# Mathematics

## Syllabus

**Full Marks: 100** **Pass Marks: 35**

Course Contents

### I. Introduction:

This course deals with the fundamentals of advanced mathematical concepts. It also tries to consolidate the concepts and skills learnt in Mathematics course in school level. It is desirable at the end of each unit sufficient related problems be solved.

### II. Specific Objectives:

On completion of this course students will be able to:

1. use principles of elementary logic to find the validity of statement;
2. state field and order axioms of Real number system;
3. define functions and illustrate them graphically;
4. sketch the curves;
5. use trigonometrical relations to find the general values, understand inverse circular functions and their properties and to find property & solution of triangle;
6. state properties of A.S., G.S. and H.S. Understand infinite series and use method of mathematical induction to establish the result;
7. define transpose, adjoint and inverse of matrix, state properties of determinants;
8. use matrix and determinant to solve system of linear equations;
9. explain the idea of a complex number, verify their properties, prove De–Moivre's theorem and use it;
10. define polynomial equations, establish fundamental theorem of algebra and quadratic equation, and find relation between roots and coefficients of a quadratic polynomials;
11. define straight lines, pair of lines in terms of co–ordinates and establish their properties;
12. define circle in terms of coordinates and establish their properties;
13. define limit of a function, establish properties of limits;
14. define continuity of a function using the concept of limit;
15. define derivative of a function and give its geometrical interpretation as rate of change;
16. use derivative to determine the nature of the function and determine the maxima and minima of a function and apply differentiation to find tangent & normal, increasing & decreasing function;
17. define anti-derivative as an inverse process of derivative and use various methods of integration; and
18. define integration as the area of the sum, and apply definite integral to find the area between the curves.

### III. Course Contents:

#### Unit 1: Sets, Real Number System and Logic

10 hrs

**Sets:** Sets and set operations, Theorems based on set operations.

**Real Number System:** Real numbers, Field axioms, Order axioms, Interval, Absolute value, Geometrical representation of the real numbers.

**Logic:** Introduction, statements, Logical connectives, Truth tables, Basic laws of logic.

#### Unit 2: Relations, Functions and Graphs

12 hrs

**Relations:** Ordered pair, Cartesian product, Geometrical representation of Cartesian product, relation, Domain and range of a relation, Inverse of a relation.

**Functions:** Definition, Domain and range of a function, Functions defined as mappings, Inverse function, Composite function, functions of special type(Identity, Constant, Absolute value, Greatest integer), Algebraic (Linear, quadratic and cubic), Trigonometric, Exponential logarithmic functions and their graphs.

#### Unit 3: Curve Sketching

10 hrs

Odd and even functions, Periodicity of a function, symmetry (about $x$ – axis, y – axis and origin) of elementary functions. Monotonocity of a function, Sketching graphs of polynomial functions $\left(\frac{1}{x}, \frac{x^2-a^2}{x-a}, \frac{1}{x+a}, x^2, x^3\right)$, Trigonometric, exponential, logarithmic functions (Simple cases only)

#### Unit 4: Trigonometry

10 hrs

Inverse circular functions, Trigonometric equations and general values, properties of a triangle (sine law, cosine law, tangent law, Projection laws, Half angle laws), the area of a triangle. Solution of a triangle (simple cases)

**Unit 5: Sequence and Series, and Mathematical Induction** 12 hrs

**Sequence and Series:** Sequence and series, type of sequences and series (Arithmetic, Geometric, Harmonic), Properties of Arithmetic, Geometric, and Harmonic sequences, A.M., G.M. And H.M. Relation among A.M., G.M. and H.M.; Sum of infinite geometric series.

**Mathematical Induction:** Sum of finite natural numbers, Sum of the squares of first n – natural numbers, Sum of cubes of first n – natural numbers. Intuition and induction, principle of mathematical induction.

**Unit 6: Matrices and Determinants** 8 hrs

Matrices and operation on matrices (Review), Transpose of a matrix and its properties, Minors and Cofactors, Adjoint, Inverse matrix. Determinant of a square matrix, properties of determinants (Without proof) upto 3 × 3.

**Unit 7: System of Linear Equations** 8 hrs

Consistency of system of linear equations, solution of a system of linear equations by Cramer's rule, Matrix method (row – equivalent and Inverse) upto three variables.

**Unit 8: Complex Number** 12 hrs

Definition of a complex number, Imaginary unit, Algebra of complex numbers, Geometric representation of a complex number, Conjugate and absolute value (Modulus) of a complex numbers and their properties, Square root of a complex number, Polar form of a complex number, product and Quotient of complex numbers. De-Moivre's theorem and its application in finding the roots of a complex number, properties of cube roots of unity.

**Unit 9: Polynomial Equations** 8 hrs

Polynomial function and polynomial equations, Fundamental theorem of algebra (without proof), Quadratic equation Nature and roots of a quadratic equation, Relation between roots and coefficients, Formation of a quadratic equation, Symmetric roots, one or both roots common.

**Unit 10: Co-ordinate Geometry** 12 hrs

**Straight line:** Review of various forms of equation of straight lines, Angle between two straight lines, condition for parallelism and perpendicularity, length of perpendicular from a given point to a given line, Bisectors of the angles between two straight lines.

**Pair of lines:** General equation of second degree in $x$ and y, condition for representing a pair of lines, Homogeneous second degree equation in $x$ and y, Angle between pair of lines, Bisectors of the angles between pair of lines.

**Unit 11: Circle** 10 hrs

Equation of a circle in various forms (Centre at origin, centre at any point, general equation of a circle, circle with a given diameter), Condition of Tangency of a line at a point to the circle, Tangent and normal to a circle.

**Unit 12: Limits and Continuity** 10 hrs

Limits of a function, Indeterminate forms, Algebraic properties of limits (without proof), Theorem on limits of algebraic, Trigonometric, Exponential and logarithmic functions

$\left(\lim_{x\to a}\frac{x^n-a^n}{x-a}, \lim_{x\to 0}\frac{\sin x}{x}, \lim_{x\to 0}\frac{e^x-1}{x}, \lim_{x\to 0}\frac{\log(1+x)}{x}\right)$ Continuity of a function, Types of discontinuity, Graph of discontinuous function.

**Unit 13: The Derivatives** 8 hrs

Derivative of a function, Derivatives of algebraic, trigonometric, exponential and logarithmic functions by definition (simple forms), Rules of differentiation, Derivatives of parametric and implicit functions, Higher order derivatives.

**Unit 14: Applications of Derivatives** 12 hrs.

Geometric interpretation of derivative, Monotonocity of a function, Interval of monotonocity, Extrema of a function, Concavity, Points of inflection, Derivative as rate measure.

**Unit 15: Antiderivatives and its Applications** 10 hrs

Antiderivative, Integration using basic integrals, Integration by substitution and by parts method, the definite integral, The definite integral as an area under the given curve, Area between two curves.

## IV. Evaluation Scheme:

| No. of questions | Marks | Total | Remarks |
|---|---|---|---|
| 15 | 2 | 30 | covering all units |
| 10 | 4 | 40 | with four OR–questions from the same |
| 5 | 6 | 30 | with two OR–questions from the same |

The questions of 6 marks will be asked from the units with 12 or more credit hours.

## V. Reference books:


1. Adhikari, D.B. and et.al. ***Element of Mathematics Part I***. Himalaya Book stall.
2. Bajracharya, D.R.; Shrestha, R.M. and et.al. ***Higher Secondary Level Basic Mathematics ( For Grade XI)***. Kathmandu: Sukunda Pustak Bhawan.

3. Bajracharya, P.M. and Basnet, G. (2008). ***Fundamentals of Mathematics for Grade XI***. Kathmandu: Buddha Academic Publishers & Distributors P. Ltd.
4. Koirala, S. and et.al. ***Fundamentals of Mathematics***. Kathmandu: Nepal Sahitya Prakashan Kendra.
5. Pant, S.R. and et.al. ***A Text–Book of Higher Secondary Mathematics (For Grade XI)***. Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
6. Uprety, K.N. and Ghimire, K.P., ***Foundation of Mathematics, (For Grade XI)***. Pigeon Educational Publisher.




## New Model Questions - 2067

**Time: 3hrs.** **Full Marks: 100**
**Pass Marks: 35**

Candidates are required to give their answers in their own words as far as possible. The figures in the margin indicate full marks.

### Group 'A' [5 × 3 × 2 = 30]

**Attempt ALL Questions:**

1. (a) Write truth table for $p \wedge q \Rightarrow p \vee q$ hence draw a conclusion from the truth table.

   (b) If A = {1, 2, 3}, find the relation on A satisfying the condition $x + y < 4$. Is this relation a function? Give reason.

   **Ans: {(1, 1), (1, 2), (2, 1)}, No**

   (c) Test periodicity and symmetricity of the function $y = \cos x$.

   **Ans: 2π, symmetric about y-axis**

2. (a) Find the value of $x$ for which $\sin x = \frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$ $(0 \le x \le 2\pi)$.

   **Ans:** $\frac{5\pi}{6}$

   (b) Using mathematical induction method, prove that $1 + 3 + 5 + \ldots + n \text{ terms} = n^2$.

   (c) Find the inverse of $\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$.  **Ans:** $\begin{pmatrix} \frac{3}{10} & \frac{-1}{10} \\ \frac{1}{20} & \frac{3}{20} \end{pmatrix}$

3. (a) Solve the following system of linear equations by Cramer's rules, if possible:
   $5x - 3y = 9;\ 10x - 6y = 16.$

   **Ans: No Soluton by Cramer's Rule**

   (b) Find the value of the real numbers $x$ and $y$ if $(x + 2) + yi = (3 + i)(1 - 2i)$.

   **Ans: x = 3, y = –5**

   (c) Determine the nature of the roots of the equation $2x^2 + 3x - 2 = 0$.

   **Ans: rational and unequal**

4. (a) Show that the points (1, 2) and (2, –3) lie on the opposite sides of the line $5x - 2y - 3 = 0$.

   (b) Find the equation of the circle concentric with $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5, 4).

   **Ans:** $x^2 + y^2 - 8x + 12y - 49 = 0$

   (c) Find the limit of $f(x) = \frac{x^2 - 4}{x - 2}$ as $x \to 2$. Is $f(x)$ continuous? If not, find the point of discontinuity.

   **Ans: Not continuous, x = 2**

5. (a) Find the derivative of $\sec^2(\tan\sqrt{x})$.

   **Ans:** $\frac{1}{\sqrt{x}}\sec^2(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2(\sqrt{x})$

   (b) For any curve $y = f(x)$, what do $f'(x) > 0$ and $f'(x) < 0$ represent?

   **Ans: Increasing, Decreasing**

   (c) Evaluate $\int \frac{x}{(1 - x^2)^{\frac{3}{2}}}dx$.  **Ans:** $\frac{1}{\sqrt{1-x^2}} + c$

### Group 'B' [5 × 2 × 4 = 40]

6. (a) If A, B and C are any three non-empty sets, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   **OR**

   Define absolute value of a real number. If a be any positive real number and $x \in R$, prove that $|x| < a \Leftrightarrow -a < x < a$.

   (b) Sketch the graph of $y = x^2 - 6x + 9$ indicating its different characteristics.

7. (a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

   **OR**

   State and prove sine law.

   (b) Prove that:

   $$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b - c)(c - a)(a - b)(a + b + c)$$

8. (a) Applying Inverse matrix method or row equivalent method, solve the system of linear equations:
   $3x + y + 2z = -1;\ 2x + 3y + z = 5;\ x + 2y - z = 8$

   **Ans: x = 1, y = 2, z = –3**

   (b) If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$.

9. (a) Prove that the line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1 + m^2}$. Also show that $3x + 4y = 20$ is tangent to the circle $x^2 + y^2 = 16$.

(b) Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x-\theta}$.

Ans: $\cot\theta + \theta\,\text{cosec}^2\theta$

**OR**

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x-3 & \text{for } x < 2 \\ 2 & \text{for } x = 2 \\ 3x-5 & \text{for } x > 2 \end{cases}$$

Is f(x) continuous at $x = 2$? If not, how can f(x) be made continuous at $x = 2$.

Ans: No, $f(x) = \begin{cases} 2x-3 & \text{for } x < 2 \\ 1 & \text{for } x = 2 \\ 3x-5 & \text{for } x > 2 \end{cases}$

). (a) Find the derivative of $\sqrt{3-2x}$ from first principles.

Ans: $\frac{-1}{\sqrt{3-2x}}$

(b) Find area between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

Ans: $\frac{16a^2}{3}$ sq. units

**Group 'C' [5 × 6 = 30]**

1. Define domain and range of a function. Find the domain and range of $f(x) = \sqrt{21 - 4x - x^2}$.

Ans: Domain = [–7, 3], Range = [0, 5]

2. If AM, GM and HM be the arithmetic, geometric and harmonic mean between two unequal positive numbers, prove that.

(i) $GM^2 = (AM.HM)$ (ii) AM > GM > HM

13. Derive the formula for the length of the perpendicular from a point $(x_1, y_1)$ to a line $x\cos\alpha + y\sin\alpha = p$. Also find the distance between the parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$.

Ans: $\frac{19}{26}$ units

**OR**

Find the condition that the general equation of second degree may represent a line pair. If $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ represents a line pair, show that the lines are perpendicular.

Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

14. Define conjugate of a complex number. Find the square root of $\frac{2-36i}{2+3i}$.

Ans: ±(1 – 3i)

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1, 2). Also examine whether the function is increasing or decreasing at $x = 0$.

Ans: Max. value = $\frac{7}{2}$ at $x = -\frac{1}{2}$ Min. value = $\frac{-25}{2}$ at $x = \frac{3}{2}$, Point of inflection is $\left(\frac{1}{2}, -\frac{9}{2}\right)$

**OR**

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 8cm/sec and that of the outer circle at the rate of 5 cm/sec. At a certain instant the radii of the inner and outer circles are respectively 24cm and 30cm. At what rate does the area between the two circles change?

Ans: Decreasing in time t at the rate 84 sq.cm/sec when $r_1$ = 24cm and $r_2$ = 30cm

## Chapter based Questions

# Unit 1: Sets, Real Number System and Logic

## A. Set Theory

### FORMULAE AND IMPORTANT POINTS

1. $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$.
2. $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$.
3. $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$.
4. $x \notin A \cap B \Leftrightarrow x \notin A$ or $x \notin B$.
5. $x \in \bar{A} \Leftrightarrow x \notin A$.
6. $x \in (A - B) \Leftrightarrow x \in A$ and $x \notin B$ or $A - B = A \cap \bar{B}$.
7. $x \in A \Delta B \Leftrightarrow x \in (A - B) \cup (B - A)$.
8. a. Every set is a subset of itself i.e., $A \subseteq A$.
   b. $\phi$ is a subset of every set.
   c. The number of subsets of a set having $n$ distinct elements is $2^n$ and number of proper subsets is $2^n - 1$.
9. Properties of Algebraic Operation on Sets
   i. Idempotent laws
      a. $A \cup A = A$ b. $A \cap A = A$
   ii. Commutative laws
      a. $A \cup B = B \cup A$ b. $A \cap B = B \cap A$
   iii. Domination laws
      a. $A \cup U = U$ b. $A \cap \phi = \phi$
   iv. Absorption laws
      a. $A \cup (A \cap B) = A$ b. $A \cap (A \cup B) = A$
   v. Associative Laws
      a. $A \cup (B \cup C) = (A \cup B) \cup C$
      b. $A \cap (B \cap C) = (A \cap B) \cap C$
   vi. Distributive Law
      a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
      b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   vii. Complementation Laws
      a. $A \cup \bar{A} = U$ b. $A \cap \bar{A} = \phi$
      c. $\bar{\phi} = U$ d. $\bar{U} = \phi$
      e. $\bar{\bar{A}} = A$
   viii. De-Morgan's Laws
      a. $(\overline{A \cup B}) = (\bar{A} \cap \bar{B})$ b. $(\overline{A \cap B}) = (\bar{A} \cup \bar{B})$

**2 Marks Questions**

1. **2074 Set A Q.No. 1a** Prove that $A - \overline{B} = A \cap B$, where A and B are any two sets: [2]
2. **2073 Supp Q.No. 1a** If A and B are two non-empty sets, prove that: $\overline{A \cup B} = \overline{A} \cap \overline{B}$. [2]
3. **2072 Set E Q.No. 1a** If A and B are subsets of U, prove that $A - B = A \cap \overline{B}$. [2]
4. **2070 Old Q.No. 1 a** If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} denotes the universal set, P = {2, 3, 5, 7} and E = {2, 4, 6, 8}, find $P^C$ and P – E. [2]
   **Ans: {1, 4, 6, 8, 9}; {3, 5, 7}**
5. **2069 [Set A] Old Q. No. 1a** Define power set. Write the power set of A = {x, y, z} [2]
   **Ans: {ϕ, {x}, {y}, {z}, {x , y}, {y, z}, {z, x}, {x, y , z}}**
6. **2069 [Set B] Old Q. No. 1a** For non empty sets A and B if $A \cap B = \Phi$, prove that: $B \subset A$. [2]
7. **2067 Q.No. 1a** Define power set. Write the power set of the set A ={a, b, c} [2]
   **Ans: {ϕ, {a}, {b}, {c}, {a,b}, {b, c}, {c, a}, {a, b, c}}**
8. **2066 Q.No. 1(a)** Given A = {1,2, 3} and B = {3, 4, 5, 6}, show that $A - (A - B) = A \cap B$. [2]
9. **2064 Q.No. 1(a)** Find $A \cup B$ if: A = {x: x=2n+1, n ≤ 5, n ∈ N} B = {x: x = 3n – 2, n ≤ 4, n ∈ N} [2]
   **Ans: {1, 3, 4, 5, 7, 9, 10, 11}**
10. **2063 Q.No. 1(a)** Find $A \cap B$ if: A = {x : x=2n+1, n ≤ 6, n∈N}, B = {x : x = 3n – 2, n ≤ 3, n∈N} [2]
    **Ans: {7}**
11. **2062 Q.No. 1(a)** If A = {1, 3, 5, 7} & B = {2, 3, 5}. Find $A \cap B$ and A – B. Show them in Venn–diagram. [2]
    **Ans: {3, 5}, {1, 7}**
12. **2061 Q.No. 1(a)** If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} be the universal set and M = {1, 3, 5, 7}, N = {2, 4, 6, 8}, then find $\overline{M \cup N}$ & $\overline{M \cap N}$. [2]
    **Ans: {9}, {1, 2, 3, 4, 5, 6, 7, 8, 9}**
13. **2060 Q.No. 1(a)** If O = {1, 3, 5, 7, 9} and P = {2, 3, 5, 7}. Find $O \cap P$ and P – O with the help of Venn diagram. [2]
    **Ans: {3, 5, 7}, {2}**
14. **2059 Q.No. 1(a)** If A = {a, e, i, o, u} and B = {a, b, c, d, e} are two given sets, find $A \cup B$ and A–B. [2]
    **Ans: A∪B = {a, b, c, d, e, I, o, u}; A – B = {I, o, u}**
15. **2058 Q.No. 1(a)** If U = {1, 2,...,10}, A = {x: x ≥ 4}, B = {x: x < 8}, Find $A \cap B$ & A – B [2]
    **Ans: A∩B = {4, 5, 6, 7}, A – B = {8, 9, 10}**
16. **2056 Q.No. 1(a)** If A = {a, e, i}, B = {e, u}, U = {a, e, i, o, u}, find $\overline{A \cup B}$ and $\overline{A \cap B}$. [2]
    **Ans: {o}, {a, i, o, u}**

**4 Marks Questions**

17. **2076 Set B Q.No. 6a** For any non-empty subsets A, B, C of universal set U, prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]
18. **2075 GIE Q.No. 6a** For non-empty sets A, B, C prove that: $A - (B \cap C) = (A - B) \cup (A - C)$. [4]
19. **2075 Set A Q.No. 6a** Let A and B be subsets of universal set U. Show that $A - (B \cap C) = (A - B) \cup (A - C)$. [4]
20. **2075 Set B Q.No. 6a** Define union of two sets. Also for any non-empty sets A, B, C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]
21. **2075 Set C Q.No. 6a** Let U be the universal set and let A and B be any subsets of U. Prove that $A \Delta B = (A \cup B) - (A \cap B)$. [4]
22. **2074 Supp Q.No. 6a** Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]
23. **2074 Set B Q.No. 6a** State De Morgan's Laws of sets. Also for any two non empty subsets A and B of universal set U, prove that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$. [4]
24. **2073 Set C Q.No. 6 OR** Define complement of a set. If A and B are the subsets of the universal set U, prove that: $(A \cup B)' = A' \cap B'$. [4]
25. **2073 Set D Q.No. 6a** For non-empty sets A, B, C, prove that: $A - (B \cup C) = (A - B) \cap (A - C)$ [4]
26. **2072 Supp Q.No. 6a** If A, B and C be any three non–empty sets, prove that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [4]
27. **2072 Set C Q.No. 6a** If A and B are the non-empty subsets of the universal set U, prove that [4]
    (i) $(A \cup B)^c = A^c \cap B^c$ (ii) $(A \cap B)^c = A^c \cup B^c$.
28. **2072 Set D Q.No. 6a** For any non-empty sets A, B and C, prove that $A - (B \cup C) = (A - B) - C$. [4]
29. **2071 Supp. Q.No. 6a** Define the symmetric difference of two sets. Prove that: $A \Delta B = (A \cup B) - (A \cap B)$ [4]
30. **2071 Set C Q.No. 6a** For any three non-empty sets A, B, C, prove that: $(A - B) - C = A - (B \cup C)$. [4]
31. **2071 Set D Q.No. 6a** Prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]
32. **2071 Old Q.No. 7a** If A and B are sub sets of a universal set U. rove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]
33. **2070 Supp Q.No. 6a** If A and B are the subsets of the universal set U prove that:
    i. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii. $\overline{A \cap B} = \overline{A} \cup \overline{B}$ [4]
34. **2070 Set C Q.No. 6 a** Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [4]
35. **2070 Set D Q.No. 6 a** Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]
36. **2070 Old Q.No. 7 a** For non-empty sets A, B, C, prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]
37. **2069 Supp Q.No. 6 a** For any non-empty sets A, B, C define distributive laws. Also prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]
38. **2069 [Set A] Q. No. 6a** Define union and intersection of two sets. If A, B and C are any three non-empty sets, prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4]
39. **2069 [Set A] Old Q. No. 7a** For any non-empty sets A, B, C show that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [4]
40. **2069 [Set B] Q. No. 6a** Define De-Morgan's law. For any non-empty sets A, B, C,
    Prove: $A - (B \cup C) = (A - B) \cap (A - C)$ [4]

41. 2068 Q.No. 6a If A, B and C be any three non-empty sets, prove that: $A - (B \cup C) = (A - B) \cap (A - C)$. [4]

42. 2067 Q.No. 7a In a group of students 30 study maths, 24 study physics, 22 study chemistry, 14 study maths only, 8 study physics only, 6 study maths and chemistry, 2 study maths and physics only and 8 study none. How many students are in the group? How many study chemistry only? How many study all three subjects? [4]

**Ans: 54, 2, 8**

43. 2066 Q.No. 7(a) Twenty three medals are awarded for folksongs, eight for Deuda songs and eleven for Maithili songs. If the total number of singers awarded is thirty two and only three singers received medals in all three types of songs, find how many singers received medals in exactly two of three types of songs. [4]

**Ans: 4**

44. 2065 Q. No. 7 a Out of group of 20 teachers in a school, 10 teach Maths, 9 teach Physics, 7 teach Chemistry, 4 teach Maths and Physics, but none teach both Maths and Chemistry:

i. How many teach Physics and Chemistry?
ii. How many teach only Physics?
iii. How many teach only Chemistry? [4]

**Ans: (i) 2 (ii) 3 (iii) 5**

45. 2064 Q.No. 7(a) A village has total population 25,000 out of which 13,000 read 'Gorkhapatra' and 10,500 read 'Kantipur' and 2500 read both papers. Find the percentage of population who read neither of these papers. [4]

**Ans: 16%**

46. 2063 Q.No. 7(a) Of the number of three athletic teams, 21 are in the basketball team, 26 in hockey team and 29 in football team, 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball, and 8 play all the games. How many members are there in all? [4]

**Ans: 43**

47. 2062 Q.No. 7(a) Define union and intersection of two sets. Illustrate them through Venn–diagrams. Let A, B, C be any non–empty subsets of U, prove that:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [4]

48. 2061 Q.No. 7(a) In a group of twenty eight teacher's of a school, 15 teach English, 15 teach Maths, 14 teach Nepali, 7 teach English and Maths, 6 teach English and Nepali, 5 teach Maths and Nepali. Find how many teach all three subjects, how many teach Maths only and Nepali only? [4]

**Ans: 2; 5; 5**

49. 2060 Q.No. 7(a) In a group of students 24 study maths, 30 study biology, 22 student physics, 8 study maths only, 14 study biology only, 6 study biology and physics only and 2 study maths and biology only. Find

(i) How many study all three subjects.
(ii) How many students were in the group? [4]

**Ans: (i) 8, (ii) 46**

50. 2059 Q.No. 7(a) In a certain village in Nepal, all the people speak Nepali or Tharu or both languages. If 90% speak Nepali and 20% Tharu language, how many speak (i) Nepali only (ii) Tharu language only and (iii) both languages. [4]

**Ans: (i) 80%, (ii)10%, (iii) 10%**

51. 2058 Q.No. 7(a) If A, B, C are subsets of a universal set U, prove that: $A - (B \cap C) = (A - B) \cup (A - C)$ [4]

52. 2057 Q.No. 7(a) Define the complement of a set. State and prove De–Morgan's laws. [4]

53. 2056 Q.No. 7(a) Define the union and the intersection of two sets. If A, B and C are subsets of universal set U, prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4]

## B. Real Number System

### FORMULAE AND IMPORTANT POINTS

1. Intervals
   i. $[a, b] = \{x : a \le x \le b\}$ : Closed interval
   ii. $(a, b) = \{x : a < x < b\}$ : Open interval
   iii. $[a, b) = \{x : a \le x < b\}$ : Right open interval
   iv. $(a, b] = \{x : a < x \le b\}$ : Left open interval
2. Absolute Value
   $$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$
3. Properties of Absolute Value
   i. For any real numbers x and y $|x + y| \le |x| + |y|$.
   ii. For any real numbers x and y $|x - y| \le |x| + |y|$.
   iii. For any real numbers x and y $|x| - |y| \le |x + y|$.
   iv. For any real numbers x and y $|x| - |y| \le |x - y|$.
   v. For any real numbers x, y and z $|x - z| \le |x - y| + |y - z|$.
   vi. If a be any positive real number and $x \in \mathbb{R}$, $|x| < a \Leftrightarrow -a < x < a$.

**2 Marks Questions**

1. 2076 Set C Q.No. 1a Given A = [– 2, 3] and B = [1, 4], find $A \cap B$. [2]

**Ans: [1, 3]**

2. 2075 Set B Q.No. 1a Represent the solution set of x of $-2 \le 3x + 4 \le 10$ in interval form. [2]

**Ans: [–2, 2]**

3. 2074 Set B Q.No. 1a If A = [–2, 3) and B = [1, 4], compute $A \cup B$. [2]

**Ans: [–2, 4]**

4. 2073 Set C Q.No. 1a Write $|x + 2| < 4$ without using modulus sign. [2]

**Ans: $-6 < x < 2$**

5. 2071 Set C Q.No. 1b For any two real numbers x and y, prove that: $|x + y| \le |x| + |y|$. [2]

6. 2071 Set D Q.No. 1b For any two real numbers x and y, prove that: $|x - y| \ge |x| - |y|$. [2]

7. 2071 Old Q.No. 1a Let A = [–1, 3 ) and B = [2, 4], compute: $A \cup B$ and $A - B$. [2]

**Ans: [–1, 4], [–1, 2)**

8. 2071 Old Q.No. 1b Rewrite, using absolute value sign for $|3x + 2| \le 1$ [2]

**Ans: $-1 \le x \le -\frac{1}{3}$**

9. 2068 Old Q.No. 1a Rewrite $|2x-1| \le 5$ without using absolute value sign. [2]

**Ans:** $-2 \le x \le 3$

10. 2065 Q. No. 1 a Given A = [–2, 4) and B = (2, 5], compute $A \cup B$ and $A \cap B$. [2]

**Ans:** [–2, 5] and (2, 4)

11. 2064 Q.No. 3(c) Write $|x-7| < 3$ without using the modulus sign. [2]

**Ans:** $4 < x < 10$

12. 206? Q.No. 3(c) Write the following by using the modulus sign: $-1 \le x \le 5$ [2]

**Ans:** $|x-2| \le 3$

13. 2057 Q.No. 1(a) Prove that for any positive real number a, $|x| < a$ implies $-a < x < a$. [2]

14. 2056 Q.No. 1(b) Rewrite, without using absolute value sign for $|3x+2| \le 1$. [2]

**Ans:** $-1 \le x \le -\frac{1}{3}$

**4 Marks Questions**

15. 2076 Set B Q.No. 6a OR Define absolute value of a real number. Also prove that $|x| - |y| \le |x+y|$, $\forall x, y \in \mathbb{R}$. [4]

16. 2076 Set C Q.No. 6a OR If a is a positive real number and $x \in \mathbb{R}$, prove that $|x| < a$ if and only if $-a < x < a$. [4]

17. 2075 GIE Q.No. 6a OR Define absolute value of a real number. If a is any positive real number prove that $|x| \le a$ if and only if $-a \le x \le a$ for all $x \in R$. [4]

18. 2075 Set A Q.No. 6a OR Solve the inequality $\frac{x+2}{x^2-3x} > 0$. [4]

**Ans:** $(-2, 0) \cup (3, \infty)$

19. 2075 Set C Q.No. 6a OR Solve the inequality $x^2 + 7x + 10 < 0$. [4]

**Ans:** (–5, –2)

20. 2074 Supp Q.No. 6a OR Solve the inequality $|2x-1| \ge 3$ and represent the solution set in the real line. [4]

**Ans:** $(-\infty, -1] \cup [2, \infty)$

21. 2074 Set A Q.No. 6a OR Solve the inequality $|2x-1| \ge 3$ and draw its graph. [4]

**Ans:** $(-\infty, -1] \cup [2, \infty)$

22. 2073 Supp Q.No. 6a OR If A = (–1, 4) and B = [3, 5), find $A \cup B$, $A \cap B$, A–B and B–A. [4]

**Ans:** (–1, 5], [3, 4), (–1, 3), [4, 5]

23. 2073 Set D Q.No. 6a OR Define absolute value of a real number. Prove that $|x+y| \le |x| + |y|$ for all $x, y \in IR$ [4]

24. 2072 Supp Q.No. 6a OR If A = [-3, 1] and B = [-2, 4], find $A \cup B$, $A \cap B$ and A – B. Present them in the graph. [4]

**Ans:** [–3, 4]; [–2, 1]; [–3, –2)

25. 2072 Set C Q.No. 6b OR Define absolute value of a real number. If $x \in R$ and a is any positive real number, prove that $|x| < a \Rightarrow -a < x < a$ and conversely. [4]

26. 2072 Set D Q.No. 6a OR Solve the inequality $|2x-1| \ge 5$ and represent the solution set in the real line. [4]

**Ans:** $\{x : x \le -2 \text{ or } x \ge 3\}$

27. 2072 Set E Q.No. 6a OR Solve: $x^2 - 2x - 3 \ge 0$. [4]

**Ans:** $\{x : x \le -1 \text{ or } x \ge 3\}$

28. 2071 Supp. Q.No. 6a OR Solve the inequality: $x^2+7x+10<0$. [4]

**Ans:** (–5, –2)

29. 2071 Set C Q.No. 6a Or Define absolute value of a real number. Also, a is a positive real number and $x \in \mathbb{R}$ then prove that: $|x| < a$ if and only if $-a < x < a$. [4]

30. 2071 Set D Q.No. 6a Or Define absolute value of a real number. If a is a positive real number and $x \in \mathbb{R}$, then prove that: $|x| < a$ if and only if $-a < x < a$. [4]

31. 2070 Supp Q.No. 6a Or Solve the inequality: $|2x+1| \ge 3$. Represent the solution in a real line. [4]

**Ans:** $\{x : x \le -2 \text{ or } x \ge 1\}$

32. 2070 Set C Q.No. 6 a Or Let A = [–3, 1] and B = [–2, 4]. Find $A \cup B$, $A \cap B$, A – B and B – A. [4]

**Ans:** [–3, 4]; [–2, 1]; [–3, –2); (1, 4]

33. 2070 Set D Q.No. 6 a Or Solve the inequality: $6 + 5x - x^2 \ge 0$. [4]

**Ans:** $x \in [-1, 6]$

34. 2069 Supp Q.No. 6 a Or If a is a positive real number and $x \in \mathbb{R}$, prove that: $|x| < a$ if and only if $-a < x < a$. [4]

35. 2069 [Set A] Q. No. 6a OR If $x \in \mathbb{R}$ and a is any positive real number, prove that: $|x| < a \Rightarrow -a < x < a$ and conversely. [4]

36. 2069 [Set B] Q. No. 6a OR Define absolute value of a real number. For any two real numbers x and y, prove that: $|x+y| \le |x| + |y|$. [4]

37. 2068 Q.No. 6a(Or) Define absolute value of a real number. Rewrite the following relation without using absolute value sign $|2x-1| \le 5$. Also, draw the graph of the inequality. [4]

**Ans:** $-2 \le x \le 3$

# C. Logic

## FORMULAE AND IMPORTANT POINTS

1. Logical Connectives

| Connective | Symbol | Name |
|---|---|---|
| " not " | ~ | negation, or denial |
| " and " | $\wedge$ | conjunction |
| " or " | $\vee$ | Disjunction or alternation |
| " If ... then " | $\Rightarrow$ (or$\rightarrow$) | implication or conditional |
| " If and only if " | $\Leftrightarrow$ (or$\leftrightarrow$) | bi-conditional |

2. Truth Table

| p | q | $p \vee q$ | $p \wedge q$ | $p \Rightarrow q$ | $p \Leftrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | T | F | F | F |
| F | T | T | F | T | F |
| F | F | F | F | T | T |

3. Proposition : $p \Rightarrow q$, (if p then q)
Converse : $q \Rightarrow p$, (if q then p)
Opposite (inverse) : $\sim p \Rightarrow \sim q$, (if not p then not q)
Contrapositive : $\sim q \Rightarrow \sim p$ (if not q then not p)

4. Tautology
A compound statement which is always true for any set of given statements is called a tautology.

5. Contradiction
A compound statement which is false for any set of given statements, is called contradiction.

6. a. $\sim (p \wedge q) \equiv (\sim p \vee \sim q)$
b. $\sim (p \vee q) \equiv (\sim p \wedge \sim q)$
c. $\sim (p \Rightarrow q) \equiv (p \wedge \sim q)$

**2 Marks Questions**

1. 2076 Set B Q.No. 1a Find the truth value of the biconditional statement of p and q where p represents '2 is an even number' and q represents '4 is an even number'. [2]

**Ans: T**

2. 2075 GIE Q.No. 1a For any statements p and q, prove that $p \vee (p \wedge q) \equiv p$ with the help of truth table. [2]
3. 2075 Set A Q.No. 1a Write the converse and contrapositive of the statement "For a function to be integrable it is sufficient that the function is continuous." [2]

**Converse: If a function is continuous, then it is differentiable.**
**Contrapositive: If a function is not continuous, then it is not differentiable.**

4. 2075 Set C Q.No. 1a For any statements p and q, prepare a truth table for $\sim(p \wedge (\sim q))$. [2]
5. 2074 Supp Q.No. 1a If p and q are two statements, construct a truth table for the compound statement $(\sim p) \vee (\sim q)$. [2]
6. 2073 Set D Q.No. 1a For any statements p and q, prepare a truth table for $q \wedge \{\sim (p \vee q)\}$. [2]
7. 2072 Supp Q.No. 1a Define conjunction of a statement. Construct a truth table for the compound statement $\sim (p \wedge q)$. [2]
8. 2072 Set C Q.No. 1a Construct a truth table of $\sim (p \vee q) \wedge q$. [2]
9. 2072 Set D Q.No. 1a Find the truth value and the negation of the statement "3 + 2 = 6 or 6 is a multiple of 3". [2]

**Ans: T; 3 + 2 ≠ 6 and 6 is not a multiple of 3.**

10. 2071 Supp. Q.No. 1a Construct a truth table for the statement $(p \vee (\sim q)) \Rightarrow q$. [2]
11. 2071 Set C Q.No. 1a Define a statement. If p is true, q is true and r is true, find the truth value of $(p \vee q) \wedge (\sim q)$. [2]

**Ans: False**

12. 2071 Set D Q.No. 1a Construct truth table of $\sim(p \vee q)$. [2]
13. 2070 Supp Q.No. 1a Find the negation of the following statements
    i. Light travels in a straight line.
    ii. 3+2 = 5 or 6 is a multiple of 5. [2]

**Ans: (i) Light does not travel in a straight line.**
**(ii) 3 + 2 ≠ 5 and 6 is not a multiple of 5.**

14. 2070 Set C Q.No. 1 a Prepare a truth table for the compound statement $p \vee \sim (p \wedge q)$. What would you conclude from the truth table? [2]

**Ans: Tautology**

15. 2070 Set D Q.No. 1 a If p and q are any two statements, prove that: $p \vee q \equiv q \vee p$. [2]
16. 2069 Supp Q.No. 1 a For any statement p show that $p \wedge \sim p$ is a contradiction. [2]
17. 2069 Set A Q. No. 1a Define negation of a statement. Construct a truth table for the compound statement $\sim(p \vee (\sim q))$. [2]
18. 2069 [Set B] Q. No. 1a Write inverse and converse of the statement 'if 3 is an odd number then 6 is not an odd number'. [2]
19. 2068 Q.No. 1a Define disjunction of two statements. Prepare a truth table for the compound statement $\sim (p \vee q)$. [2]

**4 Marks Questions**

20. 2076 Set C Q.No. 6a Define conditional and bi-conditional statements. For any statements p and q, construct the truth table for $(p \vee q) \Leftrightarrow (q \vee p)$. [4]
21. 2075 Set B Q.No. 6a OR Define connectives in logic. For the simple statements p, q, r, construct the truth table for the statement $(p \Leftrightarrow q) \vee (q \Leftrightarrow r)$. [4]
22. 2074 Set A Q.No. 6a Define conjunction of the statements. Prepare a truth table for the compound statement $(p \wedge q) \wedge \sim (p \vee q)$. Draw the conclusion about the statement from the truth table. [4]
23. 2074 Set B Q.No. 6aOR Let p, q, r be any simple statements, construct truth table for the compound statement $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$. [4]
24. 2073 Supp Q.No. 6a If p and q are two statements, prove that $p \vee \sim(p \wedge q)$ is a tautology. [4]
25. 2073 Set C Q.No. 6a Define conditional statement. Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$. What conclusion can be drawn about the statement from the truth table? [4]

**Ans: Tautology**

26. 2072 Set E Q.No. 6a Let p, q and r be statements, show that the statement $(p \wedge q \Rightarrow r) \Leftrightarrow [p \Rightarrow (q \Rightarrow r)]$ is true. [4]

# Unit 2: Relations, Functions and Graphs

## FORMULAE AND IMPORTANT POINTS

1. Equality of two ordered pairs:
   (a, b) and (c, d) are equal i.e., (a, b) = (c, d) if and only if a = b and c = d.
2. Cartesian Product
   $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.
3. Relation
   A relation from the set A to B is a subset of the cartesian product A × B.
4. Function
   A function from a set A to a set B is a rule of correspondence that assigns to each element x in A with exactly one element y in B.
5. Domain and Range
   Let $f : A \to B$ be a function. Then the set A is called the domain and B is called co-domain of f. Range of f is given by $R(f) = \{ f(x): x \in A\}$.
6. Types of Functions
   i. One to one function: $f : A \to B$ is one to one if and only if for all $x_1, x_2 \in A$, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
   Equivalently, for all $x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
   ii. Onto function: A function $f : A \to B$ is onto if for each element $y \in B$, there exists at least one element $x \in A : f(x) = y$.
   iii. Bijective function: If a function $f : A \to B$ is both injective and surjective, then it is called bijective function.
7. Inverse Function: If $f : A \to B$ defined by $y = f(x)$ for each $x \in A$ is one to one and onto, then a new function $f^{-1} : B \to A$ which associates to each element $y \in B$, a unique element $x \in A$ such that $f^{-1}(y) = x$ is called the inverse of f.
8. Composition of Functions
   If $g : A \to B$ and $f : B \to C$ be any two functions defined from A to B and B to C respectively. Then the new function defined from A to C is called composite function and is denoted by $fog : A \to C$ and it is also written as $(fog)(x) = f(g(x))$ for each $x \in A$.

9. Exponential Function
A function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = a^x$, $a \neq 1$, $a > 0$, $x \in \mathbb{R}$, is called an exponential function.
10. Logarithmic Function
If $y = a^x$, $a > 0$, $a \neq 1$, then x is called the logarithm of the number y to the base a and is written as $x = \log_a y$.
11. Properties of logarithms
For $x, y \in \mathbb{R}^+$, $a > 0$, $b > 0$ and $p \in \mathbb{R}$, we have
i. $\log_a (xy) = \log_a x + \log_a y$
ii. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
iii. $\log_a x^p = p \log_a x$
iv. $\log_a x = \log_a b \cdot \log_b x$
v. $\log_a a = 1$
vi. $\log_a 1 = 0$
vii. $\log_a a^x = x$
viii. $a^{\log_a x} = x$
12. Common logarithm
$\log_{10} x$ is written as $\log x$.
13. Natural logarithm
$\log_e x$ is written as $\ln x$.

## 2 Marks Questions

1. 2076 Set B Q.No. 1b If $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 3$, find $f^{-1}$. [2]
Ans: $\frac{x+3}{2}$

2. 2076 Set C Q.No. 1b Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 9x - 1$, $x \in \mathbb{R}$, examine whether f is one-one. [2]
Ans: One-one

3. 2075 GIE Q.No. 1b Find the domain and range for $f(x) = \frac{x}{x-1}$, $x \in \mathbb{R}$. [2]
Ans: Domain = $\mathbb{R} - \{1\}$, Range = $\mathbb{R} - \{1\}$

4. 2075 Set A Q.No. 1b Evaluate: $2 \log_5 \sqrt{5} + 3\log_2 8$. [2]
Ans: 10

5. 2075 Set B Q.No. 1b Let A = {1, 2, 3} and B = {3, 4, 5}. Show that $A \times B \neq B \times A$. [2]

6. 2075 Set C Q.No. 1b Let A = {1, 2, 3}. Find the relation in A×A satisfying the condition $x > y$ for all $(x, y) \in A \times A$. Find the domain of the relation. [2]
Ans: {(2, 1), (3, 1), (3, 2)}. Domain = {2, 3}

7. 2074 Supp Q.No. 1b Let A = {0, 1, 2, 3, 4, 5, 6) and a function $f : A \to Q$ is defined by $f(x) = \frac{x}{2}$. Find the range of f, where Q is the set of rational numbers. [2]
Ans: $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$

8. 2074 Set A Q.No. 1b Let A = {a, b), B = {b, c} and C = {c, d). Find A ×(B∪C) and A×(B∩C). [2]
Ans: { (a, b), (a, c), (a, d), (b, b), (b, c), (b, d)}; {(a, c), (b, c)}

9. 2074 Set B Q.No. 1b If A = {3, 4, 5, 6}, find the relation R in A×A, satisfying the condition $x + y = 9$; $x \in A$, $y \in A$. [2]
Ans: {(3, 6), (4, 5), (5, 4), (6, 3)}

10. 2073 Supp Q.No. 1b Let A = {1, 2, 3, 4} and B = {1,3,5}. Find the relation R from set A to set B satisfying the condition $y = 2x$. [2]
Ans: { }

11. 2073 Set C Q.No. 1b Find the domain and range of the relation R = {(1,1), (2, 2), (4, 4)}. What types of relation is this? [2]
Ans: Domain = {1, 2, 4} and Range = {1, 2, 4}

12. 2073 Set D Q.No. 1b Let A = {–1, 0, 2, 4, 6, 8} and $f: A \to \mathbb{R}$ is defined by $f(x) = \frac{x}{x+2}$. Find range of f. [2]
Ans: $\{-1, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\}$

13. 2072 Supp Q.No. 1b Let A = {a, b}, B= {b, c} and C = {c, d}, find (A × B) ∪ (A × C) and (A × B) ∩ (A × C). [2]
Ans: {(a, b), (a, c), (b, b), (b, c), (a, d), (b, d)} and {(a, c), (b, c)}

14. 2072 Set C Q.No. 1b Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = 3x - 1$, find (gof) (x) and (fog) (x). [2]
Ans: $6x + 2$, $6x - 1$

15. 2072 Set D Q.No. 1b If A = {1, 2, 3} and B = {1, 3, 5, 7}, find a relation from set A to set B determined by $x \leq y$. [2]
Ans: {(1, 1), (1, 3), (1, 5), (1, 7)}, (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7)}

16. 2072 Set E Q.No. 1b Find the value of $\log_a \sqrt{a\sqrt{a\sqrt{a}}}$. [2]
Ans: $\frac{7}{8}$

17. 2071 Supp. Q.No. 1b Prove that $x^{\ln\left(\frac{y}{z}\right)} \, y^{\ln\left(\frac{z}{x}\right)} \, z^{\ln\left(\frac{x}{y}\right)} = 1$ [2]

18. 2070 Supp Q.No. 1b If A = {1, 2, 3} and B = {a, b}, find A × B and B × A and show that A × B ≠ B × A. [2]
Ans: A× B = {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}
B × A = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

19. 2070 Set C Q.No. 1 b Let A = {1, 2, 3, 4}. Find the relation on A satisfying the condition $x + y \leq 4$. [2]
Ans: {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}

20. 2070 Set D Q.No. 1 b Let A = {a, b}, B ={b, c} and C = {c, d}. Find: A ×(B∪C) and A × (B∩C). [2]
Ans: {(a, b), (a, c), (a, d), (b, b), (b, c), (b, d)}, {(a, c), (b, c)}

21. 2070 Old Q.No. 1 c A function f is defined as $f(x) = \begin{cases} x^2, & x \leq 0 \\ \frac{1}{x}, & x > 0 \end{cases}$, find f(0), f(+2) and f $(\sqrt{3})$. [2]
Ans: $0, \frac{1}{2}, \frac{1}{\sqrt{3}}$

22. 2069 Supp Q.No. 1 b Define inverse relation and give its example. [2]

23. 2069 [Set A] Q. No. 1b Find the domain of the function $y = \sqrt{x-2}$. [2]
Ans: $[2, \infty)$

24. 2069 [Set A] Old Q. No. 1b Prove that: $\log(1 + 2 + 3) = \log 1 + \log 2 + \log 3$ [2]

25. 2069 [Set B] Q. No. 1b Find the domain, range and inverse of the relation R = {(1, 2), (2, 4), (3, 6), (4, 8)} [2]
Ans: Domain = {1, 2, 3, 4}; Range = {2, 4, 6, 8};
Inverse relation = {(2, 1), (4, 2), (6, 3) (8, 4)}

26. 2068 Q.No. 1b Let A = {1, 2, 3, 4} and B = {1, 3, 5}. Find the relation R from set A to set B determined by the condition $x > y$. [2]
Ans: {(2, 1), (3, 1), (4, 1), (4, 3)}

27. 2068 Old Q.No. 1b Let A = {-1, 0, 2, 4, 6} and a function $f : A \to R$ is defined by $y = f(x) = \frac{x}{x+2}$. Find the range of f. [2]
Ans: $\{-1, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}\}$

28. **2067 Q.No. 1b** If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that $xyz = 1$. [2]

29. **2066 Q.No. 1(b)** Prove that $\log_a x^2 - 2\log_a\sqrt{x} = \log_a x$. [2]

30. **2065 Q. No. 1 b** Let f: R → R be defined by $f(x) = 2x + 5$. Compute $f^{-1}(x)$. [2]

**Ans:** $\frac{x-5}{2}$

31. **2064 Q.No. 1(b)** Let f, g be real valued functions defined as: $f(x) = 4x + 7$, $x \in R$ and $g(x) = 5x - 2$, $x \in R$. Find $f_og(x)$ and $g_of(x)$ [2]

**Ans:** $(20x - 1)$, $(20x + 33)$

32. **2063 Q.No. 1(b)** Let f, g be real valued function defined as $f(x) = x^2 + 5x + 7$, $x \in R$ and $g(x) = 5x - 3$, $x \in R$, find fog (x) and gof(x) [2]

**Ans:** $25x^2 - 5x + 1$; $5x^2 + 25x + 32$

33. **2062 Q.No. 1(b)** Check whether the function f: [–2, 3] → R given by $f(x) = x^3$ is one to one, onto or both. [2]

**Ans:** One to one but not onto

34. **2061 Q.No. 1(b)** Let h: R→R be defined by $h(x) = 3x - 6$, find the formula that defines $h^{-1}$. [2]

**Ans:** $\frac{x-6}{3}$

35. **2060 Q.No. 1(c)** Define even and odd functions with examples. [2]

**Ans:** Even if $f(-x) = f(x)$ & odd if $f(-x) = -f(x)$

36. **2059 Q.No. 1(c)** Let f: R → R be defined by $y = f(x) = 2x - 3$, $x \in R$, find formula that defines $f^{-1}$. [2]

**Ans:** $\frac{x+3}{2}$

37. **2058 Q.No. 1(b)** Let f: A → R be given by $f(x) = 2|x| + 3$ where A = {– 2, 0, 1, 2}, find the range of f. [2]

**Ans:** {3, 5, 7}

38. **2057 Q.No. 1(b)** When does a function f: A → B become an onto and one to one? [2]

## 6 Marks Questions

39. **2076 Set B Q.No. 11** Define function. Distinguish relation and function with example. Find the domain and range of $f(x) = \sqrt{x^2 - 2x - 8}$, $x \in \mathbb{R}$. [6]

**Ans:** Domain = $(-\infty, -2] \cup [4, \infty)$; Range = $[0, \infty)$

40. **2076 Set C Q.No. 11** Define domain and range of a function. Find the domain and range of $f(x) = \sqrt{21 - 4x - x^2}$. [6]

**Ans:** Domain = [–7, 3], Range = [0, 5]

41. **2075 GIE Q.No. 11** Define one-one function and onto function. Let $f(x) = x^3 + 5$, $x \in R$. [6]
Find a formula that defines inverse function $f^{-1}$.

