Mathematics

NEW SYLLABUS

Full Marks: 100 **Teaching hours: 150**

I. Introduction:

This course is a continuation to the course of Grade XI. It further consolidates the concept learnt in Grade XI. For the completeness of the course it includes other areas in mathematics such as Mechanics, Group theory, Statistics and probability, Numerical methods.

Group (A) contains eleven units, and is compulsory while Group (B) and Group (C) each containing four units are optional. Students will be required to offer either Group (B) or Group (C).

II. Specific Objectives:

On completion of this course students will able to:

1. state basic principles of counting and find number of permutations and combinations of set of objects with various conditions;
2. prove binomial theorem for positive index, state exponential and logarithmic series, and apply them in solving problems;
3. understand group as algebraic structure and establish simple results on finite and infinite groups;
4. derive equations of parabola, ellipse and hyperbola and find tangent & normal to the parabola;
5. locate points in space and derive the equation of plane;
6. define product of vectors and give their geometrical meaning and use it to find various results of geometry and trigonometry;
7. establish the relation between continuity and differentiability of a function, compute, derivatives of exponential, logarithmic and hyperbolic and inverse circular functions, apply Hospital's rule;
8. determine standard integrals, use partial fractions of integrate rational function;
9. define differential equations and different forms of solutions and use them in application;
10. state measures of dispersion and find coefficient of correlation and equation of regression;
11. define probability, establish basic laws of probability;
12. define parallelogram of forces, composition and resolution of forces, triangle of forces, and prove Lami's theorem;
13. find resultant of like and unlike parallel forces, moment of a force and moment of couple of forces;
14. state and use Newton's laws of motion. Find Impulse, Work, Energy & Power, and acquaint with a projectile;
15. formulate linear programming problem, solve LPP graphically and by simplex method;
16. determine a root of equations by numerical methods; and
17. evaluate integrals by trapezoid and Simpson's rules.

III. Course Contents:

Group 'A'

Unit 1:Permutation and Combination. **10 hrs**

Basic principle of counting, Permutation of (a) set of objects all different (b) set of objects not all different (c) circular arrangement (d) repeated use of the same object. Combination of things all different, Properties of combination.

Unit 2:Binomial Theorem **10 hrs**

Binomial theorem for a positive integral index, general term. Binomial coefficients, Binomial theorem for any index (Without proof), Application to approximation, Euler's number. Expansion of e^x, a^x and log (1 + x) (without proof).

Unit 3:Elementary Group Theory **8 hrs**

Binary operation, Binary operation on sets of integers and their properties, Definition of a Group, Groups whose element are not numbers, Finite and infinite groups, Uniqueness of identity, Uniqueness of inverse, Cancellation law, Abelian Group.

Unit 4:Conic Sections **12 hrs**

Standard equation of parabola, Ellipse and Hyperbola, Equations of tangent and normal to a parabola at a given point.

Unit 5:Co-ordinates in Space **12 hrs**

Co-ordinate axes, Co-ordinate planes, The octants, Distance between two points, External and internal point of division. Direction cosines and ratios, fundamental relation between direction cosines, Projections, Angle between two lines. General equation of a plane, Equation of a plane in intercept and normal form, Plane through three given points, Plane through the intersection of two given planes, Parallel and perpendicular planes, angle between two planes distance of a point from a plane.

Unit 6:Vectors and its Applications **14 hrs**

Cartesian representation of vectors, Collinear and non-collinear vectors, Coplanar and non-Coplanar vectors, Linear combination of vectors.

Scalar product of two vectors, Angle between two vectors, Geometric interpretation of scalar product, Properties of Scalar Product, Condition of perpendicularity, Vector product of two vectors, Geometric interpretation of vector product, Properties of Vector Product, Application of product of vectors in plane trigonometry.

Unit 7:Derivative and its Application **14 hrs**

Derivative of inverse trigonometric, exponential and logarithmic functions by definition, Relationship between continuity and differentiability, Rules for differentiating hyperbolic function and inverse hyperbolic function, Composite function and function of the type $f(x)^{g(x)}$. L'Hospital's rule (for 0/0, ∞/∞), Differentials, Tangent and Normal, Geometric interpretation and application of Rolle's theorem and Mean value theorem.

Unit 8:Antiderivatives 7 hrs

Antiderivatives, Standard integrals, Integrals reducible to standard forms, Integrals of rational functions.

Unit 9:Differential Equations and their Applications 7 hrs

Differential equation and its order and degree, Differential equations of first order and first degree: Differential equations with separable variables, homogeneous and exact differential equations.

Unit 10: Dispersion, Correlation and Regression 12 hrs

Dispersion. Measures of dispersion (Range, Semi interpqrtlle range, Mean deviation, Standard deviation) variance, Coefficient of variation, Skewness, Karl Pearson's and Bowley's Coefficient of Skewness, Bivariate distribution, Correlation, Nature of correlation, Correlation coefficient by Karl Pearson's method. Interpretation of correlation coefficient, Properties of correlation coefficient (Without proof) Regression equation, Regression line of *y* on x and x on *y*.

Unit 11: Probability 8 hrs

Random experiment, sample space, Event, Equally likely cases, Mutually exclusive events, Exhaustive cases, Favourable cases, Independent and dependent cases, Mathematical and empirical definition of probability, Two basic laws of probability, Conditional probability (without proof), Binomial distribution, Mean and Standard deviation of binomial distribution (without proof).

Group 'B'

Unit 12: Statics 9 hrs

Forces and resultant forces, Parallelogram of forces, Composition and resolution of forces, Resultant of coplanar forces acting at a point, Triangle of forces and Lami's theorem.

Unit 13: Statics (Continued) 9 hrs

Resultant of like and unlike parallel forces, Moment of a force, Varignon's theorem, Couple and its properties (without proof).

Unit 14: Dynamics 9hrs

Motion of particle in a straight line, Motion with uniform acceleration, Motion under gravity, Motion down a smooth inclined plane. The concepts and theorems be restated and formulated as application of calculus.

Unit 15: Dynamics (Continued) 9 hrs

Newton's laws of motion, Impulse, Work, Energy and Power, Projectiles.

Group 'C'

Unit 16: Linear Programming 11 hrs

Introduction of a linear programming problem (LPP), Graphical solution of LPP in two variables, Solution of LPP by simplex method (two variables).

Unit 17: Computational Method 9 hrs

Introduction to Numerical computing (Characteristics of Numerical computing Accuracy, Rate of Convergence, Numerical Stability, Efficiency); Number systems (Decimal, Binary, Octal & Hexadecimal system conversion of one system into another), Approximations and error in computing Roots of nonlinear equation, Algebraic, polynomial & transcendental equations and their solution by bisection and Newton - Raphson Methods,

Unit 18: Computational Method (Continued) 8 hrs

Solution of system of linear equations by Gauss elimination method, Gauss-Seidel method, Ill conditioned systems, Matrix inversion method.

Unit 19: Numerical Integration 8 hrs

Trapezoidal and Simpson's rules, estimation of errors.

V. Reference books:

1. Adhikari, D.B. and et.al. ***Element of Mathematics Part II.*** Himalaya Book Stall.
2. Bajracharya, D.R.; Shrestha, R.M. and et.al. ***Higher Secondary Level Basic Mathematics (For Grade XII).*** Kathmandu: Sukunda Pustak Bhawan.
3. Bajracharya, P.M. and Basnet, G. (2008). ***Fundamentals of Mathematics (For Grade XII).*** Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
4. Balagurusamy, E., ***Numerical Methods.*** India: Tata McGraw Hill.
5. Pant, S.R. and et.al. ***A Text-Book of Higher Secondary Mathematics (For Grade XII).*** Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
6. Ranganath, G.K. and Narayan, B.S. ***A Text-Book of Computer Oriented Numerical Methods and Linear Programming.***
7. Upadhyaya, M.P., ***An Introduction to Linear Programming.*** Kathmandu: Sukunda Pustak Bhawan.
8. Uprety, K.N. and Ghimire, K.P., ***Foundation of Mathematics, (For Grade XII).*** Pigeon Educational Publisher.
9. Sitaula, K., Sharma, B., Bhatta, C.R., ***Essential Mathematics***

Evaluation Scheme

Group	Question of 2 marks		Question of 4 marks			Question of 6marks			Total marks
	Number	Total marks	Number	OR-question	Total marks	Number	OR-question	Total marks	
A	12	24	8	3	32	3	1	18	74
B	3	6	2	1	8	2	1	12	26
C	3	6	2	1	8	2	1	12	26

Remarks:
1. One question carrying 4 marks or 6 marks will be asked from each unit.
2. One question carrying 2 marks will be asked from each of the 11 units of group A.
3. One question carrying 2 marks will be asked from one of the 8 units of group A from which questions carrying 4 marks are asked
4. One question carrying 2 marks will be asked from each of any 2 units of group B from which questions carrying 4 marks are asked and one from each of any 2 units of group C from which questions carrying 4 marks are asked.
5. One question carrying 2 marks will be asked from one of any 2 units of group B and one from any two units of group C from which questions carrying 6 marks are asked.
6. OR-questions will be asked from the same unit.
7. The students are required to attempt all questions from Group A and all questions from Group B or C.

MODEL QUESTIONS

Grade: XII
Time: 3 hrs
Full Marks: 100
Pass Marks: 35

NEW MODEL QUESTIONS - 2068 (SET I)

Candidates are required to give their answer in their own words as far as practicable
The figures in the margin indicate full marks.
Attempt ALL questions of group A and group B or C.

Group A

1. a. It is required to seat 5 boys and 4 girls in a row so that the girls occupy the even places. How many such arrangements are possible? [2]
Ans: 2880 ways

b. Prove that: $\frac{1}{1.3}+\frac{1}{2.5}+\frac{1}{3.7}+\frac{1}{4.9}+\ldots = 2(1-\ln 2)$. [2]

c. Let $a * b = 3a + 2b$ for $a, b \in \mathbb{Z}$. Verify that * is a commutative binary operation on $\mathbb{Z}$. [2]

2. a. Find the equation of a hyperbola in standard position such that the length of transverse axis is 6 and it passes through (4, 2). [2]
Ans: $4x^2-7y^2=36$

b. Find the locus of points which are equidistant from the points (1, 2, 3) and (3, 2, –1). [2]
Ans: $x-2z=0$

c. Find the cosines of the angle between the vectors: $\vec{a} = (1,-2,-2)$, $\vec{b} = (2,1,-2)$. [2]
Ans: $\frac{4}{9}$

3. a. Find the derivative of $(\ln x)^{\sinh x}$. [2]
Ans: $(\ln x)^{\sinh x}\left[\frac{\sinh x}{x \ln x}+\cosh x \ln(\ln x)\right]$

b. Find the integral $\int \frac{dx}{1+2\sin x}$ [2]
Ans: $\frac{1}{\sqrt{3}}\ln\left|\frac{\tan x/2+2-\sqrt{3}}{\tan x/2+2+\sqrt{3}}\right|+C$

c. Find the integral $\int \frac{dx}{(x+7)\sqrt{2-x}}$ [2]
Ans: $\frac{1}{3}\ln\left|\frac{\sqrt{2-x}-3}{\sqrt{2-x}+3}\right|+C$

4. a. Solve the differential equation: $\frac{dy}{dx} = e^{x+y}+3x^2e^y$. [2]
Ans: $e^x+e^{-y}+x^3+C=0$

b. From the following data, calculate the expected value of Y when X = 25, and correlation coefficient r = 0.95. [2]

	X	Y
Average	5.6	12.5
Standard deviation	3.2	2.4

Ans: 26.3225

c. The average percentage of failures in a certain examination is 40. What is the probability that out of 5 candidates, at least 3 will be passed in the examination? [2]
Ans: 0.683

5. a. Show that the number of combinations of n different objects taken r at a time is given by

$$C(n, r) = \frac{n!}{(n-r)!\,r!}$$

Also, prove that $C(n, n-r) = C(n, r)$. [4]

OR

State the multiplication principle of counting. Prove that the number of circular permutations of n different objects taken all at a time is $(n-1)!$ [4]

b. What is a group? If a binary operation * is defined on a set S = {a, b, c} by the following Caley's table.

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Show that (S, *) is a group. [4]

OR

Let a, b, c and x be elements of a group G. Solve for x if $axb = c$ and $x^2b = xa^{-1}c$. [4]

6. a. Find the integral $\int \frac{x}{x^3+1}dx$. [4]
Ans: $-\frac{1}{3}\ln|x+1|+\frac{1}{6}\ln|x^2-x+1|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+C$

b. What is a linear differential equation? Solve:

$(x^2+1)\frac{dy}{dx}+2xy = 3x^2$. [4]
Ans: $y(x^2+1) = x^3+C$

7. a. State the first mean value theorem of differential calculus and interpret it geometrically. Using it to $f(x) = \sin x$ on $[0, x]$, prove that $\sin x \le x$ for $x \ge 0$. [4]

b. An urn contains four white, eight black, six red and two green marbles. If three balls are drawn at random, find the probability of getting (i) all white marbles (ii) 2 red and 1 green marbles. [4]
Ans: (i) $\frac{1}{285}$ (ii) $\frac{1}{38}$

8. a. What is a conic section? Find the equation of the tangent to the parabola $y^2 = 8x$ which is parallel to the straight line $2x - 3y + 7 = 0$. Also find its point of contact. [4]

Ans: $2x - 3y + 9 = 0$ and (9/2, 6)

b. Define linearly independent vectors. Show that the following vectors are linearly dependent.

$2\vec{i} + \vec{j} - \vec{k}, 3\vec{i} - 2\vec{j} + \vec{k}, \vec{i} + 4\vec{j} - 3\vec{k}$ [4]

OR

Prove that if θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} = ab \cos\theta$. [4]

9. For any positive integer n, prove that:

$(a + x)^n = C(n, 0)\, a^n + C(n, 1)\, a^{n-1} x + C(n, 2)\, a^{n-2} x^2 + \ldots + C(n, n)\, x^n$.

Find the term containing x^2, if any, in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$. [6]

Ans: 20/3

10. Find the direction cosines of two lines which are connected by the relations $2l + 2m - n = 0$, $mn + nl + lm = 0$. [6]

OR

Ans: $l = \frac{2}{3}, m = \frac{-1}{3}, n = \frac{2}{3}$ and $l = \frac{-1}{3}, m = \frac{2}{3}, n = \frac{2}{3}$

Prove that a plane through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Also, find the angle between planes $2x - y + z = 6$ and $x + y + 2z = 3$. [6]

Ans: $\theta = \frac{\pi}{3}$

11. Lives of two models of refrigerators turned in for new models in a recent survey are

No. of years	No. of refrigerators	
	Model A	Model B
0 – 2	5	2
2 – 4	16	7
4 – 6	13	12
6 – 8	7	19
8 – 10	5	9
10 – 12	4	1

What is the average life of each model of these refrigerators? Which model has more uniformity? [6]

Ans: Average life of model: A = 5.12 years and B = 6.16 years and Model B [CV_A (54.88%) > CV_B (36.20%)]

Group B

12. a. Three forces P, Q and R acting on a particle are in equilibrium, the angle between the P and Q is 60° and that between Q and R is 150°. Find the ratios of the forces. [2]

Ans: $1:1:\sqrt{3}$

b. A uniform beam, 4 m long, is supported in a horizontal position by two props which are 3 m apart, so that the beam projects one meter beyond one of the props. Show that the force on one of the props is double of that on the other. [2]

c. A pump having a power of 392 W pumps water at the rate of 100 litres per minute. Find the height to which the water is raised. ($g = 9.8 m/s^2$, 1 litre of water = 1kg) [2]

Ans: 24 m

13. a. A body of weight w is suspended by strings of length 3 m and 4 m attached to two points in the same horizontal line whose distance apart is 5 m. Find the tensions along the strings. [4]

Ans: $T_1 = \frac{4}{5}w$ and $T_2 = \frac{3}{5}w$

b. A body of mass 49 kg is falling freely under gravity at the rate of 20 m/s. What is the uniform force that will stop it (i) in 2 sec (ii) in 50 cm? ($g = 9.8$ ms/²) [4]

Ans: (i) 99 kg.wt. (ii) 2049 kg.wt

OR

A bullet fired into a target losses half its velocity after penetrating 3 cm. How much further will it penetrate? [4]

Ans: 1 cm

14. a. The resultant of two like parallel forces P and Q acting on rigid body is a force of magnitude P + Q in the same direction as P and Q are. If A and B are any points on the lines of action of P and Q respectively, prove that the resultant divides line segment AB internally in the inverse ratio of the forces. [6]

OR

Define the moment of a force. Forces 1, 2, 4, 5 kg-wts act along the sides of a square taken in order. Prove that their resultant is parallel to a diagonal and find where it cuts the side along which the first force acts. [6]

Ans: 2:3

15. A man travels from A to B in 45 minutes. At C, somewhere between A and B, it attains its maximum velocity of 45 m per hr. If he travels with uniform acceleration from A to C and uniform retardation from C to B, find the distance between A and B, it being supposed that the man starts from rest at A and comes to rest at B. [6]

Ans: $16\frac{7}{8}$m

Group C

16. a. Determine graphically the solution set of the following system of inequalities:

$2x + y \geq 2, 3x + 2y \leq 4, x \geq 0, y \geq 0$ [2]

b. Write a short note on accuracy of a numerical method. [2]

c. Apply the Simpons's rule to approximate the value of $\int_1^4 e^x \ln x \, dx$ with n = 3. [2]

Ans: 58.9698

17. a. Using the simplex method, maximize $p = 6x - 9y$ subject to $2x - 3y \leq 6, x + y \leq 20, x \geq 0, y \geq 0$. [4]

Ans: Max. value p = 18 at (3, 0)

b. Use Bisection method to find solution accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on the interval [1, 3.2] [4]

Ans: 2.99375

OR

Write three methods for measuring error. Approximate $\sqrt{11}$ by Newton-Raphson's method with accuracy 0.00001. [4]

Ans: 3.31662

18. Find the approximate solution of the following system of equation by matrix inversion method:

$2x - y + z = -2, x + y - 2z = -9, x + 2y + z = 9$. [6]

Ans: (x, y, z) = (−2, 3, 5)

Derive the trapezoidal rule. The capacity of a battery is a measure of $\int i\,dt$, where i is the current. Estimate, using the Trapezium rule, the capacity of a battery whose current was measured over an eight hour period with the results shown below.

Time/hours	0	1	2	3	4	5	6	7	8
Current/Amps	25.2	29.0	31.8	36.5	33.7	31.2	29.6	27.3	28.6

Ans: 246

OR

Compute an approximate value of $\int_0^1 (1+x^2)^{-1}\,dx$ by using the composite trapezoid rule with three points. Then compare with the actual value of the integral. Next, determine the error formula and numerically verify an upper bound on it. [6]

Ans: 0.785; 0.01; 0.04167

NEW MODEL QUESTIONS - 2068 (SET II)

Grade: XII **Full Marks: 100**
Time: 3 hrs **Pass Marks: 35**

Candidates are required to give their answer in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt ALL questions of group A and group B or C.

Group A

1. a. In an examination paper containing 10 questions, a candidate has to answer 7 questions. If two questions are made compulsory, in how many ways can he choose 7 questions in all? [2]

Ans: 56 ways

b. Find the middle term in the expansion of $\left(2x+\frac{1}{3x^2}\right)^9$. [2]

Ans: $\frac{448}{9}\frac{1}{x^3}$ and $\frac{224}{27}\frac{1}{x^6}$

c. Let S = {– 1, 1} and * denote the usual operation of multiplication. Represent it by Cayley's table. Show that * is a binary operation on S. [2]

2. a. Find the eccentricity and the foci of the ellipse: $x^2+4y^2-4x+24y+24=0$. [2]

Ans: $\frac{\sqrt{3}}{2}$ and $(2\pm2\sqrt{3}, -3)$

b. Find the point where the line through the points (1, 2, 3) and (4, – 4, 9) meets the zx-plane. [2]

Ans: (2, 0, 5)

c. Are the three points with position vectors $\vec{i}+2\vec{j}+4\vec{k}$, $2\vec{i}+5\vec{j}-\vec{k}$ and $3\vec{i}+8\vec{j}-6\vec{k}$ collinear? Justify your answer. [2]

3. a. Using L'Hospital's rule, evaluate $\lim_{x\to0}\frac{e^x+e^{-x}-2\cos x}{\sin^2 x}$. [2]

Ans: 2

b. Evaluate: $\int\frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}\ (\beta>\alpha)$. [2]

Ans: $2\ln\left|\sqrt{x-\alpha}+\sqrt{x-\beta}\right|+c$

c. If $\vec{a}=6\vec{i}+3\vec{j}-5\vec{k}$ and $\vec{b}=\vec{i}-4\vec{j}+2\vec{k}$ show that $\vec{a}\times\vec{b}$ is perpendicular to $\vec{a}$. [2]

4. a. Solve: $x\frac{dy}{dx}+y-1=0$. [2]

Ans: $x(y-1)=c$

b. If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. [2]

Ans: 0.59

c. Two dice are rolled once. What is the probability of getting a total of 9 or 6? [2]

Ans: $\frac{1}{4}$

5. a. In how many ways can the letters of the word "COMPUTER" be arranged so that
i. all the vowels are always together?
ii. the vowels may occupy only odd positions? [4]

Ans: (i) 4320 ways (ii) 2880 ways

b. Given the algebraic structure (G, *) with G = {1, ω, ω²} where ω represents an imaginary cube root of unity and * stands for the binary operation of multiplication, show that (G, *) is a group. [4]

6. a. Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). Express it in the slope form. [4]

OR

What is a conic section? Find the equation of the parabola in the standard form. [4]

b. Find the equation of plane through the point (2, 1, 4) and perpendicular to each of the planes. $9x-7y+6z+48=0$ and $x+y+z=0$. [4]

Ans: $13x+3y-16z+35=0$

7. a. Evaluate: $\int\frac{dx}{a+b\cos x}\ (a>b>0)$. [4]

Ans: $\frac{a-b}{|a-b|}\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)+c$

b. Solve: $x^2\frac{dy}{dx}+y^2=xy$. [4]

Ans: $x = y(\log x + c)$

OR

Solve: $(1-x^2)\frac{dy}{dx}-xy=1$.

Ans: $y\sqrt{1-x^2}=\sin^{-1}x+c$

8. a. Find Karl Pearson's coefficient of skewness from the following distribution. [4]

Marks	Above 20	Above 30	Above 40	Above 50	Above 60
No. of students	50	46	30	24	8

Ans: – 0.41

b. The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that (i) the problem will be solved if they both try (ii) A solves but B cannot. [4]

Ans: (i) $\frac{3}{4}$ (ii) $\frac{1}{12}$

OR

Suppose that in certain city 60% of all the recorded births are male. Suppose we select 5 birth records from population. What is the probability that

i. exactly three of them are male?

ii. 4 or more are male? [4]

Ans: (I) $\frac{216}{625}$ (II) $\frac{343}{3125}$

9. Show that: $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1.$ [6]

10. Define scalar product of two vectors. Find the geometrical interpretation of scalar product of two vectors. Prove vectorically that

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ [6]

11. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function.

$f(x) = x(x-1)^2$ in $[0, 1]$

Also, find the point on the curve where the tangent is parallel to the x-axis. [6]

Ans: $1, \frac{1}{3}$

OR

Find from first principle the derivative of $\ln \cos^{-1} x$. [6]

Ans: $\frac{-1}{\sqrt{1-x^2}\cos^{-1} x}$

Group B

12. a. Forces equal to 7P, 5P and 8P acting on a particle are in equilibrium. Find the angle between the latter pair of forces. [2]

Ans: 120°

b. A body is projected vertically upwards with a velocity of 19.6 m/s. How long will it take to reach a point 294 m below the point of projection? (g = 9.8 m/s) [2]

Ans: 10 sec

c. A body of mass 50 kg falling from a certain height is brought to rest after striking the ground with a speed of 5 m/s. If the resistance force of the ground is 500 N, find the duration of the contract. [2]

Ans: 0.5 sec

13. a. P and Q are two like parallel forces acting at A and B. Show that if they interchange positions, the point of application of the resultant is displaced by a distance $\frac{P-Q}{P+Q}$ AB. [4]

OR

Forces 1 N, 2 N and 3 N act at a point in direction parallel to the sides of an equilateral triangle taken in order. Find their resultant. [4]

Ans: $\sqrt{3}$ N perpendicular to the second force

b. Prove that the sum of the kinetic and potential energies of a freely falling body remains constant throughout the motion. [4]

14. The horizontal and the vertical components of the initial velocity of a projectile are U and V respectively. If R be the horizontal range and H, the greatest height attained, prove that

i. $\frac{4H}{R} = \frac{V}{U}$ ii. $\left(\frac{R}{U}\right)^2 = \frac{8H}{g}$ [6]

OR

A cat seeing a mouse at a distance of 15 m before it, starts from rest with an acceleration of 2 m/s² and pursues it. If the mouse be moving uniformly with a velocity of 14 m/s, find when and where the cat will catch the mouse. [6]

Ans: 15 sec and 210 m

15. Define the moment of a force about a point and interpret its geometrical meaning. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

Group C

16. a. If a man rides his car at 25 km/hr, he has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Formulate the above problem as a linear programming problem. [2]

Ans: $2x + 5y \le 100$; $8x + 5y \le 200$; $x \ge 0, y \ge 0$

b. Convert the decimal number 2011 into octal form. [2]

Ans: 3733_8

c. Is the following equations diagonally dominant: [2]

$12x + 3y - 5z = 1$ $x + 5y + 3z = 28$ $3x + 7y + 13z = 1$?

Ans: Yes

17. a. Using Gauss elimination method, solve the following system of equations: [4]

$x + 3y - z = -2$ $3x + 2y - z = 3$ $-6x - 4y - 2z = 18$

Ans: $(x, y, z) = (-24, 8, 2)$

OR

Solve the following equations using Gauss-seidal method: [4]

$2x_1 - x_2 = 8$; $3x_1 + 7x_2 = -5$

Ans: $x_1 = 3, x_2 = -2$

b. Evaluate the following integral using Simpson's rule:

$\int_0^1 \frac{dx}{1+x^2}$ taking 4 equal intervals (i.e. n = 4). [4]

Ans: 0.785

18. Using Simplex method, maximize $Z = 5x_1 + 7x_2$ subject to

$2x_1 + 3x_2 \le 13$ $3x_1 + 2x_2 \le 12$ $x_1, x_2 \ge 0$. [6]

Ans: Max. value of z is 31 at (2, 3)

19. Show that the equation $f(x) = x^3 - 18 = 0$ has only one positive root. Using bisection method, find the positive root correct to 3 places of decimal in the interval (2, 3). [6]

Ans: 2.621

OR

Use Newton-Raphson method to find the positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to three places of decimals. [6]

Ans: 1.154

CHAPTER BASED QUESTIONS

PERMUTATION AND COMBINATION

PERMUTATION

FORMULAE

Factorial Notation

Factorial n, denoted by, n! or $\lfloor n$ is given by

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$

Also, $0! = 1$

The total number of permutations of a set of n objects taken r at a time is given by

$^nP_r = P(n, r) = \frac{n!}{(n-r)!}, n \geq r$

The total number of permutations of a set of n objects taken all at a time, when there are p objects of one kind, q objects are of the second kind and r objects are of third kind is $\frac{n!}{p!\,q!\,r!}$

Circular permutations of n objects = $(n-1)!$

The number of permutation of n objects taken r at a time with repetition = n^r.

Marks Questions

2076 GIE Set A Q.No. 1a In how many ways the letters of the word "EXCELLENT" can be arranged so that the vowels are always together? [2]
Ans: 30240

2076 GIE Set B Q.No. 1a How many even numbers of 3 digits can be formed of the digits 1, 2, 3, 4, 5, 6 when repetition of digits is allowed? [2]
Ans: 108

2076 Set B Q.No. 1a In how many ways the letters of the word ALGEBRA can be arranged so that repeated letter are never together? [2]
Ans: 1800

2076 Set C Q.No. 1a In how many ways the letters of the word CALCULUS can be arranged so that vowels are always together? [2]
Ans: 540

2075 GIE Q.No. 1a In how many ways the letters of the word PRECARIOUS can be arranged so that all the vowels are always together? [2]
Ans: 43200

2075 Set A Q.No. 1a In how many ways can eight people be seated in a row of eight seats so that two particular persons are always together? [2]
Ans: 10080

2074 Set B Q.No. 1a In how many ways the letters of the word COMPLETE can be arranged so that the repeated letters are always together? [2]
Ans: 5040

2073 Supp Q.No. 1a In how many ways the letters of the word HEXAGON can be arranged so that vowels are always together? [2]
Ans: 720

2073 Set C Q.No. 1 How many different numbers of five digits can be formed with the digits 0, 1, 2, 3, 4 ? [2]
Ans: 96

2072 Supp. Q.No. 1a How many numbers between 3000 and 4000 can be formed with the digits 2, 3, 4, 5, 6, 7? [2]
Ans: 60

2072 Set C Q.No. 1a In how many ways can 7 students be seated in a circle? [2]
Ans: 720

2072 Set D Q.No. 1a In how many ways the letters of the word ELEMENT can be arranged so that vowels are always together? [2]
Ans: 120

2072 Set E Q.No. 1a Find the number of ways in which 4 men and 3 women can be seated in a row having seven seats so that the men and the women must alternate. [2]
Ans: 144

2071 Supp. Q.No. 1a In how many ways can eight different coloured heads be made into a bracelet? [2]
Ans: 2520

2071 Old Q.No. 2 a How many permutations are there of the letters of the word "SAARC"? [2]
Ans: 60

2070 Supp. Q.No. 1 a Six children are to be seated on a bench. How many arrangements are possible if the youngest child sits at the left end of the bench? [2]
Ans: 120

2070 Set D Q.No. 1 a In how many ways can the letters of the word "ELEMENT" be arranged? [2]
Ans: 840

2070 (Old) Q.No. 1 b How many numbers of three different digits less than 500 can be formed from the integers 1, 2, 3, 4, 5, 6, 7? [2]
Ans: 120

2069 (Set A) Q.No. 1a In how many ways can four boys and three girls be seated in a row containing seven seats if they may sit any where? [2]
Ans: 5040

2068 Q.No. 2a How many license plates consisting of 3 different digits can be made out of given integers 3, 4, 5, 6, 7? [2]
Ans: 60

2067 Q.No. 2a In how many ways letters of the word PRECARIOUS can be arranged so that all the vowels are always together? [2]
Ans: 43,200

2066 C Q.No. 2 a How many four digits odd numbers can be formed using the digits 0, 1, 2, 3, 4, 5 no digit being repeated? [2]
Ans: 144

2065 Q.No 2 a How many numbers are there between 100 and 1000 such that every digit is either 2 or 9? [2]
Ans: 8

2063 Q.No. 2a How many numbers of three different digits less than 500 can be formed from the integers 1, 2, 3, 4, 5, 6? [2]
Ans: 80

25. 2061 Q.No. 2 a In how many ways can 6 different beads be strung on a necklace? [2]
Ans: 60

26. 2060 Q.No. 2 a Find the numbers of permutation of the letters of the word 'MATHEMATICS'. [2]
Ans: 4989600

27. 2059 Q.No. 1 b How many permutations are there of the letters of the word 'mathematics' taken all together? [2]
Ans: 4989600

28. 2058 Q.No. 2 a In a certain election, there are three candidates for president, five for secretary and only two for the treasurer. Find in how many ways the election may turn out. [2]
Ans: 30 ways

4 Marks Questions

29. 2075 Set B Q.No. 5a How many words can be formed from the letters of the word 'ENGLISH'? How many of these do not begin with E? How many of these begin with E and do not end with H? [4]
Ans: 5040, 4320 and 600

30. 2074 Supp Q.No. 5a In how many ways can the letters of the world "ARRANGE" be arranged so that no two R's come together? [4]
Ans: 900

31. 2074 Set A Q.No. 5a Prove that the number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$. $(n \geq r)$ [4]

32. 2074 Set A Q.No. 5a OR How many numbers of 4 different digits can be formed from the digits 4, 5, 6, 7, 8? How many of these numbers are divisible by 5? How many of these numbers are not divisible by 5? [4]
Ans: 120,24,96

33. 2073 Set D Q.No. 5a In how many ways can the letters of the word "COMPUTER" be arranged so that (i) all the vowels are always together (ii) the vowels may occupy only odd positions. [4]
Ans: (i) 4320 (ii) 2880

34. 2072 Supp. Q.No. 5a Prove that $P(n, r) = \frac{n!}{(n-r)!}$ where the symbols have their usual meanings. [4]

35. 2071 Set C Q.No. 5 a In how many ways can the letters of the word "TUESDAY" be arranged? How many of these arrangements do not begin with T? How many begin with T and do not end with Y? [4]
Ans: 5040, 4320, 600

36. 2071 Set D Q.No. 5 a In how many ways can the letters of the word "COMPUTER" be arranged so that i) all vowels are always together? ii) the relative positions of the vowels and consonants are not changed? [4]
Ans: (i) 4320 (ii) 720

37. 2070 Set C Q.No. 5 a In how many ways can the letters of the word, 'CALCULUS' be arranged so that the two L's do not come together? [4]
Ans: 3780

38. 2070 (Old) Q.No. 7 a In how many ways can the letters of the word 'Sunday' be arranged? How many of these arrangements do not begin with S? How many begin with S and do not end with a? [4]
Ans: 720, 600, 96

39. 2069 (Set A) Q.No. 5a In how many ways can the letters of the world "ARRANGE" be arranged so that no two R is come together? [4]
Ans: 900

40. 2069 (Set A) Old Q.No. 7b In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begins with M and does not end with Y? [4]
Ans: 720; 600; 96

41. 2069 (Set B) Q.No. 5a In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begin with M and do not end with N? [4]
Ans: 720, 600, 96

42. 2069 Old (Set B) Q.No. 7b In how many ways can ten people be seated in a round table if two people insists on sitting next to each other? [4]
Ans: 80640

43. 2065 Q.No 7 b Prove that the total no. of permutations of a set of n objects taken r at a time is given by $P(n, r) = n(n-1)(n-2) \ldots (n-r+1)$, $n \geq r$. [4]

44. 2064 Q.No. 7 b Show that the number of ways in which the letters of the word "arrange" can be arranged so that two r's do not come together is 900. [4]

45. 2062 Q.No. 7 b In how many ways can 4 Art students and 4 Science students be arranged alternately at a round table? [4]
Ans: 144

46. 2061 Q.No. 7 b In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begin with M and don't end with Y? [4]
Ans: 720, 600, 96

47. 2059 Q.No. 7 b Prove that the total number of permutations of a set of n objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$. [4]

48. 2057 Q.No. 7 b In how many ways can the letters of the word ARRANGE be arranged so that no two R's come together. [4]
Ans: 900

B. COMBINATION

FORMULAE

1. ${}^nP_r = r! \cdot {}^nC_r$
2. The total number of combinations of n objects taken r at a time is ${}^nC_r = C(n, r) = \frac{n!}{(n-r)!\, r!}$, $n \geq r$
3. i. ${}^nC_r = {}^nC_{r-1}$ (Complementary combination)
   ii. If ${}^nC_r = {}^nC_{r'}$ then either $r = r'$ or $r + r' = n$
   iii. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Marks Questions

2077 Set I Q.No. 1a Find the number of ways in which 5 courses out of 8 can be selected when 3 courses are compulsory. [2]
Ans: 10

2075 Set B Q.No. 1a Find the value of r if ${}^{9}C_{2r} = {}^{9}C_{3r-1}$. [2]
Ans: 1, 2

2075 Set C Q.No. 1a In an examination, an examinee has to secure A+ grade in each of the five subjects. In how many ways can the examinee fail to secure A+ grade? [2]
Ans: 31

2074 Supp Q.No. 1a In an examination, a candidate has to pass in each of the four subjects. In how many ways can the candidate fail? [2]
Ans: 15

2074 Set A Q.No. 1a If there are 10 persons in a party and each two of them shakes hands with each other, how many hand shakes happen in the party? [2]
Ans: 45

6. **2073 Set D Q.No. 1a** From 10 persons, in how many ways can a selection of 4 be made when two particular persons are always excluded? [2]
Ans: 70

7. **2071 Set C Q.No. 1 a** Find the number of ways in which 5 courses out of 8 can be selected when 3 courses are compulsory. [2]
Ans: 10

8. **2071 Set D Q.No. 1 a** A man has 5 friends. In how many ways can he invite one or more of them to a dinner? [2]
Ans: 31

9. **2070 Set C Q.No. 1 a** From 10 persons, in how many ways can a selection of 4 be made when two particular persons are always included? [2]
Ans: 28

10. **2070 Set D Q.No. 5 a** From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least two gentlemen? [2]
Ans: 246

11. **2069 (Set A) Old Q.No. 2a** In an examination paper containing 10 questions, a candidate has to answer 7 questions only, in how many ways can he choose the questions? [2]
Ans: 120

12. **2069 (Set B) Q.No. 1a** From 10 persons, in how many ways can a selection of 4 be made if two particular persons are always excluded. [2]
Ans: 70

13. **2069 Old Set B Q.No. 1b** How many different sums of money can be made from 4 coins of different denominations? [2]
Ans: 15

14. **2066 Q.No. 2 a** From 10 persons in how many ways can a selection of 4 be made if two particular persons are always excluded? [2]
Ans: 70

15. **2064 Q.No. 2 a** A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives? [2]
Ans: 336

16. **2062 Q.No. 2 a** From 10 persons, in how many ways can a committee of 4 be made when one particular person is always included? [2]
Ans: 84

17. **2057 Q.No. 2 a** A committee is to be chosen from 12 men and 8 women and is to consist of 3 men and 2 women. How many such committee can be formed? [2]
Ans: 6160

4 Marks Questions

18. **2077 Set G Q.No. 3** In how many ways a committee of three person can be formed out of 3 men and 4 women so as to include at least one woman. [4]
Ans: 34

19. **2076 GIE Set A Q.No. 5a** From 8 gentlemen and 6 ladies, a committee of 6 is to be formed. In how many ways can this be done so as to include at least 4 ladies? [4]
Ans: 469

20. **2076 GIE Set B Q.No. 5a** An examination paper consisting of 10 questions, is divided into two groups A and B. Group A contains 6 questions. In how many ways can a student attempt 7 questions selecting at least two questions from each group? [4]
Ans: 116

21. **2076 Set B Q.No. 5a** A committee of five persons is to be formed from 5 men and 3 women. In how many ways can this be done so that at least two women are included? [4]
Ans: 40

22. **2076 Set B Q.No. 5a OR** Show that the number of combinations of 'n' different objects taken 'r' at a time is given by $C(n, r) = \frac{n!}{(n-r)!\, r!}$. Also show that $C(n, r) + C(n, r-1) = C(n+1, r)$. [4]

23. **2076 Set C Q.No. 5a** In how many ways a committee of five can be formed out of 4 men and 3 women so that it includes at least one women? [4]
Ans: 21

24. **2075 GIE Q.No. 5a** In how many ways a committee of three person can be formed out of 4 men and 3 women so that it includes at least one woman? [4]
Ans: 19

25. **2075 Set A Q.No. 5a** An examination paper consisting of 10 questions, is divided into two groups A and B. Group A contains 6 questions. In how many ways can an examinee attempt 7 questions selecting at least two questions from each group? [4]
Ans: 116

26. **2074 Set B Q.No. 5a** From 3 men and 7 women a committee of 5 is to be formed. In how many ways can this be done so as to include at least one man? [4]
Ans: 231

27. 2075 Set C Q.No. 5a If C(n, r − 1) = 36, C(n, r) = 84 and C(n, r + 1) = 126, find the value of r and n. [4]
Ans: r = 3, n = 9

28. 2073 Supp Q.No. 5a In how many ways a committee of three can be formed out of 5 men and 2 women so that it always consists at least one women? [4]
Ans: 25

29. 2073 Set C Q.No. 5a There are ten electric bulbs in the stock of a shop out of which four are defectives. In how many ways can a selection of 6 bulbs be made so that 4 of them may be good bulbs? [4]
Ans: 90

30. 2072 Supp. Q.No. 5a OR Prove that C(n, r) + C(n, r− 1) = C(n + 1,r) where C(n, r) is the combination of n things taken r at a time. [4]

31. 2072 Set C Q.No. 5a A committee of five persons is to be selected from 5 men and 4 ladies. In how many ways can this be done so that at least two ladies are always included? [4]
Ans: 105

32. 2072 Set D Q.No. 5a A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives? [4]
Ans: 336

33. 2072 Set E Q.No. 5a In a group of 10 students, 6 are boys. In how many ways can 4 students be selected for mathematical competition so as to include atmost two girls? [4]
Ans: 185

34. 2071 Supp. Q.No. 5a An examination paper consists of 12 questions divided into two parts A and B. Part A contains 7 questions, part B contains remaining questions. A candidate is required to attempt 8 questions selecting at least 3 from each part. In how many ways can the candidates select the questions? [4]
Ans: 420

35. 2071 Old Q.No. 8 b In how many ways a committee of 8 members be selected from 8 gentlemen and 6 ladies, if the committee is to include not more than three ladies. [4]
Ans: 1589

36. 2070 Supp. Q.No. 5 a A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways can this be done when at least two ladies are to be included? [4]
Ans: 186

37. 2069 (Set B) Q.No. 5a Or From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least 2 ladies? [4]
Ans: 186

38. 2068 Q.No. 7b A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives? [4]
Ans: 336

39. 2067 Q.No. 7b A committee of five is to be constituted from six boys and five girls. In how many ways can this be done so as to include at least one boy and one girl? [4]
Ans: 455

40. 2066 C Q.No. 7 b From 10 players in how many ways can a selection of 4 be made, when one particular player is always included, when two particular players are excluded? [4]
Ans: 84; 70

41. 2066 Q.No. 7 b A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite seven guests so that 5 of them may be relatives? [4]
Ans: 336

42. 2063 Q.No. 7 b A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from any group. In how many different ways can he make up his choice? [4]
Ans: 200

43. 2060 Q.No. 7 b From 6 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady? [4]
Ans: 246

44. 2058 Q.No. 7 b From 10 football players in how many ways can a selection of a 4 be made (i) when one particular player is always included (ii) when two particular players are always excluded? [4]
Ans: (i) 84 (ii) 70

2. BINOMIAL THEOREM

A. BINOMIAL THEOREM

FORMULAE

1. Binomial theorem for positive integral index n;
$(a + x)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} x + {}^nC_2\, a^{n-2} x^2 + {}^nC_3\, a^{n-3} x^3 + \ldots + {}^nC_r\, a^{n-r} x^r + \ldots + {}^nC_n\, x^n$
2. General term:
a. The general term of $(a + x)^n$:
$t_{r+1} = {}^nC_r\, a^{n-r} x^r$
b. The general term of $(a - x)^n$:
$t_{r+1} = (-1)^r\, {}^nC_r\, a^{n-r} x^r$
3. Middle term:
a. When n is even in $(a + x)^n$, the middle term is
$t_{\frac{n}{2}+1} = {}^nC_{\frac{n}{2}}\, a^{n-\frac{n}{2}} x^{\frac{n}{2}} = {}^nC_{\frac{n}{2}}\, a^{\frac{n}{2}} x^{\frac{n}{2}}$
b. When n is odd in $(a + x)^n$, the middle term is
$t_{\frac{n+1}{2}} = t_{\frac{n-1}{2}+1}$
$= {}^nC_{\frac{n-1}{2}}\, a^{n-\frac{n-1}{2}} x^{\frac{n-1}{2}}$
$= {}^nC_{\frac{n-1}{2}}\, a^{\frac{n+1}{2}} x^{\frac{n-1}{2}}$
$t_{\frac{n+1}{2}+1} = t_{\frac{n+1}{2}}\, a^{n-\frac{n+1}{2}} x^{\frac{n+1}{2}}$
$= {}^nC_{\frac{n+1}{2}}\, a^{\frac{n-1}{2}} x^{\frac{n+1}{2}}$

2 Marks Questions

1. 2077 Set G Q.No. 1a Find the term independent of x in the binomial expansion of $\left(2x + \frac{1}{2x}\right)^{10}$. [2]
Ans: $t_4 = 252$

2077 Set H Q.No. 1a Find the middle term in the expansion of $(3x + x^3)^{10}$. [2]

Ans: $61236x^{20}$

2076 GIE Set A Q.No. 1b Find the term independent of x in expansion: $\left(2x - \frac{1}{2x}\right)^{12}$ [2]

Ans: 924

2076 GIE Set B Q.No. 1b Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{18}$. [2]

Ans: $\frac{18!}{9!\,9!}$

2076 Set C Q.No. 1b Find the middle term in the expansion of $\left(x - \frac{1}{x^2}\right)^{12}$. [2]

Ans: $\frac{924}{x^6}$

2075 GIE Q.No. 1b Find the term independent of x in the expansion of $\left(x - \frac{1}{2x}\right)^6$. [2]

Ans: $-\frac{5}{2}$

7. **2075 Set A Q.No. 1b** Find the coefficient of x in the expansion of $\left(x^2 + \frac{a^2}{x}\right)^5$. [2]

Ans: $10a^6$

8. **2074 Set B Q.No. 1b** Find the coefficient of x^6 in the expansion of $\left(x^3 + \frac{1}{x}\right)^{10}$. [2]

Ans: 210

9. **2073 Set C Q.No. 1b** Find middle term in the expansion of $\left(x - \frac{1}{3x^2}\right)^{12}$. [2]

Ans: $\frac{308}{243\,x^6}$

10. **2072 Supp. Q.No. 1b** Find the middle term in the expansion of $\left(x + \frac{1}{2x}\right)^{18}$. [2]

Ans: $\frac{12155}{128}$

11. **2072 Set C Q.No. 1b** Find the coefficient of the term containing x^2 in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$. [2]

Ans: $\frac{20}{3}$

12. **2071 Set C Q.No. 1 b** Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{3x^2}\right)^{12}$. [2]

Ans: $\frac{308}{243}$

13. **2071 Set D Q.No. 1 b** Find the coefficient of x in the expansion of $\left(x^2 + \frac{a^2}{x}\right)^5$. [2]

Ans: $10a^6$

14. **2071 Old Q.No. 1 b** Find middle term or terms in the expansion of $\left(ax + \frac{1}{ax}\right)^{16}$. [2]

Ans: $\frac{16!}{8!\,8!}$

15. **2070 (Old) Q.No. 2 a** Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ [2]

Ans: 495

16. **2069 (Set A) Q.No. 1b** Which term is free from x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$? [2]

Ans: 3003

17. **2069 (Set A) Old Q.No. 1b** Find the coefficient of x^5 in the expansion of $\left(x + \frac{1}{2x}\right)^7$. [2]

Ans: $\frac{7}{2}$

18. **2068 Q.No. 1b** Find the coefficient of x^5 in the expansion of $\left(x + \frac{1}{2x}\right)^7$. [2]

Ans: $\frac{7}{2}$

19. **2067 Q.No. 1b** Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{18}$. [2]

Ans: $\frac{18!}{9!\,9!}$

20. **2066 C. Q.No. 1 b** Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{18}$. [2]

Ans: $\frac{18!}{9!\,9!}$

21. **2065 Q.No 1 b** Find the term free from x in the expansion of $\left(\frac{3x^2}{2} + \frac{1}{3x}\right)^9$. [2]

Ans: $\frac{7}{18}$

22. **2064 Q.No. 1 b** Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{18}$. [2]

Ans: $\frac{18!}{9!\cdot 9!}$

23. **2063 Q.No. 1 b** Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$. [2]

Ans: 9^{th} term = 495

24. **2062 Q.No. 1 b** If $C_0, C_1, C_2, \ldots, C_n$ are the binomial coefficients in the expansion of $(1 + x)^n$, show that: [2]

$C_0 + C_2 + C_4 + \ldots = 2^{n-1}$

25. **2061 Q.No. 1 b** Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$. [2]

Ans: 9^{th} term = 495

26 2060 Q.No. 1 b Find the coefficient of x^5 in $\left(x+\frac{1}{2x}\right)^7$. [2]

Ans: $\frac{7}{2}$

27. 2058 Q.No. 1 b Find the seventh term of $(2x+y)^{12}$. [2]

Ans: $59136\ x^6y^6$

28. 2057 Q.No. 1 b Write the middle terms in the expansion of $(a+x)^n$ when n is odd. [2]

Ans: $C\left(n,\frac{n-1}{2}\right)a^{\frac{n+1}{2}}x^{\frac{n-1}{2}}$ and $C\left(n,\frac{n+1}{2}\right)a^{\frac{n-1}{2}}x^{\frac{n+1}{2}}$

4 Marks Questions

29. 2075 Set B Q.No. 5b If $C_0, C_1, C_2, \ldots, C_n$ are binomial coefficients in the expansion of $(1+x)^n$, prove that $C_0 + 4C_1 + 7C_2 + 10C_3 + \ldots + (3n+1)C_n = (3n+2)2^{n-1}$. [4]

30. 2074 Set A Q.No. 5b Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5\ldots(2n-1)}{n!}2^n x^n$. [4]

31. 2071 Supp. Q.No. 5b If the three successive coefficient in the expansion of $(1+x)^n$ are 28, 56 and 70, find n. [4]

Ans:8

32. 2071 Old Q.No. 7 b Define the general term of the binomial expansion of $(x+a)^n$. In the expansion of $(1+x)^n$, prove that the sum of the coefficients of even terms is equal to the sum of the coefficients of odd terms and each is equal to 2^{n-1}. [4]

33. 2070 Supp. Q.No. 5 b If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, find k. [4]

Ans: 3

34. 2069 Old Set B Q.No. 8b If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that: $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{(2n)!}{(n!)^2}$. [4]

35. 2066 Q.No. 8 b If three consecutive coefficients in the expansion of $(1+x)^n$ be 165, 330 and 462, find n. [4]

Ans: 11

36. 2061 Q.No. 8 b If the three consecutive coefficients in the expansion of $(1+x)^n$ be 165, 330, 462, find n. [4]

Ans: 11

37. 2059 Q.No. 8 b If $C_0, C_1, C_2, \ldots, C_n$ are the binomial coefficients in the expansion of $(1+x)^n$ then prove that

$C_0C_n + C_1C_{n-1} + \ldots\ldots + C_nC_0 = \frac{2n!}{n!\,n!}$ [4]

38. 2058 Q.No. 8 b Find the middle term in the expansion of $(1+x)^{2n}$, where n is a positive integer. [4]

Ans: $\frac{1\cdot3\cdot5\ldots(2n-1)(2x)^n}{n!}$

6 Marks Questions

39. 2076 GIE Set A Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that:

$C_0C_2 + C_1C_3 + C_2C_4 + \ldots\ldots + C_{n-2}C_n = \frac{(2n)!}{(n-2)!\,(n+2)!}$ [6]

40. 2076 Set B Q.No. 9 Show that:

$1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + \ldots$ to $\infty = \sqrt{\frac{2}{3}}$. [6]

41. 2074 Supp Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that:

$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots + \frac{nC_n}{C_{n-1}} = \frac{n(n+1)}{2}$ [6]

42. 2073 Supp Q.No. 9 Prove that:

$C_1 - 2C_2 + 3C_3 - 4C_4 + \ldots + n(-1)^{n-1}C_n = 0$, where $C_0, C_1, \ldots, C_n$ are the binomial coefficients. [6]

43. 2073 Set D Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 \ldots\ldots + C_nx^n$, prove that: $C_0C_n + C_1C_{n-1} + \ldots + C_nC_0 = \frac{2n!}{n!n!}$ [6]

44. 2072 Set D Q.No. 9 State Binomial theorem. In the expansion of $(1+x)^n$ prove that the sum of the coefficients of the odd terms is equal to the sum of coefficients of the even terms and each equals to $2^n - 1$. [6]

45. 2072 Set E Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that: $C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \ldots + C_nC_0 = \frac{2n!}{n!n!}$ [6]

46. 2070 Set C Q.No. 9 Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5\ldots\ldots(2n-1)}{n!}(-2)^n$ [6]

47. 2070 Set D Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that: $C_0C_n + C_1C_{n-1} + \ldots + C_nC_0 = \frac{2n!}{(n!)^2}$ [6]

48. 2069 Set B Q.No. 9 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots = C_nx^n$, prove that: $C_0C_n + C_1C_{n-1} + \ldots + C_nC_0 = \frac{2n!}{n!n!}$ [6]

B. EXPONENTIAL AND LOGARITHMIC SERIES

FORMULAE

1. Expansion of e^x

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$

when n = 1, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$

when n = −1, $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots$

2. Expaniosn of a^x

$a^x = 1 + \frac{x}{1!}\log_e a + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + \ldots$

where a is any poitive number.

