsilon A}{d}$

Then,

$\frac{C_m}{C_0} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$

where ϵ_r is the relative permittivity, which is greater than 1. So, the capacitance of a capacitor increases when a dielectric is inserted between them.

Again q = CV, this shows that potential decreases on increasing capacitance.

4. 2074 Supp. Q.No. 4b **What are the factors which determine the value of capacitance of a parallel plate capacitor?** [2]

The capacitance of a parallel plate capacitor is given by $C = \frac{\varepsilon A}{d}$, where ε is the permittivity of the medium that is kept in between the plates of the capacitor, A is the common area of the plates and d is the separation between the plates. From this relation, we can see that the capacitance of a parallel plate capacitor is determined by the factors like area of plate A, distance between the plate d and permittivity of medium ε.

5. 2074 Set A Q.No. 4b **Two parallel plate capacitors identical in shape and size are filled with air and mica separately. Which one has larger capacitance? Write a proper relation in support of your answer.** [2]

The capacitance of parallel plate capacitor is given by

$C_0 = \frac{\epsilon_0 A}{d}$

Where A is the area of cross-section, d is distance between than and ϵ_0 is permittivity of free space. If a dielectric is inserted between the plate, then capacitance is

$C_m = \frac{\epsilon A}{d}$

Then,

$\frac{C_m}{C_0} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$

where ϵ_r is the relative permittivity, which is greater than 1. So, the capacitance of a capacitor increases when a dielectric is inserted between them.

The capacitor with mica will have larger capacity than the air capacitor because mica has higher dielectric constant.

6. 2073 Supp Q.No. 4b **Explain the difference between dielectric constant and dielectric strength.** [2]

The difference between dielectric constant and dielectric strength are:

Dielectric constant	Dielectric strength
a. Dielectric constant (K) of a medium is defined as the ratio of permitivity (ϵ) of the medium to permitivity (ϵ_0) of free space.	a. Dielectric strength of a medium is the maximum electric field intensity applied to it that can withstand by the medium without insulation breakdown.
b. It has no unit.	b. Its SI unit is NC^{-1} or Vm^{-1}
c. It is a pure number, so it is scalar.	c. It is a vector.

7. 2073 Set C Q.No. 4b **Can we give any desired charge to a capacitor? Explain.** [2]

No, we can't give any desired charge to a capacitor. We can charge the capacitor to its dielectric strength but not over this value. As we increase the charge to a capacitor, the potential difference between the plates also increases. A stage is reached when the electric field between the two plates attains the breakdown value of dielectric, the charges start to leak.

8. 2073 Set D Q.No. 4b **What happens to the capacity and potential difference of a charged capacitor when a dielectric medium is inserted between its plates?** [2]

Please refer to 2075 Set B Q.No. 4a

9. 2072 Set C Q.No. 4b **Can we give any desired quantity of charge to a capacitor?**

Please refer to 2073 Set C Q.No. 4b

10. 2072 Set E Q.No. 4b **Two parallel plate capacitors are identical in every aspect except the insulating dielectric media between the plates. If air and mica are kept as dielectric in respective capacitors, which one will have larger capacity?** [2]

Please refer to 2074 Set A Q.No. 4b

11. 2071 Supp Q.No. 4b **What are the uses of dielectrics in capacitors?** [2]

The capacitance of parallel plate capacitor is given by

$C_0 = \frac{\epsilon_0 A}{d}$

Where A is the area of cross-section, d is distance between than and ϵ_0 is permittivity of free space. If a dielectric is inserted between the plates, then capacitance is

$C_m = \frac{\epsilon A}{d}$

Then,

$\frac{C_m}{C_0} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$

where ϵ_r is the relative permittivity, which is greater than 1. So, the capacitance of a capacitor increases when a dielectric is inserted between them.

12. 2071 Set D Q.No. 4 a **Explain the difference between a polar and a non-polar dielectrics with one example of each.** [2]

Polar molecule: If the centre of gravity of the positive and negative electric charges does not coincide with each other, then the molecules of dielectrics are called polar molecules. Examples of polar molecules are H_2O, NH_3, HCl etc. They have both induced and permanent dipole moment. The dielectrics formed from these molecules are called polar dielectrics.

Non-polar molecules: If the centre of gravity of the positive and negative electric charges coincide with each other, than the molecules of dielectric are called non-polar molecules. Examples of non-

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polar molecules are CO_2, O_2, H_2 etc. They have only induced dipole moment but do not have permanent dipole moment. The dielectrics formed from these molecules are called non-polar dielectrics.