**Ans:** $\sqrt[3]{x-5}$

42. **2075 Set A Q.No. 11** Define domain and range of a function. Find the domain and range of the function: [6]

a. $f(x) = 5 - (x + 3)^2$  b. $f(x) = \frac{x}{|x|}$

**Ans:** (a) Domain = $(-\infty, \infty)$, Range = $(-\infty, 5]$;
(b) Domain = R – {0}, Range = {–1, 1}

43. **2075 Set B Q.No. 11** Define function, domain of a function, and range of a function. Show that $f(x) = 2x + 3$ is bijective (f : R → R). Also find $f^{-1}(2)$. [6]

**Ans:** $-\frac{1}{2}$

44. **2075 Set C Q.No. 11** Find the domain and range of the function $f(x) = \sqrt{2 - x - x^2}$. [6]

**Ans:** Domain = [–2, 1]; Range = [0, 3/2]

45. **2074 Supp Q.No. 11** Show that f: R→R defined by $f(x)=cx+d$ where c ($c \neq 0$) and d are real numbers is one to one and onto. Also find $f^{-1}$. [6]

**Ans:** $f^{-1}(x) = \frac{x-d}{c}$

46. **2074 Set A Q.No. 11** Show that f : R → R defined by $f(x) = cx + d$ where $c \neq 0$ and d are real numbers, is one to one and onto. Find $f^{-1}(x)$. [6]

**Ans:** $f^{-1}(x) = \frac{x-d}{c}$

47. **2074 Set B Q.No. 11** Let f: R → R and g: R → R be defined by $f(x) = x^3 + 2$ and $g(x) = 4x - 1$. Find (fog) (x) and (gof) (x). Is (fog)(x) = (gof)(x)? Are (fog) (x) and (gof) (x) one to one? [6]

**Ans:** $(4x-1)^3 + 2$, $4x^3 + 7$, No, Yes, Yes

48. **2073 Supp Q.No. 11** Define composite functions of two functions f and g. Let f: R → R and g: R → R be defined by $f(x) = 3x^2 - 4$ and $g(x) = 2x-5$, find (fog) (x) and (gof) (x). Are the functions (fog) (x) and (gof) (x) one to one? Give reasons. [6]

**Ans:** $12x^2 - 60x + 71$, $6x^2 - 13$, No

49. **2073 Set C Q.No. 11** Let the function $f(x) = x^3$ and $g(x) = \sin x$, $x \in R$. Find $f_og$ and $g_of$. Is $f_og = g_of$? Examine whether f is one to one and onto or not. [6]

**Ans:** $fog(x) = \sin^3 x$; $gof(x) = \sin x^3$; No; One to One and Onto

50. **2073 Set D Q.No. 11** Define function. Also define one-one onto and one-one into function. Show that the function f: [1,4]→ R defined by $f(x) = x^2$ is one-one but not onto. [6]

51. **2072 Supp Q.No. 11** Define one to one function and the onto function. Let a function f :A→ B be defined by $f(x) = \frac{x+1}{2x-1}$ with A = {-1,0,1,2,3,4} and B = {-1, 0, $\frac{4}{5}$, $\frac{5}{7}$, 1, 2, 3}. Find the range of f. Is the function f one to one and onto both? [6]

**Ans:** Range of f = {–1, 0, 4/5, 5/7, 1, 2}; one to one, not onto.

52. **2072 Set C Q.No. 11** Let a function f : A → B be defined by $f(x) = \frac{x-1}{x+2}$ with A = {–1,0,1,2,3,4} and B = $\left\{-2, 1, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$. Find the range of f. Is the function f one to one and onto both? If not, how can you make it one to one and onto both? [6]

**Ans:** Range = $\{-2, \frac{-1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{1}{2}\}$, No

53. **2072 Set D Q.No. 11** Define composite function. Let f : R → R be defined by $f(x) = 2x$ and g: R → R be defined by $g(x) = x + 1$. Find (gof) (x), (fog) (x), (fof) (x) and (gog) (x). Are the functions (gof) (x) and (fog) (x) one to one? [6]

**Ans:** 2x + 1, 2x + 2, 4x, x+2, Yes

54. **2072 Set E Q.No. 11** What condition makes a function to have its inverse? Show that f : IR – {2} → IR – {0} given by $f(x) = \frac{1}{x-2}$ is bijective. Also, find $f^{-1}$. [6]

**Ans:** $\frac{2x+1}{x}$

55. **2071 Supp. Q.No. 11** Define bijective function. Prove that f: IR→IR defined by $f(x) = 2x+3$ is bijective. [6]

56. 2071 Set C Q.No. 11 Define inverse function. If f:R→R be defined by $f(x) = x^2 - 3$, find $f^{-1}(x)$. Also determine whether $fof^{-1}(x) = f^{-1}of(x)$. [6]

57. 2071 Set D Q.No. 11 Prove that the function $f:\Re-\{3\} \to \Re - \{1\}$ given by $f(x) = \frac{x}{x-3}$ is bijective. [6]

58. 2070 Supp Q.No. 11 Define composite function of two functions. If f:R→R be defined by $f(x) = \frac{x-1}{x+1}$, $(x \neq -1)$ show that $(fof)(4) = -\frac{1}{4}$. [6]

59. 2070 Set C Q.No. 11 Let function f: A → B be defined by $f(x) = \frac{x+1}{2x-1}$. Find the range of f. Is the function f one to one and onto both? If not, how can the function be made one to one and onto both? [6]

60. 2070 Set D Q.No. 11 Define composite function of two functions f and g. Let f:R → R and g:R → R be two functions defined by $f(x) = 2x^2 - 3$ and $g(x) = 3x + 2$. Determine (fog) (x), (gof) (x), and (gog) (x). Is (fog) (x) = (gof) (x)? Are the function (fog) (x) and (gog) (x) one to one? [6]

**Ans:** $18x^2 + 24x + 5$; $6x^2 + 7$, $9x + 8$; No, No, Yes

61. 2069 Supp Q.No. 11 Define domain and range of a function. Find the domain and range of the function $y = x^2 - 4x + 3$ for $x \in R$ [6]

**Ans:** Domain $(-\infty, \infty)$, Range = $[-1, \infty)$

62. 2069 [Set A] Q. No. 11 Define one to one function and onto function. Let a function f: A → B be defined by $f(x) = \frac{x^2}{6}$ with $A = \{-2, -1, 0, 1, 2\}$ and $B = \left\{0, \frac{1}{6}, \frac{2}{3}\right\}$. Find the range of f. Is the function f one to one and onto both? [6]

**Ans:** Range of f = $\left\{0, \frac{1}{6}, \frac{2}{3}\right\}$

63. 2069 [Set B] Q. No. 11 Define function. State the condition for a function to be bijective. Given, $f(x) = x^3 + 5$, $x \in R$, find $f^{-1}$. [6]

**Ans:** $\sqrt[3]{x-5}$

64. 2068 Q.No. 11 Define the domain and the range of a function. Find the domain and the range of the function $f(x)=-x^2+4x-3$. [6]

**Ans:** Domain = $(-\infty, \infty)$, Range = $(-\infty, 1]$

## 4 Marks Questions (Old Syllabus Questions)

65. 2071 Old Q.No. 7b Let $f$ : R→R be defined by $y= f(x)=2x-3$, $x \in R$. Find a formula that defines the inverse function $f^{-1}$. [4]

**Ans:** $\frac{x+3}{2}$

66. 2070 Old Q.No. 7 b If f and g are functions from R→R defined by $f(x) = x^2 - x + 1$ and $g(x) = 2x + 3$, find (fog)x and (gof)x. [4]

**Ans:** $4x^2 + 10x + 7$; $2x^2 - 2x + 5$

67. 2069 [Set A] Old Q. No. 7b Let f: R→R and g: R→R be defined by $f(x) = 2x^2 - 3$, $g(x) = 3x + 2$. Determine the following composite functions: (i) (f o g) (x) (ii) (g o f) (x): [4]

**Ans:** (i) $18x^2 + 24x + 5$ (ii) $6x^2 - 7$

68. 2069 [Set B] Old Q. No. 7a If $A = \{x: x \in N$ and $x < 4\}$ and $B = \{x: x \in N$ and $x^2 - 9 = 0\}$. Find A × B, B × A, A × A, B × B. [4]

**Ans:** A = {1, 2, 3}; B = {3}; A × B = {(1, 3), (2, 3), (3, 3)}; B × A = {(3, 1), (3, 2), (3, 3)}; A × A = {(1, 1), (1, 2), (1, 3) (2, 1), (2, 2) (2, 3), (3, 1), (3, 2) (3,3) }; B ×B = {(3, 3)}

69. 2069 [Set B] Old Q. No. 7b If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that: $x^x y^y z^z = 1$. [4]

70. 2068 Old Q.No. 7b Let R be the set of real numbers. Show that the functions f: R→R defined by $f(x) = 3x - 2$ for all $x \in R$ is one to one and onto. Also find $f^{-1}$. [4]

**Ans:** $\frac{x+2}{3}$

71. 2067 Q.No. 7b For the function $f(x) = 2x^2 - 3$ and $g(x) = 3x+2$, $x \in R$ examine whether fog and gof are one-one. [4]

**Ans:** Both are not one-one

72. 2066 Q.No. 7(b) Let R be the set of real numbers. Show that the function f: R →R such that $f(x) = 5x - 3$ for all $x \in R$ is one to one and onto. [4]

73. 2065 Q. No. 7 b If f, g : R → R defined by $f(x) = x^3 + 2$ and $g(x) = 4x - 1$, find fog (x) and gof (x), and show that the composite function is not commutative. [4]

**Ans:** fog (x) = $64x^3 - 48x^2 + 12x + 1$ and gof (x) = $4x^3 + 7$

74. 2064 Q.No. 7(b) Let Q be the set of all rational numbers. Show that the function: f: Q → Q such that $f(x) = 3x + 5$ for all $x \in Q$ is one to one and onto. Find $f^{-1}$. [4]

**Ans:** $\frac{x-5}{3}$

75. 2063 Q.No. 7(b) Let R be the set of rational numbers. Show that the function defined by f: R → R such that $f(x) = 4x - 7$, $x \in R$ is one–one and onto. Find a formula for $f^{-1}$. [4]

**Ans:** $\frac{x+7}{4}$

76. 2062 Q.No. 7(b) When does an inverse of a function exist? If f: R → R be defined by $f(x) = x^3 + 5$, find the formula that defines $f^{-1}$. [4]

**Ans:** One to one & onto, $\sqrt[3]{x-5}$

77. 2061 Q.No. 7(b) For the functions $f(x) = 2x^2-3$ and $g(x)=3x + 2$ where $x \in R$, determine (fog) (x), gof (x). Are fog and gof one–one. [4]

**Ans:** $18x^2 + 24x + 5$; $6x^2 - 7$; No

78. 2060 Q.No. 7(b) Let f : R → R is defined by $f(x) = 2x + 3$ and g: R→R is defined by $g(x) = x^2$. Find (g of) (x) and (fog) (x). [4]

**Ans:** $(2x + 3)^2$; $(2x^2 + 3)$

79. 2059 Q.No. 7(b) Let f : R → R and g: R → R be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$, find $f^{-1}$, (gof) (x), (fog) (x). [4]

**Ans:** $\sqrt{x-1}$, $(x^2 + 1)^5$, $x^{10} + 1$

80. 2058 Q.No. 7(b) Check whether the function f : [–2, 3] → R given by $f(x) = x^3$ is one to one, onto or both. [4]

**Ans:** One to one but not onto

81. 2057 Q.No. 7(b) A function f (x) is defined as follows:

$$f(x) = \begin{cases} 4x - 2 & \text{for } x \geq 1 \\ 2x & \text{for } x < 1 \end{cases}$$

Find f(2), f(1), f(0), f(– 1), $\frac{f(h) - f(1)}{h}$ for $1 \leq h$ [4]

**Ans:** 6, 2, 0, –2 & $\frac{4(h-1)}{h}$

82. 2056 Q.No. 7(b) Let f: R→R and g: R→R be defined by $f(x) = x^2 + 1$; $g(x) = x^5$. Find $f^{-1}$: (gof) (x) and (fog) (x) [4]

**Ans:** $\sqrt{x-1}$, $(x^2+1)^5$, $x^{10}+1$

## Unit 3: Curve Sketching

### FORMULAE AND IMPORTANT POINTS

1. Even and Odd Function
If a function f satisfies f (–x) = f (x) for all x in its domain, then the function f is called an even function. Similarly, a function f is called odd function if f (–x) = –f (x) for all x.
2. Periodic Function
Let f (x) be a function and D be its domain. Then f (x) is said to be periodic if f (x + T) = f (x) for all x ∈ D where T is a positive constant and the smallest such number T is called the period of the function.
3. i. The period of f (x) = sin (ax) is $\frac{2\pi}{|a|}$.
ii. The period of f (x) = cos (ax) is $\frac{2\pi}{|a|}$.
iii. The period of f (x) = tan (ax) is $\frac{\pi}{|a|}$.
iv. The period of f (x) = cot (ax) is $\frac{\pi}{|a|}$.
4. Symmetry
i. Symmetry about y-axis: A graph is symmetrical about y-axis if replacing x by – x does not change the equation of the graph. Therefore even function are symmetrical about y-axis.
ii. Symmetry about origin: If y = f (x) satisfies f(–x) = – f(x), then the graph is symmetrical about the point origin. Thus, an odd function is always symmetrical about origin.
iii. Symmetry about x-axis: The graph of a function f (x, y) is symmetrical about x-axis if replacing y by –y does not change the equation f (x, y).

### 2 Marks Questions

1. 2076 Set B Q.No. 1c Test the even or odd nature and symmetricity of the function $y = 8x^2$. [2]
**Ans:** Even function, Symmetric about y-axis
2. 2076 Set C Q.No. 1c State the condition for which a function is symmetric with respect to origin. [2]
**Ans:** A function f is symmetric with respect to origin if f(–x) = –f(x) for all x in the domain of f.
3. 2075 GIE Q.No. 1c Examine even and odd property of $f(x) = \sqrt{x^2-1}$. [2]
**Ans:** Even function
4. 2075 Set A Q.No. 1c Test the function $f(x) = x^5 + x^3 + x$ for symmetry. [2]
**Ans:** Symmetric about origin
5. 2075 Set B Q.No. 1c Examine the symmetricity of the function $f(x) = e^x + \frac{1}{e^x}$. [2]
**Ans:** Symmetric about y-axis
6. 2075 Set C Q.No. 1c Examine whether the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd. Also examine for its symmetricity. [2]
**Ans:** Odd, symmetric about origin
7. 2074 Supp Q.No. 1c Test the periodicity of the function $f(x) = \sin \pi x$ and find its period. [2]
**Ans:** $2\pi$
8. 2074 Set A Q.No. 1c Test the periodicity of the function $f(x) = \cos\pi x$ and find its period. [2]
**Ans:** 2
9. 2074 Set B Q.No. 1c Show that the function f(x) = 5 - 2x is decreasing for all $x \in \mathbb{R}$. [2]
10. 2073 Supp Q.No. 1c Examine whether the function $f(x) = x^3 - x$ is even or odd. Also, test for symmetricity. [2]
**Ans:** Odd, Symmetric about origin
11. 2073 Set C Q.No. 1c Examine the symmetry and even or odd nature of the function $y = x^3$. [2]
**Ans:** Symmetric about origin, odd function
12. 2073 Set D Q.No. 1c Define even and odd function with suitable examples. [2]
13. 2072 Supp Q.No. 1c Test the periodicity of the function $f(x) = \tan \frac{x}{4}$ and find its period. [2]
**Ans:** $4\pi$
14. 2072 Set C Q.No. 1c Examine the even or odd nature and the symmetricity of the function $y = 10^x - 10^{-x}$. [2]
**Ans:** odd; symmetric about origin
15. 2072 Set D Q.No. 1c Find the periodicity of the function $y = \tan \frac{3x}{5}$ and find its period. [2]
**Ans:** $\frac{5\pi}{3}$
16. 2072 Set E Q.No. 1c Test for symmetry: $y = \sqrt{x^2-1}$ [2]
**Ans:** symmetric about y–axis
17. 2071 Supp. Q.No. 1c Test for symmetry: $y = x^{-3} + x^{-1}$ [2]
**Ans:** Symmetric about origin.
18. 2071 Set C Q.No. 1c Determine whether the function $f(x) = \sqrt{x^2-1}$ is even or odd. Also test its symmetricity. [2]
**Ans:** Even, symmetric about y-axis
19. 2071 Set D Q.No. 1a Define even and odd functions and give example of each. [2]
20. 2070 Supp Q.No. 1c Test the periodicity of the function $f(x) = \tan x/4$ and find its period. [2]
**Ans:** $4\pi$
21. 2070 Set C Q.No. 1 c Examine whether the function: $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd. Also examine for its symmetricity. [2]
**Ans:** Odd, symmetric about origin
22. 2070 Set D Q.No. 1 c Test the even or odd nature and the symmetricity of the function $f(x) = x^4 + 3x^2 + 1$ [2]
**Ans:** Even, symmetric about y-axis
23. 2069 Supp Q.No. 1 c Show that $y = x^3$ is an odd function and test its symmetricity. [2]
**Ans:** Symmetric about origin
24. 2069 [Set A] Q. No. 1c Test the periodicity of the function $f(x) = \sin 2x$ and find its period. [2]
**Ans:** $\pi$
25. 2069 [Set B] Q. No. 1c Examine the function $y = \cos x$ for symmetry and even or odd nature. [2]
**Ans:** Symmetric about y-axis, even function

26. 2068 Q.No. 1c Test the periodicity and the symmetricity of the function $y = \sin x$. [2]

**Ans: Period $2\pi$; symmetrical with respect to the origin**

**4 Marks Questions**

27. 2076 Set B Q.No. 6b Sketch the graph of $f(x) = (x - 1)^3$ indicating its characteristics. [4]
28. 2076 Set C Q.No. 6b Sketch the graph of $f(x) = x^2 - 3x$ indicating its characteristics. [4]
29. 2075 GIE Q.No. 6b Sketch the curve of $f(x) = x^2 + 2x + 3$ mentioning its major characteristics. [4]
30. 2075 Set A Q.No. 6b Sketch the graph of $y = 3 \sin x$ using its different characteristics. [4]
31. 2075 Set B Q.No. 6b Stating main characteristics of the function $f(x) = x^2 + 4x + 3$, sketch the graph. [4]
32. 2075 Set C Q.No. 6b Sketch the graph of $y = \left(\frac{1}{3}\right)^x$ indicating its different characteristics. [4]
33. 2074 Supp Q.No. 6b Draw the graph of the function $f(x) = x(x - 1)(x - 2)$ using its different characteristics. [4]
34. 2074 Set A Q.No. 6b Draw the graph of the function $f(x) = x - x^2$ indicating its characteristics. [4]
35. 2074 Set B Q.No. 6b Sketch the graph of the function $f(x) = x^2 - 4x + 3$ stating its main characteristics. [4]
36. 2073 Supp Q.No. 6b Sketch the graph of the function $y = x^2 + 2x - 3$ using its different characteristics. [4]
37. 2073 Set C Q.No. 6 b Draw the graph of $f(x) = x^2 - 6x + 5$ indicating its characteristics. [4]
38. 2073 Set D Q.No. 6b Stating the different characteristics sketch the curve of the function: $f(x) = x^2 + 2x - 3$. [4]
39. 2072 Supp Q.No. 6b Draw the graph of the function $y = x^2 - 5x + 6$ with its different characteristics. [4]
40. 2072 Set C Q.No. 6b Sketch the graph of $y = x - x^2$ indicating its characteristics. [4]
41. 2072 Set D Q.No. 6b Sketch the graph of $y = x^2 - 2x - 3$ with its different characteristics. [4]
42. 2072 Set E Q.No. 6b Sketch the graph of $y = 2^x + 1$ indicating its specific characteristics. [4]
43. 2071 Supp. Q.No. 6b Sketch the graph of the function $y = (x + 1)(x - 2)(x - 3)$ indicating its specific characteristics. [4]
44. 2071 Set C Q.No. 6b Sketch the graph of $y = -x^2 + 4x - 3$ indicating its characteristics. [4]
45. 2071 Set D Q.No. 6b Sketch the graph of $y = x^2 + 5x + 4$ indicating its characteristics. [4]
46. 2070 Supp Q.No. 6b Sketch the graph of $y = 2^x$ presenting its different characters. [4]
47. 2070 Set C Q.No. 6 b Using different characteristics, sketch the graph of $y = -x^2 + 4x - 3$. [4]
48. 2070 Set D Q.No. 6 b Using different characteristics, sketch the graph of $y = (x - 1)(x - 2)(x - 3)$ [4]
49. 2069 Supp Q.No. 6 b Sketch the graph of $f(x) = x^2 + 2x + 3$ indicating its characteristics. [4]
50. 2069 [Set A] Q. No. 6b Draw the graph of the function $y = x^2 - 4x + 3$ using its different characteristics. [4]
51. 2069 [Set B] Q. No. 6b Sketch the graph of $f(x) = (x - 4)^2 - 8$ indicating its characteristics. [4]
52. 2068 Q.No. 6b Draw the graph of $y = \cos x\ \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$ using its different characteristics. [4]

# Unit 4: Trigonometry

## A. Inverse Circular Functions

### FORMULAE

1. $2\sin A \cos B = \sin(A + B) + \sin(A - B)$
   $2\cos A \sin B = \sin(A + B) - \sin(A - B)$
   $2\cos A \cos B = \cos(A + B) + \cos(A - B)$
   $2\sin A \sin B = \cos(A - B) - \cos(A + B)$
2. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
   $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
   $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
   $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$
   $= -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$
3. Domain and range of inverse circular functions

| Functions | Domain (x) | Range (y) |
|---|---|---|
| $y = \sin^{-1} x$ | $-1 \le x \le 1$ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ |
| $y = \cos^{-1} x$ | $-1 \le x \le 1$ | $0 \le y \le \pi$ |
| $y = \tan^{-1} x$ | $(-\infty, \infty)$ | $-\frac{\pi}{2} < y < \frac{\pi}{2}$ |
| $y = \cot^{-1} x$ | $(-\infty, \infty)$ | $0 < y < \pi$ |
| $y = \sec^{-1} x$ | $x \ge 1$, or $x \le -1$ | $0 \le y \le \pi,\ y \ne \frac{\pi}{2}$ |
| $y = \text{cosec}^{-1} x$ | $x \ge 1$, or $x \le -1$ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \ne 0$ |

4. Self adjust property
   $\sin \sin^{-1} x = x = \sin^{-1} \sin x$
   $\cos \cos^{-1} x = x = \cos^{-1} \cos x$
   $\tan \tan^{-1} x = x = \tan^{-1} \tan x$
5. Reciprocal property
   $\sin^{-1} x = \text{cosec}^{-1}\frac{1}{x}$ $\quad \cos^{-1} x = \sec^{-1}\frac{1}{x}$
   $\tan^{-1} x = \cot^{-1}\frac{1}{x}$
6. Conversion property
   $\sin^{-1} x = \cos^{-1}\sqrt{1 - x^2}$ $\quad \sin^{-1} x = \tan^{-1}\frac{x}{\sqrt{1 - x^2}}$
   $\sin^{-1} x = \sec^{-1}\frac{1}{\sqrt{1 - x^2}}$ $\quad \sin^{-1} x = \cot^{-1}\frac{\sqrt{1 - x^2}}{x}$
7. For any numerical value of x,
   i. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ii. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
   iii. $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
8. For given numerical value of x.
   a. $\sin^{-1}(-x) = -\sin^{-1} x$ b. $\cos^{-1}(-x) = \pi - \cos^{-1} x$
   c. $\tan^{-1}(-x) = -\tan^{-1} x$ d. $\cot^{-1}(-x) = \pi - \cot^{-1} x$

9. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $xy < 1$

10. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$

11. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

## 2 Marks Questions

1. **2076 Set B Q.No. 2a** Express $\tan^{-1}x$ in terms of inverse of sine function. [2]

Ans: $\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

2. **2076 Set C Q.No. 2a** Prove that: $\sin(3\sin^{-1}x) = 3x - 4x^3$. [2]
3. **2075 GIE Q.No. 2a** Prove that $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$. [2]
4. **2075 Set B Q.No. 2a** Show that $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$. [2]
5. **2075 Set C Q.No. 2a** Prove that $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$. [2]
6. **2074 Supp Q.No. 2a** Prove that $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$. [2]
7. **2074 Set A Q.No. 2a** Prove that $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$. [2]
8. **2074 Set B Q.No. 2a** If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, prove that $x^2 + y^2 = 1$. [2]
9. **2073 Set D Q.No. 2a** Prove: $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ [2]
10. **2072 Supp Q.No. 2a** Prove that: $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$. [2]
11. **2072 Set C Q.No. 2a** Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$. [2]
12. **2071 Old Q.No. 2a** Prove that: $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$ [2]
13. **2070 Set C Q.No. 2 a** Prove that: $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$ [2]
14. **2070 Set D Q.No. 2 a** Prove that: $\cos(\sin^{-1}u + \cos^{-1}v) = v\sqrt{1-u^2} - u\sqrt{1-v^2}$ [2]
15. **2070 Old Q.No. 2 a** Express $\tan^{-1}x$ in terms of sine function. [2]

Ans: $\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

16. **2069 [Set A] Old Q. No. 2a** Show that: $\tan^{-1}x = \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. [2]
17. **2069 [Set B] Old Q. No. 2a** Prove that: $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \frac{\pi}{2} - \frac{x}{2}$. [2]
18. **2068 Old Q.No. 2a** Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ [2]
19. **2067 Q.No. 2a** Show that $\tan^{-1}x = \frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}$ [2]
20. **2066 Q.No. 2(b)** Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$ [2]
21. **2065 Q. No. 2 a** Prove that: $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$. [2]
22. **2064 Q.No. 2(a)** Prove that: $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0$ [2]
23. **2063 Q.No. 2(a)** Prove that: $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$ [2]
24. **2062 Q.No. 2(a)** Solve: $\cos(\sin^{-1}x) = \frac{1}{2}$. [2]

Ans: $x = \frac{\sqrt{3}}{2}$

25. **2061 Q.No. 2(a)** Solve: $2\tan^{-1}x = \sin^{-1}\frac{2m}{1+m^2} + \sin^{-1}\frac{2n}{1+n^2}$ [2]

Ans: $\frac{m+n}{1-mn}$

26. **2060 Q.No. 2(a)** Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$ [2]
27. **2059 Q.No. 2(a)** Show that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{6}{17}$ [2]
28. **2058 Q.No. 2(a)** Find the value of $\tan^{-1}3 + \tan^{-1}\frac{1}{3}$. [2]

Ans: $\frac{\pi}{2}$

29. **2057 Q.No. 2(a)** Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$ [2]
30. **2056 Q.No. 2(a)** Find the value of $\tan^{-1}3 + \tan^{-1}\frac{1}{3}$ [2]

Ans: $\frac{\pi}{2}$

## 4 Marks Questions

31. **2075 Set A Q.No. 7a** If $-\frac{\pi}{2} < \tan^{-1}x + \tan^{-1}y + \tan^{-1}z < \frac{\pi}{2}$, then prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx}$ [4]
32. **2073 Supp Q.No. 7a** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$. [4]
33. **2073 Set C Q.No. 7a OR** Prove that: $\sin(2\sin^{-1}x) = 2x\sqrt{1-x^2}$ [4]
34. **2072 Set D Q.No. 7a** Prove that: $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi = 2\left(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$ [4]
35. **2072 Set E Q.No. 7a** Prove that: $\cos(\sin^{-1}x + \cos^{-1}y) = y\sqrt{1-x^2} - x\sqrt{1-y^2}$. [4]
36. **2071 Supp. Q.No. 7a** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, prove that $xy + yz + zx = 1$. [4]
37. **2071 Set C Q.No. 7a** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, show that: $xy + yz + zx = 1$. [4]
38. **2071 Set D Q.No. 7a** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ then show that: $x + y + z = xyz$. [4]
39. **2071 Old Q.No. 8a** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ show that: $x^2 + y^2 + z^2 + 2xzy = 1$. [4]
40. **2070 Supp Q.No. 7a** Prove that $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$. [4]
41. **2070 Old Q.No. 8 a** If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that: $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. [4]
42. **2069 Supp Q.No. 7 a** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$. [4]
43. **2069 [Set A] Q. No. 7a** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$. [4]

**44.** **2069 [Set A] Old Q. No. 8a** If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, prove that: $yz + zx + xy = 1$. [4]

**45.** **2069 [Set B] Q. No. 7a** If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ show that: $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$. [4]

**46.** **2069 [Set B] Old Q. No. 8a** Prove that:
$\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$. [4]

**47.** **2068 Q.No. 7a** Prove that:
$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi = 2(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$ [4]

**48.** **2068 Old Q.No. 8a** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that: $x^2 + y^2 + z^2 + 2xyz = 1$. [4]

**49.** **2067 Q.No. 8a** If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$ [4]

**50.** **2066 Q.No. 8(a)** If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ show that $xy + yx + zx = 1$ [4]

**51.** **2065 Q. No. 8 a** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$. [4]

**52.** **2063 Q.No. 8(a) OR** Prove that:
$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = 2(\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3})$ [4]

**53.** **2061 Q.No. 8(a)** If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, prove that $x + y + z = xyz$ [4]

**54.** **2060 Q.No. 8(a)** Prove that $\cot^{-1}(\tan 2x) + \cot^{-1}(\tan 3x) = x$ [4]

**55.** **2059 Q.No. 8(a)** Prove: $\tan(2\tan^{-1} x) = 2\tan(\tan^{-1} x + \tan^{-1} x^3)$ [4]

**56.** **2058 Q.No. 8(a)** Prove that $\cot^{-1}3 + \mathrm{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$. [4]

**57.** **2057 Q.No. 8(a) OR** Find the value of $\cos\tan^{-1}\sin\cot^{-1} x$ [4]

Ans: $\sqrt{\frac{1+x^2}{2+x^2}}$

**58.** **2056 Q.No. 8(a)** Solve: $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1} x$ [4]

Ans: $x = \frac{a+b}{1-ab}$

## B. Trigonometric Equation and General Values

### FORMULAE AND IMPORTANT POINTS

1. The general solution of $\sin x = 0$ is $x = n\pi$, $n \in \mathbb{Z}$.
2. The general solution of $\cos x = 0$ is $x = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.
3. The general solution of $\tan x = 0$ is $x = n\pi$, $n \in \mathbb{Z}$.
4. The general solution of $\cot x = 0$ is $x = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.
5. The general solution of $\sin x = k$, $(-1 \le k \le 1)$ is $x = n\pi + (-1)^n\theta$, $n \in \mathbb{Z}$.
6. The general solution of $\cos x = k$, $(-1 \le k \le 1)$ is $x = 2n\pi \pm \theta$, $n \in \mathbb{Z}$.
7. The general solution of $\tan x = k$ is $x = n\pi + \theta$, $n \in \mathbb{Z}$.

### 2 Marks Questions

**1.** **2075 Set A Q.No. 2a** Solve the trigonometric equation. [2]
$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

Ans: $(3n+1)\frac{\pi}{9}$

**2.** **2073 Supp Q.No. 2a** Solve: $7\sin^2 x + 3\cos^2 x = 4$. [2]

Ans: $n\pi \pm \frac{\pi}{6}$

**3.** **2073 Set C Q.No. 2a** Solve: $\cot^2 x + \mathrm{cosec}^2 x = 3 (-\frac{\pi}{2} < x < \frac{\pi}{2})$. [2]

Ans: $x = \pm\frac{\pi}{4}$

**4.** **2072 Set D Q.No. 2a** Solve $\tan ax = \cot bx$. [2]

Ans: $\frac{2n+1}{a+b}\cdot\frac{\pi}{2}$

**5.** **2072 Set E Q.No. 2a** Solve: $\sin 3x + \cos 3x = 0$ [2]

Ans: $(4n+1)\frac{\pi}{12}$

**6.** **2071 Supp. Q.No. 2a** Solve: $\mathrm{cosec}\,\theta = \cot\theta + \sqrt{3}$ [2]

Ans: $2n\pi \pm \frac{\pi}{3} + \frac{\pi}{3}$

**7.** **2071 Set C Q.No. 2a** Solve for $x$: $\tan 2x = \tan x$ [2]

Ans: $2\pi$

**8.** **2071 Set D Q.No. 2a** Solve: $\sqrt{3}\sin x - \cos x = \sqrt{3}$ for $0 \le x \le 2\pi$. [2]

Ans: $\frac{\pi}{2}, \frac{5\pi}{6}$

**9.** **2070 Supp Q.No. 2a** Solve: $\tan 2x - \cot x = 0$ [2]

Ans: $(2n+1)\frac{\pi}{6}$

**10.** **2069 Supp Q.No. 2 a** Solve: $\cos 4x = \cos 2x \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$. [2]

Ans: $0°$

**11.** **2069 [Set A] Q. No. 2a** Solve: $\sin x - \cos x = \sqrt{2}$. [2]

Ans: $x = n\pi + (-1)^n\frac{\pi}{2} + \frac{\pi}{4}$

**12.** **2069 [Set B] Q. No. 2a** Solve: $\tan 2x = \tan x (-\pi \le x \le \pi)$. [2]

Ans: $x = -\pi, \pi$

**13.** **2068 Q.No. 2a** Solve: $\cot x + \tan x = 2$ $(0 \le x \le \pi)$. [2]

Ans: $x = \frac{\pi}{4}$

### 4 Marks Questions

**14.** **2076 Set B Q.No. 7a** Solve for general values of $\theta$: [4]
$\sin\theta + \sin 2\theta + \sin 3\theta = \cos\theta + \cos 2\theta + \cos 3\theta$.

Ans: $(6n \pm 2)\frac{\pi}{3}, (4n+1)\frac{\pi}{8}$

**15.** **2076 Set C Q.No. 7a** Solve for general values of $x$: $\tan^2 x = \sec x + 1$. [4]

Ans: $(2n+1)\pi$; $2n\pi \pm \frac{\pi}{3}$

**16.** **2075 GIE Q.No. 7a** Solve for general values of $x$: $\tan x - \cot x = \mathrm{cosec}\, x$. [4]

Ans: $(2n+1)\frac{\pi}{3}$ and $(2n+1)\pi$

**17.** **2075 Set B Q.No. 7a** Solve for general values of $x$: $\tan x + \tan 2x = \tan 3x$. [4]

Ans: $n\pi, \frac{n\pi}{2}, \frac{n\pi}{3}$

**18.** **2075 Set C Q.No. 7a OR** Solve: $2\sin^2 x + \sin x = 1$. [4]

Ans: $n\pi + (-1)^n\frac{\pi}{6}$, $n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

**19.** **2074 Supp Q.No. 7a** Solve: $\sin\theta - \sqrt{3}\cos\theta = 2$ [4]

Ans: $(4n+1)\frac{\pi}{2} + \frac{\pi}{3}$

20. **2074 Set A Q.No. 7a OR** Solve: sec x tan x + $\sqrt{2}$. [4]

**Ans:** $n\pi + (-1)^n \frac{\pi}{4}$

21. **2074 Set B Q.No. 7a** Solve for general values of x: $\cos^3 x - \cos x \sin x - \sin^3 x = 1$. [4]

**Ans:** $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{4}$

22. **2073 Set D Q.No. 7a** Solve: $\tan x + \tan 2x + \tan 3x = 0$. [4]

**Ans:** $x = \frac{n\pi}{3}$ and $x = n\pi + \tan^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$

23. **2072 Supp Q.No. 7a** Solve: $\sec x \tan x = \sqrt{2}$. [4]

**Ans:** $n\pi + (-1)^n \frac{\pi}{4}$

24. **2072 Set C Q.No. 7a** Solve: $\sec x . \tan x = \sqrt{2}$. [4]

**Ans:** $n\pi + (-1)^n \frac{\pi}{4}$

25. **2071 Old Q.No. 8a Or** Solve: $\sqrt{3} \sin x - \cos x = \sqrt{3}$. [4]

**Ans:** $x = n\pi + (-1)^n \frac{\pi}{3} + \frac{\pi}{6}$

26. **2070 Set C Q.No. 7 a** Solve: $\sin x + \cos x = \sqrt{2}$ $(-2\pi \le x \le 2\pi)$.[4]

**Ans:** $\frac{-7\pi}{4}, \frac{\pi}{4}$

27. **2070 Set D Q.No. 7 a** Solve: $\sin^2 \theta - 2\cos\theta + \frac{1}{4} = 0$. [4]

**Ans:** $2n\pi \pm \frac{\pi}{3}$, $n \in Z$

28. **2070 Old Q.No. 8 a Or** Solve for general values of θ, $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$. [4]

**Ans:** $2n\pi \pm \frac{\pi}{3}$, $n \in Z$

29. **2069 [Set A] Old Q. No. 8a Or** Solve for general values of x: $\cos x + \cos 3x + \cos 5x + \cos 7x = 0$. [4]

**Ans:** $x = (4n \pm 1)\frac{\pi}{2}, (4n \pm 1)\frac{\pi}{4}, (4n \pm 1)\frac{\pi}{8}$

30. **2069 [Set B] Old Q. No. 8a OR** Solve: $2 \tan 3x + \cos 2x + 1 = \tan 3x + 2\cos 2x$ [4]

**Ans:** $x = n\pi \pm \frac{\pi}{6}, (4n+1)\frac{\pi}{12}$

31. **2068 Old Q.No. 8a(Or)** Solve: $\cos\theta + \sqrt{3}\sin\theta = 2 (-2\pi \le \theta \le 2\pi)$ [4]

**Ans:** $-\frac{5\pi}{3}, \frac{\pi}{3}$

32. **2067 Q.No. 8a OR** Solve for general values of x: $2\sin^2 x + \sin^2 2x = 2$ [4]

**Ans:** $2n\pi \pm \frac{\pi}{2}, n\pi \pm \frac{\pi}{4}$

33. **2066 Q.No. 8(a) OR** Solve for general values of θ: $\tan(\theta + \alpha) . \tan(\theta - \alpha) = 1$ [4]

**Ans:** $(4n \pm 1)\frac{\pi}{4}$

34. **2065 Q. No. 8 a OR** Solve: $\sin\theta + \sin 2\theta + \sin 3\theta = \cos\theta + \cos 2\theta + \cos 3\theta$. [4]

**Ans:** $(6n \pm 2)\frac{\pi}{3}, (4n+1)\frac{\pi}{8}$

35. **2064 Q.No. 8(a)** Find the general values of x when $\cos x + \sin x = \cos 2x + \sin 2x$ [4]

**Ans:** $2n\pi, (4n+1)\frac{\pi}{6}$

36. **2064 Q.No. 8(a) OR** If $\sin 2x = 3 \sin 2y$, prove that: $2\tan(x - y) = \tan(x + y)$. [4]

37. **2063 Q.No. 8(a)** Find the general values of x, when $\sin 2x \tan x + 1 = \sin 2x + \tan x$ [4]

**Ans:** $n\pi + \frac{\pi}{4}$

38. **2062 Q.No. 8a** Solve: $\cos 3x + \cos 2x = \sin\frac{3}{2}x + \sin\frac{x}{2}$ $(0 \le x \le \pi)$.[4]

**Ans:** $\frac{\pi}{7}, \frac{5\pi}{7}, \pi$

39. **2062 Q.No. 8(a) OR** Solve: $\tan^2 x = \sec x + 1$ [4]

**Ans:** $(2n+1)\pi$

40. **2061 Q.No. 8(a) OR** Solve for general values of x: $\cos x + \cos 2x + \cos 3x = 0$. [4]

**Ans:** $(2n+1)\frac{\pi}{4}, (6n \pm 2)\frac{\pi}{3}$

41. **2060 Q.No. 8(a) OR** Solve for general values of x, $7\sin^2 x + 3\cos^2 x = 4$. [4]

**Ans:** $n\pi \pm \frac{\pi}{6}$

42. **2059 Q.No. 8(a) OR** Solve: $\cot x + \tan x = 2$ [4]

**Ans:** $n\pi + \frac{\pi}{4}$

43. **2058 Q.No. 8(a) OR** Solve: $\tan^2 x = \sec x + 1$. [4]

**Ans:** $(2n+1)\pi, 2n\pi + \frac{\pi}{3}$

44. **2057 Q.No. 8(a)** Solve: $2\sin 3x - 2\sin x + 5\cos 2x = 0$ [4]

**Ans:** $(2n+1)\frac{\pi}{4}$

45. **2056 Q.No. 8(a) OR** Solve: $\sin x + \sqrt{3}\cos x = \sqrt{2}$ [4]

**Ans:** $2n\pi + \frac{\pi}{4} + \frac{\pi}{6}$

## C. Properties of Triangle

### FORMULAE

1. The Cosine Law

   In any ΔABC,

   $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ i.e., $a^2 = b^2 + c^2 - 2bc \cos A$

   $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$ i.e., $b^2 = c^2 + a^2 - 2ca \cos B$

   $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ i.e., $c^2 = a^2 + b^2 - 2ab \cos C$

2. The Sine Law

   In any ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is the radius of the circum-circle.