3. The logarithmic series

$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ (-1 < x \le 1)$

and $\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ (-1 \le x < 1)$

Also, $\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$

2 Marks Questions

1. 2076 Set B Q.No. 1b Show that: $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots = 1$. [2]

2075 Set B Q.No. 1b If $y = \frac{x}{1!} - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots$, show that: $x = y + \frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{4} + \ldots$ [2]

2075 Set C Q.No. 1b Prove that:
$\frac{1}{1\cdot3} + \frac{1}{2\cdot5} + \frac{1}{3\cdot7} + \frac{1}{4\cdot9} + \ldots = 2(1 - \ln 2)$. [2]

2074 Supp Q.No. 1b Prove that: $\frac{1}{2}\left(e - \frac{1}{e}\right) = 1 + \frac{1}{3!} + \frac{1}{5!} + \ldots$ [2]

2074 Set A Q.No. 1b If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$, show that: $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots$ [2]

2073 Supp Q.No. 1b If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ show that: $x = y + \frac{y^2}{2!} + \frac{y^3}{3!} + \ldots$ [2]

2073 Set D Q.No. 1b Show that: $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \ldots$ to $\infty = e$ [2]

2072 Set D Q.No. 1b Prove that: $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \ldots = e$. [2]

2072 Set E Q.No. 1b Show that: $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \ldots$ to $\infty = e$. [2]

2071 Supp. Q.No. 1b If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$, show that: $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ [2]

2070 Supp. Q.No. 1 b Prove that:
$\frac{1}{1\cdot3} + \frac{1}{2\cdot5} + \frac{1}{3\cdot7} + \frac{1}{4\cdot9} + \ldots = 2(1 - \ln 2)$ [2]

2070 Set C Q.No. 1 b Show that:
$\frac{1}{2}\left(e + \frac{1}{e}\right) = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ [2]

2070 Set D Q.No. 1 b Show that: $\log_e 2 = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6} + \ldots$ [2]

2069 Set B Q.No. 1b Prove that: $\log_e 2 = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6} + \ldots$ [2]

2069 Old (Set B) Q.No. 3a Find the value of $\frac{1}{2}(e + e^{-1})$. [2]

Ans: $1 + \frac{1}{2!} + \frac{1}{4!} + \ldots$

2066 Q.No. 1 b Prove that: $\log_e 2 = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6}$ [2]

2059 Q.No. 3 a Prove that: $\log_e 2 = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6} + \ldots$ [2]

4 Marks Questions

2070 (Old) Q.No. 8b Prove that: $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots \infty = 2e$. [4]

2069 (Set A) Old Q.No. 8b If $y = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ to ∞; prove that: $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$ to ∞ [4]

2068 Q.No. 8a Prove that:
$1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \ldots \infty = \frac{3e}{2}$ [4]

2067 Q.No. 8b Prove that:
$\left(\frac{1}{3} - \frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{3^2} + \frac{1}{2^2}\right) + \frac{1}{3}\left(\frac{1}{3^3} - \frac{1}{2^3}\right) + \ldots \infty = 0$ [4]

2066 C Q.No. 8 b Prove that: $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \ldots \ldots \infty = \frac{1}{e}$ [4]

2065 Q.No 8 b Prove that: $\frac{1}{1\cdot2} + \frac{1}{2\cdot2^2} + \frac{1}{3\cdot2^3} + \ldots = \log_e 2$ [4]

2064 Q.No. 8 b If $y = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$, show that:
$x = y + \frac{1}{2!}y^2 + \frac{1}{3!}y^3 + \ldots$ [4]

2063 Q.No. 8 b If $y = \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ to ∞,
Prove that: $x = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \ldots$ to ∞ [4]

2062 Q.No. 8 b Prove that: $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \ldots$ to $\infty = \frac{1}{e}$ [4]

2060 Q.No. 8 b Prove that: $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \ldots = \frac{e}{2}$ [4]

2057 Q.No. 8 a Show that: $\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots} = \frac{e-1}{e+1}$ [4]

6 Marks Questions

2077 Set I Q.No. 6 Define exponential and logarithm series. Also sum to infinity the series: $1^2 + \frac{2^2}{2!} + \frac{3^2}{3!} + \ldots$ [6]

Ans: 2e

2076 GIE Set A Q.No. 9 OR Show that: $\sum_{x=1}^{\infty} \frac{x^2}{(x+1)^2} = e - 1$ [6]

Ans: e – 1

2076 GIE Set B Q.No. 9 Show that $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1$ [6]

2076 Set C Q.No. 9 Prove that: $\frac{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots}{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots} = \frac{e+1}{e-1}$ [6]

2075 GIE Q.No. 9 Prove that:
$\frac{5}{1\cdot2\cdot3} + \frac{7}{3\cdot4\cdot5} + \frac{9}{5\cdot6\cdot7} + \ldots$ to $\infty = -1 + 3\log 2$. [6]

2075 Set A Q.No. 9 Sum to infinity the series:
$1 + \frac{3}{1!} + \frac{5}{2!} + \frac{7}{3!} + \ldots$ [6]

Ans: 3e

35. **2075 Set C Q.No. 9** Write e^x in the series form. Using it, show that: $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots} = \frac{1-e^{-1}}{1+e^{-1}}$. [6]

36. **2074 Set B Q.No.9** Sum to infinity the series: $\frac{1^2}{1!}+\frac{2^2}{2!}+\frac{3^2}{3!}+\ldots$ [6]

Ans: 2e

37. **2073 Set C Q.No. 9a** Prove that: $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots} = \frac{e-1}{e+1}$. [6]

38. **2072 Supp. Q.No. 9** Show that: $\sum_{n=1}^{\infty}\frac{n^2}{(n+1)!} = e-1$. [6]

39. **2072 Set C Q.No. 9** Prove that:
$1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+\ldots = 2e$ [6]

40. **2071 Set C Q.No. 9** Show that: $\sum_{n=1}^{\infty}\frac{n^2}{(n+1)!} = e-1$ [6]

41. **2071 Set D Q.No. 9** Show that:
$1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots = \frac{3e}{2}$ [6]

42. **2069 (Set A) Q.No. 9** Show that: $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots} = \frac{e-1}{e+1}$ [6]

3. ELEMENTARY GROUP THEORY

FORMULAE

1. Binary Operation on a Set
   Let S be a non empty set. A mapping f from $S \times S$ to S is said to be binary operation on the set S. The image of the ordered pair (a, b) under the mapping f is denoted by a f b. Often we use symbols $*, \otimes, o, \times, -, +$, etc. to denote binary operation on a set.
2. Let G be a non empty set with binary operation 'o'. An algebraic structure (G, o) is called a group if the operation 'o' satisfies the following properties:

a. Closure property
   For all $a, b \in G$ implies that $a \circ b \in G$.

b. Associative property
   For all $a, b, c \in G$ implies that $a \circ (b \circ c) = (a \circ b) \circ c$

c. Existence of identity element
   For each element $a \in G$ there exists an identity element denoted by e such that
   $a \circ e = a = e \circ a$.

d. Existence of an inverse element
   For each element $a \in G$ there exist an inverse element of a denoted by a^{-1} in G such that
   $a \circ a^{-1} = e = a^{-1} \circ a$.

3. Commutative Group
   The group (G, o) is said to be commutative (abelian group) if $a \circ b = b \circ a$ for all $a, b \in G$.

2 Marks Questions

1. **2077 Set I Q.No. 1b** Prepare a Caley's table for $G = \{1, \omega, \omega^2\}$ where ω is the cube root of unity under multiplication. [2]
2. **2076 GIE Set A Q.No. 1c** Define binary operation with example. [2]
3. **2076 GIE Set B Q.No. 1c** Test the associative property for the binary operation* defined by $m*n = m + n + 1$, $m, n \in Z$. [2]
4. **2076 Set B Q.No. 1c** Prepare a Cayley's table for $S = \{0, 2, 3\}$, under addition modulo 4. [2]
5. **2076 Set C Q.No. 1c** Using Caley's table for $(G, *)$ where $G = \{-1, 1\}$, find inverse elements of G. [2]

Ans: -1, 1

6. **2075 GIE Q.No. 1c** Examine the commutative property for the operation '*' defined by $m * n = m + n$ on the set of integers Z; $m, n \in Z$. [2]

Ans: Commutative

7. **2075 Set A Q.No. 1c** If a and b are the elements of a group $(G, *)$ and if $a * b = e$, prove that $b = a^{-1}$. [2]
8. **2075 Set B Q.No. 1c** Solve $3x + 6 = 5$ in Z_7. [2]

Ans: 2

9. **2075 Set C Q.No. 1c** Prove that multiplication on the set Z^- of all negative integers is not a binary operation on Z^-. [2]
10. **2074 Supp Q.No. 1c** If the binary operation * on Z, the set of integers is defined by $m * n = m + n + 1$ for every $m, n \in Z$, show that * satisfies associative property. [2]
11. **2074 Set A Q.No. 1c** Let $a*b = a - b$ on Z. Show that '*' is not an associative binary operation. [2]
12. **2074 Set B Q.No. 1c** Prepare Cayley's table for the set $S = \{0, 1, 2\}$ under the operation of multiplication modulo 3. [2]
13. **2073 Supp Q.No. 1c** Show that the set $G = \{-1, 1, -i, i\}$, the fourth roots of unity satisfies the binary operation of multiplication. [2]
14. **2073 Set C Q.No. 1c** Find the inverses of the elements of $G = \{-1, 1\}$ under multiplications, if exist. [2]

Ans: -1, 1

15. **2073 Set D Q.No. 1c** Let $G = \{0, 1, 2\}$. Form a composition table for G under addition modulo 3. Find the inverse element of 2. [2]

Ans: 1

16. **2072 Supp. Q.No. 1c** Let $G = \{0, 1, 2\}$. Form a composition table for G under multiplication modulo 3. Find the inverse element of 2. [2]

Ans: 2

17. **2072 Set C Q.No. 1c** If $a * b = 3a + 2b$ for $a, b \in Z$, the set of integers, show that * is a binary operation on z. [2]
18. **2072 Set D Q.No. 1c** In a Caley's table for a finite group, does each element occur exactly once in each row and exactly once in each column? [2]
19. **2072 Set E Q.No. 1c** Test the commutative property for the operation * defined by $m * n = n$, $m, n \in Z$. [2]

Ans: Not commutative

20. **2071 Supp. Q.No. 1c** If possible, solve $2x + 1 = 6$ in Z_7 [2]
Ans: x = 6

21. **2071 Set C Q.No. 1 c** Let G = {0, 1, 2}. Form a composition table for G under the multiplication modulo 3. Find the identity element of 2. [2]
Ans: 1

22. **2071 Set D Q.No. 1 c** Show that the multiplication is a binary operation on the set S = {–1, 0, 1} [2]

23. **2070 Supp. Q.No. 1 c** Let $m*n = \sqrt{mn}$ for $m, n \in Z$. Verify that the operation * is not a binary operation on Z. [2]

24. **2070 Set C Q.No. 1 c** Let G = {0, 1, 2}. Form a composition table for G under addition modulo 3. Find the identity element of 1. [2]
Ans: 0

25. **2070 Set D Q.No. 1 c** Show that the multiplication is a binary operation on the set S = { –1, 0, 1} [2]

26. **2069 (Set A) Q.No. 1c** Let S = {–1, 1} and * denote the usual operation of multiplication. Represent it by Cayley's table. Show that the multiplication is a binary operation on S. [2]

27. **2069 (Set B) Q.No. 1c** If the binary operation * on Q, the set of rational numbers is defined by a*b = a + b + ab for every a, b∈Q show that * satisfies associative property. [2]

4 Marks Questions

28. **2077 Set H Q.No. 3** Prove that $\left\{\frac{n}{5}, n \in Z\right\}$ is a group with respect to addition. [4]

29. **2077 Set H Q.No. 3 OR** Define group. Let (G, ∗)be a group, prove that:
$(a * b)^{-1} = b^{-1} * a^{-1} \forall a, b \in G.$ [4]

30. **2076 GIE Set A Q.No. 5b** Show that the set of all positive rational numbers form an abelian group under the composition defined by $x * y = \frac{xy}{2}$ [4]

31. **2076 GIE Set A Q.No. 5b OR** Let S = {0, 1, 2, 3}. From a Caley's table for S under the addition modulo 4 ($+_4$). Show that $+_4$ satisfies the closure and associative property. Does 'S' form a group under $+_4$? [4]

32. **2076 GIE Set B Q.No. 5b** Let G = {1, – 1, i, -- i} where i is an imaginary number. Show that G forms a group under the multiplication operation. [4]

33. **2076 GIE Set B Q.No. 5b OR** Let (G, *) be a group. If a, b, c are the elements of G and (i) a * b = a * c prove that b = c (ii) b * a = c * a, prove that b = c [4]

34. **2076 Set B Q.No. 5b** Define abelian group. If (G, *) is an abelian group, prove that $(a * b)^{-1} = a^{-1} * b^{-1}, \forall a, b \in G$. [4]

35. **2076 Set C Q.No. 5b** If a and b are the elements of a group (G, *), then prove that a*x = b and x* a = b, x∈G have unique solution in (G, *). [4]

36. **2076 Set C Q.No. 5b OR** If a, b ∈ G, where (G, *) is a group, prove: [4]
i. $(a * b)^{-1} = b^{-1} * a^{-1}$
ii. $(a^{-1})^{-1} = a$

37. **2075 GIE Q.No. 5b** If a and b are the elements of a group (G, ∗), then prove that a ∗ x = b and x ∗ a = b have unique solution in (G, ∗). [4]

38. **2075 GIE Q.No. 5b OR** Define Abelian group. Also prove that if G is a group such that $(a * b)^2 = a^2 * b^2$ for all a, b ∈ G, then G is Abelian. [4]

39. **2075 Set A Q.No. 5b** Show that the set of integers Z forms a group under the operation of addition. [4]

40. **2075 Set A Q.No. 5b OR** If a and b are the elements of a group (G, *), prove that a * x = b has a unique solution in (G, *). [4]

41. **2075 Set B Q.No. 6a** Let ∗ be defined on Q^+ by $a * b = \frac{ab}{2}$. Show that $(Q^+, *)$ is an Abelian group. [4]

42. **2075 Set C Q.No. 5b** Define a group. Prove that the set of all three dimensional vectors form an infinite Abelian group under vector addition. [4]

43. **2075 Set C Q.No. 5b OR** If a and b are the elements of group (G, ∗) such that
i. a ∗ b = b prove that a = e.
ii. a ∗ b = e, prove that $b = a^{-1}$. [4]

44. **2074 Supp. Q.No. 5b** Let G = {1, –1, i, –i}. Show that G forms a group under the operation of multiplication. [4]

45. **2074 Supp Q.No. 5b OR** Let G be a group. If a, b∈(G,∗), prove that $(a*b)^{-1} = b^{-1} * a^{-1}$. [4]

46. **2074 Set A Q.No. 6a** If a and b are the elements of group (G,*), and
i. If a * b = e, prove that $b = a^{-1}$
ii. If a * b = b, prove that a = e. [4]

47. **2074 Set B Q.No. 5b** Show that $\{2^n, n \in z\}$ is an Abelian group with respect to multiplication. [4]

48. **2074 Set B Q.No. 5b OR** If a, b ∈ (G, o) where G is a group. Prove that: [4]
i. $(aob)^{-1} = b^{-1}$ o a^{-1} ii. $(a^{-1})^{-1} = a$.

49. **2073 Supp Q.No. 5b** Define abelian group. If (G, *) is an abelian group, show that $(a*b)^{-1} = a^{-1} * b^{-1}$; a, b∈G. [4]

50. **2073 Supp Q.No. 5b OR** Verify that $\{2^m : m \in Z\}$ is an abelian group with respect to multiplication, where Z is the set of integers. [4]

51. **2073 Set C Q.No. 5b** Define Abelian group. Prove that a group G is Abelian if and only if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$ for all a, b∈G. [4]

52. **2073 Set C Q.No. 5b OR** Show that the set of all positive rational numbers under the composition defined by $a*b = \frac{ab}{5}$ forms a group. [4]

53. **2073 Set D Q.No. 5b** Given the algebraic structure (G,*) with G = {1, ω, ω²} where ω represents the cube roots of unity and * stands for the binary operation of ordinary multiplication of complex numbers, show that (G, *) is a group. [4]

54. **2073 Set D Q.No. 5b OR** If a, b ∈ (G, o), prove that $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$. [4]

55. **2072 Supp. Q.No. 5b** Given that the algebraic structures (G, *) with G = {1, ω, ω²} where ω represents an imaginary cube root of unity and * stands for the binary operation of multiplication, show that (G, *) is a group. [4]

56. **2072 Set C Q.No. 5b** Show that the set of all vectors in space under addition is a group. [4]

57. **2072 Set C Q.No. 5b OR** If a, b ∈ (G, *) where G is a group. Prove. (i) $(a * b)^{-1} = b^{-1} * a^{-1}$ (ii) $(a^{-1})^{-1} = a$ [4]

58. **2072 Set D Q.No. 5a** Let (G, *) be a group. If a, b ∈ G, then prove that (i) $(a * b)^{-1} = b^{-1} * a^{-1}$ and (ii) $(a^{-1})^{-1} = a$ [4]

59. **2072 Set D Q.No. 5a OR** Define a group. Let a, b, c and x be elements of a group G. Solve the following for x: $x^2 = a^2$ and $x^5 = e$ [4]

Ans: a^{-4}

60. **2072 Set E Q.No. 5b** Show that the set T = {–1, 1} forms a group under multiplication operation. [4]

61. **2072 Set E Q.No. 5b OR** Prove that every element in a group (G, o) has unique inverse. [4]

62. **2071 Supp. Q.No. 6a** Define group. Verify that $\{2^m : m \in Z\}$ is a group with respect to multiplication. [4]

63. **2071 Set C Q.No. 5 b** Given the algebraic structure (G, *) with G = {1, ω, ω²} where ω represents the cube root of unity and * stands for the binary operation of ordinary multiplication of complex numbers, show that (G,*) is a group. [4]

64. **2071 Set C Q.No. 5 b OR** If a, b, c, are the elements of a group (G, *), prove that:

$a * b = a * c \Rightarrow b = c$ and $b * a = c * a \Rightarrow b = c$. [4]

65. **2071 Set D Q.No. 5 b** A binary operation * defined on the set S = {a, b, c} is presented in the following Cayley's table.

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Show that: (S, *) forms a group. [4]

66. **2071 Set D Q.No. 5 b OR** Let a, b, c be the elements of a group (G, *)

i. If a* b = b, prove that: a = e.

ii. If a*b = e, prove that: $b = a^{-1}$. [4]

67. **2070 Supp. Q.No. 6 a** If a and b are the elements of a group (G, *) such that

i. a * b = b, prove that a = e.

ii. a * b = e, prove that $b = a^{-1}$. [4]

68. **2070 Supp. Q.No. 6 a OR** Let G = {0, 1, 2, 3, 4}. Construct cayley's table for G under the multiplication modulo 5. Find the inverse of each element of G. [4]

69. **2070 Set C Q.No. 5 b** Show that the set T = {-1, 1} forms a group under multiplication operation. [4]

70. **2070 Set C Q.No. 5 b Or** a, b, c are the elements of a group (G,o)

i. if a o b = a o c prove that b = c.

ii. if b o a = c o a prove that b = c. [4]

71. **2070 Set D Q.No. 5 b** Show that the set of integers Z forms a group under the operation of addition. [4]

72. **2070 Set D Q.No. 5 b Or** If a and b are the elements of a group (G, 0) prove that the equation a o x = b has a unique solution in (G, o). [4]

73. **2069 (Set A) Q.No. 5b** Define group. Let G = {1, –1, i, – i} where i is an imaginary unit and * stands for the binary operation of multiplication. Show that (G, *) forms a group. [4]

74. **2069 (Set A) Q.No. 5b or** If a and b are the elements of a group (G, o) prove that: $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ [4]

75. **2069 (Set B) Q.No. 5b** Given the algebraic structure (G, *) with G = {1, w, w²} where w represents the imaginary cube root of unity and * stands for the binary operation of multiplication, show that (G, *) is a group. [4]

4. CONIC SECTIONS

A. PARABOLA

FORMULAE

1. If $e = 1$, the curve is a **parabola.**
   If $e < 1$, the curve is an **ellipse.**
   If $e > 1$, the curve is the **hyperbola.**
2. Table for Different Types of Parabola

Form of parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$	$(y-k)^2 = 4a(x-h)$	$(x-h)^2 = 4a(y-k)$
Focus	(a, 0)	(–a, 0)	(0, a)	(0, – a)	(h + a, k)	(h, k + a)
Directrix	x + a = 0	x – a = 0	y + a = 0	y – a = 0	x = h – a	y = k – a
Axis	y = 0	y = 0	x = 0	x = 0	y = k	x = h
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(h, k)	(h, k)
Tangent at vertex	x = 0	x = 0	y = 0	y = 0	x = h	y = k

3. The equation of a tangent to the parabola $y^2 = 4ax$ at (x_1, y_1) is $yy_1 = 2a(x + x_1)$
4. Equation of the tangent in slope form is $y = mx + \frac{a}{m}$.
5. Condition of tagency of a straight line $y = mx + c$ to a parabola $y^2 = 4ax$ is $c = \frac{a}{m}$. Point of contact $= \left(\frac{a}{m^2}, \frac{2a}{m}\right)$.
6. The equation of the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$ is $y - y_1 = -\frac{y_1}{2a}(x - x_1)$
7. The equation of normal to the parabola $y^2 = 4ax$ in slope form is $y = mx - 2am - am^3$.

Marks Questions

1. **2076 GIE Set A Q.No. 2a** Find the equation of the parabola with focus at (– 2, 3) and directrix x + 4 = 0. [2]
   Ans: $y^2 - 4x - 6y - 7 = 0$
2. **2075 Set B Q.No. 2a** Determine the equation of the chord joining the points t_1 and t_2 on the parabola $y^2 = 4ax$. [2]
   Ans: $2x - (t_1 + t_2)y + 2at_1t_2 = 0$
3. **2071 Supp. Q.No. 2a** Find the equation of the tangent to the parabola $y^2 = 9x$ at (4, –6). [2]
   Ans: $3x + 4y + 12 = 0$
4. **2069 (Set A) Old Q.No. 5c** Find the equation of the normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1). [2]
5. **2069 Old (Set B) Q.No. 2c** Find the equation of the tangent to the parabola $y^2 = 16x$ at the point (4, 8). [2]
   Ans: $y = x + 4$
6. **2068 Q.No. 2c** Prove that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$. [2]
7. **2068 Q.No. 2c** Prove that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$. [2]
8. **2066 Q.No. 5 c** Find the equations of the tangents from the point (–6, 9) to the parabola $y^2 = 24x$. [2]
   Ans: $2x + y + 3 = 0$, $x - 2y + 24 = 0$
9. **2064 Q.No. 5 c** Find the equation of the parabola in which the ends of the latus rectum have the coordinates (– 1, 5) and (– 1, –11) and the vertex is (– 5, – 3). [2]
   Ans: $y^2 + 6y - 16x - 71 = 0$
10. **2063 Q.No. 5 c** Find the coordinates of the vertex and the focus of the parabola whose equation is $y^2 = 6y - 12x + 45$. [2]
    Ans: vertex = $\left(\frac{9}{2}, 3\right)$ and Focus = $\left(\frac{3}{2}, 3\right)$
11. **2062 Q.No. 5 c** Determine the equation of the chord joining the points t_1 and t_2 on the parabola $y^2 = 4ax$. [2]
    Ans: $2x - (t_1 + t_2)y + 2at_1t_2 = 0$
12. **2060 Q.No. 2 c** Find the equation of the normal to the parabola $y^2 = 5x$ perpendicular to the line $x + 2y = 7$. [2]
    Ans: $y = 2x - 15$
13. **2058 Q.No. 2 c** Find the equation of the tangent to the parabola $y^2 = 16x$ at the point (4, 8). [2]
    Ans: $x - y + 4 = 0$
14. **2059 Q.No. 2 c** Find the focus and directrix of the parabola $x^2 = 12y$. [2]
    Ans: (0, 3) and $y + 3 = 0$
15. **2057 Q.No. 2 c** Find the focus and directrix of the parabola $y^2 - 4y - 8x - 20 = 0$. [2]
    Ans: (– 1, 2) and $x + 5 = 0$

4 Marks Questions

16. **2077 Set I Q.No. 3** Find equation and the point of contact of tangent to the parabola $y^2 = 12x$ which makes an angle 45° with the straight line $x - 2y + 3 = 0$. [4]
    Ans: $3x - y + 1 = 0$ and $x + 3y + 27 = 0$; $\left(\frac{1}{3}, 2\right)$, (27, –18)
17. **2077 Set I Q.No. 4** Prove that the line $lx + my + n = 0$ will be normal to the parabola $y^2 = 4ax$ if $al(2m^2 + l^2) + m^2n = 0$. [4]
18. **2076 GIE Set B Q.No. 6a** Find the condition for the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Find the equation of the tangent in the slope form. [4]
    Ans: $y = mx + \frac{a}{m}$
19. **2076 Set B Q.No. 6a** Find the condition that a line $ax + by + c = 0$ may be normal to the parabola $y^2 = 4mx$. [4]
    Ans: $am(2b^2 + a^2) + b^2c = 0$
20. **2076 Set C Q.No. 6a** Find condition that a line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. [4]
    Ans: $c = \frac{a}{m}$
21. **2075 GIE Q.No. 6a** Find the condition that the line $ax + by + c = 0$ may be tangent to the parabola $y^2 = 4x$. [4]
    Ans: $b^2 = c$
22. **2075 Set A Q.No. 6a** Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). [4]
    Ans: $yy_1 = 2a(x + x_1)$
23. **2075 Set B Q.No. 6b** Prove that the lines joining the ends of latus rectum of the parabola $y^2 = 4ax$ to the point of intersection of the directrix and the axis are at right angles. [4]
24. **2075 Set C Q.No. 6a** Find the condition of tangency of a straight line $y = mx + c$ to a parabola $y^2 = 4ax$. [4]
    Ans: $c = \frac{a}{m}$
25. **2074 Supp Q.No. 6a** Find the equation of the tangent to the parabola $y^2 = 4ax$ in the slope form. Also find the point of contact. [4]
    Ans: $y = mx + \frac{a}{m}$; $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
26. **2074 Set A Q.No. 6b** Find the condition under which the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Also find the equation of the tangent in the slope form. [4]
    Ans: $c = \frac{a}{m}$; $y = mx + \frac{a}{m}$
27. **2074 Set B Q.No. 6a** Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. [4]
28. **2073 Supp Q.No. 6a** Find the condition that a line $y=mx+c$ is a tangent to the parabola $y^2 = 4ax$. [4]
    Ans: $c = \frac{a}{m}$
29. **2073 Set C Q.No. 6a** If a tangent to the parabola $y^2 = 12x$ makes an angle 45° with the straight line $x - 2y + 3 = 0$, find the equation of the tangent. [4]
    Ans: $3x - y + 1 = 0$ and $x + 3y + 27 = 0$
30. **2073 Set D Q.No. 6a** Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). [4]
    Ans: $yy_1 = 2a(x + x_1)$
31. **2072 Supp. Q.No. 6a** Define conic section. Find the equation of the parabola in its standard form. [4]
32. **2072 Set C Q.No. 6a** Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. [4]
33. **2072 Set D Q.No. 6a** If the tangent to the parabola $y^2 = 12x$ makes an angle 45° with the straight line $x - 2y + 3 = 0$, find its equation and the point of contact. [4]
    Ans: $3x - y + 1 = 0$ and $x + 3y + 27 = 0$; $\left(\frac{1}{3}, 2\right)$, (27, –18)
34. **2072 Set E Q.No. 6b** Find the condition under which the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Find the equation of the tangent in slope form. [4]
    Ans: $c = \frac{a}{m}$, $y = mx + \frac{a}{m}$

35. 2071 Supp. Q.No. 6b Find the area of the triangle formed by the lines joining the vertex of the parabolay^2 = 12x to the ends of its latus rectum. [4]
Ans: 18 sq. units

36. 2071 Set C Q.No. 6 a Find the equation of the normal to the parabola $y^2 = 4ax$ in the slope form. [4]
Ans: $y = mx - 2am - am^3$

37. 2071 Set D Q.No. 6 a Find the equation of the parabola in the standard form $y^2 = 4ax$. [4]

38. 2071 Old Q.No. 9 a Deduce the equation of the parabola in the standard from $y^2 = 4ax$. [4]

39. 2070 Set C Q.No. 6 a Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). [4]
Ans: $yy_1 = 2a(x + x_1)$

40. 2070 Set D Q.No. 6 a Prove that the line $3x + 4y + 6 = 0$ is tangent to the parabola $2y^2 = 9x$. Find its point of contact. [4]
Ans: (2, –3)

41. 2070 (Old) Q.No. 9 b If the normal at any point $P(at_1^2, 2at_1)$ on the parabola $y^2 = 4ax$ meets the curve again in $Q(at_2^2, 2at_2)$, prove that $t_1 + \frac{2}{t_1} + t_2 = 0$. [4]

42. 2069 (Set A) Q.No. 6a Find the equation of the normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1) and express this in slope form. [4]
Ans: $y - y_1 = \frac{-y_1}{2a}(x - x_1)$ and $y = mx - 2am - am^3$

43. 2069 (Set A) Old Q.No. 9b Given an equation of the parabola $y^2 = 6y - 12x + 45$, find the focus, vertex, equation of the directrix and the length of the latus rectum. [4]
Ans: $\left(\frac{3}{2}, 3\right)$; $\left(\frac{9}{2}, 3\right)$; 2x – 15 = 0; 12.

44. 2069 (Set B) Q.No. 6a Find the condition under which the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Find the equation of tangent in slope form. Also, find the point of contact. [4]
Ans: $c = \frac{a}{m}$, $y = mx + \frac{a}{m}$; Point of contact = $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

45. 2069 Old (Set B) Q.No. 9b Find the equation of the parabola in standard form. [4]
Ans: $y^2 = 4ax$

46. 2068 Q.No. 9b Find the coordinates of the focus, the vertex, the equation of the directrix and the length of the latus rectum of the parabola $y^2 = 6y - 12x + 45$. [4]
Ans: $\left(\frac{3}{2}, 3\right)$; $\left(\frac{9}{2}, 3\right)$; 2x – 15 = 0; 12

47. 2067 Q.No. 9b Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. [4]

48. 2068 Q.No. 9b Find the coordinates of the focus, the vertex, the equation of the directrix and the length of the latus rectum of the parabola $y^2 = 6y - 12x + 45$. [4]
Ans: $\left(\frac{3}{2}, 3\right)$; $\left(\frac{9}{2}, 3\right)$; 2x – 15 = 0; 12

49. 2067 Q.No. 9b Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. [4]

50. 2066 C Q.No. 9 b Find the equation of the parabola in the standard form $y^2 = 4ax$. [4]

51. 2065 Q.No 9 b Find the equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) on the parabola. [4]
Ans: $yy_1 = 2a(x + x_1)$

52. 2064 Q.No. 9 b Deduce the equation of the parabola in the standard form $y^2 = 4ax$. [4]

53. 2063 Q.No. 9 b Prove that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$ [4]

54. 2062 Q.No. 9 b Find the equation of the normal to the parabola $y^2 = 4ax$ in the slope form. [4]
Ans: $y = mx - 2am - am^3$

55. 2061 Q.No. 9 b Prove that the latus rectum of a parabola bisects the angle between the tangent and the normal at either extremity of the latus rectum. [4]

56. 2060 Q.No. 9 b Deduce the equation of the tangent to the parabola $y^2 = 4ax$ at (x_1, y_1) on the parabola. [4]
Ans: $yy_1 = 2a(x + x_1)$

57. 2059 Q.No. 9 b Find the condition that the line $y = mx + c$ may be a tangent to the parabola $y^2 = 4ax$. [4]
Ans: $c = \frac{a}{m}$

58. 2058 Q.No. 9 b Show that the normal to the parabola $y^2 = 8x$ at (2, 4) meets the parabola again in (18, – 12). [4]

59. 2057 Q.No. 9 b Find the equation of the parabola in the standard form $y^2 = 4ax$. [4]

6 Marks Questions

60. 2070 Supp. Q.No. 9 Prove that the tangent to a parabola $y^2 = 4ax$ at a point (x_0, y_0) on the parabola is given by the equation $yy_0 = 2a(x + x_0)$. Reduce the equation in slope form. [6]

B. ELLIPSE

FORMULAE

Tables for Different Types of Ellipse

Ellipse	Center	Vertex	Focus	Major axis	Minor axis	Eccentricity (e)	Length of Latus ratum	Directrix
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 0$	(0, 0)	(±a, 0)	(±ae, 0)	2a	2b	$\sqrt{1 - \frac{b^2}{a^2}}$	$\frac{2b^2}{a}$	$x = \pm\frac{a}{e}$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b > a > 0$	(0, 0)	(0, ±b)	(0, ±be)	2b	2a	$\sqrt{1 - \frac{a^2}{b^2}}$	$\frac{2a^2}{b}$	$y = \pm\frac{b}{e}$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, $a > b > 0$	(h, k)	(h±a, k)	(h±ae, k)	2a	2b	$\sqrt{1 - \frac{b^2}{a^2}}$	$\frac{2b^2}{a}$	$x = h \pm \frac{a}{e}$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, $b > a > 0$	(h, k)	(h, k±b)	(h, k±be)	2b	2a	$\sqrt{1 - \frac{a^2}{b^2}}$	$\frac{2a^2}{b}$	$y = k \pm \frac{b}{e}$

2 Marks Questions

1. **2076 GIE Set B Q.No. 2a** Find the foci of the ellipse $3x^2 + 4y^2 = 36$ [2]
   Ans: $(\pm\sqrt{3}, 0)$
2. **2075 Set C Q.No. 2a** Find the foci of the ellipse $\frac{(x-1)^2}{25} + \frac{y^2}{16} = 1$. [2]
   Ans: $(\pm 3, 0)$
3. **2074 Supp Q.No. 2a** Find the foci of the ellipse $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$. [2]
   Ans: (0, 0), and (0, 4)
4. **2072 Supp. Q.No. 2a** Find the eccentricity and the coordinates of the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ [2]
   Ans: $\frac{\sqrt{7}}{4}$, $(0, \pm\sqrt{7})$
5. **2072 Set E Q.No. 2a** Find the eccentricity and the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$. [2]
   Ans: $\frac{4}{5}$; (0, ±4)
6. **2071 Set C Q.No. 2 a** Find the eccentricity and the foci of the ellipse $25x^2 + 4y^2 = 100$. [2]
   Ans: $\frac{\sqrt{21}}{5}$, $(0, \pm\sqrt{21})$
7. **2070 Set C Q.No. 2 a** Find the eccentricity and the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ [2]
   Ans: $\frac{\sqrt{7}}{4}$, $(0, \pm\sqrt{7})$
8. **2070 Set D Q.No. 2 a** Find the eccentricity and the foci of the ellipse $3x^2 + 4y^2 = 36$. [2]
   Ans: $\frac{1}{2}$, $(\pm\sqrt{3}, 0)$
9. **2070 (Old) Q.No. 2 b** Find the equation of the ellipse in the standard form whose focus is at (–2, 0) and vertex at (5, 0).[2]
   Ans: $\frac{x^2}{25} + \frac{y^2}{21} = 1$
10. **2069 (Set B) Q.No. 2a** Find the eccentricity and the foci of the ellipse $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$. [2]
    Ans: $e = \frac{\sqrt{7}}{4}$, $(-2 \pm \sqrt{7}, 5)$

4 Marks Questions

11. **2077 Set I Q.No. 3 OR** Find the eccentricity and coordinates of foci of: $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$ [4]
    Ans: $\frac{\sqrt{3}}{3}$, (0, 0) and (0, 4)
12. **2077 Set I Q.No. 4 OR** Find the vertices and foci of the ellipse $16x^2 + 25y^2 + 64x + 50y - 311 = 0$ [4]
    Ans: (3, – 1) and (–7, – 1); (1, – 1) and (– 5, – 1)
13. **2076 GIE Set A Q.No. 6a** Show that $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents an ellipse. Find the vertex, focus and eccentricity. [4]
    Ans: (1, –1) and (1, 5); $(1, 2 \pm \sqrt{5})$; $\frac{\sqrt{5}}{3}$
14. **2076 Set B Q.No. 6a OR** Find the vertices and foci of the ellipse $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$. [4]
    Ans: (–6, 5) and (2, 5); $(-2 \pm \sqrt{7}, 5)$
15. **2076 Set C Q.No. 6a OR** Find the vertices and foci of the curve $\frac{(x+6)^2}{4} + \frac{y^2}{36} = 1$. [4]
    Ans: $(-6, \pm 6)$, $(-6, \pm 4\sqrt{2})$
16. **2075 GIE Q.No. 6a OR** Find the eccentricity and the coordinates of the foci of the curve $9x^2 + 5y^2 - 30y = 0$.
    Ans: $\frac{2}{3}$; (0, 1) and (0, 5)
17. **2075 Set A Q.No. 6a OR** Find the equation of the ellipse in the standard form with a vertex at (0, 8) and passing through $\left(3, \frac{32}{5}\right)$. [4]
    Ans: $\frac{x^2}{25} + \frac{y^2}{64} = 1$
18. **2075 Set B Q.No. 6b OR** Find the vertices, eccentricities, foci and length of major axis of the ellipse $\frac{(x+2)^2}{2} + y^2 = 5$. [4]
    Ans: $(-2 \pm \sqrt{10}, 0)$, $\frac{1}{\sqrt{2}}$, $(-2 \pm \sqrt{5}, 0)$, $2\sqrt{10}$
19. **2074 Set A Q.No. 6b OR** Find the vertices, eccentricties, foci and length of major axis of the ellipse $\frac{x^2}{5} + \frac{(y+2)^2}{3} = 1$. [4]
    Ans: $(\pm\sqrt{5}, -2)$; $\sqrt{\frac{2}{5}}$; $(\pm\sqrt{2}, -2)$; $2\sqrt{5}$
20. **2074 Set B Q.No. 6a OR** Find the eccentricity and the coordinates of the foci of the ellipse: $x^2 + 4y^2 - 4x + 24y + 24 = 0$ [4]
    Ans: $\frac{\sqrt{3}}{2}$, $(2 \pm 2\sqrt{3}, -3)$
21. **2073 Supp Q.No. 6a OR** Find the vertices, eccentricity and foci of the ellipse $5x^2 + 9y^2 = 45$. [4]
    Ans: $(\pm 3, 0)$, $\frac{2}{3}$, $(\pm 2, 0)$
22. **2073 Set C Q.No. 6a OR** Find the equation of the ellipse whose major axis is twice its minor axis and passes through the point (0, 1). [4]
    Ans: $x^2 + 4y^2 = 4$
23. **2073 Set D Q.No. 6a OR** Find the centre, eccentricity and foci of the ellipse $9x^2 + 5y^2 - 30y = 0$. [4]
    Ans: (0, 3); $\frac{2}{3}$; (0, 5) and (0, 1)
24. **2072 Set D Q.No. 6a OR** Find the eccentricity and coordinates of the foci of the curve $\frac{(x+6)^2}{4} + \frac{y^2}{36} = 1$. [4]
    Ans: $\frac{2\sqrt{2}}{3}$, $(-6, \pm 4\sqrt{2})$
25. **2071 Supp. Q.No. 6b OR** Find the vertices, eccentricities, foci and length of major axes of the ellipse $\frac{(x+5)^2}{9} + \frac{(y-1)^2}{4} = 1$. [4]
    Ans: (–2, 1) and (–8, 1); $\frac{\sqrt{5}}{3}$; $(-5 \pm \sqrt{5}, 1)$; 6
26. **2071 Set D Q.No. 6 a OR** Find the equation of the ellipse whose distance between two foci is 8 and the semi-latus rectum is 6. [4]
    Ans: $3x^2 + 4y^2 = 192$
27. **2071 Old Q.No. 9 a OR** Find the eccentricity and the foci of the ellipse: $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$. [4]
    Ans: $\frac{\sqrt{7}}{4}$, $(-2 \pm \sqrt{7}, 5)$

28. **2069 (Set A) Q.No. 6a or** Find the eccentricity, the coordinates of the vertices and the foci of ellipse $9x^2 + 5y^2 - 30y = 0$. [4]

Ans: $e = \frac{2}{3}$, (0, 0) and (0, 6); (0, 1) and (0, 5)

29. **2069 (Set A) Old Q.No. 9b or** Show that $9x^2 + 5y^2 - 30y = 0$ represents the equation of an ellipse. Find the eccentricity, the coordinates of the centre and the foci. [4]

Ans: $\frac{2}{3}$; (0, 3); (0, 5) and (0, 1)

30. **2069 Old (Set B) Q.No. 9b Or** Find the equation of the ellipse whose latus rectum is 3 and eccentricity is $\frac{1}{\sqrt{2}}$. [4]

Ans: $x^2 + 2y^2 = 9$

31. **2068 Q.No. 9b OR** Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the ellipse $25x^2 + 4y^2 = 100$. [4]

Ans: (0, ±5), $\frac{\sqrt{21}}{5}$, (0, $\pm\sqrt{21}$)

32. **2067 Q.No. 9b OR** Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find centre, vertex and focus. [4]

Ans: (2, –3); (6, –3), (–2, – 3); (2 ±2$\sqrt{3}$, –3)

33. **2068 Q.No. 9b OR** Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the ellipse $25x^2 + 4y^2 = 100$. [4]

Ans: (0, ±5), $\frac{\sqrt{21}}{5}$, (0, $\pm\sqrt{21}$)

34. **2067 Q.No. 9b OR** Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find centre, vertex and focus. [4]

Ans: (2, –3); (6, –3), (–2, – 3); (2 ±2$\sqrt{3}$, –3)

35. **2066 C Q.No. 9 b OR** Find the eccentricity and the foci of the ellipse: $9x^2 + 5y^2 - 30y = 0$ [4]

Ans: $\frac{2}{3}$, (0, 5) and (0, 1)

36. **2066 Q.No. 9 b** Find the equation of the ellipse in the standard position whose latus rectum is equal to half its major axis and which passes through the point $(\sqrt{6}, 1)$. [4]

Ans: $x^2 + 2y^2 = 8$

37. **2065 Q.No 9 b OR** Show that: $9x^2+4y^2-18x-16y-11=0$ represents the equation of an ellipse. Find its centre, vertex and focus. [4]

Ans: (1, 2); (1, 5) and (1, –1); (1, 2±$\sqrt{5}$)

38. **2064 Q.No. 9 b OR** Find the eccentricity and the loci of the ellipse: $9x^2 + 5y^2 - 30y = 0$. [4]

Ans: $\frac{2\sqrt{2}}{3}$, (0, 3 ± 2$\sqrt{3}$)

39. **2062 Q.No. 9 b OR** Find the eccentricity and the coordinates of the foci of the ellipse: $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$. [4]

Ans: $\frac{1}{\sqrt{3}}$, (0, 0), (0, 4)

40. **2061 Q.No. 5 c** Find the equation of the ellipse in the standard position with a focus at (0, –5) and eccentricity $\frac{1}{3}$. [4]

Ans: $9x^2 + 8y^2 = 180$

41. **2060 Q.No. 9 b OR** Find the eccentricity and the foci of the ellipse: $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$. [4]

Ans: $\frac{1}{\sqrt{3}}$, (0, 4) and (0, 0)

42. **2059 Q.No. 9 b OR** Deduce the equation of the ellipse in the standard position if a focus is at (0, –5) and eccentricity is $\frac{1}{3}$. [4]

Ans: $9x^2 + 8y^2 = 180$

43. **2058 Q.No. 9 b OR** Find the eccentricity and the foci of the ellipse. $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$. [4]

Ans: $\frac{\sqrt{7}}{4}$, (–2 ±$\sqrt{7}$, 5)

44. **2057 Q.No. 9 b OR** Find the eccentricity, length of the latus rectum and coordinates of the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$. [4]

Ans: $\frac{\sqrt{3}}{2}$, 2, (±2$\sqrt{3}$, 0)

C. HYPERBOLA

FORMULAE

Tables for Different Types of Hyperbola

Equation of hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} =$
Center	(0, 0)	(0, 0)	(h, k)	(h, k)
Vertices	(± a, 0)	(0, ± b)	(h ± a, k)	(h, k ± b)
Foci	(± ae, 0)	(0, ± be)	(h ± a, k)	(h, k ± be)
Equation of transverse axis	y = 0	x = 0	x = h	y = k
Equation of conjugate axis	x = 0	y = 0	y = k	x = h
Length of transverse axis	2a	2b	2a	2b
Length of conjugate axis	2b	2a	2b	2a
Equation of directrices	$x = \pm\frac{a}{e}$	$y = \pm\frac{b}{e}$	$x = h \pm \frac{a}{e}$	$x = k \pm \frac{b}{e}$
Eccentricity (e)	$\sqrt{1+\frac{b^2}{a^2}}$	$\sqrt{1+\frac{a^2}{b^2}}$	$\sqrt{1+\frac{b^2}{a^2}}$	$\sqrt{1+\frac{a^2}{b^2}}$
Latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

2 Marks Questions

1. **2077 Set G Q.No. 1b** Determine the equation of the hyperbola with vertex (8, 0) and passing through $(8\sqrt{2}, 4)$. [2]

Ans: $\frac{x^2}{8} - \frac{y^2}{16} = 1$

2. **2077 Set H Q.No. 1b** Find the vertices and eccentricity of the hyperbola $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$ [2]

Ans: (–6, 1) and (2, 1), $\frac{5}{4}$

2076 Set B Q.No. 2a Find the eccentricity and foci of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$. [2]

Ans: $\frac{\sqrt{41}}{5}$, $(\pm\sqrt{41}, 0)$

2076 Set C Q.No. 2a Determine the equation of the hyperbola with a focus at (5, 0) and a vertex (3, 0). [2]

Ans: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

2075 GIE Q.No. 2a Show that the equation $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ represents a hyperbola. Also find its eccentricity. [2]

Ans: $\frac{5}{4}$

2075 Set A Q.No. 2a Find the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. [2]

Ans: $(\pm 5, 0)$

2074 Set A Q.No. 2a Find the equation of a hyperbola in standard position such that its transverse and conjugate axes are respectively 4 and 5. [2]

Ans: $\frac{x^2}{4} - \frac{4y^2}{25} = 1$

2074 Set B Q.No. 2a Find the equation of hyperbola with a focus at (7, 0) and a vertex at (5, 0). [2]

Ans: $\frac{x^2}{25} - \frac{y^2}{24} = 1$

2073 Supp Q.No. 2a Determine the equation of the hyperbola with a focus at (–5, 0) and a vertex at (3, 0). [2]

Ans: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

2073 Set C Q.No. 2a Find the foci and vertices of the hyperbola $9x^2 - 16y^2 = 144$. [2]

Ans: $(\pm 5, 0)$; $(\pm 4, 0)$

2073 Set D Q.No. 2a Find the equation of a hyperbola with a focus at (–7, 0) and eccentricity $\frac{7}{4}$. [2]

Ans: $\frac{x^2}{16} - \frac{y^2}{33} = 1$

2072 Set C Q.No. 2a Find eccentricity and foci of the hyperbola $\frac{x^2}{36} - \frac{y^2}{64} = 1$. [2]

Ans: $\frac{5}{3}$, $(\pm 10, 0)$

2072 Set D Q.No. 2a Find the equation of the hyperbola with vertex (8, 0) and passing through the point $(8\sqrt{2}, 4)$. [2]

Ans: $\frac{x^2}{8} - \frac{y^2}{16} = 1$

2071 Set D Q.No. 2 a Find the eccentricity and the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. [2]

Ans: 5/3, $(\pm 5, 0)$

2071 Old Q.No. 2 c Find the coordinates of vertices and eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$. [2]

Ans: $(\pm 4, 0)$, $\frac{\sqrt{5}}{2}$

2070 Supp. Q.No. 2 a Find the eccentricity of hyperbola $x^2 - 4y^2 - 12 = 0$. [2]

Ans: $\frac{\sqrt{5}}{2}$

2069 (Set A) Q.No. 2a Find the eccentricity and the foci of the hyperbola $3x^2 - 4y^2 = 36$. [2]

Ans: $\frac{\sqrt{7}}{2}$, $(\pm\sqrt{21}, 0)$

2067 Q.No. 5c Find the equation of hyperbola in the standard form with a focus at (0, 5) and a vertex at (0, – 3). [2]

Ans: $16y^2 - 9x^2 = 144$

2066 C Q.No. 5 c Find the equation to the hyperbola in the standard form with a focus at (–7, 0) and eccentricity $\frac{7}{4}$. [2]

Ans: $\frac{x^2}{16} - \frac{y^2}{33} = 1$

2065 Q.No 5 c Find the eccentricity and foci of the hyperbola $3x^2 - 4y^2 = 36$. [2]

Ans: $\frac{\sqrt{7}}{2}$, $(\pm\sqrt{21}, 0)$

4 Marks Questions

2076 GIE Set A Q.No. 6a OR Find the vertices, foci and eccentricity of the hyperbola: $5x^2 - 20y^2 - 20x = 0$ [4]

Ans: (0, 0) and (4, 0); $\frac{\sqrt{5}}{2}$; $(2 \pm \sqrt{5}, 0)$

2076 GIE Set B Q.No. 6a OR Find the vertices, eccentricity and foci of the hyperbola $9x^2 - 16y^2 + 36x + 32y - 124 = 0$

Ans: (2, 1) and (–6, 1), $\frac{5}{4}$, (–7, 1) and (3, 1)

2075 Set C Q.No. 6a OR Find the vertices, centre, eccentricity and foci of the hyperbola $9(x - 1)^2 - 16(y + 2)^2 = 144$. [4]

Ans: (5, –2), (–3, –2); (1, –2); $\frac{5}{4}$; (6, – 2), (–4, –2)

2074 Supp Q.No. 6a OR Obtain the equation to the hyperbola in the standard form with a focus at (–7, 0) and eccentricity $\frac{7}{4}$. [4]

Ans: $\frac{x^2}{16} - \frac{y^2}{33} = 1$

2072 Supp. Q.No. 6a OR Find the eccentricity, coordinates of the vertices and the foci of the hyperbola: $5x^2 - 20y^2 - 20x = 0$ [4]

Ans: $\frac{\sqrt{5}}{2}$; (4, 0) and (0, 0); $(2 \pm \sqrt{5}, 0)$,

2072 Set E Q.No. 6b OR Find the equation of the hyperbola with vertex at (0, 8) and passing through the point $(4, 8\sqrt{2})$. [4]

Ans: $\frac{x^2}{16} - \frac{y^2}{64} = -1$

2071 Set C Q.No. 6 a OR Find the equation of the hyperbola with a focus at (0,5) and a vertex at (0,–3). [4]

Ans: $16y^2 - 9x^2 = 144$

2070 Set C Q.No. 6 a or Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the hyperbola $5x^2 - 20y^2 - 20x = 0$. [4]

Ans: (4, 0) and (0, 0); $\frac{\sqrt{5}}{2}$; $(2 \pm \sqrt{5}, 0)$

29. 2070 Set D Q.No. 6 a Or Deduce the equation of a hyperbola with a focus at (6, 0) and a vertex at (4, 0) [4]

Ans: $\frac{x^2}{16} - \frac{y^2}{20} = 1$

30. 2070 (Old) Q.No. 9 b Or Find the equation to the hyperbola in standard form whose focus is at (0, 5) and vertex at (0, –3) [4]

Ans: $16y^2 - 9x^2 = 144$

31. 2069 (Set B) Q.No. 6a Or Find the equation of the hyperbola with focus at (–5, 0) and vertex at (2, 0). [4]

Ans: $21x^2 - 4y^2 = 84$

32. 2066 Q.No. 9 b OR Determine the equation of the hyperbola in the standard position with focus at (–7, 0) and eccentricity $\frac{7}{4}$. [4]

Ans: $\frac{x^2}{16} - \frac{y^2}{33} = 1$

33. 2063 Q.No. 9 b OR Find the eccentricity and the coordinates of the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ [4]

Ans: $\frac{\sqrt{5}}{2}$; $(\pm 2\sqrt{5}, 0)$

34. 2061 Q.No. 9 b OR Find the eccentricity and the foci of the hyperbola: $3x^2 - 4y^2 = 36$ Ans: $\frac{\sqrt{7}}{2}$, $(\pm\sqrt{21}, 0)$ [4]

5. CO-ORDINATES IN SPACE

A. CO-ORDINATES IN SPACE

FORMULAE

1. Distance between point (x_1, y_1, z_1) and (x_2, y_2, z_2)
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
2. Section formula:

i. $(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$ (internal division)

ii. $(x, y, z) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$ (external division)

3. Centroid of a Triangle:
$(x, y, z) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$
4. Mid point formula:
$(x, y, z) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
5. Direction Cosines of a Line
Let α, β, γ be the angles which a given directed line makes with the positive direction of the axes. Then cos α, cos β, cos γ, are called the direction cosines (dc's) of the line. The direction cosines of a line are usually denoted by l, m, n so that $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.
Also, $l^2 + m^2 + n^2 = 1$
6. Direction Ratios
Any three numbers a, b, c which are proportional to the direction cosines are called direction ratios (dr's.) of the given line. That is, if a, b, c are the direction ratios or the direction numbers of the line, then
$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$
$\therefore l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$
$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$
$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$
7. The dcs. of x axis are (1, 0, 0), dcs. of y axis are (0, 1, 0) and dcs. of z axis are (0, 0, 1).
8. Projection of the join of two points on a line
Let PQ be the line joining two given points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$. Let C'D' be a line whose direction cosines are l, m, n.
Projection of PQ on C'D' = $(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$
9. Direction Ratios and Direction Cosines of Line Joining Two Points
The line PQ, joining $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$ has its direction ratios: $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$ and direction cosines: $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$
10. Angle Between Two Lines

a. $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$
The lines are perpendicular if $l_1l_2 + m_1m_2 + n_1n_2 = 0$ and the lines are parallel if $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

b. $\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$
The lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and the lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

2 Marks Questions

1. 2077 Set H Q.No. 1c Find the angle between the lines whose direction cosines are proportional to 1, 2, 2 and 2, 3, 6. [2]

Ans: $\cos^{-1}\left(\frac{20}{21}\right)$

2. 2076 GIE Set A Q.No. 2b If l, m, n are the direction cosines of a line, prove that: $l^2 + m^2 + n^2 = 1$ [2]
3. 2076 GIE Set B Q.No. 2b If α, β and γ are the direction angles of a line, prove that: $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$ [2]
4. 2076 Set B Q.No. 2b Find the ratio in which the line joining the points P(–2, 4, 7) and Q(3, – 5, – 1) is divided by the ZX- plane. [2]

Ans: 4 : 5

5. 2076 Set C Q.No. 2b Find the direction cosines of the line joining the points (1, 2, 3) and (4, 5, 7). [2]

Ans: $\frac{3}{\sqrt{34}}, \frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}$

6. 2075 GIE Q.No. 2b Find the angle between the two lines whose direction cosines are proportional to 3, – 4, 5 and 2, 3, – 6. [2]

Ans: $\cos^{-1}\left(-\frac{18\sqrt{2}}{35}\right)$

7. 2075 Set C Q.No. 2b Find the direction cosines of the line passing through the points A(–1, 2, 5) and B(–2, 4, 3). [2]

Ans: $-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

8. 2074 Supp Q.No. 2b Find the direction cosines of a line which is equally inclined to the axes. [2]

Ans: $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

2074 Set B Q.No. 2b Find the direction cosines of the line PQ passing through P(2, 3, 4) and Q(5, 9, 13) [2]

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

2073 Supp Q.No. 2b Find the direction cosines of a line joining the points (1,2,3) and (4,5,7). [2]

Ans: $\frac{3}{\sqrt{34}}, \frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}$

2073 Set C Q.No. 2b If O is the origin, P(2, 3, 4) and Q(1, –2, 1) be any two points, show that OP is perpendicular to OQ. [2]

2073 Set D Q.No. 2b If α, β and γ are the direction cosines of a line, prove that $\cos2\alpha + \cos2\beta + \cos2\gamma + 1 = 0$. [2]

2072 Supp. Q.No. 2b Find the direction cosines of a line passing through the points P(2, 3, 4) and Q(1, 4, 6). [2]

Ans: $-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

2072 Set C Q.No. 2b Find the ratio in which the line joining the points P(–2, 4, 7) and Q(3, –5, –1) is divided by the yz- plane. [2]

Ans: 2 : 3

2072 Set D Q.No. 2b If P and Q denote the coordinates (2, 6, 2) and (4, 5, 0) respectively, find the direction cosines of the line PQ. [2]

Ans: $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

2072 Set E Q.No. 2b Find the angle between the two lines whose direction ratios are 2, 3, 4 and 1, –2, 1. [2]

Ans: $\frac{\pi^c}{2}$

2071 Set D Q.No. 2 b Find the direction cosines of a line passing through the points M(–2, 4, 3) and N(–1, 2, 5). [2]

Ans: $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$

2070 Supp. Q.No. 2 b Find the locus of a point which is equidistant from the points (1, 2, 3) and (3, 2, –1). [2]

Ans; x – 2z = 0

2070 Set C Q.No. 2 b Show that the direction cosines of a line equally inclined to the axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$. [2]