13. 2071 Set C Q.No. 4 b **Three capacitors of the same capacitance are joined all in (i) series, and (ii) all in parallel. By how much one of these combinations is larger in magnitude? Explain.** [2]

When three capacitors of equal capacitances C are connected in parallel, then equivalent capacitance C_p is

$C_p = C + C + C = 3C$

If these capacitors are connected in series, then equivalent capacitance is

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

or, $C_s = \frac{C}{3}$

Now,

$$\frac{C_p}{C_s} = 9$$

$\therefore$ C_p is 9 times greater than C_s

14. 2070 Supp (Set B) Q.No. 4 b **What happens to the capacity and potential difference of a charged capacitor when a dielectric medium is inserted between its plates?** [2]

Please refer to 2075 Set B Q.No. 4a

15. 2070 Set C Q.No. 4 a 2068 Q.No. 4 b 2060 Q.No. 8 b **Distinguish between dielectric constant and dielectric strength.** [2]

Please refer to 2073 Supp Q.No. 4b

16. 2070 Set D Q.No. 4 b **What are the factors that determine the capacitance of a parallel plate capacitor?** [2]

Please refer to 2074 Supp. Q.No. 4b

17. 2069 Supp Q.No. 4b **Two identical capacitors are connected in series. Is the resultant capacitance greater or less than that of each individual' capacitor? What if they are connected in parallel?** [2]

Please refer to 2075 GIE Q.No. 4b

18. 2069 Old (Set B) Q. No. 8c **Two identical capacitors are connected in series. Is the resulting capacitance greater or less than that of each individual capacitance? What happen if they are connected parallel?**

Please refer to 2075 GIE Q.No. 4b

19. 2068 Old Q.No. 8 c **Explain how the capacitance of a parallel plate capacitor can be increased.** [2]

The capacitance of a parallel plate capacitor is given by $C = \frac{\varepsilon A}{d}$, where ε is the permittivity of the medium that is kept in between the plates of the capacitor, A is the common area of the plates and d is the separation between the plates. From this relation, we can see that the capacitance of a parallel plate capacitor can be increased by increasing the area of each plate, keeping the dielectric of high value of dielectric constant or relative permittivity and placing the plates far each other.

20. 2067 Q.No. 4 b **Differentiate between dielectric strength and dielectric constant.** [2]

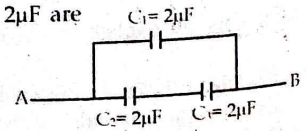
Please refer to 2073 Supp Q.No. 4b

21. 2066 Q. No. 4 b **Is dielectric constant, the same as dielectric strength? Explain with example.** [2]

Please refer to 2073 Supp Q.No. 4b

22. 2065 Q. No. 8 c **How will you arrange 3 capacitors each having the capacity of 2 μF, to get a capacitor of capacity 3μF?** [2]

Let, us consider three charges C_1, C_2 and C_3 of capacitances 2μF are joint as shown in figure.



Then, their equivalent capacitance is

$$C_{AB} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2}$$

$$= \frac{2\times2 + 2\times2 + 2\times2}{2+2} = \frac{4+4+4}{4} = \frac{12}{4} = 3\mu F$$

$\therefore$ The three capacitors each having the capacity of 2μF, to get a capacitor of capacitor 3μF are arranged such that one of capacitor is connected in parallel with series combination of other two.

23. 2062 Q.No. 8 b **Define dielectric constant and dielectric strength.** [2]

Please refer to 2073 Supp Q.No. 4b

24. 2061 Q.No. 8 c **Distinguish between polar and non-polar dielectrics.** [2]

Please refer to 2071 Set D Q.No. 4 a

Long Answer Questions

25. 2076 Set B Q.No. 8b **What is capacitance of a capacitor? Derive a relation for the energy stored in a capacitor when it is charged to a potential difference of V volts.** [4]

Capacitance of capacitor: The ability of a capacitor to store charge is called its capacitance. The rise in potential difference between conducting plates is directly proportional to the charge given to them.

i.e. $q \alpha V \Rightarrow q = CV$

where C is a constant of proportionality & is called the capacitance of the conductor. Thus,

$C = \frac{q}{V}$

When V = 1 volt, then C = q

Thus, capacitance of a capacitor is numerically equal to the charge required to rise p.d. by 1 volt. The SI unit of capacitance is Farad (F).

Energy stored of a charged Capacitor: Suppose an uncharged capacitor of capacitance C is to be charged with charge q so that its p.d. becomes V, then q = CV.

Suppose the uncharged capacitor is started to be charged. At any instant, let the charge on plate of capacitor is dq & p.d. across the plate is V.

Now, the work done in charging the charge dq is given by

$dW = V\,dq = \frac{q}{C}\,dq$ (since $q = CV$)

When the capacitor is charged to q, then the work done is charging of capacitor from 0 to q is,

$$W = \int_0^q dW = \frac{1}{C}\int_0^q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q = \frac{q^2}{2C}$$

This work done is stored as (electrical potential) energy on the capacitor. Thus, the energy stored in the capacitor is given by,

$E = \frac{q^2}{2C}$

Moreover, using $q = CV$, we get

$E = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$ and $E = \frac{1}{2}\frac{q^2}{q/V} = \frac{1}{2}qV$

Hence, $E = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{q^2}{2C}$

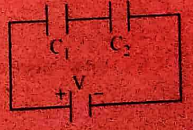
This is the required expression of energy stored by capacitor.

26. 2076 Set C Q.No. 8b **Define capacitance of a capacitor, Obtain relations for equivalent capacitance for capacitors connected in series and parallel.** [4]

Capacitance of capacitor: Please refer to 2076 Set B Q.No. 8b

Series Combination of Capacitor:

The capacitors are said to be joined in series, if the charge on each capacitor is same while the voltage across them is different. Suppose C_1 and C_2 are the capacitances of two capacitors which are connected to the terminals of a d.c. cell with p.d. V so that charge on each capacitor is q. The voltages across the capacitors are $V_1 = \frac{q}{C_1}$ and $V_2 = \frac{q}{C_2}$.