3. Projection Laws

   In any ΔABC,

   $a = c\cos B + b\cos C$

   $b = c\cos A + a\cos C$

   $c = a\cos B + b\cos A$

4. The Half Angle Formula

   In any ΔABC

   a. $\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$

   b. $\sin\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{ca}}$

c. $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{ab}}$

d. $\cos\left(\frac{A}{2}\right) = \sqrt{\frac{s(s-a)}{bc}}$

e. $\cos\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{ca}}$

f. $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{ab}}$

g. $\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

h. $\tan\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

i. $\tan\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

5. The Area of a Triangle
In any $\Delta ABC$,

a. $\Delta = \frac{abc}{4R}$

b. $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

c. $\Delta = \frac{1}{4}\sqrt{2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4}$

d. $\Delta = 2R^2 \sin A \sin B \sin C$

e. $\tan A = \frac{abc}{R} \cdot \frac{1}{b^2+c^2-a^2} = \frac{4\Delta}{b^2+c^2-a^2}$

6. The Tangent Law
In any triangle ABC, the tangent law states that

a. $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\left(\frac{A}{2}\right)$

b. $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\left(\frac{B}{2}\right)$

c. $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\left(\frac{C}{2}\right)$

## 2 Marks Questions

1. **2071 Old Q.No. 1c** Prove that: $1 - \tan\frac{A}{2}\tan\frac{B}{2} = \frac{2c}{a+b+c}$. [2]

2. **2069 [Set B] Old Q. No. 1c** In any triangle ABC, prove that: $a^2+b^2+c^2 - 2(bc\cos A + ca\cos B + ab\cos C) = 0$. [2]

3. **2067 Q.No. 1c** In any triangle if $\cos B = \frac{\sin A}{2\sin C}$, prove that the triangle is isosceles. [2]

4. **2064 Q.No. 1(c)** In a $\Delta ABC$, prove that $\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$ [2]

5. **2063 Q.No. 1(c)** In a $\Delta ABC$, prove that: $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$ [2]

6. **2061 Q.No. 1(c)** In any $\Delta ABC$, if $\cos B = \frac{\sin A}{2\sin C}$, show that the triangle is isosceles. [2]

7. **2060 Q.No. 1(b)** In any $\Delta ABC$, prove that $\sin A + \sin B + \sin C = \frac{s}{R}$. [2]

8. **2057 Q.No. 1(c)** Prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ [2]

## 4 Marks Questions

9. **2076 Set C Q.No. 7a OR** If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ are in any triangle ABC, prove that $\angle C = 60°$. What will be the relation between a, b, c if $\angle C = 90°$? [4]
Ans: $c^2 = a^2 + b^2$

10. **2075 Set A Q.No. 7a OR** Prove that: $\cos A + \cos B = \frac{2(a+b)}{c}\sin^2\frac{C}{2}$. [4]

11. **2075 Set B Q.No. 7a OR** If $2\cos A = \sin B : \sin C$, show that the triangle is isosceles. [4]

12. **2075 Set C Q.No. 7a** State cosine law. Using cosine law, prove that: $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$. [4]

13. **2074 Supp Q.No. 7a OR** Prove that: $(b+c-a)(\cot\frac{1}{2}B + \cot\frac{1}{2}C) = 2a\cot\frac{1}{2}B$. [4]

14. **2074 Set A Q.No. 7a** In any triangle ABC, prove that: $\tan\frac{1}{2}(B-C) = \frac{b-c}{b+c}\cdot\cot\frac{A}{2}$. [4]

15. **2074 Set B Q.No. 7a OR** State and prove cosine law of trigonometry. [4]

16. **2073 Supp Q.No. 7a OR** Prove, in any triangle that $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$. [4]

17. **2073 Set C Q.No. 7a** State and prove sine law in any triangle. [4]

18. **2073 Set D Q.No. 7a OR** In any triangle ABC, If $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, prove that the triangle is either isosceles or right angled. [4]

19. **2072 Supp Q.No. 7a OR** If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, prove that $C = 45°$ or $135°$ [4]

20. **2072 Set C Q.No. 7a OR** In any triangle ABC, prove that $\tan\frac{1}{2}(B-C) = \frac{b-c}{b+c}\cot\frac{A}{2}$ [4]

21. **2072 Set D Q.No. 7a OR** If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60°$. [4]

22. **2072 Set E Q.No. 7a OR** If $b - a = mc$, prove that $\cot\frac{B-A}{2} = \frac{1+m\cos B}{m\sin B}$ [4]

23. **2071 Supp. Q.No. 7a OR** If $b - a = mc$, prove that: $\cot\frac{B-A}{2} = \frac{1+m\cos B}{m\sin B}$ [4]

24. **2071 Set C Q.No. 7a Or** State sine law. Use this law to prove the projection law. [4]

25. **2071 Set D Q.No. 7a OR** In any triangle ABC, if $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, then prove that the triangle is either isosceles of right angled. [4]

26. **2070 Supp Q.No. 7a Or** State cosine law. Using cosine law, prove that: $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ [4]

27. 2070 Set C Q.No. 7 a Or State sine law. Using sine law, prove that: $\tan\frac{1}{2}(C-A)=\frac{c-a}{c+a}\cdot\cot\frac{B}{2}$ [4]

28. 2070 Set D Q.No. 7 a Or If $a^4+b^4+c^4=2c^2(a^2+b^2)$, prove that: $C=45°$ or $135°$. [4]

29. 2070 Old Q.No. 8 b In any triangle ABC, prove that: $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$. [4]

30. 2069 Supp Q.No. 7 a OR In any triangle ABC, prove that: $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$. [4]

31. 2069 [Set A] Q. No. 7a OR State sine law. Prove that: $\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\cot\frac{A}{2}$. [4]

32. 2069 [Set A] Old Q. No. 8b In any triangle ABC, prove that: $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$. [4]

33. 2068 Q.No. 7a Or State cosine law. Using cosine law, prove that: $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$ [4]

34. 2068 Old Q.No. 8b If $a^4+b^4+c^4=2c^2(a^2+b^2)$, prove that: $C=45°$ or $135°$ [4]

35. 2067 Q.No. 8b OR In any triangle ABC if $8R^2=a^2+b^2+c^2$, prove that the triangle is right angled. [4]

36. 2066 Q.No. 8 (b) OR In a $\Delta ABC$, if $(\sin A+\sin B+\sin C)(\sin A+\sin B-\sin C)=3\sin A\sin B$ then prove $\angle C=60°$. [4]

37. 2065 Q. No. 8 b In any triangle, state and prove cosine law. [4]

38. 2064 Q.No. 8(b) In any triangle, prove that: $a\cos A+b\cos B+c\cos C=4R\sin A\sin B\sin C$ [4]

39. 2062 Q.No. 8(b) In any $\Delta ABC$, prove that, $\frac{\cos B-\cos C}{\cos A+1}=\frac{c-b}{a}$ [4]

40. 2061 Q.No. 8(b) In any $\Delta ABC$, prove that $\frac{a^2\sin(B-C)}{\sin B+\sin C}+\frac{b^2\sin(C-A)}{\sin C+\sin A}+\frac{c^2\sin(A-B)}{\sin A+\sin B}=0$ [4]

41. 2060 Q.No. 8(b) Prove that in any triangle ABC: $\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2C=0$ [4]

42. 2059 Q.No. 8(b) In any triangle ABC prove: $\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\cot\frac{A}{2}$ [4]

43. 2058 Q.No. 8b In any $\Delta ABC$ prove that, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ [4]

44. 2056 Q.No. 8(b) If $a^4+b^4+c^4=2c^2(a^2+b^2)$ prove that $\angle C=45°$ or $135°$ [4]

## D. Solution of Triangle

### FORMULAE

1. $a:b:c=\sin A:\sin B:\sin C$

**2 Marks Questions**

1. 2070 Old Q.No. 1 b In any triangle ABC, if $a=3$, $b=3\sqrt{3}$, $A=30°$, find B. [2]
**Ans: B = 60° or 120°**

**4 Marks Questions**

2. 2076 Set B Q.No. 7a OR If $a=2$, $b=\sqrt{6}$ and $c=\sqrt{3}-1$, find the angles of the triangle ABC. [4]
**Ans: A = 45°, B = 120°, C = 15°**

3. 2075 GIE Q.No. 7a OR Given $a=2$, $b=\sqrt{6}$, $c=\sqrt{3}-1$, solve the triangle ABC. [4]
**Ans: A = 45°, B = 120°, C = 15°**

4. 2071 Old Q.No. 8b Or In a triangle ABC, $C=30°$; $b=\sqrt{3}$ and $a=1$. Find the other angles and the sides. [4]
**Ans: c = 1, A = 30°, B = 120°**

5. 2069 [Set A] Old Q. No. 8b OR In any triangle ABC, $a=3$, $b=3\sqrt{3}$ and $A=30°$. Solve the triangle. [4]
**Ans: B = 60°, C = 90°; c = 3 and B = 120°, C = 30°, c = 3**

6. 2069 [Set B] Q. No. 7a OR If $a=2$, $b=1+\sqrt{3}$, $C=60°$, solve the triangle ABC. [4]
**Ans: c = $\sqrt{6}$, A = 45°; B = 75°**

7. 2069 [Set B] Old Q. No. 8b OR Two sides of a triangle are $\sqrt{3}+1$ and $\sqrt{3}-1$ and the included angle is 60°, solve the triangle. [4]
**Ans: $\sqrt{6}$, 15°, 105°**

8. 2068 Old Q.No. 8b(Or) If $C=30°$, $B=45°$ and $c=6\sqrt{2}$, solve the triangle. [4]
**Ans: 105°, 12, $6(\sqrt{3}+1)$**

9. 2067 Q.No. 8b In any triangle ABC, $b=\sqrt{3}$, $c=1$ and $A=30°$, solve the triangle. [4]
**Ans: a = 1, B = 120°; C = 30°**

10. 2066 Q.No. 8 (b) If $A=30°$, $B=45°$, $a=6\sqrt{2}$, solve the triangle ABC. [4]
**Ans: C = 105°; b = 12; c = $6(\sqrt{3}+1)$**

11. 2065 Q. No. 8 OR In any triangle ABC, $b=\sqrt{3}$, $c=1$ and $\angle A=30°$, solve the triangle. [4]
**Ans: a = 1, B = 120°, C = 30°**

12. 2064 Q.No. 8(b) OR If the angles of a triangle are to one another as 1 : 2 : 3, prove that the corresponding sides are $1:\sqrt{3}:2$. [4]

13. 2062 Q.No. 8(b) OR In any $\Delta ABC$, $a=2$, $b=\sqrt{6}$ and $c=\sqrt{3}-1$, find $\angle B$. [4]
**Ans: 120°**

14. 2061 Q.No. 8(b) OR In any $\Delta ABC$, if $A=30°$ and $B=90°$, find a: b: c. [4]
**Ans: $1:2:\sqrt{3}$**

15. 2060 Q.No. 8(b) OR In any $\Delta ABC$, $b=\sqrt{3}$, $c=1$ and $A=30°$ solve the triangle. [4]
**Ans: a = 1, C = 30°, B = 120°**

16. 2059 Q.No. 8(b) OR Solve the triangle, if $a=2$, $b=\sqrt{6}$, $c=\sqrt{3}+1$. [4]
**Ans: 45°, 60° & 75°**

17. 2058 Q.No. 8(b) OR If $a=2$, $b=\sqrt{2}$, $c=\sqrt{3}+1$, solve the triangle. [4]
**Ans: 45°, 30°, 105°**

18. 2057 Q.No. 8(b) OR Solve the triangle if $a=\sqrt{6}$, $b=2$ and $c=\sqrt{3}-1$. [4]
**Ans: 120°, 45°, 15°**

19. 2056 Q.No. 8(b) OR If three sides of a triangle are in the ratio $2:\sqrt{6}:\sqrt{3}+1$, find the angles. [4]
**Ans: 45°, 60°, 75°**

# Unit 5: Sequence and Series and Mathematical Induction

## A. Sequence and Series

### FORMULAE AND IMPORTANT POINTS

1. Sequence
   A sequence of numbers is a set of numbers arranged in a definite order.
   An infinite sequence is a function $f: \mathbb{N} \to \mathbb{R}$ defined by $f(n) = a_n$, $n \in \mathbb{N}$ (the set of natural numbers)
2. Arithmetic progression:
   i. $t_n = a + (n - 1) d$
   ii. $S_n = \frac{n}{2}(a + l)$
   iii. $S_n = \frac{n}{2}[2a + (n - 1) d]$
   iv. A.M. between a and b $= \frac{a + b}{2}$
   v. To insert n A.M's between a and b, $d = \frac{b - a}{n + 1}$
3. Geometric Progression
   i. $t_n = ar^{n-1}$
   ii. $S_n = \frac{a(r^n - 1)}{r - 1}$ if $r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r}$ if $r < 1$
   iii. $S_n = \frac{lr - a}{r - 1}$, $r \neq 1$
   iv. $S_\infty = \frac{a}{1 - r}$, $|r| < 1$
   v. If a and b are two positive numbers, $G.M = \sqrt{ab}$
   vi. To insert n G.M's between a and b, $(r) = \left(\frac{b}{a}\right)^{1/n+1}$
4. Harmonic Progression
   i. $t_n = \frac{1}{a + (n - 1) d}$
   ii. If a and b are two positive numbers, $H = \frac{2ab}{a + b}$
5. Relations between A.M., G.M. and H.M.
   a. $A \times H = G^2$
   b. $A > G > H$
6. i. $1 + 2 + 3 + \cdots + n = \sum n = \frac{n(n + 1)}{2}$
   ii. Sum of first n even natural numbers = n(n + 1).
   iii. Sum of first n odd natural numbers = $n^2$.
   iv. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \sum n^2 = \frac{n(n + 1)(2n + 1)}{6}$
   v. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum n^3 = \left\{\frac{n(n + 1)}{2}\right\}^2$
7. Sum of n Terms of an Arithmetico-Geometric Series
   $S_n = \frac{a}{1 - r} + d\,r\frac{(1 - r^{n-1})}{(1 - r)^2} - \frac{\{a + (n - 1) d\} r^n}{1 - r}$
   Sum to infinity:
   $S_\infty = \frac{a}{1 - r} + \frac{d\,r}{(1 - r)^2}$, $|r| < 1$

### 2 Marks Questions

1. **2071 Supp. Q.No. 2b** If a, b, c are in A.P. and x, y, z are in G.P, show that: $x^{b-c} y^{c-a} z^{a-b} = 1$ [2]
2. **2068 Q.No. 1a** If A.M., G.M. and H.M. be the arithmetic mean, geometric mean and harmonic mean between two unequal positive numbers, prove that:
   A.M. × H.M. = $(G.M.)^2$ [2]
3. **2067 Q.No. 1a** Prove that $a^2, b^2, c^2$ are in A.P. if b + c, c + a, a + b are in H. P. [2]
4. **2066 C Q.No. 1 a** If G be the geometric mean between two distinct positive numbers a and b, show that $\frac{1}{G - a} + \frac{1}{G - b} = \frac{1}{G}$ [2]
5. **2066 Q.No. 1 a** If a, b, c are three positive numbers in harmonic sequence show that: $a^2 + c^2 > 2b^2$. [2]
6. **2065 Q.No. 1(a)** If a, b, c are in G.P., prove that $\frac{1}{a + b}, \frac{1}{2b}, \frac{1}{b + c}$ are in A.P. [2]
7. **2064 Q.No. 1(a)** If A.M. and G.M. be the arithmetic mean and geometric mean between two unequal positive numbers, prove that A.M. > G.M. [2]
8. **2063 Q.No. 1(a)** If a, b, c be in A.P.; b, c, d in G.P. and c, d, e in H.P, prove that a, c, e are in G.P. [2]
9. **2062 Q.No. 1(a)** If H be the harmonic mean between a and b, prove that: $\frac{1}{H - a} + \frac{1}{H - b} = \frac{1}{a} + \frac{1}{b}$ [2]
10. **2061 Q.No. 1(a)** If A be the A.M. and H the harmonic mean between two quantities a and b, show that: $\frac{a - A}{a - H} \times \frac{b - A}{b - H} = \frac{A}{H}$ [2]
11. **2060 Q.No. 1(a)** If $a^x = b^y = c^z$ and a, b, c are in G.P. then x, y, z are in H.P. [2]
12. **2059 Q.No. 1(a)** If p is A.M. between q and r, q is G.M. between r and p then prove that r will be H.M. between p and q. [2]
13. **2058 Q.No. 1a** Find the sum: 1.3 + 2.4 + 3.5 + ... to n terms. [2]
    **Ans:** $\frac{n(n + 1)(2n + 7)}{6}$
14. **2057 Q.No. 1(a)** Sum to infinity:
    $1 + 3x + 5x^2 + 7x^3 + \ldots (-1<x<1)$. [2]
    **Ans:** $\frac{1 + x}{(1 - x)^2}$

### 6 Marks Questions

15. **2076 Set B Q.No. 12** If A,G and H are A.M., G.M. and H.M. respectively between any two unequal positive numbers then prove that [6]
    i. A > G > H and
    ii. $G^2 = A \times H$
16. **2076 Set C Q.No. 12** If p, q, r be in A. P., q, r, x be in G. P. and r, x, y in H.P., prove that p, r, y are in G.P. [6]
17. **2075 GIE Q.No. 12** If A, G, H are A.M., G.M. and H.M. between any two unequal positive numbers, prove that (i) A > G > H and (ii) $G^2 = A \times H$. [6]
18. **2075 Set A Q.No. 12** If a, b, c are in H.P., prove that $\frac{b + c - a}{a}, \frac{c + a - b}{b}, \frac{a + b - c}{c}$ are in A.P. [6]
19. **2075 Set B Q.No. 12** Sum to n terms the series:
    $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots, \ldots$ [6]
    **Ans:** $\frac{35}{16} - \frac{12n + 7}{16 \cdot 5^{n-1}}$

20. 2075 Set C Q.No. 12 Sum to n terms to the series. [6]
$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots$
**Ans:** $\frac{35}{16} - \frac{12n+7}{16 \cdot 5^{n-1}}$

21. 2074 Supp Q.No. 12 Find the $n^{th}$ term and the sum of the n terms of the series: [6]
$3 + 7 + 13 + 21 + 31 + \dots, \dots \dots$
**Ans:** $t_n = n^2 + n + 1;\ S_n = \frac{n}{3}(n^2 + 3n + 5)$

22. 2074 Set A Q.No. 12 The sum of three numbers in A. P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G.P. Find the numbers. [6]
**Ans:** 3, 12, 21, or 63, 12, –39

23. 2074 Set B Q.No. 12 Sum to n-terms of the series:
$\frac{2}{5} + \frac{6}{5^2} + \frac{10}{5^3} + \frac{14}{5^4} + \dots, \dots, \dots$ [6]
**Ans:** $\frac{3}{4} + \frac{4n-7}{4.5^n}$

24. 2073 Supp Q.No. 12 Find the sum of n terms of the series:
$1^2 \cdot 1 + 2^2 \cdot 3 + 3^2 \cdot 5 + \dots$ [6]
**Ans:** $\frac{1}{6} n(n+1)(3n^2 + n - 1)$

25. 2073 Set C Q.No. 12 Find the general term and then find the sum of first n terms of the series $n + 2(n-1) + 3(n-2) + \dots$ [6]
**Ans:** $r(n - r + 1),\ \frac{n(n+1)(n+2)}{6}$

26. 2073 Set D Q.No. 12 Sum to n terms of the series:
$3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 \dots$ [6]
**Ans:** $2 + 2^{n+1} + (n-1)2^{n+2}$

27. 2072 Supp Q.No. 12 The sum of three numbers in A.P. is 36. When the number are increased by 1,4,43 respectively, the resulting numbers are in G.P. Find the numbers. [6]
**Ans:** 3, 12, 21 or 63, 12, –39

28. 2072 Set C Q.No. 12 The sum of three numbers in A.P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G.P. Find the numbers. [6]
**Ans:** 3, 12, 21 or 63, 12, –39

29. 2072 Set D Q.No. 13 Prove that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations: [6]
(a) $(G.M.)^2 = A.M. \times H.M.$ (b) A.M. > G.M. > H.M.
If A.M. between two numbers = 25 and H.M. = 16, find G.M.
**Ans:** 20

30. 2072 Set E Q.No. 12 Sum to n terms of the series
$1 \cdot 1^2 + 4 \cdot 2^2 + 7 \cdot 3^2 + \dots$ [6]
**Ans:** $\frac{n(n+1)(9n^2 + n - 4)}{12}$

31. 2071 Set C Q.No. 12 Sum to n terms of the series:
$\frac{2}{5} - \frac{6}{5^2} + \frac{10}{5^3} - \frac{14}{5^4} + \dots$ [6]
**Ans:** $\frac{2}{9} + \frac{(-1)^{n-1}}{9.5^{n-1}} + \frac{(-1)^{n+1}(4n-2)}{6.5^n}$

32. 2071 Set D Q.No. 12 Sum to n terms of the series:
$1.n + 2.(n-1) + 3.(n-2) + \dots$ [6]
**Ans:** $r(n - r + 1);\ \frac{n(n+1)(n+2)}{6}$

33. 2070 Supp Q.No. 12 Find the general term and also the sum of n terms of the series: $1.n + 2.(n-1) + 3.(n-2) + \dots$ [6]
**Ans:** $r(n - r + 1);\ \frac{n(n+1)(n+2)}{6}$

34. 2070 Set C Q.No. 12 Show that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the following relations. [6]
a. $(G.M.)^2 = A.M. \times H.M.$ b. A.M. > G.M. > H.M.

35. 2070 Set D Q.No. 12 Sum to infinity the following series:
$1 - 5a + 9a^2 - 13a^3 + \dots$ to $\infty$ $(-1 < a < 1)$. [6]
**Ans:** $\frac{1-3a}{(1+a)^2}$

36. 2069 Supp Q.No. 12 Define geometric mean. Find the sum of the first n terms of the series. $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \dots$ [6]

37. 2069 [Set A] Q. No. 12 Find the sum of n terms of the series
$3.1^2 + 4.2^2 + 5.3^2 + \dots$ . [6]
**Ans:** $\frac{n(n+1)(3n^2 + 11n + 4)}{12}$

38. 2069 [Set B] Q. No. 12 Find the $n^{th}$ term and then the sum of the first n-terms of the series: $1^2.2 + 2^2.3 + 3^2.4 + \dots$ [6]
**Ans:** $t_n = n^2(n+1);\ S_n = \frac{n(n+1)(3n^2 + 7n + 2)}{12}$

39. 2068 Q.No. 12 Prove that:
$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$. [6]

## 4 Marks Questions (Old Syllabus Questions)

40. 2067 Q.No. 7a The sides of a square are each 16 cm. A second square is drawn by joining the mid points of the sides, successively. In the second square the process is repeated to drawing the third square. If this process is continued indefinitely, find the sum of the areas of all the squares. [4]
**Ans:** 512 $cm^2$

41. 2066 C Q.No. 7 a Sum of n terms of series: $1 + \frac{4}{5} + \frac{7}{5^2} + \dots$ [4]
**Ans:** $\frac{35}{16} - \frac{12n+7}{16.5^{n-1}}$

42. 2066 Q.No. 7 a Sum to n terms of the series:
$1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \dots$ [4]
**Ans:** $4 - \frac{n}{2^{n-1}} - \frac{1}{2^{n-2}}$

43. 2065 Q.No. 7(a) Sum to infinity: $\frac{a}{x} + \frac{b}{x^2} + \frac{a}{x^3} + \frac{b}{x^4} + \dots$, when $|x| > 1$. [4]
**Ans:** $\frac{ax+b}{x^2-1}$

44. 2064 Q.No. 7a Sum to n terms of the series: $1^2.1 + 2^2.3 + 3^2.5 + \dots$ [4]
**Ans:** $\frac{n(n+1)(3n^2 + n - 1)}{6}$

45. 2063 Q.No. 7(a) Prove that
$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ [4]

46. 2062 Q.No. 7(a) If A.M., G.M. and H.M. are the arithmetic mean, geometric mean and harmonic mean between two unequal positive numbers a and b, prove that: A.M. > G.M. > H.M. [4]

47. 2061 Q.No. 7(a) Sum to n term the following series:
$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots$ [4]
**Ans:** $\frac{35}{16} - \frac{12n+7}{16 \times 5^{n-1}}$

48. 2060 Q.No. 7(a) Prove that the A.M., G.M. and H.M. between any two unequal positive numbers satisfy the relation A.M. > G.M. > H.M. [4]

49. 2059 Q.No. 7(a) Prove that:
$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$. [4]

50. 2058 Q.No. 7(a) If a, b, c are in H.P., prove that: 2a – b, b, 2c – b are in G.P. [4]

51. 2057 Q.No. 7(a) If a, b, c are in H.P. prove that: a(b + c), b(c + a), c(a + b) are in A.P. [4]

## B. Mathematical Induction

### FORMULAE AND IMPORTANT POINTS

1. Principle of Mathematical induction
A statement $P(n)$ is true for all $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, provided
a. P(1) is true and
b. $P(k+1)$ is true whenever $P(k)$, $k \in \mathbb{N}$ is true.

**2 Marks Questions**

1. 2076 Set B Q.No. 2b Prove by mathematical induction:
$1 + 3 + 5 + \ldots + (2n-1) = n^2$. [2]

2. 2076 Set C Q.No. 2b For the statement
$P_n: 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$, write $P_1$, $P_k$ and $P_{k+1}$. [2]
**Ans:** $P_1: 1 = \frac{1(1+1)}{2}$; $P_k: 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$,
$P_{k+1}: 1 + 2 + 3 + \ldots + k + (k+1) = \frac{(k+1)\{(k+1)+1\}}{2}$

3. 2075 GIE Q.No. 2b Using the principle of mathematical induction, prove that $1 + 3 + 5 + \ldots + (2n-1) = n^2$. [2]

4. 2075 Set A Q.No. 2b Use mathematical induction to prove that for any positive integer n, $(n+3)^2 > 2n + 7$. [2]

5. 2075 Set B Q.No. 2b Use mathematical induction to prove that the sum of first n natural numbers is $\frac{n(n+1)}{2}$. [2]

6. 2075 Set C Q.No. 2b Using the principle of mathematical induction, prove that for any positive integer, $4^n - 1$ is divisible by 3. [2]

7. 2074 Supp Q.No. 2b Using the principle of mathematical induction, prove that $1 + 3 + 5 + 7 + \ldots + (2n-1) = n^2$. [2]

8. 2074 Set A Q.No. 2b Using the principle of mathematical induction, prove that: $2 + 2^2 + 2^3 + \ldots + 2^n = 2(2^n - 1)$. [2]

9. 2074 Set B Q.No. 2b Prove by mathematical induction for all $n \in N$, $2 + 4 + 6 + \ldots + 2n = n(n+1)$. [2]

10. 2073 Supp Q.No. 2b Using the principle of mathematical induction, prove that $3^{2n} - 1$ is divisible by 8. [2]

11. 2073 Set C Q.No. 2b Using the principle of Mathematical induction prove that $1 + 3 + 5 + \ldots$ to n terms$=n^2$. [2]

12. 2073 Set D Q.No. 2b Using the principle of mathematical induction, prove that: $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$. [2]

13. 2072 Supp Q.No. 2b Prove by the principle of mathematical induction that: $1+3+5+\ldots+(2n-1) = n^2$ [2]

14. 2072 Set C Q.No. 2b Prove by the principle of mathematical induction that: $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$. [2]

15. 2072 Set D Q.No. 2b Prove by the principle of mathematical induction that $2 + 5 + 8 + \ldots + (3n-1) = \frac{n(3n+1)}{2}$. [2]

16. 2072 Set E Q.No. 2b Use mathematical induction to prove that for any positive integer n, $9^n - 1$ is divisible by 4. [2]

17. 2071 Set C Q.No. 2b Prove by mathematical induction: $1 + 3 + 5 + \ldots + n$ terms $= n^2$ [2]

18. 2071 Set D Q.No. 2b Prove by the principle of mathematical induction that: $4^n - 1$ is divisible by 3. [2]

19. 2070 Supp Q.No. 2b Using the principle of mathematical induction, prove that: $2+4+6+8+\ldots+2n=n(n+1)$ [2]

20. 2070 Set C Q.No. 2 b Using principle of mathematical induction, Prove that: $1+3+5+7+\ldots+(2n-1)=n^2$. [2]

21. 2070 Set D Q.No. 2 b Using principle of mathematical induction, prove that: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \frac{1}{2^n} = 1 - \frac{1}{2^n}$. [2]

22. 2069 Supp Q.No. 2 b Prove by the principle of mathematical induction: $1^3 + 2^3 + 3^3 + \ldots n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$ [2]

23. 2069 [Set A] Q. No. 2b Use the principle of mathematical induction: $2 + 4 + 6 + \ldots + 2n = n(n+1)$. [2]

24. 2069 [Set B] Q. No. 2b Prove by the principle of mathematical induction: $2 + 4 + 6 \ldots + 2n = n(n+1)$. [2]

25. 2068 Q.No. 2b Using the principle of mathematical induction, Prove that: $1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$. [2]

**6 Marks Questions**

26. 2071 Supp. Q.No. 12 State the principle of mathematical induction. Using it prove that: [6]
$1.3 + 2.4 + 3.5 + \ldots + n(n+2) = \frac{1}{6} n(n+1)(2n+7)$.

# Unit 6: Matrices and Determinants
## A. Matrices

### FORMULAE AND IMPORTANT POINTS

1. Matrix
A rectangular arrangement of numbers arranged in rows (horizontal lines) and columns (vertical lines) and enclosed by round or square bracket is called a matrix.
2. Multiplication of two matrices
Two matrices A and B can be multiplied only when the number of columns of A and number of rows of B are the same.
3. i. For any square matrix A, $A + A^T$ is symmetric matrix and $A - A^T$ is skew-symmetric matrix.
ii. Every square matrix A can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix as
$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.
4. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

If $C_{11}, C_{12}, C_{13}, \cdots$, be respectively the co-factors of $a_{11}, a_{12}, a_{13}, \cdots$, then the matrix $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$ is called the matrix of co-factors.

5. Singular or Non-singular Matrix: A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$ respectively.

6. $A^{-1} = \frac{1}{|A|}$ Adj. A; $|A| \neq 0$

## 2 Marks Questions

1. **2076 Set B Q.No. 2c** Find the adjoint of the matrix $\begin{pmatrix} 2 & 5 \\ 3 & -7 \end{pmatrix}$. [2]

   **Ans:** $\begin{pmatrix} -7 & -5 \\ -3 & 2 \end{pmatrix}$

2. **2076 Set C Q.No. 2c** If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, show that $(A^T)^T = A$. [2]

3. **2075 GIE Q.No. 2c** $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$, verify that $(AB)^T = B^T.A^T$. [2]

4. **2075 Set A Q.No. 2c** Define skew-symmetric matrix. For any square matrix A, show that A – A' is skew-symmetric. [2]

5. **2075 Set B Q.No. 2c** Find the adjoint of the matrix $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$. [2]

   **Ans:** $\begin{pmatrix} 2 & -3 \\ -4 & 5 \end{pmatrix}$

6. **2075 Set C Q.No. 2c** If $A = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$ be a symmetric matrix, find the value of x. [2]

   **Ans:** $x = 4$

7. **2074 Supp Q.No. 2c** If $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$, find $AA^T$. [2]

   **Ans:** $\begin{pmatrix} 58 & 36 \\ 36 & 40 \end{pmatrix}$

8. **2074 Set A Q.No. 2c** If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find $AA^T$. [2]

   **Ans:** $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

9. **2074 Set B Q.No. 2c** Find the adjoint of the matrix $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$ [2]

   **Ans:** $\begin{pmatrix} -1 & -1 \\ -4 & 2 \end{pmatrix}$

10. **2073 Supp Q.No. 2c** If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, find $B^TA^T$. [2]

    **Ans:** $\begin{pmatrix} 13 & 0 \\ 22 & -3 \end{pmatrix}$

11. **2073 Set C Q.No. 2c** If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, verify that $(A + B)^1 = A^1 + B^1$. [2]

12. **2073 Set D Q.No. 2c** If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, show that $A^T. A = I$. [2]

13. **2072 Supp Q.No. 2c** If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find $AA^T$. [2]

    **Ans:** $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

14. **2072 Set C Q.No. 2c** If $A = \begin{pmatrix} 3 & -5 \\ 4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$ find $(AB)^T$. [2]

    **Ans:** $\begin{pmatrix} -5 & 6 \\ 19 & 0 \end{pmatrix}$

15. **2072 Set D Q.No. 2c** If $A = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$, find $(A^T)^T$. [2]

    **Ans:** $\begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$

16. **2072 Set E Q.No. 2c** If $A = \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}$ and $f(x) = x^2 - 5x + 3$, find f(A). [2]

    **Ans:** $\begin{pmatrix} -13 & -6 \\ 8 & -15 \end{pmatrix}$

17. **2071 Supp. Q.No. 2c** Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, Find $(AB)^T$. [2]

    **Ans:** $\begin{pmatrix} 6 & 10 \\ 14 & 20 \end{pmatrix}$

18. **2071 Set C Q.No. 2c** Define singular matrix. Test whether the matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & -1 \\ -1 & 3 & -2 \end{pmatrix}$ is singular or not. [2]

    **Ans: Singular**

19. **2071 Set D Q.No. 2c** If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, show that $A^{-1} = \frac{1}{19}A$ [2]

20. **2070 Supp Q.No. 2c** Define symmetric matrix. If $A = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix}$ find $A + A^T$. [2]

    **Ans:** $\begin{pmatrix} 4 & 6 & 8 \\ 6 & 6 & 6 \\ 8 & 6 & 12 \end{pmatrix}$

21. **2070 Set C Q.No. 2 c** If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find $AA^T$ [2]

    **Ans:** $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

22. **2070 Set D Q.No. 2 c** If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, find $B^T A^T$. [2]

    **Ans:** $\begin{pmatrix} 13 & 0 \\ 22 & -3 \end{pmatrix}$

23. **2070 Old Q.No. 3 a** Given $A = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ show that $(A + B)^T = A^T + B^T$. [2]

24. **2069 Supp Q.No. 2c** Show that if $A = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$ then $(A')' = A$. [2]

25. 2069 Set A Q. No. 2c If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$ find $(AB)^T$. [2]

Ans: $\begin{bmatrix} 13 & 0 \\ 22 & -3 \end{bmatrix}$

26. 2069 [Set A] Old Q. No. 3a If $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ show that: $A^2 - 2A + I = 0$ where I is the unit matrix and 0 is the null matrix. [2]

27. 2069 [Set B] Q. No. 2c For the given matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$, show that $(A+B)^1 = A^1 + B^1$. [2]

28. 2069 [Set B] Old Q. No. 4b If $A=\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$, show that : $AB \neq BA$. [2]

29. 2068 Q.No. 2c Find the inverse of the matrix $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$. [2]

Ans: $\frac{1}{32}\begin{bmatrix} 2 & 3 \\ -6 & 7 \end{bmatrix}$

30. 2068 Old Q.No. 3a If $A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$, find the matrix AB – 3I where I is the unit matrix of order 2. [2]

Ans: $\begin{bmatrix} -4 & 7 \\ 5 & -2 \end{bmatrix}$

31. 2067 Q.No. 3a Construct (3 × 3) matrix with the elements given by $a_{ij} = 2i + j$ [2]

Ans: $\begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}$

32. 2066 Q.No. 3(a) If $A+B=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A-2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then determine the matrix A. [2]

Ans: $\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$

33. 2065 Q. No. 3 a Define symmetric and skew symmetric matrix with examples. [2]

34. 2064 Q.No. 4(b) If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, find $A^{-1}$. [2]

Ans: $\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$

35. 2063 Q.No. 3(a) Define a triangular matrix. How do you distinguish between upper and lower triangular matrices? [2]

36. 2062 Q.No. 3(a) If $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$, show that $(AB)^T = B^T A^T$. [2]

37. 2061 Q.No. 3(a) Construct a 3×3 matrix whose elements are $a_{ij} = 2i + j$. [2]

Ans: $\begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}$

38. 2060 Q.No. 3(a) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, show that $(AB)^T = B^T A^T$. [2]

39. 2059 Q.No. 3(a) Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$ find $(AB)^T$. [2]

Ans: $\begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$

40. 2058 Q.No. 3a Let $A = \begin{pmatrix} 2 & 1 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix}$, find $(AB)^T$. [2]

Ans: $\begin{pmatrix} 5 & 21 \\ 12 & 14 \end{pmatrix}$

41. 2057 Q.No. 3(a) Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, find the transpose of AB. [2]

Ans: $\begin{pmatrix} 6 & 10 \\ 14 & 20 \end{pmatrix}$

42. 2056 Q.No. 3(a) Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$ then prove $AB \neq BA$. [2]

**4 Marks Questions**

43. 2058 Q.No. 10(a) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, show that $A^2 - 2A - 5I = 0$ [4]

## B. Determinants

### FORMULAE AND IMPORTANT POINTS

1. If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a square matrix of order two, then the determinant of A is the number $a_{11} a_{22} - a_{21} a_{12}$.
2. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order three then the determinant of A is the number $a_{11}(a_{22}a_{33}-a_{32}a_{23})-a_{12}(a_{21}a_{33}-a_{31}a_{23})+a_{13}(a_{21}a_{32}-a_{31}a_{22})$.

**2 Marks Questions**

1. 2071 Old Q.No. 3a Without expanding the determinant prove that $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ [2]

**4 Marks Questions**

2. 2076 Set B Q.No. 7b Prove that: $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$. [4]

3. 2076 Set C Q.No. 7b Show that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ [4]

4. 2075 GIE Q.No. 7b Evaluate: $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$. [4]

Ans: $4a^2b^2c^2$

5. **2075 Set A Q.No. 7b** What is a determinant? Use properties of determinant prove that: [4]

$$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x-a)(x-b)(x+a+b).$$

6. **2075 Set B Q.No. 7b** Prove that:

$$\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix} = 1+x+y+z.$$ [4]

7. **2075 Set C Q.No. 7b** Prove that:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2.$$ [4]

8. **2074 Supp Q.No. 7b** Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & xz & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$$ [4]

9. **2074 Set A Q.No. 7b** Show that:

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-z)(z-x)(x-y).$$ [4]

10. **2074 Set B Q.No. 7b** Prove that:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}.$$ [4]

11. **2073 Supp Q.No. 7b** Prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a).$$ [4]

12. **2073 Set C Q.No. 7b** Without expanding show that

$$\begin{vmatrix} b & c & b+c \\ c & a & c+a \\ a & b & a+b \end{vmatrix} = 0.$$ [4]

13. **2073 Set D Q.No. 7b** Prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$ [4]

14. **2072 Supp Q.No. 7b** Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$ [4]

15. **2072 Set C Q.No. 7b** Show that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$$ [4]

16. **2072 Set D Q.No. 7b** Show that:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$ [4]

17. **2072 Set E Q.No. 7b** Prove that:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$ [4]

18. **2071 Supp. Q.No. 7b** Show that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right).$$ [4]

19. **2071 Set C Q.No. 7b** Without expanding the determinant show that:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} a^2 & bc & a \\ b^2 & ca & b \\ c^2 & ab & c \end{vmatrix}$$ [4]

20. **2071 Set D Q.No. 7b** Without expanding the determinant show that:

$$\begin{vmatrix} y+z & y & z \\ z+x & z & x \\ x+y & x & y \end{vmatrix} = 0$$ [4]

21. **2071 Old Q.No. 10a** Show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ [4]

22. **2070 Supp Q.No. 7b** Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ [4]

23. **2070 Set C Q.No. 7 b** Prove that:

$$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$ [4]

24. **2070 Set D Q.No. 7 b** Prove that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)$$ [4]

25. **2070 Old Q.No. 10 a** Using properties of determinant, show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ [4]

26. **2069 Supp Q.No. 7 b** Without expanding, show that:

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ [4]

27. **2069 [Set A] Q. No. 7b** Show that:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$ [4]

28. **2069 [Set A] Old Q. No. 10a** Show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$ [4]

29. **2069 [Set B] Q. No. 7b** Without expanding show that:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0.$$ [4]

30. **2069 [Set B] Old Q. No. 10a** Prove that:

$$\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = xyz\left(1+\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)$$ [4]

31. **2068 Q.No. 7b** Show that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx) \quad [4]$$

32. **2068 Old Q.No. 10a** Prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y - z)(z - x)(x - y)(xy + yz + zx) \quad [4]$$

33. **2067 Q.No. 10a** Prove that:

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2) \quad [4]$$

34. **2066 Q.No. 10 (a)** If a, b, c are non zero and

$$\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0, \text{ then show that } abc = 1. \quad [4]$$

35. **2065 Q. No. 10 a** Show that:

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2 \quad [4]$$

36. **2064 Q.No. 10(a)** Prove (without expanding):

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad [4]$$

37. **2063 Q.No. 10(a)** Prove (without expanding):

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad [4]$$

38. **2062 Q.No. 10(a)** Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 \quad [4]$$

39. **2061 Q.No. 10(a)** Without expanding show that

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad [4]$$

40. **2060 Q.No. 10(a)** Use properties of determinant to show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} \quad [4]$$

41. **2059 Q.No. 10(a)** Evaluate:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \quad [4]$$

**Ans:** (a – b) (b – c) (c – a) (a + b + c)

42. **2057 Q.No. 10(a)** Find the value of

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} \quad [4]$$

**Ans:** 0

43. **2056 Q.No. 10(a)** Evaluate:

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} \quad [4]$$

**Ans:** (xyz + yz + zx + xy)

# Unit 7: System of Linear Equations:

## A. Row Equivalent Matrix Method

### FORMULAE AND IMPORTANT POINTS

1. If the system has one solution, the system is said to be consistent and independent. If the system has no solution, the system is said to be inconsistent and independent. If the system has an infinite number of solutions, the system is said to be consistent and dependent.
2. Row Equivalent Matrix

   Let $a_1x + b_1y + c_1z = d_1$

   $a_2x + b_2y + c_2z = d_2$

   $a_3x + b_3y + c_3z = d_3$

   Augmented matrix.

   $$\begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix}$$

   of the system. By using elementary row operations, the augmented matrix is reduced to the matrix of the form

   $$\begin{bmatrix} 1 & 0 & 0 & : & p \\ 0 & 1 & 0 & : & q \\ 0 & 0 & 1 & : & r \end{bmatrix}$$

   Then x = p, y = q and z = r
3. We follow the following indicated path to reduce the augmented matrix.

   $$\begin{bmatrix} 1 & 0 & : & p \\ \downarrow & \uparrow & & \\ 0 & \rightarrow 1 & : & q \end{bmatrix}$$

   ∴ x = p and y = q

   $$\begin{bmatrix} 1 & 0 & \leftarrow 0 & : & p \\ \downarrow & & \uparrow & & \\ 0 & 1 & 0 & : & q \\ \downarrow\nearrow & \downarrow & \uparrow & & \\ 0 & 0 & \rightarrow 1 & : & r \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & : & p \\ \downarrow & \uparrow & \uparrow & & \\ 0 & 1 & 0 & : & q \\ \downarrow\nearrow & \downarrow & \searrow\uparrow & & \\ 0 & 0 & 1 & : & r \end{bmatrix} \text{ etc.}$$

   ∴ x = p, y = q and z = r.

**2 Marks Questions**

1. **2069 [Set B] Old Q. No. 3a** Solve by row equivalent method: $4x + 3y = 7$, $5x - 3y = 8$ [2]

   **Ans:** $x = \frac{5}{3}, y = \frac{1}{9}$
2. **2064 Q.No. 3(a)** Solve by row–equivalent method: $3x - 2y = 8$; $5x + 3y = 7$ [2]

   **Ans:** x = 2, y = –1

**4 Marks Questions**

3. **2075 Set A Q.No. 8a** What are elementary row operations? Use it to solve: [4]

   $x + y + z = 6$, $x - y + z = 2$, $x + y - z = 0$.

   **Ans:** x= 1, y = 2, z = 3
4. **2072 Set E Q.No. 8a** Solve the following system of equations by the row-equivalent matrix method: [4]

   $x + y + z = -2$, $2x + 8y + 5z = 5$, $x + 2y - z = 2$.

   **Ans:** x = –3, y = 2, z = –1
5. **2071 Old Q.No. 10b Or** Solve by row–equivalent matrix method: $x-y+2z=0$; $x-2y + 3z = -1$; $2x-2y+z = -3$ [4]

   **Ans:** x= 0, y = 1, z = 2
6. **2070 Old Q.No. 10 b** Solve by row equivalent matrix method: $9y - 5x = 3$, $x + z = 1$, $2y + z = 2$. [4]

   **Ans:** x = 3, y = 2, z = –2

7. 2065 Q. No. 10 b Solve by matrix method: $x + y + z = 6$; $x - y + z = 2$; $2x + y - z = 1$ [4]

Ans: $x = 1, y = 2, z = 1$

8. 2063 Q.No. 10(b) Solve by Cramer's rule or by row equivalent matrix method.
$x + y + z = 6$; $x - y + z = 2$; $2x + y - z = 1$ [4]

Ans: $x = 1, y = 2, z = 3$

9. 2062 Q.No. 10(b) Solve by row–equivalent method: $9y - 5x = 3$, $x + z = 1$ and $z + 2y = 2$ [4]

Ans: $x = 3, y = 2, z = -2$

10. 2061 Q.No. 10(b) OR Solve by "Row–equivalent matrix" method: $x - y - z = -2$, $x + 4z = 4$ & $y - 2z = 1$ [4]

Ans: $x = \frac{8}{7}, y = \frac{17}{7}, z = \frac{5}{7}$

11. 2060 Q.No. 10(b) Solve, by row equivalent matrix method: $x-2y-3z = -1$, $2x + y + z = 6$, $x + 3y - 2z = 13$ [4]

Ans: $x = 2, y = 3, z = -1$

12. 2059 Q.No. 10(b) Solve by row equivalent matrix method: $x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$ [4]

Ans: $x = 3, y = 2, z = -2$

13. 2058 .Q.No. 10(b) Solve, by row equivalent matrix method: $x + y + z = 1$, $x + 2y + 2z = 4$, $x + 3y + 7z = 13$ [4]

Ans: $x = 1, y = -3, z = 3$

14. 2057 Q.No. 10(b) Solve by using row equivalent matrix method: $x - 2y + 2z = 0$; $x-2y+3z = -1$; $2x - 2y + z = -3$. [4]

Ans: $x = -4, y = -3, z = -1$

15. 2056 Q.No. 10(b) Solve by row equivalent matrix method: $x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$ [4]

Ans: $x = 3, y = 2, z = -2$

## B. Inverse Matrix Method

### FORMULAE AND IMPORTANT POINTS

1. Inverse Matrix Method

Let $a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

The system of equations can be written in the matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \ldots(1)$$

The equation (1) can again be written as a single matrix equation as

$$AX = B, \quad \ldots(2)$$

Thus, the solution can be obtained by finding out $A^{-1}$ and multiplying it with B, where,

$$A^{-1} = \frac{1}{|A|} \text{Adj } A.$$

### 2 Marks Questions

1. 2073 Supp Q.No. 3a Using inverse matrix method, solve the following system of equations: [2]
$2x + 5y = 17$, $5x - 2y = -1$.

Ans: $x = 1, y = 3$

2. 2071 Set C Q.No. 3a Solve by Inverse matrix method: [2]
$2x + y = 7$, $x + 3y = 11$

Ans: $x = 2, y = 3$

3. 2071 Set D Q.No. 3a Solve by Inverse matrix method: [2]
$5x - 3y = 9$; $2x + 5y = 16$

Ans: $x = 3, y = 2$

4. 2069 [Set A] Old Q. No. 3b Solve by inverse matrix method: [2]
$x + y = 5$ and $x - y = 3$.

Ans: $x = 4; y = 1$

5. 2067 Q.No. 3b Solve by matrix inversion method: $x + y = 3$ and $x - y = 1$ [2]

Ans: $x = 2, y = 1$

6. 2063 Q.No. 4(b) Solve by inverse matrix method:
$3x + 2y = 5$; $7x + 5y = 12$ [2]

Ans: $x = 1, y = 1$

7. 2062 Q.No. 3(b) Solve $x = 2y + 3$ and $3x - 5y = 8$ by inverse matrix method. [2]

Ans: $x = 1, y = -1$

8. 2061 Q.No. 3(b) Solve by matrix inversion method: $x + y = 2$ & $x - y = 0$ [2]

Ans: $x = 1, y = 1$

### 4 Marks Questions

9. 2076 Set B Q.No. 8a Solve by inverse matrix or row equivalent method: [4]
$x - y = 0$, $2x - y + 4z = 18$, $-3x + z + 2 = 0$.

Ans: $x = 2, y = 2, z = 4$

10. 2076 Set C Q.No. 8a Solve by row-equivalent or inverse matrix method: [4]
$x - y = 1$, $z + x = -6$, $x + y - 2z = 3$

Ans: $x = -2, y = -3, z = -4$

11. 2075 GIE Q.No. 8a Using row equivalent method or inverse matrix method, solve: [4]
$2x - 3y + z = 0$, $x + y + z = 7$, $3x + 2y - z = 3$.

Ans: $x = 1, y = 2, z = 4$

12. 2075 Set B Q.No. 8a Solve by row-equivalent or inverse matrix method: [4]
$x + z = 1$, $z + 2y = 2$, $5x - 9y = -3$.

Ans: $x = 3, y = 2, z = -2$

13. 2075 Set C Q.No. 8a Using row-equivalent method or inverse matrix method, solve the following system of equations. [4]
$x + z = 1$; $z + 2y = 2$; $5x - 9y = -3$.