2069 (Set B) Q.No. 2b Find the direction cosines of a line which are equally inclined to the axes. [2]

Ans: $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

2068 Q.No. 5c Show that the points (3, 0, 1), (2, 2, 2), (–1, 3, 3) and (0, 1, 2) are the vertices of a parallelogram. [2]

2067 Q.No. 5 b The section of two points P(2, –4, 3) and Q(x, y, z) in the ratio 2 : 1 is (– 2, 2, –3). Find Q. [2]

Ans: (–4, 5, –6)

2066 Q.No. 5b Find the ratio in which the line joining the points (2, 4, 5) and (3, 5,–4) is divided by xy-plane. [2]

Ans: 5:4

2065 Q. No. 5 b Show that the points A (1, 2, 3), B (4, 0, 4) and C (–2, 4, 2) are collinear. [2]

2064 Q.No. 5 a Prove that the points (- 4, 9, 6), (0, 7, 10) and (- 1, 6, 6) are the vertices of a right angled isosceles triangle. [2]

2063 Q.No. 5a Find the direction cosines of a line whose direction ratios are 1, 2, 2. [2]

Ans: $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

2062 Q.No. 5b Show that P (1, 2, 3), Q(4, 0, 4) and R(- 2, 4, 2) are collinear. [2]

2061 Q.No. 5b If the section of two points P(2, -4, 3) and Q(x, y, z) in the ratio 2:1 is (- 2, 2, -3), then find Q. [2]

Ans: (- 4, 5, - 6)

2060 Q.No. 5b Find the co-ordinates of the point which divides the line joining (2, -4, 3) and (5, 5, -6) in the ratio 2:1. [2]

Ans: (4, 2, - 3)

2059 Q.No. 5b Find the angle between the lines whose direction cosines are proportional to 1, 2, 4 and -2, -9, 5. [2]

Ans: 90°

2058 Q.No. 5b Find direction cosines of a line joining the points (-1, 2, 5) and (-2, 4, 3). [2]

Ans: $\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}$

2057 Q.No. 5b What are direction cosines of a line? Prove: $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ [2]

2056 Q.No. 5b Show that the line joining the points (1, 2, 3) and (4, 5, 7) is parallel to the line joining the points (-4, 3, -6) and (2, 9, 2). [2]

4 Marks Questions

2075 Set A Q.No. 6b Show that the angle between the two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. [4]

2071 Set C Q.No. 6 b Show that the line AB is perpendicular to CD if A, B, C, D are the points (2, 3, 4), (5, 4, –1), (3, 6, 2) and (1, 2, 0) respectively. [4]

2070 Set D Q.No. 6 b Find the angle between two straight lines whose direction cosiness are l_1, m_1, n_1 and l_2, m_2, n_2. [4]

Ans: Cos $\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

2069 (Set A) Q.No. 6b Find the angle between the two lines whose direction ratios are a_1, b_1, c_1 and a_2, b_2, c_2. Also, find the condition under which the two lines are perpendicular. [4]

Ans: $\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$; $a_1a_2 + b_1b_2 + c_1c_2 = 0$

2068 Q.No. 14 a Find the angle between the two straight lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2. Also, find the condition for the two lines to be perpendicular to each other. [4]

Ans: $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$; $l_1l_2 + m_1m_2 + n_1n_2 = 0$

2068 Q.No. 14 a Or Show that the line joining the points (1, 2, 3) and (–1, –2, –3) is parallel to the line joining the points (2, 3, 4) and (5, 9, 13). [4]

2067 Q.No. 14 a Find the angle between the lines whose direction cosines are given by $l + m + n = 0$ and $2lm + 2ln - mn = 0$ [4]

Ans: 120°

2067 Q.No. 14 a OR Prove that line which makes angle, x, y, z, δ with four diagonals of a cube is

$\cos^2x + \cos^2y + \cos^2z + \cos^2\delta = \frac{4}{3}$. [4]

42. **2066 Q.No. 14 a** Prove that the straight lines whose dc's are given by ul + vm + wn = 0 and fmn + gnl + hlm = 0 are perpendicular if $\frac{f}{u} + \frac{g}{v} + \frac{h}{w} = 0$ [4]

43. **2066 Q.No. 14 a OR** A (2, 3, –1), B (5, 2, 3), C (4, 3, –5), D (–2, 1, –3) are four points in space. Find the projection of AB on CD. [4]
Ans: $\frac{-1}{\sqrt{11}}$

44. **2065 Q. No. 14 a** Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 3, –1, 1 and –3, 2, 4. [4]
Ans: $\left(\frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-1}{\sqrt{30}}\right)$

45. **2065 Q. No. 14 a OR** The projection of a line on the axis are 6, 2, 3. Find the length of the line and its direction cosines. [4]
Ans: $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

46. **2064 Q.No. 14 a** Find the direction cosines l, m, n of two lines which are connected by the relations:
l + m + n = 0 and mn - 2nl - 2lm = 0 [4]
Ans: $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

47. **2064 Q.No. 14 a OR** Find the ratio in which the line joining the points (- 3, 4, - 8) and (5, - 6, 4) is divided by the xy plane. Find also the coordinates of the point of intersection of the line with the plane. [4]
Ans: 2:1

48. **2062 Q.No. 14a** Find the direction cosines l, m, n of two lines which are connected by the relations: 4l+3m-2n = 0 and lm - mn + nl = 0. [4]
Ans: $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$ and $\left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$

49. **2062 Q.No. 14a OR** Find the ratio in which the line joining the points (- 2, 4, 7) and (3, -5, -8) is divided xy-plane. [4]
Ans: 7:8

50. **2061 Q.No. 14a** Prove that the lines whose direction cosines are given by the relation al + bm + cn = 0 and fmn + gnl + hlm = 0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$. [4]

51. **2061 Q.No. 14b OR** Prove that a line which makes angle x, y, z, t with four diagonals of a cube is
$\cos^2 x + \cos^2 y + \cos^2 z + \cos^2 t = \frac{4}{3}$. [4]

52. **2060 Q.No. 14a** The projection of a line on the axes are 6, 2, 3. Find the length of the line and its direction cosines. [4]
Ans: 7 unit, direction cosines $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$

53. **2060 Q.No. 14a OR** Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to (1, -2, -2) and (0, 2, 1). [4]
Ans: $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

54. **2059 Q.No. 14a** Find the angle between the lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) [4]
Ans: $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$

55. **2059 Q.No. 14a OR** Given three collinear points A(3, 2, -4), B(5, 4, -6) and C(9, 8, -10). Find the ratio in which B divides AC. [4]
Ans: 1:2

56. **2058 Q.No. 14a** Find the angle between the lines whose direction cosines are given by (l_1, m_1, n_1) and (l_2, m_2, n_2). [4]
Ans: $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$

57. **2058 Q.No. 14a OR** Show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. [4]

58. **2056 Q.No. 14a OR** Find the ratio in which the yz-plane divides the line joining (4, 6, 7) and (-1, 2, 5). Also find the coordinates of the point on the yz plane. [4]
Ans: 4:1; $\left(0, \frac{14}{5}, \frac{27}{5}\right)$

59. **2056 Q.No. 14a OR** Find the ratio in which the yz-plane divides the line joining (4, 6, 7) and (-1, 2, 5). Also find the coordinates of the point on the yz plane. [4]
Ans: 4:1; $\left(0, \frac{14}{5}, \frac{27}{5}\right)$

60. **2057 Q.No. 14a** Show that the angle between two diagonals of a cube is $\cos^{-1}\frac{1}{3}$. [4]

61. **2057 Q.No. 14a OR** Find the angles between the two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2). [4]
Ans: $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$

62. **2056 Q.No. 14a** Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 3, -1, 1 and -3, 2, 4. [4]
Ans: $\frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-1}{\sqrt{30}}$

6 Marks Questions

63. **2075 Set B Q.No. 9** Prove that the straight lines whose direction cosines are given by the relations al+bm + cn = 0 and fmn + gnl + hlm = 0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$. [6]

64. **2074 Set A Q.No. 9** Prove that the lines whose direction cosines are given by the relations al + bm + cn = 0 and fmn + gnl + hlm = 0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$. [6]

65. **2071 Supp. Q.No. 9** Show that the straight lines whose direction cosines are given by the equations al+bm+cn = 0 and $ul^2+vm^2+wn^2 = 0$ are perpendicular if
$a^2(v+w) + b^2(u+w) + c^2(u+v)=0$ and parallel if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$. [6]

8. PLANE

FORMULAE

1. Intercept Form of the Equation of a Plane
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
2. Normal Form of the Equation of a Plane
$lx + my + nz = p$
3. General equation of a plane that passes through a given point (x_1, y_1, z_1) is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
4. Equation of the plane passing through three given non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is
$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$
5. Angle between Two Planes
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
i. If the planes are parallel, then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
ii. If the planes are perpendicular, then $a_1a_2+b_1b_2 + c_1c_2=0$
6. Plane through the Intersection of Two Planes
The plane through the intersection of two planes
$a_1x + b_1y + c_1z + d_1 = 0, a_2x + b_2y + c_2z + d_2 = 0$ is
$(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$
where k is same constant.
7. Angle Between a Plane and a Line
$$\cos(90° - \theta) = \sin\theta = \frac{Aa_1 + Bb_1 + Cc_1}{\sqrt{A^2 + B^2 + C^2}\sqrt{a_1^2 + b_1^2 + c_1^2}}$$
where, the plane is $Ax + By + Cz + D = 0$ and a, b, c are the direction ratios of the line.

2 Marks Questions

1. **2075 Set A Q.No. 2b** Find the equation of the plane which makes equal intercepts on the axes and passes through the point (2, 3, 4). [2]
Ans: x + y + z = 9
2. **2075 Set B Q.No. 2b** Find the intercepts made by the plane $2x + 3y + 4z = 24$ on the coordinate axes. [2]
Ans: 12, 8, 6
3. **2074 Set A Q.No. 2b** Find the equation of the plane whose intercepts on the axes are 2, 3 and 4 respectively. [2]
Ans: 6x + 4y + 3z = 12
4. **2071 Supp. Q.No. 2b** Find k so that the planes $x-2y + kz = 0$ and $2x + 5y - z = 0$ are at right angles. [2]
Ans: k = –8
5. **2071 Set C Q.No. 2 b** Find the equation of the plane through (1, 2, 3) and parallel to the plane $3x-4y+5z = 0$. [2]
Ans: 3x – 4y + 5z = 10
6. **2070 Set D Q.No. 2 b** Find the equation of the plane which makes equal intercepts on the axes and passes through the point (2, 3, 4). [2]
Ans: x + y + z = 9
7. **2069 (Set A) Q.No. 2b** Find the equation of the plane through the point (3, –4, 5) and parallel to the plane $3x - 4y + 5z=7$. [2]
Ans: 3x – 4y + 5z = 50

4 Marks Questions

8. **2077 Set G Q.No. 4** Find the equation of the plane through the point (2, 2, 1) and (9, 3, 6) and normal to the plane $2x + 6y + 6z = 9$. [4]
Ans: 3x + 4y – 5z = 9
9. **2076 GIE Set A Q.No. 6b** A plane cuts the co-ordinate axes at the points A, B, C and the centroid of the triangle ABC is (1, 2, 1). Find the equation of the plane. [4]
Ans: 2x + y + 2z = 6
10. **2076 GIE Set B Q.No. 6b** Find the equation of the plane passing through the points (1, 1, 0), (– 2, 2, – 1) and (1, 2, 1). [4]
Ans: 2x + 3y – 3z = 5
11. **2076 Set B Q.No. 6b** Show that the plane $2x + 3y – 4z = 3$ is parallel to the plane $10x + 15y – 20z = 12$ and is perpendicular to $3x + 2y + 3z = 5$. [4]
12. **2076 Set C Q.No. 6b** Find the equation of the plane through the point (2, – 3, 1) and perpendicular to the line joining the two points (3, 4, – 1) and (2, – 1, 5). [4]
Ans: x + 5y – 6z + 19 = 0
13. **2075 GIE Q.No. 6b** Find the equation of the plane through the point (2, 1, 4) and perpendicular to each of the planes $9x – 7y + 6z + 48 = 0$ and $x + y + z = 0$. [4]
Ans: 13x + 3y – 16z + 35 = 0
14. **2075 Set C Q.No. 6b** Find the equation of the plane through (–2, 3, 4) and perpendicular to the planes $3x+2y + 2z – 8 = 0$ and $2x + 3y + 4z – 6 = 0$. [4]
Ans: 2x – 8y + 5z + 8 = 0
15. **2074 Supp Q.No. 6b** Show that the plane $2x + 3y – 4z = 3$ is parallel to the plane $10x + 15y –20z = 12$ and perpendicular to the plane $3x + 2y +3z = 5$. [4]
16. **2074 Set B Q.No. 6b** Find the equation of the plane through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and perpendicular to the plane $4x + 5y – 3z = 8$. [4]
Ans: x + 7y + 13z + 96 = 0
17. **2073 Supp Q.No. 6b** Find the equation of the plane through (–2, 3, 4) and perpendicular to the planes $2x + 3y + 4z = 6$ and $3x + 2y + 2z = 8$. [4]
Ans: 2x – 8y + 5z + 8 = 0
18. **2073 Set C Q.No. 6b** Find the equation of the plane through the point (2,–3, 1) and perpendicular to the line joining the two points (3, 4, –1) and (2, –1, 5). [4]
Ans: x + 5y –6z + 19 = 0
19. **2073 Set D Q.No. 6b** Find the equation of the plane passing through the points (1,1,0), (–2, 2,–1) and (1, 2, 1). [4]
Ans: 2x + 3y – 3z = 5
20. **2072 Supp. Q.No. 6b** Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and normal to the plane $2x + 6y + 6z = 9$. [4]
Ans: 3x + 4y – 5z = 9

21. 2072 Set C Q.No. 6b Find the equation of the plane through the intersection of the planes $2x + 3y + 10z = 8$ and $2x - 3y + 7z = 2$, and perpendicular to the plane $3x - 2y + 4z = 5$. [4]
Ans: $2y + z = 2$

22. 2072 Set D Q.No. 6b Show that the plane $2x + 3y - 4z = 3$ is parallel to the plane $10x + 15y - 20z = 12$ and is perpendicular to the plane $3x + 2y + 3z = 5$. [4]

23. 2072 Set E Q.No. 6a Find the equation of the plane passing through the points (1, 1 0), (–2, 2,–1) and (1, 2, 1). [4]
Ans: $2x + 3y - 3z = 5$

24. 2071 Set D Q.No. 6 b Find the equation of the plane through the points (–1, 1, 1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$. [4]
Ans: $2x + 2y - 3z + 3 = 0$

25. 2070 Set C Q.No. 6 b Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6), and normal to the plane $2x + 6y + 6z = 9$. [4]
Ans: $3x + 4y - 5z = 9$

26. 2069 (Set B) Q.No. 6b Find the equation of the plane through the points (1, 1, 0) (–2, 2, –1) and (1, 2, 1). [4]
Ans: $2x + 3y - 3z = 5$

6 Marks Questions

27. 2070 Supp. Q.No. 10 Prove that a plane through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Also, find the equation of the plane passing through (2, 2, –1), (3, 4, 2) and (7, 0, 6). [6]
Ans: $5x + 2y - 3z = 17$

6. VECTORS AND ITS APPLICATIONS

A. ELEMENTS OF VECTORS AND ITS APPLICATIONS

FORMULAE

1. Magnitude and direction of a vector
If x and y are horizontal and vertical components of a vector $\vec{a}$ then $|\vec{a}| = \sqrt{x^2 + y^2}$ is the magnitude of $\vec{a}$ and $\theta = \tan^{-1}\frac{y}{x}$ gives the direction of $\vec{a}$.

2. Unit vector
A vector whose magnitude is unity is called a unit vector.
If $\vec{a}$ is a vector then $\frac{\vec{a}}{|\vec{a}|}$ is called the unit vector of $\vec{a}$ denoted by $\hat{a}$, read as 'a cap'.

3. Parallel vectors
If $\vec{a}$ and $\vec{b}$ are two vectors then the vectors are said to be parallel if $\vec{a} = k\vec{b}$, k is any real number.

4. Collinear vectors
Vectors whose directions are either parallel or coincident are called collinear vectors. Otherwise, the vectors are non collinear.

5. Coplanar vectors
Three or more non zero vectors lying in the same plane or parallel plane, are called coplanar vectors. Otherwise, they are said to be non coplanar vectors. Two vectors are always coplanar.

6. Linear combination of vectors
A vector $\vec{r}$ is said to be a linear combination of vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}, \ldots, \vec{a_n}$ if there exist scalars $x_1, x_2, x_3, \ldots, x_n$ such that
$\vec{r} = x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} + \ldots + x_n\vec{a_n}$

7. Linearly dependent vectors
A set of vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}, \ldots, \vec{a_n}$, is said to be linearly dependent if there exist scalars $x_1, x_2, x_3, \ldots, x_n$ not all zero such that
$x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} + \ldots + x_n\vec{a_n} = 0$

8. Linearly independent vectors
A set of vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}, \ldots, \vec{a_n}$ is said to be linearly independent if there exist scalars $x_1, x_2, x_3 \ldots x_n$ all zero such that
$x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} + \ldots + x_n\vec{a_n} = 0$

2 Marks Questions

1. 2077 Set G Q.No. 1c If $3\hat{i} + \hat{j} - \hat{k}$ and $x\hat{i} - 4\hat{j} + 4\hat{k}$ are collinear vectors, find x. [2]
Ans: –12

2. 2076 GIE Set A Q.No. 2c Find $\vec{b}$ when $\vec{a} = (2, 4)$ and $2\vec{a} + 3\vec{b} = (9, 2)$ [2]
Ans: $\left(\frac{5}{3}, -2\right)$

3. 2076 GIE Set B Q.No. 2c If $3\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - 4\hat{j} + 4\hat{k}$ are collinear vectors, find the value of λ. [2]
Ans: –12

4. 2076 Set B Q.No. 2c If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$, find $|2\vec{a} + \vec{b} - \vec{c}|$. [2]
Ans: $\sqrt{206}$ units

5. 2076 Set C Q.No. 2c Show that the vectors $\hat{i} + 2\hat{j} + 4\hat{k}$, $2\hat{i} + 5\hat{j} - \hat{k}$ and $3\hat{i} + 8\hat{j} - 6\hat{k}$ are collinear. [2]

6. 2075 GIE Q.No. 2c Show that the vectors $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$ are collinear. [2]

7. 2075 Set A Q.No. 2c Express $\vec{r} = (4, 7)$ as the linear combination of $\vec{a} = (5, -4)$ and $\vec{b} = (-2, 5)$. [2]
Ans: $\vec{r} = 2\vec{a} + 3\vec{b}$

2075 Set B Q.No. 2c Show that the points $2\vec{i} + \vec{j} - \vec{k}$, $3\vec{i} - 2\vec{i} + \vec{k}$ and $\vec{i} + 4\vec{j} - 3\vec{k}$ are collinear. [2]

2074 Supp Q.No. 2c OB and OC are two straight lines and D is a point on BC such that BD : DC = m : n, show that $\vec{OD} = \frac{n\vec{OB} + m\vec{OC}}{m+n}$. [2]

2074 Set B Q.No. 2c If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$, find the unit vector along $\vec{a} - 2\vec{b} + \vec{c}$. [2]

Ans: $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

2073 Supp Q.No. 2c Show that the vectors $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} + 6\vec{k}$ are collinear. [2]

2073 Set C Q.No. 2c If $\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{b} = -\vec{i} + 2\vec{j} - 2\vec{k}$, find a unit vector along the direction of $2\vec{a} + 3\vec{b}$. [2]

Ans: $\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{k}$

2073 Set D Q.No. 2c ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point show that: $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$ [2]

2072 Supp. Q.No. 2c If $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear vectors, find the value of λ. [2]

Ans: – 12

2072 Set C Q.No. 2c Prove that the vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$ and $-7\vec{j} + 10\vec{k}$ are collinear. [2]

2072 Set D Q.No. 2c If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ find unit vector along $\vec{a} - 2\vec{b}$. [2]

Ans: $\left(\frac{7}{\sqrt{134}}, \frac{-9}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right)$

2072 Set E Q.No. 2c If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and its direction cosines. [2]

Ans: $4\vec{i} - 5\vec{j} + 11\vec{k}; \frac{4}{9\sqrt{2}}, \frac{-5}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}$

2071 Supp. Q.No. 2c Find the value of λ if the points with position vectors $\vec{i} + 2\vec{j} + \vec{k}$, $2\vec{i} - \vec{j} + 3\vec{k}$ and $5\vec{i} - 10\vec{j} + \lambda\vec{k}$ are collinear. [2]

Ans: $\lambda = 9$

2071 Set C Q.No. 2 c If $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear vectors, find the value of λ. [2]

Ans: $\lambda = -12$

2071 Set D Q.No. 2 c Show that the three points whose position vectors are $7\vec{j} + 10\vec{k}$, $-\vec{i} + 6\vec{j} + 6\vec{k}$ and $-4\vec{i} + 9\vec{j} + 6\vec{k}$ form an isosceles triangle. [2]

2071 Old Q.No. 3 b If D is the middle point of BC of the triangle ABC show that $\vec{AB} + \vec{AC} = 2\vec{AD}$. [2]

2070 Set C Q.No. 2 c ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point show that: $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$. [2]

2070 Set D Q.No. 2 c The vertices A, B, C of a triangle are (2, –1, –3), (4, 2, 3) and (6, 3, 4) respectively. Show that $\vec{AB} = (2, 3, 6)$ and AC = 9. [2]

2070 (Old) Q.No. 3 b Find the direction cosines of the vector $\vec{MN}$ where position vectors of M is $-\vec{i} + 6\vec{j} + 6\vec{k}$ and N is $-4\vec{i} + 9\vec{j} + 6\vec{k}$. [2]

Ans: $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

2069 (Set A) Q.No. 2c Show that the three points with position vectors $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$ are collinear. [2]

2069 (Set A) Old Q.No. 4a If $\vec{a} = (3, 4)$ and $3\vec{a} + 2\vec{b} = (5, 6)$ find $\vec{b}$. [2]

Ans: (–2, – 3)

2069 (Set B) Q.No. 2c If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and a unit vector along the direction of $\vec{PQ}$. [2]

Ans: $4\vec{i} - 5\vec{j} + 11\vec{k}, \frac{4}{9\sqrt{2}}\vec{i} - \frac{5}{9\sqrt{2}}\vec{j} + \frac{11}{9\sqrt{2}}\vec{k}$

2068 Q.No. 4a ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point, show that $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$. [2]

2067 Q.No. 4a Determine the unit vector of $2\vec{a} - 3\vec{b}$ where $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = -2\vec{i} - 3\vec{j}$. [2]

Ans: $\frac{14}{\sqrt{421}}\vec{i} + \frac{15}{\sqrt{421}}\vec{j}$

2066 C Q.No. 3 b Find a unit vector parallel to the sum of the vectors $2\vec{i} + 4\vec{j} - 5\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ [2]

Ans: $\frac{3}{\sqrt{61}}\vec{i} + \frac{6}{\sqrt{61}}\vec{j} - \frac{4}{\sqrt{61}}\vec{k}$

2066 Q.No. 4 a Prove that the points A, B, C are collinear, if $\vec{OA} = \vec{i} + 2\vec{j} + 4\vec{k}$, $\vec{OB} = 2\vec{i} + 5\vec{j} - \vec{k}$ and $\vec{OC} = 3\vec{i} + 8\vec{j} - 6\vec{k}$. [2]

2065 Q.No 3 b If $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear vector. Find λ. [2]

Ans: – 12

2064 Q.No. 4a If $\vec{a} = (3, 4)$ and $3\vec{a} + 2\vec{b} = (5, 6)$, find $\vec{b}$. [2]

Ans: (– 2, – 3)

34. 2063 Q.No. 3 b If $\vec{a} = (2, -3)$ and $\vec{b} = (4, -2)$, find unit vector along $4\vec{a} - 3\vec{b}$. [2]

Ans: $\left(\frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}\right)$

35. 2062 Q.No. 4 a If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} + 2\vec{j} - 4\vec{k}$ find $\vec{PQ}$ and determine its direction cosines. [2]

Ans: $4\vec{i} - 5\vec{j} + 11\vec{k}, \frac{4}{9\sqrt{2}}, \frac{-5}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}$

36. 2060 Q.No. 4 a If $\vec{a} = (2, -3)$ and $\vec{b} = (4, -2)$. Find the unit vector along $4\vec{a} - 3\vec{b}$. [2]

Ans: $\left(\frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}\right)$

37. 2059 Q.No. 3 b If $\vec{a} + \vec{b} = (5, 6)$ and $\vec{a} - \vec{b} = (3, 2)$, find $\vec{a}$ and $\vec{b}$. [2]

Ans: (4, 4) and (1, 2)

38. 2058 Q.No. 4 a If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$ find $|\vec{a} - 2\vec{b} + \vec{c}|$. [2]

Ans: 3 units

39. 2057 Q.No. 4 a ABCDEF is a regular hexagon. Express $\vec{AC}$ and $\vec{AD}$ in terms of $\vec{AB}$ and $\vec{BC}$. [2]

Ans: $\vec{AC} = \vec{AB} + \vec{BC}$, $\vec{AD} = 2\vec{BC}$

4 Marks Questions

40. 2070 (Old) Q.No. 10 a Prove that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanar. [4]

41. 2069 (Set A) Old Q.No. 10a OB and OC are two straight lines and D is a point on BC such that BD:DC = m:n, show that: $\vec{OD} = \frac{n\vec{OB} + m\vec{OC}}{m+n}$ [4]

42. 2068 Q.No. 10a Prove that the three vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are coplanor. [4]

43. 2067 Q.No. 10a Show that the three points whose position vectors are $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$ form the sides of a right angled triangle. [4]

44. 2066 C Q.No. 10 a Show that the points A, B and C with position vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-7\vec{j} + 10\vec{k}$ respectively are collinear. [4]

45. 2066 Q.No. 10 a Prove that the vectors $5\vec{a} + 6\vec{b} + 7\vec{c}$, $7\vec{a} - 8\vec{b} + 9\vec{c}$ and $3\vec{a} + 20\vec{b} + 5\vec{c}$ are coplanar. [4]

46. 2065 Q.No 10 a Show that the following vectors are linearly dependent: $5\vec{i} + 6\vec{j} + 7\vec{k}$, $7\vec{i} - 8\vec{j} + 9\vec{k}$ and $3\vec{i} + 20\vec{j} + 5\vec{k}$ [4]

47. 2064 Q.No. 10 a Show that the three points whose position vectors are $7\vec{j} + 10\vec{k}$, $-\vec{i} + 6\vec{j} + 6\vec{k}$ and $-4\vec{i} + 9\vec{j} + 6\vec{k}$ form an isosceles right angled triangle. [4]

48. 2063 Q.No. 10 a ABCD is a parallelogram G is the point of intersection of the diagonals and if O is any point, show that: $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$ [4]

49. 2062 Q.No. 10 a OB and OC are two straight lines and D is a point on BC such that BD:DC = m:n, show that $\vec{OD} = \frac{n\vec{OB} + m\vec{OC}}{m+n}$ [4]

50. 2061 Q.No. 10 a Prove that the following vectors are coplanar; $\vec{a} - 3\vec{b} + 5\vec{c}$, $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ [4]

51. 2060 Q.No. 10 a Show that the three points whose positions vectors are $7\vec{j} + 10\vec{k}$, $-\vec{i} + 6\vec{j} + 6\vec{k}$ and $-4\vec{i} + 9\vec{j} + 6\vec{k}$ form an isosceles right angled triangle. [4]

52. 2059 Q.No. 10 a Prove that the vectors $-\vec{a} + 4\vec{b} + 3\vec{c}$; $2\vec{a} - 3\vec{b} - 5\vec{c}$ and $2\vec{a} + 7\vec{b} - 3\vec{c}$ are coplanars, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are any vectors. [4]

53. 2058 Q.No. 10 a Show that the points A, B and C with position vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-7\vec{j} + 10\vec{k}$ respectively are collinear. [4]

54. 2057 Q.No. 10 a If the position vector of M and N are $3\vec{i} + \vec{j} - 3\vec{k}$ and $4\vec{i} - 2\vec{j} + \vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines. [4]

Ans: $\vec{MN} = \vec{i} - 3\vec{j} + 4\vec{k}$ $\left(\frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$

B. PRODUCT OF VECTORS

1. Scalar Product

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ then

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$

If $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ then

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

2. The angle between two vectors $\vec{a}$ and $\vec{b}$ is given by $\cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$.

3. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

4. Perpendicular or orthogonal vectors: Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$

5. $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$
$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

6. If $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$, then
$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2)\vec{i} - (a_1b_3 - a_3b_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$
$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

7. $\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}| \sin\theta)\hat{n} = ab \sin\theta \hat{n}$

8. Angle between Two Vectors
$\theta = \sin^{-1} \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

9. Vector perpendicular to $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$.
Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

10. Vector Products of $\vec{i}, \vec{j}, \vec{k}$
a. $\vec{i} \times \vec{j} = \vec{k}$, $\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$
b. $\vec{j} \times \vec{i} = -\vec{k}$, $\vec{k} \times \vec{j} = -\vec{i}$, $\vec{i} \times \vec{k} = -\vec{j}$
c. $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$

11. $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ but $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

12. Area of the parallelogram $= |\vec{a} \times \vec{b}| = ab \sin\theta$
Area of the triangle $= \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}ab \sin\theta$

13. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if $\vec{a} \times \vec{b} = 0$.

2 Marks Questions

1. **2076 GIE Set A Q.No. 3c** If $\vec{a}$ and $\vec{b}$ are perpendicular vectors and $\vec{a} = (4, 5)$, $\vec{b} = (2, k)$, find the value of k. [2]
Ans: $-\frac{8}{5}$

2. **2076 GIE Set B Q.No. 3c** Find a unit vector perpendicular to each of the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$. [2]
Ans: $\frac{\vec{i} - \vec{j} - \vec{k}}{\sqrt{3}}$

3. **2076 Set B Q.No. 3a** If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$. [2]
Ans: $-\frac{2}{\sqrt{6}}$

4. **2076 Set C Q.No. 3a** Show that the vector $\vec{a} \times \vec{b}$ is perpendicular to vectors $\vec{a}$ and $\vec{b}$. [2]

5. **2075 GIE Q.No. 3c** If $\vec{a} = (1, 1, -2)$ and $\vec{b} = (2, 1, -1)$, find the projection of $\vec{a}$ on $\vec{b}$. [2]
Ans: $\frac{5}{\sqrt{6}}$

6. **2075 Set A Q.No. 3c** Find the cosine of the angle between the two vectors $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} + 2\vec{k}$. [2]
Ans: $\frac{1}{14}$

7. **2075 Set C Q.No. 2c** If $\vec{a} = (1, 2)$ and $\vec{b} = (-3, 1)$, find the projection of $\vec{a}$ on $\vec{b}$. [2]
Ans: $-\frac{1}{\sqrt{10}}$

8. **2074 Supp Q.No. 3c** If $\vec{a} = 6\vec{i} + 3\vec{j} - 5\vec{k}$ and $\vec{b} = \vec{i} - 4\vec{j} + 2\vec{k}$, show that $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. [2]

9. **2074 Set A Q.No. 2c** Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2\vec{a} \times \vec{b}$. [2]

10. **2074 Set B Q.No. 3c** Find the area of the parallelogram determined by the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$. [2]
Ans: $6\sqrt{5}$ sq. units

11. **2073 Supp Q.No. 3c** If $\vec{a} = \vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ find the projection of $\vec{a}$ on $\vec{b}$. [2]
Ans: $\frac{-2}{\sqrt{3}}$

12. **2073 Set C Q.No. 3c** Show that vector product $\vec{a} \times \vec{b}$ is perpendicular to both vectors $\vec{a}$ and $\vec{b}$. [2]

13. **2073 Set D Q.No. 3c** If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. [2]

14. **2072 Supp. Q.No. 3c** If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ prove that $\vec{a}$ is perpendicular to $\vec{b}$. [2]

15. **2072 Set C Q.No. 3c** Find the angle between the vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$. [2]
Ans: $\theta = 90°$

16. **2072 Set D Q.No. 3c** If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$, prove that $|\vec{a}| = |\vec{b}|$. [2]

17. **2072 Set E Q.No. 3c** Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$. [2]
Ans: 20.5 sq. units

18. **2071 Set C Q.No. 3 c** For what value of m is the pair of vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} + 7\vec{j} + m\vec{k}$ orthogonal? [2]
Ans: m = 3

19. **2071 Set D Q.No. 3 c** Find a unit vector perpendicular to each of the vectors $3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$ [2]
Ans: $\frac{\vec{i} - \vec{j} - \vec{k}}{\sqrt{3}}$

20. **2071 Old Q.No. 3 c** Find the vector perpendicular to each of the vectors (1, 3, – 4) and (2, 1, –1). [2]
Ans: (1, –7, – 5)

21. **2070 Supp. Q.No. 2 c** If θ is the angle between two unit vectors $\vec{a}$ and $\vec{b}$, show that $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$. [2]

22. **2070 Set C Q.No. 3 c** If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, prove that $\vec{a}$ is perpendicular to $\vec{b}$. [2]

23. **2070 Set D Q.No. 3 c** Find the sine of the angle between the two vectors: $2\vec{i}-\vec{j}+\vec{k}$ and $3\vec{i}+4\vec{j}-\vec{k}$ [2]
Ans: $\sqrt{\frac{155}{156}}$

24. **2070 (Old) Q.No. 4 b** Find the unit vector perpendicular to the vectors $4\vec{i}-2\vec{j}+3\vec{k}$ and $5\vec{i}+\vec{j}-4\vec{k}$ [2]
Ans: $\frac{5\vec{i}+31\vec{j}+14\vec{k}}{\sqrt{1182}}$

25. **2069 (Set A) Q.No. 3c** Find the area of the parallelogram determined by the vectors: $\vec{i}+2\vec{j}+3\vec{k}$ and $-3\vec{i}-2\vec{j}+\vec{k}$. [2]
Ans: $6\sqrt{5}$ sq. units

26. **2069 (Set A) Old Q.No. 3b** If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, prove that $\vec{a}$ is perpendicular to $\vec{b}$. [2]

27. **2069 (Set B) Q.No. 3c** If $\vec{a} = \vec{i}+\vec{j}-2\vec{k}$ and $\vec{b} = 2\vec{i}-\vec{j}-\vec{k}$ are any two vectors, find the cosine of the angle between the two vectors. [2]
Ans: $\cos\theta = \frac{1}{2}$

28. **2069 Old (Set B) Q.No. 3b** For what value of x is the pair of vectors: $x\vec{i}-2\vec{j}+4\vec{k}$ and $2\vec{i}+7\vec{j}+\vec{k}$ orthogonal? [2]
Ans: 5

29. **2069 Old (Set B) Q.No. 4a** Find the area of the triangle determine by the vectors: $3\vec{i}+4\vec{j}+\vec{k}$ and $-5\vec{i}+7\vec{j}$. [2]
Ans: 20.5 sq. units

30. **2068 Q.No. 3b** Find the area of the triangle determined by the vectors $3\vec{i}+4\vec{j}$ and $-5\vec{i}+7\vec{j}$. [2]
Ans: 20.5 sq. units

31. **2067 Q.No. 3b** Given $\vec{a}$ = (3, 1, 2) and $\vec{b}$ = (2, – 2, 4), find the projection of $\vec{a}$ on $\vec{b}$. [2]
Ans: $\sqrt{6}$

32. **2066 C Q.No. 4 a** If $\vec{a} = \vec{i}+2\vec{j}+3\vec{k}$ and $\vec{b} = 2\vec{i}+3\vec{j}+4\vec{k}$, find the projection of $\vec{a}$ on $\vec{b}$. [2]
Ans: $\frac{20}{\sqrt{29}}$

33. **2066 Q.No. 3 b** For what value of m are the vectors $\vec{i}-2\vec{j}+4\vec{k}$, $2\vec{i}+7\vec{j}+m\vec{k}$ orthogonal? [2]
Ans: 3

34. **2065 Q.No 4 a** Find the value of r if the vectors $3\vec{i}-\vec{j}-2\vec{k}$ and $2\vec{i}-2\vec{j}+r\vec{k}$ are orthogonal. [2]
Ans: 4

35. **2064 Q.No. 3 b** Find the area of the triangle determined by the vectors $3\vec{i}+4\vec{j}$ and $-5\vec{i}+7\vec{j}$. [2]
Ans: 20.5 sq. units

36. **2063 Q.No. 4 a** If $\vec{i}$, $\vec{j}$, $\vec{k}$ are three mutually perpendicular unit vectors and $\vec{a} = \vec{i}-2\vec{j}+\vec{k}$, $\vec{b} = 2\vec{i}-3\vec{j}-\vec{k}$, find the cosine of the angle between the two vectors. [2]
Ans: $\sqrt{\frac{7}{12}}$

37. **2062 Q.No. 3 b** Find the area of the parallelogram determined by the vectors $\vec{i}+2\vec{j}+3\vec{k}$ and $-3\vec{i}-2\vec{j}+\vec{k}$ [2]
Ans: $6\sqrt{5}$ sq. units

38. **2061 Q.No. 3 b** Find a unit vector perpendicular to $2\vec{i}+3\vec{j}-\vec{k}$ and $\vec{i}+\vec{j}-2\vec{k}$. [2]
Ans: $\frac{-5}{\sqrt{35}}\vec{i}+\frac{3}{\sqrt{35}}\vec{j}-\frac{1}{\sqrt{35}}\vec{k}$

39. **2061 Q.No. 4 a** If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them. Show that $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$. [2]

40. **2060 Q.No. 3 b** Find the cosine of the angle between the vectors: $2\vec{i}+\vec{j}+\vec{k}$ and $4\vec{i}+3\vec{j}+5\vec{k}$. [2]
Ans: $\frac{8}{5\sqrt{3}}$

41. **2059 Q.No. 4 a** Find the angle between two vectors $\vec{a} = \vec{i}+\vec{j}-2\vec{k}$ and $\vec{b} = 2\vec{i}-\vec{j}-\vec{k}$. [2]
Ans: 60°

42. **2058 Q.No. 3 b** Show that the area of the parallelogram determined by $\vec{i}+\vec{j}-3\vec{k}$ and $-\vec{i}-2\vec{j}-3\vec{k}$ is $\sqrt{118}$ sq. units [2]

43. **2057 Q.No. 3 b** Show that the vectors $2\vec{i}+3\vec{j}-8\vec{k}$ and $2\vec{i}+4\vec{j}+2\vec{k}$ are orthogonal. [2]

4 Marks Questions

44. **2075 Set B Q.No. 7a** State and prove the sine law by the vector method. [4]

45. **2075 Set B Q.No. 7a OR** Prove that if θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} = ab\cos\theta$. [4]

46. **2071 Old Q.No. 10 b** Show that the area of the triangle PQR whose vertices are P(1, 2, 3), Q(3, 4, 5) and R (1, 4, 7) is $2\sqrt{6}$ sq. units. [4]

47. **2071 Old Q.No. 11 a** Prove by vector method: cos (A + B) = cos A cos B – sin A sin B. [4]

48. **2070 (Old) Q.No. 11 a** Prove by vector method: cos(A + B) = cos A cos B - sin A sin B. [4]

49. **2069 (Set A) Old Q.No. 11a** Prove by vector method that: sin (A – B) = sin A cos B – cos A sin B. [4]

50. **2069 Old (Set B) Q.No. 9a** Prove, analytically that the angle in a semi-circle is a right angle. [4]

51. **2069 Old (Set B) Q.No. 10a** Show that the three points whose position vectors are $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$ form the sides of a right angled triangle. Also, find the remaining two angles. [4]

Ans: $\cos^{-1}\left(\sqrt{\frac{35}{41}}\right)$ and $\cos^{-1}\left(\sqrt{\frac{6}{41}}\right)$

52. **2069 Old (Set B) Q.No. 11a** Prove vectorically that in any triangle $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [4]

53. **2068 Q.No. 11 a** Using vector method, prove that: $c^2 = a^2 + b^2 - 2ab\cos C$ [4]

54. **2067 Q.No. 11a** Prove by vector method:
$\cos(A + B) = \cos A \cos B - \sin A \sin B$. [4]

55. **2066 C Q.No. 11 a** Prove by vector method:
$\cos(A - B) = \cos A \cos B + \sin A \sin B$. [4]

56. **2066 Q.No.11 a** Use vector method to prove that, in any triangle ABC, $a = b\cos C + c\cos B$. [4]

57. **2065 Q.No 11 a** By using vectors, prove that in any ΔABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [4]

58. **2064 Q.No. 11 a** Using vector method, prove in any triangle, that: $b^2 = c^2 + a^2 - 2ac\cos B$. [4]

59. **2063 Q.No. 11 a** Using vector method, prove in any triangle that: $a = b\cos C + c\cos B$. [4]

60. **2062 Q.No. 11 a** Prove vectorically that:
$\cos(A - B) = \cos A \cos B + \sin A \sin B$. [4]

61. **2061 Q.No. 11 a** Prove, in any triangle, by vector method that: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [4]

62. **2060 Q.No. 11 a** Prove by vector method:
$\sin(A - B) = \sin A \cos B - \cos A \sin B$. [4]

63. **2059 Q.No. 11 a** Prove by vector method:
$\sin(A + B) = \sin A \cos B + \cos A \sin B$. [4]

64. **2058 Q.No. 11 a** Prove by vector method.
$\cos(A - B) = \cos A \cos B + \sin A \sin B$. [4]

65. **2057 Q.No. 11 a** Prove by vector method:
$\cos(A - B) = \cos A \cos B + \sin A \sin B$. [4]

6 Marks Questions

66. **2077 Set H Q.No. 6** Define cross product of two vectors and give its geometrical interpretation. Prove by vector method: $\cos(A - B) = \cos A \cos B + \sin A \sin B$. [6]

67. **2076 GIE Set A Q.No. 10** Define vector product at two vectors $\vec{p}$ and $\vec{q}$. Using vector product, prove that $\sin(A - B) = \sin A \cos B - \cos A \sin B$. [6]

68. **2076 GIE Set B Q.No. 10** Define scalar product of two vectors. For any triangle ABC, prove by vector method that
i. $b = c\cos A + a\cos C$
ii. $a^2 = b^2 + c^2 - 2bc\cos A$. [6]

69. **2076 Set B Q.No. 10** Define cross product of two vectors and interpret the product geometrically. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, prove that: $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$. [6]

70. **2076 Set C Q.No. 10** Define vector product of two vectors. Also prove using vector method
$\sin(A + B) = \sin A \cos B + \cos A \sin B$. [6]

71. **2075 GIE Q.No. 10** Define vector product of two vectors and geometrically interpret it. Also determine the expression for $\sin\theta$. [6]

Ans: $\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

72. **2075 Set A Q.No. 10** Define vector product of two vectors. Interpret the vector product of two vectors geometrically. Prove, in any triangle, by vector method that
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [6]

73. **2075 Set C Q.No. 10** Define a dot product. Interpret it geometrically. Let A(1, 0, –1), B(–1, 2, 0), C(2, 0, –3) and D(3, –2, –1) are four points. Show that the projection of AB on CD is equal to projection of CD on AB. Also, show that their inclination is $\cos^{-1}\left(\frac{-4}{9}\right)$. [6]

74. **2074 Supp Q.No. 10** Define scalar product of two vectors. Prove by vector method that
$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$. [6]

75. **2074 Set A Q.No. 10** Using vectors prove that
i. $b^2 = c^2 + a^2 - 2ca\cos B$.
ii. $c = a\cos B + b\cos A$ for any triangle ABC. [6]

76. **2074 Set B Q.No. 10** Define Scalar product of two vectors. Prove vectorically $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$. [6]

77. **2073 Supp Q.No. 10** Define vector product of two vectors, prove by vector method $\sin(A - B) = \sin A \cos B - \cos A \sin B$. [6]

78. **2073 Set C Q.No. 10** Define scalar product of two vectors. Prove vectorically that $\cos(A - B) = \cos A \cos B + \sin A \sin B$. [6]

79. **2073 Set D Q.No. 10** Define scalar product of two vectors. Give the geometrical interpretation of the scalar product of two vectors. Prove vectorically that, $b^2 = c^2 + a^2 - 2ca\cos B$ [6]

80. **2072 Supp. Q.No. 10** Define vector product of two vectors. Using vector method prove that
$\sin(A - B) = \sin A \cos B - \cos A \sin B$. [6]

81. **2072 Set C Q.No. 10** Define Vector product of two Vectors. Prove by Vector method
$\sin(A + B) = \sin A \cos B + \cos A \sin B$. [6]

82. **2072 Set D Q.No. 10** Define Vector product of two Vector. Prove by Vector method that in any triangle ABC,
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. [6]

83. 2072 Set E Q.No. 10 Define scalar product of two vectors. Give the geometrical interpretation of the scalar product of two vectors. In any triangle prove vectorically that $a^2 = b^2 + c^2 - 2bc \cos A$. [6]

84. 2071 Supp. Q.No. 10 Define the scalar product of two vectors. Prove by vector method: $\cos(A+B) = \cos A \cos B - \sin A \sin B$. [6]

85. 2071 Set C Q.No. 10 Define vector product of two vectors. Prove by vector method that: $\sin(A + B) = \sin A \cos B + \cos A \sin B$. [6]

86. 2071 Set D Q.No. 10 Define scalar product of two vectors. Prove by vector method that $\cos(A + B) = \cos A \cos B - \sin A \sin B$. [6]

87. 2070 Supp. Q.No. 11 Define scalar product of two vectors. Prove by vector method that $\cos(A+B)=\cos A \cos B - \sin A \sin B$. [6]

88. 2070 Supp. Q.No. 11 OR Define a vector product. Find a unit vector perpendicular to the plane of $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$. Also compute the sine of the angle between them. [6]

Ans: $\frac{1}{\sqrt{3}}(-\vec{i} - \vec{j} - \vec{k}), \frac{\sqrt{3}}{2}$

89. 2070 Set C Q.No. 10 Define vector product of two vectors. Using vector method, prove that: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [6]

90. 2070 Set D Q.No. 10 Define scalar product of two vectors. Prove by vector method that: $\cos(A - B) = \cos A \cos B + \sin A \sin B$. [6]

91. 2069 (Set A) Q.No. 10 Define scalar product of two vectors. Prove by the method of vectors that: $\cos(A - B) = \cos A \cos B + \sin A \sin B$. [6]

92. 2069 (Set B) Q.No. 10 Define vector product of two vectors. Interpret the vector product of two vectors geometrically. Prove by vector method that: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [6]

7. DERIVATIVE AND ITS APPLICATION

A. DERIVATIVES

I. DERIVATIVE BY FIRST PRINCIPLE OR DEFINITION

FORMULAE

1. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
2. Right Hand and Left Hand Derivatives

$Rf'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, $h > 0$

$Lf'(x) = \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}$, $h > 0$

4 Marks Questions

1. 2071 Old Q.No. 9 b Find the derivative of $e^{\tan x}$ from first principle. [4]
Ans: $e^{\tan x} \sec^2 x$

2. 2070 Supp. Q.No. 6 b Find, from first principle, the derivative of $\sin x^2$. [4]
Ans: $2x \cos x^2$

3. 2069 (Set A) Old Q.No. 10b Find from first principle, the derivative of $e^{\cos x}$. [4]
Ans: $-e^{\cos x} \cdot \sin x$

4. 2069 (Set B) Q.No. 11 Or Find from first principle, the derivative of $e^{\sin x}$. [4]

5. 2069 Old (Set B) Q.No. 10b Find, from definition the derivative of $\log(\sin \frac{x}{a})$. [4]
Ans: $\frac{1}{a} \cot \frac{x}{a}$

6. 2068 Q.No. 10b Find from first principle, the derivative of $e^{\sin x}$. [4]
Ans: $\cos x \, e^{\sin x}$

7. 2067 Q.No. 10b Find from first principle, the derivative of: $\sin^{-1} x$. [4]
Ans: $\frac{1}{\sqrt{1-x^2}}$

8. 2066 C Q.No. 10 b Find from first principle, the derivative of: $\log \tan x$: [4]
Ans: 2 Cosec 2x

9. 2066 Q.No. 10 b Find the derivative of Sin (log x) from first principle. [4]
Ans: $\frac{1}{x} \cos(\log x)$

10. 2065 Q.No 10 b Find from first principle, the derivative of a^x. [4]
Ans: $a^x \log a$

11. 2064 Q.No. 10 b Find from first principle, the derivative of $\log \sin x$. [4]
Ans: $\cot x$

12. 2063 Q.No. 10 b Find from first principle, the derivative of $\log \tan x$. [4]
Ans: $\frac{1}{\sin x \cos x}$

13. 2062 Q.No. 10 b Find from first principle, the derivative of $\tan^{-1} x$. [4]
Ans: $\frac{1}{1+x^2}$

14. 2061 Q.No. 10 b Find from first principle, the derivative of $e^{\sqrt{x}}$ [4]
Ans: $\frac{1}{2\sqrt{x}} e^{\sqrt{x}}$

15. 2060 Q.No. 10 b Find from first principle, the derivative of $e^{\sin x}$. [4]
Ans: $\cos x \, e^{\sin x}$

16. 2059 Q.No. 10 b Find, from definition, the derivative of $e^{\tan x}$. [4]
Ans: $\sec^2 x \, e^{\tan x}$

17. 2058 Q.No. 10 b Find from first principle, the derivatives of $e^{\tan x}$. [4]
Ans: $\sec^2 x \, e^{\tan x}$

6 Marks Questions

18. **2077 Set G Q.No. 6** Find from definition, the derivative of sin (logx). [6]
Ans: $\frac{1}{x}\cos(\log x)$

19. **2076 GIE Set B Q.No. 11 OR** Find, from first principles, the derivative of $\sin^{-1} x$. [6]

20. **2076 Set B Q.No. 11** From definition, find the derivative of $e^{\tan x}$. [6]
Ans: $e^{\tan x}\sec^2 x$

21. **2076 Set C Q.No. 11** From definition, find the derivative of $\log\cos^{-1} x$. [6]
Ans: $\frac{-1}{\cos^{-1}x\sqrt{1-x^2}}$

22. **2075 GIE Q.No. 11** Find from first principle the derivative of $\tan^{-1}x$. [6]
Ans: $\frac{1}{1+x^2}$

23. **2075 Set A Q.No. 11 OR** Find from first principle, the derivative of logsinx. [6]
Ans: cotx

24. **2075 Set B Q.No. 10** Find, from first principle, the derivative of ln $(\sin x^2)$. [6]
Ans: $2x\cot x^2$

25. **2075 Set C Q.No. 11 OR** Find from first principle, the derivative of $\cos x^2$.
Ans: $-2x\sin x^2$

26. **2074 Supp Q.No. 11 OR** Find from first principle, the derivative of $\sin x^2$. [6]
Ans: $2x\cos x^2$

27. **2074 Set A Q.No. 11** Find, from first principle, the derivative of $\ln(\cos\frac{x}{a})$. [6]
Ans: $-\frac{1}{a}\tan\frac{x}{a}$

28. **2074 Set B Q.No. 11** Find from definition the derivative of log (tan x). [6]
Ans: 2 cosec 2x

29. **2073 Supp Q.No. 11 OR** Find from first principle, the derivatives of $\cos^{-1}x$. [6]
Ans: $-\frac{1}{\sqrt{1-x^2}}$

30. **2073 Set C Q.No. 11** Find from first principle the derivative of log (sec x). [6]
Ans: tan x

31. **2073 Set D Q.No. 11 OR** Find from first principle, the derivative of sin (logx). [6]
Ans: $\frac{1}{x}\cos(\log x)$

32. **2072 Supp. Q.No. 11 OR** Find from first principle, the derivative of log (tanx). [6]
Ans: 2 cosec 2 x

33. **2072 Set C Q.No. 11 OR** Find, from first principle, the derivative of xlnx. [6]
Ans: 1 + lnx

34. **2072 Set D Q.No. 11 OR** Find from first principle. the derivative of ln $\cos^{-1} x$. [6]
Ans: $\frac{-1}{\cos^{-1}x\sqrt{1-x^2}}$

35. **2072 Set E Q.No. 11 OR** Find from first principle, the derivative of $\sin x^2$. [6]
Ans: $2x\cos x^2$

36. **2071 Supp. Q.No. 11** Find from first principle, the derivative of ln $(\cos\sqrt{x})$. [6]
Ans: $\frac{1}{2\sqrt{x}}\tan\sqrt{x}$

37. **2071 Set C Q.No. 11 OR** Find from first principle, the derivative of $\tan^{-1}x$. [6]
Ans: $\frac{1}{1+x^2}$

38. **2071 Set D Q.No. 11 OR** Find from first principle, the derivative of: $\ln(\sin\frac{x}{a})$. [6]
Ans: $\frac{1}{a}\cot\frac{x}{a}$

39. **2070 Set C Q.No. 11 or** Find from first principle, the derivative of log (tanx). [6]
Ans: 2 Cosec 2x

40. **2070 Set D Q.No. 11 Or** Find from first principle, the derivative of x^x. [6]
Ans: $x^x(1 + \log x)$

41. **2070 (Old) Q.No. 10 b** Find the derivative of log (tan x) by first principle. [6]
Ans: 2 Cosec 2 x

42. **2069 (Set A) Q.No. 11 or** Find from first principle, the derivative of sin(log x). [6]
Ans: $\frac{1}{x}\cos(\log x)$

II. DERIVATIVE USING FORMULA

FORMULAE

1. i. $\frac{d(x^n)}{dx} = nx^{n-1}$ ii. $\frac{d}{dx}(e^x) = e^x$
   iii. $\frac{d}{dx}(a^x) = a^x \ln a$ iv. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
   v. $\frac{d}{dx}(\sin x) = \cos x$ vi. $\frac{d}{dx}(\cos x) = -\sin x$
   vii. $\frac{d}{dx}(\tan x) = \sec^2 x$ viii. $\frac{d}{dx}(\sec x) = \sec x \tan x$
   ix. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
   x. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cdot \cot x$
2. i. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ ii. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
   iii. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ iv. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
   v. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ vi. $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

3. i. $\frac{d[uv]}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ii. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

4. Derivative of hyperbolic function

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\text{cosech}^2 x$

$\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$

$\frac{d}{dx}(\text{cosech } x) = -\text{cosech } x \coth x$

5. Derivative of inverse hyperbolic functions

$\frac{d(\sinh^{-1}x)}{dx} = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$ $(x > 1)$

$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$ $-1 < x < 1$

$\frac{d}{dx}(\coth^{-1}x) = \frac{-1}{x^2-1}$, $|x| > 1$

$\frac{d}{dx}(\text{sech}^{-1}x) = \frac{-1}{x\sqrt{1-x^2}}$, $|x| < 1$

$\frac{d}{dx}(\text{cosech}^{-1}x) = \frac{-1}{|x|\sqrt{1+x^2}}$, $x \in \mathbb{R} - \{0\}$

2 Marks Questions

1. **2077 Set I Q.No. 1c** Find the derivative of $\sinh^{-1}(\cosh x)$. [2]

Ans: $\frac{\sinh x}{\sqrt{1+\cosh^2 x}}$

2. **2075 Set B Q.No. 3a** Find the derivative of $x^{\sinh x}$. [2]

Ans: $x^{\sinh x}\left(\frac{\sinh x}{x} + \log x \cosh x\right)$

3. **2071 Supp. Q.No. 3a** Find the derivative of $(\sec x)^{\tan x}$. [2]

Ans: $(\sec x)^{\tan x}(\tan^2 x + \sec^2 x.\ \log \sec x)$

4. **2071 Old Q.No. 2 b** Find the derivative of $\log \sinh x$. [2]

Ans: $\coth x$

5. **2070 Supp. Q.No. 3 a** Find the derivative of $(\ln x)^{\sin hx}$. [2]

Ans: $(\ln x)^{\sin hx}\left[\frac{\sin hx}{x \ln x} + \cosh x.\ \ln(\ln x)\right]$

6. **2070 (Old) Q.No. 2 c** Find $\frac{dy}{dx}$ when $y = \text{sec h}(\tan^{-1}x)$. [2]

Ans: $-\frac{\text{sec h}(\tan^{-1}x)\tan h(\tan^{-1}x)}{1+x^2}$

7. **2069 (Set A) Old Q.No. 2b** Find the derivative of $\left(\sin h\frac{x}{a}\right)^{x^2}$. [2]

Ans: $\left(\sin h\frac{x}{a}\right)^{x^2}\left\{\frac{x^2}{a}\coth\frac{x}{a} + 2x\log\left(\sin h\frac{x}{a}\right)\right\}$

8. **2069 Old (Set B) Q.No. 2a** Find the derivative of $(\text{Cosh}x)^{\sinh^{-1}x}$. [2]

Ans: $(\cos hx)^{\sinh^{-1}x}\left(\sinh^{-1}x \tan hx + \frac{1}{\sqrt{1+x^2}}\log\cos hx\right)$

9. **2068 Q.No. 2b** Find the derivative of $\left(\cos h\frac{x}{a}\right)^{\log x}$. [2]

Ans: $\left(\cosh\frac{x}{a}\right)^{\log x}\left\{\frac{1}{a}\log x.\ \tan h\frac{x}{a} + \frac{1}{x}\log\left(\cos h\frac{x}{a}\right)\right\}$

10. **2067 Q.No. 2b** Find the derivative of $x^{\cos h\frac{x}{a}}$. [2]

Ans: $x^{\cos h\frac{x}{a}}\left(\frac{1}{a}\log x \sin h\frac{x}{a} + \frac{1}{x}\cos h\frac{x}{a}\right)$

11. **2066 C Q.No. 2 b** Find the derivative of: $\log\left(\sin h\frac{x}{a}\right)$. [2]

Ans: $\frac{1}{a}\coth\frac{x}{a}$

12. **2066 Q.No. 2 b** Find the derivative of $2\tan h^{-1}\left(\tan\frac{1}{2}x\right)$ [2]

Ans: sec x

13. **2065 Q.No 2 b** Find the derivative of Arc tan sin hx. [2]

Ans: Sec hx

14. **2064 Q.No. 2 b** Find the derivative of: $\left(\sin h\frac{x}{a}\right)^{x^2}$ [2]

Ans: $\left(\sin h\frac{x}{a}\right)^{x^2}\left[\frac{x^2}{a}\coth\frac{x}{a} + 2x\log\sin h\frac{x}{a}\right]$

15. **2063 Q.No. 2 b** Find the derivative of $x^{\cos h^2(x/a)}$. [2]

Ans: $x^{\cos h^2(x/a)}\left(\frac{1}{x}\cos h^2\frac{x}{a} + \frac{1}{a}\log\sinh\frac{2x}{a}\right)$

16. **2062 Q.No. 2 b** Find the derivative of $\log\left(\sin h\frac{x}{a}\right)$. [2]

Ans: $\frac{1}{a}\coth\frac{x}{a}$

17. **2061 Q.No. 2 b** Find the derivative of: $x^{\cos h\, x/a}$ [2]

Ans: $x^{\cosh(x/a)}\left[\frac{\cosh x/a}{x} + \frac{\log x \sinh x/a}{a}\right]$

18. **2060 Q.No. 2 b** Find the derivative of $2\tan^{-1}\left(\tan h\frac{x}{2}\right)$. [2]

Ans: sechx

19. **2059 Q.No. 2 a** Find the derivative of $e^{\cosh^{-1}x}$. [2]

Ans: $\frac{1}{\sqrt{x^2-1}}e^{\cosh^{-1}x}$

20. **2058 Q.No. 2 b** Find the derivative of $x^{\cosh x}$. [2]

Ans: $x^{\cosh x}\left(\frac{\cosh x}{x} + \log x \sinh x\right)$

21. **2057 Q.No. 2 b** Show that: $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$. [2]

4 Marks Questions

22. **2057 Q.No. 10 b** Find the derivative of: $\left(\sin h\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}$ [4]

Ans: $n\left(\sin h\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}\left[\frac{x}{a} + \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\right]$

APPLICATION OF DERIVATIVES
DIFFERENTIALS, TARGET AND NORMAL

FORMULAE

1. Differentials
$\Delta y = f(x + \Delta x) - f(x)$ is the actual change in dependent variable y.
The differential of independent variable x, denoted by dx, is defined by $dx = \Delta x$.
The differential of dependent variable y, denoted by dy, is defined by $dy = f'(x)\,dx$; which is the approximate change in y.
Error = |Actual change – Approximate change|.
Percentage error = $\left|\frac{\Delta y - dy}{y}\right| \times 100$.
2. Equation of Tangent
$$y - y_1 = -\left(\frac{dy}{dx}\right)_{(x, y) = (x_1, y_1)} (x - x_1)$$
3. Equation of Normal
$$y - y_1 = -\left(\frac{dx}{dy}\right)_{(x, y) = (x_1, y_1)} (x - x_1)$$
4. The tangent to $y = f(x)$ at P is horizontal if and only if $\frac{dy}{dx} = 0$ at P.
5. The tangent to $y = f(x)$ at P is vertical if and only if $\frac{dx}{dy} = 0$ at P.