Thus, the total voltage across them becomes,

$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2}$

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$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$, where $C = \frac{q}{V}$ is the total (or equivalent or net) capacitance of the series arrangement.

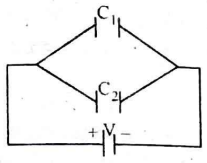
Thus, for series combination of capacitors, the reciprocal or equivalent capacitance is equal to the sum of the reciprocal of capacitance of individual capacitance and which is smaller than the smaller capacitance of individual capacitance.

Parallel Combination of Capacitors:

The capacitors are said to be joined in parallel, if each capacitor is directly connected to a source so that voltage across each capacitor remains same while the charge is different.

Suppose C_1 and C_2 are the capacitance of two capacitors which are joined in parallel across a source of emf V.

Suppose V is the p.d. between the plates of each capacitor, then charge on each capacitor is $q_1 = C_1V$ and $q_2 = C_2V$ respectively.



Thus, the total charge becomes,

$q = q_1 + q_2 = C_1V + C_2V$

$= (C_1 + C_2)\,V$

$\frac{q}{V} = C_1 + C_2$

Therefore, $C = C_1 + C_2$

where $C = \frac{q}{V}$ is the resultant (or net or total or equivalent) capacitance of the parallel combination.

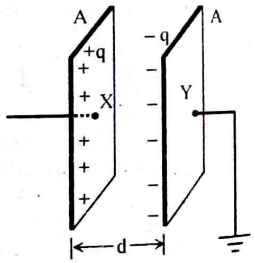
Hence, total capacitance of parallel combination is equal to the algebraic sum of the capacitance of individual capacitor.

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27. 2075 GIE Q.No. 8b **Define capacitance of a capacitor. Deduce an expression for the capacitance of a parallel plate capacitor. Discuss the action of dielectric between the plates.** [4]

Capacitance of capacitor: Please refer to 2076 Set B Q.No. 8b

Expression for the capacitance of a parallel plate capacitor: Parallel plate capacitor is a combination of two parallel plates each of equal area & placed very close to each other. One plate is given some positive charge while the other is connected to earth.



Let us consider two parallel conducing plates X and Y, each of area A, separated by a distance d. The plate X is given a charge +q and the plate Y is earthed. When charge +q is given to the plate X, then charge –q is induced on the left face of plate Y and charge +q on its right face.

When the plate Y is earthed, the charge +q on its right face flows to the earth. Charges +q and –q on plates X and Y produce an electric field between them. If each plate has a uniform surface charge density σ, then electric field E between the plates is uniform and is given as

$E = \frac{\sigma}{\epsilon_0}$

where ϵ_0 is the permittivity of free space as the plates are kept in vacuum or air. The potential difference between the plates is

$V = Ed = \frac{\sigma d}{\epsilon_0}$

Since $\sigma = \frac{q}{A}$, so

$V = \frac{qd}{\epsilon_0 A}$

or $\frac{q}{V} = \frac{\epsilon_0 A}{d}$

But $\frac{q}{V}$ is the capacitance of a capacitor. So, the capacitance of the parallel plate capacitor is,

$C = \frac{\epsilon_0 A}{d}$.

If the space between the plates is filled with a medium of absolute permittivity ϵ, then capacitance of the capacitor is given by

$C = \frac{\epsilon A}{d}$

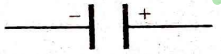
This is required expression for the capacitance of a parallel plate capacitor.

The action of dielectric (ϵ) is to increase the capacitance of the capacitor.

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28. 2075 Set A Q.No. 8b **What is a capacitor? Derive an expression for energy loss, when two charged capacitors are connected with like plates together.** [4]

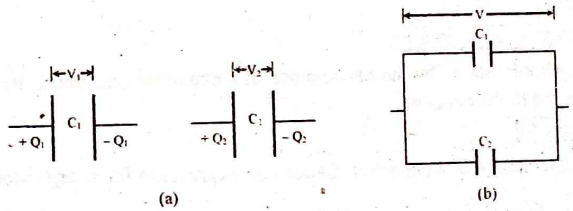
Capacitor: An electric device which is used to store charge is called capacitor. A capacitor essentially consists of two conducting surfaces separated by an insulating material called dielectric. The conducting surfaces are called plates of the capacitor and the dielectrics are mica, air, paper etc.



A capacitor in an electrical circuit which is denoted as shown in Fig, in which two vertical parallel lines represent the two conducting plates and a horizontal line attached to each plate is called lead used in connecting to the external circuit.

Expression for energy loss:

When two capacitors charged at different potential differences and are connected by a metallic wire, there is flow of charge from the capacitor at higher potential to the capacitor at lower potential till the potential of the two capacitors becomes equal. In this process, the total charge remains constant but the total energy before joining the capacitors will be greater than that after joining. This means when two charged capacitors are connected by a wire, there is always loss of energy.



Consider two charged capacitors of capacitances C_1 and C_2 that store the charges Q_1 and Q_2 respectively as shown in figure. Let V_1 and V_2 be the potentials developed on the respective capacitors.