Ans: $x = 3, y = 2, z = -2$

14. 2074 Supp Q.No. 8a Using row equivalent matrix method or inverse matrix method solve the following system of equation: [4]
$x + 4y + z = 18$, $3x + 3y - 2z = 2$, $-4y + z = -7$

15. 2074 Set A Q.No. 8a Using row-equivalent method or inverse matrix method, solve the following system of equations. [4]
$x + y + z = 1$, $x + 2y + 3z = 4$, $x + 3y + 7z = 13$.

Ans: $x = 1, y = -3, z = 3$

16. 2074 Set B Q.No. 8a Solve by row-equivalent or inverse matrix method: [4]
$x - y = 1$, $z + x = -6$, $x + y - 2z = 3$.

Ans: $x = -2, y = -3, z = -4$

17. 2073 Set C Q.No. 8a Using row equivalent or inverse matrix method, solve the following system of equations [4]
$x + z = 1$; $2y + z = 2$; $5x - 9y + 3 = 0$

Ans: $x = 3, y = 2, z = -2$

18. 2073 Set D Q.No. 8a Using inverse matrix method or row equivalent method, solve: [4]
$x + 2y + 3z = 4$, $2x + y = 4$, $3y - 2z = 0$.

Ans: $x = \frac{11}{6}, y = \frac{1}{3}$ and $z = \frac{1}{2}$

19. **2072 Supp Q.No. 8a** Using row equivalent matrix method or the inverse matrix method, solve the following system of equations: [4]
$9y - 5x = 3; x + z = 1; z + 2y = 2$

**Ans:** $x = 3, y = 2, z = -2$

20. **2072 Set C Q.No. 8a** Using row equivalent matrix or inverse matrix method, solve the following equations:
$x + 4y + z = 18, 3x + 3y - 2z = 2, -4y + z = -7$ [4]

**Ans:** $x = 1, y = 3, z = 5$

21. **2072 Set D Q.No. 8a** Using row equivalent matrix method or inverse matrix method, solve the following equations: $x + 2y - 3z = 9, 2x - y + 2z = -8, 3x - y - 4z = 3.$ [4]

**Ans:** $x = -1, y = 2, z = -2$

22. **2071 Supp. Q.No. 8a** Solve by row equivalent or inverse matrix method: $x + y + z = 6, x - y + z = 2, x + y - z = 0.$ [4]

**Ans:** $x = 1, y = 2, z = 3$

23. **2071 Old Q.No. 10b** Solve by inverse matrix method: $x - y = 2; 2x + 3y = 9.$ [4]

**Ans:** $x = 3, y = 1$

24. **2070 Supp Q.No. 8a** Applying row equivalent matrix method or the inverse matrix method, solve the following equations. $x + 4y + 3z = 6, \quad 3x + 9y = 18, -5x - 6y + 2z = -5.$ [4]

**Ans:** $x = -3, y = 3, z = -1$

25. **2070 Set C Q.No. 8 a** Using row equivalent matrix method or inverse matrix method, solve the following equations. $x + 4y + z = 18, \quad 3x + 3y - 2z = 2, \quad -4y + z = -7.$ [4]

**Ans:** $x = 1, y = 3, z = 5$

26. **2070 Set D Q.No. 8 a** Using row equivalent matrix method or inverse matrix method, solve the following equations. $9y - 5x = 3, \quad x + z = 1, \quad z + 2y = 2.$ [4]

**Ans:** $x = 3, y = 2, z = -2$

27. **2070 Old Q.No. 10 b Or** Using matrix inversion method solve: $x + 4y = 3, 3x - 2y + 5 = 0.$ [4]

**Ans:** $x = -1; y = 1$

28. **2069 Supp Q.No. 8 a** Using row equivalent or inverse matrix method, solve the following system of equations. $2x - y + 4z + 3 = 0, x - 4z = 5, 6x - y + 2z = 10.$ [4]

**Ans:** $x = 3, y = 7, z = -\frac{1}{2}$

29. **2069 [Set A] Q. No. 8a** Using row equivalent matrix method or inverse matrix method, solve the following equations. $x - 2y - z = -7; 2x + y + z = 0; 3x - 5y + 8z = 13$ [4]

**Ans:** $x = -2; y = 1; z = 3$

30. **2069 [Set B] Q. No. 8a** Using row equivalent or inverse matrix method, solve the following system of equations. $x - y = 0, 2x - y + 4z = 18, -3x + z + 2 = 0.$ [4]

**Ans:** $x = 2, y = 2, z = 4$

31. **2068 Q.No. 8a** Applying row equivalent matrix method or inverse matrix method, solve the following system of equations: $x + y + z = 1; x + 2y + 3z = 4; x + 3y + 7z = 13$ [4]

**Ans:** $x = 1, y = -3, z = 3$

32. **2068 Old Q.No. 10b** Solve the following system of equations by row equivalent matrix method or inverse matrix method: $x - y - z = -2; 5x + 10z = 20; 10y - 20z = 10$ [4]

**Ans:** $x = 2, y = 3, z = 1$

33. **2066 Q.No. 10 (b)** Solve by row equivalent or inverse matrix method: $x + z = 1; z + 2y = 2; 5x - 9y = -3$ [4]

**Ans:** $x = 3, y = 2, z = -2$

34. **2064 Q.No. 10(b)** Solve the following system of equations by inverse matrix method or Cramer's rule:
$2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$ [4]

**Ans:** $x = 1, y = 2, z = 3$

35. **2060 Q.No. 10(b) OR** Solve by matrix inversion method
$3x + 5y = 7, 5x + 9y = 7$ [4]

**Ans:** $x = 14, y = -7$

36. **2059 Q.No. 10(b) OR** Solve by using inverse matrix method:
$2x + 4y = 7, 8x - 6y = -5$ [4]

**Ans:** $x = \frac{1}{2}, y = \frac{3}{2}$

37. **2058 Q.No. 10(b) OR** Solve by inverse matrix method:
$2x + 5y = 7; 5x + 2y = -3$ [4]

**Ans:** $x = \frac{-29}{21}, y = \frac{41}{21}$

38. **2057 Q.No. 10(b) OR** Solve by inverse matrix method:
$-2x + 4y = 3; \quad 3x - 7y = 1$ [4]

**Ans:** $x = \frac{-25}{2}$ & $y = \frac{-11}{2}$

39. **2056 Q.No. 10(b) OR** Solve by using inverse matrix method:
$x - y = 2; \quad 2x + 3y = 9$ [4]

**Ans:** $x = 3, y = 1$

## C. Cramer's Rule

### FORMULAE AND IMPORTANT POINTS

1. For the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$,

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$x = \frac{D_1}{D} \text{ and } y = \frac{D_2}{D}$$

2. For the equations,
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \ \& \ D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

$$x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$$

3. If $D = 0$, we can't apply Cramer's rule.

### 2 Marks Questions

1. **2076 Set B Q.No. 3a** Using Cramer's rule, solve: $3x + 2y + 9 = 0, 2x - 3y + 6 = 0.$ [2]

**Ans:** $x = -3, y = 0$

2. **2076 Set C Q.No. 3a** Solve by Cramer's rule: $2x + y = 7, x + 3y = 11.$ [2]

**Ans:** $x = 2, y = 3$

3. **2075 GIE Q.No. 3a** Solve by Cramer's rule: $x + y = 5, 6x - 4y = 0.$ [2]

**Ans:** $x = 2; y = 3$

4. **2075 Set A Q.No. 3a** Determine consistency and independence of the system of linear equations. [2]
$3x - 2y = 3; -4y + 6x = 9.$

**Ans:** Inconsistent and Independent

5. **2075 Set B Q.No. 3a** Solve by Cramer's rule: $3x + 4y = 2$, $5x - 7y = 24$. [2]

**Ans:** $x = \frac{110}{41}$, $y = -\frac{62}{41}$

6. **2075 Set C Q.No. 3a** Using Cramer's rule, solve the following equations: $3x + 2y = -9$ and $2x - 3y = -6$. [2]

**Ans:** $x = -3$, $y = 0$

7. **2074 Supp Q.No. 3a** Using Crammer's rule, solve the following system of equations: $x - 2y = -7$ and $3x + 7y = 5$. [2]

**Ans:** $x = -3$, $y = 2$

8. **2074 Set A Q.No. 3a** Using Cramer's rule, solve the following equations: $2x + 5y = 17$, $5x - 2y = -1$. [2]

**Ans:** $x = 1$, $y = 3$

9. **2074 Set B Q.No. 3a** Solve by Cramer's rule: $x + y = 5$ and $6x - 4y = 0$. [2]

**Ans:** $x = 2$, $y = 3$

10. **2073 Set C Q.No. 3a** Use Cramer's rule to solve: $3x + 2y = 8$ and $4x + y = 9$. [2]

**Ans:** $x = 2$, $y = 1$

11. **2073 Set D Q.No. 3a** Solve by Cramer's rule: $2x + 3y = 5$ and $x + 7y = 8$. [2]

**Ans:** $x = 1$, $y = 1$

12. **2072 Supp Q.No. 3a** Using Cramer's rule, solve the following equations: $3x + 4y = -2$; $5x - 7y = 24$ [2]

**Ans:** $x = 2$, $y = -2$

13. **2072 Set C Q.No. 3a** Solve by Cramer's rule, the following equations: $3x + 4y = -2$; $5x - 7y = 24$ [2]

**Ans:** $x = 2$, $y = -2$

14. **2072 Set D Q.No. 3a** Using Cramer's rule, solve: $5x - 3y = 20$; $2x + 5y = 8$ [2]

**Ans:** $x = 8$, $y = 0$

15. **2072 Set E Q.No. 3a** Solve by Cramer's rule: $4x + 3y + 4 = 0$, $6x + 5y + 7 = 0$. [2]

**Ans:** $x = \frac{1}{2}$, $y = -2$

16. **2071 Supp. Q.No. 3a** Using Cramer's rule, solve: $\frac{4}{x} + \frac{6}{y} = 0$, $\frac{3}{x} - \frac{4}{y} = \frac{-17}{6}$ [2]

**Ans:** $x = -2$, $y = 3$

17. **2071 Old Q.No. 3b** Solve by using Cramer's Rule: $x - y = 2$; $2x + 3y = 9$ [2]

**Ans:** $x = 3$, $y = 1$

18. **2070 Supp Q.No. 3a** Using Cramer's rule, solve the following equations: $5x - 2y = -2$, $2x + 5y = -24$ [2]

**Ans:** $x = -2$, $y = -4$

19. **2070 Set C Q.No. 3 a** Using Cramer's rule, solve the following equations: $3x - 2y = 8$, $5x + 3y = 7$. [2]

**Ans:** $x = 2$, $y = -1$

20. **2070 Set D Q.No. 3 a** Applying Cramer's rule, solve the following equations: $3x + \frac{4}{y} = 10$, $-2x + \frac{3}{y} = -1$ [2]

**Ans:** $x = 2$, $y = 1$

21. **2070 Old Q.No. 3 b** Solve by Cramer's rule $x+y=3$, $x-y=1$. [2]

**Ans:** $x = 2$, $y = 1$

22. **2069 Supp Q.No. 3a** Use Cramer's rule to solve the system of equations $2x+3y=2$, $3x-2y=5$. [2]

**Ans:** $x = \frac{19}{13}$, $y = \frac{-4}{13}$

23. **2069 [Set A] Q. No. 3a** Using Cramer's rule, solve the following equations: $x - 2y = -7$; $3x + 7y = 5$ [2]

**Ans:** $x = -3$, $y = 2$

24. **2069 [Set B] Q. No. 3a** Solve by Cramer's rule: $3x + 2y + 9 = 0$, $2x - 3y + 6 = 0$. [2]

**Ans:** $x = -3$; $y = 0$

25. **2068 Q.No. 3a** Using Cramer's rule, solve the system of equations: $2x + 5y = 17$; $5x - 2y = -1$ [2]

**Ans:** $x = 1$, $y = 3$

26. **2068 Old Q.No. 3b** Using Cramer's rule, solve the following system: $3x + 4y = -2$; $5x - 7y = 24$ [2]

**Ans:** $x = 2$, $y = -2$

27. **2066 Q.No. 3(b)** Solve by Cramer's rule: $-x + y = 9$, $x-3y=5$. [2]

**Ans:** $x = -16$; $y = -7$

28. **2065 Q. No. 3 b** Solve by Crammer's rule: $3x+2y=8$; $4x+y=9$ [2]

**Ans:** $x = 2$ and $y = 1$

29. **2060 Q.No. 3(b)** Give reason why simultaneous equation $x + 2y = 5$ and $3x + 6y = 12$ are not solvable by Cramer's Rule. [2]

30. **2059 Q.No. 3(b)** Solve by Cramer's rule $2x-y=5$; $x - 2y=1$ [2]

**Ans:** $x = 3$, $y = 1$

31. **2058 Q.No. 3(b)** Solve by Cramer's rule, $x-2y=-7$, $3x+7y=5$ [2]

**Ans:** $x = -3$, $y = 2$

32. **2057 Q.No. 3(b)** Solve by Cramer's rule: $2x-y=5$, $x-2y=1$ [2]

**Ans:** $x = 3$, $y = 1$

33. **2056 Q.No. 3(b)** Solve by Cramer's rule: $-x+y=9$; $x-3y=5$ [2]

**Ans:** $x = -16$, $y = -7$

## 4 Marks Questions

34. **2073 Supp Q.No. 8a** Using row equivalent matrix method or the Cramer's rule, solve the following system of equations: $x - 2y - z = -7$, $2x + y + z = 0$, $3x - 5y + 8z = 13$. [4]

**Ans:** $x = -2$, $y = 1$, $z = 3$

35. **2071 Set C Q.No. 8a** Using Cramer's rule or row-equivalent method solve the following system: $x + y + z = 6$, $2x + 3y + 5z = 23$, $7x + 5y - 2z = 11$. [4]

**Ans:** $x = 1$, $y = 2$, $z = 3$

36. **2071 Set D Q.No. 8a** Using Cramer's rule or row-equivalent matrix method solve the following system: $x + y + z = 6$, $x - y + z = 2$, $5x - 9y = -3$. [4]

**Ans:** $x = 3$, $y = 2$, $z = 1$

37. **2069 [Set A] Old Q. No. 10b** Solve by Row-equivalent or Cramer's rule: $2x - y + 3z = 9$, $x+y + z = 6$ and $x-y+z=2$ [4]

**Ans:** $x = 1$, $y = 2$, $z = 3$

38. **2069 [Set B] Old Q. No. 10b** Solve the following system of equation by inverse matrix method Or Cramer's rule. $2x - y + 4z = -3$, $x - 4z = 5$, and $6x - y + 2z = 10$ [4]

**Ans:** $x = 3$, $y = 7$, $z = -1/2$

39. **2067 Q.No. 10 b** Solve by Cramer's rule or Row-equivalent method: $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$ [4]

**Ans:** $x = 1$, $y = 3$, $z = 5$

40. **2062 Q.No. 10(b) OR** Solve the equations by Cramer's rule: $9y - 5x = 3$, $x + z = 1$ & $z + 2y = 2$ [4]

**Ans:** $x = 3$, $y = 2$, $z = -2$

41. **2061 Q.No. 10(b)** Solve (by Cramer's rule): $x + 2y + 3z = 6$, $2x + 4y + z = 7$, $3x + 2y + 9z = 14$ [4]

**Ans:** $x = 1$, $y = 1$, $z = 1$

# Unit 8: Complex Number

## FORMULAE AND IMPORTANT POINTS

1. Complex Number
An ordered pair (a, b) of two real numbers a and b is said to be a complex number.
2. $i^2 = -1$
In general:
$i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$, where $n \in \mathbb{Z}$.
3. If $Z = a + ib$, then $\overline{Z} = a - ib$
4. If $Z = a + ib$, then $|Z| = \sqrt{a^2 + b^2}$
5. Cube Roots of Unity
$1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$ are three cube roots of unity
6. Properties of cube roots of unity
   i. Each imaginary cube root of unity is the square of the other.
   ii. The sum of the three cube roots of unity is zero.
   iii. Each complex cube root of unity is the reciprocal of the other.
   iv. The product of two imaginary cube roots of unity is 1.
7. $\omega^3 = 1$
In general $\omega^{3n} = 1$ for $n \in \mathbb{Z}$.
8. If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$
then, $z_1z_2 = r_1r_2\{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}$.
and $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$.
9. $\text{amp}(z_1z_2) = \text{amp}(z_1) + \text{amp}(z_2)$.
$\text{amp}\left(\frac{z_1}{z_2}\right) = \text{amp}(z_1) - \text{amp}(z_2)$.
10. De-Moivre's Theorem
For any positive integer n
$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$.

## 2 Marks Questions

1. 2076 Set B Q.No. 3b Express $\sqrt{3} + i$ in polar form. [2]
**Ans: $2(\cos 30° + i\sin 30°)$**
2. 2076 Set C Q.No. 3b Find the conjugate of $(3 + 5i)(7 - 5i)$. [2]
**Ans: $46 - 20i$**
3. 2075 GIE Q.No. 3b Express $\frac{2-\sqrt{-25}}{1-\sqrt{-16}}$, in a + ib form. [2]
**Ans: $\frac{22}{17} + \frac{3}{17}i$**
4. 2075 Set A Q.No. 3b Express the complex number $(1-i)^9\left(1-\frac{1}{i^3}\right)^9$ in the standard form a + bi. [2]
**Ans: $0 + (-512)i$**
5. 2075 Set B Q.No. 3b Find the conjugate of the complex number $\frac{5-7i}{5+8i}$. [2]
**Ans: $-\frac{31}{89} + \frac{75}{89}i$**
6. 2075 Set C Q.No. 3b If $\sqrt{x - iy} = a - ib$ prove that $\sqrt{x + iy} = a + ib$. [2]
7. 2074 Supp Q.No. 3b If $\omega$ is a complex cube root of unity, show that: $(1 - \omega + \omega^2)^4(1 + \omega - \omega^2)^4 = 256$ [2]
8. 2074 Set A Q.No. 3b Find the real numbers x and y if $(x - 1)i + (y + 1) = (1 + i)(4 - 3i)$. [2]
**Ans: x = 2, y = 6**
9. 2074 Set B Q.No. 3b Find the multiplicative inverse of the complex number $3 + 4i$. [2]
**Ans: $\frac{3}{25} - \frac{4}{25}i$**
10. 2073 Supp Q.No. 3b Express the complex number $\sqrt{\frac{1+i}{1-i}}$ into a + ib form. [2]
**Ans: $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$**
11. 2073 Set C Q.No. 3b Find the conjugate of $1 + i + i^2 + i^3 + i^4$. [2]
**Ans: 1**
12. 2073 Set D Q.No. 3b Find the modulus of $(3 + 4i)(4 + i)$. [2]
**Ans: $5\sqrt{17}$**
13. 2072 Supp Q.No. 3b If $x - iy = \frac{3-2i}{3+2i}$, prove that $x^2+y^2=1$. [2]
14. 2072 Set C Q.No. 3b Express the complex number: $i - \sqrt{3}$ into polar form. [2]
**Ans: $2(\cos 150° + i\sin 150°)$**
15. 2072 Set D Q.No. 3b If $x = a + b$, $y = a\omega + b\omega^2$ and $z = a\omega^2 + b\omega$, show that $x + y + z = 0$. [2]
16. 2072 Set E Q.No. 3b If z and w are complex numbers, show that: $|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2)$ [2]
17. 2071 Supp. Q.No. 3b Express the complex number $1 - \sqrt{3}i$ into polar form. [2]
**Ans: $2(\cos 300° + i\sin 300°)$**
18. 2071 Set C Q.No. 3b Prove that the modulus of a complex number and its conjugate are equal. [2]
19. 2071 Set D Q.No. 3b For any complex number Z, prove that: $Z\overline{Z} = |Z|^2$. [2]
20. 2071 Old Q.No. 4a Express $\frac{i}{i+1}$ in the polar form. [2]
**Ans: $\frac{1}{\sqrt{2}}(\cos 45° + i\sin 45°)$**
21. 2070 Supp Q.No. 3b Express the complex number $\sqrt{\frac{1+i}{1-i}}$ in the form of a + ib. [2]
**Ans: $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$**
22. 2070 Set C Q.No. 3 b If $\alpha = \frac{1}{2}(-1+\sqrt{-3})$, $\beta = \frac{1}{2}(-1-\sqrt{-3})$, show that: $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$. [2]
23. 2070 Set D Q.No. 3 b Find the values of x and y if $(x + 2) + yi = (3 + i)(1 - 2i)$. [2]
**Ans: x = 3, y = –5**
24. 2070 Old Q.No. 4 a Express the complex number in Cartesian form whose modulus is 6 and amplitude is 60°. [2]
**Ans: $3 + 3\sqrt{3}i$**
25. 2069 Supp Q.No. 3 b Express $1 + i\sqrt{3}$ in polar form. [2]
**Ans: $2(\cos 60° + i\sin 60°)$**
26. 2069 [Set A] Q. No. 3b If w be a complex cube root of unity, find the value of: $(1 - \omega + \omega^2)^4(1 + \omega - \omega^2)^4$ [2]
**Ans: 256**

27. **2069 [Set A] Old Q. No. 4a** Express the complex number $\frac{1}{1-i}$ in the form a + ib. [2]

**Ans:** $\frac{1}{2}+\frac{1}{2}i$

28. **2069 [Set B] Q. No. 3b** Express $\sqrt{3}+i$ in polar form. [2]

**Ans:** 2 (cos 30° + i sin 30°)

29. **2069 [Set B] Old Q. No. 3b** Find the values of the real number x and y if $(x+2)+yi=(3+i)(1-2i)$. [2]

**Ans:** $x = 3, y = -5$

30. **2068 Q.No. 3b** Find the cube roots of unity. [2]

**Ans:** 1, $\frac{-1+\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$

31. **2068 Old Q.No. 4a** Find the real numbers x and y if: $x+iy=(2-3i)(3-2i)$ [2]

**Ans:** $x = 0, y = -13$

32. **2067 Q.No. 4a** Express the complex number $\frac{i}{1-i}$ in the polar form. [2]

**Ans:** $\frac{1}{\sqrt{2}}$ (cos 135° + i sin 135°)

33. **2066 Q.No. 4(a)** Prove that $(2+\omega)(2+\omega^2)(2-\omega^2)(2-\omega^4)=21$ [2]

34. **2065 Q. No. 4 a** If α, β are the complex cube roots of unity then show that: $\alpha^4+\beta^4+\frac{1}{\alpha\beta}=0$ [2]

35. **2064 Q.No. 3(b)** If 1, ω, ω² be the cube roots of unity, prove that $(1+\omega^2)^3-(1+\omega)^3=0$. [2]

36. **2063 Q.No. 3(b)** Find the square roots of 7 + 24i [2]

**Ans:** $\pm(4+3i)$

37. **2062 Q.No. 4(a)** Simplify: $\left[3\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{16}$ [2]

**Ans:** $3^{16}$

38. **2061 Q.No. 4(a)** Find the conjugate of the complex number $\frac{3+4i}{3-4i}$. [2]

**Ans:** $\frac{-7}{25}-\frac{24}{25}i$

39. **2060 Q.No. 4(a)** Express the complex number $-\sqrt{2}+i\sqrt{2}$ in polar form. [2]

**Ans:** 2(cos 135° + i sin 135°)

40. **2059 Q.No. 4(a)** Express in the polar form for z = 2 + 2i. [2]

**Ans:** $2\sqrt{2}$ (cos 45° + i sin 45°)

41. **2058 Q.No. 4(a)** Express the complex number $(2, 2\sqrt{3})$ in the polar form. [2]

**Ans:** 4 (cos 60° + i sin 60°)

42. **2057 Q.No. 4(b)** If $z_1 = (3, 2)$; $z_2 = (5, 3)$, compute $z_1z_2$ and $z_1 + z_2$. [2]

**Ans:** (9, 19), (8, – 5)

43. **2056 Q.No. 4(a)** If z = 3 + 4i and w = 2 + i, find |zw| and $\left|\frac{z}{w}\right|$. [2]

**Ans:** $5\sqrt{5}, \sqrt{5}$

## 6 Marks Questions

44. **2076 Set B Q.No. 14** Define conjugate of a complex number. Using De Moivre's theorem, find the square roots of $-2+2\sqrt{3}i$. [6]

**Ans:** $\pm(1+\sqrt{3}i)$

45. **2076 Set C Q.No. 14** State De Moivre's theorem and apply it to find fourth roots of $-\frac{1}{2}+i\frac{\sqrt{3}}{2}$. [6]

**Ans:** $\pm\frac{1}{2}(\sqrt{3}+i), \pm\frac{1}{2}(1-\sqrt{3}i)$

46. **2075 GIE Q.No. 14** Define modulus of a complex number. If z and w are two complex numbers, prove that: $|z-w|\le|z|+|w|$ and $|z+w|\ge|z|-|w|$. [6]

47. **2075 Set A Q.No. 13** State De Moivre's theorem. Find the cube roots of the complex number -27i using this theorem. [6]

**Ans:** 3i, $\frac{-3\sqrt{3}}{2}-\frac{3}{2}i, \frac{3\sqrt{3}}{2}-\frac{3}{2}i$

48. **2075 Set B Q.No. 14** State De-Moivre's theorem for a positive index n. Using De-Moivre's theorem, find the fourth roots of $-\frac{1}{2}+i\frac{\sqrt{3}}{2}$. [6]

**Ans:** $\pm\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right), \pm\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

49. **2075 Set C Q.No. 14** Define conjugate of a complex number. Write its geometrical meaning. [6]

Also, find the square root of $\frac{8-6i}{1+i}$

**Ans:** $\pm\left(\sqrt{\frac{5\sqrt{2}+1}{2}}-\sqrt{\frac{5\sqrt{2}-1}{2}}\,i\right)$

50. **2074 Supp Q.No. 14** Define Modulus of a complex number. If z and w are two complex numbers, prove that: $|z+w|^2=|z|^2+|w|^2+2\,\text{Re}(z\bar{w})$. [6]

51. **2074 Set A Q.No. 14** State De Moivre's theorem. Using De Moivre's theorem, find the cube roots of unity. If $\omega=\frac{-1+\sqrt{3}i}{2}$ be a complex cube root of unity, show that $\omega^2=\frac{-1-\sqrt{3}i}{2}$. [6]

**Ans:** 1, $\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

52. **2074 Set B Q.No. 14** Define De Moivre's theorem for a positive index n. Using De Moivre's theorem find the square roots of $-1+\sqrt{3}i$. [6]

**Ans:** $\pm\frac{1}{\sqrt{2}}(1+\sqrt{3}i)$

53. **2073 Supp Q.No. 14** If z and w are two complex numbers, prove that $|z+w|\le|z|+|w|$. [6]

54. **2073 Supp Q.No. 14 OR** State De-moivre's theorem. Using De-moivre's theorem find the cube roots of unity. Also establish the properties of the cube roots of unity. [6]

**Ans:** 1, $\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

55. **2073 Set C Q.No. 14** State De Moivre's theorem for any positive index n. Using De Moivre's theorem find the square roots of $4+4\sqrt{3}i$. [6]

**Ans:** $\pm(\sqrt{6}+\sqrt{2}i)$

56. **2073 Set D Q.No. 14** State De Moivre's theorem. Using De Moivre's theorem find the cube roots of unity. Also show that sum of the three cube roots of unity is zero. [6]

57. **2072 Supp Q.No. 14** If z and w be two complex numbers, prove that: $|z+w|\le|z|+|w|$. [6]

58. 2072 Supp Q.No. 14 OR Define conjugate of a complex number. Find the square roots of the complex number $\frac{2-36i}{2+3i}$. [6]

Ans: $\pm(1-3i)$

59. 2072 Set C Q.No. 14 If z and w are two complex numbers, prove that: $|z+w|^2 = |z|^2 + |w|^2 + 2\,\text{Re}(z\bar{w})$. [6]

60. 2072 Set D Q.No. 14 State De Moivre's theorem. Using De Moivre's theorem solve $Z^6 = 1$. [6]

Ans: $\pm1, \frac{1}{2}(1 \pm \sqrt{3}\,i), \frac{1}{2}(-1 \pm \sqrt{3}\,i)$

61. 2072 Set E Q.No. 13 State and prove De Moivre's theorem. Is the theorem true for integers? Justify your answer. [6]

Ans: Yes

62. 2071 Supp. Q.No. 13 State De Moiver's theorem. Apply it to compute 4th roots of $-1$. [6]

Ans: $\pm\frac{1}{\sqrt{2}}(1+i), \pm\frac{1}{\sqrt{2}}(1-i)$

63. 2071 Set C Q.No. 14 State De Moivre's theorem. Using De Moivre's theorem find the square roots of $4+4\sqrt{3}\,i$. [6]

Ans: $\pm(\sqrt{6} + i\sqrt{2})$

64. 2071 Set D Q.No. 14 State De Moivre's theorem. Using De Moivre's theorem find the square roots of $2+2\sqrt{3}\,i$. [6]

Ans: $\pm(\sqrt{3}+i)$

65. 2070 Supp Q.No. 14 Find the square roots of the complex number $\frac{2-36i}{2+3i}$. [6]

Ans: $\pm(1-3i)$

66. 2070 Set C Q.No. 14 Find the square root of the complex number $-5+12i$. [6]

Ans: $\pm(2+3i)$

67. 2070 Set D Q.No. 14 Define absolute value of a complex number. If z and w are two complex numbers, prove that: $|z+w| \le |z| + |w|$ [6]

68. 2070 Set D Q.No. 14 Or Find the cube roots of unity. Also, establish the properties of cube roots of unity. [6]

Ans: $1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}$

69. 2069 Supp Q.No. 14 Define conjugate of a complex number and write any four of its properties. Also prove $|z_1| + |z_2| \ge |z_1+z_2|$ where $z_1$ and $z_2$ are two complex numbers. [6]

70. 2069 [Set A] Q. No. 14 If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, prove that $z_1z_2 = r_1r_2\{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}$ and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\}$. [6]

71. 2069 [Set A] Q. No. 14 OR Define complex number. Express a complex number into polar form. State De-Moivre's theorem. Using De'Moivre's theorem, find the cube roots of unity. [6]

Ans: $1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}$

72. 2069 [Set B] Q. No. 14 State De Moivre's theorem for any positive index n. Using De Moivre's theorem find the square roots of $4+4\sqrt{3}\,i$. [6]

Ans: $\pm(\sqrt{6} + \sqrt{2}\,i)$

73. 2068 Q.No. 14 State De-Moivre's theorem. Using De Moivre's theorem, find the square roots of $-2-2\sqrt{3}i$. [6]

Ans: $\pm(-1+i\sqrt{3})$

**4 Marks Questions (Old Syllabus Questions)**

74. 2071 Old Q.No. 11a State and prove De Moivre's theorem for any positive integer. [4]

75. 2070 Old Q.No. 11 a Find the square root of $\frac{5+12i}{3-4i}$. [4]

Ans: $\pm\frac{1}{5}(4+7i)$

76. 2069 [Set A] Old Q. No. 11a State De Moivre's theorem hence determines the cube roots of unity. [4]

Ans: $1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}$

77. 2068 Old Q.No. 11a Find the cube roots of unity. If ω be an imaginary cube root of unity, prove that: $1+\omega+\omega^2 = 0$ [4]

Ans: $1, \frac{-1+\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$

78. 2067 Q.No. 11a Find the square root of $\frac{5+12i}{3-4i}$. [4]

Ans: $\pm\frac{1}{5}(4+7i)$

79. 2066 Q.No. 11(a) If $\sqrt{x+iy} = a+ib$, prove that $\sqrt{x-iy} = a-ib$. [4]

80. 2065 Q. No. 11 a Find the square roots of $(-7+24i)$. [4]

Ans: $\pm(3+4i)$

81. 2064 Q.No. 12(a) Solve: $z^6 = 1$. [4]

Ans: $\pm1, \pm\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right), \pm\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$

82. 2063 Q.No. 12(b) Find the cube roots of unity and discuss their properties. [4]

Ans: $1, \frac{-1+i\sqrt{3}}{2}$

83. 2062 Q.No. 11(a) Find the cube roots of unity. Write their properties. [4]

Ans: $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

84. 2061 Q.No. 11(a) State De Moivre's theorem hence solve $z^6=1$. [4]

Ans: $\pm1, \pm\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right), \pm\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)$

85. 2060 Q.No. 11(a) Using De–Moivre's theorem, find the fourth roots of unity. [4]

Ans: $1, i, -1, -i$

86. 2059 Q.No. 11(a) Find the square roots of $z = 7-24i$ [4]

Ans: $\pm(4-3i)$

87. 2058 Q.No. 11(a) State De–Moivre's theorem. Use it to find the cube roots of 1. [4]

Ans: $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

88. 2057 Q.No. 11(b) State De–Moivre's Theorem. Use it to find the value of $(1+i)^{20}$. [4]

Ans: $-2^{10}$

89. 2056 Q.No. 11(a) State De–Moivre's theorem. Use it to find the cube roots of unity. [4]

Ans: $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

# Unit 9: Polynomial Equations

## FORMULAE AND IMPORTANT POINTS

1. A function $f(x)$ defined by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots + a_{n-1}x + a_n$, $(a_0 \neq 0)$ is said to be a polynomial of degree n in x, where $a_0, a_1, a_2, \cdots, a_n \in \mathbb{R}$ are all constants, n is a non-negative integer.
2. The roots of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Nature of the Roots of Quadratic Equations
   a. i. If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is real, hence the roots are real and distinct, for $a, b, c \in \mathbb{R}$.
      ii. If $b^2 - 4ac > 0$ and a perfect square, then the roots are rational and distinct for a, b, c rational.
      iii. If $b^2 - 4ac > 0$ and not a perfect square, then the roots are irrational and distinct.
   b. If $b^2 - 4ac = 0$, then $\sqrt{b^2 - 4ac} = 0$, the roots are real and equal for $a, b, c \in \mathbb{R}$.
   c. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is an imaginary number. So the roots are imaginary and distinct.
4. i. If a quadratic equation has an irrational root, then other root will be its conjugate. That is, if $p + \sqrt{q}$ be one root, the other root will be the conjugate irrational quantity $p - \sqrt{q}$ and conversely.
   ii. Imaginary roots always occur in conjugate pair. That is, if p + iq be one root, the other root will be the conjugate imaginary quantity p − iq and conversely.
5. Relation between Roots and Coefficients of quadratic equation $ax^2 + bx + c = 0$
   Sum of the roots $= -\dfrac{b}{a} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$
   Product of the roots $= \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$.
6. Formulation of Quadratic Equation
   $x^2 -$ (sum of roots) x + (product of roots) = 0
7. Condition for one root common
   $(bc_1 - b_1c)(ab_1 - a_1b) = (ca_1 - c_1a)^2$
   The common root is
   $\dfrac{bc_1 - b_1c}{ca_1 - c_1a}$ or, $\dfrac{ca_1 - c_1a}{ab_1 - a_1b}$
8. Condition for two roots common
   $\dfrac{a}{a_1} = \dfrac{b}{b_1} = \dfrac{c}{c_1}$

## 2 Marks Questions

1. **2076 Set B Q.No. 3c** If one root of the equation $ax^2 + bx + c = 0$ be twice the other, show that $2b^2 = 9ac$. [2]
2. **2076 Set C Q.No. 3c** Find the value of k so that the equation $9x^2 + kx + 1 = 0$ has equal roots. [2]
   **Ans: k = ±6**
3. **2075 GIE Q.No. 3c** For what values of k the equation $9x^2 + kx + 1 = 0$ has equal roots? [2]
   **Ans: k = ±6**
4. **2075 Set A Q.No. 3c** Find the value of k if the roots of the equation $3kx^2 = 4(kx - 1)$ are real and equal. [2]
   **Ans: k = 3**
5. **2075 Set B Q.No. 3c** If one root of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ be four times the other, show that $4b^2 = 25ac$. [2]
6. **2075 Set C Q.No. 3c** If a + b + c = 0, solve the equation $ax^2 + bx + c = 0$. [2]
   **Ans: $x = 1, \frac{c}{a}$**
7. **2074 Supp Q.No. 3c** If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k. [2]
8. **2074 Set A Q.No. 3c** Find the value of K so that the equation $3x^2 + 7x + 6 - K = 0$ has one root equal to zero. [2]
   **Ans: K = 6**
9. **2074 Set B Q.No. 3c** If one root of the equation $x^2 - px + q = 0$ be twice the other, show that $2p^2 = 9q$. [2]
10. **2073 Supp Q.No. 3c** Find the value of k so that the equation $3x^2 + kx - 2 = 0$ has roots whose sum is equal to 6. [2]
    **Ans: k = −18**
11. **2073 Set C Q.No. 3c** Find the value of P so that equation $5x^2 - Px + 16 = 0$ has equal roots. [2]
    **Ans: $P = \pm 8\sqrt{5}$**
12. **2073 Set D Q.No. 3c** Find a quadratic equation whose roots are twice the roots of $4x^2 + 8x - 5 = 0$. [2]
    **Ans: $x^2 + 4x - 5 = 0$**
13. **2072 Supp Q.No. 3c** If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k. [2]
    **Ans: k = 1 or 4**
14. **2072 Set C Q.No. 3c** Find the value of k so that the equation $3x^2+7x+6-k = 0$ has one root equal to zero. [2]
    **Ans: k = 6**
15. **2072 Set D Q.No. 3c** If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 3 : 4, prove that $12b^2 = 49ac$. [2]
16. **2072 Set E Q.No. 3c** If the sum of the roots of the equation $(m + 1)x^2 + (2m + 3)x + (3m + 4) = 0$, is zero find the product of its roots. [2]
    **Ans: 1**
17. **2071 Supp. Q.No. 3c** If the difference of roots of the equation $x^2+px+q = 0$ is unity, show that: $p^2 = 1+4q$. [2]
18. **2071 Set C Q.No. 3c** Find a quadratic equation whose roots are twice the roots of: $4x^2 + 8x - 5 = 0$. [2]
    **Ans: $x^2 + 4x - 5 = 0$**
19. **2071 Set D Q.No. 3c** For what values of k the equation $9x^2 + kx + 1 = 0$ has equal roots? [2]
    **Ans: k = ± 6**
20. **2071 Old Q.No. 4b** If α and β are the roots of $px^2+qx+q= 0$ prove that: $\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} + \sqrt{\dfrac{q}{p}} = 0$ [2]
21. **2070 Supp Q.No. 3c** Determine the nature of the roots of the equation $4x^2 + 8x - 5 = 0$. [2]
    **Ans: Rational and unequal**
22. **2070 Set C Q.No. 3 c** If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k. [2]
    **Ans: k = 1 or 4**
23. **2070 Set D Q.No. 3 c** Find the quadratic equation whose one root is $2 + \sqrt{3}$. [2]
    **Ans: $x^2 - 4x + 1 = 0$**

**24.** 2070 Old Q.No. 4 b If the sum of roots is $-p$ and product of roots is $q$, find the quadratic equation. [2]

**Ans:** $x^2 + px + q = 0$

**25.** 2069 Supp Q.No. 3 c Prove that the roots of $x^2 - 3x + 2 = 0$ are rational. [2]

**26.** 2069 [Set A] Q. No. 3c For what values of p will the equation $5x^2 - px + 45 = 0$ have equal roots? [2]

**Ans:** $p = \pm 30$

**27.** 2069 [Set B] Q. No. 3c If one root of the equation $ax^2+bx+c=0$ be twice the other show that: $2b^2 = 9ac$. [2]

**28.** 2068 Q.No. 3c Form a quadratic equation whose roots are $-5$ and 4. [2]

**Ans:** $x^2 + x - 20 = 0$

**29.** 2060 Q.No. 4(b) If the roots of the quadratic equations are $p + q$ and $p - q$. Find the quadratic equation. [2]

**Ans:** $x^2 - 2px + p^2 - q^2 = 0$

**30.** 2057 Q.No. 4(a) Form the quadratic equation whose one root is $3 + 4i$. [2]

**Ans:** $x^2 - 6x + 25 = 0$

**31.** 2056 Q.No. 4(b) For what value of p will the equation $5x^2 - px + 45 = 0$ have equal root? [2]

**Ans:** $\pm 30$

## 4 Marks Questions

**32.** 2076 Set B Q.No. 8b Find the equation whose roots are reciprocal of the roots of $x^2 - x + 1 = 0$. [4]

**Ans:** $x^2 - x + 1 = 0$

**33.** 2076 Set C Q.No. 8b Find the equation whose roots are reciprocal of the roots of $x^2 - 3x + 2 = 0$. [4]

**Ans:** $2x^2 - 3x + 1 = 0$

**34.** 2075 GIE Q.No. 8b If $\alpha$ and $\beta$ are the roots of the equation $ax^2 - bx + b = 0$, prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{b}{a}}$. [4]

**35.** 2075 Set A Q.No. 8b If the sum of the roots of the equation $px^2 + qx + r = 0$ are equal to the sum of their squares, show that $2pr = pq + q^2$. [4]

**36.** 2075 Set A Q.No. 8b OR The sum of roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. Prove that product of the roots is $-\frac{1}{2}(a^2 + b^2)$. [4]

**37.** 2075 Set B Q.No. 8b If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$. [4]

**38.** 2075 Set C Q.No. 8b Show that the roots of the equation $(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$ will be equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$. [4]

**39.** 2074 Supp Q.No. 8b If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that $p = q$ or $p + q + 1 = 0$. [4]

**40.** 2074 Set A Q.No. 8b If one root of the equation is the square of the other, prove that $b^3+a^2c + ac^2 = 3abc$. [4]

**41.** 2074 Set B Q.No. 8b If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$. [4]

**42.** 2073 Supp Q.No. 8b If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{c} = \frac{b}{d}$. [4]

**43.** 2073 Set C Q.No. 8b Form the equation whose roots are reciprocal of the roots of $x^2 - x + 1 = 0$. [4]

**Ans:** $x^2 - x + 1 = 0$

**44.** 2073 Set D Q.No. 8b If one of the roots of the equation $ax^2 + bx + c = 0$ is thrice the other, show that $3b^2 = 16ac$. [4]

**45.** 2072 Supp Q.No. 8b Find the condition that the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2+b^1x + c^1= 0$ may have one root common. [4]

**Ans:** $(bc' - b'c)(ab' - a'b) = (ca' - c'a)^2$

**46.** 2072 Set C Q.No. 8b If the roots of the equation $(a^2 + b^2) x^2 - 2 (ac + bd) x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$. [4]

**47.** 2072 Set D Q.No. 8b If one root of the equation $ax^2+bx+c=0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

**48.** 2072 Set E Q.No. 8b If the roots of the equation $p(q - r) x^2 + q (r - p)x + r (p - q) = 0$ are equal, show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$. [4]

**49.** 2071 Supp. Q.No. 8b If a, b, c are rational and $a+b+c =0$, show that the roots of: $(b+c-a)x^2+(c+a-b)x+(a+b-c) = 0$ are rational. [4]

**50.** 2071 Set C Q.No. 8b If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, show that $p^2m = l^2q$. [4]

**51.** 2071 Set D Q.No. 8b If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a root in common, show that: either $p=q$ or $p+q+1= 0$. [4]

**52.** 2071 Old Q.No. 11b Determine the value of p for which one root of the equation $x^2 + px + 1 = 0$ is the square of the other. [4]

**Ans:** 1, −2

**53.** 2070 Supp Q.No. 8b If $\alpha$ and $\beta$ be the roots of the equation $px^2 + qx + q = 0$, prove that: $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$ [4]

**54.** 2070 Set C Q.No. 8 b Find the condition under which the two quadratic equations $ax^2 + bx + c = 0$ and $a^1x^2+b^1x + c^1 = 0$ may have one root common. [4]

**Ans:** $(bc' - b'c)(ab' - a'b) = (ca' - c'a)^2$

**55.** 2070 Set D Q.No. 8 b If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3+a^2c+ac^2=3abc$. [4]

**56.** 2070 Old Q.No. 11 b If $\alpha$, $\beta$ be the roots of $ax^2 + bx + b = 0$, show that: $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = 0$. [4]

**57.** 2069 Supp Q.No. 8 b Determine the values of p for which the equations $3x^2 + 4px + 2 = 0$ and $2x^2 + 3x - 2 = 0$ may have a common root. [4]

**Ans:** 56, −44

**58.** 2069 [Set A] Q. No. 8b Prove that a quadratic equation cannot have more than two roots. [4]

59. **2069 [Set A] Old Q. No. 11b** If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 3:4, prove that: $12b^2 = 49ac$. [4]

60. **2069 [Set B] Q. No. 8b** Form the equation whose roots are the reciprocals of the roots of $ax^2 + bx + c = 0$. [4]

**Ans:** $cx^2 + bx + a = 0$

61. **2069 [Set B] Old Q. No. 11b** Find the condition that the roots of the quadratic equation $ax^2+cx+c=0$ may be in the ratio m:n. [4]

**Ans:** $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} + \sqrt{\frac{c}{a}} = 0$

62. **2068 Q.No. 8b** If the roots of the equation $(a^2 + b^2) x^2 - 2 (ac + bd) x + (c^2 + d^2) = 0$ are equal, then show that: $\frac{a}{b} = \frac{c}{d}$ [4]

63. **2068 Old Q.No. 11b** If the roots of the equation $(a^2 + b^2) x^2 - 2 (ac + bd) x + (c^2 + d^2) = 0$ are equal, prove that: $\frac{a}{b} = \frac{c}{d}$ [4]

64. **2067 Q.No. 11 b** If one root of the equation $ax^2 + bx + c = 0$ is triple of the other, show that $3b^2 = 16 ac$. [4]

65. **2066 Q.No. 11(b)** If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [4]

66. **2065 Q. No. 11 b** Find the equation whose roots are reciprocal to the roots of $x^2 - x + 1 = 0$. [4]

**Ans:** $x^2 - x + 1 = 0$

67. **2064 Q.No. 11(b)** Under what conditions are the roots of the quadratic equation $ax^2 + bx + c = 0$:
(i) real and unequal (ii) imaginary [4]

**Ans:** (i) $b^2 - 4ac > 0$ (ii) $b^2 - 4ac < 0$

68. **2063 Q.No. 11(b)** Under what conditions will quadratic equation $ax^2 + bx + c = 0$ has
(i) one root the reciprocal of the other.
(ii) roots equal in magnitude but opposite in sign. [4]

**Ans:** (i) $c = a$ (ii) $b = 0$

69. **2062 Q.No. 11(b)** The quadratic equation: $ax^2 + bx + c = 0$ can not have more than two roots. Prove it. [4]

70. **2061 Q.No. 11(b)** If one root of the equation $lx^2 + mx + n = 0$ be four times the other, show that $4m^2 = 25 ln$. [4]

71. **2060 Q.No. 11(b)** If the roots of $l^2x^2 + mx + n = 0$ be in the ratio 3:4 show that $12m^2 = 49 l n$. [4]

72. **2059 Q.No. 11(b)** Show that the roots of the equation $(a^2 - bc) x^2 + 2(b^2 - ca) x + (c^2 - ab) = 0$ will be equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$ [4]

73. **2058 Q.No. 11(b)** If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that, $b^3 + a^2c + ac^x = 3abc$. [4]

74. **2057 Q.No. 11(a)** Find the condition for two given quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ may have one root common and both roots common. [4]

**Ans:** $(b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

75. **2056 Q.No. 11(b)** If the roots of the equation $lx^2 + nx + n = 0$ be in the ratio p:q, find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$ [4]

**Ans:** $-\sqrt{\frac{n}{l}}$

# Unit 10: Co-ordinate Geometry

## A. Straight line

### FORMULAE AND IMPORTANT POINTS

1. Distance formula $(d) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. Section formula
   i. $(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$
   (internal division)
   ii. $(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$
   (external division)
3. Mid-point formula $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
4. Centroid formula $(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
5. Slope $(m) = \frac{y_2 - y_1}{x_2 - x_1}$
6. Slope $(m) = \tan\theta$, where θ is the angle made by the line with positive x-axis.
7. Equation of straight line
   i. Slope intercept form: $y = mx + c$
   ii. Double intercept form: $\frac{x}{a} + \frac{y}{b} = 1$
   iii. Normal form or perpendicular form
   $x \cos\alpha + y \sin\alpha - p = 0$
   iv. Point slope form: $y - y_1 = m(x - x_1)$
   v. Two points form: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
   vi. Symmetric form or distance form
   $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$
   vii. Every straight line has an equation of the form $ax + by + c = 0$, where a, b and c are constants.
8. Area of Triangle
   $$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
9. The Condition of Concurrency
   $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$
10. Angle Between the Lines
   $$\phi = \tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$$
   i. Condition of perpendicularity
   $m_1m_2 = -1$
   ii. Condition of parallelism
   $m_1 = m_2$
11. Equations of Any Line Perpendicular or Parallel to the Given Line
   i. Equation of any line perpendicular to $ax + by + c = 0$
   $bx - ay + k = 0$
   ii. Equation of any line parallel to $ax + by + c = 0$
   $ax + by + k = 0$
12. Length of Perpendicular $= \pm\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

13. Equations of the Bisectors

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

14. Acute and Obtuse Angle Bisector: Choose any one of the bisectors, say $B_1$, and any one of the lines, say $L_1$. If $|\tan\theta| < 1$, i.e., $\theta < 45°$, where $\theta$ is the angle between $L_1$ and $B_1$ then, $B_1$ is the acute angle bisector. Otherwise $B_1$ will be the obtuse angle bisector and obviously $B_2$ will be the acute angle bisector. If $|\tan\theta| > 1$, i.e., $\theta > 45°$, $B_1$ is an obtuse angle bisector.