2 Marks Questions

1. **2077 Set H Q.No. 2a** Find the equation of normal to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4). [2]
Ans: x + 4y = 18
2. **2076 GIE Set B Q.No. 3a** Find the point on the curve $4y = x^2$ where the tangent drawn makes angle 45° with the x-axis. [2]
Ans: (2, 1)
3. **2075 GIE Q.No. 3a** Find the equation of the normal to the curve $y = x^3 - 2x^2 + 4$ at (2, 4). [2]
Ans: x + 4y – 18 = 0
4. **2075 Set C Q.No. 3a** Find the equation of the tangent to $y = x^3 - 2x^2 + 4$ at (2, 4). [2]
Ans: 4x – y – 4 = 0
5. **2074 Supp Q.No. 3a** Find the equation of the normal to the curve $y = 2x^3 - 5x^2 + 8$ at the point (2, 4). [2]
Ans: x + 4y = 18
6. **2074 Set A Q.No. 3a** Find the points on the curve $x^2 + y^2 = 36$ at which the tangents are parallel to the y-axis. [2]
Ans: (6, 0), (–6, 0)
7. **2074 Set B Q.No. 3a** Find the equation of the normal of the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4). [2]
Ans: x + 4y = 18
8. **2072 Set E Q.No. 3a** Find the points on the circle $x^2 + y^2 = 16$ at which the tangents are parallel to X–axis. [2]
Ans: (0, ±4)
9. **2071 Set C Q.No. 3 a** Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent is parallel to the x-axis. [2]
Ans: (0,1) and (2, – 3)
10. **2071 Old Q.No. 6 c** Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to the x-axis. [2]
Ans: (0,1) and (2, –3)
11. **2070 Supp. Q.No. 3 b** Find the slope of the tangent to the curve $y = x^3 + 2x^2 + 3x - 10$ at (–3, 2). [2]
Ans: 16
12. **2070 Set D Q.No. 3 a** Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4) [2]
Ans: 4x – y = 4
13. **2070 (Old) Q.No. 6 c** Find the points on circle $x^2 + y^2 = 16$ at which tangents are parallel to y- axis. [2]
Ans: (4, 0), (–4, 0)
14. **2068 Q.No. 5c** At what angle does the curve $y(1+x) = x$ cut the x-axis? [2]
Ans: 45°
15. **2067 Q.No. 2c** Find the angle of intersection between the curves $y = x^2$ and $6y = 7 - x^3$ at (1, 1). [2]
Ans: 90°
16. **2066 C Q.No. 2 c** Find the points on the curve $x^2 + y^2 = 16$ at which the tangents are parallel to y–axis. [2]
Ans:(4, 0), (–4, 0)
17. **2066 Q.No. 2 c** Find the angle of intersection of the curves $y^2 = x^3$ and $y = 2x$ at the point (0, 0). [2]
Ans: $\tan^{-1}(2)$
18. **2065 Q.No 2 c** Find where the tangent is parallel to the x–axis for the curve $y = x^3 - 3x^2 - 9x + 15$. [2]
Ans: (–1, 20) and (3, –12)
19. **2064 Q.No. 2 c** Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to x–axis. [2]
Ans: (0, 1), (2, – 3)
20. **2063 Q.No. 2 c** A circular copperplate is heated so that its radius increases from 5cm to 5.06cm. Find the approximate increase in area. [2]
Ans: 0.6 π cm²
21. **2062 Q.No. 2 c** Find the angle of intersection of the curves $4y = x^2 + 12$ and $y^2 = 8x$ at (2, 4) [2]
Ans: 0°
22. **2061 Q.No. 2 c** Find the slope and inclination with the x–axis of the tangent of $y = -3x - x^4$ at $x = -1$. [2]
Ans: 1 and $\frac{\pi^c}{4}$
23. **2060 Q.No. 5 c** Find the points on the curve $4y = x^4 - 8x^2$ where the tangents are paralleled to the x–axis. [2]
Ans: (0, 0), (– 2, – 4) and (2, –4)
24. **2058 Q.No. 5 c** Find the slope and inclination with x–axis of the tangent of: $x^2 + y^2 = 36$ at (0, 6). [2]
Ans: 0 and 0°
25. **2057 Q.No. 5 c** Find the slope and inclination with x–axis of the tangent of the curve $2y = 2 - x^2$ at $x = 1$. [2]
Ans: –1, $\frac{3\pi^c}{4}$

II. L HOSPITAL'S RULE

FORMULAE

If $\phi(x)$ and $\psi(x)$ and their derivatives $\phi'(x)$ and $\psi'(x)$ are continuous at $x = a$ and if $\phi(a) = \psi(a) = 0$, then

$$\lim_{x \to a} \frac{\phi(x)}{\psi(x)} = \lim_{x \to a} \frac{\phi'(x)}{\psi'(x)} = \frac{\phi'(a)}{\psi'(a)}$$ provided $\psi'(a) \neq 0$

2 Marks Questions

1. **2076 GIE Set A Q.No. 3a** Evaluate $\lim_{x \to 2} \frac{x^4 - 16}{x^3 - 8}$ using L–Hospital's rule. [2]
Ans: $\frac{8}{3}$

2. **2076 Set B Q.No. 3b** Evaluate, using L' Hospital rule: $\lim_{x \to 0} \frac{x - \sin x \cdot \cos x}{x^3}$. [2]
Ans: $\frac{2}{3}$

3. **2076 Set C Q.No. 3c** Evaluate, using L' Hospital rule: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$. [2]
Ans: $\frac{1}{2}$

4. **2075 Set A Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$. [2]
Ans: 2

5. **2074 Set A Q.No. 4a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{x^2 - \sin^2 x}{x^2}$. [2]
Ans: 0

6. **2073 Supp. Q.No. 3a** Evaluate, using L Hospital's rule: $\lim_{x \to 0} \frac{\ln(\tan x)}{\ln x}$. [2]
Ans: 1

7. **2073 Set C Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{x - \sin x \cos x}{x^3}$. [2]
Ans: $\frac{2}{3}$

8. **2073 Set D Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{x - \sin x \cos x}{x^3}$. [2]
Ans: $\frac{2}{3}$

9. **2072 Supp. Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{xe^x - \log(1 + x)}{x^2}$. [2]
Ans: 3/2

10. **2072 Set C Q.No. 3a** Evaluate, using L' Hospital's rule: $\lim_{x \to 0} \frac{\tan ax}{\tan bx}$. [2]
Ans: $\frac{a}{b}$

11. **2072 Set D Q.No. 3b** Using L' Hospital's rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$. [2]
Ans: $\frac{1}{2}$

12. **2071 Set D Q.No. 3 a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$. [2]
Ans: 2

13. **2070 Set C Q.No. 3 a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$. [2]
Ans: 2

14. **2069 (Set A) Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$. [2]
Ans: $\frac{1}{2}$

15. **2069 (Set B) Q.No. 3a** Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{x - \sin x}{x^3}$. [2]
Ans: $\frac{1}{6}$

III. ROLLE'S THEOREM AND MEAN VALUE THEOREM

FORMULAE

1. Rolle's Theorem
If f(x) be a function defined on [a, b] such that
i. f(x) is continuous in [a, b]
ii. f(x) is derivable in (a, b)
iii. f(a) = f(b),
then there exists at least one $c \in (a, b)$ such that f'(c)=0.
2. Lagrange's Mean Value Theorem
Let f(x) be a function defined in [a, b] such that
i. f(x) is continuous in [a, b]
ii. f(x) is derivable in (a, b)
then there exists at least one value $c \in (a, b)$ such that
$f'(c) = \frac{f(b) - f(a)}{b - a}$

4 Marks Questions

1. **2070 Supp. Q.No. 6 b OR** State Rolle's theorem. Verify that the function $f(x) = x(x - 3)^2$ on [0, 3] satisfies conditions of Rolle's theorem and find c prescribed in the theorem. [4]
Ans: c = 1

6 Marks Questions

2. **2077 Set G Q.No. 6 OR** State Rolle's theorem, interpret it geometrically. Verify Rolle's theorem for $f(x) = (x + 1)(x - 2)$ in [–1, 2].

3. **2076 GIE Set A Q.No. 11** State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$ in the interval [– 1, 1]. [6]

4. **2076 GIE Set B Q.No. 11** State Mean Value Theorem. Interpret it geometrically. Verify Mean Value theorem for the function $f(x) = x^3 + x^2 - 6x$ in [– 1, 4]. [6]

2076 Set B Q.No. 11 OR State Mean value theorem. Verify it for the function $f(x) = 2x^2 - 10x + 29$ in [2, 7]. [6]

Ans: $c = \frac{9}{2}$

2076 Set C Q.No. 11 OR Using Mean Value Theorem, find a point on the curve $f(x) = x^2 - 2x$, the tangent at which is parallel to the chord joining the points (1, – 1) and (4, 8). [6]

Ans: $\left(\frac{5}{2}, \frac{5}{4}\right)$

2075 GIE Q.No. 11 OR Using Lagrange's mean value theorem, find the point on the curve $f(x) = x^2 - 2x$, the tangent at which is parallel to the chord joining the points (1, –1) and (4, 8). [6]

Ans: $\left(\frac{5}{2}, \frac{5}{4}\right)$

2075 Set A Q.No. 11 State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in [1, 3]. [6]

2075 Set B Q.No. 10 OR State the mean value theorem and interpret it geometrically. Verify that the function $f(x) = \sqrt{x}$ on [1, 4] satisfies conditions of the mean value theorem and find c prescribed in the theorem. [6]

Ans: $c = \frac{9}{4}$

10. **2075 Set C Q.No. 11** State the mean value theorem. Interpret it geometrically. Verify the mean value theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in [1, 4]. [6]
11. **2074 Supp Q.No. 11** State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x(x-1)^2$ in [0, 2]. [6]
12. **2074 Set A Q.No. 11 OR** State the Mean value theorem. Use it to find a point on the parabola $f(x) = (x - 3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1). [6]

Ans: $\left(\frac{7}{2}, \frac{1}{4}\right)$

13. **2074 Set B Q.No. 11 OR** Define Lagrange's Mean value theorem. Also verify the theorem for the function $f(x) = 2x^2 - 10x + 29$ in [2, 7]. [6]

Ans: $c = \frac{9}{2}$

14. **2073 Supp Q.No. 11** Using Mean Value theorem, find a point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1). [6]

Ans: $\left(\frac{7}{2}, \frac{1}{4}\right)$

15. **2073 Set C Q.No. 11 OR** State Rolle's theorem. Using Rolle's theorem find a point on the curve $f(x) = \cos 2x$ where the tangent is parallel to x-axis on $[-\pi, \pi]$. [6]

Ans: (0, 1)

16. **2073 Set D Q.No. 11** State Mean Value Theorem. Interpret it geometrically. Verify Lagrange's mean value theorem for the function $f(x) = x(x-1)^2$ in [0, 2]. [6]
17. **2072 Supp. Q.No. 11** State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = \sin x$, $x \in [0, \pi]$. Also find a point in the curve represented by given function where the tangent is parallel to x-axis. [6]

Ans: $c = \frac{\pi}{2}$; $\left(\frac{\pi}{2}, 1\right)$

18. **2072 Set C Q.No. 11** State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = 2x^2 - 3x + 1$ in $\left[\frac{1}{2}, 1\right]$. [6]

Ans: $c = \frac{3}{4}$

19. **2072 Set D Q.No. 11** State Mean Value theorem. Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$, $x \in [2, 4]$. Ans: $c = \sqrt{6}$ [6]
20. **2072 Set E Q.No. 11** State Rolle's theorem. What is the geometrical interpretation of Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$, $x \in [-1, 1]$. [6]
21. **2071 Supp. Q.No. 11 OR** State Rolle's theorem. Verify that the function $f(x) = \sin x + \cos x$ on $[0, 2\pi)$ satisfies the conditions of Rolle's theorem and find the constant c prescribed by the theorem. If $f(x) = (x-1)^{-2}$, show that $f(0) = f(2)$, but there is no number c in (0,2) such that $f'(c) = 0$. Why does not this contradict Rolle's theorem? [6]
22. **2071 Set C Q.No. 11** State mean value theorem. Interpret it geometrically. Verify the mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in [1, 4]. [6]

Ans: c = 3

23. **2071 Set D Q.No. 11** State Rolle's Theorem. Interpret it geometrically. Verify Rolle's Theorem for the function $f(x) = x(x-3)^2$ for $x \in [0, 3]$ [6]

Ans: c = 1

24. **2070 Set C Q.No. 11** State Rolle's theorem. Interpret it geometrically. Verify Rolle's Theorem for the function $f(x) = x(x-3)^2$ for $x \in [0, 3]$. [6]
25. **2070 Set D Q.No. 11** State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x(x-1)^2$ in [0, 2]. [6]
26. **2069 (Set A) Q.No. 11** State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x^3 + x^2 - 6x$ in [–1, 4]. [6]
27. **2069 (Set B) Q.No. 11** State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in [1, 4]. [6]

8. ANTIDERIVATIVES

FORMULAE

1. i. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

   ii. $\int \frac{1}{x} dx = \ln|x| + c, x \neq 0$

   iii. $\int e^x dx = e^x + c$

iv. $\int a^x\,dx = \frac{a^x}{\ln a} + c$

2. i. $\int \sin x\,dx = -\cos x + c$
ii. $\int \cos x\,dx = \sin x + c$
iii. $\int \tan x\,dx = \ln \sec x + c$
iv. $\int \cot x\,dx = \ln \sin x + c$
v. $\int \sec x\,dx = \ln|\sec x + \tan x| + c$
$= \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$
vi. $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c$
$= \ln \tan\frac{x}{2} + c$
vii. $\int \sec^2 x\,dx = \tan x + c$
viii. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
ix. $\int \sec x \tan x\,dx = \sec x + c$
x. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

3. i. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$

ii. $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$

iii. $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$

4. i. $\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c,\ n \neq -1.$

ii. $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$

5. $\int (uv)\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx$

6. i. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

ii. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

iii. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

7. i. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + c$

ii. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right| + c$

iii. $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left|x+\sqrt{x^2+a^2}\right| + c$

8. i. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

ii. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + c$

iii. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + c$

9. i. $\int \sinh x\,dx = \cosh x + c$
ii. $\int \cosh x\,dx = \sinh x + c$
iii. $\int \tanh x\,dx = \ln\cosh x + c$
iv. $\int \coth x\,dx = \ln\sinh x + c$
v. $\int \operatorname{sech} x\,dx = \tan^{-1}|\sinh x| + c$
vi. $\int \operatorname{cosech}\,dx = \ln\tanh\frac{x}{2} + c$
vii. $\int \operatorname{sech}^2 x\,dx = \tanh x + c$
viii. $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$
ix. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + c$
x. $\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + c$

2 Marks Questions

1. **2077 Set G Q.No. 2a** Evaluate: $\int \frac{dx}{x+\sqrt{x^2-1}}$ [2]

Ans: $\frac{x^2}{2} - \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\log\left(x+\sqrt{x^2-1}\right) + c$

2. **2077 Set I Q.No. 2a** Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$ [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + c$

3. **2076 GIE Set A Q.No. 3b** Evaluate $\int\sqrt{4x^2+4x+5}\,dx$ [2]

Ans: $\frac{1}{4}(2x+1)\sqrt{4x^2+4x+5} + \ln\left(2x+\sqrt{4x^2+4x+5}\right) + c$

4. **2076 GIE Set B Q.No. 3b** Evaluate: $\int \frac{dx}{e^x+e^{-x}}$ [2]

Ans: $\tan^{-1}(e^x) + c$

5. **2076 Set B Q.No. 3c** Evaluate: $\int \frac{x}{(x-a)(x-b)}\,dx$. [2]

Ans: $\frac{1}{a-b}\{a\log(x-a) - b\log(x-b)\} + c$

6. **2076 Set C Q.No. 3b** Evaluate: $\int \frac{dx}{1-2\cos x}$ [2]

Ans: $\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2}-1}{\sqrt{3}\tan\frac{x}{2}+1}\right) + c$

7. **2075 GIE Q.No. 3b** Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$. [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + c$

8. **2075 Set A Q.No. 3b** Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$. [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + c$

9. **2075 Set B Q.No. 3b** Find the integral $\int \frac{dx}{1+3\cos^2 x}$. [2]

Ans: $\frac{1}{2}\tan^{-1}\left(\frac{\tan x}{2}\right) + c$

10. **2075 Set C Q.No. 3b** Evaluate: $\int \frac{dx}{\sqrt{1+e^{-2x}}}$. [2]

Ans: $\log\left(e^x+\sqrt{e^{2x}+1}\right) + c$

11. **2075 Set C Q.No. 3c** Evaluate: $\int \frac{2x-11}{x^2+x-2}\,dx$. [2]

Ans: $5\log(x+2) - 3\log(x-1) + C$

12. **2074 Supp Q.No. 3b** Evaluate: $\int \frac{dx}{e^x + e^{-x}}$. [2]

Ans: $\tan^{-1}(e^x) + c$

13. **2074 Set A Q.No. 3b** Find the integral $\int \frac{dx}{\sqrt{x^2 - 6x + 13}}$. [2]

Ans: $\log(x - 3 + \sqrt{x^2 - 6x + 13}) + C$

14. **2074 Set B Q.No. 3b** Evaluate: $\int \frac{3x}{(x-a)(x-b)}\,dx$. [2]

Ans: $\frac{3}{a-b}\{a\log(x-a) - b\log(x-b)\} + C$

15. **2073 Supp Q.No. 3b** Evaluate: $\int \frac{\sin 2x}{(\sin x + \cos x)^2}\,dx$. [2]

Ans: $x + \frac{1}{\tan x + 1} + C$

16. **2073 Set C Q.No. 3b** Evaluate: $\int \frac{dx}{x + \sqrt{x^2-1}}$. [2]

Ans: $\frac{x^2}{2} - \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\log(x + \sqrt{x^2-1}) + C$

17. **2073 Set D Q.No. 3b** Evaluate: $\int \frac{dx}{x + \sqrt{x^2-1}}$. [2]

Ans: $\frac{x^2}{2} - \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\log(x + \sqrt{x^2-1}) + C$

18. **2072 Supp. Q.No. 3b** Evaluate: $\int \sqrt{2ax - x^2}\,dx$. [2]

Ans: $\frac{1}{2}(x-a)\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

19. **2072 Set C Q.No. 3b** Evaluate: $\int \frac{dx}{1 - 2\cos x}$. [2]

Ans: $\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2} - 1}{\sqrt{3}\tan\frac{x}{2} + 1}\right) + C$

20. **2072 Set D Q.No. 3a** Compute the integral $\int \frac{\coth x\,dx}{\sinh x - 9\,\text{cosech}\,x}$. [2]

Ans: $\frac{1}{6}\log\left(\frac{\sinh x - 3}{\sinh x + 3}\right) + C$

21. **2072 Set E Q.No. 3b** Evaluate: $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}\ (\beta > \alpha)$. [2]

Ans: $2\log\left(\sqrt{x-\alpha} + \sqrt{x-\beta}\right) + C$

22. **2071 Supp. Q.No. 3b** Find the integral $\int \frac{dx}{3 - 2x - x^2}$. [2]

Ans: $\frac{1}{4}\log\left(\frac{3+x}{1-x}\right) + C$

23. **2071 Set C Q.No. 3 b** Evaluate: $\int \frac{6x+1}{x^2+9}\,dx$. [2]

Ans: $3\log(x^2+9) + \frac{1}{3}\tan^{-1}\frac{x}{3} + C$

24. **2071 Set D Q.No. 3 b** Evaluate: $\int \frac{dx}{\sqrt{2ax + x^2}}$. [2]

Ans: $\log(x + a + \sqrt{2ax + x^2}) + C$

25. **2071 Old Q.No. 3 a** Show that: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$. [2]

26. **2070 Supp. Q.No. 3 c** Find the integral $\int (2x-5)\sqrt{x^2 - 5x + 1}\,dx$. [2]

Ans: $\frac{2}{3}(x^2 - 5x + 1)^{3/2} + C$

27. **2070 Set C Q.No. 3 b** Evaluate: $\int \frac{2x+3}{4x^2+1}\,dx$. [2]

Ans: $\frac{1}{4}\log(4x^2+1) + \frac{3}{2}\tan^{-1}2x + C$

28. **2070 Set D Q.No. 3 b** Evaluate: $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, $(\beta > \alpha)$. [2]

Ans: $2\log(\sqrt{x-\alpha} + \sqrt{x-\beta}$

29. **2070 (Old) Q.No. 3 a** Evaluate: $\int \frac{dx}{e^x + e^{-x}}$. [2]

Ans: $\tan^{-1}(e^x) + C$

30. **2069 (Set A) Q.No. 3b** Evaluate: $\int \frac{dx}{\sqrt{2ax - x^2}}$. [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + C$

31. **2069 (Set A) Old Q.No. 3a** Show that: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$. [2]

32. **2069 (Set B) Q.No. 3b** Evaluate: $\int \frac{dx}{e^x + e^{-x}}$. [2]

Ans: $\tan^{-1}(e^x) + C$

33. **2068 Q.No. 3a** Evaluate: $\int \frac{dx}{\sqrt{2ax - x^2}}$. [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + C$

34. **2067 Q.No. 3a** Evaluate $\int \frac{1}{x^2} e^{\frac{-1}{x}dx}$. [2]

Ans: $e^{-1/x} + C$

35. **2066 C Q.No. 3 a** Evaluate: $\int \frac{dx}{\sqrt{2ax - x^2}}$. [2]

Ans: $\sin^{-1}\left(\frac{x-a}{a}\right) + C$

36. **2066 Q.No. 3 a** Integrate: $\int \frac{dx}{\sqrt{2ax + x^2}}$. [2]

Ans: $\log(x + a + \sqrt{2ax + x^2}) + C$

37. 2065 Q.No 3 a Evaluate: $\int \frac{dx}{e^x + e^{-x}}$ [2]

Ans: $\tan^{-1}(e^x) + c$

38. 2064 Q.No. 3 a Evaluate: $\int \frac{dx}{x^2 - 16}$ [2]

Ans: $\frac{1}{8} \log \frac{x-4}{x+4} + C$

39. 2063 Q.No. 3 a Integrate: $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ ($\beta > \alpha$). [2]

Ans: $2\log(\sqrt{x-\alpha} + \sqrt{x-\beta}) + C$

40. 2062 Q.No. 3 a Integrate: $\int \frac{dx}{\sqrt{2ax - x^2}}$ [2]

Ans: $\sin^{-1}\frac{x-a}{a} + C$

41. 2061 Q.No. 3 a Evaluate: $\int \sqrt{\frac{1+x}{1-x}}\, dx$. [2]

Ans: $-\sqrt{1-x^2} + \sin^{-1}x + C$

42. 2060 Q.No. 3 a Evaluate: $\int \frac{dx}{\sqrt{a^2 - x^2}}$ [2]

Ans: $\sin^{-1}\frac{x}{a} + C$

43. 2058 Q.No. 3 a Prove that: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$. [2]

44. 2057 Q.No. 3 a Prove: $\int \operatorname{cosec} x\, dx = \log\left|\tan\frac{x}{2}\right| + c$ [2]

4 Marks Questions

45. 2077 Set H Q.No. 4 Evaluate: $\int \frac{dx}{4 + 3\cos hx}$ [4]

Ans: $\frac{1}{\sqrt{7}} \ln\left(\frac{\sqrt{7} + \tanh^2\frac{x}{2}}{\sqrt{7} - \tanh^2\frac{x}{2}}\right) + c$

46. 2077 Set H Q.No. 4 OR Evaluate: $\int \frac{5}{(x+5)(2x^2+5)} dx$ [4]

Ans: $\frac{1}{11}\ln(x+5)\,dx - \frac{1}{22}\ln(2x^2+5)\,dx + \frac{\sqrt{10}}{11}\tan^{-1}\left(\sqrt{\frac{2}{5}}x\right) + c$

47. 2076 GIE Set A Q.No. 7a Evaluate: $\int \frac{dx}{3\sin x + 4\cos x}$ [4]

Ans: $\frac{1}{5}\log\tan\left(\frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{4}{3}\right) + C$

48. 2076 GIE Set A Q.No. 7a OR Evaluate: $\int \frac{dx}{(x-1)(x^2+3)}$ [4]

Ans: $\frac{1}{4}\ln(x-1) - \frac{1}{8}\ln(x^2+3) - \frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + c$

49. 2076 GIE Set B Q.No. 7a Evaluate: $\int \frac{dx}{3\sin x - 4\cos x}$ [4]

Ans: $\frac{1}{5}\log\left(\frac{2\tan\frac{x}{2} - 1}{2\tan\frac{x}{2} + 4}\right) + c$

50. 2076 Set B Q.No. 7a Evaluate: $\int \frac{dx}{1 + \sin x + \cos x}$ [4]

Ans: $\log\left(1 + \tan\frac{x}{2}\right) + c$

51. 2076 Set C Q.No. 7a Evaluate: $\int \frac{x^2}{x^4 - 2x^2 - 15} dx$. [4]

Ans: $\frac{\sqrt{5}}{16}\log\frac{x-\sqrt{5}}{x+\sqrt{5}} + \frac{\sqrt{3}}{8}\tan^{-1}\frac{x}{\sqrt{3}} + C$

52. 2075 GIE Q.No. 7a Evaluate: $\int \frac{dx}{1 + \sin x + \cos x}$ [4]

Ans: $\log\left(1 + \tan\frac{x}{2}\right) + c$

53. 2075 Set A Q.No. 7a Evaluate: $\int \frac{dx}{\sin x + \cos x}$ [4]

Ans: $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$

54. 2075 Set B Q.No. 7b Evaluate: $\int (2x+3)\sqrt{x^2 - 2x - 3}\, dx$. [4]

Ans: $\frac{2}{3}(x^2-2x-3)^{3/2} + \frac{5}{2}(x-1)\sqrt{x^2-2x-3} - 10\log(x-1-\sqrt{x^2-2x-3}) + c$

55. 2075 Set C Q.No. 7a Evaluate: $\int \frac{1}{2\sin t + 3\cos t} dt$. [4]

Ans: $\frac{1}{\sqrt{13}}\log\left\{\tan\left(\frac{t}{2} + \frac{1}{2}\tan^{-1}\frac{3}{2}\right)\right\} + c$

56. 2074 Supp Q.No. 7a Evaluate: $\int \frac{dx}{2 + 3\cos x}$ [4]

Ans: $\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5} + \tan\frac{x}{2}}{\sqrt{5} - \tan\frac{x}{2}}\right) + c$

57. 2074 Set A Q.No. 7a Evaluate: $\int \frac{dx}{3 + 5\cos hx}$ [4]

Ans: $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\tanh\frac{x}{2}\right) + c$

58. 2074 Set B Q.No. 7a Evaluate: $\int \frac{dx}{3\sin x + 4\cos x}$ [4]

Ans: $\frac{1}{5}\log\tan\left(\frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{4}{3}\right) + C$

59. 2074 Set B Q.No. 7a OR Evaluate: $\int \sqrt{\frac{1+x}{1-x}}\, dx$. [4]

Ans: $\sin^{-1}x - \sqrt{1-x^2} + C$

60. 2073 Supp Q.No. 7a Evaluate: $\int \frac{x^2}{x^4 - 2x^2 - 15} dx$. [4]

Ans: $\frac{\sqrt{5}}{16}\log\frac{x-\sqrt{5}}{x+\sqrt{5}} + \frac{\sqrt{3}}{8}\tan^{-1}\frac{x}{\sqrt{3}} + C$

2073 Set C Q.No. 7a Evaluate: $\int \frac{dx}{1-2\cos x}$ [4]

Ans: $\frac{1}{\sqrt{3}} \log\left(\frac{\sqrt{3}\tan\frac{x}{2}-1}{\sqrt{3}\tan\frac{x}{2}+1}\right)+C$

2073 Set C Q.No. 7a OR Evaluate: $\int \frac{x^3\,dx}{2x^4-3x^2-5}$ [4]

Ans: $\frac{1}{14}\log(x^2+1)+\frac{5}{28}\log(2x^2-5)+C$

2073 Set D Q.No. 7a Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$ [4]

Ans: $\log\left(\tan\frac{x}{2}+1\right)+C$

2072 Supp. Q.No. 7a Evaluate: $\int \frac{dx}{3\sin x+4\cos x}$ [4]

Ans: $\frac{1}{5}\log\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{4}{3}\right)+C$

2072 Set C Q.No. 7a Evaluate: $\int \frac{dx}{(x-2)^2(x-3)^3}$ [4]

Ans: $\frac{-1}{2}\frac{(x-2)^2}{(x-3)^2}+\frac{3(x-2)}{x-3}-3\log\frac{x-2}{x-3}-\frac{x-3}{x-2}+C$

2072 Set D Q.No. 7a Evaluate: $\int \frac{dx}{(x-1)^2(x-2)^3}$ [4]

Ans: $-\frac{1}{2}\frac{(x-1)^2}{(x-2)^2}+3\left(\frac{x-1}{x-2}\right)-3\log\left(\frac{x-1}{x-2}\right)-\frac{x-2}{x-1}+C$

2072 Set E Q.No. 7a Evaluate: $\int \frac{dx}{2+3\cos x}$ [4]

Ans: $\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right)+C$

2071 Supp. Q.No. 7a Find the integral $\int \frac{dx}{5+4\sin x}$ [4]

Ans: $\frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}+4}{3}\right)+C$

2071 Set C Q.No. 7 a Evaluate: $\int \frac{dx}{1-2\cos x}$ [4]

Ans: $\frac{1}{\sqrt{3}}\log\left[\frac{\sqrt{3}\tan\frac{x}{2}-1}{\sqrt{3}\tan\frac{x}{2}+1}\right]+C$

2071 Set D Q.No. 7 a Evaluate: $\int \frac{dx}{1-3\sin x}$ [4]

Ans: $\frac{1}{2\sqrt{2}}\log\left(\frac{\tan\frac{x}{2}-3-2\sqrt{2}}{\tan\frac{x}{2}-3+2\sqrt{2}}\right)+C$

2071 Old Q.No. 10 a Integrate $\int \sqrt{\frac{1-x}{1+x}}\,dx$ [4]

Ans: $\sin^{-1}x-\sqrt{1-x^2}+C$

2070 Supp. Q.No. 7 a Find the integral $\int \frac{dx}{3\sin x-5\cos x}$ [4]

Ans: $\frac{1}{\sqrt{34}}\log\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\left(\frac{5}{3}\right)\right)+C$

2070 Set C Q.No. 7 a Evaluate: $\int \frac{dx}{2+\cos x}$ [4]

Ans: $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x/2}{\sqrt{3}}\right)+C$

2070 Set D Q.No. 7 a Evaluate: $\int \frac{dx}{1+2\sin x}$ [4]

Ans: $\frac{1}{\sqrt{3}}\log\left(\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}\right)+C$

2070 (Old) Q.No. 11 b Evaluate: $\int \frac{\cos x-\sin x}{\sqrt{\sin 2x}}\,dx$ [4]

Ans: $\log(\cos x+\sin x+\sqrt{\sin 2x})+C$

2069 (Set A) Q.No. 7a Evaluate: $\int \frac{dx}{3\sin x-4\cos x}$ [4]

Ans: $\frac{1}{5}\log\left(\frac{2\tan\frac{x}{2}-1}{2\tan\frac{x}{2}+4}\right)+C$

2069 (Set A) Old Q.No. 11b Evaluate: $\int \frac{dx}{\sin x+\cos x}$ [4]

Ans: $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)+C$

2069 (Set B) Q.No. 7a Evaluate: $\int \frac{dx}{2+3\cos x}$ [4]

Ans: $\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right)+C$

2069 Old (Set B) Q.No. 11b Evaluate: $\int (2-x)\sqrt{16-6x-x^2}\,dx$ [4]

Ans: $\frac{5}{2}(x+3)\sqrt{16-6x-x^2}+\frac{125}{2}\sin^{-1}\left(\frac{x+3}{5}\right)+\frac{1}{3}(16-6x-x^2)^{3/2}+C$

2068 Q.No. 11 b Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$ [4]

Ans: $\log\left(\tan\frac{x}{2}+1\right)+C$

2067 Q.No. 10b Evaluate: $\int \frac{dx}{1+2\sin x}$ [4]

Ans: $\frac{1}{\sqrt{2}}\log\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}+C$

2066 C Q.No. 11 b Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$ [4]

Ans: $\log(1+\tan x/2)+C$

83. 2066 Q.No.11 b Integrate: $\int \frac{x^2}{(x+2)(x+3)^2}\,dx$. [4]

Ans: $4\log(x+2) - 3\log(x+3) + \frac{9}{x+3} + C$

84. 2065 Q.No 11 b Integrate: $\int \frac{dx}{3+4\cos hx}$ [4]

Ans: $\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{\tan h\frac{x}{2}}{\sqrt{7}}\right) + c$

85. 2064 Q.No. 11 b Integrate: $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}}\,dx$ [4]

Ans: $\log\{(\sin x + \cos x) + \sqrt{\sin 2x}\} + C$

86. 2063 Q.No. 11 b Integrate: $\int \frac{dx}{1+\sin x+\cos x}$ [4]

Ans: $\log\left(1+\tan\frac{x}{2}\right) + C$

87. 2062 Q.No. 11 b Integrate: $\int \frac{dx}{\sin x+\cos x}$ [4]

Ans: $\frac{1}{\sqrt{2}}\log\left[\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right] + C$

88. 2061 Q.No. 11 b Find the value of: $\int \frac{dx}{3\sin x - 4\cos x}$ [4]

Ans: $\frac{1}{5}\log\frac{\tan\frac{x}{2}-\frac{1}{2}}{\tan\frac{x}{2}+2} + C$

89. 2060 Q.No. 11 b Find the value of: $\int \frac{\sin x\,.\,\cos x}{(\sin x+\cos x)^2}\,dx$ [4]

Ans: $-\frac{1}{2}\frac{1}{(\tan x+1)} + C$

90. 2059 Q.No. 11 b $\int \frac{dx}{a+b\cos x}$, $a<b$ [4]

Ans: $\frac{1}{\sqrt{b^2-a^2}}\log\left(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right) + C$

91. 2058 Q.No. 11 b Integrate: $\int \sqrt{\frac{1+x}{1-x}}\,dx$ [4]

Ans: $-\sqrt{1-x^2} + \sin^{-1}x + C$

92. 2057 Q.No. 11 b Integrate: $\int \frac{dx}{a+b\cos x}$ when $a>b$ [4]

Ans: $\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + C$

9. DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS

FORMULAE

1. Separation of variables
If the equation $Mdx + Ndy = 0$ can be put in the form $f_1(x)\,dx + f_2(y)\,dy = 0$,
then it can be solved by integrating each term separately. Thus, the solution of the above equation is $\int f_1(x)\,dx + \int f_2(y)\,dy = c$

2. Homogeneous Equations
$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$
To solve such equations, we put $y = vx$, where v is a function of x. Then,
$\frac{dx}{x} = \frac{dv}{F(v)-v}$

3.
i. $y\,dx + x\,dy = d(xy)$
ii. $\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$
iii. $\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$
iv. $\frac{y\,dx - x\,dy}{x^2+y^2} = \frac{\frac{y\,dx - x\,dy}{y^2}}{1+\left(\frac{x}{y}\right)^2} = d\left[\tan^{-1}\frac{x}{y}\right]$
v. $\frac{x\,dy - y\,dx}{x^2+y^2} = \frac{\frac{x\,dy - y\,dx}{x^2}}{1+\left(\frac{y}{x}\right)^2} = d\left[\tan^{-1}\frac{y}{x}\right]$

4. Linear equation
The expression $e^{\int P dx}$ is called the Integrating Factor (I.F.) of linear equation $\frac{dy}{dx} + Py = Q$. The solution of this equation is of the form:
$y(I.F.) = \int Q(I.F.)\,dx + c$

2 Marks Questions

1. 2077 Set G Q.No. 2b Solve: $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$ [2]
Ans: $x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$

2. 2076 GIE Set A Q.No. 4a Solve: $(x^2 - ay)\,dx - (ax - y^2)\,dy = 0$ [2]
Ans: $x^3y^3 - 3axy = c$

3. 2076 GIE Set B Q.No. 4a Solve: $\frac{dy}{dx} = \frac{e^x+1}{y}$ [2]
Ans: $y^2 = 2e^x + 2x + C$

4. 2076 Set B Q.No. 4a Solve: $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$. [2]
Ans: $\cot x = \tan y + C$

5. 2076 Set C Q.No. 4a If $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$, show that $\frac{x+y}{1-xy} = c$ where c is constant. [2]

6. 2075 GIE Q.No. 4a Solve: $(x+y^2)\,dx - 2xy\,dy = 0$. [2]
Ans: $y^2 = x(\ln x + C)$

7. 2075 Set A Q.No. 4a Solve: $xdy + (x+y)dx = 0$. [2]
Ans: $x^2 + 2xy = C$

8. **2075 Set B Q.No. 3c** Solve: $\sec^2 x \tan x\, dx + \sec^2 y \tan x\, dy = 0$. [2]
Ans: tanx + tany = C

9. **2075 Set C Q.No. 4a** Solve: $\frac{dy}{dx} = \frac{e^x}{e^y} + \frac{x^3}{e^y}$. [2]
Ans: $e^y = e^x + \frac{x^4}{4} + C$

10. **2074 Supp Q.No. 4a** Solve $\frac{dy}{dx} = e^{x-y} + x^3.e^{-y}$. [2]
Ans: $e^y = e^x + \frac{x^4}{4} + C$

11. **2074 Set A Q.No. 3c** Solve: $\frac{dy}{dx} = \frac{e^x + 1}{y}$ [2]
Ans: $y^2 = 2e^x + 2x + C$

12. **2074 Set B Q.No. 4a** Solve: $(1 + x^2)\frac{dy}{dx} = 1$. [2]
Ans: $y = \tan^{-1} x + C$

13. **2073 Supp Q.No. 4a** Solve: $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$. [2]
Ans: x – y = C(1 + xy)

14. **2073 Set C Q.No. 4a** Solve: $\frac{dx}{1 + x^2} + \frac{dy}{1 + y^2} = 0$ [2]
Ans: x + y = C(1 – xy)

15. **2073 Set D Q.No. 4a** Solve: ydx – xdy = xy dy. [2]
Ans: $\log\left(\frac{x}{y}\right) = y + C$

16. **2072 Supp. Q.No. 4a** Solve: $\frac{dy}{dx} + 4x = 2e^{2x}$. [2]
Ans: $y = e^{2x} – 2x^2 + C$

17. **2072 Set C Q.No. 4a** Solve: $\frac{dy}{dx} + \frac{y}{x} = 1$. [2]
Ans: $xy = \frac{x^2}{2} + C$

18. **2072 Set D Q.No. 4a** Solve: $\frac{dy}{dx} + \frac{1 + \cos 2y}{1 – \cos 2y} = 0$. [2]
Ans: x + tan y – y = C

19. **2072 Set E Q.No. 4a** Solve: $x\frac{dy}{dx} + y – 1 = 0$. [2]
Ans: x(y – 1) = C

20. **2071 Supp. Q.No. 3c** Solve $\frac{dy}{dx} + \sqrt{\frac{1 – y^2}{1 – x^2}} = 0$. [2]
Ans: $x\sqrt{1 – y^2} + y\sqrt{1 – x^2} = C$

21. **2071 Set C Q.No. 4 a** Solve: $e^{x-y}.dx + e^{y-x}dy = 0$ [2]
Ans: $e^{2x} + e^{2y} = C$

22. **2071 Set D Q.No. 4 a** Solve: $\frac{dy}{dx} + 4x = 2e^{2x}$ [2]
Ans: $y = e^{2x} – 2x^2 + C$

23. **2071 Old Q.No. 6 a** Solve: 2xydy – y^2dx = 0 [2]
Ans: $y^2 = Cx$

24. **2070 Supp. Q.No. 4 a** Solve the differential equation: $\frac{dy}{dx} = \frac{2x + 1}{5y^4 + 1}$ [2]
Ans: $x^2 + x – y^5 – y = C$

25. **2070 Set C Q.No. 4 a** Solve : $\frac{dy}{dx} = \frac{e^x + 1}{y}$ [2]
Ans: $y^2 = 2e^x + 2x + C$

26. **2070 Set D Q.No. 4 a** Solve : $x^2dy – y^2dx = 0$. [2]
Ans: x – y = Cxy

27. **2070 (Old) Q.No. 5 c** Solve: $(1 + x^2)\frac{dy}{dx} = 1$. [2]
Ans: $y = \tan^{-1}x + C$

28. **2069 (Set A) Q.No. 4a** Solve $\frac{dy}{dx} = e^{x-y} + x^3.e^{-y}$ [2]
Ans: $e^y = e^x + \frac{x^4}{4} + C$

29. **2069 (Set A) Old Q.No. 6c** Solve: $\frac{dy}{dx} = \frac{e^x + 1}{y}$. [2]
Ans: $y^2 = 2e^x + 2x + C$

30. **2069 (Set B) Q.No. 4a** Solve: $e^{x-y}dx + e^{y-x}.dy = 0$ [2]
Ans: $e^{2x} + e^{2y} = C$

31. **2069 Old (Set B) Q.No. 2b** Solve : 2xydx – x^2dy = 0 [2]
Ans: $x^2 = Cy$

32. **2068 Q.No. 6c** Solve: $x^2dy – y^2dx = 0$ [2]
Ans: x – y = Cxy

33. **2066 C Q.No. 6 c** Solve: $\sqrt{1 – x^2}\,dy + \sqrt{1 – y^2}dx = 0$ [2]
Ans: $x\sqrt{1–y^2} + y\sqrt{1–x^2} = C$

34. **2066 Q.No. 6 c** Solve the differential equation $(x + 2y – 3)\,dy – (2x – y + 1)\,dx = 0$ [2]
Ans: $xy + y^2 – x^2 – 3y – x = C$

35. **2065 Q.No 6 c** Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$ [2]
Ans: $(x^2 + 1)(y^2 + 1) = C$

36. **2064 Q.No. 6 c** Solve: $x\,dy + (x + y)\,dx = 0$ [2]
Ans: $x^2 + 2xy = C$

37. **2063 Q.No. 6 c** Solve: $e^{x-y}\,dx + e^{y-x}\,dy = 0$ [2]
Ans: $e^{2x} + e^{2y} = C$

38. **2062 Q.No. 6 c** Solve: $x^2dy – y^2dx = 0$ [2]
Ans: y – x = Cxy

39. **2061 Q.No. 6 c** Solve: $x^2dy – y^2dx = 0$ [2]
Ans: y – x = Cxy

40. **2060 Q.No. 6 c** Solve: $\sqrt{1 – x^2}\,dy + \sqrt{1 – y^2}\,dx = 0$. [2]
Ans: $y = \tan^{-1} x + C$

41. **2059 Q.No. 2 b** Solve: x dy – y dx = 0 [2]
Ans: y = Cx

42. **2058 Q.No. 6 c** Solve: $\frac{dy}{dx} = \frac{x^3 + 1}{y^3 + 1}$. [2]
Ans: $\frac{y^4}{4} + y = \frac{x^4}{4} + x + C$

43. **2057 Q.No. 6 c** Solve: $\frac{dy}{dx} = \frac{x^2 + x + 1}{y^2 + y + 1}$ [2]
Ans: $\frac{y^3}{3} + \frac{y^2}{2} + y = \frac{x^3}{3} + \frac{x^2}{2} + x + C$

4 Marks Questions

44. **2077 Set I Q.No. 5** Solve: $\cos^2 x\frac{dy}{dx} + y = 1$ [4]
Ans: $y = 1 + Ce^{-\tan x}$

45. **2077 Set I Q.No. 5 OR** Solve: $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ [4]
Ans: $y = cx^2(2x + y)$

46. **2076 GIE Set A Q.No. 7b** Solve: $\frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{1}{(1 + x^2)^2}$ [4]
Ans: $(1 + x^2)y = \tan^{-1}x + C$

47. **2076 GIE Set B Q.No. 7b** Solve: $\tan x\frac{dy}{dx} + y = \sec x$ [4]
Ans: y sinx = x + C

48. 2076 GIE Set B Q.No. 7b OR Solve: $x^2\frac{dy}{dx} + y^2 = xy$ [4]

Ans: $x = y(\log x + C)$

49. 2076 Set B Q.No. 7b Solve: $\cos^2 x\frac{dy}{dx} + y = 1$. [4]

Ans: $y = 1 + Ce^{-\tan x}$

50. 2076 Set B Q.No. 7b OR Solve: $(1 + x)\frac{dy}{dx} - xy = 1 - x$. [4]

Ans: $y(1 + x) = x + Ce^x$

51. 2076 Set C Q.No. 7b Solve: $\sin x\frac{dy}{dx} + y\cos x = x\sin x$. [4]

Ans: $y \sin x = \sin x - x\cos x + C$

52. 2076 Set C Q.No. 7b OR Solve: $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$. [4]

Ans: $y = \frac{1}{2}e^{\tan^{-1}x} + Ce^{-\tan^{-1}x}$

53. 2075 GIE Q.No. 7b Solve: $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$. [4]

Ans: $y = \frac{1}{2}e^{\tan^{-1}x} + ce^{-\tan^{-1}x}$

54. 2075 GIE Q.No. 7b OR Solve: $\cos^2 x\frac{dy}{dx} + y = 1$. [4]

Ans: $y = 1 + Ce^{-\tan x}$

55. 2075 Set A Q.No. 7b Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$. [4]

Ans: $\sin\left(\frac{y}{x}\right) = Cx$

56. 2075 Set A Q.No. 7b OR Solve: $\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$ [4]

Ans: $xy = 1 + x + Cxe^{1/x}$

57. 2075 Set B Q.No. 8a Solve: $\frac{dy}{dx} + \frac{x^2 - y^2}{3xy} = 0$ [4]

Ans: $(x^2 + 2y^2)^{3/4} = C\sqrt{x}$

58. 2075 Set B Q.No. 8a OR Solve: $\frac{dy}{dx} + 2y\tan x = \sin x$. [4]

Ans: $y\sec^2 x = \sec x + C$

59. 2075 Set C Q.No. 7b Solve: $\frac{dy}{dx} = \frac{(y - x)(y + x)}{2xy}$. [4]

Ans: $x^2 + y^2 = Cx$

60. 2075 Set C Q.No. 7b OR Solve: $y + (x^2 + 1)\frac{dy}{dx} = e^{\text{arc}(\tan x)}$ [4]

Ans: $y = \frac{1}{2}e^{\tan^{-1}x} + Ce^{-\tan^{-1}x}$

61. 2074 Supp Q.No. 7b Solve: $x^2\frac{dy}{dx} + y^2 = xy$. [4]

Ans: $x = y(\log x + C)$

62. 2074 Supp Q.No. 7b OR Solve: $\cos^2 x\frac{dy}{dx} + y = 1$. [4]

Ans: $y = 1 + Ce^{-\tan x}$

63. 2074 Set A Q.No. 7b Solve: $(xy - x^2)dy = y^2dx$ [4]

Ans: $y = ce^{y/x}$

64. 2074 Set A Q.No. 7b OR Solve: $x \ln x\frac{dy}{dx} + y = 2\ln x$. [4]

Ans: $y \ln x = (\ln x)^2 + C$

65. 2074 Set B Q.No. 7b Solve: $(1 + x)\frac{dy}{dx} - xy = 1 - x$. [4]

Ans: $(1 + x)y = x + Ce^x$

66. 2073 Supp Q.No. 7b Define exact differential equation hence solve $\frac{dy}{dx} = \frac{y - x + 1}{y - x + 5}$. [4]

Ans: $x^2 + y^2 - 2xy - 2x + 10y = C$

67. 2073 Supp Q.No. 7b OR $\frac{dy}{dx} + 2y\tan x = \sin x$. [4]

Ans: $y\sec^2 x = \sec x + C$

68. 2073 Set C Q.No. 7b Solve $x\frac{dy}{dx} + 2y = x^2\log x$. [4]

Ans: $y = \frac{1}{4}x^2\log x - \frac{1}{16}x^2 + \frac{C}{x^2}$

69. 2073 Set D Q.No. 6b Solve: $\frac{dy}{dx} - 2xy = x$. [4]

Ans: $y = -\frac{1}{2} + Ce^{x^2}$

70. 2073 Set D Q.No. 6b OR Solve : $x^2\frac{dy}{dx} + y^2 = xy$ [4]

Ans: $x = y(\log x + C)$

71. 2072 Supp. Q.No. 7b Solve: $(x + 1)\frac{dy}{dx} + 2y = \frac{e^x}{x + 1}$. [4]

Ans: $y(1 + x)^2 = e^x + C$

72. 2072 Supp. Q.No. 7b OR Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$. [4]

Ans: $\cot\left(\frac{y}{x}\right) = \log x + C$

73. 2072 Set C Q.No. 7b Solve: $\frac{dy}{dx} = y\tan x - 2\sin x$ [4]

Ans: $y\cos x = \frac{1}{2}\cos 2x + C$

74. 2072 Set C Q.No. 7b OR Solve: $xy\frac{dy}{dx} - y^2 = x^2$ [4]

Ans: $y^2 = 2x^2(\log x + C)$

75. 2072 Set D Q.No. 7b Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = y^2$ in linear form hence solve it. [4]

Ans: $1 + xy\log x = Cxy$

76. 2072 Set D Q.No. 7b OR Solve: $\frac{dy}{dx} = \frac{y + 1}{x + y + 1}$ [4]

Ans: $y + 1 = Ce^{\frac{x}{y+1}}$

77. 2072 Set E Q.No. 7b Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$. [4]

Ans: $\cot\left(\frac{y}{x}\right) = \log x + C$

78. 2072 Set E Q.No. 7b OR Solve: $\sin x\frac{dy}{dx} + \cos x.y = x\sin x$. [4]

Ans: $y\sin x = -x\cos x + \sin x + C$

79. 2071 Supp. Q.No. 7b Solve $x^2\ y\ dx = (x^3 + y^3)\ dy$ [4]

Ans: $x^3 = 3y^3\log\left(\frac{y}{C}\right)$

80. 2071 Supp. Q.No. 7b OR Solve $\frac{dy}{dx} + \frac{y}{x} = \sin x^2$ [4]

Ans: $xy + \frac{1}{2}\cos x^2 = C$

81. 2071 Set C Q.No. 7 b Solve : $\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$ [4]

Ans $2x = (x - y)\log Cx$

82. 2071 Set C Q.No. 7 b OR Solve: $\sin x\frac{dy}{dx} + (\cos x)y = \sin x\cos x$ [4]

Ans: $y\sin x + \frac{1}{4}\cos 2x = C$

83. 2071 Set D Q.No. 7 b Solve: $xy\frac{dy}{dx} = x^2 + y^2$ [4]
Ans: $y^2 = 2x^2(\log x + C)$

84. 2071 Set D Q.No. 7 b OR Solve: $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$ [4]
Ans: $(1+x^2)y = \tan^{-1}x + C$

85. 2071 Old Q.No. 10 a OR Solve: $\tan x \frac{dy}{dx} + y = \sec x$ [4]
Ans: $y \sin x = x + C$

86. 2070 Supp. Q.No. 7 b Solve: $x^2dy + y(x+y)dx = 0$. [4]
Ans: $x^2y = C(y + 2x)$

87. 2070 Set C Q.No. 7 b Solve: $\cos^2 x \frac{dy}{dx} + y = 1$. [4]
Ans: $y = 1 + Ce^{-\tan x}$

88. 2070 Set C Q.No. 7 b or Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$. [4]
Ans: $\cot(y/x) = \log x + C$

89. 2070 Set D Q.No. 7 b Solve $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$. [4]
Ans: $(1+x^2)y = \frac{4}{3}x^3 + C$

90. 2070 Set D Q.No. 7 b Or Solve: $(x^2+y^2)dy = xy\,dx$. [4]
Ans: $x^2 = 2y^2 \log Cy$

91. 2070 (Old) Q.No. 11 b Or Solve : $(1+x)\frac{dy}{dx} = xy - x + 1$ [4]
Ans: $(1+x)y = x + Ce^x$

92. 2069 (Set A) Q.No. 7b Solve: $\tan x \frac{dy}{dx} + y = \sec x$ [4]
Ans: $y \sin x = x + C$

93. 2069 (Set A) Q.No. 7b or Solve: $xy\frac{dy}{dx} = x^2 + y^2$ [4]
Ans: $y^2 = 2x^2(\log x + C)$

94. 2069 (Set A) Old Q.No. 11b or Solve: $\cos^2 x \frac{dy}{dx} + y = 1$. [4]
Ans: $y = 1 + Ce^{-\tan x}$

95. 2069 (Set B) Q.No. 7b Solve: $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ [4]
Ans: $y(1+x^2) = \frac{4}{3}x^3 + C$

96. 2069 (Set B) Q.No. 7b Or Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [4]
Ans: $\sin\left(\frac{y}{x}\right) = Cx$

97. 2069 Old (Set B) Q.No. 11b Or Solve: $\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^2}$ [4]
Ans: $y = 1 + Ce^{1/x}$

98. 2068 Q.No. 11 b OR Solve: $\sin x \frac{dy}{dx} + \cos x.y = x \sin x$ [4]
Ans: $y\sin x = \sin x - x\cos x + C$

99. 2067 Q.No. 6c Solve: $xdy + (x+y)dx = 0$ [4]
Ans: $x^2 + 2xy = C$

100. 2067 Q.No. 10b OR Solve: $(1-x^2)\frac{dy}{dx} = 1 + xy$ [4]
Ans: $y\sqrt{1-x^2} = \sin^{-1}x + C$

101. 2066 C Q.No. 11 b OR Solve: $(x^2 - y^2)\frac{dy}{dx} = xy$ [4]
Ans: $x^2 + 2y^2 \log Cy = 0$

102. 2066 Q.No.11 b OR Solve the differential equation: $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ [4]
Ans: $y = \frac{1}{2}e^{\tan^{-1}x} + Ce^{-\tan^{-1}x}$

103. 2065 Q.No 11 b OR Solve: $\frac{dy}{dx} + y\cot x = x$ [4]
Ans: $y \sin x = -x\cos x + \sin x + C$

104. 2064 Q.No. 11 b OR Solve: $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ [4]
Ans: $(y-x)^2 = Cxy^2$

105. 2063 Q.No. 11 b OR Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [4]
Ans: $\sin\frac{y}{x} = Cx$

106. 2062 Q.No. 11 b OR Solve: $\frac{dy}{dx} + \frac{1}{x}y = x^2$ [4]
Ans: $xy = \frac{x^4}{4} + C$

107. 2061 Q.No. 11 b OR Solve: $(x^2 - y^2)\frac{dy}{dx} = xy$ [4]
Ans: $x^2 + 2y^2 \log Cy = 0$

108. 2060 Q.No. 11 b OR Solve: $\tan x \frac{dy}{dx} + y = \sec x$ [4]
Ans: $y \sin x = x + C$

109. 2059 Q.No. 11 b OR Solve: $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ [4]
Ans: $y^2 + x^2 = Cx$

110. 2058 Q.No. 11 b OR Solve: $\cos^2 x \frac{dy}{dx} + y = 1$ [4]
Ans: $y = 1 + Ce^{-\tan x}$

111. 2057 Q.No. 11 b OR Solve: $\tan x \frac{dy}{dx} + y = \sec x$. [4]
Ans: $y \sin x = x + C$

10. DISPERSION, CORRELATION AND REGRESSION

A. DISPERSION

FORMULAE

1. Arithmetic Mean (AM)

For individual series

$\bar{X} = \frac{\Sigma X}{n}$

$\bar{X} = A + \frac{\Sigma d}{n}$, $d = X - A$ where A, the assumed mean.