Energy stored in the capacitor C_1 before joining $= \frac{1}{2} C_1 V_1^2$

and energy stored in the capacitor C_2 before joining $= \frac{1}{2} C_2 V_2^2$

$\therefore$ Total energy before joining, $E_1 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$ ... (i)

When the capacitors are connected by a wire, they will share charge till they acquire a common potential, V. Then this common potential V is given by

Common potential (V) $= \frac{\text{Total charge}}{\text{Total capacitance}}$

$= \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ ... (ii)

Total energy of the capacitors after connection is given by

$E_2 = \frac{1}{2}$ (total capacity) × (common potential)

$= \frac{1}{2}(C_1 + C_2) \times \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 = \frac{1}{2} \cdot \frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$ ... (iii)

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$\therefore$ The loss of energy due to connection is

$$E_1 - E_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2(C_1 + C_2)}\left[(C_1 + C_2)(C_1V_1^2 + C_2V_2^2) - (C_1V_1 + C_2V_2)^2\right]$$

$$= \frac{1}{2(C_1 + C_2)}\left[C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1V_1C_2V_2\right]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}\left[V_1^2 + V_2^2 - 2V_1V_2\right]$$

$$\therefore E_1 - E_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)} \quad \ldots (iv)$$

This is the expression for loss of energy after joining the charged capacitors. The quantity $(V_1 - V_2)^2$ in Equation (iv) is always positive. i.e., $E_1 - E_2 > 0$ or $E_1 > E_2$. Hence, when two charged capacitors are connected by wires, there is always loss of energy. This loss may occur in the form of heat, light and sound.

29. **2075 Set B Q.No. 8a** **Three capacitors are connected in series with a cell. The same three capacitors are connected in parallel with the same cell. Which combination of capacitors is larger in magnitude? Also find an expression for their value in series combination.** [4]

Please refer to 2076 Set C Q.No. 8b

30. **2074 Supp. Q.No. 8b** **Show that the electrical energy is always lost when two capacitors at different potentials are connected together.** [4]

Please refer to 2075 Set A Q.No. 8b

31. **2073 Supp Q.No. 8a** **What do you understand by 'One Picofarad"? Deduce an expression for the energy stored in capacitor.** [4]

One Picofarad: $1pF = 10^{-12}$ F

Second Part: Please refer to 2076 Set B Q.No. 8b

32. **2072 Supp Q.No. 8b** **Explain what is meant by the capacitance of a capacitor and define its units. Derive an expression for the energy stored in the capacitor.** [4]

Please refer to 2076 Set B Q.No. 8b

33. **2072 Set C Q.No. 8a** **Define capacitance of a capacitor. Deduce an expression for energy stored in a charged capacitor.** [4]

Please refer to 2076 Set B Q.No. 8b

34. **2072 Set D Q.No. 8b** **Obtain relations for equivalent capacitance for capacitors connected in series and parallel** [4]

Please refer to 2076 Set C Q.No. 8b

35. **2072 Set E Q.No. 8b** **How big is a microfarad? Discuss the factors on which the capacitance of a parallel plate capacitor depend. What will be the combined capacitance of two such capacitors connected in series with a battery?** [4]

1 Microfarad = 10^{-6} Farad

Second part: Please refer to 2074 Supp. Q.No. 4b

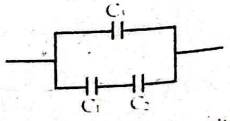
Third part: Please refer to 2076 Set C Q.No. 8b

36. **2071 Supp Q.No. 8b** **Three capacitors are arranged in such a way that two capacitors are connected in series combination and across them a third capacitor is connected in parallel combination. Find the net capacitance of the combination of the three.** [4]

In the given figure (i) two capacitors of capacitances C_1 and C_2 in series are connected in parallel with a capacitor of capacitance C_3. The total capacitance C' of C_1 and C_2 in series in given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C' = \frac{C_1C_2}{C_1 + C_2} \quad \ldots (i)$$

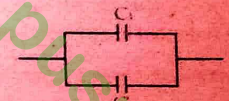


This capacitance C' is in parallel with C_3 as shown in figure (ii). Let C be the total capacitance of capacitors C' and C_3 in parallel, then

$$C = C_3 + C'$$

$$C = C_3 + \frac{C_1C_2}{C_1 + C_2}$$

$$\therefore C = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2} \quad \ldots (ii)$$



This is the required expression for the net capacitance of three capacitors arranged in such a way that one of the capacitor is connected in parallel with the series combination of other two.

37. **2071 Set C Q.No. 8 b** **2067 Supp Q.No. 8 b** **Define capacitance of a capacitor. Deduce an expression for the capacitance of a parallel plate capacitor.** [4]

First part: Please refer to 2076 Set B Q.No. 8b

Second Part: Please refer to 2075 GIE Q.No. 8b

38. **2070 Supp (Set A) Q.No. 8 b** **Deduce an expression for the effective capacitance when two capacitors C_1 and C_2 are connected in (i) parallel (ii) series.** [4]

Please refer to 2076 Set C Q.No. 8b

39. **2070 Supp (Set B) Q.No. 8 b** **Obtain an expression of energy stored in a charged capacitor and use it to derive the expression of energy density of a dielectric medium in an electric field.** [4]

First Part: Please refer to 2076 Set B Q.No. 8b

Second part:

The energy stored per unit volume of a capacitor is called energy density.

i.e.

$$\text{Energy density, } u = \frac{\text{Energy stored}}{\text{Volume}} = \frac{CV^2}{2Ad}$$

$$= \frac{\epsilon_0 A}{d}\frac{V^2}{2Ad} \qquad \left[\because C = \frac{\epsilon_0 A}{d}\right]$$

$$= \frac{\epsilon_0}{2}\frac{V^2}{d^2}$$

$$u = \frac{1}{2}\epsilon_0 E^2 \qquad \left[\because E = \frac{V}{d}\right]$$

This is energy density.