15. Bisector of the Angle Containing the Origin
The equation of the bisector of angle containing the origin is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}.$$

The other bisector not containing the origin is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$ provided that $c_1$ and $c_2$ are positive.

16. Incentre

$$x = \frac{ax_1 + bx_2 + cx_3}{a + b + c}$$

$$y = \frac{ay_1 + by_2 + cy_3}{a + b + c}.$$

17. The perpendiculars drawn from the vertices of a triangle to the opposite sides meet at a point. This point is called orthocentre of the triangle.

18. The point of intersection of perpendicular bisectors of the sides of a triangle is called the circumcentre of the triangle. The point is equidistant from the vertices. The circle is called the circum circle.

## 2 Marks Questions

1. **2076 Set B Q.No. 4a** Find the equation of the straight line that has y – intercept '2' and is parallel to $4x - 8y + 3 = 0$. [2]
**Ans: $4x - 8y + 16 = 0$**

2. **2076 Set C Q.No. 4a** Find the equation of a straight line which passes through the point (4, – 5) and is perpendicular to $x + y = 5$. [2]
**Ans: $x - y - 9 = 0$**

3. **2075 GIE Q.No. 4a** Find the equation of the line passing through the point (2, –1) and is perpendicular to $7x - 5y + 9 = 0$. [2]
**Ans: $5x + 7y - 19 = 0$**

4. **2075 Set A Q.No. 4a** Two lines passing through the point (2, 3) make an angle of 45°. If the slope of one of the line is 2, find the slope of the other. [2]
**Ans: $-3$ or $\frac{1}{3}$**

5. **2075 Set B Q.No. 4a** Find the equations of the straight line passing through the point of intersection of the lines $x - y = 0$ and $x + y = 0$ and parallel to the line $2x + 3y = 5$. [2]
**Ans: $2x + 3y = 0$**

6. **2075 Set C Q.No. 4a** Find the equation of a straight line which is parallel to the lines $2x + 3y = 5$ and making x-intercept 3 on the x-axis. [2]
**Ans: $2x + 3y = 6$**

7. **2074 Supp Q.No. 4a** Find the equation of a straight line through the middle point of the line segment joining (2, – 4) and (2, 4) and parallel to the line $3x - 2y = 4$. [2]

8. **2074 Set A Q.No. 4a** Find the equation of a straight line through the mid point of the line segment connecting (2, –4) and (2, 4) and parallel to the line $3x - 2y = 4$. [2]
**Ans: $3x - 2y = 6$**

9. **2074 Set B Q.No. 4a** Find the equation of the straight line which is perpendicular to $3x - y + 4 = 0$ and passes through the point (2, 3). [2]
**Ans: $x + 3y = 11$**

10. **2073 Supp Q.No. 4a** The length of the perpendicular from the point (a, 3) on the line $3x+4y+5=0$ is 4. Find the value of a. [2]
**Ans: $a = 1$ or $-\frac{37}{3}$**

11. **2073 Set C Q.No. 4a** Determine the value of k so that the line $kx + 3y + 10 = 0$ will be perpendicular to the line $3x - 2y = 5$. [2]
**Ans: $k = 2$**

12. **2073 Set D Q.No. 4a** Prove that the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} - \frac{y}{a} = 1$ are perpendicular to each other. [2]

13. **2072 Supp Q.No. 4a** Find the equation of the line through (4, 1) which is perpendicular to the line $x - 2y - 4 = 0$. [2]
**Ans: $2x + y - 9 = 0$**

14. **2072 Set C Q.No. 4a** Find the equation of a straight line through the centroid of the triangle with vertices at (3, –4), (–2, 1) and (5, 0) and perpendicular to the line $x - 3y = 4$. [2]
**Ans: $3x + y = 5$**

15. **2072 Set D Q.No. 4a** If p is the length of the perpendicular dropped from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$, prove that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$. [2]

16. **2072 Set E Q.No. 4a** Find the equation of the straight line passing through the point $(x_0, y_0)$ and parallel to the line $ax + by + c = 0$. [2]
**Ans: $ax + by = ax_0 + by_0$**

17. **2071 Supp. Q.No. 4a** Find the equation of the straight line passing through the point $(x_0, y_0)$ and perpendicular to the straight line $ax+by+c = 0$. [2]
**Ans: $bx - ay + ay_0 - bx_0 = 0$**

18. **2071 Set C Q.No. 4a** Find the equation of the line through (4, 2) which is parallel to $x - 2y - 4 = 0$ [2]
**Ans: $x - 2y = 0$**

19. **2071 Set D Q.No. 4a** Find the obtuse angle between the lines $x - 3y = 6$ and $y = 2x + 5$. [2]
**Ans: 135°**

20. **2071 Old Q.No. 2b** Find the length of the perpendicular from (6, 8) to the line through (1, 5) and (9, 3). [2]
**Ans: $\sqrt{17}$**

21. **2070 Supp Q.No. 4a** If p is the length of the perpendicular dropped from the (a, b) on the line $\frac{x}{a} + \frac{y}{b} = 1$, prove that: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$. [2]

22. **2070 Set C Q.No. 4 a** Find the distance between the two parallel lines: $3x + 5y = 11$ and $3x + 5y = -23$ [2]
**Ans: $\sqrt{34}$ units**

23. **2070 Set D Q.No. 4 a** Find the equation of the line passing through the middle point of the line segment connecting (2,-4) and (2, 4) and parallel to the line $3x-2y = 4$. [2]
**Ans: $3x - 2y = 6$**

24. 2070 Old Q.No. 2 b Find the equation of the line which passes through origin and is perpendicular to $4x+3y-12=0$. [2]
**Ans: $3x - 4y = 0$**

25. 2069 Supp Q.No. 4 a Find the equation to the straight line which passes through the point (4, -5) and is perpendicular to $3x+4y+5=0$. [2]
**Ans: $4x - 3y = 31$**

26. 2069 [Set A]. Q. No. 4a Find the equation of a line through (5, 4) and perpendicular to the line $4x - 3y = 10$ [2]
**Ans: $3x + 4y - 31 = 0$**

27. 2069 [Set A] Old Q. No. 2b Find the value of k so that the line whose equation is $x + y = k$ will form a triangle of area 32 square units with coordinate axes. [2]
**Ans: $k = \pm 8$**

28. 2069 [Set B] Q. No. 4a Find the equation to the straight line that has y-intercept 3 and is parallel to the straight line $8x - 4y + 9 = 0$. [2]
**Ans: $8x - 4y + 12 = 0$**

29. 2069 [Set B] Old Q. No. 2b If a line with equation $5x+6y = 2k$ together with the coordinate axes form a triangle of area 135 sq units, find the value of k. [2]
**Ans: $k = \pm 45$**

30. 2068 Q.No. 4a Find the equation of the line parallel to the line $5x + 4y = 9$ and making an intercept –5 on the x-axis. [2]
**Ans: $5x + 4y + 25 = 0$**

31. 2068 Old Q.No. 2b Find the value of 'a' for which the lines $3x + y - 2 = 0$, $ax + 2y - 3 = 0$ and $2x - y - 3 = 0$ may be concurrent. [2]
**Ans: $a = 5$**

32. 2067 Q.No. 2b Find the value of k so that the line whose equation is $x + y = k$ will form a triangle with the coordinate axes whose area is 32 sq. units. [2]
**Ans: $k = \pm 8$**

33. 2066 Q.No. 2(a) Examine whether the points (0, 11), (2, 3) and (3, –1) are collinear or not. [2]
**Ans: Collinear**

34. 2065 Q. No. 2 b Find the equation of the line through the intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$, and cutting off equal intercepts from the axes. [2]
**Ans: $23x + 23y = 11$**

35. 2064 Q.No. 2(b) If p be the perpendicular distance of the origin from a line whose intercepts on the axes are a and b, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$. [2]

36. 2063 Q.No. 2(b) Find the intercepts on the axes made by the line $2x + 3y = 5$ [2]
**Ans: x – intercept = $\frac{5}{2}$; y-intercept = $\frac{5}{3}$**

37. 2062 Q.No. 2(b) Find the distance between the parallel lines $y = 2x + 4$ and $6x - 3y = 5$ [2]
**Ans: $\frac{17}{3\sqrt{5}}$ units**

38. 2061 Q.No. 2(b) Find the equation of straight lines which have slope – 1 and form a triangle of area 8 square units with coordinates axes. [2]
**Ans: $x + y - 4 = 0$**

39. 2060 Q.No. 2(b) Find the equation of the st. line whose slope is $\frac{1}{3}$ and passes through the intersection of the lines $y = x$ and $y = -x$. [2]
**Ans: $x - 3y = 0$**

40. 2059 Q.No. 2(b) Find the equation of the line through (5, 4) and perpendicular to the line $4x - 3y = 10$ [2]
**Ans: $3x + 4y - 31 = 0$**

41. 2058 Q.No. 2(b) Write the conditions for which the straight lines given by $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ will be parallel and perpendicular. [2]
**Ans: For parallel, $\frac{A_1}{B_1} = \frac{A_2}{B_2}$; for perpendicular, $A_1A_2 + B_1B_2 = 0$**

42. 2057 Q.No. 2(b) What are the standard forms of equation of a straight line? Find the slope of the line $\frac{x}{a} - \frac{y}{b} = 1$. [2]
**Ans: $y = mx + c$; $\frac{x}{a} + \frac{y}{b} = 1$; $x \cos\alpha + y \sin\alpha = p$; slope (m) = $\frac{b}{a}$**

43. 2056 Q.No. 2(b) Find the acute angle between the lines $x - 3y - 6 = 0$ and $y = 2x + 5$. [2]
**Ans: 45°**

## 6 Marks Questions

44. 2076 Set B Q.No. 13 Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. Also prove that the bisectors are at right angle to each other, and identify the bisector which contains the origin. [6]
**Ans: $4x + 7y + 11 = 0$ (bisector containing the origin), $7x - 4y + 3 = 0$**

45. 2076 Set C Q.No. 13 Find the bisectors of the angles between the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$. [6]

46. 2075 GIE Q.No. 13 If p and p' be the lengths of perpendiculars from the origin upon the straight line whose equations are $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos2\theta$ respectively, prove that $4p^2 + p'^2 = a^2$. [6]

47. 2075 Set A Q.No. 14 Derive the length of the perpendicular from a point $(x_1, y_1)$ to a line $x \cos\alpha + y \sin\alpha = p$. Also find the distance between the parallel lines $y = mx + c$ and $y = mx + d$. [6]
**Ans: $\pm\frac{c - d}{\sqrt{1 + m^2}}$**

48. 2075 Set B Q.No. 13 If p and p' be the length of the perpendiculars from the origin upon the straight lines whose equations are $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos2\theta$, prove that $4p^2 + p'^2 = a^2$. [6]

49. 2075 Set C Q.No. 13 Find the incentre of the triangle formed by the lines $x - y + 1 = 0$, $x - 7y + 7 = 0$ and $x + y - 3 = 0$. [6]
**Ans: $\left(1, \frac{3}{2}\right)$**

50. 2074 Supp Q.No. 13 If $p_1$ and $p_2$ are the lengths of the perpendiculars drawn from the points (cosθ, sinθ) and (–secθ, cosecθ) on the line $x \sec\theta + y \operatorname{cosec}\theta = 0$, prove that $\frac{4}{p_1^2} - p_2^2 = 4$. [6]

51. 2074 Set A Q.No. 13 Find the equations of the bisectors of the angles between the lines $4x-3y+1=0$ and $12x-5y+7=0$. Also prove that the bisectors are at right angles to each other. [6]
**Ans: $4x + 7y + 11 = 0$ and $7x - 4y + 3 = 0$**

52. 2074 Set B Q.No. 13 Find the equations of the bisectors of the angles between the lines 3x+4y+2=0 and 12x–5y+3=0. Also show that bisectors are at right angle. [6]

**Ans:** $21x - 77y - 11 = 0$ and $99x + 27y + 41 = 0$

53. 2073 Supp Q.No. 13 Find the angle between the lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the condition under which the two lines are (i) parallel, (ii) perpendicular. [6]

**Ans:** $\tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$ (i) $m_1 = m_2$ (ii) $m_1.m_2 = -1$

54. 2073 Set C Q.No. 13 If $P_1$ and $P_2$ be the lengths of the perpendicular drawn from the points $(\cos\theta, \sin\theta)$ and $(-\sec\theta, \text{cosec}\theta)$ on the line $x \sec\theta + y \text{ cosec}\theta = 0$ respectively, prove that: $\frac{4}{P_1^2} - P_2^2 = 4$. [6]

55. 2073 Set D Q.No. 13 If the length of the perpendicular from the point (1,1) to the line $ax + by - c = 0$ is 1, show that $\frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{c}{2ab}$. [6]

56. 2072 Supp Q.No. 13 Find the equations of the bisectors of the angles between the lines, $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$ and prove that the two bisectors are at right angles to each other. [6]

**Ans:** $4x + 7y + 11 = 0$; $7x - 4y + 3 = 0$

57. 2072 Set C Q.No. 13 Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$, and prove that the bisectors are at right angles to each other. Also identify the bisector of the angle between the lines containing the origin. [6]

**Ans:** $4x + 7y + 11 = 0$ (containing the origin), $7x - 4y + 3 = 0$

58. 2072 Set D Q.No. 12 Find the length of the perpendicular drawn from the point (x', y') on the line whose equation is $ax + by + c = 0$. [6]

59. 2072 Set E Q.No. 14 Derive the formula for the length of the perpendicular from a point $(x_1, y_1)$ to a line $x \cos\alpha + y \sin\alpha = p$. Also, find the distance between parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$. [6]

**Ans:** 19/26

60. 2071 Supp. Q.No. 14 Derive the formula for length of the perpendicular from a point $(x_1, y_1)$ to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between the parallel lines $5x - 12y + 8 = 0$ and $10x - 24y - 3 = 0$. [6]

**Ans:** $\frac{19}{26}$

61. 2071 Set C Q.No. 13 Or Find the bisectors of the angles between the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$. Also determine the condition that the bisector of the angle in which the origin lies. [6]

62. 2071 Set D Q.No. 13 Find the angle between the liens $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the condition of parallelism and perpendicularity of the lines. Hence find the angle between the lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$. [6]

**Ans:** 60°, 120°

63. 2070 Supp Q.No. 13 Find the angle between the two lines whose equations are $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$. Also, find the condition for two lines to be (i) parallel (ii) perpendicular to each other. [6]

64. 2070 Set C Q.No. 13 Find the length of the perpendicular drawn from the point $(x^1, y^1)$ on the line whose equation is $Ax + By + C = 0$. [6]

**Ans:** $\pm\left[\frac{Ax' + By' + C}{\sqrt{A^2 + B^2}}\right]$

65. 2070 Set D Q.No. 13 Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$ and prove that line bisectors are at right angles to each other. [6]

**Ans:** $4x + 7y + 11 = 0$; $7x - 4y + 3 = 0$

66. 2069 Supp Q.No. 13 Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. And prove that the bisectors are at right angles to each other. [6]

**Ans:** $4x + 7y + 11 = 0$; $7x - 4y + 3 = 0$

67. 2069 [Set A] Q. No. 13 If P and P' be the lengths of the perpendiculars from origin upon the straight lines whose equations are $x \sec\theta + y \text{ cosec }\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$ prove that: $4p^2 + p'^2 = a^2$. [6]

68. 2069 [Set B] Q. No. 13 Prove that the perpendicular from the origin upon the straight line joining the points $(c \cos\alpha, c \sin\alpha)$ and $(c \cos\beta, c \sin\beta)$ bisects the distance between them. [6]

69. 2068 Q.No. 13 Find the angle between two straight lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the conditions under which the two straight lines will be (i) parallel (ii) perpendicular. [6]

**Ans:** $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$ (i) $m_1 = m_2$ (ii) $m_1m_2 = -1$

## 4 Marks Questions (Old Syllabus Questions)

70. 2071 Old Q.No. 9a Find the length of the perpendicular from the point $(x^1, y^1)$ on a straight line $Ax + By + C = 0$. [4]

**Ans:** $\pm\left[\frac{Ax^1 + By^1 + C}{\sqrt{A^2 + B^2}}\right]$

71. 2070 Old Q.No. 9 a Find the equation to the straight line which passes through the intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and cuts off equal intercepts from the axes. [4]

**Ans:** $23x + 23y = 11$

72. 2069 [Set A] Old Q. No. 9a Find the equation of the straight line which passes through the point of intersection of $x + y = 18$ and $3x - 2y + 1 = 0$, and is parallel to the straight line joining the points (3, 4) and (5, 6). [4]

**Ans:** $x - y + 4 = 0$

73. 2069 [Set B] Old Q. No. 9a Find the equation of the sides of an equilateral triangle whose vertex is (–1, 2) and base is $y = 0$. [4]

**Ans:** $\sqrt{3}x - y + 2 + \sqrt{3} = 0$; $\sqrt{3}x + y + \sqrt{3} - 2 = 0$

74. 2068 Old Q.No. 9a Find the angle between two straight lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the conditions for the two lines to be (i) parallel (ii) perpendicular. [4]

**Ans:** $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$ (i) $m_1 = m_2$ (ii) $m_1m_2 = -1$

75. 2067 Q.No. 9a Find the equation to the straight line which makes equal intercepts on the axes and passes through the point of intersection of the lines $2x - 3y + 1 = 0$ and $x + 2y - 2 = 0$. [4]

**Ans:** $7x + 7y = 9$

76. **2066 Q.No. 9(a)** Determine the value of m for which the straight lines $y = x + 1$, $y = 2(x + 1)$ and $y = mx + 3$ are concurrent. [4]

**Ans: m = 3**

77. **2065 Q. No. 9 a** The origin is a corner of a square and two of its sides are given by $2x + y = 0$ and $2x + y = 3$. Find the equations of the other two sides. [4]

**Ans: $x - 2y = 0$, $x - 2y + 3 = 0$**

78. **2064 Q.No. 9(b)** Find the equations of the straight lines which passes through the point (2, 3) and are inclined at 45° to the straight line $x + 3y + 4 = 0$. [4]

**Ans: $x - 2y + 4 = 0$, $2x + y - 7 = 0$**

79. **2063 Q.No. 9(a)** Prove that the equation of the straight line which passes through the point $(a\cos^3\theta, a\sin^3\theta)$ and is perpendicular to the straight line $x\sec\theta + y\operatorname{cosec}\theta = 0$ is $x\cos\theta - y\sin\theta = a\cos 2\theta$. [4]

80. **2063 Q.No. 9(a) OR** Find the equations of the bisectors of the angles between the straight lines $3x - 4y + 3 = 0$ and $12x - 5y - 1 = 0$. [4]

**Ans: $21x + 27y - 44 = 0$; $99x - 77y + 34 = 0$**

81. **2062 Q.No. 9(a)** Find the equation of the locus of a point P which is equi–distant from $3x - 4y + 2 = 0$ and the origin. [4]

**Ans: $16x^2 + 9y^2 + 24xy - 12x + 16y - 4 = 0$**

82. **2061 Q.No. 9(a)** Find the equation to the straight line which passes through the intersection of the straight lines $3x - 4y + 1 = 0$ and $5x + y = 1$ and cuts off equal intercepts from the axes. [4]

**Ans: $23x + 23y = 11$**

83. **2060 Q.No. 9(a)** Find the equation of the line through the point that divides the join of the points (–3, –4) and (7, 1) in the ratio 3:2 and perpendicular to the join. [4]

**Ans: $2x + y - 5 = 0$**

84. **2059 Q.No. 9(a)** Find the length of the perpendicular from the point $(x_1, y_1)$ on a straight line $x\cos\alpha + y\sin\alpha = p$. [4]

**Ans: $\pm(x_1\cos\alpha + y_1\sin\alpha - p)$**

85. **2058 Q.No. 9(a)** Find the length of the perpendicular from the point (h, k) on a straight line $x\cos\alpha + y\sin\alpha = p$. [4]

**Ans: $\pm(h\cos\alpha + k\sin\alpha - p)$**

86. **2057 Q.No. 9(a)** Find the length of the perpendicular from the point $(x_1, y_1)$ on a straight line $x\cos\alpha + y\sin\alpha = p$ [4]

**Ans: $\pm(x_1\cos\alpha + y_1\sin\alpha - p)$**

87. **2056 Q.No. 9(a)** Find the angles between two lines given by $y = m_1x + c_1$ and $y = m_2x + c_2$. Also state the condition for them to be perpendicular and parallel. [4]

**Ans: $\phi = \tan^{-1}\left(\pm\frac{m_1 - m_2}{1 + m_1m_2}\right)$; for $\perp$ $m_1m_2 = -1$; for // $m_1 = m_2$**

## B. Pair of Lines

### FORMULAE AND IMPORTANT POINTS

1. The second degree homogeneous equation $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines through origin

$$x = \frac{\left(-h + \sqrt{h^2 - ab}\right)}{a}y,\ x = \frac{\left(-h - \sqrt{h^2 - ab}\right)}{a}y.$$

2. Angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$

$$\theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$$

i. Condition of perpendicularity
$a + b = 0$

ii. Conditions of coincidence lines
$h^2 = ab$

3. Equations of Bisectors Represented by $ax^2 + 2hxy + by^2 = 0$ are

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

4. Condition for the general equation of second degree to represent a pair of lines
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

### 2 Marks Questions

1. **2071 Old Q.No. 2c** Find the equation of the bisectors of the angles between the lines represented by $2x^2 - 6xy - y^2 = 0$. [2]

**Ans: $x^2 + xy - y^2 = 0$**

2. **2070 Old Q.No. 2 c** Write the conditions of perpendicularity and parallelism of the lines represented by $lx^2 + 2hxy + ny^2 = 0$. [2]

**Ans: $l + n = 0$; $h^2 = ln$**

3. **2069 [Set A] Old Q. No. 2c** Find the angle between the pair of lines given by $2x^2 + 7xy + 3y^2 = 0$. [2]

**Ans: 45° or 135°**

4. **2069 [Set B] Old Q. No. 2c** Find the value of k so that: $x^2 + kxy + 2y^2 + 3x + 5y + 2 = 0$ may represent a line pair. [2]

**Ans: $k = \frac{9}{2}$ or 3**

5. **2068 Old Q.No. 2c** Find the equations of the bisectors of the angles between the lines represented by $3x^2 - 15xy + 2y^2 = 0$. [2]

**Ans: $15x^2 + 2xy - 15y^2 = 0$**

6. **2067 Q.No. 2c** Find the angle between the lines given by $x^2 - 2xy\cot\theta - y^2 = 0$ [2]

**Ans: 90°**

7. **2066 Q.No. 2(c)** For what value of K, the equation $2x^2 + 7xy + 3y^2 - 4x - 7y + K = 0$ represents a line pair? [2]

**Ans: K = 2**

8. **2065 Q. No. 2 c** Show that the equation $kx^2 + (k^2 - 1)xy - ky^2 = 0$ represents a pair of perpendicular lines for all values of k. [2]

9. **2064 Q.No. 2(c)** Find the equations of the two lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$ [2]

**Ans: $x + y + 3 = 0$, $2x + y - 1 = 0$**

10. **2063 Q.No. 2(c)** Find the angle between the lines represented by $2x^2 + 7xy + 3y^2 = 0$. [2]

**Ans: 45° or 135°**

11. **2062 Q.No. 2(c)** Find the value of k so that $2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$ may represents a pair of lines. [2]

**Ans: k = 2**

12. **2061 Q.No. 2(c)** Find the angle between the pair of lines $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$. [2]

**Ans: θ = 0°**

13. **2060 Q.No. 2(c)** Verify that whether the second degree equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents a pair of st. lines or not. [2]

**Ans: represents**

14. **2059 Q.No. 2(c)** Find the angle between the line pair $2x^2 + 7xy + 3y^2 = 0$ [2]

**Ans: 45° or 135°**

15. 2058 Q.No. 2(c) Find the angle between the line pair given by: $x^2 - 2xy \cot\theta - y^2 = 0$. [2]

Ans: $\frac{\pi}{2}$

16. 2057 Q.No. 2(c) Determine the lines represented by the equation $x^2 + 2xy + y^2 - 2x - 2y - 5 = 0$ [2]

Ans: $x + y - 5 = 0$; $x + y + 3 = 0$

17. 2056 Q.No. 2(c) Write the condition for which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a line pair. [2]

Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

**6 Marks Questions**

18. 2076 Set B Q.No. 13 OR Prove that the bisectors of the angles between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$ [6]

19. 2076 Set C Q.No. 13 OR Find the angle between a pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. Also discuss the condition of parallelism and perpendicularity of the lines. [6]

Ans: $\tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a + b}\right)$; $h^2 = ab$; $a + b = 0$

20. 2075 GIE Q.No. 13 OR Find the bisectors of angles between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$. [6]

Ans: $h(x^2 - y^2) = (a - b)xy$

21. 2075 Set A Q.No. 14 OR Prove that the straight lines joining the origin to the intersection of the straight line $kx + hy = 2hk$ and the curve $(x - h)^2 + (y - k)^2 = a^2$ are perpendicular if $h^2 + k^2 = a^2$. [6]

22. 2075 Set B Q.No. 13 OR Find the bisectors of angles between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$. [6]

Ans: $h(x^2 - y^2) = (a - b)xy$

23. 2075 Set C Q.No. 13 OR Show that the two straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy \tan\theta + y^2\sin^2\theta = 0$ make with x-axis angles such that the difference of their tangent is 2. [6]

24. 2074 Supp Q.No. 13 OR Prove that the equation $ax^2 + 2hxy + by^2 = 0$ always represent a pair of straight lines through the origin. Are they always real? [6]

25. 2074 Set A Q.No. 13 OR Find the angle between the two lines represented by $ax^2 + 2hxy + by^2 = 0$. Also find the condition under which the two lines are a) perpendicular b) coincident. [6]

Ans: $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$ (a) $a + b = 0$ (b) $h^2 = ab$

26. 2074 Set B Q.No. 13 OR Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of parallelism and perpendicularity of the lines. [6]

Ans: $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$; $h^2 = ab$; $a = b$

27. 2073 Supp Q.No. 13 OR Find the equations of the two straight lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$. Find the point of intersection and also the angle between the lines. [6]

Ans: $x + y + 3 = 0$, $2x + y - 1 = 0$, $(4, -7)$, $\tan^{-1}\left(\pm\frac{1}{3}\right)$

28. 2073 Set C Q.No. 13 OR If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove that $pq = -1$. [6]

29. 2073 Set D Q.No. 13 OR Find the angle between a pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. Also discuss the condition of parallelism and perpendicularity of the lines. [6]

Ans: $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$; $h^2 = ab$; $a = b$

30. 2072 Supp Q.No. 13 OR Find the condition under which the general equation of second degree represents a line pair. [6]

Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

31. 2072 Set C Q.No. 13 OR Deduce the condition that the general equation of second degree $ax^2+2hxy+by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight lines. [6]

Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

32. 2072 Set D Q.No. 12 OR Find the equation of the two lines represented by the equation $2x^2 + 3xy + y^2 + 5x+2y - 3 = 0$. Find their point of intersection and also the angle between them. [6]

Ans: $x + y + 3 = 0$, $2x + y - 1 = 0$; $(4, -7)$, $\tan^{-1}\left(\pm\frac{1}{3}\right)$

33. 2072 Set E Q.No. 14 OR Show that two straight lines through the origin, which make an angle of 45° with the line $px + qy + r = 0$ are given by $(p^2 - q^2)(x^2 - y^2) + 4pqxy = 0$ [6]

34. 2071 Supp. Q.No. 14 OR If θ be the angle between the lines represented by $ax^2+2hxy+by^2 = 0$ then prove that: $\tan\theta = \pm 2\frac{\sqrt{h^2 - ab}}{a + b}$ when (i) $b \neq 0$ (ii) $b = 0$. [6]

35. 2071 Set C Q.No. 13 Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of parallelism and perpendicularity of the lines. Find the angle between the lines represented by $x^2 + 9xy + 14y^2 = 0$ [6]

Ans: $\tan^{-1}\left(\pm\frac{1}{3}\right)$

36. 2071 Set D Q.No. 13 Or Find the angle between the pair of lines represented by a homogeneous equation of second degree. Also derive the condition of coincidence and perpendicularity of the lines. Find the angles between the pair of lines represented by $7x^2 + 8xy + y^2 = 0$. [6]

Ans: $\tan^{-1}\left(\pm\frac{3}{4}\right)$

37. 2070 Supp Q.No. 13 Or Find the equations of two lines represented by the equation $2x^2+3xy+y^2+5x + 2y - 3 = 0$. Find their point of intersection. [6]

Ans: $x + y + 3 = 0$, $2x + y - 1 = 0$, $(4, -7)$

38. 2070 Set C Q.No. 13 Or Find the equation to the pair of lines joining the origin to the intersection of the straight line $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1+ m^2)$. [6]

Ans: $(c^2 - a^2m^2)x^2 + 2a^2mxy + (c^2 - a^2)y^2 = 0$

39. 2070 Set D Q.No. 13 Or Find the condition under which the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of lines. [6]

Ans: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

40. 2069 Supp Q.No. 13 Or Show that the straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy \tan\theta + y^2 \sin^2\theta = 0$ make with x-axis angles such that the difference of their tangents is 2. [6]

41. 2069 [Set A] Q. No. 13 OR Show that the homogeneous equation of degree two always represents a pair of straight line passing through the origin. Also, find the angel between them. [6]

Ans: $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$

42. **2069 [Set B] Q. No. 13 OR** Prove that the bisectors of the angles between the pair of straight lines $ax^2 - 2hxy + by^2 = 0$ is given by $\frac{x^2 - y^2}{xy} = \frac{a-b}{h}$. [6]

43. **2068 Q.No. 13(Or)** Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if: $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ [6]

**4 Marks Questions (Old Syllabus Questions)**

44. **2071 Old Q.No. 9b** Prove that the straight lines joining the origin to the point of intersection of the line bx+ ay – ab = 0 and the curve $x^2 + y^2 = c^2$ are right angels if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

45. **2070 Old Q.No. 9 b** Show that the straight lines joining the origin to the points of intersection of the line kx + hy = 2hk with the curve $(x - h)^2 + (y - k)^2 = c^2$ are at right angles if $h^2 + k^2 = c^2$. [4]

46. **2069 [Set A] Old Q. No. 9b** Find the equation of two straight lines joining the origin to the points of intersection of the straight line x – y = 2 and the curve $5x^2 + 12xy - 8y^2 + 8x - 4y + 12 = 0$. [4]
**Ans: $4x^2 - y^2 = 0$**

47. **2069 [Set B] Old Q. No. 9b** Show that: the two straight lines. $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with the x-axis angles such that the difference between their tangents is 2. [4]

48. **2068 Old Q.No. 9b** Find the equation to the pair of straight lines joining the origin to the intersection of the straight lines y = mx + c and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1 + m^2)$. [4]
**Ans: $(c^2 - a^2m^2)x^2 + 2a^2mxy + (c^2 - a^2)y^2 = 0$**

49. **2067 Q.No. 9b** Find the condition so that the straight lines joining the origin to the points of intersection of the line kx + hy = 2hk with the circle $(x - h)^2 + (y - k)^2 = c^2$ are at right angle. [4]

50. **2066 Q.No. 9 (b)** Find the equation of the lines which are right angles to the lines represented by $ax^2+2hxy+by^2 = 0$. [4]

51. **2065 Q. No. 9 b** Show that pair of lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with the axis of x angle such that the difference of their tangent is 2. [4]

52. **2064 Q.No. 9(a)** Find the single equation of the lines through the origin and perpendicular to the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$. [4]
**Ans: $ay^2 - 2hxy + bx^2 = 0$**

53. **2063 Q.No. 9(b)** Find the equations of the two lines represented by $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$. Prove that the two lines are parallel. [4]
**Ans: x + 3y – 1 = 0; x + 3y + 5 = 0**

54. **2062 Q.No. 9(b)** Determine the two straight lines represented by $6x^2 - xy - 12y^2 - 8x + 29y - 14 = 0$ [4]
**Ans: 2x – 3y + 2 = 0; 3x + 4y – 7 = 0**

55. **2061 Q.No. 8(b)** For what value of c the lines joining the origin to the point of intersection of the line x – y + c = 0 and the curve $x^2 + y^2 + 4x - 6y - 36 = 0$ may be at right angles. [4]
**Ans: c = 9 or – 4**

56. **2060 Q.No. 9(b)** Show that the lines joining the points of intersection of the line x + y = 1 with the curve $4x^2 + 4y^2 + 4x - 2y - 5 = 0$ with the origin are at right angles to each other. [4]

57. **2059 Q.No. 9(b)** Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ [4]

58. **2058 Q.No. 9(b)** Prove that the pair of straight lines joining the origin to the points of intersection of the line y = mx + c and the curve $x^2 + y^2 = a^2$ are at right angles if $2c^2 = a^2(1 + m^2)$. [4]

59. **2057 Q.No. 9(b)** If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2q\,xy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove pq = – 1. [4]

60. **2056 Q.No. 9(b)** Prove that the straight lines joining the origin to the point of intersections of the line $\frac{x}{a} + \frac{y}{b} = 1$, and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [4]

# Unit 11: Circle

## FORMULAE AND IMPORTANT POINTS

1. Equation of Circle
   i. Centre at origin: $x^2 + y^2 = a^2$
   ii. Central Form: $(x - h)^2 + (y - k)^2 = a^2$
2. Particular Cases of Circles
   i. Equation of a circle with centre at (h, k) and touching the x–axis.
   $x^2 + y^2 - 2hx - 2ky + h^2 = 0$.
   ii. Equation of a circle with centre at (h, k) and touching the y-axis.
   $x^2 + y^2 - 2hx - 2ky + k^2 = 0$.
   iii. Equation of a circle with centre at (h, k) and touching both the axes.
   $x^2 + y^2 - 2ax - 2ay + a^2 = 0$.
   iv. Centre on x-axis
   $x^2 - 2hx + y^2 + h^2 - a^2 = 0$.
   v. Centre on y-axis
   $x^2 + y^2 - 2ky + k^2 - a^2 = 0$.
   vi. Diameter form
   $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
3. General Equation of a Circle
   $x^2 + y^2 + 2gx + 2fy + c = 0$
   Centre = (–g, – f)
   Radius = $\sqrt{g^2 + f^2 - c}$
4. Equation of the tangent to the circle $x^2 + y^2 = a^2$ at a point $(x_1, y_1)$ on the circle is
   $xx_1 + yy_1 = a^2$
5. Equation of the tangent at the point $(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
   $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
6. Equation of the normal at the point $(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$ is $xy_1 - yx_1 = 0$
7. Equation of the normal at the point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
   $x(y_1 + f) - y(x_1 + g) + gy_1 - fx_1 = 0$

8. Condition of tangency of a straight line $y = mx + c$ to a circle $x^2 + y^2 = a^2$ is $c = \pm a\sqrt{1+m^2}$
The equation of a tangent to the circle $x^2 + y^2 = a^2$ is $y = mx \pm a\sqrt{1+m^2}$, & the coordinates of the point of contact are $\left(-\frac{am}{\sqrt{1+m^2}}, \frac{a}{\sqrt{1+m^2}}\right)$ and $\left(\frac{am}{\sqrt{1+m^2}}, -\frac{a}{\sqrt{1+m^2}}\right)$.
9. Length of the Tangent
i. The length of the tangent from the point $(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$ is $\sqrt{x_1^2 + y_1^2 - a^2}$
ii. The length of the tangent from the point $(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

**2 Marks Questions**

1. 2076 Set B Q.No. 4b Find the equation of the circle which passes through origin and makes equal intercepts on both axes. [2]
**Ans:** $x^2 + y^2 - ax - ay = 0$
2. 2076 Set C Q.No. 4b Find the equation of the circle which passes through origin and whose center is at (1, 1). [2]
**Ans:** $x^2 + y^2 - 2x - 2y = 0$
3. 2075 GIE Q.No. 4b Find the equation of the circle whose center is at (–3, –4) and touches the x-axis. [2]
**Ans:** $x^2 + y^2 + 6x + 8y + 9 = 0$
4. 2075 Set A Q.No. 4b If one end of a diameter of circle $x^2 + y^2 - 2x + 6y - 15 = 0$ is (4, 1), find the coordinate of the other end. [2]
**Ans:** (–2, –7)
5. 2075 Set B Q.No. 4b Find the equation of the circle which touches both axes and has radius 4 units. [2]
**Ans:** $x^2 + y^2 - 8x - 8y + 16 = 0$
6. 2075 Set C Q.No. 4b Find the equation of the circle which touches both the axes and has its centre on the line $x - 2y = 3$. [2]
**Ans:** $x^2 + y^2 + 6x + 6y + 9 = 0$
7. 2074 Supp Q.No. 4b Find the equation of a circle with centre at (4, -1) and passing through the origin. [2]
**Ans:** $x^2 + y^2 - 8x + 2y = 0$
8. 2074 Set A Q.No. 4b Find the equation of a circle having radius 10 units and equations of any two diameters are $x + 2y = 8$ and $x + y = 6$. [2]
**Ans:** $x^2 + y^2 - 16x + 4y - 32 = 0$
9. 2074 Set B Q.No. 4b Find the equation of the circle which touches the x-axis at (4, 0) and passes through the point (1, 1). [2]
**Ans:** $x^2 + y^2 - 8x - 10y + 16 = 0$
10. 2073 Supp Q.No. 4b Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at (2, 3). [2]
**Ans:** $x + y - 5 = 0$
11. 2073 Set C Q.No. 4b Find the equation to the circle which touches the coordinate axes at (a,0) and (0, a). [2]
**Ans:** $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
12. 2073 Set D Q.No. 4b Find the equation of the circle touching x–axis whose center is at (4,3). [2]
**Ans:** $x^2 + y^2 - 8x - 9y + 16 = 0$
13. 2072 Supp Q.No. 4b Find the equation of the circle whose two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [2]
**Ans:** $x^2 + y^2 - 16x + 4y - 32 = 0$
14. 2072 Set C Q.No. 4b Find the equation of the circle with centre at (p, q) and radius $\sqrt{p^2 + q^2}$. [2]
**Ans:** $x^2 + y^2 - 2px - 2qy = 0$
15. 2072 Set D Q.No. 4b Find the equation of the circle with two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [2]
**Ans:** $x^2 + y^2 - 16x + 4y - 32 = 0$
16. 2072 Set E Q.No. 4b Find the equation of a circle concentric with the circle $x^2 + y^2 + 2x + 6y - 39 = 0$ and passing through (7, –7). [2]
**Ans:** $x^2 + y^2 + 2x + 6y - 70 = 0$
17. 2071 Supp. Q.No. 4b If one end of the diameter of circle $x^2+y^2-2x+6y-15=0$ is (4, 1), find the coordinates of the other end. [2]
**Ans:** (–2, –7)
18. 2071 Set C Q.No. 4b Find the equation of the circle which touches the axes at (2, 0) and (0, 2). [2]
**Ans:** $x^2 + y^2 - 4x - 4y + 4 = 0$
19. 2071 Set D Q.No. 4b Find the equation of the circle whose center is at (3, 4) and touches the x–axis. [2]
**Ans:** $x^2 + y^2 - 6x - 8y + 9 = 0$
20. 2070 Supp Q.No. 4b Prove that the line $5x + 12y + 78 = 0$ is tangent to the circle $x^2 + y^2 = 36$. [2]
21. 2070 Set C Q.No. 4 b Find the equation of the circle whose two of the diameters are $x + y = 6$ and $x + 2y = 8$ and radius 10. [2]
**Ans:** $x^2 + y^2 - 16x + 4y - 32 = 0$
22. 2070 Set D Q.No. 4 b Find the centre and the radius of the circle $x^2 + y^2 + 4x - 6y + 4 = 0$. [2]
**Ans:** Centre at (–2, 3) and radius = 3
23. 2069 Supp Q.No. 4 b Find the equation to the circle which touches the x-axis and has centre at (3, 4). [2]
**Ans:** $x^2 + y^2 - 6x - 8y + 9 = 0$
24. 2069 [Set A] Q. No. 4b Find the equation of the circle concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5, 4). [2]
**Ans:** $x^2 + y^2 - 8x + 12y - 49 = 0$
25. 2069 [Set B] Q. No. 4b Find the equation to the circle which has the points (0, –1) and (2, 3) as ends of a diameter. [2]
**Ans:** $x^2 + y^2 - 2x - 2y - 3 = 0$
26. 2068 Q.No. 1c Find the equation of a circle with centre at (4, 5) and $3x - 4y + 5 = 0$ as the line tangent to the circle. [2]
**Ans:** $25x^2 + 25y^2 - 200x - 250y + 1016 = 0$
27. 2068 Q.No. 4b Find the equation of the circle with centre at (4, –1) and passing through the origin. [2]
**Ans:** $x^2 + y^2 - 8x + 2y = 0$
28. 2067 Q.No. 1c Find the equation of the circle which is concentric to the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passes through (5, 4). [2]
**Ans:** $x^2 + y^2 - 8x + 12y - 49 = 0$
29. 2066 C Q.No. 1 c Write the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [2]
**Ans:** $(g^2 + f^2 - c)(l^2 + m^2) = (n - lg - mf)^2$
30. 2066 Q.No. 1 c Find the equation of the circle which touches the coordinate axes at (a, 0) and (0, a). [2]
**Ans:** $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
31. 2065 Q.No. 1 (c) Find the equation of the circle which has its centre on the line $y = 2$ and passes through the points (2, 0), (4, 0) [2]
**Ans:** $x^2 + y^2 - 6x - 4y + 8 = 0$

32. **2064 Q.No. 1(c)** Find the equation of a circle whose two of the diameters are $x + y = 6$ and $x + 2y = 4$ and radius 10. [2]
Ans: $x^2 + y^2 - 8x - 4y - 80 = 0$

33. **2063 Q.No. 1(c)** Find the equation of the normal to the circle $x^2 + y^2 = a^2$ at the point $(x_1, y_1)$ [2]
Ans: $x_1y = xy_1$

34. **2062 Q.No. 1(c)** Find the equation of a circle with centre at (6, 5) and touching the line $3x - 2y + 2 = 0$. [2]
Ans: $13x^2 + 13y^2 - 156x - 130y + 693 = 0$

35. **2061 Q.No. 1(c)** Find the value of k so that the straight line $4x + 3y + k = 0$ may touch the circle $x^2+y^2-4x+10y+4=0$ [2]
Ans: – 18 or 32

36. **2060 Q.No. 1(c)** Find the centre and radius of the circle $2x^2 + 2y^2 - 12x + 4y = 1$. [2]
Ans: $(3, -1)$ and $\frac{\sqrt{41}}{2}$

37. **2059 Q.No. 1(c)** Find the equation of the circle which has centre at (a, b) and touches the y–axis. [2]
Ans: $x^2 + y^2 - 2ax - 2by + b^2 = 0$

38. **2058 Q.No. 1(c)** Determine the equation of the circle if the ends of a diameter be at (1, 2) and (3, 5). [2]
Ans: $x^2 + y^2 - 4x - 6y + 8 = 0$

39. **2057 Q.No. 1(c)** Find the value of k so that the length of the tangent from (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 1. [2]
Ans: $k = -5$

## 4 Marks Questions

40. **2076 Set B Q.No. 9a** Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

41. **2076 Set C Q.No. 9a** Find the condition that the line $px + qy = r$ is tangent to the circle $x^2 + y^2 = a^2$. [4]
Ans: $a^2(p^2 + q^2) = r^2$

42. **2075 GIE Q.No. 9a** Find the equation of the circle having the chord $x - y - 1 = 0$ of the circle $2x^2 + 2y^2 - 2x - 6y - 25 = 0$ as diameter. [4]
Ans: $x^2 + y^2 - 3x - y - 10 = 0$

43. **2075 Set A Q.No. 9a** Find the value of k if the length of the tangent from the point (1, 2) to the circle $x^2+y^2+x+y-4 = 0$ and $3x^2 + 3y^2 - x - y - k = 0$ are in the ratio 4:3. [4]
Ans: $\frac{21}{4}$

44. **2075 Set B Q.No. 9a** Find the condition that the line $lx + my + n = 0$ should be tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]
Ans: $(g^2 + f^2 - c)(l^2 + m^2) = (ln - lg - mf)^2$

45. **2075 Set C Q.No. 9a** Find the equations of the tangent to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$. [4]
Ans: $4x + 3y + 19 = 0$, $4x + 3y - 31 = 0$

46. **2074 Supp Q.No. 9a** Prove that the straight line $y = x + a\sqrt{2}$ touches the circle $x^2 + y^2 = a^2$. Also find the point of contact. [4]
Ans: $\left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