For discrete series

$\bar{X} = \frac{\Sigma fX}{N}$,

$\bar{X} = A + \frac{\Sigma fd}{N}$

$\bar{X} = A + \frac{\Sigma fd'}{N} \times h$ where $d' = \frac{X - A}{h}$

For continuous series

$\bar{X} = \frac{\Sigma fX}{N}$, $N = \Sigma f$ and X-mid value

$\bar{X} = A + \frac{\Sigma fd}{N}$, $d = X - A$

$\bar{X} = A + \frac{\Sigma fd'}{N} \times h$, where $d' = \frac{X - A}{h}$

Combined mean

$\bar{X}_{12} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$

2. Median

Individual series

$M_d = \left(\frac{n+1}{2}\right)^{th}$ item

Discrete series

$M_d = \left(\frac{N+1}{2}\right)^{th}$ item

Continuous series

To find the median class, the median lies in $\left(\frac{N}{2}\right)^{th}$ item

then

$M_d = l + \frac{\frac{N}{2} - c.f.}{f} \times h$

3. Mode

Continuous series

$M_0 = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

4. Quartiles

For individual and discrete series

$Q_i = \left[\frac{i(N+1)}{4}\right]^{th}$ item, $i = 1, 2, 3$

For continuous series

$Q_i = l + \frac{\frac{iN}{4} - c.f.}{f} \times h$; $i = 1, 2, 3$

5. Empirical relation

Mode = 3 Median – 2 Mean

6. Measures of Dispersion

i. Range = L – S

Coefficient of range = $\frac{L - S}{L + S}$

ii. Inter quartile range = $Q_3 - Q_1$

Quartile deviation or semi-inter quartile = $\frac{Q_3 - Q_1}{2}$

Coefficient of Quartile Deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

iii. Mean Deviation (MD)

Calculate the mean deviation from 'M' as

$MD = \frac{1}{n}\sum_{i=1}^{n}|x_i - M| = \frac{1}{n}\sum_{i=1}^{n}|d_i|$

When we have grouped data with frequency, mean deviation is calculated using the formula

$MD = \frac{1}{N}\sum_{i=1}^{n} f_i(|x_i - M|) = \frac{1}{N}\sum_{i=1}^{n} f_i|d_i|$

Coefficient of MD from mean/median/mode

$= \frac{\text{Mean Deviation from Mean/ Median/ Mode}}{\text{Mean/ Median/ Mode}}$

iv. Root Mean Square Deviation (S) = $\sqrt{\frac{1}{N}\Sigma f_i (x_i - A)^2}$

v. Standard Deviation

Individual Series

$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ Direct method

$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$ Shortcut method

$\sigma = \sqrt{\frac{\Sigma d'^2}{n} - \left(\frac{\Sigma d'}{n}\right)^2} \times h$ Step deviation method

Discrete/ Continuous Series

$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}$ Direct method

$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$ Shortcut method

$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h$ Step deviation method

vi. Variance

$\sigma^2 = (SD)^2$

vii. Variance of the combined series

$(\sigma_{A+B})^2 = \frac{1}{n_A + n_B}[n_A(\sigma_A^2 + d_A^2) + n_B(\sigma_B^2 + d_B^2)]$

viii. Coefficient of Variation (C.V.) = $\frac{\sigma}{\bar{x}} \times 100$

2 Marks Questions

1. **2077 Set H Q.No. 2b** Calculate the semi-inter Quartile Range of 2, 5, 9, 10, 10, 9, 4. [2]

Ans: 3

2. **2076 Set B Q.No. 4b** Calculate the mean deviation from mean of the data: 3, 5, 9, 11, 7, 6 [2]

Ans: 2.17

3. **2076 Set C Q.No. 4b** Which of the following distribution is consistent from the information given below. [2]

	Distribution 'A'	Distribution 'B'
A.M.	95	90
S.d.	10	9

Ans: Distribution B

4. **2075 Set B Q.No. 4a** Find the mean deviation from mean of the data: 6, 8, 10, 13, 5. [2]

Ans: 2.48

5. **2075 Set C Q.No. 4b** If total items (n) = 10, sum of items (Σx) = 120 and the sum of square of items (Σx^2) = 1530, find the standard deviation and coefficient of variation. [2]

Ans: S.D. = 3, C.V. = 25%

6. **2074 Supp Q.No. 4b** Following are the information about the marks of two students A and B.

	A	B
Average marks	84	92
Variance of marks	16	25

Examine who has got the uniform mark. [2]

Ans: A

7. **2073 Supp Q.No. 4b** In the distribution of two sets of data, which of the distribution is consistent? [2]

	Distribution X	Distribution Y
AM	100	90
S.D.	10	18

Ans: Distribution X

2072 Set C Q.No. 4b The information about the daily temperature of two cities X and Y are as follows:

	X	Y
Average temp. (°F)	84	92
Variance of temp.	16	25

Determine which city has greater consistency in climate. [2]
Ans: City X

2072 Set D Q.No. 4b In the distribution of data 20, 25, 30, 35, 32, 43, find standard deviation. [2]
Ans: 7.4

2071 Supp. Q.No. 4a The mean of two samples of size 50 and 100 are 54.1 and 50.3 respectively and the standard deviations are 8 and 7 respectively. Obtain the mean and the standard deviation of the sample of size 150 obtained by combining the two samples. **Ans: 51.57; 7.56** [2]

2071 Set D Q.No. 4 b If $n = 10$, $\Sigma x = 120$, $\Sigma x^2 = 1530$, find the standard deviation and the coefficient of variation. [2]
S.D. = 3, C.V. = 25%

2071 Old Q.No. 4 b Find the mean deviation of the data 10, 5, 6, 12, 7 from median. [2]
Ans: 2.2

2070 Supp. Q.No. 4 b Calculate the quartile deviation from the data: 15, 7, 25, 12, 4, 22, 19, 10 [2]
Ans: 6.75

2070 (Old) Q.No. 3 c Find the coefficient of mean deviation from median of the data 5, 4, 2, 8 and 6. [2]
Ans: 0.32

2066 C Q.No. 4 b The coefficient of variation and mean of a certain frequency distribution are 50.2% and 22.8 respectively. Find the standard deviation. [2]
Ans: 11.45

2066 Q.No. 4 b Find the standard deviation from the following data: [2]

x	10	11	12	13	14
f	3	12	18	12	2

Ans: 0.93

2064 Q.No. 4 b The information about the wages distribution of the firms A and B are given below:

	Firm A	Firm B
No. of workers	500	600
Average monthly wages	Rs. 586	Rs. 575
Variance of wages distribution	81	100

In which firm is the wages distribution uniform? [2]
Ans: Firm A

2062 Q.No. 4 b Find the standard deviation of the following data: 10, 15, 20, 25, 30, 35, 40 [2]
Ans: 10

2061 Q.No. 4 b Find the mean deviation from mean of the following data: 6, 8, 10, 13 and 5. [2]
Ans: 2.48

2061 Q.No. 12 a Find out the mean and Standard Deviation from the following data: [2]

Variable	5–10	10–15	15–20	20–25	25–30	30–35
Frequency	2	9	29	54	11	5

Ans: 21.05; 4.87

21. **2060 Q.No. 4 b** Find the mean deviation from median of the numbers 5, 7, 10, 12 and 6. [2]
Ans: 2.2

22. **2059 Q.No. 4 c** The coefficient of variation and mean of a certain frequency distribution are 50.2% and 22.8 respectively. Find the s.d. [2]
Ans: 11.4456

23. **2058 Q.No. 4 b** Find the standard deviation of the following data: 100, 150, 200, 250, 300 [2]
Ans: 70.7

4 Marks Questions

24. **2074 Set B Q.No. 8a** Following are the marks obtained by the two students in 6 tests.

A	56	72	48	69	64	81
B	63	74	45	57	82	63

Which of the student will get performance award for the consistency in tests? [4]
Ans: A

25. **2069 (Set A) Q.No. 8a** Determine the standard deviation and the coefficient of variation from the following distribution. [4]

Profit (in Rs.)	0–10	10–20	20–30	30–40	40–50
No. of shops	8	13	16	8	5

Ans: Rs. 11.88 and 52.11%

26. **2069 (Set A) Old Q.No. 12a** Find the mean and the standard deviation from the following data. [4]

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	5	8	15	16	6

Ans: 27 and 11.49

27. **2069 Old (Set B) Q.No. 12b** Find out the mean and standard deviation from the following data: [4]

x	10	11	12	13	14
f	3	12	18	12	2

Ans: 11.96; 0.93

28. **2068 Q.No. 12 a** Weights of a group of individuals are given below. Find out the mean and the standard deviation. [4]

Weight (in kg)	0-10	10-20	20-30	30-40	40-50
Frequency	12	33	30	15	10

Ans: 22.8, 11.45

29. **2067 Q.No. 12b** Define standard deviation. Also prove that the root mean square deviation is not less than the standard deviation. [4]

30. **2060 Q.No. 12 a** Find the mean and S.D. from the following table: [4]

Wages (Rs.)	10–20	10–30	10–40	10–50	10–60
No. of workers	15	33	63	83	100

Ans: Mean = 35.6 and S.D. = 12.87

31. **2060 Q.No. 12 b** Prove that in a discrete distribution the standard deviation is not less than the mean deviation from the mean. [4]

B. SKEWNESS

FORMULAE

Absolute measure of skewness

1. Based on median, S_k = Mean − Median
2. Based on mode, S_k = Mean − Mode
3. Based on quartiles, $S_k = (Q_3 - Q_2) - (Q_2 - Q_1)$

Relative measure of skewness

4. Based on Karl Pearson's coefficient of skewness,

$$S_K(P) = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

Also,

$$S_K(P) = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

[∵ Mode = 3 Median − 2 Mean]

Karl Pearson's coefficient of skewness lies between −3 and 3.

5. Based on Bowley's coefficient of skewness,

$$S_K(B) = \frac{(Q_3 + Q_1) - 2Q_2}{Q_3 - Q_1}$$

Bowley's coefficient of skewness lies between −1 and 1.

Interpretation:

1. If $S_K = 0$, then frequency distribution is symmetrical.
2. If $S_K > 0$, then distribution is positive or right skewed.
3. If $S_K > 0$, then distribution is negative or left skewed.

2 Marks Questions

1. **2075 Set A Q.No. 4b** For a group of 50 items, $\Sigma x^2 = 600$, $\Sigma x = 150$ and $M_0 = 1.75$, find the pearsonian coefficient of skewness. [2]
   Ans: 0.72
2. **2075 Set B Q.No. 4b** Find the Pearson's coefficient of skewness when $\Sigma x = 735$, $\Sigma x^2 = 28730$, mode = 35.25, n = 20. [2]
   Ans: 0.16
3. **2074 Set A Q.No. 4b** Find Skewness and C.V. if mean, median and S.D. are respectively 56.80, 59.50 and 12.40. [2]
   Ans: S_K = −0.653, C.V. = 21.83%
4. **2074 Set B Q.No. 4b** The C.V, S.D. and mode of a distribution are 5%, 2 and 39 respectively. Calculate the Karl Pearson's coefficient of Skewness of the distribution. [2]
   Ans: 0.5
5. **2073 Set C Q.No. 4b** A frequency distribution gives the following results. C.V. = 5%, Mean = 40 and Mode = 39. Calculate Karl Pearson's coefficient of skewness of the distribution. [2]
   Ans: 0.5
6. **2072 Set E Q.No. 4b** For a group of 50 items; circle $\Sigma x^2 = 600$, $\Sigma x = 150$ and $m_0 = 1.75$, find the Pearsonian coefficient of skewness. [2]
   Ans: 0.72
7. **2071 Supp. Q.No. 4c** In a distribution, the difference of the two quartiles is 20 and their sum is 70 and the median is 36. Find the coefficient of skewness. [2]
   Ans: −0.1
8. **2067 Q.No. 4b** In a frequency distribution of a set of data C.V. = 5%, σ = 2 and karl Pearson coefficient of skewness = 0.5; find the mean of the data. [2]
   Ans: 40
9. **2063 Q.No. 4 b** Consider the following distribution.

	Distribution a	Distribution b
Arithmetic mean:	100	90
Median:	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the skewness? [2]
Ans: Same

4 Marks Questions

10. **2073 Set D Q.No. 8a** If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$, find Karl Pearson's coefficient of skewness. [4]
    Ans: -0.22
11. **2072 Supp. Q.No. 8a** Calculate the coefficient of skewness based on mean, mode and the standard deviation from the following data: [4]

Wages (in Rs.)	100	110	120	130	140
No. of person	2	6	10	8	4

Ans: 0.18

12. **2071 Set C Q.No. 8 a** Calculate the coefficient of Skewness based on mean, mode and standard deviation from the following data. [4]

Wages (in Rs.)	100	110	120	130	140
No. of persons	2	6	10	8	4

Ans: 0.18

13. **2070 Supp. Q.No. 8 a** Find Karl Pearson's coefficient of skewness from the given data [4]

Income	10	12	14	16	20
Frequency	5	8	15	7	5

Ans: 0.071

14. **2070 Set C Q.No. 8 a** If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$, find the skewness based on mean, mode and standard deviation. [4]
    Ans: −0.22
15. **2070 Set D Q.No. 8 a** Consider the following distribution.

	Distribution A	Distribution B
Arithmetic mean	100	90
Median	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness? [4]
Ans: C.V (A) = 10%, C.V.(B) = 11.11%, $S_K(A) = 3$, $S_K(B) = 3$

16. **2069 (Set A) Old Q.No. 12 b** For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24270$ and mode = 43.7, find the Pearsonian coefficient of Skewness. [4]
    Ans: 0.077
17. **2069 (Set B) Q.No. 8a** If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$, find the skewness based on mean, mode and standard deviation. [4]
    Ans: −0.22

2068 Q.No. 12 b Consider the following distribution:

	Distribution A	Distribution B
Arithmetic mean	100	90
Median	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skweness? [4]

2066 Q.No.12 b Calculate the coefficient of skewness from the following frequency distribution: [4]

Investment	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	12	18	20	15	10	3	2

Ans: 0.227

2065 Q.No 12 a Calculate Karl Pearson's coefficient of skewness of the data: [4]

Marks	above 0	above 10	above 20	above 30	above 40
Frequency	150	140	100	80	80

Ans: –0.89

2064 Q.No. 12 a For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24{,}270$ and mode = 43.7, find the Pearson's coefficient of skewness. [4]

Ans: 0.077

2061 Q.No. 12 b The median, mode and coefficient of skewness for a certain distribution are respectively 17.4, 15.3 and 0.35. Calculate mean and C.V. [4]

Ans: 18.45 and 48.78%

C. CORRELATION

FORMULAE

1. Karl Pearson's Coefficient of Correlation

i. $r = \frac{Cov(X, Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$

ii. $r = = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$

iii. $r = \frac{\Sigma xy}{n\sigma_x\sigma_y}$

iv. $r = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$

v. $r = \frac{n\Sigma uv - \Sigma u\Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$

2. The value of r lies between –1 and +1.

2 Marks Questions

1. **2076 GIE Set A Q.No. 4b** For a group of 50 items; $\Sigma x^2 = 600$, $\Sigma x = 150$ and mode (Mo) = 1.75, find Pearson's coefficient of skewness. [2]

Ans: 0.72

2. **2076 GIE Set B Q.No. 4b** Find the correlation coefficient between two variables x and y under the following conditions: [2]

n = 10, $\Sigma x = 60$, $\Sigma y = 60$, $\Sigma x^2 = 400$, $\Sigma y^2 = 580$ and $\Sigma xy = 415$

Ans: 0.59

3. **2075 GIE Q.No. 4b** Calculate the correlation coefficient between the two variables with the information n = 10, $\bar{x} = 5$, $\bar{y} = 3$, $\Sigma x^2 = 290$, $\Sigma y^2 = 300$, $\Sigma xy = 115$. [2]

Ans: –0.38

4. **2073 Set D Q.No. 4b** If $\Sigma(x - \bar{x})^2 = 40$, $\Sigma(y - \bar{y})^2 = 63$ and $\Sigma(x - \bar{x})(y - \bar{y}) = 35$, find the correlation coefficient between the two variables x and y. [2]

Ans: 0.697

5. **2072 Supp. Q.No. 4b** If n = 10, $\Sigma X = 18$, $\Sigma Y = 25$, $\Sigma X^2 = 90$, $\Sigma Y^2 = 120$ and $\Sigma XY = 65$, find the correlation coefficient between two variables. [2]

Ans: 0.35

6. **2071 Set C Q.No. 4 b** If n = 15, $\sigma_x = 3.2$, $\sigma_y = 3.4$ and $\Sigma(X - \bar{X})(Y - \bar{Y}) = 122$, find the correlation coefficient between the two variables. [2]

Ans: 0.75

7. **2070 Set C Q.No. 4 b** If $\Sigma(X - \bar{X})^2 = 40$, $\Sigma(Y - \bar{Y})^2 = 63$ and $\Sigma(X - \bar{X})(Y - \bar{Y}) = 35$, find the correlation coefficient between the two variables. [2]

Ans: 0.697

8. **2070 Set D Q.No. 4 b** If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. [2]

Ans: 0.59

9. **2069 (Set A) Old Q.No. 4b** If the covariance between the two variables x and y is 18, and the variances of x and y are 16 and 81 respectively, find the coefficient of correlation between them. [2]

Ans: 0.5

10. **2068 Q.No. 4b** Calculate the correlation coefficient between two variables from the following data:

$\Sigma x^2 = 114$, $\Sigma y^2 = 422$ and $\Sigma xy = 174$ [2]

Ans: 0.79

11. **2065 Q.No 4 b** If the covariance between the variable x and y is 18 and the variances of x and y are 16 and 81 respectively, find the coefficient of correlation between them. [2]

Ans: 0.5

12. **2057 Q.No. 4 b** Calculate r_{xy} if $\Sigma x^2 = 114$, $\Sigma y^2 = 442$; $\Sigma xy = 174$. [2]

Ans: 0.793

4 Marks Questions

13. **2077 Set G Q.No. 5** Calculate Karl Pearson's coefficient of correlation from the following data. [4]

X	12	9	8	10	13	7
Y	14	8	6	9	12	3

Ans: 0.946

14. **2076 GIE Set B Q.No. 8a** The following distribution gives the weights of 40 person. Find the Pearson's coefficient of skewness for the following distribution: [4]

Weight (in kg)	40-50	50-60	60-70	70-80	80-90
No. of person	5	10	15	8	2

Ans: – 0.11

15. **2076 Set B Q.No. 8a** Find correlation coefficient of the following two sets of data A and B: [4]

A	56	72	48	64	81	69
B	63	74	45	82	66	57

Ans: 0.51

16. **2076 Set C Q.No. 8a** Calculate the coefficient of correlation between the price and sales. [4]

Price	25	21	28	26	20	30
Sales	60	54	66	68	70	50

Ans: –0.32

17. **2075 Set A Q.No. 8a** Find the correlation coefficient between the two variables x and y from the following data. [4]

x	5	7	1	3	4
y	2	3	4	5	6

Ans: –0.42

18. **2074 Set A Q.No. 8a** From the following table calculate the correlation coefficient by Karl Pearson's method. [4]

X	10	12	20	?	16	14
Y	9	12	15	18	14	16

AM of X = 15.

Ans: 0.78

19. **2073 Supp Q.No. 8a** Find the karl Pearson's Coefficient of correlation from the following distribution. [4]

X	10	11	18	15	25	20	14	22
Y	9	14	15	16	22	20	18	24

Ans: 0.87

20. **2073 Set C Q.No. 8a** Calculate Karl Pearson's coefficient of correlation from the following data using product moment formula. [4]

X	12	9	8	10	11
Y	12	8	6	9	10

Ans: 0.99

21. **2072 Set D Q.No. 8a** Define correlation. Find Karl Pearson's coefficient of correlation of the marks of the following distribution. [4]

X	20	30	40	50	60
Y	50	46	30	24	8

Ans: –0.34

22. **2071 Supp. Q.No. 8a** Using the product moment formula, calculate correlation coefficient for the following series of ages of husbands (X) and wives (Y). [4]

X	41	44	45	48	40	42	44
Y	22	24	25	27	21	22	23

Ans: 0.976

23. **2071 Set D Q.No. 8 a** Calculate Karl Pearson's correlation coefficient between the two variables height (in cms) and weight (in kg) from the data given below: [4]

Height	160	162	165	161	163
Weight	63	62	64	60	61

Ans: 0.41

24. **2071 Old Q.No. 12 a** Calculate the coefficient of correlation between x and y series from the following data

	Series x	Series y
No. of observations	15	15
s.d	3.01	3.03

$\Sigma(x - \bar{x})(y - \bar{y}) = 122$ [4]

Ans: 0.89

25. **2070 (Old) Q.No. 12 b** Compute correlation and interpret about the ages of husband and wife given below: [4]

Age of husband	23	22	20	24	23	26	27	28	30	20
Age fo wife	20	18	23	20	21	21	22	24	25	26

Ans: 0.198

26. **2066 C Q.No. 12 a** Calculate the Karl Pearson's coefficient of correlation between the age and blood pressure of 8 patients: [4]

Age	23	48	43	68	70	28	35	26
Blood Pressure	115	127	123	140	145	118	121	120

Ans: 0.98

27. **2063 Q.No. 12 b** From the following table, calculate the coefficient of correlation by Karl Pearson's method.

X	6	2	10	4	8
Y	9	11	–	8	7

Arithmetic means of X and Y series are 6 and 8 respectively. [4]

Ans: – 0.92

28. **2059 Q.No. 12 b** Calculate Karl Pearson's coefficient of correlation from the following data: [4]

X	12	9	8	10	11	13	7
y	14	8	6	9	11	12	3

Ans: 0.95

D. REGRESSION

FORMULAE

1. Regression equation of Y on X

$Y - \bar{Y} = b_{YX}(X - \bar{X})$,

where, $b_{YX} = \dfrac{n\Sigma XY - \Sigma X\,\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$

2. Regression equation of X on Y

$X - \bar{X} = b_{XY}(Y - \bar{Y})$

where, $b_{XY} = \dfrac{n\Sigma XY - \Sigma X\,\Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$

3. Relation between r and b

The correlation coefficient r is the geometric mean of two regression coefficients

$r = \sqrt{b_{YX} b_{XY}}$.

Also, the regression coefficients can be expressed in terms of r and σ as,

$b_{XY} = r\dfrac{\sigma_X}{\sigma_Y}$ and $b_{YX} = r\dfrac{\sigma_Y}{\sigma_X}$.

2 Marks Questions

1. **2069 (Set A) Q.No. 4b** Find the regression equation of y on x when: $\Sigma x = 15$, $\Sigma y = 25$, $\Sigma x^2 = 55$, $\Sigma y^2 = 140$, $\Sigma xy = 78$, $n = 5$. [2]

Ans: $y = 0.3x + 4.1$

2068 (Set B) Q.No. 4b The regression coefficient of y on x is 0.32. If the arithmetic means of x and y series are 42 and 36 respectively, find the regression equation of y on x. [2]

Ans: y = 0.32x + 22.56

4 Marks Questions

2076 GIE Set A Q.No. 8a Find the regression equation y on x for the following data: [4]

x	5	7	9	10	11
y	1	2	3	4	5

Also estimate the value of y when x = 6

Ans: y = – 2.46 + 0.65x, 1.44

2075 GIE Q.No. 8a Find the regression equation of X on Y from the following data. [4]

X	5	9	13	17	21
Y	3	8	13	18	23

Ans: X = 0.8Y + 2.6

2075 Set C Q.No. 8a The regression coefficients, b_{XY} = 1.5, b_{YX} = 0.65 and arithmetic means $(\bar{X})$ = 36, $(\bar{Y})$ = 52. Find the regression equations X on Y and Y on X. Also, find the estimated value of Y when X = 60. [4]

Ans: Y = 0.65X + 28.6, X = 1.5Y – 42, 67.6

2074 Supp Q.No. 8a From the following pair of regression equations, find the correlation coefficient between the two variables x and y. [4]

4x – 5y + 33 = 0 and 20x – 9y – 107 = 0.

Ans: 0.6

2072 Set C Q.No. 8a Define regression and lines of regression. Find the correlation coefficients between the two variables when b_{xy} = 1.8 and b_{yx} = 0.35 [4]

Ans0.79

2072 Set E Q.No. 8a The regression coefficients of x on y and y on x are 0.84 and 0.32 respectively. If the arithmetic means of x and y series are 42 and 26 respectively, find two equations of lines of regression. [4]

Ans: y = 0.32x + 12.56, x= 0.84 y + 20.16

2071 Supp. Q.No. 8a OR From the following data, compute the line of regression for estimating age on weight and estimate the most probable age on a weight of 37 Kg. [4]

Age (X)	5	15	30	45	50	60
Weight (Y)	10	35	50	65	55	45

Ans: x = 0.85 y – 2.66, 28.79

2070 Supp. Q.No. 8 a OR From the following data, compute the line of regression for estimating age on weight and estimate the most probable age of a weight of 37 kg: [4]

Age (x)	5	15	30	45	50	60
Weight (y)	10	35	50	65	55	45

Ans: x = 0.85 y – 2.66, 28.79

8 Marks Questions

2075 Set B Q.No. 11 The equations of two regression lines are 3X + 4Y = 65, 3X + Y = 32

Find,

i. the mean of X and the mean of Y.
ii. the regression coefficients.
iii. the correlation coefficients between X and Y.
iv. the ratio of standard deviations of X and Y. [6]

Ans: (I) 7, 11 (II) $-\frac{3}{4}$, $-\frac{1}{3}$ (III) $-\frac{1}{2}$ (IV) 2:3

11. PROBABILITY

A. PROBABILITY (SIMPLE CASES)

FORMULAE

1. $P(E) = p = \frac{\text{Favourable number of cases}}{\text{Total number of cases}} = \frac{m}{n}$
2. Properties of Probability
   If p denotes the probability of happening of an event, then the following properties of the probability are always true.
   i. $p + q = 1$
   ii. p(certain event) = 1
   iii. p(impossible event) = 0
   iv. $0 \leq p \leq 1$.
3. Addition Theorem
   $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
4. Multiplication Theorem
   $P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)$,
   where A and B are independent events.
5. Conditional Probability
   $P(A/B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$
   $P(B/A) = \frac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$
6. $P(\bar{A}) = 1 - P(A)$

2 Marks Questions

1. **2077 Set I Q.No. 2b** Two dice are rolled simultaneously. Determine the probability of turning up the number whose sum is less than six. [2]

   Ans: $\frac{5}{18}$

2. **2076 GIE Set A Q.No. 4c** A dice is thrown once. Determine the probability of getting a number greater than or equal to 3. [2]

   Ans: $\frac{2}{3}$

3. **2076 GIE Set B Q.No. 4c** If a dice is thrown 16 times, find the mean and the standard deviation for the binomial distribution of even numbers. [2]

   Ans: 8, 2

4. **2076 Set B Q.No. 4c** The chance that A can solve the problem is $\frac{1}{3}$ and the chance that B can solve is $\frac{2}{3}$. Find the probability that the problem is solved by A and B. [2]

   Ans: $\frac{2}{9}$

5. **2076 Set C Q.No. 4c** If three coins are tossed simultaneously, what is the probability that it turns two heads? [2]

   Ans: $\frac{3}{8}$

6. **2075 GIE Q.No. 4c** Two dice are rolled simultaneously. Determine the probability turning up the number whose sum is ten. [2]

   Ans: $\frac{1}{12}$

7. **2075 Set B Q.No. 4c** An urn contains 4 white, 8 black, 6 red and 2 green marbles. If three balls are drawn at random, find the probability of getting 2 red and 1 green marbles. [2]

Ans: $\frac{1}{38}$

8. **2075 Set C Q.No. 4c** If A and B are two independent events with $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{5}$, find $P(A \cup B)$. [2]

Ans: $\frac{4}{5}$

9. **2074 Supp Q.No. 4c** If A and B are two independent events with $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5}$, find $P(A \cup B)$. [2]

Ans: $\frac{13}{15}$

10. **2074 Set B Q.No. 4c** A class consists of 30 boys and 20 girls. If two students are chosen at random what is the probability that one is boy and another is girl? [2]

Ans: $\frac{24}{49}$

11. **2073 Supp Q.No. 4c** Three coins are tossed simultaneously. Find the sample space. Find the probability that all are heads. [2]

Ans: $\frac{1}{8}$

12. **2073 Set C Q.No. 4c** The chance that A can solve the problem is $\frac{2}{3}$ and the chance that B can solve the problem is $\frac{1}{3}$. Find the probability that the problem is solved by A and B. [2]

Ans: $\frac{2}{9}$

13. **2072 Supp. Q.No. 4c** An urn contains 4 white and 8 red balls. If two balls are drawn at random, find the probability of getting one of each colour. [2]

Ans: 16/33

14. **2072 Set C Q.No. 4c** In rolling a pair of dice, determine the probability of obtaining a sum of 10. [2]

Ans: $\frac{1}{12}$

15. **2072 Set D Q.No. 4c** In a draw of a card from well shuffled deck of 52 cards what is the probability that it is a king or a queen? [2]

Ans: $\frac{2}{13}$

16. **2072 Set E Q.No. 4c** Two dice are rolled once. What is the probability of getting a total of 8 or 7? [2]

Ans: $\frac{11}{36}$

17. **2071 Supp. Q.No. 4b** Two dice are thrown together. Find the probability of getting both odd digits. [2]

Ans: $\frac{1}{4}$

18. **2071 Set C Q.No. 4 c** The chance that A can solve the problem is 3/5 and the chance that B can solve the problem is 2/3. Find the probability that the problem is solved. [2]

Ans: $\frac{13}{15}$

19. **2071 Set D Q.No. 4 c** Two coins are tossed simultaneously. Find the sample space. Find the probability that both are heads. [2]

Ans: {HH, HT, TH, TT}, $\frac{1}{4}$

20. **2071 Old Q.No. 4 c** A coin is tossed successively three times. Determine the probability of getting 2 heads and one tail. [2]

Ans: $\frac{3}{8}$

21. **2070 Supp. Q.No. 4 c** A bag contains 24 balls numbered from 1 to 24. One ball is drawn at random. What is the probability that it is a multiple of 4 and 6? [2]

Ans: $\frac{1}{12}$

22. **2070 Set C Q.No. 4 c** A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that one is boy and one girl? [2]

Ans: $\frac{16}{33}$

23. **2070 Set D Q.No. 4 c** A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is a King or a Diamond? [2]

Ans: 4/13

24. **2070 (Old) Q.No. 4 c** If three coins are tossed simultaneously, find the probability of turning all head. [2]

Ans: $\frac{1}{8}$

25. **2069 (Set A) Q.No. 4c** From 20 tickets marked from 1 to 20, one is drawn at random. Find the probability that it is a multiple of 4 or 5. [2]

Ans: $\frac{2}{5}$

26. **2069 (Set A) Old Q.No. 4c** What is the probability that an English alphabet selected at random is (i) a vowel (ii) a consonant? [2]

Ans: (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$

27. **2069 Old (Set B) Q.No. 3c** A card is drawn at random from a well shuffled deck of 52 cards. What is the probability that it is a spade? [2]

Ans: $\frac{1}{4}$

28. **2068 Q.No. 4c** A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability that is a red 8, a red 9 or a red 10? [2]

Ans: $\frac{3}{26}$

29. **2067 Q.No. 4c** Define mutually exclusive events and dependent cases with example while performing an experiment. [2]

30. **2066 Q.No. 4 c** The chance that A can solve a problem is $\frac{1}{4}$, the chance that B can solve it is $\frac{2}{3}$. Find the probability that the problems will be solved if both of them try. [2]

Ans: 3/4

31. **2068 Q.No. 4c** A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability that is a red 8, a red 9 or a red 10? [2]
Ans: $\frac{3}{26}$

32. **2067 Q.No. 4c** Define mutually exclusive events and dependent cases with example while performing an experiment. [2]

33. **2066 Q.No. 4 c** The chance that A can solve a problem is $\frac{1}{4}$ the chance that B can solve it is $\frac{2}{3}$. Find the probability that the problems will be solved if both of them try. [2]
Ans: $\frac{3}{4}$

34. **2065 Q.No 4 c** Given P (A) = 0.4, P (A∪B) = 0.56, P (B) = 0.3. Are A & B independent? [2]
Ans: Not independent

35. **2064 Q.No. 4 c** A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability of drawing (i) a white ball (ii) not a black ball. [2]
Ans: (i) $\frac{7}{20}$ (ii) $\frac{4}{5}$

36. **2063 Q.No. 4 c** The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that the problem will be solved if they both try. [2]
Ans: $\frac{3}{4}$

37. **2062 Q.No. 4 c** What is the probability that an English alphabet selected at random is (i) a vowel (ii) a consonant? [2]
Ans: (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$

38. **2060 Q.No. 4 c** Two dice are thrown. Determine the probability of getting a sum ≤ 5. [2]
Ans: $\frac{5}{18}$

39. **2059 Q.No. 3 c** Two letters are selected at random from the word "examination". Find the probability that both of them are same letters. [2]
Ans: $\frac{3}{31}$

40. **2058 Q.No. 4 c** If A and B are two independent events with P(A) = $\frac{2}{3}$ and P(B) = $\frac{3}{5}$, find P(A∪B). [2]
Ans: $\frac{13}{15}$

41. **2057 Q.No. 4 c** A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of being it (i) a red card (ii) a heart. [2]
Ans: (i) $\frac{1}{2}$ (ii) $\frac{1}{4}$

4 Marks Questions

42. **2076 GIE Set B Q.No. 8b** The chance that A can solve the problem is 3/5 and the chance that B can solve the problem is 2/3 find the probability that? [4]
i. the problem is solved
ii. none of them can solve the problem.
Ans: (i) $\frac{13}{15}$ (ii) $\frac{2}{15}$

43. **2076 Set C Q.No. 8b** The chance that A can solve the problem is 3/5 and the chance that B can solve the problem is 2/3, find the probability that:
i. the problem is solved
ii. none of them can solve the problem [4]
Ans: (i) $\frac{13}{15}$ (ii) $\frac{2}{15}$

44. **2075 Set A Q.No. 8b** A lot contains 10 items of which 3 are defective. Three items are chosen from the lot at random one after another without replacement. Find the probability that (i) all three are defective (ii) only first one is defective. [4]
Ans: (i) $\frac{1}{120}$, (ii) $\frac{7}{40}$

45. **2073 Set D Q.No. 8b** A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that (i) both are boys (ii) one boy and one girl. [4]
Ans: (i) $\frac{59}{165}$, (ii) $\frac{16}{33}$

46. **2071 Old Q.No. 12 b** State and prove the theorem of compound probability. [4]

47. **2070 (Old) Q.No. 8 a** State and prove theorem on compound probability. [4]

48. **2069 (Set A) Old Q.No. 8a** Suppose 4 cards are drawn at random from a well-shuffled deck of 52 cards.
i. What is the probability that all 4 are spade?
ii. What is the probability that all 4 are black? [4]
Ans: (i) $\frac{11}{4165}$ (ii) $\frac{46}{833}$

49. **2069 (Set B) Q.No. 8b** A bag contains 5 red and 6 white balls. Two balls are drawn at random. Find the probability that (i) both are red (ii) both are of the same colour. [4]
Ans: (i) $\frac{2}{11}$, (ii) $\frac{5}{11}$

50. **2069 Old (Set B) Q.No. 8a** State and prove the theorem of "Compound probability". [4]

51. **2068 Q.No. 8b** The chance that A can solve a certain problem is $\frac{1}{4}$. The chance that B can solve it is $\frac{2}{3}$, find the chance that the problem will be solved if they both try. [4]
Ans: $\frac{3}{4}$

52. **2067 Q.No. 8a** A class consists of 40 boys and 60 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl? [4]
Ans: (a) $\frac{26}{165}$ (b) $\frac{59}{165}$ (c) $\frac{16}{33}$

53. **2066 C Q.No. 8 a** State and prove the theorem of total probability. [4]

54. **2066 C Q.No. 8 a OR** If A, B, C are three mutually exclusive events with $\frac{1}{3}$ P(A) = $\frac{2}{3}$ P(B) = $\frac{1}{6}$ P(C). Find P(A), P(B) and P(C). [4]
Ans: P (A) = $\frac{2}{7}$, P (B) = $\frac{1}{7}$, P(C) = $\frac{4}{7}$

55. 2066 Q.No. 8 a OR Five men in a group of 20 are graduates. If three men are chosen out of 20 at random, what is the probability of at least one being graduates? [4]
Ans: $\frac{137}{228}$

56. 2068 Q.No. 8b The chance that A can solve a certain problem is $\frac{1}{4}$. The chance that B can solve it is $\frac{2}{3}$, find the chance that the problem will be solved if they both try. [4]
Ans: $\frac{3}{4}$

57. 2067 Q.No. 8a A class consists of 40 boys and 60 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl? [4]
Ans: (a) $\frac{26}{165}$ (b) $\frac{59}{165}$ (c) $\frac{16}{33}$

58. 2066 C Q.No. 8 a State and prove the theorem of total probability. [4]

59. 2066 C Q.No. 8 a OR If A, B, C are three mutually exclusive events with $\frac{1}{3}P(A) = \frac{2}{3}P(B) = \frac{1}{6}P(C)$. Find P(A), P(B) and P(C). [4]
Ans: $P(A) = \frac{2}{7}$, $P(B) = \frac{1}{7}$, $P(C) = \frac{4}{7}$

60. 2066 Q.No. 8 a OR Five men in a group of 20 are graduates. If three men are chosen out of 20 at random, what is the probability of at least one being graduates? [4]
Ans: $\frac{137}{228}$

61. 2065 Q.No 8 a State and prove the theorem of compound probability. [4]

62. 2064 Q.No. 8 a If P(A) and P(B) are the probabilities of the happening of the events A and B respectively, prove that: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $P(A \cup B)$ and $P(A \cap B)$ have the usual meanings. What will be the form of the above formula if A and B are independent events? [4]

63. 2063 Q.No. 8 a If P(A) and P(B) be the probabilities of the independent events A and B respectively, prove that: $P(A \cap B) = P(A).P(B)$ where $P(A \cap B)$ has the usual meaning. [4]

64. 2062 Q.No. 8 a State and prove "The Theorem of Total Probability". [4]

65. 2062 Q.No. 8 a OR A lot contains 10 items of which 3 are defective. Three items are chosen from the lot at random one after another without replacement. Find the probability that:
i. All three are defective.
ii. None of them are defective. [4]
Ans: (i) $\frac{1}{120}$ (ii) $\frac{119}{120}$

66. 2061 Q.No. 8 a State and prove the "Theorem of Compound Probability". [4]

67. 2061 Q.No. 8 a OR A class consists of 60 boys and 40 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl? [4]
Ans: (a) $\frac{59}{165}$, (b) $\frac{26}{165}$, (c) $\frac{16}{33}$

68. 2060 Q.No. 8 a State and prove the "Theorem of total probability." [4]

69. 2059 Q.No. 8 a If A, B, C are three mutually exclusive events with $\frac{1}{3}P(A) = \frac{2}{3}P(B) = \frac{1}{6}P(C)$, find P(A); P(B); and P(C). [4]
Ans: $\frac{2}{7}, \frac{1}{7}, \frac{4}{7}$

70. 2058 Q.No. 8 a State and prove the "Theorem of total probability" [4]

71. 2057 Q.No. 8 b State and prove the "Theorem of Total probability" [4]

B. BINOMIAL DISTRIBUTION

FORMULAE

1. Total probability for r success in n independent trials $P(r) = {}^nC_r p^r q^{n-r}$
2. Mean of the distribution is given by np.
3. Variance of the distribution is given by npq.
   Then, S.D. $= \sqrt{npq}$
4. Binomial distribution $= (q + p)^n$

2 Marks Questions

1. 2075 Set A Q.No. 4c The mean of a binomial distribution is 80 and standard deviation is 8, find the value of p, the probability of a success. [2]
Ans: $\frac{1}{6}$

2. 2074 Set A Q.No. 4c In 8 throws of a dice, turning of 1 or 6 is considered to be a success. Find the mean and standard deviation. [2]
Ans: $\frac{8}{3}, \frac{4}{3}$

3. 2073 Set D Q.No. 4c A dice is rolled 4 times. Getting an even number is considered as a success. Find the probability of getting two successes. [2]
Ans: $\frac{3}{8}$

4. 2069 (Set B) Q.No. 4c A dice is thrown 3 times. Getting a 2 or 3 is numbered as a success. Find the probabilities of getting two successes. [2]
Ans: $\frac{2}{9}$

5. 2066 C Q.No. 4 c If three dice are thrown, what is the probability of getting exactly 3 sixes? [2]
Ans: $\frac{1}{216}$

6. 2066 C Q.No. 4 c If three dice are thrown, what is the probability of getting exactly 3 sixes? [2]
Ans: $\frac{1}{216}$

7. 2061 Q.No. 4 c Find the binomial distribution having mean = 12 and variance = 8. [2]
Ans: $\left(\frac{2}{3} + \frac{1}{3}\right)^{36}$

4 Marks Questions

2077 Set H Q.No. 5 A certain manufacturing plant produces electric fuses of which 20% are defective. Find the probability that in a sample of 8 fuses selected at random there will be atleast one defective and not more that one defective. [4]

Ans: $\frac{325089}{390625}, \frac{196608}{390625}$

2076 GIE Set A Q.No. 8b The average percentage of a failure in a certain examination is 25%. What is the probability that out of 5 students.

i. 2 students will pass the examination?
ii. 1 or more students will pass the examination? [4]

Ans: (i) $\frac{135}{512}$ (ii) $\frac{781}{1024}$

2076 Set B Q.No. 8b Find the probability of getting three heads in six tosses of a coin. [4]

Ans: $\frac{5}{16}$

11. **2075 GIE Q.No. 8b** In five tosses of a coin successively, find the probability of getting (i) two heads, (ii) at least two heads. [4]

Ans: (i) $\frac{5}{16}$ (ii) $\frac{13}{16}$

12. **2075 Set B Q.No. 8b** A company produces electronics chips by a process that manufactures normally average 20% defective products. Sample of four chips is selected at random and the parts are tested for certain characteristics, what is the probability that (i) no chip is defective (ii) one chip is defective (iii) more than one chip are defective. [4]

Ans: (i) $\frac{256}{625}$ (ii) $\frac{256}{625}$ (iii) $\frac{113}{625}$

13. **2075 Set C Q.No. 8b** The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(x \geq 1)$. [4]

Ans: $\frac{728}{729}$

14. **2074 Supp Q.No. 8b** Out of 32 students in a class, 8 students are girls. If 3 students are selected, find the probability that [4]

i. one student is a boy
ii. 2 students are boys and one girl.

Ans: (i) $\frac{9}{64}$ (ii) $\frac{27}{64}$

15. **2074 Set A Q.No. 8b** A company produces electronic chips by a process that manufactures normally average 20% defective products. A sample of four chips is selected at random and the parts are tested for certain characteristics, what is the probability that (i) no chip is defective (ii) one chip is defective (iii) more than one chips are defective. [4]

Ans: (i) $\frac{256}{625}$ (ii) $\frac{256}{625}$ (iii) $\frac{113}{625}$

16. **2074 Set B Q.No. 8b** The average percentage of failure in a certain examination is 70. What is the probability that out of 5 candidates at least 3 will pass? [4]

Ans: 0.16308

17. **2073 Supp Q.No. 8b** The probability of hitting a target is 0.2. If six hits are made, find the probability that (i) exactly one will hit the target (ii) exactly two will hit the target. [4]

Ans: (i) 0.39 (ii) 0.25

18. **2073 Set C Q.No. 8b** If 4 dice are rolled simultaneously, what is the probability of getting (i) exactly 3 sixes (ii) exactly 2 sixes? [4]

Ans: (i) $\frac{5}{324}$ (ii) $\frac{25}{216}$

19. **2072 Supp. Q.No. 8b** A certain manufacturing process produce electrical fuses of which 15% are defective. Find the probability that in a sample of 10 fuses selected at random there will be (i) no defective (ii) not more than one defective. [4]

Ans: (i) 0.1969 (ii) 0.5443

20. **2072 Set C Q.No. 8b** Define Binomial distribution. Find the probability of getting (i) two heads (ii) at least two heads in 5 tosser of a coin. [4]

Ans: (i) $\frac{5}{16}$ (ii) $\frac{13}{16}$

21. **2072 Set D Q.No. 8b** The probability of hitting a target is found to be 0.25. If eight hits are made, find the probability that (i) none will hit the target (ii) exactly two will hit the target. [4]

Ans: (i) 0.1, (ii) 0.62

22. **2072 Set E Q.No. 8b** Four coins are tossed simultaneously. What is the probability of getting (i) 2 heads (ii) 4 heads. [4]

Ans: (i) $\frac{3}{8}$ (ii) $\frac{1}{16}$

23. **2071 Supp. Q.No. 8b** The probability of a man's hitting a target is $\frac{1}{4}$. If he fires 5 times, what is the probability of his hitting the target (i) exactly thrice (ii) none (iii) at least thrice. [4]

Ans: (i) 0.0879; (ii) 0.2373; (iii) 0.1035

24. **2071 Set C Q.No. 8 b** A dice is rolled 3 times. Getting 5 or 6 is numbered as success. Find the probability of getting (i) 2 successes (ii) 3 successes. [4]

Ans: (i) $\frac{2}{9}$ (ii) $\frac{1}{27}$

25. **2071 Set D Q.No. 8 b** Suppose that in a certain city 60% of all recorded births are males, suppose we select 5 birth records from the population. What is the probability that: (i) three of them are males (ii) more than 4 are males. [4]

Ans: (i) $\frac{216}{625}$ (ii) $\frac{343}{3125}$

26. **2071 Old Q.No. 12 b OR** If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random (i) 1 (ii) 0 (iii) at most 2 bulbs will be defective. [4]

Ans: (i) $\frac{256}{625}$ (ii) $\frac{256}{625}$ (iii) $\frac{608}{625}$

27. **2070 Supp. Q.No. 8 b** The probability of a man's hitting a target is $\frac{1}{4}$. If he fires 5 times, what is the probability of his hitting the target. (i) exactly thrice (ii) at least thrice [4]

Ans: (i) 0.0879 (ii) 0.1035

28. **2070 Set C Q.No. 8 b** A certain manufacturing process produces electrical fuses of which 15% are defective. Find the probability that in a sample of 10 fuses selected at random there will be (i) no defective (ii) not more than one defective. [4]

Ans: (I) 0.1969 (II) 0.6443

29. **2070 Set D Q.No. 8 b** A coin is tossed 5 times. Find the probability of getting(i) two heads (ii) at least two heads. [4]

Ans: (I) $\frac{5}{16}$ (II) $\frac{13}{16}$

30. **2070 (Old) Q.No. 8 a or** If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random (i) one (ii) zero (iii) at most 2 bulbs will be defective. [4]

Ans: (I) $\frac{256}{625}$ (II) $\frac{256}{625}$ (III) $\frac{608}{625}$

31. **2069 (Set A) Q.No. 8b** If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random (i) 1 (ii) at most 2 bulbs will be defective. [4]

Ans: (I) $\frac{256}{625}$ (II) $\frac{608}{625}$

32. **2069 (Set A) Old Q.No. 8a or** If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random (i) 1 (ii) 2 bulbs will be defective. [4]

Ans: (I) $\frac{256}{625}$ (II) $\frac{96}{625}$

33. **2069 Old (Set B) Q.No. 8a Or** If 4 dice are thrown, what is the probability of getting (i) exactly 3 sixes and (ii) no sixes.[4]

Ans: (I) $\frac{5}{324}$ (II) $\frac{625}{1296}$

34. **2068 Q.No. 8b Or** A dice is thrown 3 times. Getting a '5' or '6' is numbered a success. Find the probability of getting (i) 3 successes (ii) exactly 2 successes. [4]

Ans: (I) $\frac{1}{27}$ (II) $\frac{2}{3}$

35. **2067 Q.No. 8a Or** Define Binomial distribution, and its mean and variance, hence find the probability of getting three heads in five tosses of two coins. [4]

Ans: $\frac{45}{512}$

36. **2066 Q.No. 8 a** The probability of hitting a target is $\frac{1}{6}$, if eight hittings are made find the probability that (i) none will strike the target, (ii) exactly two will strike the target. [4]

Ans: (I) $\frac{390625}{1679616}$ (II) $\frac{109375}{419904}$

37. **2068 Q.No. 8b Or** A dice is thrown 3 times. Getting a '5' or '6' is numbered a success. Find the probability of getting (i) 3 successes (ii) exactly 2 successes. [4]

Ans: (I) $\frac{1}{27}$ (II) $\frac{2}{9}$

38. **2067 Q.No. 8a Or** Define Binomial distribution, and its mean and variance, hence find the probability of getting three heads in five tosses of two coins. [4]

Ans: $\frac{45}{512}$

39. **2066 Q.No. 8 a** The probability of hitting a target is $\frac{1}{6}$, if eight hittings are made find the probability that (i) none will strike the target, (ii) exactly two will strike the target. [4]

Ans: (I) $\frac{390625}{1679616}$ (II) $\frac{109375}{419904}$

40. **2065 Q.No 8 a OR** If three dices are thrown what is the probability of getting (i) exactly 3 sixes (ii) exactly 2 sixes [4]

Ans: (I) $\frac{1}{216}$ (II) $\frac{5}{72}$

41. **2064 Q.No. 8 a OR** Suppose that in a certain city 60% of all recorded births are males. If we select 5 births from the population, what will be the probability that:

i. none of them is male
ii. exactly three of them are male. [4]

Ans: (I) $\frac{32}{3125}$ (II) $\frac{216}{625}$

42. **2063 Q.No. 8 a OR** The incidence of occupation disease in an industry is such that the workmen have a 20% chance of suffering from it. What is the probability that out of six workmen four or more will contact the disease? [4]

Ans: $\frac{53}{3125}$

43. **2060 Q.No. 8 a OR** A sample of 100 fuses is known to have an average 5 defective fuses. Three fuses of sample are tested. What is the probability that (i) none of them is defective (ii) exactly one of them is defective? [4]

Ans: (I) $\frac{6859}{8000}$ (II) $\frac{1083}{8000}$

44. **2059 Q.No. 8 a OR** If 4 dice are thrown, what is probability of getting (i) exactly 3 sixes (ii) exactly 2 sixes and (iii) no sixes.[4]

Ans: (I) $\frac{5}{324}$ (II) $\frac{25}{216}$ (III) $\frac{625}{1296}$

45. **2058 Q.No. 8 a OR** The probability of hitting a target is 1/5. If six hittings are made, find the probability that: (i) none will strike the target (ii) exactly 2 will strike the target. [4]

Ans: (I) $\frac{4096}{15625}$ (II) $\frac{768}{3125}$

46. **2057 Q.No. 8 b OR** A dice is thrown 3 times. Getting a '5' or '6' is numbered a success. Find the probability of getting (a) 3 successes and (b) exactly 2 successes. [4]

Ans: (a) $\frac{1}{27}$ (b) $\frac{2}{9}$

12. STATICS

FORMULAE

1. Resultant of Two Forces Acting at A Point

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

2. Resolution of a Given Force in Two Given Directions

$P = \frac{F\sin\beta}{\sin(\alpha+\beta)}$, $Q = \frac{F\sin\alpha}{\sin(\alpha+\beta)}$

3. Resolved Parts of a Number of Coplanar Concurrent Forces

$R\cos\theta = P_1\cos\alpha_1 + P_2\cos\alpha_2 + P_3\cos\alpha_3 + \ldots = X$

$R\sin\theta = P_1\sin\alpha_1 + P_2\sin\alpha_2 + P_3\sin\alpha_3 + \ldots = Y$

$R = \sqrt{X^2 + Y^2}$

$\theta = \tan^{-1}\left(\frac{Y}{X}\right)$

4. Triangle of Forces: If three forces acting at a point be represented in magnitude and direction by the sides of a triangle, taken in order, they are in equilibrium.
5. Converse of the Triangle of Forces: If three forces acting at a point be in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order.
6. Lami's Theorem: If three forces acting at a point are in equilibnum, then each force is proportional to the sine of the angle between the other two.