40. **2069 Supp Q.No. 8b** **Show that there is always loss of energy when two charged capacitors are connected to each other with a conducting wire.** 4

Please refer to 2075 Set A Q.No. 8b

41. **2069 (Set A) Q. No. 8b** **How can a number of capacitors be connected to increase and decrease the effective capacitance? Find the respective expressions for the effective capacitance in each case.** [4]

Please refer to 2076 Set C Q.No. 8b

42. **2069 (Set B) Q. No. 8b** **2063 Q.No. 9a** **Derive an expression for the energy stored in a capacitor of capacitance "C" when there is potential difference "V" between the plates.** 4

Please refer to 2076 Set B Q.No. 8b

43. **2069 (Set A) Old Q. No. 9a OR** **Find the equivalent capacitance of two capacitors when they are connected (i) in series and (ii) in parallel.** [4]

Please refer to 2076 Set C Q.No. 8b

44. **2067 Q.No. 8 b** **What is a capacitor? Find the energy stored in a charged capacitor.** [4]

Please refer to 2076 Set B Q.No. 8b

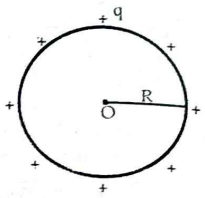
45. **2064, Q.No. 9 a** **What do you mean by capacitance? Find the expression for the capacitance of:**
(i) parallel plates capacitor and
(ii) an isolated charged sphere. [1+2+2]

i. Parallel plates capacitor: Please refer to 2076 Set C Q.No. 8b

ii. Capacitance of an isolated charged sphere.

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Let us consider an isolated spherical conductor in air or vacuum. Let its radius be R and charge on its surface be q as shown figure.



The potential at any point on its surface is given by

$$V = \frac{q}{4\pi \in_0 R}$$

or, $\frac{q}{V} = 4\pi \in_0 R$

But $\frac{q}{V} = C$, the capacitance of the isolated sphere. So,

$$C = 4\pi \in_0 R$$

This is the capacitance of an isolated charged sphere.

46. 2062 Q.No. 9 a OR **Three capacitors are arranged in such a way that one of the capacitor is connected in parallel with the series combination of other two. Find the net capacitance of the combination of the three.**

Please refer to 2071 Supp Q.No. 8b

47. 2061 Q.No. 9 a OR **Explain, what is meant by the capacitance of a capacitor and define its SI units. Derive an expression for the capacitance of a parallel plate capacitor.**

Please refer to 2075 GIE Q.No. 8b

48. 2060 Q.No. 9 a OR **What do you mean by the term capacitance? Find the equivalent capacitance of two capacitors when they are connected in (i) series (ii) parallel.** [5]

Please refer to 2076 Set C Q.No. 8b

49. 2057 Q.No. 9 a OR **What is a capacitor? Find an expression for the energy stored in a charged capacitor.**

First Part: Please refer to 2075 Set A Q.No. 8b

Second Part: Please refer to 2076 Set B Q.No. 8b

Numerical Problems

50. 2076 Set C Q.No. 12 **A 300V battery is connected across capacitors of 3μF and 6μF in parallel. Calculate the energy stored in each capacitors.** [3]

Solution

Given,

Potential (V) = 300 V

Capacitance of 1st capacitor (C_1) = 3 μ F = 3×10^{-6} F

Capacitance of 2nd capacitor (C_2) = 6 μ F = 6×10^{-6} F

Energy stored in first capacitor: $E_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 3 \times 10^{-6} \times 300^2 = 0.135$ J

Energy stored in second capacitor: $E_2 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 300^2 = 0.27$ J

51. 2074 Supp. Q.No. 12 **A parallel plate air capacitor of capacitance 245 pF has a change of magnitude 0.148 μc on each plate. Find the potential difference and electric field intensity between the plates if the distance between the plates is 5mm?** [3]

Solution

Given,

Capacitance of capacity (C) = 245×10^{-12} F

Charge on capacitor (q) = 0.148 μC = 0.148×10^{-6} C

Distance between plate (d) = 5 mm = 5×10^{-3} m

P.d. between plate (V) = ?

Electric field intensity (E) = ?

We have

$q = CV$

or, $V = \frac{q}{C} = \frac{0.148 \times 10^{-6}}{245 \times 10^{-12}} = 604$ V

Again,

$E = \frac{V}{d} = \frac{604}{5 \times 10^{-3}} = 120816$ V/m

52. 2074 Set A Q.No. 12 **Two capacitors of capacitances 5μF and 15μF respectively are connected in series with a battery of 220V. Find the charge and potential across each capacitor.** [3]

Solution

Given,

Capacitance of capacitors are, $C_1 = 5$ μF = 5×10^{-6}F

$C_2 = 15$ μF = 15×10^{-6} F

emf (V) = 220 V

Since, C_1 and C_2 are in series, charge on each capacitor is same, say q. Let, V_1 and V_2 be the potential across C_1 and C_2 respectively.