47. **2074 Set A Q.No. 9a** Find the equations of the tangent and normal to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at (2, 3). [4]
Ans: $x + y - 5 = 0$, $x - y + 1 = 0$

48. **2074 Set B Q.No. 9a** Find the conditions for the circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ to touch (i) externally (ii) internally. [4]
Ans: (i) $a + b = c$ (ii) $a \sim b = c$

49. **2073 Supp Q.No. 9a** Find the equation of a circle passing through the points (1, 2), (3, 1) and (–3, –1). [4]
Ans: $x^2 + y^2 - x + 3y - 10 = 0$

50. **2073 Set C Q.No. 9a** For what value of 'a', the line $x + 3y = a$ touches the circle $x^2 + y^2 - 3x - 3y + 2 = 0$. Find also the point of contact. [4]
Ans: a = 1, 11; (1, 0) and (2, 3)

51. **2073 Set D Q.No. 9a** Find the condition that the line $lx + my + n = 0$ should be tangent to the circle $x^2+y^2+2gx+2fy+c = 0$. [4]
Ans: $(g^2 + f^2 - c)(l^2 + m^2) = (n - lg - mf)^2$

52. **2072 Supp Q.No. 9a** Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$. [4]
Ans: $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

53. **2072 Set C Q.No. 9a** Find the equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point $(x_1, y_1)$. [4]
Ans: $xx_1 + yy_1 = a^2$

54. **2072 Set D Q.No. 9a** Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ will be tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$ if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{r^2}$ [4]

55. **2072 Set E Q.No. 9a** Find the equations of the circle and tangent to the circle perpendicular to the diameter whose end points are (3, –4) and (–4, –3). [4]
Ans: $x^2 + y^2 + x + 7y = 0$, $7x - y \pm 25 = 0$

56. **2071 Supp. Q.No. 9a** Find the value of k if the line $4x - 3y + k = 0$ is tangent to the circle. $x^2+y^2-8x+12y+3= 0$. [4]
Ans: k = 1 or –69

57. **2071 Set C Q.No. 9a** Prove that the tangents to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$. [4]

58. **2071 Set D Q.No.9a** Prove that the two circle $x^2+y^2+2ax+c=0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

59. **2070 Supp Q.No. 9a** Find the condition that the line $y=mx+c$ may be tangent to the circle $x^2 + y^2 = a^2$. Also, find the equation of the tangent in the slope form. [4]
Ans: $c = \pm a\sqrt{1 + m^2}$, $y = mx \pm a\sqrt{1 + m^2}$

60. **2070 Set C Q.No. 9 a** Find the equations of the tangent and normal to the circle $x^2 + y^2 - 3x + 10y - 5 = 0$ at the point (4, –11). [4]
Ans: Point doesn't lie on the circle

61. **2070 Set D Q.No. 9 a** Find the value of k so that the line $4x + 3y + k = 0$ may touch the circle $x^2+y^2-4x+10y + 4 = 0$. [4]
Ans: k = –18 or 32

62. **2069 Supp Q.No. 9 a** Prove that the circles $x^2+y^2+2ax+c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

63. **2069 [Set A] Q. No. 9a** Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ which are perpendicular to $3x - 4y = 14$ [4]
Ans: $4x + 3y + 5 = 0$ and $4x + 3y - 25 = 0$

64. 2069 [Set B] Q. No. 9a Show that the circles $x^2 + y^2 - 6x - 6y + 10 = 0$ and $x^2 + y^2 = 2$ touch each other at (1, 1). [4]

65. 2068 Q.No. 9a Find the equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point $(x_1, y_1)$. [4]

**Ans:** $xx_1 + yy_1 = a^2$

66. 2068 Q.No. 9a Show that the tangents to the circle $x^2+y^2=100$ at the points (6, 8) and (8, -6) are perpendicular to each other. [4]

67. 2067 Q.No. 9a Prove that the tangent to the circle $x^2 + y^2 = 5$ at the point (1,– 2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ and find the point of contact. [4]

**Ans:** (3, – 1)

68. 2066 C Q.No. 9 a A circle touches the parallel lines $3x - 4y = 7$ and $3x - 4y + 43 = 0$ and has its centre on the line $2x - 3y + 13 = 0$. Find its equation. [4]

**Ans:** $x^2 + y^2 + 4x - 6y - 12 = 0$

69. 2066 Q.No. 9 a Find the equation of the line through (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ [4]

**Ans:** $3x - 4y - 7 = 0$

70. 2065 Q.No. 9 (a) Find the equation of the line through the point (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$. [4]

**Ans:** $3x - 4y - 7 = 0$

71. 2064 Q.No. 9(a) Find the equation of the circle whose centre is at the point (h, k) and which passes through the origin. Prove that the equation of the tangent to the circle at the origin is $hx + ky = 0$. [4]

**Ans:** $x^2 + y^2 - 2xh - 2ky = 0$

72. 2063 Q.No. 9(a) Find the value of k so that the straight line $4x + 3y + k = 0$ may touch the circle $x^2 + y^2 - 4x + 10y + 4 = 0$ [4]

**Ans:** k = – 18 or 32

73. 2062 Q.No. 9(a) If the line $lx + my = 1$ touches the circle $x^2 + y^2 = a^2$ prove that the point (l, m) lies on a circle whose radius is $\frac{1}{a}$. [4]

74. 2061 Q.No. 9(a) A circle touches the parallel lines $3x-4y = 7$ and $3x - 4y + 43 = 0$ and has its centre on the line $2x - 3y + 13 = 0$. Find its equation. [4]

**Ans:** $x^2 + y^2 + 4x - 6y - 12 = 0$

75. 2060 Q.No. 9(a) Find the equation of the line through the point (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$. [4]

**Ans:** $3x - 4y - 7 = 0$

76. 2059 Q.No. 9(a) Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$. [4]

77. 2058 Q.No. 9(a) Obtain the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]

**Ans:** $(g^2 + f^2 - c)(l^2 + m^2) = (n - lg - mf)^2$

78. 2057 Q.No. 9(a) Obtain the condition for the straight line $y=mx+c$ to be a tangent to the circle $x^2 + y^2 = a^2$. [4]

**Ans:** $c = \pm a\sqrt{1 + m^2}$

# Unit 12: Limits and Continuity

## A. Limit

### FORMULAE

1. $\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
2. $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$
3. $\lim_{\theta\to 0} \sin\theta = 0$
4. $\lim_{\theta\to 0} \cos\theta = 1$
5. $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$
6. $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
7. $\lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1$

**2 Marks Questions**

1. 2076 Set B Q.No. 4c Evaluate: $\lim_{x\to 0} \frac{1 - \cos mx}{1 - \cos nx}$. [2]

**Ans:** $\frac{m^2}{n^2}$

2. 2076 Set C Q.No. 4c Evaluate: $\lim_{x\to 2} \frac{x^2 - 4x + 4}{x^2 + 3x - 10}$. [2]

**Ans:** 0

3. 2075 GIE Q.No. 4c Evaluate: $\lim_{x\to\infty} (\sqrt{x+a} - \sqrt{x})$. [2]

**Ans:** 0

4. 2075 Set B Q.No. 4c Evaluate: $\lim_{x\to\infty} \frac{3x^2 - 4}{4x^2}$. [2]

**Ans:** $\frac{3}{4}$

5. 2075 Set C Q.No. 4c Evaluate: $\lim_{x\to 0} \frac{1 - \cos x}{x^2}$. [2]

**Ans:** $\frac{1}{2}$

6. 2074 Supp Q.No. 4c Evaluate: $\lim_{x\to a} \frac{\tan(x - a)}{2x - 2a}$ [2]

**Ans:** $\frac{1}{2}$

7. 2074 Set A Q.No. 4c Evaluate: $\lim_{x\to p} \frac{x^2 - p^2}{\tan(x - p)}$. [2]

**Ans:** 2p

8. 2074 Set B Q.No. 4c Evaluate: $\lim_{x\to\infty} (\sqrt{2 + x} - \sqrt{x})$. [2]

**Ans:** 0

9. 2073 Supp Q.No. 4c Evaluate: $\lim_{x\to 64} \frac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4}$. [2]

**Ans:** $\frac{1}{4}$

10. 2073 Set D Q.No. 4c Evaluate: $\lim_{x\to 1} \frac{\sqrt{2x} - \sqrt{3 - x}}{x - 1}$. [2]

**Ans:** $\frac{3}{2\sqrt{2}}$

11. 2072 Supp Q.No. 4c Evaluate: $\lim_{x\to\pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$ [2]

Ans: 2

12. 2072 Set C Q.No. 4c Evaluate: $\lim_{x\to\infty} (\sqrt{x} - \sqrt{x-3})$ [2]

Ans: 0

13. 2072 Set D Q.No. 4c Evaluate: $\lim_{x\to 4} \frac{x^3 - 64}{x^2 - 16}$ [2]

Ans: 6

14. 2072 Set E Q.No. 4c Evaluate: $\lim_{x\to\infty} \frac{\sin x}{x}$ [2]

Ans: 0

15. 2071 Supp. Q.No. 4c Evaluate: $\lim_{x\to\infty} (\sqrt{x+1} - \sqrt{x})$. [2]

Ans: 0

16. 2071 Set C Q.No. 4c Evaluate $\lim_{x\to\infty} (\sqrt{x+a} - \sqrt{x})$ [2]

Ans; 0

17. 2071 Set D Q.No. 4c Evaluate: $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1}$ [2]

Ans: 1

18. 2071 Old Q.No. 5a Evaluate $\lim_{x\to p} \frac{x^2 - p^2}{\tan(x-p)}$ [2]

Ans: 2p

19. 2071 Old Q.No. 6a Find the limit of the function $f(x) = \begin{cases} 2x^2 + 1 & \text{for } x \le 2 \\ 4x + 1 & \text{for } x > 2 \end{cases}$ at $x = 2$ if it exists. [2]

Ans:9

20. 2070 Supp Q.No. 4c Evaluate: $\lim_{x\to\infty} \sqrt{(x-a)} - \sqrt{(x-b)}$. [2]

Ans: 0

21. 2070 Set C Q.No. 4 c Evaluate: $\lim_{x\to 4} \frac{x^3 - 64}{x^2 - 16}$ [2]

Ans: 6

22. 2070 Set D Q.No. 4 c Evaluate : $\lim_{x\to\pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$ [2]

Ans: 2

23. 2070 Old Q.No. 5 a Evaluate: $\lim_{x\to 0} \frac{1 - \cos x}{x^2}$ [2]

Ans: $\frac{1}{2}$

24. 2069 Supp Q.No. 4 c Find the limit of the function $f(x) = \left.\begin{matrix} 3x^2 - 1 & \text{when} & x < 2 \\ 4x + 3 & \text{when} & x \ge 2 \end{matrix}\right\}$ at $x = 2$. [2]

Ans: 11

25. 2069 Set A Q. No. 4c Evaluate: $\lim_{x\to\infty} (\sqrt{x} - \sqrt{x-3})$ [2]

Ans: 0

26. 2069 Set A Old Q. No. 5a Evaluate: $\lim_{x\to a} \frac{\sqrt{3x} - \sqrt{2x+a}}{2(x-a)}$ [2]

Ans: $\frac{1}{4\sqrt{3a}}$

27. 2069 Set B Q. No. 4c Evaluate: $\lim_{x\to 0} \frac{1 - \cos px}{1 - \cos qx}$ [2]

Ans: $\frac{p^2}{q^2}$

28. 2069 Set B Old Q. No. 5c Evaluate: $\lim_{x\to a} (a - x) \tan \frac{\pi x}{2a}$ [2]

Ans: $\frac{2a}{\pi}$

29. 2068 Q.No. 4c Evaluate: $\lim_{x\to a} \frac{\sin(x-a)}{x^2 - a^2}$ [2]

Ans: $\frac{1}{2a}$

30. 2068 Old Q.No. 5a Evaluate: $\lim_{x\to a} \frac{\sin(x-a)}{x^2 - a^2}$ [2]

Ans: $\frac{1}{2a}$

31. 2067 Q.No. 5a Evaluate: $\mathop{Lt}_{x\to\infty} \left(x - \sqrt{x^2 + x}\right)$ [2]

Ans: $-\frac{1}{2}$

32. 2066 Q.No. 5(a) Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x}$ [2]

Ans: 0

33. 2066 Q.No. 6(a) Determine the limit of, $f(x) \left.\begin{matrix} = 2 - x^2 & \text{for } x \le 2 \\ = x - 4 & \text{for } x > 2 \end{matrix}\right\}$ at $x = 2$, if it exists. [2]

Ans: limit exists and is equal to –2

34. 2065 Q. No. 5 a Evaluate: $\lim_{x\to a} \frac{\sin(x-a)}{x^2 - a^2}$ [2]

Ans: $\frac{1}{2a}$

35. 2064 Q.No. 5(c) Evaluate $\lim_{x\to 0} \frac{\sin x^\circ}{x}$ [2]

Ans: $\frac{\pi}{180}$

36. 2063 Q.No. 5(c) Show that: $\mathop{Lt}_{x\to 2} \frac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}} = 8$ [2]

37. 2062 Q.No. 5(a) Evaluate: $\lim_{x\to a} \frac{x^{2/3} - a^{2/3}}{x - a}$ [2]

Ans: $\frac{2}{3a^{1/3}}$

38. 2061 Q.No. 5(a) Evaluate: $\lim_{x\to 1} \frac{\sqrt{2x} - \sqrt{3 - x^2}}{x - 1}$. [2]

Ans: $\sqrt{2}$

39. 2060 Q.No. 5(a) Does the limit of the function, $f(x) = \begin{cases} x \text{ when } x > 0 \\ -x \text{ when } x < 0 \end{cases}$ exist at $x = 0$? Justify your answer. [2]

Ans: exists & equals to 0

40. 2059 Q.No. 5(a) Evaluate $\lim_{x\to 0} \frac{1 - \cos px}{1 - \cos qx}$ [2]

Ans: $\frac{p^2}{q^2}$

41. 2059 Q.No. 6(a) Does the limit of the function, $f(x) = \left.\begin{matrix} 2x + 1 & \text{for } x > 1 \\ 4x^2 - 1 & \text{for } x < 1 \end{matrix}\right\}$ at $x = 1$ exist? [2]

Ans: Yes, $\lim_{x\to 1} f(x) = 3$

42. 2058 Q.No. 5(a) Evaluate: $\lim_{x\to\infty} (\sqrt{x+a} - \sqrt{x})$ [2]

Ans: 0

43. 2057 Q.No. 5(a) Evaluate: $\lim_{x\to 1} \frac{x^2 + 3x - 4}{x - 1}$ [2]

Ans: 5

**44.** 2057 Q.No. 6(a) Find the limit of the function $f(x) = \begin{cases} x^2 + 2, & x \le 5 \\ 3x + 12, & x > 5 \end{cases}$ at $x = 5$ if it exists. [2]

**Ans: 27**

**45.** 2056 Q.No. 5(a) Evaluate: $\lim_{x\to\infty} (\sqrt{x} - \sqrt{x-3})$ [2]

**Ans: 0**

**46.** 2056 Q.No. 6(a) Determine the limit of $f(x) = \begin{cases} 2 - x^2 & \text{for } x < 2 \\ x - 4 & \text{for } x > 2 \end{cases}$ at $x = 2$, if it exists. [2]

**Ans: – 2**

## 4 Marks Questions

**47.** 2076 Set B Q.No. 9b Evaluate: $\lim_{x\to\theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$. [4]

**Ans: $\cot\theta + \theta\sin\theta$**

**48.** 2076 Set C Q.No. 9b Evaluate: $\lim_{x\to y} \frac{x\sin y - y\sin x}{x - y}$. [4]

**Ans: $\sin y - y\sin y$**

**49.** 2075 GIE Q.No. 9b Prove geometrically: $\lim_{x\to0} \frac{\sin x}{x} = 1$. [4]

**50.** 2075 Set A Q.No. 9b Evaluate: $\lim_{h\to0} \frac{\cos^2(x+h) - \cos^2 x}{h}$ [4]

**Ans: $-\sin 2x$**

**51.** 2075 Set B Q.No. 9b Evaluate: $\lim_{x\to y} \frac{x\sin y - y\sin x}{x - y}$ [4]

**Ans: $\sin y - y\cos y$**

**52.** 2075 Set C Q.No. 9b Evaluate: $\lim_{x\to0} \frac{x\cos\theta - \theta\cos x}{x - \theta}$. [4]

**Ans: $\cos\theta + \theta\sin\theta$**

**53.** 2074 Supp Q.No. 9b Evaluate: $\lim_{x\to a} \frac{\sqrt{2x} - \sqrt{3x - a}}{\sqrt{x} - \sqrt{a}}$. [4]

**Ans: $-\frac{1}{\sqrt{2}}$**

**54.** 2074 Set A Q.No. 9b Evaluate: $\lim_{x\to2} \frac{\sqrt{x} - \sqrt{6 - x^2}}{x - 2}$. [4]

**Ans: $\frac{5}{2\sqrt{2}}$**

**55.** 2074 Set B Q.No. 9b Evaluate: $\lim_{y\to x} \frac{y\sec y - x\sec x}{y - x}$ [4]

**Ans: $\sec x(1 + x\tan x)$**

**56.** 2073 Supp Q.No. 9b Evaluate: $\lim_{x\to0} \frac{\tan 2x - \sin 2x}{x^3}$ [4]

**Ans: 4**

**57.** 2073 Set C Q.No. 9b Evaluate: $\text{Lt}_{x\to\theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$. [4]

**Ans: $\cos\theta + \theta\sin\theta$**

**58.** 2073 Set D Q.No. 9b Evaluate: $\lim_{x\to y} \frac{x\cot y - y\cot x}{x - y}$. [4]

**Ans: $\cot y + y\,\text{cosec}^2 y$**

**59.** 2072 Supp Q.No. 9b Evaluate: $\lim_{x\to a} \frac{\sqrt{3a - x} - \sqrt{x + a}}{4(x - a)}$ [4]

**Ans: $-\frac{1}{4\sqrt{2a}}$**

**60.** 2072 Set C Q.No. 9b Evaluate: $\lim_{x\to\theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$ [4]

**Ans: $\cos\theta + \theta\sin\theta$**

**61.** 2072 Set D Q.No. 9b Evaluate: $\lim_{x\to0} \frac{\tan x - \sin x}{x^3}$. [4]

**Ans: $\frac{1}{2}$**

**62.** 2072 Set E Q.No. 9b Evaluate: $\lim_{x\to1} \frac{\sqrt{x+2} - \sqrt{4 - x}}{x^2 - 1}$. [4]

**Ans: $\frac{1}{2\sqrt{3}}$**

**63.** 2071 Supp. Q.No. 9b Evaluate: $\lim_{x\to y} \frac{x\sin y - y\sin x}{x - y}$ [4]

**Ans: $\sin y - y\cos y$**

**64.** 2071 Set C Q.No. 9b Prove geometrically: $\lim_{\theta\to0} \sin\theta = 0$ [4]

**65.** 2071 Set D Q.No. 9b Evaluate: $\lim_{x\to\theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$. [4]

**Ans: $\cot\theta + \theta\,\text{cosec}^2\theta$**

**66.** 2071 Old Q.No. 12b Or Evaluate: $\lim_{x\to1} \frac{1 + \cos\pi x}{\tan^2 \pi x}$ [4]

**Ans: $\frac{1}{2}$**

**67.** 2070 Supp Q.No. 9b Evaluate: $\lim_{x\to0} \frac{\tan 2x - \sin 2x}{x^3}$. [4]

**Ans: 4**

**68.** 2070 Set C Q.No. 9 b Evaluate : $\lim_{x\to\theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$. [4]

**Ans: $\cot\theta + \theta\,\text{cosec}^2\theta$**

**69.** 2070 Set D Q.No. 9 b Evaluate: $\lim_{x\to\infty} \sqrt{x}(\sqrt{x} - \sqrt{x - a})$. [4]

**Ans: $\frac{9}{2}$**

**70.** 2070 Old Q.No. 12 b Evaluate: $\lim_{x\to y} \frac{x\cos y - y\cos x}{x - y}$ [4]

**Ans: $\cos y + y\sin y$**

**71.** 2069 Supp Q.No. 9 b Prove geometrically that: $\text{Lt}_{\theta\to0} \frac{\sin\theta}{\theta} = 1$. [4]

**72.** 2069 Set A Q. No. 9b Evaluate: $\lim_{x\to\theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$ [4]

**Ans: $\cos\theta + \theta\sin\theta$**

**73.** 2069 Set A Old Q. No. 12b Evaluate: $\lim_{x\to y} \frac{x\cos y - y\cos x}{x - y}$. [4]

**Ans: $\cos y + y\sin y$**

**74.** 2069 Set B Q. No. 9b Evaluate: $\lim_{x\to2} \frac{x - \sqrt{8 - x^2}}{\sqrt{x^2 + 12} - 4}$ [4]

**Ans: 4**

**75.** 2069 Set B Old Q. No. 13b Evaluate: $\text{Lt}_{x\to1} \frac{x - \sqrt{2 - x^2}}{2x - \sqrt{2 + 2x^2}}$ [4]

**Ans: 2**

**76.** 2068 Q.No. 9b Evaluate: $\lim_{x\to a} \left(\frac{\sqrt{3x} - \sqrt{2x + a}}{2(x - a)}\right)$ [4]

**Ans: $\frac{1}{4\sqrt{3a}}$**

**77.** 2068 Old Q.No. 12b Evaluate: $\lim_{x\to a} \frac{\sqrt{3a - x} - \sqrt{x + a}}{4(x - a)}$ [4]

**Ans: $-\frac{1}{4\sqrt{2a}}$**

78. 2067 Q.No. 12 b Evaluate: $\lim\limits_{x\to 0} \frac{(a+x)\sec(a+x) - a\sec a}{x}$ [4]

Ans: $a \sin a \sec^2 a + \sec a$

79. 2066 Q.No. 12 (b) Evaluate: $\lim\limits_{x\to \theta} \frac{x\sin\theta - \theta\sin x}{x-\theta}$ [4]

Ans: $\sin\theta - \theta\cos\theta$

80. 2065 Q. No. 12 b Evaluate: $\lim\limits_{x\to \pi/2} \frac{\tan x + \cot x}{\tan x - \cot x}$ [4]

Ans: 1

81. 2064 Q.No. 13(b) Evaluate: $\lim\limits_{x\to y} \frac{\tan x - \tan y}{x-y}$ [4]

Ans: $\sec^2 y$

82. 2063 Q.No. 6(c) Find the limit of the function for $f(x) = x + 2$ when $x \geq 0$ and $f(x) = 4x+2$ when $x<0$ at $x = 0$. [4]

Ans: 2

83. 2063 Q.No. 13(b) OR Evaluate: $\lim\limits_{x\to \theta} \frac{x\cos\theta - \theta\cos x}{x-\theta}$ [4]

Ans: $\cos\theta + \theta\sin\theta$

84. 2062 Q.No. 12(b) Prove that: $\lim\limits_{x\to 1} \frac{x-\sqrt{2-x^2}}{2x-\sqrt{2+2x^2}} = 2$ [4]

85. 2062 Q.No. 12(b) OR A function is defined as: $f(x) = \begin{cases} 3x^2+2 & \text{if } x<1 \\ 2x+3 & \text{if } x \geq 1 \end{cases}$, find $\lim\limits_{x\to 1} f(x)$ [4]

Ans: 5

86. 2061 Q.No. 12(b) Evaluate: $\lim\limits_{x\to y} \frac{\tan x - \tan y}{x-y}$ [4]

Ans: $\sec^2 y$

87. 2060 Q.No. 12(b) Evaluate: $\lim\limits_{x\to 0} \frac{\tan x - \sin x}{x^3}$ [4]

Ans: $\frac{1}{2}$

88. 2059 Q.No. 12(b) Evaluate: $\lim\limits_{x\to\infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a})$ [4]

Ans: $\frac{a}{2}$

89. 2058 Q.No. 12(b) OR Prove geometrically: $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ [4]

90. 2057 Q.No. 12(b) Evaluate: $\lim\limits_{x\to y} \frac{\sin x - \sin y}{x-y}$ [4]

Ans: cos y

91. 2056 Q.No. 12(b) Prove geometrically: $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ [4]

## B. Continuity

### FORMULAE AND IMPORTANT POINTS

1. A function f (x) defined in the neighbourhood of the point x = a is said to be continuous at x = a if the following three conditions are satisfied:
   i. f (a) exists
   ii. $\lim\limits_{x\to a} f(x)$ exists
   iii. $\lim\limits_{x\to a} f(x) = f(a)$.
2. A function f (x) is said to be continuous at the point x = a if
   $\lim\limits_{x\to a+} f(x) = \lim\limits_{x\to a-} f(x) = f(a)$.
   Otherwise, the function f (x) is said to have a discontinuity at x = a.

### 2 Marks Questions

1. 2075 Set A Q.No. 4c Find the value of k for which the function [2]
   $f(x) = \begin{cases} \frac{x^2-2x}{x-2} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$
   is continuous at the point x = 3.

   Ans: 3

2. 2073 Set C Q.No. 4c Find the limit of $f(x) = \frac{x^2-4}{x-2}$ as $x\to 2$. Is f(x) continuous? If not, find the point of discontinuity. [2]

   Ans: 4; Not continuous; Point of discontinuity is x = 2

3. 2070 Old Q.No. 6 a A function f(x) is defined as follows:
   $f(x) = \begin{cases} x^2 & \text{when } x>1 \\ 2 & \text{when } x=1 \\ x & \text{when } x<1 \end{cases}$. Is the function continuous at x = 2? Justify. [2]

   Ans: Discontinuous

4. 2069 [Set A] Old Q. No. 6a Show that the function $f(x) = \begin{cases} x+1 & x\neq 1 \\ 0 & x=1 \end{cases}$ is not continuous at x = 1. [2]

5. 2069 [Set B] Old Q. No. 6c Prove that: $f(x) = \frac{2}{2+3x}$ is continuous at $x = \frac{-2}{3}$. [2]

6. 2068 Old Q.No. 6a Examine the continuity of the function $f(x) = \frac{x^2-16}{x-4}$ at x = 4. [2]

   Ans: Discontinuous

7. 2067 Q.No. 6a Test the continuity of
   $f(x) = \begin{cases} x+2 & \text{when } x\neq 2 \\ 4 & \text{when } x=2 \end{cases}$; at x = 2. [2]

   Ans: Continuous

8. 2065 Q. No. 6 a Discuss the continuity of the function $\frac{x^2-9}{x-3}$ and point out the discontinuity if exists. [2]

   Ans: if x = 3

9. 2064 Q.No. 6(c) Why the function $f(x) = \sin\frac{1}{x}$ is not continuous at x = 0? [2]

10. 2062 Q.No. 6(a) Is the function $f(x)=\frac{1}{1-x}$ continuous at x = 1? [2]

    Ans: Discontinuous

11. 2061 Q.No. 6(a) Show that the function
    $f(x) = \begin{cases} x+2 & \text{for } x\neq 2 \\ 0 & \text{for } x=2 \end{cases}$ is not continuous function at x = 2. [2]

12. 2060 Q.No. 6(a) A function is defined as
    $f(x) = \begin{cases} x^2-1 & \text{when } x<1 \\ x^2+1 & \text{when } x\geq 1 \end{cases}$
    Examine whether the function is continuous or not at x = 1. [2]

    Ans: Discontinuous

13. 2058 Q.No. 6(a) Is the function $f(x) = \frac{x^2-9}{x-3}$ continuous at x = 3? Justify your answer. [2]

    Ans: Discontinuous

**4 Marks Questions**

14. **2076 Set B Q.No. 9b OR** Given $f(x) = \begin{cases} 3-x, & 0 \le x < \frac{3}{2} \\ -3+x, & x \ge \frac{3}{2} \end{cases}$

Test f(x) for continuity at $x = \frac{3}{2}$. If f(x) is not continuous at $x = \frac{3}{2}$, how would you make it continuous? [4]

**Ans: Discontinuous**

15. **2076 Set C Q.No. 9b OR** Given $f(x) = \begin{cases} \frac{1}{2}+x, & 0 < x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{3}{2}-x, & \frac{1}{2} < x < 1 \end{cases}$

Show that f(x) has removable discontinuity at $x = \frac{1}{2}$. Define f(x) so as to make f(x) continuous at $x = \frac{1}{2}$. [4]

**Ans:** $\begin{cases} \frac{1}{2}+x, & 0 < x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2}-x, & \frac{1}{2} < x < 1 \end{cases}$

16. **2075 GIE Q.No. 9b OR** A function f(x) is defined as $f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$

Show that f(x) is continuous at x = 1 and x = 2. [4]

17. **2075 Set B Q.No. 9b OR** A function is defined as follows:

$$f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

Show that f(x) is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$. [4]

18. **2075 Set C Q.No. 9b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Is the function continuous at x = 1? If not, can it be made continuous at x = 1 ? [4]

**Ans: Discontinuous, Yes**

19. **2074 Supp Q.No. 9b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} \frac{x^2-x-6}{x^2-2x-3} & \text{for } x \ne 3 \\ \frac{5}{3} & \text{for } x = 3 \end{cases}$$

Prove that f(x) is discontinuous at x = 3. Can the definition of f(x) be modified so as to make it continuous at x = 3? If so, how? [4]

20. **2074 Set A Q.No. 9b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x+3 & \text{for } x < 1 \\ 4 & \text{for } x = 1 \\ 6x-1 & \text{for } x > 1 \end{cases}$$

Is the function continuous at x = 1? If not, how can you make it continuous? [4]

**Ans: No,** $f(x) = \begin{cases} 2x+3 & \text{for } x < 1 \\ 5 & \text{for } x = 1 \\ 6x-1 & \text{for } x > 1 \end{cases}$

21. **2074 Set B Q.No. 9b OR** A function f(x) is defined in (0, 3) as follows:

$$f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$$

Show that f(x) is continuous at x = 1 and x = 2. [4]

22. **2073 Supp Q.No. 9B OR** A function f(x) is defined in (0, 3) in the following way: [4]

$$f(x) = \begin{cases} x^2 & \text{when } 0 < x < 1 \\ x & \text{when } 1 \le x < 2 \\ \frac{1}{4}x^3 & \text{when } 2 \le x < 3 \end{cases}$$

Examine the continuity of the function f(x) at x=1 and x=2.

**Ans: Continuous at x = 1 and x = 2.**

23. **2073 Set C Q.No. 9b OR** A function f(x) is defined by

$$f(x) = \begin{cases} 3+x & \text{when } -\frac{3}{2} \le x < 0 \\ 3-x & \text{when } 0 \le x < \frac{3}{2} \\ -3+x & \text{when } x \ge \frac{3}{2} \end{cases}$$

Test the continuity of f(x) at x = 0 and $x = \frac{3}{2}$. [4]

**Ans: Continuous at x = 0 and Discontinuous at $x = \frac{3}{2}$**

24. **2073 Set D Q.No. 9b OR** A function f(x) is defined by

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ 1 & \text{when } x = \frac{1}{2} \\ 1-x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

Examine whether f(x) is continuous at $x = \frac{1}{2}$. If not, define f(x) so as to make it continuous. [4]

**Ans: Discontinuous,** $f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{when } x = \frac{1}{2} \\ 1-x & \text{when } \frac{1}{2} < x < 1 \end{cases}$

25. **2072 Supp Q.No. 9b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3+2x & \text{for } -3/2 \le x < 0 \\ 3-2x & \text{for } 0 \le x < 3/2 \\ -3-2x & \text{for } x \ge 3/2 \end{cases}$$

Show that the function f(x) is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$ [4]

26. **2072 Set C Q.No. 9b OR** A function f(x) is defined as:

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x^2 - 2x - 3}, & x \neq 3 \\ \dfrac{5}{3}, & x = 3 \end{cases}$$

Prove that f(x) is discontinuous at x = 3. Can the definition of f(x) for x = 3 be modified so as to make it continuous there? [4]

**Ans: Yes**

27. **2072 Set D Q.No. 9b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} x^2 + 2 & \text{for } x < 5 \\ 20 & \text{for } x = 5 \\ 3x + 12 & \text{for } x > 5 \end{cases}$$

Show that f(x) has removable discontinuity at x = 5. Is it possible to make the above function continuous at x = 5? If yes, how can it be made? [4]

**Ans: Yes;** $f(x) = \begin{cases} x^2 + 2 & \text{for } x < 5 \\ 27 & \text{for } x = 5 \\ 3x + 12 & \text{for } x > 5 \end{cases}$

28. **2072 Set E Q.No. 9b OR** A function f(x) is defined as:

$$f(x) = \begin{cases} x^2 - 5 & \text{for } x < 4 \\ 8 & \text{for } x = 4 \\ 2x + 3 & \text{for } x > 4 \end{cases}$$

Show that the function is discontinuous at x = 4. Is it possible to make it continuous at x = 4. If possible, how? [4]

**Ans: Yes,** $f(x) = \begin{cases} x^2 - 5 & \text{for } x < 4 \\ 11 & \text{for } x = 4 \\ 2x + 3 & \text{for } x > 4 \end{cases}$

29. **2071 Supp. Q.No. 9b OR** A function f(x) is defined as:

$$f(x) = \begin{cases} 5x^2 + 3 & \text{for } x>1 \\ 9 & \text{for } x=1 \\ 6x + 2 & \text{for } x<1 \end{cases}$$

Show that f(x) has a removable discontinuity at x = 1. [4]

30. **2071 Set C Q.No. 9bOr** Show that the function

$$f(x) = \begin{cases} x, & \text{when } 0 \le x < 1/2 \\ 1, & \text{when } x = 1/2 \\ 1 - x, & \text{when } 1/2 < x < 1 \end{cases}$$

is discontinuous at $x = \frac{1}{2}$,

Also, redefine f(x) so as to f(x) be continuous at $x = \frac{1}{2}$. [4]

**Ans:** $f(x) = \begin{cases} x, & \text{when } 0 \le x < 1/2 \\ \frac{1}{2}, & \text{when } x = 1/2 \\ 1 - x, & \text{when } 1/2 < x < 1 \end{cases}$

31. **2071 Set D Q.No. 9b OR** Show that the function

$$f(x) = \begin{cases} \dfrac{1}{2-x}, & \text{when } 0 < x < 1/2 \\ \dfrac{1}{2}, & \text{when } x = 1/2 \\ \dfrac{3}{2-x}, & \text{when } 1/2 < x < 1 \end{cases}$$

is discontinuous at x = 1/2. Also, redefine f(x) so as to f(x) be continuous at x = 1/2. [4]

32. **2071 Old Q.No. 12b** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3 + 2x & \text{for } 3/2 \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < 3/2 \\ -3 - 2x & \text{for } x \ge 3/2 \end{cases}$$

Show that f(x) is continuous at x=0 & discontinuous at $x=\frac{3}{2}$. [4]

33. **2070 Supp Q.No. 9b Or** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Is the function continuous at x = 1? If not, why? How can the function be made continuous at x = 1? [4]

**Ans: Discontinuous,** $f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$

34. **2070 Set C Q.No. 9 b Or** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3 + 2x & \text{for } \frac{-3}{2} \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < \frac{3}{2} \\ -3 - 2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

Show that f(x) is continuous at x=0 but discontinuous at $x=\frac{3}{2}$. [4]

35. **2070 Set D Q.No. 9 b Or** A function f(x) is defined below:

$$f(x) = \begin{cases} kx + 3 & \text{for } x \ge 2 \\ 3x - 1 & \text{for } x < 2 \end{cases}$$

Find the value of k so that f(x) is continuous at x = 2. [4]

**Ans: 1**

36. **2070 Old Q.No. 12 b Or**

Investigate, the following function for continuity

$$f(x) = \left.\begin{matrix} 1 + x \text{ when } x > 0 \\ 2 \text{ when } x = 0 \\ -1 - x \text{ when } x < 0 \end{matrix}\right\} \text{ at } x = 0.$$ [4]

**Ans: Discontinuous**

37. **2069 Supp Q.No. 9 b Or** Given

$$f(x) = \begin{cases} 3-x & \text{when } 0 \le x < \frac{3}{2} \\ -3 + x & \text{when } x \ge \frac{3}{2} \end{cases}$$

Is f(x) continuous at x = 3/2? If not, how would you make it continuous? [4]

**Ans: Discontinuous**

38. **2069 [Set A] Q. No. 9b OR** Define continuity of a function at a point. A function is defined as follows:

$$f(x) = \begin{cases} \dfrac{2x^2 - 18}{x - 3} & \text{for } x \neq 3 \\ k & \text{for } x=3 \end{cases}$$

Find the value of k so that f(x) is continuous at x = 3. [4]

**Ans: k = 12**

39. **2069 [Set A] Old Q. No. 12b OR** Define continuity of a function at a point.

Given $f(x) = \begin{cases} 2-x^2 & \text{for } x<2 \\ 3 & \text{for } x=2 \\ x-4 & \text{for } x>2 \end{cases}$

Test the continuity of the functions at x = 2. [4]

**Ans: Discontinuous**

**40.** **2069 [Set B] Q. No. 9b OR** Show that the function

$$f(x) = \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ 1 & \text{when } x = \frac{1}{2} \\ 1-x & \text{when } \frac{1}{2} < x < 1 \end{cases}$$

Is discontinuous at $x = \frac{1}{2}$. Also, write how it could be made continuous? [4]

**Ans:** $f(x) \begin{cases} x & \text{when } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{when } x = \frac{1}{2} \\ 1-x & \text{when } \frac{1}{2} < x < 1 \end{cases}$

**41.** **2069 [Set B] Old Q. No. 13b OR** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x+3 & \text{for } x < 1 \\ 4 & \text{for } x = 1 \\ 6x-1 & \text{for } x > 1 \end{cases}$$ is the function continuous at x=1?

If not, how can you make it continuous? [4]

**Ans: Discontinuous** $f(x) = \begin{cases} 2x+3 & \text{for } x < 1 \\ 5 & \text{for } x = 1 \\ 6x-1 & \text{for } x > 1 \end{cases}$

**42.** **2068 Q.No. 9b(Or)** A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Is the function continuous at $x = 1$? If not, can it be made continuous at $x = 1$? [4]

**Ans: Discontinuous, Yes,** $f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$

**43. 2068 Old Q.No. 12b (Or)**

A functions f(x) is defined by as follows:

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Examine whether f(x) is continuous at $x = 1$. If not, can it be made continuous at $x = 1$? [4]

**Ans: Discontinuous, Yes,** $f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$

**44.** **2067 Q.No. 12b OR** Discuss the continuity of the function:

$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{when } x \neq 0 \\ = 0 & \text{when } = 0 \end{cases}; \text{ at } x = 0.$$ [4]

**Ans: Continuous**

**45.** **2066 Q.No. 12 (b) Or** A function is defined as follows:

$$f(x) = \begin{cases} = -x & \text{when } x < 0 \\ = x & \text{when } 0<x<1 \\ = 2-x & \text{when } x \ge 1 \end{cases}$$

show that it is continuous at $x = 0$ and $x = 1$. [4]

**46.** **2065 Q. No. 12 b OR** Test the continuity of the function:

$$f(x) = \begin{cases} x, \text{ when } 0 \le x < \frac{1}{2} \\ 1, \text{ when } x = \frac{1}{2} \\ 1-x, \text{ when } \frac{1}{2} < x < 1 \end{cases} \quad \text{at } x = \frac{1}{2}$$ [4]

**Ans: Discontinuous**

**47.** **2064 Q.No. 13(b) OR** Show that the function

$$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1 & , x = 0 \end{cases}$$ is discontinuous at $x = 0$.

Redefine the function in such a way that it becomes continuous at $x = 0$. [4]

**Ans:** $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ a^2 & , x = 0 \end{cases}$

**48.** **2063 Q.No. 13(b)** Let a function f(x) be defined by

$$f(x) = \begin{cases} 2-x^2 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \\ x-4 & \text{for } x > 2 \end{cases}$$

Verify that the limit of the function exists at $x = 2$. Is the function continuous at $x = 2$? State how you can make it continuous. [4]

**Ans: Discontinuous,** $f(x) = \begin{cases} 2-x^2 & \text{for } x < 2 \\ -2 & \text{for } x = 2 \\ x-4 & \text{for } x > 2 \end{cases}$

**49.** **2061 Q.No. 12(b) OR** Discuss the continuity of the function

$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{where } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$ [4]

**Ans: Continuous**

**50.** **2060 Q.No. 12(b) OR** Discuss the continuity of the function $f(x) = |x|$ at $x = 0$. [4]

**Ans: Continuous**

**51.** **2059 Q.No. 12(b) OR** When does a function f(x) become continuous at a given point $x = a$? Test the continuity of

$$f(x) = \left.\begin{matrix} 2x+1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{matrix}\right\} \text{ at } x = 1$$ [4]

**Ans: Discontinuous**

**52.** **2058 Q.No. 12(b)** Discuss the continuity of the function

$$f(x) = \left.\begin{matrix} 2x+1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{matrix}\right\} \text{ at } x = 1$$ [4]

**Ans: Discontinuous**

**53.** **2057 Q.No. 12(b) OR** When a function f(x) becomes continuous at $x = a$?

Discuss the continuity of $f(x) = \left.\begin{matrix} 2x+1 & \text{for } x < 1 \\ 2x & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{matrix}\right\} \text{ at } x = 1$ [4]

**Ans: Discontinuous**

**54.** **2056 Q.No. 12(b) OR** When does a function f(x) become continuous at $x = a$? Is the function f(x) defined by

$$f(x) = \left.\begin{matrix} = 3+2x & -3/2 \le x < 0 \\ = 3-2x & 0 \le x < 3/2 \\ = -3-2x & x \ge 3/2 \end{matrix}\right\} \text{ continuous at } x = \frac{3}{2}?$$ [4]

**Ans: Discontinuous**

# Unit 13: The Derivatives

## A. Derivatives of Algebraic Functions

### FORMULAE

1. Power Rule
$\frac{d}{dx}(x^n) = nx^{n-1}$
2. Derivative of a Constant
$\frac{d(c)}{dx} = 0, c \in \mathbb{R}$
3. Derivative of a Constant Times a Function
$\frac{d}{dx}[k \cdot f(x)] = k\frac{d}{dx}f(x)$
4. Derivative of a Sum of Functions
$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
5. Derivative of a Product of Functions
$\frac{d}{dx}[u \cdot v] = u\frac{d(v)}{dx} + v\frac{d(u)}{dx}$
6. Derivative of a Quotient of Functions
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
7. Derivative of Composite Function (Chain Rule)
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
8. General Power Rule
$\frac{du^n}{dx} = nu^{n-1} \cdot \frac{du}{dx}$

**2 Marks Questions**

1. 2076 Set B Q.No. 5a Find $\frac{dy}{dx}$ when $x = 2at$ and $y = 2at^2$. [2]
Ans: 2t

2. 2075 GIE Q.No. 5a Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$. [2]
Ans: $\frac{ay - x^2}{y^2 - ax}$

3. 2074 Supp Q.No. 5a Find $\frac{dy}{dx}$, when $y = \frac{1}{\sqrt{x^2+a^2} - \sqrt{x^2-a^2}}$. [2]
Ans: $\frac{x}{2a^2}\left(\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2-a^2}}\right)$

4. 2073 Set C Q.No. 5a Find $\frac{dy}{dx}$ when $y = (x+1)(x+2)(x+3)$. [2]
Ans: $3x^2 + 12x + 11$

5. 2073 Set D Q.No. 5a Find $\frac{dy}{dx}$ when $x = at$, $y = \frac{a}{t}$. [2]
Ans: $-\frac{1}{t^2}$

6. 2070 Supp Q.No. 5a Find $\frac{dy}{dx}$ when $y = \frac{1}{\sqrt[3]{3x^2 - 4x - 1}}$ [2]
Ans: $-\frac{2}{3}(3x-2)(3x^2-4x-1)^{-4/3}$

7. 2070 Set C Q.No. 5 a Find $\frac{dy}{dx}$ when $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. [2]
Ans: $\frac{t^2+1}{t^2-1}$

8. 2069 Supp Q.No. 5 a Find the $\frac{dy}{dx}$; when $y = \frac{1}{\sqrt{x+a} + \sqrt{x-a}}$ [2]
Ans: $\frac{1}{4a}\left(\frac{1}{\sqrt{x+a}} - \frac{1}{\sqrt{x-a}}\right)$

9. 2069 [Set A] Q. No. 5a Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3axy = 0$ [2]
Ans: $\frac{ay - x^2}{y^2 - ax}$

10. 2068 Q.No. 5a Find the derivative of $\frac{1}{x - \sqrt{a^2 + x^2}}$ [2]
Ans: $-\frac{1}{a^2}\left(1 + \frac{x}{\sqrt{a^2+x^2}}\right)$

11. 2066 Q.No. 5(c) Find $\frac{dy}{dx}$ if $ax^2 + 2hxy + by^2 = 1$ [2]
Ans: $\frac{dy}{dx} = \frac{-(ax+hy)}{by+hx}$

**4 Marks Questions**

12. 2076 Set B Q.No. 10a Find from first principles, the derivative of $(bx - a)^n$. [4]
Ans: $nb(bx - a)^{n-1}$

13. 2076 Set C Q.No. 10a Find from definition, the derivative of $(ax + b)^n$. [4]
Ans: $na(ax + b)^{n-1}$

14. 2075 GIE Q.No. 10a Find from definition, the derivative of $\sqrt{3x - 2}$. [4]
Ans: $\frac{3}{2\sqrt{3x-2}}$

15. 2075 Set C Q.No. 10a Find from first principles the derivative of $\frac{3x+5}{\sqrt{x}}$. [4]
Ans: $\frac{3x-5}{2x^{3/2}}$

16. 2074 Supp Q.No. 10a Find from first principles the derivative of $\sqrt{2x+3}$. [4]
Ans: $\frac{1}{\sqrt{2x+3}}$

17. 2074 Set A Q.No. 10a Find from the first principles, the derivative of $\sqrt{2 - 3x}$. [4]
Ans: $-\frac{3}{2\sqrt{2-3x}}$

18. 2074 Set B Q.No. 10a Find from first principles the derivative of $\frac{1}{\sqrt{x-1}}$ [4]
Ans: $-\frac{1}{2(x-1)^{3/2}}$

19. 2073 Set D Q.No. 10a Find from definition the derivative of $\sqrt{1+x}$. [4]
Ans: $\frac{1}{2\sqrt{1+x}}$

20. 2072 Supp Q.No. 10a Find from first principles, the derivative of $\sqrt{2-3x}$. [4]
Ans: $\frac{-3}{2\sqrt{2-3x}}$

21. 2072 Set E Q.No. 10a Find the derivative of $\sqrt[5]{1+x}$ using the first principles. [4]
Ans: $\frac{1}{5(1+x)^{4/5}}$

22. 2071 Supp. Q.No. 10a Find the derivative of $\sqrt{1+x^2}$ from the first principles. [4]

Ans: $\frac{x}{\sqrt{1+x^2}}$

23. 2071 Set C Q.No. 10a Find, from definition, the derivative of $\sqrt{\frac{1}{1-x}}$. [4]

Ans: $\frac{1}{2(1-x)^{3/2}}$

24. 2071 Set D Q.No. 10a Find from first principles the derivative of $\sqrt{1/x}$. [4]