4 Marks Questions

1. **2077 Set G Q.No. 7** Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. [2]
   Ans: $R = \sqrt{P^2 + Q^2}$, $\theta = \tan^{-1}\left(\frac{Q}{P}\right)$
2. **2076 GIE Set A Q.No. 12a** If the resultant of two forces P and Q acting at a point is at right angle to P, Prove that $R^2 = Q^2 - P^2$ [2]
3. **2076 GIE Set A Q.No. 12b** A particle weighing 6N is suspended freely from the ceiling by a weightless in extensible cord. Find the tension in the cord. [2]
   Ans: 18 N
4. **2076 GIE Set B Q.No. 12a** A heavy chain has weights of 10 and 16kg wt attached to its ends and hangs in equilibrium over a smooth pulley. If the greatest tension of the chain is 20kg weight, find the weight of the chain. [2]
   Ans: 14 Kg
5. **2076 Set B Q.No. 12a** If the resultant of two equal forces is equal to the given force, find angle between the forces. [2]
   Ans: 120°
6. **2076 Set C Q.No. 12a** Two forces acting at an angle 45° have a resultant equal to $\sqrt{10}$ N.
   If one of the forces be $\sqrt{2}$ N, find the other force. [2]
   Ans: 2 N
7. **2075 GIE Q.No. 12a** Two forces acting at an angle 45° have a resultant equal to $\sqrt{10}$ N, if one of the forces be $\sqrt{2}$ N, find the other force. [2]
   Ans: 2 N
8. **2075 Set A Q.No. 12a** A force equal to 10N is inclined at an angle of 30° to the horizontal. Find its resolved parts in horizontal and vertical directions. [2]
   Ans: $5\sqrt{3}$ N, 5 N
9. **2075 Set B Q.No. 12a** Find the angle between the forces P+Q and P–Q such that their resultant may be $\sqrt{P^2 + 3Q^2}$. [2]
   Ans: 120°
10. **2075 Set C Q.No. 12a** Forces equal to 7p, 5p and 8p acting on a particle are in equilibrium. Find the angle between the latter pair of forces. [2]
    Ans: 120°
11. **2075 Set C Q.No. 12b** A particle weighing 6N is suspended freely from the ceiling by a weightless in extensible cord. Find the tension in the cord. [2]
    Ans: 18 N
12. **2074 Supp Q.No. 12a** A heavy chain has weights of 10 kg and 16 kg attached to its ends and hangs in equilibrium over a smooth pulley. If the greatest tension of the chain is 20 kg. wt., find the weight of the chain. [2]
    Ans: 14 Kg
13. **2074 Set A Q.No. 12a** Two forces 4 N and $2\sqrt{2}$ N act at an angle of 45°. Find their resultant. [2]
    Ans: $2\sqrt{10}$ N at $\tan^{-1}\left(\frac{1}{3}\right)$ with the force of 4 N
14. **2074 Set B Q.No. 12a** If the resultant R of two forces P and Q acting at a point is at right angle to P, prove that: $R^2 = Q^2 - P^2$. [2]
15. **2073 Supp Q.No. 12a** If the resultant of two equal forces is equal to the given force, find angle between the forces. [2]
    Ans: 120°
16. **2073 Set C Q.No. 12a** If the resultant of two equal forces is equal to the given force, find angle between the forces. [2]
    Ans: 120°
17. **2073 Set D Q.No. 12a** Two forces P and 2P acting at a point have a resultant $\sqrt{3}$ P. Find the angle between the two forces. [2]
    Ans: 120°
18. **2072 Supp. Q.No. 12a** Two forces P and 2P acting at a point have the resultant $\sqrt{3}$ P, find the angle between the two given forces. [2]
    Ans: 120°
19. **2072 Set C Q.No. 12 a** Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. [2]
    Ans: $R = \sqrt{P^2 + Q^2}$, $\theta = \tan^{-1}\left(\frac{Q}{P}\right)$
20. **2072 Set D Q.No. 12a** Show that the resultant of two equal forces bisects the angle between them. [2]
21. **2072 Set E Q.No. 12a** Two forces P and 2P acting at a point have the resultant $\sqrt{3}$ P. Find the angle between the two given forces. [2]
    Ans: 120°
22. **2071 Set C Q.No. 12 a** Three forces acting on a particle are in equilibrium: The angle between the first and second is 90° and that between the second and third is 120°, find the ratios of the forces. [2]
    Ans: $\sqrt{3} : 1 : 2$
23. **2071 Set D Q.No. 12 a** Forces equal to 7p, 5p and 8p acting on a particle are in equilibrium. Find the angle between latter pair of forces. [2]
    Ans: 120°
24. **2071 Old Q.No. 5 a** If a force P be resolved into two forces making angles 45° and 15° with its direction, show that the latter force is $\frac{\sqrt{6}}{3}$ P. [2]
25. **2070 Set C Q.No. 12 a** At what angle do the force equal to P+Q and P – Q act so that the resultant may be $\sqrt{P^2+Q^2}$? [2]
    Ans: $\cos^{-1}\left\{\frac{-(P^2+Q^2)}{2(P^2-Q^2)}\right\}$

26. 2070 Set D Q.No. 12 a Two forces acting at an angle of 45° have a resultant equal to $\sqrt{10}$ N; if one of the forces be $\sqrt{2}$ N, find the other force. [2]
Ans: 2 N

27. 2070 (Old) Q.No. 5 a If the resultant R of two forces P and Q acting at a point is right angle to P, then prove that $R^2 = Q^2 - P^2$ [2]

28. 2069 (Set A) Q.No. 12a Two forces whose magnitudes are P and $P\sqrt{2}$ N act on a particle in direction inclined at an angle 135° to each other, find the magnitude and the direction of the resultant. [2]
Ans: The resultant is PN acting at right angle with the first force

29. 2069 (Set A) Old Q.No. 5a Find the resultant of two forces equal to 3N and 6N acting at an angle of 120°. [2]
Ans: The resultant is a force $3\sqrt{3}$ N making an angle of 90° with the direction of force 3 N.

30. 2069 (Set B) Q.No. 12a If a force P be resolved into two forces making angles 45° and 15° with its direction; show that the latter force is $\frac{\sqrt{6}}{3}$ P. [2]

31. 2069 Old (Set B) Q.No. 5a Find the least resultant of two forces of magnitudes 12 N and 8 N respectively. [2]
Ans: 4 N

32. 2069 Old (Set B) Q.No. 5c A particle weighing 5N is suspended freely from the ceiling by a Weightless inextensible cord. Find the tension in the cord. [2]
Ans: 15 N

33. 2068 Q.No. 5a Two forces acting at an angle of 45^0 have a resultant equal to $\sqrt{10}$ N; if one of the forces be $\sqrt{2}$ N, find the other force. [2]
Ans: 2 N

34. 2067 Q.No. 5a Find the resultant of two forces equal to 3N and 6N respectively such that their diagonal is perpendicular to the first force. [2]
Ans: $3\sqrt{3}$ N

35. 2066 C Q.No. 5 b The resultant of two forces P and Q is R. If Q is doubled the new resultant is perpendicular to P. Prove that Q = R. [2]

36. 2066 Q.No. 5 a The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P. Prove that Q = R. [2]

37. 2065 Q.No 4 b The sum of two forces is 18 and the resultant whose direction is perpendicular to the smaller of the two forces is 12, find the magnitude of the forces. [2]
Ans: 5 and 13

38. 2064 Q.No. 5 b Forces equal to 7P, 5P and 8P acting on a particle are in equilibrium. Find the angle between the latter pair of forces.
Ans: 120° [2]

39. 2063 Q.No. 5 b At what angle of forces equal to (P + Q) Newton and (P – Q) newton act so that the resultant may be $\sqrt{P^2 + Q^2}$ newton? [2]
Ans: $\cos^{-1}\left\{-\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}$

40. 2062 Q.No. 6 b Two forces whose magnitudes are P and $P\sqrt{2}$ act on a particle in directions inclined at an angle of 135° to each other; find the magnitude and direction of the resultant. [2]
Ans: P Newton, 90° with P

41. 2061 Q.No. 5 a If a force P be resolved into two forces making angles of 45° and 15° with its directions. Show that the latter force is $\frac{\sqrt{6}}{3}$ P. [2]

42. 2060 Q.No. 5 a At what angle do forces equal to (P + Q) and (P – Q) act so that the resultant may be $\sqrt{P^2 + Q^2}$? [2]
Ans: $\cos^{-1}\left\{\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}$

43. 2059 Q.No. 5 a The resultant of two forces P and Q is R. If Q is doubled the new resultant is perpendicular to P. Prove that Q = R. [2]

44. 2059 Q.No. 5 c State 'Triangle of forces'. [2]

45. 2058 Q.No. 5 a At what angle do forces equal to (P + Q) N and (P – Q) N act so that the resultant may be $\sqrt{P^2 + Q^2}$? [2]
Ans: $\text{Cos}^{-1}\left\{\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}$

46. 2057 Q.No. 5 a Write the expression for the magnitude and the direction of the resultant of two forces acting at a given angle. [2]
Ans: $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$ and $\theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha}$

4 Marks Questions

47. 2077 Set G Q.No. 8 Two forces of magnitude 3P, 2P respectively have a resultant R. If the first force be doubled, the magnitude of the resultant is doubled, find the angle between the forces. [4]
Ans: 120°

48. 2077 Set G Q.No. 8 OR Two forces P and Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight w on the plane, prove that: $\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{w^2}$.

49. 2077 Set H Q.No. 8 Three forces P, Q, R acting at O along OA, OB, OC; where O is the incenter of ΔABC, are in equilibrium. Show that: $\frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$ [4]

50. 2077 Set H Q.No. 8 OR State and prove converse of the triangle of forces. [4]

51. 2076 GIE Set A Q.No. 13a State and prove λ – μ theorem. [4]

2076 GIE Set A Q.No. 13a OR Two forces equal to '2p' and 'p' respectively act on a particle. If the first be double and the second increased by 12N, the direction of the resultant is unaltered. Find the value of p. [4]
Ans: 12 N

2076 GIE Set B Q.No. 13a The resultant of two forces P and Q is equal to $\sqrt{3}$ Q and makes an angle of 30° with the direction of P. Show that P is either equal to Q or is double of Q. [4]

2076 Set B Q.No. 13a Two men carry a weight 50N between two strings fixed to the weight, one string is inclined at 30° to the vertical and the other at 60°, find the tension of each string. [4]
Ans: $25\sqrt{3}$ N and 25 N

2076 Set C Q.No. 13a State and prove triangle of forces. [4]

2076 Set C Q.No. 13a OR Forces 1 N, 2 N, 3 N act at a point in direction parallel to the sides of an equilateral triangle taken in order. Find their resultant. [4]
Ans: $\sqrt{3}$ N perpendicular to the second force

2075 GIE Q.No. 13a State and prove converse of the triangle of forces. [4]

2075 GIE Q.No. 13a OR A heavy has weights of 10 kg and 16 kg attached to its ends and hangs in equilibrium over a smooth pulley. If the greatest tension of the chain is 20 kg wt, find the weight of the chain. [4]
Ans: 14 kg

2075 Set A Q.No. 13a A body of weight 68 N is suspended by two strings of lengths 8 m and 15 m respectively and the other ends of the strings are attached to two fixed points in a horizontal line 17m apart. Find the tensions of the strings. [4]
Ans: 60 N, 32 N

2075 Set B Q.No. 13a Forces 2, 4, 6 units act at a point in directions parallel to the sides of an equilateral triangle taken in order. Find their resultant. [4]
Ans: $2\sqrt{3}$ units at an angle of 210° with 2 units force

2075 Set C Q.No. 13a OR State and prove λ – μ theorem.

2075 Set B Q.No. 13a OR A body of weight 45 N is suspended by two strings of length 30 cm and 40 cm respectively and the other ends of the strings are attached to the extremities of a rod of length 50 cm. The rod is so held that the body hangs immediately below its middle point. Find the tensions of the strings. [4]
Ans: 36 N, 27 N

2074 Supp Q.No. 13a Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2 N respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and direction of the resultant. [4]
Ans: 10 N towards the opposite angular points

2074 Supp Q.No. 13a OR Find the resultant of two forces P and Q acting at a point when the angle between them is α. [4]
Ans: $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$; $\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

2074 Set A Q.No. 13a State and prove 'Triangle Law of forces'. [4]

2074 Set B Q.No. 13a State and prove Lami's theorem. [4]

2074 Set B Q.No. 13a OR A body of weight 10kg is suspended by two strings of lengths 7 m and 24 m, their other ends being fastened to the extremities of a rod of length 25m. If the rod be so held that the body hangs immediately below its middle point, find the tensions of the strings. [4]
Ans: $\frac{14}{5}$ kg wt and $\frac{48}{5}$ kg wt

2073 Supp Q.No. 13a If a force P be resolved into two forces making angles 45° and 15° with its directions, show that the later force is $\sqrt{\frac{2}{3}}$ P. [4]

2073 Set C Q.No. 13a State and prove triangle of forces. [4]

2073 Set C Q.No. 13a OR ABCD is a square. Forces 1, 2, 3, 4 and $2\sqrt{2}$ newton's act at a point in directions AB, BC, CD, DA and AC respectively. Find the resultant. [4]
Ans: The forces are in equilibrium

2073 Set D Q.No. 13a A body of weight 100 kg is suspended by two things 7m and 24 m in length; their other ends being fastened to the extremeties of a rod of length 25m. If the rod is so held that the body hangs immediately below its middle point, find the tensions of the strings. [4]
Ans: 28 kg wt and 96 kg wt

2073 Set D Q.No. 13a OR Find the resultant of two forces P and Q acting at a point when the angle between them is α. [4]

2072 Supp. Q.No. 13a Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$, 2 N respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and the direction of the resultant. [4]
Ans: 10 N towards the opposite angular points

2072 Supp. Q.No. 13a OR State and prove Lami's theorem. [4]

2072 Set C Q.No. 13a Two men carry a weight 50N supported by two strings; one string is inclined at 30° to the vertical and other at 60°, find the tension of each string. [4]
Ans: 25 N, $25\sqrt{3}$ N

2072 Set D Q.No. 13a State and prove Lami's theorem. [4]

2072 Set D Q.No. 13a OR A body of weight 68 N is suspended by two strings of length 8 m and 15 m respectively, and the other ends of the strings are attached to two fixed points in a horizontal line 17 m apart, find the tensions of the strings. [4]
Ans: 60 N, 32 N

2072 Set E Q.No. 13a A body of weight 65 N is suspended by two strings of lengths 5 and 12 m attached to two points in the same horizontal line whose distance apart is 13m; find the tensions of the string. [4]
Ans: 60 N and 25 N

2071 Set C Q.No. 13 a The resultant of two forces P and Q acting at an angle α is equal to $(2m + 1)\sqrt{P^2 + Q^2}$. When they act at an angle $(90° - \alpha)$ the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$. Prove that: $\tan\alpha = \frac{m-1}{m+1}$ [4]

80. **2071 Set D Q.No. 13 a** The resultant of two forces P and Q is equal to $\sqrt{3}$ Q and makes an angle of 30° with the direction of P; show that P is either equal to Q or is double of Q. [4]

81. **2071 Set D Q.No. 13 a OR** State and prove Lami's theorem.[4]

82. **2071 Old Q.No. 13 a** State and prove Lami's theorem. [4]

83. **2071 Old Q.No. 13 a OR** O is the circumcentre of the triangle ABC. Forces P, Q and R acting along OA, OB and OC are in equilibrium. Show that: $\frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C}$ [4]

84. **2070 Set C Q.No. 13 a** Forces of $2, \sqrt{3}, 5, \sqrt{3}$ and 2N respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and direction of the resultant. [4]

Ans: 10 N towards the opposite angular points

85. **2070 Set D Q.No. 13 a** A body of weight 65 N is suspended by two strings of lengths 5 and 12 m attached to two points in the same horizontal line whose distance apart is 13 m; find the tensions of the string. [4]

Ans: 60 N and 25 N

86. **2070 Set D Q.No. 13 a Or** State and prove Lami's theorem.[4]

87. **2070 (Old) Q.No. 13 a Or** If the resultant R of two focus P and Q inclined to one another at any given angle, makes an angle θ with the direction of P, show that the resultant of the forces P + R and Q acting at the same given angle, will make an angle θ/2 with the direction of P + R. [4]

88. **2069 (Set A) Q.No. 13a** The resultant of two forces P and Q acting at an angle α is equal to $(2m + 1)\sqrt{P^2 + Q^2}$. When they act at an angle $(90° - \alpha)$ the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$. Prove that: $\tan\alpha = \frac{m-1}{m+1}$. [4]

89. **2069 (Set A) Old Q.No. 13a** State and prove Lami's theorem. [4]

90. **2069 (Set A) Old Q.No. 13a or** Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$, 2 newtons respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and the direction of the resultant. [4]

Ans: The resultant is 10N towards the opposite angular point.

91. **2069 (Set B) Q.No. 13a** A body of weight 65N is suspended by two strings of lengths 5m and 12m attached to two points in the same horizontal line whose distance apart is 13m, find the tension of the strings. [4]

Ans: 60N and 25N

92. **2069 (Set B) Q.No. 13a Or** Find the resultant of two forces P and Q acting at a point when the angle between them is α. [4]

Ans: $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$ and $\theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha}$

93. **2069 Old (Set B) Q.No. 13a** State and prove "Lami's theorem". [4]

94. **2069 Old (Set B) Q.No. 13a Or** If the resultant of two forces acting on a particle be at right angle to one of them and its magnitude be half of the magnitude of the other, show that the ratio of the greater force to the smaller is 2: $\sqrt{3}$. [4]

95. **2068 Q.No. 13 a** Find the resultant of two forces P and Q when the angle between them is α. [4]

Ans: $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$; $\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

96. **2068 Q.No. 13 a Or** A body of weight 65 N is suspended by two strings of length 5 m and 12 m attached to two points in the same horizontal line whose distance apart is 13 m; find the tension of the strings. [4]

Ans: 60 N, 25 N

97. **2067 Q.No. 13a** If a force P be resolved into two forces making angles of 45° and 15° with its direction; show that the latter force is $\sqrt{\frac{2}{3}}$P. [4]

98. **2067 Q.No. 13a OR** State and prove converse of the Triangle of forces. [4]

99. **2066 C Q.No. 13 a** State and prove converse of triangle of forces. [4]

100. **2066 C Q.No. 13 a OR** A body of weight 10 kg is suspended by two strings 7 and 24 metres in length, their other ends being fastened to the extremities of a rod of length 25 metres. If the rod be so hold that the body hangs immediately below its middle point. Find the tensions of the strings. [4]

Ans: $T_1 = \frac{14}{5}$ kg wt.; $T_2 = \frac{48}{5}$ kg wt

101. **2066 Q.No.13 a** Forces 2, $\sqrt{3}$, 5, $\sqrt{3}$, 2 newtons respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and direction of the resultant. [4]

Ans: 10 N towards the opposite angular point.

102. **2066 Q.No.13 a OR** State and prove the theorem on Triangle of forces for three forces acting at a point. [4]

103. **2065 Q.No 13 a** Two forces P & Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight W on the plane, prove that: $\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{W^2}$ [4]

104. **2065 Q.No 13 a OR** State and prove: λ – μ theorem. [4]

105. **2064 Q.No. 13 a** The resultant of two forces P & Q acting at an angle α is equal to $(2m + 1)\sqrt{P^2 + Q^2}$, when they act at an angle 90° – α, the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$. Prove that $\tan\alpha = \frac{m-1}{m+1}$. [4]

106. **2064 Q.No. 13 a OR** State and prove Lami's theorem. [4]

107. **2063 Q.No. 13 a** Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2 newtons respectively act at one of the angular points of a regular hexagon towards the five other angular points. Find the magnitude and direction of the resultant. [4]

Ans: 10 N towards the opposite angular point.

108. **2063 Q.No. 13 a OR** Find the resultant of two forces P and Q acting at a point. [4]

Ans: Magnitude = $\sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$ **and direction =** $\frac{Q\sin\alpha}{P + Q\cos\alpha}$

110. **2062 Q.No. 13 a** State and prove converse of triangle of forces. [4]

111. **2062 Q.No. 13 a OR** A uniform plane lamina in the form of a rhombus, one of whose angle is 120°, is supported by two forces applied at the centre in the directions of the diagonals so that one side of the rhombus is horizontal; show that if P and Q be the forces and P be the greater then $P^2 = 3Q^2$. [4]

112. **2061 Q.No. 13 a** The resultant of two forces P & Q is equal to $\sqrt{3}Q$ and making an angle of 30° with the direction of P. Show that P is either equal to Q or is double of Q. [4]

113. **2060 Q.No. 13 a** Find the resultant of n number of coplanar forces acting at a point. [4]

114. **2060 Q.No. 13 a OR** The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P, prove that R = Q. [4]

115. **2059 Q.No. 13 b** A uniform sphere of weight 3 N rests in contact with a smooth vertical wall. It is supported by a string whose length equals the radius of the sphere joining a point on the surface of the sphere to a point of the wall. Calculate the tension in the string and the reaction of the wall. [4]

Ans: $T = 2\sqrt{3}$ N and $R = \sqrt{3}$ N

116. **2059 Q.No. 13 b OR** Two forces equal to 2 P and P respectively act on a particle. If the first be doubled and the second is increased by 12N, the direction of the resultant is unaltered. Find the value of P. [4]

Ans: P = 12 N

117. **2058 Q.No. 13 a** State and prove "Triangle of forces" [4]

118. **2058 Q.No. 13 a OR** Two forces P and Q act at a point. Their resultant R is at right angles to P. Show that $Q^2 - P^2 = R^2$ and the angle between the forces is $\cos^{-1}\left(-\frac{P}{Q}\right)$. [4]

119. **2057 Q.No. 13 a** State and prove "Lami's Theorem". [4]

120. **2057 Q.No. 13 a OR** The resultant of two forces P and Q acting at an angle α is $(2m + 1)\sqrt{P^2 + Q^2}$ when they act at an angle $(90° - \alpha)$ the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$. Prove that $\tan\alpha = \frac{m-1}{m+1}$. [4]

6 Marks Questions

121. **2071 Supp. Q.No. 14** Prove that if three forces acting on a particle are in equilibrium, then each is proportional to the sine of the angle between the other two. Also, if a body of weight w is suspended by strings of length 3 m and 4 m attached to two points in the same horizontal line whose distance apart is 5m, find the tensions along the strings. [6]

Ans: $\frac{4W}{5}$ N and $\frac{3W}{5}$ N

122. **2071 Supp. Q.No. 14 OR** Let force F make angles α and β with directions OX and OY respectively. Prove that if F_x and F_y are the components of the force in the directions of OX and OY respectively then $F_x = \frac{F\sin\beta}{\sin(\alpha+\beta)}$, $F_y = \frac{F\sin\alpha}{\sin(\alpha+\beta)}$.

Also, if Q is doubled and the new resultant is perpendicular to P, prove that Q = R, where R is the resultant of forces P and Q. [6]

123. **2070 Supp. Q.No. 14** Two forces equal to 2P and P respectively act on a particle. If the first be doubled and the second increased by 12 N, the direction of the resultant is unaltered. Find the value of P. [6]

Ans: 12 N

124. **2070 Supp. Q.No. 14 OR** Let P and Q be two forces acting on a particle, whose directions include an angle of $\alpha(\neq 0 \text{ or } \pi)$. Derive the magnitude and direction of their resultant. [6]

13. STATICS (CONTINUED)

FORMULAE

1. Varignon's Theorem
   The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.
2. Moment of a force:
   $F \cdot OA = F \cdot p$

2 Marks Questions

1. **2076 Set C Q.No. 12b** Two parallel forces of 25 kg weight and 20 kg weight are acting at a distance 25cm apart. Find the point at which their resultant acts. [2]

   Ans: The resultant is 45 kg wt acting at a distance of $\frac{100}{9}$ cm from the force 25 kg wt.

2. **2075 GIE Q.No. 12b** Two parallel forces of 30 kg wt and 20 kg wt are acting at a distance 80 cms apart. Find their resultant if forces are like. [2]

   Ans: 50 kg wt

3. **2075 Set B Q.No. 12b** A straight uniform beam 1m long, when a load of 24 kg is placed at one end, it balances about a point 30 cm from that end. Find the weight of the beam. [2]

   Ans: 36 kg

4. **2074 Supp Q.No. 12b** Two like parallel forces P and Q act at points 18 m apart; if the resultant force be 9 N and acts at a distance 6m from P, find Q. [2]

   Ans: 3 N

5. **2074 Set A Q.No. 12b** Forces equal to 5N, 2N, 5N, 2N act along the sides AB, CB, CD and AD of a square ABCD whose side is 3m. Find the moment of the couple that will give equilibrium. [2]

   Ans: 9 Nm

6. **2074 Set B Q.No. 12b** A uniform beam AB is 8 m long and weights 25 kg. Weights of 10 kg and 25 kg are suspended from A and B respectively. At what point must the beam be supported so that it may rest horizontally? [2]

   Ans: At a distance of 9m from A

7. **2073 Set D Q.No. 12b** Two like parallel forces P and Q act at points 18 m apart, if the resultant force be 9 N and acts at a distance 6m from P, find Q. [2]

   Ans: 3 N

8. **2072 Supp. Q.No. 12c** Two like parallel forces P and Q act at points 18 m apart, if the resultant force be 9 N and acts at a point 6 m from P, find Q. [2]

   Ans: 3 N

9. 2072 Set C Q.No. 12b Two like parallel forces P and Q act at points 18 m apart, if the resultant force is 9 N and acts at a distance 12 m from Q, find P. [2]
Ans: 6 N

10. 2072 Set D Q.No. 12b A uniform beam AB is 16 m long and weighs 50 kg weights of 20 kg and 50 kg are suspended from A and B respectively. At what point must the beam be supported so that it may rest horizontally? [2]
Ans: 10 m

11. 2071 Supp. Q.No. 12a If one of two like parallel forces and their resultant are 18 N and 36 N respectively, find the ratio of distances of the resultant from the forces. [2]
Ans: 1: 1

12. 2071 Set D Q.No. 2 b Two unlike parallel forces, the greater of which is 75 N, have a resultant 25 N. Find the ratio of the distances of the resultant from the component forces. [2]
Ans: 2:3

13. 2071 Old Q.No. 5 b Two parallel forces of 40kg wt and 10 kg wt are acting at a distance 40 cm apart. Find their resultant if forces are unlike. [2]
Ans: 30 kg wt

14. 2070 Supp. Q.No. 12 a If two like parallel forces are 16 N and 12 N, find their resultant acting at a distance of 90 cm from the greater force and the distance between the forces. [2]
Ans: 28N, 2.1 m

15. 2070 Set D Q.No. 12 b Find two like parallel forces acting at a distance of 2.5 m apart, which are equivalent to a given force of 30 N. The lines of action of one being at a distance of 50 cm from the given force. [2]
Ans: 24 N and 6 N

16. 2069 (Set A) Q.No. 12b A straight uniform rod is 3 m long. When a load of 10 N is placed at one end it balances about a point 25 cm from that end. Find the weight of the rod. [2]
Ans: 2 N

17. 2069 (Set A) Old Q.No. 6a Two like parallel forces P and Q act at points 18 m apart. If the resultant force be 9 N and acts at a distance of 6 m from P, find Q. [2]
Ans: 3 N

18. 2069 (Set B) Q.No. 12b Replace a force of magnitude 48 kg wt by two unlike parallel forces, one at a distance of 2m and other at 8m from the given force. [2]
Ans: 64 kg.wt. and 16 kg. wt.

19. 2069 Old (Set B) Q.No. 5b Two parallel forces of 30 kg wt and 20 kg wt are acting at a distance 40 cm apart. Find their resultant and its position if forces are like. [2]
Ans: The resultant is 50 kg. wt. parallel to the given forces and acting at a distance of 16 cm from the weight of 30 kg.

20. 2068 Q.No. 6a Find two unlike parallel forces acting at a distance of 12 cm which are equivalent to a force of 20 N, the greater of the two forces being at a distance of 6 cm from the given force. [2]
Ans: 30 N, 10 N

21. 2067 Q.No. 6a Find the resultant of two parallel forces of 15 kg wt. and 10 kg wt acting at a distance 20 cm apart in the same direction. [2]
Ans: 25 kg wt at a distance of 8 cm from 15 kg wt.

22. 2066 C Q.No. 6 b Define a couple. What do you mean by arm of a couple? [2]

23. 2066 Q.No. 6 b A straight weightless rod, 48 cm in length, rests in a horizontal position between two pegs placed at a distance of 6 cm apart, one peg being at one end of the rod, and a weight of 2 kg is suspended from the other end. Find the pressures on the pegs. [2]
Ans: 14 kg wt; 16 kg wt

24. 2065 Q.No 6 b A uniform bar 4m long and weighting 3N passes over a prop and is supported in a horizontal position by a force of 1N acting vertically upwards at the other end. Find the distance of the prop from the centre of the bar. [2]
Ans: 1 m

25. 2064 Q.No. 6 b A straight uniform rod is 3 m long. When a load of 5N is placed at one end, it balances about a point 25 cm from that end. Find the weight of the rod. [2]
Ans: 1 N

26. 2063 Q.No. 6 b Find the two unlike parallel forces acting at a distance of 12 cm which are equivalent to a force of 20 N, the greater of the two forces being at a distance of 6 cm from the given force. [2]
Ans: 10 N and 30 N

27. 2062 Q.No. 5 a Define moment of a force about a point. Give the geometrical meaning of the moment of a force about a point. [2]

28. 2061 Q.No. 6 a A straight uniform rod is 3m long. When a load of 5 N is placed at one end it balances about a point 25 cm from that end. Find the weight of the rod. [2]
Ans: 1 N

29. 2061 Q.No. 6 b Forces equal to 3, 5, 3 and 5 newtons respectively act along the sides of a square taken in order, find their resultant. [2]
Ans: Couple of moment 8a Nm, where a is side of the square

30. 2060 Q.No. 6 a Find two like parallel forces, acting at a distance of 2.5 m apart, which are equivalent to a given force of 30 N, the line of action of one being at a distance of 50 cm from the given force. [2]
Ans: 24 N and 6 N

31. 2060 Q.No. 6 b Define arm of a couple and the moment of a couple. [2]

32. 2059 Q.No. 6 b A uniform bar 4 m long and weighting 3N passes over a prop and is supported in horizontal position by a force of 1 N vertically upwards at the other end. Find the distance of the prop from the centre of the bar. [2]
Ans: 1m

33. 2058 Q.No. 6 a Define a couple. What do you mean by arm of a couple? [2]

34. 2058 Q.No. 6 b A straight uniform rod is 3 m long, when a load of 5 N is placed at one end it balances about a point 25 cm from that end. Find the weight of the road. [2]
Ans: 1 N

35. 2057 Q.No. 6 a Define a couple and the moment of a couple. Express the moment of a couple mathematically. [2]
Ans: Magnitude of one of the forces × arm of the couple

2057 Q.No. 6 b Replace a force of magnitude 50 kg wt by two like parallel forces one at a distance of 2m and other at 8 m from the given force. [2]

Ans: P = 40 kg wt and Q = 10 kg wt

Marks Questions

7. 2075 Set C Q.No. 13a Two like parallel forces of magnitudes P and Q are acting at the end points A and B of a rod AB of length ℓ. If two opposite forces each of magnitude S are added to P and Q, then prove that the line of action at the new resultant will be displaced through a distance $\frac{S\ell}{P+Q}$. [4]

8. 2071 Supp. Q.No. 13a Forces 1, 2, 4, 5 kg wt act along the sides of a square taken in order. Prove that their resultant is parallel to a diagonal and find where it cuts the side along which the first force acts. [4]

9. 2071 Old Q.No. 13 b Prove that the algebraic sum of the moments of any two parallel forces about any point in their plane is equal to the moment of their resultant about the same point. [4]

10. 2070 Supp. Q.No. 13 a Forces of 6, 5, 4, 3 kg – wts respectively act along the sides of a square ABCD taken in order. Find the magnitude, direction and line of action of their resultant. [4]

Ans: $2\sqrt{2}$ kg wt. parallel to diagonal AC meets at E where BE = $\frac{7a}{2}$ where a = side of a square

11. 2070 (Old) Q.No. 6 a The resultant of two like parallel forces is 12 N and it acts at a distance 2 meter from the larger force equal to 8 N. Find the distance of the resultant from the smaller force. **Ans: 4 m** [4]

12. 2070 (Old) Q.No. 14 a Forces equal to 3,4,5,6 N respectively act along the sides of a square ABCD taken in order, find the magnitude, direction and line of action of their resultant. [4]

Ans: $2\sqrt{2}$ N, 225, x = $\frac{9}{2}$, a = side of square

13. 2069 (Set A) Old Q.No. 14a ABC is an isosceles triangle whose angle A is 120° and forces of magnitude 1, 1 and $\sqrt{3}$ N act along AB, AC and BC, show that the resultant bisects BC and is parallel to one of the other sides of the triangle. [4]

14. 2069 Old (Set B) Q.No. 13b A man carries a bundle at the end of a stick which is placed over his shoulder. If the distance between his hand and shoulder be changed, how does the pressure on his shoulder change? [4]

15. 2068 Q.No. 14 a A light rod of length 72 cm has equal weights attached to it, one at 18 cm from one end and the other at 30 cm from the other end; if it be supported by two vertical strings attached to its ends and if the strings cannot support a tension greater than the weight of 50 kg, what is the greatest magnitude of the equal weight? [4]

Ans: 42.86 kg

16. 2067 Q.No. 14a Define parallel forces. Deduce the resultant of two like parallel forces. [4]

17. 2066 C Q.No. 14 a Prove that the algebraic sum of the moments of any two forces, meeting at a point, about any point in their plane is equal to the moment of their resultant about the same point. [4]

18. 2066 Q.No.14 a Three forces P, 2P and 3P act along the sides AB, BC and CA of an equilateral triangle ABC of side a. Find the magnitude, direction and line of action of the resultant. [4]

Ans: $P\sqrt{3}$ in the direction at right angles to BC and cutting BC produced at D where CD = $\frac{a}{2}$

19. 2065 Q.No 14 a Forces equal to 3P, 4P, 5P act along the sides AB, BC and CA of an equilateral triangle ABC, find the magnitude, direction and line of action of the resultant. [4]

Ans: $P\sqrt{3}$, 90°, passes through at a distance of 3/2 BC from C.

20. 2064 Q.No. 14 a A light rod of length 72 cms has equal weight attached to it, one at 18 cms from one end and the other at 30 cms from the other end. If it is supported by two vertical strings attached to its ends and if the string can not support a tension greater than the weight of 50 kg, what is the greatest magnitude of the equal weights? [4]

Ans: $42\frac{6}{7}$ kg.

21. 2063 Q.No. 14 a Prove that the algebraic sum of the moments of any two like parallel forces about a point in their is equal to the moment of their resultant force about the same point. [4]

22. 2062 Q.No. 14 a The wire passing round a telegraph pole is horizontal and the two portion attached to the pole are inclined at an angle 60° to one another. The pole is supported by a wire attached to the middle point of the pole and inclined at 60° to the horizon; show that the tension of this wire is $4\sqrt{3}$ times that of the telegraph wire. [4]

23. 2061 Q.No. 13 a OR Find the resultant of like parallel forces. [4]

24. 2061 Q.No. 14 a ABCD is a square along AB, CB, AD and DC equal forces, P act. Show that the magnitude of their resultant is equal to double of any component and acts along DC. [4]

25. 2060 Q.No. 14 a Prove that the algebraic sum of moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. [4]

26. 2059 Q.No. 13 a Find the resultant of two like parallel forces acting on a rigid body. [4]

27. 2058 Q.No. 14 a Prove that the algebraic sum of moments of two like parallel forces, about any point in their plane is equal to the moment of their resultant about the same point. [4]

28. 2057 Q.No. 14 a Prove that the algebraic sum of the moments of any two forces, meeting at a point, about any point in their plane is equal to the moment of their resultant about the same point. [4]

6 Marks Questions

59. 2077 Set G Q.No. 9 Define moment of a force. Also interpret it geometrically. State and prove Varignon's theorem for intersecting forces. [6]

60. 2076 GIE Set A Q.No. 14 ABC is an isosceles triangle where angle A is 120° and force of magnitudes 1N, 1N and $\sqrt{3}$N act along AB, AC and BC; show that the resultant bisects BC and is parallel to one of the two sides of the triangle. [6]

61. 2076 GIE Set B Q.No. 15 Find the resultant of two like parallel forces. [6]

62. 2076 GIE Set B Q.No. 15 OR ABC is an isosceles triangle whose angle A is 120° and forces of magnitudes 1N, 1N and $\sqrt{3}$ N act along AB, AC and BC. Show that the resultant bisects BC and is parallel to one of the other sides of the triangle. [6]

63. 2076 Set B Q.No. 15 Deduce the resultant of two parallel forces. [6]

64. 2076 Set B Q.No. 15 OR Define Moment geometrically. Also state and prove the Varignon's theorem for two intersecting forces. [6]

65. 2076 Set C Q.No. 14 Forces equal to P, 2P, 3P and 4P act along the sides of a square ABCD taken in order, find the magnitude, direction and the line of action of the resultant. [6]

Ans: $2\sqrt{2}$ P, resultant is parallel to CA, $x = \frac{3a}{2}$

66. 2075 GIE Q.No. 14 Define moments of a force. Give its geometrical interpretation. Also state and prove Varignon's theorem for parallel forces. [6]

67. 2075 Set A Q.No. 15 P and Q (P>Q) are two like parallel forces acting at A and B. Show that if they interchange positions, the point of application of the resultant is displaced a distance $\frac{P-Q}{P+Q}$ AB. [6]

68. 2075 Set A Q.No. 15 OR Three forces P, 2P and 3P act along the sides AB, BC and CA of an equilateral triangle ABC of side a; find the magnitude, direction and the line of action of the resultant. [6]

Ans: $P\sqrt{3}$ in the direction at right angles to BC and cutting BC produced at D where CD = $\frac{a}{2}$, a = side of an equilateral triangle

69. 2075 Set B Q.No. 14 Prove that the algebraic sum of the moments of two intersecting forces about a point in their plane is equal to the moment of their resultant about the same point. [6]

70. 2075 Set B Q.No. 14 OR Define the moment of a force, Forces 1, 2, 4, 5 kg wt act along the sides of a square taken in order. Prove that their resultant is parallel to a diagonal and find where it cuts the side along which the first force acts. [6]

71. 2074 Supp Q.No. 15 Define moment of a force about a point. Prove that the algebraic sum of the moments of two like parallel forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

72. 2074 Set A Q.No. 14 Define the moment of a force. Prove that the algebraic sum of the moments of two parallel forces about a point in their plane is equal to the moment of their resultant about the same point. [6]

73. 2074 Set B Q.No. 14 Define moment of a force about a point. State and prove Varignon's theorem for two interesting forces. [6]

74. 2073 Supp Q.No. 14 Define parallel forces. Deduce the resultant of two parallel forces. [6]

75. 2073 Supp Q.No. 14 OR Geometrically interpret moment of a force. Also state and prove Varignon's theorem. [6]

76. 2073 Set C Q.No. 15 Define like and unlike parallel forces. A man carries a bundle at the end of a stick which is placed over his shoulder, if the distance between his hand and shoulder be changed how does the pressure on his shoulder change? [6]

77. 2073 Set D Q.No. 15 Forces equal to P, 2P, 3P and 4P act along the sides of a square ABCD taken in order. Find the magnitude, direction and the line of action of the resultant. [6]

Ans: $2\sqrt{2}$ P, resultant is parallel to CA, $x = \frac{3a}{2}$

78. 2072 Supp. Q.No. 15 Define moment of a force about a point. What does it represent geometrically? Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same points. [6]

79. 2072 Set C Q.No. 14 Three forces P, 2P and 3P act along the sides AB, BC and CA of an equilateral triangle of side a, find the magnitude, direction and line of action of the resultant. [6]

Ans: $P\sqrt{3}$ in the direction at right angles to BC and cutting BC produced at D where CD = $\frac{a}{2}$, a = side of an equilateral triangle

80. 2072 Set C Q.No. 14 OR Define moment. State and prove Varignon's theorem. [6]

81. 2072 Set D Q.No. 14 Define coplanar forces. Forces equal to P, 2P, 3P and 4P act along the sides of a square ABCD taken in order, find the magnitude, direction and the line of action of the resultant. [6]

Ans: $2\sqrt{2}$ P, resultant is parallel to CA, $x = \frac{3a}{2}$

82. 2072 Set E Q.No. 15 Find the resultant of two like parallel forces. [6]

83. 2072 Set E Q.No. 15 OR ABCD is a square; along AB, CB, AD and DC equal forces P act; show that the magnitude of their resultant is equal to double of any components and acts along DC. [6]

84. 2071 Set C Q.No. 15 Three forces p, 2p and 3p act along the sides AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant. [6]

Ans: $p\sqrt{3}$ in the direction at right angles to BC and cutting BC produced at D where CD = $\frac{a}{2}$, a = side of an equilateral triangle

85. **2071 Set C Q.No. 15 OR** Find the resultant of two unlike parallel forces. A man carries a bundle at the end of a stick 75 cm long which is placed on his shoulder. What should be the distance between his hand and shoulder, in order that the pressure on the shoulder may be three times the weight of the bundle? [6]

Ans: 25 cm

86. **2071 Set D Q.No. 15** Define moment of a force about a point. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

87. **2070 Set C Q.No. 15** Define moment of a force about a point. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

88. **2070 Set C Q.No. 15 or** P and Q are like parallel forces. If P is moved parallel to itself through a distance x, show that the resultant of P and Q moves a distance $\frac{P.x}{P+Q}$. [6]

89. **2070 Set D Q.No. 15** Define moment of a force about a point. Prove that the algebraic sum of the moments of the moment of two like parallel forces about any point in their plane is equal to the moment of their resultant about the same point. [6]

90. **2069 (Set A) Q.No. 15** ABC is an isosceles triangle whose angle A is 120° and forces of magnitudes, 1, 1 and $\sqrt{3}$ N act along AB, AC and BC; show that the resultant bisects BC and is parallel to one of the other side of the triangle. [6]

91. **2069 (Set A) Q.No. 15 or** Find the resultant of two like parallel forces. [6]

92. **2069 (Set B) Q.No. 15** Forces equal to 3p, 4p and 5p and along the sides AB, BC and CA of an equilateral triangle ABC, find the magnitude, direction and the line of action of the resultant. [6]

Ans: Resultant = $p\sqrt{3}$ in the direction at right angle to BC. If the resultant cuts BC at D then line of action is given by CD = $\frac{3BC}{2}$

14. DYNAMICS

A. MOTION IN A STRAIGHT LINE

FORMULAE

1. $a = \frac{v-u}{t}$
2. $v = u + at$
3. $s = ut + \frac{1}{2}at^2$
4. $v^2 = u^2 + 2as$
5. $s_t = u + \frac{2t-1}{2}a$

2 Marks Questions

1. **2075 GIE Q.No. 12c** A motor cyclist increases its velocity at the rate of 5 ms^{-2} to 30 ms^{-1} in 4 seconds. Find the initial velocity. [2]

Ans: 10 ms^{-1}

2. **2075 Set C Q.No. 12c** A car moving with a velocity of 20 ms^{-1} has a uniform acceleration of 2 ms^{-2}. If it moves for 2.5 sec, find the final velocity. [2]

Ans: 25 ms^{-1}

3. **2073 Set D Q.No. 12c** An aeroplane land on the runway with a velocity of 108 km/hr. If then its velocity slows down at the rate of 25 m/s^2; find the distance covered by the aeroplane before coming to rest. [2]

Ans: 18 m

4. **2072 Supp. Q.No. 12b** A cyclist travelling with a velocity of 72 km/hr accelerates at the rate of 4m/s^2 until it describes a distance of 48 m. Find the time taken. [2]

Ans: 2 secs

5. **2069 Old (Set B) Q.No. 6a** A car moving with a velocity of 15 ms^{-1} has a uniform acceleration of 2 ms^{-2}. If it moves for 2.5 sec, find its final velocity. [2]

Ans: 20m/s

6. **2063 Q.No. 5 a** A train moving with a velocity of 360 km/hr has the uniform acceleration 40 m/s^2. Obtain the distance covered by the train in $\frac{1}{2}$ minute. [2]

Ans: 18.3 km

4 Marks Questions

7. **2075 Set B Q.No. 13b** A car starting from rest, moves with uniform acceleration and describes the first kilometer in 3 minutes. If it now moves with uniform velocity, how long will it take to describe another kilometer? [4]

Ans: 1 minute 30 seconds

8. **2070 Set D Q.No. 13 b** If a, b, c be the spaces described by a particle during the p^{th}, q^{th}, r^{th} seconds of its motion respectively, prove that: $a(q-r) + b(r-p) + c(p-q) = 0$ [4]

9. **2070 (Old) Q.No. 13 b** A body moves along a straight line with uniform acceleration. The body covers a distance of 18 m in the first three seconds and 22 m in the next 5 seconds. Find the velocity at the end of 10 seconds and the distance covered in 10^{th} second. [4]

Ans: 2.6 m/sec, 2.8 m

10. **2069 (Set A) Old Q.No. 13b** Prove that for a particle moving with uniform acceleration 'a' in a straight line is $a = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t + t'}$ where s is the space described in t sec. and s' during the next t' seconds. [4]

11. **2068 Q.No. 13 b** If a, b, c be the spaces described by a particle during the p^{th}, q^{th} and r^{th} second of its motion respectively, prove that: $a(q-r) + b(r-p) + c(p-q) = 0$ [4]

12. **2066 C Q.No. 13 b** If a, b, c be the space described by a particle during the p^{th}, q^{th} and r^{th} seconds of its motion respectively, prove that: $a(q-r) + b(r-p) + c(p-q) = 0$. [4]

13. 2065 Q.No. 13 b Prove that for a particle moving with uniform acceleration a in a straight line

$a = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t + t'}$, where s is the space described in t seconds and s' during the next t' seconds [4]

14. 2064 Q.No. 13 b If a, b, c, are the spaces described by the particles during the p^{th}, q^{th}, r^{th} seconds of its motion respectively, prove that: $a(q - r) + b(r - p) + c(p - q) = 0$. [4]

15. 2062 Q.No. 13 b A railway train goes from one station to another moving during the first part of the journey with uniform acceleration f, when steam is shut off and the breaks are applied, it moves with retardation f', if 'a' be the distance between the stations, show that the time the train takes is: $\sqrt{\frac{2a(f + f')}{ff'}}$ [4]

16. 2060 Q.No. 13 b A body moves for 3 seconds with a constant acceleration during which it describes 24.30 metres, the acceleration then ceases and during the next 3 seconds, it describes 21.60 metres. Find the initial velocity and the acceleration. [4]

Ans: 9m/s and – 0.6 m/s²

17. 2059 Q.No. 14 a Prove that for a particle moving with uniform acceleration f in a straight line $f = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t + t'}$ where s is the space described in t secs and s' during the next t' secs. [4]

18. 2057 Q.No. 13 b If a, b, c be the space described by a particle during the p^{th}, q^{th} and r^{th} seconds of its motion respectively, prove that $a(q - r) + b(r - p) + c(p - q) = 0$. [4]

6 Marks Questions

19. 2076 GIE Set A Q.No. 15 Prove that for a particle moving with uniform acceleration 'a' in a straight line $a = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{(t + t')}$ where 's' is the space described in 't' seconds and s' during the next t' seconds. [6]

20. 2076 GIE Set B Q.No. 14 A car starts from rest and accelerates uniformly for 10 second to a velocity of 8m/s. It then runs at a constant velocity and is finally brought to rest in 64m with a constant retardation. The total distance covered by the car is 584m. Find the acceleration, retardation and total time taken. [6]

Ans: 0.8 ms^{-2}, 0.5 ms^{-2}, 86 seconds

21. 2075 Set A Q.No. 14 A point moving with uniform acceleration describes in the last second of its motion $\frac{9}{25}$ of the whole distance. If it started from rest, how long was it in motion and through what distance did it move, if it described 15 cms in the first second? [6]

Ans: 5 sec, 375 cms

22. 2074 Set A Q.No. 15 A bus starts from station A and stops at station B. The Velocity increases uniformly till it reaches maximum velocity v and then decreases uniformly. Show that the time taken by the bus to run from A to B is 2x/v where x is the distance between the two stations. [6]

23. 2075 Set C Q.No. 14 A point moving in a straight line with uniform acceleration describes a and b metres in successive intervals at time t_1 and t_2 seconds. Prove that the acceleration is $\frac{2(bt_1 - at_2)}{t_1t_2(t_1 + t_2)}$. [6]

24. 2071 Supp. Q.No. 15 A bus starts from station A and stops at station B. The velocity increases uniformly till it reaches maximum velocity v and then decrease uniformly, show that the time taken by the bus to run from A to B is $\frac{2x}{v}$ where x is the distance between the two stations. [6]

25. 2071 Set C Q.No. 14 If a, b, c be the spaces described by a particle during the p^{th}, q^{th}, r^{th} seconds of its motion respectively, prove that: $a(q - r) + b(r - p) + c(p - q) = 0$ [6]

26. 2070 Supp. Q.No. 15 A body starting with initial velocity of 15m/sec, moves with a uniform acceleration of 5 m/sec².
i. What is the velocity after 10 sec?
ii. How far will it in 10 sec?
iii. What will be its velocity when it has traveled 10m?
iv. What will be the distance moved in the 10^{th} second? [6]

Ans: (i) 65 m/s (ii) 400 m (iii) $5\sqrt{13}$ m/s (iv) 62.5 m

27. 2070 Set C Q.No. 14 A railway train goes from one station to another moving during the first part of the journey with uniform acceleration a; when steam is shut off and the brakes are applied, it moves with retardation a^1. If s be the distance between the stations, show that the time, the train takes is $\sqrt{\frac{2s(a + a^1)}{aa^1}}$. [6]

B. MOTION UNDER GRAVITY

FORMULAE

1. Vertically downward motion
   $v = u + gt$; $v^2 = u^2 + 2gh$;
   $h = ut + \frac{1}{2}gt^2$; $h_t = u + \frac{2t - 1}{2}g$
2. Vertically upward motion
   $v = u - gt$; $v^2 = u^2 - 2gh$;
   $h = ut - \frac{1}{2}gt^2$; $h_t = u - \frac{2t - 1}{2}g$

2 Marks Questions

1. 2076 GIE Set B Q.No. 12c A ball thrown up vertically returns to the thrower after 6 seconds. Find the height ascended. [2]

Ans: 45 m

2. 2076 Set B Q.No. 12b A ball is thrown vertically upwards at a rate of 40 ms^{-1}. Find the time taken to attain the maximum height. ($g = 10ms^{-2}$) [2]

Ans: 45

2075 Set B Q.No. 12c A body falls from rest from the top of a tower and after 5 sec it reaches the ground. Find the striking velocity of the body and height of the tower. [g = 9.8 m/s²] [2]
Ans: 49 m/sec, 122.5 m

2074 Supp Q.No. 12c A stone is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4 m. Find the velocity of the stone with which it is projected. (g = 9.8 m/s²) [2]
Ans: 39.2m/sec

2074 Set B Q.No. 12c If a ball is projected vertically upward at a rate of 40ms^{-1}, find the time taken to attain the maximum height. (g = 10 sm^{-2}) [2]
Ans: 4 sec

2073 Supp Q.No. 12b If a ball is thrown vertically upwards at a rate of 20ms^{-1}, find the time taken to attain the maximum height. (g = 10ms^{-2}) [2]
Ans: 2 sec

7. **2072 Set C Q.No. 12 c** A ball is thrown vertically upwards at a rate of 40 ms^{-2}. Find the time taken to attain the maximum height. (g = 10 ms^{-2}) [2]
Ans: 4 sec

8. **2071 Set D Q.No. 12 c** A ball is thrown vertically upwards with a velocity of 30m/s. Find the time taken by the ball to reach the ground again. (g = 10m/s²) [2]
Ans: 6 sec

9. **2070 Set D Q.No. 12 c** A ball is projected vertically upwards with a velocity of 40 m/s. Find its velocity and position at the end of 3s. (g = 10 m/s²). [2]
Ans: 10m/sec, 75 m

10. **2069 (Set A) Q.No. 12c** A body is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4m. Find the velocity of the stone with which it is projected. (g = 9.8 m/s²) [2]
Ans: u = 39.2 ms^{-1}

11. **2069 (Set A) Old Q.No. 6b** A ball thrown up vertically upwards returns to the thrower after 6 seconds. Find its position after 4 sec. (g = 10 m/s²) [2]
Ans: 5 m below the highest point

12. **2069 (Set B) Q.No. 12c** A ball thrown up vertically return to the thrower after 6 secs. Find the velocity with which it was thrown up. [2]
Ans: 30 m/sec

13. **2069 Old (Set B) Q.No. 6b** A ball is thrown vertically upward at a speed of 4ms^{-1}. Find the maximum height reached and the time taken to attain this height. [2]
Ans: $\frac{4}{5}$m; $\frac{2}{5}$ sec

14. **2068 Q.No. 6b** A stone is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4m. Find the velocity of the stone with which it is projected. (g = 9.8m/s²) [2]
Ans: 39.2 m/sec

15. **2066 Q.No. 5 b** A stone is dropped from a balloon at a height 116.4 m above the ground and it reaches the ground in 6 sec. Find the velocity with which the balloon was rising. [2]
Ans: 10ms^{-1}

16. **2064 Q.No. 5 a** A ball thrown up vertically returns to the thrower after 6 seconds. Find the velocity with which it was thrown up. (g) = 10 m/s²). [2]
Ans: 30 m/sec

17. **2062 Q.No. 5 b** A stone is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4 m. Find the velocity of the stone with which it is projected. (g = 9.8 m/s²). [2]
Ans: 39.2 m/s

18. **2058 Q.No. 5 b** A body is projected vertically with a velocity of 9.8 ms^{-1}, how long it takes to return to the point of projection? (g = 9.8 ms^{-2}). [2]
Ans: 2 sec

4 Marks Questions

19. **2077 Set I Q.No. 8** A particle is projected up from the bottom of an inclined plane with a velocity of 25m/s, while another is dropped from the highest point to slide down the plane as the same moment. If the length of the plane be 200 m and the angle of inclination of the plane with the horizon is 30°, find when and where the two particles will meet. (g = 10m/s²) [4]
Ans: 8 seconds at 40 m from the top

20. **2076 GIE Set A Q.No. 13b** A particle is projected from the bottom up and inclined plane with a velocity of 20m/s while another is dropped from the highest point to slide down the plane at the same moment. If the length of the plane be 120 m and the angle of inclination of the plane with the horizon be 30°, find when and where the two particles will meet. (g = 10m/s²). [4]
Ans: 6 seconds at 30 m from the bottom

21. **2076 Set B Q.No. 13b OR** A stone is dropped into a well and the sound of its striking the water is heard in $4\frac{2}{9}$ seconds. If the velocity of the sound is 352.8 ms^{-1}, find the depth of the well. (g = 9.8 ms^{-2}) [4]
Ans: 78.4 m

22. **2076 Set C Q.No. 13b** A stone is dropped from the top of a tower 200 m high and at the same time another is projected vertically upwards from the ground with a velocity 50ms^{-1}. Find where and when the two will meet? (g = 9.8 ms^{-2}) [4]
Ans: 121.6 m from ground, 4 sec

23. **2074 Set B Q.No. 13b** A stone is dropped from the top of a tower 200m high and at the same time another is projected vertically upwards from the ground with a velocity of 50ms^{-1}. Find where and when the two will meet? (g = 9.8 ms^{-2})[4]
Ans: 121.6 m from ground, 4 sec

24. **2073 Supp Q.No. 13b OR** A stone is dropped form the top of a tower 200 m high and at the same time another is projected vertically upwards form the ground with a velocity of 50ms^{-1}. Find where and when the two will meet? (g = 9.8 ms^{-2}) [4]
Ans: 121.6 m from ground, 4 sec

25. **2073 Set C Q.No. 13b** A body falls from rest from the top of a tower and during the last second it falls $\frac{16}{25}$ th of the whole height. Find the height of the tower. (g = 10ms^{-2}) [4]
Ans: 31.25 m

26. **2071 Old Q.No. 14 a** A stone is dropped into a well and the sound of its striking the water is heard in $4\frac{2}{9}$ seconds. If the velocity of the sound is 352.8 ms^{-1}. Find the depth of the well. ($g = 9.8\ ms^{-2}$) [4]
Ans: 78.4 m

27. **2067 Q.No. 13b** A stone is dropped from the top of a tower 200 m high and at the same time another is projected vertically upwards from the ground with a velocity of 50ms^{-1}. Find where and when the two will meet ($g = 9.8\ ms^{-2}$). [4]
Ans: 121.6 m from ground, 4 sec

28. **2063 Q.No. 13 b** A stone is dropped from the top of a tower 200 m high and at the same time another is projected vertically upwards from the ground with a velocity of 50 m/s. Find where and when the two will meet? ($g = 9.8\ m/s^2$). [4]
Ans: Two stones meet at a height of 121.6 m from the ground after 4 sec.