$\therefore V_1 = \frac{q}{C_1}$ and $V_2 = \frac{q}{C_2}$

But,

$V = V_1 + V_2$

or, $60 = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

or, $q = \frac{220}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{220}{\frac{1}{5 \times 10^{-6}} + \frac{1}{15 \times 10^{-6}}} = 8.26 \times 10^{-4}$ C

And,

$V_1 = \frac{q}{C_1} = \frac{8.26 \times 10^{-4}}{5 \times 10^{-6}} = 165$ V

$V_2 = \frac{q}{C_2} = \frac{8.26 \times 10^{-4}}{15 \times 10^{-6}} = 55$ V

53. 2074 Set B Q.No. 12 **A parallel plate air capacitor has a capacitance of 10^{-9} F. What potential difference is required for a charge of 5×10^{-5} C? What is the total energy stored in it?** [3]

Solution

Given,

Capacitance (C) = 10^{-9} F

Charge (q) = 5×10^{-5} C

Potential difference (V) = ?

Energy stored (E) = ?

We know,

$q = CV$

$\therefore V = \frac{q}{C} = \frac{5 \times 10^{-5}}{10^{-9}} = 5 \times 10^4$ V

Also,

Energy stored, $E = \frac{1}{2} CV^2 = \frac{1}{2} \times 10^{-9} (5 \times 10^{+4})^2 = 1.25$ J

54. 2073 Supp Q.No. 12 2068 Q.No. 12 **Two capacitors of capacitance 4 μF and 6 μF respectively are joined in series with a battery of e.m.f. 60 volts. The connections are broken and like terminals of the capacitors are then joined. Find the final charge of each capacitor.** [3]

Solution

Given,

Capacitances of capacitors

Capacitance of (C_1) = 4μF

First capacitor (C_2) = 6μ F

Since C_1 and C_2 are series,

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$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C'} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{C'} = \frac{3+2}{12}$$

$$C' = \frac{12}{5} = 2.4\ \mu F$$

or, $C' = 2.4 \times 10^{-6} F$

$q = C'V = 2.4 \times 10^{-6} \times 60 = 144 \times 10^{-6} C$

$q = 144\ \mu C$

In series combination,

$q_1 = q_2 = q$

$q_1 = C_1V_1 = 144\ \mu C$

or, $V_1 = \frac{144}{4} = 36V$

$V_2 = \frac{144}{6} = 24V$

When they are connected in parallel, they share their charges until acquire a common potential. Let common potential is V, then;

$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{144 + 144}{10} = 28.8V$

The charge on first capacitor

$q_1 = C_1V$

$= 4 \times 28.8 = 115.2\ \mu C$

and charge on second capacitor

$q_2 = C_2V$

$= 6 \times 28.8 = 172.8\ \mu C$

55. **2073 Set C Q.No. 12** **Two capacitors of capacitances 4μF and 12μF respectively, are connected in series. The combination is then connected momentarily across a 200V battery. Find the charge and potential difference across each capacitor.** [3]

Solution

Given,

Capacitance of first capacitor, $C_1 = 4\mu F = 4 \times 10^{-6} F$

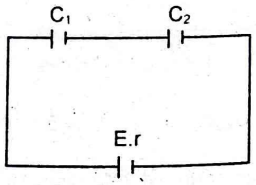
Capacitance of first capacitor, $C_2 = 12\mu F = 12 \times 10^{-6} F$

Potential difference (v) = 200 V

When the capacitors are connected in series, then equivalent capacitor C is,

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{4 \times 10^{-6} + 12 \times 10^{-6}}{(4+12) \times 10^{-6}} = 3 \times 10^{-6} F$



Charge on each capacitor,

$q = q_1 = q_2 = CV = 3 \times 10^{-6} \times 200 = 6 \times 10^{-4} C$

Now, potential on first capacitor,

$V_1 = \frac{q}{C_1} = \frac{6 \times 10^{-4}}{4 \times 10^{-6}} = 150\ V$

And potential on second capacitor

$V_2 = \frac{q}{C_2} = \frac{6 \times 10^{-4}}{12 \times 10^{-6}} = 50\ V$

56. **2073 Set D Q.No. 12** **A parallel plate capacitor with air as the dielectric has a capacitance of 6 × 10⁻⁴ μF and is charged by a 100V battery. Calculate the charge and the energy stored in the capacitor.** [3]

Solution

Given,

Capacitance of capacitor (C) = $6 \times 10^{-4}\ \mu F = 6 \times 10^{-10} F$

Potential difference (V) = 100 V

Charge (q) = ?

Energy stored (u) = ?

We have,

$q = CV = 6 \times 10^{-10} \times 100 = 6 \times 10^{-8} C = 6 \times 10^{-2}\ \mu C$

Energy stored (u) $= \frac{1}{2} CV^2 = \frac{1}{2} \times 6 \times 10^{-10} \times 100^2 = 3 \times 10^{-10} \times 10^4 = 3 \times 10^{-6} J$

57. **2072 Supp Q.No. 12** **Eight drops of water of the same size are equally and similarly charged. They combine together to form a bigger drop. Compare the capacitances of bigger drop with that of the smaller drop.** [3]

Solution,

Let r be the radius of the smaller drop and q be the charge on it, then charge on bigger drop, Q = 8q.