Ans: $-\frac{1}{2x^{3/2}}$

25. 2070 Set C Q.No. 10 a Find from first principles the derivative of $\sqrt{2-3x}$. [4]

Ans: $\frac{-3}{2\sqrt{2-3x}}$

26. 2070 Set D Q.No. 10 a Find from first principles the derivative of $\sqrt{1+x}$. [4]

Ans: $\frac{1}{2\sqrt{1+x}}$

27. 2069 [Set A] Q. No. 10a Find from first principles the derivative of $\sqrt{2x+3}$. [4]

Ans: $\frac{1}{\sqrt{2x+3}}$

28. 2069 [Set B] Q. No. 10a Find from first principles the derivative of $f(x) = \frac{1}{\sqrt{x+a}}$ [4]

Ans: $-\frac{1}{2(x+a)^{3/2}}$

29. 2068 Old Q.No. 13a Find from first principle, the derivative of $\sqrt{2x+3}$. [4]

Ans: $\frac{1}{\sqrt{2x+3}}$

30. 2067 Q.No. 13 a Find from definition, the derivative of $\frac{1}{\sqrt{x}}$. [4]

Ans: $-\frac{1}{2x^{3/2}}$

31. 2063 Q.No. 12(a) OR Find, from definition, the derivative of $\frac{1}{\sqrt{x+2}}$ [4]

Ans: $\frac{-1}{2(x+2)^{3/2}}$

32. 2061 Q.No. 13(a) Find $\frac{dy}{dx}$ from first principle when $y=x+\sqrt{x}$. [4]

Ans: $1+\frac{1}{2\sqrt{x}}$

33. 2059 Q.No. 13(a) Find from the first principles the derivative of $\frac{1}{\sqrt{3x-4}}$ [4]

Ans: $\frac{-3}{2(3x-4)^{3/2}}$

34. 2056 Q.No. 13(a) Find, from the first principles the derivative of $y = \frac{1}{\sqrt{ax+b}}$ [4]

Ans: $\frac{-a}{2(ax+b)^{3/2}}$

## B. Derivatives of Trigonometric and Inverse Circular Functions

### FORMULAE

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x.$
3. $\frac{d}{dx}(\tan x) = \sec^2 x.$
4. $\frac{d}{dx}(\sec x) = \sec x \tan x.$
5. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$
6. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
7. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1 < x < 1.$
8. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, -1 < x < 1.$
9. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, \quad x \in \mathbb{R}$
10. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}, x \in \mathbb{R}$
11. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1.$
12. $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}, |x| > 1.$

### 2 Marks Questions

1. 2076 Set C Q.No. 5a Find $\frac{dy}{dx}$ when $x^2y^2 = \sin(x+y)$. [2]

Ans: $\frac{\cos(x+y) - 2xy^2}{2x^2y - \cos(x+y)}$

2. 2075 Set B Q.No. 5a Find $\frac{dy}{dx}$, when $x = \tan t$ and $y = \sin t \cos t$. [2]

Ans: $\cos^2 t + \cos 2t$

3. 2074 Set A Q.No. 5a Find $\frac{dy}{dx}$, when $x = 2a \tan\theta$, $y = a \sec\theta$. [2]

Ans: $\tan\theta$

4. 2074 Set B Q.No. 5a Find $\frac{dy}{dx}$, when $x = a(1 - \cos t)$ and $y = a(t + \sin t)$. [2]

Ans: $\cot\frac{t}{2}$

5. 2073 Supp Q.No. 5a Find $\frac{dy}{dx}$ when $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$. [2]

Ans: $\tan t$

6. 2072 Supp Q.No. 5a Find $\frac{dy}{dx}$ if $x^2 + y^2 = \sin xy$ [2]

Ans: $\frac{2x - y\cos xy}{x\cos xy - 2y}$

7. 2072 Set D Q.No. 5a Find $\frac{dy}{dx}$ when $y = \sec^2(\tan\sqrt{x})$. [2]

Ans: $\frac{1}{\sqrt{x}}\sec^2(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2\sqrt{x}$

8. 2071 Set C Q.No. 5a Find $\frac{dy}{dx}$ when $x + y = \sin(x + y)$. [2]

Ans: – 1

9. 2071 Set D Q.No. 5a Find the derivative of tanx with respect to cotx [2]

Ans: $-\tan^2 x$

10. 2070 Set D Q.No. 5 a Find $\frac{dy}{dx}$ when $x - y = \tan xy$. [2]

Ans: $\frac{1 - y\sec^2 xy}{1 + x\sec^2 xy}$

11. 2070 Old Q.No. 5 c Find $\frac{dx}{dy}$ when $y = \tan^{-1}\left(\frac{\sin 2x}{1 + \cos 2x}\right)$. [2]

Ans: 1

12. 2069 [Set A] Old Q. No. 5c Find $\frac{dy}{dx}$, when $x = a(t + \sin t)$, $y = a(1 - \cos t)$. [2]

Ans: $\tan\frac{t}{2}$

13. 2069 [Set B] Q. No. 5a Find $\frac{dy}{dx}$ when $y = \frac{1}{\sec x - \tan x}$. [2]

Ans: $\sec x(\tan x + \sec x)$

14. 2069 [Set B] Old Q. No. 5b Find $\frac{dy}{dx}$ of $y = 2\theta - \tan\theta$ and $x = \tan\theta$ [2]

Ans: $\cos 2\theta$

15. 2068 Old Q.No. 5b Find $\frac{dy}{dx}$ when $x = a\cos^2\theta$ and $y = b\sin^2\theta$. [2]

Ans: $-\frac{b}{a}$

16. 2067 Q.No. 5c Find the derivative of $\tan^{-1}\frac{\sin 2x}{1 + \cos 2x}$ [2]

Ans: 1

17. 2065 Q. No. 5 c Find $\frac{dy}{dx}$, when $y = \sin\theta$ and $\theta = 5x^2 - 6x + 2$. [2]

Ans: $(10x - 6)\cos(5x^2 - 6x + 2)$

18. 2064 Q.No. 5(b) Find $\frac{dy}{dx}$ where $y = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$ [2]

Ans: $\frac{2}{1 + x^2}$

19. 2063 Q.No. 5(b) Find $\frac{dy}{dx}$ when $y = \sin^{-1}(3x - 4x^3)$ [2]

Ans: $\frac{3}{\sqrt{1 - x^2}}$

20. 2062 Q.No. 5(c) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\frac{2x}{1 - x^2}$ [2]

Ans: $\frac{2}{1 + x^2}$

21. 2061 Q.No. 5(c) Differentiate $\sin x$ with respect to $\tan x$. [2]

Ans: $\cos^3 x$

22. 2060 Q.No. 5(c) Find $\frac{dy}{dx}$ when $x = 2a\tan\theta$ and $y = a\sec^2\theta$. [2]

Ans: $\tan\theta$

23. 2059 Q.No. 5(c) Find $\frac{dy}{dx}$ of $x = a\sin t$, $y = a\cos t$. [2]

Ans: $-\tan t$

24. 2058 Q.No. 5(c) Find $\frac{dy}{dx}$ of $x = a\sin t$, $y = a\cos t$ [2]

Ans: $-\tan t$

## 4 Marks Questions

25. 2075 Set A Q.No. 10a Find, from the first principles, the differential coefficient of the function $\sin^2(3x - 5)$. [4]

Ans: $3\sin 2(3x - 5)$

26. 2075 Set B Q.No. 10a Find, from the first principles the derivative of $\cos^2 x$. [4]

Ans: $-\sin 2x$

27. 2073 Supp Q.No. 10a Find from first principles, the derivative of $\sqrt{\sin 2x}$. [4]

Ans: $\frac{\cos 2x}{\sqrt{\sin 2x}}$

28. 2073 Set C Q.No. 10a Find from first principles the derivative of $f(x) = \sqrt{\sin 2x}$. [4]

Ans: $\frac{\cos 2x}{\sqrt{\sin 2x}}$

29. 2072 Set C Q.No. 10a Find from the first principles, the derivative of $\tan(3x - 4)$. [4]

Ans: $3\sec^2(3x - 4)$

30. 2072 Set D Q.No. 10a Find from the first principles, the derivative of $\sin x$. [4]

Ans: $\cos x$

31. 2070 Supp Q.No. 10a Find from first principles, the derivative of $\cos 2x$. [4]

Ans: $-2\sin 2x$

32. 2070 Old Q.No. 13 a Or Find $\frac{dy}{dx}$ if $xy = \tan(x^2 + y^2)$. [4]

Ans: $\frac{y - 2x\sec^2(x^2 + y^2)}{2y\sec^2(x^2 + y^2) - x}$

33. 2069 Supp Q.No. 10 a Find from first principles the derivative of $\sec x$. [4]

Ans: $\sec x\tan x$

34. 2069 [Set A] Old Q. No. 13a Find $\frac{dy}{dx}$ of $y = \sin 2x$ from first principles. [4]

Ans: $2\cos 2x$

35. 2069 [Set B] Old Q. No. 12b OR Find from definition the derivative of $\sin^2 3x$. [4]

Ans: $3\sin 6x$

36. 2068 Q.No. 10a Find from first principle, the derivative of $\sin 4x$. [4]

Ans: $4\cos 4x$

37. 2066 Q.No. 13 (a) Find from first principles the derivatives of $\sin 2x$. [4]

Ans: $2\cos 2x$

38. 2065 Q. No. 13 a Find from first principles, the derivative of $\sqrt{\sin 2x}$. [4]

Ans: $\frac{\cos 2x}{\sqrt{\sin 2x}}$

39. 2064 Q.No. 12(b) OR Find from definition the derivative of $\cos^2 x$. [4]

Ans: $-\cos 2x$

40. 2062 Q.No. 13(a) Find from definition the derivative of $\sqrt{\tan x}$. [4]

Ans: $\frac{\sec^2 x}{2\sqrt{\tan x}}$

41. 2060 Q.No. 13(a) Find $\frac{dy}{dx}$ from first principle of $y = \sqrt{\tan x}$ [4]

Ans: $\frac{\sec^2 x}{2\sqrt{\tan x}}$

42. 2058 Q.No. 13(a) Find, from definition, the derivatives of $\sin 2x$. [4]

**Ans:** $2\cos 2x$

43. 2057 Q.No. 13(a) Find from the first principles the derivative of $y = \sqrt{\sin 2x}$ [4]

**Ans:** $\dfrac{\cos 2x}{\sqrt{\sin 2x}}$

## C. Derivatives of Logarithmic and Exponential Functions

**FORMULAE AND IMPORTANT POINTS**

1. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$
2. $\dfrac{d}{dx}(e^x) = e^x$
3. $\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

### 2 Marks Questions

1. 2075 Set A Q.No. 5a Find the derivative of $\ln(\ln\sqrt{x})$. [2]

**Ans:** $\dfrac{1}{2x\ln\sqrt{x}}$

2. 2075 Set C Q.No. 5a Find the derivative of $e^{\sqrt{\cos x}}$. [2]

**Ans:** $-\dfrac{e^{\sqrt{\cos x}}\sin x}{2\sqrt{\cos x}}$

3. 2072 Set C Q.No. 5a If $y = e^{\sin(\log x)}$, find $\dfrac{dy}{dx}$. [2]

**Ans:** $\dfrac{1}{x}e^{\sin(\log x)}.\cos(\log x)$

4. 2072 Set E Q.No. 5a Find the differential coefficient of $\ln(\sec x + \tan x)$. [2]

**Ans:** $\sec x$

5. 2071 Supp. Q.No. 5a Find the derivative of $e^{\sqrt{\sin x}}$. [2]

**Ans:** $\dfrac{\cos x}{2\sqrt{\sin x}}e^{\sqrt{\sin x}}$

6. 2071 Old Q.No. 5c Find $\dfrac{dy}{dx}$ of $y = e^{\sin(\log x)}$. [2]

**Ans:** $\dfrac{1}{x}e^{\sin(\log x)}\cos(\log x)$

7. 2057 Q.No. 5(c) Find $\dfrac{dy}{dx}$ of $y = e^{\sin(\log x)}$. [2]

**Ans:** $\dfrac{1}{x}e^{\sin(\log x)}.\cos(\log x)$

8. 2056 Q.No. 5(c) Find $\dfrac{dy}{dx}$ of $y = e^{5x}\sin(\log x)$. [2]

**Ans:** $e^{5x}\left[\dfrac{\cos(\log x)}{x} + 5\sin(\log x)\right]$

### 4 Marks Questions

9. 2071 Old Q.No. 13a Find from definition, the derivative of $\log_e x$. [4]

**Ans:** $\dfrac{1}{x}$

# Unit 14: Applications of Derivatives

## A. Maxima and Minima

**FORMULAE AND IMPORTANT POINTS**

1. The derivative of f (x) at x = a is written as

$$f'(a) = \lim_{\Delta x\to 0}\frac{f(a+\Delta x) - f(a)}{\Delta x}.$$

2. The slope of the tangent to a curve is called the slope of the curve.
3. If $f'(x) > 0$, for all x on an interval I, then f is increasing on I. If $f'(x) < 0$ for all x on I, then f is decreasing on I. The points where $f'(x) = 0$ are called stationary points.
4. Maxima and Minima of Function of One Variable
   - i. The function f (x) is said to have a local maximum at x = c if there exists h > 0 such that f (c) > f (x) for all $x \in (c - h, c + h)$, $x \neq c$.
   - ii. The function f (x) is said to have a local minimum at x = c if there exist h > 0 such that f (c) < f (x) for all $x \in (c - h, c + h)$, $x \neq c$. If f (x) has a minimum at c then f (c) is called the local minimum value of f (x).
5. A function f defined on an interval [a, b] is said to have absolute (global) maximum value at $x = x_0$ if $f(x) \le f(x_0)$ for all $x \in [a, b]$.

   Similarly, a function f defined on an interval [a, b] is said to have absolute (global) minimum value at $x = x_0$ if $f(x) \ge f(x_0)$ for all $x \in [a, b]$.
6. First Derivative Test
   - a. Find all critical points.
   - b. Locate the critical points on the number line.
   - c. Determine the sign of $f'(x)$ for each interval.
   - d. Now let x increase through each critical value x = c.
     - i. If $f'(x)$ changes from + to –, then f (x) has a relative (local) maximum value at x = c.
     - ii. If $f'(x)$ changes from – to +, then f (x) has a relative (local) minimum value at x = c.
     - iii. If $f'(x)$ does not change sign, then f (x) has neither maximum nor minimum value.
7. Concavity of Curves

   A point on a curve at which the curve changes from concave upwards to concave downwards or vice versa is called a point of inflection. At the point of inflection, $f''(x) = 0$ but $f'''(x) \neq 0$.
8. Second Derivative Test
   - a. Find all critical points.
   - b. Evaluate $f''(x)$ for each critical point x = c.
     - i. If $f''(x) < 0$ for x = c, then f (x) has a maximum value at x = c.
     - ii. If $f''(x) > 0$ for x = c, then f (x) has a minimum value at x = c.
     - iii. If $f''(x) = 0$ at x = c, then this test fails to determine maximum (minimum) value. In this case, we apply first derivative test.

### 2 Marks Questions

1. 2076 Set B Q.No. 5c Determine the interval in which the function $f(x) = \dfrac{1}{2}x^2 - x$ is increasing or decreasing. [2]

**Ans:** Decreasing on $(-\infty, 1)$ and Increasing on $(1, \infty)$

2. **2076 Set C Q.No. 5c** State what do f ' (x) > 0 and f ' (x) < 0 represent for any curve y = f(x). [2]

**Ans: Increasing function and decreasing function**

3. **2075 GIE Q.No. 5c** Test whether the function $f(x) = 2x^2 - 4x + 3$ is increasing or decreasing on the interval (1, 4]. [2]

**Ans: Increasing**

4. **2075 Set A Q.No. 5b** Find the minimum value of the function $2x^2 + 4x + 7$. [2]

**Ans: Min. value = 5 at x = –1.**

5. **2075 Set B Q.No. 5c** Examine the increasing or decreasing nature of the function $y = x - \frac{1}{x}$. [2]

**Ans: Increasing for all $x \in R$ except x = 0**

6. **2074 Supp Q.No. 5c** Show that the function $f(x)=2x^3–24 + 15$ is increasing at x = 3 and decreasing at $x = \frac{3}{2}$. [2]

7. **2074 Set A Q.No. 5c** Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [2]

**Ans: Decreasing at $x = \frac{2}{5}$, Increaseing at $x = \frac{5}{2}$**

8. **2074 Set B Q.No. 5c** Find the intervals in which the function $f(x) = 5x^3 - 135x + 22$ is increasing or decreasing. [2]

**Ans: Increasing on $(-\infty, -3) \cup (3, \infty)$ and Decreasing on (–3, 3)**

9. **2073 Set C Q.No. 5c** Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [2]

**Ans: f(x) is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$**

10. **2073 Set D Q.No. 5c** Show that $f(x) = x - \frac{1}{x}$, is increasing for all $x \in R$ except at x = 0. [2]

11. **2072 Set C Q.No. 5c** If $f(x) = 2x^3 - 6x^2 + 5$, determine where does the graph of the function concave upward. [2]

**Ans: $(1, \infty)$**

12. **2072 Set D Q.No. 5c** Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [2]

**Ans: Decreasing at $x = \frac{2}{5}$ and increasing at $x = \frac{5}{2}$**

13. **2071 Supp. Q.No. 5b** Show that the function $y = x^3 - 3x^2 + 6x + 3$ has neither maximum nor minimum values. [2]

14. **2071 Set C Q.No. 5c** Find the intervals in which $f(x) = x^2 - 2x + 10$ is increasing or decreasing. [2]

**Ans: Decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$**

15. **2071 Set D Q.No. 5c** Find the minimum of $f(x) = 3x^2 - 6x + 4$. [2]

**Ans: 1**

16. **2070 Supp Q.No. 5c** Examine whether the function $f(x) = 2x^3 - 24x + 15$ is increasing or decreasing at x = 3 and x = 3/2. [2]

**Ans: Increasing at x = 3, Decreasing at $x = \frac{3}{2}$**

17. **2070 Set C Q.No. 5 c** Find the interval in which the function $f(x) = 3x^2 - 6x + 5$ is increasing or decreasing. [2]

**Ans: Increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$**

18. **2069 Supp Q.No. 5 c** Find the interval in which the function $f(x) = x^2 - 5x + 6$ is increasing or decreasing. [2]

**Ans: Decreasing on $(-\infty, 5/2)$ and increasing on $(5/2, \infty)$**

19. **2069 [Set B] Q. No. 5c** For any curve y = f(x), what do f'(x) > 0 and f'(x) < 0 represent? [2]

**Ans: Increasing and decreasing functions.**

20. **2068 Q.No. 5c** Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. [2]

**Ans: Decreasing at $x = \frac{2}{5}$ and increasing at $x = \frac{5}{2}$**

## 6 Marks Questions

21. **2076 Set B Q.No. 15** Write the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection. [6]

**Ans: Max. value = 38 at x = 2; Min. value = 37 at x = 3; Point of inflection is at $x = \frac{5}{2}$**

22. **2076 Set C Q.No. 15** Find the absolute maxima and absolute minima of the function $f(x) = x^3 - 3x^2 + 5$ on [– 2, 2]. Also find the point of inflection, if any. [6]

**Ans: Absolute max. value = 5 at x = 0; Absolute min. value = –15 at x = –2**

23. **2075 GIE Q.No. 15** Write the criteria for the function y = f(x) to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection, if any. [6]

**Ans: Max. value = 38 at x = 2; Min. value = 37 at x = 3; Point of inflection is at $x = \frac{3}{2}$**

24. **2075 GIE Q.No. 15 OR** A window is in the form of a rectangle surmounted by a semicircle. If the total perimeter is 9 m, find the radius of the semicircle for the greatest window's area. [6]

**Ans: $\frac{9}{4 + \pi}$ m**

25. **2075 Set A Q.No. 15** Determine the intervals in which the function $f(x) = x^4 - 4x^3$ is concave upwards or downwards. Also, find the point of inflection, if they exist. [6]

**Ans: Concave upward on $(-\infty, 0) \cup (2, \infty)$; Concave downward on (0, 2); Points of inflection at x = 0, 2**

26. **2075 Set B Q.No. 15** Find the absolute maxima and absolute minima of the function $f(x) = x^3 - 3x^2 + 5$ on [–2, 2]. Also find the point of inflection, if any. [6]

**Ans: Absolute maximum = 5 at x = 0; Absolute maximum = –15 at x = –2; Point of inflection is at x = 1**

27. **2075 Set C Q.No. 15** List the criteria for a function y = f(x), to have the local maximum and local minimum at a point. Prove that $f(x) = x + \frac{1}{x}$ whose maximum value is less than its minimum value. [6]

28. **2074 Supp Q.No. 15** Determine where the graph is concave upwards and where it is concave downwards of the function $f(x) = x^4 - 8x^3 + 18x^2 - 24$ [6]

**Ans: Concave upward on $(-\infty, 1) \cup (3, \infty)$ Concave downward on (1, 3)**

29. **2074 Set A Q.No. 15** List the criteria for a function y = f(x), to have the local maxima and local minima at a point. Find the local maxima and local minima of $f(x) = 4x^3 - 15x^2 + 12x + 7$. [6]

**Ans: Max. value = $\frac{39}{4}$ at $x = \frac{1}{2}$, Min. value = 3 at x = 2**

30. **2074 Set B Q.No. 15** Write the criteria for the function $y = f(x)$ to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$. Also find the point of inflection, if any. [6]

**Ans: Max. value = 38 at x = 2, Min. value = 37 at x = 3, Point of inflection is at $x = \frac{3}{2}$**

31. **2073 Supp Q.No. 15** List the criteria at which the function $y = f(x)$ will have local maximum and local minimum values. Find the maximum and the minimum values of the function $y = 4x^3 - 6x^2 - 9x + 1$ on the interval $-1 < x < 2$. Also find the point of inflection. [6]

**Ans: Max. value = $\frac{7}{2}$ at $x = -\frac{1}{2}$, Min. value = $\frac{-25}{2}$ at $x = \frac{3}{2}$, Point of inflection is at $x = \frac{1}{2}$**

32. **2073 Set C Q.No. 15** Determine the maximum and minimum value of $f(x) = 2x^3 - 9x^2 + 12x - 4$. Also find the point of inflection, if any. [6]

**Ans: Max. value = 1 at x = 1, Min value = 0 at x = 2, Point of inflection is $x = \frac{3}{2}$**

33. **2073 Set D Q.No. 15** Define the absolute maxima and absolute minima of the function. Find the absolute maxima and absolute minima of $f(x) = x^3 - 3x^2 + 5$ on [-2,2]. Also find the point of inflection, if any. [6]

**Ans: Absolute max. value = 5 at x = 0, Absolute min. value = –15 at x = –2, Point of inflection x = 1**

34. **2072 Supp Q.No. 15** List the criteria for the function $y = f(x)$ to have the maximum and minimum value at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 5$. Also, find the point of inflection. [6]

**Ans: Max value = 33 at x = 2, Min. value = 32 at x = 3, Point of inflection at $x = \frac{5}{2}$**

35. **2072 Set C Q.No. 15** If $y = f(x)$ represents a certain curve, what do $f'(x) > 0$, $f'(x) < 0$ and $f'(x) = 0$ at a point represent? Find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing. Also, find the point of inflection. [6]

**Ans: Increasing, decreasing and stationary point; Increasing on $(-\infty, 2) \cup (3, \infty)$; decreasing on (2, 3), point of inflection at $x = \frac{5}{2}$.**

36. **2072 Set D Q.No. 15** List the criteria for the function $y = f(x)$ to have the local maxima and local minima at the point x = a. Find the local maxima and local minima of the function $f(x) = 2x^3 - 9x^2 - 24x + 3$. Also find the point of inflection. [6]

**Ans: Maximum value = 16 at x = –1, Minimum value = –109 at x = 4, point of inflection at $x = \frac{3}{2}$**

37. **2072 Set E Q.No. 15** List the criteria for a function $y = f(x)$ to have local maximum and local minimum at a point. Find the local maxima and local minima of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 6$. [6]

**Ans: Maximum value = 13 at x = 1, Minimum value = –10 at x = 2**

38. **2071 Supp. Q.No. 15** Find the local maxima and minima and points of inflection: $f(x) = 2x^3 - 15x^2 + 36x + 5$. [6]

**Ans: Max value = 33 at x = 2, Min. value = 32 at x = 3, Point of inflection at $x = \frac{5}{2}$**

39. **2071 Set C Q.No. 15** A window is in the form of a rectangle surmounted by a semi circle. If the total perimeter is 9m, find the radius of the semicircle for the greatest window area. [6]

**Ans: $\frac{9}{4 + \pi}$ m**

40. **2071 Set D Q.No. 15** Find the absolute maxima and absolute minima of the function. $f(x) = x^3 - 3x^2 + 5$ on [–2, 2]. Also find the point of inflection of f(x), if any. [6]

**Ans: Absolute max. value = 5 at x = 0, Absolute min. value = –15 at x = –2, Point of inflection x = 1**

41. **2070 Supp Q.No. 15** Find the local maxima and local minima of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1, 2). Also find the point of inflection. [6]

**Ans: Max. value = $\frac{7}{2}$ at $x = -\frac{1}{2}$; Min value = $\frac{-25}{2}$ at $x = \frac{3}{2}$; Point of inflection = $\frac{1}{2}$**

42. **2070 Set C Q.No. 15** List the criteria for the function $y = f(x)$ to have the local maxima and local minima at a point? Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. [6]

**Ans: Max. value = $\frac{39}{4}$ at $x = \frac{1}{2}$; Min value = 3 at x = 2; Point of inflection is at $x = \frac{3}{2}$**

43. **2070 Set D Q.No. 15** Write criteria for the function $y = f(x)$ to have the local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 2x^3 - 9x^2 - 24x + 3$. Also find the point of inflection. [6]

**Ans: Max. value = 16 at x = –1; Min. value = –109 at x = 4; Point of inflection is at $x = \frac{3}{2}$**

44. **2069 Supp Q.No. 15** Determine the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1,2). Also find the point of inflection. [6]

**Ans: Max. value = $\frac{7}{2}$ at $x = -\frac{1}{2}$, Min value = 3 at x = 2. The point of inflection is at $x = \frac{1}{2}$**

45. **2069 [Set A] Q. No. 15** What are the criteria for a function $y = f(x)$ to have the local maxima and local minima at a point? Find the local maxima and local minima of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (–1, 2). Also find the point of inflection. [6]

**Ans: Max value = $\frac{7}{2}$ at $x = -\frac{1}{2}$; Min value = $\frac{-25}{2}$ at $x = \frac{3}{2}$; Point of inflection = $\frac{1}{2}$**

46. **2069 [Set B] Q. No. 15** Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x - 2$. Also, find the point of inflection, if any. [6]

**Ans: Max value = 2 at x = 1; Min value = –2 at x = 3; Point of inflection x = 2**

47. **2068 Q.No. 15** List the criteria for the function $y = f(x)$ to have local maxima and local minima at a point. Find the local maxima and local minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection. [6]

**Ans: Max. value = $\frac{39}{4}$ at $x = \frac{1}{2}$; Min. value = 3 at x = 2, Point of inflection is at $x = \frac{5}{4}$**

**4 Marks Questions (Old Syllabus Questions)**

48. 2071 Old Q.No. 13a Or Determine where the graph is concave upwards and where it is concave downwards. Also find the point of inflection $f(x) = (x^2 - 1)(x^2 - 5)$. [4]

**Ans: Concave upward for $x<-1$; Concave downward for $-1<x<1$, Concave upwards for $x > 1$; Point of inflection = –1, 1**

49. 2070 Old Q.No. 13 a Examine the maxima and minima of the function $4x^3 - 6x^2 + 3x$. Also find the point of inflexion, if any. [4]

**Ans: Neither maxima nor minima and Point of inflection = $\frac{1}{2}$**

50. 2069 [Set A] Old Q. No. 13a OR Show that: $x^5 - 5x^4 + 5x^3 - 1$ is maximum when $x = 1$ and minimum when $x = 3$. [4]

51. 2069 [Set B] Old Q. No. 12b Show that: $x^5 - 5x^4 - 1 = 0$ is maximum when $x$=1, minimum when $x$=3, neither when $x$=0. [4]

52. 2068 Old Q.No. 13a(Or) Find the maximum and the minimum value of the function: $f(x)=2x^3-9x^2-24x+3$. [4]

**Ans: max. value = 16 at $x = -1$, min. value = –109 at $x = 4$. The point of inflection is at $x = \frac{3}{2}$**

53. 2067 Q.No. 13 a OR A man wishes to fence a rectangular garden with 256 meter fencing material. Find the maximum area he can enclose. [4]

**Ans: 4096 $m^2$**

54. 2066 Q.No. 13 (a) OR Using derivatives, find two numbers whose sum is 10 and sum of whose squares is minimum. [4]

**Ans: 5, 5**

55. 2065 Q. No. 13 a OR Show that the rectangle of largest possible area for a given perimeter is a square. [4]

56. 2064 Q.No. 12(b) Find the maximum area of a rectangular plot of land which can be enclosed by a rope of length 60 metres. [4]

**Ans: 225 $m^2$**

57. 2063 Q.No. 12(a) Calculate the maximum and minimum values of $x^3 - 3x^2 - 9x + 27$. [4]

**Ans: Max. value 32; min value 0**

58. 2062 Q.No. 13(a) OR Show that the rectangle of largest possible area for a given perimeter is a square. [4]

59. 2061 Q.No. 13(a) OR A man wishes to fence a rectangular garden with 256m fencing material. Find the maximum area he can enclose. [4]

**Ans: 4096 $m^2$**

60. 2060 Q.No. 13(a) OR Find the maximum and minimum value of the function $x^3 - 3x^2 + 6x + 5$, if it exists. Also, find the point of inflection. [4]

**Ans: Neither maxima nor minima; point of inflection = 1**

61. 2059 Q.No. 13(a) OR Show that the rectangle of largest possible area for a given perimeter is a square. [4]

62. 2058 Q.No. 13(a) OR Show that the rectangle of largest possible area for a given perimeter is a square. [4]

63. 2057 Q.No. 13(a) OR Find the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$. Also find the point of inflection. [4]

**Ans: Max. value = $\frac{7}{2}$ at $x = -\frac{1}{2}$, Min. value = $-\frac{25}{2}$ at $x = \frac{3}{2}$ & Point of inflection is $x = \frac{1}{2}$**

64. 2056 Q.No. 13(a) OR Determine where the graph is concave upwards or concave downwards for $f(x) = x^4 - 8x^3 + 18x^2 - 24$. Also find the point of inflection. [4]

**Ans: Concave upwards for $x < 1$ and $x > 3$; Concave downwards for $1 < x < 3$; points of inflection are $x = 1, x = 3$**

## B. Rate Measure

### FORMULAE AND IMPORTANT POINTS

1. Let $y = f(x)$ be the single valued continuous function. If $\Delta x$ and $\Delta y$ be the small increments on x and y respectively. Then $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$ represents the slope of the secant line to the curve $y = f(x)$. This can be regarded as the average rate of change of y with respect to x in the interval from x to $x + \Delta x$.
2. Physical Interpretation of Derivative

   Average velocity = $\frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{t}$,

   $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$

**2 Marks Questions**

1. 2075 Set C Q.No. 5c Find the rate of change of the volume of a cylinder of radius r and height h with respect to a change in the radius. [2]

**Ans: $2\pi rh$**

2. 2073 Supp Q.No. 5c A stone thrown into a pond produces circular ripples which expand from the point of impact. If the radius of the ripple increases at the rate of 3.5 cm/sec, how fast is the area growing when the radius is 15cm? ($\pi = 22/7$) [2]

**Ans: 330 $cm^2$ /sec**

3. 2072 Supp Q.No. 5c A stone thrown into a pond produces a circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5cm/sec, how fast is the area growing when the radius is 15cm? ($\pi = 22/7$) [2]

**Ans: 330 $cm^2$/sec**

4. 2072 Set E Q.No. 5b Find the rate of change of volume of a cylinder of radius r and height h with respect to a change in radius. [2]

**Ans: $2\pi rh$**

5. 2070 Set D Q.No. 5 c A stone thrown into a pond produces circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5 cm/sec, find how fast is the area growing when the radius is 15 cm. ($\pi = 22/7$) **Ans: 330 $cm^2$/sec** [2]

6. 2069 [Set A] Q. No. 5c The side of a square sheet is increasing at the rate of 5 cm/min. At what rate is the area increasing when the side is 12 cm. long? [2]

**Ans: 120 $cm^2$/min**

**6 Marks Questions**

7. 2076 Set B Q.No. 15 OR Water flows into an inverted conical tank at the rate of 42 $cm^3$/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

**Ans: $\frac{21}{8\pi}$ cm/ sec**

8. 2076 Set C Q.No. 15 OR The volume of a spherical balloon is increasing at the rate of 25 $cm^3$/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

**Ans: 10 $cm^2$ /sec.**

9. 2075 Set A Q.No. 15 OR A man 150 cm tall, walks away from a source of light situated at the top of a pole 5 m high at the rate of 0.7 m/s. Find the rate at which [6]
   a. his shadow is lengthening.
   b. the tip of his shadow is moving, when he is 2m away from the pole.

**Ans: (a) 0.3 m/s; (b) 1 m/s**

10. 2075 Set B Q.No. 15 OR The volume of a spherical balloon is increasing at the rate of 25 $cm^3$/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

**Ans: 10 $cm^2$/sec**

11. 2075 Set C Q.No. 15 OR Water flows into an inverted conical tank at the rate of 24$cm^3$/min. When the depth of water is 9 cm, how fast is the level rising? Assume that the height of the tank is 15 cm and the radius at the top is 5 cm. [6]

**Ans: $\frac{8}{3\pi}$ cm/min**

12. 2074 Supp Q.No. 15 OR Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 10 cm/sec and that of the outer circle at the rate of 7 cm/sec. At a certain time, the radii of the inner and the outer circles are respectively 24 cm and 30 cm. At that time, is the area between the circles increasing or decreasing? [6]

13. 2074 Set A Q.No. 15 OR A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75 $cm^3$/ min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm. [6]

**Ans: $\frac{1}{192\,\pi}$ cm/sec**

14. 2074 Set B Q.No. 15 OR The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of the (i) perimeter of the square and (ii) area of square when the side of the square is 12 cm. [6]

**Ans: (i) 0.8 cm/sec (ii) 4.8 $cm^2$/sec**

15. 2073 Set C Q.No. 15 OR If the radius of a circle increases at a uniform rate, prove that its area will increase at a rate which varies as its radius. [6]

16. 2073 Set D Q.No. 15 OR Water flows into an inverted conical tank at the rate of 42$cm^3$/sec. When the depth of water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

**Ans: $\frac{21}{8\pi}$ cm/sec**

17. 2072 Set C Q.No. 15 OR A spherical ball of salt dissolving in water decreases its value at the rate of 0.75 $cm^3$/min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm. [6]

**Ans: $\frac{1}{192\,\pi}$ cm/min**

18. 2072 Set D Q.No. 15 OR Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate 10 cm/sec and that of the outer circle at the rate of 7 cm/sec. At a certain instant, the radii of the inner and the outer circles are respectively 24 cm and 30 cm. At that time, is the area between the circles increasing or decreasing? How fast? [6]

**Ans: 60$\pi$ $cm^2$ / sec (decreasing)**

19. 2072 Set E Q.No. 15 OR A ship sailing east at 15 km/hr passed a point A at 11 AM. A second ship sailing south at 10 km/hr passed A at 9 AM. How fast was the ships separating at 1 PM? [6]

**Ans: 17 km/hr**

20. 2071 Supp. Q.No. 15 OR A ship sailing east at 15 km/hr passed a point A at 11 A.M. A second ship sailing south at 10 km/hr passed A at 9 A.M. How fast were the ships separating at 1 P.M.? [6]

**Ans: 17 km/hr**

21. 2071 Set C Q.No. 15 Or Water flows into an inverted conical tank at the rate of 42 $cm^3$/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

**Ans: $\frac{21}{8\pi}$ cm/sec**

22. 2071 Set D Q.No. 15 Or The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of the (i) perimeter of the square, and (ii) the area of the square, when the side of square is 12 cm. [6]

**Ans: (i) 0.8 cm/sec (ii) 4.8 $cm^2$/sec**

23. 2070 Supp Q.No. 15 Or Water flows into an inverted conical tank at the rate of 42$cm^3$/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. [6]

**Ans: $\frac{21}{8\pi}$ cm/sec**

24. 2070 Set C Q.No. 15 Or A spherical ball of salt is dissolving in water in such a way that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at the constant rate. [6]

25. 2069 Supp Q.No. 15 OR If the volume of an expanding cube is increasing at the rate of 5$cm^3$/min, how fast is its surface area increasing when the surface area is 24 sq cm? [6]

**Ans: 10 $cm^2$/min**

26. 2069 [Set B] Q. No. 15 Or The volume of a spherical balloon is increasing at the rate of 25 cubic cm/sec. Find the rate of change of its surface at the instant when its radius is 5 cm. [6]

**Ans: 10 $cm^2$/sec**

27. 2068 Q.No. 15(Or) A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75$cm^3$/min. Find the rate at which the radius of the salt is decreasing when its radius is 6cm. [6]

**Ans: $\frac{1}{192\pi}$ cm/min**

## 4 Marks Questions (Old Syllabus Questions)

28. 2068 Q.No. 10b Or A spherical ball of salt is dissolving in water in such a manner that the rate of decrease in volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate. [4]

29. 2067 Q.No. 10b OR Water flows into an inverted conical vessel at the rate of 24 $m^3$/m. When the depth of water depth of water is 4 m, how fast is the level rising, assuming that the height of the vessel is 8 m and the radius at the top is 2 m? [4]

**Ans: $\frac{24}{\pi}$ m/min**

30. 2066 C Q.No. 10 b OR If the volume of an expanding cube is increasing at the rate of $4ft^3/min$, how fast is its surface area increasing when the surface area is 24 sq.ft.? [4]

Ans: 8 $ft^2/min$

31. 2066 Q.No. 10 b OR If the area of a circle increases at a uniform rate prove that the rate of increase of the perimeter varies inversely as the radius. [4]

32. 2065 Q.No 10 b OR An aeroplane is flying horizontally at a height of $\frac{2}{3}$ mile with a velocity of 15m.p.h. Find the rate at which it is receding from a fixed point on the ground which it passed over two minutes ago. [4]

Ans: 9 mph

33. 2064 Q.No. 10 b OR A stone thrown into a pond produces circular ripples which expand from the point of impact. If the radius of the ripple increases at the rate of 25 cm/sec, how fast is the (i) perimeter (ii) area increasing when the radius is 80 cm? [4]

Ans: (i) 50 $\pi$ cm/sec (ii) 240 $\pi$ $cm^2$/ sec

34. 2063 Q.No. 10 b OR Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 6 cm/sec and that of the outer circle at the rate of 2.5 cm/sec. At a certain time the radius of the inner and the outer circles are respectively 20 cm and 32 cm. At that time how fast is the area between the circle increasing or decreasing? [4]

Ans: Decreasing at the rate of $80\pi$ $cm^2/sec$.

35. 2062 Q.No. 10 b OR A circular plate of metal expands by heat so that its radius increases at the rate of 0.25 cm/sec. Find the rate at which the surface is increasing when the radius is 7cm. [4]

Ans: 3.5 $\pi$ $cm^2/sec$

36. 2061 Q.No. 10 b OR If the volume of an expanding cube is increasing at the rate of 4 $ft^3/min$, how fast is its surface area increasing when the surface area is 24 sq. ft.? [4]

Ans: 8 $ft^2/min$

37. 2060 Q.No. 10 b OR A spherical balloon is inflated at the rate of 10 cubic cm/sec. At what rate is the radius increasing when the radius is 6 cm. [4]

Ans: $\frac{5}{72\pi}$ cm/sec

38. 2059 Q.No. 10 b OR Water flows into an inverted conical tank at the rate of $27ft^3/min$. When the depth of water is 2ft, how fast is the level rising? Assume that the height of the tank is 4ft and the radius of the top is 1 ft. [4]

Ans: $\frac{108}{\pi}$ ft/min

39. 2058 Q.No. 10 b OR A spherical ballon is inflated at the rate of 10 cubic cm/sec. At what rate is the radius increasing when the radius is 10 cm. [4]

Ans: $\frac{1}{40\pi}$ cm/sec

40. 2057 Q.No. 10 b OR A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x–coordinate is increasing at the rate of 4 ft/sec. Find the rate at which the distance of the point from the origin is increasing when the point is at (2, 4). [4]

Ans: $20\sqrt{5}$ ft/sec

# Unit 15: Anti–derivatives and its Applications

## A. Indefinite Integrals

### FORMULAE AND IMPORTANT POINTS

1. Rules of Integration

i. $\int 1dx = x + c$

ii. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

iii. $\int e^x dx = e^x + c$

iv. $\int e^{ax} dx = \frac{e^{ax}}{a} + c$

v. $\int a^x dx = \frac{a^x}{\ln|a|} + c$

vi. $\int \frac{1}{x} dx = \ln|x| + c \ , x \neq 0$

vii. $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$

viii. $\int \frac{1}{(ax+b)} dx = \frac{\ln|ax+b|}{a} + c$

2. Integrals of the Trigonometric Functions

i. $\int \cos x\, dx = \sin x + c$

ii. $\int \sin x\, dx = -\cos x + c$

iii. $\int \sec^2 x\, dx = \tan x + c$

iv. $\int \text{cosec}^2 x\, dx = -\cot x + c$

v. $\int \sec x \tan x\, dx = \sec x + c$

vi. $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + c$

vii. $\int \tan x\, dx = \ln|\sec x| + c$

viii. $\int \cot x\, dx = \ln|\sin x| + c$

ix. $\int \sec x\, dx = \ln|\sec x + \tan x| + c$

x. $\int \text{cosec}\, x\, dx = \ln|\text{cosec}\, x - \cot x| + c$

Similarly,

$\int \sin ax\, dx = -\frac{\cos ax}{a} + c$

$\int \cos ax\, dx = \frac{\sin ax}{a} + c$

$\int \sec^2 ax\, dx = \frac{\tan ax}{a} + c$

$\int \text{cosec}^2 ax\, dx = -\frac{\cot ax}{a} + c$

$\int \sec ax \tan ax\, dx = \frac{\sec ax}{a} + c$

$\int \text{cosec}\, ax \cot ax\, dx = -\frac{\text{cosec}\, ax}{a} + c$

$\int \tan ax\, dx = \frac{\ln|\sec ax|}{a} + c$

$\int \cot ax\, dx = \frac{\ln|\sin ax|}{a} + c$

$\int \sec ax\, dx = \frac{1}{a} \ln|\sec ax + \tan ax| + c$

$\int \text{cosec}\, ax\, dx = \frac{1}{a} \ln|\text{cosec}\, ax - \cot ax| + c.$

3. Integration by the Method of Trigonometric Substitution

i. If the expression is in the form $a^2 - x^2$, then substitute $x = a \sin\theta$ or $x = a \cos\theta$.

ii. If the expression is in the form $a^2 + x^2$, then substitute $x = a \tan\theta$ or $x = a \cot\theta$.

iii. If the expression is in the form $x^2 - a^2$, then substitute $x = a\sec\theta$ or $x = a\operatorname{cosc}\theta$.