29. **2061 Q.No. 13 b** A body falls from rest from the top of a tower and during the last second it falls $\frac{16}{25}$ th of the whole height. Find the height of the tower. ($g = 10 m/sec^2$) [4]
Ans: 31.25 m

30. **2058 Q.No. 13 b** A body falls from rest from the top of a tower and during the last second it falls $\frac{16^{th}}{25}$ of the whole height. Find the height of the tower ($g = 10ms^{-2}$). [4]
Ans: 31.25 m

6 Marks Questions

31. **2072 Set E Q.No. 14** A body is projected vertically upward with velocity u and t seconds afterwards another body is projected similarly with the same velocity. Show that they meet a height $\frac{4u^2 - g^2t^2}{8g}$ from the point of projection after $\left(\frac{u}{g} - \frac{t}{2}\right)$ secs from the instant of projection of the second body. [6]

C. MOTION DOWN AN INCLINED PLANE

FORMULAE

1. Projected downward
$v^2 = u^2 + 2g \sin\alpha \cdot l$
$v = u + g \sin\alpha \cdot t$
$l = ut + \frac{1}{2} g \sin\alpha \cdot t^2$
2. Projected upward
$v^2 = u^2 - 2g \sin\alpha \cdot l$
$v = u - g \sin\alpha \cdot t$
$l = ut - \frac{1}{2} g \sin\alpha\, t^2$

2 Marks Questions

1. **2075 Set A Q.No. 12c** A body is projected up an inclined plane with a velocity of 25 m/s. If the inclination of the plane to the horizon be 30°, what length of the plane will it cover after 4 sec? ($g = 10\ m/s^2$) [2]
Ans: 60 m

2. **2073 Set C Q.No. 12b** A particle slides down a smooth inclined plane 10 m long and acquires a velocity of $10\sqrt{2}$ ms^{-1}. Find the inclination of the plane. ($g = 10\ ms^{-2}$) [2]
Ans: 90°

3. **2072 Set D Q.No. 12c** A particle slides down a smooth inclined plane 10 m long and acquires a velocity $10\sqrt{2}$ ms^{-1}. Find the inclination of the plane. ($g = 10\ ms^{-2}$) [2]
Ans: 90°

4. **2072 Set E Q.No. 12b** A ball is projected up a smooth plane with velocity 25m/s. If the inclination of the plane to the horizon be 30°, find the velocity of the ball when it travels a distance of 22.5 m. ($g = 10\ m/s^2$) [2]
Ans: 20 m/sec

5. **2071 Supp. Q.No. 12b** A particle slides from the rest $39.2\sqrt{3}$ m in 4 seconds down a smooth inclined plane. Calculate the angle of inclination of the plane.[$g = 9.8\ m/s^2$][2]
Ans: 60°

6. **2071 Set C Q.No. 12 c** A ball is thrown up an inclined plane with a velocity of 14.7 m/s. Where will the velocity of the ball be 4.9 m/s? Assume that the inclination of the plane to the horizon is 30°. ($g = 9.8\ m/s^2$) [2]
Ans: 19.6 m

7. **2070 Supp. Q.No. 12 b** A particle, projected from the bottom of a smooth inclined plane with a velocity of 19.6 m/s, is just carried to the top in 4 sec; find the inclination of the plane to the horizon and also the length of the plane. [$g = 9.8 m/s^2$]. [2]
Ans: 30°, 39.2 m

8. **2070 Set C Q.No. 12 c** A ball is projected up a smooth inclined plane with velocity 25 m/s. If the inclination of the plane to the horizon be 30°, find the velocity of the ball when it travels a distance of 22.5 m. [$g=10\ m/s^2$]. [2]
Ans: 20 m/s

9. **2066 C Q.No. 5 a** A body slides down an inclined plane 39.24 m long and acquires a velocity of 19.6 m/sec. Find the inclination of the plane. (take $g = 10\ m/sec^2$) [2]
Ans: 29.3°

10. **2065 Q.No 5 a** A particle slides down an inclined plane 20 m long and acquires a velocity of $10\sqrt{2}$ m/sec. Find the inclination of the plane ($g = 10 m/sec^2$). [2]
Ans: 30°

11. **2059 Q.No. 5 b** A particle slides down an inclined plane 30 m. long and acquires a velocity of $\sqrt{300\sqrt{3}}$ ms^{-1}. Find the inclination of the plane. ($g = 10\ ms^{-2}$). [2]
Ans: 60°

4 Marks Questions

12. **2076 Set B Q.No. 13b** A body slides down from rest from the top of a smooth plane of height 44.1 m and inclination 30° with the horizon. Divide the plane into three parts so that the body at the top of the plane may describe each part in equal interval of time. ($g = 9.8\ ms^{-2}$) [4]
Ans: 9.8 m, 29.4 m, 49 m

2075 GIE Q.No. 13b A body is projected up an inclined plane with a velocity of 25 ms^{-1}. If the inclination of the plane to the horizon be 30°, what length of the plane will it cover after 4 seconds? [4]

Ans: 20 ms^{-1}

2066 Q.No. 13 b A particle slides down from rest from the top of a smooth plane of height 1962 cms and inclination 30° with the horizon. Divide the plane into three parts so that a particle at the top of the plane may describe each part in equal times. ($g = 981\ cm/sec^2$) [4]

Ans: 436 cm, 1308 cm, 2180 cm

4 Marks Questions

2074 Set A Q.No. 15 OR A body slides down a smooth plane whose length is 100 m and height 20 m. Find (i) the velocity of the body when it reaches the bottom of the plane, (ii) time taken by it to reach the bottom of the plane (iii) velocity of the body after 4 seconds. [$g = 10\ m/s^2$] [6]

Ans: (i) 20 m/s (ii) 10 sec (iii) 8 m/s

15. DYNAMICS (CONTINUED)

A. NEWTON'S LAW OF MOTION, IMPULSE

FORMULAE

1. Momentum = mv
   Change in momentum = mv − mu
   Average rate of change of momentum = $\frac{m(v-u)}{t}$
2. Upward force: R = m(a + g)
   Downward force: R = m(g − a)
3. Impulsive force: $F = \frac{d(mv)}{dt}$, as m is constant
4. Principle of conservation of linear momentum
   mv − MV = 0
   $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

2 Marks Questions

1. **2076 GIE Set B Q.No. 12b** A body of mass 0.5kg and initially at rest, is subjected to a force of 2N for 1 second. Find the velocity acquired during the second. [2]

   Ans: 4 ms^{-1}

2. **2076 Set B Q.No. 12c** Find the mass of an object which on earth weighs 98 N. ($g = 9.8\ ms^{-2}$) [2]

   Ans: 10 kg

3. **2076 Set C Q.No. 12c** A body of mass 0.5 kg and initially at rest is subject to a force of 2 N for 1 sec. Find the velocity acquired during the second. [2]

   Ans: 4 ms^{-1}

4. **2075 Set A Q.No. 12b** A cart is pushed on a frictionless smooth plane with an average force of 20 N for 5 seconds. If the cart with mass 50 kg is at rest in the beginning, find the velocity acquired by the cart. [2]

   Ans: 2 m/sec

5. **2074 Set A Q.No. 12c** A uniform force of 150 N change the velocity of a body moving in a straight line from 300 to 350 meters per second in 2 minutes. Find the mass of the body. [2]

6. **2073 Supp Q.No. 12c** Find the mass of an object which on earth weights 98N. ($g = 9.8\ ms^{-2}$) [2]

   Ans: 10 kg

7. **2071 Supp. Q.No. 12c** A bullet of mass 25 gm moving 250 m/s penetrating into a tree trunk and is then brought to rest in 0.02 seconds. Find impulse of the force on the bullet. [2]

   Ans: 6.25 kg m/s

8. **2071 Set C Q.No. 12 b** A body of mass 50 kg is falling from a certain height is brought to rest after striking the ground with a speed of 5 m/s. If the resistance force of the ground is 500 N, find the duration of contact. [2]

   Ans: 0.5 sec

9. **2070 Supp. Q.No. 12 c** A bullet of mass 25 gm moving 250 m/s penetrates into a tree trunk and is then brought to rest in 0.02 seconds. Find the distance of penetration of the tree-trunk. [2]

   Ans: 2.5 m

10. **2070 Set C Q.No. 12 b** A cart is pushed on a frictionless smooth plane with an average force of 20 N for 5 seconds. If the cart with mass 50 kg is at rest in the beginning, find the velocity acquired by the cart. [2]

    Ans: 2 m s^{-1}

11. **2070 (Old) Q.No. 6 b** A car of mass 1000 kg is brought to rest by applying a breaking force of 2500 N. Find the average retardation. [2]

    Ans: 2.5 m/s^2

12. **2069 (Set A) Old Q.No. 5b** A bullet of mass 2 kg is fired from a gun of mass 100 kg with a velocity 250 m/sec, find the recoil velocity of the gun. [2]

    Ans: 5 m/sec

13. **2069 Old (Set B) Q.No. 6c** A car of mass 1000 kg is brought to rest by applying a braking forces of 2500 N. Find the average retardation. [2]

    Ans: 2.5 m/sec^2

14. **2068 Q.No. 5b** A constant force of 10N acting on an object reduces its velocity from 15 m/s to 5 m/s in 2 seconds. Find the mass of the object. [2]

    Ans: 2 kg

15. **2067 Q.No. 5b** A bullet fired into a target loses half its velocity after penetrating 6 cm. How much further will it penetrate? [2]

    Ans: 2 cm

16. **2067 Q.No. 6b** State Newton's second law of motion hence define a force. [2]

17. **2066 C Q.No. 6 a** Find the velocity of a 4 kg shot that will just penetrate through a wall 16 cm thick, the resistance being 4 metric tonnes weight. [2]

    Ans: 56 m/sec

18. **2066 Q.No. 6 a** A body of mass 1 kg is falling under gravity at the rate of 28 ms^{-1}. What uniform force will stop it in 0.1 second? ($g = 9.8\ ms^{-2}$) [2]

    Ans: 29 kg wt

19. **2065 Q.No 6 a** The pull of the earth on a body is 49 N. If the acceleration due to gravity is $g = 9.8\ m/sec^2$. Find the mass of the body. [2]

    Ans: 5 kg

20. 2064 Q.No. 6 a Show that Newton's second law of motion gives the measurement of a force. [2]

21. 2062 Q.No. 6 a A constant force of 10 N acting on an object reduces its velocity from 15 ms^{-1} to 5 ms^{-1} in 2 seconds. Find the mass of the object. [2]

Ans: 2 kg

4 Marks Questions

22. 2074 Supp Q.No. 13b A shot of 2 kg is discharged by a gun of mass 400 kg with a velocity of 800 m/s. Find the constant force which would be required to stop the recoil of the gun in $1\frac{1}{4}$ sec. [4]

Ans: 1280 N

23. 2073 Set D Q.No. 13b A mass of 5 kg falls 3 m from rest and is then brought to rest by penetrating 30 cm into some sand. Find the average thrust of the sand on it. [4]

Ans: 55 kg wt

24. 2072 Supp. Q.No. 13b A balloon is rising with a acceleration f. Prove that the fraction of the weight of the balloon which must be emptied out of the balloon in order to double the acceleration is $\frac{f}{g+2f}$. [4]

25. 2072 Set C Q.No. 13b State laws of motion. A body of mass 50 kg falling from a certain height is brought to rest after striking the ground with a speed of 5 ms^{-1}. If the resistance force of ground is 500 N, find the duration of the contact. [4]

Ans: 0.5 sec

26. 2072 Set D Q.No. 13b State laws of motion. Use Newton's Law to define an absolute unit of force. [4]

27. 2072 Set E Q.No. 13b State Newton's laws of motion. Prove that Newton's second law provides the measurement of the force. [4]

28. 2071 Set D Q.No. 13 b A gun of mass 400 kg fires a shot of mass 3 kg, with a velocity of 200 m/s, find the constant force which acting on the gun would stop it after a recoil of 2.5 meters. [4]

Ans: 180 N

29. 2069 (Set A) Q.No. 13b A mass of 5 kg falls 300 cm from rest and is then brought to rest by penetrating 30 cm into some sand, find the average thrust of the sand. [4]

Ans: 55 kg wt.

30. 2069 (Set B) Q.No. 13b State Newton's laws of motion. Show that Newton's second law gives the measurement of a force. [4]

31. 2069 Old (Set B) Q.No. 14a A balloon is raising with an acceleration f. Prove that the fraction of the weight of the balloon which be emptied out of the balloon in order to double the acceleration is $\frac{f}{g+2f}$. [4]

32. 2063 Q.No. 14 b OR State Newton's laws of motion. Show that Newton's second law of motion gives the measurement of a force. [4]

33. 2060 Q.No. 14 b OR A shot whose mass is 40 kg is discharged from a 700 kg gun with a velocity of 140 ms^{-1}. Find the constant force which acts on the gun would stop it after a recoil of 6.4 m. [4]

Ans: 3500 N

34. 2069 Q.No. 14 b A body of mass 1 kg is falling under gravity at the rate of 28 ms^{-1}. What is the uniform force that will stop it in (i) 0.1 sec (ii) 20 cm ($g = 10\ ms^{-2}$). Instead of falling under gravity if the body is moving at the rate of 28 ms^{-1} along a horizontal line, what will be the force required in above two cases? [4]

Ans: (i) 290 N (ii) 80 N and (i) 280 N (ii) 70 N

6 Marks Questions

35. 2077 Set H Q.No. 9 Define laws of motion. A gun of mass 1 metric tonne force a shot of mass 14 kg and recoils up smooth inclined plane, rising to a height of 1.6 m, find the initial velocity of the projectile. [6]

Ans: 400 m /sec

36. 2075 GIE Q.No. 15 State Newton's laws of motion. A force equal to a weight of 1 kg acts on a body continuously for 10 seconds and causes it to distance 10 m in that time, find the mass of the body. [6]

Ans: 49 kg

37. 2073 Set C Q.No. 14 State Newton's Laws of Motion. A bullet of mass 10 g is fired from a gun of mass 3 kg with a velocity 300 kmh^{-1}. Find the velocity of recoil of the gun. [6]

Ans: 1 Kmh^{-1}

B. PROJECTILES

FORMULAE

1. Equations of Motion of a Projectile
   For the vertical motion,
   $v \sin\theta = u \sin\alpha - gt$
   and for the horizontal motion,
   $v \cos\theta = u \cos\alpha$
2. Time to reach the greatest height
   $t = \frac{u \sin\alpha}{g}$
3. Time of Flight and Range
   $T = \frac{2u \sin\alpha}{g}$
   $R = \frac{u^2 \sin 2\alpha}{g}$
4. Maximum horizontal range $= \frac{u^2}{g}$
5. Greatest height: $H = \frac{u^2 \sin^2\alpha}{2g}$

2 Marks Questions

1. 2071 Old Q.No. 6 c A particle is projected at an angle 75° to the horizon with a velocity of 2943 cm/sec. Find the range on a horizontal plane. [2]

Ans: 4330.62 m

2. 2059 Q.No. 6 a If u and α be the velocity and angle of projection of a projectile, then find the time of flight. [2]

Ans: $\frac{2u \sin\alpha}{g}$

Marks Questions

1. **2076 GIE Set B Q.No. 13b** A projectile thrown from a point in a horizontal plane comes back to the plane in 4 seconds at a distance of 60m in front of the point of projection. Find the velocity of projection. ($g = 10m/s^2$) [4]
   Ans: $u = 25\ ms^{-1}$

2. **2075 Set A Q.No. 13b** If R be the horizontal range and T, the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$ where α is the angle of projection. [4]

3. **2074 Set A Q.No. 13b OR** Find the velocity and the direction of the projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g = 9.8\ m/s^2$) [4]
   Ans: 70 m/s, 45°

4. **2073 Supp Q.No. 13b** Find the velocity and the direction of the projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g = 9.8\ ms^{-2}$) [4]
   Ans: 70 m/s, 45°

7. **2072 Set E Q.No. 13b OR** With what velocity must a body be projected at an angle of 45° from the top of a tower 65 m high, if it is to reach a point on the ground 180 m from the base of the tower. [4]
   Ans: 36 m/sec

8. **2071 Supp. Q.No. 13b** A body thrown from a point in a horizontal plane comes back to the plane in 4 sec at a distance of 58.8 m from the point of projection. Find the velocity of the projection. [$g = 9.8\ m/s^2$] [4]
   Ans: 24.5 m/sec

9. **2071 Set C Q.No. 13 b** A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity? [4]
   Ans: 2h, $2\sqrt{gh}$

10. **2071 Old Q.No. 14 b** A ball is thrown by a player from a height of 2 meters, at an angle of 30° with the horizon with a velocity of $18ms^{-1}$, is caught by another player at the height of 0.4 meter from the ground. How far apart were the two players? ($g = 9.8\ ms^{-2}$) [4]
    Ans: $18\sqrt{3}$ m

11. **2070 Supp. Q.No. 13 b** A ball is projected at an angle of 30° to the horizon and land on the surface of height 10 m which is $20\sqrt{3}$ m away from the point of projection. Find the velocity of projection and its striking velocity on the surface. ($g = 10\ m/s^2$) [4]
    Ans: $20\sqrt{2}$ m/s, $10\sqrt{6}$ m/s

12. **2070 Set C Q.No. 13 b** Find the velocity and the direction of projection of a shot which passes in a horizontal direction just over the top of wall which is 250 m off and 125 m high. ($g = 9.8\ m/s^2$) [4]
    Ans: 70 m/s, 45°

13. **2070 (Old) Q.No. 14 b** Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 50 meter off and 25 meter high ($g = 9.8\ ms^{-2}$). [4]
    Ans: $14\sqrt{5}$ m/sec, 45°

14. **2069 (Set A) Old Q.No. 14b** A projectile thrown from a point in a horizontal plane comes back to the plane in 4 secs at a distance of 60m infrot of the point of projection, find the velocity of projection. ($g = 10\ ms^{-2}$) [4]
    Ans: $u = 25\ ms^{-1}$

15. **2069 Old (Set B) Q.No. 14b** A ball is thrown from the top of a building towards a tall building $50\sqrt{3}$ m away. The initial velocity of the ball is $20\ ms^{-1}$ at 30° above the horizontal. How far above or below its original level will the ball strike the opposite wall? [4]
    Ans: The ball strikes the opposite wall 75 m below the original level.

16. **2068 Q.No. 14b** The horizontal and vertical components of the initial velocity of a projectile are U and V respectively. If R be the range and the H the greatest height attained, prove that: $\frac{4H}{R} = \frac{V}{U}$ [4]

17. **2067 Q.No. 14b** Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g=9.8ms^{-2}$)[4]
    Ans: 70 m/sec, 45°

18. **2066 C Q.No. 14 b** A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range of R on the horizontal plane through the point of projection is: $R = 4\sqrt{H\left(\frac{u^2}{2g} - H\right)}$ [4]

19. **2066 Q.No.14 b** Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall 250 m off and 125 m high. ($g = 9.8ms^{-2}$) [4]
    Ans: $70\ ms^{-1}$, 45°

20. **2065 Q.No 14 b** If R be the horizontal range and T the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$, where α is the angle of projection. [4]

21. **2064 Q.No. 14 b** A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity? [4]
    Ans: 2h, $2\sqrt{gh}$

22. **2063 Q.No. 14 b** A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60m in front of the point of projection. Find the velocity of projection. ($g = 10\ m/s^2$). [4]
    Ans: 25 m/sec.

23. **2062 Q.No. 14 b OR** If R be the horizontal range and T be the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$ where α is the angle of projection [4]

24. 2061 Q.No. 14 b If R be the horizontal range of a projectile and h its greatest height.
Prove that its initial velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. [4]

25. 2060 Q.No. 14 b From a point on the ground at a distance 'x' from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 'y' on the other side. Prove that the height of the wall is $\frac{xy}{x+y}$. [4]

26. 2059 Q.No. 14 b OR A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is $R = 4\sqrt{H\left(\frac{u^2}{2g}-H\right)}$. [4]

27. 2058 Q.No. 14 b If R be the horizontal range and T, the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$, where α is the angle of projection. [4]

28. 2057 Q.No. 14 b If R be the horizontal range of a projectile and h is greatest height, prove that its initial velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. [4]

6 Marks Questions

29. 2077 Set H Q.No. 9 OR A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H¹ are the greatest heights and t_1 and t_2 are the time of flight in two paths for which this is possible, prove that: [6]
a. $R^2 = 16HH^1$
b. $R = \frac{1}{2}gt_1t_2$

30. 2077 Set I Q.No. 9 A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height 'h'. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity? [6]
Ans: 2h, $2\sqrt{gh}$ at 45° to the horizon

31. 2076 Set B Q.No. 14 Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g = 9.8\ ms^{-2}$) [6]
Ans: u = 70 m/s, α = 45°

32. 2076 Set C Q.No. 15 If R be the horizontal range of a projectile and h its greatest height, prove that its initial velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. [6]

33. 2075 GIE Q.No. 15 OR A ball is thrown upwards at an angle of 30° to the horizon and lands on the top edge of a building that is $10\sqrt{3}$ m away. The top edge is 5 m above the throwing point. How fast was the ball thrown? [6]
Ans: 19.8 m/sec.

34. 2075 Set C Q.No. 14 OR A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is $R = 4\sqrt{H\left(\frac{u^2}{2g}-H\right)}$. A body is projected with a velocity of 9.8 m/sec and rises upto the height 2.45 m. Find the horizontal range. [6]
Ans: 9.8 m

35. 2074 Supp Q.No. 14 With what velocity must a body be projected at an angle of 45° from the top of a tower 65 m high, if it is to reach a point on the ground 180 m from the base of the tower. Ans: 36 m/sec [6]

36. 2074 Set B Q.No. 15 From a point on the ground at a distance x from the foot of a vertical wall a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance y on the other side. Prove that the height of the wall is $\frac{xy}{x+y}$. [6]

37. 2073 Set C Q.No. 14OR If R be the horizontal range of a projectile and h is greatest height, prove that its velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. [6]

38. 2073 Set D Q.No. 14 A projectile thrown from a point in horizontal plane comes back to the plane in 4 secs at a distance of 60 m from the point of projection. Find the velocity of the projection. ($g = 10m/s^2$) [6]
Ans: 25 m/sec

39. 2072 Supp. Q.No. 14 From a point on the ground at a distance x from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance y on the other side. Prove that the height of the wall is $\frac{xy}{x+y}$. [6]

40. 2072 Set C Q.No. 15 If R be the horizontal range of a projectile and h its greatest height, prove that its initial velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. [6]

41. 2072 Set D Q.No. 15 OR Describe motion of a projectile. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower and also find the striking velocity. Ans: 2h, $2\sqrt{gh}$ [6]

42. 2071 Set D Q.No. 14 OR A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H¹ are the greatest heights in two paths for which this is possible, prove that: $R^2 = 16HH^1$. [6]

43. 2070 Set D Q.No. 14 A projectile thrown form a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60 m in front of the point of projection. Find the velocity of projection. ($g = 10m/s^2$) Ans: 25 m/sec [6]

2069 (Set A) Q.No. 14 The horizontal and the vertical components of the initial velocity of a projectile are U and V. If R be the range and H, the greatest height attained, prove that [6]

(a) $\frac{4H}{R} = \frac{V}{U}$ (b) $\left(\frac{R}{U}\right)^2 = \frac{8H}{g}$

2069 (Set B) Q.No. 14a If R be the horizontal range of a projectile and h is greatest height, prove that its initial velocity is $\sqrt{2g\left(h + \frac{R^2}{16h}\right)}$. [6]

C. WORK, ENERGY AND POWER

FORMULAE

1. Work: $W = F \times s$
2. Power: $P = \frac{W}{t} = \frac{F \times s}{t}$
3. Energy
   - i. Kinetic Energy: $KE = \frac{1}{2} mv^2$
   - ii. Potential Energy: $PE = mgh$

2 Marks Questions

1. **2077 Set H Q.No. 7** A car covers a distance of 50 m in 5 sec. against a frictional force. If the power of the engine is 400 w, find the frictional force. [2]
   Ans: F = 400 N
2. **2077 Set I Q.No. 7** A motor boat of 5 HP working at full speed moves at the rate of 36 kmh^{-1}. What is the resistance of water to its motion? [2]
   Ans: 500 N
3. **2076 GIE Set A Q.No. 12c** Just before striking the ground, a mass of 2kg has a kinetic energy of 400 Joule. What is the velocity of the mass of that time? [2]
   Ans: 20ms^{-1}
4. **2073 Set C Q.No. 12c** How large a force is required to cover a distance of 80m if the total work done is 800J? [2]
   Ans: 10 N
5. **2072 Set E Q.No. 12c** Calculate the power of a pump which can lift 300 kg of water through a vertical height of 4m in 10 secs. ($g = 10\ m/s^2$) [2]
   Ans: 1200 w
6. **2071 Old Q.No. 6 b** A car is moving at 36km/h. What velocity will double its kinetic energy? [2]
   Ans: $10\sqrt{2}$ m/s
7. **2063 Q.No. 6a** A pump having a power of 294 w pumps water at the rate of 90 liters per minute. Find the height to which the water is raised. ($g = 9.8\ m/s^2$, 1 litre of water = 1 kg) [2]
   Ans: 20 m
8. **2061 Q.No. 5 b** A car of mass 1000 kg. moves up an incline of 30° at a constant speed of 20 m/sec. If the frictional force is 2000N, calculate the power developed by the engine. ($g = 10\ m/sec^2$). [2]
   Ans: 140 kw
9. **2060 Q.No. 5 b** A car covers a distance of 50m in 5 secs against a frictional force. If the power of the engine is 4000 watts, find the frictional force. [2]
   Ans: 400 N
10. **2059 Q.No. 6 c** A car is moving at 36 kmh^{-1}. What velocity will double its kinetic energy? [2]
    Ans: $10\sqrt{2}$ ms^{-1}
11. **2057 Q.No. 5 b** Calculate the power of a pump which can lift 300 kgs of waters through a vertical height of 4m in 10 secs. [$g = 10m\ s^{-2}$] [2]
    Ans: 1200 watt

4 Marks Questions

12. **2076 GIE Set B Q.No. 13OR** 800 kg of air, moving at 20m/s imping on the vanes of a windmill every second. At what rate in kilowatt is the energy arriving at the wind mill? What is the maximum mass of water that could be pumped each second through a vertical height of 2.5m? ($g = 10m/s^2$)
    Ans: 160 kw, 6400 kgs^{-1}
13. **2075 Set A Q.No. 13b OR** Define K.E. and P.E. of a body. Prove that the change in the K.E. of a body is equal to the work done by the force. [4]
14. **2075 Set C Q.No. 13b** A bullet of mass 100 g is fired into a target with a velocity of 500 ms^{-1}. The mass of the target is 4.9 kg and is free to move; find the loss of kinetic energy by the impact. [4]
15. **2074 Supp Q.No. 14 OR** A bullet passes through two planks in succession. Its initial velocity is 1200 m/s and it loses a velocity of 200 m/s in penetrating through each plank. Find the ratio of the thickness of the planks, assuming that they offer the same average resistance. [4]
    Ans: 11:9
16. **2074 Set A Q.No. 13b** A particle is allowed to slide down a smooth inclined plane. Show that the sum of its kinetic and potential energies is always constant throughtout its motion.[4]
17. **2072 Set C Q.No. 13b OR** State the principle of conservation of energy. Also prove that the sum of the kinetic and potential energies of a moving body remains constant throughout the motion. [4]
18. **2071 Supp. Q.No. 13b OR** A particle is allowed to slide down a smooth inclined plane. Show that the sum of its kinetic and potential energies is always constant throughout its motion.[4]
19. **2071 Set C Q.No. 13 b OR** Find the H.P. of an engine which can travel at the rate of 144 km/hr up an incline of 1 in 200, the mass of the engine and load being 15 metric tons and the resistance due to friction etc. being 15 kg weight per metric ton. ($g = 10\ m/sec^2$). **Ans: 160.83** [4]
20. **2071 Old Q.No. 14 b OR** Define work, power and energy. Prove that the sum of the kinetic and potential energy of a freely falling body remains constant throughout the motion. [4]
21. **2070 Supp. Q.No. 13 b OR** A particle is slide down a smooth inclined plane. Show that the sum of its kinetic and potential energies always constant throughout its motion. [4]
22. **2070 Set C Q.No. 13 a or** A bullet passes through two planks in succession whose initial velocity is 1200 m/s and loses a velocity of 200 m/s in penetrating each plank. Find the ratio of the thickness of the planks assuming that they offer the same average resistance. **Ans: 11:9** [4]

23. **2070 (Old) Q.No. 14 b Or** A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $\sqrt{\frac{2mE}{M(M+m)}}$. [4]

24. **2069 (Set A) Q.No. 13b or** Define work done by a force. Prove that the change in kinetic energy of a body is equal to the work done by the force. [4]

25. **2069 (Set A) Old Q.No. 14b or** Define work and energy. Prove that the sum of the kinetic and potential energies of a freely falling body at any instant is constant. [4]

26. **2069 Old (Set B) Q.No. 14b Or** What do you mean by the principle of conservation of energy? Verify its validity for a body falling under gravity. [4]

27. **2068 Q.No. 14 b Or** If a force be applied on the body, prove that the change in kinetic energy of a body is equal to the work done by the force. [4]

28. **2067 Q.No. 14b OR** A bullet of mass 200 gm is fired into a target with a velocity of 500 ms^{-1}. If the man of the target is 4.8 kg and in free to move, find the loss of kinetic energy by the impact. [4]
Ans: 24,000 Joules

29. **2066 Q.No.14 b OR** A bullet loses $1/20^{th}$ of its velocity in passing through a plank. Find how many such uniform planks it would pass through before coming to rest assuming the retardation to be uniform. [4]
Ans: $10\frac{10}{39}$

30. **2066 C Q.No. 14 b OR** An engine pumps 746 litres of water per minute from a well through an average height of 60 m. Find the horse power of the engine if 50% of the power is wasted. (1 litre of water = 1 kg, g = 10 m /sec^2) [4]
Ans: 20 HP

31. **2065 Q.No 14 b OR** State and prove the Principle of Conservation of Energy. [4]

32. **2064 Q.No. 14 b OR** If a force acts on a body, prove that the change in kinetic energy of a body is equal to the work done by the force. [4]

33. **2062 Q.No. 14 b** An engine pumps 746 liters of water per minute from a well through an average height of 60m. Find the horse power of the engine if 50% of the power is wasted. (1 liter of water = 1 kg., g = 10 m/s^2) [4]
Ans: 20 HP

34. **2061 Q.No. 14 b OR** State the principle of conservation of energy. Illustrate it with the consideration of a body sliding down a smooth inclined plane. [4]

35. **2058 Q.No. 14 b OR** "The change in kinetic energy of a body is equal to the work done by the acting force". Prove this statement. [4]

36. **2057 Q.No. 14 b OR** Define work, power and energy. Prove that the sum of the kinetic and potential energies of a freely falling body remains constant throughout the motion. [4]

6 Marks Questions

37. **2077 Set I Q.No. 9 OR** Define energy. State principle of conservation of energy. Also prove that the sum of the Kinetic and Potential energies of a falling body remains constant throughout the motion. [6]

38. **2076 GIE Set A Q.No. 15 OR** A bullet of mass 'm' moving with velocity 'u' strikes a block of mass 'm' which is free to move in the direction of the bullet and is embedded in it. Show that the loss of Kinetic Energy is $\frac{1}{2}\frac{Mm}{M+m}u^2$. [6]

39. **2076 Set C Q.No. 15 OR** A bullet passes through two planks in succession. Its initial velocity is 1200 m/s and it loses a velocity of 200 m/s in penetrating through each plank. Find the ratio of the thickness of the planks, assuming that they offer the same average resistance. [6]
Ans: 11 : 9

40. **2075 Set B Q.No. 15**
a. A rocket expels gas at the rate of 0.4 kg/s. If the velocity of the gas is 400 m/s, what is the force produced by the rocket?
b. A particle is projected with a velocity of 49 m/s, at an angle of 30° to the horizon, find the time of flight and the range.
c. A pump having a power of 392 W pumps water at the rate of 100 litres per minute. Find the height to which water is raised. ($g = 9.8\ m/s^2$, 1 litre = 1kg) [6]
Ans: (a) 160 N (b) 5 sec, 212.18 m (c) 24 m

41. **2075 Set C Q.No. 15** Define kinetic energy and potential energy with examples. A shot of mass 'm' is projected from a gun of mass 'M' by an explosion, which generates a kinetic energy E. Find i) the initial velocity of the shot ii) the velocity of the gun. [6]
Ans: (i) $\sqrt{\frac{2ME}{m(m+M)}}$ (ii) $\sqrt{\frac{2ME}{M(m+M)}}$

42. **2074 Set B Q.No. 15 OR** Define energy. State principle of conservation of energy. Also prove that the sum of the kinetic and potential energies of a moving body remains constant throughout the vertical motion. [6]

43. **2073 Supp Q.No. 15** State the principle of conservation of energy. Also prove that the sum of the Kinetic and Potential energies of a moving body remains constant throughout the motion. [6]

44. **2073 Set D Q.No. 14 OR** 800 kg of air, moving at 20 m/s, imping on the vanes of a windmill every second. At what rate in kilowatt is the energy arriving at the windmill? What is the maximum mass of water that could be pumped each second through a vertical height of 2.5 m? ($g = 10\ m/s^2$) [6]
Ans: 160 kw, 6400 kgs^{-1}

45. **2072 Supp. Q.No. 14 OR** Define kinetic and potential energies of a body. Prove that the sum of the kinetic and the potential energies of a freely falling body remains constant throughout the motion. [6]

46. **2072 Set D Q.No. 15** Define energy. State principle of conservation of energy. Also prove that the sum of the kinetic and potential energy of a moving body remains constant throughout the motion. [6]

7. **2071 Set D Q.No. 14** Define potential energy and kinetic energy of a body. Prove that the sum of the K.E. and P.E. of a freely falling body at any instant is constant. [6]

8. **2070 Set D Q.No. 14 Or** A bullet of mass 20 g is fired horizontally into a suspended stationary wooden block of mass 380 g with a velocity of 200 m/s. What is the common velocity of the bullet and the block if the bullet is embedded into the block? Find the loss of K.E. by the impact. ($g = 10\ m/s^2$) [6]

Ans: 10 m/sec, 380 J

9. **2069 (Set B) Q.No. 14a Or** A car of mass 2000 kg moves up an inclined plane at an angle 30° to the horizon at a constant speed of 20 m/s. If the frictional force is 2000 N, calculate the power developed by the engine ($g = 10\ m/s^2$). [6]

Ans: 240 kw

16. LINEAR PROGRAMMING

A. GRAPHICAL METHOD

1. If the inequality contains < or > sign, dotted line is used to represent the boundary line as it is not included in the solution set.
2. If the inequality contains ≤ or ≥ sign, dark (bold) line is used to represent the boundary line as it is included in the solution set.
3. Mathematical Model of Linear Programming Problem
   The linear programming problem can be written as
   Optimize $(z) = ax_1 + bx_2 + c$ (objective function)
   subject to the set of constraints
   $a_{11}x_1 + a_{12}x_2$ (< or ≤ or > or ≥) b_1
   $a_{21}x_1 + a_{22}x_2$ (< or ≤ or > or ≥) b_2
   The non-negative constraints
   $x_1 \geq 0, x_2 \geq 0$

2 Marks Questions

1. **2077 Set I Q.No. 10** Find the vertices of the feasible region determined by the constraints $3x + 2y \leq 24$, $x + y \leq 20$, $x \geq 0, y \geq 0$. [2]
2. **2076 GIE Set A Q.No. 16a** Shade the feasible reason under the constraints $x + y \leq 6$, $x \leq 4$, $y \leq 4$, $x, y \geq 0$ [2]
3. **2076 GIE Set B Q.No. 16a** Find the feasible region determined by the following inequalities: $x + y \leq 6$, $2x + y \geq 8$, $y \geq 0$ [2]
4. **2076 Set B Q.No. 16a** In graph shade the feasible region under the constraints $2x + y \leq 40$, $x + 2y \leq 50$, $x \geq 0, y \geq 0$. [2]
5. **2076 Set C Q.No. 16a** Graphically shade the feasible region for the constraints $x + 2y \leq 7$, $x - y \leq 4$, $x, y \geq 0$. [2]
6. **2075 GIE Q.No. 16a** Find the feasible region graphically determined by the constraints y, $x \geq 0$, $y - x \leq 4$, $1 \leq x \leq 6$. [2]
7. **2075 Set A Q.No. 16a** Find the feasible region determined by the inequalities $x + y \leq 6$, $x - y \geq -2$, $x \geq 0$, $y \geq 0$. [2]
8. **2075 Set B Q.No. 16a** Determine graphically the solution set of $2x + y \geq 2$, $x \geq 0$, $y \geq 0$. [2]
9. **2075 Set C Q.No. 16a** Shade the feasible region bounded by $x + y \leq 6$, $x, y \geq 0$. [2]
10. **2074 Supp Q.No. 16a** Find the feasible region determined by the inequalities $3x + 4y \leq 24$, $0 \leq y \leq 4$, $0 \leq x \leq 7$. [2]
11. **2074 Set A Q.No. 16a** Determine graphically the solution set of the inequality $x - 3y \leq 3$. [2]
12. **2074 Set B Q.No. 16a** Shade the feasible region under the constraints: $2x + y \leq 40$, $x + 2y \leq 50$, $x, y \geq 0$ [2]
13. **2073 Supp Q.No. 16a** Write the procedure of solving a linear programming problem by the graphical method. [2]
14. **2073 Set C Q.No. 16a** Find the vertices of the feasible region under the constraints: $3x+2y \leq 48$, $x+y \leq 20$; $x, y \geq 0$. [2]

Ans: (0, 0), (16, 0), (8, 12) and (0, 20)

15. **2073 Set D Q.No. 16a** Find the feasible region determined by the inequalities: $2x + y \leq 8$, $x + 2y \leq 10$, $x, y \geq 0$. [2]
16. **2072 Supp. Q.No. 16a** Find the vertices of the feasible region determined by the following inequalities:
    $2x + y \leq 8$, $x + 2y \leq 10$ and $x, y \geq 0$. [2]

Ans: (0, 0), (4, 0), (2, 4) and (0, 5)

17. **2072 Set C Q.No. 16a** Shade the feasible region for the constraints $x + 2y \leq 7$, $x, y \geq 0$ [2]
18. **2072 Set D Q.No. 16a** Draw the graph of the inequality: $3x - 3 \leq 5x - y$. [2]
19. **2072 Set E Q.No. 16a** Determine the feasible region bounded by the following system of inequalities:
    $x + y \leq 6$, $2x + y \geq 8$, $y \geq 0$. [2]
20. **2071 Supp. Q.No. 16a** Determine graphically the solution set of the inequality $x - 5y \leq 5$. [2]
21. **2071 Set C Q.No. 16 a** Determine the feasible region of the following system of inequalities:
    $2x + y \leq 8$, $x + 2y \leq 10$, $x, y \geq 0$ [2]
22. **2071 Set D Q.No. 16 a** Determine graphically the feasible region determined by the following inequalities:
    $3x + 4y \leq 24$, $x \geq 2$, $y \geq 1$ [2]
23. **2070 Supp. Q.No. 16 a** Determine graphically the solution set of the inequality: $2x - 3y \leq 6$. [2]
24. **2070 Set C Q.No. 16 a** Draw the graph of the following inequalities: $3x + 4y \leq 24$, $0 \leq y \leq 4$, $0 \leq x \leq 7$. [2]
25. **2070 Set D Q.No. 16 a** Draw the graph of the following inequalities: $x + y \leq 6$, $2x + y \geq 8$, $y \geq 0$. [2]
26. **2069 (Set A) Q.No. 16a** Shade the feasible region determined by the following inequalities: $3x + 2y \leq 12$, $x + y \leq 5$, $x, y \geq 0$ [2]
27. **2069 (Set B) Q.No. 16a** Shade the feasible region determined by the inequalities: $x+2y \leq 10$, $x + y \leq 6$, $x, y \geq 0$. [2]
28. **2068 Q.No. 3c** Determine the half plane represented b y the inequality $y - x \geq 1$ [2]
29. **2067 Q.No. 3 c** Solve graphically: $x - y \leq 0$ and $x \geq 0$ [2]
30. **2066 Q.No. 3c** Determine the half plane given by the inequality $y \geq -x$. [2]
31. **2065 Q. No. 3 c** Solve graphically: $y \geq 2x - 1$ [2]

32. 2062 Q.No. 3c Determine the half plane given by the inequality. $2x - y < 2$, graphically. [2]

33. 2061 Q.No. 3c Solve graphically: $x \geq y$ and $x \geq -y$ [2]

34. 2060 Q.No. 3c Graphically show the solution of $x-y-3>0$. [2]

35. 2059 Q.No. 3c Determine the solution set of $2x-1>4x+3$ [2]
Ans: $x < -2$

36. 2058 Q.No. 3c Determine the half plane given by the inequality $2x - y < 2$. [2]

37. 2057 Q.No. 3c Find the solution set of $4x + 3 \geq 2x - 1$. [2]
Ans: $\{x : x \geq -2, x \in R\}$

38. 2056 Q.No. 3c Graph the half plane given by: $y - x \geq 1$ [2]

4 Marks Questions

39. 2068 Q.No. 14 b Find the maximum and the minimum values of the objective function $F = 16x - 2y + 40$ subjected to $3x + 4y \leq 24$, $0 \leq y \leq 4$, $0 \leq x \leq 7$. [4]
Ans: Max: 152 at (7,0); Min: 32 at (0,4)

40. 2067 Q.No. 14 b Find the extreme values of the objective function $10x + 15y$ subject to constraints:
$x + 2y \leq 25$, $2x + y \leq 20$, $x \geq 3$, $y \geq 4$ [4]
Ans: Max.: 200 at (5, 10); Min: 0 at (0, 0)

41. 2066 Q.No. 14 b How does a linear inequality differ from the linear equation? Determine the maximum and minimum value of the function $\Phi(x, y) = 16x - 2y + 40$ subject to $3x + 4y \leq 24$, $0 \leq x \leq 7$, $0 \leq y \leq 4$. [4]
Ans: Max: 152 at (7,0); Min: 32 at (0,4)

42. 2065 Q. No. 14 b Determine the extreme value of the functions $F(x, y) = x + y + 100$ subject to the constraints: $y - x \geq 1$; $y - x \leq 4$ and $1 \leq x \leq 6$ [4]
Ans: Max. 116

43. 2064 Q.No. 14 b Graph the following systems of inequalities and find the vertices where they exist:
$x + 2y \leq 20$, $x + y \leq 16$, $x \geq 0$ [4]
Ans: [(16, 0) (12, 4), (0, 10); region is unbounded]

44. 2063 Q.No. 14b Graph the following system of inequalities to find maximum and minimum of the objective function
$F = 16x - 2y + 40$; $3x + 4y \leq 24$, $0 \leq y \leq 4$, $0 \leq x \leq 7$ [4]
Ans: maximum value of F = 152 at (7, 0) and minimum value of F = 32 at (0, 4)

45. 2062 Q.No. 14b Maximize: $F = 50x + 15y$, subject to $x + y \leq 60$, $5x + y \leq 100$, $x \geq 0$, $y > 0$ [4]
Ans: Max. value of F = 1250 at (10, 50) and Min. value of F = 0 at (0, 0)

46. 2061 Q.No. 14b Maximize and minimize the function $F = 34x + 6y$ subject to the constraints $x + y \geq 1$, $x + y \leq 6$ and $1 \leq x \leq 3$ [4]
Ans: Max. value of F = 120 at (3, 3); Min. value of F = 34 at (1, 0)

47. 2060 Q.No. 14b Maximize and minimize $F = 10x + 15y$ subject to $x + 2y \leq 20$, $x + y \leq 16$, $x \geq 0$, $y \geq 0$ [4]
Ans: Max. value = 180 at (12, 4); Min. value = 0 at (0, 0)

48. 2059 Q.No. 14b Maximize and minimize the function $F(x, y) = 9x + 7y$
Subject to constraints $x + 2y \leq 7$, $x - y \leq 4$; $x \geq 0$; $y \geq 0$ [4]
Ans: Max. value = 52 at (5, 1) Min. value = 0 at (0, 0)

49. 2058 Q.No. 14b Maximize and minimize the function $F = 9x + 40y$ subject to constraints $y - x \geq 1$, $x + y \leq 3$, $2 \leq x \leq 5$. [4]
Ans: Max. value = 366 at (5, 8) Min. value = 138 at (2, 3)

50. 2057 Q.No. 14b Find the extreme values of the function $G(x, y)$ defined by $G(x, y) = 10x + 15y$ over the convex polygon given by the inequalities. $x + 2y \leq 20$; $x + y \leq 16$; $x \geq 0$; $y \geq 0$ [4]
Ans: Max. value = 180 at (12, 4) Min. value = 0 at (0, 0)

51. 2056 Q.No. 14b Maximize and minimize the function $F(x, y) = 9x + 7y$.
Subject to constraints $x + 2y \leq 7$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$. [4]

6 Marks Questions

52. 2074 Set A Q.No. 10 A small industry manufactures necklaces and bracelets. The combined number of necklace and bracelets that it can handle per day is not more than 24. Each bracelet takes 1 hour of labour to make and each necklace takes a half hour. The total number of hours of labour available does not exceed 16. If the profit on the necklace is Rs. 80 and the profit on the bracelet is Rs. 50. How many of each product should be produced daily to maximize profit? [6]
Ans: 8 and 16

B. SIMPLEX METHOD

FORMULAE

Summary of the Simplex Method (Maximization)

1. Convert the constraints to equations by adding slack variables.
2. Create the initial simplex tableau.
3. Locate the most negative entry in the last row. The column for this entry is called the **entering column**. If ties occur, we can use any one column for pivot column.
4. The **departing row** corresponds to the smallest non-negative ratio $\frac{b_i}{a_{ij}}$. If all entries in the entering column are 0 or negative, then there is no maximum value.
   For ties, choose either entry. The element in the entering column and departing row is pivot.
5. Use elementary row operations to make pivot 1 and all other entries in the entering column 0.
6. If all the entries in the last row are non-negative then the optimal solution is obtained. If not, repeat step 3 to 5 until the entries in the last row are all non-negative.

The Simplex Method (Minimization)

A LPP of minimizing the objective function must have the constraints in the form of $\geq$. We solve minimization problem by converting it into maximization problem and use the method stated above.

A minimization problem in standard form is

$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ (objective function)

subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m$

where $x_i \geq 0$, i = 1, 2 ... n
$b_i \geq 0$, i = 1, 2 ... m

Step I: Write the augmented matrix

$$A = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \\ \hline c_1 & c_2 & \dots & c_n & 0 \end{array}\right]$$

Step II: Find the transpose of the matrix A.

$$A^T = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{m1} & c_1 \\ a_{12} & a_{22} & \dots & a_{m2} & c_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{1n} & a_2 & \dots & a_{mn} & c_n \\ \hline b_1 & b_2 & \dots & b_m & 0 \end{array}\right]$$

Step III: Form the dual maximization problem as follows

Maximize: $w = b_1y_1 + b_2y_2 + \dots + b_ny_n$

subject to

$a_{11}y_1 + a_{21}y_2 + \dots + am_1y_m \leq c_1$

$a_{12}y_1 + a_{22}y_2 + \dots + am_2y_m \leq c_2$

$a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m \leq c_n$

where $y_i \geq 0$, i = 1, 2 ... m

Step IV: Apply the simplex method to the dual maximization problem. The maximum value of w will be the minimum value z. The values $x_1, x_2 \dots x_n$ will occur in last row of the final simplex tableau.