Now, R be the radius of bigger drop, then,

$\frac{4}{3}\pi R^3 = 8\frac{4}{3}\pi r^3$

$R^3 = 8r^3$

$R = 2r$

Now, Capacitance of bigger drop is,

$C_b = 4\pi \epsilon_0 R$

and the capacitance of smaller drop is,

$C_s = 4\pi \epsilon_0 r$

Now,

$\frac{C_b}{C_s} = \frac{R}{r} = \frac{2r}{r} = 2$

$\therefore \frac{C_b}{C_s} = 2:1$

58. **2071 Set C Q.No. 12** **Two spheres of radii 5 cm and 10 cm are given charges 100 C and 50 C respectively and then connected by a wire. Calculate the loss of energy after connection.** [3]

Solution

Given,

Radius of 1st sphere (r_1) = 5 cm

Radius of 2nd radius (r_2) = 10 cm

Charge on first sphere (q_1) = 100 C

Charge on second sphere (q_2) = 50 C

Loss of energy (ΔE) = ?

Capacitance of first sphere (C_1) $= 4\pi\epsilon_0 \times r_1 = 4\pi \times 8.85 \times 10^{-12} \times 0.05 = 5.56 \times 10^{-12} F$

Capacitance of 2nd capacitor (C_2) $= 4\pi\epsilon_0 \times r_2 = 4\pi \times 8.85 \times 10^{-12} \times 0.1 = 1.11 \times 10^{-11} F$

Then

$V_1 = \frac{q_1}{C_1} = \frac{100}{5.56 \times 10^{-12}} = 1.8 \times 10^{13}\ V$

$V_2 = \frac{q_2}{C_2} = \frac{50}{1.11 \times 10^{-11}} = 4.5 \times 10^{12} V$

Total energy before connection,

$E_1 = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2$

$= \frac{1}{2} \times 5.56 \times 10^{-12} \times (1.8 \times 10^{13})^2 + \frac{1}{2} \times 1.11 \times 10^{-11} \times (4.5 \times 10^{12})^2$

$= 9 \times 10^{14} + 1.13 \times 10^{14} = 1.0137 \times 10^{15} J$

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Total energy after connection,

$$E_2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$= \frac{1}{2}\frac{(5.56 \times 10^{-12} \times 1.8 \times 10^{13} + 1.11 \times 10^{-11} \times 4.5 \times 10^2)^2}{5.56 \times 10^{-12} + 1.11 \times 10^{-11}}$$

$$= 6.75 \times 10^{14}\ J$$

Then loss is energy

$$\Delta E = E_1 - E_2$$

$$= 1.0137 \times 10^{15} - 6.75 \times 10^{14} = 3.38 \times 10^{14} J$$

59. 2071 Set D Q.No. 12 Three capacitors of 1 μf, 2 μf and 3 μf are connected first in series. The same capacitors are again connecting in parallel. Compare their equivalent capacitance. Which one of these combinations gives larger value of capacitance? [3]

Solution

Given,

Capacitances of there capacitors are,

$C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$ and $C_3 = 3\ \mu F$

When they are connected in series, their equivalent capacitance C_s is

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6}$$

$$\therefore C_2 = \frac{6}{11}\ \mu F$$

When they are parallel, their equivalent capacitance C_p is

$$C_p = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6\ \mu F$$

Now,

$$\frac{C_p}{C_s} = \frac{6}{\frac{6}{11}} = 11$$

$$C_p = 11\ C_s$$

∴ Equivalent capacitance in parallel combination is 11 times greater than that of series combination.

60. 2070 Set C Q.No. 12 A parallel plate air capacitor has a plate separation of 5 mm and is charged to a potential difference of 400 V. Calculate the energy density in the region between the plates. [3]

Solution

Given,

Distance between plates (d) = 5m = 5×10^{-3} m

Potential difference (V) = 400 v

Energy density (u) = ?

We have,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2} \times 8.845 \times 10^{-12}\left(\frac{400}{5 \times 10^{-3}}\right) = 0.028\ J/m^3$$

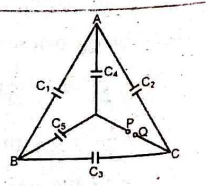
61. 2070 Set D Q.No. 12 A parallel plate air capacitor of capacitance 245 $\times 10^{-12}$ F has a charge of magnitude 0.148μC on each plate. Find the potential difference and electric field intensity between the plates if the distance between plates 5mm? [3]

Please refer to 2074 Supp. Q.No. 12

62. 2069 (Set B) Q. No. 12

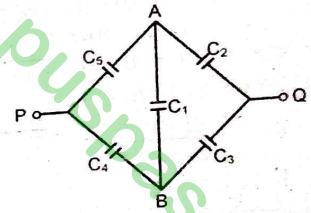
Find the equivalength capacitance of the following combination as shown in figure.