4. Integration by Parts

$$\int (uv)\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

I = Inverse trigonometric function
L = Logarithmic function
A = Algebraic function
T = Trigonometric function
E = Exponential function

**2 Marks Questions**

1. 2076 Set B Q.No. 5b Evaluate: $\int \sin 2x.\sin 3x.\,dx$. [2]
Ans: $\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + c$

2. 2076 Set C Q.No. 5b Evaluate: $\int \log x\,dx$. [2]
Ans: $x\log x - x + c$

3. 2075 GIE Q.No. 5b Evaluate: $\int \frac{\log x}{x}\,dx$. [2]
Ans: $\frac{1}{2}(\log x)^2 + C$

4. 2075 Set A Q.No. 5c Evaluate: $\int \frac{x+2}{x^2 - x - 6}\,dx$. [2]
Ans: $\log(x-3) + C$

5. 2075 Set B Q.No. 5b Evaluate: $\int \frac{x^2+1}{x+1}\,dx$. [2]
Ans: $\frac{x^2}{2} - x + 2\log(x+1) + C$

6. 2075 Set C Q.No. 5b Evaluate: $\int \frac{x+3}{x-3}\,dx$. [2]
Ans: $x + 6\log(x-3) + C$

7. 2074 Supp Q.No. 5b Evaluate: $\int xe^x dx$ [2]
Ans: $xe^x - e^x + c$

8. 2074 Set A Q.No. 5b Evaluate: $\int (3\sin x - 4)^2 \cos x\,dx$ [2]
Ans: $\frac{1}{12}(3\sin x - 4)^3 + C$

9. 2074 Set B Q.No. 5b Evaluate: $\int x\log x\,dx$. [2]
Ans: $\frac{1}{2}x^2\log x - \frac{x^2}{4} + C$

10. 2073 Supp Q.No. 5b Evaluate: $\int\left(1 + \frac{1}{x^2}\right)e^{x - \frac{1}{x}}\,dx$. [2]
Ans: $e^{x - \frac{1}{x}} + c$

11. 2073 Set C Q.No. 5b Evaluate: $\int \frac{x\,dx}{(1-x^2)^{3/2}}$. [2]
Ans: $\frac{1}{\sqrt{1-x^2}} + C$

12. 2073 Set D Q.No. 5c Evaluate: $\int \frac{dx}{x\log x}$. [2]
Ans: $\log(\log x) + C$

13. 2072 Supp Q.No. 5b Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$ [2]
Ans: $\frac{1}{12}[(2x+1)^{3/2} + (2x-3)^{3/2}] + c$

14. 2072 Set C Q.No. 5b Evaluate: $\int\left(1 - \frac{1}{x^2}\right)e^{x + \frac{1}{x}}\,dx$ [2]
Ans: $e^{x + \frac{1}{x}} + C$

15. 2072 Set D Q.No. 5b Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$ [2]
Ans: $\frac{1}{12}\{(2x+1)^{3/2} + (2x-3)^{3/2}\} + C$

16. 2072 Set E Q.No. 5c Evaluate: $\int e^{bx}(e^{ax} - e^{-ax})\,dx$. [2]
Ans: $\frac{e^{(a+b)x}}{a+b} + \frac{e^{(b-a)x}}{a-b} + c$

17. 2071 Supp. Q.No. 5c Evaluate: $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$. [2]
Ans: $2\tan\sqrt{x} + C$

18. 2071 Set C Q.No. 5b Evaluate: $\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$. [2]
Ans: $-\log(\cos x + \sin x) + c$

19. 2071 Set D Q.No. 5b Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$. [2]
Ans: $-2\cos\sqrt{x} + C$

20. 2071 Old Q.No. 6c Integrate: $\int \log x\,dx$ [2]
Ans: $x\log x - x + c$

21. 2070 Supp Q.No. 5b Evaluate: $\int (2x-1)e^{2x}\,dx$. [2]
Ans: $(x-1)e^{2x} + c$

22. 2070 Set C Q.No. 5 b Evaluate: $\int \frac{1}{\sqrt{2x+1} - \sqrt{2x-3}}\,dx$. [2]
Ans: $\frac{1}{12}[(2x+1)^{3/2} + (2x-3)^{3/2}] + C$

23. 2070 Set D Q.No. 5 c Evaluate: $\int \frac{1}{x}.\sin(\log x)\,dx$. [2]
Ans: $-\cos(\log x) + c$

24. 2070 Old Q.No. 6 b Evaluate: $\int \frac{dx}{\sin^2 x\cos^2 x}$. [2]
Ans: $\tan x - \cot x + C$

25. 2069 Supp Q.No. 5 b Evaluate: $\int\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)dx$. [2]
Ans: $\frac{x^3}{3} + \frac{1}{x} + C$

26. 2069 [Set B] Q. No. 5b Evaluate: $\int \frac{dx}{\sin^2 x.\cos^2 x}$ [2]
Ans: $\tan x - \cot x + C$

27. 2069 [Set B] Old Q. No. 6b Evaluate: $\int \cot x\,dx$. [2]
Ans: $\log(\sin x) + C$

28. 2068 Q.No. 5b Evaluate: $\int\left(1 - \frac{1}{x^2}\right)e^{x + \frac{1}{x}}\,dx$ [2]
Ans: $e^{x + \frac{1}{x}} + C$

29. 2068 Old Q.No. 6b Evaluate: $\int (a\sin x - b)^3 \cos x\, dx$. [2]

Ans: $\frac{1}{4a}(a\sin x - b)^4 + C$

30. 2069 [Set A] Q. No. 5b Evaluate: $\int \cot x (\log \sin x)^3 dx$ [2]

Ans: $\frac{(\log \sin x)^4}{4} + C$

31. 2067 Q.No. 6b Evaluate: $\int \frac{dx}{1 + \sin x}$ [2]

Ans: $\frac{-2}{\tan x/2} + C$

32. 2066 Q.No. 6(b) Evaluate: $\int \frac{dx}{\sqrt{2x+1} - \sqrt{2x-3}}$ [2]

Ans: $\frac{1}{12}[(2x+1)^{3/2} + (2x-3)^{3/2}] + c$

33. 2065 Q. No. 6 b Evaluate: $\int \log x\, dx$ [2]

Ans: $x \log x - x + c$

34. 2065 Q. No. 13 b Evaluate: $\int e^{ax} \cos bx\, dx$ [2]

Ans: $\frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2} + c$

35. 2064 Q.No. 6(b) Integrate: $\int \operatorname{cosec} x\, dx$ [2]

Ans: $\log(\operatorname{cosec} x - \cot x) + c$

36. 2063 Q.No. 6(b) Integrate: $\int \sec x\, dx$ [2]

Ans: $\log(\sec x + \tan x) + c$

37. 2061 Q.No. 6(b) Evaluate: $\int \frac{1}{x} \cos(\log x)\, dx$ [2]

Ans: $\sin(\log x) + c$

38. 2060 Q.No. 6(b) Evaluate $\int \sin^2 2x\, dx$ [2]

Ans: $\frac{x}{2} - \frac{\sin 4x}{8} + c$

39. 2059 Q.No. 6(b) Integrate $\int \sec x\, dx$ [2]

Ans: $\log(\sec x + \tan x) + c$

40. 2058 Q.No. 6(c) Integrate $\int \log x\, dx$. [2]

Ans: $x \log x - x + c$

41. 2057 Q.No. 6(b) Integrate $\int x \sin x\, dx$ [2]

Ans: $\sin x - x\cos x + c$

42. 2056 Q.No. 6(c) Integrate: $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$. [2]

Ans: $\frac{2}{3}x^{3/2} + 2x^{1/2} + c$

## 4 Marks Questions

43. 2075 Set A Q.No. 10b Evaluate: $\int (x^2 + 1)e^x dx$. [4]

Ans: $(x^2 - 2x + 3)e^x + C$

44. 2072 Set E Q.No. 10b Evaluate: $\int \frac{1}{x^2\sqrt{9 - x^2}} dx$ [4]

Ans: $\frac{-\sqrt{9 - x^2}}{9x} + c$

45. 2071 Supp. Q.No. 10b Evaluate: $\int \sin^2 ax\, dx$. [4]

Ans: $\frac{x}{2} - \frac{1}{4a}\sin 2ax + c$

46. 2071 Old Q.No. 13b Integrate: $\int \sec^3 x dx$ [4]

Ans: $\frac{1}{2}\sec x \tan x + \frac{1}{2}\log(\sec x + \tan x) + c$

47. 2069 [Set A] Old Q. No. 6b Evaluate: $\int \frac{dx}{1 + \sin x}$. [4]

Ans: $\tan x - \sec x + C$

48. 2069 [Set A] Old Q. No. 13b Integrate: $\int \cot x (\log \sin x)^3 dx$. [4]

Ans: $\frac{(\log \sin x)^4}{4} + C$

49. 2068 Old Q.No. 13b Evaluate: $\int e^{ax} \cos bx\, dx$. [4]

Ans: $\frac{e^{ax}}{a^2 + b^2}(b\sin bx + a\cos bx) + C$

50. 2067 Q.No. 13b Evaluate: $\int \sec^3 x\, dx$ [4]

Ans: $\frac{1}{2}[\sec x \tan x + \log(\sec x + \tan x)] + C$

51. 2062 Q.No. 6(b) Evaluate: $\int x \sin x\, dx$ [4]

Ans: $-x\cos x + \sin x + c$

52. 2061 Q.No. 13(b) Integrate $\int \sec^3 x\, dx$ [4]

Ans: $\frac{1}{2}[\sec x. \tan x + \log(\sec x + \tan x)] + c$

53. 2060 Q.No. 13(b) Evaluate $\int x \sin^2 x\, dx$ [4]

Ans: $\frac{1}{4}(x^2 - x\sin 2x - \frac{1}{2}\cos 2x) + c$

54. 2059 Q.No. 13(b) Evaluate: $\int e^x \cos x\, dx$ [4]

Ans: $\frac{1}{2}e^x(\sin x + \cos x) + c$

55. 2058 Q.No. 13(b) Evaluate: $\int x^2 e^{ax}\, dx$ [4]

Ans: $\frac{e^{ax}}{a}\left[x^2 - \frac{2x}{a} + \frac{2}{a^2}\right] + c$

56. 2057 Q.No. 13(b) Integrate $\int x^2 \sin x\, dx$ [4]

Ans: $-x^2\cos x + 2x\sin x + 2\cos x + c$

57. 2056 Q.No. 13(b) Evaluate: $\int \frac{dx}{\sqrt{a^2 + x^2}}$ [4]

Ans: $\log(x + \sqrt{a^2 + x^2}) + c$

## B. Definite Integrals

### FORMULAE AND IMPORTANT POINTS

1. Let $f$ be a continuous function on the interval $[a, b]$ and $F(x)$ be the antiderivative of $f(x)$. Then the definite integral of $f(x)$ on $[a, b]$ denoted by

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

**2 Marks Questions**

1. 2059 Q.No. 6(c) Evaluate $\int_1^2 \frac{\sin(\log x)}{x}\,dx.$ [2]

Ans: $1 + \cos(\log 2)$

2. 2057 Q.No. 6(c) Find the value of $\int_{\pi/3}^{-\pi/3} \cos t\,dt.$ [2]

Ans: $\sqrt{3}$

**4 Marks Questions**

3. 2070 Old Q.No. 13 b Evaluate: $\int_0^{\pi/4} \frac{\cos x}{1+\sin x}\,dx.$ [4]

Ans: $\log(2+\sqrt{2})$

4. 2069 [Set B] Old Q. No. 13 a Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ [4]

Ans: 1

5. 2066 Q.No. 13 (b) Evaluate: $\int_1^3 \frac{x\,dx}{1+x^2}$ [4]

Ans: $\log\sqrt{5}$

6. 2064 Q.No. 13(a) Evaluate $\int_2^1 \frac{\sin(\log x)}{x}\,dx$ [4]

Ans: $1 + \cos(\log 2)$

7. 2063 Q.No. 13(a) Evaluate: $\int_0^2 \frac{x\,dx}{\sqrt{x^2+4}}$ [4]

Ans: $2(\sqrt{2}-1)$

8. 2062 Q.No. 13(b) OR Evaluate: $\int_0^{-1} \frac{dx}{4-x^2}$ [4]

Ans: $\frac{-\pi}{6}$

## C. Application of Integration

### FORMULAE AND IMPORTANT POINTS

1. The area bounded by $y = f(x)$, the ordinates $x = a$ and $x = b$ is $A = \int_a^b f(x)\,dx$.
2. The area bounded by $x = f(y)$, the abscissa $y = c$ and $y = d$ is $A = \int_c^d f(y)\,dy$.
3. Area Between Two Curves

$$A = \int_a^b y_2\,dx - \int_a^b y_1\,dx = \int_a^b (y_2 - y_1)\,dx$$

**2 Marks Questions**

1. 2071 Old Q.No. 6b Find the area bounded by the curve $y^2 = 4ax$ the $x$-axis and the ordinates $x = 0$ and $x = a$. [2]

Ans: $\frac{4a^2}{3}$ sq.units

2. 2070 Old Q.No. 6 c Find the area under the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 1$. [2]

Ans: $\frac{4}{3}$ sq. units

3. 2069 [Set A] Old Q. No. 6c Find the area bounded by curve $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$. [2]

Ans: 1 sq. unit

4. 2069 [Set B] Old Q. No. 6a Find the area enclosed by $y = 3x$, the $x$-axis and ordinates $x = 0$, $x = 4$. [2]

Ans: 24 sq. units

5. 2068 Old Q.No. 6c Find the area bounded by the curve $y=3x^2-2$, $x$-axis and the two ordinates $x = 1$ and $x = 4$. [2]

Ans: 57 sq. units

6. 2067 Q.No. 6c Find the area under the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 1$. [2]

Ans: $\frac{4}{3}$ sq. units

7. 2066 Q.No. 6(c) Find the area bounded by the curve $y = \sin x$, $x = 0$, $x = \pi$. [2]

Ans: 2 sq. units

8. 2065 Q. No. 6 c Find the area of the region bounden by the curve $y = e^x$, the $x$-axis and the ordinates $x = 1$; $x = 2$. [2]

Ans: $(e^2 - e)$ sq. units

9. 2064 Q.No. 6(a) Find the area bounded by the $x$-axis and the following curve and ordinates $y = \log x$, $x = 1$, $x = e$. [2]

Ans: 1 sq. unit

10. 2063 Q.No. 6(a) Find the area bounded by the $x$-axis and the following curve and ordinates $xy = 8$; $x = 3$, $x = 8$ [2]

Ans: $8 \log \frac{8}{3}$ sq. units

11. 2062 Q.No. 6(c) Find the area bounded by the $x$-axis and curve and $y = \log(1 + x)$ and ordinates $x = 0$ and $x = 1$. [2]

Ans: $(2 \log 2 - 1)$ sq. units

12. 2061 Q.No. 6(c) Find the area bounded by curve $y = 3x^2$, $x = 1$ & $x = 3$. [2]

**Ans: 26 sq. units**

13. 2060 Q.No. 6(c) Find the area under the curve $y = x^2$ bounded by $x$–axis and between the ordinates $x = 0$ and $x = a$. [2]

**Ans: $\frac{a^3}{3}$ sq. units**

14. 2058 Q.No. 6(b) Find the area enclosed by the curve $y = 3x$, the $x$–axis and ordinates at $x = 0$ and $x = 4$. [2]

**Ans: 24 sq. units**

15. 2056 Q.No. 6(b) Find the area bounded by the exist of $x$ and the curve $y = x^3$ and the ordinates at $x = 2$ and $x = 4$. [2]

**Ans: 240 sq. units**

## 4 Marks Questions

16. 2076 Set B Q.No. 10b Find the area of the circle $x^2 + y^2 = 36$, using method of integration. [4]

**Ans: 36π square units**

17. 2076 Set C Q.No. 10b Using method of integration, find the area under the curve $4x^2 + 9y^2 = 36$. [4]

**Ans: 6π square units**

18. 2075 GIE Q.No. 10b Find the area under the curve $x^2 + y^2 = 400$, using method of integration. [4]

**Ans: 400π sq. units**

19. 2075 Set A Q.No. 10b OR Find the area bounded by the x-axis and the curve $y = (x + 1)(x – 2)(x – 3)$. [4]

**Ans: $\frac{71}{6}$ sq. units**

20. 2075 Set B Q.No. 10b Find the area of the curve enclosed by $\frac{x^2}{64} + \frac{y^2}{36} = 1$, using the method of integration. [4]

**Ans: 48π sq. units**

21. 2075 Set C Q.No. 10b Find the area of the circle $x^2 + y^2 = a^2$. [4]

**Ans: $\pi a^2$ sq. units**

22. 2074 Supp Q.No. 10b Using integration, find the area of the circle $x^2 + y^2 = a^2$. [4]

**Ans: $\pi a^2$ sq. units**

23. 2074 Set A Q.No. 10b Find the area of the region between the curve $y^2 = 4x$ and the line $x = y$. [4]

**Ans: $\frac{8}{3}$ sq. units**

24. 2074 Set B Q.No. 10b Using integration, find the area of the curve enclosed by $x^2 + y^2 = 25$. [4]

**Ans: 25π sq. units**

25. 2073 Supp Q.No. 10b Find the area bounded by the axis of coordinates, the curve $x^2 = 4a(y – 2a)$ and the ordinate of the point (h, k). [4]

**Ans: $\left(\frac{h^3}{12a} + 2ah\right)$ sq. units**

26. 2073 Set C Q.No. 10b Using integration, find the area of the circle $x^2 + y^2 = 36$. [4]

**Ans: 36π sq. units**

27. 2073 Set D Q.No. 10b Find the area enclosed by ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, using method of integration. [4]

**Ans: 12π sq. units**

28. 2072 Supp Q.No. 10b Find the area of the circle $x^2 + y^2 = a^2$ (Use the method of calculus). [4]

**Ans: $\pi a^2$ sq. units**

29. 2072 Set C Q.No. 10b Find the area of the region between the curve $y^2 = 4x$ and the line $y = x$. [4]

**Ans: $\frac{8}{3}$ sq.units**

30. 2072 Set D Q.No. 10b Using integration, find the area of the circle $x^2 + y^2 = 16$. [4]

**Ans: 16π sq.units**

31. 2072 Set E Q.No. 10b OR Find the area of the region between the curves $y = x^2$ and $x = y^2$. [4]

**Ans: $\frac{1}{3}$ sq.units**

32. 2071 Set C Q.No. 10b Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [4]

**Ans: πab sq.units**

33. 2071 Set D Q.No. 10b Using integration, find the area under the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ [4]

**Ans: 12π sq.units**

34. 2071 Old Q.No. 13b Or Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [4]

**Ans: π ab sq.units**

35. 2070 Supp Q.No. 10b Find the region between the curves $y^2 = 4ax$ and $x^2 = 4ay$. [4]

**Ans: $\frac{16}{3}a^2$ sq.units**

36. 2070 Set C Q.No. 10 b Find the area bounded by y-axis, the curve $x^2 = 4a(y – 2a)$ and $y = 6a$. [4]

**Ans: $\frac{32}{3}a^2$ sq.units**

37. 2070 Set D Q.No. 10 b Find the area enclosed by axis of $x$ and the curve $y = 3x – 5x^2$. [4]

**Ans: $\frac{9}{50}$ sq. units**

38. 2070 Old Q.No. 13 b Or Using method of integration, find the area of the portion of the circle $x^2 + y^2 = r^2$ in the first quadrant. [4]

**Ans: $\frac{\pi r^2}{4}$ sq.units**

39. 2069 Supp Q.No. 10 b Find the area of the region bounded by the curves $y = \sqrt{x}$ and $y = x$. [4]

**Ans: $\frac{1}{6}$ sq.units**

40. 2069 [Set A] Q. No. 10b Find the area of the region between the curves. $y^2 = 16x$ and the line $y = 2x$. [4]

**Ans: $5\frac{1}{3}$ sq. units**

41. 2069 [Set A] Old Q. No. 13b OR Find the area of the circle $x^2 + y^2 = 16$ using integration. [4]

**Ans: 16π sq. units**

42. 2069 [Set B] Q. No. 10b Find the area of the region bounded by the curves $x^2 = 4y$ and $x = y$. [4]

**Ans: $\frac{8}{3}$ sq. units**

43. 2069 [Set B] Old Q. No. 13 a OR Find the area bounded by y-axis, the curve $x^2 = 4a(y – 2a)$ and $y = 6a$. [4]

**Ans: $\frac{32}{3}a^2$ sq.units**

**44.** 2068 Q.No. 10b Find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$. [4]

**Ans:** $\frac{8a^2}{3}$ sq. units

**45.** 2068 Old Q.No. 13b(Or) Find the area bounded by the curve $y^2 = 4ax$ and the line $x = a$. [4]

**Ans:** $\frac{8a^2}{3}$ sq. units

**46.** 2067 Q.No. 13b OR Find the area under the curves $\frac{x^2}{16}+\frac{y^2}{25}=1$ using method of integration. [4]

**Ans:** 20 $\pi$ sq. units

**47.** 2066 Q.No. 13 (b) OR Find the area of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ [4]

**Ans:** $6\pi$ sq. units

**48.** 2065 Q. No. 13 b OR Using method of integration, find the area under the curve $x^2 + y^2 = a^2$. [4]

**Ans:** $\pi a^2$ sq. units

**49.** 2064 Q.No. 13(a) OR Find the area of the circle $x^2 + y^2 = 9$ using method of integration. [4]

**Ans:** $9\pi$ sq. units

**50.** 2063 Q.No. 13(a) OR Find the area of the circle $x^2 + y^2 = 25$, using method of integration. [4]

**Ans:** $25\pi$ sq. units

**51.** 2062 Q.No. 13(b) Find the area of ellipse: $\frac{x^2}{9}+\frac{y^2}{16}=1$. [4]

**Ans:** 12 $\pi$ sq. units

**52.** 2061 Q.No. 13(b) OR Using integration, find the area of the circle $x^2 + y^2 = a^2$ [4]

**Ans:** $\pi a^2$ sq. units

**53.** 2060 Q.No. 13(b) OR Find using method of integration the area bounded by the curve $y^2 = 4x$ and $x^2 = 4y$. [4]

**Ans:** $\frac{16}{3}$ sq. units

**54.** 2059 Q.No. 13(b) OR Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [4]

**Ans:** $\frac{16}{3}$ sq. units

**55.** 2058 Q.No. 13(b) OR Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$. [4]

**Ans:** $\frac{16a^2}{3}$ sq. units

**56.** 2057 Q.No. 13(b) OR Find the area of the circle, $x^2+y^2=25$ [4]

**Ans:** $25\pi$ sq. units

**57.** 2056 Q.No. 13(b) OR Find the area of the region between the curve $y^2 = 16x$ and the line $y = 2x$. [4]

**Ans:** $\frac{16}{3}$ sq. units

## 5 Sets Questions

### Set 1

**Group 'A' [5 × 3 × 2 = 30]**

1. a. By constructing the truth table show that $(p \wedge q) \wedge \sim(p \vee q)$ is a fallacy.
   b. Show that f: Q → Q defined by f(x) = 5x + 4 is one to one.
   c. Test the periodicity of the function f(x) = cos2x and find its period.

   **Ans:** $\frac{\pi}{2}$

2. a. Solve $\operatorname{cosec}\theta = \cot\theta + \sqrt{3}$.

   **Ans:** $2n\pi + \frac{2\pi}{3}$

   b. Using the principle of mathematical induction, prove that: 2 + 4 + 6 + ..... to n terms = n(n + 1)
   c. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$ be a 2 × 2 matrix. Find adj. A and $A^{-1}$.

   **Ans:** $\begin{pmatrix} 8 & -2 \\ -3 & 1 \end{pmatrix}, \begin{pmatrix} 4 & -1 \\ \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$

3. a. Solve by using inverse matrix method
   $2x + 4y = 7$
   $8x - 6y = -5$

   **Ans:** $x = \frac{1}{2}, y = \frac{3}{2}$

   b. Show that $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) = 9$
   c. Find the value of k so that the equation $3x^2 + (5 - k)x - 13 = 0$ has roots equal but opposite in sign.

   **Ans:** 5

4. a. Are the points (2,3) and (1,3) on the same side or on opposite sides of the line $x - 2y = -3$.

   **Ans:** Same side

   b. Find the equations of two tangents to the circle $x^2 + y^2 = 5$ which make an angle of 60° with the x-axis.

   **Ans:** $y = \sqrt{3}x \pm 2\sqrt{5}$

   c. $\lim_{x \to 0} \frac{|x|}{x}$ if it exists.

   **Ans:** does not exist

5. a. Find the derivative of $\cos^{-1}\frac{1 - x^2}{1 + x^2}$.

   **Ans:** $\frac{2}{1 + x^2}$

   b. On the curve $y = x + \frac{1}{x}$ find the points at which the tangents to the curve are parallel to the x-axis.

   **Ans:** (1, 2), (-1, -2)

   c. Integrate $\sqrt{1 + \sin 2x}$ w.r. to x.

   **Ans:** sin x − cos x + C

**Group 'B' [5 × 2 × 4 = 40]**

6. a. If A, B and C are any three subsets of U, then prove that: $A - (B \cap C) = (A - B) \cup (A - C)$
   OR
   Prove that for any positive real number a, $|x| < a$ implies $-a < x < a$. Use it to write $|3x + 2| \le 1$ without absolute value sign.

   **Ans:** $-1 \le x \le \frac{-1}{3}$

   b. Sketch the graph of y = sinx, $(-\pi \le x \le \pi)$ indicating its different characteristics.

7. a. If $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi$ show that $xy + yx + zx = 1$

OR

In a $\Delta ABC$, if $(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$ then prove $\angle C = 60°$.

b. If a, b, c are non zero and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, then show that abc = 1.

8. a. Solve by Cramer's rule or by row equivalent method.

$x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$

**Ans: x = 1, y = 2, z = 3**

b. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$.

9. a. Prove that the tangent to the circle $x^2 + y^2 = 5$ at the point (1, −2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ and find the point of contact.

**Ans: (3, −1)**

b. Evaluate: $\lim\limits_{x \to 0} \dfrac{(a + x)\sec(a + x) - a \sec a}{x}$.

**Ans: sec a + a sin a sec2a**

OR

Show that the function $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & (x \neq 0) \\ 1, & (x = 0) \end{cases}$ is discontinuous at $x = 0$.

Redefine the function in such a way that it becomes continuous at $x = 0$.

**Ans:** $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & (x \neq 0) \\ a^2, & (x = 0) \end{cases}$

10. a. Find $\dfrac{dy}{dx}$ from first principles of $y = \sqrt{\tan x}$.

**Ans:** $\dfrac{\sec 2x}{2\sqrt{\tan x}}$

b. Using integration, find the area of the circle $x^2 + y^2 = 16$.

**Ans: 16π sq. unit.**

## Group 'C' [5 × 6 = 30]

11. Define domain and range of a function. Find the domain and range of $f(x) = \sqrt{6 - x - x^2}$.

**Ans: Domain = [−3, 2], Range = [0, 5/2]**

12. Sum to n term the following series: $1 + \dfrac{4}{5} + \dfrac{7}{5^2} + \dfrac{10}{5^3} + \ldots$

**Ans:** $\dfrac{35}{15} - \dfrac{12n + 7}{16.5n - 1}$

13. The origin is a corner of a square and two of its sides are given by $2x + y = 0$ and $2x + y = 3$. Find the equations of the other two sides.

**Ans: x − 2y = 0 and x − 2y + 3 = 0**

OR

For what values of c, the lines which join the origin to the point of intersection of the line $x - y + c = 0$ and the curve $x^2 + y^2 + 4x - 6y - 36 = 0$ may be at right angle.

**Ans: c = 9 or - 4**

14. State De-Moivre's theorem. Use it to find the cube roots of unity. Write their properties.

**Ans:** $1, \dfrac{-1 + i\sqrt{3}}{2}$ and $\dfrac{-1 - i\sqrt{3}}{2}$

15. A kite is 120 m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52m/sec., find the rate at which the string is being paid out.

**Ans: 20m/sec.**

OR

Find the maximum and minimum values of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$. Also find the point of inflection.

**Ans: Max. value =** $\dfrac{7}{2}$ **Min. value =** $\dfrac{-25}{2}$

**Point of inflection is** $\left(\dfrac{1}{2}, -\dfrac{9}{2}\right)$

# Set 2

## Group 'A' [5 × 3 × 2 = 30]

1. a. Construct a truth table for the statement $(\sim p) \wedge (\sim q)$.

b. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, prove that:

$$\frac{1}{1 + x} + \frac{1}{1 + y} + \frac{1}{1 + z} = 1$$

c. Examine whether the function $f(x) = x^2 + \cos^2 x + 1$ is even, odd or neither.

**Ans: Even function**

2. a. In any triangle ABC, if A = 30°, C = 60°, find a:b:c.

**Ans:** $1 : 2 : \sqrt{3}$

b. Using the principle of mathematical induction method, show that $1 + 2 + 3 + \ldots + n = \dfrac{n(n + 1)}{2}$.

c. Define symmetric and skew-symmetric matrix with an example of each.

3. a. Solve the following equations using Cramer's rule.

$\dfrac{2x_1}{3} + x_2 = 16$ $\qquad x_1 + \dfrac{x_2}{4} = 14.$

**Ans:** $x_1 = 12, x_2 = 8$

b. If $z_1$ and $z_2$ are any two complex numbers, then prove that $\text{amp}\left(\dfrac{z_1}{z_2}\right) = \text{amp}(z_1) - \text{amp}(z_2)$.

c. Find the value of λ so that the equation $\lambda x^2 + 2(\lambda + 1)x + (3\lambda + 1) = 0$ may have reciprocal roots.

**Ans:** $\dfrac{1}{2}$

4. a. Find the distance between the lines $3x - 4y + 9 = 0$ and $6x - 8y - 17 = 0$.

**Ans:** $\dfrac{7}{2}$

b. Find the equation of a circle with centre (3, 2) and touching the x–axis.

**Ans:** $x^2 + y^2 - 6x - 4y + 9 = 0$

c. Evaluate $\lim\limits_{x \to 3} |x - 3|$, if it exists.

**Ans: 0**

5. a. Find the differential coefficient of $\tan(\sqrt{\cot 5x})$.

**Ans:** $\dfrac{-5 \sec^2(\sqrt{\cot 5x})\, \text{cosec}^2 5x}{2\sqrt{\cot 5x}}$

b. Find $f'(x)$ and $f''(x)$ of the curve $f(x) = 6x - x^2$. For what value of x is $f'(x)$ zero? What is the sign of $f''(x)$ for the same value of x?

**Ans:** $f'(x) = 6 - 2x$, $f''(x) = -2$, $x = 3$, **negative**

c. Evaluate $\int \ln x \, dx$.

**Ans:** $x \ln x - x + C$

### Group 'B' [5 × 2 × 4 = 40]

6. a. If A and B are the subsets of a universal set U, prove that,
$A \Delta B = (A \cup B) - (A \cap B)$.
OR
Define absolute value of a real number. Solve the inequality $|2x - 1| \geq 3$ and draw its graph.
**Ans:** $\{x: x \leq -1 \text{ or } x \geq 2\}$

b. Sketch the graph of $y = 3^x$ indicating its different characteristics.

7. a. In any $\Delta ABC$, if $a^2, b^2, c^2$ are in AP, prove that cot A, cot B, cot C are in A.P.
OR
If $\tan^{-1}\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \theta$ prove that $x^2 = \sin 2\theta$.

b. By using the properties of determinant, show that.
$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

8. a. Apply row–equivalent matrix method or matrix inversion method to solve the following system of linear equations.
$2x - y + 4z = -3;\ x - 4z = 5;\ 6x - y + 2z = 10$
**Ans:** $x = 3, y = 7, z = \frac{-1}{2}$

b. Find the condition for two given quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ may have one root common and both roots common.
**Ans:** $(b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2, \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

9. a. Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2+2y^2-8x-12y-9=0$.
**Ans:** $x^2 + y^2 - 4x - 6y - 87 = 0$

b. Evaluate: $\displaystyle\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$
**Ans:** $\cos\theta + \theta\sin\theta$
OR
When a function $f(x)$ is said to be continuous at a given point $x = a$? Discuss the continuity of the function.
$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x^2 - 2x - 3}, & x \neq 3 \\ \dfrac{2}{3}, & x = 3 \text{ at } x = 3 \end{cases}$$
**Ans:** Discontinuous, $f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x^2 - 2x - 3}, & x \neq 3 \\ \dfrac{5}{4}, & x = 3 \end{cases}$

10. a. Find, from first principles, the derivative of $\sqrt{\sin 2x}$.
**Ans:** $\dfrac{\cos 2x}{\sqrt{\sin 2x}}$

b. Using integration, find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.
**Ans:** $12\pi$

### Group 'C' [6 × 5 = 30]

11. Let a function f: R → R be defined by $f(x) = 2x + 3$, $x \in R$. Show that f is one to one and onto. Find a formula that defines the inverse function $f^{-1}$.
**Ans:** $\dfrac{x-3}{2}$

12. Define arithometico–geometric series. Sum to n terms the series: $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\ldots$
**Ans:** $2 - \frac{1}{2^n - 1} - \frac{n}{2^n}$

13. Find the equations of the bisectors of the angles between the lines $x = y$ and $y = 7x + 4$. Identify the bisector of the obtuse angle.
**Ans:** $6x - 3y + 2 = 0$, $x + 2y + 2 = 0$ (obtuse angle bisector)
OR
Find the angle between the line pair represented by $ax^2 + 2hxy + by^2 = 0$. Also deduce the condition of perpendicularity and coincidency.
**Ans:** $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a + b}\right)$, $a + b = 0$, $h^2 = ab$

14. Define absolute value of a complex number. Prove that for any two complex numbers z and w, $|z + w| \leq |z| + |w|$. Also verify this result for $z = 3 + 4i$ and $w = 5 - 12i$.

15. What are the criteria to be satisfied in order to have maximum and minimum values of a function? Find the extreme values of the function: $f(x) = x^5 - 5x^4 + 5x^3 - 1$.
**Ans:** Max. value = 0 at x = 1, Min value = –28 at x = 3
OR
A man 1.5 meters tall walks at a uniform speed of 5 km/hr away from a lamp-post 6 meters high. Find the rate at which the length of his shadow increases.
**Ans:** $\frac{5000}{3}$ m/hr.

## Set 3

### Group 'A' [5 × 3 × 2 = 30]

1. a. Solve the inequality $|2x - 1| \geq 3$.
**Ans:** $\{x : x \geq 2 \text{ or } x \leq -1\}$

b. Find the domain of the function $f(x) = \sqrt{9 - x^2}$.
**Ans:** [–3, 3]

c. Define odd and even function with examples.

2. a. Find the general solution of $\cot x + \tan x = 2 \operatorname{cosec} x$.
**Ans:** $2n\pi \pm \frac{\pi}{3}$

b. Find sum to n–terms the series 2.4.6+4.6.8+6.8.10+...
**Ans:** $2n(n + 1)(n + 2)(n + 3)$

c. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$, find $(AB)^T$.
**Ans:** $\begin{pmatrix} 23 & 31 \\ 34 & 46 \end{pmatrix}$

3. a. Solve by Cramer's rule: $\frac{3}{x} - \frac{7}{y} = 1, \frac{5}{x} - \frac{4}{y} = 17$.
**Ans:** $x = \frac{1}{5}$ and $y = \frac{1}{2}$

b. Express $-\sqrt{3} + i$ in polar form.
**Ans:** $2(\cos 150° + i \sin 150°)$

c. Find the quadratic equation with rational coefficient one of whose roots is $\dfrac{1}{5 + 3i}$.
**Ans:** $34x^2 - 10x + 1 = 0$

4. a. Find the point of intersection of the pair of straight lines $x + 5y + 3 = 0$ and $5x - 2y - 12 = 0$.
**Ans:** (2, –1)

b. Find the equation of a circle concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5, 4).
**Ans:** $x^2 + y^2 - 8x + 12y - 49 = 0$

c. Prove that: $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2} = 1$.

5. a. Differentiate tanx with respect to secx.

**Ans: cosecx.**

b. Find where the graph of the function $f(x) = 2x^3 - 6x^2 + 5$ is concave downward.

**Ans: x <1**

c. Evaluate $\int \sec x \, dx$.

**Ans: log(secx + tanx) + C**

**Group 'B' [5 × 2 × 4 = 40]**

6. a. Let A, B and C be the subsets of a universal set U. Then prove the followings.

i. $\overline{A \cup B} = \bar{A} \cap \bar{B}$

ii. $A \cap (B - C) = (A \cap B) - (A \cap C)$

OR

Prove that. $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$.

b. Sketch the graph of $y = \log_2 x$ stating its different characteristics.

7. a. State and prove sine law of trigonometry.

OR

If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + \sqrt{z(1 - z^2)} = 2xyz$.

b. Prove that: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

8. a. Solve the following system of equation by row equivalent matrix method or matrix inversion method.

$x + 4y + z = 18$

$3x + 3y - 2z = 2$

$-4y + z = -7$

**Ans: x = 8, y = 1, z = 2**

b. If α and β are the roots of the equation $x^2 - ax + b = 0$ Find the equation whose roots are $\alpha^2\beta^{-1}$ and $\beta^2\alpha^{-1}$.

**Ans: $bx^2 - (a^3 - 3ab)x + b^2 = 0$**

9. a. Prove that two circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

b. Evaluate $\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$.

**Ans: sinθ – θ cosθ**

OR

A function f(x) is defined as follows

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 6x - 1 & \text{for } x > 1 \end{cases}$$

. Is the function continuous at x = 1? If not how can you make it continuous?

**Ans: Discontinuous, $f(x) = \begin{cases} 2x + 3 & \text{for } x < 1 \\ 5 & \text{for } x = 1 \\ 6x - 1 & \text{for } x > 1 \end{cases}$**

10. a. Find from the first principle the derivative of y = cosx.

**Ans: –sin x**

b. Using integration, to obtain the area bounded by curve $y = x^2$ and $y = 2x$

**Ans: $\frac{4}{3}$ sq. units**

**Group 'C' [5 × 6 = 30]**

11. Let f : R → R be defined by f(x) = 2x and g : R→R defined by g(x) = x + 1. Find fog(x) and gof(x). Are the functions fog(x) and gof(x) injective?

**Ans: 2x + 2, 2x + 1, both are injective**

12. Prove by the principle of mathematical induction that

$1^2 + 2^2 + 3^3 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

13. State De Moivre's theorem and use it to compute the square root of $z = 2 + 2\sqrt{3}\,i$.

**Ans: $\pm(\sqrt{3} + i)$**

14. Find the length of the perpendicular from a point $(x_1, y_1)$ on a straight line $x \cos\alpha + y \sin\alpha = p$.

OR

Prove that the straight lines joining origin to the points of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$

15. An aeroplane is flying horizontally at a height of $\frac{2}{3}$ mile with a velocity of 15 m.p.h. Find the rate at which it is receding from a fixed point on the ground which it passed over 2-minutes ago.

**Ans: 9 m.p.h**

OR

A gardener having 120m of fencing wishes to enclose a rectangular plot of land and also to erect a fence across the land parallel to two of the sides. Find the maximum area he can enclose.

**Ans: 600 m²**

## Set 4

**Group 'A' [5 × 3 × 2 = 30]**

1. a. If A = [–2, 3] and B = [0, 4). Compute A∪B and A∩B.

**Ans: (–2, 4), [0, 3]**

b. Let A = {2, 3, 4} and B = {2, 3, 4, 6}. Find a relation from set A to set B determined by the condition that 'x divides y'.

**Ans: {(2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4)}**

c. Examine the symmetry and even or odd nature of the function $f(x) = x - x^3$.

**Ans: symmetric about origin, odd function.**

2. a. Find the value of $\cos \tan^{-1} \sin \cot^{-1} x$.

**Ans: $\frac{\sqrt{1 + x^2}}{\sqrt{2 + x^2}}$**

b. Prove by mathematical induction that $n^2 + n$ is an even number.

c. Prove that the matrices $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$ are inverse of each other.

3. a. Solve the following equations by using row-equivalent matrix method

$2x - 3y + 6 = 0$; $3x + 2y + 9 = 0$

**Ans: x = –3 and y = 0**

b. If ω be a complex cube root of unity, Show that $\frac{a+b\omega+c\omega^2}{a\omega+b\omega^2+c}+\frac{a+b\omega+c\omega^2}{a\omega^2+b+c\omega}=-1$

c. Form the quadratic equation whose one root is $\frac{1}{2+3i}$.

**Ans: $13x^2 - 4x + 1 = 0$**

4. a. Find the equation of the straight line passing through the intersection of

$x + 3y + 2 = 0$, $2x - y - 3 = 0$ and slope $= \frac{1}{3}$.

**Ans: $x - 3y - 4 = 0$**

b. Find the equation of tangents to the circle $x^2 + y^2 = 4$ and parallel to $3x + 4y = 1$.

**Ans: $3x + 4y \pm 10 = 0$**

c. Find the points of discontinuity of the function $f(x) = \frac{4x^2 - 16}{x^2 - 3x + 2}$.

**Ans: 1, 2**

5. a. Find $\frac{dy}{dx}$ of $x^3 + y^3 = 3axy$.

**Ans: $\frac{ay - x^2}{y^2 - ax}$**

b. If $y = x - \frac{1}{x}$ prove that y is always increasing for all $x \in \mathbb{R}$.

c. Evaluate $\int \sec x \, dx$.

**Ans: log (secx + tanx) + c**

## Group 'B' [5 × 2 × 4 = 40]

6. a. Define tautology and contradiction. Prove, by means of truth table, that $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction.

OR

If A, B and C are the subsets of a universal set U, prove that $A - (B \cup C) = (A - B) \cap (A - C) = (A - B) - C$.

b. Sketch the graph of $y = \frac{1}{x+3}$ indicating its different characteristics.

7. a. Find the general values of $x$ when $\cos 2x + \sin 2x = \cos x + \sin x$.

**Ans: $2n\pi$, $(4n \pm 1)\frac{\pi}{6}$**

OR

If $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$ prove that $C = 45°$ or $135°$

b. Using the properties of determinant, prove that

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

8. a. Applying Cramer's rule or inverse matrix method, solve the following system of linear equations.

$y - x + z = 0$,
$z + x - 2y = 0$
$3y - 7z + 2x = 5$

**Ans: $x = 3$, $y = -2$, $z = 1$**

b. If α and β are the roots of $ax^2 + bx + c = 0$. Find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

**Ans: $x^2 - 2\left(\frac{b^2 - 2ac}{a^2}\right)x + \frac{b^2}{a^2}\left(\frac{b^2 - 4ac}{a^2}\right) = 0$**

9. a. Prove that the two circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

b. Evaluate $\lim_{x \to \theta} \frac{x \cot \theta - \theta \cot x}{x - \theta}$

**Ans: $\cot\theta + \theta \operatorname{cosec}^2\theta$**

OR

Define continuity of a function at a point. Test the continuity of

$$f(x) = \begin{cases} x \sin\frac{1}{x} & \text{for } x \neq 0 \\ 1 \text{ for} & x = 0 \end{cases} \text{ at } x = 0$$

**Ans: Discontinuous**

10. a. Find, from definition the derivative of $y = \log_5 x$.

**Ans: $\frac{\log_5 e}{x}$**

b. Using integration, find the area of region between $y^2 = 4ax$ and $x^2 = 4ay$.

**Ans: $\frac{16a^2}{3}$**

## Group 'C' [6 × 5 = 30]

11. Show that f: R → R defined by f(x) = 3x + 4 is one to one and onto. Find $f^{-1}(x)$. Also prove that $fof^{-1}(x) = f^{-1}of(x) = x$.

12. The A.M between two number exceeds their G.M by 2 and the G.M exceeds the H.M by 1.6. Find the number.

**Ans: 4, 16 or 16, 4**

13. Show that the product of the perpendicular distances drawn rom the two points $(\pm\sqrt{a^2 - b^2}, 0)$ upon the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is $b^2$

OR

Find the equations of the two lines represented by the equation $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$. Find their points of intersection and also the angle between them.

**Ans: $x + y + 3 = 0$, $2x + y - 1 = 0$, $(4, -7)$, $\tan^{-1}(\pm\frac{1}{3})$**

14. State De Moivre's theorem. Using De- Moivre's theorem, find the sixth roots of 1.

**Ans: $\pm 1$, $\frac{1 \pm i\sqrt{3}}{2}$, $\frac{-1 \pm i\sqrt{3}}{2}$**

15. Find the local and global maximum and minimum value of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval $[-1, 2]$.

**Ans: Absolute Max. 0, Absolute Min. –9, Max. value = $\frac{7}{2}$, Min. $-\frac{25}{2}$**

OR

Sand is pouring from a pipe at the rate of 12 $\frac{cm^3}{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height sand cone increasing when the height is 4cm?

**Ans: $\frac{1}{48\pi}\frac{cm}{sec}$**

## Set 5

### Group 'A' [5 × 3 × 2 = 30]

1. a. If A and B are subjects of a universal set U, prove that $A \cap B = \phi$ implies $B \cap \bar{A} = B$

   b. Let f: R → R and g: R→ R be defined by f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3), find fog.

   **Ans: { (2, 5), (5, 2), (1, 5)}**

   c. Examine the symmetry and even or odd nature of the function $y = x^2 + x^4$.

   **Ans: Symmetry about y – axis, even function.**

2. a. Find the general solution of θ if $\sin^2\theta = \sin^2\alpha$.

   **Ans: $\theta = n\pi \pm \alpha$**

   b. Prove that: $2^{\frac{1}{2}} . 2^{\frac{1}{4}} . 2^{\frac{1}{8}} ... = 2$

   c. If $A = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$. Find a matrix X such that $AX = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

   **Ans: $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$**

3. a. Examine whether the following system of linear equations is consistent or not. Give reason for your answer.
   $-3x + 4y = 7$
   $6x - 8y = 14$

   **Ans: Inconsistent and independent**

   b. Express the complex number $\frac{(1-i)^2}{1+2i}$ into a + ib from.

   **Ans: $\frac{-4}{5} - \frac{2}{5}i$**

   c. Determine the nature of roots of equation $7x^2-6x+3=0$.

   **Ans: Imaginary and unequal**

4. a. Find the angle between the line pair $3x^2+7xy+2y^2 = 0$.

   **Ans: 45°,135°**

   b. Find the centre and radius of the circle $3x^2 + 3y^2 + 12x - 18y - 11 = 0$.

   **Ans: (–2, 3). $5\sqrt{\frac{2}{3}}$**

   c. Evaluate $\lim_{n \to \infty} \frac{1+2+3+...+n}{n^2}$

   **Ans: $\frac{1}{2}$**

5. a. Find $\frac{dy}{dx}$ of $y = x^x$.

   **Ans: $x^x(1 + \log x)$**

   b. The temperature T at time t at a place is given by $T(t) = 3t^2 - 10t + 5$. Is it getting Warmer or Cooler at time t = 2 and t = 1?

   c. Evaluate $\int \frac{x+2}{x^2+4x} dx$.

   **Ans: $\frac{1}{2}\log(x^2 + 4x) + C$**

### Group 'B' [5 × 2 × 4 = 40]

6. a. If p and q are any two statements, then by means by truth table, prove that
   i. $p \wedge q \equiv q \wedge p$
   ii. $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$

   OR

   Solve the inequality $|2x - 1| \geq 3$ and draw its graph.

   **Ans: $(-\infty, -1] \cup [2, \infty)$**

   b. Sketch the graph of the function $f(x) = x^2 - 4x + 3$ stating its main characteristics.

7. a. State and prove cosine law.

   OR

   If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then prove that:
   $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

   b. Without expanding show that
   $$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

8. a. Applying row equivalent matrix method or inverse matrix method, solve the system of linear equations.
   $3y + z = 5$
   $2x + 3 = z$
   $x + 2y + z = 7$

   **Ans: $x = \frac{8}{5}, y = \frac{2}{5}, z = \frac{31}{5}$**

   b. The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ cannot have more than two roots. Prove it.

9. a. If $y - x = 2$ is the equation of a Chord of the circle $x^2 + y^2 + 2x = 0$, Find the equation of the circle of which this chord is a diameter.

   **Ans: $x^2 + y^2 + 3x - y + 2 = 0$**

   b. Evaluate $\lim_{x \to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$.

   **Ans: $\frac{1}{2}$**

   OR

   A function f(x) is defined by
   $$f(x) = \begin{cases} x^2-1 & \text{for } x<2 \\ 2x & \text{for } x=2 \\ x+1 & \text{for } x>2 \end{cases}$$
   Is the function continuous at $x = 2$? If not how can you make it continuous at $x = 2$?

   **Ans: Discontinuous, redefining the function f(x) = 3 at x = 2**

10. a. Find, from first principle the derivative of $\sin^2x$.

    **Ans: sin2x**

    b. Using integration, find the area of the circle $x^2 + y^2 = a^2$.

    **Ans: $\pi a^2$**

### Group 'C' [5 × 6 = 30]

11. Define one–one and onto function. Let a function f: A→B be defined by
    $f(x) = \frac{x+1}{2x-1}$ with A = {–1, 0, 1, 2, 3, 4} and B {–1, 0, 1,2,3,4}
    and $B = \left\{-1, 0, \frac{4}{5}, \frac{5}{7}, 1, 2, 3\right\}$. Find the range of f. Is the function f one–one and onto both? If not, how can the function be made one-one and onto both?

12. State the principle of mathematical induction. Using the principle of mathematical induction, prove that:
$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$.
OR
The side of a given square is 10cm. The mid points of its sides are joined to form a new square. Again the mid points of the sides of this new square are joined to form another square. This process is continued indefinitely. Find the sum of the areas and the sum of the perimeters of the squares.

**Ans: 200 sq.cm, $\frac{40(2+\sqrt{2})}{1}$ cm**

13. If $P_1$ and $P_2$ are the lengths of perpendicular drawn form the points $(\cos\theta, \sin\theta)$ and $(-\sec\theta, \operatorname{cosec}\theta)$ on the line $x \sec\theta + y \operatorname{cosec}\theta = 0$ respectively, prove that $\frac{4}{P_1^2} - P_2^2 = 4$.

OR
Find the equation of bisection of the angles between the pair of lines represented by $ax^2 + 2h\,xy + by^2 = 0$,

**Ans: $h(x^2 - y^2) - (a - b)xy$.**

14. State and prove De-Moivre's theorem. Using it, find the fourth roots of $-8 - 8\sqrt{3}\,i$.

**Ans: $\pm(\sqrt{3} - i)$, $\pm(1 + \sqrt{3}\,i)$**

15. Define Concavity and Convexity of curves with figures. Determine where the graph is Concave upwards and where it is concave downwards of the function $f(x) = x^4 - 8x^3 + 18x^2 - 24$.

**Ans: concave upward for x > 3, x > 1; concave downward 1<x<3**