4 Marks Questions

1. **2075 Set B Q.No. 17a** Using the simplex method, maximize $p = x + 3y$ subject to constraints $x + y \leq 4, x - y \leq 1, x \geq 0, y \geq 0$. [4]
Ans: Max. p = 12 at x = 0, y = 4

6 Marks Questions

2. **2077 Set G Q.No. 12** Using Simplex method, maximize $Z = 3x + 5y$ subject to constraints $3x + 2y \leq 18, x \leq 4, y \leq 6$ and $x, y \geq 0$. [6]
Ans: Max. Z = 36 at x = 2 and y = 6

3. **2077 Set I Q.No. 12** Using Simplex method, maximize $Z = 50x_1 + 80x_2$ subject to constraints $x_1 + 2x_2 \leq 32, 3x_1 + 4x_2 \leq 84, x_1 \geq 0, x_2 \geq 0$. [6]
Ans: Max. Z = 1480 at x = 0, y = 6

4. **2076 GIE Set A Q.No. 18** Using Simplex method, maximize: $z = 14x + 4y$ subject to $2x + y \leq 3, x - y \leq 1, x \geq 0, y \geq 0$ [6]
Ans: Max. z = 20 at $x = \frac{4}{3}$ and $y = \frac{1}{3}$

5. **2076 GIE Set B Q.No. 18** Using Simplex method, maximize $U = 25x + 45y$ subject to $x + 3y \leq 21, 2x + 3y \leq 24, x, y \geq 0$ [6]
Ans: Max U = 345 at x = 3, y = 6

6. **2076 Set B Q.No. 18** By Simplex method maximize $F = 15x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 10, x_1 + 3x_2 \leq 10; x_1, x_2 \geq 0$. [6]
Ans: Max. F = 80 at $x_1 = 4, x_2 = 2$

7. **2076 Set C Q.No. 18** Solve by Simplex method: Maximize $z = 7x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 6, 4x_1 + 3x_2 \leq 12, x_1, x_2 \geq 0$.[6]
Ans: Max. z = 21 at $x_1 = 3, x_2 = 0$

8. **2075 GIE Q.No. 18** Using Simplex method, maximize $Z = 9x_1 + x_2$ subject to $2x_1 + x_2 \leq 8, 4x_1 + 3x_2 \leq 18; x_1, x_2 \geq 0$.[6]
Ans: Max. Z = 36 at $x_1 = 4$ and $x_2 = 0$

9. **2075 Set A Q.No. 18** Using simplex method, maximize $U = 25x + 45y$ subject to $x + 3y \leq 21, 2x + 3y \leq 24, x, y \geq 0$. [6]
Ans: Max U = 345 at x = 3, y = 6

10. **2075 Set C Q.No. 18** Using the Simplex method, minimize $W = 3x + 2y$ subject to $2x + y \geq 4, x + 2y \geq 4, x, y \geq 0$. [6]
Ans: Min. $W = \frac{20}{3}$ at $\left(\frac{4}{3}, \frac{4}{3}\right)$

11. **2074 Supp Q.No. 18** Using Simplex method, maximize $P = 50x_1 + 80x_2$ subject to $x_1 + 2x_2 \leq 32, 3x_1 + 4x_2 \leq 84, x_1, x_2 \geq 0$. [6]
Ans: Max. P = 1480 at $x_1 = 20, x_2 = 6$

12. **2074 Set A Q.No. 18 OR** Use the simplex method to maximize $P = x + y$ subject to constraints $x + 2y \leq 6, 3x + 2y \leq 12, x \geq 0, y \geq 0$. [6]
Ans: Max. P = $\frac{9}{2}$ at $x = 3, y = \frac{3}{2}$

13. **2074 Set B Q.No. 18** Apply simplex method to maximize $z = 5x + 3y$ subject to $2x + y \leq 40, x + 2y \leq 50; x, y \geq 0$. [6]
Ans: Max. z = 110 at x = 10 and y = 20

14. **2073 Supp Q.No. 18** Apply Simplex method to maximize $z = 15x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 10, x_1 + 3x_2 \leq 10, x_1, x_2 \geq 0$. [6]
Ans: Max. z = 80 at (4, 2)

15. **2073 Set C Q.No. 18** Maximize $z = 5x_1 + 7x_2$ subject to $2x_1 + 3x_2 \leq 13, 3x_1 + 2x_2 \leq 12; x_1, x_2 \geq 0$ by simplex method. [6]
Ans: Max. z = 31 at $x_1 = 2, x_2 = 3$

16. **2073 Set D Q.No. 18** Using the simplex method, maximum $z = 15x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 10; x_1 + 3x_2 \leq 10; x_1, x_2 \geq 0$. [6]
Ans: Max. z = 80 at $x_1 = 4, x_2 = 2$

17. **2072 Supp. Q.No. 18** Using simplex method, maximize $z = 5x + 3y$ subject to $2x + y \leq 40; x + 2y \leq 50; x, y \geq 0$ [6]
Ans: Max. z = 110 at x = 10, y = 10

18. **2072 Set C Q.No. 18** Solve by Simplex method, the LP problem to maximize $z = 7x + 5y$ subject to $x + 2y \leq 6, 4x + 3y \leq 12, x, y \geq 0$. [6]
Ans: Max z = 21 at (3, 0)

19. **2072 Set D Q.No. 18** Using Simplex method, find the optimal solution of $z = 7x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 6, 4x_1 + 3x_2 \leq 12, x_1, x_2 \geq 0$. [6]
Ans: 21 at (3, 0)

20. **2072 Set E Q.No. 18** Using Simplex method, Maximize $F = 5x - 3y$ subject to $3x + 2y \leq 6, -x + 3y \geq -4, x, y \geq 0$ [6]
Ans: 10 at (2, 0)

21. **2071 Supp. Q.No. 18** Using the simplex method, maximize $p = 4x + 5y$ subject to $2x + 5y \leq 25, 6x + 5y \leq 45, x \geq 0, y \geq 0$. [6]
Ans: 35 at (5, 3)

22. **2071 Set C Q.No. 18** Using simplex method, Maximize $f = 15x_1 + 10x_2$, subject to $2x_1 + x_2 \leq 10; x_1 + 3x_2 \leq 10; x_1, x_2 \geq 0$ [6]

Ans: Max. f = 80 when $x_1 = 4$, $x_2 = 2$

23. 2071 Set D Q.No. 18 Using simplex method, maximize $U = 25x + 45y$
subject to $x + 3y \leq 21$; $2x + 3y \leq 24$; $x, y \geq 0$ [6]
Ans: Max. U = 345 where x = 3, y = 6

24. 2070 Supp. Q.No. 18 Using the simplex method, maximize $P = 20x + 30y$
subject to constraints $2x + 5y \leq 20$, $2x+y \leq 12$, $x \geq 0$, $y \geq 0$ [6]
Ans: Max P = 160 when x = 5, y = 2

25. 2070 Set C Q.No. 18 Using Simplex method, Max. $z = 5x_1 + 7x_2$
subject to $2x_1 + 3x_2 \leq 13$; $3x_1 + 2x_2 \leq 12$; $x_1, x_2 \geq 0$. [6]
Ans: Max. z = 31 at $x_1 = 2$, $x_2 = 3$

26. 2070 Set D Q.No. 18 Using Simplex method, Max. $P = 50x_1 + 80x_2$
subject to $x_1 + 2x_2 \leq 32$; $3x_1 + 4x_2 \leq 84$; $x_1, x_2 \geq 0$. [6]
Ans: Max. P = 1480 at $x_1 = 20$, $x_2 = 6$

27. 2069 (Set A) Q.No. 18 Using simplex method, Maximize $Z = 7x_1 + 5x_2$
subject to $x_1 + 2x_2 \leq 6$; $4x_1 + 3x_2 \leq 12$; $x_1, x_2 \geq 0$ [6]
Ans: Max. Z = 21 when $x_1 = 3$, $x_2 = 0$

28. 2069 (Set B) Q.No. 18 Using simplex method, maximize $Z = 7x_1 + 5x_2$ [6]
subject to: $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 6$, $x_1, x_2 \geq 0$
Ans: Max. Z = 10.5 at (1.5, 0)

17. COMPUTATIONAL METHOD

A. NUMBER SYSTEM

FORMULAE

Number System in Different Form

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

2 Marks Questions

1. 2077 Set G Q.No. 10 Convert the hexadecimal number 27A to binary number. [2]
Ans: 1001111010_2

2. 2077 Set H Q.No. 10 Convert the binary number 10110010 to hexadecimal number. [2]
Ans: $B2_{16}$

3. 2076 GIE Set A Q.No. 16b Convert the hexadecimal number $34E_{16}$ into binary form. [2]
Ans: 1101001110_2

4. 2076 GIE Set B Q.No. 16b Convert hexadecimal number $12DB_{16}$ to decimal form. [2]
Ans: 1001011011011_2

5. 2076 Set B Q.No. 16b Convert the decimal number 31923 into hexadecimal number. [2]
Ans: $7CB3_{16}$

6. 2076 Set C Q.No. 16b Convert hexadecimal number 81 A into Octal number. [2]
Ans: 4032_8

7. 2075 GIE Q.No. 16b Convert the Octal number 2064 into binary form. [2]
Ans: 10000110100_2

8. 2075 Set A Q.No. 16b Convert the hexadecimal numeral $AB5_{16}$ to decimal form. [2]
Ans: 2741

9. 2075 Set C Q.No. 16b Convert the decimal number 4526_{10} to hexadecimal form. [2]
Ans: $11AE_{16}$

10. 2074 Supp Q.No. 16b Convert the decimal number 3058_{10} to hexadecimal form. [2]
Ans: $BF2_{16}$

11. 2074 Set A Q.No. 16b Convert $2B1_{16}$ into the binary number. [2]
Ans: 1010110001_2

12. 2074 Set B Q.No. 16b Convert the hexadecimal number $22F_{16}$ in to binary form. [2]
Ans: 1000101111_2

13. 2073 Supp Q.No. 16b Convert the decimal number 7593 into hexadecimal form. [2]
Ans: $1DA9_{16}$

14. 2073 Set C Q.No. 16b Convert the octal number 143_8 into hexadecimal form. [2]
Ans: 63_{16}

15. 2073 Set D Q.No. 16b Convert the decimal numeral 3058 to hexadecimal form. [2]
Ans: $BF2_{16}$

16. 2072 Supp. Q.No. 16b Convert the decimal number 1503 into hexadecimal form. [2]
Ans: $5DF_{16}$

17. 2072 Set C Q.No. 16b Convert the decimal number 3159 into hexadecimal form. [2]
Ans: $C57_{16}$

18. 2072 Set D Q.No. 16b Convert hexadecimal number $70A_{16}$ into binary form. [2]
Ans: 11101010_2

19. 2072 Set E Q.No. 16b Convert the decimal numeral 1503 into hexadecimal form. [2]
Ans: $5DF_{16}$

20. 2071 Supp. Q.No. 16b Convert the binary number 10100011000_2 into the octal number. [2]
Ans: 2430_8

21. 2071 Set C Q.No. 16 b Convert the decimal number 2567_{10} to octal form. [2]
Ans: 5007_8

22. 2071 Set D Q.No. 16 b Convert the hexdecimal number $AB5_{16}$ to the decimal number. [2]
Ans: 2741

26. **2070 Supp. Q.No. 16 b** Convert 110011_2 to the decimal number [2]
Ans: 51

27. **2070 Set C Q.No. 16 b** Convert the decimal numeral 1503 to hexadecimal form. [2]
Ans: $5DF_{16}$

28. **2070 Set D Q.No. 16 b** Convert the decimal number 3058 to hexadecimal form. [2]
Ans: $BF2_{16}$

29. **2069 (Set A) Q.No. 16b** Convert the octal numeral 3733_8 into decimal form. [2]
Ans: $(2011)_{10}$

30. **2069 (Set B) Q.No. 16b** Convert the hexadecimal numeral 2E4B into decimal form. [2]
Ans: $(11851)_{10}$

B. BISECTION METHOD

FORMULAE

1. True error = ± (True value – Approximate value)
2. Absolute error: $E_A = |X - X_1|$
3. Relative error: $E_R = \frac{E_A}{X} = \frac{|X - X_1|}{|X|}$, $X \neq 0$
4. Percentage error: $E_P = 100\ E_R = \frac{100\ |X - X_1|}{|X|}$, $X \neq 0$
5. We stop the process of bisecting the interval $[a_k, b_k]$ in one of the following three cases.
   i. Maximum number of iterations allowed.
   ii. When $f(x_k) = 0$.
   iii. When $\frac{|b_k - a_k|}{2} < \varepsilon$, where $\varepsilon > 0$ is the pre-assigned error tolerance.

2 Marks Questions

1. **2075 Set B Q.No. 16b** If f(0) = –1 and f(8) = 1, how many steps of the bisection method will be required to find an approximation to the root of f(x) accurate to 0.25? [2]
Ans: 5 or more

4 Marks Questions

2. **2077 Set I Q.No. 11** Applying the method of successive bisection find the square root of 3 within two places of decimal in (1, 2). [4]
Ans: 1.73

3. **2076 Set B Q.No. 17b** Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ in (0, 1). [4]
Ans: 0.5859

4. **2075 GIE Q.No. 17a** Apply the method of successive bisection to find the root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to three places of decimal. [4]
Ans: 2.094

5. **2074 Set A Q.No. 17a** Use the bisection method to find the solution of $x^3 - x - 1 = 0$ in the interval (1, 2) correct to three places of decimals. [4]
Ans: 1.324

6. **2074 Set B Q.No. 17a** Show that the equation $x^3 - x - 4 = 0$ has two negative roots and one positive root and find the positive root correct to 3 places of decimal by successive bisection method. [4]
Ans: 1.797

7. **2073 Set C Q.No. 17a** Apply successive bisection method to find the root of the equation $x^3 - 4x - 1 = 0$ lying between 1 and 2 correct to two places of decimal. [4]
Ans: No root lies between 1 and 2

8. **2072 Set D Q.No. 17b** Using the bisection method find the root of the equation $x^2 + x - 4 = 0$ in (1, 2) correct to two places of decimals. [4]
Ans: 1.56

9. **2071 Supp. Q.No. 17a** Find the solution of $x^2 - 10 = 0$ using the bisection method with a = 3, b = 4 and $\varepsilon = 0.01$. [4]
Ans: 3.16212

10. **2070 Supp. Q.No. 17 a** Use the Bisection method to find the solution of the equation $x - 2^{-x} = 0$ in the interval [0, 1], accurate to within 10^{-3}. [4]
Ans: 0.640625

6 Marks Questions

11. **2077 Set H Q.No. 12** Determine the number of positive roots and apply the method of successive bisection to find the roots of the equation $x^3 - 2x - 5 = 0$ in (2, 3) correct to three places of decimals. [6]
Ans: 2.094

12. **2076 GIE Set A Q.No. 19 OR** Find a root of an equation $x^3 + x - 4 = 0$ in the interval [1, 4] within an accuracy of 10^{-1} by bisection method. [6]
Ans: 1.375

13. **2076 GIE Set B Q.No. 19** Using method of Successive bisections, find a root of the equation $2x^3 - 5x + 2 = 0$ lying in between 1 and 2 correct to 4 places of decimals with error less than 0.05 [6]
Ans: 1.3125

14. **2075 Set A Q.No. 19** Find the positive root of the equation $x^3 - x - 4 = 0$ correct to 3 places of decimal with error less than 0.005. [6]
Ans: 1.796

15. **2075 Set C Q.No. 19 OR** Find the roots of the equation $f(x) = x^3 - 4x - 9$ correct to three decimal places by using bisection method. [6]
Ans: 2.706

16. **2074 Supp Q.No. 19** Applying the method of successive bisection, find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to two place of decimals. [6]
Ans: 1.86

17. **2073 Supp Q.No. 19** Show that the equation $f(x) = x^3 - 18$ has only one positive root. Using bisection method, find the positive root correct to 3 places of decimals in the interval (2, 3). [6]
Ans: 2.621

18. **2073 Set D Q.No. 19** Show that the equation $f(x) = x^3 - 3x - 8 = 0$ has only one positive root. Using bisection method, find a root in (2, 3) correct to 3 places of decimals. [6]
Ans: 2.492

19. 2072 Supp. Q.No. 19 Using the method of bisection, find the root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to 3 places of decimals. [6]
Ans: 2.094

20. 2072 Set C Q.No. 19 Apply the method of bisection to find the root of the equation $x^3 - 2x - 5 = 0$ in (2, 3) correct to three places of decimal. [6]
Ans: 2.094

21. 2072 Set E Q.No. 19 Using bisection method, find the root of the equation: $2x^3 - 5x + 2 = 0$, $x \in (1, 2)$ with error less than 10^{-2}. [6]
Ans: 1.32032

22. 2071 Set D Q.No. 19 Using the bisection method, find a root of the equation: $f(x) = 2x^3 - 5x + 2 = 0$, between 1 and 2 with error less than 10^{-2}. [6]
Ans: 1. 32032

23. 2070 Set C Q.No. 19 Find the root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to three places of decimals by successive bisection method. [6]
Ans: 2.094

24. 2070 Set D Q.No. 19 Applying the method of successive bisection, find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to 2 places of decimals. [6]
Ans: 1.86

25. 2069 (Set A) Q.No. 19 Using method of bisection, find the root of the equation $x^3 - x - 4 = 0$ lying between 1 and 2 correct to 3 places of decimals. [6]
Ans: 1.796

26. 2069 (Set B) Q.No. 19 Show that the equation $f(x) = x^3 - x - 4$ has one positive root and using the method of bisection, find the positive root correct to 3 places of decimals. [6]
Ans: 1.796

C. NEWTON RAPHSON'S METHOD

FORMULAE

1. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
2. Finding square roots by Newton–Raphson method
$f(x) = x^2 - a$
$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$.

4 Marks Questions

1. 2077 Set I Q.No. 11 OR Find a root of the equation $x^3 - x - 4 = 0$ in (1, 2) to three places of decimal by Newton Raphson method. [4]
Ans: 1.796

2. 2076 Set C Q.No. 17a Using method of successive bisection method Or Newton Raphson's method, find the root of the equation $x^2 + x - 4 = 0$ in (1, 2) correct to three places of decimals. [4]
Ans: 1.561

3. 2074 Set A Q.No. 17a OR Use Newton–Raphson's method to approximate $\sqrt[3]{2}$ with an error less than 0.00001. [4]
Ans: 1.259921

4. 2074 Set B Q.No. 17a OR Using Newton-Raphson's method, find the positive root of $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to three places of decimal. [4]
Ans: 2.094

5. 2073 Set C Q.No. 17a OR Find a root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to three places of decimal by Newton Raphson's method. [4]
Ans: 1.796

6. 2071 Supp. Q.No. 17a OR Let $f(x) = -x^3 - \cos x$ and $x_0 = -1$. Use Newton-Raphson's method to find x_2. Could $x_0 = 0$ be used? [4]
Ans: 0.8657, No

7. 2070 Supp. Q.No. 17 a OR Use Newton – Raphson method to find the solution of the equation $x^3 + x - 1 = 0$ in the interval [0, 1], accurate to within 10^{-4}. [4]
Ans: 0.68234

6 Marks Questions

8. 2077 Set H Q.No. 12 OR Using Newton Raphson method find a positive root of $x^3 + 3x - 5 = 0$ in (1, 2) correct to three places of decimals. [6]
Ans: 1.154

9. 2076 GIE Set A Q.No. 19 Using Newton-Raphson method, find a positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to 3 places of decimals. [6]
Ans: 1.154

10. 2076 GIE Set B Q.No. 19 OR Using Newton Raphson's method, find a root of the equation $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to 3 places of decimals.
Ans: 1.154

11. 2075 Set A Q.No. 19 OR Using Newton-Raphson method, find a root of the equation $x^3 + 3x - 5 = 0$ between 1 and 2 to three places of decimals. [6]
Ans: 1.154

12. 2075 Set B Q.No. 18 For $f(x) = x^3 - 4$, perform 3 iterations of Newton-Raphson's method with starting point $x_0 = 2$. Find the errors and percentage errors of x_0, x_1, x_2 and x_3. [6]
Ans: $x_0 = 2$
$x_1 = 1.66667$, $E_1 = 0.1999$, 19.99%;
$x_2 = 1.59111$, $E_2 = 0.0475$, 4.75%
$x_3 = 1.58741$, $E_3 = 0.0023$, 0.23%

13. 2075 Set C Q.No. 19 Use Newton-Raphson method (formula) to find the solutions of $f(x) = 1 - 12x + x^3$ correct upto four decimal places. [6]
Ans: 3.4216

14. 2074 Supp Q.No. 19 OR Using Newton-Raphson method, find a root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to 3 places of decimals. [6]
Ans: 2.094

15. 2073 Supp Q.No. 19 OR Use Newton Raphson's Method to find a positive root of $\cos x = x^3$. [6]
Ans: 0.99997

16. 2073 Set D Q.No. 19 OR Using Newton -Raphson method, find a root of the equation $f(x) = x^3 - x - 4 = 0$ in (1, 2) correct to 3 places of decimals. [6]
Ans: 1.796

17. 2072 Supp. Q.No. 19 OR Using Newton Raphson's method, find the root of the equation $f(x) = x^3 - x - 4 = 0$ in (1, 2) correct to 3 places of decimals. [6]
Ans: 1.796

2072 Set E Q.No. 19 OR Find a root of the equation $2x^2 - 3x - 1 = 0$, $x \in (1, 2)$ using Newton Raphson method with error less than 10^{-4}. [6]
Ans: 1.7808

2071 Set C Q.No. 19 Find a root of an equation $x^3+x-4=0$ in the interval [1, 4] within an accuracy of 10^{-1} [6]
Ans: 1.375

2071 Set C Q.No. 19 OR Find a root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to three places of decimal by Newton-Raphson method. [6]
Ans: 1.796

21. **2071 Set D Q.No. 19 OR** Derive the formula for Newton-Raphson method. Using Newton Raphson method, find a positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to three places of decimals. [6]
Ans: 1.154

22. **2070 Set C Q.No. 19 or** Solve $2x^2 - 3x - 1 = 0$ using Newton-Raphson method taking $x_0 = 1$ with error less than 10^{-4}. [6]
Ans: 1.7808

23. **2070 Set D Q.No. 19 Or** Using Newton-Raphson method, find the positive root of $x^3 - 18 = 0$ in (2, 3). [6]
Ans: 2.62

24. **2069 (Set A) Q.No. 19 or** Using Newton-Raphson's method, find the square root of 153 correct to 3 places of decimals. [6]
Ans: 12.369

25. **2069 (Set B) Q.No. 19 Or** Using Newton Raphson's method find the positive root of the equation $f(x) = x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to 3 places of decimals. [6]
Ans: 2.094

18. COMPUTATIONAL METHOD (CONTINUED)

A. GAUSS ELIMINATION METHOD

FORMULAE

1. A system of linear equation is said to be **consistent** if it has either one solution or infinitely many solutions and system is said to **inconsistent** if it has no solution.

2 Marks Questions

1. **2076 Set C Q.No. 16c** Using Gauss-elimination method, solve the equations [2]
$x + 2y = 5, 5x - 3y = -1.$
Ans: x = 1, y = 2

2. **2071 Supp. Q.No. 16c** By Gauss elimination method, solve $2x + 3y = 4$, $3x + 2y = -4$. [2]
Ans: x = –4, y = 4

3. **2070 Supp. Q.No. 16 c** Test the consistency of the following system by the Gauss elimination method:
$x - y - 2z = -1, 2x + y + z = 2, 3x + 2y + 9z = 4.$ [2]
Ans: Consistent

4 Marks Questions

4. **2076 QE Set B Q.No. 17a** Solve the following equations using Gauss – elimination method $x + 3y - 2z = 0, 2x - 3y + z = 1, 4x - 3y + z = 3$ [4]
Ans: x = 1, y = 1, z = 2

5. **2075 QE Q.No. 17b** Solve by Gauss elimination method:
$x_1 + x_2 + x_3 = -3, 3x_1 + x_2 - 2x_3 = -2, 2x_1 + 4x_2 + 7x_3 = 7.$ [4]

6. **2075 Set A Q.No. 17a** Solve the following system of equations by Gauss-elimination method. [4]
$x + 3y - z = -2, 3x + 2y - z = 3, -6x - 4y - 2z = 18.$
Ans: x = 1, y = –3, z = –6

7. **2074 Supp Q.No. 17a** Using Gauss-elimination method, solve the following system of equation: [4]
$x + 3y - 2z = 5, 3x + 5y + 6z = 7, 2x + 4y + 3z = 8.$
Ans: x = –15, y = 8, z = 2

8. **2074 Set A Q.No. 17b** Solve the following system of equations by Gauss elimination method,
$2x + 3y + 4z = 20, 3x + 4y + 5z = 26, 3x + 5y + 6z = 31.$ [4]
Ans: x = 1, y = 2, z = 3

9. **2074 Set B Q.No. 17b** Solve by Gauss elimination method:
$x + 3y - 2z = 5, 3x + 5y + 6z = 7, 2x + 4y + 3z = 8.$ [4]
Ans: x = –15, y = 8, z = 2

10. **2073 Set C Q.No. 17b** Solve by Gauss elimination method.
$3x_1 + x_2 + x_3 = 5, x_1 - 4x_2 + x_3 = -2, x_1 + x_2 - 3x_3 = -1.$ [4]
Ans: $x_1 = 1, x_2 = 1, x_3 = 1$

11. **2073 Set D Q.No. 17a** Using Gauss-elimination method, solve the following system of equations:
$2x - 3y + 3z = 27, 4x + y - 2z = 0, -6x - 4y + 2z = 0.$ [4]
Ans: x = 3, y = –2, z = 5

12. **2072 Supp. Q.No. 17a** Solve the following system of equations using Gauss elimination method: [4]
$x + 3y - 2z = 5; 3x + 5y + 6z = 7; 2x + 4y + 3z = 8.$
Ans: x = –15, y = 8, z = 2

13. **2072 Set C Q.No. 17a OR** Solve by Gauss elimination method:
$x + 3y - 2z = 5, 3x + 5y + 6z = 7, 2x + 4y + 3z = 8.$ [4]
Ans: x = –15, y = 8, z = 2

14. **2072 Set D Q.No. 17a OR** Use Gauss elimination method to solve: [4]
$4x - y + z = 8, 2x + 5y + 2z = 3, x + 2y + 4z = 11.$
Ans: x = 1, y = –1, z = 3

15. **2072 Set E Q.No. 17a** Using Gauss-elimination method, Solve the following system of equations.
$2x_2 + 3x_3 = 7, 3x_1 - 2x_2 + 2x_3 = 1, 2x_1 + 3x_2 - 3x_3 = 5.$ [4]
Ans: $x_1 = 1, x_2 = 2, x_3 = 1$

16. **2071 Set C Q.No. 17a** Using Gauss-elimination method, solve the following system of equation.
$x + 3y - z = -2, 3x + 2y - z = 3, -6x - 4y - 2z = 18$ [4]
Ans: x = 1, y = –3, z = –6

17. **2071 Set D Q.No. 17a** Using Gauss-elimination method, solve the following system of equation: $x - 2y + 3z = 2$, $2x - 3y + z = 1, 3x - y + 2z = 9.$ [4]
Ans: x = 3, y = 2, z = 1

18. **2070 Set C Q.No. 17a** Solve, using Gauss elimination method, the following equations. [4]
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$.
Ans: x = –15, y = 8, = 2

19. **2070 Set D Q.No. 17a** Solve the following system of equation by Gaussian elimination method. [4]
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$.
Ans: x –15, y = 8, z = 2

20. **2069 (Set A) Q.No. 17a** Using Gauss elimination method, solve the following system of equations: [4]
$x - 2y + 3z = 2$, $2x - 3y + z = 1$, $3x - y + 2z = 9$
Ans: x = 3, y = 2, z = 1

21. **2069 (Set B) Q.No. 17a** Using Gauss elimination method, solve the following system of equations:
$x_1 - 2x_2 + 3x_3 = 10$, $2x_1 + 3x_2 - 2x_3 = 1$, and
$-x_1 - 2x_2 + 4x_3 = 13$. [4]
Ans: $x_1 = 1$. $x_2 = 3$, $x_3 = 5$

6 Marks Questions

22. **2075 Set B Q.No. 19** What are two steps of Gauss elimination method? Find the approximate solution of the following system of equations by Gauss elimination method:
$3x - y + z = -2$, $x + 5y + 2z = 6$, $2x + 3y + z = 0$. [6]
Ans: x = –2, y = 0, z = 4

B. GAUSS-SEIDEL METHOD

FORMULAE

1. Diagonally Dominant
A system
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$ is diagonally dominant if
$|a_{11}| > |a_{12}| + |a_{13}|$
$|a_{22}| > |a_{21}| + |a_{23}|$
$|a_{33}| > |a_{31}| + |a_{32}|$.

2 Marks Questions

1. **2076 Set B Q.No. 16c** Examine whether the system of equations $3x + 12y - z = 28$, $x + 4y + 7z = 2$ and $10x + 4y - 2z = 20$ is diagonally dominant. [2]
Ans: Not diagonally dominant

2. **2075 GIE Q.No. 16c** Examine whether the system of equations $2y + 3z = 7$, $3x - 2y + 2z = 1$ and $2x + 3y - 3z = 5$ is diagonally dominant or not. [2]
Ans: Not diagonally dominant

3. **2075 Set A Q.No. 16c** Are the followings system of equation diagonally dominant? [2]
$12x_1 + 3x_2 - 5x_3 = 1$, $x_1 + 5x_2 + 3x_3 = 28$, $3x_1 + 7x_2 + 13x_3 = 1$.
Ans: Diagonally Dominant

4. **2074 Set A Q.No. 16c** Is the system $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ well-conditioned? Justify your answer. [2]
Ans: Yes

5. **2074 Set B Q.No. 16c** Examine whether the system of equations $x + 3y - 2z = 0$, $2x - 3y + z = 1$ and $4x - 3y + z = 3$ is diagonally dominant or not. [2]
Ans: No

6. **2073 Supp Q.No. 16c** Interpret geometrically that a system of equations in two variables is ill-conditioned. [2]

7. **2073 Set C Q.No. 16c** Define well-conditioned and ill-conditioned of a system of equation. [2]

8. **2072 Supp. Q.No. 16c** Examine whether the following equations are diagonally dominant: [2]
$8x_1 - 2x_2 + 3x_3 = -1$; $-3x_1 + 9x_2 - x_3 = 2$; $2x_1 - x_2 - 7x_3 = 3$
Ans: Diagonally dominant

9. **2072 Set C Q.No. 16c** Write the conditions for the system of equations $a_{11}x + a_{12}y = b_1$, $a_{21}x + a_{22}y = b_2$, to be ill conditioned. [2]

10. **2072 Set D Q.No. 16c** Test whether the system of equations $12x + 3y - 5z = 1$, $x + 5y + 3z = 28$ and $3x + 7y + 13z = 1$ is diagonally consistent? [2]
Ans: Diagonally dominant

11. **2069 (Set A) Q.No. 16c** Examine whether the following system of equations are ill conditioned.
$2x_1 + x_2 = 25$; $2.001x_1 + x_2 = 25.01$ [2]

4 Marks Questions

12. **2076 Set B Q.No. 17a** Solve by Gauss elimination or Gauss Seidel method: [4]
$2x + 2y + z = 6$, $x - y + z = 0$ and $4x + 2y + 3z = 4$.
Ans: x = 9, y = –1, z = –10

13. **2076 Set C Q.No. 17b** Use Gauss-Seidel method to solve. [4]
$3x + 4y + 8z = 7$, $x + 20y + z = -18$ and $25x + y - 5z = 19$.
Ans: x = 1, y = – 1, z = 1

14. **2075 GIE Q.No. 17b OR** Solve by Gauss Seidel method in second iteration: [4]
$3x_1 + x_2 + 2x_3 = -1$, $2x_1 + 3x_2 + x_3 = 5$, $x_1 + 2x_2 - x_3 = 8$.
Ans: $x_1 = 2.074$; $x_2 = 1.802$; $x_3 = -2.322$

15. **2073 Set D Q.No. 17a OR** Using Gauss-Seidel method, solve the following system of equations: [4]
$4x_1 + x_2 + x_3 = 7$, $2x_1 - 5x_2 + 2x_3 = 1$, $x_1 - x_2 + 3x_3 = 6$.
Ans: $x_1 = 1$, $x_2 = 1$, $x_3 = 2$

16. **2072 Supp. Q.No. 17a OR** Solve the following equation using Gauss-Seidel method: $3x_1 + x_2 = 5$; $x_1 - 3x_2 = 5$. [4]
Ans: $x_1 = 2$, $x_2 = -1$

17. **2072 Set C Q.No. 17a** Using Gauss Seidel method, solve the equations $3x + 2y = -9$, $2x - 3y = -6$. [4]
Ans: x = –3, y =0

18. **2072 Set D Q.No. 17a** Using Gauss Seidel method, solve:
$3x + 4y + 8z = 7$, $x + 20y + z = -18$, $25x + y - 5z = 19$. [4]
Ans: x = 1, y = –1, z = 1

19. **2070 Set C Q.No. 17 a or** Solve the following equation using Gauss Seidel method $3x_1 + x_2 = 5$; $x_1 + 2x_2 = 5$. [4]
Ans: $x_1 = 1$, $x_2 = 2$

20. **2070 Set D Q.No. 17 a Or** Solve the following system of equations by Gauss Seidel method
$3x + y - z = 2$, $2x - 5y + z = 20$, $x - 3y - 8z = 3$. [4]
Ans: x = 2, y – 3, z = 1

21. **2069 (Set A) Q.No. 17a or** Solve the following equations using Guess-Seidel method:
$2x_1 - x_2 = 8$; $3x_1 + 7x_2 = -5$. [4]
Ans: $x_1 = 3$, $x_2 = -2$

2069 (Set B) Q.No. 17a Or Solve the following equations using Guess-Seidel method: $3x_1 + x_2 = 6$, $x_1 - 3x_2 = 5$. [4]
Ans: $x_1 = 2$, $x_2 = -1$

4 Marks Questions

2071 Supp. Q.No. 19 Use the Gauss-Siedel method to solve the systems $4x-y+z=8$, $2x+5y+2z=3$, $x+2y+4z=11$. [6]
Ans: x = 1, y = –1, z = 3

2071 Supp. Q.No. 19 OR Given the system
$0.835x + 0.667y = 0.168$, $0.333x + 0.266y = 0.067$
Determine whether the system is ill conditioned by changing the coefficient 0.667 to 0.666. [6]
Ans: Ill-conditioned

2070 Supp. Q.No. 19 Given the system
$3x - 6y + 2z = 23$, $-4x + y - z = -8$, $x - 3y + 7z = 17$
Make it diagonally dominant and solve by Gauss-siedel method with error less than 0.005. [6]
Ans: 0.9998, – 3.0002, 0.9999; Exact solution 1, –3, 1

2070 Supp. Q.No. 19 OR Given the system
$3x + 1.52y = 1$, $2x + 1.02y = 1$
Determine whether the system is ill-conditioned by changing the coefficient 1.02 to 1.03. [6]
Ans: Ill – conditioned

C. MATRIX INVERSION METHOD

FORMULAE

1. Augment the coefficient matrix A with an identity matrix as
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & : & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & : & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & : & 0 & 0 & 1 \end{bmatrix}.$$
2. Apply the Gauss-Jordan method to the augmented matrix to reduce A to an identity matrix I as
$$\begin{bmatrix} 1 & 0 & 0 & : & a'_{11} & a'_{12} & a'_{13} \\ 0 & 1 & 0 & : & a'_{21} & a'_{22} & a'_{23} \\ 0 & 0 & 1 & : & a'_{31} & a'_{32} & a'_{33} \end{bmatrix}$$
where, $A^{-1} = \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix}$.
3. $AX = B \Rightarrow X = A^{-1}B$

4 Marks Questions

1. **2077 Set G Q.No. 11** Solve the system of equations by Gauss elimination or Matrix inversion method: $2x - 3y + z = 1$, $x - 2y + 3z = 2$, $3x - y + 2z = 9$. [4]
Ans: x = 3, y = 2, z = 1
2. **2076 GIE Set A Q.No. 17a** Solve the following equations using Gauss Elimination method or Inverse Matrix method. [4]
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$
Ans: x = – 15, y = 8, z = 2
3. **2075 Set C Q.No. 17a** Solve the following system of equations using Gauss-elimination or inverse matrix method. [4]
$x + y + z = 6$, $3x - 4y = -5$, $4z - 3x + 2y = 13$
Ans: x = 1, y = 2, z = 3
4. **2074 Supp Q.No. 17a OR** Solve the following system of equations using inverse matrix method:
$3x + y + z = 15$, $x + y + z = 3$, $y - z = -1$. [4]
Ans: x = 6, y = –2, z = –1
5. **2073 Supp Q.No. 17a** Solve by Gauss elimination or matrix inversion method the system of equations:
$3x + 12y - z = 28$, $x + 4y + 7z = 2$, and $10x + 4y - 2z = 20$. [4]
Ans: $x = \frac{22}{25}$, $y = \frac{56}{25}$, $z = \frac{-26}{25}$
6. **2072 Set E Q.No. 17a OR** Solve the following equation using matrix inversion method:
$3x + y + z = 15$, $x + y + z = 3$, $y - z = -1$ [4]
Ans: x = 6, y = –2, z = –1
7. **2071 Set C Q.No. 17 a OR** Using inverse matrix method, solve the following system of equations:
$3x + y + z = 15$, $x + y + z = 3$, $y - z = -1$ [4]
Ans: x = 6, y = –2, z = –1
8. **2071 Set D Q.No. 17 a OR** Solve the following system of equations using inverse matrix method:
$x_1 - 2x_2 - x_3 = 1$, $x_1 - x_2 + 2x_3 = 9$, $2x_1 - 3x_2 - x_3 = 4$. [4]
Ans: $x_1 = 2$, $x_2 = -1$, $x_3 = 3$

6 Marks Questions

9. **2075 Set B Q.No. 19 OR** Solve the following system of equations by matrix inversion method:
$3x + 5z = 14$, $2x + y - 3z = 3$, $x + y + z = 4$. [6]
Ans: x = 3, y = 0, z = 1

19. NUMERICAL INTEGRATION

A. TRAPEZOIDAL RULES

FORMULAE

1. Mid Point Rule (Rectangle Rule): $A \approx (b-a)\, f\left(\frac{a+b}{2}\right)$
2. Trapezoidal Rule: Simple Form
$$\int_a^b f(x)\,dx \approx \frac{1}{2}(b-a)\,[f(a) + f(b)]$$
3. Trapezoidal Rule: General Form
$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]_e$$
4. Error Analysis: $E_r \le \frac{M(b-a)^3}{12n^2}$

2 Marks Questions

1. **2076 GIE Set A Q.No. 16c** Using Composite Trapezoidal rule, compute $\int_0^2 (2x^2 - 1)\,dx$ with 4 intervals. [2]
Ans: 3.5
2. **2076 GIE Set B Q.No. 16c** Using composite Trapezoidal rule, evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx$, n = 2 [2]
Ans: 1.0531

3. **2075 Set C Q.No. 16c** Use the Trapezoidal Rule to approximate the integral $\int_1^2 \frac{1}{x} dx$. Find the error for the approximation. [2]

Ans: 0.693254, 0.000107

4. **2074 Supp Q.No. 16c** Using trapezoidal rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$, n = 2. [2]

Ans: 0.775

5. **2071 Set C Q.No. 16 c** Using the trapezoidal rule evaluate; $\int_0^1 \frac{dx}{1+x^2}$, n = 2 [2]

Ans: 0.775

6. **2071 Set D Q.No. 16 c** Using the trapezoidal rule, evaluate: $\int_0^2 (2x^2 - 1)dx$, n = 4. [2]

Ans: 3.5

7. **2070 Set C Q.No. 16 c** Using trapezoidal rule, evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$, n = 2. [2]

Ans: 1.0531

8. **2070 Set D Q.No. 16 c** Using trapezoidal rule, evaluate $\int_0^3 (3x^2 - 4x)dx$, n = 3. **Ans: 10.5** [2]

9. **2069 (Set B) Q.No. 16c** Given $I = \int_0^4 x^3\, dx$, n = 4

Estimate the value of I using Trapezoidal rule. [2]

Ans: 68

4 Marks Questions

10. **2075 Set A Q.No. 17b OR** Evaluate, using composite trapezoidal rule the integral: $\int_0^1 \frac{dx}{1+x}$, n = 5. [4]

Ans: 0.69562

11. **2075 Set B Q.No. 17b** Compute an approximate value of $\int_0^1 (1 + x^2)^{-1} dx$ by using the composite trapezoid rule with three points. Then comparing with the actual value of the integral, find the error. **Ans: 0.775, Error = 0.01** [4]

12. **2073 Set D Q.No. 17b** Using the trapezoidal rule, compute $\int_0^2 (2x^2 - 1)\, dx$ with 4 intervals. Find the absolute error of approximation from its actual value. [4]

Ans: 3.333 and 0.1667

13. **2072 Set E Q.No. 17b** Evaluate using composite trapezoidal rule, the integral $\int_0^\pi \sin x\, dx$, n = 4. [4]

Ans: 1.896

14. **2071 Supp. Q.No. 17b** Compute an approximate value of $\int_0^1 (1+x^2)^{-1}\, dx$ by the composite trapezoid rule with three points. Then comparing it with the actual value of the integral find the error. [4]

Ans: 0.775, Error = 0.01

15. **2070 Supp. Q.No. 17 b** Compute two approximate values for $\int_1^2 x^{-2}\, dx$ using $h = \frac{1}{2}$ and $h = \frac{1}{4}$ by the composite trapezoid rule. [4]

Ans: 0.5347, 0.509

16. **2069 (Set A) Q.No. 17b** Estimate the following integral using Trapezoidal rule.

$\int_0^1 \frac{dx}{1+x}$, n = 4

Estimate the error with respect to the actual value. [4]

Ans: 0.67701 and 0.01614

6 Marks Questions

17. **2076 Set B Q.No. 19 OR** Approximate the value for $\int_{-1}^1 e^x\, dx$ using Trapezoidal rule with n = 2. **Ans: 2.5431**

18. **2076 Set C Q.No. 19 OR** Evaluate: $\int_0^1 \frac{dx}{1+x}$, using Trapezoidal rule for n = 4. [6]

Ans: 0.67701

19. **2075 GIE Q.No. 19** Evaluate using Trapezoidal rule $\int_1^2 \frac{dx}{1+x}$, n = 4. Also estimate error. [6]

Ans: 0.4055; – 0.4068

20. **2074 Set A Q.No. 19** Using trapezoidal rule, evaluate $\int_0^1 \frac{dx}{1+x}$, n = 4. Estimate the error of approximation from its actual value. [6]

Ans: 0.67701 and 0.01614

21. **2074 Set B Q.No. 19** State Trapezoidal rule, hence evaluate $\int_0^2 \frac{dx}{1+x^4}$ for n = 4 correct to 3 places of decimal. [6]

Ans: 1.068

22. **2073 Set C Q.No. 19** Define Trapezoidal rule. Evaluate using Trapezoidal rule $\int_0^1 \frac{dx}{1+x}$ for n = 4. [6]

Ans: 0.67701

23. **2072 Set C Q.No. 19 OR** State and prove Trapezoidal rule of numerical approximation. [6]

2072 Set D Q.No. 19 Approximate the value using trapezoidal rule for $\int_{-1}^{1} e^x dx$ n = 2. [6]

Ans: 2.5431

8. SIMPSON'S RULE

FORMULAE

1. Simpson's $\frac{1}{3}$ Rule

$$\int_a^b y\,dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 4y_{n-1} + y_n)$$

2. Simpson's $\frac{3}{8}$ Rule

$$\int_a^b y\,dx \approx \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \ldots + 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)$$

3. Error Analysis

$$E_S \leq \frac{M(b-a)^5}{180\,n^4}$$

2 Marks Questions

1. **2075 Set B Q.No. 16c** Apply Simpson's rule to approximate the value of $\int_1^4 e^x \ln x\,dx$ with n = 3. [2]

Ans: 58.969815

2. **2073 Set D Q.No. 16c** Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^{0.2} \sqrt[3]{1-2x^2}\,dx$, n = 2. [2]

Ans: 0.1982

3. **2072 Set E Q.No. 16c** Find the approximate value of $\int_0^{0.2} \sqrt{1-2x^2}\,dx$, n = 2, using Simpson's $\frac{1}{3}$ rule. [2]

Ans: 0.1973

4 Marks Questions

4. **2077 Set H Q.No. 11** Evaluate, using Simpson's rule: $\int_0^1 \frac{dx}{1+x^2}$, n = 4 [4]

Ans: 0.7854

5. **2076 GIE Set A Q.No. 17b** Using Simpson's rule, compute $\int_0^1 \frac{dx}{1+x}$ for n = 4 correct to 4 places of decimals. [4]

Ans: 0.69562

6. **2076 GIE Set B Q.No. 17b** Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\,dx$, n = 4 [4]

Ans: 1.0819

7. **2075 Set A Q.No. 17b** Using Simpson's rule, evaluate: $\int_0^1 \frac{dx}{1+x}$, n = 4. [4]

Ans: 0.6933

8. **2075 Set B Q.No. 17b OR** Approximate $\int_0^2 2^x\,dx$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{1}{2}$. [4]

Ans: 4.3283

9. **2075 Set C Q.No. 17b** Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^2 (4x^2-4x+1)dx$, n=4. [4]

Ans: 3.3333

10. **2074 Supp Q.No. 17b** Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^2 (2x-1)^2\,dx$, n = 4. [4]

Ans: 0.785

11. **2073 Supp Q.No. 17b** Evaluate the following using Simpson's rule: $\int_0^1 \frac{dx}{1+x^2}$, n = 4 [4]

Ans: 0.785

12. **2072 Supp. Q.No. 17b** Evaluate the following integral using Simpson's rule: $\int_0^1 \frac{dx}{1+x^2}$, n = 4. [4]

Ans: 0.785

13. **2072 Set C Q.No. 17b** Using Simpson's $\frac{1}{3}$ rule, calculate $\int_1^5 x^4\,dx$ with n = 4. [4]

Ans: 625.33

14. **2071 Set C Q.No. 17 b** Using Simpson's 1/3 rule, evaluate: $\int_0^1 \sqrt{1+2x^2}\,dx$, h = 0.25 [4]

Ans: 1.2712

15. **2071 Set D Q.No. 17 b** Estimate the following integral using Simpson's 1/3 rule, $\int_0^{\pi} \sin x\,dx$, n = 6 [4]

Ans: 2.0008

16. **2070 Set C Q.No. 17 b** Using Simpson's $\frac{1}{3}$ rule evaluate $\int_0^1 \frac{dx}{1+x^2}$, n = 4. [4]

Ans: 0.785

17. **2070 Set D Q.No. 17 b** Using the Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1 \frac{dx}{1+x}$, n = 4. [4]

Ans: 0.69325

18. **2069 (Set B) Q.No. 17b** Evaluate the following integral using Simpson's rule: $\int_0^{\pi} \sin x\, dx$, n = 6. [4]

Ans: 2.0008

6 Marks Questions

19. **2076 Set B Q.No. 19** Compute $\int_0^1 (1 + x^2)^{-1}\, dx$, by Simpson's $\frac{1}{3}$ rule. Also compare it with true solution. [6]

Ans: Approximate value = 0.775, True value = 0.7854; Error = 0.0104

20. **2076 Set C Q.No. 19** Evaluate: $\int_0^1 \sqrt{1+x^3}\, dx$, using Simpson's $\frac{1}{3}$ rule with n = 4. [6]

Ans: 1.1114

21. **2075 GIE Q.No. 19 OR** Evaluate using Simpson's rule $\int_1^3 \frac{dx}{x^2}$, n = 4. Also estimate error.

Ans: 0.66667; – 0.00476

22. **2074 Set B Q.No. 19 OR** Define Simpson's rule, hence evaluate $\int_0^1 \frac{dx}{1+x}$ for n = 4 correct to 4 places of decimal. [6]

Ans: 0.6933

23. **2073 Set C Q.No. 19 OR** Using Simpson's rule, evaluate $\int_0^2 \frac{dx}{1+x^4}$ for n = 4, correct to 3 places of decimal. [6]

Ans: 1.081

24. **2072 Set D Q.No. 19 OR** Evaluate $\int_0^1 \sqrt{1+x^3}\, dx$ using Simpson's $\frac{1}{3}$ rule with n = 4. [6]

Ans: 1.111

YEARWISE QUESTIONS

2077 (Set G)

Group 'A'

Attempt all the questions.

1. a. Find the term independent of x in the binomial expansion of $\left(2x + \frac{1}{2x}\right)^{10}$. [2]

   b. Determine the equation of the hyperbola with vertex (8, 0) and passing through $(8\sqrt{2}, 4)$. [2]

   c. If $3\hat{i} + \hat{j} - \hat{k}$ and $x\hat{i} - 4\hat{j} + 4\hat{k}$ are collinear vectors, find x. [2]

2. a. Evaluate: $\int \frac{dx}{x + \sqrt{x^2 - 1}}$ [2]

   b. Solve: $\sqrt{1-x^2}\, dy + \sqrt{1-y^2}\, dx = 0$ [2]

3. In how many ways a committee of three person can be formed out of 3 men and 4 women so as to include atleast one woman. [4]

4. Find the equation of the plane through the point (2, 2, 1) and (9, 3, 6) and normal to the plane 2x + 6y + 6z = 9. [4]

5. Calculate Karl Pearson's coefficient of correlation from the following data. [4]

X	12	9	8	10	13	7
Y	14	8	6	9	12	3

6. Find from definition, the derivative of sin (logx). [6]

   OR

   State Rolle's theorem, interpret it geometrically. Verify Rolle's theorem for f(x) = (x + 1) (x – 2) in [–1, 2].

Group 'B'

7. Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. [2]

8. Two forces of magnitude 3P, 2P respectively have a resultant R. If the first force be doubled, the magnitude of the resultant is doubled, find the angle between the forces. [4]

   OR

   Two forces P and Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight w on the plane, prove that: $\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{w^2}$.

9. Define moment of a force. Also interpret it geometrically. State and prove Varignon's theorem for intersecting forces. [6]

Group 'C'

10. Convert the hexadecimal number 27A to binary number. [2]

11. Solve the system of equations by Gauss elimination or Matrix inversion method: 2x – 3y + z = 1, x – 2y + 3z = 2, 3x – y + 2z = 9. [4]

12. Using Simplex method, maximize Z = 3x + 5y subject to constraints 3x + 2y ≤ 18, x ≤ 4, y ≤ 6 and x, y ≥ 0. [6]

2077 (Set II)

Group 'A'

Attempt all the questions.

1. a. Find the middle term in the expansion of $(3x + x^3)^{10}$. [2]
   b. Find the vertices and eccentricity of the hyperbola $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$ [2]
   c. Find the angle between the lines whose direction cosines are proportional to 1, 2, 2 and 2, 3, 6. [2]
2. a. Find the equation of normal to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4). [2]
   b. Calculate the semi-inter Quartile Range of 2, 5, 9, 10, 10, 9, 4. [2]
3. Prove that $\left\{\frac{n}{5}, n \in Z\right\}$ is a group with respect to addition. [4]

   OR

   Define group. Let (G, *)be a group, prove that:
   $(a * b)^{-1} = b^{-1} * a^{-1} \forall a, b \in G$. [4]
4. Evaluate: $\int \frac{dx}{4 + 3\cos hx}$ [4]

   OR

   Evaluate: $\int \frac{5}{(x+5)(2x^2+5)} dx$ [4]
5. A certain manufacturing plant produces electric fuses of which 20% are defective. Find the probability that in a sample of 8 fuses selected at random there will be atleast one defective and not more that one defective. [4]
6. Define cross product of two vectors and give its geometrical interpretation. Prove by vector method:
   $\cos(A - B) = \cos A \cos B + \sin A \sin B$. [6]

Group 'B'

7. A car covers a distance of 50 m in 5 sec. against a frictional force. If the power of the engine is 400 w, find the frictional force. [2]
8. Three forces P, Q, R acting at O along OA, OB, OC; where O is the incenter of ΔABC, are in equilibrium. Show that: [4]
   $$\frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$$

   OR

   State and prove converse of the triangle of forces. [4]
9. Define laws of motion. A gun of mass 1 metric tonne force a shot of mass 14 kg and recoils up smooth inclined plane, rising to a height of 1.6 m, find the initial velocity of the projectile. [6]

   OR

   A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H^1 are the greatest heights and t_1 and t_2 are the time of flight in two paths for which this is possible, prove that:
   a. $R^2 = 16HH^1$ [6]
   b. $R = \frac{1}{2} gt_1t_2$

Group 'C'

10. Convert the binary number 10110010 to hexadecimal number. [2]
11. Evaluate, using Simpson's rule: $\int_0^1 \frac{dx}{1+x^2}$, $n = 4$ [4]
12. Determine the number of positive roots and apply the method of successive bisection to find the roots of the equation $x^3 - 2x - 5 = 0$ in (2, 3) correct to three places of decimals. [6]

    OR

    Using Newton Raphson method find a positive root of $x^3 + 3x - 5 = 0$ in (1, 2) correct to three places of decimals. [6]

2077 (Set I)

Group 'A'

Attempt all the questions

1. a. Find the number of ways in which 5 courses out of 8 can be selected when 3 courses are compulsory. [2]
   b. Prepare a Caley's table for $G = \{1, \omega, \omega^2\}$ where ω is the cube root of unity under multiplication. [2]
   c. Find the derivative of $\sinh^{-1}(\cosh x)$. [2]
2. a. Evaluate: $\int \frac{dx}{\sqrt{2ax - x^2}}$ [2]
   b. Two dice are rolled simultaneously. Determine the probability of turning up the number whose sum is less than six. [2]
3. Find equation and the point of contact of tangent to the parabola $y^2 = 12x$ which makes an angle 45° with the straight line $x - 2y + 3 = 0$. [4]

   OR

   Find the eccentricity and coordinates of foci of: $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$ [4]
4. Prove that the line $lx + my + n = 0$ will be normal to the parabola $y^2 = 4ax$ if $al(2m^2 + l^2) + m^2n = 0$. [4]

   OR

   Find the vertices and foci of the ellipse
   $16x^2 + 25y^2 + 64x + 50y - 311 = 0$ [4]
5. Solve: $\cos^2 x \frac{dy}{dx} + y = 1$ [4]

   OR

   Solve: $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ [4]

6. Define exponential and logarithm series. Also sum to infinity the series.

$1^2 + \frac{2^2}{2!} + \frac{3^2}{3!} + \ldots\ldots$ [6]

Group 'B'

7. A motor boat of 5 HP working at full speed moves at the rate of 36 kmh^{-1}. What is the resistance of water to its motion? [2]
8. A particle is projected up from the bottom of an inclined plane with a velocity of 25m/s, while another is dropped from the highest point to slide down the plane as the same moment. If the length of the plane be 200 m and the angle of inclination of the plane with the horizon is 30°, find when and where the two particles will meet. ($g = 10m/s^2$) [4]
9. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height 'h'. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity? [6]

OR

Define energy. State principle of conservation of energy. Also prove that the sum of the Kinetic and Potential energies of a falling body remains constant throughout the motion. [6]

Group 'C'

10. Find the vertices of the feasible region determined by the constraints

$3x + 2y \leq 24, x + y \leq 20, x \geq 0, y \geq 0.$ [2]

11. Applying the method of successive bisection find the square root of 3 within two places of decimal in (1, 2). [4]

OR

Find a root of the equation $x^3 - x - 4 = 0$ in (1, 2) to three places of decimal by Newton Raphson method. [4]

12. Using Simplex method, maximize $Z = 50x_1 + 80x_2$ subject to constraints

$x_1 + 2x_2 \leq 32, 3x_1 + 4x_2 \leq 84, x_1 \geq 0, x_2 \geq 0.$ [6]