In which $C_1 = C_2 = C_3 = C_4 = C_5 = 100\ \mu F$ **[3]**



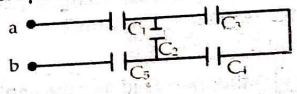
Solution

The figure can be retraced as shown. Since, values of all the capacitors are equal. So, the value of C1 can be neglected because no charge can flow through it and the points A and B will acquire same potential if a battery is applied across R and Q and hence, we find the equivalent capacitances between P & Q as



$$C_{PQ} = \frac{C_4C_2}{C_4 + C_2} + \frac{C_5 \times C_3}{C_5 + C_3} = \frac{100 \times 100}{100 + 100} + \frac{100 \times 100}{100 + 100} = 50 + 50 = 100\ \mu F$$

63. 2066 Q. No. 12 In the given capacitors circuit applied potential between a b is 220 V. What is the equivalent capacitance of the network between a and b? Given: $C_1 = C_5 = 8.4\ \mu F$ and $C_2 = C_3 = C_4 = 4.2\ \mu F$. [3]



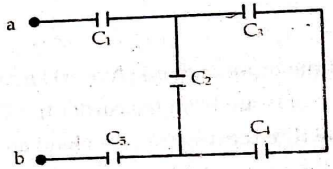
Solution

Given,

$C_1 = C_5 = 8.4\ \mu F$

$C_2 = C_3 = C_4 = 4.2 \mu F$

Since, C_3 and C_4 are in series, their equivalent C' is given by,



$$\frac{1}{C'} = \frac{1}{C_3} + \frac{1}{C_4}$$

$$\text{or, } \frac{1}{C'} = \frac{1}{4.2} + \frac{1}{4.2}$$

$$\text{or, } \frac{1}{C'} = \frac{2}{4.2}$$

$$\text{or, } C' = 2.1\ \mu F$$

Now, C' and C_2 are parallel so

$$C'' = C' + C_2$$

$$= 2.1 + 4.2 = 6.3\ \mu F$$

Again, C_1, C_5 and C'' are series,

$$\frac{1}{C'''} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C''}$$

$$\frac{1}{C'''} = \frac{1}{8.4} + \frac{1}{8.4} + \frac{1}{6.3}$$

$$\text{or, } C''' = 2.52\ \mu F.$$

64. 2059 Q.No. 9 b Two capacitors of capacitance 4 μF and 12.0 μF respectively are connected in series and the combination connected momentarily across a 200 volt battery. The charged capacitors are now isolated and connected in parallel, similar charged plates being connected together. Calculate the common potential. [3]

Solution

Given,

Capacitance of first capacitor (C_1) = 4 μF = 4×10^{-6} F

Capacitiance of second capacitor (C_2) = 12 μF = 12×10^{-6} F

Potential difference (V) = 200 V

When the capacitors are connected in series, then the equivalent capacitance (C) is obtained as

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{or, } C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{4 \times 10^{-6} \times 12 \times 10^{-6}}{(4 + 12) \times 10^{-6}} = 3 \times 10^{-6}\ F$$

Charge on each capacitor

$$q = q_1 = q_2 = CV$$

$$= 3 \times 10^{-6} \times 200 = 6 \times 10^{-4}\ C$$

Let V be the common potential difference after parallel combination of capacitors.

Then, total charge in parallel combination $= C_1V + C_2V$

$$= (C_1 + C_2)V$$

$$= (4 + 12) \times 10^{-6}\ V = 16 \times 10^{-6}\ V$$

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Since, total charge is conserved

$\therefore$ Total charge in series = Total charge in parallel

or, $q_1 + q_2 = 16 \times 10^{-6}$ V

or, $2 \times 6 \times 10^{-4} = 16 \times 10^{-6}$ V

$\therefore$ $V = \frac{12 \times 10^{-4}}{16 \times 10^{-6}}$

$\therefore$ $V = 75$ volt

65. 2056 Q.No. 9 b **Two capacitors, of capacitances 4.0μF and 12.0μF respectively, are connected in series and the combination connected momentarily across a 200V battery. The charged capacitors are now isolated and connected in parallel, similar charged plates connected together. What would be the resulting potential difference across the combinations?** [3]

Please refer to 2059 Q.No. 9 b

66. 2053 Q.No. 9 b **A thundercloud and the earth can be regarded as a parallel plate capacitor. Taking the area of the thundercloud to be 50 km^2, its height above the earth as 1 km and its potential 100 kV, calculate the energy stored. (Permittivity of vacuum is 8.85×10^{-12} C^2 N^{-1} m^{-2})**

Solution

Given,

Area of the thundercloud (A) = 50 km^2 = 50×10^6 m^2

Distance of cloud from the earth (d) = 1 km = 10^3 m

Potential difference across the cloud and the earth (V) = 100 kV = 100×10^3 V = 10^5 volt

ϵ_0 = 8.85×10^{-12} C^2 N^{-1} m^{-2}

Energy stored (E) = ?

Since, the capacitance of parallel plates capacitor is, $C = \frac{\epsilon_0 A}{d}$

$\therefore$ Energy stored (E) $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$

$= \frac{1}{2}\frac{8.85 \times 10^{-12} \times 50 \times 10^6 \times (10^5)^2}{10^3}$

$= 2212.5$ joules $\approx 2.2 \times 10^3$ joules

□□□

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